SUGGESTED PROBLEMS CH 7

7.7 Consider atmospheric air at 20°C and a velocity of 30 m/s flowing over both surfaces of a 1-m-long flat plate that is maintained at 130°C. Determine the rate of heat transfer per unit width from the plate for values of the critical Reynolds number corresponding to 10^5, 5×10^5, and 10^6.

7.8 Consider laminar, parallel flow past an isothermal flat plate of length L, providing an average heat transfer coefficient of $\bar{h}_L$. If the plate is divided into N smaller plates, each of length $L_N = L/N$, determine an expression for the ratio of the heat transfer coefficient averaged over the N plates to the heat transfer coefficient averaged over the single plate, $\bar{h}_{L,N}/\bar{h}_{L,1}$.

7.10 WP Consider a flat plate subject to parallel flow (top and bottom) characterized by $u_\infty = 5$ m/s, $T_\infty = 20°C$.

(a) Determine the average convection heat transfer coefficient, convective heat transfer rate, and drag force associated with an L = 2-m-long, w = 3-m-wide flat plate for airflow and surface temperatures of T_s = 30°C and 80°C.

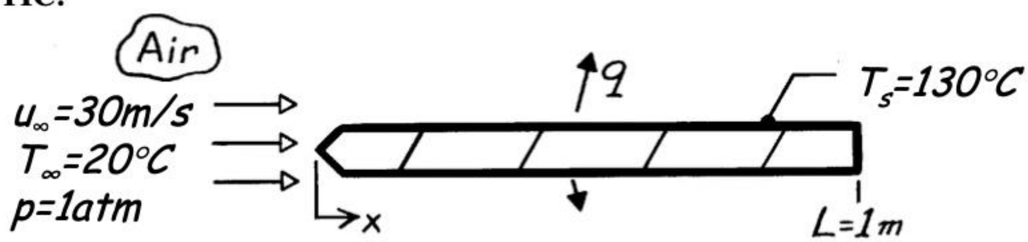
(b) Determine the average convection heat transfer coefficient, convective heat transfer rate, and drag force associated with an L = 0.1-m-long, w = 0.1-m-wide flat plate for water flow and surface temperatures of T_s = 30°C and 80°C.

PROBLEM 7.7

KNOWN: Speed and temperature of atmospheric air flowing over a flat plate of prescribed length and temperature.

FIND: Rate of heat transfer corresponding to $Re_{x,c} = 10^5$, 5×10^5 and 10^6.

SCHEMATIC:



ASSUMPTIONS: (1) Flow over top and bottom surfaces.

PROPERTIES: *Table A-4*, Air (T_f = 348K, 1 atm): ρ = 1.00 kg/m^3, $\nu = 20.72 \times 10^{-6}$ m^2/s, k = 0.0299 W/m·K, Pr = 0.700.

ANALYSIS: With

$$Re_L = \frac{u_\infty L}{\nu} = \frac{30\text{ m/s} \times 1\text{m}}{20.72 \times 10^{-6}\text{m}^2/\text{s}} = 1.45 \times 10^6$$

the flow becomes turbulent for each of the three values of $Re_{x,c}$. Hence,

$$\overline{Nu}_L = \left(0.037\, Re_L^{4/5} - A\right) Pr^{1/3}$$

$$A = 0.037\, Re_{x,c}^{4/5} - 0.664\, Re_{x,c}^{1/2}$$

$Re_{x,c}$	10^5	5×10^5	10^6
A	160	871	1671
$\overline{Nu}_L$	2645	2014	1303
$\bar{h}_L\left(W/m^2\cdot K\right)$	79.2	60.2	39
$q'(W/m)$	17,420	13,270	8595

where $q' = 2\,\bar{h}_L L(T_s - T_\infty)$ is the total heat loss per unit width of plate.

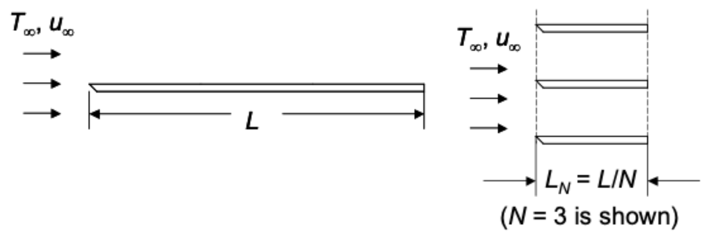
COMMENTS: Note that $\bar{h}_L$ decreases with increasing $Re_{x,c}$, as more of the surface becomes covered with a laminar boundary layer.

PROBLEM 7.8

KNOWN: Length of isothermal flat plate in parallel flow, L.

FIND: Expression for the average heat transfer coefficients for N plates each of length $L_N = L/N$ to the average coefficient for the single plate.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow, (2) Constant properties.

ANALYSIS: For the single plate, Equation 7.30 applies

$$\overline{Nu}_L = \frac{\bar{h}_{L,1}L}{k} = 0.664Re_L^{1/2}Pr^{1/3} \quad \text{or} \quad \bar{h}_{L,1} = (k/L)0.664Re_L^{1/2}Pr^{1/3} \tag{1}$$

For the multiple plates,

$$\overline{Nu}_{L,N} = \frac{\bar{h}_{L,N}L_N}{k} = 0.664Re_{L_N}^{1/2}Pr^{1/3} \quad \text{where } L_N = L/N \text{ and } Re_{L_N} = Re_L/N \tag{2a, b, c}$$

Combining Equations 2a, 2b and 2c yields

$$\bar{h}_{L,N} = \frac{kN}{L}0.664(Re_L/N)^{1/2}Pr^{1/3} \tag{3}$$

Dividing Equation 3 by Equation 1 yields

$$\bar{h}_{L,N}/\bar{h}_{L,1} = N^{1/2}$$ <

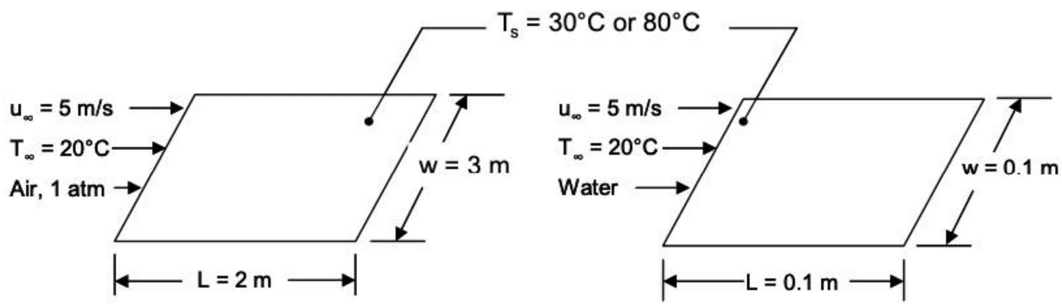
COMMENTS: (1) By breaking the single plate into shorter segments, the average boundary layer thickness is reduced, resulting in an increase of the average heat transfer coefficient. This is an effective strategy for heat transfer *enhancement*. (2) If the boundary layer over the single plate is not completely laminar, breaking it into shorter segments may or may not result in an increase in the average heat transfer coefficient since the turbulent section of the boundary layer over the single plate may be eliminated. (3) The relationship for completely turbulent flow is $\bar{h}_{L_N}/\bar{h}_L = N^{1/5}$, revealing less sensitivity to the plate length than for laminar conditions.

PROBLEM 7.10

KNOWN: Dimensions and surface temperatures of a flat plate. Velocity and temperature of air and water flow parallel to the plate.

FIND: (a) Average convective heat transfer coefficient, convective heat transfer rate, and drag force when L = 2 m, w = 2 m. (b) Average convective heat transfer coefficient, convective heat transfer rate, and drag force when L = 0.1 m, w = 0.1 m.

SCHEMATIC:



(a) Air (b) Water

ASSUMPTIONS: (1) Steady-state conditions, (2) Boundary layer assumptions are valid, (3) Constant properties, (4) Transition Reynolds number is 5×10^5.

PROPERTIES: Using *IHT*, Air (p = 1 atm, T_f = 25°C = 298 K): Pr = 0.708, k = 26.1×10^{-3} W/m·K, $\nu = 1.571 \times 10^{-5}$ m²/s, ρ = 1.171 kg/m³. Air (p = 1 atm, T_f = 50°C = 323 K): Pr = 0.704, k = 28.0×10^{-3} W/m·K, $\nu = 1.82 \times 10^{-5}$ m²/s, ρ = 1.085 kg/m³. Water (T_f = 298 K): Pr = 6.15, k = 0.610 W/m·K, $\nu = 8.99 \times 10^{-7}$ m²/s, ρ = 997 kg/m³. Water (T_f = 323 K): Pr = 3.56, k = 0.643 W/m·K, $\nu = 5.543 \times 10^{-7}$ m²/s, ρ = 988 kg/m³.

ANALYSIS:

(a) We begin by calculating the Reynolds numbers for the two different surface temperatures:

$$\mathrm{Re}_{L1} = \frac{u_\infty L}{\nu_1} = \frac{5\ \text{m/s} \times 2\ \text{m}}{1.571 \times 10^{-5}\ \text{m}^2/\text{s}} = 6.37 \times 10^5$$

$$\mathrm{Re}_{L2} = \frac{u_\infty L}{\nu_2} = \frac{5\ \text{m/s} \times 2\ \text{m}}{1.82 \times 10^{-5}\ \text{m}^2/\text{s}} = 5.49 \times 10^5$$

Therefore, in both cases the flow is turbulent at the end of the plate and the conditions in the boundary layer are "mixed."

The average drag coefficient can be calculated from Equation 7.40. For the first case,

$$\overline{C}_{f,L1} = 0.074\ \mathrm{Re}_{L1}^{-1/5} - 1742\ \mathrm{Re}_{L1}^{-1}$$

$$= 0.074(6.37 \times 10^5)^{-1/5} - 1742(6.37 \times 10^5)^{-1} = 2.37 \times 10^{-3}$$

Then

$$F_{D1} = \overline{C}_{f,L1} \frac{1}{2} \rho u_\infty^2 A_s = 2.37 \times 10^{-3} \times \frac{1}{2} \times 1.171\ \text{kg/m}^3 \times (5\ \text{m/s})^2 \times 12\ \text{m}^2$$

$$= 0.417\ \text{N}$$

<

Continued…

PROBLEM 7.10 (Cont.)

The average Nusselt number is calculated from Equation 7.38, with A = 871 for a transition Reynolds number of 5×10^5.

$$\overline{Nu}_{L1} = (0.037\, Re_L^{4/5} - 871)\, Pr^{1/3}$$

$$= \left[0.037(6.37 \times 10^5)^{4/5} - 871\right](0.708)^{1/3} = 673.$$

Then

$$\overline{h}_{L1} = \overline{Nu}_{L1} k/L = 673 \times 26.1 \times 10^{-3}\ \text{W/m}\cdot\text{K/2 m} = 8.79\ \text{W/m}^2\cdot\text{K}$$ <

and

$$q_1 = \overline{h}_{L1} A_s (T_s - T_\infty) = 8.79\ \text{W/m}^2\cdot\text{K} \times 12\ \text{m}^2 \times (30°\text{C} - 20°\text{C}) = 1055\ \text{W}$$ <

Similarly for $T_s = 80°C$ we find

$$F_{D2} = 0.227\ \text{N},\ \overline{h}_{L2} = 7.16\ \text{W/m}^2\cdot\text{K},\ q_2 = 3440\ \text{W}$$ <

(b) Repeating the calculations for water

$$Re_{L1} = \frac{u_\infty L}{\nu} = \frac{5\ \text{m/s} \times 0.1\ \text{m}}{8.99 \times 10^{-7}\ \text{m}^2/\text{s}} = 5.56 \times 10^5$$

$$Re_{L2} = 9.02 \times 10^5$$

The flow is turbulent at the end of the plate in both cases.

$$\overline{C}_{f,L1} = 0.074(5.56 \times 10^5)^{-1/5} - 1742(5.56 \times 10^5)^{-1} = 2.12 \times 10^{-3}$$

$$F_{D1} = 2.12 \times 10^{-3} \times 1/2 \times 997\ \text{kg/m}^3 \times (5\ \text{m/s})^2 \times 0.02\ \text{m}^2 = 0.528\ \text{N}$$ <

$$\overline{Nu}_L = \left[0.037(5.56 \times 10^5)^{4/5} - 871\right](6.15)^{1/3} = 1079$$

$$\overline{h}_{L1} = 1079 \times 0.610\ \text{W/m}\cdot\text{K/0.1 m} = 6583\ \text{W/m}^2\cdot\text{K}$$ <

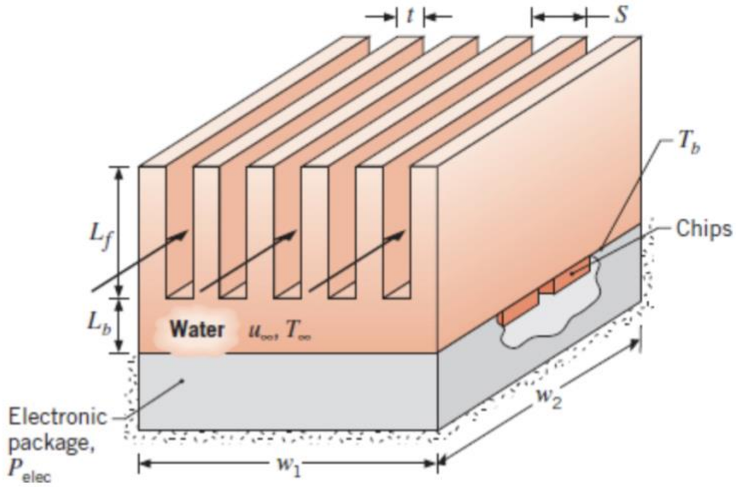
$$q_1 = 6583\ \text{W/m}^2\cdot\text{K} \times 0.02\ \text{m}^2 \times (30°\text{C} - 20°\text{C}) = 1317\ \text{W}$$ <

For the higher surface temperature,

$$F_{D2} = 0.700\ \text{N},\ \overline{h}_{L2} = 12{,}600\ \text{W/m}^2\cdot\text{K},\ q_2 = 15{,}100\ \text{W}$$ <

COMMENTS: (1) For air, kinematic viscosity increases with increasing temperature. This decreases the Reynolds number which causes the transition to turbulence to move downstream, thereby decreasing the drag force and average heat transfer coefficient. The heat transfer rate increases for the higher surface temperature, however, because of the greater temperature difference between the surface and air. (2) For water, kinematic viscosity decreases with increasing temperature, causing the opposite trends as for air. The heat transfer rate increases dramatically for the higher surface temperature because of the increases in both the heat transfer coefficient and temperature difference. (3) Even though the water flows over a plate that is 600 times smaller, the drag force and heat transfer rate are larger than for air because of the smaller viscosity and greater density, thermal conductivity, and Prandtl number. (4) The problem highlights the importance of carefully accounting for the temperature dependence of thermal properties.

7.22 WP An array of electronic chips is mounted within a sealed rectangular enclosure, and cooling is implemented by attaching an aluminum heat sink (k = 180 W/m · K). The base of the heat sink has dimensions of $w_1 = w_2$ = 100 mm, while the 6 fins are of thickness t = 10 mm and pitch S = 18 mm. The fin length is L_f = 50 mm, and the base of the heat sink has a thickness of L_b = 10 mm.



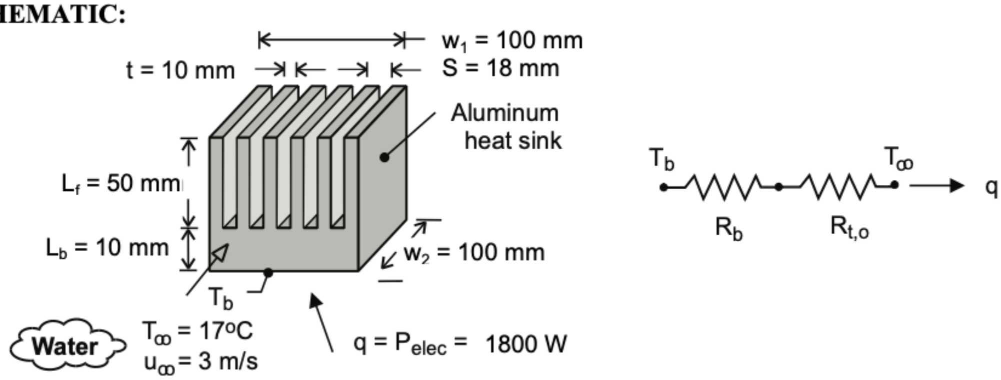
If cooling is implemented by water flow through the heat sink, with u_∞ = 3 m/s and T_∞ = 17°C, what is the base temperature T_b of the heat sink when power dissipation by the chips is P_{elec} = 1800 W? The average convection coefficient for surfaces of the fins and the exposed base may be estimated by assuming parallel flow over a flat plate. Properties of the water may be approximated as k = 0.62 W/m · K, ρ = 995 kg/m^3, c_p = 4178 J/kg · K, $\nu = 7.73 \times 10^{-7}$ m^2/s, and Pr = 5.2.

PROBLEM 7.22

KNOWN: Dimensions of aluminum heat sink. Temperature and velocity of coolant (water) flow through the heat sink. Power dissipation of electronic package attached to the heat sink.

FIND: Base temperature of heat sink.

SCHEMATIC:



ASSUMPTIONS: (1) Average convection coefficient associated with flow over fin surfaces may be approximated as that for a flat plate in parallel flow, (2) All of the electric power is dissipated by the heat sink, (3) Transition Reynolds number of $Re_{x,c} = 5 \times 10^5$, (4) Constant properties. (5) Water temperature is nearly constant as it flows through the array.

PROPERTIES: Given. Aluminum: k_{hs} = 180 W/m·K. Water: k_w = 0.62 W/m·K, $\nu = 7.73 \times 10^{-7}$ m^2/s, Pr = 5.2.

ANALYSIS: From the thermal circuit,

$$q = P_{elec} = \frac{T_b - T_\infty}{R_b + R_{t,o}}$$

where $R_b = L_b / k_{hs}(w_1 \times w_2) = 0.01\text{ m}/180\text{ W/m}\cdot\text{K}(0.10\text{ m})^2 = 5.56 \times 10^{-3}\text{ K/W}$ and, from Eqs. 3.107 and 3.108,

$$R_{t,o} = \left\{\bar{h} A_t \left[1 - \frac{NA_f}{A_t}(1-\eta_f)\right]\right\}^{-1}$$

The fin and total surface area of the array are $A_f = 2w_2(L_f + t/2) = 0.2\text{ m}(0.055\text{ m}) = 0.011\text{ m}^2$ and $A_t = NA_f + A_b = NA_f + (N-1)(S-t)w_2 = 6(0.011\text{ m}^2) + 5(0.008\text{ m})0.1\text{ m} = (0.066 + 0.004) = 0.070\text{ m}^2$.

With $Re_{w_2} = u_\infty w_2 / \nu = 3\text{ m/s} \times 0.10\text{ m}/7.73 \times 10^{-7}\text{ m}^2/\text{s} = 3.88 \times 10^5$, laminar flow may be assumed over the entire surface. Hence

$$\bar{h} = \left(\frac{k_w}{w_2}\right) 0.664\, Re_{w_2}^{1/2} Pr^{1/3} = \left(\frac{0.62\text{ W/m}\cdot\text{K}}{0.10\text{ m}}\right) 0.664\left(3.88 \times 10^5\right)^{1/2} (5.2)^{1/3} = 4443\text{ W/m}^2\cdot\text{K}$$

With $m = (2\bar{h}/k_{hs}t)^{1/2} = \left(2 \times 4443\text{ W/m}^2\cdot\text{K}/180\text{ W/m}\cdot\text{K} \times 0.01\text{ m}\right)^{1/2} = 70.3\text{ m}^{-1}$, $mL_c = 70.3\text{ m}^{-1}$ $(0.055\text{ m}) = 3.86$ and $\tanh mL_c = 0.9991$, Eq. 3.95 yields

$$\eta_f = \frac{\tanh mL_c}{mL_c} = \frac{0.9991}{3.86} = 0.259$$

Continued …

PROBLEM 7.22 (Cont.)

Hence,

$$R_{t,o} = \left\{ 4443\ W/m^2 \cdot K \times 0.070\ m^2 \left[1 - \frac{0.066\ m^2}{0.070\ m^2}(1-0.259) \right] \right\}^{-1} = 0.0107\ K/W$$

and

$$T_b = T_\infty + P_{elec}\left(R_b + R_{t,o}\right) = 17°C + 1800\ W\left(5.56 \times 10^{-3} + 0.0107\right)K/W = 46.2°C \qquad <$$

COMMENTS: (1) The boundary layer thickness at the trailing edge of the fin is $\delta = 5w_2/\left(Re_{w_2}\right)^{1/2} = 0.80\ \text{mm} << (S-t)$. Hence, the assumption of parallel flow over a flat plate is reasonable. (2) If a finned heat sink is not employed and heat transfer is simply by convection from the $w_2 \times w_2$ base surface, the corresponding convection resistance would be 0.0225 K/W, which is only twice the resistance associated with the fin array. The small enhancement by the array is attributable to the large value of $\bar{h}$ and the correspondingly small value of η_f. Were a fluid such as air or a dielectric liquid used as the coolant, the much smaller thermal conductivity would yield a smaller $\bar{h}$, a larger η_f and hence a larger effectiveness for the array. (3) The water outlet temperature may be calculated based the energy balance, $q = \dot{m}\, c_p\, (T_{m,o} - T_{m,i})$ or $T_{m,o}$ = 17°C + 1800 W/(5 × 0.008m × 0.05m × 3 m/s × 995 kg/m^3 × 4178 J/kg·K) = 17.07°C where there are N = 5 channels. (The density and specific heat are evaluated at Pr = 5.2.) The assumption of constant water mean temperature is excellent. (4) *If* the increase in the water temperature was significant, an approach described in Chapter 11 would be needed to analyze the problem. See Problem 11.69.

7.30 Consider atmospheric air at u_∞ = 2 m/s and T_∞ = 300 K in parallel flow over an isothermal flat plate of length L = 1 m and temperature T_s = 350 K.

(a) Compute the local convection coefficient at the leading and trailing edges of the hot plate with and without an unheated starting length of ξ = 1 m.

(b) Compute the average convection coefficient for the plate for the same conditions as part (a).

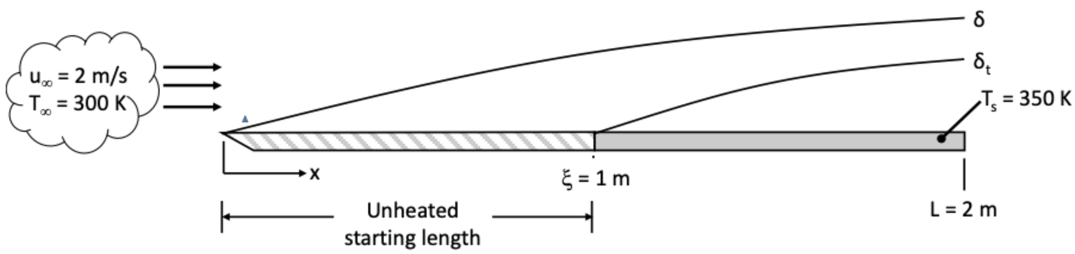
(c) Computer Icon Plot the variation of the local convection coefficient over the plate with and without an unheated starting length.

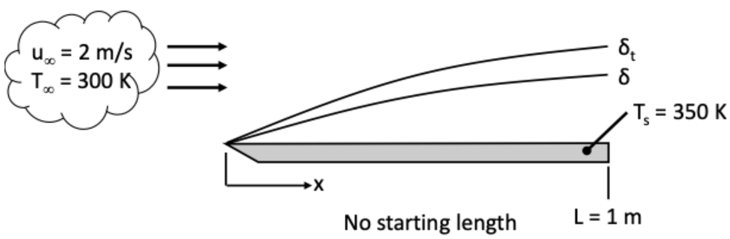
PROBLEM 7.30

KNOWN: Conditions for airflow over isothermal plate with optional unheated starting length.

FIND: (a) local coefficient, h_x, at leading and trailing edges with and without an unheated starting length, $\xi = 1$ m, (b) average convection coefficient for same conditions, (c) variation of local convection coefficient over plate with and without unheated starting length.

SCHEMATIC:





PROPERTIES: *Table A.4*, Air ($T_f = 325$ K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m²/s, Pr = 0.703, k = 0.0282 W/m·K.

ANALYSIS: (a) The Reynolds number at $\xi = 1$ m is

$$Re_\xi = \frac{u_\infty \xi}{\nu} = \frac{2\,\text{m/s} \times 1\,\text{m}}{18.4 \times 10^{-6}\,\text{m}^2/\text{s}} = 1.087 \times 10^5$$

If $Re_{x,c} = 5 \times 10^5$, flow is laminar over the entire plate (with or without the starting length). In general,

$$Nu_x = \frac{0.332\,Re_x^{1/2}\,Pr^{1/3}}{\left[1-(\xi/x)^{3/4}\right]^{1/3}} \tag{1}$$

$$h_x = \frac{\left(0.332k\,Pr^{1/3}\right)Re_x^{1/2}}{x\left[1-(\xi/x)^{3/4}\right]^{1/3}} = 0.00832\,\text{W/m}\cdot\text{K}\frac{Re_x^{1/2}}{x\left[1-(\xi/x)^{3/4}\right]^{1/3}}.$$

With Unheated Starting Length: Leading edge (x = 1 m): $Re_x = Re_\xi$, $\xi/x = 1$, $h_x = \infty$ <

Trailing Edge (x = 2 m): $Re_x = 2\,Re_\xi = 2.17 \times 10^5$, $\xi/x = 0.5$

$$h_x = 0.00832\,\text{W/m}\cdot\text{K}\frac{\left(2.17 \times 10^5\right)^{1/2}}{2\,\text{m}\left[1-(0.5)^{3/4}\right]^{1/3}} = 2.61\,\text{W/m}^2\cdot\text{K}$$ <

Continued...

PROBLEM 7.30 (Cont.)

Without Unheated Starting Length: Leading edge (x = 0): $h_x = \infty$ <

Trailing edge (x = 1 m): $Re_x = 1.087 \times 10^5$

$$h_x = 0.00832\,\text{W/m}\cdot\text{K}\,\frac{\left(1.087\times10^5\right)^{1/2}}{1\,\text{m}} = 2.74\,\text{W/m}^2\cdot\text{K}$$ <

(b) The average convection coefficient $\overline{h}_L$ for the two cases in the schematic are, from Eq. 6.14,

$$\overline{h}_L = \frac{1}{L}\int_0^L h_x(x)\,dx \qquad (2)$$

where L is the x location at the end of the heated section. Substituting Eq. (1) into Eq. (2) and numerically integrating, the results are tabulated below:

ξ (m)	h_x (L)(W/m²·K)	$\overline{h}_L$ (W/m²·K)
0	2.74	5.41
1	2.61	4.22

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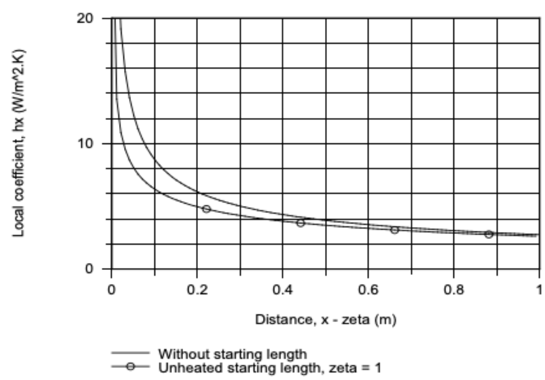
(c) The variation of the local convection coefficient over the plate, with and without the unheated starting length, using Eq. (1) is shown below. The abscissa is x - ξ.



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7.34 Consider the following fluids, each with a velocity of $V = 3$ m/s and a temperature of $T_\infty =$ 20°C, in cross flow over a 10-mm-diameter cylinder maintained at 50°C: atmospheric air, saturated water, and engine oil.

(a) Calculate the rate of heat transfer per unit length, q', using the Churchill-Bernstein correlation.

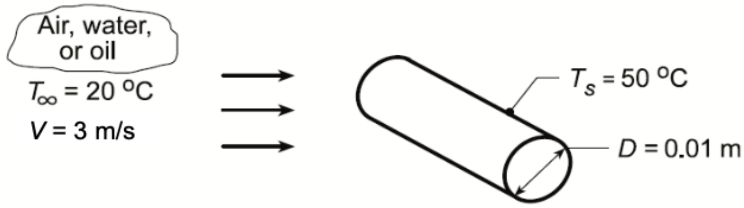
(b) C Generate a plot of q' as a function of fluid velocity for $0.5 \le V \le 10$ m/s.

PROBLEM 7.34

KNOWN: Cylinder diameter and surface temperature. Temperature and velocity of fluids in cross flow.

FIND: (a) Rate of heat transfer per unit length for the fluids: atmospheric air and saturated water, and engine oil, for velocity V = 3 m/s, using the Churchill-Bernstein correlation, and (b) Compute and plot q' as a function of the fluid velocity $0.5 \le V \le 10$ m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform cylinder surface temperature.

PROPERTIES: *Table A.4*, Air (T_f = 308 K, 1 atm): $\nu = 16.69 \times 10^{-6}$ m²/s, k = 0.0269 W/m·K, Pr = 0.706; *Table A.6*, Saturated Water (T_f = 308 K): ρ = 994 kg/m³, $\mu = 725 \times 10^{-6}$ N·s/m², k = 0.625 W/m·K, Pr = 4.85; *Table A.5*, Engine Oil (T_f = 308 K): $\nu = 340 \times 10^{-6}$ m²/s, k = 0.145 W/m·K, Pr = 4000.

ANALYSIS: (a) For each fluid, calculate the Reynolds number and use the Churchill-Bernstein correlation, Eq. 7.54,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+\left(0.4/Pr\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

Fluid: Atmospheric Air

$$Re_D = \frac{VD}{\nu} = \frac{(3\,m/s)0.01\,m}{16.69\times10^{-6}\,m^2/s} = 1797$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(1797)^{1/2}(0.706)^{1/3}}{\left[1+(0.4/0.706)^{2/3}\right]^{1/4}}\left[1+\left(\frac{1797}{282,000}\right)^{5/8}\right]^{4/5} = 21.5$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0269\,W/m\cdot K}{0.01\,m}21.5 = 57.9\,W/m^2\cdot K$$

$$q' = \bar{h}\pi D(T_s - T_\infty) = 57.9\,W/m^2\cdot K\,\pi(0.01\,m)(50-20)^\circ C = 54.6\,W/m \quad <$$

Fluid: Saturated Water

$$Re_D = \frac{VD}{\nu} = \frac{(3\,m/s)0.01\,m}{725\times10^{-6}\,N\cdot s/m^2/994\,kg/m^3} = 41,130$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(41,130)^{1/2}(4.85)^{1/3}}{\left[1+(0.4/4.85)^{2/3}\right]^{1/4}}\left[1+\left(\frac{41,130}{282,000}\right)^{5/8}\right]^{4/5} = 252$$

Continued...

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.625\,\mathrm{W/m\cdot K}}{0.01\,\mathrm{m}}252 = 15,730\,\mathrm{W/m^2\cdot K} \qquad q' = 14,830\,\mathrm{W/m}$$ <

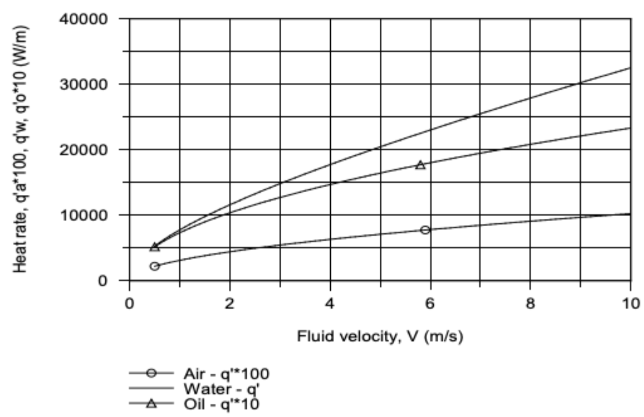
Fluid: Engine Oil

$$Re_D = \frac{VD}{\nu} = \frac{(3\,\mathrm{m/s})0.01\,\mathrm{m}}{340\times10^{-6}\,\mathrm{m^2/s}} = 88.2$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(88.2)^{1/2}(4000)^{1/3}}{\left[1+(0.4/4000)^{2/3}\right]^{1/4}}\left[1+\left(\frac{88.2}{282,000}\right)^{5/8}\right]^{4/5} = 93.2$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.145\,\mathrm{W/m\cdot K}}{0.01\,\mathrm{m}}93.2 = 1350\,\mathrm{W/m^2\cdot K} \qquad q' = 1270\,\mathrm{W/m}$$ <

(b) Using the *IHT Correlations Tool*, *External Flow*, *Cylinder*, along with the *Properties Tool* for each of the fluids, the heat rates, q', were calculated for the range $0.5 \le V \le 10$ m/s. Note the q' scale multipliers for the air and oil fluids which permit easy comparison of the three curves.



COMMENTS: (1) Note the inapplicability of the Zukauskas relation, Eq. 7.53, since $Pr_{oil} > 500$.

(2) In the plot above, recognize that the heat rate for the water is more than 10 times that with oil and 300 times that with air. How do changes in the velocity affect the heat rates for each of the fluids?

7.36 A long, cylindrical, electrical heating element of diameter $D = 12$ mm, thermal conductivity $k = 240$ W/m · K, density $\rho = 2700$ kg/m3, and specific heat $c_p = 900$ J/kg · K is installed in a duct for which air moves in cross flow over the heater at a temperature and velocity of 30°C and 8 m/s, respectively.

(a) Neglecting radiation, estimate the steady-state surface temperature when, per unit length of the heater, electrical energy is being dissipated at a rate of 1000 W/m.

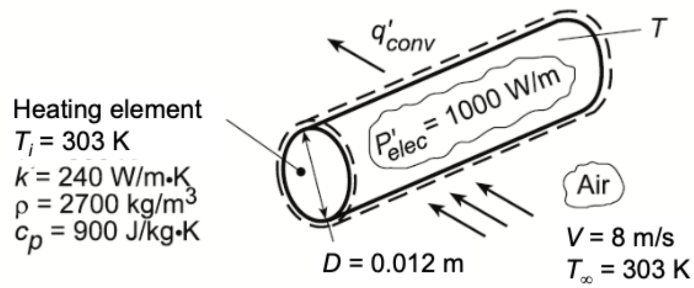
(b) If the heater is activated from an initial temperature of 30°C, estimate the time required for the surface temperature to come within 10°C of its steady-state value.

PROBLEM 7.36

KNOWN: Initial temperature, power dissipation, diameter, and properties of heating element. Velocity and temperature of air in cross flow.

FIND: (a) Steady-state temperature, (b) Time to come within 10°C of steady-state temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform heater temperature, (2) Negligible radiation.

PROPERTIES: *Table A.4*, air (assume $T_f \approx 450$ K): $\nu = 32.39 \times 10^{-6}$ m²/s, k = 0.0373 W/m·K, Pr = 0.686.

ANALYSIS: (a) Performing an energy balance for steady-state conditions, we obtain

$$q'_{conv} = \bar{h}(\pi D)(T - T_\infty) = P'_{elec} = 1000\,\text{W/m}$$

With

$$Re_D = \frac{VD}{\nu} = \frac{(8\,\text{m/s})0.012\,\text{m}}{32.39 \times 10^{-6}\,\text{m}^2/\text{s}} = 2964$$

the Churchill and Bernstein correlation, Eq. 7.54, yields

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(2964)^{1/2}(0.686)^{1/3}}{\left[1 + (0.4/0.686)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{2964}{282{,}000}\right)^{5/8}\right]^{4/5} = 27.6$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0373\,\text{W/m}\cdot\text{K}}{0.012\,\text{m}}\,27.6 = 85.7\,\text{W/m}^2\cdot\text{K}$$

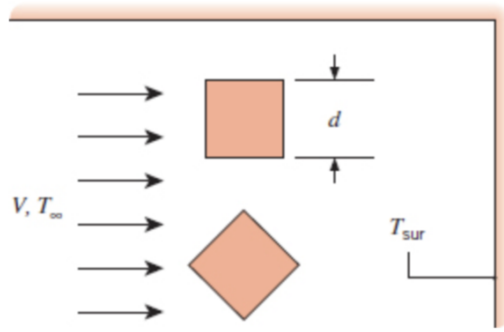
Hence, the steady-state temperature is

$$T = T_\infty + \frac{P'_{elec}}{\pi D \bar{h}} = 303\,\text{K} + \frac{1000\,\text{W/m}}{\pi(0.012\,\text{m})85.7\,\text{W/m}^2\cdot\text{K}} = 612\,\text{K}$$ <

(b) With $Bi = \bar{h}r_o/k$ = 85.7 W/m²·K(0.006 m)/240 W/m·K = 0.0021, a lumped capacitance analysis may be performed. The time response of the heater is given by Eq. 5.25, which, for $T_i = T_¥$, reduces to

$$T = T_\infty + (b/a)\left[1 - \exp(-at)\right]$$

7.43 In a manufacturing process, long aluminum rods of square cross section with $d = 25$ mm are cooled from an initial temperature of $T_i = 400°C$. Which configuration in the sketch should be used to minimize the time needed for the rods to reach a *safe-to-handle* temperature of 60°C when exposed to air in cross flow at $V = 8$ m/s, $T_\infty = 30°C$? What is the required cooling time for the preferred configuration? The emissivity of the rods is $\varepsilon = 0.10$ and the surroundings temperature is $T_{sur} = 20°C$.

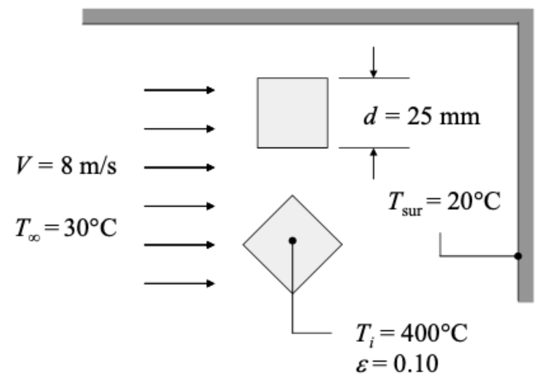


PROBLEM 7.43

KNOWN: Dimension and initial temperature of long aluminum rods of square cross-section. Velocity and temperature of air in cross flow. Rod emissivity and surroundings temperature.

FIND: Which orientation of the rod relative to the cross flow should be used to minimize the time needed for the rods to reach a temperature of 60°C. Required cooling time for preferred configuration.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties.

PROPERTIES: *Table A-4*, Air ($T = 400$ K): $\nu = 26.41 \times 10^{-6}$ m²/s, $k = 0.0338$ W/m·K, $Pr = 0.690$. *Table A-1*, Pure aluminum ($T = 500$ K): $\rho_s = 2702$ kg/m³, $c_{p,s} = 991$ J/kg·K, $k_s = 235$ W/m·K.

ANALYSIS: The heat transfer coefficient can be calculated from Equation 7.52, with the dimension D defined differently for the two configurations, as shown in Table 7.3. When the air flows perpendicular to a face of the rod,

$D = d = 0.025$ m, $Re_D = VD/\nu = 8 \text{ m/s} \times 0.025 \text{ m}/26.41 \times 10^{-6} \text{ m}^2/\text{s} = 7573$, and from Table 7.3, $C = 0.158$ and $m = 0.66$. Thus,

$$\bar{h} = \frac{k}{D} C Re_D^m Pr^{1/3} = \frac{0.0338 \text{ W/m}\cdot\text{K}}{0.025 \text{ m}} 0.158(7573)^{0.66}(0.69)^{1/3} = 68.6 \text{ W/m}^2\cdot\text{K}$$

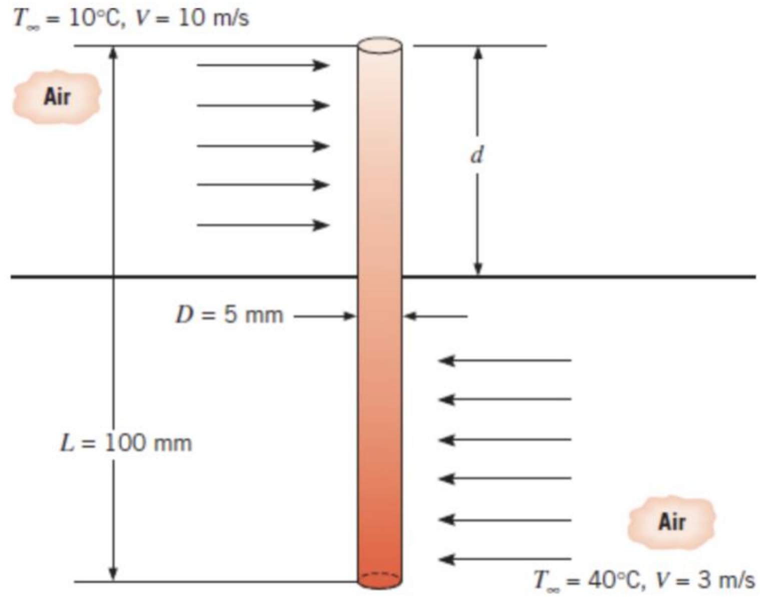
When the rod is rotated so that it presents an edge to the flow, $D = 2^{1/2}d = 2^{1/2} \times 0.025 \text{ m} = 0.0354$ m, $Re_D = VD/\nu = 8 \text{ m/s} \times 0.0354 \text{ m}/26.41 \times 10^{-6} \text{ m}^2/\text{s} = 10{,}710$, and from Table 7.3, $C = 0.304$ and $m = 0.59$. Thus,

$$\bar{h} = \frac{k}{D} C Re_D^m Pr^{1/3} = \frac{0.0338 \text{ W/m}\cdot\text{K}}{0.0354 \text{ m}} 0.304(10{,}710)^{0.59}(0.69)^{1/3} = 61.2 \text{ W/m}^2\cdot\text{K}$$

Radiation will affect both rods in the same way, therefore the rod with the larger value of convection heat transfer coefficient will cool faster. The rod should be oriented with a face perpendicular to the flow in order to minimize the cooling time. <

The importance of radiation can be estimated by calculating the radiation heat flux at the initial time, $q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.1 \times 5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4 \times \left[(673 \text{ K})^4 - (293 \text{ K})^4\right] = 1120 \text{ W/m}^2$. The initial convection heat transfer flux is $q''_{conv} = \bar{h}(T_s - T_\infty) = 68.6 \text{ W/m}^2\cdot\text{K} \times (400°\text{C} - 30°\text{C}) = 25{,}380 \text{ W/m}^2$. Since the radiation heat transfer rate is only around 4% initially, and will decrease in relative importance with time, radiation can be neglected in a calculation of the cooling time.

7.49 To augment heat transfer between two flowing fluids, it is proposed to insert a 100-mm-long, 5-mm-diameter 2024 aluminum pin fin through the wall separating the two fluids. The pin is inserted to a depth of d into fluid 1. Fluid 1 is air with a mean temperature of 10°C and velocity of 10 m/s. Fluid 2 is air with a mean temperature of 40°C and velocity of 3 m/s.



(a) Determine the rate of heat transfer from the warm air to the cool air through the pin fin for d = 50 mm.

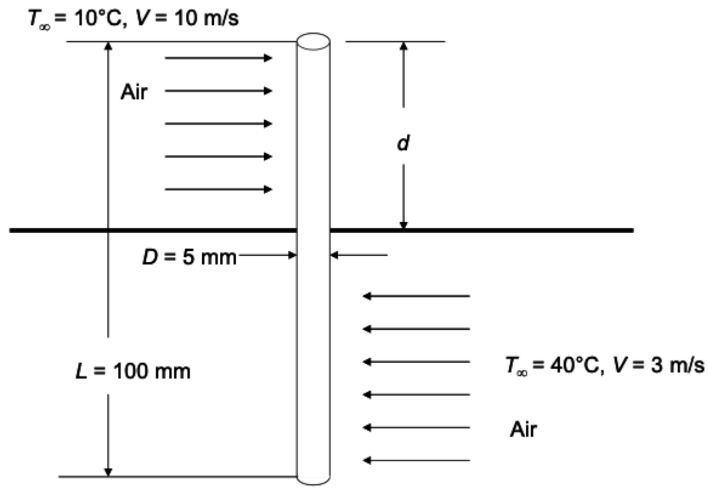
(b) C Plot the variation of the heat transfer rate with the insertion distance, d. Does an optimal insertion distance exist?

PROBLEM 7.49

KNOWN: Velocities and temperatures of two air streams separated by a wall. Dimensions of an aluminum pin fin inserted through the wall. Distance it extends into the upper fluid.

FIND: (a) Heat transfer rate between the fluids via the pin fin, when it extends 50 mm into the upper fluid. (b) Heat transfer rate as a function of the distance it extends into the upper fluid.

SCHEMATIC:



ASSUMPTIONS: (1) Velocity is uniform – decreased velocity near wall can be neglected, (2) For the purpose of evaluating properties, the fin temperature is equal to the average of the two fluid temperatures, $T_s = 25°C$.

PROPERTIES: Table A-4, Air 1 ($T_{f1} = 17.5°C \cong 290.5$ K): $\nu_1 = 1.504 \times 10^{-5}$ m²/s, $k_1 = 0.02554$ W/m·K, $Pr_1 = 0.710$. Air 2 ($T_{f2} = 32.5°C \cong 305.5$ K): $\nu_2 = 1.644 \times 10^{-5}$ m²/s, $k_2 = 0.02671$ W/m·K, $Pr_2 = 0.706$. Table A-1, Aluminum 2024 ($T_s = 25°C \approx 300$ K): k = 177 W/m·K.

ANALYSIS:

(a) The heat transfer coefficients between the air and the fin are analyzed as flow past a cylinder using the Churchhill-Bernstein correlation:

$$Re_{D1} = \frac{V_1 D}{\nu_1} = \frac{10\text{ m/s} \times 0.005\text{ m}}{1.504 \times 10^{-5}\text{m}^2\text{/s}} = 3320$$

$$Re_{D2} = \frac{V_2 D}{\nu_2} = \frac{3\text{ m/s} \times 0.005\text{ m}}{1.644 \times 10^{-5}\text{m}^2\text{/s}} = 912.$$

From Equation 7.54,

$$Nu_{D1} = 0.3 + \frac{0.62\, Re_{D1}^{1/2}\, Pr^{1/3}}{\left[1 + (0.4/Pr_1)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_{D1}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62 \times (3320)^{1/2} \times (0.710)^{1/3}}{\left[1 + (0.4/0.710)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{3320}{282{,}000}\right)^{5/8}\right]^{4/5} = 29.7$$

and $$h_1 = \frac{Nu_{D1} k_1}{D} = \frac{29.7 \times 0.02554\text{ W/m}\cdot\text{K}}{0.005\text{ m}} = 152\text{ W/m}^2\cdot\text{K}$$

Continued…

Similarly, $Nu_{D2} = 15.3$, $h_2 = 81.5\ W/m^2 \cdot K$.

Next we analyze heat transfer along the rod as if it were two fins joined at their base – the location where the fin passes through the wall. Thus, using the corrected fin length approach, Equation 3.94,

$$q_1 = M_1 \tanh m_1 L_{c1}$$
$$q_2 = M_2 \tanh m_2 L_{c2}$$

where

$$M_i = \sqrt{h_i P k A_c}\,\theta_{bi} = \sqrt{h_i D k}\,\frac{\pi D}{2}\,(T_b - T_{\infty i})$$
$$m_i = \sqrt{h_i P / k A_c} = 2\sqrt{h_i / kD}$$

and $L_{ci} = L_i + D/4$. In this expression, $L_1 = d$ and $L_2 = L - d$. Finally, since heat leaving one rod enters the other,

$$q_1 = -q_2$$
$$\sqrt{h_1 D k}\,\frac{\pi D}{2}\,(T_b - T_{\infty 1}) \tanh m_1 L_{c1} = -\sqrt{h_2 D k}\,\frac{\pi D}{2}\,(T_b - T_{\infty 2}) \tanh m_2 L_{c2}$$

Solving for T_b:

$$T_b = \frac{\sqrt{h_1}\;T_{\infty 1}\tanh(m_1 L_{c1}) + \sqrt{h_2}\;T_{\infty 2}\tanh(m_2 L_{c2})}{\sqrt{h1}\;\tanh(m_1 L_{c1}) + \sqrt{h_2}\;\tanh(m_2 L_{c2})} \qquad (1)$$

We calculate

$$m_1 = 2\sqrt{h_1/kD} = 2\sqrt{152\ W/m^2 \cdot K/(177\ W/m \cdot K \times 0.005\ m)} = 26.2\ m^{-1}$$

and similarly, $m_2 = 19.2\ m^{-1}$. Also, $L_{c1} = L_{c2} = d + D/4 = 0.05\ m + 0.005\ m/4 = 0.05125\ m$.

Thus

$$T_b = \left[\begin{array}{l}\sqrt{152\ W/m^2 \cdot K} \times 10°C \times \tanh(26.2\ m^{-1} \times 0.05125\ m) \\ \quad + \sqrt{81.5\ W/m^2 \cdot K} \times 40°C \times \tanh(19.2\ m^{-1} \times 0.05125\ m)\end{array}\right] \Bigg/ \left[\begin{array}{l}\sqrt{152\ W/m^2 \cdot K} \times \tanh(26.2\ m^{-1} \times 0.05125\ m) \\ \quad + \sqrt{81.5\ W/m^2 \cdot K} \times \tanh(19.2\ m^{-1} \times 0.05125\ m)\end{array}\right] = 21.6°C$$

Finally

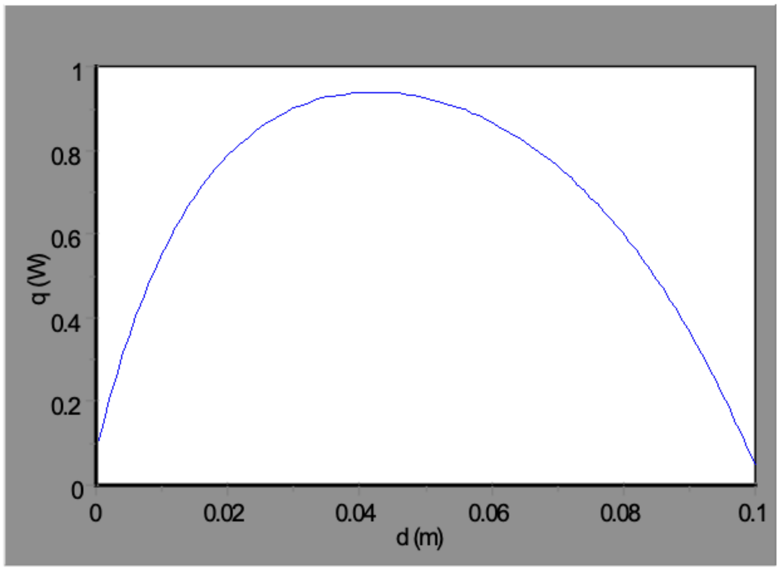
$$q = q_1 = -q_2 = \sqrt{h_1 D k}\,\frac{\pi D}{2}\,(T_b - T_{\infty 1}) \tanh(m_1 L_{c1}) \qquad (2)$$
$$= \sqrt{152\ W/m^2 \cdot K \times 0.005\ m \times 177\ W/m \cdot K} \times \frac{\pi(0.005\ m)}{2}$$
$$\times (21.6°C - 10°C) \tanh(26.2\ m^{-1} \times 0.05125\ m)$$
$$= 0.924\ W$$

<

Continued…

PROBLEM 7.49 (Cont.)

(b) With $L_{c1} = d + D/4$ and $L_{c2} = L - d + D/4$, we vary d in the range $0 \le d \le 0.1$ m and solve Equations (1) and (2). The results for q are plotted below.



We see that there is an optimal insertion distance, $d \cong 40$ mm. A longer fin length (≈ 60 mm) is needed in fluid 2 to compensate for its smaller heat transfer coefficient.

COMMENTS: It is of interest to compare the heat transfer between the two fluids via the fin to the heat transfer through the wall. In Chapter 8 we will see how to calculate heat transfer coefficients for flow in a channel. Assuming that the channel widths are both approximately 50 mm, the heat transfer coefficients between the fluid and the wall are roughly 40 $W/m^2 \cdot K$ and 10 $W/m^2 \cdot K$ for the faster and slower streams, respectively. Then $q'' = \dfrac{T_2 - T_1}{1/h_1 + 1/h_2} \cong 240\ W/m^2$. A wall area of $4 \times 10^{-3}\ m^2$, for example a 60 mm-square area, would be required to transfer the same amount of heat as the fin (in part a), 0.924 W.

7.55 Consider a sphere with a diameter of 20 mm and a surface temperature of 60°C that is immersed in a fluid at a temperature of 30°C and a velocity of 2.5 m/s. Calculate the drag force and the heat rate when the fluid is (a) water and (b) air at atmospheric pressure. Explain why the results for the two fluids are so different.

7.56 A spherical, underwater instrument pod used to make soundings and to measure conditions in the water has a diameter of 100 mm and dissipates 400 W.

(a) Estimate the surface temperature of the pod when suspended in a bay where the current is 1 m/s and the water temperature is 15°C.

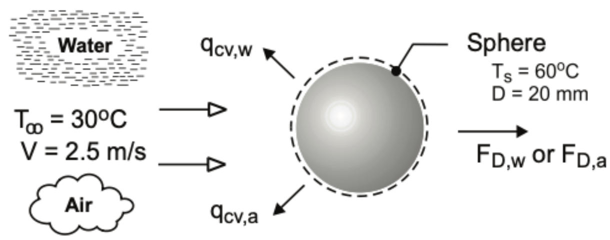
(b) Inadvertently, the pod is hauled out of the water and suspended in ambient air without deactivating the power. Estimate the surface temperature of the pod if the air temperature is 15°C and the wind speed is 3 m/s.

PROBLEM 7.55

KNOWN: Sphere with a diameter of 20 mm and a surface temperature of 60°C that is immersed in a fluid at a temperature of 30°C with a velocity of 2.5 m/s.

FIND: The drag force and the heat rate when the fluid is (a) water and (b) air at atmospheric pressure. Explain why the results for the two fluids are so different.

SCHEMATIC:



ASSUMPTIONS: (1) Flow over a smooth sphere, (2) Constant properties.

PROPERTIES: *Table A-6*, Water (T_∞ = 30°C = 303 K): $\mu = 8.034 \times 10^{-4}$ N·s/m^2, $\nu = 8.068 \times 10^{-7}$ m^2/s, k = 0.6172 W/m·K, Pr = 5.45; Water (T_s = 333 K): $\mu_s = 4.674 \times 10^{-4}$ N·s/m^2; *Table A-4*, Air (T_∞ = 30°C = 303 K, 1 atm): $\mu = 1.86 \times 10^{-5}$ N·s/m^2, $\nu = 1.619 \times 10^{-5}$ m^2/s, k = 0.0265 W/m·K, Pr = 0.707; Air (T_∞ = 333 K): $\mu_s = 2.002 \times 10^{-5}$ N·s/m^2.

ANALYSIS: The drag force, F_o, for the sphere is determined from the drag coefficient, Eq. 7.50,

$$C_D = \frac{F_D}{A_f\left(\rho V^2/2\right)}$$

where $A_f = \pi D^2/4$ is the frontal area. C_D is a function of the Reynolds number $Re_D = VD/\nu$ as represented in Figure 7.9. For the convection rate equation,

$$q = \bar{h}_D A_s\left(T_s - T_\infty\right)$$

where $A_s = \pi D^2$ is the surface area and the convection coefficient is estimated using the Whitaker correlation, Eq. 7.56,

$$\overline{Nu}_D = 2 + \left[0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right] Pr^{0.4}\left(\mu/\mu_s\right)^{1/4}$$

where all properties except μ_s are evaluated at T_∞. For convenience we will evaluate properties required for the drag force at T_∞. The results of the analyses for the two fluids are tabulated below. <

Fluid	Re_D	C_D	F_D (N)	$\overline{Nu}_D$	$\bar{h}_D\left(W/m^2\cdot K\right)$	q(W)
water	6.198×10^4	0.5	0.489	439	13,540	510
air	3.088×10^3	0.4	0.452×10^{-3}	31.9	42.3	1.59

The frontal and surface areas, respectively, are $A_f = 3.142 \times 10^{-4}$ m^2 and $A_s = 1.257 \times 10^{-3}$ m^2.

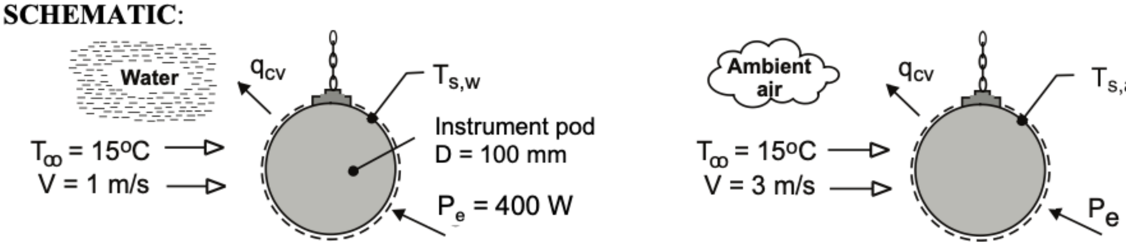
COMMENTS: (1) The Reynolds number is the ratio of inertia to viscous forces. We associate higher viscous shear and heat transfer with larger Reynolds numbers. The drag force also depends upon the fluid density, which further explains why F_D for water is much larger, by a factor of 1000, than for air. Nu_D is dependent upon Re_D^n where n is 1/2 to 2/3, and represents the dimensionless temperature gradient at the surface. Since the thermal conductivity of water is nearly 20 times that of air, we expect a significant difference between $\bar{h}_D$ and q for the two fluids. (2) For air, the viscosity ratio is outside of the range for which the Whitaker correlation was developed.

PROBLEM 7.56

KNOWN: An underwater instrument pod having a spherical shape with a diameter of 100 mm dissipating 400 W.

FIND: Estimate the surface temperature of the pod for these conditions: (a) when submersed in a bay where the water temperature is 15°C and the current is 1 m/s, and (b) after being hauled out of the water *without deactivating the power* and suspended in the ambient where the air temperature is 15°C and the wind speed is 3 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Flow over a smooth sphere, (3) Uniform surface temperatures, (4) Negligible radiation heat transfer, and (5) Constant properties ($\mu = \mu_s$) for water.

PROPERTIES: *Table A-6*, Water (T_∞ = 15°C = 288 K): $\mu = 0.001053\ N{\cdot}s/m^2$, $\nu = 1.139 \times 10^{-6}\ m^2/s$, k = 0.5948 W/m·K, Pr = 8.06; *Table A-4*, Air (T_∞ = 288 K, 1 atm): $\mu = 1.788 \times 10^{-5}\ N{\cdot}s/m^2$, $\nu = 1.482 \times 10^{-5}\ m^2/s$, k = 0.02534 W/m·K, Pr = 0.710 ; Air (T_s = 945 K): $\mu_s = 4.099 \times 10^{-5}\ N{\cdot}s/m^2$.

ANALYSIS: The energy balance for the submersed-in-water (w) and suspended-in-air (a) conditions are represented in the schematics above and have the form

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = -q_{cv} + P_e = 0 \qquad (1)$$

$$-\bar{h}_D A_s \left(T_s - T_\infty\right) + P_e = 0$$

where $A_s = \pi D^2$ and $\bar{h}_D$ is estimated using the Whitaker correlation, Eq. 7.56,

$$\overline{Nu}_D = 2 + \left[0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right] Pr^{0.4} \left(\mu / \mu_s\right)^{1/4} \qquad (2)$$

where all properties except μ_s are evaluated at T_∞. The results are tabulated below.

Condition	Re_D	$\overline{Nu}_D$	$\bar{h}_D$ (W/m²·K)	T_s (°C)
(w) water	8.78×10^4	548	3261	19.1
(a) air	2.02×10^4	73.9	18.7	695

COMMENTS: (1) While submerged and dissipating 400 W, the pod is safely operating at a temperature slightly above that of the water. When hauled from the water and suspended in air, the pod temperature increases to a destruction temperature (695°C).

(2) The assumption that $\mu/\mu_s \approx 1$ is appropriate for the water (w) condition. For the air (a) condition, $\mu/\mu_s = 0.436$ and the final term of the correlation is significant. Unfortunately, the Whitaker correlation is restricted to $\mu/\mu_s > 1$, so the results here should be seen as an estimate only.
(3) Recognize that radiation exchange with the surroundings for the air condition should be considered for an improved estimate.

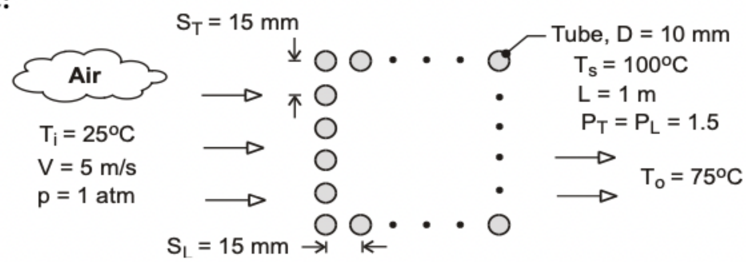
7.70 Consider the in-line tube bank of Problem 7.69 ($D = 10$ mm, $L = 1$ m, and $ST = SL = 15$ mm), with condensing steam used to heat atmospheric air entering the tube bank at $Ti = 25°C$ and $V = 5$ m/s. In this case, however, the desired outlet temperature, not the number of tube rows, is known. What is the minimum value of NL needed to achieve an outlet temperature of $To \geq 75°C$? What is the corresponding pressure drop across the tube bank?

PROBLEM 7.70

KNOWN: Surface temperature and geometry of a tube bank. Inlet velocity and inlet and outlet temperatures of air in cross flow over the tubes.

FIND: Number of tube rows needed to achieve the prescribed outlet temperature and corresponding pressure of drop of air.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible temperature drop across tube wall and uniform outer surface temperature, (3) Constant properties, (4) $C_2 \approx 1$, (5) Negligible radiation and incompressible flow.

PROPERTIES: *Table A-4*, Atmospheric air. $(\overline{T} = (T_i + T_o)/2 = 323K)$: $\rho = 1.085\,kg/m^3$, $c_p = 1007\,J/kg\cdot K$, $\nu = 18.2\times10^{-6} m^2/s$, $k = 0.028\,W/m\cdot K$, $Pr = 0.707$; $(T_i = 298K)$: $\rho = 1.17\,kg/m^3$; $(T_s = 373K)$: $Pr_s = 0.695$.

ANALYSIS: The temperature difference $(T_s - T)$ decreases exponentially in the flow direction, and at the outlet

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\pi D N_L \overline{h}}{\rho V S_T c_p}\right)$$

where $N_L = N/N_T$. Hence,

$$N_L = -\frac{\rho V S_T c_p}{\pi D \overline{h}} \ell n\left(\frac{T_s - T_o}{T_s - T_i}\right) \qquad (1)$$

With $V_{max} = [S_T/(S_T - D)]V = 15\,m/s, Re_{D,max} = V_{max}D/\nu = 8240$. Hence, with $S_T/S_L = 1 > 0.7$, $C_1 = 0.27$ and $m = 0.63$ from Table 7.5, and the Zukauskas correlation yields

$$\overline{Nu}_D = C_1 C_2 Re_{D,max}^m Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4} = 0.27\times1(8240)^{0.63}(0.707)^{0.36}(0.707/0.695)^{1/4} = 70.1$$

$$\overline{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.028\,W/m\cdot K}{0.01m}70.1 = 196.3\,W/m^2\cdot K$$

Hence,

$$N_L = -\frac{1.17\,kg/m^3(5\,m/s)0.015m(1007\,J/kg\cdot K)}{\pi(0.01m)196.3\,W/m^2\cdot K}\ell n\left(\frac{25}{75}\right) = 15.7$$

and 16 tube rows should be used $\qquad N_L = 16 \qquad$ <

With $Re_{D,max} = 8240$, $P_L = 1.5$ and $(P_T - 1)/(P_L - 1) = 1$, $f \approx 0.35$ and $\chi = 1$ from Fig. 7.14. Hence,

$$\Delta p \approx N_L \chi\left(\frac{\rho V_{max}^2}{2}\right) f = 16\left[\frac{1.085\,kg/m^3\times(15\,m/s)^2}{2}\right]0.35 = 684\,N/m^2$$ <

COMMENTS: (1) With $C_2 = 0.99$ for $N_L = 16$ from Table 7.6, assumption 4 is appropriate. (2) Note use of the density evaluated at $T_i = 298K$ in Eq. (1).

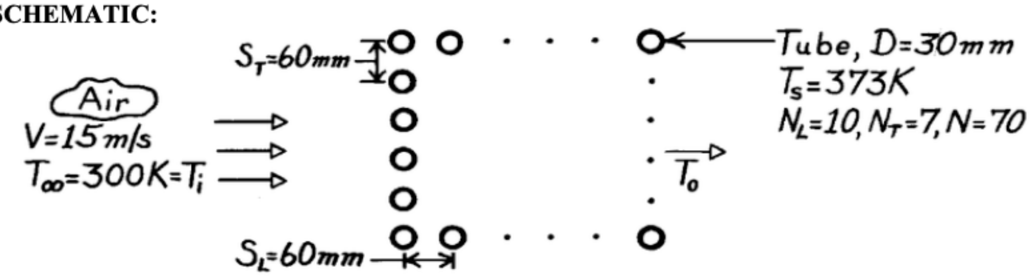
7.73 A tube bank uses an aligned arrangement of 30-mm-diameter tubes with $S_T = S_L = 60$ mm and a tube length of 1 m. There are 10 tube rows in the flow direction ($N_L = 10$) and 7 tubes per row ($N_T = 7$). Air with upstream conditions of $T_\infty = 27°C$ and $V = 15$ m/s is in cross flow over the tubes, while a tube wall temperature of 100°C is maintained by steam condensation inside the tubes. Determine the temperature of air leaving the tube bank, the pressure drop across the bank, and the fan power requirement.

PROBLEM 7.73

KNOWN: Surface temperature and geometry of a tube bank. Velocity and temperature of air in cross-flow.

FIND: (a) Air outlet temperature, (b) Pressure drop and fan power requirements.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Air pressure is approximately one atmosphere, (4) Uniform surface temperature.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): $\rho = 1.1614$ kg/m^3, $c_p = 1007$ J/kg·K, $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707; (373K): Pr = 0.695.

ANALYSIS: (a) The air temperature increases exponentially, with

$$T_o = T_s - \left(T_s - T_i\right)\exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right).$$

With $V_{max} = \dfrac{S_T}{S_T - D}V = \dfrac{60}{30}15\dfrac{m}{s} = 30\dfrac{m}{s}$; $Re_{D,max} = \dfrac{30\text{ m/s} \times 0.03\text{ m}}{15.89 \times 10^{-6}\text{ m}^2/\text{s}} = 56,639.$

Tables 7.5 and 7.6 give $C_1 = 0.27$, m = 0.63 and $C_2 = 0.97$. Hence from the Zukauskas correlation,

$$\overline{Nu}_D = 0.27(0.97)(56,639)^{0.63}(0.707)^{0.36}(0.707/0.695)^{1/4} = 229$$

$$\bar{h} = \overline{Nu}_D k/D = 229 \times 0.0263\text{ W/m}\cdot\text{K}/0.03\text{ m} = 201\text{ W/m}^2\cdot\text{K}.$$

Hence,

$$T_o = 373\text{K} - (373 - 300)\text{K}\exp\left(-\frac{\pi \times 0.03\text{ m} \times 70 \times 201\text{ W/m}^2\cdot\text{K}}{1.1614\text{ kg/m}^3 \times 15\text{ m/s} \times 7 \times 0.06\text{ m} \times 1007\text{ J/kg}\cdot\text{K}}\right)$$

$$T_o = 373\text{K} - 73\text{K} \times 0.835 = 312\text{K} = 39^\circ\text{C}.$$ <

(b) With $Re_{D,max} = 5.66 \times 10^4$, $P_L = 2$, $(P_T - 1)/(P_L - 1) = 1$, Fig. 7.14 yields $f \approx 0.19$ and $\chi = 1$. Hence,

$$\Delta p = N_L \chi \left(\frac{\rho V_{max}^2}{2}\right) f = 10\left(\frac{1.1614\text{ kg/m}^3 \times (30\text{ m/s})^2}{2}\right)0.19 = 993\text{ N/m}^2 = 0.00993\text{ bar}.$$ <

The fan power requirement is

$$P = \dot{m}_a \Delta p/\rho = \rho V N_T S_T L\,\Delta p/\rho = 15\text{ m/s} \times 7 \times 0.06\text{ m} \times 1\text{m} \times 993\text{ N/m}^2 = 6.26\text{ kW}.$$ <

COMMENTS: The heat rate is

$$q = \dot{m}_a c_p\left(T_o - T_i\right) = \rho V N_T S_T L\, c_p\left(T_o - T_i\right)$$

$$q = 1.1614\text{ kg/m}^3 \times 15\text{ m/s} \times 7 \times 0.06\text{m} \times 1\text{m} \times 1007\text{ J/kg}\cdot\text{K}\,(312 - 300)\text{K} = 88.4\text{ kW}.$$