Fundamentals of Heat and Mass Transfer

Chapter 1

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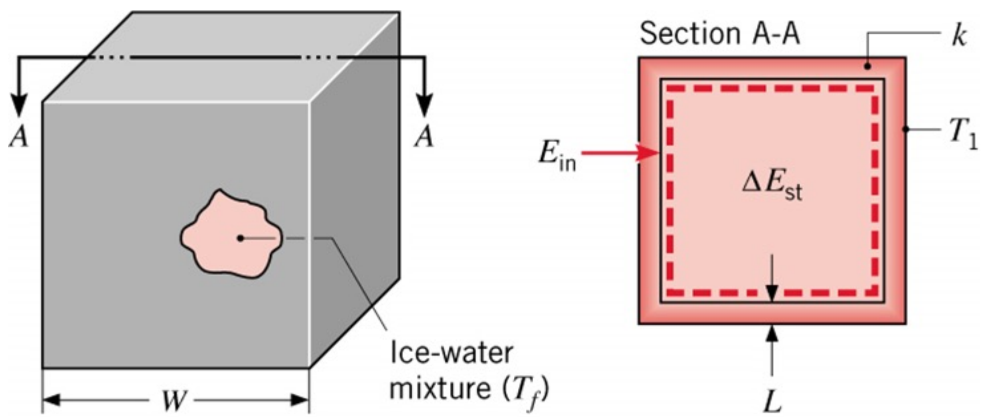
Problem: Solid-liquid Phase Change

Example 1.5: Application to isothermal solid-liquid phase change in a container:

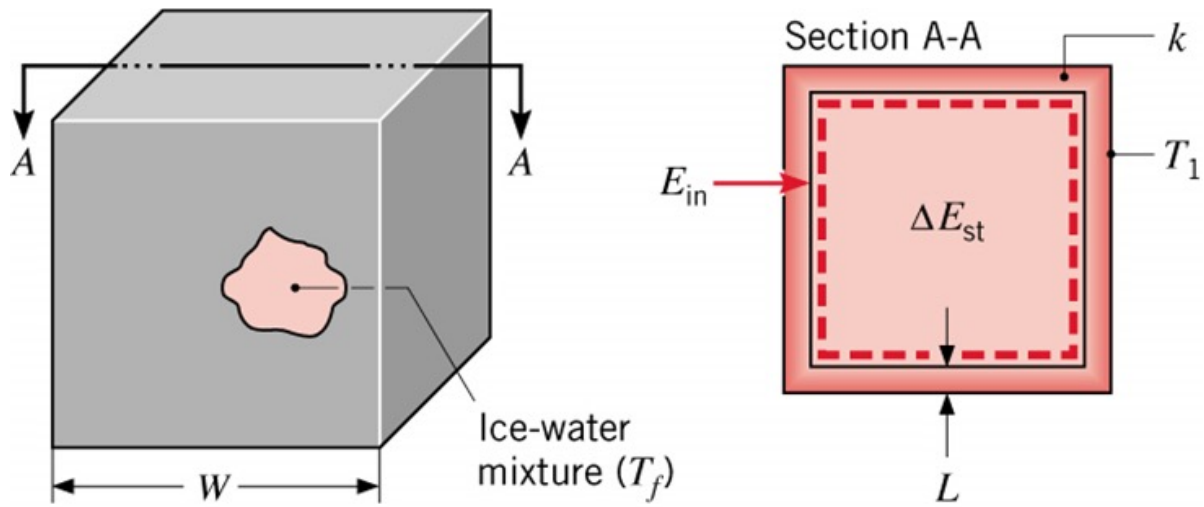


$$\Delta U_t = \Delta U_{lat} = m h_{sf}$$

Latent heat of fusion (h_{sf})

Latent Heat of Fusion:
The amount of heat energy released or absorbed under the phase change process at atmospheric pressure at its melting point.
(Melting and Evaporation→ Endothermic)
(Freezing→ Exothermic)

Consider a mass m of ice at the fusion temperature ($T_f = 0°C$) that is enclosed in a cubical container of width W on a side. The container wall is of thickness L and thermal conductivity k. If the outer surface of the wall is heated to a temperature $T_1 > T_f$ to melt the ice, obtain an expression for the time needed to melt the entire mass of ice.



SOLUTION

Known: Mass and temperature of ice. Dimensions, thermal conductivity, and outer surface temperature of containing wall.

Find: Expression for time needed to melt the ice.

Assumptions:

1. Inner surface of wall is at T_f throughout the process.
2. Constant properties.
3. Steady-state, one-dimensional conduction through each wall.
4. Conduction area of one wall may be approximated as W^2 ($L \ll W$).

$$E_{\text{in}} = \Delta E_{\text{st}} = \Delta U_{\text{lat}}$$

$$q_{\text{cond}} = k(6W^2)\frac{T_1 - T_f}{L}$$

$$E_{\text{in}} = \left[k(6W^2)\frac{T_1 - T_f}{L}\right]t_m$$

$$\Delta E_{\text{st}} = mh_{sf} \qquad t_m = \frac{mh_{sf}L}{6W^2 k(T_1 - T_f)}$$

Comments:

1. Several complications would arise if the ice were initially subcooled. The storage term would have to include the change in sensible (internal thermal) energy required to take the ice from the subcooled to the fusion temperature. During this process, temperature gradients would develop in the ice. Subcooling to -10 ^oC then heating at 30 ^oC (Sensible plus latent in the stored energy)

2. Consider a cavity of width W = 100 mm on a side, wall thickness L = 5 mm, and thermal conductivity k = 0.05 W/m · K. The mass of the ice in the cavity is $m = \rho_s (W - 2L)^3 = 920\ \text{kg/m}^3 \times (0.100 - 0.01)^3\ \text{m}^3 = 0.67\ \text{kg}$

If the outer surface temperature is T_1 = 30°C, the time required to melt the ice is

$$t_m = \frac{0.67\ \text{kg} \times 334{,}000\ \text{J/kg} \times 0.005\ \text{m}}{6(0.100\ \text{m})^2 \times 0.05\ \text{W/m} \cdot \text{K}\ (30 - 0)°\text{C}} = 12{,}430\ \text{s} = 207\ \text{min}$$

The density and latent heat of fusion of the ice are ρ_s = 920 kg/m3 and h_{sf}= 334 kJ/kg, respectively.

The Surface Energy Balance

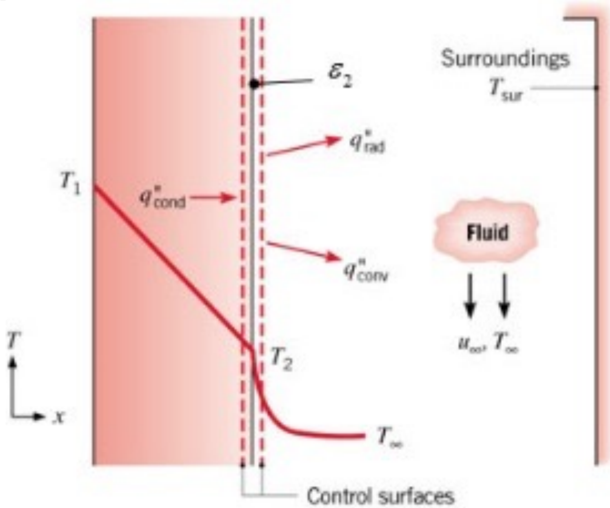
A special case for which no volume or mass is encompassed by the control surface.

Conservation of Energy (Instant in Time):

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad (1.13)$$

- With no mass and volume, energy storage and generation are not pertinent to the energy balance, even if they occur in the medium bounded by the surface. Applies for steady-state and transient conditions.

Consider surface of wall with heat transfer by conduction, convection and radiation.



$$q''_{cond} - q''_{conv} - q''_{rad} = 0$$

$$k\frac{T_1 - T_2}{L} - h(T_2 - T_\infty) - \varepsilon_2\sigma\left(T_2^4 - T_{sur}^4\right) = 0$$

<u>EXAMPLE 1.6</u>

Humans are able to control their rates of heat production and heat loss to maintain a nearly constant core temperature of $T_c = 37°C$ under a wide range of environmental conditions. This process is called thermoregulation.

From the perspective of calculating heat transfer between a human body and its surroundings, we focus on a layer of skin and fat, with its outer surface exposed to the environment and its inner surface at a temperature slightly less than the core temperature, $T_i = 35°C = 308$ K. Consider a person with a skin/fat layer of thickness $L = 3$ mm and effective thermal conductivity $k = 0.3$ W/m · K. The person has a surface area $A = 1.8$ m^2 and is dressed in a bathing suit. The emissivity of the skin is $\varepsilon = 0.95$.

1. When the person is in still air at $T_\infty = 297$ K, what is the skin surface temperature and rate of heat loss to the environment? Convection heat transfer to the air is characterized by a free convection coefficient of $h = 2\ W/m^2 \cdot K$.

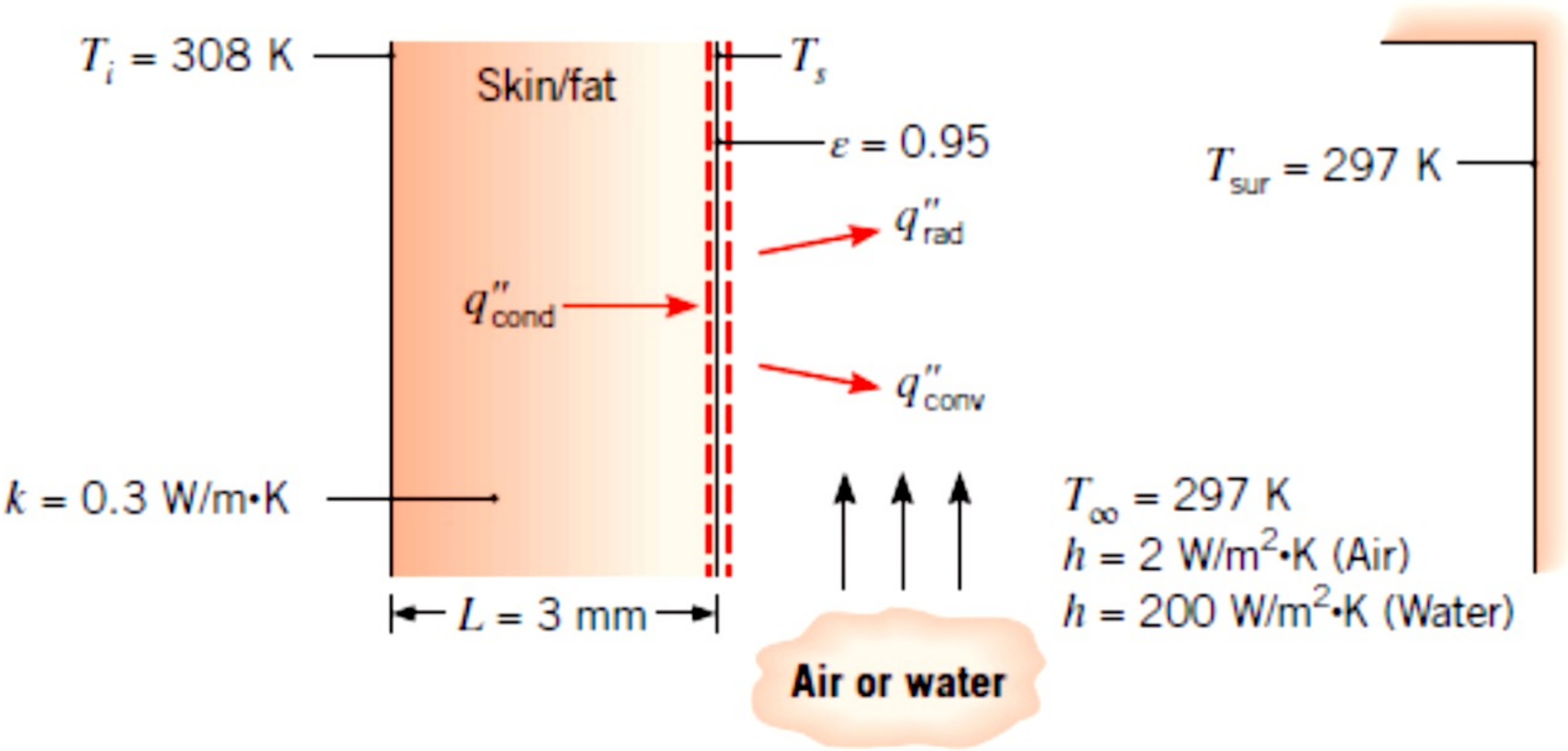
2. When the person is in water at $T_\infty = 297$ K, what is the skin surface temperature and rate of heat loss? Heat transfer to the water is characterized by a convection coefficient of $h = 200\ W/m^2 \cdot K$.

SOLUTION

Known: Inner surface temperature of a skin/fat layer of known thickness, thermal conductivity, emissivity, and surface area. Ambient conditions.

Find: Skin surface temperature and rate of heat loss for the person in air and the person in water.

Schematic:



T_i = 308 K
Skin/fat
T_s
ε = 0.95
T_{sur} = 297 K
q''_{rad}
q''_{cond}
q''_{conv}
k = 0.3 W/m•K
T_∞ = 297 K
h = 2 W/m^2•K (Air)
h = 200 W/m^2•K (Water)
L = 3 mm
Air or water

Assumptions:

1. Steady-state conditions.
2. One-dimensional heat transfer by conduction through the skin/fat layer.
3. Thermal conductivity is uniform.
4. In part 1, radiation exchange between the skin surface and the surroundings is between a small surface and a large enclosure at the air temperature.
5. Liquid water is opaque to thermal radiation (An opaque body in thermal radiation is a body that does not allow thermal radiation to pass through it)
6. Bathing suit has no effect on heat loss from body.
7. Solar radiation is negligible.
8. Body is completely immersed in water in part 2.

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = 0$$

$$q''_{\text{cond}} - q''_{\text{conv}} - q''_{\text{rad}} = 0$$

$$k\frac{T_i - T_s}{L} = h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_{\text{sur}}^4)$$

$$k\frac{T_i - T_s}{L} = h(T_s - T_\infty) + h_r(T_s - T_{\text{sur}})$$

$$T_s = \frac{\dfrac{kT_i}{L} + (h + h_r)T_\infty}{\dfrac{k}{L} + (h + h_r)}$$

1- We estimate h_r with a guessed value of T_s = 305 K and T_∞ = 297 K, to yield h_r = 5.9 W/m^2 · K. Then, substituting numerical values into the preceding equation, we find

$$T_s = \frac{\dfrac{0.3\ \text{W/m}\cdot\text{K} \times 308\ \text{K}}{3\times10^{-3}\ \text{m}} + (2+5.9)\ \text{W/m}^2\cdot\text{K}\times 297\ \text{K}}{\dfrac{0.3\ \text{W/m}\cdot\text{K}}{3\times10^{-3}\ \text{m}} + (2+5.9)\ \text{W/m}^2\cdot\text{K}} = 307.2\ \text{K}$$

With this new value of T_s, we can recalculate h_r and T_s, which are unchanged. Thus, the skin temperature is 307.2 K ≅ 34°C.

$$q_s = kA\frac{T_i - T_s}{L} = 0.3\ \text{W/m}\cdot\text{K}\times 1.8\ \text{m}^2 \times \frac{(308-307.2)\ \text{K}}{3\times10^{-3}\ \text{m}} = 146\ \text{W}$$

2- Since liquid water is opaque to thermal radiation, heat loss from the skin surface is by convection only. Using the previous expression with $h_r = 0$, we find

$$T_s = \frac{\dfrac{0.3\ \text{W/m}\cdot\text{K} \times 308\ \text{K}}{3 \times 10^{-3}\ \text{m}} + 200\ \text{W/m}^2\cdot\text{K} \times 297\ \text{K}}{\dfrac{0.3\ \text{W/m}\cdot\text{K}}{3 \times 10^{-3}\ \text{m}} + 200\ \text{W/m}^2\cdot\text{K}} = 300.7\ \text{K}$$

$$q_s = kA\frac{T_i - T_s}{L} = 0.3\ \text{W/m}\cdot\text{K} \times 1.8\ \text{m}^2 \times \frac{(308 - 300.7)\ \text{K}}{3 \times 10^{-3}\ \text{m}} = 1320\ \text{W}$$

Comments:

1. When using energy balances involving radiation exchange, the temperatures appearing in the radiation terms must be expressed in kelvins, and it is good practice to use kelvins in all terms to avoid confusion.

2. In part 1, the rates of heat loss due to convection and radiation are 37 W and 109 W, respectively. Thus, it would not have been reasonable to neglect radiation. Care must be taken to include radiation when the heat transfer coefficient is small (as it often is for natural convection to a gas), even if the problem statement does not give any indication of its importance.

Comments:

3. A typical rate of metabolic heat generation is 100 W. If the person stayed in the water too long, the core body temperature would begin to fall. The large rate of heat loss in water is due to the higher heat transfer coefficient,

4. The skin temperature of 34°C in part 1 is comfortable, but the skin temperature of 28°C in part 2 is uncomfortably cold.