Suggested Problems
Heat Transfer I
Ch2 & Ch3

2.26 A one-dimensional plane wall of thickness 2L = 80 mm experiences uniform thermal energy generation of $\dot{q}$=1000 W/m^3 and is convectively cooled at x = ±40 mm by an ambient fluid characterized by
T_∞ = 30°C. If the steady-state temperature distribution within the wall is T(x) = a(L2 − x2) + b where a = 15°C/m2 and b = 40°C.

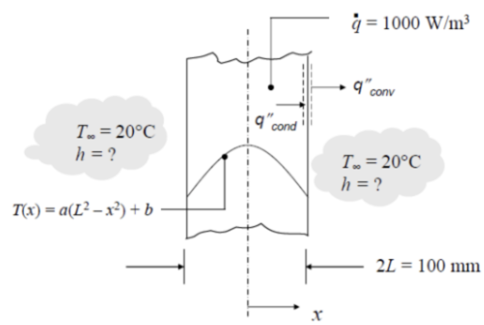
1- What is the thermal conductivity of the wall, k?
2- What is the value of the convection heat transfer coefficient, h?

PROBLEM 2.26

KNOWN: Wall thickness. Thermal energy generation rate. Temperature distribution. Ambient fluid temperature.

FIND: Thermal conductivity. Convection heat transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation.

ANALYSIS: Under the specified conditions, the heat equation, Equation 2.21, reduces to

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

With the given temperature distribution, $d^2T/dx^2 = -2a$. Therefore, solving for k gives

$$k = \frac{\dot{q}}{2a} = \frac{1000\ \text{W/m}^3}{2 \times 15°\text{C/m}^2} = 33.3\ \text{W/m}\cdot\text{K} \quad <$$

The convection heat transfer coefficient can be found by applying the boundary condition at $x = L$ (or at $x = -L$),

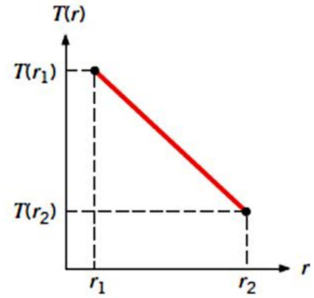
$$-k\left.\frac{dT}{dx}\right|_{x=L} = h\left[T(L) - T_\infty\right]$$

Therefore

$$h = \frac{-k\left.\frac{dT}{dx}\right|_{x=L}}{\left[T(L) - T_\infty\right]} = \frac{2kaL}{b - T_\infty} = \frac{2 \times 33.3\ \text{W/m}\cdot\text{K} \times 15°\text{C/m}^2 \times 0.04\ \text{m}}{40°\text{C} - 30°\text{C}} = 4\ \text{W/m}^2\cdot\text{K} \quad <$$

COMMENTS: (1) In Chapter 3, you will learn how to determine the temperature distribution. (2) The heat transfer coefficient could also have been found from an energy balance on the wall. With $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$, we find $-2hA[T(L) - T_\infty] + 2\dot{q}LA = 0$. This yields the same result for h.

2.30 One-dimensional, steady-state conduction with no energy generation is occurring in a spherical shell of inner radius r_1 and outer radius r_2. Under what condition is the linear temperature distribution shown below possible?

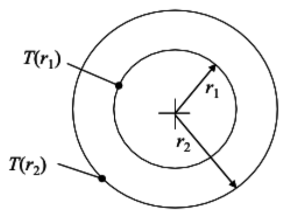


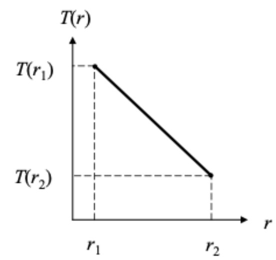
PROBLEM 2.30

KNOWN: Spherical shell under steady-state conditions with no energy generation.

FIND: Under what conditions is a linear temperature distribution possible.

SCHEMATIC:





ASSUMPTIONS: (1) Steady state, (2) One-dimensional, (3) No heat generation.

ANALYSIS: Under the stated conditions, the heat equation in spherical coordinates, Equation 2.29, reduces to

$$\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)=0$$

If the temperature distribution is a linear function of r, then the temperature gradient is constant, and this equation becomes

$$\frac{d}{dr}\left(kr^2\right)=0$$

which implies kr^2 = constant, or $k \sim 1/r^2$. The only way there could be a linear temperature distribution in the spherical shell is if the thermal conductivity were to vary inversely with r^2. <

COMMENTS: It is unlikely to encounter or even create a material for which k varies inversely with the spherical radial coordinate r in the manner necessary to develop a linear temperature distribution. Assuming linear temperature distributions in radial systems is nearly always both fundamentally incorrect and physically implausible.

2.31 The steady-state temperature distribution in a one-dimensional wall of thermal conductivity *k* and thickness *L* is of the form $T = ax^2 + bx + c$. Derive expressions for the heat fluxes at the two wall faces ($x = 0, L$), and the energy generation rate in the wall per unit wall area.

PROBLEM 2.31

KNOWN: Steady-state temperature distribution in a one-dimensional wall is $T(x) = Ax^2 + Bx + C$, thermal conductivity, thickness.

FIND: Expressions for the heat fluxes at the two wall faces (x = 0,L) and the heat generation rate in the wall per unit area.

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Homogeneous medium.

ANALYSIS: The appropriate form of the heat diffusion equation for these conditions is

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \quad \text{or} \quad \dot{q} = -k\frac{d^2T}{dx^2}.$$

Hence, the generation rate is

$$\dot{q} = -k\frac{d}{dx}\left[\frac{dT}{dx}\right] = -k\frac{d}{dx}\left[2Ax + B + 0\right]$$

$$\dot{q} = -k\left[2A\right]$$ <

which is constant. The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x = -k\frac{dT}{dx} = -k\left[2Ax + B\right]$$

using the expression for the temperature gradient derived above. Hence, the heat fluxes are:

Surface x=0:

$$q''_x(0) = -kB$$ <

Surface x=L:

$$q''_x(L) = -k\left[2AL + B\right].$$ <

COMMENTS: (1) From an overall energy balance on the wall, find

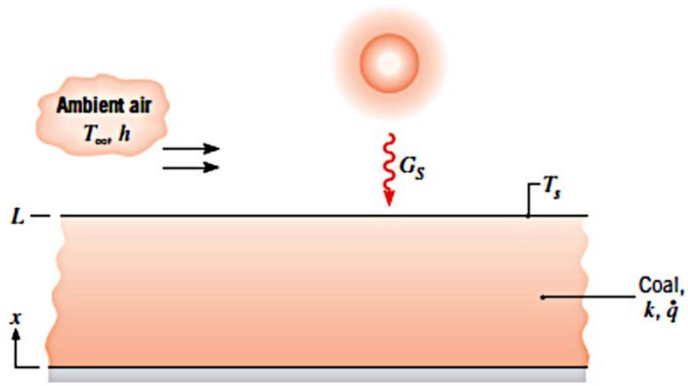
$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}''_g = 0$$
$$q''_x(0) - q''_x(L) + \dot{E}''_g = (-kB) - (-k)\left[2AL + B\right] + \dot{E}''_g = 0$$
$$\dot{E}''_g = -2AkL.$$

From integration of the volumetric heat rate, we can also find $\dot{E}''_g$ as

$$\dot{E}''_g = \int_0^L \dot{q}(x)dx = \int_0^L -k\left[2A\right]dx = -k\left[2AL\right]$$

which agrees with the above, as it should.

2.33 A plane layer of coal of thickness L = 1 m experiences uniform volumetric generation at a rate of $\dot{q}$=10 W/m^3 due to slow oxidation of the coal particles. Averaged over a daily period, the top surface of the layer transfers heat by convection to ambient air for which h = 8 W/m^2 · K and T∞ = 30°C, while receiving solar irradiation in the amount GS = 500 W/m^2. Irradiation from the atmosphere may be neglected. The solar absorptivity and emissivity of the surface are each αS = ε = 0.95.



(a) Write the steady-state form of the heat diffusion equation for the layer of coal. Verify, by direct substitution, that this equation is satisfied by a temperature distribution of the form

$$T(x) = T_s + \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right)$$

From this distribution, what can you say about conditions at the bottom surface (x = 0)? Sketch the temperature distribution and label key features.

(b) Obtain an expression for the rate of heat transfer by conduction per unit area at x = L. Applying an energy balance to a control surface about the top surface of the layer, obtain an expression for Ts. Evaluate Ts and T(0) for the prescribed conditions.

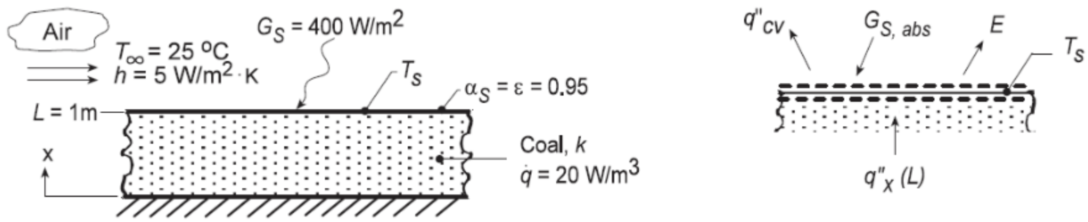
(c) Daily average values of GS and h depend on a number of factors, such as time of year, cloud cover, and wind conditions. For h = 8 W/m2 · K, compute and plot TS and T(0) as a function of GS for 50 ≤ GS ≤ 500 W/m2. For GS = 500 W/m2, compute and plot TS and T(0) as a function of h for 5 ≤ h ≤ 50 W/m2 · K.

PROBLEM 2.33

KNOWN: Coal pile of prescribed depth experiencing uniform volumetric generation with convection, absorbed irradiation and emission on its upper surface.

FIND: (a) The appropriate form of the heat diffusion equation (HDE) and whether the prescribed temperature distribution satisfies this HDE; conditions at the bottom of the pile, $x = 0$; sketch of the temperature distribution with labeling of key features; (b) Expression for the conduction heat rate at the location $x = L$; expression for the surface temperature T_s based upon a surface energy balance at $x = L$; evaluate T_S and $T(0)$ for the prescribed conditions; (c) Based upon typical daily averages for G_S and h, compute and plot T_S and $T(0)$ for (1) $h = 5\ W/m^2 \cdot K$ with $50 \le G_S \le 500\ W/m^2$, (2) $G_S = 400\ W/m^2$ with $5 \le h \le 50\ W/m^2 \cdot K$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Uniform volumetric heat generation, (3) Constant properties, (4) Negligible irradiation from the surroundings, and (5) Steady-state conditions.

PROPERTIES: *Table A.3*, Coal (300K): $k = 0.26\ W/m \cdot K$

ANALYSIS: (a) For one-dimensional, steady-state conduction with uniform volumetric heat generation and constant properties the heat diffusion equation (HDE) follows from Eq. 2.22,

$$\frac{d}{dx}\left(\frac{dT}{dx}\right) + \frac{\dot{q}}{k} = 0 \qquad (1) <$$

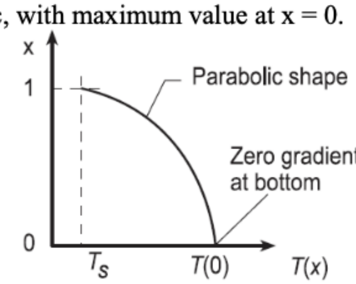
Substituting the temperature distribution into the HDE, Eq. (1),

$$T(x) = T_S + \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) \qquad \frac{d}{dx}\left[0 + \frac{\dot{q}L^2}{2k}\left(0 - \frac{2x}{L^2}\right)\right] + \frac{\dot{q}}{k} ? = ?0 \qquad (2,3)$$

we find that it does indeed satisfy the HDE for all values of x. <

From Eq. (2), note that the temperature distribution must be quadratic, with maximum value at $x = 0$. At $x = 0$, the heat flux is

$$q''_x(0) = -k\left.\frac{dT}{dx}\right)_{x=0} = -k\left[0 + \frac{\dot{q}L^2}{2k}\left(0 - \frac{2x}{L^2}\right)\right]_{x=0} = 0$$



so that the gradient at $x = 0$ is zero. Hence, the bottom is insulated.

(b) From an overall energy balance on the pile, the conduction heat flux at the surface must be

$$q''_x(L) = \dot{E}''_g = \dot{q}L \qquad <$$

Continued...

From a surface energy balance per unit area shown in the schematic above,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q''_x(L) - q''_{conv} + G_{S,abs} - E = 0$$

$$\dot{q}L - h(T_s - T_\infty) + 0.95G_S - \varepsilon\sigma T_s^4 = 0 \tag{4}$$

$$10\,W/m^3 \times 2\,m - 8\,W/m^2{\cdot}K(T_s - 303\,K) + 0.95 \times 500\,W/m^2 - 0.95 \times 5.67 \times 10^{-8}\,W/m^2{\cdot}K^4 T_s^4 = 0$$

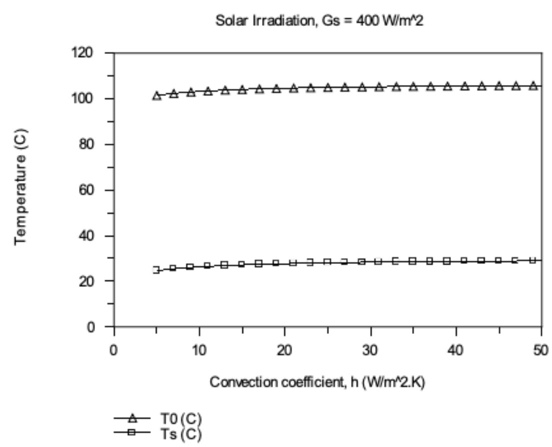
$$T_s = 305.6\,K = 32.6°C$$ <

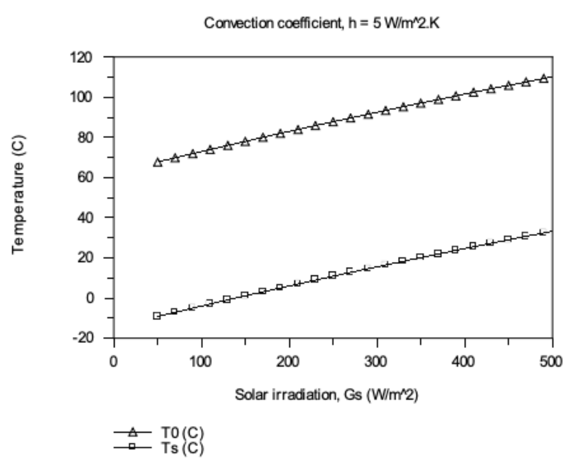
From Eq. (2) with x = 0, find

$$T(0) = T_s + \frac{\dot{q}L^2}{2k} = 32.6°C + \frac{10\,W/m^3 \times (2\,m)^2}{2 \times 0.26\,W/m \cdot K} = 109.5°C \tag{5}$$ <

where the thermal conductivity for coal was obtained from Table A.3.

(c) Two plots are generated using Eq. (4) and (5) for T_s and T(0), respectively; (1) with h = 5 W/m^2·K for 50 ≤ G_S ≤ 500 W/m^2 and (2) with G_S = 400 W/m^2 for 5 ≤ h ≤ 50 W/m^2·K.





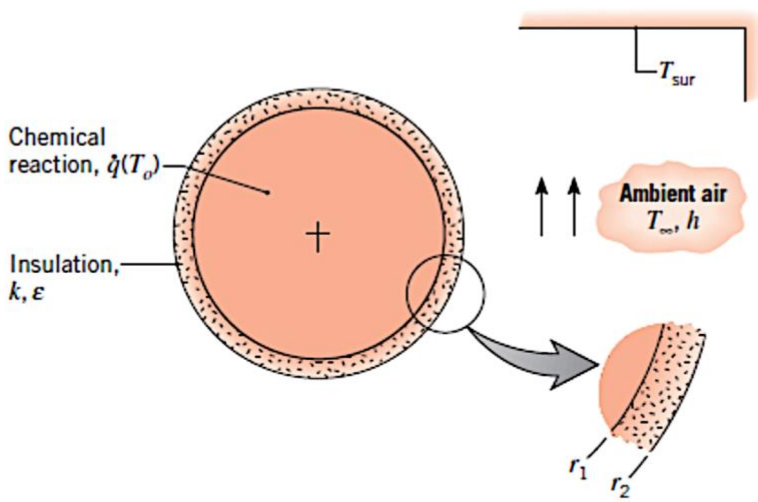
From the T vs. h plot with $G_S = 400\ W/m^2$, note that the convection coefficient does not have a major influence on the surface or bottom coal pile temperatures. From the T vs. G_S plot with $h = 5\ W/m^2\cdot K$, note that the solar irradiation has a very significant effect on the temperatures. The fact that T_s is less than the ambient air temperature, T_∞, and, in the case of very low values of G_S, below freezing, is a consequence of the large magnitude of the emissive power E.

COMMENTS: In our analysis we ignored irradiation from the sky, an environmental radiation effect you'll consider in Chapter 12. Treated as large isothermal surroundings, $G_{sky} = \sigma T_{sky}^4$ where T_{sky} = -30°C for very clear conditions and nearly air temperature for cloudy conditions. For low G_S conditions we should consider G_{sky}, the effect of which will be to predict higher values for T_s and T(0).

2.41 A chemically reacting mixture is stored in a thin-walled spherical container of radius r1 = 200 mm, and the exothermic reaction generates heat at a uniform, but temperature-dependent volumetric rate of

$$\dot{q} = \dot{q}_o \exp(-A/T_o),$$

Where $\dot{q}$=5000 W/m³, A = 75 K, and To is the mixture temperature in kelvins. The vessel is enclosed by an insulating material of outer radius r2, thermal conductivity k, and emissivity ε. The outer surface of the insulation experiences convection heat transfer and net radiation exchange with the adjoining air and large surroundings, respectively.



(a) Write the steady-state form of the heat diffusion equation for the insulation. Verify, by direct substitution, that this equation is satisfied by the temperature distribution
Sketch the temperature distribution, T(r), labeling key features.

$$T(r) = T_{s,1} - (T_{s,1} - T_{s,2})\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$$

(b) Applying Fourier's law, show that the rate of heat transfer by conduction through the insulation may be expressed as
Applying an energy balance to a control surface about the container, obtain an alternative expression for q_r, expressing your result in terms of $\dot{q}$ and r1.

$$q_r = \frac{4\pi k(T_{s,1} - T_{s,2})}{(1/r_1) - (1/r_2)}$$

(c) Applying an energy balance to a control surface placed around the outer surface of the insulation, obtain an expression from which Ts,2 may be determined as a function of $\dot{q}$, r1, h, T_∞, ε, and T_{sur}.

(d) The process engineer wishes to maintain a reactor temperature of T_o = T(r1) = 95°C under conditions for which k = 0.05 W/m · K, r2 = 208 mm, h = 5 W/m2 · K, ε = 0.9, T_∞ = 25°C, and T_{sur} = 35°C. What is the actual reactor temperature and the outer surface temperature Ts,2 of the insulation?

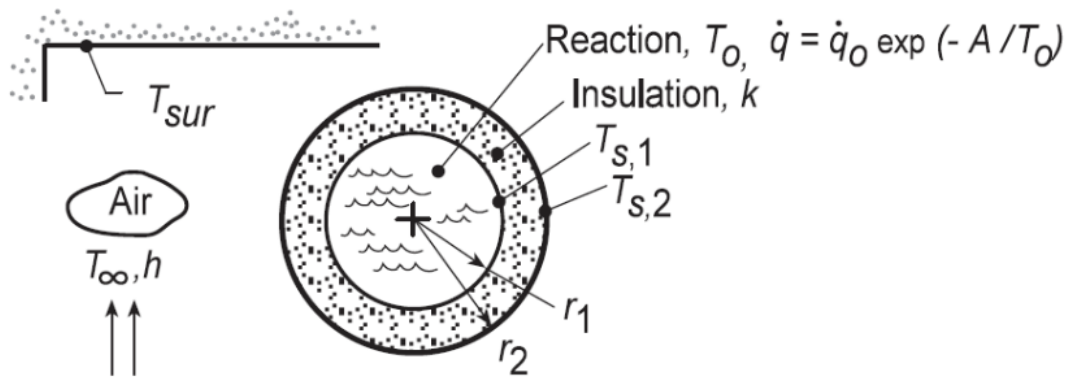
(e) Computer Icon Compute and plot the variation of Ts,2 with r2 for 201 ≤ r2 ≤ 210 mm. The engineer is concerned about potential burn injuries to personnel who may come into contact with the exposed surface of the insulation. Is increasing the insulation thickness a practical solution to maintaining Ts,2 ≤ 45°C? What other parameter could be varied to reduce Ts,2?

PROBLEM 2.41

KNOWN: Spherical container with an exothermic reaction enclosed by an insulating material whose outer surface experiences convection with adjoining air and radiation exchange with large surroundings.

FIND: (a) Verify that the prescribed temperature distribution for the insulation satisfies the appropriate form of the heat diffusion equation; sketch the temperature distribution and label key features; (b) Applying Fourier's law, verify the conduction heat rate expression for the insulation layer, q_r, in terms of $T_{s,1}$ and $T_{s,2}$; apply a surface energy balance to the container and obtain an alternative expression for q_r in terms of $\dot{q}$ and r_1; (c) Apply a surface energy balance around the outer surface of the insulation to obtain an expression to evaluate $T_{s,2}$; (d) Determine $T_{s,2}$ for the specified geometry and operating conditions; (e) Compute and plot the variation of $T_{s,2}$ as a function of the outer radius for the range $201 \le r_2 \le 210$ mm; explore approaches for reducing $T_{s,2} \le 45°C$ to eliminate potential risk for burn injuries to personnel.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial spherical conduction, (2) Isothermal reaction in container so that $T_o = T_{s,1}$, (2) Negligible thermal contact resistance between the container and insulation, (3) Constant properties in the insulation, (4) Surroundings large compared to the insulated vessel, and (5) Steady-state conditions.

ANALYSIS: The appropriate form of the heat diffusion equation (HDE) for the insulation follows from Eq. 2.29,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \qquad (1) <$$

The temperature distribution is given as

$$T(r) = T_{s,1} - \left(T_{s,1} - T_{s,2}\right)\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right] \qquad (2)$$

Continued...

Substitute T(r) into the HDE to see if it is satisfied:

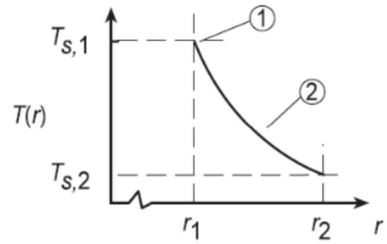
$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\left[0-\left(T_{s,1}-T_{s,2}\right)\frac{0+\left(r_1/r^2\right)}{1-\left(r_1/r_2\right)}\right]\right)=0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(+\left(T_{s,1}-T_{s,2}\right)\frac{r_1}{1-\left(r_1/r_2\right)}\right)=0$$ <

and since the expression in parenthesis is independent of r, T(r) does indeed satisfy the HDE. The temperature distribution in the insulation and its key features are as follows:

(1) $T_{s,1} > T_{s,2}$

(2) Decreasing gradient with increasing radius, r, since the heat rate is constant through the insulation.



(b) Using Fourier's law for the radial-spherical coordinate, the heat rate through the insulation is

$$q_r=-kA_r\frac{dT}{dr}=-k\left(4\pi r^2\right)\frac{dT}{dr}$$ <

and substituting for the temperature distribution, Eq. (2),

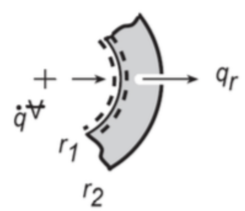
$$q_r=-4k\pi r^2\left[0-\left(T_{s,1}-T_{s,2}\right)\frac{0+\left(r_1/r^2\right)}{1-\left(r_1/r_2\right)}\right]$$

$$q_r=\frac{4\pi k\left(T_{s,1}-T_{s,2}\right)}{\left(1/r_1\right)-\left(1/r_2\right)} \qquad (3)$$ <

Applying an energy balance to a control surface about the container at $r = r_1$,

$$\dot{E}_{in}-\dot{E}_{out}=0$$

$$\dot{q}\forall - q_r = 0$$



where $\dot{q}\forall$ represents the generated heat in the container,

$$q_r=(4/3)\pi r_1^3\dot{q} \qquad (4)$$ <

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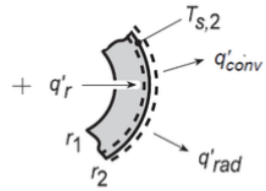
(c) Applying an energy balance to a control surface placed around the outer surface of the insulation,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$



$$q_r - q_{conv} - q_{rad} = 0$$

$$q_r - hA_s\left(T_{s,2} - T_\infty\right) - \varepsilon A_s \sigma\left(T_{s,2}^4 - T_{sur}^4\right) = 0 \qquad (5) <$$

where

$$A_s = 4\pi r_2^2 \qquad (6)$$

These relations can be used to determine $T_{s,2}$ in terms of the variables $\dot{q}$, r_1, r_2, h, T_∞, ε and T_{sur}.

(d) Consider the reactor system operating under the following conditions:

$r_1 = 200$ mm	$h = 5$ W/m²·K	$\varepsilon = 0.9$
$r_2 = 208$ mm	$T_\infty = 25°C$	$T_{sur} = 35°C$
$k = 0.05$ W/m·K		

The heat generated by the exothermic reaction provides for a volumetric heat generation rate,

$$\dot{q} = \dot{q}_o \exp\left(-A/T_o\right) \qquad q_o = 5000\ \mathrm{W/m^3} \qquad A = 75\,\mathrm{K} \qquad (7)$$

where the temperature of the reaction is that of the inner surface of the insulation, $T_o = T_{s,1}$. The following system of equations will determine the operating conditions for the reactor.

Conduction rate equation, insulation, Eq. (3),

$$q_r = \frac{4\pi \times 0.05\ \mathrm{W/m \cdot K}\left(T_{s,1} - T_{s,2}\right)}{\left(1/0.200\,\mathrm{m} - 1/0.208\,\mathrm{m}\right)} \qquad (8)$$

Heat generated in the reactor, Eqs. (4) and (7),

$$q_r = 4/3\pi\left(0.200\,\mathrm{m}\right)^3 \dot{q} \qquad (9)$$

$$\dot{q} = 5000\ \mathrm{W/m^3} \exp\left(-75\,\mathrm{K}/T_{s,1}\right) \qquad (10)$$

Surface energy balance, insulation, Eqs. (5) and (6),

$$q_r - 5\ \mathrm{W/m^2 \cdot K}\, A_s\left(T_{s,2} - 298\,\mathrm{K}\right) - 0.9 A_s 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\left(T_{s,2}^4 - \left(308\,\mathrm{K}\right)^4\right) = 0 \qquad (11)$$

$$A_s = 4\pi\left(0.208\,\mathrm{m}\right)^2 \qquad (12)$$

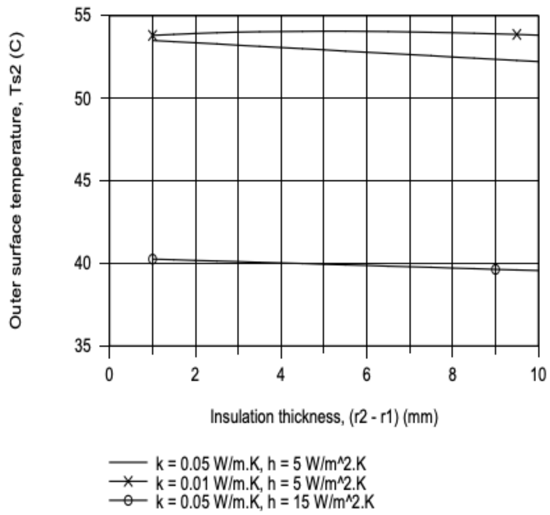
Continued...

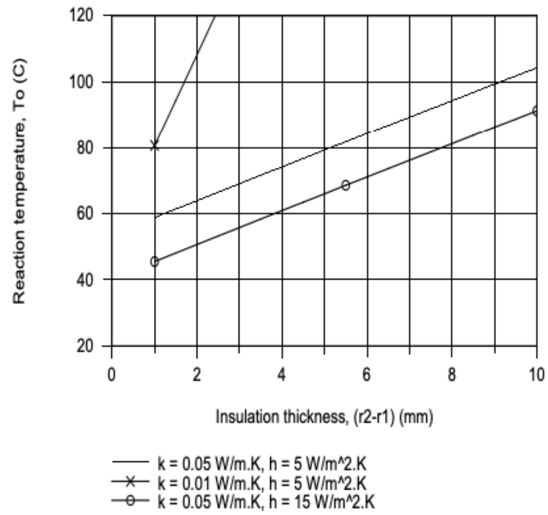
Solving these equations simultaneously, find that

$$T_{s,1} = 94.3^\circ C \qquad T_{s,2} = 52.5^\circ C$$ <

That is, the reactor will be operating at $T_o = T_{s,1} = 94.3°C$, very close to the desired 95°C operating condition.

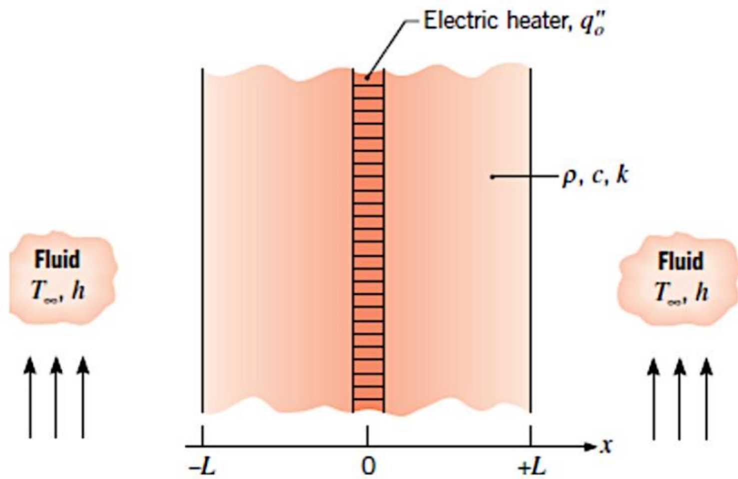
(e) Using the above system of equations, Eqs. (8)-(12), we have explored the effects of changes in the convection coefficient, h, and the insulation thermal conductivity, k, as a function of insulation thickness, $t = r_2 - r_1$.





In the $T_{s,2}$ vs. $(r_2 - r_1)$ plot, note that decreasing the thermal conductivity from 0.05 to 0.01 W/m·K slightly increases $T_{s,2}$ while increasing the convection coefficient from 5 to 15 $W/m^2\cdot K$ markedly decreases $T_{s,2}$. Insulation thickness only has a minor effect on $T_{s,2}$ for either option. In the T_o vs. $(r_2 - r_1)$ plot, note that, for all the options, the effect of increased insulation is to increase the reaction temperature. With k = 0.01 W/m·K, the reaction temperature increases beyond 95°C with less than 2 mm insulation. For the case with h = 15 $W/m^2\cdot K$, the reaction temperature begins to approach 95°C with insulation thickness around 10 mm. We conclude that by selecting the proper insulation thickness and controlling the convection coefficient, the reaction could be operated around 95°C such that the outer surface temperature would not exceed 45°C.

2.42 A thin electrical heater dissipating 4000 W/m2 is sandwiched between two 25-mm-thick plates whose exposed surfaces experience convection with a fluid for which T_∞ = 20°C and h = 400 W/m2. K. The thermophysical properties of the plate material are ρ = 2500 kg/m3, c = 700 J/kg · K, and k = 5 W/m · K.



(a) On T − x coordinates, sketch the steady-state temperature distribution for −L ≤ x ≤ +L. Calculate values of the temperatures at the surfaces, x = ±L, and the midpoint, x = 0. Label this distribution as Case 1, and explain its salient features.

(b) Consider conditions for which there is a loss of coolant and existence of a nearly adiabatic condition on the x = +L surface. On the T − x coordinates used for part (a), sketch the corresponding steady-state temperature distribution and indicate the temperatures at x = 0, ±L. Label the distribution as Case 2, and explain its key features.

(c) With the system operating as described in part (b), the surface x = −L also experiences a sudden loss of coolant. This dangerous situation goes undetected for 15 min, at which time the power to the heater is deactivated. Assuming no heat losses from the surfaces of the plates, what is the eventual (t → ∞), uniform, steady-state temperature distribution in the plates? Show this distribution as Case 3 on your sketch, and explain its key features. Hint: Apply the conservation of energy requirement on a time-interval basis, Eq. 1.12b, for the initial and final conditions corresponding to Case 2 and Case 3, respectively.

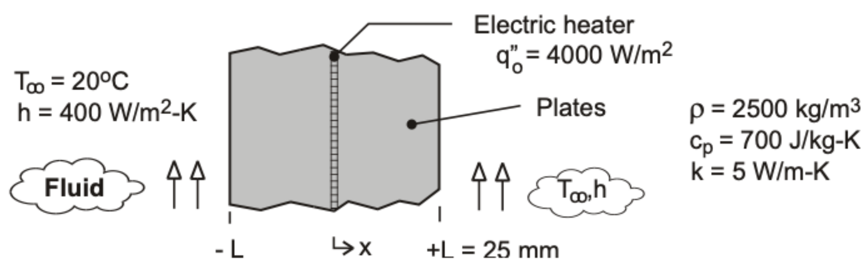
(d) On T − t coordinates, sketch the temperature history at the plate locations x = 0, ±L during the transient period between the distributions for Case 2 and Case 3. Where and when will the temperature in the system achieve a maximum value?

PROBLEM 2.42

KNOWN: Thin electrical heater dissipating 4000 W/m^2 sandwiched between two 25-mm thick plates whose surfaces experience convection.

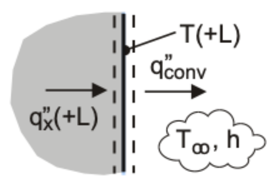
FIND: (a) On T-x coordinates, sketch the steady-state temperature distribution for $-L \le x \le +L$; calculate values for the surfaces x = L and the mid-point, x = 0; label this distribution as Case 1 and explain key features; (b) Case 2: sudden loss of coolant causing existence of adiabatic condition on the x = +L surface; sketch temperature distribution on same T-x coordinates as part (a) and calculate values for x = 0, ± L; explain key features; (c) Case 3: further loss of coolant and existence of adiabatic condition on the x = - L surface; situation goes undetected for 15 minutes at which time power to the heater is deactivated; determine the eventual ($t \to \infty$) uniform, steady-state temperature distribution; sketch temperature distribution on same T-x coordinates as parts (a,b); and (d) On T-t coordinates, sketch the temperature-time history at the plate locations x = 0, ± L during the transient period between the steady-state distributions for Case 2 and Case 3; at what location and when will the temperature in the system achieve a maximum value?

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal volumetric generation in plates, and (3) Negligible thermal resistance between the heater surfaces and the plates.

ANALYSIS: (a) Since the system is symmetrical, the heater power results in equal conduction fluxes through the plates. By applying a surface energy balance on the surface x = +L as shown in the schematic, determine the temperatures at the mid-point, x = 0, and the exposed surface, x + L.



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_x(+L) - q''_{conv} = 0 \qquad \text{where} \qquad q''_x(+L) = q''_o/2$$

$$q''_o/2 - h\left[T(+L) - T_\infty\right] = 0$$

$$T_1(+L) = q''_o/2h + T_\infty = 4000\,\text{W/m}^2/\left(2\times 400\,\text{W/m}^2\cdot\text{K}\right) + 20°\text{C} = 25°\text{C} \qquad <$$

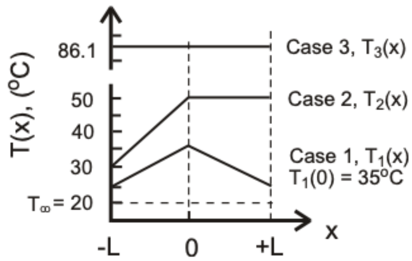
From Fourier's law for the conduction flux through the plate, find T(0).

$$q''_x = q''_o/2 = k\left[T(0) - T(+L)\right]/L$$

$$T_1(0) = T_1(+L) + q''_o L/2k = 25°\text{C} + 4000\,\text{W/m}^2\cdot\text{K}\times 0.025\text{m}/\left(2\times 5\,\text{W/m}\cdot\text{K}\right) = 35°\text{C} \qquad <$$

The temperature distribution is shown on the T-x coordinates below and labeled Case 1. The key features of the distribution are its symmetry about the heater plane and its linear dependence with distance.

Continued …



(b) Case 2: sudden loss of coolant with the existence of an adiabatic condition on surface x = +L. For this situation, all the heater power will be conducted to the coolant through the left-hand plate. From a surface energy balance and application of Fourier's law as done for part (a), find

$$T_2(-L) = q_o''/h + T_\infty = 4000\ \text{W/m}^2 / 400\ \text{W/m}^2\cdot\text{K} + 20°\text{C} = 30°\text{C} \qquad <$$

$$T_2(0) = T_2(-L) + q_o'' L/k = 30°\text{C} + 4000\ \text{W/m}^2 \times 0.025\ \text{m} / 5\ \text{W/m}\cdot\text{K} = 50°\text{C} \qquad <$$

The temperature distribution is shown on the T-x coordinates above and labeled Case 2. The distribution is linear in the left-hand plate, with the maximum value at the mid-point. Since no heat flows through the right-hand plate, the gradient must zero and this plate is at the maximum temperature as well. The maximum temperature is higher than for Case 1 because the heat flux through the left-hand plate has increased two-fold.

(c) Case 3: sudden loss of coolant occurs at the x = -L surface also. For this situation, there is no heat transfer out of either plate, so that for a 15-minute period, Δt_o, the heater dissipates 4000 W/m^2 and then is deactivated. To determine the eventual, uniform steady-state temperature distribution, apply the conservation of energy requirement on a time-interval basis, Eq. 1.12b. The initial condition corresponds to the temperature distribution of Case 2, and the final condition will be a uniform, elevated temperature $T_f = T_3$ representing Case 3. We have used T_∞ as the reference condition for the energy terms.

$$E_{in}'' - E_{out}'' + E_{gen}'' = \Delta E_{st}'' = E_f'' - E_i'' \qquad (1)$$

Note that $E_{in}'' - E_{out}'' = 0$, and the dissipated electrical energy is

$$E_{gen}'' = q_o'' \Delta t_o = 4000\ \text{W/m}^2 (15 \times 60)\text{s} = 3.600 \times 10^6\ \text{J/m}^2 \qquad (2)$$

For the final condition,

$$E_f'' = \rho c (2L)[T_f - T_\infty] = 2500\ \text{kg/m}^3 \times 700\ \text{J/kg}\cdot\text{K}(2 \times 0.025\text{m})[T_f - 20]°\text{C}$$
$$E_f'' = 8.75 \times 10^4 [T_f - 20]\ \text{J/m}^2 \qquad (3)$$

where $T_f = T_3$, the final uniform temperature, Case 3. For the initial condition,

$$E_i'' = \rho c \int_{-L}^{+L} [T_2(x) - T_\infty] dx = \rho c \left\{ \int_{-L}^{0} [T_2(x) - T_\infty] dx + \int_{0}^{+L} [T_2(0) - T_\infty] dx \right\} \qquad (4)$$

where $T_2(x)$ is linear for $-L \le x \le 0$ and constant at $T_2(0)$ for $0 \le x \le +L$.

$$T_2(x) = T_2(0) + [T_2(0) - T_2(L)] x/L \qquad -L \le x \le 0$$
$$T_2(x) = 50°\text{C} + [50 - 30]°\text{C}x / 0.025\text{m}$$
$$T_2(x) = 50°\text{C} + 800x \qquad (5)$$

Substituting for $T_2(x)$, Eq. (5), into Eq. (4)

Continued …

$$E_i'' = \rho c\left\{\int_{-L}^{0}\left[50+800x-T_\infty\right]dx+\left[T_2(0)-T_\infty\right]L\right\}$$

$$E_i'' = \rho c\left\{\left[50x+400x^2-T_\infty x\right]_{-L}^{0}+\left[T_2(0)-T_\infty\right]L\right\}$$

$$E_i'' = \rho c\left\{-\left[-50L+400L^2+T_\infty L\right]+\left[T_2(0)-T_\infty\right]L\right\}$$

$$E_i'' = \rho c L\left\{+50-400L-T_\infty+T_2(0)-T_\infty\right\}$$

$$E_i'' = 2500\,\text{kg/m}^3\times 700\,\text{J/kg}\cdot\text{K}\times 0.025\,\text{m}\left\{+50-400\times 0.025-20+50-20\right\}\text{K}$$

$$E_i'' = 2.188\times 10^6\,\text{J/m}^2 \qquad (6)$$

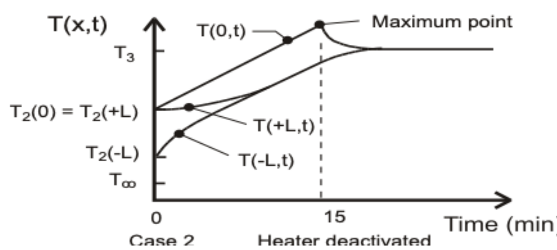
Returning to the energy balance, Eq. (1), and substituting Eqs. (2), (3) and (6), find $T_f = T_3$.

$$3.600\times 10^6\,\text{J/m}^2 = 8.75\times 10^4\left[T_3-20\right]-2.188\times 10^6\,\text{J/m}^2$$

$$T_3 = (66.1+20)^\circ\text{C} = 86.1^\circ\text{C}$$ <

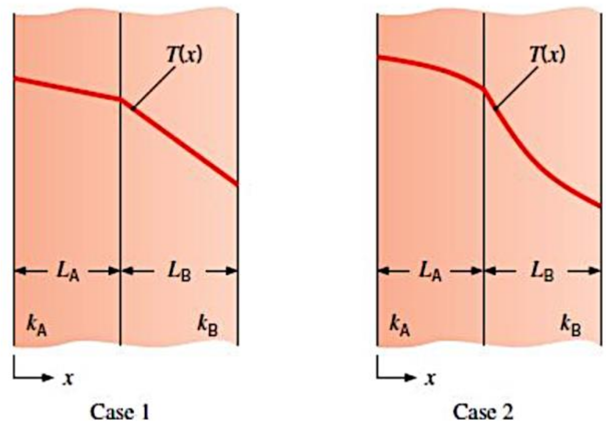
The temperature distribution is shown on the T-x coordinates above and labeled Case 3. The distribution is uniform, and considerably higher than the maximum value for Case 2.

(d) The temperature-time history at the plate locations x = 0, ± L during the transient period between the distributions for Case 2 and Case 3 are shown on the T-t coordinates below.



Note the temperatures for the locations at time t = 0 corresponding to the instant when the surface x = - L becomes adiabatic. These temperatures correspond to the distribution for Case 2. The heater remains energized for yet another 15 minutes and then is deactivated. The midpoint temperature, T(0,t), is always the hottest location and the maximum value slightly exceeds the final temperature T_3.

2.44 Consider the steady-state temperature distributions within a composite wall composed of Material A and Material B for the two cases shown. There is no internal generation, and the conduction process is one-dimensional.



Answer the following questions for each case. Which material has the higher thermal conductivity? Does the thermal conductivity vary significantly with temperature? If so, how? Describe the heat flux $q''_x(x)$ distribution through the composite wall. If the thickness and thermal conductivity of each material were both doubled and the boundary temperatures remained the same, what would be the effect on the heat flux distribution?

Case 1. Linear temperature distributions exist in both materials, as shown.

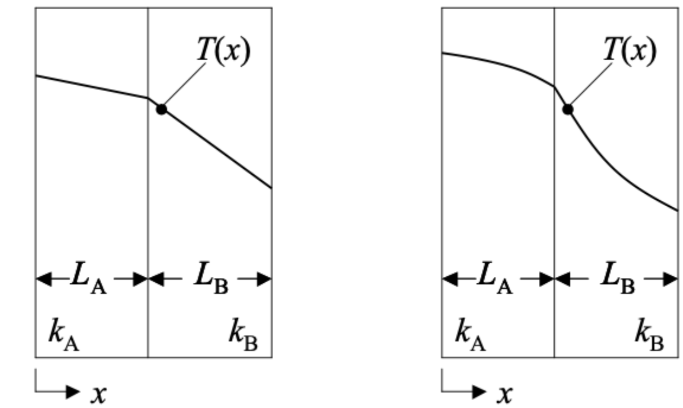
Case 2. Nonlinear temperature distributions exist in both materials, as shown.

PROBLEM 2.44

KNOWN: Qualitative temperature distributions in two cases.

FIND: For each of two cases, determine which material (A or B) has the higher thermal conductivity, how the thermal conductivity varies with temperature, description of the heat flux distribution through the composite wall, effect of simultaneously doubling the wall thickness and thermal conductivity.

SCHEMATIC:



Case 1. Case 2.

ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Negligible contact resistances, (3) No internal energy generation.

ANALYSIS: Under steady-state conditions with no internal generation, the conservation of energy requirement dictates that the heat flux through the wall must be constant. <

For Materials A and B, Fourier's law is written $q''_A = -k_A \dfrac{dT_A}{dx} = q''_B = -k_B \dfrac{dT_B}{dx}$. Therefore,

$$\frac{k_A}{k_B} = \frac{dT_B/dx}{dT_A/dx} > 1 \quad \text{and} \quad k_B < k_A \text{ for both cases.}$$ <

Since the heat flux through the wall is constant, Fourier's law dictates that lower thermal conductivity material must exist where temperature gradients are larger. For Case 1, the temperature distributions are linear. Therefore, the temperature gradient is constant in each material, and the thermal conductivity of each material must not vary significantly with temperature. For Case 2, Material A, the temperature gradient is larger at lower temperatures. Hence, for Material A the thermal conductivity increases with increasing material temperature. For Case 2, Material B, the temperature gradient is smaller at lower temperatures. Hence, for Material B the thermal conductivity decreases with increases in material temperature. <

COMMENTS: If you were given information regarding the relative values of the thermal conductivities and how the thermal conductivities vary with temperature in each material, you should be able to sketch the temperature distributions provided in the problem statement.

CHAPTER 3

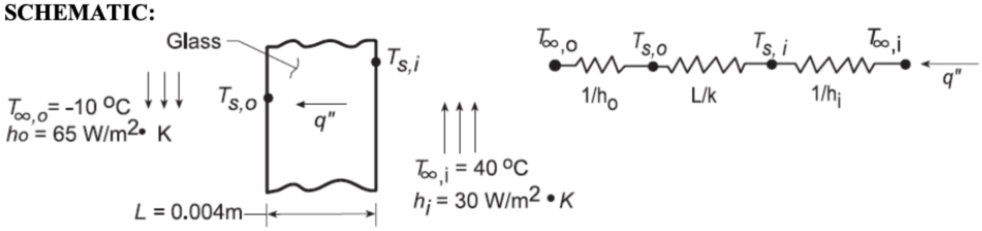
3.3 The rear window of an automobile is defogged by passing warm air over its inner surface.
(a) If the warm air is at $T_{\infty,i}$ = 40°C and the corresponding convection coefficient is h_i = 30 W/m2 · K, what are the inner and outer surface temperatures of 4-mm-thick window glass, if the outside ambient air temperature is $T_{\infty,o}$ = −10°C and the associated convection coefficient is h_o = 65 W/m2 · K?
(b)Computer Icon In practice $T_{\infty,o}$ and h_o vary according to weather conditions and car speed. For values of h_o = 2, 65, and 100 W/m2 · K, compute and plot the inner and outer surface temperatures as a function of $T_{\infty,o}$ for $-30 \le T_{\infty,o} \le 0$°C.

PROBLEM 3.3

KNOWN: Temperatures and convection coefficients associated with air at the inner and outer surfaces of a rear window.

FIND: (a) Inner and outer window surface temperatures, $T_{s,i}$ and $T_{s,o}$, and (b) $T_{s,i}$ and $T_{s,o}$ as a function of the outside air temperature $T_{\infty,o}$ and for selected values of outer convection coefficient, h_o.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation effects, (4) Constant properties.

PROPERTIES: *Table A-3*, Glass (300 K): k = 1.4 W/m·K.

ANALYSIS: (a) The heat flux may be obtained from Eqs. 3.11 and 3.12,

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}} = \frac{40^\circ C - \left(-10^\circ C\right)}{\frac{1}{65\,W/m^2\cdot K} + \frac{0.004\,m}{1.4\,W/m\cdot K} + \frac{1}{30\,W/m^2\cdot K}}$$

$$q'' = \frac{50^\circ C}{\left(0.0154 + 0.0029 + 0.0333\right)m^2\cdot K/W} = 969\,W/m^2 .$$

Hence, with $q'' = h_i\left(T_{\infty,i} - T_{\infty,o}\right)$, the inner surface temperature is

$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40^\circ C - \frac{969\,W/m^2}{30\,W/m^2\cdot K} = 7.7^\circ C \qquad <$$

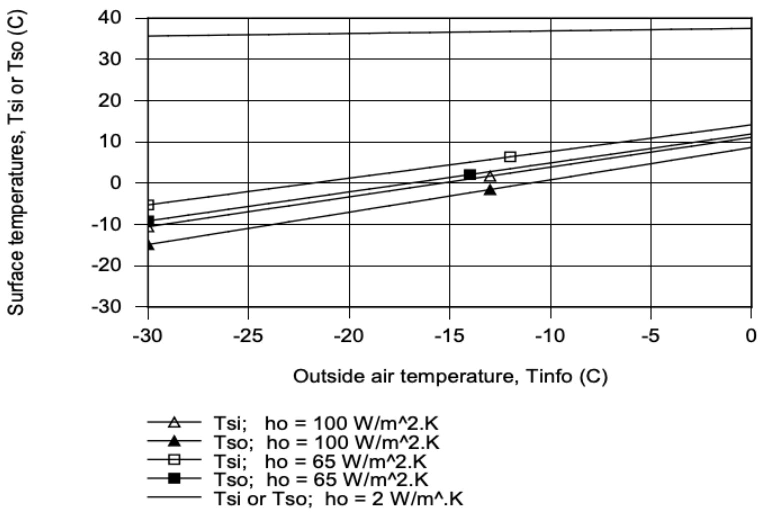
Similarly for the outer surface temperature with $q'' = h_o\left(T_{s,o} - T_{\infty,o}\right)$ find

$$T_{s,o} = T_{\infty,o} + \frac{q''}{h_o} = -10^\circ C + \frac{969\,W/m^2}{65\,W/m^2\cdot K} = 4.9^\circ C \qquad <$$

(b) Using the same analysis, $T_{s,i}$ and $T_{s,o}$ have been computed and plotted as a function of the outside air temperature, $T_{\infty,o}$, for outer convection coefficients of h_o = 2, 65, and 100 W/m^2·K. As expected, $T_{s,i}$ and $T_{s,o}$ are linear with changes in the outside air temperature. The difference between $T_{s,i}$ and $T_{s,o}$ increases with increasing convection coefficient, since the heat flux through the window likewise increases. This difference is larger at lower outside air temperatures for the same reason. Note that with h_o = 2 W/m^2·K, $T_{s,i}$ - $T_{s,o}$, is too small to show on the plot.

Continued …

PROBLEM 3.3 (Cont.)



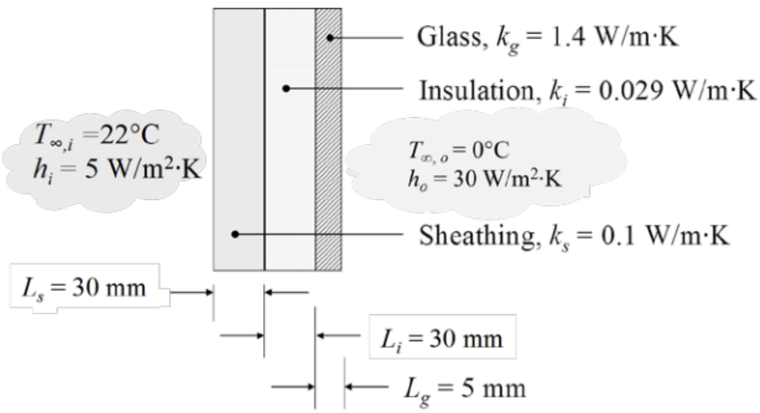
3.4 A dormitory at a large university, built 50 years ago, has exterior walls constructed of Ls = 30-mm-thick sheathing with a thermal conductivity of ks = 0.1 W/m · K. To reduce heat losses in the winter, the university decides to encapsulate the entire dormitory by applying an Li = 30-mm-thick layer of extruded insulation characterized by ki = 0.029 W/m · K to the exterior of the original sheathing. The extruded insulation is, in turn, covered with an Lg = 5-mm-thick architectural glass with kg = 1.4 W/m · K. Determine the heat flux through the original and retrofitted walls when the interior and exterior air temperatures are T∞,i = 22°C and T∞,o = 0°C, respectively. The inner and outer convection heat transfer coefficients are hi = 5 W/m2 · K and ho = 30 W/m2 · K, respectively.

PROBLEM 3.4

KNOWN: Thermal conductivities and thicknesses of original wall, insulation layer, and glass layer. Interior and exterior air temperatures and convection heat transfer coefficients.

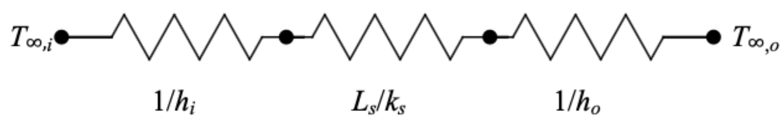
FIND: Heat flux through original and retrofitted walls.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties, (4) Negligible contact resistances.

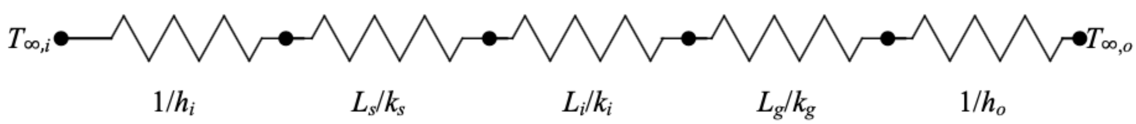
ANALYSIS: The original wall with convection inside and outside can be represented by the following thermal resistance network, where the resistances are each for a unit area:



Thus the heat flux can be expressed as

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_i} + \frac{L_s}{k_s} + \frac{1}{h_o}} = \frac{22°C - 0°C}{\frac{1}{5\ \text{W/m}^2\cdot\text{K}} + \frac{0.030\ \text{m}}{0.1\ \text{W/m}\cdot\text{K}} + \frac{1}{30\ \text{W/m}^2\cdot\text{K}}} = 41.3\ \text{W/m}^2$$ <

The retrofitted wall has three layers. The thermal circuit can be represented as follows:



Thus the heat flux can be expressed as

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_i} + \frac{L_s}{k_s} + \frac{L_i}{k_i} + \frac{L_g}{k_g} + \frac{1}{h_o}}$$ <

$$= \frac{22°C - 0°C}{\frac{1}{5\ \text{W/m}^2\cdot\text{K}} + \frac{0.030\ \text{m}}{0.1\ \text{W/m}\cdot\text{K}} + \frac{0.030\ \text{m}}{0.029\ \text{W/m}\cdot\text{K}} + \frac{0.005\ \text{m}}{1.4\ \text{W/m}\cdot\text{K}} + \frac{1}{25\ \text{W/m}^2\cdot\text{K}}} = 14.0\ \text{W/m}^2$$

COMMENTS: The heat flux has been reduced to approximately one-third of the original value because of the increased resistance, which is mainly due to the insulation layer.

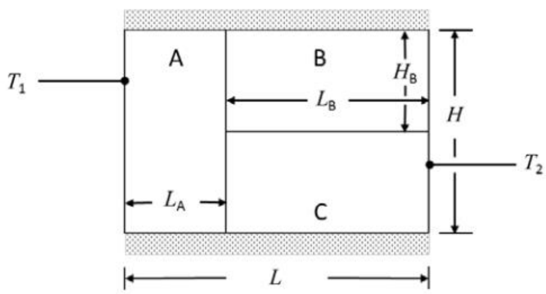
3.13 Consider a composite wall of overall height H = 20 mm and thickness L = 30 mm. Section A has thickness LA = 10 mm, and sections B and C each have height HB = 10 mm and thickness LB = 20 mm. The temperatures of the left and right faces of the composite wall are T1 = 50°C and T2 = 20°C, respectively. If the top and bottom of the wall are insulated, determine the heat rate per unit wall depth for each of the following three cases. Which case yields the largest heat rate per unit depth? Which yields the smallest heat rate per unit depth?

PROBLEM 3.13

KNOWN: Composite wall with known dimensions and thermal conductivities.

FIND: Heat rate per unit wall depth, and which of three cases yields the largest and smallest heat rates.

SCHEMATIC:

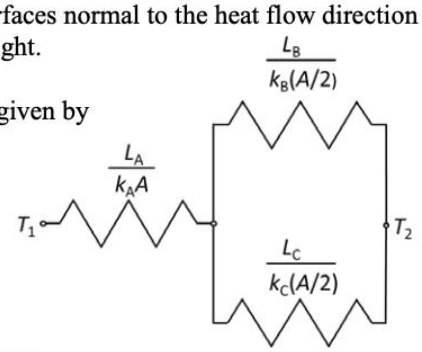


H = 20 mm
L = 30 mm
L_A = 10 mm
H_B = 10 mm, L_B = 20 mm
T_1 = 50°C, T_2 = 20°C.

Case	k_A (W/m·K)	k_B (W/m·K)	k_C (W/m·K)
1	1	2	3
2	2	3	1
3	3	1	2

ASSUMPTIONS: (1) Surfaces normal to heat flow direction are isothermal *or* surfaces parallel to the heat flow direction are adiabatic, (2) Steady-state conditions, (3) Negligible contact resistance.

ANALYSIS: This scenario is similar to the composite wall of Figure 3.3, but without the right-most region labeled H. If it is assumed that surfaces normal to the heat flow direction are isothermal, the resistance network is shown to the right.



In this case, the total resistance (for a unit depth) is given by

$$R'_{tot} = \frac{L_A}{k_A H} + \left(\frac{k_B H/2}{L_B} + \frac{k_C H/2}{L_C}\right)^{-1}$$

and the heat rate per unit depth is

$$q' = \frac{(T_1 - T_2)}{R'_{tot}} = \frac{(T_1 - T_2)}{\frac{L_A}{k_A H} + \left(\frac{k_B H/2}{L_B} + \frac{k_C H/2}{L_C}\right)^{-1}}$$

Substituting values for Case 1 yields

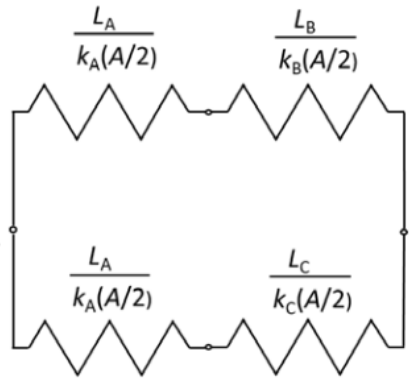
$$q'_1 = \frac{(50-20)°C}{\frac{0.01\ m}{1\ W/m\cdot K \times 0.02\ m} + \left(\frac{2\ W/m\cdot K \times 0.02\ m/2}{0.02\ m} + \frac{3\ W/m\cdot K \times 0.02\ m/2}{0.02\ m}\right)^{-1}} = 33.3\ W/m \ <$$

Continued…

Repeating the calculation for the two other cases yields

$$q_2' = 40.0 \text{ W/m}, \quad q_3' = 36.0 \text{ W/m}$$ <

On the other hand, if it is assumed that surfaces parallel to the heat flow direction are adiabatic, the resistance network is shown to the right.



In this case, the total resistance (for a unit depth) is given by

$$R'_{tot} = \left[\left(\frac{L_A}{k_A H/2} + \frac{L_B}{k_B H/2} \right)^{-1} + \left(\frac{L_A}{k_A H/2} + \frac{L_B}{k_C H/2} \right)^{-1} \right]^{-1}$$

and the heat rate per unit depth is

$$q' = \frac{(T_1 - T_2)}{R'_{tot}}$$

$$= (T_1 - T_2) \left[\left(\frac{L_A}{k_A H/2} + \frac{L_B}{k_B H/2} \right)^{-1} + \left(\frac{L_A}{k_A H/2} + \frac{L_B}{k_C H/2} \right)^{-1} \right]$$

Substituting values for Case 1 yields

$$q_1' = (50 - 20)°\text{C} \times \left[\begin{array}{l} \left(\dfrac{0.01 \text{ m}}{1 \text{ W/m}\cdot\text{K} \times 0.02 \text{ m}/2} + \dfrac{0.02 \text{ m}}{2 \text{ W/m}\cdot\text{K} \times 0.02 \text{ m}/2} \right)^{-1} \\ + \left(\dfrac{0.01 \text{ m}}{1 \text{ W/m}\cdot\text{K} \times 0.02 \text{ m}/2} + \dfrac{0.02 \text{ m}}{3 \text{ W/m}\cdot\text{K} \times 0.02 \text{ m}/2} \right)^{-1} \end{array} \right] = 33.0 \text{ W/m}$$ <

Repeating the calculation for the two other cases yields

$$q_2' = 37.7 \text{ W/m}, \quad q_3' = 35.4 \text{ W/m}$$ <

COMMENTS: The results are quite close between the two different methods for approximating the thermal resistance network. This is because the thermal conductivities of the three materials are the same order of magnitude. If the thermal conductivities differed more dramatically, the results of the two methods wouldn't agree as well.