Objective

- To study the characteristics of convergent parallel nozzle with variable inlet conditions.

Experimental Setup

The experimental setup is shown in figure (1). The convergent parallel nozzle device is connected to a compressed air source to supply it with compressed air. A control valve (1) controls the amount of air flowing into the device. A chest is fitted to the nozzle inlet to prevent the fluctuations in the pressure of the compressed air, so that it enters the nozzle with uniform inlet pressure P_o. This inlet pressure is measured using a pressure gauge (1) connected to the chest. The value of P_o is adjusted using another control valve (2).

A pressure probe is installed inside the nozzle to measure the pressure at certain positions along the axis of the nozzle. The value of the pressure measured by the probe is displayed on a pressure gauge (2). A position indicator is used to adjust the position of the probe inside the nozzle at certain locations by rotating it. These locations are designated by position numbers ranging from 7 (entrance of the nozzle) to 32, each of which corresponds to a certain $\frac{Position}{Length\ of\ the\ nozzle}$ ratio. The position numbers are labeled on the circumference of the position indicator as shown in figure (1).

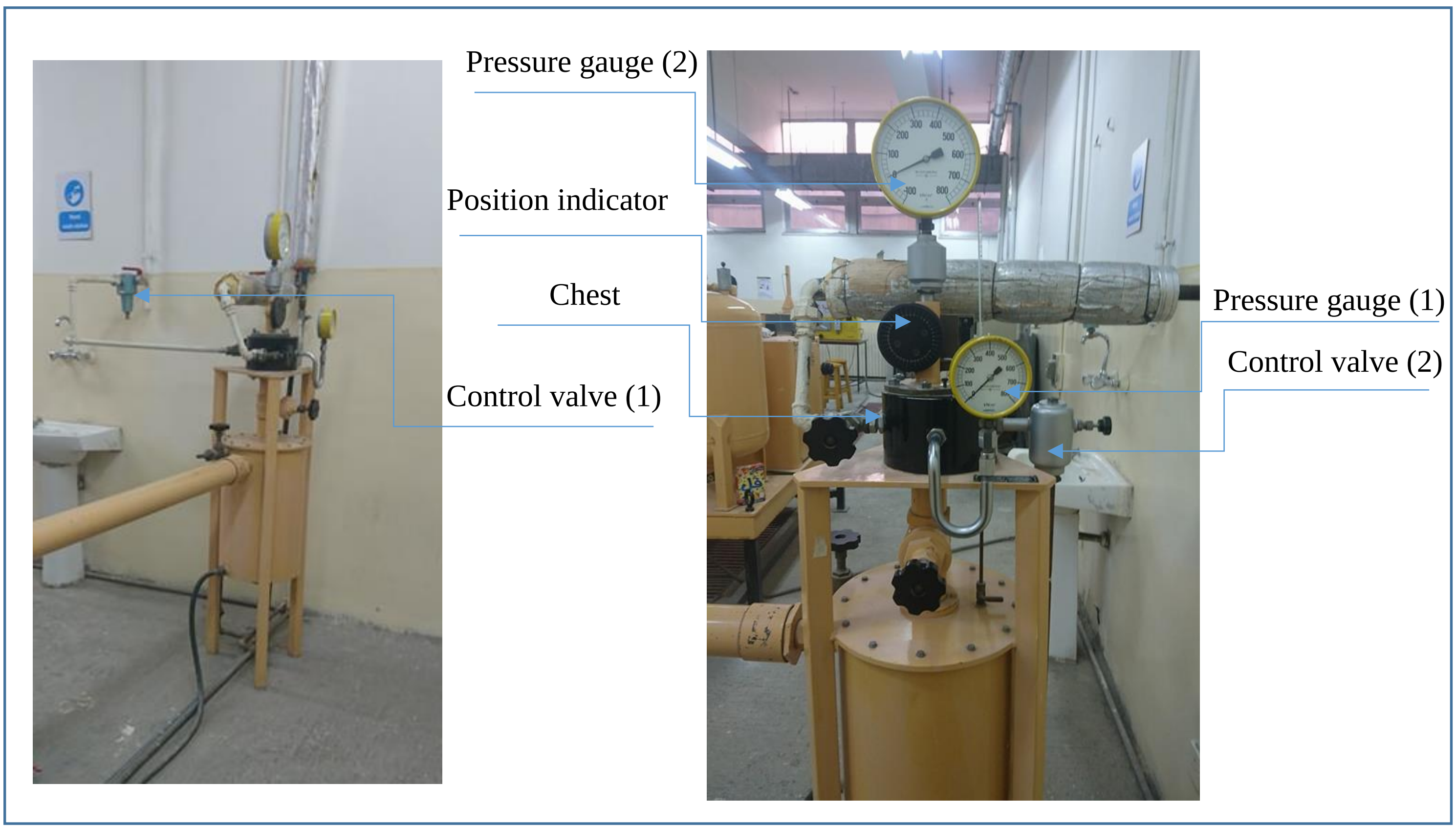


Figure (1): Convergent-Parallel nozzle device

Start-up Procedure

1. Open the control valve (1) to allow the compressed air to flow from its source to the device.
2. Adjust the position indicator such that it indicates position number 7.
3. The experiment now is ready to carry on.

Experimental Procedure

1. Perform the start-up procedure.
2. Rotate the control valve (2) to set the value of the inlet pressure to P_o = 200 kPa.
3. Rotate the position indicator and record the pressure at each position number.
4. Repeat the steps with P_o = 400 kPa and P_o = 600 kPa.

Given Data

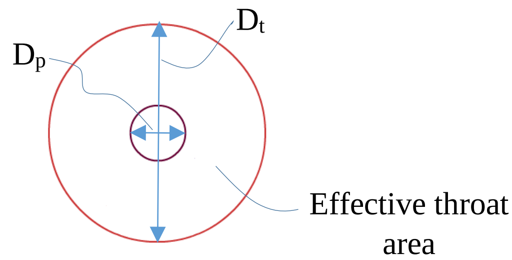
- Atmospheric pressure P_{atm} = 90.1 kPa.
- Atmospheric Temperature T_{atm} = 20.5 ^{o}C.
- Specific heat ratio for air k_{air} = 1.4 [1].
- Air gas constant R_{air} = 287 J/kg.K [1].
- Throat diameter D_t = 4.77 mm.
- Probe diameter D_p = 3.33 mm.

Observed Data

Table (1): Observed Data

Position No.	Position-to-length ratio	Chest (Stagnation) pressure(P_o) (kPa)	Chest (Stagnation) pressure(P_o) (kPa)	Chest (Stagnation) pressure(P_o) (kPa)
	(X/L)	P_o = 200 kPa	P_o = 400 kPa	P_o = 600 kPa
7 (Entrance)	0	200	400	600
8	0.25	200	400	600
9	0.5	195	395	590
10	0.75	180	385	570
11 (Throat)	1	165	340	510
12	1.25	140	300	455
13	1.5	138	290	440
14	1.75	135	285	430
15	2	130	282	420
16	2.25	125	275	400
17	2.5	120	265	400
18	2.75	118	260	395
19	3	110	258	390
20	3.25	110	255	380
21	3.5	105	242	370
22	3.75	102	240	360
23	4	102	230	345
24	4.25	101	225	340
25	4.5	100	218	330
26	4.75	95	208	320
27	5	83	200	310
28	5.25	80	190	295
29	5.5	79	180	285
30	5.75	65	175	270
31	6	50	150	240
32	6.25	5	110	195

Sample Calculations



Step(1): Find the effective throat area

$A_t = \frac{\pi}{4} \times (D_t^2 - D_p^2)$

$$= \frac{\pi}{4} \times \left(\left(\frac{4.77}{1000}\right)^2 - \left(\frac{4.77}{1000}\right)^2\right) = 9.16088 \times 10^{-6}\ \mathrm{m}^2$$

Step(2): Find sonic velocity

$$C = \sqrt{k_{air} \times R_{air} \times T_{atm}}\ \text{, where } T_{atm} \text{ is in Kelvins} \quad [1]$$

$$T_{atm}\,(K) = T_{atm}\,(°C) + 273.15 \quad [1]$$

$$T_{atm}\,(K) = 20.5 + 273.15 = 293.65\,K$$

$$C = \sqrt{1.4 \times 287 \times 293.65} = 343.495\,m/s$$

Step(3): Find the critical pressure

$$P^* = \left(\frac{2}{k_{air}+1}\right)^{\frac{k_{air}}{k_{air}-1}} \times P_o \quad [1]$$

Take column 2 from Table(1) as a sample for calculations :

P_o must be in kPa_Abs

$$P_{o_{abs}} = P_{o_{gauge}} + P_{atm} \quad [1]$$

$$P_{o_{abs}} = 400 + 90.1 = 490.1\,kPa\_Abs$$

$$P^* = \left(\frac{2}{1.4+1}\right)^{\frac{1.4}{1.4-1}} \times 490.1 = 258.91\,kPa\_Abs$$

Step(4): Find the air velocity at the throat

$$V_t = \sqrt{\frac{2k_{air}}{k_{air}-1} \times R_{air} \times T_{chest} \times \left[1 - \left(\frac{P_t}{P_o}\right)^{\frac{k_{air}-1}{k_{air}}}\right]}\ ,\ T_{chest} = T_{atm} \quad [1]$$

$$V_t = \sqrt{\frac{2 \times 1.4}{1.4 - 1} \times 287 \times 293.65 \times \left[1 - \left(\frac{340 + 90.1}{400 + 90.1}\right)^{\frac{1.4-1}{1.4}}\right]} = 146.9908\,m/s$$

Step(5): Find Mach Number

Mach No. = $\frac{V_t}{C}$ [1]

$$= \frac{146.9908}{343.495} = 0.427$$

Step(6) : Calculate the mass flow rate for the flowing air

$$\dot{m}_{air} = A_t \times P_o \times \left(\frac{P_t}{P_o}\right)^{\frac{1}{k_{air}}} \times \sqrt{\frac{2 \times k_{air}}{k_{air}-1} \times \frac{1}{R_{air} \times T_{chest}} \times \left[1 - \left(\frac{P_t}{P_o}\right)^{\frac{k_{air}-1}{k_{air}}}\right]} \quad [1]$$

$$= 9.16088 \times 10^{-6} \times 490.1 \times \left(\frac{430.1}{490.1}\right)^{\frac{1}{1.4}} \times \sqrt{\frac{2\times1.4}{1.4-1} \times \frac{1}{287\times293.65} \times \left[1 - \left(\frac{430.1}{490.1}\right)^{\frac{1.4-1}{1.4}}\right]}$$

$$= 7.13 \times 10^{-6}\ kg/s$$

Table (2) :Data Calculated

Calculated Parameters	Experiment 1 P_o = 200 kPa	Experiment 2 P_o = 400 kPa	Experiment 3 P_o = 600 kPa
Critical pressure (P^*) (kPa_Abs)	153.25	258.91	364.56
Air velocity at the throat (V_t) (m/s)	145.8696	146.9908	151.9537
Mach Number	0.424	0.427	0.442
Mass flow rate of air ($\dot{m}_{air}$)(kg/s)	4.20×10^{-6}	7.13×10^{-6}	1.03×10^{-5}

Uncertainty Analysis

- For a calculated quantity x that is dependent on another quantities $x_1, x_2, x_3, \ldots, x_n$

  $$x = f(x_1, x_2, x_3, \ldots, x_n)$$

  The uncertainty of x (w_x) is given by :

$$w_x = \pm\sqrt{\left(\frac{\partial x}{\partial x_1} \times w_{x1}\right)^2 + \left(\frac{\partial x}{\partial x_2} \times w_{x2}\right)^2 + \left(\frac{\partial x}{\partial x_3} \times w_{x3}\right)^2 + \cdots + \left(\frac{\partial x}{\partial x_n} \times w_{xn}\right)^2} \quad [3]$$

- In this experiment, the calculated P^* value is dependent on P_o
  i.e
  $P^* = f\ (P_o)$

  The uncertainty of P^* is given by

$$w_{P^*} = \pm\sqrt{\left(\frac{\partial P^*}{\partial P_o} \times w_{P_o}\right)^2}$$

- The uncertainty of an observed quantity measured using a device , is the value of one-half the smallest division of the device. [3] The uncertainty of P_o are as follows :
  $w_{P_o} = \pm 10\ kPa$

- The following quantity is found by differentiating $P^* = \left(\frac{2}{k_{air}+1}\right)^{\frac{k_{air}}{k_{air}-1}} \times P_o$:

$$\frac{\partial P^*}{\partial P_o} = \frac{dP^*}{dP_o} = \left(\frac{2}{k_{air}+1}\right)^{\frac{k_{air}}{k_{air}-1}}$$

⇨ Take experiment 1 from Table(2) as a sample for calculations.

$$w_{P^*} = \pm\sqrt{\left(\left(\frac{2}{1.4+1}\right)^{\frac{1.4}{1.4-1}} \times 10\right)^2}$$

= $\pm$ 5.28 kPa = $\pm$ 3.4%

⇨ The value of uncertainty $w_{P^*}$for the second experiment is

w_{P^*}= $\pm$2.04%

⇨ The value of uncertainty $w_{P^*}$for the third experiment is

w_{P^*}= $\pm$1.45%

Results & Discussion

Nozzles are mechanical devices with varying cross-section that increase the velocity of a fluid at the expense of pressure. From Table(1), it is obvious that there is a great reduction in pressure between the entrance of the nozzle and the throat caused by the increase in the velocity of the fluid. Air pressure continues to decrease slightly in the parallel portion of the nozzle due to friction.

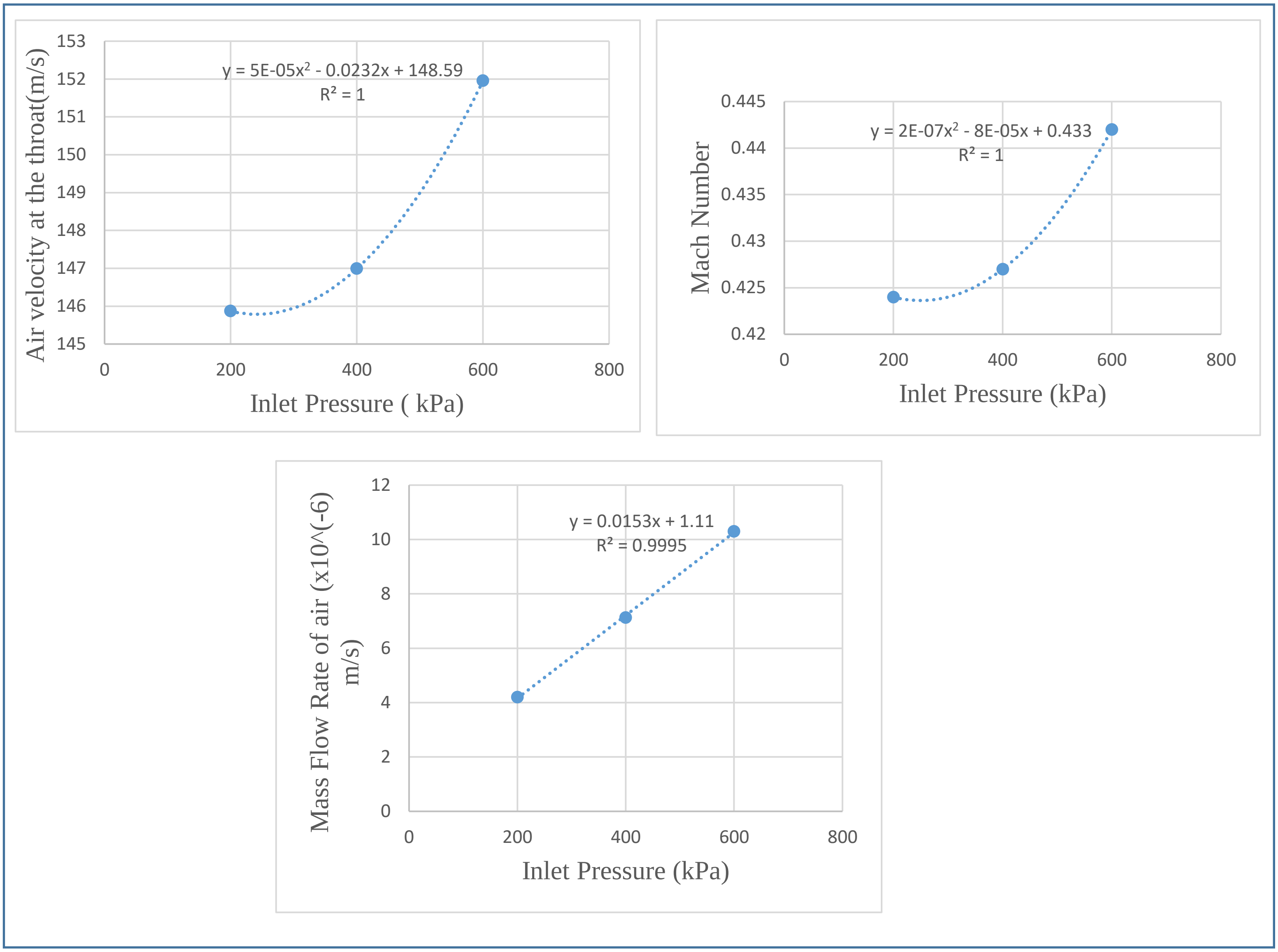


Figure (2): The Relationship between the inlet pressure and $\dot{m}_{air}$, Mach No., V_t

Since the objective of this experiment is to study the characteristics of convergent parallel nozzle with variable inlet conditions, several conclusions can be drawn from the results obtained from calculations. First, the air velocity at the throat has increased with the increase of the inlet pressure P_o . It is obvious from Table(1) that the difference between the pressure at the inlet and the pressure at the throat is increasing as the inlet pressure increases. According to Bernoulli's equation, more pressure energy is converted into kinetic energy. As a result, the air velocity rises causing the Mach number to increase. Since the mass flow rate of the air is related to its velocity, the increase in the mass flow rate is justified by the increase in the velocity of the air.

Finally, it is noticeable that the pressure at the throat is greater than the critical pressure. Critical pressure is the value of pressure that if it is reached, the velocity of the air inside the convergent nozzle will reach sonic speed . At this point, the velocity of the air will not continue increasing (i.e it will stay fixed), unless the convergent nozzle is followed by a divergent portion. This phenomenon is called choked flow for compressible fluids. In the experiment, the values of Mach number didn't reach or approach the unity. Thus choked flow didn't occur.

Sources of Error

Errors in this experiment are caused by several factors such as: the pressure gauge is not accurately calibrated, since zero gauge pressure is not labeled. This type of error is called zero scale error. Furthermore, each division in the pressure gauge represents 20 kPa, which is considered as large step size and hence it leads to human error in recording the values of pressure .

Summery & Conclusions

Overall, the experiment shows that nozzles are devices the increase the velocity of fluids at the expense of pressure. This justifies the reduction in pressure through the convergent part of the nozzle. Furthermore, it was shown that increasing the inlet pressure causes the velocity at a certain point to increase and hence cause the Mach number at that location to increase.

References

[1] Çengel, Y. A., & Boles, M. A. (2015). Thermodynamics: an engineering approach (8th ed.). New York: McGraw-Hill Education.

[2] Chapra, S. C., & Canale, R. P. (2010). Numerical Methods for Engineers (6th ed.). New York: McGraw-Hill Education.

[3] Holman J. P. (2012). Experimental Methods for Engineers (8th ed.). New York: McGraw-Hill Education.