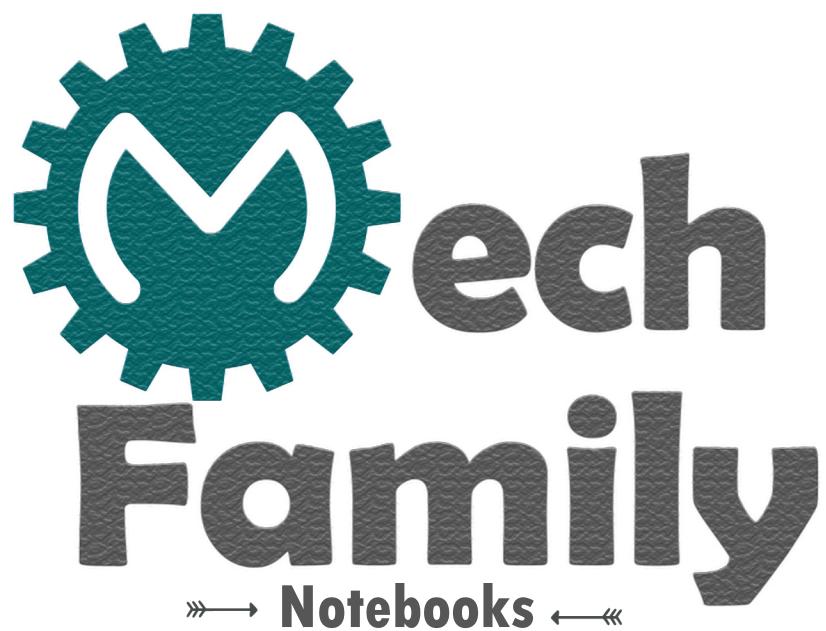


# **Machines**

**Dr. Sahban Naser**

**1st Semester 2017**

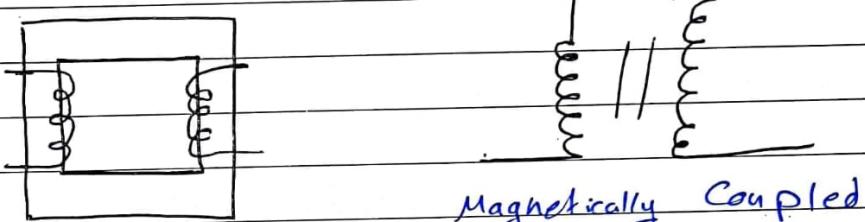


25/9/2017

Machines

→ Motor (electrical  $\rightarrow$  Mechanical)  $\rightarrow$  Magnetic field  
Generator (mechanical  $\rightarrow$  electrical)  
Transformer (converts AC voltage from one level to another)

Transformers



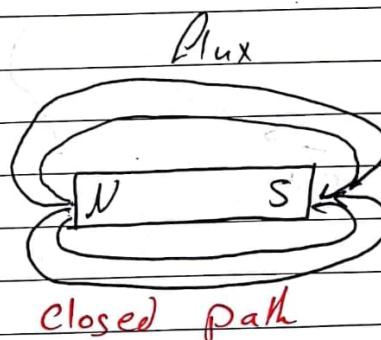
- Magnetic field

\* Permanent magnet

- Flux "lines"  $\phi$  web

- Flux density  $B$   $\frac{\text{Web}}{\text{m}^2} = \frac{\phi}{\text{A}}$

- Flux intensity ( $H$ ) =  $\left(\frac{\text{A}}{\text{m}}\right)$

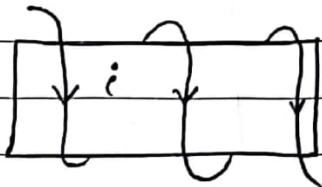


## Electro magnet

$$i \rightarrow \phi$$

التيار  $i$  يولد میدفون  $\phi$

$N$  Turns



\* Maxwell's Cork Screw rule

\* Right-handed Cork screw  $\rightarrow i$

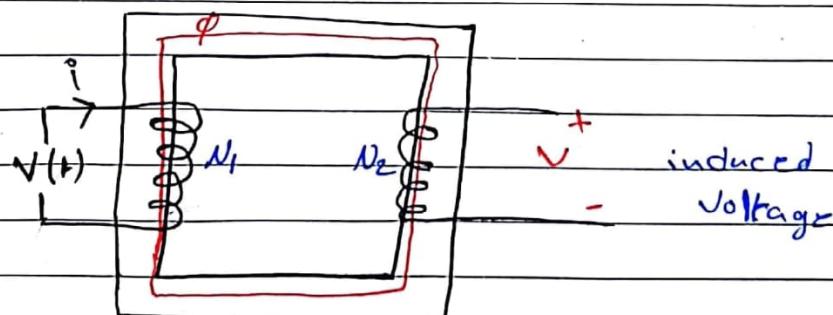
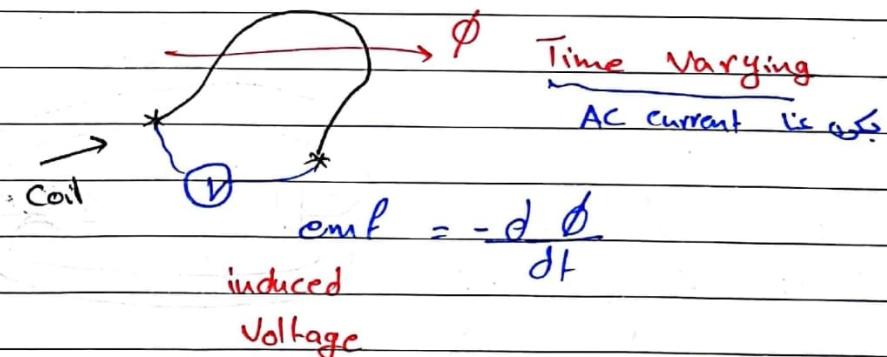
تحتوى اتجاه التيار

\* Thum b  $\rightarrow \phi$

## -Principles of magnetic field

\* A current carrying conductor produces a magnetic field

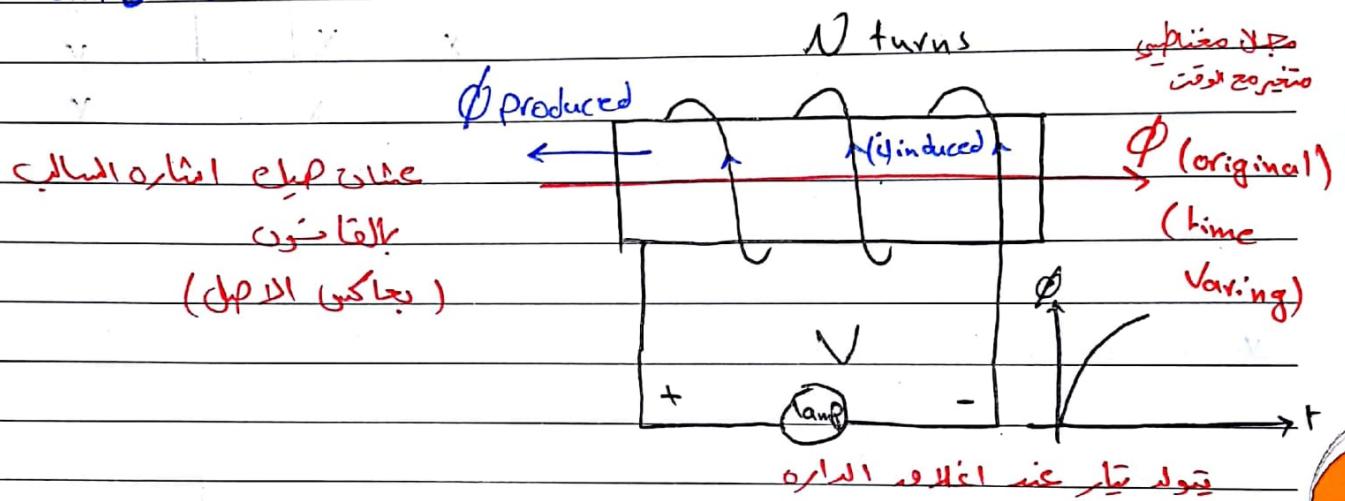
\* Transformers actions Farady's law



$$\text{emf} = -\frac{d\phi}{dt} \quad \text{For 1 turn}$$

$$\text{emf} = -N \frac{d\phi}{dt} \quad \text{For } N \text{ turns}$$

- Lent's law:

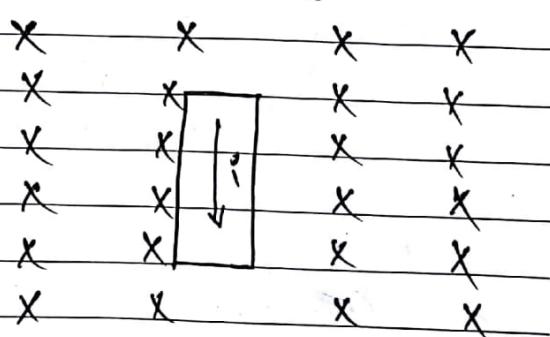


## Motor Action (Force on a wire)

## ✓ Magnetic Field

~~X~~ → in

0 → out  $\uparrow$  Direction



$$F = I \times B$$

↑ ↑ ↑

الثمار cross product  $\times$

## اتجاه الابهام

## اتجاه اول صبايج

لهم اكثرنا في حملة الارض

الآن يهتمون

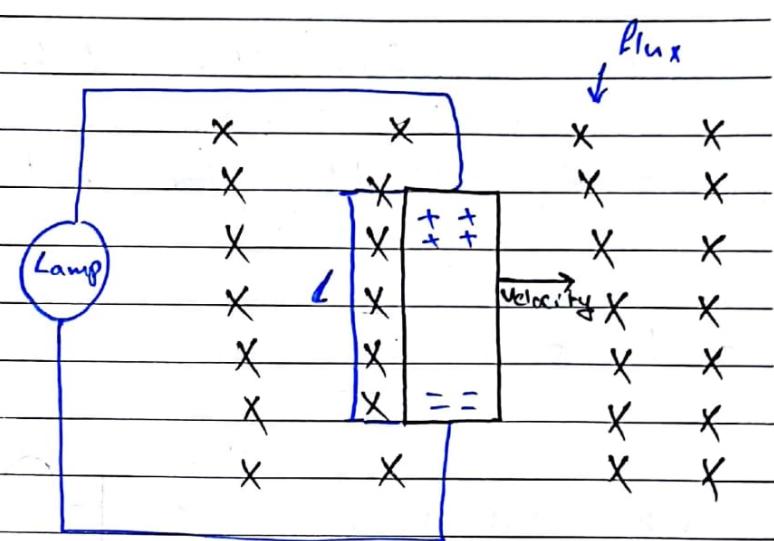
input is current  $\rightarrow$  Flux

## Output Force Movement

## Generator action

input: velocity, Flux

$$\text{output: } e = (\vec{V} \times \vec{B}) L$$



## \* Magnetic circuits

### - Ampers law

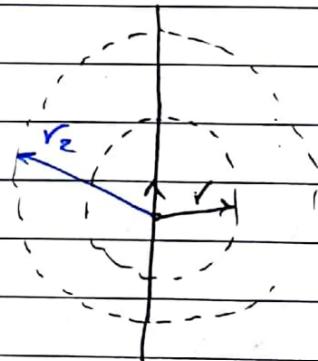
closed path  $\oint H dL = I_{\text{net}}$   
(Line integral)

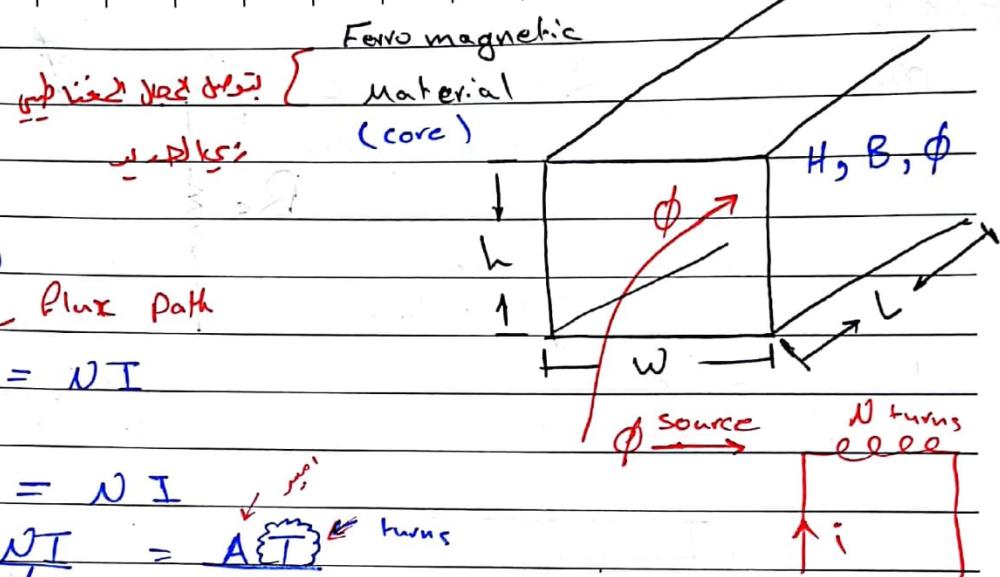
$$\Rightarrow H (2\pi r) = I_{\text{net}}$$

$$\Rightarrow H = \frac{I_{\text{net}}}{2\pi r} = \frac{A}{m}$$

فهي تتناسب مع المسافة

$$H = \frac{I}{2\pi r} = \frac{I}{2\pi r_1}$$





web/m<sup>2</sup>  
Tesla

$$B = \mu H \leftarrow \frac{AT}{m}$$

Permeability  $\left( \frac{H}{m} \right)$

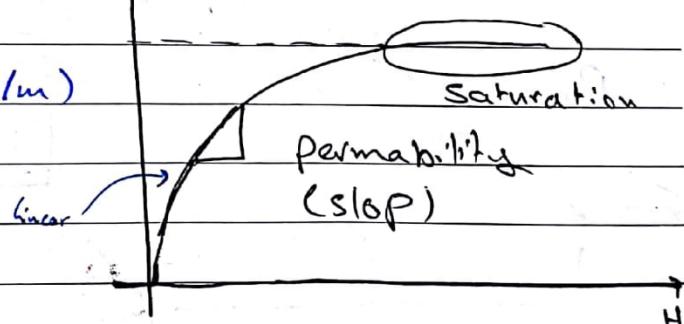
Material

$E \rightarrow \square \square$	$G \rightarrow \square \square \square \square$
$H \rightarrow \square \square \square \square$	

$$\mu = \mu_r \mu_0 \leftarrow \text{Free space} \quad (4\pi \times 10^{-7} \text{ H/m})$$

relative

B-H curve



$$\Phi = BA = H A$$

$$\Phi = \frac{NI}{L} A \quad \text{Web}$$

Magnetic motive force

$$\Phi = \frac{NI}{L} = \frac{F}{R}$$

: MA

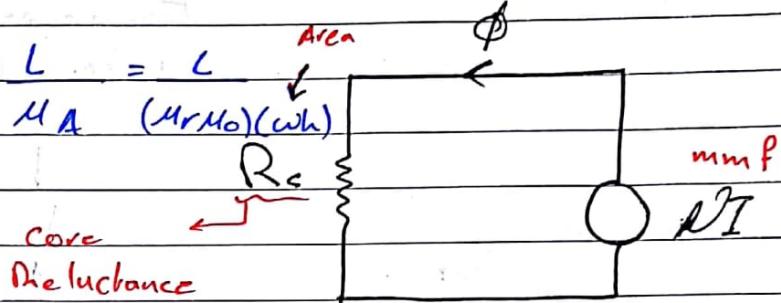
Reluctance

area

27/9/2017

← Cont.

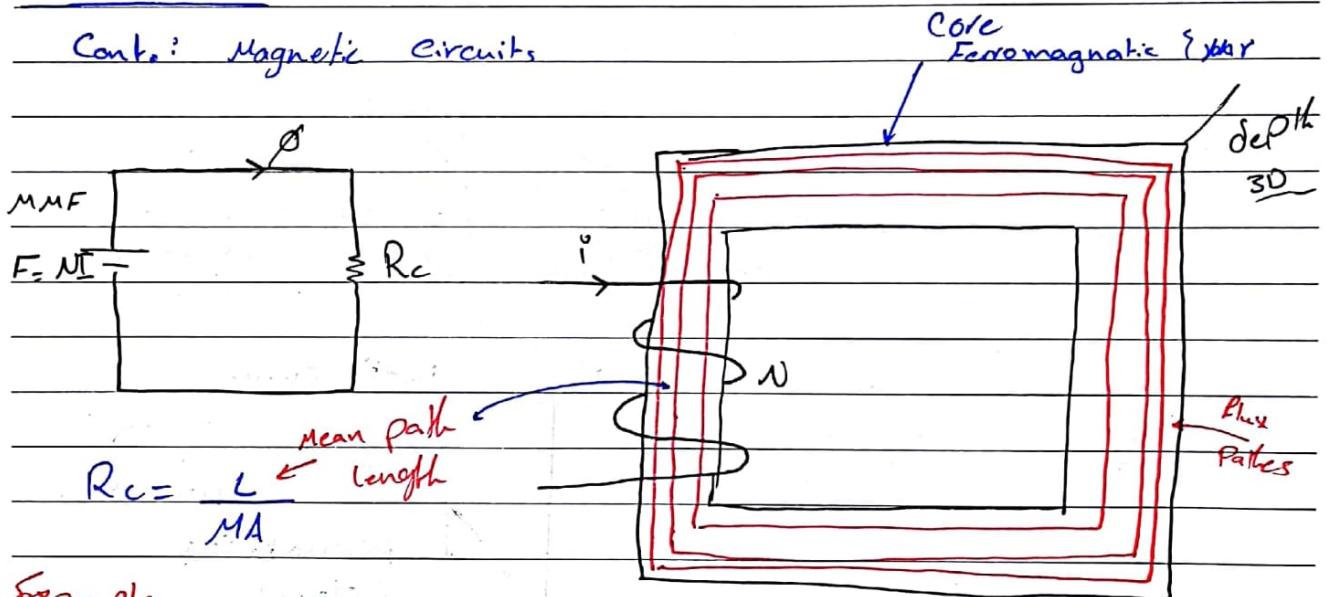
$$\phi = \frac{NI}{R} \quad , \quad R = \frac{L}{M_A} = \frac{L}{(MrMo)(wh)} \quad \text{Area}$$



$$R = \frac{NI}{\emptyset} = \frac{A \cdot I}{wb}$$

$$i = \frac{V}{R}$$

## Contd.: Magnetic Circuits



## Example

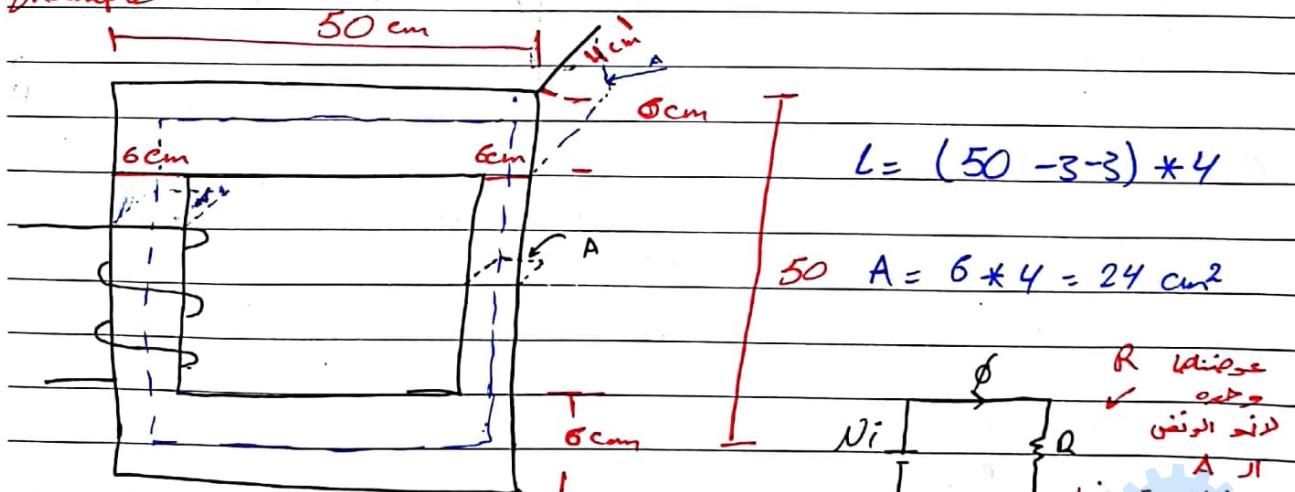
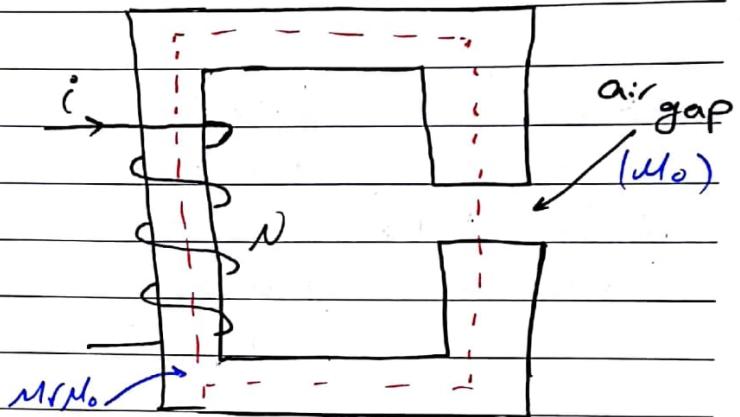


Diagram of a series circuit with a resistor  $R$  and an inductor  $L$ . The current flows through the loop in a clockwise direction. The voltage across the inductor is labeled as the voltage drop across it.

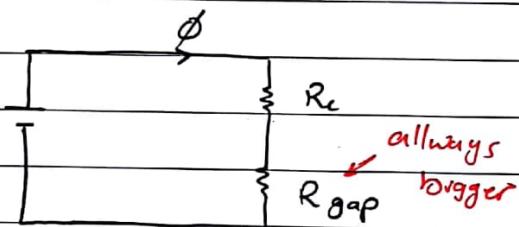
## Magnetic Circuits with an air gap

Mean path  $l_c$

Area core  $A_c$



$$R = \frac{l}{M₀A}$$



$$\oint H \cdot dL = NI$$

$$NI = H_c^{\text{core}} l_c + H_g^{\text{gap}} l_g$$

$$NI = \frac{B_c l_c}{M_c} + \frac{B_g l_g}{M_g}$$

$$\Phi = B_c A_c \leftrightarrow B_g A_g$$

$$B = \mu A$$

$$H = \frac{B}{\mu}$$

مختلط دو دو

$$B_g = \frac{\Phi}{A_c} \Rightarrow B_g = \frac{\Phi}{A_g}$$

نیتانی A دل نیسانی Flux x

یاری Flux intensity

$$NI = \frac{\Phi l_c}{M_c A_c} + \frac{\Phi l_g}{M_g A_g}$$

$$NI = \Phi \left( \frac{l_c}{M_c A_c} + \frac{l_g}{M_g A_g} \right) \Rightarrow NI = \Phi (R_c + R_g)$$

## \* Approximation

- Mean path length
- Leakage flux

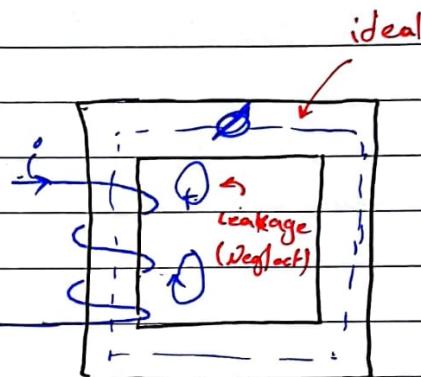
- Fringing

کم جزو کس

سینی A نمی

$$A = (a + l_g)(b + l_g)$$

Fringing نیز باید ایجاد شود -  
ideal بنحو



Leakage

Fringing

$l_g$

Effective  
Area of  
Air gap

$$A = 16 \text{ cm}^2$$

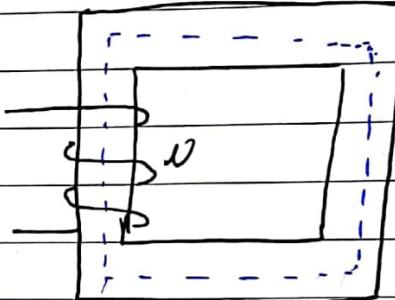
$$L_c = 40 \text{ cm} \quad \leftarrow \text{mean length}$$

$$N = 350 \text{ turns}$$

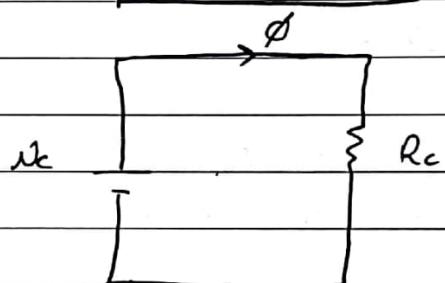
$$M_r = 50000$$

$$B = 1.5 \text{ T}$$

Find current in the coil



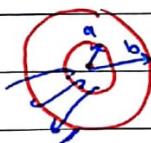
$$R_c = \frac{L}{\mu A} = \frac{40 \times 10^{-2}}{(50000 \times \mu_0)(16 \times 10^{-4})} = 4 \times 10^3 \text{ ohms}$$



$$R_c = 3979 \text{ ohms}$$

$$\phi = 1.5 \times 16 \times 10^{-4} \text{ wb}$$

$$N \phi i = \phi R \Rightarrow i = \frac{\phi R}{N} = \frac{1.5 \times 16 \times 10^{-4} \times 3979}{350} = 27.3 \text{ mA}$$



$$\text{إذا كانت دائرة متاخدة (L) بمسافة } (2\pi r)$$

$5 \times 8$

$$N = 350$$

$$i = 1.2 \text{ A}$$

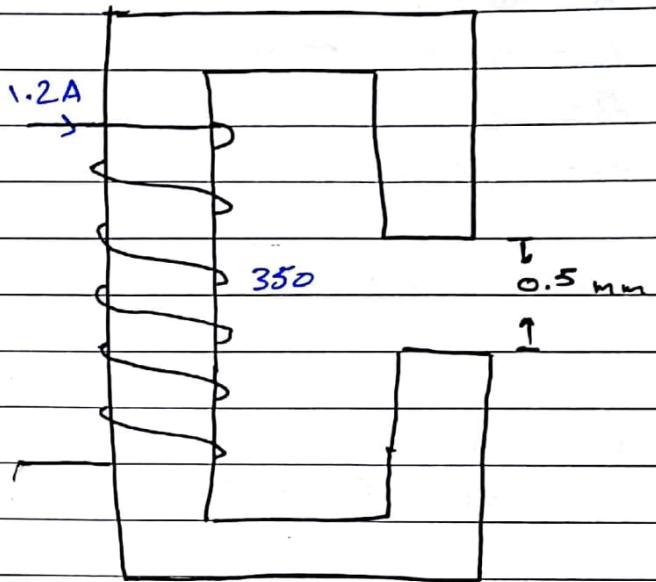
$$A_c = 16 \text{ cm}^2$$

$$L_c = 40 \text{ cm}$$

$$\mu_r = 50000$$

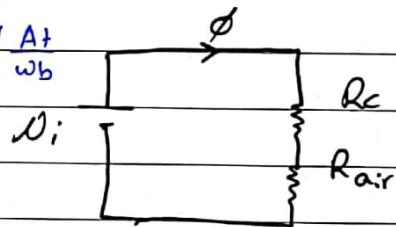
Find flux density

in the core and air gap



ما اختلف كثير عن المقابل

$$R_c = \frac{40 \times 10^{-2} - 0.5 \times 10^{-3}}{50000 \text{ M}_0 (16 \times 10^{-4})} = 3974 \text{ At}$$



$$R_g = \frac{0.5 \times 10^{-3}}{M_0 (16 \times 10^{-4})} = 248680 \frac{\text{At}}{w_b}$$

$$\phi = \frac{N_i}{R_c + R_{airgap}} = 1.66 \times 10^{-3} \text{ wb}$$

$$B = \frac{1.66 \times 10^{-3}}{16 \times 10^{-4}} = 1.04 \text{ T}$$

Ex: Find  $\phi$

$$M_r = 2000$$

$$N = 1000$$

$$A_{eff} i = 1A$$

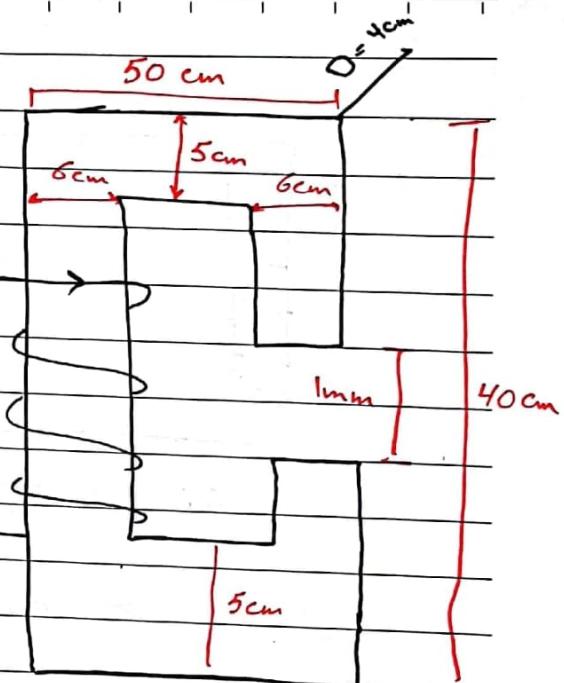
$$A \Rightarrow A_h = 5 \text{ cm} \times 4 \text{ cm} = 20 \text{ cm}^2$$

$$A_v = 6 \text{ cm} \times 4 \text{ cm} = 24 \text{ cm}^2$$

$$A_g = 6 \text{ cm} \times 4 \text{ cm} = 24 \text{ cm}^2$$

$$R = 2$$

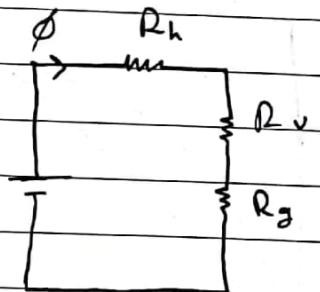
$$M(A) \leftarrow \text{add to}$$



$$L_h = 44 \times 2 = 88 \text{ cm}$$

$$L_v = 70 \text{ cm}$$

$$L_g = 1 \text{ mm}$$



$$R_h = 700 \frac{\text{m}}{\text{A}}$$

$$R_h = \frac{88 \times 10^{-2}}{2000 \mu_0 \times 10^{-4}} = 175070.4$$

$$R_v = 464000 \frac{\text{A}}{\text{m}}$$

$$R_v = \frac{70 \times 10^{-2}}{2000 \times 10^{-4} \times 24 \times 10^{-4}} = 116050.5$$

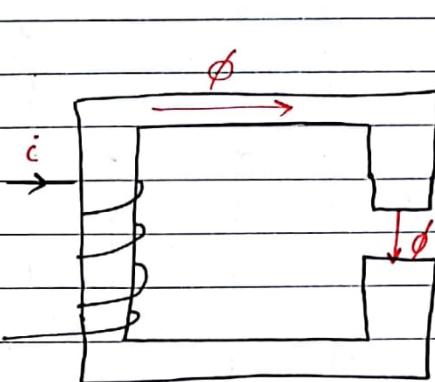
$$R_g = 331000 \frac{\text{A}}{\text{m}}$$

$$R_g = \frac{1 \times 10^{-3}}{\mu_0 \times 10^{-4}} = 331572.8$$

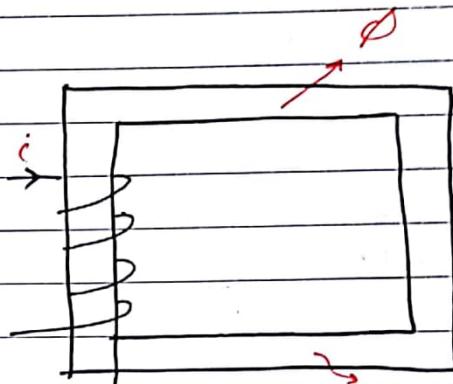
$$\phi = \frac{NI}{\Sigma R} = 7 \times 10^{-3} \text{ wb}$$

$$\phi = \frac{1000 \times 1}{R_{total}} = 1.6 \times 10^{-3} \text{ wb}$$

(21/10/2017)



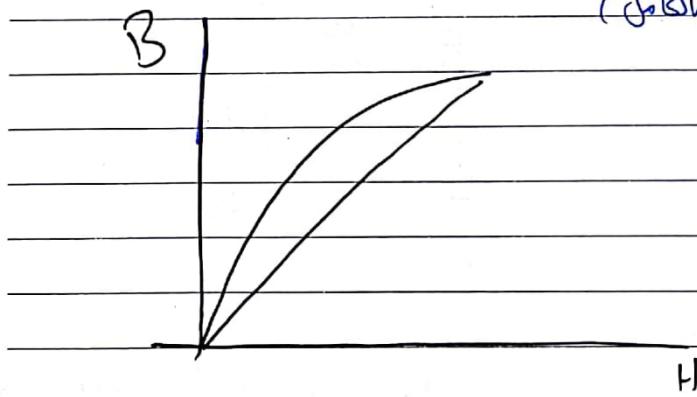
with air gap



without air gap

توضيح "Air gaps" حتى لا ي saturation ( saturation ) يتحقق في جميع المكونات

(عثمان ماجد عاصم الـ (Bakar) pizza core (يتاخر))



$\phi R$

$$BA = \frac{L}{MA} H$$

2 materials

fringing "بسبور" Argaps (ذيل) يدخلون داخلا

التيار :  $\phi$

$$\frac{BL}{M} = HL$$

$$\text{Ampers law} : \oint H \cdot dL = NI$$

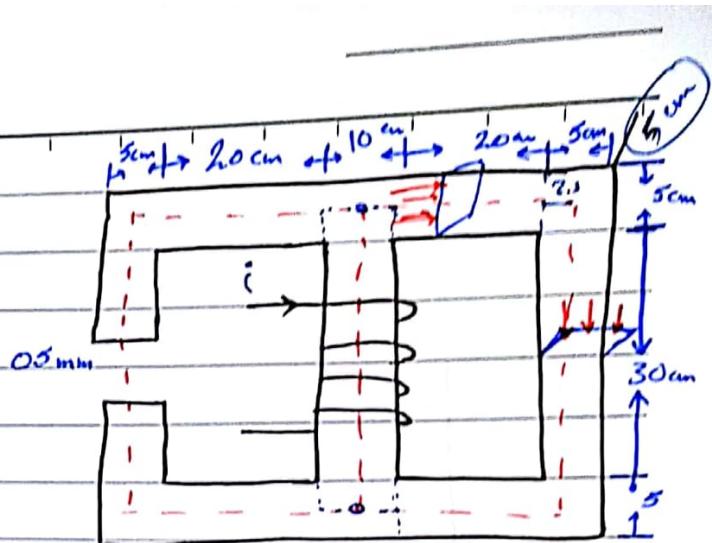
Ex)

$$\underline{M_r = 2000}$$

$$N_{\text{eff}} = 1000$$

$$\phi_g = \ln w_b$$

$$e_P = ?$$



## Solutions

$$R_1 = \frac{61}{144} = \frac{30 + 2.5 + 2.5}{2000 \text{ N/m} (10 \times 5) \times 10^{-4}} \text{ cm}$$

$$R_2 = \frac{L_2}{4A} = \frac{(5+20+2.5) \times 2 + (2.5 + 30 + 25) \times 2}{2000 \text{ H}_0 (5 \times 5) \times 10^{-4}}$$

$$= 1432.4$$

$$R_3 = \frac{(5+20+25) * 2 + (25 + 30 + 25)) \text{ cm}}{2000 \text{ N} \cdot (5\text{cm} * 5\text{cm})} - 0.5 \text{ mm}$$

$$= 1431.6$$

$$F_{ab} = \phi_1 (R_B + R_g) \quad , \quad \phi_2 = F_{ab} / R_2$$

$$\phi = \phi_1 + \phi_2 = 3.1 \text{ mwb}$$

$$-N_i + \phi Q_i + F_{ab} = 0$$

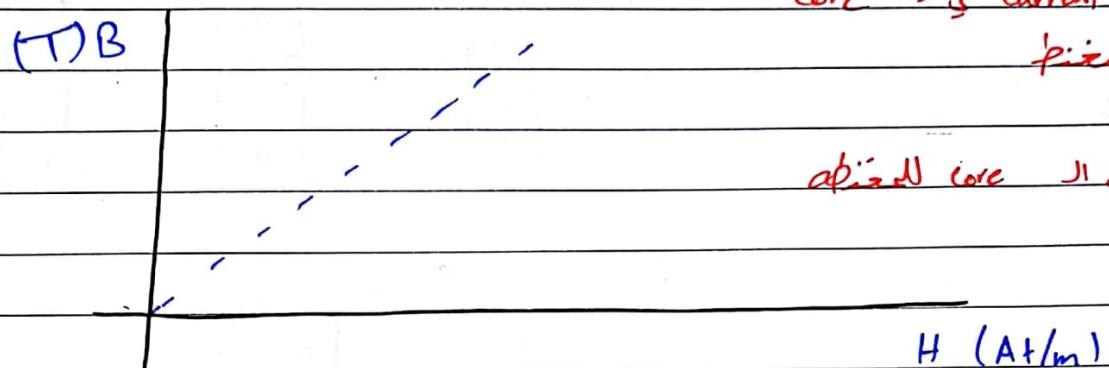
$$-N_i + \phi Q_i + F_{ab} = 6$$

$$i = 0.38 A$$

## Magnetic behavior of Ferro magnetic material:

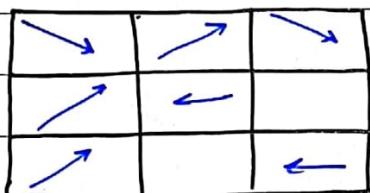
core  $\rightarrow$  current  $\parallel$  میدان مغناطیسی  
پیوسته باشد

آبیزدیل کوئری  $\parallel$  میدان  $\Rightarrow H$



Random

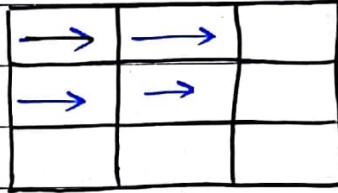
$H=0$



(a)

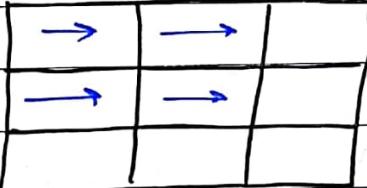
alignment

$H=H_1$



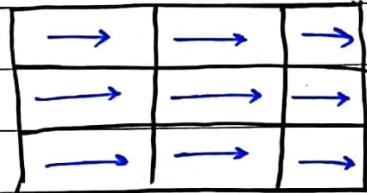
(b)

$H=H_2$



(c)

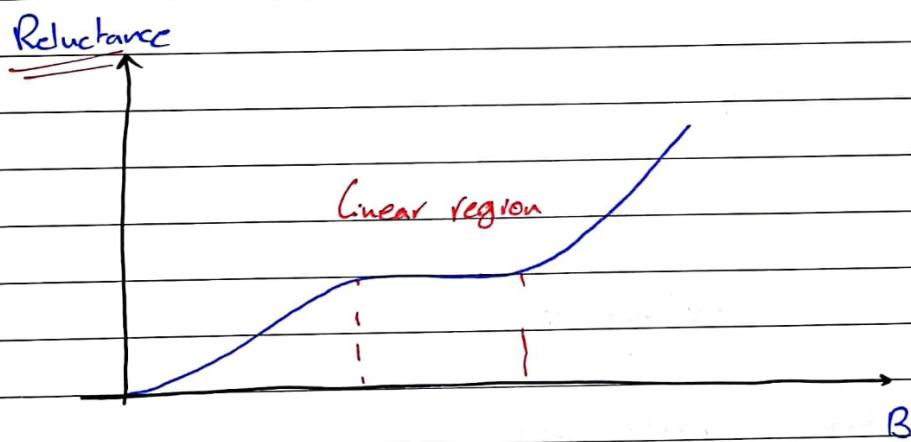
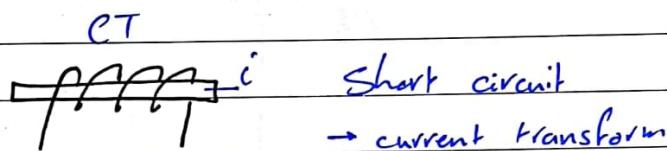
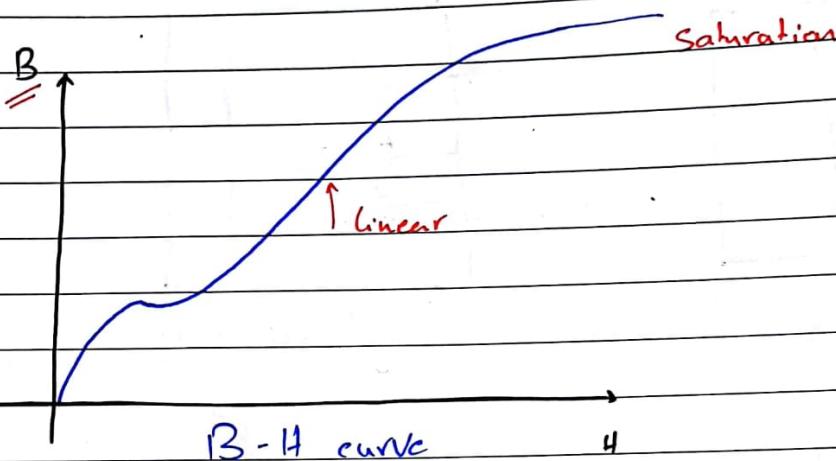
$H=H_3$



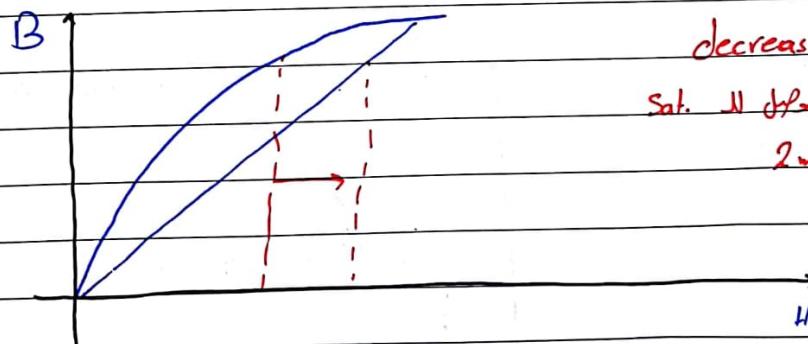
(d)

$B_2 > B_1, H_2 > H_1$

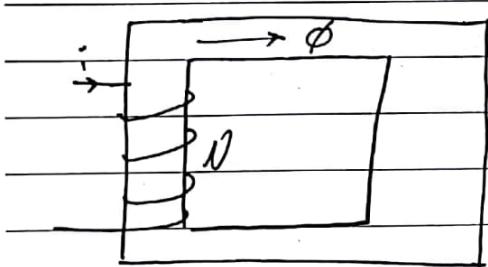
No change in  $B$   
even with increasing  $H$



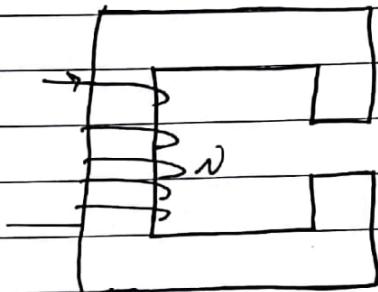
Q) How to delay saturation?



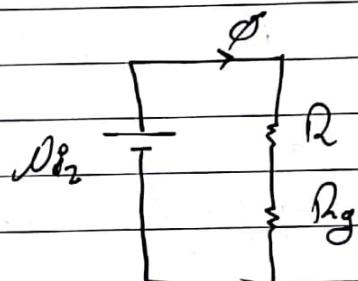
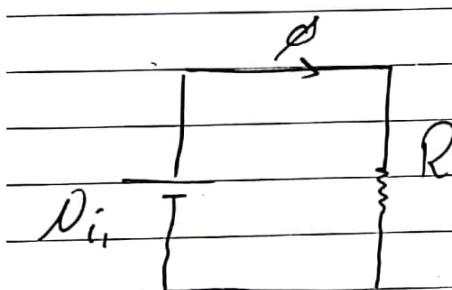
decrease  $M \rightarrow$  increase  $R_c$   
 Sat.  $M \downarrow \rightarrow$  refl. delay curve  $M \downarrow \rightarrow$   
 2mA  $\downarrow \rightarrow$  3mA  $\downarrow \rightarrow$   
 (Delay) airgap



without air gap



with air gap



assume at linear region

Prob: equation

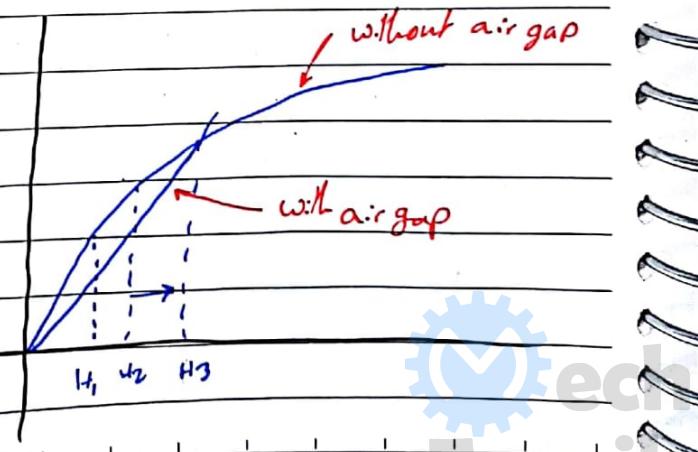
$$\Phi = (N_1) / R \quad \text{without air gap}$$

$$\Phi = (N_1) / (R + R_g) \quad \text{with air gap}$$

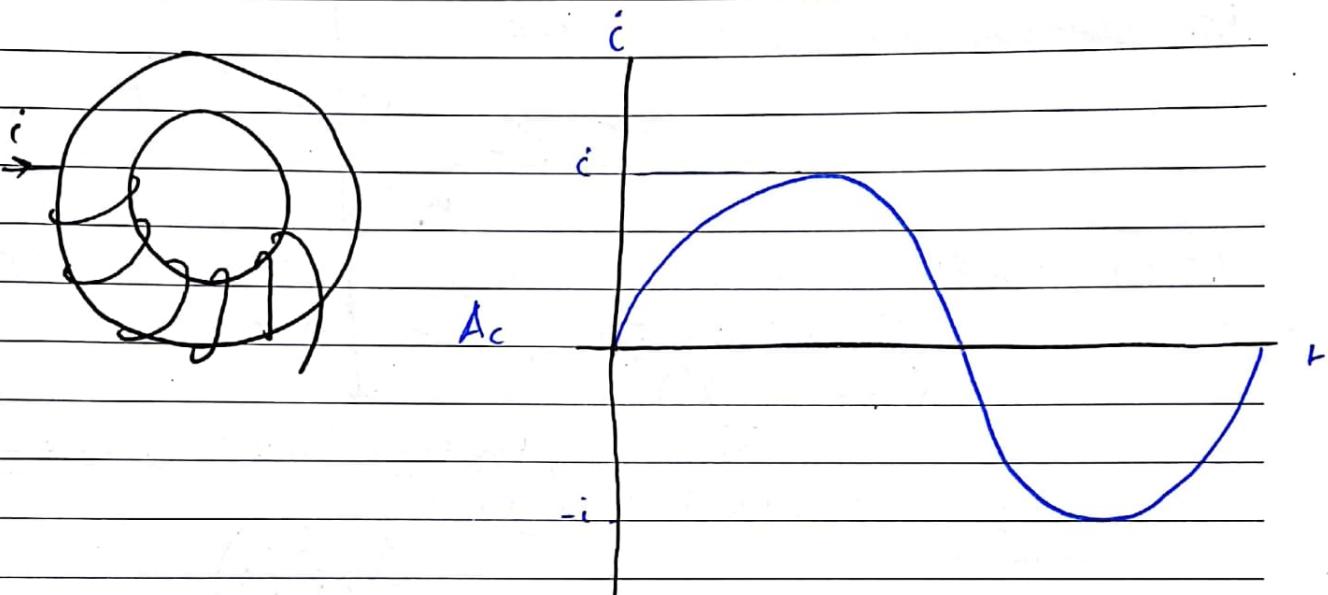
$$\frac{N_1}{R} = \frac{N_1}{R + R_g}$$

$$\frac{i_2}{i_1} = \frac{R + R_g}{R} > 1$$

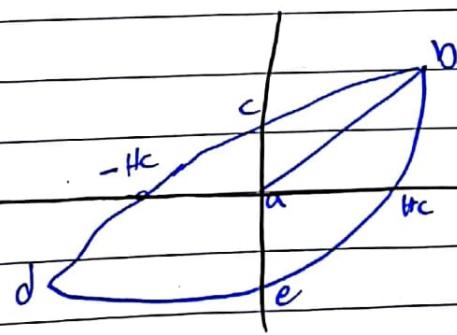
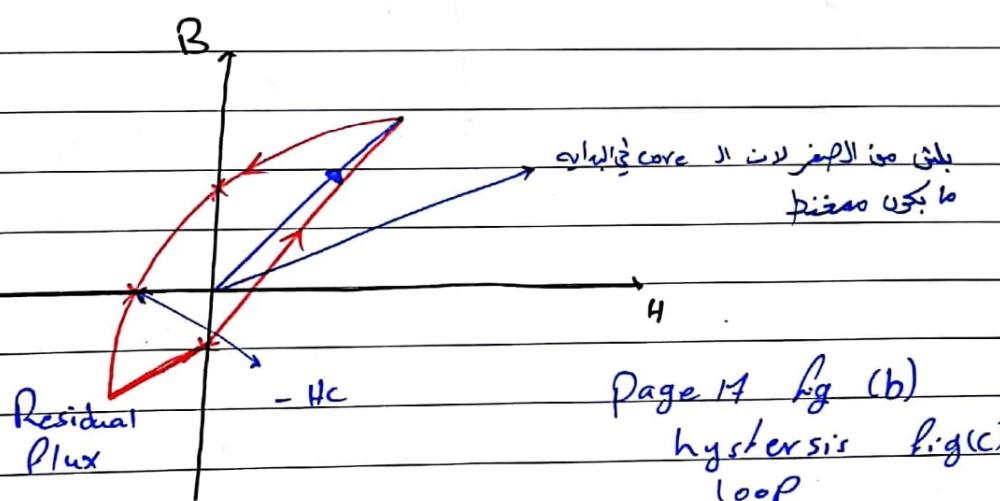
$$\frac{i_2}{i_1} = 1 + \frac{R_g}{R}$$



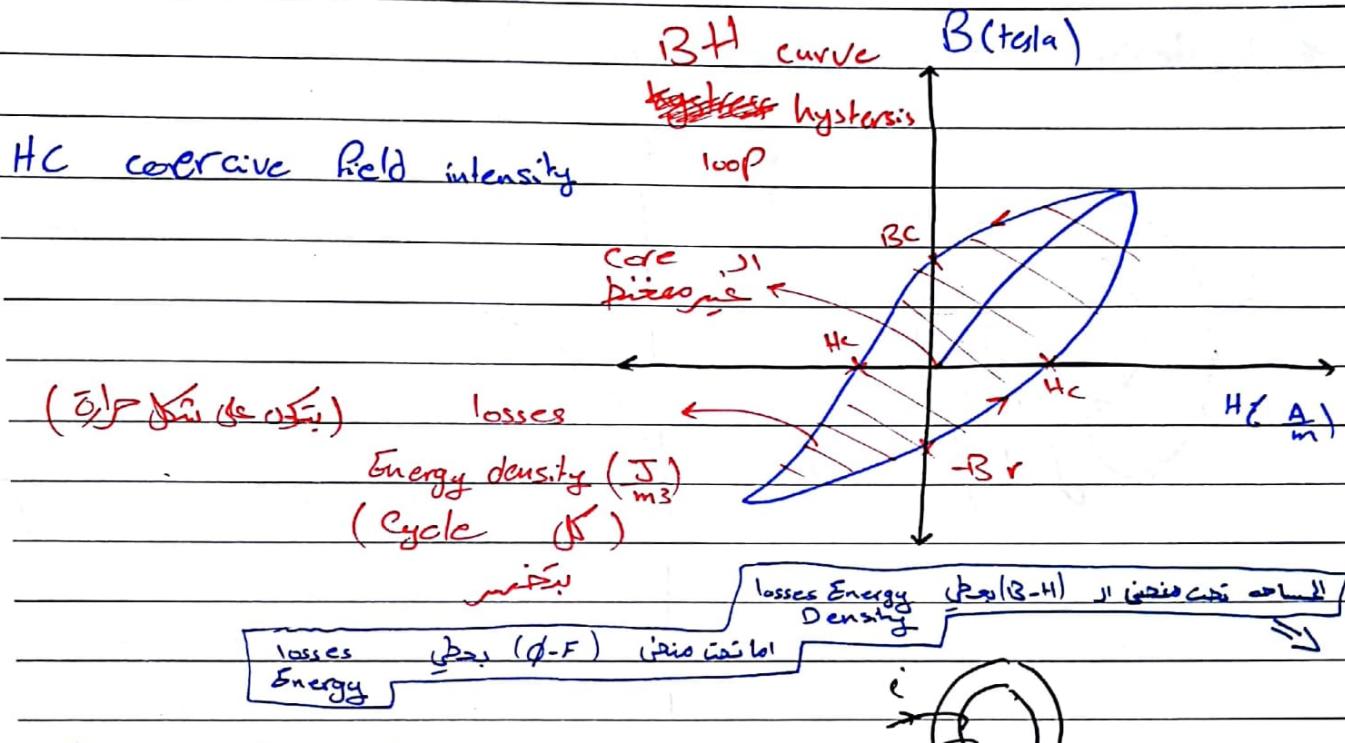
\* hysteresis loss and eddy current loss



عند إزالة التيار فإن المغناطيسية تأخذ قيمتين ازالة المغناطيسية



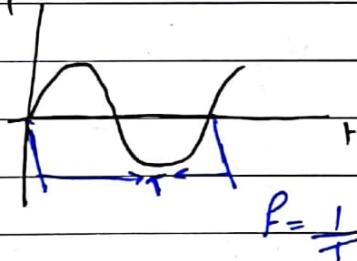
٤/١٠/٢٠١٧



$$= (\text{Area } (B-H) * V_{core} * P)$$

$\frac{1}{T}$

Volume



Rev.

$$F = cB L$$

$$\Rightarrow B = \frac{N}{A \cdot m} \Rightarrow \frac{J}{A \cdot m^2}$$

$$F \cdot \text{distance} = \omega$$

$$N \cdot m = J$$

$$N = \frac{J}{m}$$

$$H = \frac{A}{m} \Rightarrow H \cdot B \Rightarrow \frac{A}{m} \cdot \frac{J}{A \cdot m^2} = \frac{J}{m^3}$$

losses (Area under the curve)

General electric company!

GE  $\Rightarrow$  Approximation

ما انتو ايجاد المساحة تحت المنحنى

ضرر انتظاري تقدير المساحة فاحدوا على انتظاري

$$\text{Power} = \frac{KH}{T} * (B_{\max})^n * P = \frac{W}{ms}$$

density  $\frac{W}{T}$   $\hookrightarrow$  max flux density

\* Area B-H  
 $KH * (B_{\max})^n$

(for losses) Constant (material)

Area  $\phi F$

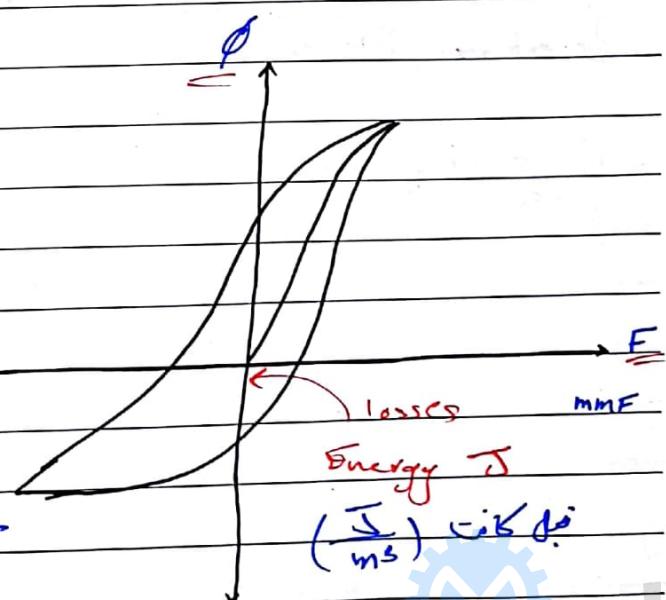
$$e = \frac{d\phi}{dt} * \omega \Rightarrow \phi = \frac{V \cdot \text{sec}}{\text{Turn}}$$

$$\hookrightarrow \text{voltage} = N \frac{\Delta \phi}{\Delta t}$$

$$F = A \cdot \text{Turn}$$

$$\Rightarrow \phi F =$$

$$A \cdot \text{Turn} \cdot \frac{V \cdot \text{sec}}{\text{Turn}} = \text{Energy } J$$



\* Core losses

~~Hysteresis~~ hysteresis losses

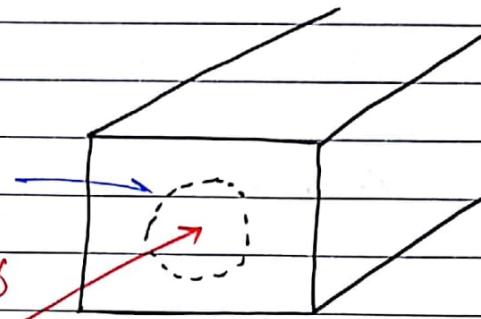
$$P_h = KH \times B_{\max}^2 \times F \quad \text{W/m}^3$$

Eddy current losses

$$P_e = K_c \cdot B_{\max}^2 \cdot F^2 \quad \text{W/m}^3$$

Materials and  
Laminations

Plux  $\phi$  will change  
with time  
(Varying)



$$\text{Losses} = (i_{\text{eddy}})^2 R$$

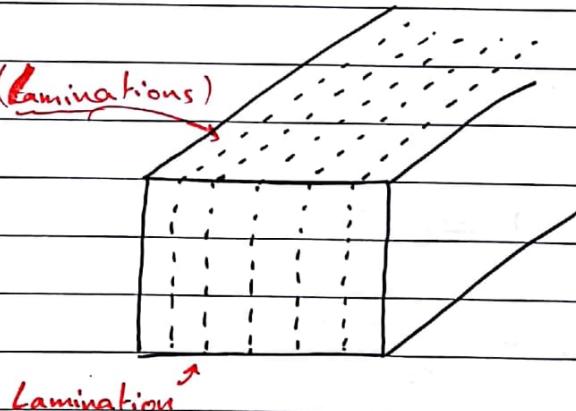
(الخالي)

(حقل التيار)

Power losses (الخالي) = (حقل التيار)  $\times$  (Laminations)

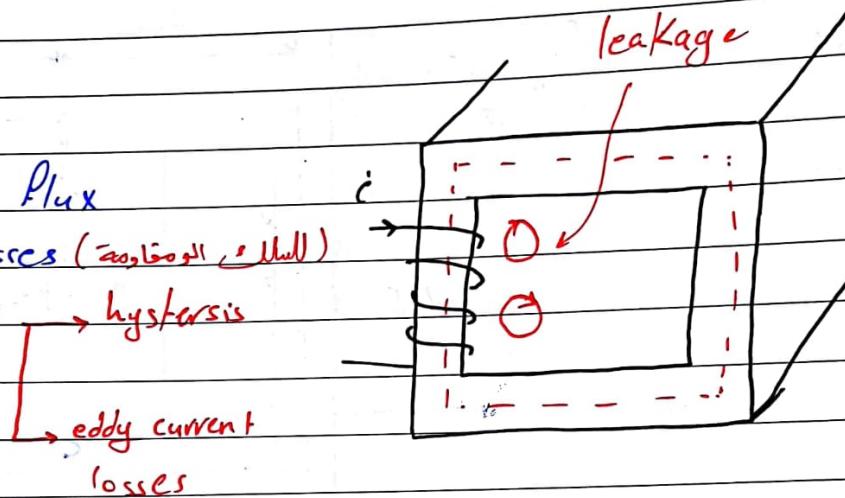
Resistive losses (اجزاء المكون)

$$P_{\text{losses}} = (i_{\text{eddy}})^2 R$$

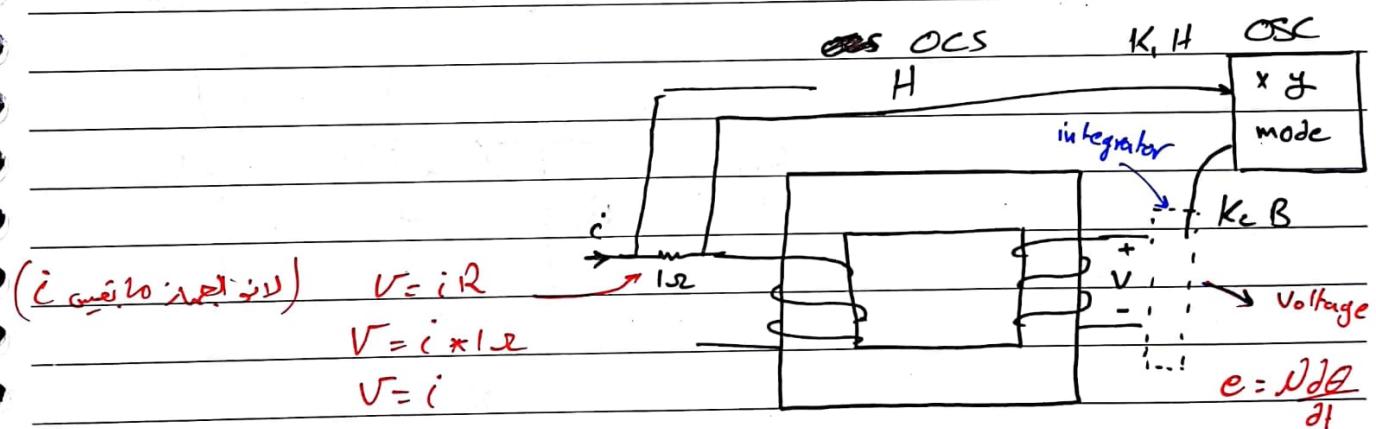


losses here

- 1) leakage Flux
- 2) copper losses (الخسارة المعدنية)
- 3) core losses  $\rightarrow$  hysteresis



measuring B-H curve of un known core ~~core~~ (بزاو ایکس (پریزیت))  
~~بزاو ایکلیپس~~

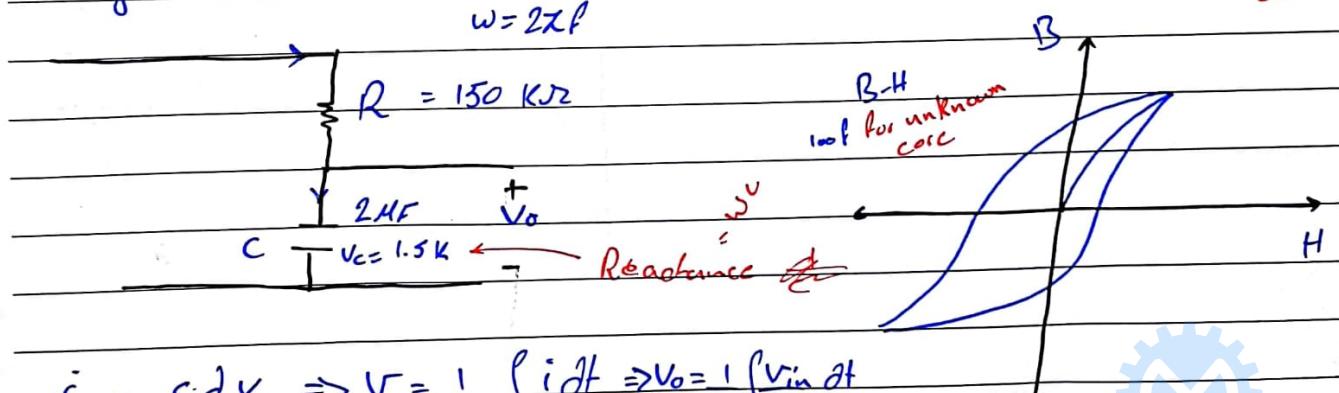


integrator

$$f = 50 \text{ Hz}$$

$$\omega = 2\pi f$$

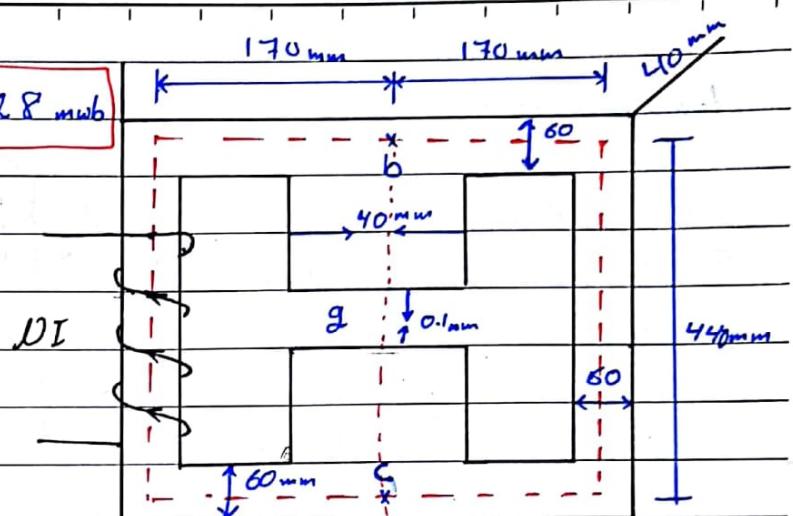
$$\Rightarrow V \propto \frac{d\theta}{dt}$$



$$i_c = \frac{C dV}{dt} \Rightarrow V = \frac{1}{C} \int i dt \Rightarrow V_o = \frac{1}{C} \int V_m dt$$

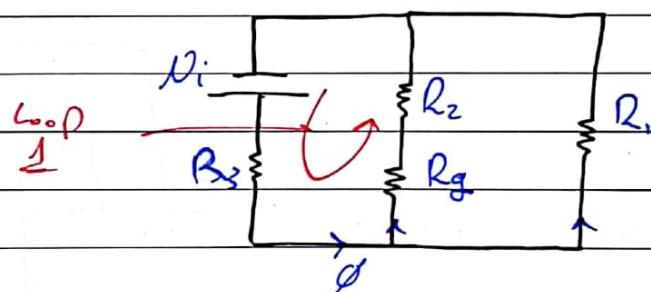
Ex: Find (I) so that  $\phi_g = 1.28 \text{ mwb}$

$$U = 200 \text{ V}$$

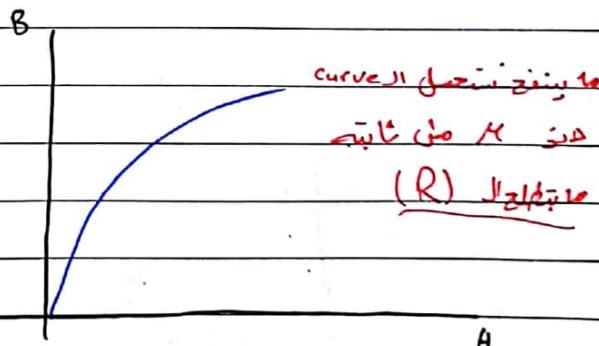


$B$ (T)	0.39	0.8	0.925	
$H$ (A/m)	363.7	500	562	

\* No Pringing  $\rightarrow$  Hc same flux density



$$\phi R \Rightarrow H_L$$



Solution %

 $\phi R_2$ , Ampere's law  $\oint H dL = I$ 

$$R_2 = L$$

 $\frac{NA}{\mu_0 A}$  given (air gap  $\approx 0.1$  mm)

$$\phi_g = \phi_{core} (R_2)$$

$$B_g = B_{core} \text{ (No fringing)}$$

$$\phi_g = 1.28 \text{ mwb}, B_g = \frac{\phi_g}{A_g}$$

$$B = \frac{\phi}{A}, H = \frac{B}{\mu}$$

$$\Rightarrow \frac{\phi_g}{A_g} = \frac{1.28}{40 \times 10^{-3} \times 40 \times 10^{-3}} = 0.8 T \quad \left| \begin{array}{l} \phi \rightarrow B \rightarrow H \\ \cancel{mwb} \end{array} \right.$$

$$B_{core} (R_2) = 0.8 T$$

$$H_{air \ gap} = \frac{B_g}{\mu_0} = 63.66 \times 10^4 \text{ AT/m}$$

$$H_{core} \Rightarrow \text{Table} \Rightarrow H_{core} = 500 \text{ AT/m}$$

loop 1

KVL

$$-N_i + H_3 L_2 + H_g L_g + H_2 L_2 = 0$$

Voltage drop

 $\phi R, H_L$ 

$$\oint H dL = NI = H_1 L_1 + H_2 L_2 + H_3 L_3 \dots$$

To find the unknown

$$\phi = \phi_1 + \phi_2$$

Total mmF For core ( $R_2$ ) and Air gap

$$\begin{aligned} &= H_g L_g + H_2 L_2 \\ &= 63.66 \times 10^4 (0.1 \times 10^{-3}) + 500 (440 - 0.1) \times 10^{-3} \\ &= 283.66 \text{ AT} \end{aligned}$$

$$- 283.66 = H_1 L_1$$

$$283.66 = H_1 \times (170 + 2 + 440) \times 10^{-3}$$

$$H_1 = 363.67 \frac{\text{AT}}{\text{m}}$$

$$B_1 = 0.389 \text{ T} \quad (\text{Depth } 40)$$

$$\phi_1 = B_1 \times A = 0.39 \times 60 \times 10^{-2} \times 40 \times 10^{-3} = 0.94 \times 10^{-3} \text{ wb}$$

$$\Rightarrow \phi_1 = \phi_1 + \phi_2 \Rightarrow \phi = 2.22 \times 10^{-3} \text{ wb}$$

$$B_3 = \frac{\phi}{A} = \frac{2.22 \times 10^{-3}}{60 \times 10^{-2} \times 40 \times 10^{-3}} = 0.92 \text{ T}$$

$$H_3 = 562.5$$

From Table

$$-N_i + H_3 L_3 + H_g L_g + H_2 L_2 = 0 \Rightarrow i = 3.61 \text{ A}$$

Ex) hysteresis and eddy current losses in a certain equipment

$$P_h = 840 \text{ W}, P_e = 642, (240 \text{ V}, 25 \text{ Hz})$$

Determine hysteresis and eddy current losses if the core is connected to a 60 Hz as to core flux density 62% of rated value. ~~Assume~~ Assume  $n = 1.4$

Sol:

$$P_h = K_h * f * B_m^n, P_e = K_e * f^2 * B_m^2$$

Recall  $V = \frac{N d\theta}{dt} = NA \frac{dB}{dt}, B = B_m \sin \omega t$  assume

$$V(t) = NA B_m \omega \cos \omega t$$

$V, B$  in phase shift  $\frac{\pi}{2}$  rad

$$V(t) = NA B_m (2\pi f) \cos \omega t$$

$$\Rightarrow V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{NA B_m (2\pi f)}{\sqrt{2}}$$

$$V_{rms} = 4.44 * f * N * A * B_{max}$$

$$V \propto f * B_{max}$$

$$\frac{V}{f} = K * B_{max}$$

$$V = K * f * B_{max}$$

Cont. Sol.

$$P_{h_1} = 846 \text{ W}, P_0 = 642 \text{ W}, [240 \text{ V}, 25 \text{ Hz}], B_{\max}$$

$$P_{h_2} = ??, P_{0_2} = ??, [V = ??, 60 \text{ Hz}], B_{\max} = 0.62 B_{\max_1}$$

$$V_1 = K \cdot f_1 \cdot B_{\max_1}$$

$$V_2 = K \cdot f_2 \cdot \underbrace{B_{\max_2}}_{0.62 \cdot B_{\max_1}}$$

$$\frac{V_1}{V_2} = \frac{f_1}{\frac{P_2}{P_1} \cdot 0.62} \Rightarrow \frac{240}{V_1} = \frac{25}{50 \cdot 0.62} \Rightarrow V_2 = \frac{240 \cdot 50 \cdot 0.62}{25}$$

$$\Rightarrow V_2 = 357 \text{ V}$$

$$P_{h_1} = K_h \cdot 25 \cdot B_{\max_1}^{1.4} = 846$$

$$P_{h_2} = K_h \cdot 60 \cdot (0.62 B_{\max_1})^{1.4} = P_{h_2}$$

$$\frac{P_{h_2}}{846} = \frac{60}{25} \cdot \frac{(0.62 B_{\max})^{1.4}}{(B_{\max})^{1.4}}$$

$$\Rightarrow P_{h_2} = 1039.75$$

$$642 = K_c * (25)^2 * B_m^2$$

$$P_{e2} = K_c * 60^2 * (0.62 B_m)^2$$

$$\Rightarrow B_{e2} = 1421.48$$

E(x) A core was connected to :

(No need for (n))

$$P_e + P_h \quad P_{core} = 500 \text{ W} @ 25 \text{ Hz}, 240 \text{ V} \quad \text{First time}$$

$$P_{core} = 1400 \text{ W} @ 50 \text{ Hz}, 480 \text{ V} \quad \text{Second time}$$

$$\text{Find } P_{e1}, P_{e2} \\ P_{h1}, P_{h2}$$

Sol:

$$P_e + P_h = 500 \quad \dots \textcircled{1}$$

$$P_{e2} + P_{h2} = 1400 \quad \dots \textcircled{2}$$

$$\frac{P_{h1}}{P_{h2}} = \frac{25}{50} \Rightarrow P_{h2} = 2P_{h1} \quad \dots \textcircled{3}$$

$$\frac{P_{e1}}{P_{e2}} = \left(\frac{25}{50}\right)^2 \Rightarrow P_{e2} = 4P_{e1} \quad \dots \textcircled{4}$$

$$P_e + P_h = 500 \\ 4P_{e1} + 2P_{h1} = 1400 \quad \boxed{\text{Solve}}$$

$$\Rightarrow P_{e1} = 200 \quad P_{h1} = 300 \\ P_{e2} = 800 \quad P_{h2} = 600$$

11/10/2017

## Transformer 8

\* Notes:

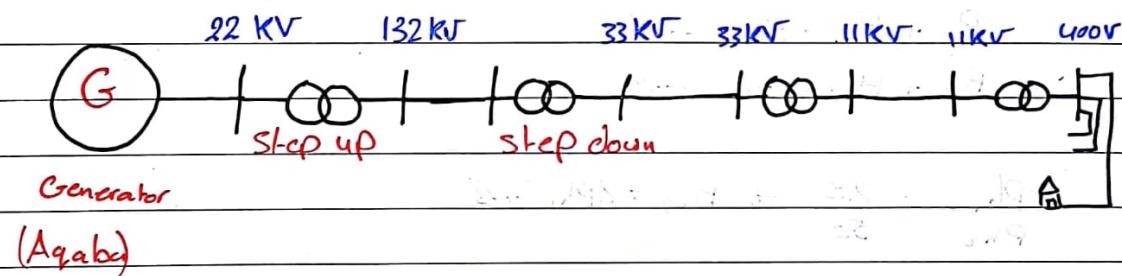
with out bring ing :

$$\phi_{core} = \phi_g \text{ and } B_{core} = B_g$$

with out bring ing %

$$B_{core} \neq B_g$$

\* Used in : Power system (Peak demand in Jordan 3GW)



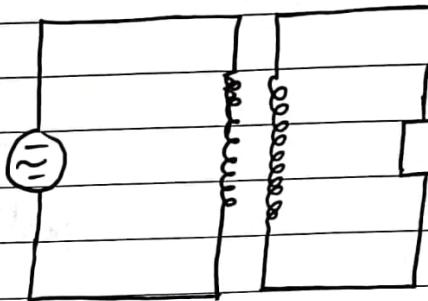
## 2-winding Transformer

"Primary and Secondary"

High voltage, low voltage

HV and LV side

Step up and step down



## Classification

\* Single Phase transformer (used in networks rarely)

\* Three Phase transformer (widely used)

## Name plate

- Rated primary voltage
- Rated secondary voltage
- Rated Frequency
- Rated Power

## Single Phase transformers

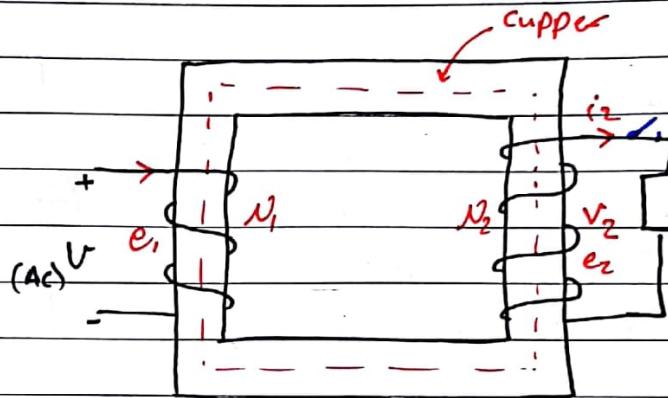
1- ideal transformers (No losses)

2- Real transformers

## Ideal Transformers

Faradays law

$$V = -N \frac{d\phi}{dt}$$



$$V_1 = -N_1 \frac{d\phi}{dt}$$

$$\boxed{\phi = \frac{1}{\omega_1} \int V_1(t) dt}$$

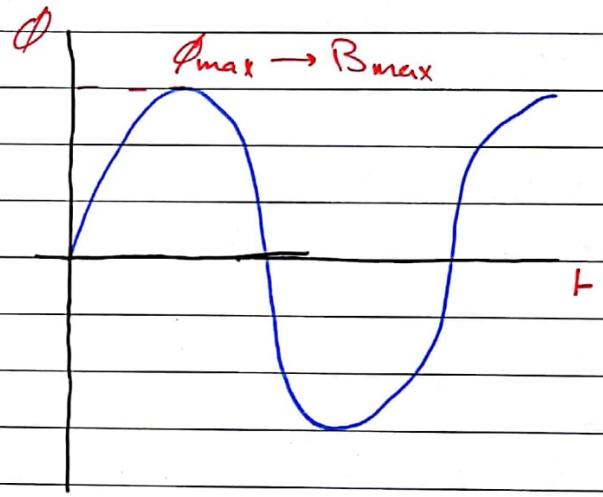
$$V_{rms} = 4.44 N A F B_{max}$$

Open circuit  $L_2=0$  (case 1)

$$V_2 = N_2 \frac{d\phi}{dt} \leftarrow \text{mutual flux}$$

$$V_1 = N_1 \frac{d\phi}{dt}$$

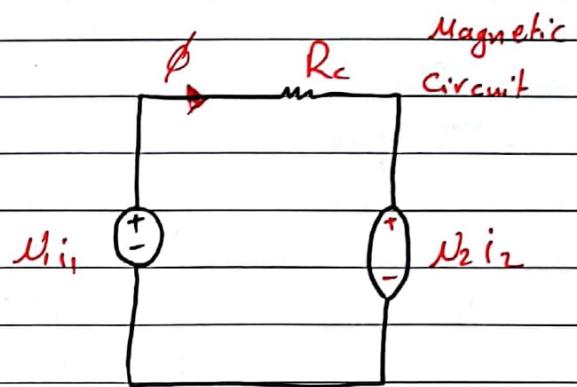
$$\frac{V_2}{V_1} = \left( \frac{N_2}{N_1} \right) \text{ turn ratio}$$



ideal  $\rightarrow (\mu \rightarrow \infty, R = \frac{L}{mA} = 0)$

$i_1 = 0$  an ideal case

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \Rightarrow \frac{e_2}{e_1} = \frac{N_2}{N_1}$$

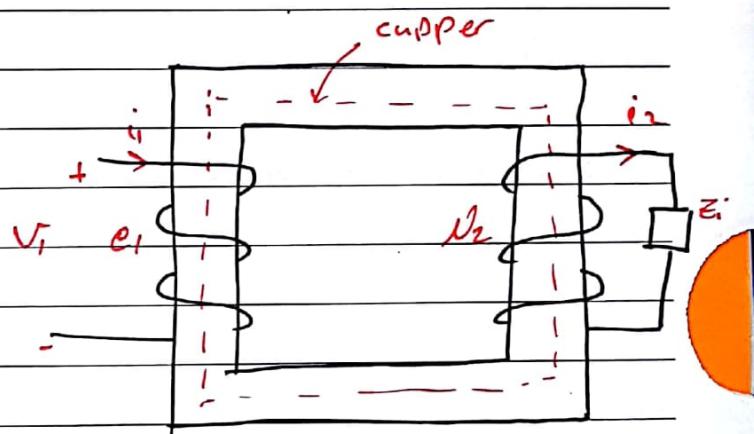


ideal transformer under load

$$R = 0, M = \infty$$

$$U_1 i_1 = U_2 i_2$$

$$\frac{N_1}{N_2} = \frac{i_2}{i_1}, \quad \frac{U_2}{U_1} = \frac{N_2}{N_1}$$



$$P_{\text{primary}} = P_{\text{secondary}}$$



Speed = Voltage

Elec.  $\rightarrow$  Mech.

Torque = current

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

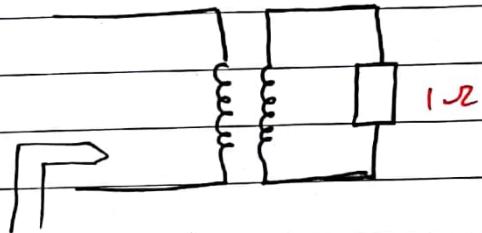
$$ZL = ZL * \left( \frac{N_1}{N_2} \right)^2$$

$$\frac{i_2}{i_1} = \frac{N_1}{N_2}$$

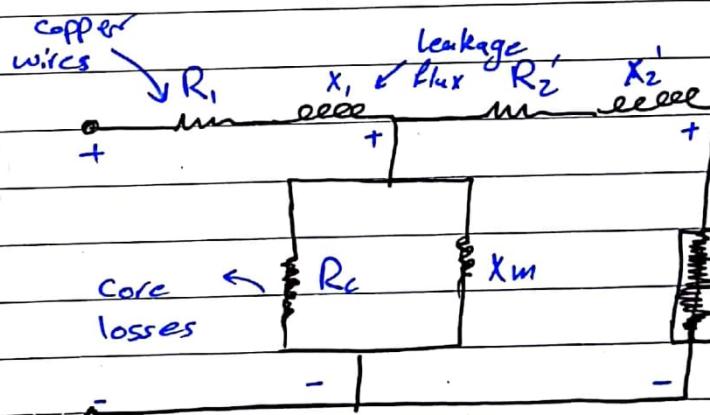
Ex:

Single-Phase transformer

1 : 4



$$ZL' = 1 * \left( \frac{1}{4} \right)^2$$



- Real transformer equivalent CKT

## modeling real transformer

\*) Copper losses

\*)漏磁 Flux

\*) Core losses

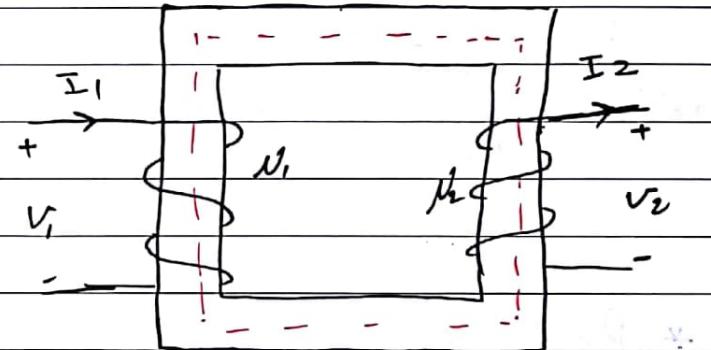
$R_c$

漏磁

hysteresis

eddy current

Losses

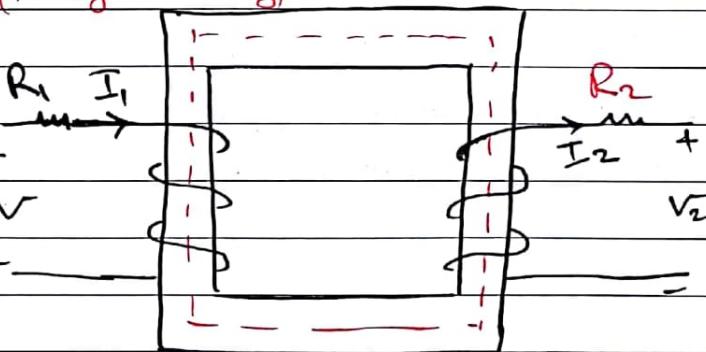


Magnetic coupling

$$L = \frac{N\Phi}{i}$$

Copper losses ( $R$ )

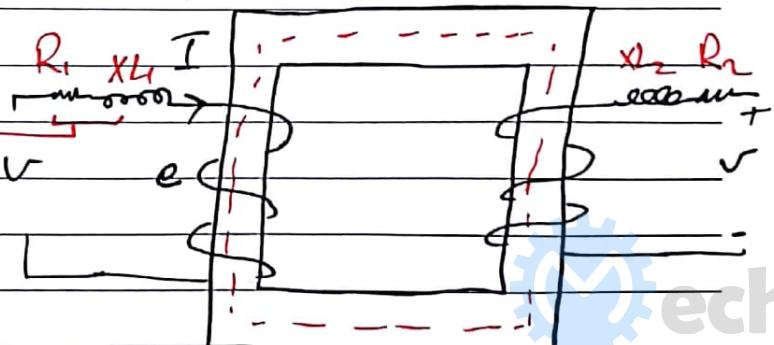
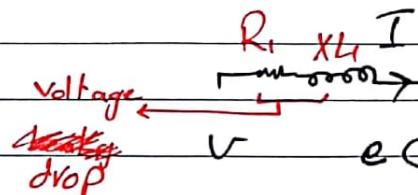
(Primary and secondary)



Leakage Flux

$$e = \frac{N\Phi}{dt}$$

$$\Phi_m = \frac{1}{m} \text{per unit}$$



Recall

$$|X_L| = \omega L = 2\pi f L \quad \text{---} \quad \text{eeee} \quad \angle \omega = 90^\circ (j)$$
$$(X_C = 1/\omega C = 1/2\pi f C) \quad \text{---} \quad \text{ffff} \quad \angle \omega = -90^\circ (-j)$$

$$Z = R + jX$$

impedance  $\downarrow$  reactance  
 $\downarrow$  Resistance

$$Y = \frac{g}{Y} + j\frac{b}{Y}$$

admittance  $\downarrow$  susceptance  
 $\downarrow$  conductance

$$Z = \frac{1}{Y}$$

\* Real transformer under no-load

\*  $I_2 = 0 \Rightarrow I_1 \neq 0$  (Excitation current)

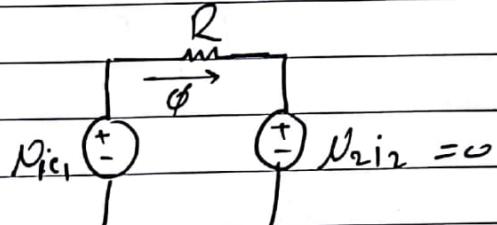
2 components

- 1) Magnetization current  $\rightarrow$  Primary  $\downarrow$  current  $i_{mp}$  (جي)
- 2) core losses  $\rightarrow$  Secondary current  $i_{sp}$  (جي)

⇒ Magnetization  $\uparrow$  current

$$N_{ii} = \emptyset R \Rightarrow i \neq 0$$

$i$  is in phase  $\phi$



میدان مغناطیسی (Magnetic field)

$$e = \frac{D \partial \phi}{\partial t}$$

$$\phi = \phi_m \sin \omega t = \phi \cos(\omega t - 90^\circ)$$

$$e = m D_w \phi_m \cos \omega t$$

$\Rightarrow e$  leads  $\phi$

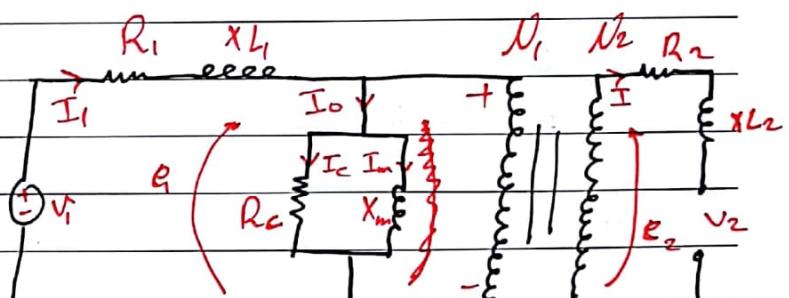
e Lead  $i \Rightarrow$  inductor  $\Rightarrow$  Magnetization current

A black right-pointing arrow symbol, indicating a continuation or next step in the sequence.

## Modelling real transformer

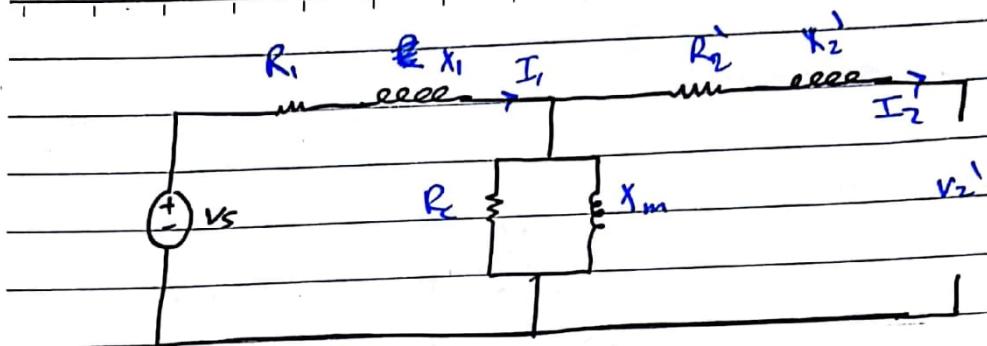
ideal transformer + external component:

- 1) copper losses ( $R_c$ )
- 2) Leackage Flux ( $\chi_L$ )
- 3) core losses ( $R_c$ )
- 4) Magnetization current ( $\chi_m$ )



$I_0$  = excitation current

$I_m$  = Magnetization current

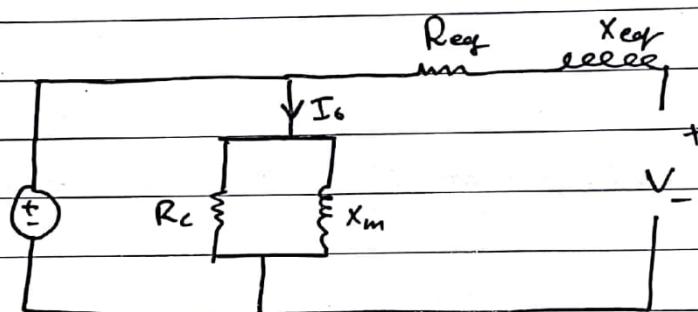


$$R_2' = R_2 \left( \frac{N_1}{N_2} \right)^2$$

نقل السيجنال الى المخرج

$$X_2' = X_2 \left( \frac{N_1}{N_2} \right)^2$$

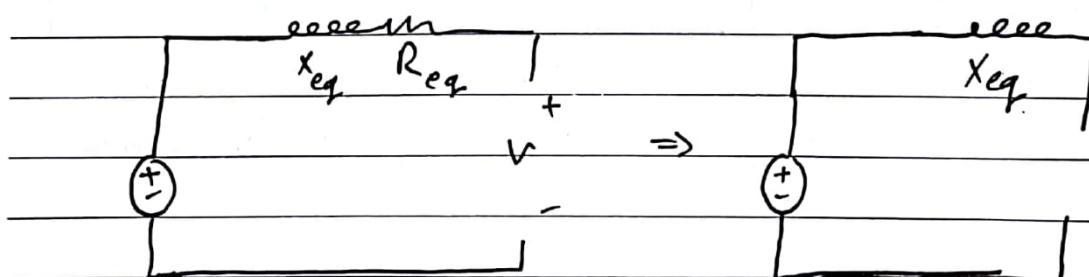
L-equivalent Approximation



$$R_{eq} = R_1 + R_2'$$

$$X_{eq} = X_1 + X_2'$$

$I_0 \approx (5-8) \cdot I$  Full load



one transformer

$R_{eq}$  very small

## Transformer Rating and Nameplate

10 KVA, 1100 / 110 V

complex power  $S = P + jQ$

transformer  $\Delta I = \frac{P}{R_w} \times 1000$

$\Rightarrow$  2 windings

HV side 1100 V      Rated

LV side 110      Voltage

turns ratio  $\frac{1100}{110} = 10$

Rated power @ HV side = 10 KVA

Rated power @ LV side = 10 KVA

Rated current @ HV side =  $10 \text{ KVA} / 1100 = 9.09 \text{ A}$

Rated current @ LV side =  $10 \text{ KVA} / 110 = 90.9 \text{ A}$

$$S = P + jQ$$

$\downarrow$  Real power (W, KW)

Complex (apparent power)

KVA

MVA

$Q$ : Reactive power

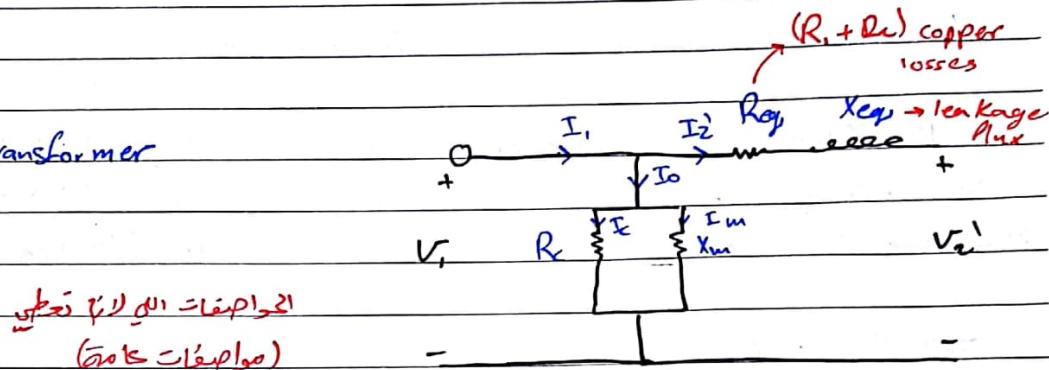
( $-90^\circ$ ,  $-11^\circ$ )

$$\text{P.F.} = \frac{P}{S}$$

18/Oct/2017

Recall

model transformer



Power copper losses @ rated load

Power core losses (No load losses) @ rated voltage

(load always causes core losses)

$$\text{Power copper losses} = \underline{\underline{(I_2')^2}} \text{Req}$$

Dependent on  $I_2'$  (load)

$$\text{Power core losses} = \frac{V^2}{R_c} \leftarrow \begin{array}{l} \text{high } V \text{ causes} \\ \text{high } I \text{ in } R_c \text{ series} \\ (\text{low } V \text{ causes low } I) \end{array}$$

: Tests to find core losses

transform test

→ No load test

(open circuit test)

→ No load losses

$$\begin{array}{l} \rightarrow R_c \\ \rightarrow X_m \end{array}$$

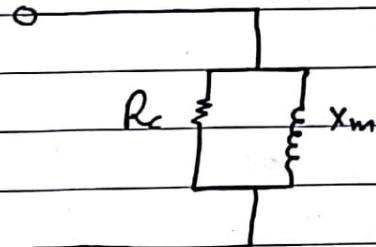
→ short circuit test

→ load losses

$$\rightarrow R_{eq}$$

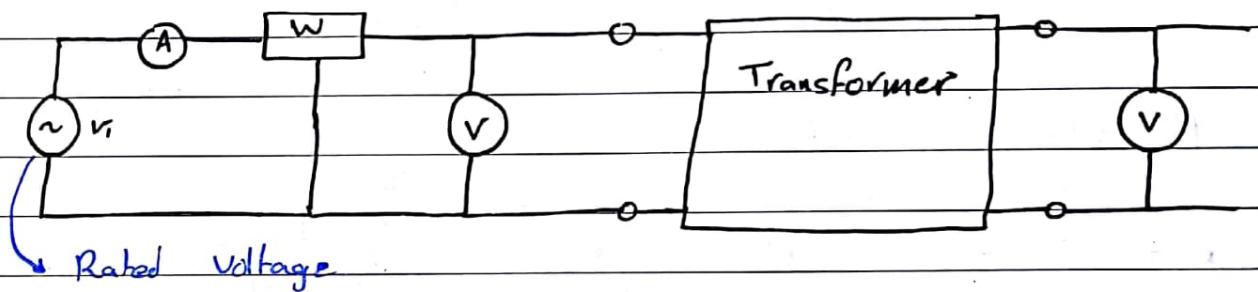
$$\rightarrow X_{eq}$$

No load test



equivalent ckt

watt meter



$$W = VI$$

$$W = VI \times \text{P.F}$$

Power Factor

Real Power  $\rightarrow$  P

Reactive Power  $\rightarrow$  Q

Wattmeter  $\rightarrow$  Complex Power  $\rightarrow$  S

$$S = VI^* \Rightarrow |S| = |V||I|$$

$$S = P + \underline{Q}$$

Complex power  $\rightarrow$  ohmic load

Auxiliary inductor  $\rightarrow$  ~~II~~



$\Rightarrow$  No load losses,  $R_p$ ,  $X_p$  ??

$\rightarrow$  inputs : Voltmeter, Ammeter, Wattmeter

Q ) where the test is done ? L.V side or H.V side ??

$$\Rightarrow 1 \text{ kVA}, 1000/100 \text{ V}$$

open circuit test

$N$	1000 V
$I$	---
$P$	---

$(R_c, X_m)$  referred to HV

HV side excited

LV side open circuit

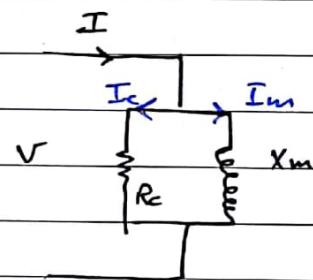
H.V Jis lii l 101000 Rated ~~V~~ V Jis lii Test Jis lii

Voltage

Back to the test!

$$P = \frac{V^2}{R_c} \Rightarrow R_c = \frac{V^2}{P} \quad \text{Power} \quad \textcircled{1}$$

$$I_c = \frac{V}{R_c} \quad \textcircled{2}$$



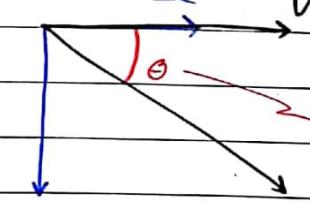
$$I_m = \sqrt{I^2 - I_c^2} \quad \text{---} \textcircled{3}$$

Voltage J is in phase with I

$$X_m = \frac{V}{I_m} \quad \text{---} \textcircled{4}$$

$$I_c$$

$$V$$



$$P.F = \frac{P}{V I_m} = \frac{P}{V^2}$$

loads are inductor J, J

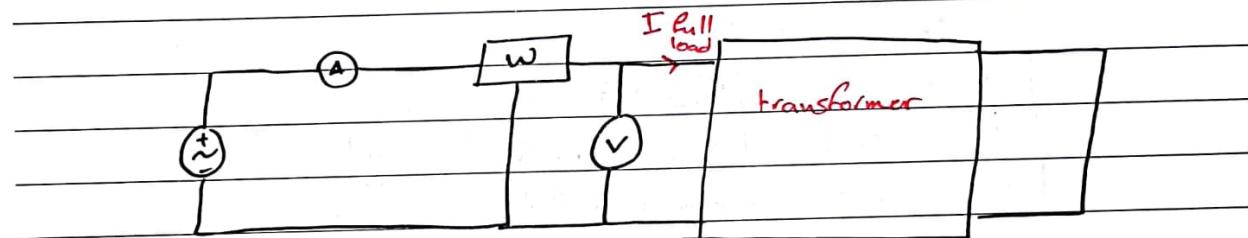
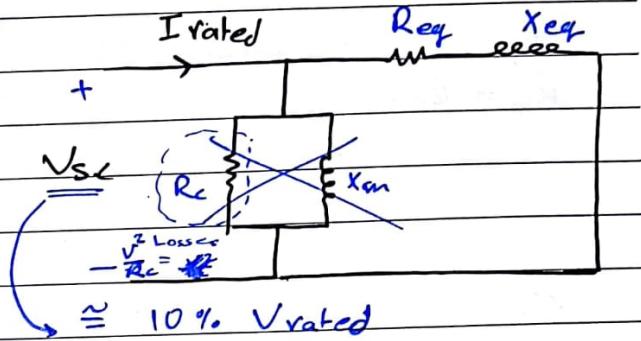
Voltage J

ج

## Short circuit test

to find load losses @ Rated current

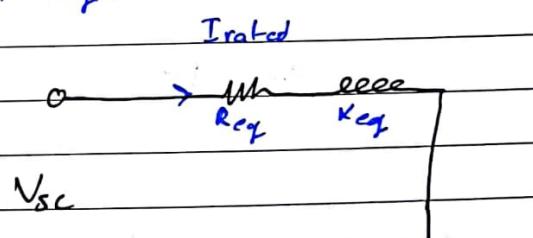
to reach the rated current  
we set the short circuit  
voltage at 10% of the  
Rated Voltage



Wattmeter = copper losses + core losses = 0  
 @ rated current @ 10% rated voltage  
 Full load  $(V_{\text{sc}})$

now we have to find  $(R_{\text{eq}}, X_{\text{eq}})$   
 $(P, V, I)$  known

$$P = I^2 R_{\text{eq}} \Rightarrow R_{\text{eq}} = \frac{P}{I^2} \quad \text{--- (1)}$$



$$Z_{\text{eq}} = \frac{V}{I}$$

$$Z_{\text{eq}} = R_{\text{eq}} + jX_{\text{eq}}$$

$$X_{\text{eq}} = \sqrt{Z_{\text{eq}}^2 - R_{\text{eq}}^2}$$

$$R_{eq} = R_1 + R_2$$

$$X_{eq} = X_1 + X_2$$

جيلاس، ١٢٥٢١

$$R_1 \text{ (can be measured)}$$

$$R_2 = R_{eq} - R_1$$

$$X_1 \approx X_2$$

$$X_1 = X_{eq}/2$$

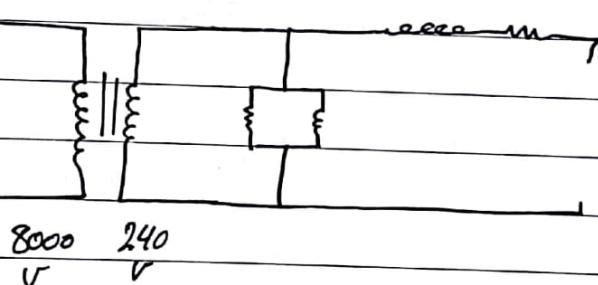
$$X_2 = X_{eq}/2$$

Ex) 20 kVA, 8000 / 240 V, 60 Hz transformer

Open ckt	Short ckt
$V = 8000 \text{ V}$	$V_{sc} = 489 \text{ V}$
$I = 0.214 \text{ A}$	$I_{sc} = 2.5 \text{ A}$
$P = 400 \text{ W}$	$P_{sc} = 240 \text{ W}$

H.V side  
L.V side

Draw the equivalent ckt of transformer



Open ckt  $\rightarrow R_c$  referred to  
 $\rightarrow X_m$  high voltage side

$$I_{rated/HV} = \frac{20 \text{ kVA}}{240}$$

$$I_{rated/HV} = \frac{20 \text{ kVA}}{8000}$$

$$= 2.5 \text{ A}$$

short ckt  $\rightarrow R_{eq}$  referred to  
 $\rightarrow X_{eq}$  HV side

سولہ  
No load losses  $\rightarrow$   $8100 = (V)$   $\parallel$   $R_c$   $\parallel$   $X_m$

$$8000 \rightarrow 400$$

$$8100 \rightarrow ??$$

$$P = \frac{V^2}{R_c} \rightarrow R_c \text{ is fixed or}$$

پھر 10% of  $P_{no\ load}$  کی وجہ سے  $8000V$  Rated Voltage پر  $z_{pp}$   
(No load losses  $\rightarrow$  losses  $\parallel$   $R_c$   $\parallel$   $X_m$  بوجوں پر تھے)

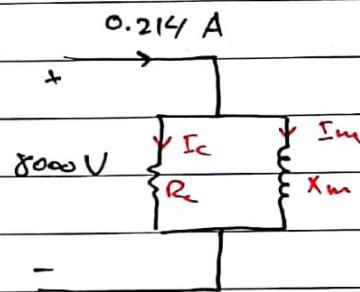
cont sol:

$$P = \frac{V^2}{R_c} \rightarrow P = \frac{(8000)^2 - 400}{R_c} \Rightarrow R_c = 159 \Omega$$

$$I_c = \frac{8000}{159} =$$

$$T_m = \sqrt{0.214^2 + (I_c)^2}$$

$$X_m = \frac{8000}{I_m} = 38.3 \text{ kN}$$



$$R_c / L_V = 159 * \left( \frac{240}{8000} \right)^2 =$$

$$X_m / L_V = 38.3 \left( \frac{240}{8000} \right)^2 =$$

$$Y_{o.c} = \frac{1}{Z} = \frac{1}{R_c} + \frac{1}{jX_m}$$

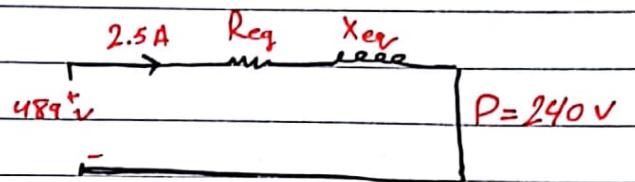
open circuit

Power losses  
Active  $\rightarrow$  heat  
Reactive  $\rightarrow$  Flux generation  
all losses  $\parallel$   $R_c$   $\parallel$   $X_m$   
Voltage  $\parallel$   $R_c$   $\parallel$   $X_m$

## Short ckt. test

$$240 = (2.5) A \times R_{eq}$$

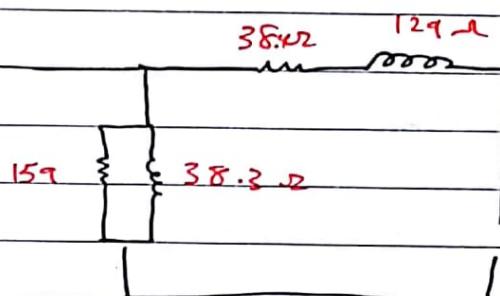
$$R_{eq} = 38.4 \Omega$$



$$Z_{eq} = \frac{489}{25}$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = 192.52$$

$$\frac{X}{R} - \frac{192}{38} \equiv 5$$



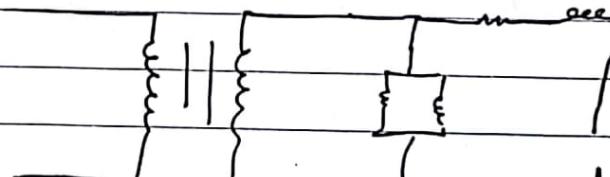
نقطي القمة على H.V.S او على L.V.S

short skt  $\rightarrow$  H.V.S.C

open skt  $\rightarrow$  L.V side  $\int d\omega^i$

موده يكون طالب على الـ

L.V.S.3



نیپی، H. V. side الگویی

$$\left(\frac{990}{8000}\right)^2$$

8000 / 240v  L.V side

## Analysis of the transformer

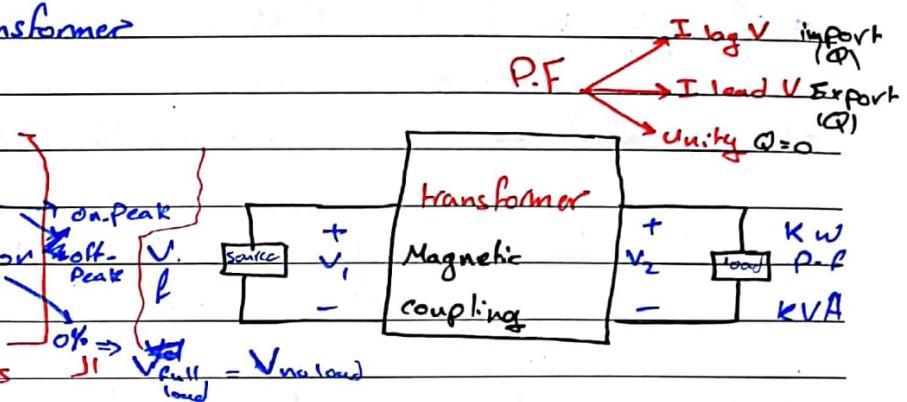
\* Voltages

\* Voltages drop

\* Voltage Regulation

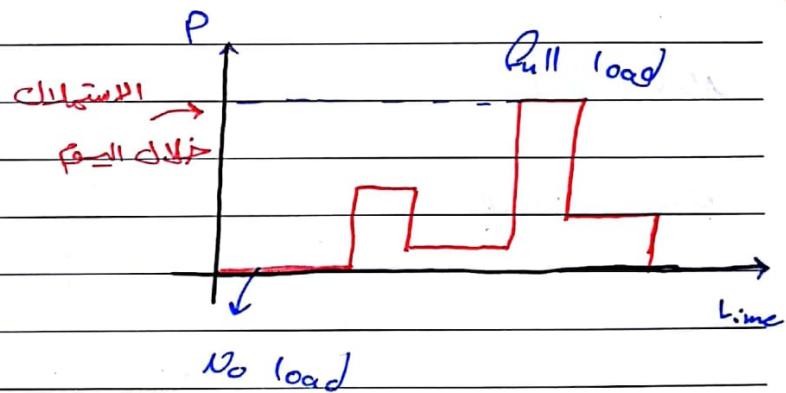
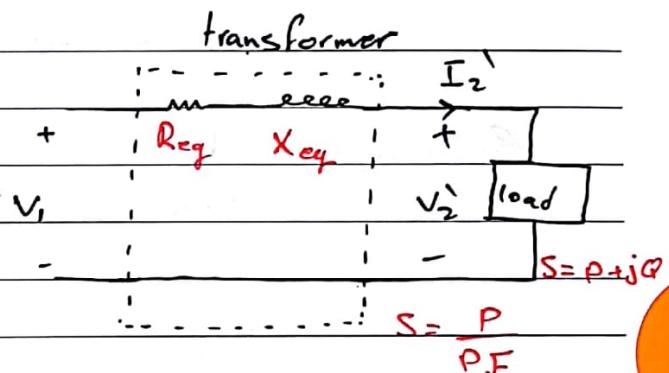
\* Efficiency

Winding Analysis



- It's aimed to keep voltage at the load side during
  - No load
  - Full load

close to  $|V_2'|$



$V_1$  Fixed,  $V_2'$  depends on the load

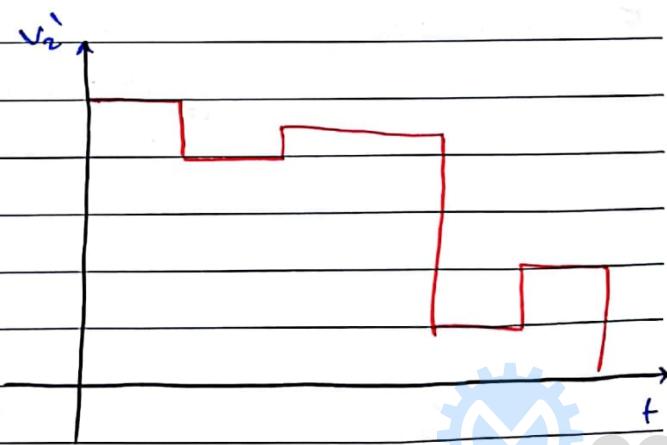
$$V_2' = V_1 - I_1' (R_{\text{req}} + X_{\text{req}})$$

$$S = V_1 I_1'$$

مقدار  $I_1'$  برابر با  $I_1$  است

$$|I_1| = \frac{|S|}{V_1}$$

مقدار  $V_2'$  برابر با  $V_2$  است



\* if  $V_2'$  is fixed ( $V_1$  is controlled)

$\rightarrow V_2$  limit

at no load  $\Rightarrow V_1 = V_2$  limit  $\Rightarrow V.D = 0$

at full load  $\Rightarrow V_1 = V_2$  limit +  $I (R_{eq} + jX_{eq})$

$$\Rightarrow V.D = V_1 - V_2 \text{ limit}$$

$$V.R. = \frac{|V_1|_{\text{no load}} - |V_1|_{\text{full load}}}{|V_1|_{\text{full load}}} \times 100 \%$$

$$\text{Efficiency} = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{losses}}$$

$$= \frac{P_{out}}{P_{out} + P_{core} + P_{copper}} \quad P_{core} = \frac{V^2}{R_C}$$
$$P_{core} = (I_A)^2 R_{eq}$$

Max efficiency

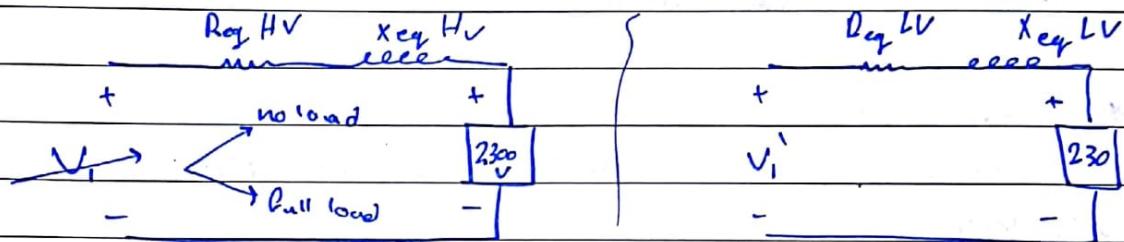
$$P_{core} = P_{copper}$$

$$\text{Energy efficiency} = \frac{\sum_{i=1}^N P_{out,i} \times \Delta t_i}{\sum_{i=1}^N P_{in,i} \times \Delta t_i}$$

25/Oct/2017

$$VR\% = \frac{|V_i \text{ no load}| - |V_i \text{ full load}|}{|V_i \text{ full load}|} \times 100\%$$

Ex) 2300 / 230 V



L.V side

$$\eta = \frac{P_o}{P_o + P_{core} + P_{cu}}$$

Ex) Unity, PF load, Kep load voltage fixed - what is the impact on core losses

- No load

- Full load

$$P_{core} = \frac{V^2}{R_c} \quad (\text{fixed voltage @ full load})$$

$\Rightarrow V_i \uparrow \rightarrow P_{core} \uparrow$

Ex) 15 KVA, 2300/230 transformer

Open ckt	Short ckt
$V_{o.c} = 2300 \text{ V}$	$V_{sc} = 47 \text{ V}$
$I_{oc} = 0.21 \text{ A}$	$I_{sc} = 6 \text{ A}$
$P_{oc} = 50 \text{ W}$	$P_{sc} = 160 \text{ W}$

\* Equivalent ckt referred to the H.V side

\* Equivalent ckt referred to the L.V side

$110 \text{ (15 KVA, 230 V)} \leftarrow 110 \text{ (230 V)}$

\* Full load voltage ~~regulation~~ regulation

at 0.8 lagging power factor

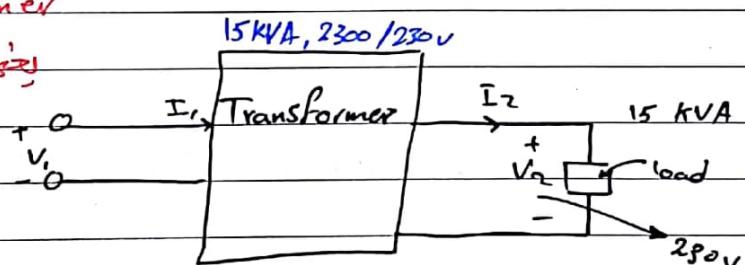
at unity P.F

at 0.8 P.F lead

\* efficiency at full load with 0.8 PF lag.

(Step down transformer)

((L.V.S) II SC (S))

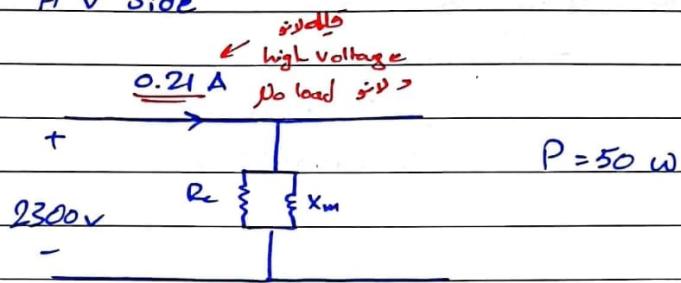


$$\text{Rated current } I_{H.V.} = \frac{15 \text{ KVA}}{2300} = 6 \text{ A}$$

H.V Side

$$\text{Rated current } I_{L.V.} = \frac{15 \text{ KVA}}{230} = -$$

\* HV Side



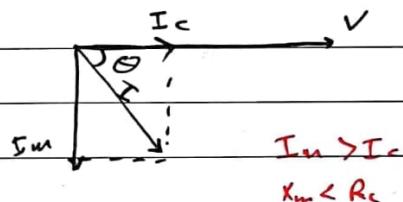
$$P = \frac{V^2}{R_L} \Rightarrow R_L = \frac{(2300)^2}{50} = 105 \text{ k}\Omega$$

$$I_C = \frac{V}{R_L}, \quad I_M = \sqrt{I^2 + -I_C^2}$$

$$\Rightarrow |X_M| = \frac{V}{I_M} = 11 \text{ k}\Omega, \quad X_M = 11 \text{ k}\Omega \angle 90^\circ$$

$$R_L \text{ HV} = 105 \text{ k}\Omega$$

$$X_M \text{ HV} = 11 \text{ k}\Omega$$



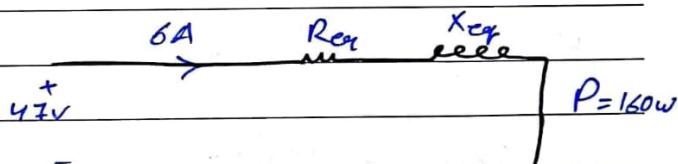
short ckt

$$P = I^2 R_{eq}$$

$$R_{eq} = \frac{160}{6^2} = 4.45 \Omega$$

$$Z_{eq} = \frac{V}{I} =$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = 6.45 \Omega$$



$$R_c \text{ HV} = 105 \text{ k}\Omega \quad R_c \text{ LV} = R_c \text{ HV} \times \frac{230}{220} \quad R_c \text{ LV} = 1050 \text{ }\Omega$$

$$X_m \text{ HV} = 11 \text{ k}\Omega \quad \Rightarrow \quad X_m \text{ LV} = 110 \text{ }\Omega$$

$$R_{eq} \text{ HV} = 4.45 \text{ }\Omega$$

$$R_{eq} \text{ LV} = 0.4445 \text{ }\Omega$$

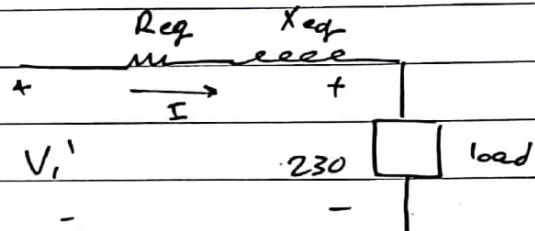
$$X_{eq} \text{ HV} = 6.45 \text{ }\Omega$$

$$X_{eq} \text{ LV} = 0.645 \text{ }\Omega$$

\* VRx. (15 KVA, 230V, unity PF)  
Load

1) No load  $I = 0$

$$\Rightarrow V_1' = 230 \text{ V}$$



2) Full load  $S = V I^*$

$$S = V I^*$$

$$I = \frac{15 \text{ kVA}}{230} = 65.2 \text{ A}$$

a) unity P.F  $Q = 0$

V and I in phase

$$T = 62.2 \angle 0^\circ$$

b) lagging (0.8 P.F)

$$I = 62.2 \angle -\cos^{-1} \text{ P.F}$$

c) leading (0.8 P.F)

$$I = 62.2 \angle \cos^{-1} \text{ P.F}$$

$$V_1' = 230 \angle 0^\circ + I (R_{eq} + j X_{eq})$$

\* Unity PF  $\Rightarrow V_1' = 232.94 \angle 21.04^\circ$

$$V_2 = (230 \angle 20^\circ)^* \quad \begin{array}{l} \text{Real} \\ \text{Power} \end{array}$$

Reactive Power  $\downarrow$   $\text{Q} = 0$

load  $\parallel$  capacitor  $\Rightarrow 15 \text{ kVA, unity P.F}$   
 $\Rightarrow P = 15 \text{ kW, } Q = 0$   $\begin{array}{l} \text{مقدار} \\ \text{متحدة} \end{array}$

\* lagging power factor (0.8)  $\begin{array}{l} \text{مقدار} \\ \text{متحدة} \end{array}$  من  $Q$   $\Rightarrow$

$$I = 65.2 \angle -\cos^{-1} 0.8$$

$$V_1' = 230 \angle 0^\circ + I (R_{eq} + j X_{eq})$$

$$V_1' = 234.85 \angle 0.4^\circ \text{ V}$$

! Q ~~lagging~~ <sup>leading</sup>

\* leading power factor

$$I = 65.2 \angle +\cos^{-1} 0.8$$

$V_1'$   $\begin{array}{l} \text{مقدار} \\ \text{متحدة} \end{array}$   $\begin{array}{l} \text{مقدار} \\ \text{متحدة} \end{array}$   $\begin{array}{l} \text{مقدار} \\ \text{متحدة} \end{array}$

أقل من  $180^\circ$   $\begin{array}{l} \text{مقدار} \\ \text{متحدة} \end{array}$  unity  $\parallel$  load

! Voltage

$$V_1' = 234.85 \angle 1.27^\circ$$

$$0.95 = \text{PF} \quad \begin{array}{l} \text{مقدار} \\ \text{متحدة} \end{array}$$

Efficiency @ full load (15 kVA, 230 V, 0.8 pf lagging)

Energy efficiency  $\parallel$   $\text{متحدة}$

Point 1  $\Rightarrow$  Efficiency  $\parallel$   $\text{متحدة}$

losses  $\parallel$   $\text{متحدة}$

$$\eta = \frac{P_o}{P_o + P_{core} + P_{cu}} * 100\%$$

$$P_o = 15 * 0.8 = 12 \text{ kW}$$

$$P_{core} = \frac{V^2}{R_c} = \frac{(234.85)^2}{1050}$$

$$P_{cu} = I^2 R_{eq} = (65.2)^2 * (0.045)$$

### \* Voltage Regulation :

$$V.R \% = \frac{|V_2, \text{ no load}| - |V_2, \text{ full load}|}{|V_2, \text{ full load}|} \times 100\%$$

$V_1$  fixed,  $V_1$  rated,  $V_2$  full load

By definition Full load  $\Rightarrow V_1 = V_2$  rated

$$1) I \rho_l \Rightarrow |I| = \frac{|KVA|}{V_2 \text{ rated}} \Rightarrow I = |I_{FL}| \angle \begin{cases} +\cos^{-1} \rho_f & \rightarrow \text{PF leading} \\ -\cos \rho_f & \rightarrow \text{PF lagging} \\ 0 & \rightarrow \text{PF unity} \end{cases}$$

$$2) V_1' = V_2 + I (R_{eq} + jX_{eq})$$

$$3) V.R \% = \frac{|V_1'| - |V_2 \text{ rated}|}{|V_2 \text{ rated}|} \times 100\% \quad \begin{matrix} \text{drop voltage at full load} \\ \text{no load} \\ V_2 \text{ rated} \end{matrix}$$

$$0.8 \text{ lagging PF} \Rightarrow |V_1'| = 234 \text{ V}$$

$$\Rightarrow V.R \% = \frac{234 - 230}{230} \times 100\% = \quad \begin{cases} @ \text{no load} \\ V_2 = V_1' \end{cases}$$

$$\text{unity PF} \Rightarrow |V_1'| = 232 \text{ V}$$

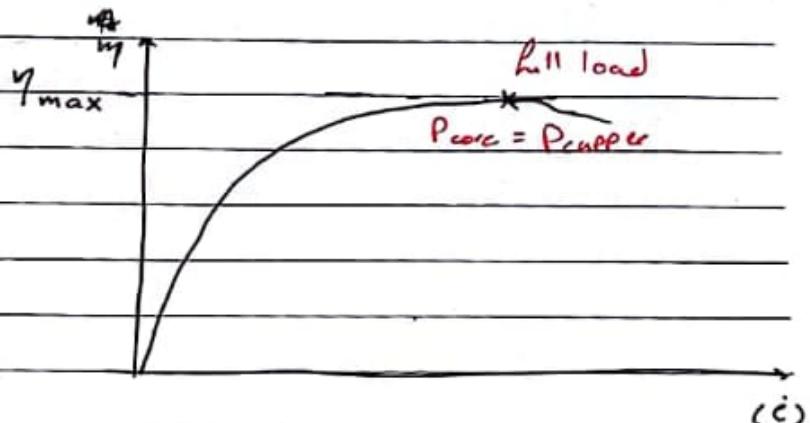
$$V.R \% = \frac{232 - 230}{230} \times 100\% =$$

$$0.8 \text{ leading PF} \Rightarrow |V_1'| = 229.85$$

$$V.R \% = \frac{229.85 - 230}{230} \times 100\%$$

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{P_o}{P_o + P_{core} + P_{copper}}$$

full load better



all day efficiency  $\Rightarrow$  Energy Perspective

$$M \text{ volt} * \text{PF} = M \text{ W}$$

$$M \text{ volt Amt} * \text{PF}$$

$$\text{E}_o = 3 * 8 + 1 * 10 + 10 * 6$$

$$\text{output} = 94 \text{ MWh}$$

energy

assume  $P_{core} = 500 \text{ W}$ ,  $V$  is fixed all

per day

$$E_{core \text{ losses}} = 0.5 * 24 = 12 \text{ MWh}$$

$$E_{losses \text{ copper}} = (I^2 * R_{eq}) * 8 + (I_2^2 * R_{eq}) * 6 + (I_3^2 * R_{eq}) * 10$$

Ex) Three identical single-phase transformer, each rated 10 KVA,  $2400/1120$  V, 60 Hz. They are connected to form a  $4160/1208$  V  $3\phi$  transformer. The equivalent impedance of each transformer referred to (H.V) side is  $(10 + 25j) \Omega$   $3\phi$  load (27 KW, 208 V, 0.9 PF reading)

determining the connection

$\text{Y/Y} \rightarrow$  from the voltage  
 $2160 \quad 208$

$\text{3\phi } 4160/120$   
 $\text{Y/N}$

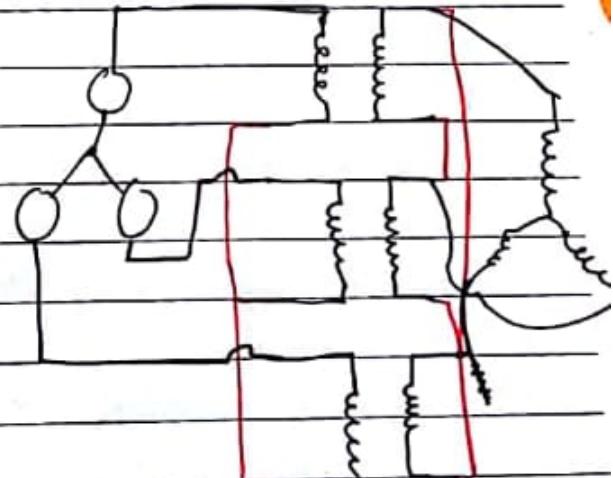
$$V_{LL} = \sqrt{3} V_{LN} \text{ Y side}$$

Balanced  $\Rightarrow$  per phase analysis

$$V_{LL} = V_{LN} \Delta \text{ side}$$

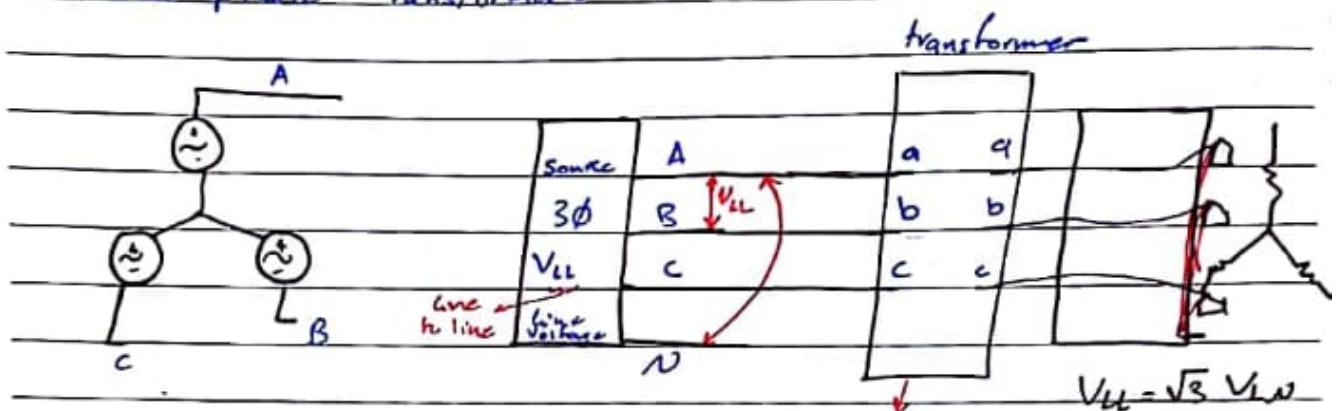
$(10 + 25)$

$U_1$   $U_2$   
 $9\text{KW}$   
 $208$   
 $V_L$



$(Y/\Delta)$

## Three Phase Transformers

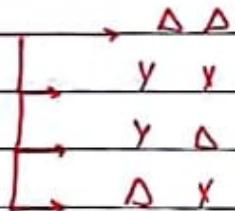


\* 3 $\phi$  Rated KVA

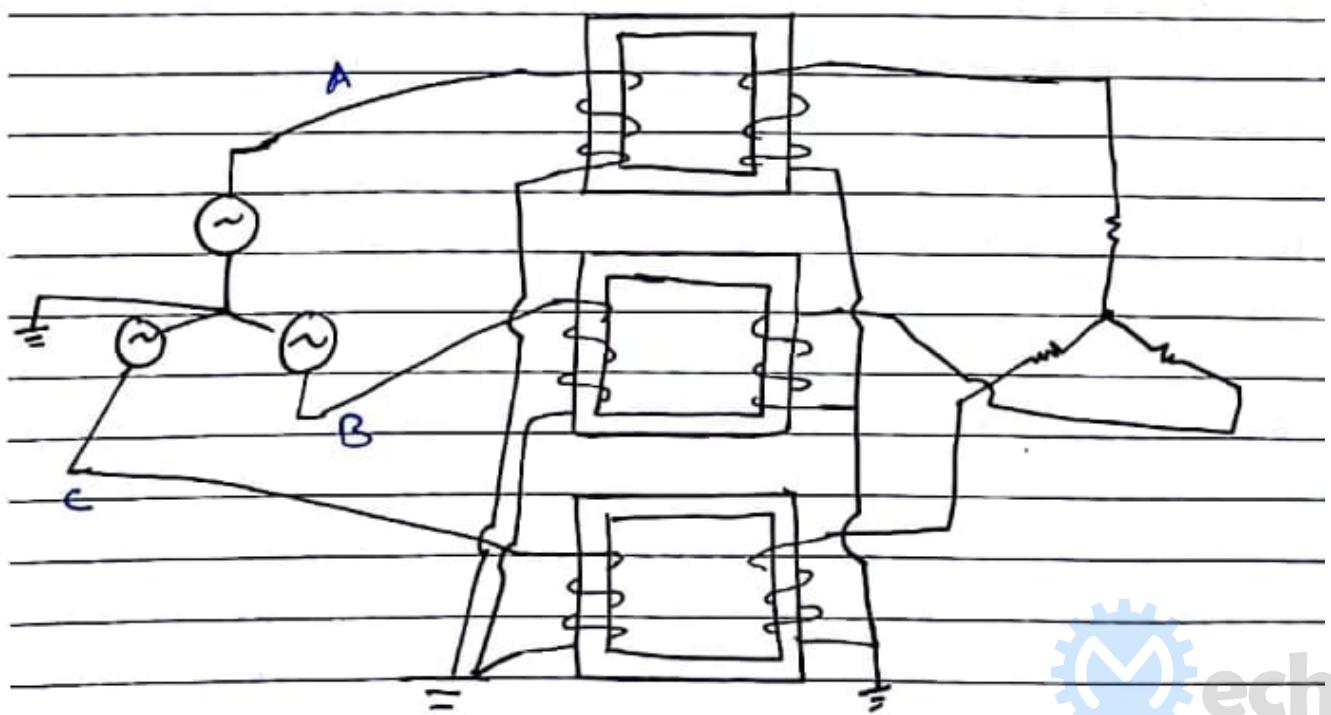
\*  $V_{LL}$  (HV) rated

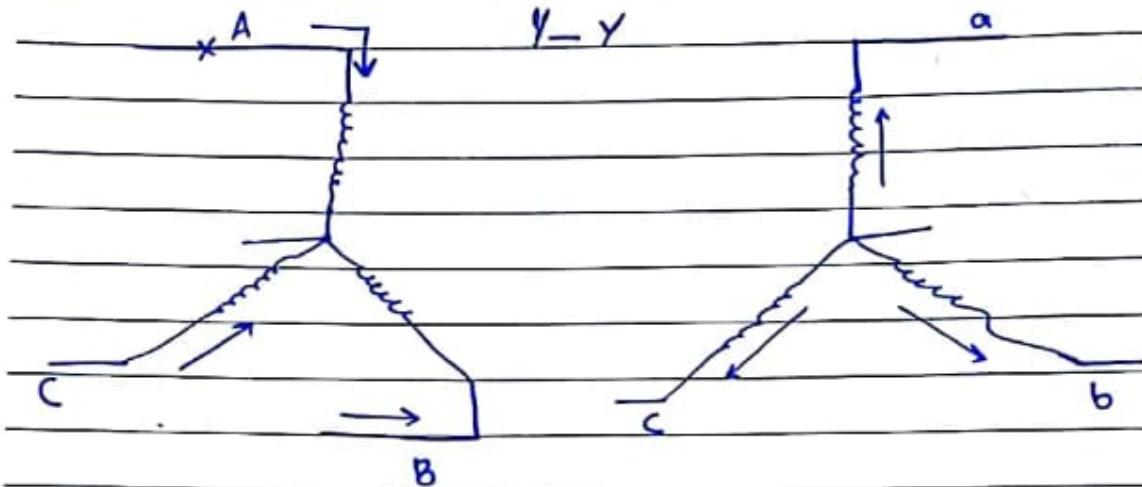
\*  $V_{LL}$  (LV) Rated

\* connection



## 3 $\phi$ transformer Y-Y





$$a = \text{turns Ratio} = \frac{V_p(HV)}{V_p(LV)}$$

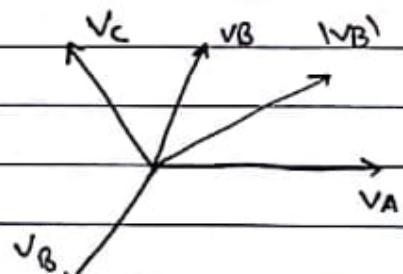
$$a_T = \text{Transformation Ratio} = \frac{V_{LL}(HV)}{V_{LL}(LV)} \quad V_{AB} = V_A - V_B$$

$$V_{LL}(HV) = \sqrt{3} |V_p(HV)|$$

$$(V_{LL} LV) = \sqrt{3} |V_p LV|$$

$$\Rightarrow a_T = \frac{\sqrt{3}}{\sqrt{3}} \frac{|V_p(HV)|}{|V_p(LV)|} = a_s = \text{turns Ratio (Y-Y)}$$

$$I_e^{HV} = I_e LV \times \frac{|V_p \text{ Rated LV}|}{|V_p \text{ rated HV}|}$$



turns Ratio  $\rightarrow$  converts from phase to phase

Transformation Ratio  $\rightarrow$  converts from line to line

$\Rightarrow$  find  $I_1$  on LV

$$|I_1| = T_0 * \left( \frac{0.4/\sqrt{3}}{11} \right), \text{ phase to phase (turns Rat)}$$

$$I_2 = \sqrt{3} * 6 * \frac{0.4}{\sqrt{3} * 11}$$

$R_{\text{mid}}$   $R'$

$$R_{\text{HV}} = R_{\text{LV}} \left( \frac{11}{0.4} \right)^2$$

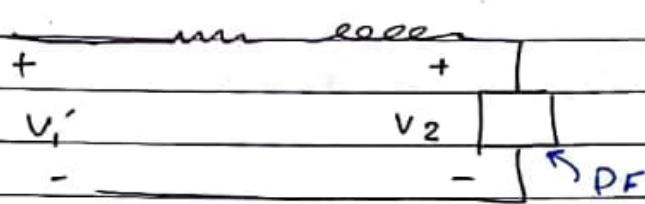
$$T_{\text{L HV}} = T_{\text{L LV}} * \left( \frac{0.4}{11} \right)$$

$$V_{\text{L HV}} = V_{\text{L LV}} * \left( \frac{11}{0.4} \right)$$

$$Y \underline{R_{\text{HV}}} = Y \underline{R_{\text{LL}}} * \left( \frac{11}{0.4} \right)^2$$

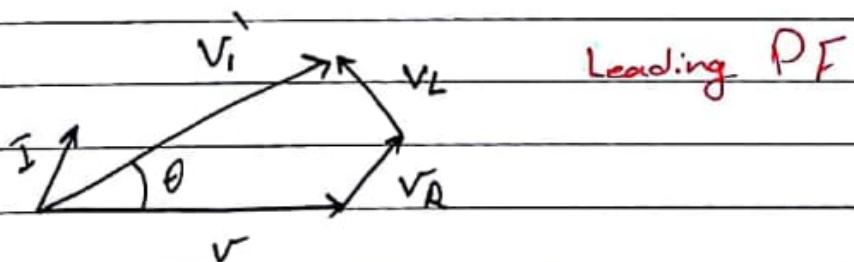
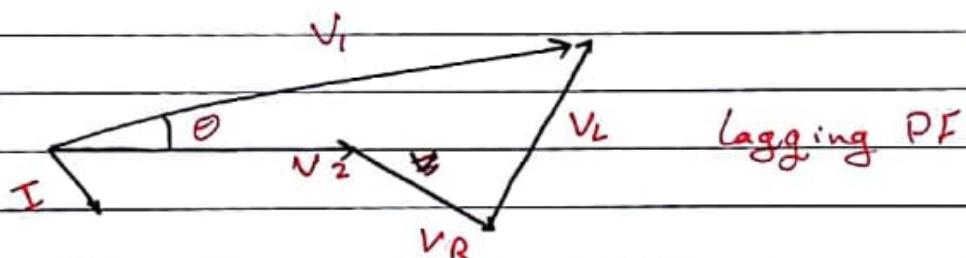
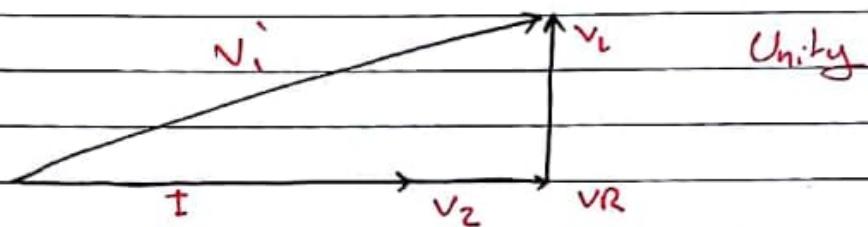
$Y_{\text{mid}} = Y_{\text{mid lin}}$

Ex ) on phasor diagram :



$V_R$  and  $I$  in phase  
 $V_L$  leads  $I$  by  $90^\circ$   
 $V_L$  leads  $V_R$  by  $90^\circ$

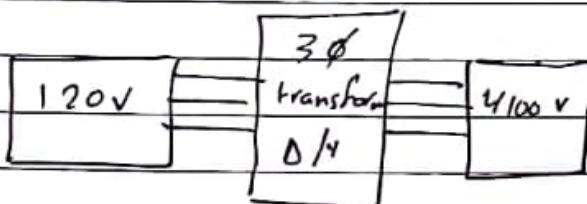
Draw phasor diagram (Leading and lagging and unity)



$$P = |V| |I| \cos \theta \quad S = P + Q; \\ Q = |V| |I| \sin \theta$$

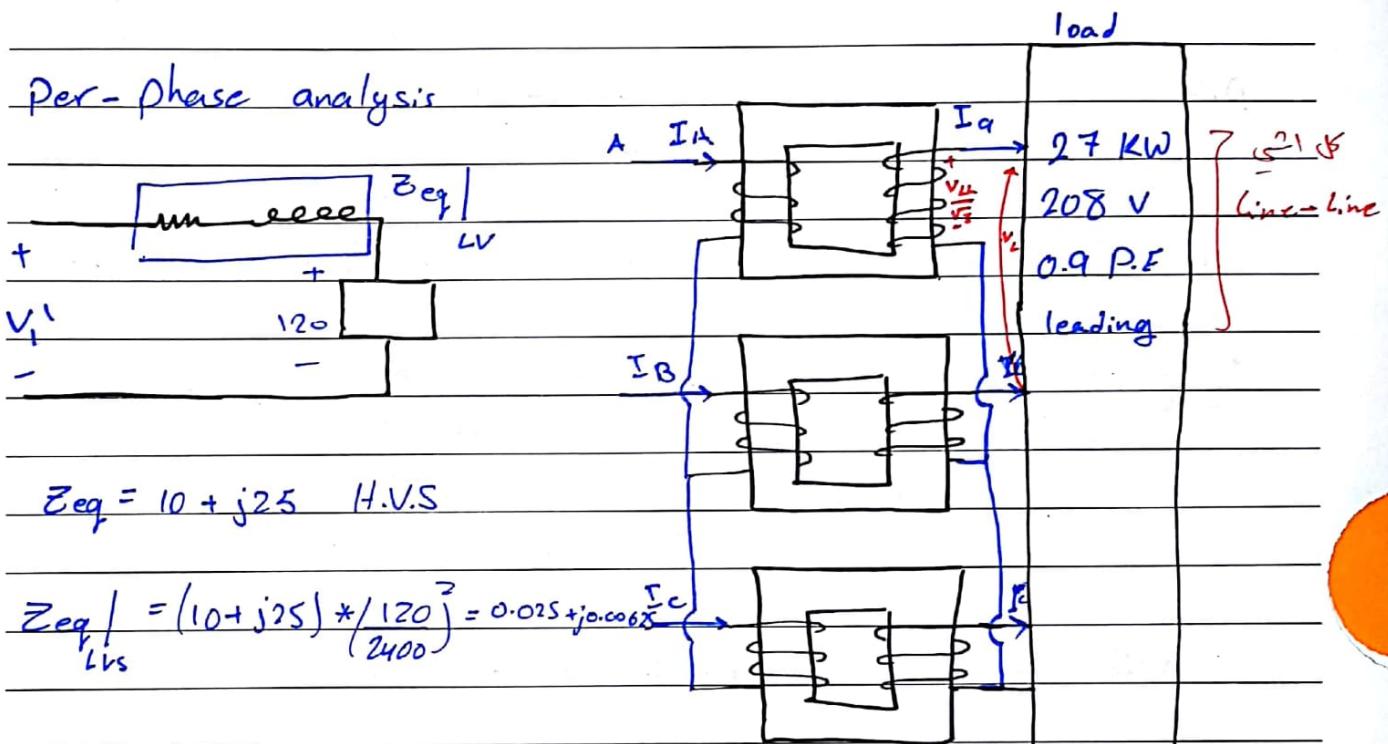
Ex) 3 Identical single phase transformer, each  
10 KVA, 2400 / 120 V

① Y/Δ  $\rightarrow$  30 KVA,  $2400\sqrt{3}$  / 120 V

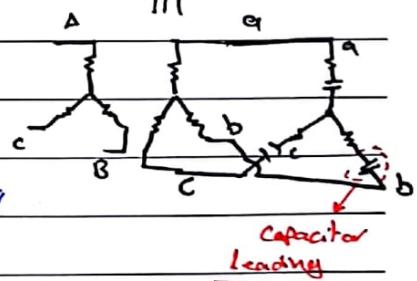


Ex) Three identical single-phase transformer, 10 KVA, 2400/120 V, 60 Hz; are connected to form 4160/208 V, the equivalent impedance at (HVS) is:  $(10 + j25) \Omega$ , load 27 kW, 208, 0.9 PF leading. Find VR%?

## Per-phase analysis



$$I_{\text{phase}} = \frac{|S|}{|V|} = \frac{9000 * 0.9}{120} \text{ A}$$



$$\left| \frac{I_{\text{phase}}}{HVS} \right| = 83.34 \times \frac{(120)}{2400} = 4.16 \angle + \cos^7 0.9$$

To find current ?

$$S = \sqrt{3} V_L I_L$$

$$6 \frac{27}{0.9} = \sqrt{3} * 708 * T$$

$$|T| = 83.34 L + \cos^{-1} 0.9$$

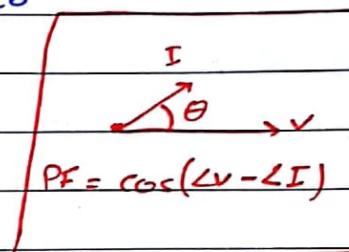
$$\left. \begin{aligned} S_{3\phi} &= 3S_1\phi \\ S_{3\phi} &= 3\sqrt{P}I_P \\ &= \frac{3}{\sqrt{3}} V_L I_L = \sqrt{3} V_L I_L \end{aligned} \right\}$$

$\theta = 2.67^\circ$

$$V_1' = 83.34 \cos^{-1} 0.9 (0.025 + 0.0625) + 120 \cos 0$$

$$V_1' =$$

$$V_1 = 2392.6 \angle 2.67^\circ \text{ V (LN)}$$



$$\text{P.F. @ source} = \cos(2.67^\circ - \cos^{-1} 0.9) \leftarrow \text{leading}$$

الزاوية الايجابية

Transformation Ratio equals the Turns ratio  $Y-Y$  نسبة

$$\frac{I_{\text{HVS}}}{I_{\text{LVS}}} = \frac{I(208)}{4160}$$

Transformation Ratio

$$= \frac{83.34 \times (208)}{4160}$$

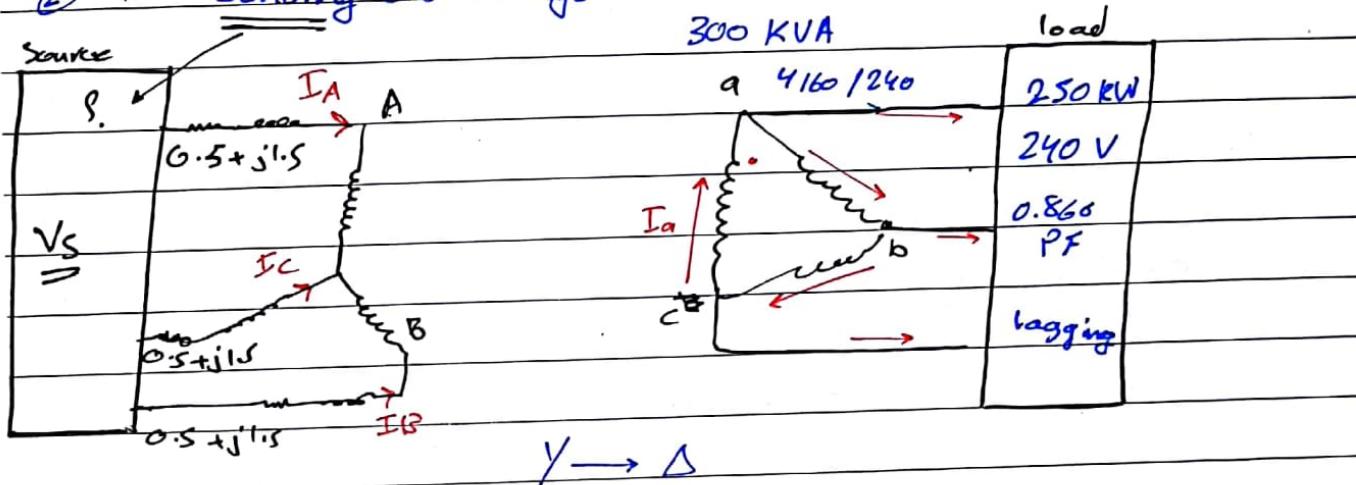
$$\text{VR \%} = \frac{|V_1'| - |V_2| \times 100\%}{V_2} = -0.31\%$$

Ems ~~not~~

Ex) Three single phase transformers 100 KVA, 2400 / 240, 60 Hz  
 $Z_{phase} = 0.045 + j 0.16 \Omega$ , The transformers are connected to the  
 source ( $0.5 + j 1.5 \Omega$  / phase) load ~~250 KW @ 240V, 0.866 lag~~  
 $\frac{250}{240}$   
 $[3\phi : 4160 / 240]$

① determine winding current

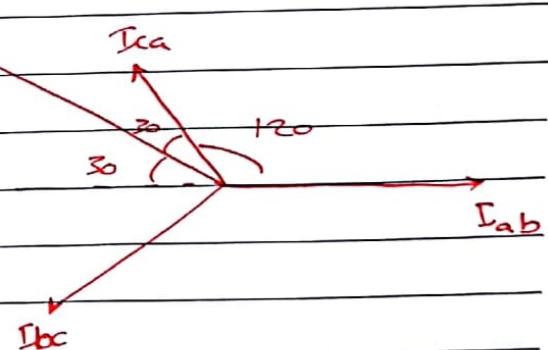
② " sending end voltage "



$$S_{3\phi} = \sqrt{3} V_L I_L = \frac{250 \times 240 \times I_L}{0.866} \Rightarrow |I_L| = 694.5 \text{ A}$$

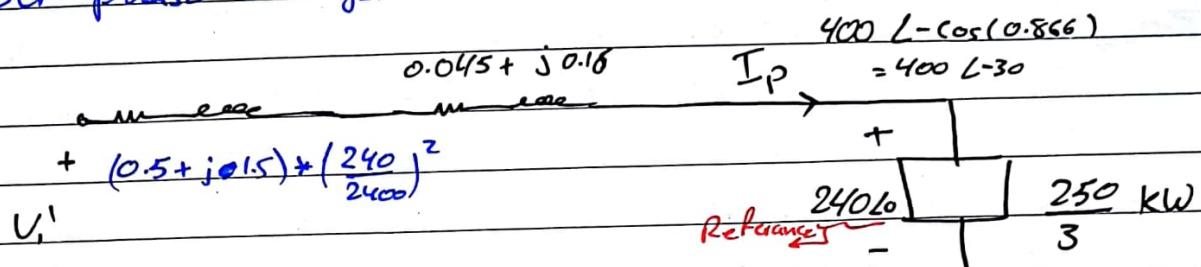
Rated current in ~~3\phi~~ 300  $\leftarrow$  250  $\downarrow$  ~~line~~  $\leftarrow$  ~~line~~

$$|I_p| = \frac{I_L}{\sqrt{3}} = \frac{694.5}{\sqrt{3}} \text{ A}$$



110 No second Ys

Per-Phase analysis



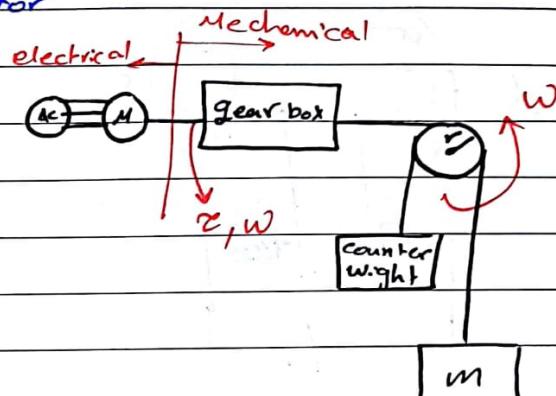
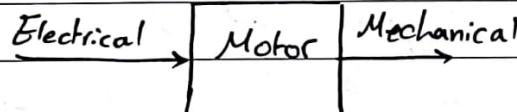
$$V_1' = (E K V L)$$

$$V_1 = V_1' * \left(\frac{240}{240}\right)$$

$$V_{1LL} = \sqrt{3} V_1 = 5138.5$$

8th Nov/2017

\* Dc Machines  $\xrightarrow{\text{Motor}}$   $\xrightarrow{\text{Generator}}$



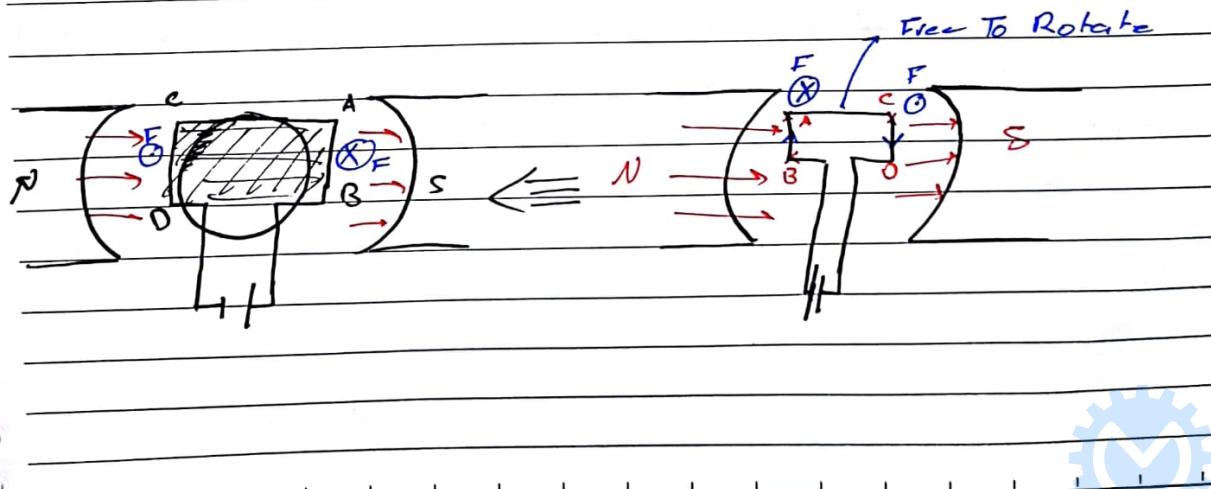
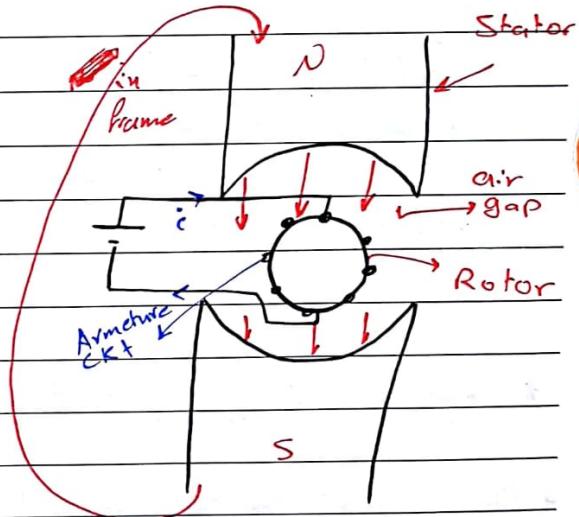
$$P = T \omega$$

$$w \downarrow \Rightarrow P \downarrow \Rightarrow B, I \downarrow$$

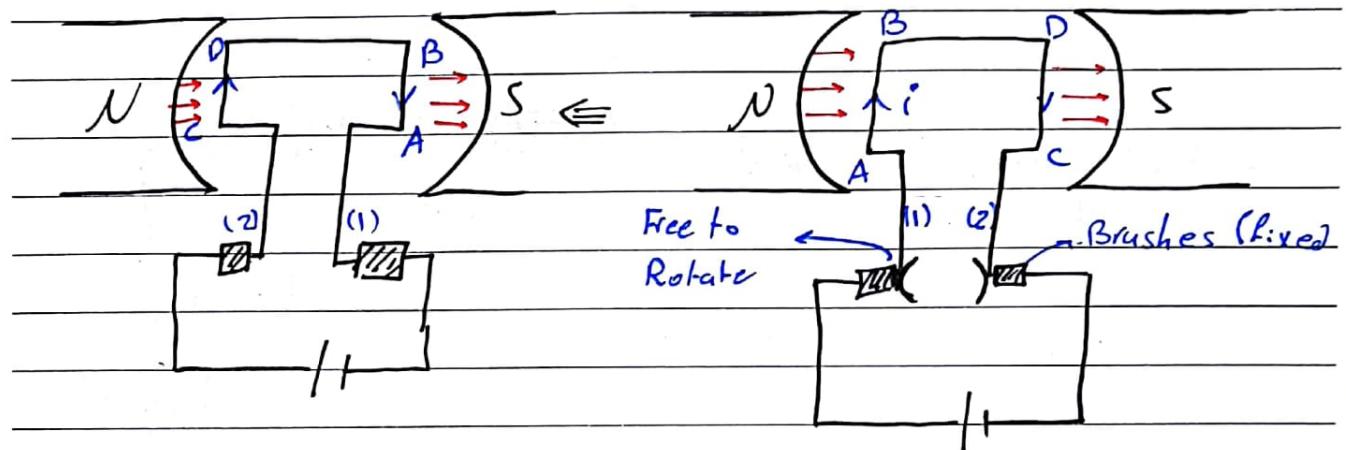
$$F = mg, T = mgr$$

④  $F = i \times B L$  ?

Force      Current      Flux



## \* Commutator



## \* Stator , Rotor

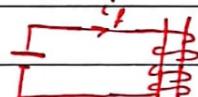
field ckt  $\rightarrow \phi$

Armature ckt  $\rightarrow \underline{i_f}$

برفقه اللد !

$$\Rightarrow F = i_f \times \phi$$

$\phi \rightarrow$  Permanent magnet  
 $\underline{i_f} \rightarrow$  Electromagnet

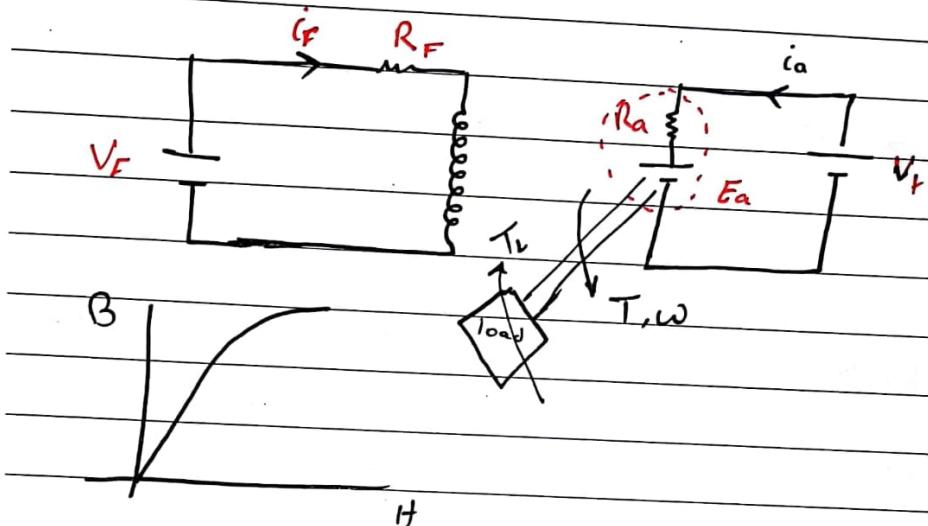


$$* \text{ emf} = N \frac{d\phi}{dt}, \text{ emf} = \underline{\phi} \times B L \quad (\text{back emf})$$

## \* DC Motor classification (for electro magnetic Flux)

- Separately excited DC Motor (source Wb, & armature 1, field 2)
- Shunt DC Motor
- Series DC Motor
- Compound DC Motor

## \* Separately-exited DC Motor



$$F = B \ell i$$

$$\text{Torque} = K_1 \phi i_a$$

$$K_1 = K_2 \ell$$

$$\text{emf} = B \ell v$$

$$E_a = K_2 \phi \omega$$

\* if  $\phi$  is constant

$$E_a = K_2 \phi \omega$$

$$E_a = K_V \omega$$

$$\left. \begin{aligned} T &= K_V i_a \\ K_V &\Rightarrow \frac{\omega_m}{A} \Rightarrow K_V = K \ell \end{aligned} \right\}$$

$$\text{Unit. } \left. \begin{aligned} K_V &= V/\text{rad/s} \\ &= V \cdot \text{s} \end{aligned} \right\}$$

$$\text{N.m ?? (VA.s)?}$$

$$J = J \text{ vs}$$

$$\frac{\text{power motor}}{\text{airgap}} = E_a i_a$$

MP class

$$\text{mechanical power} = T_w$$

$$E_a i_a = T_w$$

$$k_2 \phi_w i_a = k_1 \phi_i a_w$$

$$k_1 = k_2 \quad \#$$

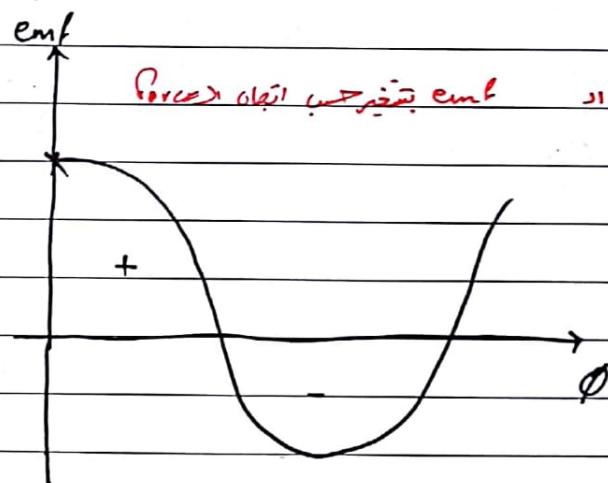
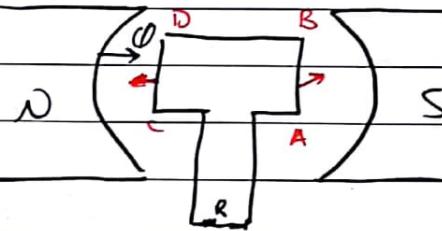
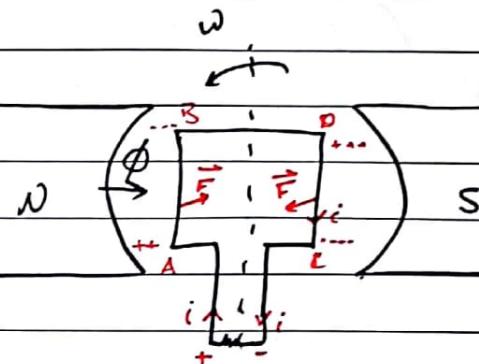
13/Nov/2017

## DC Generator

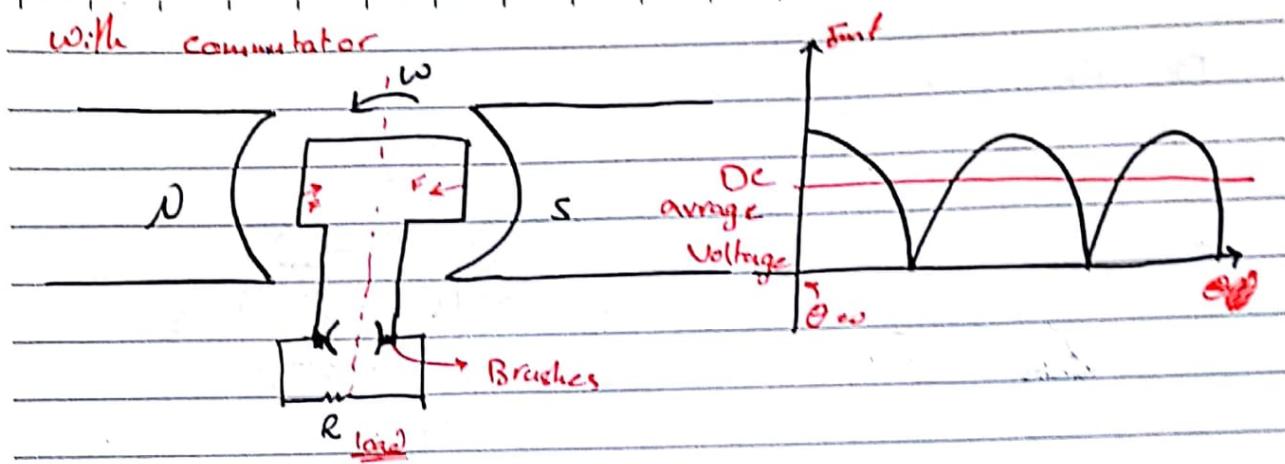
$$\text{emf} = v L \times B$$

$$= \cancel{v} L B \sin \theta$$

velocity  $\cancel{v}$  and  $B$

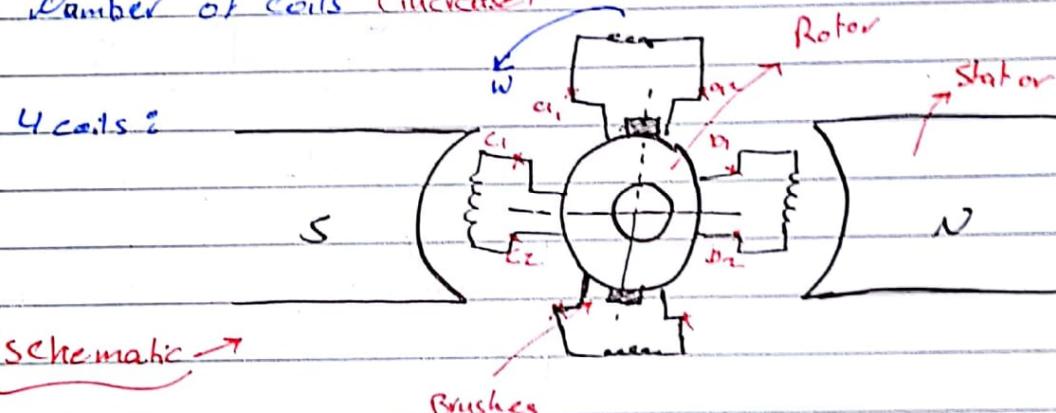


with commutator



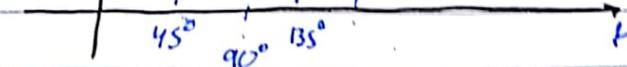
- How to improve wave form

\* Number of coils (increased)



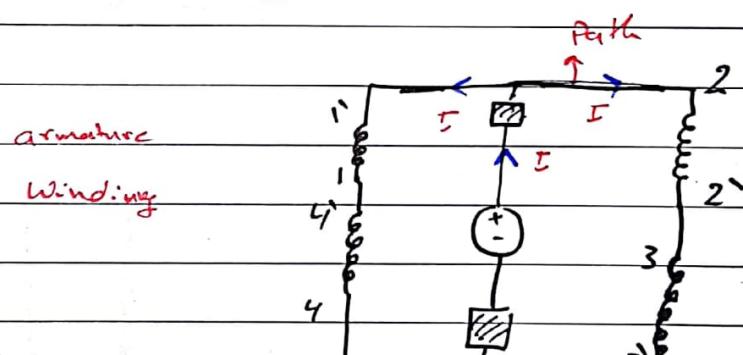
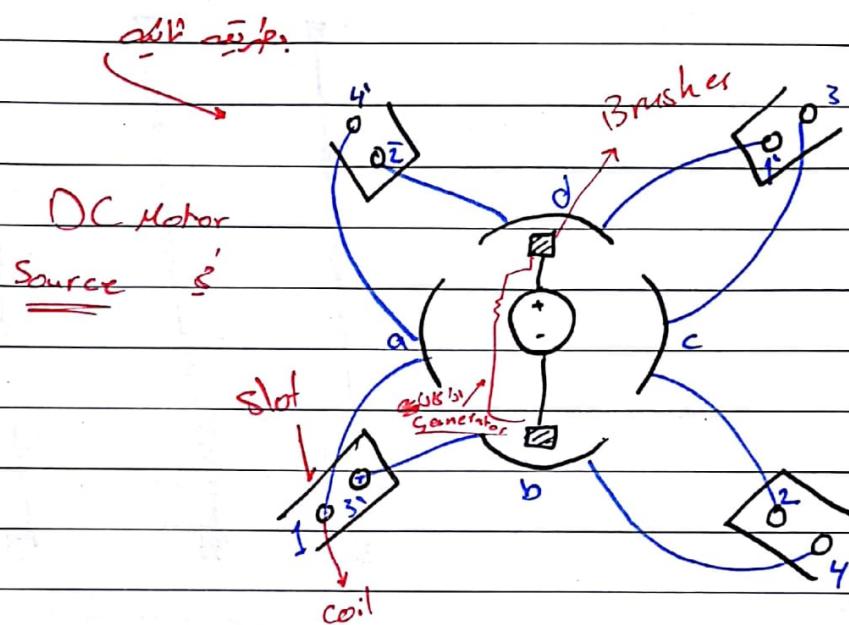
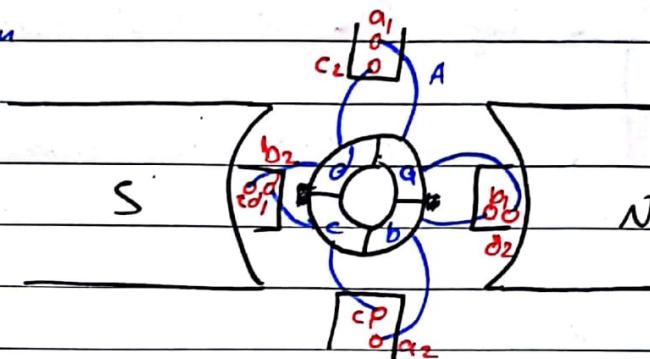
emf

$45^\circ$   $90^\circ$   $135^\circ$



Actual construction

2 coils / slot

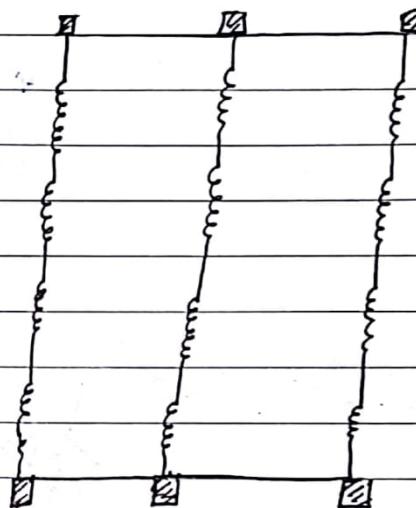


eq.  $BKT$

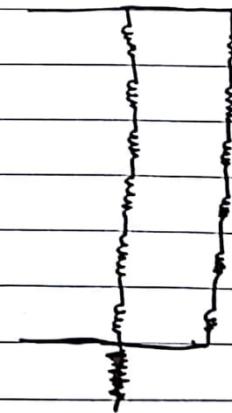
wiring  $\rightarrow$  lap ( $\rightarrow$  LV, high current)

$\rightarrow$  wave (HV, low current)

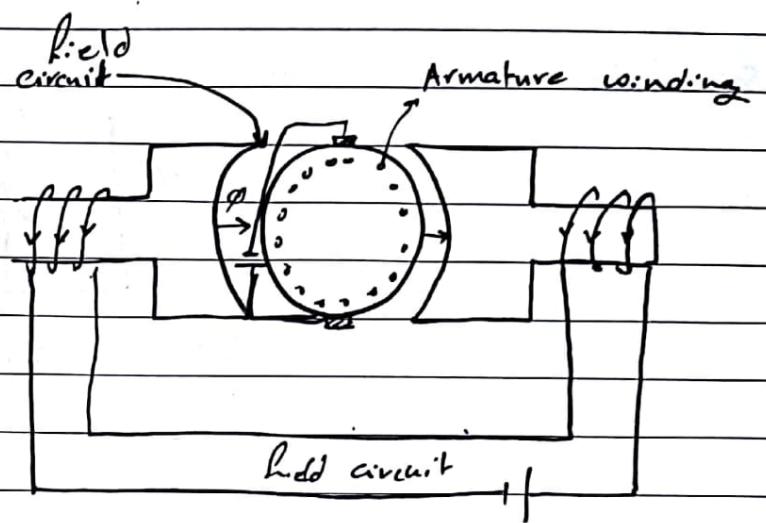
lap winding



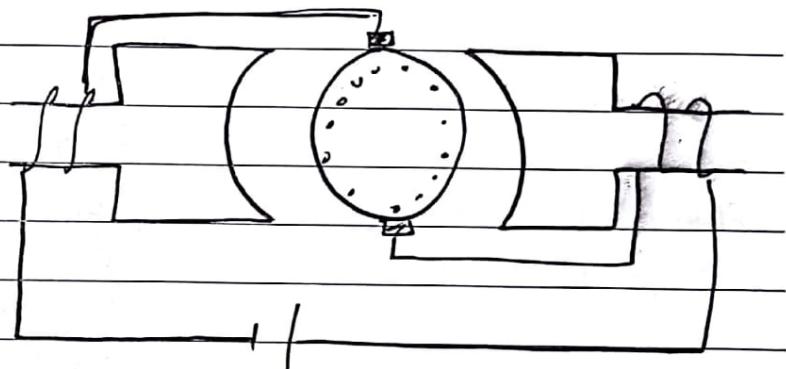
wave winding



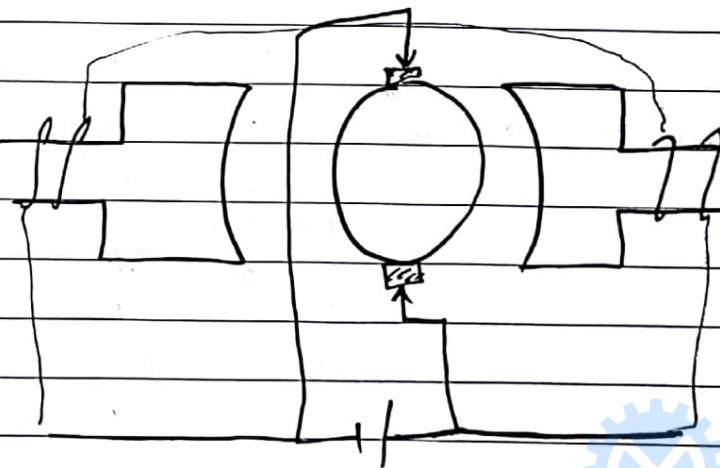
Separately excited DC motor



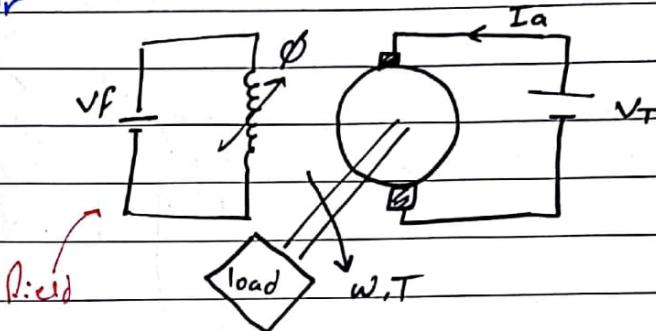
Series dc Motor



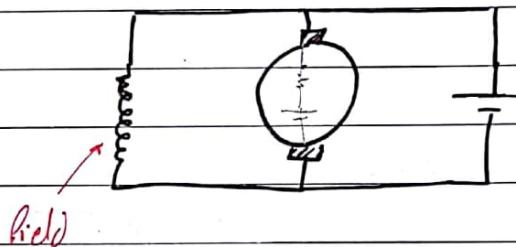
Shunt



## Separately excited Dc Motor

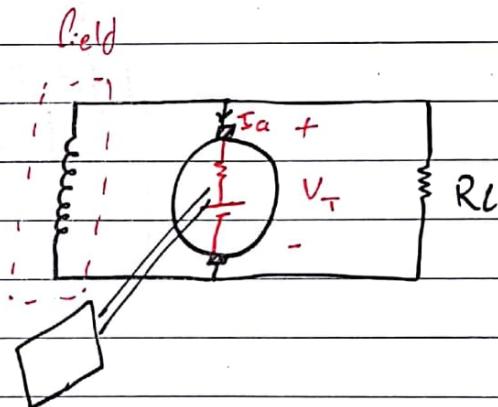


## Shunt Dc motor

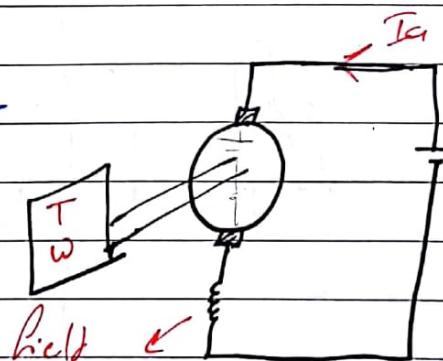


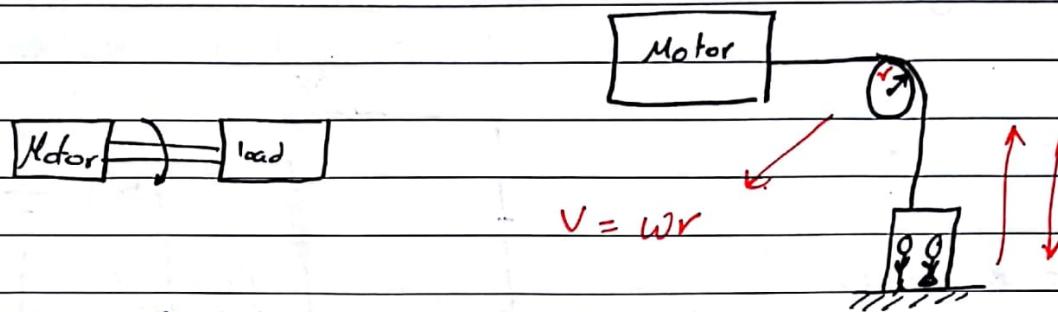
## Shunt Dc Generator

(self-excited generator)



## Pole Series Dc motor





$$T_L = \sum_m x_m^q x_r N_m$$

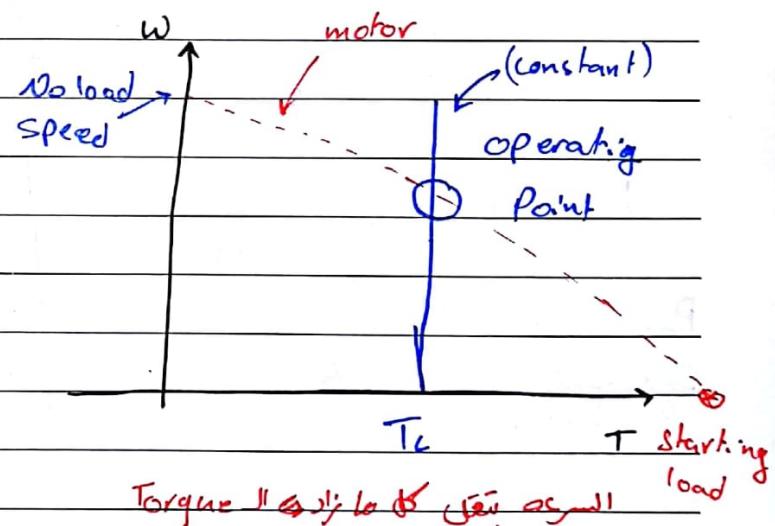
$$\frac{J_{dwg}}{d} = T_m - T_L$$

$$@ \text{Steady state} \Rightarrow T_m = T_c$$

acceleration = 0

$\Rightarrow$  Need very high torque

عندما ينبع خارج دلزم يكتنفه  
أقل من 1000 board  
Motor II



$$T_m = T_L + \int \frac{d\omega}{\delta t}$$

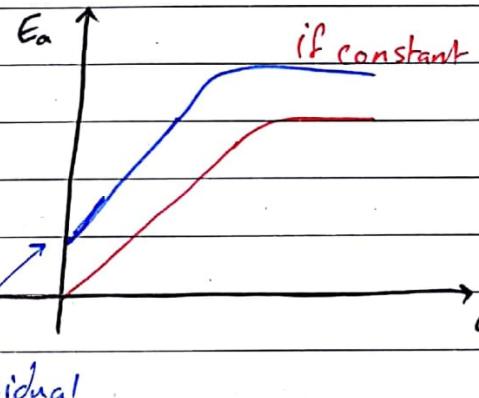
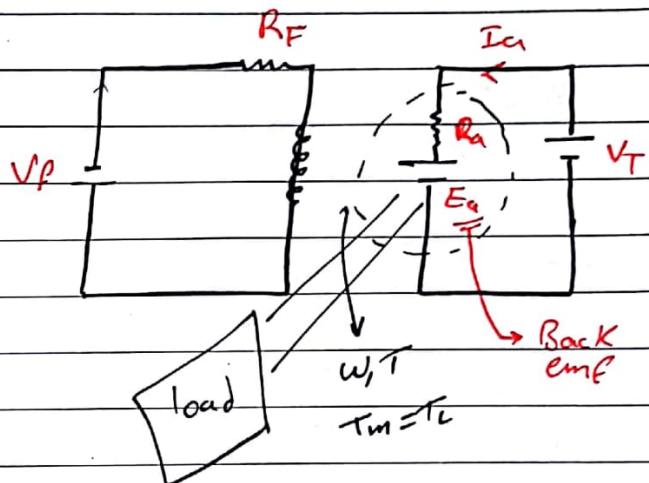
## Separately excited Dc Motor

$$E_a \text{ emf} = K_1 \phi w$$

$$= K' I_a w$$

at starting  $\Rightarrow (w=0)$

$$E_a = \text{zero}$$



$$T = K_2 \phi i_a$$

$$K = 0.3 \text{ V/rpm}$$

$$E_a = K \frac{w}{60} \rightarrow \text{Rotational speed}$$

$$w = \frac{2\pi n}{60}$$

$$P_{\text{mechanical}} = T w$$

$$\text{Developed power} = E_a T_a$$

by the motor

$$T_w = E_a I_a$$

$$T_w = K_1 \phi w \frac{T}{K_2 \phi}$$

$$T_w = \frac{K_1}{K_2} w T \Rightarrow K_1 = K_2$$

@ Rest (starting, stand)

$$\omega = 0$$

$$E_a = \text{zero}$$

$$I_a = \frac{V_t}{R_a} \uparrow$$

$$T_m = K \phi I_a \uparrow$$

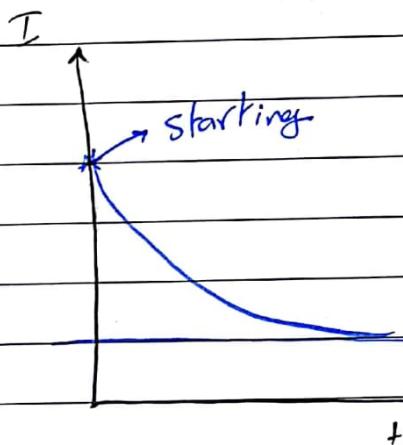
$$J \frac{d\omega}{dt} = T_m - T_L$$

acceleration

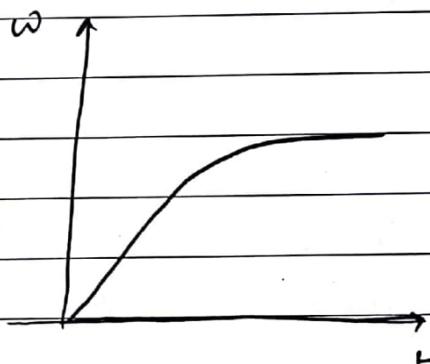
$$E_a \uparrow, I_a \downarrow, T_m \downarrow$$

$$T_m = T_L$$

$$K \phi I_a = T_L$$



$$T_{\text{starting}} = K \phi I_{a_{\text{starting}}}$$



20/12/2017

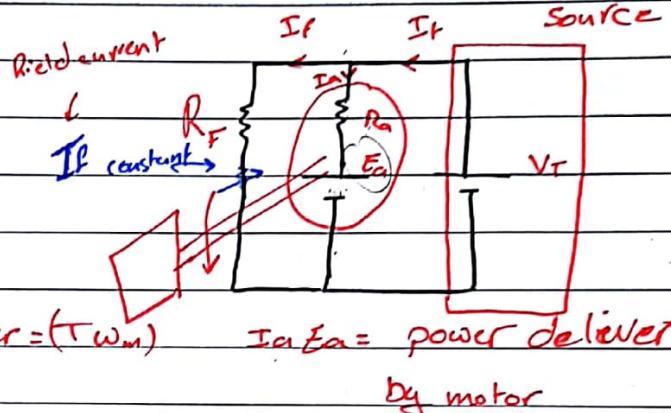
## ② Shunt DC motor

### Shunt DC motor:

$$E_a = k\phi w_m$$

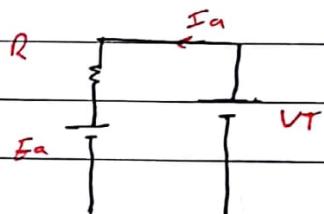
$$V_t - I_a R_a = k\phi w_m$$

$$w_m = \frac{V_t - R_a I_a}{k\phi}$$



$$\text{Power} = (T w_m)$$

$I_a E_a = \text{power delivered}$   
by motor



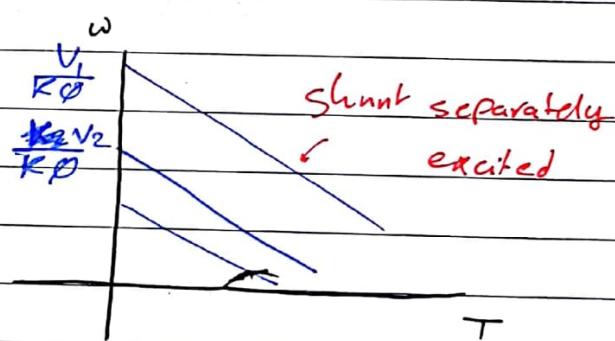
$$E_a = k\phi w_m$$

$$V_t - I_a R_a = k\phi w_m$$

$$w_m = \frac{V_t}{k\phi} - \frac{R_a}{k\phi} w_m$$

shunt  $\Rightarrow$  ip curve  $\propto$   $\frac{V_t}{R_f}$   $\propto$   $\frac{V_t}{R_f}$

not  $\propto$  separately



IP constant

\* Series DC Motor

$$I_R = I_a$$

$$I_a$$

$$E_a = K\phi w_m$$

$$E_a = V_T - I_a (R_a + R_f)$$

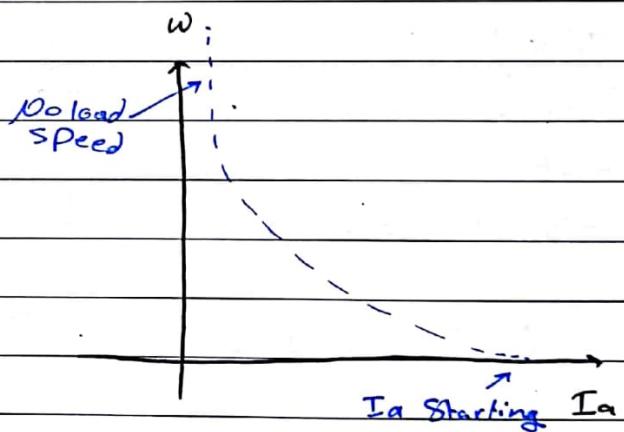
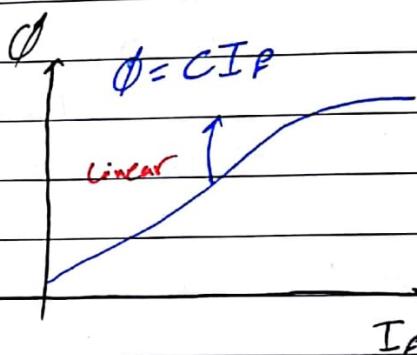
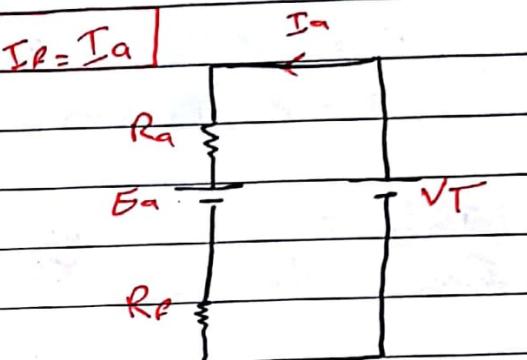
$$V_T - I_a (R_a + R_f) = K\phi m$$

$$w_m = \frac{V_T}{K\phi} - I_a \frac{(R_a + R_f)}{K\phi}$$

$$\phi = C I_a$$

$$w_m = \frac{V_T}{K C I_a} - I_a \frac{(R_a + R_f)}{K C I_a}$$

$$w_m = \frac{V_T}{K C I_a} - \frac{R_a + R_f}{K C}$$



$$T = K \phi I_a = K C I_a I_a \quad T = K \phi I_a = K C I_a I_e$$

$$T = K C I_a^2$$

$$I_{a,sh} = \frac{V_T}{R_a + R_F} \quad w=0 \quad I_a=0$$

at starting

$$I_{a,sh} = \frac{V_T}{R_a}, \quad I_F = \frac{V_T}{R_F}$$

$$T_{sh} = K C \frac{V_T}{R_F} \frac{V_T}{R_a}$$

$$T_{st} = \frac{K C (V_T)^2}{(R_a + R_F)^2}$$

$$T_{st} = \frac{K C (V_T)^2}{R_a R_F}$$

$R_p$  series  $\ll R_F$  shunt

$\Rightarrow T_{st, \text{series}} \gg T_{st, \text{shunt}}$

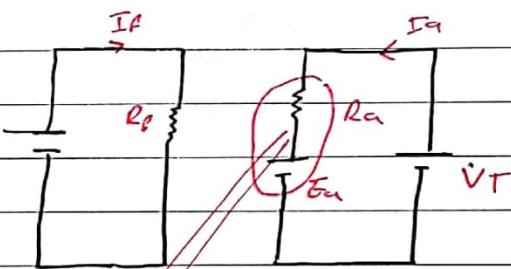
# Speed control - separately excited

\* Voltage,  $I_F$  constant

$$T_L = K \phi I_a$$

constant, constant, constant

$$\Rightarrow I_a = \text{constant}$$



$\Rightarrow V_T \uparrow, I_a \text{ constant}, E_a ??$

$$E_a = V_T - \frac{I_a R_s}{I_p} \Rightarrow E_a \uparrow$$

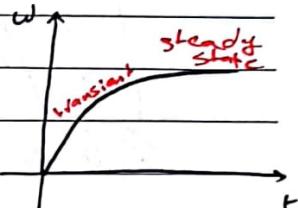
$$E_a = K \phi \omega \Rightarrow \omega \uparrow$$

④ Starting

$$\omega = 0 \Rightarrow E_a = 0$$

$$I_a \uparrow \Rightarrow T_m \uparrow \Rightarrow \frac{d\omega}{dt} = T_m - T_L \Rightarrow E_a \uparrow \Rightarrow I_a \uparrow$$

$$\Rightarrow \text{steady state } T_a \Rightarrow T_L = K\phi I_a$$



\* Field - weakening ( $|I_a|$ ),  $V_T$  fixed, speed  $\uparrow$

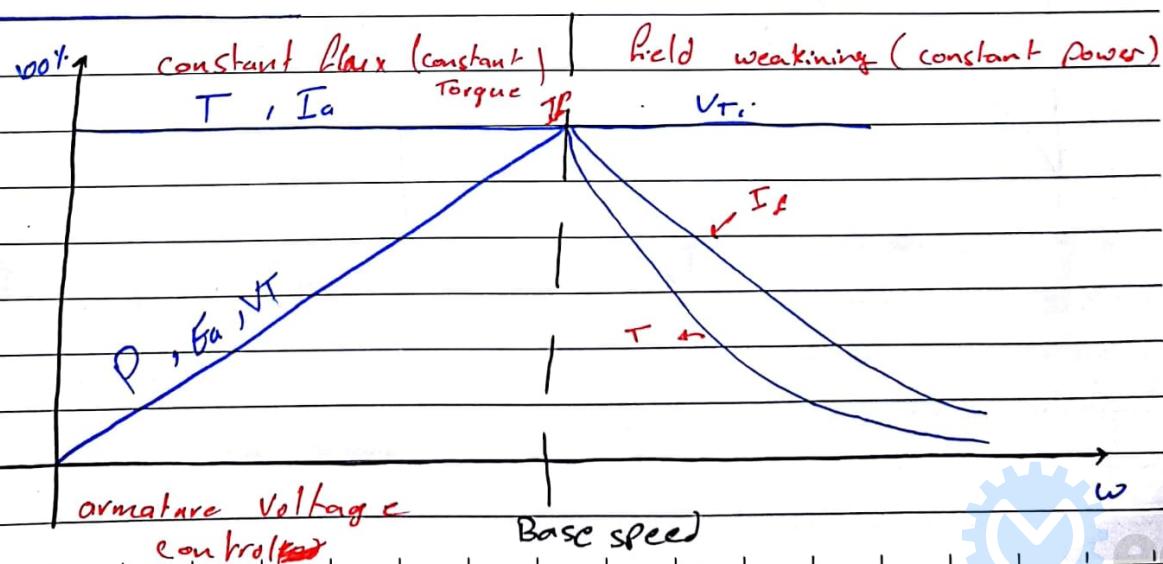
$$\omega = \frac{V_T}{K\phi} - \frac{R_a}{K\phi} I_a$$

$$\text{constant } T = K\phi T_a, \phi \downarrow \Rightarrow I_a \uparrow$$

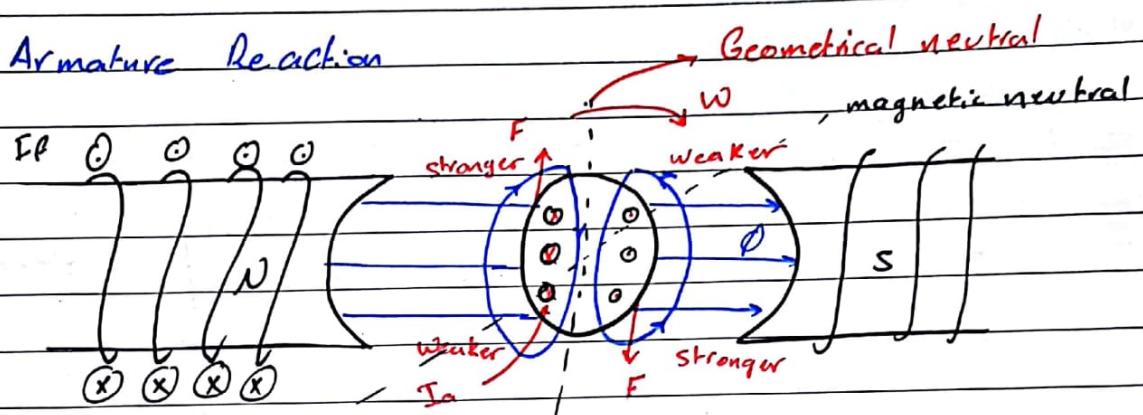
To torque

$$E_a = V_T - I_a R_a \quad (\text{almost fixed})$$

$$\frac{E_a}{\text{constant}} = K\phi \omega_n \Rightarrow \omega_n \uparrow$$

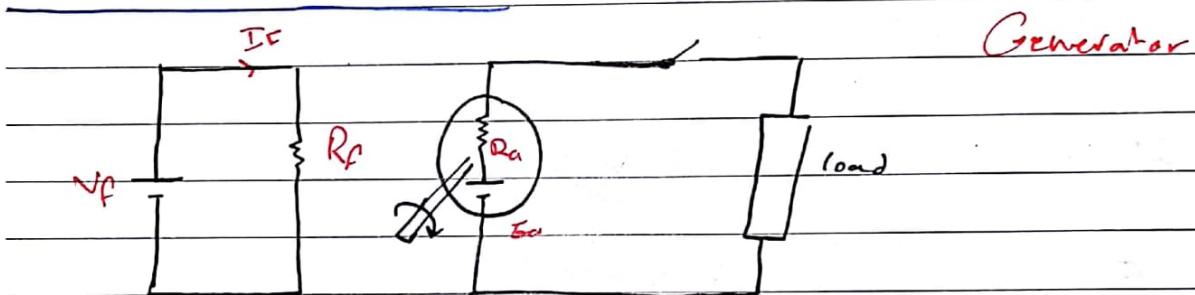


## Armature Reaction

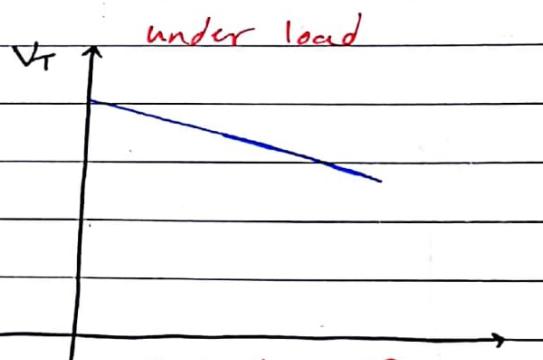
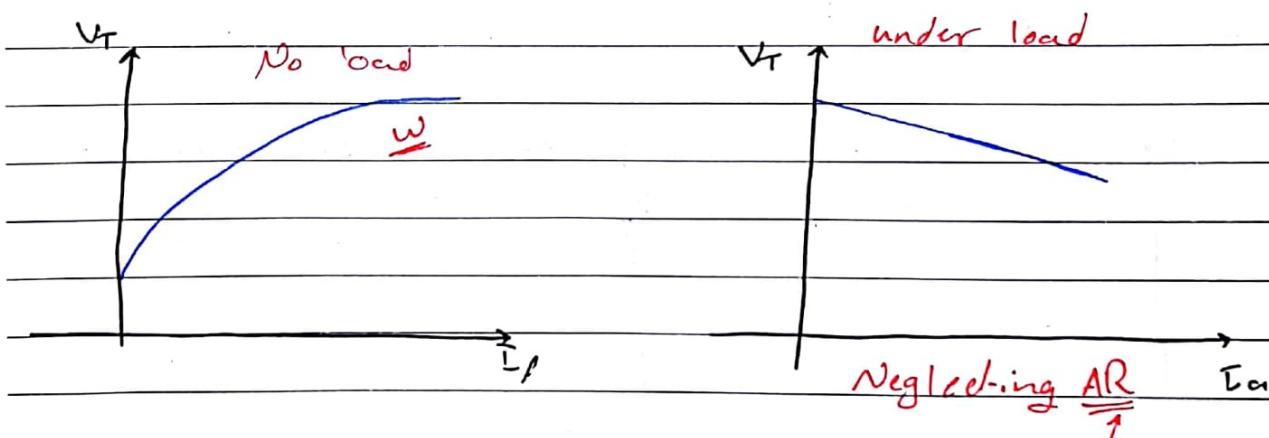


## Generator

$\Rightarrow \phi \downarrow$   $\leftarrow$  field flux



(Switch open) No load  $\Rightarrow V_T = E_a$ ;  $I_a = 0$



Neglecting AR  
 $\uparrow$

Armature  
Reaction

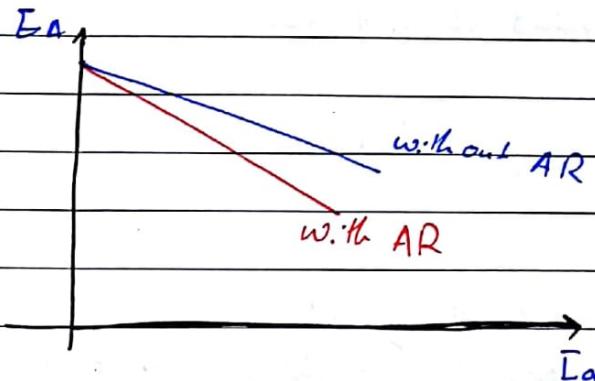
under load  

$$V_T = E_a - I_a R_a$$

with Armature Reaction (AR)

AR  $\Rightarrow$  field weakening

$E_a \downarrow$



Ex<sup>o</sup> 220 V, dc shunt motor

AR  $\leftarrow$  WCP to

$R_a = 0.2 \Omega$ ,  $R_f = 110 \Omega$

Neglect  $J_m$

at No load  $\Rightarrow$  <sup>No load speed</sup> 1000 rpm, total line current 7 A

at Full load  $\Rightarrow$  input Power 11 kW

\* air gap flux fixed at its value at no load

V speed, speed regulation, developed Torque at Full load

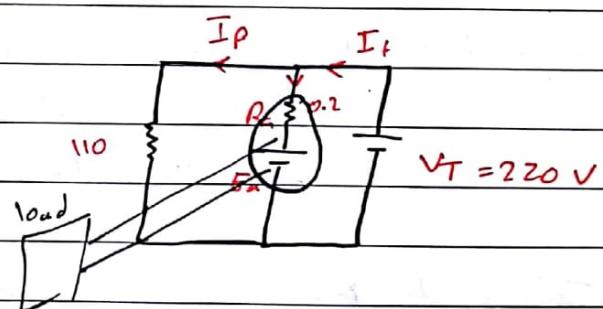
at no load

$\frac{V_T}{I_P} = 7-2$

$$I_P = \frac{220}{110} = 2 A \Rightarrow I_a = 5 A$$

Fixed

at Full load (Power = 11 kW)



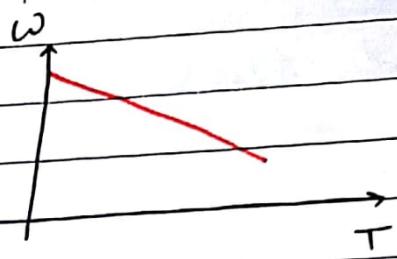
$$P_{terminal} = V_T I_T$$

$$11kW = 220 \times I_T \Rightarrow I_T = 50 A$$

$$|I_A|_{\text{Full load}} = 48$$

$\uparrow$   
50 - 2

at no load ; speed = 1000 rpm



1) speed at full load

$$\omega = \frac{V_T}{K\phi} - \frac{R_a}{K\phi} I_a$$

at no load <sup>rad/s</sup>

$$\frac{1000 \times 2\pi}{60} = \frac{220}{K\phi} - \frac{0.2 \times 5}{K\phi} \Rightarrow K\phi = 11$$

$$\frac{\omega_{full}}{full \text{ load}} = \frac{220}{K\phi} - \frac{0.2 \times 48}{K\phi} = 960 \text{ rpm}$$

another way to solve:

$$\frac{\omega_{full}}{\omega_{no}} = \frac{220 - 0.2 \times 48}{220 - 0.2 \times 5} \Rightarrow \frac{\omega_{full}}{full} = \frac{2104}{219} \times \frac{1000 \times 2\pi}{60} = 960 \text{ rpm}$$

\* speed regulation =  $\frac{|\omega_{no}| - |\omega_{full}|}{|\omega_{full}|} \times 100\%$

$$= \frac{1000 - 960}{960} = 4.09\%$$

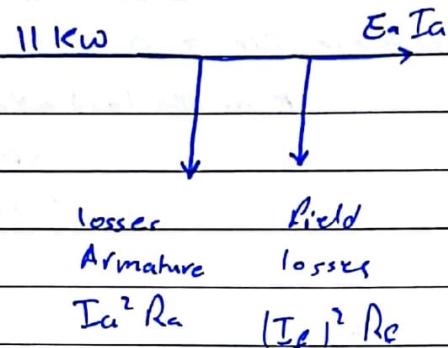
\* Torque developed by the motor at full load

$$T_d = \underline{K\phi} I_a \underline{\text{full}} = 100 \times 3.8 \text{ N.m}$$

another way

Power developed by the motor  $P_{full}$   
=  $I_a V_a$

$$* E_a = 220 - (48 \times 0.2) = 20.4 \text{ V}$$
$$\Rightarrow$$



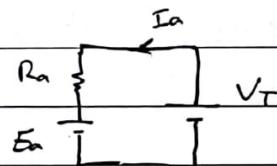
$$\text{Power} = T \omega = E_a I_a \Rightarrow T = \frac{E_a I_a}{\omega} = \frac{210.4 \times 48}{960 \times 2 \pi / 60} = 100.38 \text{ N.m}$$

$\downarrow = 10.1 \text{ kW}$

\* ID separately excited and  $P_0$  was delivered

$$(V_T I_a - I_a^2 R_a = P_0) \leftarrow \stackrel{\text{loop}}{=} \quad \text{نقطة المخرج}$$

$$220 I_a - 0.2 I_a^2 = 10.1$$



\* Starting Torque if  $I_a$  is limited to 150% of full load

$$T_{st} = K \phi T_a$$

$$I_a = 1.5 \times 48 = 72 \text{ A}$$

$$T_{st} = K \phi T_a = 150.57 \text{ N.m}$$

another way

$$T_{st} = K \phi I_a / \text{start}$$

$$T_{st} = K \phi I_a / I_{full}$$

$$\Rightarrow \frac{T_{st}}{T_{st}} = 1.5$$

$$\Rightarrow T_{st} = 1.5 \times 100.38 = 150.57 \text{ N.m}$$

with out limitation

$$I_a / = 220 / 0.2 = 1100 \text{ A}$$

starting

$$W = 0 \Rightarrow E_a = 0$$

$\Rightarrow 23$  times  $I_{full}$  load

in large sizes it is often 1.5 to 2.0

coil may be directly connected

\* Consider Armature Reaction, it reduces air gap Flux by 50%  
 calculate Speed at full load \* at No load  $\rightarrow$  the same speed

$$w = \frac{V_T - R_a I_a}{K\phi}$$

(Ans)

$$w_{old} = \frac{220 - 0.2 \times 48}{K\phi} \text{ without AR}$$

$$w_{new} = \frac{220 - 0.2 \times \frac{48}{2}}{0.5 K\phi} \text{ with AR} \rightarrow \text{Torque Reduced by half}$$

$w \rightarrow$  increased

$$\frac{w_{new}}{w_{old}} = 2$$

\*  $\eta = \frac{\text{Power output}}{\text{Power input}} \times 100\% \text{, all day } \eta = ??$   
 $= \eta + \text{time}$

1 Rotational losses = 500 W  
 No load  $\rightarrow$

$$\Rightarrow \eta = \frac{10.1 \times 10^3 - 500}{11 \times 10^3} \times 100\%$$

\* Speed can be reduced by adding Resistance

$$w = \frac{V_T}{K\phi} = \frac{R_a I}{(K\phi)^2}$$

27/Nov/2017

3)  
4)  
5)

4) 440 V DC shunt machine  $R_F = 110 \Omega$ ,  $R_a = 0.15 \Omega$

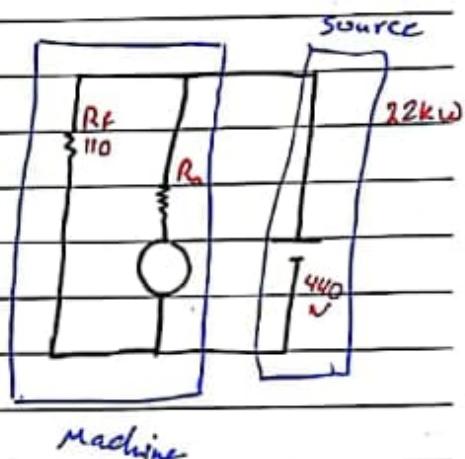
5) ① Power delivered by the machine if it absorbs 22 kW  
~~Rated 110V 440V~~ motor

6) ② Power delivered by the machine if it supplies 22 kW @ 440V

$$I_F = \frac{22 \text{ kW}}{440} = 50 \text{ A}$$

$$I_F = \frac{440}{110} = 4 \text{ A}$$

$$I_a = 50 - 4 = 46 \text{ A}$$



1) Power delivered =  $E_a I_a$

$$E_a = 440 - 46 \times 0.15 = 433.1 \text{ V}$$

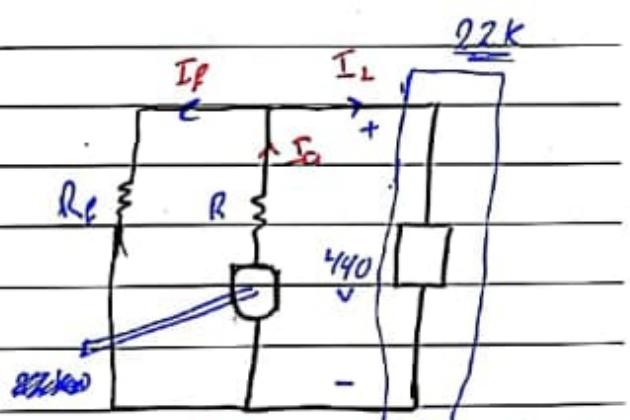
$$\text{Power delivered} = 433.1 \times 46 = 19.9 \text{ kW}$$

$$\eta = \frac{19.9}{22} =$$

2)  $I_L = \frac{22 \text{ kW}}{440} \rightarrow I_L = 50 \text{ A}$

$$I_F = \frac{440}{110} = 4 \text{ A}$$

$$I_a = 50 + 4 = 54 \text{ A}$$



$$E_a = 54(0.15) + 440 = 448 - 440 + 8 = 8 \leftarrow \text{Voltage drop +}$$

$$P = 448 \times 54 = 24.9 \text{ kW}$$

$$\eta = \frac{22 \text{ kW}}{24.9} \times 100\% =$$

$$\text{W uj } E_a \text{ J uj } 0.15 \text{ +}$$

$$\text{Flux } (\phi) \text{ J uj } 0.15 \text{ +}$$

Ex) 230 V, DC motor (series) :  $R_a = 0.2 \Omega$

$$R_d = 0.05 \Omega$$

$$\text{Series} \rightarrow T_a = T_p$$

$$\leftarrow T_A = 20 \text{ A}, 1500 \text{ rpm}$$

Proportional losses = 400 W

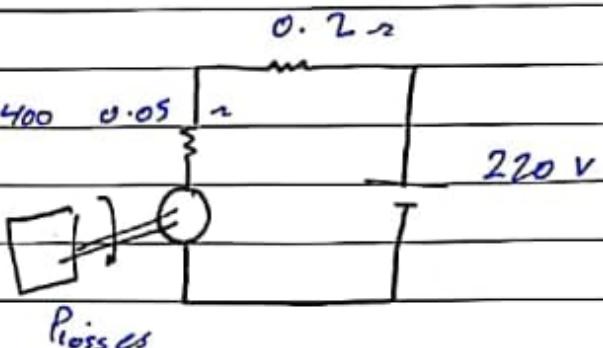
$$\eta = ??$$

$$P_{in} = 230 \times (20) = 4.6 \text{ kW}$$

$$P_o = P_{in} - P_{\text{losses}} - \text{Proportional}$$

$$\Rightarrow 4600 - (20^2 \times 0.2 + 20^2 \times 0.05) - 400 = 4.6 \text{ kW}$$

$$\eta = \frac{P_o}{P_{in}} \times 100\% = 89.1\%$$



Ex) 230 V, DC shunt,  $R_a = 0.05 \Omega$ ,  $R_f = 75 \Omega$

$I_f$  A \* @ 1120 rpm  $\rightarrow$  motor draws  $7 \text{ A}$

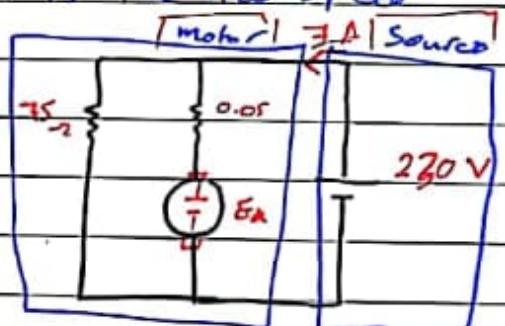
$I_f$  A \* @ certain constant operating  $\rightarrow$  line current  $4.6 \text{ A}$

3)  $\rightarrow$   $\eta$  is  $\rightarrow$  motor speed ??

2)  $R_a$  is increased to  $100 \Omega$ , what is the new speed

$$I_f = 230 \text{ V} = 3.07 \text{ A}$$

$$75 \Omega$$



$$E_a = 230 - (3.07 \times 0.05)$$

mechanical speed rpm

$$E_a = K \Phi \omega \Rightarrow E_a = K' I_f N$$

$$K' = \frac{E_a}{I_f N} \Rightarrow K' = 0.0668$$

B

$$E_a = K' T_p N_m$$

$$I_a = 46 - 307 = 42.93 A$$

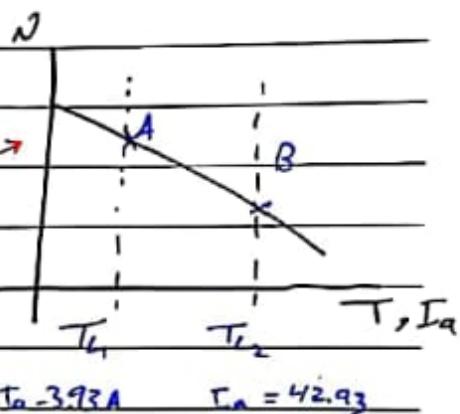
$$E_a = 220 - (42.93 \times 0.05) \Rightarrow N_m = \frac{1111}{1473} \text{ rpm}$$

(1120 rpm) A وأعلى عزم دوران

$$\omega = \frac{V_T}{K\phi} - \frac{R_a}{K\phi} T_a$$

$$\omega = \frac{V_T}{K\phi} - \left( \frac{R_a}{(K\phi)} \right) T$$

slope

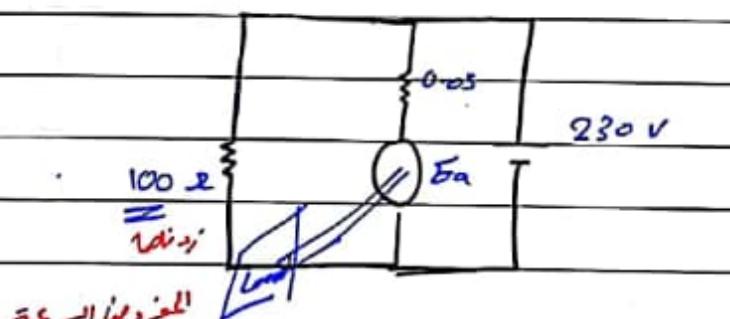


$$T_a = 3.93 A \quad T_m = 42.93$$

$$R_p \uparrow \Rightarrow I_p \downarrow \Rightarrow \omega \uparrow$$

$$I_p = \frac{230}{100} = 2.3$$

$$I_a = 46 - 2.3 = 43.7$$



$$E_a = 230 - (43.7)(0.05)$$

between the V and ground Field winding

Power at no load

زد current or load resistance

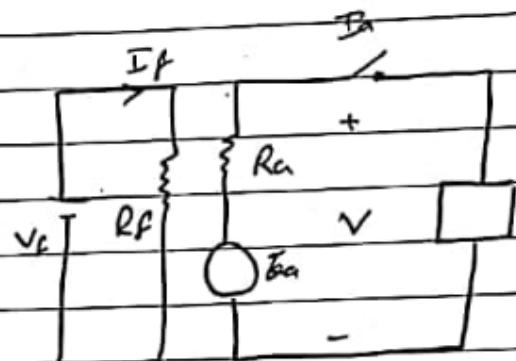
$$P = E_a I_a$$

(losses and load current) Rate of losses and load current  $P = T \omega$

Separately excited DC

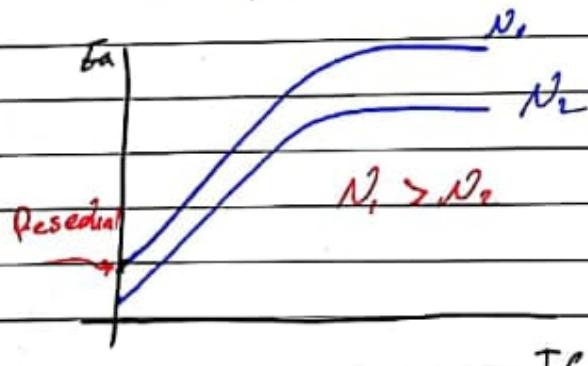
open circuit C/s

load C/s



$$If \rightarrow \phi$$

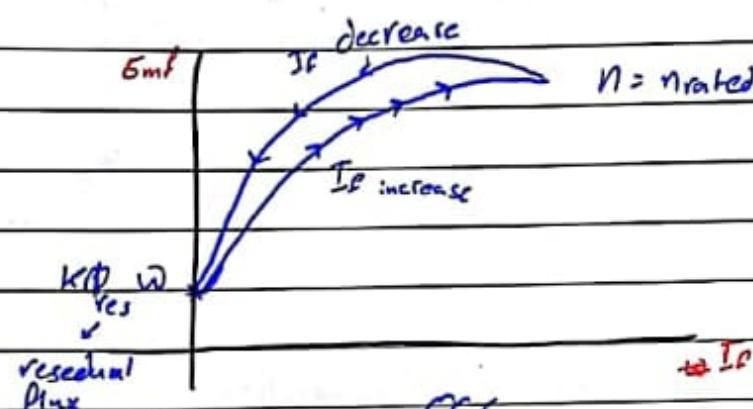
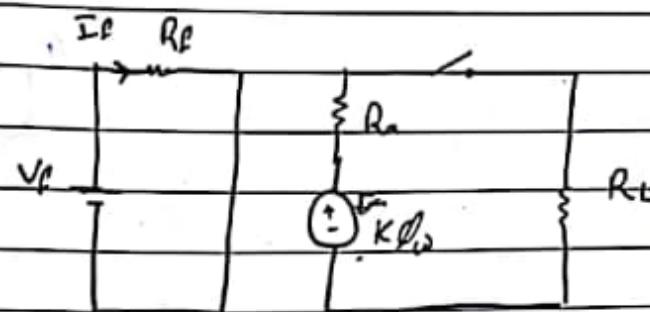
$$E_a = \phi \text{ and } \omega$$



$$E_a = K \phi \omega$$

$$E_a = K C I_f \omega$$

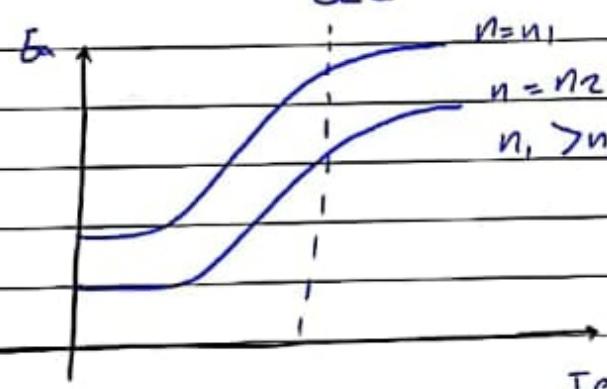
## Open circuit C/s



$$\mathcal{E}_a = K\Phi_w$$

Same  $I_F$

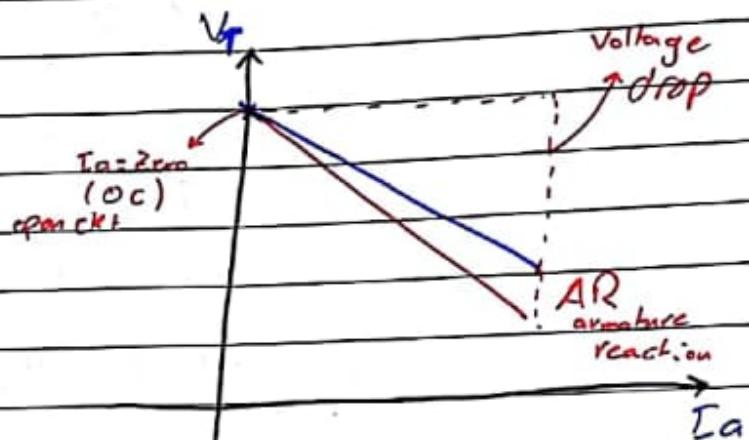
$$\frac{\mathcal{E}_{a2}}{\mathcal{E}_{a1}} = \frac{\omega_2}{\omega_1} - \frac{2V_n}{n_1}$$



load c/s

$$\frac{V_R}{N_L} = \frac{V_R}{F_L} - \frac{V_R}{F_L}$$

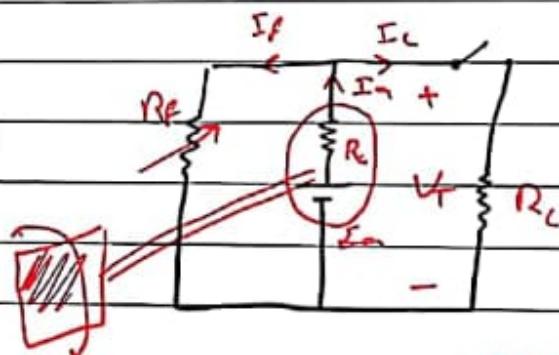
$V_R$  at no load  
 $F_L$  at full load  
Voltage Reversing



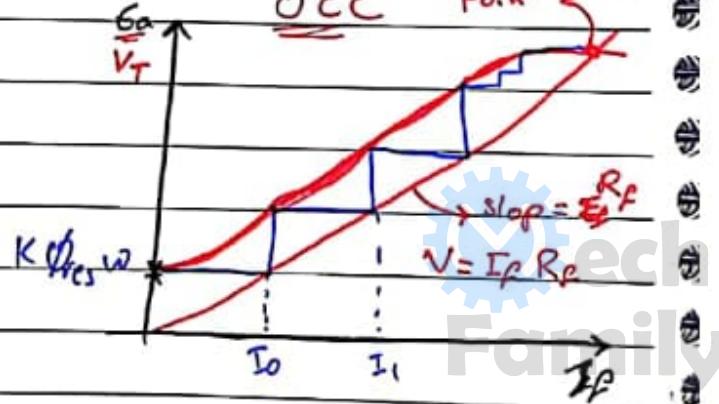
\* armature reaction increases voltage drop

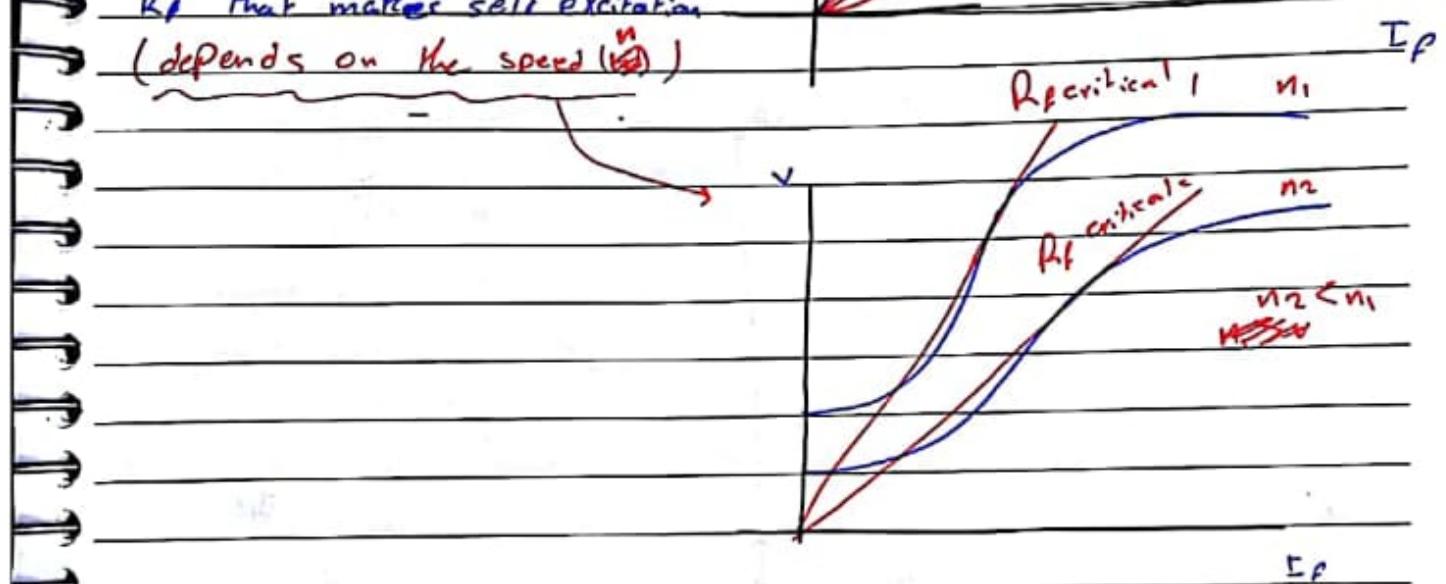
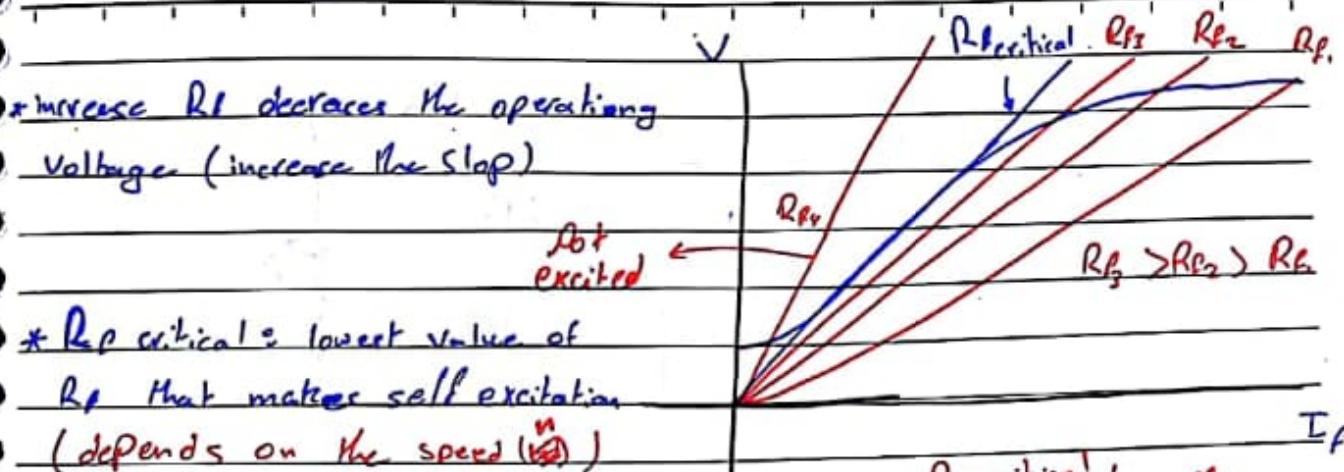
Self-excited DC generator

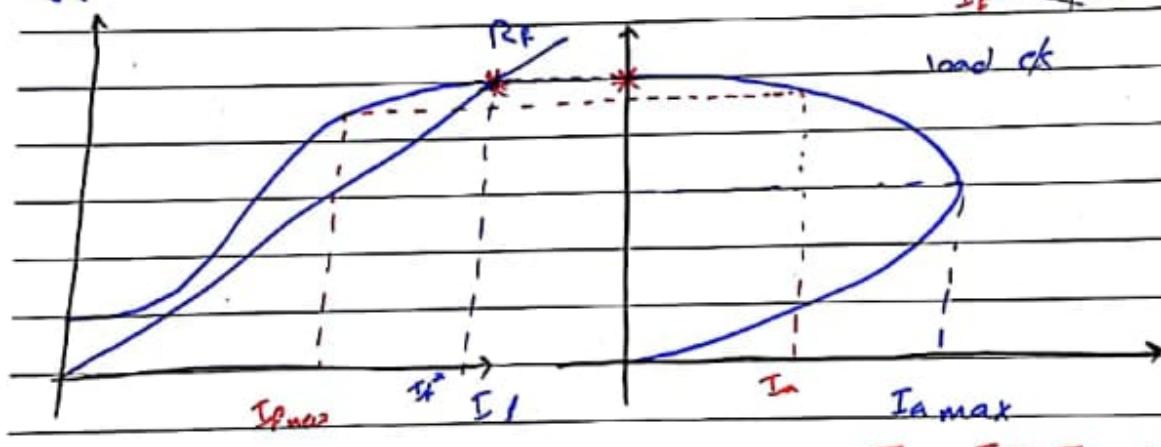
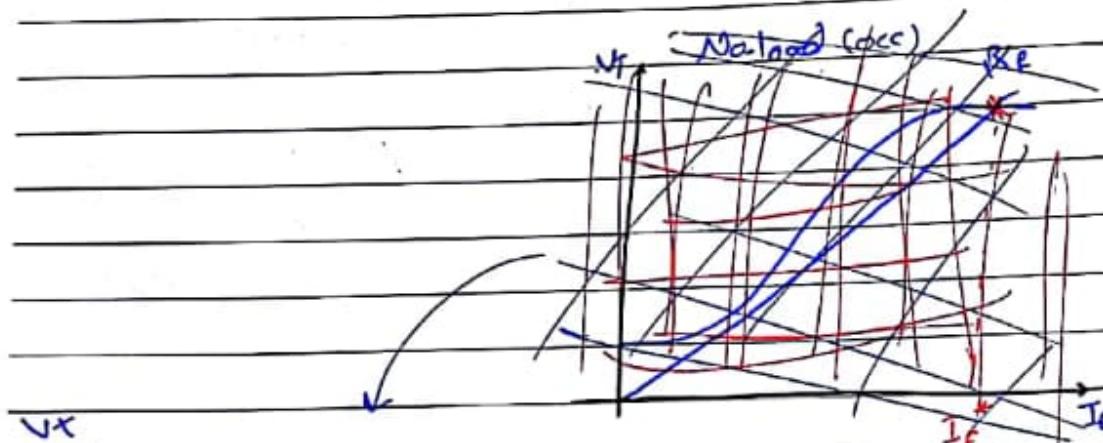
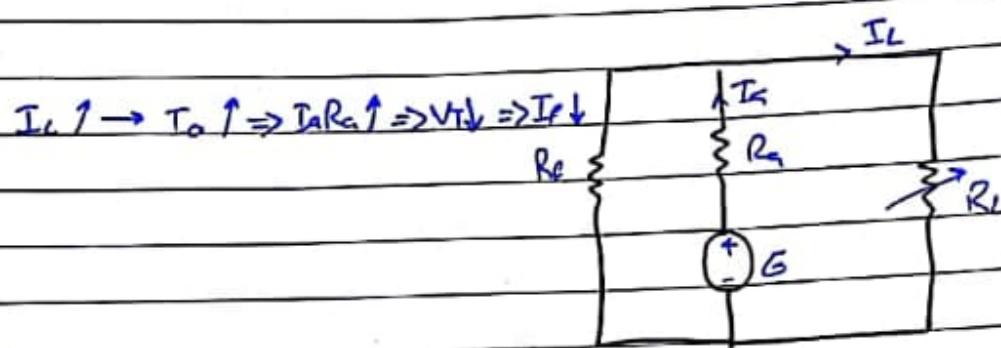
No need for independent source for the excitation (I\_e) excitation current



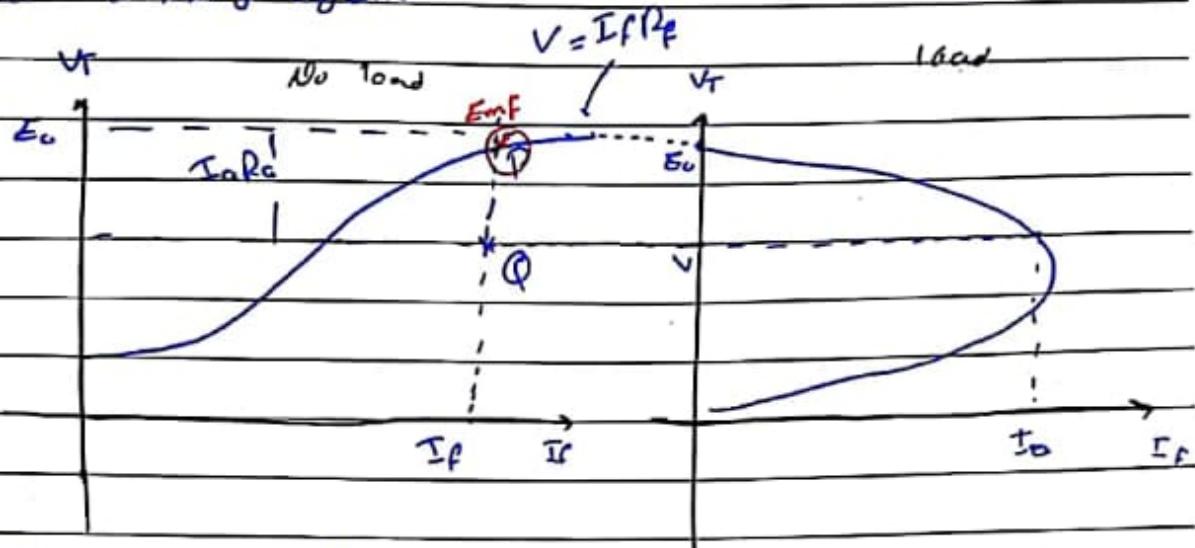
operating point controlled by  $R_f$







The same Mag again



- 1)\* Assume  $V$  and find  $I_a$
- 2)\* Draw horizontal line and find  $I_f$  // point Q //
- 3)\*  $Emf \{ P \}$
- 4)\*  $P - Q \Rightarrow I_a R_a$
- 5)\*  $R_a$  Known  $\Rightarrow I_a$  will be Known

Ex) OCC of a DC shunt generator 300 rpm

$I_F$ (A)	0	2	3	4	5	6	7
armature voltage	75	92	132	162	183	190	212

① Plot OCC at 375 rpm

② Determine the voltage to which the machine will excite if field circuit resistance is 40  $\Omega$

③ What additional  $R$  in the field ckt to reduce voltage to 200V at 375 rpm

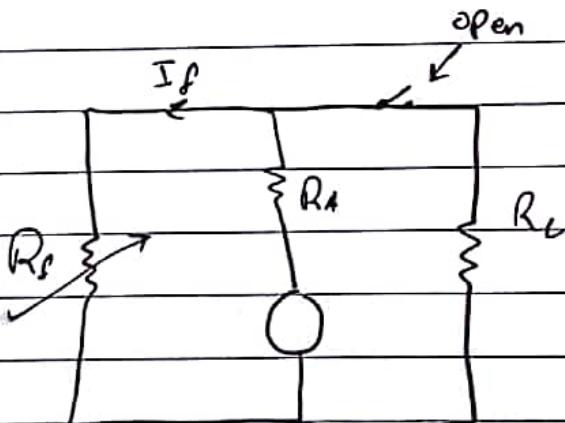
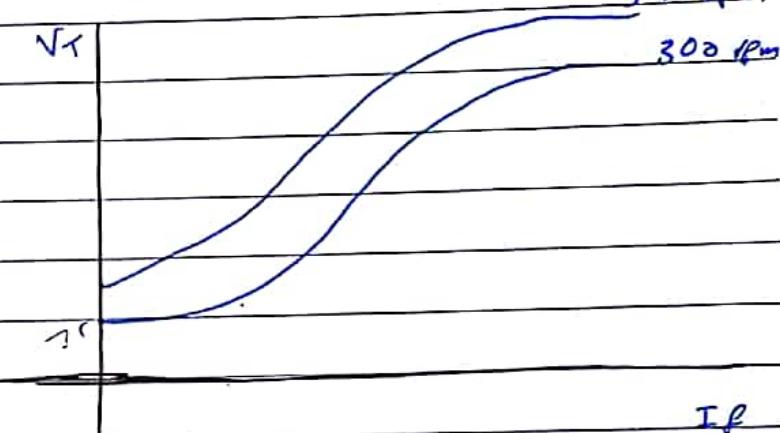
$$\mathcal{E}_a = K\phi w_m = K' I_F w_m$$

$$\frac{V_T, \text{new}}{V_T, \text{old}} = \frac{I_F, \text{new} w_m, \text{new}}{I_F, \text{old} w_m, \text{old}}$$

$$\Rightarrow I_F = 3A \Rightarrow 300 \text{ rpm} \rightarrow V_T = 132V$$

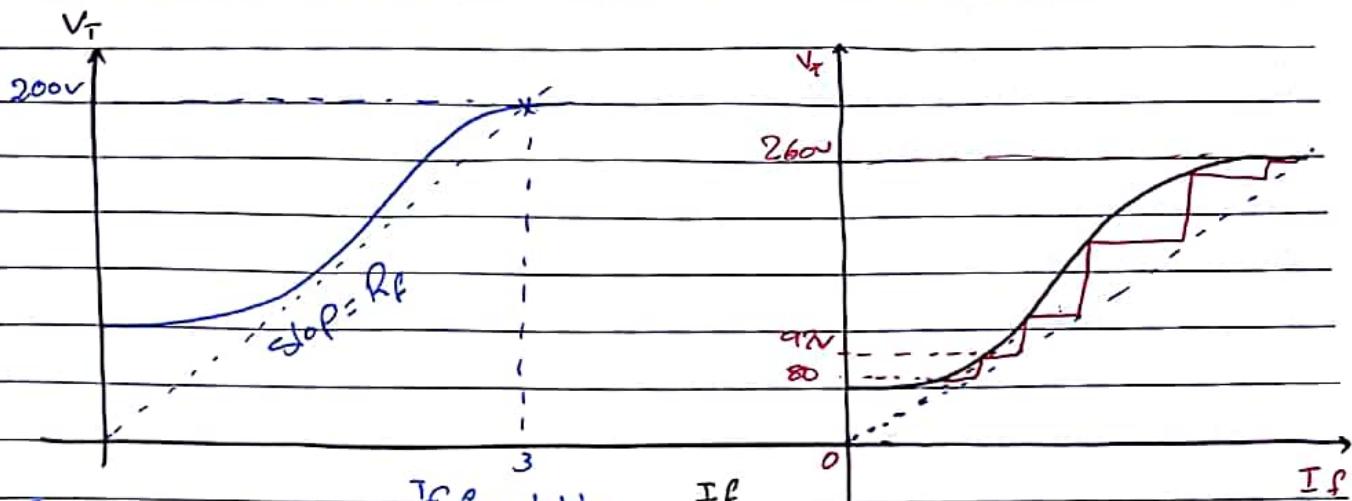
$$375 \text{ rpm} \rightarrow V_T = ?$$

$$\rightarrow V_T, \text{new} = 132 + \left( -\frac{375}{300} \right)$$



at 375

$I_f$ (A)	0	2	3	4	5	6	7
$V_T$ (V)	9.4	115	202.5	228.8	248.3	265	



$$R_f = 52.6 \Omega$$

$$52.6 - 40 = 12.6 \Omega$$

additional ( $R$ )

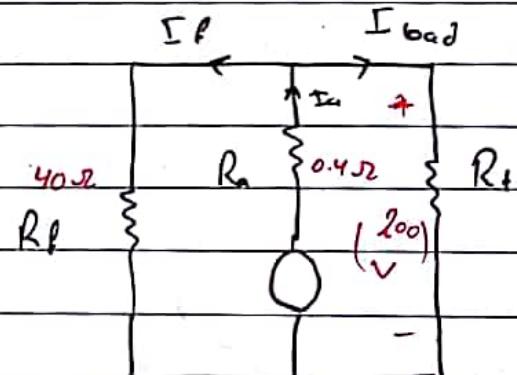
# with out additional Resistance, Determine load current supplied by the generator with its terminal voltage 200 v? assume  $R_a = 0.4$

$$375 \text{ rpm} \quad \text{field current} = \frac{200}{40} = 50 \text{ A}$$

$$\mathcal{E}_{mf} = 228.8 \text{ V}$$

$$I_a = \frac{228.8 - 200}{0.4} = 72 \text{ A}$$

$$I_{load} = 72 - 5 = 67 \text{ A}$$



$I_f$	0	2	3	4	5	6	7
$V$	9.4	115	150	202.5	228.8	248.8	265

Ex) dc shunt generator (table @ 600 rpm)

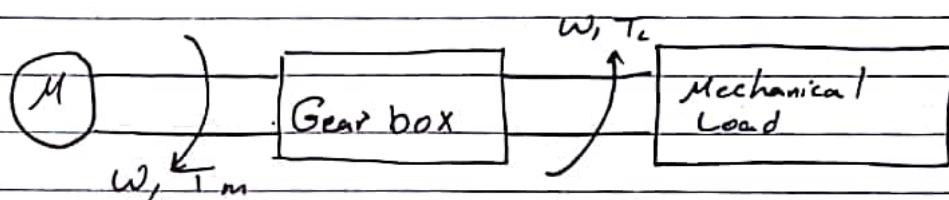
$I_F$	1	2	3	4	5	6	7	8
$V$	23	45	67	85	100	117	121	126

$R_f = 15 \Omega$ ,  $V_f = 120 V$ , load current ??

$$R_a = 0.02 \Omega$$

$$I_L = 292 A$$

What is the gear box?

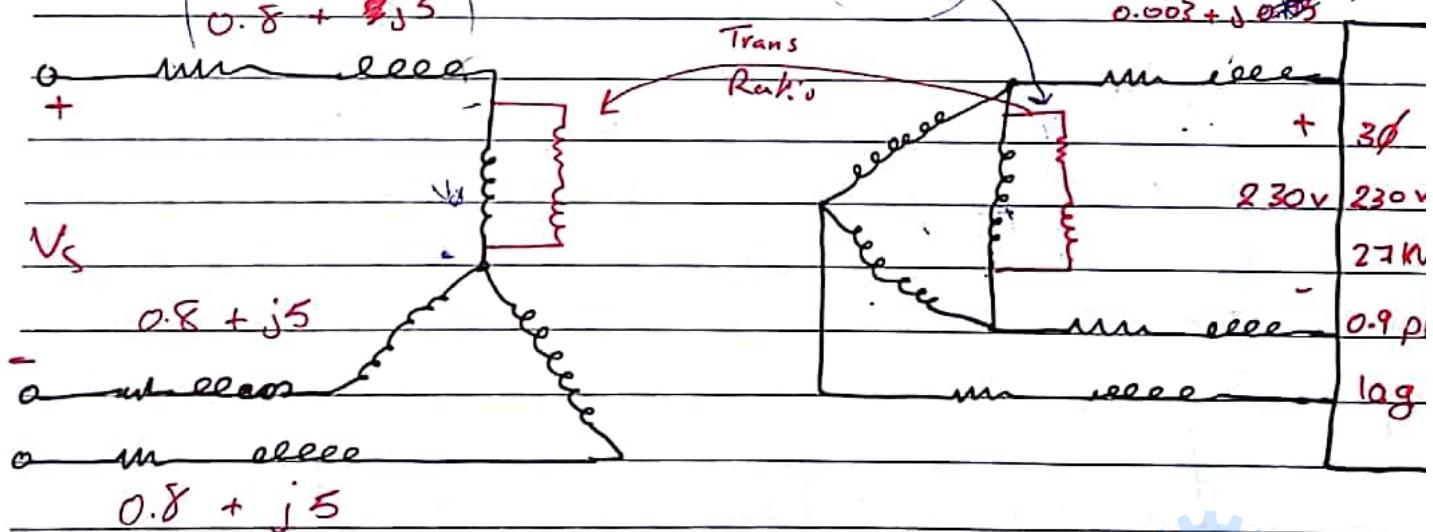


## ✳ Transformer

Ex) Three phase, 1φ (1330, 230 V),  $\frac{Z_1}{1\phi \text{ LV}} = 0.12 + 0.25 j \Omega$

$$| 0.8 + j5 |$$

$$| 0.003 + j 0.005 |$$



$$V_{LL} = \sqrt{3} V_p, \quad V_{LL} = V_p$$

$$Z \text{ seen from source} = (0.003 + 0.015j) * \frac{(1330 + \sqrt{3})^2}{230} +$$

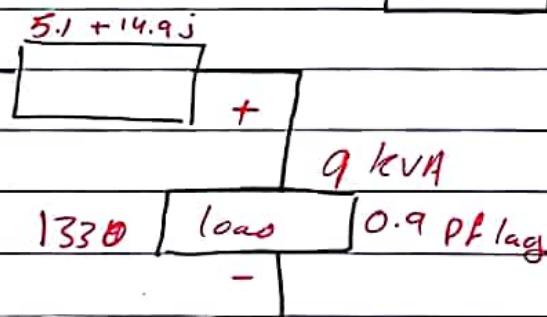
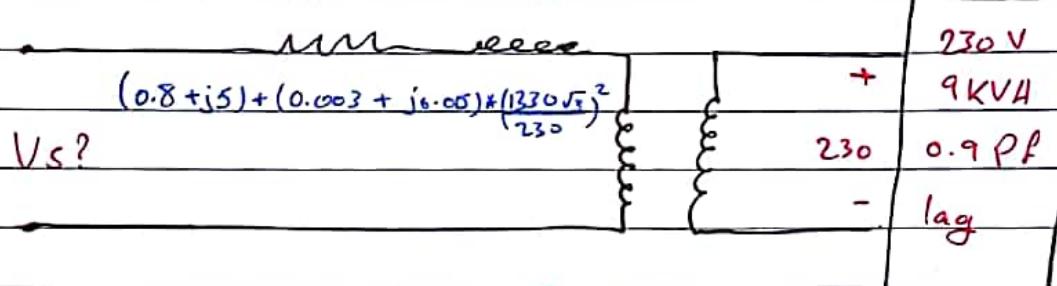
$$(0.12 + 0.25j) * \frac{(1330)^2}{230} + (0.8 + j5) = 51 + 14.9j \Omega$$

$$(3, 10) (1330 / 230) Y - \Delta$$

$$1330 * \sqrt{3} / 230 \text{ V}$$

### Single phase

$$5.1 + 14.9j$$



$$I = 6.8 \angle -25.8^\circ$$

$$|V_s| = 1407 \text{ V}$$

Phase

$$|V_s| = 2434 \text{ V}$$

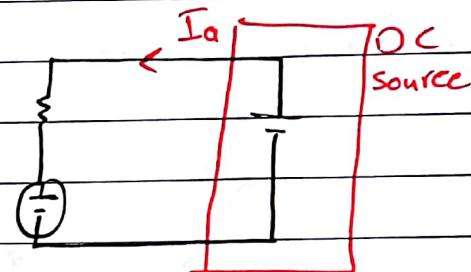
11/Dec/2017

Monday

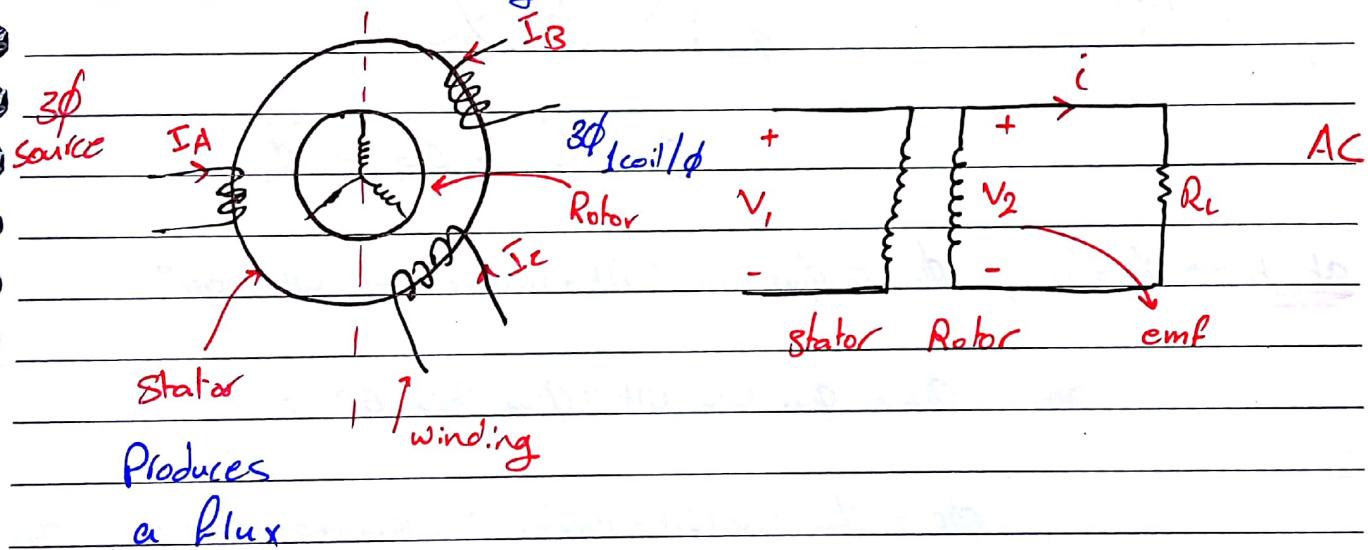
## Induction motor (3Ø)

AC  $\rightarrow$  inexpensive / easy to maintain / reliable

DC motor  $\rightarrow$  power is conducted directly to the armature  
easy controlled

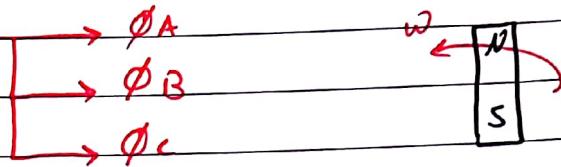


\* Induction  $\rightarrow$  Rotating transformer



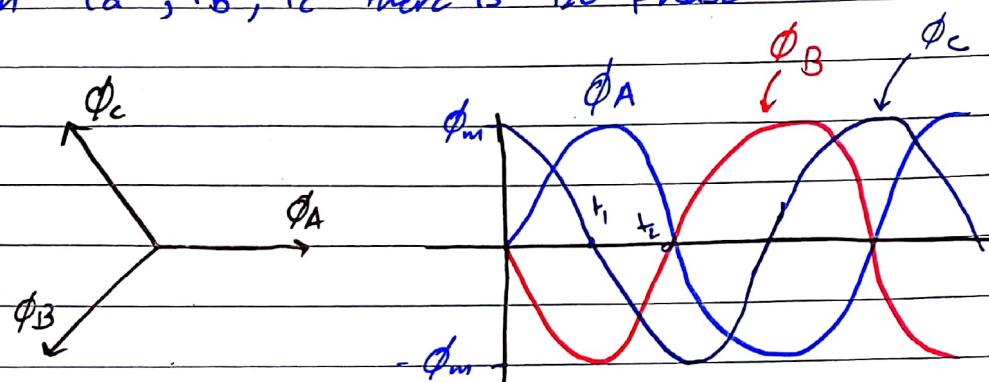
## \* Rotating Field

$$i \rightarrow \phi \Rightarrow \phi_{\text{resultant}}$$



\* Stator has 3 winding ( $120^\circ$  between each one)

\* Between  $i_a, i_b, i_c$  there is  $120^\circ$  phase

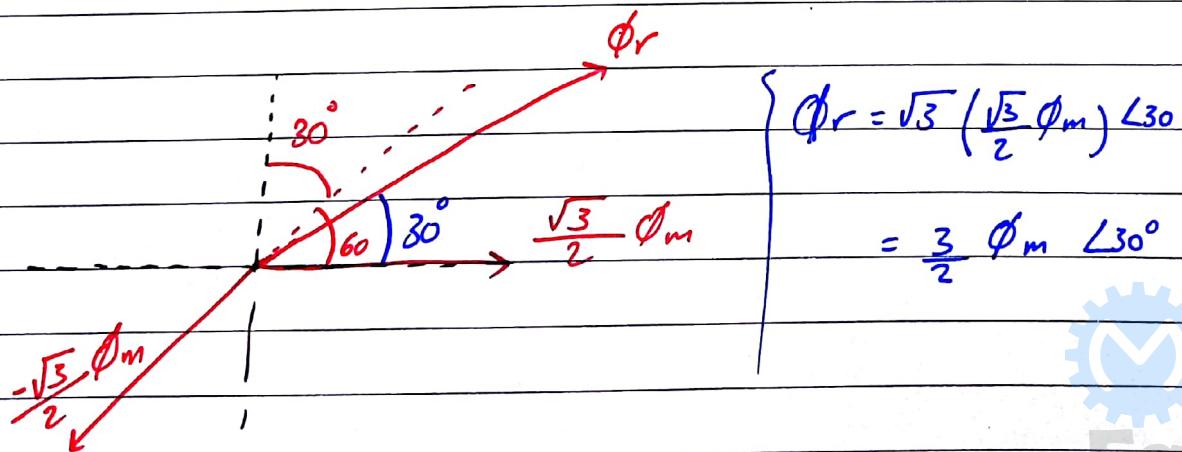


$$\phi_r = \vec{\phi}_a + \vec{\phi}_b + \vec{\phi}_c$$

at  $t_1$   $\rightarrow \phi_c = 0, \phi_c = \phi_m \sin(\omega t + 120) = 0 \Rightarrow \omega t = 60^\circ$

$$\phi_a = \phi_m \sin \omega t = \phi_m \sin 60 = \frac{\sqrt{3}}{2} \phi_m$$

$$\phi_b = \phi_m \sin(\omega t - 120) = \phi_m \sin(60 - 120) = -\frac{\sqrt{3}}{2} \phi_m$$

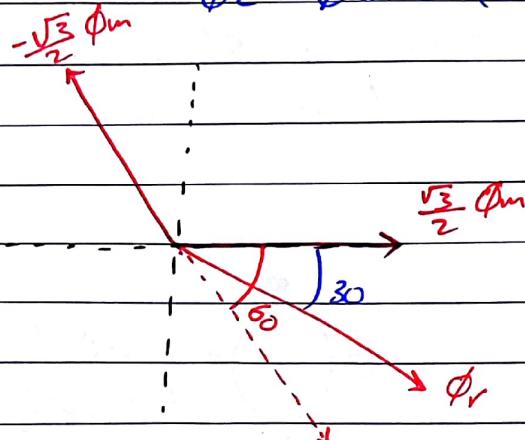


$$\left. \begin{aligned} \phi_r &= \sqrt{3} \left( \frac{\sqrt{3}}{2} \phi_m \right) \angle 30 \\ &= \frac{3}{2} \phi_m \angle 30^\circ \end{aligned} \right\}$$

$$\text{at } t_2 : \quad \phi_B = 0 \rightarrow \phi_B = \phi_m \sin(\omega t - 120^\circ) = 0 \Rightarrow \omega t = 120^\circ$$

$$\phi_A = \phi_m \sin \omega t = \phi_m \sin 120^\circ = \frac{\sqrt{3}}{2} \phi_m$$

$$\phi_C = \phi_m \sin(\omega t + 120^\circ) = \phi_m \sin(240^\circ) = -\frac{\sqrt{3}}{2} \phi_m$$

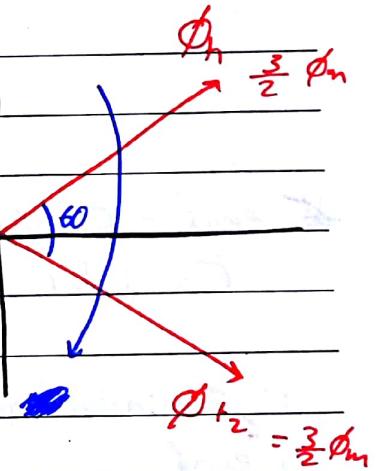


$$\phi_r = \phi_A + \phi_B + \phi_C$$

$$= \sqrt{3} \left( \frac{\sqrt{3}}{2} \phi_m \right) \angle -30^\circ$$

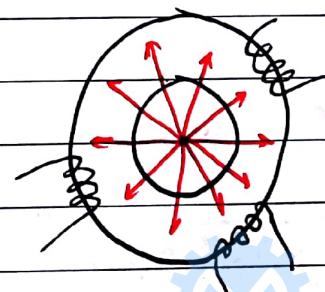
$$= \frac{3}{2} \phi_m \angle -30^\circ$$

\* Rotating Field  
(advantage of  $3\phi$ )

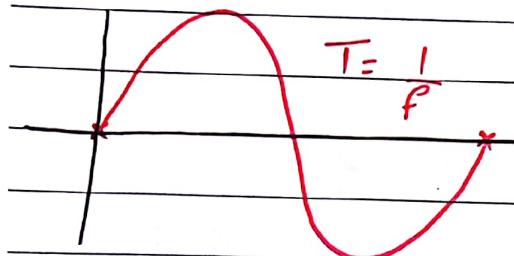


\* Air gap Plus

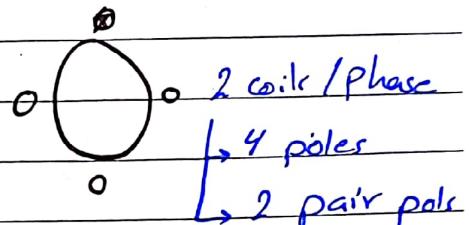
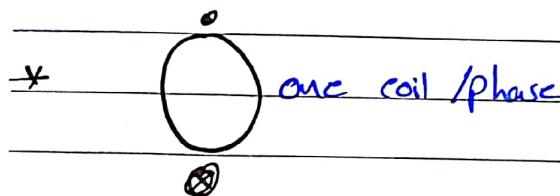
- constant Magnitude ( $\frac{3}{2} \phi_m$ )
- Angle is changing
- C.W



→ Speed of air gap Flux = 1 mechanical revolution (1/f sec)  
 (Synchronous speed)  
 $= P_{rev} / \text{sec}$   
 $= 60 f \text{ rev/min}$



For 2 poles or 1 pair pole  
 (North and south)



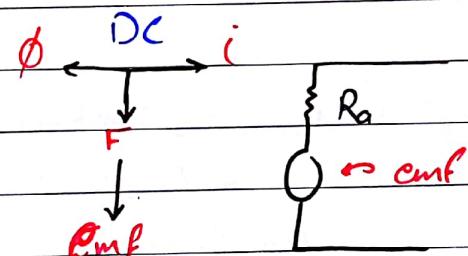
synchronous speed (Ns)

$$= \frac{60 f}{P_p} = \frac{120 f}{P_p}$$

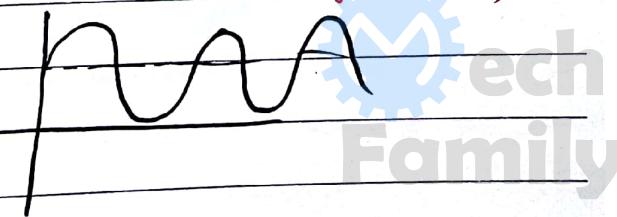
Number of Poles  
 $= 2P_p$

~~emf = BLV~~  $\rightarrow \text{emf} = f(\phi, \Delta n)$   
 $F = BLi$

- 1)  $\phi$  (induction Rotating Flux)
- 2) emf
- 3) i
- 4) F, T



- 1)  $\phi$
- 2)  $\text{Emf} = \Delta n$
- 3)  $i = \frac{\text{emf}}{\sqrt{R_a^2 + (X_a)^2}}$
- 4) T average :  $\tau \frac{d\omega_m}{dt} = T_m - T_L$



\* Running operation

TL <sup>Causes</sup> → Speed Drop

Prove: Current will change to meet the torque

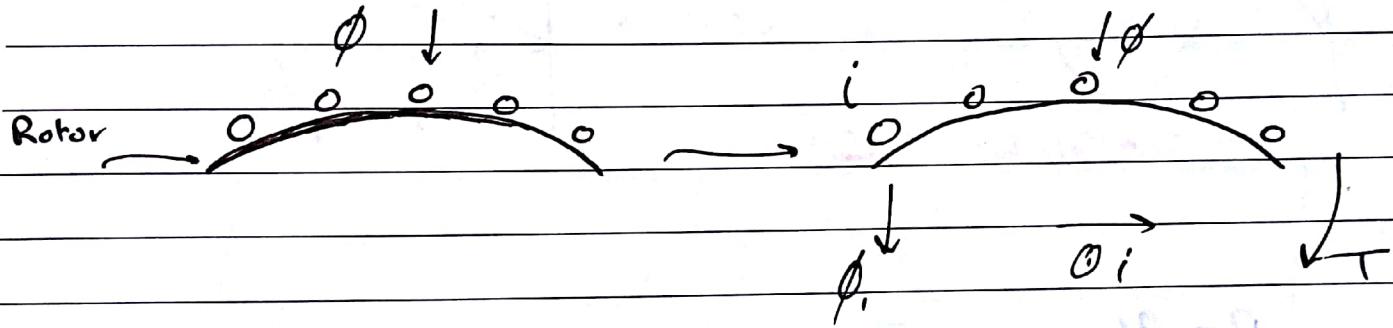
- emf ↑

- emf  $\propto \Delta n \rightarrow \Delta n \uparrow$

-  $\Delta n = N_s - N_m \rightarrow N_m \downarrow$

can't be zero

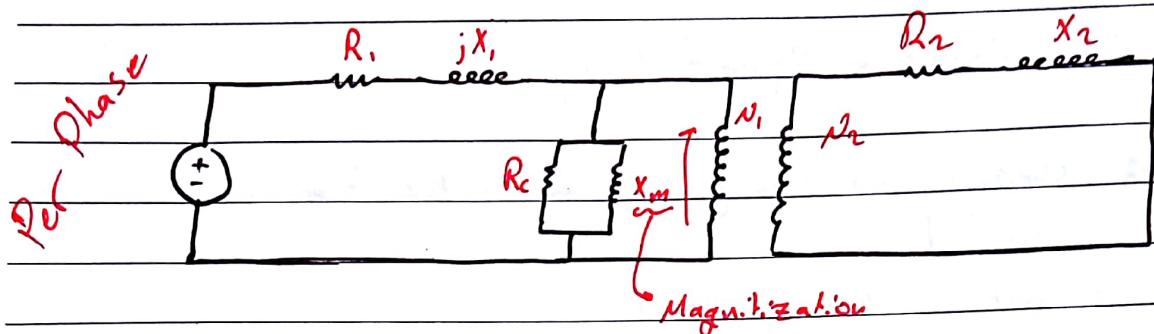
$$* \text{Slip (s)} = \frac{\Delta n}{N_s} = \frac{(N_s - N_m)}{N_m} \leftarrow \begin{array}{l} \text{Rotor speed} \\ \text{Rotating field speed} \end{array}$$



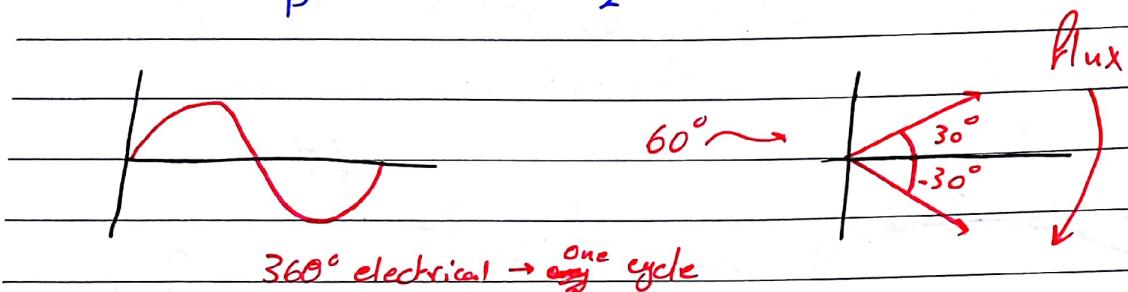
13/12/2017

Wednesday

## Equivalent circuit



$$N_s = 120 P, \theta_e = \frac{P}{2} \theta_m, P=2$$



$$P = 2f \rightarrow 720 \text{ electrical one cycle}$$

$\Rightarrow$  Number of poles increase; it will have slower speed

$I_m \sim$  stationary, stand still

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

\*  $\phi_2$

\*  $E_2 = \text{emf} / \text{at stand still}$

$$\text{emf} = BLV$$

Rotor

$i_i$   
 $T$

\* Running  $\phi$   $E_2 \propto N_s$

$$E_r \propto N_s - N_m$$

$$\frac{E_r}{E_2} = \frac{N_s - N_m}{N_s} = \frac{s}{s - \text{slip}}$$

$$\Rightarrow E_r = s E_2$$

\* Stand still ( $s=1$ )  $\Rightarrow E_r = E_2$

$f_r$  = Frequency of rotor current

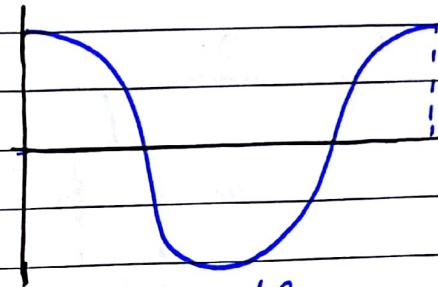
\* when we move with the Rotor, we will see the difference in speed

Relative speed

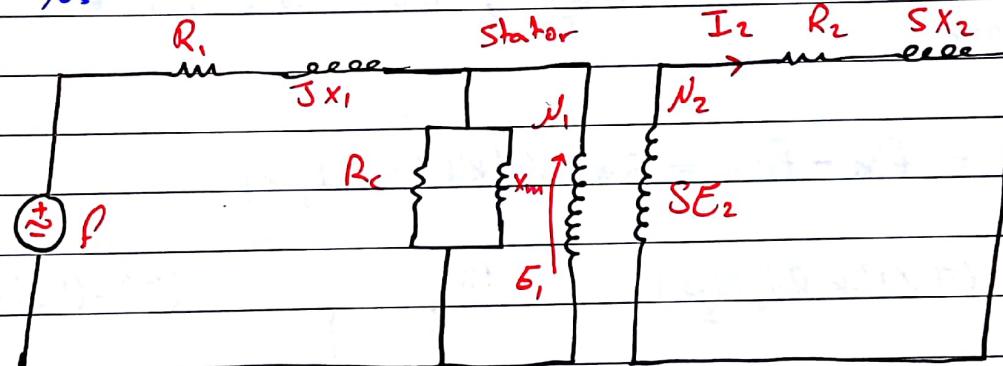
$$f_r = ?? \rightarrow \frac{120}{P} f_r = N_s - N_m$$

↑  
synchronous speed  
↓  
Mechanical speed

Relative speed



$$s = \frac{N_s - N_m}{N_s} \Rightarrow 120 f_r = s N_s \rightarrow \frac{120}{P} f_r = s \Rightarrow f_r = s f$$

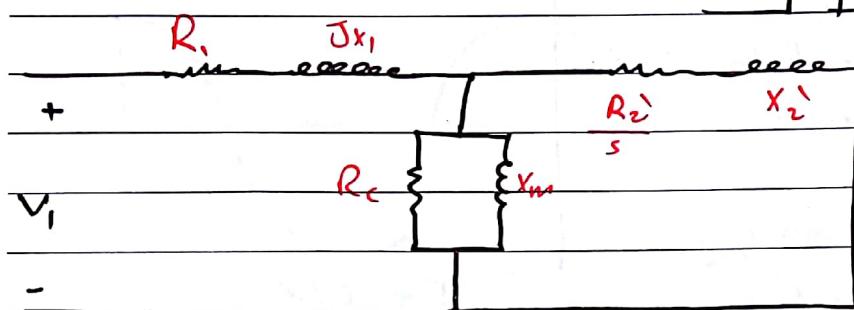
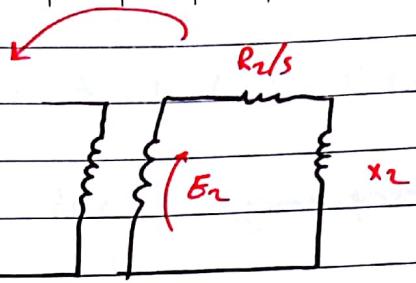


after adjusting

$$X = \omega r L = 2\pi f_r L = 2\pi s f L = (2\pi f L) s \Rightarrow f_r = s f$$

$$I_2 = \frac{SE_2}{R_2 + j s X_2} = \frac{E_2}{R_2 + j X_2}$$



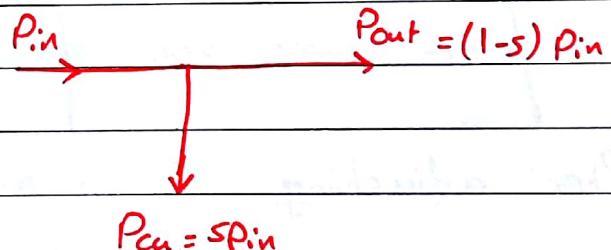


$$\frac{R_2'}{s} = \frac{R_2}{s} \times \left(\frac{N_1}{N_2}\right)^2, \quad x_2' = x_2 \times \left(\frac{N_1}{N_2}\right)^2$$

$$P_{in} \text{ Rotor} = 3 \times (I_2')^2 \times \frac{R_2'}{s}, \quad P_{cu} = 3 \times (I_2')^2 R_2'$$

$$P_{output} = P_{in} - P_{cu} = 3 \times (I_2')^2 \times \left(\frac{R_2'}{s}\right) \left(\frac{1-s}{s}\right)$$

$$* P_0 = 3 \times (I_2')^2 \times R_2' \left(\frac{1-s}{s}\right)$$



$$* \eta = \frac{P_0}{P_{in}} - \frac{(1-s)}{s} P_{in} = (1-s) = \eta$$

*	1410 rpm	1490 rpm	Mechanical speed
S	$s_1$	$s_2$	$\frac{N_r}{P} = \frac{120 R}{P} = 120 (50)$

P	2	4
$N_r$	300	1500

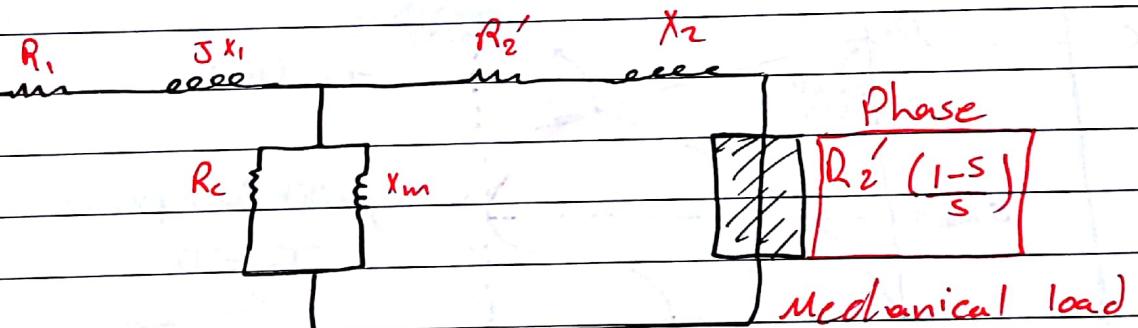
we assume  $N_s = 1500$

Since 1490 rpm is closer ~~than~~ to 1500 rpm

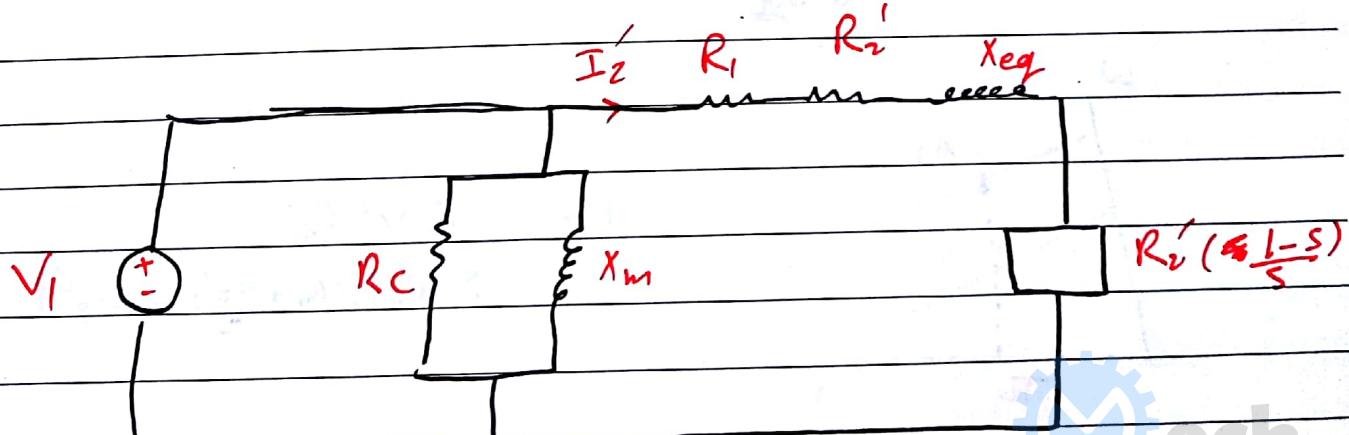
$$\Rightarrow \boxed{s_1 > s_2}$$

$$\boxed{m_1 < m_2}$$

$$* \frac{R_2'}{s} = R_2' + R_2' \left( \frac{1-s}{s} \right)$$



Multiply by 3 because 3Ø



Torque - Speed C/s Im:

$$\text{Torque} : P = TW$$

$$T = \frac{P}{\omega} = 3 \frac{(I_i)^2 (L_i) \left(\frac{1-s}{s}\right)}{\omega}$$

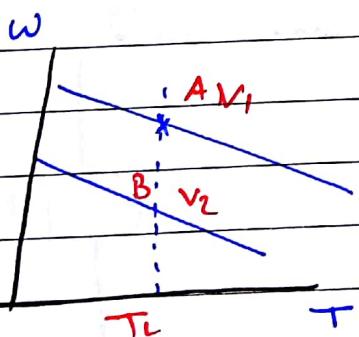
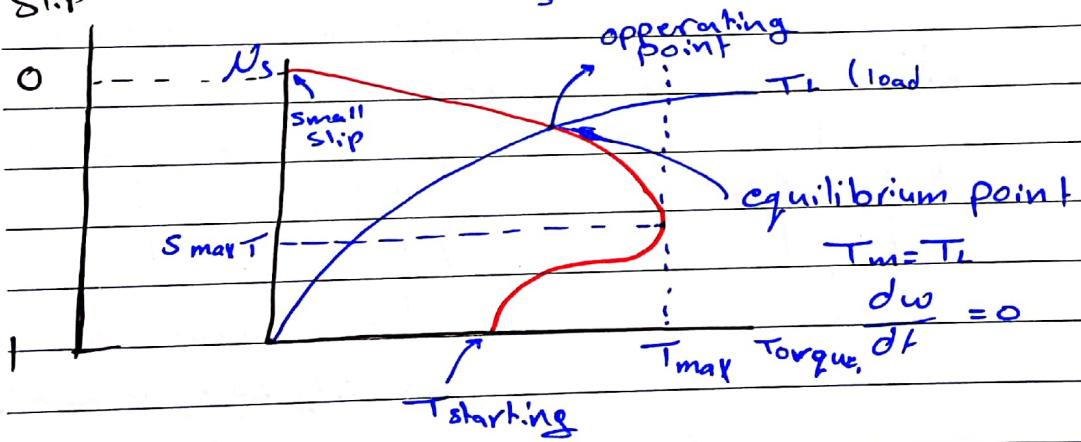
$$w = ws(1-s)$$

$$\Rightarrow I_2' = \frac{V}{\sqrt{\left(\frac{R_1 + \frac{R_2}{s}}{s}\right)^2 + (x_{eq})^2}}$$

$$-V_1 + I_2' R_1 + I_2' x_{eq} s + I_2' R_2' + I_2' (R_2' \frac{1-s}{s})$$

$$\Rightarrow T = \underline{3 V^2 R_2'}$$

$$SWS \left( (R_i + \frac{R_2'}{S})^2 + x_{eq}^2 \right)^{\frac{1}{2}}$$



$$T = \frac{3V^2 R_2'}{5ws \left[ \frac{(R_1 + R_2')^2}{2} + x_{eq}^2 \right]}$$

\* 3 Regions

- at large slip :  $x_{eq} \gg \frac{R'_i}{s} + R_i$

$$T = \frac{3 \nu^2 R'_i}{s w_s x_{eq}^2}, \quad T_{St.} = \frac{3 \nu^2 R'_i}{w_s x_{eq}^2} \quad \text{at } s=1$$

- at small slip :  $\frac{R'_i}{s} \gg R_i, \quad \frac{R'_i}{s} \gg x_{eq}$

\* Stability (Need to be avoided)

$$\tau \frac{dw_m}{dt} = T_m - T_L$$

$$w \uparrow \Rightarrow T_m \downarrow \Rightarrow \frac{dw_m}{dt} \text{ (-ve) de-acceleration}$$

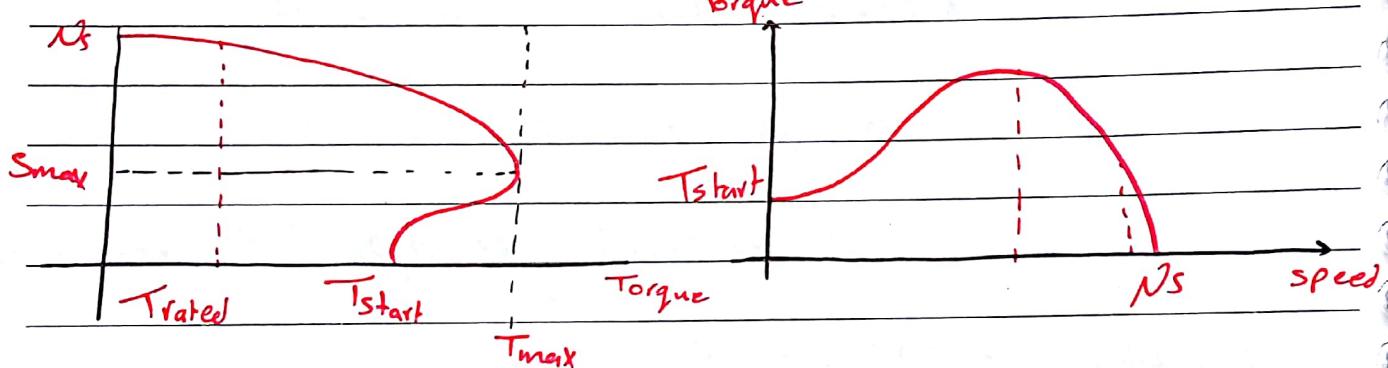
per phase :  $T = \frac{3 \nu^2 s}{w_s R'_i}$

Thursday  
14/Dec/2017

$$* S_{max} = \frac{R_2'}{\sqrt{R_1^2 + X_{eq}^2}}$$

$$T_{max} = \frac{3V^2}{2w_c [R_1 + \sqrt{R_1^2 + X_{eq}^2}]}$$

$$T_d = \frac{3V^2 R_2'}{speed \cdot 2w_c [(R_1 + \frac{R_2'}{s})^2 + X_{eq}^2]}$$



If we used load with Torque higher than  $T_{st}$   
it won't work

3 cases

\* large slip

$$(R_1 + \frac{R_2'}{s})^2 \ll x_{eq}^2$$

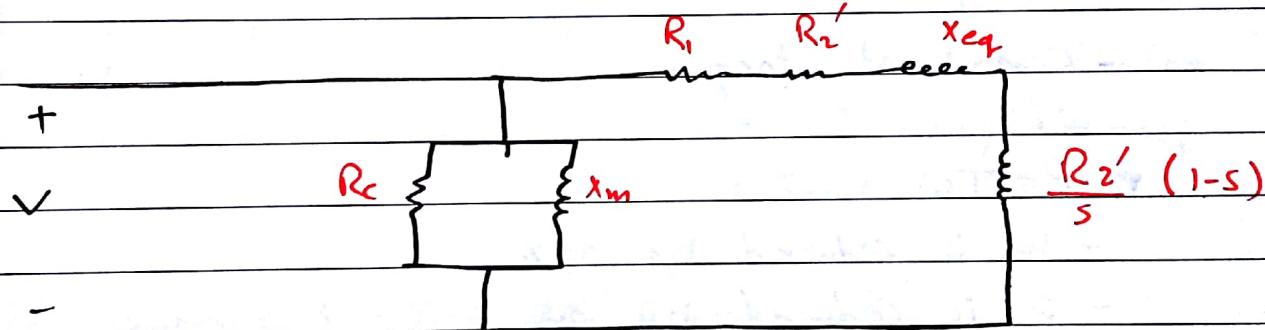
$$T_d = \frac{3v^2 R_2'}{s w_r x_{eq}^2}$$

$$s=1 \Rightarrow T_d = \frac{3v^2 R_2'}{w_s x_{eq}^2}$$

\* small slip

$$R_1 \ll \frac{R_2'}{s} \gg x_{eq}$$

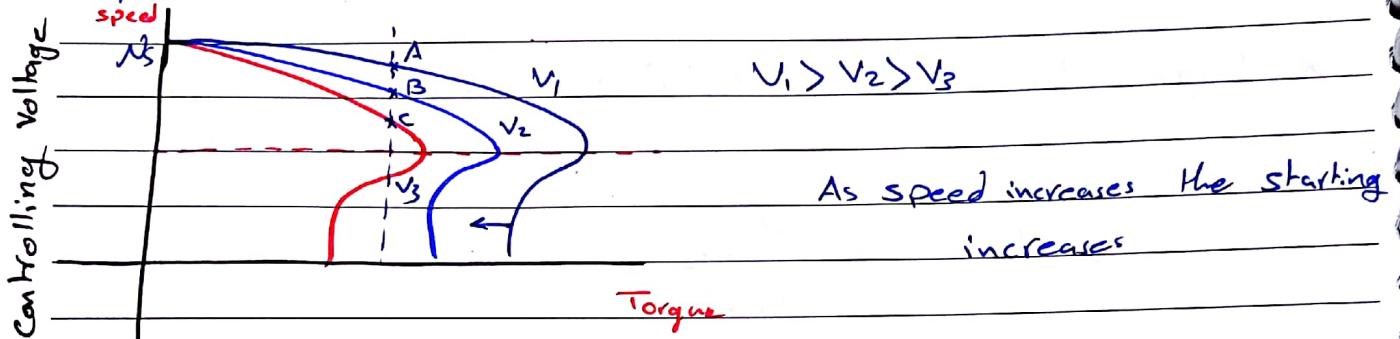
$$T_d = \frac{3v^2 s}{w_s R_2'}$$



at maximum torque  $T_{max} = \frac{3v^2}{2w_s [R_1 + \sqrt{R_1^2 + x_{eq}^2}]}$

$$s_{max} = \frac{R_2'}{\sqrt{R_1^2 + x_{eq}^2}}$$

## Speed control IM (Induction Motor)



$$\text{Power} = TW$$

Fan  $\rightarrow$  variable Torque  $\propto \omega^n$

- $\omega$  is reduced by 50%
- $T$  is reduced by 0.25
- $P$  is reduced by  $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

- constant torque

$\omega \downarrow$

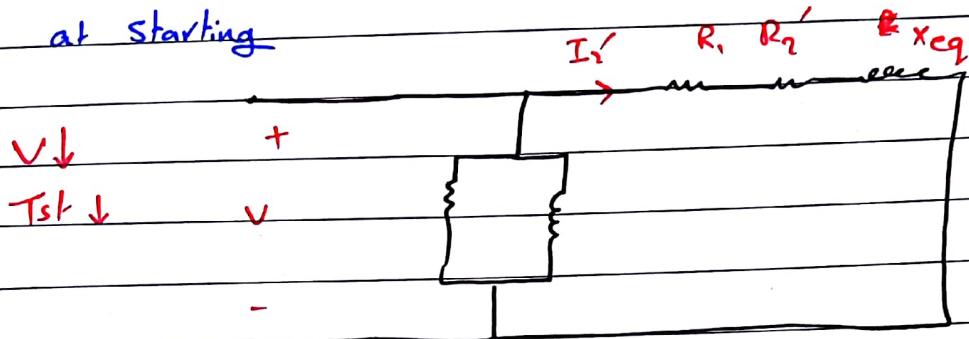
$$\nabla P = TW$$

- $\omega$  is reduced by 50%
- $P$  is reduced by 0.5

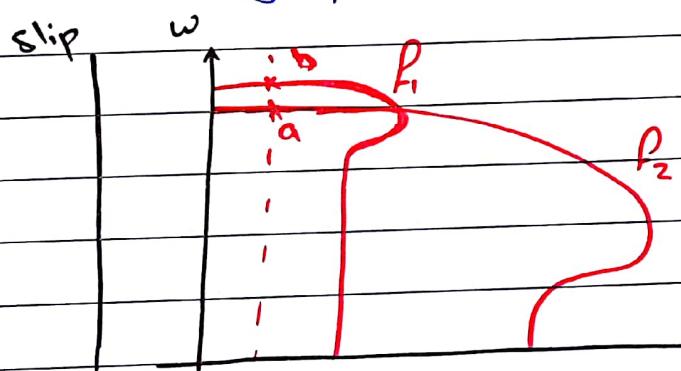
\* controlling speed by adjusting starting voltage

10% reduction in voltage  $\rightarrow$  19% reduction in torque

at Starting



\* controlling speed by adjusting Frequency



$$P_1 > P_2, \quad s_{max} = \frac{R_2'}{\sqrt{R_2'^2 + X_{eq}^2}}$$

$$N_s = \frac{120F}{P}$$

$$T_{St} = \frac{3V^2 R_2}{s}$$

$$W_s X_{eq}$$

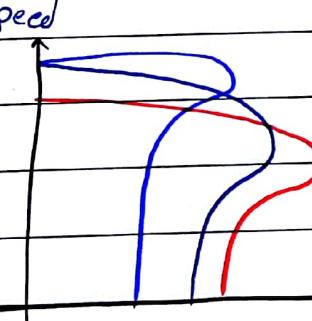
\*  $P \uparrow \rightarrow X_{eq} \uparrow \rightarrow s_{max} \downarrow$

$\propto T_{St} \downarrow, T_{max} \downarrow$

\* V/f control:  $V = 4.44 NAP B_{max} f \propto P$

$\frac{V}{f}$  constant  $\rightarrow$  constant flux  $\rightarrow$  constant torque

$$T_{max} = \frac{3V^2}{2W_s [R_2 + \sqrt{R_2^2 + X_{eq}^2}]} \quad X_{eq} \gg R_2$$



$$T_{max} = \frac{3V^2}{2W_s X_{eq}} \quad T_{max} \propto \left(\frac{V}{f}\right)^2$$

$$\left| \begin{array}{l} V \uparrow \rightarrow I_{St} \uparrow \\ f \uparrow \rightarrow T_{St} \downarrow = \frac{V}{f} \\ X \uparrow \end{array} \right.$$

18/Dec/2017

Monday

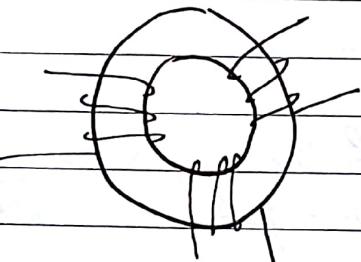
## \* Synchronous machines

→ Asynchronous machines "induction"

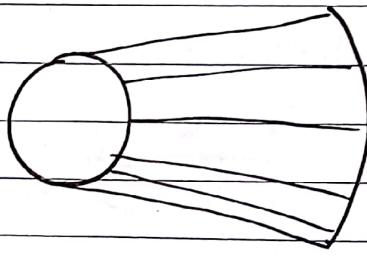
$\omega_s > \omega_m \rightarrow \text{Slip}$

→ Synchronous machines

$$\omega_s = \omega_m$$



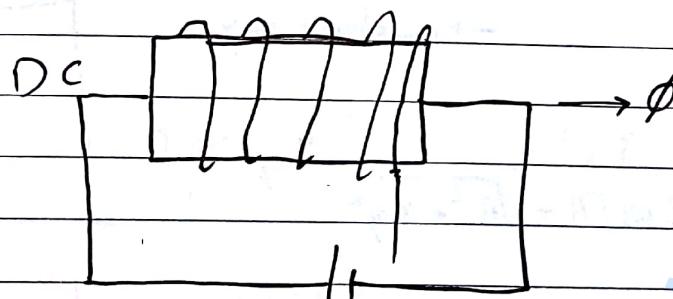
Stator



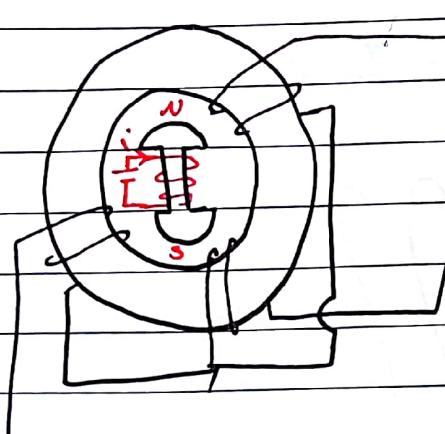
Rotor (Squirrel Cage)

\* Same stator 3Ø AC "motor"

Rotor → Permanent magnet  
→ Electro magnet



## Synchronous Generator



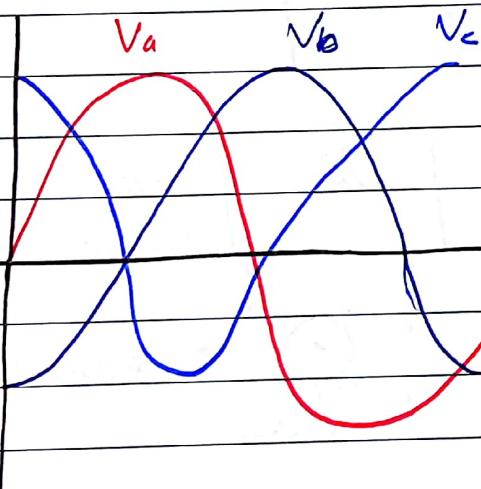
Gr.ij

Motion of

Rotor

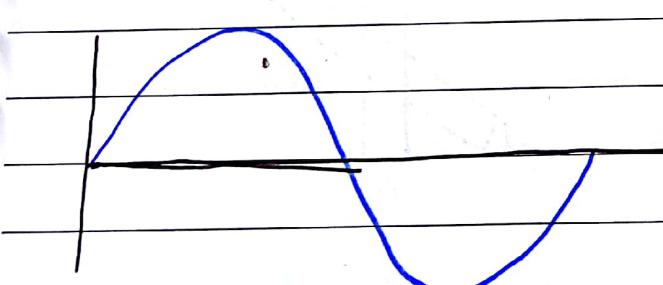
by gas turbine

Moving Rotor  $\rightarrow \frac{d\theta}{dt}$  [stator]  $V = \frac{N \phi \theta}{dt}$



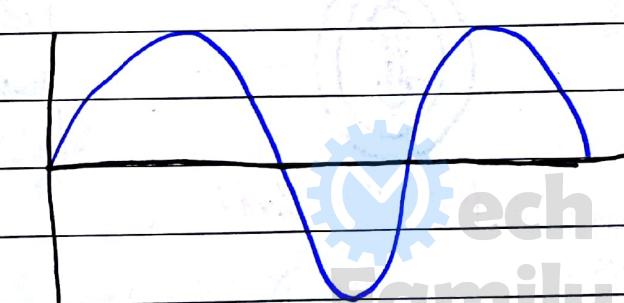
$$* f_s = \frac{120f}{P}$$

$$\theta_e = \frac{P}{2} \theta_m$$



one cycle Mechanical

$$P=2$$



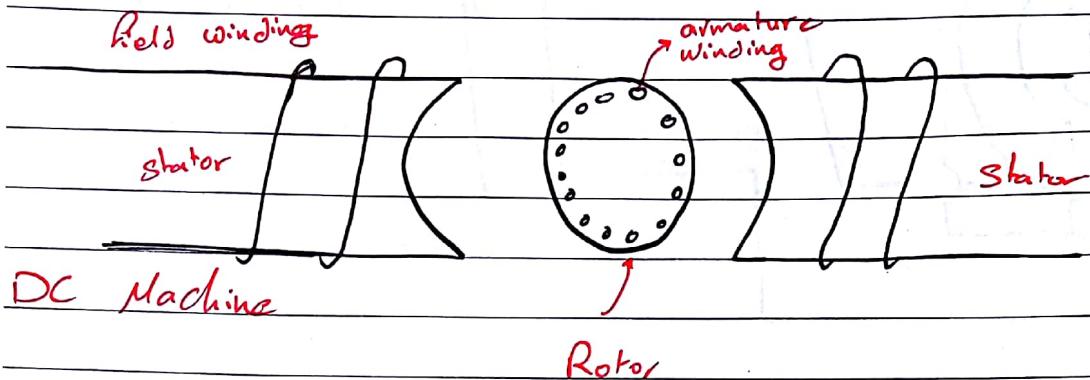
one cycle Mechanical

$$\theta_e = 2 \theta_m$$

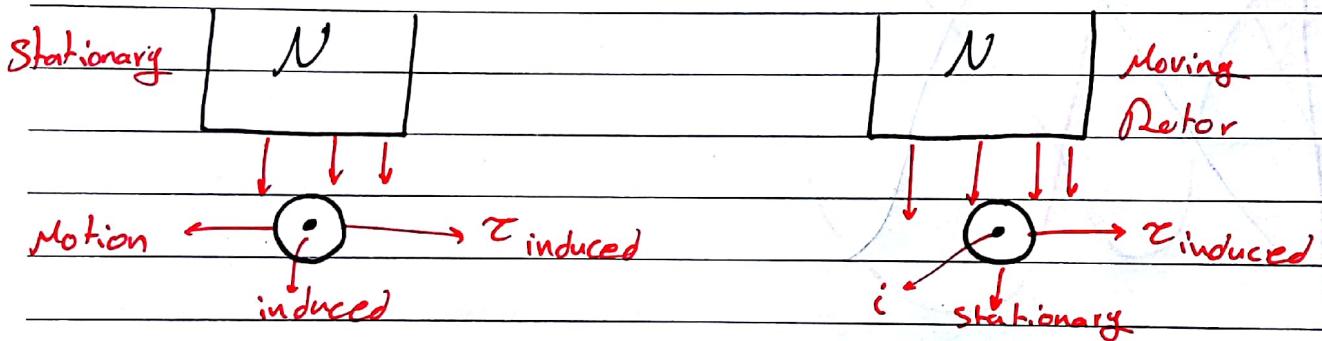
$$P=4$$

$$* f = 50 \text{ Hz}, P = 2 \rightarrow N_s = 3000 \text{ rpm}$$

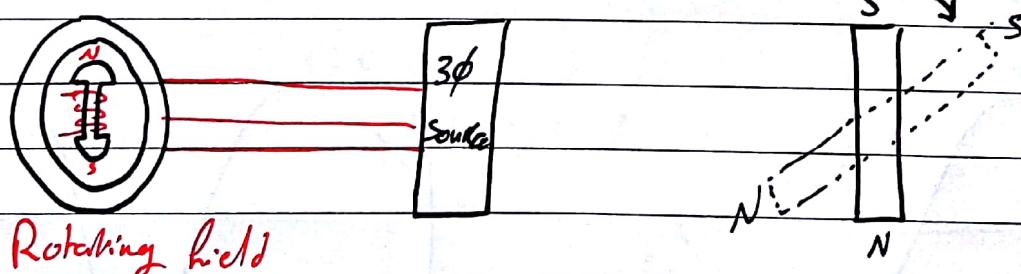
\* we use slip rings to connect power to the rotor



	DC Machine	Synchronous Machine
Stator	<del>Field winding</del>	Armature winding
Rotor	Armature winding	Field winding



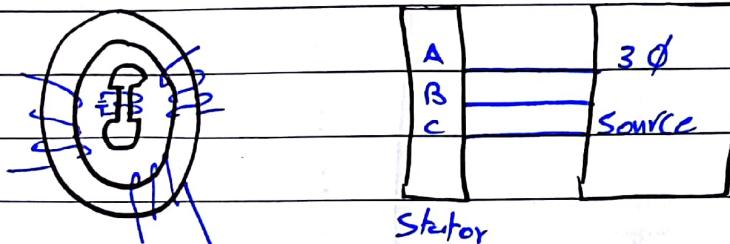
\* Difference Between Synchronous and Induction speed



\* As Flux rotating Rotor will start following Flux at the same speed

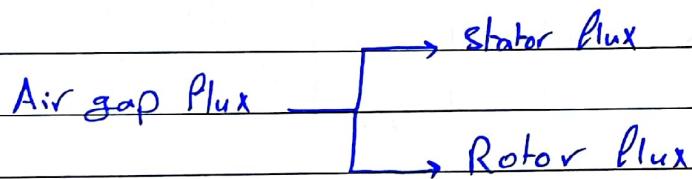
Wednesday  
20/Dec/2017

## Synchronous Motor

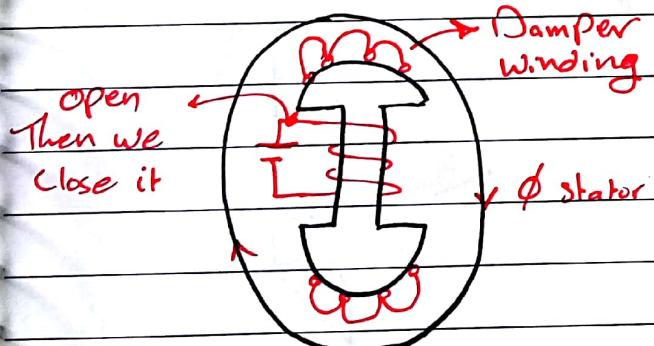


Rotating field  $\rightarrow$  Speed =  $N_s = \frac{120F}{P}$        $N_m = N_s$

④ Starting (There is a problem in starting)



in starting we should increase the rotor speed (by adding a prime mover or by using induction motor)



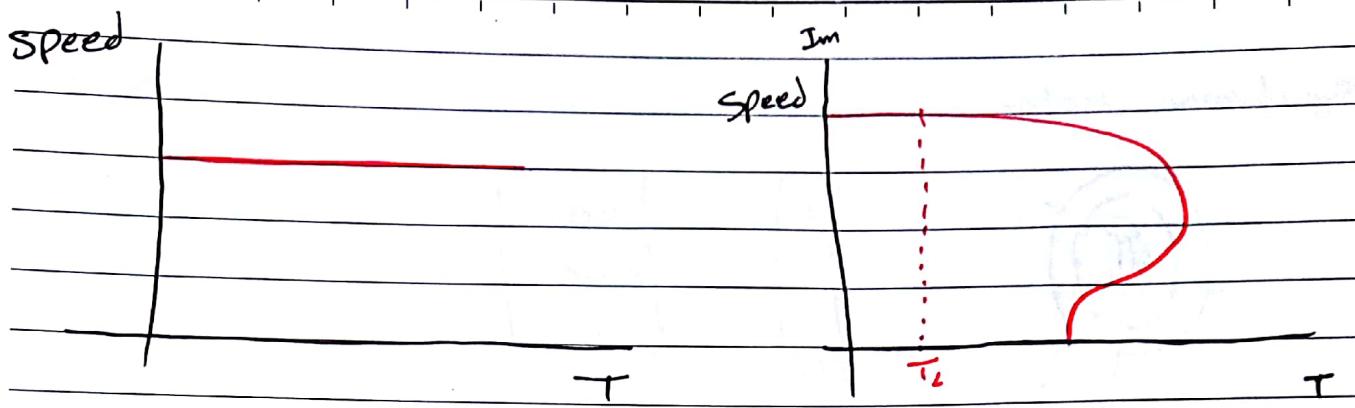
① Rotating field  $\phi$  stator

② damper winding  
 $i \rightarrow T \rightarrow$  rotor speed  $\uparrow$

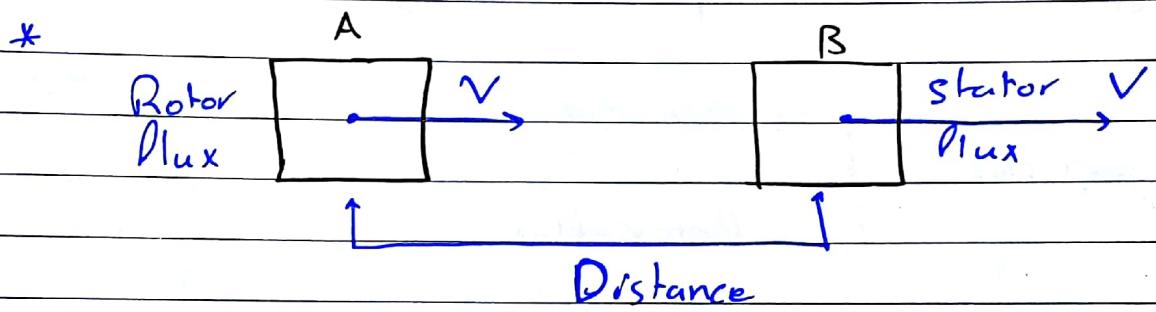
at starting  $N_s = 3000$  rpm,  $N_m = 0$

$$P = 2, N_s = \frac{120F}{P}$$

\* low frequency  $\rightarrow f = 0.1$  Hz  $\Rightarrow N_s = 6$  rpm  
 increase  $P$  gradually



### Synchronous Motor



There is a phase shift between them, as the distance increase the phase increases (angle phase  $\delta$ )

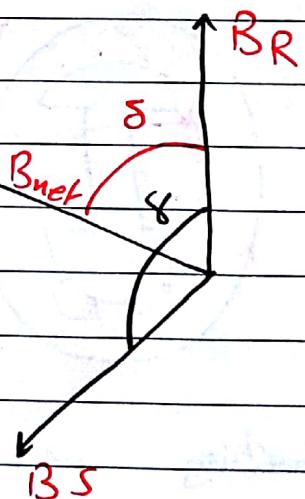
$$* T = K |B_R| |B_s| \sin \delta$$

$$= K |B_R| \times |B_s|$$

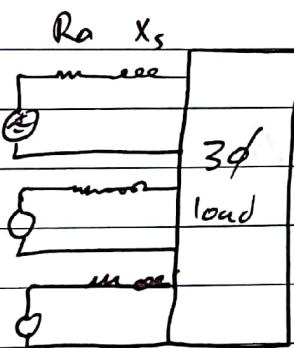
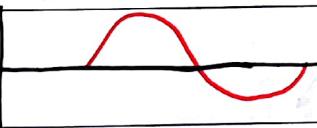
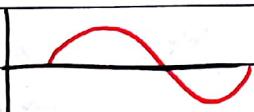
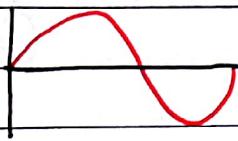
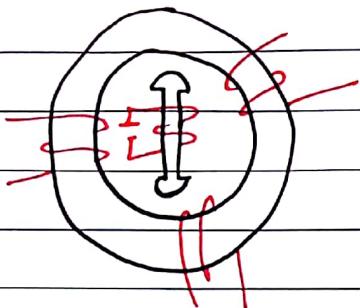
$$T = \vec{B_R} \times (\vec{B_{net}} - \vec{B_R})$$

$$T = \vec{B_R} \times \vec{B_{net}}$$

$$T = B_R B_{net} \sin \delta$$

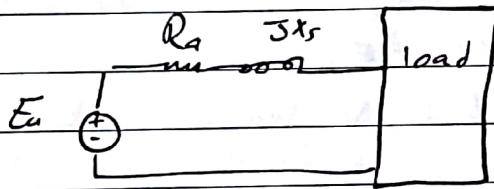


## \* Equivalent Circuit of Synchronous Generator



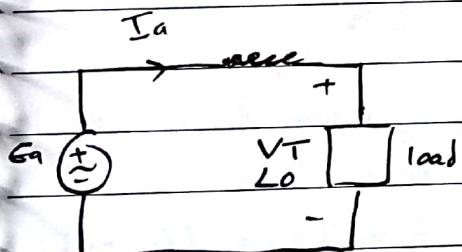
$R_a$  = Armature Resistance

$X_s$  = Synchronous Resistance



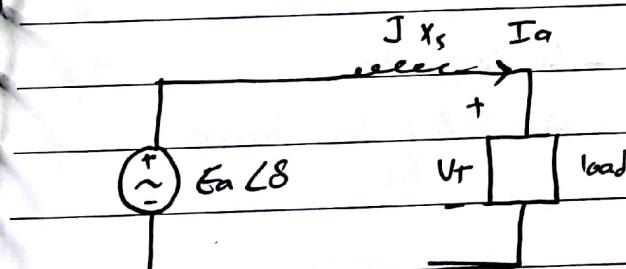
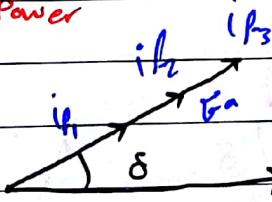
$$E_a = K \phi \omega$$

$$E_a - I_a (R_a + jX_s) = V_T$$



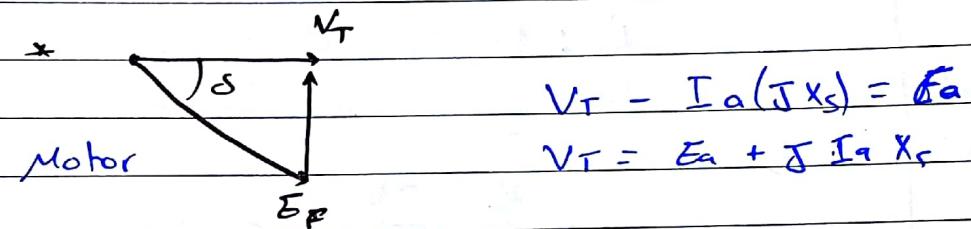
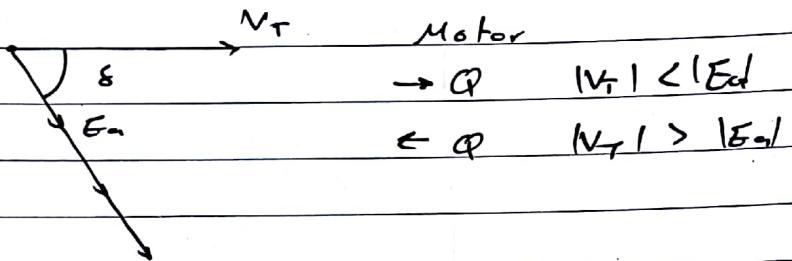
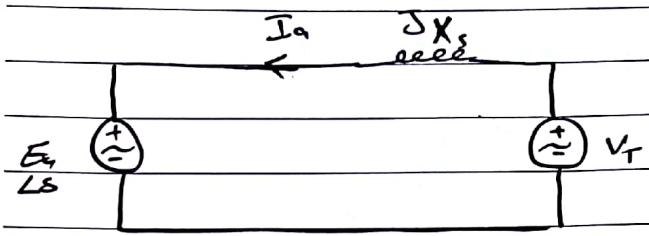
$$S = P + jQ$$

Real Power      Reactive Power

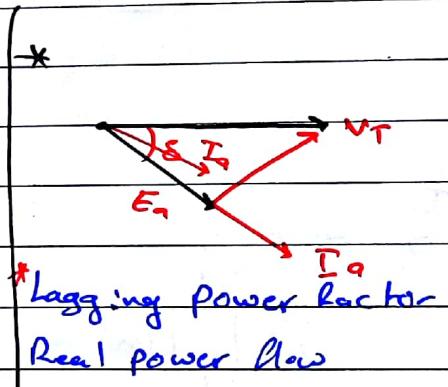
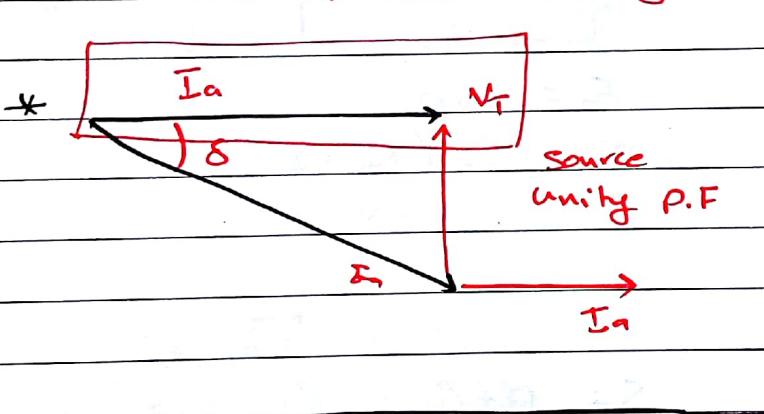


$|E_a| > |V_T| \rightarrow$  Generator Export Q

$|E_a| < |V_T| \rightarrow$  Generator Import Q

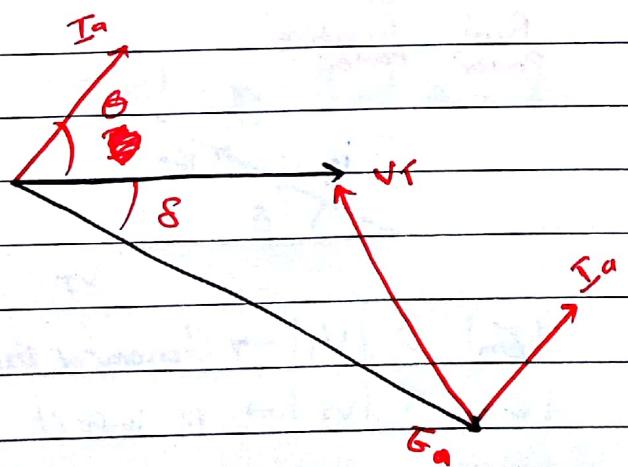


L: V leads I by  $90^\circ$



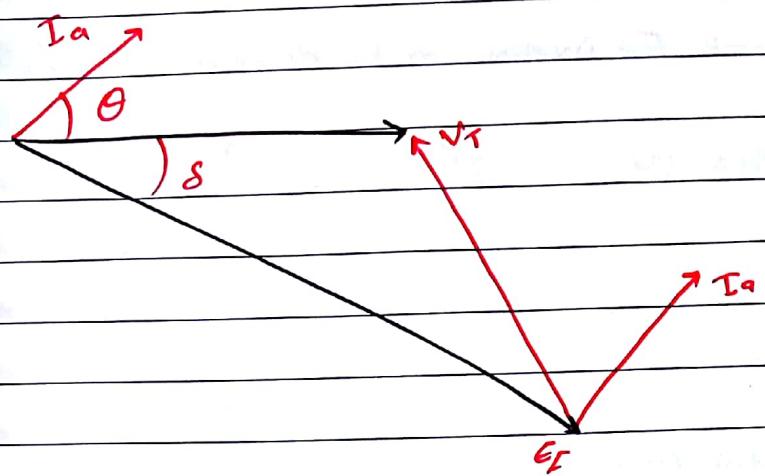
$$P = 3 V_T I_a \cos (L V - 2 \delta)$$

$$P = 3 V_T I_a \cos \theta$$



- Leading PF

$$P = 3 E_a V_T \sin \delta$$



Phasor diagram

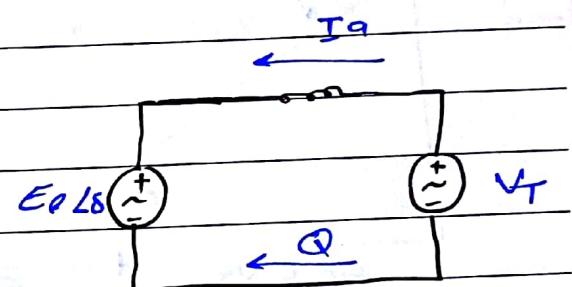
$I_a$  leads  $V_T$

injects reactive power

$$Q = 3 V_T I_a \sin \theta$$

$$E_F \cos \delta - V_T = I_a X_s \sin \theta$$

$$E_F \cos \delta - V_T = \frac{Q}{3 V_T} X_s$$

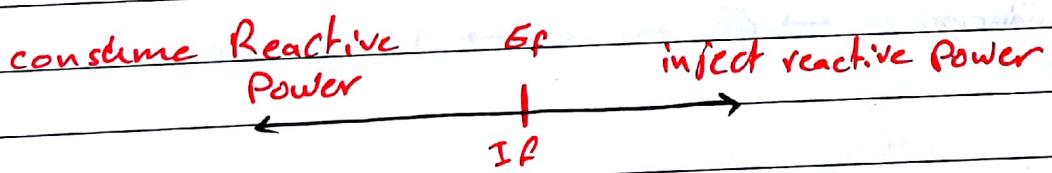


$$\Rightarrow Q = \frac{3 V_T}{X_s} (E_F \cos \delta - V_T)$$

No load ( $\delta = 0$ )  $\Rightarrow E_F > V_T \rightarrow Q (+ve)$  inject  $Q$

$E_F = V_T \rightarrow$  unity PF  $Q = 0$

$E_F < V_T \rightarrow$  consume  $Q$

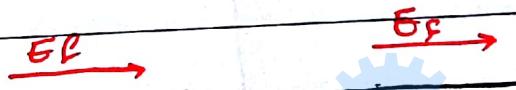


$I_a$  No load

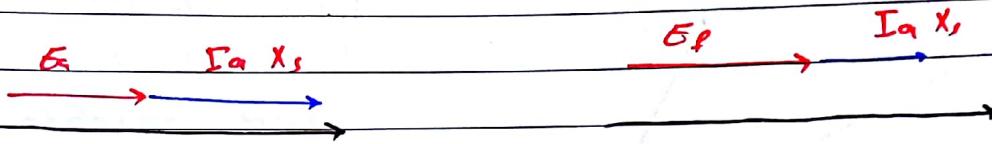
a)  $Q = 0$  unity P.F

$I_a = 0$

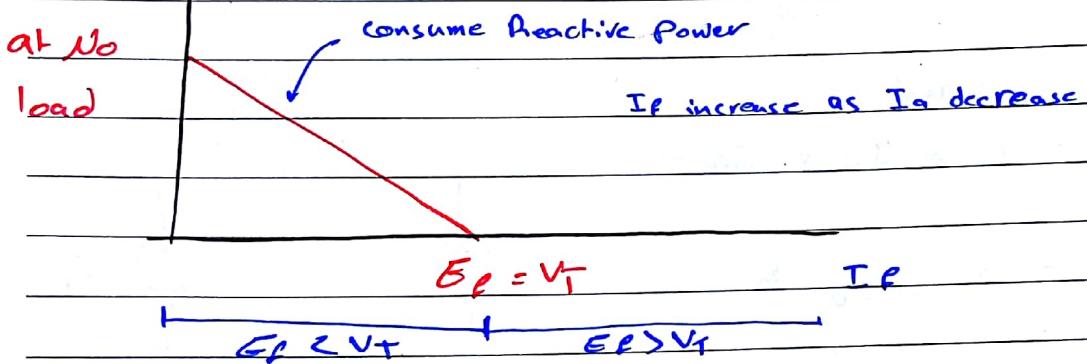
b) Motor consume Reactive Power



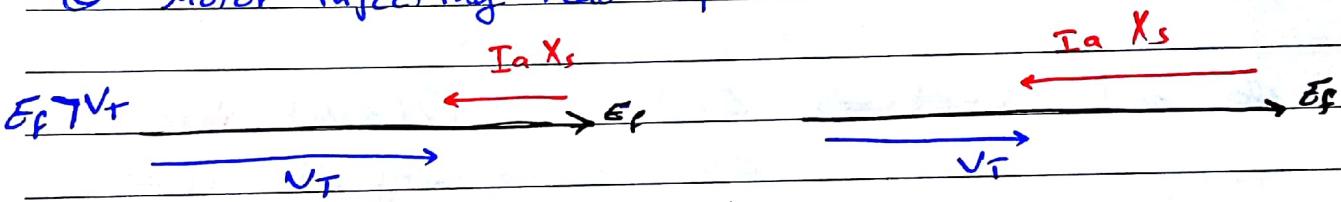
as  $I_F$  increases  $\rightarrow E_F$  increase  $\rightarrow I_a$  decrease ( $T_a X_s \downarrow$ )



$I_a$



② Motor injecting reactive power



as  $I_F$  increase  $\rightarrow E_F$  increase  $\rightarrow I_a$  increase

at No load

injecting Reactive power

$$E_F < V_T \quad | \quad E_F > V_T$$

$$E_F = V_T$$

$I_a$

\* load  $\Rightarrow \delta$

$$Q=0 \rightarrow E_F \cos \delta - V_T = 0 \Rightarrow \frac{V_T}{\cos \delta} = E_F$$

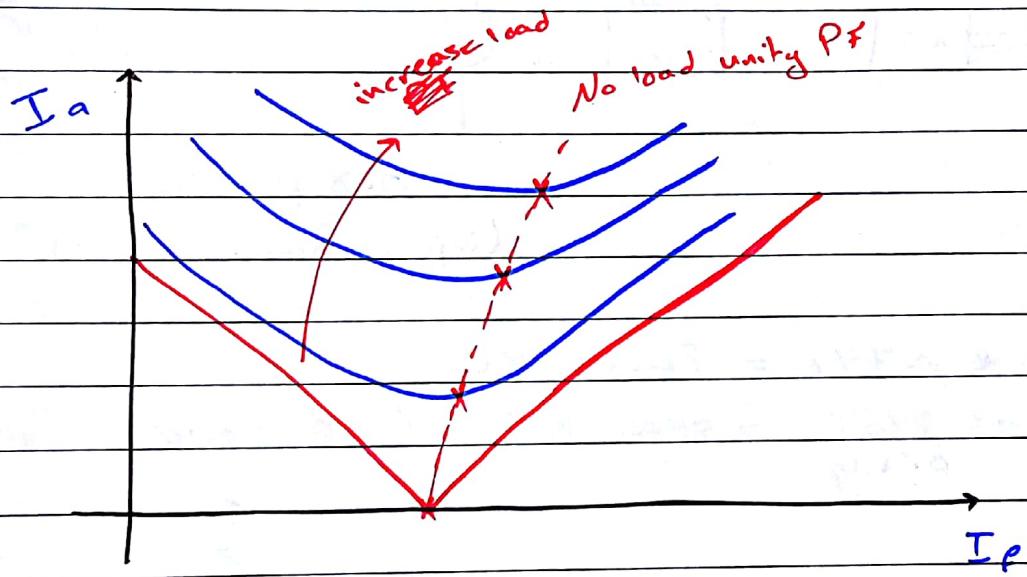
\* No load

- unity P.F
- zero reactive
- $E_{F1} = V_T$
- $I_{F1} = 0$

\* load

- unity P.F
- $E_F \cos \delta = V_T$
- $E_{F2} = V_T / \cos \delta$

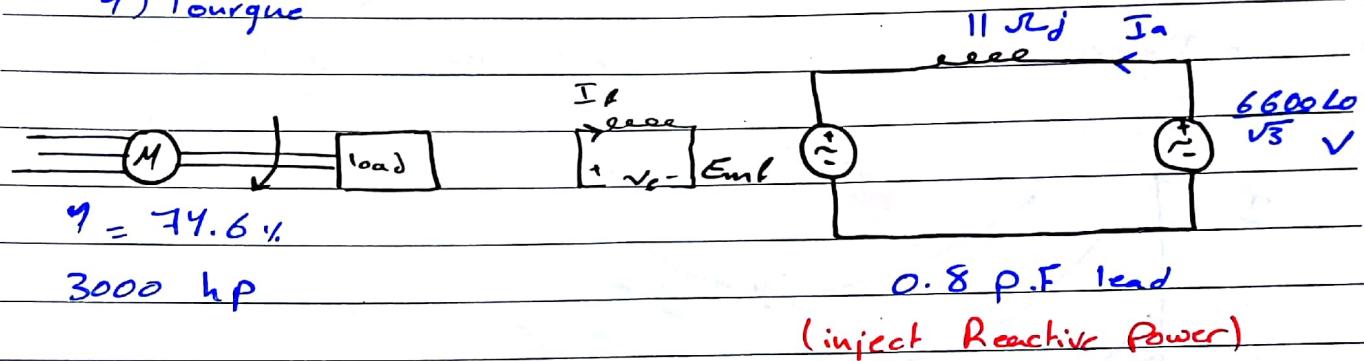
$$E_{F2} > E_F, I_{F2} > I_{F1}$$



Ex) 3000 hp, 6600 V, 3φ, Y connected synchronous motor operates at full load at a leading P.F of 0.8 and efficiency  $\eta = 74.6\%$ .

$$X_s = 11 \Omega$$

- 1) apparent power per phase
- 2) line current
- 3) Emf
- 4) Torque



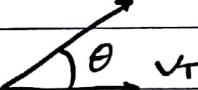
$$-P_o = 3000 * 0.746 = 22.38 \text{ KW}$$

$$-P_i = \frac{P_o}{\eta} = \frac{2238}{0.746} = 3000 \text{ KW} \quad (3\phi \text{ power})$$

$$P_{in} = \sqrt{3} V_L I_L \cos \phi$$

$$3000 = \sqrt{3} * 6600 * I_L \cos(0.8) \Rightarrow I_L = 328 / \cos^{-1} 0.8 = 328 / 36.9 \text{ A}$$

$I_a$



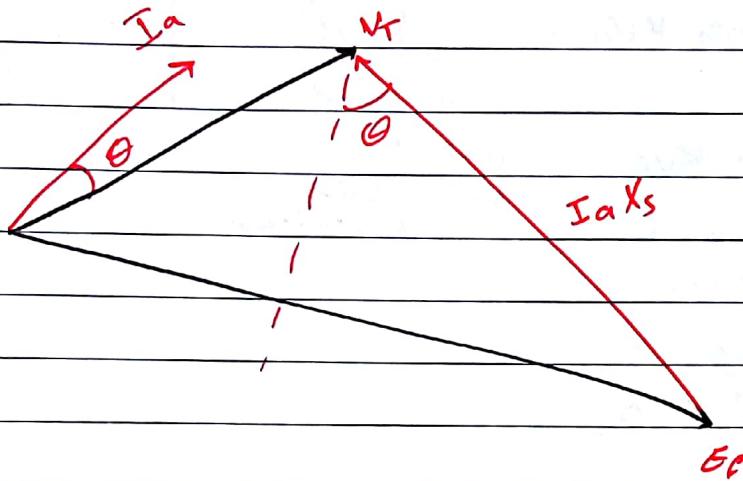
$$Emf = V_T - I_a (11j)$$

$$= \frac{6600 L0}{\sqrt{3}} - (328 L 36.9)(11j) = 1 \quad 1 \quad L$$

$$\Rightarrow Emf = 6636.8 L - 26.8$$

## Real power

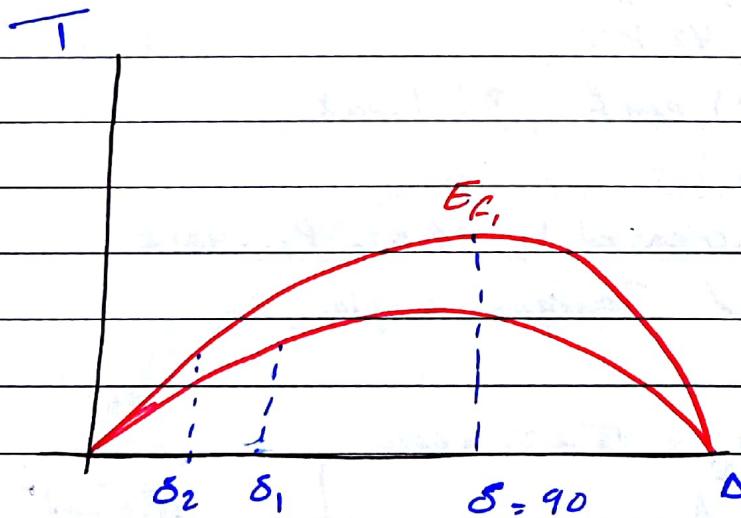
$$P = 3 V_T I_a \cos \theta$$



$$I_a \sin \theta = I_a X_s \cos \theta$$

$$I_a \cos \theta = E_F \sin \theta$$

$$P = \frac{3 V_T E_F \sin \theta}{X_s}, \quad P = \frac{3 V_T E_F \sin \theta}{X_s}$$



$$T = \frac{P}{\omega_s}$$

$$T = \frac{3 V_T E_F \sin \theta}{X_s \omega_s}$$

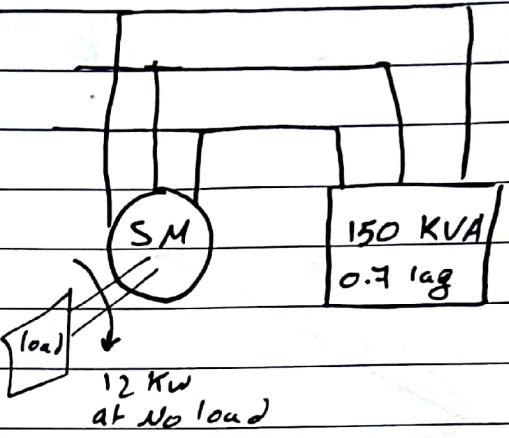
Ex) Calculate the (SM) rating to bring the overall P. F to unity

$$P = 150 * 0.7 = 105 \text{ kW}$$

$$Q = \sqrt{(150)^2 - (105)^2} = 107 \text{ kVA}$$

$$SM | = 12 \text{ kV} - j107 \text{ kVA}$$

Output      injecting Reactive Power



at unity  $P.F \rightarrow Q = 0$

$$\Rightarrow Q_{sm} - Q_{load} = 0$$

$$|S_M|_{\text{rating}} = \sqrt{(12)^2 + (10\sqrt{2})^2} = 107.67 \text{ kVA}$$

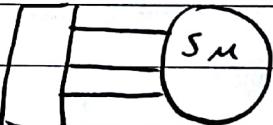
→ the SM works 0.5 capacitor "condenser"

Ex)  $X_s = 252$  /phase

6600 V, 50 Hz, 6 poles

0.8 P.F lag, 400 KW

Find: 1)  $I_L$ , 2)  $emf$ , 3)  $T_{max}$

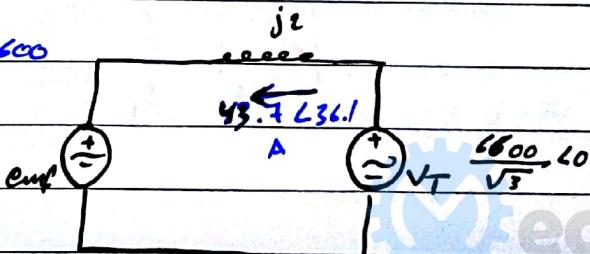


4) E.M.F is increased by 25%,  $P_m = 400 \text{ kW}$

Find Torque angle

$$1) S = \sqrt{3} V_L I_L \Rightarrow \frac{400}{0.8} = \sqrt{3} * I_L * 6600$$

$$\Rightarrow I_L = 43.7 L-36.1 A$$



$$\mathcal{E} \text{ Emf} = V_T - I_a (j_2) = 3758.6 \text{ } \underline{\text{A}}$$

$V_T$  leads

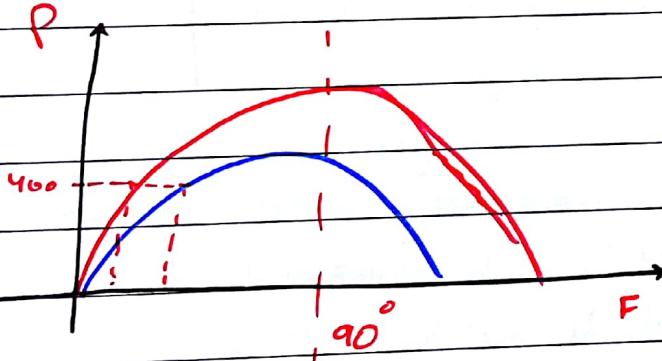
$$\frac{\text{emf}}{L_L} = 65$$

$$3) T_{120f} = \frac{3 V_T \text{ emf}}{w_s x_s} \sin \delta$$

$$T_{\max f} = 3 \left( \frac{6600}{\sqrt{3}} \right) * 3758 \sin 90^\circ = 205.15 \text{ kN.m}$$

$$\Rightarrow w_s = \frac{120f}{P} + \frac{z}{60} = \text{---}$$

$$4) P_f = \frac{3 V_T \text{ Emf}}{w_s x_s} \sin \delta$$



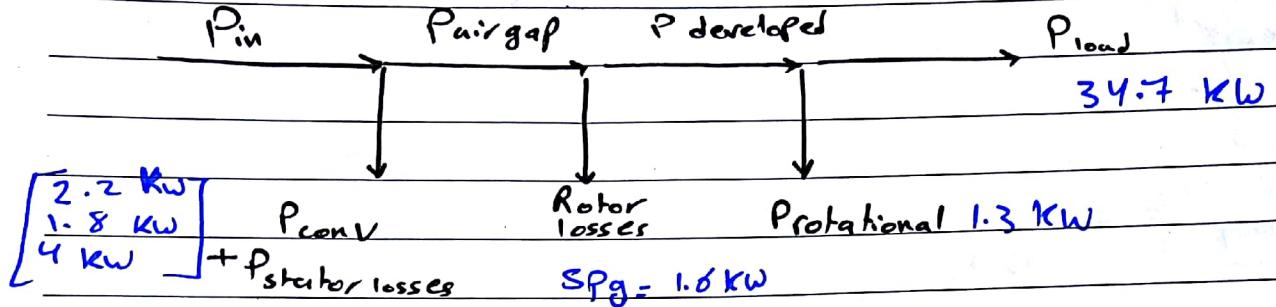
$$P_1 = P_2 = \frac{3 V_T \text{ Emf} \sin \delta_1}{w_s x_s} = \frac{3 V_T \text{ emf}_2 \sin \delta_2}{w_s x_s}$$

$$\text{Emf}_1 \sin \delta_1 = \text{Emf}_2 \sin \delta_2$$

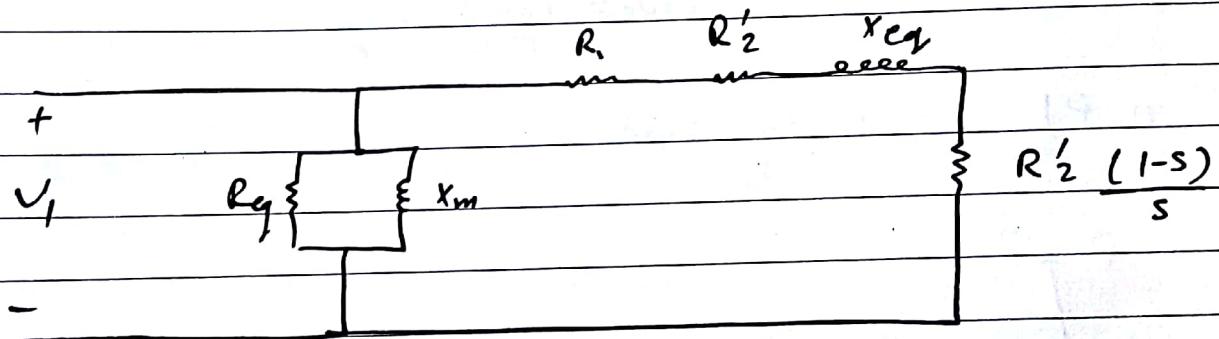
$$\sin( \text{---} ) = 1.25$$

$$\delta_2 = -0.8^\circ$$

\* Im



$$P_g = \frac{36}{1-5} = \underline{36} = 37.6 \text{ kW}$$



Ex) 440 V, 50 Hz, 6 poles, 3Ø, Im

Full load  $\rightarrow$  Slip = 4.3%  
 Stator P.F = 0.87 lagging  
 Developed Power 36 kW  
 Stator copper losses  
 $P_{core} = 2.2 \text{ kW}$   
 Shaft friction = 1.3 kW

Find : 1)  $N_s$  (Synchronous speed)  $\Rightarrow N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$

2) Full load motor speed

$$\Rightarrow N_m = N_s(1-s) = 1000 \times (1-0.043) \\ = 957 \text{ rpm}$$

Ex) 3Ø, 480 V, 12 pole, Y connection,  $R_1 = 1 \Omega$

$$R_2' = 0.5 \Omega, X_{eq} = 10 \Omega, X_m = 100 \Omega$$

Find 8) Starting Torque

2) Torque at full load at 100% slip

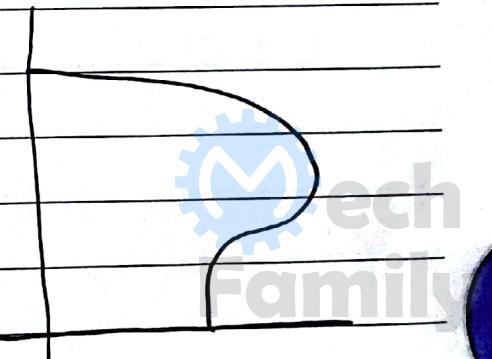
3) Motor speed at full load

4) slip / max torque

5)  $T_{max}$

$$T = \frac{3 V_1^2}{5 w_s} \frac{R_2'}{\left[ \left( R_1 + \frac{R_2'}{3} \right)^2 + X_{eq}^2 \right]}$$

$$V_1 = 480 \sqrt{3}, w_s = \frac{120f}{P} \times \frac{2\pi}{60}$$



\* Starting  $S=1 \Rightarrow T_{ST} = 179 \text{ N.m}$

\* Full load  $S=0.01 \Rightarrow T_{FL} = 67.9 \text{ N.m}$

\* Motor speed at full load  $= N_s (1 - S_{FL})$

$$\text{Slip} / \text{max torque} = \frac{R_2'}{\sqrt{R_1^2 + X_{eq}^2}}$$