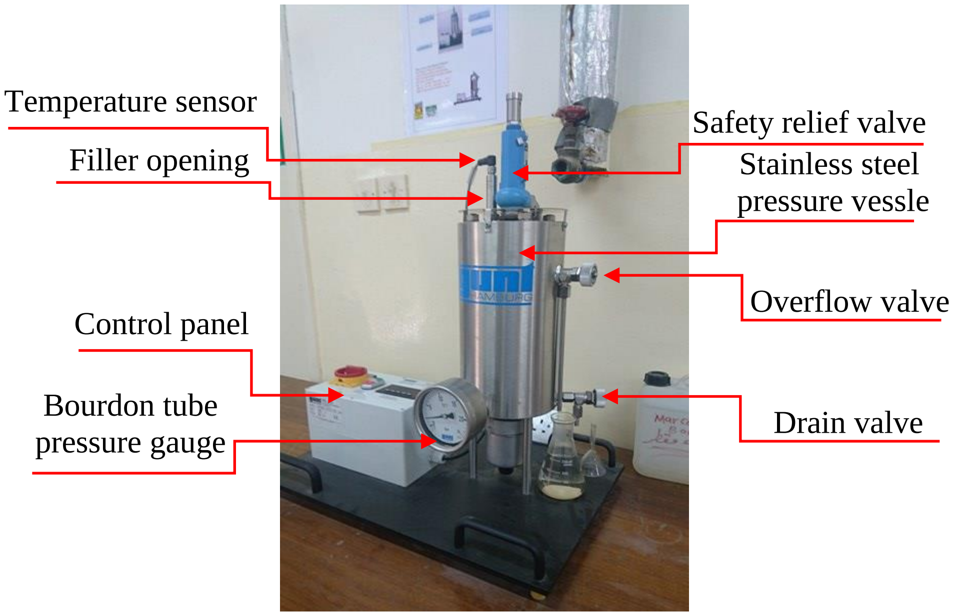
Objective

- To investigate the relationship between pressure and temperature of saturated steam in equilibrium with water .
- To verify Clapeyron’s equation .

Experimental Setup

The experimental setup is shown in figure(1) . The main element of marcet boiler is the stainless steel pressure vessel. An electric heater is installed inside the vessel to heat the water. A temperature sensor and a filler opening can be found at the top of the vessel .The white box to the left of the boiler represents the control panel , which contains the device switch botton(on-off knob) ,the heater switch botton and a screen displaying the temperature of water. Bourdon tube pressure gauge is used to measure the pressure of the steam. At the top of the vessel, there is a safety relief valve ,which allows the steam to escape if the pressure inside the vessel exceeds a certain value, to ensure that no explosions will occur. Two valves are located on the right side of the vessel . The overflow valve , the one at the top, ensures that the vessel is filled to the correct level and allows the air to escape from the vessel.The other valve is the drain valve, which is used to drain the vessel.



Figure(1):Marcet boiler

Start-up Procedure

1. Open the flow valve , and remove the filler plug.
2. Pour clean water into the boiler through the filler opening until it reaches the level of the flow valve(water spilled from the valve confirms that the correct level is reached)
3. In order to be sure that the boiler will contain steam and water only through the experiment, leave the overflow valve open until boiling occurs. Generated steam will push the air out of the boiler unit. This step is important as the accuracy of the experimental results will be significantly affected when air is present .
4. Insert the temperature sensor into the filler opening.

5. Plug the boiler into a walll socket.
6. At the control panel, rotate the knob to switch on thr boiler unit and press on the green botton to switch on the heater.
7. The experiment now is ready to carry on .

Experimental Procedure

1. Perform the Start-up Procedure.
2. Wait until the water is boiling (Steam coming out of the flow valve emphasizes that boiling point is reached) . Record the temperature at this point. This temperature represents T_{sat} at zero guage pressure .
3. Close the overflow valve, and start taking simultaneous readings of pressure and temperature in increments of 1 bar until the pressure reaches 10 bar .

Warning !
Never open the overflow valve once it is closed , as the pressurized steam can cause skin burns .

4. Press on the red botton at the control panel to switch off the heater.
5. As the pressure drops , take simultaneous readings of pressure and temperature in decrements of 1 bar until the pressure reaches 0 bar . (Note: In the experiment performed in the lab, readings are taken until the pressure reaches 6 bar , and extrapolation was done to estimate the rest of the values of temperature in the range of 5-0 bar , since it will take a very long time for the pressure to drop to 0 bar).

Given Data

- Atmospheric pressure P_{atm} = 89.85 kPa .
- Atmospheric temperature T_{atm} = 21.5 ^{o}C .

Data Observed

Table (1) :Data Observed

S. No.	Guage steam pressure (bar)	Steam temperature (^{o}C)	
		Increasing pressure	Decreasing pressure
1	0	97.0	144.6
2	1	115.0	147.1
3	2	128.5	150.1
4	3	139.1	153.1
5	4	148.3	156.6
6	5	155.5	160.1
7	6	161.7	164.1
8	7	167.8	169.7
9	8	172.4	174.8
10	9	177.1	179.7
11	10	181.2	181.2

Sample Calculations

Step(1) : Find the mean temperature at every value of pressure and convert it from ^{o}C to K .
Taking the first row from Table(1)

1	0	97.0	144.6

Mean temperature = $\frac{97.0+144.6}{2} = 120.8$ ^{o}C

T(K) = T(^{o}C) + 273 [1]
T(K) = 120.8 +273 = 393.8 K

Step(2) : Convert the values of steam gauge pressure from bar to kPa and from guage pressure to absolute pressure. Taking the first row from Table1 :

1 bar = 100 kPa [1]
P_{gauge} = 0 bar = 0 kPa
P_{abs} = P_{gauge} + P_{atm} [1]
P_{abs} = 0 + 89.85 = 89.85 kPa

Table (2.a) :Data Calculated

S. No.	Guage steam pressure		Absolute steam pressure (kPa)
	bar	kPa	
1	0	0	89.85
2	1	100	189.85
3	2	200	289.85
4	3	300	389.85
5	4	400	489.85
6	5	500	589.85
7	6	600	689.85
8	7	700	789.85
9	8	800	889.85
10	9	900	989.85
11	10	1000	1089.85

Steam temperature (^{o}C)		Mean Temperature	
Increasing pressure	Decreasing pressure	^{o}C	K
97.0	144.6	120.8	393.8
115.0	147.1	131.05	404.05
128.5	150.1	139.3	412.3
139.1	153.1	146.1	419.1
148.3	156.6	152.45	425.45
155.5	160.1	157.8	430.8
161.7	164.1	162.9	435.9
167.8	169.7	168.75	441.75
172.4	174.8	173.6	446.6
177.1	179.7	178.4	451.4
181.2	181.2	181.2	454.2

Step (3) : Calculate the slope $\left(\frac{dT}{dP}\right)_{sat}$ using experimental data . Central Difference Method (DCM) will be used to find the slope .

Step (3-a) : Let T_{i-1} = T_{sat} (at P_{abs}= 89.85 kPa) = 393.8 K
T_i = T_{sat} (at P_{abs} = 189.85 kPa) = 404.05 K
T_{i+1} = T_{sat} (at P_{abs} = 289.85 kPa) = 412.3 K

Step (3-b) :

$$\left(\frac{dT}{dP}\right)_{sat} = \frac{T_{i+1} - T_{i-1}}{P_{i+1} - P_{i-1}} \quad [2]$$

$$\left(\frac{dT}{dP}\right)_{sat} = \frac{412.3 - 393.8}{289.85 - 89.85} = 0.0925\ \mathrm{K/kPa}$$

Step (4) : Calculate the slope $\left(\frac{dT}{dP}\right)_{sat}$ using steam tables and Clausius-Clapeyron equation .

Clausius-Clapeyron equation : $\left(\frac{\mathrm{dP}}{\mathrm{dT}}\right)_{\mathrm{sat}} = \frac{h_{fg}}{T\times v_{fg}}$ [1] => $\left(\frac{dT}{dP}\right)_{sat} = \frac{T\times v_{fg}}{h_{fg}} \approx \frac{T\times v_{g}}{h_{fg}}$

Step (4-a) : Find h_{fg} at $T_i = T_{sat}$ (at P_{abs} = 189.85 kPa) = 404.05 K =131.05 °C by simple interpolation (from Saturated water-Temperature table) .

Temp. T °C	Specific Enthalpy h_{fg} kJ/kg
130	2173.7
131.05	2170.6
135	2159.1

$$\frac{2159.1-2173.7}{135-130} = \frac{h_{fg}-2173.7}{131.05-130}$$

$$h_{fg} = 2170.6\ kJ/kg$$

Table (3.a) : Temperature – Specific enthalpy data for water [1]

Step (4-b) : Find v_g at $T_i = T_{sat}$ (at P_{abs} = 189.85 kPa) = 404.05 K =131.05 °C by simple interpolation (from Saturated water-Temperature table).

Temp. T °C	Specific volume v_g m^3/kg
130	0.66808
131.05	0.64996
135	0.58179

$$\frac{0.58179-0.66808}{135-130} = \frac{v_g-0.66808}{131.05-130}$$

$$v_g = 0.64996\ m^3/kg$$

Table (3.b) : Temperature – Specific volume data for water [1]

Step (4-c) : Find $\left(\frac{\mathrm{dT}}{\mathrm{dP}}\right)_{\mathrm{sat}}$ at $T_i = T_{sat}$ (at P_{abs} = 189.85 kPa) = 404.05 K =131.05 °C using Clapeyron's equation

$$\left(\frac{\mathrm{dT}}{\mathrm{dP}}\right)_{\mathrm{sat}} = \frac{T\times v_g}{h_{fg}} = \frac{404.05\times 0.64996}{2170.6} = 0.1209\ K/kPa$$

Step(5) : Calculate the error between the two methods .

$$Eror\ \% = \left|\frac{Theoretical\ value\ of\ slope - Experimental\ value\ of\ slope}{Theoretical\ value\ of\ slope}\right| \times 100\% \quad [2]$$

$$= \left|\frac{0.1209-0.0925}{0.1209}\right| \times 100\% = 23.4\%$$

Uncertainty Analysis

- For a calculated quantity x that is dependent on another quantities $x_1, x_2, x_3, \dots., x_n$

$$x = f(x_1, x_2, x_3, \dots., x_n)$$

The uncertainty of x (w_x) is given by :

$$w_x = \pm\sqrt{\left(\frac{\partial x}{\partial x_1} \times w_{x1}\right)^2 + \left(\frac{\partial x}{\partial x_2} \times w_{x2}\right)^2 + \left(\frac{\partial x}{\partial x_3} \times w_{x3}\right)^2 + \dots + \left(\frac{\partial x}{\partial x_n} \times w_{xn}\right)^2} \quad [3]$$

- In this experiment, the calculated quantity $\left(\frac{dT}{dP}\right)_{sat}$ is dependent on T_{i-1} , T_{i+1} , P_{i+1} ,P_{i-1} i.e

$$\left(\frac{dT}{dP}\right)_{sat} = f\,(\,T_{i+1}, T_{i-1}, P_{i-1}, P_{i+1}\,)$$

The uncertainty of $\left(\frac{dT}{dP}\right)_{sat}$ is given by :

$$w_{\left(\frac{dT}{dP}\right)_{sat}} =$$

$$\pm\sqrt{\left(\frac{\partial\left(\frac{dT}{dP}\right)_{sat}}{\partial T_{i+1}} \times w_{T_{i+1}}\right)^2 + \left(\frac{\partial\left(\frac{dT}{dP}\right)_{sat}}{\partial T_{i-1}} \times w_{T_{i-1}}\right)^2 + \left(\frac{\partial\left(\frac{dT}{dP}\right)_{sat}}{\partial P_{i-1}} \times w_{P_{i-1}}\right)^2 + \left(\frac{\partial\left(\frac{dT}{dP}\right)_{sat}}{\partial P_{i+1}} \times w_{P_{i+1}}\right)^2}$$

- The uncertainty of an observed quantity measured using a device , is the value of one-half the smallest division of the device. [3] The uncertainties of T_{i-1} , T_{i+1} , P_{i+1} ,P_{i-1} are as follows :

$$w_{T_{i+1}} = w_{T_{i-1}} = \pm 0.05\ ^{o}\mathrm{C} = \pm\ 0.05\ \mathrm{K}$$
$$w_{P_{i+1}} = w_{T_{P-1}} = \pm\ 0.25\ \mathrm{bar} = \pm\ 25\ \mathrm{kPa}$$

- The following quantities are found by differentiating $\left(\frac{\mathrm{dT}}{\mathrm{dP}}\right)_{\mathrm{sat}} = \frac{\mathrm{T_{i+1} - T_{i-1}}}{\mathrm{P_{i+1} - P_{i-1}}}$ partially :

$$\frac{\partial\left(\frac{dT}{dP}\right)_{sat}}{\partial T_{i+1}} = \frac{1}{\mathrm{P_{i+1} - P_{i-1}}} \qquad \frac{\partial\left(\frac{dT}{dP}\right)_{sat}}{\partial T_{i-1}} = \frac{-1}{\mathrm{P_{i+1} - P_{i-1}}}$$

$$\frac{\partial\left(\frac{dT}{dP}\right)_{sat}}{\partial\, P_{i+1}} = \frac{-(\mathrm{T_{i+1} - T_{i-1}})}{(\mathrm{P_{i+1} - P_{i-1}})^2} \qquad \frac{\partial\left(\frac{dT}{dP}\right)_{sat}}{\partial\, P_{i-1}} = \frac{(\mathrm{T_{i+1} - T_{i-1}})}{(\mathrm{P_{i+1} - P_{i-1}})^2}$$

- Take the third row from Table(2.b) as a sample for calculations

189.85	404.05	0.0925	0.1209	23.4%	± 0.0163

$$w_{\left(\frac{dT}{dP}\right)_{sat}} =$$

$$\pm\sqrt{\left(\frac{1}{289.85 - 89.85} \times 0.05\right)^2 + \left(\frac{-1}{289.85 - 89.85} \times 0.05\right)^2 + \left(\frac{(412.3 - 393.8)}{(289.85 - 89.85)^2} \times 25\right)^2 + \left(\frac{-(412.3 - 393.8)}{(289.85 - 89.85)^2} \times 25\right)^2}$$

$$w_{\left(\frac{dT}{dP}\right)_{sat}} = \pm 0.0163$$

Table (2.b) :Data Calculated

Absolute steam pressure (kPa)	Mean Temperature (K)	Slope dT/dP using CDM (K/kPa)	Slope dT/dP using Clapeyron's equation (K/kPa)	Error %	Uncertainty $w_{\left(\frac{dT}{dP}\right)_{sat}}$
89.85	393.8	-	-	-	
189.85	404.05	0.0925	0.1209	23.4%	± 0.0163
289.85	412.3	0.0753	0.0996	24.3%	±0.0133
389.85	419.1	0.0657	0.0856	23.2%	±0.0116
489.85	425.45	0.0585	0.0747	21.6%	±0.0103
589.85	430.8	0.0523	0.0668	21.7%	±0.0093
689.85	435.9	0.0548	0.0603	9.12%	±0.0097
789.85	441.75	0.0535	0.0538	0.558%	±0.0095
889.85	446.6	0.0483	0.0491	1.63%	±0.0085
989.85	451.4	0.0380	0.0449	15.3%	±0.0067
1089.85	454.2	-	-	-	

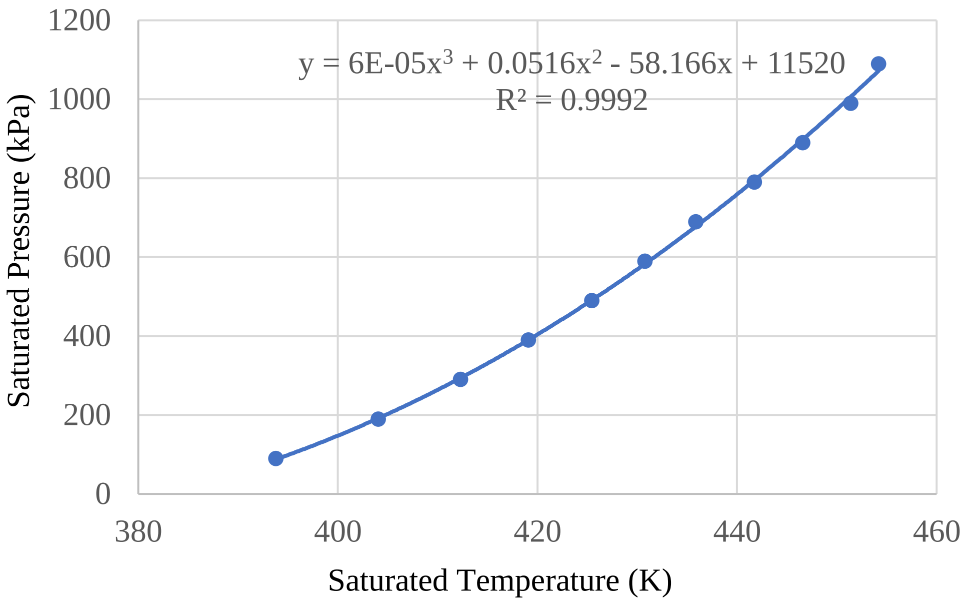
Results & Discussion

Water molecules are always moving. When heat is added , they move faster, and if enough heat is added , they will spread out and push the molecules around them away forming bubbles of gas . These bubbles rise to the surface and that is called boiling.

Boiling is strongly influenced by the atmospheric pressure. Scientifically, Boiling point is defined as the temperature at which the vapor pressure of the liquid equals the atmospheric pressure. Atmospheric pressure can be thought as the collisions of air molecules against the surface of water. The more collisions , the greater the pressure. On the other hand , the pressure created by the water molecules escaping the liquid is called vapor pressure. As these fast moving water molecules escape, they collide and push against the air molecules. This reduces the amount of pressure felt by the water.

If the atmospheric pressure increases , it will be harder for water molecules to spread out and form bubbles unless they are supplied with enough energy.As more heat is added , vapor pressure increases as molecules from the water escape and collide against the air molecules and effectively reduce some of the pressure on the surface of the water. This makes it easier for the liquid molecules to spread out and form bubbles and for boiling to take place . It can be concluded that as the pressure increases, saturated temperature (boiling point) increases .

Figure (2) shows the graph obtained from plotting the experimental values of pressure and temperature of steam in equilibrium with water . It's obvious that increasing pressure is always accompanied with an increase in saturated temperature. Furthermore , the relationship between pressure and temperature is non-linear as shown in figure(2), which is consistent with the transition cuve between liquid and water in the T-P diagram for water.



Figure(2) : Relationship between pressure and temperature of steam in equilibrium with water

The slope of the tangent line at any point on the curve is given by the Clausius-Clapeyron's equation: $\left(\frac{\mathrm{dP}}{\mathrm{dT}}\right)_{\mathrm{sat}} = \frac{h_{fg}}{T\times v_{fg}}$. It represents the rate of increase in vapor pressure per unit increase in temperature.It is apparent from the right side of the equation that the value of $\left(\frac{\mathrm{dP}}{\mathrm{dT}}\right)_{\mathrm{sat}}$ is always positive, which means that any increase in pressure will lead into increase in the saturated temperature and vice versa . In this experiment , the value of $\left(\frac{\mathrm{dT}}{\mathrm{dP}}\right)_{\mathrm{sat}}$ was calculated using experimental data and compared with that value obtained using Clapeyron's equation . An average error of 15.6% results from the human error in recording the true value of temperature due to the fast change in temperature reading comapred to the change in pressure reading.

Sources of Error

Errors in this experiment are caused by several factors such as : extrapolation was done to estimate the values of temperature in the range of 5-0 bar , human error in recording the values of temperature and computational errors due to approximation.

Summery & Conclusions

Overall, The experiment shows that the increase in the pressure causes water to boil at higher temperature. The positive value of the slope confirms this relation between pressure and saturated temperature. When compared to the theoretical slope, the experimental slope shows a small deviation between them because of certain errors.

References

[1] Çengel, Y. A., & Boles, M. A. (2015). Thermodynamics: an engineering approach (8th ed.). New York: McGraw-Hill Education.
[2] Chapra, S. C., & Canale, R. P. (2010). Numerical Methods for Engineers (6th ed.). New York: McGraw-Hill Education.
[3] Holman J. P. (2012). Experimental Methods for Engineers (8th ed.). New York: McGraw-Hill Education.