SUGGESTED PROBLEMS FOR CH 6

6.3 In a particular application involving airflow over a surface, the boundary layer temperature distribution may be approximated as

$$\frac{T - T_s}{T_\infty - T_s} = 1 - \exp\left(-\mathrm{Pr}\frac{u_\infty y}{\nu}\right)$$

where y is the distance normal to the surface and the Prandtl number, $Pr = c_p\mu/k = 0.7$, is a dimensionless fluid property. If $T_\infty = 400$ K, $T_s = 300$ K, and $u_\infty/\nu = 5000\ \mathrm{m}^{-1}$, what is the surface heat flux?

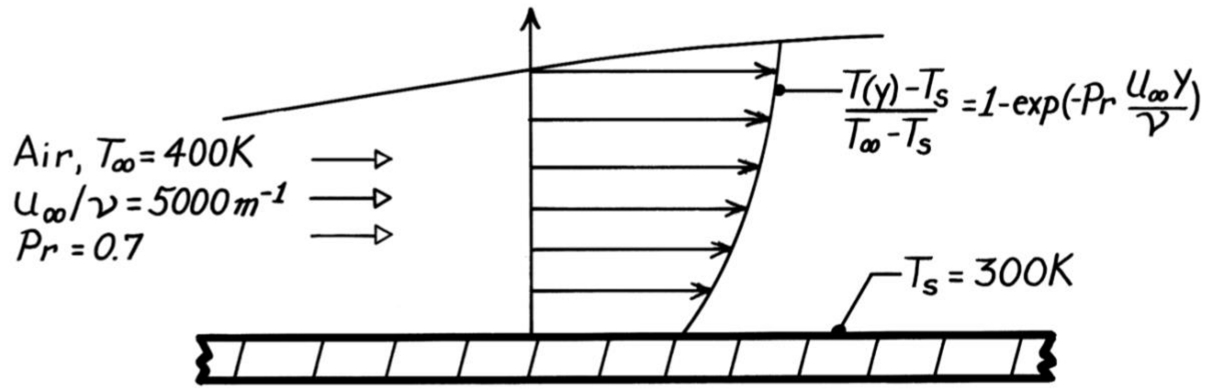
6.4 Water at a temperature of $T_\infty = 25°\mathrm{C}$ flows over one of the surfaces of a stainless steel wall (AISI 302) whose temperature is $T_{s,1} = 40°\mathrm{C}$. The wall is 0.05 m thick, and its other surface temperature is $T_{s,2} = 100°\mathrm{C}$. For steady-state conditions what is the convection coefficient associated with the water flow? What is the temperature gradient in the wall and in the water that is in contact with the wall? Sketch the temperature distribution in the wall and in the adjoining water.

PROBLEM 6.3

KNOWN: Boundary layer temperature distribution.

FIND: Surface heat flux.

SCHEMATIC:



PROPERTIES: *Table A-4*, Air (T_s = 300K): k = 0.0263 W/m·K.

ANALYSIS: Applying Fourier's law at y = 0, the heat flux is

$$q_s'' = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = -k(T_\infty - T_s)\left[Pr\frac{u_\infty}{\nu}\right]\exp\left[-Pr\frac{u_\infty y}{\nu}\right]\bigg|_{y=0}$$

$$q_s'' = -k(T_\infty - T_s)Pr\frac{u_\infty}{\nu}$$

$$q_s'' = -0.0263\ \text{W/m}\cdot\text{K}(100\text{K})0.7\times 5000\ 1/\text{m}.$$

$$q_s'' = -9205\ \text{W/m}^2.$$ <

COMMENTS: (1) Negative flux implies convection heat transfer to the surface.

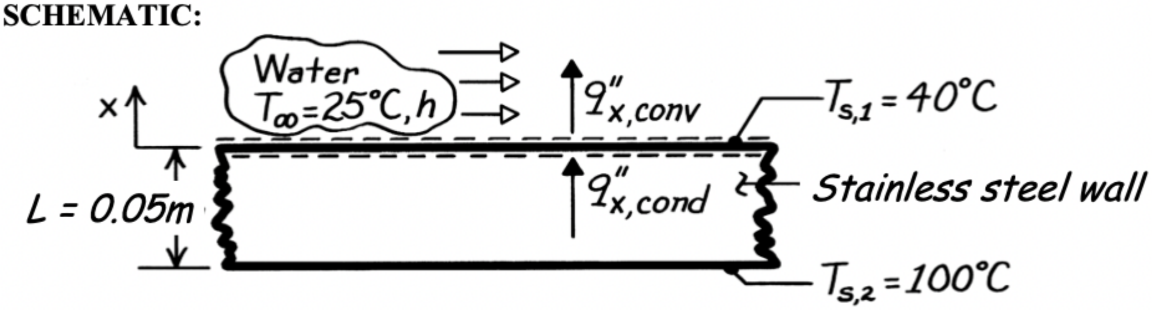
(2) Note use of k at T_s to evaluate q_s'' from Fourier's law.

PROBLEM 6.4

KNOWN: Surface temperatures of a steel wall and temperature of water flowing over the wall.

FIND: (a) Convection coefficient, (b) Temperature gradient in wall and in water at wall surface.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in x, (3) Constant properties.

PROPERTIES: *Table A-1*, Stainless steel Type AISI 302 (70°C = 343K), $k_s = 17.3$ W/m·K; *Table A-6*, Water (40°C = 313 K), $k_f = 0.632$ W/m·K.

ANALYSIS: (a) Applying an energy balance to the control surface at x = 0, it follows that

$$q''_{x,cond} - q''_{x,conv} = 0$$

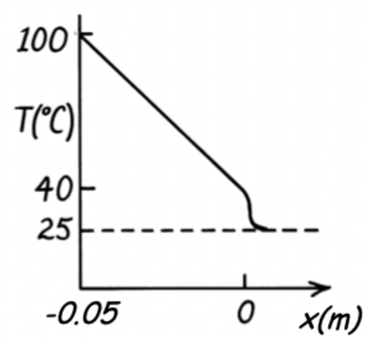
and using the appropriate rate equations,

$$k_s \frac{T_{s,2} - T_{s,1}}{L} = h\left(T_{s,1} - T_\infty\right).$$

Hence,

$$h = \frac{k_s}{L}\frac{T_{s,2} - T_{s,1}}{T_{s,1} - T_\infty} = \frac{17.3\ \text{W/m}\cdot\text{K}}{0.05\text{m}}\frac{60^\circ\text{C}}{15^\circ\text{C}} = 1380\ \text{W/m}^2\cdot\text{K}.$$ <

(b) The gradient in the wall at the surface is



$$\left(dT/dx\right)_s = -\frac{T_{s,2} - T_{s,1}}{L} = -\frac{60^\circ\text{C}}{0.05\text{m}} = -1200^\circ\text{C/m}.$$

In the water at x = 0, the definition of h gives

$$\left(dT/dx\right)_{f,x=0} = -\frac{h}{k_f}\left(T_{s,1} - T_\infty\right)$$

where k_f is evaluated at x = 0 (T = 40°C).

$$\left(dT/dx\right)_{f,x=0} = -\frac{1380\ \text{W/m}^2\cdot\text{K}}{0.632\ \text{W/m}\cdot\text{K}}\left(15^\circ\text{C}\right) = -32{,}800^\circ\text{C/m}.$$ <

COMMENTS: Note the relative magnitudes of the gradients. Why is there such a large difference?

6.5 For laminar flow over a flat plate, the local heat transfer coefficient h_x is known to vary as $x^{-1/2}$, where x is the distance from the leading edge ($x = 0$) of the plate. What is the ratio of the average coefficient between the leading edge and some location x on the plate to the local coefficient at x?

6.6 A flat plate is of planar dimension 1 m × 0.75 m. For parallel laminar flow over the plate, calculate the ratio of the average heat transfer coefficients over the entire plate, $\bar{h}_{L,1}/\bar{h}_{L,2}$, for two cases. In Case 1, flow is in the short direction (L = 0.75 m); in Case 2, flow is in the long direction (L = 1 m). Which orientation will result in the larger heat transfer rate? See Problem 6.5.

6.8 Laminar flow normally persists on a smooth flat plate until a critical Reynolds number value is reached. However, the flow can be *tripped* to a turbulent state by adding roughness to the leading edge of the plate. For a particular situation, experimental results show that the local heat transfer coefficients for laminar and turbulent conditions are

$$h_{\text{lam}}(x) = 1.74\ \text{W/m}^{1.5} \cdot \text{K}\ x^{-0.5}$$
$$h_{\text{turb}}(x) = 3.98\ \text{W/m}^{1.8} \cdot \text{K}\ x^{-0.2}$$

Calculate the average heat transfer coefficients for laminar and turbulent conditions for plates of length L = 0.1 m and 1 m.

PROBLEM 6.5

KNOWN: Variation of h_x with x for laminar flow over a flat plate.

FIND: Ratio of average coefficient, $\bar{h}_x$, to local coefficient, h_x, at x.

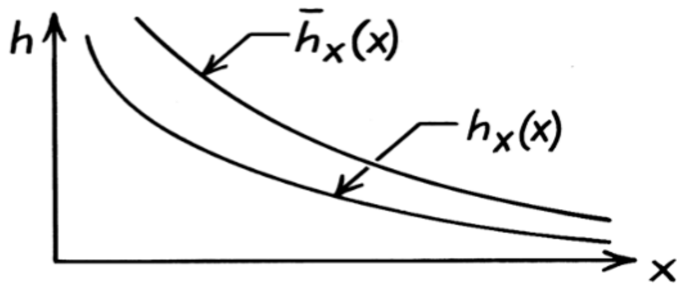
SCHEMATIC:

ANALYSIS: The average value of h_x between 0 and x is

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx = \frac{C}{x}\int_0^x x^{-1/2} dx$$
$$\bar{h}_x = \frac{C}{x} 2x^{1/2} = 2Cx^{-1/2}$$
$$\bar{h}_x = 2h_x.$$

Hence, $$\frac{\bar{h}_x}{h_x} = 2.$$ <

COMMENTS: Both the local and average coefficients decrease with increasing distance x from the leading edge, as shown in the sketch below.

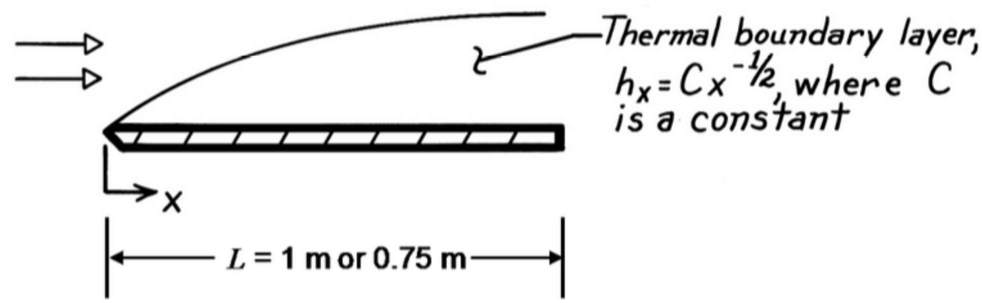


PROBLEM 6.6

KNOWN: Variation of local heat transfer coefficient with x. Length of plate.

FIND: Ratio of heat transfer coefficients for flow oriented in short and long directions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Laminar flow, (3) Incompressible flow.

ANALYSIS: The local heat transfer coefficient varies with x according to

$$h_x = Cx^{-1/2}$$

The average heat transfer coefficient over the entire plate is given by Eq. 6.14:

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x dx = \frac{1}{L}\int_0^L Cx^{-1/2} dx = 2CL^{-1/2}$$

Therefore the ratio of average heat transfer coefficients for the two different flow orientations is

$$\frac{\bar{h}_{L,1}}{\bar{h}_{L,2}} = \left(\frac{L_2}{L_1}\right)^{1/2}$$

The average heat transfer coefficient is larger when the flow is oriented in the short direction because local heat transfer coefficients are largest near the leading edge. Therefore the heat transfer rate will be larger when flow is oriented in the short direction. <

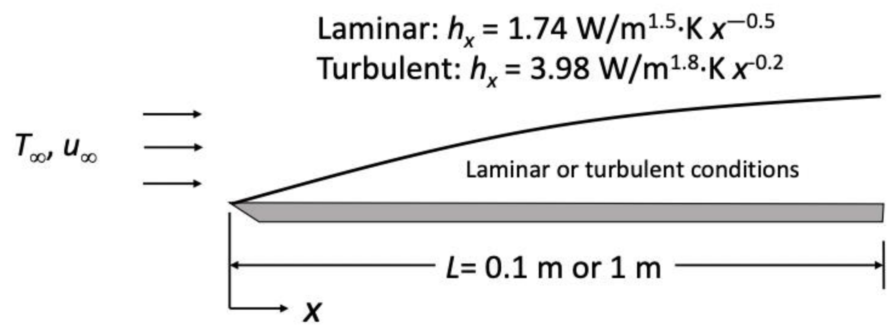
COMMENTS: Many engineering devices that are affected by, or utilize convection heat transfer in their operation incorporate short sections of surfaces in order to take advantage of the high local heat transfer coefficients that exist near the leading edges of such surfaces.

PROBLEM 6.8

KNOWN: Smooth flat plate with either laminar or turbulent conditions starting at the leading edge. Corresponding heat transfer coefficient correlations.

FIND: Average heat transfer coefficients for both conditions for plates of length $L = 0.1$ m and 1 m.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional flow and heat transfer.

ANALYSIS: The average heat transfer coefficient is defined as:

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x dx$$

For laminar flow, $h_x = A_{\text{lam}} x^{-0.5}$ where $A_{\text{lam}} = 1.74\ \text{W/m}^{1.5} \cdot \text{K}$. Thus,

$$\bar{h}_{L,\text{lam}} = \frac{A_{\text{lam}}}{L}\int_0^L x^{-0.5} dx = \frac{2A_{\text{lam}}}{L} x^{0.5}\Big|_0^L = \frac{2A_{\text{lam}}}{L^{0.5}} = \frac{3.48\ \text{W/m}^{1.5} \cdot \text{K}}{L^{0.5}}$$

The results for the two plate lengths are:

$$\bar{h}_{L,\text{lam}} = \begin{cases} 11.0\ \text{W/m}^2 \cdot \text{K}, & L = 0.1\ \text{m} \\ 3.48\ \text{W/m}^2 \cdot \text{K}, & L = 1\ \text{m} \end{cases}$$

<

For turbulent flow, $h_x = A_{\text{turb}}\ x^{-0.2}$ where $A_{\text{turb}} = 3.98\ \text{W/m}^{1.8} \cdot \text{K}$. Thus,

$$\bar{h}_{L,\text{turb}} = \frac{A_{\text{turb}}}{L}\int_0^L x^{-0.2} dx = \frac{A_{\text{turb}}}{0.8L} x^{0.8}\Big|_0^L = \frac{A_{\text{turb}}}{0.8L^{0.2}} = \frac{4.98\ \text{W/m}^{1.8} \cdot \text{K}}{L^{0.2}}$$

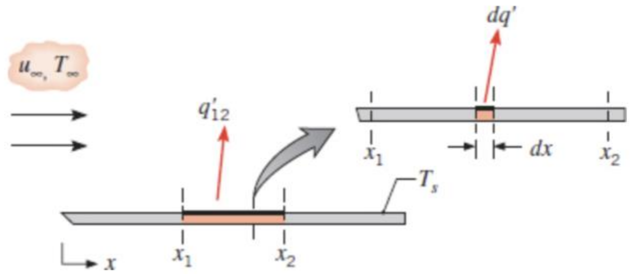
The results for the two plate lengths are:

$$\bar{h}_{L,\text{turb}} = \begin{cases} 7.88\ \text{W/m}^2 \cdot \text{K}, & L = 0.1\ \text{m} \\ 4.98\ \text{W/m}^2 \cdot \text{K}, & L = 1\ \text{m} \end{cases}$$

<

COMMENTS: For $L = 1$ m, the turbulent heat transfer coefficient is larger than the laminar one, as is usually the case. However, for $L = 0.1$ m, the situation is reversed.

6.12 The heat transfer rate per unit width (normal to the page) from a longitudinal section, $x_2 - x_1$, can be expressed as $q'_{12} = \bar{h}_{12}(x_2 - x_1)(T_s - T_\infty)$, where $\bar{h}_{12}$ is the average coefficient for the section of length $(x_2 - x_1)$. Consider laminar flow over a flat plate with a uniform temperature T_s. The spatial variation of the local convection coefficient is of the form $h_x = Cx^{-1/2}$, where C is a constant.



(a) Beginning with the convection rate equation in the form $dq' = h_x\,dx(T_s - T_\infty)$, derive an expression for $\bar{h}_{12}$ in terms of C, x_1, and x_2.

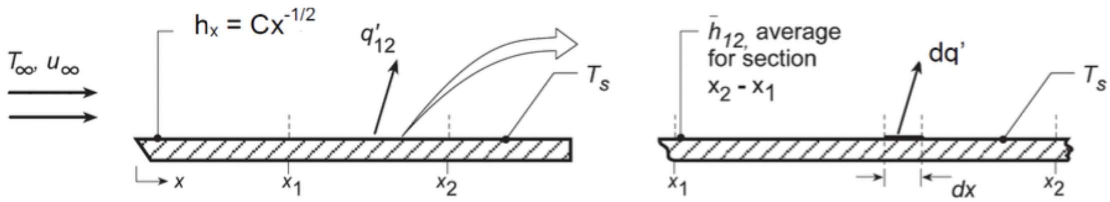
(b) Derive an expression for $\bar{h}_{12}$ in terms of x_1, x_2, and the average coefficients $\bar{h}_1$ and $\bar{h}_2$, corresponding to lengths x_1 and x_2, respectively.

PROBLEM 6.12

KNOWN: Variation of local convection coefficient with distance x from a heated plate with a uniform temperature T_s.

FIND: (a) An expression for the average coefficient $\bar{h}_{12}$ for the section of length $(x_2 - x_1)$ in terms of C, x_1 and x_2, and (b) An expression for $\bar{h}_{12}$ in terms of x_1 and x_2, and the average coefficients $\bar{h}_1$ and $\bar{h}_2$, corresponding to lengths x_1 and x_2, respectively.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow over a plate with uniform surface temperature, T_s, and (2) Spatial variation of local coefficient is of the form $h_x = Cx^{-1/2}$, where C is a constant.

ANALYSIS: (a) The heat transfer rate per unit width from a longitudinal section, $x_2 - x_1$, can be expressed as

$$q'_{12} = \bar{h}_{12}(x_2 - x_1)(T_s - T_\infty) \tag{1}$$

where $\bar{h}_{12}$ is the average coefficient for the section of length $(x_2 - x_1)$. The heat rate can also be written in terms of the local coefficient, Eq. (6.11), as

$$q'_{12} = \int_{x_1}^{x_2} h_x dx (T_s - T_\infty) = (T_s - T_\infty)\int_{x_1}^{x_2} h_x dx \tag{2}$$

Combining Eq. (1) and (2),

$$\bar{h}_{12} = \frac{1}{(x_2 - x_1)}\int_{x_1}^{x_2} h_x dx \tag{3}$$

and substituting for the form of the local coefficient, $h_x = Cx^{-1/2}$, find that

$$\bar{h}_{12} = \frac{1}{(x_2 - x_1)}\int_{x_1}^{x_2} Cx^{-1/2}dx = \frac{C}{x_2 - x_1}\left[\frac{x^{1/2}}{1/2}\right]_{x_1}^{x_2} = 2C\frac{x_2^{1/2} - x_1^{1/2}}{x_2 - x_1} \tag{4}$$ <

(b) The heat rate, given as Eq. (1), can also be expressed as

$$q'_{12} = \bar{h}_2 x_2 (T_s - T_\infty) - \bar{h}_1 x_1 (T_s - T_\infty) \tag{5}$$

which is the difference between the heat rate for the plate over the section $(0 - x_2)$ and over the section $(0 - x_1)$. Combining Eqs. (1) and (5), find,

$$\bar{h}_{12} = \frac{\bar{h}_2 x_2 - \bar{h}_1 x_1}{x_2 - x_1} \tag{6}$$ <

COMMENTS: (1) Note that, from Eq. 6.6,

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx = \frac{1}{x}\int_0^x Cx^{-1/2}dx = 2Cx^{-1/2} \tag{7}$$

or $\bar{h}_x = 2h_x$. Substituting Eq. (7) into Eq. (6), see that the result is the same as Eq. (4).

6.15 Consider airflow over a flat plate of length L = 1 m under conditions for which transition occurs at xc = 0.5 m based on the critical Reynolds number, Rex, c = 5 × 105.

(a) Evaluating the thermophysical properties of air at 350 K, determine the air velocity.

(b) In the laminar and turbulent regions, the local convection coefficients are, respectively,

$$h_{\text{lam}}\,(x) = C_{\text{lam}}\,x^{-0.5} \quad \text{and} \quad h_{\text{turb}} = C_{\text{turb}}\,x^{-0.2}$$

where, at T = 350 K, C_{lam} = 8.845 W/m$^{3/2}$ · K, C_{turb} = 49.75 W/m1.8 · K, and x has units of m. Develop an expression for the average convection coefficient, $h_{lam}\,(x)$, as a function of distance from the leading edge, x, for the laminar region, 0 ≤ x ≤ xc.

(c) Develop an expression for the average convection coefficient, $h_{turb}\,(x)$, as a function of distance from the leading edge, x, for the turbulent region, xc ≤ x ≤ L.

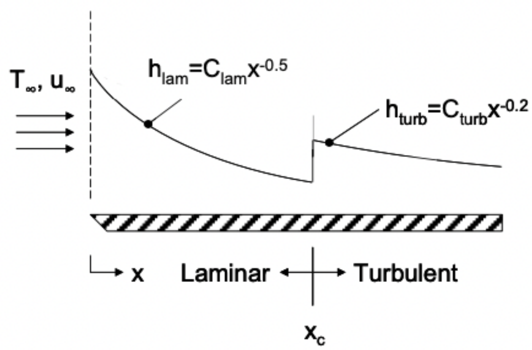
(d) On the same coordinates, plot the local and average convection coefficients, h_x and $\bar{h}_x$, respectively, as a function of x for 0 ≤ x ≤ L.

PROBLEM 6.15

KNOWN: Air flow over a flat plate of known length, location of transition from laminar to turbulent flow, value of the critical Reynolds number.

FIND: (a) Free stream velocity with properties evaluated at T = 350 K, (b) Expression for the average convection coefficient, $\bar{h}_{lam}(x)$, as a function of the distance x from the leading edge in the laminar region, (c) Expression for the average convection coefficient $\bar{h}_{turb}(x)$, as a function of the distance x from the leading edge in the turbulent region, (d) Compute and plot the local and average convection coefficients over the entire plate length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

PROPERTIES: Table A.4, air (T = 350 K): k = 0.030 W/m·K, $\nu = 20.92 \times 10^{-6}$ m²/s, Pr = 0.700.

ANALYSIS:

(a) Using air properties evaluated at 350 K with $x_c = 0.5$ m,

$$Re_{x,c} = \frac{u_\infty x_c}{\nu} = 5\times10^5$$

$$u_\infty = 5\times10^5\,\nu/x_c = 5\times10^5 \times 20.92\times10^{-6}\,\text{m}^2/\text{s}/0.5\,\text{m} = 20.9\,\text{m/s}$$ <

(b) From Eq. 6.13, the average coefficient in the laminar region, $0 \le x \le x_c$, is

$$\bar{h}_{lam}(x) = \frac{1}{x}\int_0^x h_{lam}(x)\,dx = \frac{1}{x}C_{lam}\int_0^x x^{-0.5}dx = \frac{1}{x}C_{lam}x^{0.5} = 2C_{lam}x^{-0.5} = 2h_{lam}(x) \quad (1)$$ <

(c) The average coefficient in the turbulent region, $x_c \le x \le L$, is

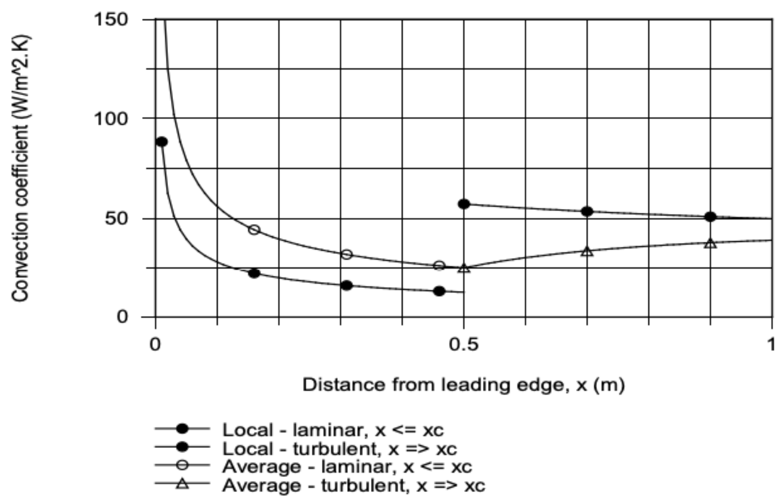
$$\bar{h}_{turb}(x) = \frac{1}{x}\left[\int_0^{x_c} h_{lam}(x)\,dx + \int_{x_c}^{x} h_{turb}(x)\,dx\right] = \left[C_{lam}\left.\frac{x^{0.5}}{0.5}\right|_0^{x_c} + C_{turb}\left.\frac{x^{0.8}}{0.8}\right|_{x_c}^{x}\right]$$

Continued…

PROBLEM 6.15 (Cont.)

$$\overline{h}_{turb}(x) = \frac{1}{x}\left[2C_{lam}x_c^{0.5} + 1.25C_{turb}\left(x^{0.8} - x_c^{0.8}\right)\right] \qquad (2) <$$

(d) The local and average coefficients, Eqs. (1) and (2) are plotted below as a function of x for the range $0 \le x \le L$.



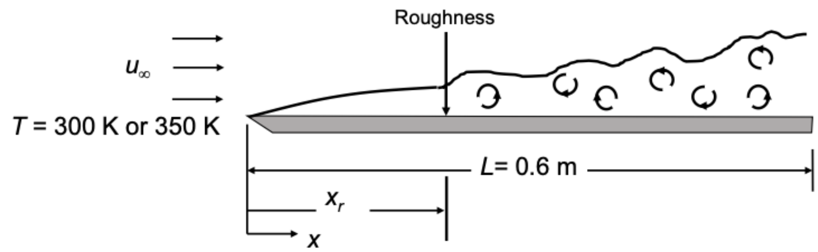
6.17 Consider the conditions of Example 6.4. A laminar boundary layer can be *tripped* to a turbulent condition at $x = x_r$ by roughening the surface of the plate at x_r. Calculate the minimum and maximum possible average convection coefficients for the plate. At which temperature, T = 300 K or T = 350 K, do each of the extreme values of h occur? What are the corresponding values of x_r?

PROBLEM 6.17

KNOWN: Velocity and temperature of water flowing over a flat plate. Length of plate. Variation of local convection coefficient with x for laminar and turbulent flow.

FIND: Minimum and maximum average convection coefficient for roughness applied over the range $x_r \le x \le L$. Temperature at which extreme values of average convection coefficient occur and corresponding values of x_r.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Transition occurs at a critical Reynolds number of 5×10^5 for the smooth plate, (4) Incompressible flow.

PROPERTIES: *Table A.6*, Liquid water (T = 300 K): $\rho = v_f^{-1} = 997$ kg/m^3, $\mu = 855 \times 10^{-6}$ N·s/m^2; Liquid water (T = 350 K): $\rho = v_f^{-1} = 974$ kg/m^3, $\mu = 3655 \times 10^{-6}$ N·s/m^2.

ANALYSIS: The smooth plate transition location, x_c, was found in Example 6.4 to be 0.43 m and 0.19 m, for T = 300 K and 350 K, respectively. For roughness applied over the range $0 \le x_r \le x_c$, transition occurs at x_r. From Eq. 6.14,

$$\overline{h} = \frac{1}{L}\int_0^L h dx = \frac{1}{L}\left[\int_0^{x_r} h_{\text{lam}} dx + \int_{x_r}^{L} h_{\text{turb}} dx\right] = \frac{1}{L}\left[\frac{C_{\text{lam}}}{0.5}x^{0.5}\Big|_0^{x_r} + \frac{C_{\text{turb}}}{0.8}x^{0.8}\Big|_{x_r}^{L}\right]$$
$$= \frac{1}{L}\left[\frac{C_{\text{lam}}}{0.5}x_r^{0.5} + \frac{C_{\text{turb}}}{0.8}\left(L^{0.8} - x_r^{0.8}\right)\right] \qquad (1)$$

Roughness applied over the range $x_r > x_c$ has no effect on the transition since the transition occurs at x_c for the smooth plate. Thus, from Example 6.4, $\overline{h} = 1620\ \text{W/m}^2\cdot\text{K}$ or $3710\ \text{W/m}^2\cdot\text{K}$ for T = 300 K or 350 K, respectively for $x_r > x_c$.

The critical locations $x_{r,c}$ that give rise to minimum or maximum values of $\overline{h}$ can be found by differentiating Eq. (1) and setting the result to zero:

$$\frac{1}{L}\left[C_{\text{lam}}x_r^{-0.5} - C_{\text{turb}}x_r^{-0.2}\right] = 0, \quad x_{r,c} = \left(\frac{C_{\text{lam}}}{C_{\text{turb}}}\right)^{1/0.3} \qquad (2)$$

Evaluating $x_{r,c}$ for the two temperatures yields:

$$x_{r,c} = \left(\frac{395\ \text{W/m}^{1.5}\cdot\text{K}}{2330\ \text{W/m}^{1.8}\cdot\text{K}}\right)^{10/3} = 0.00270\ \text{m} = 2.70\ \text{mm for } T = 300\ \text{K}$$

and

Continued…

PROBLEM 6.17 (Cont.)

$$x_{r,c} = \left(\frac{477\ \text{W/m}^{1.5} \cdot \text{K}}{3600\ \text{W/m}^{1.8} \cdot \text{K}} \right)^{10/3} = 0.00119\ \text{m} = 1.19\ \text{mm for } T = 350\ \text{K}$$

These values are both less than their respective turbulence transition locations, x_c, therefore Eq. (1) holds at these locations. Substituting these values into Eq. (1), the average heat transfer coefficients for roughness applied at these locations can be calculated.

For $T = 300$ K,

$$\bar{h}_{\max} = \frac{1}{0.6\text{m}} \left[\frac{395 \dfrac{\text{W}}{\text{m}^{1.5} \cdot \text{K}}}{0.5} (0.00270\text{m})^{0.5} + \frac{2330 \dfrac{\text{W}}{\text{m}^{1.8} \cdot \text{K}}}{0.8} \left((0.6\text{m})^{0.8} - (0.00270\text{m})^{0.8} \right) \right] = 3250 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \quad <$$

For $T = 350$ K,

$$\bar{h}_{\max} = \frac{1}{0.6\text{m}} \left[\frac{477 \dfrac{\text{W}}{\text{m}^{1.5} \cdot \text{K}}}{0.5} (0.00119\text{m})^{0.5} + \frac{3600 \dfrac{\text{W}}{\text{m}^{1.8} \cdot \text{K}}}{0.8} \left((0.6\text{m})^{0.8} - (0.00119\text{m})^{0.8} \right) \right] = 5000 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \quad <$$

The subscript max is used above to indicate that these are the maximum values, since they are larger than the respective values for $x_r > x_c$, namely:

For $T = 300$ K,

$$\bar{h}_{\min} = 1620\ \text{W/m}^2 \cdot \text{K} \quad <$$

For $T = 350$ K

$$\bar{h}_{\min} = 3710\ \text{W/m}^2 \cdot \text{K} \quad <$$

Thus the minimum value of $\bar{h}$ is $\bar{h}_{\min} = 1620\ \text{W/m}^2 \cdot \text{K}$ which occurs for $T = 300$ K, for any $x_r >$ 0.43 m. The maximum value of $\bar{h}$ is $\bar{h}_{\max} = 5000\ \text{W/m}^2 \cdot \text{K}$ which occurs for $T = 350$ K, for $x_r =$ 1.19 mm.

COMMENTS: (1) The maximum value of $\bar{h}$ exists when transition occurs very close to the leading edge of the plate. It does not occur exactly at the leading edge because the laminar heat transfer coefficient equation yields a slightly higher value than the turbulent heat transfer coefficient equation very near $x = 0$. (2) Turbulent heat transfer coefficients are usually (but not always) larger than laminar heat transfer coefficients. Therefore, tripping the transition to turbulence at or near the leading edge results in enhanced heat transfer. (3) The conclusion that the laminar heat transfer coefficient is slightly higher than the turbulent heat transfer coefficient very near $x = 0$ may not be accurate. Turbulent heat transfer coefficient measurements are usually not performed very close to the leading edge since, in most cases, turbulence develops further downstream. (4) Adding roughness at x locations downstream of where the transition to turbulence would normally occur has no influence on the transition or the average heat transfer rate.

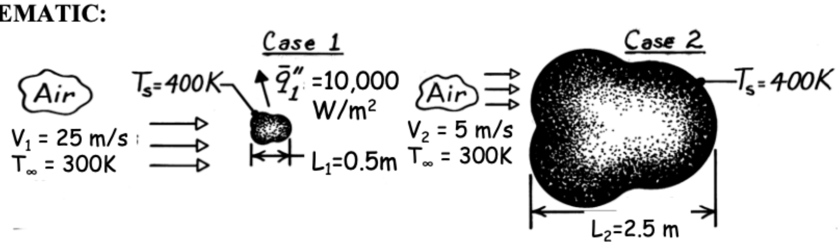
6.23 WP An object of irregular shape has a characteristic length of $L = 0.5$ m and is maintained at a uniform surface temperature of $T_s = 400$ K. When placed in atmospheric air at a temperature of $T_\infty = 300$ K and moving with a velocity of $V = 25$ m/s, the average heat flux from the surface to the air is 10,000 W/m². If a second object of the same shape, but with a characteristic length of $L = 2.5$ m, is maintained at a surface temperature of $T_s = 400$ K and is placed in atmospheric air at $T_\infty = 300$ K, what will the value of the average convection coefficient be if the air velocity is $V = 5$ m/s?

PROBLEM 6.23

KNOWN: Characteristic length, surface temperature and average heat flux for an object placed in an airstream of prescribed temperature and velocity.

FIND: Average convection coefficient if characteristic length of object is increased by a factor of five and air velocity is decreased by a factor of five.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

ANALYSIS: For a particular geometry,

$$\overline{Nu}_L = f(Re_L, Pr).$$

The Reynolds numbers for each case are

Case 1: $$Re_{L,1} = \frac{V_1 L_1}{\nu_1} = \frac{(25\,m/s)0.5m}{\nu_1} = \frac{12.5\ m^2/s}{\nu_1}$$

Case 2: $$Re_{L,2} = \frac{V_2 L_2}{\nu_2} = \frac{(5\,m/s)2.5m}{\nu_2} = \frac{12.5\ m^2/s}{\nu_2}.$$

Hence, with $\nu_1 = \nu_2$, $Re_{L,1} = Re_{L,2}$. Since $Pr_1 = Pr_2$, it follows that

$$\overline{Nu}_{L,2} = \overline{Nu}_{L,1}.$$

Hence,

$$\bar{h}_2 L_2 / k_2 = \bar{h}_1 L_1 / k_1$$
$$\bar{h}_2 = \bar{h}_1 \frac{L_1}{L_2} = 0.2\,\bar{h}_1.$$

For *Case 1*, using the rate equation, the convection coefficient is

$$q_1 = \bar{h}_1 A_1 (T_s - T_\infty)_1$$
$$\bar{h}_1 = \frac{(q_1/A_1)}{(T_s - T_\infty)_1} = \frac{q_1''}{(T_s - T_\infty)_1} = \frac{10{,}000\ W/m^2}{(400-300)K} = 100\ W/m^2\cdot K.$$

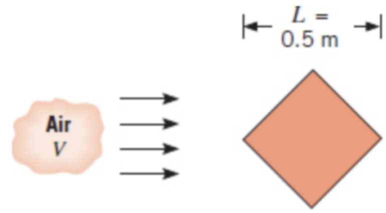
Hence, it follows that for *Case 2*

$$\bar{h}_2 = 0.2 \times 100\ W/m^2\cdot K = 20\ W/m^2\cdot K.$$ <

COMMENTS: If $Re_{L,2}$ were *not* equal to $Re_{L,1}$, it would be necessary to know the specific form of f(Re_L, Pr) before $\bar{h}_2$ could be determined.

6.25 Experimental measurements of the convection heat transfer coefficient for a square bar in cross flow yielded the following values:

$$\bar{h}_1 = 50 \text{ W/m}^2 \cdot \text{K} \text{ when } V_1 = 20 \text{ m/s}$$
$$\bar{h}_2 = 40 \text{ W/m}^2 \cdot \text{K} \text{ when } V_2 = 15 \text{ m/s}$$



Assume that the functional form of the Nusselt number is $\text{Nu} = C\text{Re}^m\,\text{Pr}^n$, where C, m, and n are constants.

(a) What will be the convection heat transfer coefficient for a similar bar with $L = 1$ m when $V = 15$ m/s?

(b) What will be the convection heat transfer coefficient for a similar bar with $L = 1$ m when $V = 30$ m/s?

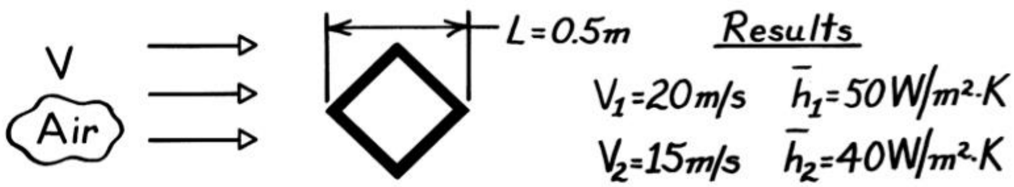
(c) Would your results be the same if the side of the bar, rather than its diagonal, were used as the characteristic length?

PROBLEM 6.25

KNOWN: Experimental measurements of the heat transfer coefficient for a square bar in cross flow.

FIND: (a) $\bar{h}$ for the condition when L = 1m and V = 15m/s, (b) $\bar{h}$ for the condition when L = 1m and V = 30m/s, (c) Effect of defining a side as the characteristic length.

SCHEMATIC:



ASSUMPTIONS: (1) Functional form $\overline{Nu} = C Re^m Pr^n$ applies with C, m, n being constants, (2) Constant properties.

ANALYSIS: (a) For the experiments and the condition L = 1m and V = 15m/s, it follows that Pr as well as C, m, and n are constants. Hence

$$\bar{h}L \propto (VL)^m.$$

Using the experimental results, find m. Substituting values

$$\frac{\bar{h}_1 L_1}{\bar{h}_2 L_2} = \left[\frac{V_1 L_1}{V_2 L_2}\right]^m \qquad \frac{50 \times 0.5}{40 \times 0.5} = \left[\frac{20 \times 0.5}{15 \times 0.5}\right]^m$$

giving m = 0.782. It follows then for L = 1m and V = 15m/s,

$$\bar{h} = \bar{h}_1 \frac{L_1}{L}\left[\frac{V \cdot L}{V_1 \cdot L_1}\right]^m = 50 \frac{W}{m^2 \cdot K} \times \frac{0.5}{1.0}\left[\frac{15 \times 1.0}{20 \times 0.5}\right]^{0.782} = 34.3 W/m^2 \cdot K.$$ <

(b) For the condition L = 1m and V = 30m/s, find

$$\bar{h} = \bar{h}_1 \frac{L_1}{L}\left[\frac{V \cdot L}{V_1 \cdot L_1}\right]^m = 50 \frac{W}{m^2 \cdot K} \times \frac{0.5}{1.0}\left[\frac{30 \times 1.0}{20 \times 0.5}\right]^{0.782} = 59.0 W/m^2 \cdot K.$$ <

(c) If the characteristic length were chosen as a side rather than the diagonal, the value of C would change. However, the coefficients m and n would not change.

COMMENTS: The foregoing Nusselt number relation is used frequently in heat transfer analysis, providing appropriate scaling for the effects of length, velocity, and fluid properties on the heat transfer coefficient.