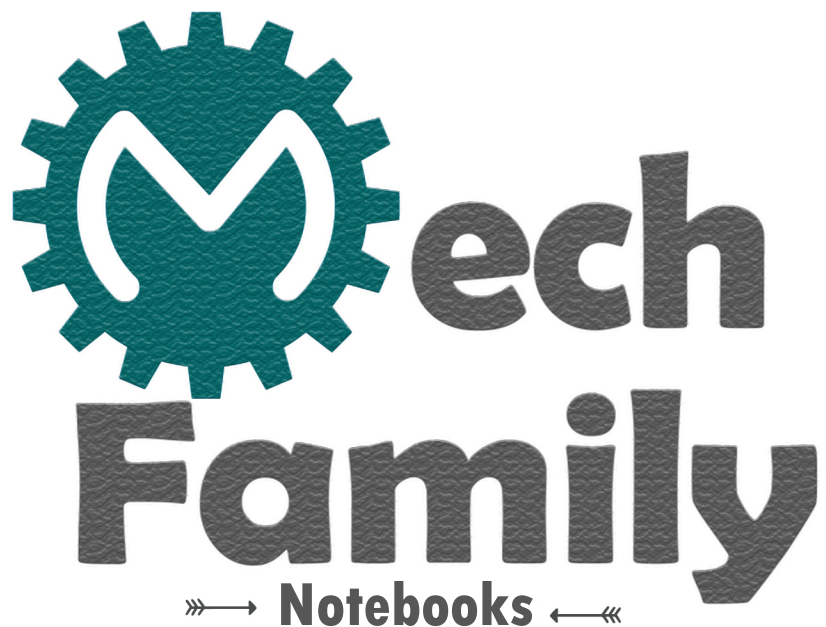


# **Machines**

**Dr. Sahban Naser**

**1st Semester 2017**



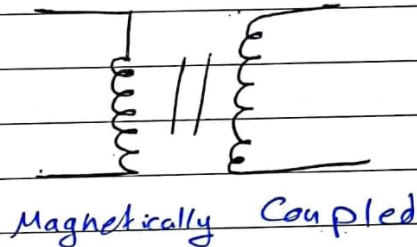
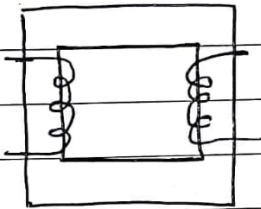
25/9/2017

Machines

- Motor (electrical → Mechanical)
- Generator (mechanical → electrical)
- Transformer (convert AC voltage from one level ~~to~~ another)

Magnetic Field

Transformers:



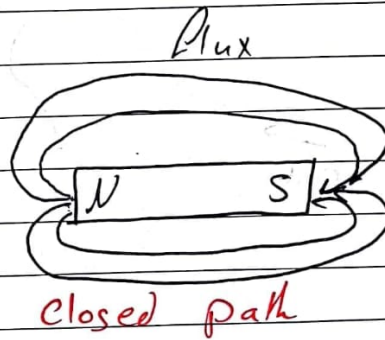
- Magnetic Field

\* permanent magnet

- Flux "lines"  $\phi$  web

- Flux density  $B = \frac{\text{web}}{m^2} = \frac{\phi}{A}$

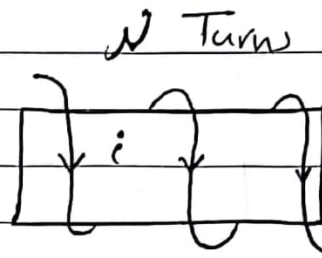
- Flux intensity  $(H) = \left( \frac{A}{m} \right)$





## Electro magnet

$i \rightarrow \phi$   
تيار      ديس



\* Maxwells cork screw rule

\* Right-handed cork screw  $\rightarrow i$

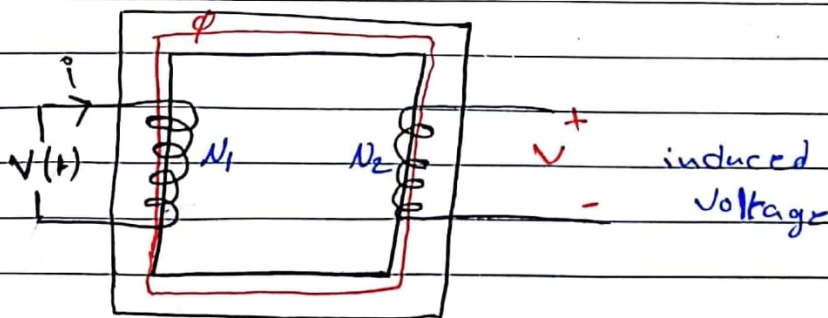
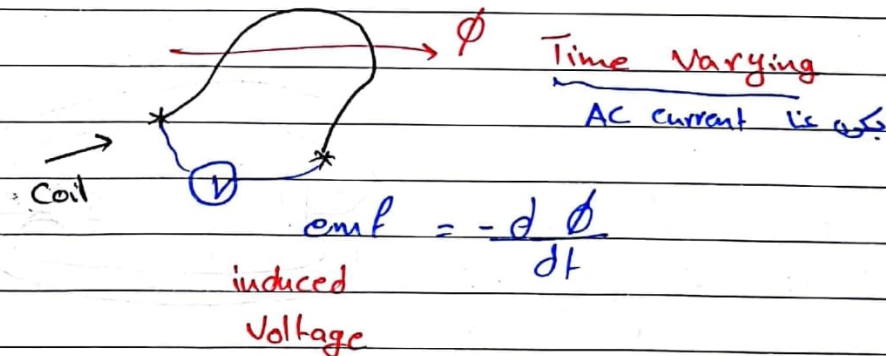
تصديق اتجاه التيار

\* Thumb  $\rightarrow \phi$

## -Principles of magnetic field

\* A current carrying conductor produces a magnetic field

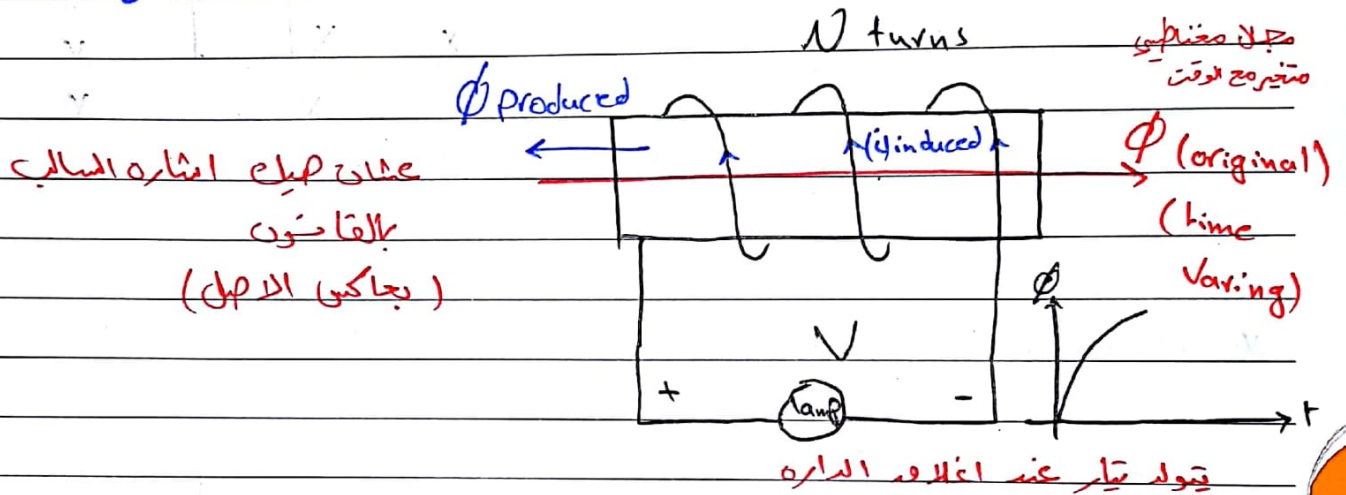
\* Transformers actions Farady's law



$$emf = - \frac{d\phi}{dt} \quad \text{For 1 turn}$$

$$emf = - N \frac{d\phi}{dt} \quad \text{For } N \text{ turns}$$

- Lenz's law :



Motor Action (Force on a wire)

↙ Magnetic Field

X → in

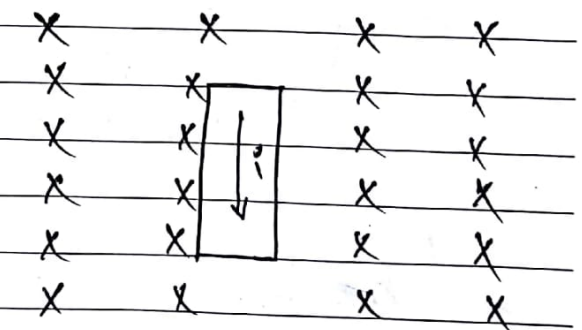
⊙ → out → Direction

$$F = I \times B$$

↑ ↑ ↑  
التيار cross Product direction

↑ اتجاه التيار    ↑ اتجاه الحقل    ↑ قاعده اليد اليمنى

بظهر اليد يمين اتجاه الحقل  
بالخمس اليد اليمنى !



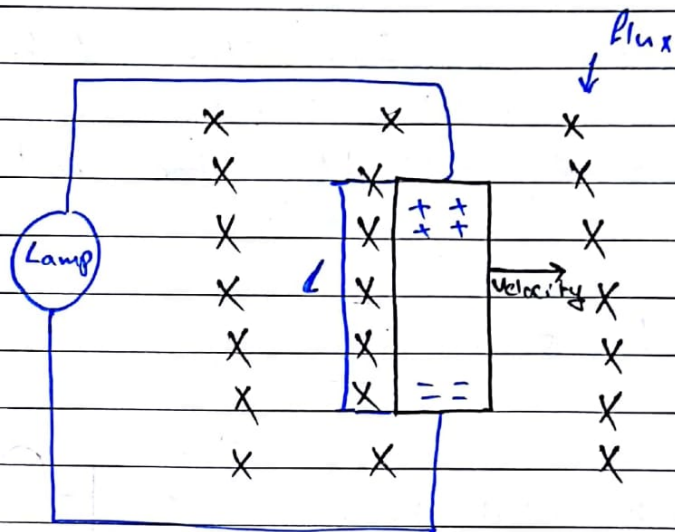
input : current , flux

output : Force Movement

## Generator action

input : velocity, Flux

output :  $\mathcal{E} = (\vec{v} \times \vec{B}) L$



## \* Magnetic circuits

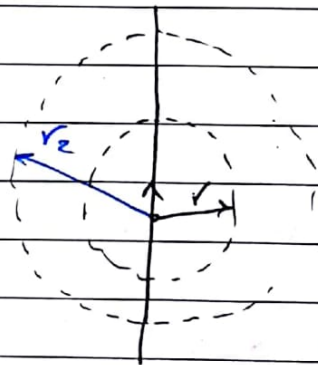
- Ampers law :

closed Path  $\oint H \cdot dL = I_{net}$   
(Line integral)

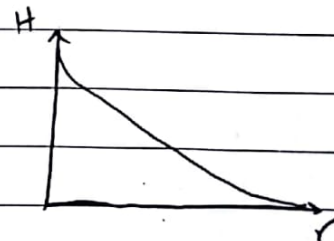
$$\Rightarrow H (2\pi r) = I_{net}$$

$$\Rightarrow H = \frac{I_{net}}{2\pi r} = \frac{A}{m}$$

قوة التيار الناتج مجال مغناطيسي  
الخط



$$H = \frac{I}{2\pi r_2} = \frac{I}{2\pi r_1}$$





فیرو مغناطیسی مواد (core)  
 Ferromagnetic material (core)

Ampere's law

Flux path

$$\oint H \cdot dl = NI$$

$$\Rightarrow HL = NI$$

$$\Rightarrow H = \frac{NI}{L} = \frac{AT}{m}$$

turns

weber / m<sup>2</sup>  
 Tesla

$$B = \mu H \leftarrow \frac{AT}{m}$$

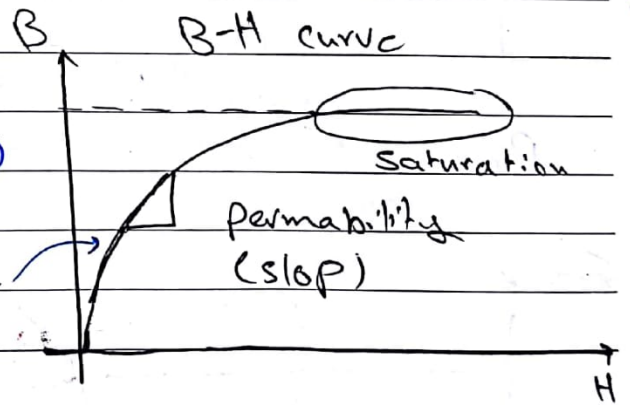
Permeability  $\left(\frac{H}{m}\right)$   
 هیرگی

Material

E	→ — —
G	→ —mm—
H	→ —ooo—

$$\mu = \mu_r \mu_0 \leftarrow \text{Free space } (4\pi \times 10^{-7} \text{ H/m})$$

relative



$$\phi = BA = \mu HA$$

$$\phi = \mu \frac{NI}{L} A$$

Web

Magnetomotive force

$$\phi = \frac{NI}{\frac{L}{\mu A}} = \frac{F}{R}$$

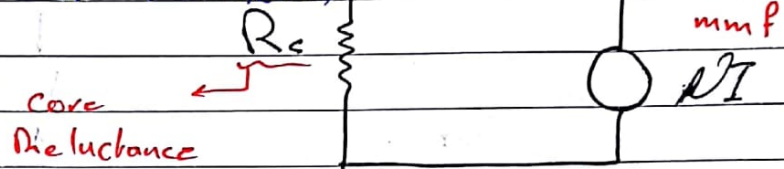
Reluctance  
 عریضی



27/9/2017

Cont.

$$\phi = \frac{NI}{R}, \quad R = \frac{L}{\mu A} = \frac{L}{(\mu_r \mu_0)(wh)}$$

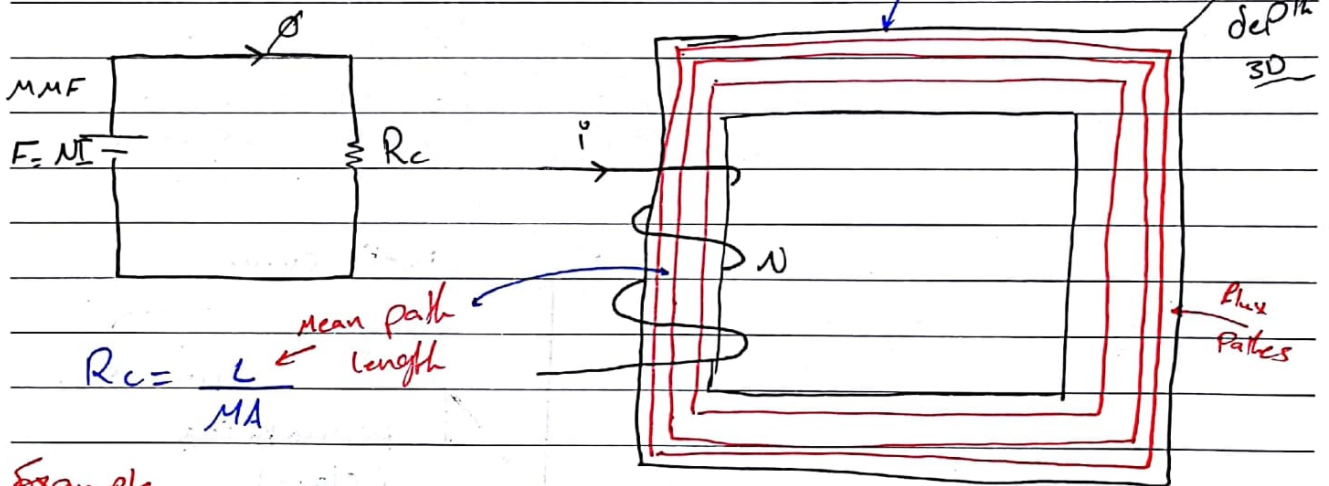


$$R = \frac{NI}{\phi} = \frac{At}{wb}$$

مثل الدارة الكهربائية

$$i = \frac{V}{R}$$

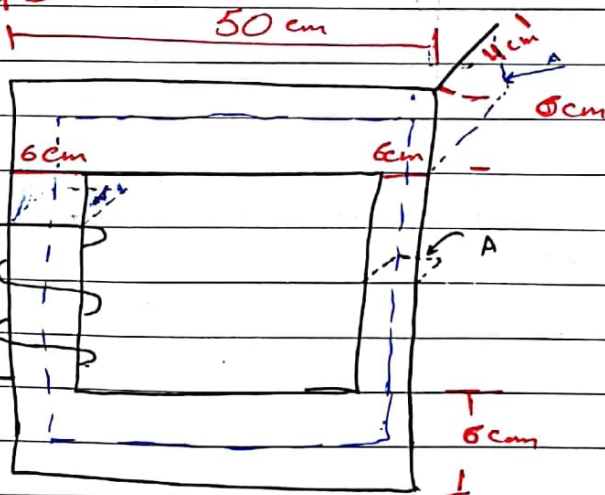
Cont.: Magnetic Circuits



$$R_c = \frac{L}{\mu A}$$

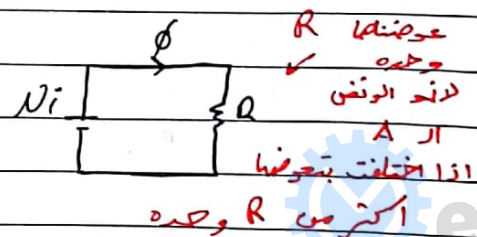
mean path length

Example



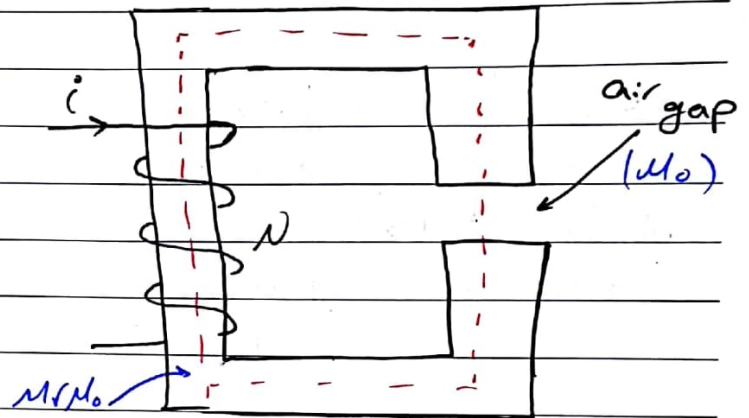
$$L = (50 - 3 - 3) \times 4$$

$$A = 6 \times 4 = 24 \text{ cm}^2$$

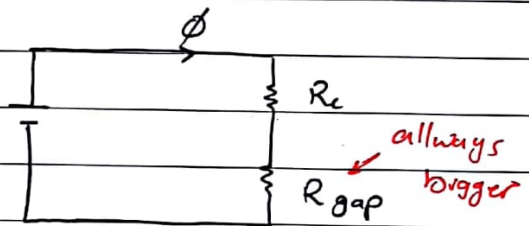


## Magnetic circuits with an air gap

Mean path  $l_c$   
Area core  $A_c$



$$R = \frac{L}{\mu A}$$



$$\oint H \cdot dl = NI$$

$$NI = \underbrace{H_c l_c}_{\text{core}} + \underbrace{H_g l_g}_{\text{gap}}$$

$$NI = \frac{B_c l_c}{\mu_c} + \frac{B_g l_g}{\mu_g}$$

$$B = \mu H$$

$$H = \frac{B}{\mu}$$

$$\Phi = B_c A_c = B_g A_g$$

ممكن ديك مكنه لال

$$B_g = \frac{\Phi}{A_c} \Rightarrow B_g = \frac{\Phi}{A_g}$$

Flux تقسم على A بتقارن  
Flux intensity تقارن

$$NI = \frac{\Phi l_c}{\mu_c A_c} + \frac{\Phi l_g}{\mu_g A_g}$$

$$NI = \Phi \left( \frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g} \right) \Rightarrow NI = \Phi (R_c + R_g)$$

## \* Approximation

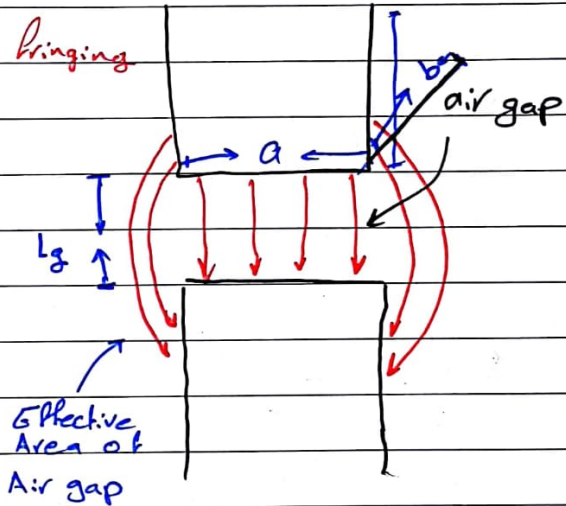
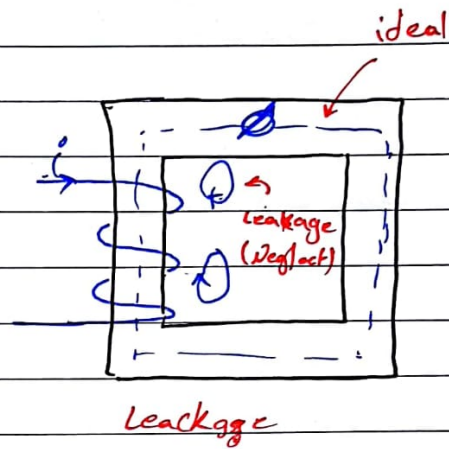
- Mean path length
- Leakage flux

## - Fringing

ممکن ہے کہ ایک  
نیزہ لا A نسبت

$$A = (a + l_g)(b + l_g)$$

او  
- انہی کے لیے یہ ہے کہ  
ideal بنیاد





$$A = 16 \text{ cm}^2$$

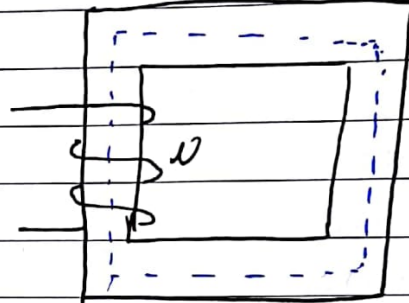
$$l_c = 40 \text{ cm} \leftarrow \text{mean length}$$

$$N = 850 \text{ turns}$$

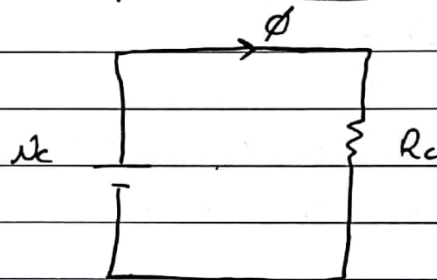
$$\mu_r = 50000$$

$$B = 1.5 \text{ T}$$

Find current in the coil



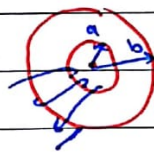
$$R_c = \frac{l}{\mu A} = \frac{40 \times 10^{-2} \text{ m}}{(50000 \times \mu_0)(16 \times 10^{-4} \text{ m}^2)}$$



$$R_c = 3979 \frac{\text{A} \cdot \text{t}}{\text{Wb}}$$

$$\phi = 1.5 \times 10^{-4} \text{ Wb}$$

$$\mathcal{N} i = \phi R \Rightarrow i = \frac{\phi R_c}{N} = \frac{1.5 \times 10^{-4} \times 3979}{850} = 27.3 \text{ mA}$$



اذا كانت دائرة يتأخذ  $(2\pi r)$  المحيط



Ex 6

$$N = 350$$

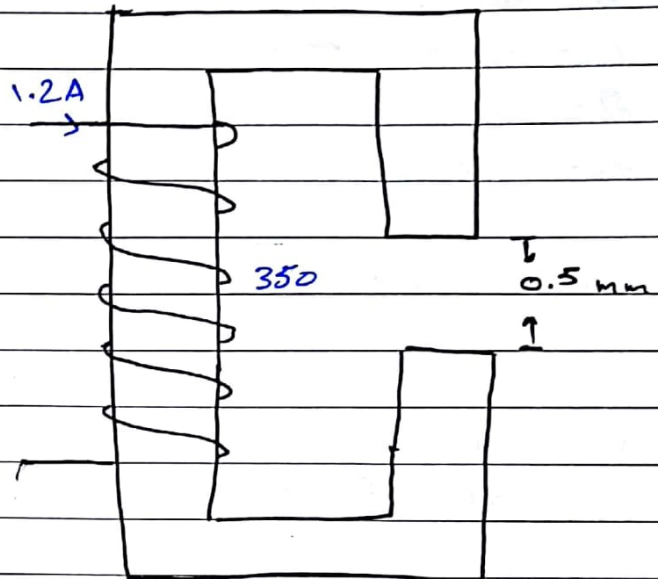
$$i = 1.82 \text{ A}$$

$$A_c = 16 \text{ cm}^2$$

$$L_c = 40 \text{ cm}$$

$$\mu_r = 50000$$

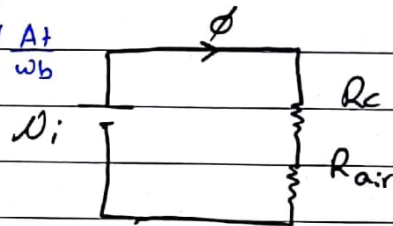
Find flux density  
in the core and air gap



ما اختلفت من قبل الى قبل

$$R_c = \frac{40 \times 10^{-2}}{50000 \mu_0 (16 \times 10^{-4})} = 3974 \frac{\text{At}}{\text{wb}}$$

$$R_g = \frac{0.5 \times 10^{-3}}{\mu_0 (16 \times 10^{-4})} = 248680 \frac{\text{At}}{\text{wb}}$$



$$\phi = \frac{N_i}{R_c + R_{\text{airgap}}} = 1.66 \times 10^{-3} \text{ wb}$$

$$B = \frac{1.66 \times 10^{-3}}{16 \times 10^{-4}} = 1.04 \text{ T}$$

Ex: Find  $\phi$

$$\mu_r = 2000$$

$$N = 1000$$

$$i = 1 \text{ A}$$

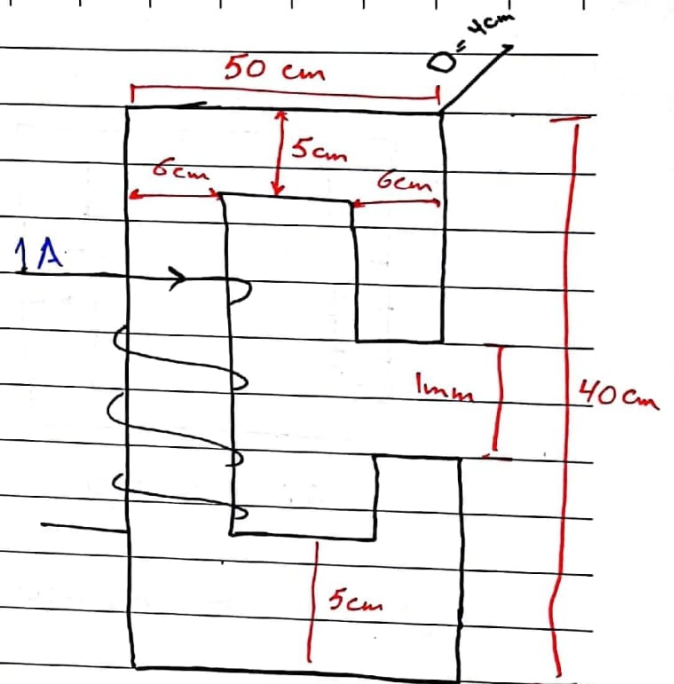
$$A \Rightarrow A_h = 5 \text{ cm} \times 4 \text{ cm} = 20 \text{ cm}^2$$

$$A_v = 6 \text{ cm} \times 4 \text{ cm} = 24 \text{ cm}^2$$

$$A_g = 6 \text{ cm} \times 4 \text{ cm} = 24 \text{ cm}^2$$

$$R = \frac{l}{\mu A}$$

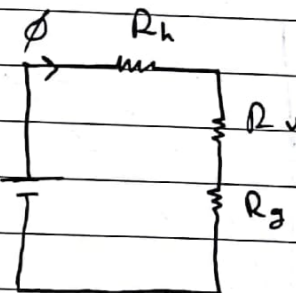
$\mu(A) \leftarrow \text{silico}$



$$L_h = 44 \times 2 = 88 \text{ cm}$$

$$L_v = 70 \text{ cm}$$

$$L_g = 1 \text{ mm}$$



$$R_h = 700200 \frac{\text{At}}{\text{wb}}$$

$$R_h = \frac{88 \times 10^{-2}}{2000 \mu_0 \times 20 \times 10^{-4}} = 175070.4$$

$$R_v = 2464000 \frac{\text{At}}{\text{wb}}$$

$$R_v = \frac{70 \times 10^{-2}}{2000 \times \mu_0 \times 24 \times 10^{-4}} = 116050.5$$

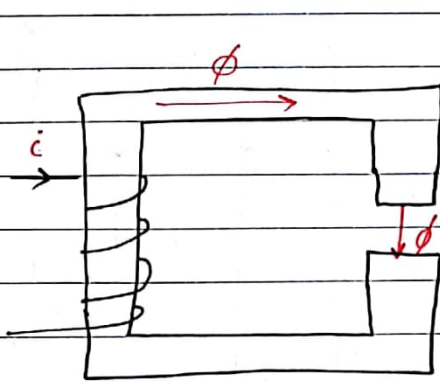
$$R_g = 331000 \frac{\text{At}}{\text{wb}}$$

$$R_g = \frac{1 \times 10^{-3}}{\mu_0 \times 10^{-4} \times 24} = 331572.8$$

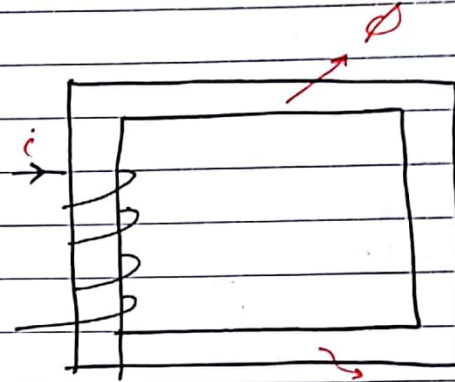
$$\phi = \frac{NI}{\sum R} = 7 \times 10^{-3} \text{ wb}$$

$$\phi = \frac{1000 \times 1}{R_{\text{total}}} = 1.6 \times 10^{-3} \text{ wb}$$

(2/10/2017)

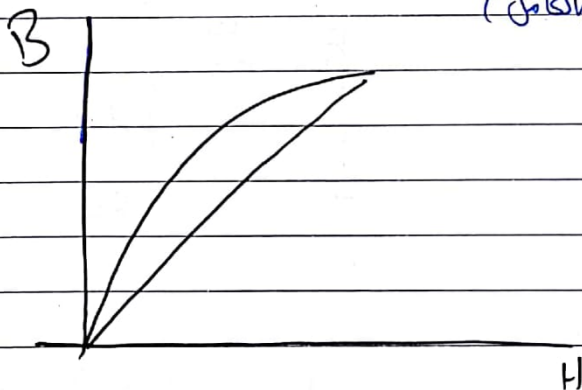


with air gap



without air gap

توضیح: " Air gaps حتى: آخر الـ (Saturation) حالة الاشباع  
 وهي جبهة في بعض الحالات  
 (عشان ما يغير الـ core piece بالكلية)  
 (بتأخر)



$\phi R$

$$BA = \frac{L}{MA}$$

$$\frac{BL}{M} = HL$$

H: دائمة متغيرة لانه دائما في 2 materials  
 B: يتغير اذا كان في Air gaps و حسب الـ Ringing  
 φ: ثابت

Amper's Law:  $\oint H \cdot dl = NI$

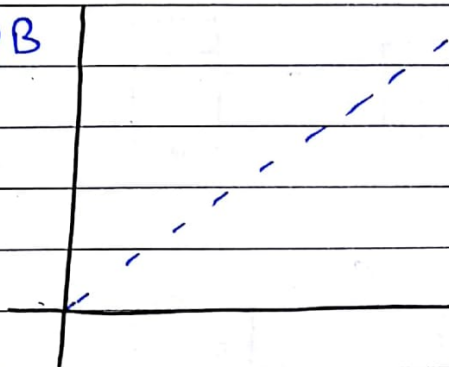






# Magnetic behavior of ferro magnetic material :

(T) B



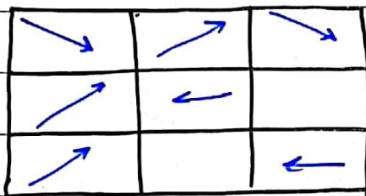
core کے لیے current کی  $\mu$  core  
پہلے سے ہے

$\mu$  : قابلية ال core

H (A/m)

Random

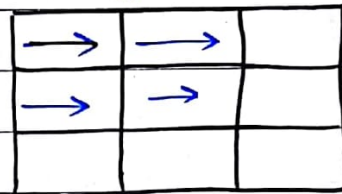
H=0



(a)

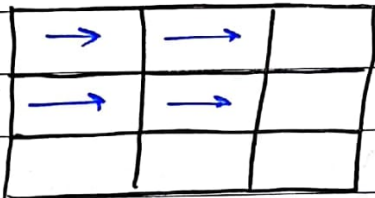
alignment

H=H<sub>1</sub>



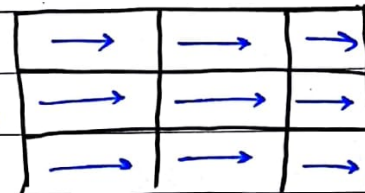
(b)

H=H<sub>2</sub>



(c)

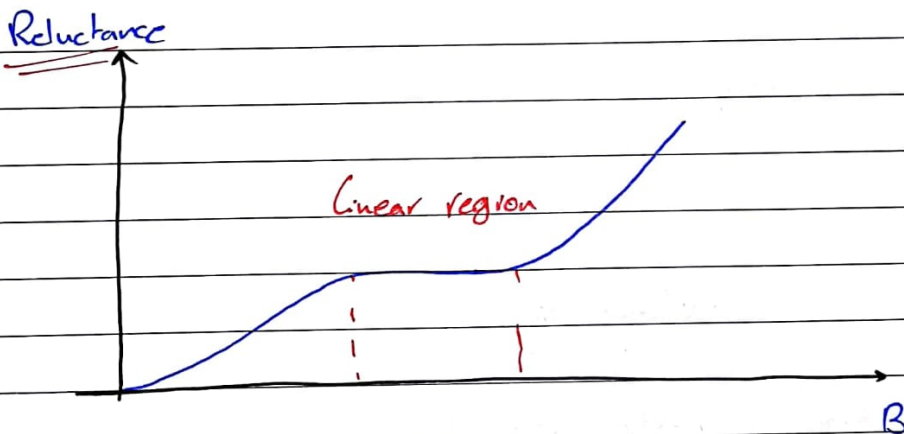
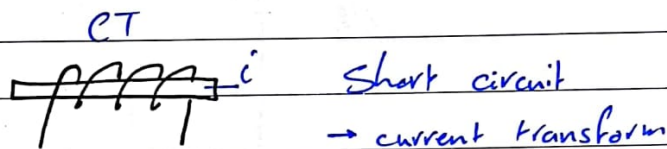
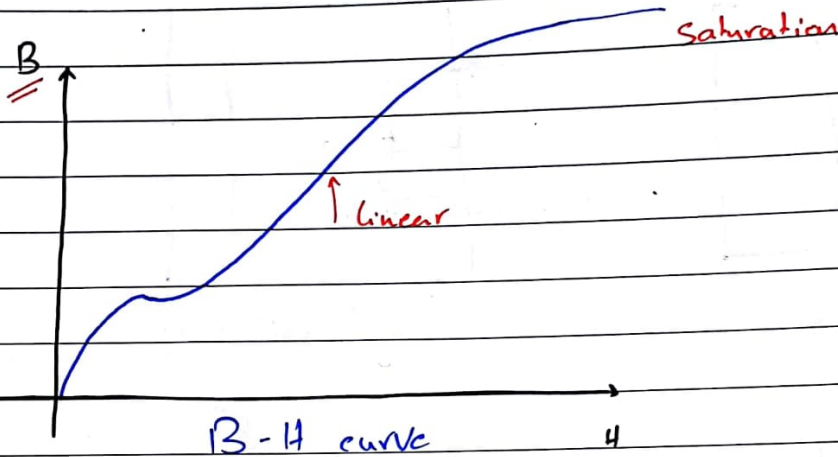
H=H<sub>3</sub>



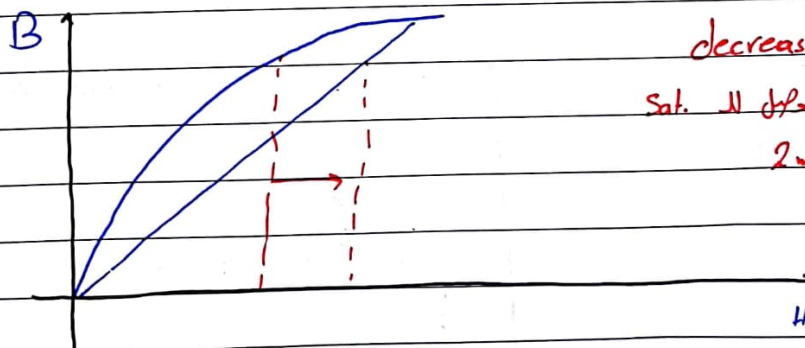
(d)

$B_2 > B_1$  ,  $H_2 > H_1$

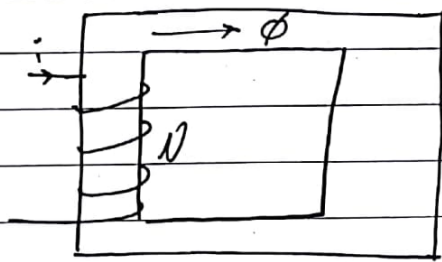
No change in B  
even with increasing H



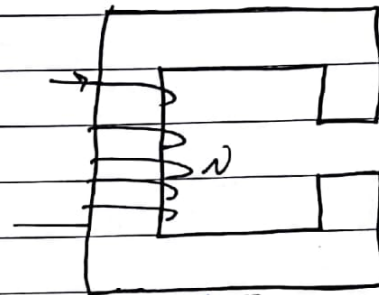
Q) How to delay saturation?



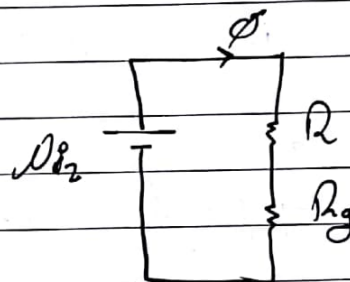
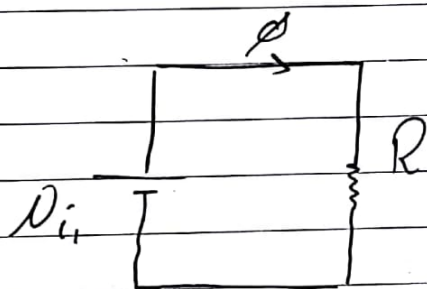
decrease  $\mu \rightarrow$  increase  $R_c$   
 Sat.  $\mu$   $\rightarrow$  delay curve  $\mu$  \*  
 2mH  $\rightarrow$  3mH  $\mu$  \*  
 (Delay) airgap



without air gap



with air gap



assume at linear region

Proof: equation

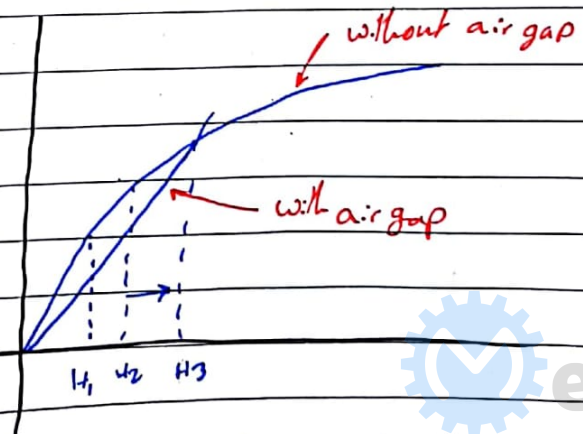
$$\phi = (Ni) / R \quad \text{without air gap}$$

$$\phi = (Ni2) / (R + Rg) \quad \text{with air gap}$$

$$\frac{Ni1}{Rg} = \frac{Ni2}{R + Rg}$$

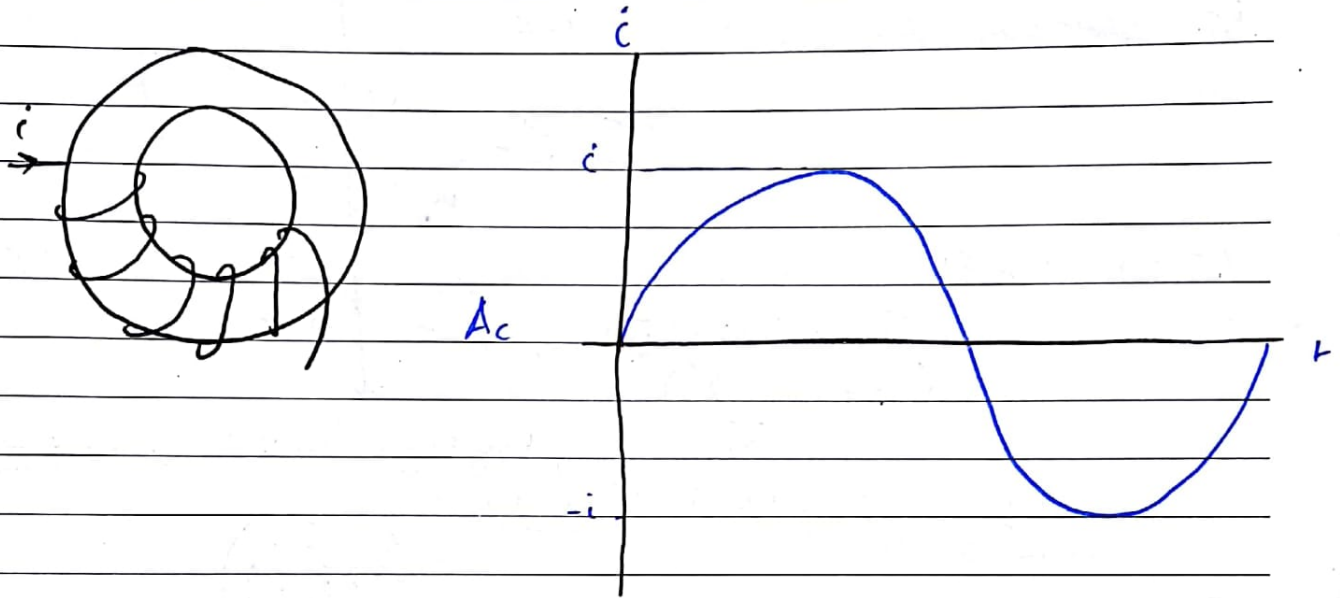
$$\frac{i2}{i1} = \frac{R + Rg}{R} > 1$$

$$\frac{i2}{i1} = 1 + \frac{Rg}{R}$$

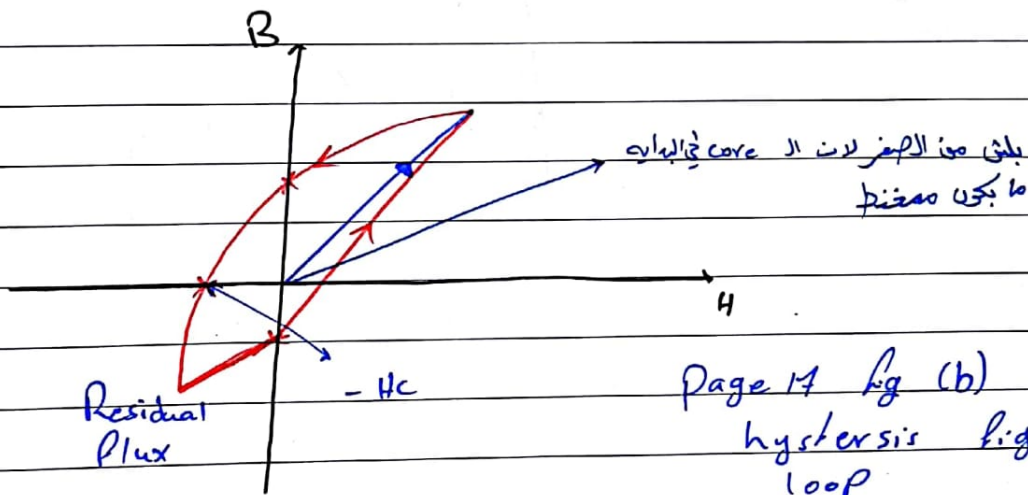




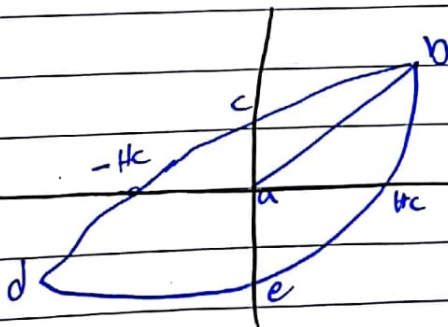
\* hysteresis loss and eddy current loss



عند إزالة التيار فإن المادة تحتاج عملية إزالة المغناطيسية

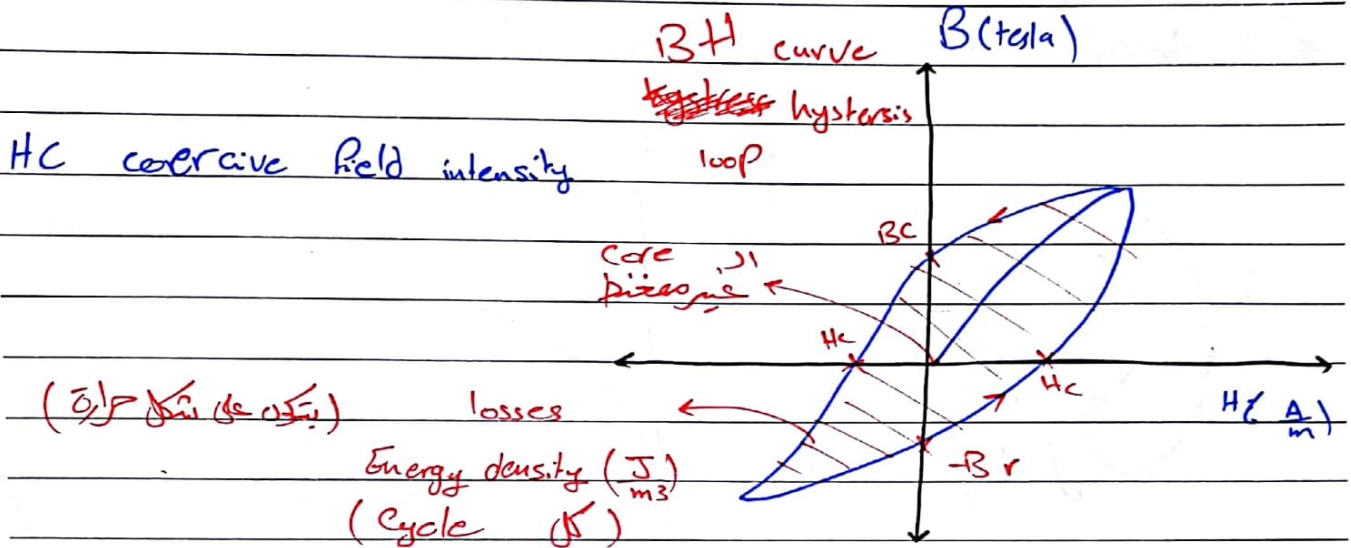


Page 17 fig (b)  
hysteresis fig(c)  
loop





4/10/2017



losses Energy Density ( $J/m^3$ ) (cycle)

الطاقة المفقودة لكل دورة (الطاقة المفقودة لكل دورة)

losses Energy Density ( $J/m^3$ ) (cycle)

الطاقة المفقودة لكل دورة (الطاقة المفقودة لكل دورة)

losses Energy Density ( $J/m^3$ ) (cycle)

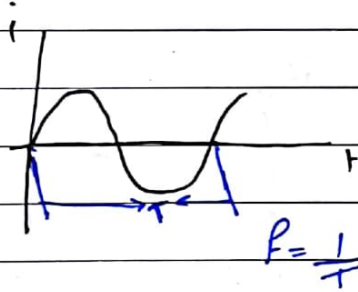
الطاقة المفقودة لكل دورة (الطاقة المفقودة لكل دورة)

Power = Energy / time

$= (\text{Area } BH) \times V_{\text{core}} \times f$

Volume

$f = \frac{1}{T}$



Rev.

$$F = i B L$$

$$\Rightarrow B = \frac{N}{A \cdot m} \Rightarrow \frac{J}{A m^2}$$

$$F \cdot distance = W$$

$$N \cdot m = J$$

$$N = \frac{J}{m}$$

$$H = \frac{A}{m}$$

$$\Rightarrow H \cdot B$$

$$\Rightarrow \frac{A}{m} \cdot \frac{J}{A m^2}$$

$$= \frac{J}{m^3}$$

losses (Area under The curve)

General electric company!

GE  $\Rightarrow$  Approximation

بما اننا ايجاد مساحة تحت المنحنى

فردا يحطوا بتقريب للمساحة فادرسوا طي الحساب

$$\text{Power density} = \underbrace{KH}_{\text{Constant (material)}} * (B_{max})^n * f = \frac{W}{m^3}$$

(for losses)

Constant (material)

$$* \frac{\text{Area B-H}}{KH * (B_{max})^n}$$

Area  $\phi$  F

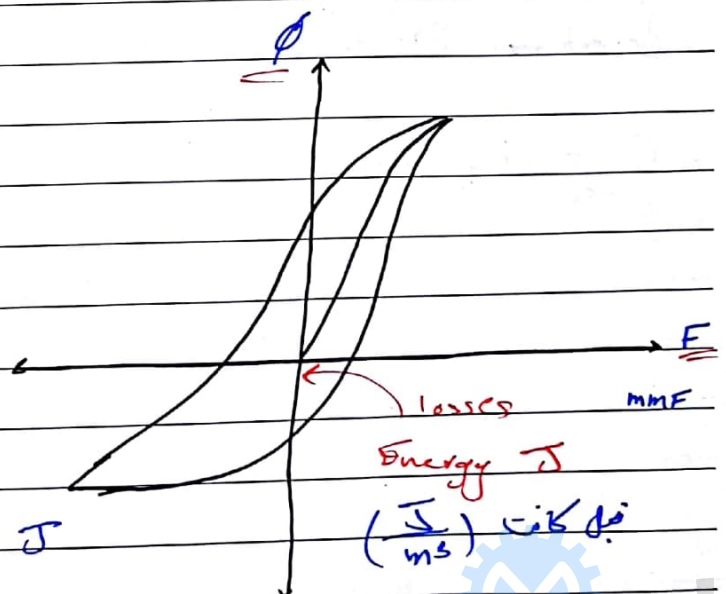
$$e = \frac{d\phi}{dt} * N \Rightarrow \phi = \frac{V \cdot sec}{Turn}$$

$$\text{voltage} = N \frac{\Delta\phi}{\Delta t}$$

$$F = A \cdot Turn$$

$$\Rightarrow \phi F =$$

$$A \cdot Turn \cdot \frac{V \cdot sec}{Turn} = \text{Energy } J$$



## \* Core losses

hysteresis losses

$$P_h = K_h \times B_{\max} \times F \quad \text{W/m}^3$$

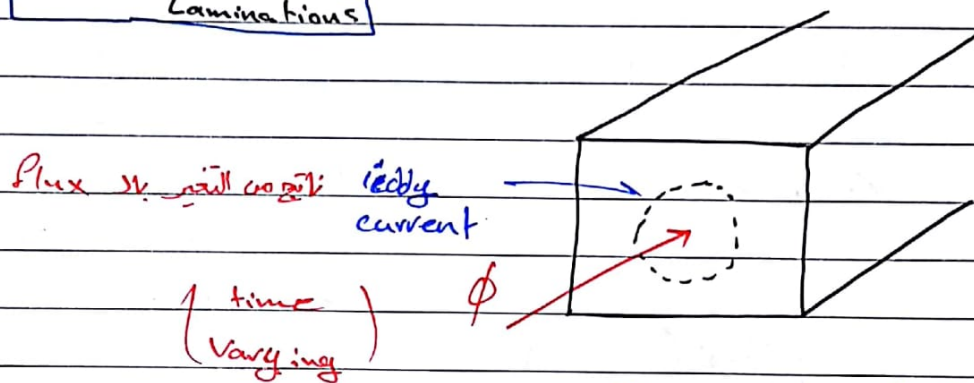
Eddy current losses

$$P_e = K_e \cdot B_{\max}^2 \cdot f^2 \quad \frac{\text{W}}{\text{m}^3}$$

Materials and

Laminations

depends on



$$P_{\text{losses}} = (i_{\text{eddy}})^2 R$$

(زيادة المقاومة بزيادة التردد)

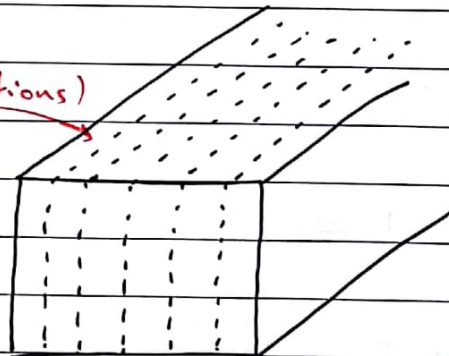
أقل التيار (less current)

Power losses أقل التيار على الـ Resistor  
أكثر من تأثير المقاومة

(Laminations)

$$P_{\text{losses}} = (i_{\text{eddy}})^2 R$$

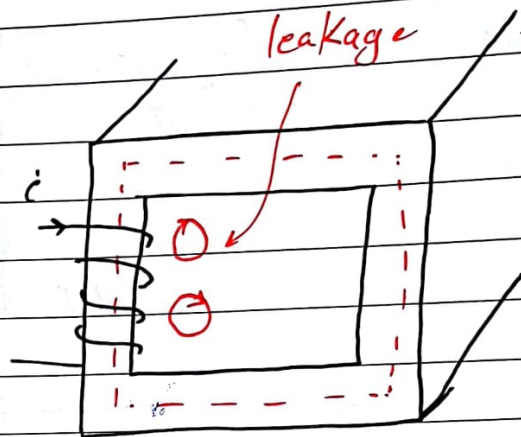
Lamination



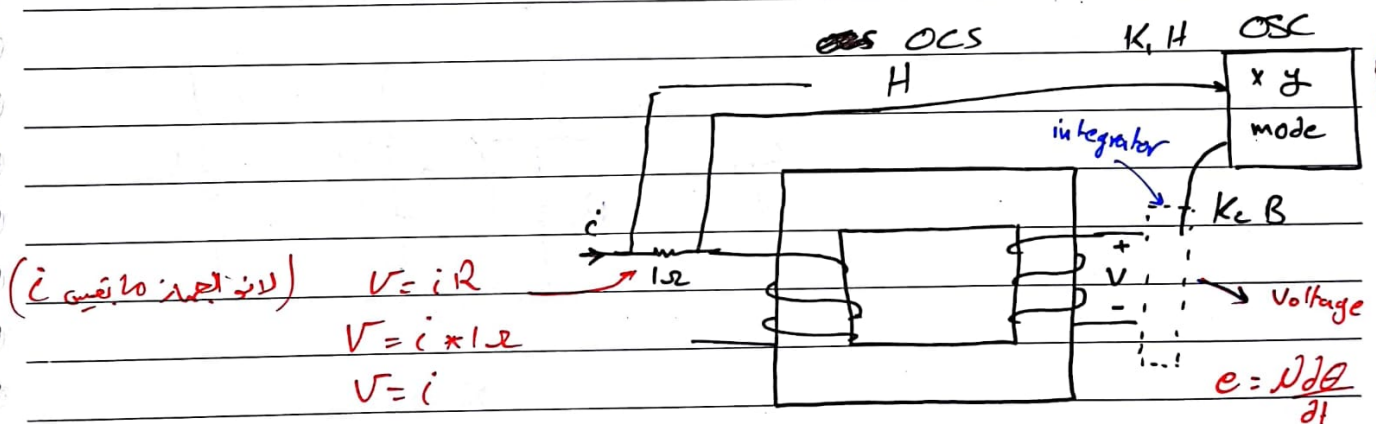


losses here

- 1) leakage flux
- 2) copper losses (السخونة النحاسية)
- 3) core losses
  - hysteresis
  - eddy current losses



measuring B-H curve of an known core (المعروف) يكون معطي لل core  
ب اننا ناك ان يكون معطي

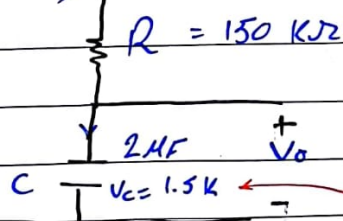


integrator

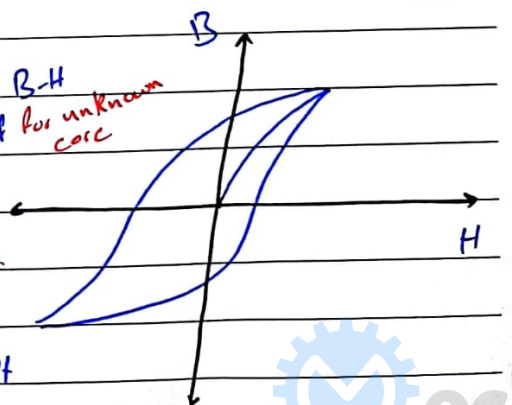
$$f = 50 \text{ Hz}$$

$$\omega = 2\pi f$$

$$\Rightarrow V \propto \frac{dB}{dt}$$



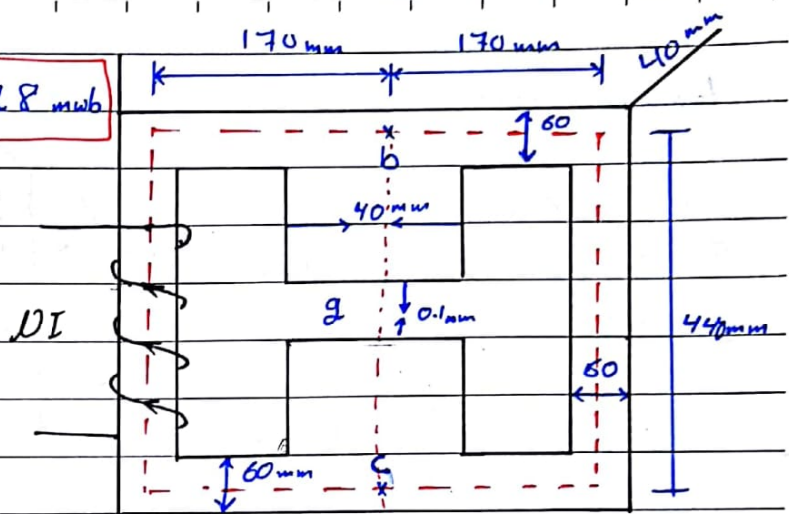
B-H  
loop for unknown core



$$i_c = \frac{C}{R} \frac{dV}{dt} \Rightarrow V = \frac{1}{C} \int i dt \Rightarrow V_0 = \frac{1}{C} \int \frac{V_{in}}{R} dt$$

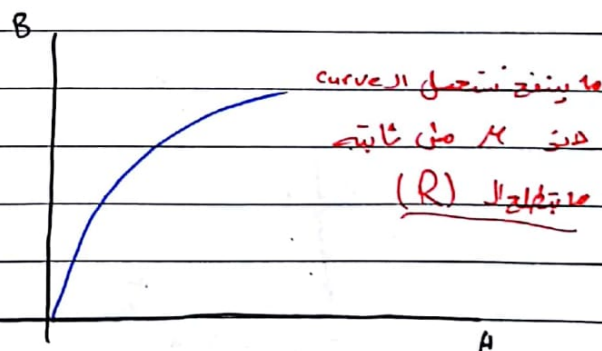
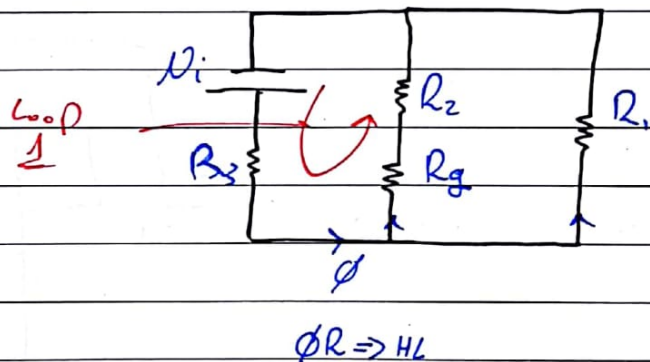
Ex: Find (I) so that  $\phi_g = 1.28 \text{ mwb}$

$N = 200 \text{ T}$



B (T)	0.39	0.8	0.925
H (A/m)	363.7	500	562

\* No Fringing  $\rightarrow$  the same flux density



9/10/2017

Solution 2

$\phi R$ , Ampere's law  $\oint H dl = I$

$$R = \frac{L}{\mu A} \quad \text{(air gap)} \quad \text{(core)} \quad \text{(air gap)}$$

$$\phi_g = \phi_{\text{core}} \quad (R_2)$$

$$B_g = B_{\text{core}} \quad (\text{No fringing})$$

$$\phi_g = 1.28 \text{ mwb}, \quad B_g = \frac{\phi_g}{A_g}$$

$$B = \frac{\phi}{A}, \quad H = \frac{B}{\mu}$$

$$\Rightarrow \phi_g = \frac{1.28 \text{ mwb} \times 10^{-3}}{40 \times 10^{-3} \times 40 \times 10^{-3}} = 0.8 \text{ T} \quad \phi \rightarrow B \rightarrow H$$

$$B_{\text{core}} (R_2) = 0.8 \text{ T}$$

$$H_{\text{air gap}} = \frac{B_g}{\mu_0} = 63.66 \times 10^4 \frac{\text{AT}}{\text{m}}$$

$$H_{\text{core}} \Rightarrow \text{Table} \Rightarrow H_{\text{core}} (R_2) = 500 \frac{\text{AT}}{\text{m}}$$

loop 1

KVL

$$- \frac{N_1}{?} + \frac{H_2 L_2}{?} + H_g L_g + H_2 L_2 = 0$$

Voltage drop

$$\phi R_1 H_1$$

$$\oint H dl = NI = H_1 L_1 + H_2 L_2 + H_2 L_2 \dots$$

mmf



To find the unknown

$$\Phi = \Phi_1 + \Phi_2$$

Total mmF For core ( $\mu_r$ ) and Air gap

$$\begin{aligned} &= H_g L_g + H_2 L_2 \\ &= 63.66 \times 10^4 (0.1 \times 10^{-3}) + 500 (440 - 0.1) \times 10^{-3} \\ &= 283.66 \text{ AT} \end{aligned}$$

$$- 283.66 = H_1 L_1$$

مغناطيسية  
(مقدار القوة)

$$283.66 = H_1 \times (170 \times 2 + 440) \times 10^{-3}$$

$$H_1 = 363.67 \frac{\text{AT}}{\text{m}}$$

$$B_1 = 0.389 \text{ T} \quad (\text{من الجدول})$$

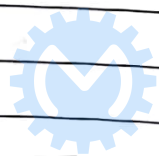
$$\Phi_1 = B_1 \times A = 0.39 \times 60 \times 10^{-3} \times 40 \times 10^{-3} = 0.94 \times 10^{-3} \text{ wb}$$

$$\Rightarrow \Phi_2 = \Phi_1 + \Phi_2 \Rightarrow \Phi = 2.22 \times 10^{-3} \text{ wb}$$

$$B_2 = \frac{\Phi}{A} = \frac{2.22 \times 10^{-3}}{60 \times 10^{-3} \times 40 \times 10^{-3}} = 0.92 \text{ T}$$

$$H_3 = 562.5 \quad \text{From Table}$$

$$-N_1 i + H_3 L_3 + H_g L_g + H_2 L_2 = 0 \Rightarrow i = 3.61 \text{ A}$$



Five Apple

Ex) hysteresis and eddy current losses in a certain equipment

$$P_h = 846 \text{ W}, P_e = 642, (240 \text{ V}, 25 \text{ Hz})$$

Determine hysteresis and eddy current losses if the core is connected to a 60 Hz as to core flux density 62% of Rated value. ~~Assume~~ Assume  $n = 1.4$

Sol:

$$P_h = K_h * f * B_m^n, P_e = K_e * f^2 * B_m^2$$

Recall  $V = \frac{N d\phi}{dt} = NA \frac{dB}{dt}$ ,  $B = B_m \sin \omega t$  <sup>assume</sup>

$$V(t) = NA B_m \omega \cos \omega t$$

$$V(t) = NA B_m (2\pi f) \cos \omega t$$

$V, B$  in phase shift  $\pi/2$

$$\Rightarrow V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{NA B_m (2\pi f)}{\sqrt{2}}$$

$$V_{rms} = 4.44 * f * N_2 * A * B_{max}$$

$$V \propto f * B_{max}$$

$$\frac{V}{f} = K * B_{max}$$

$$V = K * f * B_{max}$$

Cont. Sol.

$$p_{h1} = 8460 \text{ W}, \quad P_{g1} = 642 \text{ W}, \quad [240 \text{ V}, 25 \text{ Hz}], B_{\max}$$
$$p_{h2} = ??, \quad P_{g2} = ??, \quad [V = ??, 60 \text{ Hz}], B_{\max} = 0.62 B_{\max 1}$$

$$V_1 = K \times f_1 \times B_{\max 1}$$

$$V_2 = K \times f_2 \times \underbrace{B_{\max 2}}_{0.62 \times B_{\max 1}}$$

$$\frac{V_1}{V_2} = \frac{f_1}{f_2 \times 0.62} \Rightarrow \frac{240}{V_2} = \frac{25}{50 \times 0.62} \Rightarrow V_2 = \frac{240 \times 50 \times 0.62}{25}$$

$$\Rightarrow V_2 = 357 \text{ V}$$

$$p_h = K_h \times 25 \times B_{m1}^{1.4} = 846$$

$$p_{h2} = K_h \times 60 \times (0.62 B_{m1})^{1.4} = p_{h2}$$

$$\frac{p_{h2}}{846} = \left(\frac{60}{25}\right) \times \frac{(0.62 B_m)^{1.4}}{(B_m)^{1.4}}$$

$$\Rightarrow p_{h2} = 1039.75$$



$$642 = K_e * (25)^2 * B_m^2$$

$$P_{e2} = K_e * 60^2 * (0.62 B_m)^2$$

$$\Rightarrow B_{e2} = 1421.48$$

Ex) A core was connected to:

(No need for m)

$P_e + P_h$        $P_{core} = 500 \text{ W @ } 25 \text{ Hz, } 240 \text{ V}$       First time  
 $P_{core} = 1400 \text{ W @ } 50 \text{ Hz, } 480 \text{ V}$       Second time

$P_{ind}$        $P_{e1}, P_{e2}$   
 $P_{h1}, P_{h2}$

Sol:

$$P_{e1} + P_{h1} = 500 \dots (1)$$

$$P_{e2} + P_{h2} = 1400 \dots (2)$$

$$\frac{P_{h1}}{P_{h2}} = \frac{25}{50} \Rightarrow P_{h2} = 2P_{h1} \dots (3)$$

$$\frac{P_{e1}}{P_{e2}} = \left(\frac{25}{50}\right)^2 \Rightarrow P_{e2} = 4P_{e1} \dots (4)$$

$$\left. \begin{array}{l} P_{e1} + P_{h1} = 500 \\ 4P_{e1} + 2P_{h1} = 1400 \end{array} \right\} \text{ solve}$$

$$\Rightarrow \begin{array}{ll} P_{e1} = 200 & P_{h1} = 300 \\ P_{e2} = 800 & P_{h2} = 600 \end{array}$$

11/10/2017

## Transformer 8

### \* Notes:

with out fringing:

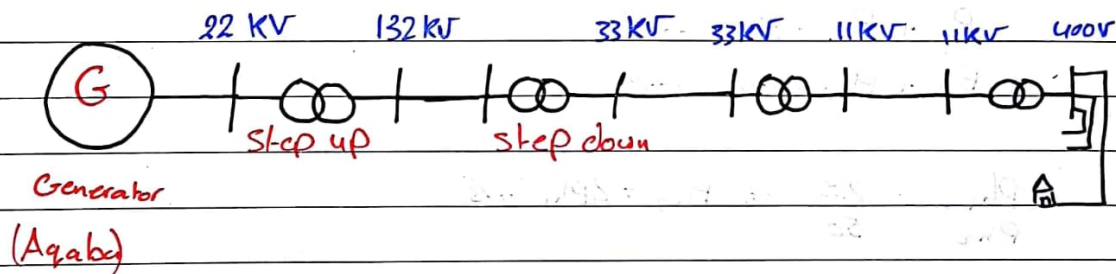
$$\phi_{\text{core}} = \phi_g \text{ and } B_{\text{core}} = B_g$$

gap

with out fringing:

$$B_{\text{core}} \neq B_g$$

\* Used in: Power system (Peak demand in Jordan 3GW)



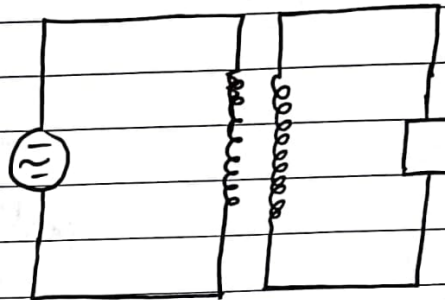
## 2-winding Transformer

"Primary and Secondary"

high voltage      low voltage

HV and LV side

Step up and step down



## Classification

- \* Single Phase transformer (used in networks rarely)
- \* Three Phase transformer (widely used)

## Name plate

- Rated primary voltage
- Rated secondary voltage
- Rated frequency
- Rated power

## Single Phase transformers

- 1- ideal transformers (No losses)
- 2- Real transformers



## Ideal Transformers

Faradays law

$$V = -N \frac{d\phi}{dt}$$

$$V_1 = -N_1 \frac{d\phi}{dt}$$

$$\phi = \frac{1}{N_1} \int V_1(t) dt$$

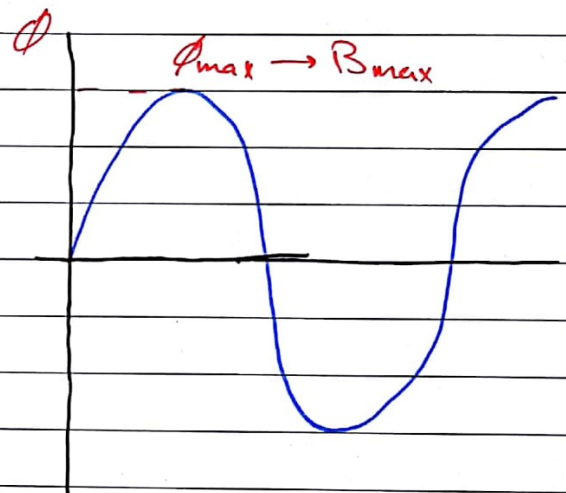
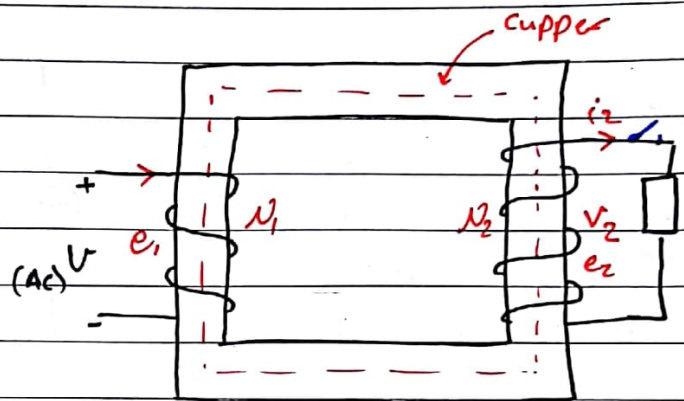
$$V_{rms} = 4.44 N A F B_{max}$$

Open circuit  $L_2 = 0$  (case 1)

$$V_2 = N_2 \frac{d\phi}{dt} \leftarrow \text{mutual flux}$$

$$V_1 = N_1 \frac{d\phi}{dt}$$

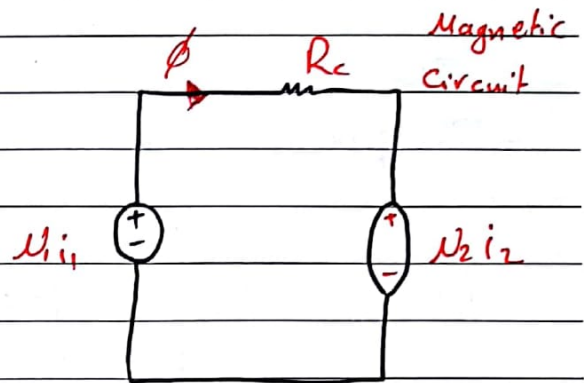
$$\frac{V_2}{V_1} = \left( \frac{N_2}{N_1} \right) \text{ turn ratio}$$



ideal  $\rightarrow (\mu \rightarrow \infty, R = \frac{L}{\mu A} = 0)$

$i_1 = 0$  an ideal case

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \Rightarrow \frac{e_2}{e_1} = \frac{N_2}{N_1}$$

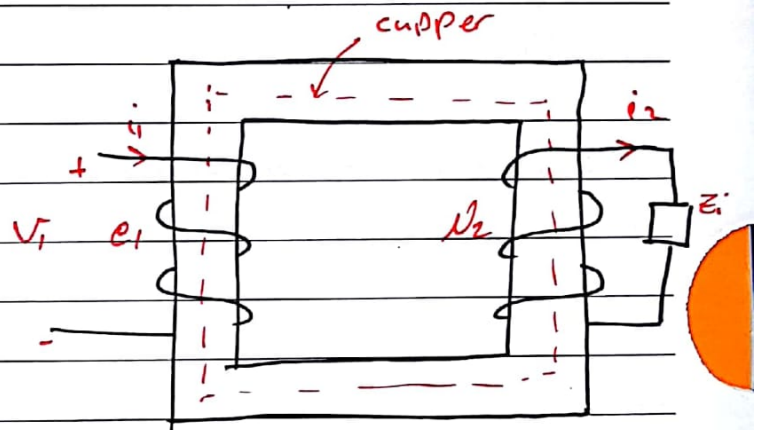


ideal transformer under load

$R=0, \mu=\infty$

$$N_1 i_1 = N_2 i_2$$

$$\frac{N_1}{N_2} = \frac{i_2}{i_1}, \quad \frac{V_2}{V_1} = \frac{N_2}{N_1}$$



$P_{primary} = P_{secondary}$



Speed = Voltage  
Torque = current

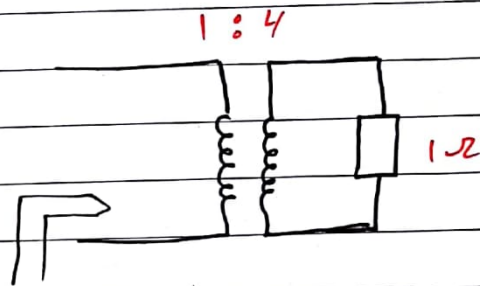
Elec.  $\rightarrow$  Mech.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}, \quad Z_L = Z_L \times \left(\frac{N_1}{N_2}\right)^2$$

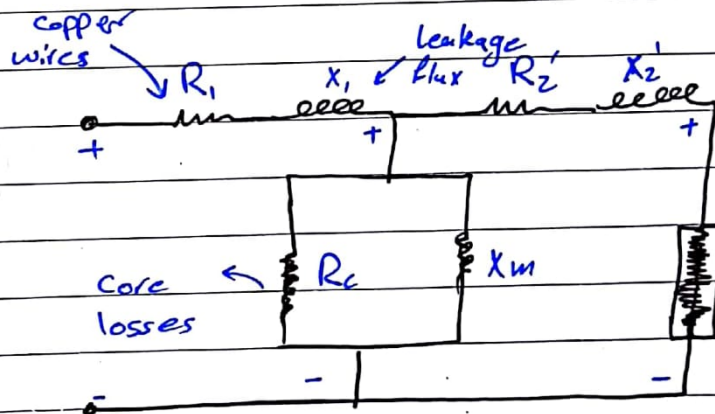
$$\frac{i_2}{i_1} = \frac{N_1}{N_2}$$

Ex:

Single-phase transformer



$$Z_L' = 1 \times \left(\frac{1}{4}\right)^2$$



- Real transformer equivalent ckt



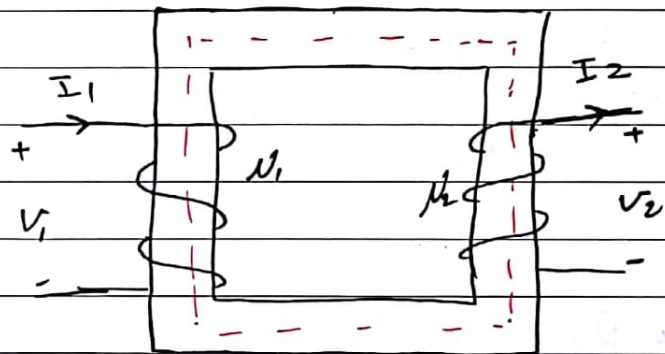
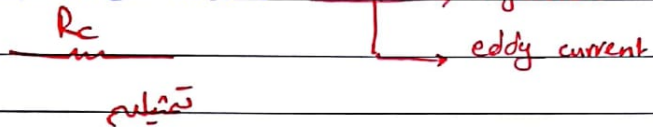
16/oct/2016

## modeling real transformer

\* Copper losses

\* Leakage flux

\* Core losses

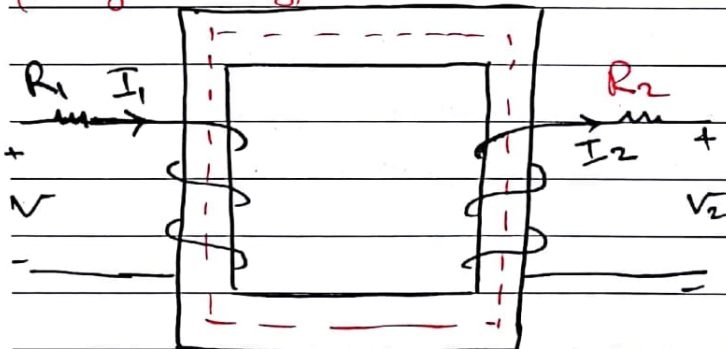


Magnetic coupling

$$L = \frac{N\Phi}{i}$$

Copper losses ( $R$ )

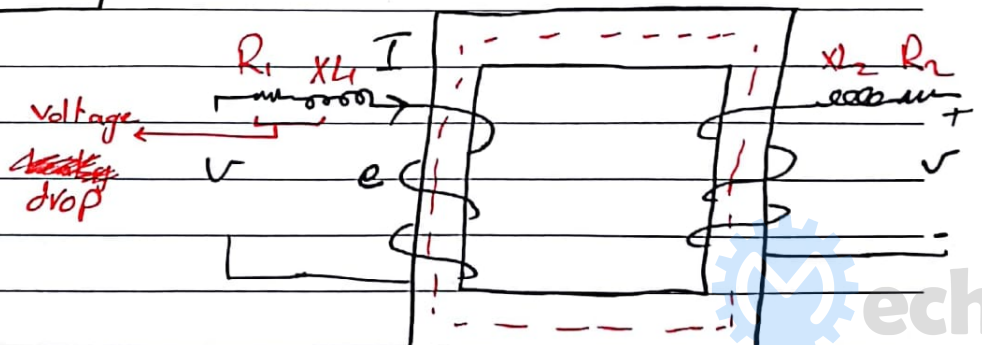
(Primary and secondary)  $V_1 \rightarrow 50-50$



Leakage flux

$$e = \frac{N\Phi}{dt}$$

$$\Phi_m = \frac{1}{m} \int e \cdot dt$$



Recall

$$|X_L| = \omega L = 2\pi f L \quad \text{eeee}^L$$

$$\angle \omega = 90^\circ (j)$$

$$|X_C| = 1/\omega C = 1/2\pi f C \quad \text{---}^C \text{---}$$

$$\angle X_C = -90^\circ (-j)$$

$$\underbrace{Z}_{\text{impedance}} = \underbrace{R}_{\text{Resistance}} + j \underbrace{X}_{\text{reactance}}$$

$$Z = \frac{1}{Y}$$

$$\underbrace{Y}_{\text{admittance}} = \underbrace{g}_{\text{conductance}} + j \underbrace{b}_{\text{susceptance}}$$

\* Real transformer under no-load

$$I_2 = 0 \Rightarrow I_1 \neq 0 \text{ (Excitation current)}$$

2 components

1) Magnetization current

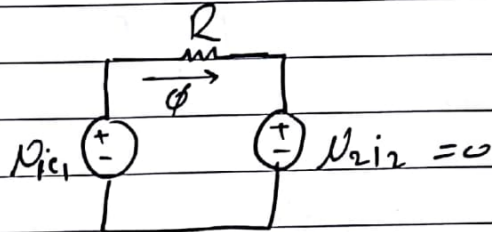
2) core losses

التي بدور في Current في ال Primary  
مزانو ال secondary current  
سيف

$\Rightarrow$  Magnetization ~~loss~~ current

$$N_1 i_1 = \Phi R \Rightarrow i \neq 0$$

$i$  is in phase  $\phi$



مغناطیسی ہمارے لائو کا سبب بنو گی  
field

$$e = N \frac{d\Phi}{dt}$$

$$\phi = \phi_m \sin \omega t = \phi \cos(\omega t - 90^\circ)$$

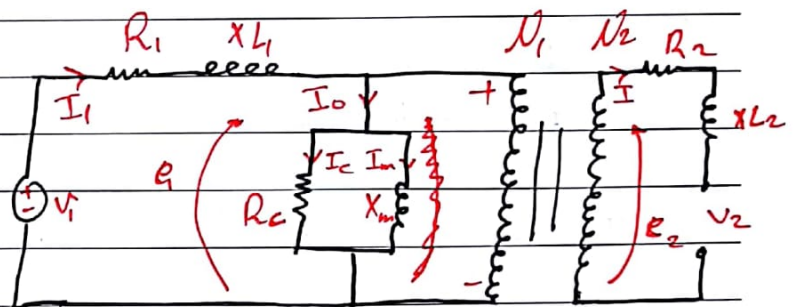
$$e = -N \omega \phi_m \cos \omega t$$

$\Rightarrow e$  leads  $\phi$

$e$  lead  $i \Rightarrow$  inductor  $\Rightarrow$  Magnetization current

$\Rightarrow$

Modelling real transformer:  
ideal transformer + external  
component:



1) copper losses ( $R_1$ )

2) Leakage flux ( $X_L$ )

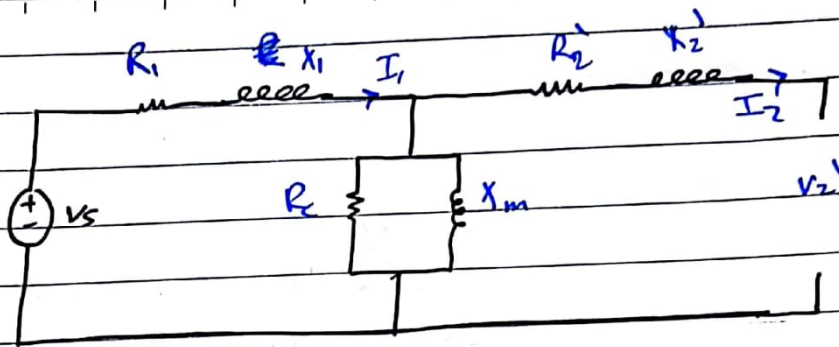
3) core losses ( $R_c$ )

4) Magnetization current ( $X_m$ )

$I_o \equiv$  excitation current

$I_m \equiv$  Magnetization current



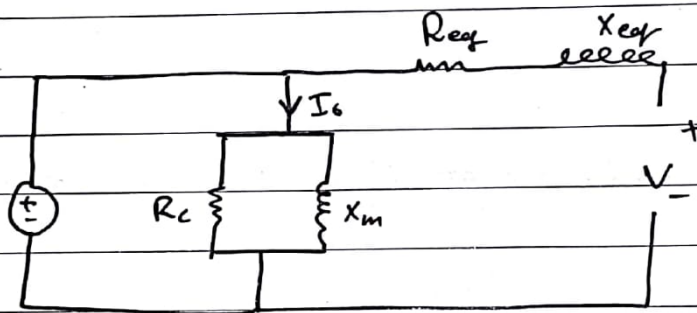


$$R_2' = R_2 \left( \frac{N_1}{N_2} \right)^2$$

نقل الـ  $R_2$  على الـ  $N_1$

$$X_2' = X_2 \left( \frac{N_1}{N_2} \right)^2$$

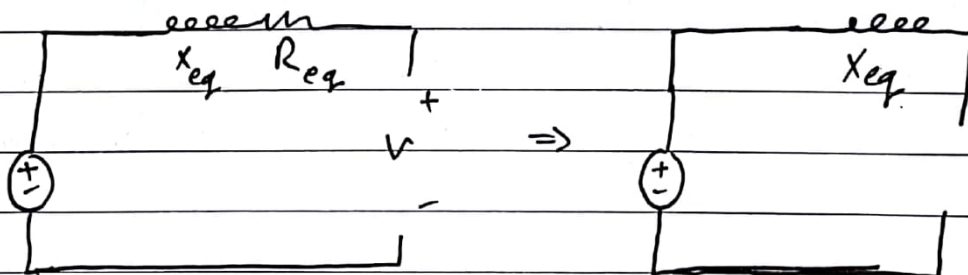
1. equivalent Approximation



$$R_{eq} = R_1 + R_2'$$

$$X_{eq} = X_1 + X_2'$$

$I_0 \approx (5-8)\%$  Full load



one transformer  
 $R_{eq}$  very small

## Transformer Rating and Nameplate

10 KVA , 1100 / 110 V

complex power  $\rightarrow$  اكبر  
transformer 1  $\rightarrow$  10 KVA

$\Rightarrow$  2 windings

HV side 1100 V } Rated  
LV side 110 } Voltage

turns ratio  $\frac{1100}{110} = 10$

Rated power @ HV side = 10 KVA

Rated power @ LV side = 10 KVA

Rated current @ HV side =  $10 \text{ KVA} / 1100 = 9.09 \text{ A}$

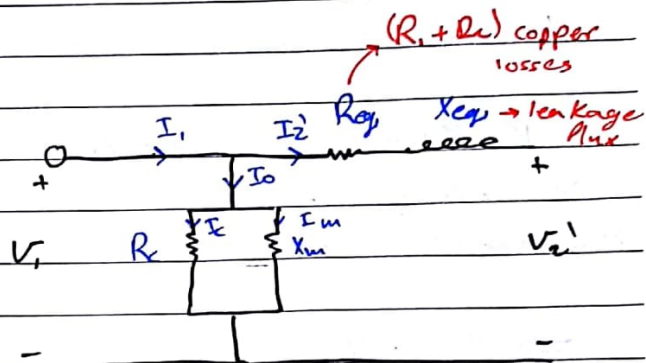
Rated current @ LV side =  $10 \text{ KVA} / 110 = 90.9 \text{ A}$

$S = P + jQ$   
 $\downarrow$  Real Power (W, KW)  
complex (apparent power)  
KVA  
MVA  
 $Q$ : Reactive power  
( $-\infty$  to  $+\infty$ )  
P.F. =  $P/S$

18/oct/2017

Recall

model transformer



التيار في الدائرة  
(المقاومة في الدائرة)

Power copper losses @ rated load

Power core losses @ (No load losses) @ rated voltage

! Load also affects core losses

$$P_{\text{copper losses}} = (I_1')^2 R_{eq}$$

Dependent  $I_2'$  (load)

$$P_{\text{core losses}} = \frac{V^2}{R_c} \leftarrow \text{high } V \text{ تأثير}$$

high  $R_c$  يعني  
والجهد العالي (لازم في المولدات)

Tests: قبل الموافقة على الجهاز بنسبة الى Losses ويجب ان يكون

transform test

No load test

(open circuit test)

No load losses

$R_c$

$X_m$

short circuit test

Load losses

$R_{eq}$

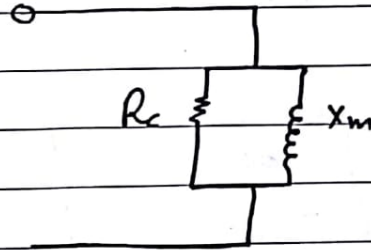
$X_{eq}$

Five Apple

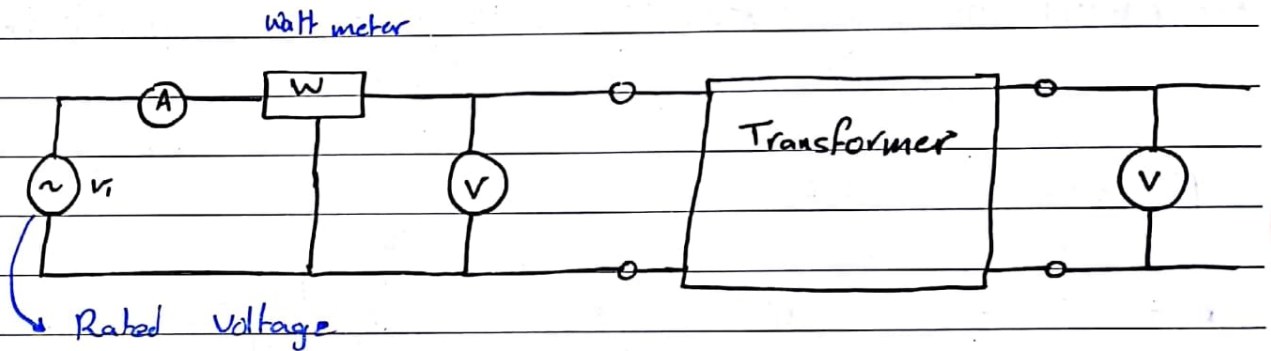
Family



## No load test



equivalent ckt



$$W = (V)(I) \times$$

$$W = (V)(I) \times \text{P.F.}$$

Power factor

Real power

Power loss in the transformer

Watt meter complex power meter

$$S = VI^* \Rightarrow |S| = |V||I|$$

$$S = P + jQ$$

Complex power

Aux inductor



$\Rightarrow$  No load losses,  $R_c$ ,  $X$  ??

$\rightarrow$  inputs: volt meter, Ameter, Watt meter

Q) where the test is done? L.V side or H.V side ??

$\Rightarrow 1 \text{ kVA}, 1000/100 \text{ V}$

open circuit test

V	1000 V
I	---
P	---

$(R_c, X_m)$  referred to HV

HV side excited

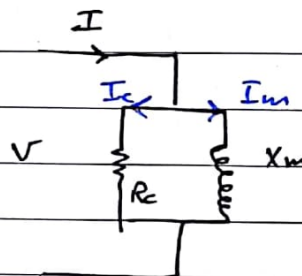
LV side open circuit

H.V. Jigidi lola, Rated Voltage V Test Jigidi Voltage

Back to the test!

$$P = \frac{V^2}{R_c} \Rightarrow R_c = \frac{V^2}{\text{Power}} \quad (1)$$

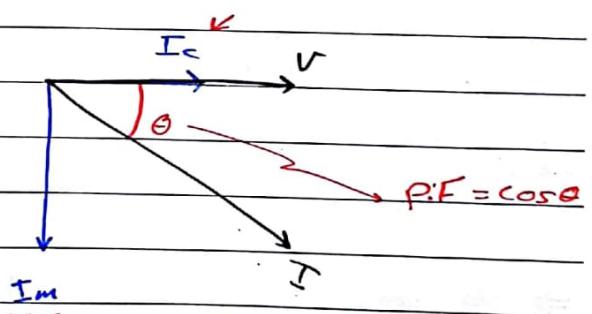
$$I_c = \frac{V}{R_c} \quad (2)$$



$$I_m = \sqrt{I^2 - I_c^2} \quad (3)$$

Voltage is in phase with  $I_c$

$$X_m = \frac{V}{I_m} \quad (4)$$



leads inductor  $X_m$

Voltage is

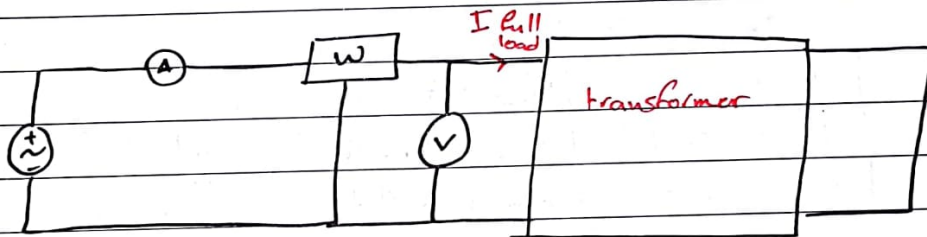
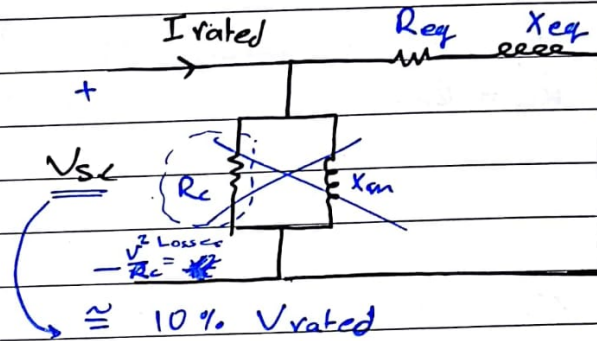
(سلف)

$$P.F. = \frac{P}{VI} = \frac{P}{S}$$

## Short circuit test

to find load losses @ Rated current

to reach the rated current we set the short circuit voltage at 10% of the Rated Voltage



Watt meter = copper losses + core losses = 0  
 @ rated current @ 10% rated voltage  
 Full load ( $V_{sc}$ )

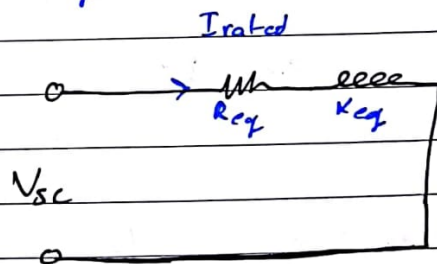
now we have to find ( $R_{eq}$ ,  $X_{eq}$ )  
 ( $P$ ,  $V$ ,  $I$ ) known

$$P = I^2 R_{eq} \Rightarrow R_{eq} = \frac{P}{I^2} \quad (1)$$

$$Z_{eq} = \frac{V}{I}$$

$$Z_{eq} = R_{eq} + jX_{eq}$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$





$$R_{eq} = R_1 + R_2'$$

$$X_{eq} = X_1 + X_2'$$

ايزع صغيرات د صلا والين

$R_1$  ( can be measured )  
 $R_2 = R_{eq} - R_1$

نفرجات

$$X_1 \cong X_2'$$

$$X_1 = X_{eq} / 2$$

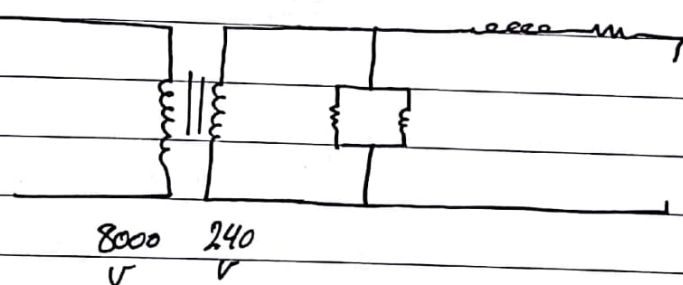
$$X_2' = X_{eq} / 2$$

Ex) 20 KVA , 8000 / 240 V , 60 Hz transformer

Open ckt	Short ckt
V = 8000 V	V <sub>sc</sub> = 489 V
I = 0.214 A	I <sub>sc</sub> = <u>2.5 A</u>
P = 400 W	P <sub>sc</sub> = 240 W

H.V ال جانب  
side  
L.V ال جانب  
side

Draw the equivalent ckt of transformer



Open ckt  $\rightarrow R_c$  Referred to  
 $\rightarrow X_m$  high voltage side

Short ckt  $\rightarrow R_{eq}$  Referred to  
 $\rightarrow X_{eq}$  HV side

$$I_{rated/LV} = \frac{20 \text{ KVA}}{240}$$

$$I_{rated/HV} = \frac{20 \text{ KVA}}{8000}$$

$$= \underline{2.5 \text{ A}}$$

23/oct/2017

مسألة

إذا كانت الـ  $8100 = (V)$  في الـ No load losses

نسبة وتساوي

$$8000 \rightarrow 400$$

$$8100 \rightarrow ??$$

$$P = \frac{V^2}{R_c} \text{ او } P = \frac{V^2}{R_c} \text{ وبتحل انو}$$

Suppose Rated Voltage  $8000V$  يعني ممكن بتحل اكثر من  $10\%$  تقريباً  
 يعني بتكون الـ حساب الـ losses (بتزيد No load losses)

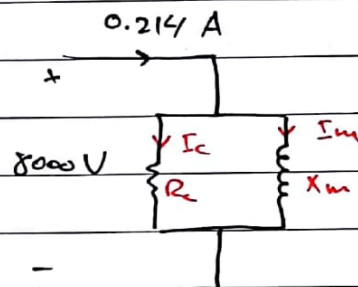
cont sol :

$$P = \frac{V^2}{R_c} \Rightarrow P = \frac{(8000)^2}{R_c} = 400 \Rightarrow R_c = 159 \Omega$$

$$I_c = \frac{8000}{159} =$$

$$I_m = \sqrt{(0.214)^2 + (I_c)^2}$$

$$X_m = \frac{8000}{I_m} = 38.3 \text{ k}\Omega$$



$$R_c / L_v = 159 * \left( \frac{240}{8000} \right)^2 =$$

$$X_m / L_v = 38.3 * \left( \frac{240}{8000} \right)^2 =$$

$$Y_{o.c} = \frac{1}{Z} = \frac{1}{R_c} + \frac{1}{jX_m}$$

open circuit

Power losses

Active  $\rightarrow$  heat,

Reactive  $\rightarrow$  Flux generation

all losses will be given

leakage flux

Short ckt test

$$240 = (2.5) A \times R_{eq}$$

$$R_{eq} = 38.4 \Omega$$

$$Z_{eq} = \frac{489}{2.5}$$

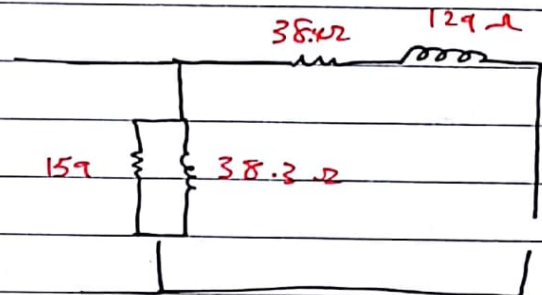
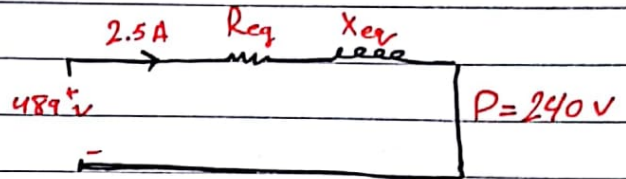
$$X_{eq_{HV}} = \sqrt{Z_{eq}^2 - R_{eq}^2} = 192 \Omega$$

$$\frac{X}{R} = \frac{192}{38} \approx 5$$

مقاومة دفرنا على السبب جده اتي

في ونطال منها قيم

نفس القسم على H.V.s او على ال L.V.s



H V Side

short skt → H.V side  
open skt → L.V side

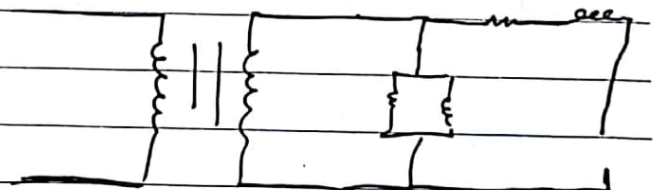
دوره يكون طالب على ال

L.V. side

مقاومة دفرنا على ال H.V side ونطال

$$\left( \frac{240}{8000} \right)^2$$

8000 / 240V L.V side





## Analysis of the transformer

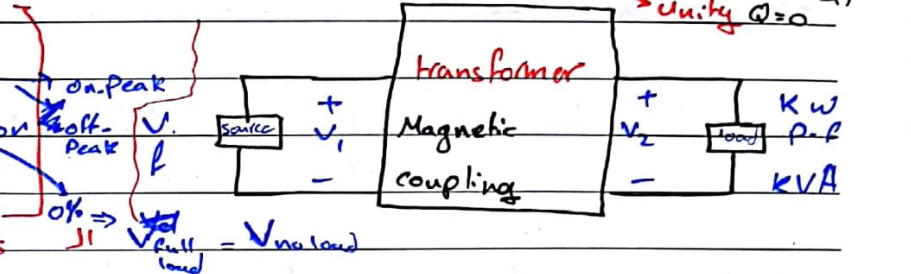
\* voltages

\* voltages drop

\* Voltage Regulation

\* efficiency

Losses & Efficiency Analysis

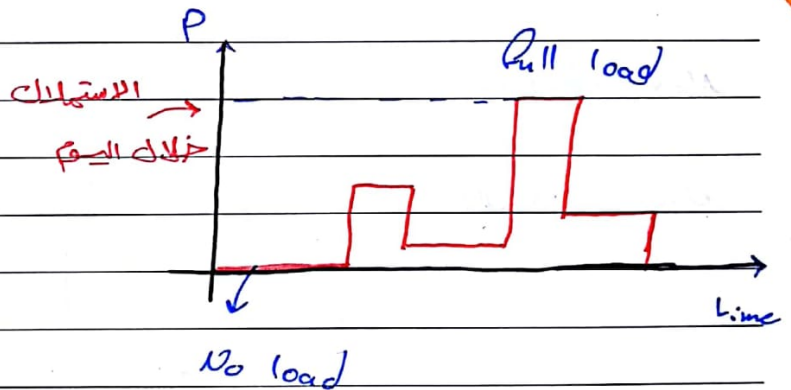
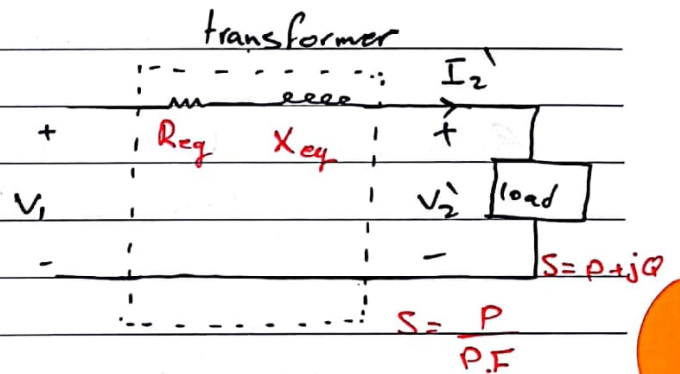


- It's aimed to keep voltage at the load side during

① No load

② Full load

close to  $(V_2')$



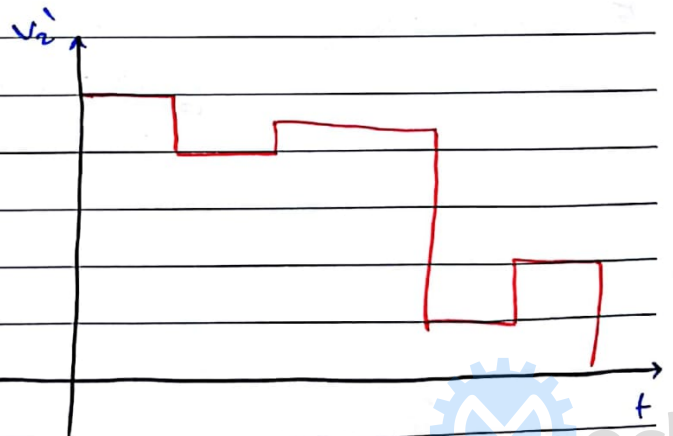
$V_1$  Fixed,  $V_2'$  depends on the load

$$V_2' = V_1 - I_2' (R_{eq} + X_{eq})$$

$$S = V I^*$$

$$|I| = \frac{|S|}{|V|}$$

كلما زاد الحمل قل  $V_2'$  (كلما زاد الحمل قل  $V_2'$ )



\* if  $V_2'$  is fixed ( $V_1$  is controlled)

$\rightarrow V_2$  limit

at no load  $\Rightarrow V_1 = V_2 \text{ limit} \Rightarrow V.D = 0$

at full load  $\Rightarrow V_1 = V_2 \text{ limit} + I (R_{eq} + jX_{eq})$

$$\Rightarrow V.D = V_1 - V_2 \text{ limit}$$

$$V.R. = \frac{|V_{1, \text{no load}}| - |V_{1, \text{full load}}|}{|V_{1, \text{full load}}|} \times 100 \%$$

$$\text{Efficiency} = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{losses}}$$

$$= \frac{P_{out}}{P_{out} + P_{core} + P_{copper}}$$

$$P_{core} = \frac{V^2}{R_c}$$
$$P_{cu} = (I_2')^2 R_{eq}$$

Max efficiency

$$P_{core} = P_{copper}$$

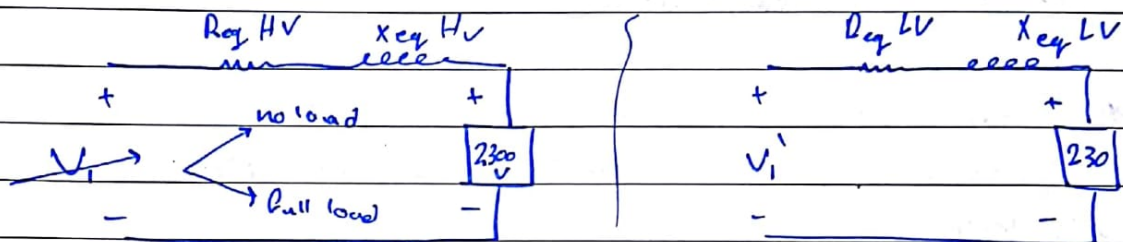
$$\text{Energy efficiency} = \frac{\sum_{i=1}^N P_{o,i} \times \Delta t_i}{\sum_{i=1}^N P_{i,i} \times \Delta t_i}$$

$N = 24 \text{ hours}$

25/oct/2017

$$VR \% = \frac{|V_{i, \text{no load}}| - |V_{i, \text{full load}}|}{|V_{i, \text{full load}}|} \times 100 \%$$

Ex) 2300/230 V



L.V side

$$\eta = \frac{P_o}{P_o + P_{core} + P_{cu}}$$

Ex) Unity, PF load, Keep load voltage fixed. what is the impact on core losses

- No load
- Full load

$$P_{core} = \frac{V^2}{R_c} \quad (\text{fixed voltage @ full load})$$

$$\rightarrow V_i \uparrow \rightarrow P_{core} \uparrow$$



Ex) 15 KVA, 2300/230 transformer

Open ckt	Short ckt
$V_{oc} = 2300 \text{ V}$	$V_{sc} = 47 \text{ V}$
$I_{oc} = 0.21 \text{ A}$	$I_{sc} = 6 \text{ A}$
$P_{oc} = 50 \text{ W}$	$P_{sc} = 160 \text{ W}$

\* Equivalent ckt referred to the H.V side

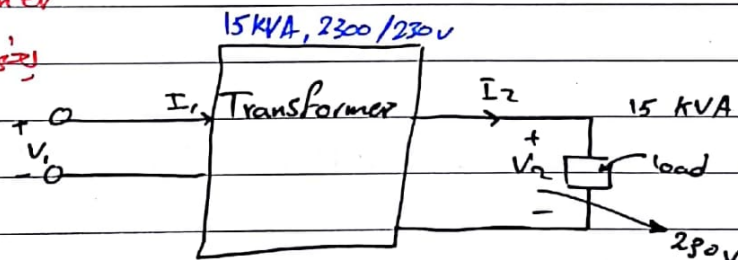
\* Equivalent ckt referred to the L.V side

Q.10 (15 KVA, 230 V) ← 15 KVA, 230 V

- \* Full load voltage ~~regulation~~ regulation
  - at 0.8 lagging power factor
  - at unity P.F
  - at 0.8 P.F lead

\* efficiency at full load with 0.8 PF lag.

(step down transformer  
(H.V.S) → L.V.S)

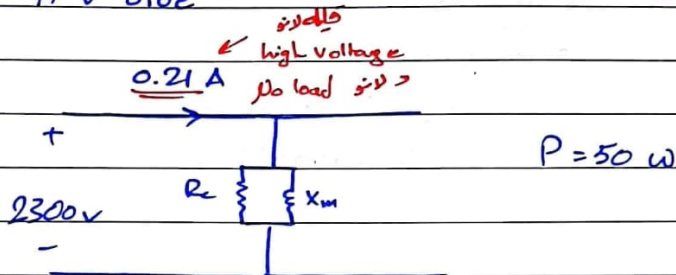


$$\text{Rated current}_{HV} = \frac{15 \text{ KVA}}{2300} = 6.5 \text{ A}$$

H.V Side

$$\text{Rated current}_{LV} = \frac{15 \text{ KVA}}{230} = 65 \text{ A}$$

\* H V Side



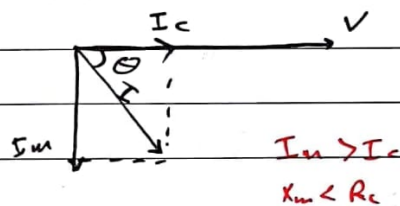
$$P = \frac{V^2}{R_c} \Rightarrow R_c = \frac{(2300)^2}{50} = 105 \text{ k}\Omega$$

$$I_c = \frac{V}{R_c}, \quad I_m = \sqrt{I^2 - I_c^2}$$

$$\Rightarrow |X_m| = \frac{V}{I_m} = 11 \text{ k}\Omega, \quad X_m = 11 \text{ k}\Omega \angle 90^\circ$$

$$R_c \text{ HV} = 105 \text{ k}\Omega$$

$$X_m \text{ HV} = 11 \text{ k}\Omega$$



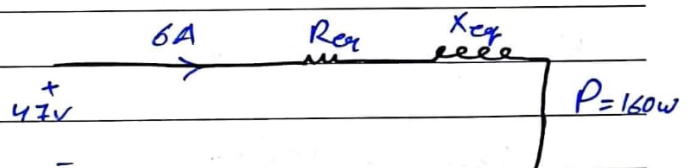
Short ckt

$$P = I^2 R_{eq}$$

$$R_{eq} = \frac{160}{6^2} = 4.45 \Omega$$

$$Z_{eq} = \frac{V}{I} =$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = 6.45 \Omega$$



$$R_{e\text{ HV}} = 105 \text{ k}\Omega$$

$$X_{m\text{ HV}} = 11 \text{ k}\Omega$$

$$R_{eq\text{ HV}} = 4.45 \Omega$$

$$X_{eq\text{ HV}} = 6.45 \Omega$$

$$R_{e\text{ LV}} = R_{e\text{ HV}} \left( \frac{230}{6200} \right)^2 = 1050 \Omega$$

$$\Rightarrow X_{m\text{ LV}} = 110 \Omega$$

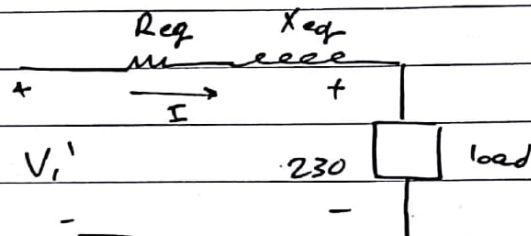
$$R_{eq\text{ LV}} = 0.6445 \Omega$$

$$X_{eq\text{ LV}} = 0.0645 \Omega$$

\* VRY (15 kVA, 230V, unity PF)  
Load

$$1) \text{ No load } I = 0$$

$$\Rightarrow V_1' = 230 \text{ V}$$



$$2) \text{ Full load } S = V I^*$$

$$|S| = |V| |I|$$

$$I = \frac{15 \text{ k}}{230} = 65.2 \text{ A}$$

$$a) \text{ unity P.F. } Q = 0$$

V and I in phase

$$I = 62.2 \angle 0^\circ$$

$$b) \text{ lagging (0.8 P.F.)}$$

$$I = 62.2 \angle -\cos^{-1} \text{ P.F.}$$

$$c) \text{ leading (0.8 P.F.)}$$

$$I = 62.2 \angle \cos^{-1} \text{ P.F.}$$



$$V_1' = 230 \angle 0 + I(R_{eq}) + I j(X_{eq})$$

\* Unity PF  $\Rightarrow V_1' = 232.94 \angle 21.4^\circ$   
 $V_2 = 230 \angle 0^\circ$   
 Real Power  
 Reactive Power

load 11 clploo  $\Rightarrow 15 \text{ KVA, unity P.F}$   
 $\Rightarrow P = 15 \text{ kW, } Q = 0$

\* lagging power factor (0.8)  $\Rightarrow$   $Q$  كثر من  $P$   $\Rightarrow$   $Q$  كثر من  $P$

$$I = 65.2 \angle -\cos^{-1} 0.8$$

$$V_1' = 230 \angle 0 + I(R_{eq} + jX_{eq})$$

$$V_1' = 234.85 \angle 0.4^\circ \text{ V}$$

\* Leading

\* leading power factor  $\Rightarrow V_1'$  كثر من  $V_2$   $\Rightarrow$   $V_1'$  كثر من  $V_2$

$$I = 65.2 \angle +\cos^{-1} 0.8$$

Voltage كثر

$$V_1' = 229.85 \angle 1.27^\circ$$

$$0.85 = \text{PF} \Rightarrow \text{PF} = 0.85$$

Efficiency @ full load (15 KVA, 230 V, 0.8 pf lagging)

Energy efficiency  $\Rightarrow$   $\eta$  كثر من 1

Point 11  $\Rightarrow$  Efficiency  $\Rightarrow$   $\eta$  كثر من 1

Watt كثر من 1000

$$\eta = \frac{P_o}{P_o + P_{core} + P_{cu}} \times 100\%$$

$$P_o = 15 \times 0.8 = 12 \text{ kW}$$

$$P_{core} = \frac{V^2}{R_c} = \frac{(234.85)^2}{1050}$$

$$P_{cu} = I^2 R_{eq} = (65.2)^2 \times (0.0145)$$

30/10/2017

\* Voltage Regulation :

$$V.R \% = \frac{|V_2, \text{no load}| - |V_2, \text{full load}|}{|V_2, \text{full load}|} \times 100\%$$

$V_1$  fixed,  $V_2$  no load,  $V_2$  full load

By definition Full load  $\Rightarrow V_2 = V_2 \text{ rated}$

$$1) I_{PL} \Rightarrow |I| = \frac{|KVA|}{V_2 \text{ rated}} \Rightarrow I = |I_{FL}| \begin{cases} + \cos \phi \rightarrow \text{PF leading} \\ - \cos \phi \rightarrow \text{PF lagging} \\ 0 \rightarrow \text{PF unity} \end{cases}$$

$$2) V_1' = V_2 + I(R_{eq} + jX_{eq})$$

$$3) V.R \% = \frac{(|V_1'| - |V_2 \text{ rated}|)}{|V_2 \text{ rated}|} \times 100\%$$

← drop voltage at full load

0.8 lagging PF  $\Rightarrow |V_1'| = 234 \text{ V}$

$$\Rightarrow V.R \% = \frac{234 - 230}{230} \times 100\% =$$

unity PF  $\Rightarrow |V_1'| = 232 \text{ V}$

$$V.R \% = \frac{232 - 230}{230} \times 100\% =$$

0.8 leading PF  $\Rightarrow |V_1'| = 229.85$

$$V.R \% = \frac{229.85 - 230}{230} \times 100\%$$

@ no load  
 $V_2 = V_1'$

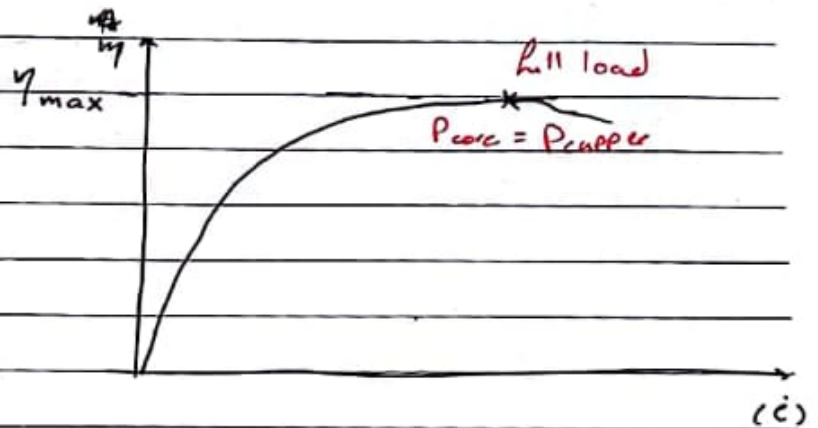
@ Full load  
 $V_2 = |V_2 \text{ rated}|$

$V_1 \Rightarrow$  unknown

eg: Rated 11 kv line  
82.5%

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{P_o}{P_o + P_{\text{core}} + P_{\text{copper}}}$$

full load better



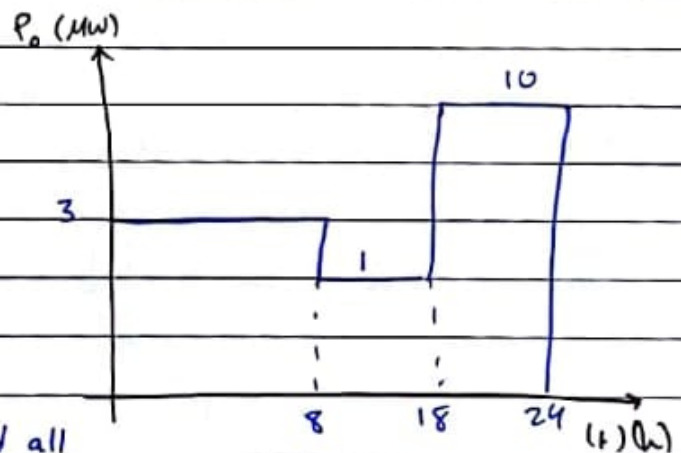
all day efficiency  $\Rightarrow$  Energy Perspective

$$M \text{ volt} * PF = MW$$

$$M \text{ volt Am} * PF$$

$$E_o = 3 * 8 + 1 * 10 + 10 * 6$$

output energy = 94 MWh



assume  $P_{\text{core}} = 500 \text{ W}$ ,  $V_i$  is fixed all the day

$$E_{\text{core loss}} = 0.5 * 24 = 12 \text{ MWh}$$

$$E_{\text{losses copper}} = (I_1^2 * R_{eq}) * 8 + (I_2^2 * R_{eq}) * 6 + (I_3^2 * R_{eq}) * 6$$



Ex) Three identical single-phase transformer, each rated 10 KVA, <sup>Phase</sup> 2400 / <sup>Phase</sup> 1120 V, 60 Hz. They are connected to form a 3 $\phi$  4160 / 208 V 3 $\phi$  transformer. The equivalent impedance of each transformer referred to (H.V) side is  $(10 + j25) \Omega$  3 $\phi$  load (27 KW, 208 V, 0.9 PF leading)

determining the connection

$\Delta / Y \Rightarrow$  From the voltage  
2160 208

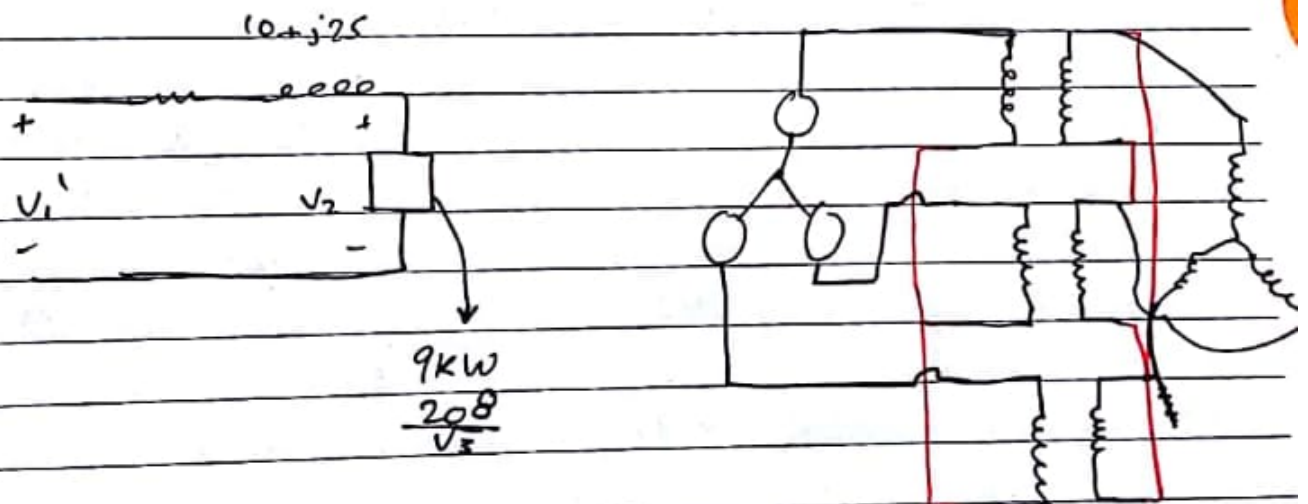
3 $\phi$  4160 / 120

Y /  $\Delta$

$V_{LL} = \sqrt{3} V_{LN}$  Y

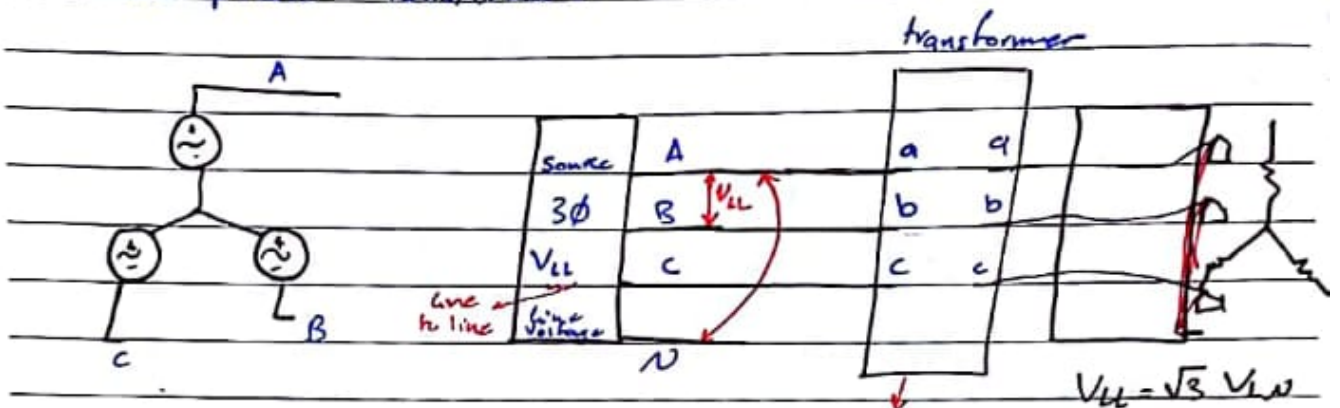
$V_{LL} = V_{LN}$   $\Delta$

Balanced  $\Rightarrow$  per Phase Analysis



(Y /  $\Delta$ )

## Three phase transformers

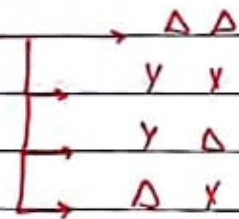


\* 3 $\phi$  Rated KVA

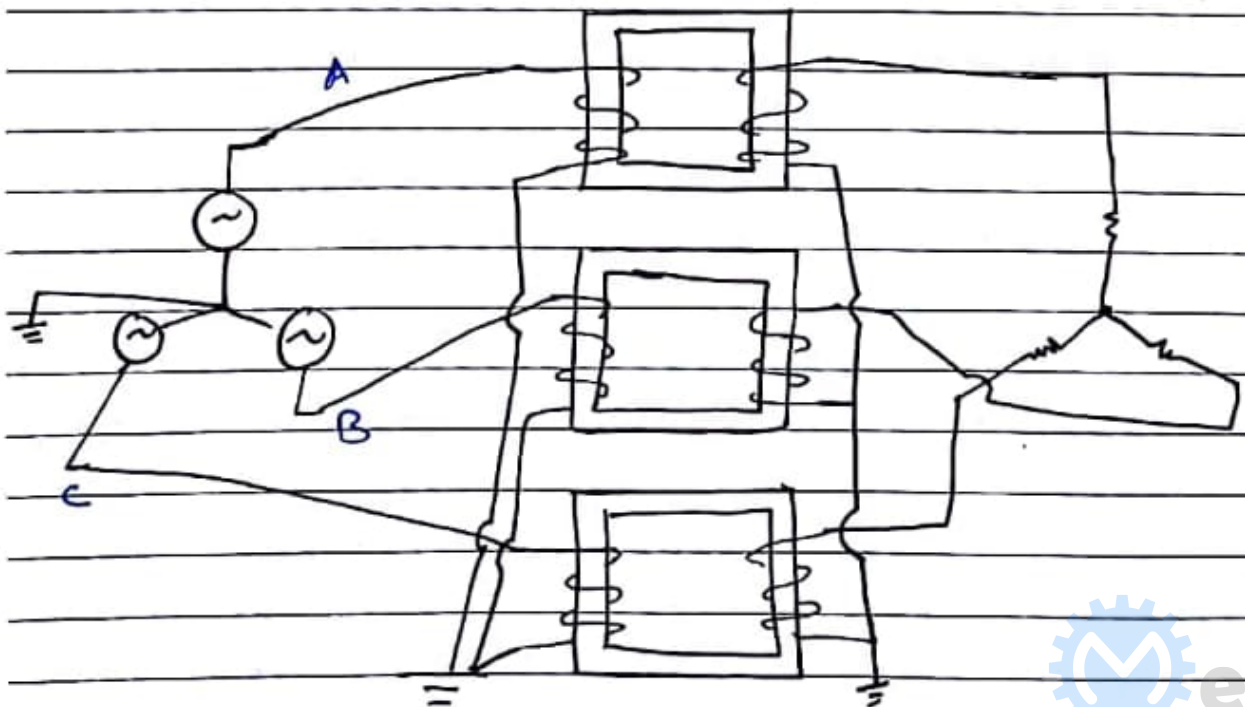
\*  $V_{LL}$  (HV) rated

\*  $V_{LL}$  (LV) Rated

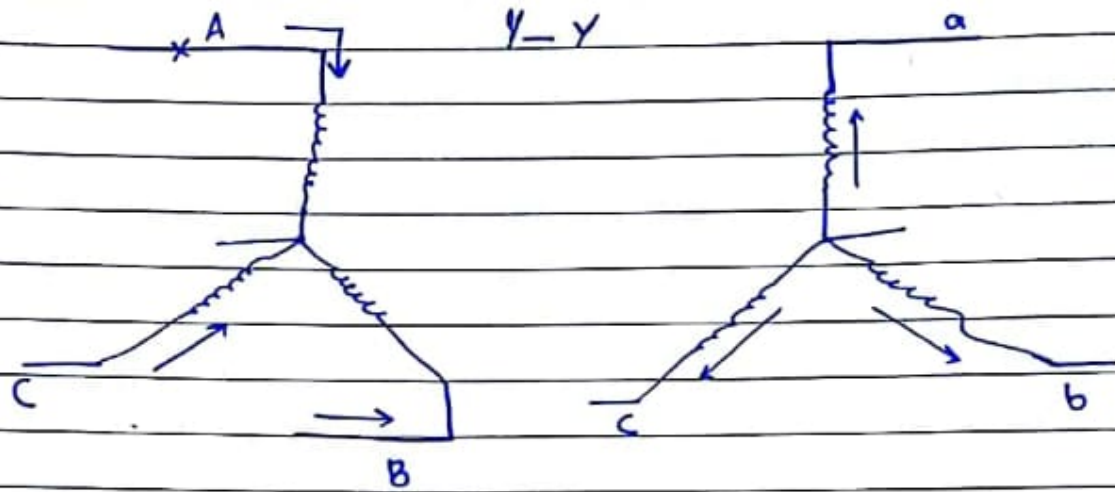
\* connection



## 3 $\phi$ transformer Y-Y



1/Nov/2017



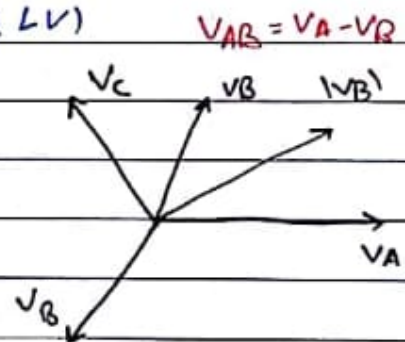
$$a = \text{turns Ratio} = \frac{V_p (HV)}{V_p (LV)}$$

$$a_T = \text{Transformation Ratio} = \frac{V_{LL} (HV)}{V_{LL} (LV)}$$

$$V_{LL} (HV) = \sqrt{3} \cdot V_p (HV)$$

$$V_{LL} (LV) = \sqrt{3} \cdot V_p (LV)$$

$$\Rightarrow a_T = \frac{\sqrt{3} \cdot V_p (HV)}{\sqrt{3} \cdot V_p (LV)} = a_o = \text{turns Ratio (Y-Y)}$$



$$I_L^{HV} = I_L^{LV} \times \left( \frac{V_p \text{ Rated LV}}{V_p \text{ rated HV}} \right)$$



turns Ratio  $\rightarrow$  converts from phase to phase

Transformation Ratio  $\rightarrow$  converts from line to line

$\Rightarrow$  find  $I_1$  on LV

$$|I_1| = I_0 * \left( \frac{0.4/\sqrt{3}}{11} \right), \text{ phase to phase (turns Rat)}$$

$$I_2 = \sqrt{3} * I_0 * \frac{0.4}{\sqrt{3} * 11}$$

find  $R'$

$$R_{H,V} = R_{L,V} \left( \frac{11}{0.4} \right)^2$$

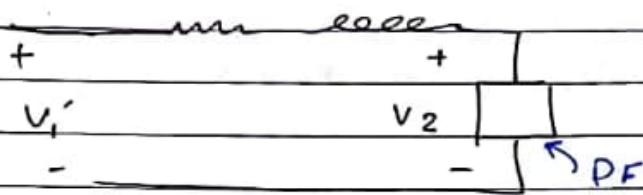
$$I_{L,HV} = I_{L,LV} * \left( \frac{0.4}{11} \right)$$

$$V_{L,HV} = V_{L,LV} * \left( \frac{11}{0.4} \right)$$

$$Y_{R,HV} = Y_{R,L,LV} * \left( \frac{11}{0.4} \right)^2$$

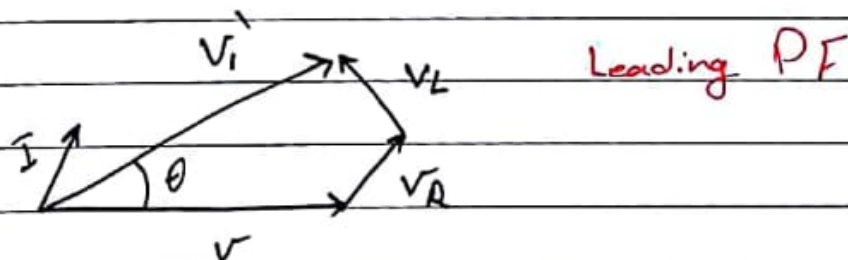
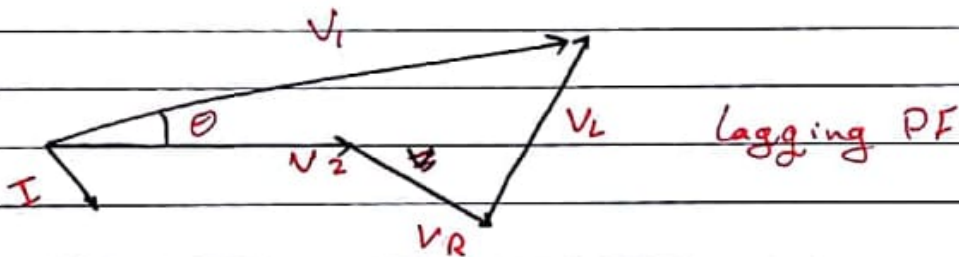
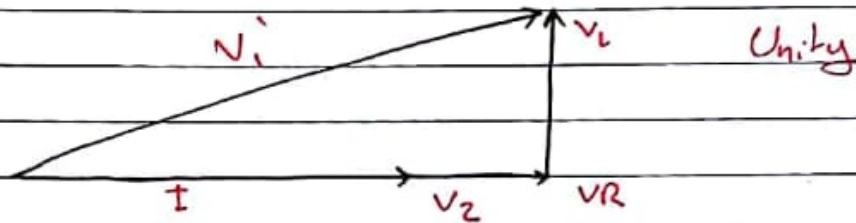
$Y_{L,HV} = Y_{L,LV} * \left( \frac{11}{0.4} \right)^2$

Ex) on phasor diagram:



$V_R$  and  $I$  in phase  
 $V_L$  leads  $I$  by  $90^\circ$   
 $V_L$  leads  $V_R$  by  $90^\circ$

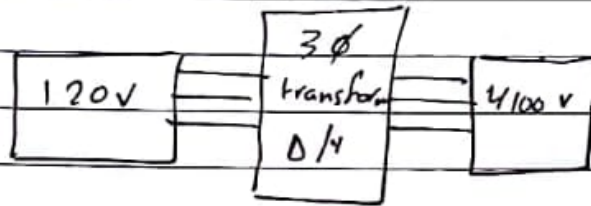
Draw phasor diagram (leading and lagging and unity)



$$\begin{aligned} P &= |V||I| \cos \theta \\ Q &= |V||I| \sin \theta \end{aligned} \quad \left. \vphantom{\begin{aligned} P &= |V||I| \cos \theta \\ Q &= |V||I| \sin \theta \end{aligned}} \right\} S = P + jQ$$

Ex) 3 Identical single phase transformer, each  
10 KVA , 2400 / 120 V

① Y/Δ  $\rightarrow$  30 KVA ,  $2400\sqrt{3}$  / 120 V

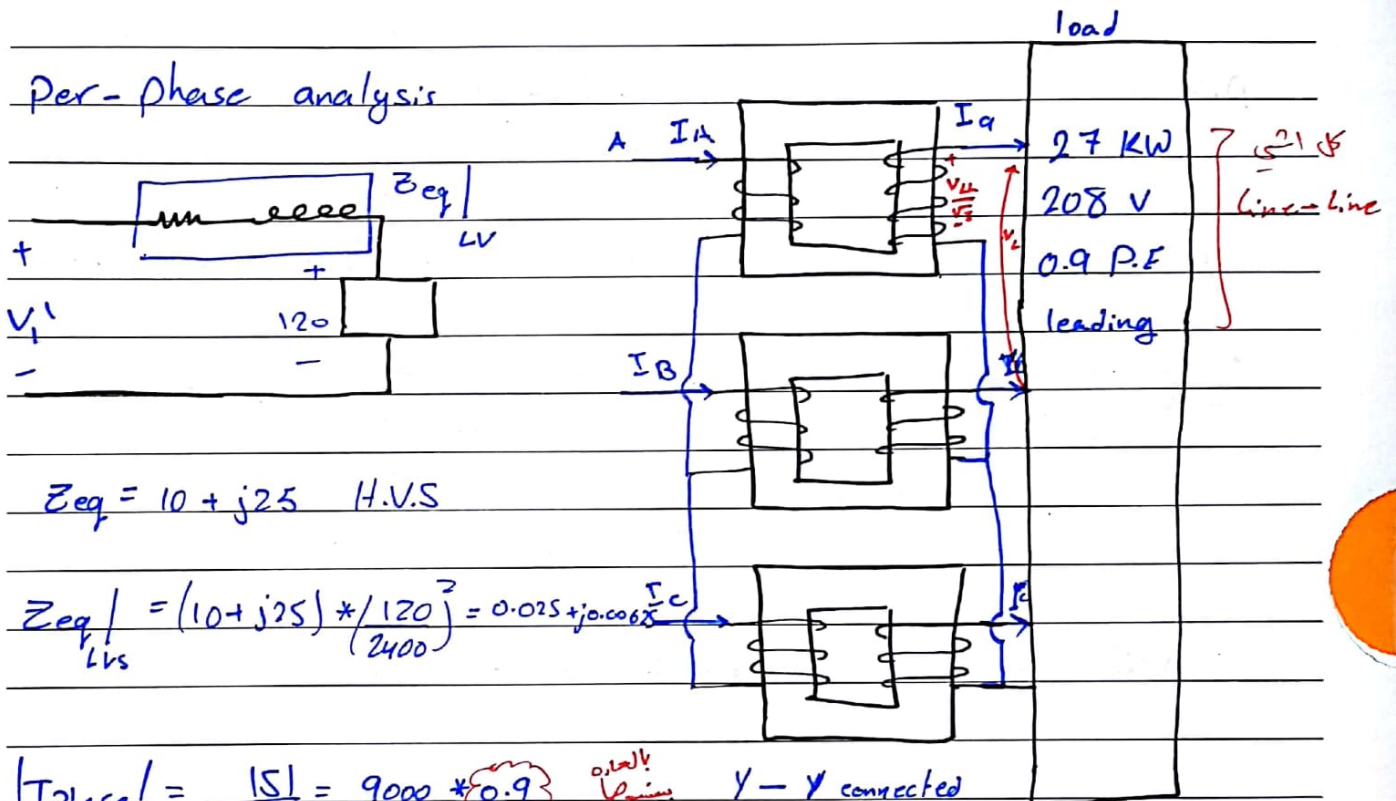




6th Nov/2018

Ex) Three identical single-phase transformer, 10 KVA, 2400/120 V, 60 Hz; are connected to form 4160/208 V, the equivalent impedance at (H.V.S) is:  $(10 + j25) \Omega$ , load 27 KW, 208, 0.9 PF leading. Find VR %?

per-phase analysis



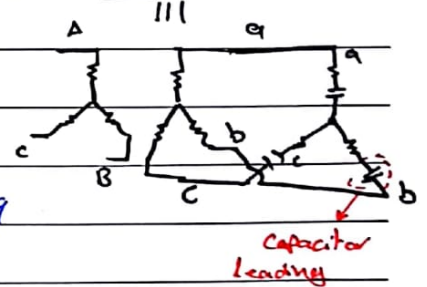
$$Z_{eq} = 10 + j25 \text{ H.V.S}$$

$$Z_{eq|_{Lvs}} = (10 + j25) \left( \frac{120}{2400} \right)^2 = 0.025 + j0.0625 \Omega$$

$$|I_{phase}| = \frac{|S|}{|V|} = \frac{9000 \times 0.9}{120} \quad \text{Y-Y connected}$$

$$= 83.34 \angle + \cos^{-1} 0.9 \text{ A}$$

$$\frac{|I_{phase}|}{\text{HVS}} = 83.34 \times \left( \frac{120}{2400} \right) = 4.16 \angle + \cos^{-1} 0.9$$



To find current

$$S = \sqrt{3} V_L I_L$$

$$\frac{27}{0.9} = \sqrt{3} \times 208 \times I$$

$$|I| = 83.34 \angle + \cos^{-1} 0.9$$

$$S_{3\phi} = 3 S_{1\phi}$$

$$S_{3\phi} = 3 V_P I_P$$

$$= \frac{3}{\sqrt{3}} V_L I_L = \sqrt{3} V_L I_L$$

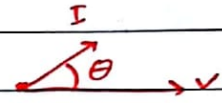
Mech Family

Five Apple

$$V_1' = 83.34 \angle \cos^{-1} 0.9 (0.025 + j0.0625) + 120 \angle 0$$

$$V_1' =$$

$$V_1 = 2392.6 \angle 2.67^\circ \text{ V (LN)}$$



$$PF = \cos(\angle V - \angle I)$$

$$P.F. @ \text{source} = \cos(2.67 - \cos^{-1} 0.9) \leftarrow \text{leading}$$

الزاوية الأكبر يكون lead

Transformation Ratio equals the Turns ratio  $V_1 - V_2$  لن

$$I_{HVS} = I \left( \frac{208}{4160} \right) = 83.34 * \left( \frac{208}{4160} \right)$$

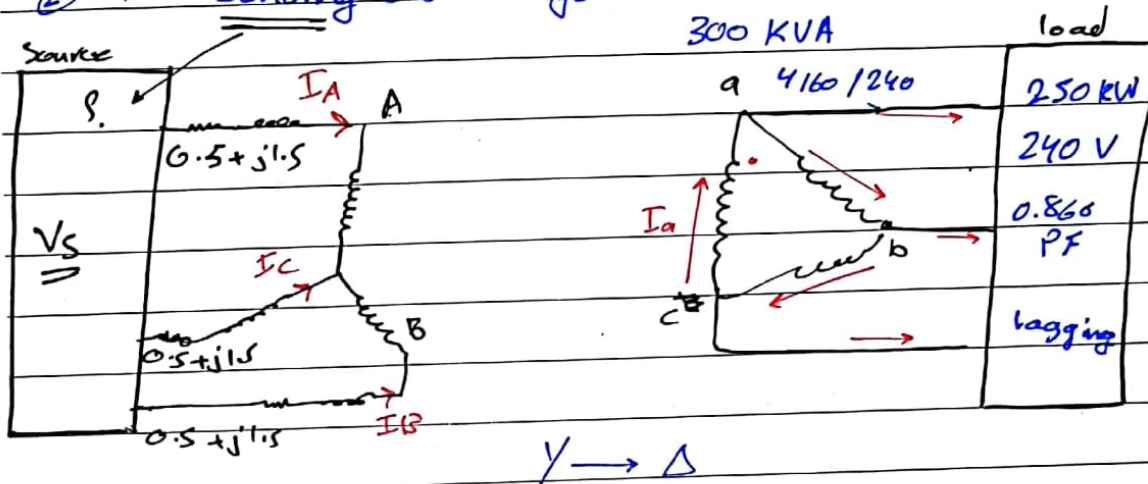
Transformation Ratio

$$VR\% = \frac{V_1' - |V_2|}{V_2} * 100\% = -0.31\%$$

Ex) Three single phase transformers 100 KVA, 2400/240, 60 Hz  
 $Z_{\text{phase}} = 0.045 + j0.16 \Omega$ , The transformers are connected to the  
 source ( $0.5 + j1.5 \Omega$  / phase) load ~~4160~~ 250 kW @ 240V, 0.866 lag  
 $[3\phi : 4160 / 240]$

① determine winding current

② " sending end voltage

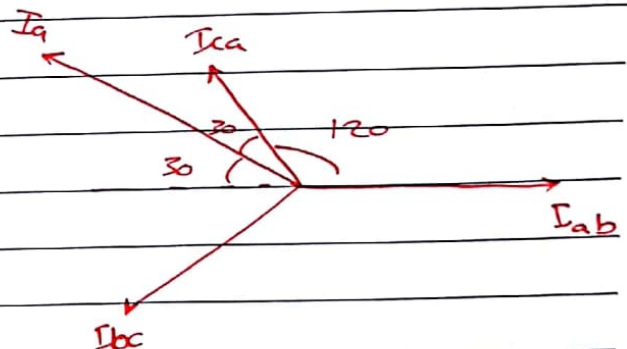


$$S_{3\phi} = \sqrt{3} V_L I_L = \frac{250 \times 240 \times I_L}{0.866} \Rightarrow |I_L| = 694.5 \text{ A}$$

line current

Rated current at 240V: 300 ← 250 kW, 0.866 PF

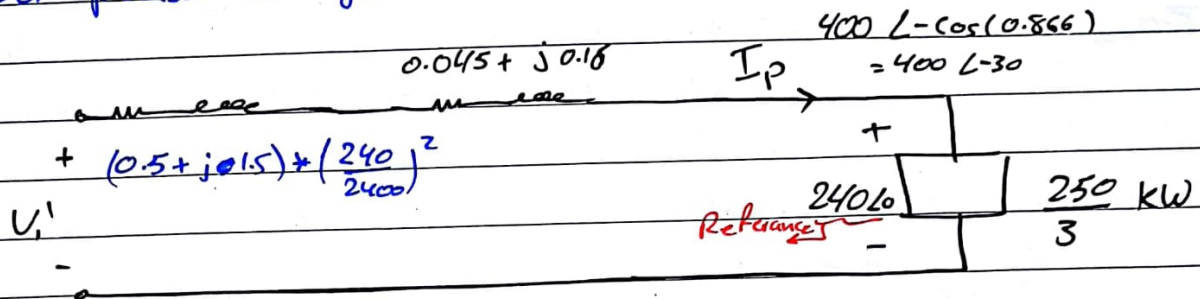
$$|I_p| = \frac{I_L}{\sqrt{3}} = \frac{694.5}{\sqrt{3}} \text{ A}$$





! 10/10 second 1/5

per-phase analysis



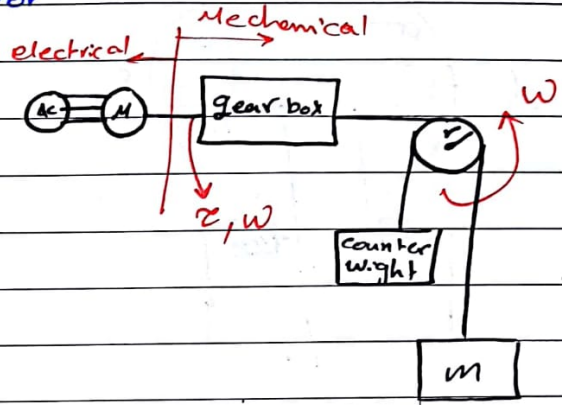
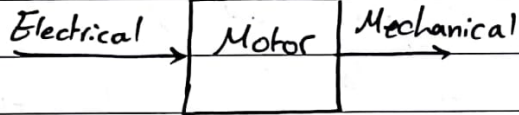
$$V_1' = (KVL)$$

$$V_1 = V_1' * \left( \frac{2400}{240} \right)$$

$$V_{1 \text{ LL}} = \sqrt{3} V_1 = 5138.5$$

8th Nov/2017

\* Dc Machines  $\begin{cases} \rightarrow \text{Motor} \\ \rightarrow \text{Generator} \end{cases}$



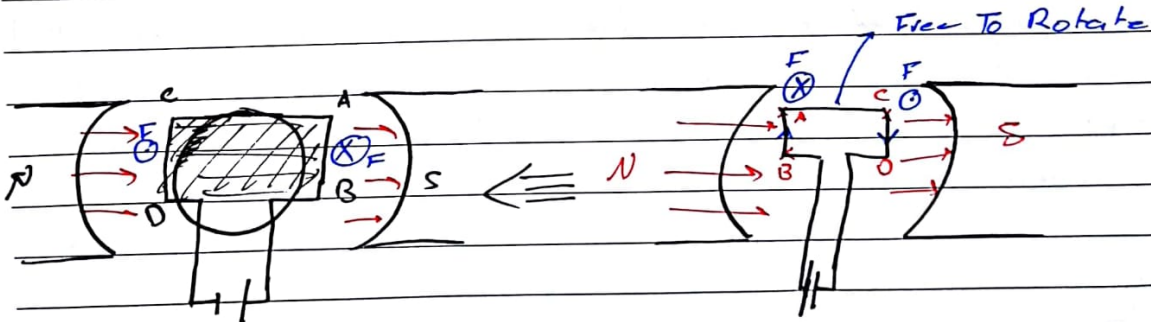
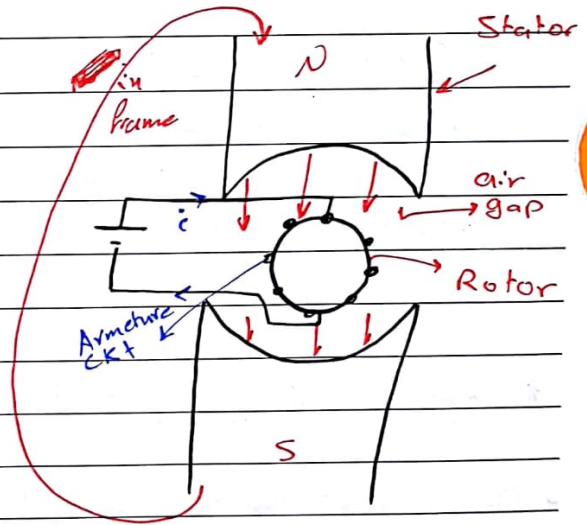
$$p = T \omega$$

$$w \downarrow \Rightarrow P \downarrow \Rightarrow B, I \downarrow$$

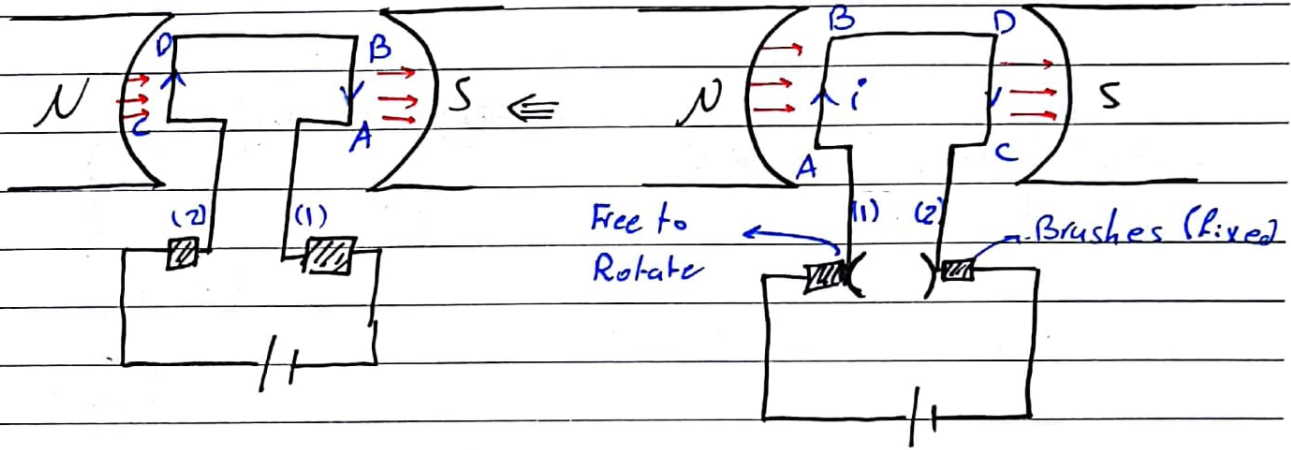
$$F = mg, \tau = mgr$$

$$\textcircled{*} \quad \underline{F} = \underline{i} \times \underline{B} L \leftarrow ?$$

Force                      current                      flux

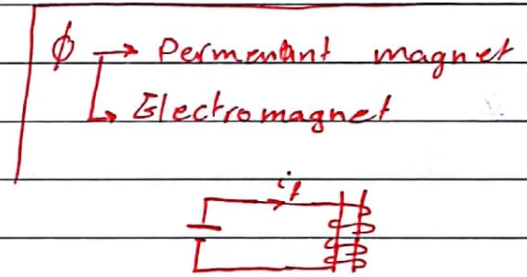


## \* Commutator



## \* Stator , Rotor

Field ckt  $\rightarrow \phi$   
 Armature ckt  $\rightarrow i$   
 $\Rightarrow F = i \times \phi$



$$* \text{ emf} = N \frac{d\phi}{dt}, \text{ emf} = v \times BL \quad (\text{back emf})$$

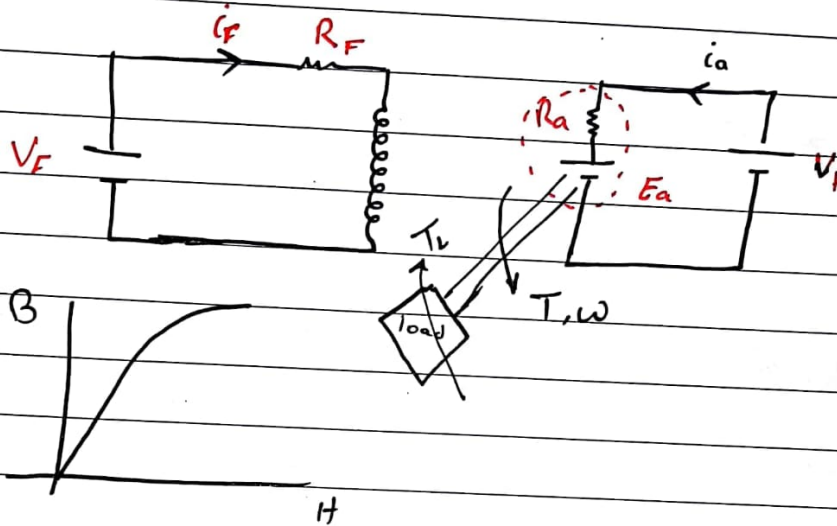
## \* DC Motor classification (for electro magnetic flux)

- Separately excited DC Motor (source WLP of armature & field is 1)
- Shunt DC Motor
- Series DC Motor
- Compound DC Motor





## \* Separately - excited DC Motor



$$F = B L i$$

$$T_{\text{torque}} = K_1 \Phi i_a$$

$$K_1 = K_2$$

$$e_m f = B L v$$

$$E_a = K_2 \Phi \omega$$

\* if  $\Phi$  is constant

$$E_a = K_2 \Phi \omega$$

$$E_a = K_v \omega$$

$$\text{unit: } \left\{ \begin{array}{l} K_v = V/\text{rad/s} \\ = V.s \end{array} \right.$$

$$T = K_t i_a$$

$$K_t \Rightarrow \frac{N.m}{A} \Rightarrow K_v = K_t$$

$$N.m \text{ ?? } (V.A.s) ?$$

$$J = J \quad \checkmark$$

$$\text{power motor} \Big|_{\text{air gap}} = E_a i_a \quad \text{W} \text{ or } \text{J/s}$$

$$\text{Mechanical power} = T \omega$$

$$E_a i_a = T \omega$$

$$K_2 \phi \omega i_a = K_1 \phi i_a \omega$$

$$K_1 = K_2 \quad \#$$

13/Nov/2017

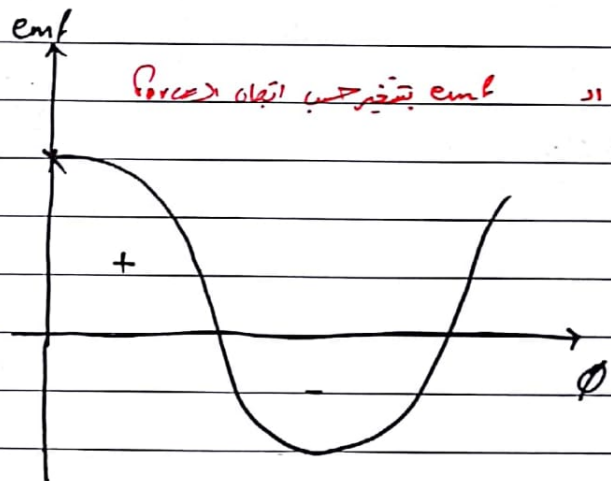
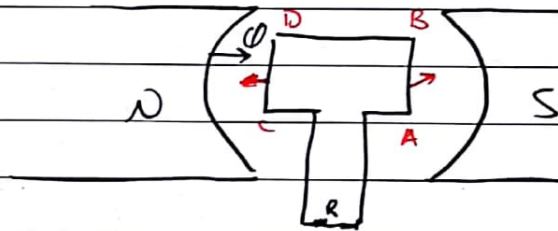
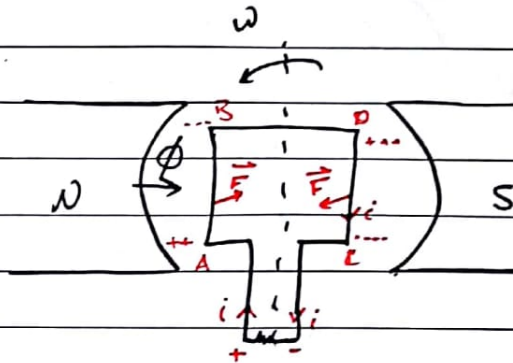
## DC Generator

$$emf = v \times B$$

$$= v \times B \sin \theta$$

velocity

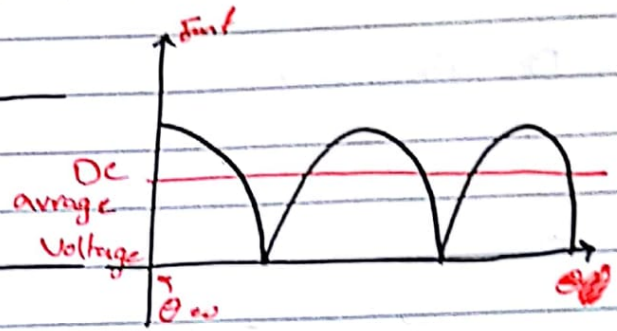
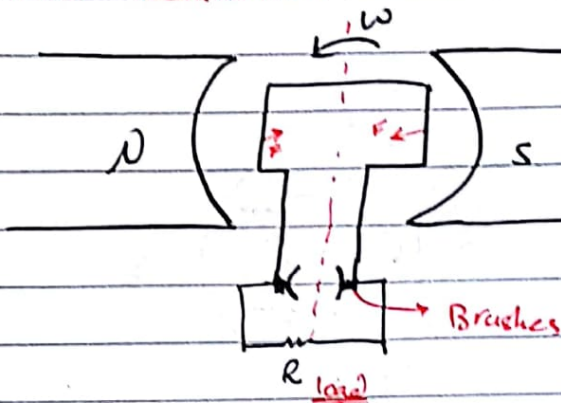
F and B



Five Apple



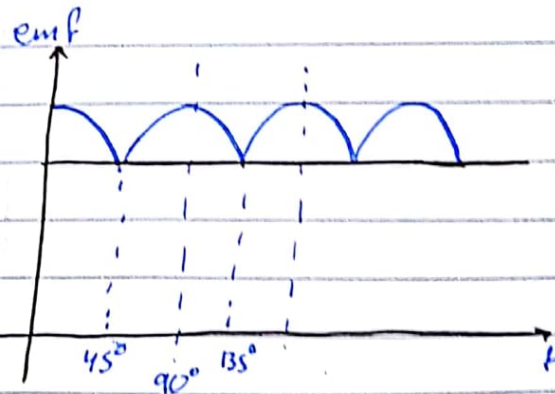
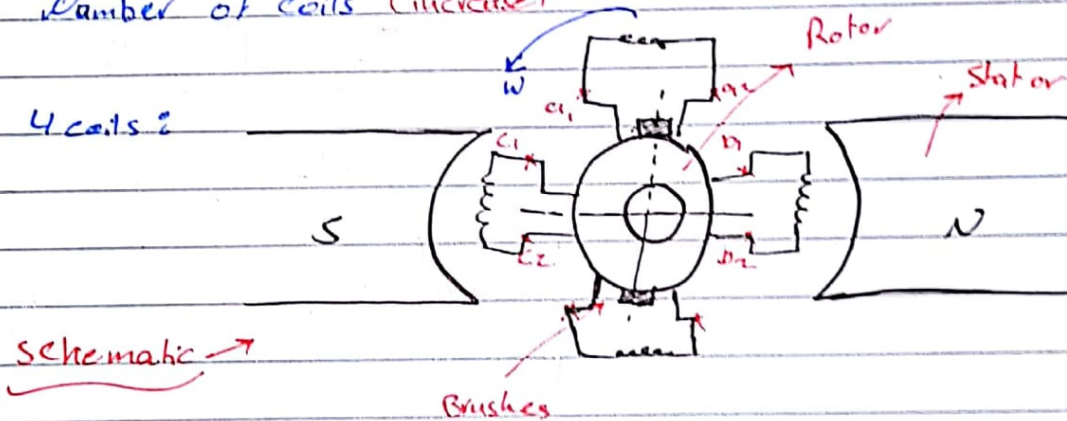
with commutator



- How to improve wave form

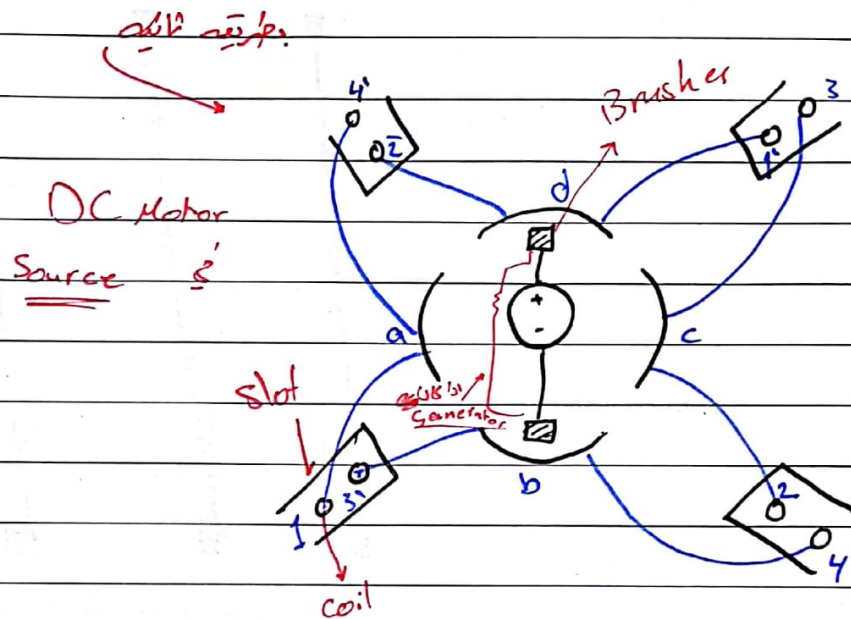
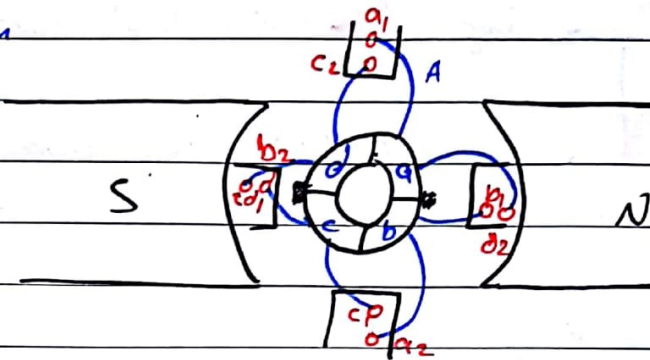
\* Number of coils (increase)

4 coils

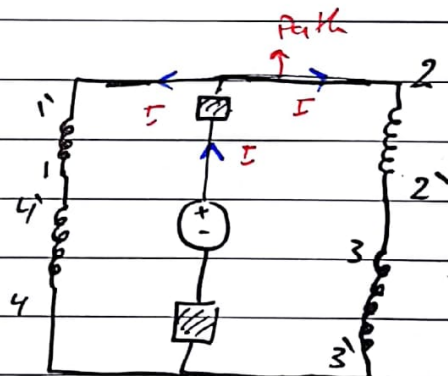


Actual construction

2 coils / slot



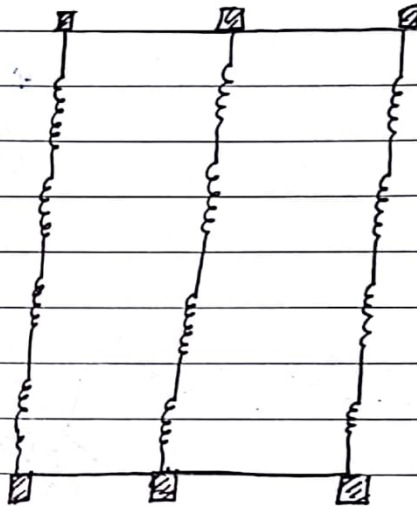
armature winding



eq. SKT

wiring → lap (LV, high current)  
→ wave (HV, low current)

lap winding

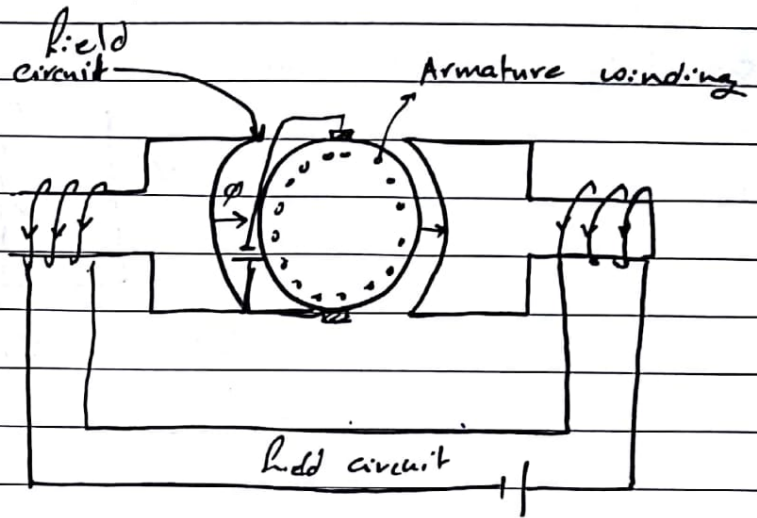


wave winding

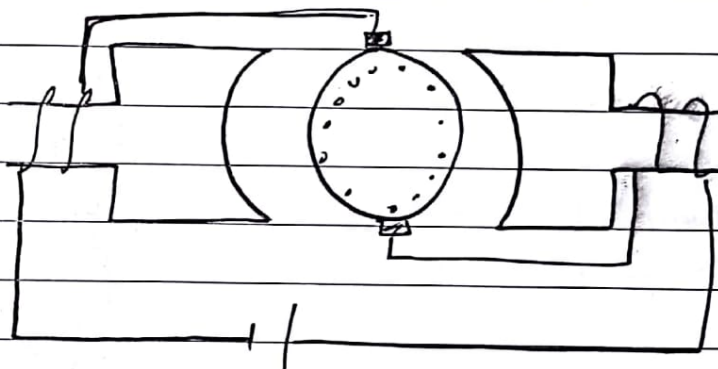




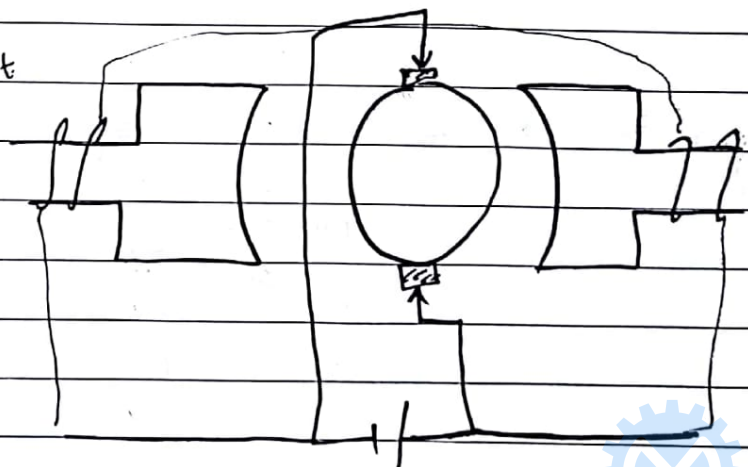
Separately  
excited DC motor



Series dc Motor

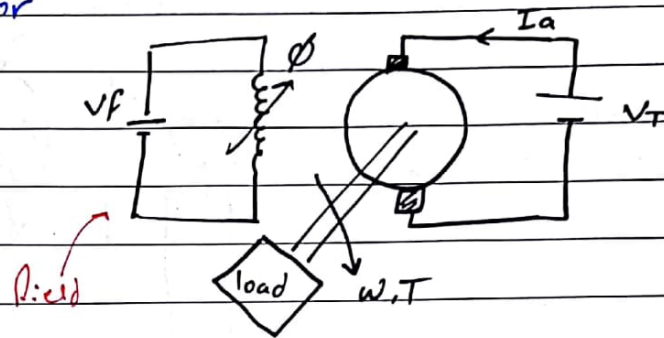


Shunt

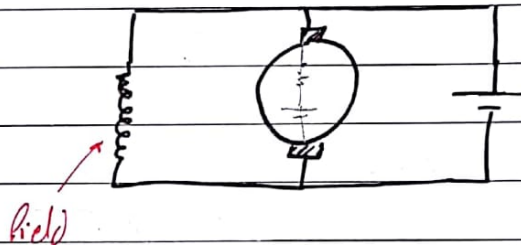


15/Nov/2017

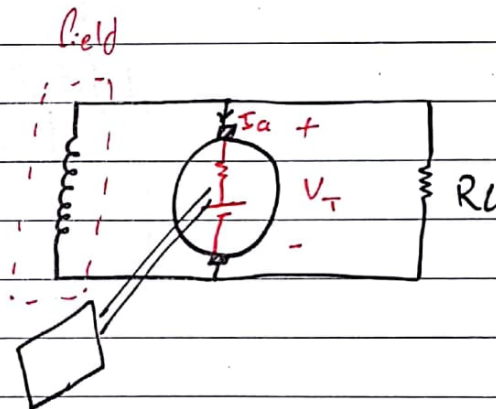
Separately excited Dc Motor



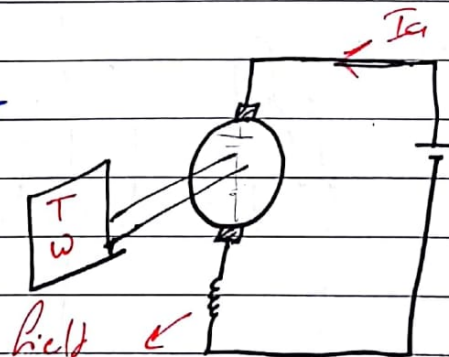
Shunt Dc motor



Shunt Dc Generator  
(self-excited generator)

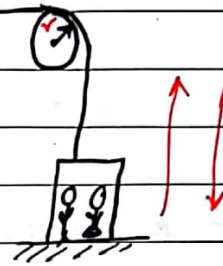


Series Dc motor





$$V = \omega r$$

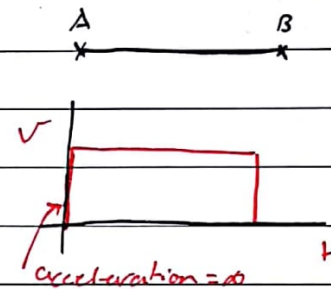


$$T_L = \sum m \times g \times r \text{ N.m}$$

$$J \frac{d\omega}{dt} = T_m - T_L$$

@ Steady state  $\Rightarrow T_m = T_L$

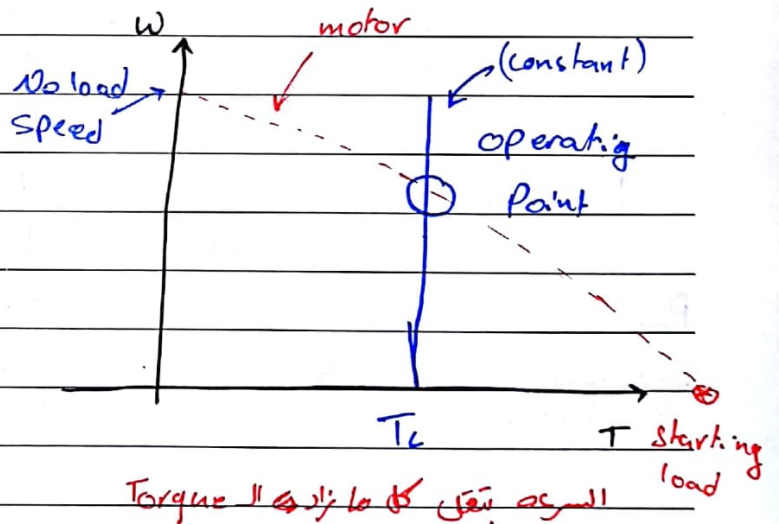
acceleration = 0



$\Rightarrow$  Need very high torque

السرعة في البداية تكون صفر  
starting load & steady load  
Motor

$$T_m = T_L + J \frac{d\omega}{dt}$$



السرعة تبقى كما هي لا تتغير  
Torque



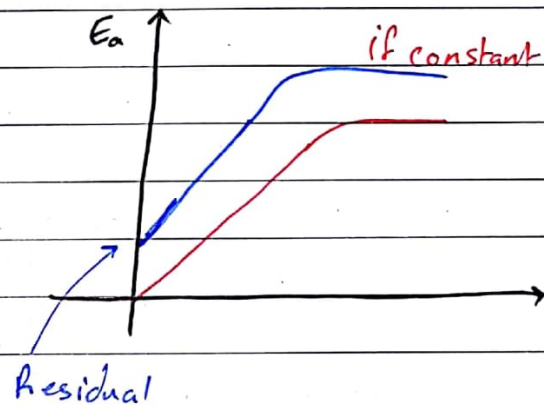
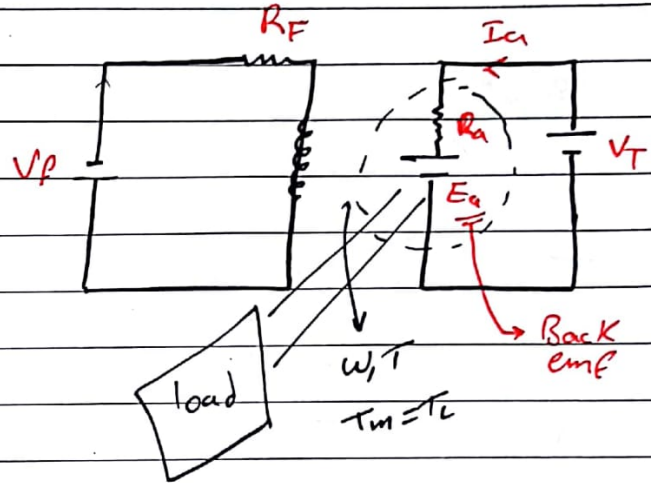
## ⊗ Separately - excited Dc Motor

$$E_a \text{ emf} = K_1 \phi \omega$$

$$= K' I_f \omega$$

at starting  $\Rightarrow (\omega=0)$

$E_a = \text{Zero}$



$$T = K_2 \phi i_a$$

$$P_{\text{mechanical}} = T \omega$$

$$\text{Developed power by the motor} = E_a I_a$$

$$T \omega = E_a I_a$$

$$T \omega = K_1 \phi \omega \frac{T}{K_2 \phi}$$

$$T \omega = \frac{K_1}{K_2} \omega T \Rightarrow K_1 = K_2$$

$$K = 0.3 \text{ V/rpm}$$

$$E_a = K \omega \rightarrow \text{Rotational speed}$$

$$\omega = \frac{2\pi n}{60}$$

@ Rest (starting, stand)

$$\omega = 0$$

$$E_a = \text{zero}$$

$$I_a = \frac{V_t}{R_a} \quad \uparrow \uparrow$$

$$T_m = K \phi I_a \quad \uparrow \uparrow$$

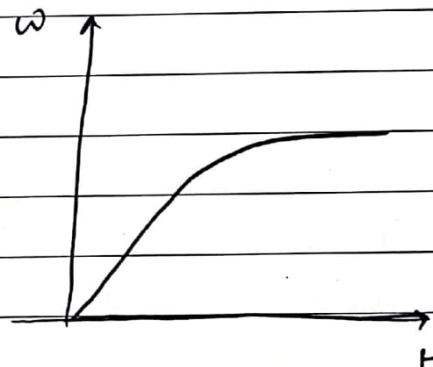
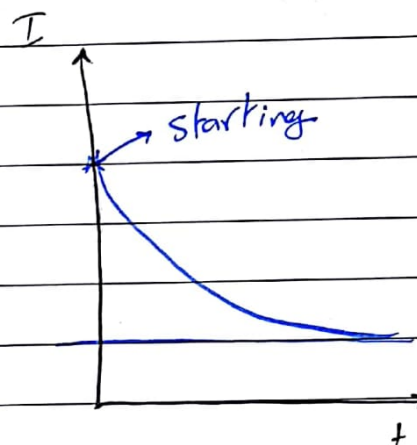
$$J \frac{d\omega}{dt} = T_m - T_L$$

$\underbrace{\quad}_{\text{acceleration}}$

$$E_a \uparrow, I_a \downarrow, T_m \downarrow$$

$$T_m = T_L$$

$$K \phi I_a = T_L$$



$$T_{\text{starting}} = K \phi I_{a \text{ starting}}$$

20/12/2017

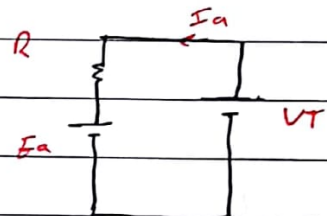
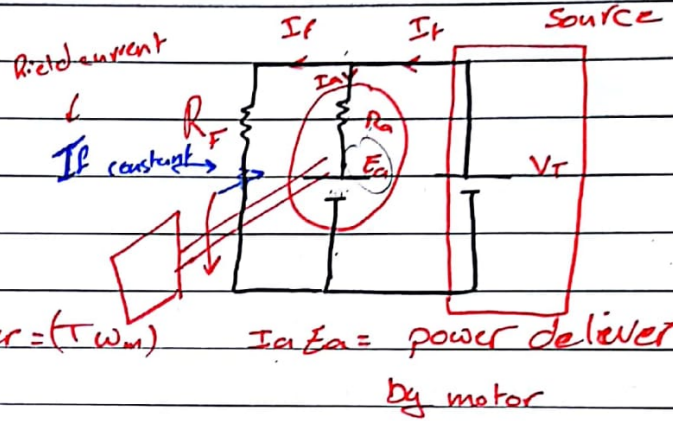
# \* Shunt DC motor

shunt DC motor:

$$E_a = k\phi \omega_m$$

$$V_t - I_a R_a = k\phi \omega_m$$

$$\omega_m = \frac{V_t}{k\phi} - \frac{R_a}{k\phi} I_a$$

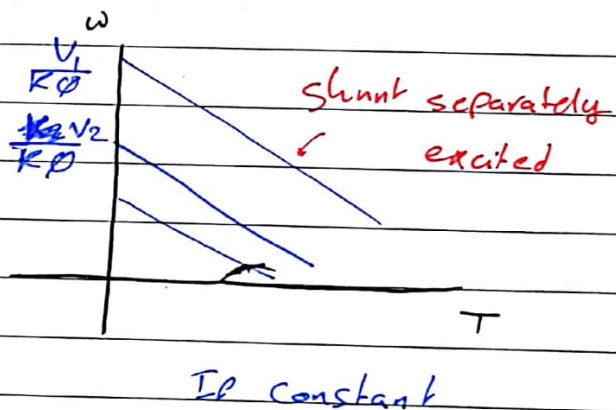


$$E_a = k\phi \omega_m$$

$$V_t - I_a R_a = k\phi \omega_m$$

$$\omega_m = \frac{V_t}{k\phi} - \frac{R_a}{k\phi} \omega_m$$

shunt DC curve is plotted as follows  
separately





## \* Series DC Motor

$$E_a = k \phi \omega_m$$

$$E_a = V_T - I_a (R_a + R_f)$$

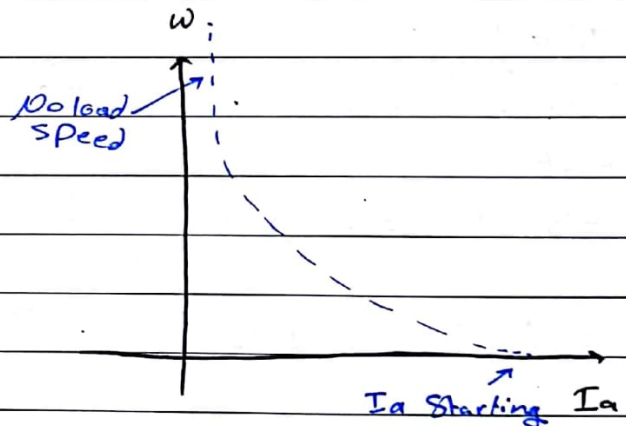
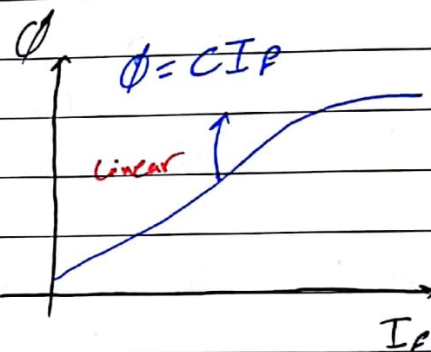
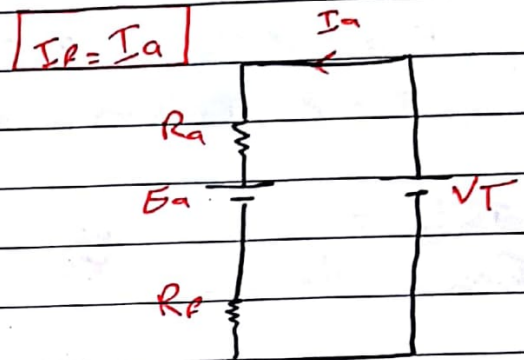
$$V_T - I_a (R_a + R_f) = k \phi \omega_m$$

$$\omega_m = \frac{V_T}{k \phi} - \frac{I_a (R_a + R_f)}{k \phi}$$

$$\phi = C I_a \quad \text{--- } I_f = I_a$$

$$\omega_m = \frac{V_T}{k C I_a} - \frac{I_a (R_a + R_f)}{k C I_a}$$

$$\omega_m = \frac{V_T}{k C I_a} - \frac{R_a + R_f}{k C}$$



$$T = k \phi I_a = K_c I_a I_a \quad | \quad T = k \phi I_a = K_c I_a I_f$$

$$T = K_c I_a^2$$

at starting  $I_{a, \text{st}} = \frac{V_T}{R_a + R_f}$   $\omega = 0$   
 $E_a = 0$

$$I_{a, \text{st}} = \frac{V_T}{R_a}, \quad I_f = \frac{V_T}{R_f}$$

$$T_{\text{st}} = K_c \frac{V_T}{R_f} \frac{V_T}{R_a}$$

$$T_{\text{st}} = \frac{K_c (V_T)^2}{(R_a + R_f)^2}$$

$$T_{\text{st}} = \frac{K_c (V_T)^2}{R_a R_f}$$

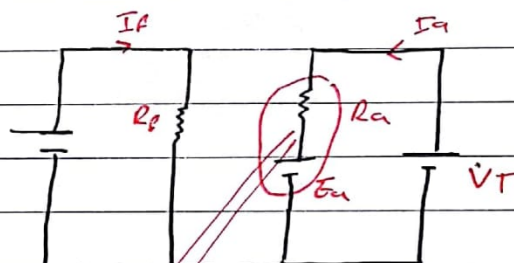
$R_{\text{series}} \ll R_{\text{shunt}}$

$\Rightarrow T_{\text{st, series}} \gg T_{\text{st, shunt}}$

# Speed control - separately excited

\* Voltage  $\nearrow$ ,  $I_f$  constant

$T_e = K \phi I_a$   
 $\nearrow$  constant  $\nearrow$  constant  
 constant  
 $\Rightarrow I_a = \text{constant}$



$\Rightarrow V_T \uparrow, I_a \text{ constant}, E_a ??$

$T_e, \text{ constant}$

$$E_a = V_T - \underbrace{I_a R_a}_{\text{drop}} \Rightarrow E_a \uparrow$$

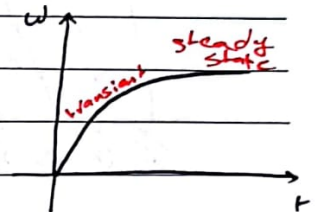
$$E_a = k \phi \omega \Rightarrow \omega \uparrow$$

⊗ starting

$$\omega = 0 \Rightarrow E_a = 0$$

$$I_a \uparrow \Rightarrow T_m \uparrow \Rightarrow \frac{J d\omega}{dt} = T_m - T_L \Rightarrow E_a \uparrow \Rightarrow I \downarrow$$

$$\Rightarrow \text{steady state } I_a \Rightarrow T_L = K\phi I_a$$



\* Field - weakening ( $I_f \downarrow$ ),  $V_t$  fixed, speed  $\uparrow$

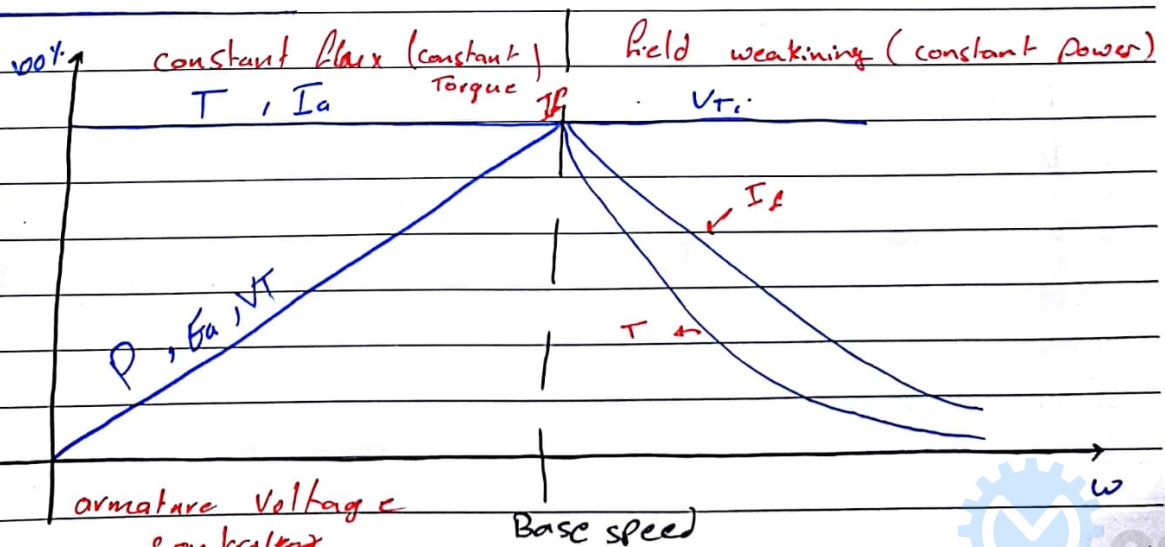
$$\omega = \frac{V_t}{K\phi} - \frac{R_a}{K\phi} I_a$$

constant Torque  $\Rightarrow T = K\phi I_a$ ,  $\phi \downarrow \Rightarrow I_a \uparrow$

$$E_a = \frac{V_t}{\text{constant small}} - I_a R_a \quad (\text{almost fixed})$$

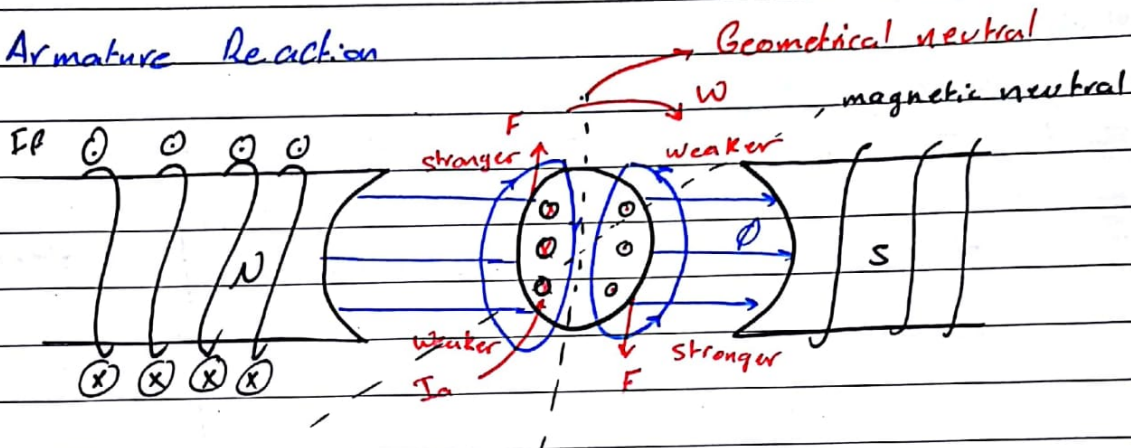
$$E_a = K\phi \omega_n \Rightarrow \omega_n \uparrow$$

constant  $\downarrow$  cda



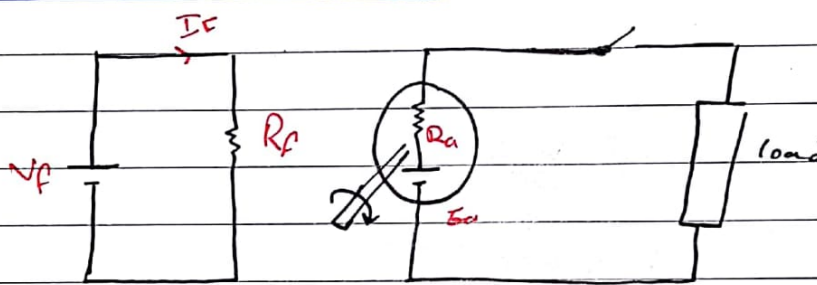


## Armature Reaction



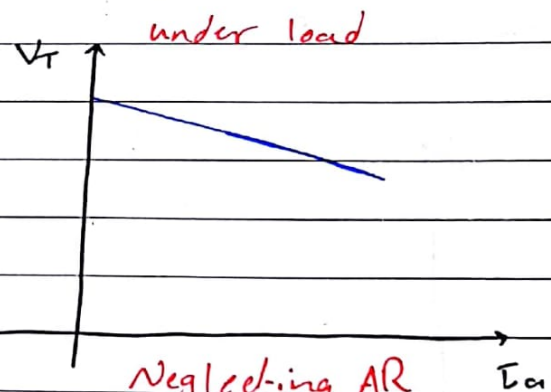
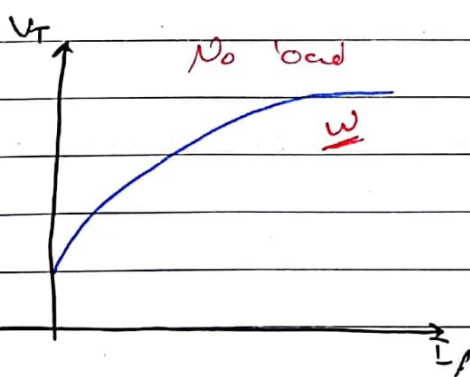
## Generator

$\Rightarrow \Phi \downarrow \leftarrow$  field flux



## Generator

(Switch open) No load  $\Rightarrow V_t = E_a ; I_a = 0$



Neglecting  $\frac{AR}{I_a}$

Armature Reaction

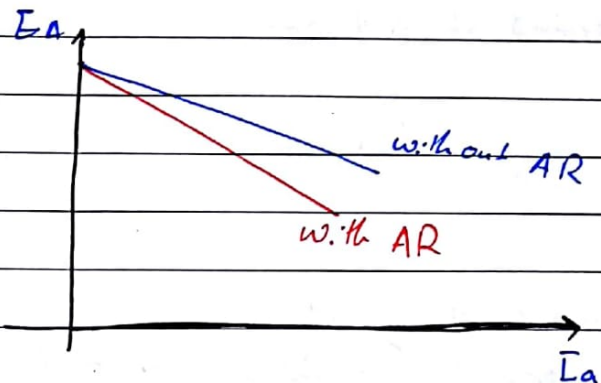
under load

$$V_t = E_a - I_a R_a$$

with Armature Reaction (AR)

AR  $\Rightarrow$  field weakening

$E_a \downarrow$



Ex: 220 V, dc shunt motor

AR  $\Rightarrow$  field weakening

$R_a = 0.2 \Omega$ ,  $R_F = 110 \Omega$

Neglect  $I_a$

at No load  $\Rightarrow$  <sup>No load speed</sup> 1000 rpm, total line current 7 A

at full load  $\Rightarrow$  input power 11 kW

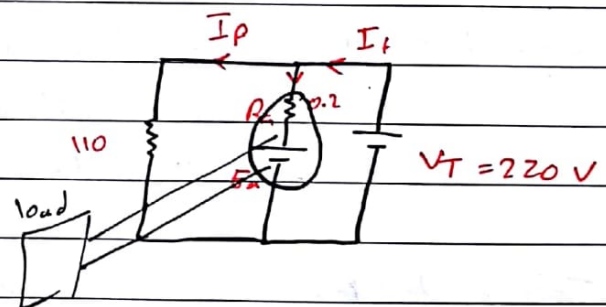
\* air gap flux fixed at its value at no load

V speed, speed regulation, developed Torque at full load

at no load

$$I_F = \frac{220}{110} = 2 \text{ A} \Rightarrow I_a = 5 \text{ A}$$

Fixed



at full load (power = 11 kW)

$$P_{\text{terminal}} = V_T I_T$$

$$11 \text{ kW} = 220 \times I_T \Rightarrow I_T = 50 \text{ A}$$

$$I_a \big|_{\text{full load}} = 48$$

$\uparrow$  50 - 2

at no load ; Speed = 1000 rpm

1) speed at full load

$$\omega = \frac{V_t}{K\phi} - \frac{R_a}{K\phi} I_a$$

at no load <sup>rad/s</sup>

$$\frac{1000 \times 2\pi}{60} = \frac{220}{K\phi} - \frac{0.2 \times (5)}{K\phi} \Rightarrow K\phi = \sqrt{1}$$

$$\omega_{full\ load} = \frac{220}{K\phi} - \frac{0.2 \times (48)}{K\phi} = 960\ rpm$$

another way to solve:

$$\frac{\omega_{full}}{\omega_{no}} = \frac{220 - 0.2(48)}{220 - 0.2(5)} \Rightarrow \omega_{full} = \frac{2104}{219} \times \frac{1000 \times 2\pi}{60} = 960\ rpm$$

$$\begin{aligned} * \text{ speed regulation} &= \frac{|\omega_{no}| - |\omega_{full}|}{|\omega_{full}|} \times 100\% \\ &= \frac{1000 - 960}{960} = 4.09\% \end{aligned}$$

\* Torque developed by the motor at full load

$$T_d = \frac{K\phi}{\omega} I_a = 100.38\ N.m$$

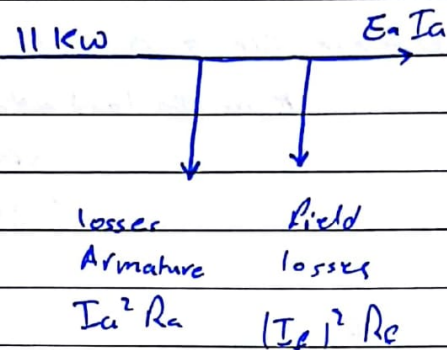
another way:

Power developed by the motor  $P_{dev}$

$$= I_a V_a$$

$$* E_a = 220 - (48 \times 0.2) = 20.4\ V$$





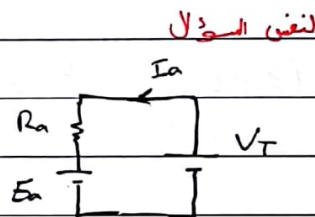
$$\text{Power} = T\omega = E_a I_a \Rightarrow T = \frac{E_a I_a}{\omega} = \frac{210.4 \times 48}{\frac{960 \times 2\pi}{60}} = 100.38 \text{ N.m}$$

$\downarrow$   $= 10.1 \text{ kW}$

\* If separately excited and  $P_D$  was delivered

$$(V_t I_a - I_a^2 R_a = P_D) \leftarrow \text{Loop}$$

$$220 I_a - 0.2 I_a^2 = 10.1$$



\* Starting Torque if  $I_a$  is limited to 150% full load

$$T_{st} = K \phi I_a$$

$$I_a = 1.5 \times 48 = 72 \text{ A}$$

$$T_{st} = K \phi I_a = 150.57 \text{ N.m}$$

another way

$$T_{st} = K \phi I_a / \text{start}$$

$$T_{fl} = K \phi I_a / \text{full}$$

$$\Rightarrow \frac{T_{st}}{T_{fl}} = 1.5$$

$$\Rightarrow T_{st} = 1.5 \times 100.38 = 150.57 \text{ N.m}$$

with out limitation

$$I_a / = \frac{220 - 0}{0.2} = 1100 \text{ A}$$

starting

$$W = 0 \Rightarrow E_a = 0$$

$\Rightarrow$  23 times  $I_{full \text{ load}}$

بما ان تيار الحث في تيار الحث هو 1100  
فان تيار الحث في تيار الحث هو 1100

\* Consider Armature Reaction, it reduces air gap flux by 50%  
calculate speed at full load

\* at no load  $\rightarrow$  the same speed  
(initial)

$$W = \frac{V_T}{K\phi} - \frac{R_a}{K\phi} I_a$$

$$W_{old} = \frac{220}{K\phi} - \frac{0.2}{K\phi} (48) \quad \text{without AR}$$

$$W_{new} = \frac{220}{0.5 K\phi} - \frac{0.2}{\frac{K\phi}{2}} \times 48 \quad \text{with AR} \rightarrow \text{Torque Reduced by half}$$

$W \rightarrow$  increased

$$\frac{W_{new}}{W_{old}} = 2$$

\*  $\eta = \frac{\text{Power output}}{\text{Power input}} \times 100\%$

, all day  $\eta = ??$   
 $= \eta \times \text{time}$

At Rotational losses = 500 W  
No load it is

$$\Rightarrow \eta = \frac{10.1 \times 10^3 - 500}{11 \times 10^3} \times 100\%$$

\* Speed can be reduced by adding Resistance

$$W = \frac{V_T}{K\phi} = \frac{R_a I}{(K\phi)^2}$$

27/Nov/2017

5A

440 V DC shunt machine  $R_f = 110 \Omega$ ,  $R_a = 0.15 \Omega$

① Power delivered by the machine if it absorbs 22 kW

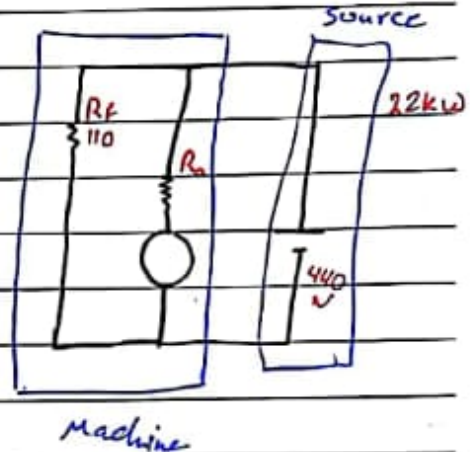
Rated voltage = 440 V & 22 kW motor

② power delivered by the machine if it supplies 22 kW @ 440V

$$I_L = \frac{22 \text{ kW}}{440} = 50 \text{ A}$$

$$I_f = \frac{440}{110} = 4 \text{ A}$$

$$I_a = 50 - 4 = 46 \text{ A}$$



1) Power delivered =  $E_a I_a$

$$E_a = 440 - 46 \times 0.15 = 433.1 \text{ V}$$

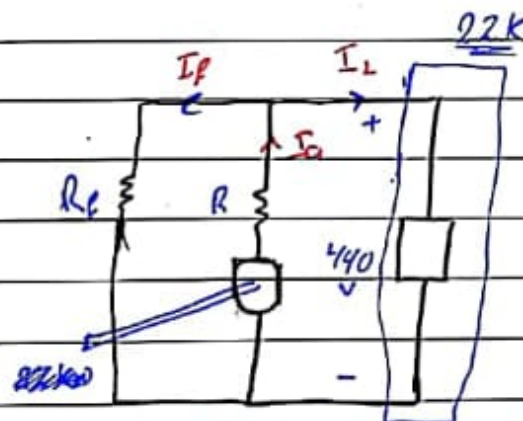
$$\text{power delivered} = 433.1 \times 46 = 19.9 \text{ kW}$$

$$\eta = \frac{19.9}{22} \times 100 =$$

2)  $I_L = \frac{22 \text{ kW}}{440} \rightarrow I_L = 50 \text{ A}$

$$I_f = \frac{440}{110} = 4 \text{ A}$$

$$I_a = 50 + 4 = 54 \text{ A}$$



$$E_a = 54(0.15) + 440 = 448 \quad -440 + 448 = 8 \leftarrow \text{voltage drop}$$

$$P = 448 \times 54 = 24.4 \text{ kW}$$

$$\eta = \frac{22 \text{ kW}}{24.4} \times 100\%$$

و از  $E_a$  و  $I_a$  می توانیم محاسبه کنیم  
Flux (فیلد) و از  $I_f$  می توانیم محاسبه کنیم



Ex) 230 V, DC motor (series) :  $R_a = 0.2 \Omega$

$$R_f = 0.05 \Omega$$

Series  $\rightarrow I_a = I_f$

$I_a = 20 \text{ A}$ , 1500 rpm

Rotational losses = 400 W

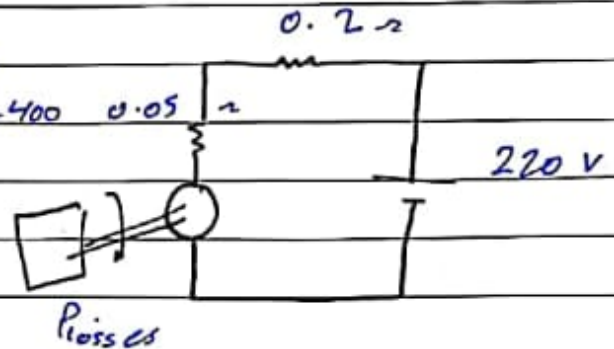
$\eta = ??$

$$P_{in} = 230 \times (20) = 4.6 \text{ kW}$$

$$P_o = P_{in} - P_{copper} - P_{rotational}$$

$$\Rightarrow 4600 - (20^2 \times 0.2 + 20^2 \times 0.05) - 400$$

$$\eta = \frac{P_o}{P_{in}} \times 100\% = 89.1\%$$



Ex) 230 V, DC shunt,  $R_a = 0.05 \Omega$ ,  $R_f = 75 \Omega$

A \* @ 1120 rpm  $\rightarrow$  motor draws 7 A

B \* @ certain ~~condition~~ operating  $\rightarrow$  line current 46 A

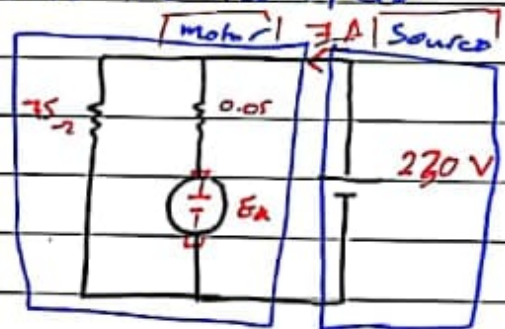
condition  
1) motor speed ??

2)  $R_a$  is increased to  $100 \Omega$ , what is the new speed

$$I_f = \frac{230 \text{ V}}{75 \Omega} = 3.07 \text{ A}$$

$$I_a = 7 - 3.07 = 3.93 \text{ A}$$

$$E_a = 230 - (3.93 \times 0.05)$$



$$E_a = K \phi \omega \Rightarrow E_a = K' I_f N$$

$$K' = \frac{E_a}{I_f N} \Rightarrow K' = 0.0668$$

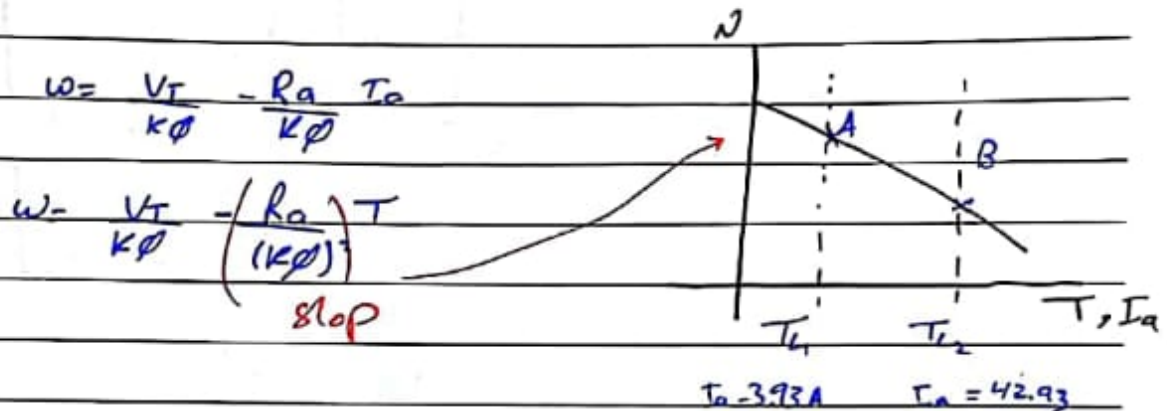
B

$$E_a = K' \omega_r N_m$$

$$I_a = 46 - 30.7 = 42.93 \text{ A}$$

$$E_a = 220 - (42.93 \times 0.05) \Rightarrow N_m = \frac{1111}{42.93} \text{ rpm}$$

(1120 rpm) A يكون أقل من



$$R_f \uparrow \Rightarrow I_f \downarrow \Rightarrow \omega \uparrow$$

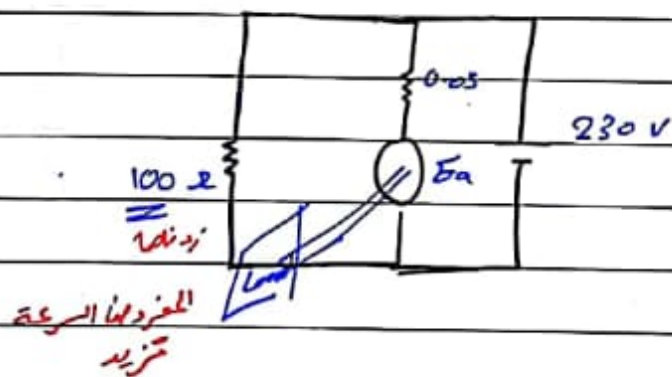
$$I_f = \frac{230}{100} = 2.3$$

$$I_a = \frac{46}{100} - 2.3 = 43.7$$

$$E_a = 230 - (43.7)(0.05)$$

$$E_a \propto N_m I_f$$

$$N_m = 1483 \text{ rpm}$$



كل field weakness ثقتنا ال لا و غيرنا ال

ولنا ان شكل بال power

زادت السرعة اذا زدنا ال T بتقدير P

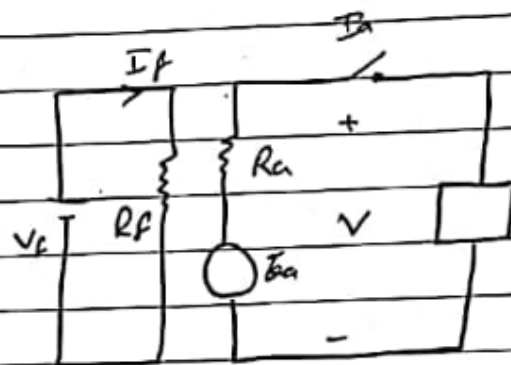
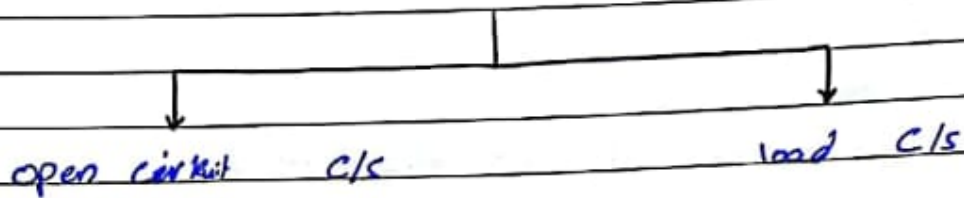
و يمكن تقدير لفرقة ال Rated

$$P = T \omega$$

(عناط طمان تستخدم 0.5) (السرعة العالي)

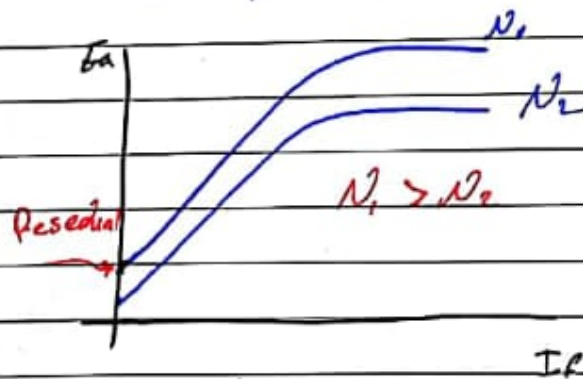
$$P = E_a I_a$$

Separately excited DC



$$I_f \rightarrow \phi$$

$$E_a = \phi \text{ and } \omega$$



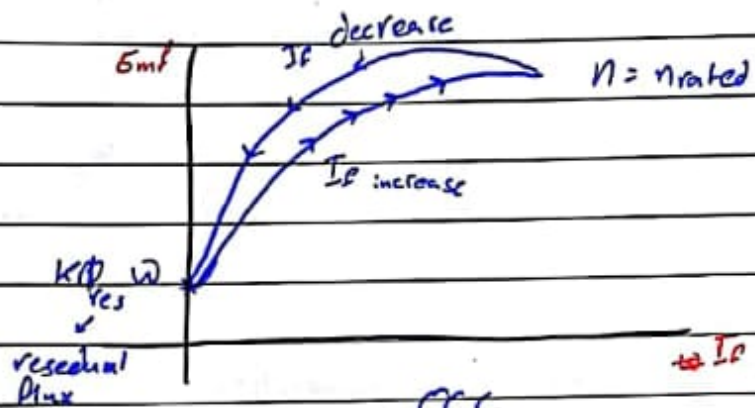
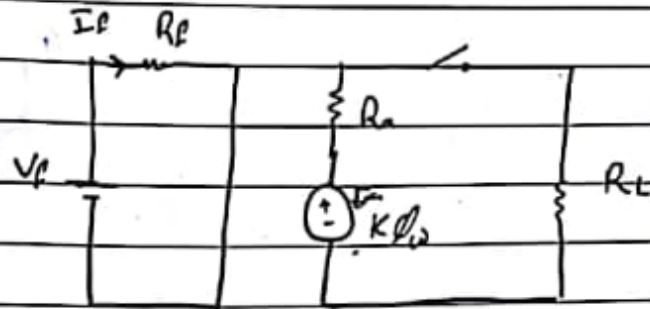
$$E_a = K \phi \omega$$

$$E_a = K_c I_f \omega$$



29/Nov/2017

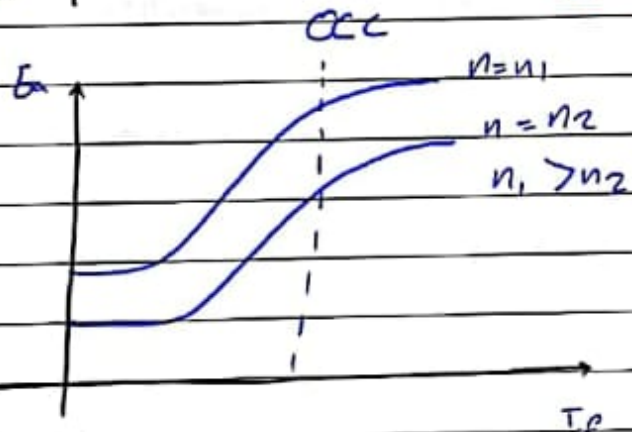
Open circuit c/s



$$E_a = K\phi\omega$$

same  $I_f$

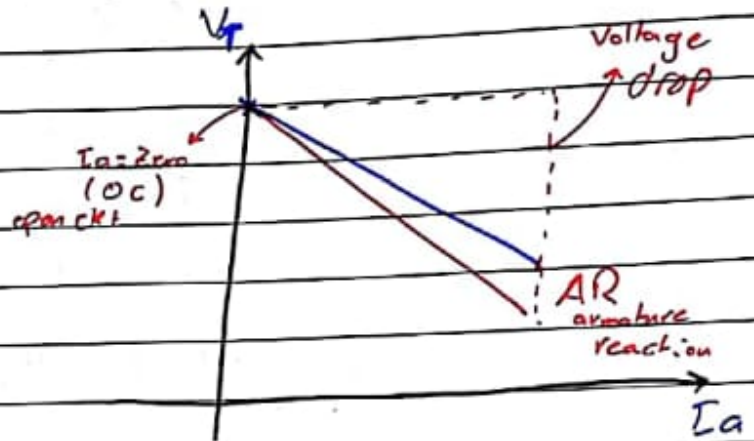
$$\frac{E_{a2}}{E_{a1}} = \frac{\omega_2}{\omega_1} = \frac{n_2}{n_1}$$



load c/s

$$V.R.\% = \frac{V_R}{V_L} \times 100 - \frac{V_R}{V_L}$$

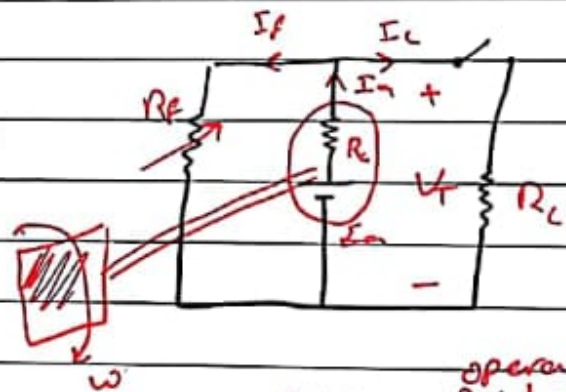
$V_R$  at full load  
voltage receiving



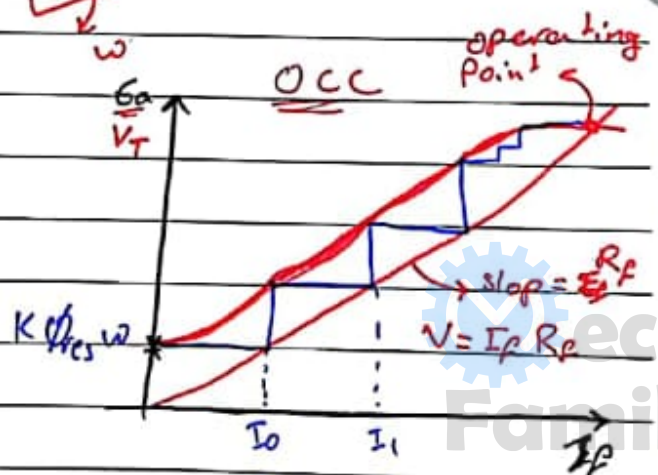
\* armature reaction increases voltage drop

Self-excited dc generator

No need for independent source for the excitation ( $I_e$ )  
excitation current

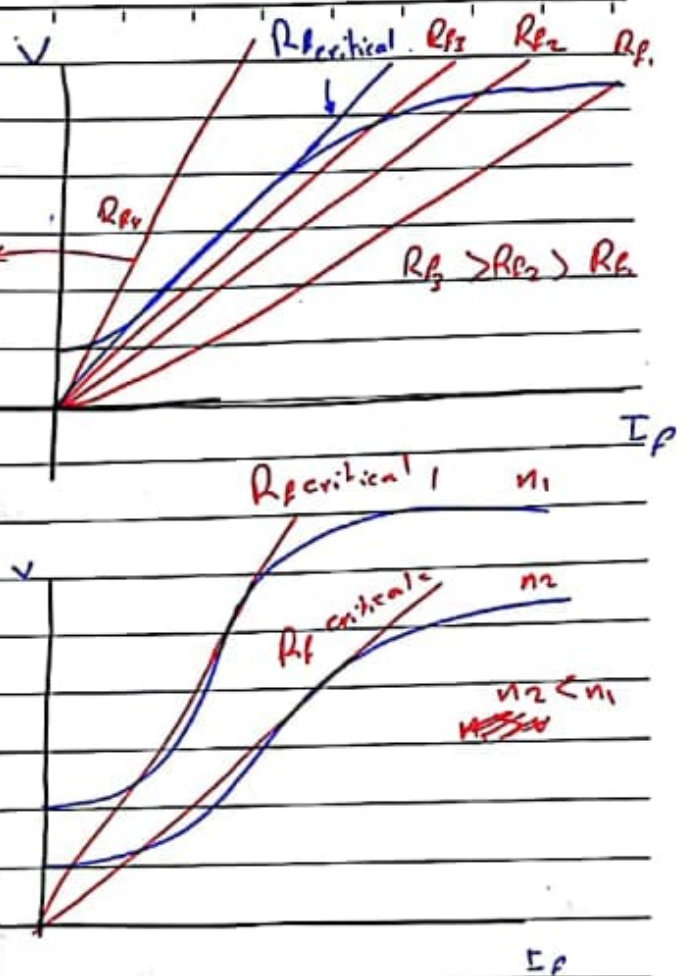


operating point controlled by  $R_f$



\* increase  $R_f$  decreases the operating voltage (increase the slope)

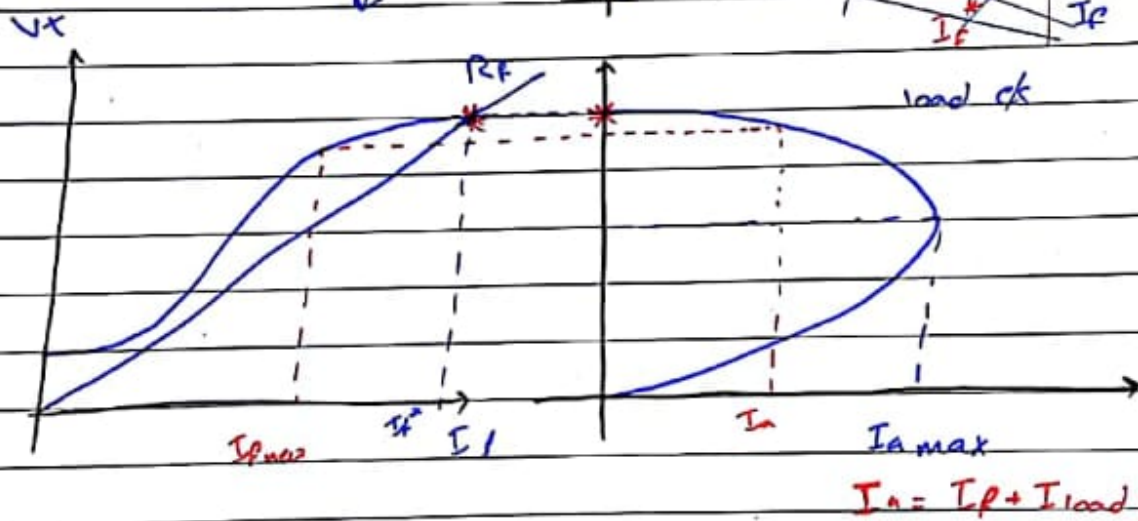
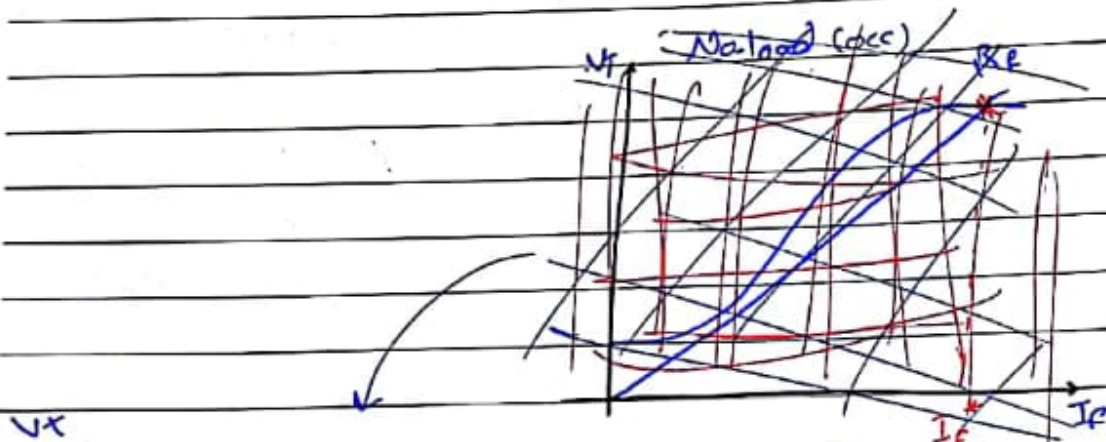
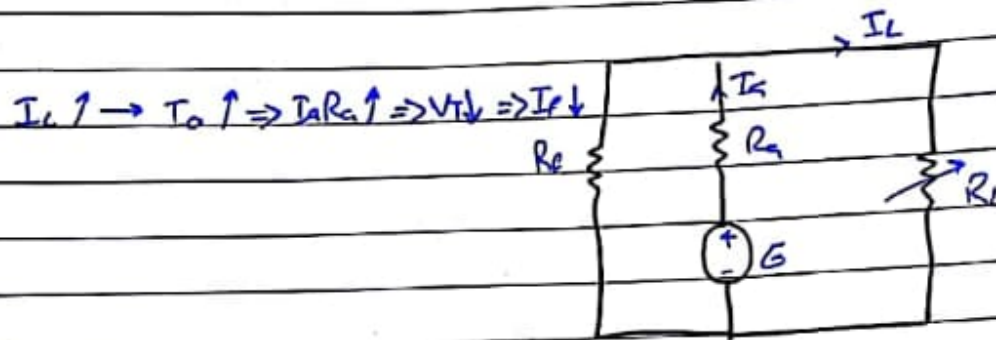
\*  $R_{f \text{ critical}}$  is lowest value of  $R_f$  that makes self excitation (depends on the speed ( $n$ ))



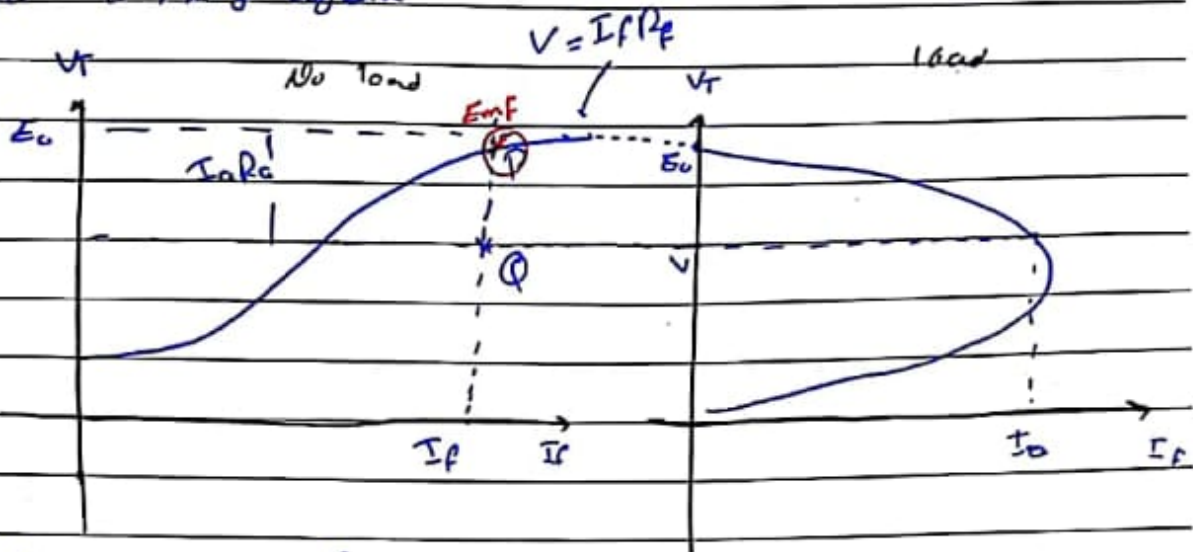
\* Failure to self-excite

- 1) in-sufficient residual flux
- 2) Polarity of the field
- 3)  $R_f > R_{f \text{ critical}}$
- 4) the speed is below the critical value





The same thing again



- 1)\* Assume  $V$  and find  $I_a$
- 2)\* Draw horizontal line and find  $I_f$  // point Q
- 3)\* Emf {P}
- 4)\*  $P-Q \Rightarrow I_a R_a$
- 5)\*  $R_a$  known  $\Rightarrow I_a$  will be known

4/12/2017

Ex) OCC of a DC shunt generator 300 rpm

$I_f (A)$	0	2	3	4	5	6	7
armature voltage	75	92	132	162	183	190	212

- plot OCC at 375 rpm
- Determine the voltage to which the machine will excite if field circuit resistance is 40  $\Omega$
- what additional R in the field ckt to reduce voltage to 200V @ 375 rpm

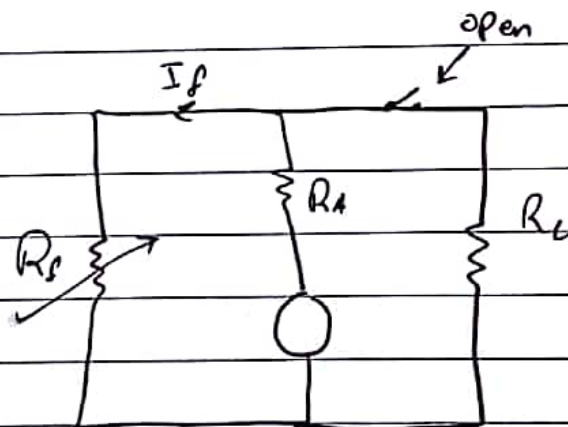
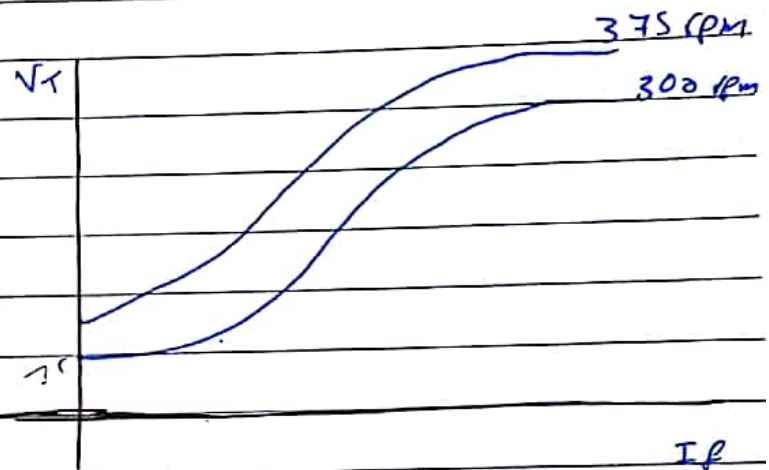
$$E_a = K \phi \omega_m = K' I_f \omega_m$$

$$\frac{V_{t, \text{new}}}{V_{t, \text{old}}} = \frac{I_{f, \text{new}} \omega_{m, \text{new}}}{I_{f, \text{old}} \omega_{m, \text{old}}}$$

$$\Rightarrow I_f = 3A \Rightarrow 300 \text{ rpm} \rightarrow V_t = 132V$$

$$375 \text{ rpm} \rightarrow V_t = ?$$

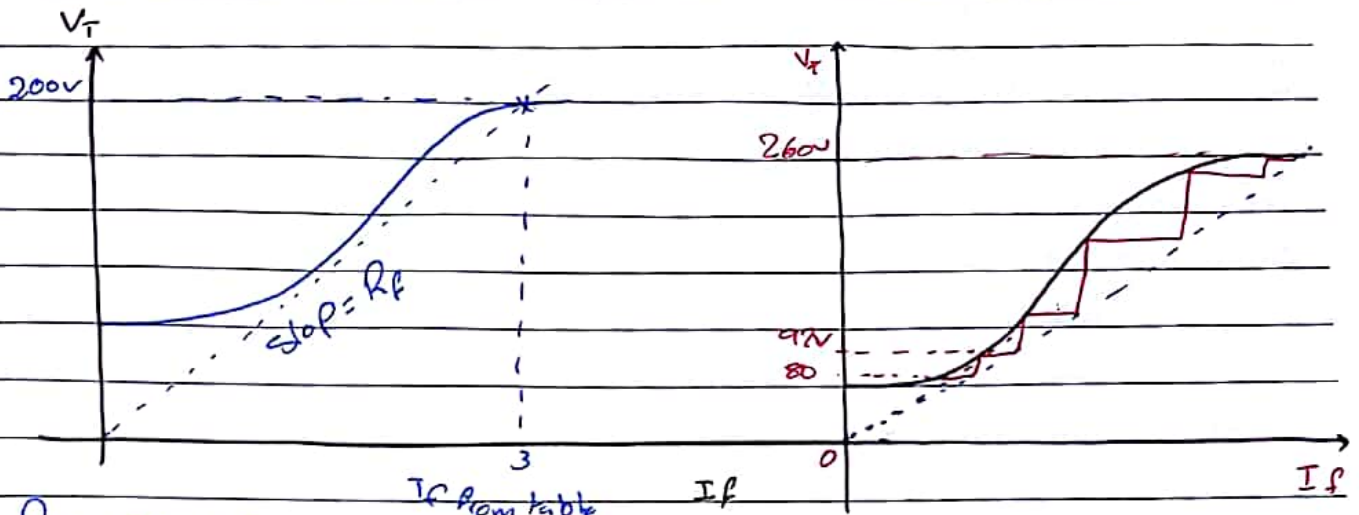
$$\rightarrow V_{t, \text{new}} = 132 * \left( \frac{375}{300} \right)$$





at 375

$I_f$ (A)	0	2	3	4	5	6	7
$V_T$ (V)	9.4	115	202.5	228.8	248.3	265	



$$R_f = 52.6 \Omega$$

$$52.6 - 40 = 12.6 \Omega$$

additional (R)

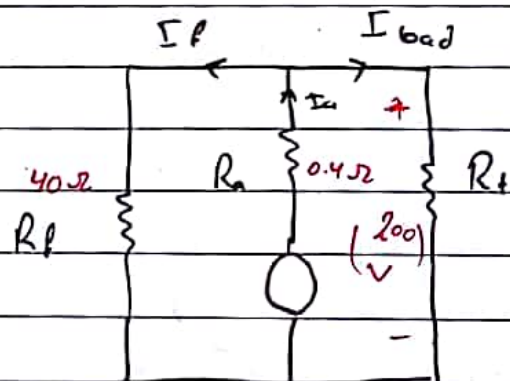
# with out additional Resistance, Determine load current supplied by the generator with its Terminal voltage 200 v? assume  $R_a = 0.4$

375 rpm Field current =  $\frac{200}{40} = 50$  A

$$E_{imp} = 228.8 \text{ V}$$

$$I_a = \frac{228.8 - 200}{0.4} = 72 \text{ A}$$

$$I_{load} = 72 - 5 = 67 \text{ A}$$



$I_f$	0	2	3	4	5	6	7
V	9.4	115	150	202.5	228.8	248.8	265

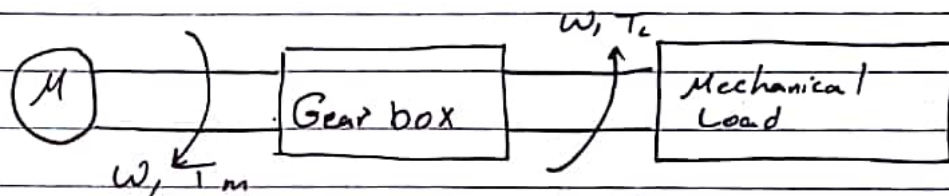
Ex) dc shunt generator (table @ 600 rpm)

IP	1	2	3	4	5	6	7	8
V	23	45	67	85	100	112	121	126

$R_F = 15 \Omega$ ,  $V_T = 120 \text{ V}$ , load current ??

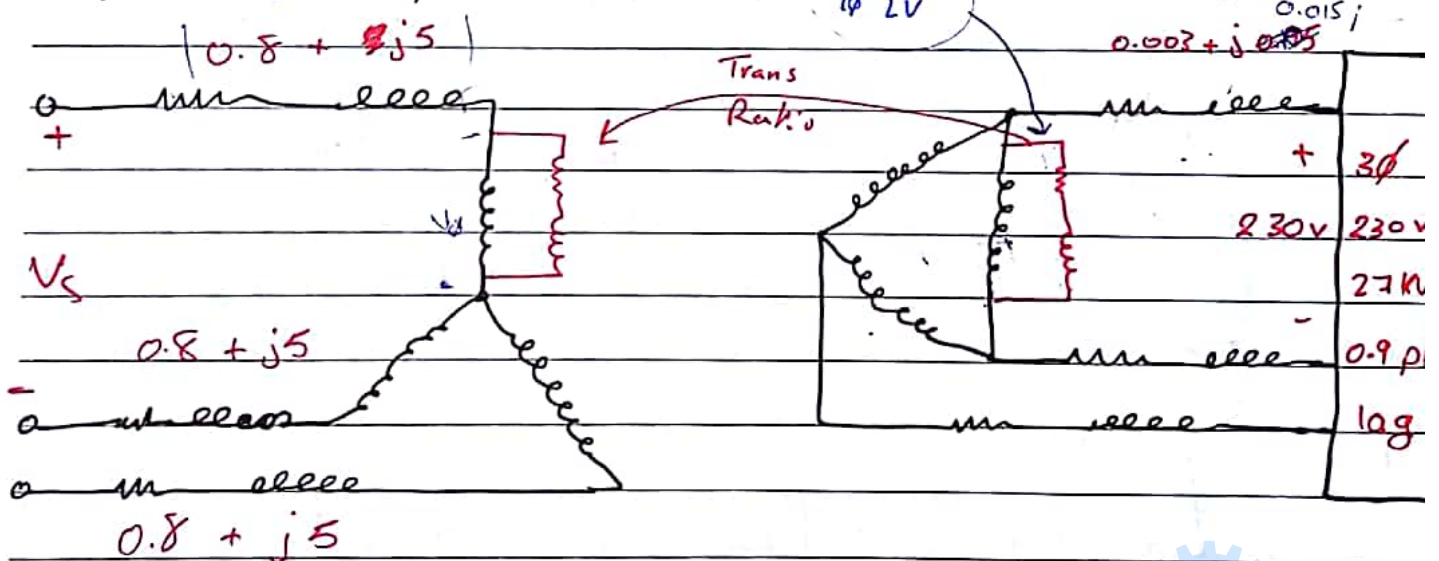
$$R_a = 6.02 \Omega$$
$$I_L = 292 \text{ A}$$

What is the gear box?



⑥ Transformer

Ex) Three phase, 1 $\phi$  (1330, 230 V),  $Z_1 = 0.12 + 0.25j \Omega$



$$V_{LL} = \frac{\sqrt{3}}{Y} V_p, \quad V_{LL} = V_p \quad \Delta$$

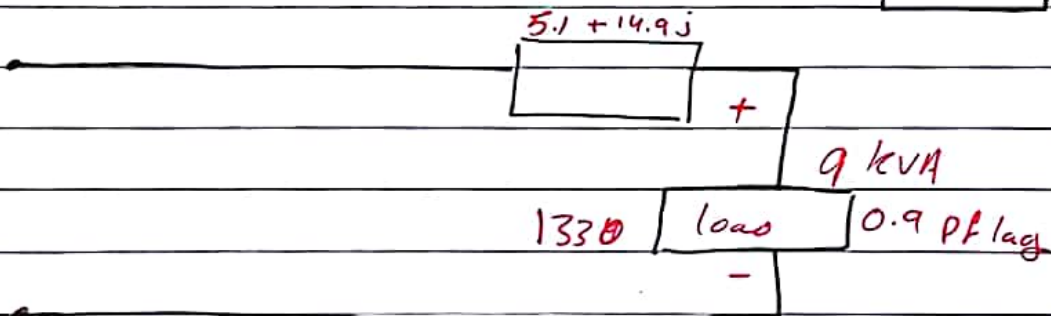
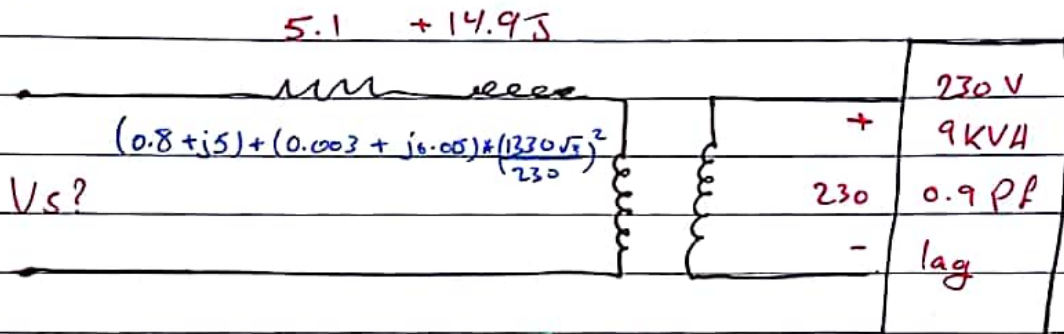
$$Z_{\text{seen from source}} = (0.003 + 0.015j) * \left( \frac{1330 * \sqrt{3}}{230} \right)^2 +$$

$$(0.12 + 0.25j) * \left( \frac{1330}{230} \right)^2 + (0.8 + j5) = 5.1 + 14.9j \, \Omega$$

$$(3, \phi) (1330 / 230) \, Y-\Delta$$

$$1330 * \sqrt{3} / 230 \, V$$

Single phase



$$I = 6.8 \angle -25.8$$

$$V_s = 1407 \, V$$

Phase

$$V_s = 2434 \, V$$

Line



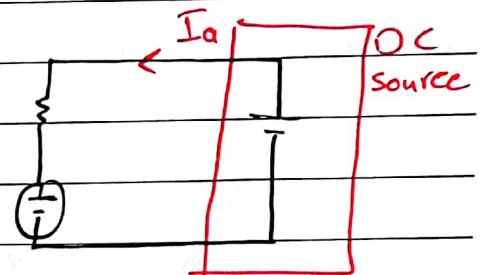
11/Dec/2017

Monday

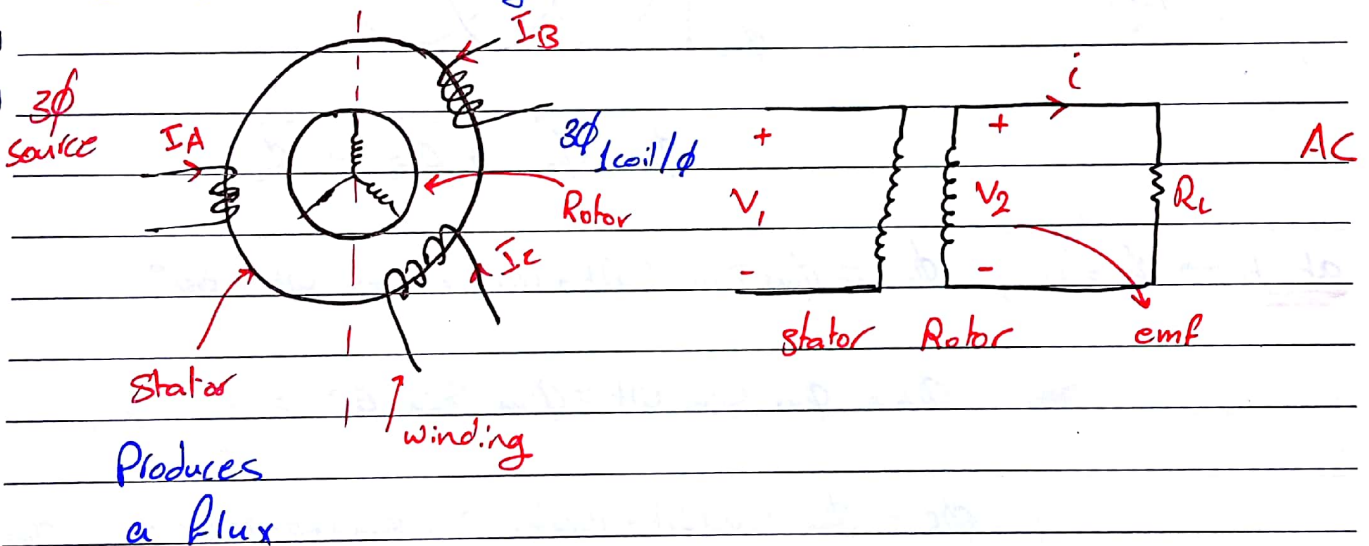
Induction motor (3 $\phi$ )

AC  $\rightarrow$  inexpensive / easy to maintain / reliable

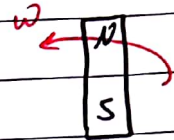
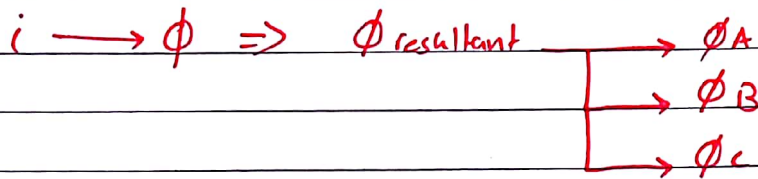
DC motor  $\rightarrow$  power is conducted directly to the armature  
easy controlled



\* Induction  $\rightarrow$  Rotating transformer

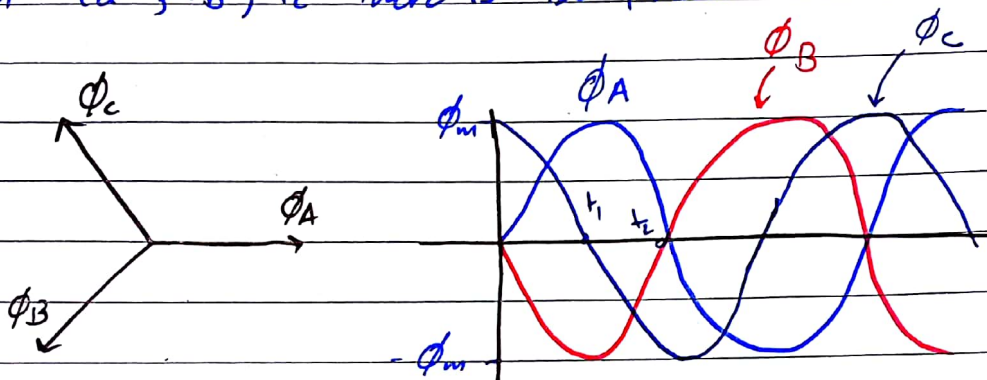


## \* Rotating Field



\* Stator has 3 winding (120° between each one)

\* Between  $i_a, i_b, i_c$  there is 120° phase

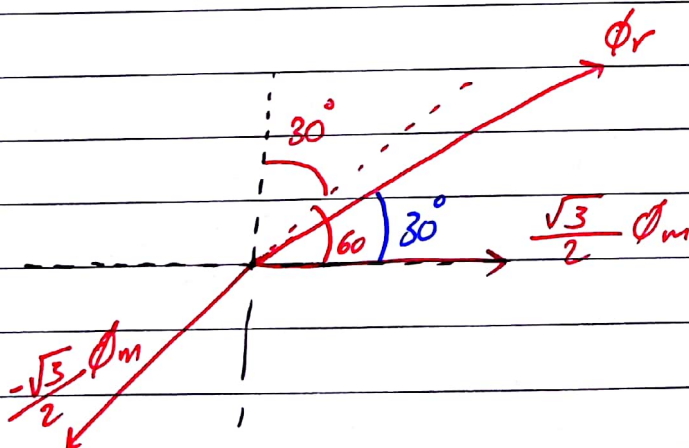


$$\phi_r = \vec{\phi}_A + \vec{\phi}_B + \vec{\phi}_C$$

at  $t_1$   $\rightarrow \phi_C = 0$ ,  $\phi_C = \phi_m \sin(\omega t + 120) = 0 \Rightarrow \omega t = 60^\circ$

$$\phi_A = \phi_m \sin \omega t = \phi_m \sin 60 = \frac{\sqrt{3}}{2} \phi_m$$

$$\phi_B = \phi_m \sin(\omega t - 120) = \phi_m \sin(60 - 120) = -\frac{\sqrt{3}}{2} \phi_m$$

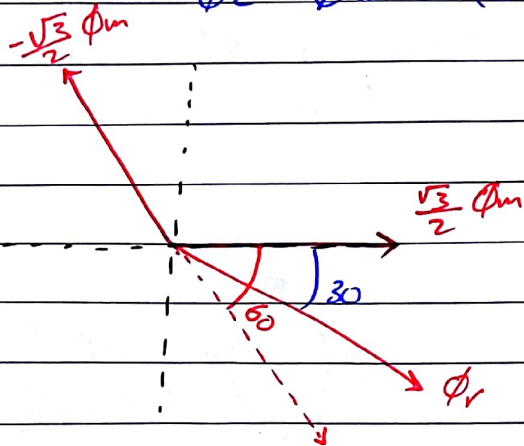


$$\left\{ \begin{aligned} \phi_r &= \sqrt{3} \left( \frac{\sqrt{3}}{2} \phi_m \right) \angle 30^\circ \\ &= \frac{3}{2} \phi_m \angle 30^\circ \end{aligned} \right.$$

at  $t_2$  :  $\phi_B = 0 \rightarrow \phi_B = \phi_m \sin(\omega t - 120) = 0 \Rightarrow \omega t = 120^\circ$

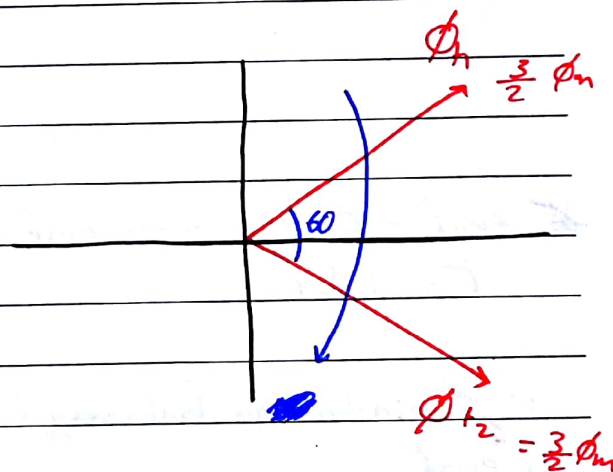
$\phi_A = \phi_m \sin \omega t = \phi_m \sin 120 = \frac{\sqrt{3}}{2} \phi_m$

$\phi_C = \phi_m \sin(\omega t + 120) = \phi_m \sin(240) = -\frac{\sqrt{3}}{2} \phi_m$

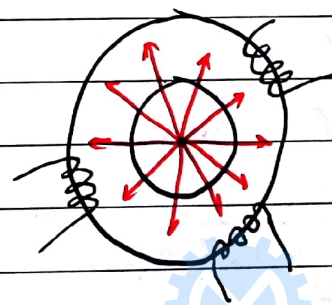


$$\begin{aligned}\phi_r &= \phi_A + \phi_B + \phi_C \\ &= \sqrt{3} \left( \frac{\sqrt{3}}{2} \phi_m \right) \angle 30^\circ \\ &= \frac{3}{2} \phi_m \angle 30^\circ\end{aligned}$$

\* Rotating Field  
(advantage of  $3\phi$ )



\* Air gap Flux  $\rightarrow$  constant Magnitude ( $\frac{3}{2} \phi_m$ )  
 $\rightarrow$  Angle is changing  
 $\rightarrow$  C.W

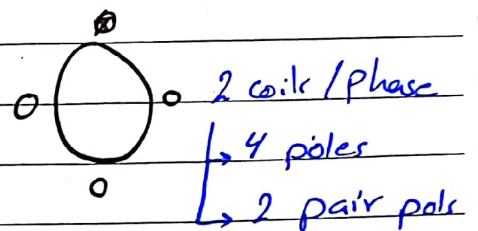
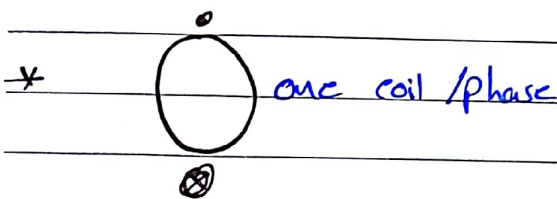
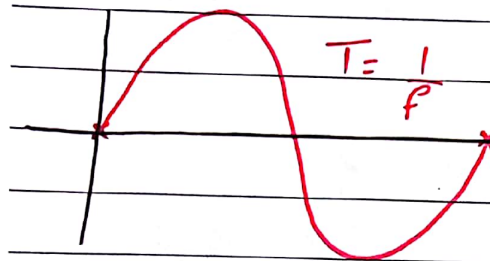




\* Speed of air gap Flux = 1 mechanical revolution (1/f sec)  
(Synchronous speed) =  $f$  rev/sec

$$= 60 f \text{ rev/min}$$

For 2 poles or 1 pair pole  
(North and south)



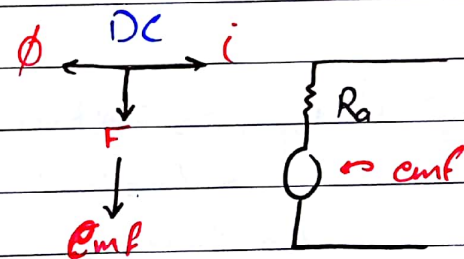
Synchronous speed ( $N_s$ )

$$= \frac{60 f}{PP} = \frac{120 f}{PP}$$

Number of Poles  
 $= 2PP$

\*  $emf = BLV \rightarrow emf = f(\phi, \Delta n)$   
 $F = BLi$

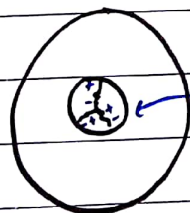
- 1)  $\phi$  (induction Rotating Flux)
- 2)  $emf$
- 3)  $i$
- 4)  $F, T$



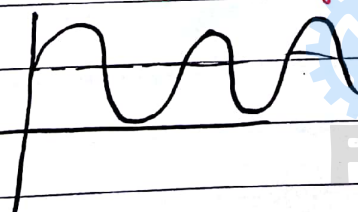
- 1)  $\phi$
- 2)  $Emf = \Delta n$

$$3) i = \frac{emf}{\sqrt{R_2^2 + (X_2)^2}}$$

$$4) T_{average} = J \frac{d\omega_m}{dt} = T_m - T_L$$



Rotor  $\Delta n$  Rotating field speed ( $N_s$ )  
Rotor speed ( $N_m$ )



## \* Running operation

TL causes → Speed Drop

Prove: current will change to meet the torque

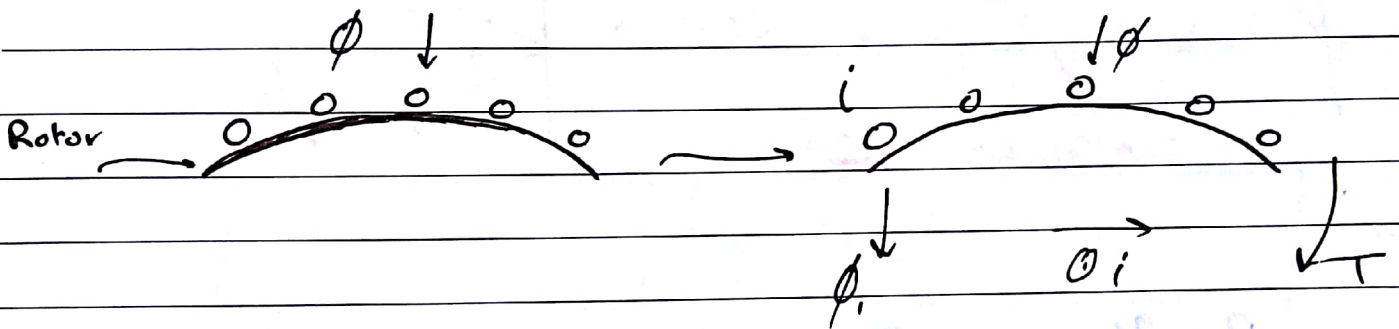
-  $\text{emf} \uparrow$

-  $\text{emf} \propto \Delta n \rightarrow \Delta n \uparrow$

-  $\Delta n = N_s - N_m \rightarrow N_m \downarrow$

can't be zero

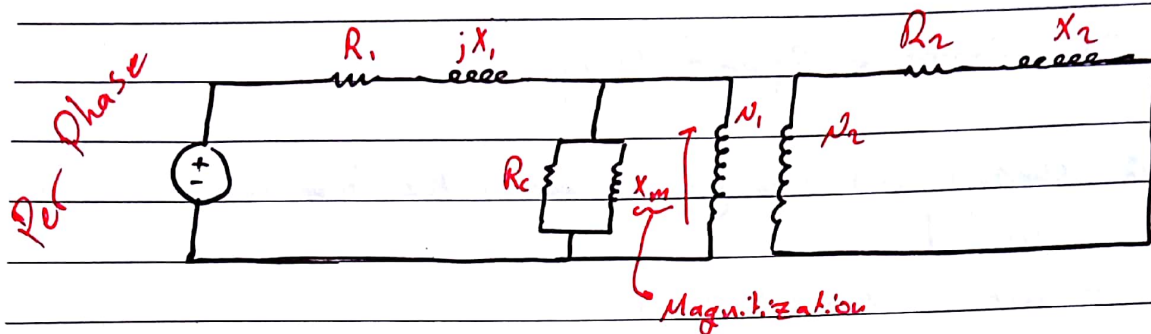
$$* \text{Slip } (s) = \frac{\Delta n}{N_s} = \frac{N_s - N_m}{N_m} \leftarrow \begin{array}{l} \text{Rotor speed} \\ \text{Rotating field speed} \end{array}$$



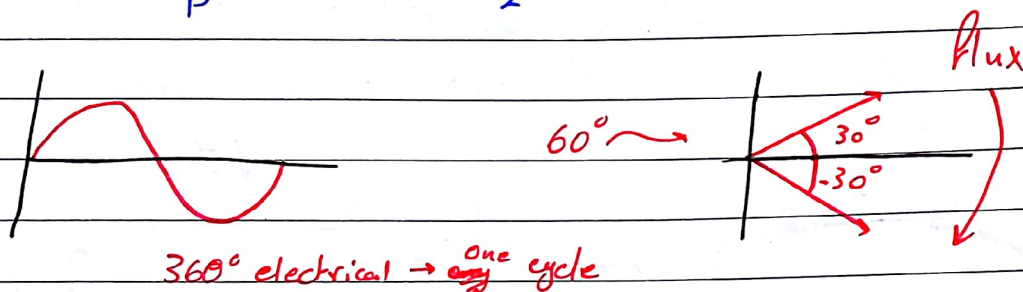
13/12/2017

wednesday

## Equivalent circuit



$$N_s = \frac{120}{P} P, \theta_c = \frac{P}{2} \theta_m, P=2$$



$P=2$  → 720 electrical one cycle

⇒ Number of poles increase; it will have slower speed

$I_m$  → stationary, stand still

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

\*  $\phi$   
emf = BLV  
 $i$   
 $T$

\*  $E_2 = \text{emf}$  at stand still  
Rotor

\* Running  $\phi$   $E_2 \propto N_s$   
 $E_r \propto N_s - N_m$

$$\frac{E_r}{E_2} = \frac{N_s - N_m}{N_s} = s = \text{slip}$$

$$\Rightarrow E_r = s E_2$$

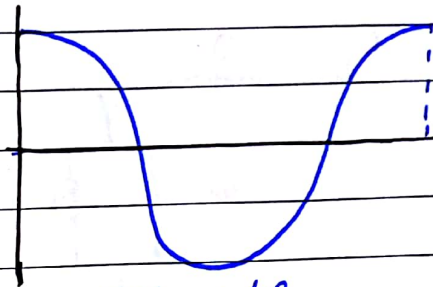


\* Stand still ( $s=1$ )  $\Rightarrow E_r = E_2$

$f_r$  = Frequency of rotor current

\* when we move with the Rotor, we will see the difference in speed

Relative speed



$$T = 1/f$$

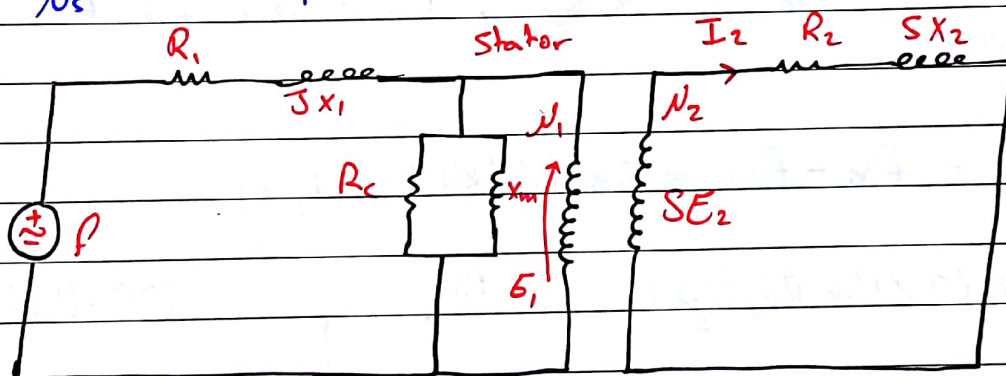
$$f_r = ?? \rightarrow \frac{120 f_r}{P} = N_s - N_m$$

synchronous speed

Mechanical speed

Relative speed

$$s = \frac{N_s - N_m}{N_s} \Rightarrow \frac{120 f_r}{P} = s N_s \rightarrow \frac{120 f_r}{P} = s * \frac{120 f}{P} \Rightarrow \underline{f_r = s f}$$

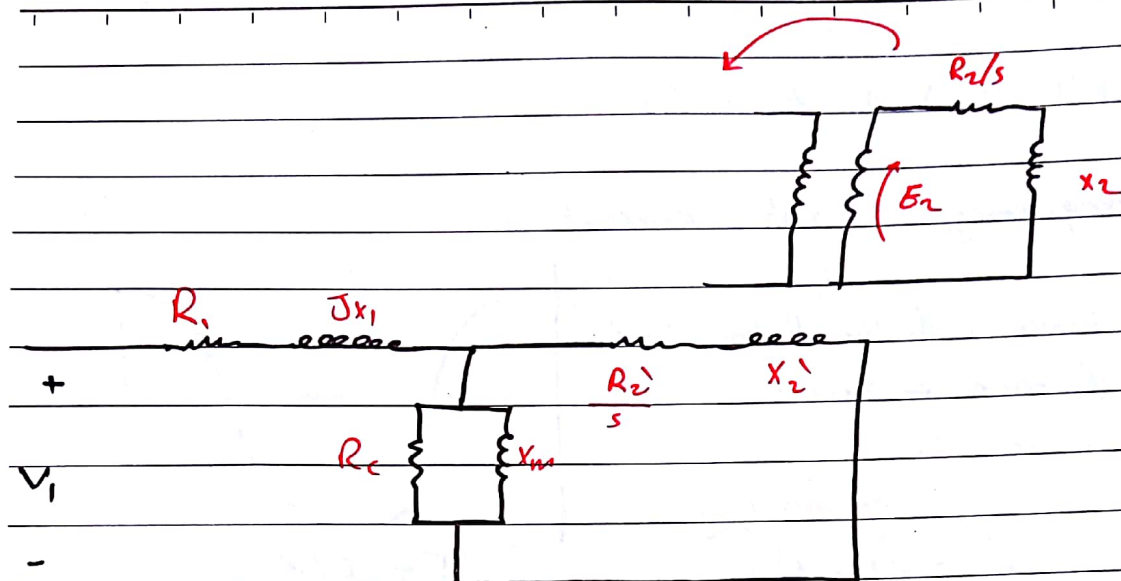


after adjusting

$$X = \omega_r L = 2\pi f_r L = 2\pi s f L = (2\pi f L) s \Rightarrow f_r = s f$$

$$I_2 = \frac{s E_2}{R_2 + j s X_2} = \frac{E_2}{\frac{R_2}{s} + j X_2}$$



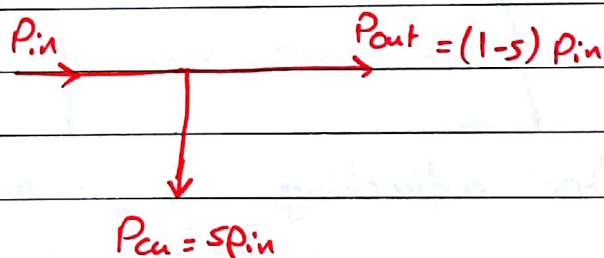


$$\frac{R_2'}{s} = \frac{R_2}{s} \times \left(\frac{N_1}{N_2}\right)^2, \quad x_2' = x_2 \times \left(\frac{N_1}{N_2}\right)^2$$

$$P_{in} \text{ Rotor} = 3 \times (I_2')^2 \times \frac{R_2'}{s}, \quad P_{cu} = 3 \times (I_2')^2 R_2'$$

$$P_{output} = P_{in} - P_{cu} = 3 \times (I_2')^2 \times (R_2') \left(\frac{1-s}{s}\right)$$

$$* P_o = 3 (I_2')^2 \times R_2' \left(\frac{1-s}{s}\right)$$



$$* \eta = \frac{P_o}{P_{in}} = \frac{(1-s) P_{in}}{P_{in}} = (1-s) = \eta$$

*		1410 rpm	1490 rpm	Mechanical speed
	S	S <sub>1</sub>	S <sub>2</sub>	$N_s = \frac{120 f}{P} = \frac{120 (50)}{P}$

P	2	4
N <sub>r</sub>	300	1500

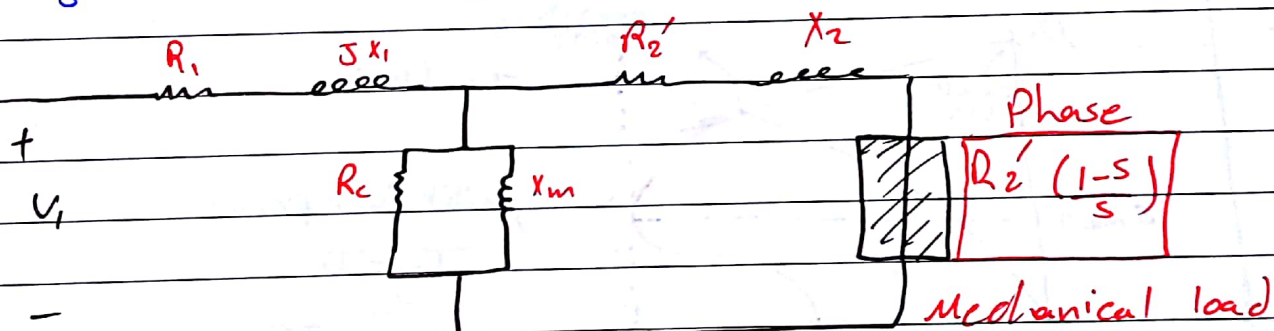
we assume  $N_s = 1500$

Since 1490 rpm is closer ~~than~~ to 1500 rpm

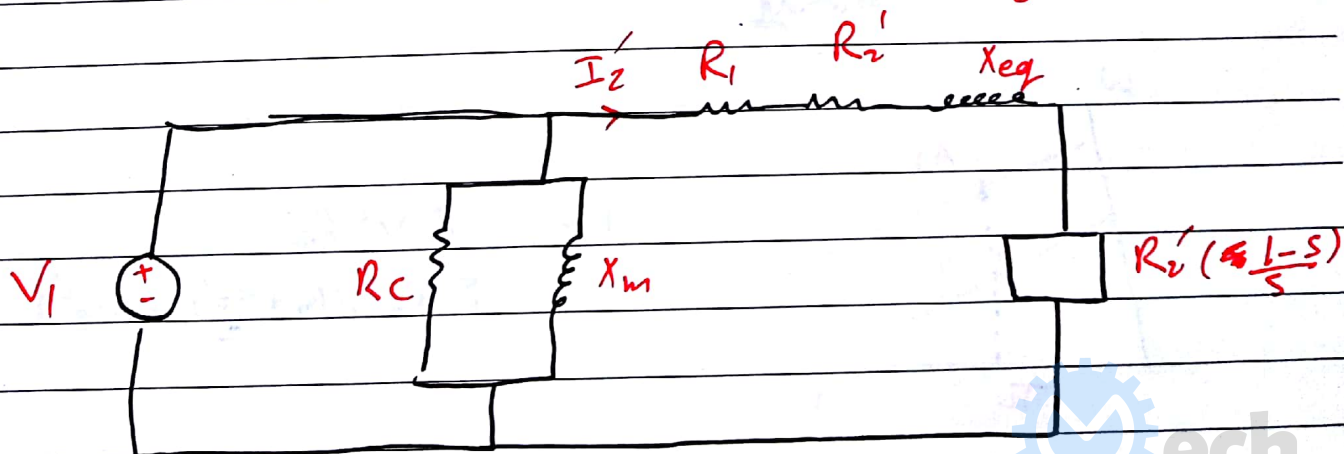
$$\Rightarrow \boxed{S_1 > S_2}$$

$$\boxed{\eta_1 < \eta_2}$$

$$* \frac{R_2'}{s} = R_2' + R_2' \left( \frac{1-s}{s} \right)$$



Mechanical load  
Multiply by 3 because 3φ





Torque - Speed ch Im:

Torque :  $P = T\omega$

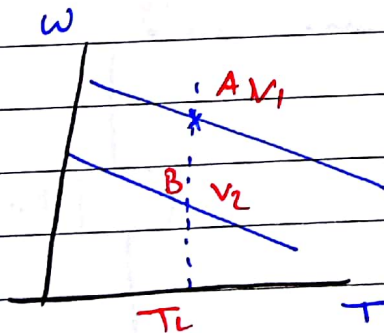
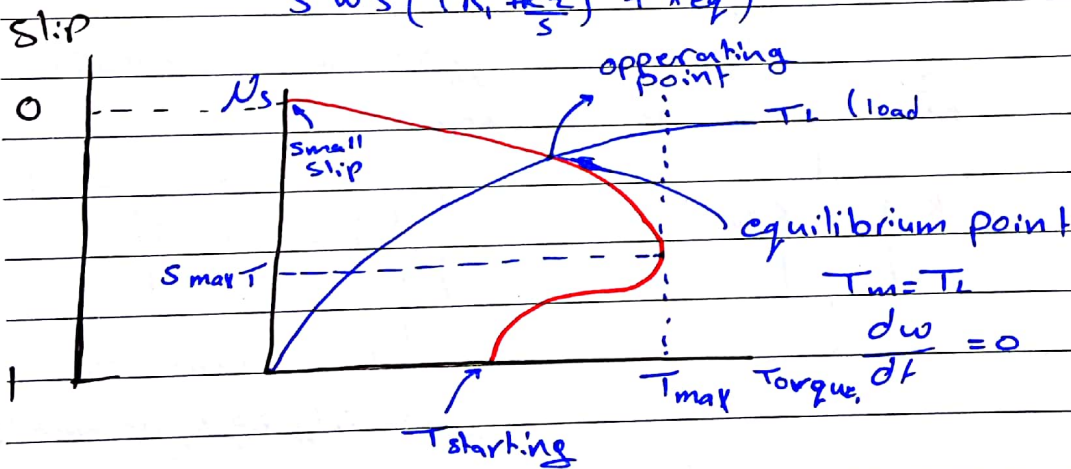
$$T = \frac{P}{\omega} = \frac{3 (I_2')^2 (R_2') \left(\frac{1-s}{s}\right)}{\omega}$$

$$\omega = \omega_s (1-s)$$

$$\Rightarrow I_2' = \frac{V}{\sqrt{(R_1 + \frac{R_2'}{s})^2 + (X_{eq})^2}}$$

$-V_1 + I_2' R_1 + I_2' X_{eq} s + I_2' R_2' + I_2' (R_2' \frac{1-s}{s})$

$$\Rightarrow T = \frac{3 V^2 R_2'}{s \omega_s [(R_1 + \frac{R_2'}{s})^2 + X_{eq}^2]}$$



$$T = \frac{3 V^2 R_2'}{s \omega_s [(R_1 + \frac{R_2'}{s})^2 + X_{eq}^2]}$$

\* 3 Regions

- at large slip :  $x_{eq} \gg \frac{R_2'}{s} + R_1$

$$T = \frac{3 V^2 R_2'}{s \omega_s x_{eq}^2}, \quad T_{st.} = \frac{3 V^2 R_2'}{\omega_s x_{eq}^2} \quad \text{at } s=1$$

- at small slip :  $\frac{R_2'}{s} \gg R_1$ ,  $\frac{R_2'}{s} \gg x_{eq}$

\* Stability (Need to be avoided)

$$J \frac{d\omega_m}{dt} = T_m - T_L$$

$$\omega \uparrow \Rightarrow T_m \downarrow \Rightarrow \frac{d\omega_m}{dt} (-ve) \quad \text{de-acceleration}$$

$$\text{per phase : } T = \frac{3 V^2 s}{\omega_s R_2'}$$

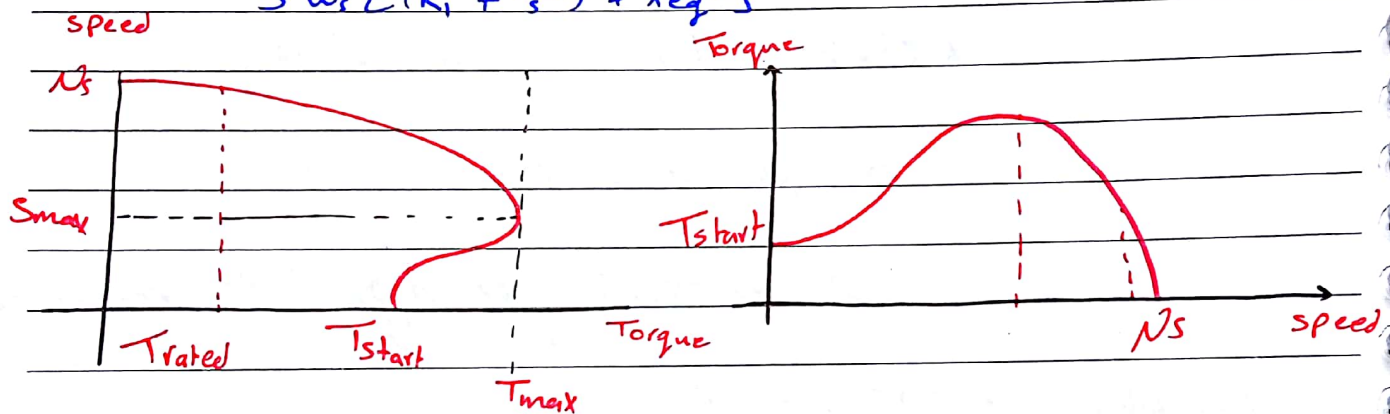
Thursday

14/Dec/2017

$$* S_{max} = \frac{R_1'}{\sqrt{R_1'^2 + X_{eq}^2}}$$

$$T_{max} = \frac{3 V^2}{2 \omega_s [R_1 + \sqrt{R_1'^2 + X_{eq}^2}]}$$

$$T = \frac{3 V^2 R_1'}{s \omega_s [(R_1 + \frac{R_1'}{s})^2 + X_{eq}^2]}$$



If we used load with Torque higher than  $T_{st}$   
it won't work



3 varies

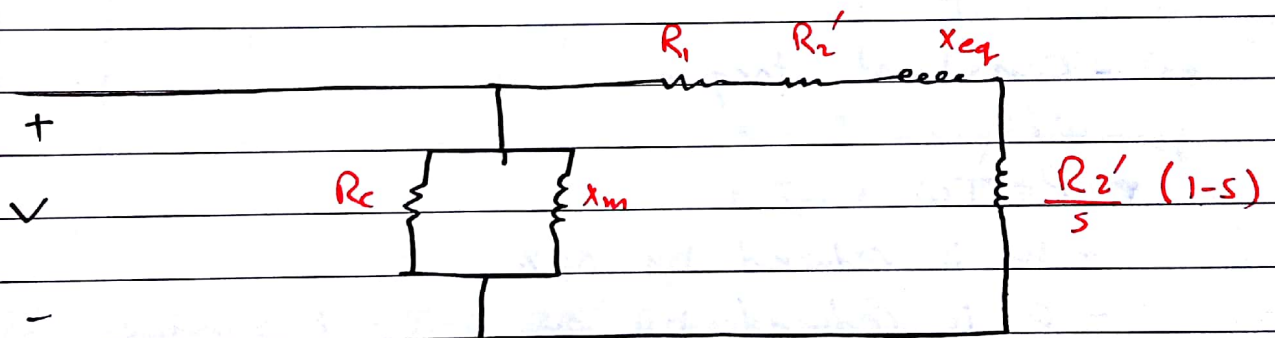
\* large slip  $(R_1 + \frac{R_2'}{s})^2 \ll X_{eq}^2$

$$T_d = \frac{3 V^2 R_2'}{s \omega_s X_{eq}^2}$$

$s=1 \Rightarrow (T_d)_{\text{starting}} = \frac{3 V^2 R_2'}{\omega_s X_{eq}^2}$

\* Small slip:  $R_1 \ll \frac{R_2'}{s} \gg X_{eq}$

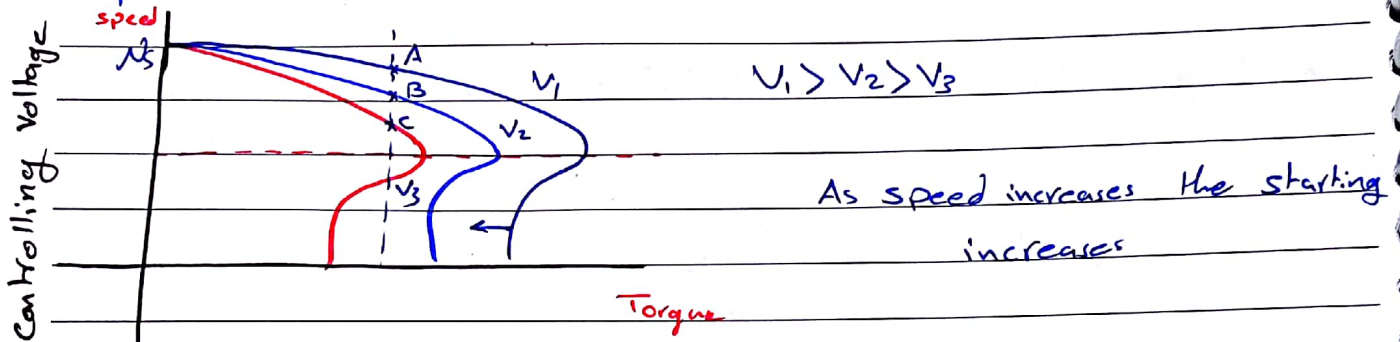
$$T_d = \frac{3 V^2 s}{\omega_s R_2'}$$



at maximum torque  $T_{max} = \frac{3 V^2}{2 \omega_s [R_1 + \sqrt{R_1^2 + X_{eq}^2}]}$

$$s_{max} = \frac{R_2'}{\sqrt{R_1^2 + X_{eq}^2}}$$

## Speed control I M (Induction Motor)



$$\text{Power} = T \omega$$

Fan  $\rightarrow$  variable Torque  $T \propto \omega^2$

- $\omega$  is reduced by 50%
- $T$  is reduced by 0.25
- $P$  is reduced by  $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

- Constant torque  
 $\omega \downarrow$

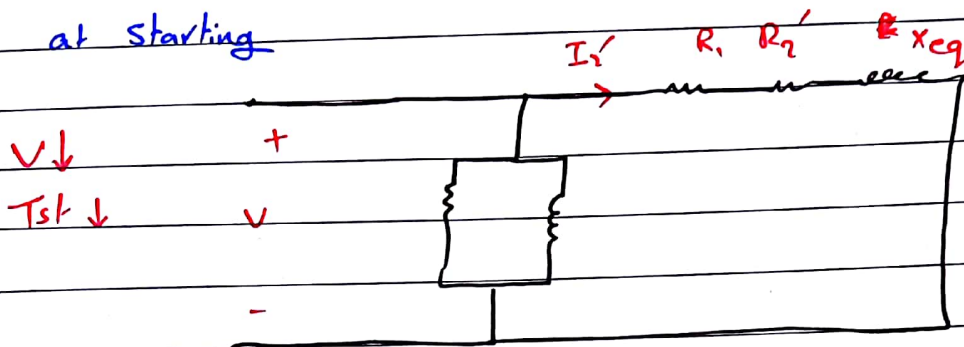
$$P = T \omega$$

- $\omega$  is reduced by 50%
- $P$  is reduced by 0.5

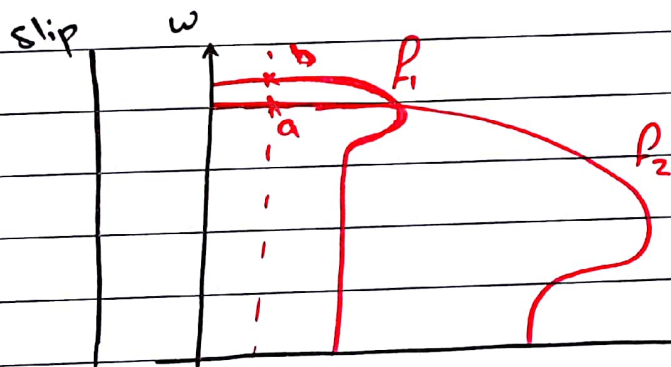
\* controlling speed by adjusting starting voltage

10% reduction in voltage  $\rightarrow$  19% reduction in torque

at starting



\* controlling speed by adjusting frequency



$$P_1 > P_2, \quad s_{max} = \frac{R_2'}{\sqrt{R_1^2 + X_{eq}^2}}$$

$$N_s = \frac{120f}{P}$$

$$T_{st} = \frac{3V^2 R_2}{\omega_s X_{eq}}$$

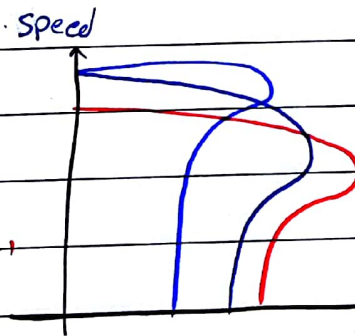
\*  $P \uparrow \rightarrow X_{eq} \uparrow \rightarrow s_{max} \downarrow$   
 $\times T_{st} \downarrow, T_{max} \downarrow$

\* V/f control:  $V = 4.44 N A \Phi B_{max} P \uparrow$

$\frac{V}{f}$  constant  $\rightarrow$  constant flux  
 $\rightarrow$  constant torque

$$T_{max} = \frac{3V^2}{2\omega_s [R_1 + \sqrt{R_1^2 + X_{eq}^2}]}$$

$X_{eq} \gg R_1$



$$T_{max} = \frac{3V^2}{2\omega_s X_{eq}}$$

$T_{max} \propto \left(\frac{V}{f}\right)^2$

$V \uparrow \rightarrow I_{st} \uparrow$   
 $f \uparrow \rightarrow T_{st} \downarrow = \frac{V}{z}$   
 $x \uparrow$



18/Dec/2017

Monday

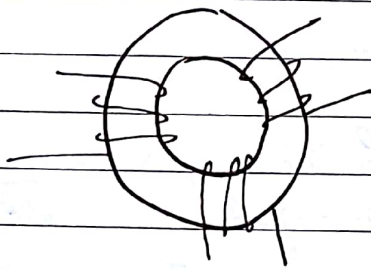
## \* Synchronous machines

→ Asynchronous machines "induction"

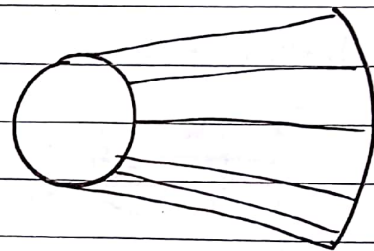
$$\omega_s > \omega_m \rightarrow \text{Slip}$$

→ Synchronous machines

$$\omega_s = \omega_m$$



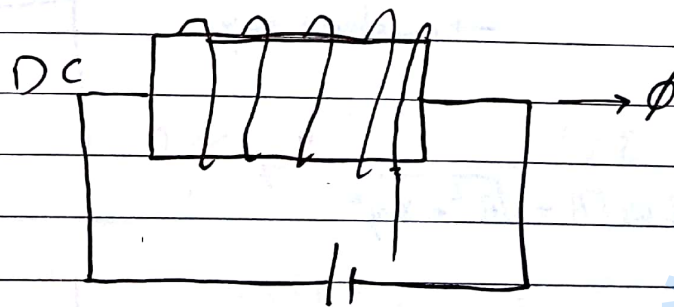
Stator



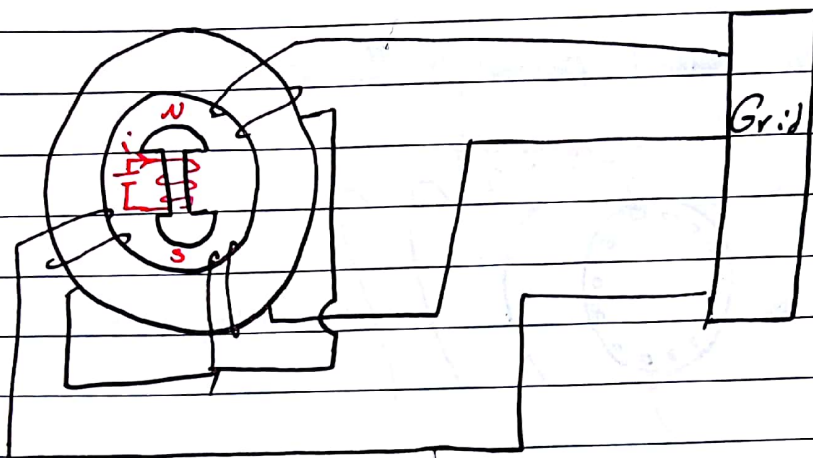
Rotor (Squirrel Cage)

\* Same stator 3 $\phi$  AC "motor"

Rotor  $\rightarrow$  Permanent magnet  
 $\rightarrow$  Electro magnet

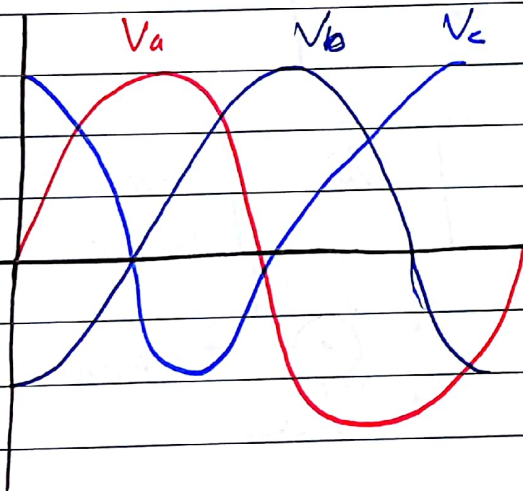


## Synchronous Generator



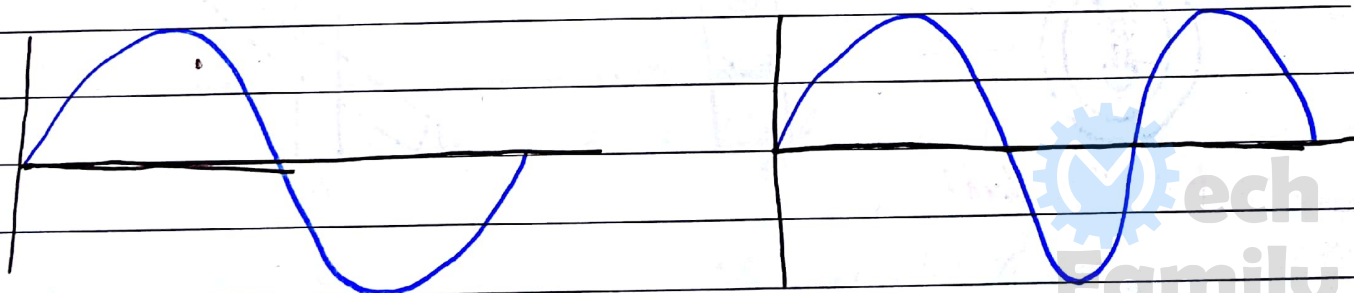
Motion of Rotor  
by gas turbine

Moving Rotor  $\rightarrow \frac{d\phi}{dt}$  [Stator]  $V = N \frac{d\phi}{dt}$



$$* f_s = \frac{120f}{p}$$

$$\theta_e = \frac{p}{2} \theta_m$$



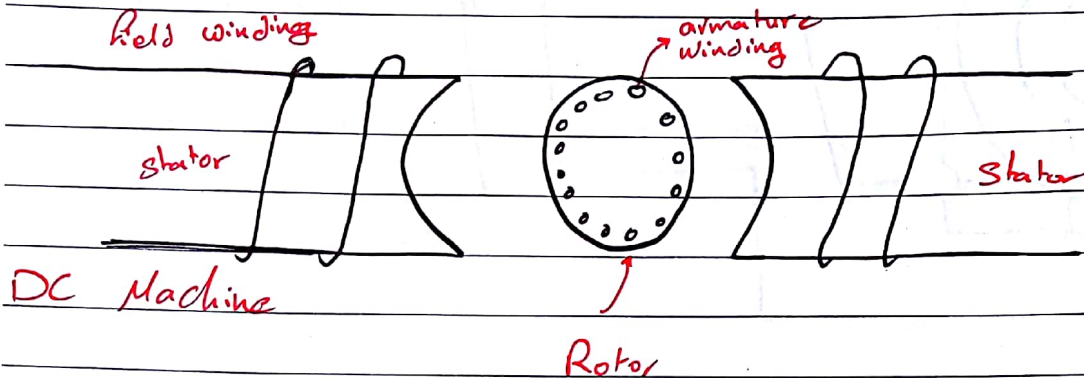
one cycle Mechanical  $p=2$

one cycle Mechanical  $p=4$   
 $\theta_e = 2\theta_m$

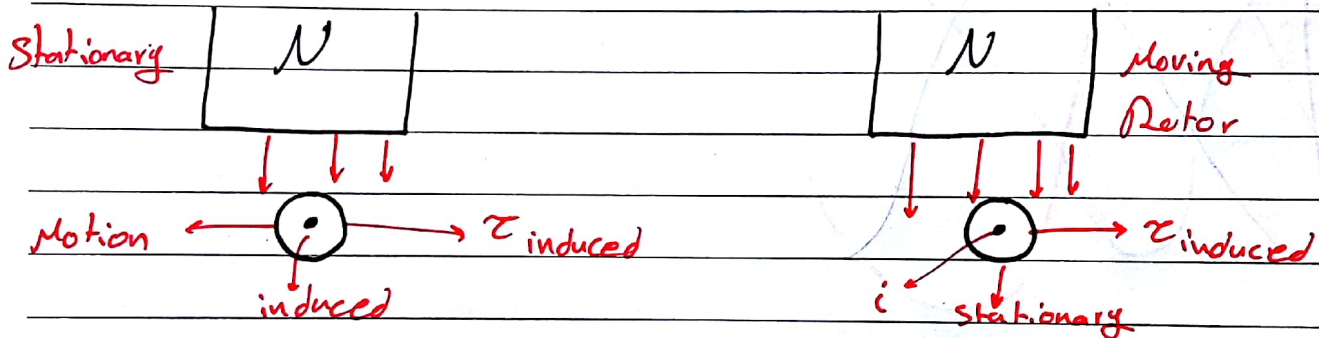


\*  $f = 50 \text{ Hz}$ ,  $P = 2 \rightarrow N_s = 3000 \text{ rpm}$

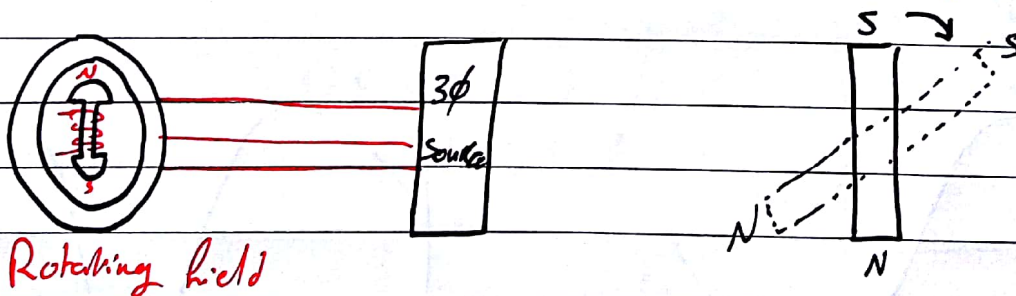
\* We use slip rings to connect power to the rotor



	DC Machine	Synchronous Machine
Stator	Field winding	Armature winding
Rotor	Armature winding	Field winding



\* Difference Between Synchronous and Induction speed

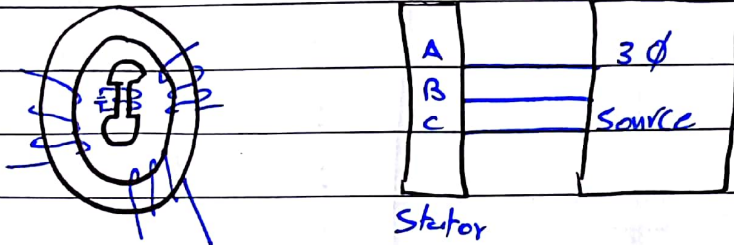


\* As flux rotating rotor will start following flux @ the same speed



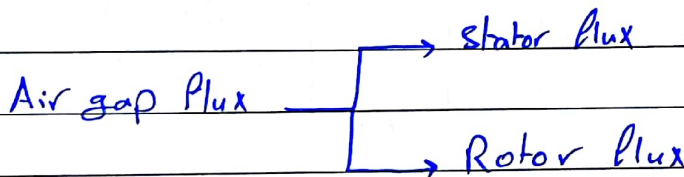
wednesday  
20/Dec/2024

## Synchronous Motor

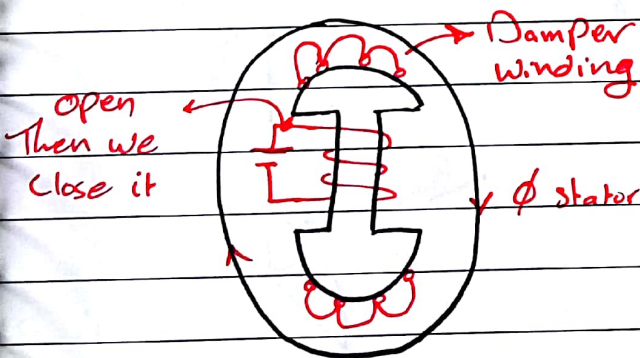


Rotating field  $\rightarrow$  Speed  $= N_s = \frac{120f}{P}$   $N_m = N_s$

⊗ Starting (There is a problem in starting)



in starting we should increase the rotor speed (by adding a prime mover or by using induction motor)



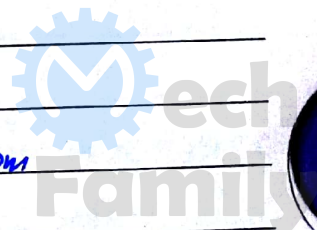
① Rotating field  $\phi$  stator

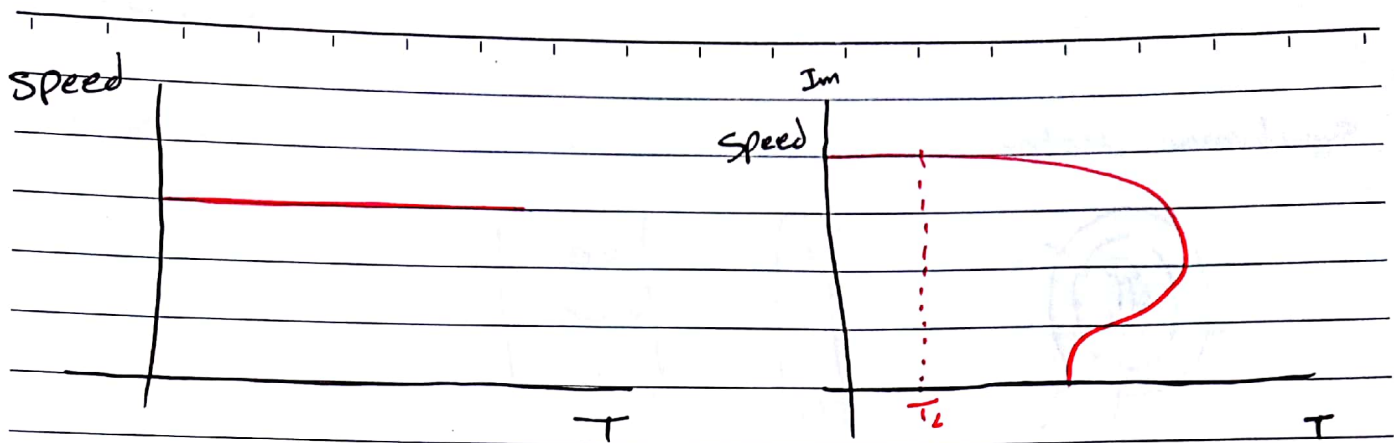
② damper winding

$i \rightarrow T \rightarrow$  rotor speed  $\uparrow$

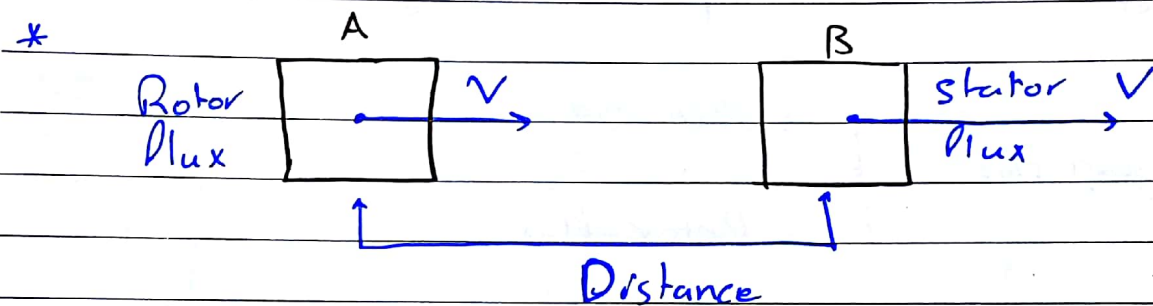
at starting  $N_s = 3000 \text{ rpm}$ ,  $N_m = 0$   
 $P = 2$ ,  $N_s = \frac{120f}{P}$

\* low frequency  $\rightarrow f = 0.1 \text{ Hz} \Rightarrow N_s = 6 \text{ rpm}$   
increase  $f$  gradually





Synchronous Motor



There is a phase shift between them, as the distance increase the phase increases (angle phase  $\delta$ )

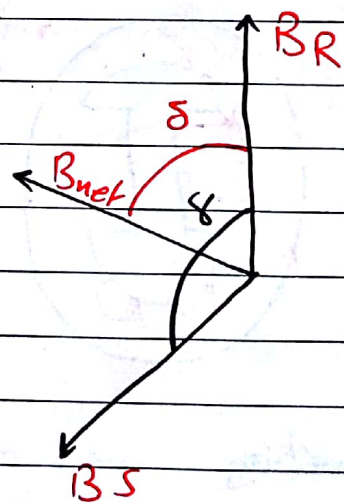
$$* T = K |B_R| |B_S| \sin \delta$$

$$= K |B_R| \times |B_S|$$

$$T = \vec{B_R} \times (\vec{B_{net}} - \vec{B_R})$$

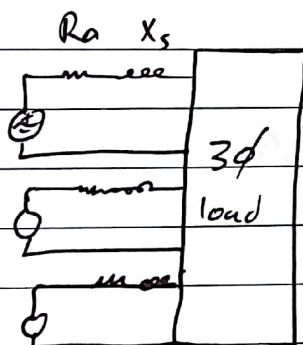
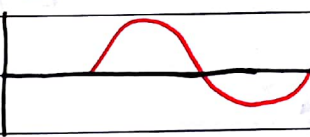
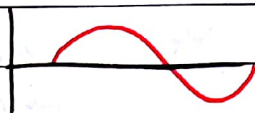
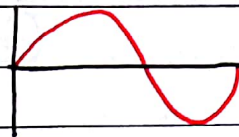
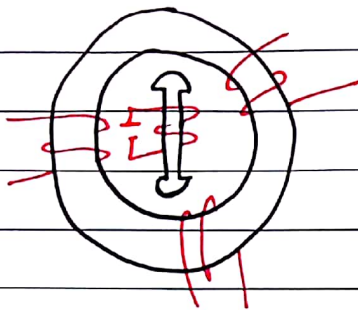
$$T = \vec{B_R} \times \vec{B_{net}}$$

$$T = B_R B_{net} \sin \delta$$

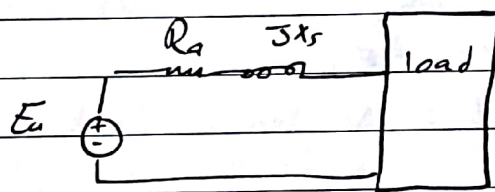




## \* Equivalent Circuit of Synchronous Generator

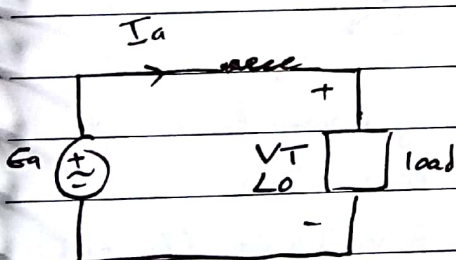


$R_a$  : Armature Resistance  
 $X_s$  : Synchronous Reactance



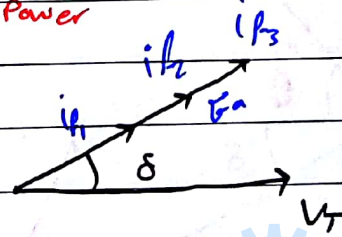
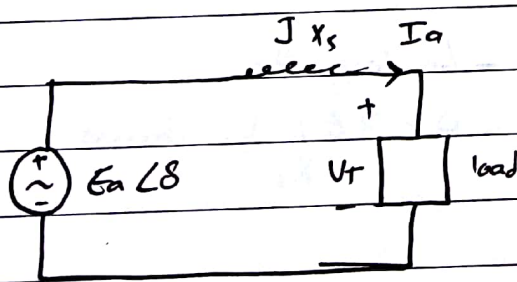
$$E_a = K \phi \omega$$

$$E_a - I_a (R_a + jX_s) = V_t$$



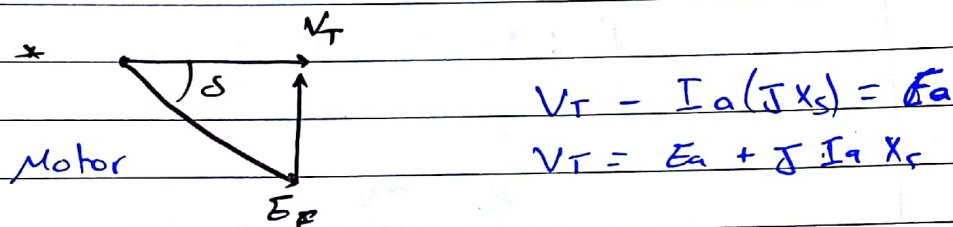
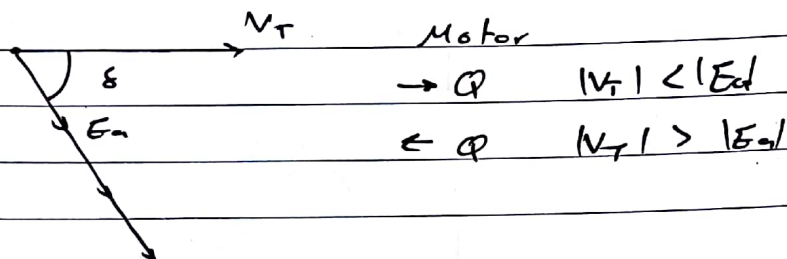
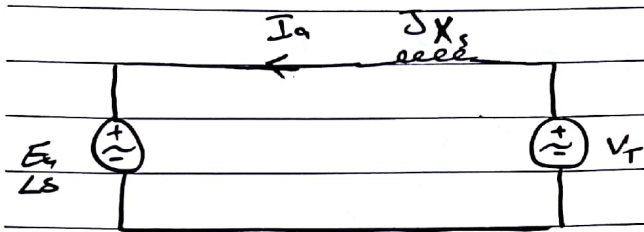
$E_a \angle \delta$

$$S = \underbrace{P}_{\text{Real Power}} + j \underbrace{Q}_{\text{Reactive Power}}$$

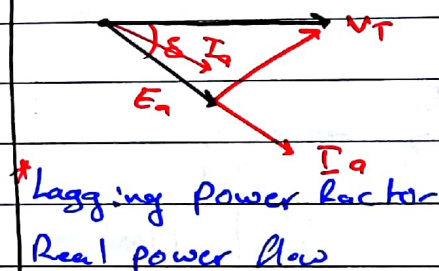
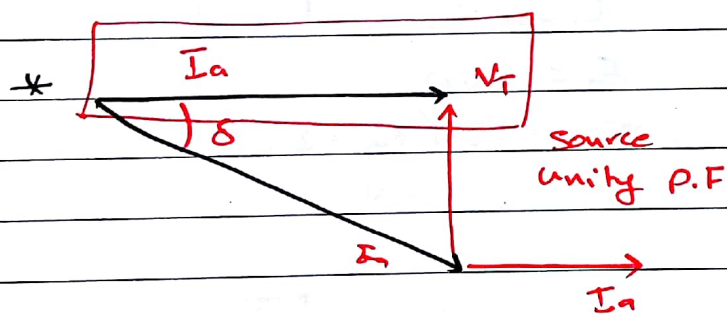


$|E_a| > |V_t| \rightarrow \text{Generator Export } Q$   
 $|E_a| < |V_t| \rightarrow \text{import } Q$



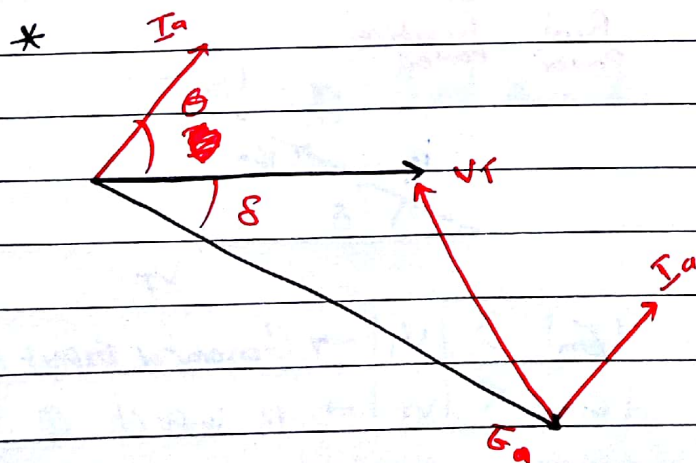


L: V leads I by  $90^\circ$



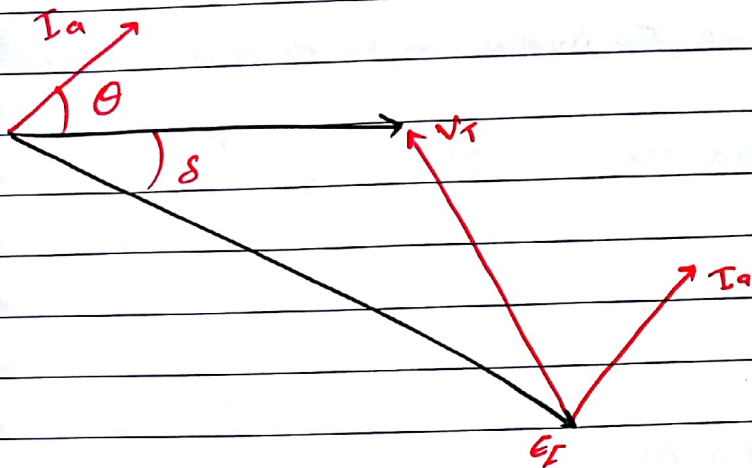
$$P = 3 V_T I_a \cos(\angle V - \angle I)$$

$$P = 3 V_T I_a \cos \theta$$



- Leading PF

$$P = \frac{3 E_f V_T \sin \delta}{X_s}$$



Phasor diagram

$I_a$  leads  $V_T$

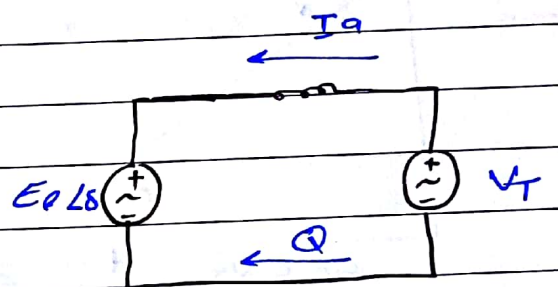
injects reactive power

$$Q = 3 V_T I_a \sin \theta$$

$$E_f \cos \delta - V_T = I_a X_s \sin \theta$$

$$E_f \cos \delta - V_T = \frac{Q}{3 V_T} X_s$$

$$\Rightarrow Q = \frac{3 V_T (E_f \cos \delta - V_T)}{X_s}$$

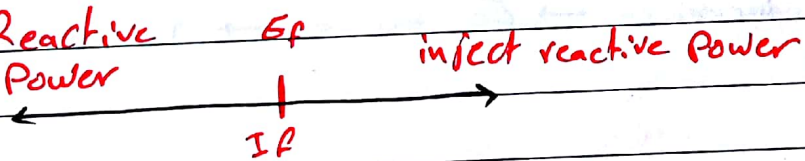


No load ( $\delta=0$ )  $\Rightarrow E_f > V_T \rightarrow Q(+ve)$  inject  $Q$

$E_f = V_T \rightarrow$  unity PF  $Q=0$

$E_f < V_T \rightarrow$  consume  $Q$

consume Reactive Power

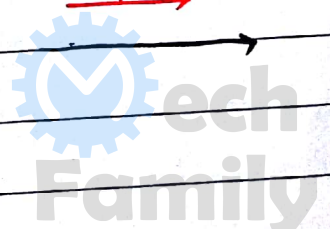
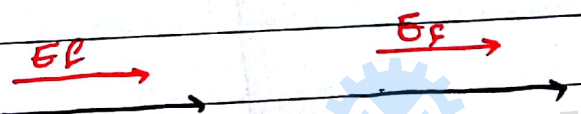


$I_a$  No load

(a)  $Q=0$  unity P.F

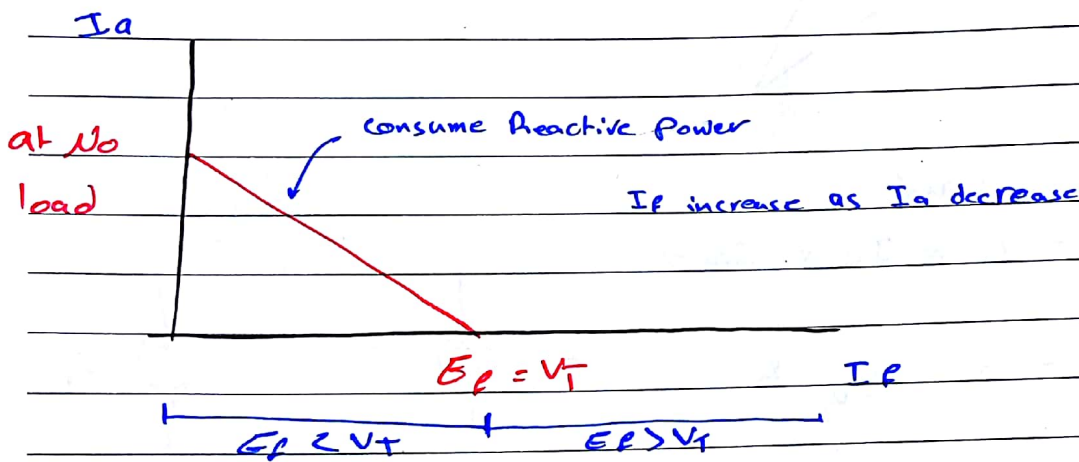
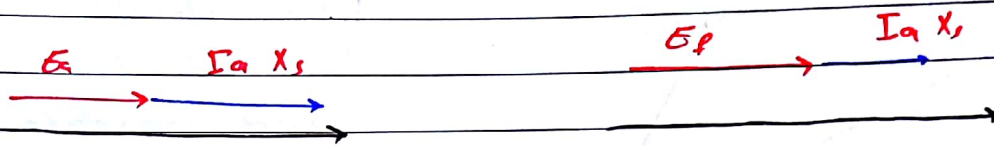
$I_a=0$

(b) Motor consume Reactive Power

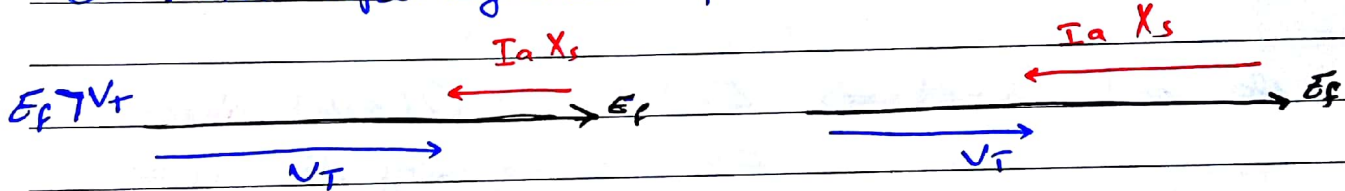




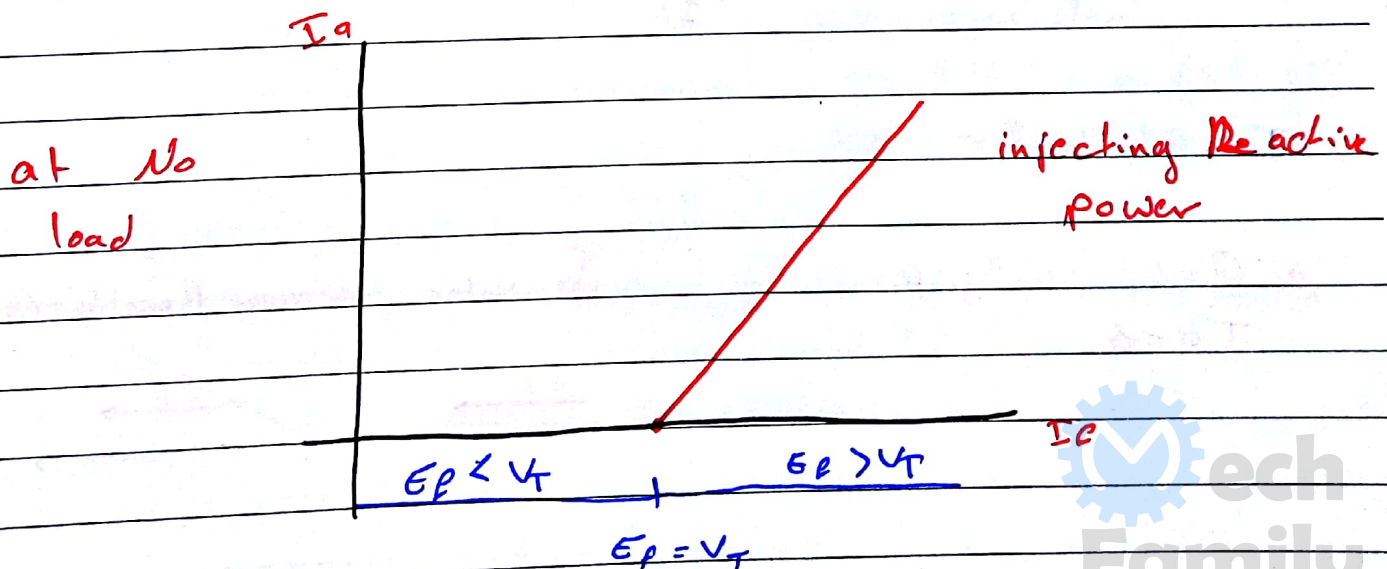
as  $I_f$  increases  $\rightarrow E_f$  increase  $\rightarrow I_a$  decrease ( $I_a X_s \downarrow$ )



© Motor injecting reactive power



as  $I_f$  increase  $\rightarrow E_f$  increase  $\rightarrow I_a$  increase





\* load  $\Rightarrow \delta$

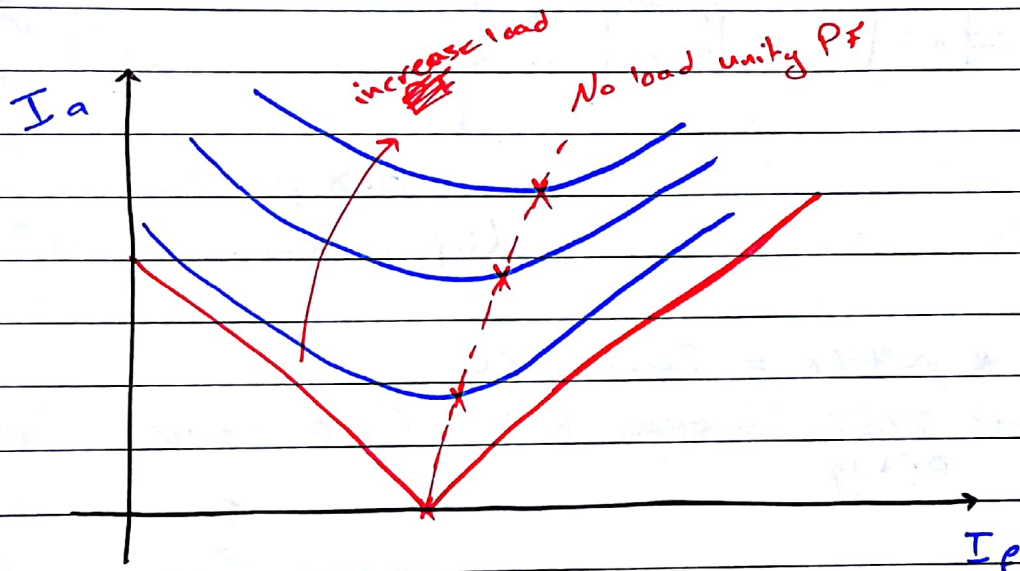
$$Q = 0 \rightarrow E_f \cos \delta - V_T = 0 \Rightarrow \frac{V_T}{\cos \delta} = E_f$$

\* No load

- unity P.F
- Zero reactive
- $E_{f1} = V_T$
- $I_{f1} = 0$

\* load

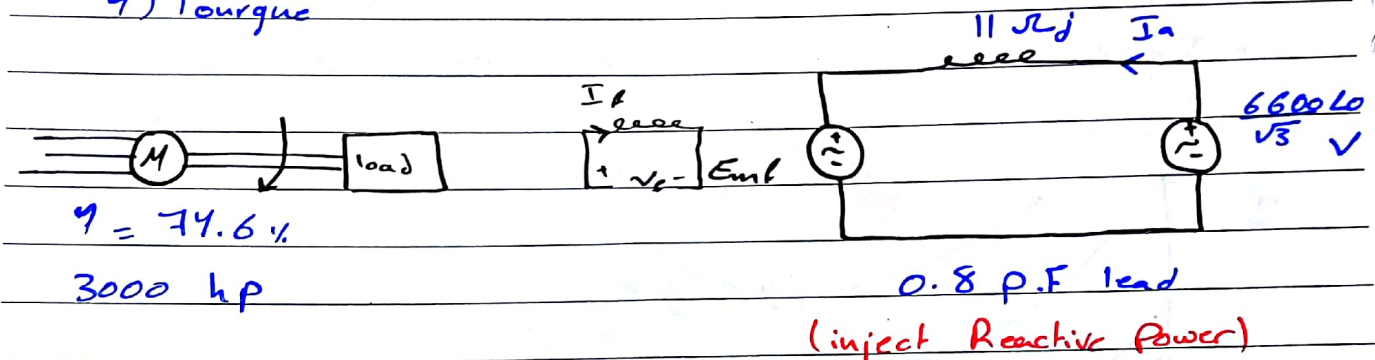
- unity P.F
- $E_{f2} \cos \delta = V_T$
- $E_{f2} = V_T / \cos \delta$
- $E_{f2} > E_{f1}, I_{f2} > I_{f1}$



Ex) 3000 hp, 6600 V, 3 $\phi$ , Y connected synchronous motor operates at full load at a leading P.F of 0.8 and efficiency  $\approx 74.6\%$ .

$$X_s = 11 \Omega$$

- 1) apparent power per phase
- 2) line current
- 3) Emf
- 4) Torque



$$- P_o = 3000 \times 0.746 = 22.38 \text{ KW}$$

$$- P_i = \frac{P_o}{\eta} = \frac{2238}{0.746} = 3000 \text{ KW} \quad (3\phi \text{ power})$$

$$P_{in} = \sqrt{3} V_L I_L \cos \phi$$

$$3000 = \sqrt{3} \times 6600 \times I_L \cos(0.8) \Rightarrow I_L = 328 \angle \cos^{-1} 0.8 = 328 \angle 36.9^\circ \text{ A}$$

$$E_{mf} = V_T - I_a (11 j)$$

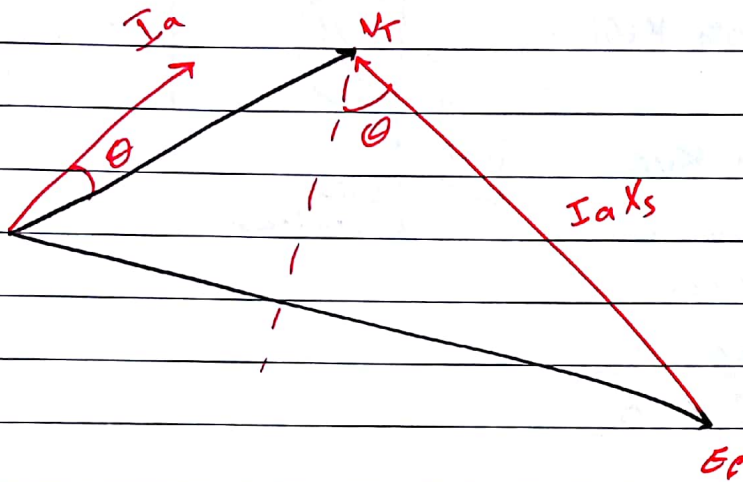
$$= \frac{6600}{\sqrt{3}} \angle 0^\circ - (328 \angle 36.9^\circ)(11 j) = 1 \angle 1 \text{ L}$$

$$\Rightarrow E_{mf} = 6636.8 \angle -26.8^\circ$$



Real power

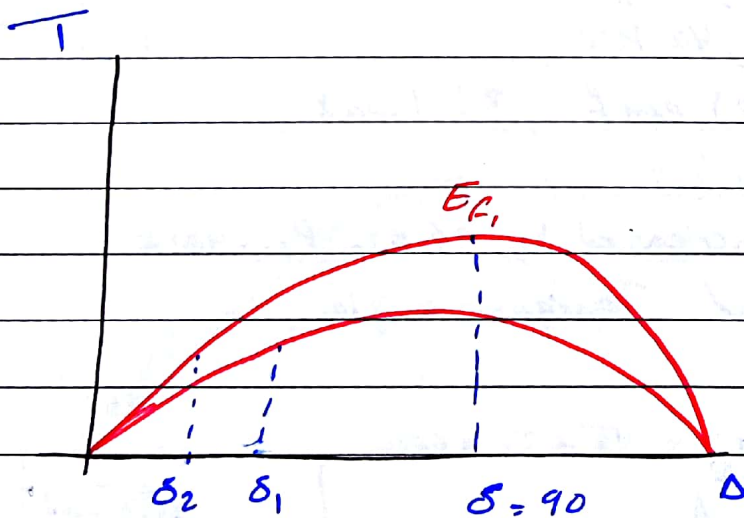
$$P = 3 V_T I_a \cos \theta$$



$$E_f \sin \delta = I_a X_s \cos \theta$$

$$I_a \cos \theta = E_f \sin \delta$$

$$P = \frac{3 V_T E_f \sin \delta}{X_s}, \quad P = \frac{3 V_T E_f \sin \delta}{X_s}$$



$$T = \frac{P}{\omega_s}$$

$$T = \frac{3 V_T E_f \sin \delta}{X_s \omega_s}$$



24/Dec/2017

Sunday

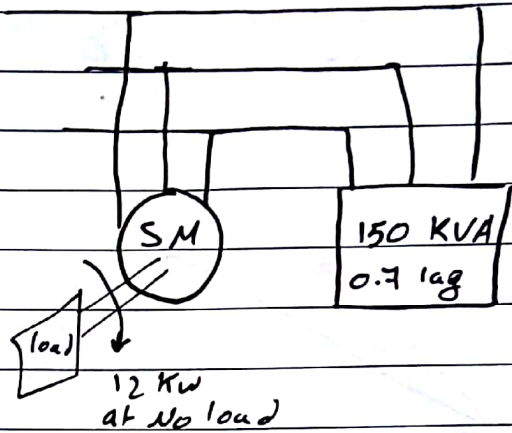
Ex) Calculate the (SM) rating to bring the overall P.F to unity

$$P = 150 \times 0.7 = 105 \text{ KW}$$

$$Q = \sqrt{(150)^2 - (105)^2} = 107 \text{ KVA}$$

$$SM / \text{output} = 12 \text{ KW} - j107 \text{ KVA}$$

injecting Reactive Power



at unity P.F  $\rightarrow Q = 0$

$$\Rightarrow Q_{sm} - Q_{load} = 0$$

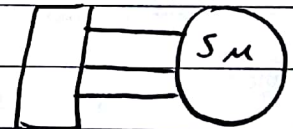
$$|SM|_{rating} = \sqrt{(12)^2 + (107)^2} = 107.67 \text{ KVA}$$

$\rightarrow$  The SM works 0.5 capacitor "condenser"

Ex)  $X_s = 2 \Omega$  / Phase

6600 V, 50 Hz, 6 poles

0.8 P.F lag, 400 KW

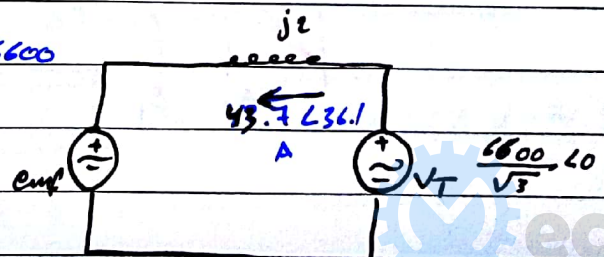


Find: 1)  $I_L$ , 2) emf, 3)  $T_{max}$

4) Emf is increased by 25%,  $P_m = 400 \text{ KW}$   
Find Torque angle

$$1) S = \sqrt{3} V_L I_L \Rightarrow \frac{400}{0.8} = \sqrt{3} \times I_L \times 6600$$

$$\Rightarrow I_L = 43.7 \angle -36.1^\circ \text{ A}$$



$$Q \quad E_{mf} = V_T - I_a (jx) = 3758.6 \text{ V}$$

$V_T$  leads

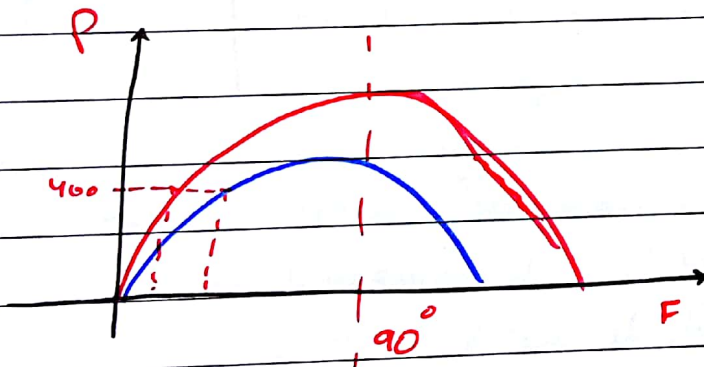
$$\frac{E_{mf}}{L} = 65$$

$$3) \quad T_{\text{max}} = \frac{3 V_T E_{mf} \sin \delta}{\omega_s X_s}$$

$$T_{\text{max}} \big|_{\delta=90} = 3 \left( \frac{6600}{\sqrt{3}} \right) * 3758 \sin 90^\circ = 265.15 \text{ kN.m}$$

$$\Rightarrow W_s = \frac{120 \text{ kJ}}{P} * \frac{\pi}{60} = \dots$$

$$4) \quad P = \frac{3 V_T E_{mf} \sin \delta}{\omega_s X_s}$$



$$P_1 = P_2 = \frac{3 V_T E_{mf1} \sin \delta_1}{\omega_s X_s} = \frac{3 V_T E_{mf2} \sin \delta_2}{X_s \omega_s}$$

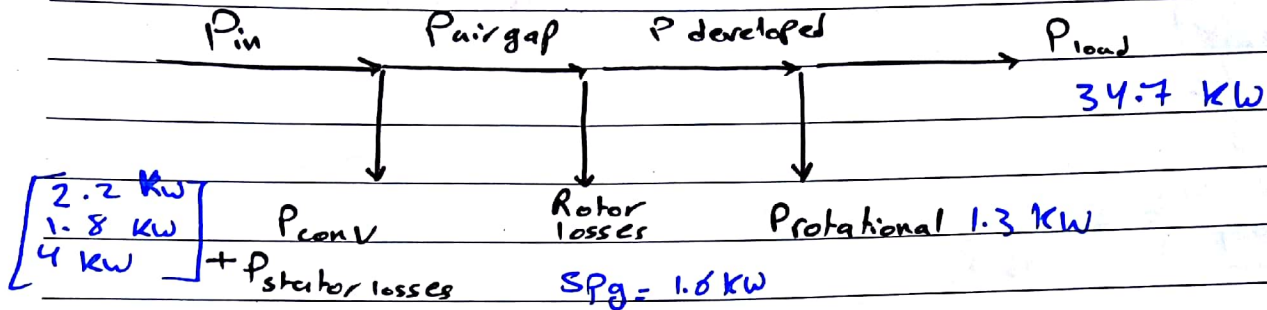
$$E_{mf1} \sin \delta_1 = E_{mf2} \sin \delta_2$$

$$\sin ( \quad ) = 1.25$$

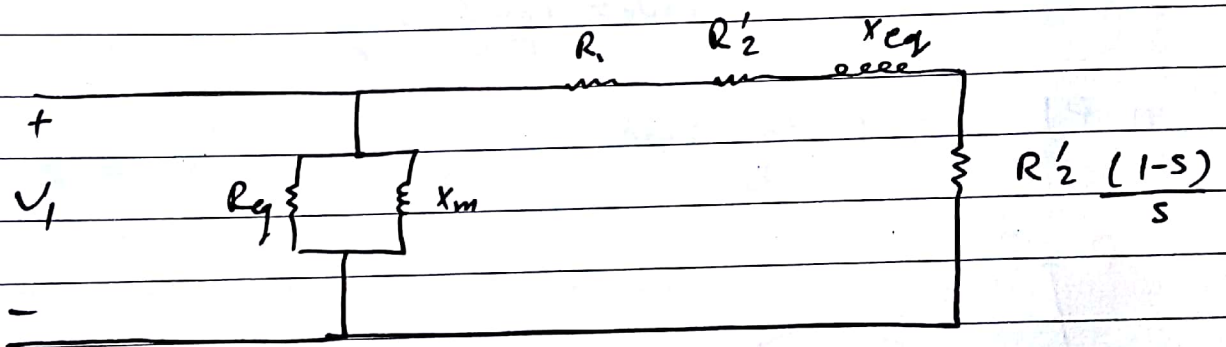
$$\delta_2 = -0.8^\circ$$



\*  $I_m$



$$P_g = \frac{36}{1-s} = \frac{36}{1-0.043} = 37.6\ kW$$





Ex) 440 V, 50 Hz, 6 poles, 3 $\phi$ , Im

Full load  $\rightarrow$  slip = 4.3%  
 $\rightarrow$  Stator P.F. = 0.87 lagging  
 $\rightarrow$  Developed power 36 kW  
 $\rightarrow$  Stator copper losses  
 $\rightarrow$  P<sub>core</sub> = 2.2 kW  
 $\rightarrow$  ~~Friction~~ Friction = 1.3 kW

Find : 1) N<sub>s</sub> (Synchronous speed)  $\Rightarrow N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$   
2) Full load motor speed  
 $\Rightarrow N_m = N_s(1-s) = 1000 \times (1-0.043)$   
 $= 957 \text{ rpm}$

Ex) 3 $\phi$ , 480 V, 12 pole, Y connection, R<sub>1</sub> = 1  $\Omega$   
R'<sub>2</sub> = 0.5  $\Omega$ , X<sub>eq</sub> = 10  $\Omega$ , X<sub>m</sub> = 100  $\Omega$

Find 8) Starting Torque

- 2) Torque at full load at 100% slip
- 3) Motor speed at full load
- 4) slip / max torque
- 5) T<sub>max</sub>

$$T = \frac{3 V_1^2 R_2'}{5 \omega_s [(R_1 + \frac{R_2'}{s})^2 + X_{eq}^2]}$$

$$V_1 = 480 \sqrt{3}, \omega_s = \frac{120f}{P} \times \frac{2\pi}{60}$$



\* Starting  $S=1 \Rightarrow T_{ST} = 179 \text{ N.m}$

\* Full load  $S=0.01 \Rightarrow T_{FL} = 67.9 \text{ N.m}$

\* Motor speed at full load  $= N_s (1 - S_{FL})$

$$\frac{\text{Slip}}{\text{max torque}} = \frac{R_2'}{\sqrt{R_1^2 + X_{eq}^2}}$$