Objective

- To determine the specific heat ratio (k) for air through certain processes.

Experimental Setup

The experimental setup is shown in figure (1). The main element of this apparatus is the air vessel. A mechanical air pump is connected to the vessel to charge it with air. A manometer is used to measure the pressure of the air inside the vessel. At the left side of the vessel, there is a valve, which allows for the trapped air to escape from the vessel.

Figure(1):Air specific heat ratio Apparatus

Start-up Procedure

1. Check that the mechanical pump is properly connected to the vessel, to be sure that no air leakage will occur while charging the vessel with the air.
2. Check that the manometer is properly connected to the vessel, to be sure that the pressure readings taken through the experiment are correct.
3. Open the valve to allow for any trapped air to escape from the vessel, and then close it.
4. The experiment now is ready to carry on.

Experimental Procedure

1. Perform the Start-up procedure.
2. Charge the vessel with any amount of air using the mechanical air pump. (The more amount of air pumped into the vessel, the greater the pressure of air will be, the better the adiabatic expansion will occur).

3. Record the pressure (P_1) of the air inside the vessel when thermal equilibrium between the air and the surrounding is reached ($T_1= T_{amb}$).
4. Open the valve and close it promptly to allow for the air to expand.Since this process is rapid, no heat is exchanged with the surroundings. As a result, this process can be assumed to be adiabatic. Record the air pressure P_2 (In the experiment performed in the lab, the air pressure P_2 is assumed to be equal to P_{atm}). The air temperature T_2 is less than the room temperature, since work is done by the trapped air against the surrounding during the adiabatic expansion process.
5. Wait for a few minutes, until thermal equilibrium between the trapped air and the surrounding is attained ($T_3= T_{amb}$), then record the air pressure P_3 .

Given Data

- Atmospheric pressure P_{atm} = 90 kPa .
- Atmospheric temperature T_{atm} = 21.5 ^{o}C .
- Density of manometer fluid ρ_{Hg} = 13600 kg/m^3

Data Observed

The data observed from the experiment is summarized below in Table (1)

Table (1) :Data Observed

S. No.	Initial Manometer Reading (H_1) (cm Hg)	Manometer reading at the end of Constant Volume Expansion (H_3) (cm Hg)
1	6	2
2	8	3.8
3	10	4.6
4	18.4	6.8
5	5.8	1

Sample Calculations

Step(1): Convert the manometer readings (H_1,H_3) from cm Hg to kPa and from gauge pressure to absolute pressure.

Take the first row from Table(1) as a sample for calculation :

1	6	2

$P_1 = \rho_{Hg}\times g\times H_1 + P_{atm}$ [1]

$= 13600\times9.81\times0.06\times10^{-3} + 90 = 98.005$ kPa

$P_3 = \rho_{Hg}\times g\times H_3 + P_{atm}$

$= 13600\times9.81\times0.02\times10^{-3} + 90 = 92.668$ kPa

Step(2) : Find the values of $\ln\left(\frac{P_1}{P_2}\right)$ and $\ln\left(\frac{P_1}{P_3}\right)$

$$\ln\left(\frac{P_1}{P_2}\right) = \ln\left(\frac{98.005}{90}\right) = 0.0852$$

$$\ln\left(\frac{P_1}{P_3}\right) = \ln\left(\frac{98.005}{92.668}\right) = 0.0559$$

Step(3): Find the specific heat ratio k

$$k = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{P_1}{P_3}\right)}$$ [1]

$$k = \frac{0.0852}{0.0559} = 1.52415$$

Step(5) : Calculate the error between the experimental value and the theoretical value .

The theoretical value of the specific heat ratio for air $k_{air} = 1.4$ [1]

$$\text{Eror \%} = \left|\frac{\text{Theoretical value} - \text{Experimental value}}{\text{Theoretical value}}\right| \times 100\%$$ [2]

$$= \left|\frac{1.4-1.52415}{1.4}\right| \times 100\% = 8.86\%$$

Uncertainty Analysis

- For a calculated quantity x that is dependent on another quantities $x_1, x_2, x_3, \dots, x_n$
$$x = f(x_1, x_2, x_3, \dots, x_n)$$
The uncertainty of x (w_x) is given by :

$$w_x = \pm\sqrt{\left(\frac{\partial x}{\partial x_1} \times w_{x1}\right)^2 + \left(\frac{\partial x}{\partial x_2} \times w_{x2}\right)^2 + \left(\frac{\partial x}{\partial x_3} \times w_{x3}\right)^2 + \cdots + \left(\frac{\partial x}{\partial x_n} \times w_{xn}\right)^2} \quad [3]$$

- In this experiment, the calculated quantity k is dependent on P_1 , P_2, P_3
i.e
$$k = f(P_1, P_2, P_3)$$
The uncertainty of k is given by :

$$w_k = \pm\sqrt{\left(\frac{\partial k}{\partial P_1} \times w_{P1}\right)^2 + \left(\frac{\partial k}{\partial P_2} \times w_{P2}\right)^2 + \left(\frac{\partial k}{\partial P_3} \times w_{P3}\right)^2}$$

- The uncertainty of an observed quantity measured using a device , is the value of one-half the smallest division of the device. [3] The uncertainties of P_1 , P_2, $P_3$are as follows :
$w_{P_1} = w_{P_3} = \pm 0.5$ mm Hg = $\pm$ 0.067 kPa
w_{P_2} =$\pm$ 0.25 mbar = $\pm$ 0.025 kPa

- The following quantities are found by differentiating $k = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{P_1}{P_3}\right)}$ partially :

$$\frac{\partial k}{\partial P_1} = \frac{\ln\left(\frac{P_1}{P_3}\right) \times \frac{1/P_2}{P_1/P_2} - \ln\left(\frac{P_1}{P_2}\right) \times \frac{1/P_3}{P_1/P_3}}{\left(\ln\left(\frac{P_1}{P_3}\right)\right)^2} \qquad \frac{\partial k}{\partial P_2} = \frac{-1}{P_2 \times \ln\left(\frac{P_1}{P_3}\right)}$$

$$\frac{\partial k}{\partial P_3} = \frac{\ln\left(\frac{P_1}{P_2}\right)}{P_3 \times \left(\ln\left(\frac{P_1}{P_3}\right)\right)^2}$$

- Take the first row from Table(2) as a sample for calculations

1	98.005	90	92.668	0.0852	0.0559	1.52415	8.86%	$\pm$0.02131

w_k

$$= \pm\sqrt{\left(\frac{(0.0559 - 0.0852) \times 1/98.005}{0.0559^2} \times 0.067\right)^2 + \left(\frac{-1}{90 \times 0.0559} \times 0.025\right)^2 + \left(\frac{0.0852}{92.668 \times 0.0559^2} \times 0.067\right)^2}$$

= $\pm$0.02131

Table (2) :Data Calculated

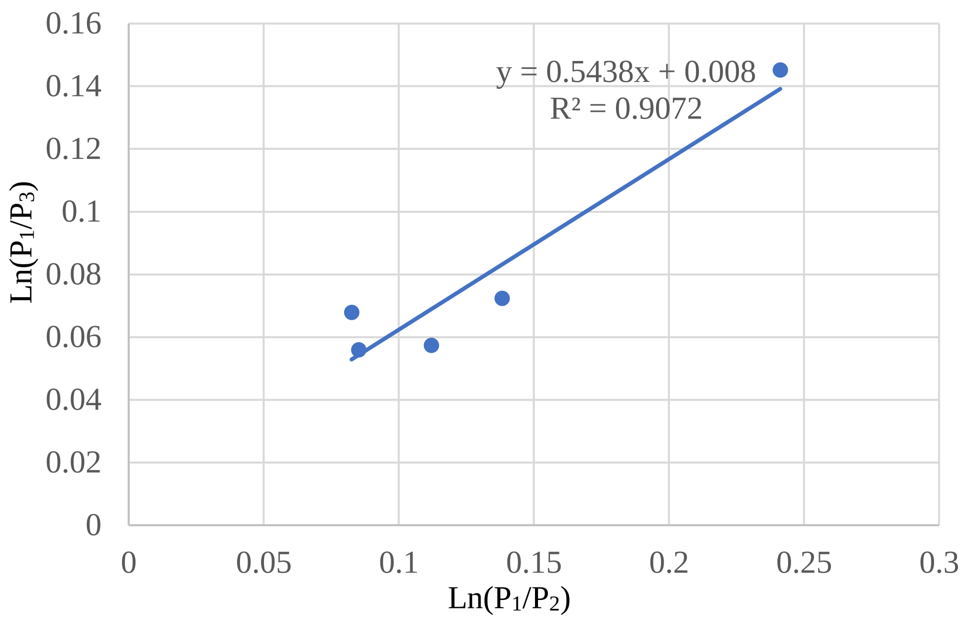
S. No.	Initial Pressure P_1 (kPa,abs)	Pressure at the end of the Adiabatic Expansion P_2 (kPa,abs)	Final pressure P_3 (kPa,abs)	$\ln\left(\frac{P_1}{P_2}\right)$	$\ln\left(\frac{P_1}{P_3}\right)$	$k = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{P_1}{P_3}\right)}$	Error (%)	Uncertainty w_k
1	98.005	90	92.668	0.0852	0.0559	1.52415	8.86%	± 0.02132
2	100.673	90	95.069	0.1121	0.0573	1.95636	39.74%	± 0.02694
3	103.342	90	96.137	0.1382	0.0723	1.91147	36.53%	± 0.02052
4	114.549	90	99.072	0.2412	0.1451	1.662301	18.73%	± 0.00842
5	97.738	90	91.334	0.0825	0.0678	1.21681	13.085%	± 0.01296
					Average	1.65422		

Results & Discussion

In Thermodynamics, Specific heat ratio (k) is the ratio of the specific heat at constant pressure (C_p) to the specific heat at constant volume (C_v). Physically, the specific heat at constant pressure can be viewed as the energy required to raise the temperature of a unit mass of a substance by one degree as the pressure is maintained constant. The energy required to do the same as the volume is maintained constant is the specific heat at constant volume. The specific heat at constant pressure C_p is always greater than C_v because at constant pressure, the system is allowed to expand and the energy for this expansion work must be supplied to the system. As a result, the specific heat ratio is always greater than 1. The values of specific heat ratio for air obtained in this experiment are consistent with the previous fact .

The exact value of specific heat ratio for air is k=1.4 . Most of the results are greater than the theoretical value. One of the reasons that causes this increase in the experimental values of k, is that P_2 is assumed to be equal to the atmospheric pressure P_2. This assumption is not accurate. If the valve was opened at state 2 through the experiment , some air will escape from the vessel, which means that the actual pressure inside the vessel P_2 is greater than P_{atm} . Thus , as P_2 is assumed to be less than its actual value , $\ln\left(\frac{P_1}{P_2}\right)$ will increase and as a result k would increase.

Figure (2) shows the graph obtained from plotting the experimental values of $\ln\left(\frac{P_1}{P_2}\right)$and $\ln\left(\frac{P_1}{P_3}\right)$. It is obvious that the relation between these two quantities is linear, as they are related to each other by the constant (k).



Figure(2) : Relationship between the experimental values of $Ln(P_1/P_2)$ and $Ln(P_1/P_3)$

Sources of Error

Errors in this experiment are caused by several factors such as : pressure at the end of the adiabatic expansion is assumed to be equal to the atmospheric pressure, human error in recording the values of pressure and computational errors due to approximation.

Summery & Conclusions

Overall, The experiment shows that the specific heat ratio for air can be obtained by allowing the air to undergo adiabatic expansion process and constant volume process. When compared to the theoretical value, the experimental value of specific heat ratio for air shows a small deviation between them because of certain errors.

References

[1] Çengel, Y. A., & Boles, M. A. (2015). Thermodynamics: an engineering approach (8th ed.). New York: McGraw-Hill Education.

[2] Chapra, S. C., & Canale, R. P. (2010). Numerical Methods for Engineers (6th ed.). New York: McGraw-Hill Education.

[3] Holman J. P. (2012). Experimental Methods for Engineers (8th ed.). New York: McGraw-Hill Education.