Fundamentals of Heat and Mass Transfer

Chapter 2

Fourier's Law and the Heat Equation

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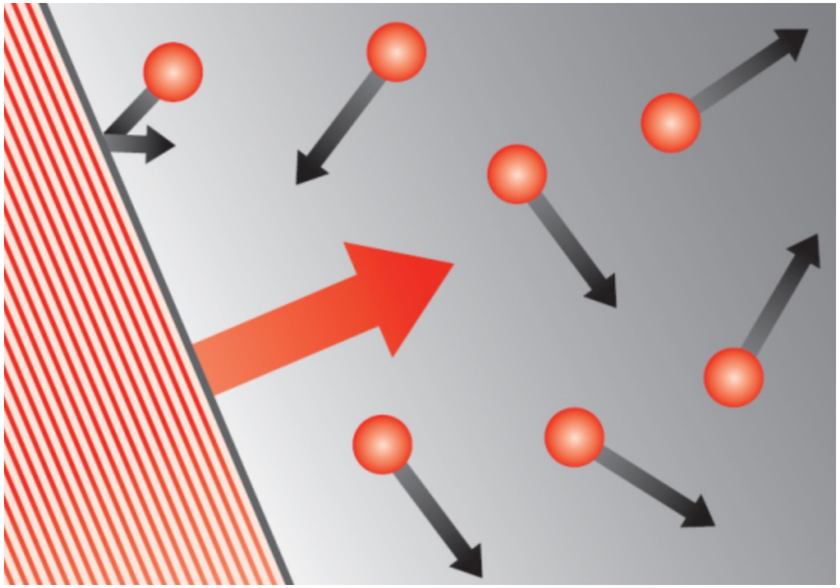
Introduction



- **Conduction is the transport of energy in a medium due to a temperature gradient, and the physical mechanism is one of random atomic or molecular activity.**

- Conduction heat transfer is **governed by Fourier's law** and that use of the law to determine the heat flux depends on knowledge of the manner in which temperature varies within the medium (the temperature distribution).

- **We restricted our attention to simplified conditions (one-dimensional, steady-state conduction in a plane wall).**

Introduction

Fourier's law is applicable to transient, multidimensional conduction in complex geometries.

The objectives of this chapter:

1- A deeper understanding of Fourier's law.

- What is its origins?
- What form does it take for different geometries?
- How does its proportionality constant (the thermal conductivity) depend on the physical nature of the medium?

Introduction

2- To develop, **the general equation**, termed the heat equation, which governs the temperature distribution in a medium in which conduction is the only mode of heat transfer.

- The solution to this equation provides knowledge of the temperature distribution, which may then be used with Fourier's law to determine the heat flux.

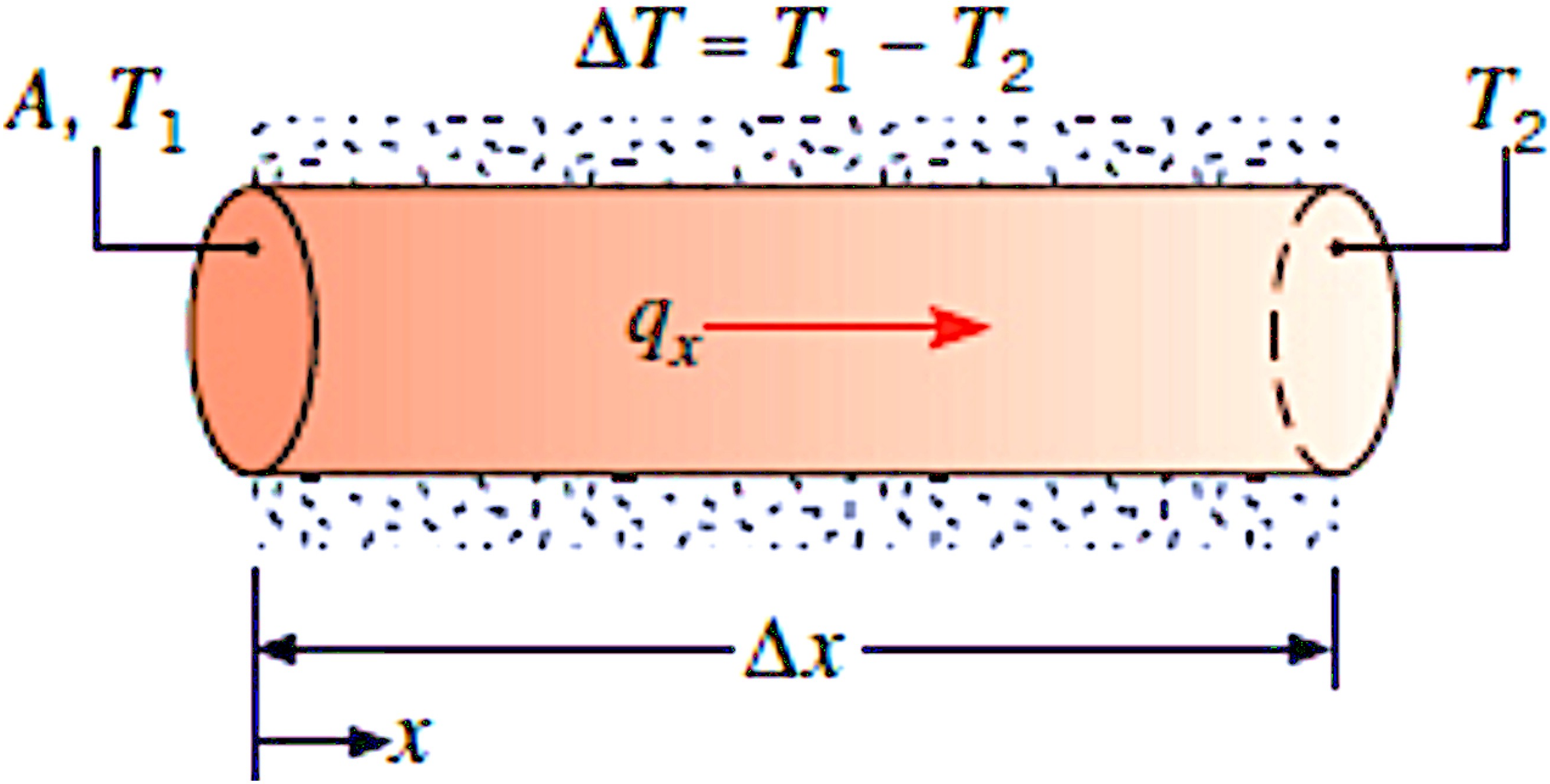
Fourier’s Law

- Fourier's law is developed from observed phenomena rather than being derived from first principles.

- Hence, we view the rate equation as a generalization based on much experimental evidence.

- For example, consider the steady-state conduction experiment of the figure in the next slide.

Fourier's Law



Fourier’s Law

- A cylindrical rod of known material is insulated on its lateral surface, while its end faces are maintained at different temperatures, with $T1 > T2$.

- The temperature difference causes conduction heat transfer in the positive x-direction.

- We are able to measure the heat transfer rate q_x, and we seek to determine how q_x depends on the following variables: ΔT, the temperature difference; Δx, the rod length; and A, the cross-sectional area.

Fourier's Law

- We might imagine first holding ΔT and Δx constant and varying A.
- If we do so, we find that q_x is directly proportional to A.
- Similarly, holding ΔT and A constant, we observe that q_x varies inversely with Δx.
- Finally, holding A and Δx constant, we find that q_x is directly proportional to ΔT. The collective effect is then:

$$q_x \propto A \frac{\Delta T}{\Delta x}$$

Fourier's Law

- In changing the material (e.g., from a metal to a plastic), we would find that this proportionality remains valid.

- However, we would also find that, for equal values of A, Δx, and ΔT, the value of q_x would be smaller for the plastic than for the metal.

- This suggests that the proportionality may be converted to an equality by introducing a coefficient that is a measure of the material behavior. Hence, we write

$$q_x = kA\frac{\Delta T}{\Delta x}$$

Fourier's Law

- where k, the thermal conductivity (W/m · K), is an important property of the material.

- Evaluating this expression in the limit as $\Delta x \rightarrow 0$, we obtain for the heat rate

$$q_x = -kA\frac{dT}{dx}$$

For for the heat flux

$$q''_x = \frac{qx}{A} = -k\frac{dT}{dx}$$

- The minus sign is necessary because heat is always transferred in the direction of decreasing temperature.

Fourier’s Law

- Fourier's law implies that the heat flux is a directional quantity.

- In particular, the direction of q''_x is normal to the cross-sectional area A.
- Or, more generally, the direction of heat flow q''_x will always be normal to a surface of constant temperature, called an isothermal surface.

- Figure 2.2 illustrates the direction of heat flow in a plane wall for which the temperature gradient dT/dx is negative.

- From Equation 2.2, it follows that is positive. Note that the isothermal surfaces are planes normal to the x-direction.

Fourier's Law

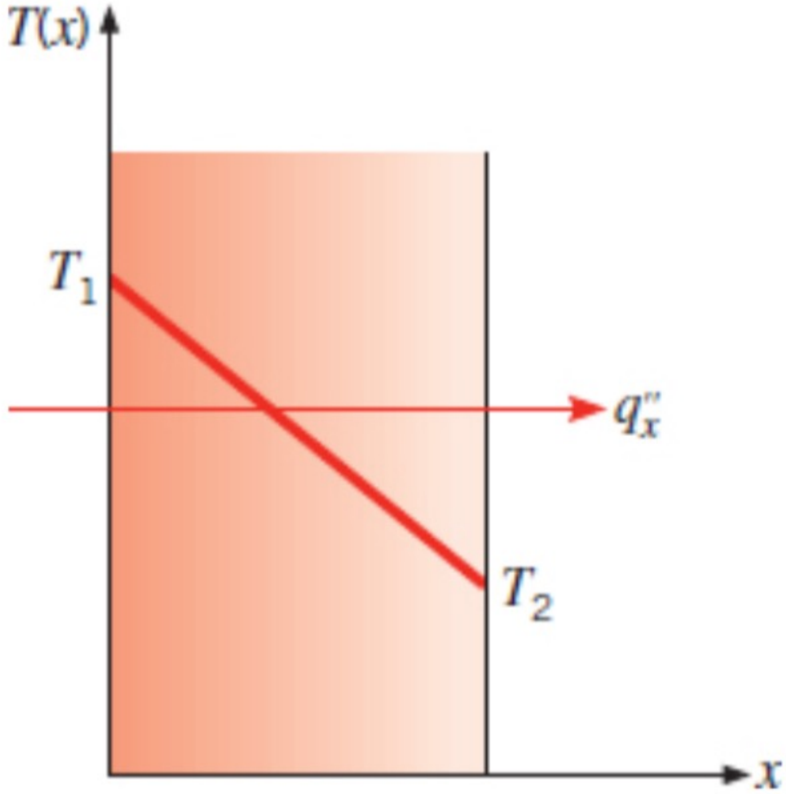


FIGURE 2.2 The relationship between coordinate system, heat flow direction, and temperature gradient in one dimension.

Fourier's Law

- Recognizing that the heat flux is a vector quantity, we can write a more general statement of the conduction rate equation (Fourier's law) as follows:

$$\boldsymbol{q}'' = -k\nabla T = -k\left(\boldsymbol{i}\frac{\partial T}{\partial x} + \boldsymbol{j}\frac{\partial T}{\partial y} + \boldsymbol{k}\frac{\partial T}{\partial z}\right)$$

- where ∇ is the three-dimensional del operator, i, j, and k are the unit vectors in the x, y, and z directions, and T(x, y, z) is the scalar temperature field.