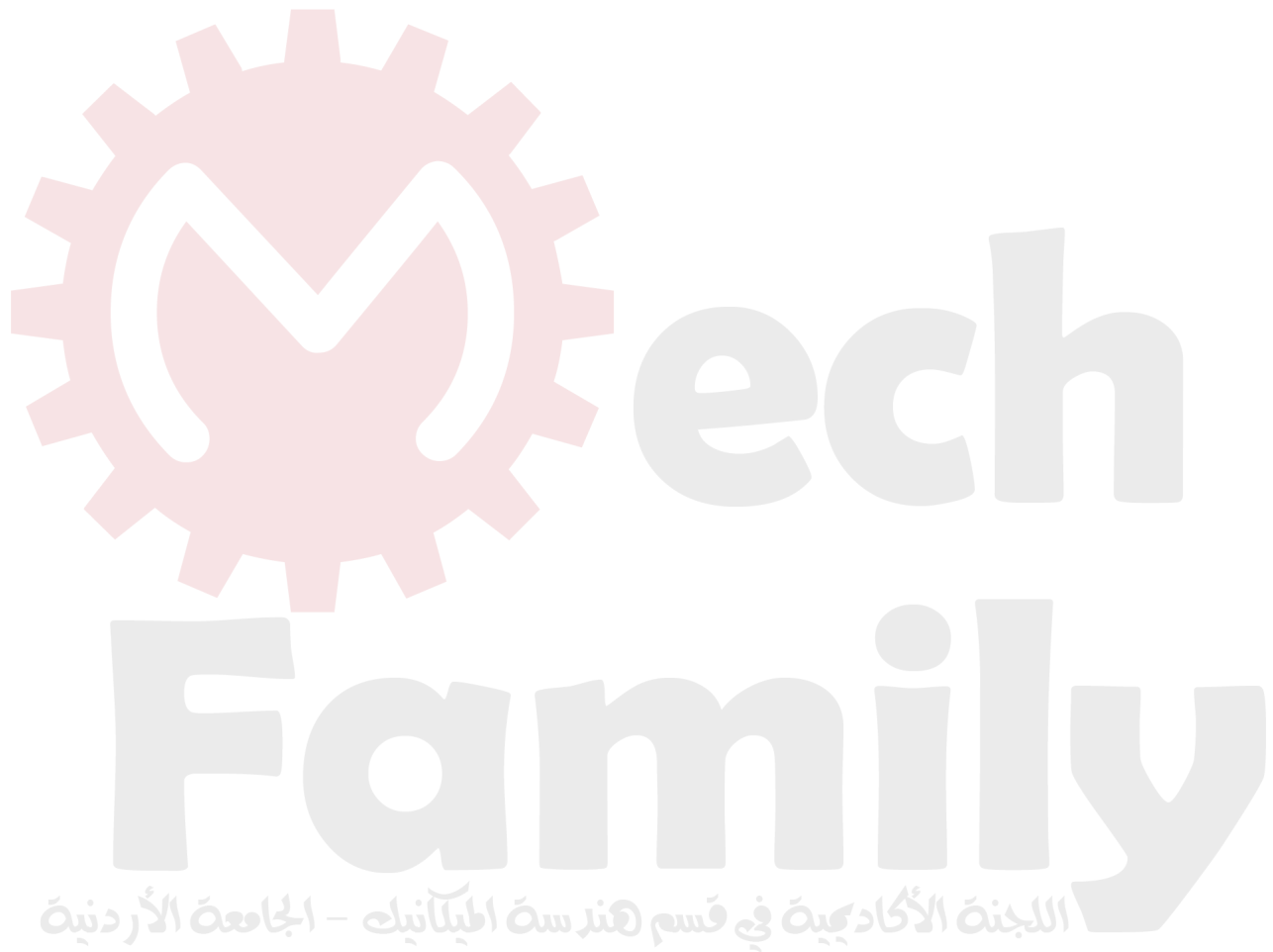


Chapter 1 Instructor Notes

Chapter 1 is introductory in nature, establishing some rationale for studying electrical engineering methods, even though the students' primary interest may lie in other areas. The material in this chapter should be included in every syllabus, and can typically be thoroughly covered in a single-day introductory lecture. Oftentimes, reading of this material is left up to the discretion of the student.



Chapter 1 problem solutions

1.1 A few examples are:

Bathroom

ventilation fan
electric toothbrush
hair dryer
electric shaver
electric heater fan

Kitchen

microwave fan
microwave turntable
mixer
food processor
blender
coffee grinder
garbage disposal
ceiling fan
electric clock
exhaust fan
refrigerator compressor
dish washer

Utility Room

clothes washer
dryer
air conditioner
furnace blower
pump

Family Room

VCR drive
cassette tape drive
reel-to-reel tape drive
record turntable drive
computer fan

treadmill

Miscellaneous

lawn tools
power tools

1.2

Several examples are listed below for each system:

a) A ship

Circuit Analysis

design of the ship's
electrical system

Electromagnetics

radar

Solid-State Electronics

radio
sonar

Electric Machines

pump
elevator

Electric Power Systems

lighting
generators

Digital Logic Circuits

elevator control

Computer Systems

navigation

Communication Systems

radio
telephone

Electro-Optics

Morse light
bridge displays

Instrumentation	Control Systems
compass	rudder
speed indicator	flaps
Control Systems	
rudder	
HVAC	
b) A Commercial Passenger Aircraft	c) Household
Circuit Analysis	Circuit Analysis
Design of the plane's electrical system	design of the home's electrical system
Electromagnetics	Electromagnetics
radar	microwave oven
microwave oven	stereo speakers
Solid-State Electronics	Solid-State Electronics
radio	television
Electric Machines	stereo
turbines	VCR
fans	Electric Machines
Electric Power Systems	appliances
lighting	power tools
HVAC	fans
Digital Logic Circuits	Electric Power Systems
seat belts	lighting
Computer Systems	HVAC
navigation	receptacles
Communication Systems	Digital Logic Circuits
radio	clocks
telephone	timers
Electro-Optics	Computer Systems
cockpit displays	microwave oven
Instrumentation	programmable
compass	VCR
air speed indicator	Communication Systems
inclinometer	telephone
altimeter	CB radio
	television
	radio
	Electro-Optics

digital clocks

Instrumentation

electric meter

Control Systems

thermostat

1.3

Some examples are:

a) HVAC

lighting

office equipment

typewriter

computer

copy machine

clock

stapler

shredder

elevator

b) conveyor

punch press

lighting

ventilation

drill press

hoist

lathe

c) power saw

drill

lighting

elevator

pump

compressor

Chapter 2 Instructor Notes

Chapter 2 develops the foundations for the first part of the book. Coverage of the entire Chapter would be typical in an introductory course. The first four sections provide the basic definitions and cover Kirchhoff's Laws and the passive sign convention; the box *Focus on Methodology: The Passive Sign Convention* (p. 35) and two examples illustrate the latter topic. The sidebars *Make The Connection: Mechanical Analog of Voltage Sources* (p. 20) and *Make The Connection: Hydraulic Analog of Current Sources* (p. 22) present the concept of analogies between electrical and other physical domains; these analogies will continue through the first six chapters.

Sections 2.5 and 2.6 introduce the i - v characteristic and the resistance element. Tables 2.1 and 2.2 on p. 41 summarize the resistivity of common materials and standard resistor values; Table 2.3 on p. 44 provides the resistance of copper wire for various gauges. The sidebar *Make The Connection: Electric Circuit Analog of Hydraulic Systems – Fluid Resistance* (p. 40) continues the electric-hydraulic system analogy.

Finally, Sections 2.7 and 2.8 introduce some basic but important concepts related to ideal and non-ideal current sources, and measuring instruments.

The Instructor will find that although the material in Chapter 2 is quite basic, it is possible to give an applied flavor to the subject matter by emphasizing a few selected topics in the examples presented in class. In particular, a lecture could be devoted to *resistance devices*, including the resistive displacement transducer of *Focus on Measurements: Resistive throttle position sensor* (pp. 52-54), the resistance strain gauges of *Focus on Measurements: Resistance strain gauges* (pp. 54-55), and *Focus on Measurements: The Wheatstone bridge and force measurements* (pp. 55-56). The instructor wishing to gain a more in-depth understanding of resistance strain gauges will find a detailed analysis in¹.

Early motivation for the application of circuit analysis to problems of practical interest to the non-electrical engineer can be found in the *Focus on Measurements: The Wheatstone bridge and force measurements*. The Wheatstone bridge material can also serve as an introduction to a laboratory experiment on strain gauges and the measurement of force (see, for example²). Finally, the material on practical measuring instruments in Section 2.8b can also motivate a number of useful examples.

The homework problems include a variety of practical examples, with emphasis on instrumentation. Problem 2.36 illustrates analysis related to fuses; problems 2.44-47 are related to wire gauges; problem 2.52 discusses the thermistor; problems 2.54 and 2.55 discuss moving coil meters; problems 2.52 and 2.53 illustrate calculations related to temperature sensors; and problems 2.56-66 present a variety of problems related to practical measuring devices.

It has been the author's experience that providing the students with an early introduction to practical applications of electrical engineering to their own disciplines can increase the interest level in the course significantly.

Learning Objectives

1. Identify the principal *elements of electrical circuits*: nodes, loops, meshes, branches, and voltage and current sources.
2. Apply *Kirchhoff's Laws* to simple electrical circuits and derive the basic circuit equations.
3. Apply the *passive sign convention* and *compute power* dissipated by circuit elements.
4. Apply the *voltage and current divider laws* to calculate unknown variables in simple series, parallel and series-parallel circuits.
5. Understand the rules for connecting *electrical measuring instruments* to electrical circuits for the measurement of voltage, current, and power.

¹ E. O. Doebelin, *Measurement Systems – Application and Design*, 4th Edition, McGraw-Hill, New York, 1990.

² G. Rizzoni, *A Practical Introduction to Electronic Instrumentation*, 3rd Edition, Kendall-Hunt, 1998.

Section 2.1: Definitions**Problem 2.1****Solution:****Known quantities:**

Initial Coulombic potential energy, $V_i = 17 \text{ kJ} / \text{C}$; initial velocity, $U_i = 93 \text{ M} \frac{\text{m}}{\text{s}}$; final Coulombic potential energy, $V_f = 6 \text{ kJ} / \text{C}$.

Find:

The change in velocity of the electron.

Assumptions:

$$\Delta PE_g \ll \Delta PE_c$$

Analysis:

Using the first law of thermodynamics, we obtain the final velocity of the electron:

$$Q_{\text{heat}} - W = \Delta KE + \Delta PE_c + \Delta PE_g + \dots$$

Heat is not applicable to a single particle. $W=0$ since no external forces are applied.

$$\Delta KE = -\Delta PE_c$$

$$\frac{1}{2} m_e (U_f^2 - U_i^2) = -Q_e (V_f - V_i)$$

$$U_f^2 = U_i^2 - \frac{2Q_e}{m_e} (V_f - V_i)$$

$$= \left(93 \text{ M} \frac{\text{m}}{\text{s}} \right)^2 - \frac{2(-1.6 \times 10^{-19} \text{ C})}{9.11 \times 10^{-31} \text{ kg}} (6 \text{ kV} - 17 \text{ kV})$$

$$= 8.649 \times 10^{15} \frac{\text{m}^2}{\text{s}^2} - 3.864 \times 10^{15} \frac{\text{m}^2}{\text{s}^2}$$

$$U_f = 6.917 \times 10^7 \frac{\text{m}}{\text{s}}$$

$$|U_f - U_i| = 93 \text{ M} \frac{\text{m}}{\text{s}} - 69.17 \text{ M} \frac{\text{m}}{\text{s}} = 23.83 \text{ M} \frac{\text{m}}{\text{s}}.$$

Problem 2.2**Solution:****Known quantities:**

MKSQ units.

Find:

Equivalent units of volt, ampere and ohm.

Analysis:

$$\text{Voltage} = \text{Volt} = \frac{\text{Joule}}{\text{Coulomb}} \quad V = \frac{J}{C}$$

$$\text{Current} = \text{Ampere} = \frac{\text{Coulomb}}{\text{second}} \quad a = \frac{C}{s}$$

$$\text{Resistance} = \text{Ohm} = \frac{\text{Volt}}{\text{Ampere}} = \frac{\text{Joule} \times \text{second}}{\text{Coulomb}^2} \quad \Omega = \frac{J \cdot s}{C^2}$$

$$\text{Conductance} = \text{Siemen or Mho} = \frac{\text{Ampere}}{\text{Volt}} = \frac{C^2}{J \cdot s}$$

Problem 2.3**Solution:****Known quantities:**

Battery nominal rate of 100 A-h.

Find:

- Charge potentially derived from the battery
- Electrons contained in that charge.

Assumptions:

Battery fully charged.

Analysis:

a)

$$100 \text{ A} \times 1 \text{ hr} = \left(100 \frac{\text{C}}{\text{s}}\right) (1 \text{ hr}) \left(3600 \frac{\text{s}}{\text{hr}}\right) \\ = 360000 \text{ C}$$

b)

$$\text{charge on electron: } -1.602 \times 10^{-19} \text{ C}$$

no. of electrons =

$$\frac{360 \times 10^3 \text{ C}}{1.602 \times 10^{-19} \text{ C}} = 224.7 \times 10^{22}$$

Problem 2.4**Solution:****Known quantities:**

Two-rate charge cycle shown in Figure P2.4.

Find:

- a) The charge transferred to the battery
- b) The energy transferred to the battery.

Analysis:

a) To find the charge delivered to the battery during the charge cycle, we examine the charge-current relationship:

$$i = \frac{dq}{dt} \quad \text{or} \quad dq = i \cdot dt$$

thus:

$$\begin{aligned} Q &= \int_{t_0}^{t_1} i(t) dt \\ Q &= \int_0^{5 \text{ hrs}} 50 \text{ mA} dt + \int_{5 \text{ hrs}}^{10 \text{ hrs}} 20 \text{ mA} dt \\ &= \int_0^{18000 \text{ s}} 0.05 \text{ A} dt + \int_{18000}^{36000 \text{ s}} 0.02 \text{ A} dt \\ &= 900 + 360 \\ &= 1260 \text{ C} \end{aligned}$$

b) To find the energy transferred to the battery, we examine the energy relationship

$$p = \frac{dw}{dt} \quad \text{or} \quad dw = p(t) dt$$

$$w = \int_0^{t_1} p(t) dt = \int_0^{t_1} v(t) i(t) dt$$

observing that the energy delivered to the battery is the integral of the power over the charge cycle. Thus,

$$\begin{aligned} w &= \int_0^{18000} 0.05 \left(1 + \frac{0.75t}{18000} \right) dt + \int_{18000}^{36000} 0.02 \left(1 + \frac{0.25t}{18000} \right) dt \\ &= \left(0.05t + \frac{0.75}{36000} t^2 \right) \Big|_0^{18000} + \left(0.02t + \frac{0.25}{36000} t^2 \right) \Big|_{18000}^{36000} \end{aligned}$$

$$\boxed{w = 1732.5 \text{ J}}$$

Problem 2.5**Solution:****Known quantities:**

Rated voltage of the battery; rated capacity of the battery.

Find:

- a) The rated chemical energy stored in the battery
- b) The total charge that can be supplied at the rated voltage.

Analysis:

a)

$$\Delta V \equiv \frac{\Delta PE_c}{\Delta Q} \quad I = \frac{\Delta Q}{\Delta t}$$

$$\text{Chemical energy} = \Delta PE_c$$

$$= \Delta V \cdot \Delta Q$$

$$= \Delta V \cdot (I \cdot \Delta t)$$

$$= 12 \text{ V} \cdot 350 \text{ A} \cdot 3600 \frac{\text{s}}{\text{hr}}$$

$$= 15.12 \text{ MJ.}$$

As the battery discharges, the voltage will decrease below the rated voltage. The remaining chemical energy stored in the battery is less useful or not useful.

b) ΔQ is the total charge passing through the battery and gaining 12 J/C of electrical energy.

$$\Delta Q = I \cdot \Delta t = 350 \text{ A} \cdot 3600 \frac{\text{s}}{\text{hr}} = 1.26 \text{ MC.}$$

Problem 2.6**Solution:****Known quantities:**

Resistance of external circuit.

Find:

- a) Current supplied by an ideal voltage source
- b) Voltage supplied by an ideal current source.

Assumptions:

Ideal voltage and current sources.

Analysis:

a) An ideal voltage source produces a constant voltage at or below its rated current. Current is determined by the power required by the external circuit (modeled as R).

$$I = \frac{V_s}{R} \quad P = V_s \cdot I$$

b) An ideal current source produces a constant current at or below its rated voltage. Voltage is determined by the power demanded by the external circuit (modeled as R).

$$V = I_s \cdot R \quad P = V \cdot I_s$$

A real source will overheat and, perhaps, burn up if its rated power is exceeded.

Sections 2.2, 2.3: KCL, KVL**Problem 2.7****Solution:****Known quantities:**

Circuit shown in Figure P2.7 with currents $I_0 = -2$ A, $I_1 = -4$ A, $I_S = 8$ A, and voltage source $V_S = 12$ V.

Find:

The unknown currents.

Analysis:

Applying KCL to node (a) and node (b):

$$\begin{cases} I_0 + I_1 + I_2 = 0 \\ I_0 + I_S + I_1 - I_3 = 0 \end{cases} \Rightarrow \begin{cases} I_2 = -(I_0 + I_1) = 6 \text{ A} \\ I_3 = I_0 + I_S + I_1 = 2 \text{ A} \end{cases}$$

Section 2.4: Sign Convention

Focus on Methodology: Passive Sign Convention

1. Choose an arbitrary direction of current flow.
2. Label polarities of all active elements (current and voltage sources).
3. Assign polarities to all passive elements (resistors and other loads); for passive elements, current always flows into the positive terminal.
4. Compute the power dissipated by each element according to the following rule: If positive current flows into the positive terminal of an element, then the power dissipated is positive (i.e., the element absorbs power); if the current leaves the positive terminal of an element, then the power dissipated is negative (i.e., the element delivers power).

Problem 2.8

Solution:

Known quantities:

Direction and magnitude of the current through the elements in Figure P2.8; voltage at the terminals.

Find:

Class of the components A and B.

Analysis:

The current enters the negative terminal of element B and leaves the positive terminal: its coulombic potential energy increases.

Element B is a power source. It must be either a voltage source or a current source.

The reverse is true for element A. The current loses energy as it flows through element A.

Element A could be 1. a resistor or 2. a power source through which current is being forced to flows 'backwards'.

Problem 2.9

Solution:

Known quantities:

Current absorbed by the heater; voltage at which the current is supplied; cost of the energy.

Find:

- a) Power consumption
- b) Energy dissipated in 24 hr.
- c) Cost of the Energy

Assumptions:

The heater works for 24 hours continuously.

Analysis:

- a)

$$P = VI = 110 \text{ V} (23 \text{ A}) = 2.53 \text{ K} \frac{\text{J}}{\text{A s}} = 2.53 \text{ KW}$$

b)

$$W = Pt = 2.53 \text{ K} \frac{\text{J}}{\text{s}} 24 \text{ hr} 3600 \frac{\text{s}}{\text{hr}} = 218.6 \text{ MJ}$$

c)

$$\text{Cost} = (\text{Rate})W = 6 \frac{\text{cents}}{\text{KW hr}} (2.53 \text{ KW})(24 \text{ hr}) = 364.3 \text{ cents} = \$3.64$$

Problem 2.10

Solution:

Known quantities:

Current through elements A, B and C shown in Figure P2.10; voltage across elements A, B and C.

Find:

Which components are absorbing power, which are supplying power; verify the conservation of power.

Analysis:

$$\text{A absorbs } (35 \text{ V})(15 \text{ A}) = 525 \text{ W}$$

$$\text{B absorbs } (15 \text{ V})(15 \text{ A}) = 225 \text{ W}$$

$$\text{C supplies } (50 \text{ V})(15 \text{ A}) = 750 \text{ W}$$

$$\text{Total power supplied} = P_C = 750 \text{ W}$$

$$\text{Total power absorbed} = P_B + P_A = 225 \text{ W} + 525 \text{ W} = 750 \text{ W}$$

Total power supplied = Total power absorbed, so conservation of power is satisfied.

Problem 2.11

Solution:

Known quantities:

Circuit shown in Figure P2.11 with voltage source, $V_s = 12\text{V}$; internal resistance of the source, $R_s = 5\text{k}\Omega$; and resistance of the load, $R_L = 7\text{k}\Omega$.

Find:

The terminal voltage of the source; the power supplied to the circuit, the efficiency of the circuit.

Assumptions:

Assume that the only loss is due to the internal resistance of the source.

Analysis:

$$KVL: -V_S + I_T R_S + V_T = 0$$

$$OL: V_T = I_T R_L \quad \therefore I_T = \frac{V_T}{R_L}$$

$$-V_S + \frac{V_T}{R_L} R_S + V_T = 0$$

$$V_T = \frac{V_S}{1 + \frac{R_S}{R_L}} = \frac{12 \text{ V}}{1 + \frac{5 \text{ K}\Omega}{7 \text{ K}\Omega}} = 7 \text{ V} \quad \text{or} \quad V_D : V_T = \frac{V_S R_L}{R_S + R_L} = \frac{12 \text{ V} \cdot 7 \text{ K}\Omega}{5 \text{ K}\Omega + 7 \text{ K}\Omega} = 7 \text{ V}.$$

$$P_L = \frac{V_T^2}{R_L} = \frac{(7 \text{ V})^2}{7 \text{ K}\Omega} = 7 \text{ mW}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{R_L}}{P_{R_S} + P_{R_L}} = \frac{I_T^2 R_L}{I_T^2 R_S + I_T^2 R_L} = \frac{7 \text{ K}\Omega}{5 \text{ K}\Omega + 7 \text{ K}\Omega} = 0.5833 \quad \text{or} \quad 58.33\%.$$

Problem 2.12**Solution:****Known quantities:**

Circuit shown in Figure P2.12; Current through elements D and E; voltage across elements B, C and E.

Find:

- Which components are absorbing power and which are supplying power
- Verify the conservation of power.

Analysis:

a)

By KCL, the current through element B is 5 A, to the right. By KVL,

$$v_D = v_E = 10 \text{ V} \quad (\text{positive at the top})$$

$$v_A + 5 - 10 - 10 = 0$$

Therefore the voltage across element A is

$$v_A = 15 \text{ V} \quad (\text{positive on top})$$

$$A \text{ supplies } (15 \text{ V})(5 \text{ A}) = 75 \text{ W}$$

$$B \text{ supplies } (5 \text{ V})(5 \text{ A}) = 25 \text{ W}$$

$$C \text{ absorbs } (10 \text{ V})(5 \text{ A}) = 50 \text{ W}$$

$$D \text{ absorbs } (10 \text{ V})(4 \text{ A}) = 40 \text{ W}$$

$$E \text{ absorbs } (10 \text{ V})(1 \text{ A}) = 10 \text{ W}$$

b)

$$\text{Total power supplied} = P_A + P_B = 75 \text{ W} + 25 \text{ W} = 100 \text{ W}$$

Total power absorbed = $P_C + P_D + P_E = 50 \text{ W} + 40 \text{ W} + 10 \text{ W} = 100 \text{ W}$

Total power supplied = Total power absorbed, so conservation of power is satisfied.

Problem 2.13

Solution:

Known quantities:

Headlights connected in parallel to a 24-V automotive battery; power absorbed by each headlight.

Find:

Resistance of each headlight; total resistance seen by the battery.

Analysis:

Headlight no. 1:

$$P = v \times i = 100 \text{ W} = \frac{v^2}{R} \quad \text{or}$$

$$R = \frac{v^2}{100} = \frac{576}{100} = 5.76 \Omega$$

Headlight no. 2:

$$P = v \times i = 75 \text{ W} = \frac{v^2}{R} \quad \text{or}$$

$$R = \frac{v^2}{75} = \frac{576}{75} = 7.68 \Omega$$

The total resistance is given by the parallel combination:

$$\frac{1}{R_{TOTAL}} = \frac{1}{5.76 \Omega} + \frac{1}{7.68 \Omega} \quad \text{or} \quad R_{TOTAL} = 3.29 \Omega$$

Problem 2.14

Solution:

Known quantities:

Headlights and 24-V automotive battery of problem 2.13 with 2 15-W taillights added in parallel; power absorbed by each headlight; power absorbed by each taillight.

Find:

Equivalent resistance seen by the battery.

Analysis:

The resistance corresponding to a 75-W headlight is:

$$R_{75W} = \frac{v^2}{75} = \frac{576}{75} = 7.68 \Omega$$

For each 15-W tail light we compute the resistance:

$$R_{15W} = \frac{v^2}{15} = \frac{576}{15} = 38.4\Omega$$

Therefore, the total resistance is computed as:

$$\frac{1}{R_{TOTAL}} = \frac{1}{7.68\Omega} + \frac{1}{7.68\Omega} + \frac{1}{38.4\Omega} + \frac{1}{38.4\Omega} \text{ or } R_{TOTAL} = 3.2\Omega$$

Problem 2.15

Solution:

Known quantities:

Circuit shown in Figure P2.15 with voltage source, $V_s = 20V$; and resistor, $R_o = 5\Omega$.

Find:

The power absorbed by variable resistor R (ranging from 0 to 20 Ω).

Analysis:

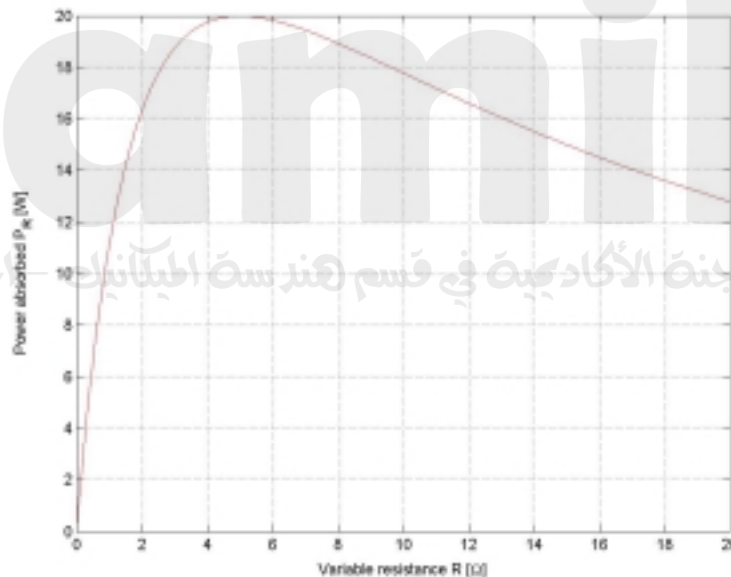
The current flowing clockwise in the series circuit is:

$$i = \frac{20}{5 + R}$$

The voltage across the variable resistor R , positive on the left, is:

$$v_R = Ri = \frac{20R}{R + 5}$$

$$\text{Therefore, } P_R = v_R i = \left(\frac{20}{5 + R} \right)^2 R$$



Problem 2.16

Solution:

Known quantities:

Circuit shown in Figure P2.16 with source voltage, $V_s = 12V$; internal resistance of the source, $R_s = 0.3\Omega$. Current, $I_T = 0, 5, 10, 20, 30$ A.

Find:

- The power supplied by the ideal source as a function of current
- The power dissipated by the nonideal source as a function of current
- The power supplied by the source to the circuit
- Plot the terminal voltage and power supplied to the circuit as a function of current

Assumptions:

There are no other losses except that on R_s .

Analysis:

a) P_s = power supplied by the source $= V_s I_s = V_s I_T$.

b) R_s = equivalent resistance for internal losses

$$P_{loss} = I_T^2 R_s$$

c) V_T = voltage at the battery terminals:

$$VD: V_T = V_s - R_s I_T$$

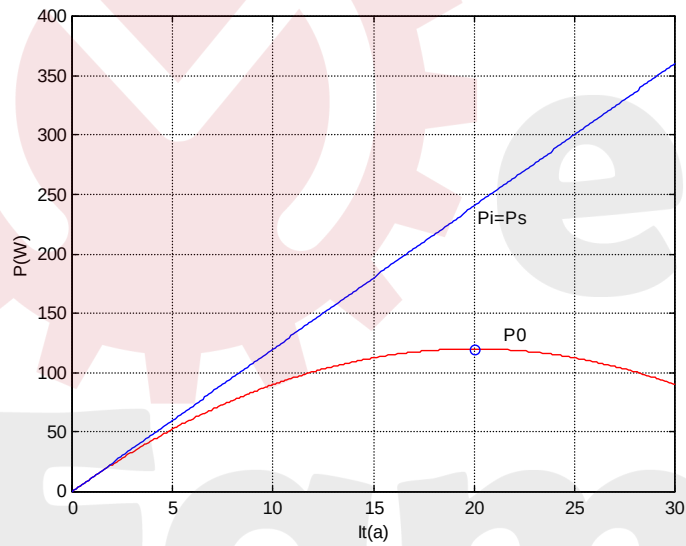
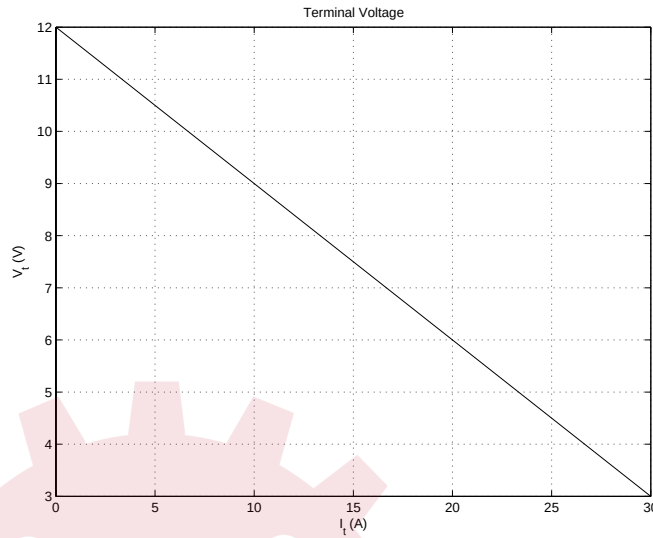
P_0 = power supplied to the circuit (R_L in this case) $= I_T V_T$.

Conservation of energy:

$$P_s = P_{loss} + P_0 .$$

$I_T (A)$	$P_s (W)$	$P_{loss} (W)$	$V_T (V)$	$P_0 (W)$
0	0	0	0	0
2	30	1.875	11.4	28.13
5	60	7.5	10.5	52.5
10	120	30	9	90
20	240	120	6	120
30	360	270	3	90

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Note that the power supplied to the circuit is maximum when $I_T = 20a$.

$$R_L = \frac{P_0}{I_T^2} = \frac{120 Va}{(20a)^2} = 30 m \frac{V}{a} = 30 m\Omega$$

$$R_S = \frac{P_{loss}}{I_T^2} = \frac{120 Va}{(20a)^2} = 30 m\Omega$$

$$R_L = R_S$$

Problem 2.17

Solution:

Known quantities:

Circuit shown in Figure P2.17 if the power delivered by the source is 40 mW; the voltage $v = v_1/4$; and $R_1 = 8k\Omega$, $R_2 = 10k\Omega$, $R_3 = 12k\Omega$

Find:

The resistance R, the current i and the two voltages v and v_1

Analysis:

$$P = v \cdot i = 40 \text{ mW} \quad (\text{eq. 1})$$

$$v_1 = R_2 \cdot i = 10000 \cdot i = \frac{v}{4} \quad (\text{eq. 2})$$

From eq.1 and eq.2, we obtain:

$$i = 1.0 \text{ mA} \quad \text{and} \quad v = 40 \text{ V.}$$

Applying KVL for the loop:

$$-v + 8000i + 10000i + Ri + 12000i = 0 \quad \text{or,} \quad 0.001R = 10$$

Therefore,

$$R = 10k\Omega \quad \text{and} \quad v_1 = 10V.$$

Problem 2.18

Solution:

Known quantities:

Rated power; rated optical power; operating life; rated operating voltage; open-circuit resistance of the filament.

Find:

- The resistance of the filament in operation
- The efficiency of the bulb.

Analysis:

a)

$$P = VI \quad \therefore I = \frac{P_R}{V_R} = \frac{60 \text{ Va}}{115 \text{ V}} = 521.7 \text{ ma}$$

$$\text{OL: } R = \frac{V}{I} = \frac{V_R}{I} = \frac{115 \text{ V}}{521.7 \text{ ma}} = 220.4 \Omega$$

b)

Efficiency is defined as the ratio of the useful power dissipated by or supplied by the load to the total power supplied by the source. In this case, the useful power supplied by the load is the optical power. From any handbook containing equivalent units: 680 lumens=1 W

$$P_{o,out} = \text{Optical Power Out} = 820 \text{ lum} \frac{W}{680 \text{ lum}} = 1.206 W$$

$$\eta = \text{efficiency} = \frac{P_{o,out}}{P_R} = \frac{1.206 W}{60 W} = 0.02009 = 2.009 \%$$

Problem 2.19

Solution:

Known quantities:

Rated power; rated voltage of a light bulb.

Find:

The power dissipated by a series of three light bulbs connected to the nominal voltage.

Assumptions:

The resistance of each bulb doesn't vary when connected in series.

Analysis:

When connected in series, the voltage of the source will divide equally across the three bulbs. The across each bulb will be 1/3 what it was when the bulbs were connected individually across the source. Power dissipated in a resistance is a function of the voltage squared, so the power dissipated in each bulb when connected in series will be 1/9 what it was when the bulbs were connected individually, or 11.11 W:

$$\text{Ohm's Law: } P = IV_B = I^2 R_B = \frac{V_B^2}{R_B}$$

$$V_B = V_S = 110 V$$

$$R_B = \frac{V_B^2}{P} = \frac{(110 V)^2}{100 W} = 121 \Omega$$

Connected in series and assuming the resistance of each bulb remains the same as when connected individually:

$$\text{KVL: } -V_S + V_{B1} + V_{B2} + V_{B3} = 0$$

$$\text{OL: } -V_S + IR_{B3} + IR_{B2} + IR_{B1} = 0$$

$$I = \frac{V_S}{R_{B1} + R_{B2} + R_{B3}} = \frac{110 V}{121 + 121 + 121 \frac{V}{a}} = 303 \text{ ma}$$

$$P_{B1} = I^2 R_{B1} = (303 \text{ ma})^2 \left(121 \frac{V}{a} \right) = 11.11 W = \frac{1}{9} 100 W.$$

Problem 2.20

Solution:

Known quantities:

Rated power and rated voltage of the two light bulbs.

Find:

The power dissipated by the series of the two light bulbs.

Assumptions:

The resistance of each bulb doesn't vary when connected in series.

Analysis:

When connected in series, the voltage of the source will divide equally across the three bulbs. The across each bulb will be 1/3 what it was when the bulbs were connected individually across the source. Power dissipated in a resistance is a function of the voltage squared, so the power dissipated in each bulb when connected in series will be 1/9 what it was when the bulbs were connected individually, or 11.11 W:

$$\text{Ohm's Law: } P = IV_B = I^2 R_B = \frac{V_B^2}{R_B}$$

$$V_B = V_S = 110 \text{ V}$$

$$R_{60} = \frac{V_B^2}{P_{60}} = \frac{(110 \text{ V})^2}{60 \text{ W}} = 201.7 \, \Omega$$

$$R_{100} = \frac{V_B^2}{P_{100}} = \frac{(110 \text{ V})^2}{100 \text{ W}} = 121 \, \Omega$$

When connected in series and assuming the resistance of each bulb remains the same as when connected individually:

$$\text{KVL: } -V_S + V_{B60} + V_{B100} = 0$$

$$\text{OL: } -V_S + IR_{B60} + IR_{B100} = 0$$

$$I = \frac{V_S}{R_{B60} + R_{B100}} = \frac{110 \text{ V}}{201.7 + 121 \, \Omega} = 340.9 \text{ mA}$$

$$P_{B60} = I^2 R_{B60} = (340.9 \text{ mA})^2 \left(201.7 \, \Omega \right) = 23.44 \text{ W}$$

$$P_{B100} = I^2 R_{B100} = (340.9 \text{ mA})^2 \left(121 \, \Omega \right) = 14.06 \text{ W}$$

Notes: 1. It's strange but it's true that a 60 W bulb connected in series with a 100 W bulb will dissipate more power than the 100 W bulb. 2. If the power dissipated by the filament in a bulb decreases, the temperature at which the filament operates and therefore its resistance will decrease. This made the assumption about the resistance necessary.

Problem 2.21**Solution:****Known quantities:**

Schematic of the circuit shown in Figure P2.21.

Find:

The resistor values, including the power rating, necessary to achieve the indicated voltages for:

$$\text{a) } V = 30\text{V}, R_1 = 10\text{k}\Omega, v_{out} = 10\text{V}$$

b) $V = 12V, R_1 = 140k\Omega, v_{out} = 8.5V$

Assumptions:

Resistors are available in $\frac{1}{8}$ - $\frac{1}{4}$ - $\frac{1}{2}$ -, and 1-W ratings.

Analysis:

(a)

$$v_{out} = \left(\frac{R_2}{R_2 + R_1} \right) \cdot V = \frac{R_2}{R_2 + 10000} \cdot (30) = 10$$

$$R_2(30 - 10) = 10 \cdot 10 \cdot 10^3$$

$$R_2 = 5 \text{ k}\Omega$$

$$P_2 = I^2 R = \left(\frac{30}{15000} \right)^2 \cdot (5000) = 20 \text{ mW}$$

$$P_{R_2} = \frac{1}{8} \text{ W}$$

$$P_1 = I^2 R_1 = 40 \text{ mW}$$

$$P_{R_1} = \frac{1}{8} \text{ W}.$$

(b)

$$v_{out} = \left(\frac{R_2}{R_2 + R_1} \right) \cdot V = 12 \cdot \left(\frac{140}{140 + R_1} \right) = 8.5$$

$$R_1 = 57 \Omega$$

$$I = \frac{V}{R_1 + R_2} = \frac{12 \text{ V}}{57 \Omega + 140 \Omega} = 61 \text{ ma} \Rightarrow P_1 = I^2 R_1 = 212.1 \text{ mW}$$

$$P_2 = I^2 R_2 = 520.9 \text{ mW}$$

$$P_{R_1} = \frac{1}{4} \text{ W}$$

$$P_{R_2} = 1 \text{ W}$$

Problem 2.22

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.22 with resistances, $R_1 = 1.6k\Omega, R_2 = 4.3k\Omega$; and voltages, $V = 110V, v_{out} = 64.3V$.

Find:

The resistor values, including the power rating, necessary to achieve the indicated voltages.

Assumptions:

Resistors are available in $\frac{1}{8}$ - $\frac{1}{4}$ - $\frac{1}{2}$ -, and 1-W ratings.

Analysis:

$$v_{out} = \frac{R_2}{R_0 + R_1 + R_2} V = 110 \cdot \left(\frac{4300}{1600 + R_1 + 4300} \right) = 64.3$$

$$R_1 = 1.45 k\Omega$$

$$I = \frac{V}{R_0 + R_1 + R_2} = \frac{110 \text{ V}}{1600 \Omega + 1450 \Omega + 4300 \Omega} = 15 \text{ mA}$$

$$P_0 = I^2 R_0 = 360 \text{ mW} \Rightarrow P_{R_0} = \frac{1}{2} \text{ W}$$

$$P_1 = I^2 R_1 = 326.25 \text{ mW} \Rightarrow P_{R_1} = \frac{1}{2} \text{ W}$$

$$P_2 = I^2 R_2 = 967.5 \text{ mW} \Rightarrow P_{R_2} = 1 \text{ W}$$

Problem 2.23**Solution:****Known quantities:**

Schematic of the circuit shown in Figure P2.23 with source voltage, $v = 24\text{V}$; and resistances, $R_0 = 8\Omega$, $R_1 = 10\Omega$, $R_2 = 2\Omega$.

Find:

- The equivalent resistance seen by the source
- The current i
- The power delivered by the source
- The voltages v_1 and v_2
- The minimum power rating required for R_1

Analysis:

- a) The equivalent resistance seen by the source is

$$R_{eq} = R_0 + R_1 + R_2 = 8 + 10 + 2 = 20\Omega$$

- b) Applying KVL:

$$V - R_{eq} i = 0, \text{ therefore } i = \frac{V}{R_{eq}} = \frac{24\text{V}}{20\Omega} = 1.2\text{A}$$

- c)

$$P_{source} = Vi = 24\text{V} \cdot 1.2\text{A} = 28.8 \text{ W}$$

- d) Applying Ohm's law:

$$v_1 = R_1 i = 10\Omega \cdot 1.2\text{A} = 12 \text{ V}, \text{ and } v_2 = R_2 i = 2\Omega \cdot 1.2\text{A} = 2.4 \text{ V}$$

- e)

$$P_1 = R_1 i^2 = 10\Omega \cdot (1.2A)^2 = 14.4 \text{ W}, \text{ therefore the minimum power rating for } R_1 \text{ is } 16 \text{ W}.$$

Problem 2.24

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.24 with resistors,

$$R_1 = 25\Omega, R_2 = 10\Omega, R_3 = 5\Omega, R_4 = 7\Omega.$$

Find:

- The currents i_1 and i_2
- The power delivered by the 3-A current source and the 12-V voltage source
- The total power dissipated by the circuit.

Analysis:

- KCL at node 1 requires that:

$$\frac{v_1}{R_2} + \frac{v_1 - 12 \text{ V}}{R_3} - 3 \text{ A} = 0$$

Solving for v_1 we have

$$v_1 = 3 \frac{(4 + R_3)R_2}{R_2 + R_3} = 18 \text{ V}$$

Therefore,

$$i_1 = -\frac{v_1}{R_2} = -\frac{18}{10} = -1.8 \text{ A}$$

$$i_2 = \frac{12 - v_1}{R_3} = -\frac{6}{5} = -1.2 \text{ A}$$

- The power delivered by the 3-A source is:

$$P_{3\text{-A}} = (v_{3\text{-A}})(3)$$

Thus, we can compute the voltage across the 3-A source as

$$v_{3\text{-A}} = 3R_1 + v_1 = 3 \cdot 25 + 18 = 93 \text{ V}$$

Thus,

$$P_{3\text{-A}} = (93)(3) = 279 \text{ W}.$$

Similarly, the power supplied by the 12-V source is:

$$P_{12\text{-V}} = (12)(I_{12\text{-V}})$$

We have $I_{12\text{-V}} = \frac{12}{R_4} + i_2 = 514.3 \text{ mA}$, thus:

$$P_{12\text{-V}} = (12)(I_{12\text{-V}}) = 6.17 \text{ W}$$

- Since the power dissipated equals the total power supplied:

$$P_{\text{diss}} = P_{3-A} + P_{12-V} = 279 + 6.17 = 285.17 \text{ W}$$

Problem 2.25

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.25.

Find:

The power delivered by the dependent source.

Analysis:

$$i = \frac{24V}{(7+5)\Omega} = \frac{24}{12} \text{ A} = 2 \text{ A}$$

$$i_{\text{source}} = 0.5i^2 = 0.5 \cdot (4) = 2 \text{ A}$$

The voltage across the dependent source (+ ref. taken at the top) can be found by KVL:

$$-v_D + (2A)(15\Omega) + 24V = 0 \Rightarrow v_D = 54 \text{ V}$$

Therefore, the power delivered by the dependent source is

$$P_D = v_D i_{\text{source}} = 54 \cdot 2 = 108 \text{ W.}$$

Problem 2.26

Solution:

Known quantities:

Schematic of the circuit in Figure P2.26.

Find:

- If $V_1 = 12.0V$, $R_1 = 0.15\Omega$, $R_L = 2.55\Omega$, the load current and the power dissipated by the load
- If a second battery is connected in parallel with battery 1 with $V_2 = 12.0V$, $R_2 = 0.28\Omega$, determine the variations in the load current and in the power dissipated by the load due to the parallel connection with a second battery.

Analysis:

a)

$$I_L = \frac{V_1}{R_1 + R_L} = \frac{12}{0.15 + 2.55} = \frac{12}{2.7} = 4.44 \text{ A}$$

$$P_{\text{Load}} = I_L^2 R_L = 50.4 \text{ W.}$$

b) with another source in the circuit we must find the new power dissipated by the load. To do so, we write

KVL twice using mesh currents to obtain 2 equations in 2 unknowns:

$$\begin{cases} I_2 R_2 - V_1 + V_2 - I_1 R_1 = 0 \\ (I_1 + I_2) R_L + I_2 R_2 = V_2 \end{cases} \Rightarrow \begin{cases} 0.28 \cdot I_2 - 0.15 \cdot I_1 = 0 \\ 2.55 \cdot (I_1 + I_2) + 0.28 \cdot I_2 = 12 \end{cases}$$

Solving the above equations gives us:

$$I_1 = 2.95 \text{ A}, \quad I_2 = 1.58 \text{ A} \Rightarrow I_L = I_1 + I_2 = 4.53 \text{ A}$$

$$P_{Load} = I_L^2 R_L = 52.33 \text{ W}$$

This is an increase of 1%.

Problem 2.27

Solution:

Known quantities:

Open-circuit voltage at the terminals of the power source is 50.8 V; voltage drop with a 10-W load attached is to 49 V.

Find:

- The voltage and the internal resistance of the source
- The voltage at its terminals with a 15-Ω load resistor attached
- The current that can be derived from the source under short-circuit conditions.

Analysis:

(a)

$$\frac{(49\text{V})^2}{R_L} = 10\text{W} \Rightarrow R_L = 240\Omega$$

$v_s = 50.8\text{V}$, the open-circuit voltage

$$\frac{R_L}{R_s + R_L} v_s = 49 \Rightarrow \frac{240}{R_s + 240} 50.8 = 49$$

$$R_s = \frac{(240)(50.8)}{49} - 240 = 8.82\Omega$$

(b)

$$v = \frac{R_L}{R_s + R_L} v_s = \frac{15}{8.82 + 15} 50.8 = 32.0\text{V}$$

(c)

$$i_{CC}(R_L = 0) = \frac{v_s}{R_s} = \frac{50.8}{8.82} = 5.76 \text{ A}$$

Problem 2.28

Solution:

Known quantities:

Voltage of the heater, maximum and minimum power dissipation; number of coils, schematics of the configurations.

Find:

- a) The resistance of each coil
- b) The power dissipation of each of the other two possible arrangements.

Analysis:

(a) For the parallel connection, $P = 2000$ W. Therefore,

$$2000 = \frac{(220)^2}{R_1} + \frac{(220)^2}{R_2}$$

$$= (220)^2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

or,

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{5}{121}.$$

For the series connection, $P = 300$ W. Therefore,

$$300 = \frac{(220)^2}{R_1 + R_2}$$

or,

$$\frac{1}{R_1 + R_2} = \frac{3}{484}.$$

Solving, we find that $R_1 = 131.6\Omega$ and $R_2 = 29.7\Omega$.

(b) the power dissipated by R_1 alone is:

$$P_{R_1} = \frac{(220)^2}{R_1} = 368W$$

and the power dissipated by R_2 alone is

$$P_{R_2} = \frac{(220)^2}{R_2} = 1631W.$$

Section 2.5, 2.6: Resistance and Ohm's Law**Problem 2.29****Solution:****Known quantities:**

Diameter of the cylindrical substrate; length of the substrate; conductivity of the carbon.

Find:

The thickness of the carbon film required for a resistance R of $33\text{ K}\Omega$.

Assumptions:

Assume the thickness of the film to be much smaller than the radius
Neglect the end surface of the cylinder.

Analysis:

$$R = \frac{d}{\sigma \cdot A} \cong \frac{d}{\sigma \cdot 2\pi a \cdot \Delta t}$$

$$\Delta t = \frac{d}{R \cdot 2\pi a \cdot \sigma} = \frac{9 \cdot 10^{-3} \text{ m}}{33 \cdot 10^3 \Omega \cdot 2.9 \cdot 10^6 \frac{\text{S}}{\text{m}} \cdot 2\pi \cdot 1 \cdot 10^{-3} \text{ m}}$$

Problem 2.30**Solution:****Known quantities:**

The constants A and k ; the open-circuit resistance.

Find:

The rated current at which the fuse blows, showing that this happens at:

$$I = \frac{1}{\sqrt{AkR_0}}.$$

Assumptions:

Here the resistance of the fuse is given by:

$$R = R_0 [1 + A(T - T_0)]$$

where T_0 , room temperature, is assumed to be 25°C .

We assume that:

$$T - T_0 = kP$$

where P is the power dissipated by the resistor (fuse).

Analysis:

$$R = R_0 (1 + A \cdot \Delta T) = R_0 (1 + AkP) = R_0 (1 + AkI^2 R)$$

$$R - R_0 AkI^2 R = R_0$$

$$R = \frac{R_0}{1 - R_0 A k I^2} \rightarrow \infty \quad \text{when} \quad I - R_0 A k I^2 \rightarrow 0$$

$$I = \frac{1}{\sqrt{A k R_0}} = (0.7 \frac{m}{^\circ C} 0.35 \frac{^\circ C}{V a} 0.11 \frac{V}{a})^{-\frac{1}{2}} = 6.09 \text{ a.}$$

Problem 2.31

Solution:

Known quantities:

Circuit shown in Figure P2.31 with voltage source, $V_s = 10V$ and resistors,

$$R_1 = 20\Omega, R_2 = 40\Omega, R_3 = 10\Omega, R_4 = R_5 = R_6 = 15\Omega.$$

Find:

The current in the 15- Ω resistors.

Analysis:

Since the 3 resistors must have equal currents,

$$I_{15\Omega} = \frac{1}{3} \cdot I$$

and,

$$I = \frac{V_s}{R_1 + R_2 \parallel R_3 + R_4 \parallel R_5 \parallel R_6} = \frac{10}{20 + 8 + 5} = \frac{10}{33} = 303 \text{ mA}$$

$$\text{Therefore, } I_{15\Omega} = \frac{10}{99} = 101 \text{ mA}$$

Problem 2.32

Solution:

Known quantities:

Schematic of the circuit in Figure P2.7 with currents $I_0 = -2 \text{ A}$, $I_1 = -4 \text{ A}$, $I_s = 8 \text{ A}$, voltage source $V_s = 12 \text{ V}$, and resistance $R_0 = 2 \Omega$.

Find:

The unknown resistances R_1 , R_2 , R_3 , R_4 and R_5 .

Assumption:

In order to solve the problem we need to make further assumptions on the value of the resistors. For

example, we may assume that $R_4 = \frac{2}{3} R_1$ and $R_2 = \frac{1}{3} R_1$.

Analysis:

We can express each current in terms of the adjacent node voltages:

$$I_0 = \frac{v_a - v_b}{R_0 + R_4} = \frac{v_a - v_b}{2 + \frac{2}{3}R_1} = -2$$

$$I_1 = \frac{v_a - v_b}{R_1} = -4$$

$$I_2 = \frac{v_a}{R_2} = \frac{v_a}{\frac{1}{3}R_1} = 6$$

$$I_3 = \frac{v_b}{R_3} = 2$$

$$I_S = \frac{V_S - v_b}{R_5} = \frac{12 - v_b}{R_5} = 8$$

Solving the system we obtain:

$$v_a = 3 \text{ V}, v_b = 9 \text{ V}, R_1 = 1.5 \Omega, R_2 = 0.5 \Omega, R_3 = 4.5 \Omega, R_4 = 1 \Omega \text{ and } R_5 = 0.375 \Omega.$$

Problem 2.33

Solution:

Known quantities:

Schematic of the circuit in Figure P2.7 with resistors $R_1 = 2\Omega$, $R_2 = 5\Omega$, $R_3 = 4\Omega$, $R_4 = 1\Omega$, $R_5 = 3\Omega$, voltage source $V_S = 54 \text{ V}$, and current $I_2 = 4 \text{ A}$.

Find:

The unknown currents I_0 , I_1 , I_3 , I_S and the resistor R_0 .

Analysis:

We can express each current in terms of the adjacent node voltages:

$$I_0 = \frac{v_a - v_b}{R_0 + R_4}$$

$$I_1 = \frac{v_a - v_b}{R_1}$$

$$I_2 = \frac{v_a}{R_2} = 4 \Rightarrow v_a = 4 \cdot 5 = 20 \text{ V}$$

$$I_3 = \frac{v_b}{R_3}$$

$$I_S = \frac{V_S - v_b}{R_5}$$

Applying KCL to node (a) and (b) :

$$\begin{cases} I_0 + I_1 + I_2 = 0 \\ I_0 + I_S + I_1 - I_3 = 0 \end{cases} \Rightarrow \begin{cases} \frac{20 - v_b}{R_0 + 1} + \frac{20 - v_b}{2} + 4 = 0 \\ \frac{20 - v_b}{R_0 + 1} + \frac{54 - v_b}{3} + \frac{20 - v_b}{2} - \frac{v_b}{4} = 0 \end{cases}$$

Solving the system we obtain: $v_b = 24 \text{ V}$ and $R_0 = 1 \Omega$.

Problem 2.34

Solution:

Known quantities:

NOTE: Typo in Problem Statement for units of R_3

Schematic of the circuit shown in Figure P2.34 with resistors $R_0 = 2\Omega$, $R_1 = 1\Omega$, $R_2 = 4/3\Omega$, $R_3 = 6\Omega$ and voltage source $V_S = 12 \text{ V}$.

Find:

- The mesh currents i_a , i_b , i_c
- The current through each resistor.

Analysis:

Applying KVL to mesh (a), mesh (b) and mesh (c):

$$\begin{cases} i_a R_0 + (i_a - i_b) R_1 = 0 \\ (i_a - i_b) R_1 - i_b R_2 + (i_c - i_b) R_3 = 0 \\ V_S = (i_c - i_b) R_3 \end{cases} \Rightarrow \begin{cases} 2i_a + (i_a - i_b) = 0 \\ (i_a - i_b) - \frac{4}{3}i_b + 6(i_c - i_b) = 0 \\ 6(i_c - i_b) = 12 \end{cases}$$

Solving the system we obtain:

$$\begin{cases} i_a = 2 \text{ A} \\ i_b = 6 \text{ A} \\ i_c = 8 \text{ A} \end{cases} \Rightarrow \begin{cases} I_{R_0} = i_a = 2 \text{ A} & \text{(positive in the direction of } i_a) \\ I_{R_1} = i_b - i_a = 4 \text{ A} & \text{(positive in the direction of } i_b) \\ I_{R_2} = i_b = 6 \text{ A} & \text{(positive in the direction of } i_b) \\ I_{R_3} = i_c - i_b = 2 \text{ A} & \text{(positive in the direction of } i_c) \end{cases}$$

Problem 2.35

Solution:

Known quantities:

NOTE: Typo in Problem Statement for units of R_3

Schematic of the circuit shown in Figure P2.34 with resistors $R_0 = 2\Omega$, $R_1 = 2\Omega$, $R_2 = 5\Omega$, $R_3 = 4\Omega$ and voltage source $V_S = 24\text{ V}$.

Find:

- The mesh currents i_a , i_b , i_c
- The current through each resistor.

Analysis:

Applying KVL to mesh (a), mesh (b) and mesh (c):

$$\begin{cases} i_a R_0 + (i_a - i_b) R_1 = 0 \\ (i_a - i_b) R_1 - i_b R_2 + (i_c - i_b) R_3 = 0 \\ V_S = (i_c - i_b) R_3 \end{cases} \Rightarrow \begin{cases} 2i_a + 2(i_a - i_b) = 0 \\ 2(i_a - i_b) - 5i_b + 4(i_c - i_b) = 0 \\ 4(i_c - i_b) = 24 \end{cases}$$

Solving the system we obtain:

$$\begin{cases} i_a = 2\text{ A} \\ i_b = 4\text{ A} \\ i_c = 10\text{ A} \end{cases} \Rightarrow \begin{cases} V_{R_0} = R_0 i_a = 4\text{ V} & (\oplus \text{ up}) \\ V_{R_1} = R_1 (i_b - i_a) = 4\text{ V} & (\oplus \text{ down}) \\ V_{R_2} = R_2 i_b = 20\text{ V} & (\oplus \text{ up}) \\ V_{R_3} = R_3 (i_c - i_b) = 24\text{ V} & (\oplus \text{ up}) \end{cases}$$

Problem 2.36

Solution:

Known quantities:

NOTE: Typo in Problem Statement for units of R_3

Schematic of the circuit shown in Figure P2.34 with resistors $R_0 = 1\Omega$, $R_1 = 3\Omega$, $R_2 = 2\Omega$, $R_3 = 4\Omega$ and of the current source $I_S = 12\text{ A}$.

Find:

The voltage across each resistance.

Analysis:

Applying KVL to mesh (a), mesh (b) and mesh (c):

$$\begin{cases} i_a R_0 + (i_a - i_b) R_1 = 0 \\ (i_a - i_b) R_1 - i_b R_2 + (i_c - i_b) R_3 = 0 \\ i_c = I_S \end{cases} \Rightarrow \begin{cases} i_a + 3(i_a - i_b) = 0 \\ 3(i_a - i_b) - 2i_b - 4i_b + 48 = 0 \\ i_c = 12\text{ A} \end{cases}$$

Solving the system we obtain:

$$\begin{cases} i_a = \frac{16}{3} \text{ A} \\ i_b = \frac{64}{9} \text{ A} \\ i_c = 12 \text{ A} \end{cases} \Rightarrow \begin{cases} V_{R_0} = R_0 i_a = 5.33 \text{ V} & (\oplus \text{ up}) \\ V_{R_1} = R_1 (i_b - i_a) = 5.33 \text{ V} & (\oplus \text{ down}) \\ V_{R_2} = R_2 i_b = 14.22 \text{ V} & (\oplus \text{ up}) \\ V_{R_3} = R_3 (i_c - i_b) = 19.55 \text{ V} & (\oplus \text{ up}) \end{cases}$$

Problem 2.37

Solution:

Known quantities:

Schematic of voltage divider network shown of Figure P2.37.

Find:

- The worst-case output voltages for ± 10 percent tolerance
- The worst-case output voltages for ± 5 percent tolerance

Analysis:

a) 10% worst case: low voltage

$$R_2 = 4500 \, \Omega, R_1 = 5500 \, \Omega$$

$$V_{OUT,MIN} = \frac{4500}{4500 + 5500} 5 = 2.25 \text{ V}$$

10% worst case: high voltage

$$R_2 = 5500 \, \Omega, R_1 = 4500 \, \Omega$$

$$V_{OUT,MAX} = \frac{5500}{4500 + 5500} 5 = 2.75 \text{ V}$$

b) 5% worst case: low voltage

$$R_2 = 4750 \, \Omega, R_1 = 5250 \, \Omega$$

$$V_{OUT,MIN} = \frac{4750}{4750 + 5250} 5 = 2.375 \text{ V}$$

5% worst case: high voltage

$$R_2 = 5250 \, \Omega, R_1 = 4750 \, \Omega$$

$$V_{OUT,MAX} = \frac{5250}{5250 + 4750} 5 = 2.625 \text{ V}$$

Problem 2.38

Solution:

Known quantities:

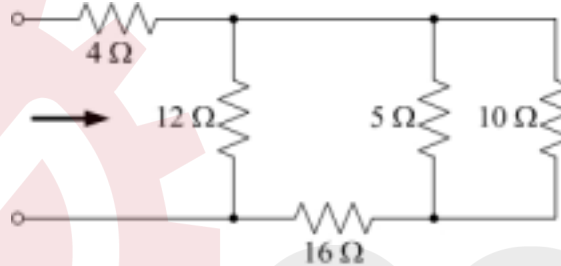
Schematic of the circuit shown in figure P2.38 with resistances, $R_0 = 4\Omega$, $R_1 = 12\Omega$, $R_2 = 8\Omega$, $R_3 = 2\Omega$, $R_4 = 16\Omega$, $R_5 = 5\Omega$.

Find:

The equivalent resistance of the circuit.

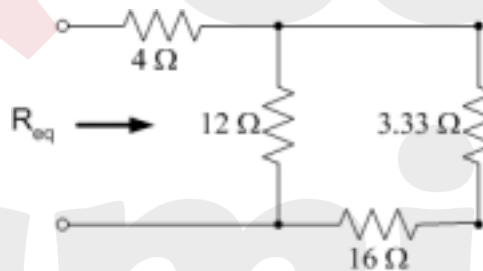
Analysis:

Starting from the right side, we combine the two resistors in series:

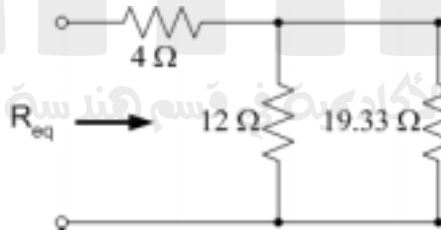


Then, we can combine the two parallel resistors, namely the 5 Ω resistor and 10 Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{5} + \frac{1}{10} (\Omega^{-1}) \Rightarrow R_{parallel} = \frac{10}{3} \Omega$$

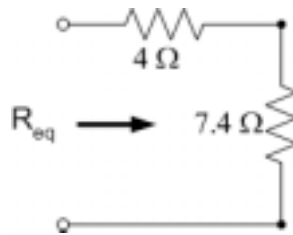


Then, we can combine the two resistors in series, namely the 3.33 Ω and the 16 Ω resistor:



Then, we can combine the two parallel resistors, namely the 12 Ω resistor and 19.33 Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{12} + \frac{1}{19.33} (\Omega^{-1}) \Rightarrow R_{parallel} = 7.4 \Omega$$



Therefore, $R_{eq} = 4 + 7.4 = 11.4 \Omega$.

Problem 2.39

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.39 with source voltage, $V_s = 12V$; and resistances,

$$R_0 = 4\Omega, R_1 = 2\Omega, R_2 = 50\Omega, R_3 = 8\Omega, R_4 = 10\Omega, R_5 = 12\Omega, R_6 = 6\Omega.$$

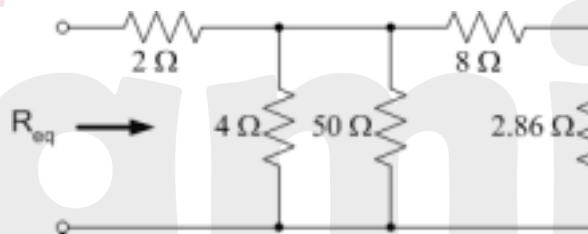
Find:

The equivalent resistance of the circuit seen by the source; the current i through the resistance R_2 .

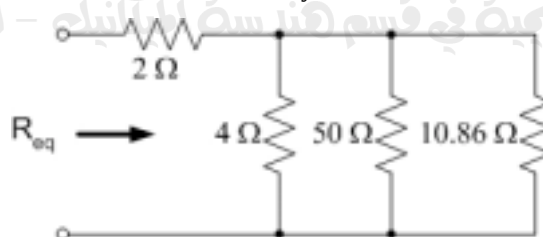
Analysis:

Starting from the right side, we can combine the three parallel resistors, namely the 10Ω resistor, the 12Ω resistor and the 6Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{10} + \frac{1}{12} + \frac{1}{6} (\Omega^{-1}) \Rightarrow R_{parallel} = \frac{20}{7} \Omega$$

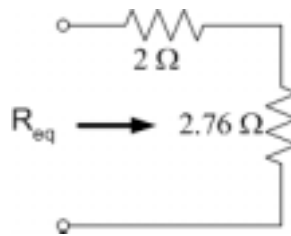


Then, we can combine the two resistors in series, namely the 8Ω and the 2.86Ω resistor:



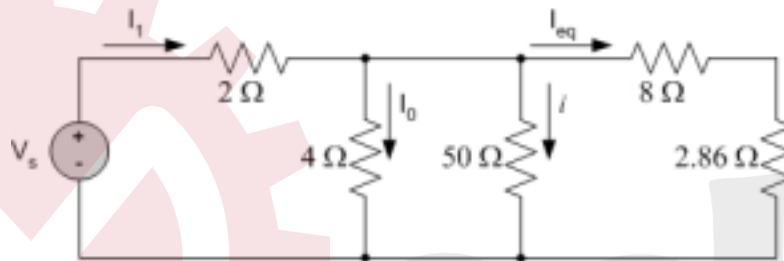
Then, we can combine the three parallel resistors, namely the 4Ω resistor, the 50Ω resistor and the 10.86Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{4} + \frac{1}{50} + \frac{1}{10.86} (\Omega^{-1}) \Rightarrow R_{parallel} = 2.76 \Omega$$



Therefore, $R_{eq} = 2 + 2.76 = 4.76 \Omega$.

Looking at the following equivalent circuit:



We can apply KVL and KCL to the above circuit:

$$\begin{cases} V_s - 2I_1 - 4I_0 = 0 \\ I_1 = I_0 + i + I_{eq} \\ 4I_0 = 50i = 2.86I_{eq} \end{cases} \Rightarrow \begin{cases} I_0 = 12.5i \\ I_1 = 6 - 25i \\ I_{eq} = 17.48i \\ i = I_1 - I_0 - I_{eq} \end{cases} \Rightarrow i = 107 \text{ mA}$$

Problem 2.40

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.40 with source voltage, $V_s = 50V$; resistances, $R_1 = 20\Omega$, $R_2 = 5\Omega$, $R_3 = 2\Omega$, $R_4 = 8\Omega$, $R_5 = 8\Omega$, $R_6 = 30\Omega$; and power absorbed by the 20Ω -resistor.

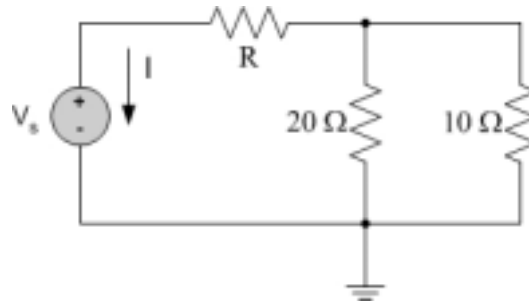
Find:

The resistance R .

Analysis:

Starting from the right side, we can replace resistors R_i ($i=2..6$) with a single equivalent resistors:

$$R_{eq} = R_2 + (R_3 + (R_4 \parallel R_5)) \parallel R_6 = 10 \Omega$$



The same voltage appears across both R_1 and R_{eq} and, therefore, these elements are in parallel. Applying the voltage divider rule:

$$V_{R_1} = \frac{R_1 \parallel R_{eq}}{R + R_1 \parallel R_{eq}} V_s = \frac{1000}{3R + 20}$$

The power absorbed by the 20-Ω resistor is:

$$P_{20\Omega} = \frac{(V_{R_1})^2}{R_1} = \frac{1}{20} \left(\frac{1000}{3R + 20} \right)^2 = \frac{50000}{(3R + 20)^2} = 20 \Rightarrow R = 10 \Omega$$

Problem 2.41

Solution:

Known quantities:

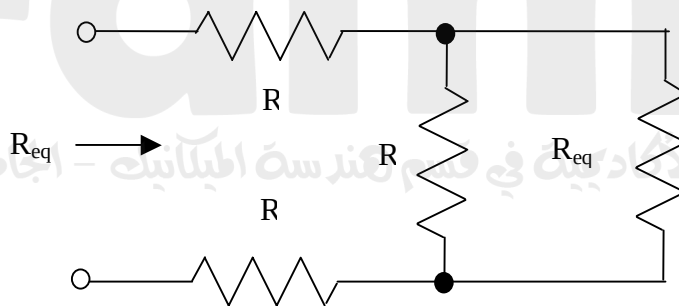
Schematic of the circuit shown in Figure P2.41.

Find:

The equivalent resistance R_{eq} of the infinite network of resistors.

Analysis:

We can see the infinite network of resistors as the equivalent to the circuit in the picture:



Therefore,

$$R_{eq} = R + (R \parallel R_{eq}) + R = 2R + \frac{RR_{eq}}{R + R_{eq}} \Rightarrow R_{eq} = (1 + \sqrt{3})R$$

Problem 2.42

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.42 with source voltage, $V_s = 110V$; and resistances,

$R_1 = 90\Omega$, $R_2 = 50\Omega$, $R_3 = 40\Omega$, $R_4 = 20\Omega$, $R_5 = 30\Omega$, $R_6 = 10\Omega$, $R_7 = 60\Omega$, $R_8 = 80\Omega$.

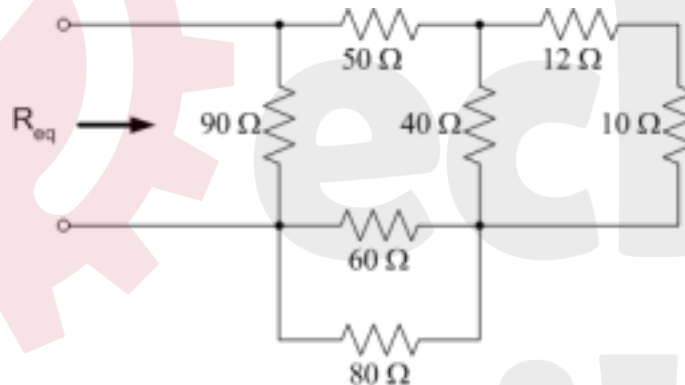
Find:

- The equivalent resistance of the circuit seen by the source.
- The current through and the power absorbed by the resistance 90Ω resistance.

Analysis:

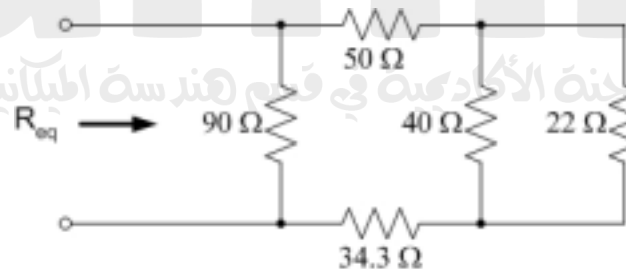
a) Starting from the right side, we can combine the two parallel resistors, namely the 20Ω resistor and the 30Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{20} + \frac{1}{30} (\Omega^{-1}) \Rightarrow R_{parallel} = 12\Omega$$



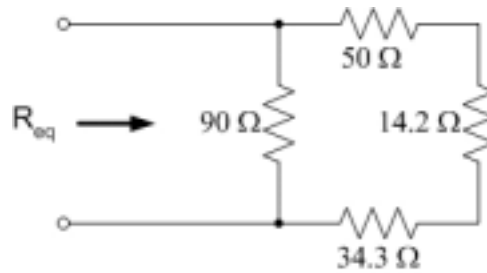
Then we can combine the two parallel resistors in the bottom, namely the 60Ω resistor and the 80Ω , and the two resistor in series:

$$\frac{1}{R_{parallel}} = \frac{1}{60} + \frac{1}{80} (\Omega^{-1}) \Rightarrow R_{parallel} = 34.3\Omega$$



Then we can combine the two parallel resistors on the right, namely the 40Ω resistor and the 22Ω :

$$\frac{1}{R_{parallel}} = \frac{1}{40} + \frac{1}{22} (\Omega^{-1}) \Rightarrow R_{parallel} = 14.2\Omega$$



Therefore, $\frac{1}{R_{eq}} = \frac{1}{90} + \frac{1}{(50 + 14.2 + 34.3)} (\Omega^{-1}) \Rightarrow R_{eq} = 47 \Omega$.

b) The current through and the power absorbed by the 90-Ω resistor are:

$$I_{90\Omega} = \frac{V_s}{R_1} = \frac{110}{90} = 1.22 \text{ A}$$

$$P_{90\Omega} = \frac{(V_s)^2}{R_1} = \frac{110^2}{90} = 134.4 \text{ W}$$

Problem 2.43

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.43.

Find:

The equivalent resistance at terminals a,b in the case that terminals c,d are a) open b) shorted; the same for terminals c,d with respect to terminals a,b.

Analysis:

With terminals c-d open, $R_{eq} = (360 + 540) \parallel (180 + 540) \Omega = 400 \Omega$,

with terminals c-d shorted, $R_{eq} = (360 \parallel 180) + (540 \parallel 540) \Omega = 390 \Omega$,

with terminals a-b open, $R_{eq} = (540 + 540) \parallel (360 + 180) \Omega = 360 \Omega$,

with terminals a-b shorted, $R_{eq} = (360 \parallel 540) + (180 \parallel 540) \Omega = 351 \Omega$.

Problem 2.44

Solution:

Known quantities:

Layout of the site shown in Figure P2.44; characteristics of the cables; rated voltage of the generator; range of voltages and currents absorbed by the engine at full load.

Find:

The minimum AWG gauge conductors which must be used in a rubber insulated cable.

Analysis:

The cable must meet two requirements:

1. The conductor current rating must be greater than the rated current of the motor at full load. This requires AWG #14.
2. The voltage drop due to the cable resistance must not reduce the motor voltage below its minimum rated voltage at full load.

$$\text{KVL: } -V_G + V_{RC1} + V_{M-\text{Min}} + V_{RC2} = 0$$

$$-V_G + I_{M-FL} R_{C1} + V_{M-\text{Min}} + I_{M-FL} R_{C2} = 0$$

$$R_{C1} + R_{C2} = \frac{V_G - V_{M-\text{Min}}}{I_{M-FL}} =$$

$$= \frac{110 \text{ V} - 105 \text{ V}}{7.103 \text{ A}} = 703.9 \text{ m}\Omega$$

$$R_{\text{Max}} = \frac{R_{C1}}{d} = \frac{R_{C2}}{d} = \frac{\frac{1}{2} [703.9 \text{ m}\Omega]}{150 \text{ m}} = 2.346 \text{ m} \frac{\Omega}{\text{m}}$$

Therefore, AWG #8 or larger wire must be used.

Problem 2.45**Solution:****Known quantities:**

Layout of the building shown in Figure P2.45; characteristics of the cables; rated voltage of the generator; total electrical load in the building.

Find:

The minimum AWG gauge conductors which must be used in a rubber insulated cable.

Analysis:

The cable must meet two requirements:

1. The conductor current rating must be greater than the rated current of the motor at full load. This requires AWG #4.
2. The voltage drop due to the cable resistance must not reduce the motor voltage below its minimum rated voltage at full load.

$$\text{KVL: } -V_S + V_{RC1} + V_{L-\text{Min}} + V_{RC2} = 0$$

$$-V_S + I_{L-FL} R_{C1} + V_{L-\text{Min}} + I_{L-FL} R_{C2} = 0$$

$$R_{C1} + R_{C2} = \frac{V_S - V_{L-\text{Min}}}{I_{L-FL}} =$$

$$= \frac{450 \text{ V} - 446 \text{ V}}{51.57 \text{ A}} = 77.6 \text{ m}\Omega$$

$$R_{\text{Max}} = \frac{R_{C1}}{d} = \frac{R_{C2}}{d} = \frac{\frac{1}{2} [77.6 \text{ m}\Omega]}{85 \text{ m}} = 0.4565 \text{ m} \frac{\Omega}{\text{m}}$$

Therefore, AWG #0 or larger wire must be used.

Problem 2.46

Solution:

Known quantities:

Layout of the site shown in Figure P2.46; characteristics of the cables; rated voltage of the generator; electrical characteristics of the engine.

Find:

The maximum length of a rubber insulated cable with AWG #14 which can be used to connect the motor and the generator.

Analysis:

The voltage drop due to the cable resistance must not reduce the motor voltage below its minimum rated voltage at full load.

$$\begin{aligned}
 \text{KVL: } -V_G + V_{RC1} + V_{M-\text{Min}} + V_{RC2} &= 0 \\
 -V_G + I_{M-\text{FL}} R_{C1} + V_{M-\text{Min}} + I_{M-\text{FL}} R_{C2} &= 0 \\
 R_{C1} + R_{C2} &= \frac{V_G - V_{M-\text{Min}}}{I_{M-\text{FL}}} = \\
 &= \frac{110 \text{ V} - 105 \text{ V}}{7.103 \text{ A}} = 703.9 \text{ m}\Omega \\
 d_{\text{Max}} = \frac{R_{C1}}{R_{\text{rated}}} = \frac{R_{C2}}{R_{\text{rated}}} &= \frac{\frac{1}{2} [703.9 \text{ m}\Omega]}{8.285 \text{ m} \frac{\Omega}{\text{m}}} = 42.48 \text{ m}
 \end{aligned}$$

Problem 2.47

Solution:

Known quantities:

Layout of the building shown in Figure P2.47; characteristics of the cables; rated voltage of the generator; total electrical load in the building.

Find:

The maximum length of a rubber insulated cable with AWG #4 which can be used to connect the source to the load.

Analysis:

The voltage drop due to the cable resistance must not reduce the motor voltage below its minimum rated voltage at full load.

$$\begin{aligned}
 \text{KVL: } -V_S + V_{RC1} + V_{L-Min} + V_{RC2} &= 0 \\
 -V_S + I_{L-FL} R_{C1} + V_{L-Min} + I_{L-FL} R_{C2} &= 0 \\
 R_{C1} + R_{C2} &= \frac{V_S - V_{L-Min}}{I_{L-FL}} = \\
 &= \frac{450 \text{ V} - 446 \text{ V}}{51.57 \text{ A}} = 77.6 \text{ m}\Omega
 \end{aligned}$$

$$d_{Max} = \frac{R_{C1}}{R_{rated}} = \frac{R_{C2}}{R_{rated}} = \frac{\frac{1}{2}[77.6 \text{ m}\Omega]}{0.8153 \text{ m} \frac{\Omega}{m}} = 47.59 \text{ m}$$

Problem 2.48

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.48 with resistances, $R_1 = 2.2\text{k}\Omega$, $R_2 = 18\text{k}\Omega$, $R_3 = 220\text{k}\Omega$, $R_4 = 3.3\text{k}\Omega$.

Find:

The equivalent resistance between A and B.

Analysis:

Shorting nodes C and D creates a single node to which all four resistors are connected.

$$R_{eq1} = R_1 \parallel R_3 = \frac{R_1 R_3}{R_1 + R_3} = \frac{[2.2 \text{ K}\Omega][220 \text{ K}\Omega]}{2.2 + 220 \text{ K}\Omega} = 1.499 \text{ K}\Omega$$

$$R_{eq2} = R_2 \parallel R_4 = \frac{R_2 R_4}{R_2 + R_4} = \frac{[18 \text{ K}\Omega][3.3 \text{ K}\Omega]}{18 + 3.3 \text{ K}\Omega} = 2.789 \text{ K}\Omega$$

$$R_{eq} = R_{eq1} + R_{eq2} = 1.499 + 2.789 \text{ K}\Omega = 4.288 \text{ K}\Omega$$

Problem 2.49

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.49 with source voltage, $V_s = 12\text{V}$; and resistances,

$$R_1 = 11\text{k}\Omega, R_2 = 220\text{k}\Omega, R_3 = 6.8\text{k}\Omega, R_4 = 0.22\text{m}\Omega$$

Find:

The voltage between nodes A and B.

Analysis:

The same current flows through R1 and R3. Therefore, they are connected in series. Similarly, R2 and R4 are connected in series.

SPECIFY THE ASSUMED POLARITY OF THE VOLTAGE BETWEEN NODES A AND B. THIS WILL HAVE TO BE A WILD GUESS AT THIS POINT.

Specify the polarities of the voltage across R3 and R4 which will be determined using voltage division. The actual polarities are not difficult to determine. Do so.

$$VD: V_{R3} = \frac{V_S R_3}{R_1 + R_3} = \frac{[12 V][6.8 k\Omega]}{11 + 6.8 k\Omega} = 4.584 V$$

$$VD: V_{R4} = \frac{V_S R_4}{R_2 + R_4} = \frac{[12 V][0.22 \times 10^{-6} k\Omega]}{(220 + 0.22 \times 10^{-6}) k\Omega} = 1.20 \times 10^{-8} V \approx 0$$

$$KVL: -V_{R3} + V_{AB} + V_{R4} = 0 \therefore V_{AB} = V_{R3} - V_{R4} = 4.584 V$$

The voltage is negative indicating that the polarity of V_{AB} is opposite of that specified.

A solution is not complete unless the assumed positive direction of a current or assumed positive polarity of a voltage IS SPECIFIED ON THE CIRCUIT.

Problem 2.50**Solution:****Known quantities:**

Schematic of the circuit shown in Figure P2.49 with source voltage, $V_s = 5V$; and resistances,

$$R_1 = 2.2k\Omega, R_2 = 18k\Omega, R_3 = 4.7k\Omega, R_4 = 3.3k\Omega$$

Find:

The voltage between nodes A and B.

Analysis:

The same current flows through R1 and R3. Therefore, they are connected in series. Similarly, R2 and R4 are connected in series.

SPECIFY THE ASSUMED POLARITY OF THE VOLTAGE BETWEEN NODES A AND B. THIS WILL HAVE TO BE A WILD GUESS AT THIS POINT.

Specify the polarities of the voltage across R3 and R4 which will be determined using voltage division. The actual polarities are not difficult to determine. Do so.

$$VD: V_{R3} = \frac{V_S R_3}{R_1 + R_3} = \frac{[5 V][4.7 K\Omega]}{2.2 K\Omega + 4.7 K\Omega} = 3.406 V$$

$$VD: V_{R4} = \frac{V_S R_4}{R_2 + R_4} = \frac{[5 V][3.3 K\Omega]}{18 K\Omega + 3.3 K\Omega} = 0.917 V$$

$$KVL: -V_{R3} + V_{AB} + V_{R4} = 0 \Rightarrow V_{AB} = V_{R3} - V_{R4} = 2.489 V$$

A solution is not complete unless the assumed positive direction of a current or assumed positive polarity of a voltage IS SPECIFIED ON THE CIRCUIT.

Problem 2.51**Solution:****Known quantities:**

Schematic of the circuit shown in Figure P2.51 with source voltage, $V_s = 12V$; and resistances,

$$R_1 = 1.7m\Omega, R_2 = 3k\Omega, R_3 = 10k\Omega$$

Find:

The voltage across the resistance R_3 .

Analysis:

The same voltage appears across both R_2 and R_3 and, therefore, these element are in parallel. Applying the voltage divider rule:

$$V_{R_3} = \frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3} V_s = \frac{2.3k\Omega}{1.7m\Omega + 2.3k\Omega} 12 V = 11.999991 V \quad (\oplus \text{ down})$$

Note that since $R_1 \ll R_2 \parallel R_3$, then $V_{R_3} \cong V_s$.

Sections 2.7, 2.8: Practical Sources and Measuring Devices

Problem 2.52

Solution:

Known quantities:

Parameters $R_0 = 300 \, \Omega$ (resistance at temperature $T_0 = 298 \, \text{K}$), and $\beta = -0.01 \, \text{K}^{-1}$, value of the second resistor.

Find:

- Plot $R_{th}(T)$ versus T in the range $350 \leq T \leq 750 \, [^\circ\text{K}]$
- The equivalent resistance of the parallel connection with the $250\text{-}\Omega$ resistor; plot $R_{eq}(T)$ versus T in the range $350 \leq T \leq 750 \, [^\circ\text{K}]$ for this case on the same plot as part a.

Assumptions:

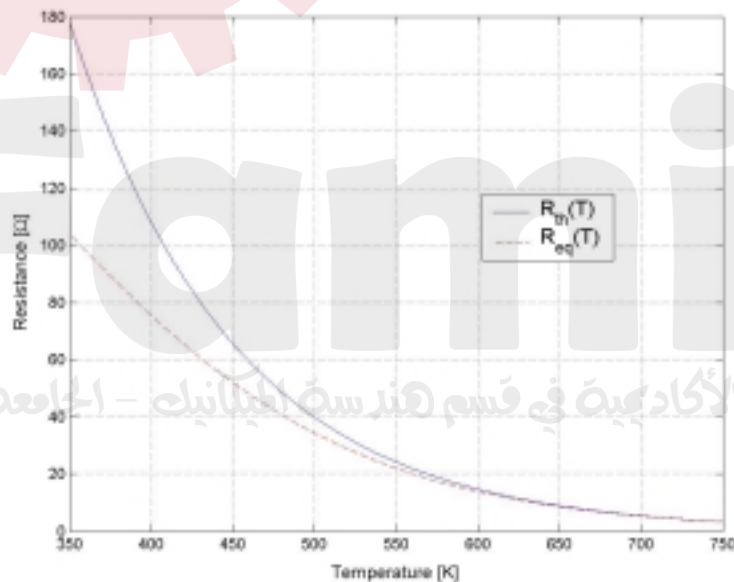
$$R_{th}(T) = R_0 e^{-\beta(T-T_0)}.$$

Analysis:

$$\text{a) } R_{th}(T) = 300 e^{-0.01(T-298)}$$

$$\text{b) } R_{eq}(T) = R_{th}(T) \parallel 250 \, \Omega = \frac{1500 e^{-0.01(T-298)}}{5 + 6 e^{-0.01(T-298)}}$$

The two plots are shown below.



In the above plot, the solid line is for the thermistor alone; the dashed line is for the thermistor-resistor combination.

Problem 2.53

Solution:

Known quantities:

A potentiometer shown in the circuit of Figure P.253 with The value of the resistance R_m , the total length of the resistor x_T and voltage source V_S .

Find:

- The expression for $v_{out}(x)$. Plot v_{out}/V_S versus x/x_T .
- The distance x when $v_{out} = 5\text{ V}$.
- Assuming the resistance R_m becomes infinite, repeat parts a and b

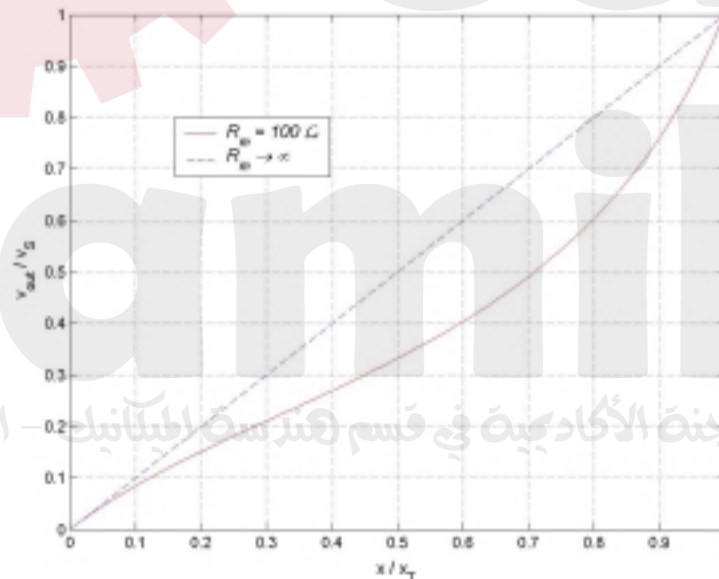
Assumptions:

$$\frac{v_{out}}{V_S} = \frac{1}{1/(x/x_T) + (R_p/R_m)(1 - x/x_T)}$$

$$R_p(x) = 200e^x$$

Analysis:

$$a) v_{out}(x) = \frac{10}{1/(x/0.02) + (200e^x/100)(1 - x/0.02)} = 500 \frac{x}{1 - 10^2 e^x x + 5 \cdot 10^3 e^x x^2}$$



In the above plot, the solid line is for $R_m = 100\ \Omega$; the dashed line is for $R_m \rightarrow \infty$.

$$b) x(v_{out} = 5\text{ V}) = 14.18\text{ cm}$$

$$c) \text{ Now } R_m \rightarrow \infty.$$

$$v_{out}(x) = \frac{10}{1/(x/0.02) + (200e^x/\infty)(1-x/0.02)} \rightarrow 500x$$

$$x(v_{out} = 5 \text{ V}) = 10 \text{ cm}$$

Problem 2.54

Solution:

Known quantities:

Meter resistance of the coil; meter current for full scale deflection; max measurable pressure.

Find:

- The circuit required to indicate the pressure measured by a sensor
- The value of each component of the circuit; the linear range
- The maximum pressure that can accurately be measured.

Assumptions:

Sensor characteristics follow what is shown in Figure P2.54

Analysis:

- A series resistor to drop excess voltage is required.
- At full scale, meter:

$$I_{\hat{m}FS} = 10 \mu A$$

$$r_{\hat{m}} = 200 \Omega$$

$$0.L.: V_{\hat{m}FS} = I_{\hat{m}FS} r_{\hat{m}} = 2 \text{ mV}.$$

at full scale, sensor (from characteristics):

$$P_{FS} = 100 \text{ kPa}$$

$$V_{TFS} = 9.5 \text{ mV}$$

$$KVL: -V_{TFS} + V_{RFS} + V_{\hat{m}FS} = 0$$

$$V_{RFS} = V_{TFS} - V_{\hat{m}FS} = 9.5 \text{ mV} - 2 \text{ mV} = 7.5 \text{ mV}$$

$$I_{RFS} = I_{TFS} = I_{\hat{m}FS} = 10 \mu A$$

$$\text{Ohm law: } R = \frac{V_{RFS}}{I_{RFS}} = \frac{7.5 \text{ mV}}{10 \mu A} = 750 \Omega.$$

- from sensor characteristic: 30 kPa – 110 kPa.

Problem 2.55

Solution:

Known quantities:

Meter resistance of the coil; meter current for full scale deflection; max measurable pressure.

Find:

- Redesign the circuit to meet the specification.
- The value of each component of the circuit.
- The linear range of the system.

Assumptions:

Sensor characteristics follow what is shown in Figure P2.55

Analysis:

a) A series resistor to drop excess voltage is required.

b) At full scale, meter:

$$I_{\hat{m}FS} = 50 \mu A$$

$$r_{\hat{m}} = 1.8 k\Omega$$

$$0.L.: V_{\hat{m}FS} = I_{\hat{m}FS} r_{\hat{m}} = 90 mV.$$

at full scale, sensor (from characteristics):

$$V_{TFS} = 9.5 V$$

$$KVL: -V_{TFS} + V_{RFS} + V_{\hat{m}FS} = 0$$

$$V_{RFS} = V_{TFS} - V_{\hat{m}FS} = 9.5 V - 90 mV = 9.41 V$$

$$I_{RFS} = I_{TFS} = I_{\hat{m}FS} = 50 \mu A$$

$$\text{Ohm law: } R = \frac{V_{RFS}}{I_{RFS}} = \frac{9.41 V}{50 \mu A} = 188.2 k\Omega.$$

c) from sensor characteristic: 20 kPa – 110 kPa.

Problem 2.56**Solution:****Known quantities:**

Meter resistance of the coil; meter voltage for full scale deflection; max measurable temperature.

Find:

- The circuit required to meet the specifications of the new sensor.
- The value of each component of the circuit.
- The linear range of the system.

Assumptions:

Sensor characteristics follow what is shown in Figure P2.56

Analysis:

a) A parallel resistor is required to shunt (bypass) the excess current.

b) At full scale, meter:

$$V_{\hat{m}FS} = 250 mV$$

$$r_{\hat{m}} = 2.5 k\Omega$$

$$0.L.: I_{\hat{m}FS} = \frac{V_{\hat{m}FS}}{r_{\hat{m}}} = 100 \mu A.$$

at full scale, sensor (from characteristics):

$$T_{FS} = 400 ^\circ C$$

$$I_{TFS} = 8.5 mA$$

$$KCL: -I_{TFS} + I_{RFS} + I_{\hat{m}FS} = 0$$

$$I_{RFS} = I_{TFS} - I_{\hat{m}FS} = 8.5 \text{ mA} - 100 \text{ } \mu\text{A} = 8.4 \text{ mA}$$

$$V_{RFS} = V_{TFS} = V_{\hat{m}FS} = 250 \text{ mV}$$

$$\text{Ohm law: } R = \frac{V_{RFS}}{I_{RFS}} = \frac{250 \text{ mV}}{8.4 \text{ mA}} = 29.76 \text{ } \Omega.$$

c) from sensor characteristic: 220 °C – 410 °C.

Problem 2.57

Solution:

Known quantities:

Meter resistance of the coil; meter voltage at full scale; max measurable temperature.

Find:

- The circuit required to meet the specifications of the new sensor.
- The value of each component of the circuit
- The linear range of the system.

Assumptions:

Sensor characteristics follow what is shown in Figure P2.57

Analysis:

- A parallel resistor is required to shunt (bypass) the excess current.
- At full scale, meter:

$$V_{\hat{m}FS} = 250 \text{ mV}$$

$$r_{\hat{m}} = 2.5 \text{ k}\Omega$$

$$\text{O.L.: } I_{\hat{m}FS} = \frac{V_{\hat{m}FS}}{r_{\hat{m}}} = 100 \text{ } \mu\text{A}.$$

at full scale, sensor (from characteristics):

$$T_{FS} = 400 \text{ } ^\circ\text{C}$$

$$I_{TFS} = 8.5 \text{ mA}$$

$$\text{KCL: } -I_{TFS} + I_{RFS} + I_{\hat{m}FS} = 0$$

$$I_{RFS} = I_{TFS} - I_{\hat{m}FS} = 8.5 \text{ mA} - 100 \text{ } \mu\text{A} = 8.4 \text{ mA}$$

$$V_{RFS} = V_{TFS} = V_{\hat{m}FS} = 250 \text{ mV}$$

$$\text{Ohm law: } R = \frac{V_{RFS}}{I_{RFS}} = \frac{250 \text{ mV}}{8.4 \text{ mA}} = 29.76 \text{ } \Omega.$$

c) from sensor characteristic: 220 °C – 410 °C.

Problem 2.58

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.58; voltage at terminals with switch open and closed for fresh battery; same voltages for the same battery after 1 year.

Find:

The internal resistance of the battery in each case.

Analysis:

a)

$$V_{out} = \left(\frac{10}{10 + r_B} \right) V_{oc}$$

$$r_B = 10 \left(\frac{V_{oc}}{V_{out}} - 1 \right) = 10 \left(\frac{2.28}{2.27} - 1 \right)$$

$$= 0.044 \Omega$$

b)

$$r_B = 10 \left(\frac{V_{oc}}{V_{out}} - 1 \right) = 10 \left(\frac{2.2}{0.31} - 1 \right)$$

$$= 60.97 \Omega$$

Problem 2.59**Solution:****Known quantities:**

Ammeter shown in Figure P2.59; Current for full-scale deflection; desired full scale values.

Find:

Value of the resistors required for the given full scale ranges.

Analysis:

We desire R_1 , R_2 , R_3 such that $I_a = 30 \mu A$ for $I = 10 \text{ mA}$, 100 mA , and 1 A , respectively. We use conductances to simplify the arithmetic:

$$G_a = \frac{1}{R_a} = \frac{1}{1000} \text{ S}$$

$$G_{1,2,3} = \frac{1}{R_{1,2,3}}$$

By the current divider rule:

$$I_a = \frac{G_a}{G_a + G_x} I$$

or:

$$G_x = G_a \left(\frac{I}{I_a} \right) - G_a \text{ or } \frac{1}{G_x} = \frac{1}{G_a} \left(\frac{I_a}{I - I_a} \right)$$

$$R_x = R_a \left(\frac{I_a}{I - I_a} \right).$$

We can construct the following table:

x	I	R_x (Approx.)
1	10^{-2} A	3Ω
2	10^{-1} A	0.3Ω
3	10^0 A	0.03Ω

Problem 2.60

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.60; for part b: value of R_p and current displayed on the ammeter.

Find:

The current i ; the internal resistance of the meter.

Assumptions:

$$r_a \ll 50 \text{ k}\Omega$$

Analysis:

a) Assuming that $r_a \ll 50 \text{ k}\Omega$

$$i \approx \frac{V_s}{R_s} = \frac{12}{50000} = 240 \mu\text{A}$$

b) With the same assumption as in part a)

$$i_{\text{meter}} = 150 \cdot (10)^{-6} = \frac{R_p}{r_a + R_p} i$$

or:

$$150 \cdot (10)^{-6} = \frac{15}{r_a + 15} 240 \cdot 10^{-6}.$$

Therefore, $r_a = 9 \Omega$.

Problem 2.61

Solution:

Known quantities:

Voltage read at the meter; schematic of the circuit shown in Figure P2.61 with source voltage,

$V_s = 12\text{V}$ and source resistance, $R_s = 25\text{k}\Omega$.

Find:

The internal resistance of the voltmeter.

Analysis:

Using the voltage divider rule:

$$V = 11.81 = \frac{r_m}{r_m + R_s} (12)$$

Therefore, $r_m = 1.55 \text{ M}\Omega$.

Problem 2.62**Solution:****Known quantities:**

Circuit shown in Figure P2.61 with source voltage, $V_s = 24\text{V}$; and ratios between R_s and r_m .

Find:

The meter reads in the various cases.

Analysis:

By voltage division:

$$V = \frac{r_m}{r_m + R_s} (24)$$

R_s	V
$0.2 r_m$	20 V
$0.4 r_m$	17.14 V
$0.6 r_m$	15 V
$1.2 r_m$	10.91 V
$4 r_m$	4.8 V
$6 r_m$	3.43 V
$10 r_m$	2.18 V

For a voltmeter, we always desire $r_m \gg R_s$.

Problem 2.63**Solution:****Known quantities:**

Schematic of the circuit shown in Figure P2.63, values of the components.

Find:

The voltage across R_4 with and without the voltmeter for the following values:

- a) $R_4 = 100\Omega$
- b) $R_4 = 1k\Omega$
- c) $R_4 = 10k\Omega$
- d) $R_4 = 100k\Omega$.

Assumptions:

The voltmeter behavior is modeled as that of an ideal voltmeter in parallel with a 120- k Ω resistor.

Analysis:

We develop first an expression for V_{R_4} in terms of R_4 . Next, using current division:

$$\begin{cases} I_{R_1} = I_S \left(\frac{R_S}{R_S + R_1 + R_2 \parallel (R_3 + R_4)} \right) \\ I_{R_4} = I_{R_1} \left(\frac{R_2}{R_2 + R_3 + R_4} \right) \end{cases}$$

Therefore,

$$\begin{aligned} I_{R_4} &= I_S \left(\frac{R_S}{R_S + R_1 + R_2 \parallel (R_3 + R_4)} \right) \cdot \left(\frac{R_2}{R_2 + R_3 + R_4} \right) \\ V_{R_4} &= I_{R_4} R_4 \\ &= I_S \left(\frac{R_S R_4}{R_S + R_1 + R_2 \parallel (R_3 + R_4)} \right) \cdot \left(\frac{R_2}{R_2 + R_3 + R_4} \right) \\ &= \frac{66000 \cdot R_4}{R_4 + 2.1352 \cdot 10^6} \end{aligned}$$

Without the voltmeter:

- a) $V_{R_4} = 3.08 \text{ V}$
- b) $V_{R_4} = 30.47 \text{ V}$
- c) $V_{R_4} = 269.91 \text{ V}$
- d) $V_{R_4} = 1260.7 \text{ V}$.

Now we must find the voltage drop across R_4 with a 120-k Ω resistor across R_4 . This is the voltage that the voltmeter will read.

$$\begin{aligned} I_{R_4} &= I_S \left(\frac{R_S}{R_S + R_1 + R_2 \parallel (R_3 + (R_4 \parallel 120k\Omega))} \right) \cdot \left(\frac{R_2}{R_2 + R_3 + (R_4 \parallel 120k\Omega)} \right) \\ V_{R_4} &= I_{R_4} R_4 \\ &= I_S \left(\frac{R_S R_4}{R_S + R_1 + R_2 \parallel (R_3 + (R_4 \parallel 120k\Omega))} \right) \cdot \left(\frac{R_2}{R_2 + R_3 + (R_4 \parallel 120k\Omega)} \right) \\ &= 82.5 \frac{(120000 + R_4) \cdot R_4}{7319R_4 + 320.28 \cdot 10^6} \end{aligned}$$

With the voltmeter:

- a) $V_{R_4} = 3.08 \text{ V}$
- b) $V_{R_4} = 30.47 \text{ V}$
- c) $V_{R_4} = 272.57 \text{ V}$
- d) $V_{R_4} = 1724.99 \text{ V}$.

Problem 2.64

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.64, value of the components.

Find:

The current through R_5 both with and without the ammeter, for the following values of the resistor R_5 :

- a) $R_5 = 1k\Omega$
- b) $R_5 = 100\Omega$
- c) $R_5 = 10\Omega$
- d) $R_5 = 1\Omega$.

Analysis:

First we should find an expression for the current through R_5 in terms of R_5 and the meter resistance, R_m . By the voltage divider rule we have:

$$V_{R_3} = \frac{R_3 \parallel (R_4 + R_5 + R_m) V_s}{R_3 \parallel (R_4 + R_5 + R_m) + R_2 + (R_1 \parallel R_s)}$$

and

$$I_{R_3} = \frac{V_{R_3}}{R_4 + R_5 + R_m}$$

Therefore,

$$\begin{aligned} I_{R_3} &= \frac{R_3 \parallel (R_4 + R_5 + R_m) V_s}{R_3 \parallel (R_4 + R_5 + R_m) + R_2 + (R_1 \parallel R_s)} \cdot \frac{1}{R_4 + R_5 + R_m} \\ &= \frac{5904}{208350 + 373 \cdot (R_m + R_s)} \end{aligned}$$

Using the above equation will give us the following table:

	with meter in circuit	without meter in circuit
a	10.15 mA	9.99 mA
b	24.03 mA	23.15 mA
c	27.84 mA	26.67 mA
d	28.29 mA	27.08 mA

Problem 2.65

Solution:

Known quantities:

Schematic of the circuit and geometry of the beam shown in Figure P2.65, characteristics of the material, reads on the bridge.

Find:

The force applied on the beam.

Assumptions:

Gage Factor for Strain gauge is 2

Analysis:

R_1 and R_2 are in series; R_3 and R_4 are in series.

$$\text{Voltage Division: } V_{R_2} = \frac{V_S R_2}{R_1 + R_2} = \frac{V_S (R_0 - \Delta R)}{R_0 + \Delta R + R_0 - \Delta R} = \frac{V_S (R_0 - \Delta R)}{2R_0}$$

$$\text{Voltage Division: } V_{R_4} = \frac{V_S R_4}{R_3 + R_4} = \frac{V_S (R_0 + \Delta R)}{R_0 - \Delta R + R_0 + \Delta R} = \frac{V_S (R_0 + \Delta R)}{2R_0}$$

$$\text{KVL: } -V_{R_2} - V_{BA} + V_{R_4} = 0$$

$$V_{BA} = V_{R_4} - V_{R_2} = \frac{V_S (R_0 + \Delta R)}{2R_0} - \frac{V_S (R_0 - \Delta R)}{2R_0} = \frac{V_S 2\Delta R}{2R_0} = V_S GF \epsilon = \frac{V_S (2)(6)LF}{wh^2 Y}$$

$$F = \frac{V_{BA} wh^2 Y}{V_S 12L} = \frac{0.050 \text{ V} (0.025 \text{ m})(0.100 \text{ m})^2 69 \times 10^9 \frac{\text{N}}{\text{m}^2}}{12 \text{ V}(12) 0.3 \text{ m}} = 19.97 \text{ kN}.$$

Problem 2.66

Solution:

Known quantities:

Schematic of the circuit and geometry of the beam shown in Figure P2.65, characteristics of the material, reads on the bridge.

Find:

The force applied on the beam.

Assumptions:

Gage Factor for Strain gauge is 2

Analysis: R_1 and R_2 are in series; R_3 and R_4 are in series.

$$\text{VD: } V_{R_2} = \frac{V_S R_2}{R_1 + R_2} = \frac{V_S (R_0 - \Delta R)}{R_0 + \Delta R + R_0 - \Delta R} = \frac{V_S (R_0 - \Delta R)}{2R_0}$$

$$\text{VD: } V_{R_4} = \frac{V_S R_4}{R_3 + R_4} = \frac{V_S (R_0 - \Delta R)}{R_0 - \Delta R + R_0 + \Delta R} = \frac{V_S (R_0 + \Delta R)}{2R_0}$$

$$\text{KVL: } -V_{R_2} - V_{BA} + V_{R_4} = 0$$

$$V_{BA} = V_{R_4} - V_{R_2} = \frac{V_S (R_0 + \Delta R)}{2R_0} - \frac{V_S (R_0 - \Delta R)}{2R_0} = \frac{V_S 2\Delta R}{2R_0} = V_S GF \epsilon = \frac{V_S (2)(6)LF}{wh^2 y}$$

$$F = 1.3 \times 10^6 \text{ N} = \frac{V_{BA} wh^2 y}{V_S 12L} = \frac{V_{BA} (0.03 \text{ m})(0.07 \text{ m})^2 200 \times 10^9 \frac{\text{N}}{\text{m}^2}}{24V(12)1.7 \text{ m}}$$

$$V_{BA} = \frac{1.3 \times 10^6 \text{ N} \times 24V(12)1.7 \text{ m}}{(0.03 \text{ m})(0.07 \text{ m})^2 200 \times 10^9 \frac{\text{N}}{\text{m}^2}} = 21.6 \text{ V}$$

Chapter 3 Instructor Notes

Chapter 3 presents the principal topics in the analysis of resistive (DC) circuits. The presentation of node voltage and mesh current analysis is supported by several solved examples and drill exercises, with emphasis placed on developing consistent solution methods, and on reinforcing the use of a systematic approach. The aim of this style of presentation, which is perhaps more detailed than usual in a textbook written for a non-majors audience, is to develop good habits early on, with the hope that the orderly approach presented in Chapter 3 will facilitate the discussion of AC and transient analysis in Chapters 4 and 5. *Make The Connection* sidebars (pp. 75-77) introduce analogies between electrical and thermal circuit elements. These analogies are encountered again in Chapter 5. A brief discussion of the principle of superposition precedes the discussion of Thévenin and Norton equivalent circuits. Again, the presentation is rich in examples and drill exercises, because the concept of equivalent circuits will be heavily exploited in the analysis of AC and transient circuits in later chapters. The *Focus on Methodology* boxes (p.76 – Node Analysis; p. 86 – Mesh Analysis; pp. 103, 107, 111 – Equivalent Circuits) provide the student with a systematic approach to the solution of all basic network analysis problems.

After a brief discussion of maximum power transfer, the chapter closes with a section on nonlinear circuit elements and load-line analysis. This section can be easily skipped in a survey course, and may be picked up later, in conjunction with Chapter 9, if the instructor wishes to devote some attention to load-line analysis of diode circuits. Finally, those instructors who are used to introducing the op-amp as a circuit element, will find that sections 8.1 and 8.2 can be covered together with Chapter 3, and that a good complement of homework problems and exercises devoted to the analysis of the op-amp as a circuit element is provided in Chapter 8.

The homework problems present a graded variety of circuit problems. Since the aim of this chapter is to teach solution techniques, there are relatively few problems devoted to applications. We should call the instructor's attention to the following end-of-chapter problems: 3.8 and 3.19 on the Wheatstone bridge; 3.21, 3.22, 3.23, on three-wire residential distribution service; 3.24, 3.25, 3.26 on AC three-phase electrical distribution systems; 3.28-3.31 on fuses; 3.62-66 on various nonlinear resistance devices.

Learning Objectives

1. Compute the solution of circuits containing linear resistors and independent and dependent sources using *node analysis*.
2. Compute the solution of circuits containing linear resistors and independent and dependent sources using *mesh analysis*.
3. Apply the *principle of superposition* to linear circuits containing independent sources.
4. Compute *Thévenin and Norton equivalent circuits* for networks containing linear resistors and independent and dependent sources.
5. Use equivalent circuits ideas to compute the *maximum power transfer* between a source and a load.
6. Use the concept of equivalent circuit to determine voltage, current and power for nonlinear loads using *load-line analysis* and analytical methods.

Sections 3.1, 3.2, 3.3, 3.4: Nodal and Mesh Analysis

I

Focus on Methodology: Node Voltage Analysis Method

1. Select a reference node(usually ground). This node usually has most elements tied to it. All other nodes will be referenced to this node.
2. Define the remaining $n-1$ node voltages as the independent or dependent variables. Each of the m voltage sources in the circuit will be associated with a dependent variable. If a node is not connected to a voltage source, then its voltage is treated as an independent variable.
3. Apply KCL at each node labeled as an independent variable, expressing each current in terms of the adjacent node voltages.

Focus on Methodology: Mesh Current Analysis Method

1. Define each mesh current consistently. Unknown mesh currents will be always defined in the clockwise direction; known mesh currents (i.e., when a current source is present) will always be defined in the direction of the current source.
2. In a circuit with n meshes and m current sources, $n-m$ independent equations will result. The unknown mesh currents are the $n-m$ independent variables.
3. Apply KVL to each mesh containing an unknown mesh current, expressing each voltage in terms of one or more mesh currents..
4. Solve the linear system of $n-m$ unknowns.

Problem 3.1**Solution:****Known quantities:**

Circuit shown in Figure P2.1 with mesh currents: $I_1 = 5 \text{ A}$, $I_2 = 3 \text{ A}$, $I_3 = 7 \text{ A}$.

Find:

The branch currents through:

- a) R_1 ,
- b) R_2 ,
- c) R_3 .

Analysis:

a) Assume a direction for the current through R_1 (e.g., from node A to node B). Then summing currents at node A:

$$KCL: \quad -I_1 + I_{R1} + I_3 = 0$$

$$I_{R1} = I_1 - I_3 = -2 \text{ A}$$

This can also be done by inspection noting that the assumed direction of the current through R_1 and the direction of I_1 are the same.

b) Assume a direction for the current through R_2 (e.g., from node B to node A). Then summing currents at node B:

$$KCL: \quad I_2 + I_{R2} - I_3 = 0$$

$$I_{R2} = I_3 - I_2 = 4 \text{ A}$$

This can also be done by inspection noting that the assumed direction of the current through R_2 and the direction of I_3 are the same.

c) Only one mesh current flows through R_3 . If the current through R_3 is assumed to flow in the same direction, then:

$$I_{R1} = I_3 = 7 \text{ A}.$$

Problem 3.2

Solution:

Known quantities:

Circuit shown in Figure P3.1 with source and node voltages:

$$V_{S1} = V_{S2} = 110 \text{ V}, \quad V_A = 103 \text{ V}, \quad V_B = -107 \text{ V}.$$

Find:

The voltage across each of the five resistors.

Analysis:

Assume a polarity for the voltages across R_1 and R_2 (e.g., from ground to node A, and from node B to ground). R_1 is connected between node A and ground; therefore, the voltage across R_1 is equal to this node voltage. R_2 is connected between node B and ground; therefore, the voltage across R_2 is equal to the negative of this voltage.

$$V_{R1} = V_A = 103 \text{ V}, \quad V_{R2} = -V_B = +107 \text{ V}$$

The two node voltages are with respect to the ground which is given.

Assume a polarity for the voltage across R_3 (e.g., from node B to node A). Then:

$$\text{KVL: } V_A + V_{R3} + V_B = 0$$

$$V_{R3} = V_A - V_B = 210 \text{ V}$$

Assume polarities for the voltages across R_4 and R_5 (e.g., from node A to ground, and from ground to node B):

$$\text{KVL: } -V_{S1} + V_{R4} + V_A = 0$$

$$V_{R4} = V_{S1} - V_A = 7 \text{ V}$$

$$\text{KVL: } -V_{S2} - V_B - V_{R5} = 0$$

$$V_{R5} = -V_{S2} - V_B = -3 \text{ V}$$

Problem 3.3

Solution:

Known quantities:

Circuit shown in Figure P3.3 with known source currents and resistances, $R_1 = 3\Omega$, $R_2 = 1\Omega$, $R_3 = 6\Omega$.

Find:

The currents I_1 , I_2 using node voltage analysis.

Analysis:

At node 1:

$$v_1 \cdot \left(\frac{1}{3} + 1 \right) + v_2 \cdot (-1) = 1$$

At node 2:

$$v_1 \cdot (-1) + v_2 \cdot \left(1 + \frac{1}{6} \right) = -2$$

Solving, we find that:

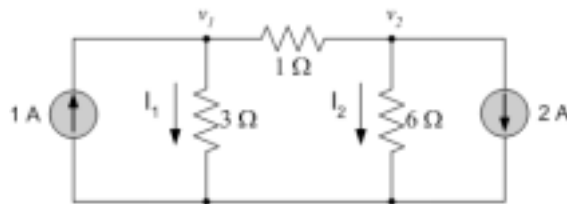
$$v_1 = -1.5 \text{ V}$$

$$v_2 = -3 \text{ V}$$

Then,

$$i_1 = \frac{v_1}{3} = -0.5 \text{ A}$$

$$i_2 = \frac{v_2}{6} = -0.5 \text{ A}$$

**Problem 3.4****Solution:****Known quantities:**

Circuit shown in Figure P3.3 with known source currents and resistances, $R_1 = 3\Omega$, $R_2 = 1\Omega$, $R_3 = 6\Omega$.

Find:

The currents I_1 , I_2 using mesh analysis.

Analysis:

At mesh (a):

$$i_a = 1 \text{ A}$$

At mesh (b):

$$3(i_b - i_a) + i_b + 6(i_b - i_c) = 0$$

At mesh (c):

$$i_c = 2 \text{ A}$$

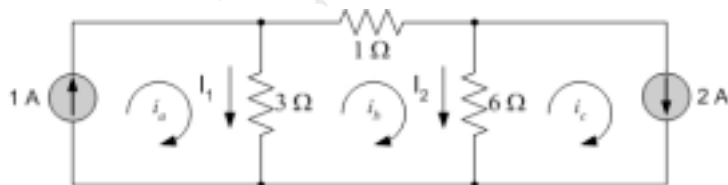
Solving, we find that:

$$i_b = 1.5 \text{ A}$$

Then,

$$i_1 = (i_a - i_b) = -0.5 \text{ A}$$

$$i_2 = (i_b - i_c) = -0.5 \text{ A}$$



Problem 3.5**Solution:****Known quantities:**

Circuit shown in Figure P3.5 with resistance values, current and voltage source values.

Find:

The current, i , through the voltage source using node voltage analysis.

Analysis:

At node 1:

$$\frac{v_1}{200} + \frac{v_1 - v_2}{5} + \frac{v_1 - v_3}{100} = 0$$

At node 2:

$$\frac{v_2 - v_1}{5} + i + 0.2 = 0$$

At node 3:

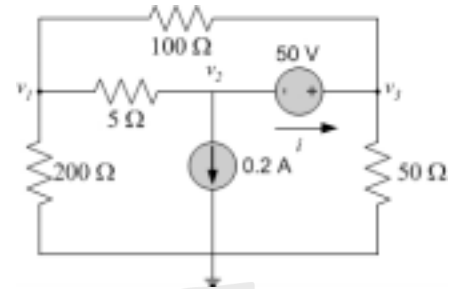
$$-i + \frac{v_3 - v_1}{100} + \frac{v_3}{50} = 0$$

For the voltage source we have:

$$v_3 - v_2 = 50 \text{ V}$$

Solving the system, we obtain:

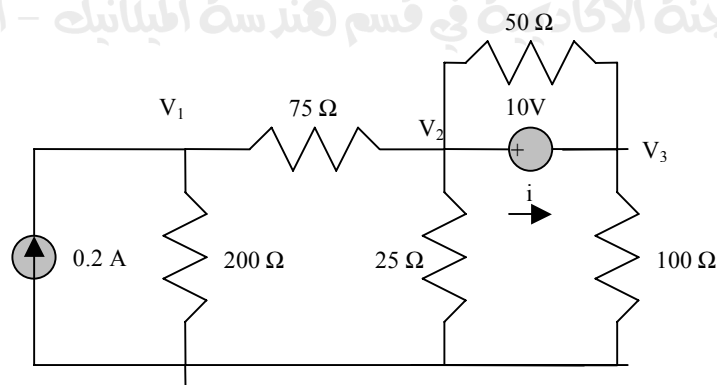
$$v_1 = -45.53 \text{ V}, v_2 = -48.69 \text{ V}, v_3 = 1.31 \text{ V} \text{ and, finally, } i = 491 \text{ mA}.$$

**Problem 3.6****Solution:****Known quantities:**

The current source value, the voltage source value and the resistance values for the circuit shown in Figure P3.6.

Find:

The three node voltages indicated in Figure P3.6 using node voltage analysis.

Analysis:

At node 1:

$$\frac{v_1}{200} + \frac{v_1 - v_2}{75} = 0.2 \text{ A}$$

At node 2:

$$\frac{v_2 - v_1}{75} + \frac{v_2}{25} + \frac{v_2 - v_3}{50} + i = 0$$

At node 3:

$$-i + \frac{v_3 - v_2}{50} + \frac{v_3}{100} = 0$$

For the voltage source we have:

$$v_3 + 10 = v_2$$

Solving the system, we obtain:

$$v_1 = 14.24 \text{ V}, v_2 = 4.58 \text{ V}, v_3 = -5.42 \text{ V} \text{ and, finally, } i = -254 \text{ mA}.$$

Problem 3.7

Solution:

Known quantities:

The voltage source value, 3 V, and the five resistance values, indicated in Figure P3.7.

Find:

The current, i , drawn from the independent voltage source using node voltage analysis.

Analysis:

At node 1:

$$\frac{v_1 - 3}{0.5} + \frac{v_1}{0.5} + \frac{v_1 - v_2}{0.25} = 0$$

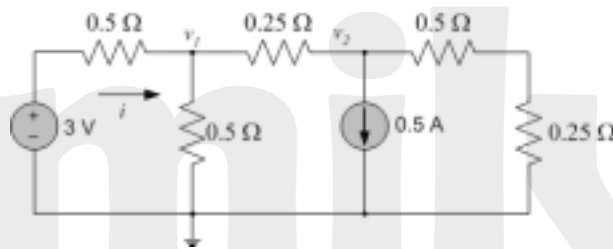
At node 2:

$$\frac{v_2 - v_1}{0.25} + \frac{v_2}{0.75} + 0.5 = 0$$

Solving the system, we obtain:

$$v_1 = 1.125 \text{ V}, v_2 = 0.75 \text{ V}$$

$$\text{Therefore, } i = \frac{3 - v_1}{0.5} = 3.75 \text{ A}.$$



Problem 3.8

Solution:

Known quantities:

The voltage source value, 15 V, and the four resistance values, indicated in Figure P3.8.

Find:

The voltage at nodes a and b , V_a and V_b , and their difference, $V_a - V_b$ using node voltage analysis.

Analysis:

Using nodal analysis at the two nodes a and b , we write the equations

$$\frac{V_b - 15}{18} + \frac{V_b}{20} = 0$$

$$\frac{V_a - 15}{36} + \frac{V_a}{20} = 0$$

Rearranging the equations,

$$38V_b - 300 = 0$$

$$14V_a - 75 = 0$$

Solving for the two unknowns,

$$V_a = 5.36 \text{ V} \quad \text{and} \quad V_b = 7.89 \text{ V}$$

Therefore,

$$V_a - V_b = -2.54 \text{ V}$$

Problem 3.9**Solution:****Known quantities:**

The voltage source value, 15 V, and the four resistance values, indicated in Figure P3.8.

Find:

The voltage at nodes a and b , V_a and V_b , and their difference, $V_a - V_b$ using mesh analysis.

Analysis:

Using mesh analysis at the two meshes a and b , we write the equations

$$36(i_a - i_b) + 20(i_a - i_b) = 15$$

$$18i_b + 20i_b + 20(i_b - i_a) + 36(i_b - i_a) = 0$$

Rearranging the equations,

$$i_a = \frac{15}{56} + i_b$$

$$94i_b - 56i_a = 0$$

Solving for the two unknowns,

$$i_a = 662 \text{ mA} \quad \text{and} \quad i_b = 395 \text{ mA}$$

Therefore,

$$V_a = 20(i_b - i_a) = 5.36 \text{ V}, \quad V_b = 20i_b = 7.89 \text{ V} \quad \text{and} \quad V_a - V_b = -2.54 \text{ V}.$$

Problem 3.10**Solution:****Known quantities:**

Circuit of Figure P3.10 with voltage source, V_S , current source, I_S , and all resistances.

Find:

- The node equations required to determine the node voltages.
- The matrix solution for each node voltage in terms of the known parameters.

Analysis:

a) Specify the nodes (e.g., A on the upper left corner of the circuit in Figure P3.10, and B on the right corner). Choose one node as the reference or ground node. If possible, ground one of the sources in the circuit. Note that this is possible here. When using KCL, assume all unknown current flow out of the node. The direction of the current supplied by the current source is specified and must flow into node A.

KCL:

$$-I_s + \frac{V_a - V_s}{R_2} + \frac{V_a - V_b}{R_1} = 0$$

$$V_a \left(\frac{1}{R_2} + \frac{1}{R_1} \right) + V_b \left(-\frac{1}{R_1} \right) = I_s + \frac{V_s}{R_2}$$

$$\frac{V_b - V_a}{R_1} + \frac{V_b - V_s}{R_3} + \frac{V_b - 0}{R_4} = 0$$

KCL:

$$V_a \left(-\frac{1}{R_1} \right) + V_b \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right) = \frac{V_s}{R_3}$$

b) Matrix solution:

$$V_a = \frac{\begin{vmatrix} I_s + \frac{V_s}{R_2} & -\frac{1}{R_1} \\ \frac{V_s}{R_3} & \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \end{vmatrix}}{\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \end{vmatrix}} = \frac{\left(I_s + \frac{V_s}{R_2} \right) \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right) - \left(\frac{V_s}{R_3} \right) \left(-\frac{1}{R_1} \right)}{\left(\frac{1}{R_1} + \frac{1}{R_2} \right) \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right) - \left(-\frac{1}{R_1} \right) \left(-\frac{1}{R_1} \right)}$$

$$V_b = \frac{\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} & I_s + \frac{V_s}{R_2} \\ -\frac{1}{R_1} & \frac{V_s}{R_3} \end{vmatrix}}{\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \end{vmatrix}} = \frac{\left(\frac{1}{R_1} + \frac{1}{R_2} \right) \left(\frac{V_s}{R_3} \right) - \left(-\frac{1}{R_1} \right) \left(I_s + \frac{V_s}{R_2} \right)}{\left(\frac{1}{R_1} + \frac{1}{R_2} \right) \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right) - \left(-\frac{1}{R_1} \right) \left(-\frac{1}{R_1} \right)}$$

Notes:

- The denominators are the same for both solutions.
- The main diagonal of a matrix is the one that goes to the right and down.
- The denominator matrix is the "conductance" matrix and has certain properties:
 - The elements on the main diagonal [i(row) = j(column)] include all the conductance connected to node i = j.

- b) The off-diagonal elements are all negative.
- c) The off-diagonal elements are all symmetric, i.e., the i j -th element = j i -th element. This is true only because there are no controlled (dependent) sources in this circuit.
- d) The off-diagonal elements include all the conductance connected between node i [row] and node j [column].

Problem 3.11

Solution:

Known quantities:

Circuit shown in Figure P3.11

$$V_{S1} = V_{S2} = 110 \text{ V}$$

$$R_1 = 500 \text{ m}\Omega \quad R_2 = 167 \text{ m}\Omega$$

$$R_3 = 700 \text{ m}\Omega$$

$$R_4 = 200 \text{ m}\Omega \quad R_5 = 333 \text{ m}\Omega$$

Find:

- a. The most efficient way to solve for the voltage across R_3 . Prove your case.
- b. The voltage across R_3 .

Analysis:

a) There are 3 meshes and 3 mesh currents requiring the solution of 3 simultaneous equations. Only one of these mesh currents is required to determine, using Ohm's Law, the voltage across R_3 .

In the terminal (or node) between the two voltage sources is made the ground (or reference) node, then three node voltages are known (the ground or reference voltage and the two source voltages). This leaves only two unknown node voltages (the voltages across R_1 , V_{R1} , and across R_2 , V_{R2}). Both these voltages are required to determine, using KVL, the voltage across R_3 , V_{R3} .

A difficult choice. Choose node analysis due to the smaller number of unknowns. Specify the nodes. Choose one node as the ground node. In KCL, assume unknown currents flow out.

b)

$$\text{KCL: } \frac{V_{R1} - V_{S1}}{R_4} + \frac{V_{R1} - 0}{R_1} + \frac{V_{R1} - V_{R2}}{R_3} = 0$$

$$\text{KCL: } \frac{V_{R2} - (-V_{S2})}{R_5} + \frac{V_{R2} - 0}{R_2} + \frac{V_{R2} - V_{R1}}{R_3} = 0$$

$$V_{R1} \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right) + V_{R2} \left(-\frac{1}{R_3} \right) = \frac{V_{S1}}{R_4}$$

$$V_{R1} \left(-\frac{1}{R_3} \right) + V_{R2} \left(\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} \right) = -\frac{V_{S2}}{R_5}$$

$$\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{500 \cdot 10^{-3}} + \frac{1}{700 \cdot 10^{-3}} + \frac{1}{200 \cdot 10^{-3}} = 8.43 \Omega^{-1}$$

$$\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{333 \cdot 10^{-3}} + \frac{1}{167 \cdot 10^{-3}} + \frac{1}{700 \cdot 10^{-3}} = 10.42 \Omega^{-1}$$

$$\frac{1}{R_3} = \frac{1}{700 \cdot 10^{-3}} = 1.43 \Omega^{-1}$$

$$\frac{V_{S1}}{R_4} = \frac{110}{200 \cdot 10^{-3}} = 550 \text{ A} \quad \frac{V_{S2}}{R_5} = \frac{110}{333 \cdot 10^{-3}} = 330 \text{ A}$$

$$V_{R1} = \frac{\begin{vmatrix} 550 & -1.43 \\ -330 & 10.42 \end{vmatrix}}{\begin{vmatrix} 8.43 & -1.43 \\ -1.43 & 10.42 \end{vmatrix}} = \frac{(5731) - (472)}{(87.84) - (2.04)} = 61.30 \text{ V}$$

$$V_{R1} = \frac{\begin{vmatrix} 8.429 & 550 \\ -1.429 & -330 \end{vmatrix}}{85.790} = \frac{(-2782) - (-786)}{85.80} = -23.26 \text{ V}$$

$$\text{KVL: } -V_{R1} + V_{R3} + V_{R2} = 0 \quad V_{R3} = V_{R1} - V_{R2} = 84.59 \text{ V}$$

Problem 3.12

Solution:

Known quantities:

Circuit shown in Figure P3.12

$$V_{S2} = kT \quad k = 10 \text{ V/}^\circ\text{C}$$

$$V_{S1} = 24 \text{ V} \quad R_S = R_1 = 12 \text{ k}\Omega$$

$$R_2 = 3 \text{ k}\Omega \quad R_3 = 10 \text{ k}\Omega$$

$$R_4 = 24 \text{ k}\Omega \quad V_{R3} = -2.524 \text{ V}$$

The voltage across R_3 , which is given, indicates the temperature.

Find:

The temperature, T .

Analysis:

Specify nodes (A between R_1 and R_3 , C between R_3 and R_2) and polarities of voltages (V_A from ground to A, V_C from ground to C, and V_{R3} from C to A). When using KCL, assume unknown currents flow out.

$$\text{KVL: } -V_A + V_{R3} + V_C = 0$$

$$V_C = V_A - V_{R3}$$

Now write KCL at node C, substitute for V_C , solve for V_A :

$$\text{KCL: } \frac{V_C - V_{S1}}{R_2} + \frac{V_C - V_A}{R_3} + \frac{V_C}{R_4} = 0$$

$$-\frac{V_A}{R_3} - \frac{V_{S1}}{R_2} + (V_A - V_{R3}) \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) = 0$$

$$V_A = \frac{\frac{V_{S1}}{R_2} + V_{R3} \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)}{\frac{1}{R_2} + \frac{1}{R_4}} = \frac{\frac{24}{3 \cdot 10^3} + (-2.524) \left(\frac{1}{3 \cdot 10^3} + \frac{1}{10 \cdot 10^3} + \frac{1}{24 \cdot 10^3} \right)}{\frac{1}{3 \cdot 10^3} + \frac{1}{24 \cdot 10^3}} = 18.14 \text{ V}$$

$$V_C = V_A - V_{R3} = 18.14 - (-2.524) = 20.66 \text{ V}$$

Now write KCL at node A and solve for V_{S2} and T:

$$\text{KCL: } \frac{V_A - V_{S1}}{R_1} + \frac{V_A - V_{S2}}{R_5} + \frac{V_A - V_C}{R_3} = 0$$

$$V_{S2} = V_A + \frac{R_5}{R_1} (V_A - V_{S1}) + \frac{R_5}{R_3} (V_A - V_C) =$$

$$= 18.14 + \frac{12 \cdot 10^3}{12 \cdot 10^3} (18.14 - 24) + \frac{12 \cdot 10^3}{10 \cdot 10^3} (18.14 - 20.66) = 9.26 \text{ V}$$

$$T = \frac{V_{S2}}{k} = \frac{9.26}{10} = 0.926 \text{ } ^\circ\text{C}$$

Problem 3.13

Solution:

Known quantities:

Circuit shown in Figure P3.13

$$V_S = 5 \text{ V} \quad A_V = 70 \quad R_1 = 2.2 \text{ k}\Omega$$

$$R_2 = 1.8 \text{ k}\Omega \quad R_3 = 6.8 \text{ k}\Omega \quad R_4 = 220 \Omega$$

Find:

The voltage across R_4 using KCL and node voltage analysis.

Analysis:

A node analysis is not a method of choice because the dependent source is [1] a voltage source and [2] a floating source. Both factors cause difficulties in a node analysis. A ground is specified. There are three unknown node voltages, one of which is the voltage across R_4 . The dependent source will introduce two additional unknowns, the current through the source and the controlling voltage (across R_1) that is not a node voltage. Therefore 5 equations are required:

$$[1] KCL \quad \frac{V_1 - V_S}{R_1} + \frac{V_1 - V_3}{R_3} + \frac{V_1 - V_2}{R_2} = 0$$

$$[2] KCL \quad \frac{V_2 - V_1}{R_2} - I_{CS} = 0$$

$$[3] KCL \quad \frac{V_3 - V_1}{R_3} + I_{CS} + \frac{V_3}{R_4} = 0$$

$$[4] KVL \quad -V_S + V_{R1} + V_1 = 0 \quad V_{R1} = V_S - V_1$$

$$[5] KVL \quad -V_3 - A_V V_{R1} + V_2 = 0 \quad V_2 = V_3 + A_V V_{R1} = V_3 + A_V (V_S - V_1)$$

Substitute using Equation [5] into Equations [1], [2] and [3] and eliminate V_2 (because it only appears twice in these equations). Collect terms:

$$V_1 \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2} + \frac{A_V}{R_2} \right) + V_3 \left(-\frac{1}{R_3} - \frac{1}{R_2} \right) + I_{CS}(0) = \frac{V_S}{R_1} + \frac{V_S A_V}{R_2}$$

$$V_1 \left(-\frac{1}{R_2} - \frac{A_V}{R_2} \right) + V_3 \left(\frac{1}{R_2} \right) + I_{CS}(-1) = -\frac{V_S A_V}{R_2}$$

$$V_1 \left(-\frac{1}{R_3} \right) + V_3 \left(\frac{1}{R_3} + \frac{1}{R_4} \right) + I_{CS}(+1) = 0$$

$$\frac{1}{R_2} = \frac{1}{1.8 \cdot 10^3} = 555.6 \cdot 10^{-6} \Omega^{-1}$$

$$\frac{1}{R_3} = \frac{1}{6.8 \cdot 10^3} = 147.1 \cdot 10^{-6} \Omega^{-1}$$

$$\frac{1}{R_3} + \frac{1}{R_2} = \frac{1}{6.8 \cdot 10^3} + \frac{1}{1.8 \cdot 10^3} = 702.6 \cdot 10^{-6} \Omega^{-1}$$

$$\frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{6.8 \cdot 10^3} + \frac{1}{0.22 \cdot 10^3} = 4.69 \cdot 10^{-3} \Omega^{-1}$$

$$\frac{1}{R_2} + \frac{A_V}{R_2} = \frac{1+70}{1.8 \cdot 10^3} = 39.44 \cdot 10^{-3} \Omega^{-1}$$

$$\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2} + \frac{A_V}{R_2} = \frac{1}{2.2 \cdot 10^3} + \frac{1}{6.8 \cdot 10^3} + \frac{1+70}{1.8 \cdot 10^3} = 40.05 \cdot 10^{-3} \Omega^{-1}$$

$$\frac{V_S A_V}{R_2} = \frac{(5)(70)}{1.8 \cdot 10^3} = 194.4 \text{ mA}$$

$$\frac{V_S}{R_1} + \frac{V_S A_V}{R_2} = \frac{5}{2.2 \cdot 10^3} + \frac{(5)(70)}{1.8 \cdot 10^3} = 196.7 \text{ mA}$$

Solving, we have:

$$V_{R4} = V_3 = 5.1 \text{ mV}$$

Note:

1. This solution was not difficult in terms of theory, but was terribly long and arithmetically cumbersome. This was because the wrong method was used. There are only 2 mesh currents in the circuit; the sources were voltage sources; therefore, a mesh analysis is the method of choice.
2. In general, a node analysis will have fewer unknowns (because one node is the ground or reference node) and will, in such cases, be preferable.

Problem 3.14**Solution:****Known quantities:**

The values of the resistors and of the voltage sources (see Figure P3.14).

Find:

The voltage across the $10\ \Omega$ resistor in the circuit of Figure P3.14 using mesh current analysis.

Analysis:

For mesh (a):

$$i_a(50 + 20 + 20) - i_b(20) - i_c(20) = 12$$

For mesh (b):

$$-i_a(20) + i_b(20 + 10) - i_c(10) + 5 = 0$$

For mesh (c):

$$-i_a(20) - i_b(10) + i_c(20 + 10 + 15) = 0$$

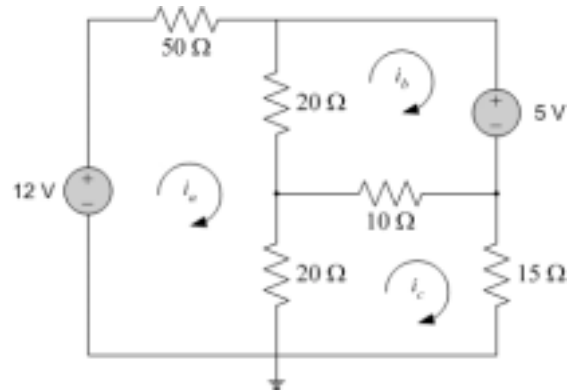
Solving,

$$i_a = 127.5\text{ mA}$$

$$i_b = -67.8\text{ mA}$$

$$i_c = 41.6\text{ mA}$$

and $v_{R_4} = 10(i_b - i_c) = 10(-0.109) = -1.09\text{ V}.$

**Problem 3.15****Solution:****Known quantities:**

The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.15.

Find:

The voltage across the current source using mesh current analysis.

Analysis:

For mesh (a):

$$i_a(20 + 30) + i_b(-30) = 3$$

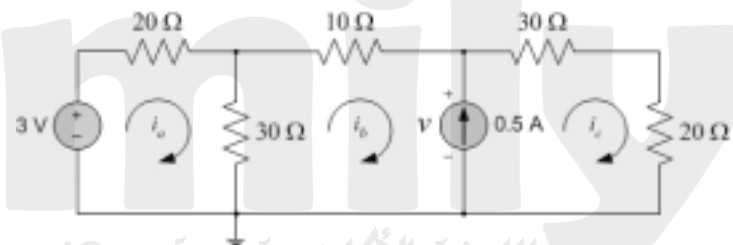
For meshes (b) and (c):

$$i_a(-30) + i_b(10 + 30) + i_c(30 + 20) = 0$$

For the current source:

$$i_c - i_b = 0.5$$

Solving,



$$i_a = -133 \text{ mA}, i_b = -322 \text{ mA} \text{ and } i_c = 178 \text{ mA}.$$

Therefore,

$$v = i_c(30 + 20) = 8.89 \text{ V}.$$

Problem 3.16

Solution:

Known quantities:

The values of the resistors and of the voltage source in the circuit of Figure P3.16.

Find:

The current i through the resistance R_4 mesh current analysis.

Analysis:

For mesh (a):

$$i_a(50 + 1200) + i_b(-1200) = 5.6$$

For meshes (b) and (c):

$$i_a(-1200) + i_b(1200 + 330) + i_c(440) = 0$$

For the current source:

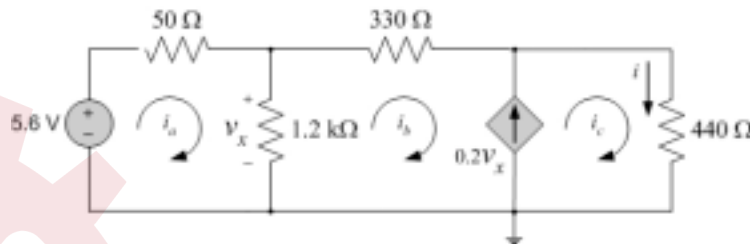
$$i_c - i_b = 0.2v_x = 0.2(1200(i_a - i_b)) = 240(i_a - i_b)$$

Solving,

$$i_a = 136 \text{ mA}, i_b = 137 \text{ mA} \text{ and } i_c = -106 \text{ mA}.$$

Therefore,

$$i = i_c = -106 \text{ mA}.$$



Problem 3.17

Solution:

Known quantities:

The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.5.

Find:

The current through the voltage source using mesh current analysis.

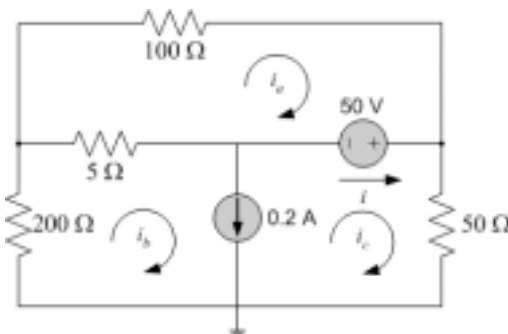
Analysis:

For mesh (a):

$$i_a(100 + 5) + i_b(-5) + 50 = 0$$

For the current source:

$$i_b - i_c = 0.2$$



For meshes (b) and (c):

$$-i_a(5) + i_b(200 + 5) + i_c(50) = 50$$

Solving,

$$i_a = -465 \text{ mA}, i_b = 226 \text{ mA} \text{ and } i_c = 26 \text{ mA}.$$

Therefore,

$$i = i_c - i_a = 491 \text{ mA}.$$

Problem 3.18

Solution:

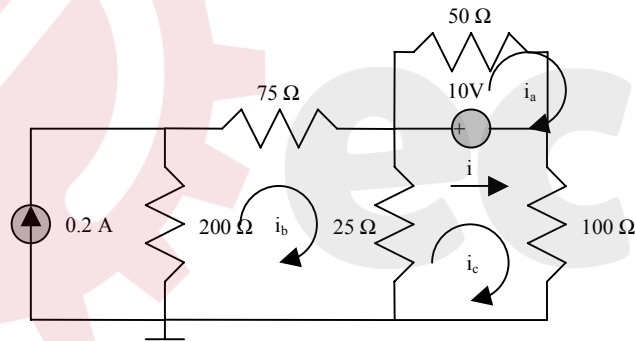
Known quantities:

The values of the resistors and of the current source in the circuit of Figure P3.6.

Find:

The current through the voltage source in the circuit of Figure P3.6 using mesh current analysis.

Analysis:



For mesh (a):

$$i_a(100) + 10 = 0$$

For mesh (b):

$$i_b(200 + 75 + 25) + i_c(-25) + 0.2(-200) = 0$$

For mesh (c):

$$i_b(-25) + i_c(50 + 25) = 10$$

Solving,

$$i_a = -100 \text{ mA}, i_b = 148 \text{ mA} \text{ and } i_c = 183 \text{ mA}.$$

Therefore,

$$i = i_c - i_a = 283 \text{ mA}$$

Problem 3.19

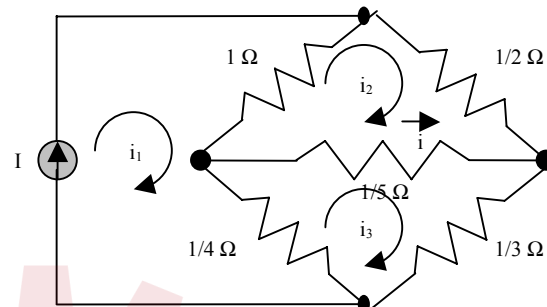
Solution:

Known quantities:

The values of the resistors in the circuit of Figure P3.19.

Find:

The current in the circuit of Figure P3.19 using mesh current analysis.

Analysis:

Since I is unknown, the problem will be solved in terms of this current.

For mesh #1, it is obvious that:

$$i_1 = I$$

For mesh #2:

$$i_1(-1) + i_2\left(1 + \frac{1}{2} + \frac{1}{5}\right) + i_3\left(-\frac{1}{5}\right) = 0$$

For mesh #3:

$$i_1\left(-\frac{1}{4}\right) + i_2\left(-\frac{1}{5}\right) + i_3\left(\frac{1}{4} + \frac{1}{3} + \frac{1}{5}\right) = 0$$

Solving,

$$i_2 = 0.645I$$

$$i_3 = 0.483I$$

Then, $i = i_3 - i_2$

and $i = 0.483I - 0.645I = -0.163I$

Problem 3.20**Solution:****Known quantities:**

The values of the resistors of the circuit in Figure P3.20.

Find:

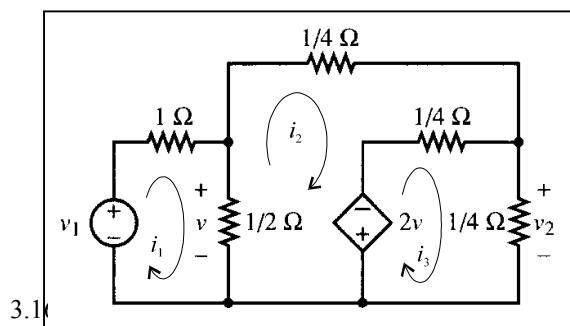
The voltage gain, $A_v = \frac{v_2}{v_1}$, in the circuit of Figure P3.20

using mesh current analysis.

Analysis:

Note that $v = \frac{i_1 - i_2}{2}$

For mesh #1:



$$i_1 \left(1 + \frac{1}{2} \right) + i_2 \left(-\frac{1}{2} \right) + i_3(0) = v_1$$

For mesh #2:

$$i_1 \left(-\frac{1}{2} \right) + i_2 \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{4} \right) + i_3 \left(-\frac{1}{4} \right) = 2v$$

or

$$i_1(-1.5) + i_2(2) + i_3(-0.25) = 0$$

For mesh #3:

$$i_1(0) + i_2 \left(-\frac{1}{4} \right) + i_3 \left(\frac{1}{4} + \frac{1}{4} \right) = -2v$$

or

$$i_1(1) + i_2(-1.25) + i_3(0.5) = 0$$

Solving, $i_3 = -0.16v_1$

from which $v_2 = \frac{1}{4}i_3 = -0.04v_1$

and $A_v = \frac{v_2}{v_1} = -0.04$

Problem 3.21

Solution:

Known quantities:

Circuit in Figure P3.21 and the values of the voltage sources, $V_{S1} = V_{S2} = 450 \text{ V}$, and the values of the 5 resistors:

$$R_1 = 8 \Omega \quad R_2 = 5 \Omega$$

$$R_4 = R_5 = 0.25 \Omega \quad R_3 = 32 \Omega$$

Find:

The voltages across R_1 , R_2 and R_3 using KCL and node analysis.

Analysis:

Choose a ground/reference node. The node common to the two voltage sources is the best choice. Specify polarity of voltages and direction of the currents.

$$\text{KCL: } \frac{V_{R1} - V_{S1}}{R_4} + \frac{V_{R1} - 0}{R_1} + \frac{V_{R1} - V_{R2}}{R_3} = 0$$

$$\text{KCL: } \frac{V_{R2} - (-V_{S2})}{R_5} + \frac{V_{R2} - 0}{R_2} + \frac{V_{R2} - V_{R1}}{R_3} = 0$$

Collect terms in terms of the unknown node voltages:

$$V_{R1} \left(\frac{1}{R_4} + \frac{1}{R_1} + \frac{1}{R_3} \right) + V_{R2} \left(-\frac{1}{R_3} \right) = \frac{V_{S1}}{R_4}$$

$$V_{R1} \left(-\frac{1}{R_3} \right) + V_{R2} \left(\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} \right) = -\frac{V_{S2}}{R_5}$$

Evaluate the coefficients of the unknown node voltages:

$$\frac{V_{S1}}{R_4} = \frac{V_{S2}}{R_5} = \frac{450}{0.25} = 1.8 \text{ kA} \quad \frac{1}{R_3} = \frac{1}{32} = 0.03125 \Omega^{-1}$$

$$\frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_3} = \frac{1}{0.25} + \frac{1}{8} + \frac{1}{32} = 4.14 \Omega^{-1}$$

$$\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{0.25} + \frac{1}{5} + \frac{1}{32} = 4.23 \Omega^{-1}$$

$$V_{R1} = \frac{\begin{vmatrix} 1800 & -31.25 \cdot 10^{-3} \\ -1800 & 4.23 \end{vmatrix}}{\begin{vmatrix} 4.16 & -31.25 \cdot 10^{-3} \\ -31.25 \cdot 10^{-3} & 4.23 \end{vmatrix}} = 429.5 \text{ V}$$

$$V_{R2} = \frac{\begin{vmatrix} 4.156 & 1800 \\ -31.25 \cdot 10^{-3} & -1800 \end{vmatrix}}{17.59} = -422.2 \text{ V}$$

KVL: $-V_{R1} + V_{R3} + V_{R2} = 0$

$$V_{R3} = V_{R1} - V_{R2} = 852.0 \text{ V}$$

Problem 3.22

Solution:

Known quantities:

Circuit in Figure P3.22 with the values of the voltage sources, $V_{S1} = V_{S2} = 115 \text{ V}$, and the values of the 5 resistors:

$$R_1 = R_2 = 5 \Omega \quad R_3 = 10 \Omega$$

$$R_4 = R_5 = 200 \text{ m}\Omega$$

Find:

The new voltages across R_1 , R_2 and R_3 , in case F_1 "blows" or opens using KCL and node analysis.

Analysis:

Specify polarity of voltages. The ground is already specified. The current through the fuse F_1 is zero.

$$\text{KCL: } 0 + \frac{V_{R1} - 0}{R_1} + \frac{V_{R1} - V_{R2}}{R_3} = 0$$

$$\text{KCL: } \frac{V_{R2} - (-V_{S2})}{R_5} + \frac{V_{R2} - 0}{R_2} + \frac{V_{R2} - V_{R1}}{R_3} = 0$$

Collect terms in unknown node voltages:

$$V_{R1} \left(\frac{1}{R_1} + \frac{1}{R_3} \right) + V_{R2} \left(-\frac{1}{R_3} \right) = 0$$

$$V_{R1} \left(-\frac{1}{R_3} \right) + V_{R2} \left(\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} \right) = -\frac{V_{S2}}{R_5}$$

$$\frac{1}{R_3} = \frac{1}{10} = 0.1 \Omega^{-1}$$

$$\frac{1}{R_1} + \frac{1}{R_3} = 0.3 \Omega^{-1}$$

$$\frac{V_{S2}}{R_5} = \frac{115}{200 \cdot 10^{-3}} = 575 \text{ A}$$

$$\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} = 5.3 \Omega^{-1}$$

$$V_{R1} = \frac{\begin{vmatrix} 0 & -0.1 \\ -575 & 5.3 \end{vmatrix}}{\begin{vmatrix} 0.3 & -0.1 \\ -0.1 & 5.3 \end{vmatrix}} = \frac{(0) - (57.5)}{(1.59) - (0.01)} = -36.39 \text{ V}$$

$$V_{R2} = \frac{\begin{vmatrix} 0.3 & 0 \\ -0.1 & -575 \end{vmatrix}}{1.58} = \frac{(-172.5) - (0)}{1.58} = -109.2 \text{ V}$$

$$\begin{aligned} \text{KVL: } -V_{R1} + V_{R3} + V_{R2} &= 0 \\ V_{R3} &= V_{R1} - V_{R2} = 72.81 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{KVL: } -V_{S1} + V_{R4} + V_F + V_{R1} &= 0 & V_{R4} &= I_1 R_4 = 0 \\ V_F &= 115 - 0 - (-36.39) = 151.4 \text{ V} \end{aligned}$$

Note the voltages are strongly dependent on the loads (R_1 , R_2 and R_3) connected at the time the fuse blows. With other loads, the result will be quite different.

Problem 3.23

Solution:

Known quantities:

Circuit in Figure P3.22 and the values of the voltage sources, $V_{S1} = V_{S2} = 120 \text{ V}$, and the values of the 5 resistors:

$$R_1 = R_2 = 2 \Omega \quad R_3 = 8 \Omega$$

$$R_4 = R_5 = 250 \text{ m}\Omega$$

Find:

The voltages across R_1 , R_2 , R_3 , and F_1 in case F_1 "blows" or opens using KCL and node analysis.

Analysis:

Specify polarity of voltages. The ground is already specified. The current through the fuse F_1 is zero.

$$\text{KCL: } 0 + \frac{V_{R1} - 0}{R_1} + \frac{V_{R1} - V_{R2}}{R_3} = 0$$

$$KCL: \quad \frac{V_{R2} - (-V_{S2})}{R_5} + \frac{V_{R2} - 0}{R_2} + \frac{V_{R2} - V_{R1}}{R_3} = 0$$

$$V_{R1} \left(\frac{1}{R_1} + \frac{1}{R_3} \right) + V_{R2} \left(-\frac{1}{R_3} \right) = 0$$

$$V_{R1} \left(-\frac{1}{R_3} \right) + V_{R2} \left(\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} \right) = -\frac{V_{S2}}{R_5}$$

$$\frac{1}{R_3} = \frac{1}{8} = 0.125 \Omega^{-1} \quad \frac{1}{R_1} + \frac{1}{R_3} = 0.625 \Omega^{-1}$$

$$\frac{V_{S2}}{R_5} = \frac{120}{250 \cdot 10^{-3}} = 480 \text{ A} \quad \frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} = 4.625 \Omega^{-1}$$

$$V_{R1} = \frac{\begin{vmatrix} 0 & -0.125 \\ -480 & 4.625 \end{vmatrix}}{\begin{vmatrix} 0.625 & -0.125 \\ -0.125 & 4.625 \end{vmatrix}} = \frac{(0) - (60)}{(2.89) - (0.016)} = -20.87 \text{ V}$$

$$V_{R2} = \frac{\begin{vmatrix} 0.625 & 0 \\ -0.125 & -480 \end{vmatrix}}{2.87} = \frac{(-300) - (0)}{2.87} = -104.35 \text{ V}$$

$$KVL: \quad -V_{R1} + V_{R3} + V_{R2} = 0$$

$$V_{R3} = V_{R1} - V_{R2} = 83.48 \text{ V}$$

$$KVL: \quad -V_{S1} + V_{R4} + V_F + V_{R1} = 0 \quad V_{R4} = I_1 R_4 = 0$$

$$V_F = 120 - 0 - (-20.87) = 140.9 \text{ V}$$

Problem 3.24

Solution:

Known quantities:

The values of the voltage sources, $V_{S1} = V_{S2} = V_{S3} = 170 \text{ V}$, and the values of the 6 resistors in the circuit of Figure P3.24:

$$R_{W1} = R_{W2} = R_{W3} = 0.7 \Omega$$

$$R_1 = 1.9 \Omega \quad R_2 = 2.3 \Omega \quad R_3 = 11 \Omega$$

Find:

- The number of unknown node voltages and mesh currents.
- Unknown node voltages.

Analysis:

If the node common to the three sources is chosen as the ground/reference node, and the series resistances are combined into single equivalent resistances, there is only one unknown node voltage. On the other

hand, there are two unknown mesh currents. A node analysis is the method of choice! Specify polarity of voltages and direction of currents.

$$R_{eq1} = R_{W1} + R_1 = 2.6 \, \Omega \quad R_{eq2} = R_{W2} + R_2 = 3.0 \, \Omega$$

$$R_{eq3} = R_{W3} + R_3 = 11.7 \, \Omega$$

KCL:

$$\frac{V_N - V_{S1}}{R_{eq1}} + \frac{V_N - (-V_{S2})}{R_{eq2}} + \frac{V_N - V_{S3}}{R_{eq3}} = 0$$

$$V_N = \frac{\frac{V_{S1}}{R_{eq1}} - \frac{V_{S2}}{R_{eq2}} + \frac{V_{S3}}{R_{eq3}}}{\frac{1}{R_{eq1}} + \frac{1}{R_{eq2}} + \frac{1}{R_{eq3}}} = \frac{\frac{170}{2.6} - \frac{170}{3.0} + \frac{170}{11.7}}{\frac{1}{2.6} + \frac{1}{3.0} + \frac{1}{11.7}} = 28.94 \, \text{V}$$

KVL:

$$-V_{S1} + I_1 R_{W1} + I_1 R_1 + V_N = 0$$

$$I_1 = \frac{V_{S1} - V_N}{R_{W1} + R_1} = \frac{170 - 28.94}{2.6} = 54.26 \, \text{A}$$

Problem 3.25

Solution:

Known quantities:

The values of the voltage sources, $V_{S1} = V_{S2} = V_{S3} = 170 \, \text{V}$, the common node voltage, $V_N = 28.94 \, \text{V}$, and the values of the 6 resistors in the circuit of Figure P3.24:

$$R_{W1} = R_{W2} = R_{W3} = 0.7 \, \Omega$$

$$R_1 = 1.9 \, \Omega \quad R_2 = 2.3 \, \Omega \quad R_3 = 11 \, \Omega$$

Find:

The current through and voltage across R_1 .

Analysis:

KVL:

$$-V_{S1} + I_1 R_{W1} + I_1 R_1 + V_N = 0$$

$$I_1 = \frac{V_{S1} - V_N}{R_{W1} + R_1} = \frac{170 - 28.94}{2.6} = 54.26 \, \text{A}$$

$$OL: \quad V_{R1} = I_1 R_1 = (54.26)(1.9) = 103.1 \, \text{V}$$

Problem 3.26

Solution:

Known quantities:

The values of the voltage sources, $V_{S1} = V_{S2} = V_{S3} = 170 \, \text{V}$, and the values of the 6 resistors in the circuit of Figure P3.24:

$$R_{W1} = R_{W2} = R_{W3} = 0.7 \, \Omega$$

$$R_1 = 1.9 \, \Omega \quad R_2 = 2.3 \, \Omega \quad R_3 = 11 \, \Omega$$

Find:

The mesh (or loop) equations and any additional equation required to determine the current through R_1 in the circuit shown in Figure P3.24.

Analysis:

$$\text{KVL:} \quad -V_{S1} + I_1 R_{W1} + I_1 R_1 + (I_1 - I_2) R_2 + (I_1 - I_2) R_{W2} - V_{S2} = 0$$

$$\text{KVL:} \quad V_{S2} + (I_2 - I_1) R_{W2} + (I_2 - I_1) R_2 + I_2 R_3 + I_2 R_{W3} + V_{S3} = 0$$

$$I_1 (R_1 + R_{W1} + R_2 + R_{W2}) + I_2 (-R_2 - R_{W2}) = V_{S1} + V_{S2}$$

$$I_1 (-R_2 - R_{W2}) + I_2 (R_2 + R_{W2} + R_3 + R_{W3}) = -V_{S2} - V_{S3}$$

$$I_{R1} = I_1 = \frac{\begin{vmatrix} (V_{S1} + V_{S2}) & -(R_2 + R_{W2}) \\ -(V_{S2} + V_{S3}) & (R_2 + R_{W2} + R_3 + R_{W3}) \end{vmatrix}}{\begin{vmatrix} (R_1 + R_{W1} + R_2 + R_{W2}) & -(R_2 + R_{W2}) \\ -(R_2 + R_{W2}) & (R_2 + R_{W2} + R_3 + R_{W3}) \end{vmatrix}}$$

Problem 3.27**Solution:****Known quantities:**

The values of the voltage sources, $V_{S1} = 90 \, \text{V}$, $V_{S2} = V_{S3} = 110 \, \text{V}$, and the values of the 6 resistors in the circuit of Figure P3.24:

$$R_{W1} = R_{W2} = R_{W3} = 1.3 \, \Omega$$

$$R_1 = 7.9 \, \Omega \quad R_2 = R_3 = 3.7 \, \Omega$$

Find:

The branch currents, using KVL and loop analysis.

Analysis:

Three equations are required. Voltages will be summed around the 2 loops that are meshes, and KCL at the common node between the resistances. Assume directions of the branch currents and the associated polarities of the voltages. After like terms are collected:

$$\text{KVL:} \quad -V_{S1} + I_1 R_{W1} + I_1 R_1 + (I_1 - I_2) R_2 + (I_1 - I_2) R_{W2} - V_{S2} = 0$$

$$\text{KVL:} \quad V_{S2} + (I_2 - I_1) R_{W2} + (I_2 - I_1) R_2 + I_2 R_3 + I_2 R_{W3} - V_{S3} = 0$$

$$\text{KCL:} \quad I_3 = I_1 - I_2$$

Plugging in the given parameters results in the following system of equations:

$$14.2I_1 - 5.0I_2 = 200$$

$$5.0I_1 - 10.0I_2 = 0$$

$$I_3 = I_1 - I_2$$

Solving the system of equations gives:

$$I_1 = 17.09 \, \text{A}$$

$$I_2 = 8.55 \, \text{A}$$

$$I_3 = 8.55 \text{ A}$$

Hence, the assumed polarity of the second and third branch currents is actually reversed.

Problem 3.28

Solution:

Known quantities:

The values of the voltage sources, $V_{s1} = V_{s2} = 115 \text{ V}$, and the values of the 5 resistors in the circuit of Figure P3.22:

$$\begin{array}{llll} R_1 & R_2 & 5 & \\ R_3 & 10 & R_4 & R_5 \quad 200 \text{ m} \end{array}$$

Find:

The voltages across R_1 , R_2 and R_3 , under normal conditions, i.e., no blown fuses using KVL and a mesh analysis.

Analysis:

KVL:

$$\begin{array}{llllll} I_1 R_1 & R_4 & R_1 I_3 & V_{s1} & & \\ I_2 R_2 & R_5 & R_2 I_3 & V_{s2} & & \\ -I_1 R_1 & I_2 R_2 & R_1 & R_2 & R_3 & I_3 & 0 \end{array}$$

Rearranging the above equations:

$$5.2I_1 - 5I_3 = 115$$

$$5.2I_2 - 5I_3 = 115$$

$$5I_1 + 5I_2 - 20I_3 = 0$$

$$I_1 = I_2 \quad I_1 = 2I_3$$

$$I_1 = I_2 = \frac{\begin{vmatrix} 115 & 0 & -5 \\ 115 & 5.2 & -5 \\ 0 & 5 & -20 \end{vmatrix}}{\begin{vmatrix} 5.2 & 0 & -5 \\ 0 & 5.2 & -5 \\ 5 & 5 & -20 \end{vmatrix}} = \frac{-11960}{-280.8} = 42.6 \text{ A} \quad I_3 = 21.3 \text{ A}$$

$$V_{R1} = R_1(I_1 - I_3) = 106.5 \text{ V}$$

$$V_{R2} = R_2(I_3 - I_2) = -106.5 \text{ V}$$

$$V_{R3} = R_3 I_3 = 213 \text{ V}$$

Problem 3.29**Solution:****Known quantities:**

The values of the voltage sources, $V_{S1} = V_{S2} = 110 \text{ V}$, and the values of the 5 resistors in the circuit of Figure P3.22:

$$R_1 = 100 \, \Omega \quad R_2 = 22 \, \Omega \quad R_3 = 70 \, \Omega$$

$$R_4 = R_5 = 13 \, \Omega$$

Find:

The voltage across R_1 using KVL and mesh analysis.

Analysis:

KVL:

$$I_1(R_1 + R_4) - R_1 I_3 = V_{S1}$$

$$I_2(R_2 + R_5) - R_2 I_3 = V_{S2}$$

$$-I_1 R_1 - I_2 R_2 + (R_1 + R_2 + R_3) I_3 = 0$$

Rearranging the above equations:

$$113 I_1 - 100 I_3 = 110$$

$$35 I_2 - 22 I_3 = 110$$

$$100 I_1 + 22 I_2 - 192 I_3 = 0$$

$$I_1 = \frac{\begin{vmatrix} 110 & 0 & -100 \\ 110 & 35 & -22 \\ 0 & 22 & -192 \end{vmatrix}}{\begin{vmatrix} 113 & 0 & -100 \\ 0 & 35 & -22 \\ 100 & 22 & -192 \end{vmatrix}} = \frac{-935660}{-354668} = 2.64 \text{ A} \quad I_3 = 1.88 \text{ A}$$

$$V_{R1} = R_1(I_1 - I_3) = 75.89 \text{ V}$$

Problem 3.30**Solution:****Known quantities:**

The values of the voltage sources, $V_{S1} = V_{S2} = 115 \text{ V}$, and the values of the 5 resistors in the circuit of Figure P3.22:

$$R_1 = 5 \, \Omega \quad R_2 = 5 \, \Omega \quad R_3 = 10 \, \Omega$$

$$R_4 = R_5 = 0.2 \, \Omega$$

Find:

The voltage across R_1 using KVL and mesh analysis.

Analysis:

Specify polarity of voltages. The ground is already specified. The current through the fuse F_1 is zero.

$$I_1 = 0$$

$$\text{KVL: } -V_{S2} + (I_2 - I_3)R_2 + I_2R_5 = 0$$

$$\text{KVL: } I_3R_1 + I_3R_3 + (I_3 - I_2)R_2 = 0$$

$$I_2(R_2 + R_5) + I_3(-R_2) = V_{S2}$$

$$I_2(-R_2) + I_3(R_1 + R_2 + R_3) = 0$$

$$R_2 + R_5 = 5.2 \, \Omega$$

$$R_1 + R_2 + R_3 = 20 \, \Omega$$

$$I_2 = \frac{\begin{vmatrix} 115 & -5 \\ 0 & 20 \end{vmatrix}}{\begin{vmatrix} 5.2 & -5 \\ -5 & 20 \end{vmatrix}} = \frac{(2300) - (0)}{(104) - (25)} = 29.11 \, \text{A}$$

$$I_3 = \frac{\begin{vmatrix} 5.2 & 115 \\ -5 & 0 \end{vmatrix}}{\begin{vmatrix} 5.2 & -5 \\ -5 & 20 \end{vmatrix}} = \frac{(0) - (-575)}{79} = 7.28 \, \text{A}$$

$$V_{R1} = I_{R1}R_1 = -I_3R_1 = -36.39 \, \text{V}$$

$$V_{R2} = I_{R2}R_2 = (I_3 - I_2)R_2 = -109.16 \, \text{V}$$

$$V_{R3} = I_{R3}R_3 = I_3R_3 = 72.78 \, \text{V}$$

$$\begin{aligned} \text{KVL: } -V_{S1} + I_1R_4 + V_F + V_{R1} &= 0 \\ V_F &= 151.39 \, \text{V} \end{aligned}$$

Problem 3.31**Solution:****Known quantities:**

The values of the voltage sources, $V_{S1} = V_{S2} = 115 \, \text{V}$, and the values of the 5 resistors in the circuit of Figure P3.22:

$$R_1 = 4 \, \Omega \quad R_2 = 7.5 \, \Omega \quad R_3 = 12.5 \, \Omega$$

$$R_4 = R_5 = 1 \, \Omega$$

Find:

The voltages across R_1 , R_2 , R_3 , and across the open fuse using KVL and mesh analysis.

Analysis:

Specify polarity of voltages. The ground is already specified. The current through the fuse F_1 is zero.

$$I_1 = 0$$

KVL:

$$I_2(R_2 + R_5) - R_2 I_3 = V_{S2}$$

$$-I_2 R_2 + (R_1 + R_2 + R_3) I_3 = 0$$

Rearranging the above equations:

$$8.5 I_2 - 7.5 I_3 = 115$$

$$7.5 I_2 - 24 I_3 = 0$$

$$I_2 = \frac{\begin{vmatrix} 115 & -7.5 \\ 0 & -24 \end{vmatrix}}{\begin{vmatrix} 8.5 & -7.5 \\ 7.5 & -24 \end{vmatrix}} = \frac{-2760}{-147.75} = 18.68 \text{ A} \quad I_3 = 5.84 \text{ A}$$

$$V_{R1} = -I_3 R_1 = -23.35 \text{ V}$$

$$V_{R2} = R_2 (I_3 - I_2) = -96.3 \text{ V}$$

$$V_{R3} = R_3 I_3 = 73 \text{ V}$$

$$V_F = V_{S1} - V_{R1} = 138.35 \text{ V}$$

Section 3.5: Superposition

Problem 3.32

Solution:

Known quantities:

The values of the voltage sources, $V_{S1} = 110 \text{ V}$, $V_{S2} = 90 \text{ V}$ and the values of the 3 resistors in the circuit of Figure P3.32:

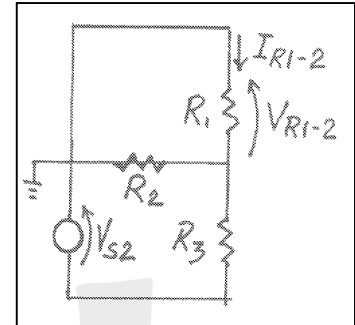
$$R_1 = 560 \Omega \quad R_2 = 3.5 \text{ k}\Omega \quad R_3 = 810 \Omega$$

Find:

The current through R_1 due only to the source V_{S2} .

Analysis:

Suppress V_{S1} . Redraw the circuit. Specify polarity of V_{R1} . Choose ground.



$$\begin{aligned} \text{KCL: } \frac{-V_{R1-2} - 0}{R_1} + \frac{-V_{R1-2} - 0}{R_2} + \frac{-V_{R1-2} - (-V_{S2})}{R_3} &= 0 \\ V_{R1-2} &= \frac{\frac{V_{S2}}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{\frac{90}{810}}{\frac{1}{560} + \frac{1}{3500} + \frac{1}{810}} = 33.61 \text{ V} \\ \text{OL: } I_{R1-2} &= \frac{V_{R1-2}}{R_1} = \frac{33.61}{560} = 60.02 \text{ mA} \end{aligned}$$

Problem 3.33

Solution:

Known quantities:

The values of the current source, of the voltage source and of the resistors in the circuit of Figure P3.33:

$$\begin{aligned} I_B &= 12 \text{ A} & R_B &= 1 \Omega \\ V_G &= 12 \text{ V} & R_G &= 0.3 \Omega \\ R &= 0.23 \Omega \end{aligned}$$

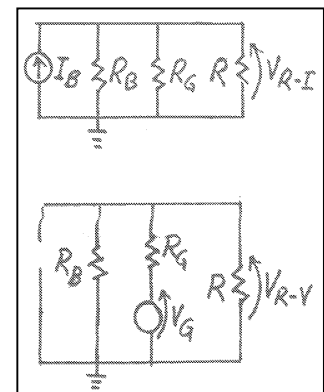
Find:

The voltage across R_1 using superposition.

Analysis:

Specify a ground node and the polarity of the voltage across R . Suppress the voltage source by replacing it with a short circuit. Redraw the circuit.

$$\text{KCL: } -I_B + \frac{V_{R-I}}{R_B} + \frac{V_{R-I}}{R_G} + \frac{V_{R-I}}{R} = 0$$



$$V_{R-I} = \frac{I_B}{\frac{1}{R_B} + \frac{1}{R_G} + \frac{1}{R}} = \frac{12}{\frac{1}{1} + \frac{1}{0.3} + \frac{1}{0.23}} = 1.38 \text{ V}$$

Suppress the current source by replacing it with an open circuit.

$$\text{KCL: } \frac{V_{R-V}}{R_B} + \frac{V_{R-V} - V_G}{R_G} + \frac{V_{R-V}}{R} = 0$$

$$V_{R-V} = \frac{\frac{V_G}{R_G}}{\frac{1}{R_B} + \frac{1}{R_G} + \frac{1}{R}} = \frac{\frac{12}{0.3}}{\frac{1}{1} + \frac{1}{0.3} + \frac{1}{0.23}} = 4.61 \text{ V}$$

$$V_R = V_{R-I} + V_{R-V} = 5.99 \text{ V}$$

Note: Superposition essentially doubles the work required to solve this problem. The voltage across R can easily be determined using a single KCL.

Problem 3.34

Solution:

Known quantities:

The values of the voltage sources and of the resistors in the circuit of Figure P3.34:

$$V_{S1} = V_{S2} = 12 \text{ V}$$

$$R_1 = R_2 = R_3 = 1 \text{ k}\Omega$$

Find:

The voltage across R_2 using superposition.

Analysis:

Specify the polarity of the voltage across R_2 . Suppress the voltage source V_{S1} by replacing it with a short circuit. Redraw the circuit.

$$R_{eq} = R_1 \parallel R_3 = \frac{1}{2} 1 \text{ k}\Omega = 0.5 \text{ k}\Omega$$

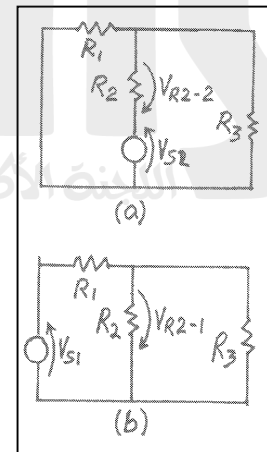
$$V_{R2-2} = V_{S2} \frac{R_2}{R_2 + R_{eq}} = \frac{(12)(1000)}{1000 + 500} = 8 \text{ V}$$

Suppress the voltage source V_{S2} by replacing it with a short circuit. Redraw the circuit.

$$R_{eq} = R_2 \parallel R_3 = \frac{1}{2} 1 \text{ k}\Omega = 0.5 \text{ k}\Omega$$

$$V_{R2-1} = -V_{S1} \frac{R_{eq}}{R_1 + R_{eq}} = \frac{(12 \text{ V})(0.5 \text{ k}\Omega)}{1 \text{ k}\Omega + 0.5 \text{ k}\Omega} = -4 \text{ V}$$

$$V_{R2} = V_{R2-1} + V_{R2-2} = -4 \text{ V} + 8 \text{ V} = 4 \text{ V}$$



Note: Although superposition is necessary to solve some circuits, it is a very inefficient and very cumbersome way to solve a circuit. This method should, if at all possible, be avoided. It must be used when the sources in a circuit are AC sources with different frequencies, or where some sources are DC and others are AC.

Problem 3.35

Solution:

Known quantities:

The values of the voltage sources and of the resistors in the circuit of Figure P3.35:

$$V_{S1} = V_{S2} = 450 \text{ V}$$

$$R_1 = 7 \Omega \quad R_2 = 5 \Omega \quad R_3 = 10 \Omega \quad R_4 = R_5 = 1 \Omega$$

Find:

The component of the current through R_3 that is due to V_{S2} , using superposition.

Analysis:

Suppress V_{S1} by replacing it with a short circuit. Redraw the circuit. A solution using equivalent resistances looks reasonable. R_1 and R_4 are in parallel:

$$R_{14} = \frac{R_1 R_4}{R_1 + R_4} = \frac{(7)(1)}{7 + 1} = 0.875 \Omega$$

R_{14} is in series with R_3 :

$$R_{143} = R_{14} + R_3 = 0.875 + 10 = 10.875 \Omega$$

$$R_{eq} = R_5 + (R_2 \parallel R_{143}) = R_5 + \frac{R_2 R_{143}}{R_2 + R_{143}} = 1 + \frac{(5)(10.875)}{5 + 10.875} = 4.425 \Omega$$

$$OL: \quad I_S = \frac{V_{S2}}{R_{eq}} = \frac{450}{4.425} = 101.695 \text{ A}$$

$$CD: \quad I_{R3-2} = \frac{I_S R_2}{R_2 + R_{143}} = \frac{(101.695)(5)}{5 + 10.875} = 32.03 \text{ A}$$

Problem 3.36

Solution:

Known quantities:

The values of the voltage sources and of the resistors in the circuit of Figure P3.24:

$$V_{S1} = V_{S2} = V_{S3} = 170 \text{ V}$$

$$R_{W1} = R_{W2} = R_{W3} = 0.7 \Omega$$

$$R_1 = 1.9 \Omega \quad R_2 = 2.3 \Omega \quad R_3 = 11 \Omega$$

Find:

The current through R_1 , using superposition.

Analysis:

$$R_{eq1} = R_{W1} + R_1 = 2.6 \, \Omega \quad R_{eq2} = R_{W2} + R_2 = 3 \, \Omega \quad R_{eq3} = R_{W3} + R_3 = 11.7 \, \Omega$$

Specify the direction of I_1 . Suppress V_{S2} and V_{S3} . Redraw circuit.

$$R_{eq} = R_{eq1} + \frac{R_{eq2} R_{eq3}}{R_{eq2} + R_{eq3}} = 4.99 \, \Omega$$

$$I_{I-1} = \frac{V_{S1}}{R_{eq}} = 34.08 \, \text{A}$$

Suppress V_{S1} and V_{S3} . Redraw circuit.

$$\text{KCL: } \frac{V_A - (-V_{S2})}{R_{eq2}} + \frac{V_A}{R_{eq1}} + \frac{V_A}{R_{eq3}} = 0$$

$$V_A = -\frac{V_{S2}}{1 + \frac{R_{eq2}}{R_{eq1}} + \frac{R_{eq2}}{R_{eq3}}} = -70.54 \, \text{V}$$

$$I_{I-2} = -\frac{V_A}{R_{eq1}} = 27.13 \, \text{A}$$

Suppress V_{S1} and V_{S2} . Redraw circuit.

$$\text{KCL: } \frac{V_A - (-V_{S3})}{R_{eq3}} + \frac{V_A - 0}{R_{eq1}} + \frac{V_A - 0}{R_{eq2}} = 0$$

$$V_A = -\frac{V_{S3}}{1 + \frac{R_{eq3}}{R_{eq1}} + \frac{R_{eq3}}{R_{eq2}}} = 18.09 \, \text{V}$$

$$I_{I-3} = -\frac{V_A}{R_{eq1}} = -6.96 \, \text{A}$$

$$I = I_{I-1} + I_{I-2} + I_{I-3} = 54.25 \, \text{A}$$

Note: Superposition should be used only for special conditions, as stated in the solution to Problem 3.34. In the problem above a better method is:

- mesh analysis using KVL (2 unknowns)
- node analysis using KCL (1 unknown but current must be obtained using OL).

Problem 3.37**Solution:****Known quantities:**

The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.5.

Find:

The current through the voltage source using superposition.

Analysis:

- Suppress voltage source V . Redraw the circuit.

For mesh (a):

$$i_a(100+5)+i_b(-5)=0$$

For the current source:

$$i_b - i_c = 0.2$$

For meshes (b) and (c):

$$-i_a(5)+i_b(200+5)+i_c(50)=0$$

Solving,

$$i_a = 2 \text{ mA}, i_b = 39 \text{ mA} \text{ and } i_c = -161 \text{ mA}.$$

Therefore,

$$i_1 = i_c - i_a = -163 \text{ mA}.$$

(2) Suppress current source I . Redraw the circuit.

For mesh (a):

$$i_a(100+5)+i_b(-5)+50=0$$

For mesh (b):

$$-i_a(5)+i_b(200+5+50)=50$$

Solving,

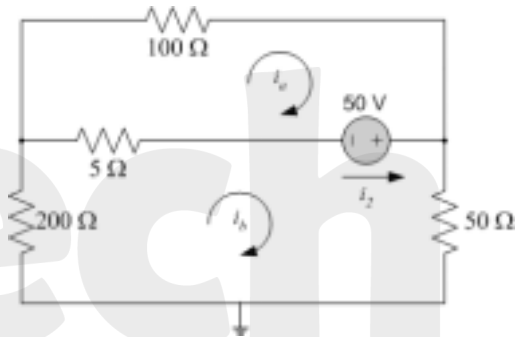
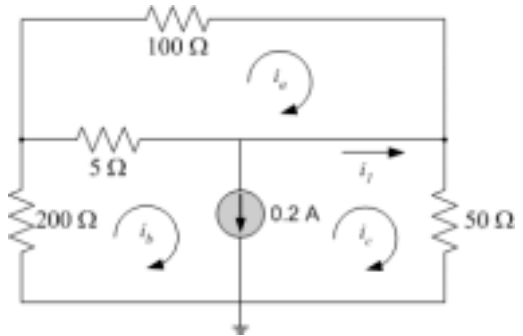
$$i_a = -467 \text{ mA} \text{ and } i_b = 187 \text{ mA}.$$

Therefore,

$$i_2 = i_b - i_a = 654 \text{ mA}.$$

Using the principle of superposition,

$$i = i_1 + i_2 = 491 \text{ mA}$$



Problem 3.38

Solution:

Known quantities:

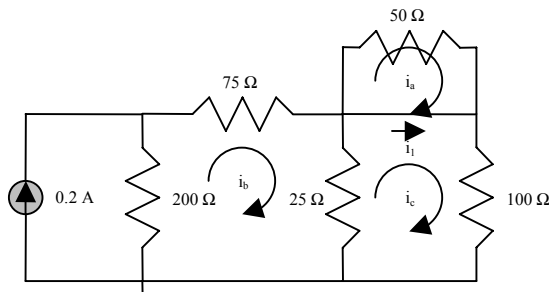
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.6.

Find:

The current through the voltage source using superposition.

Analysis:

(1) Suppress voltage source V . Redraw the circuit.



For mesh (a):

$$i_a = 0$$

For mesh (b):

$$i_b(200 + 75 + 25) + i_c(-25) - 40 = 0$$

For mesh (c):

$$i_b(-25) + i_c(25 + 100) = 0$$

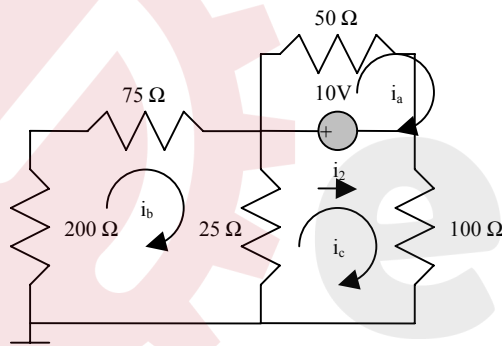
Solving,

$$i_b = 136 \text{ mA} \text{ and } i_c = 27 \text{ mA}.$$

Therefore,

$$i_1 = i_c = 27 \text{ mA}.$$

(2) Suppress current source I . Redraw the circuit.



For mesh (a):

$$i_a(50) - 10 = 0$$

For mesh (b):

$$i_b(200 + 75 + 25) + i_c(-25) = 0$$

For mesh (c):

$$i_b(-25) + i_c(25 + 100) = -10$$

Solving,

$$i_a = 200 \text{ mA}, i_b = -6.8 \text{ mA} \text{ and } i_c = -81 \text{ mA}.$$

Therefore,

$$i_2 = i_c - i_a = -281 \text{ mA}.$$

Using the principle of superposition,

$$i = i_1 + i_2 = -254 \text{ mA}$$

Problem 3.39**Solution:****Known quantities:**

The voltage source value, 3 V, and the five resistance values, indicated in Figure P3.7.

Find:

The current, i , drawn from the independent voltage source using superposition.

Analysis:

(1) Suppress voltage source V . Redraw the circuit.

At node 1:

$$\frac{v_1}{0.5} + \frac{v_1}{0.5} + \frac{v_1 - v_2}{0.25} = 0$$

At node 2:

$$\frac{v_2 - v_1}{0.25} + \frac{v_2}{0.75} + 0.5 = 0$$

Solving the system, we obtain:

$$v_1 = -0.075 \text{ V}, \quad v_2 = -0.15 \text{ V}$$

$$\text{Therefore, } i_1 = -\frac{v_1}{0.5} = 150 \text{ mA.}$$

(2) Suppress current source I . Redraw the circuit.

At node 1:

$$\frac{v_1 - 3}{0.5} + \frac{v_1}{0.5} + \frac{v_1}{(0.25 + 0.5 + 0.25)} = 0$$

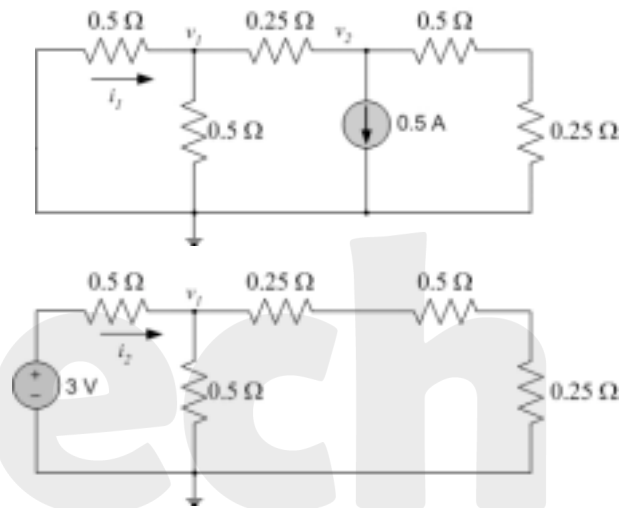
Solving,

$$v_1 = 1.2 \text{ V}$$

$$\text{Therefore, } i_1 = \frac{3 - v_1}{0.5} = 3.6 \text{ A.}$$

Using the principle of superposition,

$$i = i_1 + i_2 = 3.75 \text{ A}$$

**Problem 3.40****Solution:****Known quantities:**

Circuit in Figure P3.12

$$V_{S2} = kT \quad k = 10 \text{ V/}^\circ\text{C}$$

$$V_{S1} = 24 \text{ V} \quad R_S = R_1 = 12 \text{ k}\Omega$$

$$R_2 = 3 \text{ k}\Omega \quad R_3 = 10 \text{ k}\Omega$$

$$R_4 = 24 \text{ k}\Omega \quad V_{R3} = -2.524 \text{ V}$$

The voltage across R_3 , which is given, indicates the temperature.

Find:

The temperature, T using superposition.

Analysis:

(1) Suppress voltage source V_{S2} . Redraw the circuit.

For mesh (a):

$$i_a(24k) + i_b(-12k) + i_c(-12k) = 24$$

For mesh (b):

$$i_a(-12k) + i_b(46k) + i_c(-10k) = 0$$

For mesh (c):

$$i_a(-12k) + i_b(-10k) + i_c(25k) = 0$$

Solving,

$$i_a = 2.08 \text{ mA}, i_b = 0.83 \text{ mA} \text{ and } i_c = 1.33 \text{ mA}.$$

Therefore,

$$V_{R3,S2} = 10000(i_b - i_c) = -5 \text{ V}.$$

(2) Suppress voltage source V_{S1} . Redraw the circuit.

For mesh (a):

$$i_a(24k) + i_b(-12k) + i_c(-12k) + 10T = 0$$

For mesh (b):

$$i_a(-12k) + i_b(46k) + i_c(-10k) = 10T$$

For mesh (c):

$$i_a(-12k) + i_b(-10k) + i_c(25k) = 0$$

Solving,

$$i_a = -0.52T \text{ mA}, i_b = 0.029T \text{ mA} \text{ and } i_c = -0.2381T \text{ mA}.$$

Therefore,

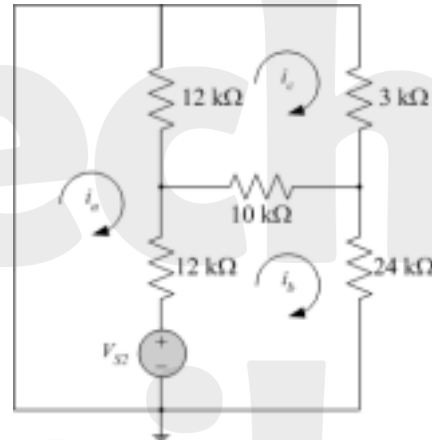
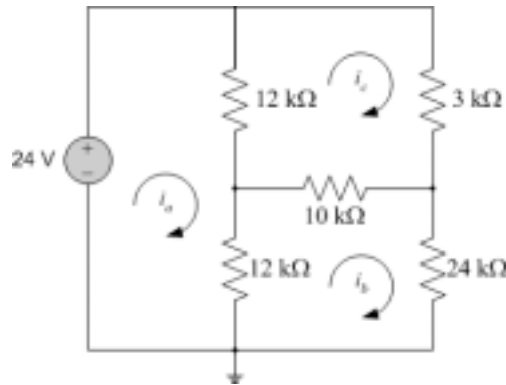
$$V_{R3,S1} = 10000(i_b - i_c) = 2.671T \text{ V}.$$

Using the principle of superposition,

$$V_{R3} = V_{R3,S2} + V_{R3,S1} = -5 + 2.671T = -2.524 \text{ V}$$

Therefore,

$$T = 0.926 \text{ } ^\circ\text{C}.$$



Problem 3.41

Solution:

Known quantities:

The values of the resistors and of the voltage sources (see Figure P3.14).

Find:

The voltage across the 10Ω resistor in the circuit of Figure P3.14 using superposition.

Analysis:

(1) Suppress voltage source V_{S1} . Redraw the circuit.

For mesh (a):

$$i_a(50 + 20 + 20) - i_b(20) - i_c(20) = 0$$

For mesh (b):

$$-i_a(20) + i_b(20 + 10) - i_c(10) + 5 = 0$$

For mesh (c):

$$-i_a(20) - i_b(10) + i_c(20 + 10 + 15) = 0$$

Solving,

$$i_a = -73.8 \text{ mA}, i_b = -245 \text{ mA} \text{ and } i_c = -87.2 \text{ mA}.$$

Therefore,

$$V_{10\Omega, S1} = 10(i_b - i_c) = -1.578 \text{ V}.$$

(2) Suppress voltage source V_{S2} . Redraw the circuit.

For mesh (a):

$$i_a(50 + 20 + 20) - i_b(20) - i_c(20) = 12$$

For mesh (b):

$$-i_a(20) + i_b(20 + 10) - i_c(10) = 0$$

For mesh (c):

$$-i_a(20) - i_b(10) + i_c(20 + 10 + 15) = 0$$

Solving,

$$i_a = 201 \text{ mA}, i_b = 177 \text{ mA} \text{ and}$$

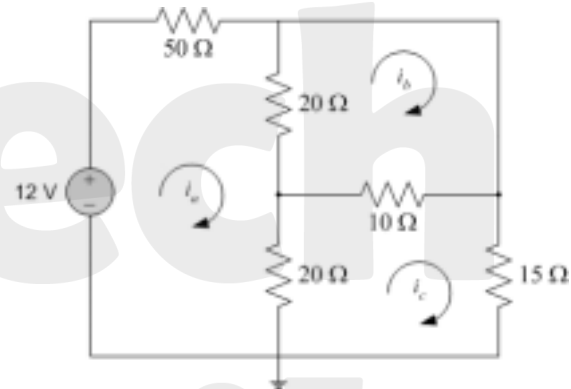
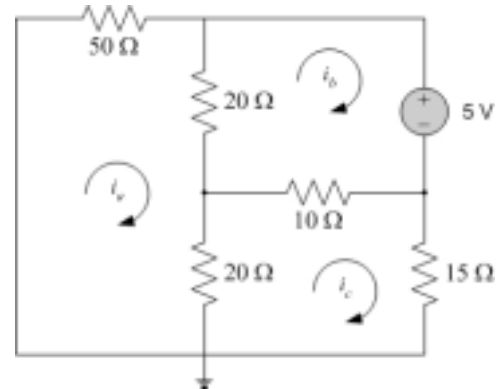
$$i_c = 129 \text{ mA}.$$

Therefore,

$$V_{10\Omega, S1} = 10(i_b - i_c) = 0.48 \text{ V}.$$

Using the principle of superposition,

$$V_{10\Omega} = V_{10\Omega, S2} + V_{10\Omega, S1} = -1.09 \text{ V}.$$

**Problem 3.42****Solution:****Known quantities:**

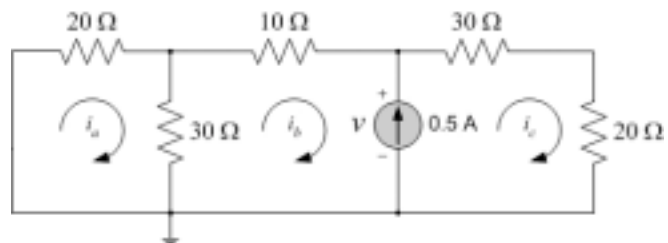
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.15.

Find:

The voltage across the current source using superposition.

Analysis:

(1) Suppress voltage source. Redraw the circuit.



For mesh (a):

$$i_a(20 + 30) + i_b(-30) = 0$$

For meshes (b) and (c):

$$i_a(-30) + i_b(10 + 30) + i_c(30 + 20) = 0$$

For the current source:

$$i_c - i_b = 0.5$$

Solving,

$$i_a = -208 \text{ mA}, i_b = -347 \text{ mA} \text{ and } i_c = 153 \text{ mA}.$$

Therefore,

$$v_V = i_c(30 + 20) = 7.65 \text{ V}.$$

(2) Suppress current source. Redraw the circuit.

For mesh (a):

$$i_a(20 + 30) + i_b(-30) = 3$$

For mesh (b):

$$i_a(-30) + i_b(90) = 0$$

Solving,

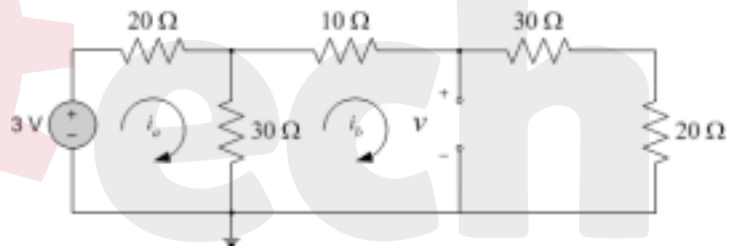
$$i_a = 75 \text{ mA and } i_b = 25 \text{ mA}.$$

Therefore,

$$v_I = i_b(30 + 20) = 1.25 \text{ V}.$$

Using the principle of superposition,

$$v = v_V + v_I = 8.9 \text{ V}.$$



Section 3.6: Equivalent circuits

Focus on Methodology: Computation of equivalent resistance of a one-port network that does not contain dependent sources

1. Remove the load, leaving the load terminals open circuited.
2. Zero all independent voltage and current sources
3. Compute the total resistance between load terminals, *with the load removed*. This resistance is equivalent to that which would be encountered by a current source connected to the circuit in place of the load.

Focus on Methodology: Computing the Thevenin voltage

1. Remove the load, leaving the load terminals open circuited.
2. Define the open-circuit voltage v_{OC} across the open load terminals.
3. Apply any preferred method (e.g.: nodal analysis) to solve for v_{OC} .
4. The Thevenin voltage is $v_T = v_{OC}$.

Focus on Methodology: Computing the Norton current

1. Replace the load with a short circuit.
2. Define the short-circuit current i_{SC} to be the Norton equivalent current.
3. Apply any preferred method (e.g.: nodal analysis) to solve for i_{SC} .
4. The Norton current is $i_N = i_{SC}$.

Problem 3.43

Solution:

Known quantities:

The schematic of the circuit (see Figure P3.1).

Find:

The Thévenin equivalent resistance seen by resistor R_3 , the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_3 is the load.

Analysis:

- (1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

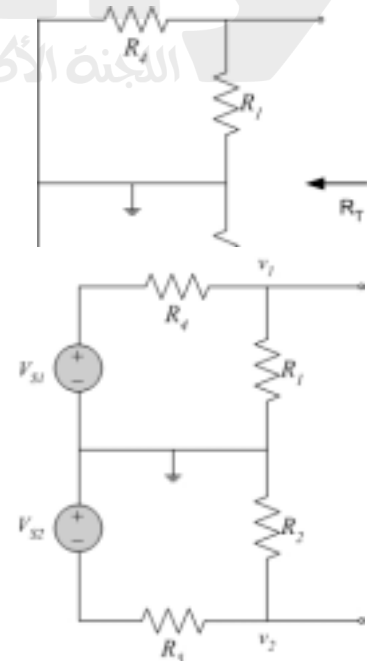
$$R_T = R_1 \parallel R_4 + R_2 \parallel R_5 = \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_5}{R_2 + R_5}$$

- (2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:

$$\frac{v_1}{R_1} + \frac{v_1 - V_{S1}}{R_4} = 0$$

For node #2:



$$\frac{v_2}{R_2} + \frac{v_2 + V_{S2}}{R_5} = 0$$

Solving the system,

$$v_1 = \frac{R_1}{R_1 + R_4} V_{S1}$$

$$v_2 = -\frac{R_2}{R_2 + R_5} V_{S2}$$

Therefore,

$$v_{OC} = v_1 - v_2 = \frac{R_1}{R_1 + R_4} V_{S1} + \frac{R_2}{R_2 + R_5} V_{S2}$$

(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a):

$$i_a(R_1 + R_4) - R_1 i_c = V_{S1}$$

For mesh (b):

$$i_b(R_2 + R_5) - R_2 i_c = V_{S2}$$

For mesh (c):

$$-R_1 i_a - R_2 i_b + i_c(R_1 + R_2) = 0$$

Solving the system,

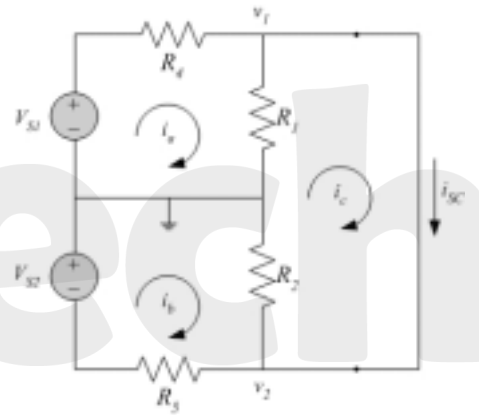
$$i_a = \frac{(R_1 R_2 + R_1 R_5 + R_2 R_5) V_{S1} + R_1 R_2 V_{S2}}{R_1 R_4 (R_2 + R_5) + R_2 R_5 (R_1 + R_4)}$$

$$i_b = \frac{R_1 R_2 V_{S1} + (R_1 R_2 + R_1 R_4 + R_2 R_4) V_{S2}}{R_1 R_4 (R_2 + R_5) + R_2 R_5 (R_1 + R_4)}$$

$$i_c = \frac{R_1 (R_2 + R_5) V_{S1} + R_2 (R_1 + R_4) V_{S2}}{R_1 R_4 (R_2 + R_5) + R_2 R_5 (R_1 + R_4)}$$

Therefore,

$$i_{SC} = i_c = \frac{R_1 (R_2 + R_5) V_{S1} + R_2 (R_1 + R_4) V_{S2}}{R_1 R_4 (R_2 + R_5) + R_2 R_5 (R_1 + R_4)}$$



Problem 3.44

Solution:

Known quantities:

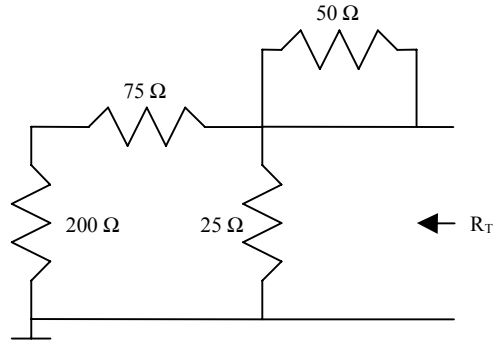
The schematic of the circuit (see Figure P3.6).

Find:

The Thévenin equivalent resistance seen by resistor R_5 , the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_5 is the load.

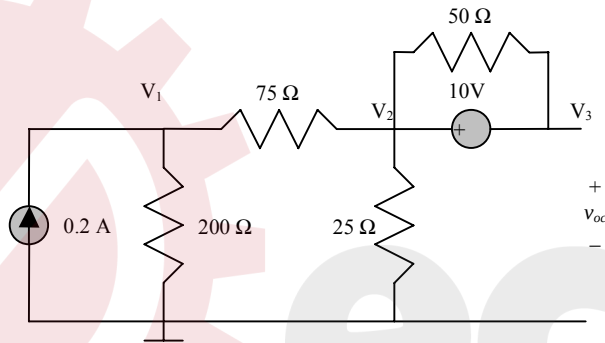
Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.



$$R_T = 25 \, \Omega \parallel (75 \, \Omega + 200 \, \Omega) = 22.92 \, \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.



For node #1:

$$\frac{v_1}{200} + \frac{v_1 - v_2}{75} = 0.2$$

For node #2:

$$\frac{v_2 - v_1}{75} + \frac{v_2}{25} + \frac{v_2 - v_3}{50} + i_{10V} = 0$$

For node #3:

$$\frac{v_3 - v_2}{50} = i_{10V}$$

For the voltage source:

$$v_3 + 10 = v_2$$

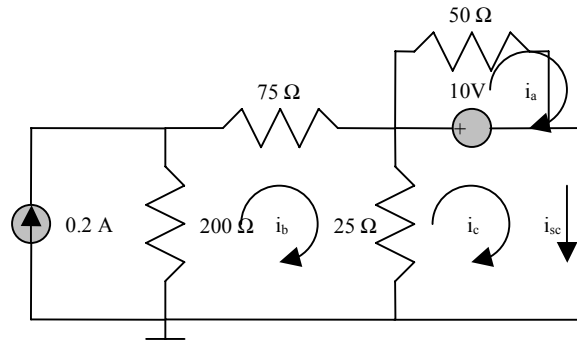
Solving the system,

$$v_1 = 13.33 \, \text{V}, \, v_2 = 3.33 \, \text{V} \text{ and } v_3 = -6.67 \, \text{V}.$$

Therefore,

$$v_{OC} = v_3 = -6.67 \, \text{V}.$$

(3) Replace the load with a short circuit. Redraw the circuit.



For mesh (a):

$$i_a(50) = 10$$

For mesh (b):

$$i_b(300) - i_c(25) = 40$$

For mesh (c):

$$i_b(25) - i_c(25) = 10$$

Solving the system,

$$i_a = 200 \text{ mA}, i_b = 109 \text{ mA} \text{ and } i_c = -291 \text{ mA}.$$

Therefore,

$$i_{sc} = i_c = -291 \text{ mA}.$$

Problem 3.45

Solution:

Known quantities:

The schematic of the circuit (see Figure P3.7).

Find:

The Thévenin equivalent resistance seen by resistor R_5 , the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_5 is the load.

Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 0.5 \Omega + 0.25 \Omega + (0.5 \Omega \parallel 0.5 \Omega) = 1 \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

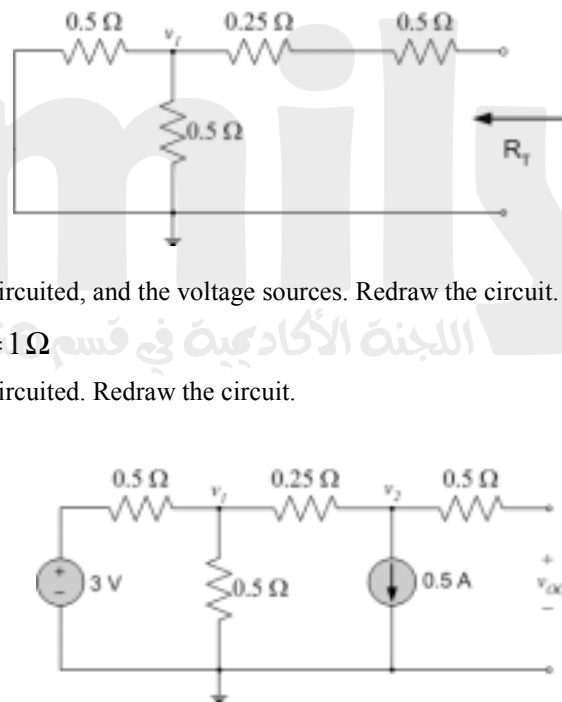
For node #1:

$$\frac{v_1 - 3}{0.5} + \frac{v_1}{0.5} + \frac{v_1 - v_2}{0.25} = 0$$

For node #2:

$$\frac{v_2 - v_1}{0.25} + 0.5 = 0$$

Solving the system,



$$v_1 = 1.375 \text{ V} \text{ and } v_2 = 1.25 \text{ V} .$$

Therefore,

$$v_{OC} = v_2 = 1.25 \text{ V} .$$

(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a):

$$i_a(0.5 + 0.5) - i_b(0.5) = 3$$

For meshes (b) and (c):

$$-i_a(0.5) + i_b(0.5 + 0.25) + i_c(0.5) = 0$$

For the current source:

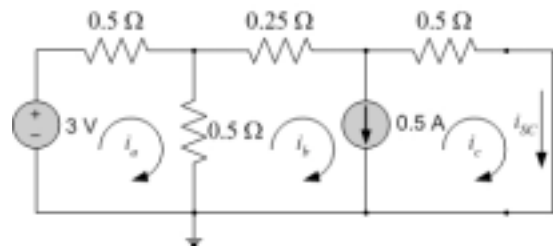
$$i_b - i_c = 0.5$$

Solving the system,

$$i_a = 3.875 \text{ A}, i_b = 1.75 \text{ A} \text{ and } i_c = 1.25 \text{ A} .$$

Therefore,

$$i_{SC} = i_c = 1.25 \text{ A} .$$



Problem 3.46

Solution:

Known quantities:

The schematic of the circuit (see Figure P3.12).

Find:

The Thévenin equivalent resistance seen by resistor R_3 , the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_3 is the load.

Assumption:

As in P3.12, we assume $T = 0.926^\circ\text{C}$, so that $V_{S2} = 9.26 \text{ V}$.

Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 12 \text{ k}\Omega \parallel 12 \text{ k}\Omega + 3 \text{ k}\Omega \parallel 24 \text{ k}\Omega = 8.67 \text{ k}\Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:

$$\frac{v_1 - 24}{12000} + \frac{v_1 - 9.26}{12000} = 0$$

For node #2:

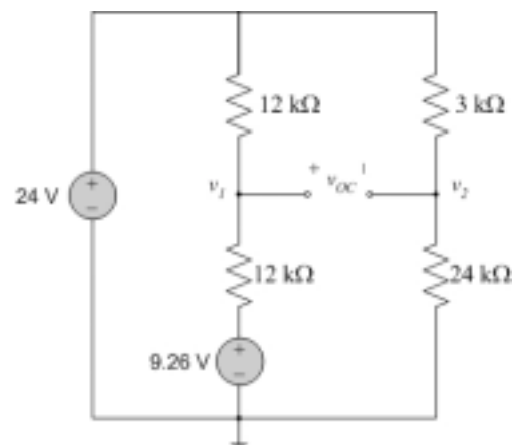
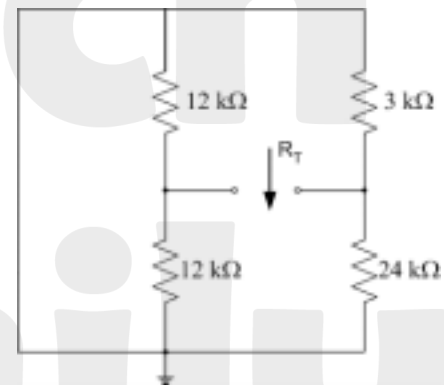
$$\frac{v_2 - 24}{3000} + \frac{v_2}{24000} = 0$$

Solving the system,

$$v_1 = 16.63 \text{ V} \text{ and } v_2 = 21.33 \text{ V} .$$

Therefore,

$$v_{OC} = v_1 - v_2 = -4.7 \text{ V} .$$



(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a):

$$i_a(24k) - i_b(12k) - i_c(12k) = 24 - 9.26$$

For mesh (b):

$$-i_a(12k) + i_b(36k) = 9.26$$

For mesh (c):

$$-i_a(12k) + i_c(15k) = 0$$

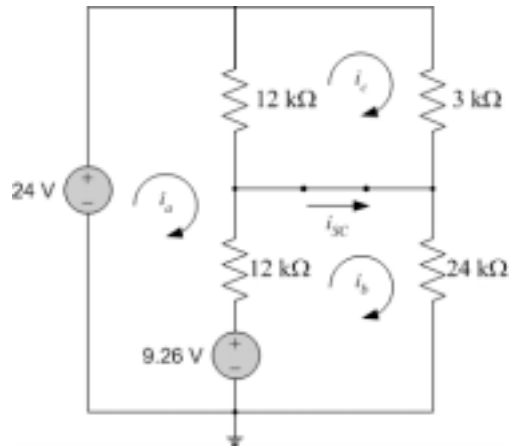
Solving the system,

$$i_a = 1.71 \text{ mA}, i_b = 0.83 \text{ mA} \text{ and}$$

$$i_c = 1.37 \text{ mA}.$$

Therefore,

$$i_{SC} = i_b - i_c = -0.54 \text{ mA}.$$



Problem 3.47

Solution:

Known quantities:

The schematic of the circuit (see Figure P3.14).

Find:

The Thévenin equivalent resistance seen by resistor R_4 , the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_4 is the load.

Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = R_2 \parallel (R_3 + (R_1 \parallel R_5)) = 20 \Omega \parallel (20 \Omega + (50 \Omega \parallel 15 \Omega)) = 12.24 \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:

$$\frac{v_1 - 12}{50} + \frac{v_1 - v_2}{20} + i_{5V} = 0$$

For node #2:

$$\frac{v_2 - v_1}{20} + \frac{v_2}{20} = 0$$

For node #3:

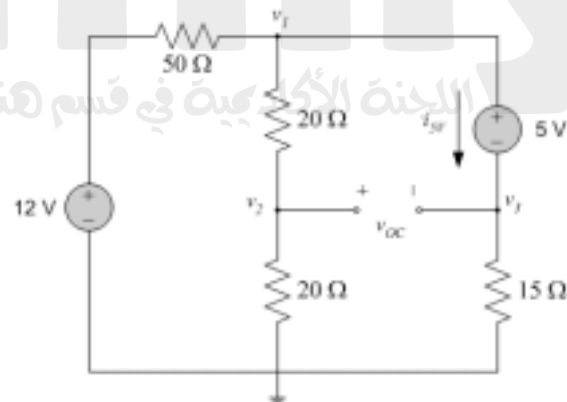
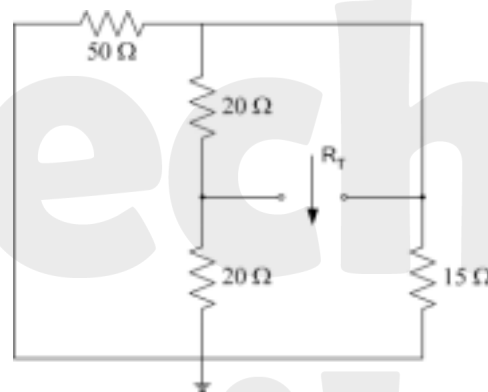
$$\frac{v_3}{15} - i_{5V} = 0$$

For the 5-V voltage source:

$$v_1 - v_3 = 5$$

Solving the system,

$$v_1 = 5.14 \text{ V}, v_2 = 2.57 \text{ V}, v_3 = 0.13 \text{ V} \text{ and } i_{5V} = 8.95 \text{ mA}.$$



Therefore,

$$v_{OC} = v_2 - v_3 = 2.44 \text{ V}.$$

(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a):

$$i_a(90) - i_b(20) - i_c(20) = 12$$

For mesh (b):

$$-i_a(20) + i_b(20) + 5 = 0$$

For mesh (c):

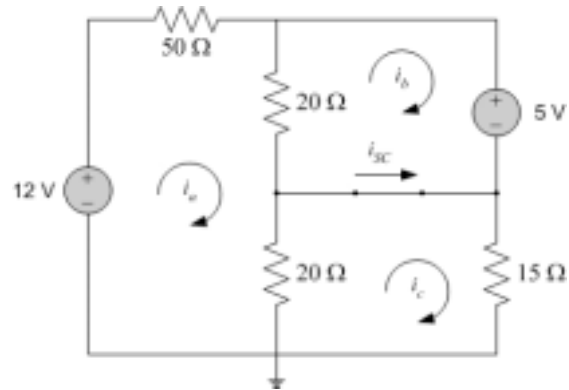
$$-i_a(20) + i_c(35) = 0$$

Solving the system,

$$i_a = 119.5 \text{ mA}, i_b = -130.5 \text{ mA} \text{ and } i_c = 68.3 \text{ mA}.$$

Therefore,

$$i_{SC} = i_c - i_b = 198.8 \text{ mA}.$$



Problem 3.48

Solution:

Known quantities:

The schematic of the circuit (see Figure P3.15).

Find:

The Thévenin equivalent resistance seen by resistor R_5 , the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_5 is the load.

Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 30 \Omega + 10 \Omega + (20 \Omega \parallel 30 \Omega) = 52 \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:

$$\frac{v_1 - 3}{20} + \frac{v_1}{30} + \frac{v_1 - v_2}{10} = 0$$

For node #2:

$$\frac{v_2 - v_1}{10} = 0.5$$

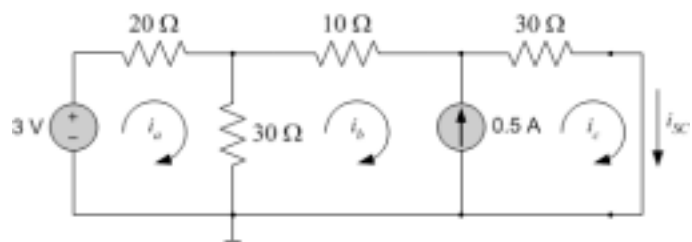
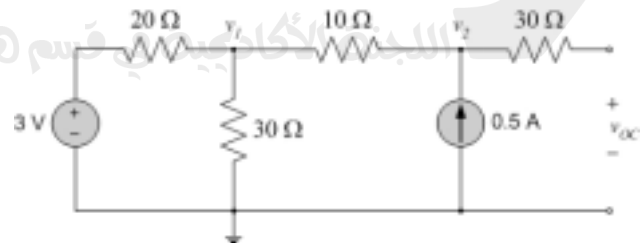
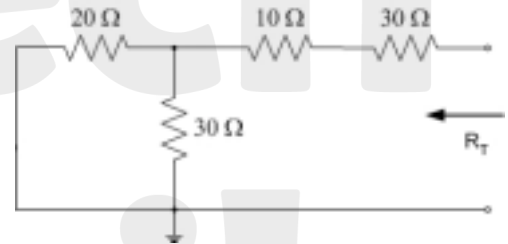
Solving the system,

$$v_1 = 7.8 \text{ V} \text{ and } v_2 = 12.8 \text{ V}.$$

Therefore,

$$v_{OC} = v_2 = 12.8 \text{ V}.$$

(3) Replace the load with a short circuit. Redraw the circuit.



For mesh (a):

$$i_a(20 + 30) - i_b(30) = 3$$

For meshes (b) and (c):

$$-i_a(30) + i_b(30 + 10) + i_c(30) = 0$$

For the current source:

$$i_c - i_b = 0.5$$

Solving the system,

$$i_a = -92 \text{ mA}, i_b = -254 \text{ mA} \text{ and } i_c = 246 \text{ mA}.$$

Therefore,

$$i_{SC} = i_c = 246 \text{ mA}.$$

Problem 3.49

Solution:

Known quantities:

The schematic of the circuit (see Figure P3.33).

Find:

The Thévenin equivalent resistance seen by resistor R , the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R is the load.

Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 1 \Omega \parallel 0.3 \Omega = 0.23 \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:

$$\frac{v_1}{1} + \frac{v_1 - 12}{0.3} = 12$$

Solving,

$$v_1 = 12 \text{ V}.$$

Therefore,

$$v_{OC} = v_1 = 12 \text{ V}.$$

(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a):

$$i_a(1 + 0.3) - i_b(0.3) - 12(1) + 12 = 0$$

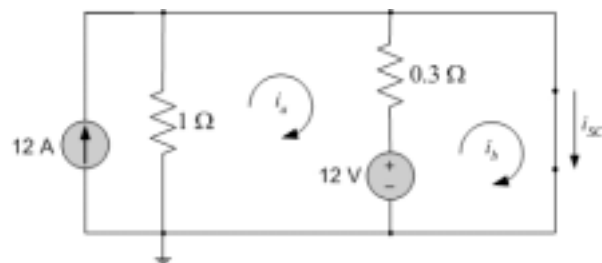
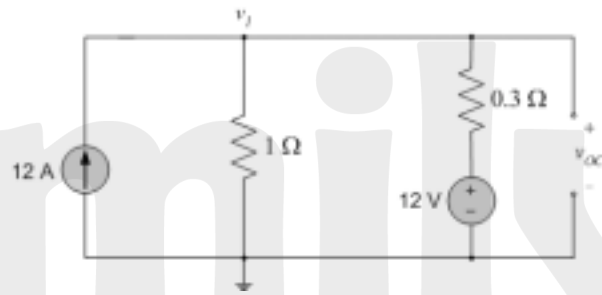
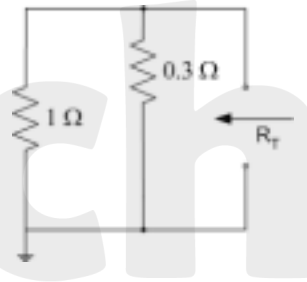
For mesh (b):

$$-i_a(0.3) + i_b(0.3) = 12$$

Solving the system,

$$i_a = 12 \text{ A} \text{ and } i_b = 52 \text{ A}.$$

Therefore,



$$i_{SC} = i_b = 52 \text{ A} .$$

Problem 3.50

Solution:

Known quantities:

The schematic of the circuit (see Figure P3.35).

Find:

The Thévenin equivalent resistance seen by resistor R_3 , the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_3 is the load.

Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 1 \Omega \parallel 7 \Omega + 1 \Omega \parallel 5 \Omega = 1.71 \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:

$$\frac{v_1 - 450}{1} + \frac{v_1}{7} = 0$$

For node #2:

$$\frac{v_2 + 450}{1} + \frac{v_2}{5} = 0$$

Solving the system,

$$v_1 = 393.75 \text{ V} \text{ and } v_2 = -375 \text{ V} .$$

Therefore,

$$v_{OC} = v_1 - v_2 = 768.75 \text{ V} .$$

(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a):

$$i_a(1+7) - i_c(7) = 450$$

For mesh (b):

$$i_b(5+1) - i_c(5) = 450$$

For mesh (c):

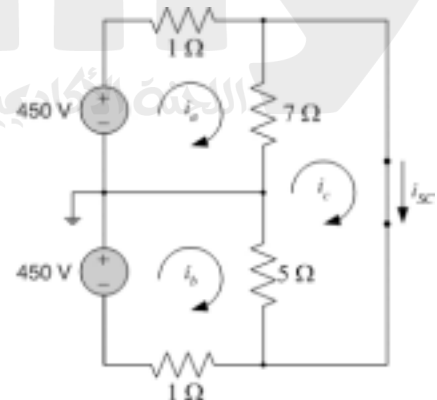
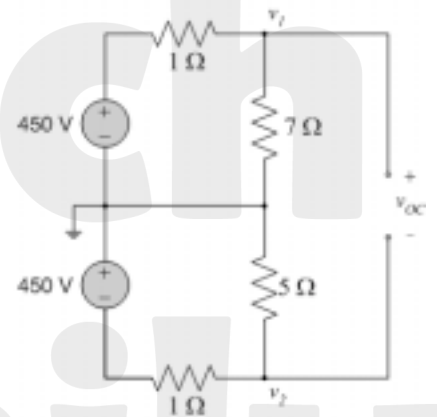
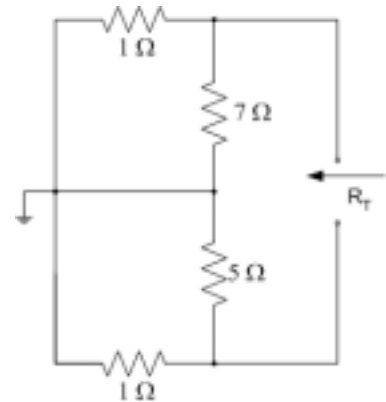
$$-i_a(7) - i_b(5) + i_c(7+5) = 0$$

Solving the system,

$$i_a = 450 \text{ A} , i_b = 450 \text{ A} \text{ and } i_c = 450 \text{ A} .$$

Therefore,

$$i_{SC} = i_c = 450 \text{ A} .$$



Problem 3.51**Solution:****Known quantities:**

The values of the voltage source, $V_S = 110 \text{ V}$, and the values of the 4 resistors in the circuit of Figure P3.51:

$$\begin{aligned} R_1 &= R_2 = 930 \text{ m}\Omega & R_3 &= 100 \text{ m}\Omega \\ R_S &= 19 \text{ m}\Omega \end{aligned}$$

Find:

The change in the voltage across the total load, when the customer connects the third load R_3 in parallel with the other two loads.

Analysis:

Choose a ground. If the node at the bottom is chosen as ground (which grounds one terminal of the ideal source), the only unknown node voltage is the required voltage. Specify directions of the currents and polarities of voltages.

Without R_3 :

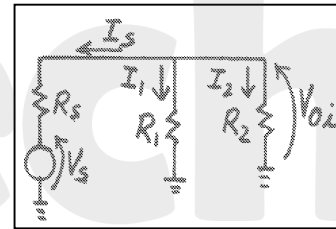
$$\text{KCL: } I_S + I_1 + I_2 = 0$$

OL:

$$\frac{V_{RS}}{R_S} + \frac{V_{R1}}{R_1} + \frac{V_{R2}}{R_2} = 0$$

$$\frac{V_{Oi} - V_S}{R_S} + \frac{V_{Oi} - 0}{R_1} + \frac{V_{Oi} - 0}{R_2} = 0$$

$$V_{Oi} = \frac{\frac{V_S}{R_S}}{\frac{1}{R_S} + \frac{1}{R_1} + \frac{1}{R_2}} = \frac{110}{1.04086} = 105.7 \text{ V}$$



With R_3 :

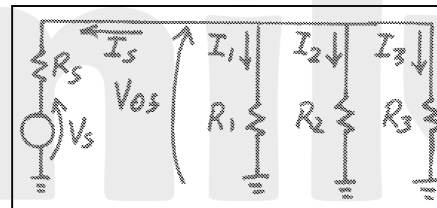
$$\text{KCL: } I_S + I_1 + I_2 + I_3 = 0$$

OL:

$$\frac{V_{RS}}{R_S} + \frac{V_{R1}}{R_1} + \frac{V_{R2}}{R_2} + \frac{V_{R3}}{R_3} = 0$$

$$\frac{V_{Of} - V_S}{R_S} + \frac{V_{Of} - 0}{R_1} + \frac{V_{Of} - 0}{R_2} + \frac{V_{Of} - 0}{R_3} = 0$$

$$V_{Of} = \frac{\frac{V_S}{R_S}}{\frac{1}{R_S} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{110}{1.04086 + 0.19} = 89.37 \text{ V}$$



Therefore, the voltage decreased by:

$$\Delta V_o = V_{Of} - V_{Oi} = -16.33 \text{ V}$$

Notes:

1. "Load" to an EE usually means current rather than resistance.
2. Additional load reduces the voltage supplied to the customer because of the additional voltage dropped across the losses in the distribution system.

Problem 3.52

Solution:

Known quantities:

The values of the voltage source, $V_S = 450 \text{ V}$, and the values of the 4 resistors in the circuit of Figure P3.52

$$R_1 = R_2 = 1.3 \Omega \quad R_3 = 500 \text{ m}\Omega$$

$$R_S = 19 \text{ m}\Omega$$

Find:

The change in the voltage across the total load, when the customer connects the third load R_3 in parallel with the other two loads.

Analysis:

See Solution to Problem 3.40 for a detailed mathematical analysis.

$$\Delta V_o = V_{of} - V_{oi} = -15.6 \text{ V}$$

Problem 3.53

Solution:

Known quantities:

The circuit shown in Figure P3.53, the values of the terminal voltage, V_T , before and after the application of the load, respectively $V_T = 20 \text{ V}$ and $V_T = 18 \text{ V}$, and the value of the load resistor $R_L = 2.7 \text{ k}\Omega$.

Find:

The internal resistance and the voltage of the ideal source.

Analysis:

KVL: $-V_S + I_T R_S + V_T = 0$

If $I_T = 0$: $V_S = V_T = 20 \text{ V}$

If $V_T = 18 \text{ V}$:

OL: $I_T = \frac{V_T}{R_L} = 6.67 \text{ mA}$ and $R_S = \frac{V_S - V_T}{I_T} = 300 \Omega$

Note that R_S is an equivalent resistance, representing the various internal losses of the source and is not physically a separate component. V_S is the voltage generated by some internal process. The source voltage can be measured directly by reducing the current supplied by the source to zero, i.e., no-load or open-circuit conditions. The source resistance cannot be directly measured; however, it can be determined, as was done above, using the interaction of the source with an external load.

Problem 3.54**Solution:****Known quantities:**

The values of the voltage source, $V_{CC} = 20 \text{ V}$, and the values of the 2 resistors in the circuit of Figure P3.54:

$$R_1 = 1.3 \text{ M}\Omega \quad R_2 = 220 \text{ k}\Omega$$

Find:

The Thévenin equivalent circuit with respect to the port shown in Figure P3.54.

Analysis:

The Thévenin equivalent voltage is the open circuit voltage [with $I = 0$] between the terminals of the port. Specify the polarity of the voltage.

KCL:

$$\frac{V_{TH} - V_{CC}}{R_1} + \frac{V_{TH}}{R_2} + I = 0$$

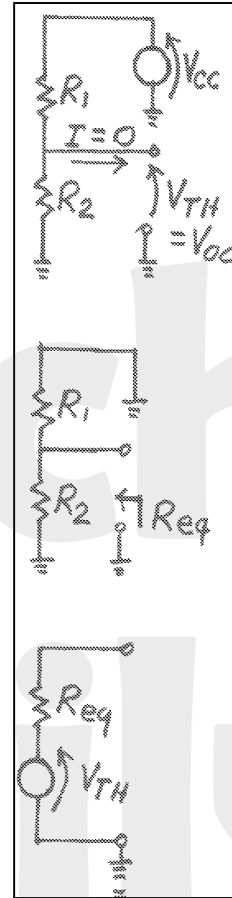
$$V_{TH} = \frac{\frac{V_{CC}}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} = 2.895 \text{ V}$$

Note that, since $I = 0$, R_1 and R_2 are effectively in series,

using a VD relation would be easier.

Suppress the ideal, independent voltage source, by shorting it. Determine the equivalent resistance with respect to the terminals of the port.

$$R_{EQ} = R_1 \parallel R_2 = \frac{(1.3 \cdot 10^6)(220 \cdot 10^3)}{1.3 \cdot 10^6 + 220 \cdot 10^3} = 188.2 \text{ k}\Omega$$

**Problem 3.55****Solution:****Known quantities:**

The values of the battery voltage, $V_B = 11 \text{ V}$, the value of the generator voltage, $V_G = 12 \text{ V}$, and the values of the 3 resistors in the circuit of Figure P3.55:

$$R_B = 0.7 \Omega \quad R_G = 0.3 \Omega \quad R_L = 7.2 \Omega$$

Find:

- The Thévenin equivalent of the circuit to the right of the terminal pair of port x-x'.
- The terminal voltage of the battery, i.e., the voltage between x and x'.

Analysis:

- a. Specify the polarity of the voltage:

$$VD: \quad V_{TH} = \frac{V_G R_L}{R_G + R_L} = \frac{(12)(7.2)}{0.3 + 7.2} = 11.52 \text{ V}$$

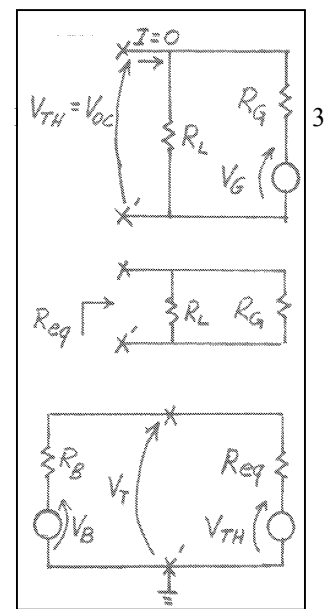
Suppress generator source:

$$R_{EQ} = R_L \parallel R_G = \frac{(7.2)(0.3)}{0.3 + 7.2} = 288 \text{ m}\Omega$$

- b. Specify the polarity of the terminal voltage. Choose a ground.

$$KCL: \quad \frac{V_T - V_B}{R_B} + \frac{V_T - V_{TH}}{R_{EQ}} = 0$$

$$V_T = \frac{\frac{V_B}{R_B} + \frac{V_{TH}}{R_{EQ}}}{\frac{1}{R_B} + \frac{1}{R_{EQ}}} = 11.37 \text{ V}$$

**Problem 3.56****Solution:**

The values of the battery voltage, $V_B = 11 \text{ V}$, the value of the generator voltage, $V_G = 12 \text{ V}$, and the values of the 3 resistors in the circuit of Figure P3.56:

$$R_B = 0.7 \Omega \quad R_G = 0.3 \Omega \quad R_L = 7.2 \Omega$$

Find:

- The Thévenin equivalent of the circuit to the left of the terminal pair of port y-y'.
- The terminal voltage of the generator, i.e., the voltage between y and y'.

Analysis:

- a. Specify the polarity of the Thévenin equivalent voltage:

$$VD: \quad V_{TH} = \frac{V_B R_L}{R_B + R_L} = \frac{(11)(7.2)}{0.7 + 7.2} = 10.03 \text{ V}$$

Suppress generator source:

$$R_T = R_L \parallel R_B = \frac{(7.2)(0.7)}{0.7 + 7.2} = 638 \text{ m}\Omega$$

- b. Specify the polarity of the terminal voltage. Choose a ground.

$$KCL: \quad \frac{V_T - V_G}{R_G} + \frac{V_T - v_T}{R_T} = 0$$

$$V_T = \frac{\frac{V_G}{R_G} + \frac{v_T}{R_T}}{\frac{1}{R_G} + \frac{1}{R_T}} = 11.37 \text{ V}$$

Section 3.7: Maximum power transfer

Problem 3.57

Solution:

Known quantities:

The values of the voltage and of the resistor in the equivalent circuit of Figure P3.57:

$$V_{TH} = 12 \text{ V}$$

$$R_{eq} = 8 \Omega$$

Assumptions:

Assume the conditions for maximum power transfer exist.

Find:

- The value of R_L .
- The power developed in R_L .
- The efficiency of the circuit, that is the ratio of power absorbed by the load to power supplied by the source.

Analysis:

- For maximum power transfer:

$$R_L = R_{eq} = 8 \Omega$$

$$\text{b. } V_{RL} = \frac{V_{TH} R_L}{R_{eq} + R_L} = \frac{(12)(8)}{8 + 8} = 6 \text{ V}$$

$$P_{RL} = \frac{V_{RL}^2}{R_L} = \frac{(6)^2}{8} = 4.5 \text{ W}$$

$$\text{c. } \eta = \frac{P_0}{P_S} = \frac{P_{RL}}{P_{Req} + P_{RL}} = \frac{I_S^2 R_L}{I_S^2 R_{eq} + I_S^2 R_L} = \frac{R_L}{R_{eq} + R_L} = 0.5 = 50\%$$

Problem 3.58

Solution:

Known quantities:

The values of the voltage and of the resistor in the equivalent circuit of Figure P3.57:

$$V_{TH} = 300 \text{ V}$$

$$R_{eq} = 600 \Omega$$

Assumptions:

Assume the conditions for maximum power transfer exist.

Find:

- The value of R_L .
- The power developed in R_L .
- The efficiency of the circuit, that is the ratio of power absorbed by the load to power supplied by the source.

Analysis:

- For maximum power transfer:

$$R_L = R_{eq} = 600 \, \Omega$$

b. VD:
$$V_{RL} = \frac{V_{TH} R_L}{R_{eq} + R_L} = \frac{(35)(600)}{600 + 600} = 17.5 \, \text{V}$$

$$P_{RL} = \frac{V_{RL}^2}{R_L} = \frac{(17.5)^2}{600} = 510.4 \, \text{mW}$$

c.
$$\eta = \frac{P_0}{P_S} = \frac{P_{RL}}{P_{Req} + P_{RL}} = \frac{I_S^2 R_L}{I_S^2 R_{eq} + I_S^2 R_L} = \frac{R_L}{R_{eq} + R_L} = 0.5 = 50\%$$

Problem 3.59

Solution:

Known quantities:

The values of the voltage source, $V_S = 12 \, \text{V}$, and of the resistance representing the internal losses of the source, $R_S = 0.3 \, \Omega$, in the circuit of Figure P3.59.

Find:

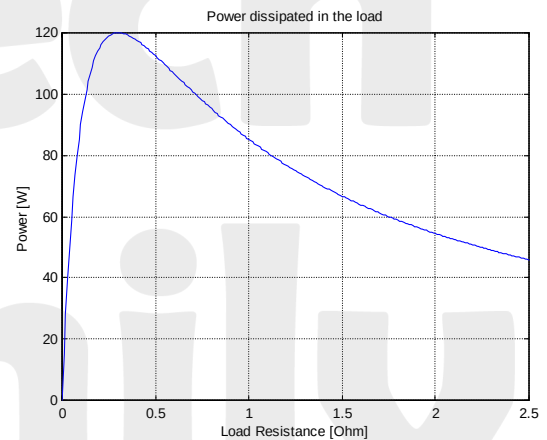
- Plot the power dissipated in the load as a function of the load resistance. What can you conclude from your plot?
- Prove, analytically, that your conclusion is valid in all cases.

Analysis:

a. KVL:
$$-V_S = IR_S + IR = 0$$

$$I = \frac{V_S}{R_S + R} \quad P_R = I^2 R$$

$R \, [\Omega]$	$I \, [\text{A}]$	$P_R \, [\text{W}]$
0	40	0.0
0.1	30	90.0
0.3	20	120.0
0.9	10	90.0
2.1	5	52.5



b.
$$P_R = I^2 R = \frac{V_S^2 R}{(R + R_S)^2} = V_S^2 R (R + R_S)^{-2}$$

$$\frac{dP_R}{dR} = V_S^2 (1)(R + R_S)^{-2} + V_S^2 (R)(-2)(R + R_S)^{-3}(1) = 0$$

$$(R + R_S)^1 - 2R = 0 \Rightarrow R = R_S$$

Section 3.8: Nonlinear circuit elements

Problem 3.60

Solution:

Known quantities:

The two nonlinear resistors, in the circuit of Figure P3.60, are characterized by:

$$i_a = 2v_a^3 \quad i_b = v_b^3 + 10v_b$$

Find:

The node voltage equations in terms of v_1 and v_2 .

Analysis:

At node 1, $\frac{v_1}{1} + i_a = 1 \Rightarrow v_1 + 2v_a^3 = 1$

At node 2, $i_b - i_a = 26 \Rightarrow v_b^3 + 10v_b - 2v_a^3 = 26$

But $v_a = v_1 - v_2$ and $v_b = v_2$. Therefore, the node equations are

$$v_1 + 2(v_1 - v_2)^3 = 1$$

and

$$v_2^3 + 10v_2 - 2(v_1 - v_2)^3 = 26$$

Problem 3.61

Solution:

Known quantities:

The characteristic curve I - V shown in Figure P3.61, and the values of the voltage, $V_T = 15 \text{ V}$, and of the resistance, $R_T = 200 \Omega$, in the circuit of Figure P3.61.

Find:

- The operating point of the element that has the characteristic curve shown in Figure P3.61.
- The incremental resistance of the nonlinear element at the operating point of part a.
- If V_T were increased to 20 V , find the new operating point and the new incremental resistance.

Analysis:

$$-15 + 200I + V = 0$$

a. KVL: $-15 + 200(0.0025V^2) + V = 0$

Solving for V and I ,

$$I = 52.2 \text{ mA} \quad V = 4.57 \text{ V} \quad \text{or} \quad -6.57 \text{ V}$$

The second voltage value is physically impossible.

b. $R_{\text{inc}} = 10(0.0522)^{-0.5} = 43.8 \Omega$

c. $I = 73 \text{ mA} \quad V = 5.40 \text{ V} \quad R_{\text{inc}} = 37 \Omega$

Problem 3.62**Solution:****Known quantities:**

The characteristic curve I - V shown in Figure P3.62, and the values of the voltage, $V_S = 450$ V, and of the resistance, $R = 9$ Ω , in the circuit of Figure P3.62.

Find:

The current through and the voltage across the nonlinear device.

Analysis:

The I - V characteristic for the nonlinear device is given. Plot the circuit I - V characteristic, i.e., the DC load line.

$$\text{KVL: } -V_S + I_D R + V_D = 0$$

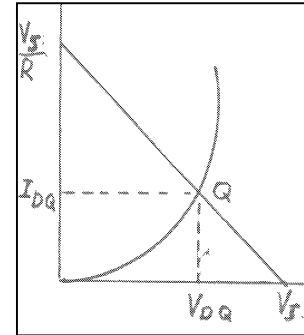
$$I_D = \frac{V_S - V_D}{R} = \frac{450 - V_D}{9} =$$

$$= 0 \text{ A} \quad \text{if } V_D = 450 \text{ V}$$

$$= 50 \text{ A} \quad \text{if } V_D = 0$$

The DC load line [circuit characteristic] is linear. Plotting the two intercepts above and connecting them with a straight line gives the DC load line. The solution for V and I is at the intersection of the device and circuit characteristics:

$$I_{DQ} \approx 26 \text{ A} \quad V_{DQ} \approx 210 \text{ V}.$$

**Problem 3.63****Solution:****Known quantities:**

The I - V characteristic shown in Figure P3.63, and the values of the voltage, $V_S = V_T = 1.5$ V, and of the resistance, $R = R_{eq} = 60$ Ω , in the circuit of Figure P3.63.

Find:

The current through and the voltage across the nonlinear device.

Analysis:

The solution is at the intersection of the device and circuit characteristics.

The device I - V characteristic is given. Determine and plot the circuit I - V characteristic.

$$\text{KVL: } -V_S + I_D R + V_D = 0$$

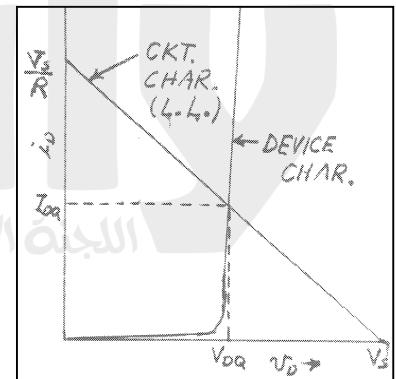
$$I_D = \frac{V_S - V_D}{R} = \frac{1.5 \text{ V} - V_D}{60 \Omega} =$$

$$= 0 \text{ A} \quad \text{if } V_D = 1.5 \text{ V}$$

$$= 25 \text{ mA} \quad \text{if } V_D = 0$$

The DC load line [circuit characteristic] is linear. Plotting the two intercepts above and connecting them with a straight line gives the DC load line. The solution is at the intersection of the device and circuit characteristics, or "Quiescent", or "Q", or "DC operating" point:

$$I_{DQ} \approx 12 \text{ mA} \quad V_{DQ} \approx 0.77 \text{ V}.$$



Problem 3.64**Solution:****Known quantities:**

The I - V characteristic shown in Figure P3.64 as a function of pressure.

$$V_S = V_T = 2.5 \text{ V} \quad R = R_{eq} = 125 \Omega$$

Find:

The DC load line, the voltage across the device as a function of pressure, and the current through the nonlinear device when $p = 30$ psig.

Analysis:

Circuit characteristic [DC load line]:

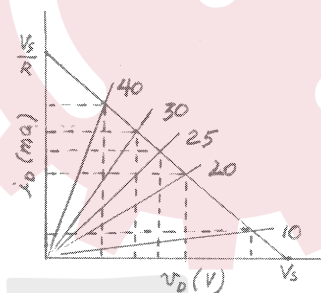
$$\text{KVL: } -V_S + I_D R + V_D = 0$$

$$I_D = \frac{V_S - V_D}{R} = \frac{2.5 \text{ V} - V_D}{125 \Omega}$$

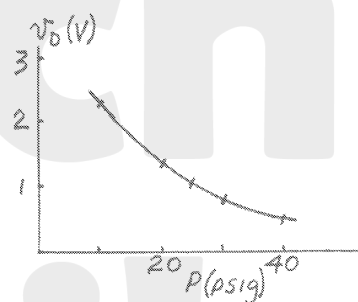
$$= 0 \text{ A} \quad \text{if } V_D = 2.5 \text{ V}$$

$$= 20 \text{ mA} \quad \text{if } V_D = 0$$

The circuit characteristic is a linear relation. Plot the two intercepts and connect with a straight line to plot the DC load line. Solutions are at the intersections of the circuit with the device characteristics, i.e.:



p [psig]	V_D [V]
10	2.14
20	1.43
25	1.18
30	0.91
40	0.60



The plot is nonlinear.

At $p = 30$ psig: $V_D = 1.08 \text{ V}$ $I_D = 12.5 \text{ mA}$.

Problem 3.65**Solution:****Known quantities:**

The I - V characteristic of the nonlinear device in the circuit shown in Figure P3.65:

$$I_D = I_0 e^{\frac{V_D}{V_T}}$$

$$I_0 = 10^{-15} \text{ A} \quad V_T = 26 \text{ mV}$$

$$V_S = V_T = 1.5 \text{ V}$$

$$R = R_{eq} = 60 \Omega$$

Find:

An expression for the DC load line. The voltage across and current through the nonlinear device.

Analysis:

Circuit characteristic [DC load line]:

$$KVL: -V_S + I_D R + V_D = 0$$

$$[1] \quad I_D = \frac{V_S - V_D}{R} = \frac{1.5 - V_D}{60}$$

$$[2] \quad V_D = V_T \ln\left(\frac{I_D}{I_0}\right) = 0.026 \cdot \ln\left(\frac{I_D}{10^{-15}}\right)$$

Iterative procedure:

Initially guess $V_D = 750 \text{ mV}$. Note this voltage must be between zero and the value of the source voltage.

Then:

- Use Equation [1] to compute a new I_D .
- Use Equation [2] to compute a new V_D .
- Iterate, i.e., go step a. and repeat.

$V_D \text{ [mV]}$	$I_D \text{ [mA]}$
750	12.5
784.1	11.93
782.9	11.95
782.9	11.95
...	...

$$I_{DQ} \approx 11.95 \text{ mA} \quad V_{DQ} \approx 782.9 \text{ mV}.$$

Problem 3.66**Solution:****Known quantities:**The I - V characteristic shown in Figure P3.66 as a function of pressure.

$$V_S = V_T = 2.5 \text{ V} \quad R = R_{eq} = 125 \Omega$$

Find:The DC load line, and the current through the nonlinear device when $p = 40 \text{ psig}$.**Analysis:**

Circuit characteristic [DC load line]:

$$KVL: -V_S + I_D R + V_D = 0$$

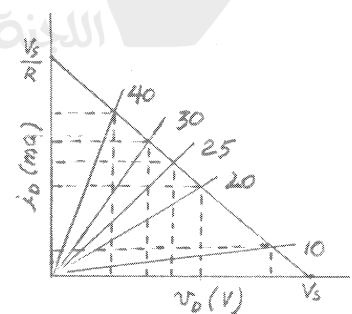
$$I_D = \frac{V_S - V_D}{R} = \frac{2.5 \text{ V} - V_D}{125 \Omega}$$

$$= 0 \text{ A} \quad \text{if } V_D = 2.5 \text{ V}$$

$$= 20 \text{ mA} \quad \text{if } V_D = 0$$

The circuit characteristic is a linear relation that can be plotted by plotting the two intercepts and connecting them with a straight line. Solutions are at the intersections of the circuit and device characteristics.

$$\text{At } p = 40 \text{ psig:} \quad V_D = 0.60 \text{ V} \quad I_D = 15.2 \text{ mA}$$



Chapter 4 Instructor Notes

The chapter starts by developing the dynamic equations for energy storage elements. The analogy between electrical and hydraulic circuits (*Make The Connection: Fluid (hydraulic) Capacitance*, p. 138, *Make The Connection: Fluid (hydraulic) Inertance*, p. 150, Table 4.2) is introduced early to permit a connection with ideas that may already be familiar to the student from a course in fluid mechanics, such as mechanical, civil, chemical and aerospace engineers are likely to have already encountered. A *Focus on Measurements* box: *Capacitive displacement transducer and microphones*, pp. 147-148, permits approaching the subject of capacitance in a pragmatic fashion, if so desired. The instructor wishing to gain a more in-depth understanding of such transducers will find a detailed analysis in¹.

Next, signal sources are introduced, with special emphasis on sinusoids. The material in this section can also accompany a laboratory experiment on signal sources. The emphasis placed on sinusoidal signals is motivated by the desire to justify the concepts of phasors and impedance, which are introduced next. The author has found that presenting the impedance concept early on is an efficient way of using the (invariably too short) semester or quarter. The chapter is designed to permit a straightforward extension of the resistive circuit analysis concepts developed in Chapter 3 to the case of dynamic circuits excited by sinusoids. The ideas of nodal and mesh analysis, and of equivalent circuits, can thus be reinforced at this stage. The treatment of AC circuit analysis methods is reinforced by the usual examples and drill exercises, designed to avoid unnecessarily complicated circuits. Two *Focus on Methodology* boxes (pp. 165 and 180) provide the student with a systematic approach to the solution of basic AC analysis problems using phasor and impedance concepts.

The capacitive displacement transducer example is picked up again in *Focus on Measurements: Capacitive displacement transducer* (pp.175-177) to illustrate the use of impedances in a bridge circuit. This type of circuit is very common in mechanical measurements, and is likely to be encountered at some later time by some of the students.

The homework problems in this chapter are mostly exercises aimed at mastery of the techniques.

Learning Objectives

1. Compute currents, voltages and energy stored in capacitors and inductors.
2. Calculate the average and root-mean-square value of an arbitrary (periodic) signal.
3. Write the differential equation(s) for circuits containing inductors and capacitors.
4. Convert time-domain sinusoidal voltages and currents to phasor notation, and vice-versa, and represent circuits using impedances.
5. Apply the circuit analysis methods of Chapter 3 to AC circuits in phasor form.

¹ E. O. Doebelin, *Measurement Systems – Application and Design*, 4th Edition, McGraw-Hill, New York, 1990.

Section 4.1: Energy Storage Elements

Problem 4.1

Solution:

Known quantities:

Inductance value, $L = 0.5 \text{ H}$; the current through the inductor as a function of time.

Find:

The voltage across the inductor, (Eq. 4.9), as a function of time.

Assumptions:

$$i_L(t \leq 0) = 0$$

Analysis:

Using the differential relationship for the inductor, we may obtain the voltage by differentiating the current:

$$\begin{aligned} v_L(t) &= L \frac{di_L(t)}{dt} = 0.5 \frac{di_L(t)}{dt} = 0.5 \times \left[-377 \times 2 \sin\left(377t + \frac{\pi}{6}\right) \right] \\ &= 377 \sin\left(377t + \frac{\pi}{6} - \pi\right) = 377 \sin\left(377t - \frac{5\pi}{6}\right) \text{ V} \end{aligned}$$

Problem 4.2

Solution:

Known quantities:

Capacitance value $C = 100 \mu\text{F}$; capacitor terminal voltage as a function of time.

Find:

The current through the capacitor as a function of time for each case:

- $v_c(t) = 40 \cos(20t - \pi/2) \text{ V}$
- $v_c(t) = 20 \sin(100t) \text{ V}$
- $v_c(t) = -60 \sin(80t + \pi/6) \text{ V}$
- $v_c(t) = 30 \cos(100t + \pi/4) \text{ V}$.

Assumptions:

The capacitor is initially discharged: $v_c(t=0) = 0$

Analysis:

Using the defining differential relationship for the capacitor, (Eq. 4.4), we may obtain the current by differentiating the voltage:

$$i_C(t) = C \frac{dv_C(t)}{dt} = 100 \times 10^{-6} \frac{dv_C(t)}{dt} = 10^{-4} \frac{dv_C(t)}{dt}$$

a)

$$\begin{aligned}
 i_C(t) &= 10^{-4} \left[-20 \times 40 \sin \left(20t - \frac{\pi}{2} \right) \right] = -0.08 \sin \left(20t - \frac{\pi}{2} \right) \\
 &= 0.08 \sin \left(20t - \frac{\pi}{2} + \pi \right) = 0.08 \sin \left(20t + \frac{\pi}{2} \right) \text{ A}
 \end{aligned}$$

b)

$$i_C(t) = 10^{-4} [100 \times 20 \cos 100t] = 0.2 \cos 100t \text{ A}$$

c)

$$\begin{aligned}
 i_C(t) &= 10^{-4} \left[-80 \times 60 \cos \left(80t + \frac{\pi}{6} \right) \right] = -0.48 \cos \left(80t + \frac{\pi}{6} \right) \\
 &= 0.48 \cos \left(80t + \frac{\pi}{6} - \pi \right) = 0.48 \cos \left(80t - \frac{5\pi}{6} \right) \text{ A}
 \end{aligned}$$

d)

$$\begin{aligned}
 i_C(t) &= 10^{-4} \left[-100 \times 30 \sin \left(100t + \frac{\pi}{4} \right) \right] = -0.3 \sin \left(100t + \frac{\pi}{4} \right) \\
 &= 0.3 \sin \left(100t + \frac{\pi}{4} - \pi \right) = 0.3 \sin \left(100t - \frac{3\pi}{4} \right) \text{ A}
 \end{aligned}$$

Problem 4.3

Solution:

Known quantities:

Inductance value, $L = 250 \text{ mH}$; the current through the inductor, as a function of time.

Find:

The voltage across the inductor as a function of time for each case

- a) $i_L(t) = 5 \sin 25t \text{ A}$
- b) $i_L(t) = -10 \cos 50t \text{ A}$
- c) $i_L(t) = 25 \cos(100t + \pi/3) \text{ A}$
- d) $i_L(t) = 20 \sin(10t - \pi/12) \text{ A}$.

Assumptions:

$$i_L(t \leq 0) = 0$$

Analysis:

Using the differential relationship for the inductor, (Eq. 4.9), we may obtain the voltage by differentiating the current:

$$v_L(t) = L \frac{di_L(t)}{dt} = 250 \times 10^{-3} \frac{di_L(t)}{dt} = 0.25 \frac{di_L(t)}{dt}$$

a)

$$v_L(t) = 0.25 [25 \times 5 \cos 25t] = 31.25 \cos 25t \text{ V}$$

b)

$$v_L(t) = 0.25 [-50 \times (-10 \sin 50t)] = 125 \sin 50t \text{ V}$$

c)

$$\begin{aligned}
 v_L(t) &= 0.25 \left[-100 \times 25 \sin \left(100t + \frac{\pi}{3} \right) \right] = -625 \sin \left(100t + \frac{\pi}{3} \right) \\
 &= 625 \sin \left(100t + \frac{\pi}{3} - \pi \right) = 625 \sin \left(100t - \frac{2\pi}{3} \right) \text{ V}
 \end{aligned}$$

d)

$$v_L(t) = 0.25 \left[10 \times 20 \cos \left(10t - \frac{\pi}{12} \right) \right] = 50 \cos \left(10t - \frac{\pi}{12} \right) \text{ V}$$

Problem 4.4**Solution:****Known quantities:**

Inductance value; resistance value; the current through the circuit shown in Figure P4.4 as a function of time.

Find:

The energy stored in the inductor as a function of time.

Analysis:

The magnetic energy stored in an inductor may be found from, (Eq. 4.16):

$$w_L(t) = \frac{1}{2} Li(t)^2 = \frac{1}{2} (2)i^2(t) = i^2(t)$$

For $-\infty < t < 0$,

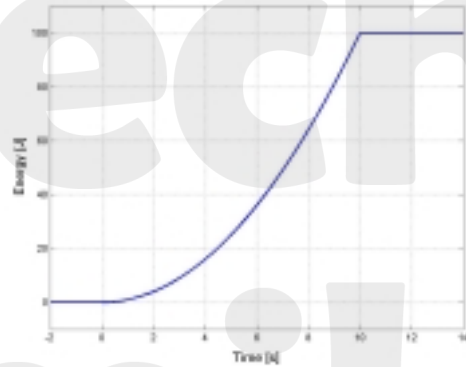
$$w_L(t) = 0$$

For $0 \leq t < 10 \text{ s}$

$$w_L(t) = t^2 \text{ J}$$

For $10 \text{ s} \leq t < +\infty$

$$w_L(t) = 100 \text{ J}$$

**Problem 4.5****Solution:****Known quantities:**

Inductance value; resistance value; the current through the circuit in Figure P4.4 as a function of time.

Find:

The energy delivered by the source as a function of time.

Analysis:

The energy delivered by the source is the sum of the energy absorbed by the resistance and the energy stored in the inductor:

$$w_s(t) = w_R(t) + w_L(t) = Ri^2(t) + \frac{1}{2} Li^2(t)$$

$$= (1)i^2(t) + \frac{1}{2}(2)i^2(t) = 2i^2(t)$$

For $-\infty < t < 0$,

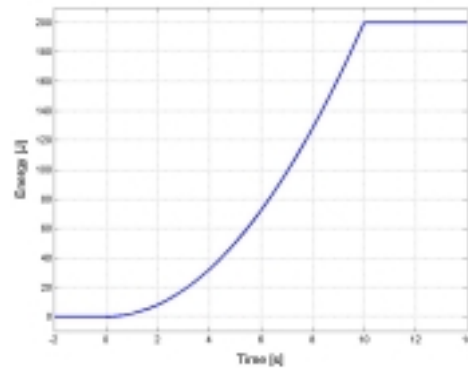
$$w_s(t) = 0$$

For $0 \leq t < 10$ s

$$w_s(t) = 2t^2 \text{ J}$$

For $10 \leq t < +\infty$

$$w_s(t) = 200 \text{ J}$$



Problem 4.6

Solution:

Known quantities:

Inductance value; resistance value; the current through the circuit shown in Figure P4.4 as a function of time.

Find:

The energy stored in the inductor and the energy delivered by the source as a function of time.

Analysis:

The magnetic energy stored in an inductor may be found from, (Eq. 4.16):

$$w_L(t) = \frac{1}{2}Li(t)^2 = \frac{1}{2}(2)i^2(t) = i^2(t)$$

For $-\infty < t < 0$,

$$w_L(t) = 0$$

For $0 \leq t < 10$ s

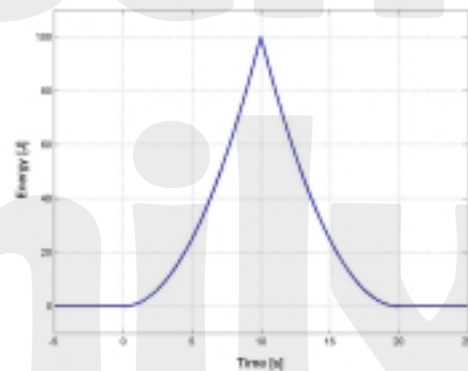
$$w_L(t) = t^2 \text{ J}$$

For $10 \leq t < 20$ s

$$w_L(t) = (20 - t)^2 = 400 - 40t + t^2 \text{ J}$$

For $20 \leq t < +\infty$

$$w_L(t) = 0 \text{ J}$$



The energy delivered by the source is the sum of the energy absorbed by the resistance and the energy stored in the inductor:

$$w_s(t) = w_R(t) + w_L(t) = Ri^2(t) + \frac{1}{2}Li^2(t)$$

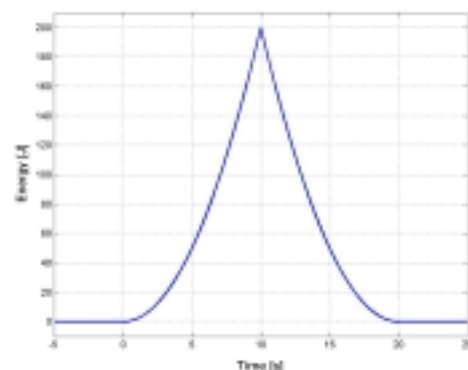
$$= (1)i^2(t) + \frac{1}{2}(2)i^2(t) = 2i^2(t)$$

For $-\infty < t < 0$,

$$w_s(t) = 0$$

For $0 \leq t < 10$ s

$$w_s(t) = 2t^2 \text{ J}$$



For $10 \leq t < 20 \text{ s}$

$$w_L(t) = 2(20 - t)^2 = 800 - 80t + 2t^2 \text{ J}$$

For $20 \text{ s} \leq t < +\infty$

$$w_L(t) = 0 \text{ J}$$

Problem 4.7

Solution:

Known quantities:

Capacitance value; resistance value; the voltage applied to the circuit shown in Figure P4.7 as a function of time.

Find:

The energy stored in the capacitor as a function of time.

Analysis:

The energy stored in a capacitor may be found from:

$$w_C(t) = \frac{1}{2} C v(t)^2 = \frac{1}{2} (0.1) v(t)^2 = 0.05 v(t)^2$$

For $-\infty < t < 0$

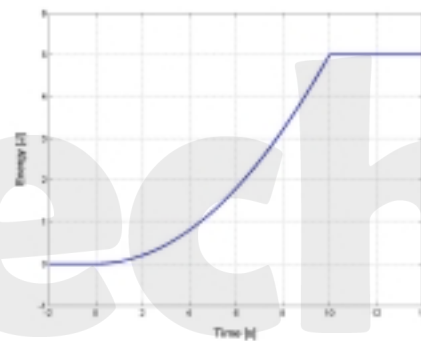
$$w_C(t) = 0$$

For $0 \leq t < 10 \text{ s}$

$$w_C(t) = 0.05 t^2 \text{ J}$$

For $10 \text{ s} \leq t < +\infty$

$$w_C(t) = 0.05(10)^2 = 5 \text{ J}$$



Problem 4.8

Solution:

Known quantities:

Capacitance value; resistance value; the voltage applied to the circuit shown in Figure P4.7 as a function of time.

Find:

The energy delivered by the source as a function of time.

Analysis:

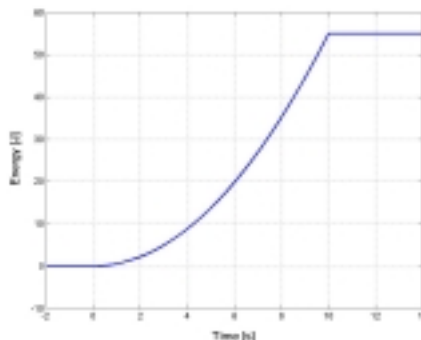
The energy delivered by the source is the sum of the energy absorbed by the resistance and the energy stored in the capacitor:

$$\begin{aligned} w_S(t) &= w_R(t) + w_C(t) = \frac{v^2(t)}{R} + \frac{1}{2} C v^2(t) \\ &= \frac{1}{2} v^2(t) + \frac{1}{2} (0.1) v^2(t) = 0.55 i^2(t) \end{aligned}$$

For $-\infty < t < 0$,

$$w_S(t) = 0$$

For $0 \leq t < 10 \text{ s}$



$$w_s(t) = 0.55t^2 \text{ J}$$

For $10 \leq t < +\infty$

$$w_s(t) = 55 \text{ J}$$

Problem 4.9

Solution:

Solution:

Known quantities:

Capacitance value; resistance value; the voltage applied to the circuit shown in Figure P4.7 as a function of time.

Find:

The energy stored in the capacitor and the energy delivered by the source as a function of time.

Analysis:

The energy stored in a capacitor may be found from:

$$w_C(t) = \frac{1}{2} C v(t)^2 = \frac{1}{2} (0.1) v(t)^2 = 0.05 v(t)^2$$

For $-\infty < t < 0$

$$w_C(t) = 0$$

For $0 \leq t < 10 \text{ s}$

$$w_C(t) = 0.05 t^2 \text{ J}$$

For $10 \leq t < 20 \text{ s}$

$$w_C(t) = 0.05 (20 - t)^2 = 20 - 2t + 0.05 t^2 \text{ J}$$

For $10 \leq t < +\infty$

$$w_C(t) = 0$$

The energy delivered by the source is the sum of the energy absorbed by the resistance and the energy stored in the capacitor:

$$\begin{aligned} w_s(t) &= w_R(t) + w_C(t) = \frac{v^2(t)}{R} + \frac{1}{2} C v^2(t) \\ &= \frac{1}{2} v^2(t) + \frac{1}{2} (0.1) v^2(t) = 0.55 i^2(t) \end{aligned}$$

For $-\infty < t < 0$

$$w_s(t) = 0$$

For $0 \leq t < 10 \text{ s}$

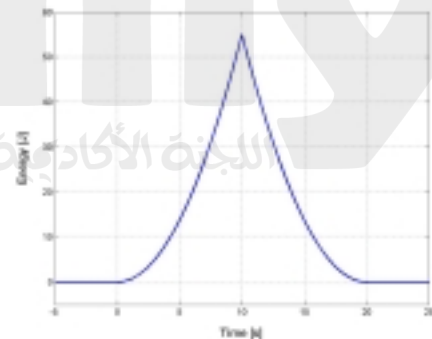
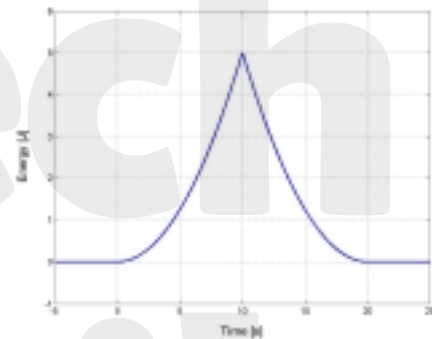
$$w_s(t) = 0.55 t^2 \text{ J}$$

For $10 \leq t < 20 \text{ s}$

$$w_s(t) = 0.55 (20 - t)^2 = 220 - 22t + 0.55 t^2 \text{ J}$$

For $10 \leq t < +\infty$

$$w_s(t) = 0$$



Problem 4.10**Solution:****Known quantities:**

Capacitance, resistance and inductance values; the source voltage $v_s = 6 \text{ V}$ applied to the circuit shown in Figure P4.10.

Find:

The energy stored in each capacitor and inductor.

Analysis:

Under steady-state conditions, all the currents are constant, no current can flow through the capacitors, and the voltage across any inductor is equal to zero.

$$v_{2F} = v_{4\Omega} \Rightarrow \frac{6 - v_{4\Omega}}{2} = \frac{v_{4\Omega}}{4} + \frac{v_{4\Omega}}{8} \Rightarrow v_{4\Omega} = 3.43 \text{ V}$$

$$\Rightarrow w_{2F} = \frac{1}{2} C_{2F} v_{2F}^2 = \frac{1}{2} (2 \text{ F})(3.43 \text{ V})^2 = 11.76 \text{ J}$$

$$v_{1F} = v_{2H} = 0 \Rightarrow w_{1F} = \frac{1}{2} C_{1F} v_{1F}^2 = \frac{1}{2} (1 \text{ F})(0)^2 = 0$$

$$i_{2H} = \frac{v_{4\Omega}}{8} = 0.43 \text{ A} \Rightarrow w_{2H} = \frac{1}{2} L_{2H} i_{2H}^2 = \frac{1}{2} (2 \text{ H})(0.43 \text{ A})^2 = 0.18 \text{ J}$$

$$v_{3F} = v_{4\Omega} = 3.43 \text{ V} \Rightarrow w_{3F} = \frac{1}{2} C_{3F} v_{3F}^2 = \frac{1}{2} (3 \text{ F})(3.43 \text{ V})^2 = 17.65 \text{ J}$$

Problem 4.11**Solution:****Known quantities:**

Capacitance, resistance and inductance values; the voltage $v_A = 12 \text{ V}$ applied to the circuit shown in Figure P4.11.

Find:

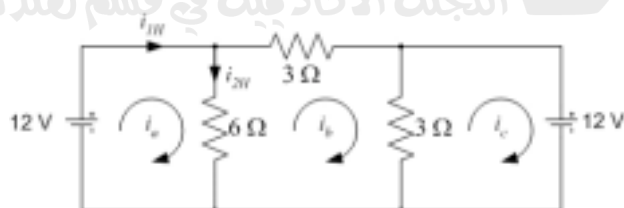
The energy stored in each capacitor and inductor.

Analysis:

Under steady-state conditions, all the currents are constant, no current can flow across the capacitors, and the voltage across any inductor is equal to zero.

The voltage for the 1-F capacitor is equal to the 12-Volt input.

Since the voltage is the same on either end of the 3- Ω resistor in parallel with the 2-F capacitor, there is no voltage drop through either component.



Finally, since there is no voltage drop through the 3- Ω resistor in parallel with the 2-F capacitor, there is no current flow through the resistor, and the current through the 1-H inductor is equal to the current through the 2-H inductor.

Therefore,

$$v_{1F} = v_A = 12 \text{ V} \Rightarrow w_{1F} = \frac{1}{2} C_{1F} v_{1F}^2 = \frac{1}{2} (1 \text{ F})(12 \text{ V})^2 = 72 \text{ J}$$

$$i_{1H} = \frac{12 \text{ V}}{6 \Omega} = 2 \text{ A} \Rightarrow w_{1H} = \frac{1}{2} L_{1H} i_{1H}^2 = \frac{1}{2} (1 \text{ H})(2 \text{ A})^2 = 2 \text{ J}$$

$$i_{2H} = i_{1H} = 2 \text{ A} \Rightarrow w_{2H} = \frac{1}{2} L_{2H} i_{2H}^2 = \frac{1}{2} (2 \text{ H})(2 \text{ A})^2 = 4 \text{ J}$$

$$v_{2F} = 0 \text{ V} \Rightarrow w_{2F} = \frac{1}{2} C_{2F} v_{2F}^2 = \frac{1}{2} (2 \text{ F})(0 \text{ V})^2 = 0 \text{ J}$$

Problem 4.12

Solution:

Known quantities:

Capacitance value $C = 80 \mu\text{F}$; the voltage applied to the capacitor as a function of time as shown in Figure P4.12.

Find:

The current through the capacitor as a function of time.

Analysis:

Since the voltage waveform is piecewise continuous, the derivative must be evaluated over each continuous segment.

For $0 < t < 5 \text{ ms}$

$$v_C(t) = m_{v_C} t + q_{v_C}$$

where:

$$m_{v_C} = \frac{[-10 \text{ V}] - [+20 \text{ V}]}{[5 \text{ ms}] - [0]} = -6 \frac{\text{V}}{\text{ms}}$$

$$q_{v_C} = +20 \text{ V}$$

$$i_C = C \frac{dv_C(t)}{dt} = C \frac{d}{dt} [m_{v_C} t + q_{v_C}] = C m_{v_C} = (80 \mu\text{F}) \left(-6 \frac{\text{V}}{\text{ms}} \right) = -480 \text{ mA}$$

For $5 \text{ ms} < t < 10 \text{ ms}$

$$v_C(t) = -10 \text{ V}$$

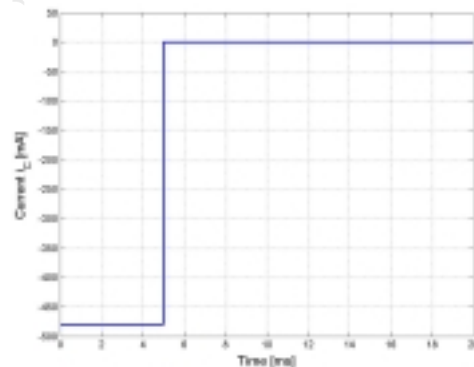
$$i_C = C \frac{dv_C(t)}{dt} = C \frac{d}{dt} [-10 \text{ V}] = 0$$

For $t > 10 \text{ ms}$

$$v_C(t) = 0$$

$$i_C = C \frac{dv_C(t)}{dt} = C \frac{d}{dt} [0] = 0$$

A capacitor is fabricated from two conducting plates separated by a dielectric constant. Dielectrics are also isolators; therefore, current cannot really flow through a capacitor. Positive charge, however, entering one plate exerts a repulsive force on and forces positive carriers to exit the other plate. Current then *appears* to



flow through the capacitor. Such currents are called *electric displacement currents*.

Problem 4.13

Solution:

Known quantities:

Inductance value, $L = 35 \text{ mH}$; the voltage applied to the inductor as shown in Figure P4.12; the initial condition for the current $i_L(0) = 0$.

Find:

The current across the inductor as a function of time.

Analysis:

Since the voltage waveform is piecewise continuous, integration must be performed over each continuous segment. Where not indicated t is supposed to be expressed in seconds.

For $0 < t \leq 5 \text{ ms}$

$$v_L(t) = m_{v_L} t + q_{v_L}$$

where:

$$m_{v_L} = \frac{[-10 \text{ V}] - [+20 \text{ V}]}{[5 \text{ ms}] - [0]} = -6 \frac{\text{V}}{\text{ms}}$$

$$q_{v_L} = +20 \text{ V}$$

$$\begin{aligned} i_L &= i_L(0) + \frac{1}{L} \int_0^t v_L(\tau) d\tau = 0 + \frac{1}{L} \int_0^t (m_{v_L} \tau + q_{v_L}) d\tau = \frac{1}{L} \left[\frac{1}{2} m_{v_L} \tau^2 + q_{v_L} \tau \right]_0^t = \\ &= \frac{1}{2L} (m_{v_L} t^2 + q_{v_L} t) = \frac{1}{2 \cdot 35 \text{ mH}} \left(-6 \frac{\text{V}}{\text{ms}} \cdot t^2 + 20 \text{ V} \cdot t \right) = (-85.71 \cdot 10^3 t^2 + 571.4 t) \text{ A} \end{aligned}$$

$$i_L(t = 5 \text{ ms} = 0.005 \text{ s}) = (-85.71 \cdot 10^3 \cdot (0.005)^2 + 571.4 \cdot 0.005) \text{ A} = 714.3 \text{ mA}$$

For $5 \text{ ms} < t \leq 10 \text{ ms}$

$$v_L(t) = c_{v_L} = -10 \text{ V}$$

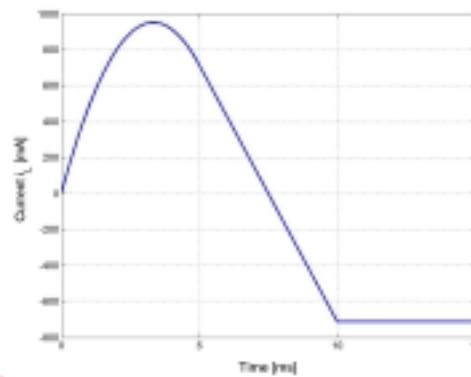
$$\begin{aligned} i_L &= i_L(0.005) + \frac{1}{L} \int_{0.005}^t v_L(\tau) d\tau = i_L(0.005) + \frac{1}{L} \int_{0.005}^t (c_{v_L}) d\tau = i_L(0.005) + \frac{1}{L} [c_{v_L} \tau]_{0.005}^t = \\ &= 714.3 \text{ mA} - \frac{1}{35 \text{ mH}} \cdot (-10 \text{ V}) \cdot (t - 0.005 \text{ s}) = (2.143 - 285.7 t) \text{ A} \end{aligned}$$

$$i_L(t = 10 \text{ ms} = 0.01 \text{ s}) = (2.143 - 285.7(0.01)) \text{ A} = -713.5 \text{ mA}$$

For $t > 10 \text{ ms}$

$$v_L(t) = c_{v_L} = 0$$

$$\begin{aligned} i_L &= i_L(0.01) + \frac{1}{L} \int_{0.01}^t v_L(\tau) d\tau = i_L(0.01) + \frac{1}{L} \int_{0.01}^t (0) d\tau = i_L(0.01) + \frac{1}{L} [0]_{0.005}^t = \\ &= i_L(0.01) = -713.5 \text{ mA} \end{aligned}$$



Problem 4.14

Solution:

Known quantities:

Inductance value $L = 0.75 \text{ mH}$; the voltage applied to the inductor as a function of time as shown in Figure P4.14.

Find:

The current through the inductor at the time $t = 15 \mu\text{s}$.

Assumptions:

$$i_L(t \leq 0) = 0$$

Analysis:

Since the voltage waveform is a piecewise continuous function of time, integration must be performed over each continuous segment. Where not indicated, t is expressed in seconds.

For $0 < t \leq 5 \mu\text{s}$

$$v_L(t) = m_{v_L} t + q_{v_L}$$

where:

$$m_{v_L} = \frac{[3.5 \text{ V}] - [0 \text{ V}]}{[5 \mu\text{s}] - [0]} = 0.7 \frac{\text{V}}{\mu\text{s}}$$

$$q_{v_L} = 0 \text{ V}$$

For $t > 5 \mu\text{s}$

$$v_L(t) = c_{v_L} = -1.9 \text{ V}$$

Therefore:

$$\begin{aligned} i_L(t = 15 \mu\text{s}) &= \frac{1}{L} \int_{-\infty}^{15 \mu\text{s}} v_L(\tau) d\tau = i_L(0) + \frac{1}{L} \int_0^{5 \mu\text{s}} (m_{v_L} \tau) d\tau + \frac{1}{L} \int_{5 \mu\text{s}}^{15 \mu\text{s}} (c_{v_L}) d\tau = \\ &= i_L(0) + \frac{1}{L} \left[\frac{1}{2} m_{v_L} \tau^2 \right]_0^{5 \mu\text{s}} + \frac{1}{L} [c_{v_L} \tau]_{5 \mu\text{s}}^{15 \mu\text{s}} = 0 + \frac{1}{0.75 \text{ mH}} \cdot \frac{0.7 \frac{\text{V}}{\text{ms}}}{2} \cdot ((5 \mu\text{s})^2 - 0) + \\ &+ \frac{1}{0.75 \text{ mH}} \cdot (-1.9 \text{ V}) \cdot (15 \mu\text{s} - 5 \mu\text{s}) = -13.67 \text{ mA} \end{aligned}$$

Problem 4.15**Solution:****Known quantities:**

Capacitance value $C = 680 \text{ nF}$; the periodic voltage applied to the capacitor as shown in Figure P4.15:

$$v_{Peak} = 20 \text{ V}, T = 40 \mu\text{s}.$$

Find:

The waveform and the plot for the current through the capacitor as a function of time.

Analysis:

Since the voltage waveform is not a continuous function of time, differentiation can be performed only over each continuous segment. In the discontinuity points the derivative of the voltage will assume an infinite value, the sign depending on the sign of the step. Where not indicated, t is expressed in seconds.

For each period $0 < t < T$, $T < t < 2T$, ... the behavior of the capacitor will be the same; thus, we consider only the first period:

For $0 < t < T$

$$v_C(t) = m_{v_C} t + q_{v_C}$$

where:

$$m_{v_C} = \frac{[v_{Peak}] - [0]}{[T] - [0]} = 0.5 \frac{\text{V}}{\mu\text{s}}$$

$$q_{v_C} = 0 \text{ V}$$

$$i_C = C \frac{dv_C(t)}{dt} = C \frac{d}{dt} [m_{v_C} t] = C m_{v_C} = \left(480 \frac{\text{nAs}}{\text{V}} \right) \left(0.5 \frac{\text{V}}{\mu\text{s}} \right) = 340 \text{ mA}$$

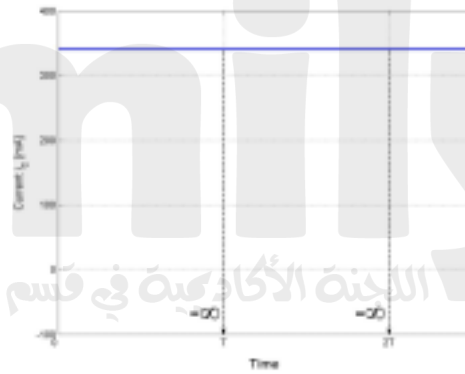
For $t = T, 2T, \dots$

$$i_C = C \frac{dv_C(t)}{dt} = C[-\infty]$$

Figure P 4.15 shows the current waveform.

Note: For the voltage across the capacitor to decrease instantaneously to zero at $t = T, 2T, \dots$, the charge on the plates of the capacitor should be instantaneously discharged. This requires an infinite current which is not physically possible.

If this were a practical waveform, the slope at $t = T, 2T, \dots$, would be finite, not infinite. A large negative spike of current over a finite period of time would result instead of the infinite spike over zero time. These large spike of current (or voltage) degrade the performance of many circuits.

**Problem 4.16****Solution:****Known quantities:**

Inductance value, $L = 16 \mu\text{H}$; the voltage applied to the inductor as a function of time as shown in

Figure P4.16; the initial condition for the current $i_L(0) = 0$.

Find:

The current through the inductor at $t = 30 \mu\text{s}$.

Analysis:

Since the voltage waveform is piecewise continuous, the integration can be performed over each continuous segment. Where not indicated t is supposed to be expressed in seconds.

$$\begin{aligned} i_L(t = 30 \mu\text{s}) &= \frac{1}{L} \int_{-\infty}^{30 \mu\text{s}} v_L(\tau) d\tau = i_L(0) + \frac{1}{L} \int_0^{20 \mu\text{s}} v_L(\tau) d\tau + \frac{1}{L} \int_{20 \mu\text{s}}^{30 \mu\text{s}} v_L(\tau) d\tau = \\ &= i_L(0) + \frac{1}{L} \left[\frac{3}{3} \tau^3 \frac{V}{s^2} \right]_0^{20 \mu\text{s}} + \frac{1}{L} [1.2 \tau \text{ nV}]_{20 \mu\text{s}}^{30 \mu\text{s}} = 0 + \frac{1}{16 \mu\text{H}} \cdot 1 \frac{V}{s^2} \cdot ((20 \mu\text{s})^3 - 0) + \\ &+ \frac{1}{16 \mu\text{H}} \cdot (1.2 \text{ nV}) \cdot (30 \mu\text{s} - 20 \mu\text{s}) = 1.250 \text{ nA} \end{aligned}$$

Problem 4.17**Solution:****Known quantities:**

Resistance value $R = 7 \Omega$; inductance value $L = 7 \text{ mH}$; capacitance value $C = 0.5 \mu\text{F}$; the voltage across the components as shown in Figure P4.17.

Find:

The current through each component.

Assumptions:

$$i_R(t \leq 0) = i_L(t \leq 0) = i_C(t \leq 0) = 0$$

Analysis:

Since the voltage waveform is piecewise continuous, integration and differentiation can only be performed over each continuous segment. Where not indicated, t is expressed in seconds.

For $t \leq 0$:

$$i_R(t) = i_L(t) = i_C(t) = 0$$

For $0 < t < 5 \text{ ms}$:

$$v(t) = m_v t + q_v$$

where:

$$m_v = \frac{[15 \text{ V}] - [0]}{[5 \text{ ms}] - [0]} = 3 \frac{\text{V}}{\text{ms}}$$

$$q_{v_c} = 0 \text{ V}$$

$$i_R(t) = \frac{v(t)}{R} = \frac{m_v \cdot t}{R} = \frac{3 \frac{\text{V}}{\text{ms}} \cdot t}{7 \Omega} = 428.6 \cdot t \text{ A}$$

$$\begin{aligned} i_L(t) &= \frac{1}{L} \int_0^t v(\tau) d\tau = \frac{1}{L} \int_0^t m_v \cdot \tau d\tau = \frac{1}{L} \left[\frac{1}{2} m_v \cdot \tau^2 \right]_0^t = \frac{1}{2L} m_v \cdot t^2 = \\ &= \frac{1}{2 \cdot 7 \text{ mH}} \cdot 3 \frac{\text{V}}{\text{ms}} \cdot t^2 = 214.3 \cdot 10^3 \cdot t^2 \text{ A} \end{aligned}$$

$$i_C(t) = C \frac{dv_C(t)}{dt} = C \frac{d}{dt}[m_v t] = C m_v = \left(0.5 \frac{\mu\text{As}}{\text{V}}\right) \left(3 \frac{\text{V}}{\text{ms}}\right) = 1.5 \text{ mA}$$

For $t = 5 \text{ ms}$:

$$i_R(5 \cdot 10^{-3}) = 428.6 \cdot t \text{ A} = 428.6 \cdot 5 \cdot 10^{-3} \text{ A} = 2.143 \text{ A}$$

$$i_L(5 \cdot 10^{-3}) = 214.3 \cdot 10^3 \cdot t^2 \text{ A} = 214.3 \cdot 10^3 \cdot (5 \cdot 10^{-3})^2 \text{ A} = 5.357 \text{ A}$$

$$i_C(5 \cdot 10^{-3}) = 1.5 \text{ mA}$$

For $5 \text{ ms} < t < 10 \text{ ms}$:

$$v(t) = c_v = 15 \text{ V}$$

$$i_R(t) = \frac{v(t)}{R} = \frac{c_v}{R} = \frac{15 \text{ V}}{7 \Omega} = 2.143 \text{ A}$$

$$\begin{aligned} i_L(t) &= i_L(5 \cdot 10^{-3}) + \frac{1}{L} \int_{5 \cdot 10^{-3}}^t v(\tau) d\tau = i_L(5 \cdot 10^{-3}) + \frac{1}{L} \int_{5 \cdot 10^{-3}}^t c_v d\tau = \\ &= i_L(5 \cdot 10^{-3}) + \frac{1}{L} [c_v \cdot \tau]_{5 \cdot 10^{-3}}^t = i_L(5 \cdot 10^{-3}) + \frac{1}{L} \cdot c_v \cdot (t - 5 \cdot 10^{-3}) = \\ &= 5.357 \text{ A} + \frac{1}{7 \text{ mH}} \cdot 15 \text{ V} \cdot (t - 5 \cdot 10^{-3} \text{ s}) = -5.357 + 2.143 \cdot 10^3 \cdot t \text{ A} \end{aligned}$$

$$i_C(t) = C \frac{dv_C(t)}{dt} = C \frac{d}{dt}[c_v] = 0$$

For $t = 10 \text{ ms}$:

$$i_R(0.01) = 2.143 \text{ A}$$

$$i_L(0.01) = -5.357 + 2.143 \cdot 10^3 \cdot t \text{ A} = -5.357 + 2.143 \cdot 10^3 \cdot 0.01 \text{ A} = 16.07 \text{ A}$$

$$i_C(0.01) = 0$$

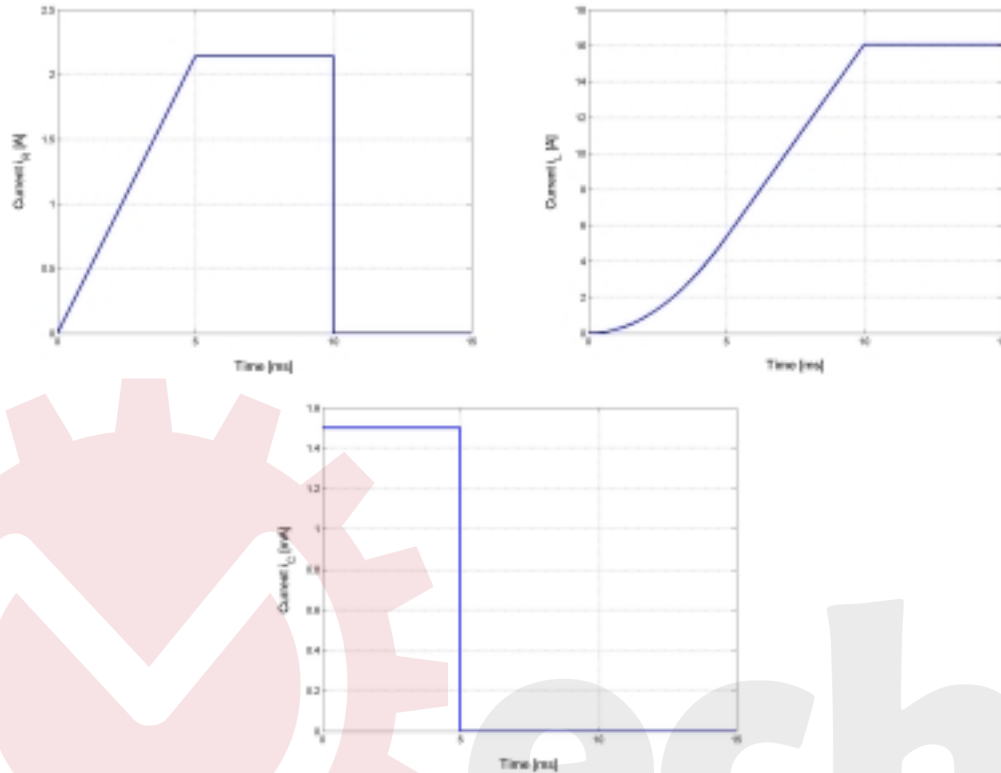
For $t > 10 \text{ ms}$:

$$v(t) = 0$$

$$i_R(t) = \frac{v(t)}{R} = \frac{0}{R} = 0$$

$$\begin{aligned} i_L(t) &= i_L(10 \cdot 10^{-3}) + \frac{1}{L} \int_{10 \cdot 10^{-3}}^t v(\tau) d\tau = i_L(10 \cdot 10^{-3}) + \frac{1}{L} \int_{10 \cdot 10^{-3}}^t 0 d\tau = \\ &= i_L(10 \cdot 10^{-3}) + \frac{1}{L} [0]_{10 \cdot 10^{-3}}^t = i_L(10 \cdot 10^{-3}) = 16.07 \text{ A} \end{aligned}$$

$$i_C(t) = C \frac{dv_C(t)}{dt} = C \frac{d}{dt}[0] = 0$$



Problem 4.18

Solution:

Known quantities:

The voltage across and the current through an ideal capacitor as shown in Figure P4.18.

Find:

The capacitance of the capacitor.

Analysis:

Considering the period: $-2.5 \mu\text{s} < t < +2.5 \mu\text{s}$:

$$i_c = C \frac{dv_c}{dt} = C \frac{\Delta v_c}{\Delta t}, \text{ since the voltage has a linear waveform.}$$

Substituting:

$$12 \text{ A} = C \frac{[+10 \text{ V}] - [-10 \text{ V}]}{5 \mu\text{s}} \Rightarrow C = 12 \text{ A} \cdot \frac{5 \mu\text{s}}{20 \text{ V}} = 3 \mu\text{F}$$

Problem 4.19

Solution:

Known quantities:

The voltage across and the current through an ideal inductor as shown in Figure P4.19.

Find:

The inductance of the inductor.

Analysis:

$$v_L = L \frac{di_L}{dt} = L \frac{\Delta i_L}{\Delta t}, \text{ since the current has a linear waveform.}$$

Substituting:

$$2 \text{ V} = L \frac{[2 \text{ A}] - [1 \text{ A}]}{10 \text{ ms} - 5 \text{ ms}} \Rightarrow L = 2 \text{ V} \cdot \frac{5 \text{ ms}}{1 \text{ A}} = 10 \text{ mH}$$

Problem 4.20**Solution:****Known quantities:**

The voltage across and the current through an ideal capacitor as shown in Figure P4.20.

Find:

The capacitance of the capacitor.

Analysis:

Considering the period: $0 < t < 5 \text{ ms}$:

$$i_c = C \frac{dv_c}{dt} = C \frac{\Delta v_c}{\Delta t}, \text{ since the voltage has a linear waveform.}$$

Substituting:

$$1.5 \text{ mA} = C \frac{[15 \text{ V}] - [0]}{5 \text{ ms}} \Rightarrow C = 1.5 \text{ mA} \cdot \frac{5 \text{ ms}}{15 \text{ V}} = 0.5 \mu\text{F}$$

Problem 4.21**Solution:****Known quantities:**

The voltage across and the current through an ideal capacitor as shown in Figure P4.21.

Find:

The capacitance of the capacitor.

Analysis:

Considering the period: $0 < t < 5 \text{ ms}$:

$$i_c = C \frac{dv_c}{dt} = C \frac{\Delta v_c}{\Delta t}, \text{ since the voltage has a linear waveform.}$$

Substituting:

$$3 \text{ mA} = C \frac{[7 \text{ V}] - [0]}{5 \text{ ms}} \Rightarrow C = 3 \text{ mA} \cdot \frac{5 \text{ ms}}{7 \text{ V}} = 2.14 \mu\text{F}$$

Section 4.2: Time-Dependent Signals

Problem 4.22

Solution:

Known quantities:

The signal $x(t) = 2\cos(\omega t) + 2.5$.

Find:

The average and rms value of the signal.

Analysis:

The average value is:

$$\begin{aligned}\langle v(t) \rangle &= \frac{\omega}{2\pi} \left[\int_0^{\frac{2\pi}{\omega}} 2\cos(\omega t) dt + \int_0^{\frac{2\pi}{\omega}} (2.5) dt \right] = \frac{1}{2\pi} \left[-\sin(\omega t) \Big|_0^{\frac{2\pi}{\omega}} + 2.5t \Big|_0^{\frac{2\pi}{\omega}} \right] \\ &= \frac{1}{2\pi} [\sin(0) - \sin(2\pi)] + 2.5 = \frac{1}{2\pi} [0 - 0] + 2.5 = 2.5\end{aligned}$$

The rms value is:

$$\begin{aligned}x_{\text{rms}} &= \sqrt{\frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} (2\cos(\omega t) + 2.5)^2 dt} = \sqrt{\frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} [4 \cdot [\cos(\omega t)]^2 + 10 \cdot \cos(\omega t) + 6.25] dt} = \\ &= \sqrt{\frac{\omega}{2\pi} \left[4 \cdot \int_0^{\frac{2\pi}{\omega}} [\cos(\omega t)]^2 dt + 10 \cdot \int_0^{\frac{2\pi}{\omega}} \sin(\omega t) dt + 6.25 \cdot \int_0^{\frac{2\pi}{\omega}} dt \right]} = \\ &= \sqrt{\frac{\omega}{2\pi} \left[4 \cdot \frac{1}{2} \cdot \frac{2\pi}{\omega} + 10 \cdot 0 + 6.25 \cdot \frac{2\pi}{\omega} \right]} = \sqrt{8.25} = 2.87\end{aligned}$$

Note: The integral of a sinusoid over an integer number of period is identically zero. This is a useful and important result.

Problem 4.23

Solution:

Known quantities:

The sinusoidal voltage $v(t)$ of 110 V rms shown in Figure P4.23.

Find:

The average and rms voltage.

Analysis:

The rms value of a sinusoidal is equal to 0.707 times the peak value:

$$V_{\text{peak}} = 110\sqrt{2}$$

The average value is:

$$\begin{aligned}
 \langle v(t) \rangle &= \frac{1}{2\pi} \left[\int_0^\theta 110\sqrt{2} \sin(t) dt + \int_{2\pi-\theta}^{2\pi} 110\sqrt{2} \sin(t) dt \right] = \\
 &= \frac{1}{2\pi} \left[-110\sqrt{2} \cos(t) \Big|_0^\theta - 110\sqrt{2} \cos(t) \Big|_{2\pi-\theta}^{2\pi} \right] = \\
 &= -\frac{50\sqrt{2}}{\pi} [\cos(\theta) - 1 + \cos(2\pi) - \cos(2\pi - \theta)] = 0
 \end{aligned}$$

The rms value is:

$$\begin{aligned}
 v_{rms} &= \left\{ \frac{1}{2\pi} \left(\int_0^\theta (110\sqrt{2} \sin(t))^2 dt + \int_{2\pi-\theta}^{2\pi} (110\sqrt{2} \sin(t))^2 dt \right) \right\}^{1/2} = \\
 &= \left\{ \frac{12100}{\pi} \left(\frac{1}{2} (-\cos(t)\sin(t) + t) \Big|_0^\theta + \frac{1}{2} (-\cos(t)\sin(t) + t) \Big|_{2\pi-\theta}^{2\pi} \right) \right\}^{1/2} = \\
 &= \left\{ \frac{6050}{\pi} (-\cos(\theta)\sin(\theta) + \theta + 2\pi - (-\cos(2\pi - \theta)\sin(2\pi - \theta) + 2\pi - \theta)) \right\}^{1/2} = \\
 &= \left\{ \frac{6050}{\pi} (2\theta) \right\}^{1/2} = 110 \sqrt{\frac{\theta}{\pi}}
 \end{aligned}$$

Problem 4.24

Solution:

Known quantities:

The sinusoidal voltage $v(t)$ of 110 V rms shown in Figure P4.23.

Find:

The angle θ that correspond to delivering exactly one-half of the total available power in the waveform to a resistive load.

Analysis:

From $v_{rms} = 110 \sqrt{\frac{\theta}{\pi}}$, we obtain:

$$v_{rms}^2 = 110^2 \frac{\theta}{\pi} = \frac{110^2}{2} \Rightarrow \theta = \frac{\pi}{2}$$

Problem 4.25

Solution:

Known quantities:

The signal $v(t)$ shown in Figure P4.25.

Find:

The ratio between average and rms value of the signal.

Analysis:

The average value is:

$$\langle v \rangle = \frac{1}{0.004} \left[\int_0^{0.002} (-9) dt + \int_{0.002}^{0.004} (1) dt \right] = 250(-0.018 + 0.002) = -4 \text{ V}$$

The rms value is:

$$v_{rms} = \sqrt{\frac{1}{0.004} \left[\int_0^{0.002} (-9)^2 dt + \int_{0.002}^{0.004} (1)^2 dt \right]} = \sqrt{250 \cdot [81 \cdot 0.002 + 0.004 - 0.002]} = 6.40 \text{ V}$$

Therefore,

$$\frac{\langle v \rangle}{v_{rms}} = -\frac{4}{6.40} = -0.625$$

Problem 4.26**Solution:****Known quantities:**

The signal $i(t)$ shown in Figure P4.26.

Find:

The power dissipated by a $1\text{-}\Omega$ resistor.

Analysis:

The rms value is:

$$i_{rms} = \sqrt{\frac{1}{p} \int_0^p (10 \cdot \sin^2(t))^2 dt} = \sqrt{\frac{1}{p} \int_0^p 100 \cdot \sin^4(t) dt} = \sqrt{\frac{1}{p} \cdot 100 \cdot \frac{3p}{8}} = 6.12 \text{ A}$$

Therefore, the power dissipated by a $1\text{-}\Omega$ resistor is:

$$P_{1\Omega} = Ri_{rms}^2 = (1)(6.12)^2 \text{ W} = 37.5 \text{ W}$$

Problem 4.27**Solution:****Known quantities:**

The signal $x(t)$ shown in Figure P4.27.

Find:

The average and rms value of the signal.

Analysis:

The average value is:

$$\langle V \rangle = \frac{1}{T} \int_0^{t_0+\tau} V_m dt = \frac{t}{T} V_m$$

where t_0 is the left-hand side of the pulse.

The rms value is:

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^{T_0+\tau} V_m^2 dt} = \sqrt{\frac{t}{T}} V_m$$

Therefore,

$$\frac{\langle V \rangle}{V_{rms}} = \sqrt{\frac{t}{T}}$$

Problem 4.28

Solution:

Known quantities:

The signal $i(t)$ shown in Figure P4.28.

Find:

The rms value of the signal.

Analysis:

The rms value is:

$$\begin{aligned} i_{rms} &= \sqrt{\frac{1}{T} \left[\int_0^{\frac{T}{4}} \left(\frac{8}{T} t \right)^2 dt + \int_{\frac{T}{4}}^{\frac{3T}{4}} \left(-\frac{8}{T} t + 4 \right)^2 dt + \int_{\frac{3T}{4}}^T \left(\frac{8}{T} t - 8 \right)^2 dt \right]} = \\ &= \sqrt{\frac{1}{T} \left[\int_0^{\frac{T}{4}} \left(\frac{64}{T^2} t^2 \right) dt + \int_{\frac{T}{4}}^{\frac{3T}{4}} \left(\frac{64}{T^2} t^2 - \frac{64}{T} t + 16 \right) dt + \int_{\frac{3T}{4}}^T \left(\frac{64}{T^2} t^2 - \frac{128}{T} t + 64 \right) dt \right]} = \\ &= \sqrt{\frac{1}{T} \left[\frac{1}{3} T + 9T - 18T + 12T - \frac{1}{3} T + 2T - 4T + \frac{64}{3} T - 64T + 64T - 9T + 36T - 48T \right]} = \\ &= \sqrt{\frac{1}{T} \left[\frac{4}{3} T \right]} = \frac{2}{\sqrt{3}} = 1.15 \text{ A} \end{aligned}$$

Problem 4.29

Solution:

Known quantities:

The signal $v(t)$.

Find:

The rms value of the signal.

Analysis:

The rms value is:

$$v_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (v(t))^2 d(\omega t)}$$

$$\begin{aligned}
 v_{rms}^2 &= \frac{1}{2\pi} \int_0^{2\pi} (V_{DC} + V_0 \cos(\omega t))^2 d(\omega t) = \frac{1}{2\pi} \int_0^{2\pi} (V_{DC}^2 + 2V_{DC}V_0 \cos(\omega t) + V_0^2 \cos^2(\omega t)) d(\omega t) = \\
 &= \frac{1}{2\pi} \int_0^{2\pi} \left(V_{DC}^2 + 2V_{DC}V_0 \cos(\omega t) + \frac{V_0^2}{2} + \frac{V_0^2}{2} \cos^2(\omega t) \right) d(\omega t) = \\
 &= \frac{1}{2\pi} \left(V_{DC}^2 [\omega t]_0^{2\pi} + 0 + \frac{V_0^2}{2} [\omega t]_0^{2\pi} + 0 \right) = \frac{1}{2\pi} \left(V_{DC}^2 (2\pi - 0) + 0 + \frac{V_0^2}{2} (2\pi - 0) + 0 \right) \\
 v_{rms} &= \sqrt{V_{DC}^2 + \frac{V_0^2}{2}} = \sqrt{(50 \text{ V})^2 + \frac{1}{2}(70.7 \text{ V})^2} = 70.7 \text{ V}
 \end{aligned}$$

Note:

1. T = period in units of time and ωt = period in angular units, i.e., 2π radians. Considering ωt as a single variable is useful when dealing with sinusoids.
2. A sinusoid integrated over one or more whole periods gives 0 which is very useful.

Section 4.4: Phasor Solution of Circuits with Sinusoidal Excitation

Focus on Methodology: Phasors

- Any sinusoidal signal may be mathematically represented in one of two ways: a **time domain form**: $v(t) = A \cos(\omega t + \phi)$, and a frequency domain form: $\mathbf{V}(j\omega) = A e^{j\phi}$. Note the $j\omega$ in the notation $\mathbf{V}(j\omega)$, indicating the $e^{j\omega t}$ dependence of the phasor. In the remainder of this chapter, bold uppercase quantities indicate phasor voltages and currents.
- A phasor is a complex number, expressed in polar form, consisting of a *magnitude* equal to the peak amplitude of the sinusoidal signal and a *phase angle* equal to the phase shift of the sinusoidal signal *referenced to a cosine signal*.
- When one is using phasor notation, it is important to note the specific frequency ω of the sinusoidal signal, since this is not explicitly apparent in the phasor expression.

Problem 4.30

Solution:

Known quantities:

The current through and the voltage across a component.

Find:

- Whether the component is a resistor, capacitor, inductor
- The value of the component in ohms, farads, or henrys.

Analysis:

- The current and the voltage can be expressed in phasor form:

$$\mathbf{I} = 17 \angle -15^\circ \text{ mA}, \quad \mathbf{V} = 3.5 \angle 75^\circ \text{ V}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{3.5 \angle 75^\circ \text{ V}}{17 \angle -15^\circ \text{ mA}} = 205.9 \angle 90^\circ \Omega = 0 + j \cdot 205.9 \Omega$$

The impedance has a positive imaginary or reactive component and a positive angle of 90 degree indicating that this is an inductor (see Fig. 4.39).

$$\text{b) } \mathbf{Z}_L = j \cdot X_L = j \cdot \omega L = j \cdot 205.9 \Omega \Rightarrow L = \frac{205.9 \Omega}{628.3 \frac{\text{rad}}{\text{s}}} = 327.7 \text{ m} \frac{\text{Vs}}{\text{A}} = 327.7 \text{ mH}$$

Problem 4.31

Solution:

Known quantities:

The waveform of a signal shown in Figure P4.31.

Find:

The sinusoidal description of the signal.

Analysis:

From the graph of Figure P4.31:

$$\phi = +\frac{\pi}{3} \frac{180^\circ}{\pi} = 60^\circ, V_0 = 170 \text{ V}, \omega = 2\pi f = \frac{2\pi}{T} \dots \text{not given.}$$

$$v_r(t) = V_0 \cos(\omega t + \phi) = 170 \cos(\omega t + 60^\circ) \text{ V}$$

Phasor form:

$$V = V_0 \angle \phi = 170 \angle 60^\circ \text{ V} = 170 \text{ V} \cdot e^{j60^\circ}$$

Problem 4.32

Solution:

Known quantities:

The waveform of a signal shown in Figure P4.32.

Find:

The sinusoidal description of the signal.

Analysis:

From graph:

$$\phi = -\frac{3\pi}{4} \frac{180^\circ}{\pi} = -135^\circ, I_0 = 8 \text{ mA}, \omega = 2\pi f = \frac{2\pi}{T} = 1571 \frac{\text{rad}}{\text{s}}$$

$$i(t) = I_0 \cos(\omega t + \phi) = 8 \cos\left(1571 \frac{\text{rad}}{\text{s}} \cdot t - 135^\circ\right) \text{ mA}$$

Phasor form:

$$I = I_0 \angle \phi = 8 \angle -135^\circ \text{ mA} = 8 \text{ mA} \cdot e^{-j135^\circ}$$

Problem 4.33

Solution:

Known quantities:

The waveform of a signal shown in Figure P4.33.

Find:

The sinusoidal description of the signal.

Analysis:

From graph:

$$\phi = -\frac{3\pi}{4} \frac{180^\circ}{\pi} = -135^\circ, I_0 = 8 \text{ mA}, \omega = 2\pi f = \frac{2\pi}{T} = 1571 \frac{\text{rad}}{\text{s}}$$

$$i(t) = I_0 \cos(\omega t + \phi) = 8 \cos\left(1571 \frac{\text{rad}}{\text{s}} \cdot t - 135^\circ\right) \text{ mA}$$

Phasor form:

$$I = I_0 \angle \phi = 8 \angle -135^\circ \text{ mA} = 8 \text{ mA} \cdot e^{-j135^\circ}$$

Problem 4.34**Solution:****Known quantities:**

The current through $i(t) = I_0 \cos(\omega t + 45^\circ)$, $I_0 = 3 \text{ mA}$, $\omega = 6.283 \frac{\text{rad}}{\text{s}}$, and the voltage across

$v(t) = V_0 \cos(\omega t)$, $V_0 = 700 \text{ mV}$, $\omega = 6.283 \frac{\text{rad}}{\text{s}}$ an electrical component.

Find:

- Whether the component is inductive or capacitive.
- The waveform of the instantaneous power $p(t)$ as a function of ωt over the range $0 < \omega t < 2\pi$.
- The average power dissipated as heat in the component.
- The same as b. and c. with the phase of the current equal to zero.

Analysis:

- Phasor notation:

$$\mathbf{I} = 3 \angle 45^\circ \text{ mA}, \mathbf{V} = 700 \angle 0^\circ \text{ mV}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{700 \angle 0^\circ \text{ mV}}{3 \angle 90^\circ \text{ mA}} = 233.3 \angle -45^\circ \Omega = 165.0 - j165.0 \Omega$$

The component is inductive because it is lagging.

-

$$\begin{aligned} p(t) &= v(t)i(t) = V_0 I_0 \cos(\omega t + 45^\circ) \cos(\omega t) = \frac{1}{2} V_0 I_0 (\cos(2\omega t + 45^\circ) + \cos(45^\circ)) = \\ &= \frac{1}{2} (700 \text{ mV})(3 \text{ mA})(\cos(2\omega t + 45^\circ) + 0.707) = (1050 \cos(2\omega t + 45^\circ) + 742.4) \mu\text{W} \end{aligned}$$

-

$$\begin{aligned} P &= \frac{1}{\omega T} \int_0^{\omega T} p(t) d\omega t = \frac{1}{\omega T} \int_0^{\omega T} (1050 \mu\text{W} \cos(2\omega t + 45^\circ) + 742.4) d\omega t = \\ &= \frac{1}{\omega T} (1050 \mu\text{W}) \frac{1}{2} [\sin(2\omega t + 45^\circ)]_0^{2\pi} + \frac{1}{\omega T} 742.4 (\omega t)_0^{2\pi} = \\ &= \frac{1}{\omega T} (1050 \mu\text{W}) \frac{1}{2} [\sin(765^\circ) - \sin(45^\circ)] + 742.4 = \\ &= \frac{1}{2\pi} (1050 \mu\text{W}) \frac{1}{2} [0 - 0] + 742.4 = 742.4 \mu\text{W} \end{aligned}$$

-

$$\begin{aligned} p(t) &= v(t)i(t) = \frac{1}{2} V_0 I_0 \cos(\omega t) \cos(\omega t) = \frac{1}{2} V_0 I_0 (\cos(2\omega t) + \cos(0^\circ)) = \\ &= \frac{1}{2} (700 \text{ mV})(3 \text{ mA})(\cos(2\omega t) + 1) = 1050(\cos(2\omega t) + 1) \mu\text{W} \end{aligned}$$

$$\begin{aligned}
 P &= \frac{1}{\omega T} \int_0^{\omega T} p(t) d\omega t = \frac{1}{\omega T} \int_0^{\omega T} (1050 \mu\text{W} (\cos(2\omega t) + 1)) d\omega t = \\
 &= \frac{1}{\omega T} (1050 \mu\text{W}) \left(\frac{1}{2} [\sin(2\omega t)]_0^{2\pi} + [\omega t]_0^{2\pi} \right) = \\
 &= \frac{1}{2\pi} (1050 \mu\text{W}) \left(\frac{1}{2} (0 - 0) + (2\pi - 0) \right) = 1050 \mu\text{W}
 \end{aligned}$$

Problem 4.35

Solution:

Known quantities:

The values of the impedance, $R_1 = 2.3 \text{ k}\Omega$, $R_2 = 1.1 \text{ k}\Omega$, $L = 190 \text{ mH}$, $C = 55 \text{ nF}$ and the voltage applied to the circuit shown in Figure P4.35, $v_s(t) = 7 \cos(3000t + 30^\circ) \text{ V}$.

Find:

The equivalent impedance of the circuit.

Analysis:

$$X_L = \omega L = \left(3 \text{ k} \frac{\text{rad}}{\text{s}} \right) (190 \text{ mH}) = 0.57 \text{ k}\Omega \Rightarrow Z_L = +j \cdot X_L = +j \cdot 0.57 \text{ k}\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(3 \text{ k} \frac{\text{rad}}{\text{s}} \right) (55 \text{ nF})} = 6.061 \text{ k}\Omega \Rightarrow Z_C = -j \cdot X_C = -j \cdot 6.061 \text{ k}\Omega$$

$$Z_{eq1} = Z_{R1} + Z_L = R_1 + jX_L = 2.3 + j \cdot 0.57 \text{ k}\Omega = 2.37 \angle 13.92^\circ \text{ k}\Omega$$

$$Z_{eq2} = Z_{R2} + Z_C = R_2 - jX_C = 1.1 - j \cdot 6.061 \text{ k}\Omega = 6.16 \angle -79.71^\circ \text{ k}\Omega$$

$$\begin{aligned}
 Z_{eq} &= \frac{Z_{eq1} \cdot Z_{eq2}}{Z_{eq1} + Z_{eq2}} = \frac{(2.37 \angle 13.92^\circ \text{ k}\Omega) \cdot (6.16 \angle -79.71^\circ \text{ k}\Omega)}{(2.3 + j \cdot 0.57 \text{ k}\Omega) + (1.1 - j \cdot 6.061 \text{ k}\Omega)} = \\
 &= \frac{14.60 \angle -65.79^\circ \text{ k}\Omega^2}{3.4 - j \cdot 5.491 \text{ k}\Omega} = \frac{14.60 \angle -65.79^\circ \text{ k}\Omega^2}{6.458 \angle -58.23^\circ \text{ k}\Omega} = 2.261 \angle -7.56^\circ \text{ k}\Omega
 \end{aligned}$$

Problem 4.36

Solution:

Known quantities:

The values of the impedance, $R_1 = 3.3 \text{ k}\Omega$, $R_2 = 22 \text{ k}\Omega$, $L = 1.90 \text{ H}$, $C = 6.8 \text{ nF}$ and the voltage applied to the circuit shown in Figure P4.35, $v_s(t) = 636 \cos(3000t + 15^\circ) \text{ V}$.

Find:

The equivalent impedance of the circuit.

Analysis:

$$X_L = \omega L = \left(3 \text{ k} \frac{\text{rad}}{\text{s}}\right)(1.90 \text{ H}) = 5.7 \text{ k}\Omega \Rightarrow Z_L = +j \cdot X_L = +j \cdot 5.7 \text{ k}\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(3 \text{ k} \frac{\text{rad}}{\text{s}}\right)(6.8 \text{ nF})} = 49.02 \text{ k}\Omega \Rightarrow Z_C = -j \cdot X_C = -j \cdot 49.02 \text{ k}\Omega$$

$$Z_{eq1} = Z_{R1} + Z_L = R_1 + jX_L = 3.3 + j \cdot 5.7 \text{ k}\Omega = 6.59 \angle 59.93^\circ \text{ k}\Omega$$

$$Z_{eq2} = Z_{R1} + Z_C = R_1 - jX_C = 22 - j \cdot 49.02 \text{ k}\Omega = 53.73 \angle -65.83^\circ \text{ k}\Omega$$

$$\begin{aligned} Z_{eq} &= \frac{Z_{eq1} \cdot Z_{eq2}}{Z_{eq1} + Z_{eq2}} = \frac{(6.59 \angle 59.93^\circ \text{ k}\Omega) \cdot (53.73 \angle -65.83^\circ \text{ k}\Omega)}{(3.3 + j \cdot 5.7 \text{ k}\Omega) + (22 - j \cdot 49.02 \text{ k}\Omega)} = \\ &= \frac{354.08 \angle -5.9^\circ \text{ k}\Omega^2}{25.3 - j \cdot 43.32 \text{ k}\Omega} = \frac{354.08 \angle -5.9^\circ \text{ k}\Omega^2}{50.17 \angle -59.71^\circ \text{ k}\Omega} = 7.05 \angle 53.81^\circ \text{ k}\Omega \end{aligned}$$

Problem 4.37**Solution:****Known quantities:**

The current in the circuit, $i_s(t) = I_0 \cos(\omega t + 30^\circ)$, $I_0 = 13 \text{ mA}$, $\omega = 1000 \frac{\text{rad}}{\text{s}}$, and the value of the capacitance present in the circuit shown in Figure P4.37 $C = 0.5 \mu\text{F}$.

Find:

- The phasor notation for the source current.
- The impedance of the capacitor.
- The voltage across the capacitor, showing all the passages and using phasor notation only.

Analysis:

- Phasor notation:

$$I_s = I_0 \angle \phi = 13 \angle 30^\circ \text{ mA}$$

-

$$Z_C = -jX_C = -j \frac{1}{\omega C} = -j \frac{1}{\left(1000 \frac{\text{rad}}{\text{s}}\right)(0.5 \mu\text{F})} = 0 - j2 \text{ k}\Omega = 2 \angle -90^\circ \text{ k}\Omega$$

-

$$V_C = I_s \cdot Z_C = (13 \angle 30^\circ \text{ mA}) \cdot (2 \angle -90^\circ \text{ k}\Omega) = 26 \angle -60^\circ \text{ V}$$

$$v_C(t) = 26 \cos(1000t - 60^\circ) \text{ V}$$

Note that conversion from phasor notation to time notation or vice versa can be done at any time.

Problem 4.38**Solution:****Known quantities:**

The values of two currents in the circuit shown in Figure P4.38: $i_1(t) = 141.4 \cos(\omega t + 135^\circ) \text{ mA}$,
 $i_2(t) = 50 \cos(\omega t + 53.13^\circ) \text{ mA}$, $\omega = 377 \frac{\text{rad}}{\text{s}}$.

Find:

The current $i_3(t)$.

Analysis:

A solution using trigonometric identities is possible but inefficient, cumbersome, and takes a lot of time. Phasors are better! Note that one current is described with a sine and the other with a cosine function. When using phasors, all currents and voltages must be described with either sine functions or cosine functions. Which does not matter, but it is a good idea to adopt one and use it consistently. Therefore, first converts to cosines.

$$\begin{aligned} \text{KCL: } -i_1(t) + i_2(t) + i_3(t) &= 0 \Rightarrow +i_3(t) = i_1(t) - i_2(t) \\ i_3(t) &= 141.4 \cos(\omega t + 135^\circ) \text{ mA} - 50 \sin(\omega t - 53.13^\circ) \text{ mA} = \\ &= 141.4 \cos(\omega t + 135^\circ) \text{ mA} - 50 \cos(\omega t - 53.13^\circ - 90^\circ) \text{ mA} \\ I_3 &= 141.4 \text{ mA} \angle 135^\circ - 50 \text{ mA} \angle -143.13^\circ = \\ &= (-99.98 + j \cdot 99.98) \text{ mA} - (-40.00 - j \cdot 30.00) \text{ mA} = \\ &= (-59.98 + j \cdot 129.98) \text{ mA} = 143.2 \text{ mA} \angle 114.8^\circ \\ i_3(t) &= 143.2 \cos(\omega t + 114.8^\circ) \text{ mA} \end{aligned}$$

If sine functions were used, the result in phasor notation would differ in phase by 90 degrees.

Problem 4.39**Solution:****Known quantities:**

The values of the impedance, $Z_1 = 5.9 \angle 7^\circ \text{ k}\Omega$, $Z_2 = 2.3 \angle 0^\circ \Omega$, $Z_3 = 17 \angle 11^\circ \Omega$ and the voltages applied to the circuit shown in Figure P4.39, $v_{s1}(t) = v_{s2}(t) = 170 \cos(377t) \text{ V}$.

Find:

The current through Z_3 .

Analysis:

$$\begin{aligned} V_{s1} &= V_{s2} = 170 \angle 0^\circ \text{ V} = (170 + j0) \text{ V} \\ \text{KVL: } -V_{s1} - V_{s2} + I_3 Z_3 &= 0 \\ I_3 &= \frac{V_{s1} + V_{s2}}{Z_3} = \frac{170 \angle 0^\circ \text{ V} + 170 \angle 0^\circ \text{ V}}{17 \angle 11^\circ \Omega} = \frac{340 \angle 0^\circ \text{ V}}{17 \angle 11^\circ \Omega} = 20 \angle -11^\circ \text{ A} \end{aligned}$$

$$i_3(t) = 20 \cos\left(377 \frac{\text{rad}}{\text{s}} \cdot t - 11^\circ\right) \text{ A}$$

Note also:

$$\text{KVL: } -\mathbf{V}_{s1} + \mathbf{I}_1 Z_1 = 0 \Rightarrow \mathbf{I}_1 = \frac{\mathbf{V}_{s1}}{Z_1}, \quad -\mathbf{V}_{s2} - \mathbf{I}_2 Z_2 = 0 \Rightarrow \mathbf{I}_2 = -\frac{\mathbf{V}_{s2}}{Z_2}$$

Problem 4.40

Solution:

Known quantities:

The values of the impedance in the circuit shown in Figure P4.40, $Z_s = (13000 + j\omega 3) \Omega$,
 $R = 120 \Omega$, $L = 19 \text{ mH}$, $C = 220 \text{ pF}$.

Find:

The frequency such that the current $\mathbf{I}_i$ and the voltage $\mathbf{V}_0$ are in phase.

Analysis:

Z_s is not a factor in this solution. Only R, L, and C will determine if the voltage across this combination is in phase with the current through it. If the voltage and current are in phase, then, the equivalent impedance must have an "imaginary" or reactive part which is zero!

$$\begin{aligned} Z_{eq} &= \frac{\mathbf{V}_0}{\mathbf{I}_i} = |Z_{eq}| \angle 0^\circ = R_{eq} + jX_{eq}, \quad X_{eq}(\omega) = 0 \\ Z_{eq} &= \frac{(Z_R + Z_L) \cdot Z_C}{Z_R + Z_L + Z_C} = \frac{(R + jX_L) \cdot (-jX_C)}{R + jX_L - jX_C} = \frac{X_L X_C - jRX_C}{R + j(X_L - X_C)} \cdot \frac{R - j(X_L - X_C)}{R - j(X_L - X_C)} = \\ &= \frac{[X_L X_C R - RX_C(X_L - X_C)] - j[R^2 X_C + X_L X_C(X_L - X_C)]}{R^2 + (X_L - X_C)^2} \end{aligned}$$

At the resonant frequency the reactive component of this impedance must equal zero:

$$X_{eq}(\omega) = \frac{R^2 X_C + X_L X_C (X_L - X_C)}{R^2 + (X_L - X_C)^2} = 0 \Rightarrow R^2 + X_L (X_L - X_C) = 0$$

$$R^2 + \omega L \left(\omega L - \frac{1}{\omega C} \right) = 0 \Rightarrow \omega^2 L^2 = \frac{L}{C} - R^2$$

$$\begin{aligned} \omega &= \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{(19 \text{ mH})(220 \text{ pF})} - \frac{(120 \Omega)^2}{(19 \text{ mH})^2}} = \\ &= \sqrt{293.3 \text{ G} \frac{\text{rad}}{\text{s}} - 39.89 \text{ M} \frac{\text{rad}}{\text{s}}} = 489.1 \text{ k} \frac{\text{rad}}{\text{s}} \end{aligned}$$

Notes:

1. To separate the equivalent impedance into real (resistive) and "imaginary" (reactive) components, the denominator had to be "rationalized". This was done by multiplying numerator and denominator by the complex conjugate of the denominator, and multiplying term by term. Remember that $j^2 = -1$, etc.
2. The term with R had a negligible effect on the resonant frequency in this case. If R is sufficiently large, however, it will significantly affect the answer.

Problem 4.41**Solution:****Known quantities:**

The circuit shown in Figure P4.40

Find:

The frequency such that the current $\mathbf{I}_i$ and the voltage $\mathbf{V}_0$ are in phase.

Analysis:

If the voltage and current are in phase, then, the equivalent impedance must have an "imaginary" or reactive part which is zero!

$$Z_{eq} = \frac{\mathbf{V}_0}{\mathbf{I}_i} = |Z_{eq}| \angle 0^\circ = R_{eq} + jX_{eq}, \quad X_{eq}(\omega) = 0$$

$$Z_{eq} = \frac{(Z_R + Z_L) \cdot Z_C}{Z_R + Z_L + Z_C} = \frac{(R + jX_L) \cdot (-jX_C)}{R + jX_L - jX_C} = \frac{X_L X_C - jRX_C}{R + j(X_L - X_C)} \cdot \frac{R - j(X_L - X_C)}{R - j(X_L - X_C)} =$$

$$= \frac{[X_L X_C R - RX_C(X_L - X_C)] - j[R^2 X_C + X_L X_C(X_L - X_C)]}{R^2 + (X_L - X_C)^2}$$

At the resonant frequency the reactive component of this impedance must equal zero:

$$X_{eq}(\omega) = \frac{R^2 X_C + X_L X_C(X_L - X_C)}{R^2 + (X_L - X_C)^2} = 0 \Rightarrow R^2 + X_L(X_L - X_C) = 0$$

$$R^2 + \omega L \left(\omega L - \frac{1}{\omega C} \right) = 0 \Rightarrow \omega^2 L^2 = \frac{L}{C} - R^2$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Problem 4.42**Solution:****Known quantities:**

The values of the impedance, $R_s = 50 \, \Omega$, $R_c = 40 \, \Omega$, $L = 20 \, \mu\text{H}$, $C = 1.25 \, \text{nF}$, and the voltage

applied to the circuit shown in Figure P4.42, $v_s(t) = V_0 \cos(\omega t + 0^\circ)$, $V_0 = 10 \, \text{V}$, $\omega = 6 \, \text{M} \frac{\text{rad}}{\text{s}}$.

Find:

The current supplied by the source.

Analysis:

Assume clockwise currents:

$$X_L = \omega L = \left(6 \, \text{M} \frac{\text{rad}}{\text{s}} \right) (20 \, \mu\text{H}) = 1203 \, \Omega \Rightarrow Z_L = 0 + j120 \, \Omega = 120 \angle 90^\circ \, \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(6 \, \text{M} \frac{\text{rad}}{\text{s}} \right) (1.25 \, \text{nF})} = 133.3 \, \Omega \Rightarrow Z_C = 0 - j133.3 \, \Omega = 133.3 \angle -90^\circ \, \Omega$$

$$Z_{R_c} = 40 - j \Omega = 40 \angle 0^\circ \Omega, Z_{R_s} = 50 - j \Omega = 50 \angle 0^\circ \Omega$$

Equivalent impedances:

$$Z_{eq1} = Z_{R_c} + Z_L = 40 + j120 \Omega = 126.5 \angle 71.56^\circ \Omega$$

$$\begin{aligned} Z_{eq} &= Z_{R_s} + \frac{Z_C \cdot Z_{eq1}}{Z_C + Z_{eq1}} = 50 + j0 \Omega + \frac{(133.3 \angle -90^\circ \Omega) \cdot (126.5 \angle 71.56^\circ \Omega)}{133.3 \angle -90^\circ \Omega + 126.5 \angle 71.56^\circ \Omega} = \\ &= 50 + j0 \Omega + \frac{16.87 \angle -18.44^\circ \text{ k}\Omega^2}{42.161 \angle -18.44^\circ \Omega} = 50 \angle 0^\circ \Omega + 400 \angle 0^\circ \Omega = 450 \angle 0^\circ \Omega \end{aligned}$$

$$\text{OL: } I_s = \frac{V_s}{Z_{eq}} = \frac{10 \angle 0^\circ \text{ V}}{450 \angle 0^\circ \Omega} = 22.22 \angle 0^\circ \text{ mA} \Rightarrow i_s(t) = 22.22 \cos(\omega t + 0^\circ) \text{ mA}$$

Note:

The equivalent impedance of the parallel combination is purely resistive; therefore, the frequency given is the resonant frequency of this network.

Problem 4.43

Solution:

Known quantities:

The values of the impedance and the voltage applied to the circuit shown in Figure P4.43.

Find:

The current in the circuit.

Analysis:

Assume clockwise currents:

$$\omega = 3 \frac{\text{rad}}{\text{s}}, V_s = 12 \angle 0^\circ \text{ V}$$

$$Z_C = \frac{1}{j\omega C} = -j \Omega, Z_L = j\omega L = j9 \Omega \Rightarrow Z_{total} = 3 + j9 - j = 3 + j8 \Omega$$

$$I = \frac{12}{3 + j8} = 0.4932 - j1.3151 \text{ A} = 1.4045 \angle -69.44^\circ \text{ A},$$

$$i(t) = 1.4 \cos(\omega t - 69.4^\circ) \text{ A}$$

Problem 4.44

Solution:

Known quantities:

The values of the impedance and the current source shown in Figure P4.44.

Find:

The voltage.

Analysis:

Assume clockwise currents:

$$\omega = 2 \frac{\text{rad}}{\text{s}}, I_s = 10 \angle 0^\circ \text{ A}, Z_L = j\omega L = j6 \Omega, Z_C = \frac{1}{j\omega C} = -j1.5 \Omega$$

$$Z_{eq} = \frac{1}{\frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}} = \frac{1}{\frac{1}{3} - j\frac{1}{6} + j\frac{2}{3}} = \frac{1}{0.33 + j0.5} = 0.9231 - j1.3846 \, \Omega$$

$$V = I_S Z_{eq} = 10 \, \text{A} \cdot (0.9231 - j1.3846) \, \Omega = 9.231 - j13.846 \, \text{A} \cdot 10 \, \text{V} = 16.641 \angle -56.31^\circ \, \text{V}$$

Problem 4.45

Solution:

Known quantities:

The values of the impedance and the current source for the circuit shown in Figure P4.45.

Find:

The current $\mathbf{I}_1$.

Analysis:

Specifying the positive directions of the currents as in figure P4.45:

$$Z_{eq} = \frac{1}{\frac{1}{2} + \left(\frac{1}{-j4} \right)} = 1.79 \angle 26.56^\circ \, \Omega$$

$$V_S = I_S Z_{eq} = (10 \angle -22.5^\circ) \, \text{A} \cdot (1.79 \angle 26.56^\circ) \, \Omega = 17.9 \angle 4.06^\circ \, \text{V}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_S}{R} = 8.95 \angle 4.06^\circ \, \text{A}$$

Problem 4.46

Solution:

Known quantities:

The values of the impedance and the voltage source for circuit shown in Figure P4.46.

Find:

The voltage $\mathbf{V}_2$.

Analysis:

Specifying the positive directions as in figure P4.46:

$$Z_L = j\omega L = j12 \, \Omega$$

$$V_2 = \frac{R_{6\Omega} - R_{6\Omega}}{R_{12\Omega} + Z_L + R_{6\Omega}} V = \frac{6 \, \Omega}{(12 + j12 + 6) \, \Omega} 25 \angle 0^\circ \, \text{V} = \frac{150 \angle 0^\circ \, \Omega}{18 + j12 \, \Omega} \text{V} = 6.93 \angle -33.7^\circ \, \text{V}$$

Problem 4.47

Solution:

Known quantities:

The values of the impedance and the current source of circuit shown in Figure P4.44.

Find:

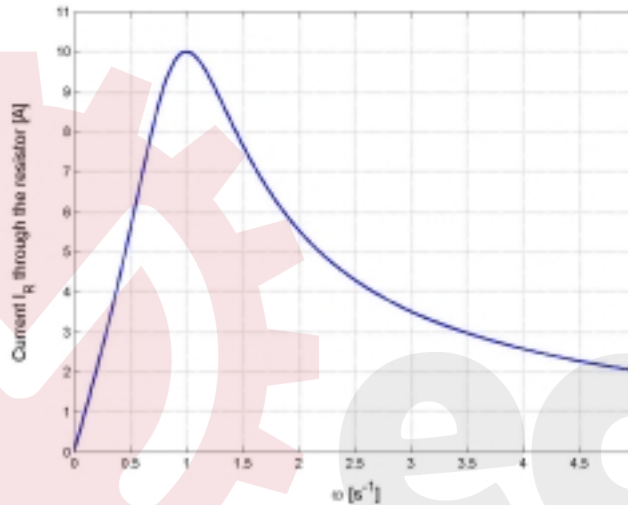
The value of ω for which the current through the resistor is maximum.

Analysis:

Assume clockwise currents:

$$I_s = 10\angle 0^\circ \text{ A}, Z_L = j\omega L = j3\omega \Omega, Z_C = \frac{1}{j\omega C} = -j\frac{3}{\omega} \Omega$$

$$I_R = \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}} I_s = \frac{\frac{1}{3}}{\frac{1}{3} - j\frac{1}{3\omega} + j\frac{\omega}{3}} 10\angle 0^\circ = \frac{10\omega}{\omega + j(\omega^2 - 1)}$$



The maximum of I_R is obtained for $\omega = 1$, therefore $i_R(t) = 10\cos(t)$.

Problem 4.48**Solution:****Known quantities:**

The values of the impedance and the current source for circuit shown in Figure P4.48.

Find:

The current through the resistor.

Analysis:

Specifying the positive directions as in figure P4.48:

By current division:

$$\begin{aligned} I_R &= -\frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{Z_C}} \cdot I_s = -\frac{1}{1 + \frac{R}{j\omega C}} \cdot I_s = -\frac{1}{1 + j\omega RC} \cdot I_s = -\frac{1 - j\omega RC}{1 + (\omega RC)^2} \cdot I_s = \\ &= -(0.0247 - j0.1552) \text{ A} \cdot 1\angle 0^\circ \text{ A} = 157 \cdot 10^{-3} \angle 99.04^\circ \text{ A} \\ i_R(t) &= 157 \cos(200\pi t + 99.04^\circ) \text{ mA} \end{aligned}$$

Problem 4.49**Solution:****Known quantities:**

The values of the reactance $X_L = 1 \text{ k}\Omega$, $X_C = 10 \text{ k}\Omega$, and the current source $\mathbf{I} = 10\angle 45^\circ \text{ mA}$ for circuit shown in Figure P4.49.

Find:

The voltage v_{out} .

Analysis:

Specifying the positive directions of the currents as in figure P4.49:

$$\begin{aligned} \mathbf{V}_{out} &= \mathbf{Z}_{eq} \mathbf{I} = (\mathbf{Z}_L + \mathbf{Z}_C) \mathbf{I} = (0 + jX_L + 0 - jX_C) \mathbf{I} = (j1 \text{ k}\Omega - j10 \text{ k}\Omega) \cdot 10\angle 45^\circ \text{ mA} = \\ &= (-j9 \text{ k}\Omega) \cdot 10\angle 45^\circ \text{ mA} = 9\angle -90^\circ \text{ k}\Omega \cdot 10\angle 45^\circ \text{ mA} = 90\angle -45^\circ \text{ V} \\ v_{out} &= 90 \cos(\omega t - 45^\circ) \text{ V} \end{aligned}$$

Problem 4.50**Solution:****Known quantities:**

The circuit shown in Figure P4.50, the values of the resistance, $R = 2 \Omega$, capacitance, $C = 1/8 \text{ F}$, inductance, $L = 1/4 \text{ H}$, and the frequency $\omega = 4 \frac{\text{rad}}{\text{s}}$.

Find:

The impedance Z .

Analysis:

$$\begin{aligned} Z_L &= j\omega L = j4 \frac{1}{4} \Omega = j \Omega, \quad Z_C = \frac{1}{j\omega C} = -j \frac{1}{\omega C} = -j \frac{1}{4 \cdot (1/8)} = -j2 \Omega \\ Z &= Z_L + Z_C \parallel R = Z_L + \frac{1}{\frac{1}{Z_C} + \frac{1}{R}} = j + \frac{1}{\frac{1}{-j2} + \frac{1}{2}} = j + \frac{j2}{-1 + j} = j + \frac{(j2)(-1 - j)}{(-1 + j)(-1 - j)} \\ &= j + \frac{j2(-1 - j)}{1 + 1} = j - j + 1 = 1 \Omega \end{aligned}$$

Problem 4.51**Solution:****Known quantities:**

Circuit shown in Figure P4.51, the values of the resistance, $R = 3 \Omega$, capacitance, $C = 1/10 \text{ F}$, inductance, $L = 4/5 \text{ H}$, and the frequency $\omega = 5 \frac{\text{rad}}{\text{s}}$.

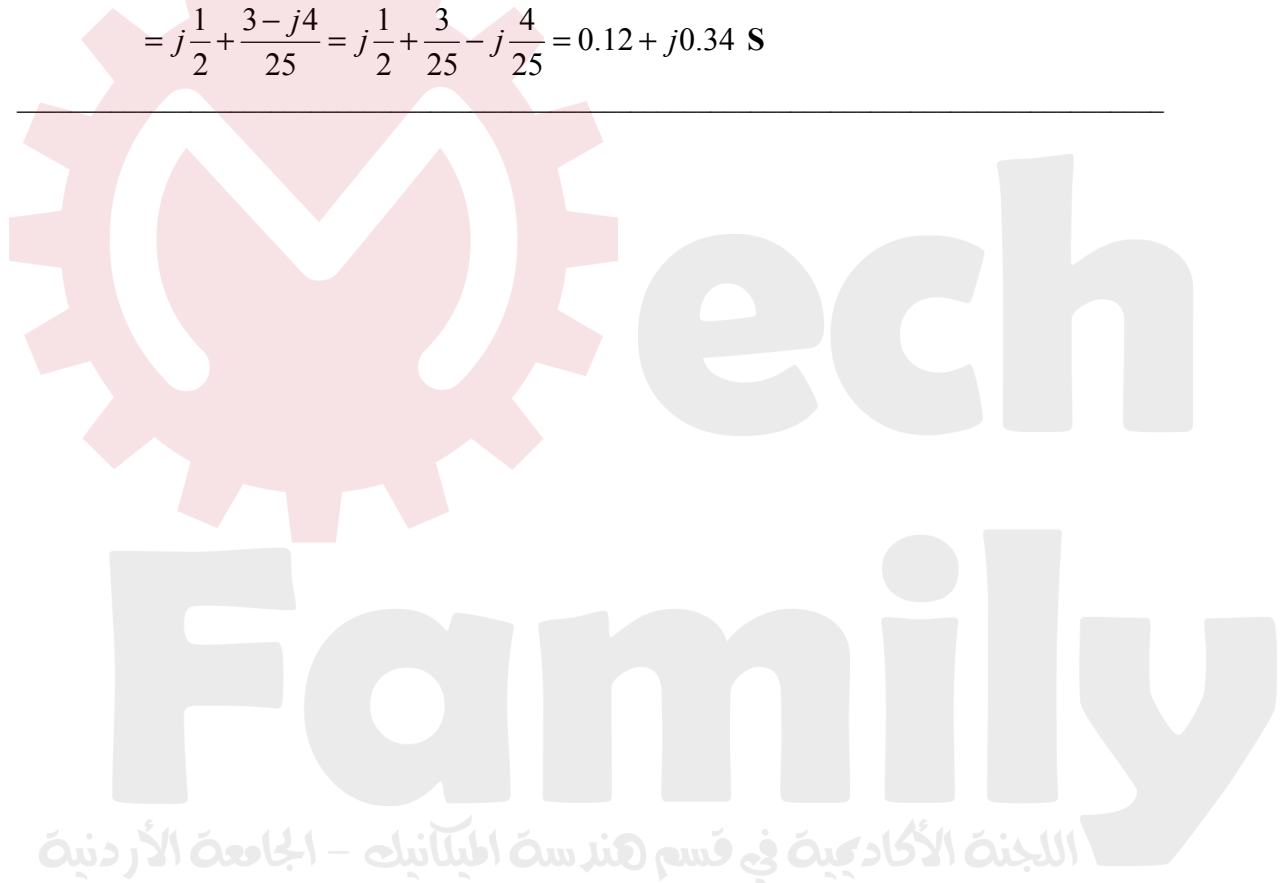
Find:The admittance Y .**Analysis:**

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j5 \cdot (1/10)} = -j2 \Omega, Z_L = j5 \cdot \frac{4}{5} = j4 \Omega$$

$$Y = \frac{1}{Z} = \frac{1}{Z_C \parallel (R + Z_L)} = \frac{1}{\frac{1}{\frac{1}{Z_C} + \frac{1}{R + Z_L}}} = \frac{1}{Z_C} + \frac{1}{R + Z_L} = \frac{1}{-j2} + \frac{1}{3 + j4}$$

$$= j\frac{1}{2} + \frac{(1)(3 - j4)}{(3 + j4)(3 - j4)} = j\frac{1}{2} + \frac{3 - j4}{9 + 16}$$

$$= j\frac{1}{2} + \frac{3 - j4}{25} = j\frac{1}{2} + \frac{3}{25} - j\frac{4}{25} = 0.12 + j0.34 \text{ S}$$



Section 4.5: AC Circuit Analysis methods

Focus on Methodology: AC Circuit Analysis

1. Identify the sinusoidal source(s) and note the excitation frequency.
2. Convert the source(s) to phasor form.
3. Represent each circuit element by its impedance.
4. Solve the resulting phasor circuit, using appropriate circuit analysis tools.
5. Convert the (phasor-form) answer to its time-domain equivalent, using equation 4.50.

Problem 4.52

Solution:

Known quantities:

Circuit shown in Figure P4.52, the values of the resistance, $R = 9\ \Omega$, capacitance, $C = 1/18\ \text{F}$, inductance, $L_1 = 3\ \text{H}$, $L_2 = 3\ \text{H}$, $L_3 = 3\ \text{H}$, and the voltage source $v_s(t) = 36 \cos\left(3t - \frac{\pi}{3}\right)\ \text{V}$.

Find:

The voltage across the capacitance v using phasor techniques.

Analysis:

$$\omega = 3\ \frac{\text{rad}}{\text{s}}, \quad V_s = 36 \angle -60^\circ\ \text{V}$$

$$Z_{L_2} = j\omega L_2 = j3 \cdot 3 = j9\ \Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j3 \cdot (1/18)} = -j6\ \Omega$$

$$Z_{L_3} = j\omega L_3 = j3 \cdot 3 = j9\ \Omega$$

$$Z_{eq} = \frac{1}{Z_{L_3} \parallel (Z_{L_2} + Z_C)} = \frac{1}{\frac{1}{Z_{L_3}} + \frac{1}{(Z_{L_2} + Z_C)}} = \frac{1}{\frac{1}{j9} + \frac{1}{j3}} = \frac{j9}{4} = 2.25 \angle 90^\circ\ \Omega$$

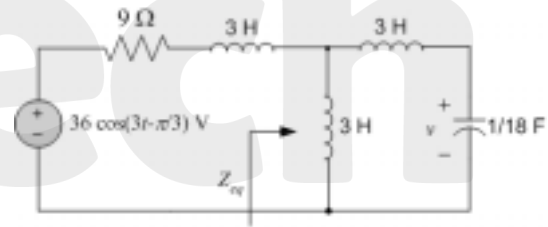
$$Z_T = Z_R + Z_{L_1} + Z_{eq} = 9 + j3 \cdot 3 + j2.25 = 9 + j11.25 = 14.407 \angle 51.34^\circ\ \Omega$$

$$I = \frac{V_s}{Z_T} = \frac{36 \angle -60^\circ\ \text{V}}{14.407 \angle 51.34^\circ\ \Omega} = 2.499 \angle -111.34^\circ\ \text{A}$$

$$V_{eq} = I Z_{eq} = (2.499 \angle -111.34^\circ)(2.25 \angle 90^\circ) = 5.623 \angle -21.34^\circ\ \text{V}$$

$$V = \frac{Z_C}{(Z_{L_2} + Z_C)} V_{eq} = \frac{-j6}{j3} 5.623 \angle -21.34^\circ = 11.25 \angle 158.66^\circ\ \text{V}$$

$$v = 11.25 \cos(3t - 158.66^\circ)\ \text{V}$$



Problem 4.53**Solution:****Known quantities:**

Circuit shown in Figure P4.53, the values of the resistance, $R = 5 \Omega$, capacitance, $C = 1/2 \text{ F}$, inductance, $L_1 = 0.5 \text{ H}$, $L_2 = 1 \text{ H}$, $L_3 = 10 \text{ H}$, and the current source $i_s(t) = 6 \cos(2t) \text{ A}$.

Find:

The current through the inductance i_{L_2} .

Analysis:

$$\omega = 2 \frac{\text{rad}}{\text{s}}, Z_{L_2} = j\omega L_2 = j2 \Omega$$

$$Z_C = \frac{1}{j\omega C} = -j \Omega, Z_{L_3} = j\omega L_3 = j20 \Omega$$

$$I = \frac{Z_{L_3} + Z_C}{(Z_{L_3} + Z_C) + (R + Z_{L_2})} I_s = \frac{j20 - j}{(j20 - j) + (5 + j2)} 6 \angle 0^\circ$$

$$= \frac{j19}{5 + j21} 6 \angle 0^\circ = 5.28 \angle 13.4^\circ \text{ A}$$

$$i = 5.28 \cos(2t + 13.4^\circ) \text{ A}$$

Problem 4.54**Solution:****Known quantities:**

Circuit shown in Figure P4.54 the values of the resistance, $R_s = R_L = 500 \Omega$, $R = 1 \text{ k}\Omega$ and the inductance, $L = 10 \text{ mH}$.

Find:

- The Thévenin equivalent circuit if the voltage applied to the circuit is $v_s(t) = 10 \cos(1,000t)$.
- The Thévenin equivalent circuit if the voltage applied to the circuit is $v_s(t) = 10 \cos(1,000,000t)$.

Analysis:

$$\text{a) } Z_L = j\omega L = j1000 \frac{\text{rad}}{\text{s}} \cdot 10 \text{ mH} = j10 \Omega,$$

The equivalent impedance is:

$$Z_T = \frac{Z_L \cdot R}{(Z_L + R)} + R_s = \frac{(j10)1000}{j10 + 1000} + 500 = 500 + \frac{j10^3}{100 + j} = 500.1 + j9.999 \Omega$$

The equivalent Thévenin voltage is:

$$V_T = V_s = 10 \angle 0^\circ \text{ V}$$

$$\text{b) } Z_L = j\omega L = j10^6 \frac{\text{rad}}{\text{s}} \cdot 10 \text{ mH} = j10^4 \Omega,$$

The equivalent impedance is:

$$Z_T = \frac{Z_L \cdot R}{(Z_L + R)} + R_S = \frac{(j10^4)1000}{j10^4 + 1000} + 500 = 500 + \frac{j10^4}{1 + j10} = 1490.1 + j99.01 \Omega$$

The equivalent Thévenin voltage is:

$$V_T = V_S = 10 \angle 0^\circ \text{ V}$$

Problem 4.55

Solution:

Circuit shown in Figure P4.55 the values of the impedance, $L = 0.1 \text{ H}$, capacitance, $C = 100 \mu\text{F}$, and the voltage source $v_{in}(t) = 12 \cos(10t) \text{ V}$.

Find:

The Thévenin equivalent of the circuit as seen by the load resistor R_L .

Analysis:

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j10 \frac{\text{rad}}{\text{s}} \cdot 100 \mu\text{F}} = -j1000 \Omega$$

$$Z_L = j\omega L = j10 \frac{\text{rad}}{\text{s}} \cdot 0.1 \text{ H} = j1 \Omega$$

The equivalent impedance is:

$$Z_T = Z_L \parallel Z_C = \frac{Z_L \cdot Z_C}{Z_L + Z_C} = \frac{j(-j1000)}{j - j1000} = \frac{1000}{-j999} = 1.001 \angle 90^\circ \Omega = j1.001 \Omega$$

The Thévenin voltage is:

$$V_T = \frac{Z_C}{Z_L + Z_C} V_{in} = \frac{-j1000}{j - j1000} \cdot 12 \angle 0^\circ = \frac{1000}{999} \cdot 12 \angle 0^\circ = 12.012 \angle 0^\circ \text{ V}$$

Problem 4.56

Solution:

Known quantities:

Circuit shown in Figure P4.56 the values of the resistance, $R_1 = 4 \Omega$, $R_2 = 4 \Omega$, capacitance, $C = 1/4 \text{ F}$, inductance, $L = 2 \text{ H}$, and the voltage source $v_s(t) = 2 \cos(2t) \text{ V}$.

Find:

The current in the circuit $i_L(t)$ using phasor techniques.

Analysis:

$$V_s(t) = 2 \angle 0^\circ \text{ V}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j2 \frac{1}{4}} = -j2 \Omega$$

$$Z_L = j\omega L = j2 \cdot 2 = j4 \Omega$$

Applying the voltage divider rule:

$$V_L = \frac{(Z_L \parallel (Z_C + Z_2))}{Z_1 + (Z_L \parallel (Z_C + Z_2))} V_s = \frac{4 \angle 36.8^\circ}{4 \angle 0^\circ + 4 \angle 36.8^\circ} 2 \angle 0^\circ = 1.05 \angle 18.4^\circ \text{ V}$$

Therefore, the current is:

$$I_L = \frac{V_L}{Z_L} = \frac{1.05 \angle 18.4^\circ}{4 \angle 90^\circ} = 0.2635 \angle -71.6^\circ \text{ A}$$

$$i_L(t) = 0.2635 \cos(2t - 71.6^\circ) \text{ A}$$

Problem 4.57

Solution:

Known quantities:

Circuit shown in Figure P4.57, the values of the resistance, $R_1 = 75 \Omega$, $R_2 = 100 \Omega$, capacitance, $C = 1 \mu\text{F}$, inductance, $L = 0.5 \text{ H}$, and the voltage source $v_s(t) = 15 \cos(1,500t) \text{ V}$.

Find:

The currents in the circuit $i_1(t)$ and $i_2(t)$.

Analysis:

In the phasor domain:

$$Z_C = \frac{-j}{1500(1 \times 10^{-6})} = -j \frac{2000}{3} = -j666.7 \Omega, Z_L = j(1500)(0.5) = j750 \Omega$$

By applying KVL in the first loop, we have

$$V_s = R_1 I_1 + Z_C (I_1 - I_2)$$

By applying KVL in the second loop, we have

$$0 = (Z_C)(I_2 - I_1) + (Z_L + R_2) I_2$$

That is:

$$\begin{cases} 15 \angle 0^\circ = \left(75 - j \frac{2000}{3}\right) I_1 + j \frac{2000}{3} I_2 \\ 0 = j \frac{2000}{3} I_1 + \left(100 + j \frac{250}{3}\right) I_2 \end{cases}$$

By solving above equations, we have

$$I_1 = 3.8 \cdot 10^{-3} \angle 46.6^\circ \text{ A}$$

$$I_2 = 19.6 \cdot 10^{-3} \angle -83.2^\circ \text{ A}$$

$$i_1(t) = 3.8 \cos(1,500t + 46.6^\circ) \text{ mA}$$

$$i_2(t) = 19.6 \cos(1,500t - 83.2^\circ) \text{ mA}$$

Problem 4.58

Solution:

Known quantities:

Circuit shown in Figure P4.58, the values of the resistance, $R_1 = 40 \Omega$, $R_2 = 10 \Omega$, capacitance, $C = 500 \mu\text{F}$, inductance, $L = 0.2 \text{ H}$, and the current source $i_s(t) = 40 \cos(100t) \text{ A}$.

Find:

The voltages in the circuit $v_1(t)$ and $v_2(t)$.

Analysis:

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{100 \cdot 500 \cdot 10^{-6}} = -j20 \, \Omega, \quad Z_L = j\omega L = j100 \cdot 0.2 = j20 \, \Omega$$

Applying KCL at node 1, we have:

$$I_s = \frac{V_1}{R_1} + \frac{V_1 - V_2}{Z_C} \Rightarrow I_s = \left(\frac{1}{R_1} + \frac{1}{Z_C} \right) V_1 - \frac{1}{Z_C} V_2 \Rightarrow 40 \angle 0^\circ = \left(\frac{1}{40} + \frac{j}{20} \right) V_1 - \frac{j}{20} V_2$$

Applying KCL at node 2, we have

$$\frac{V_1 - V_2}{Z_C} = \frac{V_2}{R_2} + \frac{V_2}{Z_L} \Rightarrow \frac{V_1}{Z_C} = \left(\frac{1}{R_2} + \frac{1}{Z_L} + \frac{1}{Z_C} \right) V_2 \Rightarrow j \frac{V_1}{20} = \left(\frac{1}{10} - j \frac{1}{20} + j \frac{1}{20} \right) V_2$$

Therefore:

$$\begin{cases} 40 \angle 0^\circ = \left(\frac{1}{40} + \frac{j}{20} \right) V_1 - \frac{j}{20} V_2 \\ j \frac{V_1}{20} = \left(\frac{1}{10} \right) V_2 \end{cases} \Rightarrow \begin{cases} 40 \angle 0^\circ = \left(\frac{1}{40} + \frac{j}{20} \right) (-j2V_2) - \frac{j}{20} V_2 \\ V_1 = -j2V_2 \end{cases}$$

$$\begin{cases} 40 \angle 0^\circ = -\frac{j}{20} V_2 + \frac{1}{10} V_2 - \frac{j}{20} V_2 = \left(\frac{1}{10} - \frac{j}{10} \right) V_2 \\ V_1 = -j2V_2 \end{cases}$$

$$V_2 = \frac{40 \angle 0^\circ}{\left(\frac{1}{10} - \frac{j}{10} \right)} = 282.84 \angle 45^\circ \, \text{V}, \quad V_1 = -j2V_2 = 565.68 \angle -45^\circ \, \text{V}$$

$$v_2(t) = 282.84 \cos(100t + 45^\circ) \, \text{V}, \quad v_1(t) = 568.68 \cos(100t - 45^\circ) \, \text{V}$$

Problem 4.59**Solution:****Known quantities:**

The circuit called Wheatstone bridge shown in Figure P4.59.

- The balanced status for the bridge: $v_{ab} = 0$.
- The values of the resistance, $R_1 = 100 \, \Omega$, $R_2 = 1 \, \Omega$, the capacitance, $C_3 = 4.7 \, \mu\text{F}$, the inductance, $L_3 = 0.098 \, \text{H}$, that are necessary to balance the bridge: $v_{ab} = 0$, and the voltage applied to the bridge, $v_s = 24 \sin(2,000t) \, \text{V}$.

Find:

- The unknown reactance X_4 in terms of the circuit elements.
- The value of the unknown reactance X_4 .
- The source frequency that should be avoided in this circuit.

Analysis:

- Assuming a balanced circuit, we have $v_{ab} = 0$, that is, $v_a = v_b$

From the voltage divider:
$$\frac{R_2}{jX_{L_3} - jX_{C_3} + R_2} = \frac{jX_4}{R_1 + jX_4} \Rightarrow \frac{R_2}{j\omega L_3 - \frac{j}{\omega C_3} + R_2} = \frac{jX_4}{R_1 + jX_4}$$

Inverting both sides and equating imaginary parts:

$$R_1 R_2 = \left(-\omega L_3 + \frac{1}{\omega C_3} \right) X_4 \Rightarrow X_4 = \frac{R_1 R_2}{\left(\frac{1}{\omega C_3} - \omega L_3 \right)}$$

b)

$$X_4 = \frac{100 \cdot 1}{\left(\frac{1}{2000 \cdot 4.7 \cdot 10^{-6}} - 2000 \cdot 0.098 \right)} = -1.116 \Omega$$

Negative reactance implies that the component is a capacitor.

$$\frac{1}{\omega C} = 1.116 \Omega \Rightarrow C = \frac{1}{\omega \cdot 1.116} = 448 \mu\text{F}$$

c)

If the reactances of L_3 and C_3 cancel, the bridge can not measure X_4 . Thus, the condition to be avoided is:

$$\omega L_3 - \frac{1}{\omega C_3} = 0 \Rightarrow L_3 C_3 = \frac{1}{\omega^2} \Rightarrow \omega = \frac{1}{\sqrt{L_3 C_3}} = \frac{1}{\sqrt{0.098 \cdot 4.7 \cdot 10^{-6}}} = 1473 \frac{\text{rad}}{\text{s}}$$

$$f = 234.5 \text{ Hz}$$

Problem 4.60

Solution:

Known quantities:

Circuit shown in Figure P4.56, the values of the resistance, $R_1 = 4 \Omega$, $R_2 = 4 \Omega$, capacitance, $C = 1/4 \text{ F}$, inductance, $L = 2 \text{ H}$, and the voltage source $v_s(t) = 2 \cos(2t) \text{ V}$.

Find:

The Thévenin impedance seen by resistor R_2 .

Analysis:

$$Z_T = (R_1 \parallel Z_L) + (Z_C) = (4 \parallel j4) + (-j2) = j2(1-j) - j2 = 2 + j2 + (-j2) = 2 \Omega$$

Problem 4.61

Solution:

Known quantities:

Circuit shown in Figure P4.58, the values of the resistance, $R_1 = 10 \Omega$, $R_2 = 40 \Omega$, capacitance, $C = 500 \mu\text{F}$, inductance, $L = 0.2 \text{ H}$, and the current source $i_s(t) = 40 \cos(100t) \text{ A}$.

Find:

The Thévenin voltage seen by inductance L .

Analysis:

The Thévenin equivalent voltage source is the open-circuit voltage at the load terminals:

$$\mathbf{V}_T = R_2 \mathbf{I}_2 = 40 \mathbf{I}_2$$

From the current division, we have

$$\mathbf{I}_2 = \frac{R_1}{(R_2 + Z_C) + R_1} \mathbf{I}_S = \frac{10}{(40 - j20) + 10} 40 \angle 0^\circ = 7.43 \angle 21.8^\circ \text{ A}$$

$$\mathbf{V}_T = R_2 \mathbf{I}_2 = 40 \cdot 7.43 \angle 21.8^\circ = 297 \angle 21.8^\circ \text{ V}$$

$$v_T(t) = 297 \cos(100t + 21.8^\circ) \text{ V}$$

Problem 4.62

Solution:

Known quantities:

Circuit shown in Figure P4.62, the values of the impedance, $R = 8 \Omega$, $Z_C = -j8 \Omega$, $Z_L = j8 \Omega$, and the voltage source $\mathbf{V}_s = 5 \angle -30^\circ \text{ V}$.

Find:

The Thévenin equivalent circuit seen from the terminals a-b.

Analysis:

The Thévenin equivalent circuit is given by:

$$\mathbf{V}_{TH} = \left(\frac{8 + j8}{8 + j8 - j8} \right) 5 \angle -30^\circ = (1 + j) 5 \angle -30^\circ = 7.07 \angle 15^\circ \text{ V}$$

$$\mathbf{Z}_{TH} = \frac{(8 + j8)(-j8)}{8 + j8 - j8} = (8 - j8) = 8\sqrt{2} \angle -45^\circ \Omega$$

Problem 4.63

Solution:

Known quantities:

Circuit shown in Figure P4.56, the values of the resistance, $R_1 = 4 \Omega$, $R_2 = 4 \Omega$, capacitance, $C = 1/4 \text{ F}$, inductance, $L = 2 \text{ H}$, and the voltage source $v_s(t) = 2 \cos(2t) \text{ V}$.

Find:

The Thévenin equivalent voltage seen by the resistor R_2 .

Analysis:

The Thévenin equivalent circuit is given by:

$$\mathbf{V}_T = \frac{j4}{4 + j4} 2 \angle 0^\circ = (1 + j) = \sqrt{2} \angle 45^\circ = 1.414 \angle 45^\circ \text{ V}$$

$$v_T(t) = 1.414 \cos(2t + 45^\circ) \text{ V}$$

Problem 4.64**Solution:****Known quantities:**

Circuit shown in Figure P4.56, the values of the resistance, $R_1 = 4 \Omega$, $R_2 = 4 \Omega$, capacitance, $C = 1/4 \text{ F}$, inductance, $L = 2 \text{ H}$, and the voltage source $v_s(t) = 2 \cos(2t) \text{ V}$.

Find:

The Norton equivalent circuit seen by the resistor R_2 .

Analysis:

From the result of Problem 4.60, we have $Z_T = 2 \Omega$. From the current divider:

$$\mathbf{I}_N = \frac{j4}{j4-j2} \mathbf{I} = 2\mathbf{I}$$

and

$$j4 \parallel -j2 = \frac{(-j2)(j4)}{j2} = -j4$$

The current is:

$$\mathbf{I} = \frac{2 \angle 0^\circ}{4-j4} = \frac{\sqrt{2}}{4} \angle 45^\circ = 0.353 \angle 45^\circ \text{ A}$$

Therefore:

$$\mathbf{I}_N = 2\mathbf{I} = 0.707 \angle 45^\circ \text{ A}$$

Problem 4.65**Solution:****Known quantities:**

Circuit shown in Figure P4.66.

Find:

The equations required to solve for the loop currents in the circuit in:

- Integral-differential form;
- Phasor form.

Analysis:

$$\text{KVL: } -v_s + i_1 R_s + v_c(0) + \frac{1}{C} \int_0^t (i_1 - i_2) dt + (i_1 - i_2) R_1 = 0$$

$$\text{KVL: } (i_2 - i_1) R_1 - v_c(0) + \frac{1}{C} \int_0^t (i_2 - i_1) dt + L \frac{di_2}{dt} + i_2 R_2 = 0$$

Note: The initial voltage across the capacitor must, in general, be considered. It is modeled as an ideal voltage source in series with the capacitor.

$$\text{KVL: } -\mathbf{V}_s + \mathbf{I}_1 R_s + (\mathbf{I}_1 - \mathbf{I}_2) Z_C + (\mathbf{I}_1 - \mathbf{I}_2) R_1 = 0$$

$$\text{KVL: } (\mathbf{I}_2 - \mathbf{I}_1) R_1 + (\mathbf{I}_2 - \mathbf{I}_1) Z_C + \mathbf{I}_2 Z_L + \mathbf{I}_2 R_2 = 0$$

Note:

- The i - v characteristics of the inductor and capacitor, i.e. the integral and derivative, have been replaced here by the impedance.
- This form of the equation is applicable only when the waveforms of the currents and voltages are sinusoids!

Problem 4.66**Solution:****Known quantities:**

Circuit shown in Figure P4.65.

Find:

The node equations required to solve for all currents and voltages in the circuit.

Analysis:

$$\mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_3$$

$$\mathbf{V}_S - \mathbf{I}_1 R_S = \mathbf{I}_2 (Z_C + R_1) + \mathbf{I}_3 (Z_L + R_2)$$

Problem 4.67**Solution:****Known quantities:**

The voltages at the nodes of the circuit shown in Figure P4.67, $\mathbf{V}_a = 450 \angle 0^\circ \text{ V}$, $\mathbf{V}_b = 440 \angle 30^\circ \text{ V}$, $\mathbf{V}_c = 420 \angle -200^\circ \text{ V}$, $\mathbf{V}_{bc} = 779.5 \angle 5.621^\circ \text{ V}$, $\mathbf{V}_{cd} = 153.9 \angle 68.93^\circ \text{ V}$, $\mathbf{V}_{ba} = 230.6 \angle 107.4^\circ \text{ V}$, and the voltage sources, $v_{s1} = 450 \cos(\omega t) \text{ V}$, $v_{s2} = 450 \cos(\omega t) \text{ V}$.

Find:

The new values of $\mathbf{V}_b$ and $\mathbf{V}_{bc}$, if the ground is moved from Node e to Node d.

Analysis:

A node voltage is defined as the voltage between a node and the ground node. If the ground node is changed, then all node voltages in the circuit will change. With the ground at Node d:

$$\begin{aligned} \mathbf{V}_b &= \mathbf{V}_{bd} = \mathbf{V}_{be} + \mathbf{V}_{ed} = \mathbf{V}_{be} + \mathbf{V}_{s2} = 440 \angle 30^\circ \text{ V} + 450 \angle 0^\circ \text{ V} = \\ &= (381.1 + j220.0) \text{ V} + (450 + j0) \text{ V} = 831.1 + j220.0 \text{ V} = 859.6 \angle 14.83^\circ \text{ V} \end{aligned}$$

The voltages between any two nodes in a circuit do not depend on which is the ground node; therefore, the voltage between Node b and Node c remains the same when the ground is moved from Node e to Node d:

$$\mathbf{V}_{bc} = 779.5 \angle 5.621^\circ \text{ V}$$

Problem 4.68**Solution:****Known quantities:**

Circuit shown in Figure P4.68, the values of the resistance, $R_L = 120 \Omega$, the capacitance,

$C = 12.5 \mu\text{F}$, and the inductance, $L = 60 \text{ mH}$, and the voltage source,

$$v_i = 4 \cos(1,000t + 30^\circ) \text{ V}.$$

Find:

The new value of $\mathbf{V}_0$.

Analysis:

The circuit has 3 unknown mesh currents but only 1 unknown node voltage.

$$Z_C = -jX_C = -j \frac{1}{\omega C} = -j \frac{1}{\left(1,000 \text{ k} \frac{\text{rad}}{\text{s}}\right)(12.5 \mu\text{F})} = -j80 \Omega = 80 \angle -90^\circ \Omega$$

$$Z_L = jX_L = j\omega L = j \left(1,000 \text{ k} \frac{\text{rad}}{\text{s}}\right)(60 \text{ mH}) = j60 \Omega = 60 \angle 90^\circ \Omega$$

Reference phasor: $V_i = 4 \angle 30^\circ \text{ V}$

$$\text{KCL: } \frac{V_0 - 0}{Z_{R_L}} + \frac{V_0 - 0}{Z_C} + \frac{V_0 - V_i}{Z_L} = 0$$

$$\begin{aligned} V_0 &= \frac{\frac{V_i}{Z_L}}{\frac{1}{Z_{R_L}} + \frac{1}{Z_C} + \frac{1}{Z_L}} = \frac{V_i}{\frac{Z_L}{Z_{R_L}} + \frac{Z_L}{Z_C} + 1} = \frac{4 \angle 30^\circ \text{ V}}{\frac{60 \angle 90^\circ \Omega}{120 \angle 0^\circ \Omega} + \frac{60 \angle 90^\circ \Omega}{80 \angle -90^\circ \Omega} + 1} = \\ &= \frac{4 \angle 30^\circ \text{ V}}{0.5 \angle 90^\circ + 0.75 \angle 180^\circ + 1} = \frac{4 \angle 30^\circ \text{ V}}{(0 + j0.5) + (-0.75 + j0) + (1 + j0)} = \\ &= \frac{4 \angle 30^\circ \text{ V}}{0.25 + j0.5} = \frac{4 \angle 30^\circ \text{ V}}{0.559 \angle 63.43^\circ} = 7.155 \angle -33.43^\circ \text{ V} \\ v_0(t) &= 7.155 \cos(\omega t - 33.43^\circ) \text{ V} \end{aligned}$$

Problem 4.69

Solution:

Known quantities:

Circuit shown in Figure P4.69, the mesh currents and node voltages,

$$i_1(t) = 3.127 \cos(\omega t - 47.28^\circ) \text{ A}, \quad i_2(t) = 3.914 \cos(\omega t - 102.0^\circ) \text{ A},$$

$$i_3(t) = 1.900 \cos(\omega t + 37.50^\circ) \text{ A}, \quad v_1(t) = 130.0 \cos(\omega t + 10.08^\circ) \text{ V},$$

$$v_2(t) = 130.0 \cos(\omega t - 25.00^\circ) \text{ V}, \quad \text{where } \omega = 377.0 \frac{\text{rad}}{\text{s}}.$$

Find:

One of the following: L_1 , C_2 , R_3 , L_3 .

Analysis:

$$\text{KCL: } -\mathbf{I}_1 + \mathbf{I}_{Z_1} + \mathbf{I}_3 = 0$$

$$\mathbf{I}_{Z_1} = \mathbf{I}_1 - \mathbf{I}_3 = (2.121 - j2.297) \text{ A} - (1.507 + j1.157) \text{ A} = 3.508 \angle -79.92^\circ \text{ A}$$

$$\text{OL: } Z_1 = \frac{V_1}{\mathbf{I}_{Z_1}} = \frac{130 \angle 10.08^\circ \text{ V}}{3.508 \angle -79.92^\circ \text{ A}} = 37.05 \angle 90^\circ \Omega = \omega L_1 \angle 90^\circ$$

$$L_1 = \frac{37.05 \Omega}{377 \frac{\text{rad}}{\text{s}}} = 98.29 \text{ mH}$$

$$\text{KCL: } \mathbf{I}_2 + \mathbf{I}_{Z_2} - \mathbf{I}_3 = 0$$

$$I_{Z_2} = I_3 - I_2 = (1.507 - j1.157) \text{ A} - (-0.8138 - j3.828) \text{ A} = 5.499 \angle 65.03^\circ \text{ A}$$

$$\text{OL: } Z_2 = \frac{V_2}{I_{Z_2}} = \frac{130 \angle 24.97^\circ \text{ V}}{5.499 \angle 65.03^\circ \text{ A}} = 23.64 \angle -90^\circ \Omega = \frac{1}{\omega C_2} \angle -90^\circ$$

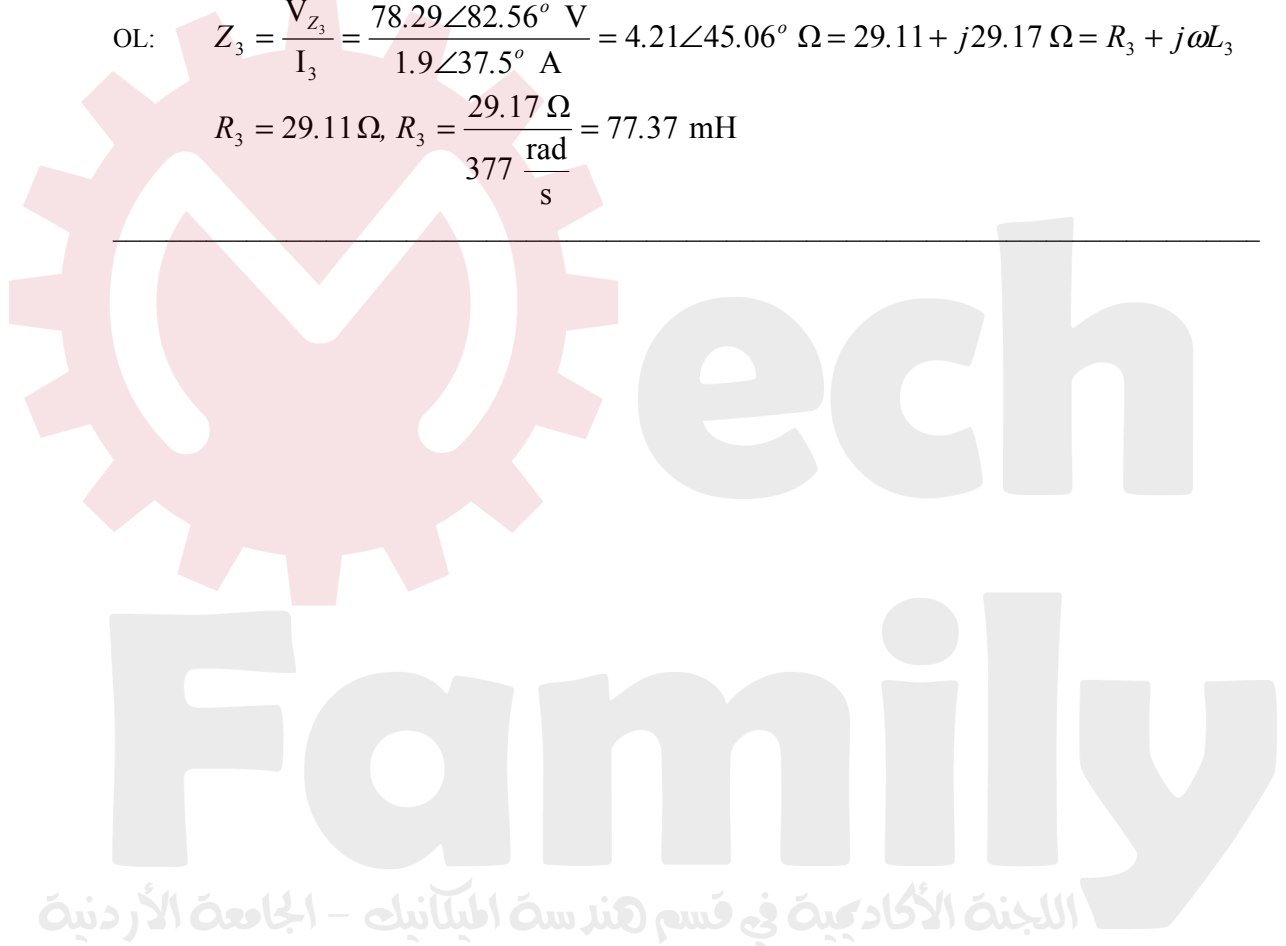
$$C_2 = \frac{1}{\left(377 \frac{\text{rad}}{\text{s}}\right)(23.64 \Omega)} = 112.2 \mu\text{F}$$

$$\text{KVL: } V_2 - V_1 + V_{Z_3} = 0$$

$$V_{Z_3} = V_1 - V_2 = (128.0 + j22.75) \text{ V} - (117.8 - j54.88) \text{ V} = 78.29 \angle 82.56^\circ \text{ V}$$

$$\text{OL: } Z_3 = \frac{V_{Z_3}}{I_3} = \frac{78.29 \angle 82.56^\circ \text{ V}}{1.9 \angle 37.5^\circ \text{ A}} = 4.21 \angle 45.06^\circ \Omega = 29.11 + j29.17 \Omega = R_3 + j\omega L_3$$

$$R_3 = 29.11 \Omega, R_3 = \frac{29.17 \Omega}{377 \frac{\text{rad}}{\text{s}}} = 77.37 \text{ mH}$$



Chapter 5 Instructor Notes

Chapter 5 has been reorganized in response to a number of suggestions forwarded by users of the third edition of this book. The material is now divided into three background sections (overview of transient analysis, writing differential equations, and DC steady-state solution) and two major sections: first and second order transients. A few examples have been added, and all previous examples have been reorganized to follow the methodology outlined in the text.

In the section on first order transients, a *Focus on Methodology: First Order Transient Response* (p. 215) clearly outlines the methodology that is followed in the analysis of first order circuits; this methodology is then motivated and explained, and is applied to eight examples, including four examples focusing on engineering applications (5.8 - Charging a camera flash; 5.9 and 5.11 dc motor transients; and 5.12, transient response of supercapacitor bank). The analogy between electrical and thermal systems that was introduced in Chapter 3 is now extended to energy storage elements and transient response (*Make The Connection: Thermal Capacitance*, p. 204; *Make The Connection: Thermal System Dynamics*, p. 205; *Make The Connection: First-Order Thermal System*, p. 218-219;); similarly, the analogy between hydraulic and electrical circuits, begun in Chapter 2 and continued in Chapter 4, is continued here (*Make The Connection: Hydraulic Tank*, pp. 214-215). The box *Focus on Measurements: Coaxial Cable Pulse Response* (pp. 230-232) illustrates an important transient analysis computation (this problem was suggested many years ago by a Nuclear Engineering colleague).

The section on second order transients summarizes the analysis of second order circuits in the boxes *Focus on Methodology: Roots of Second-Order System* (p. 240) and *Focus on Methodology: Second Order Transient Response* (pp. 244-245). These boxes clearly outline the methodology that is followed in the analysis of second order circuits; the motivation and explanations in this section are accompanied by five very detailed examples in which the methodology is applied step by step. The last of these examples takes a look at an automotive ignition circuit (with many thanks to my friend John Auzins, formerly of Delco Electronics, for suggesting a simple but realistic circuit). The analogy between electrical and mechanical systems is explored in *Make The Connection: Automotive Suspension*, pp. 239-240 and pp. 245-246.

The homework problems are divided into four sections, and contain a variety of problems ranging from very basic to the fairly advanced. The focus is on mastering the solution methods illustrated in the chapter text and examples.

Learning Objectives

1. Write differential equations for circuits containing inductors and capacitors.
2. Determine the DC steady state solution of circuits containing inductors and capacitors.
3. Write the differential equation of first order circuits in standard form and determine the complete solution of first order circuits excited by switched DC sources.
4. Write the differential equation of second order circuits in standard form and determine the complete solution of second order circuits excited by switched DC sources.
5. Understand analogies between electrical circuits and hydraulic, thermal and mechanical systems.

Section 5.2: Writing Differential Equations for Circuits Containing Inductors and Capacitors

Problem 5.1

Solution:

Known quantities:

$$L = 0.9 \text{ mH}, V_s = 12 \text{ V}, R_1 = 6 \text{ k}\Omega, R_2 = 6 \text{ k}\Omega, R_3 = 3 \text{ k}\Omega.$$

Find:

The differential equation for $t > 0$ (switch open) for the circuit of P5.21.

Analysis:

Apply KCL at the top node (nodal analysis) to write the circuit equation. Note that the top node voltage is the inductor voltage, v_L .

$$\frac{v_L}{R_1 + R_2} + i_L + \frac{v_L}{R_3} = 0$$

Next, use the definition of inductor voltage to eliminate the variable v_L from the nodal equation:

$$\begin{aligned} \frac{L}{R_1 + R_2} \frac{di_L}{dt} + i_L + \frac{L}{R_3} \frac{di_L}{dt} &= 0 \\ \frac{di_L}{dt} + \frac{(R_1 + R_2)R_3}{L(R_1 + R_2 + R_3)} i_L &= 0 \end{aligned}$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{di_L}{dt} + 2.67 \cdot 10^6 i_L = 0$$

Problem 5.2

Solution:

Known quantities:

$$V_1 = 12 \text{ V}, C = 0.5 \text{ }\mu\text{F}, R_1 = 0.68 \text{ k}\Omega, R_2 = 1.8 \text{ k}\Omega.$$

Find:

The differential equation for $t > 0$ (switch closed) for the circuit of P5.23.

Analysis:

Apply KCL at the top node (nodal analysis) to write the circuit equation. Note that the top node voltage is the capacitor voltage, v_C .

$$i_C + \frac{v_C}{R_2} + \frac{v_C - V_1}{R_1} = 0$$

Next, use the definition of capacitor current to eliminate the variable i_C from the nodal equation:

$$C \frac{dv_C}{dt} + \frac{R_1 + R_2}{R_1 R_2} v_C = \frac{V_1}{R_1} \Rightarrow \frac{dv_C}{dt} + \frac{R_1 + R_2}{C(R_1 R_2)} v_C = \frac{V_1}{C R_1}$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{dv_C}{dt} + (4052)v_C - 35292 = 0$$

Problem 5.3

Solution:

Known quantities:

$$V_1 = 12\text{ V}, R_1 = 0.68\text{ k}\Omega, R_2 = 2.2\text{ k}\Omega, R_3 = 1.8\text{ k}\Omega, C = 0.47\text{ }\mu\text{F}.$$

Find:

The differential equation for $t > 0$ (switch closed) for the circuit of P5.27.

Analysis:

Apply KCL at the two node (nodal analysis) to write the circuit equation. Note that the node #1 voltage is the capacitor voltage, v_C .

For node #1:

$$i_C + \frac{v_C - v_2}{R_2} = 0$$

For node #2:

$$\frac{v_2 - v_C}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - V_1}{R_1} = 0$$

Solving the system:

$$v_C = \frac{R_3}{R_1 + R_3} V_1 - \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 + R_3} i_C$$

$$v_2 = \frac{R_3}{R_1 + R_3} V_1 - \frac{R_1 R_3}{R_1 + R_3} i_C$$

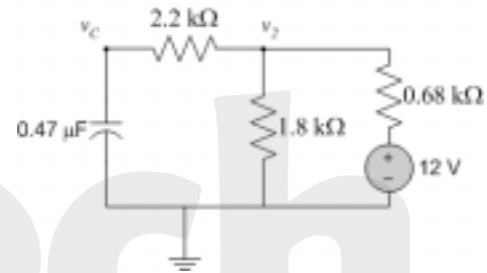
Next, use the definition of capacitor current to eliminate the variable i_C from the nodal equation:

$$\frac{C(R_1 R_2 + R_1 R_3 + R_2 R_3)}{R_1 + R_3} \frac{dv_C}{dt} + v_C = \frac{R_3}{R_1 + R_3} V_1$$

$$\frac{dv_C}{dt} + \frac{R_1 + R_3}{C(R_1 R_2 + R_1 R_3 + R_2 R_3)} v_C = \frac{R_3}{C(R_1 R_2 + R_1 R_3 + R_2 R_3)} V_1$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{dv_C}{dt} + (790)v_C - 6876 = 0$$



Problem 5.4

Solution:

Known quantities:

$$V_{S2} = 13\text{ V}, L = 170\text{ mH}, R_2 = 4.3\text{ k}\Omega, R_3 = 29\text{ k}\Omega.$$

Find:

The differential equation for $t > 0$ (switch open) for the circuit of P5.29.

Analysis:

Applying KVL we obtain:

$$(R_2 + R_3)i_L + v_L + V_{S2} = 0$$

Next, using the definition of inductor voltage to eliminate the variable v_L from the nodal equation:

$$(R_2 + R_3)i_L + L \frac{di_L}{dt} + V_{S2} = 0$$

$$\frac{di_L}{dt} + \frac{(R_2 + R_3)}{L}i_L + \frac{V_{S2}}{L} = 0$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{di_L}{dt} + 1.96 \cdot 10^5 i_L + 76.5 = 0$$

Problem 5.5**Solution:****Known quantities:**

$$I_0 = 17 \text{ mA}, C = 0.55 \mu\text{F}, R_1 = 7 \text{ k}\Omega, R_2 = 3.3 \text{ k}\Omega.$$

Find:

The differential equation for $t > 0$ for the circuit of P5.32.

Analysis:

Using the definition of capacitor current:

$$C \frac{dv_C}{dt} = I_0 \Rightarrow \frac{dv_C}{dt} = \frac{I_0}{C}$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{dv_C}{dt} - 30909 = 0$$

Problem 5.6**Solution:****Known quantities:**

$$V_{S1} = V_{S2} = 11 \text{ V}, C = 70 \text{ nF}, R_1 = 14 \text{ k}\Omega, R_2 = 13 \text{ k}\Omega, R_3 = 14 \text{ k}\Omega.$$

Find:

The differential equation for $t > 0$ (switch closed) for the circuit of P5.34.

Analysis:

Apply KCL at the top node (nodal analysis) to write the circuit equation.

$$\frac{v_1 - V_{S2}}{R_1} + i_C + \frac{v_1}{R_3} = 0$$

Note that the node voltage v_1 is equal to:

$$v_1 = R_2 i_C + v_C$$

Substitute the node voltage v_1 in the first equation:

$$\left(\frac{1}{R_1} + \frac{1}{R_3}\right)v_C + \left(\frac{R_2}{R_1} + \frac{R_2}{R_3} + 1\right)i_C - \frac{V_{s2}}{R_1} = 0$$

Next, use the definition of capacitor current to eliminate the variable i_C from the nodal equation:

$$\left(\frac{1}{R_1} + \frac{1}{R_3}\right)v_C + \left(\frac{R_2}{R_1} + \frac{R_2}{R_3} + 1\right)C \frac{dv_C}{dt} - \frac{V_{s2}}{R_1} = 0$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{dv_C}{dt} + 714.3v_C - 3929 = 0$$

Problem 5.7

Solution:

Known quantities:

$$V_S = 20\text{ V}, R_1 = 5\ \Omega, R_2 = 4\ \Omega, R_3 = 3\ \Omega, R_4 = 6\ \Omega, C_1 = 4\text{ F}, C_2 = 4\text{ F}, I_S = 4\text{ A}$$

Find:

The differential equation for $t > 0$ (switch closed) for the circuit of P5.41.

Analysis:

Apply KCL at the two node (nodal analysis) to write the circuit equation. Note that the node #1 voltage is equal to the two capacitor voltages, $v_{C1} = v_{C2} = v_C$.

For node #1:

$$i_{C1} + i_{C2} + \frac{v_C - v_2}{R_2} = 0$$

For node #2:

$$\frac{v_2 - v_C}{R_2} + \frac{v_2}{R_3} + \frac{v_2}{R_4} - I_S = 0$$

Solving the system:

$$v_2 = \frac{R_3 R_4}{R_3 + R_4} (i_{C1} + i_{C2} - I_S)$$

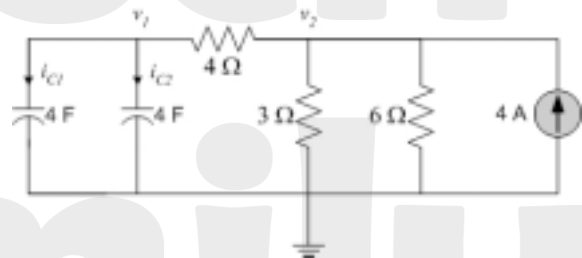
$$v_C = -\left(R_2 + \frac{R_3 R_4}{R_3 + R_4}\right)(i_{C1} + i_{C2}) + \frac{R_3 R_4}{R_3 + R_4} I_S$$

Next, use the definition of capacitor current to eliminate the variables i_{Ci} from the nodal equation:

$$v_C + \left(R_2 + \frac{R_3 R_4}{R_3 + R_4}\right)(C_1 + C_2) \frac{dv_C}{dt} - \frac{R_3 R_4}{R_3 + R_4} I_S = 0$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{dv_C}{dt} + \frac{1}{48}v_C - \frac{1}{6} = 0$$



Problem 5.8**Solution:****Known quantities:**

$$C = 1\mu F, R_s = 15k\Omega, R_3 = 30k\Omega.$$

Find:

The differential equation for $t > 0$ (switch closed) for the circuit of P5.47.

Assume:

Assume that $V_s = 9V$, $R_1 = 10k\Omega$ and $R_2 = 20k\Omega$.

Analysis:

Apply KCL at the top node (nodal analysis) to write the circuit equation.

- Before the switch opens. Apply KCL at the top node (nodal analysis) to write the circuit equation.

$$\frac{v_C - V_s}{R_s} + \frac{v_C}{R_1} + i_C + \frac{v_C}{R_2} + \frac{v_C}{R_3} = 0 \Rightarrow \left(\frac{1}{R_s} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) v_C + i_C - \frac{V_s}{R_s} = 0$$

Next, use the definition of capacitor current to eliminate the variable i_C from the nodal equation:

$$\left(\frac{1}{R_s} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) v_C + C \frac{dv_C}{dt} - \frac{V_s}{R_s} = 0$$

Substituting numerical value, we obtain the following differential equation:

$$\frac{dv_C}{dt} + 250v_C - 600 = 0$$

- After the switch opens. Apply KCL at the top node (nodal analysis) to write the circuit equation.

$$\frac{v_C - V_s}{R_s} + \frac{v_C}{R_1} + i_C = 0 \Rightarrow \left(\frac{1}{R_s} + \frac{1}{R_1} \right) v_C + i_C - \frac{V_s}{R_s} = 0$$

Next, use the definition of capacitor current to eliminate the variable i_C from the nodal equation:

$$\left(\frac{1}{R_s} + \frac{1}{R_1} \right) v_C + C \frac{dv_C}{dt} - \frac{V_s}{R_s} = 0$$

Substituting numerical values, we obtain the following differential equation:

$$\frac{dv_C}{dt} + \frac{500}{3} v_C - 600 = 0$$

Problem 5.9**Solution:****Known quantities:**

Values of the voltage source, of the inductance and of the resistors.

Find:

The differential equation for $t > 0$ (switch open) for the circuit of P5.49.

Analysis:

Apply KCL at the top node (nodal analysis) to write the circuit equation.

$$\frac{v_1 - 100}{10} + \frac{v_1}{5} + i_L = 0 \Rightarrow 0.3v_1 + i_L - 10 = 0$$

Note that the node voltage v_1 is equal to:

$$v_1 = 2.5i_L + v_L$$

Substitute the node voltage v_1 in the first equation:

$$(1.75)i_L + (0.3)v_L - 10 = 0$$

Next, use the definition of inductor voltage to eliminate the variable v_L from the nodal equation:

$$(0.3)(0.1)\frac{di_L}{dt} + (1.75)i_L - 10 = 0 \Rightarrow \frac{di_L}{dt} + (58.33)i_L - 333 = 0$$

Problem 5.10

Solution:

Known quantities:

$$I_s = 5 \text{ A}, L_1 = 1 \text{ H}, L_2 = 5 \text{ H}, R = 10 \text{ k}\Omega.$$

Find:

The differential equation for $t > 0$ (switch open) for the circuit of P5.52.

Analysis:

Applying KCL at the top node (nodal analysis) to write the circuit equation.

$$-I_s + \frac{v_1}{R} + i_L = 0$$

Note that the node voltage v_1 is equal to:

$$v_1 = v_{L1} + v_{L2}$$

Substitute the node voltage v_1 in the first equation:

$$\frac{v_{L1} + v_{L2}}{R} + i_L - I_s = 0$$

Next, use the definition of inductor voltage to eliminate the variable v_L from the nodal equation:

$$\frac{(L_1 + L_2)}{R} \frac{di_L}{dt} + i_L - I_s = 0$$

Substituting numerical value, we obtain the following differential equation:

$$\frac{di_L}{dt} + \frac{5000}{3} i_L - \frac{25000}{3} = 0$$

Section 5.3: DC Steady-State Solution of Circuits Containing Inductors and Capacitors - Initial and Final Conditions

Problem 5.11

Solution:

Known quantities:

$$L = 0.9 \text{ mH}, V_s = 12 \text{ V}, R_1 = 6 \text{ k}\Omega, R_2 = 6 \text{ k}\Omega, R_3 = 3 \text{ k}\Omega.$$

Find:

The initial and final conditions for the circuit of P5.21.

Analysis:

Before opening, the switch has been closed for a long time. Thus we have a steady-state condition, and we treat the inductor as a short circuit. The voltages across the resistances R_1 and R_3 is equal to zero, since they are in parallel to the short circuit, so all the current flow through the resistor R_2 :

$$i_L(0) = \frac{V_s}{R_2} = 2 \text{ mA}.$$

After the switch has been opened for a long time, we have again a steady-state condition, and we treat the inductor as a short circuit. When the switch is open, the voltage source is not connected to the circuit. Thus,

$$i_L(\infty) = 0 \text{ A}.$$

Problem 5.12

Solution:

Known quantities:

$$V_1 = 12 \text{ V}, C = 0.5 \text{ }\mu\text{F}, R_1 = 0.68 \text{ k}\Omega, R_2 = 1.8 \text{ k}\Omega.$$

Find:

The initial and final conditions for the circuit of P5.23.

Analysis:

Before closing, the switch has been opened for a long time. Thus we have a steady-state condition, and we treat the capacitor as an open circuit. When the switch is open, the voltage source is not connected to the circuit. Thus,

$$v_C(0) = 0 \text{ V}.$$

After the switch has been closed for a long time, we have again a steady-state condition, and we treat the capacitor as an open circuit. The voltage across the capacitor is equal to the voltage across the resistance R_2 :

$$v_C(\infty) = \frac{R_2}{R_1 + R_2} V_1 = \frac{1800}{1800 + 680} 12 = 8.71 \text{ V}.$$

Problem 5.13**Solution:****Known quantities:**

$$V_1 = 12 \text{ V}, R_1 = 0.68 \text{ k}\Omega, R_2 = 2.2 \text{ k}\Omega, R_3 = 1.8 \text{ k}\Omega, C = 0.47 \text{ }\mu\text{F}.$$

Find:

The initial and final conditions for the circuit of P5.27.

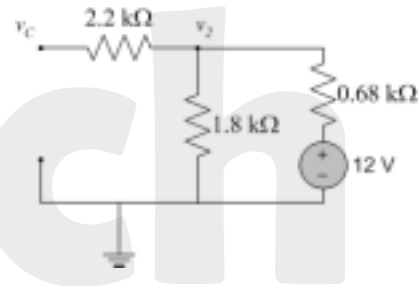
Analysis:

Before closing, the switch has been opened for a long time. Thus we have a steady-state condition, and we treat the capacitor as an open circuit. When the switch is open, the voltage source is not connected to the circuit. Thus,

$$v_C(0) = 0 \text{ V}.$$

After the switch has been closed for a long time, we have again a steady-state condition, and we treat the capacitor as an open circuit. Since the current flowing through the resistance R_2 is equal to zero, the voltage across the capacitor is equal to the voltage across the resistance R_3 :

$$v_C(\infty) = \frac{R_3}{R_1 + R_3} V_1 = \frac{1800}{1800 + 680} 12 = 8.71 \text{ V}.$$

**Problem 5.14****Solution:****Known quantities:**

$$V_{S1} = V_{S2} = 13 \text{ V}, L = 170 \text{ mH}, R_2 = 4.3 \text{ k}\Omega, R_3 = 29 \text{ k}\Omega.$$

Find:

The initial and final conditions for the circuit of P5.29.

Analysis:

In a steady-state condition we can treat the inductor as a short circuit. Before the switch changes, applying the KVL we obtain:

$$V_{S1} - V_{S2} = (R_1 + R_2) i_L(0) \Rightarrow i_L(0) = \frac{V_{S1} - V_{S2}}{R_1 + R_2} = 0 \text{ mA}.$$

After the switch has changed for a long time, we have again a steady-state condition, and we treat the inductor as a short circuit. Thus, applying the KVL we have:

$$i_L(\infty) = -\frac{V_{S2}}{R_2 + R_3} = -0.39 \text{ mA}.$$

Problem 5.15**Solution:****Known quantities:**

$$I_0 = 17 \text{ mA}, C = 0.55 \mu\text{F}, R_1 = 7 \text{ k}\Omega, R_2 = 3.3 \text{ k}\Omega.$$

Find:

The initial and final conditions for the circuit of P5.32.

Analysis:

Before the switch changes, the capacitor is not connected to the circuit, so we don't have any information about its initial voltage.

After the switch has changed, the current source and the capacitor will be in series so the current to the capacitor will be constant at I_0 . Therefore, the rate at which charge accumulates on the capacitor will also be constant and, consequently, the voltage across the capacitor will rise at a constant rate, without reaching any equilibrium state.

Problem 5.16**Solution:****Known quantities:**

$$V_{S1} = 17 \text{ V}, V_{S2} = 11 \text{ V}, C = 70 \text{ nF}, R_1 = 14 \text{ k}\Omega, R_2 = 13 \text{ k}\Omega, R_3 = 14 \text{ k}\Omega.$$

Find:

The initial and final conditions for the circuit of P5.34.

Analysis:

In a steady-state condition we can treat the capacitor as an open circuit. Before the switch changes, applying the KVL we have that the voltage across the capacitor is equal to the source voltage V_{S1} :

$$v_C(0) = V_{S1} = 17 \text{ V}.$$

After the switch has changed for a long time, we have again a steady-state condition, and we treat the capacitor as an open circuit. Since the current flowing through the resistance R_2 is equal to zero, the voltage across the capacitor is equal to the voltage across the resistance R_3 :

$$v_C(\infty) = \frac{R_3}{R_1 + R_3} V_{S2} = \frac{14000}{14000 + 14000} 11 = 5.5 \text{ V}.$$

Problem 5.17 – اللجنة الأكاديمية في قسم هندسة الميكانيك –**Solution:****Known quantities:**

$$V_S = 20 \text{ V}, R_1 = 5 \Omega, R_2 = 4 \Omega, R_3 = 3 \Omega, R_4 = 6 \Omega, C_1 = 4 \text{ F}, C_2 = 4 \text{ F}, I_S = 4 \text{ A}.$$

Find:

The initial and final conditions for the circuit of P5.41.

Analysis:

The switch S_1 is always open and the switch S_2 closes at $t = 0$. Before closing, the switch S_2 has been opened for a long time. Thus we have a steady-state condition, and we treat the capacitors as open circuits. When the switch is open, the current source is not connected to the circuit. Thus,

$$v_{C1}(0) = 0 \text{ V},$$

$$v_{C2}(0) = 0 \text{ V}.$$

After the switch S_2 has been closed for a long time, we have again a steady-state condition, and we treat the capacitors as open circuits. The voltages across the capacitors are both equal to the voltage across the resistance R_3 :

$$v_{C1}(\infty) = v_{C2}(\infty) = (R_3 \parallel R_4) I_S = \frac{6 \cdot 3}{6 + 3} 4 = 8 \text{ V}.$$

Problem 5.18

Solution:

Known quantities:

$$C = 1 \mu\text{F}, R_S = 15 \text{ k}\Omega, R_3 = 30 \text{ k}\Omega.$$

Find:

The initial and final conditions for the circuit of P5.47.

Assume:

$$\text{Assume that } V_S = 9 \text{ V}, R_1 = 10 \text{ k}\Omega \text{ and } R_2 = 20 \text{ k}\Omega.$$

Analysis:

Before opening, the switch has been closed for a long time. Thus, we have a steady-state condition, and we treat the capacitor as an open circuit. The voltage across the capacitor is equal to the voltage across the resistances R_1 , R_2 , and R_3 . Thus,

$$v_C(0) = \frac{R_1 \parallel (R_2 \parallel R_3)}{R_S + R_1 \parallel (R_2 \parallel R_3)} V_S = \frac{10 \text{ k} \parallel 12 \text{ k}}{15 \text{ k} + 10 \text{ k} \parallel 12 \text{ k}} 9 = 2.4 \text{ V}.$$

After opening, the switch has been opened for a long time. Thus we have a steady-state condition, and we treat the capacitor as an open circuit. The voltage across the capacitor is equal to the voltage across the resistance R_1 . Thus,

$$v_C(\infty) = \frac{R_1}{R_1 + R_S} V_S = \frac{10000}{10000 + 15000} 9 = 3.6 \text{ V}.$$

Problem 5.19

Solution:

Known quantities:

Values of the voltage source, of the inductance and of the resistors.

Find:

The initial and final conditions for the circuit of P5.49.

Analysis:

Before the switch changes, apply KCL at the top node (nodal analysis) to write the following circuit equation.

$$\frac{v_1 - 100}{1000} + \frac{v_1}{5} + \frac{v_1}{2.5} = 0 \Rightarrow v_1 = \frac{100}{601} = 0.165 \text{ V}.$$

$$i_L(0) = \frac{v_1}{2.5} = \frac{40}{601} = 66 \text{ mA.}$$

After the switch has changed, apply KCL at the top node (nodal analysis) to write the following circuit equation.

$$\frac{v_1 - 100}{10} + \frac{v_1}{5} + \frac{v_1}{2.5} = 0 \Rightarrow v_1 = \frac{100}{7} = 14.285 \text{ V.}$$

$$i_L(\infty) = \frac{v_1}{2.5} = \frac{40}{7} = 5.714 \text{ A.}$$

Problem 5.20

Solution:

Known quantities:

$$I_s = 5 \text{ A}, L_1 = 1 \text{ H}, L_2 = 5 \text{ H}, R = 10 \text{ k}\Omega.$$

Find:

The initial and final conditions for the circuit of P5.52.

Analysis:

Before closing, the switch has been opened for a long time. Thus we have a steady-state condition, and we treat the inductors as short circuits. The values of the two resistors are equal so the current flowing through the inductors is:

$$i_L(0) = \frac{I_s}{2} = 2.5 \text{ A.}$$

After the switch has been closed for a long time, we have again a steady-state condition, and we treat the inductors as short circuits. In this case the resistors are short-circuited and so all the current is flowing through the inductors.

$$i_L(\infty) = I_s = 5 \text{ A.}$$

Section 5.4: Transient Response of First-Order Circuits

Focus on Methodology

First-order transient response

1. Solve for the steady-state response of the circuit before the switch changes state ($t = 0^-$), and after the transient has died out ($t \rightarrow \infty$). We shall generally refer to these responses as $x(0^-)$ and $x(\infty)$.
2. Identify the initial condition for the circuit, $x(0^+)$, using continuity of capacitor voltages and inductor currents ($v_C(0^+) = v_C(0^-)$, $i_L(0^+) = i_L(0^-)$), as illustrated in Section 5.4.
3. Write the differential equation of the circuit for $t = 0^+$, that is, immediately after the switch has changed position. The variable $x(t)$ in the differential equation will be either a capacitor voltage, $v_C(t)$, or an inductor current, $i_L(t)$. It is helpful at this time to reduce the circuit to Thévenin or Norton equivalent form, with the energy storage element (capacitor or inductor) treated as the load for the Thévenin (Norton) equivalent circuit. Reduce this equation to standard form (Equation 5.8).
4. Solve for the time constant of the circuit: $\tau = R_T C$ for capacitive circuits, $\tau = L/R_T$ for inductive circuits.
5. Write the complete solution for the circuit in the form:

$$x(t) = x(\infty) + [x(0^+) - x(\infty)] e^{-t/\tau}$$

Problem 5.21

Solution:

Known quantities:

Circuit shown in Figure P5.21, $L = 0.9 \text{ mH}$, $V_s = 12 \text{ V}$, $R_1 = 6 \text{ k}\Omega$, $R_2 = 6 \text{ k}\Omega$, $R_3 = 3 \text{ k}\Omega$.

Find:

If the steady-state conditions exist just before the switch was opened.

Assumptions:

$i_L = 1.70 \text{ mA}$ before the switch is opened at $t = 0$.

Analysis:

Determine the steady state current through the inductor at $t < 0$. If this current is equal to the current specified, steady-state conditions did exist; otherwise, opening the switch interrupted a transient in progress. To determine the steady-state current before the switch was opened, replace the inductor with an equivalent DC short-circuit and compute the steady-state current through the short circuit.

At steady state, the inductor is modeled as a short circuit:

$$V_{R_3}(0^-) = 0 \quad i_{R_3}(0^-) = 0$$

Thus, the short-circuit current through the inductor is simply the current through R_2 which can be found by applying Kirchhoff's Laws and Ohm's Law.

Apply KVL:

$$-V_s + i_{R_3}(0^-)R_2 = 0 \quad i_{R_2}(0^-) = \frac{V_s}{R_2} = \frac{12}{6 \times 10^3} = 2 \text{ mA}$$

Apply KCL:

$$-i_{R_2}(0^-) + i_L(0^-) + i_{R_3}(0^-) = 0, \quad i_L(0^-) = i_{R_2}(0^-) = 2 \text{ mA}$$

The actual steady state current through the inductor is larger than the current specified. Therefore, the circuit is not in a steady state condition just before the switch is opened.

Problem 5.22

Solution:

Known quantities:

Circuit shown in Figure P5.22,

$$V_{S1} = 35V, V_{S2} = 130V, C = 11\mu F, R_1 = 17k\Omega, R_2 = 7k\Omega, R_3 = 23k\Omega.$$

Find:

At $t = 0^+$ the initial current through R_3 just after the switch is changed.

Assumptions:

None.

Analysis:

To solve this problem, find the steady state voltage across the capacitor before the switch is thrown. Since the voltage across a capacitor cannot change instantaneously, this voltage will also be the capacitor voltage immediately after the switch is thrown. At that instant, the capacitor may be viewed as a DC voltage source.

At $t = 0^-$:

Determine the voltage across the capacitor. At steady state, the capacitor is modeled as an open circuit:

$$i_{R1}(0^-) = i_{R2}(0^-) = 0$$

Apply KVL:

$$V_{S1} + 0 - V_C(0^-) + 0 - V_{S2} = 0$$

$$V_C(0^-) = V_{S1} - V_{S2} = -95V$$

At $t = 0^+$:

$$V_C(0^+) = V_C(0^-) = -95V$$

$$i_{R2}(0^+) = i_{R3}(0^+)$$

Apply KVL:

$$V_{S2} - i_{R3}(0^+)R_2 + V_C(0^+) - i_{R3}(0^+)R_3 = 0$$

$$i_{R3}(0^+) = \frac{V_{S2} + V_C(0^+)}{R_2 + R_3} = \frac{130 - 95}{7 \times 10^3 + 23 \times 10^3} = 1.167 \text{ mA}$$

Problem 5.23

Solution:

Known quantities:

Circuit shown in Figure P5.23, $V_1 = 12V, C = 0.5\mu F, R_1 = 0.68k\Omega, R_2 = 1.8k\Omega$.

Find:

The current through the capacitor just before and just after the switch is closed.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

At $t = 0^-$, assume steady state conditions exist. If charge is stored on the plates of the capacitor, then there will be energy stored in the electric field of the capacitor and a voltage across the capacitor. This will cause a current flow through R_2 which will dissipate energy until no energy is stored in the capacitor at which time the current ceases and the voltage across the capacitor is zero. These are the steady state conditions, i.e.:

$$i_C(0^-) = 0 \quad V_C(0^-) = 0$$

At $t = 0^+$, the switch is closed and the transient starts. Continuity requires:

$$V_C(0^+) = V_C(0^-) = 0$$

At this instant, treat the capacitor as a DC voltage source of strength zero i.e. a short-circuit. Therefore, all of the voltage V_1 is across the resistor R_1 and the resulting current through R_1 is the current into the capacitor.

Apply KCL: (Sum of the currents out of the top node)

$$i_C(0^+) + \frac{V_C(0^+) - 0}{R_2} + \frac{V_C(0^+) - V_1}{R_1} = 0$$

$$i_C(0^+) = \frac{V_1}{R_1} = \frac{12}{0.68 \times 10^3} = 17.65 \text{ mA}$$

The CURRENT through the capacitor is NOT continuous but changes from 0 to 17.65 mA when the switch is closed. The voltage across the capacitor is continuous because the stored energy CANNOT CHANGE INSTANTANEOUSLY.

Problem 5.24**Solution:****Known quantities:**

Circuit shown in Figure P5.23, $V_1 = 12 \text{ V}$, $C = 150 \mu\text{F}$, $R_1 = 400 \text{ m}\Omega$, $R_2 = 2.2 \text{ k}\Omega$.

Find:

The current through the capacitor just before and just after the switch is closed.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

At $t = 0^-$, assume steady state conditions exist. If charge is stored on the plates of the capacitor, then there will be energy stored in the electric field of the capacitor and a voltage across the capacitor. This will cause a current flow through R_2 which will dissipate energy until no energy is stored in the capacitor at which time the current ceases and the voltage across the capacitor is zero. These are the steady state conditions, i.e.:

$$i_C(0^-) = 0 \quad V_C(0^-) = 0$$

At $t = 0^+$, the switch is closed and the transient starts. Continuity requires:

$$V_C(0^+) = V_C(0^-) = 0$$

At this instant, treat the capacitor as a DC voltage source of strength zero i.e. a short-circuit. Therefore, all of the voltage V_1 is across the resistor R_1 and the resulting current through R_1 is the current into the capacitor.

Apply KCL: (Sum of the currents out of the top node)

$$i_C(0^+) + \frac{V_C(0^+) - 0}{R_2} + \frac{V_C(0^+) - V_1}{R_1} = 0$$

$$i_C(0^+) = \frac{V_1}{R_1} = \frac{12}{400 \times 10^{-3}} = 30 \text{ A}$$

The CURRENT through the capacitor is NOT continuous but changes from 0 to 30 A when the switch is closed. The voltage across the capacitor is continuous because the stored energy CANNOT CHANGE INSTANTANEOUSLY.

Problem 5.25

Solution:

Known quantities:

Circuit shown in Figure P5.21, $V_s = 12 \text{ V}$, $L = 0.9 \text{ mH}$, $R_1 = 6 \text{ k}\Omega$, $R_2 = 6 \text{ k}\Omega$, $R_3 = 3 \text{ k}\Omega$.

Find:

The voltage across R_3 just after the switch is open.

Assumptions:

$i_L = 1.70 \text{ mA}$ before the switch is opened at $t = 0$.

Analysis:

When the switch is opened the voltage source is disconnected from the circuit and plays no role. Since the current through the inductor cannot change instantaneously the current through the inductor at $t = 0^+$ is also 1.70 mA. At this instant, treat the inductor as a DC current source and solve for the voltage across R_3 by current division or KCL and Ohm's Law.

Specify the polarity of the voltage across R_3 .

$$i_L(0^+) = i_L(0^-) = 1.7 \text{ mA}$$

$$R_{eq} = R_1 + R_2 = 12 \text{ k}\Omega$$

Apply KCL: (Sum of the currents out of the top node)

$$\frac{V_{R3}(0^+)}{R_{eq}} + i_L(0^+) + \frac{V_{R3}(0^+)}{R_3} = 0$$

$$V_{R3}(0^+) = -\frac{i_L(0^+)}{\frac{1}{R_{eq}} + \frac{1}{R_3}} = -\frac{1.7 \times 10^{-3}}{\frac{1}{12 \times 10^3} + \frac{1}{3 \times 10^3}} = -4.080 \text{ V}$$

Problem 5.26

Solution:

Known quantities:

Circuit shown in Figure P5.26, $V_1 = 12 \text{ V}$, $R_s = 0.7 \text{ }\Omega$, $R_1 = 22 \text{ k}\Omega$, $L = 100 \text{ mH}$.

Find:

The voltage through the inductor just before and just after the switch is changed.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

In steady-state the inductor acts like a short-circuit so it has no voltage across it for $t < 0$. However, its current is non-zero and is equal to the current out of the source V_S and through R_S . At the instant the switch is changed the current through the inductor is unchanged since the current through an inductor cannot change instantaneously. Also notice that after the switch is changed the current through R_1 is always equal to the inductor current and the voltage across R_1 is always equal to the inductor voltage. Thus, at $t = 0^+$ the voltage across the inductor must be non-zero. That's fine since the voltage across an inductor can change instantaneously (or relatively so.)

Assume a polarity for the voltage across the inductor.

$t = 0^-$: Steady state conditions exist. The inductor can be modeled as a short circuit with:

$$V_L(0^-) = 0$$

Apply KVL;

$$-V_S + i_L(0^-)R_S + V_L(0^-) = 0$$

$$i_L(0^-) = \frac{V_S}{R_S} = \frac{12}{0.7} = 17.14 \text{ A}$$

At $t = 0^+$, the transient commences. Continuity requires:

$$i_L(0^+) = i_L(0^-)$$

Apply KVL:

$$i_L(0^+)R_1 + V_L(0^+) = 0$$

$$V_L(0^+) = -i_L(0^+)R_1 = -17.14 \times 22 \times 10^3 = -337.1 \text{ kV}$$

Problem 5.27**Solution:****Known quantities:**

Circuit shown in Figure P5.27, $V_1 = 12 \text{ V}$, $R_1 = 0.68 \text{ k}\Omega$, $R_2 = 2.2 \text{ k}\Omega$, $R_3 = 1.8 \text{ k}\Omega$, $C = 0.47 \mu\text{F}$.

Find:

The current through the capacitor at $t = 0^+$, just after the switch is closed.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

For $t < 0$, the switch is open and no power source is connected to the left half of the circuit. In steady state, by definition, the voltage across the capacitor and the current out of it must be constant. However, without a power source to replenish the energy dissipated by the resistors, that constant must be zero. Otherwise, current would flow out of the capacitor, its voltage would drop as it lost charge, and the energy of that charge would be dissipated by the resistors. This process would continue until no net charge remained on the capacitor and its voltage was zero. At steady state, then, the voltage across the capacitor is zero.

At $t = 0^+$, the voltage across the capacitor is still zero since the voltage across a capacitor cannot change instantaneously. At that instant, the capacitor can be treated as a voltage source of strength zero (i.e. a short-circuit.) However, the current through the capacitor can change instantaneously (or relatively so) from 0 to a new value. In this problem it will change as the switch is closed because the voltage source V_1 will drive current through R_1 and the parallel combination of R_2 and R_3 . The current through R_2 is the capacitor current.

$$V_C(0^+) = V_C(0^-) = 0$$

Apply KCL:

$$\frac{V_{R3}(0^+) - 0}{R_2} + \frac{V_{R3}(0^+)}{R_3} + \frac{V_{R3}(0^+) - V_1}{R_1} = 0$$

$$V_{R3}(0^+) = \frac{\frac{V_1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{V_1}{1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}} = \frac{12}{1 + \frac{0.68}{2.2} + \frac{0.68}{1.8}} = 7.114V$$

Recall that the voltage across the capacitor (Volts = Joules/Coulomb) represents the energy stored in the electric field between the plates of the capacitor. The electric field is due to the amount of charge stored in the capacitor and it is not possible to instantaneously remove charge from the capacitor's plates. Therefore, the voltage across the capacitor cannot change instantaneously when the circuit is switched.

However, the **rate** at which charge is removed from the plates of the capacitor (i.e. the capacitor current) **can** change instantaneously (or relatively so) when the circuit is switched.

Note also that these conditions hold only at the instant $t = 0^+$. For $t > 0^+$, the capacitor is gaining charge, all voltages and currents exponentially approach their final or steady state values.

Apply KVL:

$$-V_C(0^+) + i_C(0^+)R_2 + V_{R3}(0^+) = 0$$

$$i_C(0^+) = \frac{V_{R3}(0^+) - V_C(0^+)}{R_2} = \frac{7.114}{2.2 \times 10^3} = 3.234mA$$

Problem 5.28

Solution:

Known quantities:

NOTE: Typo in problem statement: should read "At $t < 0$, ..."

Circuit shown in Figure P5.22,

$$V_{S1} = 35V, V_{S2} = 130V, C = 11\mu F, R_1 = 17k\Omega, R_2 = 7k\Omega, R_3 = 23k\Omega.$$

Find:

The time constant of the circuit for $t > 0$.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

For $t > 0$, the transient is in progress. The time constant is a relative measure of the rate at which the voltages and currents are changing in the transient phase. The time constant for a single capacitor system is $R_{eq}C$ where R_{eq} is the Thévenin equivalent resistance as seen by the capacitor, i.e., with respect to the port or terminals of the capacitor. To calculate R_{eq} turn off (set to zero) the ideal independent voltage source and ask the question "What is the net equivalent resistance encountered in going from one terminal of the capacitor to the other through the network?" Then:

$$R_{eq} = R_2 + R_3 = 7 \times 10^3 + 23 \times 10^3 = 30 \text{ k}\Omega$$

$$\tau = R_{eq}C = 30 \times 10^3 \times 11 \times 10^{-6} = 330.0 \text{ ms}$$

Problem 5.29

Solution:

Known quantities:

Circuit shown in Figure P5.29,

$$V_{S1} = 13 \text{ V}, V_{S2} = 13 \text{ V}, L = 170 \text{ mH}, R_1 = 2.7 \text{ k}\Omega, R_2 = 4.3 \text{ k}\Omega, R_3 = 29 \text{ k}\Omega.$$

Find:

The time constant of the circuit for $t > 0$.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

For $t > 0$, the transient is in progress. The time constant is a relative measure of the rate at which the voltages and currents are changing in the transient phase. The time constant for a single inductor system is L/R_{eq} where R_{eq} is the Thévenin equivalent resistance as seen by the inductor, i.e., with respect to the port or terminals of the inductor. To calculate R_{eq} turn off (set to zero) the ideal independent voltage source and ask the question “What is the net equivalent resistance encountered in going from one terminal of the inductor to the other through the network?” Then:

$$R_{eq} = R_2 + R_3 = 4.3 \times 10^3 + 29 \times 10^3 = 33.30 \text{ k}\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{170 \times 10^{-3}}{33.30 \times 10^3} = 5.105 \mu\text{s}$$

Problem 5.30

Solution:

Known quantities:

Circuit shown in Figure P5.27, $V_1 = 12 \text{ V}, C = 0.47 \mu\text{F}, R_1 = 680 \Omega, R_2 = 2.2 \text{ k}\Omega, R_3 = 1.8 \text{ k}\Omega$.

Find:

The time constant of the circuit for $t > 0$.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

For $t > 0$, the transient is in progress. The time constant is a relative measure of the rate at which the voltages and currents are changing in the transient phase. The time constant for a single capacitor system is $R_{eq}C$ where R_{eq} is the Thévenin equivalent resistance as seen by the capacitor, i.e., with respect to the port or terminals of the capacitor. To calculate R_{eq} turn off (set to zero) the ideal independent voltage source and ask the question “What is the net equivalent resistance encountered in going from one terminal of the capacitor to the other through the network?” Then:

$$R_{eq} = R_2 + \frac{R_3 R_1}{R_3 + R_1} = 2.2 \times 10^3 + \frac{1.8 \times 10^3 \times 0.68 \times 10^3}{1.8 \times 10^3 + 0.68 \times 10^3} = 2.694 \text{ k}\Omega$$

$$\tau = R_{eq}C = 2.694 \times 10^3 \times 0.47 \times 10^{-6} = 1.266 \text{ ms}$$

Problem 5.31**Solution:****Known quantities:**

Circuit shown in Figure P5.21, $V_S = 12\text{ V}$, $L = 0.9\text{ mH}$, $R_1 = 6\text{ k}\Omega$, $R_2 = 6\text{ k}\Omega$, $R_3 = 3\text{ k}\Omega$.

Find:

The time constant of the circuit for $t > 0$.

Assumptions:

The current through the inductor is $i_L = 1.70\text{ mA}$ before the switch is opened at $t = 0$.

Analysis:

For $t > 0$, the transient is in progress. The time constant is a relative measure of the rate at which the voltages and currents are changing in the transient phase. The time constant for a single inductor system is L/R_{eq} where R_{eq} is the Thévenin equivalent resistance as seen by the inductor, i.e., with respect to the port or terminals of the inductor. To calculate R_{eq} turn off (set to zero) the ideal independent voltage source and ask the question “What is the net equivalent resistance encountered in going from one terminal of the inductor to the other through the network?” Then:

$$R_{eq} = \frac{R_3(R_1 + R_2)}{R_3 + (R_1 + R_2)} = \frac{3 \times 10^3 (6 \times 10^3 + 6 \times 10^3)}{3 \times 10^3 + (6 \times 10^3 + 6 \times 10^3)} = 2.400\text{ k}\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{0.9 \times 10^{-3}}{2.4 \times 10^3} = 0.3750\text{ }\mu\text{s}$$

Problem 5.32**Solution:****Known quantities:**

Circuit shown in Figure P5.32, $V_c(0^-) = -7\text{ V}$, $I_0 = 17\text{ mA}$, $C = 0.55\text{ }\mu\text{F}$, $R_1 = 7\text{ k}\Omega$, $R_2 = 3.3\text{ k}\Omega$.

Find:

The voltage $V_c(t)$ across the capacitor for $t > 0$.

Assumptions:

Before the switch is thrown the voltage across the capacitor is -7 V .

Analysis:

The current source and the capacitor will be in series so the current to the capacitor will be constant at I_0 . Therefore, the rate at which charge accumulates on the capacitor will also be constant and, consequently, the voltage across the capacitor will rise at a constant rate. The integral form of the capacitor i-V relationship best expresses this accumulation process. The continuity of the voltage across the capacitor requires:

$$\begin{aligned}
 V_C(0^+) &= V_C(0^-) = -7V \\
 i_C(t) &= I_0 = 17mA \\
 V_C(t) &= \frac{1}{C} \int_{-\infty}^t i_C(t) dt = \frac{1}{C} \left(\int_{-\infty}^0 i_C(t) dt + \int_0^t I_0 dt \right) \\
 &= V_C(0^+) + \frac{I_0}{C} \int_0^t dt = V_C(0^+) + \frac{I_0}{C} t \Big|_0^t \\
 &= -7 + \frac{17 \times 10^{-3}}{0.55 \times 10^{-6}} t = -7 + 30.91 \times 10^3 t
 \end{aligned}$$

Problem 5.33

Solution:

Known quantities:

Circuit shown in Figure P5.29,

$$V_{S1} = 23V, V_{S2} = 20V, L = 23mH, R_1 = 0.7\Omega, R_2 = 13k\Omega, R_3 = 330k\Omega.$$

Find:

The current $i_{R3}(t)$ through resistor R_3 for $t > 0$.

Assumptions:

The circuit is in steady-state conditions for $t < 0$. It is a resistive circuit with one storage element (e.g. inductor) so an assumed solution is of the form

$$i_{R3}(t) = I_{SS} + (I_0 - I_{SS})e^{-t/\tau}$$

Analysis:

The approach here is to first find the initial condition at $t = 0^+$ for the inductor and use it to determine the initial condition on the current through resistor R_3 . Second, find the final steady-state condition of the circuit for $t > 0$. To do so, simply apply DC circuit analysis to solve for the current through resistor R_3 i.e. replace the inductor with a short-circuit. Finally, solve for the time constant of the circuit for $t > 0$. Each of these three results is needed to construct the complete transient solution.

At $t = 0^-$:

Assume steady state conditions exist. At steady state, the inductor is modeled as a short-circuit:

Apply KVL:

$$-V_{S2} + i_L(0^-)R_1 + i_L(0^-)R_2 + V_{S1} = 0$$

$$i_L(0^-) = \frac{V_{S2} - V_{S1}}{R_1 + R_2} = \frac{20 - 23}{0.7 + 13} = -0.22A$$

This current is flowing in the direction from the inductor to the switch.

Find I_0 at $t = 0^+$:

Continuity of the current through the inductor requires that:

$$i_L(0^+) = i_L(0^-) = -0.22A$$

$$I_0 = i_{R3}(0^+) = i_L(0^+) = -0.22A$$

Find I_{SS} at $t = \text{infinity}$:

Assume that enough time has elapsed for steady state conditions to return. In steady state the inductor is modeled as a short circuit; therefore, the voltage across the inductor is zero. The result is a simple series

connection of resistors R_2 and R_3 . The current across R_3 is found directly from Ohm's Law in this case.

$$I_{SS} = i_{R_3} = \frac{V_{S2}}{R_2 + R_3} = \frac{20 \text{ V}}{343 \Omega} \cong 58 \text{ mA}$$

Find τ for $t > 0$:

To find the time constant t one first needs to determine the Thevenin equivalent resistance R_{TH} across the terminals of the inductor. To do so, set all independent ideal sources to zero and determine the equivalent resistance "seen" by the inductor, i.e. with respect to the port or terminals of the inductor:

$$R_{eq} = R_2 + R_3 = 13 + 330 \times 10^3 = 330.0 \text{ k}\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{23 \times 10^{-3}}{330.0 \times 10^3} = 69.70 \text{ ns}$$

The complete response can now be written using the solution for transient voltages and currents in first-order circuits:

$$\begin{aligned} i_{R_3}(t) &= i_{R_3}(\infty) + (i_{R_3}(0^+) - i_{R_3}(\infty))e^{-t/\tau} \\ &= 0.058 + (-0.22 - 0.058)e^{-t/69.70 \times 10^{-9}} \\ &= 0.058 - 0.28 e^{-t/69.70 \times 10^{-9}} \text{ A} \end{aligned}$$

Problem 5.34

Solution:

Known quantities:

Circuit shown in Figure P5.34,

$$V_{S1} = 17 \text{ V}, V_{S2} = 11 \text{ V}, R_1 = 14 \text{ k}\Omega, R_2 = 13 \text{ k}\Omega, R_3 = 14 \text{ k}\Omega, C = 70 \text{ nF}.$$

Find:

- $V(t)$ for $t > 0$.
- The time for $V(t)$ to change by 98% of its total change in voltage after the switch is operated.

Assumptions:

The circuit is in steady-state conditions for $t < 0$. It is a resistive circuit with one storage element (e.g. capacitor) so an assumed solution is of the form

$$V_{R_3}(t) = V_{SS} + (V_0 - V_{SS})e^{-t/\tau}$$

Analysis:

In general, the approach in problems such as this one is to first find the initial condition at $t = 0^+$ for the capacitor and use it to determine the initial condition on the voltage across resistor R_3 . Second, find the final steady-state condition of the circuit for $t > 0$. To do so, simply apply DC circuit analysis to solve for the voltage across the resistor R_3 (i.e. replace the capacitor with an open-circuit and solve.) Finally, solve for the time constant of the circuit for $t > 0$ by finding the Thevenin equivalent resistance R_{TH} across the terminals of the capacitor. Each of these three results is needed to construct the complete transient solution.

a) At $t = 0^-$:

Steady state conditions are specified. At steady state, the capacitor is modeled as an open circuit:

$$i_C(0^-) = 0$$

Apply KVL:

$$V_{S1} = i_C(0^-)R_2 + V_C(0^-)$$

$$V_C(0^-) = V_{S1} = 17V$$

At $t = 0^+$:

The voltage across the capacitor remains the same:

$$V_C(0^+) = V_C(0^-) = 17V$$

Apply KCL:

$$\begin{aligned} \frac{V(0^+) - (V_{S2})}{R_1} + \frac{V(0^+) - V_C(0^+)}{R_2} + \frac{V(0^+) - 0}{R_3} &= 0 \\ V(0^+) &= \frac{\frac{V_C(0^+)}{R_2} + \frac{V_{S2}}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \\ &= \frac{\frac{17}{13 \times 10^3} + \frac{11}{14 \times 10^3}}{\frac{1}{14 \times 10^3} + \frac{1}{13 \times 10^3} + \frac{1}{14 \times 10^3}} = 9.525V \end{aligned}$$

At $t > 0$:

Determine the equivalent resistance as "seen" by the capacitor, ie, with respect to the port or terminals of the capacitor. Suppress the independent ideal voltage source:

$$\begin{aligned} R_{eq} &= R_2 + (R_1 \parallel R_3) = R_2 + \frac{R_1 R_3}{R_1 + R_3} = 13 \times 10^3 + \frac{14 \times 10^3 \times 14 \times 10^3}{14 \times 10^3 + 14 \times 10^3} \\ &= 20.00 k\Omega \end{aligned}$$

$$\tau = R_{eq} C = 20 \times 10^3 \times 70 \times 10^{-9} = 1.400 ms$$

At $t = \text{infinity}$:

Steady state is again established. At steady state the capacitor is once again modeled as an open circuit:

$$i_C(\infty) = 0$$

Since the current through the capacitor branch is zero in steady state the voltage across R_3 may be found quickly by voltage division

$$\begin{aligned} V(\infty) &= \frac{V_{S2} R_3}{R_3 + R_1} \\ &= \frac{11 \times 14 \times 10^3}{14 \times 10^3 + 14 \times 10^3} = 5.500V \end{aligned}$$

The complete response for $t > 0$ is then:

$$\begin{aligned}
 V(t) &= V(\infty) + (V(0^+) - V(\infty))e^{-t/\tau} \\
 &= 5.5 + (9.525 - 5.5)e^{-t/1.4 \times 10^{-3}} \\
 &= 5.5 + 4.025e^{-t/1.4 \times 10^{-3}} \text{ V}
 \end{aligned}$$

b)

The time required for $V(t)$ to reach 98% of its final value is found from

$$0.98 = \frac{V(t) - V(0^+)}{V(\infty) - V(0^+)}$$

or

$$\begin{aligned}
 \Delta V &= V(\infty) - V(0^+) = 5.5 - (9.525) \\
 &= -4.025 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 V(t_1) &= V(0^+) + 0.98\Delta V \\
 &= 9.525 + 0.98 \times (-4.025) \\
 &= 5.5805 = 5.5 + 4.025e^{-t_1/1.4 \times 10^{-3}}
 \end{aligned}$$

$$t_1 = -1.4 \times 10^{-3} \ln\left(\frac{5.5805 - 5.5}{4.025}\right) = 5.477 \text{ ms}$$

Problem 5.35

Solution:

Known quantities:

Circuit shown in Figure P5.35, $V_G = 12 \text{ V}$, $R_G = 0.37 \Omega$, $R = 1.7 \text{ k}\Omega$.

Find:

The value of L and R_1 .

Assumptions:

The voltage across the spark plug gap V_R just after the switch is changed is 23 kV and the voltage will change exponentially with a time constant $\tau = 13 \text{ ms}$.

Analysis:

At $t = 0^-$:

Assume steady state conditions exist. At steady state the inductor is modeled as a short circuit:

$$V_L(0^-) = 0$$

The current through the inductor at this point is given directly by Ohm's Law:

$$i_L(0^-) = \frac{V_G}{R_G + R_1}$$

At $t = 0^+$:

Continuity of the current through the inductor requires that:

$$\begin{aligned}
 i_L(0^+) &= i_L(0^-) = \frac{V_G}{R_G + R_1} \\
 V_R(0^+) &= -i_L(0^+)R = -\frac{V_G R}{R_G + R_1} \\
 R_1 &= -\frac{V_G R}{V_R(0^+)} - R_G \\
 &= -\frac{12 \times 1.7 \times 10^3}{-23 \times 10^3} - 0.37 = 0.5170 \Omega
 \end{aligned}$$

Note that the voltage across the gap V_R was written as -23 kV since the current from the inductor flows opposite to the polarity shown for V_R ; that is, the actual polarity of the voltage across R is opposite that shown.

At $t > 0$:

Determine the Thevenin equivalent resistance as "seen" by the inductor, ie, with respect to the port or terminals of the inductor:

$$R_{eq} = R_1 + R$$

$$\tau = \frac{L}{R_{eq}} = \frac{L}{R_1 + R}$$

$$L = \tau(R_1 + R) = 13 \times 10^{-3} \times (0.5170 + 1.7 \times 10^3) = 22.11 H$$

Problem 5.36

Solution:

Known quantities:

Circuit shown in Figure P5.36, when $i_L \geq +2$ mA, the relay functions.

$$V_S = 12 V, L = 10.9 mH, R_1 = 3.1 k\Omega.$$

Find:

R_2 so that the relay functions at $t = 2.3$ s.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

In this problem the current through the inductor is clearly zero before the switch is thrown. The task is determine the value of the resistance R_2 such that the current through the inductor will need 2.3 seconds to rise to 2 mA. Once again, we must find the complete transient solution, this time for the current through the inductor. Assume a solution of the form

$$i_L(t) = i_\infty + (i_0 - i_\infty)e^{-t/\tau}$$

At $t = 0^-$:

The current through the inductor is zero since no source is connected.

$$i_L(0^-) = 0$$

At $t = 0^+$:

$$i_0 = i_L(0^+) = i_L(0^-) = 0$$

At $t > 0$:

Determine the Thevenin equivalent resistance as "seen" by the inductor, i.e., with respect to the port or terminals of the inductor:

$$R_{TH} = (R_1 \parallel R_2) = \frac{R_1 R_2}{R_1 + R_2}$$

And so

$$\tau = \frac{L}{R_{TH}} = \frac{L(R_1 + R_2)}{R_1 R_2}$$

At $t = \text{infinity}$:

Steady state is again established. At steady state the inductor is again modeled as a short circuit. Thus, the current through R_2 is zero and the current through the inductor is given by

$$i_\infty = i_L(\infty) = \frac{V_s}{R_1} = \frac{12}{3.1 \times 10^3} = 3.87 \text{ mA}$$

Plug in the above quantities to the complete solution and set the current through the inductor equal to 2 mA and the time equal to 2.3 s.

$$2 \times 10^{-3} = 3.87 \times 10^{-3} (1 - e^{-2.3/\tau})$$

or

$$\frac{-2.3}{\tau} = \ln \left[1 - \frac{2}{3.87} \right]$$

or

$$\tau \cong 3.16 \text{ seconds}$$

Solving for R_2 :

$$3.16 = \frac{L(R_1 + R_2)}{R_1 R_2}$$

or

$$R_2 = \frac{LR_1}{3.16R_1 - L} \cong 3.4 \text{ m}\Omega$$

Problem 5.37

Solution:

Known quantities:

Circuit shown in Figure P5.37, $V_1 = 12 \text{ V}$, $R_1 = 400 \text{ m}\Omega$, $R_2 = 2.2 \text{ k}\Omega$, $C = 150 \mu\text{F}$.

Find:

The current through the capacitor just before and just after the switch is closed.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

At $t = 0^-$, assume steady state conditions exist. If charge is stored on the plates of the capacitor, then there will be energy stored in the electric field of the capacitor and a voltage across the capacitor. This will cause a current flow through R_2 which will dissipate energy until no energy is stored in the capacitor at which time the current ceases and the voltage across the capacitor is zero. Thus, in this circuit, and others like it that contain no connected sources prior to the switch being thrown, the steady state condition is zero currents and zero voltages.

$$i_C(0^-) = 0 \quad V_C(0^-) = 0$$

At $t = 0^+$, the switch is closed; the transient starts. Continuity requires:

$$V_C(0^+) = V_C(0^-) = 0$$

Since the voltage across the capacitor is zero at this instant, the voltage across the resistor R_2 is also zero at this instant which implies that no current passes through the resistor at $t = 0^+$. Therefore, at $t = 0^+$, the current out of the source must be the current into the capacitor.

$$i_C(0^+) = \frac{V_1}{R_1} = \frac{12}{0.4} = 30 \text{ A}$$

The CURRENT through the capacitor is NOT continuous but changes from 0 to 30A when the switch is closed. The voltage across the capacitor is continuous because the stored energy CANNOT CHANGE INSTANTANEOUSLY.

Problem 5.38**Solution:****Known quantities:**

Circuit shown in Figure P5.38, $V_S = 12 \text{ V}$, $R_S = 0.24 \Omega$, $R_1 = 33 \text{ k}\Omega$, $L = 100 \text{ mH}$.

Find:

The voltage across the inductor before and just after the switch is changed.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

The solution to this problem can be visualized by noting that before the switch is thrown the circuit is presumed to be in a DC steady state. Therefore, before the switch is thrown the inductor may be modeled as a short circuit such that the voltage across it is zero. Moreover, the current through the inductor before the switch is thrown is simply V_S/R_S . Immediately after the switch is thrown the current through the inductor must be this same value since the current through an inductor cannot change instantaneously. However, after the switch is thrown this current must also be the current drawn through R_1 since that resistor and the inductor are then in series. Finally, the voltage across the inductor after the switch is thrown is always equal to the voltage across R_1 , which is simply the product of i_L and R_1 . Thus, the voltage across the inductor immediately after the switch is thrown must be $i_L(t=0^+)$ times R_1 .

In quantitative terms, at $t = 0^-$,

$$v_L(0^-) = 0$$

$$i_L(0^-) = \frac{V_S}{R_S} = \frac{12}{0.24} = 50 \text{ A}$$

At $t = 0^+$,

$$i_L(0^+) = i_L(0^-) = 50 \text{ A}$$

and

$$v_L(0^+) = i_L(0^+)R_1 = (50 \text{ A})(33 \text{ k}\Omega) = 1.65 \text{ MV}$$

This high side of this voltage is located at the ground terminal due to the direction of current flow through R_1 .

ANSWER: $0 \text{ V}, -1.65 \text{ MV}$

Problem 5.39

Solution:

Known quantities:

Circuit shown in Figure P5.27, $V_1 = 12 \text{ V}$, $C = 150 \mu\text{F}$, $R_1 = 4 \text{ M}\Omega$, $R_2 = 80 \text{ M}\Omega$, $R_3 = 6 \text{ M}\Omega$.

Find:

The time constant of the circuit for $t > 0$.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

At $t = 0^+$, just after the switch is closed, a transient starts. Since this is a first order circuit (a single independent capacitance), the transient will be exponential with some time constant. That time constant is the product of the capacitance of the capacitor and the Thevenin equivalent resistance seen across the terminals of the capacitor.

To find the Thevenin equivalent resistance, suppress the independent, ideal voltage source which is equivalent to replacing it with a short circuit. Then with respect to the terminals of the capacitor:

$$R_{TH} = R_2 + (R_1 \parallel R_3) = 80 + (4 \parallel 6) = 82.4 \text{ M}\Omega$$

The time constant is simply

$$\tau = R_{TH}C = (82.4 \text{ M}\Omega)(150 \mu\text{F}) = 12360 \text{ sec} = 206 \text{ minutes} = 3.43 \text{ hours} +$$

Notice that this value is quite independent of the magnitude of the voltage source.

ANSWER: $2.36 \text{ ks} = 206.0 \text{ min} = 3.433 \text{ hr}$

Problem 5.40

Solution:

Known quantities:

Circuit shown in Figure P5.21, $V_S = 12 \text{ V}$, $L = 100 \text{ mH}$, $R_1 = 400 \Omega$, $R_2 = 400 \Omega$, $R_3 = 600 \Omega$.

Find:

The time constant of the circuit for $t > 0$.

Assumptions:

$i_L = 1.70 \text{ mA}$ just before the switch is opened at $t = 0$.

Analysis:

At $t = 0^+$, just after the switch is opened, a transient starts. Since this is a first order circuit (a single independent capacitance), the transient will be exponential with some time constant. That time constant is

the product of the inductance of the inductor and the Thevenin equivalent resistance seen across the terminals of the inductor.

In this problem, the Thevenin equivalent resistance is particularly easy to find since there are no sources connected. With respect to the terminals of the inductor:

$$R_{TH} = (R_1 + R_2) \parallel R_3 = 800 \parallel 600 \cong 343 \, \Omega$$

The time constant is simply

$$\tau = \frac{L}{R_{TH}} = \frac{0.1}{343} \cong 292 \, ns$$

ANSWER: 291.7 ns

Problem 5.41

Solution:

Known quantities:

Circuit shown in Figure P5.41,

$$V_S = 20 \, V, R_1 = 5 \, \Omega, R_2 = 4 \, \Omega, R_3 = 3 \, \Omega, R_4 = 6 \, \Omega, C_1 = 4 \, F, C_2 = 4 \, F, I_S = 4 \, A.$$

Find:

- The capacitor voltage $V_C(t)$ at $t = 0^+$.
- The time constant τ for $t \geq 0$.
- The expression for $V_C(t)$ and sketch the function.
- Find $V_C(t)$ for each of the following values of t : $0, \tau, 2\tau, 5\tau, 10\tau$.

Assumptions:

Switch S_1 is always open and switch S_2 closes at $t = 0$.

Analysis:

- Without any power sources connected the steady state voltages are zero due to relentless dissipation of energy in the resistors.

$$V_C(0^-) = V_C(0^+) = 0 \, V$$

When the initial condition on a transient is zero, the general solution for the transient simplifies to

$$V_C(t) = V(\infty)(1 - e^{-t/\tau})$$

- The two capacitors in parallel can be combined into one 8 F equivalent capacitor. The Thevenin equivalent resistance seen by the 8 F capacitance is found by suppressing the current source (i.e. replacing it with an open circuit) and computing $R_2 + R_3 \parallel R_4$.

$$R_{TH} = 4 + (3 \parallel 6) = 6 \, \Omega$$

$$\tau = R_{TH} C = (6)(8) = 48 \, s$$

- The long-term steady state voltage across the capacitors is found by replacing them with DC open circuits and solving for the voltage across R_3 . This voltage is found readily by current division

$$V_C(\infty) = \frac{6 \, \Omega}{3 \, \Omega + 6 \, \Omega} (4 \, A)(3 \, \Omega) = 8 \, V$$

Plug in to the generalized solution given above to find

$$V_C(t) = 8(1 - e^{-t/48}) \quad , \quad t \geq 0$$

$$V_C(t) = 0 \quad , \quad t \leq 0$$

d)

$$V_C(0) = 0V; \quad V_C(\tau) = 5.06V;$$

$$V_C(2\tau) = 6.9V; \quad V_C(5\tau) = 7.95V;$$

$$V_C(10\tau) = 8.0V$$

Problem 5.42

Solution:

Known quantities:

Circuit shown in Figure P5.41,

$$V_S = 20V, R_1 = 5\Omega, R_2 = 4\Omega, R_3 = 3\Omega, R_4 = 6\Omega, C_1 = 4F, C_2 = 4F, I_S = 4A.$$

Find:

- The capacitor voltage $V_C(t)$ at $t = 0^+$.
- The time constant τ for $t \geq 0$.
- The expression for $V_C(t)$ and sketch the function.
- Find $V_C(t)$ for each of the following values of t : $0, \tau, 2\tau, 5\tau, 10\tau$.

Assumptions:

Switch S_1 has been open for a long time and closes at $t = 0$; conversely, switch S_2 has been closed for a long time and opens at $t = 0$.

Analysis:

- The capacitor voltage immediately after the switch S_1 is closed and the switch S_2 is opened is equal to that when the switches were the first opened and the second closed respectively. Since the second switch was closed for a long time one can assume that the capacitor voltage had reached a long-term steady state value. This value is found by replacing both capacitors with DC open circuits and solving for the voltage across R_3 . This voltage is found readily by current division

$$V_C(0^+) = V_C(0^-) = \frac{6\Omega}{3\Omega + 6\Omega} (4A)(3\Omega) = 8V$$

- The Thevenin equivalent resistance seen by the parallel capacitors is $R_1 \parallel (R_2 + R_3)$.

$$\tau = R_{TH}C = \left(\frac{1}{5} + \frac{1}{4+3}\right)^{-1} \times (4+4) = \frac{70}{3} s$$

- The generalized solution for the transient is

$$V_C(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau}$$

The long-term steady state voltage across the capacitors is found by replacing them with DC open circuits and solving for the voltage across R_3 . This voltage is found readily by voltage division. Thus,

$$V_C(\infty) = \frac{R_2 + R_3}{R_1 + R_2 + R_3} V_S = \frac{4\Omega + 3\Omega}{5\Omega + 4\Omega + 3\Omega} (20V) = \frac{35}{3} V$$

Plug in to the generalized solution given above to find

$$V_C(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau} = \frac{35}{3} + \left(8 - \frac{35}{3}\right)e^{-3t/70} = \frac{35}{3} - \frac{11}{3}e^{-3t/70}, \quad t \geq 0$$

$$V_C(t) = 8, \quad t \leq 0$$

d)

$$V_C(0) = 8 \text{ V}; \quad V_C(\tau) = 10.318 \text{ V};$$

$$V_C(2\tau) = 11.170 \text{ V}; \quad V_C(5\tau) = 11.642 \text{ V};$$

$$V_C(10\tau) = 11.666 \text{ V}$$

Problem 5.43

Solution:

Known quantities:

Circuit shown in Figure P5.41,

$$V_S = 20 \text{ V}, R_1 = 5 \Omega, R_2 = 4 \Omega, R_3 = 3 \Omega, R_4 = 6 \Omega, C_1 = 4 \text{ F}, C_2 = 4 \text{ F}, I_S = 4 \text{ A}$$

Find:

- The capacitor voltage $V_C(t)$ at $t = 0^+$.
- The expression for $V_C(t)$ and sketch the function.

Assumptions:

Switch S_2 is always open; switch S_1 has been closed for a long time, and opens at $t = 0$. At

$t = t_1 = 3\tau$, switch S_1 closes again.

Analysis:

The approach here is to find the transient solution in the interval $0 < t < 3$ seconds and use that solution to determine the initial condition (the capacitor voltage) for the new transient after the switch S_1 closes again.

- S_1 has been closed for a long time and in DC steady state the capacitors can be replaced with open circuits. Thus, by voltage division

$$V_C(0^+) = V_C(0^-) = \frac{7}{12} \times 20 = 11.67 \text{ V}$$

- As mentioned above, to find the complete transient solution for $t > 0$ it is necessary to find the capacitor voltage when switch S_1 closes at $t = 3$ seconds. To do so it is first necessary to find the complete transient solution for when the switch is open (i.e. as if the switch never closed again.)

The long-term steady state capacitor voltage when the switch is held open is zero. The time constant is simply $R_{TH} C_{EQ} = 56$ seconds. Thus, the complete transient solution for the first 3 seconds is

$$V_C(t) = V(0^+)e^{-t/56} = 11.67e^{-t/56}, \quad 0 \leq t \leq 3$$

At $t = 3$ seconds, the capacitor voltage is

$$V_C(t = 3^-) = 11.67e^{-3/56} = 11.06$$

At $t = 3$ seconds the switch S_1 closes again. Continuity of voltage across the capacitors still holds so

$$V_C(t = 3^+) = V_C(t = 3^-) = 11.06$$

With the switch closed the long-term steady state capacitor voltage is the same as that found in part a.

$$V_C(\infty) = \frac{7}{12} \times 20 = 11.67 \text{ V}$$

The new time constant is found after suppressing the independent voltage source (i.e. replacing it with a short circuit) and finding the new Thevenin equivalent resistance seen by the capacitors.

$$R_{TH} = (4 + 3) \parallel 5 = 2.92 \, \Omega$$

and

$$\tau = R_{TH} C = (2.92)(4 + 4) = 23.3 \text{ seconds}$$

Finally, the transient solution for $t > 3$ is

$$V_C(t) = 11.67 + [11.06 - 11.67]e^{-t/23.3} = 11.67 - 0.61e^{-(t-3)/23.3}, \quad t \geq 3$$

Notice the use of the shifted time scale $(t-3)$ in the exponent.

Problem 5.44

Solution:

Known quantities:

Circuit shown in Figure P5.41,

$$V_S = 20 \text{ V}, R_1 = 5 \, \Omega, R_2 = 4 \, \Omega, R_3 = 3 \, \Omega, R_4 = 6 \, \Omega, C_1 = 4 \text{ F}, C_2 = 4 \text{ F}, I_S = 4 \text{ A}.$$

Find:

- The capacitor voltage $V_C(t)$ at $t = 0^+$.
- The time constant τ for $t \geq 0$.
- The expression for $V_C(t)$ and sketch the function.
- Find $V_C(t)$ for each of the following values of t : $0, \tau, 2\tau, 5\tau, 10\tau$.

Assumptions:

Both switches S_1 and S_2 close at $t = 0$.

Analysis:

- Without any power sources connected the steady state voltages are zero due to the complete dissipation of all circuit energy by the resistors.

$$V_C(0^-) = V_C(0^+) = 0 \text{ V}$$

When the initial condition on a transient is zero, the general solution for the transient simplifies to

$$V_C(t) = V(\infty)(1 - e^{-t/\tau})$$

- The two capacitors in parallel can be combined into one 8 F equivalent capacitor. The Thevenin equivalent resistance seen by the 8 F capacitance is found by suppressing the independent sources (i.e. by replacing the current source with an open circuit and the voltage source with a short circuit) and computing $R_1 \parallel (R_2 + R_3 \parallel R_4)$.

$$R_{TH} = [5 \parallel (4 + (3 \parallel 6))] = [5 \parallel 6] = \frac{30}{11} \approx 2.73 \, \Omega$$

$$\tau = R_{TH}C = \frac{30}{11}8 = \frac{240}{11} \cong 21.8 \text{ s}$$

- c) At this point only the long-term steady state capacitor voltage is needed to write down the complete transient solution. In DC steady state the capacitors can be modeled as open circuits. Furthermore, $R_3 \parallel R_4$ can be replaced with an equivalent resistance. This resistance is in parallel with the independent current source and the two can be replaced with a Thevenin source transformation of an appropriate voltage source in series with the same resistance. Once this replacement is made it is a simple matter of voltage division to determine the capacitor voltage.

$$R_3 \parallel R_4 = 3 \parallel 6 = 2 \, \Omega$$

The source transformation results in an 8V voltage source in series with this resistance. Then, by voltage division,

$$V_C(\infty) = 8 + \frac{4+2}{4+2+5}(20-8) = \frac{160}{11} \cong 14.55 \text{ V}$$

Now plug in to the general form of the transient solution to find

$$V_C(t) \cong 14.5 \left[1 - e^{-11t/240} \right], \quad t \geq 0$$

d)

$$V_C(0) = 0 \text{ V}; \quad V_C(\tau) = 9.17 \text{ V};$$

$$V_C(2\tau) = 12.5 \text{ V}; \quad V_C(5\tau) = 14.4 \text{ V};$$

$$V_C(10\tau) = 14.5 \text{ V}$$

Problem 5.45

Solution:

Known quantities:

Circuit shown in Figure P5.41,

$$V_S = 20 \text{ V}, R_1 = 5 \, \Omega, R_2 = 4 \, \Omega, R_3 = 3 \, \Omega, R_4 = 6 \, \Omega, C_1 = 4 \text{ F}, C_2 = 4 \text{ F}, I_S = 4 \text{ A}$$

Find:

- The capacitor voltage $V_C(t)$ at $t = 0^+$.
- The time constant τ for $0 \leq t \leq 48 \text{ s}$.
- The expression for $V_C(t)$ valid for $0 \leq t \leq 48 \text{ s}$.
- The time constant τ for $t > 48 \text{ s}$.
- The expression for $V_C(t)$ valid for $t > 48 \text{ s}$.
- Plot $V_C(t)$ for all time.

Assumptions:

Switch S_1 opens at $t = 0$; switch S_2 opens at $t = 48 \text{ s}$.

Analysis:

The approach here is to find the transient solution in the interval $0 < t < 48$ seconds and use that solution to determine the initial condition (the capacitor voltage) for the new transient after the switch S_2 opens.

- a) S_1 and S_2 have been closed for a long time and in DC steady state the capacitors can be replaced with open circuits. Thus, by node analysis and voltage division

$$\frac{V_s - V_C(0^-)}{R_1} = \frac{V_C(0^-)R_2}{R_2(R_2 + R_3 + R_4)} \Rightarrow V_C(0^-) = \frac{V_s(R_2 + R_3 + R_4)}{(R_1 + R_2 + R_3 + R_4)}$$

$$V_C(0^+) = V_C(0^-) = \frac{20(4 + 3 + 6)}{(5 + 4 + 3 + 6)} = \frac{260}{18}V \cong 14.4V$$

- b) The two capacitors in parallel can be combined into one 8 F equivalent capacitor. The Thevenin equivalent resistance seen by the 8 F capacitance is found by suppressing the independent sources (i.e. by replacing the current source with an open circuit) and computing $(R_2 + R_3 || R_4)$.

$$R_{TH} = R_2 + (R_3 || R_4) = 4 + (3 || 6) = 4 + 2 = 6\Omega$$

$$\tau = R_{TH}(C_1 + C_2) = 6 \cdot 8 = 48s$$

- c) As mentioned above, to find the complete transient solution for $t > 0$ it is necessary to find the capacitor voltage when switch S_2 opens at $t = 48s$. To do so it is first necessary to find the complete transient solution for when only the switch S_1 is open (i.e. as if the switch S_2 never opens.). The generalized solution for the transient is

$$V_C(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau}$$

The long-term steady state voltage across the capacitors is found by replacing them with DC open circuits and solving for the voltage across R_3 . This voltage is found readily by current division. Thus,

$$V_C(\infty) = \frac{6\Omega}{3\Omega + 6\Omega} (4A)(3\Omega) = 8V$$

Plug in to the generalized solution given above to find

$$V_C(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau} = 8 + (14.4 - 8)e^{-t/48} = 8 + 6.4e^{-t/48}, \quad 0 \leq t \leq 48$$

At $t = 48s$, the capacitor voltage is

$$V_C(t = 48^-) = 8 + 6.4e^{-1} = 10.35V$$

Continuity of voltage across the capacitors still holds so

$$V_C(48^+) = V_C(48^-) = 10.35V$$

- d) The two capacitors in parallel can be combined into one 8 F equivalent capacitor. When both the switches are opened, there are no independent sources connected to the circuit. Thus, the Thevenin equivalent resistance seen by the 8 F capacitance is found by computing $(R_2 + R_3)$.

$$R_{TH} = R_2 + R_3 = 4 + 3 = 7\Omega$$

$$\tau = R_{TH}(C_1 + C_2) = 7 \cdot 8 = 56s$$

- e) The generalized solution for the transient is

$$V_C(t) = V(\infty) + [V(t_0^+) - V(\infty)]e^{-(t-t_0)/\tau}$$

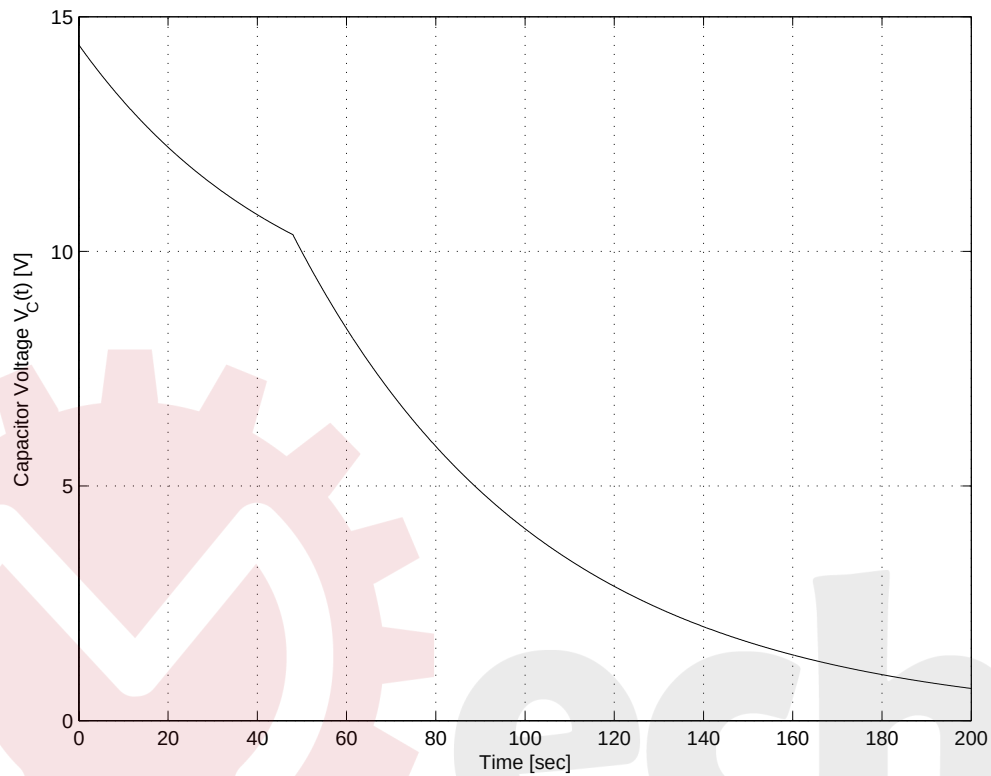
The long-term steady state capacitor voltage after the switch has been opened is zero since no independent sources are connected and all the initial energy in the circuit is eventually dissipated by the resistors. Thus,

$$V_C(\infty) = 0V$$

Plug in to the generalized solution given above to find

$$V_C(t) = V(48^+)e^{-t/\tau} = 10.35e^{-(t-48)/56}, \quad t > 48$$

- f) The plot of $V_C(t)$ for all time is shown in the following figure.



Problem 5.46

Solution:

Known quantities:

Circuit shown in Figure P5.41,

$$V_S = 20\text{ V}, R_1 = 5\ \Omega, R_2 = 4\ \Omega, R_3 = 3\ \Omega, R_4 = 6\ \Omega, C_1 = 4\text{ F}, C_2 = 4\text{ F}, I_S = 4\text{ A}$$

Find:

- The capacitor voltage $V_C(t)$ at $t = 0^+$.
- The time constant τ for $0 \leq t \leq 96\text{ s}$.
- The expression for $V_C(t)$ valid for $0 \leq t \leq 96\text{ s}$.
- The time constant τ for $t > 96\text{ s}$.
- The expression for $V_C(t)$ valid for $t > 96\text{ s}$.
- Plot $V_C(t)$ for all time.

Assumptions:

Switch S_1 opens at $t = 96\text{ s}$; switch S_2 opens at $t = 0$.

Analysis:

The approach here is to find the transient solution in the interval $0 < t < 96$ seconds and use that solution to determine the initial condition (the capacitor voltage) for the new transient after the switch S_1 opens.

- a) S_1 and S_2 have been closed for a long time and in DC steady state the capacitors can be replaced with open circuits. Thus, by node analysis and voltage division

$$\frac{V_S - V_C(0^-)}{R_1} = \frac{V_C(0^-)R_2}{R_2(R_2 + R_3 + R_4)} \Rightarrow V_C(0^-) = \frac{V_S(R_2 + R_3 + R_4)}{(R_1 + R_2 + R_3 + R_4)}$$

$$V_C(0^+) = V_C(0^-) = \frac{20(4 + 3 + 6)}{(5 + 4 + 3 + 6)} = \frac{260}{18}V \cong 14.4V$$

- b) The two capacitors in parallel can be combined into one 8 F equivalent capacitor. The Thevenin equivalent resistance seen by the 8 F capacitance is found by suppressing the independent sources (i.e. by replacing the current source with an open circuit) and computing $R_1 \parallel (R_2 + R_3)$.

$$R_{TH} = R_1 \parallel (R_2 + R_3) = 5 \parallel (4 + 3) = 2.92\Omega$$

$$\tau = R_{TH}(C_1 + C_2) = 2.92 \cdot 8 = 23.3s$$

- c) As mentioned above, to find the complete transient solution for $t > 0$ it is necessary to find the capacitor voltage when switch S_1 opens at $t = 96$ s. To do so it is first necessary to find the complete transient solution for when only the switch S_2 is open (i.e. as if the switch S_1 never opens.). The generalized solution for the transient is

$$V_C(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau}$$

The long-term steady state voltage across the capacitors is found by replacing them with DC open circuits and solving for the voltage across R_2 and R_3 . This voltage is found readily by voltage division. Thus,

$$V_C(\infty) = \frac{4\Omega + 3\Omega}{5\Omega + 4\Omega + 3\Omega}(20V) = 11.67V$$

Plug in to the generalized solution given above to find

$$V_C(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau} = 11.67 + (14.4 - 11.67)e^{-t/23.3}$$

$$V_C(t) = 11.67 + 2.73e^{-t/23.3}, \quad 0 \leq t \leq 96$$

At $t = 96$ s, the capacitor voltage is

$$V_C(t = 96^-) = 11.67 + 2.73e^{-96/23.3} = 11.71V$$

Continuity of voltage across the capacitors still holds so

$$V_C(96^+) = V_C(96^-) = 11.71V$$

- d) The two capacitors in parallel can be combined into one 8 F equivalent capacitor. When both the switches are opened, there are no independent sources connected to the circuit. Thus, the Thevenin equivalent resistance seen by the 8 F capacitance is found by computing $(R_2 + R_3)$.

$$R_{TH} = R_2 + R_3 = 4 + 3 = 7\Omega$$

$$\tau = R_{TH}(C_1 + C_2) = 7 \cdot 8 = 56s$$

- e) The generalized solution for the transient is

$$V_C(t) = V(\infty) + [V(t_0^+) - V(\infty)]e^{-(t-t_0)/\tau}$$

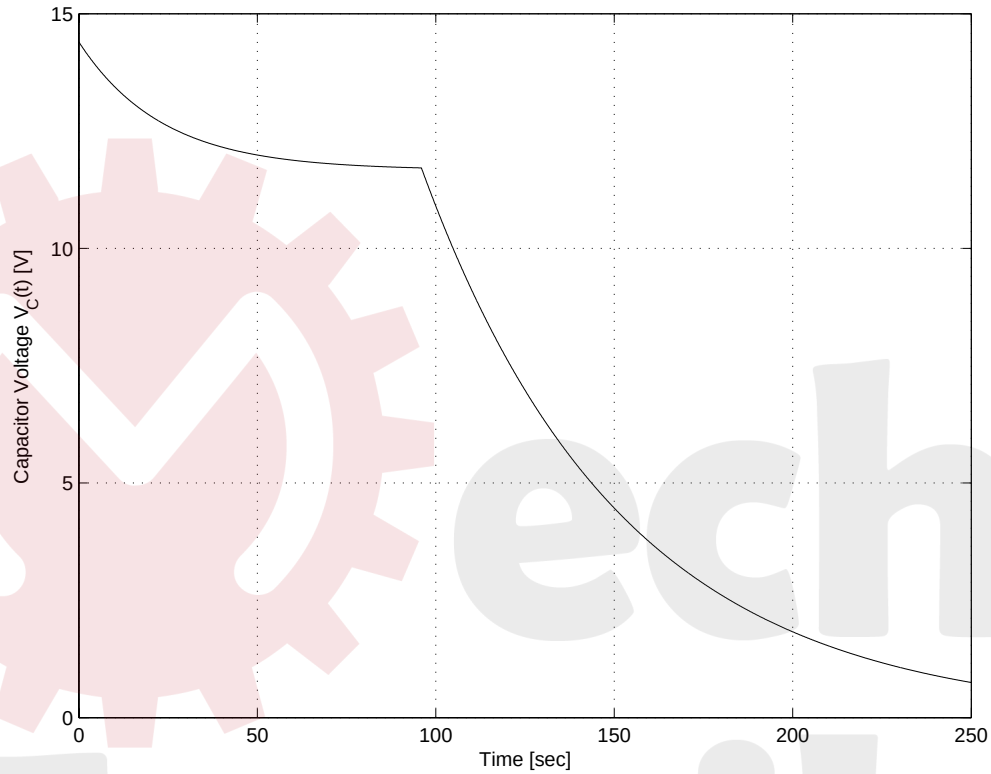
The long-term steady state capacitor voltage after the switch has been opened is zero since no independent sources are connected and all the initial energy in the circuit is eventually dissipated by the resistors. Thus,

$$V_C(\infty) = 0V$$

Plug in to the generalized solution given above to find

$$V_C(t) = V(96^+)e^{-t/\tau} = 11.71e^{-(t-96)/56}, \quad t > 96$$

f) The plot of $V_C(t)$ for all time is shown in the following figure.



Problem 5.47

Solution:

Known quantities:

$R_S = 15\text{ k}\Omega$, $\tau = 1.5\text{ ms}$, $\tau' = 10\text{ ms}$, $R_3 = 30\text{ k}\Omega$, $C = 1\text{ }\mu\text{F}$.

Find:

The value of resistors R_1 and R_2 .

Assumptions:

None.

Analysis:

Before the switch opens:

$$R_{eq} = R_S // R_1 // R_2 // R_3$$

$$\tau = R_{eq} C = 1.5\text{ ms}$$

After the switch opens:

$$R'_{eq} = R_S // R_1$$

$$\tau' = R'_{eq} C = 10 \text{ ms}$$

Solving the system of equations we have,

$$R_1 = \frac{R_S \tau'}{R_S C - \tau'} = \frac{15000 \cdot 0.01}{15000 \cdot 10^{-6} - 0.01} = 30 \text{ k}\Omega$$

$$R_2 = \left(\frac{C}{\tau} - \frac{1}{R_S} - \frac{1}{R_1} - \frac{1}{R_3} \right)^{-1} = \left(\frac{10^{-6}}{0.0015} - \frac{1}{15000} - \frac{1}{30000} - \frac{1}{30000} \right)^{-1} = 1875 \Omega$$

Problem 5.48

Solution:

Known quantities:

Circuit shown in Figure P5.47, $V_S = 100 \text{ V}$, $R_S = 4 \text{ k}\Omega$, $R_1 = 2 \text{ k}\Omega$, $R_2 = R_3 = 6 \text{ k}\Omega$, $C = 1 \mu\text{F}$.

Find:

The value of the voltage across the capacitor after $t = 2.666 \text{ ms}$.

Assumptions:

None.

Analysis:

Before opening, the switch has been closed for a long time. Thus we have a steady-state condition, and we treat the capacitor as an open circuit. The voltage across the capacitor is equal to the voltage across the resistance R_1 . Thus,

$$V_C(t_0^-) = V_C(t_0^+) = \frac{R_S // R_1 // R_2 // R_3}{R_S} V_S = \frac{300}{13} V \cong 23.077 \text{ V}$$

After the switch opens, the time constant of the circuit is

$$R_{eq} = R_S // R_1 = \frac{4000}{3} \Omega$$

$$\tau = R_{eq} C = \frac{1}{750} \text{ ms} \cong 1.3 \text{ ms}$$

the generalized solution for the transient is

$$V_C(t) = V(\infty) + [V(t_0^+) - V(\infty)] e^{-(t-t_0)/\tau}$$

The long-term steady state voltage across the capacitors is found by replacing them with DC open circuits and solving for the voltage across R_1 . This voltage is found readily by voltage division. Thus,

$$V_C(\infty) = \frac{R_1}{R_S + R_1} V_S = \frac{2000 \Omega}{4000 \Omega + 2000 \Omega} 20 \text{ V} = \frac{20}{3} \text{ V} \cong 6.67 \text{ V}$$

Plug in to the generalized solution given above to find

$$V_C(t) = V(\infty) + [V(t_0^+) - V(\infty)] e^{-(t-t_0)/\tau} = \frac{20}{3} + \left(\frac{300}{13} - \frac{20}{3} \right) e^{-750(t-t_0)} = \frac{20}{3} + \frac{640}{39} e^{-750(t-t_0)}$$

Finally,

$$V_C(t_0 + 2.666ms) = \frac{20}{3} + \frac{640}{39} e^{-750(0.002666)} = 8.888V$$

Problem 5.49

Solution:

Known quantities:

As described in Figure P5.49.

Find:

The time at which the current through the inductor is equal to 5 A, and the expression for $i_L(t)$ for $t \geq 0$.

Assumptions:

None.

Analysis:

At $t < 0$:

Using the current divider rule:

$$i_L(0^-) = \left(\frac{100}{1000 + 5 // 2.5} \right) \left(\frac{5}{5 + 2.5} \right) = 66.5 \text{ mA}$$

At $t > 0$:

Using the current divider rule:

$$i_L(\infty) = \left(\frac{100}{10 + 5 // 2.5} \right) \left(\frac{5}{5 + 2.5} \right) = 5.71 \text{ A}$$

To find the time constant for the circuit we must find the Thevenin resistance seen by the inductor:

$$R_{eq} = 10 // 5 + 2.5 = 5.83 \Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{0.1}{5.83} = 17.1 \text{ ms}$$

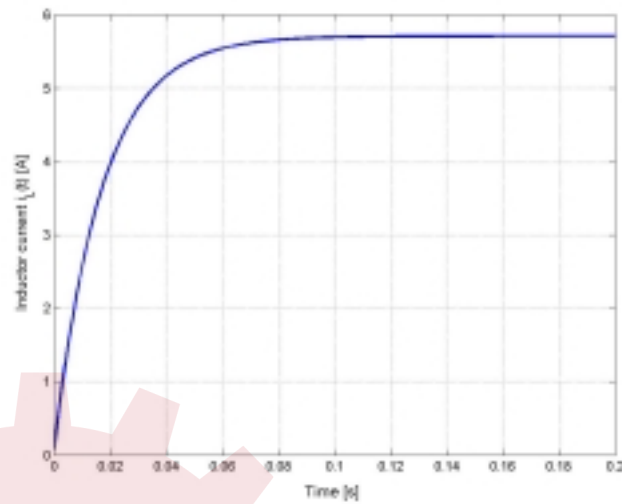
Finally, we can write the solution:

$$\begin{aligned} i_L(t) &= i_L(\infty) - (i_L(\infty) - i_L(0))e^{-t/\tau} \\ &= 5.71 - (5.71 - 0.0665)e^{-t/17.1 \times 10^{-3}} \\ &= 5.71 - 5.64e^{-t/17.1 \times 10^{-3}} \text{ A} \end{aligned}$$

Solving the equation we have,

$$i_L(\hat{t}) = 5.71 - 5.64e^{-\hat{t}/17.1 \times 10^{-3}} = 5 \Rightarrow \hat{t} = 35.437 \text{ ms}$$

The plot of $i_L(t)$ for all time is shown in the following figure.



Problem 5.50

Solution:

Known quantities:

As described in Figure P5.49.

Find:

The expression for $i_L(t)$ for $0 \leq t \leq 5\text{ms}$. The maximum voltage between the contacts during the 5-ms duration of the switch.

Assumptions:

The mechanical switching action requires 5 ms.

Analysis:

a) At $t < 0$:

Using the current divider rule:

$$i_L(0^-) = \left(\frac{100}{1000 + 5 \parallel 2.5} \right) \left(\frac{5}{5 + 2.5} \right) = 66.5 \text{ mA}$$

For $0 \leq t \leq 5\text{ms}$:

The long term steady state inductor current after the switch has been opened is zero since no independent source is connected to the circuit and all the initial energy in the circuit is eventually dissipated by the resistors. Thus,

$$i_L(\infty) = 0 \text{ A}$$

To find the time constant for the circuit we must find the Thevenin resistance seen by the inductor:

$$R_{eq} = 5 + 2.5 = 7.5 \Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{0.1}{7.5} = 13.33 \text{ ms}$$

Finally, we can write the solution:

$$i_L(t) = i_L(0) e^{-t/\tau} = (0.0665) e^{-t/13.33 \times 10^{-3}} \text{ A} \quad (0 \leq t \leq 5\text{ms})$$

b) The voltage between the contacts during the 5-ms duration of the switching is equal to:

$$V_{cont}(t) = V_S - V_{5\Omega} = 100 - (V_L + 2.5i_L)$$

where,

$$V_L(t) = L \frac{di_L(t)}{dt} = -\frac{(0.1)(0.0665)}{13.33 \times 10^{-3}} e^{-t/13.33 \times 10^{-3}} = (-0.5) e^{-t/13.33 \times 10^{-3}} V$$

Therefore,

$$V_{cont}(t) = 100 - [(2.5)(0.0665) - 0.5] e^{-t/13.33 \times 10^{-3}} = 100 + (0.33) e^{-t/13.33 \times 10^{-3}} V$$

Thus, the maximum voltage between the contacts during the 5-ms duration of the switch is:

$$V_{cont}^{MAX} = V_{cont}(t=0) = 100.33V$$

Problem 5.51

Solution:

Known quantities:

As described in Figure P5.51. The switch closes when the voltage across the capacitor voltage reaches v_M^C ; The switch opens when the voltage across the capacitor voltage reaches $v_M^O = 1V$. The period of the capacitor voltage waveform is 200 ms.

Find:

The voltage v_M^C .

Assumptions:

The initial capacitor voltage is 1V and the switch has just opened.

Analysis:

With the switch open:

$$V_C(\infty) = 10V$$

$$\tau = RC = 0.15s$$

$$V_C(t) = V_C(\infty) - [V_C(\infty) - V_C(0)] e^{-t/\tau}$$

$$= 10 - (10 - 1) e^{-t/0.15}$$

$$= 10 - 9e^{-t/0.15} V$$

Now we must determine the time when $V_C(t) = v_M^C$.

Using the expression for the capacitor voltage:

$$v_M^C = 10 - 9e^{-t_0/0.15} \Rightarrow e^{-t_0/0.15} = \frac{10 - v_M^C}{9} \Rightarrow t_0 = -0.15 \ln\left(\frac{10 - v_M^C}{9}\right)$$

With the switch closed, the capacitor sees the Thevenin equivalent defined by:

$$V_{eq} = V_C(\infty) = \frac{10}{10 + 10000} \times 10 \approx 1 \times 10^{-2} V \quad (\text{voltage division})$$

$$R_{eq} = 10k\Omega \parallel 10\Omega \approx 10\Omega$$

$$\tau = R_{eq} C = 0.15ms$$

The initial value of this part of the transient is v_M^C at $t = t_0$. With these values we can write the expression for the capacitor voltage:

$$V_C(t) = V_C(\infty) - [V_C(\infty) - V_C(t_0)]e^{-(t-t_0)/\tau}$$

$$= 0.01 + (v_M^C - 0.01)e^{-(t-t_0)/0.15 \times 10^{-3}} \text{ V}$$

The end of one full cycle of the waveform across the 10Ω resistor occurs when the second transient reaches $v_M^O = 1\text{V}$. If we call the time at which this event occurs t_1 , then:

$$V_C(t) = 1\text{V} \quad \text{at } t = t_1 = 200\text{ms}$$

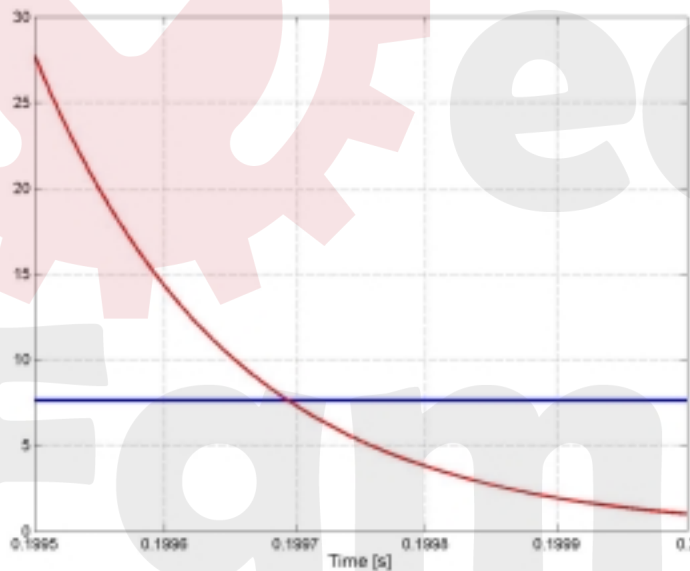
and so

$$1 = 0.01 + (v_M^C - 0.01)e^{-(t_1-t_0)/0.15 \times 10^{-3}}$$

Graphically, the solution is the intersection between the following function:

$$v_M^C = 10 - 9e^{-t_0/0.15} \quad (\text{blue line})$$

$$v_M^C = 0.01 + \frac{(1 - 0.01)}{e^{-(0.2-t_0)/0.15 \times 10^{-3}}} \quad (\text{red line})$$



which correspond to $v_M^C = 7.627\text{ V}$ and $t_0 \cong 0.1997\text{ ms}$.

Problem 5.52

Solution:

Known quantities:

As describes in Figure P5.52. At $t = 0$, the switch closes.

Find:

a) $i_L(t)$ for $t \geq 0$.

b) $V_{L1}(t)$ for $t \geq 0$.

Assumptions:

$$i_L(0) = 0 \text{ A}.$$

Analysis:

a) In the long-term DC steady state after the switch is closed the inductors may be modeled as short circuits and so all of the current from the source will travel through the inductors.

$$i_L(\infty) = 5 \text{ A}$$

With the current source suppressed (treated as an open circuit) the Thevenin equivalent resistance seen by the inductors in series and the associated time constant are

$$R_{eq} = 10 \text{ k}\Omega$$

$$L_{eq} = L_1 + L_2 = 6 \text{ H}$$

$$\tau = \frac{L_{eq}}{R_{eq}} = 0.6 \text{ ms}$$

$$\begin{aligned} i_L(t) &= i_L(\infty) \left[1 - e^{-t/\tau} \right] \\ &= 5 \left[1 - e^{-t/0.6 \times 10^{-3}} \right] \text{ A} \end{aligned}$$

b) The voltage across either of the inductors is derived directly from the differential relationship between current and voltage for an inductor.

$$\begin{aligned} V_{L1}(t) &= L_1 \frac{di_L(t)}{dt} \\ &= (1)(5) \frac{d}{dt} (1 - e^{-t/0.6 \times 10^{-3}}) \\ &= 5 \left(\frac{1}{0.6 \times 10^{-3}} e^{-t/0.6 \times 10^{-3}} \right) \\ &= 8.333 e^{-t/0.6 \times 10^{-3}} \text{ kV} \end{aligned}$$

Problem 5.53

Solution:

Known quantities:

As describes in Figure P5.52. At $t = 0$, the switch closes.

Find:

The voltage across the $10\text{-k}\Omega$ resistor in parallel with the switch for $t \geq 0$.

Assumptions:

None.

Analysis:

When the switch closes at $t = 0$, the $10\text{-k}\Omega$ resistor is in parallel with a short circuit, so its voltage is equal to zero for all time ($t \geq 0$).

Section 5.5: Transient Response of Second-Order Circuits

Focus on Methodology – roots of second order systems

- Case 1: **Real and distinct roots.** This case occurs when $\zeta > 1$, since the term under the square root is positive in this case, and the roots are: $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_n^2}$. This leads to an **overdamped response**.
- Case 2: **Real and repeated roots.** This case holds when $\zeta = 1$, since the term under the square root is zero in this case, and $s_{1,2} = -\alpha$. This leads to a **critically damped response**.
- Case 3: **Complex conjugate roots.** This case holds when $\zeta < 1$, since the term under the square root is negative in this case, and $s_{1,2} = -\alpha \pm j\omega_n \sqrt{1 - \zeta^2}$. This leads to an **underdamped response**.

Focus on Methodology

Second-order transient response

1. Solve for the steady-state response of the circuit before the switch changes state ($t = 0^-$), and after the transient has died out ($t \rightarrow \infty$). We shall generally refer to these responses as $x(0^-)$ and $x(\infty)$.
2. Identify the initial conditions for the circuit, $x(0^+)$, and $\dot{x}(0^+)$ using continuity of capacitor voltages and inductor currents ($v_C(0^+) = v_C(0^-)$, $i_L(0^+) = i_L(0^-)$), and circuit analysis. This will be illustrated by examples.
3. Write the differential equation of the circuit for $t = 0^+$, that is, immediately after the switch has changed position. The variable $x(t)$ in the differential equation will be either a capacitor voltage, $v_C(t)$, or an inductor current, $i_L(t)$. Reduce this equation to standard form (Equation 5.9, or 5.48).
4. Solve for the parameters of the second-order circuit: ω_n and ζ .
5. Write the complete solution for the circuit in one of the three forms given below, as appropriate:

Overdamped case ($\zeta > 1$):

$$x(t) = x_N(t) + x_F(t) = e^{-\alpha_1 t} + e^{-\alpha_2 t} x(0^-) \quad t \geq 0$$

Critically damped case ($\zeta = 1$):

$$x(t) = x_N(t) + x_F(t) = e^{-\alpha t} + t e^{-\alpha t} x(0^-) \quad t \geq 0$$

Underdamped case ($\zeta < 1$):

$$x(t) = x_N(t) + x_F(t) = e^{-\alpha t} \left[\cos(\omega_d t) + \frac{\alpha}{\omega_d} \sin(\omega_d t) \right] x(0^-) \quad t \geq 0$$

6. Apply the initial conditions to solve for the constants α_1 and α_2 .

Problem 5.54**Solution:****Known quantities:**

Circuit shown in Figure P5.54,

$$V_{S1} = 15\text{ V}, V_{S2} = 9\text{ V}, R_{S1} = 130\ \Omega, R_{S2} = 290\ \Omega, R_1 = 1.1\text{ k}\Omega, R_2 = 700\ \Omega, L = 17\text{ mH}, C = 0.35\ \mu\text{F}.$$

Find:

The voltage across the capacitor and the current through the inductor and R_{S2} as t approaches infinity.

Assumptions:

The circuit is in DC steady-state conditions for $t < 0$.

Analysis:

The conditions that exist at $t < 0$ have no effect on the long-term DC steady state conditions at $t \rightarrow \infty$. In the long-term DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. In this case, the inductor short circuits the R_1 branch and the R_2 C branch. Thus, the voltage across these branches and the current through them are zero. In other words all of the current produced by the 9V source travels through the inductor in this case.

$$i_L(\infty) = \frac{V_{S2}}{R_{S2}} = \frac{9}{290} = 31.03\text{ mA}$$

Of course, this current is also the current traveling through the 290 Ω resistor.

$$i_{RS2}(\infty) = i_L(\infty) = 31.03\text{ mA}$$

And since the voltage across the inductor in the long-term DC steady state is zero (short circuit)

$$0 + V_C(\infty) + i_{R2}(\infty)R_2 = 0$$

$$V_C(\infty) = 0$$

Problem 5.55**Solution:****Known quantities:**

Circuit shown in Figure P5.54,

$$V_{S1} = 12\text{ V}, V_{S2} = 12\text{ V}, R_{S1} = 50\ \Omega, R_{S2} = 50\ \Omega, R_1 = 2.2\text{ k}\Omega, R_2 = 600\ \Omega, L = 7.8\text{ mH}, C = 68\ \mu\text{F}.$$

Find:

The voltage across the capacitor and the current through the inductor as t approaches infinity.

Assumptions:

The circuit is in DC steady-state conditions for $t < 0$.

Analysis:

The conditions that exist at $t < 0$ have no effect on the long-term DC steady state conditions at $t \rightarrow \infty$. In the long-term DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. In this case, the inductor short circuits the R_1 branch and the R_2 C branch. Thus, the voltage across these branches and the current through them are zero. In other words all of the current produced by the 12V source travels through the inductor in this case.

$$i_L(\infty) = \frac{V_{S2}}{R_{S2}} = \frac{12}{50} = 240\text{ mA}$$

Of course, this current is also the current traveling through the 290 Ω resistor.

$$i_{RS2}(\infty) = i_L(\infty) = 240\text{ mA}$$

And since the voltage across the inductor in the long-term DC steady state is zero (short circuit)

$$0 + V_C(\infty) + i_{R2}(\infty)R_2 = 0$$

$$V_C(\infty) = 0$$

Problem 5.56

Solution:

Known quantities:

Circuit shown in Figure P5.56,

$$V_S = 170 \text{ V}, R_S = 7 \text{ k}\Omega, R_1 = 2.3 \text{ k}\Omega, R_2 = 7 \text{ k}\Omega, L = 30 \text{ mH}, C = 130 \text{ }\mu\text{F}.$$

Find:

The current through the inductor and the voltage across the capacitor and R_1 at steady state.

Assumptions:

None.

Analysis:

As $t \rightarrow \infty$, the circuit will return to DC steady state conditions. In the long-term DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. Therefore, in this case, the current through R_2 is zero and thus the voltage across the capacitor must be equal to the voltage across R_1 . Furthermore, the current through the inductor and R_1 is simply $V_S/(R_S + R_1)$.

$$i_C(\infty) = 0 \quad i_L(\infty) = i_{R1}(\infty) = \frac{V_S}{R_S + R_1} = \frac{170}{7000 + 2300} \cong 18.3 \text{ mA}$$

$$V_{R1}(\infty) = i_L(\infty)R_1 = 18.28 \times 10^{-3} \times 2.3 \times 10^3 = 42.04 \text{ V}$$

and

$$V_C(\infty) = V_{R1}(\infty) = 42.04 \text{ V}$$

Problem 5.57

Solution:

Known quantities:

Circuit shown in Figure P5.57,

$$V_S = 12 \text{ V}, C = 130 \text{ }\mu\text{F}, R_1 = 2.3 \text{ k}\Omega, R_2 = 7 \text{ k}\Omega, L = 30 \text{ mH}.$$

Find:

The current through the inductor and the voltage across the capacitor and R_1 at steady state.

Assumptions:

None.

Analysis:

As $t \rightarrow \infty$, the circuit will return to DC steady state conditions (practically after about 5 time constants.) In the long-term DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. Therefore, in this case, the voltage across the capacitor must be equal to the voltage across R_2 . Furthermore, the current through the inductor and R_2 is simply $V_S/(R_1 + R_2)$.

$$i_C(\infty) = 0 \quad V_L(\infty) = 0$$

$$i_L(\infty) = i_{R2}(\infty) = \frac{V_S}{R_1 + R_2} = \frac{12}{9.3 \times 10^3} = 1.29 \text{ mA}$$

$$V_{R1}(\infty) = i_S(\infty)R_1 = 1.29 \times 10^{-3} \times 2.3 \times 10^3 = 2.968 \text{ V}$$

And by observation

$$V_C(\infty) = V_{R_2}(\infty) = (1.29 \times 10^{-3})(7 \times 10^3) = 9.03 \text{ V}$$

All answers are positive indicating that the directions of the currents and polarities of the voltages assumed initially are correct. (You did do that, didn't you?)

Problem 5.58

Solution:

Known quantities:

Circuit shown in Figure P5.58,

$$V_S = 12 \text{ V}, C = 0.5 \mu\text{F}, R_1 = 31 \text{ k}\Omega, R_2 = 22 \text{ k}\Omega, L = 0.9 \text{ mH}.$$

Find:

The current through the inductor and the voltage across the capacitor at steady state.

Assumptions:

None.

Analysis:

As $t \rightarrow \infty$, the circuit will return to DC steady state conditions (practically after about 5 time constants).

In the long-term DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. Therefore, in this case, the voltage across the capacitor must be equal to the voltage across R_2 . Furthermore, the current through the inductor and R_2 is simply $V_S/(R_1 + R_2)$.

$$i_C(\infty) = 0 \quad V_L(\infty) = 0$$

$$i_L(\infty) = i_{R_2}(\infty) = \frac{V_S}{R_1 + R_2} = \frac{12}{(31 + 22) \times 10^3} \cong 226 \mu\text{A}$$

$$V_{R_2}(\infty) = i_{R_2}(\infty) R_2 = (226 \times 10^{-3})(22 \times 10^3) \cong 4.98 \text{ V}$$

And by observation

$$V_C(\infty) = V_{R_2}(\infty) = 4.98 \text{ V}$$

Theoretically, when the switch is OPENED, the current through the inductor must continue to flow, at least momentarily. However, the inductor is in series with an OPEN switch through which current CANNOT flow. What the theory does not predict is that a very large voltage is developed across the gap and this causes an arc with a current (kind of like a teeny, weeny lightning bolt). The energy stored in the magnetic field of the inductor is rapidly dissipated in the arc. The same effect will be important later when discussing transistors as switches.

Problem 5.59

Solution:

Known quantities:

Circuit shown in Figure P5.59,

$$V_s = 12\text{ V}, C = 3300\text{ }\mu\text{F}, R_1 = 9.1\text{ k}\Omega, R_2 = 4.3\text{ k}\Omega, R_3 = 4.3\text{ k}\Omega, L = 16\text{ mH}, .$$

Find:

The initial voltage across R_2 just after the switch is changed.

Assumptions:

At $t < 0$ the circuit is at steady state and the voltage across the capacitor is $+7\text{ V}$.

Analysis:

It is important to remember that only the values of the capacitor voltage and the inductor current are guaranteed continuity from immediately before the switch is thrown to immediately afterward. Therefore, to determine the initial voltage across R_2 it is necessary to first determine the initial voltage across the capacitor and the initial current through the inductor. Assume that before the switch was thrown DC steady state conditions existed. In DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. The initial voltage across the capacitor is given as $+7\text{ V}$. The initial current through the inductor is equal to the current through R_3 , which is given by Ohm's Law.

$$i_L(0^+) = i_L(0^-) = \frac{V_s}{R_3} = \frac{12}{4.3 \times 10^3} = 2.791\text{ mA}$$

and

$$V_C(0^+) = V_C(0^-) = 7\text{ V}$$

Apply KCL:

$$\begin{aligned} \frac{V_{R2}(0^+) - V_C(0^+)}{R_1} + \frac{V_{R2}(0^+)}{R_2} + i_L(0^+) &= 0 \\ V_{R2}(0^+) &= \frac{\frac{V_C(0^+)}{R_1} - i_L(0^+)}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{(V_C(0^+) - i_L(0^+)R_1)R_2}{R_2 + R_1} \\ &= \frac{(7 - 2.791 \times 10^{-3} \times 9.1 \times 10^3) \times 4.3 \times 10^3}{4.3 \times 10^3 + 9.1 \times 10^3} = -5.93\text{ V} \end{aligned}$$

One could also solve for V_{R2} by superposition.

$$V_{R2}(0^+) = \frac{R_2}{R_1 + R_2}(7\text{ V}) - \frac{R_1 R_2}{R_1 + R_2}(2.8\text{ mA}) = -5.93\text{ V}$$

Problem 5.60

Solution:

Known quantities:

Circuit shown in Figure P5.60,

$$V_{S1} = 15\text{ V}, V_{S2} = 9\text{ V}, R_{S1} = 130\ \Omega, R_{S2} = 290\ \Omega, R_1 = 1.1\text{ k}\Omega, R_2 = 700\ \Omega, L = 17\text{ mH}, C = 0.35\ \mu\text{F}.$$

Find:

The current through and the voltage across the inductor and the capacitor and the current through R_{S2} at $t = 0^+$.

Assumptions:

The circuit is in DC steady-state conditions for $t < 0$.

Analysis:

Since this was not done in the specifications above, you must note on the circuit the assumed polarities of voltages and directions of currents.

At $t = 0^-$:

Assume that steady state conditions exist. At steady state the inductor is modeled as a short circuit and the capacitor as an open circuit. Choose a ground. Note that because the inductor is modeled as a short circuit, there is no voltage drop from the top node to the bottom node and so before the switch there is no current through R_1 .

$$V_L(0^-) = 0 \quad i_C(0^-) = 0$$

Apply KCL:

$$\frac{0 - V_{S1}}{R_{S1}} + i_L(0^-) + \frac{0}{R_1} + 0 + \frac{0 - V_{S2}}{R_{S2}} = 0$$

$$\begin{aligned} i_L(0^-) &= \frac{V_{S1}}{R_{S1}} + \frac{V_{S2}}{R_{S2}} = \\ &= \frac{15}{130} + \frac{9}{290} = 146.4\text{ mA} \end{aligned}$$

Apply KVL:

$$V_C(0^-) + i_C(0^-)R_2 = 0$$

$$V_C(0^-) = 0$$

At $t = 0^+$:

$$i_L(0^+) = i_L(0^-) = 146.4\text{ mA}$$

$$V_C(0^+) = V_C(0^-) = 0$$

Apply KVL:

$$-V_L(0^+) + 0 + i_C(0^+)R_2 = 0$$

$$i_C(0^+) = \frac{V_L(0^+)}{R_2}$$

Apply KCL:

$$\begin{aligned}
 i_L(0^+) + \frac{V_L(0^+)}{R_1} + \frac{V_L(0^+)}{R_2} + \frac{V_L(0^+) - V_{S2}}{R_{S2}} &= 0 \\
 V_L(0^+) &= \frac{\frac{V_{S2} - i_L(0^+)}{R_{S2}}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{S2}}} = \frac{V_{S2} - i_L(0^+) R_{S2}}{\frac{R_{S2}}{R_1} + \frac{R_{S2}}{R_2} + 1} \\
 &= \frac{9 - 146.4 \times 10^{-3} \times 0.29 \times 10^3}{\frac{0.29}{1.1} + \frac{0.29}{0.7} + 1} = -19.94 \text{ V} \\
 i_C(0^+) &= \frac{V_L(0^+)}{R_2} = \frac{-19.94}{0.7 \times 10^3} = -28.49 \text{ mA}
 \end{aligned}$$

Apply KVL again:

$$\begin{aligned}
 -V_L(0^+) + i_{RS2}(0^+) R_{S2} + V_{S2} &= 0 \\
 i_{RS2}(0^+) &= \frac{V_L(0^+) - V_{S2}}{R_{S2}} = \frac{-19.94 - 9}{0.29 \times 10^3} = -99.79 \text{ mA}
 \end{aligned}$$

Problem 5.61

Solution:

Known quantities:

Circuit shown in Figure P5.60,

$$V_{S1} = 12 \text{ V}, V_{S2} = 12 \text{ V}, R_{S1} = 50 \, \Omega, R_{S2} = 50 \, \Omega, R_1 = 2.2 \text{ k}\Omega, R_2 = 600 \, \Omega, L = 7.8 \text{ mH}, C = 68 \, \mu\text{F}.$$

Find:

The voltage across the capacitor and the current through the inductor as t approaches infinity.

Assumptions:

The circuit is in DC steady-state conditions for $t < 0$.

Analysis:

The conditions that exist at $t < 0$ have no effect on steady state conditions as $t \rightarrow \infty$. In the long-term DC steady state the inductor may be modeled as a short circuit and the capacitor as an open circuit. In this case, the inductor short circuits the R_1 branch and the R_2 C branch. Thus, the voltage across these branches and the current through them are zero. In other words all of the current produced by the 12V source travels through the inductor in this case.

Apply KCL;

$$\begin{aligned}
 i_L(\infty) + i_{R1}(\infty) + i_{R2}(\infty) + \frac{0 - V_{S2}}{R_{S2}} &= 0 \\
 i_{R1}(\infty) = i_{R2}(\infty) &= 0 \\
 i_L(\infty) &= \frac{V_{S2}}{R_{S2}} = \frac{12}{50} = 240 \text{ mA}
 \end{aligned}$$

Apply KVL:

$$0 + V_C(\infty) + i_{R2}(\infty)R_2 = 0$$

$$V_C(\infty) = 0$$

Problem 5.62

Solution:

Known quantities:

As described in Figure P5.62.

Find:

An expression for the inductor current for $t \geq 0$.

Assumptions:

The switch has been closed for a long time. It is suddenly opened at $t = 0$ and then reclosed at $t = 5$ s

Analysis:

For $0 \leq t \leq 5$:

Define clockwise mesh currents, i_1 in the lower loop, and i_2 in the upper loop. Then the mesh equations are:

$$(5 + 5s)I_1 - 3I_2 = 0$$

$$-3I_1 + (3 + \frac{1}{4s})I_2 = 0$$

from which we determine that

$$(5 + 5s)(3 + \frac{1}{4s}) - 9 = 0$$

$$s = 0.242 \pm j0.158$$

Therefore, the inductor current is of the form:

$$i(t) = e^{-0.242t} [A \cos(0.158t) + B \sin(0.158t)]$$

From the initial conditions:

$$i(0) = \frac{6}{3} = 2 = A$$

$$L \frac{di}{dt} \Big|_{t=0} = 5 \frac{di}{dt} \Big|_{t=0} = V_C(0^+) = -10$$

$$\Rightarrow \frac{di}{dt} \Big|_{t=0} = -2 = -0.242A + 0.158B$$

Solving the above equations:

$$A = 2 \quad B = -9.59$$

$$i(t) = e^{-0.242t} [2 \cos(0.158t) - 9.59 \sin(0.158t)] \text{ A} \quad \text{for } 0 \leq t \leq 5 \text{ s}$$

The solution for capacitor voltage will have the same form.

$$V_C(t) = e^{-0.242t} [A \cos(0.158t) + B \sin(0.158t)] \text{ V}$$

From the initial conditions:

$$V_C(0) = 6 = A$$

$$\frac{dV_C}{dt} \Big|_{t=0} = \frac{1}{C} i_C(0) = 0 \Rightarrow -0.242A + 0.158B = 0$$

Solving the above equations:

$$A = 6 \quad B = 9.18$$

$$V_C(t) = e^{-0.242t} [6 \cos(0.158t) + 9.18 \sin(0.158t)] V \quad \text{for } 0 \leq t \leq 5 \text{ s}$$

From the above results:

$$V_C(5) = 3.2807 V$$

$$i(5) = -1.641 A$$

These are the initial conditions for the solution after the switch recloses.

For $t \geq 5$:

The mesh equations are:

$$\begin{aligned} (3 + 5s)I_1 - 3I_2 &= \frac{6}{s} + 5i(5) \\ -3I_1 + (3 + \frac{1}{4s})I_2 &= -\frac{V_C(5)}{s} \end{aligned}$$

from which we determine that

$$\begin{aligned} 60s^2 + 5s + 3 &= 0 \\ s &= 0.041 \pm j0.220 \end{aligned}$$

Therefore, the inductor current is of the form:

$$i(t) = 2 + e^{-0.041t} \{A \cos[0.220(t-5)] + B \sin[0.220(t-5)]\}$$

From the initial conditions:

$$2 + A = -1.641 \Rightarrow A = -3.641$$

$$\begin{aligned} \frac{di}{dt} \Big|_{t=5} &= \frac{V_L(5)}{5} = \frac{6 - 238}{5} = 0.543 \\ \Rightarrow -0.41A + 0.220B &= 0.543 \end{aligned}$$

Solving the above equations:

$$A = -3.641 \quad B = 1.77$$

$$i(t) = 2 + e^{-0.041t} \{-3.641 \cos[0.220(t-5)] + 1.77 \sin[0.220(t-5)]\} A \quad \text{for } t \geq 5 \text{ s}$$

This, together with the previous result, gives the complete solution to the problem.

Problem 5.63

Solution:

Known quantities:

As described in Figure P5.63.

Find:

Determine if the circuit is underdamped or overdamped. The capacitor value that results in critical dumping.

Assumptions:

The circuit initially stores no energy. The switch is closed at $t = 0$.

Analysis:

a) For $t \geq 0$:

The characteristic polynomial is:

$$Ls^2 + Rs + \frac{1}{C} = 0$$

The damping ratio is:

$$\xi = \frac{RC}{2} \sqrt{\frac{1}{LC}} = \frac{400 \cdot 10^{-8}}{2} \sqrt{10^{10}} = 0.2 < 1$$

The system is underdamped, in fact we have the following complex conjugate roots:

$$s_{1,2} = -2 \times 10^4 \pm j(9.79 \times 10^4)$$

b) The capacitor value that results in critical damping is:

$$\xi = \frac{RC}{2} \sqrt{\frac{1}{LC}} = 1 \Rightarrow 200C \sqrt{\frac{100}{C}} = 1$$

$$200^2 C^2 \frac{100}{C} = 1 \Rightarrow C = \frac{1}{4 \times 10^6} F = 0.25 \mu F$$

Problem 5.64

Solution:

Known quantities:

As described in Figure P5.63.

Find:

- The capacitor voltage as t approaches infinity
- The capacitor voltage after $20 \mu s$
- The maximum capacitor voltage.

Assumptions:

The circuit initially stores no energy. The switch is closed at $t = 0$.

Analysis:

For $t \geq 0$:

The characteristic polynomial is:

$$0.01s^2 + 400s + 10^8 = 0$$

$$s = -2 \times 10^4 \pm j(9.79 \times 10^4)$$

Therefore, the solution is of the form:

$$V_C(t) = 10 + e^{-2 \times 10^4 t} [A \cos(9.79 \times 10^4 t) + B \sin(9.79 \times 10^4 t)]$$

From the initial conditions:

$$10 + A = 0$$

$$-2 \times 10^4 A + 9.79 \times 10^4 B = 0$$

Solving the above equations:

$$A = -10$$

$$B = -2.04$$

$$V_C(t) = 10 + e^{-2 \times 10^4 t} [-10 \cos(9.79 \times 10^4 t) - 2.04 \sin(9.79 \times 10^4 t)] V$$

a) The capacitor voltage as t approaches infinity is:

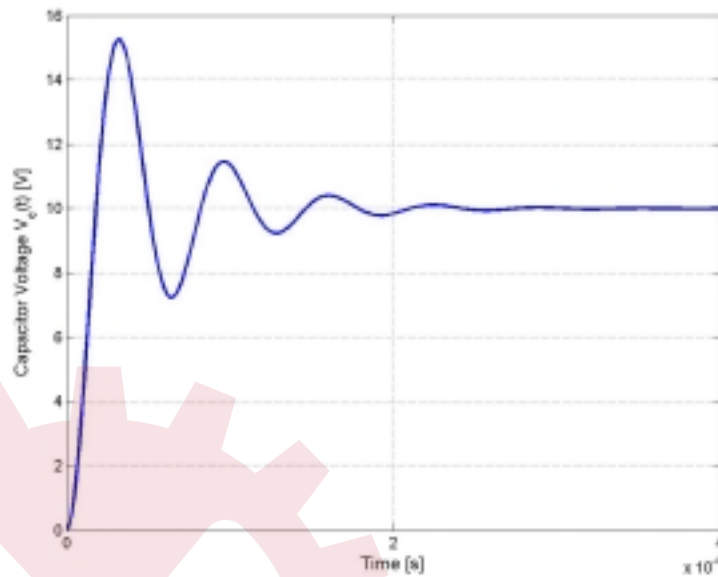
$$V_C(\infty) = 10V$$

b) The capacitor voltage after $20 \mu s$ is:

$$V_C(20 \mu s) = 11.26V$$

c) Graphically, the maximum capacitor voltage is:

$$V_C^{\max} \cong 15V$$



Problem 5.65

Solution:

Known quantities:

As described in Figure P5.65.

Find:

An expression for the capacitor voltage for $t \geq 0$.

Assumptions:

The circuit initially stores no energy, the switch S_1 is open and the switch S_2 is closed. The switch S_1 is closed at $t = 0$ and the switch is opened S_2 at $t = 5 \text{ s}$.

Analysis:

This circuit has the same configuration during the interval $0 \leq t \leq 5 \text{ s}$ as the one for Problem 5.39 did for $t > 5 \text{ s}$. Therefore, the roots of the characteristic polynomial will be the same as those determined in that problem. They are:

$$s = -0.041 \pm j0.220$$

Ad the general form of the capacitor voltage is

$$V_C(t) = 6 + e^{-0.041t} [A \cos(0.220t) + B \sin(0.220t)]$$

For $0 \leq t \leq 5 \text{ s}$:

The initial conditions are:

$$V_C(0) = 0 \Rightarrow 6 + A = 0$$

$$\left. \frac{dV_C}{dt} \right|_{t=0} = \frac{1}{C} i_C(0) = 0$$

$$\Rightarrow -0.41A + 0.220B = 0$$

Solving the above equations:

$$A = -6 \quad B = -0.904$$

$$V_C(t) = 6 + e^{-0.041t} [-6 \cos(0.220t) - 0.904 \sin(0.220t)] V \quad \text{for } 0 \leq t \leq 5 \text{ s}$$

Note that $V_C(5) = 3.127 V$.

For $t > 5 \text{ s}$:

We have a simple RC decay:

$$V_C(t) = 3.127 e^{\frac{t-5}{12}} V$$

Problem 5.66

Solution:

Known quantities:

Circuit shown in Figure P5.66, $C = 1.6 \text{ nF}$; After the switch is closed at $t = 0$, the capacitor voltage reaches an initial peak value of $70 V$ when $t = 5\pi/3 \mu\text{s}$, a second peak value of $53.2 V$ when $t = 5\pi \mu\text{s}$, and eventually approaches a steady-state of $50 V$.

Find:

The values of R and L .

Assumptions:

The circuit is underdamped and the circuit initially stores no energy.

Analysis:

Using the given characteristics of the circuit step response and the assumption that the circuit is underdamped, the damping ratio and natural frequency can be determined as follows:

$$\zeta = \frac{1}{\sqrt{(\pi / \ln(a / A))^2 + 1}}$$

where a is the overshoot distance and A is the steady-state value

$$\omega_n = \frac{2\pi}{T\sqrt{1-\zeta^2}}$$

where T is the period of oscillation

In our case,

$$a = 70 - 50 = 20 \text{ and } A = 50$$

$$\zeta = \frac{1}{\sqrt{(\pi / \ln(20 / 50))^2 + 1}} = 0.28$$

The period of the waveform is

$$T = \left(5\pi - \frac{5\pi}{3}\right) \times 10^{-6} = \frac{10\pi}{3} \times 10^{-6}$$

$$\omega_n = \frac{2\pi}{\frac{10\pi}{3} \times 10^{-6} \sqrt{1-0.28^2}} = 6.25 \times 10^5$$

Implying the characteristic polynomial for the circuit is

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

Compare this with the standard form of the characteristic polynomial for a series RLC circuit:

$$s^2 + \frac{R}{L}s + \frac{1}{LC}$$

Matching terms yields $L = 1.6 \mu H, R = 0.56 \Omega$.

Problem 5.67

Solution:

Known quantities:

Same as P5.66, but the first two peaks occur at $5\pi \mu s$ and $15\pi \mu s$

Find:

Explain how to modify the circuit to meet the requirements.

Assumptions:

The capacitor value C cannot be changed.

Analysis:

Assuming we wish to retain the same peak amplitudes, we proceed as follows:

The new period is

$$T = 15\pi \times 10^{-6} - 5\pi \times 10^{-6} = 10\pi \times 10^{-6}$$

Using the given characteristics of the circuit step response and the assumption that the circuit is underdamped, the damping ratio and natural frequency can be determined as follows:

$$\zeta = \frac{1}{\sqrt{(\pi / \ln(a / A))^2 + 1}}$$

where a is the overshoot distance and A is the steady-state value

$$\omega_n = \frac{2\pi}{T\sqrt{1-\zeta^2}}$$

where T is the period of oscillation

In our case,

$$a = 70 - 50 = 20 \text{ and } A = 50$$

$$\zeta = \frac{1}{\sqrt{(\pi / \ln(20 / 50))^2 + 1}} = 0.28$$

$$\omega_n = \frac{2\pi}{10\pi \times 10^{-6} \sqrt{1-0.28^2}} = 2.17 \times 10^5$$

Implied the characteristic polynomial for the circuit is

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

Compare this with the standard form of the characteristic polynomial for a series RLC circuit:

$$s^2 + \frac{R}{L}s + \frac{1}{LC}$$

Matching terms yields $L = 13.3 \mu H, R = 1.61 \Omega$.

Note that the frequency for this problem is one-third that of Problem 5.66, the inductance is 3^2 times that of Problem 5.66, and the resistance is 3 times that of Problem 5.66.

Problem 5.68

Solution:

Known quantities:

Circuit shown in Figure P5.68, $i(0) = 0$ A, $V(0) = 10$ V.

Find:

$i(t)$ for $t > 0$.

Assumptions:

None.

Analysis:

The initial condition for the capacitor voltage is $V(0^-) = 10$ V. Applying KCL,

$$i_R + i_C + i = 0$$

where

$$i_R = \frac{V}{1\Omega}, \quad i_C = 0.5 \frac{dV}{dt}$$

Therefore,

$$i + V + 0.5 \frac{dV}{dt} = 0$$

where

$$V = 2 \frac{di}{dt} + 4i$$

Thus,

$$\frac{d^2 i}{dt^2} + 4 \frac{di}{dt} + 5i = 0$$

Solving the differential equation:

$$i(t) = k_1 e^{(-2+j)t} + k_2 e^{(-2-j)t} \quad t > 0$$

$$i(0) = k_1 + k_2 = 0$$

$$V(0) = 2 \frac{di(0)}{dt} + 4i(0) = -(4-j2)k_1 - (4+j2)k_2 = 10$$

Solving for k_1 and k_2 and substituting, we have

$$i(t) = -j \frac{5}{2} e^{(-2+j)t} + j \frac{5}{2} e^{(-2-j)t} \text{ A} \quad \text{for } t > 0$$

Problem 5.69**Solution:****Known quantities:**

As described in Figure P5.69.

Find:

The maximum value of V .

Assumptions:

The circuit is in steady state at $t = 0^-$.

Analysis:

$$V(0^-) = V(0^+) = 0$$

Applying KVL:

$$\frac{d^2V}{dt^2} + 4\frac{dV}{dt} + 4V = 48$$

Solving the differential equation:

$$V = k_1 e^{-2t} + k_2 t e^{-2t} + 12$$

From the initial condition:

$$V(0) = 0 \Rightarrow k_1 = -12$$

$$i_L(0) = C \frac{dV(0)}{dt} \Rightarrow 6 + \frac{k_2}{4} = 3 \Rightarrow k_2 = -12$$

$$V(t) = -12e^{-2t} - 12te^{-2t} + 12V \quad \text{for } t > 0$$

The maximum value of V is:

$$V_{\max} = V(\infty) = 12V$$

Problem 5.70**Solution:****Known quantities:**

As described in Figure P5.70.

Find:

The value of t such that $i = 2.5 \text{ A}$.

Assumptions:

The circuit is in steady state at $t = 0^-$.

Analysis:

In steady state, the inductors behave as short circuits. Using mesh analysis, we can find the initial conditions.

$$i(0^-) = i(0^+) = 5 \text{ A}$$

$$V(0^-) = 0 \text{ V}$$

After the switch is closed, the circuit is modified.

Applying nodal analysis:

$$V(0^+) = 0 \text{ V}$$

$$\frac{d^2 i}{dt^2} + 7 \frac{di}{dt} + 6i = 0$$

Solving the differential equation:

$$i(t) = k_1 e^{-t} + k_2 e^{-6t} \quad \text{for } t > 0$$

$$i(0) = k_1 + k_2 = 5$$

$$V(0) = -k_1 - 6k_2 = 0$$

Solving for the unknown constants, using the initial conditions, we have:

$$i(t) = 6e^{-t} - e^{-6t} \text{ A} \quad \text{for } t > 0$$

Therefore,

$$i(\bar{t}) = 6e^{-\bar{t}} - e^{-6\bar{t}} = 2.5 \Rightarrow \bar{t}_1 = 873 \text{ ms}$$

Problem 5.71

Solution:

Known quantities:

As described in Figure P5.71.

Find:

The value of t such that $i = 6 \text{ A}$.

Assumptions:

The circuit is in steady state at $t = 0^-$.

Analysis:

In steady state, the inductors behave as short circuits. Using mesh analysis, we can find the initial conditions.

$$i(0^-) = i(0^+) = 12.5 \text{ A}$$

$$V(0^-) = 0 \text{ V}$$

After the switch is closed, the circuit is modified.

Applying nodal analysis:

$$V(0^+) = -15 \text{ V}$$

$$\frac{d^2 i}{dt^2} + 7 \frac{di}{dt} + 6i = 0$$

Solving the differential equation:

$$i(t) = k_1 e^{-t} + k_2 e^{-6t} \quad \text{for } t > 0$$

$$i(0) = k_1 + k_2 = 12.5$$

$$V(0) = -k_1 - 6k_2 = -15$$

Solving for the unknown constants, using the initial conditions, we have:

$$i(t) = 12e^{-t} + 0.5e^{-6t} \text{ A} \quad \text{for } t > 0$$

Therefore,

$$i(\bar{t}) = 12e^{-\bar{t}} + 0.5e^{-6\bar{t}} = 6 \Rightarrow \bar{t}_1 = 694 \text{ ms}$$

Problem 5.72**Solution:****Known quantities:**

As described in Figure P5.72.

Find:

The value of t such that $V = 7.5V$.

Assumptions:

The circuit is in steady state at $t = 0^-$.

Analysis:

The circuit at $t = 0^-$ has the capacitors replaced by open circuits.
By current division:

$$i_{2\Omega} = \frac{3}{3+5} \times 20 = 7.5 \text{ A}$$

$$V(0^-) = V(0^+) = 2i_2 = 15V$$

$$i(0^-) = 0A$$

After the switch opens, apply KCL:

$$i(0^+) = 0A$$

$$i + i_1 + i_2 = 0$$

$$i = C \frac{dV}{dt} = \frac{1}{6} \frac{dV}{dt}, \quad i_2 = \frac{V}{2}$$

$$i_1 = -\frac{1}{6} \frac{dV}{dt} - \frac{V}{2}$$

Applying KVL:

$$V = 3i_1 + V_1 = 3\left(-\frac{1}{6} \frac{dV}{dt} - \frac{V}{2}\right) + V_1$$

$$V_1 = V - 3\left(-\frac{1}{6} \frac{dV}{dt} - \frac{V}{2}\right)$$

$$i_1 = \frac{1}{6} \frac{dV_1}{dt} = \frac{1}{12} \frac{d^2V}{dt^2} + \frac{5}{12} \frac{dV}{dt}$$

Applying KCL:

$$\frac{1}{12} \frac{d^2V}{dt^2} + \frac{5}{12} \frac{dV}{dt} + \frac{V}{2} + \frac{1}{6} \frac{dV}{dt} = 0$$

$$\Rightarrow \frac{d^2V}{dt^2} + 7 \frac{dV}{dt} + 6V = 0$$

Solving the differential equation,

$$V(t) = k_1 e^{-t} + k_2 e^{-6t} \quad \text{for } t > 0$$

$$V(0) = k_1 + k_2 = 15$$

$$i(0) = \frac{1}{6}(-k_1 - 6k_2) = 0$$

Solving for the unknown constants, using the initial conditions, we have:

$$V(t) = 18e^{-t} - 3e^{-6t} \text{ V for } t > 0$$

Therefore,

$$V(\bar{t}) = 18e^{-\bar{t}} - 3e^{-6\bar{t}} = 7.5 \Rightarrow \bar{t}_1 = 873 \text{ ms}$$

Problem 5.73

Solution:

Known quantities:

As described in Figure P5.73.

Find:

The maximum value of V and the maximum voltage between the contacts of the switches.

Assumptions:

The circuit is in steady state at $t = 0^-$. $L = 3 \text{ H}$.

Analysis:

At $t = 0^-$:

$$i(0^-) = i(0^+) = \frac{10}{5} = 2 \text{ A}$$

$$V(0^-) = V(0^+) = 0 \text{ V}$$

After the switch is closed:

Applying KVL:

$$\begin{aligned} \frac{1}{L} \int_{-\infty}^t V dt + \frac{1}{12} \frac{dV}{dt} + \frac{V}{3} &= 0 \\ \Rightarrow \frac{d^2V}{dt^2} + 4 \frac{dV}{dt} + \frac{12}{L} V &= 0 \end{aligned}$$

The particular response is zero for $t > 0$ because the circuit is source-free.

$$L = 3 \text{ H} \Rightarrow s^2 + 4s + 4 = 0$$

$$s_1 = s_2 = -2$$

$$V(t) = e^{-2t} (A + Bt) \quad \text{for } t > 0$$

From the initial condition:

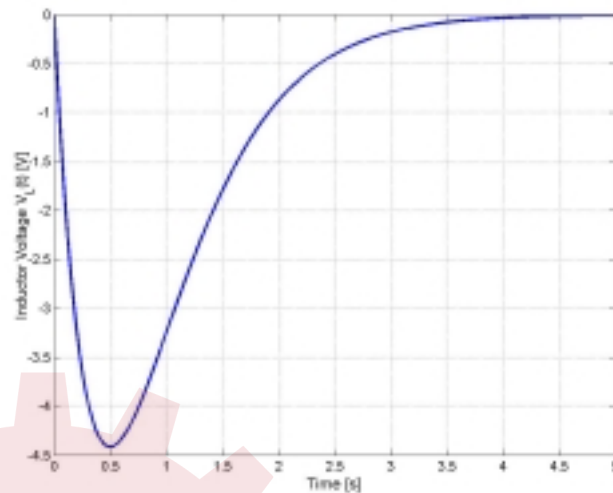
$$V(0^+) = 0 = e^0 (A + B(0)) \Rightarrow A = 0$$

Substitute the solution into the original KCL equation and evaluate at $t = 0^+$:

$$\frac{1}{3}(0) + \frac{1}{12} \frac{d}{dt} [e^{-2t} (A + Bt)]|_{t=0} + 2 = 0$$

$$0 + \frac{1}{12} [Be^{-2t} - 2Bte^{-2t}]|_{t=0} + 2 = 0 \Rightarrow B = -24$$

$$V(t) = -24te^{-2t} \text{ V for } t > 0$$



The maximum absolute value of V is:

$$V_{\max} = V(t = 0.5s) = \left| -\frac{12}{e} \right| V \cong 4.414V$$

The maximum voltage between the contacts of the switches is:

$$V_{\text{switch}}^{\text{MAX}} = V_S = 10V$$

since the voltage between the contacts of the switches is a constant.

Problem 5.74

Solution:

Known quantities:

As described in Figure P5.74.

Find:

V at $t > 0$.

Assumptions:

The circuit is in steady state at $t = 0^-$.

Analysis:

At $t = 0^-$:

$$V(0^-) = 12V$$

$$i_L(0^-) = i_L(0^+) = 6A$$

$$i_C(0^-) = 0A$$

$$V_C(0^-) = V_C(0^+) = 4V$$

$$V_L(0^-) = 0V$$

For $t > 0$:

$$i_C(0^+) = -2A$$

$$V_L(0^+) = 4V$$

$$V(0^+) = 8V$$

$$\frac{dV}{dt}(0^+) = -\frac{dV_C}{dt}(0^+) = -\frac{i_C}{C} = -\frac{-2}{1/4} = 8$$

Using KVL and KCL, we can find the differential equation for the voltage in the resistor:

$$12 - 2i - 0.8 \frac{di_L}{dt} = 0$$

$$i = i_L + i_C \Rightarrow i_L = i - i_C$$

$$i_C = C \frac{dV_C}{dt} = \frac{1}{4} \frac{d}{dt}(12 - V) = -\frac{1}{4} \frac{dV}{dt}$$

$$12 - 2i - 0.8 \frac{d}{dt}(i - i_C) = 0 \Rightarrow 12 - 2i - 0.8 \frac{di}{dt} + 0.8 \frac{d}{dt}\left(-\frac{1}{4} \frac{dV}{dt}\right) = 0$$

$$i = V/2 \Rightarrow 12 - V - 0.4 \frac{dV}{dt} - 0.2 \frac{d^2V}{dt^2} = 0$$

$$0.2 \frac{d^2V_{2\Omega}}{dt^2} + 0.4 \frac{dV_{2\Omega}}{dt} + V_{2\Omega} = 12$$

$$\Rightarrow \frac{d^2V_{2\Omega}}{dt^2} + 2 \frac{dV_{2\Omega}}{dt} + 5V_{2\Omega} = 60$$

Solving the differential equation:

Homogeneous Solution:

$$V_{2\Omega,h} = K_1 e^{(-1+2j)t} + K_2 e^{(-1-2j)t} \quad t > 0$$

$$V(0) = 8V, \frac{dV}{dt}(0) = 8$$

$$K_1 + K_2 = 8$$

$$(-1+2j)K_1 - (1+2j)K_2 = 8$$

$$K_1 = 4 - 4j$$

$$K_2 = 4 + 4j$$

$$V_{2\Omega,h} = (4 - 4j)e^{(-1+2j)t} + (4 + 4j)e^{(-1-2j)t} \quad t > 0$$

Particular Solution:

$$V_{2\Omega,p} = (-6 + 3j)e^{(-1+2j)t} + (-6 - 3j)e^{(-1-2j)t} + 12 \quad t > 0$$

The Total Solution:

$$V_{2\Omega} = V_{2\Omega,h} + V_{2\Omega,p}$$

$$V_{2\Omega} = (4 - 4j)e^{(-1+2j)t} + (4 + 4j)e^{(-1-2j)t} + (-6 + 3j)e^{(-1+2j)t} + (-6 - 3j)e^{(-1-2j)t} + 12$$

$$V_{2\Omega} = (-2 - j)e^{(-1+2j)t} + (-2 + j)e^{(-1-2j)t} + 12 \quad t > 0$$

Chapter 6 Instructor Notes

Chapter 6 has also seen substantive revisions with respect to the Third Edition. A section on Fourier analysis has been added (Section 6.2), and the material on Bode plots has been expanded (Section 6.4). The material on Laplace transforms has been moved to the Appendix B. These changes were prompted by comments and suggestions made by numerous users. Chapter 6 can be covered immediately following Chapter 4, or after completing Chapter 5. There is no direct dependence of Chapter 6 on Chapter 5.

After the first section briefly introduces the notion of sinusoidal frequency response and motivates the use of sinusoidal signals, the Fourier Series method of representing signals is described in detail in Section 6.2. Further, the text and examples also illustrate the effect of a multi-components signal propagating through a linear system. Four examples accompany this presentation. Instructors who use this material will find some computing tools available on the website that will assist the students in developing an intuitive understanding of Fourier series.

Section 6.3 introduces filters, and outlines the basic characteristics of low-, high- and band-pass filters. The concept of resonance is treated in greater depth than in the previous edition, and a connection is made with the natural response of second order circuits, which may be useful to those instructors who have already covered transient response of second-order circuits. Four detailed examples are included in this section. Further, the boxes *Focus on Measurements: Wheatstone bridge filter* (pp. 310-303), *Focus on Measurements: AC line interference filter* (pp. 303-305), and *Focus on Measurements: Seismic displacement transducer* (pp. 305-308) touch on additional application examples. The first and last of these boxes can be linked to related material in Chapters 2, 3, and 4.

The instructor who has already introduced the operational amplifier as a circuit element will find that section 8.3, on active filters, is an excellent vehicle to reinforce both the op-amp concept and the frequency response ideas. Another alternative (employed by this author) consists of introducing the op-amp at this stage, covering sections 8.1 through 8.3.

Finally, Section 6.4 expands the previous coverage of Bode plots, and illustrates how to create approximate Bode plots using the straight-line asymptotic approximation. The box *Focus on Methodology: Bode Plots* clearly outlines the method, which is further explained in two examples.

The homework problems present several frequency response, Fourier Series, filter and Bode plot exercises of varying difficulty. The instructor who wishes to use one of the many available software aids (e.g., MATLAB® or Electronics Workbench®) to analyze the frequency response of more complex circuits and to exploit more advanced graphics capabilities, will find that several advanced problems lend themselves nicely to such usage. More advanced problems could be used as a vehicle to introduce modern computer aids. The computer aided example solutions found in the Virtual Lab CD-ROM will guide the student in the solution of these more advanced problems.

Learning Objectives

1. Understand the physical significance of frequency domain analysis, and compute the frequency response of circuits using AC circuit analysis tools.
2. Compute the Fourier spectrum of periodic signals using the Fourier series representation, and use this representation in connection with frequency response ideas to compute the response of circuits to periodic inputs.
3. Analyze simple first- and second-order electrical filters, and determine their frequency response and filtering properties.
4. Compute the frequency response of a circuit and its graphical representation in the form of a Bode plot.

Section 6.1: Sinusoidal Frequency Response

Problem 6.1

Solution:

Known quantities:

Resistance and inductance values, in the circuit of Figure P6.1, $R = 200 \text{ k}\Omega$ and $L = 0.5 \text{ H}$, respectively.

Find:

- The frequency response for the circuit of Figure P6.1.
- Plot magnitude and phase of the circuit using a linear scale for frequency.
- Repeat part b., using semilog paper.
- Plot the magnitude response using semilog paper with magnitude in dB.

Analysis:

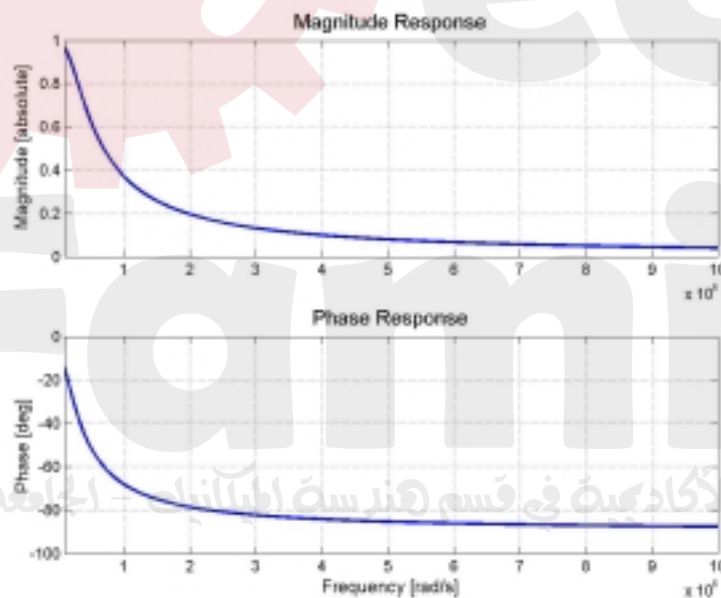
$$a) \quad \frac{v_{out}}{v_{in}}(j\omega) = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L/R} = \frac{1}{1 + j(2.5 \times 10^{-6} \omega)}$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{\sqrt{1 + (\omega L/R)^2}} = \frac{1}{\sqrt{1 + 6.25 \times 10^{-12} \omega^2}}$$

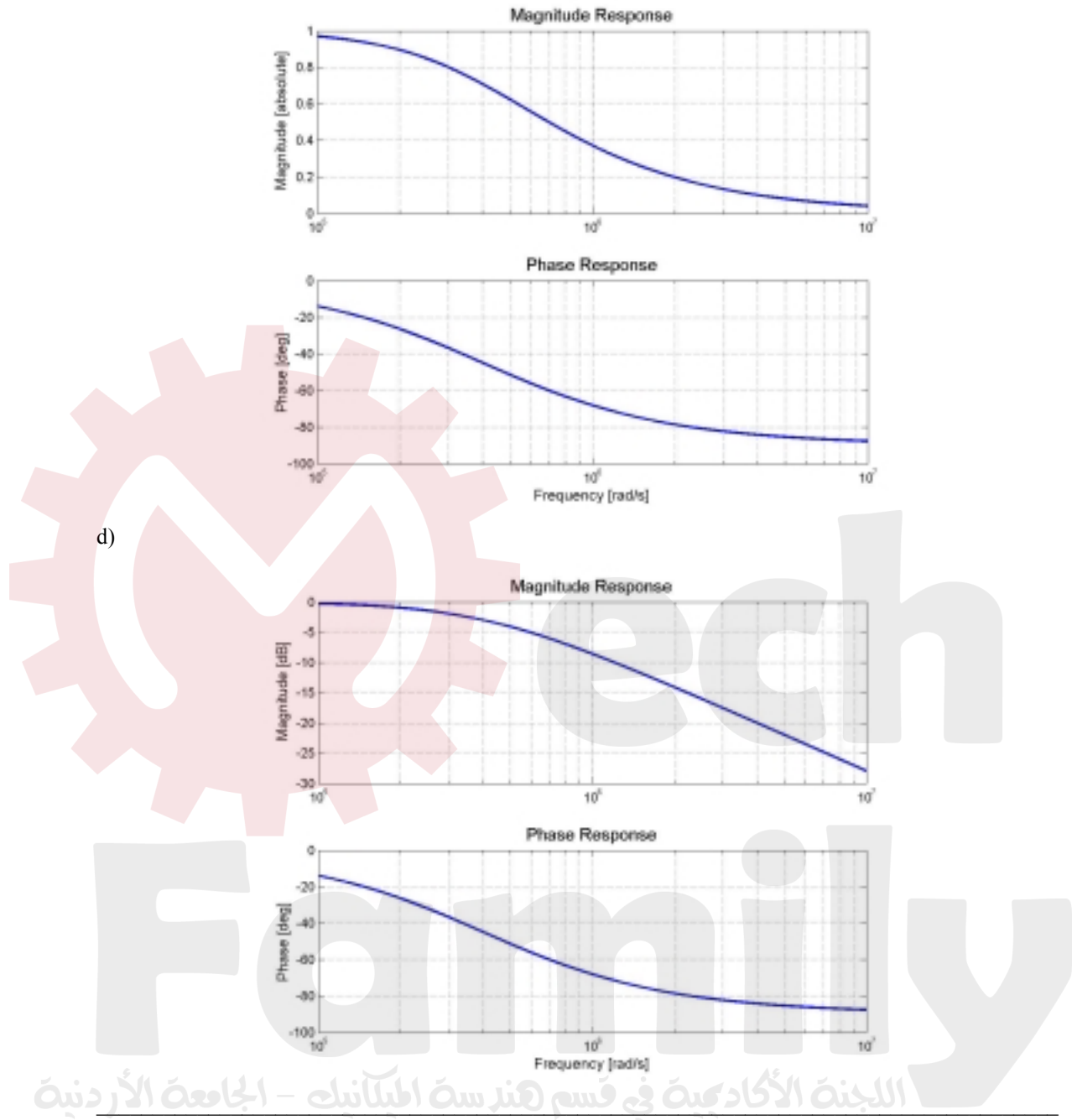
$$\phi(\omega) = -\arctan(2.5 \times 10^{-6} \omega)$$

The plots obtained using Matlab are shown below:

b)



c)



Problem 6.2

Solution:

Known quantities:

Resistance and capacitance values, in the circuit of Figure P6.2.

Find:

- The frequency response for the circuit of Figure P6.2.
- Plot magnitude and phase of the circuit using a linear scale for frequency.
- Repeat part b., using semilog paper.
- Plot the magnitude response using semilog paper with magnitude in dB.

Analysis:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$R_T = 500 \parallel 500 = 250 \, \Omega$$

and

$$v_{OC} = \frac{500}{500 + 500} v_{in} = \frac{v_{in}}{2}$$

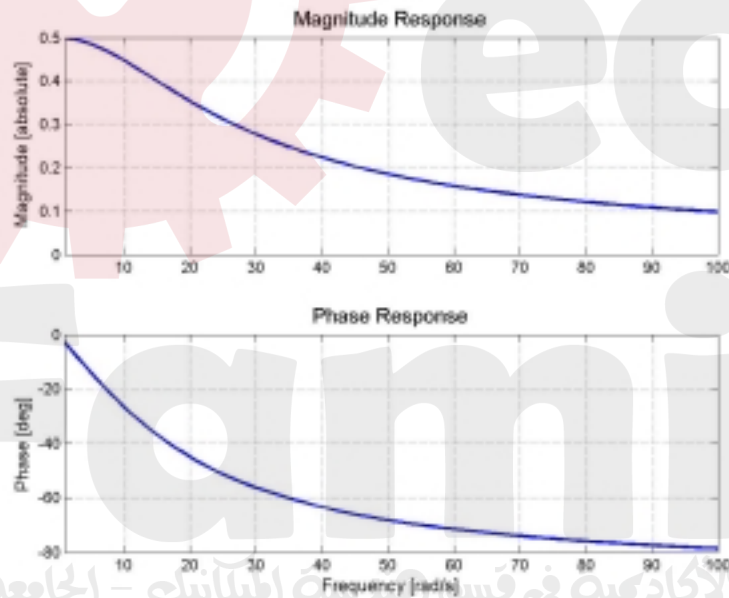
$$a) \quad \frac{v_{out}}{v_{OC}} = \frac{1/j\omega C}{R_T + 1/j\omega C} = \frac{1}{1 + j\omega R_T C} = \frac{1}{1 + j(0.05\omega)}$$

$$\left| \frac{v_{out}}{v_{OC}} \right| = \frac{1}{\sqrt{1 + 0.0025\omega^2}}$$

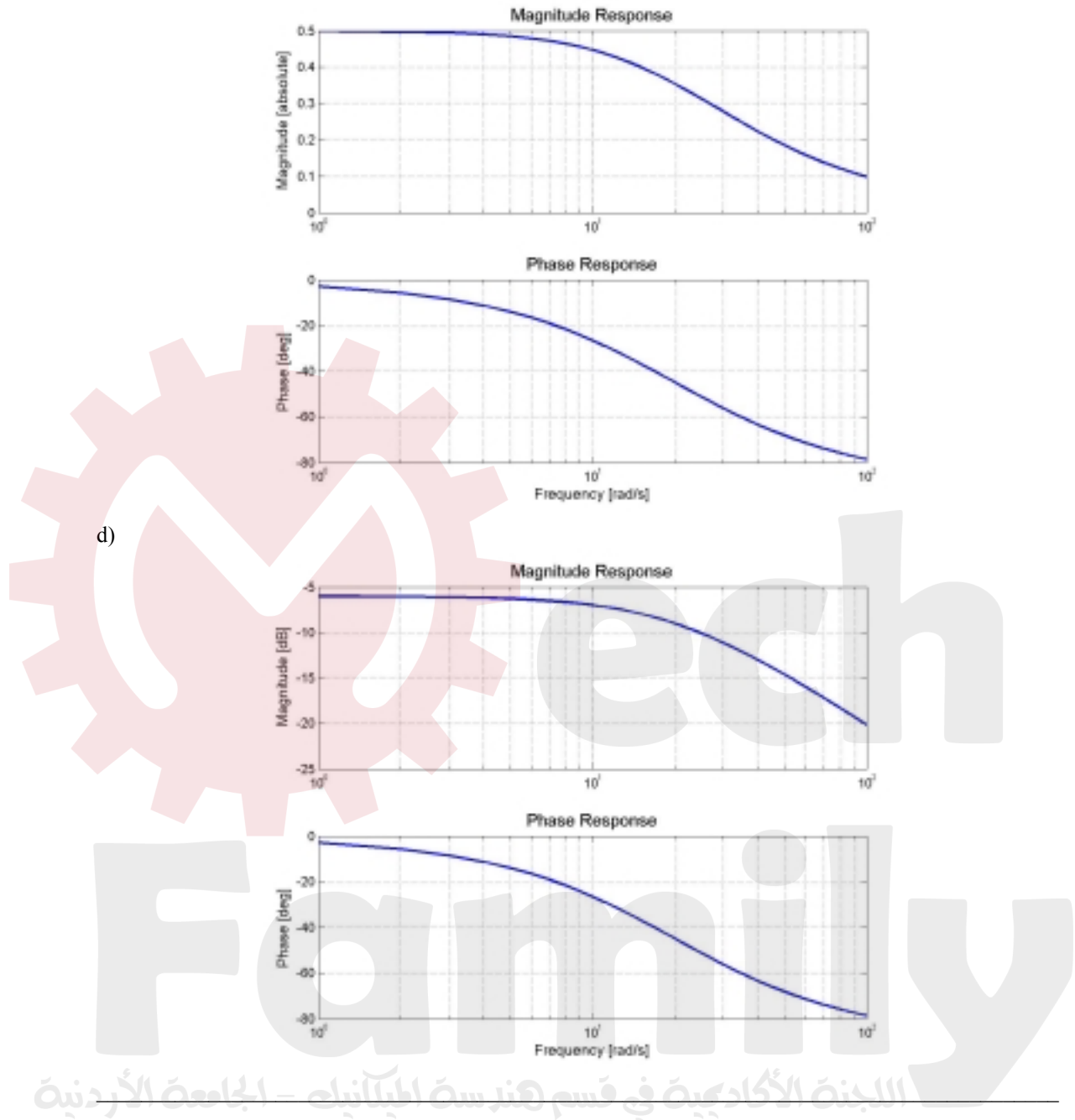
$$\left| \frac{v_{out}}{v_{in}} \right| = \frac{1}{2} \left| \frac{v_{out}}{v_{OC}} \right| = \frac{1}{\sqrt{4 + 0.01\omega^2}}$$

$$\phi(\omega) = -\arctan(0.05\omega)$$

b) The plots obtained using Matlab are shown below:



c)



Problem 6.3

Solution:

Known quantities:

Resistance and capacitance values, in the circuit of Figure P6.3.

Find:

- The frequency response for the circuit of Figure P6.3.
- Plot magnitude and phase of the circuit using a linear scale for frequency.
- Repeat part b., using semilog paper.
- Plot the magnitude response using semilog paper with magnitude in dB.

Analysis:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$R_T = 2000 \parallel 2000 + 1000 = 2000 \quad \Omega$$

and

$$v_{OC} = \frac{2000}{2000 + 2000} v_{in} = \frac{v_{in}}{2}$$

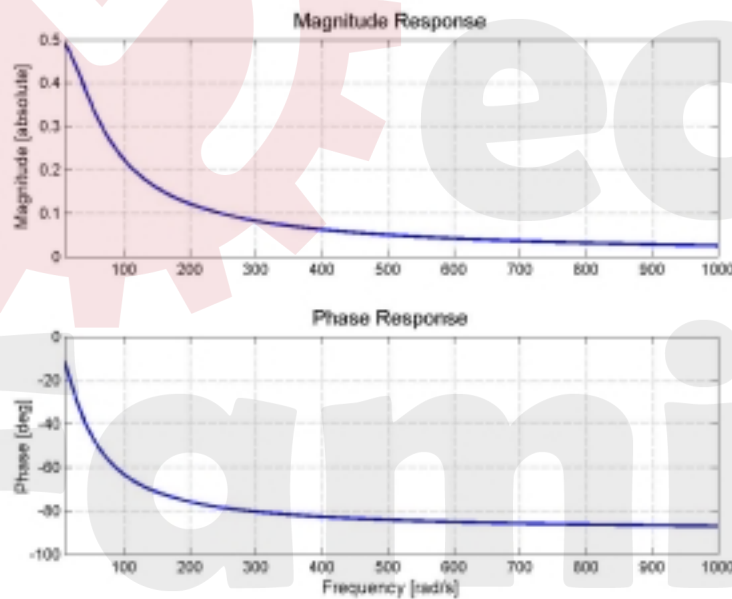
$$a) \quad \frac{v_{out}}{v_{OC}} = \frac{1/j\omega C}{R_T + 1/j\omega C} = \frac{1}{1 + j\omega R_T C} = \frac{1}{1 + j(0.02\omega)}$$

$$\left| \frac{v_{out}}{v_{OC}} \right| = \frac{1}{\sqrt{1 + 4 \times 10^{-4} \omega^2}}$$

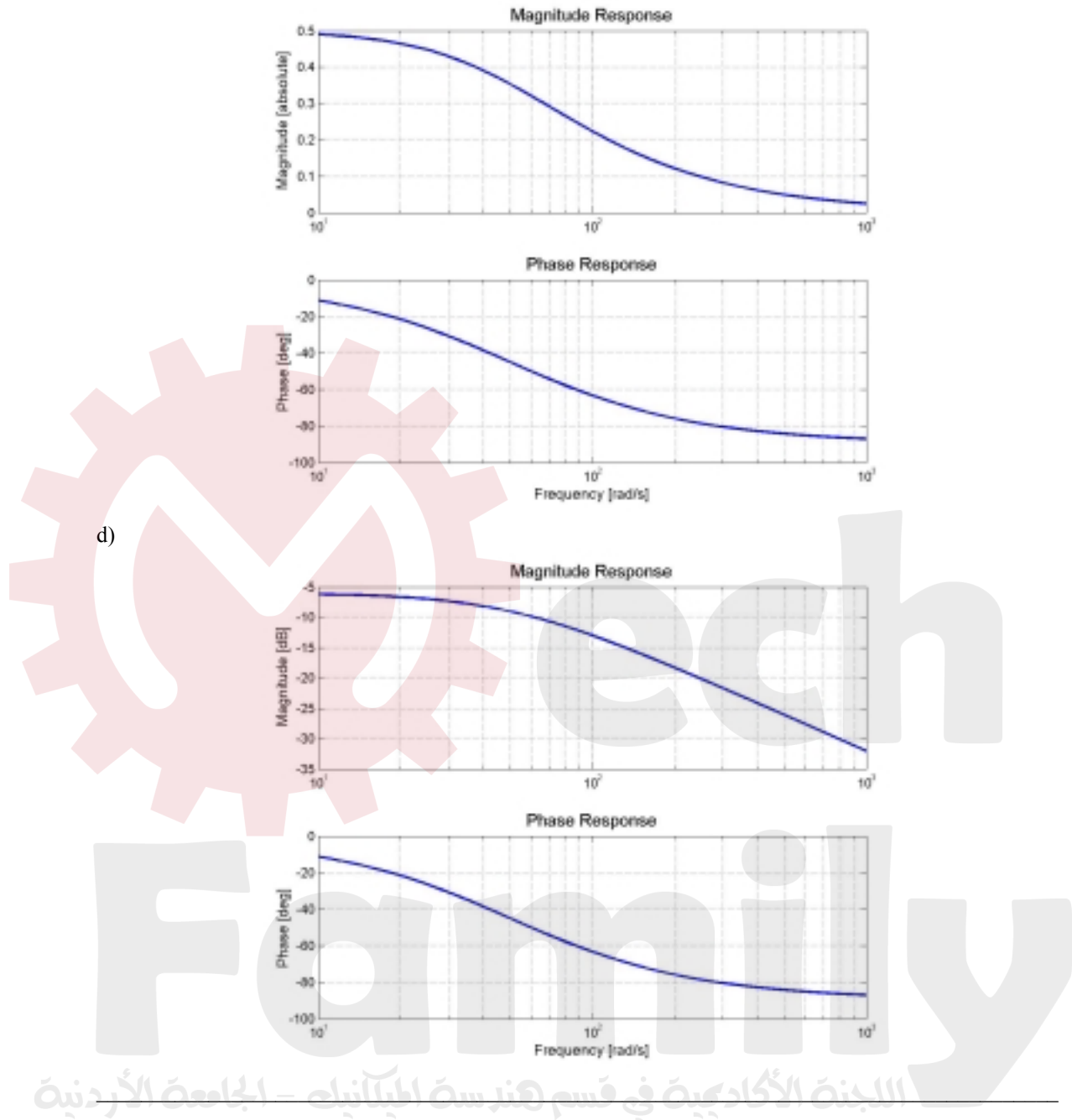
$$\left| \frac{v_{out}}{v_{in}} \right| = \frac{1}{2} \left| \frac{v_{out}}{v_{OC}} \right| = \frac{0.5}{\sqrt{1 + 4 \times 10^{-4} \omega^2}}$$

$$\phi(\omega) = -\arctan(0.02\omega)$$

b) The plots obtained using Matlab are shown below:



c)



Problem 6.4

Solution:

Known quantities:

Resistance, inductance and capacitance values, in the circuit of Figure P6.4.

Find:

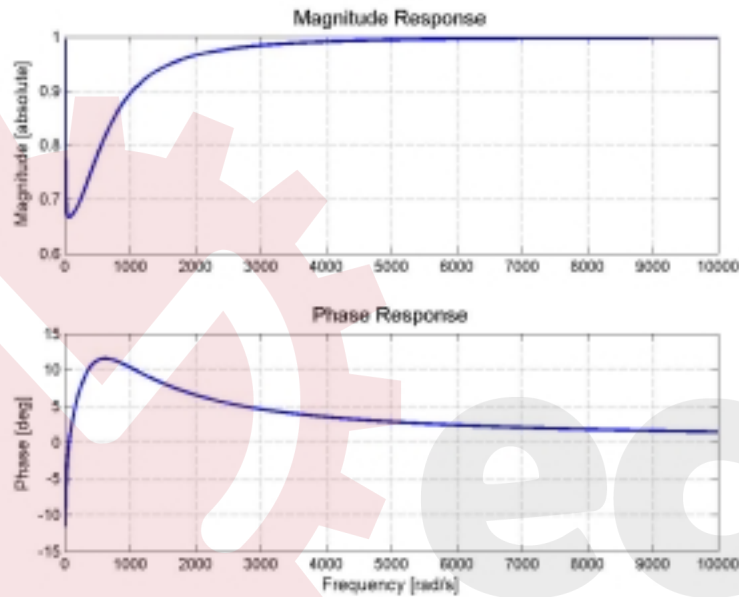
- The frequency response for the circuit of Figure P6.4.
- Plot magnitude and phase of the circuit using a linear scale for frequency.
- Repeat part b., using semilog paper.
- Plot the magnitude response using semilog paper with magnitude in dB.

Analysis:

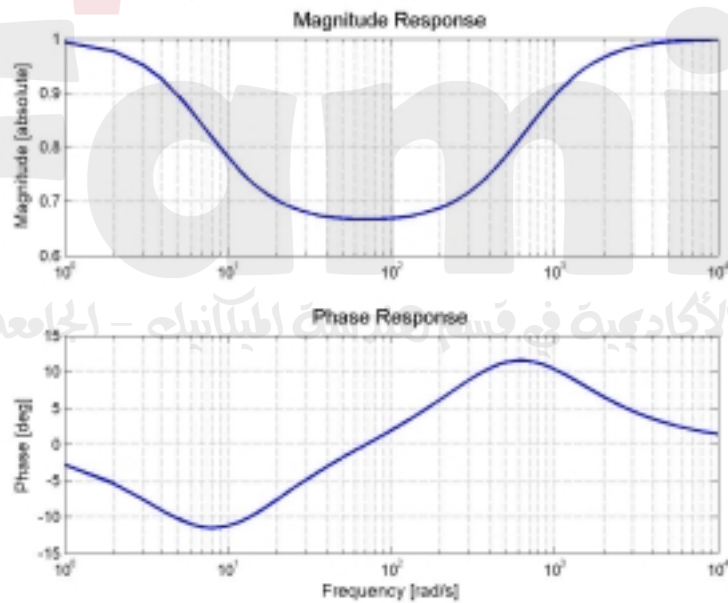
a)

$$\frac{v_{out}}{v_{in}}(j\omega) = \frac{R_2 + j\omega L + 1/j\omega C}{R_1 + R_2 + j\omega L + 1/j\omega C} = \frac{1 + j\omega CR_2 + (j\omega)^2 LC}{1 + j\omega C(R_1 + R_2) + (j\omega)^2 LC} = \frac{1 - 0.0002\omega^2 + j(0.1)\omega}{1 - 0.0002\omega^2 + j(0.15)\omega}$$

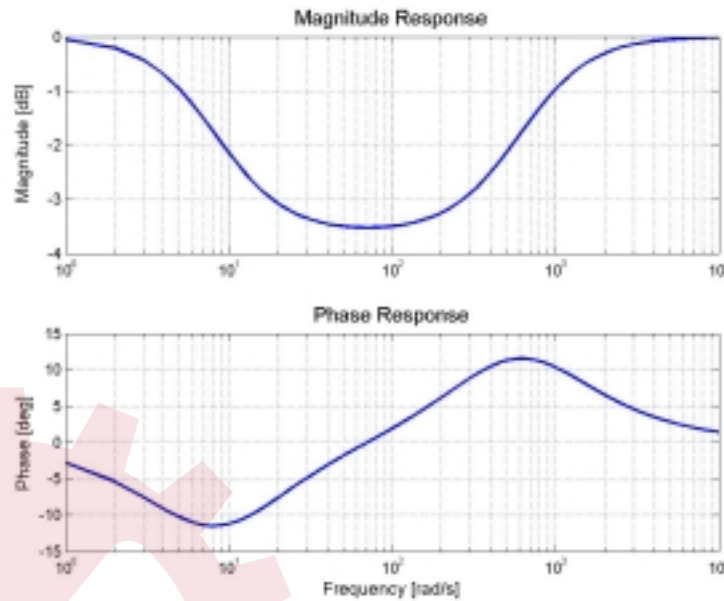
b) The plots obtained using Matlab are shown below:



c)



d)



Problem 6.5

Solution:

Known quantities:

Resistance, inductance and capacitance values, in the circuit of Figure P6.5

Find:

- The frequency response for the circuit of Figure P6.5
- Plot magnitude and phase of the circuit using a linear scale for frequency.
- Repeat part b., using semilog paper.
- Plot the magnitude response using semilog paper with magnitude in dB.

Assume:

Assume that the output voltage is the voltage across the capacitor.

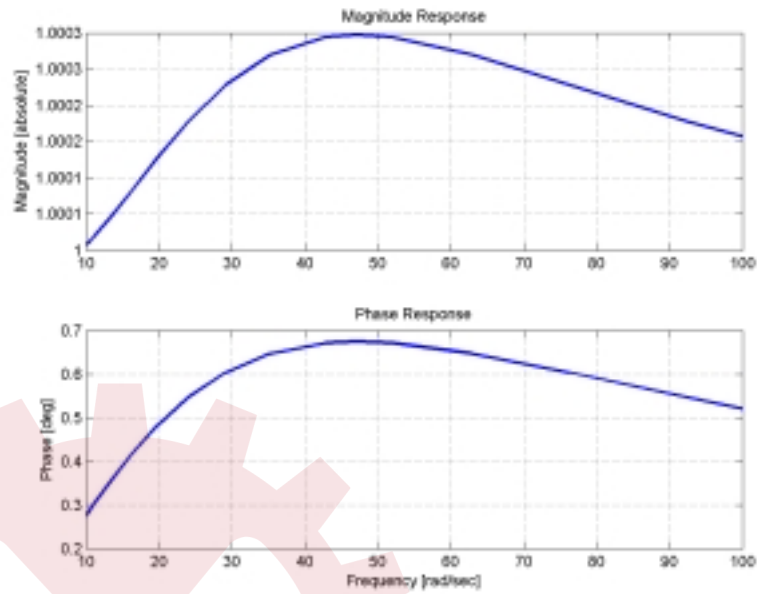
Analysis:

$$\begin{aligned}
 \text{a) } \frac{v_{out}}{v_{in}}(j\omega) &= \frac{(Z_L \parallel Z_C) + R_2}{(Z_L \parallel Z_C) + R_2 + R_1} = \frac{\left(\frac{1}{j\omega L} + j\omega C\right)^{-1} + R_2}{\left(\frac{1}{j\omega L} + j\omega C\right)^{-1} + R_2 + R_1} \\
 \frac{v_{out}}{v_{in}}(j\omega) &= \frac{\left(\frac{j\omega L}{1 - CL\omega^2}\right) + R_2}{\left(\frac{j\omega L}{1 - CL\omega^2}\right) + R_2 + R_1} = \frac{R_2 - CLR_2\omega^2 + j\omega L}{R_2 + R_1 - CL(R_2 + R_1)\omega^2 + j\omega L}
 \end{aligned}$$

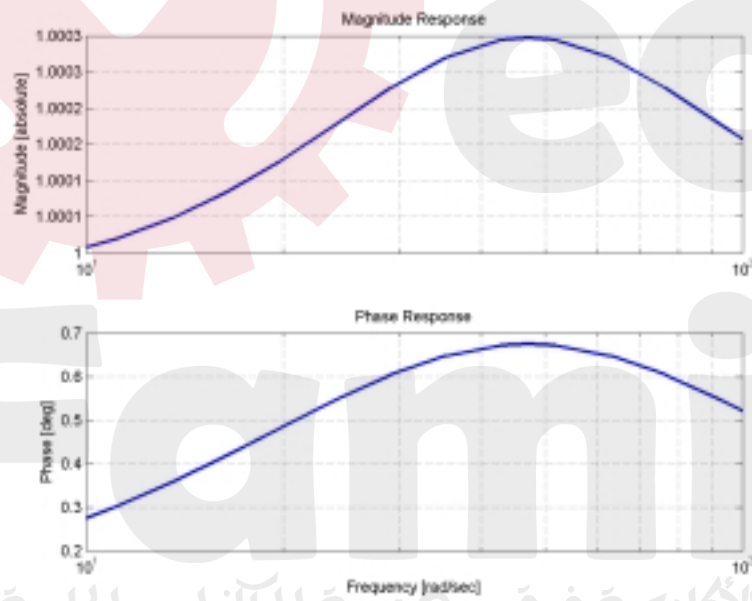
Substituting the numerical values:

$$\frac{v_{out}}{v_{in}}(j\omega) = \frac{(1 - 4.5 \times 10^{-4} \omega^2) + j(0.0015)\omega}{(1 - 4.5 \times 10^{-4} \omega^2) + j(0.0010)\omega}$$

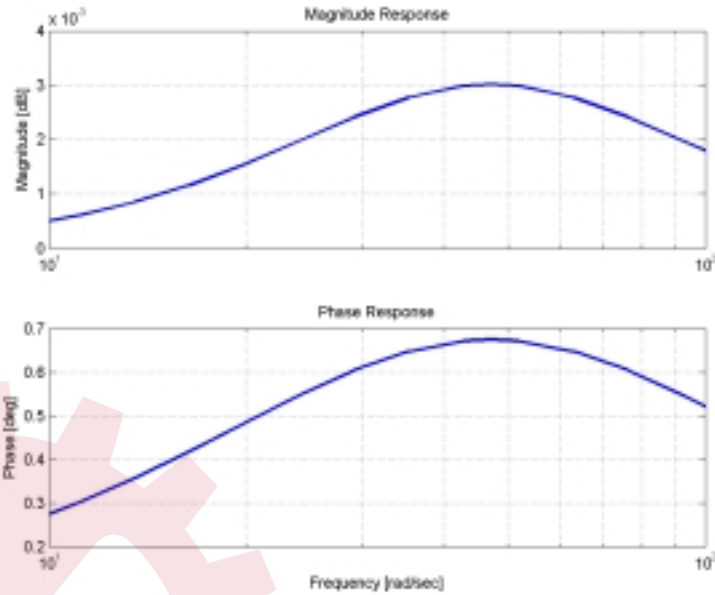
- The plots obtained using Matlab are shown below:



c)



d)



Problem 6.6

Solution:

Known quantities:

Resistance, inductance and capacitance values, in the circuit of Figure P6.6

Find:

- The frequency response for the circuit of Figure P6.6
- Plot magnitude and phase of the circuit using a linear scale for frequency.
- Repeat part b., using semilog paper.
- Plot the magnitude response using semilog paper with magnitude in dB.

Analysis:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$Z_T = Z_{R2} + (Z_{C1} \parallel Z_{R1}) = R_2 + \frac{R_1 / j\omega C_1}{1/j\omega C_1 + R_1} = R_2 + \frac{R_1}{1 + j\omega C_1 R_1}$$

and

$$v_{OC} = \frac{Z_{R1}}{Z_{R1} + Z_{C1}} v_{in} = \frac{R_1}{R_1 + 1/j\omega C_1} v_{in} = \frac{j\omega C_1 R_1}{1 + j\omega C_1 R_1} v_{in}$$

$$a) \quad \frac{v_{out}}{v_{OC}} = \frac{Z_{C2}}{Z_T + Z_{C2}} = \frac{1/j\omega C_2}{\left(R_2 + \frac{R_1}{1 + j\omega C_1 R_1}\right) + 1/j\omega C_2} = \frac{1}{1 + \left(R_2 + \frac{R_1}{1 + j\omega C_1 R_1}\right) j\omega C_2}$$

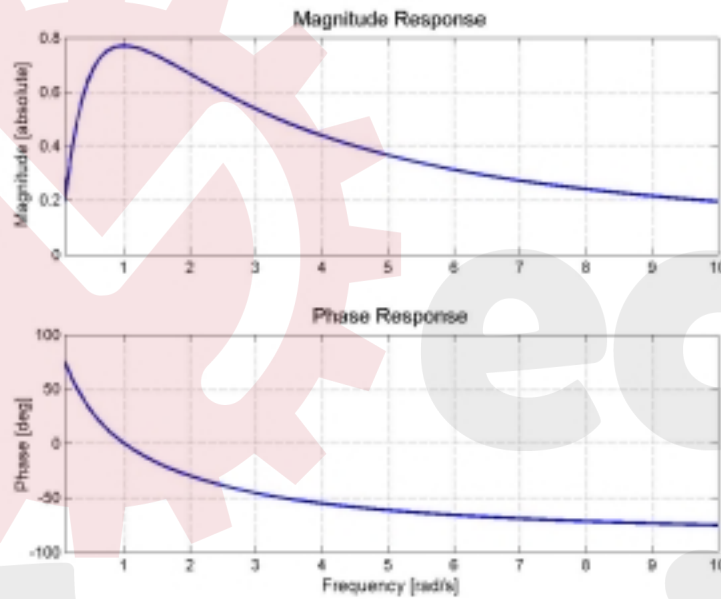
Therefore,

$$\begin{aligned}\frac{v_{out}}{v_{in}} &= \frac{j\omega C_1 R_1}{1 + j\omega C_1 R_1} \cdot \frac{1}{1 + \left(R_2 + \frac{R_1}{1 + j\omega C_1 R_1} \right) j\omega C_2} \\ &= \frac{j\omega C_1 R_1}{1 + j\omega [C_1 R_1 + C_2 (R_1 + R_2)] + (j\omega)^2 C_1 C_2 R_1 R_2}\end{aligned}$$

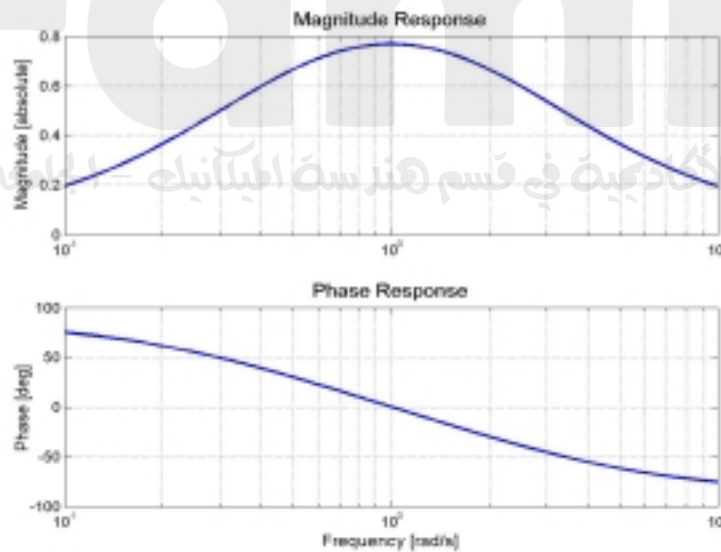
Substituting the numerical values:

$$\frac{v_{out}}{v_{in}} = \frac{j(2)\omega}{(1 - \omega^2) + j(2.6)\omega}$$

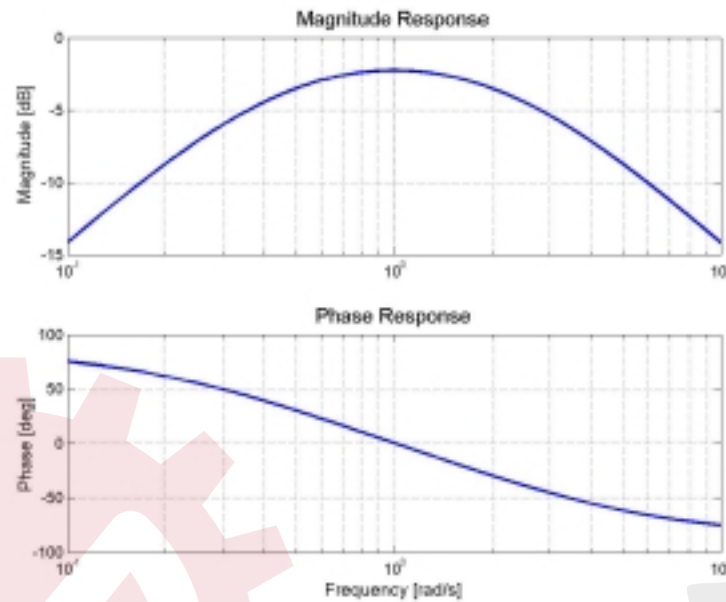
b) The plots obtained using Matlab are shown below:



c)



d)



Problem 6.7

Solution:

Known quantities:
Figure P6.7.

Find:

- How the "driving point" impedance, $Z[j\omega] = \frac{V_i[j\omega]}{I_i[j\omega]}$, behaves at extremely high or low frequencies.
- An expression for the input (or driving point) impedance.
- Show that this expression can be manipulated into the form:

$$Z[j\omega] = Z_o (1 \pm j f[\omega])$$

Where: $Z_o = R$ $f[\omega] = \frac{1}{\omega RC}$ $C = 0.5 \mu\text{F}$ $R = 2 \text{ k}\Omega$
- Determine the "cutoff" frequency $\omega = \omega_c$ at which $f[\omega_c] = 1$.
- Determine the magnitude and angle of $Z[j\omega]$ at $\omega = 100 \text{ rad/s}$, 1000 rad/s , and $10,000 \text{ rad/s}$.
- Predict (without computing) the magnitude and angle of $Z[j\omega]$ at $\omega = 10 \text{ rad/s}$ and $100,000 \text{ rad/s}$.

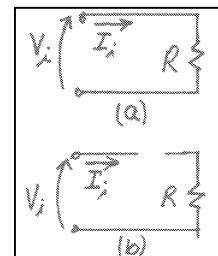
Analysis:

a)

$$\text{As } \omega \rightarrow \infty, Z_C \rightarrow 0 \Rightarrow \text{Short} \Rightarrow Z \rightarrow R$$

$$\text{As } \omega \rightarrow 0, Z_C \rightarrow \infty \Rightarrow \text{Open} \Rightarrow Z \rightarrow \infty$$

b)



$$KVL : -V_i + I_i Z_C + I_i Z_R = 0$$

$$Z[j\omega] = \frac{V_i}{I_i} = Z_C + Z_R = \frac{1}{j\omega C} + R$$

$$c) \quad Z[j\omega] = R + j \frac{1}{j^2 \omega C} = R \left[1 - j \frac{1}{\omega RC} \right]$$

$$d) \quad f[\omega_c] = \frac{1}{\omega_c RC} = 1 \Rightarrow \omega_c = \frac{1}{RC} = \frac{1}{[2000] [0.5 \cdot 10^{-6}]} = 1000 \frac{\text{rad}}{\text{s}}$$

e)

$$Z[100 \frac{\text{rad}}{\text{s}}] = R \left(1 - j \frac{1}{[100] [2000] [0.5 \cdot 10^{-6}]} \right)$$

$$= 2000 [1 - j10] = 20.10 \text{ k}\Omega \angle -84.29^\circ$$

$$Z[1000 \frac{\text{rad}}{\text{s}}] = R \left(1 - j \frac{1}{[1000] [2000] [0.5 \cdot 10^{-6}]} \right)$$

$$= 2000 [1 - j1] = 2.828 \text{ k}\Omega \angle -45.00^\circ$$

$$Z[10 \text{ k} \frac{\text{rad}}{\text{s}}] = R \left(1 - j \frac{1}{[10000] [2000] [0.5 \cdot 10^{-6}]} \right)$$

$$= 2000 [1 - j0.1] = 2.010 \text{ k}\Omega \angle -5.71^\circ$$

f)

$$Z[10 \frac{\text{rad}}{\text{s}}] \approx 200 \text{ k}\Omega \angle -90^\circ$$

$$Z[100 \text{ k} \frac{\text{rad}}{\text{s}}] \approx 2 \text{ k}\Omega \angle 0^\circ$$

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Problem 6.8

Solution:

Known quantities:

Figure P6.8.

Find:

- a) How the "driving point" impedance, $Z[j\omega] = \frac{V_i[j\omega]}{I_i[j\omega]}$, behaves at extremely high or low frequencies.

- b) An expression for the driving point impedance.
 c) Show that this expression can be manipulated into the form:

$$Z[j\omega] = Z_o(1 + j f[\omega])$$

$$\text{Where: } Z_o = R \quad f[\omega] = \frac{\omega L}{R} \quad L = 2 \text{ mH} \quad R = 2 \text{ k}\Omega$$

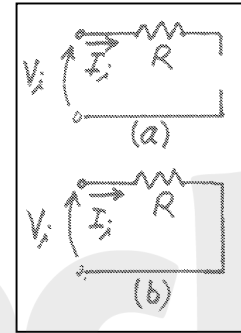
- d) Determine the "cutoff" frequency $\omega = \omega_c$ at which $f[\omega_c] = 1$.
 e) Determine the magnitude and angle of $Z[j\omega]$ at $\omega = 100 \text{ krad/s}$, $1,000 \text{ krad/s}$, and $10,000 \text{ krad/s}$.
 f) Predict (without computing it) the magnitude and angle of $Z[j\omega]$ at $\omega = 10 \text{ krad/s}$ and $100,000 \text{ krad/s}$.

Analysis:

a)

$$\text{As } \omega \rightarrow \infty, Z_L \rightarrow 0 \Rightarrow \text{Open} \Rightarrow Z \rightarrow \infty$$

$$\text{As } \omega \rightarrow 0, Z_L \rightarrow 0 \Rightarrow \text{Short} \Rightarrow Z \rightarrow R$$



b)

$$\text{KVL: } -V_i + I_i Z_R + I_i Z_L = 0$$

$$Z[j\omega] = \frac{V_i}{I_i} = Z_L + Z_R = j\omega L + R$$

c) In standard form: $Z[j\omega] = R + j\omega L = R \left[1 + j \frac{\omega L}{R} \right]$

d) $f[\omega_c] = \frac{\omega_c L}{R} = 1 \Rightarrow \omega_c = \frac{R}{L} = \frac{2000}{2 \cdot 10^{-3}} = 1000 \text{ k} \frac{\text{rad}}{\text{s}}$

e) The standard form can now be rewritten as:

$$Z[j\omega] = R \left[1 + j \frac{\omega}{\omega_c} \right] = 2000 \left[1 + j \frac{\omega}{1 \cdot 10^6} \right]$$

$$Z[100 \text{ k} \frac{\text{rad}}{\text{s}}] = R \left(1 + j \frac{100 \cdot 10^3}{1 \cdot 10^6} \right) = 2000 [1 + j 0.1] = 2.01 \text{ k}\Omega \angle 5.71^\circ$$

$$Z[1 \text{ M} \frac{\text{rad}}{\text{s}}] = R \left(1 + j \frac{1 \cdot 10^6}{1 \cdot 10^6} \right) = 2000 [1 + j 1] = 2.82 \text{ k}\Omega \angle 45.00^\circ$$

$$Z[10 \text{ M} \frac{\text{rad}}{\text{s}}] = R \left(1 + j \frac{10 \cdot 10^6}{1 \cdot 10^6} \right) = 2000 [1 + j 10] = 20.10 \text{ k}\Omega \angle 84.29^\circ$$

Note, in particular, the behavior of the impedance one decade below and one decade above the cutoff frequency.

f)

$$Z[100 \text{ M} \frac{\text{rad}}{\text{s}}] \approx 200 \text{ k}\Omega \angle 90^\circ$$

$$Z[10 \text{ k} \frac{\text{rad}}{\text{s}}] \approx 2 \text{ k}\Omega \angle 0^\circ$$

Problem 6.9

Solution:

Known quantities:

With reference to Figure P6.9:

$$L = 190 \text{ mH} \quad R_1 = 2.3 \text{ k}\Omega$$

$$C = 55 \text{ nF} \quad R_2 = 1.1 \text{ k}\Omega$$

Find:

- How the "driving point" impedance, $Z[j\omega] = \frac{V_i[j\omega]}{I_i[j\omega]}$, behaves at extremely high or low frequencies.
- An expression for the driving point impedance in the form:

$$Z[j\omega] = Z_o \left[\frac{1 + j f_1[\omega]}{1 + j f_2[\omega]} \right] \quad Z_o = R_1 + \frac{L}{R_2 C}$$

$$f_1[\omega] = \frac{\omega^2 R_1 L C - R_1 - R_2}{\omega [R_1 R_2 C + L]} \quad f_2[\omega] = \frac{\omega^2 L C - 1}{\omega C R_2}$$
- Determine the four cutoff frequencies at which $f_1[\omega] = +1$ or -1 and $f_2[\omega] = +1$ or -1 .
- Determine the resonant frequency of the circuit.
- Plot the magnitude of the impedance [in dB] as a function of the Log of the frequency, i.e., a Bode plot.

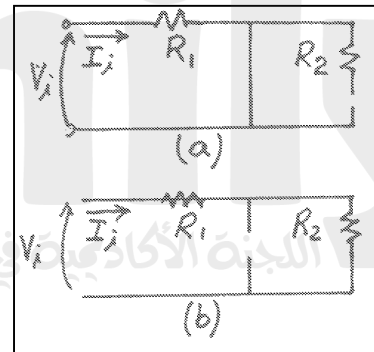
Analysis:

$$\text{As } \omega \rightarrow \infty, Z_L \rightarrow \infty \Rightarrow \text{Open}, Z_C \rightarrow 0$$

$$\Rightarrow \text{Short} \Rightarrow Z \rightarrow R_1$$

$$\text{As } \omega \rightarrow 0, Z_C \rightarrow \infty \Rightarrow \text{Open}, Z_L \rightarrow 0$$

$$\Rightarrow \text{Short} \Rightarrow Z \rightarrow R_1 + R_2$$



b)

$$\begin{aligned}
 Z[j\omega] &= \frac{V[j\omega]}{I[j\omega]} = Z_{R1} + \frac{Z_C [Z_{R2} + Z_L]}{Z_C + [Z_{R2} + Z_L]} = R_1 + \frac{[\frac{1}{j\omega C}] [R_2 + j\omega L]}{\frac{1}{j\omega C} + [R_2 + j\omega L]} \cdot \frac{j\omega C}{j\omega C} = \\
 &= R_1 + \frac{R_2 + j\omega L}{1 - \omega^2 LC + j\omega R_2 C} = \frac{(R_1 [1 - \omega^2 LC] + R_2) + j(\omega R_1 R_2 C + \omega L)}{[1 - \omega^2 LC] + j\omega R_2 C} \cdot \frac{(-j)}{(-j)} \Rightarrow \\
 \Rightarrow Z[j\omega] &= \frac{\omega(R_1 R_2 C + L) + j(\omega^2 R_1 LC - R_1 - R_2)}{\omega R_2 C + j(\omega^2 LC - 1)} = \frac{R_1 R_2 C + L}{R_2 C} \cdot \frac{1 + j \frac{\omega^2 R_1 LC - R_1 - R_2}{\omega(R_1 R_2 C + L)}}{1 + j \frac{\omega^2 LC - 1}{\omega R_2 C}}
 \end{aligned}$$

c) Both $f_1[\omega]$ and $f_2[\omega]$ can be positive or negative, and therefore equal to plus or minus one depending on the frequency; therefore, both cases must be considered.

$$f_1[\omega_c] = \frac{\omega_c [R_1 R_2 C + L]}{R_1 [1 - \omega_c^2 LC] + R_2} = \pm 1$$

$$\omega_c^2 \pm \left[\frac{R_2}{L} + \frac{1}{R_1 C} \right] \omega_c - \frac{R_1 + R_2}{R_1 LC} = 0$$

$$\frac{R_2}{L} + \frac{1}{R_1 C} = \frac{1100}{0.19} + \frac{1}{[2300][55 \cdot 10^{-9}]} = 13.69 \text{ k} \frac{\text{rad}}{\text{s}}$$

$$\frac{R_1 + R_2}{R_1 LC} = \frac{3400}{[2300][0.19][55 \cdot 10^{-9}]} = 141.46 \text{ M} \frac{\text{rad}}{\text{s}^2}$$

$$\omega_c = -\frac{1}{2} [\pm 13.69 \cdot 10^3] \pm \frac{1}{2} ([\pm 13.69 \cdot 10^3]^2 - 4[1][-141.5 \cdot 10^6])^{1/2}$$

$$= \pm 6.845 \cdot 10^3 \pm 13.724 \cdot 10^3 \Rightarrow \omega_{c1} = 6.879 \text{ k} \frac{\text{rad}}{\text{s}} \quad \omega_{c4} = 20.569 \text{ k} \frac{\text{rad}}{\text{s}}$$

Where only the positive answers are physically valid, i.e., a negative frequency is physically impossible.

$$f_2[\omega_c] = \frac{\omega_c^2 LC - 1}{\omega_c R_2 C} = \pm 1 \Rightarrow \omega_c^2 \pm \left[\frac{R_2}{L} \right] \omega_c - \frac{1}{LC} = 0$$

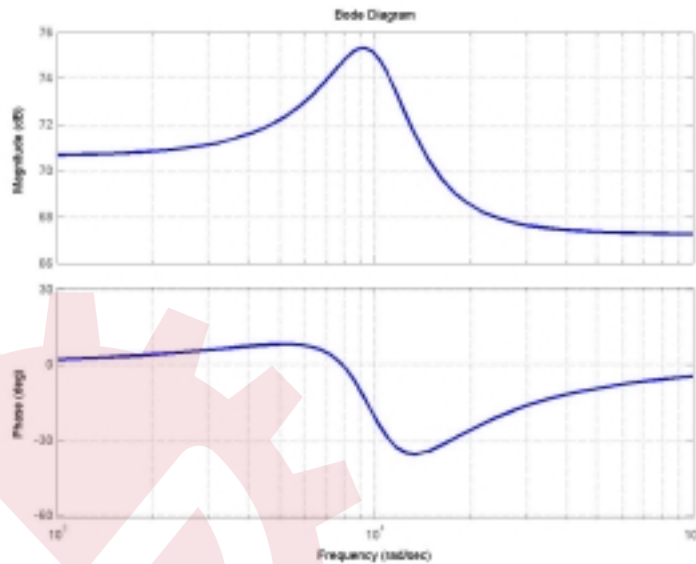
$$\frac{R_2}{L} = \frac{1100}{0.19} = 5.79 \text{ k} \frac{\text{rad}}{\text{s}} \quad \frac{1}{LC} = \frac{1}{[0.19][55 \cdot 10^{-9}]} = 95.69 \text{ M} \frac{\text{rad}}{\text{s}^2}$$

$$\omega_c = -\frac{1}{2} [\pm 5790] \pm \frac{1}{2} ([\pm 5790]^2 + 4[1][95.69 \cdot 10^6])^{1/2} = \pm 2895 \pm 10201$$

$$\Rightarrow \omega_{c2} = 7.31 \text{ k} \frac{\text{rad}}{\text{s}} \quad \omega_{c3} = 13.09 \text{ k} \frac{\text{rad}}{\text{s}}$$

Again, the negative roots were rejected because they are physically impossible.

d) Plotting the response in a Bode Plot:



Problem 6.10

Solution:

Known quantities:

In the circuit of Figure P6.10:

$$R_1 = 1.3 \text{ k}\Omega \quad R_2 = 1.9 \text{ k}\Omega \quad C = 0.5182 \mu\text{F}$$

Find:

- a) How the voltage transfer function: $H_v[j\omega] = \frac{V_o[j\omega]}{V_i[j\omega]}$ behaves at extremes of high and low frequencies.

- b) An expression for the voltage transfer function, showing that it can be manipulated into the form:

$$H_v[j\omega] = \frac{H_o}{1 + j f[\omega]} \quad \text{Where: } H_o = \frac{R_2}{R_1 + R_2} \quad f[\omega] = \frac{\omega R_1 R_2 C}{R_1 + R_2}$$

- c) The "cutoff" frequency at which $f[\omega] = 1$ and the value of H_o in dB.

Analysis:

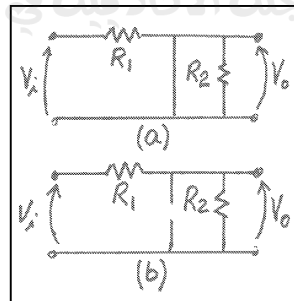
a)

$$\text{As } \omega \rightarrow \infty: Z_C \rightarrow 0 \angle -90^\circ \Rightarrow \text{Short}$$

$$\text{VD: } H_v \rightarrow 0 \angle -90^\circ$$

$$\text{As } \omega \rightarrow 0: Z_C \rightarrow \infty \angle -90^\circ \Rightarrow \text{Open}$$

$$\text{VD: } H_v \rightarrow \frac{R_2}{R_1 + R_2} \angle 0^\circ$$



$$b) \quad Z_{eq} = \frac{Z_C Z_{R2}}{Z_C + Z_{R2}} = \frac{\left[\frac{1}{j\omega C} \right] [R_2]}{\frac{1}{j\omega C} + R_2} \frac{j\omega C}{j\omega C} = \frac{R_2}{1 + j\omega R_2 C}$$

$$VD: H_v[j\omega] = \frac{V_o[j\omega]}{V_i[j\omega]} = \frac{Z_{eq}}{Z_{R1} + Z_{eq}} = \frac{\frac{R_2}{1 + j\omega R_2 C}}{R_1 + \frac{R_2}{1 + j\omega R_2 C}} \frac{1 + j\omega R_2 C}{1 + j\omega R_2 C} =$$

$$= \frac{R_2}{R_1 + R_2 + j\omega R_1 R_2 C} = \frac{R_2}{R_1 + R_2} \frac{1}{1 + j \frac{\omega R_1 R_2 C}{R_1 + R_2}}$$

$$c) \quad f[\omega_c] = \frac{\omega_c R_1 R_2 C}{R_1 + R_2} = 1 \quad \omega_c = \frac{1300 + 1900}{[1300][1900][0.5182 \cdot 10^{-6}]} = 2.5 \text{ k} \frac{\text{rad}}{\text{s}}$$

$$H_o = \frac{R_2}{R_1 + R_2} = \frac{1900}{1300 + 1900} = 0.5938 = 20 \cdot \text{Log}[0.5938] = -4.527 \text{ dB}$$

Problem 6.11

Solution:

Known quantities:

Figure P6.11.

Find:

- The behavior of the voltage transfer function or gain at extremely high and low frequencies.
- The output voltage V_o if the input voltage has a frequency where:
 $V_i = 7.07 \text{ V} \angle 45^\circ \quad R_1 = 2.2 \text{ k}\Omega \quad R_2 = 3.8 \text{ k}\Omega \quad X_C = 5 \text{ k}\Omega \quad X_L = 1.25 \text{ k}\Omega$
- The output voltage if the frequency of the input voltage doubles so that:
 $X_C = 2.5 \text{ k}\Omega \quad X_L = 2.5 \text{ k}\Omega$
- The output voltage if the frequency of the input voltage again doubles so that:
 $X_C = 1.25 \text{ k}\Omega \quad X_L = 5 \text{ k}\Omega$

Analysis:

a)

$$\text{As } \omega \rightarrow 0 \quad Z_C \rightarrow \infty \Rightarrow \text{Open}$$

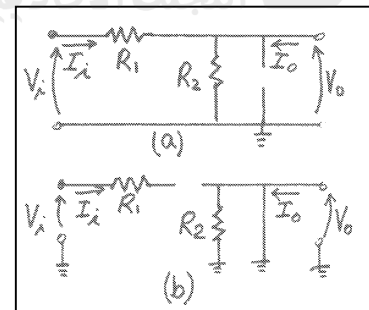
$$Z_L \rightarrow 0 \Rightarrow \text{Short}$$

$$V_o \rightarrow 0$$

$$\text{As } \omega \rightarrow \infty \quad Z_C \rightarrow 0 \Rightarrow \text{Short}$$

$$Z_L \rightarrow \infty \Rightarrow \text{Open}$$

$$VD: V_o = \frac{V_i R_2}{R_1 + R_2} = \frac{[7.07][3800]}{2200 + 3800} = 4.478 \text{ V} \angle 45^\circ$$



b)

$$Z_{eq1} = Z_{R1} + Z_C = R_1 - j X_C \quad Z_{eq2} = \frac{Z_{R2} Z_L}{Z_{R2} + Z_L} = \frac{[R_2][j X_L]}{R_2 + j X_L}$$

$$VD: V_o = \frac{V_i Z_{eq2}}{Z_{eq1} + Z_{eq2}} = V_i \frac{\frac{j R_2 X_L}{R_2 + j X_L}}{R_1 - j X_C + \frac{j R_2 X_L}{R_2 + j X_L}} \Rightarrow$$

$$\Rightarrow V_o = V_i \frac{j R_2 X_L}{[R_1 R_2 + X_C X_L] + j [X_L (R_1 + R_2) - X_C R_2]}$$

$$V_i \cdot [j R_2 X_L] = [7.07 \text{ V } \angle 45^\circ][(3.8 \text{ k}\Omega)(1.25 \text{ k}\Omega) \angle 90^\circ] = 33.58 \cdot 10^6 \angle 135^\circ$$

$$R_1 R_2 + X_C X_L = [2200][3800] + [5000][1250] = 14.61 \cdot 10^6$$

$$X_L [R_1 + R_2] - X_C R_2 = [1250][6000] - [5000][3800] = -11.50 \cdot 10^6$$

$$V_o = \frac{33.58 \cdot 10^6 \angle 135^\circ}{14.61 \cdot 10^6 - j 11.50 \cdot 10^6} = \frac{33.58 \angle 135^\circ}{18.59 \angle -38.2^\circ} = 1.806 \text{ V } \angle 173.2^\circ$$

c)

$$V_i \cdot [j R_2 X_L] = [7.07 \text{ V } \angle 45^\circ][(3800)(2500) \angle 90^\circ] = 67.17 \cdot 10^6 \angle 135^\circ$$

$$R_1 R_2 + X_C X_L = [2200][3800] + [2500][2500] = 14.61 \cdot 10^6$$

$$X_L [R_1 + R_2] + X_C R_2 = [2500][6000] - [2500][3800] = 5.50 \cdot 10^6$$

$$V_o = \frac{67.17 \cdot 10^6 \angle 135^\circ}{14.61 \cdot 10^6 + j 5.50 \cdot 10^6} = \frac{67.17 \text{ V } \angle 135^\circ}{15.61 \angle 20.6^\circ} = 4.303 \text{ V } \angle 114.4^\circ$$

d)

$$V_i \cdot [j R_2 X_L] = [7.07 \text{ V } \angle 45^\circ][(3800)(5000) \angle 90^\circ] = 134.34 \cdot 10^6 \angle 135^\circ$$

$$R_1 R_2 + X_C X_L = [2200][3800] + [1250][5000] = 14.61 \cdot 10^6$$

$$X_L [R_1 + R_2] + X_C R_2 = [5000][6000] - [1250][3800] = 25.25 \cdot 10^6$$

$$V_o = \frac{134.34 \cdot 10^6 \angle 135^\circ}{14.61 \cdot 10^6 + j 25.25 \cdot 10^6} = \frac{134.34 \text{ V } \angle 135^\circ}{29.17 \angle 59.94^\circ} = 4.605 \text{ V } \angle 75.05^\circ$$

Problem 6.12

Solution:

Known quantities:

Figure P6.12.

Find:

a) The voltage transfer function in the form:
$$H_v[j\omega] = \frac{V_o[j\omega]}{V_i[j\omega]} = \frac{H_{vo}}{1 \pm j f[\omega]}.$$

b) Plot the Bode diagram, i.e., a semilog plot where the magnitude [in dB] of the transfer function is plotted on a linear scale as a function of frequency on a log scale.

Assume:

The values of the resistors and of the capacitor in the circuit of Figure P6.12:

$$R_1 = 16 \Omega \quad R_2 = 16 \Omega \quad C = 0.47 \mu\text{F}$$

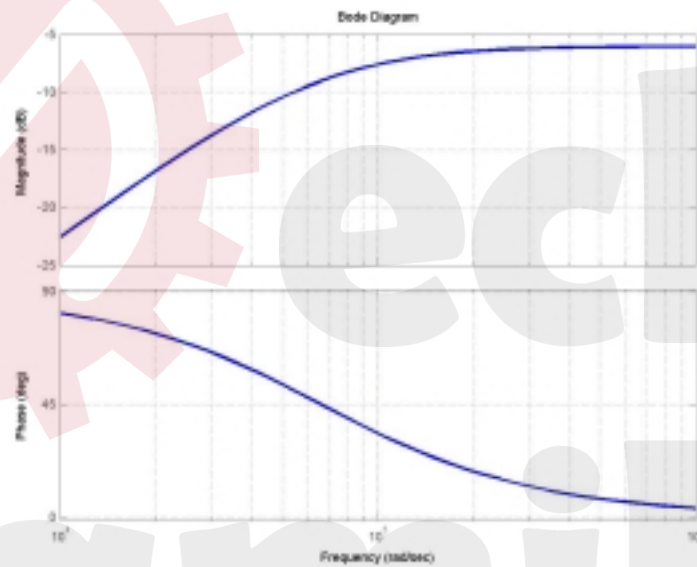
Analysis:

a)

$$VD: V_o = V_i \frac{Z_{R2}}{Z_{R1} + Z_C + Z_{R2}} = V_i \frac{R_2}{R_1 + \frac{1}{j\omega C} + R_2}$$

$$H_v[j\omega] = \frac{V_o[j\omega]}{V_i[j\omega]} = \frac{R_2}{R_1 + R_2} \frac{1}{1 - j \frac{1}{\omega C [R_1 + R_2]}}$$

b)

**Problem 6.13****Solution:****Known quantities:**

The values of the resistors and of the capacitor in the circuit of Figure P6.13:

$$R_1 = 100 \Omega \quad R_L = 100 \Omega \quad R_2 = 50 \Omega \quad C = 80 \text{ nF}$$

Find:

Compute and plot the frequency response function.

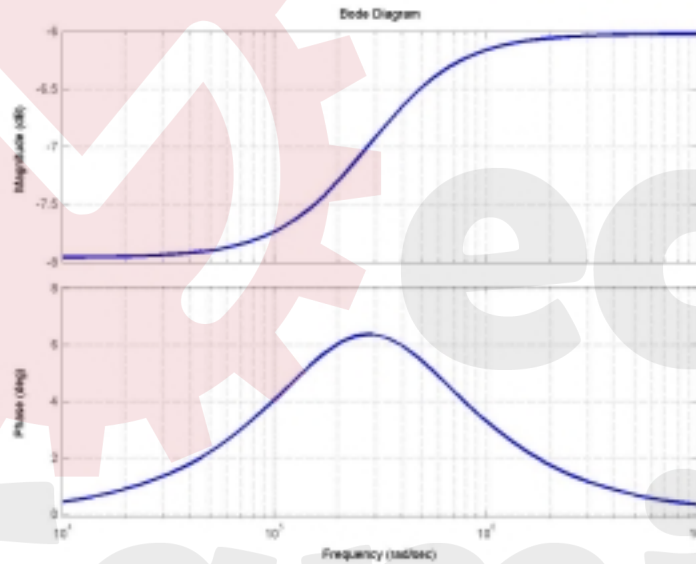
Analysis:

Using voltage division:

$$Z_{eq} = \frac{Z_{R2} Z_C}{Z_{R2} + Z_C} = \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{1 + j\omega R_2 C}$$

$$\begin{aligned} VD: H_v[j\omega] &= \frac{V_o[j\omega]}{V_i[j\omega]} = \frac{Z_{RL}}{Z_{R1} + Z_{eq} + Z_{RL}} = \frac{R_L}{R_1 + \frac{R_2}{1 + j\omega R_2 C} + R_L} \frac{1 + j\omega R_2 C}{1 + j\omega R_2 C} = \\ &= \frac{R_L [1 + j\omega R_2 C]}{R_1 + R_2 + R_L + j [R_1 + R_L] \omega R_2 C} = \frac{R_L}{R_1 + R_2 + R_L} \frac{1 + j\omega R_2 C}{1 + j \frac{[R_1 + R_L] \omega R_2 C}{R_1 + R_2 + R_L}} \end{aligned}$$

Plotting the response in a Bode Plot:



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Section 6.2: Fourier Analysis

Problem 6.14

Solution:

Find:

Use trigonometric identities to show that the equalities in equations 6.16 and 6.17 hold.

Analysis:

Looking at figure 6.8, we can write the following equations:

$$a_n = c_n \sin(\theta_n)$$

$$b_n = c_n \cos(\theta_n)$$

and using the trigonometric identities $\sin^2(\theta_n) + \cos^2(\theta_n) = 1$:

$$a_n^2 + b_n^2 = c_n^2 \sin^2(\theta_n) + c_n^2 \cos^2(\theta_n) = c_n^2 \Rightarrow c_n = \sqrt{a_n^2 + b_n^2}$$

Finally,

$$\frac{b_n}{a_n} = \frac{c_n \cos(\theta_n)}{c_n \sin(\theta_n)} = \cot(\theta_n) = \tan(\psi_n)$$

where,

$$\psi_n = \frac{\pi}{2} - \theta_n.$$

Problem 6.15

Solution:

Known quantities:

The square wave of Figure 6.11(a) in the text.

Find:

A general expression for the Fourier series coefficients.

Assume:

None

Analysis:

The square wave is a function of time as follows:

$$x(t) = \begin{cases} A & (n - \frac{1}{4})T \leq t \leq (n + \frac{1}{4})T, \quad n = \pm 0, \pm 1, \pm 2, \dots \\ 0 & (n + \frac{1}{4})T \leq t \leq (n + \frac{3}{4})T, \quad n = \pm 0, \pm 1, \pm 2, \dots \end{cases}$$

We can compute the Fourier series coefficient using the integrals in equations (6.20), (6.21) and (6.22):

$$a_0 = \frac{1}{T} \int_0^T x(t) dt = \frac{1}{T} \int_{T/4}^{T/4} A dt = \frac{A}{2}$$

$$\begin{aligned}
 a_n &= \frac{2}{T} \int_0^T x(t) \cos\left(n \frac{2\pi}{T} t\right) dt = \frac{2}{T} \int_{-T/4}^{T/4} A \cos\left(n \frac{2\pi}{T} t\right) dt = \\
 &= \frac{2A}{T} \left[\sin\left(n \frac{2\pi}{T} t\right) \frac{T}{2n\pi} \right]_{-T/4}^{T/4} = \frac{A}{n\pi} \left[\sin\left(\frac{n\pi}{2}\right) - \sin\left(-\frac{n\pi}{2}\right) \right] = 0 \quad (\forall n) \\
 b_n &= \frac{2}{T} \int_0^T x(t) \sin\left(n \frac{2\pi}{T} t\right) dt = \frac{2}{T} \int_{-T/4}^{T/4} A \sin\left(n \frac{2\pi}{T} t\right) dt = \\
 &= \frac{2A}{T} \left[-\cos\left(n \frac{2\pi}{T} t\right) \frac{T}{2n\pi} \right]_{-T/4}^{T/4} = \frac{A}{n\pi} \left[-\cos\left(\frac{n\pi}{2}\right) + \cos\left(-\frac{n\pi}{2}\right) \right] = \\
 &= \frac{A}{n\pi} \left[-2 \cos\left(\frac{n\pi}{2}\right) \right] = \begin{cases} \frac{2A}{n\pi} & (n \text{ even}) \\ 0 & (n \text{ odd}) \end{cases}
 \end{aligned}$$

Problem 6.16

Solution:

Known quantities:

The periodic function shown in Figure P6.16 and defined as:

$$x(t) = \begin{cases} A & 0 \leq t \leq \frac{T}{3} \\ 0 & \frac{T}{3} \leq t \leq T \end{cases}$$

Find:

A general expression for the Fourier series coefficients.

Analysis:

We can compute the Fourier series coefficient using the integrals in equations (6.20), (6.21) and (6.22):

$$\begin{aligned}
 a_0 &= \frac{1}{T} \int_0^T x(t) dt = \frac{1}{T} \int_0^{T/3} A dt = \frac{A}{3} \\
 a_n &= \frac{2}{T} \int_0^T x(t) \cos\left(n \frac{2\pi}{T} t\right) dt = \frac{2}{T} \int_0^{T/3} A \cos\left(n \frac{2\pi}{T} t\right) dt = \\
 &= \frac{2A}{T} \left[\sin\left(n \frac{2\pi}{T} t\right) \frac{T}{2n\pi} \right]_0^{T/3} = \frac{A}{n\pi} \left[\sin\left(\frac{2}{3} n\pi\right) - 0 \right] = \frac{A}{n\pi} \sin\left(\frac{2}{3} n\pi\right) \\
 b_n &= \frac{2}{T} \int_0^T x(t) \sin\left(n \frac{2\pi}{T} t\right) dt = \frac{2}{T} \int_0^{T/3} A \sin\left(n \frac{2\pi}{T} t\right) dt = \\
 &= \frac{2A}{T} \left[-\cos\left(n \frac{2\pi}{T} t\right) \frac{T}{2n\pi} \right]_0^{T/3} = \frac{A}{n\pi} \left[1 - \cos\left(\frac{2}{3} n\pi\right) \right]
 \end{aligned}$$

Thus, the Fourier series expansion of the function is:

$$x(t) = \frac{A}{3} + \sum_{n=1}^{\infty} \frac{A}{n\pi} \sin\left(\frac{2}{3}n\pi\right) \cos\left(n\frac{2\pi}{T}t\right) + \sum_{n=1}^{\infty} \frac{A}{n\pi} \left[1 - \cos\left(\frac{2}{3}n\pi\right)\right] \sin\left(n\frac{2\pi}{T}t\right)$$

Problem 6.17

Solution:

Known quantities:

The periodic function shown in Figure P6.17 and defined as:

$$x(t) = \begin{cases} \cos\left(\frac{2\pi}{T}t\right) & -\frac{T}{4} \leq t \leq \frac{T}{4} \\ 0 & \text{else} \end{cases}$$

Find:

A general expression for the Fourier series coefficients.

Analysis:

The function in Figure P6.17 is an even function. Thus, we only need to compute the a_n coefficients.

We can compute the Fourier series coefficient using the integrals in equations (6.20) and (6.21):

$$\begin{aligned} a_0 &= \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt = \frac{1}{T} \int_{-T/4}^{T/4} \cos\left(\frac{2\pi}{T}t\right) dt = \frac{1}{2\pi} \left[\sin\left(\frac{2\pi}{T}t\right) \right]_{-T/4}^{T/4} = \\ &= \frac{1}{2\pi} \left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) \right] = \frac{1}{\pi} \\ a_n &= \frac{2}{T} \int_{-T/2}^{T/2} x(t) \cos\left(n\frac{2\pi}{T}t\right) dt = \frac{2}{T} \int_{-T/4}^{T/4} \cos\left(\frac{2\pi}{T}t\right) \cos\left(n\frac{2\pi}{T}t\right) dt = \\ &= -\frac{2}{\pi} \frac{\cos\left(\frac{n\pi}{2}\right)}{n^2 - 1} = \begin{cases} (-1)^{\frac{n}{2}-1} \frac{2}{\pi(n^2 - 1)} & (n \text{ even}) \\ 0 & (n \text{ odd}) \end{cases} \end{aligned}$$

Problem 6.18

Solution:

Known quantities:

The periodic function shown in Figure P6.18 and defined as:

$$x(t) = \begin{cases} \frac{2A}{T}t & 0 \leq t \leq \frac{T}{2} \\ A & \frac{T}{2} \leq t \leq T \end{cases}$$

Find:

Compute the Fourier series expansion.

Analysis:

We can compute the Fourier series coefficient using the integrals in equations (6.20), (6.21) and (6.22):

$$a_0 = \frac{1}{T} \int_0^T x(t) dt = \frac{1}{T} \left(\int_0^{T/2} \frac{2A}{T} t dt + \int_{T/2}^T A dt \right) = \frac{A}{T} \left(\left[\frac{t^2}{T} \right]_0^{T/2} + T - \frac{T}{2} \right) = \frac{3A}{4}$$

$$\begin{aligned} a_n &= \frac{2}{T} \int_0^T x(t) \cos\left(n \frac{2\pi}{T} t\right) dt = \frac{2}{T} \left(\int_0^{T/2} \frac{2A}{T} t \cos\left(n \frac{2\pi}{T} t\right) dt + \int_{T/2}^T A \cos\left(n \frac{2\pi}{T} t\right) dt \right) = \\ &= \frac{2}{T} \left(\left[\frac{TA}{2(n\pi)^2} \left(\cos\left(n \frac{2\pi}{T} t\right) + n \frac{2\pi}{T} t \sin\left(n \frac{2\pi}{T} t\right) \right) \right]_0^{T/2} + \left[\frac{TA}{2n\pi} \sin\left(n \frac{2\pi}{T} t\right) \right]_{T/2}^T \right) = \\ &= \frac{A}{(n\pi)^2} [\cos(n\pi) - 1 + 2n\pi \sin(n\pi) \cos(n\pi)] = \frac{A}{(n\pi)^2} [\cos(n\pi) - 1] \end{aligned}$$

$$\begin{aligned} b_n &= \frac{2}{T} \int_0^T x(t) \sin\left(n \frac{2\pi}{T} t\right) dt = \frac{2}{T} \left(\int_0^{T/2} \frac{2A}{T} t \sin\left(n \frac{2\pi}{T} t\right) dt + \int_{T/2}^T A \sin\left(n \frac{2\pi}{T} t\right) dt \right) = \\ &= \frac{2}{T} \left(\left[\frac{TA}{2(n\pi)^2} \left(\sin\left(n \frac{2\pi}{T} t\right) - n \frac{2\pi}{T} t \cos\left(n \frac{2\pi}{T} t\right) \right) \right]_0^{T/2} + \left[-\frac{TA}{2n\pi} \cos\left(n \frac{2\pi}{T} t\right) \right]_{T/2}^T \right) = \\ &= \frac{A}{(n\pi)^2} [\sin(n\pi) - 2n\pi \cos^2(n\pi) + n\pi] = \frac{A}{n\pi} [1 - 2\cos^2(n\pi)] \end{aligned}$$

Thus, the Fourier series expansion of the function is:

$$x(t) = \frac{3A}{4} + \sum_{n=1}^{\infty} \frac{A}{(n\pi)^2} [\cos(n\pi) - 1] \cos\left(n \frac{2\pi}{T} t\right) + \sum_{n=1}^{\infty} \frac{A}{n\pi} [1 - 2\cos^2(n\pi)] \sin\left(n \frac{2\pi}{T} t\right)$$

Problem 6.19

Solution:

Known quantities:

The periodic function shown in Figure P6.19 and defined as:

$$x(t) = \begin{cases} \sin\left(\frac{2\pi}{T} t\right) & 0 \leq t \leq \frac{T}{2} \\ 0 & \frac{T}{2} \leq t \leq T \end{cases}$$

Find:

Compute the Fourier series expansion.

Analysis:

The function in Figure P6.19 is an even function. Thus, we only need to compute the a_n coefficients.

We can compute the Fourier series coefficient using the integrals in equations (6.20) and (6.21):

$$\begin{aligned}
 a_0 &= \frac{1}{T} \int_0^T x(t) dt = \frac{1}{T} \int_0^{T/2} \sin\left(\frac{2\pi}{T}t\right) dt = \frac{1}{2\pi} \left[-\cos\left(\frac{2\pi}{T}t\right) \right]_0^{T/2} = \\
 &= \frac{1}{2\pi} [-\cos(\pi) + \cos(0)] = \frac{1}{\pi} \\
 a_n &= \frac{2}{T} \int_0^T x(t) \cos\left(n \frac{2\pi}{T}t\right) dt = \frac{2}{T} \int_0^{T/2} \sin\left(\frac{2\pi}{T}t\right) \cos\left(n \frac{2\pi}{T}t\right) dt = \\
 &= -\frac{\cos(n\pi) + 1}{\pi(n^2 - 1)} = \begin{cases} -\frac{2}{\pi(n^2 - 1)} & (n \text{ even}) \\ 0 & (n \text{ odd}) \end{cases}
 \end{aligned}$$

Thus, the Fourier series expansion of the function is:

$$x(t) = \frac{1}{\pi} - \sum_{n=1}^{\infty} \frac{\cos(n\pi) + 1}{\pi(n^2 - 1)} \cos\left(n \frac{2\pi}{T}t\right)$$

Problem 6.20

Solution:

Known quantities:

The periodic function shown in Figure P6.20.

Find:

A complete expression for the function $x(t)$ and the Fourier coefficients.

Analysis:

The periodic function shown in Figure P6.20 can be defined as:

$$x(t) = \begin{cases} \frac{V}{t_1}t & 0 \leq t \leq t_1 \\ V & t_1 \leq t \leq T - t_1 \\ \frac{V}{t_1}(T-t) & T - t_1 \leq t \leq T + t_1 \\ -V & T + t_1 \leq t \leq 2T - t_1 \\ \frac{V}{t_1}(t-2T) & 2T - t_1 \leq t \leq 2T \end{cases}$$

The function in Figure P6.19 is an odd function with period equal to $2T$. Thus, we only need to compute the b_n coefficients.

We can compute the Fourier series coefficient using the integrals in equation (6.22):

$$\begin{aligned}
b_n &= \frac{2}{2T} \int_0^{2T} x(t) \sin\left(n \frac{2\pi}{T} t\right) dt = \frac{1}{T} \left(\int_0^{t_1} \frac{V}{t_1} t \sin\left(n \frac{2\pi}{T} t\right) dt + \int_{t_1}^{T-t_1} V \sin\left(n \frac{2\pi}{T} t\right) dt + \right. \\
&\quad \left. + \int_{T-t_1}^{T+t_1} \frac{V}{t_1} (T-t) \sin\left(n \frac{2\pi}{T} t\right) dt - \int_{T+t_1}^{2T-t_1} V \sin\left(n \frac{2\pi}{T} t\right) dt + \int_{2T-t_1}^{2T} \frac{V}{t_1} (t-2T) \sin\left(n \frac{2\pi}{T} t\right) dt \right) \\
&= \frac{V}{2(n\pi)^2 t_1} \left[4T \sin\left(n \frac{\pi}{T} t_1\right) \cos\left(n \frac{\pi}{T} t_1\right) - 2\pi n t_1 \cos\left(n \frac{\pi}{T} t_1\right) + \pi n t_1 \left(1 + \cos\left(n \frac{2\pi}{T} t_1\right)\right) \right. \\
&\quad \left. - 12T \cos^2(n\pi) \sin\left(n \frac{\pi}{T} t_1\right) \cos\left(n \frac{\pi}{T} t_1\right) + 8T \cos^4(n\pi) \sin\left(n \frac{\pi}{T} t_1\right) \cos\left(n \frac{\pi}{T} t_1\right) \right]
\end{aligned}$$

Problem 6.21

Solution:

Known quantities:

The periodic function shown in Figure P6.21.

Find:

A complete expression for the function $x(t)$ and the Fourier coefficients.

Analysis:

The periodic function shown in Figure P6.21 can be defined as:

$$x(t) = \begin{cases} A & 0 \leq t \leq \frac{T}{4} \\ -A & T - \frac{T}{4} \leq t \leq T \end{cases}$$

The function in Figure P6.19 is an odd function. Thus, we only need to compute the b_n coefficients.

We can compute the Fourier series coefficient using the integrals in equation (6.22):

$$\begin{aligned}
b_n &= \frac{2}{T} \int_{T/2}^{T/2} x(t) \sin\left(n \frac{2\pi}{T} t\right) dt = \frac{2}{T} \left(- \int_{T/4}^0 A \sin\left(n \frac{2\pi}{T} t\right) dt + \int_0^{T/4} A \sin\left(n \frac{2\pi}{T} t\right) dt \right) = \\
&= \frac{2A}{n\pi} \left(1 - \cos\left(n \frac{\pi}{2}\right) \right) = \frac{2A}{n\pi}
\end{aligned}$$

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Problem 6.22

Solution:

Known quantities:

The periodic function defined as:

$$x(t) = 10 \cos(10t + \pi/6)$$

Find:

All Fourier series coefficients.

Analysis:

Using trigonometric identities we can expand the function $x(t)$ in the following way:

$$\begin{aligned}x(t) &= 10 \cos(10t + \pi/6) = 10[\cos(10t)\cos(\pi/6) - \sin(10t)\sin(\pi/6)] = \\&= 5\sqrt{3} \cos(10t) - 5 \sin(10t)\end{aligned}$$

Now the function is already in Fourier series form, since it contains only sinusoidal terms! We recognize the following parameters:

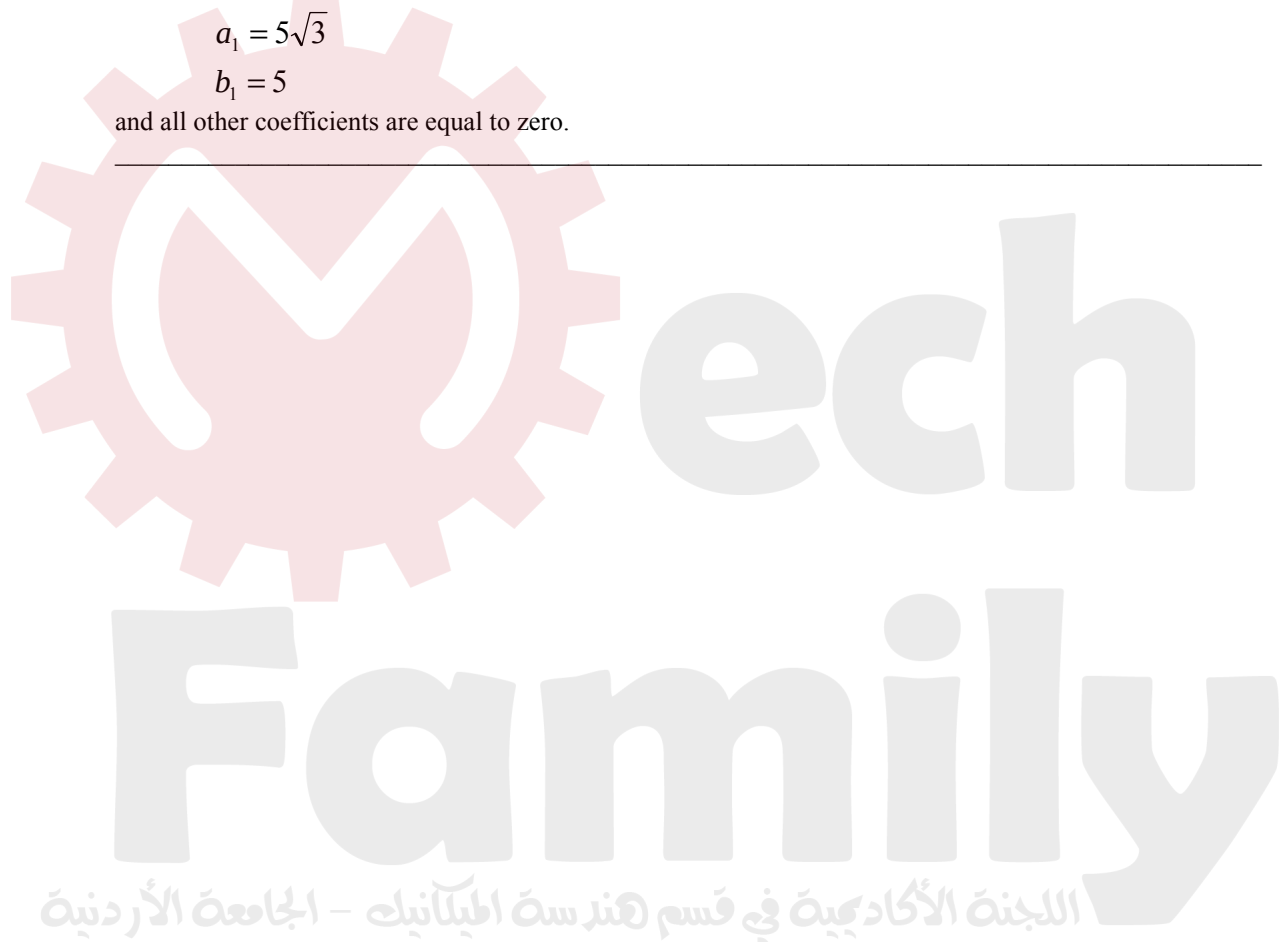
$$\omega_0 = 10$$

$$a_0 = 0$$

$$a_1 = 5\sqrt{3}$$

$$b_1 = 5$$

and all other coefficients are equal to zero.



Section 6.3: Filters

Problem 6.23

Solution:

Known quantities:

The resistance of the RC high-pass filter.

Find:

Design an RC high-pass filter with a breakpoint at 200 kHz.

Analysis:

The frequency response of the RC high-pass filter is:

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{j\omega CR}{1 + j\omega CR}$$

The cutoff frequency is:

$$\omega_0 = \frac{1}{RC}$$

Thus,

$$\omega_0 = \frac{1}{RC} = 2\pi \times 200000 \Rightarrow C = \frac{1}{R\omega_0} = \frac{1}{2\pi \cdot 15000 \cdot 200000} \cong 53 \text{ pF}$$

Problem 6.24

Solution:

Known quantities:

The resistance of the RC low-pass filter.

Find:

Design an RC low-pass filter that would attenuate a 120-Hz sinusoidal voltage by 20 dB with respect to the DC gain.

Analysis:

The frequency response of the RC low-pass filter is:

$$H_v(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{1 + j\omega CR}$$

The response of the circuit to the periodic input $u(t) = A\sin(\hat{\omega}t + \varphi)$ is:

$$y_\infty(t) = |H_v(j\hat{\omega})| A\sin(\hat{\omega}t + \varphi + \angle H_v(j\hat{\omega}))$$

In order to attenuate the sinusoidal input by 20 dB (a factor of 10) with respect to the DC gain,

$$|H_v(j\hat{\omega})| = \frac{1}{\sqrt{1 + (\hat{\omega}CR)^2}} = 0.1 \Rightarrow C = \frac{1}{R\hat{\omega}} \sqrt{10^2 - 1} = \frac{\sqrt{99}}{500 \cdot 2\pi \times 120} \cong 26.4 \mu\text{F}$$

Problem 6.25**Solution:****Known quantities:**

The resistance and the inductance of the parallel LC resonant circuit.

Find:

Design a parallel LC resonant circuit to resonate at 500-kHz.

Analysis:

The frequency response of the parallel LC resonant circuit is:

$$H_v(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1 + (j\omega)^2 LC}{1 + j\omega \frac{L}{R} + (j\omega)^2 LC}$$

The resonant frequency of the circuit is:

$$\omega_n = \sqrt{\frac{1}{LC}} = 2\pi \times 500 \text{ kHz}$$

Thus,

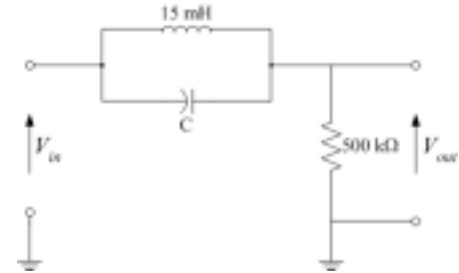
$$C = \frac{1}{L(\pi^2 \times 10^{12})} \cong 1 \text{ pF}$$

The damping ratio is,

$$\xi = \frac{1/RC}{2\omega_n} = \frac{\pi^2 \times 10^{11}}{500000 \cdot 2(2\pi \times 500000)} \cong 0.3142$$

The quality factor is,

$$Q = \frac{1}{2\xi} \cong 1.5915$$

**Problem 6.26****Solution:****Find:**

In an RLC circuit, show that $Q = \frac{\omega_n}{\Delta\omega}$.

Analysis:

The frequency response of an RLC circuit is:

$$H_v(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{\left(2\xi/\omega_n\right)j\omega}{1 + j\omega\left(2\xi/\omega_n\right) + \left(j\omega/\omega_n\right)^2}$$

We can compute the half-power frequencies ω_1 and ω_2 by equating the magnitude of the band-pass filter frequency response to $1/\sqrt{2}$ (this will result in a quadratic equation in ω , which can be solved for the

two frequencies). Defining $\Omega = \frac{\omega}{\omega_n}$, we can write the following equation:

$$|H_v(j\omega)| = \left| \frac{2\xi(j\omega)}{1 + 2\xi(j\omega) + (j\omega)^2} \right| = \frac{2\xi\Omega}{\sqrt{(1-\Omega^2)^2 + (2\xi\Omega)^2}} = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\Rightarrow \Omega^4 - 2(1 + 2\xi^2)\Omega^2 + 1 = 0 \Rightarrow \Omega^2 = 1 + 2\xi^2 \pm 2\xi\sqrt{1 + \xi^2}$$

Finally, discarding the negative solutions:

$$\Omega_{1,2} = \pm\xi + \sqrt{1 + \xi^2} \Rightarrow \omega_{1,2} = (\pm\xi + \sqrt{1 + \xi^2})\omega_n$$

Thus,

$$\Delta\omega = \omega_2 - \omega_1 = (\xi + \sqrt{1 + \xi^2})\omega_n - (-\xi + \sqrt{1 + \xi^2})\omega_n = 2\xi\omega_n$$

and,

$$\frac{\omega_n}{\Delta\omega} = \frac{\omega_n}{2\xi\omega_n} = \frac{1}{2\xi} = Q$$

Problem 6.27

Solution:

Known quantities:

The resistance, inductance and capacitance of a series RLC resonant circuit.

Find:

- Show that the impedance at the resonance frequency becomes a value of Q times the resistance at the resonance frequency. **NOTE:** The word inductive should not be in the problem statement.
- Determine the impedance at the resonance frequency, assuming $L = 280 \text{ mH}$, $C = 0.1 \mu\text{F}$ and $R = 25 \Omega$.

Assumptions:

The circuit is as shown in the figure below with the output impedance across the inductor of the RLC circuit.

Also, the output impedance is the impedance of interest.

Analysis:

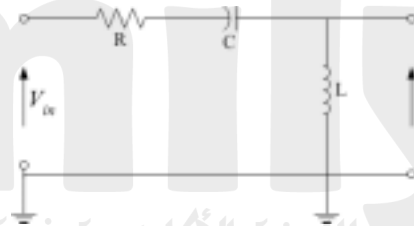
- The output impedance of the circuit is:

$$\begin{aligned} Z_{out}(j\omega) &= \left(R + \frac{1}{j\omega C} \right) \parallel j\omega L = \\ &= \frac{1 + j\omega CR}{j\omega C} \parallel j\omega L = \frac{(1 + j\omega CR)j\omega L}{1 + j\omega CR + (j\omega)^2 LC} \end{aligned}$$

and the quality factor is:

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{\omega_n L}{R} = \frac{1}{\omega_n RC}$$

Thus, for $\omega \rightarrow \omega_n$:



$$\begin{aligned}
 Z_{out}(j\omega_n) &= \frac{(1 + j\omega_n CR)j\omega_n L}{1 + j\omega_n CR + (j\omega_n)^2 LC} = \frac{j\omega_n L - \omega_n^2 LRC}{1 - \omega_n^2 LC + j\omega_n CR} = \\
 &= R \frac{j\omega_n L / R - (\omega_n L / R)(\omega_n RC)}{1 - (\omega_n L / R)(\omega_n RC) + j\omega_n CR} = R \frac{jQ - Q/Q}{1 - Q/Q + j/Q} = \\
 &= R \frac{jQ - 1}{j/Q} = R(Q^2 + jQ) = RQ(Q + j)
 \end{aligned}$$

For a high quality factor circuit, we have

$$|Z_{out}(j\omega_n)| = RQ\sqrt{1 + Q^2} \cong RQ^2 = R\left(\omega_n \frac{L}{R}\right)Q = \omega_n LQ$$

Finally, the impedance at the resonance frequency becomes a value of Q times the inductive resistance at the resonance frequency.

b) The quality factor is:

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{25} \sqrt{2.8 \times 10^6} \cong 67$$

The impedance at the resonance frequency is:

$$|Z_{out}(j\omega_n)| = RQ\sqrt{1 + Q^2} = 25 \cdot 67 \sqrt{1 + 67^2} = 112.01 \text{ k}\Omega$$

while, Q times the inductive resistance at the resonance frequency is:

$$\omega_n LQ = \frac{1}{\sqrt{2.8 \times 10^{-8}}} \cdot 0.28 \cdot 67 = 112 \text{ k}\Omega$$

Problem 6.28

Solution:

Known quantities:

Frequency response $H_v(j\omega)$ of the circuit of Example 6.7.

Find:

The frequency at which the phase shift introduced by the circuit is equal to -10° .

Analysis:

The frequency response of the circuit is:

$$H_v(j\omega) = \frac{1}{1 + j\omega CR}$$

From Example 6.7:

$$\omega_0 = \frac{1}{CR} = 2,128 \text{ rad/sec}$$

The phase shift introduced by the circuit is:

$$\angle H_v(j\omega) = -\arctan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$\angle H_v(j\omega) = -\arctan^{-1}\left(\frac{\omega}{2128}\right)$$

Thus,

$$\angle H_v(j\bar{\omega}) = -\arctan\left(\frac{\bar{\omega}}{2128}\right) = -10 \Rightarrow \bar{\omega} = 2128 \tan(10) = 375.2 \text{ rad/s}$$

Problem 6.29

Solution:

Known quantities:

Frequency response $H_v(j\omega)$ of the circuit of Example 6.7.

Find:

The frequency at which the output of the circuit is attenuated by 10 percent.

Analysis:

The frequency response of the circuit is:

$$H_v(j\omega) = \frac{1}{1 + j\omega CR}$$

From Example 6.7:

$$\omega_0 = \frac{1}{CR} = 2,128 \text{ rad/sec}$$

The attenuation introduced by the circuit is:

$$|H_v(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$$

Thus,

$$|H_v(j\bar{\omega})| = \frac{1}{\sqrt{1 + (\bar{\omega}/2128)^2}} = 0.9 \Rightarrow \bar{\omega} = (2128) \sqrt{\left(\frac{1}{0.9}\right)^2 - 1} = 1031 \text{ rad/s}$$

Problem 6.30

Solution:

Known quantities:

Frequency response $H_v(j\omega)$ of the circuit of Example 6.11.

Find:

The frequency at which the output of the circuit is attenuated by 10 percent.

Analysis:

The frequency response of the circuit is:

$$H_v(j\omega) = \frac{2}{1 + 0.2j\omega}$$

The attenuation introduced by the circuit is:

$$|H_v(j\omega)| = \frac{2}{\sqrt{1 + (0.2\omega)^2}}$$

Thus,

$$|H_v(j\bar{\omega})| = \frac{2}{\sqrt{1+(0.2\bar{\omega})^2}} = 0.9 \Rightarrow \bar{\omega} = \frac{1}{0.2} \sqrt{\left(\frac{2}{0.9}\right)^2 - 1} = 9.9225 \text{ rad/s}$$

Problem 6.31

Solution:

Known quantities:

Frequency response $H_v(j\omega)$ of the circuit of Example 6.11.

Find:

The frequency at which the phase shift introduced by the circuit is equal to 20° .

Analysis:

The frequency response of the circuit is:

$$H_v(j\omega) = \frac{j\omega CR}{1 + j\omega CR + (j\omega)^2 LC}$$

The phase shift introduced by the circuit is:

$$\angle H_v(j\omega) = \frac{\pi}{2} - \arctan\left(\frac{\omega CR}{1 - \omega^2 LC}\right)$$

Thus,

$$\angle H_v(j\bar{\omega}) = \frac{\pi}{2} - \arctan\left(\frac{\bar{\omega} CR}{1 - \bar{\omega}^2 LC}\right) = 20^\circ \Rightarrow \frac{\bar{\omega} CR}{1 - \bar{\omega}^2 LC} = \tan 25^\circ$$

a) $R = 1k\Omega, C = 10\mu F, L = 5mH :$

$$\bar{\omega} = 46.6 \text{ rad/sec}$$

b) $R = 10k\Omega, C = 10\mu F, L = 5mH :$

$$\bar{\omega} = 4.66 \text{ rad/sec}$$

Problem 6.32

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.1, the period $T = 10\mu s$ and the peak amplitude $A = 1$ for the sawtooth waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first two terms of the Fourier series expansion of the sawtooth waveform of Example 6.3, we have

$$x(t) = \frac{2A}{\pi} \sin\left(\frac{2\pi}{T}t\right) + \frac{A}{\pi} \sin\left(\frac{4\pi}{T}t\right) = \frac{2}{\pi} \sin(2 \times 10^5 \pi t) + \frac{1}{\pi} \sin(4 \times 10^5 \pi t)$$

Thus, for this problem,

$$c_1 = \frac{2}{\pi} \quad \omega_1 = 2 \times 10^5 \pi = 6.28 \times 10^5 \text{ rad/s}$$

and,

$$c_2 = \frac{1}{\pi} \quad \omega_2 = 4 \times 10^5 \pi = 12.56 \times 10^5 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{1}{1 + j(2.5 \times 10^{-6} \omega)} = \frac{1}{\sqrt{1 + 6.25 \times 10^{-12} \omega^2}} \angle -\arctan(2.5 \times 10^{-6} \omega)$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 and ω_2 analytically:

$$|H_v(j\omega_1)| = \frac{1}{\sqrt{1 + 6.25 \times 10^{-12} \omega_1^2}} = 0.537$$

$$\Phi(j\omega_1) = -\arctan(2.5 \times 10^{-6} \omega_1) = -1.0039 \text{ rad} = -57.52^\circ$$

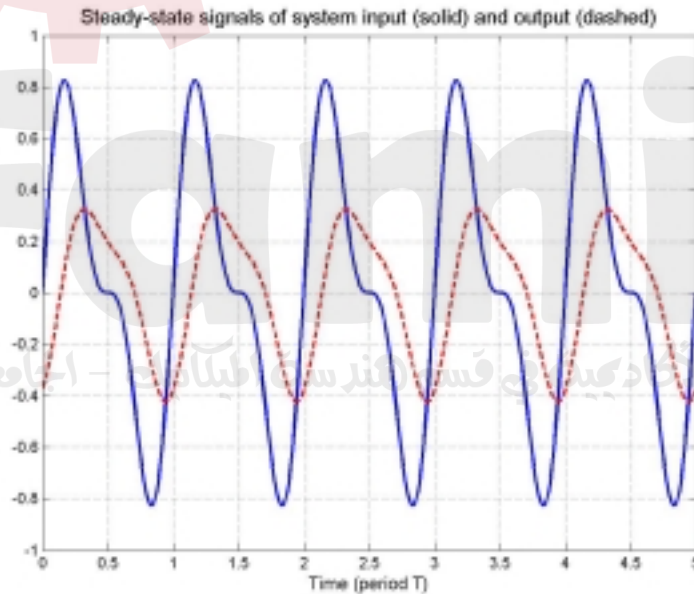
$$|H_v(j\omega_2)| = \frac{1}{\sqrt{1 + 6.25 \times 10^{-12} \omega_2^2}} = 0.3033$$

$$\Phi(j\omega_2) = -\arctan(2.5 \times 10^{-6} \omega_2) = -1.2626 \text{ rad} = -72.34^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= \sum_{n=1}^2 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= 0.537 \frac{2}{\pi} \sin(2 \times 10^5 \pi t - 1.0039) + 0.3033 \frac{1}{\pi} \sin(4 \times 10^5 \pi t - 1.2626) \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.33**Solution:****Known quantities:**

The frequency response $H_v(j\omega)$ of the circuit of P6.1, the period $T = 10 \mu\text{s}$ and the peak amplitude $A = 1$ for the square wave of Figure 6.11(a).

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

The square wave can be defined as:

$$x(t) = \begin{cases} A & (n - \frac{1}{4})T \leq t \leq (n + \frac{1}{4})T, \quad n = \pm 0, \pm 1, \pm 2, \dots \\ 0 & (n + \frac{1}{4})T \leq t \leq (n + \frac{3}{4})T, \quad n = \pm 0, \pm 1, \pm 2, \dots \end{cases}$$

We can compute the Fourier series coefficient using the integrals in equations (6.20), (6.21) and (6.22):

$$a_0 = \frac{1}{T} \int_0^T x(t) dt = \frac{1}{T} \int_{-T/4}^{T/4} A dt = \frac{A}{2}$$

$$\begin{aligned} a_n &= \frac{2}{T} \int_0^T x(t) \cos\left(n \frac{2\pi}{T} t\right) dt = \frac{2}{T} \int_{-T/4}^{T/4} A \cos\left(n \frac{2\pi}{T} t\right) dt = \\ &= \frac{2A}{T} \left[\sin\left(n \frac{2\pi}{T} t\right) \frac{T}{2n\pi} \right]_{-T/4}^{T/4} = \frac{A}{n\pi} \left[\sin\left(\frac{n\pi}{2}\right) - \sin\left(-\frac{n\pi}{2}\right) \right] = 0 \quad (\forall n) \end{aligned}$$

$$\begin{aligned} b_n &= \frac{2}{T} \int_0^T x(t) \sin\left(n \frac{2\pi}{T} t\right) dt = \frac{2}{T} \int_{-T/4}^{T/4} A \sin\left(n \frac{2\pi}{T} t\right) dt = \\ &= \frac{2A}{T} \left[-\cos\left(n \frac{2\pi}{T} t\right) \frac{T}{2n\pi} \right]_{-T/4}^{T/4} = \frac{A}{n\pi} \left[-\cos\left(\frac{n\pi}{2}\right) + \cos\left(-\frac{n\pi}{2}\right) \right] = \\ &= \frac{A}{n\pi} \left[-2 \cos\left(\frac{n\pi}{2}\right) \right] = \begin{cases} \frac{2A}{n\pi} & (n \text{ even}) \\ 0 & (n \text{ odd}) \end{cases} \end{aligned}$$

Using the first two terms of the Fourier series expansion of the square waveform, we have

$$x(t) = \frac{A}{2} + \frac{A}{\pi} \sin\left(\frac{4\pi}{T} t\right) = \frac{1}{2} + \frac{1}{\pi} \sin(4 \times 10^5 \pi t)$$

Thus, for this problem,

$$c_0 = \frac{1}{2} \quad \omega_0 = 0 \text{ rad/s}$$

$$c_1 = 0 \quad \omega_1 = 2 \times 10^5 \pi = 6.28 \times 10^5 \text{ rad/s}$$

and,

$$c_2 = \frac{1}{\pi} \quad \omega_2 = 4 \times 10^5 \pi = 12.56 \times 10^5 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{1}{1 + j(2.5 \times 10^{-6} \omega)} = \frac{1}{\sqrt{1 + 6.25 \times 10^{-12} \omega^2}} \angle -\arctan(2.5 \times 10^{-6} \omega)$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_0 and ω_2 analytically:

$$|H_v(j\omega_0)| = 1$$

$$|H_v(j\omega_2)| = \frac{1}{\sqrt{1 + 6.25 \times 10^{-12} \omega_2^2}} = 0.3033$$

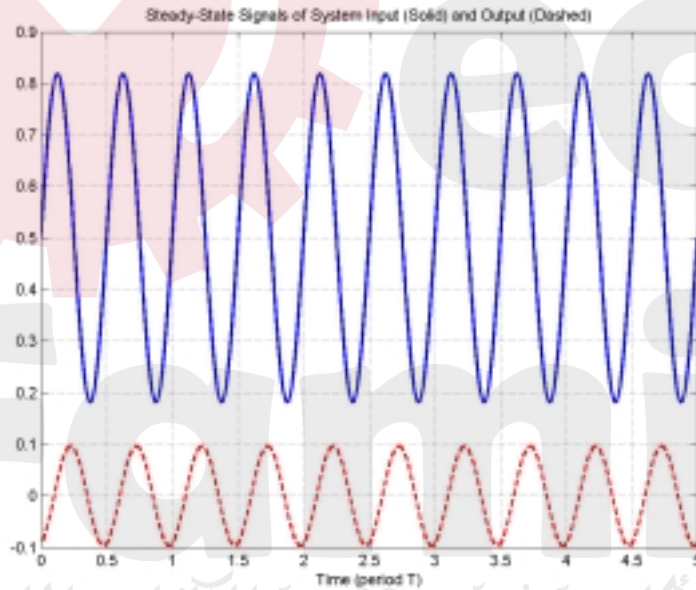
$$\Phi(j\omega_0) = -\arctan(2.5 \times 10^{-6} \omega_0) = 0 \text{ rad} = 0^\circ$$

$$\Phi(j\omega_2) = -\arctan(2.5 \times 10^{-6} \omega_2) = -1.2626 \text{ rad} = -72.34^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= \sum_{n=1}^2 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= \frac{1}{2} \sin(0) + \frac{0.3033}{\pi} \sin(4 \times 10^5 \pi t - 1.2626) = \frac{0.3033}{\pi} \sin(4 \times 10^5 \pi t - 1.2626) \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.34

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.1, the period $T = 10 \mu\text{s}$ and the peak amplitude $A = 1$ for the pulse waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first two terms of the Fourier series expansion of the pulse waveform of Example 6.4, we have

$$x(t) = 0.2 + 0.3027 \cos(2 \times 10^5 \pi t) + 0.2199 \sin(2 \times 10^5 \pi t) \\ + 0.0935 \cos(4 \times 10^5 \pi t) + 0.2879 \sin(4 \times 10^5 \pi t)$$

Thus, for this problem,

$$c_0 = 0.2$$

$$c_1 = 0.3742, \quad \theta_1 = 0.9425 \text{ rad} = 54^\circ \quad \omega_1 = 2 \times 10^5 \pi = 6.28 \times 10^5 \text{ rad/s}$$

and,

$$c_2 = 0.3027, \quad \theta_2 = 0.3140 \text{ rad} = 18^\circ \quad \omega_2 = 4 \times 10^5 \pi = 12.56 \times 10^5 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{1}{1 + j(2.5 \times 10^{-6} \omega)} = \frac{1}{\sqrt{1 + 6.25 \times 10^{-12} \omega^2}} \angle -\arctan(2.5 \times 10^{-6} \omega)$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 and ω_2 analytically:

$$|H_v(j\omega_1)| = \frac{1}{\sqrt{1 + 6.25 \times 10^{-12} \omega_1^2}} = 0.537$$

$$\Phi(j\omega_1) = -\arctan(2.5 \times 10^{-6} \omega_1) = -1.0039 \text{ rad} = -57.52^\circ$$

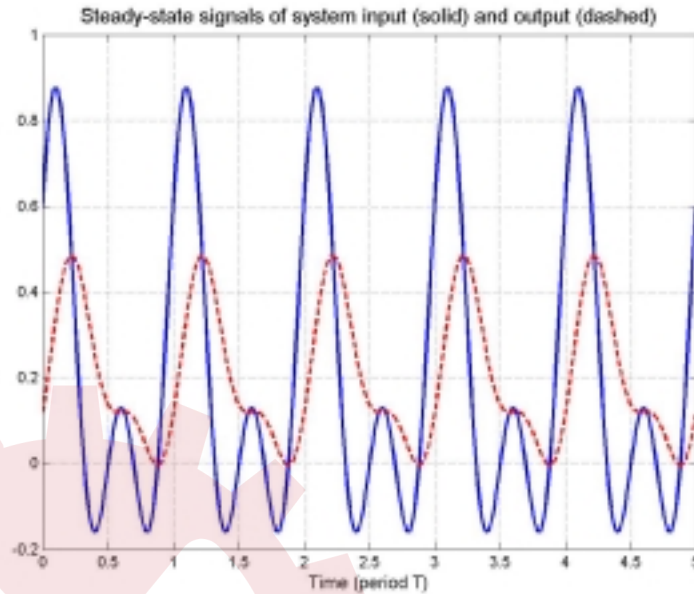
$$|H_v(j\omega_2)| = \frac{1}{\sqrt{1 + 6.25 \times 10^{-12} \omega_2^2}} = 0.3033$$

$$\Phi(j\omega_2) = -\arctan(2.5 \times 10^{-6} \omega_2) = -1.2626 \text{ rad} = -72.34^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$y(t) = c_0 |H_v(0)| + \sum_{n=1}^2 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ = 0.2 + 0.2009 \sin(2 \times 10^5 \pi t - 0.0614) + 0.0918 \sin(4 \times 10^5 \pi t - 0.9484)$$

The input and output steady state signals plot is shown below:



Problem 6.35

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.2, the period $T = 0.5$ s and the peak amplitude $A = 2$ for the sawtooth waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first three terms of the Fourier series expansion of the sawtooth waveform of Example 6.3, we have

$$\begin{aligned} x(t) &= \frac{2A}{\pi} \sin\left(\frac{2\pi}{T}t\right) + \frac{A}{\pi} \sin\left(\frac{4\pi}{T}t\right) + \frac{2A}{3\pi} \sin\left(\frac{6\pi}{T}t\right) = \\ &= \frac{4}{\pi} \sin(4\pi t) + \frac{2}{\pi} \sin(8\pi t) + \frac{4}{3\pi} \sin(12\pi t) \end{aligned}$$

Thus, for this problem,

$$c_1 = \frac{4}{\pi} \quad \omega_1 = 4\pi = 12.5664 \text{ rad/s}$$

$$c_2 = \frac{2}{\pi} \quad \omega_2 = 8\pi = 25.1327 \text{ rad/s}$$

$$c_3 = \frac{4}{3\pi} \quad \omega_3 = 12\pi = 37.6991 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{1}{2(1 + 0.05j\omega)} = \frac{1}{\sqrt{4 + 0.01\omega^2}} \angle -\arctan(0.05\omega)$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 , ω_2 and ω_3 analytically:

$$|H_v(j\omega_1)| = \frac{1}{\sqrt{4 + 0.01\omega_1^2}} = 0.4234$$

$$\Phi(j\omega_1) = -\arctan(0.05\omega_1) = -0.5610 \text{ rad} = -32.14^\circ$$

$$|H_v(j\omega_2)| = \frac{1}{\sqrt{4 + 0.01\omega_2^2}} = 0.3113$$

$$\Phi(j\omega_2) = -\arctan(0.05\omega_2) = -0.8986 \text{ rad} = -51.49^\circ$$

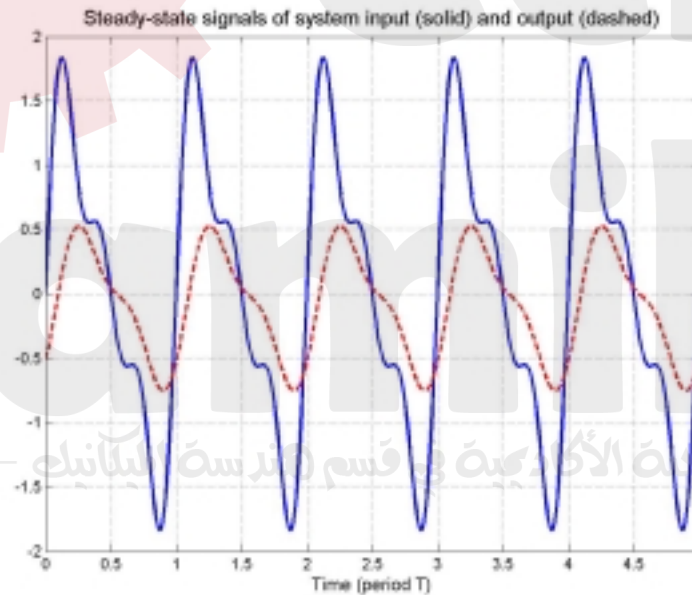
$$|H_v(j\omega_3)| = \frac{1}{\sqrt{4 + 0.01\omega_3^2}} = 0.2343$$

$$\Phi(j\omega_3) = -\arctan(0.05\omega_3) = -1.0830 \text{ rad} = -62.05^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= \sum_{n=1}^3 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= 0.4234 \frac{4}{\pi} \sin(4\pi t - 0.561) + 0.3113 \frac{2}{\pi} \sin(8\pi t - 0.8986) \\ &\quad + 0.2343 \frac{4}{3\pi} \sin(12\pi t - 1.083) \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.36**Solution:****Known quantities:**

The frequency response $H_v(j\omega)$ of the circuit of P6.2, the period $T = 0.5$ s and the peak amplitude $A = 2$ for the square wave.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first three terms of the Fourier series expansion of the square waveform of P6.33, we have

$$x(t) = \frac{4A}{\pi} \sin\left(\frac{2\pi}{T}t\right) + \frac{4A}{3\pi} \sin\left(\frac{6\pi}{T}t\right) = \frac{8}{\pi} \sin(4\pi t) + \frac{8}{3\pi} \sin(12\pi t)$$

Thus, for this problem,

$$\begin{aligned} c_1 &= \frac{8}{\pi} & \omega_1 &= 4\pi = 12.5664 \text{ rad/s} \\ c_2 &= 0 & \omega_2 &= 8\pi = 25.1327 \text{ rad/s} \\ c_3 &= \frac{8}{3\pi} & \omega_3 &= 12\pi = 37.6991 \text{ rad/s} \end{aligned}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{1}{2(1 + 0.05j\omega)} = \frac{1}{\sqrt{4 + 0.01\omega^2}} \angle -\arctan(0.05\omega)$$

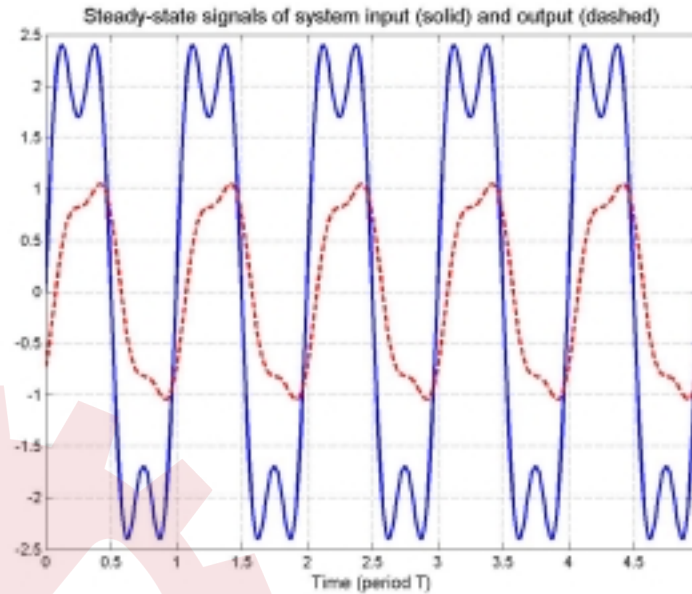
At this point, we could evaluate the frequency response of the system at the frequencies ω_1 and ω_3 analytically:

$$\begin{aligned} |H_v(j\omega_1)| &= \frac{1}{\sqrt{4 + 0.01\omega_1^2}} = 0.4234 \\ \Phi(j\omega_1) &= -\arctan(0.05\omega_1) = -0.5610 \text{ rad} = -32.14^\circ \\ |H_v(j\omega_3)| &= \frac{1}{\sqrt{4 + 0.01\omega_3^2}} = 0.2343 \\ \Phi(j\omega_3) &= -\arctan(0.05\omega_3) = -1.0830 \text{ rad} = -62.05^\circ \end{aligned}$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= \sum_{n=1}^3 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= 0.4234 \frac{8}{\pi} \sin(4\pi t - 0.561) + 0.2343 \frac{8}{3\pi} \sin(12\pi t - 1.083) \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.37

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.2, the period $T = 0.5$ s and the peak amplitude $A = 2$ for the pulse waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first three terms of the Fourier series expansion of the pulse waveform of Example 6.4, we have

$$x(t) = 0.4 + 0.6054 \cos(4\pi t) + 0.4398 \sin(4\pi t) + 0.1870 \cos(8\pi t) \\ + 0.5758 \sin(8\pi t) - 0.1248 \cos(12\pi t) + 0.3838 \sin(12\pi t)$$

Thus, for this problem,

$$c_0 = 0.4$$

$$c_1 = 0.7484, \quad \theta_1 = 0.9425 \text{ rad} = 54^\circ \quad \omega_1 = 4\pi = 12.5664 \text{ rad/s}$$

$$c_2 = 0.6054, \quad \theta_2 = 0.3140 \text{ rad} = 18^\circ \quad \omega_2 = 8\pi = 25.1327 \text{ rad/s}$$

$$c_3 = 0.4036, \quad \theta_3 = -0.3144 \text{ rad} = -18^\circ \quad \omega_3 = 12\pi = 37.6991 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{1}{2(1 + 0.05j\omega)} = \frac{1}{\sqrt{4 + 0.01\omega^2}} \angle -\arctan(0.05\omega)$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 , ω_2 and ω_3 analytically:

$$|H_v(j\omega_1)| = \frac{1}{\sqrt{4 + 0.01\omega_1^2}} = 0.4234$$

$$\Phi(j\omega_1) = -\arctan(0.05\omega_1) = -0.5610 \text{ rad} = -32.14^\circ$$

$$|H_v(j\omega_2)| = \frac{1}{\sqrt{4 + 0.01\omega_2^2}} = 0.3113$$

$$\Phi(j\omega_2) = -\arctan(0.05\omega_2) = -0.8986 \text{ rad} = -51.49^\circ$$

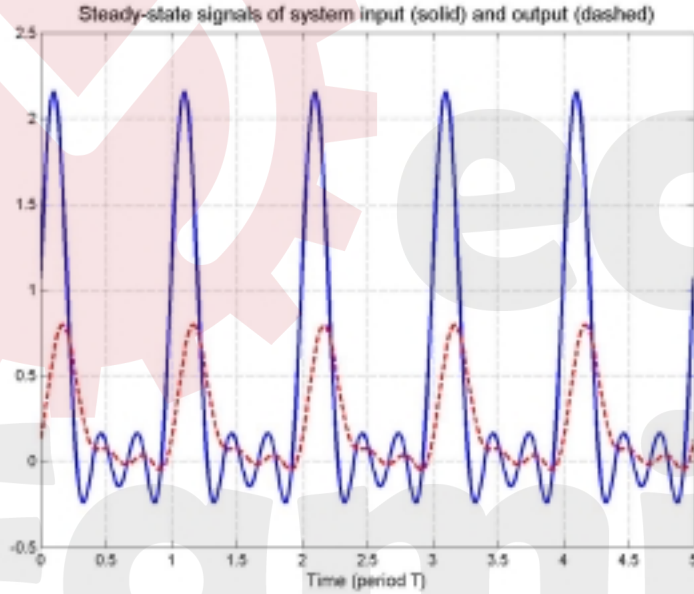
$$|H_v(j\omega_3)| = \frac{1}{\sqrt{4 + 0.01\omega_3^2}} = 0.2343$$

$$\Phi(j\omega_3) = -\arctan(0.05\omega_3) = -1.0830 \text{ rad} = -62.05^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= c_0 |H_v(0)| + \sum_{n=1}^3 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= 0.2 + 0.3168 \sin(4\pi t + 0.3815) + 0.1885 \sin(8\pi t - 0.5846) + 0.0946 \sin(12\pi t - 1.3974) \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.38

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.3, the period $T = 0.1 \text{ s}$ and the peak amplitude $A = 1$ for the sawtooth waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first four terms of the Fourier series expansion of the sawtooth waveform of Example 6.3, we have

$$\begin{aligned}
 x(t) &= \frac{2A}{\pi} \sin\left(\frac{2\pi}{T}t\right) + \frac{A}{\pi} \sin\left(\frac{4\pi}{T}t\right) + \frac{2A}{3\pi} \sin\left(\frac{6\pi}{T}t\right) + \frac{A}{2\pi} \sin\left(\frac{8\pi}{T}t\right) \\
 &= \frac{2}{\pi} \sin(20\pi t) + \frac{1}{\pi} \sin(40\pi t) + \frac{2}{3\pi} \sin(60\pi t) + \frac{1}{2\pi} \sin(80\pi t)
 \end{aligned}$$

Thus, for this problem,

$$c_1 = \frac{2}{\pi} \quad \omega_1 = 20\pi = 62.8 \text{ rad/s}$$

$$c_2 = \frac{1}{\pi} \quad \omega_2 = 40\pi = 125.6 \text{ rad/s}$$

$$c_3 = \frac{2}{3\pi} \quad \omega_3 = 60\pi = 188.4 \text{ rad/s}$$

and,

$$c_4 = \frac{1}{2\pi} \quad \omega_4 = 80\pi = 251.2 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{1}{2(1 + 0.02j\omega)} = \frac{1}{\sqrt{4 + 0.0016\omega^2}} \angle -\arctan(0.02\omega)$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 , ω_2 , ω_3 and ω_4 analytically:

$$|H_v(j\omega_1)| = \frac{1}{\sqrt{4 + 0.0016\omega_1^2}} = 0.3113$$

$$\Phi(j\omega_1) = -\arctan(0.02\omega_1) = -0.8986 \text{ rad} = -51.49^\circ$$

$$|H_v(j\omega_2)| = \frac{1}{\sqrt{4 + 0.0016\omega_2^2}} = 0.1848$$

$$\Phi(j\omega_2) = -\arctan(0.02\omega_2) = -1.1921 \text{ rad} = -68.3^\circ$$

$$|H_v(j\omega_3)| = \frac{1}{\sqrt{4 + 0.0016\omega_3^2}} = 0.1282$$

$$\Phi(j\omega_3) = -\arctan(0.02\omega_3) = -1.3115 \text{ rad} = -75.14^\circ$$

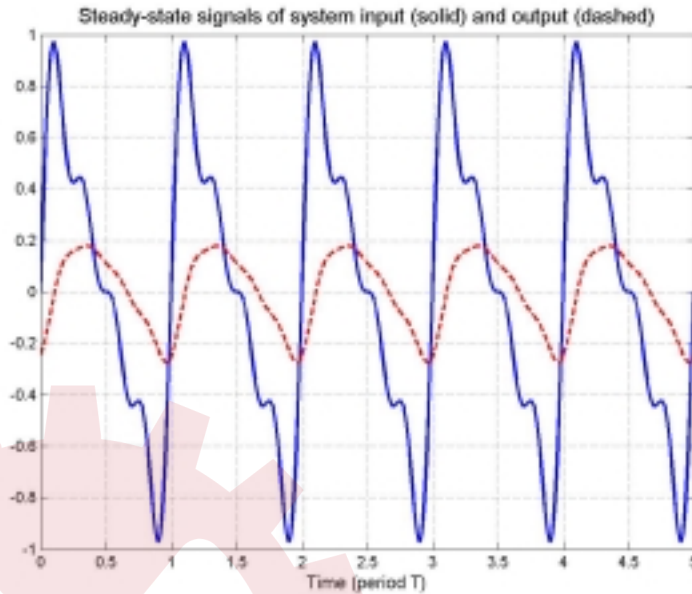
$$|H_v(j\omega_4)| = \frac{1}{\sqrt{4 + 0.0016\omega_4^2}} = 0.0976$$

$$\Phi(j\omega_4) = -\arctan(0.02\omega_4) = -1.3744 \text{ rad} = -78.75^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned}
 y(t) &= \sum_{n=1}^4 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\
 &= 0.1982 \sin(20\pi t - 0.8986) + 0.0588 \sin(40\pi t - 1.1921) \\
 &\quad + 0.0272 \sin(60\pi t - 1.3115) + 0.0155 \sin(80\pi t - 1.3744)
 \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.39

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.3, the period $T = 0.1$ s and the peak amplitude $A = 1$ for the square waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first four terms of the Fourier series expansion of the square waveform of P6.33, we have

$$x(t) = \frac{4A}{\pi} \sin\left(\frac{2\pi}{T}t\right) + \frac{4A}{3\pi} \sin\left(\frac{6\pi}{T}t\right) = \frac{4}{\pi} \sin(20\pi t) + \frac{4}{3\pi} \sin(60\pi t)$$

Thus, for this problem,

$$c_1 = \frac{4}{\pi} \quad \omega_1 = 20\pi = 62.8 \text{ rad/s}$$

$$c_3 = \frac{4}{3\pi} \quad \omega_3 = 60\pi = 188.4 \text{ rad/s}$$

$$c_2 = c_4 = 0$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{1}{2(1 + 0.02j\omega)} = \frac{1}{\sqrt{4 + 0.0016\omega^2}} \angle -\arctan(0.02\omega)$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 and ω_3 analytically:

$$|H_v(j\omega_1)| = \frac{1}{\sqrt{4 + 0.0016\omega_1^2}} = 0.3113$$

$$\Phi(j\omega_1) = -\arctan(0.02\omega_1) = -0.8986 \text{ rad} = -51.49^\circ$$

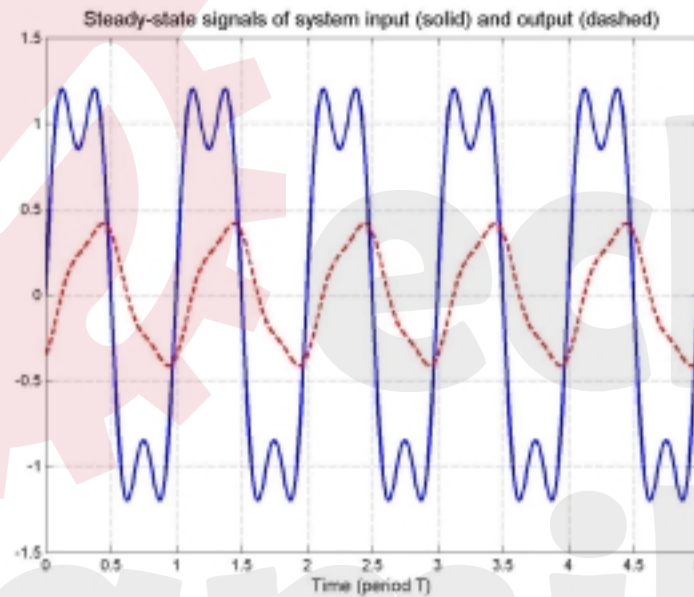
$$|H_v(j\omega_3)| = \frac{1}{\sqrt{4 + 0.0016\omega_3^2}} = 0.1282$$

$$\Phi(j\omega_3) = -\arctan(0.02\omega_3) = -1.3115 \text{ rad} = -75.14^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= \sum_{n=1}^4 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= 0.3964 \sin(20\pi t - 0.8986) + 0.0544 \sin(60\pi t - 1.3115) \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.40

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.3, the period $T = 0.1 \text{ s}$ and the peak amplitude $A = 1$ for the pulse waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first four terms of the Fourier series expansion of the pulse waveform of Example 6.4, we have

$$\begin{aligned} x(t) &= 0.2 + 0.3027 \cos(20\pi t) + 0.2199 \sin(20\pi t) + 0.0935 \cos(40\pi t) + 0.2879 \sin(40\pi t) \\ &\quad - 0.0624 \cos(60\pi t) + 0.1919 \sin(60\pi t) - 0.0757 \cos(80\pi t) + 0.055 \sin(80\pi t) \end{aligned}$$

Thus, for this problem,

$$c_0 = 0.2$$

$$c_1 = 0.3742, \quad \theta_1 = 0.9425 \text{ rad} = 54^\circ \quad \omega_1 = 20\pi = 62.8 \text{ rad/s}$$

$$c_2 = 0.3027, \quad \theta_2 = 0.3140 \text{ rad} = 18^\circ \quad \omega_2 = 40\pi = 125.6 \text{ rad/s}$$

$$c_3 = 0.2018, \quad \theta_3 = -0.3144 \text{ rad} = -18^\circ \quad \omega_3 = 60\pi = 188.4 \text{ rad/s}$$

and,

$$c_4 = 0.0935, \quad \theta_4 = -0.9425 \text{ rad} = -54^\circ \quad \omega_4 = 80\pi = 251.2 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{1}{2(1+0.02j\omega)} = \frac{1}{\sqrt{4+0.0016\omega^2}} \angle -\arctan(0.02\omega)$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 , ω_2 , ω_3 and ω_4 analytically:

$$|H_v(j\omega_1)| = \frac{1}{\sqrt{4+0.0016\omega_1^2}} = 0.3113$$

$$\Phi(j\omega_1) = -\arctan(0.02\omega_1) = -0.8986 \text{ rad} = -51.49^\circ$$

$$|H_v(j\omega_2)| = \frac{1}{\sqrt{4+0.0016\omega_2^2}} = 0.1848$$

$$\Phi(j\omega_2) = -\arctan(0.02\omega_2) = -1.1921 \text{ rad} = -68.3^\circ$$

$$|H_v(j\omega_3)| = \frac{1}{\sqrt{4+0.0016\omega_3^2}} = 0.1282$$

$$\Phi(j\omega_3) = -\arctan(0.02\omega_3) = -1.3115 \text{ rad} = -75.14^\circ$$

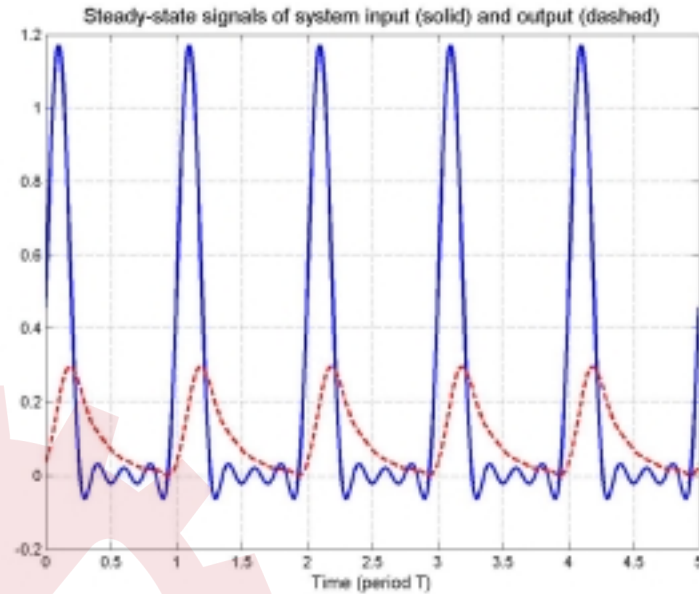
$$|H_v(j\omega_4)| = \frac{1}{\sqrt{4+0.0016\omega_4^2}} = 0.0976$$

$$\Phi(j\omega_4) = -\arctan(0.02\omega_4) = -1.3744 \text{ rad} = -78.75^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= c_0 |H_v(0)| + \sum_{n=1}^4 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= 0.1 + 0.1165 \sin(20\pi t + 0.0439) + 0.0559 \sin(40\pi t - 0.8781) \\ &\quad + 0.0259 \sin(60\pi t - 1.6259) + 0.00915 \sin(80\pi t - 2.3169) \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.41

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.4, the period $T = 50 \text{ ms}$ and the peak amplitude $A = 2$ for the sawtooth waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first two terms of the Fourier series expansion of the sawtooth waveform of Example 6.3, we have

$$x(t) = \frac{2A}{\pi} \sin\left(\frac{2\pi}{T}t\right) + \frac{A}{\pi} \sin\left(\frac{4\pi}{T}t\right) = \frac{4}{\pi} \sin(40\pi t) + \frac{2}{\pi} \sin(80\pi t)$$

Thus, for this problem,

$$c_1 = \frac{4}{\pi} \quad \omega_1 = 40\pi = 125.6 \text{ rad/s}$$

and,

$$c_2 = \frac{2}{\pi} \quad \omega_2 = 80\pi = 251.2 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$\begin{aligned} H_v(j\omega) &= \frac{1 - 0.0002\omega^2 + 0.1j\omega}{1 - 0.0002\omega^2 + 0.15j\omega} = \\ &= \frac{\sqrt{[1 - 0.0002\omega^2]^2 + 0.01\omega^2}}{\sqrt{[1 - 0.0002\omega^2]^2 + 0.0225\omega^2}} \angle \left[\arctan\left(\frac{0.1\omega}{1 - 0.0002\omega^2}\right) - \arctan\left(\frac{0.15\omega}{1 - 0.0002\omega^2}\right) \right] \end{aligned}$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 and ω_2 analytically:

$$|H_v(j\omega_1)| = \frac{\sqrt{[1 - 0.0002\omega_1^2]^2 + 0.01\omega_1^2}}{\sqrt{[1 - 0.0002\omega_1^2]^2 + 0.0225\omega_1^2}} = 0.6720$$

$$\Phi(j\omega_1) = \arctan\left(\frac{0.1\omega_1}{1 - 0.0002\omega_1^2}\right) - \arctan\left(\frac{0.15\omega_1}{1 - 0.0002\omega_1^2}\right) = 0.0561 \text{ rad} = 3.21^\circ$$

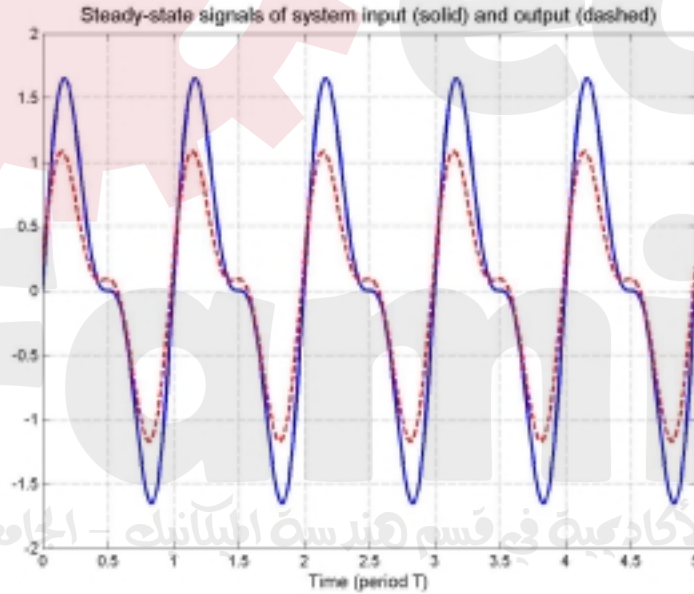
$$|H_v(j\omega_2)| = \frac{\sqrt{[1 - 0.0002\omega_2^2]^2 + 0.01\omega_2^2}}{\sqrt{[1 - 0.0002\omega_2^2]^2 + 0.0225\omega_2^2}} = 0.7020$$

$$\Phi(j\omega_2) = \arctan\left(\frac{0.1\omega_2}{1 - 0.0002\omega_2^2}\right) - \arctan\left(\frac{0.15\omega_2}{1 - 0.0002\omega_2^2}\right) = 0.1342 \text{ rad} = 7.69^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= \sum_{n=1}^2 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= 0.6720 \frac{4}{\pi} \sin(40\pi t + 0.0561) + 0.7020 \frac{2}{\pi} \sin(80\pi t + 0.1342) \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.42

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.4, the periods $T_1 = 0.5 \text{ s}$ and $T_2 = 5 \text{ ms}$ and the peak amplitude $A = 2$ for the sawtooth waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$. Compare the plots with the one obtained in P6.41.

Analysis:

According to the Fourier series definitions of the previous section, and using the first two terms of the Fourier series expansion of the sawtooth waveform of Example 6.3, we have

$$x(t) = \frac{2A}{\pi} \sin\left(\frac{2\pi}{T}t\right) + \frac{A}{\pi} \sin\left(\frac{4\pi}{T}t\right) \Rightarrow \begin{aligned} x_1(t) &= \frac{4}{\pi} \sin(4\pi t) + \frac{2}{\pi} \sin(8\pi t) \\ x_2(t) &= \frac{4}{\pi} \sin(400\pi t) + \frac{2}{\pi} \sin(800\pi t) \end{aligned}$$

Thus, for this problem,

$$c_{1,1} = c_{1,2} = \frac{4}{\pi} \quad \omega_{1,1} = 4\pi = 12.56 \text{ rad/s} \quad \omega_{1,2} = 400\pi = 1256 \text{ rad/s}$$

and,

$$c_{2,1} = c_{2,2} = \frac{2}{\pi} \quad \omega_{2,1} = 8\pi = 25.12 \text{ rad/s} \quad \omega_{2,2} = 800\pi = 2512 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$\begin{aligned} H_v(j\omega) &= \frac{1 - 0.0002\omega^2 + 0.1j\omega}{1 - 0.0002\omega^2 + 0.15j\omega} = \\ &= \frac{\sqrt{[1 - 0.0002\omega^2]^2 + 0.01\omega^2}}{\sqrt{[1 - 0.0002\omega^2]^2 + 0.0225\omega^2}} \angle \left[\arctan\left(\frac{0.1\omega}{1 - 0.0002\omega^2}\right) - \arctan\left(\frac{0.15\omega}{1 - 0.0002\omega^2}\right) \right] \end{aligned}$$

At this point, we could evaluate the frequency response of the system at the frequencies $\omega_{1,1}$, $\omega_{2,1}$, $\omega_{1,2}$ and $\omega_{2,2}$ analytically:

$$|H_v(j\omega_{1,1})| = 0.7486, \quad \Phi(j\omega_{1,1}) = -0.1820 \text{ rad} = -10.43^\circ$$

$$|H_v(j\omega_{2,1})| = 0.6876, \quad \Phi(j\omega_{2,1}) = -0.1068 \text{ rad} = -6.12^\circ$$

and,

$$|H_v(j\omega_{1,2})| = 0.9238, \quad \Phi(j\omega_{1,2}) = 0.1597 \text{ rad} = 9.15^\circ$$

$$|H_v(j\omega_{2,2})| = 0.9770, \quad \Phi(j\omega_{2,2}) = 0.0937 \text{ rad} = 5.37^\circ$$

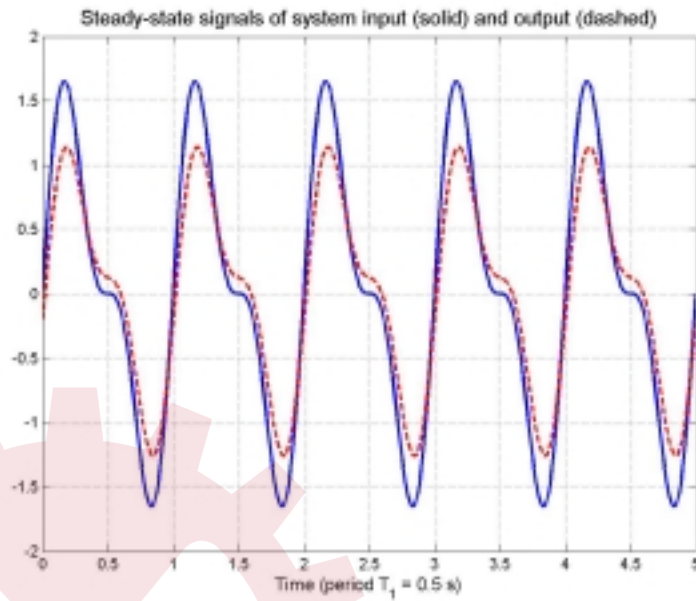
Finally, we can compute the steady-state periodic outputs of the system:

$$\begin{aligned} y_1(t) &= \sum_{n=1}^2 |H_v(j\omega_{n,1})| c_{n,1} \sin[\omega_{n,1}t + \theta_{n,1} + \Phi(j\omega_{n,1})] = \\ &= 0.7486 \frac{4}{\pi} \sin(4\pi t - 0.182) + 0.6876 \frac{2}{\pi} \sin(8\pi t - 0.1068) \end{aligned}$$

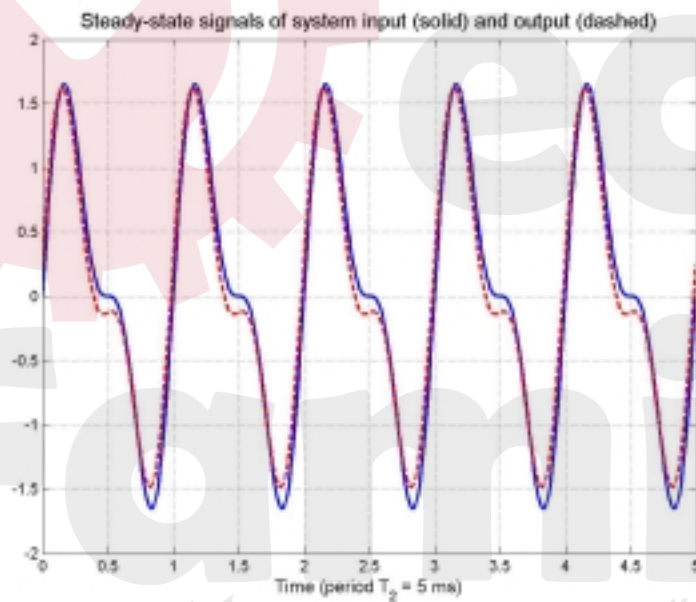
and,

$$\begin{aligned} y_2(t) &= \sum_{n=1}^2 |H_v(j\omega_{n,2})| c_{n,2} \sin[\omega_{n,2}t + \theta_{n,2} + \Phi(j\omega_{n,2})] = \\ &= 0.9238 \frac{4}{\pi} \sin(400\pi t + 0.1597) + 0.977 \frac{2}{\pi} \sin(800\pi t + 0.0937) \end{aligned}$$

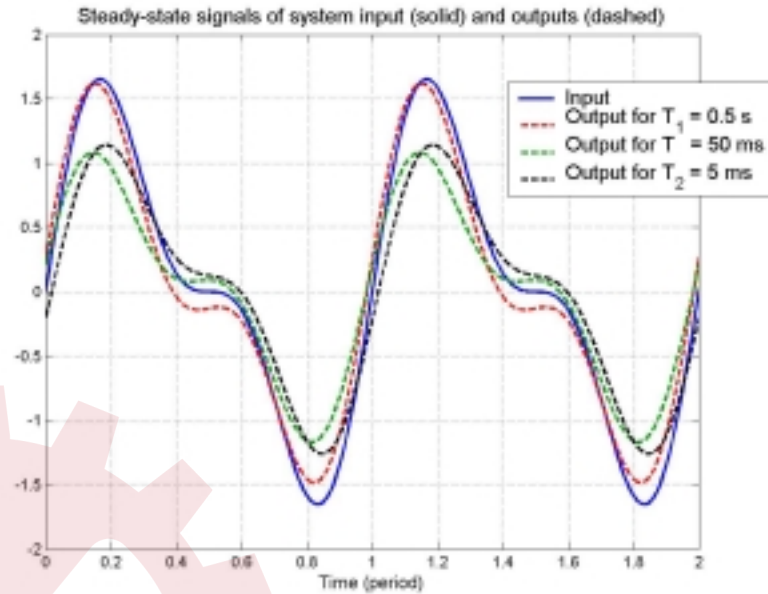
The input and output steady state signals plot for $T_1 = 0.5 \text{ s}$ is shown below:



The input and output steady state signals plot for $T_2 = 5$ ms is shown below:



Comparing the results with $T = 50$ ms, we have:



Problem 6.43

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.4, the period $T = 50 \text{ ms}$ and the peak amplitude $A = 2$ for the square wave.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first two terms of the Fourier series expansion of the square waveform of P6.33, we have

$$x(t) = \frac{4A}{\pi} \sin\left(\frac{2\pi}{T}t\right) = \frac{8}{\pi} \sin(40\pi t)$$

Thus, for this problem,

$$c_1 = \frac{8}{\pi} \quad \omega_1 = 40\pi = 125.6 \text{ rad/s}$$

and,

$$c_2 = 0$$

The frequency response of the system can be expressed in magnitude and phase form:

$$\begin{aligned} H_v(j\omega) &= \frac{1 - 0.0002\omega^2 + 0.1j\omega}{1 - 0.0002\omega^2 + 0.15j\omega} = \\ &= \frac{\sqrt{[1 - 0.0002\omega^2]^2 + 0.01\omega^2}}{\sqrt{[1 - 0.0002\omega^2]^2 + 0.0225\omega^2}} \angle \left[\arctan\left(\frac{0.1\omega}{1 - 0.0002\omega^2}\right) - \arctan\left(\frac{0.15\omega}{1 - 0.0002\omega^2}\right) \right] \end{aligned}$$

At this point, we could evaluate the frequency response of the system at the frequency ω_1 analytically:

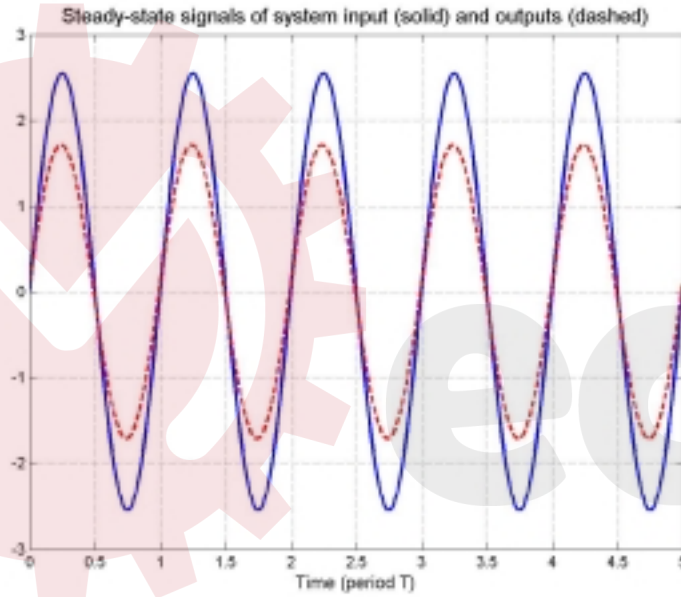
$$|H_v(j\omega_1)| = \frac{\sqrt{[1 - 0.0002\omega_1^2]^2 + 0.01\omega_1^2}}{\sqrt{[1 - 0.0002\omega_1^2]^2 + 0.0225\omega_1^2}} = 0.6720$$

$$\Phi(j\omega_1) = \arctan\left(\frac{0.1\omega_1}{1 - 0.0002\omega_1^2}\right) - \arctan\left(\frac{0.15\omega_1}{1 - 0.0002\omega_1^2}\right) = 0.0561 \text{ rad} = 3.21^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$y(t) = \sum_{n=1}^2 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = 0.672 \frac{8}{\pi} \sin(40\pi t + 0.0561)$$

The input and output steady state signals plot is shown below:



Problem 6.44

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.4, the period $T = 50 \text{ ms}$ and the peak amplitude $A = 2$ for the pulse waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first two terms of the Fourier series expansion of the pulse waveform of Example 6.4, we have

$$x(t) = 0.4 + 0.6054 \cos(40\pi t) + 0.4398 \sin(40\pi t) + 0.1870 \cos(80\pi t) + 0.5758 \sin(80\pi t)$$

Thus, for this problem,

$$c_0 = 0.4$$

$$c_1 = 0.7484, \quad \theta_1 = 0.9425 \text{ rad} = 54^\circ \quad \omega_1 = 40\pi = 125.6 \text{ rad/s}$$

and,

$$c_2 = 0.6054, \quad \theta_2 = 0.3140 \text{ rad} = 18^\circ \quad \omega_2 = 80\pi = 251.2 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$\begin{aligned} H_v(j\omega) &= \frac{1 - 0.0002\omega^2 + 0.1j\omega}{1 - 0.0002\omega^2 + 0.15j\omega} = \\ &= \frac{\sqrt{[1 - 0.0002\omega^2]^2 + 0.01\omega^2}}{\sqrt{[1 - 0.0002\omega^2]^2 + 0.0225\omega^2}} \angle \left[\arctan\left(\frac{0.1\omega}{1 - 0.0002\omega^2}\right) - \arctan\left(\frac{0.15\omega}{1 - 0.0002\omega^2}\right) \right] \end{aligned}$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 and ω_2 analytically:

$$|H_v(j\omega_1)| = \frac{\sqrt{[1 - 0.0002\omega_1^2]^2 + 0.01\omega_1^2}}{\sqrt{[1 - 0.0002\omega_1^2]^2 + 0.0225\omega_1^2}} = 0.6720$$

$$\Phi(j\omega_1) = \arctan\left(\frac{0.1\omega_1}{1 - 0.0002\omega_1^2}\right) - \arctan\left(\frac{0.15\omega_1}{1 - 0.0002\omega_1^2}\right) = 0.0561 \text{ rad} = 3.21^\circ$$

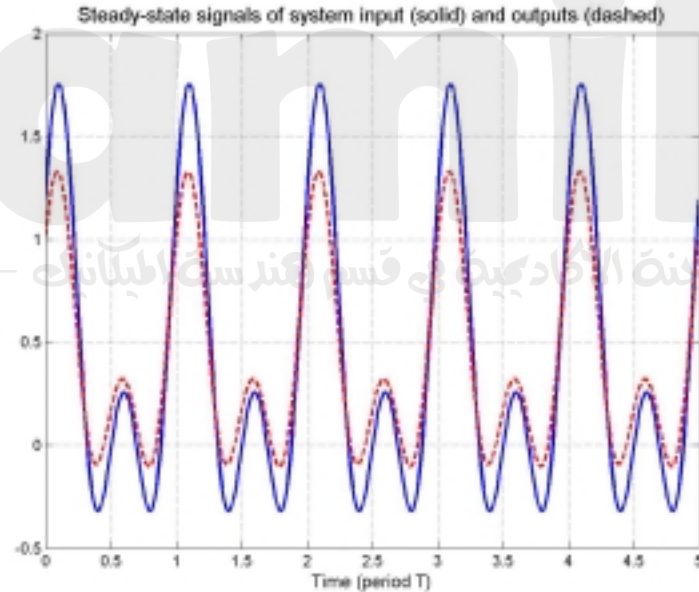
$$|H_v(j\omega_2)| = \frac{\sqrt{[1 - 0.0002\omega_2^2]^2 + 0.01\omega_2^2}}{\sqrt{[1 - 0.0002\omega_2^2]^2 + 0.0225\omega_2^2}} = 0.7020$$

$$\Phi(j\omega_2) = \arctan\left(\frac{0.1\omega_2}{1 - 0.0002\omega_2^2}\right) - \arctan\left(\frac{0.15\omega_2}{1 - 0.0002\omega_2^2}\right) = 0.1342 \text{ rad} = 7.69^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= c_0 |H_v(0)| + \sum_{n=1}^2 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= 0.4 + 0.503 \sin(40\pi t + 0.9986) + 0.425 \sin(80\pi t + 0.4482) \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.45**Solution:****Known quantities:**

The frequency response $H_v(j\omega)$ of the circuit of P6.6, the period $T = 5$ s and the peak amplitude $A = 1$ for the sawtooth waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first three terms of the Fourier series expansion of the sawtooth waveform of Example 6.3, we have

$$\begin{aligned} x(t) &= \frac{2A}{\pi} \sin\left(\frac{2\pi}{T}t\right) + \frac{A}{\pi} \sin\left(\frac{4\pi}{T}t\right) + \frac{2A}{3\pi} \sin\left(\frac{6\pi}{T}t\right) \\ &= \frac{2}{\pi} \sin(0.4\pi t) + \frac{1}{\pi} \sin(0.8\pi t) + \frac{2}{3\pi} \sin(1.2\pi t) \end{aligned}$$

Thus, for this problem,

$$c_1 = \frac{2}{\pi} \quad \omega_1 = 0.4\pi = 1.256 \text{ rad/s}$$

$$c_2 = \frac{1}{\pi} \quad \omega_2 = 0.8\pi = 2.512 \text{ rad/s}$$

and,

$$c_3 = \frac{2}{3\pi} \quad \omega_3 = 1.2\pi = 3.768 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{2j\omega}{(1-\omega^2) + 2.6j\omega} = \frac{2\omega}{\sqrt{(1-\omega^2)^2 + 6.76\omega^2}} \angle \left[90^\circ - \arctan\left(\frac{2.6\omega}{1-\omega^2}\right) \right]$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 , ω_2 and ω_3 analytically:

$$|H_v(j\omega_1)| = \frac{2\omega_1}{\sqrt{(1-\omega_1^2)^2 + 6.76\omega_1^2}} = 0.7575$$

$$\Phi(j\omega_1) = 90^\circ - \arctan\left(\frac{2.6\omega_1}{1-\omega_1^2}\right) = -0.1750 \text{ rad} = -10^\circ$$

$$|H_v(j\omega_2)| = \frac{2\omega_2}{\sqrt{(1-\omega_2^2)^2 + 6.76\omega_2^2}} = 0.5969$$

$$\Phi(j\omega_2) = 90^\circ - \arctan\left(\frac{2.6\omega_2}{1-\omega_2^2}\right) = -0.6826 \text{ rad} = -39.11^\circ$$

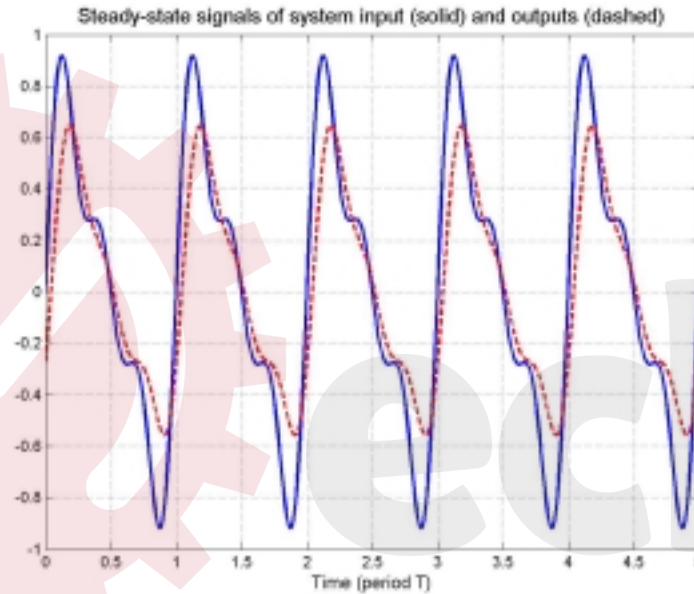
$$|H_v(j\omega_3)| = \frac{2\omega_3}{\sqrt{(1-\omega_3^2)^2 + 6.76\omega_3^2}} = 0.4585$$

$$\Phi(j\omega_3) = 90^\circ - \arctan\left(\frac{2.6\omega_3}{1-\omega_3^2}\right) = -0.9322 \text{ rad} = -53.41^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= \sum_{n=1}^3 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= 0.4822 \sin(0.4\pi t - 0.175) + 0.19 \sin(0.8\pi t - 0.6826) + 0.0973 \sin(1.2\pi t - 0.9322) \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.46

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.6, the period $T = 50$ s and the peak amplitude $A = 1$ for the sawtooth waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first three terms of the Fourier series expansion of the sawtooth waveform of Example 6.3, we have

$$\begin{aligned} x(t) &= \frac{2A}{\pi} \sin\left(\frac{2\pi}{T}t\right) + \frac{A}{\pi} \sin\left(\frac{4\pi}{T}t\right) + \frac{2A}{3\pi} \sin\left(\frac{6\pi}{T}t\right) \\ &= \frac{2}{\pi} \sin(0.04\pi t) + \frac{1}{\pi} \sin(0.08\pi t) + \frac{2}{3\pi} \sin(0.12\pi t) \end{aligned}$$

Thus, for this problem,

$$c_1 = \frac{2}{\pi} \quad \omega_1 = 0.04\pi = 0.1256 \text{ rad/s}$$

$$c_2 = \frac{1}{\pi} \quad \omega_2 = 0.08\pi = 0.2512 \text{ rad/s}$$

and,

$$c_3 = \frac{2}{3\pi} \quad \omega_3 = 0.12\pi = 0.3768 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{2j\omega}{(1-\omega^2) + 2.6j\omega} = \frac{2\omega}{\sqrt{(1-\omega^2)^2 + 6.76\omega^2}} \angle \left[90^\circ - \arctan\left(\frac{2.6\omega}{1-\omega^2}\right) \right]$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 , ω_2 and ω_3 analytically:

$$|H_v(j\omega_1)| = \frac{2\omega_1}{\sqrt{(1-\omega_1^2)^2 + 6.76\omega_1^2}} = 0.2422$$

$$\Phi(j\omega_1) = 90^\circ - \arctan\left(\frac{2.6\omega_1}{1-\omega_1^2}\right) = 1.2504 \text{ rad} = 71.64^\circ$$

$$|H_v(j\omega_2)| = \frac{2\omega_2}{\sqrt{(1-\omega_2^2)^2 + 6.76\omega_2^2}} = 0.4399$$

$$\Phi(j\omega_2) = 90^\circ - \arctan\left(\frac{2.6\omega_2}{1-\omega_2^2}\right) = 0.9620 \text{ rad} = 55.12^\circ$$

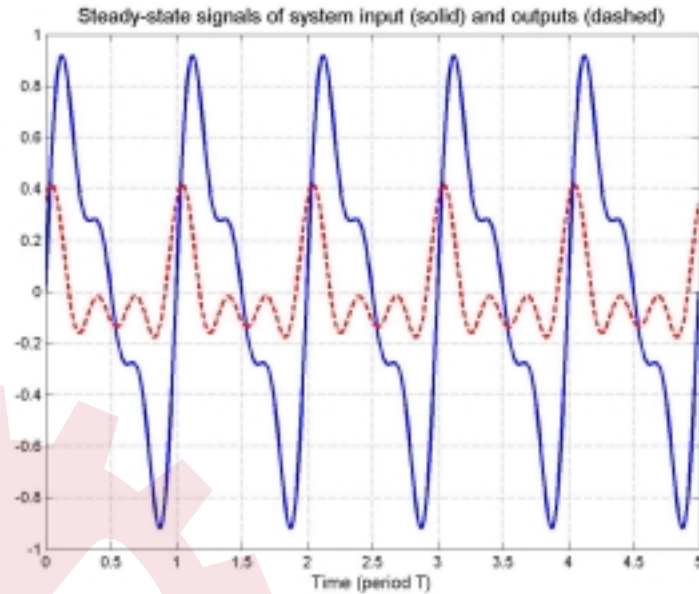
$$|H_v(j\omega_3)| = \frac{2\omega_3}{\sqrt{(1-\omega_3^2)^2 + 6.76\omega_3^2}} = 0.5787$$

$$\Phi(j\omega_3) = 90^\circ - \arctan\left(\frac{2.6\omega_3}{1-\omega_3^2}\right) = 0.7193 \text{ rad} = 41.21^\circ$$

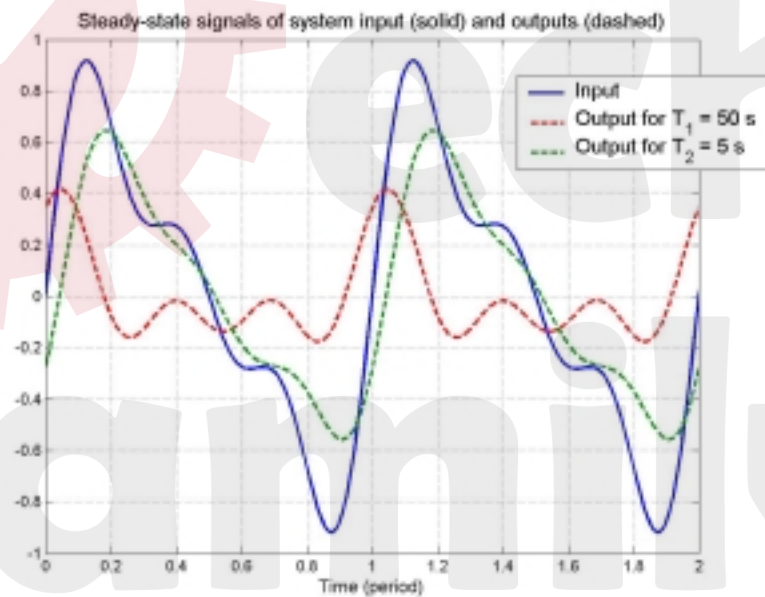
Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= \sum_{n=1}^3 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= 0.1542 \sin(0.4\pi t + 1.2504) + 0.14 \sin(0.8\pi t + 0.9620) + 0.1228 \sin(1.2\pi t + 0.7193) \end{aligned}$$

The input and output steady state signals plot is shown below:



Comparing the results with $T = 50$ s, we have:



Problem 6.47

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.6, the period $T = 5$ s and the peak amplitude $A = 1$ for the square waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first three terms of the Fourier series expansion of the square waveform of P6.33, we have

$$x(t) = \frac{4A}{\pi} \sin\left(\frac{2\pi}{T}t\right) + \frac{4A}{3\pi} \sin\left(\frac{6\pi}{T}t\right) = \frac{4}{\pi} \sin(0.4\pi t) + \frac{4}{3\pi} \sin(1.2\pi t)$$

Thus, for this problem,

$$c_1 = \frac{4}{\pi} \quad \omega_1 = 0.4\pi = 1.256 \text{ rad/s}$$

$$c_3 = \frac{4}{3\pi} \quad \omega_3 = 1.2\pi = 3.768 \text{ rad/s}$$

$$c_2 = 0$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{2j\omega}{(1-\omega^2) + 2.6j\omega} = \frac{2\omega}{\sqrt{(1-\omega^2)^2 + 6.76\omega^2}} \angle \left[90^\circ - \arctan\left(\frac{2.6\omega}{1-\omega^2}\right) \right]$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 and ω_3 analytically:

$$|H_v(j\omega_1)| = \frac{2\omega_1}{\sqrt{(1-\omega_1^2)^2 + 6.76\omega_1^2}} = 0.7575$$

$$\Phi(j\omega_1) = 90^\circ - \arctan\left(\frac{2.6\omega_1}{1-\omega_1^2}\right) = -0.1750 \text{ rad} = -10^\circ$$

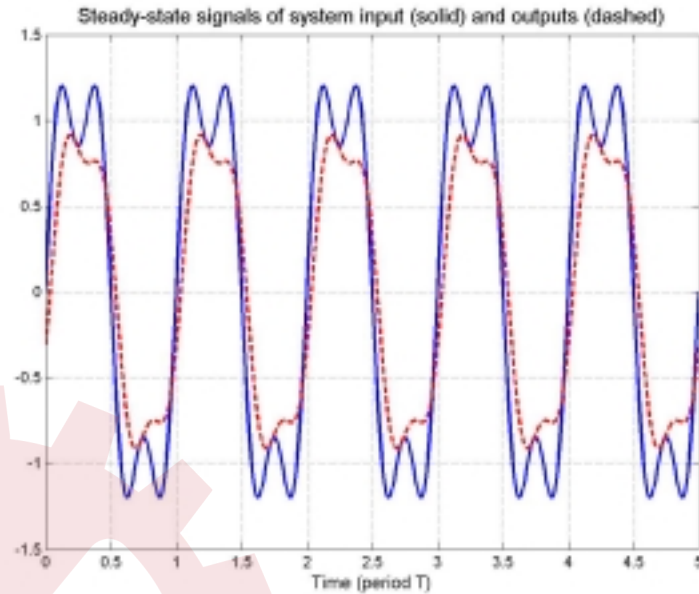
$$|H_v(j\omega_3)| = \frac{2\omega_3}{\sqrt{(1-\omega_3^2)^2 + 6.76\omega_3^2}} = 0.4585$$

$$\Phi(j\omega_3) = 90^\circ - \arctan\left(\frac{2.6\omega_3}{1-\omega_3^2}\right) = -0.9322 \text{ rad} = -53.41^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= \sum_{n=1}^3 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= 0.9644 \sin(0.4\pi t - 0.175) + 0.1946 \sin(1.2\pi t - 0.9322) \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.48

Solution:

Known quantities:

The frequency response $H_v(j\omega)$ of the circuit of P6.6, the period $T = 5$ s and the peak amplitude $A = 1$ for the pulse waveform.

Find:

Output of system $y(t)$ in response to input $x(t)$.

Analysis:

According to the Fourier series definitions of the previous section, and using the first three terms of the Fourier series expansion of the pulse waveform of Example 6.4, we have

$$x(t) = 0.2 + 0.3027 \cos(0.4\pi t) + 0.2199 \sin(0.4\pi t) + 0.0935 \cos(0.8\pi t) \\ + 0.2879 \sin(0.8\pi t) - 0.0624 \cos(1.2\pi t) + 0.1919 \sin(1.2\pi t)$$

Thus, for this problem,

$$c_0 = 0.2$$

$$c_1 = 0.3742, \quad \theta_1 = 0.9425 \text{ rad} = 54^\circ \quad \omega_1 = 0.4\pi = 1.256 \text{ rad/s}$$

$$c_2 = 0.3027, \quad \theta_2 = 0.3140 \text{ rad} = 18^\circ \quad \omega_2 = 0.8\pi = 2.512 \text{ rad/s}$$

and,

$$c_3 = 0.2018, \quad \theta_3 = -0.3144 \text{ rad} = -18^\circ \quad \omega_3 = 1.2\pi = 3.768 \text{ rad/s}$$

The frequency response of the system can be expressed in magnitude and phase form:

$$H_v(j\omega) = \frac{2j\omega}{(1-\omega^2) + 2.6j\omega} = \frac{2\omega}{\sqrt{(1-\omega^2)^2 + 6.76\omega^2}} \angle \left[90^\circ - \arctan\left(\frac{2.6\omega}{1-\omega^2}\right) \right]$$

At this point, we could evaluate the frequency response of the system at the frequencies ω_1 , ω_2 and ω_3 analytically:

$$|H_v(j\omega_1)| = \frac{2\omega_1}{\sqrt{(1-\omega_1^2)^2 + 6.76\omega_1^2}} = 0.7575$$

$$\Phi(j\omega_1) = 90^\circ - \arctan\left(\frac{2.6\omega_1}{1-\omega_1^2}\right) = -0.1750 \text{ rad} = -10^\circ$$

$$|H_v(j\omega_2)| = \frac{2\omega_2}{\sqrt{(1-\omega_2^2)^2 + 6.76\omega_2^2}} = 0.5969$$

$$\Phi(j\omega_2) = 90^\circ - \arctan\left(\frac{2.6\omega_2}{1-\omega_2^2}\right) = -0.6826 \text{ rad} = -39.11^\circ$$

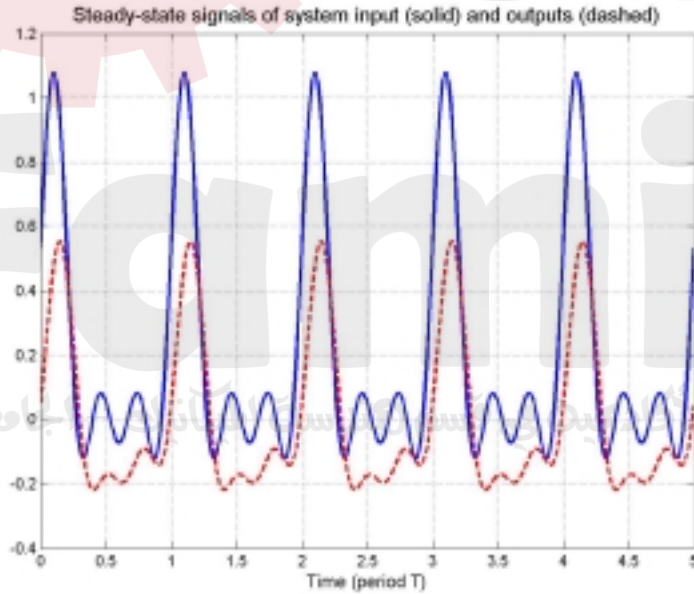
$$|H_v(j\omega_3)| = \frac{2\omega_3}{\sqrt{(1-\omega_3^2)^2 + 6.76\omega_3^2}} = 0.4585$$

$$\Phi(j\omega_3) = 90^\circ - \arctan\left(\frac{2.6\omega_3}{1-\omega_3^2}\right) = -0.9322 \text{ rad} = -53.41^\circ$$

Finally, we can compute the steady-state periodic output of the system:

$$\begin{aligned} y(t) &= c_0 |H_v(0)| + \sum_{n=1}^3 |H_v(j\omega_n)| c_n \sin[\omega_n t + \theta_n + \Phi(j\omega_n)] = \\ &= 0.2834 \sin(0.4\pi t + 0.7675) + 0.1807 \sin(0.8\pi t - 0.3686) + 0.0925 \sin(1.2\pi t - 1.2466) \end{aligned}$$

The input and output steady state signals plot is shown below:



Problem 6.49**Solution:****Known quantities:**

Resistance, capacitance, and inductance values, in the circuit of Figure P6.49.

Find:

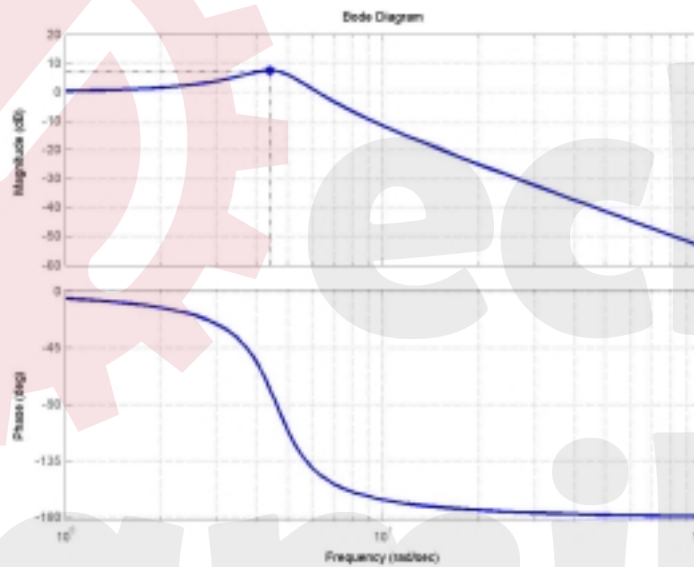
The resonant frequency and the bandwidth for the circuit.

Analysis:

Taking the output as the voltage across the parallel R-C subcircuit,

$$\frac{V_o}{V_s} = \frac{1/LC}{(j\omega)^2 + j\omega \frac{1}{RC} + \frac{1}{LC}} = \frac{3/64}{(j\omega)^2 + j\omega 2 + 3/64} \left(= \frac{\omega_n^2 \mu}{(j\omega)^2 + j\omega(2\xi\omega_n) + \omega_n^2} \right)$$

The corresponding Bode diagrams are shown below:



In this circuit, as frequency increases, the impedance of the capacitor decreases and the impedance of the inductor increases. Both effects cause the magnitude of the output voltage to decrease so this is a 2nd order low pass filter.

The resonance frequency is,

$$\omega_n = \sqrt{\frac{1}{LC}} = \sqrt{\frac{64}{3}} \cong 4.6188 \text{ rad/s.}$$

The damping ratio is,

$$\xi = \frac{1/RC}{2\omega_n} = \frac{\sqrt{3}}{8} \cong 0.2165$$

The quality factor is,

$$Q = \frac{1}{2\xi} = \frac{4}{\sqrt{3}} \cong 2.3094$$

The bandwidth is,

$$B = \frac{\omega_n}{Q} = \frac{8}{\sqrt{3}} \frac{1}{4/\sqrt{3}} = 2 \text{ rad/s.}$$

Problem 6.50

Solution:

Known quantities:

Figure P6.50.

Find:

What kind of filters are the ones shown in Figure P6.50.

Analysis:

In a), as frequency increases, the impedance of the capacitor decreases and the impedance of the inductor increases. Both effects cause the magnitude of the output voltage to decrease so this is a 2nd order low pass filter. Note that L and C are connected neither in series nor parallel and do not form a resonant circuit.

In b), L and C are connected in series and form a series resonant circuit with an impedance which is minimum at the resonant frequency and larger above and below the resonant frequency. This series resonant circuit is in series with the output giving, because of voltage division, a maximum output voltage at the resonant frequency and less at higher and lower frequencies. Therefore, b) is a band-pass filter.

In c), L and C are connected in parallel and form a parallel resonant circuit with an impedance which is maximum at the resonant frequency and smaller above and below the resonant frequency. This parallel resonant circuit is in parallel with the output giving, because of voltage division, a maximum output at the resonant frequency and less at higher and lower frequencies. Therefore, c) is a band-pass filter.

Problem 6.51

Solution:

Known quantities:

Figure P6.51.

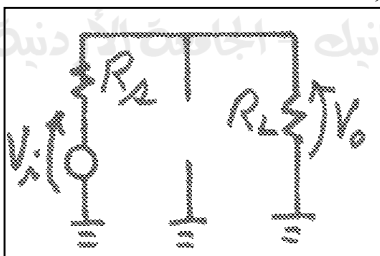
Find:

What kind of filters are the ones shown in Figure P6.51.

Analysis:

None of the inductors or capacitors is in series or parallel with any other. Therefore, there are no series or parallel resonant circuits and none of the circuits shown is band pass or band stop filters.

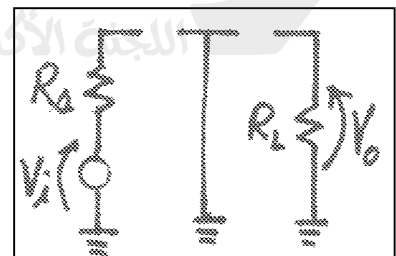
Circuits a) and d): As frequency approaches infinity, the inductors can be modeled as open circuits and the capacitors as short circuits. Therefore, the voltage transfer function approaches zero.



As frequency approaches zero, the inductors can be modeled as short circuits and the capacitors as open circuits.

$$\text{Then: } VD: H_v \rightarrow \frac{R_L}{R_s + R_L}$$

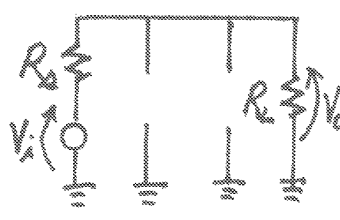
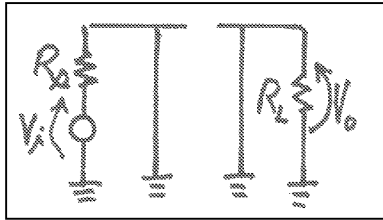
Therefore, circuits a) and d) are low pass filters.



Circuits b) and c) As frequency approaches infinity, the inductors can be modeled as open circuits and the

capacitors as short circuits. Then, $VD: H_v \rightarrow \frac{R_L}{R_s + R_L}$

Therefore:



As frequency approaches zero, inductors can be modeled as short circuits and the capacitors as open circuits. The voltage transfer function approaches zero. Therefore, circuits b) and c) are high pass filters.

Note: Multiple capacitors and inductors give higher order low and high pass filter. Better performance is obtained outside the pass band where the response for these circuits decreases by 60 dB/decade. In first order filters, the response decreases by only 20 dB/decade.

Problem 6.52

Solution:

Known quantities:

Figure P6.52.

Find:

- If this is a low-pass, high-pass, band-pass, or band-stop filter.
- Compute and plot the frequency response function if:

$$L = 11 \text{ mH} \quad C = 0.47 \text{ nF} \quad R_1 = 2.2 \text{ k}\Omega \quad R_2 = 3.8 \text{ k}\Omega$$

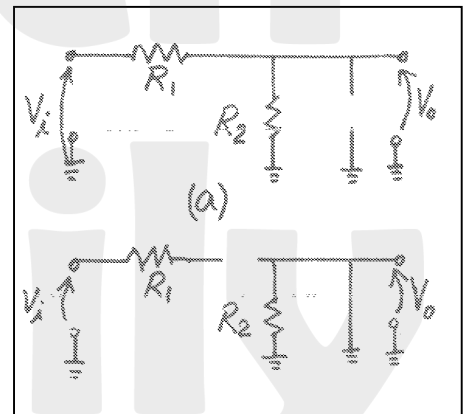
Analysis:

a)

$$\begin{aligned} \text{As } \omega \rightarrow 0: \quad Z_L &\rightarrow 0 \Rightarrow \text{Short} \\ Z_C &\rightarrow \infty \Rightarrow \text{Open} \\ \Rightarrow \text{VD: } H_v &= \frac{V_o}{V_i} \rightarrow \frac{R_2}{R_1 + R_2} \end{aligned}$$

$$\begin{aligned} \text{As } \omega \rightarrow \infty: \quad Z_L &\rightarrow \infty \Rightarrow \text{Open} \\ Z_C &\rightarrow 0 \Rightarrow \text{Short} \\ \Rightarrow H_v &\rightarrow 0 \end{aligned}$$

The filter is a low pass filter.



- First, we find the Thévenin equivalent circuit seen by the capacitor:

$$Z_T = (Z_{R1} + Z_L) \parallel Z_{R2} = \left(\frac{1}{R_1 + j\omega L} + \frac{1}{R_2} \right)^{-1} = \frac{(R_1 + j\omega L)R_2}{R_1 + j\omega L + R_2}$$

and

$$V_{OC} = \frac{Z_{R2}}{Z_{R1} + Z_L + Z_{R2}} V_{in} = \frac{R_2}{R_1 + j\omega L + R_2} V_{in}$$

$$\frac{v_{out}}{v_{OC}} = \frac{Z_C}{Z_T + Z_C} = \frac{\frac{1}{j\omega C}}{\frac{(R_1 + j\omega L)R_2}{R_1 + j\omega L + R_2} + \frac{1}{j\omega C}} = \frac{R_1 + j\omega L + R_2}{R_1 + j\omega L + R_2 + (R_1 + j\omega L)j\omega CR_2}$$

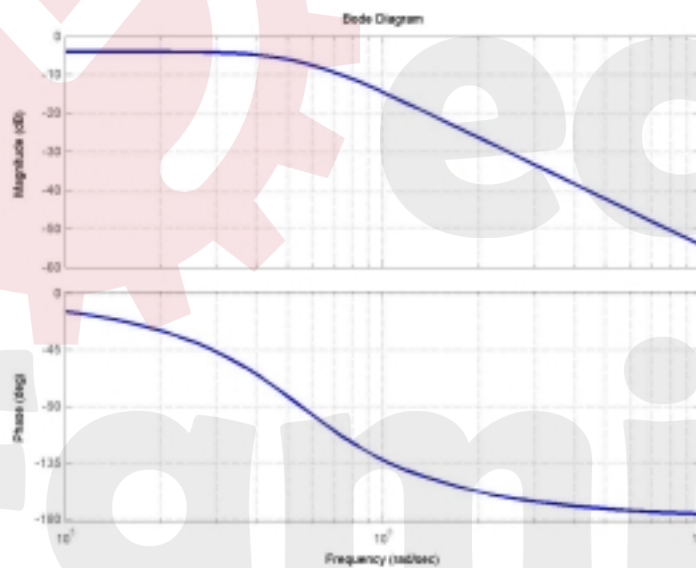
Therefore,

$$\begin{aligned} \frac{v_{out}}{v_{in}} &= \frac{R_2}{R_1 + j\omega L + R_2} \cdot \frac{R_1 + j\omega L + R_2}{R_1 + j\omega L + R_2 + (R_1 + j\omega L)j\omega CR_2} \\ &= \frac{1}{1 + \frac{R_1}{R_2} + j\omega \left(\frac{L}{R_2} + CR_1 \right) + (j\omega)^2 LC} \end{aligned}$$

Substituting the numerical values:

$$\frac{v_{out}}{v_{in}} = \frac{1}{(1.579 - 5.17 \times 10^{-12} \omega^2) + j(3.929 \times 10^{-6})\omega}$$

The corresponding Bode diagrams are shown below:



Problem 6.53

Solution:

Known quantities:

The values of the resistors, of the capacitance and of the inductance in the circuit of Figure P6.53.

Find:

Compute and plot the frequency response function. What type of filter is this?

Analysis:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$Z_T = Z_C \parallel [(Z_{R_S} + Z_L) \parallel (Z_{R_C} + Z_L)] = \left(j\omega C + \frac{1}{R_S + j\omega L} + \frac{1}{R_C + j\omega L} \right)^{-1}$$

$$= \frac{(R_S + j\omega L)(R_C + j\omega L)}{j\omega C(R_S + j\omega L)(R_C + j\omega L) + (R_S + R_C + j2\omega L)}$$

and, by node analysis,

$$v_{OC} = \frac{Z_C \parallel [(Z_{R_S} + Z_L) \parallel (Z_{R_C} + Z_L)]}{Z_{R_S} + Z_L} v_{in} = \frac{(R_C + j\omega L)}{j\omega C(R_S + j\omega L)(R_C + j\omega L) + (R_S + R_C + j2\omega L)} v_{in}$$

$$\frac{v_{out}}{v_{OC}} = \frac{Z_{R_L}}{Z_T + Z_{R_L}} = \frac{R_L}{Z_T + R_L} = \frac{1}{1 + \frac{Z_T}{R_L}} =$$

$$\frac{j\omega C(R_S + j\omega L)(R_C + j\omega L) + (R_S + R_C + j2\omega L)}{j\omega C(R_S + j\omega L)(R_C + j\omega L) + (R_S + R_C + j2\omega L) + (R_S + j\omega L)(R_C + j\omega L)/R_L}$$

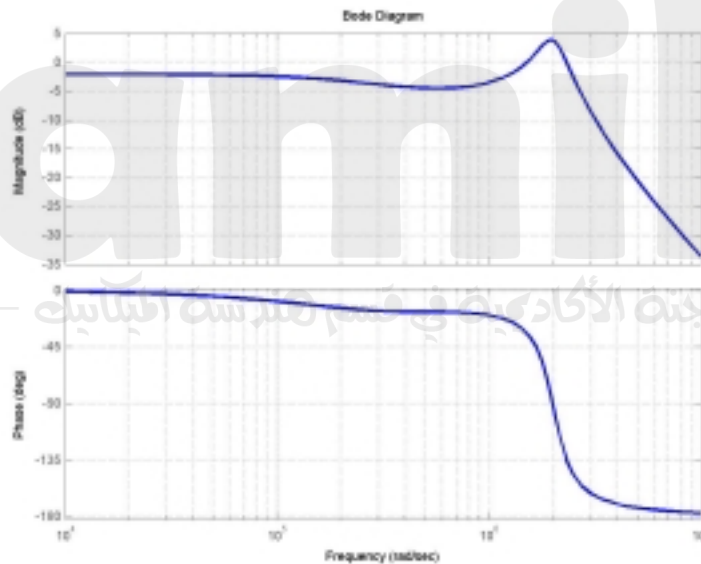
Therefore,

$$\frac{v_{out}}{v_{in}} = \frac{(R_C + j\omega L)}{j\omega C(R_S + j\omega L)(R_C + j\omega L) + (R_S + R_C + j2\omega L) + (R_S + j\omega L)(R_C + j\omega L)/R_L}$$

Substituting the numerical values:

$$\frac{v_{out}}{v_{in}} = \frac{8 \times 10^{17} + j(2 \times 10^{12})\omega}{(j\omega)^3 + (j\omega)^2 9 \times 10^5 + j(4.24 \times 10^{12})\omega + 1.016 \times 10^{18}}$$

The corresponding Bode diagrams are shown below:



The magnitude of the voltage transfer function is highest at the resonant frequency and decreases at higher and lower frequencies. Therefore, this is a band-pass filter.

However, it is not a particularly good filter since the voltage gain [or actually insertion loss] is not very different at the resonant and lower frequencies. This is due to the large inductor losses modeled here as the equivalent resistance R_C . This causes a low "Q" circuit.

Problem 6.54

Solution:

Known quantities:

The values of the resistors, of the capacitance and of the inductance in the circuit of Figure P6.53.

Find:

Compute and plot the frequency response function. What type of filter is this?

Analysis:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$Z_T = Z_C \parallel [(Z_{R_S} + Z_L) \parallel (Z_{R_C} + Z_L)] = \left(j\omega C + \frac{1}{R_S + j\omega L} + \frac{1}{R_C + j\omega L} \right)^{-1}$$

$$= \frac{(R_S + j\omega L)(R_C + j\omega L)}{j\omega C(R_S + j\omega L)(R_C + j\omega L) + (R_S + R_C + j2\omega L)}$$

and, by node analysis,

$$v_{OC} = \frac{Z_C \parallel [(Z_{R_S} + Z_L) \parallel (Z_{R_C} + Z_L)]}{Z_{R_S} + Z_L} v_{in} = \frac{(R_C + j\omega L)}{j\omega C(R_S + j\omega L)(R_C + j\omega L) + (R_S + R_C + j2\omega L)} v_{in}$$

$$\frac{v_{out}}{v_{OC}} = \frac{Z_{R_L}}{Z_T + Z_{R_L}} = \frac{R_L}{Z_T + R_L} = \frac{1}{1 + \frac{Z_T}{R_L}} =$$

$$\frac{j\omega C(R_S + j\omega L)(R_C + j\omega L) + (R_S + R_C + j2\omega L)}{j\omega C(R_S + j\omega L)(R_C + j\omega L) + (R_S + R_C + j2\omega L) + (R_S + j\omega L)(R_C + j\omega L)/R_L}$$

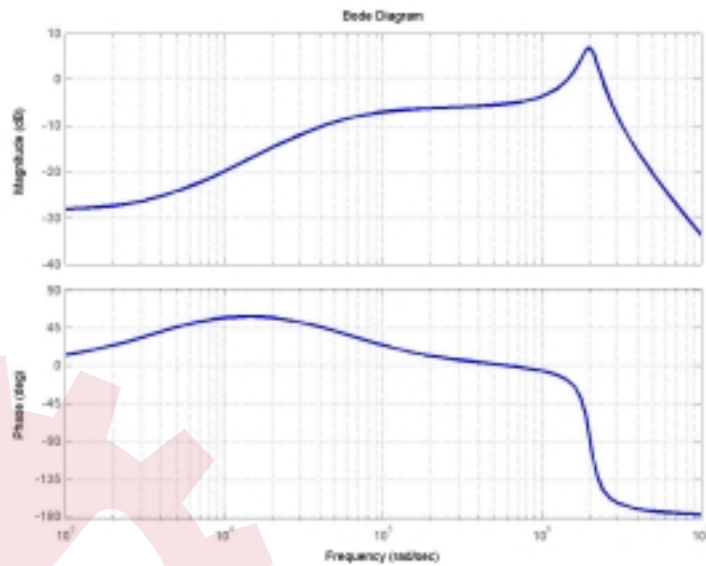
Therefore,

$$\frac{v_{out}}{v_{in}} = \frac{(R_C + j\omega L)}{j\omega C(R_S + j\omega L)(R_C + j\omega L) + (R_S + R_C + j2\omega L) + (R_S + j\omega L)(R_C + j\omega L)/R_L}$$

Substituting the numerical values:

$$\frac{v_{out}}{v_{in}} = \frac{8 \times 10^{15} + j(2 \times 10^{12})\omega}{(j\omega)^3 + (j\omega)^2 5.04 \times 10^5 + j(4.042 \times 10^{12})\omega + 2.082 \times 10^{17}}$$

The corresponding Bode diagrams are shown below:



The magnitude of the voltage transfer function is highest at the resonant frequency and decreases at higher and lower frequencies. Therefore, this is a band-pass filter.

Note: The inductor or coil loss is much smaller [4Ω] in this circuit, which gives much better band-pass filter performance or a higher "Q" circuit. The magnitude of the voltage transfer ratio or voltage gain [or insertion loss] is much higher at resonance than at higher or lower frequencies.

Problem 6.55

Solution:

Known quantities:

The values of the resistors, of the capacitance and of the inductance in the circuit of Figure P6.55:

$$R_s = 5 \text{ k}\Omega \quad C = 56 \text{ nF} \quad R_L = 100 \text{ k}\Omega \quad L = 9 \mu\text{H}$$

Find:

- An expression for the voltage transfer function: $H_v(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$
- The resonant frequency.
- The half-power frequencies.
- The bandwidth and Q.

Analysis:

a)

$$Z_{eq} = \frac{1}{\frac{1}{Z_C} + \frac{1}{Z_L} + \frac{1}{Z_{R_L}}} = \frac{1}{j\omega C + \frac{1}{j\omega L} + \frac{1}{R_L}} = \frac{j\omega L R_L}{(j\omega)^2 L C R_L + j\omega L + R_L}$$

$$VD: H_v(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{Z_{eq}}{Z_{R_s} + Z_{eq}} = \frac{j\omega L R_L}{R_s + \frac{j\omega L R_L}{(j\omega)^2 L C R_L + j\omega L + R_L}}$$

$$H_v(j\omega) = \frac{j\omega L R_L}{(j\omega)^2 L C R_s R_L + j\omega L (R_L + R_s) + R_s R_L} = \frac{1}{R_s} \frac{j\omega L}{(j\omega)^2 L C + j\omega L \left(\frac{R_L + R_s}{R_s R_L} \right) + 1}$$

b) The resonance frequency is,

$$\omega_n = \sqrt{\frac{1}{LC}} \cong 1.4086 \text{ Mrad/s.}$$

c)

$$\left| H_v(j\omega_{hp}) \right| = \left| \frac{1}{R_s} \frac{j\omega_{hp} L}{(j\omega_{hp})^2 L C + j\omega_{hp} L \left(\frac{R_L + R_s}{R_s R_L} \right) + 1} \right| = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\Rightarrow \frac{1}{R_s} \frac{\omega_{hp} L}{\sqrt{(1 - \omega_{hp}^2 L C)^2 + \left(\omega_{hp} L \left(\frac{R_L + R_s}{R_s R_L} \right) \right)^2}} = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\Rightarrow \omega_{hp1} \cong 1.4069 \text{ Mrad/s and } \omega_{hp2} \cong 1.41028 \text{ Mrad/s.}$$

d) The damping ratio is,

$$\xi = \frac{\omega_n}{2} L \left(\frac{R_L + R_s}{R_s R_L} \right) \cong 0.0013$$

The quality factor is,

$$Q = \frac{1}{2\xi} \cong 375.62$$

The bandwidth is,

$$B = \frac{\omega_n}{Q} \cong 3.75 \text{ Krad/s.}$$

Notes:

1. The absence of coil resistance caused the gain at the resonant frequency to be much higher than at high and low frequencies.
2. The bandwidth is small compared with the resonant frequency and the "Q" is quite large. These are dependent on the "loading" or power dissipation of the source and load resistors and the capacitance.
3. A circuit with a high Q is "selective" since it will pass a very narrow band of frequencies. "High" Q circuits have a Q = 10 or more.

Problem 6.56**Solution:****Known quantities:**

The values of the resistors, of the capacitance and of the inductance in the circuit of Figure P6.55:

$$R_s = 5 \text{ k}\Omega \quad C = 0.5 \text{ nF} \quad R_L = 100 \text{ k}\Omega \quad L = 1 \text{ mH}$$

Find:

a) An expression for the voltage transfer function:

$$H_v(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$$

b) The resonant frequency.

c) The half-power frequencies.

d) The bandwidth and Q.

Analysis:

a)

$$Z_{eq} = \frac{1}{\frac{1}{Z_C} + \frac{1}{Z_L} + \frac{1}{Z_{R_L}}} = \frac{1}{j\omega C + \frac{1}{j\omega L} + \frac{1}{R_L}} = \frac{j\omega L R_L}{(j\omega)^2 L C R_L + j\omega L + R_L}$$

$$VD: H_v(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{Z_{eq}}{Z_{R_s} + Z_{eq}} = \frac{\frac{j\omega L R_L}{(j\omega)^2 L C R_L + j\omega L + R_L}}{R_s + \frac{j\omega L R_L}{(j\omega)^2 L C R_L + j\omega L + R_L}}$$

$$H_v(j\omega) = \frac{j\omega L R_L}{(j\omega)^2 L C R_s R_L + j\omega L (R_L + R_s) + R_s R_L} = \frac{1}{R_s} \frac{j\omega L}{(j\omega)^2 L C + j\omega L \left(\frac{R_L + R_s}{R_s R_L} \right) + 1}$$

b) The resonance frequency is,

$$\omega_n = \sqrt{\frac{1}{LC}} \cong 1.4142 \text{ Mrad/s.}$$

c)

$$\left| H_v(j\omega_{hp}) \right| = \left| \frac{1}{R_s} \frac{j\omega_{hp} L}{(j\omega_{hp})^2 L C + j\omega_{hp} L \left(\frac{R_L + R_s}{R_s R_L} \right) + 1} \right| = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\Rightarrow \frac{1}{R_s} \frac{\omega_{hp} L}{\sqrt{(1 - \omega_{hp}^2 L C)^2 + \left(\omega_{hp} L \left(\frac{R_L + R_s}{R_s R_L} \right) \right)^2}} = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\Rightarrow \omega_{hp1} \cong 1.24 \text{ Mrad/s and } \omega_{hp2} \cong 1.62 \text{ Mrad/s.}$$

d) The damping ratio is,

$$\xi = \frac{\omega_n}{2} L \left(\frac{R_L + R_s}{R_s R_L} \right) \cong 0.1485$$

The quality factor is,

$$Q = \frac{1}{2\xi} \cong 3.3672$$

The bandwidth is,

$$B = \frac{\omega_n}{Q} \cong 420 \text{ Krad/s.}$$

Problem 6.57

Solution:

Known quantities:

The values of the resistors, of the capacitance and of the inductance in the circuit of Figure P6.57.

Find:

Compute and plot the voltage frequency response function. What type of filter is this?

Analysis:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$\begin{aligned} Z_T &= Z_{R_S} + Z_C \parallel (Z_{R_C} + Z_L) = R_S + \left(j\omega C + \frac{1}{R_C + j\omega L} \right)^{-1} \\ &= R_S + \frac{(R_C + j\omega L)}{j\omega C(R_C + j\omega L) + 1} = \frac{(R_C + j\omega L) + R_S[1 + j\omega C(R_C + j\omega L)]}{j\omega C(R_C + j\omega L) + 1} \end{aligned}$$

and

$$V_{OC} = V_{in}$$

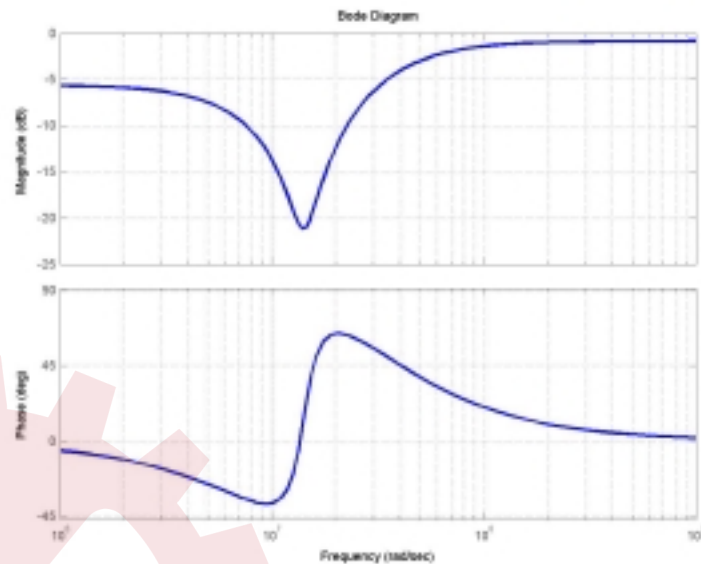
Therefore,

$$\begin{aligned} \frac{V_{out}}{V_{in}} &= \frac{Z_{R_L}}{Z_T + Z_{R_L}} = \frac{R_L}{Z_T + R_L} = \frac{1}{1 + \frac{Z_T}{R_L}} = \\ &= \frac{j\omega C(R_C + j\omega L) + 1}{j\omega C(R_C + j\omega L) + 1 + \{(R_C + j\omega L) + R_S[1 + j\omega C(R_C + j\omega L)]\}/R_L} = \\ &= \frac{1 + j\omega C R_C + (j\omega)^2 LC}{\left(1 + \frac{R_C + R_S}{R_L}\right) + j\omega \left[CR_C \left(1 + \frac{R_S}{R_L}\right) + \frac{L}{R_L}\right] + (j\omega)^2 LC \left(1 + \frac{R_S}{R_L}\right)} \end{aligned}$$

Substituting the numerical values:

$$\frac{V_{out}}{V_{in}} = \frac{1 + j(2 \times 10^{-8})\omega + (j\omega)^2 5 \times 10^{-15}}{(j\omega)^2 5.5 \times 10^{-15} + j(2.22 \times 10^{-7})\omega + 1.9}$$

The corresponding Bode diagrams are shown below:



The magnitude of the voltage transfer function is lowest at the resonant frequency and increases at higher and lower frequencies. Therefore, this is a band stop or "notch" filter.

At its resonant frequency, a parallel resonant circuit has a high equivalent resistance that is resistive. Connected here in series with the load, this high impedance reduces the magnitude of the voltage transfer function [or voltage gain or insertion loss] at the resonant frequency.

The loading due to the inductor losses, modeled here as an equivalent "coil" resistance, is fairly small giving a substantially lower gain at the resonant frequency compared with the gain at higher or lower frequencies. Therefore this is a high "Q" circuit with good performance and selectivity. The inductor losses also affect only slightly the resonant frequency.

The cutoff frequencies are difficult [but not impossible] to determine in circuits containing a parallel resonant circuit which includes inductor losses, so no attempt was made to do so.

Problem 6.58

Solution:

Known quantities:

The values of the resistors, of the capacitance and of the inductance in the circuit of Figure P6.58.

Find:

Compute and plot the frequency response function.

Analysis:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$Z_T = Z_{R_s} + Z_C \parallel Z_L = R_s + \left(j\omega C + \frac{1}{j\omega L} \right)^{-1} = \frac{j\omega L + R_s [1 + (j\omega)^2 LC]}{1 + (j\omega)^2 LC}$$

and

$$V_{OC} = V_{in}$$

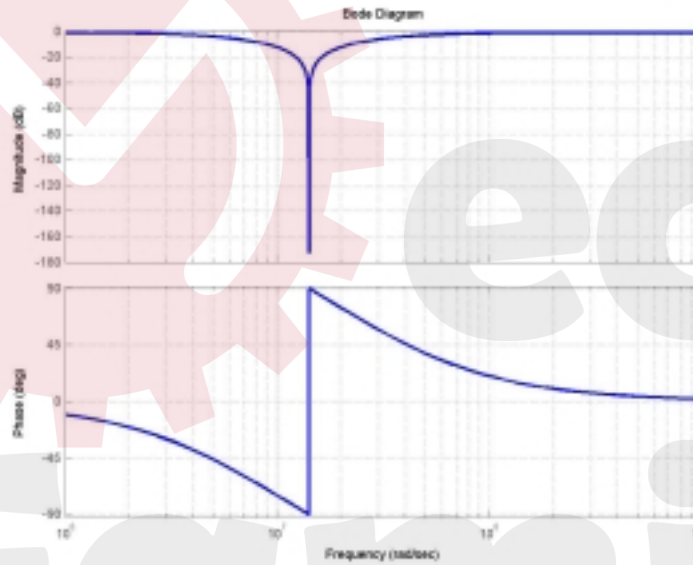
Therefore,

$$\begin{aligned}\frac{v_{out}}{v_{in}} &= \frac{Z_{R_L}}{Z_T + Z_{R_L}} = \frac{R_L}{Z_T + R_L} = \frac{1}{1 + \frac{Z_T}{R_L}} = \frac{1 + (j\omega)^2 LC}{1 + (j\omega)^2 LC + \left\{ j\omega L + R_S \left[1 + (j\omega)^2 LC \right] \right\} / R_L} = \\ &= \frac{1 + (j\omega)^2 LC}{\left(1 + \frac{R_S}{R_L} \right) + j\omega \frac{L}{R_L} + (j\omega)^2 LC \left(1 + \frac{R_S}{R_L} \right)}\end{aligned}$$

Substituting the numerical values:

$$\frac{v_{out}}{v_{in}} = \frac{1 + (j\omega)^2 5 \times 10^{-15}}{(j\omega)^2 5.5 \times 10^{-15} + j(2 \times 10^{-7})\omega + 1.1}$$

The corresponding Bode diagrams are shown below:



Problem 6.59

Solution:

Known quantities:

The filter circuit shown in Figure P6.58.

Find:

The equation for the voltage transfer function in standard form. Then, if:

$$R_S = 500 \, \Omega \quad R_L = 5 \, \text{k}\Omega \quad \omega_n = 12.1278 \, \text{M} \frac{\text{rad}}{\text{s}} \quad C = 68 \, \text{nF} \quad L = 0.1 \, \mu\text{H}$$

determine the cutoff frequencies, the bandwidth, BW, and Q.

Analysis

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$Z_T = Z_{R_S} + Z_C \parallel Z_L = R_S + \left(j\omega C + \frac{1}{j\omega L} \right)^{-1} = \frac{j\omega L + R_S [1 + (j\omega)^2 LC]}{1 + (j\omega)^2 LC}$$

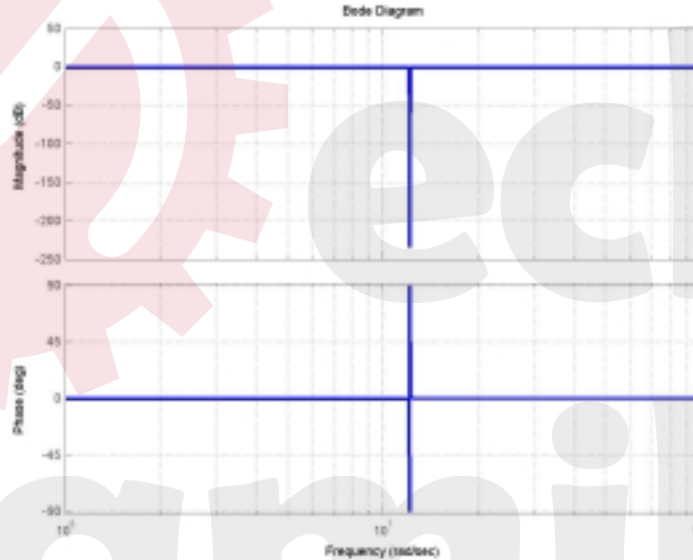
and

$$v_{OC} = v_{in}$$

Therefore,

$$\begin{aligned} \frac{v_{out}}{v_{in}} &= \frac{Z_{R_L}}{Z_T + Z_{R_L}} = \frac{R_L}{Z_T + R_L} = \frac{1}{1 + \frac{Z_T}{R_L}} = \frac{1 + (j\omega)^2 LC}{\left(1 + \frac{R_S}{R_L}\right) + j\omega \frac{L}{R_L} + (j\omega)^2 LC \left(1 + \frac{R_S}{R_L}\right)} \\ &= \frac{R_L}{R_S + R_L} \frac{1 - \omega^2 LC}{1 - \omega^2 LC + j\omega \frac{L}{R_S + R_L}} = \frac{R_L}{R_S + R_L} \frac{1}{1 + j\omega \frac{L}{(R_S + R_L)(1 - \omega^2 LC)}} \end{aligned}$$

The corresponding Bode diagrams are shown below:



Then, in order to calculate the half-power frequencies, we have to solve:

$$\left| H_v(j\omega_{hp}) \right| = \frac{R_L}{R_S + R_L} \frac{1 - \omega_{hp}^2 LC}{\sqrt{(1 - \omega_{hp}^2 LC)^2 + \left(\omega_{hp} \frac{L}{R_S + R_L} \right)^2}} = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\Rightarrow \omega_{hp1} \cong 12.061 \text{ Mrad/s and } \omega_{hp2} \cong 12.194 \text{ Mrad/s.}$$

The damping ratio is,

$$\xi = \frac{\omega_n}{2} \frac{L}{R_L + R_S} \cong 1.1024 \times 10^{-4}$$

The quality factor is,

$$Q = \frac{1}{2\xi} \cong 4535.4$$

The bandwidth is,

$$B = \frac{\omega_n}{Q} \cong 2.6738 \text{ Krad/s.}$$

Problem 6.60

Solution:

Known quantities:

The filter circuit shown in Figure P6.58.

Find:

The equation for the voltage transfer function in standard form. Then, if:

$$R_s = 4.4 \text{ k}\Omega \quad R_L = 60 \text{ k}\Omega \quad \omega_r = 25 \text{ M} \frac{\text{rad}}{\text{s}} \quad C = 0.8 \text{ nF} \quad L = 2 \mu\text{H}$$

determine the cutoff frequencies, the bandwidth, BW, and Q.

Analysis

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$Z_T = Z_{R_s} + Z_C \parallel Z_L = R_s + \left(j\omega C + \frac{1}{j\omega L} \right)^{-1} = \frac{j\omega L + R_s [1 + (j\omega)^2 LC]}{1 + (j\omega)^2 LC}$$

and

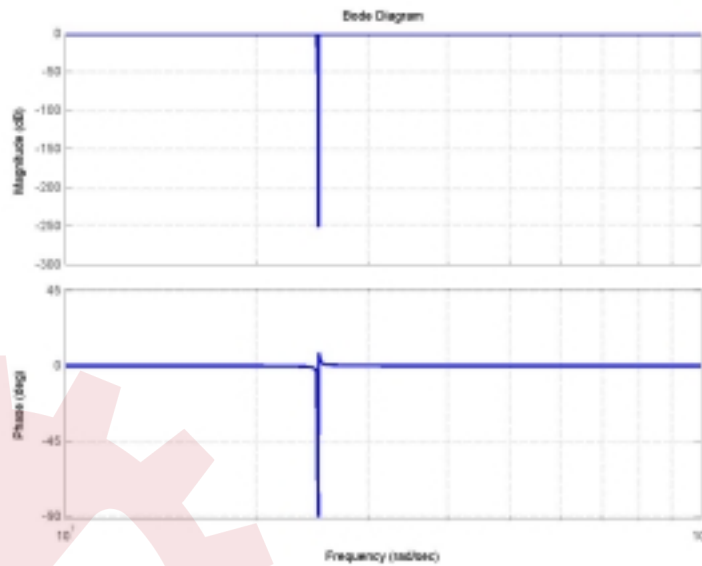
$$V_{OC} = V_{in}$$

Therefore,

$$\begin{aligned} \frac{V_{out}}{V_{in}} &= \frac{Z_{R_L}}{Z_T + Z_{R_L}} = \frac{R_L}{Z_T + R_L} = \frac{1}{1 + \frac{Z_T}{R_L}} = \frac{1 + (j\omega)^2 LC}{\left(1 + \frac{R_s}{R_L} \right) + j\omega \frac{L}{R_L} + (j\omega)^2 LC \left(1 + \frac{R_s}{R_L} \right)} \\ &= \frac{R_L}{R_s + R_L} \frac{1 - \omega^2 LC}{1 - \omega^2 LC + j\omega \frac{L}{R_s + R_L}} = \frac{R_L}{R_s + R_L} \frac{1}{1 + j\omega \frac{L}{(R_s + R_L)(1 - \omega^2 LC)}} \end{aligned}$$

The corresponding Bode diagrams are shown below:

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Then, in order to calculate the half-power frequencies, we have to solve:

$$|H_v(j\omega_{hp})| = \frac{R_L}{R_S + R_L} \frac{1 - \omega_{hp}^2 LC}{\sqrt{(1 - \omega_{hp}^2 LC)^2 + \left(\omega_{hp} \frac{L}{R_S + R_L}\right)^2}} = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\Rightarrow \omega_{hp1} \cong 24.8609 \text{ Mrad/s and } \omega_{hp2} \cong 24.066 \text{ Mrad/s.}$$

The damping ratio is,

$$\xi = \frac{\omega_n}{2} \frac{L}{R_L + R_S} \cong 3.882 \times 10^{-4}$$

The quality factor is,

$$Q = \frac{1}{2\xi} = 1288$$

The bandwidth is,

$$B = \frac{\omega_n}{Q} \cong 19.41 \text{ Krad/s.}$$

Problem 6.61

Solution:

Known quantities:

The bandstop filter circuit shown in Figure P6.61, where:

$$L = 0.4 \text{ mH} \quad R_c = 100\Omega \quad C = 1 \text{ pF} \quad R_s = R_L = 3.8 \text{ k}\Omega$$

Find:

- a) An expression for the voltage transfer function or gain in the form:

$$H_v[j\omega] = \frac{V_o[j\omega]}{V_i[j\omega]} = H_o \frac{1 + j f_1[\omega]}{1 + j f_2[\omega]}$$

- b) The magnitude of the function at high and low frequencies and at the resonant frequency.
 c) The resonant frequency.
 d) The half power frequencies.

Analysis

a)

$$Z_{eq1} = Z_{Rc} + Z_L + Z_C = R_c + j \left[\omega L - \frac{1}{\omega C} \right]$$

$$Z_{eq} = \frac{Z_{eq1} Z_{RL}}{Z_{eq1} + Z_{RL}} = \frac{(R_c + j [\omega L - \frac{1}{\omega C}]) R_L}{(R_c + j [\omega L - \frac{1}{\omega C}]) + R_L} = \frac{R_c R_L + j [\omega L - \frac{1}{\omega C}] R_L}{R_c + R_L + j R_L [\omega L - \frac{1}{\omega C}]} = \frac{N}{D}$$

$$VD: H_v[j\omega] = \frac{V_o[j\omega]}{V_i[j\omega]} = \frac{Z_{eq}}{Z_{Rs} + Z_{eq}} = \frac{\frac{N}{D}}{R_s + \frac{N}{D}} = \frac{N}{R_s D + N} \quad (TRICKY)$$

$$\Rightarrow H_v[j\omega] = \frac{R_c R_L + j R_L [\omega L - \frac{1}{\omega C}]}{R_s (R_c + R_L + j [\omega L - \frac{1}{\omega C}]) + (R_c R_L + j R_L [\omega L - \frac{1}{\omega C}])} =$$

$$= \frac{R_c R_L + j R_L [\omega L - \frac{1}{\omega C}]}{R_s [R_c + R_L] + R_c R_L + j [R_s + R_L] [\omega L - \frac{1}{\omega C}]} =$$

$$H_v[j\omega] = \frac{R_c R_L}{R_s [R_c + R_L] + R_c R_L} \frac{1 + j \frac{1}{R_c} [\omega L - \frac{1}{\omega C}]}{1 + j \frac{[R_s + R_L] [\omega L - \frac{1}{\omega C}]}{R_c [R_s + R_L] + R_s R_L}}$$

b)

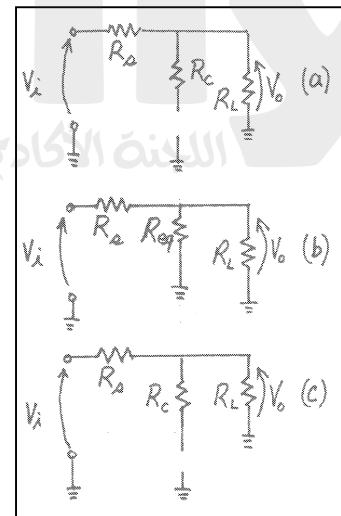
As $\omega \rightarrow \infty$: $Z_L \rightarrow \infty \Rightarrow$ Open $Z_C \rightarrow 0 \Rightarrow$ Short

$$VD: H_v[j\omega] \rightarrow \frac{R_L}{R_s + R_L} = 0.5 = -6.021 \text{ dB}$$

As $\omega \rightarrow 0$: $Z_C \rightarrow \infty \Rightarrow$ Open $Z_L \rightarrow 0 \Rightarrow$ Short

$$VD: H_v[j\omega] \rightarrow \frac{R_L}{R_s + R_L} = 0.5 = -6.021 \text{ dB}$$

At resonance:



$$\begin{aligned}
 H[j\omega_r] &= H_0 = \frac{R_c R_L}{R_s [R_c + R_L] + R_c R_L} = \\
 &= \frac{[100][3800]}{[3800][100 + 3800] + [100][3800]} = 0.025 = -32.00 \text{ dB}
 \end{aligned}$$

- c) At the resonant frequency, the transfer function is real. This requires the two functions of frequency to be equal:

$$\begin{aligned}
 f_1[\omega_r] &= f_2[\omega_r] \Rightarrow \frac{1}{R_c} \left[\omega_r L - \frac{1}{\omega_r C} \right] = \frac{[R_s + R_L]}{R_c [R_s + R_L] + R_s R_L} \left[\omega_r L - \frac{1}{\omega_r C} \right] \\
 \Rightarrow \omega_r L - \frac{1}{\omega_r C} &= 0 \Rightarrow \omega_r = [LC]^{-1/2} = ([0.4 \cdot 10^{-3}][1 \cdot 10^{-12}])^{-1/2} = 50 \text{ M} \frac{\text{rad}}{\text{s}}
 \end{aligned}$$

- d) Then, in order to calculate the half-power frequencies, we have to solve:

$$\begin{aligned}
 |H_v(j\omega_{hp})| &= \frac{R_c R_L}{R_s (R_c + R_L) + R_c R_L} \frac{\sqrt{1 + \left(\frac{1}{R_c} \left(\omega L - \frac{1}{\omega C} \right) \right)^2}}{\sqrt{1 + \left(\frac{(R_c + R_L) \left(\omega L - \frac{1}{\omega C} \right)}{R_c (R_c + R_L) + R_s R_L} \right)^2}} = \frac{1}{\sqrt{2}} \Rightarrow \\
 \Rightarrow \omega_{hp1} &\cong 44.9922 \text{ Mrad/s and } \omega_{hp2} \cong 55.565 \text{ Mrad/s.}
 \end{aligned}$$

Problem 6.62

Solution:

Known quantities:

The values of the resistors, of the capacitance and of the inductance in the circuit of Figure P6.55:

$$R_s = 5 \text{ k}\Omega \quad C = 5 \text{ nF} \quad R_L = 50 \text{ k}\Omega \quad L = 2 \text{ mH}$$

Find:

- An expression for the voltage transfer function: $H_v(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$
- The resonant frequency.
- The half-power frequencies.
- The bandwidth and Q.
- Plot $H_v(j\omega)$.

Analysis:

-

$$Z_{eq} = \frac{1}{\frac{1}{Z_C} + \frac{1}{Z_L} + \frac{1}{Z_{R_L}}} = \frac{1}{j\omega C + \frac{1}{j\omega L} + \frac{1}{R_L}} = \frac{j\omega L R_L}{(j\omega)^2 L C R_L + j\omega L + R_L}$$

$$VD: H_v(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{Z_{eq}}{Z_{R_s} + Z_{eq}} = \frac{\frac{j\omega L R_L}{(j\omega)^2 L C R_L + j\omega L + R_L}}{R_L + \frac{j\omega L R_L}{(j\omega)^2 L C R_L + j\omega L + R_L}}$$

$$H_v(j\omega) = \frac{j\omega L R_L}{(j\omega)^2 L C R_s R_L + j\omega L (R_L + R_s) + R_s R_L} = \frac{1}{R_s} \frac{j\omega L}{(j\omega)^2 L C + j\omega L \left(\frac{R_L + R_s}{R_s R_L} \right) + 1}$$

b) The resonance frequency is,

$$\omega_n = \sqrt{\frac{1}{LC}} \cong 316.23 \text{ Krad/s.}$$

c)

$$\begin{aligned} |H_v(j\omega_{hp})| &= \left| \frac{1}{R_s} \frac{j\omega_{hp} L}{(j\omega_{hp})^2 L C + j\omega_{hp} L \left(\frac{R_L + R_s}{R_s R_L} \right) + 1} \right| = \frac{1}{\sqrt{2}} \Rightarrow \\ \Rightarrow \frac{1}{R_s} \frac{\omega_{hp} L}{\sqrt{\left(1 - \omega_{hp}^2 L C \right)^2 + \left(\omega_{hp} L \left(\frac{R_L + R_s}{R_s R_L} \right) \right)^2}} &= \frac{1}{\sqrt{2}} \Rightarrow \end{aligned}$$

$$\Rightarrow \omega_{hp1} \cong 1.4069 \text{ Mrad/s and } \omega_{hp2} \cong 1.41028 \text{ Mrad/s.}$$

d) The damping ratio is,

$$\xi = \frac{\omega_n}{2} L \left(\frac{R_L + R_s}{R_s R_L} \right) \cong 0.0696$$

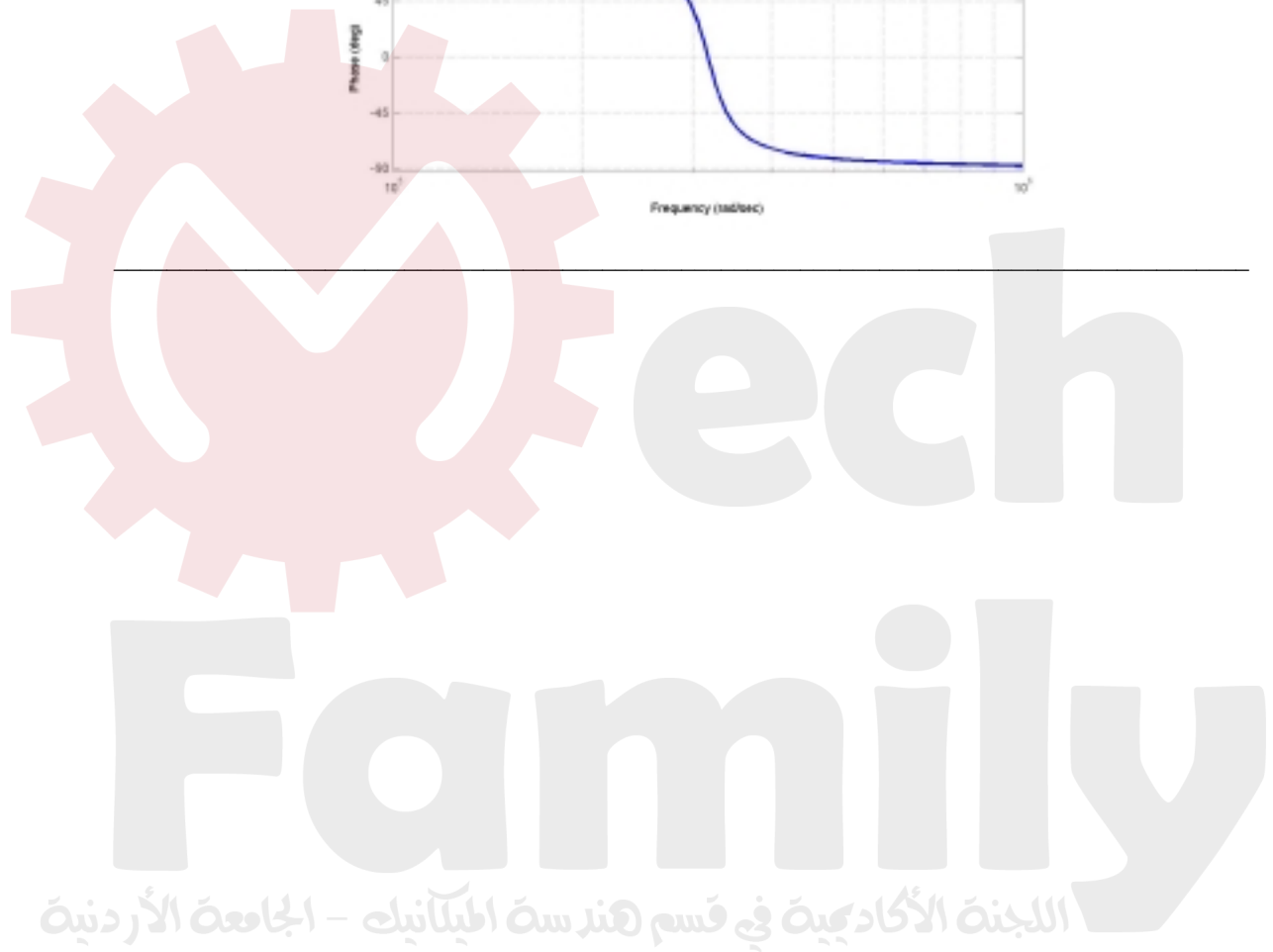
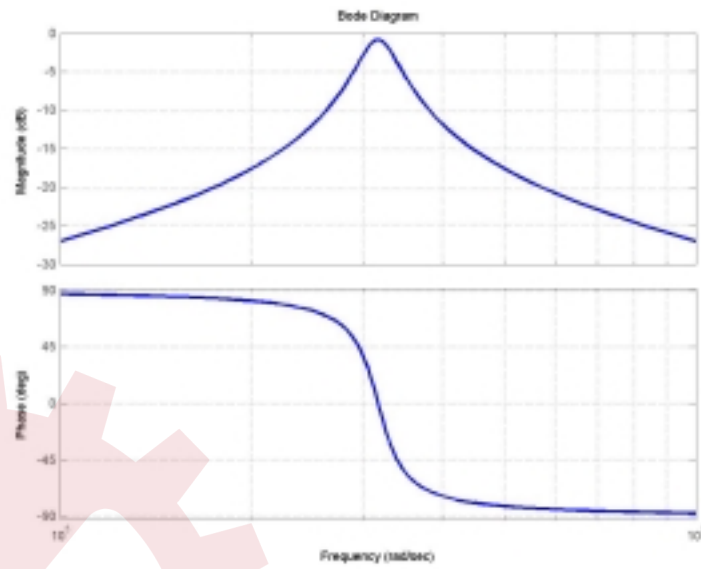
The quality factor is,

$$Q = \frac{1}{2\xi} \cong 7.187$$

The bandwidth is,

$$B = \frac{\omega_n}{Q} \cong 40 \text{ Krad/s.}$$

e) The Bode diagrams are shown below:



Section 6.4: Bode Plots

Focus on Methodology

Bode plots

This box illustrates the Bode plot asymptotic approximation construction procedure. The method assumes that there are no complex conjugate factors in the response, and that both the numerator and denominator can be factored into first-order terms with real roots.

1. Express the frequency response function in factored form, resulting in an expression similar to equation 6.57:

$$H(j\omega) = K \frac{\prod_{i=1}^m \frac{j\omega}{\omega_{p_i}}}{\prod_{j=1}^n \frac{j\omega}{\omega_{z_j}}}$$

2. Select the appropriate frequency range for the semi-logarithmic plot, extending at least a decade below the lowest 3-dB frequency and a decade above the highest 3-dB frequency.
3. Sketch the magnitude and phase response asymptotic approximations for each of the first-order factors using the techniques illustrated in Figures 6.36 and 6.37.
4. Add, graphically, the individual terms to obtain a composite response.
5. If desired, apply the correction factors of Table 6.2.

Problem 6.63

Solution:

Known quantities:

The values of the resistors, of the capacitance and of the inductance in the circuit of Figure P6.63:

$$R_1 = R_2 = 1 \text{ k}\Omega \quad C_1 = 1 \text{ }\mu\text{F} \quad C_2 = 1 \text{ mF} \quad L = 1 \text{ H}$$

Find:

- a) The frequency response function $H_v(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$ for the circuit of Figure P6.63.
- b) Manually sketch a magnitude and phase Bode plot of the system, using a five-cycle semilog paper.
- c) Use Matlab and the Bode command to generate the same plot.

Analysis:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$a) \quad Z_T = Z_{R_2} + Z_{C_1} \parallel Z_L \parallel Z_{R_1} = R_2 + \left(j\omega C_1 + \frac{1}{j\omega L} + \frac{1}{R_1} \right)^{-1} = \frac{(j\omega)^2 LC_1 R_2 + j\omega L \left(1 + \frac{R_2}{R_1} \right) + R_2}{(j\omega)^2 LC_1 + j\omega \frac{L}{R_1} + 1}$$

and

$$V_{OC} = \frac{j\omega \frac{L}{R_1}}{(j\omega)^2 LC_1 + j\omega \frac{L}{R_1} + 1} V_{in}$$

$$\frac{V_{out}}{V_{OC}} = \frac{1/j\omega C_2}{Z_T + 1/j\omega C_2} = \frac{(j\omega)^2 LC_1 + j\omega \frac{L}{R_1} + 1}{(j\omega)^3 LC_1 C_2 R_2 + (j\omega)^2 L \left[C_1 + C_2 \left(1 + \frac{R_2}{R_1} \right) \right] + j\omega \left(\frac{L}{R_1} + C_2 R_2 \right) + 1}$$

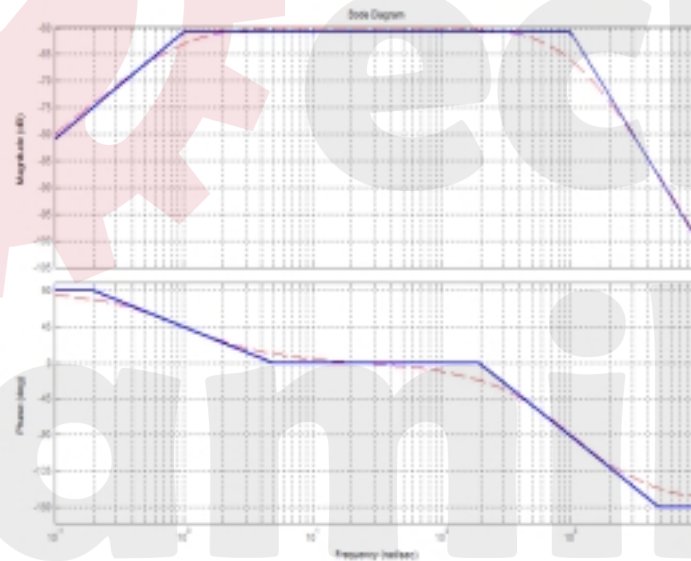
Thus,

$$\frac{V_{out}}{V_{in}} = \frac{j\omega \frac{L}{R_1}}{(j\omega)^3 LC_1 C_2 R_2 + (j\omega)^2 L \left[C_1 + C_2 \left(1 + \frac{R_2}{R_1} \right) \right] + j\omega \left(\frac{L}{R_1} + C_2 R_2 \right) + 1}$$

- b) Substituting the numerical values and expressing the frequency response function in factored form, we have:

$$H_v(j\omega) = 10^{-3} \frac{j\omega}{(j\omega + 1) \left(\frac{j\omega}{968.361} + 1 \right) \left(\frac{j\omega}{1031.638} + 1 \right)}$$

- c) The sketch plots and the ones obtained using Matlab are shown below:



Problem 6.64

Solution:

Known quantities:

The values of the resistors, of the capacitance and of the inductance in the circuit of Figure P6.63:

$$R_1 = R_2 = 1 \text{ k}\Omega \quad C_1 = 1 \text{ }\mu\text{F} \quad C_2 = 1 \text{ mF} \quad L = 1 \text{ H}$$

Find:

- a) The frequency response function $H_v(j\omega) = \frac{I_{out}(j\omega)}{V_{in}(j\omega)}$ for the circuit of Figure P6.63.

- b) Manually sketch a magnitude and phase Bode plot of the system, using a five-cycle semilog paper.
 c) Use Matlab and the Bode command to generate the same plot.

Analysis:

- a) The frequency response function $H_v(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$ is (see P6.63 for details):

$$\frac{V_{out}}{V_{in}} = \frac{j\omega \frac{L}{R_1}}{(j\omega)^3 LC_1 C_2 R_2 + (j\omega)^2 L \left[C_1 + C_2 \left(1 + \frac{R_2}{R_1} \right) \right] + j\omega \left(\frac{L}{R_1} + C_2 R_2 \right) + 1}$$

and,

$$I_{out} = j\omega C_2 V_{out}$$

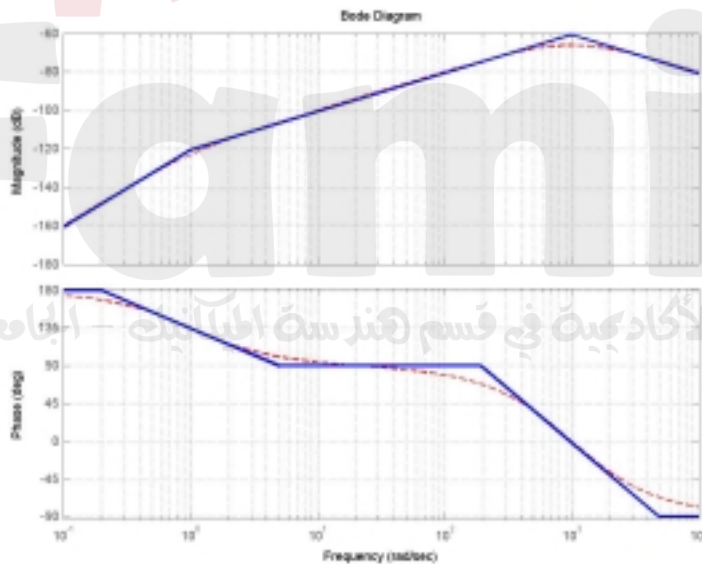
Thus,

$$\frac{I_{out}}{V_{in}} = \frac{(j\omega)^2 \frac{LC_2}{R_1}}{(j\omega)^3 LC_1 C_2 R_2 + (j\omega)^2 L \left[C_1 + C_2 \left(1 + \frac{R_2}{R_1} \right) \right] + j\omega \left(\frac{L}{R_1} + C_2 R_2 \right) + 1}$$

- b) Substituting the numerical values and expressing the frequency response function in factored form, we have:

$$H(j\omega) = 10^{-6} \frac{(j\omega)^2}{(j\omega + 1) \left(\frac{j\omega}{968.361} + 1 \right) \left(\frac{j\omega}{1031.638} + 1 \right)}$$

- c) The sketch plots and the ones obtained using Matlab are shown below:



Problem 6.65**Solution:****Known quantities:**

The values of the resistors, of the capacitance and of the inductance in the circuit of Figure P6.65:

$$R_1 = R_2 = 1 \text{ k}\Omega \quad C = 1 \text{ mF} \quad L = 1 \text{ H}$$

Find:

- The frequency response function $H_v(j\omega) = \frac{V_{out}(j\omega)}{I_{in}(j\omega)}$ for the circuit of Figure P6.65.
- Manually sketch a magnitude and phase Bode plot of the system, using a five-cycle semilog paper.
- Use Matlab and the Bode command to generate the same plot.

Analysis:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$Z_T = (Z_{R_1} + Z_{C_1}) \parallel Z_L = \left(R_1 + \frac{1}{j\omega C} \right) \parallel j\omega L = \left(\frac{j\omega C}{1 + j\omega C R_1} + \frac{1}{j\omega L} \right)^{-1} =$$

$$= \frac{(j\omega)^2 L C R_1 + j\omega L}{(j\omega)^2 L C + j\omega C R_1 + 1}$$

and

$$\frac{V_{OC}}{I_{in}} = \frac{j\omega L}{(j\omega)^2 L C + j\omega C R_1 + 1}$$

$$\frac{V_{out}}{V_{OC}} = \frac{R_2}{Z_T + R_2} = \frac{(j\omega)^2 L C + j\omega C R_1 + 1}{(j\omega)^2 L C \left(1 + \frac{R_1}{R_2} \right) + j\omega \left(C R_1 + \frac{L}{R_2} \right) + 1}$$

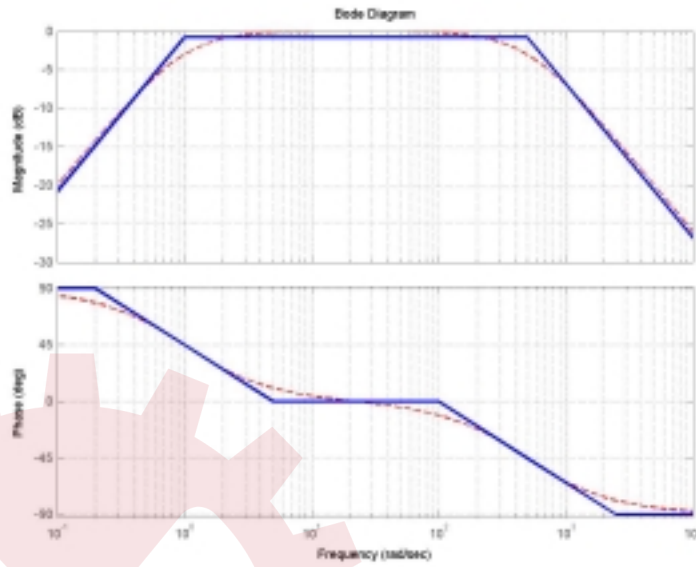
Thus,

$$\frac{V_{out}}{I_{in}} = \frac{j\omega L}{(j\omega)^2 L C \left(1 + \frac{R_1}{R_2} \right) + j\omega \left(C R_1 + \frac{L}{R_2} \right) + 1}$$

- Substituting the numerical values and expressing the frequency response function in factored form, we have:

$$H_v(j\omega) = \frac{j\omega}{(j\omega + 1) \left(\frac{j\omega}{499.5} + 1 \right)}$$

- The sketch plots and the ones obtained using Matlab are shown below:



Problem 6.66

Solution:

Known quantities:

The values of the resistors, of the capacitance and of the inductance in the circuit of Figure P6.65:

$$R_1 = R_2 = 1 \text{ k}\Omega \quad C = 1 \text{ mF} \quad L = 1 \text{ H}$$

Find:

- The frequency response function $H_v(j\omega) = \frac{I_{out}(j\omega)}{I_{in}(j\omega)}$ for the circuit of Figure P6.65.
- Manually sketch a magnitude and phase Bode plot of the system, using a five-cycle semilog paper.
- Use Matlab and the Bode command to generate the same plot.

Analysis:

- The frequency response function $H_v(j\omega) = \frac{V_{out}(j\omega)}{I_{in}(j\omega)}$ is (see P6.65 for details):

$$\frac{V_{out}}{I_{in}} = \frac{j\omega L}{(j\omega)^2 LC \left(1 + \frac{R_1}{R_2}\right) + j\omega \left(CR_1 + \frac{L}{R_2}\right) + 1}$$

and,

$$I_{out} = \frac{V_{out}}{R_2}$$

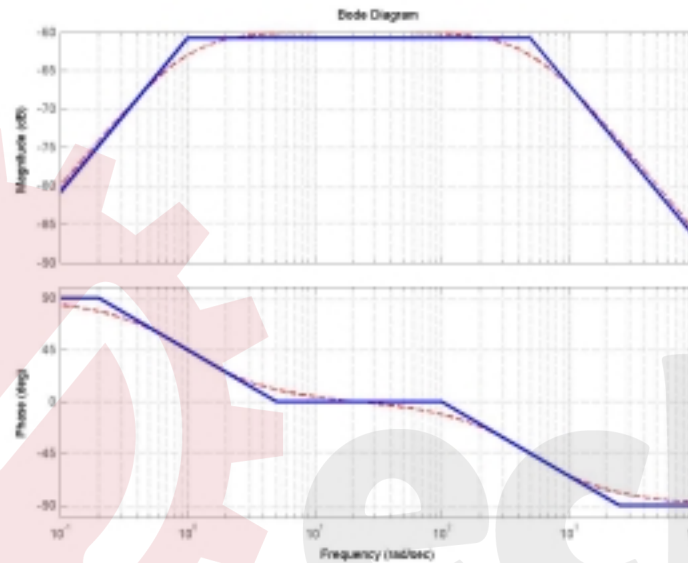
Thus,

$$\frac{I_{out}}{I_{in}} = \frac{j\omega \frac{L}{R_2}}{(j\omega)^2 LC \left(1 + \frac{R_1}{R_2}\right) + j\omega \left(CR_1 + \frac{L}{R_2}\right) + 1}$$

- b) Substituting the numerical values and expressing the frequency response function in factored form, we have:

$$H_v(j\omega) = 10^{-3} \frac{j\omega}{(j\omega + 1) \left(\frac{j\omega}{499.5} + 1 \right)}$$

- c) The sketch plots and the ones obtained using Matlab are shown below:



Problem 6.67

Solution:

Known quantities:

The values of the resistors and of the capacitances in the circuit of Figure P6.67:

$$R_1 = R_2 = 1 \text{ k}\Omega \quad C_1 = 1 \text{ }\mu\text{F} \quad C_2 = 1 \text{ mF}$$

Find:

- The frequency response function $H_v(j\omega) = \frac{V_{out}(j\omega)}{I_{in}(j\omega)}$ for the circuit of Figure P6.65.
- Manually sketch a magnitude and phase Bode plot of the system, using a five-cycle semilog paper.
- Use Matlab and the Bode command to generate the same plot.

Analysis:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$a) \quad Z_T = Z_{R_2} + Z_{C_1} \parallel Z_{R_1} = R_2 + \left(j\omega C_1 + \frac{1}{R_1} \right)^{-1} = R_2 + \frac{R_1}{1 + j\omega C_1 R_1} = \frac{j\omega C_1 R_1 R_2 + R_1 + R_2}{1 + j\omega C_1 R_1}$$

and

$$\frac{V_{OC}}{I_{in}} = \frac{R_1}{1 + j\omega C_1 R_1}$$

$$\begin{aligned}\frac{V_{out}}{V_{OC}} &= \frac{1/j\omega C_2}{Z_T + 1/j\omega C_2} = \frac{j\omega C_2}{1 + j\omega C_2 \left[\frac{j\omega C_1 R_1 R_2 + R_1 + R_2}{1 + j\omega C_1 R_1} \right]} \\ &= \frac{(1 + j\omega C_1 R_1)j\omega C_2}{(j\omega)^2 C_1 C_2 R_1 R_2 + j\omega [C_1 R_1 + C_2 (R_1 + R_2)] + 1}\end{aligned}$$

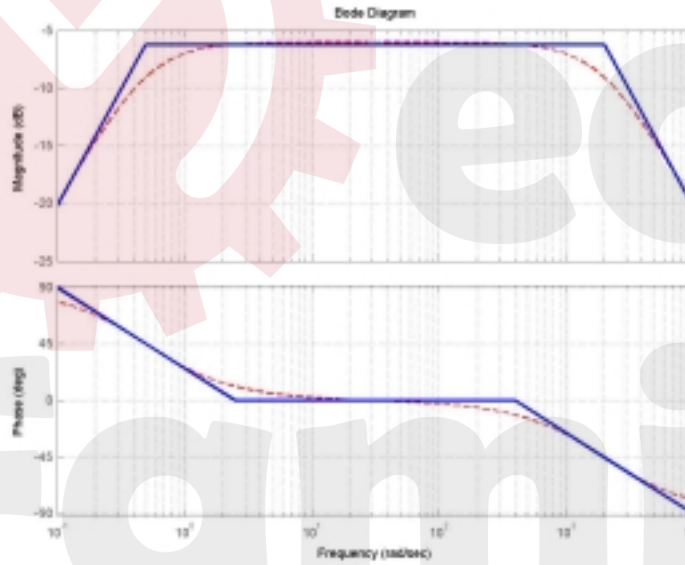
Thus,

$$\frac{V_{out}}{I_{in}} = \frac{j\omega C_2 R_1}{(j\omega)^2 C_1 C_2 R_1 R_2 + j\omega [C_1 R_1 + C_2 (R_1 + R_2)] + 1}$$

- b) Substituting the numerical values and expressing the frequency response function in factored form, we have:

$$H_v(j\omega) = \frac{j\omega}{\left(\frac{j\omega}{0.5} + 1\right)\left(\frac{j\omega}{2000} + 1\right)}$$

- c) The sketch plots and the ones obtained using Matlab are shown below:



Problem 6.68

Solution:

Known quantities:

The values of the resistors and of the capacitances in the circuit of Figure P6.67:

$$R_1 = R_2 = 1 \text{ k}\Omega \quad C_1 = 1 \text{ }\mu\text{F} \quad C_2 = 1 \text{ mF}$$

Find:

- The frequency response function $H_v(j\omega) = \frac{V_{out}(j\omega)}{I_{in}(j\omega)}$ for the circuit of Figure P6.65.
- Manually sketch a magnitude and phase Bode plot of the system, using a five-cycle semilog paper.
- Use Matlab and the Bode command to generate the same plot.

Analysis:

- a) The frequency response function $H_v(j\omega) = \frac{V_{out}(j\omega)}{I_{in}(j\omega)}$ is (see P6.67 for details):

$$\frac{V_{out}}{I_{in}} = \frac{j\omega C_2 R_1}{(j\omega)^2 C_1 C_2 R_1 R_2 + j\omega[C_1 R_1 + C_2(R_1 + R_2)] + 1}$$

and,

$$I_{out} = j\omega C_2 V_{out}$$

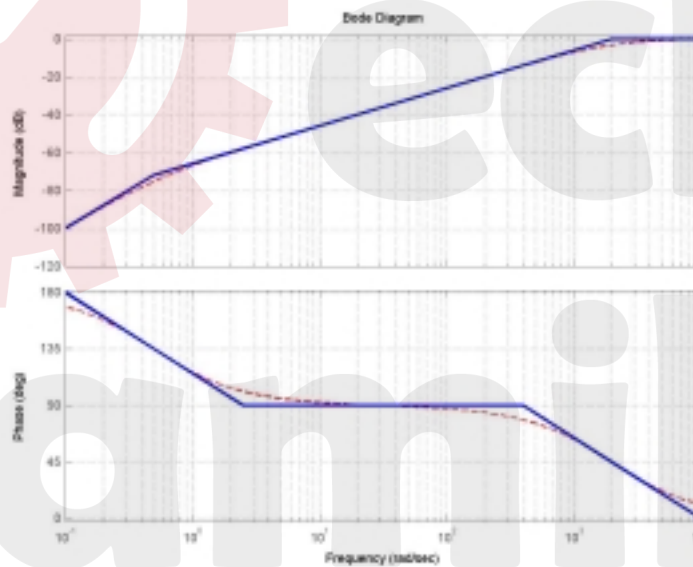
Thus,

$$\frac{I_{out}}{I_{in}} = \frac{(j\omega C_2)^2 R_1}{(j\omega)^2 C_1 C_2 R_1 R_2 + j\omega[C_1 R_1 + C_2(R_1 + R_2)] + 1}$$

- b) Substituting the numerical values and expressing the frequency response function in factored form, we have:

$$H_v(j\omega) = 10^{-3} \frac{(j\omega)^2}{\left(\frac{j\omega}{0.5} + 1\right)\left(\frac{j\omega}{2000} + 1\right)}$$

- c) The sketch plots and the ones obtained using Matlab are shown below:

**Problem 6.69****Solution:****Known quantities:**

Resistance, inductance and capacitance values, in the circuit of Figure P6.4.

Find:

- Manually sketch a magnitude and phase Bode plot of the system, using a five-cycle semilog paper.
- Use Matlab and the Bode command to generate the same plot.

Analysis:

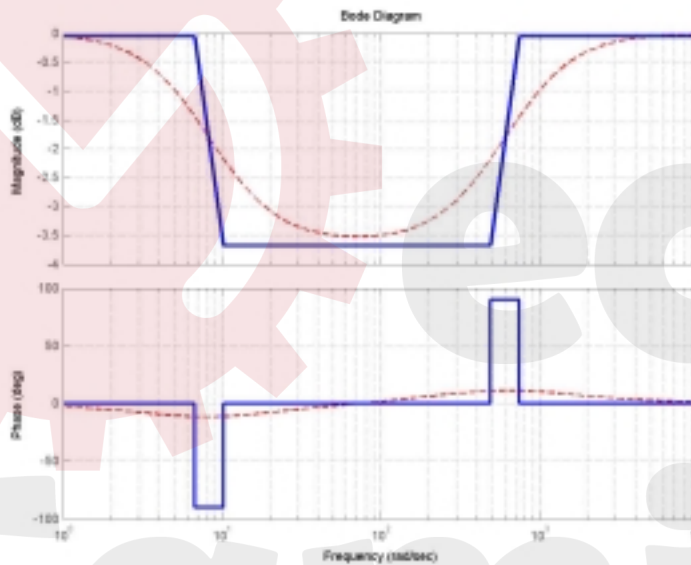
The frequency response function $H_v(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$ is (see P6.4 for details):

$$\frac{V_{out}}{V_{in}}(j\omega) = \frac{1 + j\omega CR_2 + (j\omega)^2 LC}{1 + j\omega C(R_1 + R_2) + (j\omega)^2 LC}$$

a) Substituting the numerical values and expressing the frequency response function in factored form, we have:

$$H_v(j\omega) = \frac{\left(\frac{j\omega}{489.79} + 1\right)\left(\frac{j\omega}{10.21} + 1\right)}{\left(\frac{j\omega}{743.27} + 1\right)\left(\frac{j\omega}{6.72} + 1\right)}$$

b) The sketch plots and the ones obtained using Matlab are shown below:

**Problem 6.70****Solution:****Known quantities:**

Resistance, inductance and capacitance values, in the circuit of Figure P6.5.

Find:

- Manually sketch a magnitude and phase Bode plot of the system, using a five-cycle semilog paper.
- Use Matlab and the Bode command to generate the same plot.

Assume:

Assume that the output voltage is the voltage across the resistor.

Analysis:

The frequency response function $H_V(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$ is (see P6.5 for details):

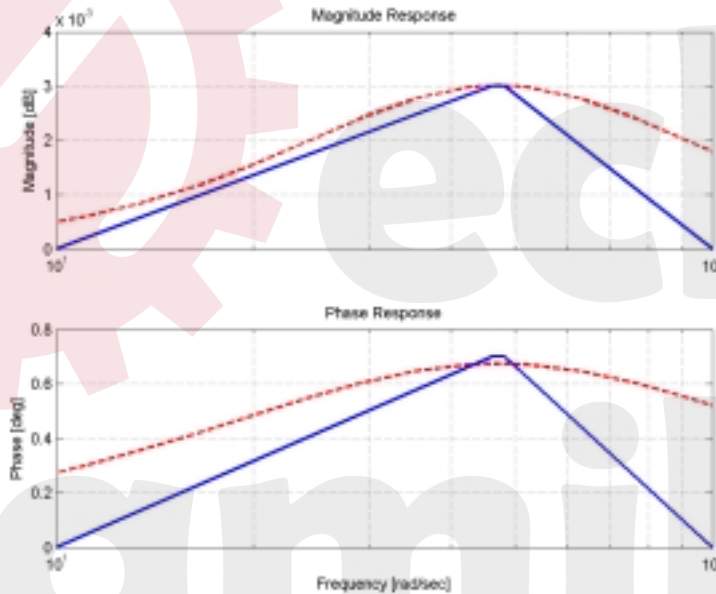
$$H_V(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{R_2 - CLR_2\omega^2 + j\omega L}{R_2 + R_1 - CL(R_2 + R_1)\omega^2 + j\omega L}$$

- a) Substituting the numerical values and expressing the frequency response function in factored form, we have:

$$H_V(j\omega) = \frac{(1 - 4.5 \times 10^{-4} \omega^2) + j(0.0015)\omega}{(1 - 4.5 \times 10^{-4} \omega^2) + j(0.0010)\omega}$$

$$H_V(j\omega) = \frac{-1.004 \left(\frac{j\omega}{48.84} + 1 \right) \left(\frac{j\omega}{45.54} + 1 \right)}{\left(\frac{j\omega}{48.26} + 1 \right) \left(\frac{j\omega}{46.04} + 1 \right)}$$

- b) The sketch plots and the ones obtained using Matlab are shown below:

**Problem 6.71****Solution:****Known quantities:**

Resistance, inductance and capacitance values, in the circuit of Figure P6.5.

Find:

- Manually sketch a magnitude and phase Bode plot of the system, using a five-cycle semilog paper.
- Use Matlab and the Bode command to generate the same plot.

Analysis:

The frequency response function $H_v(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$ is (see P6.5 for details):

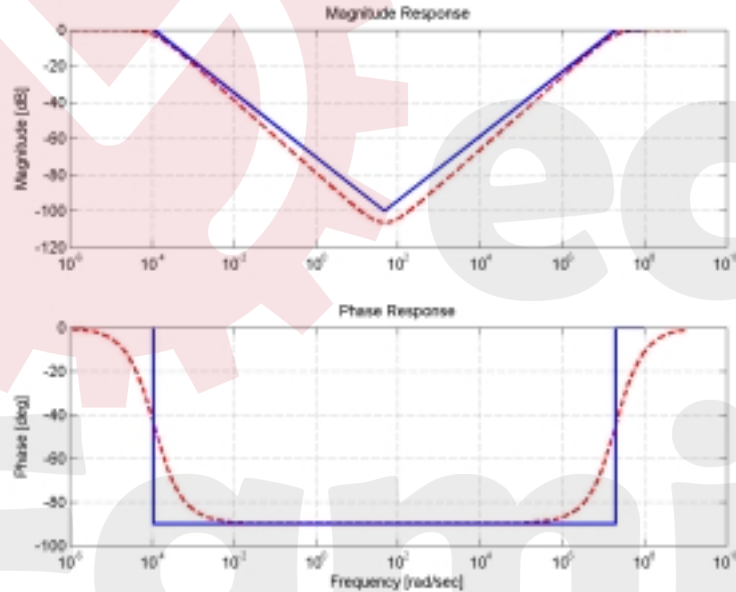
$$\frac{V_{out}}{V_{in}}(j\omega) = \frac{V_C}{V_{in}}(j\omega) = \frac{1 - CL\omega^2}{1 - CL\omega^2 + j\omega L(R_2 + R_1)}$$

- a) Substituting the numerical values and expressing the frequency response function in factored form, we have:

$$H_v(j\omega) = \frac{(1 - 4.5 \times 10^{-4} \omega^2)}{(1 - 4.5 \times 10^{-4} \omega^2) + j(9000)\omega}$$

$$H_v(j\omega) = \frac{\left(\frac{j\omega}{47.14} + 1\right)\left(\frac{j\omega}{47.14} + 1\right)}{\left(\frac{j\omega}{2e7} + 1\right)\left(\frac{j\omega}{1.11e-4} + 1\right)}$$

- b) The sketch plots and the ones obtained using Matlab are shown below:

**Problem 6.72****Solution:****Known quantities:**

Resistance, inductance and capacitance values, in the circuit of Figure P6.4.

Find:

- Manually sketch a magnitude and phase Bode plot of the system, using a five-cycle semilog paper.
- Use Matlab and the Bode command to generate the same plot.

Analysis:

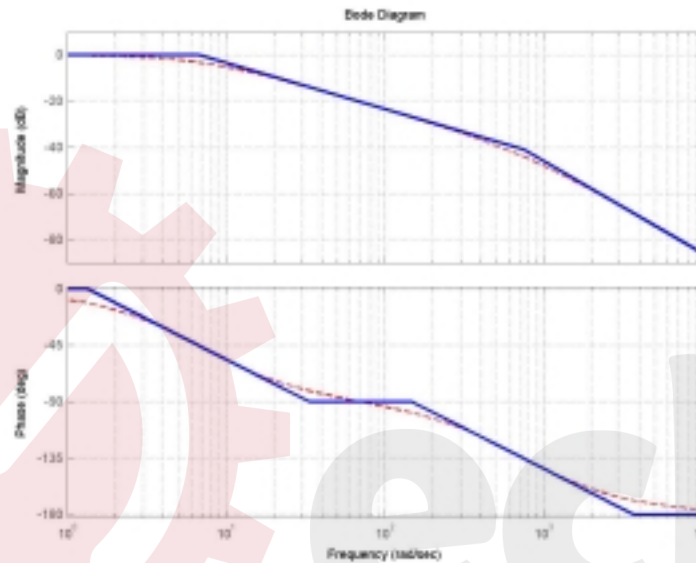
The frequency response function is:

$$\frac{V_{out}}{V_{in}}(j\omega) = \frac{1}{1 + j\omega C(R_1 + R_2) + (j\omega)^2 LC}$$

- a) Substituting the numerical values and expressing the frequency response function in factored form, we have:

$$H_v(j\omega) = \frac{1}{\left(\frac{j\omega}{743.27} + 1\right)\left(\frac{j\omega}{6.72} + 1\right)}$$

- b) The sketch plots and the ones obtained using Matlab are shown below:



Problem 6.73

Solution:

Known quantities:

Resistance and capacitance values, in the circuit of Figure P6.6.

Find:

- Manually sketch a magnitude and phase Bode plot of the system, using a five-cycle semilog paper.
- Use Matlab and the Bode command to generate the same plot.

Analysis:

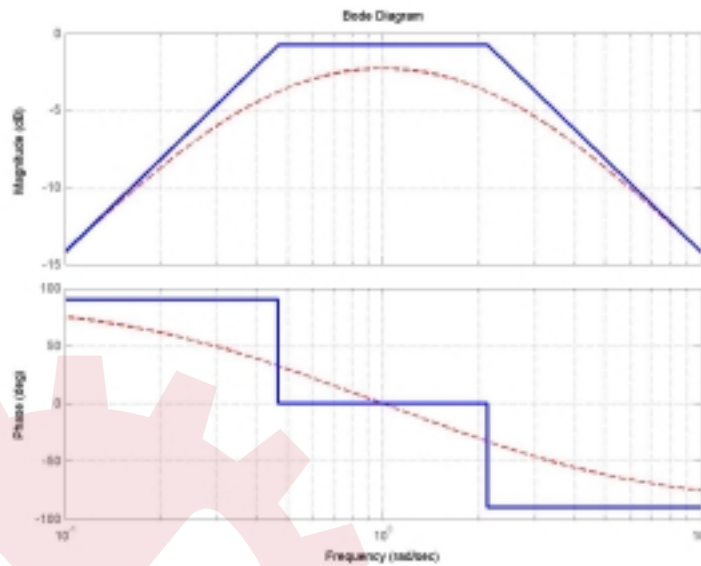
The frequency response function $H_v(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$ is (see P6.6 for details):

$$\frac{V_{out}}{V_{in}}(j\omega) = \frac{j\omega C_1 R_1}{1 + j\omega[C_1 R_1 + C_2(R_1 + R_2)] + (j\omega)^2 C_1 C_2 R_1 R_2}$$

- a) Substituting the numerical values and expressing the frequency response function in factored form, we have:

$$H_v(j\omega) = \frac{2j\omega}{(j\omega)^2 + 2.6j\omega + 1} = 2 \frac{j\omega}{\left(\frac{j\omega}{2.13} + 1\right)\left(\frac{j\omega}{0.47} + 1\right)}$$

- b) The sketch plots and the ones obtained using Matlab are shown below:



Problem 6.74

Solution:

Known quantities:

Ratio of output amplitude to input amplitude, being proportional to $\frac{1}{\omega^3}$ in a certain frequency range.

Find:

The slope of the Bode plot in this frequency range, expressed in dB per decade.

Analysis:

If $\left| \frac{v_{out}}{v_{in}} \right| \propto \frac{1}{\omega^3}$, it is seen that the amplitude is reduced by a factor of 1000, or multiplied by $\frac{1}{1000}$,

every time the frequency increases by a factor of 10. Since $\frac{1}{1000}$ is a -60 dB gain, we speak of the

transfer function rolling off at a $-60 \frac{\text{dB}}{\text{decade}}$ slope. The term “decade” refers to a frequency factor of

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Problem 6.75

Solution:

The output amplitude of a given circuit as a function of frequency:

$$|V| = \frac{A\omega + B}{\sqrt{C + D\omega^2}}$$

Find:

- The break frequency.
- The slope of the Bode plot (in dB per decade) above the break frequency.

- c) The slope of the Bode plot below the break frequency.
 d) The high-frequency limit of V .

Analysis:

- a) Given $|V| = \frac{A\omega + B}{\sqrt{C + D\omega^2}}$, this is seen to rise from $\frac{B}{\sqrt{C}}$ at zero frequency to $\frac{A}{\sqrt{D}}$ at high

frequencies. The corresponding complex phasor function is:

$$V = \frac{j\omega A + B}{\sqrt{C + j\sqrt{D}\omega}} = \frac{\frac{B}{\sqrt{C}} + j\omega \frac{A}{\sqrt{C}}}{1 + j\sqrt{\frac{D}{C}}\omega} = \frac{\frac{B}{\sqrt{C}} + j\omega \frac{A}{\sqrt{C}}}{1 + \frac{j\omega}{\sqrt{\frac{C}{D}}}} = \frac{\left(1 + j\omega \frac{A}{B}\right) \frac{B}{\sqrt{C}}}{1 + \frac{j\omega}{\sqrt{\frac{C}{D}}}}$$

which we recognize to have a break frequency (or cut-off frequency, or half-power frequency) of:

$$\omega_{co} = \sqrt{\frac{C}{D}}$$

- b) At high frequencies the slope is zero and the magnitude is equal to $\frac{A}{\sqrt{D}}$
 c) At low frequencies the slope is zero and the magnitude is equal to $\frac{B}{\sqrt{C}}$
 d) At high frequencies, $|V| \rightarrow \frac{A}{\sqrt{D}}$

Problem 6.76

Solution:

Known quantities:

Figures P6.76a and P6.76b.

Find:

An expression for the equivalent impedance in Figure P6.76a in standard form. Choose the Bode plot, from Figure P6.76b, that best describes the behavior of the impedance as a function of frequency and describe how (a simple one line statement and no analysis is sufficient) you would obtain the resonant and cutoff frequencies and the magnitude of the impedance where it is constant over some frequency range. Label the Bode plot to indicate which feature you are discussing.

Analysis:

In standard form.

$$\begin{aligned} Z[j\omega] &= \frac{Z_C [Z_{R_c} + Z_L]}{Z_C + Z_{R_c} + Z_L} = \frac{\frac{1}{j\omega C} [R_c + j\omega L]}{\frac{1}{j\omega C} + R_c + j\omega L} \frac{j\omega C}{j\omega C} = \\ &= \frac{R_c + j\omega L}{[1 - \omega^2 LC] + j\omega R_c C} = \frac{R_c}{1 - \omega^2 LC} \frac{1 + j\frac{\omega L}{R_c}}{1 + j\frac{\omega R_c C}{1 - \omega^2 LC}} = Z_o \frac{1 + j f_1[\omega]}{1 + j f_2[\omega]} \end{aligned}$$

Bode Plot [b]. The circuit is a parallel resonant circuit and should exhibit maxima of impedance and minima of impedance at low and high frequencies.

1. At the resonant frequency, the impedance is real, i.e., the reactive part is zero. 1. $f_1[\omega_r] = f_2[\omega_r] \Rightarrow$ Solve for ω_r

2. The magnitude of the impedance at the resonant frequency is Z_o evaluated at the resonant frequency.

$$2. Z_o = \frac{R_c}{1 - \omega_r^2 LC}$$

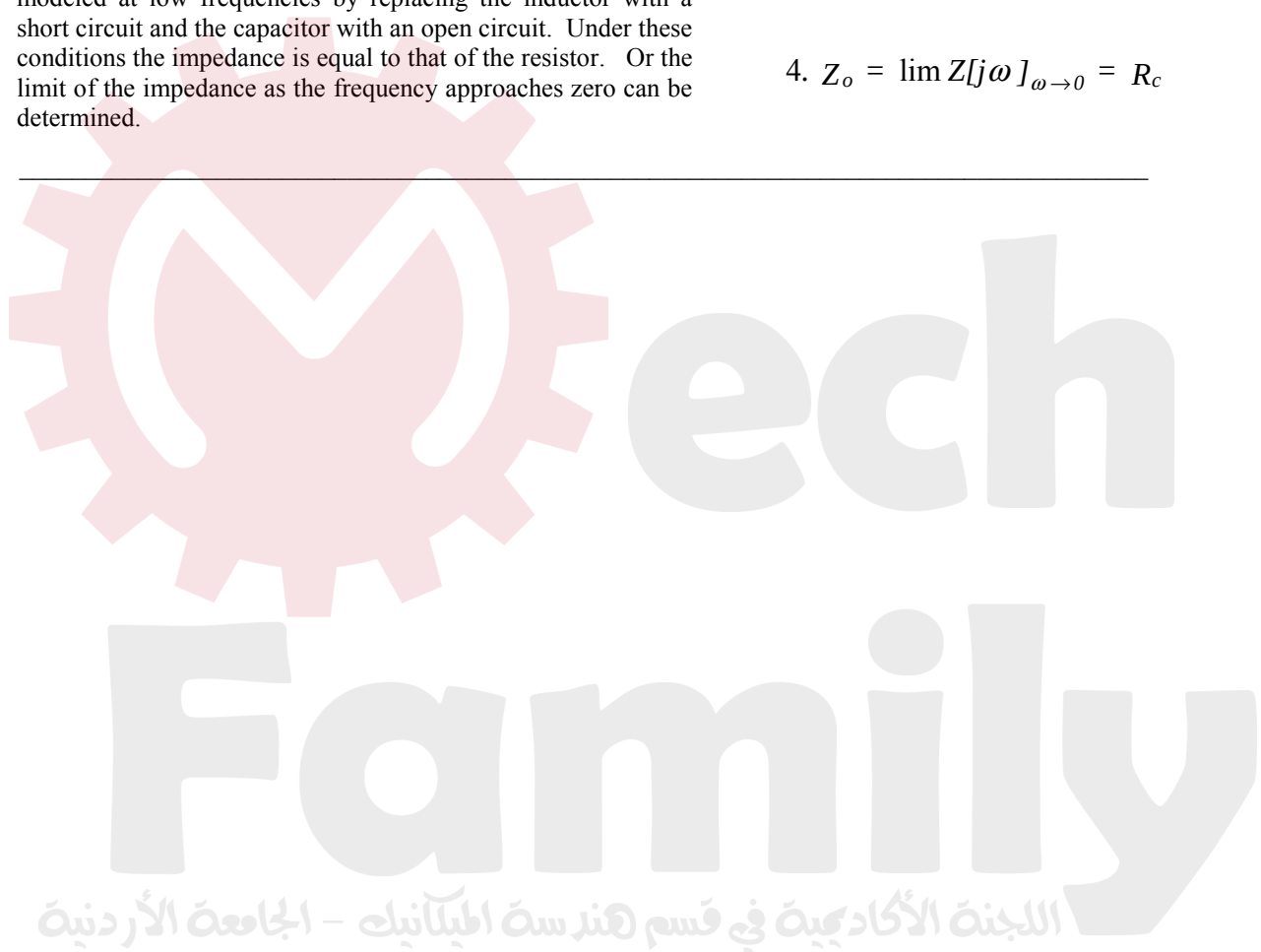
3. There are three cutoff frequencies [the 3 dB] frequencies evaluated by making the functions of frequency equal to +1 or - 1.

$$3. f_1[\omega_c] = 1 \text{ Gives } \omega_{c3}.$$

4. The magnitude of the impedance when the frequency is low can be determined in two ways. First, the circuit can be modeled at low frequencies by replacing the inductor with a short circuit and the capacitor with an open circuit. Under these conditions the impedance is equal to that of the resistor. Or the limit of the impedance as the frequency approaches zero can be determined.

$$f_2[\omega_c] = \pm 1 \text{ Gives } \omega_{c1} \text{ and } \omega_{c2}.$$

$$4. Z_o = \lim_{\omega \rightarrow 0} Z[j\omega] = R_c$$



Chapter 7 Instructor Notes

Chapter 7 surveys all important aspects of electric power. Coverage of Chapter 7 can take place immediately following Chapter 4, or as part of a later course on energy systems or electric machines. The material in this chapter will be of particular importance to Aerospace, Civil, Industrial, and Mechanical engineers, who are concerned with the utilization of electric power.

The chapter permits very flexible coverage, with sections 7.1 and 7.2 describing basic single-phase AC power ideas. A survey course might only use this introductory material. The next two sections discuss transformers and three-phase power. Two descriptive sections are also provided to introduce the ideas of residential wiring, grounding and safety, and the generation and distribution of AC power. These sections can be covered independent of the transformer and three-phase material.

The section *Focus on Measurements: The wattmeter* provides a practical look at the measurement of power. The section *Focus on Measurements: How Hall-effect current transducers work* may be useful in the context of a power systems course, while *Focus on Measurements: Power factor correction* proposes a more applied look at the problem of improving the power factor of an industrial load.

The homework problems present a few simple applications in addition to the usual exercises meant to reinforce the understanding of the fundamentals. Problems 7.19, 20, 22-25 present a variety of power factor correction problems. Two advanced problems (7.35, 7.36) discuss transformer test methods; these problems may be suitable in a second course in energy systems. Problem 7.48 illustrates the billing penalties incurred when electric loads have insufficient power factors (this problem is based on actual data supplied by Detroit Edison).

Those instructors who plan to integrate the three-phase material into a course on power systems and electric machines, will find that problems 7.47 and 7.49-51 can be assigned in conjunction with the material covered in Chapter 17, as part of a more in-depth look at three-phase machines.

Problem 7.1**Solution:****Known quantities:**

Resistance value, $R = 30\ \Omega$, and the voltage across the soldering iron, $\tilde{V} = 117\text{ V}$.

Find:

The power dissipated in the soldering iron.

Analysis:

The power dissipated in the soldering iron is:

$$P = \frac{\tilde{V}^2}{R} = \frac{117^2}{30} = 456.3\text{ W}$$

Problem 7.2**Solution:****Known quantities:**

Rated power, $P = 1000\text{ W}$, and the voltage across the heating element, $\tilde{V} = 240\text{ V}$.

Find:

The resistance of the heating element.

Analysis:

The power dissipated in the electric heater is:

$$R = \frac{\tilde{V}^2}{P} = \frac{240^2}{1000} = 57.6\ \Omega$$

Problem 7.3**Solution:****Known quantities:**

Resistance value, $R = 50\ \Omega$ of the resistor.

Find:

The power dissipated in the resistor if the current source connected to the resistor is:

- $i(t) = 5\cos(50t)\text{ A}$
- $i(t) = 5\cos(50t - 45^\circ)\text{ A}$
- $i(t) = 5\cos(50t) - 2\cos(50t - 50^\circ)\text{ A}$
- $i(t) = 5\cos(50t) - 2\text{ A}$

Analysis:

The average power can be expressed as:

$$P_{av} = \frac{1}{2} I^2 R$$

$$\text{a) } P_{av} = \frac{5^2 \cdot 50}{2} = 625\text{ W}$$

$$b) \quad P_{av} = \frac{5^2 \cdot 50}{2} = 625 \text{ W}$$

c) By using phasor techniques:

$$\mathbf{I} = 5\angle 0^\circ - 2\angle -50^\circ = 5 - 1.2856 + j1.5321 = 3.7144 + j1.5321 = 4.0180\angle 22.41^\circ \text{ A}$$

Then, the instantaneous current can be expressed as:

$$i(t) = 4.0180 \cos(100t + 22.41^\circ) \text{ A}$$

Therefore, the average power is:

$$P_{av} = \frac{4.0180^2 \cdot 50}{2} = 403.6 \text{ W}$$

d) The instantaneous voltage can be expressed as:

$$v(t) = Ri(t) = 250 \cos(50t) - 100 \text{ V}$$

Then, the instantaneous power can be written as:

$$\begin{aligned} p(t) &= v(t) \cdot i(t) = [250 \cos(50t) - 100] \cdot [5 \cos(50t) - 2] \\ &= 1250 \cos^2(50t) - 1000 \cos(50t) + 200 \\ &= 625 + 625 \cos(100t) - 1000 \cos(50t) + 200 \text{ W} \end{aligned}$$

Therefore, the average power is:

$$P_{av} = 625 + 200 = 825 \text{ W}$$

Problem 7.4

Solution:

Known quantities:

The current values.

Find:

The rms value of each of the following currents.

- $\cos 450t + 2 \cos 450t$
- $\cos 5t + \sin 5t$
- $\cos 450t + 2$
- $\cos 5t + \cos(5t + \pi/3)$
- $\cos 200t + \cos 400t$

Analysis:

The rms current can be expressed as:

$$I_{rms} = \tilde{I} = \frac{I}{\sqrt{2}} \text{ if the current is periodic or if the current can be converted to a phasor quantity.}$$

Otherwise, the rms current must be calculated using integration techniques.

- a) Summing the common cosine terms leads to

$$I = 3 \cos 450t$$

$$\tilde{I} = \frac{3}{\sqrt{2}} = 2.1213 \text{ A}$$

- b) Using phasor analysis:

$$I = \cos 5t + \cos(5t - 90^\circ) = 1\angle 0^\circ + 1\angle -90^\circ = 1 - j = \sqrt{2}\angle -45^\circ \text{ A}$$

$$\tilde{I} = \frac{\sqrt{2}}{\sqrt{2}} = 1 \text{ A}$$

- c) Since the second term is not periodic, integration techniques must be used:

$$\begin{aligned}\tilde{I} &= \sqrt{\frac{2\pi}{450} \int_0^{2\pi} (\cos 450t + 2)^2 dt} = \sqrt{\frac{2\pi}{450} \int_0^{2\pi} (\cos^2 450t + 4 \cos 450t + 4) dt} = \\ \tilde{I} &= \sqrt{\frac{2\pi}{450} \int_0^{2\pi} (\cos^2 450t + 4 \cos 450t + 4) dt} = \sqrt{\frac{2\pi}{450} \int_0^{2\pi} \left(\frac{1}{2} + \frac{1}{2} \cos 900t + 4 \cos 450t + 4 \right) dt} = \\ \tilde{I} &= \sqrt{\frac{2\pi}{450} \frac{450}{2\pi} \left[\frac{9}{4} - \frac{1}{1800} \sin 900t - \frac{4}{450} \sin 450t \right]} = \frac{3}{\sqrt{2}} = 2.1213 \text{ A}\end{aligned}$$

- d) Using phasor analysis:

$$I = 1\angle 0^\circ + 1\angle 60^\circ = 1 + 0.5 + j0.866 = 1.732\angle 30^\circ \text{ A}$$

$$\tilde{I} = \frac{1.732}{\sqrt{2}} = 1.225 \text{ A}$$

- e) Can't use phasor analysis because phasor analysis does not work for different frequencies. Must integrate as in part c:

$$\tilde{I} = \frac{2}{\sqrt{2}} = 1.414 \text{ A}$$

Problem 7.5

Solution:

Known quantities:

The current rms value, 4 A, the voltage source rms value, 110 V, the lag between the current and the voltage, 60° .

Find:

The power dissipated by the circuit and the power factor

Analysis:

The average power drawn by the circuit is:

$$P = \frac{VI}{2} \cos(\theta) = \frac{110\sqrt{2} \cdot 4\sqrt{2}}{2} \cos(60^\circ) = 220 \text{ W}$$

The power factor is:

$$pf = \cos(60^\circ) = 0.5$$

Problem 7.6

Solution:

Known quantities:

The voltage source rms value, 120 V, the source frequency, 60 Hz, the power consumption, 1.2 kW, and the power factor, 0.8.

Find:

- The rms current.
- The phase angle.
- The impedance.
- The resistance.

Analysis:

- a) The power is expressed as:

$$P = \tilde{V}\tilde{I} \cos(\theta)$$

Thus, the rms current is:

$$\tilde{I} = \frac{P}{\tilde{V} \cos(\theta)} = \frac{1200}{120 \cdot 0.8} = 12.5 \text{ A}$$

- b) The power factor is:

$$pf = \cos(\theta)$$

Thus, the phase angle θ is:

$$\theta = \cos^{-1}(0.8) = 36.87^\circ$$

- c) The impedance
- Z
- is:

$$Z = \frac{\tilde{V}}{\tilde{I}} = \frac{120}{12.5} = 9.6 \Omega$$

- d) The resistance
- R
- is:

$$R = Z \cos(\theta) = 7.68 \Omega$$

Problem 7.7**Solution:****Known quantities:**

The rms values of the supply voltage and current, 110 V and 14 A, the power requirement, 1 kW, the machine efficiency, 90%, and the power factor, 0.8.

Find:

The AC machine efficiency.

Analysis:

The efficiency is:

$$\eta_{motor} = \frac{\text{Mechanical Power}}{\text{Electrical Power}} = \frac{1 \text{ kW} / 0.9}{\tilde{V}\tilde{I} \cos(\theta)} = \frac{1111 \text{ W}}{1232 \text{ W}} = 0.9$$

Problem 7.8**Solution:****Known quantities:**

The waveform of a voltage source shown in Figure P7.8.

Find:

- The steady DC voltage that would cause the same heating effect across a resistance.
- The average current supplied to a 10- Ω resistor connected across the voltage source.
- The average power supplied to a 1- Ω resistor connected across the voltage source.

Analysis:

$$a) \quad V_{DC} = \tilde{V} = \sqrt{\frac{(1 \cdot 1) + (9 \cdot 1)}{2}} = \sqrt{\frac{10}{2}} = 2.24 \text{ V}$$

$$b) \quad I_{av} = \frac{(1 \cdot 1) + (-3 \cdot 1)}{2 \cdot 10} = \frac{-2}{20} = -0.1 \text{ A}$$

$$c) \quad P_{av} = \frac{\tilde{V}^2}{R} = \frac{2}{1} = 5 \text{ W}$$

Problem 7.9

Solution:

Known quantities:

The current and the voltage values.

Find:

The average power, the reactive power and the complex power.

Analysis:

$$a) \quad P = \tilde{V}\tilde{I} \cos(\theta) = \frac{450}{\sqrt{2}} \cdot \frac{50}{\sqrt{2}} \cdot \cos(20^\circ) = 21140 \text{ W}$$

$$Q = \tilde{V}\tilde{I} \sin(\theta) = \frac{450}{\sqrt{2}} \cdot \frac{50}{\sqrt{2}} \cdot \sin(20^\circ) = 7696 \text{ VAR}$$

$$S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^* = 11250 \angle 20^\circ \text{ VA}$$

$$b) \quad P = \tilde{V}\tilde{I} \cos(\theta) = 140 \cdot 5.85 \cdot \cos(-30^\circ) = 709.3 \text{ W}$$

$$Q = \tilde{V}\tilde{I} \sin(\theta) = 140 \cdot 5.85 \cdot \sin(-30^\circ) = -409.5 \text{ VAR}$$

$$S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^* = 819 \angle -30^\circ \text{ VA}$$

$$c) \quad P = \tilde{V}\tilde{I} \cos(\theta) = 50 \cdot 19.2 \cdot \cos(-45.8^\circ) = 668.8 \text{ W}$$

$$Q = \tilde{V}\tilde{I} \sin(\theta) = 50 \cdot 19.2 \cdot \sin(-45.8^\circ) = -688.7 \text{ VAR}$$

$$S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^* = 960 \angle -45.8^\circ \text{ VA}$$

$$d) \quad P = \tilde{V}\tilde{I} \cos(\theta) = 740 \cdot 10.8 \cdot \cos(-85.9^\circ + 45^\circ) = 6040.8 \text{ W}$$

$$Q = \tilde{V}\tilde{I} \sin(\theta) = 740 \cdot 10.8 \cdot \sin(-85.9^\circ + 45^\circ) = -5232.7 \text{ VAR}$$

$$S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^* = 7992 \angle -40.9^\circ \text{ VA}$$

Problem 7.10

Solution:

Known quantities:

The current and the voltage values or the impedance.

Find:

The power factor and state if it is leading or lagging.

Analysis:

$$a) \quad pf = \cos(\theta_i - \theta_v) = \cos(21.2^\circ) = 0.932 \quad \text{Leading}$$

$$b) \quad pf = \cos(\theta_i - \theta_v) = \cos(-40.6^\circ) = 0.759 \quad \text{Lagging}$$

$$c) \quad i_L(t) = 48.7 \sin(\omega t + 2.74) = 48.7 \sin(\omega t + 2.74 - 90^\circ)$$

$$pf = \cos(\theta_i - \theta_v) = \cos(67^\circ) = 0.391 \quad \text{Leading}$$

$$d) \quad \theta = \tan^{-1}\left(\frac{8}{12}\right) = 33.7^\circ \Rightarrow pf = \cos(\theta_i - \theta_v) = \cos(-33.7^\circ) = 0.832 \quad \text{Lagging}$$

Problem 7.11

Solution:

Known quantities:

The power factor or the values of the current and the voltage.

Find:

The kind of the load (capacitive or inductive).

Analysis:

- a) Capacitive.
- b) Capacitive.
- c) Since $i_L(t) = 1.8 \cos(\omega t - 90^\circ)$, Inductive.
- d) Since the phase difference is zero, Resistive.

Problem 7.12

Solution:

Known quantities:

Circuit shown in Figure P7.12, the values of the resistance, $R = 4 \Omega$, the capacitance, $C = 1/18 \mu\text{F}$, the inductance, $L = 2 \text{ H}$, and the voltage source.

Find:

The real and reactive power supplied by the following sources.

- a) $v_S(t) = 10 \cos(3t) \text{ V}$
- b) $v_S(t) = 10 \cos(9t) \text{ V}$

Analysis:

$$a) \quad \omega = 3, Z_T = j6 - j6 + 4 = 4 + j0 \Omega, \tilde{I} = \frac{10}{\sqrt{2} \cdot 4} = 1.77 \text{ A}$$

$$P = \tilde{I}^2 R = 12.5 \text{ W}, Q = \tilde{I}^2 X = 0 \text{ VAR}$$

$$b) \quad \omega = 9, Z_T = j18 - j2 + 4 = 4 + j16 \Omega, \tilde{I} = \frac{10}{\sqrt{2} \cdot 16.5} = 0.42 \text{ A}$$

$$P = \tilde{I}^2 R = 0.7 \text{ W}, Q = \tilde{I}^2 X = 2.82 \text{ VAR}$$

Problem 7.13

Solution:

Known quantities:

Circuit shown in Figure P7.13, the values of the resistances, $R_1 = 8 \Omega$, $R_2 = 6 \Omega$, the reactances,

$X_C = -12 \Omega$, $X_L = 6 \Omega$, and the voltage sources, $\tilde{V}_{S1} = 36 \angle -\pi/3 \text{ V}$, $\tilde{V}_{S2} = 24 \angle 0.644 \text{ V}$.

Find:

- a) The active and reactive current for each source

b) The total real power.

Analysis:

a) From Figure P7.13:

$$\begin{aligned} \mathbf{V}_{S1} &= R_1 \mathbf{I}_1 + jX_L (\mathbf{I}_1 - \mathbf{I}_2) = (8 + j6) \mathbf{I}_1 - j6 \mathbf{I}_2 \\ -\mathbf{V}_{S2} &= -jX_L (\mathbf{I}_1 - \mathbf{I}_2) + R_2 \mathbf{I}_2 + X_C \mathbf{I}_2 = -j6 \mathbf{I}_1 + (6 - j6) \mathbf{I}_2 \end{aligned}$$

Substituting the values for the voltages sources gives:

$$\begin{cases} 18 - j31.2 = (8 + j6) \mathbf{I}_1 - j6 \mathbf{I}_2 \\ -19.2 - j14.4 = -j6 \mathbf{I}_1 + (6 - j6) \mathbf{I}_2 \end{cases}$$

Solving for $\mathbf{I}_1$ and $\mathbf{I}_2$ yields:

$$\begin{cases} \mathbf{I}_1 = 0.398 - j3.38 \text{ A} \\ \mathbf{I}_2 = 1.091 - j0.911 \text{ A} \end{cases}$$

Therefore, the active and reactive currents for each source are:

$$\begin{cases} \mathbf{I}_{A1} = 0.398 \text{ A} \\ \mathbf{I}_{A2} = 1.091 \text{ A} \end{cases} \quad \text{and} \quad \begin{cases} \mathbf{I}_{R1} = 3.38 \text{ A} \\ \mathbf{I}_{R2} = 0.91 \text{ A} \end{cases}$$

b) $P = R_2 \mathbf{I}_2^2 + R_1 \mathbf{I}_1^2 = 6 \cdot 1.421^2 + 8 \cdot 3.403^2 = 105 \text{ W}$

Problem 7.14

Solution:

Known quantities:

Circuit shown in Figure P7.14, the values of the resistors, $R_L = 25 \Omega$, $R = 1 \Omega$, the capacitor, $C = 0.1 \mu\text{F}$, the voltage source, $\tilde{V}_S = 230 \text{ V}$, and the frequency, $f = 60 \text{ Hz}$.

Find:

- The source power factor.
- The current $\mathbf{I}_S$.
- The apparent power delivered to the load.
- The apparent power supplied by the source.
- The power factor of the load.

Analysis:

a) $pf_{source} = \frac{R}{Z} = \frac{R_{line} + R_{load}}{\sqrt{(R_{line} + R_{load})^2 + X_C^2}} = \frac{26}{\sqrt{676 + 7.036e^8}} = 0.00098 \quad \text{Leading}$

b) $\tilde{I}_S = \frac{\tilde{V}_S}{Z} = \frac{230}{26525 \angle -90^\circ} = 8.67 \text{ mA}$

Therefore, $\mathbf{I}_S = 8.67 \angle -90^\circ \text{ mA}$

c) $S_{load} = \sqrt{P_{load}^2 + Q_{load}^2} = \tilde{I}_S^2 \sqrt{R_{load}^2 + X_C^2} = 1.994 \text{ VA}$

d) $S_{source} = \tilde{I}_S \tilde{V}_S = 1.994 \text{ VA}$

e) $pf_{load} = \frac{R_{load}}{Z_{load}} = \frac{25}{26525} = 0.00094$

Problem 7.15**Solution:****Known quantities:**

Circuit shown in Figure P7.14, the values of the resistors, $R_L = 25\Omega$, $R = 1\Omega$, the inductor, $L = 0.1\text{ H}$, the voltage source, $\tilde{V}_S = 230\text{ V}$, and the frequency, $f = 60\text{ Hz}$.

Find:

- The apparent power supplied by the source.
- The apparent power delivered to the load.
- The power factor of the load.

Analysis:

$$\text{a) } S = \frac{\tilde{V}^2}{Z} = \frac{\tilde{V}^2}{\sqrt{(R_{line} + R_{load})^2 + X_L^2}} = \frac{230^2}{\sqrt{26^2 + 37.7^2}} = 1.155\text{ kVA}$$

$$\text{b) } S_{load} = \tilde{I}_S^2 Z_{load} = \frac{\tilde{V}^2}{Z^2} \sqrt{R_{load}^2 + X_L^2} = \left(\frac{230}{45.8}\right)^2 \sqrt{25^2 + 37.7^2} = 1.141\text{ kVA}$$

$$\text{c) } pf_{load} = \frac{R_{load}}{Z_{load}} = \frac{25}{45.2} = 0.55$$

Problem 7.16**Solution:****Known quantities:**

Circuit shown in Figure P7.14, the values of the resistors, $R_L = 25\Omega$, $R = 1\Omega$, the capacitor, $C = 0.1\text{ mF}$, the inductor, $L = 70.35\text{ mH}$, the voltage source, $\tilde{V}_S = 230\text{ V}$, and the frequency, $f = 60\text{ Hz}$.

Find:

- The apparent power delivered to the load.
- The real power supplied by the source.
- The power factor of the load.

Analysis:

$$\text{a) } S = \frac{\tilde{V}^2}{Z} = \frac{\tilde{V}^2}{\sqrt{(R_{line} + R_{load})^2 + (X_L - X_C)^2}} = \frac{230^2}{\sqrt{26^2 + (26.5 - 26.5)^2}} = 2.03\text{ kVA}$$

$$\text{b) } X_C = X_L \Rightarrow P = \frac{\tilde{V}^2}{R} = \frac{230^2}{25 + 1} = 2.03\text{ kW}$$

$$\text{c) } X_C = X_L \Rightarrow pf = 1$$

Problem 7.17**Solution:****Known quantities:**

Circuit shown in Figure P7.17, the values of the resistor, $R = 20\Omega$, the capacitor, $C = 0.1\text{ mF}$, the voltage source, $\tilde{V}_s = 50\text{ V}$.

Find:

The apparent power, the real power, and the reactive power; draw the power triangle.

Analysis:

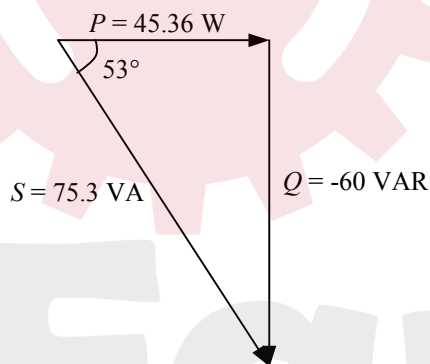
$$S = \frac{\tilde{V}^2}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{50^2}{\sqrt{20^2 + 26.5^2}} = \frac{2500}{33.2} = 75.3\text{ VA}$$

$$P = S \cdot \cos(\theta) = S \cdot \frac{R}{Z} = 75.3 \frac{20}{33.2} = 45.36\text{ W}$$

$$Q = \sqrt{S^2 - P^2} = -60\text{ VAR}$$

For the power triangle, $\theta = \cos^{-1}\left(\frac{R}{Z}\right) = 53^\circ$

Therefore, the power triangle can be drawn as shown below:

**Problem 7.18****Solution:****Known quantities:**

Circuit shown in Figure P7.17, the values of the resistor, $R = 20\Omega$, the capacitor, $C = 0.1\text{ mF}$, the voltage source, $\tilde{V}_s = 50\text{ V}$.

Find:

The apparent power, the real power, and the reactive power, in the cases of $f = 50$ and 0 Hz .

Analysis:

For the frequency of 0 Hz ,

$$S_{0\text{ Hz}} = 0, P_{0\text{ Hz}} = 0, Q_{0\text{ Hz}} = 0$$

For the frequency of 50 Hz ,

$$S_{50\text{Hz}} = \frac{\tilde{V}^2}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{2500}{37.2} = 67.15 \text{ VA}$$

$$P_{50\text{Hz}} = S \cdot \cos(\theta) = S \cdot \frac{R}{Z} = 67.15 \cdot \frac{20}{37.2} = 36 \text{ W}$$

$$Q_{50\text{Hz}} = \sqrt{S^2 - P^2} = 56.7 \text{ VA}$$

Problem 7.19

Solution:

Known quantities:

A single-phased motor connected across a 220-V source at 50 Hz as shown in Figure P7.19, power factor $pf = 1.0$, $I = 20 \text{ A}$, and $I_l = 25 \text{ A}$.

Find:

The capacitance required to give a unity power factor when connected in parallel with the load.

Analysis:

The magnitude of the current $\tilde{I}_2$ is:

$$\tilde{I}_2 = \sqrt{\tilde{I}_1^2 - \tilde{I}^2} = \sqrt{625 - 400} = 15 \text{ A}$$

The voltage source can be expressed as:

$$\tilde{V} = \tilde{I}_2 \cdot X_C$$

Therefore, the required capacitor is:

$$C = \frac{\tilde{I}_2}{\tilde{V} \cdot \omega} = \frac{15}{220 \cdot 314} = 217 \mu\text{F}$$

Problem 7.20

Solution:

Known quantities:

The currents and voltages required by an air-conditioner, a freezer, a refrigerator, and their power factors.

Find:

The power to be supplied by an emergency generator to run all the appliances.

Analysis:

In this problem we will use the following equations:

$$P = \tilde{I} \tilde{V} \cos(\theta), Q = \tilde{I} \tilde{V} \sin(\theta), pf = \cos(\theta)$$

The real and reactive power used by the air conditioner are:

$$P_1 = 9.6 \cdot 120 \cdot 0.9 = 1036.8 \text{ W}, Q_1 = 9.6 \cdot 120 \cdot \sin(\cos^{-1}(0.9)) = 502.15 \text{ VAR}$$

The real and reactive power used by the freezer are:

$$P_2 = 4.2 \cdot 120 \cdot 0.87 = 438.48 \text{ W}, Q_2 = 4.2 \cdot 120 \cdot \sin(\cos^{-1}(0.87)) = 248.5 \text{ VAR}$$

The real and reactive power used by the refrigerator are:

$$P_3 = 3.5 \cdot 120 \cdot 0.8 = 336 \text{ W}, Q_3 = 3.5 \cdot 120 \cdot \sin(\cos^{-1}(0.8)) = 252 \text{ VAR}$$

The total real and reactive power P are:

$$P = P_1 + P_2 + P_3 = 1811.28 \text{ W}, Q = Q_1 + Q_2 + Q_3 = 1002.65 \text{ VAR}$$

Therefore, the following power must be supplied:

$$S = P + jQ = 1811.28 + j1002.65 \text{ VA} = 2070.3 \angle 28.97^\circ \text{ VA}$$

Problem 7.21

Solution:

Known quantities:

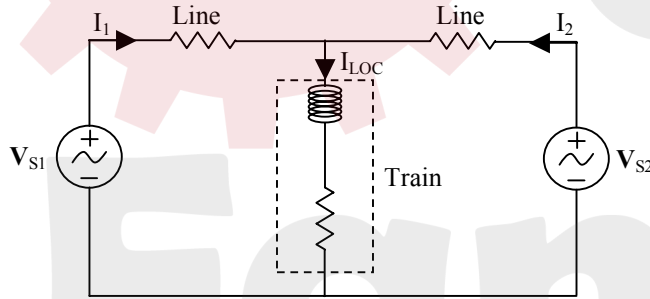
The schematics of the power supply module consisting of two 25-kV single-phase power stations shown in Figure P7.21, the power consumption by the train, the DC power supply at a low speed operation, the average power factor in AC operation, the over-head line equivalent specific resistance, and negligible rail resistance.

Find:

- The equivalent circuit.
- The locomotive current in the condition of a 10% voltage drop.
- The reactive power.
- The supplied real power, over-head line losses, and the maximum distance between two power station supplied in the condition of a 10% voltage drop when the train is located at the half distance between the stations.
- Over-head line losses in the condition of a 10% voltage drop when the train is located at the half distance between the stations, assuming $pf = 1$ (The French TGV is designed with the state of art power compensation system).
- The maximum distance between the two power station supplied in the condition of a 10% percent voltage drop when the train is located at the half distance between the stations, assuming the DC (1.5 kV) operation at a quarter power.

Analysis:

- The equivalent circuit is:



- The locomotive current for the 10% voltage drop is:

$$\tilde{I}_{LOC} = \tilde{I}_1 + \tilde{I}_2 = 2\tilde{I}_1 = 2\tilde{I}_2 = \frac{P_{LOC}}{(\tilde{V}_S - 10\%) \cos(\theta)} = \frac{11 \text{ MW}}{22.5 \text{ kV} \cdot 0.8} = 611 \text{ A}$$

- The reactive power is:

$$\begin{aligned} Q &= \sqrt{S_{LOC}^2 - P_{LOC}^2} = \sqrt{(\tilde{V}_S - 10\%)^2 I_{LOC}^2 - P_{LOC}^2} \\ &= \sqrt{(13.75 \text{ MVA})^2 - (11 \text{ MW})^2} = 8.25 \text{ MVAR} \end{aligned}$$

- The supplied real power is:

$$P = \sqrt{S^2 - Q^2} = \sqrt{(\tilde{V}_S \cdot \tilde{I})^2 - Q^2} = 12.85 \text{ MW}$$

The over-head line power loss is:

$$P_{Line} = -P_{LOC} + \sqrt{(\tilde{V}_S \cdot \tilde{I})^2 - Q^2}$$

$$= -11 \text{ MW} + \sqrt{(15.27 \text{ MVA})^2 - (8.25 \text{ MVAR})^2} = 1.85 \text{ MW}$$

The maximum distance between the two power stations is:

$$R_{Line} || R_{Line} = \frac{P_{Line}}{\tilde{I}_{LOC}^2} = 5 \Omega \Rightarrow \text{Distance}_{\max} = \frac{2R_{Line}}{0.2 \Omega/\text{km}} = 100 \text{ km}$$

e) The over-head line power loss is:

$$P_{LOC} = 10\% \cdot \tilde{V}_S \cdot I_{LOC} \cdot \cos(\theta) = 2500 \text{ V} \cdot 489 \text{ A} = 1.22 \text{ MW}$$

$$\text{f) } I_{LOC} = \frac{0.25 P_{LOC}}{90\% \cdot V_{S,DC}} = \frac{2.75 \text{ MW}}{1350 \text{ V}} = 2037 \text{ A}$$

$$P_{Line} = 10\% \cdot V_{S,DC} \cdot I_{LOC} = 305 \text{ kW}$$

$$R_{Line} || R_{Line} = \frac{P_{Line}}{I_{LOC}^2} = 0.0735 \Omega$$

The maximum distance between the two power stations is:

$$\text{Distance}_{\max} = \frac{2R_{Line}}{0.2 \Omega/\text{km}} = 1.5 \text{ km}$$

Problem 7.22

Solution:

Known quantities:

One hundred 40-W lamps supplied by a 120-V and 60-Hz source, the power factor of 0.65, the penalty at billing, and the average prices of the power supply and the capacitors.

Find:

Number of days of operation for which the penalty billing covers the price of the power factor correction capacitor.

Analysis:

The capacitor value for $pf = 0.85$ is:

$$C = \frac{\tilde{I}_C}{\tilde{V}_C \cdot \omega} = \frac{(\tilde{I}_{X,0.65} - \tilde{I}_{X,0.85})}{\omega \cdot V_C} = \frac{\sqrt{\left(\frac{P}{\tilde{V} \cdot 0.65}\right)^2 - \left(\frac{P}{V}\right)^2} - \sqrt{\left(\frac{P}{\tilde{V} \cdot 0.85}\right)^2 - \left(\frac{P}{V}\right)^2}}{\omega \cdot V_C}$$

$$= \frac{\sqrt{51.3^2 - 33.3^2} - \sqrt{39.2^2 - 33.3^2}}{337 \cdot 120} = \frac{19}{377 \cdot 120} = 420 \mu\text{F}$$

Therefore, the number of days of operation for which the penalty billing covers the price of the power factor correction capacitor is:

$$\text{Number of Days} = \frac{420 \mu\text{F} \cdot 50 \frac{\$}{\text{mF}}}{\frac{4 \text{ kW}}{4} \cdot 1 \text{ hr} \cdot 0.01 \frac{\$}{\text{kW}} \cdot 24 \frac{\text{hr}}{\text{day}}} = 88 \text{ Days}$$

Problem 7.23**Solution:****Known quantities:**

Reference to the problem 7.22, and the network current decreasing with the power factor correction.

Find:

- The capacitor value for the unity power factor.
- The maximum number of lamps that can be installed supplementary without changing the cable network if a local compensation capacitor is used.

Analysis:

$$a) \quad |\tilde{\mathbf{I}}_C| = |\tilde{\mathbf{I}}_L|$$

$$C = \frac{|\tilde{\mathbf{I}}_L|}{\tilde{V}\omega} = \frac{\sqrt{\left(\frac{P}{\tilde{V} \cdot 0.65}\right)^2 - \left(\frac{P}{\tilde{V}}\right)^2}}{\tilde{V}\omega} = \frac{\sqrt{51.3^2 - 33.3^2}}{377 \cdot 120} = 862 \mu\text{F}$$

- b) Initial cable network is:

$$\tilde{I} = \frac{P}{\tilde{V} \cos(\theta)} = \frac{4000}{120 \cdot 0.65} = 51.3 \text{ A}$$

One lamp current for pf = 1 is:

$$I_{Lamp} = \frac{P_{Lamp}}{\tilde{V}} = \frac{40}{120} = 0.333 \text{ A}$$

$$\text{The total number of lamps} = \frac{\tilde{I}}{I_{Lamp}} = 154$$

Therefore, the number of supplementary lamps = $154 - 100 = 54$

Problem 7.24**Solution:****Known quantities:**

The voltage and the current supplied by a source, $\tilde{\mathbf{V}}_S = 7 \angle 50^\circ \text{ V}$, $\tilde{\mathbf{I}}_S = 13 \angle -20^\circ \text{ A}$.

Find:

- The power supplied by the source which is dissipated as heat or work in the load
- The power stored in reactive components in the load.
- Determine if the circuit is an inductive or a capacitive load.

Analysis:

$$a) \quad S = \tilde{\mathbf{V}}_S \tilde{\mathbf{I}}_S^* = (7 \angle 50^\circ \text{ V})(13 \angle -20^\circ \text{ A})^* = 91 \angle 70^\circ \text{ VA} = 31.12 + j85.51 \text{ VA}$$

$$P_{av} = 31.12 \text{ W}$$

$$b) \quad Q = 85.51 \text{ VAR}$$

$$c) \quad \theta = (\theta_I - \theta_V) = -70^\circ \Rightarrow pf = \cos(\theta) = 0.342 \quad \text{Lagging}$$

The load is inductive.

Problem 7.25**Solution:****Known quantities:**

Circuit shown in Figure P7.25, the voltage supplied by a power plant,

$v_S(t) = 450 \cos(\omega t) \text{ V}$, $\omega = 377 \text{ rad/s}$, and the impedances of the plant, $Z = 7 + j\Omega$ and of the power plant, $Z_G = 3 + j0.11\Omega$.

Find:

Determine C so that the plant power factor is corrected to 1.

Analysis:

Note: Z_G influences only the phase difference between $\mathbf{V}_S$ and $\mathbf{V}_0$ and not the one between $\mathbf{V}_0$ and $\mathbf{Z}$.

For this reason, the result does not depend from Z_G .

$$\begin{aligned} Z_{eq} &= \frac{ZZ_C}{Z + Z_C} = \frac{(R + jX)(-jX_C)}{(R + jX) + (-jX_C)} = \frac{XX_C - jRX_C}{R + j(X - X_C)} \frac{R - j(X - X_C)}{R - j(X - X_C)} = \\ &= \frac{(XX_C R - RX_C(X - X_C)) - j(R^2 X_C - XX_C(X - X_C))}{R^2 + (X - X_C)^2} = R_{eq} - jX_{eq} \end{aligned}$$

$$\mathbf{I}_S = \frac{\mathbf{V}_0}{Z_{eq}}$$

If $\mathbf{I}_S$ and $\mathbf{V}_0$ are in phase, they have the same phase angle. For this reason, it must be:

$$Z_{eq} = R_{eq} - jX_{eq} = R_{eq} - j0 \Rightarrow X_{eq} = 0 \Rightarrow R^2 X_C - XX_C(X - X_C) = 0$$

$$X_C = \frac{R^2 + X^2}{X} = 50\Omega = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{(377 \text{ rad/s})(50\Omega)} = 51.3 \mu\text{F}$$

Problem 7.26**Solution:****Known quantities:**

Circuit shown in Figure P7.25, the voltage supplied by a power plant,

$v_S(t) = 450 \cos(\omega t) \text{ V}$, $\omega = 377 \text{ rad/s}$, and the impedances of the plant, $Z = 7 \angle 10^\circ \Omega$.

Find:

Determine C so that the plant power factor is corrected to 1.

Analysis:

Note: Z_G influences only the phase difference between $\mathbf{V}_S$ and $\mathbf{V}_0$ and not the one between $\mathbf{V}_0$ and $\mathbf{Z}$.

For this reason, the result does not depend from Z_G .

$$Z = 7 \angle 10^\circ \Omega = 6.89 + j1.21\Omega$$

$$\begin{aligned} Z_{eq} &= \frac{ZZ_C}{Z + Z_C} = \frac{(R + jX)(-jX_C)}{(R + jX) + (-jX_C)} = \frac{XX_C - jRX_C}{R + j(X - X_C)} \frac{R - j(X - X_C)}{R - j(X - X_C)} = \\ &= \frac{(XX_C R - RX_C(X - X_C)) - j(R^2 X_C - XX_C(X - X_C))}{R^2 + (X - X_C)^2} = R_{eq} - jX_{eq} \end{aligned}$$

$$\mathbf{I}_S = \frac{\mathbf{V}_0}{Z_{eq}}$$

If $\mathbf{I}_S$ and $\mathbf{V}_0$ are in phase, they have the same phase angle. For this reason, it must be:

$$Z_{eq} = R_{eq} - jX_{eq} = R_{eq} - j0 \Rightarrow X_{eq} = 0 \Rightarrow R^2 X_C - X X_C (X - X_C) = 0$$

$$X_C = \frac{R^2 + X^2}{X} = 40.31 \Omega = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{(377 \text{ rad/s})(40.31 \Omega)} = 65.8 \mu\text{F}$$

Problem 7.27

Solution:

Known quantities:

Circuit shown in Figure P7.25, the voltage across a plant, $\tilde{\mathbf{V}}_0 = 450 \angle 0^\circ \text{ V}$, $f = 60 \text{ Hz}$, and the current through it without the capacitance in parallel with the plant, $\tilde{\mathbf{I}}_S = 17 \angle -10^\circ \text{ A}$, the value of the capacitance in parallel with the plant, $C = 17.40 \mu\text{F}$.

Find:

The reduction of current which resulted from connecting the capacitor into the circuit.

Assumptions:

The impedance of the power plant is very small, $Z_G \approx 0$.

Analysis:

Without capacitor:

$$\tilde{\mathbf{I}}_S = \tilde{\mathbf{I}}_g = 17 \angle -10^\circ \text{ A}, \tilde{\mathbf{V}}_0 = 450 \angle 0^\circ \text{ V}.$$

With capacitor, being $Z_G \approx 0$, the voltage across the plant does not change, as well as the current:

$$\tilde{\mathbf{V}}_0 = 450 \angle 0^\circ \text{ V} \Rightarrow \tilde{\mathbf{I}}_g = 17 \angle -10^\circ \text{ V}$$

$$\text{KCL: } -\tilde{\mathbf{I}}_S + \tilde{\mathbf{I}}_g + \tilde{\mathbf{I}}_C = 0$$

$$\begin{aligned} \tilde{\mathbf{I}}_S &= \tilde{\mathbf{I}}_g + \tilde{\mathbf{I}}_C = \tilde{\mathbf{I}}_g + \frac{\tilde{\mathbf{V}}_0}{Z_C} = 17 \angle -10^\circ + \frac{450 \angle 0^\circ}{1/j\omega C} \\ &= 16.74 - j2.952 + j(377 \cdot 17.4 \cdot 10^{-6}) = 16.74 + j0 \text{ A} = 16.74 \angle 0^\circ \text{ A} \end{aligned}$$

Problem 7.28

Solution:

Known quantities:

Circuit shown in Figure P7.25, the voltage across a plant, $v_0(t) = 170 \cos(\omega t) \text{ V}$, $f = 60 \text{ Hz}$, the current through it without the capacitance in parallel with the plant, $i_S(t) = 130 \cos(\omega t - 11^\circ) \text{ A}$, and the value of the capacitance in parallel with the plant, $C = 387 \mu\text{F}$.

Find:

The reduction of current which resulted from connecting the capacitor into the circuit.

Assumptions:

The impedance of the power plant is very small, $Z_G \approx 0$.

Analysis:

Without capacitor:

$$\tilde{\mathbf{I}}_S = \tilde{\mathbf{I}}_g = 130\angle -11^\circ \text{ A}, \quad \hat{\mathbf{V}}_0 = 170\angle 0^\circ \text{ V}.$$

With capacitor, being $Z_G \approx 0$, the voltage across the plant does not change, as well as the current:

$$\tilde{\mathbf{V}}_0 = 170\angle 0^\circ \text{ V} \Rightarrow \tilde{\mathbf{I}}_g = 130\angle -11^\circ \text{ V}$$

$$\text{KCL: } -\tilde{\mathbf{I}}_S + \tilde{\mathbf{I}}_g + \tilde{\mathbf{I}}_C = 0$$

$$\begin{aligned} \tilde{\mathbf{I}}_S &= \tilde{\mathbf{I}}_g + \tilde{\mathbf{I}}_C = \tilde{\mathbf{I}}_g + \frac{\tilde{\mathbf{V}}_0}{Z_C} = 130\angle -11^\circ + \frac{170\angle 0^\circ}{1/j\omega C} \\ &= 127.61 - j24.81 + j(377 \cdot 17.4 \cdot 10^{-6}) = 127.61 + j0 \text{ A} = 127.61\angle 0^\circ \text{ A} \end{aligned}$$

Problem 7.29**Solution:****Known quantities:**

Circuit shown in Figure P7.29, the values of the voltages and all the impedances.

Find:

The total average power, the real power dissipated and the reactive power stored in each of the impedances.

Analysis:

$$S_1 = \frac{\tilde{\mathbf{V}}_{S1}^2}{Z_1^*} = \frac{(170/\sqrt{2} \text{ V})^2}{0.7\angle -30^\circ \Omega} = 20.643\angle 30^\circ \text{ kVA} = 17.88 + j10.32 \text{ kVA} = P_{av1} + jQ_1$$

$$S_2 = \frac{\tilde{\mathbf{V}}_{S2}^2}{Z_2^*} = \frac{(170/\sqrt{2} \text{ V})^2}{1.5\angle -7^\circ \Omega} = 9.633\angle 7^\circ \text{ kVA} = 9.56 + j1.17 \text{ kVA} = P_{av2} + jQ_2$$

$$\begin{aligned} S_3 &= \frac{(\tilde{\mathbf{V}}_{S1} + \tilde{\mathbf{V}}_{S2})^2}{Z_3^*} = \frac{(170\angle 0^\circ + 170\angle 90^\circ \text{ V})^2}{0.3 - j0.4 \Omega} \\ &= \frac{(240.42\angle 45^\circ \text{ V})^2}{0.5\angle -53.13^\circ \Omega} = 57.8\angle 53.13^\circ \text{ kVA} = 34.68 - j46.24 \text{ kVA} = P_{av3} + jQ_3 \end{aligned}$$

Problem 7.30**Solution:****Known quantities:**The voltage and the current supplied by a source, $\tilde{\mathbf{V}}_S = 170\angle -9^\circ \text{ V}$, $\tilde{\mathbf{I}}_S = 13\angle 16^\circ \text{ A}$.**Find:**

- The power supplied by the source which is dissipated as heat or work in the load
- The power stored in reactive components in the load.
- Determine if the circuit is an inductive or a capacitive load.

Analysis:

$$a) \quad S = \tilde{\mathbf{V}}_S \tilde{\mathbf{I}}_S^* = (170\angle -9^\circ \text{ V})(13\angle 16^\circ \text{ A})^* = 2210\angle -25^\circ \text{ VA} = 2003 - j934.0 \text{ VA}$$

$$P_{av} = 2003 \text{ W}$$

- b) $Q = -934 \text{ VAR}$
 c) $\theta = (\theta_I - \theta_V) = 25^\circ \Rightarrow pf = \cos(\theta) = 0.906$ Leading
 The load is capacitive.

Problem 7.31

Solution:

Known quantities:

Circuit shown in Figure P7.31, 3ach secondary connected to 5-kW resistive load, the primary connected to 120-V rms.

Find:

- a) Primary power.
 b) Primary current.

Analysis:

- a) $P_{prim} = P_{sec1} + P_{sec2} = 10 \text{ kW}$
 b) $\tilde{I}_{prim} = \frac{P_{prim}}{\tilde{V}} = \frac{10000}{120} = 83.3 \text{ A}$
 $\tilde{I}_{prim} = 83.3 \angle 0^\circ \text{ A}$

Problem 7.32

Solution:

Known quantities:

Circuit shown in Figure P7.31, the ratio between the secondary and the primary, $\frac{\tilde{V}_{sec}}{\tilde{V}_{prim}} = n$ and

$$\tilde{V}_{sec1} = \tilde{V}_{sec2} = \frac{1}{2} \tilde{V}_{sec}$$

Find:

- a) V_{sec} and V_{sec1} if $V_{prim} = 220 \text{ V rms}$ and $n = 11$.
 b) n if $V_{prim} = 110 \text{ V rms}$ and $V_{sec2} = 5 \text{ V rms}$.

Analysis:

- a) $V_{sec} = \frac{V_{prim}}{n} = \frac{220}{11} = 20 \text{ V}$
 $V_{sec1} = \frac{V_{sec}}{2} = 10 \text{ V}$

- b) $n = \frac{V_{prim}}{2 \cdot V_{sec2}} = \frac{110}{2 \cdot 5} = 11$

Problem 7.33**Solution:****Known quantities:**

The circuit shown in Figure P7.33 and $v_g = 120$ V rms.

Find:

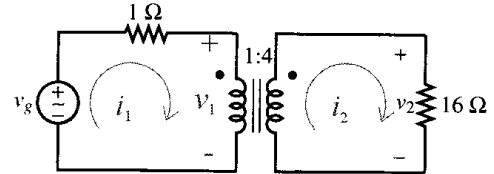
- The total resistance seen by the voltage source.
- The primary current.
- The primary power.

Analysis:

From the circuit shown on the right hand side:

$$\begin{aligned} v_g &= i_1 R_1 + v_1 & i_2 &= v_2 / R_2 \\ v_2 &= n v_1 & i_1 &= n i_2 \end{aligned}$$

- $R_{tot} = \frac{v_g}{i_1} = \frac{R_1 i_1 + v_1}{i_1} = R_1 + \frac{R_2}{n^2} = 2 \Omega$
- $\tilde{I}_1 = \frac{v_g}{R_{tot}} = \frac{120}{2} = 60$ A
- $P_1 = \tilde{I}_1 \cdot \tilde{V}_1 = 7.2$ kW

**Problem 7.34****Solution:****Known quantities:**

The circuit shown in Figure P7.33 and $v_g = 120$ V rms.

Find:

- The secondary current.
- The installation efficiency P_{load} / P_{source} .
- The value of the load resistance which can absorb the maximum power from the given source.

Analysis:

From the circuit shown on the right hand side:

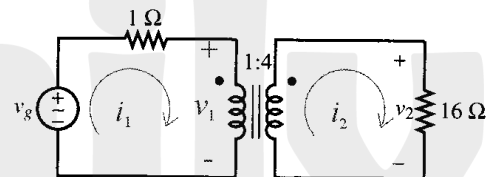
$$a) \quad I_2 = \frac{I_1}{4} = \frac{60}{4} = 15$$
 A

$$b) \quad \eta = \frac{P_{load}}{P_{source}} = \frac{I_2^2 \cdot R_2}{v_g \cdot I_1} = \frac{225 \cdot 16}{120 \cdot 60} = \frac{3.6 \text{ kW}}{7.2 \text{ kW}} = 0.5$$

- For the maximum power transfer:

$$R_{prim} = R'_{sec}$$

$$R_1 = \frac{1}{n^2} R_{load} \Rightarrow R_{load} = R_1 \cdot n^2 = 1 \Omega \cdot 16 = 16 \Omega$$



Problem 7.35**Solution:****Known quantities:**

Circuit shown in Figure P7.35, the voltage and the power that a transformer is rated to deliver to a customer, $\tilde{V}_1 = 380 \text{ V}$, $P_{in} = 460 \text{ kW}$.

Find:

- The current that the transformer supply to the customer.
- The maximum power that the customer can receive if the load is purely resistive.
- The maximum power that the customer can receive if the power factor is 0.8, lagging.
- The maximum power that the customer can receive if the power factor is 0.7, lagging.
- The minimum power factor to operate if the customer requires 300 kW.

Analysis:

- a) From $S_{in} = \tilde{V}_1 \tilde{I}_1 = \tilde{V}_2 \tilde{I}_2 = S_{out}$:

$$I_1 = \frac{S_{in}}{\tilde{V}_1} = \frac{460 \cdot 1000}{380} = 1.21 \text{ kA}$$

- b) For an ideal transformer:

$$P_{out} = P_{in} \cos(\theta) = \tilde{V}_1 \tilde{I}_1 \cos(\theta)$$

For $\cos(\theta) = 1$:

$$P_{out} = \tilde{V}_1 \tilde{I}_1 = 460 \text{ kW}$$

- c) For $\cos(\theta) = 0.8$, the maximum power is:

$$P_{out} = \tilde{V}_1 \tilde{I}_1 \cos(\theta) = 368 \text{ kW}$$

- d) For $\cos(\theta) = 0.7$, the maximum power is:

$$P_{out} = \tilde{V}_1 \tilde{I}_1 \cos(\theta) = 322 \text{ kW}$$

- e) For $P_{out} = 300 \text{ kW}$, the minimum power factor is:

$$\cos(\theta) = \frac{P_{out}}{P_{in}} = \frac{300 \text{ kW}}{460 \text{ kW}} = 0.65$$

Problem 7.36**Solution:****Known quantities:**

Circuit shown in Figure P7.36, the voltage, $v_S(t) = 294 \cos(377t) \text{ V}$, the resistances in a circuit

containing a transformer and the ratio $n = \frac{v_0(t)}{v_S(t)} = \frac{1}{2.5}$.

Find:

- Primary current.
- $v_0(t)$.
- Secondary power.
- The installation efficiency P_{load} / P_{source} .

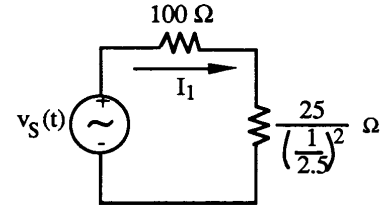
Analysis:

- a) The primary circuit is described in the figure left-hand side.
The primary current is:

$$\tilde{V}_S = \tilde{I}_1(R_1 + R'_2) = \tilde{I}_1(R_1 + n^2 R_2)$$

$$\tilde{I}_1 = \frac{\tilde{V}_1}{R_1 + n^2 R_2} = \frac{294}{\sqrt{2}} \cdot \frac{1}{100 + 6.25 \cdot 25} = 0.82 \text{ A}$$

$$i(t) = 1.144 \cos(377t) \text{ A}$$



- b) The output voltage is:

$$\tilde{V}_0 = \frac{\tilde{V}_1}{n} = \frac{\tilde{V}_S - R_1 \tilde{I}_1}{n} = \frac{208 - 100 \cdot 0.82}{2.5} = 50.4 \text{ V}$$

$$v_0(t) = 71 \cos(377t) \text{ V}$$

- c) For the secondary power, if $pf = 1$:

$$P_2 = \tilde{I}_2 \tilde{V}_0 = \tilde{I}_1 \cdot \tilde{V}_0 \cdot n = 0.82 \cdot 50.4 \cdot 2.5 = 103.3 \text{ W}$$

- d) The installation efficiency is:

$$\eta = \frac{P_{load}}{P_{source}} = \frac{P_2}{\tilde{V}_S \cdot \tilde{I}_1} = \frac{103.3 \text{ W}}{208 \text{ V} \cdot 0.82 \text{ A}} = 0.6$$

Problem 7.37**Solution:****Known quantities:**

Circuit shown in Figure P7.37, the resistances, $R_S = 1800 \Omega$, $R_L = 8 \Omega$.

Find:

The turn's ratio that will provide the maximum power transfer to the load.

Analysis:

From Equation (7.41) for the reflected source impedance circuit, we have: $R_{Seq} = N^2 R_S$

Therefore, the power is maximized if:

$$R_{Seq} = R_L \Rightarrow N = 1/n = \sqrt{R_L/R_S} = 0.067$$

$$\therefore n = 15$$

Problem 7.38**Solution:****Known quantities:**

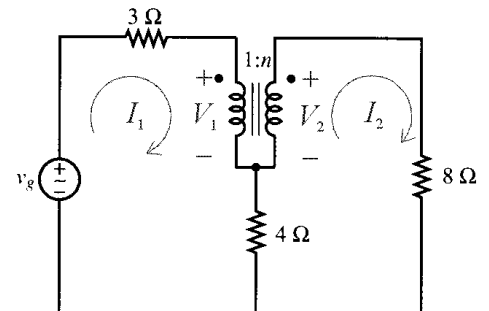
The voltage source and the resistances in the circuit shown in Figure P7.38.

Find:

- Maximum power dissipated by the load.
- Maximum power absorbing from the source.
- The installation efficiency.

Analysis:

All the impedances are resistances, and therefore it is possible to consider the modules of voltages and currents.



- a) To maximize the power delivered to the $8\text{-}\Omega$ resistance, n must be selected to maximize the load current $I_2 > 0$.

Note that $V_1 = \frac{1}{n}V_2$ and $I_1 = nI_2$.

KVL Mesh 1:

$$v_g = 3I_1 + \frac{1}{n}V_2 + 4(I_1 - I_2) = 3nI_2 + \frac{1}{n}V_2 + 4(nI_2 - I_2)$$

KVL Mesh 2:

$$V_2 = 8I_2 + 4(I_2 - I_1) = 8I_2 + 4(I_2 - nI_2)$$

Rearranging the two mesh equations:

$$\begin{cases} (7n-4)I_2 + \frac{1}{n}V_2 = v_g \\ (12-4n)I_2 = V_2 \end{cases} \Rightarrow (7n-4)I_2 + \frac{1}{n}(12-4n)I_2 = v_g$$

$$I_2 = \frac{n}{7n^2 - 8n + 12} v_g$$

$$\frac{d}{dn} I_2 = \frac{(7n^2 - 8n + 12) - n(14n - 8)}{(7n^2 - 8n + 12)^2} v_g = \frac{-7n^2 + 12}{(7n^2 - 8n + 12)^2} v_g$$

For the maximum value of the load current I_2 , $\frac{dI_2}{dn} = 0$:

$$-7n^2 + 12 = 0 \Rightarrow n = \sqrt{\frac{12}{7}} = 1.31$$

The maximum load current is:

$$I_2 = \frac{n}{7n^2 - 8n + 12} v_g = 0.122 v_g = 13.47 \text{ A}$$

The maximum power dissipated by the load is:

$$P_{load} = R_{load} I_2^2 = 2.45 \text{ kW}$$

- b) The maximum power absorbing from the source is:

$$P_{source} = P_{load} + 3\Omega \cdot I_1^2 + 4\Omega \cdot (I_1 - I_2)^2$$

$$P_{source} = P_{load} + 3\Omega \cdot nI_2^2 + 4\Omega \cdot (nI_2 - I_2)^2$$

$$P_{source} = 2540 \text{ W} + 934 \text{ W} + 70 \text{ W} = 3.54 \text{ kW}$$

- c) The installation efficiency is:

$$\eta = \frac{P_{load}}{P_{source}} = \frac{2.45 \text{ kW}}{3.54 \text{ kW}} = 0.7$$

Problem 7.39

Solution:

Known quantities:

The current and the voltage delivered by the transformer, and the circuit of the transformer shown in Figure P7.39.

Find:

The efficiency of the installation.

Analysis:

$$\tilde{V}_W = n\tilde{V}_{sec} = 2\tilde{V}_{sec} = 220 \text{ V rms}$$

$$\tilde{I}_W = \frac{1}{n}\tilde{I}_{sec} = \frac{1}{2}\tilde{I}_{sec} = 25 \text{ A rms}$$

$$\tilde{I}_L = \frac{\tilde{V}_W}{j20} = \frac{220}{20} \angle -90^\circ = 11 \angle -90^\circ \text{ A rms}$$

Since the currents are exactly 90° out of phase, the current is the square root of the sum of the phasor magnitudes squared:

$$\tilde{I}_S = \sqrt{\tilde{I}_L^2 + \tilde{I}_W^2} = 27.3 \text{ A rms}$$

$$P_S = 1\Omega \cdot \tilde{I}_S^2 + P_{sec} = 745 \text{ W} + 5.5 \text{ kW} = 6.245 \text{ kW}$$

Therefore, the efficiency of the installation is:

$$\eta = \frac{P_{load}}{P_{source}} = \frac{P_{sec}}{P_{source}} = \frac{5.5 \text{ kW}}{6.245 \text{ kW}} = 0.88$$

Problem 7.40**Solution:****Known quantities:**

The model for the circuit of a transformer shown in Figure P7.40 and the results of two tests performed at $\omega = 377 \text{ rad/s}$:

1. Open-circuit test: $\tilde{V}_{oc} = 241 \text{ V}$, $\tilde{I}_{oc} = 0.95 \text{ A}$, $P_{oc} = 32 \text{ W}$.
2. Short-circuit test: $\tilde{V}_{sc} = 5 \text{ V}$, $\tilde{I}_{sc} = 5.25 \text{ A}$, $P_{sc} = 26 \text{ W}$.

Find:

The value of the impedances in the equivalent circuit.

Analysis:

The power factor during the open circuit test is:

$$pf_{oc} = \cos(\theta_{oc}) = \frac{P_{oc}}{\tilde{V}_{oc}\tilde{I}_{oc}} = 0.1398 \quad \text{Lagging}$$

The excitation admittance is given by:

$$Y_c = \frac{\tilde{I}_{oc}}{\tilde{V}_{oc}} \angle \cos^{-1}(pf_{oc}) = \frac{0.95}{241} \angle -81.96^\circ \text{ S} = 0.0005511 - j0.003903 \text{ S}$$

$$\begin{cases} R_c = 1.8 \text{ k}\Omega \\ X_c = 256.2 \Omega \end{cases} \Rightarrow \begin{cases} R_c = 1.8 \text{ k}\Omega \\ L_c = \frac{X_c}{\omega} = 0.68 \text{ H} \end{cases}$$

The power factor during the short circuit test is:

$$pf_{sc} = \cos(\theta_{sc}) = \frac{P_{sc}}{\tilde{V}_{sc}\tilde{I}_{sc}} = 0.9905 \quad \text{Leading}$$

The series impedance is given by:

$$Z_w = \frac{\tilde{V}_{sc}}{\tilde{I}_{sc}} \angle \cos^{-1}(pf_{sc}) = \frac{5}{5.25} \angle 7.914^\circ \Omega = 0.9476 + j0.1311 \Omega$$

$$\begin{cases} R_w = 0.9476 \Omega \\ X_w = 0.1311 \Omega \end{cases} \Rightarrow \begin{cases} R_w = 0.9476 \Omega \\ L_w = \frac{X_w}{\omega} = 0.348 \text{ mH} \end{cases}$$

Problem 7.41

Solution:

Known quantities:

The model for the circuit shown in Figure P7.40 of a 460 kVA transformer and the results of two tests performed at $f = 60 \text{ Hz}$:

1. Open-circuit test: $\tilde{V}_{oc} = 4600 \text{ V}$, $\tilde{I}_{oc} = 0.7 \text{ A}$, $P_{oc} = 200 \text{ W}$.
2. Short-circuit test: $\tilde{V}_{sc} = 5.2 \text{ V}$, $P_{sc} = 50 \text{ W}$.

Find:

The value of the impedances in the equivalent circuit.

Analysis:

The power factor during the open circuit test is:

$$pf_{oc} = \cos(\theta_{oc}) = \frac{P_{oc}}{\tilde{V}_{oc} \tilde{I}_{oc}} = 0.062 \quad \text{Lagging}$$

The excitation admittance is given by:

$$Y_c = \frac{\tilde{I}_{oc}}{\tilde{V}_{oc}} \angle \cos^{-1}(pf_{oc}) = \frac{0.7}{4600} \angle -86.45^\circ \text{ S} = 9.4 \cdot 10^{-6} - j0.152 \cdot 10^{-3} \text{ S}$$

$$\begin{cases} R_c = 106.38 \text{ k}\Omega \\ X_c = 6.58 \text{ k}\Omega \end{cases} \Rightarrow \begin{cases} R_c = 106.38 \text{ k}\Omega \\ L_c = \frac{X_c}{\omega} = 17.46 \text{ H} \end{cases}$$

The power factor during the short circuit test:

$$pf_{sc} = \cos(\theta_{sc}) = \frac{P_{sc}}{S_{sc}} \approx 1$$

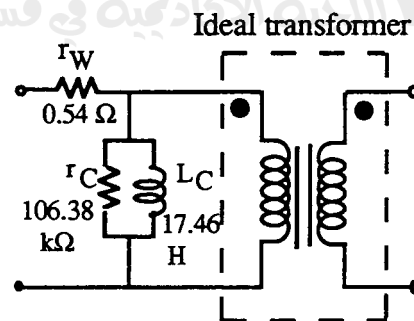
since it is a high power transformer.

The series impedance has therefore imaginary part ≈ 0 :

$$Z_w = \frac{\tilde{V}_{sc}^2}{P_{sc}} \angle \cos^{-1}(pf_{sc}) = 0.54 \Omega$$

$$\begin{cases} R_w = 0.54 \Omega \\ X_w = 0 \Omega \end{cases} \Rightarrow \begin{cases} R_w = 0.54 \Omega \\ L_w = \frac{X_w}{\omega} = 0 \end{cases}$$

Therefore, the equivalent circuit is shown besides.



Problem 7.42**Solution:****Known quantities:**

Circuit shown in Figure P7.42 of the single-phase transformer with the high voltage regulation from five different slots in the primary winding, the secondary voltage regulation in the range of 10%, and the number of turns in the secondary coil.

Find:

The number of turns for each slot.

Analysis:

The secondary voltages are:

$$\tilde{V}_{21} = \tilde{V}_{23} - 0.12 \text{ V} = 1.08 \text{ V rms}$$

$$\tilde{V}_{22} = \tilde{V}_{23} - 0.06 \text{ V} = 1.14 \text{ V rms}$$

$$\tilde{V}_{23} = 1.20 \text{ V rms}$$

$$\tilde{V}_{24} = \tilde{V}_{23} + 0.06 \text{ V} = 1.26 \text{ V rms}$$

$$\tilde{V}_{25} = \tilde{V}_{23} + 0.12 \text{ V} = 1.32 \text{ V rms}$$

Therefore, the number of turns for each slot is:

$$n_{prim} = \frac{\tilde{V}_{prim}}{\tilde{V}_{sec}} \cdot n_{sec} \Rightarrow \begin{cases} n_{11} = 203.7 \times 2 \cong 408 \text{ turns} \\ n_{12} = 192.9 \times 2 \cong 386 \text{ turns} \\ n_{13} = 183.3 \times 2 \cong 367 \text{ turns} \\ n_{14} = 174.6 \times 2 \cong 349 \text{ turns} \\ n_{15} = 166.6 \times 2 \cong 333 \text{ turns} \end{cases}$$

Problem 7.43**Solution:****Known quantities:**

The pipe's resistance = 0.0002 Ω , the secondary resistance = 0.00005 Ω , the primary current = 28.8 A, and $pf = 0.91$.

Find:

- The slot number. **NOTE:** Typo in problem statement.
- The secondary reactance.
- The installation efficiency.

Analysis:

- The secondary current is:

$$P_{prim} = P_{sec} \Rightarrow \tilde{V}_{prim} \cdot \tilde{I}_{prim} \cdot \cos(\theta) = R_{sec} \cdot \tilde{I}_{sec}^2$$

$$I_{sec} = \sqrt{\frac{\tilde{V}_{prim} \cdot \tilde{I}_{prim} \cdot \cos(\theta)}{R_{sec}}} = \sqrt{\frac{220 \cdot 28.8 \cdot 0.91}{0.00025}} = 4800 \text{ A}$$

Therefore, the slot number is:

$$N = \frac{I_{sec}}{I_{prim}} = \frac{4800}{28.8} = 166.6 \Rightarrow \text{Slot Number 5}$$

- The secondary reactance is:

$$pf_{prim} = pf_{sec} = \frac{R_{sec}}{\sqrt{R_{sec}^2 + X_{sec}^2}}$$

$$X_{sec} = R_{sec} \sqrt{\frac{1}{pf^2} - 1} = 0.00025 \sqrt{\frac{1}{0.91^2} - 1} = 114 \mu\Omega$$

c) The installation efficiency is:

$$\eta = \frac{P_{load}}{P_{prim}} = \frac{R_{load} \cdot I_{sec}^2}{R_{sec} \cdot I_{sec}^2} = \frac{R_{load}}{R_{sec}} = \frac{200 \mu\Omega}{250 \mu\Omega} = 0.8$$

Problem 7.44

Solution:

Known quantities:

A single-phase transformer converting 6 kV to 230 V with 0.95 efficiency, the pf of 0.8, and the primary apparent power of 30 KVA.

Find:

- The secondary current.
- The transformer's ratio.

Analysis:

a) The secondary current is:

$$P_{sec} = P_{prim} \cdot \eta = S_{prim} \cdot \cos(\theta) \cdot \eta = 30 \cdot 0.8 \cdot 0.95 = 22.8 \text{ kW}$$

$$\tilde{I}_{sec} = \frac{P_{sec}}{\tilde{V}_{sec} \cdot \cos(\theta)} = 124 \text{ A}$$

b) The primary current is:

$$\tilde{I}_{prim} = \frac{S_{prim}}{\tilde{V}_{prim}} = \frac{30000}{6000} = 5 \text{ A}$$

Therefore, the transformer's ratio is:

$$\frac{1}{N} = \frac{\tilde{I}_{sec}}{\tilde{I}_{prim}} = 24.8 \Rightarrow N = 0.04$$

Problem 7.45

Solution:

Known quantities:

The magnitude of the phase voltage of a three-phase wye system, 220 V rms.

Find:

The expression of each phase in both polar and rectangular coordinates.

Analysis:

The phase voltages in polar form are:

$$\tilde{V}_{an} = 220 \angle 0^\circ \text{ V}, \tilde{V}_{bn} = 220 \angle -120^\circ \text{ V}, \tilde{V}_{cn} = 220 \angle 120^\circ \text{ V}$$

The rectangular forms are:

$$\tilde{V}_{an} = 220 \text{ V}, \tilde{V}_{bn} = -110 - j190.52 \text{ V}, \tilde{V}_{cn} = -110 + j190.5 \text{ V}$$

The line voltages in polar form are:

$$\tilde{\mathbf{V}}_{ab} = \sqrt{3}\tilde{\mathbf{V}}_{an}\angle 30^\circ = 380\angle 30^\circ \text{ V}$$

$$\tilde{\mathbf{V}}_{bc} = 380\angle -90^\circ \text{ V}$$

$$\tilde{\mathbf{V}}_{ca} = 380\angle 150^\circ \text{ V}$$

The line voltages in rectangular form are:

$$\tilde{\mathbf{V}}_{ab} = 329 + j190 \text{ V}, \tilde{\mathbf{V}}_{bc} = -j380 \text{ V}, \tilde{\mathbf{V}}_{ca} = -329 + j190 \text{ V}$$

Problem 7.46

Solution:

Known quantities:

The phase currents, $\tilde{\mathbf{I}}_{an} = 10\angle 0^\circ$, $\tilde{\mathbf{I}}_{bn} = 12\angle 150^\circ$, $\tilde{\mathbf{I}}_{cn} = 8\angle 165^\circ$.

Find:

The current in the neutral wire.

Analysis:

The neutral current is:

$$\tilde{\mathbf{I}}_n = \tilde{\mathbf{I}}_{an} + \tilde{\mathbf{I}}_{bn} + \tilde{\mathbf{I}}_{cn} = 5\angle 0^\circ + 12\angle 150^\circ + 8\angle 165^\circ = -13.11 + j8.07 = 15.39\angle 148.4^\circ \text{ A}$$

Problem 7.47

Solution:

Known quantities:

Circuit shown in Figure P7.47, the voltage sources, $\tilde{\mathbf{V}}_R = 120\angle 0^\circ \text{ V}$, $\tilde{\mathbf{V}}_W = 120\angle 120^\circ \text{ V}$,

$$\tilde{\mathbf{V}}_B = 120\angle 240^\circ \text{ V}.$$

Find:

- The voltages, $\tilde{\mathbf{V}}_{RW}$, $\tilde{\mathbf{V}}_{WB}$, $\tilde{\mathbf{V}}_{BR}$.
- The voltages, $\tilde{\mathbf{V}}_{RW}$, $\tilde{\mathbf{V}}_{WB}$, $\tilde{\mathbf{V}}_{BR}$, using $\tilde{\mathbf{V}}_{xy} = \tilde{\mathbf{V}}_x\sqrt{3}\angle -30^\circ$.
- Compare the results obtained in a and b.

Analysis:

$$\text{a) } \tilde{\mathbf{V}}_{RW} = \tilde{\mathbf{V}}_R - \tilde{\mathbf{V}}_W = 120\angle 0^\circ - 120\angle 120^\circ = 120 + 60 - j103.92 = 207.8\angle -30^\circ \text{ V}$$

$$\tilde{\mathbf{V}}_{WB} = \tilde{\mathbf{V}}_W - \tilde{\mathbf{V}}_B = 120\angle 120^\circ - 120\angle 240^\circ$$

$$= -60 + j103.92 + 60 - j103.92 = 207.8\angle 90^\circ \text{ V}$$

$$\tilde{\mathbf{V}}_{BR} = \tilde{\mathbf{V}}_B - \tilde{\mathbf{V}}_R = 120\angle 240^\circ - 120\angle 0^\circ$$

$$= -60 - j103.92 - 120 = 207.8\angle -150^\circ \text{ V}$$

$$\text{b) } \tilde{\mathbf{V}}_{RW} = \tilde{\mathbf{V}}_R\sqrt{3}\angle -30^\circ = 120\sqrt{3}\angle -30^\circ = 207.8\angle -30^\circ \text{ V}$$

$$\tilde{\mathbf{V}}_{WB} = \tilde{\mathbf{V}}_W\sqrt{3}\angle -30^\circ = 120\angle 120^\circ\sqrt{3}\angle -30^\circ = 207.8\angle 90^\circ \text{ V}$$

$$\tilde{\mathbf{V}}_{BR} = \tilde{\mathbf{V}}_B\sqrt{3}\angle -30^\circ = 120\angle 240^\circ\sqrt{3}\angle -30^\circ = 207.8\angle 210^\circ \text{ V} = 207.8\angle -150^\circ \text{ V}$$

- The two calculations are identical.

Problem 7.48**Solution:****Known quantities:**

Circuit shown in Figure P7.48, the voltage sources, $\tilde{\mathbf{V}}_R = 110\angle 0^\circ \text{ V}$, $\tilde{\mathbf{V}}_W = 110\angle 120^\circ \text{ V}$, $\tilde{\mathbf{V}}_B = 110\angle 240^\circ \text{ V}$, and the three loads, $Z_R = 50\Omega$, $Z_W = -j20\Omega$, $Z_B = j45\Omega$.

Find:

- The current in the neutral wire.
- The real power.

Analysis:

$$\text{a) } \tilde{\mathbf{I}}_R = \frac{\tilde{\mathbf{V}}_R}{Z_R} = \frac{110\angle 0^\circ}{50} = 2.2\angle 0^\circ \text{ A}$$

$$\tilde{\mathbf{I}}_B = \frac{\tilde{\mathbf{V}}_B}{Z_B} = \frac{110\angle 240^\circ}{j45} = 2.44\angle 150^\circ \text{ A}$$

$$\tilde{\mathbf{I}}_W = \frac{\tilde{\mathbf{V}}_W}{Z_W} = \frac{110\angle 120^\circ}{-j20} = 5.5\angle 210^\circ \text{ A}$$

$$\tilde{\mathbf{I}}_N = \tilde{\mathbf{I}}_R + \tilde{\mathbf{I}}_W + \tilde{\mathbf{I}}_B = 2.2 + 5.5\angle 210^\circ + 2.44\angle 150^\circ = 4.92\angle -161.9^\circ \text{ A}$$

$$\text{b) } P = R \cdot \tilde{\mathbf{I}}_R^2 = 50 \cdot 2.2^2 = 242 \text{ W}$$

Problem 7.49**Solution:****Known quantities:**

Circuit shown in Figure P7.49, the voltage sources, $\tilde{\mathbf{V}}_R = 220\angle 0^\circ \text{ V}$, $\tilde{\mathbf{V}}_W = 220\angle 120^\circ \text{ V}$, $\tilde{\mathbf{V}}_B = 220\angle 240^\circ \text{ V}$, and the impedances, $R_W = R_B = R_R = 10\Omega$.

Find:

- The current in the neutral wire.
- The real power.

Analysis:

$$\text{a) } \tilde{\mathbf{I}}_R = \frac{\tilde{\mathbf{V}}_R}{R_R} = \frac{220\angle 0^\circ}{10} = 22\angle 0^\circ \text{ A}$$

$$\tilde{\mathbf{I}}_W = \frac{\tilde{\mathbf{V}}_W}{R_W} = \frac{220\angle 120^\circ}{10} = 22\angle 120^\circ \text{ A}$$

$$\tilde{\mathbf{I}}_B = \frac{\tilde{\mathbf{V}}_B}{R_B} = \frac{220\angle 240^\circ}{10} = 22\angle 240^\circ \text{ A}$$

Therefore, the current in the neutral wire is:

$$\tilde{\mathbf{I}}_N = \tilde{\mathbf{I}}_R + \tilde{\mathbf{I}}_W + \tilde{\mathbf{I}}_B = 0 \text{ A}$$

- The real power is:

$$P = \tilde{I}_R^2 \cdot R + \tilde{I}_W^2 \cdot R + \tilde{I}_B^2 \cdot R = 3R \cdot \tilde{I}_R^2 = 3 \frac{\tilde{V}_R^2}{R} = 3 \frac{220^2}{10} = 14.52 \text{ kW}$$

Problem 7.50

Solution:

Known quantities:

A three-phase electric oven with a phase resistance of $10 \, \Omega$, connected at $3 \times 380 \text{ V AC}$.

Find:

- The current flowing through the resistors in Y and Δ connections.
- The power of the oven in Y and Δ connections.

Analysis:

- In Y-connection:

$$\tilde{I}_{RY} = \frac{\tilde{V}_{\text{phase}}}{R} = \frac{380/\sqrt{3}}{10} = \frac{220}{10} = 22 \text{ A}$$

- In Δ -connection:

$$\tilde{I}_{RD} = \frac{\tilde{V}_{\text{line}}}{R} = \frac{380}{30} = 12.7 \text{ A}$$

- In Y-connection:

$$P = \sqrt{3} \cdot \tilde{V}_{\text{line}} \cdot \tilde{I}_{\text{line}} = \sqrt{3} \cdot \tilde{V}_{\text{line}} \cdot \tilde{I}_{RY} = \sqrt{3} \cdot 380 \cdot 22 = 14.5 \text{ kW}$$

- In Δ -connection:

$$P = \sqrt{3} \cdot \tilde{V}_{\text{line}} \cdot \tilde{I}_{\text{line}} = \sqrt{3} \cdot \tilde{V}_{\text{line}} \cdot \tilde{I}_{RD} = \sqrt{3} \cdot 380 \cdot 12.7 = 8.36 \text{ kW}$$

Problem 7.51

Solution:

Known quantities:

Apparent power of 50 kVA and supplied voltage of 380 V for a synchronous generator.

Find:

The phase currents, the active powers, and the reactive powers if:

- The power factor is 0.85.
- The power factor is 1.

Analysis:

- For the power factor of 0.85:

$$S = \sqrt{3} \tilde{V} \tilde{I} \Rightarrow \tilde{I} = \frac{S}{\sqrt{3} \tilde{V}} = \frac{50000}{\sqrt{3} \cdot 380} = 76 \text{ A}$$

$$P = S \cdot \cos(\theta) = 50000 \cdot 0.85 = 42.5 \text{ kW}$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{50^2 - 42.5^2} = 26.3 \text{ kVAR}$$

- For the power factor of 1.00:

$$S = P \Rightarrow \tilde{I} = 76 \text{ A}$$

$$P = S \cdot \cos(\theta) = 50000 \cdot 1.00 = 50.0 \text{ kW}$$

$$Q = \sqrt{S^2 - P^2} = 0$$

Problem 7.52**Solution:****Known quantities:**

Circuit shown in Figure P7.52, the voltage sources, $v_{s1}(t) = 170\cos(\omega t)\text{V}$, $v_{s2}(t) = 170\cos(\omega t + 120^\circ)\text{V}$, $v_{s3}(t) = 170\cos(\omega t - 120^\circ)\text{V}$, and the impedances, $Z_1 = 0.5\angle 20^\circ\Omega$, $Z_2 = 0.35\angle 0^\circ\Omega$, $Z_3 = 1.7\angle -90^\circ\Omega$, the frequency, $f = 60\text{Hz}$.

Find:

The current through Z_1 , using:

- Loop/mesh analysis.
- Node analysis.
- Superposition.

Analysis:

- Applying KVL in the upper mesh:

$$\tilde{V}_{s2} - \tilde{V}_{s1} + \tilde{I}_1 Z_1 + (\tilde{I}_1 - \tilde{I}_2) Z_2 = 0 \Rightarrow \tilde{I}_1 (Z_1 + Z_2) + \tilde{I}_2 (-Z_2) = \tilde{V}_{s1} - \tilde{V}_{s2}$$

Applying KVL in the lower mesh:

$$\tilde{V}_{s3} - \tilde{V}_{s2} + (\tilde{I}_2 - \tilde{I}_1) Z_2 + \tilde{I}_2 Z_3 = 0 \Rightarrow \tilde{I}_1 (-Z_2) + \tilde{I}_2 (Z_2 + Z_3) = \tilde{V}_{s2} - \tilde{V}_{s3}$$

For each mesh equation:

$$\tilde{V}_{s1} - \tilde{V}_{s2} = 170\angle 0^\circ - 170\angle 120^\circ = 170 - (-85 + j147) = 294\angle -30^\circ\text{V}$$

$$\tilde{V}_{s2} - \tilde{V}_{s3} = 170\angle 120^\circ - 170\angle -120^\circ = (-85 + j147) - (-85 - j147) = 294\angle 90^\circ\text{V}$$

$$Z_1 + Z_2 = 0.47 + j0.171 + 0.35 = 0.838\angle 11.8^\circ\Omega$$

$$Z_2 + Z_3 = 0.35 - j1.7 = 1.74\angle -78.4^\circ\Omega$$

Therefore, the current through Z_1 is:

$$\begin{aligned} \tilde{I}_1 &= \frac{\begin{vmatrix} \tilde{V}_{s1} - \tilde{V}_{s2} & -Z_2 \\ \tilde{V}_{s2} - \tilde{V}_{s3} & Z_2 + Z_3 \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_3 \end{vmatrix}} = \frac{\begin{vmatrix} 294\angle -30^\circ & -0.35\angle 0^\circ \\ 294\angle 90^\circ & 1.74\angle -78.4^\circ \end{vmatrix}}{\begin{vmatrix} 0.838\angle 11.8^\circ & -0.35\angle 0^\circ \\ -0.35\angle 0^\circ & 1.74\angle -78.4^\circ \end{vmatrix}} \\ &= \frac{512\angle -108.4^\circ + 103\angle 90^\circ}{1.46\angle -66.6^\circ - 0.123\angle 0^\circ} = \frac{415.9\angle -112.9^\circ}{1.416\angle -71.2^\circ} = 293\angle -41.8^\circ\text{A} \end{aligned}$$

- Choose the ground at the center of the three voltage source, and let a be the center of the three loads.

The voltage between the node a and the ground is unknown.

Applying KCL at the node a :

$$\frac{\tilde{V}_a - \tilde{V}_{s1}}{Z_1} + \frac{\tilde{V}_a - \tilde{V}_{s2}}{Z_2} + \frac{\tilde{V}_a - \tilde{V}_{s3}}{Z_3} = 0$$

Rearranging the equation:

$$\begin{aligned} \tilde{V}_a &= \frac{\frac{\tilde{V}_{s1}}{Z_1} + \frac{\tilde{V}_{s2}}{Z_2} + \frac{\tilde{V}_{s3}}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{\frac{170\angle 0^\circ}{0.5\angle 20^\circ} + \frac{170\angle 120^\circ}{0.35\angle 10^\circ} + \frac{170\angle -120^\circ}{1.7\angle -90^\circ}}{\frac{1}{0.5\angle 20^\circ} + \frac{1}{0.35\angle 10^\circ} + \frac{1}{1.7\angle -90^\circ}} \\ &= \frac{340\angle -20^\circ + 486\angle 120^\circ + 100\angle 330^\circ}{2\angle -20^\circ + 2.86\angle 0^\circ + 0.59\angle 90^\circ} = \frac{303\angle 57.3^\circ}{4.74\angle -1.2^\circ} = 63.9\angle 58.5^\circ\text{V} \end{aligned}$$

Applying KVL, the current through Z_1 is:

$$\tilde{\mathbf{V}}_a - \tilde{\mathbf{V}}_{s1} - \tilde{\mathbf{I}}_1 Z_1 = 0 \Rightarrow \tilde{\mathbf{I}}_1 = \frac{\tilde{\mathbf{V}}_{s1} - \tilde{\mathbf{V}}_a}{Z_1} = \frac{170\angle 0^\circ - 63.9\angle 58.5^\circ}{0.5\angle 20^\circ} = 293\angle -41.8^\circ \text{ A}$$

c) Superposition is not the method of choice for its great complexity.

Problem 7.53

Solution:

Known quantities:

Circuit shown in Figure P7.13, the voltage sources, $v_1(t) = 170\cos(\omega t)\text{V}$, $v_2(t) = 170\cos(\omega t + 120^\circ)\text{V}$, $v_3(t) = 170\cos(\omega t - 120^\circ)\text{V}$, and the impedances, $R = 100\Omega$, $C = 0.47\mu\text{F}$, $L = 100\text{mH}$, the frequency, $f = 400\text{Hz}$.

Find:

The current through R .

Analysis:

For each impedance:

$$Z_1 = 100\Omega = 100\angle 0^\circ\Omega$$

$$Z_2 = -j\frac{1}{\omega C} = -j\frac{1}{2\pi f \cdot C} = -j846.6\Omega = 846.6\angle -90^\circ\Omega$$

$$Z_3 = j\omega L = j2\pi f \cdot L = j251.3\Omega = 251.3\angle 90^\circ\Omega$$

Applying KVL in the upper mesh:

$$\tilde{\mathbf{V}}_2 - \tilde{\mathbf{V}}_1 + \tilde{\mathbf{I}}_1 Z_1 + (\tilde{\mathbf{I}}_1 - \tilde{\mathbf{I}}_2) Z_2 = 0 \Rightarrow \tilde{\mathbf{I}}_1 (Z_1 + Z_2) + \tilde{\mathbf{I}}_2 (-Z_2) = \tilde{\mathbf{V}}_1 - \tilde{\mathbf{V}}_2$$

Applying KVL in the lower mesh:

$$\tilde{\mathbf{V}}_3 - \tilde{\mathbf{V}}_2 + (\tilde{\mathbf{I}}_2 - \tilde{\mathbf{I}}_1) Z_2 + \tilde{\mathbf{I}}_2 Z_3 = 0 \Rightarrow \tilde{\mathbf{I}}_1 (-Z_2) + \tilde{\mathbf{I}}_2 (Z_2 + Z_3) = \tilde{\mathbf{V}}_2 - \tilde{\mathbf{V}}_3$$

For each mesh equation:

$$\tilde{\mathbf{V}}_1 - \tilde{\mathbf{V}}_2 = 170\angle 0^\circ - 170\angle 120^\circ = 170 - (-85 + j147) = 294\angle -30^\circ \text{ V}$$

$$\tilde{\mathbf{V}}_2 - \tilde{\mathbf{V}}_3 = 170\angle 120^\circ - 170\angle -120^\circ = (-85 + j147) - (-85 - j147) = 294\angle 90^\circ \text{ V}$$

$$Z_1 + Z_2 = 100 - j846.6 = 852.5\angle -83.3^\circ\Omega$$

$$Z_2 + Z_3 = -j846.6 + j251.3 = 595.3\angle -90^\circ\Omega$$

Therefore, the current through R is:

$$\begin{aligned} \tilde{\mathbf{I}}_1 &= \frac{\begin{vmatrix} \tilde{\mathbf{V}}_1 - \tilde{\mathbf{V}}_2 & -Z_2 \\ \tilde{\mathbf{V}}_2 - \tilde{\mathbf{V}}_3 & Z_2 + Z_3 \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_3 \end{vmatrix}} = \frac{\begin{vmatrix} 294\angle -30^\circ & -846.6\angle -90^\circ \\ 294\angle 90^\circ & 595.3\angle -90^\circ \end{vmatrix}}{\begin{vmatrix} 852.5\angle -83.3^\circ & -846.6\angle -90^\circ \\ -846.6\angle -90^\circ & 595.3\angle -90^\circ \end{vmatrix}} \\ &= \frac{175.0 \cdot 10^3 \angle -120^\circ + 248.9 \cdot 10^3 \angle 0^\circ}{507.5 \cdot 10^3 \angle -173.3^\circ - 716.7 \cdot 10^3 \angle -180^\circ} = \frac{221.4\angle -43.2^\circ}{220.8\angle -15.6^\circ} \\ &= 1.003\angle -27.6^\circ \text{ A} \end{aligned}$$

Problem 7.54**Solution:****Known quantities:**

Circuit shown in Figure P7.54, the voltage sources, $v_{s1}(t) = 170 \cos(\omega t) \text{ V}$, $v_{s2}(t) = 170 \cos(\omega t + 120^\circ) \text{ V}$, $v_{s3}(t) = 170 \cos(\omega t - 120^\circ) \text{ V}$, and the impedances, $Z_1 = 3 \angle 0^\circ \Omega$, $Z_2 = 7 \angle 90^\circ \Omega$, $Z_3 = 0 - j11 \Omega$, the frequency, $f = 60 \text{ Hz}$.

Find:

The currents, $\tilde{\mathbf{I}}_1$, $\tilde{\mathbf{I}}_2$, $\tilde{\mathbf{I}}_3$.

Analysis:

Applying KVL in the upper mesh:

$$\tilde{\mathbf{V}}_{s2} - \tilde{\mathbf{V}}_{s1} + \tilde{\mathbf{I}}_1 Z_1 = 0$$

$$\tilde{\mathbf{I}}_1 = \frac{\tilde{\mathbf{V}}_{s1} - \tilde{\mathbf{V}}_{s2}}{Z_1} = \frac{170 \angle 0^\circ - 170 \angle 120^\circ}{3 \angle 0^\circ} = \frac{294 \angle -30^\circ}{3 \angle 0^\circ} = 98.1 \angle -30^\circ \text{ A}$$

Applying KVL in the right-side mesh:

$$\tilde{\mathbf{V}}_{s3} - \tilde{\mathbf{V}}_{s1} + \tilde{\mathbf{I}}_2 Z_2 = 0$$

$$\tilde{\mathbf{I}}_2 = \frac{\tilde{\mathbf{V}}_{s1} - \tilde{\mathbf{V}}_{s3}}{Z_2} = \frac{170 \angle 0^\circ - 170 \angle -120^\circ}{7 \angle 90^\circ} = \frac{294 \angle 30^\circ}{7 \angle 90^\circ} = 42.1 \angle -60^\circ \text{ A}$$

Applying KVL in the lower mesh:

$$\tilde{\mathbf{V}}_{s3} - \tilde{\mathbf{V}}_{s2} + \tilde{\mathbf{I}}_3 Z_3 = 0$$

$$\tilde{\mathbf{I}}_3 = \frac{\tilde{\mathbf{V}}_{s2} - \tilde{\mathbf{V}}_{s3}}{Z_3} = \frac{170 \angle 120^\circ - 170 \angle -120^\circ}{11 \angle -90^\circ} = \frac{294 \angle 90^\circ}{11 \angle -90^\circ} = 26.8 \angle -180^\circ \text{ A}$$

Problem 7.55**Solution:****Known quantities:**

Circuit shown in Figure P7.55, the voltage sources, $\tilde{\mathbf{V}}_{RW} = 416 \angle -30^\circ \text{ V}$, $\tilde{\mathbf{V}}_{WB} = 416 \angle 210^\circ \text{ V}$, $\tilde{\mathbf{V}}_{BR} = 416 \angle 90^\circ \text{ V}$, and the impedances, $R_1 = R_2 = R_3 = 40 \Omega$, $L_1 = L_2 = L_3 = 5 \text{ mH}$. The frequency of each of the sources, $f = 60 \text{ Hz}$.

Find:

The currents, $\tilde{\mathbf{I}}_W$, $\tilde{\mathbf{I}}_B$, $\tilde{\mathbf{I}}_R$, $\tilde{\mathbf{I}}_N$.

Analysis:

The line voltages are:

$$\tilde{\mathbf{V}}_{RW} = 416 \angle -30^\circ \text{ V}, \tilde{\mathbf{V}}_{WB} = 416 \angle 210^\circ \text{ V}, \tilde{\mathbf{V}}_{BR} = 416 \angle 90^\circ \text{ V}$$

The phase voltages are:

$$\tilde{\mathbf{V}}_R = 240 \angle 0^\circ \text{ V}, \tilde{\mathbf{V}}_W = 240 \angle 120^\circ \text{ V}, \tilde{\mathbf{V}}_B = 240 \angle -120^\circ \text{ V}$$

The currents are:

$$\begin{aligned}\tilde{\mathbf{I}}_R &= \frac{\tilde{\mathbf{V}}_R}{Z_1} = \frac{\tilde{\mathbf{V}}_R}{R_1 + j\omega L_1} = \frac{\tilde{\mathbf{V}}_R}{R_1 + j2\pi L_1} = \frac{240\angle 0^\circ}{40 + j377 \cdot 5 \cdot 10^{-3}} = 6\angle -2.7^\circ \text{ A} \\ \tilde{\mathbf{I}}_W &= \frac{\tilde{\mathbf{V}}_W}{Z_2} = \frac{240\angle 120^\circ}{40 + j377 \cdot 5 \cdot 10^{-3}} = 6\angle 117.3^\circ \text{ A} \\ \tilde{\mathbf{I}}_B &= \frac{\tilde{\mathbf{V}}_B}{Z_3} = \frac{240\angle -120^\circ}{40 + j377 \cdot 5 \cdot 10^{-3}} = 6\angle -122.7^\circ \text{ A} \\ \tilde{\mathbf{I}}_N &= \tilde{\mathbf{I}}_R + \tilde{\mathbf{I}}_W + \tilde{\mathbf{I}}_B = 0\end{aligned}$$

Problem 7.56**Solution:****Known quantities:**

Circuit shown in Figure P7.54, the voltage sources, $\tilde{\mathbf{V}}_{RW} = 416\angle -30^\circ \text{ V}$, $\tilde{\mathbf{V}}_{WB} = 416\angle 210^\circ \text{ V}$, $\tilde{\mathbf{V}}_{BR} = 416\angle 90^\circ \text{ V}$, and the impedances, $R_1 = R_2 = R_3 = 40\Omega$, $L_1 = L_2 = L_3 = 5 \text{ mH}$. The frequency of each of the sources, $f = 60 \text{ Hz}$.

Find:

- The power delivered to the motor.
- The motor's power factor.
- The reason for which it is common in industrial practice not to connect the ground lead to motors of this type.

Analysis:

- The power delivered to the motor is:

$$P = 3\tilde{V}_R\tilde{I}_R\cos(\theta) = 3 \cdot 240 \cdot 6 \cdot \cos(-2.7^\circ) = 4315 \text{ W}$$

- The motor's power factor is:

$$pf = \cos(-2.7^\circ) = 0.9988 \quad \text{Lagging}$$

- The circuit is balanced and no neutral current flows; thus the connection is unnecessary.

Problem 7.57**Solution:****Known quantities:**

A three-phase induction motor designed not only for Y connection operation in general but also for Δ connection at the nominal Y voltage for a short time operation.

Find:

The ratio between the powers.

Analysis:

The power for Y connection operation is:

$$P_Y = 3 \cdot \tilde{\mathbf{V}}_{\text{phase}_{MOTOR}} \cdot \tilde{\mathbf{I}}_{\text{line}_Y} = 3 \cdot \frac{\tilde{\mathbf{V}}_{\text{line}}}{\sqrt{3}} \cdot \tilde{\mathbf{I}}_{\text{line}_Y} = \sqrt{3} \cdot \tilde{\mathbf{V}}_{\text{line}} \cdot \tilde{\mathbf{I}}_{\text{line}_Y}$$

The power for Δ connection operation is:

$$\begin{aligned}
 P_{\Delta} &= 3 \cdot \tilde{V}_{line} \cdot \tilde{I}_{phase_{MOTOR}} = 3 \cdot \tilde{V}_{line} \cdot \frac{\tilde{V}_{line}}{Z_{phase_{MOTOR}}} \\
 &= 3 \cdot \tilde{V}_{line} \cdot \sqrt{3} \cdot \frac{\tilde{V}_{phase_{MOTOR}}}{Z_{phase_{MOTOR}}} = 3 \cdot \tilde{V}_{line} \cdot \sqrt{3} \cdot \tilde{I}_{line_Y} = 3 \cdot P_Y
 \end{aligned}$$

Therefore, the ratio between the powers is:

$$Ratio = \frac{P_{\Delta}}{P_Y} = 3$$

Problem 7.58

Solution:

Known quantities:

Circuit shown in Figure P7.58, the voltage sources, $\tilde{V}_R = 120 \angle 0^\circ \text{ V}$, $\tilde{V}_W = 120 \angle 120^\circ \text{ V}$, $\tilde{V}_B = 120 \angle 240^\circ \text{ V}$, and the impedances, $R_1 = R_2 = R_3 = 5 \Omega$, $X_{L_1} = X_{L_2} = X_{L_3} = 6 \Omega$ of the motor.

Find:

- The total power supplied to the motor.
- The power converted to mechanical energy if the motor is 80% efficient.
- The power factor.
- The risk for the company to face a power factor penalty if all the motors in the factory are similar to this one.

Analysis:

- By virtue of the symmetry of the circuit, we can solve the problem by considering just one phase.

The current $\tilde{I}_R$ is:

$$\tilde{I}_R = \frac{\tilde{V}_R}{R_1 + jX_{L_1}} = \frac{120 \angle 0^\circ}{5 + j6} = 15.36 \angle -50.19^\circ \text{ A}$$

The total power supplied to the motor is:

$$P = 3 \tilde{V}_R \tilde{I}_R \cos(\theta) = 3 \cdot 120 \cdot 15.36 \cdot \cos(50.19^\circ) = 3541.3 \text{ W}$$

- The mechanical power is:

$$P_m = 0.8P = 2832.23 \text{ W}$$

- The power factor is:

$$pf = \cos(\theta) = 0.64$$

- The company will face a 25% penalty.

Problem 7.59

Solution:

Known quantities:

The voltage source at 220 V rms of a residential four-wire system supplying power to the single-phase appliances; ten 75-W bulbs on the 1st phase, one 750-W vacuum cleaner with $pf = 0.87$ on the 2nd phase, ten 40-W lamps with $pf = 0.64$ on the 3rd phase.

Find:

- The current in the neutral wire.
- The real, reactive, and apparent power for each phase.

Analysis:

a) The current in the neutral wire is:

$$\tilde{\mathbf{I}}_A = \frac{P_A}{\tilde{\mathbf{V}}_{phase}} = \frac{\sqrt{3} \cdot P_A}{\tilde{\mathbf{V}}_{line}} = \frac{\sqrt{3} \cdot 10 \cdot 75}{380} = 3.4 \angle 0^\circ \text{ A}$$

$$\tilde{\mathbf{I}}_B = \frac{P_B}{\tilde{\mathbf{V}}_{phase}} = \frac{\sqrt{3} \cdot P_B}{\tilde{\mathbf{V}}_{line} \cdot \cos(\theta)} = \frac{\sqrt{3} \cdot 750}{380 \cdot 0.87} = 3.92 \angle -150^\circ \text{ A} = -3.4 - j1.95 \text{ A}$$

$$\tilde{\mathbf{I}}_C = \frac{P_C}{\tilde{\mathbf{V}}_{phase}} = \frac{\sqrt{3} \cdot P_C}{\tilde{\mathbf{V}}_{line} \cdot \cos(\theta)} = \frac{\sqrt{3} \cdot 10 \cdot 40}{380 \cdot 0.64} = 2.8 \angle -290^\circ \text{ A} = 0.95 + j2.63 \text{ A}$$

$$\tilde{\mathbf{I}}_N = \tilde{\mathbf{I}}_A + \tilde{\mathbf{I}}_B + \tilde{\mathbf{I}}_C = 0.95 - j0.68 \text{ A} \Rightarrow \tilde{I}_N = 1.16 \text{ A}$$

b) The real, reactive, and apparent powers for the 1st phase are:

$$S_A = P_A = 750 \text{ W}, Q_A = 0$$

The real, reactive, and apparent powers for the 2nd phase are:

$$P_B = 750 \text{ W}, S_B = \frac{P_B}{\cos(\theta_B)} = \frac{750}{0.87} = 862 \text{ VA}, Q_B = S_B \cdot \sin(\theta_B) = 431 \text{ VAR}$$

The real, reactive, and apparent powers for the 3rd phase are:

$$P_C = 400 \text{ W}, S_C = \frac{P_C}{\cos(\theta_C)} = \frac{400}{0.64} = 625 \text{ VA}, Q_C = S_C \cdot \sin(\theta_C) = 478 \text{ VAR}$$

Chapter 8 Instructor Notes

Chapter 8 introduces the notion of integrated circuit electronics through the most common building block of electronic instrumentation, the operational amplifier. This is, in practice, the area of modern electronics that is most likely to be encountered by a practicing non-electrical engineer. Thus, the aim of the chapter is to present a fairly complete functional description of the operational amplifier, including a discussion of the principal limitations of the op-amp and of the effects of these limitations on the performance of op-amp circuits employed in measuring instruments and in signal conditioning circuits. The material presented in this chapter lends itself particularly well to a series of laboratory experiments (see for example¹), which can be tied to the lecture material quite readily.

After a brief introduction, in which ideal amplifier characteristics are discussed, open- and closed- loop models of the op-amp are presented in section 8.2; the use of these models is illustrated by application of the basic circuit analysis methods of Chapters 2 and 3. Thus, the Instructor who deems it appropriate can cover the first two sections in conjunction with the circuit analysis material. A brief, intuitive discussion of feedback is also presented to explain some of the properties of the op-amp in a closed-loop configuration. The closed-loop models include a fairly detailed introduction to the inverting, non-inverting and differential amplifier circuits; however, the ultimate aim of this section is to ensure that the student is capable of recognizing each of these three configurations, so as to be able to quickly determine the closed loop gain of practical amplifier circuits, summarized in Table 8.1 (p. 402). The section is sprinkled with various practical examples, introducing practical op-amp circuits that are actually used in practical instruments, including the summing amplifier (p. 938), the voltage follower (p. 400), a differential amplifier (*Focus on Measurements: Electrocardiogram (EKG) Amplifier*, pp. 402-404), the instrumentation amplifier (pp. 404-405), the level shifter (p.406), and a transducer calibration circuit (*Focus on Measurements: Sensor calibration circuit*, pp. 407-409). Two features, new in the third edition, will assist the instructor in introducing practical design considerations: the box *Practical Op-Amp Design Considerations* (p. 410) illustrates some standard design procedures, providing an introduction to a later section on op-amp limitations; the box *Focus on Methodology: Using Op-amp Data Sheets* (pp. 410-412) illustrates the use of device data sheets for two common op-amps. The use of Device Data Sheets is introduced in this chapter for the first time. In a survey course, the first two sections might be sufficient to introduce the device.

Section 8.3 presents the idea of active filters; this material can also be covered quite effectively together with the frequency response material of Chapter 6 to reinforce these concepts. Section 8.4 discusses integrator and differentiator circuits, and presents a practical application of the op-amp integrator in the charge amplifier (*Focus on Measurements: Charge Amplifiers*, pp. 420-421). The latter example is of particular relevance to the non-electrical engineer, since charge amplifiers are used to amplify the output of piezo-electric transducers in the measurement of strain, force, torque and pressure (for additional material on piezo-electric transducers, see, for example²). A brief section (8.5) is also provided on analog computers, since these devices are still used in control system design and evaluation.

Coverage of sections 8.4 and 8.5 is not required to complete section 8.6.

The last section of the chapter, 8.6, is devoted to a discussion of the principal performance limits of the operational amplifier. Since the student will not be prepared to fully comprehend the reason for the saturation, limited bandwidth, limited slew rate, and other shortcomings of practical op-amps, the section focuses on describing the effects of these limitations, and on identifying the relevant parameters on the data sheets of typical op-amps. Thus, the student is trained to recognize these limits, and to include them in the design of practical amplifier circuits. Since some of these limitations are critical even in low frequency applications, it is easy (and extremely useful) to supplement this material with laboratory exercises. The box *Focus on Methodology: Using Op-amp Data Sheets – Comparison of LM 741 and LMC 6061*, pp. 441-446) further reinforces the value of data sheets in realizing viable designs.

The homework problems present a variety of interesting problems at varying levels of difficulty; many of these problems extend the ideas presented in the text, and present practical extensions of the circuits discussed in the examples. In a one-semester course, Chapter 8 can serve as a very effective capstone to a first course in circuit analysis and electronics by stimulating curiosity towards integrated circuit electronics,

¹ Rizzoni, G., *A Practical Introduction to Electronic Instrumentation*, 3rd Ed. Kendall-Hunt, 1998

² Doebelin E. O., *Measurement Systems*, McGraw-Hill, Fourth Edition, 1987.

and by motivating the student to pursue further study in electronics or instrumentation. In many respects this chapter is the centerpiece of the book.

Learning Objectives

1. Understand the properties of ideal amplifiers, and the concepts of gain, input impedance, and output impedance. [Section 1.](#)
2. Understand the difference between open-loop and closed-loop op-amp configuration, and compute the gain (or complete the design of) simple inverting, non-inverting, summing and differential amplifiers using ideal op-amp analysis. Analyze more advanced op-amp circuits, using ideal op-amp analysis, and identify important performance parameters in op-amp data sheets. [Section 2.](#)
3. Analyze and design simple active filters. Analyze and design ideal integrator and differentiator circuits. [Sections 3 and 4.](#)
4. Understand the structure and behavior of analog computers, and design analog computer circuits to solve simple differential equations. [Section 5.](#)
5. Understand the principal physical limitations of an op-amp. [Section 6.](#)

Section 8.1: Ideal Amplifiers

Problem 8.1

Solution:

Known quantities:

For the circuit shown in Figure P8.1:

$$G = \frac{P_o}{P_s} \quad R_s = 0.6 \text{ k}\Omega \quad R_L = 0.6 \text{ k}\Omega$$

$$R_{i1} = 3 \text{ k}\Omega \quad R_{i2} = 3 \text{ k}\Omega \quad R_{o1} = 2 \text{ k}\Omega \quad R_{o2} = 2 \text{ k}\Omega$$

$$A_{vo1} = 100 \quad G_{M2} = 350 \text{ m}\Omega^{-1} \quad V_{i2} = V_{i2} \quad V_{i1} = V_{i1}$$

Find:

The power gain $G = P_o/P_s$, in dB.

Analysis:

Starting from the last stage and going backward, we get

$$V_o = G_{M2} V_{i2} \frac{R_{o2} R_L}{R_{o2} + R_L} = 161.5 V_{i2} \Rightarrow P_o = V_o I_o = \frac{V_o^2}{R_L} = 43.49 V_{i2}^2$$

$$V_{i2} = A_{v01} V_{i1} \frac{R_{i2}}{R_{i2} + R_{o1}} = 60 V_{i1} \Rightarrow P_o = 1.566 \cdot 10^5 V_{i1}^2$$

$$V_{i1} = V_s \frac{R_{i1}}{R_{i1} + R_s} = 0.833 V_s \Rightarrow P_o = 1.0875 \cdot 10^5 V_s^2$$

$$I_s = V_s \frac{1}{R_{i1} + R_s} = 2.778 \cdot 10^{-4} V_s \Rightarrow P_s = V_s I_s = 2.778 \cdot 10^{-4} V_s^2$$

$$G = \frac{P_o}{P_s} = \frac{1.0875 \cdot 10^5}{2.778 \cdot 10^{-4}} = 3.915 \cdot 10^8 = 20 \text{ dB } \text{Log}_{10}[3.915 \cdot 10^8] = 171.85 \text{ dB}$$

Problem 8.2

Solution:

Known quantities:

The temperature sensor shown in Figure P8.2 produces a no load (i.e., sensor current = 0) voltage:

$$v_s = V_{so} \cos \omega t \quad R_s = 400 \Omega \quad V_{so} = 500 \text{ mV} \quad \omega = 6.28 \text{ k} \frac{\text{rad}}{\text{s}}$$

The temperature is monitored on a display (the load) with a vertical line of light emitting diodes. Normal conditions are indicated when a string of the bottommost diodes 2 cm in length are on. This requires that a voltage be supplied to the display input terminals where:

$$R_L = 12 \text{ k}\Omega \quad v_o = V_o \cos \omega t \quad V_o = 6 \text{ V}.$$

The signal from the sensor must therefore be amplified. Therefore, a voltage amplifier is connected between the sensor and CRT with: $R_i = 2 \text{ k}\Omega$ $R_o = 3 \text{ k}\Omega$.

Find:

The required no load gain of the amplifier.

Analysis:

The overall loaded voltage gain, using the amplitudes of the sensor voltage and the specified CRT voltage must be:

$$A_v = \frac{V_o}{V_{so}} = \frac{6 \text{ V}}{500 \text{ mV}} = 12$$

An expression for the overall voltage gain can also be obtained using two voltage divider relationships:

$$V_o = A_{vo} V_{io} \frac{R_L}{R_o + R_L} = A_{vo} \left[V_{so} \frac{R_i}{R_i + R_s} \right] \frac{R_L}{R_o + R_L}$$

$$A_{vo} = \frac{V_o}{V_{so}} = \frac{A_v [R_s + R_i] [R_o + R_L]}{R_i R_L} = \frac{[12][0.4 + 2][3 + 12]}{[2][12]} = 18$$

The loss in gain due to the two voltage divisions or "loading" is characteristic of all practical amplifiers. Ideally, there is no reduction in gain due to loading. This would require an ideal signal source with a source resistance equal to zero and a load resistance equal to infinity.

Problem 8.3**Solution:****Known quantities:**

For the circuit shown in Figure P8.3:

$$G = \frac{P_o}{P_i} \quad P_i = \frac{v_{I1}^2}{R_{i1}} \quad R_s = 0.7 \text{ k}\Omega \quad R_L = 16 \Omega$$

$$R_{i1} = 1.1 \text{ k}\Omega \quad R_{i2} = 19 \text{ k}\Omega \quad R_{o1} = 2.9 \text{ k}\Omega \quad R_{o2} = 22 \Omega$$

$$A_{vo1} = 65 \quad G_{m2} = 130 \text{ mS}$$

Find:The power gain $G = P_o/P_i$, in dB.**Analysis:**

$$OL: v_o = G_{m2} v_{I2} \frac{R_{o2} R_L}{R_{o2} + R_L} \Rightarrow 1.204 \cdot v_{I2}$$

$$VD: v_o = \left[A_{vo1} v_{I1} \frac{R_{i2}}{R_{o1} + R_{i2}} \right] [1.204] \Rightarrow \frac{v_o}{v_{I1}} = \frac{[65][19]}{2.9 + 19} [1.204] = 67.91$$

$$G = \frac{\frac{v_o^2}{R_L}}{\frac{v_{I1}^2}{R_{i1}}} = \left[\frac{v_o}{v_{I1}} \right]^2 \frac{R_{i1}}{R_L} = [67.91]^2 \frac{1100}{16} = 317.1 \cdot 10^3 = 20 \text{ dB} \text{ Log}_{10}[317.1 \cdot 10^3] = 110.02 \text{ dB}$$

Problem 8.4**Solution:****Known quantities:**

For the circuit shown in Figure P8.4:

$$R_s = 0.3 \text{ k}\Omega \quad R_L = 2 \text{ k}\Omega \quad R_{i1} = R_{i2} = 7.7 \text{ k}\Omega \quad R_{o1} = R_{o2} = 1.3 \text{ k}\Omega$$

$$A_{vo1} = A_{vo2} = 17 \quad \frac{v_o}{v_{I1}} = 149.9$$

Find:

- The power gain in, dB.
- The overall voltage gain v_o/v_s .

Analysis:

a)

$$G = \frac{P_o}{P_i} = \frac{\frac{v_o^2}{R_L}}{\frac{v_{I1}^2}{R_{i1}}} = \left[\frac{v_o}{v_{I1}} \right]^2 \frac{R_{i1}}{R_L} = [149.9]^2 \frac{7.7}{2} = 86509 = 20 \text{ Log}_{10}[86509] = 98.74 \text{ dB}$$

- b) The input voltage of the first stage can be determined in terms of the source voltage using voltage division:

$$A_v = \frac{v_O}{v_S} = \frac{v_{O1}}{v_{I1}} \frac{v_{I1}}{v_S} = \frac{v_O}{v_{I1}} \frac{v_S \frac{R_{i1}}{R_{i1} + R_S}}{v_S} = 149.9 \frac{7.7}{0.3 + 7.7} = 144.3 = 20 \log_{10}[144.3] = 43.18 \text{ dB}$$

Problem 8.5

Solution:

Known quantities:

Figure P8.5.

Find:

What approximations are usually made about the voltages and currents shown for the ideal operational amplifier [op-amp] model.

Analysis:

$$i_P \approx 0 \quad i_N \approx 0 \quad v_D \approx 0.$$

Problem 8.6

Solution:

Known quantities:

Figure P8.6.

Find:

What approximations are usually made about the circuit components and parameters shown for the ideal operational amplifier [op-amp] model.

Analysis:

$$r_i \approx \infty \quad \mu \approx \infty \quad r_o \approx 0.$$

Section 8.2: The Operational Amplifier

Problem 8.7

Solution:

Known quantities:

The circuit shown in Figure P8.7.

Find:

The resistor R_s that will accomplish the nominal gain requirement, and state what the maximum and minimum values of R_s can be. Will a *standard* 5 percent tolerance resistor be adequate to satisfy this requirement?

Analysis:

For a non-inverting amplifier the voltage gain is given by:

$$A_v = \frac{v_{out}}{v_{in}} = \frac{R_f + R_s}{R_s} = \frac{R_f}{R_s} + 1 \quad ; \quad A_{vnom} = 16 = \frac{R_f}{R_s} + 1 \Rightarrow R_s = \frac{R_f}{16-1} = 1\text{ k}\Omega$$

To find the maximum and minimum R_s we note that $R_s \propto \frac{1}{A_v}$, so to find the maximum R_s we consider

$$\text{the minimum } A_v: R_{s\max} = \frac{15\text{ k}\Omega}{16(1-0.02)-1} = 1.02\text{ k}\Omega.$$

$$\text{Conversely, to find the minimum } R_s \text{ we consider the maximum } A_v: R_{s\min} = \frac{15\text{ k}\Omega}{16(1+0.02)-1} = 980\Omega.$$

Since a standard 5% tolerance 1-k Ω resistor has resistance $950 < R < 1050$, a standard resistor will not suffice in this application.

Problem 8.8

Solution:

Known quantities:

The values of the two 10 percent tolerance resistors used in an inverting amplifier:

$$R_F = 33\text{ k}\Omega \quad ; \quad R_S = 1.2\text{ k}\Omega.$$

Find:

- The nominal gain of the amplifier.
- The maximum value of $|A_v|$.
- The minimum value of $|A_v|$.

Analysis:

$$\text{a. The gain of the inverting amplifier is: } A_v = -\frac{R_f}{R_s} = \frac{-33}{1.2} = -27.5.$$

- First we note that the gain of the amplifier is proportional to R_f and inversely proportional to R_s . This tells us that to find the maximum gain of the amplifier we consider the maximum R_f and the minimum R_s .

$$|A_v|_{\max} = \frac{R_{f\max}}{R_{s\min}} = \frac{33 + 0.1(33)}{1.2 - 0.1(1.2)} = 33.6.$$

c. To find $|A_v|_{\min}$ we consider the opposite case: $|A_v|_{\min} = \frac{R_{f\min}}{R_{S\max}} = \frac{33 - 0.1(33)}{1.2 + 0.1(1.2)} = 22.5$.

Problem 8.9

Solution:

Known quantities:

For the circuit shown in Figure P8.9, let

$$v_1(t) = 10 + 10^{-3} \sin(\omega t) \text{ V}, \quad R_F = 10 \text{ k}\Omega, \quad V_{batt} = 20 \text{ V}.$$

Find:

- The value of R_S such that no DC voltage appears at the output.
- The corresponding value of $v_{out}(t)$.

Analysis:

- The circuit may be modeled as shown:

Applying the principle of superposition:

For the 20-V source:

$$v_o|_{20} = \frac{-10 \text{ k}\Omega}{R_S} (20)$$

For the 10-V source:

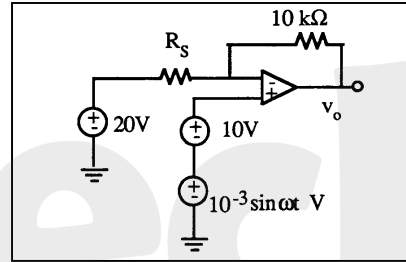
$$v_o|_{10} = \left(\frac{10 \text{ k}\Omega}{R_S} + 1 \right) (10)$$

The total DC output is: $v_o|_{DC} = v_o|_{20} + v_o|_{10} = -\frac{10,000}{R_S} (20) + \left(\frac{10,000}{R_S} + 1 \right) (10) = 0$

Solving for R_S : $\frac{10,000}{R_S} (10 - 20) = -10 \Rightarrow R_S = 10 \text{ k}\Omega$.

- Since we have already determined R_S such that the DC component of the output will be zero, we can simply treat the amplifier as if the AC source were the only source present. Therefore,

$$v_o(t) = 0.001 \sin(\omega t) \cdot \left(\frac{R_f}{R_S} + 1 \right) = 0.001 \sin(\omega t) \cdot (1 + 1) = 2 \cdot 10^{-3} \sin(\omega t) \text{ V}.$$



Problem 8.10

Solution:

Known quantities:

For the circuit shown in Figure P8.10, let:

$$R = 2 \text{ M}\Omega \quad A_{v(OL)} = 200,000 \quad R_0 = 50 \Omega$$

$$R_S = 1 \text{ k}\Omega \quad R_1 = 1 \text{ k}\Omega \quad R_2 = 100 \text{ k}\Omega \quad R_{LOAD} = 10 \text{ k}\Omega$$

Find:

The gain $A_V = \frac{v_o}{v_i}$

Analysis:

The op-amp has a very large input resistance, a very large "open loop gain" $[A_{V(OL)}]$, and a very small output resistance. Therefore, it can be modeled with small error as an ideal op-amp.

The amplifier shown in Figure P8.10 is a noninverting amplifier, so we have

$$A_V = 1 + \frac{R_2}{R_1} = 1 + \frac{100 \cdot 10^3}{1 \cdot 10^3} = 101$$

Problem 8.11**Solution:****Known quantities:**

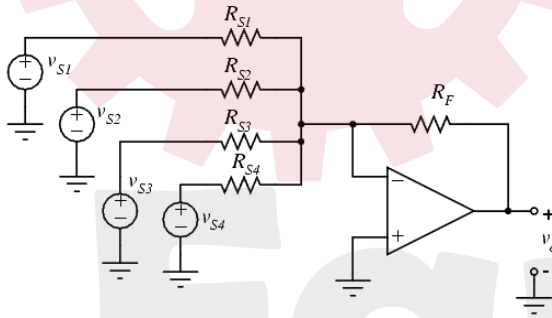
$$v_{out}(t) = -(2 \sin \omega_1 t + 4 \sin \omega_2 t + 8 \sin \omega_3 t + 16 \sin \omega_4 t) \text{ V}, \quad R_F = 5 \text{ k}\Omega.$$

Find:

Design an inverting summing amplifier to obtain $v_{out}(t)$ and determine the required source resistors.

Analysis:

The inverting summing amplifier is shown in the following figure.



By superposition and by selecting $R_{Si} = \frac{R_F}{2^i}$, $v_{Si} = \sin \omega_i t$, $i = 1 \dots 4$, the output voltage is

$$v_{out} = -\sum_{i=1}^4 \frac{R_F}{R_{Si}} v_{Si} = -\sum_{i=1}^4 2^i v_{Si} = -\sum_{i=1}^4 2^i \sin \omega_i t$$

that coincides with the desired output voltage.

So, the required source resistors are

$$R_{S1} = \frac{R_F}{2} = 2.5 \text{ k}\Omega, \quad R_{S2} = \frac{R_F}{4} = 1.25 \text{ k}\Omega, \quad R_{S3} = \frac{R_F}{8} = 625 \Omega, \quad R_{S4} = \frac{R_F}{16} = 312.5 \Omega.$$

Problem 8.12**Solution:****Known quantities:**

For the circuit shown in Figure P8.12:

$$r_i = 2 \text{ M}\Omega \quad \mu = 200,000 \quad r_o = 25 \Omega$$

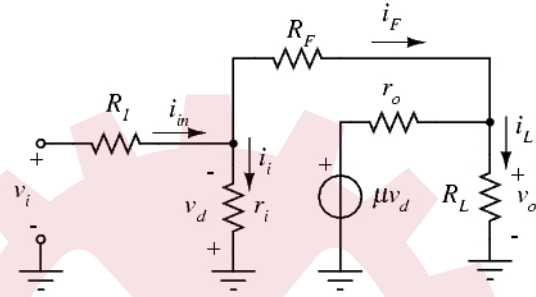
$$R_S = 2.2 \text{ k}\Omega \quad R_1 = 1 \text{ k}\Omega \quad R_F = 8.7 \text{ k}\Omega \quad R_L = 20 \Omega$$

Find:

- An expression for the input resistance v_i/i_i including the effects of the op-amp.
- The value of the input resistance including the effects of the op-amp.
- The value of the input resistance with ideal op-amp.

Analysis:

- The circuit in Figure P8.12 can be modeled as in the following figure



where

$$v_i = -v_d + R_1 i_i,$$

$$v_d = -r_i(i_i - i_F),$$

$$i_F = -\frac{v_d + v_o}{R_F},$$

$$v_o = \mu v_d + r_o(i_F - \frac{v_o}{R_L}).$$

Substituting the expression for v_d in all the other equations, we obtain

$$v_i = (R_1 + r_i)i_i - r_i i_F,$$

$$i_F = \frac{r_i i_i - v_o}{R_F + r_i},$$

$$v_o(1 + \frac{r_o}{R_L}) = -\mu r_i i_i + (\mu r_i + r_o)i_F.$$

By solving the second equation above for v_o and substituting in the third equation

$$v_i = (R_1 + r_i)i_i - r_i i_F,$$

$$i_F = \frac{\left(1 + \frac{r_o}{R_L} + \mu\right)r_i}{\left(1 + \frac{r_o}{R_L}\right)(R_F + r_i) + \mu r_i + r_o} i_i$$

and finally,

$$\frac{v_i}{i_i} = (R_1 + r_i) - \frac{\left(1 + \frac{r_0}{R_L} + \mu\right) r_i^2}{\left(1 + \frac{r_0}{R_L}\right)(R_F + r_i) + \mu r_i + r_0}$$

b. The value of the input resistance is

$$\begin{aligned} \frac{v_i}{i_i} &= (R_1 + r_i) - \frac{\left(1 + \frac{r_0}{R_L} + \mu\right) r_i^2}{\left(1 + \frac{r_0}{R_L}\right)(R_F + r_i) + \mu r_i + r_0} = 2.001 \cdot 10^6 - \frac{\left(1 + \frac{25}{20} + 2 \cdot 10^5\right) 4 \cdot 10^{12}}{\left(1 + \frac{25}{20}\right) 2.0087 \cdot 10^6 + 4 \cdot 10^{11} + 25} = \\ &= 1.0001 \text{ k}\Omega \end{aligned}$$

c. In the ideal case:

$$\frac{v_i}{i_i} = R_1 = 1 \text{ k}\Omega$$

Problem 8.13

Solution:

Known quantities:

For the circuit shown in Figure P8.13:

$$v_s(t) = 0.02 + 10^{-3} \cos(\omega t) \text{ V}, \quad R_F = 220 \text{ k}\Omega, \quad R_1 = 47 \text{ k}\Omega, \quad R_2 = 1.8 \text{ k}\Omega.$$

Find:

- expression for the output voltage.
- The corresponding value of $v_o(t)$.

Analysis:

- the circuit is a noninverting amplifier, then

$$v_o = \left(1 + \frac{R_F}{R_2}\right) v_s$$

$$\text{b) } v_o = \left(1 + \frac{220}{1.8}\right) v_s = 2.464 + 0.1232 \cos(\omega t) \text{ V}$$

Problem 8.14

Solution:

Known quantities:

For the circuit shown in Figure P8.13:

$$v_s(t) = 0.05 + 30 \cdot 10^{-3} \cos(\omega t) \text{ V}, \quad R_s = 50 \Omega, \quad R_L = 200 \Omega.$$

Find:

The output voltage v_o .

Analysis:

It is a particular case of a noninverting amplifier where $v_o = v_S$.

Problem 8.15**Solution:****Known quantities:**

For the circuit shown in Figure P8.15:

$$v_{S1}(t) = 2.9 \cdot 10^{-3} \cos(\omega t) \text{ V}, \quad R_1 = 1 \text{ k}\Omega, \quad R_2 = 3.3 \text{ k}\Omega$$

$$v_{S2}(t) = 3.1 \cdot 10^{-3} \cos(\omega t) \text{ V}, \quad R_3 = 10 \text{ k}\Omega, \quad R_4 = 18 \text{ k}\Omega$$

Find:

The output voltage v_o and a numerical value.

Analysis:

By using superposition,

$$\begin{aligned} v_o &= -\frac{R_3}{R_1} v_{S1} + \frac{R_4}{R_2 + R_4} \left(1 + \frac{R_3}{R_1} \right) v_{S2} = \left(1 + \frac{10}{1} \right) \frac{18}{18 + 3.3} 3.1 \cdot 10^{-3} \cos(\omega t) - 2.9 \cdot 10^{-2} \cos(\omega t) = \\ &= -1.83 \cdot 10^{-3} \cos(\omega t) \text{ V} \end{aligned}$$

Problem 8.16**Solution:****Known quantities:**

For the circuit shown in Figure P8.15:

$$v_{S1} = 5 \text{ mV}, \quad R_1 = 1 \text{ k}\Omega, \quad R_2 = 15 \text{ k}\Omega$$

$$v_{S2} = 7 \text{ mV}, \quad R_3 = 72 \text{ k}\Omega, \quad R_4 = 47 \text{ k}\Omega$$

Find:

The output voltage v_o analytically and numerically.

Analysis:

From the solution of Problem 8.15,

$$v_o = -\frac{R_3}{R_1} v_{S1} + \frac{R_4}{R_2 + R_4} \left(1 + \frac{R_3}{R_1} \right) v_{S2} = (1 + 72) \frac{47}{15 + 47} 7 \cdot 10^{-3} - 3.60 \cdot 10^{-1} = 27.37 \text{ mV}$$

Problem 8.17**Solution:****Known quantities:**

If, in the circuit shown in Figure P8.17:

$$v_{S1} = v_{S2} = 7 \text{ mV}, \quad R_1 = 850 \Omega, \quad R_2 = 1.5 \text{ k}\Omega, \quad R_F = 2.2 \text{ k}\Omega$$

Op Amp: *Motorola MC1741C*

$$r_i = 2 \text{ M}\Omega, \quad \mu = 200,000, \quad r_o = 25 \Omega$$

Find:

- The output voltage.
- The voltage gain for the two input signals.

Analysis:

- The op amp has a very large input resistance, a very large "open loop gain" $[A]$, and a very small output resistance. Therefore, it can be modeled with small error as an ideal op amp with:

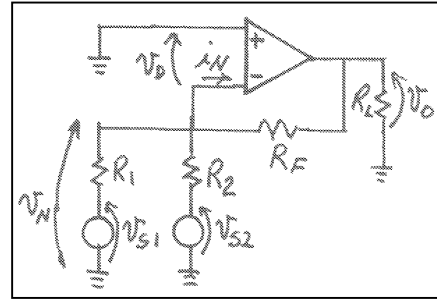
$$KVL: v_D + v_N = 0$$

$$v_D \approx 0 \Rightarrow v_N \approx 0$$

$$KCL:$$

$$i_N + \frac{v_N - v_{S2}}{R_2} + \frac{v_N - v_{S1}}{R_1} + \frac{v_N - v_O}{R_F} = 0$$

$$v_N \approx 0 \quad i_N \approx 0$$



$$\begin{aligned} \Rightarrow v_O &= -\frac{R_F}{R_1} v_{S1} - \frac{R_F}{R_2} v_{S2} = \left[-\frac{2.2}{0.85}\right][7 \text{ mV}] + \left[-\frac{2.2}{1.5}\right][7 \text{ mV}] \\ &= [-2.588][7 \text{ mV}] + [-1.467][7 \text{ mV}] = -28.38 \text{ mV} \end{aligned}$$

- Using the results above: $A_{V1} = -2.588$ $A_{V2} = -1.467$.

Note: The output voltage and gain are not dependent on either the op amp parameters or the load resistance. This result is extremely important in the majority of applications where amplification of a signal is required.

Problem 8.18**Solution:****Known quantities:**

For the circuit shown in Figure P8.15:

$$v_{S1} = kT_1, \quad R_1 = 11 \text{ k}\Omega, \quad R_2 = 27 \text{ k}\Omega$$

$$v_{S2} = kT_2, \quad R_3 = 33 \text{ k}\Omega, \quad R_4 = 68 \text{ k}\Omega$$

$$T_1 = 35^\circ \text{C}, \quad T_2 = 100^\circ \text{C}, \quad k = 50 \text{ mV}/^\circ \text{C}$$

Find:

- The output voltage.
- The conditions required for the output voltage to depend only on the difference between the two temperatures.

Analysis:

- From the solution of Problem 8.15,

$$\begin{aligned} v_O &= -\frac{R_3}{R_1} kT_1 + \frac{R_4}{R_2 + R_4} \left(1 + \frac{R_3}{R_1}\right) kT_2 = \left(1 + \frac{33}{11}\right) \frac{68}{68 + 27} 5 - \frac{33}{11} 1.75 = \\ &= 9.065 \text{ V} \end{aligned}$$

- For $R_3 = R_4$, $R_1 = R_2 = k R_3$, the output voltage is

$$v_0 = -\frac{R_3}{R_1} kT_1 + \frac{R_4}{R_2 + R_4} \left(1 + \frac{R_3}{R_1}\right) kT_2 = -\frac{1}{k} kT_1 + \frac{R_3}{R_1 + R_3} \left(\frac{R_1 + R_3}{R_1}\right) kT_2 = T_2 - T_1$$

Problem 8.19

Solution:

Find:

In a differential amplifier, if: $A_{v1} = -20$ $A_{v2} = +22$, derive expressions for and then determine the value of the common and differential mode gains.

Analysis:

There are several ways to do this. Using superposition:

$$\text{If } v_{S-D} = 0: v_O = v_{O-C} + v_{O-D} = v_{O-C} \Rightarrow A_{v-C} = \frac{v_{O-C}}{v_{S-C}}$$

$$\text{If } v_{S-C} = 0: v_O = v_{O-C} + v_{O-D} = v_{O-D} \Rightarrow A_{v-D} = \frac{v_{O-D}}{v_{S-D}}$$

Assume that signal source #2 is connected to the non-inverting input of the op-amp. The common-mode output voltage can be obtained using the gains given above but assuming the signal voltages have only a common-mode component:

$$v_O = v_{S1} A_{v1} + v_{S2} A_{v2} \quad v_{S1} = v_{S-C} - \frac{1}{2} v_{S-D} \quad v_{S2} = v_{S-C} + \frac{1}{2} v_{S-D}$$

$$\text{Let } v_{S-D} = 0$$

$$v_{O-C} = v_{S-C} A_{v1} + v_{S-C} A_{v2} = v_{S-C} [A_{v1} + A_{v2}] = v_{S-C} A_{v-C}$$

$$\Rightarrow A_{v-C} = A_{v1} + A_{v2} = [-20] + [+22] = 2$$

The difference-mode output voltage can be obtained using the gains given but assuming the signal voltage have only a difference-mode component:

$$\text{Let } v_{S-C} = 0$$

$$v_{O-D} = \left[-\frac{1}{2} v_{S-D}\right] A_{v1} + \left[\frac{1}{2} v_{S-D}\right] A_{v2} = v_{S-D} \frac{1}{2} [-A_{v1} + A_{v2}] = v_{S-D} A_{v-D}$$

$$A_{v-D} = \frac{1}{2} [A_{v2} - A_{v1}] = \frac{1}{2} ([+22] - [-20]) = +21$$

Note: If signal source #1 were connected to the non-inverting input of the op amp, then the difference mode gain would be the negative of that obtained above.

Problem 8.20

Solution:

Known quantities:

For the circuit shown in Figure P8.15:

$$v_{S1} = 1.3V, \quad R_1 = R_2 = 4.7k\Omega$$

$$v_{S2} = 1.9V, \quad R_3 = R_4 = 10k\Omega$$

Find:

- The output voltage.
- The common-mode component of the output voltage.

c) The differential-mode component of the output voltage.

Analysis:

a) From the solution of Problem 8.15,

$$v_0 = -\frac{R_3}{R_1} v_{S1} + \frac{R_4}{R_2 + R_4} \left(1 + \frac{R_3}{R_1} \right) v_{S2} = \left(1 + \frac{10000}{4700} \right) \frac{10000}{4700 + 10000} 1.9 - \frac{10000}{4700} 1.3 = 1.28V$$

b) The common-mode component is zero.

c) The differential-mode component is v_0 .

Problem 8.21

Solution:

Known quantities:

For the circuit shown in Figure P8.15:

$$v_{S1,2} = A + BP_{1,2} \quad , \quad A = 0.3 \quad , \quad B = 0.7 \text{ V/psi}$$

$$R_1 = R_2 = 4.7\text{k}\Omega \quad , \quad R_3 = R_4 = 10\text{k}\Omega \quad , \quad R_L = 1.8\text{k}\Omega$$

$$P_1 = 6\text{kPa} \quad , \quad P_2 = 5\text{kPa}$$

Find:

- The common-mode input voltage.
- The differential-mode input voltage.

Analysis:

a) The common-mode input voltage is

$$v_{in+} = \frac{v_{S2} + v_{S1}}{2} = A + B \frac{P_1 + P_2}{2} = 0.3 + 0.7 \cdot 1.4504 \cdot 10^{-4} \frac{11 \cdot 10^3}{2} = 0.858V$$

b) The differential-mode input voltage is

$$v_{in-} = v_{S2} - v_{S1} = B(P_2 - P_1) = -0.7 \cdot 1.4504 \cdot 10^{-4} \cdot 10^3 = -0.1015V$$

Problem 8.22

Solution:

Known quantities:

A linear potentiometer [variable resistor] R_p is used to sense and give a signal voltage v_Y proportional to the current y position of an x - y plotter. A reference signal v_R is supplied by the software controlling the plotter. The difference between these voltages must be amplified and supplied to a motor. The motor turns and changes the position of the pen and the position of the "pot" until the signal voltage is equal to the reference voltage [indicating the pen is in the desired position] and the motor voltage = 0. For proper operation the motor voltage must be 10 times the difference between the signal and reference voltage. For rotation in the proper direction, the motor voltage must be negative with respect to the signal voltage for the polarities shown. An additional requirement is that $i_P = 0$ to avoid "loading" the pot and causing an erroneous signal voltage.

Find:

- Design an op amp circuit which will achieve the specifications given. Redraw the circuit shown in Figure P8.22 replacing the box [drawn with dotted lines] with your circuit. Be sure to show how the signal voltage and output voltage are connected in your circuit.
- Determine the value of each component in your circuit. The op amp is an MC1741.

Analysis:

- a) The output voltage to the motor must be dependent on the difference between two input voltages. A difference amp is required. The signal voltage must be connected to the inverting input of the amplifier. However, the feedback path is also connected to the inverting input and this will cause loading of the input circuit, ie, cause an input current. This can be corrected by adding an isolation stage [or voltage follower] between the input circuit and the inverting input of the 2nd stage. The isolation stage must have a gain = 1 and an input current = 0. The difference amplifier, the second stage, must give an output voltage:

$$v_{O1} = \left(\frac{10\text{k}\Omega}{R_S} + 1 \right) (10)$$

$$v_M = 10 [v_R - v_Y] = [10] v_R + [-10] v_Y$$

$$\Rightarrow A_{vr} = 10 \quad A_{vy} = -10$$

The circuit configuration shown will satisfy these specifications.

- b) In this first approximation analysis, assume the op amps can be modeled as ideal op amps:

$$\Rightarrow v_D \approx 0 \quad i_P = i_N \approx 0$$

Consider the first or isolation or voltage follower stage:

$$\text{Ideal: } \Rightarrow i_P \approx 0 \quad \text{KVL: } -v_Y + v_D + v_{O1} = 0 \quad v_D \approx 0 \quad \Rightarrow v_{O1} = v_Y$$

$$\text{KVL: } -v_N - v_D + v_P = 0 \quad v_D \approx 0 \quad \Rightarrow v_N = v_P$$

$$\text{KCL: } \frac{v_N - v_{O1}}{R_1} + \frac{v_N - v_M}{R_2} + i_N = 0 \quad i_N \approx 0 \quad \Rightarrow v_N = \frac{\frac{v_{O1}}{R_1} + \frac{v_M}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} \left(\frac{v_{O1}}{R_1} + \frac{v_M}{R_2} \right)$$

$$\text{KCL: } \frac{v_P - v_R}{R_3} + \frac{v_P - 0}{R_4} + i_P = 0 \quad i_P \approx 0 \quad \Rightarrow v_P = \frac{\frac{v_R}{R_3}}{\frac{1}{R_3} + \frac{1}{R_4}} = \frac{R_3 R_4}{R_3 + R_4} v_R$$

The second stage:

$$v_{O1} = v_Y \quad v_N = v_P \quad \Rightarrow \frac{v_{O1} R_2}{R_2 + R_1} + \frac{v_M R_1}{R_2 + R_1} = \frac{v_R R_4}{R_4 + R_3}$$

$$v_M = v_Y \left[-\frac{R_2}{R_1} \right] + v_R \frac{R_4 [R_2 + R_1]}{R_1 [R_4 + R_3]} = v_Y A_{vy} + v_R A_{vr}$$

$$A_{vy} = -\frac{R_2}{R_1} = -10 \quad \text{Choose: } R_2 = 100\text{ k}\Omega \quad R_1 = 10\text{ k}\Omega$$

$$A_{vr} = \frac{R_4 [R_2 + R_1]}{R_1 [R_4 + R_3]} = 10 \quad \text{Choose: } R_3 = 10\text{ k}\Omega \quad R_4 = 100\text{ k}\Omega$$

The resistances chosen are standard values and there is some commonality in the choices. Moderately large values were chosen to reduce currents and the resistor power ratings. Cost of the resistors will be determined primarily by power rating and tolerance.

Problem 8.23**Solution:****Known quantities:**

For the circuit shown in Figure P8.15:

$$v_{S1} = 13\text{mV} \quad , \quad R_1 = 1\text{k}\Omega \quad , \quad R_2 = 13\text{k}\Omega$$

$$v_{S2} = 19\text{mV} \quad , \quad R_3 = 81\text{k}\Omega \quad , \quad R_4 = 56\text{k}\Omega$$

Find:

The output voltage v_o .

Analysis:

From the solution of Problem 8.15,

$$v_o = \frac{R_4}{R_2 + R_4} \left(1 + \frac{R_3}{R_1} \right) v_{S2} - \frac{R_3}{R_1} v_{S1} = (1 + 81) \frac{56}{56 + 13} 19 \cdot 10^{-3} - 81 \cdot 13 \cdot 10^{-3} = 0.211 \text{ V}$$

Problem 8.24**Solution:****Known quantities:**

The circuit shown in Figure P8.24.

Find:

Show that the current I_{out} through the light-emitting diode is proportional to the source voltage V_S as long as :

$$V_S > 0.$$

Analysis:

$$\text{Assume the op amp is ideal: } \left. \begin{array}{l} V^- \approx V^+ = V_S \\ I^- \approx 0 \end{array} \right\} \Rightarrow \text{if } V_S > 0 \quad I_{out} = \frac{V^-}{R_2} = \frac{V_S}{R_2}.$$

Problem 8.25**Solution:****Known quantities:**

The circuit shown in Figure P8.25.

Find:

Show that the voltage V_{out} is proportional to the current generated by the CdS solar cell. Show that the transimpedance of the circuit V_{out}/I_S is $-R$.

Analysis:

$$\text{Assuming an ideal op-amp: } v^+ \cong v^- = 0 \Rightarrow V_{out} = -RI_S$$

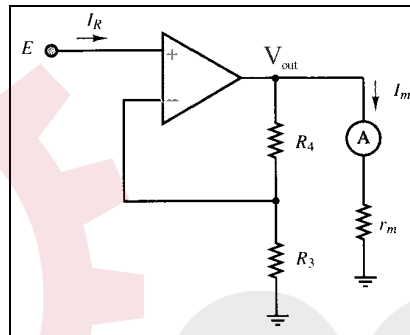
$$\text{The transimpedance is given by } R_{trans} = \frac{V_{out}}{I_S} = -R$$

Problem 8.26**Solution:****Known quantities:**

The op-amp voltmeter circuit shown in Figure P8.26 is required to measure a maximum input of $E = 20$ mV. The op-amp input current is $I_B = 0.2 \mu\text{A}$, and the meter circuit has $I_m = 100 \mu\text{A}$ full-scale deflection and $r_m = 10 \text{ k}\Omega$.

Find:

Determine suitable values for R_3 and R_4 .

Analysis:

$$V_{out} = I_m r_m = (100 \mu\text{A})(10 \text{ k}\Omega) = 1 \text{ V}$$

From KCL at the inverting input, $\frac{E}{R_3} + \frac{E - V_{out}}{R_4} = 0$ or $\frac{V_{out}}{E} = \frac{R_4 + R_3}{R_3} = 1 + \frac{R_4}{R_3}$.

Then, $\frac{R_4}{R_3} = \frac{V_{out}}{E} - 1 = \frac{1}{20 \times 10^{-3}} - 1 = 49$. Now, choose R_3 and R_4 such that

$$I_B = \frac{E}{R_3 \parallel (R_4 + r_m)} \leq 0.2 \mu\text{A}.$$

At the limit, $\frac{20 \times 10^{-3}}{R_3 \parallel (49R_3 + 10 \times 10^3)} = 0.2 \times 10^{-6}$. Solving for R_3 , we have $R_3 \approx 102 \text{ k}\Omega$.

Therefore, $R_4 \approx 5 \text{ M}\Omega$.

Problem 8.27**Solution:****Known quantities:**

Circuit in Figure P8.27.

Find:

The output voltage v_o .

Analysis:

The circuit is a cascade of a noninverting op-amp with an inverting op-amp. Assuming ideal op-amps, the input-output voltage gain is equal to the product of the single gains, therefore

$$v_0 = -\frac{R_{F2}}{R_{S2}} \left(1 + \frac{R_{F1}}{R_{S1}} \right) v_S$$

Problem 8.28

Solution:

Known quantities:

Circuit in Figure P8.27.

Find:

Select appropriate components using standard 5% resistor values to obtain a gain of magnitude approximately equal to 1,000. How closely can you approximate the gain? Compute the error in the gain assuming that the resistors have the nominal value.

Analysis:

From the solution of Problem 8.27, the gain is given by

$$A_v = -\frac{R_{F2}}{R_{S2}} \left(1 + \frac{R_{F1}}{R_{S1}} \right)$$

From Table 2.2, if we select

$$R_{S1} = 1.8\text{k}\Omega, \quad R_{S2} = 1\text{k}\Omega$$

$$R_{F1} = 8.2\text{k}\Omega, \quad R_{F2} = 180\text{k}\Omega$$

we obtain

$$A_v = -\frac{R_{F2}}{R_{S2}} \left(1 + \frac{R_{F1}}{R_{S1}} \right) = -\frac{180}{1} \left(\frac{1.8 + 8.2}{1.8} \right) = -1000$$

So, we can obtain a nominal error equal to zero.

Problem 8.29

Solution:

Known quantities:

Circuit in Figure P8.27.

Find:

Same as in Problem 8.28, but use the $\pm 5\%$ tolerance range to compute the possible range of gains for this amplifier.

Analysis:

From the solution of Problem 8.27, the gain is given by

$$A_v = -\frac{R_{F2}}{R_{S2}} \left(1 + \frac{R_{F1}}{R_{S1}} \right)$$

From Table 2.2, if we select

$$R_{S1n} = 1.8\text{k}\Omega, \quad R_{S2n} = 1\text{k}\Omega$$

$$R_{F1n} = 8.2\text{k}\Omega, \quad R_{F2n} = 180\text{k}\Omega$$

we obtain

$$A_{V_n} = -\frac{R_{F2n}}{R_{S2n}} \left(1 + \frac{R_{F1n}}{R_{S1n}} \right) = -\frac{180}{1} \left(\frac{1.8 + 8.2}{1.8} \right) = -1000$$

So, we can obtain a nominal error equal to zero.

The maximum gain is given by

$$A_{V^+} = -\frac{R_{F2}}{R_{S2}} \left(1 + \frac{R_{F1}}{R_{S1}} \right) = -\frac{R_{F2n}(1+0.05)}{R_{S2n}(1-0.05)} \left(1 + \frac{R_{F1n}(1+0.05)}{R_{S1n}(1-0.05)} \right) = -1200$$

while the minimum gain is

$$A_{V^-} = -\frac{R_{F2}}{R_{S2}} \left(1 + \frac{R_{F1}}{R_{S1}} \right) = -\frac{R_{F2n}(1-0.05)}{R_{S2n}(1+0.05)} \left(1 + \frac{R_{F1n}(1-0.05)}{R_{S1n}(1+0.05)} \right) = -834$$

Problem 8.30

Solution:

Known quantities:

For the circuit in Figure P8.30,

$$R_L = 20\text{k}\Omega, \quad v_0 = 0 \div 10 \text{ V}$$

$$i_{in_{\max}} = 1\text{mA}$$

Find:

The resistance R such that $i_{in_{\max}} = 1\text{mA}$.

Analysis:

$$i_{in} = \frac{v_0}{R} \Rightarrow R = \frac{v_{0_{\max}}}{i_{in_{\max}}} = \frac{10}{10^{-3}} = 10 \text{ k}\Omega$$

Problem 8.31

Solution:

Known quantities:

Circuit in Figure P8.13.

Find:

Select appropriate components using standard 5% resistor values to obtain a gain of magnitude approximately equal to 200. How closely can you approximate the gain? Compute the error in the gain assuming that the resistors have the nominal value.

Analysis:

From the solution of Problem 8.13, the gain is given by

$$A_V = \left(1 + \frac{R_F}{R_2} \right)$$

From Table 2.2, if we select

$$R_2 = 33\Omega$$

$$R_F = 6.8\text{k}\Omega$$

we obtain

$$A_{V_n} = \left(1 + \frac{R_F}{R_2}\right) = 1 + \frac{6800}{33} = 207$$

The error in the gain is

$$\varepsilon_{A_V} = \frac{207 - 200}{200} = 3.5\%$$

Problem 8.32

Solution:

Known quantities:

Circuit in Figure P8.13.

Find:

Same as in Problem 8.31, but use the $\pm 5\%$ tolerance range to compute the possible range of gains for this amplifier.

Analysis:

From the solution of Problem 8.31, the gain is given by

$$A_V = \left(1 + \frac{R_F}{R_2}\right)$$

From Table 2.2, if we select

$$R_{2n} = 33\Omega$$

$$R_{Fn} = 6.8k\Omega$$

we obtain

$$A_{V_n} = \left(1 + \frac{R_{Fn}}{R_{2n}}\right) = 1 + \frac{6800}{33} = 207$$

The maximum gain is given by

$$A_{V^+} = \left(1 + \frac{R_{Fn}(1+0.05)}{R_{2n}(1-0.05)}\right) = 1 + \frac{6800 \cdot 1.05}{33 \cdot 0.95} = 228.75$$

while the minimum gain is

$$A_{V^-} = \left(1 + \frac{R_{Fn}(1-0.05)}{R_{2n}(1+0.05)}\right) = 1 + \frac{6800 \cdot 0.95}{33 \cdot 1.05} = 187.44$$

Problem 8.33

Solution:

Known quantities:

For the circuit in Figure P8.15,

$$R_1 = R_2 \quad , \quad R_3 = R_4$$

Find:

Select appropriate components using standard 1% resistor values to obtain a differential amplifier gain of magnitude approximately equal to 100. How closely can you approximate the gain? Compute the error in the gain assuming that the resistors have nominal value.

Analysis:

From the solution of Problem 8.15, the gain is given by

$$A_v = \frac{R_3}{R_1}$$

From Table 2.2, if we select

$$R_1 = 1\text{k}\Omega, \quad R_3 = 100\text{k}\Omega$$

we obtain

$$A_v = \frac{R_3}{R_1} = 100$$

and the error for the gain with nominal values is zero.

Problem 8.34

Solution:

Known quantities:

For the circuit in Figure P8.15,

$$R_1 = R_2, \quad R_3 = R_4$$

Find:

Same as in Problem 8.33, but use the $\pm 1\%$ tolerance range to compute the possible range of gains for this amplifier.

Analysis:

From the solution of Problem 8.33

$$A_v = \frac{R_3}{R_1} \Rightarrow A_{v \max} = \frac{100(1+0.01)}{1(1-0.01)} = 102$$

$$A_v = \frac{R_3}{R_1} \Rightarrow A_{v \min} = \frac{100(1-0.01)}{1(1+0.01)} = 98$$

Section 8.3: Active Filters

Problem 8.35

Solution:

Known quantities:

For the circuit shown in Figure P8.35: $C = 1\ \mu\text{F}$ $R = 10\text{ k}\Omega$ $R_L = 1\text{ k}\Omega$.

Find:

- The gain [in dB] in the pass band.
- The cutoff frequency.
- If this is a low or high pass filter.

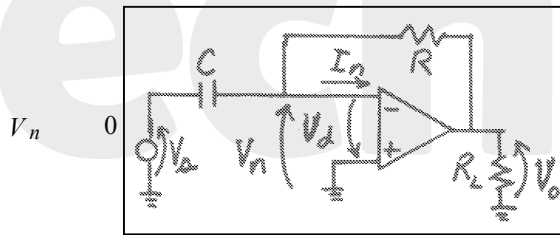
Analysis:

- Assume the op-amp is ideal. Determine the transfer function in the form:

$$H_v[j\omega] = \frac{V_o[j\omega]}{V_s[j\omega]} = H_o \frac{1}{1 + j\omega RC}$$

$$\text{KVL: } -V_n - V_d = 0 \quad V_d = 0$$

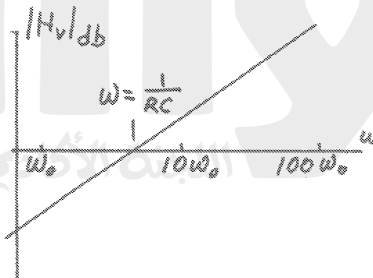
$$\text{KCL: } \frac{V_n - V_s}{\frac{1}{j\omega C}} + I_n + \frac{V_n - V_o}{R} = 0$$



$$I_n = 0 \quad V_n = 0 \quad H_v[j\omega] = \frac{V_o[j\omega]}{V_s[j\omega]} = j\omega RC$$

This is not in the standard form desired but is the best that can be done. There are no cutoff frequencies and no clearly defined pass band. The gain [i.e., the magnitude of the transfer function] and output voltage increases continuously with frequency, at least until the output voltage tries to exceed the DC supply voltages and clipping occurs. In a normal high pass filter, the gain will increase with frequency until the cutoff frequency is reached above which the gain remains constant.

- There is no cutoff frequency.
- This filter is best called a high pass filter; however, since the output will be clipped and severely distorted above some frequency, it is not a particularly good high pass filter. It could even be called a terrible filter with few redeeming graces.



Problem 8.36

Solution:

Known quantities:

For the circuit shown in Figure P8.36: $C = 1\ \mu\text{F}$ $R_1 = 1.8\text{ k}\Omega$ $R_2 = 8.2\text{ k}\Omega$ $R_L = 333\Omega$.

Find:

- a) Whether the circuit is a low- or high-pass filter.
- b) The gain V_0/V_S in decibel in the passband.
- c) The cutoff frequency.

Analysis:

- a) From Figure 8.24, it results that the amplifier in Figure P8.36 is a high-pass filter.

In fact, the output voltage is

$$\mathbf{V}_0(j\omega) = -\frac{j\omega CR_2}{1 + j\omega CR_1} \mathbf{V}_S(j\omega)$$

$$\text{b) } \lim_{\omega \rightarrow \infty} \left| \frac{\mathbf{V}_0(j\omega)}{\mathbf{V}_S(j\omega)} \right|_{dB} = 20 \text{Log} \frac{R_2}{R_1} = 20 \text{Log} \frac{8.2}{1.8} = 13.17 \text{ dB}$$

$$\text{c) } \omega_0 = \frac{1}{CR_1} = \frac{1}{1 \cdot 10^{-6} \cdot 1.8 \cdot 10^3} = 5555 \text{ rad/s}$$

Problem 8.37**Solution:****Known quantities:**

For the circuit shown in Figure P8.36: $C = 200 \text{ pF}$ $R_1 = 10 \text{ k}\Omega$ $R_2 = 220 \text{ k}\Omega$ $R_L = 1 \text{ k}\Omega$.

Find:

- a) Whether the circuit is a low- or high-pass filter.
- b) The gain V_0/V_S in decibel in the passband.
- c) The cutoff frequency.

Analysis:

- a) From Figure 8.24, it results that the amplifier in Figure P8.36 is a high-pass filter.

In fact, the output voltage is

$$\mathbf{V}_0(j\omega) = -\frac{j\omega CR_2}{1 + j\omega CR_1} \mathbf{V}_S(j\omega)$$

$$\text{b) } \lim_{\omega \rightarrow \infty} \left| \frac{\mathbf{V}_0(j\omega)}{\mathbf{V}_S(j\omega)} \right|_{dB} = 20 \text{Log} \frac{R_2}{R_1} = 20 \text{Log} \frac{220}{10} = 26.84 \text{ dB}$$

$$\text{c) } \omega_0 = \frac{1}{CR_1} = \frac{1}{200 \cdot 10^{-12} \cdot 10 \cdot 10^3} = 5 \cdot 10^5 \text{ rad/s}$$

Problem 8.38**Solution:****Known quantities:**

For the circuit shown in Figure P8.38: $C = 100 \text{ pF}$ $R_1 = 4.7 \text{ k}\Omega$ $R_2 = 68 \text{ k}\Omega$ $R_L = 220 \text{ k}\Omega$.

Find:

Determine the cutoff frequencies and the magnitude of the voltage frequency response function at very low and at very high frequencies.

Analysis:

The output voltage in the frequency domain is

$$\mathbf{V}_o(j\omega) = \left(1 + \frac{R_2}{\frac{1}{j\omega C} + R_1} \right) \mathbf{V}_i(j\omega) = \frac{1 + j\omega C(R_1 + R_2)}{1 + j\omega CR_1} \mathbf{V}_i(j\omega)$$

The circuit is a high-pass filter. The cutoff frequencies are

$$\omega_1 = \frac{1}{CR_1} = \frac{1}{100 \cdot 10^{-12} \cdot 4.7 \cdot 10^3} = 2.127 \cdot 10^6 \text{ rad/s}$$

$$\omega_2 = \frac{1}{C(R_1 + R_2)} = \frac{1}{100 \cdot 10^{-12} \cdot 72.7 \cdot 10^3} = 1.375 \cdot 10^5 \text{ rad/s}$$

For high frequencies we have

$$A_\infty = \lim_{\omega \rightarrow \infty} \left| \frac{\mathbf{V}_o(j\omega)}{\mathbf{V}_i(j\omega)} \right| = 1 + \frac{R_2}{R_1} = 1 + \frac{68}{4.7} = 15.46$$

At low frequencies

$$A_0 = \lim_{\omega \rightarrow 0} \left| \frac{\mathbf{V}_o(j\omega)}{\mathbf{V}_i(j\omega)} \right| = 1$$

Problem 8.39

Solution:

Known quantities:

For the circuit shown in Figure P8.39: $C = 20 \text{ nF}$ $R_1 = 1 \text{ k}\Omega$ $R_2 = 4.7 \text{ k}\Omega$ $R_3 = 80 \text{ k}\Omega$.

Find:

- An expression for $H_v(j\omega) = \frac{\mathbf{V}_o(j\omega)}{\mathbf{V}_i(j\omega)}$.
- The cutoff frequencies.
- The passband gain.
- The Bode plot.

Analysis:

- The frequency response is

$$H_v(j\omega) = \frac{\mathbf{V}_o(j\omega)}{\mathbf{V}_i(j\omega)} = 1 + \frac{R_3 \parallel \frac{1}{j\omega C}}{R_2} = \frac{R_2 + R_3 + j\omega CR_2 R_3}{R_2 + j\omega CR_2 R_3} = \left(1 + \frac{R_3}{R_2} \right) \frac{1 + j\omega C \frac{R_2 R_3}{R_2 + R_3}}{1 + j\omega CR_3}$$

- The cutoff frequencies are

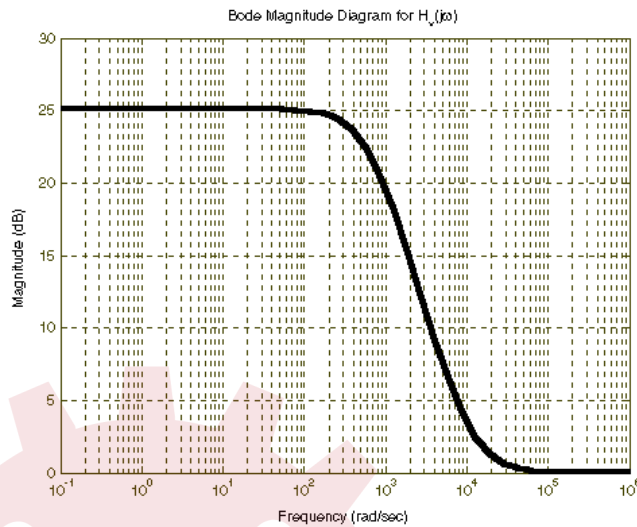
$$\omega_1 = \frac{1}{CR_3} = \frac{1}{20 \cdot 10^{-9} \cdot 80 \cdot 10^3} = 625 \text{ rad/s}$$

$$\omega_2 = \frac{(R_2 + R_3)}{C R_2 R_3} = \frac{84.7 \cdot 10^3}{20 \cdot 10^{-9} \cdot 80 \cdot 10^3 \cdot 4.7 \cdot 10^3} = 11263 \text{ rad/s}$$

- The passband gain is obtained by evaluating the frequency response at low frequencies,

$$A_0 = \lim_{\omega \rightarrow 0} |H_v(j\omega)| = 1 + \frac{R_3}{R_2} = 1 + \frac{80}{4.7} = 18$$

- The magnitude Bode plot for the given amplifier is as shown.



Problem 8.40

Solution:

Known quantities:

For the circuit shown in Figure P8.40: $C = 0.47 \mu\text{F}$ $R_1 = 9.1\text{k}\Omega$ $R_2 = 22\text{k}\Omega$ $R_L = 2.2\text{k}\Omega$.

Find:

- Whether the circuit is a low- or high-pass filter.
- An expression in standard form for the voltage transfer function.
- The gain in decibels in the passband, that is, at the frequencies being passed by the filter, and the cutoff frequency.

Analysis:

- From Figure 8.21, it results that the amplifier in Figure P8.40 is a low-pass filter. In fact, the output voltage is

$$\mathbf{V}_0(j\omega) = -\frac{R_2}{R_1} \frac{1}{1 + j\omega CR_2} \mathbf{V}_s(j\omega)$$

$$\text{b) } H_v(j\omega) = \frac{\mathbf{V}_0(j\omega)}{\mathbf{V}_i(j\omega)} = -\frac{R_2}{R_1} \frac{1}{1 + j\omega CR_2}$$

- The gain in decibel is obtained by evaluating $|H_v(j\omega)|$ at $\omega=0$, i.e.

$$|H_v(j0)|_{dB} = 20 \text{Log} \frac{R_2}{R_1} = 20 \text{Log} \frac{22}{9.1} = 7.66 \text{ dB}.$$

The cutoff frequency is

$$\omega_0 = \frac{1}{CR_2} = \frac{1}{0.47 \cdot 10^{-6} \cdot 22 \cdot 10^3} = 96.71 \text{ rad/s}$$

Problem 8.41**Solution:****Known quantities:**

For the circuit shown in Figure P8.40: $C = 0.47 \text{ nF}$ $R_1 = 2.2 \text{ k}\Omega$ $R_2 = 68 \text{ k}\Omega$ $R_L = 1 \text{ k}\Omega$.

Find:

- An expression in standard form for the voltage frequency response function.
- The gain in decibels in the passband, that is, at the frequencies being passed by the filter, and the cutoff frequency.

Analysis:

- From Figure 8.21, it results that the amplifier in Figure P8.40 is a low-pass filter. The voltage frequency response function is

$$H_v(j\omega) = \frac{\mathbf{V}_o(j\omega)}{\mathbf{V}_i(j\omega)} = -\frac{R_2}{R_1} \frac{1}{1 + j\omega CR_2}$$

- The gain in decibel is obtained by evaluating $|H_v(j\omega)|$ at $\omega=0$, i.e.

$$|H_v(j0)|_{dB} = 20 \text{ Log} \frac{R_2}{R_1} = 20 \text{ Log} \frac{68}{2.2} = 29.8 \text{ dB}.$$

The cutoff frequency is

$$\omega_0 = \frac{1}{CR_2} = \frac{1}{0.47 \cdot 10^{-9} \cdot 68 \cdot 10^3} = 31289 \text{ rad/s}$$

Problem 8.42**Solution:****Known quantities:**

For the circuit shown in Figure P8.42: $C_1 = C_2 = 0.1 \text{ }\mu\text{F}$ $R_1 = R_2 = 10 \text{ k}\Omega$.

Find:

- The passband gain.
- The resonant frequency.
- The cutoff frequencies.
- The circuit Q.
- The Bode plot.

Analysis:

- From Figure 8.26, we have

$$A_{BP}(j\omega) = -\frac{\mathbf{Z}_2}{\mathbf{Z}_1} = -\frac{j\omega C_1 R_2}{(1 + j\omega C_1 R_1)(1 + j\omega C_2 R_2)} = -\frac{j\omega C_1 R_1}{(1 + j\omega C_1 R_1)^2} = -\frac{j\omega C_1 R_1}{(j\omega)^2 (C_1 R_1)^2 + 2j\omega C_1 R_1 + 1}$$

The magnitude of the frequency response is $|A_{BP}(j\omega)| = \frac{\omega C_1 R_1}{1 + \omega^2 (C_1 R_1)^2}$

The passband gain is the maximum over ω of the magnitude of the frequency response, i.e.

$$\left| A_{BP}\left(j \frac{1}{C_1 R_1}\right) \right| = \frac{1}{2}$$

- The resonant frequency is $\omega_n = \frac{1}{R_1 C_1} = 1000 \text{ rad/s}$

- The cutoff frequencies are obtained by solving with respect to ω the equation

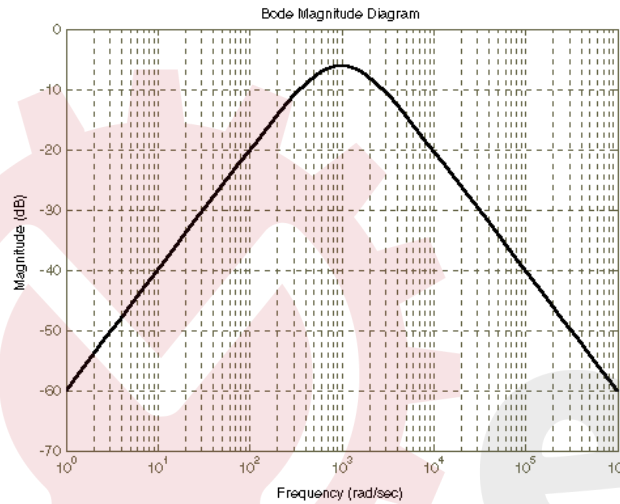
$$|A_{BP}(j\omega)| = \frac{\omega C_1 R_1}{1 + \omega^2 (C_1 R_1)^2} = \frac{1}{2\sqrt{2}} \Rightarrow \omega^2 - 2\sqrt{2}\omega_n \omega + \omega_n^2 = 0 \Rightarrow \omega_{1,2} = \omega_n(\sqrt{2} \pm 1)$$

$$\omega_1 = 2414 \text{ rad/s}$$

$$\omega_2 = 414 \text{ rad/s}$$

d) In this case $\zeta=1$, which implies $Q = \frac{1}{2\zeta} = \frac{1}{2}$

e) The Bode plot of the frequency response is as shown.



Problem 8.43

Solution:

Known quantities:

For the circuit shown in Figure P8.43: $C = 0.47 \text{ nF}$ $R_1 = 220\Omega$ $R_2 = 68\text{k}\Omega$ $R_L = 1\text{k}\Omega$.

Find:

- An expression in standard form for the voltage frequency response function.
- The gain in decibels in the passband, that is, at the frequencies being passed by the filter, and the cutoff frequency.

Analysis:

- a) The voltage frequency response function is

$$H_v(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{R_2}{R_1} \frac{1}{1 + j\omega C R_2}$$

- b) The gain in decibel is obtained by evaluating $|H_v(j\omega)|$ at $\omega=0$, i.e.

$$|H_v(j0)|_{dB} = 20 \text{Log} \frac{R_2}{R_1} = 20 \text{Log} \frac{68000}{220} = 49.8 \text{ dB}.$$

The cutoff frequency is

$$\omega_0 = \frac{1}{C R_2} = \frac{1}{0.47 \cdot 10^{-9} \cdot 68 \cdot 10^3} = 31289 \text{ rad/s}$$

Problem 8.44**Solution:****Known quantities:**

For the circuit shown in Figure P8.44: $C_1 = 2.2 \mu\text{F}$ $C_2 = 1 \text{ nF}$ $R_1 = 2.2 \text{ k}\Omega$ $R_2 = 100 \text{ k}\Omega$.

Find:

Determine the passband gain.

Analysis:

The voltage frequency response is

$$A_{BP}(j\omega) = -\frac{Z_2}{Z_1} = -\frac{j\omega C_1 R_2}{(1 + j\omega C_1 R_1)(1 + j\omega C_2 R_2)} \Rightarrow |A_{BP}(j\omega)| = \frac{\omega C_1 R_2}{\sqrt{1 + \omega^2 (C_1 R_1)^2} \sqrt{1 + \omega^2 (C_2 R_2)^2}}$$

The cutoff frequencies are

$$\omega_1 = \frac{1}{C_1 R_1} = \frac{1}{2.2 \cdot 10^{-6} \cdot 2.2 \cdot 10^3} = 206.6 \text{ rad/s}$$

$$\omega_2 = \frac{1}{C_2 R_2} = \frac{1}{1 \cdot 10^{-9} \cdot 100 \cdot 10^3} = 10000 \text{ rad/s}$$

The passband gain can be calculated approximately by evaluating the magnitude of the frequency response at frequencies greater than ω_1 and smaller than ω_2 , i.e.

$$\omega_1 \ll \omega \ll \omega_2 \Rightarrow 1 + \omega^2 (C_1 R_1)^2 \approx \omega^2 (C_1 R_1)^2, \quad 1 + \omega^2 (C_2 R_2)^2 \approx 1$$

$$|A_{BP}| \approx \frac{\omega C_1 R_2}{\sqrt{\omega^2 (C_1 R_1)^2}} = \frac{R_2}{R_1} = \frac{100}{2.2} = 45.45$$

Problem 8.45**Solution:****Known quantities:**

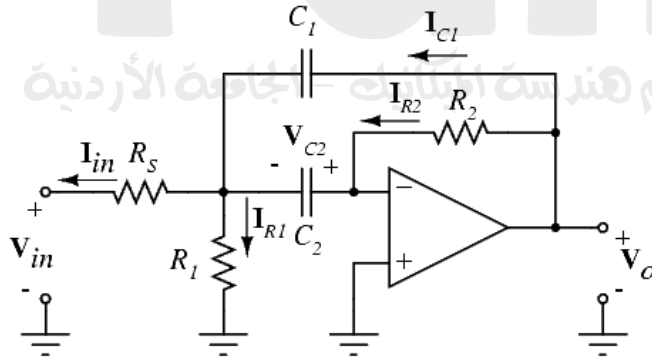
For the circuit shown in Figure P8.45, let $C = C_1 = C_2 = 220 \mu\text{F}$ $R = R_1 = R_2 = R_S = 2 \text{ k}\Omega$.

Find:

Determine the frequency response.

Analysis:

With reference to the Figure shown below, we have



$$\mathbf{I}_{R_2}(j\omega) = \frac{1}{R_2} \mathbf{V}_o(j\omega) \Rightarrow \mathbf{V}_{C_2}(j\omega) = \frac{1}{j\omega C_2} \mathbf{I}_{R_2}(j\omega) = \frac{1}{j\omega C_2 R_2} \mathbf{V}_o(j\omega),$$

$$\mathbf{I}_{R_1}(j\omega) = -\frac{1}{R_1} \mathbf{V}_{C_2}(j\omega) = -\frac{1}{j\omega C_2 R_2 R_1} \mathbf{V}_o(j\omega),$$

$$\mathbf{I}_{C_1}(j\omega) = \frac{\left(R_2 + \frac{1}{j\omega C_2}\right) \frac{1}{R_2} \mathbf{V}_o(j\omega)}{\frac{1}{j\omega C_1}} = \left(1 + \frac{1}{j\omega C_2 R_2}\right) j\omega C_1 \mathbf{V}_o(j\omega),$$

$$\begin{aligned} \mathbf{I}_{in}(j\omega) &= \mathbf{I}_{C_1}(j\omega) + \mathbf{I}_{R_2}(j\omega) - \mathbf{I}_{R_1}(j\omega) = \left(j\omega C_1 + \frac{C_1}{C_2 R_2} + \frac{1}{R_2} + \frac{1}{j\omega C_2 R_2 R_1}\right) \mathbf{V}_o(j\omega) = \\ &= \left(\frac{2}{R} + j\omega C + \frac{1}{j\omega C R^2}\right) \mathbf{V}_o(j\omega), \end{aligned}$$

$$\begin{aligned} \mathbf{V}_{in}(j\omega) &= -\mathbf{V}_{C_2}(j\omega) - R_s \mathbf{I}_{in}(j\omega) = -\frac{1}{j\omega C R} \mathbf{V}_o(j\omega) - \left(2 + j\omega C R + \frac{1}{j\omega C R}\right) \mathbf{V}_o(j\omega) = \\ &= -\left(2 + j\omega C R + \frac{2}{j\omega C R}\right) \mathbf{V}_o(j\omega) \Rightarrow \end{aligned}$$

$$H_v(j\omega) = \frac{\mathbf{V}_o(j\omega)}{\mathbf{V}_{in}(j\omega)} = -\frac{1}{2 + j\omega C R + \frac{2}{j\omega C R}} = -\frac{j\omega C R}{(j\omega)^2 (C R)^2 + 2j\omega C R + 2}$$

Problem 8.46

Solution:

Known quantities:

The inverting amplifier shown in Figure P8.46.

Find:

- The frequency response of the circuit.
- If $R_1 = R_2 = 100 \text{ k}\Omega$ and $C = 0.1 \text{ }\mu\text{F}$, compute the attenuation in dB at $\omega = 1,000 \text{ rad/s}$.
- Compute gain and phase at $\omega = 2,500 \text{ rad/s}$.
- Find range of frequencies over which the attenuation is less than 1 dB.

Analysis:

- Applying KCL at the inverting terminal:

$$\frac{v_{OUT}}{v_{IN}} = -\frac{R_2 + \frac{1}{j\omega C}}{R_1} = -\frac{1 + j\omega R_2 C}{j\omega R_1 C}$$

- Gain = 0.043 dB
- Gain = 0.007 dB ; Phase = 177.71°
- To find the desired frequency range we need to solve the equation:

$$\left| \frac{1 + j\omega R_2 C}{j\omega R_1 C} \right| < 0.8913 \text{ since } 20 \log_{10}(0.8913) = -1 \text{ dB}.$$

This yields a quadratic equation in ω , which can be solved to find $\omega > 196.5 \text{ rad/s}$.

Problem 8.47

Solution:

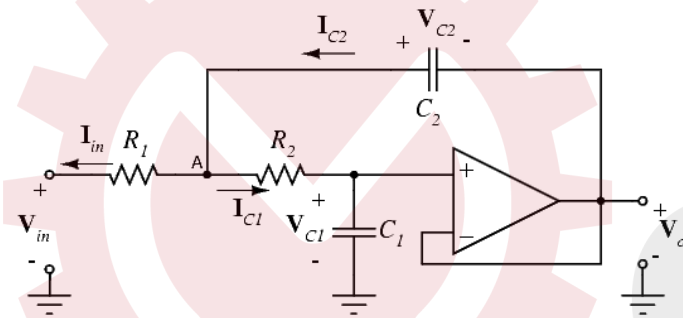
Known quantities:

For the circuit shown in Figure P8.47, let $C = C_1 = C_2 = 100 \mu\text{F}$ $R_1 = 3\text{k}\Omega$ $R_2 = 2\text{k}\Omega$.

Find:

Determine an expression for the gain.

Analysis:



With reference to the Figure shown above, we have

$$\mathbf{V}_{C1}(j\omega) = \mathbf{V}_o(j\omega) \Rightarrow \mathbf{I}_{C1}(j\omega) = j\omega C_1 \mathbf{V}_o(j\omega),$$

$$\mathbf{V}_A(j\omega) = R_2 \mathbf{I}_{C1}(j\omega) + \mathbf{V}_o(j\omega) = (1 + j\omega C_1 R_2) \mathbf{V}_o(j\omega),$$

$$\mathbf{I}_{C2}(j\omega) = j\omega C_2 (\mathbf{V}_o(j\omega) - \mathbf{V}_A(j\omega)) = -(j\omega)^2 C_1 C_2 R_2 \mathbf{V}_o(j\omega),$$

$$\mathbf{V}_{in}(j\omega) = \mathbf{V}_A(j\omega) - R_1 (\mathbf{I}_{C2}(j\omega) - \mathbf{I}_{C1}(j\omega)) = (1 + j\omega C_1 (R_1 + R_2) + (j\omega)^2 C_1 C_2 R_1 R_2) \mathbf{V}_o(j\omega)$$

And finally, the expression for the gain is

$$A_v(j\omega) = \frac{\mathbf{V}_o(j\omega)}{\mathbf{V}_{in}(j\omega)} = \frac{1}{1 + j\omega C(R_1 + R_2) + (j\omega)^2 C^2 R_1 R_2} = \frac{1}{(1 + j\omega C R_1)(1 + j\omega C R_2)}$$

Problem 8.48

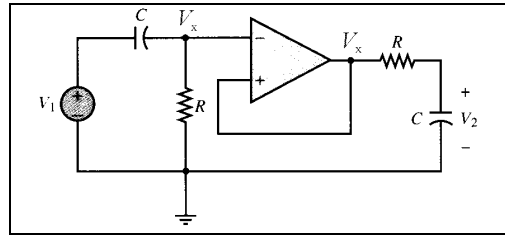
Solution: اللجنة الأكاديمية في قسم هندسة الميكانيك - الجامعة الأردنية

Known quantities:

The circuit shown in Figure P8.48.

Find:

Sketch the amplitude response of V_2 / V_1 , indicating the half-power frequencies. Assume the op-amp is ideal.

Analysis:

From KCL at the inverting input:

$$\frac{V_x - V_1}{Z_C} + \frac{V_x}{R} = 0 \quad V_x \left(j\omega C + \frac{1}{R} \right) = j\omega C V_1 \quad V_x \left(1 + \frac{1}{j\omega RC} \right) = V_1 \quad V_x = V_1 \left(\frac{1}{1 + \frac{1}{j\omega RC}} \right)$$

Similarly, from KCL at the output of the op-amp:

$$\frac{V_2 - V_x}{R} + \frac{V_2}{Z_C} = 0 \quad V_2 \left(\frac{1}{R} + j\omega C \right) = \frac{V_x}{R} \quad V_x = V_2 (1 + j\omega RC)$$

Combining the above results, we find $\frac{V_2}{V_1} = \frac{j\omega RC}{(1 + j\omega RC)^2}$

or $\left| \frac{V_2}{V_1} \right| = \frac{\omega RC}{1 + (\omega RC)^2}$

This function has the form of a band-pass filter, with maximum value determined as follows:

$$G = \left| \frac{V_2}{V_1} \right| = \frac{\omega RC}{1 + (\omega RC)^2} \quad \frac{dG}{d\omega} = \frac{[1 + (\omega RC)^2]RC - \omega RC[2\omega(RC)^2]}{[1 + (\omega RC)^2]^2}$$

Setting the derivative equal to zero and solving for the center frequency,

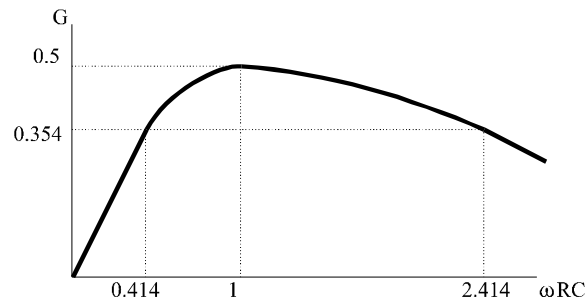
$$RC + \omega^2 R^3 C^3 - 2\omega^2 R^3 C^3 = 0 \quad 1 - \omega^2 R^2 C^2 = 0 \quad \omega = \frac{1}{RC}$$

Then $G_{\max} = \frac{1}{1+1} = \frac{1}{2}$, and the half-power frequencies are given by:

$$\frac{\omega RC}{1 + (\omega RC)^2} = \frac{1}{\sqrt{2}} G_{\max} = \frac{1}{\sqrt{2}} \frac{1}{2} \quad \omega^2 R^2 C^2 + 1 = 2\sqrt{2}\omega RC \quad R^2 C^2 \omega^2 - 2\sqrt{2}RC\omega + 1 = 0$$

$$\omega = \frac{2\sqrt{2}RC \pm \sqrt{8R^2 C^2 - 4R^2 C^2}}{2R^2 C^2} = \frac{\sqrt{2} \pm 1}{RC}$$

The curve is sketched below.



Problem 8.49

Solution:

Known quantities:

The circuit shown in Figure P8.49.

Find:

Determine an analytical expression for the output voltage for the circuit shown in Figure P8.49. What kind of filter does this circuit implement?

Analysis:

The resistance R_{F1} does not influence the output voltage, so

$$\mathbf{V}_o(j\omega) = -\frac{1}{j\omega C_F R_{S1}} \mathbf{V}_{S1}(j\omega) = -\frac{1}{j\omega C_F R_{S1}} \mathbf{V}_{S1}(j\omega)$$

The circuit is an integrator (low-pass filter).

Problem 8.50

Solution:

Known quantities:

The circuit shown in Figure P8.50.

Find:

Determine an analytical expression for the output voltage for the circuit shown in Figure P8.50. What kind of filter does this circuit implement?

Analysis:

Figure P8.50 shows a noninverting amplifier, so the output voltage is given by

$$\mathbf{V}_o(j\omega) = \left(1 + \frac{R_F + \frac{1}{j\omega C_F}}{\frac{1}{j\omega C_S}} \right) \mathbf{V}_s(j\omega) = \left(1 + \frac{C_S}{C_F} + j\omega C_S R_F \right) \mathbf{V}_s(j\omega)$$

The filter is clearly a high-pass filter.

Sections 8.4, 8.5: Integrator and Differentiator Circuits, Analog Computers

Problem 8.51

Solution:

Known quantities:

Figure P8.50.

Find:

If: $C = 1 \mu\text{F}$ $R = 10 \text{ k}\Omega$ $R_L = 1 \text{ k}\Omega$, determine an expression for and plot the output voltage as a function of time.

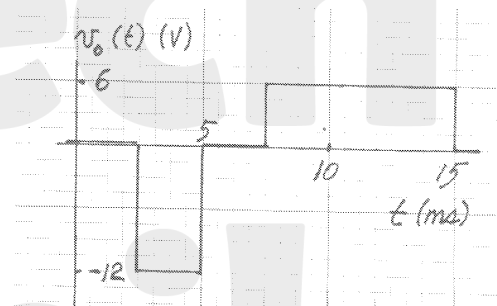
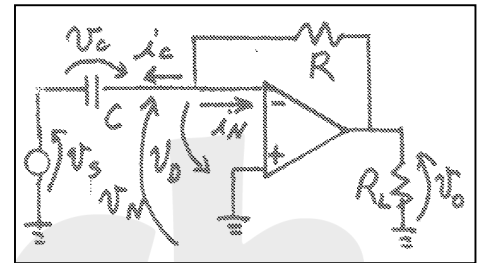
Analysis:

$$\text{KVL: } -v_N - v_D = 0 \quad v_D \approx 0 \Rightarrow v_N \approx 0$$

$$i_C = C \frac{dv_C}{dt} = C \frac{d[v_N - v_S]}{dt}$$

$$\text{KCL: } C \frac{d[v_N - v_S]}{dt} + i_N + \frac{v_N - v_O}{R} = 0$$

$$i_N \approx 0 \quad v_N \approx 0 \Rightarrow v_O = -RC \frac{dv_S}{dt}$$



The derivative is the slope of the curve for the source voltage, which is zero for:

$t < 2.5 \text{ ms}$, $5 \text{ ms} < t < 7.5 \text{ ms}$, and $t > 15 \text{ ms}$

$$\text{For } 2.5 \text{ ms} < t < 5 \text{ ms: } v_O = [10 \cdot 10^3] [1 \cdot 10^{-6}] \frac{3 - 0}{5 \cdot 10^{-3} - 2.5 \cdot 10^{-3}} = -12 \text{ V}$$

$$\text{For } 7.5 \text{ ms} < t < 15 \text{ ms: } v_O = -10 \cdot 10^3 \frac{[-1.5] - [+3]}{15 \cdot 10^{-3} - 7.5 \cdot 10^{-3}} = +6 \text{ V}$$

Problem 8.52

Solution:

Known quantities:

Figure P8.52(a) and Figure P8.52(b).

Find:

If: $C = 1 \mu\text{F}$ $R = 10 \text{ k}\Omega$ $R_L = 1 \text{ k}\Omega$

a) An expression for the output voltage.

- b) The value of the output voltage at $t = 5, 7.5, 12.5, 15$, and 20 ms and a plot of the output voltage as a function of time.

Analysis:

- a) As usual, assume the op amp is ideal so:

$$KVL: -v_N - v_D = 0 \quad v_D \approx 0 \Rightarrow v_N \approx 0$$

$$i_C = C \frac{d[v_N - v_O]}{dt}$$

$$KCL: \frac{v_N - v_S}{R} + i_N + C \frac{d[v_N - v_O]}{dt} = 0$$

$$v_N \approx 0 \quad i_N \approx 0 \Rightarrow d v_O = -\frac{1}{RC} v_S dt$$

$$\text{Integrating: } v_O[t_f] = v_O[t_i] - \frac{1}{RC} \int_{t_i}^{t_f} v_S dt$$

- b) Integrating gives the area under a curve. Recall area of triangle = $1/2[\text{base} \times \text{height}]$ and area of rectangle = base $\times$ height.

Integrating the source voltage when it is constant gives an output voltage which is a linear function of time.

Integrating the source voltage when it is a linear function of time gives an output voltage which is a quadratic function of time.

$$\frac{1}{RC} = \frac{1}{[10 \cdot 10^3][1 \cdot 10^{-6}]} = 100 \frac{\text{rad}}{\text{s}}$$

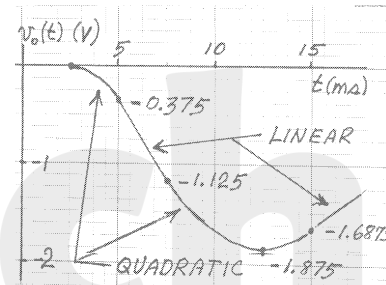
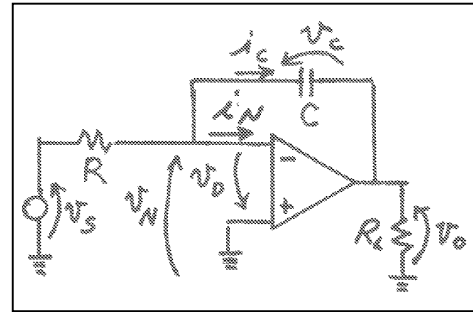
$$v_O[5\text{ms}] = 0 \text{ V} - [100] \frac{1}{2} [2.5 \cdot 10^{-3}] [3 \text{ V}] = -375 \text{ mV}$$

$$v_O[7.5\text{ms}] = -375 \cdot 10^{-3} - [100] [2.5 \cdot 10^{-3}] [3] = -1125 \text{ mV}$$

$$v_O[12.5\text{ms}] = -1125 \cdot 10^{-3} - [100] \frac{1}{2} [5 \cdot 10^{-3}] [3] = -1875 \text{ mV}$$

$$v_O[15\text{ms}] = -1875 \cdot 10^{-3} - [100] \frac{1}{2} [2.5 \cdot 10^{-3}] [-1.5] = -1687.5 \text{ mV}$$

$$v_O[20\text{ms}] = -1687.5 \cdot 10^{-3} - [100] [5 \cdot 10^{-3}] [-1.5] = -937.5 \text{ mV}$$

**Problem 8.53****Solution:****Known quantities:**

In the circuit shown in Figure P8.53, the capacitor is initially uncharged, and the source voltage is:

$$v_{in}(t) = 10 \cdot 10^{-3} + \sin(2,000\pi) \text{ V}.$$

Find:

- a) At $t = 0$, the switch S_I is closed. How long does it take before clipping occurs at the output if $R_S = 10 \text{ k}\Omega$ and $C_F = 0.008 \mu\text{F}$?
- b) At what times does the integration of the DC input cause the op-amp to saturate fully?

Analysis:

$$v_{out} = -\frac{1}{R_S C_F} \int v_{in}(t) dt = -\frac{1}{R_S C_F} \int_0^{\tau} [0.01 + \sin(2000\pi t)] dt$$

a)

$$= -\frac{1}{R_S C_F} \int_0^{\tau} 0.01 dt - \frac{1}{R_S C_F} \int_0^{\tau} \sin(2000\pi t) dt$$

The peak amplitude of the AC portion of the output is: $v_p = \frac{1}{R_S C_F} \left(\frac{1}{2000\pi} \right) = 1.989 \text{ V}$

The output will begin to clip when: $v_0(DC) - v_p = -15 \text{ V}$ so we need to find at what time the

condition: $-\frac{1}{R_S C_F} \int_0^{\tau} 0.01 dt = -13 \text{ V}$ is satisfied. The answer is found below:

$$-\frac{1}{R_S C_F} 0.01\tau = -13 \Rightarrow \tau = \frac{13 R_S C_F}{0.01} = 104 \text{ ms}$$

b) Using the results obtained in part a.: $\tau = \frac{15 R_S C_F}{0.01} = 120 \text{ ms}$

Problem 8.54**Solution:****Known quantities:**

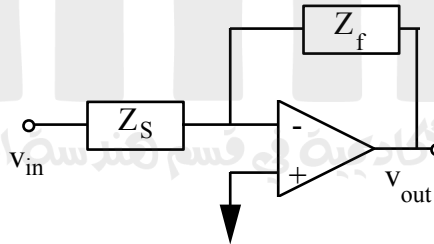
The circuit shown in Figure 8.21.

Find:

- If $R_S = 10 \text{ k}\Omega$, $R_F = 2 \text{ M}\Omega$, $C_F = 0.008 \mu\text{F}$, and $v_S(t) = 10 + \sin(2,000\pi t) \text{ V}$, find v_{out} using phasor analysis.
- Repeat part a if $R_F = 200 \text{ k}\Omega$, and if $R_F = 20 \text{ k}\Omega$.
- Compare the time constants with the period of the waveform for part a and b. What can you say about the time constant and the ability of the circuit to integrate?

Analysis:

- a) Replacing the circuit elements with the corresponding impedances:



$$Z_f = -\frac{R_f}{1 + j\omega R_f C_f} \quad Z_S = R_S$$

For the signal component at $\omega = 2,000\pi$:

$$v_{out} = -\frac{R_f}{R_s} \left(\frac{1}{1 + j\omega R_f C_f} \right) v_{in} = -200 \left(\frac{1}{1 + j\omega/62.5} \right) v_{in}$$

$$= v_{in} \frac{200}{\sqrt{1 + (\omega/62.5)^2}} \angle (180^\circ - \arctan(\omega/62.5)) = 1.9839 \angle 90.57^\circ \text{ V}$$

For the signal component at $\omega = 0$ (DC): $v_{out} = -200v_{in} = -2,000 \text{ V}$. Thus,

$$v_{out}(t) = -2000 + 1.9839 \sin(2,000\pi t + 90.57^\circ) \text{ V} \approx -2000 + 2 \cos(2,000\pi t) \text{ V}.$$

b) $R_F = 200 \text{ k}\Omega$

For the signal component at $\omega = 2,000\pi$:

$$v_{out} = -\frac{R_f}{R_s} \left(\frac{1}{1 + j\omega R_f C_f} \right) v_{in} = 1.9797 \angle 95.68^\circ \text{ V}$$

For the signal component at $\omega = 0$ (DC): $v_{out} = -20v_{in} = -200 \text{ V}$. Thus,

$$v_{out}(t) = -200 + 1.9797 \sin(2,000\pi t + 95.68^\circ) \text{ V} \approx -200 + 2 \cos(2,000\pi t) \text{ V}$$

$R_F = 20 \text{ k}\Omega$

For the signal component at $\omega = 2,000\pi$:

$$v_{out} = -\frac{R_f}{R_s} \left(\frac{1}{1 + j\omega R_f C_f} \right) v_{in} = 1.41 \angle 134.8^\circ \text{ V}$$

For the signal component at $\omega = 0$ (DC): $v_{out} = -2v_{in} = -20 \text{ V}$. Thus,

$$v_{out}(t) = -20 + 1.41 \sin(2,000\pi t + 135^\circ) \text{ V} \approx -20 + 1.41 \cos(2,000\pi t + 45^\circ) \text{ V}.$$

c)

R_f	τ	T
2 M Ω	16 ms	1 ms
200 k Ω	1.6 ms	1 ms
20 k Ω	0.16 ms	1 ms

In order to have an ideal integrator, it is desirable to have $\tau \gg T$.

Problem 8.55

Solution:

Known quantities:

For the circuit of Figure 8.26, assume an ideal op-amp with $v_S(t) = 10 \cdot 10^{-3} \sin(2,000\pi t) \text{ V}$,

$C_S = 100 \mu\text{F}$, $C_F = 0.008 \mu\text{F}$, $R_F = 2 \text{ M}\Omega$, and $R_S = 10 \text{ k}\Omega$.

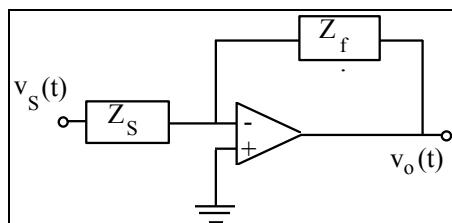
Find:

a) The frequency response, $\frac{v_0}{v_S}(j\omega)$.

b) Use superposition to find the actual output voltage (remember that DC = 0 Hz).

Analysis:

a)



$$Z_S = R_S + \frac{1}{j\omega C_S} \quad Z_f = \frac{R_f}{1 + R_f C_f j\omega}$$

$$\frac{v_o}{v_S} = -\frac{Z_f}{Z_S} = -\frac{\frac{R_f}{1 + j\omega R_f C_f}}{R_S + \frac{1}{j\omega C_S}} = -\frac{j\omega R_f C_S}{(j\omega R_S C_S + 1)(j\omega R_f C_f + 1)}$$

$$\frac{v_o}{v_S}(j\omega) = -\frac{j\omega}{5 \cdot 10^{-3} \left((j\omega + 1) \left(\frac{j\omega}{62.5} + 1 \right) \right)}$$

b) $v_o = v_S \frac{v_o}{v_S}(j\omega)$. By superposition, $v_o|_{10\text{mV}} = v_S \frac{j0v_S}{1+1} = 0\text{V}$

$$v_o|_{\omega=2000\pi} = \frac{j1.257 \cdot 10^6 v_S}{(1 + j6283)(1 + j100)} = 20 \cdot 10^{-3} \angle -89.43^\circ \text{ V}$$

$$v_o(t) = 20 \cdot 10^{-3} \sin(2000\pi t - 89.43^\circ) \text{ V}$$

We can say that the practical differentiator is a good approximation of the ideal differentiator.

Problem 8.56**Solution:****Find:**

Derive the differential equation corresponding to the analog computer simulation circuit of Figure P8.56.

Analysis:

$$x(t) = -200 \int z \, dt \quad \text{or} \quad z = -\frac{1}{200} \frac{dx}{dt} \quad \text{Also, } z = -20y.$$

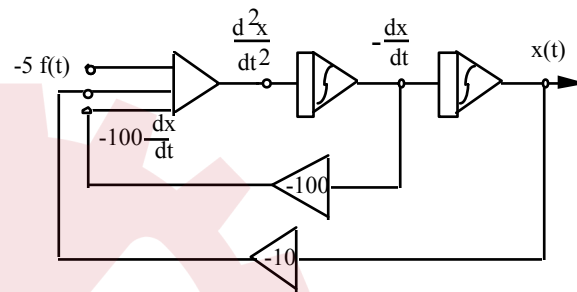
$$\text{Therefore, } y = \frac{1}{4000} \frac{dx}{dt} \quad \text{Also, } y = \int (4f(t) + x(t)) \, dt \quad \text{or} \quad \frac{dy}{dt} = 4f(t) + x(t).$$

$$\text{Therefore} \quad \frac{1}{4000} \frac{d^2x}{dt^2} = 4f(t) + x(t) \quad \text{or} \quad \frac{d^2x}{dt^2} - 4000x(t) - 16000f(t) = 0.$$

Problem 8.57**Solution:****Find:**

Construct the analog computer simulation corresponding to the following differential equation:

$$\frac{d^2x}{dt^2} + 100\frac{dx}{dt} + 10x = -5f(t).$$

Analysis:

Section 8.6: Physical Limitations of Operational Amplifiers

Problem 8.58

Solution:

Known quantities:

For the circuit shown in Figure 8.8: $R_S = R_F = 2.2\text{k}\Omega$.

Find:

Find the error introduced in the output voltage if the op-amp has an input offset voltage of 2 mV. Assume zero bias currents and that the offset voltage appears as in Figure 8.48.

Analysis:

By superposition, the error is given by

$$\Delta v_O = \left(1 + \frac{R_F}{R_S}\right) V_{\text{Offset}} = 4\text{mV}$$

Problem 8.59

Solution:

Known quantities:

For the circuit shown in Figure 8.8: $R_S = R_F = 2.2\text{k}\Omega$.

Find:

Repeat Problem 8.58 assuming that in addition to the input offset voltage, the op-amp has an input bias current of 1 μA . Assume that the bias currents appear as in Figure 8.49.

Analysis:

By superposition, the effect of the bias currents on the output is

$$v_+ = -RI_{B+} = v_-$$

$$\frac{\Delta v_{O,IB} - v_-}{R_F} - \frac{v_-}{R_S} = I_{B-} \Rightarrow \Delta v_{O,IB} = R_F \left(I_{B-} - R \left(\frac{1}{R_F} + \frac{1}{R_S} \right) I_{B+} \right)$$

$$\text{If } R = R_F \parallel R_S \text{ then } \Delta v_{O,IB} = R_F (I_{B-} - I_{B+}) = -R_F I_{OS} = -2200 \cdot 1 \cdot 10^{-6} = -2.2\text{mV}$$

The effect of the bias voltage is

$$\Delta v_{O,VB} = \left(1 + \frac{R_F}{R_S}\right) V_{\text{Offset}} = 4\text{mV}$$

So, the total error on the output voltage is

$$\Delta v_O = \Delta v_{O,VB} + \Delta v_{O,IB} = 1.8\text{mV}$$

Problem 8.60

Solution:

Known quantities:

The circuit shown in Figure P8.60.

The input bias currents are equal and the input bias voltage is zero.

Find:

The value of R_x that eliminates the effect of the bias currents in the output voltage.

Analysis:

We can write

$$v_+ = -R_x I_{B+} = v_- \quad , \quad I_{B+} = I_{B-}$$

$$\frac{\Delta v_{O,IB} - v_-}{R_F} - \frac{v_-}{R_1} = I_{B-} \Rightarrow \Delta v_{O,IB} = R_F \left(I_{B-} - R_x \left(\frac{1}{R_F} + \frac{1}{R_1} \right) I_{B+} \right)$$

By selecting $R_x = R_F \parallel R_1$, a zero output voltage error is obtained.

Problem 8.61**Solution:****Known quantities:**

For the circuit shown in Figure P8.60: $R_F = 3.3\text{k}\Omega$ $R_1 = 1\text{k}\Omega$ $v_s = 1.5 \sin(\omega t)$ V

Find:

The highest-frequency input that can be used without exceeding the slew rate limit of $1\text{V}/\mu\text{s}$.

Analysis:

The maximum slope of a sinusoidal signal at the output of the amplifier is

$$S_0 = |A| \cdot \omega = \frac{R_F}{R_1} \omega = 10^6 \frac{\text{V}}{\text{s}} \Rightarrow \omega_{\max} = \frac{1}{3.3} 10^6 = 3.03 \cdot 10^5 \text{ rad/s}$$

Problem 8.62**Solution:****Known quantities:**

The Bode plot shown in Figure 8.45: $A_0 = 10^6$ $\omega_0 = 10\pi \text{ rad/s}$

Find:

The approximate bandwidth of a circuit that uses the op-amp with a closed loop gain of $A_1 = 75$ and $A_2 = 350$.

Analysis:

The product of gain and bandwidth in any given op-amp is constant, so

$$\omega_1 = \frac{A_0 \omega_0}{A_1} = \frac{10^7 \pi}{75} = 4.186 \cdot 10^5 \text{ rad/s}$$

$$\omega_2 = \frac{A_0 \omega_0}{A_2} = \frac{10^7 \pi}{350} = 8.971 \cdot 10^4 \text{ rad/s}$$

Problem 8.63**Solution:****Known quantities:**

For the practical charge amplifier circuit shown in Figure P8.63, the user is provided with a choice of three time constants - $\tau_{long} = R_L C_F$, $\tau_{medium} = R_M C_F$, $\tau_{short} = R_S C_F$, which can be selected by means of a switch. Assume that $R_L = 10 \text{ M}\Omega$, $R_M = 1 \text{ M}\Omega$, $R_S = 0.1 \text{ M}\Omega$, and $C_F = 0.1 \mu\text{F}$.

Find:

Analyze the frequency response of the practical charge amplifier for each case, and determine the lowest input frequency that can be amplified without excessive distortion for each case. Can this circuit amplify a DC signal?

Analysis:

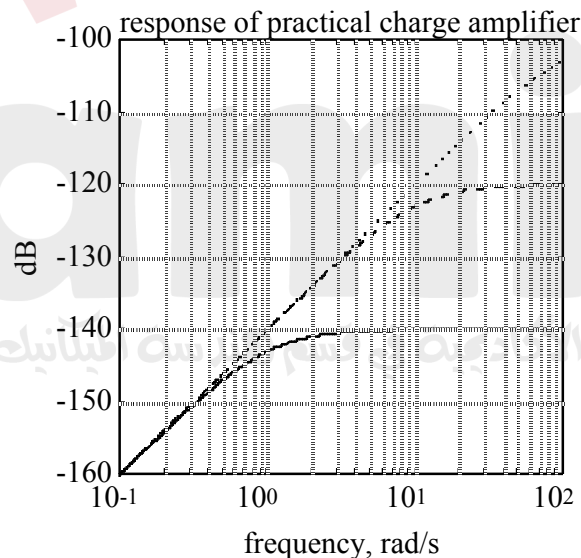
Applying KCL at the inverting terminal:

$$i + \frac{V_{out}}{R + \frac{1}{j\omega C}} = 0$$

or,
$$\frac{V_{out}}{i} = - \frac{j\omega C}{1 + j\omega RC}$$

This response is clearly that of a high-pass filter, therefore the charge amplifier will never be able to amplify a DC signal.

The low end of the (magnitude) frequency response is plotted below for the three time constants. The figure illustrates how as the time constant decreases the cut-off frequency moves to the right (solid line: $R = 10 \text{ M}\Omega$; dashed line: $R = 1 \text{ M}\Omega$; dotted line: $R = 0.1 \text{ M}\Omega$).



From the frequency response plot one can approximate the minimum useful frequency for distortionless response to be (nominally) 1 Hz for the $10 \text{ M}\Omega$ case, 10 Hz for the $1 \text{ M}\Omega$ case, and 100 Hz for the $0.1 \text{ M}\Omega$ case.

Problem 8.64**Solution:****Find:**

Consider a differential amplifier. We would desire the common-mode output to be less than 1% of the differential-mode output. Find the minimum dB common-mode rejection ratio ($CMRR$) that fulfills this requirement if the differential mode gain $A_{dm} = 1,000$. Let

$$v_1 = \sin(2,000\pi t) + 0.1\sin(120\pi t) \text{ V}$$

$$v_2 = \sin(2,000\pi t + 180^\circ) + 0.1\sin(120\pi t) \text{ V}$$

$$v_0 = A_{dm}(v_1 - v_2) + A_{cm}\left(\frac{v_1 + v_2}{2}\right)$$

Analysis:

We first determine which is the common mode and which is the differential mode signal:

$$v_1 - v_2 = 2\sin(2,000\pi t)$$

$$\frac{v_1 + v_2}{2} = 0.1\sin(120\pi t)$$

$$\text{Therefore, } v_{out} = A_{dm} 2\sin(2000\pi t) + A_{cm} 2\sin(120\pi t)$$

Since we desire the common mode output to be less than 1% of the differential mode output, we require:

$$A_{cm}(0.1) \leq 0.01(2) \text{ or } A_{cm} \leq 0.2.$$

$$CMRR = \frac{A_{dif}}{A_{cm}} \quad \text{So} \quad CMRR_{\min} = \frac{1000}{0.2} = 5000 = 74 \text{ dB}.$$

Problem 8.65**Solution:****Known quantities:**

As indicated in Figure P8.65, the rise time, t_r , of the output waveform is defined as the time it takes for the waveform to increase from 10% to 90% of its final value, i.e.,

$$t_r \equiv t_b - t_a = -\tau(\ln 0.1 - \ln 0.9) = 2.2\tau, \text{ where } \tau \text{ is the circuit time constant.}$$

Find:

Estimate the slew rate for the op-amp.

Analysis:

$$\left| \frac{dv_{out}}{dt} \right|_{\max} = \frac{V_m(0.9 - 0.1)}{(14.5 - 10.1) \times 10^{-6}} = \frac{15 \times 0.8 \text{ V}}{4.4 \text{ } \mu\text{s}} \approx 2.73 \frac{\text{V}}{\mu\text{s}}. \text{ Therefore, the slew rate is approximately}$$

$$2.73 \frac{\text{V}}{\mu\text{s}}.$$

Problem 8.66**Solution:****Find:**

Consider an inverting amplifier with open-loop gain 10^5 . With reference to Equation 8.18,

- a) If $R_S = 10\text{ k}\Omega$ and $R_F = 1\text{ M}\Omega$, find the voltage gain $A_{V(CL)}$.
- b) Repeat part a if $R_S = 10\text{ k}\Omega$ and $R_F = 10\text{ M}\Omega$.
- c) Repeat part a if $R_S = 10\text{ k}\Omega$ and $R_F = 100\text{ M}\Omega$.
- d) Using the resistors values of part c, find $A_{V(CL)}$ if $A_{V(OL)} \rightarrow \infty$.

Analysis:

$$A_{CL} = -\frac{R_F}{R_S} \frac{1}{1 + \frac{R_F + R_S}{R_S A_{OL}}}$$

- a) $A_{CL} = -99.899$
- b) $A_{CL} = -990$
- c) $A_{CL} = -9091$
- d) As $A_{OL} \rightarrow \infty$, $A_{CL} = -10,000$.

Problem 8.67

Solution:

Known quantities:

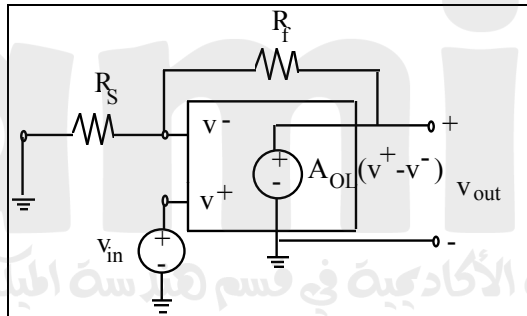
Figure P8.67.

Find:

- a) If the op-amp shown in Figure P8.67 has an open-loop gain of 45×10^5 , find the closed-loop gain for $R_S = R_F = 7.5\text{ k}\Omega$.
- b) Repeat part a if $R_F = 5R_S = 37.5\text{ k}\Omega$.

Analysis:

a)



$$v_{in} = v^+ \quad \text{and} \quad v_0 = A_{OL}(v_{in} - v^-) \Rightarrow v^- = -\left(\frac{v_0}{A_{OL}} - v_{in}\right).$$

$$\text{Writing KCL at } v^-: \quad \frac{v^- - 0}{R_S} + \frac{v^- - v_0}{R_F} = 0.$$

Substituting,

$$\frac{-\frac{v_0}{A_{OL}} + v_{in}}{R_S} + \frac{-\frac{v_0}{A_{OL}} + v_{in}}{R_F} = \frac{v_0}{R_F} \Rightarrow v_0 \left(-\frac{1}{A_{OL} R_S} - \frac{1}{A_{OL} R_F} - \frac{1}{R_F} \right) = -v_{in} \left(\frac{1}{R_S} + \frac{1}{R_F} \right)$$

$$\frac{v_0}{v_{in}} = \frac{-\left(\frac{1}{R_S} + \frac{1}{R_F}\right)}{-\frac{1}{A_{OL}R_S} - \frac{1}{A_{OL}R_F} - \frac{1}{R_F}} = A_{OL}R_S R_F \left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$\text{where: } \begin{aligned} K_1 &= R_F R_S + R_S^2 + R_S A_{OL} R_F \\ K_2 &= R_F R_S + R_F^2 + R_S A_{OL} R_F \end{aligned} \rightarrow \frac{v_0}{v_{in}} = \frac{R_F}{R_S} \left(\frac{1}{\frac{R_S + R_F}{A_{OL}R_S} + 1} \right) + \left(\frac{1}{\frac{R_S + R_F}{A_{OL}R_S} + 1} \right)$$

For the conditions of part a we obtain:

$$A_{CL} = \frac{1}{2/45 \times 10^5 + 1} + \frac{1}{2/45 \times 10^5 + 1} = 1.999$$

$$\text{b) } A_{CL} = 5 \frac{1}{6/45 \times 10^5 + 1} + \frac{1}{6/45 \times 10^5 + 1} = 5.999.$$

Problem 8.68

Solution:

Find:

Given the unity-gain bandwidth for an ideal op-amp equal to 5.0 MHz, find the voltage gain at frequency of $f = 500$ kHz.

Analysis:

$$A_0 \omega_0 = K = A_1 \omega_1$$

$$\Rightarrow K = 1 \times 2\pi \times 5.0 \text{ MHz} = 10\pi \times 10^6$$

$$A_1 = \frac{K}{\omega_1} = \frac{10\pi \times 10^6}{2\pi \times 500} = 10,000$$

Problem 8.69

Solution:

Find:

Determine the relationship between a finite and frequency-dependent open-loop gain $A_{V(OL)}(\omega)$ and the closed-loop gain $A_{V(CL)}(\omega)$ of an inverting amplifier as a function of frequency. Plot $A_{V(CL)}$ versus ω .

Analysis:

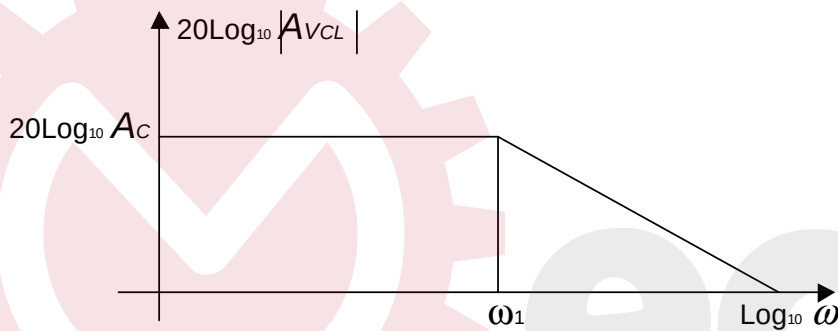
As shown in Equation 8.84, if we consider a real op-amp:

$$\frac{A_{V(OL)}(\omega)}{A_{V(CL)}(\omega)} = \frac{A_0}{1 + j\omega/\omega_0} \cdot \frac{A_C}{1 + j\omega/\omega_1}, \text{ where } A_C = -R_F/R_S, \text{ and } A_0 \text{ is the low-frequency open-loop gain.}$$

If we choose $A_0 = 10^6$, and $\omega_0 = 10\pi$, and since $A_0\omega_0 = A_C\omega_1$:

$$\omega_1 = \frac{10^6 \cdot 10\pi}{R_F/R_S}.$$

Therefore:



Problem 8.70

Solution:

Find:

A sinusoidal sound (pressure) wave impinges upon a condenser microphone of sensitivity S (mV / Pa). The voltage output of the microphone v_S is amplified by two cascaded inverting amplifiers to produce an amplified signal v_0 . Determine the peak amplitude of the sound wave (in dB) if $v_0 = 5 \text{ V}_{\text{RMS}}$. Estimate the maximum peak magnitude of the sound wave in order that v_0 not contain any saturation effects of the op-amps.

Analysis:

$$\left. \begin{aligned} p(t) &= P_0 \sin(\omega t) \text{ Pa} \quad ; \quad v_S(t) = S \cdot p(t) \text{ mV} \\ v_0(t) &= \frac{A_1 A_2}{1000} v_S(t) \text{ V} = \frac{A_1 A_2}{1000} S \cdot P_0 \sin(\omega t) \text{ V} \\ v_0 &= 5 \text{ V}_{\text{RMS}} \end{aligned} \right\} \Rightarrow A_1 A_2 \cdot S \cdot P_0 = 5000 \text{ mV} \Rightarrow$$

$$\Rightarrow P_0 = 20 \log_{10} \frac{5000}{A_1 A_2 \cdot S} \text{ dB}$$

$$\frac{A_1 A_2 \cdot S \cdot P_0}{1000} \leq 12 \text{ V} \Rightarrow P_0 \leq 12,000 \cdot A_1 A_2 \cdot S \text{ Pa}.$$

Problem 8.71**Solution:****Known quantities:**

If, in the circuit shown in Figure P8.71

$$v_{S1} = 2.8 + 0.01 \cos(\omega t) \text{ V} \quad ; \quad v_{S2} = 3.5 - 0.007 \cos(\omega t) \text{ V}$$

$$A_{v1} = -13 \quad ; \quad A_{v2} = 10 \quad ; \quad \omega = 4 \frac{\text{krad}}{\text{s}}$$

Find:

- The common and difference mode input signals.
- The common and differential mode gains.
- The common and difference mode components of the output voltage.
- The total output voltage.
- The common mode rejection ratio.

Analysis:

a)

$$\begin{aligned} v_{S-C} &= \frac{1}{2} [v_{S1} + v_{S2}] = \frac{1}{2} ([2.8 \text{ V} + 10 \text{ mV} \cos \omega t] + [3.5 \text{ V} - 7 \text{ mV} \cos \omega t]) = \\ &= 3.15 \text{ V} + 1.5 \text{ mV} \cos \omega t \\ v_{S-D} &= v_{S1} - v_{S2} = [2.8 \text{ V} + 10 \text{ mV} \cos \omega t] - [3.5 \text{ V} - 7 \text{ mV} \cos \omega t] = \\ &= -0.7 \text{ V} + 17 \text{ mV} \cos \omega t \end{aligned}$$

Note that the expression for the differential input voltage (and the differential gain below) depends on which of the two sources (in this case, v_{S1}) is connected to the non-inverting input.

$$\text{b) } A_{vc} = A_{v1} + A_{v2} = -3 \quad A_{vd} = \frac{1}{2} [A_{v1} - A_{v2}] = -11.5$$

$$\begin{aligned} \text{c) } v_{O-C} &= v_{S-C} A_{vc} = [3.15 \text{ V} + 1.5 \text{ mV} \cos \omega t] [-3] = -9.450 \text{ V} - 4.5 \text{ mV} \cos \omega t \\ v_{O-D} &= v_{S-D} A_{vd} = [-0.7 \text{ V} + 17 \text{ mV} \cos \omega t] [-11.5] = 8.050 \text{ V} - 195.5 \text{ mV} \cos \omega t \end{aligned}$$

$$\text{d) } v_O = v_{O-C} + v_{O-D} = -1.4 \text{ V} - 200.0 \text{ mV} \cos \omega t$$

$$\text{e) } CMRR = 20 \text{ dB} \log_{10} \left[\frac{|A_{vc}|}{|A_{vd}|} \right] = 20 \text{ dB} \log_{10} \left[\frac{3}{11.5} \right] = -11.67 \text{ dB}.$$

Problem 8.72**Solution:****Known quantities:**

If, in the circuit shown in Figure P8.71:

$$v_{S1} = 3.5 + 0.01 \cos \omega t \text{ V} \quad v_{S2} = 3.5 - 0.01 \cos \omega t$$

$$A_{vc} = 10 \text{ dB} \quad A_{vd} = 20 \text{ dB} \quad \omega = 4 \frac{\text{krad}}{\text{s}}$$

Find:

- The common and differential mode input voltages.
- The voltage gains for v_{S1} and v_{S2} .
- The common mode component and differential mode components of the output voltage.
- The common mode rejection ratio [CMRR] in dB.

Analysis:

a)

$$v_{S-C} = \frac{1}{2} [v_{S1} + v_{S2}] = \frac{1}{2} ([3.5 \text{ V} + 10 \text{ mV} \cos \omega t] + [3.5 \text{ V} - 10 \text{ mV} \cos \omega t]) = 3.5 \text{ V}$$

$$v_{S-D} = v_{S1} - v_{S2} = [3.5 \text{ V} + 10 \text{ mV} \cos \omega t] - [3.5 \text{ V} - 10 \text{ mV} \cos \omega t] = 20 \text{ mV} \cos \omega t$$

Note the expression for the difference mode voltage depends on which signal [in this case, v_{S1}] is connected to the non-inverting input. This is also true for the expression for the differential gain below.

b)

$$A_{vc} = 10^{\frac{10 \text{ dB}}{20 \text{ dB}}} = 3.162 \quad A_{vd} = 10^{\frac{20 \text{ dB}}{20 \text{ dB}}} = 10$$

$$A_{vc} = A_{v1} + A_{v2} \Rightarrow A_{v1} = \frac{1}{2} A_{vc} + A_{vd} = \frac{1}{2} [3.162] + 10 = 11.581$$

$$A_{vd} = \frac{1}{2} [A_{v1} - A_{v2}] \Rightarrow A_{v2} = \frac{1}{2} A_{vc} - A_{vd} = \frac{1}{2} [3.162] - 10 = -8.419$$

$$\text{c) } v_{O-C} = v_{S-C} A_{vc} = [3.5 \text{ V}] [3.162] = 11.07 \text{ V}$$

$$v_{O-D} = v_{S-D} A_{vd} = [20 \text{ mV} \cos \omega t] [10] = 200 \text{ mV} \cos \omega t$$

$$\text{d) } CMRR = 20 \text{ dB} \log_{10} \left| \frac{A_{vc}}{A_{vd}} \right| = 20 \text{ dB} \log_{10} \left[\frac{3.162}{10} \right] = -10 \text{ dB}.$$

Problem 8.73**Solution:****Known quantities:**

If, in the circuit shown in Figure P8.73, the two voltage sources are temperature sensors with $T =$ Temperature [Kelvin] and:

$$v_{S1} = k T_1 \quad v_{S2} = k T_2$$

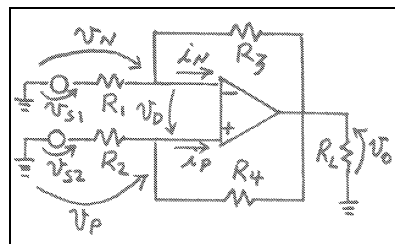
$$\text{Where: } k = 120 \mu \frac{\text{V}}{\text{K}} \quad R_1 = R_3 = R_4 = 5 \text{ k}\Omega \quad R_2 = 3 \text{ k}\Omega \quad R_L = 600 \Omega$$

Find:

- The voltage gains for the two input voltages.
- The common mode and differential mode input voltage.
- The common mode and difference mode gains.
- The common mode component and the differential mode component of the output voltage.
- The common mode rejection ratio [CMRR] in dB.

Analysis:

a)



Assume the op amp is ideal.: $KVL: -v_N - v_D + v_P = 0$
 $v_D \approx 0 \Rightarrow v_N = v_P$

$$KCL: i_N + \frac{v_N - v_{S1}}{R_1} + \frac{v_N - v_O}{R_3} = 0 \quad i_N \approx 0: \Rightarrow v_N = \frac{\frac{v_{S1}}{R_1} + \frac{v_O}{R_3}}{\frac{1}{R_1} + \frac{1}{R_3}}$$

$$KCL: i_P + \frac{v_P - v_{S2}}{R_2} + \frac{v_P - v_O}{R_4} = 0 \quad i_P \approx 0: \Rightarrow v_P = \frac{\frac{v_{S2}}{R_2} + \frac{v_O}{R_4}}{\frac{1}{R_2} + \frac{1}{R_4}}$$

Equating: $\frac{v_{S1} R_3 + v_O R_1}{R_3 + R_1} = \frac{v_{S2} R_4 + v_O R_2}{R_4 + R_2}$

$$\begin{aligned} v_O &= \frac{v_{S2} \frac{R_4}{R_4 + R_2} + v_{S1} \left[-\frac{R_3}{R_3 + R_1} \right]}{\frac{R_1}{R_3 + R_1} - \frac{R_2}{R_4 + R_2}} \frac{[R_4 + R_2][R_3 + R_1]}{[R_4 + R_2][R_3 + R_1]} \\ &= v_{S2} \frac{R_4 [R_3 + R_1]}{R_1 [R_4 + R_2] - R_2 [R_3 + R_1]} + v_{S1} \left[-\frac{R_3 [R_4 + R_2]}{R_1 [R_4 + R_2] - R_2 [R_3 + R_1]} \right] \\ A_{v2} &= \frac{R_4 [R_3 + R_1]}{R_1 [R_4 + R_2] - R_2 [R_3 + R_1]} = \frac{[5][10]}{[5][8] - [3][10]} = 5 \\ A_{v1} &= \frac{-R_3 [R_4 + R_2]}{R_1 [R_4 + R_2] - R_2 [R_3 + R_1]} = \frac{-[5][8]}{[5][8] - [3][10]} = -4 \end{aligned}$$

b)

$$v_{S1} = k T_1 = \left[120 \mu \frac{V}{K} \right] [310 K] = 37.20 \text{ mV}$$

$$v_{S2} = k T_2 = \left[120 \mu \frac{V}{K} \right] [335 K] = 40.20 \text{ mV}$$

$$v_{S-C} = \frac{1}{2} [v_{S1} + v_{S2}] = \frac{1}{2} [37.20 \text{ mV} + 40.20 \text{ mV}] = 38.70 \text{ mV}$$

$$v_{S-D} = v_{S2} - v_{S1} = 40.20 \text{ mV} - 37.2 \text{ mV} = +3 \text{ mV}$$

Note that the expression for the differential mode voltage (and the differential mode gain below) depends on Source #2 being connected to the non-inverting input.

c)

$$\begin{aligned} v_O &= v_{S2} A_{v2} + v_{S1} A_{v1} = \left[v_{S-C} + \frac{1}{2} v_{S-D} \right] A_{v2} + \left[v_{S-C} - \frac{1}{2} v_{S-D} \right] A_{v1} \\ &= v_{S-C} [A_{v2} + A_{v1}] + v_{S-D} \frac{1}{2} [A_{v2} - A_{v1}] = v_{S-C} A_{v-c} + v_{S-D} A_{v-d} \\ \Rightarrow \begin{cases} A_{v-c} = A_{v2} + A_{v1} = 5 + [-4] = 1 \\ A_{v-d} = \frac{1}{2} [A_{v2} - A_{v1}] = \frac{1}{2} [5 - (-4)] = 4.5 \end{cases} \end{aligned}$$

- d) $v_{O-C} = v_{S-C} A_{vc} = [38.70 \text{ mV}][1] = 38.70 \text{ mV}$
 $v_{O-D} = v_{S-D} A_{vd} = [3 \text{ mV}][4.5] = 13.5 \text{ mV}$
 e) An ideal difference amplifier would eliminate all common mode output. This did not happen here. A figure of merit for a differential amplifier is the Common Mode Rejection Ratio [CMRR]:

$$CMRR = 20 \text{ dB} \log_{10} \left[\frac{A_{vc}}{A_{vd}} \right] = 20 \text{ dB} \log_{10} \left[\frac{1}{4.5} \right] = -13.06 \text{ dB}.$$

Problem 8.74

Solution:

Known quantities:

If, for the differential amplifier shown in Figure P8.73:

$$v_{S1} = 13 \text{ mV} \quad v_{S2} = 9 \text{ mV} \quad v_O = v_{OC} + v_{OD}$$

$$v_{OC} = 33 \text{ mV} \quad v_{OD} = 18 \text{ V}$$

Find:

- The common mode gain.
- The differential mode gain.
- The common mode rejection ratio in dB.

Analysis:

a)

$$v_{SC} = \frac{1}{2} [v_{S1} + v_{S2}] = \frac{1}{2} [13 \text{ mV} + 9 \text{ mV}] = 11 \text{ mV}$$

$$\Rightarrow A_{VC} = \frac{v_{OC}}{v_{SC}} = \frac{33 \text{ mV}}{11 \text{ mV}} = 3$$

$$v_{SD} = v_{S2} - v_{S1} = 9 \text{ mV} - 13 \text{ mV} = -4 \text{ mV}$$

b) $\Rightarrow A_{VD} = \frac{v_{OD}}{v_{SD}} = \frac{18 \text{ V}}{-4 \text{ mV}} = -4500$

c) $CMRR = 20 \text{ dB} \log_{10} \left[\frac{|A_{VC}|}{|A_{VD}|} \right] = -63.52 \text{ dB}.$

Chapter 9 Instructor Notes

After a brief introduction to semiconductors materials in Section 9.1, Section 9.2 introduces the *pn* junction and the semiconductor diode equation and *i-v* curves. These sections could be bypassed in favor of a more intuitive presentation, provided in Section 9.3 with an explanation of the basic circuit behavior of the diode, beginning with large-signal models. This explanation is supplemented by the sidebar *Make the Connection: Hydraulic Check Valves* (pp. 466-467), which continues the electric-hydraulic analogy introduced in earlier chapters, and provides an intuitive explanation of the operation of the semiconductor diodes¹. The box *Focus on Methodology: Determining the Conduction State of a Diode* (p. 467) helps the student understand how diode state of conduction can be determined. The large-signal circuit model material is probably sufficient for the purposes of most introductory courses.

Instructors who are interested in introducing the subject of small-signal models, in preparation for the study of transistor small-signal amplifiers, may find the rest of the section, covering small-signal diode models (pp. 474-483), of interest. The concept of operating point is introduced, with a review of the load-line equation and the definition of the quiescent operating point of a device. The box *Focus on Methodology: Determining the Operating Point of a Diode* (p. 475) summarizes this material. Section 3.8 can be recalled (or introduced for the first time) at this point. The solution methods used in Examples 9.5, 9.6, 9.7 emphasize the use of simple circuit models for the diode, together with the use of Thévenin equivalent circuits; this method is quite general, and will reinforce the importance (and understanding) of the concept of equivalent circuits in analyzing more advanced circuits.

The use of Device Data Sheets is continued in this chapter with a summary data sheet on a general purpose diode. The box *Focus on Methodology: Using device data sheets* (pp. 476-477) is designed to familiarize the student with the basic contents and function of device data sheets. The instructor may wish to expand on this introductory presentation by asking students to locate data sheets in the accompanying CD-ROM or on the web, and to identify specific devices and their parameters for use in homework problems.

Section 9.4 discusses various diode rectifier circuits; Section 9.5 introduces DC power supplies; Zener diode circuits and voltage regulation. Section 9.6 analyzes various signal processing circuits, and introduces two application examples in the boxes *Focus on Measurements: Peak Detector Circuit for Capacitive Displacement Transducer* (pp. 499-501), which is tied to two earlier boxes on the capacitive displacement transducer (pp. 147-148 and pp. 175-177), and *Focus on Measurements: Diode Thermometer* (pp. 502-503). The latter example can be tied to a laboratory experiment². Finally, Section 9.7 introduces photodiodes and solar cells and includes the box *Focus on Measurements: Opto-Isolators* (p. 506).

The homework problems present a graded variety of problems, mostly related to the 17 examples and application examples presented in the text.

Learning Objectives

1. Understand the basic principles underlying the physics of semiconductor devices in general, and of the *pn* junction in particular. Become familiar with the diode equation and *i-v* characteristic. [Sections 1, 2.](#)
2. Use various circuit models of the semiconductor diode in simple circuits. These are divided into two classes: large signal model, useful to study rectifier circuits, and small signal models, useful in signal processing applications. [Section 2.](#)
3. Study practical full-wave rectifier circuits and learn to analyze and determine the practical specifications of a rectifier using large-signal diode models. [Section 3.](#)
4. Understand the basic operation of Zener diodes as voltage references, and use simple circuit models to analyze elementary voltage regulators. [Section 4.](#)
5. Use the diode models presented in Section 2 to analyze the operation of various practical diode circuits in signal processing applications. [Section 5.](#)
6. Understand the basic principle of operation of photodiodes, including solar cells, photosensors and light-emitting diodes. [Section 6.](#)

¹ With many thanks to Bill Ribbens, who first suggested this idea to me some 20 years ago.

² G. Rizzoni, A Practical Introduction to Electronic Instrumentation, 3rd Edition, Kendall-Hunt, 1998.

Section 9.1: Electrical Conduction in Semiconductor Devices**Section 9.2: The pn Junction and the Semiconductor Diode****Problem 9.1****Solution:****Known quantities:**

The Ionized Acceptor Density for a doped Silicon:

$$N_a = N_a^- = 10^{17} \frac{1}{\text{m}^3}, N_d = 0$$

Find:

- If this material is an N or P type extrinsic semiconductor.
- Which are the majority and which the minority charge carriers.
- The density of majority and minority carriers.

Analysis:

- Each acceptor dopant atom introduces an additional positive charge carrier and a negative atomic ion. The ion is NOT a charge carrier. The density of positive carriers [holes] increases because of the doping so the material is extrinsic P type Silicon.
- The majority carriers are the positive carriers or valence band holes; the minority carriers are the negative carriers or conduction band free electrons.

$$\text{c) } CNE : p_{po} + 0 - n_{po} - N_a^- = 0, \quad CPE : n_{po} = \frac{n_{io}^2}{p_{po}}$$

$$p_{po} - \frac{n_{io}^2}{p_{po}} - N_a^- = 0, \quad p_{po}^2 + p_{po}(-N_a^-) + (-n_{io}^2) = 0$$

Using the quadratic equation:

$$p_{po} = -\frac{1}{2}(-N_a^-) \pm \frac{1}{2} \left((-N_a^-)^2 - 4(-n_{io}^2) \right)^{1/2} = -\frac{1}{2}(-10^{17}) \pm \frac{1}{2} \left((-10^{17})^2 - 4(-2.25 \cdot 10^{32}) \right)^{1/2} =$$

$$= 510^{16} \pm 5.22 \cdot 10^{16} = 1.022 \cdot 10^{17} \frac{1}{\text{m}^3}$$

where the negative answer is physically impossible.

Now use the CPE to obtain the minority carrier density:

$$n_{po} = \frac{n_{io}^2}{p_{po}} = \frac{2.25 \cdot 10^{32}}{1.022 \cdot 10^{17}} = 2.202 \cdot 10^{15} \frac{1}{\text{m}^3}$$

Note that because of the doping, the hole density is now about 100 times the electron density. The thermally produced carriers present in the intrinsic Silicon before doping has a small effect on the carrier densities in the extrinsic Silicon. At higher doping levels, the effect becomes negligible.

As temperature increases, the densities of the thermally produced carriers increase and their effect on the final carrier densities increase. At very high temperatures [about 175 C for Silicon] the thermally produced carriers primarily determine the final carrier densities and the doping has a negligible effect [ie, the semiconductor behaves as an intrinsic material]. This is why semiconductors cannot operate in high temperature environments.

Problem 9.2

Solution:

Known quantities:

The intrinsic Silicon is doped:

$$N_a = N_a^- = 10^{17} \frac{1}{\text{m}^3}, \quad N_d = N_d^+ = 510^{18} \frac{1}{\text{m}^3}$$

Find:

- If the silicon is an N or P type extrinsic semiconductor.
- Which are the majority and which the minority charge carriers.
- The density of majority and minority carriers.

Analysis:

- N Type
- Majority = Conduction band free electrons = Negative carriers.
Minority = Valence band holes = Positive carriers.

$$c) \quad n_{no} \approx 4.910^{18} \frac{1}{\text{m}^3}, \quad p_{no} = 4.5910^{13} \frac{1}{\text{m}^3}$$

Problem 9.3

Solution:

Find:

Describe the microscopic structure of semiconductor materials. What are the three most commonly used semiconductor materials?

Analysis:

Semiconductor materials are crystalline with the atoms arranged in a repeated three dimensional array. The distance between atoms in the array is the "lattice constant".

Each atom of a semiconductor has four valence electrons. These electrons participate in covalent bonds with the valence electrons of other atoms.

For certain materials with the properties above, quantum/wave mechanics predicts that the valence electrons may have a total energy [kinetic plus Coulombic potential energy] within certain "allowed" bands. The two most important bands are the valence band containing the valence electrons in covalent bonds and the conduction band containing conduction or free electrons which have obtained enough energy to escape from its covalent bond.

Separating these two allowed bands is the "energy gap" extending over those energies which the electrons are "forbidden" to have.

For semiconductor materials, the energy gap is on the order of 1 electron-Volt [eV].

Silicon is the most common semiconductor material and is used in a variety of applications and devices.

Germanium is used in some optical devices and other special purpose devices.

Gallium Arsenide is a compound III-V semiconductor material. One atom has three and the other has five valence electrons giving an average of four per atom. It is used in microwave, optical, and very high speed digital devices.

Problem 9.4

Solution:

Find:

Describe the thermal production of charge carriers in a semiconductor and how this process limits the operation of a semiconductor device.

Analysis:

At a temperature of absolute zero, ALL valence electrons in a semiconductor are contained in a covalent bond and there are NO charge carriers.

The internal or thermal energy of a solid material is caused by the vibration of the atoms and electrons about their equilibrium position. As the temperature of the material increases, its thermal vibrational energy increases. Some electrons will gain sufficient energy to escape the covalent bond in the valence band, and "jump" past the energy gap into the conduction band. As a consequence, TWO charge carriers are generated. The conduction or free electron in the conduction band is a negative charge carrier. The vacancy in the valence band covalent bond or "hole" is a positive charge carrier.

A conduction band electron may also give up energy and recombine with a valence band hole.

The generation and recombination rates both increase with temperature. At any particular temperature, they are equal and produce equal equilibrium densities of electrons and holes. The equilibrium carrier densities increase with temperature. For Silicon at $T = 300\text{ K}$ [approximately room temperature]:

$$n_{io} = p_{io} = 1.510^{10} \frac{\text{carriers}}{\text{cm}^3} = 1.510^{16} \frac{\text{carriers}}{\text{m}^3}$$

[A number of carriers is a dimensionless quantity and may be omitted from the units.]

Almost all semiconductor devices are "doped" to achieve DIFFERENT densities of positive and negative carriers. A "P-type" semiconductor has a higher density of positive carriers and an "N-type" semiconductor has a higher density of negative carriers. However, at high temperatures the density of thermally produced carriers becomes very large and significantly reduces or nullifies the effects of the doping, ie, the positive and negative carrier densities become nearly equal. For this reason, semiconductor devices cannot be used in high temperature applications. The limit in temperature depends on the semiconductor material.

Problem 9.5**Solution:****Find:**

Describe the properties of donor and acceptor dopant atoms and how they affect the densities of charge carriers in a semiconductor material.

Analysis:

An "intrinsic" semiconductor material is undoped. When dopant atoms are added the material becomes an "extrinsic" semiconductor. Doping results in the replacement of an intrinsic atom with a dopant atom. As few as one out of every million intrinsic atoms may be replaced.

A "donor" dopant atom has 5 valence electrons. Only 4 are required to complete the bonding structure in the semiconductor material. The 5th electron requires very little energy to escape to the conduction band and become a negative charge carrier. This leaves behind a donor atom with one missing electron or a negative atomic ion. The ion is immobile and cannot move through the material; therefore, IT IS NOT A CHARGE CARRIER.

Each donor contributes an additional negative carrier to the material. The increased density of negative carriers results in an increased recombination rate which reduces the density of positive carriers. [The PRODUCT of the two densities remains constant.] Materials doped with donor atoms are N type extrinsic semiconductors. The majority carriers are conduction band electrons, the minority carriers are valence band holes.

An "acceptor" dopant atom has 3 valence electrons; however, 4 are required to complete the bonding structure in the semiconductor material. The "missing" 4th electron causes a vacancy or hole in the bonding structure. Another valence electron may move to and occupy this hole thus eliminating it and generating another hole and a negative atomic ion. The ion is immobile and cannot move through the material; therefore, IT IS NOT A CHARGE CARRIER.

Each acceptor contributes an additional positive carrier to the material. The increased density of positive carriers results in an increased recombination rate which reduces the density of negative carriers. [The PRODUCT of the two densities remains constant.] Materials doped with acceptor atoms are P type

extrinsic semiconductors. The majority carriers are valence band holes, the minority carriers are conduction band electrons.

Problem 9.6

Solution:

Find:

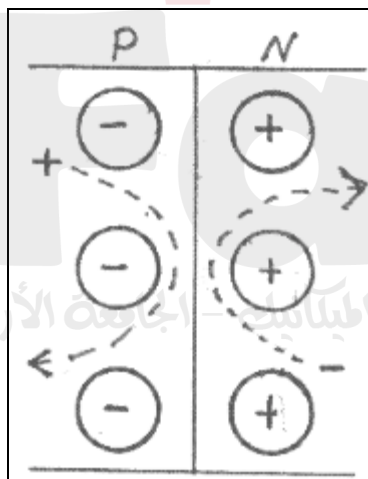
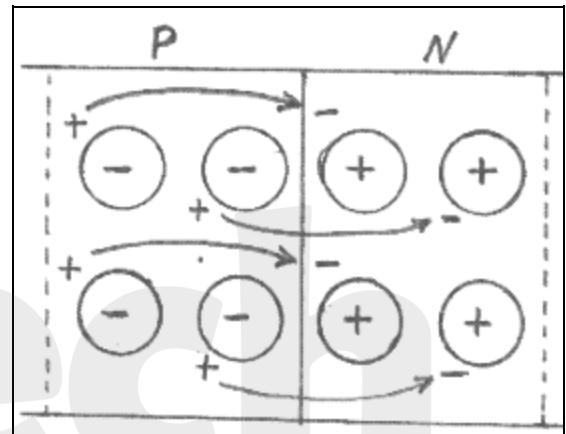
Physically describe the behavior of the charge carriers and ionized dopant atoms in the vicinity of a semiconductor PN junction that causes the potential [energy] barrier that tends to prevent charge carriers from crossing the junction.

Analysis:

Semiconductor atoms are not shown in the two figures. The circles represent ionized dopant atoms and the uncircled plus and minus signs represent charge carriers. The dotted line in the first figure represents how far the depletion/space charge region extends into the P and N regions.

Near the junction, the negative carriers in the N material recombine with the positive carriers in the P material. This forms a small region on either side of the junction which is depleted of charge carriers; however, the ionized dopant atoms are immobile and remain. Therefore, in the N material the region has a net positive charge and in the P material the region has a net negative charge. This region is called either a "depletion" region [depleted of carriers] or a "space charge" region [due to the dopant ions].

If a conduction band free electron [a majority carrier] in the N material tries to cross into the P material, it encounters and is repelled by the net negative charge [due to the ionized acceptor atoms] in the depleted part of the P material.



If a valence band hole [a majority carrier] in the P material tries to cross into the the N material, it encounters and is repelled by the net positive charge [due to the ionized donor atoms] in the depleted part of the N material.

The repulsion of the carriers is characterized as a Coulombic "potential [energy] barrier". With no voltage applied across the junction, Ohm's law requires the current to be zero. Actually, very, very small equal but opposite currents do flow across the junction but the net current is zero.

The barrier can be decreased by applying a "forward bias" voltage across the junction. This allows more carriers to cross the junction and when this voltage is greater than a certain value [0.7 V for Silicon] a significant current [milliamps] flows.

The barrier can be increased by applying a "reverse bias" voltage across the junction. This increases the barrier and fewer majority carriers have sufficient energy to cross the junction, ie, the current essentially ceases. However, there is a VERY, VERY small

"reverse saturation current" [in the femtoamps range] due to the minority carriers. Since the minority carriers are thermally produced, this current is dependent on temperature.

Section 9.3: Circuit Models for the Semiconductor Diode

Focus on Methodology: Determining the conduction state of an ideal diode

1. Assume a diode conduction state (on or off).
2. Substitute the ideal diode circuit model into the circuit (short circuit if “on”, open circuit if “off”).
3. Solve for diode current and voltage using linear circuit analysis techniques.
4. If the solution is consistent with the assumption, then the initial assumption was correct; if not, the diode conduction state is opposite to that initially assumed. For example, if the diode has been assumed “off”, but the diode voltage computed after replacing the diode with an open circuit is a forward bias, then it must be true that the actual state of the diode is “on”.

Focus on Methodology: Determining the operating point of a diode

1. Reduce the circuit to a Thévenin or Norton equivalent circuit, with the diode as the load.
 2. Write the load line equation, 9.15.
 3. Solve numerically two simultaneous equations in two unknowns (the load-line equation and the diode equation) for the diode current and voltage.
- or
4. Solve graphically by finding the intersection of the diode curve (e.g., from a data sheet) with the loadline. The intersection of the two curves is the diode operating point.

Problem 9.7

Solution:

Known quantities:

The circuit of Figure P9.7.

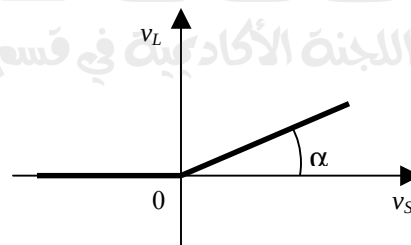
Find:

A plot of v_L versus v_S .

Analysis:

For $v_S < 0$, the diode is reverse biased, and $v_L = 0$. For $v_S \geq 0$, the diode is forward biased, and

$$v_L = v_S \left(\frac{R_L \parallel R_1}{R_S + R_L \parallel R_1} \right).$$



Where $\alpha = \tan^{-1} \left(\frac{R_L \parallel R_1}{R_S + R_L \parallel R_1} \right)$

Problem 9.8**Solution:****Known quantities:**

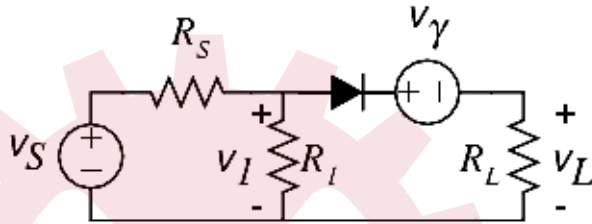
The circuit of Figure P9.7 using the offset diode model.

Find:

A plot of v_L versus v_S .

Analysis:

The circuit can be represented as shown in the following figure.



For $v_I < V_\gamma$, the diode is reverse biased, and $v_L = 0$.

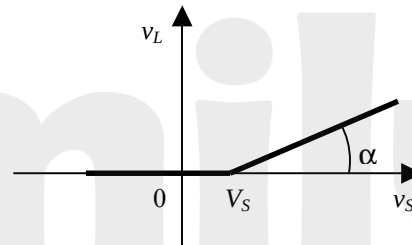
In term of v_S , we have

$$v_L = 0 \Leftrightarrow v_S < V_\gamma \left(1 + \frac{R_S}{R_L} \right)$$

For $v_I \geq V_\gamma$, i.e. $v_S \geq V_\gamma \left(1 + \frac{R_S}{R_L} \right)$ the diode is forward biased, and

$$v_I = \frac{\frac{v_S}{R_S} + \frac{V_\gamma}{R_L}}{\frac{1}{R_S} + \frac{1}{R_L} + \frac{1}{R_L}}$$

$$v_L = v_I - V_\gamma = \frac{\frac{v_S}{R_S} - V_\gamma \left(\frac{1}{R_S} + \frac{1}{R_L} \right)}{\frac{1}{R_S} + \frac{1}{R_L} + \frac{1}{R_L}}$$



Where $V_S = V_\gamma \left(1 + \frac{R_S}{R_L} \right)$, $\alpha = \tan^{-1} \left(\frac{\frac{1}{R_S}}{\frac{1}{R_S} + \frac{1}{R_L} + \frac{1}{R_L}} \right)$.

Problem 9.9**Solution:****Known quantities:**

For the circuit of Figure P9.7: $v_S = 6 \text{ V}$, and the resistances $R_1 = R_S = R_L = 1 \text{ k}\Omega$.

Find:

Determine i_D and v_D graphically.

Assumptions:

Use the diode characteristic of the 1N461A.

Analysis:

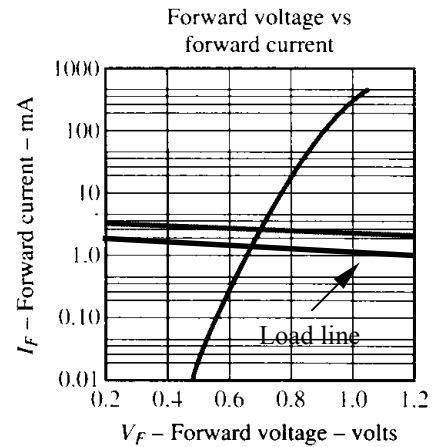
Replace the diode with an open circuit, and $v_{D_{oc}} = 3 \text{ V}$.

Replace the diode with a short circuit, and $i_{D_{sc}} = \frac{1}{2} \frac{6}{1500} = 2 \text{ mA}$

These are the end points of the load line.

The load line is superimposed on the diode characteristic in the figure.

From the intersection of the load line and the diode characteristic, we see that $i_D \approx 1.5 \text{ mA}$ and $v_D \approx 0.75 \text{ V}$.

**Problem 9.10****Solution:****Known quantities:**

The current $I = 1 \text{ mA}$, that make the diode to be above the knee of its $i-v$ characteristic.

Find:

- The value of R to establish a 5 mA current in the circuit.
- With the value of R established in the preceding part, what is the minimum value to which the voltage E could be reduced and still maintain diode current above the knee.

Assumptions:

$V_\gamma = 0.7 \text{ V}$.

Analysis:

a)

$$R = \frac{5 - 0.7}{5 \cdot 10^{-3}} = 860 \, \Omega$$

b)

$$I = \frac{E_{min} - 0.7}{860} = 1 \cdot 10^{-3}$$

$$\Rightarrow E_{min} = 0.86 + 0.7 = 1.56 \text{ V}$$

Problem 9.11**Solution:****Known quantities:**

The circuit of Figure P9.11 driven by a sinusoidal source of 50 V rms , $V_\gamma = 0.7 \text{ V}$, $R = 220 \, \Omega$.

Find:

- The maximum forward current.

b) The peak inverse voltage across the diode.

Analysis:

$$a) I_{F_{max}} = \frac{50\sqrt{2} - 0.7}{220} = 318 \text{ mA}$$

$$b) V_{rev_{max}} = 50\sqrt{2} = 70.7 \text{ V}$$

Problem 9.12

Solution:

Known quantities:

The configurations shown in Figure P9.12.

Find:

Which diode are forward biased, and which are reverse biased.

Analysis:

- a) reverse-biased
- b) forward-biased
- c) reverse-biased
- d) forward-biased
- e) forward-biased

Problem 9.13

Solution:

Known quantities:

The configuration shown in Figure P9.13.

Find:

The range of V_{in} for which D_1 is forward-biased.

Analysis:

The diode D_1 is clearly forward-biased for any $V_{in} > 0$.

Problem 9.14

Solution:

Known quantities:

The configurations of Figure P9.14.

Find:

Determine which diodes are forward-biased and which are reverse-biased. Determine the output voltage.

Assumptions:

The drop across each forward biased diode is 0.7 V .

Analysis:

a) D_2 and D_4 are forward biased; D_1 and D_3 are reverse biased. $v_{out} = -5 + 0.7 = -4.3 \text{ V}$

b) D_1 and D_2 are reverse biased; D_3 is forward biased. $v_{out} = -10 + 0.7 = -9.3 \text{ V}$

c) D_1 is reverse biased; D_2 is reverse biased.

Problem 9.15**Solution:****Known quantities:**

The circuit of Figure P9.15; $v_S(t) = 10 \sin(2,000\pi t)$.

Find:

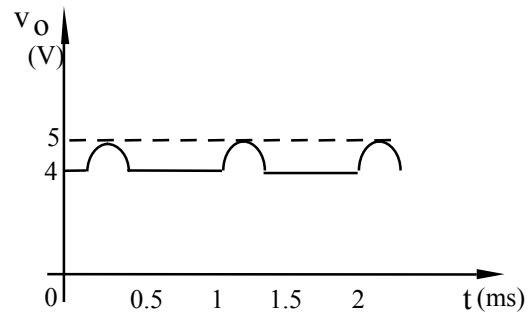
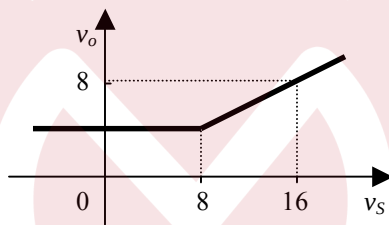
The output waveform and the voltage transfer characteristic.

Assumptions:

The diode is ideal.

Analysis:

For $v_S < 8$ V, $v_o = 4$ V. For $v_S \geq 8$ V, $v_o = v_S/2$.
The voltage transfer characteristic is

**Problem 9.16****Solution:****Known quantities:**

The circuit of Figure P9.15; $v_S(t) = 10 \sin(2,000\pi t)$.

Find:

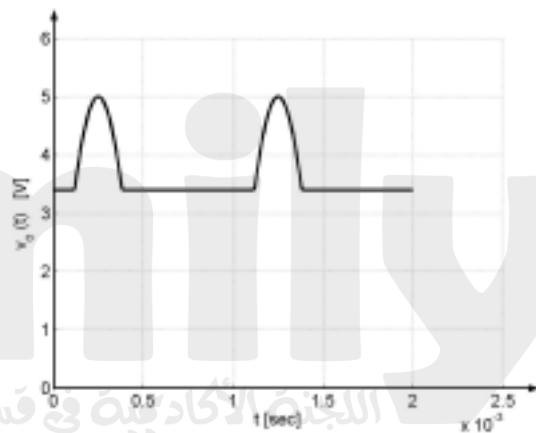
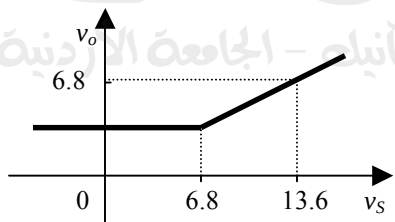
The output waveform and the voltage transfer characteristic.

Assumptions:

The diode has an offset $V_\gamma = 0.6$ V.

Analysis:

For $v_S < 6.8$ V, $v_o = 4 - V_\gamma = 3.4$ V. For $v_S \geq 6.8$ V, $v_o = v_S/2$.
The voltage transfer characteristic is



Problem 9.17**Solution:****Known quantities:**

Same as Problem 9.15 but with $v_s(t) = 1.5 \sin(2,000\pi t)$, the battery equal to 1 V and the resistors of 1k Ω .

Find:

The output waveform and the voltage transfer characteristic.

Assumptions:

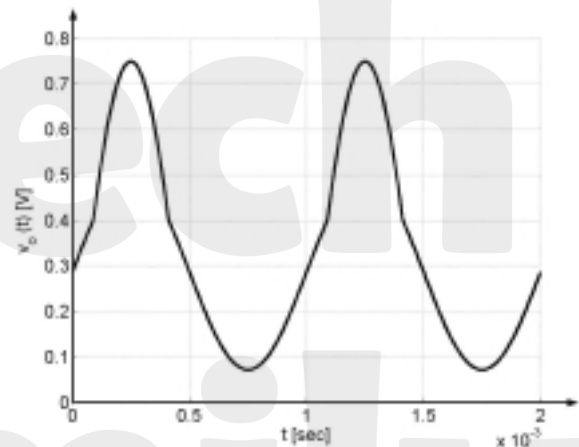
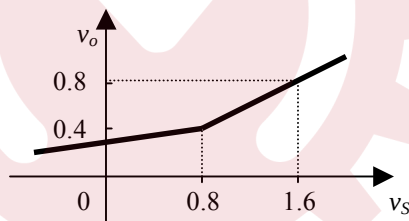
The diode has an offset $V_\gamma = 0.6$ V, and $r_D = 200$ Ω .

Analysis:

$$\text{For } v_s < 2(1 - V_\gamma) = 0.8 \text{ V, } v_o = \frac{\frac{v_s}{1000} + \frac{1 - V_\gamma}{r_D}}{\frac{1}{1000} + \frac{1}{r_D} + \frac{1}{1000}} = \frac{v_s + 2}{7}$$

For $v_s \geq 0.8$ V, $v_o = v_s / 2$.

The voltage transfer characteristic is

**Problem 9.18****Solution:****Known quantities:**

The circuit of Figure P9.18; the diode is fabricated from Silicon and $I_D = I_0 \left(e^{\frac{V_D}{V_T}} - 1 \right)$. At

$$T = 300 \text{ K, } I_0 = 250 \cdot 10^{-12} \text{ A, } V_T = \frac{kT}{q} \approx 26 \text{ mV, } v_s = 4.2 \text{ V} + 110 \cos(\omega t) \text{ mV,}$$

$$\omega = 377 \frac{\text{rad}}{\text{s}}, R = 7 \text{ k}\Omega.$$

Find:

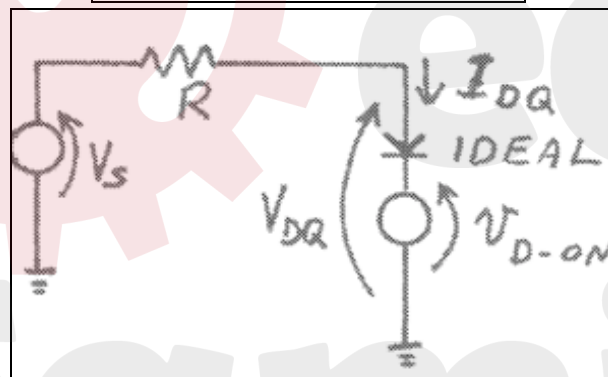
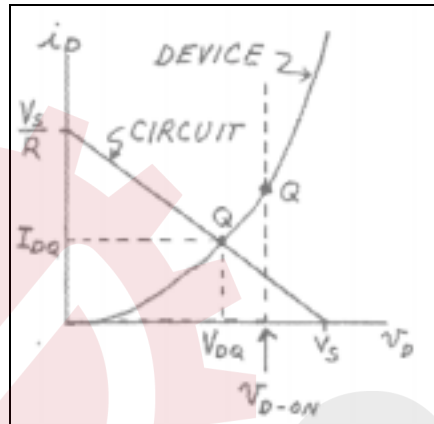
Determine, using superposition, the DC or Q-point current through the diode:

- Using the DC offset model for the diode.
- By iteratively solving the circuit characteristic and device characteristic.

Analysis:

a) Suppress the AC component of the source voltage. Construct the DC equivalent circuit using the threshold (or offset voltage model). The Q point is at the intersection of the device (diode) characteristic and the circuit characteristic given by the KVL below. Here, the device characteristic is approximated by the threshold voltage model giving the approximate Q point at the upper right. Assume the diode is on. Then:

$$V_D = V_{D-on} = 0.7 \text{ V}, \text{ KVL } -V_S + I_{DQ}R + V_D = 0, I_{DQ} = \frac{V_S - V_{D-on}}{R} = 0.5 \text{ mA}$$



b) In the forward biased region with significant conduction:

$$\text{Device: } I_D = I_0 e^{\frac{V_D}{V_T}}, V_D = V_T \ln \frac{I_D}{I_0},$$

$$\text{Circuit, KVL: } -V_S + I_D R + V_D = 0,$$

$$I_D = \frac{V_S - V_D}{R}$$

A simultaneous solution (the lower left Q point) of the device and circuit characteristics is required. To do this iteratively, initially assume a value for the diode voltage, say 0.7 V for a Silicon device. Then:

1. Using the initial or new diode voltage and the circuit characteristic, determine a new diode current.
2. Using this new diode current and the device characteristic, determine a new diode voltage.

ITERATE or REPEAT until convergence is obtained.

Voltage	New Current	New Voltage
-----	-----	-----
700 mV	0.5 ma	377.2 mV
377.2 mV	0.5461 ma	379.5 mV
379.5 mV	0.5458 ma	379.5 mV <<< Convergence

Convergence is obtained after only 3 iterations and:

$$I_{DQ} = 546 \mu\text{A}, V_{DQ} = 379.5 \text{ mV}$$

This is a much more accurate solution than that of Part a). The two solutions differ significantly because the reverse saturation current given is atypically large.

Problem 9.19

Solution:

Known quantities:

The circuit of Figure P9.18; the diode is fabricated from Silicon and $I_D = I_0 \left(e^{\frac{V_D}{V_T}} - 1 \right)$. At

$$T = 300 \text{ K}, I_0 = 2.03 \cdot 10^{-15} \text{ A}, V_T = \frac{kT}{q} \approx 26 \text{ mV}, v_s = 5.3 \text{ V} + 7 \cos(\omega t) \text{ mV},$$

$$\omega = 377 \frac{\text{rad}}{\text{s}}, R = 4.6 \text{ k}\Omega.$$

Find:

Determine, using superposition and the offset voltage model for the diode, the DC or Q-point current through the diode.

Analysis:

Suppress the AC component of the source voltage. Construct the DC equivalent circuit using the threshold (or offset voltage model). The Q point is at the intersection of the device (diode) characteristic and the circuit characteristic given by the KVL below. Here, the device characteristic is approximated by the threshold voltage model giving the approximate Q point at the upper right.

The DC source voltage will tend to make the diode conduct. Assume the diode is on. Then:

$$V_D = V_{D-on} = 0.7 \text{ V}, \text{ KVL } -V_s + I_{DQ}R + V_D = 0, I_{DQ} = \frac{V_s - V_{D-on}}{R} = 1.0 \text{ mA}$$

The current is positive so the assumption above that the diode is on is valid.

Problem 9.20

Solution:

Known quantities:

The circuit of Figure P9.18; the diode is fabricated from Silicon and $I_D = I_0 \left(e^{\frac{V_D}{V_T}} - 1 \right)$. At

$$T = 300 \text{ K}, I_0 = 250 \cdot 10^{-12} \text{ A}, V_T = \frac{kT}{q} \approx 26 \text{ mV}, v_s = 4.2 \text{ V} + 110 \cos(\omega t) \text{ mV},$$

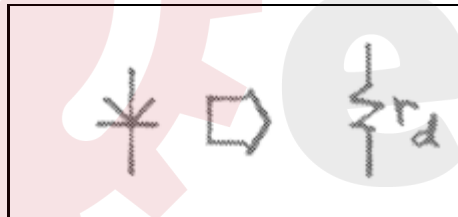
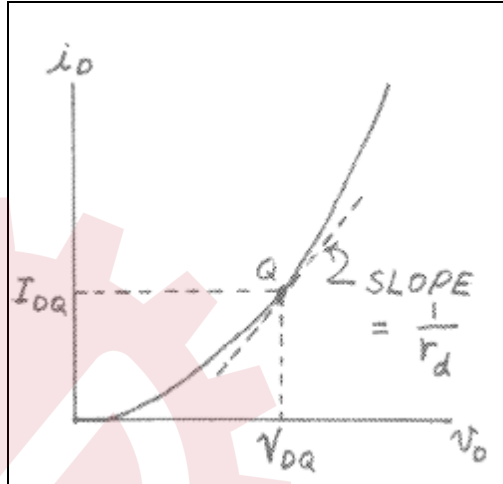
$$\omega = 377 \frac{\text{rad}}{\text{s}}, R = 7 \text{ k}\Omega. \text{ The DC operating point is: } I_{DQ} = 0.546 \text{ mA}, V_{DQ} = 379.5 \text{ mV}.$$

Find:

The equivalent small-signal AC resistance of the diode at room temperature at the Q point given.

Analysis:

$$r_D = \left. \frac{dV_D}{dI_D} \right|_Q = \frac{1}{\left. \frac{dI_D}{dV_D} \right|_Q} = \frac{1}{\left. \frac{1}{V_T} I_0 e^{\frac{V_D}{V_T}} \right|_Q} \approx \left. \frac{V_T}{I_D} \right|_Q = \frac{V_T}{I_{DQ}} = 47.64 \, \Omega$$

**Problem 9.21****Solution:****Known quantities:**

The circuit of Figure P9.18; the diode is fabricated from Silicon and $I_D = I_0 \left(e^{\frac{V_D}{V_T}} - 1 \right)$. At

$T = 300 \, \text{K}$, $I_0 = 2.03 \cdot 10^{-15} \, \text{A}$, $V_T = \frac{kT}{q} \approx 26 \, \text{mV}$, $v_s = 5.3 \, \text{V} + 70 \cos(\omega t) \, \text{mV}$,

$\omega = 377 \, \frac{\text{rad}}{\text{s}}$, $R = 4.6 \, \text{k}\Omega$. The DC operating point is: $I_{DQ} = 1.0 \, \text{mA}$, $V_{DQ} = 0.7 \, \text{V}$.

Find:

The equivalent small-signal AC resistance of the diode at room temperature at the Q point given.

Analysis:

$$r_D = \left. \frac{dV_D}{dI_D} \right|_Q = \frac{1}{\left. \frac{dI_D}{dV_D} \right|_Q} = \frac{1}{\left. \frac{1}{V_T} I_0 e^{\frac{V_D}{V_T}} \right|_Q} \approx \left. \frac{V_T}{I_D} \right|_Q = \frac{V_T}{I_{DQ}} = 26 \, \Omega$$

Problem 9.22

Solution:

Known quantities:

A diode with the i - v characteristic in Figure 9.8, connected to a 5 V source and a load resistance of 200 Ω .

Find:

- The load current and voltage
- The power dissipated by the diode.
- The load current and voltage if the load is changed to 100 Ω and 500 Ω .

Analysis:

- The operating point can be determined by using the load-line analysis

The load line is

$$i_D = \frac{5 - v_D}{R_L} = \frac{5}{200} - \frac{v_D}{200}$$

The load voltage is

$$v_L = 5 - v_D \cong 5 - 0.74 = 4.26 \text{ V}$$

The load current is obtained by the figure as intersection of the two characteristics and is equal to 0.021 A.

- The power dissipated by the diode is

$$P_D = v_D i_D = 0.74 \cdot 0.021 = 15 \text{ mW}$$

- For $R_L = 100 \Omega$, we have

$$v_D \cong 0.757 \text{ V}$$

$$i_D = i_L = \frac{5 - 0.757}{100} = 0.0424 \text{ A}$$

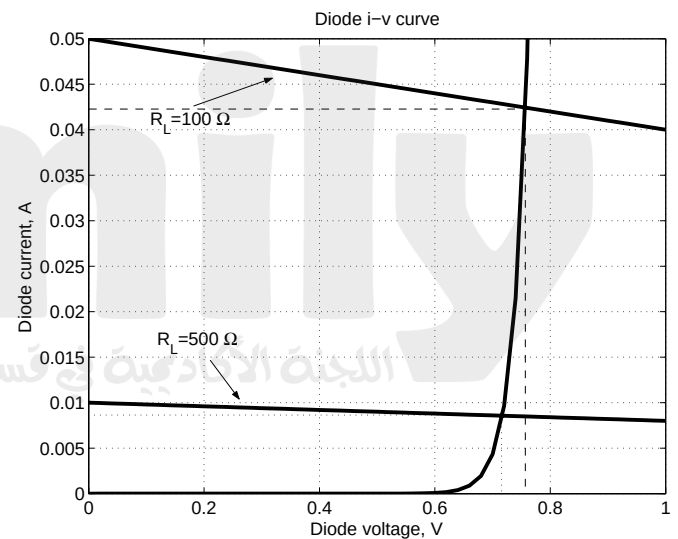
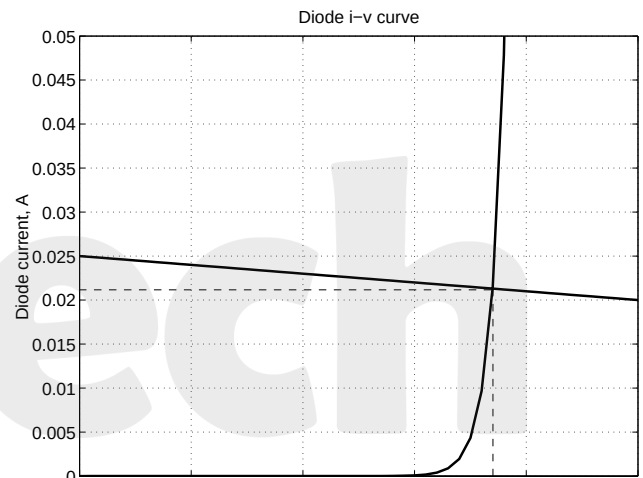
$$v_L = 5 - 0.757 = 4.24 \text{ V}$$

Similarly, for $R_L = 500 \Omega$, we have

$$v_D \cong 0.717 \text{ V}$$

$$i_D = i_L = \frac{5 - 0.717}{100} = 0.04283 \text{ A}$$

$$v_L = 5 - 0.717 = 4.283 \text{ V}$$



Problem 9.23**Solution:****Known quantities:**

A diode with the i - v characteristic in Figure 9.32, connected in series to a 2 V source and a load resistance of 200 Ω .

Find:

- The load current and voltage
- The power dissipated by the diode.
- The load current and voltage if the load is changed to 100 Ω and 300 Ω .

Analysis:

- The operating point can be determined by using the load-line analysis
The load line is

$$i_D = \frac{2 - v_D}{R_L} = \frac{2}{200} - \frac{v_D}{200} = 0.01 - 0.005 v_D$$

By drawing this line on the top of Figure 9.32, the following operating point is obtained

$$i_D = i_L \approx 6 \text{ mA}; \quad v_D \approx 0.73 \text{ V} \Rightarrow v_L = 2 - v_D = 2 - 0.73 = 1.27 \text{ V}$$

$$\text{b) } P_D = v_D i_D = 0.73 \cdot 0.006 = 4.38 \text{ mW}$$

- For $R_L = 100 \Omega$, we have
 $v_D \approx 0.825 \text{ V}$

$$i_D = i_L \approx \frac{2 - 0.825}{100} = 11.75 \text{ mA}$$

$$v_L \approx 2 - 0.825 = 1.175 \text{ V}$$

For $R_L = 300 \Omega$, we have

$$v_D \approx 0.7 \text{ V}$$

$$i_D = i_L \approx \frac{2 - 0.7}{300} = 4.3 \text{ mA}$$

$$v_L \approx 2 - 0.7 = 1.3 \text{ V}$$

Problem 9.24**Solution:****Known quantities:**

The circuit of Figure P9.18; the diode is fabricated from Silicon and $I_D = I_0 \left(e^{\frac{v_D}{V_T}} - 1 \right)$. At

$$T = 300 \text{ K}, \quad I_0 = 250 \cdot 10^{-12} \text{ A}, \quad V_T = \frac{kT}{q} \approx 26 \text{ mV}, \quad v_s = 4.2 \text{ V} + 110 \cos(\omega t) \text{ mV},$$

$\omega = 377 \frac{\text{rad}}{\text{s}}$, $R = 7 \text{ k}\Omega$. The DC operating point is: $I_{DQ} = 0.548 \text{ mA}$, $V_{DQ} = 0.365 \text{ V}$,
 $r_d = 47.45 \Omega$.

Find:

Determine, using superposition the AC voltage across the diode and the AC current through it.

Analysis:

Suppress the DC component of the source voltage. Replace the diode with its AC equivalent resistance; then:

$$V_d = V_s \frac{r_d}{R + r_d}$$

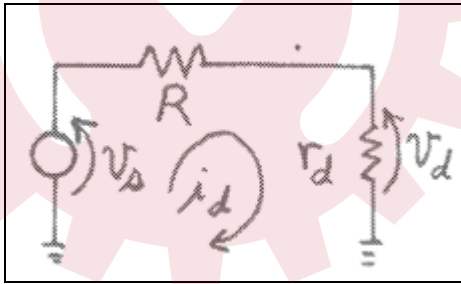
$$V_d = 110 \angle 0^\circ \frac{47.45}{7,000 + 47.45} = 740.6 \angle 0^\circ \mu \text{ V}, v_d(t) = 740.6 \cos(\omega t) \mu \text{ V}$$

$$I_d = \frac{V_d}{r_d} = \frac{740.6 \angle 0^\circ}{47.45} = 15.61 \angle 0^\circ \mu \text{ A}, i_d = 15.61 \cos(\omega t) \mu \text{ A}.$$

The total solution is then:

$$i_d = 0.548 \cdot 10^{-3} + 15.61 \cos(\omega t) \cdot 10^{-6} \text{ A}$$

$$v_d(t) = 0.365 + 740.6 \cos(\omega t) \cdot 10^{-6} \text{ V}$$

**Problem 9.25****Solution:****Known quantities:**

The circuit of Figure P9.25. The diode is fabricated from Silicon and $R = 2.2 \text{ k}\Omega$, $V_{S2} = 3 \text{ V}$.

Find:

The minimum value of V_{S1} at and above which the diode will conduct with a significant current.

Analysis:

A diode fabricated from silicon will conduct with a significant current if it has a forward bias equal to or larger than about 0.7 V .

$$\text{KVL} \quad -V_{S1} + I_D R + V_D + V_{S2} = 0, \quad V_{S1} = I_D R + V_D + V_{S2}.$$

At point of conduction:

$$I_D = 0, \quad V_D = V_{D-on} = 0.7 \text{ V}, \quad V_{S1} = 0 + 0.7 + 3 = 3.7 \text{ V}$$

Section 9.4: Rectifier Circuits

Problem 9.26

Solution:

Known quantities:

The circuit of Figure P9.26. The input voltage is sinusoidal with an amplitude of 5 V.

Find:

The average value of the output voltage.

Assumptions:

$$V_\gamma = 0.7 \text{ V}.$$

Analysis:

The capacitor will charge to $5 \text{ V} - 0.7 \text{ V} = 4.3 \text{ V}$ and, therefore, the input sine wave will be shifted up 4.3 V to produce the output. As a result, after the cycle (the capacitor builds up its stored charge during the third quarter cycle), the average value of the output will be 4.3 V.

Problem 9.27

Solution:

Known quantities:

The rectifier circuit of Figure P9.27; $v(t) = A \sin(2\pi 100t) \text{ V}$. The conduction must begin during each positive half-cycle at an angle no greater than 5° .

Find:

The minimum peak value A that the AC source must produce.

Assumptions:

$$V_\gamma = 0.7 \text{ V}.$$

Analysis:

$$A_{\min} \sin(5^\circ) = 0.7 \Rightarrow A_{\min} = \frac{0.7}{\sin(5^\circ)} = \frac{0.7}{0.0872} = 8.03 \text{ V}$$

Problem 9.28

Solution:

Known quantities:

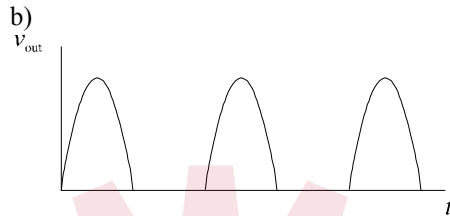
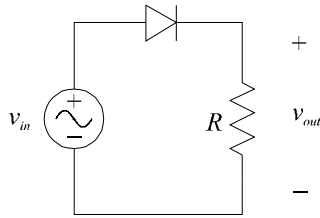
A half-wave rectifier is to provide an average voltage of 50 V at its output.

Find:

- Draw a schematic diagram of the circuit.
- Sketch the output voltage waveshape.
- Determine the peak value of the input voltage.
- Sketch the input voltage waveshape.
- The rms voltage at the input.

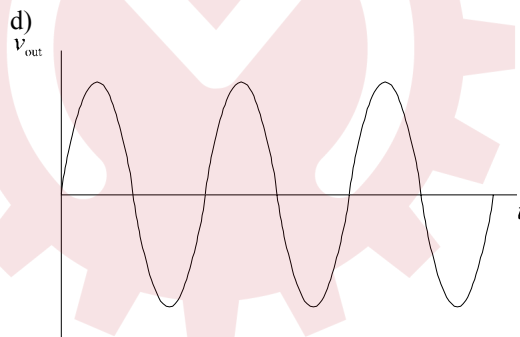
Analysis:

-



c)

$$v_{ave} = 0.318 v_{peak} = 50 \Rightarrow v_{peak} = 157.2 \text{ V}$$



e)

$$V_{in_{rms}} = \frac{157.2}{\sqrt{2}} = 111.2 \text{ V}$$

Problem 9.29

Solution:

Known quantities:

A rectifier circuit similar to that of Figure 9.25. Load Resistance 100Ω , AC source voltage 30 V (rms) .

Find:

The peak and average current in the load.

Assumptions:

Ideal diode.

Analysis:

The peak voltage is

$$V_{R_{peak}} = 30\sqrt{2} \text{ V} \Rightarrow I_{R_{peak}} = \frac{V_{R_{peak}}}{R} = \frac{30\sqrt{2}}{100} = 0.424 \text{ A}$$

The average current in the load is

$$I_{R_{AV}} = \frac{I_{R_{peak}}}{\pi} = 0.135 \text{ A}$$

Problem 9.30

Solution:

Known quantities:

A rectifier circuit similar to that of Figure 9.25. Load Resistance 220Ω , AC source voltage 25 V (rms) .

Find:

The peak and average current in the load.

Assumptions:

Ideal diode.

Analysis:

The peak voltage is

$$V_{R_{peak}} = 25\sqrt{2} \text{ V} \Rightarrow I_{R_{peak}} = \frac{V_{R_{peak}}}{R} = \frac{25\sqrt{2}}{220} = 0.161 \text{ A}$$

The average current in the load is

$$I_{R_{AV}} = \frac{I_{R_{peak}}}{\pi} = 51.8 \text{ mA}$$

Problem 9.31

Solution:

Known quantities:

The full-wave power supply of Figure P9.31. The diodes are 1N461 with a rated peak reverse voltage equal to 25 V , and are fabricated from Silicon. $n = 0.05883$, $C = 80 \mu\text{F}$, $V_{line} = 170 \cos(377t) \text{ V}$.

Find:

- The actual peak reverse voltage across each diode.
- The reasons for which these diodes are or are not suitable for the specification given.

Analysis:

- At $\omega t = 0$, $D1$ is on. At $\omega t = \pi$, $D1$ is off and the reverse voltage across it is maximum.

$$V_{so} = V_{io} n = 170 \cdot 0.05883 = 10 \text{ V}$$

$$\text{KVL: } -v_{s1}(t) + v_{D1} + v_L(t) = 0$$

$$\text{At } \omega t = 0, -V_{so} + V_{D-on} + V_m = 0 \Rightarrow V_m = V_{so} - V_{D-on} = 10 - 0.7 = 9.3 \text{ V}$$

$$\text{At } \omega t = \pi, -(-V_{so}) + V_{D1} + V_m = 0 \Rightarrow V_{D1} = -V_{so} - V_m = -10 - 9.3 = -19.3 \text{ V}$$

- The actual peak reverse voltage (19.3 V) is less than the rated peak reverse voltage (25 V) by a barely adequate margin of safety. Therefore, the diodes are suitable for the specifications given.

Problem 9.32

Solution:

Known quantities:

The full-wave power supply of Figure P9.31; $n = 0.05883$, $C = 80 \mu\text{F}$, $V_{line} = 170 \cos(377t) \text{ V}$.

The diodes are 1N4727 switching diodes fabricated from Silicon and with the following rated performances: $P_{max} = 500 \text{ mW}$ at $T = 25^\circ\text{C}$; derated $3 \frac{\text{mW}}{^\circ\text{C}}$ $T = 25 \div 125^\circ\text{C}$ and $4 \frac{\text{mW}}{^\circ\text{C}}$ $T = 125 \div 175^\circ\text{C}$; $V_{pk-rev} = 30 \text{ V}$.

Find:

- The actual peak reverse voltage across each diode.
- The reasons for which these diodes are or are not suitable for the specification given.

Analysis:

a) At $\omega t = 0$, $D1$ is on. At $\omega t = \pi$, $D1$ is off and the reverse voltage across it is maximum.

$$V_{so} = V_{io} n = 1700.1 = 17 \text{ V}$$

$$\text{KVL: } -v_{s1}(t) + v_{D1} + v_L(t) = 0$$

$$\text{At } \omega t = 0, -V_{so} + V_{D-on} + V_m = 0 \Rightarrow V_m = V_{so} - V_{D-on} = 17 - 0.7 = 16.3 \text{ V}$$

$$\text{At } \omega t = \pi, -(-V_{so}) + V_{D1} + V_m = 0 \Rightarrow V_{D1} = -V_{so} - V_m = -17 - 16.3 = -33.3 \text{ V}$$

b) The actual peak reverse voltage (33.3 V) is greater than the rated peak reverse voltage (30 V). Therefore, the diodes are not suitable for the specifications given.

Problem 9.33**Solution:****Known quantities:**

The full-wave DC power supply of Figure P9.31; the load voltage of Figure P9.33; $I_L = 60 \text{ mA}$,

$$V_L = 5 \text{ V}, V_r = 5\%, v_{line} = 170 \cos(\omega t) \text{ V}, \omega = 377 \frac{\text{rad}}{\text{s}}.$$

Find:

- The turns ratio.
- The capacitor C .

Analysis:

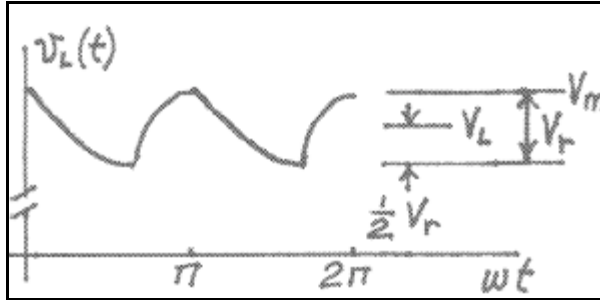
$$\text{a) } V_m = V_L + \frac{1}{2} V_r = 5 + 0.125 = 5.125 \text{ V}, V_{L-min} = V_L - \frac{1}{2} V_r = 5 - 0.125 = 4.875 \text{ V},$$

$$V_{s1} = V_{s2} = V_{s0} \cos(\omega t).$$

$$\text{KVL: } -v_{s1}(t) + v_{D1} + v_L(t) = 0$$

$$\text{At } \omega t = 0, -V_{so} + V_{D-on} + V_L = 0 \Rightarrow V_{s0} = V_{D-on} + V_m = 0.7 + 5.125 = 5.825 \text{ V}$$

$$n = \frac{V_{so}}{V_{io}} = 0.0343$$



b) KVL : $v_{s2}(t) + v_{D2} + v_L(t) = 0$.

At $t = t_2$, $V_{so} \cos(\omega t_2) = -V_{D2-on} - V_{L-min} \cdot t_2 = \frac{1}{\omega} \cos^{-1} \left(-\frac{V_{D2-on} + V_{L-min}}{V_{so}} \right) = 7.533 \text{ ms}$.

The exponential discharge of the capacitor can be expressed:

$$v_L(t) = v_L(\infty) + (v_L(0) - v_L(\infty))e^{-\frac{t}{TC}} = 0 + (V_m - 0)e^{-\frac{t}{R_L C}} = V_m e^{-\frac{I_L t}{V_L C}}$$

$$v_L(t_2) = V_{L-min} = V_m e^{-\frac{I_L t_2}{V_L C}}, C = -\frac{I_L t_2}{V_L \ln \left(\frac{V_{L-min}}{V_m} \right)} = 1812 \mu\text{F}$$

Note: An approximate but conservative value of C can be obtained by using the approximation:

$$\omega t_2 \approx \pi. \text{ Then } C \approx -\frac{I_L(\omega t_2)}{\omega V_L \ln \left(\frac{V_{L-min}}{V_m} \right)} = 2000 \mu\text{F}.$$

This value is conservative because it gives a smaller ripple voltage than that specified.

Problem 9.34

Solution:

Known quantities:

The full-wave DC power supply of Figure P9.31; $I_L = 600 \text{ mA}$, $V_L = 50 \text{ V}$, $V_r = 8 \%$,

$$v_{line} = 170 \cos(\omega t) \text{ V}, \omega = 377 \frac{\text{rad}}{\text{s}}.$$

Find:

- The turns ratio.
- The capacitor C.

Analysis:

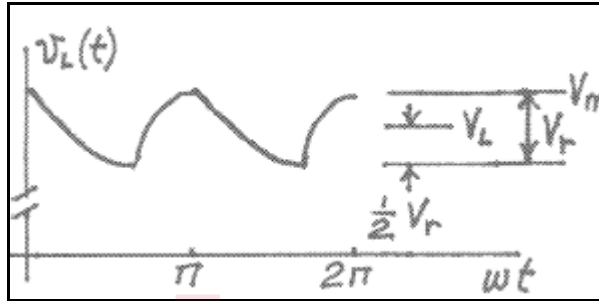
$$\text{a) } V_m = V_L + \frac{1}{2} V_r = 50 + 2 = 52 \text{ V}, V_{L-min} = V_L - \frac{1}{2} V_r = 50 - 2 = 48 \text{ V},$$

$$V_{s1} = V_{s2} = V_{s0} \cos(\omega t).$$

$$\text{KVL : } -v_{s1}(t) + v_{D1} + v_L(t) = 0$$

$$\text{At } \omega t = 0, -V_{so} + V_{D-on} + V_L = 0 \Rightarrow V_{s0} = V_{D-on} + V_m = 0.7 + 52 = 52.7 \text{ V}$$

$$n = \frac{V_{so}}{V_{io}} = 0.31$$



b) KVL : $v_{s2}(t) + v_{D2} + v_L(t) = 0$.

$$\text{At } t = t_2, V_{so} \cos(\omega t_2) = -V_{D2-on} - V_{L-min}. \quad t_2 = \frac{1}{\omega} \cos^{-1} \left(-\frac{V_{D2-on} + V_{L-min}}{V_{so}} \right) = 7.29 \text{ ms}.$$

The exponential discharge of the capacitor can be expressed:

$$v_L(t) = v_L(\infty) + (v_L(0) - v_L(\infty))e^{-\frac{t}{TC}} = 0 + (V_m - 0)e^{-\frac{t}{R_L C}} = V_m e^{-\frac{I_L t}{V_L C}}$$

$$v_L(t_2) = V_{L-min} = V_m e^{-\frac{I_L t_2}{V_L C}}, \quad C = -\frac{I_L t_2}{V_L \ln \left(\frac{V_{L-min}}{V_m} \right)} = 1093 \mu\text{F}$$

Problem 9.35

Solution:

Known quantities:

The full-wave DC power supply of Figure P9.31; $I_L = 5 \text{ mA}$, $V_L = 10 \text{ V}$, $V_r = 20 \%$,

$$v_{line} = 170 \cos(\omega t) \text{ V}, \quad \omega = 377 \frac{\text{rad}}{\text{s}}.$$

Find:

- The turns ratio.
- The capacitor C .

Analysis:

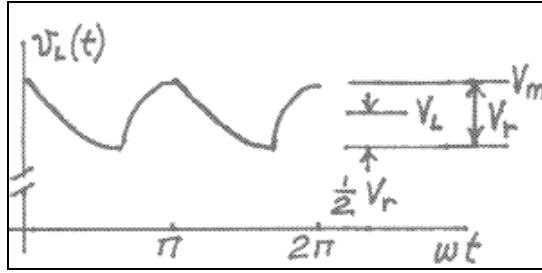
$$\text{a) } V_m = V_L + \frac{1}{2} V_r = 10 + 1 = 11 \text{ V}, \quad V_{L-min} = V_L - \frac{1}{2} V_r = 10 - 1 = 9 \text{ V},$$

$$V_{s1} = V_{s2} = V_{s0} \cos(\omega t).$$

$$\text{KVL : } -v_{s1}(t) + v_{D1} + v_L(t) = 0$$

$$\text{At } \omega t = 0, -V_{s0} + V_{D-on} + V_L = 0 \Rightarrow V_{s0} = V_{D-on} + V_m = 0.7 + 1 = 6.7 \text{ V}$$

$$n = \frac{V_{so}}{V_{io}} = 0.0688$$



b) KVL : $v_{s2}(t) + v_{D2} + v_L(t) = 0$.

At $t = t_2$, $V_{so} \cos(\omega t_2) = -V_{D2-on} - V_{L-min} \cdot t_2 = \frac{1}{\omega} \cos^{-1} \left(-\frac{V_{D2-on} + V_{L-min}}{V_{so}} \right) = 6.76 \text{ ms}$.

The exponential discharge of the capacitor can be expressed:

$$v_L(t) = v_L(\infty) + (v_L(0) - v_L(\infty))e^{-\frac{t}{TC}} = 0 + (V_m - 0)e^{-\frac{t}{R_L C}} = V_m e^{-\frac{I_L t}{V_L C}}$$

$$v_L(t_2) = V_{L-min} = V_m e^{-\frac{I_L t_2}{V_L C}}, \quad C = -\frac{I_L t_2}{V_L \ln \left(\frac{V_{L-min}}{V_m} \right)} = 16.84 \mu\text{F}$$

Note: An approximate but conservative value of C can be obtained by using the approximation:

$$\omega t_2 \approx \pi. \text{ Then } C \approx -\frac{I_L(\omega t_2)}{\omega V_L \ln \left(\frac{V_{L-min}}{V_m} \right)} = 20.76 \mu\text{F}.$$

This value is conservative because it gives a smaller ripple voltage than that specified.

Problem 9.36

Solution:

Known quantities:

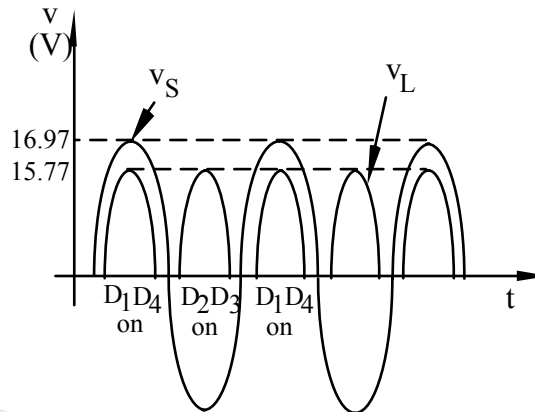
The full-wave rectifier of Figure P9.36, with a 12 V rms supply.

Find:

- Sketch the input source voltage $v_s(t)$, and the output voltage $v_L(t)$, and state which diodes are on and which are off if the diodes have an offset voltage of 0.6 V and the frequency of the source is 60 Hz.
- Sketch the output voltage if $R_L = 1,000 \Omega$ and a capacitor, placed across R_L to provide some filtering, has a value of $8 \mu\text{F}$.
- As part b, with the capacitance equal to $100 \mu\text{F}$.

Analysis:

- The input source voltage is shown below, together with the rectified load voltage. (12 V rms = 16.97 V peak)



b) The time constant, $\tau = CR$, is: $CR = 1000 \cdot 8 \cdot 10^{-6} = 8 \text{ ms}$.

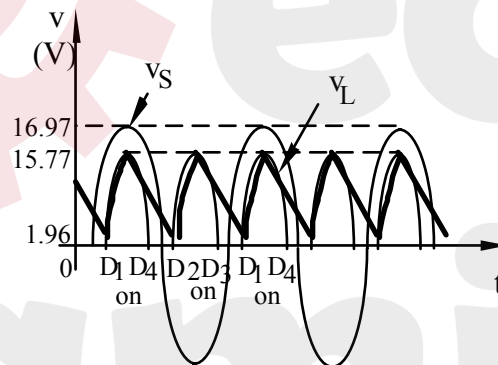
The period of the input sinusoid is: $T = \frac{1}{f} = \frac{1}{60} = 16.7 \text{ ms}$.

Since the capacitor initial voltage is: $v_C(0) = 16.97 - 1.2 = 15.77 \text{ V}$, and the final value is $v_C(\infty) = 0 \text{ V}$

$v_C(t)$ is given by: $v_C(t) = 15.77 e^{-t/\tau}$.

Therefore, at $t = T$, we have $v_C(T) = 15.77 e^{-T/\tau} = 1.96 \text{ V}$.

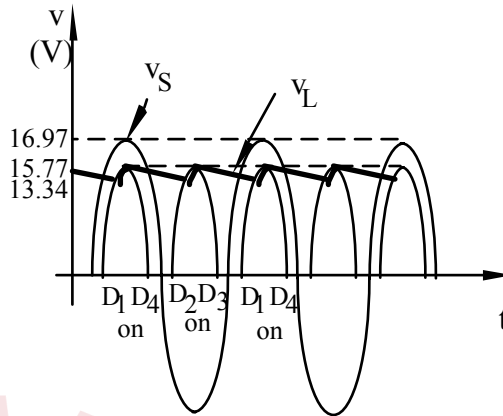
The output waveform is shown below:



(c) The time constant is $CR = 1000 \cdot 100 \cdot 10^{-6} = 100 \text{ ms}$.

Note that $CR \gg T$; $v_C(0) = 16.97 - 1.2 = 15.77 \text{ V}$, and the final value is $v_C(\infty) = 0 \text{ V}$; $v_C(t)$ is given by: $v_C(t) = 15.77 e^{-t/\tau}$ and therefore $v_C(T) = 15.77 e^{-T/\tau} = 13.34 \text{ V}$.

The output waveform is shown below.



Problem 9.37

Solution:

Known quantities:

The full-wave bridge power supply of Figure P9.37; the diodes are 1N659 with a rated peak reverse voltage 50 V. $n = 0.2941$, $R_L = 2.5 \text{ k}\Omega$, $C = 700 \mu\text{F}$, $v_{line} = 170 \cos(\omega t) \text{ V}$, $\omega = 377 \frac{\text{rad}}{\text{s}}$.

Find:

- The actual peak reverse voltage across the diodes.
- Explain why these diodes are or are not suitable for the specifications given.

Analysis:

$$\text{a) } v_{line} = n v_{line} = 50 \cos(\omega t) \text{ V}, V_{so} = 50 \text{ V}$$

At $\omega t = 0$, the source voltage has the polarity shown; therefore, $D1$ and $D3$ are conducting, and $D2$ and $D4$ are off. At $\omega t = \pi$, $D1$ and $D3$ are off and the voltage across them is the peak reverse voltage.

$$\text{KVL: } -v_s(t) + v_{D1} + v_L(t) + v_{D3} = 0$$

$$\text{At } \omega t = 0: v_L(0) = V_m = V_{D1} + V_{D3} + V_{so} = 48.6 \text{ V}$$

$$\text{At } \omega t = \pi: v_{D1}(\pi) + v_{D3}(\pi) = V_{so} \cos(\pi) - v_L(\pi) = -V_{so} - V_m$$

$$V_{D1} = V_{D3}, V_{D1,3} = \frac{1}{2}(-50 - 48.6) = -49.3 \text{ V}$$

- The diodes are not suitable because the rated and actual peak reverse voltages are about the same.

Problem 9.38

Solution:

Known quantities:

The full-wave bridge power supply of Figure P9.37; the diodes are T151 with a rated peak reverse voltage 10 V and are fabricated from Silicon. $n = 0.0423$, $V_L = 5.1 \text{ V}$, $V_r = 0.2 \text{ V}$, $I_L = 2.5 \text{ mA}$,

$$v_{line} = 156 \cos(\omega t) \text{ V}, \omega = 377 \frac{\text{rad}}{\text{s}}.$$

Find:

- The actual peak reverse voltage across the diodes.
- Explain why these diodes are or are not suitable for the specifications given.

Analysis:

$$a) \quad V_{so} = nV_{io} = 6.6 \text{ V}, \quad v_s(t) = 6.6 \cos(\omega t) \text{ V}. \quad V_m = V_L + \frac{1}{2}V_r = 5.2 \text{ V}$$

At $\omega t = 0$, the source voltage has the polarity shown; therefore, $D1$ and $D3$ are conducting, and $D2$ and $D4$ are off.

$$KVL: \quad -v_s(t) - v_{D4} - v_L(t) - v_{D2} = 0$$

$$\text{At } \omega t = 0: \quad v_{D2}(0) + v_{D4}(0) = -v_L(0) - V_{so} \cos(0) = -V_m - V_{so}$$

$$V_{D2} = V_{D4}, \quad V_{D2,4} = \frac{1}{2}(-6.6 - 5.2) = -5.9 \text{ V}$$

b) The diodes are suitable because the actual PRV (5.9 V) is significantly less than the rated PRV (10 V).

Problem 9.39**Solution:****Known quantities:**

The full-wave bridge power supply of Figure P9.37; the diodes are fabricated from Silicon.

$$\phi = 23.66^\circ, \quad V_L = 10 \text{ V}, \quad V_r = 1 \text{ V}, \quad I_L = 650 \text{ mA}, \quad v_{line} = 170 \cos(\omega t) \text{ V}, \quad \omega = 377 \frac{\text{rad}}{\text{s}}.$$

Find:

The value of the average and the peak current through each diode.

Analysis:

Diodes $D1$ and $D3$ will conduct half of the load current and Diodes $D2$ and $D4$ will conduct the other half. Therefore:

$$I_{D-ave} = \frac{1}{2} I_L = 325 \text{ mA}$$

The waveforms of the diode currents are complex but can be roughly approximated as triangular (recall area of triangle = $bh/2$):

$$I_L = (I_{D1,3} + I_{D2,4})_{ave} = \frac{1}{2\pi} \int_0^{2\pi} (I_{D1,3}(\omega t) + I_{D2,4}(\omega t)) d(\omega t) = \frac{1}{2\pi} \left(\frac{\phi I_{D-pk}}{2} + \frac{\phi I_{D-pk}}{2} \right)$$

$$I_{D-pk} = \frac{2\pi I_L}{\frac{1}{2}\phi + \frac{1}{2}\phi} = \frac{2\pi I_L}{\phi} = 1.28 \text{ A}$$

Problem 9.40**Solution:****Known quantities:**

The full-wave bridge power supply of Figure P9.37; the diodes are fabricated from Silicon. $V_L = 5.3 \text{ V}$,

$$V_r = 0.6 \text{ V}, \quad I_L = 85 \text{ mA}, \quad v_{line} = 156 \cos(\omega t) \text{ V}, \quad \omega = 377 \frac{\text{rad}}{\text{s}}.$$

Find:

a) The turns ratio n .

b) The capacitor C .

Analysis:

a) First determine the maximum and minimum voltage across the load resistance and capacitor:

$$V_m = V_L + \frac{1}{2}V_r = 5.3 + 0.3 = 5.6 \text{ V}, \quad V_{L-\min} = V_L - \frac{1}{2}V_r = 5.3 - 0.3 = 5 \text{ V}.$$

The amplitude of the supply voltage can now be determined.

$$KVL: -V_{so} \cos(\omega t) + v_{D1} + v_{D3} + v_L(t) = 0$$

$$\text{At } t = 0, -V_{so} + V_{D-\text{on}} + V_{D-\text{on}} + V_L = 0 \Rightarrow V_{so} = 2V_{D-\text{on}} + V_m = 0.7 + 0.7 + 5.6 = 7 \text{ V}$$

$$n = \frac{V_{so}}{V_{io}} = 0.04487$$

$$b) KVL: v_s(t) + v_{D2} + v_{D4} + v_L(t) = 0.$$

$$\text{At } t = t_2, V_{so} \cos(\omega t_2) = -V_{D-\text{on}} - V_{D-\text{on}} - V_{L-\min}.$$

$$\omega t_2 = \cos^{-1}\left(-\frac{2V_{D-\text{on}} + V_{L-\min}}{V_{so}}\right) = 2.725 \text{ rad}.$$

The exponential discharge of the capacitor can be expressed:

$$v_L(t) = v_L(\infty) + (v_L(0) - v_L(\infty))e^{-\frac{t}{TC}} = 0 + (V_m - 0)e^{-\frac{t}{R_L C}} = V_m e^{-\frac{I_L \omega t}{\omega V_L C}}$$

$$v_L(t_2) = V_{L-\min} = V_m e^{-\frac{I_L \omega t_2}{\omega V_L C}}, \quad C = -\frac{I_L \omega t_2}{\omega V_L \ln\left(\frac{V_{L-\min}}{V_m}\right)} = 1023 \mu\text{F}$$

Problem 9.41

Solution:

Known quantities:

The full-wave bridge power supply of Figure P9.37; the diodes are fabricated from Silicon. $V_L = 10 \text{ V}$,

$$V_r = 2.4 \text{ V}, \quad I_L = 250 \text{ mA}, \quad v_{\text{line}} = 156 \cos(\omega t) \text{ V}, \quad \omega = 377 \frac{\text{rad}}{\text{s}}.$$

Find:

a) The turns ratio n .

b) The capacitor C .

Analysis:

a) First determine the maximum and minimum voltage across the load resistance and capacitor:

$$V_m = V_L + \frac{1}{2}V_r = 10 + 1.2 = 11.2 \text{ V}, \quad V_{L-\min} = V_L - \frac{1}{2}V_r = 10 - 1.2 = 8.8 \text{ V}.$$

The amplitude of the supply voltage can now be determined.

$$KVL: -V_{so} \cos(\omega t) + v_{D1} + v_{D3} + v_L(t) = 0$$

$$\text{At } t = 0, -V_{so} + V_{D-\text{on}} + V_{D-\text{on}} + V_L = 0 \Rightarrow V_{so} = 2V_{D-\text{on}} + V_m = 0.7 + 0.7 + 11.2 = 12.6 \text{ V}$$

$$n = \frac{V_{so}}{V_{io}} = 0.08077$$

b) KVL : $v_s(t) + v_{D2} + v_{D4} + v_L(t) = 0.$

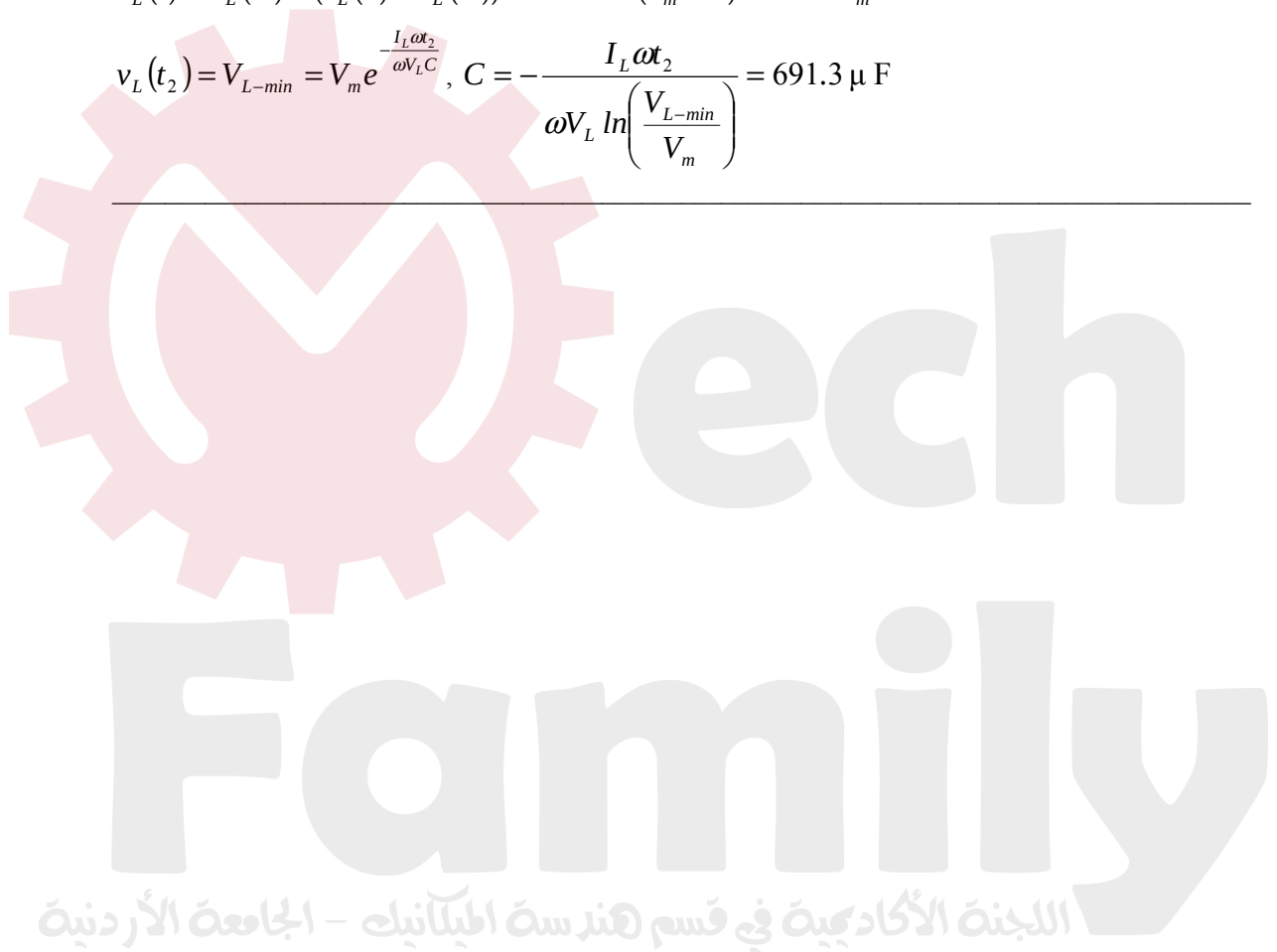
At $t = t_2$, $V_{so} \cos(\omega t_2) = -V_{D-on} - V_{D-on} - V_{L-min}.$

$$\omega t_2 = \cos^{-1} \left(-\frac{2V_{D2-on} + V_{L-min}}{V_{so}} \right) = 1.505 \text{ rad}.$$

The exponential discharge of the capacitor can be expressed:

$$v_L(t) = v_L(\infty) + (v_L(0) - v_L(\infty))e^{-\frac{t}{\tau}} = 0 + (V_m - 0)e^{-\frac{t}{R_L C}} = V_m e^{-\frac{I_L \omega t}{\omega V_L C}}$$

$$v_L(t_2) = V_{L-min} = V_m e^{-\frac{I_L \omega t_2}{\omega V_L C}}, C = -\frac{I_L \omega t_2}{\omega V_L \ln \left(\frac{V_{L-min}}{V_m} \right)} = 691.3 \mu F$$



Section 9.5: DC Power Supplies, Zener Diodes and Voltage Regulations

Problem 9.42

Solution:

Known quantities:

The piecewise characteristic that passes through the points $(-10 \text{ V}, -5 \mu\text{A})$, $(0, 0)$, $(0.5 \text{ V}, 5 \text{ mA})$ and $(1 \text{ V}, 50 \text{ mA})$.

Find:

Determine the piecewise linear model, and, using that model, solve for i and v .

Analysis:

Assume that the diode is forward-biased, and operating in the region between $(0.5 \text{ V}, 5 \text{ mA})$ and $(1 \text{ V}, 50 \text{ mA})$. If this is true, then the diode can be modeled by the resistance

$$r_D = \frac{\Delta v_D}{\Delta i_D} = \frac{1 - 0.5}{(50 - 5)10^{-3}} = \frac{0.5}{4510^{-3}} = 11.11 \Omega \text{ in series with a battery having value}$$

$$V_{bat} = 0.5 - 510^{-3}(11.11) = 0.444 \text{ V}.$$

$$\text{Then, } i = \frac{2 - 0.444}{111.11} = 14 \text{ mA and } v = 0.444 + 0.014(11.11) = 0.6 \text{ V}.$$

This solution is within the range initially assumed, justifying the assumption.

Problem 9.43

Solution:

Known quantities:

The output voltage at 5.6 V .

Find:

Determine the minimum value of R_L for which the output voltage remains at just 5.6 V .

Analysis:

$$\frac{R_{L_{min}}}{R_{L_{min}} + 1800}(18) = 5.6 \Rightarrow 12.4R_{L_{min}} = 10080 \Rightarrow R_{L_{min}} = 812.9 \Omega$$

Problem 9.44

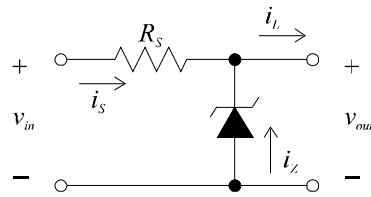
Solution:

Known quantities:

The output voltage 25 V . The input voltage that varies from 35 to 40 V . The maximum load current is 75 mA . The maximum current, 250 mA , for the Zener diode used.

Find:

Determine the minimum and the maximum value for the series resistance.

Analysis:

$$i_{s_{max}} = \frac{40 - 25}{R_{s_{min}}}, \quad i_{s_{min}} = \frac{35 - 25}{R_{s_{max}}}, \quad i_{Z_{min}} = 0, \quad i_{Z_{max}} = -250 \text{ mA}, \quad i_L = 75 \text{ mA}$$

$$i_{s_{max}} = 250 + 75 = 325 \text{ mA} \Rightarrow R_{s_{min}} = \frac{15}{325 \times 10^{-3}} = 462 \Omega,$$

$$i_{s_{min}} = 0 + 75 = 75 \text{ mA} \Rightarrow R_{s_{max}} = \frac{10}{75 \times 10^{-3}} = 1333 \Omega$$

Problem 9.45**Solution:****Known quantities:**

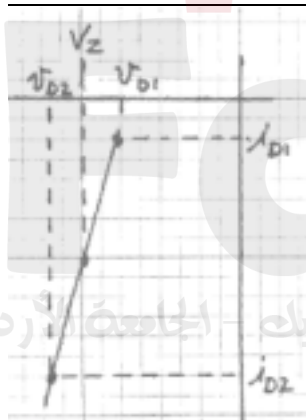
The i - v characteristic of a semiconductor; the minimum current at the "knee" of the curve, 5 mA, and the maximum current, 90 mA.

Find:

Determine the Zener resistance and Zener voltage of the diode.

Analysis:

The Zener voltage is evaluated at the middle of the rated region of operation, ie, midway between the knee of the curve and maximum rated current.



$$V_Z = 5 \text{ V}$$

The Zener resistance is determined from the slope of the i - v characteristic over the rated region of operation.

$$r_Z = \frac{1}{\text{Slope}} = \frac{1}{\frac{\Delta i_D}{\Delta v_D}} = \frac{\Delta v_D}{\Delta i_D} = \frac{v_{D1} - v_{D2}}{i_{D1} - i_{D2}} = \frac{2}{810^{-3}} = 25 \Omega$$

Note: The maximum Zener current is directly related to the maximum power the Zener diode can dissipate without breaking out in smoke and flames. In the specification sheet, either or both may be specified.

Problem 9.46**Solution:****Known quantities:**

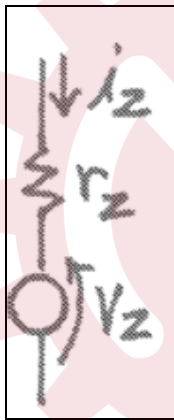
The Zener diode used, the 1N5231B; the ripple component of the source voltage, obtained from a DC power supply, $v_s = V_s + V_r$, where $V_s = 20 \text{ V}$, $V_r = 250 \text{ mV}$, $R = 220 \Omega$, $I_L = 65 \text{ mA}$, $V_L = 5.1 \text{ V}$, $R_Z = 17 \Omega$, $P_{\text{Rated}} = 0.5 \text{ W}$, $I_{Z-\min} = 10 \text{ mA}$.

Find:

Determine the maximum rated current the diode can handle without exceeding its power limitation.

Analysis:

Only the rated power is required from the information given. Use the DC model of the Zener. Note that power is dissipated in the equivalent Zener resistance and in the equivalent source. Note the direction of the current and the polarity of the source.



$$P = I_Z^2 R_Z + I_Z V_Z$$

$$\text{Let } P = P_{\text{Rated}}$$

$$\text{Then: } I_Z = I_{Z-\max}$$

$$P_{\text{Rated}} = I_{Z-\max}^2 R_Z + I_{Z-\max} V_Z$$

$$I_{Z-\max} = \frac{1}{2} \left(-\frac{V_Z}{R_Z} \right) \pm \frac{1}{2} \left(\left(\frac{V_Z}{R_Z} \right)^2 - 4 \frac{P_{\text{Rated}}}{R_Z} \right)^{1/2} = -150 \pm 227.8 = 77.8 \text{ mA}$$

where the negative answer is rejected because the Zener current by definition flows in the direction shown.

Problem 9.47**Solution:****Known quantities:**

The Zener diode used, the 1N963; $V_Z = 12 \text{ V}$, $R_Z = 11.5 \Omega$, $P_{\text{Rated}} = 0.4 \text{ W}$, and at the knee of the curve, $i_{ZK} = 0.25 \text{ mA}$, $r_{ZK} = 700 \Omega$.

Find:

Determine the maximum rated current the diode can handle without exceeding its power limitation.

Analysis:

Only the rated power is required from the information given. Use the DC model of the Zener. Note that power is dissipated in the equivalent Zener resistance and in the equivalent source. Note the direction of the current and the polarity of the source.

$$P = I_Z^2 R_Z + I_Z V_Z. \text{ Let } P = P_{\text{Rated}}$$

$$\text{Then: } I_Z = I_{Z-\max}, P_{\text{Rated}} = I_{Z-\max}^2 R_Z + I_{Z-\max} V_Z$$

$$I_{Z-\max} = \frac{1}{2} \left(-\frac{V_Z}{R_Z} \right) \pm \frac{1}{2} \left(\left(\frac{V_Z}{R_Z} \right)^2 - 4 \frac{P_{\text{Rated}}}{R_Z} \right)^{1/2} = -521.5 \pm 554.1 = 32.6 \text{ mA}$$

where the negative answer is rejected because the Zener current by definition flows in the direction shown.

Problem 9.48**Solution:****Known quantities:**

$$V_Z = 5 \text{ V} \pm 10\%, R_Z = 15 \Omega, I_{Z-\min} = 3.5 \text{ mA}, I_{Z-\max} = 80 \text{ mA}, V_S = 12 \pm 3 \text{ V}, \\ I_L = 70 \pm 20 \text{ mA}.$$

Find:

Determine the maximum and minimum value of R to maintain the Zener diode current within its specified limits.

Analysis:

Construct DC equivalent circuit:

$$\text{KCL} \quad \frac{V_L - V_S}{R} + I_Z + I_L = 0$$

$$\text{KVL} \quad -V_Z - I_Z R_Z + V_L = 0$$

Then:

$$V_L = V_Z + I_Z R_Z$$

$$R = \frac{V_S - V_L}{I_Z + I_L} = \frac{V_S - V_Z - I_Z R_Z}{I_Z + I_L}$$

A maximum Zener current is caused by:

Minimum value of R
Maximum source voltage
Minimum load current
Minimum Zener voltage.

$$R_{\min} = \frac{V_{S-\max} - V_{Z-\min} - I_{Z-\max} R_Z}{I_{Z-\max} + I_{L-\min}} = 71.54 \Omega$$

A minimum Zener current is caused by:

Maximum value of R
Minimum source voltage
Maximum load current
Maximum Zener voltage.

$$R_{\max} = \frac{V_{S-\min} - V_{Z-\max} - I_{Z-\min} R_Z}{I_{Z-\min} + I_{L-\max}} = 36.87 \Omega$$

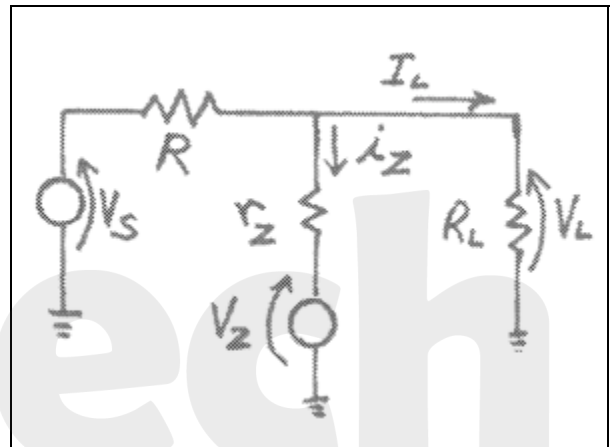
Note that the minimum value of R **EXCEEDS** the maximum value of R. This means that there is no value of R for which all the specifications will be met under all conditions. A value of R can be chosen but conditions may occur where the Zener current exceeds its maximum value or falls below its minimum value.

This problem can be solved by:

1. Choosing another Zener diode with different minimum and maximum currents.
2. Relaxing the specifications on source voltage and load current.

Note that the relationships between minimum and maximum values ALWAYS STARTS WITH THE QUESTION OF WHAT WILL CAUSE A MINIMUM OR MAXIMUM ZENER CURRENT !!! If the Zener current exceeds its maximum rated value, it will burn up; if it falls below its minimum value, the diode will leave the Zener region and cease to regulate the voltage. These represent WORST CASE conditions, a procedure frequently used in design.

Note that a Zener diode acts like an electrical "surge tank" for current; however, the analogy is not exact. A liquid surge tank regulates, ie, maintains constant, pressure and fluid flow rates. It does this by temporarily storing excess fluid when flow rates increase or supplying extra fluid when flow rates decrease. The Zener "surge tank" primarily regulates load voltage as the load resistance [and therefore the load current] or source voltage [and therefore the current supplied by the source] changes. It temporarily "stores"



excess current when load current decreases or source voltage increases and "supplies" extra current when load current increases or source voltage decreases.

Problem 9.49

Solution:

Known quantities:

Circuit shown in Figure P9.46,

$$V_Z = 12 \text{ V} \pm 10\%, R_Z = 9 \Omega, I_{Z-\min} = 3.25 \text{ mA}, I_{Z-\max} = 80 \text{ mA}, V_S = 25 \pm 1.5 \text{ V}, \\ I_L = 31 \pm 21.5 \text{ mA}.$$

Find:

Determine the maximum and minimum value of R to maintain the Zener diode current within its specified limits.

Analysis:

Construct DC equivalent circuit:

$$\text{KCL } \frac{V_L - V_S}{R} + I_Z + I_L = 0, \text{ KVL } -V_Z - I_Z R_Z + V_L = 0$$

Then:

$$V_L = V_Z + I_Z R_Z, R = \frac{V_S - V_L}{I_Z + I_L} = \frac{V_S - V_Z - I_Z R_Z}{I_Z + I_L}$$

A maximum Zener current is caused by:

Minimum value of R
Minimum load current

Maximum source voltage
Minimum Zener voltage.

$$R_{\min} = \frac{V_{S-\max} - V_{Z-\min} - I_{Z-\max} R_Z}{I_{Z-\max} + I_{L-\min}} = 14.98 \Omega$$

A minimum Zener current is caused by:

Maximum value of R
Maximum load current

Minimum source voltage
Maximum Zener voltage.

$$R_{\max} = \frac{V_{S-\min} - V_{Z-\max} - I_{Z-\min} R_Z}{I_{Z-\min} + I_{L-\max}} = 184.2 \Omega$$

Problem 9.50

Solution:

Known quantities:

The diode used, 1N4740A, in the circuit shown in Figure P9.46. $V_Z = 10 \text{ V} \pm 5\%$, $R_Z = 7 \Omega$,

$$I_{Z-\min} = 10 \text{ mA}, I_{Z-\max} = 91 \text{ mA}, V_S = 14 \pm 2 \text{ V}, R = 19.8 \Omega, P_{\text{Rated}} = 1 \text{ W}.$$

Find:

Determine the maximum and minimum value of the load current to maintain the Zener diode current within its specified limits.

Analysis:

Construct DC equivalent circuit:

$$\text{KCL } \frac{V_L - V_S}{R} + I_Z + I_L = 0, \text{ KVL } -V_Z - I_Z R_Z + V_L = 0$$

Then:

$$V_L = V_Z + I_Z R_Z, I_L = \frac{V_S - V_L}{R} - I_Z = \frac{V_S - V_Z - I_Z R_Z}{R} - I_Z$$

A minimum and maximum Zener current is caused by [respectively]:

Minimum source voltage	Maximum source voltage
Maximum load current	Minimum load current
Maximum Zener voltage	Minimum Zener voltage

Substituting with these extreme values and solving for the load current:

$$I_{L-max} = \frac{V_{S-min} - V_{Z-max} - I_{Z-min} R_Z}{R} - I_{Z-min} = 62.22 \text{ mA}$$

$$I_{L-min} = \frac{V_{S-max} - V_{Z-min} - I_{Z-max} R_Z}{R} - I_{Z-max} = 205.1 \text{ mA}$$

The minimum load current **EXCEEDS** the maximum load current.

Problem 9.51

Solution:

Known quantities:

The diode used, 1N963, in circuit shown in Figure P9.46. $V_Z = 12 \text{ V} \pm 10\%$, $R_Z = 11.5 \Omega$, $I_{Z-min} = 2.5 \text{ mA}$, $I_{Z-max} = 32.6 \text{ mA}$, $V_S = 25 \pm 2 \text{ V}$, $R = 470 \Omega$, $P_{Rated} = 0.4 \text{ W}$.

Find:

Determine the maximum and minimum value of the load current to maintain the Zener diode current within its specified limits.

Analysis:

Construct DC equivalent circuit:

$$KCL \quad \frac{V_L - V_S}{R} + I_Z + I_L = 0, \quad KVL \quad -V_Z - I_Z R_Z + V_L = 0$$

Then:

$$V_L = V_Z + I_Z R_Z, I_L = \frac{V_S - V_L}{R} - I_Z = \frac{V_S - V_Z - I_Z R_Z}{R} - I_Z$$

A minimum and maximum Zener current is caused by [respectively]:

Minimum source voltage	Maximum source voltage
Maximum load current	Minimum load current
Maximum Zener voltage	Minimum Zener voltage

Substituting with these extreme values and solving for the load current:

$$I_{L-max} = \frac{V_{S-min} - V_{Z-max} - I_{Z-min} R_Z}{R} - I_{Z-min} = 18.29 \text{ mA}$$

$$I_{L-min} = \frac{V_{S-max} - V_{Z-min} - I_{Z-max} R_Z}{R} - I_{Z-max} = 1.07 \text{ mA}$$

Problem 9.52

Solution:

Known quantities:

The diode used, 1N4740A, in circuit shown in Figure P9.46. $V_Z = 10 \text{ V} \pm 5\%$, $R_Z = 7 \Omega$, $I_{Z-min} = 10 \text{ mA}$, $I_{Z-max} = 91 \text{ mA}$, $I_L = 35 \pm 10 \text{ mA}$, $R = 80 \Omega \pm 5\%$, $P_{Rated} = 1 \text{ W}$.

Find:

Determine the maximum and minimum value of the source voltage to maintain the Zener diode current within its specified limits.

Analysis:

Construct DC equivalent circuit:

$$KCL \quad \frac{V_L - V_S}{R} + I_Z + I_L = 0, \quad KVL \quad -V_Z - I_Z R_Z + V_L = 0$$

Then:

$$V_L = V_Z + I_Z R_Z, \quad V_S = V_L + R(I_Z + I_L) = V_Z + I_Z R_Z + R(I_Z + I_L)$$

Minimum and Maximum Zener currents are caused by [respectively]:

Minimum source voltage.

Maximum source voltage.

Maximum load current.

Minimum load current.

Maximum Zener voltage.

Minimum Zener voltage.

Maximum R.

Minimum R.

Substituting with these extreme values and solving for the source voltage:

$$V_{S-\min} = V_{Z-\max} + I_{Z-\min} R_Z + R_{\max} (I_{Z-\min} + I_{L-\max}) = 15.19 \text{ V}$$

$$V_{S-\max} = V_{Z-\min} + I_{Z-\max} R_Z + R_{\min} (I_{Z-\max} + I_{L-\min}) = 18.95 \text{ V}$$

Problem 9.53**Solution:****Known quantities:**

The diode used, 1N4740A, in circuit shown in Figure P9.46. The source voltage is obtained from a DC power supply that has a DC and a ripple component $v_S = V_S + V_r$, where $V_S = 16 \text{ V}$, $V_r = 2 \text{ V}$,

$$I_{Z-\min} = 10 \text{ mA}, \quad I_{Z-\max} = 91 \text{ mA}, \quad I_L = 35 \text{ mA}, \quad R_Z = 7 \Omega, \quad R = 80 \Omega, \quad V_L = 10 \text{ V},$$

$$V_Z = 10 \text{ V}.$$

Find:

Determine the ripple voltage across the load.

Analysis:

Construct the AC equivalent circuit. Suppress the DC component of all voltages and currents in the circuit. Suppress also the DC source in the Zener diode model:

$$R_L = \frac{V_L}{I_L} = 285.7 \Omega, \quad R_{eq} = \frac{R_Z R_L}{R_Z + R_L} = 6.83 \Omega$$

Note: $v_S = V_r$.

$$V_L = V_r \frac{R_{eq}}{R + R_{eq}} = 157.4 \text{ mV}$$

Problem 9.54**Solution:****Known quantities:**

The diode used, 1N5231B, in circuit shown in Figure P9.46. The source voltage is obtained from a DC power supply that has a DC and a ripple component $v_S = V_S + V_r$, where $V_S = 20 \text{ V}$,

$$V_r = 250 \text{ mV}, I_{Z-\min} = 10 \text{ mA}, P_{\text{Rated}} = 0.5 \text{ W}, I_L = 65 \text{ mA}, R_Z = 17 \Omega, R = 220 \Omega, \\ V_L = 5.1 \text{ V}, V_Z = 5.1 \text{ V}.$$

Find:

Determine the ripple voltage across the load.

Analysis:

Construct the AC equivalent circuit. Note that in this case the load resistance is much smaller than the Zener resistance and will cause a small reduction in the ripple voltage across the load.

$$R_L = \frac{V_L}{I_L} = 78.46 \Omega, R_{eq} = \frac{R_Z R_L}{R_Z + R_L} = 13.97 \Omega$$

$$\text{Note: } v_S = V_r.$$

$$V_l = V_L \frac{R_{eq}}{R + R_{eq}} = 14.93 \text{ mV}$$

Problem 9.55**Solution:****Known quantities:**

The diode used, 1N970, in circuit shown in Figure P9.46. The source voltage is obtained from a DC power supply that has a DC and a ripple component $v_S = V_S + V_r$, where $V_S = 30 \text{ V}$, $V_r = 3 \text{ V}$,

$$I_{Z-\min} = 1.5 \text{ A}, I_{Z-\max} = 15 \text{ A}, I_L = 8 \text{ A}, R_Z = 33 \Omega, R = 1 \Omega, V_L = 24 \text{ V}, V_Z = 24 \text{ V}.$$

Find:

Determine the ripple voltage across the load.

Analysis:

Construct the AC equivalent circuit. Note that in this case the load resistance is much smaller than the Zener resistance and will cause a small reduction in the ripple voltage across the load.

$$R_L = \frac{V_L}{I_L} = 3 \Omega, R_{eq} = \frac{R_Z R_L}{R_Z + R_L} = 2.750 \Omega$$

$$\text{Note: } v_S = V_r.$$

$$V_l = V_L \frac{R_{eq}}{R + R_{eq}} = 2.2 \text{ V}$$

Section 9.6: Signal Processing Applications**Section 9.7: Photodiodes****Problem 9.56****Solution:****Known quantities:**

The range for the input voltage $0 \leq v \leq 10 \text{ V}$ for the circuit of Figure P9.56.

Find:

Determine the i - v characteristic of the circuit.

Analysis:

With the variables defined in the circuit below, we can compute the following currents:

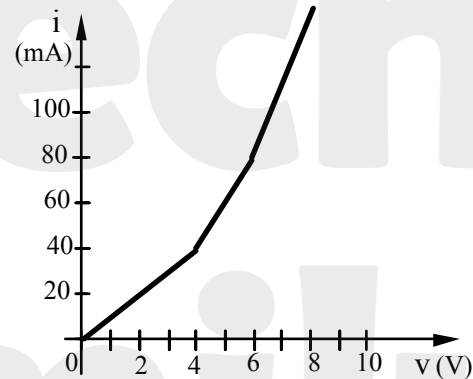
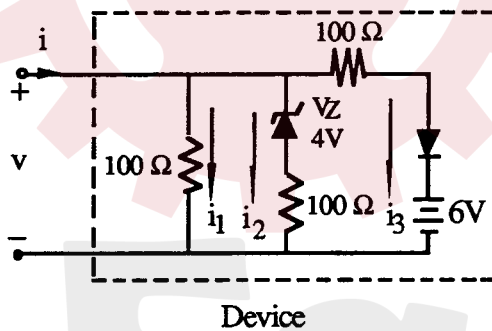
$$I_1 = \frac{v}{100}, I_2 = \frac{v}{100} \text{ for } v \geq 4 \text{ V}, I_3 = \frac{v}{100} \text{ for } v \geq 6 \text{ V}$$

For $0 \leq v \leq 4 \text{ V}$, $I = 0.01v$

For $4 \leq v \leq 6 \text{ V}$, $I = 0.02v - 0.04$

For $6 \leq v \leq 10 \text{ V}$, $I = 0.03v - 0.1$

The resulting i - v characteristic is shown below:

**Problem 9.57****Solution:****Known quantities:**

The input voltage waveform and the circuit of Figure P9.57.

Find:

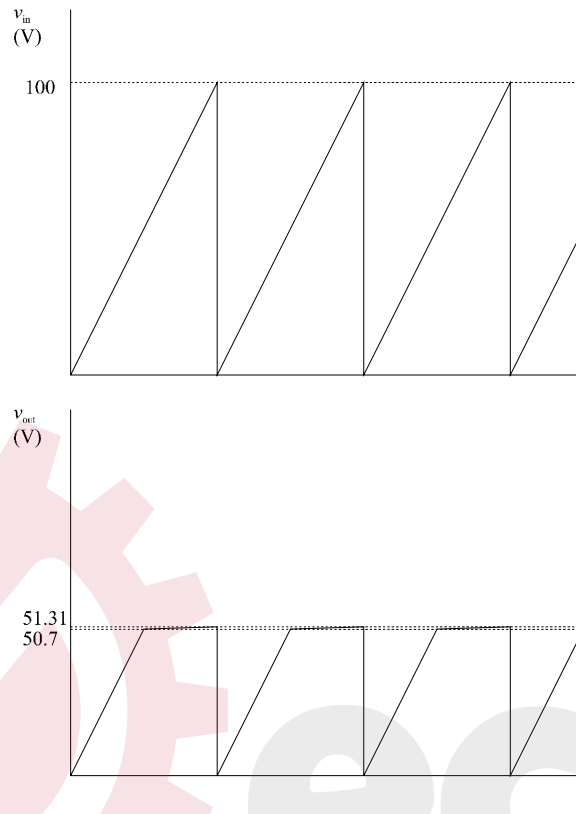
Determine the output voltage.

Analysis:

For $v_{in} < 50.7 \text{ V}$, $v_{out} = v_{in}$.

When $v_{in} \geq 50.7 \text{ V}$, $v_{out} = 50.7 + \frac{0.6}{98 + 0.6} v_{in} = 50.7 + (6.085 \times 10^{-3}) v_{in}$

The input and output voltage waveforms are sketched below:



Problem 9.58

Solution:

Known quantities:

The circuit of Figure P9.58 where the switch is close at time $t = t_1$.

Find:

The currents I_S , I_B , I_{SW} for the following conditions:

a) $t = t_1^-$, b) $t = t_1^+$, c) what will happen to the battery after the switch closes.

Repeat all considering the circuit of Figure P9.58(b) if the diode has an offset voltage of 0.6 V.

Analysis:

For Figure P9.58 (a):

a) At $t = t_1^-$, before the switch S_1 closes, we have

$$I_{SW} = 0, I_S = I_B = \frac{V_S - V_{Battery}}{R_S + R_B} = 0.31 \text{ A}$$

b) At $t = t_1^+$, we have

$$I_S = 13 \text{ A}, I_B = -0.96 \text{ A}, I_{SW} = I_S - I_B = 13.96 \text{ A}$$

c) The battery voltage will drop quickly because of the small resistance in the circuit.

For Figure P9.58 (b):

a) At $t = t_1^-$, we have

$$I_{SW} = 0, I_S = I_B = \frac{V_S - V_{Battery} - V_\gamma}{R_S + R_B} = 0.25 \text{ A}$$

b) At $t = t_1^+$, we have

$$I_S = I_{SW} = 13 \text{ A}, I_B = 0$$

c) The battery will not be drained, because of the large reverse resistance of the diode.

Problem 9.59

Solution:

Known quantities:

The circuit in Figure P9.59. The diode has $V_\gamma = 0.7 \text{ V}$. The input voltage is sinusoidal with an amplitude of 6, 1.5, 0.4 V and zero average value.

Find:

Determine the average value of the output voltage.

Analysis:

The capacitor will charge to $V_S - 0.7 \text{ V}$.

6 V:

Therefore, the input sine wave will be shifted up 5.3 V to produce the output. As a result, after the cycle (the capacitor builds up its stored charge during the third quarter cycle), the average value of the output will be 5.3 V.

1.5 V:

Therefore, the input sine wave will be shifted up 0.8 V to produce the output. As a result, after the cycle (the capacitor builds up its stored charge during the third quarter cycle), the average value of the output will be 0.8 V.

0.4 V:

Since the input is less than 0.7 V, the capacitor does not charge and the output voltage is 0 V.

Problem 9.60

Solution:

Known quantities:

The LED circuit in Figure 9.14. Diode operating point: $V_{LED} = 1.7 \text{ V}$; $I_{LED} = 20 \text{ mA}$; $V_S = 5 \text{ V}$.

Find:

Determine the LED power consumption and the power required by the voltage source.

Analysis:

The power consumption is

$$P_{LED} = V_{LED} I_{LED} = 34 \text{ mW}$$

The power required at the source is

$$P_S = V_S I_{LED} = 100 \text{ mW}$$

Problem 9.61

Solution:

Known quantities:

The LED circuit in Figure 9.14. Diode operating point: $V_{LED} = 1.5 \text{ V}$; $I_{LED} = 30 \text{ mA}$; $V_S = 5 \text{ V}$.

Find:

Determine the LED power consumption and the power required by the voltage source.

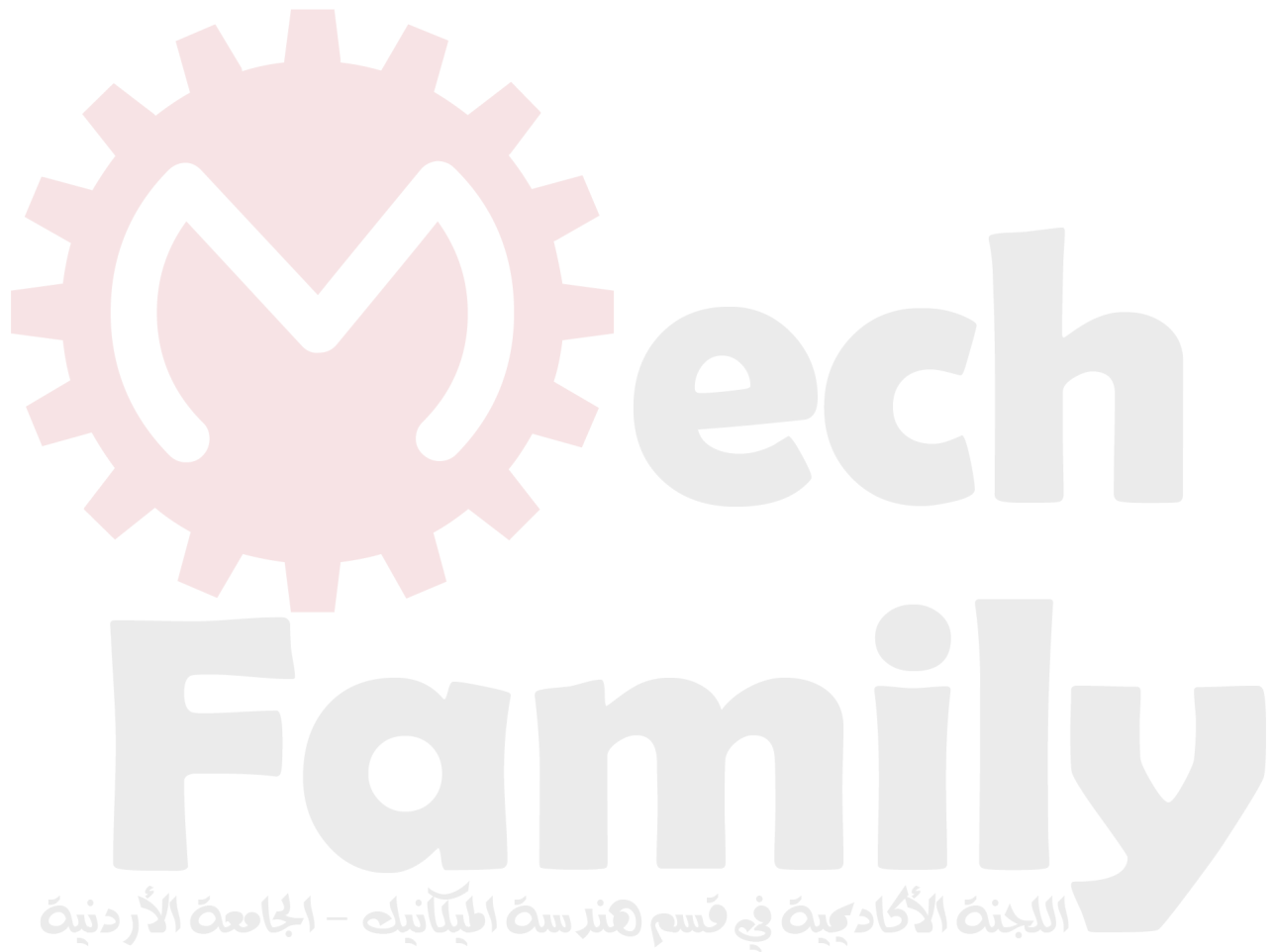
Analysis:

The power consumption is

$$P_{\text{LED}} = V_{\text{LED}} I_{\text{LED}} = 45 \text{ mW}$$

The power required at the source is

$$P_S = V_S I_{\text{LED}} = 150 \text{ mW}$$



Chapter 10 Instructor Notes

Chapter 10 introduces bipolar junction transistors. The material on transistors has been reorganized in this 4th Edition, and is now divided into two independent chapters, one on bipolar devices, and one on field-effect devices. The two chapters are functionally independent, except for the fact that Section 10.1, introducing the concept of transistors as amplifiers and switches, can be covered prior to starting Chapter 11 if the instructor decides to only teach field-effect devices.

Section 10.2 introduces the fundamental ideas behind the operation of bipolar transistors, and illustrates the calculation of the state and operating point of basic transistor circuits. The discussion of the properties of the BJT in Section 10.2 is centered around a description of the base and collector characteristics, and purposely avoids a detailed description of the physics of the device, with the intent of providing an intuitive understanding of the transistor as an amplifier and electronic switch. Example 10.5 introduces the use of Electronics Workbench (EWB) as a tool for analyzing electronics circuits; the CD-ROM contains an introduction to the software package and a number of solved problems.

Section 10.3 introduces large-signal models of the BJT, and also includes the box *Focus on Methodology: Using device data sheets* (pp. 535-537). Example 10.7 (LED Driver) and the box *Focus on Measurements: Large Signal Amplifier for Diode Thermometer* (pp. 539-541) provide two application examples that include EWB solutions.

Section 8.4 introduces the analysis of BJT switches and presents TTL gates.

The end-of-chapter problems are straightforward applications of the concepts illustrated in the chapter.

Learning Objectives

1. Understand the basic principles of amplification and switching. Section 1.
2. Understand the physical operation of bipolar transistors; determine and select the operating point of a bipolar transistor circuit; understand the principle of small signal amplifiers. Section 2.
3. Understand the large-signal model of the bipolar transistor, and apply it to simple amplifier circuits. Section 3.
4. Understand the operation of bipolar transistor as a switch and analyze basic analog and digital gate circuits. Section 4.

Section 10.2: Operation of the Bipolar Junction Transistor

Problem 10.1

Solution:

Known quantities:

Transistor diagrams, as shown in Figure P10.1:

- (a) *pn*p, $V_{EB} = 0.6 \text{ V}$ and $V_{EC} = 4.0 \text{ V}$
- (b) *n*p*n*, $V_{CB} = 0.7 \text{ V}$ and $V_{CE} = 0.2 \text{ V}$
- (c) *n*p*n*, $V_{BE} = 0.7 \text{ V}$ and $V_{CE} = 0.3 \text{ V}$
- (d) *pn*p, $V_{BC} = 0.6 \text{ V}$ and $V_{EC} = 5.4 \text{ V}$

Find:

For each transistor shown in Figure P10.1, determine whether the BE and BC junctions are forward or reverse biased, and determine the operating region.

Analysis:

- (a) $V_{BE} = -0.6 \text{ V}$ for a *pn*p transistor implies that the BE junction is forward-biased.
 $V_{BC} = V_{EC} - V_{EB} = 3.4 \text{ V}$. The CB junction is reverse-biased. Therefore, the transistor is in the active region.
- (b) $V_{BC} = -0.7 \text{ V}$ for a *n*p*n* transistor implies that the CB junction is reverse-biased.
 $V_{BE} = V_{BC} - V_{EC} = -0.5 \text{ V}$. The BE junction is reverse-biased. Therefore, the transistor is in the cutoff region.
- (c) $V_{BE} = 0.7 \text{ V}$ for a *n*p*n* transistor implies that the BE junction is forward-biased.
 $V_{BC} = V_{EC} - V_{EB} = 0.4 \text{ V}$. The CB junction is forward-biased. Therefore, the transistor is in the saturation region.
- (d) $V_{BC} = 0.6 \text{ V}$ for a *pn*p transistor implies that the CB junction is reverse-biased.
 $V_{BE} = V_{BC} - V_{EC} = -4.8 \text{ V}$. The BE junction is forward-biased. Therefore, the transistor is in the active region.

Problem 10.2

Solution:

Known quantities:

Transistor type and operating characteristics:

- a) *n*p*n*, $V_{BE} = 0.8 \text{ V}$ and $V_{CE} = 0.4 \text{ V}$
- b) *n*p*n*, $V_{CB} = 1.4 \text{ V}$ and $V_{CE} = 2.1 \text{ V}$
- c) *pn*p, $V_{CB} = 0.9 \text{ V}$ and $V_{CE} = 0.4 \text{ V}$
- d) *n*p*n*, $V_{BE} = -1.2 \text{ V}$ and $V_{CB} = 0.6 \text{ V}$

Find:

The region of operation for each transistor.

Analysis:

- a) Since $V_{BE} = 0.8 \text{ V}$, the BE junction is forward-biased. $V_{CB} = V_{CE} + V_{EB} = -0.4 \text{ V}$. Thus, the CB junction is forward-biased. Therefore, the transistor is in the saturation region.

- b) $V_{BE} = V_{BC} + V_{CE} = 0.7 \text{ V}$. The BE junction is forward-biased.
 $V_{CB} = 1.4 \text{ V}$. The CB junction is reverse-biased. Therefore, the transistor is in the active region.
- c) $V_{CB} = 0.9 \text{ V}$ for a *pnp* transistor implies that the CB junction is forward-biased.
 $V_{BE} = V_{BC} - V_{CE} = -1.3 \text{ V}$. The BE junction is forward-biased. Therefore, the transistor is in the saturation region.
- d) With $V_{BE} = -1.2 \text{ V}$, the BE junction is reverse-biased.
 $V_{CB} = -0.6 \text{ V}$. The CB junction is reverse-biased. Therefore, the transistor is in the cutoff region.

Problem 10.3

Solution:

Known quantities:

The circuit of Figure P10.3. $\beta = \frac{I_C}{I_B} = 100$.

Find:

The operating point and the state of the transistor.

Analysis:

$V_{BE} = 0.6 \text{ V}$ and the BE junction is forward biased.

$$I_B = \frac{V_{CC} - V_{BE}}{R_1} = \frac{12 - 0.6}{820} = 13.9 \mu\text{A}$$

$$I_C = \beta \cdot I_B = 1.39 \text{ mA}$$

Writing KVL around the right-hand side of the circuit:

$$-V_{CC} + I_C R_C + V_{CE} + I_E R_E = 0 \quad \Rightarrow$$

$$\begin{aligned} V_{CE} &= V_{CC} - I_C R_C - (I_B + I_C) R_E \\ &= 12 - (1.39)(2.2) - (1.39 + 0.0139)(0.910) \\ &= 7.664 \text{ V} \end{aligned}$$

$$V_{BC} = V_{BE} + V_{CE} = 0.6 + 7.664 = 8.264 \text{ V}$$

$V_{CE} > V_{BE} \Rightarrow$ The transistor is in the active region.

Problem 10.4**Solution:****Known quantities:**

The magnitude of a *pnp* transistor's emitter and base current, and the magnitudes of the voltages across the emitter-base and collector-base junctions:

$$I_E = 6 \text{ mA}, I_B = 0.1 \text{ mA} \text{ and } V_{EB} = 0.65 \text{ V}, V_{CB} = 7.3 \text{ V}.$$

Find:

- V_{CE} .
- I_C .
- The total power dissipated in the transistor, defined as $P = V_{CE} I_C + V_{BE} I_B$.

Analysis:

- $V_{CE} = V_{CB} - V_{EB} = 7.3 - 0.65 = 6.65 \text{ V}$.
- $I_C = I_E - I_B = 6 - 0.1 = 5.9 \text{ mA}$.
- The total power dissipated in the transistor can be found to be: $P \approx V_{CE} I_C = 6.65 \times 5.9 \times 10^{-3} = 39 \text{ mW}$.

Problem 10.5**Solution:****Known quantities:**

The circuit of Figure P10.5, assuming the BJT has $V_\gamma = 0.6 \text{ V}$.

Find:

The emitter current and the collector-base voltage.

Analysis:

Applying KVL to the right-hand side of the circuit,
$$I_E = -\left(\frac{V_{BE} + 15}{30000}\right) = -\frac{(0.6 + 15)}{30000} = -520 \mu\text{A}$$

Then, on the left-hand side, assuming $\beta \gg 1$:

$$\begin{aligned} V_{CB} &= 10 - I_C \cdot R_C \\ -10 + I_C R_C + V_{CB} &= 0 \Rightarrow 10 - (-520) \cdot (15) \cdot 10^{-3} \\ &= 17.8 \text{ V} \end{aligned}$$

Problem 10.6**Solution:****Known quantities:**

The circuit of Figure P10.6, assuming the BJT has $V_{BE} = 0.6 \text{ V}$ and $\beta = 150$.

Find:

The operating point and the region in which the transistor operates.

Analysis:

Define $R_C = 3.3\text{k}\Omega$, $R_E = 1.2\text{k}\Omega$, $R_1 = 62\text{k}\Omega$, $R_2 = 15\text{k}\Omega$, $V_{CC} = 18\text{ V}$

By applying Thevenin's theorem from base and mass, we have

$$R_B = R_1 \parallel R_2 = 12.078\text{ k}\Omega$$

$$V_{BB} = \frac{R_2}{R_1 + R_2} V_{CC} \cong 3.5\text{ V}$$

$$I_B = \frac{V_{BB} - V_{BE}}{R_B + R_E(1 + \beta)} \cong 15\text{ }\mu\text{A}$$

$$I_C = \beta I_B = 2.25\text{ mA}$$

$$V_{CE} = V_{CC} - R_C I_C - R_E I_E = 18 - 3300 \cdot 2.25 \cdot 10^{-3} - 1200 \cdot 151 \cdot 15 \cdot 10^{-6} = 7.857\text{ V}$$

From the value of V_{CE} it is clear that the BJT is in the active region.

Problem 10.7**Solution:****Known quantities:**

The circuit of Figure P10.7, assuming the BJT has $V_{\gamma} = 0.6\text{ V}$.

Find:

The emitter current and the collector-base voltage.

Analysis:

Applying KVL to the right-hand side of the circuit,

$$-V_{CC} + I_E R_E + V_{EB} = 0$$

$$I_E = \frac{V_{CC} - V_{EB}}{R_E} = \frac{20 - 0.6}{39 \cdot 10^3} = 497.4\text{ }\mu\text{A} \text{ . Since } \beta \gg 1, I_C \approx I_E = 497.4\text{ }\mu\text{A}$$

Applying KVL to the left-hand side: $V_{CB} + I_C R_C - V_{DD} = 0$

$$V_{CB} = V_{DD} - I_C R_C = 20 - 497.4 \cdot 20 \cdot 10^{-3} = 10.05\text{ V}$$

Problem 10.8**Solution:****Known quantities:**

The circuit of Figure P10.7, assuming the emitter resistor is changed to $22\text{ k}\Omega$ and the BJT has

$$V_{\gamma} = 0.6\text{ V} \text{ .}$$

Find:

The operating point of the transistor.

Analysis:

$$I_E = \frac{V_{CC} - V_{EB}}{R_E} = \frac{20 - 0.6}{22 \cdot 10^3} = 881.8\text{ }\mu\text{A} \text{ , } I_C \approx I_E = 881.8\text{ }\mu\text{A}$$

$$V_{CB} = V_{DD} - I_C R_C = 20 - 881.8 \cdot 20 \cdot 10^{-3} = 2.364 \text{ V}$$

Problem 10.9

Solution:

Known quantities:

The collector characteristics for a certain transistor, as shown in Figure P10.9.

Find:

- The ratio I_C/I_B for $V_{CE} = 10 \text{ V}$ and $I_B = 100 \mu\text{A}$, $200 \mu\text{A}$, and $600 \mu\text{A}$
- V_{CE} , assuming the maximum allowable collector power dissipation is 0.5 W for $I_B = 500 \mu\text{A}$.

Analysis:

- For $I_B = 100 \mu\text{A}$ and $V_{CE} = 10 \text{ V}$, from the characteristics, we have $I_C = 17 \text{ mA}$. The ratio I_C/I_B is 170.

For $I_B = 200 \mu\text{A}$ and $V_{CE} = 10 \text{ V}$, from the characteristics, we have $I_C = 33 \text{ mA}$. The ratio I_C/I_B is 165.

For $I_B = 600 \mu\text{A}$ and $V_{CE} = 10 \text{ V}$, from the characteristics, we have $I_C = 86 \text{ mA}$. The ratio I_C/I_B is 143.

- For $I_B = 500 \mu\text{A}$, and if we consider an average β from a., we have $I_C = 159 \cdot 500 \cdot 10^{-3} = 79.5 \text{ mA}$. The power dissipated by the transistor is $P = V_{CE} I_C + V_{BE} I_B \approx V_{CE} I_C$, therefore:

$$V_{CE} \approx \frac{P}{I_C} = \frac{0.5}{79.5 \cdot 10^{-3}} = 6.29 \text{ V}.$$

Problem 10.10

Solution:

Known quantities:

Figure P10.10, assuming both transistors are silicon-based with $\beta = 100$.

Find:

- I_{C1} , V_{C1} , V_{CE1} .
- I_{C2} , V_{C2} , V_{CE2} .

Analysis:

- From KVL: $-30 + I_{B1} R_{B1} + V_{BE1} = 0 \Rightarrow I_{B1} = \frac{30 - 0.7}{750 \cdot 10^3} = 39.07 \mu\text{A}$

$$I_{C1} = \beta \cdot I_{B1} = 3.907 \text{ mA} \Rightarrow$$

$$V_{C1} = 30 - R_{C1} I_{C1} = 30 - 3.907 \cdot 6.2 = 5.779 \text{ V}$$

$$V_{CE1} = V_{C1} = 5.779 \text{ V}.$$

- Again, from KVL: $-5.779 + V_{BE2} + I_{E2} R_{E2} = 0 \Rightarrow I_{E2} = \frac{5.779 - 0.7}{4.7 \cdot 10^3} = 1.081 \text{ mA}$

$$\text{and } I_{C2} = I_{E2} \left(\frac{\beta}{\beta + 1} \right) = 1.081 \cdot \left(\frac{100}{101} \right) = 1.07 \text{ mA}.$$

$$\text{Also, } -30 + I_{C2}(R_{C2} + R_{E2}) + V_{CE2} = 0 \Rightarrow V_{CE2} = 30 - (1.07) \cdot (20 + 4.7) = 3.574 \text{ V}.$$

$$\text{Finally, } I_{C2} = \frac{30 - V_{CE2}}{R_{C2}} \Rightarrow V_{CE2} = 30 - (1.07) \cdot (20) = 8.603 \text{ V}.$$

Problem 10.11

Solution:

Known quantities:

Collector characteristics of the 2N3904 npn transistor, see data sheet pg. 536.

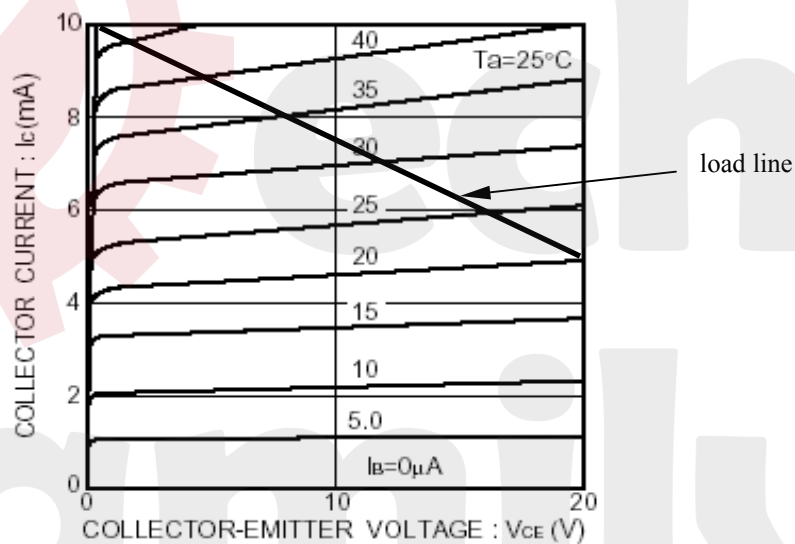
Find:

The operating point of the transistor in Figure P10.11, and the value of β at this point.

Analysis:

Construct a load line. Writing KVL, we have: $-50 + 5000 \cdot I_C + V_{CE} = 0$.

Then, if $I_C = 0$, $V_{CE} = 50 \text{ V}$; and if $V_{CE} = 0$, $I_C = 10 \text{ mA}$. The load line is shown superimposed on the collector characteristic below:



The operating point is at the intersection of the load line and the $I_B = 20 \mu\text{A}$ line of the characteristic.

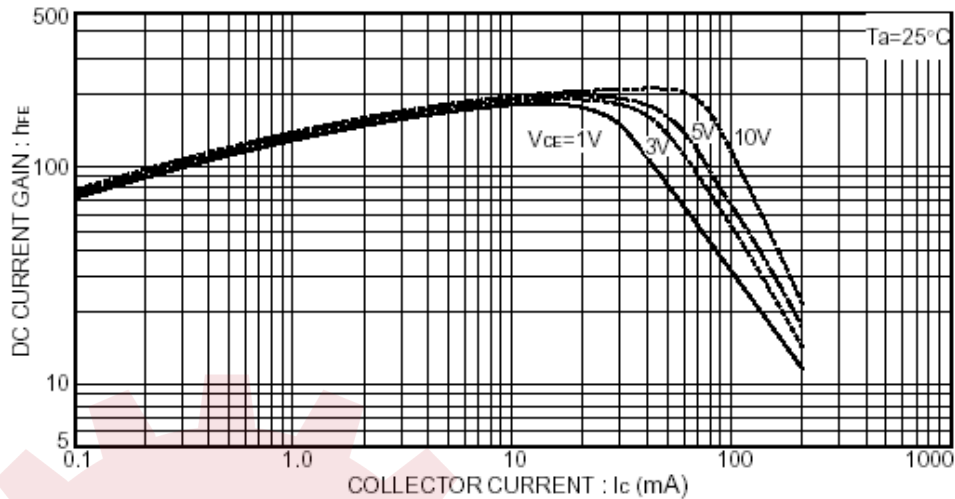
Therefore, $I_{CQ} \approx 5 \text{ mA}$ and $V_{CEQ} \approx 20 \text{ V}$.

Under these conditions, an $5 \mu\text{A}$ increase in I_B yields an increase in I_C of approximately

$6 - 5 = 1 \text{ mA}$. Therefore,

$$\beta \approx \frac{\Delta I_C}{\Delta I_B} = \frac{1 \cdot 10^{-3}}{5 \cdot 10^{-6}} = 200$$

The same result can be obtained by checking the h_{FE} gain from the data-sheets corresponding to 5 mA.



Problem 10.12

Solution:

Known quantities:

The circuit shown in Figure P10.12. With reference to Figure 10.20, assume $V_\gamma = 0.6\text{ V}$, $V_{sat} = 0.2\text{ V}$.

Find:

The operating point of the transistor, by computing the ratio of collector current to base current.

Analysis:

$$V_{CE} = V_{sat} = 0.2\text{ V}, \text{ therefore } I_C = \frac{10 - 0.2}{R_C} = 9.8\text{ mA}$$

$$V_{BE} = V_\gamma = 0.6\text{ V}, \text{ therefore } I_B = \frac{5.7 - 0.6}{R_B} = 102\text{ }\mu\text{A}$$

$$\frac{I_C}{I_B} = \frac{9.8 \cdot 10^{-3}}{102 \cdot 10^{-6}} = 96.08 \ll \beta$$

Problem 10.13

Solution: اللجنة الأكاديمية في قسم هندسة الميكانيك - الجامعة الأردنية

Known quantities:

For the circuit shown in Figure P10.13, $V_E = 1\text{ V}$ and $V_\gamma = 0.6\text{ V}$.

Find:

a) V_B	d) I_C
b) I_B	e) β
c) I_E	f) α

Analysis:

$$\text{a) } V_{EB} = V_E - V_B = V_\gamma = 0.6\text{ V} \Rightarrow V_B = V_E - V_{EB} = 1 - 0.6 = 0.4\text{ V}$$

- b) $I_B = \frac{V_B}{R_B} = \frac{0.4}{20 \cdot 10^3} = 20 \mu\text{A}$
- c) $I_E = \frac{5 - V_E}{R_E} = \frac{5 - 1}{5000} = 800 \mu\text{A}$
- d) $I_C = I_E - I_B = 800 - 20 = 780 \mu\text{A}$
- e) $\beta = \frac{I_C}{I_B} = \frac{780}{20} = 39$
- f) $\alpha = \frac{I_C}{I_E} = \frac{780}{800} = 0.975$.

Problem 10.14

Solution:

Known quantities:

For the circuit shown in Figure P10.14:

$$V_{CC} = 20 \text{ V} \quad \beta = 130 \quad R_1 = 1.8 \text{ M}\Omega \quad R_2 = 300 \text{ k}\Omega \quad R_C = 3 \text{ k}\Omega \quad R_E = 1 \text{ k}\Omega$$

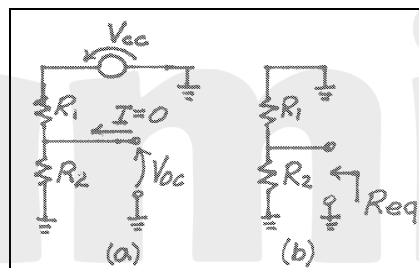
$$R_L = 1 \text{ k}\Omega \quad R_S = 0.6 \text{ k}\Omega \quad v_S = \cos[6.28 \times 10^3 \cdot t] \text{ mV}.$$

Find:

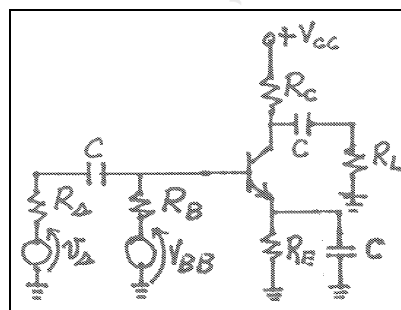
The Thévenin equivalent of the part of the circuit containing R_1 , R_2 , and V_{CC} with respect to the terminals of R_2 . Redraw the schematic using the Thévenin equivalent.

Analysis:

Extracting the part of the circuit specified, the Thévenin equivalent voltage is the open circuit voltage. The equivalent resistance is obtained by suppressing the ideal independent voltage source:



Note that V_{CC} must remain in the circuit because it supplies current to other parts of the circuit:



Problem 10.15**Solution:****Known quantities:**

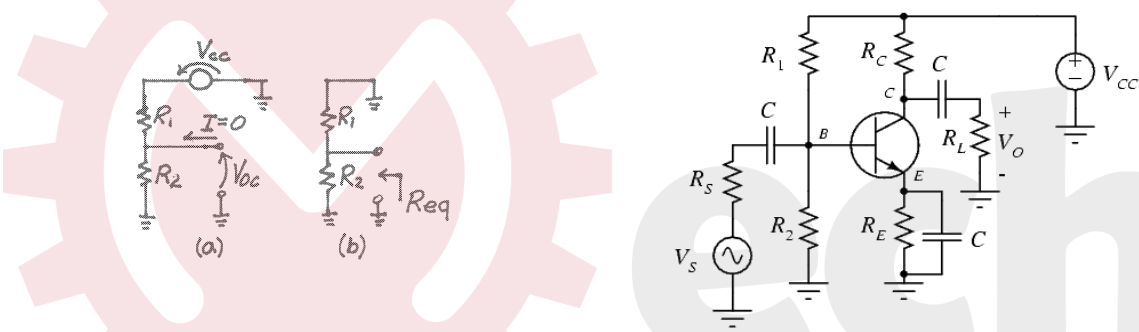
For the circuit shown in Figure P10.14:

$$V_{CC} = 15\text{ V} \quad \beta = 100 \quad R_1 = 68\text{ k}\Omega \quad R_2 = 11.7\text{ k}\Omega \quad R_C = 200\Omega \quad R_E = 200\Omega$$

$$R_L = 1.5\text{ k}\Omega \quad R_S = 0.9\text{ k}\Omega \quad v_S = \cos[6.28 \times 10^3 \cdot t] \text{ mV}.$$

Find: V_{CEQ} and the region of operation.**Analysis:**

Simplify the circuit by obtaining the Thévenin equivalent of the biasing network in the base circuit:



$$\text{VD: } V_{BB} = V_{TH} = \frac{V_{CC} R_2}{R_1 + R_2} = \frac{15 \cdot 11.7}{68 + 11.7} = 2.202 \text{ V}$$

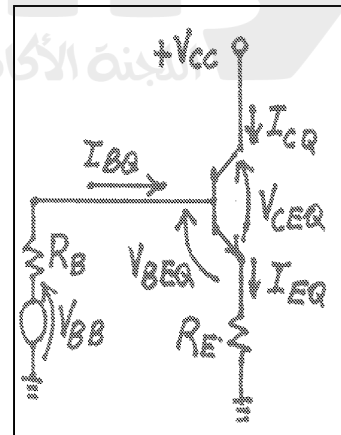
$$\text{Suppress } V_{CC}: \quad R_B = R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{68 \cdot 11.7}{68 + 11.7} = 9.982 \text{ k}\Omega$$

Redraw the circuit using the Thévenin equivalent. The "DC blocking" or "AC coupling" capacitors act as open circuits for DC; therefore, the signal source and load can be neglected since this is a DC problem. Specify directions of current and polarities of voltages. Assume the transistor is operating in its active region; then, the base-emitter junction is forward biased.

$$V_{BEQ} \approx 700 \text{ mV} \quad [\text{Si}]$$

$$I_{CQ} = \beta \cdot I_{BQ} \quad I_{EQ} = (\beta + 1) I_{BQ}$$

Then:



$$\text{KVL: } -V_{BB} + I_{BQ}R_B + V_{BEQ} + I_{EQ}R_E = 0 \Rightarrow -V_{BB} + I_{BQ}R_B + V_{BEQ} + [\beta + 1]I_{BQ}R_E = 0$$

$$I_{BQ} = \frac{V_{BB} - V_{BEQ}}{R_B + [\beta + 1]R_E} = \frac{2.202 - 0.7}{9982 + (100 + 1)(200)} = 49.76 \mu\text{A}$$

$$I_{CQ} = \beta I_{BQ} = 100 \cdot 49.76 \cdot 10^{-6} = 4.976 \text{ mA}$$

$$I_{EQ} = (\beta + 1) \cdot I_{BQ} = (100 + 1) \cdot 49.76 \cdot 10^{-6} = 5.026 \text{ mA}$$

$$\text{KVL: } -I_{EQ}R_E - V_{CEQ} - I_{CQ}R_C + V_{CC} = 0$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E = 15 - 4.976 \cdot 0.2 - 5.026 \cdot 0.2 = 13.00 \text{ V}$$

The collector-emitter voltage is greater than its saturation value (0.3 V for Silicon). Therefore the initial assumption (operation in the active region) was correct and the solution is valid.

Notes:

1. If the collector-emitter voltage were less than its saturation value, the transistor would be operating in its "saturation" or "ohmic" region. In this case, the collector-emitter voltage is equal to its saturation value (0.3 V for Silicon). The solution for the base current remains valid. However, the parameter β and the solution for the collector and emitter currents become invalid. In the saturation region, the base-emitter junction is still forward biased and its voltage remains the same. A solution for the collector and emitter currents is possible using the collector-emitter saturation voltage (0.3 V for Silicon) in a KVL.
2. In the saturation region, the base current has no control over the collector or emitter currents (since β is invalid). Therefore, amplification is impossible.

Problem 10.16

Solution:

Known quantities:

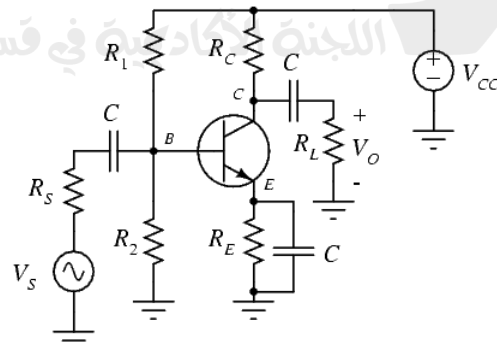
For the circuit shown in Figure P10.14:

$$V_{CC} = 15 \text{ V} \quad \beta = 100 \quad R_1 = 68 \text{ k}\Omega \quad R_2 = 11.7 \text{ k}\Omega \quad R_C = 4 \text{ k}\Omega \quad R_E = 200 \Omega$$

$$R_L = 1.5 \text{ k}\Omega \quad R_S = 0.9 \text{ k}\Omega \quad v_s = \cos[6.28 \times 10^3 \cdot t] \text{ mV}$$

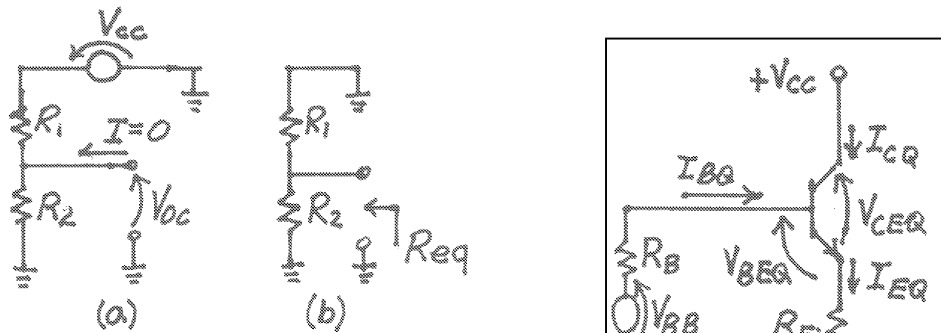
Find:

V_{CEQ} and the region of operation.



Analysis:

Simplify the circuit by obtaining the Thévenin equivalent of the biasing network in the base circuit:



Redraw the circuit using the Thévenin equivalent. The "DC blocking" or "AC coupling" capacitors act as open circuits for DC; therefore, the signal source and load can be neglected since this is a DC problem. Specify directions of current and polarities of voltages. Assume the transistor is operating in its active region. Then, the base-emitter junction is forward biased.

$$V_{BEQ} \approx 700 \text{ mV [Si]} \quad I_{CQ} = \beta \cdot I_{BQ} \quad I_{EQ} = (\beta + 1) I_{BQ}$$

$$\text{KVL: } -V_{BB} + I_{BQ} R_B + V_{BEQ} + I_{EQ} R_E = 0 \Rightarrow -V_{BB} + I_{BQ} R_B + V_{BEQ} + [\beta + 1] I_{BQ} R_E = 0$$

$$I_{BQ} = \frac{V_{BB} - V_{BEQ}}{R_B + [\beta + 1] R_E} = \frac{2.202 - 0.7}{9982 + (100 + 1)(200)} = 49.76 \mu\text{A}$$

$$I_{CQ} = \beta I_{BQ} = 100 \cdot 49.76 \cdot 10^{-6} = 4.976 \text{ mA}$$

$$I_{EQ} = (\beta + 1) I_{BQ} = (100 + 1) \cdot 49.76 \cdot 10^{-6} = 5.026 \text{ mA}$$

$$\text{KVL: } -I_{EQ} R_E - V_{CEQ} - I_{CQ} R_C + V_{CC} = 0$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C - I_{EQ} R_E = 15 - 4.976 \cdot 4 - 5.026 \cdot 0.2 = -5.91 \text{ V}$$

The collector-emitter voltage is less (more negative) than its saturation value (+ 0.3 V for Silicon). Therefore the initial assumption (operation in the active region) was incorrect and the solution is not valid. The device is operating in the saturation region with a saturation collector-emitter voltage equal to 0.3 V. The solutions for the collector and emitter currents are invalid.

THE VALID SOLUTION:

$$\text{KVL: } -I_{EQ} R_E - V_{CE-SAT} - I_{CQ} R_C + V_{CC} = 0 \quad I_{EQ} = I_{BQ} + I_{CQ}$$

$$- [I_{BQ} + I_{CQ}] R_E - V_{CE-SAT} - I_{CQ} R_C + V_{CC} = 0$$

$$I_{CQ} = \frac{V_{CC} - I_{BQ} R_E - V_{CE-SAT}}{R_E + R_C} = \frac{15 - 49.76 \cdot 0.2 \cdot 10^{-3} - 0.3}{200 + 4000} = 3.498 \text{ mA}$$

The solution for the base current is valid. The value of beta given is not valid.

Problem 10.17**Solution:****Known quantities:**

For the circuit shown in Figure P10.17:

$$V_{CC} = 12\text{ V} \quad \beta = 130 \quad R_1 = 82\text{ k}\Omega \quad R_2 = 22\text{ k}\Omega \quad R_E = 0.5\text{ k}\Omega \quad R_L = 16\Omega.$$

Find:

V_{CEQ} at the DC operating point.

Analysis:

Simplify the circuit by obtaining the Thévenin equivalent of the biasing network (R_1 , R_2 , V_{CC}) in the base circuit:

$$\text{VD: } V_{BB} = V_{TH} = V_{OC} = \frac{V_{CC} R_2}{R_1 + R_2} = \frac{12 \cdot 22}{82 + 22} = 2.538\text{ V}$$

$$\text{Suppress } V_{CC}: \quad R_B = R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{82 \cdot 22}{82 + 22} = 17.35\text{ k}\Omega$$

Redraw the circuit using the Thévenin equivalent. The "DC blocking" or "AC coupling" capacitors act as open circuits for DC; therefore, the signal source and load can be neglected since this is a DC problem. Specify directions of current and polarities of voltages.

Assume the transistor is operating in its active region. Then, the base-emitter junction is forward biased.

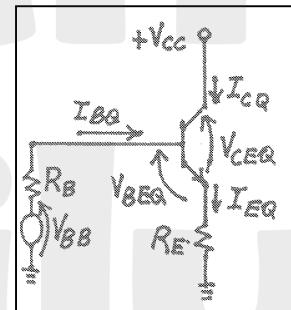
$$V_{BEQ} \approx 700\text{ mV [Si]} \quad I_{EQ} = [\beta + 1] I_{BQ}$$

$$\text{KVL: } -V_{BB} + I_{BQ} R_B + V_{BEQ} + I_{EQ} R_E = 0$$

$$-V_{BB} + I_{BQ} R_B + V_{BEQ} + [\beta + 1] I_{BQ} R_E = 0$$

$$I_{BQ} = \frac{V_{BB} - V_{BEQ}}{R_B + (\beta + 1) R_E} = \frac{2.538 - 0.7}{17350 + (130 + 1) \cdot 500} = 22.18\mu\text{A}$$

$$I_{EQ} = (\beta + 1) I_{BQ} = (130 + 1) \cdot 22.18 \cdot 10^{-6} = 2.906\text{ mA}$$



$$\text{KVL: } -I_{EQ} R_E - V_{CEQ} + V_{CC} = 0$$

$$V_{CEQ} = V_{CC} - I_{EQ} R_E = 12 - 2.906 \cdot 0.5 = 10.55\text{ V}$$

The collector-emitter voltage is greater than its saturation value (0.3 V for Silicon). Therefore the initial assumption (operation in the active region) was correct and the solution is valid.

Problem 10.18**Solution:****Known quantities:**

For the circuit shown in Figure P10.18:

$$V_{CC} = 12 \text{ V} \quad V_{EE} = 4 \text{ V} \quad \beta = 100 \quad R_B = 100 \text{ k}\Omega \quad R_C = 3 \text{ k}\Omega \quad R_E = 3 \text{ k}\Omega$$

$$R_L = 6 \text{ k}\Omega \quad R_s = 0.6 \text{ k}\Omega \quad v_s = \cos[6.28 \times 10^3 \cdot t] \text{ mV}.$$

Find:

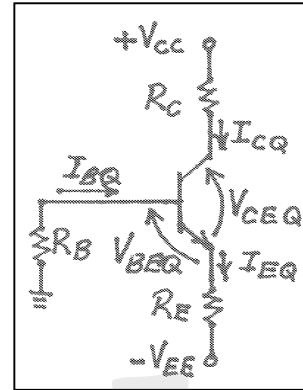
V_{CEQ} and the region of operation.

Analysis:

The "DC blocking" or "AC coupling" capacitors act as open circuits for DC; therefore, the signal source and load can be neglected since this is a DC problem. Specify directions of current and polarities of voltages. Assume the transistor is operating in its active region; then, the base-emitter junction is forward biased and:

$$V_{BEQ} \approx 700 \text{ mV} \quad [\text{Si}]$$

$$I_{CQ} = \beta \cdot I_{BQ} \quad I_{EQ} = (\beta + 1) I_{BQ}$$



$$\text{KVL: } -V_{EE} + I_{BQ} R_B + V_{BEQ} + I_{EQ} R_E = 0 \Rightarrow$$

$$\Rightarrow I_{BQ} = \frac{V_{EE} - V_{BEQ}}{R_B + [\beta + 1] R_E} = \frac{4 - 0.7}{100000 + (100 + 1)(3000)} = 8.189 \mu\text{A}$$

$$I_{CQ} = \beta \cdot I_{BQ} = (100) \cdot 8.189 \cdot 10^{-6} = 818.9 \text{ mA}$$

$$I_{EQ} = (\beta + 1) \cdot I_{BQ} = (100 + 1) \cdot 8.189 \cdot 10^{-6} = 827.0 \text{ mA}$$

$$\text{KVL: } +V_{EE} - I_{EQ} R_E - V_{CEQ} - I_{CQ} R_C + V_{CC} = 0$$

$$V_{CEQ} = V_{EE} + V_{CC} - I_{CQ} R_C - I_{EQ} R_E = 4 + 12 - 818.9 \cdot 3 \cdot 10^{-3} - 827.0 \cdot 3 \cdot 10^{-3} = 11.06 \text{ V}$$

The collector-emitter voltage is greater (more positive) than its saturation value (+ 0.3 V for Silicon).

Therefore the initial assumption (operation in the active region) was correct and the solution is valid.

Notes:

1. DC power may be supplied to an *npn* BJT circuit by connecting the positive terminal of a DC source to the collector circuit, or, by connecting the negative terminal of a DC source to the emitter circuit, or, as was done here, both.
2. In a *pnp* BJT circuit the polarities of the sources must be reversed. Negative to collector and positive to emitter.

Problem 10.19

Solution:

Known quantities:

For the circuit shown in Figure P10.19:

$$V_{CC} = 12 \text{ V} \quad \beta = 130 \quad R_B = 325 \text{ k}\Omega \quad R_C = 1.9 \text{ k}\Omega \quad R_E = 2.3 \text{ k}\Omega$$

$$R_L = 10\text{ k}\Omega \quad R_s = 0.5\text{ k}\Omega \quad v_s = \cos[6.28 \times 10^3 \cdot t] \text{ mV}.$$

Find:

V_{CEQ} and the region of operation.

Analysis:

The "DC blocking" or "AC coupling" capacitors act as open circuits for DC; therefore, the signal source and load can be neglected since this is a DC problem. Specify directions of current and polarities of voltages. Assume the transistor is operating in its active region; then, the base-emitter junction is forward biased. The base and collector currents both flow through the collector resistor in this circuit.

$$V_{BEQ} \approx 700 \text{ mV} \quad [\text{Si}]$$

$$I_{CQ} = \beta I_{BQ} \quad I_{EQ} = [\beta + 1] I_{BQ}$$

$$\text{KCL: } I_{BQ} + I_{CQ} - I_{RC} = 0$$

$$I_{RC} = I_{CQ} + I_{BQ} = \beta I_{BQ} + I_{BQ} = [\beta + 1] I_{BQ}$$

$$\text{KVL: } -I_{EQ} R_E - V_{BEQ} - I_{BQ} R_B - I_{RC} R_C + V_{CC} = 0$$

$$-[\beta + 1] I_{BQ} [R_E + R_C] - V_{BEQ} - I_{BQ} R_B + V_{CC} = 0$$

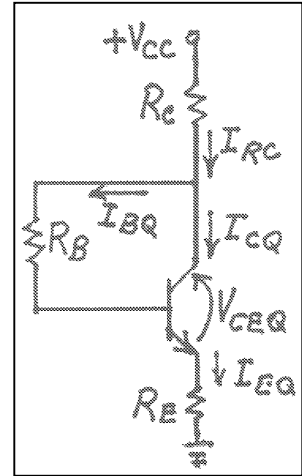
$$I_{BQ} = \frac{V_{CC} - V_{BEQ}}{R_B + [\beta + 1] [R_E + R_C]} = \frac{12 - 0.7}{[325 + (130 + 1) \cdot (2.3 + 1.9)] \cdot 10^3} = 12.91 \mu\text{A}$$

$$I_{RC} = I_{EQ} = [\beta + 1] I_{BQ} = (130 + 1) \cdot 12.91 \cdot 10^{-6} = 1.691 \text{ mA}$$

$$\text{KVL: } -I_{EQ} R_E - V_{CEQ} - I_{RC} R_C + V_{CC} = 0 \Rightarrow$$

$$\Rightarrow V_{CEQ} = V_{CC} - I_{RC} R_C - I_{EQ} R_E = 12 - 1.691 \cdot 1.9 - 1.691 \cdot 2.3 = 4.896 \text{ V}$$

The collector-emitter voltage is greater than its saturation value (0.3 V for Silicon). Therefore the initial assumption (operation in the active region) was correct and the solution is valid.



Problem 10.20

Solution:

Known quantities:

For the circuit shown in Figure P10.19:

$$V_{CC} = 15 \text{ V} \quad C = 0.5 \mu\text{F} \quad \beta = 170 \quad R_B = 22 \text{ k}\Omega \quad R_C = 3.3 \text{ k}\Omega \quad R_E = 3.3 \text{ k}\Omega$$

$$R_L = 1.7 \text{ k}\Omega \quad R_s = 70 \Omega \quad v_s = \cos[6.28 \times 10^3 \cdot t] \text{ mV}.$$

Find:

V_{CEQ} and the region of operation.

Analysis:

When β is very high (say, greater than 50) the "high beta approximation" can be used, i.e.:

$$I_C \approx I_E \approx \beta I_B$$

Using this approximation in this problem:

$$I_{BQ} \approx 12.5\mu\text{A} \quad I_{CQ} \approx I_{EQ} \approx 2.125\text{mA} \quad V_{CEQ} \approx 0.975\text{V}$$

The collector-emitter voltage is greater than the saturation value (but not by much) so the original assumption (operation in the active region) and the solution are valid. However, when a signal is introduced into the circuit, the collector-emitter voltage will vary about its Q point value. Since the Q point is close to saturation, saturation and severe distortion due to clipping is likely.

Problem 10.21**Solution:****Known quantities:**

- For the circuit shown in Figure P10.14:

$$V_{CC} = 15\text{V} \quad C = 0.47\mu\text{F} \quad R_1 = 220\text{k}\Omega \quad R_2 = 55\text{k}\Omega \quad R_C = 3\text{k}\Omega$$

$$R_E = 710\Omega$$

$$R_L = 3\text{k}\Omega \quad R_S = 0.6\text{k}\Omega \quad v_S = V_{i0} \cdot \sin(\omega t) \quad V_{i0} = 10\text{mV}$$

- DC operating point:

$$I_{BQ} = 19.9\mu\text{A} \quad V_{CEQ} = 7.61\text{V}$$

- Transfer characteristic and β of the *npn* silicon transistor:

$$I_C \approx I_S e^{\frac{V_{BE}}{V_T}} = I_S e^{\frac{V_{BEQ} + v_{be}}{V_T}} \quad \beta = 100$$

- Device i-v characteristic plotted in Figure P9.21.

Find:

- The no-load large signal gain $[v_0/v_i]$.
- Sketch the waveform of the output voltage as a function of time.
- How the output voltage is distorted compared with the input waveform.

Analysis:

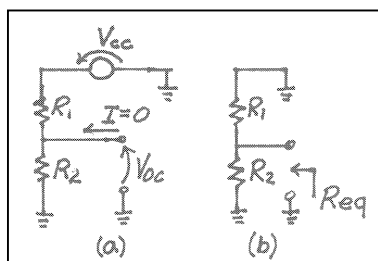
- Simplify the circuit by determining the Thévenin equivalent of the biasing network in the base circuit.

$$V_D: V_{BB} = V_{TH} = \frac{V_{CC} R_2}{R_1 + R_2} = \frac{15 \cdot 55}{220 + 55} = 3.00\text{V}$$

Suppress V_{CC} :

$$R_B = R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{220 \cdot 55 \cdot 10^3}{220 + 55} = 44.0\text{k}\Omega$$

First, determine the collector and emitter Q point:



$$\text{KVL: } -I_{EQ} R_E - V_{CEQ} - I_{CQ} R_C + V_{CC} = 0$$

$$V_{CEQ} = -2.010 \cdot 0.710 - 1.990 \cdot 3 + 15 = 7.603 \text{ V}$$

$$I_{CQ} = \beta I_{BQ} = 100 \cdot 19.9 \cdot 10^{-6} = 1.99 \text{ mA}$$

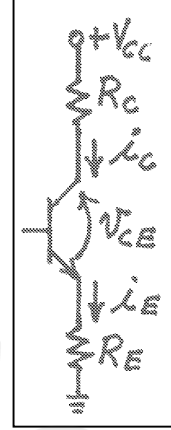
$$I_{EQ} = [\beta + 1] I_{BQ} = 101 \cdot 19.9 \cdot 10^{-6} = 2.01 \text{ mA}$$

Now, determine the DC load line (circuit characteristic).

$$\text{KVL: } -i_E R_E - v_{CE} - i_C R_C + V_{CC} = 0 \quad i_E = \frac{\beta + 1}{\beta} i_C$$

$$i_C = \frac{V_{CC} - v_{CE}}{\frac{\beta + 1}{\beta} R_E + R_C} = \frac{V_{CC} - v_{CE}}{3717}$$

$$= \begin{cases} = 0 & \text{If: } v_{CE} = V_{CC} = 15 \text{ V} \quad [\text{Intercept 1}] \\ = \frac{15}{3717} = 4.035 \text{ mA} & \text{If: } v_{CE} = 0 \quad [\text{Intercept 2}] \end{cases}$$



Plot the DC load line on the same plot as the transistor i - v characteristic. Note that the collector current is a linear function of the collector-emitter voltage. Only two points (the two intercepts shown as Points 1 and 2 on the plot) are required to plot a linear function. The DC load line should pass through the Q point.

The AC load line can now be plotted. It will pass through the Q point as does the DC load line; however, its intercepts cannot be directly determined. Instead, its slope will be determined. The slope and the Q point can then be used to plot the AC load line.

$$\text{Slope}_{AC} = -\frac{1}{R_C} = -\frac{1}{3000} = -0.3333 \frac{\text{mA}}{\text{V}} = -\frac{2 \text{ mA}}{6 \text{ V}} = \frac{\Delta i_C}{\Delta v_{CE}}$$

For AC the capacitors act as short circuits. The bypass capacitor in parallel with the emitter resistor shorts out any AC voltage across it. The collector coupling capacitor acts as a short and connects the load resistor to the circuit. However, no-load conditions are specified, i.e., the load current is zero or the load resistor is replaced by an open circuit.

With no signal, the circuit operates at its Q point. When a signal is introduced, the operating point changes along the AC load line as a function of the signal voltage. Unfortunately, the device i - v characteristic is plotted as a function of input (base) current instead of voltage.

But, there is the static characteristic that relates collector current to base-emitter voltage:

$$i_C = I_s e^{\frac{v_{BE}}{V_T}} = I_s e^{\frac{V_{BEQ} + v_{be}}{V_T}} = [I_s e^{\frac{V_{BEQ}}{V_T}}] e^{\frac{v_{be}}{V_T}} = I_{CQ} e^{\frac{v_{be}}{V_T}}$$

$$i_B = \frac{i_C}{\beta} = \frac{I_{CQ}}{\beta} e^{\frac{v_{be}}{V_T}} = I_{BQ} e^{\frac{v_{be}}{V_T}}$$

The base-emitter signal voltage is related to the input signal voltage. At sufficiently high frequencies (the "mid"-frequency range) the capacitors can be modeled as short circuits for AC. This means that there is no AC voltage across either the AC coupling (DC blocking) capacitor or the capacitor bypassing the emitter resistor. Using superposition to sum only AC voltages around the loop:

$$\text{KVL: } -v_i + v_c + v_{be} + v_e = 0 \quad v_c = v_e = 0 \Rightarrow v_{be} = v_i$$

$$i_B = I_{BQ} e^{\frac{v_{be}}{V_T}} = [19.9 \mu\text{A}] e^{\frac{v_i}{26 \text{ mV}}}$$

This transfer function can now be used to give the base current as a function of the input voltage. The intersection of the base current curves with the AC load line give the corresponding values of the collector current and collector-emitter voltage (points 3, Q and 4 on the plot):

ωt [rad]	v_i [mV]	i_B [μA]	i_C [mA]	v_{CE} [V]	v_o [V]
$0, \pi, 2\pi$	0	19.9	1.99	7.61	0
$\pi/2$	+10	29.23	2.92	4.55	-3.06
$3\pi/2$	-10	13.54	1.35	9.15	+1.54

The capacitor connecting the load to the circuit is a DC blocking [and AC coupling] capacitor. Remember that for no load conditions, the load resistor is assumed to be open. Again using superposition to sum only AC voltages around the loop and modeling the capacitors as shorts:

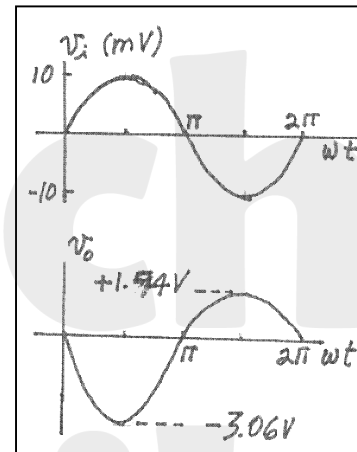
$$-v_e + v_{ce} + v_c + v_o = 0 \quad v_e = v_c = 0 \Rightarrow v_o = v_{ce} = v_{CE} - V_{CEQ}$$

The no-load voltage gain can be determined using the maximum excursion in the input and output voltages:

$$A_{vo} = \frac{\Delta v_o}{\Delta v_i} = \frac{(-1.54) - (+3.06)}{(10 \cdot 10^{-3}) - (-10 \cdot 10^{-3})} = -230$$

When the input voltage becomes more positive, the output voltage becomes more negative and vice versa. This is called "phase inversion" and is why the gain is negative. In normal amplification this is not important. If the loading effect of the load resistance is included, the AC load line will be steeper and the gain will be reduced.

- b) A plot is a large number of calculated points plotted and connected by a smooth curve. In a sketch, only the most significant points [maxima, minima, intercepts, etc] are calculated, plotted, and connected by a carefully drawn smooth curve.
- c) The output waveform is very distorted, i.e., it is not a true linearly amplified copy of the input waveform. This is most obvious when the positive and negative peaks of the two curves are compared:
- | | | |
|----------------|---------|---------|
| Input voltage | +10 mV | -10 mV |
| Output voltage | -3.06 V | +1.94 V |



The two peaks of the input voltage are equal; however, the positive peak is amplified less than the negative peak. The nonlinear (exponential) behavior of the transistor causes the amplification to be nonlinear and the output waveform to be very distorted. This is characteristic of large signal amplifiers.

Even more serious distortion could occur if the input voltage (also called the "excitation" or "drive") were increased so that saturation or cutoff occurs. This causes "clipping" and a severely distorted output.

Section 10.3: BJT Large-Signal Model

Problem 10.22

Solution:

Known quantities:

For the circuit shown in Figure 10.24:

$$V_{off} = 0 \text{ V}, V_{on} = 5 \text{ V}, I_B = 5 \text{ mA}, R_B = 1 \text{ k}\Omega, V_{CC} = 5 \text{ V}, V_\gamma = 0.7 \text{ V}, V_{CEsat} = 0.2 \text{ V}, \beta = 95,$$

$$V_{\gamma LED} = 1.4 \text{ V}, I_{LED} \geq 10 \text{ mA}, P_{max} = 100 \text{ mW}$$

Find:

Range of R_C .

Analysis:

$$R_C = \frac{V_{CC} - V_{\gamma LED} - V_{CEsat}}{I_{LED}} \leq \frac{5 - 1.4 - 0.2}{0.01} = 340 \Omega$$

From the maximum power

$$I_{LEDmax} = \frac{P_{max}}{V_{\gamma LED}} = \frac{0.1}{1.4} = 71 \text{ mA}$$

$$R_C > \frac{V_{CC} - V_{\gamma LED} - V_{CEsat}}{I_{LEDmax}} = 47 \Omega$$

Therefore, $R_C \in [47, 340] \Omega$

Problem 10.23

Solution:

Known quantities:

For the circuit shown in Figure 10.26:

$$V_D = 1.1 \text{ V}, R_B = 33 \text{ k}\Omega, V_{CC} = 12 \text{ V}, V_{BE} = 0.75 \text{ V}, V_{CEQ} = 6 \text{ V}, \beta = 188.5, R_S = 500 \Omega$$

Find:

The resistance R_C .

Analysis:

The current through the resistance R_B is given by

$$I_B = \frac{V_D - V_{BEQ}}{R_B} = \frac{1.1 - 0.75}{33000} = 10.6 \mu\text{A}$$

The current through R_S is

$$I_S = \frac{V_{CEQ} - V_D}{R_S} = \frac{6 - 1.1}{500} = 9.8 \text{ mA}$$

It follows that the current through the resistance R_C is

$$I_{CQ} = \beta I_B + I_S = 11.8 \text{ mA}$$

Finally,

$$R_C = \frac{V_{CC} - V_{CEQ}}{I_{CQ}} = \frac{12 - 6}{0.0118} = 508.5 \Omega$$

Problem 10.24**Solution:****Known quantities:**

For the circuit shown in Figure 10.24:

$$V_{off} = 0 \text{ V}, V_{on} = 5 \text{ V}, I_{B\max} = 5 \text{ mA}, R_C = 340 \Omega, V_{CC} = 5 \text{ V}, V_\gamma = 0.7 \text{ V}, V_{CEsat} = 0.2 \text{ V}, \beta = 95, \\ V_{\gamma LED} = 1.4 \text{ V}, I_{LED} \geq 10 \text{ mA}, P_{\max} = 100 \text{ mW}$$

Find:

Range of R_B .

Analysis:

If the BJT is in saturation

$$I_C = \frac{V_{CC} - V_{\gamma LED} - V_{CEsat}}{R_C} = 10 \text{ mA}$$

In order to guarantee that the BJT is in saturation

$$R_B \leq \frac{V_{on} - V_\gamma}{I_C / \beta} = \frac{5 - 0.7}{\frac{0.01}{95}} = 40.85 \text{ k}\Omega$$

$$R_B \geq \frac{V_{on} - V_\gamma}{I_{B\max}} = 860 \Omega$$

Problem 10.25**Solution:****Known quantities:**

For the circuit shown in Figure 10.24:

$$V_{off} = 0 \text{ V}, V_{on} = 5 \text{ V}, I_{B\max} = 5 \text{ mA}, R_B = 10 \text{ k}\Omega, R_C = 340 \Omega, V_{CC} = 5 \text{ V}, V_\gamma = 0.7 \text{ V}, V_{CEsat} = 0.2 \text{ V}, \\ V_{\gamma LED} = 1.4 \text{ V}, I_{LED} \geq 10 \text{ mA}, P_{\max} = 100 \text{ mW}$$

Find:

Minimum value of β that will ensure the correct operation of the LED.

Analysis:

$$I_B = \frac{V_{on} - V_\gamma}{R_B} = \frac{4.3}{10000} = 0.43 \text{ mA}$$

$$\beta_{\min} = \frac{I_{LED\min}}{I_B} = \frac{0.01}{0.43 \cdot 10^{-3}} = 23.25$$

Problem 10.26**Solution:****Known quantities:**

For the circuit shown in Figure 10.24:

$$V_{off} = 0 \text{ V}, V_{on} = 3.3 \text{ V}, I_{B\max} = 5 \text{ mA}, R_B = 10 \text{ k}\Omega, R_C = 340 \Omega, V_{CC} = 5 \text{ V}, V_\gamma = 0.7 \text{ V}, V_{CEsat} = 0.2 \text{ V}, \\ V_{\gamma LED} = 1.4 \text{ V}, I_{LED} \geq 10 \text{ mA}, P_{\max} = 100 \text{ mW}$$

Find:

Minimum value of β that will ensure the correct operation of the LED.

Analysis:

$$I_B = \frac{V_{on} - V_\gamma}{R_B} = \frac{3.3 - 0.7}{10000} = 0.26 \text{ mA}$$

$$\beta_{\min} = \frac{I_{LED\min}}{I_B} = \frac{0.01}{0.26 \cdot 10^{-3}} = 38.5$$

Problem 10.27**Solution:****Known quantities:**

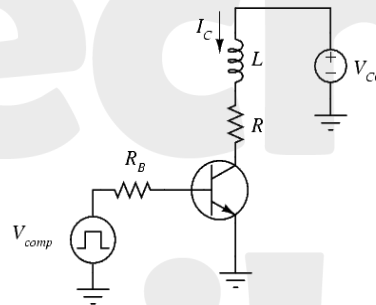
For the circuit shown in Figure 10.24:

$V_{off} = 0 \text{ V}$, $V_{on} = 5 \text{ V}$, $I_{B\max} = 1 \text{ mA}$, $R_B = 1 \text{ k}\Omega$, $R = 12 \Omega$, $V_{CC} = 13 \text{ V}$, $V_\gamma = 0.7 \text{ V}$, $V_{CEsat} = 1 \text{ V}$,

$$I_C \geq 1 \text{ A}$$

Find:

Minimum value of β that will ensure the correct operation of the fuel injector.

Analysis:

$$I_C = \frac{V_{CC} - V_{CEsat}}{R} = \frac{13 - 1}{12} = 1 \text{ A}$$

$$\beta_{\min} = \frac{I_C}{I_{B\max}} = \frac{1}{1 \cdot 10^{-3}} = 1000$$

Problem 10.28**Solution:****Known quantities:**

For the circuit shown in Figure 10.24:

$V_{off} = 0 \text{ V}$, $V_{on} = 5 \text{ V}$, $I_{B\max} = 1 \text{ mA}$, $\beta = 2000$, $R = 12 \Omega$, $V_{CC} = 13 \text{ V}$, $V_\gamma = 0.7 \text{ V}$, $V_{CEsat} = 1 \text{ V}$,

$$I_C \geq 1 \text{ A}$$

Find:

The range of R_B that will ensure the correct operation of the fuel injector.

Analysis:

If the BJT is in saturation

$$I_C = \frac{V_{CC} - V_{CEsat}}{R} = 1 \text{ A}$$

Because this is the minimum value allowed for the current to drive the fuel injector, it is necessary to guarantee that the BJT is in saturation.

In order to guarantee that the BJT is in saturation

$$R_B \leq \frac{V_{on} - V_\gamma}{I_C / \beta} = \frac{5 - 0.7}{\frac{1}{2000}} = 8.6 \text{ k}\Omega$$

$$R_B \geq \frac{V_{on} - V_\gamma}{I_{Bmax}} = 4.3 \text{ k}\Omega$$

Problem 10.29**Solution:****Known quantities:**

For the circuit shown in Figure 10.24:

$$V_{off} = 0 \text{ V}, V_{on} = 3.3 \text{ V}, I_{Bmax} = 1 \text{ mA}, \beta = 2000, R = 12 \Omega, V_{CC} = 13 \text{ V}, V_\gamma = 0.7 \text{ V}, V_{CEsat} = 1 \text{ V},$$

$$I_C \geq 1 \text{ A}$$

Find:

The range of R_B that will ensure the correct operation of the fuel injector.

Analysis:

If the BJT is in saturation

$$I_C = \frac{V_{CC} - V_{CEsat}}{R} = 1 \text{ A}$$

Because this is the minimum value allowed for the current to drive the fuel injector, it is necessary to guarantee that the BJT is in saturation.

In order to guarantee that the BJT is in saturation

$$R_B \leq \frac{V_{on} - V_\gamma}{I_C / \beta} = \frac{3.3 - 0.7}{\frac{1}{2000}} = 5.2 \text{ k}\Omega$$

$$R_B \geq \frac{V_{on} - V_\gamma}{I_{Bmax}} = 2.6 \text{ k}\Omega$$

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Section 10.4: BJT Switches and Gates

Problem 10.30

Solution:

Known quantities:

The circuit given in Figure P10.30.

Find:

Show that the given circuit functions as an OR gate if the output is taken at v_{o1} .

Analysis:

Construct a state table. This table clearly describes an AND gate when the output is taken at v_{o1} .

v_1	v_2	Q_1	Q_2	Q_3	v_{o1}	v_{o2}
0	0	off	off	on	0	5V
0	5V	off	on	off	5V	0
5V	0	on	off	off	5V	0
5V	5V	on	on	off	5V	0

Problem 10.31

Solution:

Known quantities:

The circuit given in Figure P10.30.

Find:

Show that the given circuit functions as a NOR gate if the output is taken at v_{o2} .

Analysis:

See the state table constructed for Problem 10.30. This table clearly describes a NOR gate when the output is taken at v_{o2} .

Problem 10.32

Solution:

Known quantities:

The circuit given in Figure P10.32.

Find:

Show that the given circuit functions as an AND gate if the output is taken at v_{o1} .

Analysis:

Construct a state table. This table clearly describes an AND gate when the output is taken at v_{o1} .

v_1	v_2	Q_1	Q_2	Q_3	v_{o1}	v_{o2}
0	0	off	off	on	0	5V
0	5V	off	on	on	0	5V
5V	0	on	off	on	0	5V
5V	5V	on	on	off	5V	0

Problem 10.33

Solution:

Known quantities:

The circuit given in Figure P10.32.

Find:

Show that the given circuit functions as a NAND gate if the output is taken at v_{o2} .

Analysis:

See the state table constructed for Problem 10.32. This table clearly describes a NAND gate when the output is taken at v_{o2} .

Problem 10.34

Solution:

Known quantities:

In the circuit given in Figure P10.34 the minimum value of v_{in} for a high input is 2.0 V. Assume that the transistor Q_1 has a β of at least 10.

Find:

The range for resistor R_B that can guarantee that the transistor is on.

Analysis:

$$i_c = \frac{5 - 0.2}{2000} = 2.4 \text{ mA}, \text{ therefore, } i_B = i_c / \beta = 0.24 \text{ mA}.$$

$$(v_{in})_{\min} = 2.0 \text{ V and } (v_{in})_{\max} = 5.0 \text{ V, therefore, applying KVL: } -v_{in} + R_B i_B + 0.6 = 0$$

$$\text{or } R_B = \frac{v_{in} - 0.6}{i_B}. \text{ Substituting for } (v_{in})_{\min} \text{ and } (v_{in})_{\max}, \text{ we find the following range for } R_B:$$

$$5.833 \text{ k}\Omega \leq R_B \leq 18.333 \text{ k}\Omega$$

Problem 10.35

Solution:

Known quantities:

For the circuit given in Figure P10.35: $R_{1C} = R_{2C} = 10 \text{ k}\Omega$, $R_{1B} = R_{2B} = 27 \text{ k}\Omega$.

Find:

- v_B , v_{out} , and the state of the transistor Q_1 when v_{in} is low.
- v_B , v_{out} , and the state of the transistor Q_1 when v_{in} is high.

Analysis:

- a) v_{in} is low $\Rightarrow Q_1$ is cutoff $\Rightarrow v_B = 5 \text{ V} \Rightarrow Q_2$ is in saturation $\Rightarrow v_{out} = \text{low} = 0.2 \text{ V}$.
 b) v_{in} is high $\Rightarrow Q_1$ is in saturation $\Rightarrow v_B = 0.2 \text{ V} \Rightarrow Q_2$ is cutoff $\Rightarrow v_{out} = \text{high} = 5 \text{ V}$.

Problem 10.36**Solution:****Known quantities:**

For the inverter given in Figure P10.36: $R_{C1} = R_{C2} = 2 \text{ k}\Omega$, $R_B = 5 \text{ k}\Omega$.

Find:

The minimum values of β_1 and β_2 to ensure that Q_1 and Q_2 saturate when v_{in} is high.

Analysis:

$$i_c = \frac{5 - 0.2}{2000} = 2.4 \text{ mA} , \text{ therefore, } i_c = \frac{2.5}{\beta} \text{ mA} . \text{ Applying KVL:}$$

$$-5 + R_B i_{B1} + 0.6 + 0.6 + 0.6 = 0$$

$$\text{Therefore, } i_{B1} = 0.64 \text{ mA. } i_{E1} = \beta_1 \cdot i_{B1} = \frac{600}{500} + i_{B2} \text{ or } 0.64 \cdot \beta_1 = 1.2 + \frac{2.5}{\beta_2}$$

$$\text{Choose } \beta_2 = 10 \Rightarrow \beta_1 = 2.27.$$

Problem 10.37**Solution:****Known quantities:**

For the inverter given in Figure P10.36: $R_{C1} = 2.5 \text{ k}\Omega$, $R_{C2} = 2 \text{ k}\Omega$, $\beta_1 = \beta_2 = 4$.

Find:

Show that Q_1 saturates when v_{in} is high. Find a condition for R_{C2} to ensure that Q_2 also saturates.

Analysis:

$$i_{B1} = \frac{3.2}{4000} = 0.8 \text{ mA} \Rightarrow i_{C1} = 3.2 \text{ mA}$$

$$\text{Applying KCL: } \frac{600}{500} + i_{B2} = 3.2 \Rightarrow i_{B2} = 2 \text{ mA} ; i_{C2} = \beta \cdot i_{B2} = 8 \text{ mA}$$

$$\text{Applying KVL: } 5 - 0.2 = 0.008 \cdot R_{C2} \Rightarrow R_{C2} = 600 \Omega$$

Problem 10.38**Solution:****Known quantities:**

The basic circuit of a TTL gate, shown in Figure P10.38.

Find:

The logic function performed by this circuit.

Analysis:

The circuit performs the function of a 2-input NAND gate. The analysis is similar to Example 10.8.

Problem 10.39**Solution:****Known quantities:**

The circuit diagram of a three-input TTL NAND gate, given in Figure P10.39.

Find:

v_{B1} , v_{B2} , v_{B3} , v_{C2} , and v_{out} , assuming that all the input voltages are high.

Analysis:

Q_2 and Q_3 conduct, while Q_4 is cutoff. $v_{B1} = 1.8$ V, $v_{B2} = 1.2$ V, $v_{B3} = 0.6$ V, and $v_{C2} = v_{out} = 0.2$ V.

Problem 10.40**Solution:****Known quantities:**

Figure P10.40.

Find:

Show that when two or more emitter-follower outputs are connected to a common load, as shown in Figure P10.54, the OR operation results; that is, $v_o = v_1$ OR v_2 .

Analysis:

v_2	v_1	Q_1	Q_2	v_o
L	L	L	L	L
L	H	H	L	H
H	L	L	H	H
H	H	H	H	H

L : Low; H : High.

Chapter 11 Instructor Notes

Chapter 11 introduces field-effect transistors. The material on transistors has been reorganized in this 4th Edition, and is now divided into two independent chapters, one on bipolar devices, and one on field-effect devices. The two chapters are functionally independent, except for the fact that Section 10.1, introducing the concept of transistors as amplifiers and switches, can be covered prior to starting Chapter 11 if the instructor decides to only teach field-effect devices.

Section 11.1 briefly reviews the classification and symbols for the major families of field-effect devices. Section 11.2 introduces the fundamental ideas behind the operation of N-channel field-effect enhancement-mode transistors, and illustrates the calculation of the state and operating point of basic field-effect transistor circuits. A brief explanation of P-channel devices is also presented in this section. Section 11.3 briefly outlines the operation of MOSFET amplifiers. Section 11.4 introduces the analysis of MOSFET switches and presents CMOS gates. The box *Focus on Measurements: MOSFET bidirectional analog gate* (pp. 572-573) presents an analog application of CMOS technology.

The end-of-chapter problems are straightforward applications of the concepts illustrated in the chapter.

Learning Objectives

1. Understand the classification of field-effect transistors. [Section 1.](#)
2. Learn the basic operation of enhancement-mode MOSFETs by understanding their *i-v* curves and defining equations. [Section 2.](#)
3. Learn how enhancement-mode MOSFET circuits are biased. [Section 2.](#)
4. Understand the concept and operation of FET amplifiers. [Section 3](#)
5. Understand the concept and operation of FET switches. [Section 4.](#)
6. Analyze FET switches and digital gates. [Section 4.](#)

Section 11.2: n-channel MOSFET Operation

Problem 11.1

Solution:

Known quantities:

For the transistors shown in Figure P11.1, $|V_T| = 3 \text{ V}$.

Find:

The operating state of each transistor.

Analysis:

- a) This is an n-channel enhancement MOSFET, with $V_T = -3 \text{ V}$. To operate in the triode region, the condition is: $v_{DS} < v_{GS} - V_T$. To operate in the saturation region, the condition is: $v_{DS} \geq v_{GS} - V_T$. To turn the transistor on, the condition is: $v_{GS} > V_T$.

We can compute: $v_{GS} = -2.5 \text{ V}$ $v_{DS} = 2.5 \text{ V}$

$$v_{GS} - V_T = -2.5 + 3 = 0.5 \text{ V}$$

$$v_{DS} = 2.5 \text{ V} > v_{GS} - V_T = 0.5 \text{ V}.$$

Therefore, the transistor is in the saturation region.

- b) This is a p-channel enhancement MOSFET, with $V_T = 3 \text{ V}$. To operate in the triode region, the condition is: $v_{DS} > v_{GS} - V_T$. To operate in the saturation region, the condition is: $v_{DS} \leq v_{GS} - V_T$. To turn the transistor on, the condition is: $v_{GS} < V_T$.

We can compute: $v_{GS} = 2 \text{ V}$ $v_{DS} = -1 \text{ V}$

$$v_{GS} - V_T = 2 - 3 = -1 \text{ V}$$

$$v_{DS} = -1 \text{ V} = v_{GS} - V_T = -1 \text{ V}.$$

Therefore, the transistor is in the saturation region.

- c) This is a p-channel enhancement MOSFET, with $V_T = -3 \text{ V}$. To operate in the triode region, the condition is: $v_{DS} > v_{GS} - V_T$. To operate in the saturation region, the condition is: $v_{DS} \leq v_{GS} - V_T$. To turn the transistor on, the condition is: $v_{GS} < V_T$.

We can compute: $v_{GS} = -5 \text{ V}$ $v_{DS} = -1 \text{ V}$

$$v_{GS} - V_T = -5 + 3 = -2 \text{ V}$$

$$v_{DS} = -1 \text{ V} > v_{GS} - V_T = -2 \text{ V}.$$

Therefore, the transistor is in the triode region.

- d) This is an n-channel enhancement MOSFET, with $V_T = -3 \text{ V}$. To operate in the triode region, the condition is: $v_{DS} < v_{GS} - V_T$. To operate in the saturation region, the condition is: $v_{DS} \geq v_{GS} - V_T$. To turn the transistor on, the condition is: $v_{GS} > V_T$.

We have: $v_{GS} = -2 \text{ V} > V_T$.

$$v_{GS} - V_T = -2 + 3 = 1 \text{ V}$$

$$v_{DS} = 6 \text{ V} > v_{GS} - V_T = 1 \text{ V}$$

Therefore, the transistor is in the saturation region.

Problem 11.2**Solution:****Known quantities:**

The potentials of an n-channel enhancement-mode MOSFET (4, 5, and 10 V respectively).

Find:

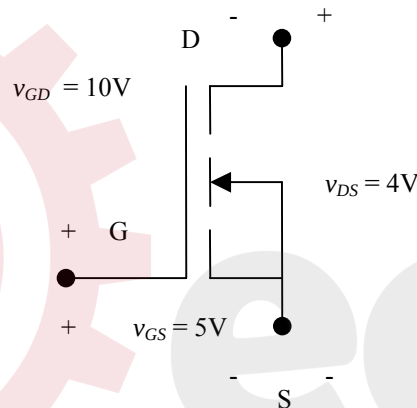
The circuit symbol, if the device is operating:

- In the ohmic state.
- In the active region.

Analysis:

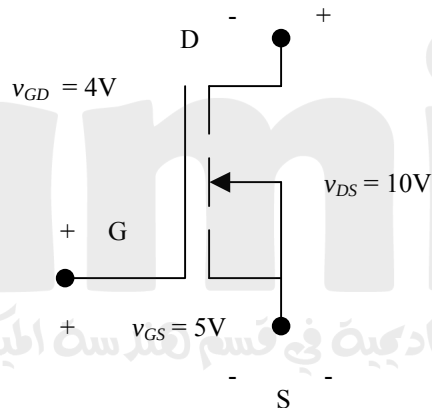
- To operate in the ohmic region, the condition is: $v_{DS} < v_{GS} - V_T$ and $V_T > 0$, $v_{DS} > 0$.

The circuit for operation in the ohmic region is shown below.



- To operate in the active region, the condition is: $v_{DS} \geq v_{GS} - V_T$ and $V_T > 0$, $v_{DS} > 0$.

The circuit for operation in the active region is shown below.

**Problem 11.3****Solution:****Known quantities:**

The threshold voltage, $V_T = 2$ V, of an enhancement-type NMOS that has its source grounded and a 3 V DC source connected to the gate.

Find:

The operating state if:

- a) $v_D = 0.5 \text{ V}$.
- b) $v_D = 1 \text{ V}$.
- c) $v_D = 5 \text{ V}$

Analysis:

$$v_{DS} = v_D = 0.5 \text{ V}$$

$$\text{a) } v_{GS} - V_T = 3 - 2 = 1 \text{ V}$$

$$v_{DS} < v_{GS} - V_T$$

The transistor is in the triode region.

$$v_{DS} = v_D = 1 \text{ V}$$

$$\text{b) } v_{GS} - V_T = 3 - 2 = 1 \text{ V}$$

$$v_{DS} = v_{GS} - V_T$$

The transistor is either in the triode or in the saturation region.

$$v_{DS} = v_D = 5 \text{ V}$$

$$\text{c) } v_{GS} - V_T = 3 - 2 = 1 \text{ V}$$

$$v_{DS} > v_{GS} - V_T$$

The transistor is in the saturation region.

Problem 11.4**Solution:****Known quantities:**

The threshold voltage, $V_T = 2 \text{ V}$, of the p-channel transistor shown in Figure P11.4. $k = 10 \text{ mA/V}^2$.

Find:

R and v_D for $i_d = 0.4 \text{ mA}$.

Analysis:

The device shown is a p-channel enhancement mode MOSFET, with $V_T = 2 \text{ V}$ and $V_{DG} = 0 \text{ V}$. To operate in the saturation region we require: $v_{DS} \geq v_{GS} - V_T$.

Since $v_{DG} = v_{DS} - v_{GS} = 0 > -V_T = -2 \text{ V}$, the transistor is in the saturation region. Knowing

$k = 10 \text{ mA/V}^2$, we can write: $0.4 = 10 \cdot (v_{GS} - 2)^2$ and determine $v_D = v_{DS} = v_{GS} = 2.2 \text{ V}$. R can be found as follows:

$$R = \frac{20 - v_D}{i_D} = \frac{20 - 2.2}{0.4 \cdot 10^{-3}} = 44.5 \text{ k}\Omega$$

Problem 11.5**Solution:****Known quantities:**

The threshold voltage, $V_T = 2 \text{ V}$, of an enhancement-type NMOS transistor. $i_D = 1 \text{ mA}$ when $v_{GS} = v_{DS} = 3 \text{ V}$.

Find:

The value of i_D for $v_{GS} = 4 \text{ V}$.

Analysis:

Because $v_{DS} > v_{GS} - V_T$, the transistor is in the saturation region:

$$i_D = k \cdot (v_{GS} - V_T)^2 = k \cdot (3 - 2)^2 = 0.001 \text{ A}$$

$$\Rightarrow k = 0.001.$$

For $v_{GS} = 4 \text{ V}$ we have:

$$i_D = 0.001 \cdot (4 - 2)^2 = 4 \text{ mA}.$$

Problem 11.6**Solution:****Known quantities:**

Characteristics of an n-channel enhancement-mode MOSFET operated in the ohmic region:

$$v_{DS} = 0.4 \text{ V}, V_T = 3.2 \text{ V}. \text{ Effective resistance of the channel, given by: } R_{DS} = \frac{500}{(v_{GS} - 3.2)} \Omega.$$

Find:

The value of i_D when $v_{GS} = 5 \text{ V}$, $R_{DS} = 500 \Omega$, and $v_{GD} = 4 \text{ V}$.

Analysis:

Since $V_{DS} = 0.4 < v_{GS} - V_T = 5 - 3.2 = 1.8 \text{ V}$ the transistor is operating in the ohmic region. The

effective resistance is: $R_{DS} = \frac{500}{(5 - 3.2)} = 277.78 \Omega$. Since $R_{DS} = \frac{V_{DS}}{i_D}$, we have:

$$i_D = \frac{V_{DS}}{R_{DS}} = 1.44 \text{ mA}.$$

Problem 11.7**Solution:****Known quantities:**

The threshold voltage, $V_T = 2.5 \text{ V}$, of an enhancement-type NMOS that has its source grounded and a 4 V DC source connected to the gate.

Find:

The operating state if:

a) $v_D = 0.5 \text{ V}$

b) $v_D = 1.5 \text{ V}$.

Analysis:

a) $v_{DS} = 0.5 < v_{GS} - V_T = 4 - 2.5 = 1.5 \text{ V}$, therefore the transistor is in the triode region.

b) $v_D = 1.5 \text{ V} = v_{DS}$, therefore the transistor is at the border of the saturation and triode regions.

Problem 11.8**Solution:****Known quantities:**

The threshold voltage, $V_T = 4 \text{ V}$, of an enhancement-type NMOS. $i_D = 1 \text{ mA}$ when $v_{GS} = v_{DS} = 6 \text{ V}$.

Find:

The value of i_D when $v_{GS} = 5 \text{ V}$.

Analysis:

From $0.001 = k(6 - 4)^2$, we have $k = 0.25 \times 10^{-3}$

For $v_{GS} = 5 \text{ V}$, and assuming active operation: $i_D = 0.25 \times 10^{-3}(5 - 4)^2 = 0.25 \text{ mA}$.

Problem 11.9**Solution:****Known quantities:**

The threshold voltage, $V_T = 1.5 \text{ V}$, of the NMOS transistor shown in Figure P11.9. $k = 0.4 \text{ mA/V}^2$.

Find:

The voltage levels of the pulse signal at the drain output, if v_G is a pulse with 0 V to 5 V .

Analysis:

Since $V_T = 1.5 \text{ V}$, with $v_G = 0 \text{ V}$, $v_{GS} < V_T$, the transistor is cut off. Therefore, $v_D = 5 \text{ V}$.

When $v_G = 5 \text{ V}$, and assuming that the transistor is in the active region:

$i_D = k(v_{GS} - V_T)^2 = 0.4(5 - 1.5)^2 = 4.9 \text{ mA}$. Therefore, $v_D = 5 - 4.9 \times 1 = 0.1 \text{ V}$.

Section 11.3: *n*-channel MOSFET Amplifiers

Problem 11.10

Solution:

Known quantities:

The i - v characteristic of Figure P11.10(a), and the circuit in Figure P11.10(b):

$$V_{GG} = 7 \text{ V}, V_{DD} = 10 \text{ V}, R_D = 5 \Omega$$

Find:

The current i_{DQ} the voltage v_{DSQ} , and the region of operation of the MOSFET.

Analysis:

The operating point can be determined using the load line method.

$$i_D = \frac{V_{DD}}{R_D} - \frac{v_{DS}}{R_D} = 2 - 0.2 v_{DS}$$

By superimposing the load line on Figure P11.10(a), and by noticing that $V_{GS} = V_{GG} = 7 \text{ V}$, we obtain

$$i_{DQ} = 0.8 \text{ A}, v_{DSQ} = 6 \text{ V}$$

The MOSFET is in the saturation region.

Problem 11.11

Solution:

Known quantities:

The circuit in Figure P11.10(b):

$$V_{GG} = 7 \text{ V}, V_{DD} = 20 \text{ V}, V_T = 3 \text{ V}, R_D = 5 \Omega, K = 50 \text{ mA/V}^2$$

Find:

The current i_{DQ} the voltage v_{DSQ} , and the region of operation of the MOSFET.

Analysis:

Assuming that the MOSFET is in the saturation region, the quiescent drain current is

$$i_{DQ} = K(v_{GSQ} - V_T)^2 = 0.05(7 - 3)^2 = 0.8 \text{ A}$$

The drain-to-source voltage is

$$v_{DSQ} = V_{DD} - R_D i_{DQ} = 20 - 5 \cdot 0.8 = 16 \text{ V}$$

Since $v_{DSQ} = v_{DS} - v_{GS} = 9 \text{ V} > V_T \Rightarrow$ hypothesis was correct

Problem 11.12

Solution:

Known quantities:

The circuit in Figure 11.12:

$$V_{DD} = 36 \text{ V}, V_T = 4 \text{ V}, R_D = 10 \text{ k}\Omega, R_1 = R_2 = 2 \text{ M}\Omega, K = 0.1 \text{ mA/V}^2$$

Find:

The current i_{DQ} , the voltage v_{DSQ} , the resistance R_S , and the operating region of the MOSFET.

Analysis:

Using Thevenin equivalent,

$$V_{GG} = \frac{R_2}{R_1 + R_2} V_{DD} = 18 \text{ V}$$

We can write the equations

$$V_{GG} = v_{GSQ} + R_S i_{DQ} = 18,$$

$$V_{DD} = (R_D + R_S) i_{DQ} + v_{DSQ} = R_D i_{DQ} + 18 - v_{GSQ} + v_{DSQ} = 36 \Rightarrow R_D i_{DQ} + v_{DSQ} = 18 + v_{GSQ}$$

Assuming saturation conditions, the current i_D can be written as

$$i_{DQ} = K(v_{GSQ} - V_T)^2 \Rightarrow v_{GSQ} + R_S K(v_{GSQ} - V_T)^2 = 18$$

and

$$R_D K(v_{GSQ} - V_T)^2 + v_{DSQ} = 18 + v_{GSQ}$$

Notice that the problem has more unknown than equations; we can impose the v_{DSQ} to ensure saturation conditions as

$$v_{DSQ} = V_{DD} / 2 = 18 \text{ V} \Rightarrow$$

$$(v_{GSQ} - V_T)^2 = v_{GSQ} \Rightarrow v_{GSQ}^2 - 9v_{GSQ} + 16 = 0 \Rightarrow v_{GSQ} = 6.56 \text{ V}$$

Remark: The other solution of the algebraic equation is not acceptable because $< V_T$.

The resistance R_S is given by

$$R_S = \frac{18 - v_{GSQ}}{K(v_{GSQ} - V_T)^2} = \frac{18 - 6.56}{0.1 \cdot 10^{-3} (6.56 - 4)^2} = 17.45 \text{ k}\Omega$$

and the drain current

$$i_{DQ} = K(v_{GSQ} - V_T)^2 = 0.655 \text{ mA}$$

Problem 11.13

Solution:

Known quantities:

n -channel MOSFET operating in saturation region with

$$v_{DSQ} = 5 \text{ V}, v_{GSQ} = 3 \text{ V}, V_T = 1 \text{ V}, K = 0.1 \text{ mA/V}^2$$

Find:

The transconductance g_m .

Analysis:

The drain current is

$$i_{DQ} = K(v_{GSQ} - V_T)^2 = 0.4 \text{ mA}$$

The transconductance is

$$g_m = 2\sqrt{K i_{DQ}} = 2\sqrt{0.04} \cdot 10^{-3} = 0.4 \text{ mA/V}^2$$

Problem 11.14

Solution:

Known quantities:

The circuit in Figure 11.12:

$$V_{DD} = 12 \text{ V}, V_T = 1 \text{ V}, R_S = R_D = 10 \text{ k}\Omega, R_1 = R_2 = 2 \text{ M}\Omega, K = 1 \text{ mA/V}^2$$

Find:

The current i_{DQ} , the voltage v_{DSQ} , and the voltage v_{GSQ} .

Analysis:

Using Thevenin,

$$V_{GG} = \frac{R_2}{R_1 + R_2} V_{DD} = 6 \text{ V}$$

We can write the equations

$$V_{GG} = v_{GSQ} + R_S i_{DQ} = 6,$$

$$V_{DD} = (R_D + R_S) i_{DQ} + v_{DSQ} = 2R_D i_{DQ} + v_{DSQ} = 12$$

Assuming saturation conditions, the current i_D can be written as

$$i_{DQ} = K(v_{GSQ} - V_T)^2 \Rightarrow v_{GSQ} + R_S K(v_{GSQ} - V_T)^2 = 6 \Rightarrow 10v_{GSQ}^2 - 19v_{GSQ} + 4 = 0 \Rightarrow v_{GSQ} = 1.66 \text{ V}$$

The other solution is not acceptable because less than V_T .

It follows

$$i_{DQ} = \frac{6 - v_{GSQ}}{R_S} = 0.434 \text{ mA},$$

$$v_{DSQ} = 12 - 2R_D i_{DQ} = 3.32 \text{ V}$$

Section 11.4: MOSFET Switches

Problem 11.15

Solution:

Known quantities:

The CMOS NAND gate of Figure 11.21.

Find:

Identify the state of each transistor for $v_1 = v_2 = 5\text{ V}$.

Analysis:

The two transistors at the top are cut off and the two at the bottom are on.

Problem 11.16

Solution:

Known quantities:

The CMOS NAND gate of Figure 11.21.

Find:

Identify the state of each transistor for $v_1=5\text{V}$, $v_2=0\text{V}$.

Analysis:

The transistor at the bottom and the first on the top are off, the other two are on.

Problem 11.17

Solution:

Find:

Draw the schematic diagram of a two-input CMOS OR gate.

Analysis:

The output of the circuit of Figure 11.18 is connected as an input to the circuit of Figure 11.14.

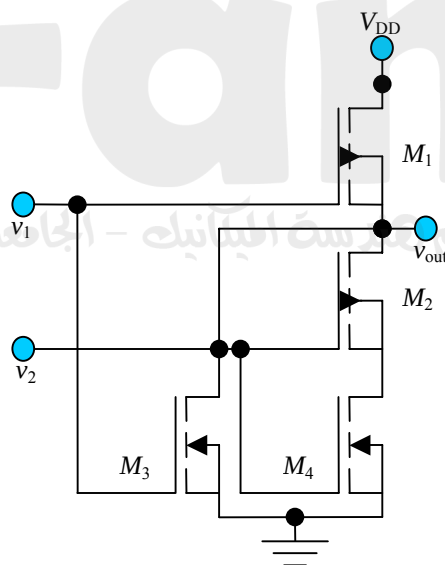


Figure 11.18

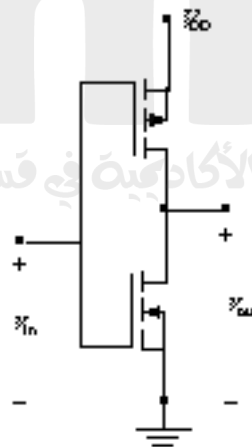


Figure 11.14

Problem 11.18**Solution:****Find:**

Draw the schematic diagram of a two-input CMOS AND gate.

Analysis:

The output of the circuit of Figure 11.21 is connected as an input to the circuit of Figure 11.14.

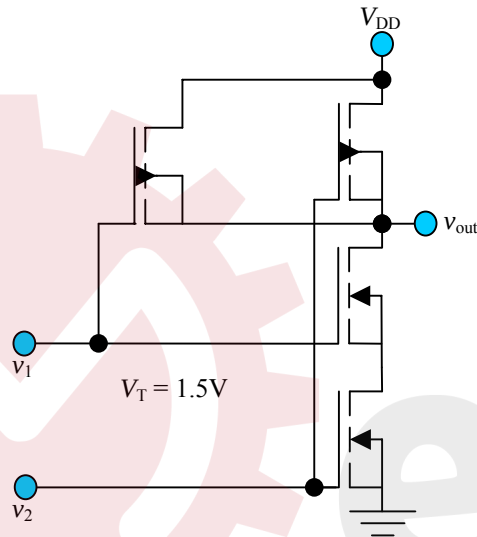


Figure 11.21

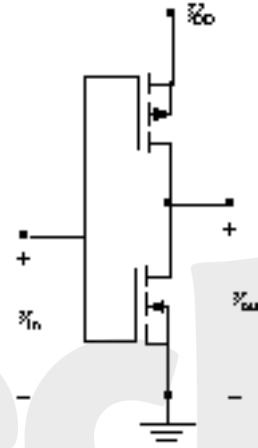


Figure 11.14

Problem 11.19**Solution:****Find:**

Draw the schematic diagram of a two-input CMOS NOR gate.

Analysis:

The circuit of Figure 11.18

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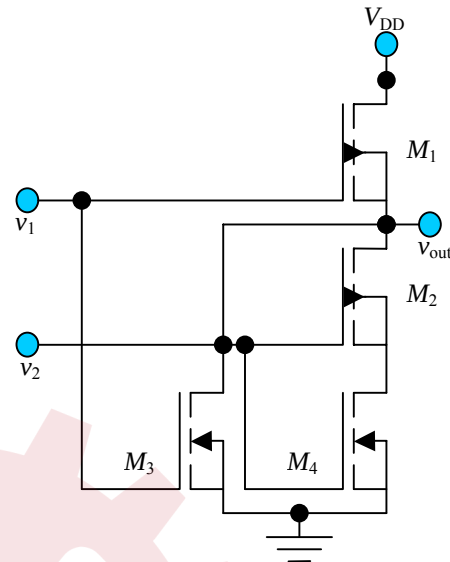


Figure 11.18

Problem 11.20**Solution:****Find:**

Draw the schematic diagram of a two-input CMOS NAND gate.

Analysis:

The circuit of Figure 11.21.

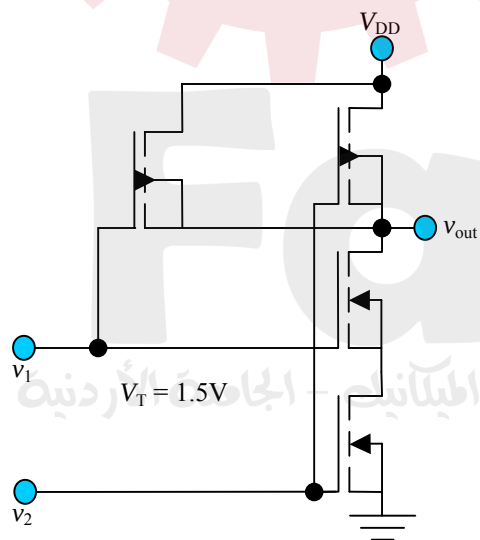


Figure 11.21

Problem 11.21**Solution:****Known quantities:**

The circuit of Figure P11.21.

Find:

Show that the given circuit functions as a logic inverter.

Analysis:

Construct a state table:

v_{in}	Q_1	Q_2	v_{out}
low	resistive	open	high
high	open	resistive	low

This table clearly describes an inverter.

Problem 11.22**Solution:****Known quantities:**

The circuit of Figure P11.22.

Find:

Show that the given circuit functions as a NOR gate.

Analysis:

Construct a state table:

v_1	v_2	Q_1	Q_2	v_{out}
0	0	off	off	high
0	high	off	on	low
high	0	on	off	low
high	high	on	on	low

This table clearly describes a NOR gate.

Problem 11.23**Solution:****Known quantities:**

The circuit of Figure P11.23.

Find:

Show that the given circuit functions as a NAND gate.

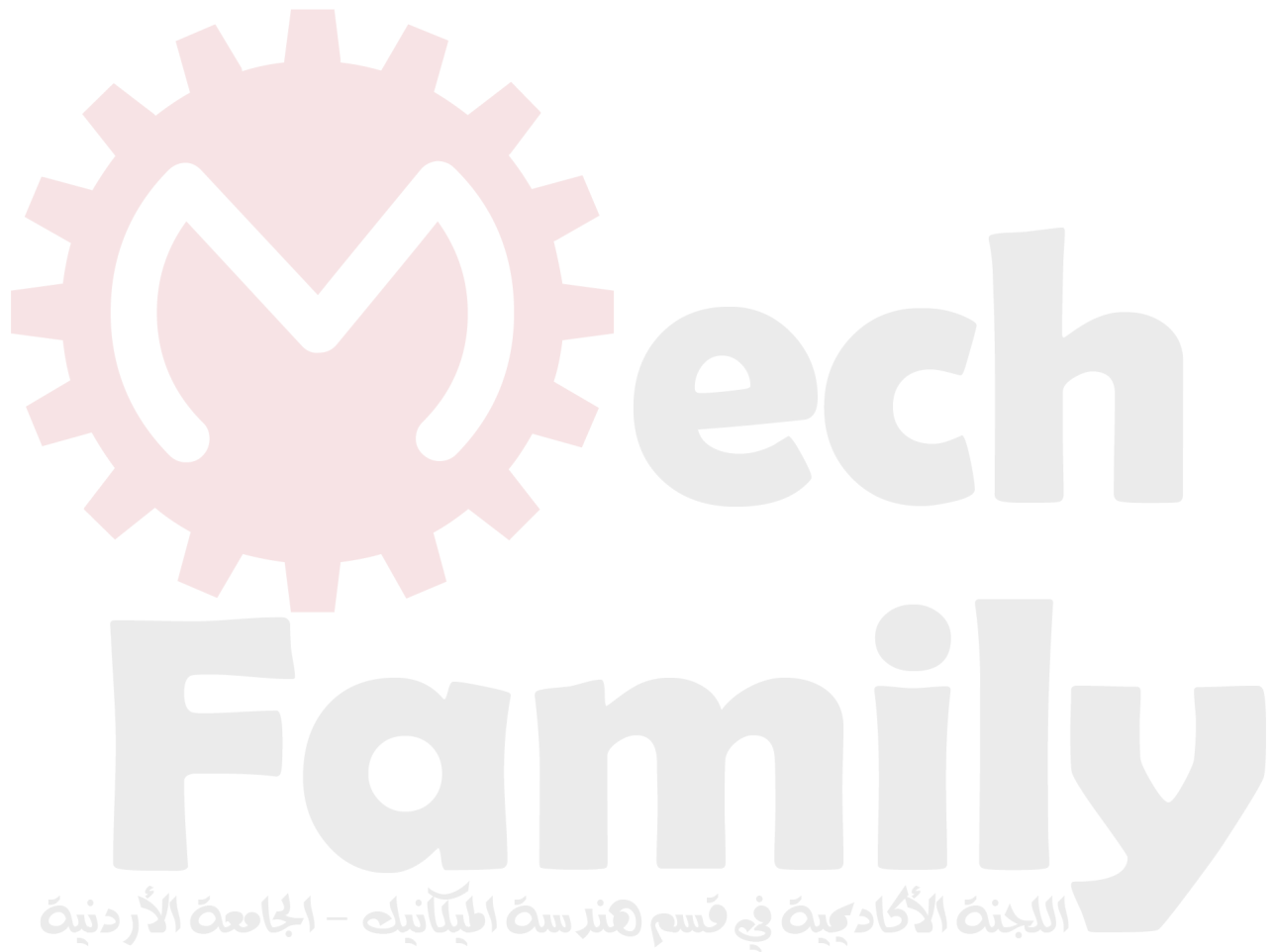
Analysis:

Construct a state table:

v_1	v_2	Q_1	Q_2	v_{out}
0	0	off	off	high
0	high	off	on	high

high	0	on	off	high
high	high	on	on	low

This table clearly describes a NAND gate.



Chapter 12 Instructor Notes

Chapter 12 introduces the subject of power electronics. The importance of power electronics cannot be overemphasized, considering the widespread industrial application of electric machines, and other high current loads in practical engineering applications. The chapter discusses the basic characteristics and limitations of power amplifiers, practical voltage regulators, inductive loads (such as electric motors), and SCRs. The aim is to give the student sufficient understanding of the device characteristics to be able to complete simple "order of magnitude" calculations to be able to size a device for a given application. This chapter is much more practically oriented than some of the others in the text, and may be used to accompany a course in electric power and machines based on Chapters 7, 16, 17 and 18.

After Sections 12.1 and 12.2 present a classification of power electronic devices (Figure 12.1, p. 578) and circuits (Table 12.1, p. 579), the discussion is divided into the topics of Voltage Regulators (Section 12.3), Power Amplifiers and Transistor Switches (Section 12.4), Rectifiers and Controlled Rectifiers (Section 12.5), and Electric Motor Drives (Section 12.6)

Homework problems are divided into three major sections. The first, on voltage regulators, includes problems two different voltage regulator circuits (12.2, 12.3) The second, on rectifiers and controlled, illustrates a battery charging circuit (12.7) and two simple motor speed control problems (12.9, 12.10). The last section, on drives, introduces choppers, and more advanced problems on DC motor supplies based on controlled rectifiers (12.23 and 12.24).

Learning Objectives

1. Learn the classification of power electronic devices and circuits. Sections 1 and 2.
2. Analyze the operation of practical voltage regulators. Section 3.
3. Understand the principal limitations of transistor power amplifiers. Section 4.
4. Analyze the operation of single- and three-phase controlled rectifier circuits. Section 5.
5. Understand the operation of power converters used in electric motor control, and perform simplified analysis on DC-DC converters. Section 6.

Section 12.3: Voltage Regulators

Problem 12.1

Solution:

Find:

Repeat Example 12.1 for a 7-V Zener diode.

Analysis:

Calculating the collector and base currents according to:

$$I_E = \frac{7 - 1.3}{10} = 0.57 \text{ A}$$

$$I_B = \frac{I_E}{11} = 51.8 \text{ mA}$$

We find:

$$I_R = \frac{20 - 7}{47} = 0.277 \text{ A}$$

$$I_Z = I_R - I_B = 0.225 \text{ A}$$

$$V_{CE} = 20 - V_L = 20 - 5.7 = 14.3 \text{ V} > 0.6 \text{ V}$$

Thus, the transistor is in the active region. The Zener power is: $I_Z \cdot V_Z = 1.576 \text{ W}$.

Problem 12.2

Solution:

Known quantities:

The current regulator circuit shown in Figure P12.2.

Find:

The expression for R_S .

Analysis:

Assuming that the Zener voltage is V_Z , that $V_{BE} = V_\gamma = 0.6 \text{ V}$, and that the required current is I , we have:

$$R_S = \frac{V_Z + V_{BE}}{I} = \frac{V_Z + 0.6}{I}.$$

Problem 12.3

Solution:

Known quantities:

The shunt-type voltage regulator shown in Figure P12.3.

Find:

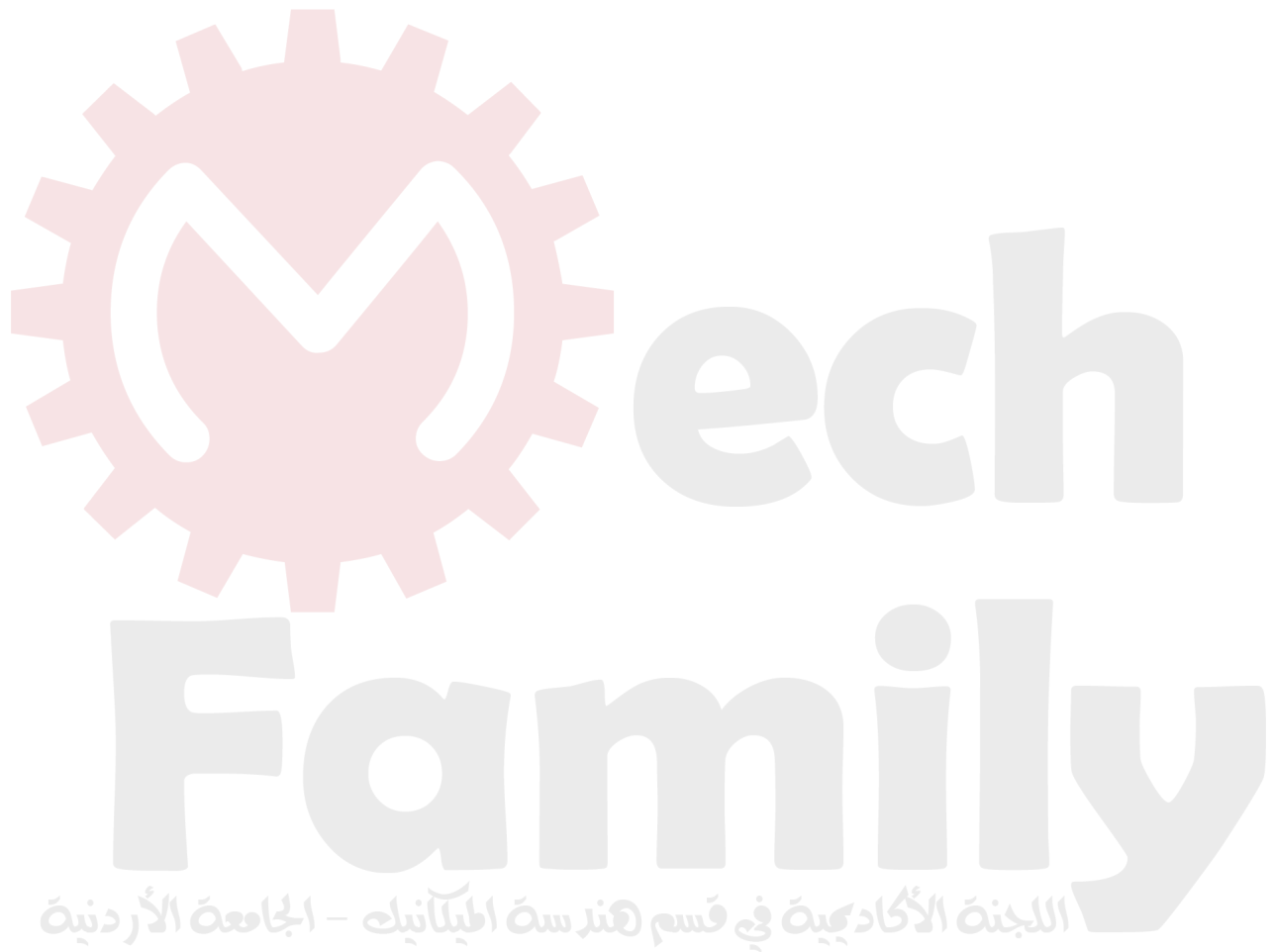
The expression for the output voltage, V_{out} .

Analysis:

If the Zener diode is to be in the regulator mode, the CB junction must be forward biased; in this case, both the CB and the BE junctions are forward biased, since a substantial base current will be generated through

the Zener diode (depending on the value of the shunt resistor in the output circuit). Thus, the collector-emitter voltage is equal to: $V_{CEsat} \approx 0.2 \text{ V}$, and the source current will be: $I_S = \frac{V_S - V_{CEsat}}{R_S} = I_C + I_Z$.

The voltage across the shunt resistor will therefore be V_Z and the output voltage is: $V_{out} = V_Z + V_\gamma$.



Section 12.5: Rectifiers and Controlled Rectifiers (AC-DC Converters)

Problem 12.4

Solution:

Known quantities:

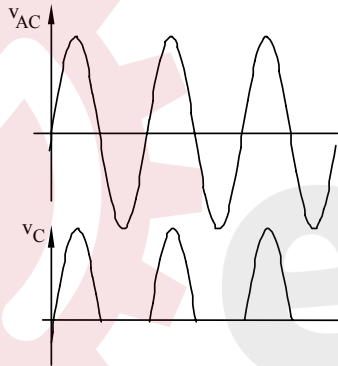
The circuit shown in Figure 12.17.

Find:

If the LR load is replaced by a capacitor, draw the output waveform and label the values.

Analysis:

When the sinusoidal source voltage is in the positive half cycle, the series diode conducts, and the shunt diode is an open circuit; thus, the positive half cycle appears directly across the capacitor (assuming ideal diodes). During the negative half cycle, the series diode is open, and therefore the voltage across the capacitor remains zero, as shown in the sketches below.



Problem 12.5

Solution:

Known quantities:

The circuit shown in Figure 12.17.

Find:

If the diode forward resistance is $50\ \Omega$, the forward bias voltage is 0.7 V , and the load consists of a resistor $R = 10\ \Omega$ and an inductor $L = 2\text{ H}$, draw $v_L(t)$ and label the values for the given circuit.

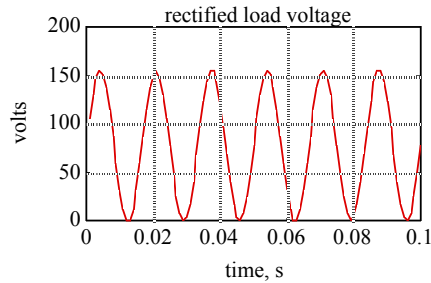
Analysis:

To obtain exact numerical values, we assume a $111\text{ V}_{\text{rms}}$ source, $R = 10\ \Omega$, and $L = 2\text{ H}$, then:

$v_{AC}(t) = A \sin(\omega t) = 155.6 \cdot \sin(377t)$, and from Equation 12.6, the average load current is:

$$I_L = \frac{155.6}{\pi R} = 4.95\text{ A}. \text{ Using the approximation: } v_L(t) \approx \frac{A}{2} + \frac{A}{2} \sin(\omega t) \text{ we have:}$$

$$v_L(t) \approx \frac{A}{2} + \frac{A}{2} \sin(\omega t) = 77.8 + 77.8 \sin(377t). \text{ The waveform is shown below:}$$



Problem 12.6

Solution:

Known quantities:

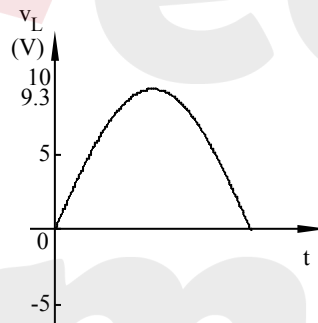
For the circuit shown in Figure P12.6, v_{AC} is a sinusoid with 11 V peak amplitude, $R = 2 \text{ k}\Omega$ and the forward-conducting voltage of D is 0.7 V.

Find:

- Sketch the waveform of $v_L(t)$.
- Find the average value of $v_L(t)$.

Analysis:

- Assume $v_{AC}(t) = 10 \sin(\omega t) \text{ V}$. The output voltage is: $v_L(t) = (10 - 0.7) \sin(\omega t)$. The waveform is shown below:



$$\text{b. } \langle v_L \rangle = \frac{1}{2\pi} \int_0^\pi 9.3 \sin(\omega t) d(\omega t) = \frac{9.3}{\pi} = 2.96 \text{ V}.$$

Problem 12.7

Solution:

Known quantities:

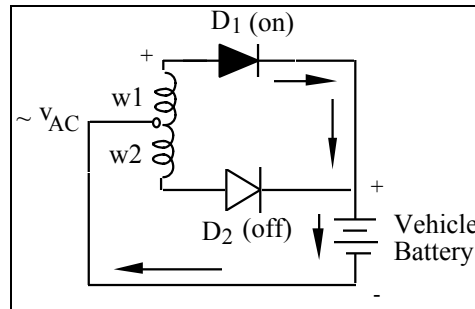
The vehicle battery charge circuit shown in Figure P12.7.

Find:

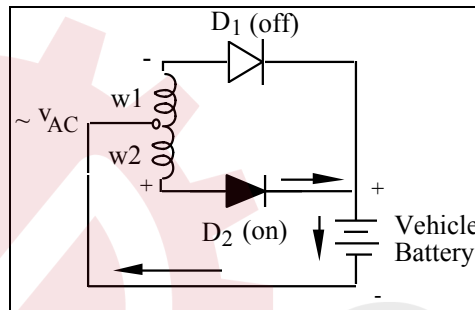
Describe the circuit, and draw the output waveform (L_1 and L_2 represent the inductances of the windings of the alternator).

Analysis:

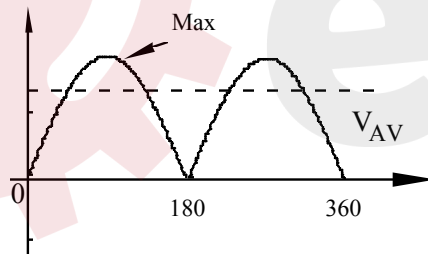
The positive half cycle from w1 is conducted by diode D_1 . Diode D_2 does not conduct due to negative bias at w2. The first half cycle is passed through to the battery.



The second half cycle finds w2 positive and diode D₂ conducts current to the battery while diode D₁ is negatively biased and is off.



The full-wave rectified output waveform is shown below.



The average DC value V_{AV} is 63% of the peak value.

Problem 12.8

Solution:

Find:

NOTE: Typo in problem statement, referring to the wrong example problem.

Repeat Example 12.6 for $\alpha = \pi/3$ and $\pi/6$.

Analysis:

a) For $\alpha = \pi/3$ we have:
$$v_L\left(\frac{\pi}{3}\right) = \frac{120\sqrt{2}}{2} \sqrt{1 - \frac{1}{3} + \sin \frac{2\pi}{3}} = 105 \text{ V}.$$

The power is:
$$P = \frac{v_L^2}{R} = 45.94 \text{ W}.$$

b) For $\alpha = \pi/6$ we have:
$$v_L\left(\frac{\pi}{6}\right) = \frac{120\sqrt{2}}{2} \sqrt{1 - \frac{1}{6} + \sin \frac{\pi}{3}} = 110.6 \text{ V}.$$

The power is: $P = \frac{v_L^2}{R} = 50.97 \text{ W}.$

Problem 12.9

Solution:

Known quantities:

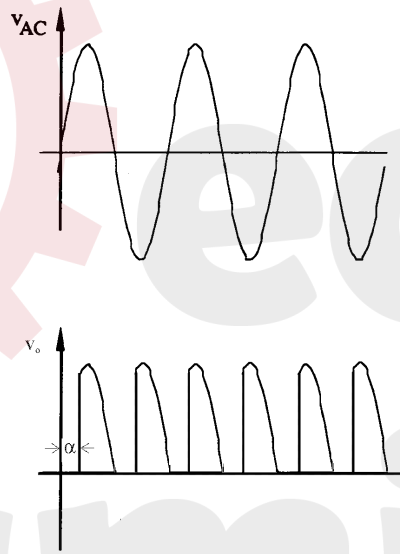
For the circuit shown in Figure P12.9, assume the thyristors are fired at $\alpha = 60^\circ$ and that the motor current is 20 A and is ripple free. The supply is 111 V_{AC} (rms).

Find:

- Sketch the output voltage waveform, v_o .
- Compute the power absorbed by the motor.
- Determine the volt-amperes generated by the supply.

Analysis:

a)



$$\alpha = 60^\circ \times \frac{\pi}{180} = \frac{\pi}{3}, \quad R_a = 0.2 \quad .$$

$$\text{b) } V_{o_{rms}} = \frac{120\sqrt{2}}{2} \left[1 - \frac{1}{3} + \sin 120^\circ \right]^{\frac{1}{2}} = 105 \text{ V} \rightarrow P_m = I_o V_{o_{rms}} = (20 \text{ A})(105 \text{ V}) = 2.1 \text{ kW} .$$

$$\text{c) } P_R = I_o^2 R_a = (20)^2 (0.2) = 80 \text{ W} ; \quad P_S = P_m + P_R = 2180 \text{ W} .$$

Problem 12.10

Solution:

Known quantities:

The circuit of Figure 12.2, replacing the resistive load with a DC motor. The motor operates at 110 V and absorbs 4 kW of power. The AC supply is 80 V, 60 Hz. Assume that the motor inductance is very large (i.e., the motor current is ripple free), and that the motor constant is 0.055 V/rev/min.

Find:

If the motor runs at 1,000 rev/min at rated current:

- Determine the firing angle of the converter.
- Determine the rms value of the supply current.

Analysis:

$$\text{Back emf} = k_T \times N = 0.055 \times 1000 = 55 \text{ V}$$

$$I_{DC} = \frac{P}{V} = \frac{4000}{110} = 36.4 \text{ A}$$

$$V = 110 \text{ V} . \text{ Assume } R = 1 . \quad I_{DC} = \frac{1}{\pi R} \left[\sqrt{2} V (\cos \alpha) - V_B (\pi - \alpha) \right]$$

$$\text{a) } 36.4 = \frac{1}{\pi} \left[\sqrt{2} (110) \cos \alpha - 55 (\pi - \alpha) \right]. \text{ Solving yields: } \alpha \approx \frac{7}{12} \pi \text{ rad.}$$

$$\text{b) With zero ripple, } I_{rms} = I_{DC} = 36.4 \text{ A} .$$

Problem 12.11**Solution:****Find:**

NOTE: Typo in problem statement, referring to the wrong example problem.

For the light dimmer circuit of Example 12.6, determine the load power at firing angles

$\alpha = 0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 180^\circ$, and plot the load power as a function of α .

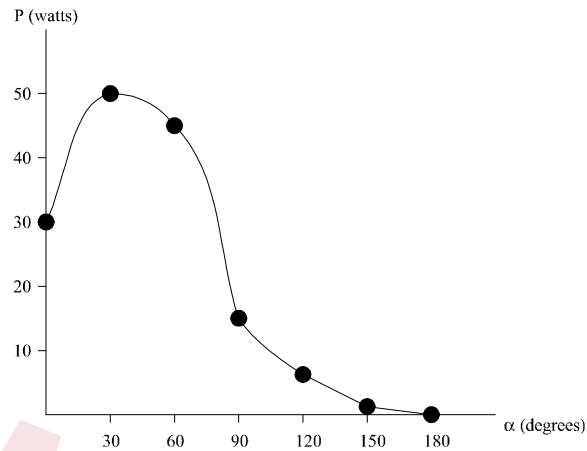
Analysis:

$$\text{Note that } V_{L_{rms}} = \frac{120}{\sqrt{2}} \sqrt{1 - \frac{\alpha}{\pi} + \sin 2\alpha} , \text{ for } \alpha \leq 90^\circ ,$$

$$\text{and } V_{L_{rms}} = \frac{120}{\sqrt{2}} \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}} , \text{ for } \alpha > 90^\circ$$

α	$V_{L_{rms}}$	$P = \frac{V_{L_{rms}}^2}{R_{BULB}}$
0°	84.85	30
30°	111.60	50.98
60°	115.00	45.98
90°	60.00	15.00
120°	37.53	5.87
150°	14.41	0.87
180°	0	0

A sketch of power vs. firing angle is shown below:



Problem 12.12

Solution:

Known quantities:

For the circuit shown in Figure P12.12:

$$V_L = 10 \text{ V} \quad V_r = 10\% = 1 \text{ V}$$

$$I_L = 650 \text{ mA} \quad v_{line} = 170 \cos \omega t \text{ V}$$

$$\omega = 2513 \frac{\text{rad}}{\text{s}}$$

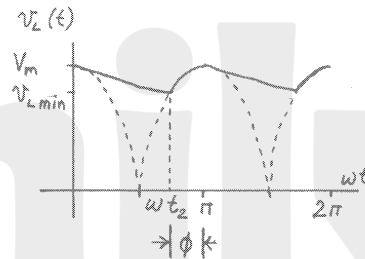
Find:

Determine the conduction angle of the diodes, if the diodes are fabricated from silicon.

Analysis:

$$V_m = V_L + \frac{1}{2} V_r = 10 \text{ V} + \frac{1}{2} [1 \text{ V}] = 10.5 \text{ V}$$

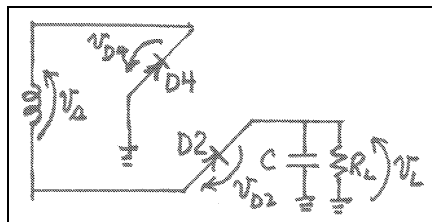
$$v_{L-\min} = V_L - \frac{1}{2} V_r = 10 \text{ V} - \frac{1}{2} [1 \text{ V}] = 9.5 \text{ V}$$



D2 and D4 conduct during $\omega t_2 < \omega t < \pi$.

First, determine the amplitude of the source voltage. [The secondary of the transformer acts as a source.]

Then use the same KVL to determine the angle at which the diodes start conducting.



$$KVL: +v_{D4} + v_s + v_{D2} + v_L = 0$$

$$\text{At } \omega t = \pi: v_{D2} = v_{D4} = v_{D-on} = 0.7 \text{ V [Si]}$$

$$v_s = -V_{so} \quad v_L = V_m$$

$$V_{so} = v_{D-on} + v_{D-on} + V_m = 0.7 \text{ V} + 0.7 \text{ V} + 10.5 \text{ V} = 11.9 \text{ V}$$

$$\text{At } \omega t = \omega t_2: v_{D4} = v_{D2} = v_{D-on} = 0.7 \text{ V} \quad v_s = V_{so} \cos \omega t_2 \quad v_L = v_{L-\min}$$

$$\cos \omega t_2 = -\frac{v_{D-on} + v_{D-on} + v_{L-\min}}{V_{so}} = -\frac{0.7 \text{ V} + 0.7 \text{ V} + 9.5 \text{ V}}{11.9 \text{ V}} = -0.91597$$

$$\omega t_2 = 156.3^\circ \quad \therefore \phi = 180^\circ - 156.3^\circ = 23.66^\circ$$

Problem 12.13

Solution:

Known quantities:

For the circuit shown in Figure P12.13, assume that the conduction angle of the diodes shown (which are Silicon) is:

$$\Phi = 23^\circ$$

$$v_{s1}(t) = v_{s2}(t) = 8 \cos(\omega t) \text{ V}$$

$$\omega = 377 \frac{\text{rad}}{\text{s}} \quad R_L = 20 \text{ k}\Omega \quad C = 0.5 \mu\text{F}$$

Find:

The ripple voltage.

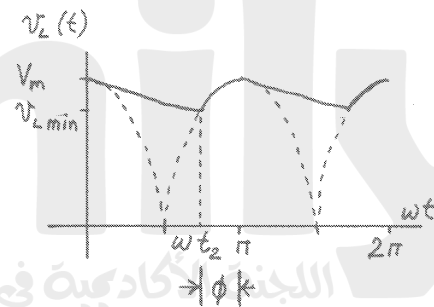
Analysis:

$$KVL: -v_{s1}(t) + v_{D1} + v_L(t) = 0$$

$$\text{At } t = 0:$$

$$v_L(0) = V_m = V_{so} - v_{D-on} = 8 \text{ V} - 0.7 \text{ V} = 7.3 \text{ V}$$

$$\omega t_2 = \pi - \Phi \Rightarrow t_2 = \frac{\pi - 23^\circ}{377 \frac{\text{rad}}{\text{s}}} \frac{180^\circ}{\pi} = 7.268 \text{ ms}$$



$$v_L(t) = v_L(\infty) + (v_L(0) - v_L(\infty))e^{-\frac{t}{T_c}} = 0 + [V_m - 0]e^{-\frac{t}{R_L C}}$$

$$v_L(t_2) = v_{L-\min} = 7.3 \cdot e^{-\frac{7.286 \cdot 10^{-3}}{[20 \cdot 10^3][0.5 \cdot 10^{-6}]}} = 3.529 \text{ V}$$

$$V_r = V_m - v_{L-\min} = 7.3 - 3.529 = 3.771 \text{ V}$$

Note the ripple is quite large. This is primarily due to the very small (for this type circuit) value of the capacitance. Also the conduction angle assumed above is not correct for this circuit.

Problem 12.14**Solution:****Known quantities:**

The diodes in the full-wave DC power supply shown are Silicon. If:

$$I_L = 85 \text{ mA} \quad V_L = 5.3 \text{ V} \quad V_r = 0.6 \text{ V} \quad \omega = 377 \frac{\text{rad}}{\text{s}} \quad v_{\text{line}} = 156 \cos \omega t \text{ V}$$

$$C = 1023 \mu\text{F} \quad \phi = \text{Conduction Angle} = 23.90^\circ$$

Find:

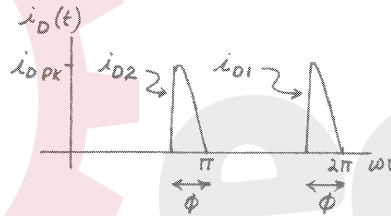
The value of the average and peak current through each diode.

Analysis:

Diodes D1 and D3 will conduct half of the load current and Diodes D2 and D4 will conduct the other half.

Therefore: $i_{D\text{-ave}} = \frac{1}{2} I_L = \frac{1}{2} [85 \text{ mA}] = 42.5 \text{ mA}$

The waveforms of the diode currents are complex but can be roughly approximated as triangular [recall area of triangle = bh/2]:



$$I_L = [i_{D1,3} + i_{D2,4}]_{\text{ave}} = \frac{1}{2\pi} \int_0^{2\pi} [i_{D1,3}(\omega t) + i_{D2,4}(\omega t)] d[\omega t] = \frac{1}{2\pi} \left[\frac{\phi i_{D\text{-pk}}}{2} + \frac{\phi i_{D\text{-pk}}}{2} \right]$$

$$i_{D\text{-pk}} = \frac{2\pi I_L}{\frac{1}{2}\phi + \frac{1}{2}\phi} = \frac{2\pi I_L}{\phi} = \frac{[2\pi \text{ rad}][85 \text{ mA}]}{[23.90^\circ][\frac{\pi \text{ rad}}{180^\circ}]} = 1.280 \text{ A}$$

Problem 12.15**Solution:****Known quantities:**

The diodes in the full-wave DC power supply shown in Figure P12.13 are Silicon. If:

$$I_L = 600 \text{ mA} \quad V_L = 50 \text{ V} \quad V_r = 8\% = 4 \text{ V} \quad v_{\text{line}} = 170 \cos \omega t \text{ V} \quad \omega = 377 \frac{\text{rad}}{\text{s}}$$

Find:

The value of the conduction angle for the diodes and the average and peak current through the diodes. The load voltage waveform is shown in Figure P12.15.

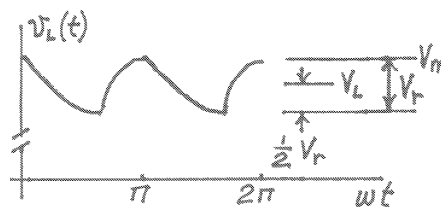
Analysis:

$$v_{L\text{-min}} = V_L - \frac{1}{2} V_r = 50 - \frac{1}{2} 4 = 48 \text{ V}$$

$$V_m = V_L + \frac{1}{2} V_r = 50 + \frac{1}{2} 4 = 52 \text{ V}$$

$$\text{KVL: } -v_{s1}(t) + v_{D1} + v_L(t) = 0$$

$$\text{At } t = 0: -V_{so} \cos(0) + v_{D\text{-on}} + V_m = 0 \Rightarrow V_{so} = 0.7 + 52 = 52.7 \text{ V}$$



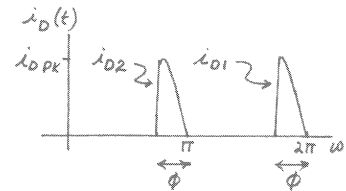
$$KVL: +v_{s2}(t) + v_{D2} + v_L(t) = 0$$

$$\text{At } t = t_2: +V_{so} \cos(\omega t_2) + v_{D-on} + v_{L-min} = 0$$

$$\omega t_2 = \cos^{-1} \left[-\frac{v_{D-on} + V_{L-min}}{V_{so}} \right] = \cos^{-1} \left[-\frac{0.7V + 48V}{52.7V} \right] = 2.749 \text{ rad}$$

$$\Phi = \pi - \omega t_2 = \pi - 2.749 \text{ rad} = 392.1 \text{ mrad} \frac{180^\circ}{\pi \text{ rad}} = 22.47^\circ$$

The waveforms of the diode currents are complex but can be roughly approximated as triangular [recall the area of triangle = bh/2].



$$i_{D1-ave} = i_{D2-ave} = \frac{1}{2} I_L = \frac{1}{2} 600 \text{ mA} = 300 \text{ mA}$$

$$[i_{D1} + i_{D2}]_{ave} = I_L$$

$$\frac{1}{\omega T} \int_0^{2\pi} [i_{D1}(\omega t) + i_{D2}(\omega t)] d[\omega t] = I_L$$

$$\frac{1}{\omega T} \left[\frac{1}{2} \Phi i_{D-pk} + \frac{1}{2} \Phi i_{D-pk} \right] = I_L$$

$$i_{D-pk} = \frac{\omega T I_L}{\Phi} = \frac{[2\pi \text{ rad}][600 \text{ mA}]}{392.1 \text{ mrad}} = 9.615 \text{ A}$$

Section 12.6: Electric Motor Drives

Problem 12.16

Solution:

Known quantities:

For the chopper of Figure P12.35, the supply voltage is 120 V, and the armature resistance of the motor is 0.15 Ω . The motor back emf constant is 0.05 V/rev/min and the chopper frequency is 250 Hz.

Assume that the motor current is free of ripple and equal to 125 A at 120 rev/min.

Find:

- The duty cycle of the chopper, δ , and the chopper-on time, t_1 .
- The power absorbed by the motor.
- The power generated by the supply.

Analysis:

a)

$$v_o = i_o R_a + E_a = (125)(0.15) + 6 = 24.75 \text{ V}$$

$$24.75 = \delta(120) \Rightarrow \delta = \frac{24.75}{120} = 0.2063$$

$$\delta = \frac{t_1}{T} \Rightarrow t_1 = \delta T = (0.2063) \left(\frac{1}{250} \right) = 825 \mu\text{s}$$

b)

$$P_m = E_a i_o = (6)(125) = 750 \text{ W}$$

$$P_R = R_a i_o^2 = (0.15)(125)^2 = 2.344 \text{ kW}$$

c)

$$P_S = P_m + P_R = 3.094 \text{ kW} \quad \text{or} \quad P_S = \delta \cdot V_S \cdot i_o = (0.2063)(120)(125) = 3.094 \text{ kW}.$$

Problem 12.17

Solution:

Known quantities:

For the circuit shown in Figure 12.39, the motor constant is 0.3 V/rev/min, the supply voltage is 600 V, and the armature resistance is $R_a = 0.2 \Omega$.

Find:

If the motor speed is 800 rev/min and the motor current is 300 A, determine:

- The duty cycle of the chopper, δ .
- The power fed back to the supply.

Analysis:

$$a) \quad v_o = E_a + i_o R_a = 240 + (-300)(0.2) = 180 \text{ V} \Rightarrow \delta = \frac{180}{600} = 0.300.$$

$$b) \quad \left. \begin{aligned} P_m &= E_a i_o = (240)(-300) = -72.0 \text{ kW} \\ P_R &= R_a i_o^2 = (0.2)(-300)^2 = 18 \text{ kW} \end{aligned} \right\} \Rightarrow P_S = P_m + P_R = -54.0 \text{ kW}$$

$$\text{or} \quad P_S = \delta \cdot V_S \cdot i_o = (0.300)(600)(-300) = -54.0 \text{ kW}.$$

Problem 12.18

Solution:**Known quantities:**

For the two quadrant chopper of Figure 12.40, assume the thyristors S_1 and S_2 are turned on for time t_1 and off for time $T - t_1$ (T is the chopping period).

Find:

An expression for the average output voltage in terms of the supply voltage, V_S , and the duty cycle, δ .

Analysis:

$$\langle v_o \rangle = \frac{t_1}{T} V_S = \frac{t_1}{t \cdot t_1} V_S = \frac{1}{t} V_S = \delta \cdot V_S.$$

Problem 12.19**Solution:****Known quantities:**

Supply voltage; chopper duty cycle.

Find:

Average and rms value of ideal switched supply voltage.

Analysis:

The duty cycle is $\delta = \frac{t_1}{T} = \frac{1}{2.5} = 0.4$. The average value is therefore $\langle V_{\text{supply}} \rangle = \delta \times V_{\text{supply}} = 40 \text{ V}$.

To compute the rms value we use the definition of eq. 4.24:

$$\tilde{V}_{\text{supply}} = \sqrt{\int_0^{\delta} V_{\text{supply}}^2 dt} = \sqrt{\delta \cdot 100^2} = 63.25 \text{ V}$$

Problem 12.20**Solution:****Known quantities:**

Load resistance and inductance; ideal supply voltage; duty cycle.

Find:

Average values of current and voltage; power drawn from battery supply.

Analysis:

From the data given, $\delta = 0.333$. Since the period of the waveform is 3 ms, we can calculate the switching frequency to be:

$$f = \frac{1}{T} = \frac{1}{3 \cdot 10^{-3}} = 333.33 \text{ Hz}$$

$$\omega = 2,094.4 \text{ rad/s}$$

The time constant of the load impedance is $\tau = \frac{L}{R} = \frac{10^{-3} \text{ H}}{0.5} = 2 \text{ ms}$.

The average load voltage is: $\langle V_L \rangle = \delta \times V_{\text{supply}} = 33.33 \text{ V}$.

The average load current is $I_L = \delta \cdot \frac{V_{\text{supply}}}{R} = 0.33 \cdot \frac{100 \text{ V}}{0.5} = 66.67 \text{ A}$.

To compute the power drawn from the battery supply (which is assumed equal to the load power if switching losses are held negligible), we really need to compute the rms load current, since $P_L = \tilde{I}_L^2 R$; however, this calculation cannot be completed without knowing exactly the shape of the load current.

Without further analysis we can only state that the power drawn from the battery is greater than $\langle P_L \rangle = \langle V_L \rangle \langle I_L \rangle = 2.22 \text{ kW}$.

Problem 12.21

Solution:

Known quantities:

The converter of Problem 12.20 with a DC motor as load.

$$R_a = 0.2 \Omega, L_a = 1 \text{ mH}, E_a = 10 \text{ V}, T = 3 \text{ ms}, \delta = \frac{1}{3}$$

Find:

The average load current and voltage.

Analysis:

The average load voltage is: $\langle V_L \rangle = \delta \times V_{\text{supply}} = 33.33 \text{ V}$.

$$\text{The average current is: } \langle I_L \rangle = \frac{\langle V_L \rangle - E_a}{R_a} = \frac{33.33 - 10}{0.2} = 116.5 \text{ A}$$

Problem 12.22

Solution:

Known quantities:

Load resistance and inductance; load current; motor armature constant; DC supply and desired rpm range.

Find:

Range of duty cycles required.

Analysis:

Assume steady-state operation, so that the effects of the load inductance may be neglected. When the rpm is zero, the back emf is also 0, so

$$V_L - I_a R_a = 25 - 0.3 \times 7.5 \text{ V}.$$

The motor emf constant is $k_a \phi = 0.00167 \text{ V-s/rev}$, or $60 \times 0.00167 = 0.1004 \text{ V-min/rev}$

At 2000 rpm, the back emf is:

$$E_a = k_a \phi n = 200.4 \text{ V}.$$

Thus, the total load voltage is

$$V_L - I_a R_a - E_a = 7.5 - 200.4 = 207.9 \text{ V}$$

From the range of voltages required by the motor for its proper operation, we conclude that the range of required duty cycles is:

$$\delta_{\min} = \frac{V_{\min}}{V_{\text{supply}}} = \frac{7.5}{220} = 0.0341$$

$$\delta_{\max} = \frac{V_{\max}}{V_{\text{supply}}} = \frac{207.9}{220} = 0.943$$

Problem 12.23

Solution:

Known quantities:

Motor ratings; motor armature resistance and armature constant; power supply ratings.

Find:

Motor speed, power factor and efficiency for $\alpha = 0^\circ$ and $\alpha = 20^\circ$.

Analysis:

The nominal torque from the rated data is

$$T_m = \frac{P}{\omega_m} = \frac{10000}{\frac{2\pi \cdot 1000}{60}} = 95.49 \text{ Nm}$$

It follows that the DC current is

$$I_a = \frac{T_m}{K_a} = \frac{95.49}{2} = 47.75 \text{ A}$$

The average load voltage for firing angle of 0° is

$$\langle V_{L,0^\circ} \rangle = \frac{2\sqrt{2}}{\pi} V_s = \frac{2\sqrt{2}}{\pi} 240 = 216 \text{ V}$$

The speed at zero degree of firing angle is given by

$$\langle V_{L,0^\circ} \rangle = E_{a,0^\circ} + R_a I_a \Rightarrow E_{a,0^\circ} = 216 - 0.42 \cdot 47.75 = 196 \text{ V}$$

$$\omega_{m,0^\circ} = \frac{E_{a,0^\circ}}{K_a} = \frac{196}{2} = 97.97 \text{ rad/s}$$

The efficiency is

$$\eta = \frac{E_{a,0^\circ} I_a}{E_{a,0^\circ} I_a + R_a I_a^2} = \frac{E_{a,0^\circ}}{E_{a,0^\circ} + R_a I_a} = \frac{196}{196 + 0.42 \cdot 47.75} = 91\%$$

The rms voltage is

$$V_{rms,L} = V_s = 240$$

The power factor can be calculate as follows

$$\text{pf}(0^\circ) \cong \frac{E_{a,0^\circ} I_a + R_a I_a^2}{V_{rms,L} I_a} = \frac{E_{a,0^\circ} + R_a I_a}{V_{rms,L}} = \frac{196 + 0.42 \cdot 47.75}{240} = 0.9$$

In the case of firing angle of 20° , we have

$$\langle V_{L,20^\circ} \rangle = \frac{2\sqrt{2}}{\pi} V_s \cos 20^\circ = 203 \text{ V}$$

$$E_{a,20^\circ} = \langle V_{L,20^\circ} \rangle - R_a I_a = 203 - 0.42 \cdot 47.75 = 182.95 \text{ V}$$

$$\omega_{m,20^\circ} = \frac{E_{a,20^\circ}}{K_a} = \frac{182.95}{2} = 91.47 \text{ rad/s}$$

$$\eta = \frac{E_{a,20^\circ}}{E_{a,20^\circ} + R_a I_a} = \frac{182.95}{182.95 + 0.42 \cdot 47.75} = 90\%$$

$$\text{pf}(20^\circ) \cong \frac{E_{a,20^\circ} + R_a I_a}{V_{rms,L}} = \frac{182.95 + 0.42 \cdot 47.75}{240} = 0.846$$

Problem 12.24

Solution:**Known quantities:**

Separately excited DC motor:

$$P = 10 \text{ kW}, V = 300 \text{ V}, \omega = 1000 \text{ rev/min}, R_a = 0.2 \Omega, K_a = 1.38 \text{ V} \cdot \text{s/rad}$$

Power supply: $V_s = 220 \text{ (rms)V}$, $f = 60 \text{ Hz}$

Three phase controlled bridge rectifier.

Find:Speed, power factor, and efficiency for a firing angle of 30° .**Assumptions:**Load torque is constant, and the DC motor deliver the power P at 0° of firing angle.

Additional inductance is present to ensure continuous conduction.

Analysis:The average voltage supplied by the rectifier at 0° of firing angle is

$$\langle V_{L,0^\circ} \rangle = \frac{3\sqrt{3}}{\pi} \sqrt{2} V_s = 514.6 \text{ V}$$

It follows

$$\langle V_{L,0^\circ} \rangle = R_a I_a + E_{a,0^\circ} = R_a \frac{P}{E_{a,0^\circ}} + E_{a,0^\circ} \Rightarrow E_{a,0^\circ}^2 - \langle V_{L,0^\circ} \rangle E_{a,0^\circ} + R_a P = 0 \Rightarrow E_{a,0^\circ}^2 - 514.6 E_{a,0^\circ} + 2000 = 0$$

$$E_{a,0^\circ} = 510.68 \text{ V} \Rightarrow I_a = I_{a,0^\circ} = I_{a,30^\circ} = \frac{P}{E_{a,0^\circ}} = \frac{10000}{510.68} = 19.58 \text{ A}$$

The average voltage supplied by the rectifier at 30° of firing angle is

$$\langle V_{L,30^\circ} \rangle = \langle V_{L,0^\circ} \rangle \cdot \cos 30^\circ = \frac{3\sqrt{3}}{\pi} \frac{\sqrt{3}}{2} \sqrt{2} V_s = 445.65 \text{ V}$$

The emf of the DC motor in this condition is given by

$$E_{a,30^\circ} = \langle V_{L,30^\circ} \rangle - R_a I_a = 445.65 - 0.2 \cdot 19.58 = 441.74 \text{ V}$$

Finally, the speed is given by

$$\omega_m = \frac{E_{a,30^\circ}}{K_a} = 320.1 \frac{\text{rad}}{\text{s}}$$

If we assume that the inductance is big enough, then the current ripple is negligible.

Under this conditions

$$\eta = \frac{E_{a,30^\circ} I_a}{E_{a,30^\circ} I_a + R_a I_a^2} = \frac{8649.27}{8725.94} = 99.1\%$$

The rms voltage supplied for a firing angle of 30° is

$$V_{L,rms} = 453.03 \text{ V}$$

The power factor is

$$\text{pf} = \frac{P_{out}}{V_{L,rms} I_{L,rms}} \cong \frac{E_{a,30^\circ} I_a + R_a I_a^2}{V_{L,rms} I_a} = \frac{E_{a,30^\circ} + R_a I_a}{V_{L,rms}} = \frac{441.74 + 3.92}{453.03} = 0.984$$

Chapter 13 Instructor Notes

Chapter 13 is a stand-alone chapter that does not require much more than a general introduction to the idea of analog and digital signals as a prerequisite. Thus, the chapter could be covered as early as desired. Some instructors may find it desirable to first introduce the basics of electronic switching circuits by covering the appropriate sections of Chapters 9-11.

The first section introduces the ideas of analog and digital signals, and the concepts of sampling and quantization, in an intuitive fashion. Section 13.2 introduces the binary number system, and binary codes; the box *Focus on Measurements: Digital Position Encoders* (pp. 622-623) discusses optical position encoders of the type commonly encountered in many industrial applications (e.g., robotics). The third section presents the foundations of Boolean algebra, and defines the properties of logic gates; the box *Focus on Measurements: Fail-safe Autopilot Logic* (p. 629) illustrates a simple application of digital logic to motivate the content of the section from a more practical perspective. Combinational logic design through the use of Karnaugh maps is presented in section 13.4; the boxes *Focus on Methodology: Sum-of-Products Realizations* (p. 639) and *Focus on Methodology: Products-of-Sums Realizations* (p. 644) summarize design procedures for sum-of-products and product-of-sums circuits. The box *Focus on Measurements: Safety Circuit for the Operation of a Stamping Press* (pp. 646-648) demonstrates of the usefulness of even the simplest logic circuits in an industrial setting. A brief survey of digital logic could stop here, if desired.

Section 13.5 describes more advanced combinational logic modules; The box *Focus on Measurements: EPROM-based Look-up Table for Automotive Fuel Injection System Control* (pp. 654-655) is centered around the air-to-fuel ratio control problem in an internal combustion engine, and illustrates a truly wide-spread application of digital logic, since this type of circuit is present in virtually every modern automobile.

The end-of-chapter problems are divided into four sections. The first contains a few simple exercises related to number systems; the second section on combinational logic offers a selection of simple problems that are extensions of the examples given in the text, and also includes 5 applied problems (13.19-13.23) that demonstrate the use of Boolean logic in five every-day situations. The third section covers logic design, and also includes a couple of problems with an applied flavor (13.38, 13.53), in addition to a variety of more traditional design problems. Problems 13.48 - 13.52 are related to number codes. The fourth section contains a few problems related to combinational logic modules.

Learning Objectives

1. Understand the concepts of analog and digital signals and of quantization. [Section 1.](#)
2. Convert between decimal and binary number system and use the hexadecimal system and BCD and Gray codes. [Section 2.](#)
3. Write truth tables, realize logic functions from truth tables using logic gates. [Section 3.](#)
4. Systematically design logic functions using Karnaugh maps. [Section 4.](#)
5. Study various combinational logic modules, including multiplexers, memory and decoder elements, and programmable logic arrays. [Section 5.](#)

Section 13.2: The Binary Number System

Problem 13.1

Solution:**Known quantities:**

The base 10 representation of five numbers: 401_{10} , 273_{10} , 15_{10} , 38_{10} , 56_{10} .

Find:

The hex and the binary representation for these numbers.

Analysis:

Using the methodologies introduced in paragraph 13.2:

- a) 191_{16} , 110010001_2 b) 111_{16} , 100010001_2 c) F_{16} , 1111_2 d) 26_{16} , 100110_2 e) 38_{16} , 111000_2
-

Problem 13.2

Solution:**Known quantities:**

The hex representation of five numbers: A_{16} , 66_{16} , 47_{16} , 21_{16} , 13_{16} .

Find:

The base 10 and the binary representation for these numbers.

Analysis:

Using the methodologies introduced in paragraph 13.2:

- a) 10_{10} , 1010_2 b) 102_{10} , 1100110_2 c) 71_{10} , 1000111_2 d) 33_{10} , 100001_2 e) 19_{10} , 10011_2
-

Problem 13.3

Solution:**Known quantities:**

The base 10 representation of four numbers: 271.25_{10} , 53.375_{10} , 37.32_{10} , 54.27_{10} .

Find:

The binary representation for these numbers.

Analysis:

Using the methodologies introduced in paragraph 13.2:

- a) 100001111.01_2 b) 110101.011_2 c) 100101.01010_2 d) 110110.010001_2
-

Problem 13.4

Solution:**Known quantities:**

The binary representation of six numbers: 1111_2 , 1001101_2 , 1100101_2 , 1011100_2 , 11101_2 , 101000_2 .

Find:

The hex and the base 10 representation for these numbers.

Analysis:

Using the methodologies introduced in paragraph 13.2:

- a) F_{16} , 15_{10} b) $4D_{16}$, 77_{10} c) 65_{16} , 101_{10} d) $5C_{16}$, 92_{10} e) $1D_{16}$, 29_{10} f) 28_{16} , 40_{10}
-

Problem 13.5**Solution:****Known quantities:**

Three couples of binary numbers.

Find:

The addition for each couple.

Analysis:

Using the methodologies introduced in paragraph 13.2:

- a) 11111010 b) 100010100 c) 110000100
-

Problem 13.6**Solution:****Known quantities:**

Three couples of binary numbers.

Find:

The subtraction for each couple.

Analysis:

Using the methodologies introduced in paragraph 13.2:

- a) 11100 b) 1101110 c) 1000
-

Problem 13.7**Solution:****Known quantities:**

Three eight-bit binary numbers in sign-magnitude form.

Find:

The decimal value of these numbers.

Analysis:

Using the methodologies introduced in paragraph 13.2:

- a) -120 b) -31 c) 121
-

Problem 13.8**Solution:****Known quantities:**

Three decimal numbers.

Find:

The sign-magnitude form binary representation.

Analysis:

Using the methodologies introduced in paragraph 13.2:

- a) 01111110 b) 11111110 c) 01101100 d) 11100010
-

Problem 13.9**Solution:****Known quantities:**

Four binary numbers.

Find:

The two's complement of these four numbers.

Analysis:

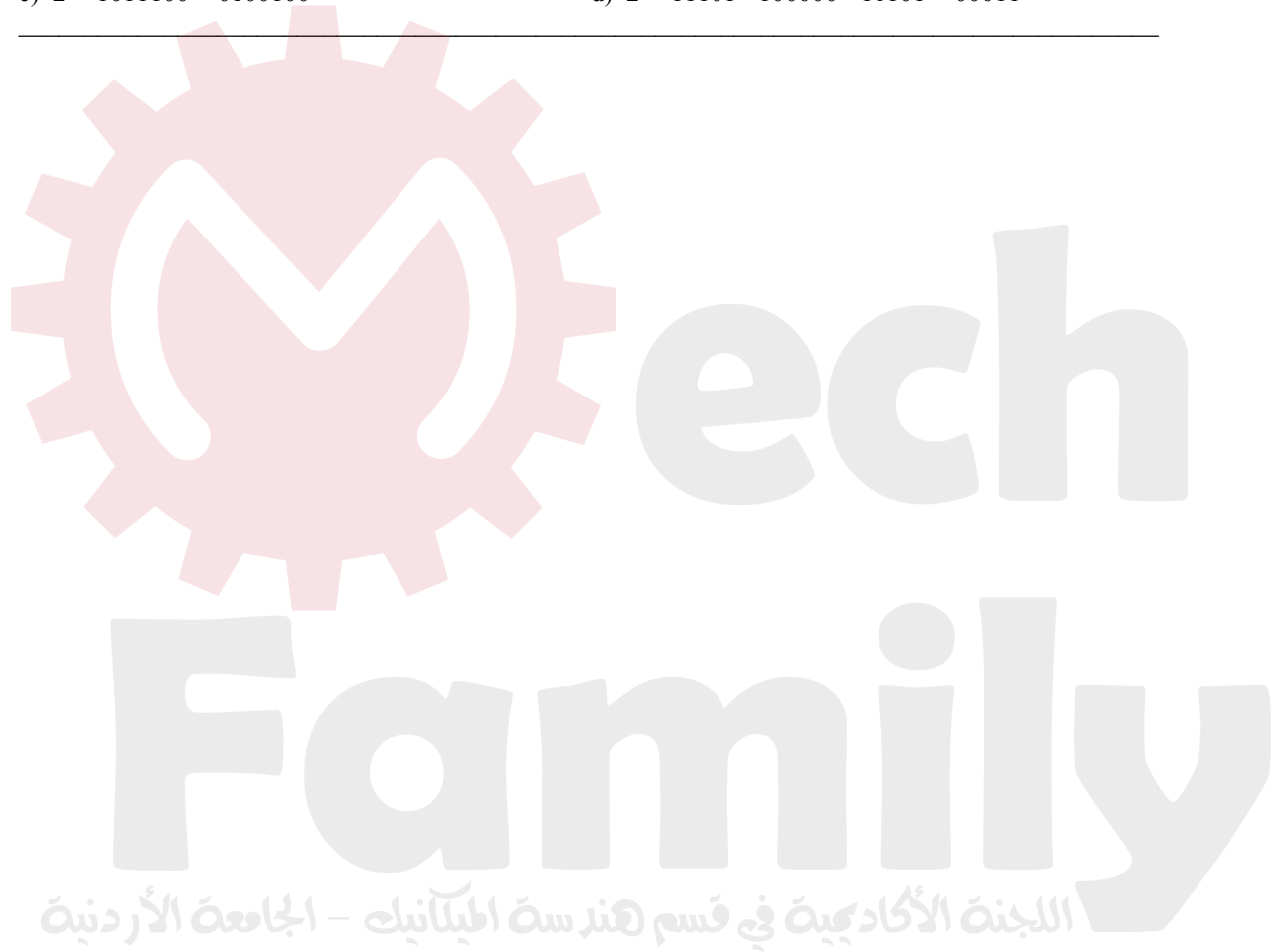
Using the methodologies introduced in paragraph 13.2:

a) $2^4 - 1111 = 10000 - 1111 = 0001$

b) $2^7 - 1001101 = 0110011$

c) $2^7 - 1011100 = 0100100$

d) $2^5 - 11101 = 100000 - 11101 = 00011$



Section 13.3: Boolean Algebra

Problem 13.10

Solution:

Known quantities:

The expression $B = AB + \bar{A}B$.

Find:

The truth table that proves that the expression is true.

Analysis:

Using a truth table as explained in paragraph 13.3:

A	B	$\bar{A}$	AB	$\bar{A}B$	$AB + \bar{A}B$
0	0	1	0	0	0
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	0	1

we prove that the expression is true.

Problem 13.11

Solution:

Known quantities:

The expression $BC + B\bar{C} + \bar{B}A = A + B$.

Find:

The truth table that proves that the expression is true.

Analysis:

Using a truth table as explained in paragraph 13.3:

A	B	C	BC	$B\bar{C}$	$\bar{B}A$	$BC + B\bar{C} + \bar{B}A$	$A + B$
1	1	1	1	0	0	1	1
1	1	0	0	1	0	1	1
1	0	1	0	0	1	1	1
1	0	0	0	0	1	1	1
0	1	1	1	0	0	1	1
0	1	0	0	1	0	1	1
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0

we prove that the expression is true.

Problem 13.12**Solution:****Known quantities:**

The expression $(X + Y) \cdot (\bar{X} + X \cdot Y) = Y$.

Find:

The proof that the expression is true using the perfect induction method.

Analysis:

Using a truth table as explained in paragraph 13.3:

X	Y	$X + Y$	$\bar{X}$	$X \cdot Y$	$\bar{X} + X \cdot Y$	$(X + Y) \cdot (\bar{X} + X \cdot Y)$
0	0	0	1	0	1	0
0	1	1	1	0	1	1
1	0	1	0	0	0	0
1	1	1	0	1	1	1

we prove that the expression is true.

Problem 13.13**Solution:****Known quantities:**

The logic function $F(X, Y, Z) = \bar{X} \cdot \bar{Y} \cdot \bar{Z} + \bar{X} \cdot Y \cdot Z + X \cdot (\bar{Y} + \bar{Z})$.

Find:

The simplification of the expression using the rules of Boolean algebra and De Morgan's theorems.

Analysis:

Applying De Morgan's theorems, $F(X, Y, Z) = \bar{X} \cdot \bar{Y} \cdot \bar{Z} + \bar{X} \cdot Y \cdot Z + X \cdot \bar{Y} \cdot \bar{Z}$

Applying the rules of Boolean algebra,

$$F(X, Y, Z) = \bar{X} \cdot \bar{Y} \cdot \bar{Z} + X \cdot \bar{Y} \cdot \bar{Z} + \bar{X} \cdot Y \cdot Z = \bar{Y} \cdot \bar{Z} + \bar{X} \cdot Y \cdot Z$$

Problem 13.14**Solution:****Known quantities:**

The logic function $f(A, B, C, D) = A \cdot B \cdot C + \bar{A} \cdot C \cdot D + \bar{B} \cdot C \cdot D$.

Find:

The simplification of the expression using the rules of Boolean algebra and De Morgan's theorems.

Analysis:

$$\begin{aligned} f(A, B, C, D) &= ABC + \bar{A}CD + \bar{B}CD \\ &= ABC + CD(\bar{A} + \bar{B}) \leftarrow \text{Distributive} \end{aligned}$$

Problem 13.15**Solution:****Known quantities:**

The logic function $F(A, B, C) = \bar{A} \cdot B \cdot \bar{C} + \bar{A} \cdot B \cdot C + A \cdot B \cdot \bar{C} + A \cdot B \cdot C$.

Find:

The simplification of the expression using the rules of Boolean algebra.

Analysis:

Applying the rules of Boolean algebra,

$$F(A, B, C) = \bar{A} \cdot B \cdot (\bar{C} + C) + A \cdot B \cdot (\bar{C} + C) = \bar{A} \cdot B + A \cdot B = (\bar{A} + A) \cdot B = B$$

Problem 13.16**Solution:****Known quantities:**

The truth table in Figure P13.16 for a logic function.

Find:

The expression for the logic function.

Analysis:

We first find the Karnaugh map for the function.

From the map, we can see that: $F = A + C$

		BC			
A		00	01	11	10
		0	1	1	0
	1	1	1	1	1

Problem 13.17**Solution:****Known quantities:**

The circuit of Figure P13.17.

Find:

The Boolean function describing the operation of the circuit.

Analysis:

$$F = \overline{AB \cdot CD \cdot E} = \overline{AB + CD + E}$$

where the second expression is a result of applying DeMorgan's theorem to the first.

Problem 13.18**Solution:****Known quantities:**

The circuit of Figure P13.18.

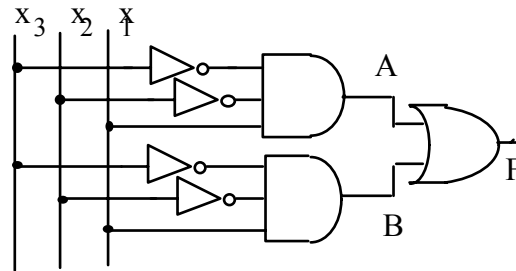
Find:

The truth table of the circuit.

Analysis:

The truth table can be obtained from the circuit considering the steps A, B and F, as reported in the figure of the circuit.

x3	x2	x1	A	B	F
0	0	0	0	0	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	0	0	0
1	1	1	0	0	0

**Problem 13.19****Solution:****Known quantities:**

The rules that have to be followed in order to implement a strategy able to decide when the steal sign has to be given, for a baseball team.

The steal sign has to be given if:

- there are no other runners, the pitcher is right-handed and the runner is fast, or
- there is no one other runner on third-base, and one of the runner is fast, or
- there is one other runner on second-base, the pitcher is left-handed, and both runners are fast.
- Under no circumstances should the steal sign be given if all three bases have runners.

Find:

The circuit that implements these rules.

Analysis:

The table below lists the 8 possible conditions under which the steal sign should be given, using the following logic notations:

A runner on a base is 1, no runner is 0

A fast runner on a base is 1, a non-fast runner is 0

A right-handed pitcher is 1, a left-handed pitcher is 0

Base			Runner			Pitcher	Output
b_1	b_2	b_3	f_1	f_2	f_3	P	Light On, y
1	0	0	1	0	0	1	1
1	0	1	1	0	0	0	1
1	0	1	0	1	0	0	1
1	0	1	1	1	1	0	1
1	0	1	1	0	1	1	1
1	0	1	0	1	1	1	1
1	0	1	1	1	0	1	1
1	1	0	1	0	0	0	1

For all other conditions, the output is off.

Note: The circles in the figure are nodes, not inversions.

Problem 13.20

Solution:

Known quantities:

The rules to pass a law in a small county.

Find:

The logic circuit that takes three votes as inputs and lights either a green or red light to indicate whether or not a measure passed.

Assumptions:

Green is enlightened if the law passes.

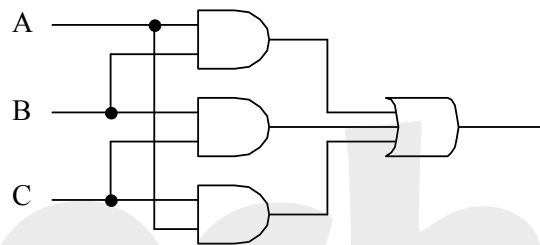
Analysis:

The function that implements the rules is:

$$f(A, B, C) = AB + BC + AC$$

If $f(A, B, C)$ is equal to 1 then the green light is lighted, otherwise the red one.

The circuit that implement this function is reported in the figure besides.



Problem 13.21

Solution:

Known quantities:

The set-up for a water purification plant.

Find:

The logic circuit that sounds an alarm if a dangerous situation occurs.

Analysis:

The four variables are the flow and the height of water in the tank A and in the tank B:

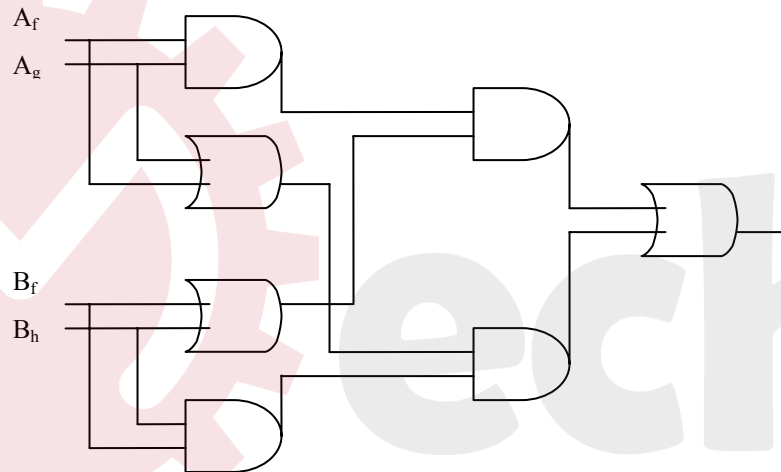
A_f, A_h, B_f, B_h .

From the Karnaugh map reported below we can find the minimum expression:

$$A_f A_h (B_f + B_h) + B_f B_h (A_f + A_h)$$

and the following realization of the circuit.

$A_f A_h$ \ $B_f B_h$	$B_f B_h$			
	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	0	1	0	1
10	0	0	1	0



Problem 13.22

Solution:

Known quantities:

The rules for an alert system designed for cars.

Find:

The logic circuit that implements these functions.

Analysis:

Let's design two different circuits for the buzzer and the starting conditions.

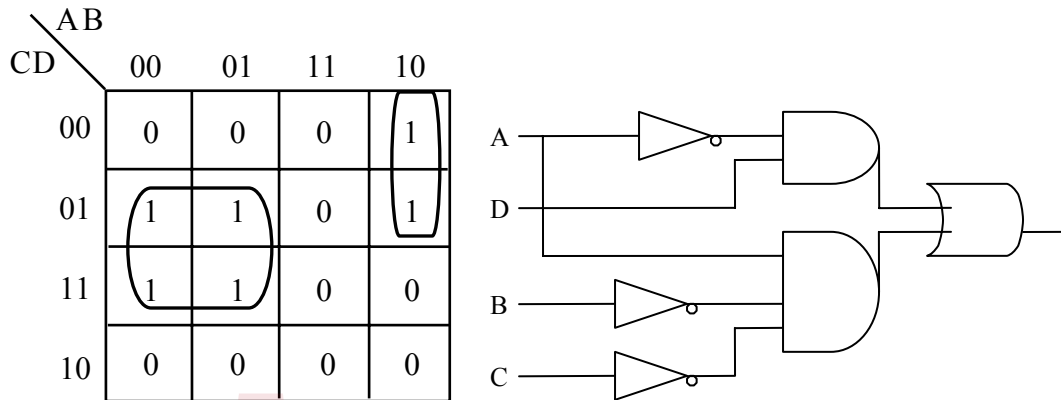
For both circuits we find first the Karnaugh map and then the minimum expression and the circuit to implement it.

A = Ignition key (1 if turned); B = Door (1 if closed); C = Seat belt (1 if fasten); D = Lights (1 if on);

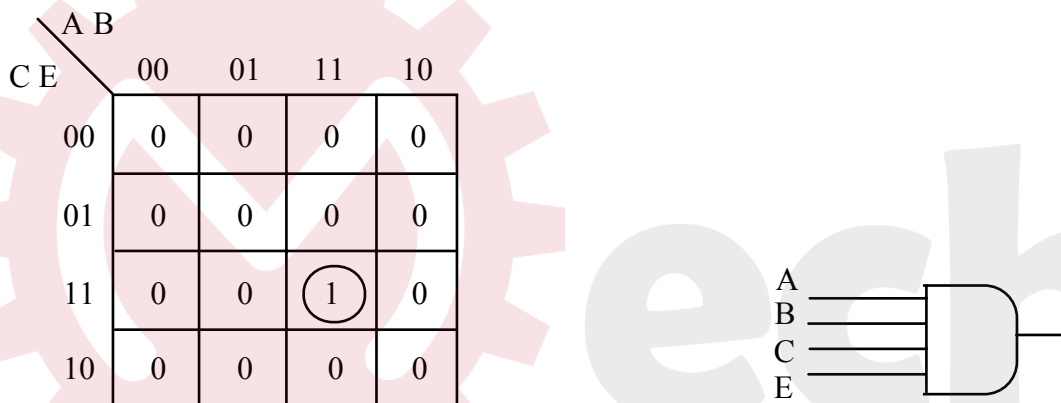
E = Park (1 if on);

For the first circuit:

$$\text{Buzzer} = \overline{A} \overline{B} \overline{C} + \overline{A} D$$



Start = ABCE



Problem 13.23

Solution:

Known quantities:

The on/off strategy for the compressor motor of a large commercial air conditioning unit.

Find:

The logic diagram that incorporates the state of four devices (S, D, T and M) and produces the correct on/off condition for the motor startup.

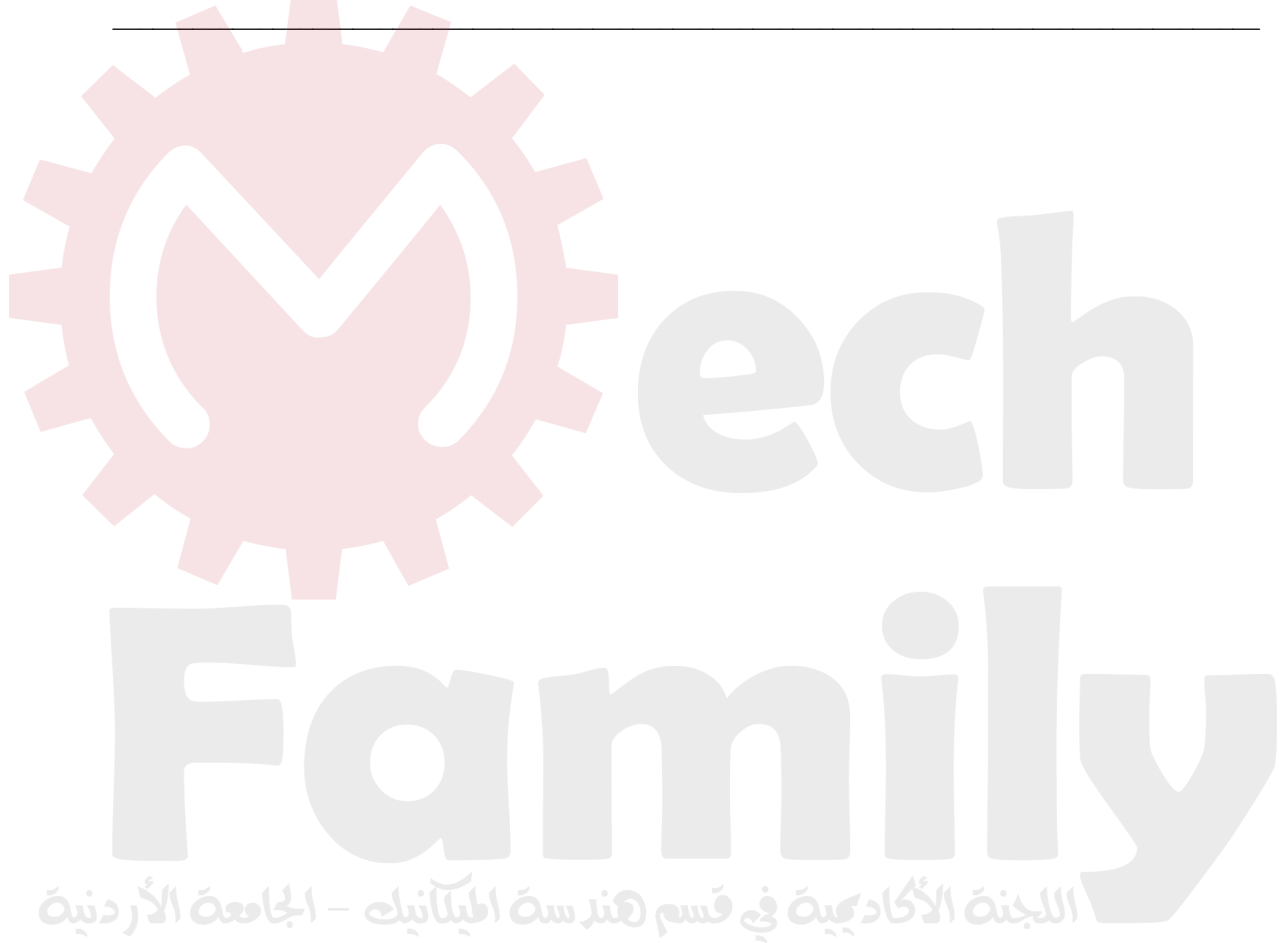
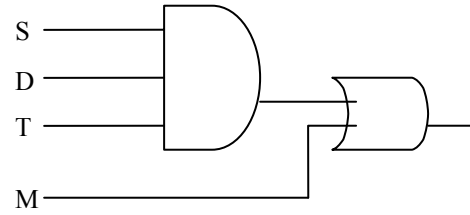
Analysis:

From the Karnaugh map we determine the minimum expression:

$$SDT + M$$

that is implemented by the following logic diagram.

S D T M		00	01	11	10
		00	01	11	10
00		0	0	0	0
01		1	1	1	1
11		1	1	1	1
10		0	0	1	0



Section 13.4: Karnaugh Maps and Logic Design

Focus on Methodology: Sum-of-Products Realizations

1. Begin with isolated cells. These must be used as they are, since no simplification is possible.
2. Find all cells that are adjacent to only one cell, forming two-cell subcubes.
3. Find cells that form four-cell subcubes, eight-cell subcubes, and so forth.
4. The minimal expression is formed by the collection of the *smallest number of maximal subcubes*.

Focus on Methodology: Products-of Sums Realizations

1. Solve for the 0s exactly as for the 1s in the sum-of-products expressions.
2. Complement the resulting expression.

Problem 13.24

Solution:

Known quantities:

The truth table of Figure P13.24.

Find:

The logic function corresponding to the truth table.

Analysis:

From the map, we can see that:

$$F = A \cdot B + \bar{B} \cdot \bar{C}$$

BC		00	01	11	10
A	0	1	0	0	0
	1	1	0	1	1

Problem 13.25

Solution:

Known quantities:

The circuit shown in Figure P13.25.

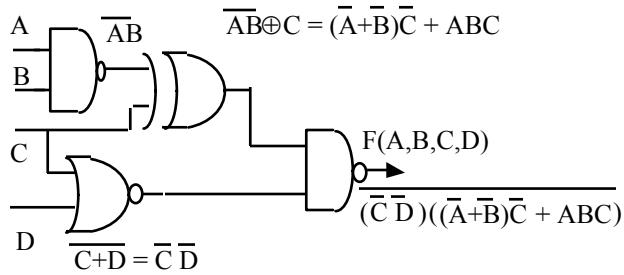
Find:

The minimum expression for the output.

Analysis:

The logic circuit gives the following expression:

$$\begin{aligned}
 F &= (\overline{C+D})(\overline{A \cdot B \oplus C}) = (\bar{C} \cdot \bar{D})(\overline{A \cdot B + C}) \cdot (\overline{A \cdot B \cdot C}) = (\bar{C} \cdot \bar{D})(\overline{A \cdot B + C}) \cdot (A \cdot B \cdot \bar{C}) = \\
 &= (\bar{C} \cdot \bar{D})(A \cdot B \cdot C + \overline{A \cdot B} \cdot \bar{C}) = (\bar{C} \cdot \bar{D})(A \cdot B \cdot C + (\bar{A} + \bar{B}) \cdot \bar{C})
 \end{aligned}$$



Problem 13.26

Solution:

Known quantities:

The logic function $f(A, B, C) = A \cdot B \cdot C + A \cdot B \cdot \overline{C} + A \cdot \overline{B} \cdot \overline{C}$.

Find:

The minimum expression for the function.

Analysis:

Using a Karnaugh map (reported besides) we determine:

$$f = AB + A\overline{C}$$

BC \ A	A	
	0	1
00	0	1
01	0	0
11	0	1
10	0	1

Problem 13.27

Solution:

Known quantities:

The truth table of Figure P13.27.

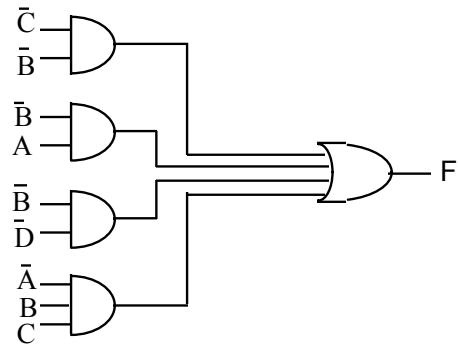
Find:

- The Karnaugh map for the logic function defined by the truth table.
- The minimum expression for the function.
- The realization of the function using AND, OR and NOT gates.

Analysis:

- The Karnaugh map is shown below.

	AB			
CD	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	1	0	1
10	1	1	0	1



b) The map leads to the expression:

$$f = \bar{B} \cdot \bar{C} + A \cdot \bar{B} + \bar{A} \cdot B \cdot C + \bar{B} \cdot D$$

c) and to the gate realization shown above.

Problem 13.28

Solution:

Known quantities:

The truth table of Figure P13.28.

Find:

The Karnaugh map and the minimum expression for the function defined by the truth table.

Analysis:

The Karnaugh map is shown besides.

	BC			
A	00	01	11	10
0	0	1	0	1
1	1	0	1	0

The minimum expression is reported below.

$$f(A, B, C) = \bar{A} \cdot \bar{B} \cdot C + \bar{A} \cdot B \cdot \bar{C} + A \cdot \bar{B} \cdot \bar{C} + A \cdot B \cdot C$$

Problem 13.29

Solution:

Known quantities:

The rules that define a logic function.

Find:

The minimum expression for this function and a sketch of a circuit to implement this function using only AND and NOT gates.

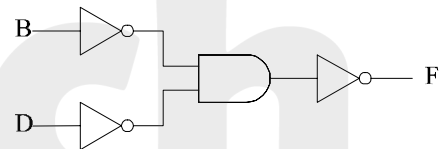
Analysis:

The truth table for the function is reported below as well as the Karnaugh map the minimum expression for the function and its realization using AND and NOT gates.

A	B	C	D	F
0	0	0	0	0
0	0	0	1	X
0	0	1	0	0
0	0	1	1	1
0	1	0	0	X
0	1	0	1	X
0	1	1	0	1
0	1	1	1	X
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	X
1	1	0	0	1
1	1	0	1	X
1	1	1	0	X
1	1	1	1	1

AB \ CD	00	01	11	10
00	0	X	1	0
01	X	X	X	1
11	1	X	1	X
10	0	1	X	0

$$F = B + D$$



Problem 13.30

Solution:

Known quantities:

The truth table that defines a logic function shown in Figure P13.30.

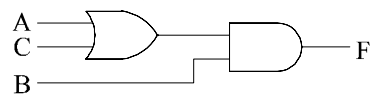
Find:

The construction of a logic circuit describing the function using only two gates.

Analysis:

$$\text{SOP: } F = AB + BC \Rightarrow 3 \text{ gates}$$

$$\text{POS: } \overline{F} = \overline{B} + \overline{A} \cdot \overline{C} \Rightarrow F = B(A + C) \Rightarrow 2 \text{ gates}$$



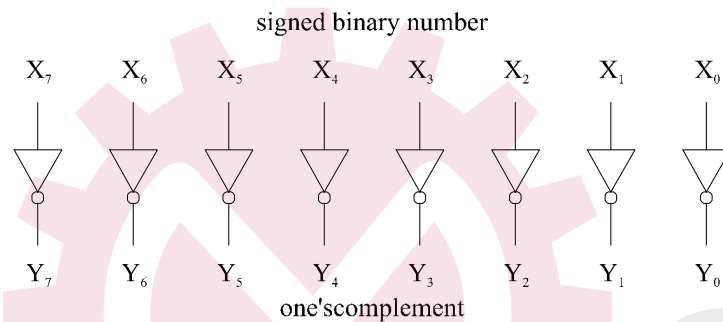
BC \ A	0	1
00	0	0
01	0	0
11	1	X
10	0	1

Problem 13.31**Solution:****Known quantities:**

The requirement for the circuit to be designed: to produce the one's complement of an 8-bit signed binary number.

Find:

The design of the logic circuit.

Analysis:**Problem 13.32****Solution:****Known quantities:**

The truth table of Figure P13.32.

Find:

The Karnaugh map and the minimum expression for the logic function.

Analysis:

The Karnaugh map and the expression for the function are reported below.

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	1	0
11	1	1	0	1
10	1	0	0	1

$$F = \bar{B} \cdot \bar{D} + A \cdot \bar{D} + A \cdot B \cdot \bar{C} + \bar{A} \cdot B \cdot C \cdot D$$

Problem 13.33**Solution:****Known quantities:**

The requirement for the circuit to be designed: to produce the two's complement of an 8-bit signed binary number.

Find:

The design of the logic circuit.

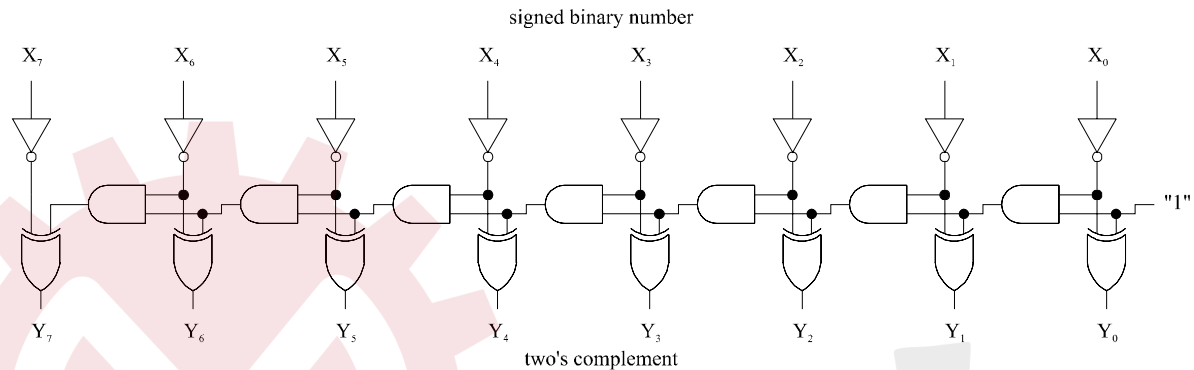
Analysis:

The two's complement is the one's complement plus one.

X	C _{IN}	SUM	C _{OUT}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$SUM = X \oplus C_{IN}$$

$$C_{OUT} = X \cdot C_{IN}$$



Problem 13.34

Solution:

Known quantities:

The circuit of Figure P13.34.

Find:

The minimum output expression for the circuit.

Analysis:

$$f = A \cdot \overline{B} + B + \overline{C} = \overline{A \cdot \overline{B} \cdot B + \overline{C}} = \overline{A \cdot \overline{B} \cdot B + \overline{C}} = A \cdot B \cdot (B + C) = A \cdot B(1 + C) = A \cdot B$$

Problem 13.35

Solution:

Known quantities:

The combinational logic to be designed: the addition between two 4-bit binary numbers.

Find:

The circuit to implement this operation.

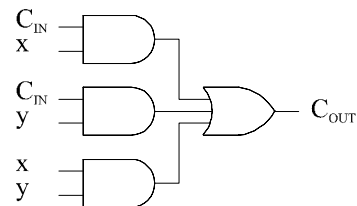
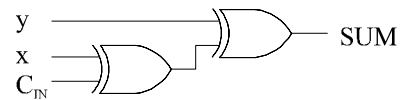
Analysis:

A one-bit adder truth table is as follows, and from this table we find the following expression, and the two circuits for those functions:

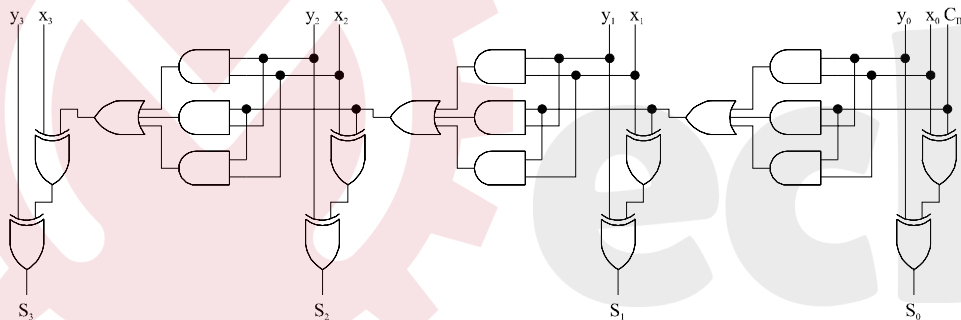
C_{IN}	x	y	SUM	C_{OUT}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$SUM = C_{IN} \oplus x \oplus y$$

$$C_{OUT} = C_{IN} + xy$$



The complete 4-bit adder can be constructed as shown below:



Note that this circuit assumes a carry-in for the lsb. If this is not necessary, then the circuit can be reduced correspondingly.

Problem 13.36

Solution:

Known quantities:

The truth table of Figure P13.36.

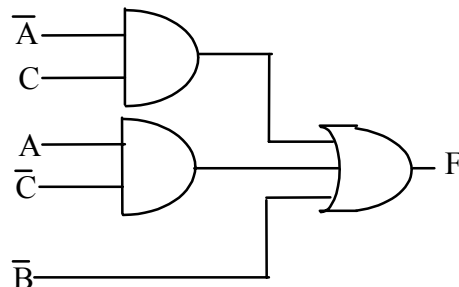
Find:

The minimum output expression and the circuit for the function.

Analysis:

Reported below are the Karnaugh map, the expression for the function and circuit realization.

BC					
A		00	01	11	10
0		1	1	1	0
1		1	1	0	1



$$F = \overline{B} + A \cdot \overline{C} + \overline{A} \cdot C$$

Problem 13.37

Solution:

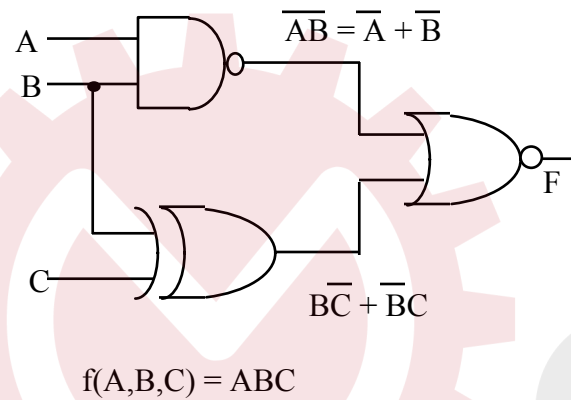
Known quantities:

The logic circuit of Figure P13.37.

Find:

The minimum output expression for the circuit.

Analysis:



Problem 13.38

Solution:

Known quantities:

The rules for blood donation.

Find:

The circuit which will approve or disapprove any particular transfusion between donator and receiver.

Analysis:

Let WX represent a 2-bit code for the donor blood type, and let yz represent a 2-bit code for the recipient blood type. Then WXYZ will represent a donor-recipient pair. Let F be true if a transfusion can be made. Blood type codes may be assigned as follows:

	WX	YZ
A	00	00
B	01	01
AB	10	10
O	11	11

The truth table is:

Donor–Recipient	WX	YZ	F
A-A	00	00	1
A-B	00	01	0
A-AB	00	10	1
A-O	00	11	0
B-A	01	00	0
B-B	01	01	1
B-AB	01	10	1
B-O	01	11	0
AB-A	10	00	0
AB-B	10	01	0
AB-AB	10	10	1
AB-O	10	11	0
O-A	11	00	1
O-B	11	01	1
O-AB	11	10	1
O-O	11	11	1

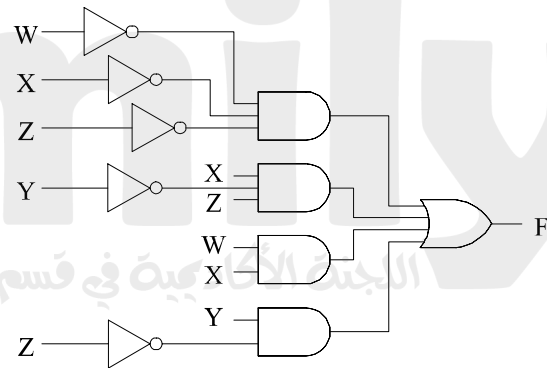
And the Karnaugh map, then, is as follows:

		WX			
		00	01	11	10
YZ	00	1	0	1	0
	01	0	1	1	0
	11	0	0	1	0
	10	1	1	1	1

From the Karnaugh map,

$$F = \overline{W} \cdot \overline{X} \cdot \overline{Z} + X \cdot \overline{Y} \cdot Z + WX + Y\overline{Z}$$

and the resulting circuit is shown above.



Problem 13.39**Solution:****Known quantities:**

The logic circuit of Figure P13.39.

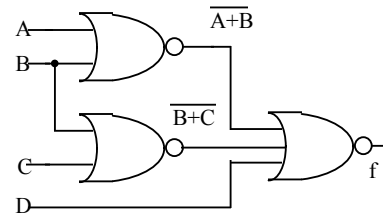
Find:

The minimum expression for the logic function.

Analysis:

The minimum expression is:

$$\begin{aligned} F &= \overline{A+B} + \overline{B+C} + D = (A+B) \cdot (B+C) \cdot \overline{D} \\ &= (B + A \cdot C) \cdot \overline{D} = B \cdot \overline{D} + A \cdot C \cdot \overline{D} \end{aligned}$$

**Problem 13.40****Solution:****Known quantities:**

The combinational logic that have to be followed.

Find:

The circuit that implements this logic.

Analysis:

The appropriate truth table can be constructed as follows:

A B C D	F4	F3	F2	F1	F0
0 0 0 0	0	0	0	0	0
0 0 0 1	0	0	0	1	0
0 0 1 0	0	0	0	0	1
0 0 1 1	0	0	1	1	0
0 1 0 0	0	0	0	1	0
0 1 0 1	0	1	0	1	0
0 1 1 0	0	0	0	1	1
0 1 1 1	0	1	1	1	0
1 0 0 0	0	0	1	0	0
1 0 0 1	1	0	0	1	0
1 0 1 0	0	0	1	0	1
1 0 1 1	1	0	1	1	0
1 1 0 0	0	0	1	1	0
1 1 0 1	1	1	0	1	0
1 1 1 0	0	0	1	1	1
1 1 1 1	1	1	1	1	0

Next, we construct a Karnaugh map for each bit of the output, and determine its corresponding function.

		F ₄			
CD	AB	00	01	11	10
00		0	0	0	0
01		0	0	1	1
11		0	0	1	1
10		0	0	0	0

$$F_4 = AD$$

		F ₃			
CD	AB	00	01	11	10
00		0	0	0	0
01		0	1	1	0
11		0	1	1	0
10		0	0	0	0

$$F_3 = BC$$

		F ₂			
CD	AB	00	01	11	10
00		0	0	1	1
01		0	0	0	0
11		1	1	1	1
10		0	0	1	1

$$F_2 = AB + A\bar{D}$$

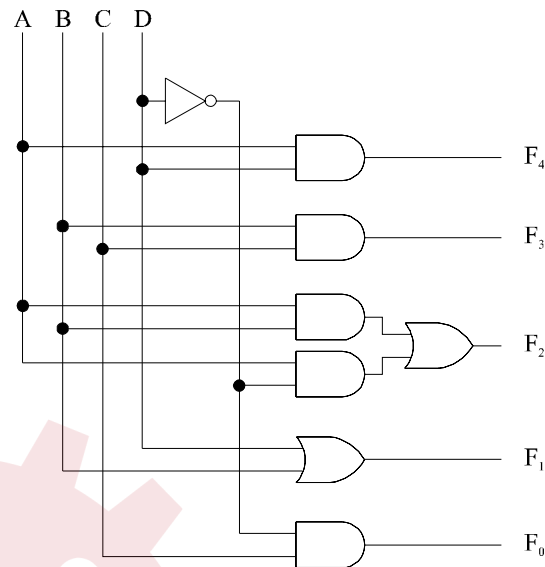
		F ₁			
CD	AB	00	01	11	10
00		0	1	1	0
01		1	1	1	1
11		1	1	1	1
10		0	1	1	0

$$F_1 = B + D$$

		F ₀			
CD	AB	00	01	11	10
00		0	0	0	0
01		0	0	0	0
11		0	0	0	0
10		1	1	1	1

$$F_0 = C\bar{D}$$

This completes the design.



Problem 13.41

Solution:

Known quantities:

The truth table of Figure P13.41.

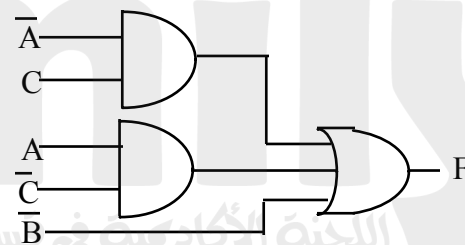
Find:

- The Karnaugh map for the function defined in the truth table.
- The minimum expression for the function.
- The circuit using AND, OR, NOT gates.

Analysis:

The answers are reported in the following plots.

		BC			
		00	01	11	10
A	0	1	1	1	0
	1	1	1	0	1



$$f = \bar{B} + \bar{A}C + A\bar{C}$$

Problem 13.42

Solution:

Known quantities:

The truth table of Figure P13.42.

Find:

- The Karnaugh map for the function defined in the truth table.
- The minimum expression for the function.

Analysis:

The answers are reported in the following plots.

a)

AB \ CD				
	00	01	11	10
00	1	0	0	1
01	0	0	1	0
11	1	1	0	1
10	1	0	0	1

b)

$$F = \bar{B} \cdot \bar{D} + A \cdot \bar{D} + A \cdot B \cdot \bar{C} + \bar{A} \cdot B \cdot C \cdot D$$

Problem 13.43**Solution:****Known quantities:**

The truth table of Figure P13.43.

Find:

- The Karnaugh map for the function defined in the truth table.
- The minimum expression for the function.
- The circuit implementation using only NAND gates.

Analysis:

The answers are reported in the following plots.

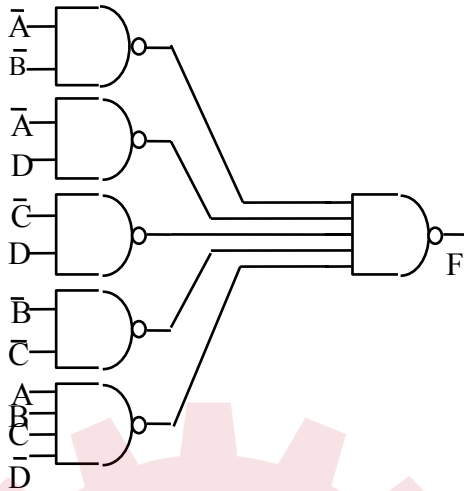
a)

AB \ CD				
	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	1	0	1
10	1	1	0	0

b)

$$F = \bar{A} \cdot \bar{B} + \bar{A} \cdot D + \bar{C} \cdot D + \bar{B} \cdot \bar{C} + A \cdot B \cdot C \cdot \bar{D}$$

c)



Problem 13.44

Solution:

Known quantities:

The logic that defines a function.

Find:

- The Karnaugh map for the function and the truth table.
- The minimum expression for the function.
- The circuit using only AND, OR, and NOT gates.

Analysis:

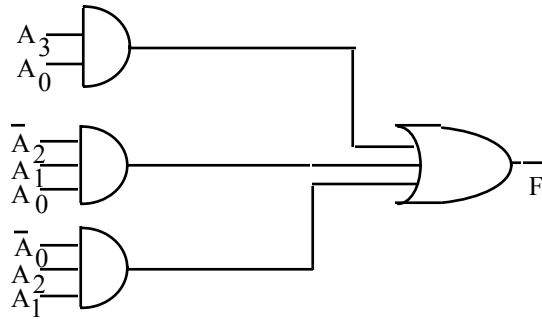
a)

A3	A2	A1	A0	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	d
1	0	1	1	d
1	1	0	0	d
1	1	0	1	d
1	1	1	0	d
1	1	1	1	d

		A ₁ A ₀			
A ₃ A ₂		00	01	11	10
		00	01	11	10
00		0	0	1	0
01		0	0	0	1
11		d	d	d	d
10		0	1	0	d

b) From K-Map groupings, $F = A_3 \cdot A_0 + A_2 \cdot A_1 \cdot \overline{A_0} + A_1 \cdot A_0 \cdot \overline{A_2}$

c) The circuit for this function is:



Problem 13.45

Solution:

Known quantities:

The Karnaugh map reported in Figure P13.45.

Find:

The simplified sum-of-products representation of the function.

Analysis:

AB \ CD		AB			
		00	01	11	10
CD	00	0	1	0	0
	01	1	1	0	0
	11	0	x	1	0
	10	0	0	1	0

$$F = \bar{A} \cdot \bar{C} \cdot D + \bar{A} \cdot B \cdot \bar{C} + A \cdot B \cdot C$$

Problem 13.46

Solution:

Known quantities:

The simplification for the circuit of Problem 13.40 if it is known that the input represents a BCD (binary-coded decimal) number.

Find:

The simplified circuit or the reason for which it is not possible to simplify the circuit.

Analysis:

For this problem, the truth table is as follows:

A B C D	F ₄	F ₃	F ₂	F ₁	F ₀
0 0 0 0	0	0	0	0	0
0 0 0 1	0	0	0	1	0
0 0 1 0	0	0	0	0	1
0 0 1 1	0	0	1	1	0
0 1 0 0	0	0	0	1	0
0 1 0 1	0	1	0	1	0
0 1 1 0	0	0	0	1	1
0 1 1 1	0	1	1	1	0
1 0 0 0	0	0	1	0	0
1 0 0 1	1	0	0	1	0
1 0 1 0	X	X	X	X	X
1 0 1 1	X	X	X	X	X
1 1 0 0	X	X	X	X	X
1 1 0 1	X	X	X	X	X
1 1 1 0	X	X	X	X	X
1 1 1 1	X	X	X	X	X

Now, we construct the Karnaugh maps and determine the corresponding functions.

F₄

AB \ CD	00	01	11	10
00	0	0	X	0
01	0	0	X	1
11	0	0	X	X
10	0	0	X	X

$$F_4 = AD$$

F₃

AB \ CD	00	01	11	10
00	0	0	X	0
01	0	1	X	0
11	0	1	X	X
10	0	0	X	X

$$F_3 = BC$$

F₂

AB \ CD	00	01	11	10
00	0	0	X	1
01	0	0	X	0
11	1	1	X	X
10	0	0	X	X

$$F_2 = AB + \overline{AD}$$

F₁

AB \ CD	00	01	11	10
00	0	1	X	0
01	1	1	X	1
11	1	1	X	X
10	0	1	X	X

$$F_1 = B + D$$

		F_0			
CD	AB	00	01	11	10
00		0	0	X	0
01		0	0	X	0
11		0	0	X	X
10		1	1	X	X

$$F_0 = C\bar{D}$$

These expressions are identical to those obtained in Problem 12.36. Surprisingly, the presence of the don't cares did not change (or simplify) the solution.

Problem 13.47

Solution:

Known quantities:

The Karnaugh map shown in Figure P13.47.

Find:

The simplified sum-of-product representation of the function.

Analysis:

CD	AB	00	01	11	10
00		0	1	x	0
01		0	1	x	0
11		0	1	0	1
10		x	x	1	0

$$F = A \cdot \bar{B} \cdot C \cdot D + B \cdot \bar{D} + \bar{A} \cdot B$$

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Problem 13.48**Solution:****Known quantities:**

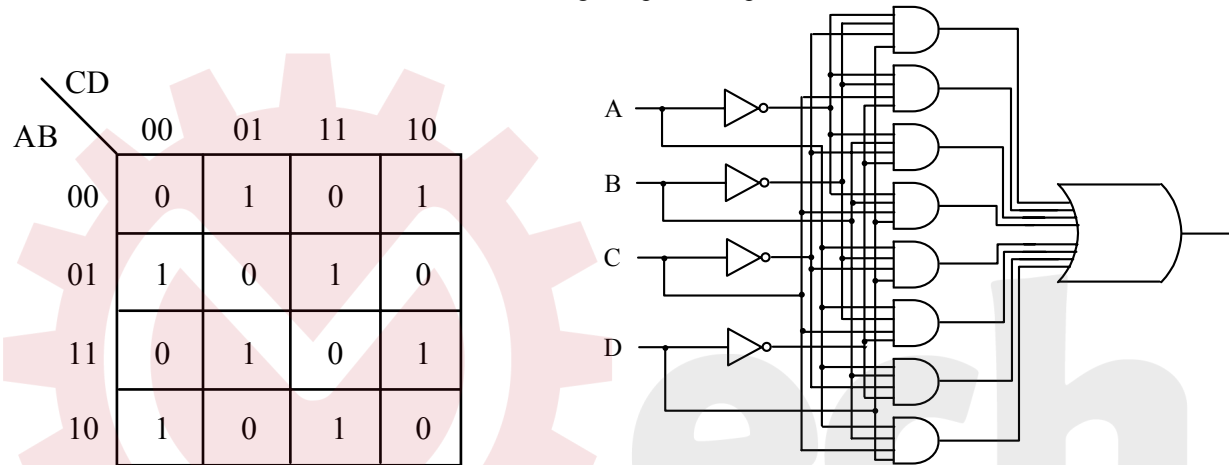
The parity bit method to ensure reliability in data transmission systems.

Find:

The logic circuit that checks the nibble of data and transmits the proper parity bit for even-parity systems.

Analysis:

If we write the nibble in the form ABCD, the Karnaugh map for this problem is:



Hence the function that gives the parity bit can be written as:

$$F = \overline{A}BCD + A\overline{B}CD + AB\overline{C}D + ABC\overline{D} + \overline{A}BC\overline{D} + A\overline{B}C\overline{D} + AB\overline{C}\overline{D} + ABC\overline{D}$$

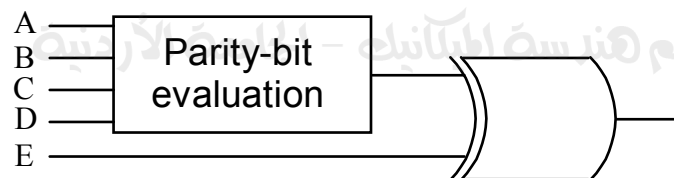
The logic circuit is shown above.

Problem 13.49**Solution:****Find:**

The logic circuit that check a nibble of data and its parity-bit.

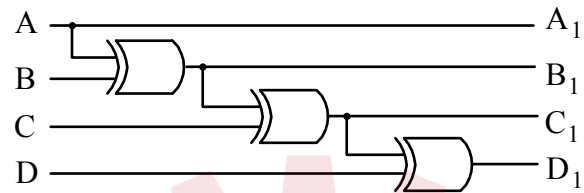
Analysis:

If we write the nibble in the form ABCD and E is the parity-bit, the logic circuit is:



Problem 13.50**Solution:****Find:**

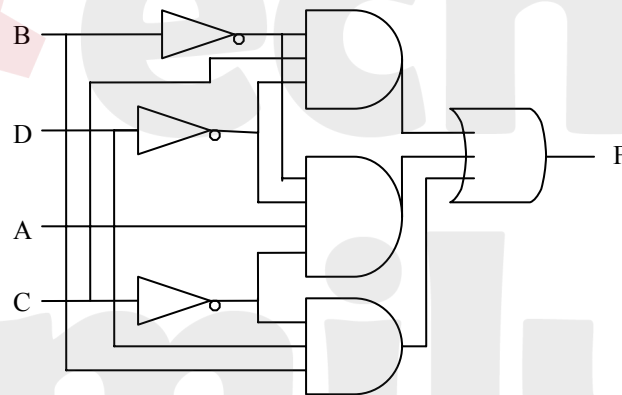
A logic circuit that takes a 4-bit gray code input into a 4-bit nibble of BCD code.

Analysis:**Problem 13.51****Solution:****Find:**

A logic circuit that takes a 4-bit gray-code

Analysis:

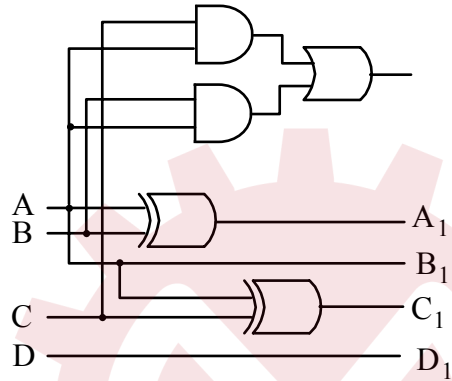
AB \ CD	00	01	11	10
00	0	0	0	1
01	0	1	1	0
11	0	0	0	0
10	1	0	0	1



$$F = \overline{B}C\overline{D} + A\overline{B}C\overline{D} + B\overline{C}D$$

Problem 13.52**Solution:****Find:**

A logic circuit that takes a BCD nibble as input and converts it into its 4221 equivalent, and reports an error if the BCD value exceeds 1001.

Analysis:**Problem 13.53****Solution:****Find:**

A logic circuit that reports an error if the 4-bit outputs of two sensors differ by more than one part per 30 seconds period.

Analysis:

The output of the logic circuit will be one when a difference greater or equal to 2 is detected between the two sensor outputs.

By defining with a_1, a_2, a_3, a_4 the four bits of the first sensor, and with b_1, b_2, b_3, b_4 the bits of the second sensor, the following Karnaugh maps can be derived.

From the maps, considering the zeros, the following expression can be obtained

$$\begin{aligned}
 y = & (a_1 + a_2 + a_3 + b_1 + b_2 + b_3)(a_1 + a_2 + \bar{a}_3 + b_1 + b_2 + \bar{b}_3)(a_1 + \bar{a}_2 + \bar{a}_3 + b_1 + \bar{b}_2 + \bar{b}_3) \\
 & (a_1 + \bar{a}_2 + a_3 + b_1 + \bar{b}_2 + b_3)(\bar{a}_1 + \bar{a}_2 + a_3 + \bar{b}_1 + \bar{b}_2 + b_3)(\bar{a}_1 + \bar{a}_2 + \bar{a}_3 + \bar{b}_1 + \bar{b}_2 + \bar{b}_3) \\
 & (\bar{a}_1 + a_2 + \bar{a}_3 + \bar{b}_1 + b_2 + \bar{b}_3)(\bar{a}_1 + a_2 + a_3 + \bar{b}_1 + b_2 + b_3)(a_1 + a_2 + \bar{a}_3 + a_4 + b_1 + b_2 + \bar{b}_4) \\
 & (a_1 + a_2 + \bar{a}_4 + b_1 + b_2 + \bar{b}_3 + b_4)(a_1 + \bar{a}_2 + \bar{a}_4 + b_1 + \bar{b}_2 + \bar{b}_3 + b_4)(a_1 + \bar{a}_2 + \bar{a}_3 + a_4 + b_1 + \bar{b}_2 + \bar{b}_4) \\
 & (\bar{a}_1 + a_2 + \bar{a}_3 + a_4 + \bar{b}_1 + b_2 + \bar{b}_4)(\bar{a}_1 + a_2 + \bar{a}_4 + \bar{b}_1 + b_2 + \bar{b}_3 + b_4)(\bar{a}_1 + \bar{a}_2 + \bar{a}_4 + \bar{b}_1 + \bar{b}_2 + \bar{b}_3 + b_4) \\
 & (\bar{a}_1 + \bar{a}_2 + \bar{a}_3 + a_4 + \bar{b}_1 + \bar{b}_2 + \bar{b}_4)(a_1 + \bar{a}_2 + a_3 + a_4 + b_1 + b_2 + \bar{b}_3 + \bar{b}_4)(a_1 + a_2 + \bar{a}_3 + \bar{a}_4 + b_1 + \bar{b}_2 + b_3 + b_4) \\
 & (\bar{a}_1 + \bar{a}_2 + a_3 + a_4 + \bar{b}_1 + b_2 + \bar{b}_3 + \bar{b}_4)(\bar{a}_1 + a_2 + \bar{a}_3 + \bar{a}_4 + \bar{b}_1 + \bar{b}_2 + b_3 + b_4) \\
 & (a_1 + \bar{a}_2 + \bar{a}_3 + \bar{a}_4 + \bar{b}_1 + b_2 + b_3 + b_4)(\bar{a}_1 + a_2 + a_3 + a_4 + b_1 + \bar{b}_2 + \bar{b}_3 + \bar{b}_4)
 \end{aligned}$$

$a_1 = b_1 = 0$

a_2	a_3	a_4	b_2	b_3	b_4	000	001	011	010	110	111	101	100
000	0	0	0	0	0	0	0						
001	0	0	0	0	0			0					
011			0	0	0								0
010			0	0	0								
110			0	0	0								
111			0	0	0								
101			0	0	0								
100			0	0	0								

$a_1 = 0, b_1 = 1$

a_2	a_3	a_4	b_2	b_3	b_4	000	001	011	010	110	111	101	100
000											0		
001													
011													
010													
110													
111													
101													
100													

$a_1 = 1, b_1 = 0$

a_2	a_3	a_4	b_2	b_3	b_4	000	001	011	010	110	111	101	100
000													
001													
011													
010													
110													
111	0												
101													
100													

$a_1 = b_1 = 1$

a_2	a_3	a_4	b_2	b_3	b_4	000	001	011	010	110	111	101	100
000	0	0	0	0	0	0	0						
001	0	0	0	0	0			0					
011			0	0	0								0
010			0	0	0								
110			0	0	0								
111			0	0	0								
101			0	0	0								
100			0	0	0								

The expression can be simplified as follows

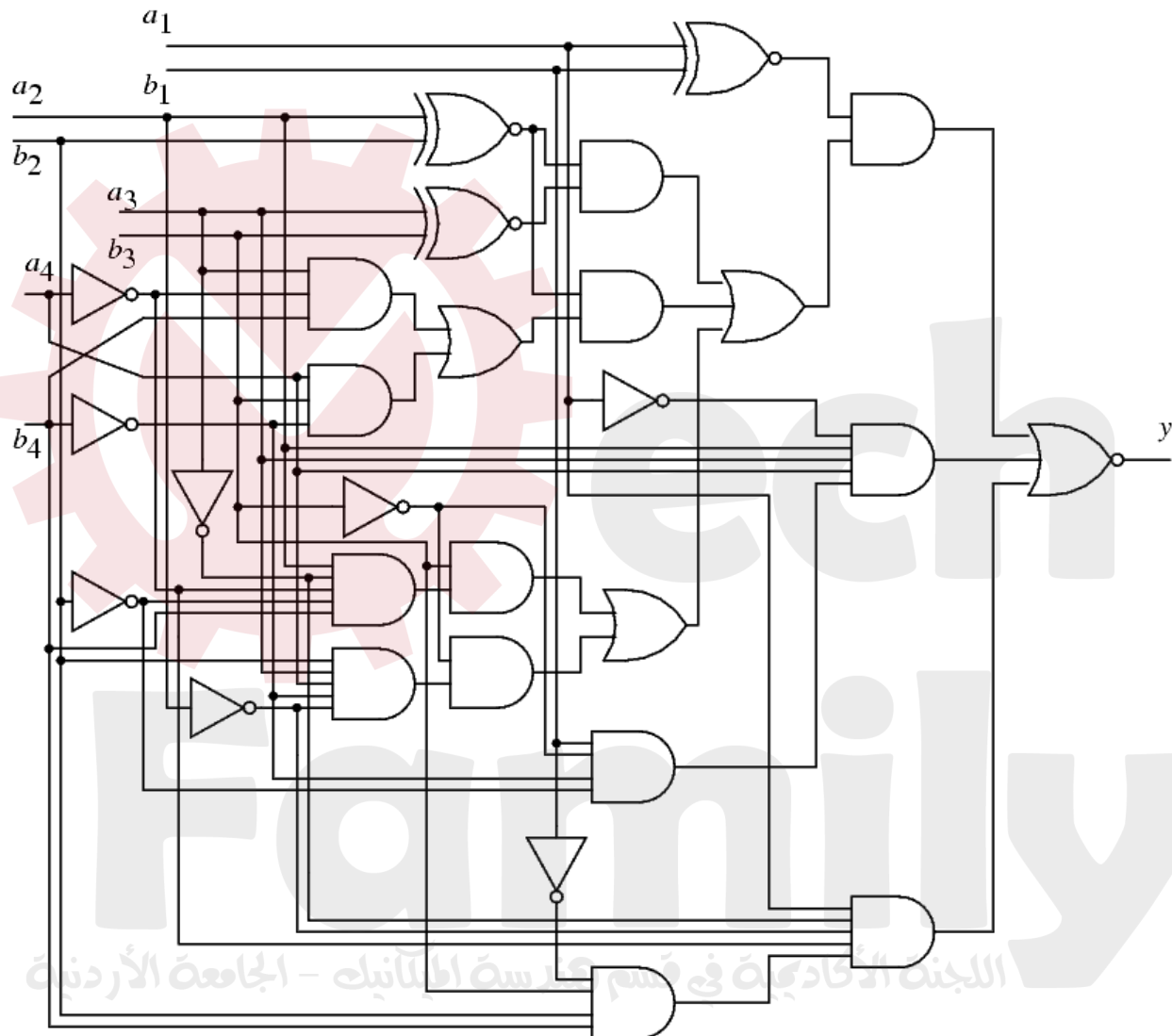
$$\begin{aligned}
 y &= (a_1 \oplus b_1 + a_2 + a_3 + b_2 + b_3)(a_1 \oplus b_1 + a_2 + \bar{a}_3 + b_2 + \bar{b}_3)(a_1 \oplus b_1 + \bar{a}_2 + \bar{a}_3 + \bar{b}_2 + \bar{b}_3) \\
 &= (a_1 \oplus b_1 + \bar{a}_2 + \bar{a}_3 + \bar{b}_2 + \bar{b}_3)(a_1 \oplus b_1 + a_2 + \bar{a}_3 + a_4 + b_2 + \bar{b}_4)(a_1 \oplus b_1 + a_2 + \bar{a}_4 + b_2 + \bar{b}_3 + b_4) \\
 &= (a_1 \oplus b_1 + \bar{a}_2 + \bar{a}_4 + \bar{b}_2 + \bar{b}_3 + b_4)(a_1 \oplus b_1 + \bar{a}_2 + \bar{a}_3 + a_4 + \bar{b}_2 + \bar{b}_4)(a_1 \oplus b_1 + \bar{a}_2 + a_3 + a_4 + b_2 + \bar{b}_3 + \bar{b}_4) \\
 &= (a_1 \oplus b_1 + a_2 + \bar{a}_3 + \bar{a}_4 + \bar{b}_2 + b_3 + b_4)(a_1 + \bar{a}_2 + \bar{a}_3 + \bar{a}_4 + \bar{b}_1 + b_2 + b_3 + b_4)(\bar{a}_1 + a_2 + a_3 + a_4 + b_1 + \bar{b}_2 + \bar{b}_3 + \bar{b}_4)
 \end{aligned}$$

Using De Morgan's Theorem

$$y = (\overline{a_1 \otimes b_1})(\overline{\bar{a}_2 \bar{a}_3 \bar{b}_2 \bar{b}_3} + \overline{\bar{a}_2 a_3 \bar{b}_2 b_3} + \overline{a_2 a_3 b_2 b_3} + \overline{a_2 \bar{a}_3 b_2 \bar{b}_3} + \overline{\bar{a}_2 a_3 \bar{a}_4 \bar{b}_2 b_4} + \overline{\bar{a}_2 a_4 \bar{b}_2 b_3 \bar{b}_4} + \overline{a_2 a_4 b_2 b_3 \bar{b}_4} + \overline{a_2 a_4 \bar{b}_2 \bar{b}_3 \bar{b}_4})$$

$$\begin{aligned}
& + a_2 a_3 \bar{a}_4 b_2 b_4 + a_2 \bar{a}_3 \bar{a}_4 \bar{b}_2 b_3 b_4 + \bar{a}_2 a_3 a_4 b_2 \bar{b}_3 \bar{b}_4) + \bar{a}_1 a_2 a_3 a_4 b_1 \bar{b}_2 \bar{b}_3 \bar{b}_4 + a_1 \bar{a}_2 \bar{a}_3 \bar{a}_4 \bar{b}_1 b_2 b_3 b_4 \\
& = (\bar{a}_1 \otimes b_1)((a_2 \otimes b_2)(a_3 \otimes b_3) + (a_2 \otimes b_2)(a_3 \bar{a}_4 b_4 + a_4 b_3 \bar{b}_4) + a_2 \bar{a}_3 \bar{a}_4 \bar{b}_2 b_3 b_4 + \bar{a}_2 a_3 a_4 b_2 \bar{b}_3 \bar{b}_4) + \\
& + \bar{a}_1 a_2 a_3 a_4 b_1 \bar{b}_2 \bar{b}_3 \bar{b}_4 + a_1 \bar{a}_2 \bar{a}_3 \bar{a}_4 \bar{b}_1 b_2 b_3 b_4
\end{aligned}$$

From the expression above, it is possible to derive the following logic circuit



Section 13.5: Combinational Logic Modules

Problem 13.54

Solution:

Known quantities:

The truth table for a function.

Find:

- The Karnaugh map for the function;
- Its minimum expression
- Realize the function using a 1-of-8 multiplexer

Analysis:

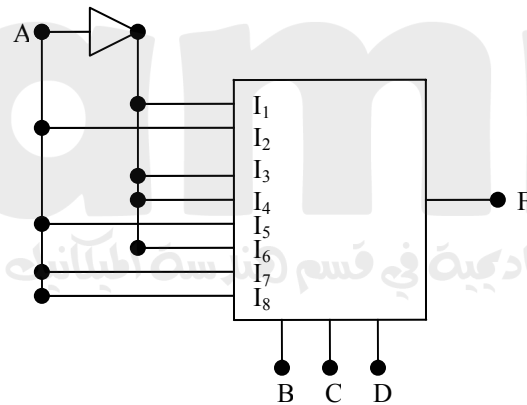
a)

AB \ CD	00	01	11	10
00	1	0	1	0
01	0	1	0	1
11	1	0	1	0
10	1	0	1	0

b)

$$F = \overline{A}\overline{B}\overline{D} + \overline{A}\overline{B}C + \overline{A}B\overline{C}D + A\overline{B}\overline{D} + ABC + \overline{A}\overline{B}CD = (\overline{A}\overline{B} + AB)(C + \overline{D}) + \overline{C}D(\overline{A}\overline{B} + AB)$$

c)



Problem 13.55**Solution:****Known quantities:**

The multiplexer circuit shown in Figure P13.55.

Find:

- The truth table for the multiplexer
- The binary function performed by the multiplexer.

Analysis:

a)

x	y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

b) Binary Addition - S is the sum, and C is the carry.

Problem 13.56**Solution:****Known quantities:**

A circuit that can operate as a 4-to-16 decoder is shown in Figure P13.56.

Find:

The operation of the 4-to-16 decoder, and the role of the logic variable A.

Analysis:

Assuming that the enable input (EN) is active high, when EN is logic 0 (A is logic 1), all decoder outputs of the first decoder are forced to logic 1 independent of the inputs. However, when EN is logic 1 (A is logic 0), all decoder outputs of the second decoder are forced to logic 1 independent of the select inputs. Therefore, A functions as the fourth bit of the select inputs. Thus, the circuit operates as a 4 of 16 decoder.

Problem 13.57**Solution:****Known quantities:**

The circuit of Figure P13.57.

Find:

The ability of the circuit of performing a conversion from 4-bit binary numbers to 4-bit Gray code.

Analysis:

We construct the truth table for this circuit as shown below:

Binary Input $B_3 B_2 B_1 B_0$	G_3	G_2 $B_2 \oplus B_3$	G_1 $B_1 \oplus B_2$	G_0 $B_0 \oplus B_1$
0 0 0 0	0	0	0	0
0 0 0 1	0	0	0	1
0 0 1 0	0	0	1	1
0 0 1 1	0	0	1	0
0 1 0 0	0	1	1	0
0 1 0 1	0	1	1	1
0 1 1 0	0	1	0	1
0 1 1 1	0	1	0	0
1 0 0 0	1	1	0	0
1 0 0 1	1	1	0	1
1 0 1 0	1	1	1	1
1 0 1 1	1	1	1	0
1 1 0 0	1	0	1	0
1 1 0 1	1	0	1	1
1 1 1 0	1	0	0	1
1 1 1 1	1	0	0	0

The output is clearly a Gray code since each number only changes by one bit relative to the previous number.

Problem 13.58

Solution:

Known quantities:

Four Boolean expressions.

Find:

- Show that these four expressions represent the conversion from 4-bit Gray code to 4-bit binary numbers.
- Draw the circuit which implements the conversion.

Analysis:

- Note that:

$$B_3 = G_3$$

$$B_2 = G_3 \oplus G_2 = B_3 \oplus G_2$$

$$B_1 = G_3 \oplus G_2 \oplus G_1 = B_2 \oplus G_1$$

$$B_0 = G_3 \oplus G_2 \oplus G_1 \oplus G_0 = B_1 \oplus G_0$$

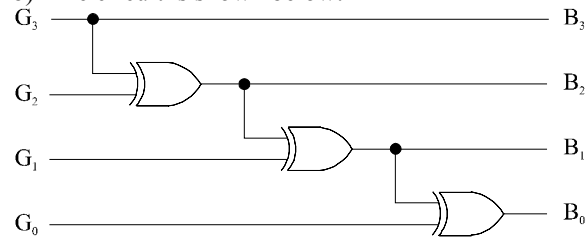
Then, the truth table is:

$G_3 G_2 G_1 G_0$	B_3 G_3	B_2 $G_3 \oplus G_2$	B_1 $B_2 \oplus G_1$	B_0 $B_1 \oplus G_0$
0 0 0 0	0	0	0	0
0 0 0 1	0	0	0	1
0 0 1 0	0	0	1	0
0 0 1 1	0	0	1	1
0 1 0 0	0	1	0	0
0 1 0 1	0	1	0	1
0 1 1 0	0	1	1	0

0 1 1 1	0	1	1	1
1 0 0 0	1	0	0	0
1 0 0 1	1	0	0	1
1 0 1 0	1	0	1	0
1 0 1 1	1	0	1	1
1 1 0 0	1	1	0	0
1 1 0 1	1	1	0	1
1 1 1 0	1	1	1	0
1 1 1 1	1	1	1	1

The table verifies that the claim is correct.

b) The circuit is shown below:



Problem 13.59

Solution:

Known quantities:

The function $f(A, B, C) = \overline{A}B\overline{C} + A\overline{B}\overline{C} + AC$

Find:

The inputs for a 4-input multiplexer to implement the function.

Assumptions:

The inputs I_0, I_1, I_2, I_3 correspond to $\overline{A}\overline{B}$, $\overline{A}B$, $A\overline{B}$ and AB respectively, and each input may be 0, 1, $\overline{C}$ or C .

Analysis:

$$f = \overline{A}B\overline{C} + A\overline{B}\overline{C} + AC$$

		A	
		0	1
BC	00	0	1
	01	0	1
	11	0	1
	10	1	0

From the truth table it is clear that:

$$I_0 = 0$$

$$I_1 = \overline{C}$$

$$I_2 = 1$$

and

$$I_3 = C$$

Problem 13.60**Solution:****Known quantities:**

The function $f(A, B, C, D) = \sum (2, 5, 6, 8, 9, 10, 11, 13, 14)_{10}$.

Find:

The inputs for an 8-bit multiplexer to implement the function.

Assumptions:

The inputs I_0 through I_7 correspond to $\overline{A}\overline{B}\overline{C}$, $\overline{A}\overline{B}C$, $\overline{A}B\overline{C}$, $\overline{A}BC$, $A\overline{B}\overline{C}$, $A\overline{B}C$, $AB\overline{C}$ and ABC respectively, and each input may be 0, 1, $\overline{D}$ or D .

Analysis:

AB \ CD		00	01	11	10
CD	00	0	0	0	1
	01	0	1	1	1
	11	0	0	0	1
	10	1	1	1	1

From the truth table it is clear that:

$$I_0 = 0$$

$$I_1 = \overline{D}$$

$$I_2 = D$$

$$I_3 = \overline{D}$$

$$I_4 = 1$$

$$I_5 = 1$$

$$I_6 = D$$

and

$$I_7 = \overline{D}$$

Chapter 14 Instructor Notes

Chapter 14 logically follows the material on combinational digital logic circuits introduced in Chapter 13. Section 14.1 contains a discussion of sequential logic modules; the box *Focus on Measurements: Digital Measurement of Angular Position and Velocity* (pp. 679-681) illustrates the use of a counter to measure the speed of rotation of a slotted wheel; this is a very common measurement in mechanical systems; the box *Focus on Measurements: Seven-Segment Display* (pp. 682-683) draws attention to a very common logic module. Section 14.2 provides an elementary introduction to sequential logic design, and Section 14.3 introduces microcomputers. This, and the two following sections on microcomputer architecture and microcontrollers, have been extensively revised, and are designed to provide an introduction to the concept of mechatronic design (see box on p. 688), recognizing that it is impossible to include a serious coverage of microcontrollers in a single chapter. Thus, the treatment is focused on an overview of the organization of microcontrollers, including a brief, qualitative illustration of computer interface issues presented in *Focus on Measurements: Reading Sensor Data By Using Interrupts* (pp. 697-698). The last section, 14.6, is dedicated to an example (courtesy of Delphi – Delco Electronic Systems) that describes a current automotive engine microcontroller. The example is a qualitative description, but will permit the instructor who desires to do so to motivate further study of this topic. The Find-It-On-The-Web references will provide students and instructor with additional reference material on the subject of automotive engine control.

In recent years mechanical and industrial engineering programs have seen a significant growth in courses related to mechatronics, or more specifically to microcontroller applications in industrial and mechanical systems. The objective of Chapter 14 is to serve as an introduction to such courses.

The homework problems are mostly devoted to sequential logic circuits; the last 6 problems review simple concepts related to the architecture and functions of microcomputers.

Learning Objectives

1. Analyze the operation of sequential logic circuits. [Section 1.](#)
2. Understand the operation of digital counters. [Section 1.](#)
3. Design simple sequential circuits using state transition diagrams. [Section 2.](#)
4. Study the basic architecture of microprocessors and microcomputers. [Sections 3,4, 5.](#)

Section 14.1: Sequential Logic Modules

Section 14.2: Sequential Logic Design

Problem 14.1

Solution:

Known quantities:

For the circuit shown in Figure P14.1, the input is a square wave having a period of 2 s, maximum value of 5 V, and minimum value of 0 V. Assume all flip-flops are initially in the RESET state.

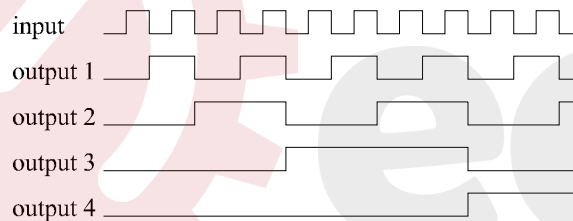
Find:

- Explain what the circuit does.
- Sketch the timing diagram, including the input and all four outputs.

Analysis:

- The device is called a MOD-16 ripple counter. It can count clock pulses from 0 to (2^4-1) . The outputs divide the frequency by 2^1 , 2^2 , 2^3 , and 2^4 respectively. Therefore, you can use this circuit as a divide by N counter, where N is 2, 4, 8 and 16.

b)



Problem 14.2

Solution:

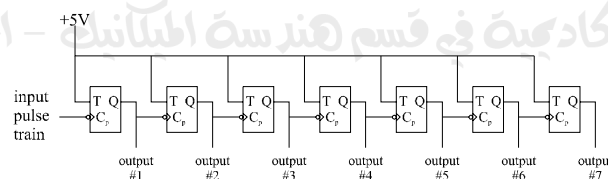
Find:

- How many flip-flops would be required to construct a binary pulse counter, which can count up to 100_{10} , by interconnecting T-type flip-flops in an appropriate manner.
- Sketch the circuit needed to implement this counter.

Analysis:

- $100_{10} = 1100100_2 \Rightarrow 7 \text{ flip flops required.}$

b)



This circuit could be modified with combinational logic if it is desired to have it reset at 100_{10} and start counting again from 0000000_2 .

Problem 14.3

Solution:**Known quantities:**

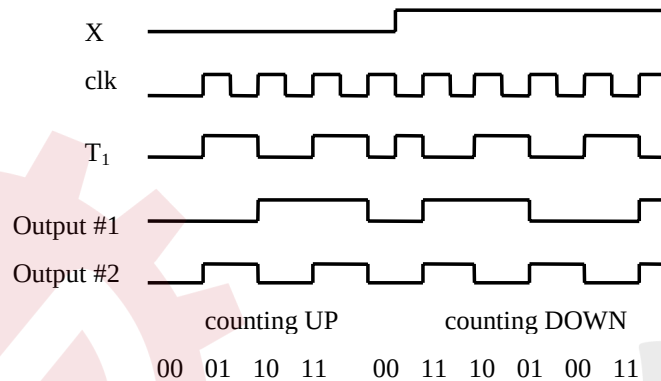
The circuit shown in Figure P14.3.

Find:

Explain what the circuit does and how it works.

Analysis:

The basic operation of the circuit is to count up when $X = 0$, and to count down when $X = 1$.



Clock	X	Output #2	T ₁	Output #1
↑	0	0	0	No change
↑	1	1	0	No change
↑	0	1	1	toggle
↑	1	0	1	toggle

Problem 14.4**Solution:****Known quantities:**

If a circuit is constructed from 3 *D*-type flip-flops,

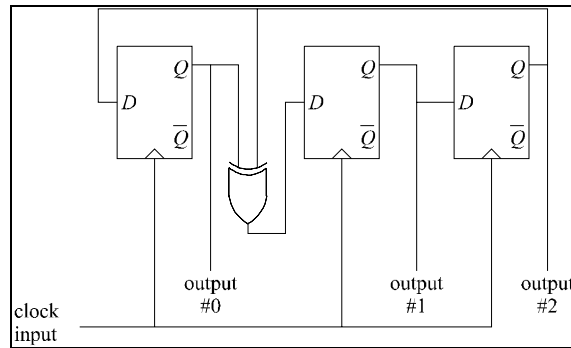
with: $D_0 = Q_2$ $D_1 = Q_2 \oplus Q_0$ $D_2 = Q_1$.

Find:

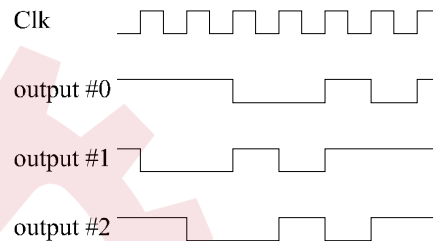
- Draw the circuit diagram.
- Assuming the circuit starts with all flip-flops SET, sketch a timing diagram which shows the outputs of all three flip-flops.

Analysis:

-



b)



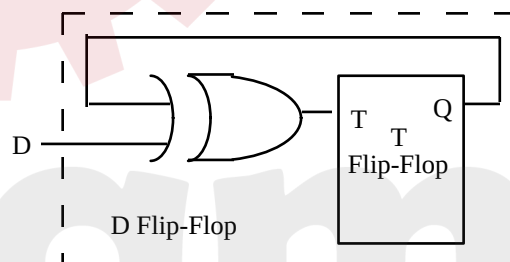
Problem 14.5

Solution:

Find:

Assuming you have all the logic gates available, make a D flip-flop using a T flip-flop and some logic gate(s).

Analysis:



Assume that Q is logic 0 initially. If the input D is logic 0, the output of the gate is logic 0 which means that the output of the T flip-flop will not be toggled (i.e., will remain logic 0). When the input D is logic 1, the output of the gate will be logic 1. Therefore, the output of the flip-flop will be 1. Thus, the circuit will operate as a D flip-flop.

Problem 14.6

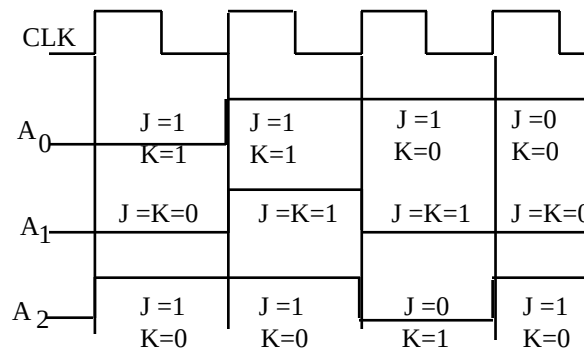
Solution:

Known quantities:

For the circuit shown in Figure P14.6, assume that all the initial values are 0. Note that all the flip-flops are negative edge-triggered.

Find:

Draw a timing diagram (four complete clock cycles) for A_0 , A_1 , and A_2 .

Analysis:**Problem 14.7****Solution:****Known quantities:**

Assume that the slotted encoder shown in Figure P14.7 has a length of 1 meter and a total of 1,000 slots (i.e., there is one slot per millimeter).

Find:

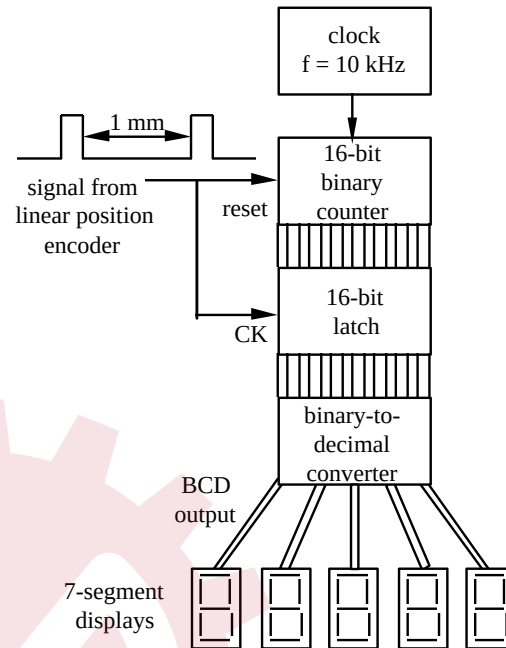
If a counter is incremented by 1 each time a slot goes past a sensor, design a digital counting system that determines the speed of the moving encoder (in meters per second).

Analysis:

Assuming a maximum speed of 10 m/s and a minimum speed of 1 mm/s, we can calculate the instantaneous speed of the slotted encoder by counting the number of clock pulses between slots using a fixed frequency clock. This resolution should be sufficient to measure the speed of the encoder over the range of interest. The figure depicts the arrangement: A 10 kHz clock increments a 16-bit binary counter.

The choice for a 16-bit counter is due to the maximum speed requirement: $2^{16} = 65,536$, will be the maximum count between slots. At a speed of 10 m/s, the time for one slot to go by is 10^{-4} s, thus the number of counts would be 1 count; at the minimum speed of 1 mm/s the number of counts would be 10^4 . A 14-bit counter would be sufficient, but in practice it is easier to cascade two 8-bit counters; thus the choice of a 16-bit counter. The count is held by a latch, and then converted to BCD for use with seven-segment displays. The details of the seven-segment display encoders are not shown (see Focus on Measurements: Seven-Segment Display). If a decimal point is placed to the right of the second seven-segment display (starting from the left in the figure), the display will read the speed in m/s, up to a maximum of 10 m/s, with a resolution of 1 mm/s.

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Problem 14.8

Solution:

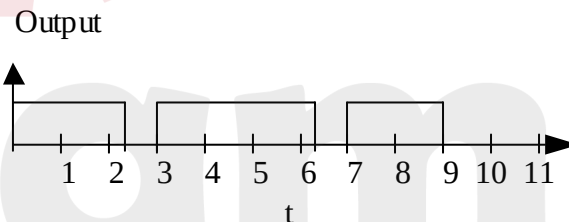
Known quantities:

The circuit shown in Figure P14.8.

Find:

The output Q for the given circuit.

Analysis:



Problem 14.9

Solution:

Find:

Describe how the ripple counter works. Why is it so named? What disadvantages can you think of for this counter?

Analysis:

This is briefly discussed in the digital counters section (see Figure 14.12).

Problem 14.10

Solution:**Find:**

Write the truth table for an RS flip-flop with enable (E), preset (P), and clear (C) lines.

Analysis:

Knowing that an input to the R or S line will be effective only when the enable input is 1, and the outputs are initially 0, the truth table for an RS flip-flop with set and preset is as follows:

S	R	P	C	Q
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	1	0	0	0
0	1	0	1	0
1	0	0	0	1
1	0	1	0	1

Problem 14.11**Solution:****Known quantities:**

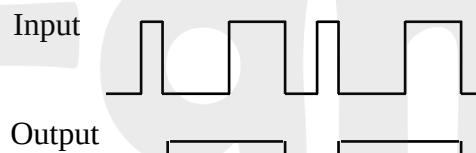
The JK flip-flop shown in Figure P14.11, with a given input signal.

Find:

Assuming that Q is at logic 0 initially and the trailing edge triggering is effective, sketch the output Q .

Analysis:

J_n	K_n	Q_{n+1}
1	1	$\overline{Q_n}$ (toggle)

**Problem 14.12****Solution:****Known quantities:**

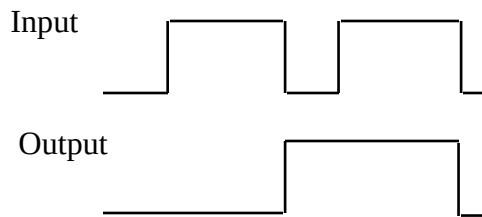
With reference to the JK flip-flop shown in figure P14.11, assume that the output at the Q terminal is made to serve as the input to a second JK flip-flop wired exactly as the first.

Find:

Sketch the output Q of the second flip-flop.

Analysis:

J_n	K_n	Q_{n+1}
1	1	$\overline{Q_n}$ (toggle)

**Problem 14.13****Solution:****Known quantities:**

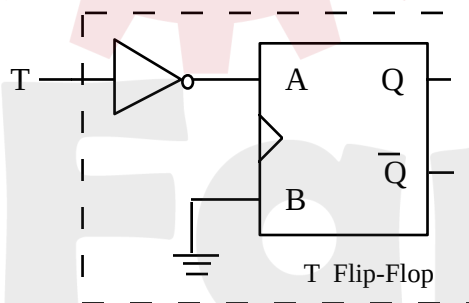
The flip-flop with the characteristic given in Figure P 14.13, where A and B are the inputs to the flip-flop and Q is the next state output.

Find:

Using necessary logic gates, make a T flip-flop from the given flip-flop.

Analysis:

T	A	B	Q
0	1	0	q
1	0	0	$\overline{q}$

**Section 14.3: Microprocessors****Section 14.4: Computer System Architecture****Section 14.5: Microcontrollers****Problem 14.14****Solution:****Find:**

If a typical PC has 32 Mbytes of standard memory:

- a) How many words is this?

- b) How many nibbles is this?
- c) How many bits is this?

Analysis:

$$32 \text{ Mbytes} = 32 \times 2^{20} \text{ bytes} = 33554432 \text{ bytes}$$

- a) $33554432 \text{ bytes} \times \frac{1 \text{ word}}{2 \text{ bytes}} = 16777216 \text{ words}$
 - b) $33554432 \text{ bytes} \times \frac{2 \text{ nibbles}}{1 \text{ byte}} = 67108864 \text{ nibbles}$
 - c) $33554432 \text{ bytes} \times \frac{8 \text{ bits}}{1 \text{ byte}} = 268435456 \text{ bits}$
-

Problem 14.15

Solution:

Find:

Suppose a microprocessor has n registers.

- a) How many control lines do you need to connect each register to all other registers?
- b) How many control lines do you need if a bus is used?

Analysis:

- a) $n \cdot (n - 1)$.
 - b) $2n$.
-

Problem 14.16

Solution:

Find:

Suppose it is desired to implement a 4K 16-bit memory.

- a) How many bits are required for the memory address register?
- b) How many bits are required for the memory data register?

Analysis:

- a) We need $2^N = 4\text{Kbytes} = 4096 \text{ bits} \Rightarrow N = 12$

Therefore, we need 12 bits for the memory address register.

- b) The data register must be at least as large as each word in memory. Therefore, the data register must be 16 bits in length.
-

Problem 14.17

Solution:

Find:

What is the distinction between volatile and nonvolatile memory.

Analysis:

"Volatile" memory is memory whose contents are lost when the power is turned off. This is the RAM in a computer. "Nonvolatile" means that the information in the memory is not lost when the power is off. This is a magnetic disk, magnetic tape or ROM in a computer.

Problem 14.18**Solution:****Known quantities:**

Suppose that a particular magnetic tape can be formatted with 8 tracks per centimeter of tape width. The recording density is 200 bits/cm, and the transport mechanism moves the tape past the read heads at a velocity of 25 cm/s.

Find:

How many bytes/s can be read from a 2-cm-wide tape.

Analysis:

$$\frac{8 \text{ tracks}}{\text{cm}} \times 2 \text{ cm} = 16 \text{ tracks}$$

$$16 \text{ tracks} \times 200 \frac{\text{bits}}{\text{cm}} = 3200 \frac{\text{bits}}{\text{cm}}$$

$$3200 \frac{\text{bits}}{\text{cm}} \times \frac{1 \text{ byte}}{8 \text{ bits}} = 400 \frac{\text{bytes}}{\text{cm}}$$

$$400 \frac{\text{bytes}}{\text{cm}} \times 25 \frac{\text{cm}}{\text{s}} = 10000 \frac{\text{bytes}}{\text{s}}$$

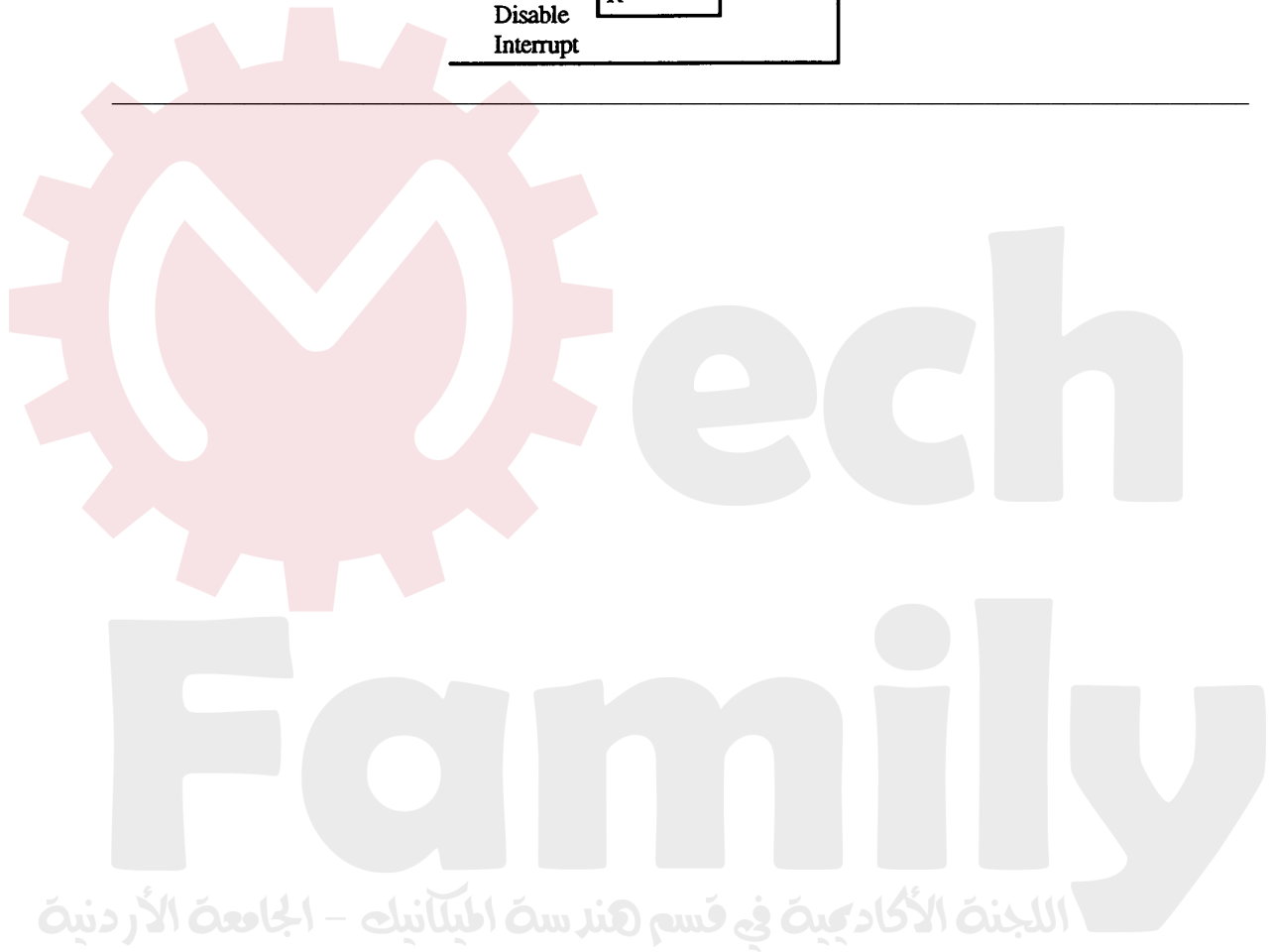
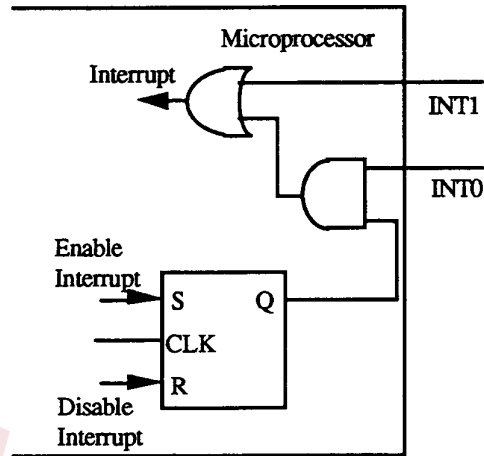
Problem 14.19**Solution:****Find:**

Draw a block diagram of a circuit that will interface two interrupts, INT0 and INT1, to the INT input of a CPU so that INT1 has the higher priority and INT0 has the lower. In other words, a signal on INT1 is to be able to interrupt the CPU even when the CPU is currently handling an interrupt generated by INT0, but not vice versa.

Analysis:

There are two types of interrupts: non-maskable, and maskable. When a logic signal is applied to a maskable interrupt input (INT1 in this case), the microprocessor is immediately interrupted. When a logic signal is applied to a maskable input (INT0 in this case), the microprocessor is interrupted only if that particular input is enabled. Maskable interrupts are disabled or enabled under program control.

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Chapter 15 Instructor Notes

Chapter 15 continues the discussion of integrated circuit electronics begun in Chapter 8 with op-amps. The Chapter is extremely modular, and the degree of coverage can vary widely, depending on the requirements of each individual Instructor. The first two sections cover measurement systems and transducers, providing a summary in Table 15.1 (p. 712), and noise problems. The third covers the instrumentation amplifier (which had been introduced in Chapter 8) in greater depth. This section might be of interest by itself as an extension of Chapter 8, and can be covered immediately following Section 8.2. Section 15.3 also discusses practical active filters, focusing on Butterworth and Chebyshev designs, and is a logical continuation of Section 8.3 on active filters; the material in this section is fairly advanced, and will require the student to have had a rigorous introduction to the Laplace Transform and to systems concepts. Thus, this section will be appropriate for a second course on electronics and instrumentation. The active filter design material would be nicely complemented by laboratory exercises; the importance of filter design and analysis in instrumentation problems is not to be underestimated. This material could also be supplemented very effectively by a review of filter design and analysis procedures using computer aids (e.g., MATLAB® or Electronics Workbench®).

The material in Section 15.4, on the subject of signal interface (A/D, D/A conversion and sample-and-hold amplifiers) is, on the other hand, much more quantitative, and can be presented separately, for example, in conjunction with Chapter 8, or with Chapters 13 and 14. There are excellent possibilities for very useful laboratory experiments in connection with this material. The emphasis is on illustrating the important parameters and performance limitations in the application of commercial ADCs, DACs, and sample-and-hold amplifiers. A commercial data acquisition board is also described in *Focus on Measurements: Data Acquisition Card for Personal Computer* (pp. 747-750). It is this Author's opinion that this material is of great practical importance to non-majors, many of whom will at some point make use of a microcomputer-based digital data acquisition system, regardless of their specialty.

Section 15.5 can also be viewed as an extension of Chapter 8, and could be covered in a first course. Again, the section is independent of the other sections in the chapter. In this section, the op-amp comparator and the Schmitt trigger are introduced first. The section closes with a functional description of timing circuits including a one-shot IC (74123) and the NE 555 timer IC.

Section 15.6 provides a summary of other IC instrumentation circuits, to give the student a flavor of the capabilities of modern integrated electronics. The box *Focus on Measurements: Using the ADXL202 Accelerometer as a Multifunction Sensor in Car Alarms* (pp. 762-766) provides a detailed look at an application note (Courtesy: Analog Devices). The aim of this box is to present Application Notes as a valuable design resource for practicing engineers.

The last section, 15.7, is somewhat unusual for a textbook of this nature in covering the basic elements of digital data transmission. However, given the pervasive presence of digital instruments and microcomputers in engineering laboratories and in the field, a brief survey of the IEEE 488 and RS-232 standards can prove extremely useful in practice. New in the 4th Edition are a brief overview of the USB and CAN bus standards.

The homework problems present a variety of analysis and design problems on instrumentation amplifiers and active filters. Several design problems are also given to complement the section on timing circuits; a few of the problems require the student to explore the data sheets for the AD 625 instrumentation amplifier, the 555 timer, and the 74123 one-shot. Although these problems are fairly simple, they can be used to educate the student to search for design parameters in the data sheets. The data sheets are provided in the CD-ROM and on the website.

Also included is a series of problems on DAC and ADC analysis and design. Some emphasis is again placed on reading and understanding the device data sheets of commercial ADCs and DACs. Issues in sampling frequency selection and resolution are approached in a few applied problems, where practical measurement situations pertaining to the measurement of angular position (problem 15.59), torque (problem 15.61), and altitude (problem 15.62) are described. The chapter problems end with a few simple problems on data transmission and coding.

Learning Objectives

1. Review the major classes of sensors. [Section 1.](#)
2. Learn how to properly ground circuits, and methods for noise shielding and reduction. [Section 2.](#)
3. Design signal conditioning amplifiers and filters. [Section 3.](#)
4. Understand A/D and D/A conversion, and select the specifications of a the appropriate conversion system for a given application. [Section 4.](#)
5. Analyze and design simple comparator and timing circuits using integrated circuits. Review other common instrumentation integrated circuits. [Section 5, 6.](#)
6. Learn basics of data transmission in digital instruments. [Sections 6.](#)

Section 15.1: Measurement Systems and Transducers

Problem 15.1

Solution:**Find:**

Explain the differences between tachometers and speedometers.

Analysis:

Frequency - engine speed is normally measured right at the crankshaft prior to any gearing in rpm - typically several thousands. The transducers used with speedometers measure speed at the axle in rpm - typically much lower than at the engine output.

Scale factors - the tachometer would require none, the speedometer requires a conversion factor from rpm of the axle (rotational) to mph (linear).

Problem 15.2

Solution:**Find:**

Explain the differences between the engineering specifications you would write for a transducer to measure the frequency of an audible sound wave and a transducer to measure the frequency of a visible light wave.

Analysis:

Audio frequencies: $0 < f \leq 15 \text{ kHz}$

Visible frequencies: $3.9 \times 10^{14} \text{ Hz} \leq f \leq 7.9 \times 10^{14} \text{ Hz}$

Devices used to measure quantities at audio frequencies will be incapable of sensing or measuring accurately those same quantities at frequencies in the visual range.

Various types of photocells are available for use as light sensors. For audio frequencies, more conventional devices like bridges may be used for measuring signals.

Problem 15.3

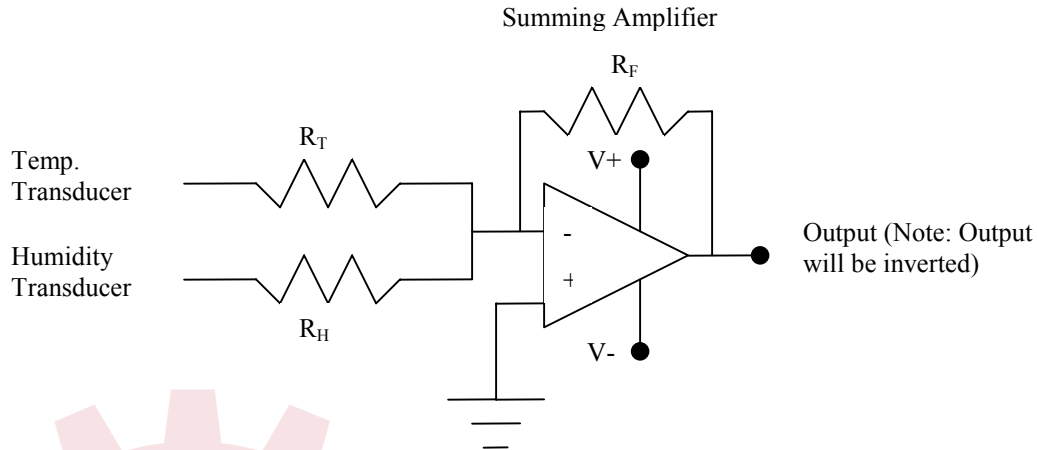
Solution:**Find:**

A simple schematic diagram of a circuit to compute the sum of a temperature and a percentage relative humidity signal.

Analysis:

Use a transducer that will convert temperature in degrees Fahrenheit to volts between the values of the 2-sided supply voltage.

Similarly measure the percentage relative humidity and convert the transducer output to the requisite voltage.



Problem 15.4

Solution:

Known quantities:

The capacity of a capacitive displacement transducer $C = \frac{0.255 A}{d} \text{ F}$, where A is the cross-sectional area of the transducer plate (in^2), and d the air-gap length (in).

Find:

The change in voltage (Δv_0) when the air-gap changes from 0.01 in to 0.015 in.

Analysis:

$$i = C \frac{\Delta v_o}{\Delta t}, \Delta q = C \Delta v_o, \frac{\Delta q}{C} = \Delta v_o$$

Assume no change in charge.

$$C_{new} = \frac{2}{3} C_{old} \Rightarrow \Delta v_o = \frac{3}{2} \text{ V}$$

Problem 15.5

Solution:

Known quantities:

The circuit of Figure P15.5 in which $i_D/H = 0.5 \cdot 10^{-6} \mu \text{ A m}^2/\text{W}$.

Find:

- Show that the output voltage varies linearly with H .
- If $H = 1,500 \text{ W/m}^2$, $V_D = 7.5 \text{ V}$, and an output voltage of 1 V is desired, determine an appropriate value for R_L .

Analysis:

$$a) \quad V_{out} = i_D R_L$$

Given a large enough value of V_D ,

$$V_{out} = 0.5 \cdot 10^{-6} H R_L; \text{ hence, varies linearly with } H.$$

$$b) \quad V_{out} = i_D/H R_L \Rightarrow 1 = 0.5 \cdot 10^{-6} (1500) R_L, R_L = 1333 \Omega$$

Problem 15.6**Solution:****Known quantities:**

The value of the constant G for quartz in compressive stress, $0.055 \frac{\text{V} \cdot \text{m}}{\text{N}}$, and for polyvinylidene fluoride in axial stress, $0.22 \frac{\text{V} \cdot \text{m}}{\text{N}}$.

The quartz element of a force sensor is 0.25 in thick and has a rectangular cross section of 0.09 in^2 .

The fluoride film of a piezoelectric sensor is $30 \mu\text{m}$ thick, 1.5 cm wide, and 2.5 cm in the axial direction.

Find:

- The output of the force sensor in V/N.
- The output of the load sensor in V/N.

Analysis:

$$\text{a) } V_{\text{out}} = 0.055 \frac{\text{V} \cdot \text{m}}{\text{N}} \frac{1 \text{ in}}{0.0254 \text{ m}} \times \frac{1}{0.25 \text{ in}} = 8.66 \frac{\text{V}}{\text{N}}$$

$$\text{b) } 0 \frac{\text{V}}{\text{N}}$$

Problem 15.7**Solution:****Known quantities:**

The allowable levels of error in the measurement of K , m and ξ : $\pm 5\%$, $\pm 2\%$, $\pm 10\%$. The expression for b , $b = 2\xi\sqrt{Km}$.

Find:

The percentage error limit for b .

Analysis:

$$\text{error} = \pm \left(10 + \frac{5+2}{2} \right) = \pm 13.5\%$$

Problem 15.8**Solution:****Known quantities:**

The measurements taken by a sensor that measure the thickness of a wet pulp layer every 2 feet along the sheet: 8.2, 9.8, 9.92, 10.1, 9.98, 10.2, 10.2, 10.16, 10.0, 9.94, 9.9, 9.8, 10.1, 10.0, 10.2, 10.3, 9.94, 10.14, 10.22, 9.8. The roller speed is adjusted based on the last 20 measurements unless the probability that the mean thickness lies within $\pm 2\%$ of the sample mean exceeds 0.99.

Find:

The adjustment of the speed roller.

Analysis:

$$\text{mean} = \frac{\sum \text{measurements}}{20} = 9.945 \pm 2 \%, \quad \text{max.} = 10.144, \quad \text{min.} = 9.746$$

$$\text{average deviation} = \frac{\sum |\text{deviations}|}{20} = 0.225$$

$$\text{standard deviation} = \sqrt{\frac{\sum |\text{deviations}|^2}{20}} = 0.427$$

Measurement #1 exceeds the standard deviation σ probability < 0.99 $\Rightarrow$ roller speed will be adjusted

Problem 15.9**Solution:****Find:**

Discuss:

- Measurement accuracy.
- Instrument accuracy.
- Measurement error.
- Precision.

Analysis:

- This term and instrument accuracy are used interchangeably if only one instrument is involved and if the measurement method is appropriate. Basically, the accuracy of the measurement is given by the instrument's specifications (ordinarily in terms of percent of indicated value or full scale value).
- See answer to part a).
- Measurement error can be synonymous with measurement accuracy, but can also refer to sloppy methods of data acquisition, use of multiple transducers and/or instruments whose individual errors combine, or simply a lack of reliable, multiple data points.
- Precision and resolution are interchangeable terms and refer to the smallest increment of measured quantity that can be detected by the instrument.

Problem 15.10**Solution:****Known quantities:**

Four sets of measurements shown in Figure P15.10 taken on the same response variable of a process using four different sensors.

Find:

Rank these data sets with respect to:

- Precision.
- Accuracy.

Analysis:

- (b) and (c) are precise, (a) and (d) are not.
 (a) and (c) are accurate, (b) and (d) are not.

Section 15.3: Signal Conditioning

Problem 15.11

Solution:

Known quantities:

The resistances for the instrumentation amplifier of Figure P15.11, $R_1 = 1 \text{ k}\Omega$, $R_2 = 5 \text{ k}\Omega$.

Find:

The gain of the input stage.

Analysis:

$$A = 1 + \frac{2R_2}{R_1} = 1 + \frac{2(5 \text{ k}\Omega)}{1 \text{ k}\Omega} = 1 + 10 = 11$$

Problem 15.12

Solution:

Known quantities:

The resistance $R_1 = 1 \text{ k}\Omega$ for the instrumentation amplifier of Figure P15.11, and the value of the gain for the input stage, 50.

Find:

The resistance R_2 to make that gain for the amplifier.

Analysis:

$$A = 1 + \frac{2R_2}{R_1} \Rightarrow R_2 = \frac{1}{2} R_1 (A - 1) = \frac{1}{2} (1 \text{ k}\Omega)(50 - 1) = 24.5 \text{ k}\Omega$$

Problem 15.13

Solution:

Known quantities:

The resistance $R_2 = 10 \text{ k}\Omega$ for the instrumentation amplifier of Figure P15.11, and the value of the gain for the input stage, 16.

Find:

The resistance R_1 to make that gain for the amplifier.

Analysis:

$$A = 1 + \frac{2R_2}{R_1} \Rightarrow R_1 = \frac{2R_2}{A - 1} = \frac{2(10 \text{ k}\Omega)}{16 - 1} \approx 1333 \Omega$$

Problem 15.14

Solution:

Known quantities:

The resistances for the instrumentation amplifier of Figure 15.16, $R_1 = 1 \text{ k}\Omega$, $R_2 = 10 \text{ k}\Omega$.

Find:

The gain of the input stage.

Analysis:

$$A = 1 + \frac{2R_2}{R_1} = 1 + \frac{2(10 \text{ k}\Omega)}{1 \text{ k}\Omega} = 1 + 20 = 21$$

Problem 15.15**Solution:****Known quantities:**

The resistances for the instrumentation amplifier of Figure 15.16, $R_1 = 1.5 \text{ k}\Omega$, $R_2 = 80 \text{ k}\Omega$.

Find:

The gain of the input stage.

Analysis:

$$A = 1 + \frac{2R_2}{R_1} = 1 + \frac{2(80 \text{ k}\Omega)}{1.5 \text{ k}\Omega} = 1 + 106.7 = 107.7$$

Problem 15.16**Solution:****Known quantities:**

The resistances for the instrumentation amplifier of Figure 15.16, $R_1 = R' = R = 1 \text{ k}\Omega$, $R_2 = 5 \text{ k}\Omega$, $R_f = 10 \text{ k}\Omega$.

Find:

The differential gain for the IA.

Analysis:

$$A_{dif} = A \frac{R_f}{R} = \left(1 + \frac{2R_2}{R_1}\right) \frac{R_f}{R} = \left(1 + \frac{2(5)}{1}\right) \frac{10}{1} = 110$$

Problem 15.17**Solution:****Known quantities:**

The resistances for the instrumentation amplifier of Figure P15.11, $R = 1 \text{ k}\Omega$, $R_f = 200 \text{ k}\Omega$,

$\Delta R = 2 \% \text{ of } R$.

Find:

The CMMR of the IA.

Analysis:

$$CMRR_{dB} = 20 \log_{10} \left| \frac{A_{dif}}{\frac{R_f}{R} \left(\frac{R + R_f}{R_f + R + \Delta R} - 1 \right) A} \right|$$

$$A_{dif} = A \frac{R_f}{R}, \Delta R = 0.02 \cdot R = 0.02 \cdot 1 \text{ k}\Omega = 20 \Omega$$

$$CMRR_{dB} = 20 \log_{10} \left| \frac{A \frac{R_f}{R}}{\frac{R_f}{R} \left(\frac{R + R_f}{R_f + R + \Delta R} - 1 \right) A} \right| = 20 \log_{10} \left| \frac{1}{\frac{R + R_f}{R_f + R + \Delta R} - 1} \right| \approx 80 \text{ dB}$$

Problem 15.18**Solution:****Known quantities:**

The resistances for the instrumentation amplifier of Figure P15.11, $R = 1 \text{ k}\Omega$, $R_f = 200 \text{ k}\Omega$, $\Delta R = 2 \%$ of R .

Find:

The mismatch in gains for the differential components in dB.

Analysis:

Assume $A = 10$. Then $A_{dif} = \frac{R_f}{R} A = 2000$

$$20 \log_{10} A_{dif} \approx 66 \text{ dB}$$

$$CMRR_{dB} = 80 \text{ dB} \quad (\text{from Problem 15.17})$$

$$20 \log_{10} A_{dif} - CMRR_{dB} = -14 \text{ dB}$$

Problem 15.19**Solution:****Known quantities:**

The resistances for the instrumentation amplifier of Figure 15.16, $R_1 = 2 \text{ k}\Omega$, $R_f = 10 \text{ k}\Omega$, and the differential gain, 900.

Find:

The resistances R and R_2 to achieve that gain.

Analysis:

$$A_{dif} = \frac{R_f}{R} \left(1 + \frac{2R_2}{R_1} \right) = \frac{10}{R} \left(1 + \frac{2R_2}{2} \right) = \frac{10}{R} (1 + R_2) = 900$$

Thus, $1 + R_2 = 90R$

Choose: $R = 1 \text{ k}\Omega \Rightarrow R_2 = 89 \text{ k}\Omega$

Problem 15.20

Solution:

Known quantities:

The cutoff specification of Example 15.3, $\omega_c = 10 \text{ rad/s}$.

Find:

The order of the filter required to achieve 40 dB attenuation at $\omega_s = 24 \text{ rad/s}$.

Analysis:

$$20 \log_{10} \sqrt{1 + \omega_s^{2n}} \leq 40 \quad @ \quad \omega_s = 25 \text{ rad/s}$$

Solving the equation, we obtain $n = 1.43$. Thus $n = 2$ is desired.

Problem 15.21

Solution:

Known quantities:

The circuit for a low-pass filter shown in Figure P15.21 with gain.

Find:

- The relationship between output amplitude and input amplitude.
- The relationship between output phase angle and input phase angle.

Analysis:

This is an inverting amplifier circuit, with

$$V_{out} = -\frac{Z_F}{Z_{in}} V_{in}, \text{ where: } Z_F = R_F \parallel \frac{1}{j\omega C_F} = \frac{R_F \cdot \frac{1}{j\omega C_F}}{R_F + \frac{1}{j\omega C_F}} = \frac{R_F}{1 + j\omega R_F C_F} \text{ and } Z_{in} = R_{in}.$$

$$\text{Therefore, } V_{out} = -\frac{R_F}{R_{in}} \frac{1}{1 + j\omega R_F C_F} V_{in}$$

$$\text{a) } \left| \frac{V_{out}}{V_{in}} \right| = \frac{R_F}{R_{in}} \frac{1}{\sqrt{1 + (\omega R_F C_F)^2}}$$

$$\text{b) } \angle V_{out} - \angle V_{in} = \pi - \tan^{-1}(\omega R_F C_F) \text{ rad, or } \angle V_{out} - \angle V_{in} = 180^\circ - \tan^{-1}(\omega R_F C_F) \text{ deg.}$$

Problem 15.22

Solution:

Known quantities:

The circuit for a low-pass filter shown in Figure P15.21 with gain, with $R_{in} = 20 \text{ k}\Omega$, $R_F = 100 \text{ k}\Omega$, $C_F = 100 \text{ pF}$, $v_{in} = 2 \sin(2,000\pi t) \text{ V}$.

Find:

The expression of v_{out} .

Analysis:

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{R_F}{R_{in}} \frac{1}{\sqrt{1 + (\omega R_F C_F)^2}} = 4.99$$

$$\angle V_{out} - \angle V_{in} = \pi - \tan^{-1}(2000\pi \cdot 100 \cdot 10^3 \cdot 100 \cdot 10^{-12}) = \pi - \tan^{-1}(62.8 \cdot 10^{-3}) = 3.079 \text{ rad}$$

$$v_{out}(t) = 9.98 \sin(2000\pi t + 3.079) \text{ V}$$

Problem 15.23**Solution:****Known quantities:**

The circuit of the low-pass filter of Figure 15.22.

Find:

The frequency response of the filter.

Analysis:

The circuit is shown below:

We have:

$$V_{AB} = \frac{R_B}{R_A + R_B} V_{out}$$

$$i_1 = \frac{V_s - V}{R_1}, i_2 = \frac{V - V_{AB}}{R_2}, i_3 = j\omega C_1 (V_{out} - V),$$

$$i_4 = j\omega C_2 V_{AB}.$$

$$\text{From } i_2 = i_4, \text{ we have: } V = V_{AB} (1 + j\omega R_2 C_2)$$

$$\text{From } i_1 + i_3 = i_2, \text{ we have:}$$

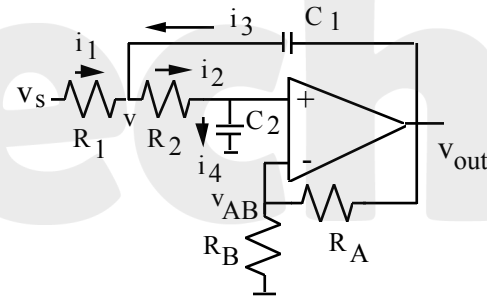
$$V_{out} \left(-j\omega C_1 + \frac{R_B}{R_A + R_B} \left(\left(j\omega C_1 + \frac{1}{R_1} + \frac{1}{R_2} \right) (1 + j\omega R_2 C_2) \right) - \frac{1}{R_2} \right) = \frac{V_s}{R_1}$$

$$\frac{V_{out}}{V_s}(j\omega) = \frac{1}{R_1 \left(-j\omega C_1 + \frac{R_B}{R_A + R_B} \left(\left(j\omega C_1 + \frac{1}{R_1} + \frac{1}{R_2} \right) (1 + j\omega R_2 C_2) \right) - \frac{1}{R_2} \right)}$$

Therefore, the frequency response is:

$$\frac{V_{out}}{V_s}(j\omega) = \frac{K(1/R_1 R_2 C_1 C_2)}{(j\omega)^2 + \left[\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} (1 - K) \right] (j\omega) + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$\text{where: } K = 1 + \frac{R_A}{R_B}$$



Problem 15.24**Solution:****Known quantities:**

The circuit of the high-pass filter of Figure 15.22.

Find:

The frequency response of the filter.

Analysis:

The circuit is shown below:

We have:

$$V_{AB} = \frac{R_B}{R_A + R_B} V_{out}$$

$$i_1 = j\omega C_1 (V_s - V), \quad i_2 = j\omega C_2 (V - V_{AB}),$$

$$i_3 = \frac{V_{out} - V}{R_1}, \quad i_4 = \frac{V_{AB}}{R_2}.$$

$$\text{From } i_2 = i_4, \text{ we have: } V = V_{AB} \left(1 + \frac{1}{j\omega R_2 C_2} \right)$$

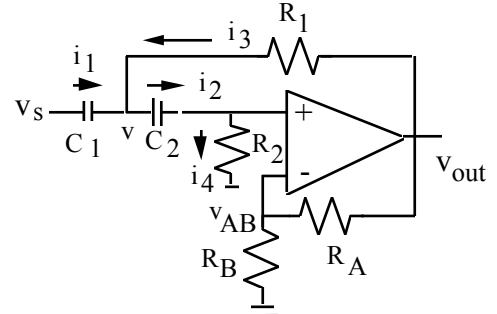
$$\text{From } i_1 + i_3 = i_2, \text{ we have:}$$

$$j\omega C_1 V_s = V_{out} \left\{ \left(\frac{R_B}{R_A + R_B} \right) \left[\left(\frac{1}{R_1} + j\omega C_1 + j\omega C_2 \right) \left(\frac{1}{j\omega R_2 C_2} + 1 \right) - j\omega C_2 \right] - \frac{1}{R_1} \right\}$$

Therefore, the frequency response is:

$$\frac{V_{out}}{V_s}(j\omega) = \frac{K(j\omega)^2}{(j\omega)^2 + \left[\frac{1}{R_1 C_1} (1 - K) + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} \right] (j\omega) + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$\text{where: } K = 1 + \frac{R_A}{R_B}$$

**Problem 15.25****Solution:****Known quantities:**

The circuit of the band-pass filter of Figure 15.22.

Find:

The frequency response of the filter.

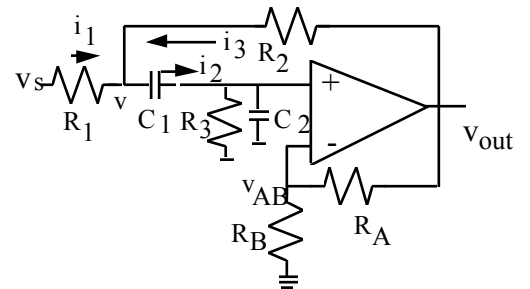
Analysis:

The circuit is shown below:

We have:

$$V_{AB} = \frac{R_B}{R_A + R_B} V_{out}$$

$$i_1 = \frac{V_s - V}{R_1}, \quad i_2 = j\omega C_1 (V - V_{AB}), \quad i_3 = \frac{V_{out} - V}{R_2}.$$



From $i_2 = V_{AB} \left(\frac{1}{R_3} + j\omega C_2 \right)$, we have: $V = V_{AB} \left(1 + \frac{C_2}{C_1} + \frac{1}{j\omega R_3 C_1} \right)$

From $i_1 + i_3 = i_2$, we have:

$$\frac{V_s}{R_1} = V_{out} \left\{ \left(\frac{R_B}{R_A + R_B} \right) \left[\left(\frac{R_1 + R_2}{R_1} + j\omega C_1 \right) \left(\frac{1}{j\omega C_1 R_3} + \frac{C_1 + C_2}{C_1} \right) - j\omega C_1 \right] - \frac{1}{R_2} \right\}$$

Therefore, the frequency response is:

$$\frac{V_{out}}{V_s}(j\omega) = \frac{Kj\omega \frac{1}{R_1 C_2}}{(j\omega)^2 + \left[\frac{1}{R_1 C_1} + \frac{1}{R_1 C_2} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} (1 - K) \right] (j\omega) + \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}$$

where: $K = 1 + \frac{R_A}{R_B}$

Problem 15.26

Solution:

Known quantities:

The circuit of Figure P15.21, where $C_F = 100 \text{ pF}$, and the desired cutoff frequency and gain magnitude are respectively 20 kHz and 5.

Find:

The appropriate values of R_F and R_{in} .

Analysis:

$$f_c = 20 \text{ kHz} \Rightarrow \omega_c = 2\pi f_c = 40\pi \text{ k} \frac{\text{rad}}{\text{s}}$$

$$\omega_c = \frac{1}{R_F C_F} \Rightarrow R_F = \frac{1}{\omega_c C_F} = \frac{1}{40\pi \cdot 10^3 \cdot 100 \cdot 10^{-12}} = 79.6 \text{ k}\Omega$$

$$\frac{R_F}{R_{in}} = 5 \Rightarrow R_{in} = \frac{R_F}{5} = 15.9 \text{ k}\Omega$$

Problem 15.27

Solution:

Known quantities:

The cutoff frequency, 10 kHz, and the DC gain, 10, for a second-order Butterworth high pass filter, with $Q = 5$ and $V_s = \pm 15 \text{ V}$.

Find:

The design of the filter.

Analysis:

$$K = 1 + \frac{R_A}{R_B} = 10; \text{ choosing } R_A = 9 \text{ k}\Omega \Rightarrow R_B = 1 \text{ k}\Omega$$

$$f = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} = 10 \text{ kHz}$$

$$\left[\sqrt{\frac{R_2 C_2}{R_1 C_1}} + \sqrt{\frac{R_1 C_2}{R_2 C_1}} - 9 \sqrt{\frac{R_1 C_1}{R_2 C_2}} \right] = 0.2$$

Choose $C_1 = C_2 = 0.01 \mu\text{F}$ and solve for $R_1 = 540 \Omega$ and $R_2 = 4.7 \text{ k}\Omega$. Then, substitute the values thus obtained in the high-pass filter of Figure 15.22.

Problem 15.28

Solution:

Known quantities:

The cutoff frequency, 25 kHz, and the DC gain, 15, for a second-order Butterworth high pass filter, with $Q = 10$ and $V_s = \pm 15 \text{ V}$.

Find:

The design of the filter.

Analysis:

$$K = 1 + \frac{R_A}{R_B} = 15; \text{ choosing } R_A = 14 \text{ k}\Omega \Rightarrow R_B = 1 \text{ k}\Omega$$

$$f = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} = 25 \text{ kHz}$$

$$\left[\sqrt{\frac{R_2 C_2}{R_1 C_1}} + \sqrt{\frac{R_1 C_2}{R_2 C_1}} - 14 \sqrt{\frac{R_1 C_1}{R_2 C_2}} \right] = 0.1$$

Choose $C_1 = C_2 = 1 \mu\text{F}$ and solve for $R_1 = 1.8 \Omega$ and $R_2 = 23 \Omega$. Then, substitute the values thus obtained in the high-pass filter of Figure 15.22.

Problem 15.29

Solution:

Known quantities:

The circuit of Figure P15.29.

Find:

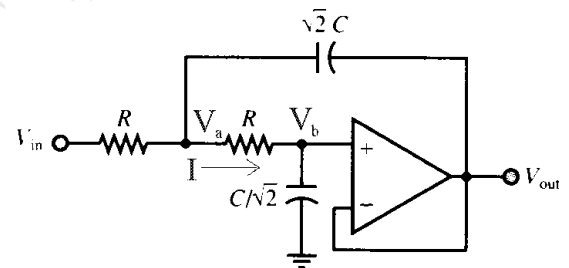
The characteristic of the filter.

Analysis:

Note that $V_b \approx V_{out}$. Therefore,

$$I = \frac{V_b}{\left(\frac{1}{j\omega \frac{C}{\sqrt{2}}} \right)} = j\omega \frac{C}{\sqrt{2}} V_{out}$$

and



$$V_a = RI + V_b = R \left(j\omega \frac{C}{\sqrt{2}} V_{out} \right) + V_{out} = \left(1 + j \frac{\omega RC}{\sqrt{2}} \right) V_{out}$$

Now, writing a KCL equation at node a,

$$\frac{V_a - V_{in}}{R} + I + \frac{V_a - V_{out}}{\left(\frac{1}{j\omega\sqrt{2}C} \right)} = 0$$

$$\frac{\left(1 + j \frac{\omega RC}{\sqrt{2}} \right) V_{out} - V_{in}}{R} + j\omega \frac{C}{\sqrt{2}} V_{out} + \frac{\left(1 + j \frac{\omega RC}{\sqrt{2}} \right) V_{out} - V_{out}}{\left(\frac{1}{j\omega\sqrt{2}C} \right)} = 0$$

$$\frac{1}{R} V_{in} = \left[\frac{\left(1 + j \frac{\omega RC}{\sqrt{2}} \right)}{R} + j\omega \frac{C}{\sqrt{2}} + \frac{\left(1 + j \frac{\omega RC}{\sqrt{2}} - 1 \right)}{\left(\frac{1}{j\omega\sqrt{2}C} \right)} \right] V_{out} = \left[\frac{1}{R} + j \frac{\omega C}{\sqrt{2}} + j \frac{\omega C}{\sqrt{2}} - \omega^2 RC^2 \right] V_{out}$$

$$V_{in} = \left(1 + j\sqrt{2}\omega RC - \omega^2 R^2 C^2 \right) V_{out} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{1 - (\omega RC)^2 + j\sqrt{2}\omega RC}$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{\sqrt{1 - 2(\omega RC)^2 + (\omega RC)^4 + 2(\omega RC)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^4}}$$

which is a second-order Butterworth low-pass function with cutoff frequency $\omega_c = \frac{1}{RC}$

Problem 15.30

Solution:

Known quantities:

The cutoff frequency, 15 kHz, and the DC gain, 15, for a second-order Butterworth low pass filter, with $Q = 5$ and $V_s = \pm 15$ V.

Find:

The design of the filter.

Analysis:

$$K = 1 + \frac{R_A}{R_B} = 15; \text{ choosing } R_A = 14 \text{ k}\Omega \Rightarrow R_B = 1 \text{ k}\Omega$$

$$f = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} = 15 \text{ kHz}$$

$$\left[\sqrt{\frac{R_2 C_2}{R_1 C_1}} + \sqrt{\frac{R_1 C_2}{R_2 C_1}} - 14 \sqrt{\frac{R_1 C_1}{R_2 C_2}} \right] = 0.2$$

Choose $C_1 = C_2 = 1 \mu\text{F}$ and solve for $R_1 = 2.6 \Omega$ and $R_2 = 43.9 \Omega$. Then, substitute the values thus obtained in the high-pass filter of Figure 15.22.

Problem 15.31

Solution:

Known quantities:

The low cutoff frequency, 200 Hz, the high cutoff frequency, 1 kHz, and the pass band gain, 4.

Find:

The value of Q for the filter, and the approximate frequency response of this filter.

Analysis:

$$Q = \frac{\sqrt{f_H f_L}}{f_H - f_L} = 0.6$$

$$K = 1 + \frac{R_A}{R_B} = 2; \text{ choosing } R_A = 1 \text{ k}\Omega \Rightarrow R_B = 1 \text{ k}\Omega$$

$$f = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} = 1 \text{ kHz}$$

Choose $R_1 = R_2$ and $C_1 = C_2 = 1 \mu\text{F}$

$R_1 = R_2 = 160 \Omega$. Then, substitute the obtained values in the high-pass filter of Figure 15.22.

$$K = 1 + \frac{R_A}{R_B} = 2; \text{ choosing } R_A = 1 \text{ k}\Omega \Rightarrow R_B = 1 \text{ k}\Omega$$

$$f = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} = 200 \text{ Hz}$$

Choose $R_1 = R_2$ and $C_1 = C_2 = 1 \mu\text{F}$

$R_1 = R_2 = 800 \Omega$. Then, substitute the obtained values in the high-pass filter of Figure 15.22.

By connecting the output of the high-pass filter to the input of the low-pass filter, we obtain the desired filter.

Problem 15.32

Solution:

Known quantities:

The circuit of Figure P15.29 and the cutoff frequency, 10 Hz, for a second-order Butterworth low pass filter.

Find:

The design of the filter.

Analysis:

$$f_c = 10 \text{ Hz} = \frac{1}{2\pi RC} \Rightarrow \omega_c = \frac{1}{RC} = 2\pi \cdot 10 = 20\pi \frac{\text{rad}}{\text{s}}$$

$$\text{Choose } R = 20 \text{ k}\Omega. \text{ Then, } \frac{1}{(20 \text{ k}\Omega)C} = 20\pi \Rightarrow C = \frac{1}{20 \cdot 10^3 \cdot 20\pi} = 796 \text{ nF}$$

and the two capacitors have values given by

$$\sqrt{2}C = 1.125 \mu\text{F} \quad \text{and} \quad \frac{C}{\sqrt{2}} = 563 \text{ nF}.$$

Problem 15.33

Solution:

Known quantities:

The low pass Sallen Key filter of Figure P15.33.

Find:

The voltage gain $\frac{V_{out}}{V_{in}}$ as a function of frequency and generate its Bode magnitude plot. Show that the cutoff frequency is $\frac{1}{2\pi RC}$ and that the low frequency gain is $\frac{R_4}{R_3}$.

Analysis:

We have:

$$V_{AB} = \frac{R_4}{R_3 + R_4} V_{out}$$

$$i_1 = j\omega C(V_{out} - V), \quad i_2 = \frac{V_s - V}{R}, \quad i_3 = \frac{V - V_{AB}}{R}.$$

From $i_2 = j\omega C V_{AB}$, we have: $V = V_{AB}(1 + j\omega RC)$

From $i_1 + i_3 = i_2$, we have:

$$V_s = V_{out} \left\{ \left(\frac{R_4}{R_3 + R_4} \right) [(2 + j\omega RC)(1 + j\omega RC) - 1] - j\omega RC \right\}$$

Therefore, the frequency response is:

$$\frac{V_{out}}{V_s}(j\omega) = \frac{K \frac{1}{(RC)^2}}{(j\omega)^2 + \left[\frac{1}{RC}(3 - K) \right](j\omega) + \frac{1}{(RC)^2}} \quad \text{where: } K = 1 + \frac{R_3}{R_4}$$

Problem 15.34

Solution:

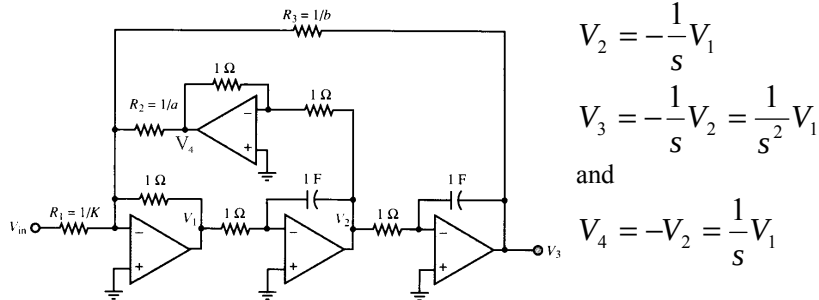
Known quantities:

The circuit shown in Figure P15.34

Find:

The transfer functions relating each of the three outputs to the input V_{in} .

Analysis:



$$V_2 = -\frac{1}{s} V_1$$

$$V_3 = -\frac{1}{s} V_2 = \frac{1}{s^2} V_1$$

and

$$V_4 = -V_2 = \frac{1}{s} V_1$$

Using Laplace transforms, note that

Writing a KCL equation at the inverting input to the leftmost op-amp,

$$-KV_{in} - V_1 - aV_4 - bV_3 = 0 \text{ or } -KV_{in} - V_1 - \frac{a}{s}V_1 - \frac{b}{s^2}V_1 = 0 \Rightarrow \frac{V_1}{V_{in}} = -\frac{Ks^2}{s^2 + as + b}$$

which is a second-order high-pass function.

Also,

$$\frac{V_2}{V_{in}} = -\frac{1}{s} \frac{V_1}{V_{in}} = -\frac{Ks}{s^2 + as + b}$$

which is a bandpass function, and

$$\frac{V_3}{V_{in}} = \frac{1}{s^2} \frac{V_1}{V_{in}} = -\frac{K}{s^2 + as + b}$$

which is a low-pass function.

Problem 15.35

Solution:

Known quantities:

The filter shown in Figure P15.35.

Find:

Verify that the filter's frequency response has the following expression:

$$H(j\omega) = \frac{-(1/R_3 R_2 C_1 C_2) R_3 / R_1}{(j\omega)^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_3 C_1} \right) j\omega + \frac{1}{R_3 R_2 C_1 C_2}}$$

Analysis:

The circuit is shown below:

We have:

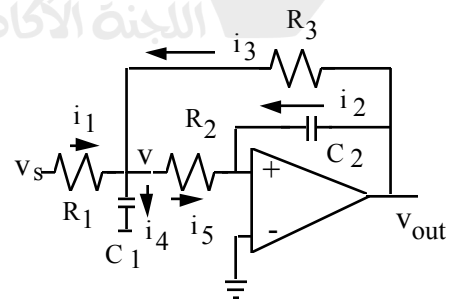
$$i_1 = \frac{V_s - V}{R_1}, \quad i_2 = j\omega C_2 V_{out}, \quad i_3 = \frac{V_{out} - V}{R_3}, \quad i_4 = j\omega C_1 V,$$

$$i_5 = \frac{V}{R_2}.$$

From $i_5 = -i_2$, we have: $V = -j\omega R_2 C_2 V_{out}$

From $i_1 + i_3 = i_5 + i_4$, we have:

$$\frac{V_s}{R_1} + \frac{j\omega C_2 R_2}{R_1} V_{out} + \frac{V_{out}}{R_3} + \frac{j\omega C_2 R_2}{R_3} V_{out} = -(j\omega)^2 C_1 C_2 R_2 V_{out} - j\omega C_2 V_{out}$$



Therefore, the frequency response is:

$$H(j\omega) = \frac{-(1/R_3 R_2 C_1 C_2) R_3 / R_1}{(j\omega)^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_3 C_1} \right) j\omega + \frac{1}{R_3 R_2 C_1 C_2}}$$

Problem 15.36

Solution:

Known quantities:

The filter shown in Figure P15.36.

Find:

Verify that the filter's frequency response has the following expression:

$$H(j\omega) = \frac{j\omega K / R_1 C_1}{(j\omega)^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_3 C_2} + \frac{1}{R_3 C_1} + \frac{1-K}{R_2 C_1} \right) j\omega + \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}$$

Analysis:

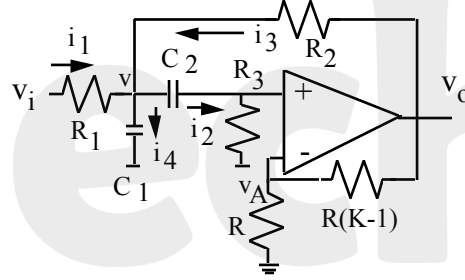
The circuit is shown below:

We have:

$$V_{AB} = \frac{R}{R + R(K-1)} V_{out} = \frac{V_{out}}{K}$$

$$i_1 = \frac{V_i - V}{R_1}, i_2 = j\omega C_2 (V - V_A) = \frac{V_A}{R_3},$$

$$i_3 = \frac{V_{out} - V}{R_2}, i_4 = j\omega C_1 V.$$



From $i_2 = j\omega C_2 (V - V_A) = \frac{V_A}{R_3}$, we have: $V = V_A \left(1 + \frac{1}{j\omega R_3 C_2} \right) = \frac{V_{out}}{K} \left(1 + \frac{1}{j\omega R_3 C_2} \right)$

From $i_1 + i_3 = i_2 + i_4$, we have:

$$\frac{V_i}{R_1} - \left(\frac{1}{j\omega C_2 R_3} + 1 \right) \frac{V_{out}}{K R_1} + \frac{V_{out}}{R_2} - \left(\frac{1}{j\omega C_2 R_3} + 1 \right) \frac{V_{out}}{K R_2} = \frac{V_{out}}{K R_3} + \left(\frac{j\omega C_1}{j\omega C_2 R_3} + j\omega C_1 \right) \frac{V_{out}}{K}$$

Therefore, the frequency response is:

$$H(j\omega) = \frac{j\omega K / R_1 C_1}{(j\omega)^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_3 C_2} + \frac{1}{R_3 C_1} + \frac{1-K}{R_2 C_1} \right) j\omega + \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}$$

where: $K = 1 + \frac{R_A}{R_B}$

Problem 15.37**Solution:****Known quantities:**

The filter shown in Figure P15.35.

Find:

Verify that for the filter:

$$\frac{1}{Q} = \sqrt{R_2 R_3} \frac{C_2}{C_1} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

Analysis:

From the expression, we can see that

$$\omega_c = \sqrt{\frac{1}{C_1 C_2 R_2 R_3}} \quad \text{and} \quad \frac{\omega_c}{Q} = \frac{1}{C_1 R_1} + \frac{1}{C_1 R_2} + \frac{1}{C_1 R_3}$$

$$\text{or } \frac{1}{Q} = \sqrt{C_1 C_2 R_2 R_3} \left(\frac{1}{C_1 R_1} + \frac{1}{C_1 R_2} + \frac{1}{C_1 R_3} \right)$$

$$\text{or } \frac{1}{Q} = \sqrt{R_2 R_3} \frac{C_2}{C_1} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

Section 15.4: Analog-to-Digital and Digital-to-Analog Conversion

Problem 15.38

Solution:

Find:

List two advantages of digital signal processing over analog signal processing.

Analysis:

1. Digital signals are less subject to noise, since one only needs to discriminate between two voltages.
2. Digital signals are directly compatible with digital computers, and can therefore be easily stored on a disk, or exchanged between computers. Thus, digital signals are intrinsically more portable than analog signals.

Problem 15.39

Solution:

Find:

Discuss the role of a multiplexer in a data acquisition system.

Analysis:

It sequentially switches a set of analog inputs to the system input.

Problem 15.40

Solution:

Known quantities:

The circuit shown in Figure P15.40.

Find:

Explain the operation of the circuit.

Analysis:

Op-amp #1 is an input buffer. The JFET behaves as a low-leakage diode which enables and disables the RC holding circuit, and op-amp #2 is a voltage-follower whose purpose is to isolate the circuit from the load.

Problem 15.41

Solution:

Known quantities:

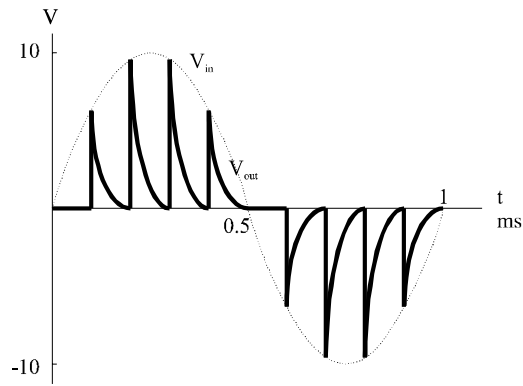
The circuit shown in Figure P15.40. The input is a 1 kHz sinusoidal signal with 0° phase angle, 0 V DC offset and 20 V peak to peak amplitude. V_G is a rectangular pulse train with 10 μs , and a period 100 μs , with a leading edge of the first pulse at $t=0$.

Find:

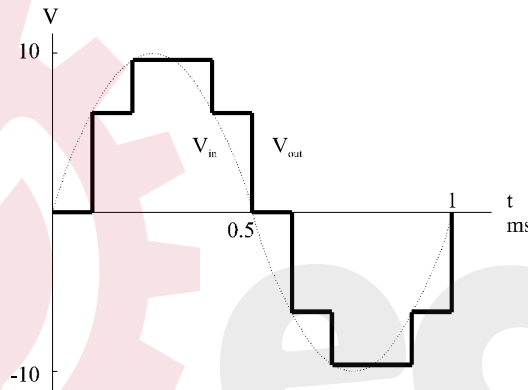
- a) Sketch V_{out} if the RC circuit has a time constant equal to 20 μs .
- b) Sketch V_{out} if the RC circuit has a time constant equal to 1 ms.

Analysis:

- a)



b)

**Problem 15.42****Solution:****Known quantities:**

The input to a four-digit DAC, 12_{10} , given that $R_F = R_0/15$, logic 0 corresponds to 0 V, and logic 1 corresponds to 4.5 V.

Find:

- The output of the DAC.
- The maximum voltage that can be outputted from the DAC.
- The resolution over the range 0 to 4.5 V.
- The number of bits required if an improved resolution of 20 mV is desired.

Analysis:

$$V_a = -4.5 \frac{R_F}{R_0} [2^3 b_3 + 2^2 b_2 + 2^1 b_1 + b_0]$$

$$\text{a) } V_a = -4.5 \frac{1}{15} [12] = -3.6 \text{ V}$$

$$\text{b) } (V_a)_{\max} = -4.5 \frac{1}{15} (15) = -4.5 \text{ V}$$

$$\text{c) } \delta V_a = 4.5 \frac{1}{15} = 0.3 \text{ V}$$

$$d) \quad n \geq \frac{\log \left(\frac{|(V_a)_{\max} - (V_a)_{\min}|}{(\delta V_a)_{\text{req}}} + 1 \right)}{\log 2} = 7.82$$

Therefore, we choose $n = 8$.

Problem 15.43

Solution:

Known quantities:

The input to a eight-digit DAC, 215_{10} , given that $R_F = R_0/255$, logic 0 corresponds to 0 V, and logic 1 corresponds to 10 V.

Find:

- The output of the DAC.
- The maximum voltage that can be outputted from the DAC.
- The resolution over the range 0 to 10 V.
- The number of bits required if an improved resolution of 3 mV is desired.

Analysis:

$$215_{10} = 11010111_2$$

$$V_a = -10 \frac{R_F}{R_0} [2^7 b_7 + 2^6 b_6 + \dots + 2^1 b_1 + b_0]$$

$$a) \quad V_a = -10 \frac{1}{255} [215] = -8.341 \text{ V}$$

$$b) \quad (V_a)_{\max} = -10 \frac{1}{255} [255] = -10 \text{ V}$$

$$c) \quad \delta V_a = 10 \frac{1}{255} = 39.2 \text{ mV}$$

$$d) \quad n \geq \frac{\log \left(\frac{|(V_a)_{\max} - (V_a)_{\min}|}{(\delta V_a)_{\text{req}}} + 1 \right)}{\log 2} = 11.703$$

Therefore, we choose $n = 12$.

Problem 15.44

Solution:

Known quantities:

The circuit of Figure P15.45, a simple 4-bit DAC. In the circuit if the bit is 1, the corresponding switch is up, if the bit is 0 the switch is down.

Find:

If the digital number is represented by $b_3 b_2 b_1 b_0$, determine an expression relating v_0 to the binary input bits..

Analysis:

This circuit is just a summing amplifier, with

$$\begin{aligned}
 v_O &= -\frac{R_2}{R_1}b_3V - \frac{R_2}{2R_1}b_2V - \frac{R_2}{4R_1}b_1V - \frac{R_2}{8R_1}b_0V = -\frac{R_2V}{R_1}\left(b_3 + \frac{b_2}{2} + \frac{b_1}{4} + \frac{b_0}{8}\right) \\
 &= -\frac{R_2V}{8R_1}(8b_3 + 4b_2 + 2b_1 + b_0)
 \end{aligned}$$

Problem 15.45

Solution:

Known quantities:

The input to a eight-digit DAC, 98_{10} , given that $R_F = R_0/255$, logic 0 corresponds to 0 V, and logic 1 corresponds to 4.5 V.

Find:

- The output of the DAC.
- The maximum voltage that can be outputted from the DAC.
- The resolution over the range 0 to 4.5 V.
- The number of bits required if an improved resolution of 0.5 mV is desired.

Analysis:

$$a) \quad V_a = 4.5 \frac{1}{255} [98] = 1.729 \text{ V}$$

$$b) \quad (V_a)_{\max} = 4.5 \frac{1}{255} [255] = 4.5 \text{ V}$$

$$c) \quad \delta V_a = 4.5 \frac{1}{255} = 17.6 \text{ mV}$$

$$d) \quad n \geq \frac{\log \left(\frac{|(V_a)_{\max} - (V_a)_{\min}|}{(\delta V_a)_{\text{req}}} + 1 \right)}{\log 2} = 13.136$$

Therefore, we choose $n = 14$.

Problem 15.46

Solution:

Known quantities:

The four-digit DAC of Figure P15.46, with an output range $-10 \leq V_O \leq 0 \text{ V}$.

Find:

The value of R_F that will give that output.

Assumptions:

The logic 0 = 0 V and the logic 1 = 5 V.

Analysis:

For the circuit of Figure P15.46, $R_O = 1 \text{ k}\Omega$ and $n = 4$.

Therefore, $(V_a)_{\max} = -10 \text{ V}$, and $-10 = -5 \frac{R_f}{1000} 15$, or $R_f = 133.3 \Omega$.

$$d) \quad n \geq \frac{\log \left(\frac{|(V_a)_{\max} - (V_a)_{\min}|}{(\delta V_a)_{\text{req}}} + 1 \right)}{\log 2} = 14.288$$

Therefore, we choose a 15-bit ADC.

Problem 15.50

Solution:

Known quantities:

The four-digit DAC of Figure P15.46, with an output range $-15 \leq V_o \leq 0 \text{ V}$.

Find:

The value of R_F that will give that output.

Assumptions:

The logic 0 = 0 V and the logic 1 = 5 V.

Analysis:

Here, $-15 \leq V_o \leq 0 \text{ V}$. For the circuit of Figure P15.46, $R_o = 1 \text{ k}\Omega$ and $n = 4$.

Therefore, $(V_a)_{\max} = -15 \text{ V}$, and $-15 = -5 \frac{R_f}{1000} 15$, or $R_f = 200 \Omega$.

Problem 15.51

Solution:

Known quantities:

The circuit of Figure P15.44, and the desired output of the 4-bit DAC,

$$V_o = -\frac{1}{10} (8b_3 + 4b_2 + 2b_1 + b_0) \text{ V}.$$

Find:

The design of the DAC.

Analysis:

From the results of Problem 15.44, we see that we must choose V_0 , R_2 and R_1 such that: $\frac{R_2 \cdot V_0}{8 \cdot R_1} = \frac{1}{10}$

One possible choice is to let $V = 15 \text{ V}$, $R_1 = 30 \text{ k}\Omega$ and $R_2 = 1.6 \text{ k}\Omega$.

Problem 15.52

Solution:

Known quantities:

The range, $\pm 15 \text{ V}$, and the resolution, 0.01 V , of a data acquisition system.

Fin

The number of bits of the DAC.

Analysis:

$$n \geq \log_2 \left(\frac{15 - (-15)}{0.01} + 1 \right) = 11.55. \text{ Choose } n = 12.$$

Problem 15.53**Solution:****Known quantities:**

The range, ± 10 V, and the resolution, 0.04 V, of a data acquisition system.

Find:

The number of bits of the DAC.

Analysis:

$$n \geq \log_2 \left(\frac{10 - (-10)}{0.04} + 1 \right) = 8.96. \text{ Choose } n = 9.$$

Problem 15.54**Solution:****Known quantities:**

The range, $-10 - 15$ V, and the resolution, 0.004 V, of a data acquisition system.

Find:

The number of bits of the DAC.

Analysis:

$$n \geq \log_2 \left(\frac{15 - (-10)}{0.004} + 1 \right) = 12.6. \text{ Choose } n = 13.$$

Problem 15.55**Solution:****Known quantities:**

The range, 0 – 2,500 rev/min, and the resolution, 1 rev/min, of a DAC used to deliver velocity commands to a motor.

Find:

The number of bits of the DAC.

Analysis:

$$n \geq \log_2 \left(\frac{2500 - (0)}{1} + 1 \right) = 11.29. \text{ Choose } n = 12.$$

With this choice we compute the following resolution: $res = \frac{2500}{2^{12}} = 0.61 \text{ rev/min}$

Problem 15.56**Solution:****Known quantities:**

The range, 0 – 10 V, for an ADC.

Find:

- a) The resolution if this is a 3-bit device.
- b) The resolution if this is a 8-bit device.
- c) A general comment about the relationship between the number of bits and the resolution.

Analysis:

- a) $res = 2^{-3} \cdot 10 \text{ V} = 1.25 \text{ V}$.
- b) $res = 2^{-8} \cdot 10 \text{ V} = 39.0625 \text{ mV}$.
- c) more bits give better resolution.

Problem 15.57**Solution:****Known quantities:**

The range, $-5 - 15 \text{ V}$, and the resolution required, 0.05% , for a DAC.

Find:

The number of bits required.

Analysis:

The range is $15 - (-5) = 20 \text{ V}$

Thus, $n \geq \log_2 \left(\frac{20}{20 \cdot 0.0005} + 1 \right) = 10.97$. Choose $n = 11$.

Problem 15.58**Solution:****Known quantities:**

The number of channels of an ADC, eight, the time required for ADC conversion, $100 \mu\text{s}$, the time required for computation and output time for four of the channels, $500 \mu\text{s}$, and for the other four, $250 \mu\text{s}$.

Find:

The number of bits required.

Analysis:

We assume a data acquisition system of the type shown in Figure 15.32. Therefore, each channel will be sampled at $\frac{1}{8}$ of the external clock rate and the slowest channels will determine the rate.

Thus, $\text{sampling rate} = 8(100 \mu\text{s} + 500 \mu\text{s}) = 4.8 \text{ ms}$.

Thus, $f_s = \frac{1}{4.8 \cdot 10^{-3}} = 208.3 \text{ Hz}$ and $f_{\max} = \frac{f_s}{2} = 104.15 \text{ Hz}$

Problem 15.59**Solution:****Known quantities:**

The range of the potentiometer, 270° and 10 V , and the maximum displacement to be measured, 180° .

Find:

- a) The voltage to be resolved by an ADC to resolve an angular displacement of 0.5° , and the number of bits to do that.

- b) If the ADC requires a 10 V input voltage find the optimum amplifier gain to take advantage of the full range of the ADC.

Analysis:

For a dynamic range of 10 V for 270° of rotation, we compute the following resolution:

$$res = \frac{10}{270} 0.5 = 18.52 \text{ mV}$$

- a) Finding the range for 180° rotation it is possible to determine the bits requirement for the ADC.

$$n = \log_2 \left(\frac{10 \left(\frac{180}{270} \right)}{18.52 \cdot 10^{-3}} + 1 \right) = 8.50, \text{ and we choose } n = 9.$$

- b) The voltage gain of the amplifier is

$$gain = \frac{270}{180} = 1.5$$

Problem 15.60**Solution:****Known quantities:**

The maximum frequency of the signal to be digitized, 250 kHz, and the number of bits, 10, of the successive approximation ADC used.

Find:

The maximum permissible conversion time.

Analysis:

The conversion time should be no more than 10% of the signal period. For this case, the signal period is

$$T = \frac{1}{250 \cdot 10^3} = 4 \mu\text{s}$$

Therefore, the conversion time should be no longer than 400 ns.

Problem 15.61**Solution:****Known quantities:**

The maximum frequency of the signal to be digitized, a torque signal from a torque sensor mounted on a farm tractor engine, is twice the shaft rotation frequency; the rotational speed of the crankshaft is 800 rpm.

Find:

The minimum sampling period according to Nyquist criterion.

Analysis:

Shaft rotation frequency is 800 rpm or 13.33 rev/sec; therefore, the fluctuation frequency is 26.67 Hz:

$$T_s = \frac{1}{26.67 \cdot 2} = 18.75 \text{ ms}$$

Problem 15.62**Solution:****Known quantities:**

The range of an aircraft altimeter, from 0 V at 0 m to 10 V at 10000m, and the allowable error in sensing, 10 m.

Find:

The minimum number of bits for the ADC.

Analysis:

The allowable error is ± 10 m. Therefore, an equivalent 20 m step size is allowable and

$$n = \log_2 \left(\frac{10000}{20} + 1 \right) = 8.97$$

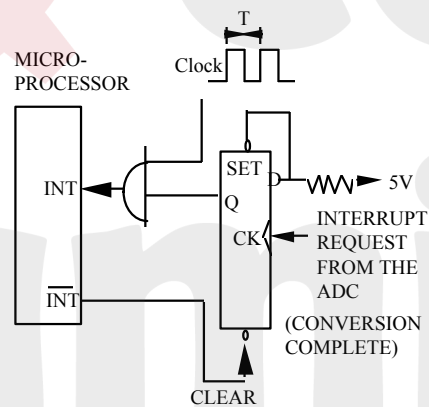
Thus, a 9-bit ADC is required.

Problem 15.63**Solution:****Known quantities:**

The characteristic of the circuit needed, a circuit that generates interrupts at fixed time intervals.

Find:

The design of the circuit using a square wave that has a period equal to the desired time interval between interrupts.

Analysis:**Problem 15.64****Solution:****Find:**

The minimum number of bits required to digitize an analog signal with a resolution of:

- a) 5%. b) 2%. c) 1%.

Analysis:

a) 5% $\Rightarrow 2^{-n} \leq 0.05 \Rightarrow n = 5$

b) 2% $\Rightarrow 2^{-n} \leq 0.02 \Rightarrow n = 6$

c) 1% $\Rightarrow 2^{-n} \leq 0.01 \Rightarrow n = 7$

Section 15.5: Comparator and Timing Circuits

Problem 15.65

Solution:

Known quantities:

The window comparator circuit of Figure P15.65.

Find:

Show that $V_{out} = 0$ whenever $V_{low} \leq V_{in} \leq V_{high}$, and that $V_{out} = V_{in}$ otherwise.

Analysis:

If $V_{in} \leq V_{low}$,

$$\left. \begin{array}{l} V_1 = 0 \\ V_2 = V^+ \end{array} \right\} \Rightarrow V_{out} = V^+$$

If $V_{low} \leq V_{in} \leq V_{high}$,

$$\left. \begin{array}{l} V_1 = 0 \\ V_2 = 0 \end{array} \right\} \Rightarrow V_{out} = 0$$

If $V_{in} \geq V_{high}$,

$$\left. \begin{array}{l} V_1 = V^+ \\ V_2 = 0 \end{array} \right\} \Rightarrow V_{out} = V^+$$

Problem 15.66

Solution:

Known quantities:

The noise peak amplitude, ± 150 mV, the reference value around which the circuit is to switch, -1 V, and the characteristic of the op-amp, ± 10 V supplies ($V_{sat} = 8.5$ V).

Find:

The design of the circuit.

Analysis:

This is very similar to Example 15.14, Therefore,

$$v^+ = \frac{R_2}{R_2 + R_1} v_{out} + \frac{R_2}{R_2 + R_3} v_s^+$$

Since the required noise protection level is ± 150 mV, R_1 and R_2 can be computed from:

$$\frac{\Delta v}{2} = \frac{R_2}{R_2 + R_1} v_{sat} = \frac{R_2}{R_2 + R_1} 8.5 = 0.15 \text{ V where } \Delta v = 300 \text{ mV}$$

Assuming $R_1 = 100 \text{ k}\Omega$, R_2 can be calculated to be approximately $1.8 \text{ k}\Omega$. Since the required reference voltage is -1 V, we can find R_3 by solving the equation

$$\frac{R_2}{R_2 + R_3} v_s^- = \frac{R_2}{R_2 + R_3} 10 = 1 \text{ V to obtain: } R_3 = 16.2 \text{ k}\Omega.$$

Problem 15.67

Solution:

Known quantities:

The circuit of Figure P15.67; $R_1 = 100 \text{ }\Omega$, $R_2 = 56 \text{ k}\Omega$, $R_i = R_1 \parallel R_2$, and v_{in} is a 1 V peak to peak sine wave.

Find:

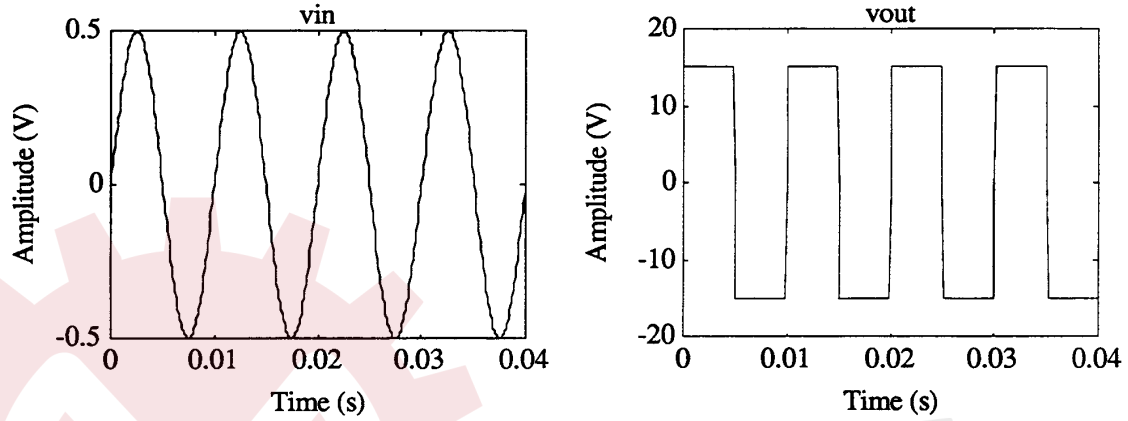
The threshold voltages and the output waveform.

Assumptions:

The supply voltages are ± 15 V.

Analysis:

Applying KCL: $\frac{V_o}{V_{in}} = 1 + \frac{R_2}{R_1}$. Therefore, $\frac{V_o}{V_{in}} = 561$

**Problem 15.68****Solution:****Known quantities:**

The circuit of Figure P15.68.

Find:

Explain the operation of the circuit.

Analysis:

Operation of this circuit depends on the magnitude of the input voltage, V_{in} , being much larger than zero.

When $V_{in} \gg 0$, the op-amp will be saturated, and V_{out} will be at its saturation limit, V_R (positive rail).

When the op-amp is saturated, the approximation $V_+ = V_-$ is no longer valid. Writing KCL at the non-inverting input, we have

$$\frac{V_+ - V_{in}}{R_{in}} + \frac{V_+ - V_{out}}{R_F} = 0 \quad \text{or} \quad V_+ \left(\frac{1}{R_{in}} + \frac{1}{R_F} \right) = \frac{1}{R_{in}} V_{in} + \frac{1}{R_F} V_{out}$$

The output changes from positive rail ($+V_R$) to negative rail ($-V_R$) when $V_+ = V_- = 0$. When V_{in} is

small, then $V_+ \approx V_- = 0$, and $V_{out} = -\left(\frac{R_F}{R_{in}}\right)V_{in}$

$$\text{If } V_{out} = V_R, \text{ then: } V_{in} = -\left(\frac{R_{in}}{R_F}\right)V_R$$

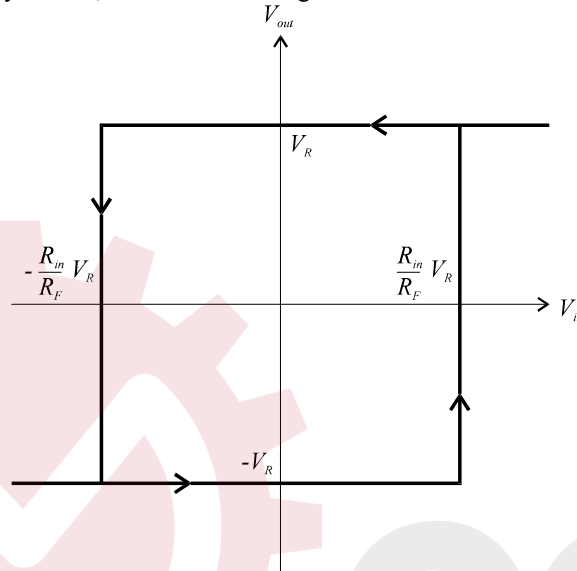
What this means is the following: As V_{in} drops from a large positive value, through zero, to a negative

value, the output of the op-amp “switches” (from $+V_R$ to $-V_R$) at the point $V_{in} = -\left(\frac{R_{in}}{R_F}\right)V_R$.

Similarly, as V_{in} increases in the opposite direction, the “switch” from $-V_R$ to $+V_R$ occurs at

$$V_{in} = +\left(\frac{R_{in}}{R_F}\right)V_R.$$

This is a form of voltage hysteresis, as shown in the figure below.



Problem 15.69

Solution:

Known quantities:

The circuit of Figure P15.68. The op-amp is a LM741 with $\pm 15\text{ V}$, $R_F = 104\text{ k}\Omega$, and V_{in} is a 1 kHz sinusoidal signal with 1 V amplitude.

Find:

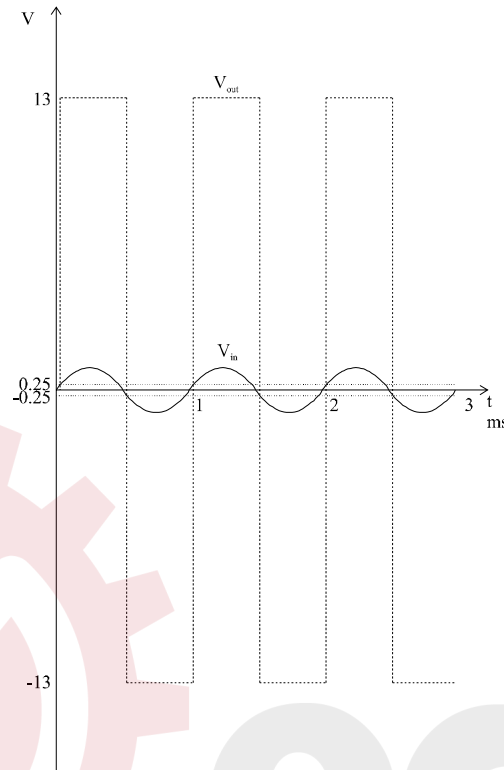
- The appropriate value for R_{in} if the output is to be high whenever $|V_{in}| \geq 0.25\text{ V}$.
- Sketch the input and the output waveforms.

Analysis:

a) We know that $V_R \approx 13\text{ V}$ when the LM741 op-amp is used with $\pm 15\text{ V}$ bias supplies. Then, from the discussion in the answer to Problem 15.69,

$$V_{in} = -\left(\frac{R_{in}}{R_F}\right)V_{out} \Rightarrow R_{in} = \left(\frac{V_{in}}{V_{out}}\right)R_F = \left(\frac{0.25}{13}\right)(104\text{ k}\Omega) = 2\text{ k}\Omega$$

- The input and output waveforms are sketched below.



Problem 15.70

Solution:

Known quantities:

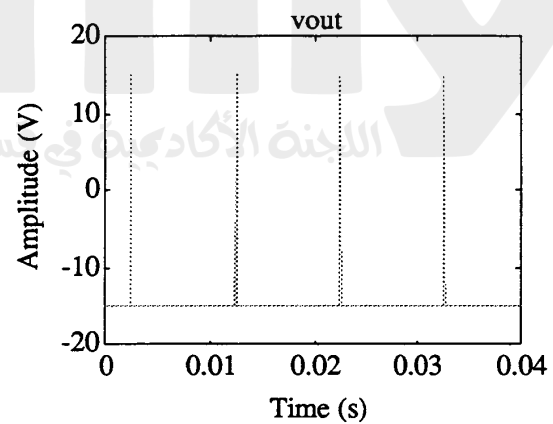
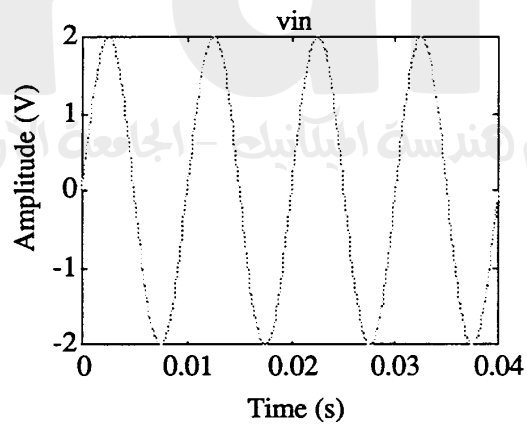
The circuit of Figure P15.70.

Find:

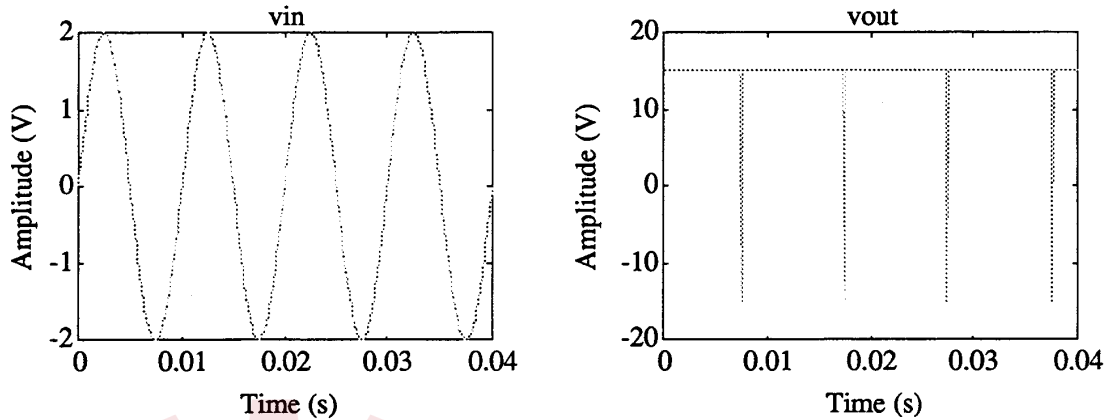
- The output waveform for v_{in} a 4 V peak to peak sine wave at 100 Hz and $V_{ref} = 2$ V.
- The output waveform for v_{in} a 4 V peak to peak sine wave at 100 Hz and $V_{ref} = -2$ V.

Analysis:

a)



b)



Problem 15.71

Solution:

Known quantities:

The go-no go detector application circuit of Figure P15.71.

Find:

- Explain how the circuit works.
- Design a circuit such that the green LED will turn on when V_{in} exceeds 5 V, and the LED will be on whenever V_{in} is less than 5 V.

Assumptions:

Only 15 V supplies are available.

Analysis:

- Define $V_2 = \frac{R_1}{R_2 + R_1} V$ as the voltage at the inverting input of the op-amp. Then:

When $V_{in} > V_2$ the output of the op-amp will be positive and the green LED will turn on (*go*).

When $V_{in} < V_2$ the output of the op-amp will be negative and the red LED will turn on (*no go*).

- For this design, $V_2 = 5$ V and $V = 15$ V.

$$\frac{R_1}{R_2 + R_1} 15 \text{ V} = 5 \text{ V} \Rightarrow \frac{R_1}{R_2 + R_1} = \frac{1}{3}$$

$$\text{or } \frac{R_2}{R_1} + 1 = 3 \Rightarrow \frac{R_2}{R_1} = 2$$

Choose $R_1 = 10 \text{ k}\Omega$ and $R_2 = 20 \text{ k}\Omega$ to complete the design.

Problem 15.72

Solution:

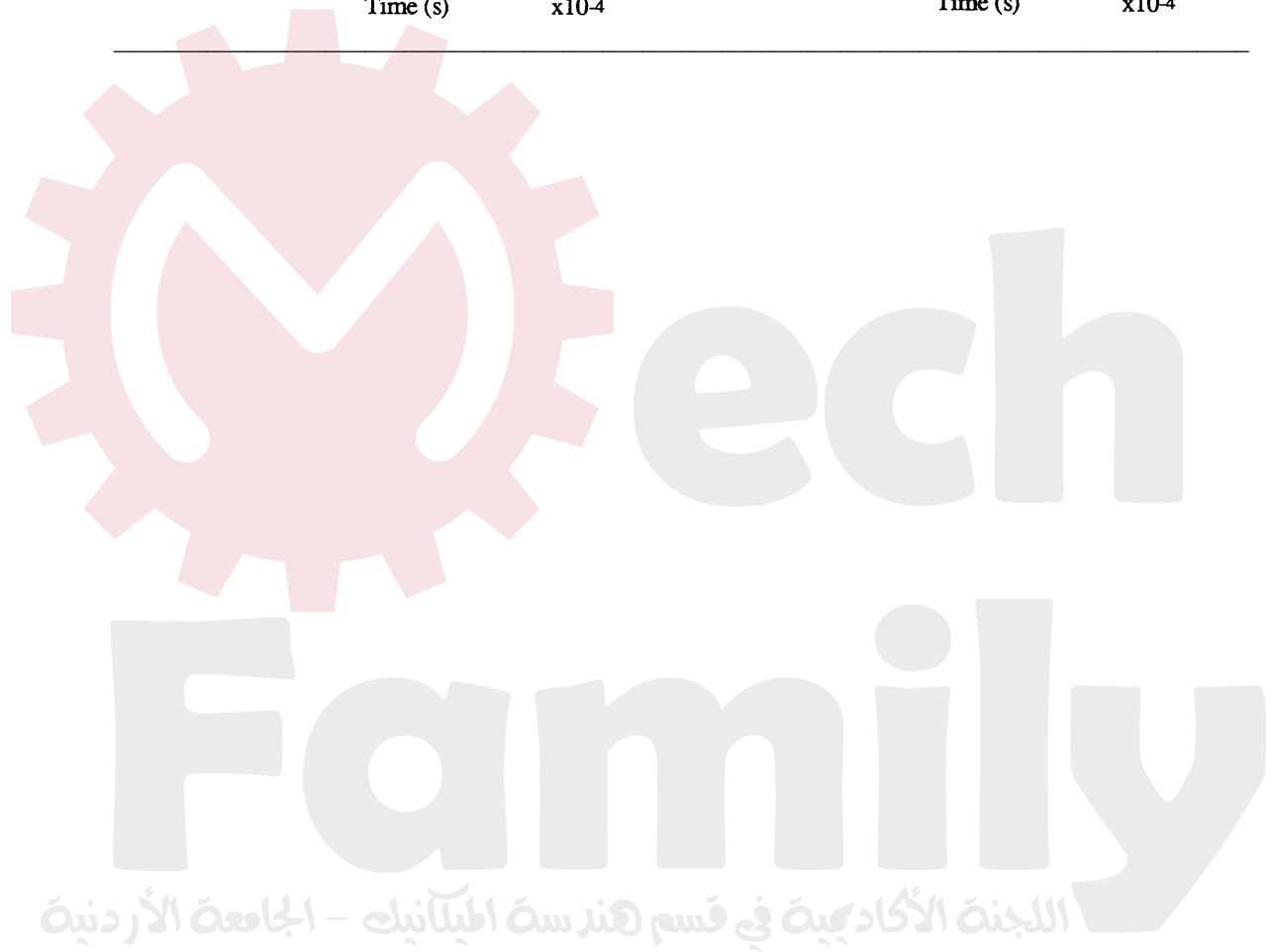
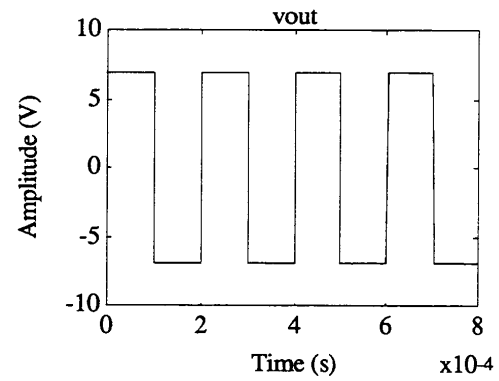
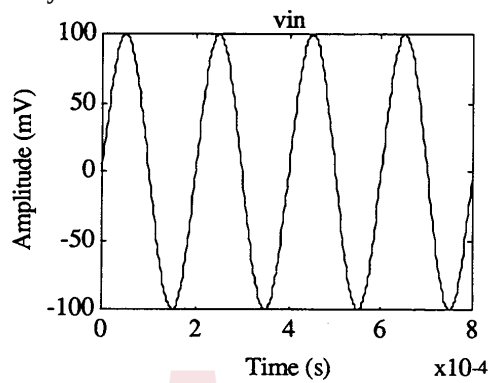
Known quantities:

The circuit of Figure P15.72, where v_{in} is a 100 mV peak sine wave at 5 kHz, $R = 10 \text{ k}\Omega$, and $D1$ and $D2$ are 6.2 V Zener diodes.

Find:

Draw the output voltage waveform.

Analysis:



Section 15.7: Data Transmission in Digital Instruments

Problem 15.73

Solution:

Known quantities:

An ASCII (hex) encoded message.

Find:

Decode the message.

Analysis:

ASCII decoding is easy!

Problem 15.74

Solution:

Known quantities:

An ASCII (binary) encoded message.

Find:

Decode the message.

Analysis:

This is a time-consuming problem.

Problem 15.75

Solution:

Known quantities:

Some decimal numbers.

Find:

The ASCII form for the numbers.

Analysis:

<u>Decimal</u>	<u>ASCII</u>
12	31 32
345.2	33 34 35 2E 32
43.5	34 33 2E 35

Problem 15.76

Solution:

Known quantities:

Some words.

Find:

The ASCII form for the words.

Analysis:

- a) 44 69 67 69 74 61 6C
- b) 43 6F 6D 70 75 74 65 72
- c) 41 73 63 69 69

d) 41 53 43 49 49

Problem 15.77

Solution:

Find:

Explain why data transmission over long distances is usually done via a serial scheme rather than parallel.

Analysis:

Serial data transmission requires only a single data path. Parallel requires 16 (or more, depending on word length), and would, therefore, be much more expensive.

Problem 15.78

Solution:

Known quantities:

The on-board memory of an automated data-logging, 16 K-words, that samples the variable of interest once every 5 min.

Find:

How often must the data be downloaded and the memory cleared in order to avoid losing any data.

Analysis:

$$\begin{aligned} \text{Longest possible delay} &= 16 \cdot 1024 \cdot 5 = 81920 \text{ min} = \\ &= 81920 \text{ min} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ hr}} \approx 56.9 \text{ days} \end{aligned}$$

Problem 15.79

Solution:

Find:

Explain why three wires are required for the handshaking technique employed by IEEE 488 bus system.

Analysis:

Three lines are used for handshaking in the IEEE 488 bus to accomplish the following functions: One line is used to declare the bus ready to accept data; another line to declare that data has been accepted, and a third one to declare that the data was indeed valid.

Problem 15.80

Solution:

Known quantities:

The information held in a CD-ROM, 650 MB. The CD-ROM are packaged 50 per box, and 100 boxes are shipped. The distance for the trip is 2,500 miles and the airplane speed is 400 mi/hr.

Find:

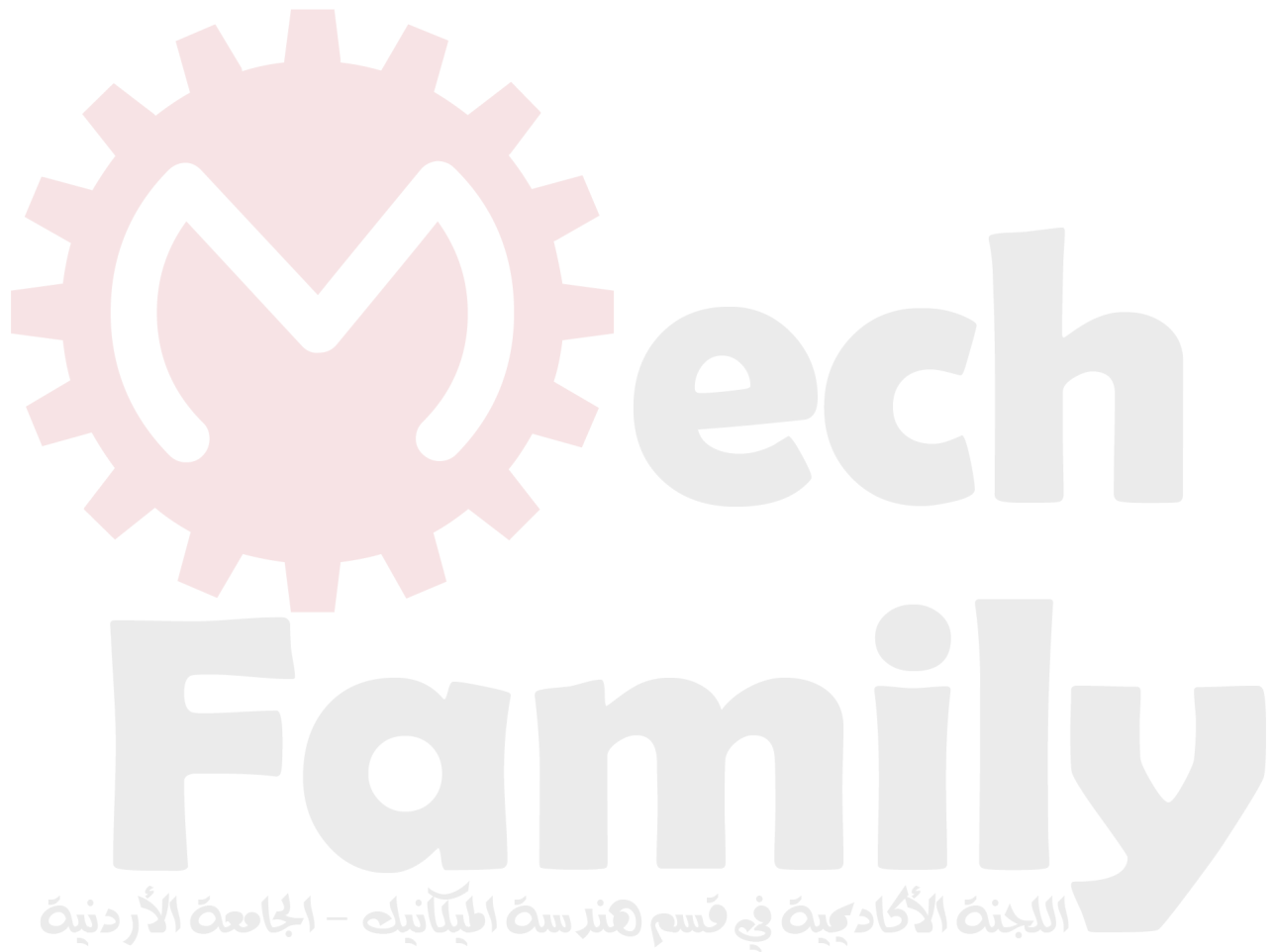
The transmission rate between the two cities in bits/s.

Analysis:

$$\frac{650 \cdot 1024 \cdot 1024 \frac{\text{bytes}}{\text{CD}} \cdot 50 \frac{\text{CDs}}{\text{box}} \cdot 100 \text{ boxes} \cdot 400 \frac{\text{mi}}{\text{hr}}}{2500 \text{ mi}} = 5.4525952 \cdot 10^{11} \frac{\text{bytes}}{\text{hr}}$$

$$5.4525952 \cdot 10^{11} \frac{\text{bytes}}{\text{hr}} \cdot 8 \frac{\text{bits}}{\text{byte}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \approx 1.21 \cdot 10^9 \frac{\text{bits}}{\text{s}}$$

or approximately $1.13 \frac{\text{Gbits}}{\text{s}}$.



Chapter 16 Instructor Notes

The last part of the book presents an introduction to electro-magneto-mechanical systems. Some of the foundations needed for this material (AC power) were discussed in Chapter 7; the polyphase AC power material in Chapter 7 may be introduced prior to covering Chapter 17, or together with the AC machine material of Chapter 17.

The emphasis in this chapter (and the next two) is on preparing the student for the use of electro-magneto-mechanical systems as practical actuators for industrial applications. Thus, more emphasis is placed on describing the performance characteristics of linear motion actuators and of rotating machines than on a description of their construction details. The material in Chapters 16-18 has been used by several instructors over the last several years in a second (quarter-length) course in system dynamics (System Dynamics and Electromechanics) designed for mechanical engineering juniors.

Section 16.1 reviews basic laws of electricity and magnetism, which should already be familiar to the student from an earlier Physics course. The box *Focus on Measurements: Linear Variable Differential Transformer* (pp. 795-796) presents an example related to sensors with a discussion of the LVDT as a position transducer. Section 16.2 discusses approximate linear magnetic circuits and the idea of reluctance, and introduces magnetic structures with air gaps and simple electro-magnets. The box *Focus on Methodology: Magnetic Structures and Magnetic Equivalent Circuits* (p. 804) summarizes the analysis methods used in this section. A magnetic reluctance position sensor is presented in *Focus on Measurements: Magnetic Reluctance Position Sensor* (pp. 811-812) and *Focus on Measurements: Voltage Calculation in Magnetic Reluctance Position Sensor* (pp. 812-814). The non-ideal properties of magnetic materials are presented in Section 16.3, where hysteresis, saturation, and eddy currents are discussed qualitatively. Section 16.4 introduces simple models for transformers; more advanced topics are presented in the homework problems.

Section 16.5 is devoted to the analysis of forces and motion in electro-magneto-mechanical structures characterized by linear motion. The boxes *Focus on Methodology: Analysis of Moving-Iron Electromechanical Transducers* (pp. 823-824) and *Focus on Methodology: Analysis of Moving-Coil Electromechanical Transducers* (p. 836) summarize the analysis methods used in this section. The author has found that it is pedagogically advantageous to introduce the *Bli* and *Blu* laws for linear motion devices before covering these concepts for rotating machines: the student can often visualize these ideas more clearly in the context of a loudspeaker or of a vibration shaker. Example 16.9 (pp. 824-825) analyzes the forces in a simple electromagnet, and Examples 16.10 (pp. 825-827) and 16.12 (p.831) extend this concept to a solenoid and a relay. Example 16.11 (p. 827-828) ties the material presented in this chapter to the transient analysis topics of Chapter 5. Example 16.13 (pp. 836-839) performs a dynamic analysis of a loudspeaker, showing how the frequency response of a loudspeaker can be computed from an electromechanical analysis of its dynamics. Finally, The box *Focus on Measurements: Seismic Transducer* (pp.839-840) presents the dynamic analysis of an electromechanical seismic transducer.

The homework problems are divided into four sections. The first reviews basic concepts in electricity and magnetism; the second presents basic and more advanced problems related to the concept of magnetic reluctance; the third offers some problems related to transformers. Section 4 contains a variety of applied problems related to electromechanical transducers; some of these problems emphasize dynamic analysis (16.40, 16.41, 16.42, 16.47-50) and are aimed at a somewhat more advanced audience.

Learning Objectives

1. Review the basic principles of electricity and magnetism. Section 1.
2. Use the concepts of reluctance and magnetic circuit equivalents to compute magnetic flux and currents in simple magnetic structures. Section 2.
3. Understand the properties of magnetic materials and their effects on magnetic circuit models. Section 3.
4. Use magnetic circuit models to analyze transformers. Section 4.
5. Model and analyze force generation in electro-magneto-mechanical systems. Analyze moving iron transducers (electromagnets, solenoids, relays), and moving-coil transducers (electro-dynamic shakers, loudspeakers, seismic transducers. Section 5.

Section 16.1: Electricity and Magnetism

Problem 16.1

Solution:

Known quantities:

As shown in Figure P16.1.

Find:

- The flux density in the core.
- Sketch the magnetic flux lines and indicate their direction.
- The north and south poles of the magnet.

Assumptions:

None.

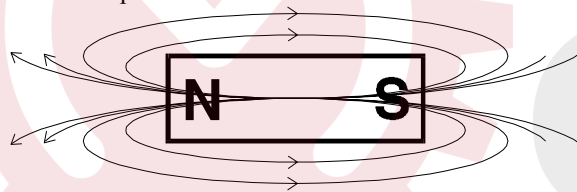
Analysis:

a)

$$B = \frac{\phi}{A} = \frac{4 \times 10^{-4}}{0.01} = 0.04 \text{ T}$$

b)

Viewed from the top:



c)

See above.

Problem 16.2

Solution:

Known quantities:

As shown in Figure P16.2.

Find:

If there is a resultant force on the single coil? If so, in what direction? Why?

Assumptions:

None.

Analysis:

Yes, the resultant force on the single coil is in the downward direction. If the coils are thought of as electromagnets, there is a north pole from the lower coil attracting a south pole from the upper coil.

Problem 16.3

Solution:

Known quantities:

A LVDT is connected to a resistive load R_L .

Find:

The LVDT equations.

Assumptions:

None.

Analysis:

Assume both secondary windings have resistance R_S and inductance L_S , if M_S is the mutual coupling, we have:

$$\begin{aligned} V_1 &= M_1 \frac{di}{dt} - (R_S i_L + L \frac{di_L}{dt}) + M_S \frac{di_L}{dt} \\ V_2 &= M_2 \frac{di}{dt} + (R_S i_L + L \frac{di_L}{dt}) - M_S \frac{di_L}{dt} \\ \therefore V_{out} &= V_1 - V_2 \\ &= (M_1 - M_2) \frac{di}{dt} - 2(R_S i_L + L \frac{di_L}{dt}) + 2M_S \frac{di_L}{dt} \end{aligned}$$

$$i_L = \frac{V_{out}}{R_L}$$

$$\therefore (2(L - M_S)s + 1 + 2\frac{R_S}{R_L})V_{out} = (M_1 - M_2)sI_L$$

Therefore, the transfer function is:

$$\frac{V_{out}}{I_L} = \frac{(M_1 - M_2)s}{2(L - M_S)s + 1 + 2\frac{R_S}{R_L}}$$

Problem 16.4**Solution:****Known quantities:**

Equations of "Focus on Measurements: Linear Variable Differential Transformer", and the results of Problem 16.3.

Find:

The frequency response of the LVDT and the range of frequencies for which the device will have maximum sensitivity for a given excitation.

Assumptions:

None.

Analysis:

We have $V_{ex} = L_p \frac{di}{dt} + R_p i$

$$\therefore V_{ex}(s) = (L_p s + R_p)I(s)$$

$$V_{out}(s)(2(L - M)s + 1 + 2\frac{R_S}{R_L}) = M_S I(s)$$

where $M = M_1 - M_2$. Therefore,

$$\frac{V_{out}(s)}{V_{ex}(s)} = \frac{M_S}{(L_p s + R_p)(2(L - M)s + 1 + 2\frac{R_S}{R_L})}$$

To determine the maximum sensitivity of the output voltage to the excitation we could compute the

derivative of $H(s) = \frac{V_{out}(s)}{V_{ex}(s)}$ with respect to s , set $\frac{\partial H(s)}{\partial s} = 0$, and solve for s . By setting $s = j\omega$, this

procedure will yield the excitation frequency for which the sensitivity of the output is maximum. It may, however, be more useful to compute the frequency response $H(j\omega)$ numerically, to visualize the range of frequencies over which the sensitivity is acceptable.

Problem 16.5

Solution:

Known quantities:

$$i = \frac{\lambda}{0.5 + \lambda}$$

a.

$$\lambda = 1 \text{ V} \cdot \text{s}$$

b.

$$R = 1 \Omega$$

$$i(t) = 0.625 + 0.01 \sin 400t \text{ A}$$

Find:

- The energy, coenergy, and incremental inductance.
- The voltage across the terminals on the inductor.

Assumptions:

None.

Analysis:

a)

For $\lambda = 1 \text{ V} \cdot \text{s}$:

The current is:

$$i = \frac{\lambda}{0.5 + \lambda} = 0.667 \text{ A}$$

The energy is:

$$W_m = \int_0^{1.0} \left(\frac{\lambda}{0.5 + \lambda} \right) d\lambda = \int_0^{1.0} \left(1 - \frac{0.5}{0.5 + \lambda} \right) d\lambda = \left(\lambda - 0.5 \ln|0.5 + \lambda| \right) \Big|_0^{1.0}$$

$$W_m = 1.0 - 0.5 \ln|0.5 + 1.0| - 0 + 0.5 \ln|0.5| = 0.4507 \text{ J}$$

The coenergy is:

$$W_m' = i\lambda - W_m = 0.2163 \text{ J}$$

The incremental inductance is:

$$L_\Delta = \frac{d\lambda}{di} \Big|_{\lambda=0.625}$$

$$\frac{di}{d\lambda} = \frac{0.5 + \lambda - \lambda}{(0.5 + \lambda)^2} = \frac{d\lambda}{di} = \frac{(0.5 + \lambda)^2}{0.5}$$

$$L_\Delta = \frac{d\lambda}{di} \Big|_{\lambda=0.625} = \frac{(0.5 + \lambda)^2}{0.5} \Big|_{\lambda=0.625} = 4.5 \text{ H}$$

b)

To compute the voltage, we must add the contribution of the voltage across the resistive part of the inductor plus that generated by the inductance:

$$V_L(t) = Ri(t) + L_\Delta \frac{di}{dt}$$

$$\frac{di}{dt} = 4 \cos(400t)$$

$$V_L(t) = (1\Omega)(0.625 + 0.01\sin(400t)) + (4.5H)(4\cos(400t))$$

$$V_L(t) = 0.625 + 18\sin(400t + 90^\circ)$$

It is important to observe that this is the inductor terminal voltage only for values of flux linkage in the neighborhood of $1V \cdot s$.

Problem 16.6

Solution:

Known quantities:

$$i = \frac{\lambda^2}{0.5 + \lambda^2}$$

a) $\lambda = 1V \cdot s$

b) $R = 1\Omega$

$$i(t) = 0.625 + 0.01\sin 400t \text{ A}$$

Find:

- a) The energy, coenergy, and incremental inductance.
- b) The voltage across the terminals on the inductor.

Assumptions:

None.

Analysis:

a)

For $\lambda = 1V \cdot s$:

The current is:

$$i = \frac{\lambda^2}{0.5 + \lambda^2} = 0.667 \text{ A}$$

The energy is:

$$W_m = \int_0^{1.0} \left(\frac{\lambda^2}{0.5 + \lambda^2} \right) d\lambda = \int_0^{1.0} \left(1 - \frac{0.5}{0.5 + \lambda^2} \right) d\lambda$$

Integral of form:

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + c$$

$$W_m = \int_0^{1.0} \left(1 - \frac{0.5}{0.5 + \lambda^2} \right) d\lambda = \left(\lambda - \frac{0.5}{\sqrt{0.5}} \tan^{-1} \left(\frac{\lambda}{\sqrt{0.5}} \right) \right) \Big|_0^{1.0}$$

$$W_m = 1.0 - \frac{0.5}{\sqrt{0.5}} \tan^{-1} \left(\frac{1}{\sqrt{0.5}} \right) - 0 + \frac{0.5}{\sqrt{0.5}} \tan^{-1} \left(\frac{0}{\sqrt{0.5}} \right) = 0.3245 \text{ J}$$

The coenergy is:

$$W_m' = i\lambda - W_m = 0.3425 \text{ J}$$

The incremental inductance is:

$$L_\Delta = \frac{d\lambda}{di} \Big|_{\lambda=0.625}$$

$$\frac{di}{d\lambda} = \frac{2\lambda(0.5 + \lambda^2) - 2\lambda(\lambda^2)}{(0.5 + \lambda^2)^2} \quad \frac{d\lambda}{di} = \frac{(0.5 + \lambda^2)^2}{\lambda}$$

$$L_\Delta = \frac{d\lambda}{di} \Big|_{\lambda=0.625} = \frac{(0.5 + \lambda^2)^2}{\lambda} \Big|_{\lambda=0.625} = 2.25H$$

b)

To compute the voltage, we must add the contribution of the voltage across the resistive part of the inductor plus that generated by the inductance:

$$V_L(t) = Ri(t) + L_{\Delta} \frac{di}{dt}$$

$$\frac{di}{dt} = 4 \cos(400t)$$

$$V_L(t) = (1\Omega)(0.625 + 0.01 \sin(400t)) + (2.25H)(4 \cos(400t))$$

$$V_L(t) = 0.625 + 9 \sin(400t + 89.9^\circ)$$

It is important to observe that this is the inductor terminal voltage only for values of flux linkage in the neighborhood of $1 \text{ V} \cdot \text{s}$.

Problem 16.7

Solution:

Known quantities:

Characteristic plot shown in Figure P16.7.

Find:

- The energy and the incremental inductance for $i = 1.0 \text{ A}$.
- The voltage across the terminals of the inductor when $R = 2\Omega$, $i(t) = 0.5 \sin 2\pi t$.

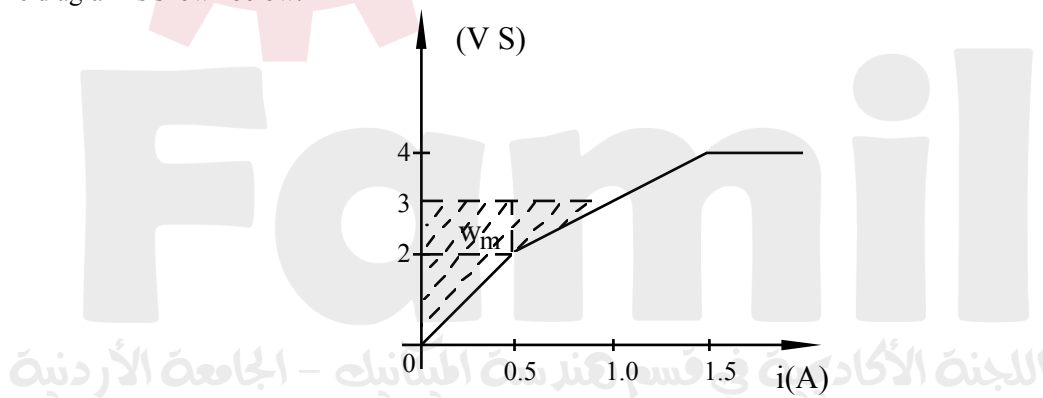
Assumptions:

None.

Analysis:

a)

The diagram is shown below:



W_m is the area at the left of the curve as shown.

$$W_m = \frac{1}{2} \times 0.5 \times 2 + 0.5 \times 1 + \frac{1}{2} \times 0.5 \times 1 = 1.25 \text{ J}$$

The incremental inductance is:

$$L_{\Delta} = \frac{\Delta \lambda}{\Delta i} \Big|_{i=1} = \frac{2}{1} = 2 \text{ H}$$

b)

For $i = 0.5 \sin 2\pi t$ and $R = 2\Omega$:

$$V = Ri + \frac{d\lambda}{dt}$$

For $|i| < 0.5, \lambda = 4i$:

$$\begin{aligned} V_L(t) &= \sin(2\pi t) + 4 \times 0.5 \times 2\pi \cos(2\pi t) \\ &= \sin(2\pi t) + 4\pi \cos(2\pi t) \end{aligned}$$

Problem 16.8

Solution:

Known quantities:

Structure of Figure 16.12

$$A = 0.1 \text{ m}^2$$

$$\mu_r = 2000$$

Find:

The reluctance of the structure.

Assumptions:

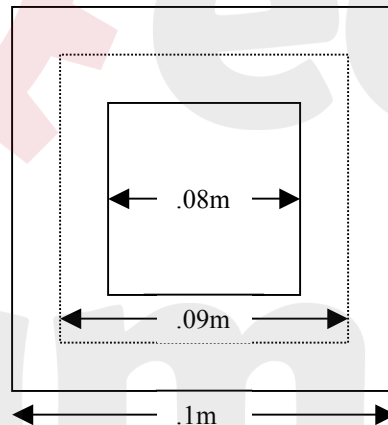
Each leg is 0.1 m in length

Mean magnetic path runs through the exact center of the structure

Analysis:

Calculation of mean path:

Using the assumption that the mean magnetic path runs through the exact center of the structure, and since the structure is square, the mean path is determined using the following figure:



$$l_c = 4 \times 0.09 \text{ m} = 0.36 \text{ m}$$

Calculation of Reluctance:

$$\mathfrak{R} = \frac{l_c}{\mu A} = \frac{l_c}{\mu_r \mu_o A} = \frac{0.36}{2000 \times 4\pi \times 10^{-7} \times 0.1} = 1432 \text{ A-turns / Wb}$$

Section 16.2: Magnetic Circuits

Focus on Methodology: Magnetic Structures and Equivalent Magnetic Circuits

Direct Problem

Given – The structure geometry and the coil parameters.

Calculate – The magnetic flux in the structure.

1. Compute the mmf.
2. Determine the length and cross section of the magnetic path for each continuous *leg* or section of the path.
3. Calculate the equivalent reluctance of each *leg*.
4. Generate the equivalent magnetic circuit diagram, and calculate the total equivalent reluctance.
5. Calculate the flux, flux density, and magnetic field intensity, as needed.

Direct Problem

Given – The desired flux or flux density and structure geometry.

Calculate – The necessary coil current and number of turns.

1. Calculate the total equivalent reluctance of the structure from the desired flux.
2. Generate the equivalent magnetic circuit diagram.
3. Determine the mmf required to establish the desired flux.
4. Choose the coil current and number of turns required to establish the desired mmf.

Problem 16.9

Solution:

Known quantities:

- a) $\phi = 4.2 \times 10^{-4} \text{ Wb}, \text{mmf} = 400 \text{ A} \cdot t$.
- b) $l = 6 \text{ in.}$

Find:

- a) The reluctance of a magnetic circuit.
- b) The magnetizing force in SI units.

Assumptions:

None.

Analysis:

a)

$$\mathfrak{R} = \frac{F}{\phi} = \frac{400}{4.2 \times 10^{-4}} = 9.52 \times 10^5 \frac{\text{A} \cdot t}{\text{Wb}}$$

b)

$$H = \frac{F}{6 \times 0.0254 \frac{\text{m}}{\text{in}}} = 2625 \frac{\text{A} \cdot t}{\text{m}}$$

Problem 16.10**Solution:****Known quantities:**

As shown in Figure P16.10.

Find:

- The reluctance values and show the magnetic circuit when $\mu = 3000\mu_0$.
- The inductance of the device.
- The new value of inductance when a gap of 0.1 mm is cut in the arm of length l_3 .
- The limiting value of inductance when the gap is increased in size (length).

Assumptions:

Neglect leakage flux and fringing effects.

Analysis:

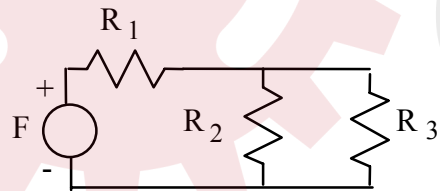
a)

$$\mathfrak{R}_1 = \frac{l_1}{\mu A_1} = \frac{0.3}{(3000)4\pi \cdot 10^{-7}(0.01)} = 7.96 \cdot 10^3 \text{ H}^{-1}$$

$$\mathfrak{R}_2 = \frac{l_2}{\mu A_2} = \frac{0.1}{(4\pi \cdot 10^{-7})3000(25 \cdot 10^{-4})} = 10.671 \cdot 10^3 \text{ H}^{-1}$$

$$\mathfrak{R}_3 = \mathfrak{R}_1 = 7.96 \cdot 10^3 \text{ H}^{-1}$$

The circuit is shown below:



$$\mathfrak{R}_T = \mathfrak{R}_1 + \frac{\mathfrak{R}_2 \mathfrak{R}_3}{\mathfrak{R}_2 + \mathfrak{R}_3} = 12.51 \times 10^3 \text{ H}^{-1}$$

b)

$$L = \frac{N^2}{\mathfrak{R}_T} = \frac{100^2}{12.51 \times 10^3} = 0.8 \text{ H}$$

c)

We have

$$\mathfrak{R}_g = \frac{0.0001}{(4\pi \times 10^{-7})(100 \times 10^{-4})} = 7.96 \times 10^3 \text{ H}^{-1}$$

$\mathfrak{R}_g$ is in series with $\mathfrak{R}_3$ and thus:

$$\mathfrak{R}_T = \mathfrak{R}_1 + \frac{\mathfrak{R}_2(\mathfrak{R}_3 + \mathfrak{R}_g)}{\mathfrak{R}_2 + \mathfrak{R}_3 + \mathfrak{R}_g} = 14.33 \times 10^3 \text{ H}^{-1}$$

$$L = \frac{N^2}{\mathfrak{R}_T} = 0.7 \text{ H}$$

d)

As the gaps get longer, $\mathfrak{R}_g$ will get larger and as an extreme case the circuit is made of $\mathfrak{R}_1$ and $\mathfrak{R}_2$ in series, therefore:

$$\mathfrak{R}_T = 18.57 \times 10^3 \text{ H}^{-1}$$

$$L = \frac{N^2}{\mathfrak{R}_T} = 0.54 \text{ H}$$

Problem 16.11

Solution:

Known quantities:

$$N = 1000 \text{ turns}, i = 0.2 \text{ A}, l_{g1} = 0.02 \text{ cm}, l_{g2} = 0.04 \text{ cm}.$$

Find:

The flux and flux density in each of the legs of the magnetic circuit.

Assumptions:

Neglect fringing at the air gaps and any leakage fields. Assume the reluctance of the magnetic core to be negligible.

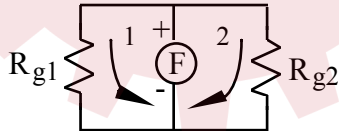
Analysis:

Calculate Reluctance in each air gap

$$\begin{aligned} \mathfrak{R}_{g1} &= \frac{0.0002}{4\pi \times 10^{-7} \times (0.01)^2} \\ &= 1.59 \times 10^6 \end{aligned}$$

$$\mathfrak{R}_{g2} = 2\mathfrak{R}_{g1} = 3.18 \times 10^6$$

Assume the reluctance of the material can be neglected when compared to the reluctance of the air gaps; the analogous circuit is shown below:



$$\phi_1 = \frac{Ni}{\mathfrak{R}_{g1}} = 1.26 \times 10^{-4} \text{ Wb}$$

$$B_1 = \frac{\phi_1}{A} = 1.26 \text{ Wb/m}^2$$

$$\phi_2 = \frac{1}{2} \phi_1 = 0.63 \times 10^{-4} \text{ Wb}$$

$$B_2 = \frac{1}{2} B_1 = 0.63 \text{ Wb/m}^2$$

Problem 16.12

Solution:

Known quantities:

$$\phi = 3 \times 10^{-4} \text{ Wb}, l_{\text{iron}} = l_{\text{steel}} = 0.3 \text{ m}, A = 5 \times 10^{-4} \text{ m}^2, N = 100 \text{ turns}.$$

Find:

The current needed to establish the flux.

Assumptions:

None

Analysis:

Reluctance for each material is calculated as follows:

$$\mathfrak{R} = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o}$$

The total reluctance of the structure is the sum of the reluctances for each material.

From Table 16.1:

Cast Iron: $\mu_r = 5195$ Cast Steel: $\mu_r = 1000$

Cast Iron:

$$\mathfrak{R}_{CI} = \frac{l_{CI}}{\mu_r \mu_o A} = \frac{0.3m}{(5195)(4\pi \times 10^{-7})(5 \times 10^{-4} m^2)} = 9.1909 \times 10^4 A - turns / Wb$$

Cast Steel:

$$\mathfrak{R}_{CS} = \frac{l_{CS}}{\mu_r \mu_o A} = \frac{0.3m}{(1000)(4\pi \times 10^{-7})(5 \times 10^{-4} m^2)} = 4.7746 \times 10^5 A - turns / Wb$$

Note: Cast Steel is less permeable than cast iron

Total Reluctance:

$$\begin{aligned}\mathfrak{R}_T &= \mathfrak{R}_{CI} + \mathfrak{R}_{CS} = 9.1909 \times 10^4 A - turns / Wb + 4.7746 \times 10^5 A - turns / Wb = \\ \mathfrak{R}_T &= 5.6937 \times 10^5 A - turns / Wb\end{aligned}$$

From $\phi = \frac{Ni}{\mathfrak{R}_T}$, we can compute the current.

$$i = \frac{\phi \mathfrak{R}_T}{N} = \frac{(3 \times 10^{-4} Wb)(5.6937 \times 10^5 A - turns / Wb)}{100 turns} = 1.71 A$$

Problem 16.13**Solution:****Known quantities:**

As shown in Figure P16.13.

Find:The magnetic flux ϕ .**Assumptions:**

None.

Analysis:

$$I = 2 \text{ A}, r = 0.08 \text{ m}, N = 100, A_{\text{cross}} = 0.009 \text{ m}^2, \mu_r = 1000$$

$$l = 2\pi r = 0.50265 \text{ m}, \mu = \mu_r \mu_0$$

$$R = \frac{1}{\mu A_{\text{cross}}} = 4.44444 \times 10^4 \frac{\text{A}}{\text{Wb}}$$

$$\text{mmf} = I \cdot N$$

$$\therefore \phi = \frac{\text{mmf}}{R} = 0.0045 \text{ Wb}$$

Problem 16.14**Solution:****Known quantities:**

$$\phi = 2.4 \times 10^{-4} \text{ Wb}, l_{ab} = l_{ef} = 0.05 \text{ m}, l_{af} = l_{be} = 0.02 \text{ m}, l_{bc} = l_{dc}, A = 2 \times 10^{-4} \text{ m}^2.$$

The material is sheet steel.

Find:

- The current required to establish the flux.
- Compare the mmf drop across the air gap to that across the rest of the magnetic circuit and discuss your results using the value of μ for each material.

Assumptions:

None.

Analysis:

a)

Assume the material is cast steel.

$$B = \frac{2.4 \times 10^{-4}}{2 \times 10^{-4}} = 12 \text{ T}$$

$$H_{CS} = 1400 \frac{\text{A} \cdot \text{t}}{\text{m}}$$

$$H_{AG} = 9.55 \times 10^5 \frac{\text{A} \cdot \text{t}}{\text{m}}$$

$$100I = 1400(0.1 + 0.02 + 0.017) + (9.55 \times 10^5)(0.003)$$

$$\Rightarrow I = 30.6 \text{ A}$$

b)

$$F_{CS} = 191.8 \text{ A} \cdot \text{t}$$

$$F_{AG} = 2865 \text{ A} \cdot \text{t}$$

Note:

$$\mu_{CS} = 0.0009$$

$$\mu_{AG} = 0.00000126$$

$$\frac{\mu_{CS}}{\mu_{AG}} \approx 700$$

$$\mu_{AG}$$

Problem 16.15**Solution:****Known quantities:**

Magnet of Figure P16.15,

$$\phi = 2 \times 10^{-4} \text{ Wb}, l_{ab} = l_{bg} = l_{gh} = l_{ha} = 0.2 \text{ m}, l_{bc} = l_{fg} = 0.1 \text{ m}, l_{cd} = l_{ef} = 0.099 \text{ m}.$$

The material is sheet steel.

Find:

The value of I required to establish the flux.

Assumptions:

None.

Analysis:

$$A_1 = 2 \times 10^{-4} \text{ m}^2, A_2 = 5 \times 10^{-4} \text{ m}^2$$

$$\mu_r = 4000$$

$$l_{gap} = l_{ha} - 2l_{cd} = 0.002 \text{ m}$$

$$R_{gap} = \frac{l_{gap}}{\mu_0 A_2} = 3.1831 \times 10^6 \frac{\text{A}}{\text{Wb}}$$

$$\text{mmf}_{gap} = \phi_1 R_{gap} = 636.61977 \text{ A}$$

$$R_{ef} = R_{cd} = \frac{l_{cd}}{\mu A_2} = 3.93908 \times 10^4 \frac{\text{A}}{\text{Wb}}$$

$$\text{mmf}_{cd} = \phi_1 R_{cd} = 7.87817 \text{ A}$$

$$R_{fg} = R_{bc} = \frac{l_{bc}}{\mu A_2} = 3.97887 \times 10^4 \frac{\text{A}}{\text{Wb}}$$

$$\text{mmf}_{bc} = \phi_1 R_{bc} = 7.95775 \text{ A}$$

To find the mmf in the rightmost leg of the magnetic circuit,

$$R_{ef} = R_{cd} = \frac{l_{cd}}{\mu A_2} = 3.93908 \times 10^4 \frac{\text{A}}{\text{Wb}}$$

$$R_{ab} = \frac{l_{ab}}{\mu A_2} = 7.95775 \times 10^4 \frac{\text{A}}{\text{Wb}}$$

$$R_{series} = 3R_{ab} = 2.38732 \times 10^5 \frac{\text{A}}{\text{Wb}}$$

$$\text{mmf}_{series} = \phi_T R_{series} = 848.69641 \text{ A}$$

$$\text{mmf}_{total} = \text{mmf}_{series} + \text{mmf}_{parallel} = 1.51799 \times 10^3 \text{ A}$$

$$\therefore i = \frac{\text{mmf}_{total}}{N} = 7.59 \text{ A}$$

$$\text{mmf}_{parallel} = \text{mmf}_{gap} + 2\text{mmf}_{cd} + 2\text{mmf}_{bc} = 66829161 \text{ A}$$

$$R_{bg} = \frac{l_{bg}}{\mu A_1} = 1.98944 \times 10^5 \frac{\text{A}}{\text{Wb}}$$

$$\phi_2 = \frac{\text{mmf}_{parallel}}{R_{bg}} = 0.00336 \text{ Wb}$$

$$\phi_T = \phi_1 + \phi_2 = 0.00356 \text{ Wb}$$

Problem 16.16**Solution:****Known quantities:**

Actuator of Figure P16.16, $N = 2000 \text{ turns}$, $g = 10 \text{ mm}$, $B = 1.2 \text{ T}$, the air gap is fixed.

Find:

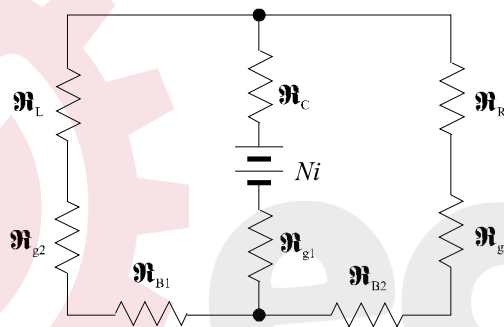
- The coil current.
- The energy stored in the air gaps.
- The energy stored in the steel.

Assumptions:

None.

Analysis:

- The equivalent circuit is:



where:

From Table 16.1, the relative permeability of sheet steel is 4000.

$$\mathfrak{R}_L = \mathfrak{R}_R = \frac{80 \times 10^{-3}}{4000(4\pi \times 10^{-7})(35 \times 17.5) \times 10^{-6}} = 25,984 \text{ A} \cdot \text{t/Wb}$$

$$\mathfrak{R}_C = \frac{80 \times 10^{-3}}{4000(4\pi \times 10^{-7})(35 \times 35) \times 10^{-6}} = 12,992 \text{ A} \cdot \text{t/Wb}$$

$$\mathfrak{R}_{B1} = \mathfrak{R}_{B2} = \frac{61.25 \times 10^{-3}}{4000(4\pi \times 10^{-7})(35 \times 17.5) \times 10^{-6}} = 19,894 \text{ A} \cdot \text{t/Wb}$$

$$\mathfrak{R}_{g1} = \frac{10 \times 10^{-3}}{(4\pi \times 10^{-7})(35 \times 35) \times 10^{-6}} = 6,496,120 \text{ A} \cdot \text{t/Wb}$$

$$\mathfrak{R}_{g2} = \frac{10 \times 10^{-3}}{(4\pi \times 10^{-7})(35 \times 17.5) \times 10^{-6}} = 12,955,225 \text{ A} \cdot \text{t/Wb}$$

$$\mathfrak{R}_T = \mathfrak{R}_C + \mathfrak{R}_{g1} + \frac{\mathfrak{R}_R + \mathfrak{R}_{g2} + \mathfrak{R}_{B2}}{2} = 13.01 \times 10^6 \text{ A} \cdot \text{t/Wb}$$

$$\phi_T = BA = 1.2(35 \times 35) \times 10^{-6} = 1.47 \times 10^{-3} \text{ Wb}$$

$$2000I = \mathfrak{R}_T \phi_T \Rightarrow I = 9.56 \text{ A}$$

b)

$$H_{lg} = \frac{1.2}{4\pi \times 10^{-7}} (10 \times 10^{-3}) = 9549.3 \text{ A} \cdot t$$

$$w_{g1} = \frac{1}{2} (1.47 \times 10^{-3}) (9549.3) = 7.02 \text{ J}$$

$$w_{g2} = \frac{1}{2} \left(\frac{1.47 \times 10^{-3}}{2} \right) (9549.3) = 3.51 \text{ J}$$

$$w_g = w_{g1} + 2w_{g2} = 14.04 \text{ J}$$

c)

$$w_T = \frac{1}{2} (1.47 \times 10^{-3}) (2000) (9.56) = 14.05 \text{ J}$$

$$w_{ST} = w_T - w_g = 0.01 \text{ J}$$

Problem 16.17

Solution:

Known quantities:

$$\mu_r = 2000, N = 100.$$

Find:

- The current needed to produce $\phi = 0.4 \text{ Wb/m}^2$ in the center leg.
- The current needed to produce $\phi = 0.8 \text{ Wb/m}^2$ in the center leg.

Assumptions:

None.

Analysis:

With $l_1 = 34 \text{ cm}$, $l_2 = l_3 = 90 \text{ cm}$ and $A = (8 \times 10^{-2})^2 \text{ cm}^2$, we compute:

$$\mathfrak{R}_1 = \frac{0.34}{2000 \times 4\pi \times 10^{-7} \times (8 \times 10^{-2})^2} = 2.114 \times 10^4 \text{ H}^{-1}$$

$$\mathfrak{R}_2 = 5.595 \times 10^4 \text{ H}^{-1} = \mathfrak{R}_3$$

$$\mathfrak{R}_T = \mathfrak{R}_1 + \frac{\mathfrak{R}_2 \mathfrak{R}_3}{\mathfrak{R}_2 + \mathfrak{R}_3} = 4.91 \times 10^4 \text{ H}^{-1}$$

$$\phi_T = 0.4 \times (0.08)^2 = 2.56 \times 10^{-3} \text{ Wb}$$

From $\phi_T = BA = \frac{Ni}{\mathfrak{R}_T}$, we have $i = \frac{B A \mathfrak{R}_T}{N}$.

a)

$$i = \frac{2.56 \times 10^{-3} \times 4.91 \times 10^4}{100} = \frac{125.7}{100} = 1.257 \text{ A}$$

b)

Since the current is directly proportional to B, the current will have to be doubled.

Section 16.4: Transformers

Problem 16.18

Solution:

Known quantities:

$$N = 1000 \text{ turns}, l_1 = 16 \text{ cm}, A_1 = 4 \text{ cm}^2, l_2 = 22 \text{ cm}, A_2 = 4 \text{ cm}^2, l_3 = 5 \text{ cm}, A_3 = 2 \text{ cm}^2, \mu_r = 1500.$$

Find:

- Construct the equivalent magnetic circuit and find the reluctance associated with each part of the circuit.
- The self-inductance and mutual-inductance for the pair of coils.

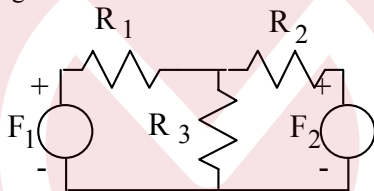
Assumptions:

None.

Analysis:

a)

The analogous circuit is shown below:



The individual reluctances are:

$$\mathfrak{R}_1 = \frac{16 \times 10^{-2}}{4\pi \times 10^{-7} \times 1500 \times 4 \times 10^{-4}} = 2.12 \times 10^5 \text{ H}^{-1}$$

$$\mathfrak{R}_2 = \frac{22 \times 10^{-2}}{4\pi \times 10^{-7} \times 1500 \times 4 \times 10^{-4}} = 2.92 \times 10^5 \text{ H}^{-1}$$

$$\mathfrak{R}_3 = \frac{5 \times 10^{-2}}{4\pi \times 10^{-7} \times 1500 \times 2 \times 10^{-4}} = 1.33 \times 10^5 \text{ H}^{-1}$$

b)

The inductance can be computed as follows:

$$L_{m1} = \frac{N^2}{\mathfrak{R}_1} = 4.72 \text{ H}$$

$$L_{m2} = \frac{N^2}{\mathfrak{R}_2} = 3.43 \text{ H}$$

$$L_{m3} = \frac{N^2}{\mathfrak{R}_3} = 7.54 \text{ H}$$

$$\text{let } L_T = L_{m1} + L_{m2} + L_{m3} = 15.68 \text{ H}$$

$$L_m = \frac{L_{m1}L_{m2}}{L_T} = 1.03 \text{ H} = L_{12} = L_{21} = M$$

$$L_1 = \frac{L_{m1}L_{m3}}{L_T} = 2.27 \text{ H}$$

$$L_2 = \frac{L_{m2}L_{m3}}{L_T} = 1.65 \text{ H}$$

$$L_{11} = L_1 + L_m = 3.3 \text{ H}$$

$$L_{22} = L_2 + L_m = 2.68 \text{ H}$$

Problem 16.19

Solution:

Known quantities:

A 300Ω resistive load referred to the primary is 7500Ω .

$r_1 = 20\Omega, L_1 = 1.0 \text{ mH}, L_m = 25 \text{ mH}, r_2 = 20\Omega, L_2 = 1.0 \text{ mH}.$

Find:

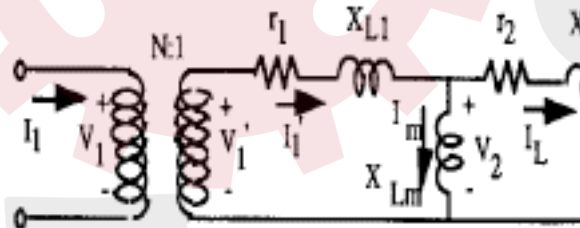
- The turns ratio.
- The input voltage, current, and power and the efficiency when this transformer is delivering 12 W to the 300Ω load at a frequency $f = 10,000/2\pi \text{ Hz}$.

Assumptions:

Core losses are negligible.

Analysis:

The equivalent circuit is:



a)

From $7500 = N^2 \times 300$, we have $N = 5$.

b)

$$X_{L1} = 2\pi f L_1 = 10 = X_{L2}$$

$$X_{Lm} = 250$$

From $I_L^2 R_L = 12 \text{ W}, I_L^2 = 0.04$, we have:

$$I_L = 0.2 \angle 0^\circ \text{ A}$$

$$V_L = 60 \angle 0^\circ \text{ V}$$

$$V_2 = I_L(R_L + r_2 + jX_{L2}) = 64 + j2$$

$$= 64.03 \angle 1.79^\circ \text{ V}$$

$$I_m = 0.256 \angle -88.21^\circ = 0.008 - j0.2559 \text{ A}$$

$$I_1' = I_m + I_L = 0.33 \angle -50.9^\circ \text{ A}$$

$$V_1' = I_1'(r_1 + jX_{L1}) + V_2$$

$$= 70.72 - j1.04 = 70.72 \angle -0.84^\circ \text{ V}$$

$$V_1 = NV_1' = 353.6 \angle -0.84^\circ \text{ V}$$

$$I_1 = \frac{1}{N} I_1' = 0.066 \angle -50.9^\circ \text{ A}$$

$$P_{in} = V_1 I_1 \cos \theta = 14.98 \text{ W}$$

$$\therefore \text{efficiency} = \eta = \frac{P_{out}}{P_{in}} = 80.1\%$$

Problem 16.20

Solution:

Known quantities:

A 220/20V transformer has 50 turns on its low-voltage side.

Find:

- The number of turns on its high side.
- The turns ratio α when it is used as a step-down transformer.
- The turns ratio α when it is used as a step-up transformer.

Assumptions:

None.

Analysis:

$$\alpha = \frac{220}{20} = 11$$

a) The primary has $N_P = 50 \times 11 = 550$ turns

b) $\alpha = 11$ is a step-down transformer.

c) $\alpha = \frac{1}{11}$ is a step-up transformer.

Problem 16.21

Solution:

Known quantities:

The high-voltage side of the transformer has 750 turns, and the low-voltage side 50 turns. The high side is connected to a rated voltage of 120V. A rated load of 40A is connected to the low side.

Find:

- The turns ratio.
- The secondary voltage.

- c) The resistance of the load.

Assumptions:

No internal transformer impedance voltage drops

Analysis:

- a)

$$\alpha = \frac{750}{50} = 15$$

- b)

$$V_2 = \frac{1}{\alpha} V_1 = \frac{120}{15} = 8V$$

- c)

$$R_L = \frac{8}{40} = 0.2\Omega$$

Problem 16.22**Solution:****Known quantities:**

A transformer is used to match an 8Ω loudspeaker to a 500Ω audio line.

Find:

- The turns ratio of the transformer.
- The voltages at the primary and secondary terminals when $10W$ of audio power is delivered to the speaker.

Assumptions:

The speaker is a resistive load and the transformer is ideal.

Analysis:

- a)

From $\alpha^2 R_L = 500$, we have $\alpha = 7.91$

- b)

From $10 = \frac{V_2^2}{R_L}$, we have $V_2 = 8.94V$

- c)

$$V_1 = \alpha V_2 = 70.7V$$

Problem 16.23**Solution:****Known quantities:**

It is a step-down transformer. The high-voltage and low-voltage sides have 800 turns and 100 turns respectively. 240VAC voltage is applied to the high side. The impedance of the low side is 3Ω .

Find:

- The secondary voltage and current.
- The primary current.
- The primary input impedance from the ratio of primary voltage and current.
- The primary input impedance.

Assumptions:

None.

Analysis:

We have $\alpha = \frac{N_1}{N_2} = 8$.

a)

$$V_2 = \frac{1}{\alpha} V_1 = 30 \text{ V}$$

$$I_2 = \frac{V_2}{R_L} = 10 \text{ A}$$

b)

$$I_1 = \frac{1}{\alpha} I_2 = 1.25 \text{ A}$$

c)

$$Z_{in} = \frac{240}{1.25} = 192$$

d)

$$Z_{in} = \alpha^2 R_L = 192$$

Problem 16.24**Solution:****Known quantities:**

It is a step-up transformer. All the others are the same as Problem 16.23.

Find:

The transformer ratio of the transformer.

Assumptions:

None.

Analysis:

$$\alpha = \frac{100}{800} = \frac{1}{8}$$

Problem 16.25**Solution:****Known quantities:**

It is a 2,300/240-V, 60-Hz, 4.6-kVA transformer. It has an induced emf of 2.5V/turn.

Find:

- The number of high-side turns N_h and low-side turns N_l .
- The rated current of the high-voltage side I_h .
- The transformer ratio when the device is used as a step-up transformer.

Assumptions:

It is an ideal transformer.

Analysis:

a)

$$N_h = \frac{2300}{2.5} = 920 \text{ turns}$$

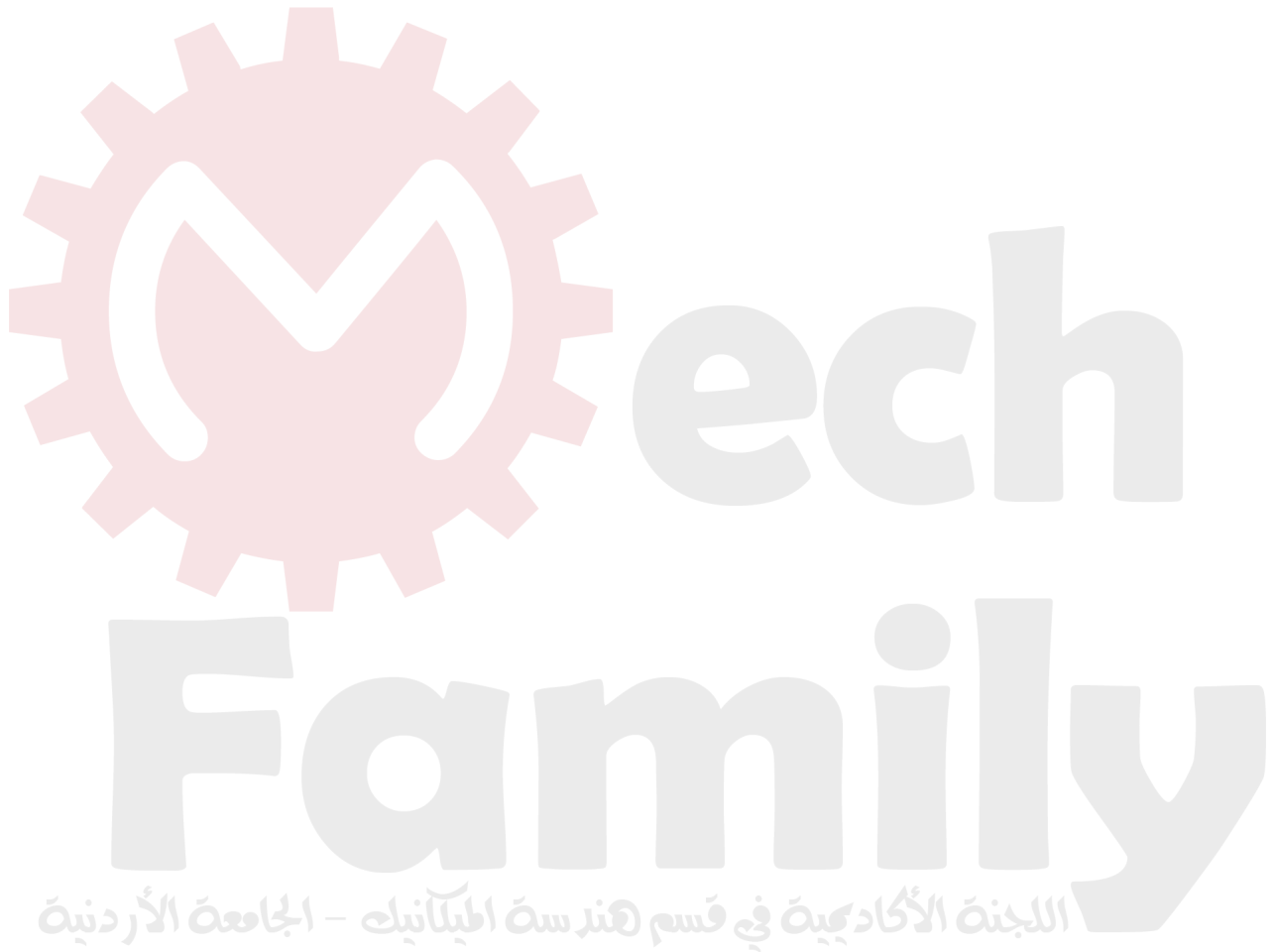
$$N_l = \frac{240}{2.5} = 96 \text{ turns}$$

b)

$$I_h = \frac{4.6 \times 10^3}{2300} = 2 \text{ A}$$

c)

$$\alpha = \frac{N_l}{N_h} = 0.1044$$



Section 16.5: Electromechanical Transducers

a)

Focus on Methodology

Analysis of moving-iron electromechanical transducers

a. Calculation of current required to generate a given force

1. Derive an expression for the reluctance of the structure, as a function of air gap displacement: $R(x)$.
2. Express the magnetic flux in the structure, as a function of the mmf (i.e., of the current, I) and of the reluctance, $R(x)$:
$$\frac{F(i)}{R(x)}$$
3. Compute an expression for the force using the known expressions for the flux and for the reluctance:
$$|f| = \frac{1}{2} \frac{dR(x)}{dx}$$
4. Solve the expression in 3. for the unknown current, i .

b. Calculation of force generated by a given transducer geometry and mmf

Repeat steps 1-3 above, substituting the known current to solve for the force, f .

Problem 16.26

Solution:

Known quantities:

Electromagnet of Example 16-9 (Figure 16.38)

$$N = 700 \text{ turns}; f_{\text{restore}} = 8,900 \text{ N}; A_{\text{gap}} = 0.01 \text{ m}^2; L = 1 \text{ m}; \mu_r = 1000$$

Find:

- a) The current required to keep the bar in place
- b) Initial current to lift the magnet if the bar is initially 0.1 m away from the electromagnet

Assumptions:

- a) Air gap becomes zero and the iron reluctance cannot be neglected
- b) Neglect the iron reluctance

Analysis:

- a) To compute the current we need to derive an expression for the force in the air gap.

Without neglecting the iron reluctance, we can write the expression for the reluctance as follows:

$$(x) \quad \frac{L}{r_0 A} + \frac{2x}{r_0 A}$$

where L is the total length of the iron magnetic path (excluding the air gap).

Knowing the reluctance we can calculate the magnetic flux in the structure as a function of the coil current:

$$(x) \quad \frac{Ni}{L} = \frac{Ni}{L} \frac{r_0 A}{2x}$$

Then, the magnitude of the force in the air gap is given by the expression

$$|f| = \frac{1}{2} \frac{d}{dx} \left(\frac{1}{L} \frac{Ni^2}{4L} \frac{r_0 A^2}{4x^2} \right) = \frac{2}{L^2} \frac{Ni^2}{4L} \frac{r_0 A^2}{4x^2}$$

As x approaches zero, we can calculate the force to be:

$$|f(x=0)| = \frac{(Ni\mu_r)^2 \mu_0 A}{L^2}$$

and the current required to maintain the given force is:

$$|f(x=0)| = \frac{(Ni\mu_r)^2 \mu_0 A}{L^2}$$

$$i = \pm \sqrt{\frac{L^2 |f(x=0)|}{(N\mu_r)^2 \mu_0 A}} = \pm \frac{L}{N\mu_r} \sqrt{\frac{|f(x=0)|}{\mu_0 A}}$$

Assuming that the total length of the magnetic path is $L=1$ m and that $\mu_r = 1,000$, we can calculate a value for the current to be 1.20 A.

b) Since the bar is initially 0.1 m away from the structure, the reluctance of the air dominates the reluctance of the structure. The reluctance is calculated as:

$$\mathfrak{R}(x) = \frac{2x}{\mu_0 A}$$

Knowing the reluctance we can calculate the magnetic flux in the structure as a function of the coil current:

$$\phi = \frac{Ni}{\mathfrak{R}(x)} = Ni \frac{\mu_0 A}{2x}$$

Then, the magnitude of the force in the air gap is given by the expression

$$|f| = \frac{\phi^2}{2} \frac{d\mathfrak{R}(x)}{dx} = \frac{1}{2} \frac{(Ni\mu_0 A)^2}{4x^2} \frac{2}{\mu_0 A} = \frac{(Ni)^2 \mu_0 A}{4x^2}$$

Finally, the current required is:

$$i = \pm \frac{2x}{N} \sqrt{\frac{|f|}{\mu_0 A}} = \pm \frac{2(0.1m)}{700} \sqrt{\frac{8900N}{4\pi \times 10^{-7} \times 0.01m^2}} = \pm 240.3A$$

Note that the holding current from part a is significantly smaller than the current required to lift the bar from the initial distance of 0.1 m.

Problem 16.27

Solution:

Known quantities:

$$\mathfrak{R}(x) = 7 \times 10^8 (0.002 + x) H^{-1}, N = 980t, R = 30\Omega, V_a = 120V$$

Find:

- The energy stored in the magnetic field for $x = 0.005$ m.
- The magnetic force for $x = 0.005$ m.

Assumptions:

None.

Analysis:

a)

$$\text{We have } L(x) = \frac{N^2}{\mathfrak{R}(x)}.$$

$$W_m' = W_m = \frac{L(x)i^2}{2}$$

The current is:

$$I_{DC} = \frac{120}{30} = 4 \text{ A}$$

$$\therefore W_m = \frac{980^2 \times 4^2}{2 \times 7 \times 10^8 \times 0.007} = 1.568 \text{ J}$$

b)

$$f = -\frac{i^2}{2} \frac{N^2}{(\mathcal{R}(x))^2} \frac{d\mathcal{R}(x)}{dx} = -224 \text{ N}$$

The minus sign indicates that the force f is in a direction opposite to that indicated in the figure.

Problem 16.28

Solution:

Known quantities:

Solenoid of Example 16.10 (Figure 16.40)

Find:

The best combination of current magnitude and wire diameter to reduce the volume of the solenoid coil.

Will this minimum volume result in the lowest possible resistance?

How does the power dissipation of the coil change with the wire gauge and current value?

Assumptions:

Use of Copper wire in solenoid

Analysis:

In order to access the effects of the wire diameter and current magnitude to the volume, resistance, and power dissipated, mathematical expressions need to be developed for each variable.

Volume:

$$V_{coil} = l_{coil} A_{coil}$$

The length of the coil is given by the circumference:

$$l_{coil} = \pi N d_{coil}$$

The area of the coil:

$$A_{coil} = \frac{\pi}{4} d_{coil}^2$$

$$V_{coil} = \pi N d_{coil} \frac{\pi}{4} d_{coil}^2 = \frac{\pi^2 N d_{coil}^3}{4}$$

From Example 16.10, the relationship between the current and the number of turns is given as:

$$Ni = 56.4 \text{ A-turns}$$

Hence:

$$V_{coil} = \frac{(56.4)^2 \pi^2 d_{coil}^3}{4i}$$

Resistance:

The resistance of the wire is given as:

$$R = \frac{\rho l_{coil}}{A_{coil}}$$

where ρ is the resistivity of the wire, which is assumed to be copper with a $\rho = 1.725 \times 10^{-8} \Omega/m$

Using the previous derivations:

$$R = \frac{\rho \pi N d_{coil}}{\frac{\pi}{4} d_{coil}^2} = \frac{4 \rho N}{d_{coil}} = \frac{(4)(56.4)\rho}{d_{coil}}$$

Power:

The dissipated power is given by:

$$P = i^2 R = \frac{(4)(56.4)i\rho}{d_{coil}}$$

Using a chart for AWG wire gauge and current carrying capacity, a table can be developed relating the wire gauge and current carrying capacity to the volume, resistance, and power dissipated.

AWG Gauge	Wire Diameter [in]	Wire Diameter [m]	Current Capacity [A]	Required Number of Turns [turns]	Coil Volume [m ³]	Resistance [Ω]	Power Dissipated [W]
10	1.019E-01	2.588E-02	30.0	2	8.041E-05	5.012E-06	4.511E-03
12	8.081E-02	2.053E-02	20.0	3	6.017E-05	9.480E-06	3.792E-03
14	6.408E-02	1.628E-02	15.0	4	4.001E-05	1.594E-05	3.586E-03
16	5.082E-02	1.291E-02	10.0	6	2.993E-05	3.015E-05	3.015E-03
18	4.030E-02	1.024E-02	5.0	11	2.986E-05	7.603E-05	1.901E-03
20	3.196E-02	8.118E-03	3.3	17	2.256E-05	1.453E-04	1.582E-03
22	2.530E-02	6.426E-03	2.5	23	1.477E-05	2.422E-04	1.514E-03
24	2.010E-02	5.105E-03	1.25	45	1.481E-05	6.098E-04	9.528E-04
26	1.590E-02	4.039E-03	0.83	68	1.111E-05	1.168E-03	7.950E-04
28	1.260E-02	3.200E-03	0.63	90	7.299E-06	1.946E-03	7.600E-04
30	1.000E-02	2.540E-03	0.31	180	7.297E-06	4.903E-03	4.788E-04
32	8.000E-03	2.032E-03	0.21	273	5.661E-06	9.286E-03	3.950E-04
34	6.300E-03	1.600E-03	0.16	361	3.649E-06	1.556E-02	3.800E-04
36	5.000E-03	1.270E-03	7.81E-02	722	3.649E-06	3.922E-02	2.394E-04
38	4.000E-03	1.016E-03	5.16E-02	1094	2.831E-06	7.428E-02	1.975E-04
40	3.100E-03	7.874E-04	3.91E-02	1444	1.739E-06	1.265E-01	1.931E-04
42	2.500E-03	6.350E-04	1.95E-02	2888	1.824E-06	3.138E-01	1.197E-04
44	2.000E-03	5.080E-04	1.29E-02	4375	1.415E-06	5.943E-01	9.875E-05
46	1.600E-03	4.064E-04	9.77E-03	5775	9.565E-07	9.806E-01	9.351E-05
48	1.200E-03	3.048E-04	4.88E-03	11551	8.070E-07	2.615E+00	6.234E-05
50	1.000E-03	2.540E-04	3.22E-03	17501	7.076E-07	4.754E+00	4.938E-05

As the wire gauge increases, the wire diameter and current carrying capacity both decrease, and in turn the number of turns required increases, and the coil volume decreases. However, the resistance also increases with an increase in wire gauge. Hence, the minimum volume will not result in the lowest possible resistance. Finally, the power dissipation decreases with the increase of wire gauge due to decrease in current capacity.

Problem 16.29

Solution:

Known quantities:

Solenoid of Example 16.10 (Figure 16.40)

$$a = 0.01\text{m}, l_{gap} = 0.001\text{m}, K = 10\text{N/m}$$

Find:

f , mmf using equation 16.46 and equation 16.30

Assumptions:

The reluctance of the iron is negligible, Neglect fringing

At $x = 0$, the plunger is in the gap by an infinitesimal displacement ε

Analysis:

From Example 16.10:

$$\mathfrak{R}(x) = \frac{2l_{gap}}{\mu_o ax}$$

Compute inductance if the magnetic circuit as a function of reluctance (equation 16.30):

$$L = \frac{N^2}{\mathfrak{R}(x)} = \frac{N^2 \mu_o ax}{2l_{gap}}$$

Compute stored magnetic energy:

$$W_m = \frac{1}{2} Li^2 = \frac{1}{2} \frac{N^2 i^2 \mu_o ax}{2l_{gap}}$$

Finally, use equation 16.46 to write the expression for the magnetic force:

$$f_e = -\frac{dW_m}{dx} = -\frac{N^2 i^2 \mu_o a}{4l_{gap}}$$

This matches the derivation from example 16.10 using the relationship between the magnetic flux, the reluctance of the structure, and the magnetic force.

The calculation for the required mmf is identical to example 16.10:

$$f_{gap} = kx = ka = (10N/m) \times (0.01m) = 0.1N$$

Problem 16.30**Solution:****Known quantities:**

Solenoid of Example 16.11 (Figure 16.40)

$$a = 0.01m, l_{gap} = 0.001m, K = 10N/m, N = 1000turns, V = 12V, R_{coil} = 5\Omega$$

Find:

Current and magnetic force response as a function of time using equation 16.46 and equation 16.30 in the derivation

Assumptions:

The reluctance of the iron is negligible, Neglect fringing

The inductance of the solenoid is approximately constant and is equal to the midrange value (plunger displacement equal to $a/2$).

Analysis:

From Example 16.11:

$$\mathfrak{R}_{gap}(x) = \frac{2l_{gap}}{\mu_o ax}$$

Compute inductance if the magnetic circuit as a function of reluctance (equation 16.30):

$$L = \frac{N^2}{\mathfrak{R}_{gap}(x)} = \frac{N^2 \mu_o ax}{2l_{gap}}$$

Compute stored magnetic energy:

$$W_m = \frac{1}{2} Li(t)^2 = \frac{1}{2} \frac{N^2 i(t)^2 \mu_o ax}{2l_{gap}}$$

Finally, use equation 16.46 to write the expression for the magnetic force:

$$f_{gap} = -\frac{dW_m}{dx} = -\frac{N^2 i(t)^2 \mu_o a}{4l_{gap}} = \frac{(1000)^2 \times 4\pi \times 10^{-7} \times 0.01}{4 \times 0.001} i(t)^2 = \pi i(t)^2$$

With the assumption that the inductance is constant with $x = a/2$:

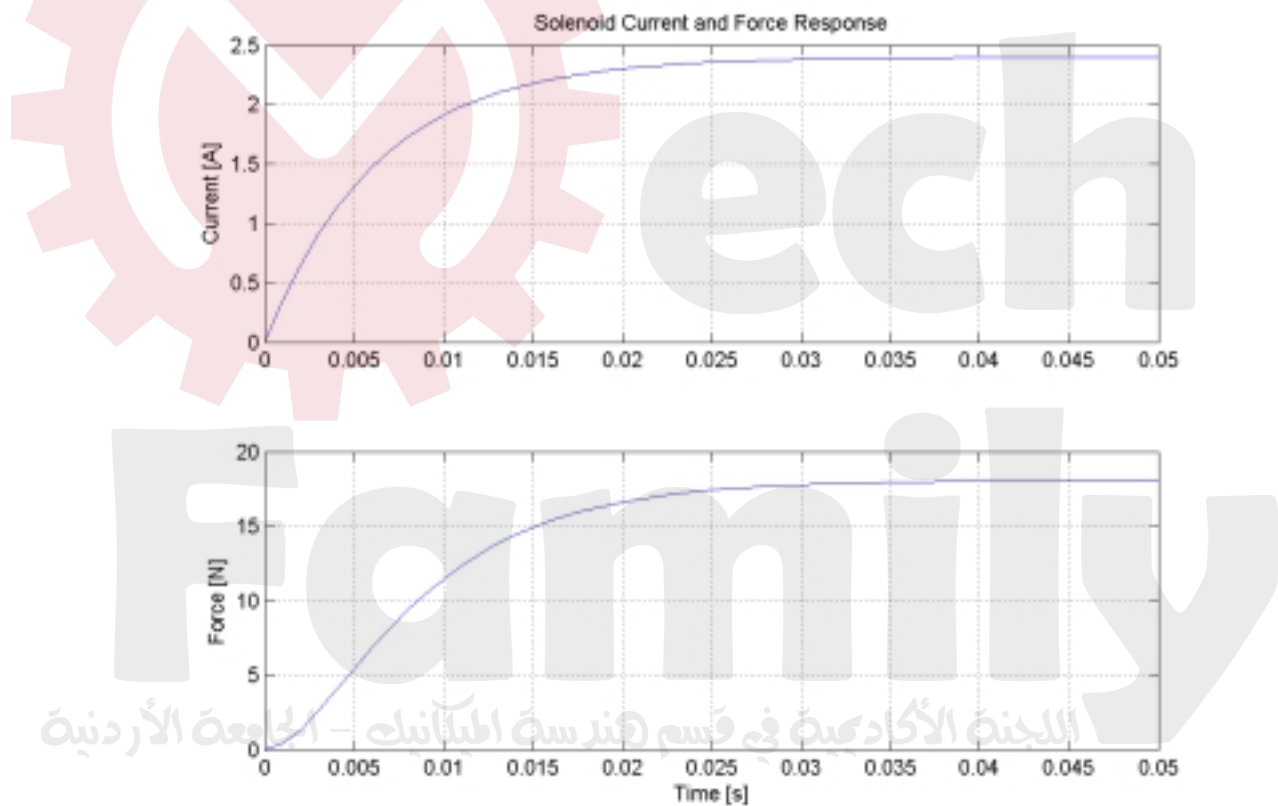
$$L \approx \frac{N^2}{\mathfrak{R}_{gap}(x)} = \frac{N^2 \mu_o a^2}{4l_{gap}} = \frac{(1000)^2 \times 4\pi \times 10^{-7} \times (0.01)^2}{4 \times 0.001} = 31.4mH$$

From Example 16.11:

$$i(t) = \frac{V}{R} \left(1 - e^{-Rt/L} \right) = \frac{12}{5} \left(1 - e^{-t/6.3 \times 10^{-3}} \right) A$$

$$f_{gap} = \pi \left[\frac{12}{5} \left(1 - e^{-t/6.3 \times 10^{-3}} \right) \right]^2$$

This matches the derivation from example 16.11 using the relationship between the magnetic flux, the reluctance of the structure, and the magnetic force. The response curves are shown below:



Problem 16.31

Solution:

Known quantities:

Solenoid of Example 16.11 (Figure 16.40)

$$a = 0.01m, l_{gap} = 0.001m, K = 10N/m, N = 1000turns, V = 12V, R_{coil} = 5\Omega$$

$$m = 0.5\text{kg}$$

Find:

Generate a simulation program that accounts for the fact that the solenoid inductance is not constant, but is a function of plunger position

Compare graphically the current and force step responses of this system to the step response obtained in Example 16.11

Assumptions:

The reluctance of the iron is negligible, Neglect fringing

Neglect damping on the plunger

Analysis:

From Example 16.11:

$$\mathfrak{R}_{gap}(x) = \frac{2l_{gap}}{\mu_o ax}$$

The inductance is now a function of plunger position

$$L = \frac{N^2}{\mathfrak{R}_{gap}(x)} = \frac{N^2 \mu_o ax}{2l_{gap}}$$

The differential equation for the current using the equivalent circuit shown below:



$$V(t) = Ri(t) + L \frac{di}{dt}$$

To find the equation of the force, compute stored magnetic energy as a function of current:

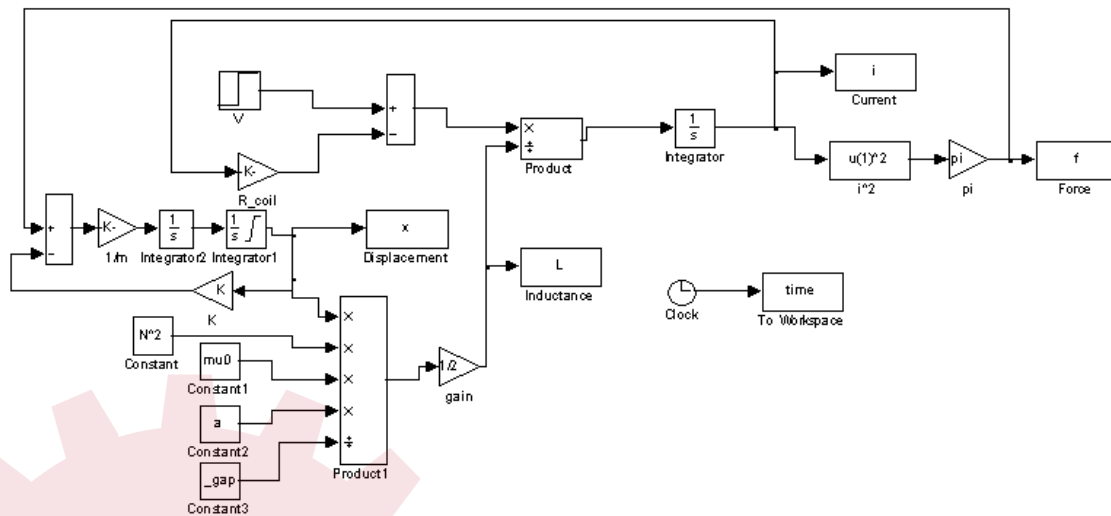
$$W_m = \frac{1}{2} Li(t)^2 = \frac{1}{2} \frac{N^2 i(t)^2 \mu_o ax}{2l_{gap}}$$

Finally, use equation 16.46 to write the expression for the magnetic force:

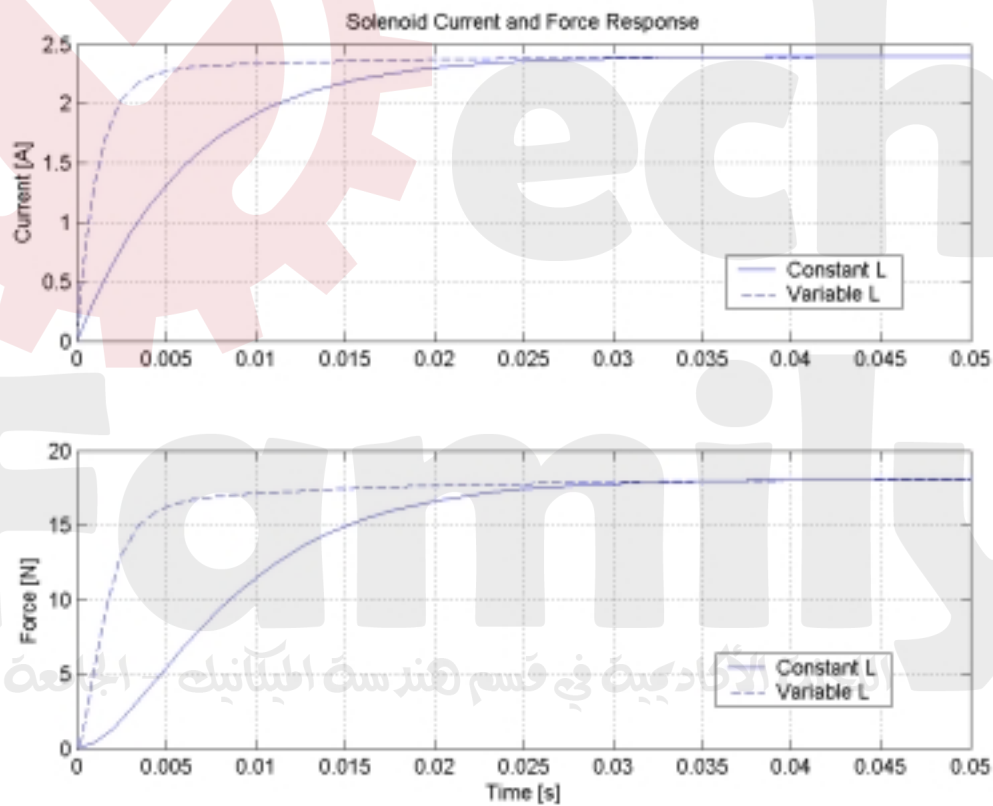
$$f_{gap} = -\frac{dW_m}{dx} = -\frac{N^2 i(t)^2 \mu_o a}{4l_{gap}} = \frac{(1000)^2 \times 4\pi \times 10^{-7} \times 0.01}{4 \times 0.001} i(t)^2 = \pi i(t)^2$$

This matches the derivation from example 16.11 using the relationship between the magnetic flux, the reluctance of the structure, and the magnetic force.

Simulink Block Diagram:



Comparison of Step Responses for Constant Inductance and Variable Inductance Systems:



Note the quicker response of the variable inductance system due to the smaller inductance initially. The larger constant inductance results in a delayed response.

Problem 16.32**Solution:****Known quantities:**

Relay of Example 16.12 (Figure 16.46)

$$N = 10,000 \text{ turns}; f_{\text{restore}} = 5N; A_{\text{gap}} = (0.01 \text{ m})^2; x = 0.05 \text{ m}; \mu_r = 1000$$

Find:

Required holding current to keep relay closed

Assumptions:

Air gap becomes zero and the iron reluctance cannot be neglected

Analysis:

To compute the current we need to derive an expression for the force in the air gap.

Without neglecting the iron reluctance, we can write the expression for the reluctance as follows:

$$(x) \quad \frac{L}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A}$$

where L is the total length of the iron magnetic path (excluding the air gap).

Knowing the reluctance we can calculate the magnetic flux in the structure as a function of the coil current:

$$(x) \quad \frac{Ni}{\frac{L}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A}}$$

Then, the magnitude of the force in the air gap is given by the expression

$$|f| = \frac{1}{2} \frac{d}{dx} \left(\frac{N^2 \mu_0 A^2}{L^2 + 4Lx + 4x^2} \right) = \frac{N^2 \mu_0 A^2}{L^2 + 4Lx + 4x^2} \cdot \frac{2x}{L^2 + 4Lx + 4x^2}$$

As x approaches zero, we can calculate the force to be:

$$|f(x=0)| = \frac{(Ni\mu_r)^2 \mu_0 A}{L^2}$$

and the current required to maintain the given force is:

$$|f(x=0)| = \frac{(Ni\mu_r)^2 \mu_0 A}{L^2}$$

$$i = \pm \sqrt{\frac{L^2 |f(x=0)|}{(N\mu_r)^2 \mu_0 A}} = \pm \frac{L}{N\mu_r} \sqrt{\frac{|f(x=0)|}{\mu_0 A}}$$

The total length of the magnetic path:

$$L = 0.05 \text{ m} + 0.05 \text{ m} + 0.10 \text{ m} + 0.10 \text{ m} = 0.30 \text{ m}$$

The current:

$$i = \pm \frac{(0.30)}{(10000)(1000)} \sqrt{\frac{(5)}{(4\pi \times 10^{-7})(0.01)^2}} = \pm 0.0060 \text{ A} = \pm 6.0 \text{ mA}$$

Problem 16.33**Solution:****Known quantities:**

Relay Circuit shown in Figure P16.33

$$N = 500 \text{ turns}; A_{\text{gap}} = 0.001 \text{ m}^2; L = 0.02 \text{ m}; k = 1000 \text{ N/m}; R = 18 \Omega$$

Find:Minimum DC supply voltage v for which the relay will make contact when the electrical switch is closed

Assumptions:

Neglect the iron reluctance

Analysis:

The reluctance of the gap:

$$\mathfrak{R}_{gap}(x) = \frac{2x}{\mu_0 A_{gap}}$$

$$\frac{d\mathfrak{R}_{gap}(x)}{dx} = \frac{2}{\mu_0 A_{gap}}$$

Magnetic Flux:

$$\phi = \frac{Ni}{\mathfrak{R}(x)} = \frac{Ni\mu_0 A_{gap}}{2x}$$

Magnetic force:

$$f_{gap} = \frac{\phi^2}{2} \frac{d\mathfrak{R}(x)}{dx} = \left(\frac{Ni\mu_0 A_{gap}}{2x} \right)^2 \frac{1}{2} \frac{2}{\mu_0 A_{gap}} = \frac{(Ni)^2 \mu_0 A_{gap}}{4x^2}$$

The force that must be overcome is the spring force, f_k :

$$f_{gap} = f_k = kx$$

Equating the two force equations and solving for the current:

$$i = \pm \frac{2}{N} \sqrt{\frac{kx^3}{\mu_0 A_{gap}}}$$

The voltage is determined using Ohm's law, and $x = L$:

$$v = iR = \pm \frac{2R}{N} \sqrt{\frac{kx^3}{\mu_0 A_{gap}}} = \pm \frac{2(18)}{(500)} \sqrt{\frac{(1000)(0.02)^3}{4\pi \times 10^{-7} (0.001)}} = 182.0V$$

Problem 16.34**Solution:****Known quantities:**

The simplified representation of a surface roughness sensor shown in Figure P16.34

Find:Derive an expression for the displacement x as a function of the various parameters of the magnetic circuit and of the measured emf e **Assumptions:**The flux $\phi = \beta / \mathfrak{R}(x)$ where β is a known constant

Frictionless contact between the moving plunger and the magnetic structure

The plunger is restrained to vertical motion only

Analysis:

$$e = N \frac{d}{dt} \left(\frac{L}{r_0} \frac{x}{A} + \frac{x}{r_0 A} + \frac{1}{r_0 A} L \right) = N \frac{d}{dt} \left(\frac{L}{r_0} \frac{x}{A} + \frac{x}{r_0 A} + \frac{1}{r_0 A} L \right)$$

$$e = N \frac{d}{dt} \left(\frac{L}{r_0} \frac{x}{A} + \frac{x}{r_0 A} + \frac{1}{r_0 A} L \right) = N \frac{d}{dt} \left(\frac{L}{r_0} \frac{x}{A} + \frac{x}{r_0 A} + \frac{1}{r_0 A} L \right)$$

Problem 16.35

Solution:

Known quantities:

As shown in Figure P16.35. The air gap between the shell and the plunger is uniform and equal to 1 mm . The diameter is $d = 25 \text{ mm}$. The exciting current is 7.5 A . $N = 200$.

Find:

The force acting on the plunger when $x = 2 \text{ mm}$.

Assumptions:

None.

Analysis:

The cross-section area A is:

$$A = \pi \left(\frac{25 \times 10^{-3}}{2} \right)^2 \text{ m}^2$$

From this expression we can compute the variable reluctance of the air gap:

$$\mathcal{R}_x = \frac{x}{4\pi \times 10^{-7} \times \frac{\pi(25 \times 10^{-3})^2}{4}} = 1621 \times 10^6 x$$

and the resulting force is:

$$f = \frac{i^2}{2} \frac{N^2}{R_x^2} \frac{dR_x}{dx}$$

$$= \frac{7.5^2}{2} \frac{200^2}{(1621 \times 10^6 x)^2} \times 1621 \times 10^6 \Big|_{x=2 \times 10^{-3}}$$

$$= 173.5 \text{ N}$$

Problem 16.36

Solution:

Known quantities:

As shown in Figure P16.36, the double-excited electromechanical system moves horizontally. The cross section of the structure is $w \times w$.

Find:

- a) The reluctance of the magnetic circuit.
- b) The magnetic energy stored in the air gap.
- c) The force on the movable part as a function of its position.

Assumptions:

Resistance, magnetic leakage and fringing are negligible. The permeability of the core is very large.

Analysis:

a)

The reluctance of the magnetic circuit is:

$$\mathfrak{R} = \frac{x}{\mu_0 w^2} + \frac{l_g}{\mu_0 w(w-x)}$$

b)

The magnetic energy stored in the air gap is:

$$W_m = \frac{(N_1 + N_2)^2 i^2}{2\mathfrak{R}}$$

c)

The magnetic force is:

$$f = \frac{i^2}{2} \frac{(N_1 + N_2)^2 \left(\frac{x}{\mu_0 w^2} + \frac{l_g}{\mu_0 w(w-x)^2} \right)}{\left(\frac{x}{\mu_0 w^2} + \frac{l_g}{\mu_0 w(w-x)} \right)^2}$$

Problem 16.37**Solution:****Known quantities:**

The flux density in the cast steel pathway is $1.1T$. The diameter of the plunger is 10 mm .

Find:

The force f between the faces of the poles.

Assumptions:

Air gap is negligible between walls and plunger

Since the pathway is cast steel, $\mu_r = 1000$ (from Table 16.1).

Analysis:

Using Equation 16.50:

$$f = \frac{\phi^2}{2} \frac{d\mathfrak{R}(x)}{dx}$$

The flux is determined from the flux density and the area:

$$\phi = BA$$

Since the plunger is cylindrical and the air gap between the plunger and the coil is negligible, the reluctance is calculated as:

$$\mathfrak{R}(x) = \frac{x}{\mu A}$$

where x is the gap between the plunger and back wall of the solenoid. The reluctance of the cast steel pathway can be neglected due to the low reluctance of the structure.

The derivative of the reluctance is:

$$\frac{d\mathcal{R}(x)}{dx} = \frac{1}{\mu A}$$

The area is the cross-sectional area of the plunger:

$$A = \frac{\pi}{4} d^2$$

Combining all of the equations into the force equation:

$$f = \frac{\phi^2}{2} \frac{d\mathcal{R}(x)}{dx} = \frac{(BA)^2}{2} \frac{1}{\mu A} = \frac{\pi d^2 B^2}{(2)(4)} \frac{1}{\mu_o} = \frac{\pi(0.010)^2 (1.1)^2}{8(4\pi \times 10^{-7})} = 37.8 N$$

Problem 16.38

Solution:

Known quantities:

A force of $10,000 N$ is required to support the weight. The cross-sectional area of the magnetic core is $0.01 m^2$. The coil has $1000 turns$.

Find:

The minimum current that can keep the weight from falling for $x = 1.0 mm$.

Assumptions:

Negligible reluctance for the steel parts and negligible fringing in the air gaps.

Analysis:

The variable reluctance is given by:

$$\begin{aligned} \mathcal{R}_T(x) = 2\mathcal{R}(x) &= \frac{2x}{4\pi \times 10^{-7} (0.01)} \\ &= 159.15 \times 10^6 x \end{aligned}$$

The force is related to the reluctance by:

$$f = -10000 = -\frac{N^2 i^2}{2\mathcal{R}_T^2(x)} \frac{d\mathcal{R}_T(x)}{dx}$$

Therefore,

$$i = \sqrt{3.18} = 1.784 A$$

Problem 16.39

Solution:

Known quantities:

The 12-VDC control relay is made of sheet steel. Average length of the magnetic circuit is $12 cm$. The average cross section of the magnetic circuit is $0.60 cm^2$. The coil has $250 turns$ and carries $50 mA$.

Find:

- The flux density B in the magnetic circuit of the relay when the coil is energized.
- The force F exerted on the armature to close it when the coil is energized.

Assumptions:

None.

Analysis:

-

$$NI = HI$$

$$250(250 \times 10^{-3}) = H(12 \times 10^{-2}) \Rightarrow H = 104.2$$

from the curve, $B = 0.75 T$.

b.

$$F = \frac{1}{2} \frac{AB^2}{\mu_0} = \frac{1}{2} \frac{(0.6 \times 10^{-4})(0.75)^2}{4\pi \times 10^{-7}} = 13.4 N$$

Problem 16.40

Solution:

Known quantities:

As shown in Figure P16.40.

Find:

The differential equations describing the system.

Assumptions:

None.

Analysis:

The equation for the electrical system is:

$$v = iR + L(x) \frac{di}{dt}$$

where:

$$L(x) = \frac{N^2}{\mathfrak{R}_T(x)} = \frac{N^2 \mu_0 A}{2x}$$

The equation for the mechanical system is:

$$F_m = m \frac{d^2 x}{dt^2} + kx$$

where F_m is the magnetic pull force. To calculate this force we use the following equation:

$$F_m = - \frac{dW_m}{dx}$$

where W_m is the energy stored in the magnetic field.

Let F and $\mathfrak{R}$ be the magnetomotive force acting on the structure and its reluctance, respectively; then:

$$W_m = \frac{\phi^2 \mathfrak{R}(x)}{2} = \frac{F^2}{2\mathfrak{R}(x)} = \frac{N^2 i^2 \mu A}{4x}$$

$$F_m = - \frac{dW_m}{dx} = \frac{N^2 i^2 \mu A}{4x^2}$$

Finally, the differential equations governing the system are:

$$v = iR + \frac{N^2 \mu_0 A}{2x} \frac{di}{dt}$$

$$m \frac{d^2 x}{dt^2} + kx = \frac{N^2 i^2 \mu A}{4x^2}$$

This system of equations could be solved using a numerical simulation, since it is nonlinear.

Problem 16.41**Solution:****Known quantities:**

As shown in Figure P16.41. The solenoid has a cross section of 5 cm^2 .

Find:

- The force exerted on the plunger when the distance x is 2 cm and the current is 5 A . $N = 100\text{ turns}$. Relative permeabilities of the magnetic material and the nonmagnetic sleeve are $2,000$ and 1 .
- Develop a set of differential equations governing the behavior of the solenoid.

Assumptions:

The fringing and leakage effects are negligible.

Analysis:

a.

$$\mathfrak{R}_x = \frac{x}{\mu_0 A_x}$$

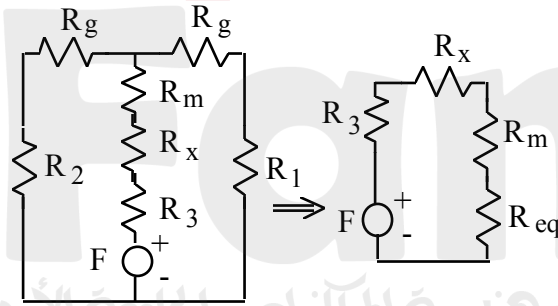
$$\mathfrak{R}_g = \frac{l_g}{\mu_0 A_g} = \frac{0.005}{(4\pi \times 10^{-7})(5 \times 10^{-4})} = 7.96 \times 10^6 H^{-1}$$

$$\mathfrak{R}_1 = \mathfrak{R}_2 = \frac{l_1}{\mu_r \mu_0 A_g} = \frac{0.335}{2000(4\pi \times 10^{-7})(5 \times 10^{-4})} = 2.67 \times 10^5 H^{-1}$$

$$\mathfrak{R}_3 = \frac{l_3}{\mu_r \mu_0 A_x} = \frac{0.055}{2000(4\pi \times 10^{-7})(10 \times 10^{-4})} = 2.19 \times 10^4 H^{-1}$$

$$\mathfrak{R}_m = \frac{l_x}{\mu_r \mu_0 A_x} = \frac{l_x}{2000(4\pi \times 10^{-7})(10 \times 10^{-4})} = (3.98 \times 10^5)(0.095 - x) H^{-1}$$

The circuit is shown below:



$$\mathfrak{R}_{eq} = \frac{\mathfrak{R}_g + \mathfrak{R}_1}{2} = \frac{l_g}{2\mu_0 A_g} + \frac{l_1}{2\mu_r \mu_0 A} = 4.11 \times 10^6 H^{-1}$$

$$\mathfrak{R}_T = \mathfrak{R}_3 + \mathfrak{R}_x + \mathfrak{R}_m + \mathfrak{R}_{eq} = \frac{l_3}{\mu_r \mu_0 A_x} + \frac{x}{\mu_0 A_x} + \frac{(0.095 - x)}{\mu_r \mu_0 A_x} + \frac{l_g}{2\mu_0 A_g} + \frac{l_1}{2\mu_r \mu_0 A_g}$$

Since $A_x = 2A_g$:

$$\begin{aligned}
\mathfrak{R}_T &= \frac{l_3 + x\mu_r + (0.095 - x) + l_g\mu_r + l_1}{2\mu_r\mu_0A_g} = \\
&= \frac{0.055 + 0.095 - x + 2000x + (0.005)(2000) + 0.335}{2(2000)(4\pi \times 10^{-7})(5 \times 10^{-4})} = \frac{10.485 + 1999x}{2.51 \times 10^{-6}} = \\
&= 4.18 \times 10^6 + 7.96 \times 10^8 x \\
f_m &= -\frac{\phi^2}{2} \frac{\partial \mathfrak{R}_T(x)}{\partial x} = -\frac{N^2 i^2}{2R_T^2} \frac{d\mathfrak{R}_T}{dx} = -\frac{N^2 i^2 (2\mu_r\mu_0A_g)^2}{2(l_3 + x\mu_r + (0.095 - x) + l_g\mu_r + l_1)^2} \frac{(\mu_r - 1)}{2\mu_r\mu_0A_g} = \\
&= -\frac{\mu_r\mu_0A_g N^2 i^2 (\mu_r - 1)}{(l_3 + x\mu_r + (0.095 - x) + l_g\mu_r + l_1)^2} = \\
&= -\frac{(2000)(4\pi \times 10^{-7})(5 \times 10^{-4})(100)^2 (5)^2 (2000 - 1)}{(0.055 + 0.095 - x + 2000x + (0.005)(2000) + 0.335)^2}
\end{aligned}$$

For $x = 0.02$, $f = -0.25N$.

b.

Electrical subsystem:

$$v_s(t) = L(x) \frac{di(t)}{dt} + Ri(t)$$

Mechanical subsystem:

$$m\ddot{x}(t) = f_m(x) - d \frac{dx(t)}{dt} - \frac{k}{l} x(t)$$

Reluctance:

$$\mathfrak{R}_T = \frac{l_3 + x\mu_r + (0.095 - x) + l_g\mu_r + l_1}{2\mu_r\mu_0A_g}$$

Flux:

$$\phi = \frac{Ni(t)}{\mathfrak{R}_m(x)} = \frac{2\mu_r\mu_0A_g Ni(t)}{l_3 + x\mu_r + (0.095 - x) + l_g\mu_r + l_1}$$

Magnetic force:

$$\begin{aligned}
f_m(x) &= \frac{\phi^2}{2} \frac{\partial \mathfrak{R}_T(x)}{\partial x} = -\frac{N^2 i^2}{2R_T^2} \frac{d\mathfrak{R}_T}{dx} = -\frac{N^2 i^2 (2\mu_r\mu_0A_g)^2}{2(l_3 + x\mu_r + (0.095 - x) + l_g\mu_r + l_1)^2} \frac{(\mu_r - 1)}{2\mu_r\mu_0A_g} = \\
&= \frac{\mu_r\mu_0A_g N^2 i^2 (\mu_r - 1)}{(l_3 + x\mu_r + (0.095 - x) + l_g\mu_r + l_1)^2}
\end{aligned}$$

Inductance:

$$L(x) = \frac{N^2}{\mathfrak{R}_T(x)} = \frac{N^2 2\mu_r\mu_0A_g}{l_3 + x\mu_r + (0.095 - x) + l_g\mu_r + l_1}$$

Substituting the expressions for f_m and $L(x)$ in the two differential equations, we have the final answer.

Electrical subsystem:

$$v_s(t) = \left(\frac{N^2 2\mu_r \mu_0 A_g}{l_3 + x\mu_r + (0.095 - x) + l_g \mu_r + l_1} \right) \frac{di(t)}{dt} + Ri(t)$$

Mechanical subsystem:

$$m\ddot{x}(t) = \frac{\mu_r \mu_0 A_g N^2 i^2 (\mu_r - 1)}{(l_3 + x\mu_r + (0.095 - x) + l_g \mu_r + l_1)^2} - d \frac{dx(t)}{dt} - \frac{k}{l} x(t)$$

Note that these equations are very nonlinear!

Problem 16.42

Solution:

Known quantities:

The relay shown in Figure P16.42

Find:

Derive the differential equations (electrical and mechanical) for the relay

Assumptions:

The inductance is a function of x

The iron reluctance is negligible

Analysis:

Electrical subsystem:

$$v(t) = L(x) \frac{di(t)}{dt} + Ri(t)$$

Mechanical subsystem:

$$m\ddot{x}(t) = f_m(x) - b \frac{dx(t)}{dt} - kx(t)$$

Next, we calculate the magnetic force and inductance as functions of x .

Reluctance:

$$\mathcal{R}(x) = \frac{2x}{A}$$

Flux:

$$\Phi(x) = \frac{Ni(t)}{\mathcal{R}(x)} = \frac{Ni(t)}{2x} A$$

Magnetic force:

$$f_m(x) = \frac{\phi^2}{2} \frac{\partial \mathcal{R}(x)}{\partial x} = \frac{N^2 i^2 \mu A}{4x^2}$$

Inductance:

$$L(x) = \frac{N^2}{\mathcal{R}(x)} = \frac{N^2 \mu A}{2x}$$

Substituting the expressions for f_m and $L(x)$ in the two differential equations, we have the final answer.

Electrical subsystem:

$$v(t) = \frac{N^2 \mu A}{2x(t)} \frac{di(t)}{dt} + Ri(t)$$

Mechanical subsystem:

$$m\ddot{x}(t) = \frac{N^2 i^2 \mu A}{4x(t)^2} - b \frac{dx(t)}{dt} - kx(t)$$

Note that these equations are very nonlinear!

b) Moving-Coil Transducers

Focus on Methodology: Analysis of moving-coil electromechanical transducers

1. Apply KVL to write the differential equation for the electrical subsystem, including the back emf, $e = Blu$, term.
2. Apply Newton's Second Law to write the differential equation for the mechanical subsystem, including the magnetic force $f = Bli$, term.
3. Laplace transform the two coupled differential equations to formulate a system of linear algebraic equations, and solve for the desired mechanical and electrical variables.

Problem 16.43

Solution:

Known quantities:

Length of the wire is 20 cm ; Flux density is 0.1 T ; The position of the wire is $x(t) = 0.1 \sin 10t$ m

Find:

The induced emf across the length of the wire as a function of time.

Assumptions:

None.

Analysis:

From $e = Blu$, we have

$$e(t) = Bl \frac{dx}{dt} = 0.02 \cos(10t) \text{ V}$$

Problem 16.44**Solution:****Known quantities:**

Emf: $e_1(t) = 0.02 \cos 10t$; the length of the second wire: 0.1 m ; the position of the second wire: $x(t) = 1 - 0.1 \sin 10t$.

Find:

The induced emf $e(t)$ defined by the difference in emf's $e_1(t)$ and $e_2(t)$.

Assumptions:

None.

Analysis:

We have

$$e(t) = e_1(t) - e_2(t)$$

$$e_2(t) = (0.1)(0.1)(-1 \cos 10t) V$$

$$e(t) = 0.02 \cos(10t) + 0.01 \cos(10t) = 0.03 \cos(10t) V$$

Problem 16.45**Solution:****Known quantities:**

$$I = 4\text{ A}, B = 0.3\text{ Wb/m}^2.$$

Find:

The magnitude and direction of the force induced on the conducting bar.

Assumptions:

None.

Analysis:

$$f = Bli = 0.3 \times l \times 4 = 1.2\text{ N}$$

Force will be to the left if current flows upward.

Problem 16.46**Solution:****Known quantities:**

$$B = 0.3\text{ Wb/m}^2.$$

Find:

The magnitude and direction of the induced voltage in the wire.

Assumptions:

None.

Analysis:

$$e = Blu \cos 45^\circ = 2.83\text{ V}$$

Problem 16.47**Solution:****Known quantities:**

The electrodynamic shaker shown in Figure P16.47

Shaker mass, m ; air gap dimension, d ; number of coil turns, N ; spring parameter, k ; armature resistance, R and inductance, L

Find:

- Reluctance of the structure and Flux density B
- Dynamic equations of the shaker
- Transfer function and frequency response of the shaker velocity to input voltage V_s .

Assumptions:

Iron reluctance is negligible. Neglect fringing. Assume no damping in this system.

Analysis:

- Reluctance of structure:

$$\frac{2d}{\mu_0 A}$$

Compute flux density:

$$B = \frac{\mu_0 N I_f}{2d}$$

- Electrical Subsystem, using KVL:

$$V_s = Ri + L \frac{di}{dt} + Bl \frac{dx}{dt}$$

Mechanical Subsystem:

$$m \frac{d^2 x}{dt^2} + Bl i = kx$$

- Laplace Transform:

$$\begin{aligned} V_s(s) &= (R + Ls)I(s) + BlsX(s) \\ 0 &= BlI(s) + (ms^2 + k)X(s) \end{aligned}$$

$$V_s(s) = \frac{(R + Ls)(ms^2 + k)}{Bl} X(s)$$

$$\frac{X(s)}{V_s(s)} = \frac{Bl}{mLs^3 + mRs^2 + (kL + B^2 l^2)s + Rk}$$

$$\frac{U(s)}{V_s(s)} = \frac{SX(s)}{V_s(s)} = \frac{Bl s}{mLs^3 + mRs^2 + (kL + B^2 l^2)s + Rk}$$

Frequency Response: $s = j\omega$

$$\frac{U(j\omega)}{V_s(j\omega)} = \frac{Blj\omega}{mL(j\omega)^3 + mR(j\omega)^2 + (kL + B^2 l^2)j\omega + Rk}$$

$$\frac{U(j\omega)}{V_s(j\omega)} = \frac{Blj\omega}{R(k - m\omega^2) + j[(kL + B^2 l^2)\omega - mL\omega^3]}$$

Problem 16.48**Solution:****Known quantities:**

The electrodynamic shaker shown in Figure P16.47 is used to perform vibration testing of an electrical connector.

$B = 1,000 \text{ Wb/m}^2$; $l = 5 \text{ m}$; $k = 1000 \text{ N/m}$; $m = 1 \text{ kg}$;

$b = 5 \text{ N-s/m}$; $L = 0.8 \text{ H}$; $R = 0.5 \text{ } \Omega$;

The test consists of shaking the connector at the frequency $\omega = 2\pi \times 100 \text{ rad/s}$

Find:

The peak amplitude of sinusoidal voltage V_s required to generate an acceleration of $5g(49 \text{ m/s}^2)$ under the stated test conditions

Assumptions:

Connector has negligible mass when compared to the platform.

Analysis:

Applying KVL around the coil circuit:

$$L \frac{di}{dt} + Ri + Bl \frac{dx}{dt} = V_s$$

Next, we apply Newton's Second Law:

$$Bli + m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

To derive the frequency response, Laplace transform the two equations to obtain:

$$sL + R + Bls \quad I(s) = V_s(s)$$

$$BlI(s) + (ms^2 + bs + k)X(s) = 0$$

We can write the above equations in matrix form and resort to Cramer's rule to solve for $U(s)$ as a function of $V(s)$:

$$\begin{bmatrix} sL + R & Bls \\ Bl & ms^2 + bs + k \end{bmatrix} \begin{bmatrix} I(s) \\ X(s) \end{bmatrix} = \begin{bmatrix} V(s) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} sL + R & Bls \\ Bl & ms^2 + bs + k \end{bmatrix} \begin{bmatrix} I(s) \\ X(s) \end{bmatrix} = \begin{bmatrix} V(s) \\ 0 \end{bmatrix}$$

with solution

$$X(s) = \frac{\det \begin{bmatrix} Ls + R & V(s) \\ Bl & 0 \end{bmatrix}}{\det \begin{bmatrix} Ls + R & Bls \\ Bl & ms^2 + bs + k \end{bmatrix}} = \frac{-BlV(s)}{(Ls + R)(ms^2 + bs + k) - Bl^2s}$$

To obtain the acceleration response, we multiply the numerator by s^2 :

$$\frac{s^2 X(s)}{V(s)} = \frac{\ddot{X}(s)}{V(s)} = \frac{Bl s^2}{(Lm)s^3 + (bL + Rm)s^2 + (bR + kL + (Bl)^2)s + (kR)}$$

$$\frac{\ddot{X}(j\omega)}{V(j\omega)} = \frac{-Bl\omega^2}{[kR - (bL + Rm)\omega^2] + j[(bR + kL + (Bl)^2)\omega - (Lm)\omega^3]}$$

The magnitude of this complex number, evaluated at $\omega = 2\pi \times 100$ is 0.62.

Thus, to obtain the desired acceleration (peak) of 49 m/s^2 , we wish to have a peak voltage amplitude $|V_s| = 49/0.62 \approx 78 \text{ V}$.

Problem 16.49**Solution:****Known quantities:**

As described in Example 16.13.

Find:

Derive and sketch the frequency response of the loudspeaker in the following two cases. Describe qualitatively how the loudspeaker frequency response changes as the spring stiffness k increases and decreases. Find the limit of the frequency response and the kind of the speaker as k approaches zero.

a. $k = 50,000 \text{ N/m}$.

b. $k = 5 \times 10^6 \text{ N/m}$.

Assumptions:

None.

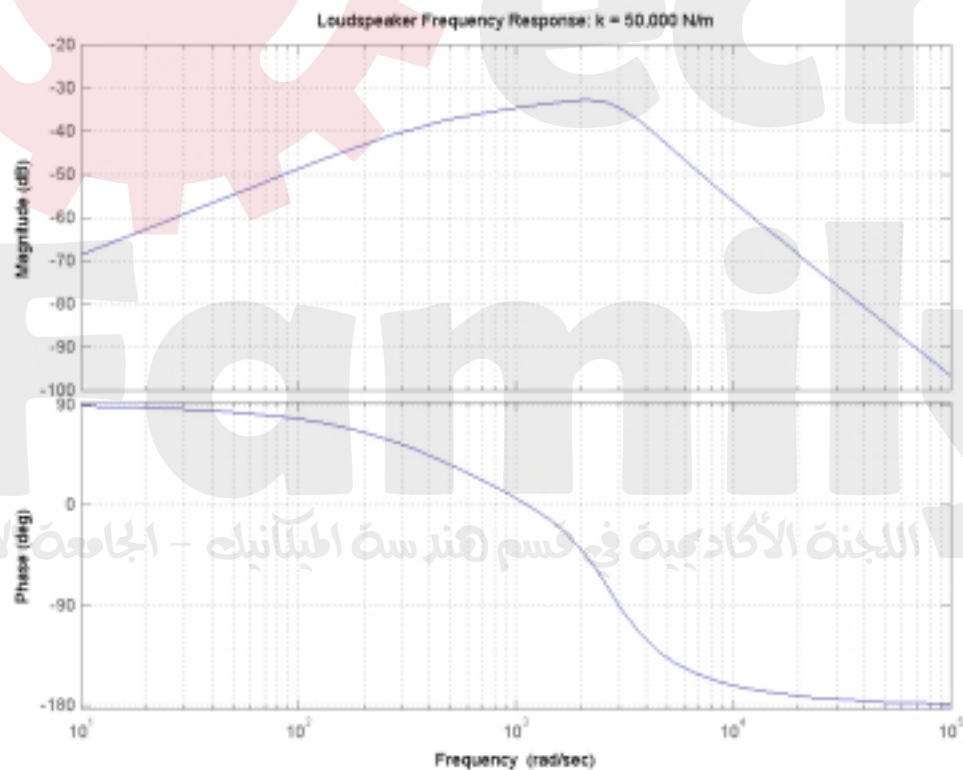
Analysis:

a.

For $k = 50,000 \text{ N/m}$, the transfer function is:

$$\frac{U}{V} = \frac{1.478 \times 10^5 s}{s^3 + 3075s^2 + 9 \times 10^6 s + 4 \times 10^9}$$

The magnitude frequency response is plotted below:



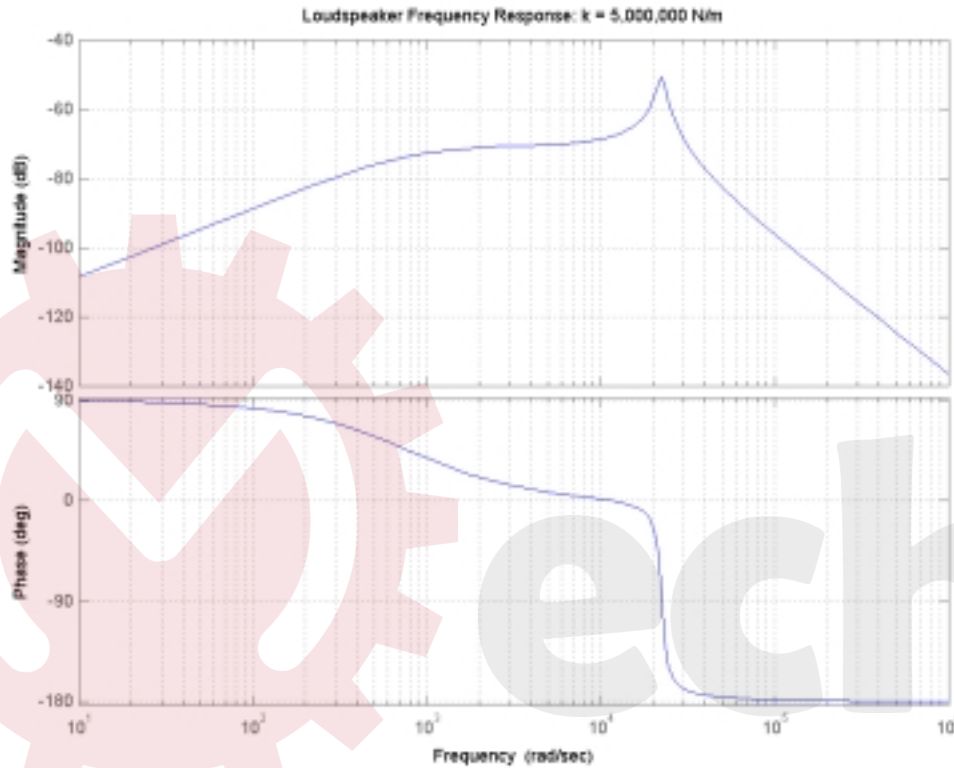
This response would correspond to a midrange speaker.

b.

For $k = 5 \times 10^6 \text{ N/m}$, the transfer function is :

$$\frac{U}{V} = \frac{1.478 \times 10^5 s}{s^3 + 3075s^2 + 5.04 \times 10^8 s + 4 \times 10^{11}}$$

The magnitude frequency response is plotted below:



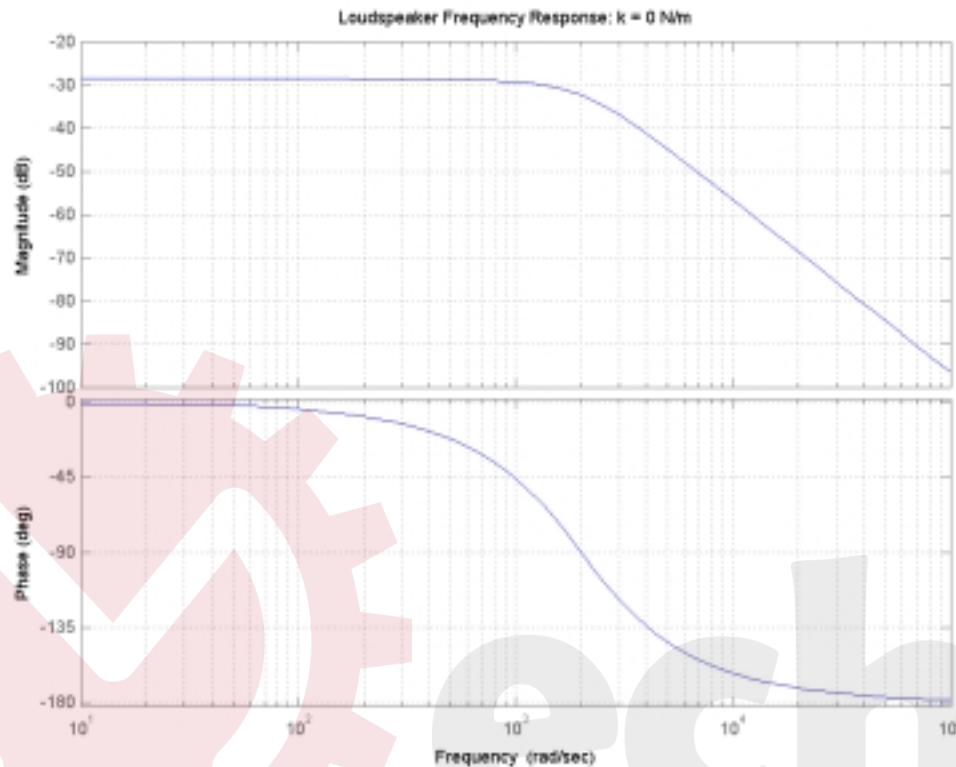
It should be apparent that this response would enhance the treble range, and is the response of a "tweeter".

c. For $k = 0$, the transfer function is :

$$\frac{U}{V} = \frac{1.478 \times 10^5}{s^2 + 3075s + 4 \times 10^6}$$

The magnitude frequency response is plotted below:

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In this case, the speaker acts as a "woofer", emphasizing the low frequency range. In practice, k cannot be identically zero, so the actual response of a woofer would resemble that of a midrange speaker, shifted towards the lower frequencies.

Problem 16.50

Solution:

Known quantities:

Loudspeaker of Example 16.13 (Figure 16.52)

$$r_{\text{coil}} = 0.05 \text{ m}; L = 10 \text{ mH}; R = 8 \Omega; N = 47; B = 1 \text{ T};$$

$$m = 0.01 \text{ kg}; b = 22.75 \text{ N} \cdot \text{s} / \text{m}; k = 5 \times 10^4 \text{ N} / \text{m}$$

Find:

Modify the parameters of the loudspeaker (mass, damping, and spring rate), so as to obtain a loudspeaker with a bass response centered on 400 Hz.

Demonstrate that your design accomplishes the intended task, using frequency response plots.

Assumptions:

None.

Analysis:

From Example 16.13, the system transfer function is:

$$\frac{U(s)}{V(s)} = \frac{Bl s}{(Lm)s^3 + (Rm + Lb)s^2 + [Rb + kL + (Bl)^2]s + kR}$$

The frequency response is:

$$\frac{U(j\omega)}{V(j\omega)} = \frac{jBl\omega}{kR - (Rm + Lb)\omega^2 + j\{[Rb + kL + (Bl)^2]\omega - (Lm)\omega^3\}}$$

where: $l = 2\pi N r_{coil}$

Using MATLAB, it is easy to adjust the mechanical parameters one-by-one in order to see each parameters' effect on the system frequency response.

Mass: Increasing the mass decreases the frequency response center frequency.

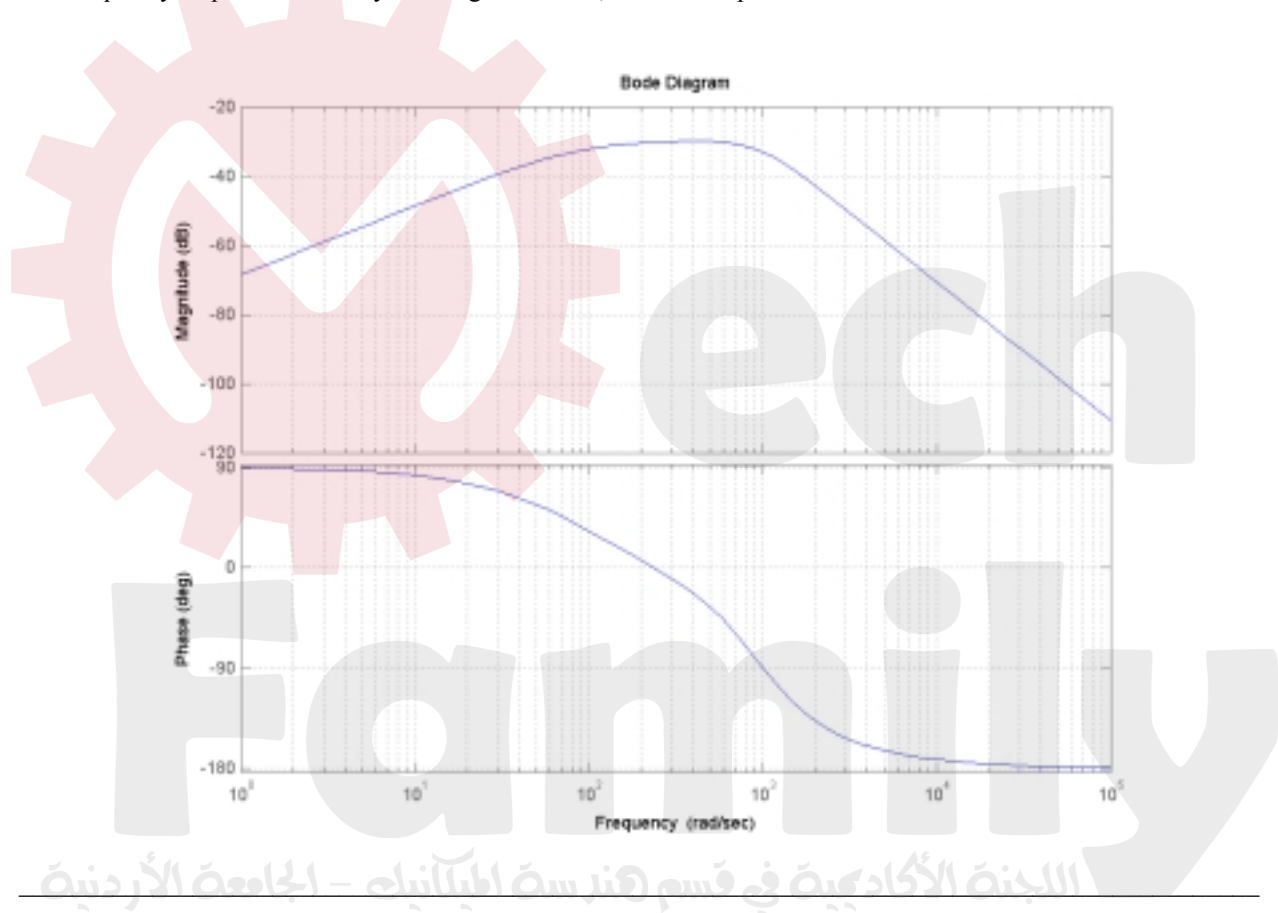
Damping: Increasing the damping also decreases the center frequency, but also widens the response bandwidth.

Spring Rate: Decreasing the spring rate also decreases the center frequency.

There are many possible combinations of mechanical parameters that could be substituted to generate the desired response.

One such set of parameters is: $m = 0.05\text{kg}$; $b = 30\text{N} \cdot \text{s} / \text{m}$; $k = 5 \times 10^3 \text{N} / \text{m}$

The frequency response for this system is given below, with the response centered over 400 Hz:



Chapter 17 Instructor Notes

The objective of Chapter 17 is to introduce the foundations for the analysis of rotating electric machines. In Section 17.1, rotating electric machines are classified on the basis of their energy conversion characteristics and of the nature of the electric power they absorb (or generate). Section 17.2 reviews the physical structure of a DC machine and presents a simple general circuit model that is valid for both motors and generators, including dynamic equations. Section 17.3 contains a brief discussion of DC generators. Section 17.4 describes the characteristics of the various configurations of DC motors, both of the wound stator and permanent magnet types. The torque speed characteristics of the different configurations are compared, and the dynamic equations are given for each type of motor. The section ends with a brief qualitative discussion of speed control in DC motors.

The second half of the chapter is devoted to the analysis of AC machines. In Section 17.5, we introduce the concept of a rotating magnetic field. The next two sections describe synchronous generators and motors; the discussion is brief, but includes the analysis of circuit models of synchronous machines and a few examples. Circuit models for the induction motor, as well as general performance characteristics of this class of machines are discussed in Section 17.8, including a brief treatment of AC machine speed and torque control. Although the discussion of the AC machines is not particularly detailed, all of the important concepts that a non-electrical engineer would be interested in to evaluate the performance characteristics of these machines are introduced in the chapter, and reinforced in the homework problem set. The homework problems include a mix of traditional electric machinery problems based on circuit models and of more system-oriented problems. Problems 17.24-36 deal with the performance and dynamics of systems including DC motors. These problems are derived from the author's experience in teaching a Mechanical Engineering System Dynamics course with emphasis on electromechanics, and are somewhat unusual (although relevant and useful for non-electrical engineers) in this type of textbook. These problems are well suited to a more mature audience that has already been exposed to a first course in system dynamics. Problem 17.39 provides a link to the power electronics topics covered in Chapter 12. All other problems are based on the content of the chapter.

Learning Objectives

1. Understand the basic principles of operation of rotating electric machines, their classification, and basic efficiency and performance characteristics. [Section 1.](#)
2. Understand the operation and basic configurations of separately-excited, permanent-magnet, shunt and series DC machines. [Section 2.](#)
3. Analyze DC generators at steady-state. [Section 3.](#)
4. Analyze DC motors under steady-state and dynamic operation. [Section 4.](#)
5. Understand the operation and basic configuration of AC machines, including the synchronous motor and generator, and the induction machine. [Sections 5, 6, 7, 8.](#)

Section 17.1: Rotating Electric Machines

Problem 17.1

Solution:

Known quantities:

The relationship of the power rating and the ambient temperature is shown in the table. A motor with $P_e = 10\text{ kW}$ is rated up to 85°C .

Find:

The actual power for the following conditions.

- Ambient temperature is 50°C .
- Ambient temperature is 25°C .

Assumptions:

None.

Analysis:

a)

The power at ambient temperature 50°C :

$$P_e' = 10 - 10 \times 0.125 = 8.75\text{ kW}$$

b)

The power at ambient temperature 30°C :

$$P_e' = 10 + 10 \times 0.08 = 10.8\text{ kW}$$

Problem 17.2

Solution:

Known quantities:

The speed-torque characteristic of an induction motor is shown in the table. The load requires a starting torque of $4\text{ N} \cdot \text{m}$ and increase linearly with speed to $8\text{ N} \cdot \text{m}$ at 1500 rev/min .

Find:

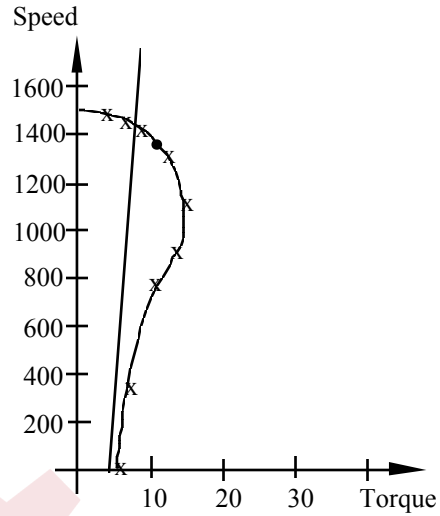
- The steady state operating point of the motor.
- The change in voltage if the load torque increases to $10\text{ N} \cdot \text{m}$.

Assumptions:

None.

Analysis:

The characteristic is shown below:



a)

The operating point is:

$$n_m = 1425 \text{ rev/min}, \quad T = 7 \text{ N} \cdot \text{m}$$

b)

From the following equation:

$$\frac{T_{\text{new}}}{T_{\text{old}}} = \left| \frac{V_{s,\text{new}}}{V_{s,\text{old}}} \right|^2$$

$$\Rightarrow \frac{10}{7} = \left| \frac{KV_s}{V_s} \right|^2 = K^2$$

$$\Rightarrow K = 1.195$$

$$\therefore V_{s,\text{new}} = 1.195 V_{s,\text{old}}$$

Section 17.2: Direct Current Machines

Problem 17.3

Solution:

Known quantities:

Each conductor of the DC motor is 6 in. long. The current is 90 A. The field density is $5.2 \times 10^{-4}\text{ Wb/in}^2$.

Find:

The force exerted by each conductor on the armature.

Assumptions:

None.

Analysis:

$$F = BI \times l = 5.2 \times 10^{-4} \frac{\text{Wb}}{\text{in}^2} \times \frac{\text{in}^2}{(0.0254\text{ m})^2} \times 90 \times 6\text{ in} \times \frac{0.0254\text{ m}}{\text{in}}$$

$$= 11.06\text{ Nt}$$

Problem 17.4

Solution:

Known quantities:

The air-gap flux density of the DC machine is 4 Wb/m^2 . The area of the pole face is $2\text{ cm} \times 4\text{ cm}$.

Find:

The flux per pole in the machine.

Assumptions:

None.

Analysis:

With $B = 4\text{ kG} = 0.4\text{ T} = 0.4\text{ Wb/m}^2$, we can compute the flux to be:

$$\phi = BA = 0.4 \times 0.02 \times 0.04 = 0.32\text{ mWb}$$

Section 17.3: Direct Current Generators

Problem 17.5

Solution:

Known quantities:

A 120V, 10A shunt motor. The armature resistance is 0.6Ω . The shunt field current is 2A.

Find:

The LVDT equations.

Assumptions:

None.

Analysis:

V_L at full load is 120V and

$$E_b = 120 + (2 + 10) \times 0.6 = 127.2V$$

$$R_f = \frac{120}{2} = 60\Omega$$

Assuming E_b to be constant, we have:

$$i_a = i_f = \frac{127.2}{0.6 + 60} = 2.1A$$

Therefore:

$$V_L = 127.2 - 2.1 \times 0.6 = 125.9V$$

$$\text{Voltage reg.} = \frac{125.9 - 120}{120} = 0.049 = 4.9\%$$

Problem 17.6

Solution:

Known quantities:

A 20kW, 230V separately excited generator. The armature resistance is 0.2Ω . The load current is 100A.

Find:

a) The generated voltage when the terminal voltage is 230V.

b) The output power.

Assumptions:

None.

Analysis:

If we assume rated output voltage, that is $V_L = 230V$, we have

a)

The generated voltage is 230V.

b)

The output power is 23kW.

If we assume rated output power, that is $P_{out} = 20kW$, we have

a)

The generated voltage is $200V$.

b)

The output power is $20kW$.If we assume $E_b = 230V$, and compute the output voltage to be:

$$V_L = 230 - 100 \times 0.2 = 210V$$

We have:

a)

The generated voltage is $210V$.

b)

The output power is $21kW$.

Problem 17.7

Solution:

Known quantities:

A $10kW, 120VDC$ series generator. The armature resistance is 0.1Ω and a series field resistance is 0.05Ω .

Find:

a) The armature current.

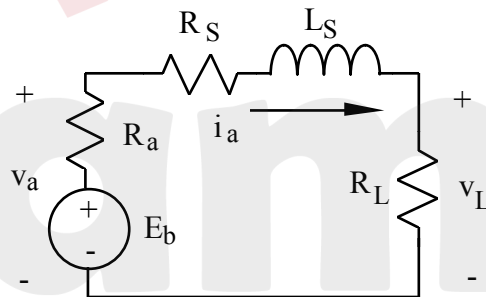
b) The generated voltage.

Assumptions:

The generator is delivering rated current at rated speed.

Analysis:

The circuit is shown below:



a)

$$i_a = \frac{P}{V_L} = \frac{10 \times 10^3}{120} = 83.33 A$$

b)

$$V_a = 120 + i_a R_s = 124.17V$$

Problem 17.8**Solution:****Known quantities:**

A 30 kW , 440 V shunt generator. The armature resistance is $0.1\ \Omega$ and a series field resistance is $200\ \Omega$.

Find:

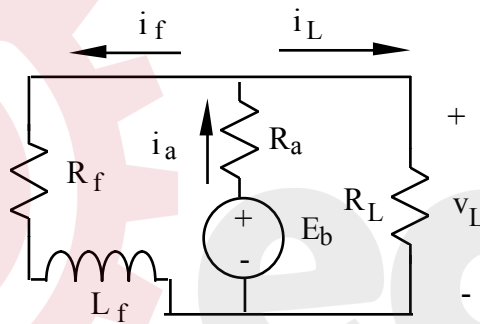
- The power developed at rated load.
- The load, field, and armature currents.
- The electrical power loss.

Assumptions:

None.

Analysis:

The circuit is shown below:



$$i_L = \frac{30 \times 10^3}{440} = 68.2\text{ A}$$

$$i_f = \frac{440}{200} = 2.2\text{ A}$$

$$i_a = 70.4\text{ A}$$

a)

$$E_b = V_L + i_a R_a = 440 + 70.4 \times 0.1 = 447.04\text{ V}$$

$$P = E_b i_a = 31.471\text{ kW}$$

b)

$$i_L = 62.8\text{ A}$$

$$i_f = 2.2\text{ A}$$

$$i_a = 70.4\text{ A}$$

c)

$$P_{\text{loss}} = i_a^2 R_a + i_f^2 R_f = 1464\text{ W}$$

Problem 17.9**Solution:****Known quantities:**

A four-pole 450 kW , 4.6 kV shunt generator. The armature resistance is $2\ \Omega$ and a series field resistance is $333\ \Omega$. The generator is operating at the rated speed of 3600 rev/min .

Find:

The no-load voltage of the generator and terminal voltage at half load.

Assumptions:

None.

Analysis:

For $n = 3600\text{ rev/min}$, $\omega_m = 377\text{ rad/sec}$:

$$i_L = \frac{450 \times 10^3}{4.6 \times 10^3} = 97.8\text{ A}$$

$$i_f = \frac{4.6 \times 10^3}{333} = 13.8\text{ A}$$

$$i_a = i_f + i_L = 111.6\text{ A}$$

Using the relation:

$$E_b = V_L + i_a R_a = 4823.2\text{ V}$$

At no-load,

$$V_L = E_b - i_a R_a = 4820.4\text{ V}$$

At half load,

$$i_L = 48.9\text{ A}$$

$$i_a = i_f + i_L = 62.7\text{ A}$$

$$V_L = E_b - i_a R_a = 4810.7\text{ V}$$

Problem 17.10**Solution:****Known quantities:**

A 30 kW , 240 V generator is running at half load at 1800 rev/min with efficiency of 85 percent.

Find:

The total losses and input power.

Assumptions:

None.

Analysis:

$$P_{out} = \frac{1}{2} \text{ rated load} = 15\text{ kW}$$

At an efficiency of 0.85, the input power can be computed to be:

$$P_{in} = \frac{15 \times 10^3}{0.85} = 17.647\text{ kW}$$

The total loss is:

$$P_{loss} = P_{in} - P_{out} = 2.647 \text{ kW}$$

Problem 17.11

Solution:

Known quantities:

A self excited DC shunt generator. At 200 rev/min , it delivers 20 A to a 100 V line. The armature resistance is 1.0Ω and a series field resistance is 100Ω . The magnetization characteristic is shown in Figure P17.11. When the generator is disconnected from the line, the drive motor speed up to 220 rad/s .

Find:

The terminal voltage.

Assumptions:

None.

Analysis:

From the figure, for $I_f > 0.5 \text{ A}$, $\omega = 200 \text{ rad/sec}$

$$E_b = 40I_f + 100$$

For $\omega = 220 \text{ rad/sec}$, we have:

$$\frac{E'_b}{220} = \frac{100 + 40I_f}{200}$$

Therefore,

$$E'_b = \frac{220}{200}(100 + 40I_f) = 110 + 44I_f$$

For no load, $I_a = I_f$. Therefore,

$$110 + 44I_f = 101I_f$$

$$\therefore I_f = 1.93 \text{ A}$$

The terminal voltage is:

$$V = I_f R_f = 193 \text{ V}$$

Section 17.4: Direct Current Motors

Problem 17.12

Solution:

Known quantities:

A 220V shunt motor. The armature resistance is $0.32\ \Omega$. A field resistance is $110\ \Omega$. At no load the armature current is 6 A and the speed is 1800 rpm.

Find:

- The speed of the motor when the line current is 62 A.
- The speed regulation of the motor.

Assumptions:

The flux does not vary with load. Assume a $8\ \text{N} \cdot \text{m}$ brush drop.

Analysis:

a)

$$1800 = \frac{220 - 2 - 6(0.32)}{K_a \phi}$$

$$\Rightarrow K_a \phi = 0.12$$

$$\therefore n = \frac{220 - 2 - 6(0.32)}{K_a \phi} = 1657\ \text{rpm}$$

b)

$$\% \text{reg} = \frac{1800 - 1657}{1657} \times 100 = 8.65\%$$

Problem 17.13

Solution:

Known quantities:

A 50 hp, 550 volt shunt generator. The armature resistance including brushes is $0.36\ \Omega$. Operating at rated load and speed, the armature current is 75 A.

Find:

What resistance should be inserted in the armature circuit to get a 20 percent speed reduction when the motor is developing 70 percent of rated torque.

Assumptions:

There is no flux change.

Analysis:

$$T = K_a \phi I_a \Rightarrow I_a = 0.7(75) = 52.5\ \text{A}$$

$$n_R = \frac{550 - 75(0.36)}{K_a \phi} \Rightarrow K_a \phi n_R = 523$$

$$0.8n_R = \frac{550 - 52.5R_T}{K_a \phi} \Rightarrow 0.8 \times 523 = 550 - 52.5R_T$$

$$\therefore R_T = 2.51\ \Omega$$

$$R_{add} = 2.51 - 0.36 = 2.15\ \Omega$$

Problem 17.14

Solution:

Known quantities:

A 100 kW , 440 V shunt DC motor. The armature resistance is $0.2\ \Omega$ and a series field resistance is $400\ \Omega$. The generator is operating at the rated speed of 1200 rev/min . The full-load efficiency is 90 percent.

Find:

- The motor line current.
- The field and armature currents.
- The counter emf at rated speed.
- The output torque.

Assumptions:

None.

Analysis:

At $n = 1200\text{ rev/min}$, $\omega_m = 125.7\text{ rad/sec}$, the output power is $100\text{ hp} = 74.6\text{ kW}$.

From full-load efficiency of 0.9, we have:

$$P_{in} = \frac{74.6}{0.9} = 82.9\text{ kW}$$

a)

From $P_{in} = i_s V_s = 82.9\text{ kW}$, we have:

$$i_s = \frac{82.9 \times 10^3}{440} = 188.4\text{ A}$$

b)

$$i_f = \frac{440}{400} = 1.1\text{ A}$$

$$i_a = 187.3\text{ A}$$

c)

$$E_b = V_L - i_a R_a = 402.5\text{ V}$$

d)

$$T_{out} = \frac{P_{out}}{\omega_m} = 593.5\text{ N} \cdot \text{m}$$

Problem 17.15

Solution:

Known quantities:

A 240 V series motor. The armature resistance is $0.42\ \Omega$ and a series field resistance is $0.18\ \Omega$. The speed is 500 rev/min when the current is 36 A .

Find:

What is the motor speed when the load reduces the line current to 21 A .

Assumptions:

A 3 volts brush drop and the flux is proportional to the current.

Analysis:

$$500 = \frac{240 - 3 - 36(0.6)}{K_a \phi} \Rightarrow K_a \phi = 0.431$$

$$n = \frac{240 - 3 - 21(0.6)}{\left(\frac{21}{36}\right)(0.431)} = 893 \text{ rpm}$$

Problem 17.16**Solution:****Known quantities:**

A 220VDC shunt motor. The armature resistance is 0.2Ω . The rated armature current is 50 A.

Find:

- The voltage generated in the armature.
- The power developed.

Assumptions:

None.

Analysis:

a)

$$E_b = V_L - i_a R_a = 220 - 50 \times 0.2 = 210 \text{ V}$$

b)

$$P = E_b i_a = 210 \times 50 = 10.5 \text{ kW} = 14.07 \text{ hp}$$

Problem 17.17**Solution:****Known quantities:**

A 550V series motor. The armature resistance is 0.15Ω . The speed is 820 rev/min when the current is 112 A and the load is 75 hp.

Find:

The horsepower output of the motor when the current drops to 84 A.

Assumptions:

The flux is reduced by 15 percent.

Analysis:

$$HP = \frac{2\pi \cdot n \cdot T}{33,000}$$

$$75 = \frac{2\pi(820)T}{33,000} \Rightarrow T = 480.4 \text{ lb} \cdot \text{ft}$$

$$T = K_a \phi I_a \Rightarrow 480.4 = K_a \phi (112) \Rightarrow K_a \phi = 4.29$$

$$T_n = 4.29(0.85)(84) = 306.2 \text{ lb} \cdot \text{ft}$$

$$n_n = \frac{550 - 84(0.15)}{0.85(0.65)} = 973 \text{ rpm}$$

$$HP_n = \frac{2\pi(973)(306.2)}{33,000} = 56.7 \text{ hp}$$

Problem 17.18**Solution:****Known quantities:**

A 220VDC shunt motor. The armature resistance is 0.1Ω and a series field resistance is 100Ω .

The speed is 1100 rev/min when the current is 4 A and there is no load.

Find:

E and the rotational losses at 1100 rev/min .

Assumptions:

The stray-load losses can be neglected.

Analysis:

Since $n = 1100 \text{ rev/min}$ corresponds to $\omega = 115.2 \text{ rad/sec}$, we have:

$$i_s = 4 \text{ A}$$

$$i_f = \frac{200}{100} = 2 \text{ A}$$

$$i_a = i_s - i_f = 2 \text{ A}$$

Also,

$$E_b = 200 - 2 \times 0.1 = 199.8 \text{ V}$$

The power developed by the motor is:

$$P = P_{in} - P_{copper\_loss}$$

$$= 200 \times 4 - (2^2 \times 100 + 2^2 \times 0.1)$$

$$= 399.6 \text{ W}$$

Problem 17.19
Solution:**Known quantities:**

A 230VDC shunt motor. The armature resistance is 0.5Ω and a series field resistance is 75Ω . At 1100 rev/min, $P_{rot} = 500W$. When loaded, the current is 46 A.

Find:

- a) The speed P_{dev} and T_{sh} .
 b) $i_a(t)$ and $\omega_m(t)$ if $L_f = 25H$, $L_a = 0.008H$ and the terminal voltage has a 115V change.

Assumptions:

None.

Analysis:

$$i_f = \frac{230}{75} = 3.07 A$$

$$i_a = i_L - i_f = 42.93 A$$

$$\omega_m = 117.3 \text{ rad/sec}$$

At no load, $117.3 = \frac{230}{K_a \phi}$, therefore,

$$K_a \phi = 1.96$$

At full load,

$$\omega_m = \frac{230 - 0.5 \times 42.93}{K_a \phi}$$

The back emf is:

$$E_b = 230 - 0.5 \times 42.93 = 208.5V$$

The power developed is:

$$P_{dev} = E_b I_a = 8.952 \text{ kW}$$

The power available at the shaft is:

$$P_o = P_{dev} - P_{rot} = 8952 - 500 = 8452 W$$

The torque available at the shaft is:

$$T_{sh} = \frac{P_o}{\omega_m} = 72.1 N \cdot m$$

Problem 17.20**Solution:****Known quantities:**

A 200VDC shunt motor. The armature resistance is 0.1Ω and a series field resistance is 100Ω . At 955 rev/min with no load, $P_{rot} = 500W$, the line current is 5 A.

Find:

The motor speed, the motor efficiency, total losses, and the load torque when the motor draws 40 A from the line.

Assumptions:

Rotational power losses are proportional to the square of shaft speed.

Analysis:

$$i_s = 5 A$$

$$i_f = \frac{200}{100} = 2 A$$

$$i_a = i_s - i_f = 3 A$$

The copper loss is:

$$P_{copper} = i_f^2 R_f + i_a^2 R_a = 400.9 W$$

The input power is:

$$P_{in} = 5 \times 200 = 1 kW$$

Therefore,

$$P_{rot} + P_{SL} = 1000 - 409 = 599.1 W$$

$$\text{at } \omega_m = 2\pi \frac{955}{60} = 100 \text{ rad/sec.}$$

Also, E_b at no load is:

$$E_b = 200 - 3 \times 0.1 = 199.7 V$$

$$K_a \phi = 1.997$$

When $i_s = 40 A$ with $i_f = 2 A$ and $i_a = 38 A$,

$$E_b = 200 - 38 \times 0.1 = 196.2 V$$

$$\omega_m = \frac{E_b}{K_a \phi} = 98.25 \text{ rad/sec} = 938.2 \text{ rev/min}$$

The power developed is:

$$P = E_b I_a = 196.2 \times 38 = 7456 W$$

The copper loss is:

$$P_{copper} = i_f^2 R_f + i_a^2 R_a = 544.4 W$$

The input power is:

$$P_{in} = 40 \times 200 = 8 kW$$

And

$$P_{SH} = 7456 - \frac{98.25}{100} \times 599.1 = 6867.4 \text{ W}$$

$$T_{SH} = \frac{6867.4}{98.25} = 69.9 \text{ N} \cdot \text{m}$$

Finally, the efficiency is:

$$\text{eff} = \frac{P_{SH}}{P_{in}} = 85.84\%$$

Problem 17.21

Solution:

Known quantities:

A 50 hp, 230V shunt motor operates at full load when the line current is 181 A at 1350 rev/min.

The field resistance is 17.7 Ω. To increase the speed to 1600 rev/min, a resistance of 5.3 Ω is cut in via the field rheostat. The line current is increased to 190 A.

Find:

- The power loss in the field and its percentage of the total power input for the 1350 rev/min speed.
- The power losses in the field and the field rheostat for the 1600 rev/min speed.
- The percent losses in the field and in the field rheostat at 1600 rev/min speed.

Assumptions:

None.

Analysis:

a)

$$I_f = \frac{230}{17.7} = 13.0 \text{ A}$$

$$P_f = (230)(13.0) = 2988.7 \text{ W}$$

$$\frac{P_f}{P_m} = \frac{2988.7}{(230)(181)} = 0.072 = 7.2\%$$

b)

$$I_f = \frac{230}{(17.7 + 5.3)} = 10 \text{ A}$$

$$P_f = 10^2 (17.7) = 1770 \text{ W}$$

$$P_R = 10^2 (5.3) = 530 \text{ W}$$

c)

$$P_{in} = (230)(190) = 43,700 \text{ W}$$

$$\%P_f = \frac{1770}{43700} \times 100 = 4.05\%$$

$$\%P_R = \frac{530}{43700} \times 100 = 1.21\%$$

Problem 17.22**Solution:****Known quantities:**

A 10 hp, 230 V shunt-wound motor. The armature resistance is 0.26Ω and a series field resistance is 225Ω . The rated speed is 1000 rev/min . The full-load efficiency is 86 percent.

Find:

The effect on counter emf, armature current and torque when the motor is operating under rated load and the field flux is very quickly reduced to 50 percent of its normal value. The effect on the operation of the motor and its speed when stable operating conditions have been regained.

Assumptions:

None.

Analysis:

$E_C = K\phi n$; counter emf will decrease.

$I_a = \frac{V - E_C}{r_a}$; armature current will increase.

$T = K\phi I_a$; effect on torque is indeterminate.

Operation of a dc motor under weakened field conditions is frequently done when speed control is an important factor and where decreased efficiency and less than rated torque output are lesser considerations.

$$n = \frac{V - I_a r_a}{K\phi} \Rightarrow 1000 = \frac{V - I_a r_a}{K\phi}$$

$$n_{new} = \frac{V - I_a r_a}{0.5K\phi}$$

Assume small change in the steady-state value of I_a . Then:

$$\frac{1000}{n_{new}} = \frac{0.5}{1} \Rightarrow n_{new} = 2000 \text{ rpm}$$

Problem 17.23**Solution:****Known quantities:**

The machine is the same as that in Example 17.7. The circuit is shown in Figure P17.23. The armature resistance is 0.2Ω and the field resistance is negligible. $n = 120 \text{ rev/min}$, $I_a = 8 \text{ A}$. In the operating region, $\phi = kI_f$, $k = 200$.

Find:

- The number of field winding turns necessary for full-load operation.
- The torque output for the following speeds:
 - $n' = 2n$
 - $n' = 3n$
 - $n' = n/2$
 - $n' = n/4$
- Plot the speed-torque characteristic for the conditions of part b.

Assumptions:

None.

Analysis:

in Example 17.7, $i_f = 0.6 A$, the mmf F is:

$$F = 200 \times 0.6 = 120 \text{ At}$$

For a series field winding with:

a)

$i_{series} = i_a = 8 A$, we have:

$$N_{series} = \frac{120}{8} = 15 \text{ turns}$$

b)

$$n_m = 120 \text{ rev/min}$$

$$\omega_m = 12.57 \text{ rad/sec}$$

$$E_b = V_s - i_a (R_a + R_s)$$

Neglecting R_s , we have

$$E_b = 7.2 - 8 \times 0.2 = 5.6 \text{ V} = k_a \phi \omega_m$$

$$k_a \phi = \frac{5.6}{12.57} = 0.446$$

From $T = k_T \phi i_a$ and $k_T = k_a$,

$$T = 0.446 \times 8 = 3.56 \text{ N} \cdot \text{m}$$

By using $\phi = k i_a$, we have:

$$E_b = k_a k i_a \omega_m = V_s - i_a R_a$$

$$T = k_T (k i_a) i_a = k_a k i_a^2$$

$$\text{From } i_a = \frac{V_s}{R_a + K \omega_m}, \text{ where } K = k_a k = \frac{5.6}{8 \times 12.57} = 0.056$$

And $T \propto \left(\frac{1}{R_a + K \omega_m} \right)^2$, we have:

$$\frac{T_X}{T} = \left(\frac{R_a + K \omega_m}{R_a + K \omega_X} \right)^2 = \left(\frac{\frac{R_a}{K} + \omega_m}{\frac{R_a}{K} + \omega_X} \right)^2$$

$$\therefore \frac{R_a}{K} = 3.59$$

$$\therefore T_X = 3.56 \left(\frac{3.59 + \omega_m}{3.59 + \omega_X} \right)^2$$

$$1. \text{ at } \omega_X = 2\omega_m = 25.12 \text{ rad/sec},$$

$$T_X = 1.13 \text{ N} \cdot \text{m}$$

$$2. \text{ at } \omega_X = 3\omega_m = 37.71 \text{ rad/sec},$$

$$T_X = 0.55 \text{ N} \cdot \text{m}$$

$$3. \text{ at } \omega_X = 0.5\omega_m = 6.28 \text{ rad/sec},$$

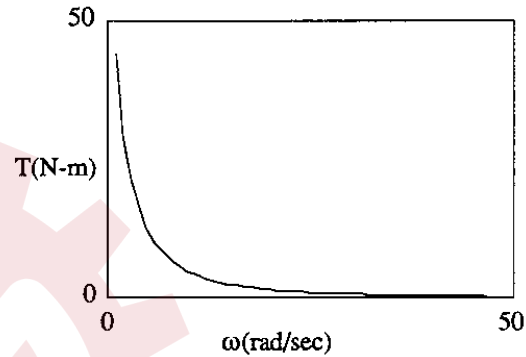
$$T_x = 9.54 \text{ N} \cdot \text{m}$$

$$4. \text{ at } \omega_x = 0.25\omega_m = 3.14 \text{ rad/sec},$$

$$T_x = 20.53 \text{ N} \cdot \text{m}$$

c)

The diagram is shown below:



Problem 17.24

Solution:

Known quantities:

PM DC motor circuit model; mechanical load model. Example 17.9.

Find:

Voltage-step response of motor.

Assumptions:

None.

Analysis:

Applying KVL and equation 17.47 to the electrical circuit we obtain:

$$V_L(t) - R_a I_a(t) - L_a \frac{dI_a(t)}{dt} - E_b(t) = 0$$

or

$$L_a \frac{dI_a(t)}{dt} + R_a I_a(t) - K_{aPM} \omega_m(t) = V_L(t)$$

Applying Newton's Second Law and equation 17.46 to the load inertia, we obtain:

$$J \frac{d\omega_m(t)}{dt} + T(t) - T_{Load}(t) - b \omega_m(t) = 0$$

or

$$K_{TPM} I_a(t) - J \frac{d\omega_m(t)}{dt} - b \omega_m(t) = 0$$

since the load torque is assumed to be zero. To derive the transfer function from voltage to speed, we use the result of Example 17.9 with $T_{load} = 0$:

$$\omega_m(s) = \frac{K_{TPM}}{sL_a + R_a + (sJ + b) K_{aPM} K_{TPM}} V_L(s)$$

The step response of the system can be computed by assuming a unit step input in voltage:

$$\omega_m(s) = \frac{K_{TPM}}{sL_a + R_a + (sJ + b) K_{aPM} K_{TPM}} \frac{1}{s}$$

$$F(s) = \frac{K_{TPM}}{(sL_a + R_a)(sJ + b) + K_{aPM}K_{TPM}} \frac{1}{s}$$

$$F(s) = \frac{K_{TPM}}{s[JL_a s^2 + (L_a b + R_a J)s + R_a b + K_a K_T]}$$

$$F(s) = \frac{K_{TPM}}{s \left[s^2 + \frac{(L_a b + R_a J)}{JL_a} s + \frac{(R_a b + K_a K_T)}{JL_a} \right]}$$

Set:

$$m = \frac{(L_a b + R_a J)}{JL_a} \quad n = \frac{(R_a b + K_a K_T)}{JL_a}$$

$$F(s) = \frac{K_{TPM}}{s[s^2 + ms + n]} = \frac{K_{TPM}}{s \left[\left(s^2 + ms + \frac{m^2}{4} \right) + n - \frac{m^2}{4} \right]} = \frac{K_{TPM}}{s \left[\left(s + \frac{m}{2} \right)^2 + \left(n - \frac{m^2}{4} \right) \right]}$$

For

$$F(s) = \frac{1}{s[(s-a)^2 + b_o^2]} \quad f(t) = \frac{1}{b_o^2} \frac{1}{bb_o} e^{at} \sin(bt)$$

$$\text{where: } a = \tan^{-1} \frac{b}{a} \quad \text{and } b_o = \sqrt{b^2 - a^2}$$

Therefore:

$$a = \frac{m}{2} = \frac{(L_a b + R_a J)}{2JL_a} \quad b = \sqrt{n - \frac{m^2}{4}} = \sqrt{\frac{R_a b + K_a K_T}{JL_a} - \left(\frac{L_a b + R_a J}{JL_a} \right)^2 \frac{1}{4}}$$

$$b_o^2 = \left[\frac{(L_a b + R_a J)}{2JL_a} \right]^2 + \frac{R_a b + K_a K_T}{JL_a} - \left(\frac{L_a b + R_a J}{JL_a} \right)^2 \frac{1}{4} = \frac{R_a b + K_a K_T}{JL_a}$$

$$\phi = \tan^{-1} \frac{\sqrt{\frac{R_a b + K_a K_T}{JL_a} - \left(\frac{L_a b + R_a J}{JL_a} \right)^2 \frac{1}{4}}}{-\frac{(L_a b + R_a J)}{2JL_a}}$$

Thus giving the step response:

$$\Omega_m(t) = \frac{JL_a}{R_a b + K_a K_T} + \frac{1}{\left(\sqrt{\frac{R_a b + K_a K_T}{JL_a} - \left(\frac{L_a b + R_a J}{JL_a} \right)^2 \frac{1}{4}} \right) \left(\sqrt{\frac{R_a b + K_a K_T}{JL_a}} \right)} e^{-\left[\frac{(L_a b + R_a J)}{2JL_a} \right] t} * \sin \left[\left(\sqrt{\frac{R_a b + K_a K_T}{JL_a} - \left(\frac{L_a b + R_a J}{JL_a} \right)^2 \frac{1}{4}} \right) t - \tan^{-1} \frac{\sqrt{\frac{R_a b + K_a K_T}{JL_a} - \left(\frac{L_a b + R_a J}{JL_a} \right)^2 \frac{1}{4}}}{-\frac{(L_a b + R_a J)}{2JL_a}} \right]$$

Expressions for the natural frequency and damping ratio of the second-order system may be derived by comparing the motor voltage-speed transfer function to a standard second-order system transfer function:

$$H(s) = \frac{K_S}{s^2 \frac{1}{2\zeta\omega_n} s + \omega_n^2}$$

The motor transfer function is:

$$\frac{m(s)}{V_L(s)} = \frac{K_{TPM}}{sL_a R_a (sJ + b) K_{aPM} K_{TPM}} = \frac{K_{TPM}}{JL_a s^2 + JR_a bL_a s + R_a b K_{aPM} K_{TPM}}$$

$$= \frac{\frac{K_{TPM}}{JL_a}}{s^2 + \frac{JR_a bL_a}{JL_a} s + \frac{R_a b K_{aPM} K_{TPM}}{JL_a}}$$

Comparing terms, we determine that:

$$\omega_n^2 = \frac{R_a b K_{aPM} K_{TPM}}{JL_a}$$

$$2\zeta\omega_n = \frac{JR_a bL_a}{JL_a}$$

or

$$\omega_n = \sqrt{\frac{R_a b K_{aPM} K_{TPM}}{JL_a}}$$

$$\zeta = \frac{1}{2} \frac{JR_a bL_a}{JL_a} \sqrt{\frac{JL_a}{R_a b K_{aPM} K_{TPM}}}$$

From these expressions, we can see that both natural frequency and damping ratio are affected by each of the parameters of the system, and that one cannot predict the nature of the damping without knowing numerical values of the parameters.

Problem 17.25

Solution:

Known quantities:

Torque-speed curves of motor and load: $T_m = a\omega + b$ (motor), $T_L = c\omega^2 + d$ (load),

Find:

Equilibrium speeds and their stability.

Assumptions:

All coefficients of torque-speed curve functions are positive constants

Analysis:

The first consideration is that the motor static torque, $T_{0,m} = b$ must exceed the load static torque, $T_{0,L} = d$; thus, the first condition is $b > d$.

The next step is to determine the steady state speed of the motor-load pair. If we set the motor torque equal to the load torque, the resulting angular velocities will be the desired solutions.

$$T_m = T_L \quad a\omega + b = c\omega^2 + d$$

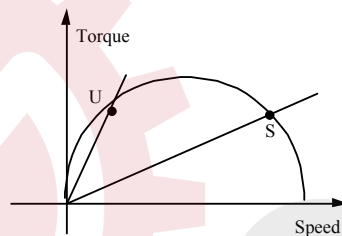
resulting in the quadratic equation

$$c\omega^2 + a\omega + d - b = 0$$

with solution

$$\omega = \frac{-a \pm \sqrt{a^2 - 4c(b-d)}}{2c}$$

Both solutions are positive, and therefore physically acceptable. The question of stability can be addressed by considering the following sketch.



In the figure, we see that the intersection of a line with a quadratic function when both solutions are positive leads to two possible situations: the line intersecting the parabola when the rate of change of both curves is positive, and the line intersecting the parabola when the rate of change of torque w.r. to speed of the latter is negative. The first case leads to an unstable operating point; the second case to a stable operating point (you can argue each case qualitatively by assuming a small increase in load torque and evaluating the consequences). We can state this condition mathematically by requiring that the following steady-state stability condition hold:

$$\frac{dT_L}{d\omega} > \frac{dT_m}{d\omega}$$

Evaluating this for our case, we see that

$$\frac{dT_L}{d\omega} = 0 > \frac{dT_m}{d\omega} = 2c\omega + a$$

From the expression we obtained earlier,

$$\omega = \frac{-a \pm \sqrt{a^2 - 4c(b-d)}}{2c}$$

it is clear that $2c\omega > -a$ always holds, since the term under the square root is a positive constant. Thus, this motor-load pair always leads to stable solutions. To verify this conclusion intuitively, you might wish to plot the motor and load torque-speed curves and confirm that the condition $\frac{dT_L}{d\omega} > \frac{dT_m}{d\omega}$ is always satisfied (note that the sketch above is **not** an accurate graphical representation of the two curves).

Problem 17.26**Solution:****Known quantities:**

Expression for friction and windage torque, T_{FW} , functional form of motor torque, T , or load torque, T_L .

Find:

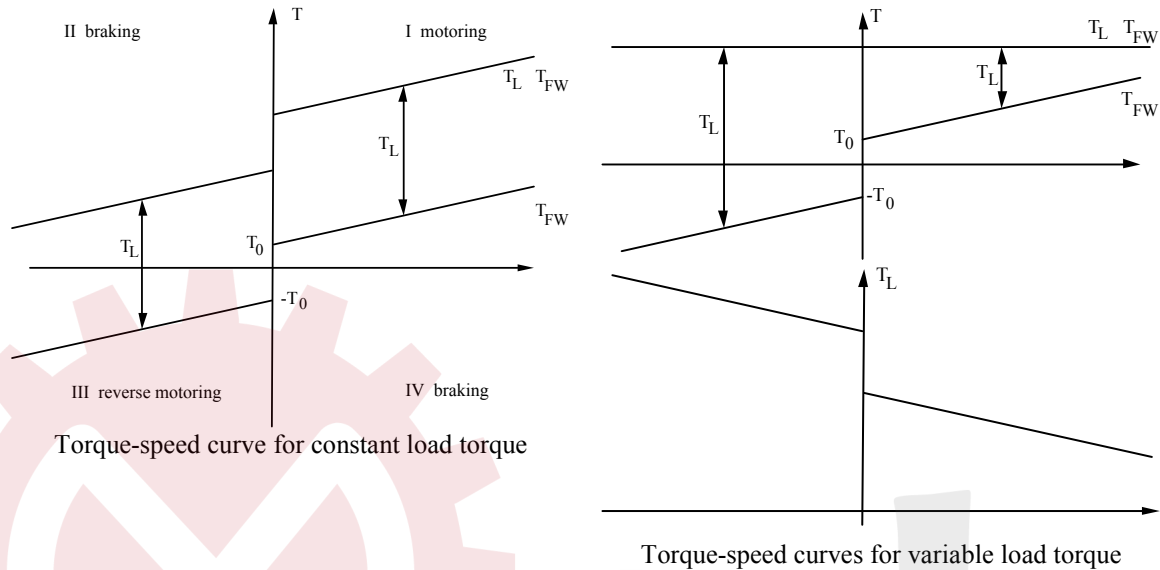
Sketch torque-speed curve

Assumptions:

None.

Analysis:

The sketches are shown below.

**Problem 17.27****Solution:****Known Quantities:**

A PM DC motor and parameters when 1) in steady-state, no load conditions, and 2) connected to a pump

Find:

- A damping coefficient, sketch of the motor, the dynamic equations, the transfer function, and 3 dB bandwidth.
- A sketch of the motor, dynamic equations, transfer function, and 3 dB bandwidth.

Assumptions: k_t k_a k_{PM} **Analysis:**

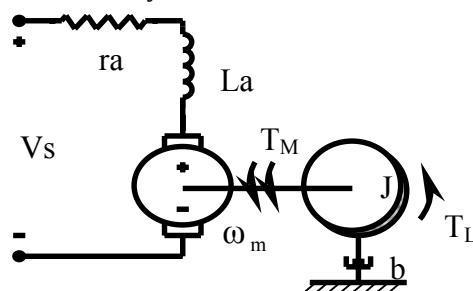
a)

The magnetic torque balanced the damping torque gives s:

$$k_t * i_a - b * \omega_m$$

or

$$b = \frac{k_{PM} * i_a}{\omega_m} = \frac{7 * 10^{-3} \frac{\text{N} \cdot \text{m}}{\text{A}} * .15 \text{ A}}{\frac{3350 \text{ rev}}{\text{min}} * \frac{2 \text{ rad}}{\text{rev}} * \frac{1 \text{ min}}{60 \text{ sec}}} = 2.993 * 10^{-6} \text{ N} \cdot \text{m} \cdot \text{sec}$$

Sketch: **PM DC Motor-Load System**

Dynamic equations:

$$\begin{aligned}
 V_s &= L_a \frac{di_a}{dt} + r_a i_a + E_b \\
 E_b &= k_{PM} \omega_m \\
 V_s &= L_a \frac{di_a}{dt} + r_a i_a + k_{PM} \omega_m \\
 J \frac{d\omega_m}{dt} &= T_m - T_L - b \omega_m \\
 T_m &= k_{PM} i_a \\
 T_L &= J \frac{d\omega_m}{dt} + b \omega_m + k_{PM} i_a
 \end{aligned}$$

To get the transfer function:

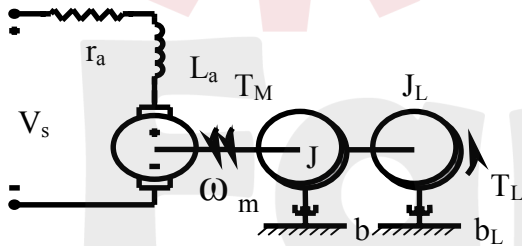
$$\begin{bmatrix} (L_a s + r_a) & k_{PM} \\ -k_{PM} & (J s + b) \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} = \begin{bmatrix} V_s \\ -T_L \end{bmatrix}$$

$$\omega_m(s) = \frac{\det \begin{bmatrix} (L_a s + r_a) & V_s \\ -k_{PM} & -T_L \end{bmatrix}}{\det \begin{bmatrix} (L_a s + r_a) & k_{PM} \\ -k_{PM} & (J s + b) \end{bmatrix}}$$

$$\left. \frac{\omega_m(s)}{V_s} \right|_{T_L=0} = \frac{k_{PM}}{(J L_a s^2 + (r_a J + L_a b) s + r_a b + k_{PM}^2)}$$

b)

Sketch:



Dynamic Equations:

$$\begin{aligned}
 V_s &= L_a \frac{di_a}{dt} + r_a i_a + E_b \\
 E_b &= k_{PM} \omega_m \\
 V_s &= L_a \frac{di_a}{dt} + r_a i_a + k_{PM} \omega_m \\
 (J + J_L) \frac{d\omega_m}{dt} &= T_m - T_L - (b + b_L) \omega_m \\
 T_m &= k_{PM} i_a \\
 T_L &= (J + J_L) \frac{d\omega_m}{dt} + (b + b_L) \omega_m + k_{PM} i_a
 \end{aligned}$$

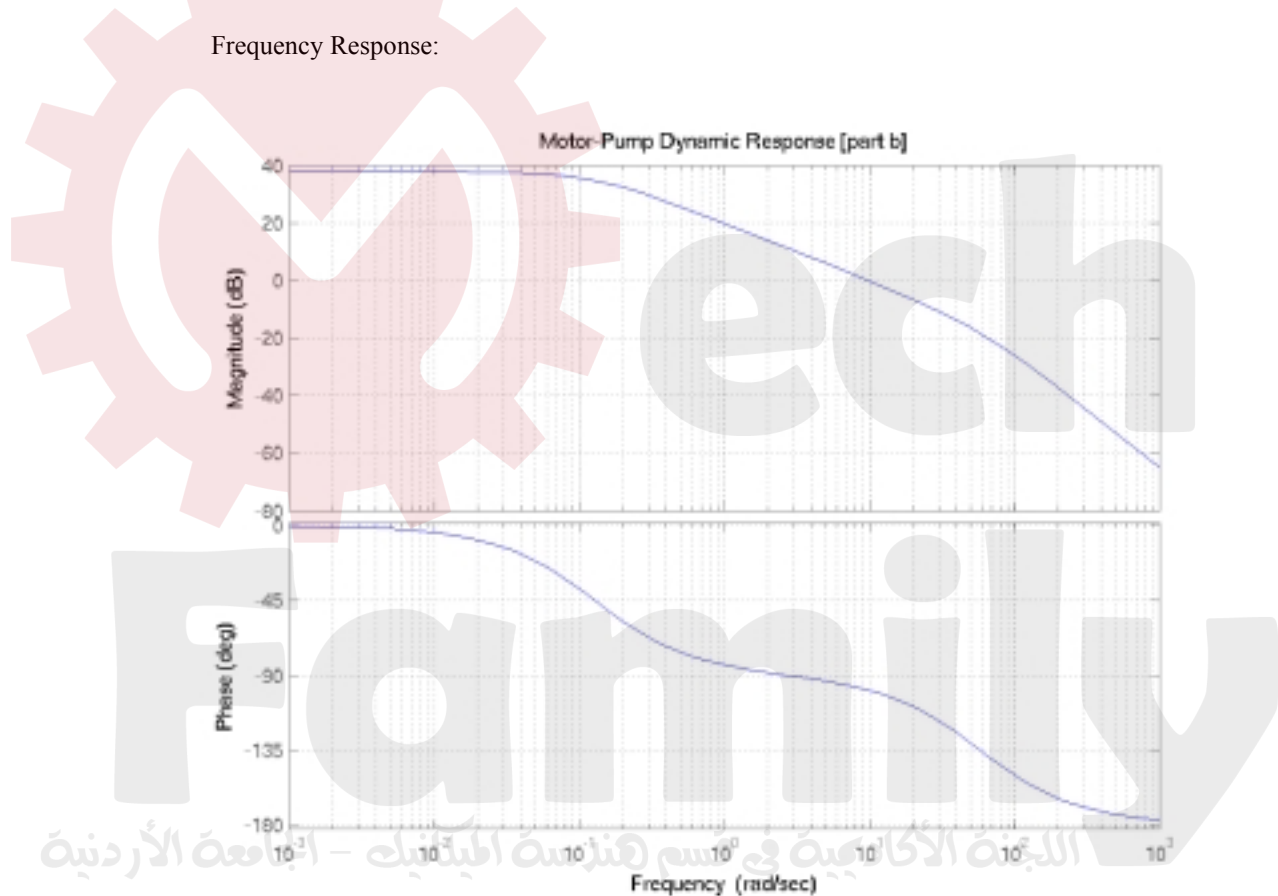
To get the transfer function:

$$\begin{bmatrix} (L_a s + r_a) & k_{PM} \\ -k_{PM} & ((J + J_L)s + (b + b_L)) \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} = \begin{bmatrix} V_s \\ -T_L \end{bmatrix}$$

$$\omega_m(s) = \frac{\det \begin{bmatrix} (L_a s + r_a) & V_s \\ -k_{PM} & -T_L \end{bmatrix}}{\det \begin{bmatrix} (L_a s + r_a) & k_{PM} \\ -k_{PM} & ((J + J_L)s + (b + b_L)) \end{bmatrix}}$$

$$\left. \frac{\omega_m(s)}{V_s} \right|_{T_L=0} = \frac{k_{PM}}{((J + J_L) * L_a) s^2 + (r_a * (J + J_L) + L_a * (b + b_L)) s + r_a * (b + b_L) + k_{PM}^2}$$

Frequency Response:



Problem 17.28

Solution:

Known Quantities:

A PM DC motor is used to power a pump

Find:

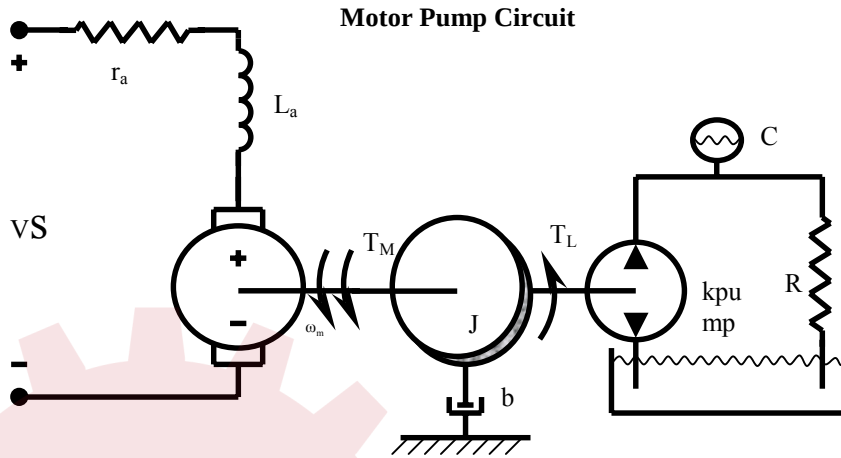
The dynamic equations of the system and the transfer function between the motor voltage and pressure.

Assumptions:

The inertia and damping of the motor and pump can be lumped together.

Analysis:

Sketch:



$$\begin{aligned}
 V_s &= L_a \frac{di_a}{dt} + r_a i_a + E_b \\
 V_s &= L_a \frac{di_a}{dt} + r_a i_a + k_{PM} \omega_m \\
 J \frac{d\omega_m}{dt} &= T_m - T_L - b \omega_m \\
 T_L &= J \frac{d\omega_m}{dt} + b \omega_m + k_{PM} i_a \\
 k_p \frac{dp}{dt} &= \frac{p}{R} - C_{acc} \frac{dp}{dt} \\
 C_{acc} \frac{dp}{dt} &= \frac{p}{R} - k_p \omega_m
 \end{aligned}$$

To get the transfer function:

$$\begin{bmatrix} L_a s + r_a & k_{PM} & 0 \\ -k_{PM} & Js + b & k_p \\ 0 & -Rk_p & RC_{acc}s + 1 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ p \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix}$$

$$p(s) = \frac{\det \begin{bmatrix} L_a s + r_a & -k_{PM} & V_s \\ -k_{PM} & Js + b & 0 \\ 0 & -Rk_p & 0 \end{bmatrix}}{\det \begin{bmatrix} L_a s + r_a & k_{PM} & 0 \\ -k_{PM} & Js + b & k_p \\ 0 & -Rk_p & RC_{acc}s + 1 \end{bmatrix}}$$

$$\frac{p}{V_s}(s) = \frac{k_{PM} R k_p}{(L_a s + r_a)(Js + b)(RC_{acc}s + 1) + k_{PM}^2 (RC_{acc}s + 1) + R k_p^2 (L_a s + r_a)}$$

Problem 17.29**Solution:****Known quantities:**

Motor circuit shown in Figure P17.29 and magnetization parameters, load parameters. Operating point.

Note correction to the operating point: $I_{a0} = 186.67 \text{ A}$;

Note correction to the parameter: $k_f = 0.12 \text{ V-s/A-rad}$

Find:

- System differential equations in symbolic form
- Linearized equations

Assumptions:

The dynamics of the field circuit are negligible.

Analysis:

- Differential equations

Applying KVL and equation 17.47 to the electrical circuit we obtain:

$$L_f \frac{dI_f(t)}{dt} + R_f I_f(t) = V_S(t) \quad \text{field circuit}$$

or

$$L_a \frac{dI_a(t)}{dt} + R_a I_a(t) - k_f I_f(t) = m(t) V_S(t) \quad \text{armature circuit}$$

Applying Newton's Second Law and equation 17.46 to the load inertia, we obtain:

$$J \frac{d m(t)}{dt} + T_m(t) - T_L(t) = b_m$$

or

$$k_f I_f(t) I_a(t) - J \frac{d m(t)}{dt} - b_m m(t) = T_L(t)$$

Since the dynamics of the field circuit are much faster than those of the armature circuit (time constant

$\frac{L_f}{R_f}$ vs $\frac{L_a}{R_a}$), we can write $I_f \approx \frac{V_S}{R_f}$ and the system of equations is now:

$$L_a \frac{dI_a(t)}{dt} + R_a I_a(t) - k_f \frac{V_S(t)}{R_f} = m(t) V_S(t)$$

$$k_f \frac{V_S(t)}{R_f} I_a(t) - J \frac{d m(t)}{dt} - b_m m(t) = T_L(t)$$

- Linearization

Define perturbation variables:

$$\begin{aligned} I_a(t) &= \bar{I}_a + \tilde{I}_a(t) \\ m(t) &= \bar{m} + \tilde{m}(t) \\ V_S(t) &= \bar{V}_S + \tilde{V}_S(t) \end{aligned}$$

Next, we write the steady-state equations (all derivatives equal to zero):

$$R_a \bar{I}_a - k_f \frac{\bar{V}_S}{R_f} \bar{m} = \bar{V}_S$$

$$k_f \frac{\bar{V}_S}{R_f} \bar{I}_a - b_m \bar{m} = \bar{T}_L$$

These equations must be satisfied at the operating point. We can verify this using numerical values:

$$0.75 \quad \bar{I}_a \quad 0.12 \frac{150}{60} 200 \quad 200$$

resulting in

$$\bar{I}_a \quad \frac{1}{0.75} 200 \quad 0.12 \frac{150}{60} 200 \quad 186.67$$

$$0.12 \frac{150}{60} 186.67 \quad 0.6 \quad 200 \quad \bar{T}_L$$

resulting in

$$\bar{T}_L \quad 56 \quad 120 \quad 64 \text{ N} \cdot \text{m}$$

Now, given that the system is operating at the stated operating point, we can linearize the differential equation for the perturbation variables around the operating point. To linearize the equation we recognize

the nonlinear terms: $k_f \frac{V_S(t)}{R_f} m(t)$ and $k_f \frac{V_S(t)}{R_f} I_a(t)$.

To linearize these terms, we use the first-order term in the Taylor series expansion:

$$k_f \frac{V_S(t)}{R_f} m(t) \approx \left. \frac{k_f}{R_f} \frac{V_S(t)}{V_S} m(t) \right|_{\bar{V}_S, \bar{m}} V_S(t) \frac{k_f}{R_f} \frac{V_S(t)}{m} m(t) \bigg|_{\bar{V}_S, \bar{m}}$$

$$\frac{k_f}{R_f} \bar{m} V_S(t) - \bar{V}_S m(t)$$

$$k_f \frac{V_S(t)}{R_f} I_a(t) \approx \left. \frac{k_f}{R_f} \frac{V_S(t) I_a(t)}{V_S} \right|_{\bar{V}_S, \bar{I}_a} V_S(t) \frac{k_f}{R_f} \frac{V_S(t) I_a(t)}{I_a} \bigg|_{\bar{V}_S, \bar{I}_a} \bar{I}_a(t)$$

$$\frac{k_f}{R_f} \bar{I}_a V_S(t) - \bar{V}_S I_a(t)$$

Now we can write the linearized differential equations in the perturbation variables:

$$L_a \frac{d I_a(t)}{dt} = R_a I_a(t) + \frac{k_f}{R_f} \bar{V}_S m(t) - V_S(t) \frac{k_f}{R_f} \bar{m} V_S(t)$$

$$\frac{k_f}{R_f} \bar{V}_S I_a(t) - J \frac{d m(t)}{dt} = b m(t) - \bar{I}_a V_S(t) - T_L(t)$$

This set of equations is now linear, and numerical values can be substituted to obtain a numerical answer, valid in the neighborhood of the operating point, for given voltage and load torque inputs to the system.

Problem 17.30

Solution:

Known Quantities:

$$T_L = 5 + 0.05 \omega + 0.001 \omega^2$$

$$k_{\text{TPM}} = k_{\text{APM}} = 2.42$$

$$R_a = 0.2$$

$$V_S = 50\text{V}$$

Find:

What will the speed of rotation be of the fan?

Assumptions:

The fan is operating at constant speed.

Analysis:

Applying KVL for a PM DC motor (note at constant current short inductor)

$$V_s = i_a R_a + E_b \quad (\text{eqn. 17.48})$$

$$i_a = \frac{T}{k_{TPM}}$$

$$E_b = k_{aPM} \omega_m$$

$$V_s = \frac{T}{k_{TPM}} R_a + k_{aPM} \omega_m$$

$$T = T_L = 5 \text{ N} \cdot \text{m} = 0.001 \text{ m}^2$$

$$V_s = \frac{5 \text{ N} \cdot \text{m} \cdot 0.001 \text{ m}^2}{k_{TPM}} R_a + k_{aPM} \omega_m$$

Plug in the known variables and solving for ω_m .

$$50 \text{ V} = 5 \text{ N} \cdot \text{m} \cdot 0.001 \text{ m}^2 \cdot \frac{0.02}{2.42 \text{ N} \cdot \text{m} / \text{Amp}} + 2.42 \text{ V} \cdot \text{sec} / \text{rad} \cdot \omega_m$$

$$8.26 \times 10^{-5} \text{ m}^2 \cdot 2.42 + 4.13 \times 10^{-3} \text{ m} \cdot \omega_m = 50 \text{ V} \quad (50 - 413) = 0$$

$$\omega_m = 20.5, \text{ or } -29318 \text{ rad/sec}$$

$$\omega_m = 20.5 \text{ rad/sec}$$

$$N_m = 20.5 \text{ rad/sec} \cdot \frac{60 \text{ sec}}{\text{min}} \cdot \frac{\text{rev}}{2} = 196 \text{ RPM}$$

Problem 17.31

Solution:

Known Quantities:

A separately excited DC motor

$$R_a = 0.1 \Omega, R_f = 100 \Omega, L_a = 0.2 \text{ H}, L_f = 0.02 \text{ H}, K_a = 0.8, K_f = 0.9$$

$$J = 0.5 \text{ kg} \cdot \text{m}^2, b = 2 \text{ N} \cdot \text{m} \cdot \text{rad} / \text{s}$$

Find:

- A sketch of the system and its three differential equations
- Sketch a simulation block diagram
- Put the diagram into Simulink
- Run the simulation with *Armature Control* with a constant field voltage $V_f = 100 \text{ V}$
Plot the current and angular speed responses
Run the simulation with *Field Control* with a constant armature voltage $V_f = 100 \text{ V}$
Plot the current and angular speed responses

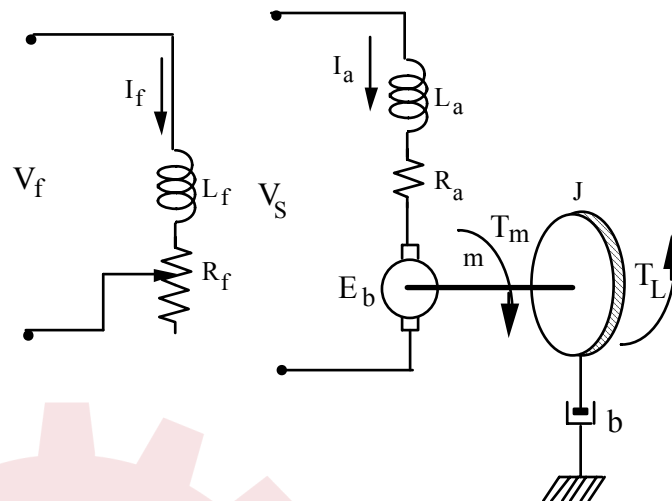
Assumptions:

No external load torque is applied

Analysis:

- Sketch:

Separately Excited DC Motor



The three dynamic equations are:

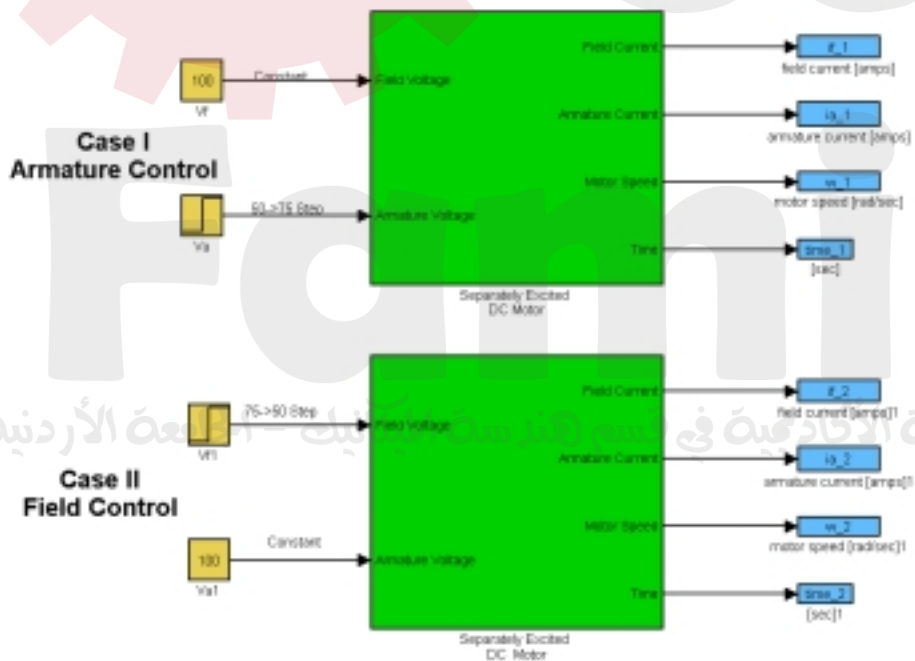
$$V_f = L_f \frac{dI_f}{dt} + R_f I_f$$

$$V_a = L_a \frac{dI_a}{dt} + R_a I_a + E_b$$

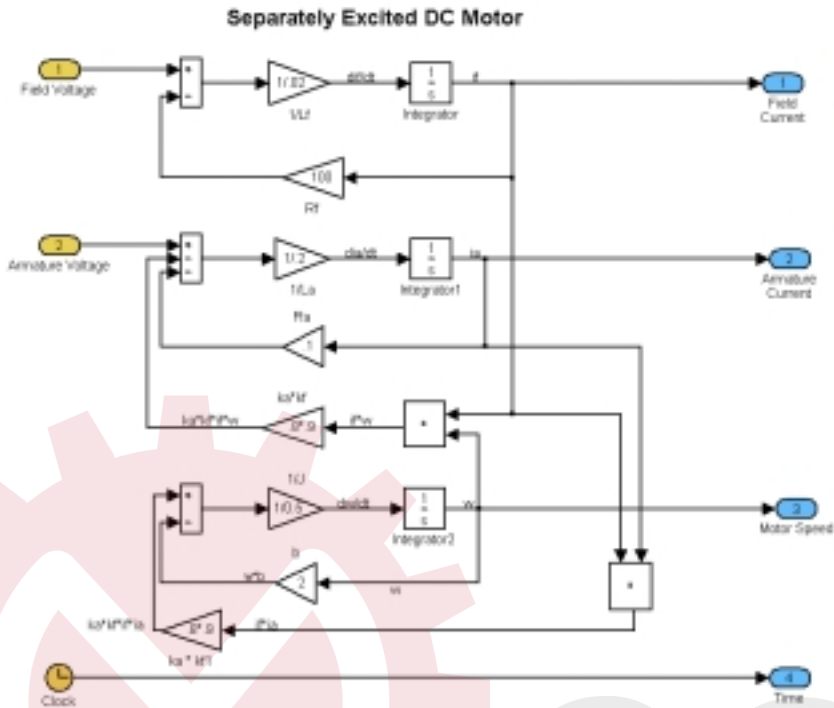
$$J \frac{d\omega_m}{dt} + b \omega_m = T_m = k_a I_a$$

b) Simulink block diagram

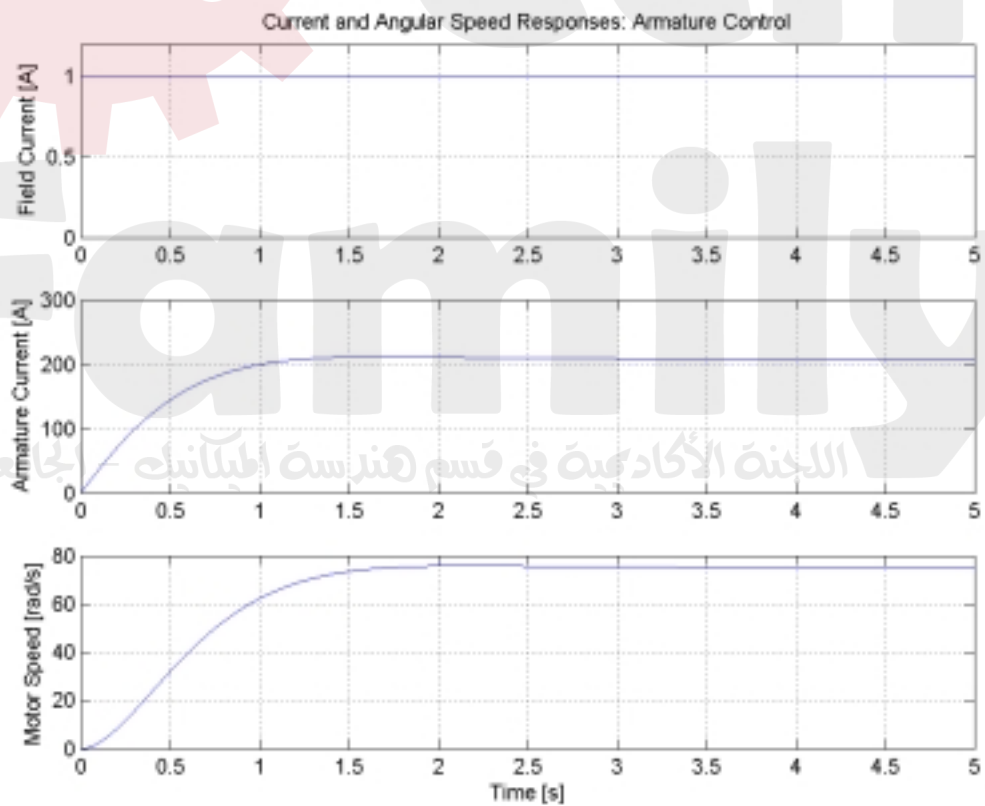
Problem 17.31 - Separately Excited DC Motor



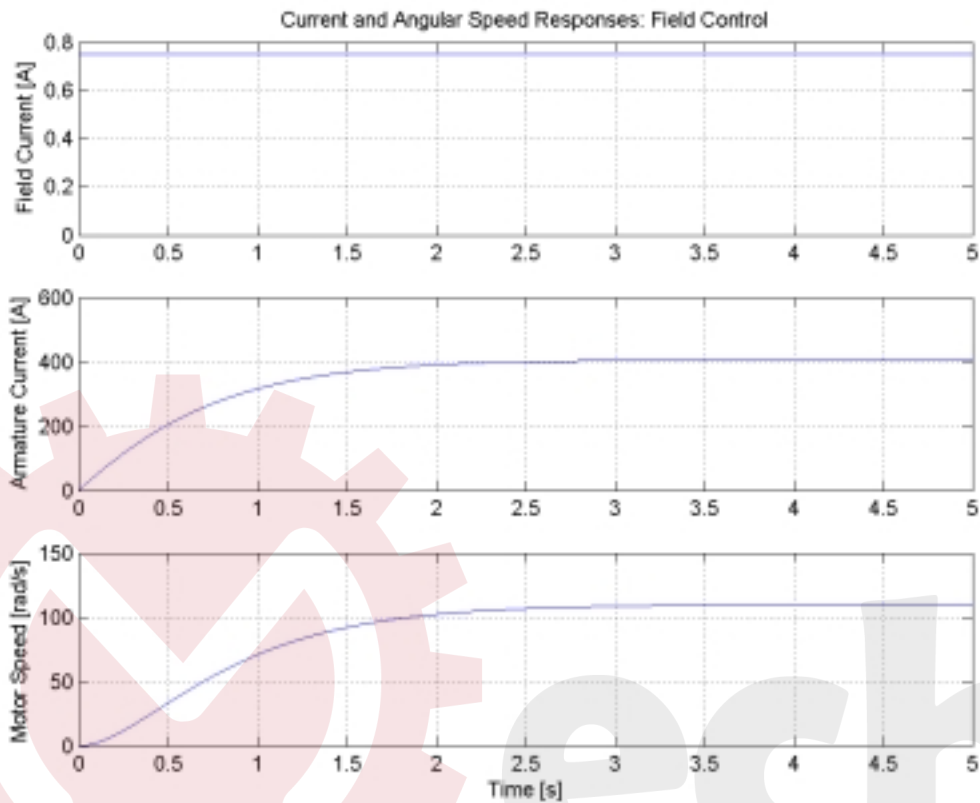
Simulink DC Motor Subsystem



d) Simulink Responses:
Armature Control:



Field Control:



Problem 17.32

Solution:

Known quantities:

$$R_a, L_a, k_a = k_T, J_m, b_m, J, b, T_L.$$

Find:

Transfer functions from armature voltage to angular velocity and from load torque to angular velocity.

Schematics, diagrams, circuits and given data.

See equations 17.16-18 and Figure 17.20.

Assumptions:

Analysis:

Applying KVL and equation 17.47 to the electrical circuit we obtain:

$$V_a(t) - R_a I_a(t) - L_a \frac{dI_a(t)}{dt} - E_b(t) = 0$$

or

$$L_a \frac{dI_a(t)}{dt} + R_a I_a(t) - k_a \omega_m(t) = V_a(t)$$

Applying Newton's Second Law and equation 17.46 to the load inertia, we obtain:

$$J_m \frac{d}{dt} (T_m(t) - T_L(t)) = b_m \dot{\theta}_m - b \dot{\theta}_L$$

or

$$k_T I_a(t) - J_m \frac{d}{dt} (T_m(t) - T_L(t)) = b_m \dot{\theta}_m - b \dot{\theta}_L$$

To derive the transfer function, we Laplace transform the two equations to obtain:

$$sL_a R_a I_a(s) - k_a (s T_m(s) - T_L(s)) = V_a(s)$$

$$k_a I_a(s) - s J_m (T_m(s) - T_L(s)) = b_m T_m(s) - b T_L(s)$$

We can write the above equations in matrix form and resort to Cramer's rule to solve for $\Omega_m(s)$ as a function of $V_a(s)$ and $T_L(s)$.

$$\begin{bmatrix} sL_a R_a & -k_a \\ k_a & -s J_m + b_m - b \end{bmatrix} \begin{bmatrix} I_a(s) \\ T_m(s) \end{bmatrix} = \begin{bmatrix} V_a(s) \\ T_L(s) \end{bmatrix}$$

with solution

$$T_m(s) = \frac{\det \begin{bmatrix} sL_a R_a & V_a(s) \\ k_a & T_L(s) \end{bmatrix}}{\det \begin{bmatrix} sL_a R_a & -k_a \\ k_a & -s J_m + b_m - b \end{bmatrix}}$$

or

$$T_m(s) = \frac{sL_a R_a T_L(s) - k_a V_a(s)}{sL_a R_a (-s J_m + b_m - b) + k_a^2}$$

and finally

$$\frac{T_m(s)}{T_L(s)} = \frac{sL_a R_a}{sL_a R_a (-s J_m + b_m - b) + k_a^2}$$

$$\frac{T_m(s)}{V_a(s)} = \frac{-k_a}{sL_a R_a (-s J_m + b_m - b) + k_a^2}$$

Problem 17.33

Solution:

Known Quantities:

A PM DC motor that is coupled to a pump with a long shaft.

Find:

The dynamic equations of the system and the transfer function from input voltage to load inertia speed.

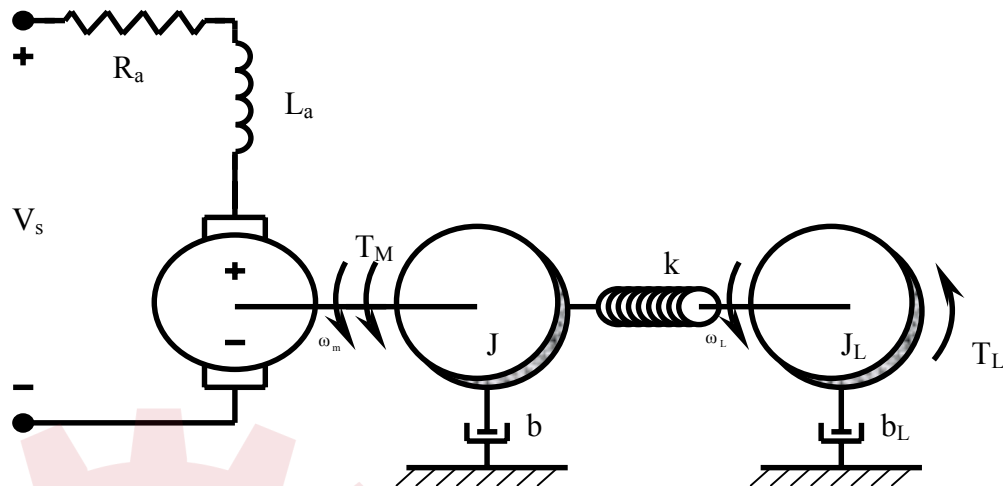
Assumptions:

The energy conversion is ideal.

Analysis:

Sketch:

PM DC Motor-Load Coupling



Knowing:

$$\frac{d}{dt} \quad \frac{1}{s}$$

Then the three dynamic equations for the system are:

$$V_a - L_a \frac{di_a}{dt} - R_a i_a - E_a = L_a \frac{di_a}{dt} - R_a i_a - k_a \omega_m$$

$$J_m \frac{d\omega_m}{dt} - b_m \omega_m - K \frac{\omega_m}{s} - k_a i_a = K \frac{\omega_m}{s} - T_L$$

$$J_L \frac{d\omega_L}{dt} - b_L \omega_L - K \frac{\omega_L}{s} - K \frac{\omega_m}{s} = T_L$$

Putting them in matrix form:

$$\begin{bmatrix} L_a s + R_a & k_a & 0 \\ -k_a & J_m s^2 + b_m s + K/s & -K/s \\ 0 & -K/s & J_L s^2 + b_L s + K/s \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \omega_L \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ -T_L \end{bmatrix}$$

$$\left. \frac{\omega_L}{V_a} \right|_{T_L=0} = \frac{k_a K/s}{(L_a s + R_a)(J_m s + b_m + K/s)(J_L s + b_L + K/s) - (-K/s)^2 (L_a s + R_a) + (J_L s + b_L + K/s)(-k_a)(k_a)}$$

Problem 17.34

Solution:

Known quantities:

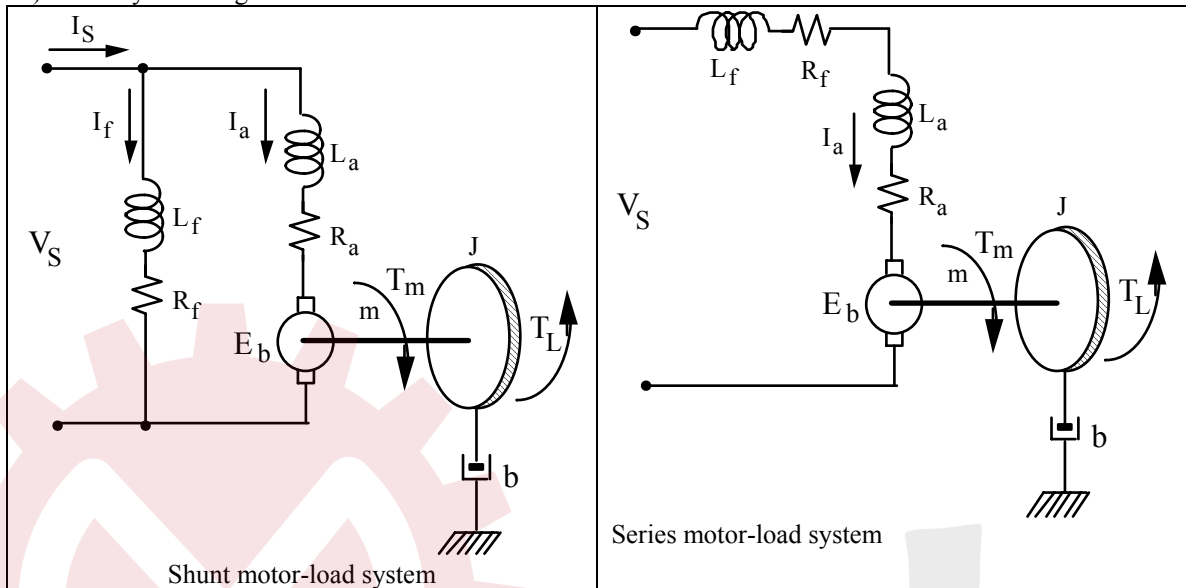
Field and armature circuit parameters; magnetization and armature constants; motor and load inertia and damping coefficients.

Find:

- sketch system diagrams for shunt and series configuration
- write expression for torque-speed curves for each configuration
- write the differential equations for each configuration
- determine whether equations are linear or nonlinear and how they could be linearized

Assumptions:**Analysis:**

a) System diagrams



b) Write expressions for the torque-speed curves

Shunt configuration

Applying KVL and Newton's Second Law for the steady-state system we write:

$$V_S = R_f I_f \quad \text{field circuit}$$

and

$$V_S = R_a I_a + k_f I_f \omega_m \quad \text{armature circuit}$$

or

$$V_S = R_a I_a + k_f \frac{V_S}{R_f} \omega_m$$

$$T_m = k_f I_f I_a = k_f \frac{V_S}{R_f} I_a \quad \omega_m = T_L$$

To obtain the torque-speed curve of the motor (there will also be a load torque-speed equation, but we do not have any information on the nature of the load), we write:

$$T_m = k_f I_f I_a$$

$$I_a = \frac{T_m}{k_f I_f} = \frac{T_m R_f}{k_f V_S}$$

and substitute the expression for I_a in the electrical circuit equation:

$$V_S = R_a I_a + k_f \frac{V_S}{R_f} \omega_m = \frac{R_a R_f}{k_f V_S} T_m + k_f \frac{V_S}{R_f} \omega_m$$

or

$$T_m = \frac{k_f V_S}{R_a R_f} V_S - k_f \frac{V_S}{R_f} \omega_m = \frac{k_f V_S^2}{R_a R_f} - \frac{k_f^2 V_S^2}{R_a R_f^2} \omega_m$$

Series configuration

Applying KVL and Newton's Second Law for the steady-state system we write:

$$V_S = R_a I_a + R_f I_a + k_f I_a^2 \omega_m$$

$$T_m = k_f I_a^2 \omega_m - b \omega_m - T_L$$

To obtain the torque-speed curve of the motor (there will also be a load torque-speed equation, but we do not have any information on the nature of the load), we write:

$$I_a = \sqrt{\frac{T_m}{k_f}}$$

$$V_S = R_a I_a + R_f I_a + k_f I_a^2 \omega_m = R_a I_a + R_f \sqrt{\frac{T_m}{k_f}} + T_m \omega_m$$

which leads to a quadratic equation in T_m and ω_m .

c) Write the differential equations

Shunt configuration

Applying KVL and equation 17.47 to the electrical circuit we obtain:

$$L_f \frac{dI_f(t)}{dt} + R_f I_f(t) = V_S(t) \quad \text{field circuit}$$

or

$$L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_f I_f(t) \omega_m(t) = V_S(t) \quad \text{armature circuit}$$

Applying Newton's Second Law and equation 17.46 to the load inertia, we obtain:

$$J \frac{d\omega_m(t)}{dt} = T_m(t) - T_L(t) - b \omega_m(t)$$

or

$$k_f I_f(t) I_a(t) - J \frac{d\omega_m(t)}{dt} - b \omega_m(t) = T_L(t)$$

Note that we have three differential equations that must be solved simultaneously. If the dynamics of the field circuit are much faster than those of the armature circuit (time constant $\frac{L_f}{R_f} \ll \frac{L_a}{R_a}$, as is often the

case) one can assume that the field current varies instantaneously with the supply voltage, leading to

$$I_f = \frac{V_S}{R_f} \quad \text{and to the equations:}$$

$$L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_f \frac{V_S(t)}{R_f} \omega_m(t) = V_S(t)$$

$$k_f \frac{V_S(t)}{R_f} I_a(t) - J \frac{d\omega_m(t)}{dt} - b \omega_m(t) = T_L(t)$$

Series configuration

Applying KVL and equation 17.47 to the electrical circuit we obtain:

$$L_a \frac{dI_a(t)}{dt} + L_f \frac{dI_a(t)}{dt} + R_a I_a(t) + R_f I_a(t) + k_f I_a(t) \omega_m(t) = V_S(t)$$

Applying Newton's Second Law and equation 17.46 to the load inertia, we obtain:

$$J \frac{d\omega_m(t)}{dt} = T_m(t) - T_L(t) - b \omega_m(t)$$

or

$$k_f I_a^2(t) - J \frac{d\omega_m(t)}{dt} - b \omega_m(t) = T_L(t)$$

d) Determine whether the equations are nonlinear

Both systems of equations are nonlinear. In the shunt case, we have product terms in I_f and ω_m , and in I_f and I_a (or in V_S and ω_m and in V_S and I_a if we use the simplified system of two equations). In the series case, we have a quadratic term in I_a^2 and a product term in I_f and ω_m . In either case, no simple assumption leads

to a linear set of equations; thus either linearization or nonlinear solution methods (e.g.: numerical simulation) must be employed.

Problem 17.35

Solution:

Known quantities:

A shunt-connected DC motor shown in Figure P17.35

Motor parameters: k_a, k_T = armature and torque reluctance constant and k_f = field flux constant

Find:

Derive the differential equations describing the electrical and mechanical dynamics of the motor

Draw a simulation block diagram of the system

Assumptions:

None

Analysis:

Electrical subsystem

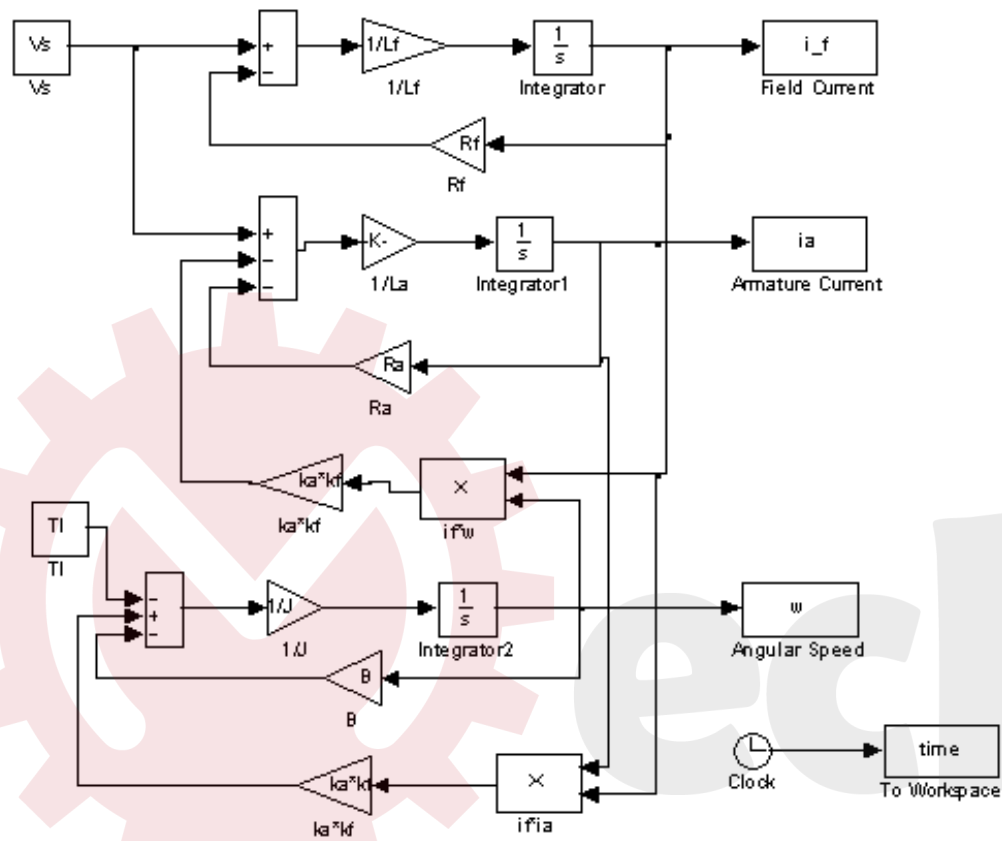
$$V_S(t) = L_f \frac{dI_f(t)}{dt} + R_f I_f(t) \quad \text{field}$$

$$V_S(t) = L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_a \phi \omega_m(t) = L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_a k_f I_f(t) \omega_m(t) \quad \text{armature}$$

Mechanical subsystem

$$J \frac{d\omega_m(t)}{dt} = T_m(t) - T_L(t) - b\omega_m(t) = k_a \phi I_a(t) - T_L(t) - b\omega_m(t) = k_a k_f I_f(t) I_a(t) - T_L(t) - b\omega_m(t)$$

Simulation block diagram:



Problem 17.36

Solution: اللجنة الأكاديمية في قسم هندسة الميكانيك - الجامعة

Known quantities:

A series-connected DC motor shown in Figure P17.36

Motor parameters: k_a, k_t = armature and torque reluctance constant and k_f = field flux constant

Find:

Derive the differential equations describing the electrical and mechanical dynamics of the motor
Draw a simulation block diagram of the system

Assumptions:

None

Analysis:

Electrical subsystem

$$V_S(t) - L_a - L_f \frac{dI_a(t)}{dt} - R_a - R_f I_a(t) - k_a \omega_m(t)$$

$$V_S(t) - L \frac{dI_a(t)}{dt} - R I_a(t) - k_a k_f I_a(t) - \omega_m(t)$$

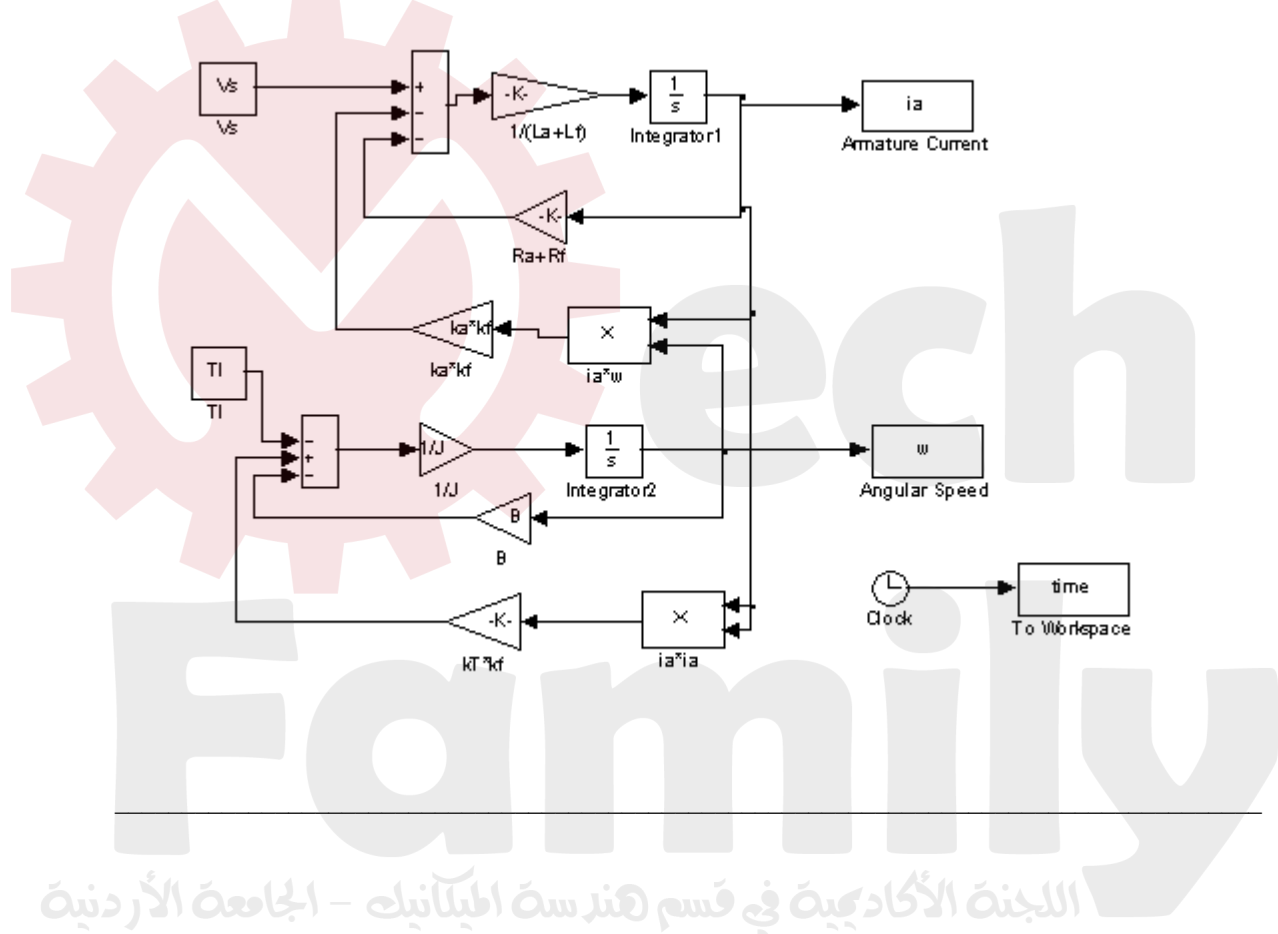
$$\frac{dI_a(t)}{dt} = \frac{1}{L} V_S(t) - \frac{R}{L} I_a(t) - \frac{k_a k_f}{L} I_a(t) - \omega_m(t)$$

Mechanical subsystem

$$J \frac{d\omega_m(t)}{dt} = T_m(t) - T_L(t) - b \omega_m(t) - k_T I_a(t) - T_L(t) - b \omega_m(t) - k_T k_f I_a^2(t) - T_L(t) - b \omega_m(t)$$

$$\frac{d\omega_m(t)}{dt} = \frac{k_T k_f}{J} I_a^2(t) - \frac{1}{J} T_L(t) - \frac{b}{J} \omega_m(t)$$

Simulation block diagram:



Section 17.6: The Alternator (Synchronous Generator)

Problem 17.37

Solution:

Known quantities:

A $550V \cdot A, 20V$ rated automotive alternator. At rated $V \cdot A$, the power factor is 0.85 . The resistance per phase is 0.05Ω . The field takes $2A$ at $12V$. The friction and windage loss is $25W$ and core loss is $30W$.

Find:

The percent efficiency under rated conditions.

Assumptions:

None.

Analysis:

$$I_a = \frac{500}{20} = 25A$$

$$P_a = I_a^2 R_a = 31.25W$$

$$P_{out} = 500(0.85) = 425W$$

$$P_f = 2(12) = 24W$$

$$P_{in} = P_{out} + P_a + 25 + 30 + 24 = 535.25W$$

$$\% = \frac{425}{535.25} \times 100 = 79.4\%$$

Problem 17.38

Solution:

Known quantities:

A three-phase $2300V, 500kW \cdot A$ synchronous generator. $X_s = 8.0\Omega, r_a = 0.1\Omega$. The machine is operating at rated load and voltage at a power factor of 0.867 lagging.

Find:

The generated voltage per phase and the torque angle.

Assumptions:

None.

Analysis:

$$I = \frac{500k}{\sqrt{3}(2300)} = 125.5 \text{ A}$$

$$\begin{aligned} E &= \frac{2300}{\sqrt{3}} \angle 0^\circ + 125.5 \angle -30^\circ (0.1 + j0.8) \\ &= 1327.9 + 101.2 \angle 52.9^\circ \\ &= 1389 + j80.7 \\ &= 1391.3 \angle 3.3^\circ \text{ V} \end{aligned}$$

$$\therefore E = 1391.3 \text{ V}$$

$$\delta = 3.3^\circ$$

Problem 17.39**Solution:****Known quantities:**

As shown in Figure P17.39.

Find:

Explain the function of Q , D , Z , and SCR .

Assumptions:

None.

Analysis:

Q : The setting of R_1 determines the biasing of Q . When Q conducts, the SCR will fire, energizing the alternator's field.

D : This diode serves as a "free-wheeling" element, allowing the field current to circulate without interfering with the commutation of the SCR .

Z : The Zener diode provides a fixed reference voltage at the emitter of transistor Q ; i.e., determination of when Q conducts is controlled solely by the setting of R_1 .

SCR : The SCR acts as a half-wave rectifier, providing field excitation for the alternator. Without the field, of course, the alternator cannot generate.

Section 17.7: The Synchronous Motor

Problem 17.40

Solution:

Known quantities:

A non-salient pole, Y-connected, three phase, two-pole synchronous machine. The synchronous reactance is $7\ \Omega$ and the resistance and rotational losses are negligible. One point on the open-circuit characteristic is given by $V_0 = 400\text{ V}$ (phase voltage) for a field current of 3.32 A . The machine operates as a motor, with a terminal voltage of 400 V (phase voltage). The armature current is 50 A , with power factor 0.85 leading.

Find:

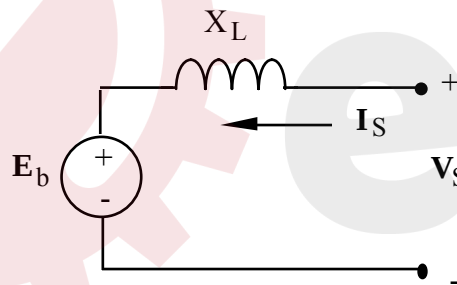
E_b , field current, torque developed, and power angle δ .

Assumptions:

None.

Analysis:

The per phase circuit is shown below:



Since the power factor is 0.85 , we have:

$$\theta = 31.79^\circ$$

$$\omega_m = \frac{2\pi}{60} 3600 = 377\text{ rad/sec}$$

From $V_{OC} = 400\text{ V}$, we have

$$E_b = 400\text{ V (open circuit)} = k\omega_m i_f$$

$$\text{Therefore } k = \frac{400}{377 \times 3.32} = 0.3196$$

$$E_b = 400\angle 0^\circ - 50\angle 31.79^\circ \times 7\angle 90^\circ$$

$$= 400 + 184.38 - j297.49$$

$$= 655.74\angle -26.98^\circ\text{ V}$$

$$i_f = \frac{E_b}{120.48} = 5.44\text{ A}$$

$$\theta_T = 31.79^\circ + 26.98^\circ = 58.77^\circ$$

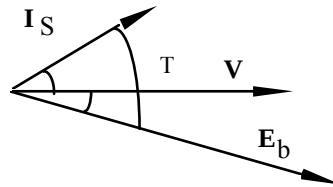
The torque developed is:

$$T = \frac{3}{377} |E_b| |I_s| \cos \theta_T = 135.27\text{ N} \cdot \text{m}$$

δ is the angle from V to E_b :

$$\delta = -26.98^\circ$$

The phase diagram is shown below:



Problem 17.41

Solution:

Known quantities:

A factory load of 900 kW at 0.6 power factor lagging is increased by adding a 450 kW synchronous motor.

Find:

The power factor this motor operates at and the KVA input if the overall power factor is 0.9 lagging.

Assumptions:

None.

Analysis:

$$P_{old} = 900\text{ kW} \quad Q_{old} = 1200\text{ kVAR}$$

$$P_m = 450\text{ kW}$$

$$P_T = 1350\text{ kW} \quad Q_T = 653.8\text{ kVAR}$$

$$Q_m = 653.8 - 1200 = -546.2\text{ kVAR}$$

$$pf_m = \cos(\tan^{-1} \frac{Q_m}{P_m}) = 0.636 \text{ leading}$$

$$S_m = \frac{P_m}{pf_m} = 708\text{ kVA}$$

Problem 17.42

Solution:

Known quantities:

A non-salient pole, Y-connected, three phase, two-pole synchronous generator is connected to a 400 V (line to line), 60 Hz , three-phase line. The stator impedance is $0.5 + j1.6$ (per phase). The generator is delivering rated current 36 A at unity power factor to the line.

Find:

The power angle for this load and the value of E_b for this condition. Sketch the phasor diagram, showing E_b , I_S , and V_S .

Assumptions:

None.

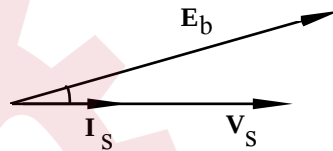
Analysis:

$$V_L = \frac{400}{\sqrt{3}} \angle 0^\circ = 230.9 \angle 0^\circ \text{ V}$$

$$I_L = 36 \angle 0^\circ \text{ A}$$

$$Z_S = 0.5 + j1.6 = 1.676 \angle 72.65^\circ \Omega$$

$$\begin{aligned} E_b &= V_L + I_L Z_S = 248.9 + j57.6 \\ &= 255.5 \angle 13.03^\circ \text{ V} \end{aligned}$$

The power angle is 13.03° .**Problem 17.43****Solution:****Known quantities:**

A non-salient pole, three phase, two-pole synchronous generator is connected in parallel with a three-phase, Y-connected load. The equivalent circuit is shown in Figure P17.43. The parallel combination is connected to a 220V (line to line), three-phase line. The load current is 25 A at a power factor of 0.866 inductive. $X_S = 2 \Omega$. The motor is operating with $I_f = 1 \text{ A}$, $T = 50 \text{ N} \cdot \text{m}$ at a power angle of -30° .

Find:

I_S, P_{in} (to the motor), the overall power factor and the total power drawn from the line.

Assumptions:

Neglect all losses for the motor.

Analysis:

The phasor per-phase voltage is:

$$V_S = 127 \angle 0^\circ \text{ V}$$

$$T_{dev} = 50 \text{ N} \cdot \text{m} = -\frac{3}{377} \frac{|E_b| |V_S|}{X_S} \sin \delta$$

Therefore,

$$|E_b| = -\frac{50(377)2}{3(127) \sin(-30^\circ)} = 197.9 \text{ V}$$

$$E_b = 197.9 \angle -30^\circ \text{ V}$$

For $i_f = 1 \text{ A}$,

$$I_S = 49.47 + j22.2 = 54.23 \angle 24.16^\circ \text{ A}$$

The load current is:

$$I_L = 25 \angle -\cos^{-1} 0.866 = 21.65 - j12.5$$

and

$$I_1 = I_L + I_S = 71.12 + j9.7 = 71.78 \angle 7.77^\circ \text{ A}$$

$$P_{in\_motor} = 3 \times 54.23 \times 127 \times \cos 24.16^\circ = 18.85 \text{ kW}$$

$$P_{in\_total} = 3 \times 71.78 \times 127 \times \cos 7.77^\circ = 27.10 \text{ kW}$$

The power factor is:

$$pf = \cos 7.77^\circ = 0.991 \text{ leading}$$

Problem 17.44

Solution:

Known quantities:

A non-salient pole, Y-connected, three phase, four-pole synchronous machine. The synchronous reactance is 10Ω . It is connected to a $230\sqrt{3} \text{ V}$ (line to line), 60 Hz , three-phase line. The load requires a torque of $T_{shaft} = 30 \text{ N} \cdot \text{m}$. The line current is 15 A leading the phase voltage.

Find:

The power angle δ and E for this condition. The line current when the load is removed. Is it leading or lagging the voltage.

Assumptions:

All losses can be neglected.

Analysis:

At $\omega_m = 188.5 \text{ rad/sec}$, we can calculate

$$P_{out} = 30 \times 188.5 = 5655 \text{ W}$$

Since $P_{in} = P_{out}$ and $P'_{in} \text{ (per phase)} = 1885 \text{ W} = 230 \times 15 \cos \theta$, we calculate

$$\theta = \cos^{-1} 0.5464 = 56.88^\circ$$

Since $V_S = 230 \angle 0^\circ \text{ V}$, $I_S = 15 \angle 56.88^\circ \text{ A}$

$$E_b = 355.6 - j81.96 = 364.92 \angle -12.98^\circ \text{ V}$$

The power angle is:

$$\delta = -12.98^\circ$$

If the load is removed, the power angle is 0° and from

$$364.92 \angle 0^\circ = 230 \angle 0^\circ - 10 \angle 90^\circ$$

$$\Rightarrow I = 13.495 \angle 90^\circ \text{ A}$$

The current is leading the voltage.

Problem 17.45**Solution:****Known quantities:**

A 10 hp, 230 V, 60 Hz Y-connected, three phase synchronous motor delivers full load at a power factor of 0.8 leading. The synchronous reactance is 6Ω . The rotational loss is 230 W, and the field loss is 50 W.

Find:

- The armature current.
- The motor efficiency.
- The power angle.

Assumptions:

Neglect the stator winding resistance.

Analysis:

$$P_{out} = 10 \text{ hp} = 7460 \text{ W}$$

$$P_{in} = P_{out} + P_r + P_{copper} = 7740$$

$$\therefore P_{in} (\text{per phase}) = 2580 = V_s I_s 0.8$$

$$V_s = \frac{230}{\sqrt{3}} = 132.8 \text{ V}$$

$$\therefore I_s = \frac{2580}{132.8 \times 0.8} = 24.3 \text{ A}$$

That is:

$$V_s = 132.8 \angle 0^\circ \text{ V}, \quad I_s = 24.3 \angle 36.87^\circ \text{ A}$$

$$E_b = V_s - I_s (6 \angle 90^\circ) = 249.2 \angle -27.9^\circ \text{ V}$$

a)

$$I_s = 24.3 \angle 36.87^\circ \text{ A}$$

b)

$$\text{efficiency} = \frac{7460}{7740} = 0.964 = 96.4\%$$

c)

$$\text{power angle} = -27.9^\circ$$

Problem 17.46**Solution:****Known quantities:**

A three-phase 2300 V, 60 Hz, 30 poles, 2000 hp, unity power factor synchronous motor.

$X_s = 1.95\Omega$ per phase.

Find:

The maximum power and torque.

Assumptions:

Neglect all losses.

Analysis:

$$n_s = \frac{3600}{15} = 240 \text{ rev/min}$$

$$\omega_s = 25.13 \text{ rad/sec}$$

At full load,

$$P_{in} = 746 \times 2000 = 1.492 \text{ MW}$$

$$V_s = \frac{2300}{\sqrt{3}} = 1327.9 \angle 0^\circ \text{ V}$$

For unity power factor,

$$I_s = 374.5 \angle 0^\circ \text{ A}$$

$$\begin{aligned} E_b = V_s - I_s jX_s &= 1327.9 - j730.3 \\ &= 1515.5 \angle -28.2^\circ \text{ V} \end{aligned}$$

The maximum power and torque are:

$$P_{\max} = 3 \frac{|E_b| |V_s|}{X_s} = 3.096 \text{ MW}$$

$$T_{\max} = \frac{P_{\max}}{\omega_s} = 123.2 \text{ kN} \cdot \text{m}$$

Problem 17.47**Solution:****Known quantities:**

A 1200V Y-connected, three phase synchronous motor takes 110kW when operated under a certain load at 1200 rev/min. The back emf of the motor is 2000V. The synchronous reactance is 10Ω per phase.

Find:

The line current and the torque developed by the motor.

Assumptions:

Winding resistance is negligible.

Analysis:

$$V_s = \frac{1200}{\sqrt{3}} = 692.8 \angle 0^\circ$$

The input power per phase is:

$$L = \frac{N^2}{\mathfrak{R}_T} = \frac{100^2}{12.51 \times 10^3} = 0.8 \text{ H}$$

The power developed is:

$$P = -3 \frac{|E_b| |V_s|}{X_s} \sin \delta$$

$$\therefore \sin \delta = -0.2646$$

$$\delta = -15.34^\circ$$

The torque developed is:

$$T = \frac{P}{\omega_s} = 875.1 \text{ N} \cdot \text{m}$$

Problem 17.48

Solution:

Known quantities:

A 600V Y-connected, three phase synchronous motor takes 24 kW at a leading power factor of 0.707. The per-phase impedance is $5 + j50 \Omega$.

Find:

The induced voltage and the power angle of the motor.

Assumptions:

None.

Analysis:

$$V_s = \frac{600}{\sqrt{3}} = 346.4 \angle 0^\circ \text{ V}$$

$$Z_s = 5 + j50 = 50.25 \angle 84.29^\circ \Omega$$

From $pf = 0.707$, we have $\theta = 45^\circ$.

From $P_{in} = 3|V_s||I_s| \cos \theta$, we have

$$|I_s| = \frac{24 \times 10^3}{3 \times 346.4} \times 0.707 = 32.67 \text{ A}$$

$$I_s = 32.67 \angle 45^\circ \text{ A}$$

$$E_b = V_s - I_s Z_s = 1385 - j1270.6 \\ = 1880.3 \angle -42.51^\circ \text{ V}$$

The power angle is:

$$\delta = -42.51^\circ$$

The power developed and the copper loss are:

$$P_{dev} = 3|E_b||I_s| \cos 87.51^\circ = 8.006 \text{ kW}$$

$$P_{loss} = 3|I_s|^2 R_s = 16.01 \text{ kW}$$

Section 17.8: The Induction Motor

Problem 17.49

Solution:

Known quantities:

A 74.6 kW three-phase, 440 V (line to line), four-pole, 60 Hz induction motor. The equivalent circuit parameters are:

$$R_S = 0.06\ \Omega \quad R_R = 0.08\ \Omega$$

$$X_S = 0.3\ \Omega \quad X_R = 0.3\ \Omega \quad X_m = 5\ \Omega$$

The no-load power input is 3240 W at a current of 45 A .

Find:

The line current, the input power, the developed torque, the shaft torque, and the efficiency at $s = 0.02$.

Assumptions:

None.

Analysis:

$$V_S = \frac{400}{\sqrt{3}} = 254\angle 0^\circ\text{ V}$$

$$\begin{aligned} Z_{in} &= 0.06 + j0.3 + \frac{j5(4 + j0.3)}{4 + j5.3} \\ &= 2.328 + j2.294 = 3.268\angle 44.59^\circ\ \Omega \end{aligned}$$

$$I_S = 77.7\angle -44.59\text{ A}$$

$$P_{in} = 3 \times 254 \times 77.7 \cos(-44.59^\circ) = 42.16\text{ kW}$$

$$I_2 = \frac{j5}{4 + j5.3} I_S = 58.51\angle -7.55^\circ\text{ A}$$

The total power transferred to the rotor is:

$$P_T = 3 \frac{R_S}{S} |I_2|^2 = 41.1\text{ kW}$$

$$\begin{aligned} P_m &= P_T - P_{\text{copper\_loss\_in\_rotor}} \\ &= 41.1 \times 10^3 (1 - s) = 40.25\text{ kW} \end{aligned}$$

$$\omega_m = (1 - s)\omega_s = 0.98 \times 188.5 = 184.7\text{ rad/sec}$$

Therefore, the torque developed is:

$$\begin{aligned} T_{dev} &= \frac{P_m}{184.7} = 218\text{ N} \cdot \text{m} \\ &= 1880.3\angle -42.51^\circ\text{ V} \end{aligned}$$

The rotational power and torque losses are:

$$P_{rot} = 3240 - 3 \times 45^2 \times 0.06 = 2875.5 W$$

$$T_{rot} = 15.56 N \cdot m$$

The shaft torque is:

$$T_{sh} = 218 - 15.56 = 202.4 N \cdot m$$

Efficiency is:

$$T_{sh} = \frac{P_{out}}{P_{in}} = \frac{202.4 \times 184.7}{42.16 \times 10^3} = 0.887$$

Problem 17.50

Solution:

Known quantities:

A 60 Hz, four-pole, Y-connected induction motor is connected to a three-phase, 400V (line to line), 60 Hz line. The equivalent circuit parameters are:

$$\begin{aligned} R_S &= 0.2 \Omega & R_R &= 0.1 \Omega \\ X_S &= 0.5 \Omega & X_R &= 0.2 \Omega & X_m &= 20 \Omega \end{aligned}$$

When the machine is running at 1755 rev/min, the total rotational and stray-load losses are 800 W.

Find:

The slip, input current, total input power, mechanical power developed, shaft torque and efficiency.

Assumptions:

None.

Analysis:

From $n_s = 1800 \text{ rev/min}$, we have

$$s = 0.025$$

$$\frac{R_R}{s} = 4$$

$$\begin{aligned} Z_{in} &= 0.2 + j0.5 + \frac{j20(4 + j0.2)}{4 + j20.2} \\ &= 3.972 + j1.444 = 4.226 \angle 19.98^\circ \Omega \end{aligned}$$

Therefore,

$$I_s = 54.6 \angle -19.98^\circ \text{ A}$$

$$P_{in} = 3(54.6) \left(\frac{400}{\sqrt{3}} \cos(-19.98^\circ) \right) = 35.6 \text{ kW}$$

$$P_t = P_{in} = 3|I_s|^2 R_s = 35.6 \times 10^3 - 3(54.6)^2 \times 0.2 = 33.81 \text{ kW}$$

$$P_m = (1 - s)P_t = 32.97 \text{ kW}$$

$$P_{sh} = P_{out} = P_m - 800 = 32.17 \text{ kW}$$

$$\omega_m = 183.8 \text{ rad/sec}$$

$$T_{sh} = 175 \text{ N} \cdot \text{m}$$

$$\text{efficiency} = \frac{32.17}{35.6} = 0.904$$

Problem 17.51

Solution:

Known quantities:

A three-phase, 60 Hz, eight-pole induction motor operates with a slip of 0.05 for a certain load.

Find:

- The speed of the rotor with respect to the stator.
- The speed of the rotor with respect to the stator magnetic field.
- The speed of the rotor magnetic field with respect to the rotor.
- The speed of the rotor magnetic field with respect to the stator magnetic field.

Assumptions:

None.

Analysis:

$$n_s = 900 \text{ rev/min}, \quad \omega_s = 94.25 \text{ rad/sec}$$

a)

$$n_m = (1 - s)n_s = 855 \text{ rev/min}$$

b)

The speed of the stator field is 900 rev/min, the rotor speed relative to the stator field is -45 rev/min.

c)

$$45 \text{ rev/min}$$

d)

$$0 \text{ rev/min}$$

Problem 17.52**Solution:****Known quantities:**

A three-phase, 60 Hz, 400 V (per phase), two-pole induction motor develops $P_m = 37 \text{ kW}$ at a certain speed.

The rotational loss at this speed is 800 W.

Find:

- a) The slip and the output torque if the total power transferred to the rotor is 40 kW.
 b) I_s and the power factor if $P_m = 45 \text{ kW}$, $R_s = 0.5 \Omega$.

Assumptions:

Stray-load loss is negligible.

Analysis:

a)

$$P_m = 3(1-s)P_t = 37 \text{ kW}$$

$$1-s = 0.925 \Rightarrow s = 0.075$$

$$n_s = 3600 \text{ rev/min}, \quad \omega_s = 377 \text{ rad/sec}$$

$$\omega_m = (1-s)\omega_s = 348.7 \text{ rad/sec}$$

$$P_{sh} = P_{out} = 37 - 0.8 = 36.2 \text{ kW}$$

$$T_{sh} = \frac{P_{sh}}{\omega_m} = 103.8 \text{ N} \cdot \text{m}$$

b)

$$P_{in} = 3|I_s|^2 R_s + P_t$$

$$3|I_s|^2 R_s = 5 \text{ kW}$$

$$\therefore |I_s| = 57.7 \text{ A}$$

$$P_{in} = 3|V_s||I_s|\cos\theta = 45 \text{ kW}$$

The power factor is:

$$\cos\theta = 0.65 \text{ lagging}$$

Problem 17.53**Solution:****Known quantities:**

The nameplate speed of a 25 Hz induction motor is 720 rev/min. The speed at no load is 745 rev/min.

Find:

- a) The slip.
 b) The percent regulation.

Assumptions:

None.

Analysis:

a)

$$p \approx \frac{120(25)}{720} = 4.17 \Rightarrow p = 4$$

$$n_{sync} = \frac{120(25)}{4} = 750 \text{ rpm}$$

$$slip = \frac{750 - 720}{750} = 0.04 = 4\%$$

b)

$$reg = \frac{745 - 720}{720} = 0.035 = 3.5\%$$

Problem 17.54**Solution:****Known quantities:**

The name plate of a squirrel-cage four-pole induction motor has 25 hp, 220 V, 60 Hz, 830 rev/min, 64 A, three-phase line current. The motor draws 20,800 W when operating at a full load.

Find:

- a) slip.
- b) Percent regulation if the no-load speed is 895 rpm.
- c) Power factor.
- d) Torque.
- e) Efficiency.

Assumptions:

None.

Analysis:

a)

$$n_{sync} = 900 \text{ rpm}$$

$$slip = \frac{900 - 830}{900} = 0.078 = 7.8\%$$

b)

$$reg = \frac{895 - 830}{830} = 0.078 = 7.8\%$$

c)

$$pf = \frac{20,800}{\sqrt{3}(220)(64)} = 0.853 \text{ lagging}$$

d)

$$T = \frac{7.04(25 \times 746)}{830} = 158.2 \text{ lb} \cdot \text{ft}$$

e)

$$eff = \frac{25 \times 746}{20,800} = 0.897 = 89.7\%$$

Problem 17.55**Solution:****Known quantities:**

A 60 Hz, four-pole, Y-connected induction motor is connected to a 200 V (line to line), three-phase, 60 Hz line. The equivalent circuit parameters are:

$$\begin{aligned} R_S &= 0.48 \Omega & \text{Rotational loss torque} &= 3.5 \text{ N} \cdot \text{m} \\ X_S &= 0.8 \Omega & R_R &= 0.42 \Omega \text{ (referred to the stator)} \\ X_m &= 30 \Omega & X_R &= 0.8 \Omega \text{ (referred to the stator)} \end{aligned}$$

The motor is operating at slip $s = 0.04$.

Find:

The input current, input power, mechanical power, and shaft torque.

Assumptions:

Stray-load losses are negligible.

Analysis:

$$V_S = 115.5 \text{ V}$$

$$\omega_m = (1 - s)188.5 = 181 \text{ rad/sec}$$

$$\begin{aligned} Z_{in} &= 0.48 + j0.8 + \frac{j30(10.5 + j0.8)}{10.5 + j30.8} \\ &= 9.404 + j4.63 = 10.48 \angle 26.2^\circ \end{aligned}$$

$$\therefore I_S = 11.02 \angle -26.2^\circ \text{ A}$$

$$P_{in} \text{ (per phase)} = 115.5 \times 11.02 \times \cos(-26.2^\circ)$$

$$P_{in} \text{ (total)} = 3426 \text{ W}$$

$$P_t = P_{in} \text{ (total)} - 3R_S |I_S|^2 = 3251 \text{ W}$$

$$\therefore P_m = (1 - s)P_t = 3121 \text{ W}$$

$$T_{sh} = \frac{3121}{181} = 17.24 \text{ N} \cdot \text{m}$$

Problem 17.56**Solution:****Known quantities:**

a) A three-phase, 220 V, 60 Hz induction motor runs at 1140 rev/min.

b) A three-phase squirrel-cage induction motor is started by reducing the line voltage to $V_S/2$ in order to reduce the starting current.

Find:

a) The number of poles (for minimum slip), the slip, and the frequency of the rotor currents.

b) The factor the starting torque and the starting current reduced.

Assumptions:

None.

Analysis:

a)

For minimum slip, the synchronous speed, $\frac{3600}{p/2}$, should be as close as possible to 1140 rev/min , therefore,

$$n_s = 1200, \quad p = 6 \text{ poles}$$

$$s = \frac{1200 - 1140}{1200} = 0.05$$

$$f_{\text{rotor}} = 3 \text{ Hz}$$

b)

If the line voltage is reduced to half, the starting current is reduced by a factor of 2. The developed torque is proportional to $|I_s|^2$. Therefore, the starting torque is reduced by a factor of 4.

Problem 17.57**Solution:****Known quantities:**

A six-pole induction machine has a 50 kW rating and is 85 percent efficient. If the supply is 220 V at 60 Hz .

6 poles
60 Hz
50 kW
85% efficient
220 Volt
4% slip

Find:

The motor speed and torque at a slip $s = 0.04$.

Assumptions:

None.

Analysis:

$$a) \quad n_s = \frac{120f}{p} = \frac{120 \cdot 60}{6} = 1200 \text{ rev/min}$$

@ slip of 4%

$$n = n_s (1 - s) = 1200 \text{ rev/min} (1 - 0.04) = 1152 \text{ rev/min}$$

$$P_{\text{out}} = P_{\text{in}} \times \text{efficiency} = (50 \text{ kW})(0.85) = 42.5 \text{ kW}$$

$$b) \quad T_{\text{out}} = \frac{P_{\text{out}}}{\omega} = \frac{42500 \text{ W}}{1152 \text{ rev/min}} * \frac{1 \text{ rev}}{2\pi \text{ rad}} \frac{60 \text{ sec}}{\text{min}} = 352.3 \text{ N} \cdot \text{m}$$

Problem 17.58**Solution:**

Known quantities:

6 poles

60 Hz
 240 Volt rms
 10% slip
 Torque = 60 N-m

Find:

- The speed and the slip of the induction machine if a load torque of 50 N-m opposes the motor.
- The rms current when the induction machine is operating under the load conditions of part a.

Assumptions:

The speed torque curve is linear in the region of our interests.

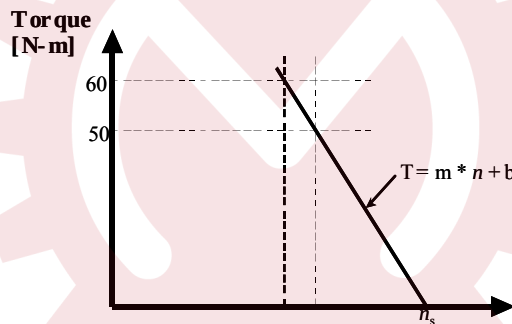
Analysis:

a)

$$n_s = \frac{120 f}{p} = \frac{120}{6} \frac{60}{1} = 1200 \text{ rev/min}$$

@ slip of 4%

$$n = n_s (1 - s) = 1200 \text{ rev/min} (1 - 0.04) = 1152 \text{ rev/min}$$



torque

$$T = m n + b$$

$$m = \frac{60 - 0 \text{ N-m}}{1080 - 1200 \text{ rev/min}} = 0.5 \frac{\text{N-m}}{\text{rev/min}}$$

$$b = T - m n = 60 \text{ N-m} - 0.5 \frac{\text{N-m}}{\text{rev/min}} (1080 \text{ rev/min}) = 600 \text{ N-m}$$

motor speed (@ 50 N-m)

$$n = \frac{T - b}{m} = \frac{50 \text{ N-m} - 600 \text{ N-m}}{0.5 \text{ N-m}} = 1000 \text{ rev/min}$$

slip (@ 50 N-m)

$$s = \frac{n_s - n}{n_s} = \frac{1200 - 1100}{1200} = 0.0833 = 8.33\%$$

b)

Output Power

$$P_{out} = T \omega = 50 \text{ N-m} \cdot 1100 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 5760 \text{ W}$$

Input Power

$$P_{in} = \frac{P_{out}}{\text{efficiency}} = I_{rms} V_{rms}$$

Current

$$I_{rms} = \frac{P_{out}}{\text{efficiency} \cdot V_{rms}} = \frac{5760 \text{ W}}{0.92 \cdot 240 \text{ volts}} = 26.1 \text{ amps}$$

Problem 17.59**Solution:****Known quantities:**A three-phase, 5 hp, 220 V, 60 Hz induction motor. $V = 8V, I = 18 A, P = 610W$.**Find:**

- The equivalent stator resistance per phase, R_S .
- The equivalent rotor resistance per phase, R_R .
- The equivalent blocked-rotor reactance per phase, X_R .

Assumptions:

None.

Analysis:

a)

$$R_S = \frac{1}{2} \frac{P_{BR}}{3I_{BR}^2} = 0.314 \Omega$$

b)

$$R_R = 0.314 \Omega$$

c)

$$Z_S = \frac{V_{BR}/\sqrt{3}}{I_{BR}} = \frac{48/\sqrt{3}}{18} = 1.54 \Omega$$

$$X_R = \sqrt{Z_S^2 - R^2} = \sqrt{(1.54)^2 - (0.628)^2} = 1.4 \Omega$$

Problem 17.60**Solution:****Known quantities:**

The starting torque equation is:

$$T = \frac{m}{\omega_e} \cdot V_s^2 \cdot \frac{R_R}{(R_R + R_S)^2 + (X_R + X_S)^2}$$

Find:

- The starting torque when it is started at 220 V.
- The starting torque when it is started at 110 V.

Assumptions:

None.

Analysis:

a)

$$T = \frac{1}{\omega_s} \frac{q_1 V_1^2 (R_R/s)}{R^2 + X^2}; \quad s = 1$$

$$\therefore T = \frac{1}{377} \frac{3(127)^2 (0.314)}{(0.628)^2 + (1.4)^2} = 17.1 \text{ N} \cdot \text{m}$$

b)

$$T = \frac{1}{377} \frac{3(63.5)^2 (0.314)}{(0.628)^2 + (1.4)^2} = 4.28 \text{ N} \cdot \text{m}$$

Problem 17.61**Solution:****Known quantities:**

A four-pole, three-phase induction motor drives a turbine load with torque-speed characteristic given by

$$T_L = 20 + 0.006\omega^2$$

At a certain operating point, the machine has 4% slip and 87% efficiency.

Find:

Torque at the motor-turbine shaft
 Total power delivered to the turbine
 Total power consumed by the motor

Assumptions:

Motor run by 60-Hz power supply

Analysis:

Synchronous speed of four-pole induction motor at 60-Hz:

$$n_s = \frac{120 f}{P} = \frac{60 \text{ s/min} \times 60 \text{ r/s}}{4/2} = 1800 \text{ r/min}$$

$$\omega_s = 1800 \text{ rev/min} \times \frac{2\pi \text{ rad/rev}}{60 \text{ s/min}} = 188.5 \text{ rad/s}$$

Rotor mechanical speed at 4% slip:

$$\omega_m = (1 - s)\omega_s = (1 - 0.04)(188.5 \text{ rad/s}) = 181.0 \text{ rad/s}$$

Load torque at the shaft:

$$T_L = 20 + 0.006(181.0 \text{ rad/s})^2 = 216 \text{ N} \cdot \text{m}$$

Total power delivered to the turbine:

$$P = T_L \omega_m = (216 \text{ N} \cdot \text{m})(181.0 \text{ rad/s}) = 39.1 \text{ kW}$$

Total power consumed by the motor:

$$P_m = \frac{P}{\eta} = \frac{39.1 \text{ kW}}{0.87} = 44.9 \text{ kW}$$

Problem 17.62**Solution:****Known quantities:**

A four-pole, three-phase induction motor rotates at 1700 r/min when the load is 100 N-m.

The motor is 88% efficient.

Find:

- Slip
- For a constant-power, 10-kW load, the operating speed of the machine
- Total power consumed by the motor
- Sketch the motor and load torque-speed curves on the same graph. Show numerical values.

Assumptions:

Motor run by 60-Hz power supply

Analysis:

- Synchronous speed of four-pole induction motor at 60-Hz:

$$n_s = \frac{120 f}{P} = \frac{60 \text{ s/min} \times 60 \text{ r/s}}{4/2} = 1800 \text{ r/min}$$

Slip:

$$s = \frac{n_s - n}{n_s} = \frac{1800 \text{ r/min} - 1700 \text{ r/min}}{1800 \text{ r/min}} = 0.056 = 5.6\%$$

- Operating speed of machine for a constant-power load of 10-kW

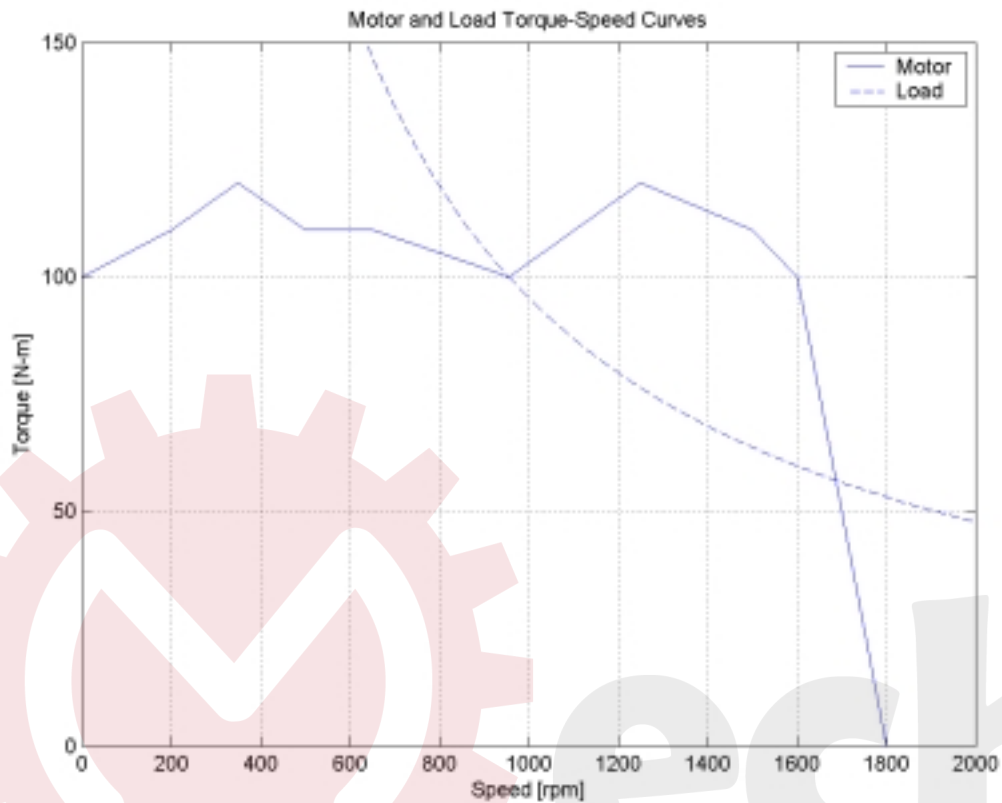
$$\omega = \frac{P}{T_L} = \frac{10000 \text{ W}}{100 \text{ N-m}} = 100 \text{ rad/sec}$$

$$n = \omega \frac{60 \text{ s/min}}{2\pi \text{ rad/rev}} = 955 \text{ r/min}$$

- Total power consumed by the motor

$$P_m = \frac{P}{\eta} = \frac{10 \text{ kW}}{0.88} = 11.4 \text{ kW}$$

- Sketch of motor and load torque-speed curves on the same graph, with the operating point at the first intersection:



Problem 17.63

Solution:

Known quantities:

A six-pole, three-phase motor.

Find:

The speed of the rotating field when the motor is connected to:

- a 60 Hz line.
- a 50 Hz line.

Assumptions:

None.

Analysis:

a)

$$\text{For } 60 \text{ Hz, } \omega_m = \frac{4\pi f}{P} = 125.7 \text{ rad/sec, } n_m = 1200 \text{ rev/min}$$

b)

$$\text{For } 50 \text{ Hz, } \omega_m = 104.72 \text{ rad/sec, } n_m = 1000 \text{ rev/min}$$

Problem 17.64**Solution:****Known quantities:**

A six-pole, three-phase, 440V, 60 Hz induction motor. The model impedances are:

$$R_S = 0.8 \Omega \quad X_S = 0.7 \Omega$$

$$R_R = 0.3 \Omega \quad X_R = 0.7 \Omega$$

$$X_m = 35 \Omega$$

Find:

The input current and power factor of the motor for a speed of 1200 rev/min.

Assumptions:

None.

Analysis:

$$V_S = \frac{440}{\sqrt{3}} = 254 \angle 0^\circ \text{ V}$$

For $n_m = n_s = 1200 \text{ rev/min}$, $s = 0$ (no load).

$$\begin{aligned} Z_{in} &= R_S + j(X_S + X_m) = 0.8 + j35.7 \\ &= 35.71 \angle 88.7^\circ \Omega \end{aligned}$$

$$I_S = 7.11 \angle -88.7^\circ \text{ A}$$

The power factor is:

$$\cos 88.7^\circ = 0.0224 \text{ lagging}$$

$$P_{in} = 3|I_S||V_S|\cos\theta = 121.4 \text{ W}$$

Problem 17.65**Solution:****Known quantities:**

A eight-pole, three-phase, 220V, 60 Hz induction motor. The model impedances are:

$$R_S = 0.78 \Omega \quad X_S = 0.56 \Omega$$

$$R_R = 0.28 \Omega \quad X_R = 0.84 \Omega$$

$$X_m = 32 \Omega$$

Find:

The input current and power factor of the motor for $s = 0.02$.

Assumptions:

None.

Analysis:

For 8 poles,

$$n_s = \frac{3600}{4} = 900 \text{ rev/min}$$

$$\omega_s = 94.25 \text{ rad/sec}$$

$$\omega_m = (1 - s)\omega_s = 92.4 \text{ rad/sec}$$

By using the equivalent circuit, we have:

$$Z_{in} = 0.78 + j0.56 + \frac{j32(\frac{0.28}{0.02} + j0.84)}{14 + j32.84}$$

$$= 12.03 + j6.17 = 13.52 \angle 27.15^\circ \Omega$$

$$V_s = 127 \angle 0^\circ \text{ V}$$

$$I_s = 9.39 \angle -27.15^\circ \text{ A}$$

$$pf = \cos(-27.15^\circ) = 0.8898 \text{ lagging}$$

Problem 17.66

Solution:

Known quantities:

The nameplate is as given in Example 17.2.

Find:

The rated torque, rated volt amperes, and maximum continuous output power for this motor.

Assumptions:

None.

Analysis:

The speed is:

$$n_m = 3565 \text{ rev/min}$$

$$\omega_m = \frac{2\pi \times 3565}{60} = 373.3 \text{ rad/sec}$$

The rated *volt · amperes* is:

$$\sqrt{3} \times (230 \text{ V}) \times (106 \text{ A}) = 42.23 \text{ kVA}$$

$$\text{or } \sqrt{3} \times (460 \text{ V}) \times (53 \text{ A}) = 42.23 \text{ kVA}$$

The maximum continuous output power is:

$$P_o = 40 \times 746 = 29840 \text{ W}$$

The rated output torque is:

$$T = \frac{P_o}{\omega_m} = 79.93 \text{ N} \cdot \text{m}$$

Problem 17.67**Solution:****Known quantities:**

At rated voltage and frequency, the 3-phase induction machine has a starting torque of 140 percent and a maximum torque of 210 percent of full-load torque.

Find:

- a) The slip at full load.
- b) The slip at maximum torque.
- c) The rotor current at starting as a percent of a full-load rotor current.

Assumptions:

Neglect stator resistance and rotational losses. Assume constant rotor resistance.

Analysis:

a)

$$T_R = \frac{KV^2(R_2/s_R)}{(R_2/s_R)^2 + X^2}$$

$$T_{ST} = 1.4T_R \quad s_{ST} = 1.0$$

$$T_{MT} = 2.1T_R \quad s_{MT} = \frac{R_2}{X}$$

The above leads to 3 equations in 3 unknowns:

$$(1) \quad 4.2XR_2 = 1.4R_2^2 + 1.4X^2$$

$$(2) \quad \frac{R_2^2}{s_R} + s_R X^2 = 1.4R_2^2 + 1.4X^2$$

$$(3) \quad 4.2 \frac{XR_2}{s_R} = \left(\frac{R_2}{s_R}\right)^2 + X^2$$

Solving the equations, we have:

$$\frac{R_2}{X} = 0.382$$

$$s_R = 0.097$$

b)

$$s_{MT} = \frac{R_2}{X} = 0.382$$

c)

$$I_R = \frac{KV}{4.06}, \quad I_{ST} = \frac{KV}{1.07}$$

$$\frac{I_{ST}}{I_R} \times 100 = 379\%$$

Chapter 18 Instructor Notes

The content of Chapter 18 is somewhat unusual for a textbook of this nature. The intent of this chapter is to provide a reasonably quantitative overview of the operation of small electric machines (mostly motors). In many practical industrial applications, ranging from servos for robots, to drug delivery systems, to actuation devices for control systems, to manufacturing equipment, to fluid power systems, small motors find widespread application.

The first section discusses the brushless DC motor, including the basics of the electronic circuits that make its operation possible. The second section introduces the stepper motor and its drive. In section 18.3, the switched reluctance machine is introduced; this is a new section in the third edition, motivated by the increasing interest in this family of machines for industrial applications. Next, single phase AC motors are discussed in Section 18.4, starting with the universal motor, and continuing with a classification of single phase induction motors, which includes split-phase, capacitor-type and shaded-pole motors. The presentation detail is sufficient to permit quantitative analysis of these motors using circuit models. The final section, 18.5, on motor selection and application, introduces some of the basic ideas behind motor selection and performance calculations. This section, which describes calculations related to reflected load inertias in the presence of mechanical gear reductions, and calculations of acceleration, torque, efficiency, and thermal loading, could be covered at any point in Chapter 17 or 18, even if the material in Sections 18.1-18.4 is not covered.

The examples given in the chapter are supplemented by over thirty homework problems, some of which are extensions of the examples presented in the text. Problems 18.5 and 18.9 require some background in digital logic circuits (Chapter 13); problems 18.7, 18.36 and 18.37 require some background in system dynamics; all remaining problems can be solved strictly on the basis of the material covered in the chapter.

Learning Objectives

1. Understand the basic principles of operation of brushless DC motors, and the trade-offs between these and brush-type DC motors. [Section 1.](#)
2. Understand the operation and basic configurations of step motors, and understand step sequences for the different classes of step motors. [Section 2.](#)
3. Understand the operating principles of switched-reluctance machines. [Section 3.](#)
4. Classify and analyze single-phase AC motors, including the universal motor and various types of single-phase induction motors, using simple circuit models. [Section 4.](#)
5. Outline the selection process for an electric machine given an application; perform calculations related to load inertia, acceleration, efficiency, and thermal characteristics. [Section 5.](#)

Section 18.1: Brushless DC Motors

Problem 18.1

Solution:

Known quantities:

A permanent magnet six-pole two-phase synchronous machine. $\lambda_m = 0.1 \text{ V} \cdot \text{s}$.

Find:

The amplitude of the open-circuit phase voltage measured when the rotor is turned at $n = 60 \text{ rev/sec}$.

Assumptions:

None.

Analysis:

We know that

$$\lambda_m = 0.1 \text{ V} \cdot \text{s}$$

$$p = 6$$

$$m = 2$$

$$\omega_m = 60 \text{ rev/s} = 2\pi \times 60 \text{ rad/s}$$

Let flux linkage $\lambda = \lambda_m \sin \omega t$, where

$$\omega = \frac{p}{2} \omega_m = 3 \times 60 = 180 \text{ rev/s}$$

$$= 2\pi \times 180 \text{ rad/s} = 360\pi \text{ rad/s}$$

Then, the generated voltage:

$$V = e = \left| \frac{d\lambda}{dt} \right| = \frac{d(\lambda_m \sin \omega t)}{dt} = \omega \lambda_m \cos \omega t$$

$$= V_m \cos \omega t$$

$$V_m = \omega \lambda_m = 360\pi \times 0.1 = 113.1 \text{ V}$$

Problem 18.2

Solution:

Known quantities:

A four-pole two-phase brushless dc motor. $n = 3600 \text{ rev/min}$. The open-circuit voltage across one of the phases is 50 V .

Find:

a) λ .

b) The no-load rotor speed ω in rad/s when the mechanical source is removed and

$$V_a = \sqrt{2} 25 \cos \theta, V_b = \sqrt{2} 25 \sin \theta, \text{ where } \theta = \omega_e t.$$

Assumptions:

None.

Analysis:

We know that

$$p = 4$$

$$m = 2$$

$$\omega_m = 3600 \text{ rev/min}$$

$$V_n = 50 \text{ V}$$

a)

$$\text{let } e = V = \sqrt{2}V \sin \theta = \sqrt{2}V \sin \omega t$$

$$\omega = \frac{p}{2} \omega_m \cdot 2\pi \text{ rad/min} = 2 \times 60 \times 2\pi \text{ rad/s}$$

$$\lambda = \int_0^t e dt = \int_0^t \sqrt{2}V \sin \omega t dt = \frac{\sqrt{2}}{\omega} V \cos \omega t$$

$$= \frac{\sqrt{2}}{240\pi} 50 \cos 240\pi t = 0.094 \cos \omega t$$

b)

Symmetric voltages in symmetric windings produce a rotational field in voltage with frequency f_s .

Let $f_s = 3600 \text{ rev/min}$. Then, the rotor speed is

$$\omega_m = \frac{\omega}{p/2} = \frac{3600}{2} = 1800 \text{ rev/min} = \frac{2\pi}{60} \times 1800 \text{ rad/s} = 60\pi \text{ rad/s}$$

Problem 18.3

Solution:

Known quantities:

$T_1 = T_3 = 1 \text{ s}$ (see Figure 18.7); maximum motor rpm, $n_{\max} = 1,800 \text{ rev/min}$.

Find:

T_2 .

Assumptions:

The motor covers 0.5 m in 100 revolutions.

Analysis:

The maximum rotational velocity of the motor is: $v = n_{\max}/60 = 1,800/60 = 30 \text{ rev/s}$. Using the expression derived in Example 18.2, we know that the maximum motor rotational velocity is:

$$v = \frac{d}{\frac{1}{2}T_1 + T_2 + \frac{1}{2}T_3}$$

and we can calculate T_2 as follows:

$$T_2 = \frac{d}{v} - \frac{1}{2}T_1 - \frac{1}{2}T_3 = \frac{100 \text{ rev}}{30 \text{ rev/s}} - 1 \text{ s} - 1 \text{ s} = 2.33 \text{ s}$$

Thus, the total trapezoidal profile has been shortened by 2/3 s.

Problem 18.4

Solution:

Known quantities:

Desired load motion profile (Figure P18.4). The motor covers 0.5 m (100 revolutions) in 3 s.

Find:

Maximum motor speed, acceleration and deceleration times.

Assumptions:

Assume a triangular speed profile..

Analysis:

To simplify the analysis, choose a symmetrical speed profile; thus, the motor will accelerate for 1.5 s and decelerate for 1.5 s, or $T_1 = T_2 = 1.5$ s.

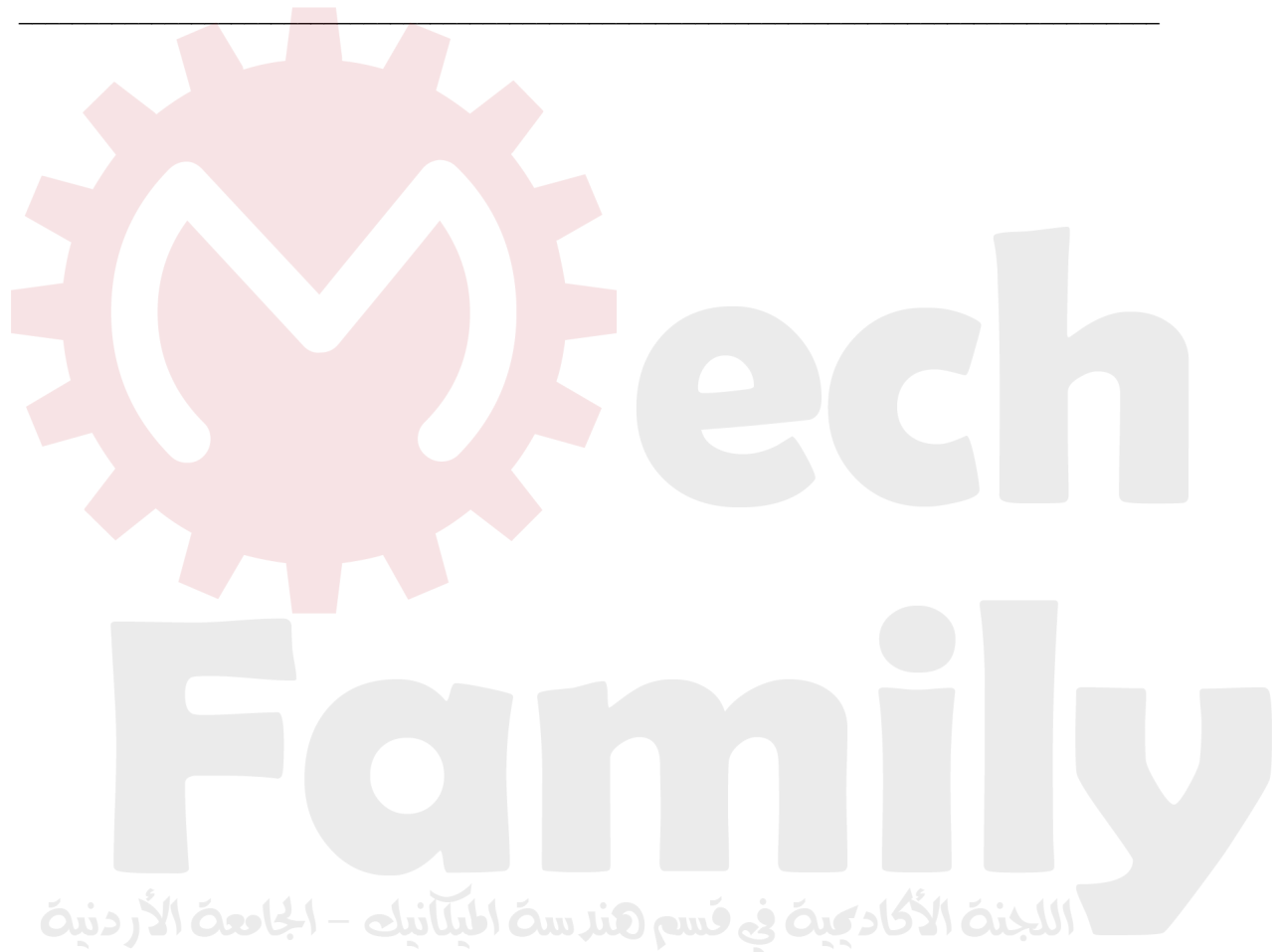
Using the results of Example 18.2, if we set the flat portion of the speed profile (T_2 in Example 18.2) to zero, we can write an expression for the total motor travel.

$$d = v \frac{1}{2} T_1 + \frac{1}{2} T_2$$

and calculate the maximum motor speed to be:

$$v = \frac{d}{\frac{1}{2} T_1 + \frac{1}{2} T_2} = \frac{100 \text{ rev}}{\frac{1}{2} 1.5 + \frac{1}{2} 1.5 \text{ s}} = 66.67 \text{ rev/s}$$

which corresponds to $n_{\max} = 66.67 \times 60 = 4,000 \text{ rev/min}$.



Section 18.2: Stepping Motors

Problem 18.5

Solution:

Known quantities:

Variable-reluctance step motor of Example 18.4 (Figure 18.11)

Find:

Design a logic circuit to achieve the step sequence given in Table 18.4 (see below)

Assumptions:

Hint: Use a counter and logic gates

Analysis:

Table 18.4: Current Excitation Sequence for VR Step Motor

S_A	S_B	S_C	S_D	Rotor Position
1	0	0	0	0°
1	1	0	0	45°
0	1	0	0	90°
0	1	1	0	135°
0	0	1	0	180°
0	0	1	1	225°
0	0	0	1	270°
1	0	0	1	315°
1	0	0	0	360°

There are eight possible configurations for the motor. Hence, a 3-bit binary counter was chosen that pulses every 45°. The table below lists the corresponding logic for the binary counter.

Binary Counter			Step Motor Response				
b_0	b_1	b_2	S_A	S_B	S_C	S_D	Rotor Position
0	0	0	1	0	0	0	0°
1	0	0	1	1	0	0	45°
0	1	0	0	1	0	0	90°
1	1	0	0	1	1	0	135°
0	0	1	0	0	1	0	180°
1	0	1	0	0	1	1	225°
0	1	1	0	0	0	1	270°
1	1	1	1	0	0	1	315°
0	0	0	1	0	0	0	360°

Next, convert the truth table to a logical expression for each of the outputs:

$$S_A = \bar{b}_0 \cdot \bar{b}_1 \cdot \bar{b}_2 + b_0 \cdot \bar{b}_1 \cdot \bar{b}_2 + b_0 \cdot b_1 \cdot b_2 = \bar{b}_1 \cdot \bar{b}_2 (\bar{b}_0 + b_0) + b_0 \cdot b_1 \cdot b_2$$

$$S_A = \bar{b}_1 \cdot \bar{b}_2 + b_0 \cdot b_1 \cdot b_2$$

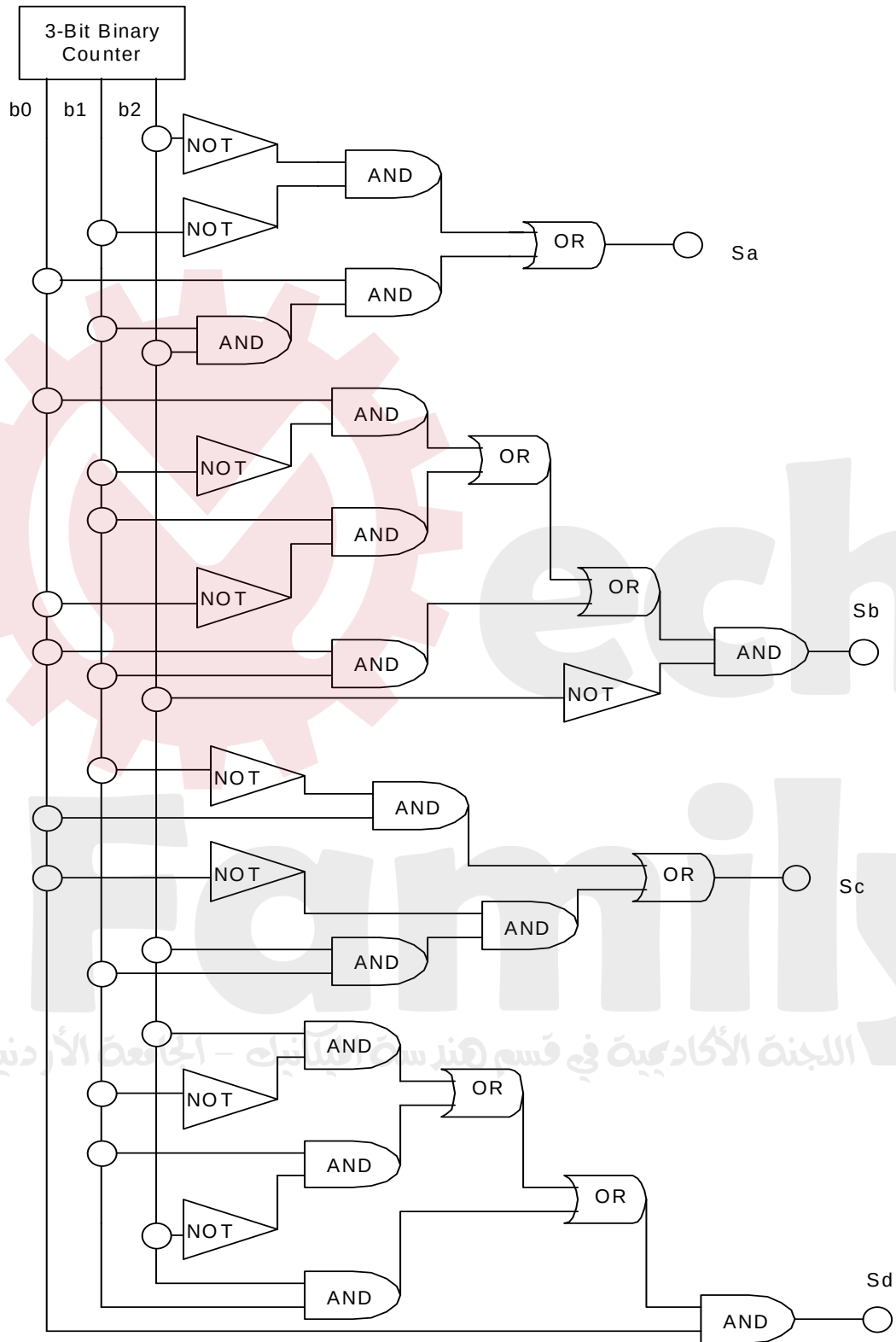
$$S_B = b_0 \cdot \bar{b}_1 \cdot \bar{b}_2 + \bar{b}_0 \cdot b_1 \cdot \bar{b}_2 + b_0 \cdot b_1 \cdot \bar{b}_2 = \bar{b}_2 (b_0 \cdot \bar{b}_1 + \bar{b}_0 \cdot b_1 + b_0 \cdot b_1)$$

$$S_C = b_0 \cdot b_1 \cdot \bar{b}_2 + \bar{b}_0 \cdot \bar{b}_1 \cdot b_2 + b_0 \cdot \bar{b}_1 \cdot b_2 = \bar{b}_1 \cdot b_2 (\bar{b}_0 + b_0) + b_0 \cdot b_1 \cdot \bar{b}_2$$

$$S_C = \bar{b}_1 \cdot b_2 + b_0 \cdot b_1 \cdot \bar{b}_2$$

$$S_D = b_0 \cdot \bar{b}_1 \cdot b_2 + \bar{b}_0 \cdot b_1 \cdot b_2 + b_0 \cdot b_1 \cdot b_2 = b_2 (b_0 \cdot \bar{b}_1 + \bar{b}_0 \cdot b_1 + b_0 \cdot b_1)$$

From these four expressions, a logic circuit diagram can be made for each individual output, and tied together to achieve the desired step response.



Problem 18.6**Solution:****Known quantities:**

PM stepper motor with 6 poles, bipolar supply,

Find:

Smallest achievable step size.

Assumptions:

None

Analysis:

With reference to Example 18.3, we see that the half-step sequence for the 2-phase 4-pole motor leads to 45-degree steps. The addition of two poles will reduce the step size by 50%, resulting in 30-degree steps.

Problem 18.7**Solution:****Known quantities:**

$$J_m, J_L, D, T_f.$$

Find:

The dynamic equation for a stepping motor coupled to a load.

Assumptions:

None.

Analysis:

The equation will have the following form:

$$V = Ri + L \frac{di}{dt} + K_E \omega$$

$$T = K_T i = (J_m + J_L) \frac{d\omega}{dt} + D\omega + T_F + T_L$$

Problem 18.8**Solution:****Known quantities:**

A hybrid stepper motor capable of 18° steps.

Find:

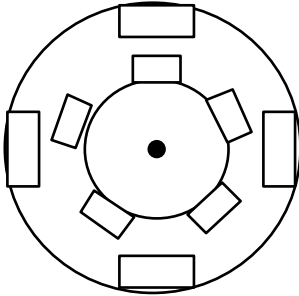
Sketch the rotor-stator configuration of the motor.

Assumptions:

None.

Analysis:

The rotor and stator configuration is shown below:



The motor has 5 rotor teeth and 4 stator teeth (two phases).

Problem 18.9

Solution:

Known quantities:

Shown in Check Your understanding on page 928.

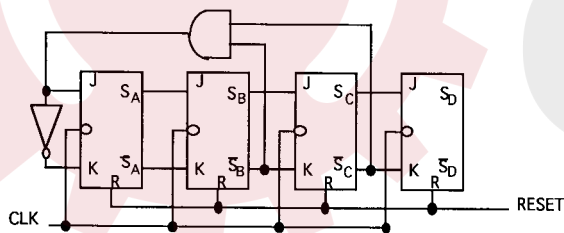
Find:

A binary counter and logic gates to implement the stepping motor binary sequence.

Assumptions:

None.

Analysis:



Problem 18.10

Solution:

Known quantities:

A two-phase permanent magnet stepper motor has 50 rotor teeth. When driven at $\omega = 100 \text{ rad/s}$, the measured open circuit phase peak-to-peak voltage is 25 V .

Find:

- Calculate λ
- Express the developed torque when $i_a = 1\text{ A}$ and $i_b = 0$.

Assumptions:

The winding resistance is 0.1Ω .

Analysis:

- a)

We know that

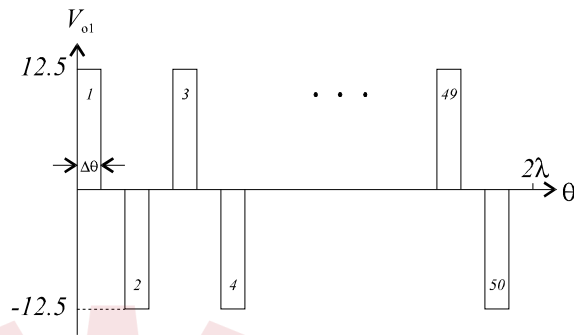
$p = 50$

$$m = 2$$

$$\omega_m = 100 \text{ rad/s}$$

Strikes per rev.:

$$N = p \cdot m = 100$$



$$\Delta\theta = \frac{2\pi}{N} = \frac{2\pi}{100} = \frac{2\pi}{50}$$

$$\begin{aligned}\lambda &= \int_0^{\pi} e dt = \int_0^{\pi} \frac{V_0}{\omega_m} d\theta \\ &= \frac{V_0}{\omega_m} \cdot \frac{\pi}{50} = \frac{12.5}{100} \cdot \frac{\pi}{50} = 0.00785 \text{ V} \cdot \text{s}\end{aligned}$$

b)

Let the winding resistance be represented by R_w , then

$$V = k_a \omega_m + R_w I$$

$$T = k_T I$$

where $k_a = k_T = \frac{V - R_w I}{\omega_m}$

Then $T = k_a I = \frac{V - R_w I}{\omega_m} N \cdot m$

Problem 18.11

Solution:

Known quantities:

The schematic diagram of a four-phase, two-pole PM stepper motor is shown in Figure P18.11. The phase coils are excited in sequence by means of a logic circuit.

Find:

The no-load voltage of the generator and terminal voltage at half load.

Assumptions:

- The logic schedule for full-stepping of this motor.
- The displacement angle of the full step.

Analysis:

a)

For full step clockwise rotation is:

$$\text{Phase 1} \rightarrow \text{phase 4} \rightarrow \text{phase 3} \rightarrow \text{phase 2} \rightarrow \text{phase 1}$$

b)

The displacement angle of the full step sequence is 90° .

Problem 18.12

Solution:

Known quantities:

A PM stepper motor provides a full-step angle of 15° .

Find:

The number of stator and rotor poles.

Assumptions:

None.

Analysis:

The motor will require 24 stator teeth and 2 rotor teeth.

Problem 18.13

Solution:

Known quantities:

A bridge driver scheme for a two-phase stepping motor is as shown in Figure P18.13.

Find:

The excitation sequences of the bridge operation.

Assumptions:

None.

Analysis:

CK	R	1	2	3	4	5	6	7	8
S1	1	1	0	0	0	0	0	1	1
S2	0	0	0	1	1	1	0	0	0
S3	0	0	0	1	1	1	0	0	0
S4	1	1	0	0	0	0	0	1	1
S5	0	1	1	1	0	0	0	0	0
S6	0	0	0	0	0	1	1	1	0
S7	0	0	0	0	0	1	1	1	0
S8	0	1	1	1	0	0	0	0	0

Where "1" means switch is closed.

Problem 18.14**Solution:****Known quantities:**

A PM stepper motor provides a full-step angle of 15° . It is used to directly drive a 0.100 in. lead screw.

Find:

- The resolution of the stepper motor in steps/revolution.
- The distance the screw travels in inches for each step 15° of motor.
- The number of full 15° steps required to move the lead screw and the stepper motor shaft through 17.5 revolution.
- The shaft speed (in rev/min) when the stepping frequency is 220 pps .

Assumptions:

None.

Analysis:

a)

$$\text{steps/revolution} = \frac{360}{15} = 24$$

b)

$$d = 0.1" \times \frac{15^\circ}{360^\circ} = 0.0042"$$

c)

$$\text{steps} = 17.5 \text{ rev} \times 24 \frac{\text{steps}}{\text{rev}} = 420 \text{ steps}$$

d)

$$\# \text{ pulses} = \# \text{ steps}$$

$$\Rightarrow n_{SH} = 220 \frac{\text{steps}}{\text{s}} \times \frac{1 \text{ rev}}{24 \text{ steps}} \times \frac{60 \text{ s}}{\text{min}} = 550 \text{ rpm}$$

Section 18.4: Single-Phase AC Motors**Problem 18.15****Solution:****Known quantities:**

The motor data are the following:

- a) $\frac{3}{4}hp, 900 rev/min$
- b) $1\frac{1}{2}hp, 3600 rev/min$
- c) $\frac{3}{4}hp, 1800 rev/min$
- d) $1\frac{1}{2}hp, 6000 rev/min$

Find:

Whether the following motors are integral- or fractional-horse power motors.

Assumptions:

None.

Analysis:

a)

The power is $0.75 \frac{1800}{900} = 1.5hp$.

Integral.

b)

The power is $1.5 \frac{1800}{3600} = 0.75hp$.

Fractional.

c)

The power is $0.75 \frac{1800}{1800} = 0.75hp$.

Fractional.

d)

The power is $1.5 \frac{1800}{6000} = 0.45hp$.

Fractional.

Problem 18.16**Solution:****Known quantities:**

$$F_1 = F_{1(peak)} \cos \theta, \quad F_{1(peak)} = F_{1(max)} \cos \theta, .$$

Find:

The expression for F_1 and verify that for a single-phase winding, both forward and backward components are present.

Assumptions:

None.

Analysis:The stator mmf F_1 can be expressed as:

$$\begin{aligned}
 F_1 &= F_{1\max} \cos(\omega t) \cos \theta \\
 &= \frac{1}{2} F_{1\max} \cos \theta \cos(\omega t) - \frac{1}{2} F_{1\max} \cos \theta \cos(\omega t) \\
 &\quad + \frac{1}{2} F_{1\max} \cos \theta \cos(\omega t) + \frac{1}{2} F_{1\max} \cos \theta \cos(\omega t) \\
 &= F_{CW} + F_{CCW}
 \end{aligned}$$

where:

 F_{CW} is a clockwise-rotating mmf. F_{CCW} is a counter clockwise-rotating magnetic mmf.**Problem 18.17****Solution:****Known quantities:**

A 200V, 60 Hz, 10 hp single-phase induction motor operates at an efficiency of 0.86 a power factor of 0.9.

Find:

The capacitor that should be placed in parallel with the motor so that the feeder supplying the motor will operate at unity power factor.

Assumptions:

None.

Analysis:

We have:

$$P_{out} = 746 \times 10 = 7460 \text{ W}$$

$$P_{in} = \frac{P_{out}}{eff.} = 8674.4 \text{ W}$$

From $P_{in} = V_S I_S \cos \theta_S = 8674.4 \text{ W}$, we have:

$$|I_S| = 48.2 \text{ A}, \quad \theta = 25.84^\circ \text{ lagging}$$

Therefore,

$$I_S = 48.2 \angle -25.84^\circ = 43.38 - j21.01 \text{ A}$$

To get unit power factor, $I_C = j21.01$

$$\begin{aligned}
 \text{From } j21.01 &= \frac{-200}{-j \frac{1}{\omega C}}, \quad \omega = 377 \text{ rad/s, we have:} \\
 C &= 278.6 \mu\text{F}
 \end{aligned}$$

Problem 18.18**Solution:****Known quantities:**

A 230V, 50 Hz single-phase two-pole induction motor operates at 3 percent slip.

Find:

The slip in the opposite direction of rotation. Find the speed of the motor in the normal direction of rotation.

Assumptions:

None.

Analysis:

For a 2-pole machine, the synchronous speed is 3000 rev/min for an excitation frequency of 50 Hz. From $s = 0.03$, the slip in the opposite direction of rotation is 0.97, the motor speed is 2910 rev/min = 304.7 rad/s.

Problem 18.19**Solution:****Known quantities:**

A stepper motor with a 15° step angle operates in one-phase excitation mode.

Find:

The amount of time to take for the motor to rotate through 28 rev when the pulse rate is 180 pps.

Assumptions:

None.

Analysis:

$$\omega = 180 \times 15 = 2700^\circ / s = 7.5 \text{ rev/s}$$

$$t = \frac{28 \text{ rev}}{7.5 \text{ rev/s}} = 3.73 \text{ s}$$

Problem 18.20**Solution:****Known quantities:**

A $\frac{1}{4}$ hp, 110V, 60 Hz, four-pole capacitor-start motor has the following parameters:

$$\begin{array}{ll} R_S = 2.02 \Omega & X_S = 2.8 \Omega \\ R_R = 4.12 \Omega & X_R = 2.12 \Omega \\ X_m = 66.8 \Omega & s = 0.05 \end{array}$$

Find:

- The stator current.
- The mechanical power.
- The rotor speed.

Assumptions:

None.

Analysis:

a)

$$0.5Z_b = 0.991 + j1.057 = 0.5(R_b + jX_b)$$

$$0.5Z_f = 15.93 + j20.07 = 0.5(R_f + jX_f)$$

$$Z_{in} = 18.94 + j23.93 = 30.52 \angle 51.64^\circ$$

$$\therefore I_1 = \frac{V_1}{Z_{in}} = 3.60 \angle -51.64^\circ \text{ A}$$

b)

$$P_f = I_1^2 (0.5R_f) = 207.0 \text{ W}$$

$$P_b = I_1^2 (0.5R_b) = 12.84 \text{ W}$$

$$P_{mech} = (1-s)(P_f - P_b) = 184.45 \text{ W}$$

c)

For a 4-pole machine,

$$\omega_s = 188.5 \text{ rad/s}, \quad \omega_m = 179.1 \text{ rad/s}$$

Thus, the rotor speed is:

$$179.1 \text{ rad/s} = 1710 \text{ rev/min}$$

Problem 18.21**Solution:****Known quantities:**

A $\frac{1}{4}$ hp, 110V, 60 Hz, four-pole, single-phase induction motor has the following parameters:

$$R_S = 1.86 \Omega \quad X_S = 2.56 \Omega$$

$$R_R = 3.56 \Omega \quad X_R = 2.56 \Omega$$

$$X_m = 53.5 \Omega \quad s = 0.05$$

Find:

The mechanical power output.

Assumptions:

None.

Analysis:

$$0.5Z_b = 0.830 + j1.248$$

$$0.5Z_f = 12.41 + j16.98$$

$$Z_{in} = 15.1 + j20.79 = 25.7 \angle 54.0^\circ$$

$$I_1 = \frac{V_1}{Z_{in}} = 4.28 \angle -54.0^\circ \text{ A}$$

$$P_f = I_1^2 (0.5R_f) = 227.5 \text{ W}$$

$$P_b = I_1^2 (0.5R_b) = 15.2 \text{ W}$$

$$P_{mech} = (1-s)(P_f - P_b) = 201.68 \text{ W}$$

Problem 18.22**Solution:****Known quantities:**

A 115V, 60 Hz, four-pole, one-phase induction motor has the following parameters:

$$R_S = 0.5 \Omega \quad X_S = 0.4 \Omega$$

$$R_R = 0.25 \Omega \quad X_R = 0.4 \Omega$$

$$X_m = 35 \Omega$$

Find:

The input current and developed torque when the motor speed is 1,730 rev/min .

Assumptions:

None.

Analysis:

The synchronous speed is 1,800 rev/min for 1,730 rev/min .

$(1-s) = 0.961$, therefore the slip $s = 0.039$. We have:

$$0.5Z_b = 0.064 + j0.2$$

$$0.5Z_f = 3.034 + j0.747$$

$$Z_{in} = 3.842 \angle 20.52^\circ$$

$$I_S = 29.9 \angle -20.52^\circ \text{ A}$$

$$P_f = 2717.5 \text{ W}$$

$$P_b = 57.22 \text{ W}$$

$$P_{mech} = 2660.3 \text{ W}$$

$$\omega_m = 181.2 \text{ rad/s}$$

The torque developed is:

$$T_{dev} = \frac{2551.6}{181.2} = 14.68 \text{ N} \cdot \text{m}$$

Problem 18.23**Solution:****Known quantities:**

No-load test of a single-phase induction motor at rated voltage and rated frequency.

Find:

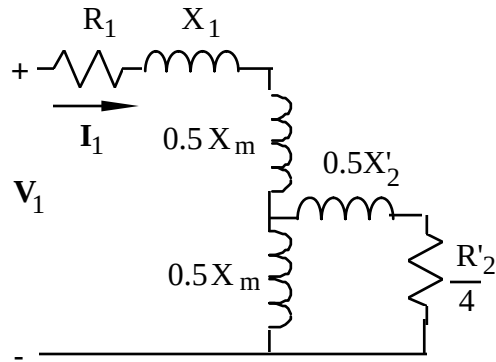
The equivalent circuit of a single-phase induction motor for the no-load test.

Assumptions:

None.

Analysis:

At no load, $s \approx 0$. The circuit model is shown below:

**Problem 18.24****Solution:****Known quantities:**

The locked-rotor test of the single-phase induction motor.

Find:

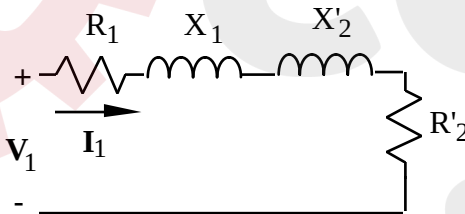
The equivalent circuit.

Assumptions:

Neglect the magnetizing current.

Analysis:

For locked rotor, $\omega_m = 0$, $s = 1$. The circuit is shown below:

**Problem 18.25****Solution:****Known quantities:**

A $\frac{1}{8}$ hp, 115V two-pole universal motor has the effective resistances of the armature and series field as 4Ω and 6Ω . The output torque is $0.17N \cdot m$ when the motor is drawing rated current of $1.5A$ at a power factor of 0.88 at rated speed.

Find:

- The full-load efficiency.
- The rated speed.
- The full-load copper losses.
- The combined windage, friction, and iron losses.
- The motor speed when the rms current is $0.5A$.

Assumptions:

Phase differences and saturation.

Analysis:

From $P_f = 0.88$, we have $\theta = -28.4^\circ$ and $I_S = 1.5 \angle -28.4^\circ \text{ A}$.

The rated speed is: $\frac{1/8 \times 746}{0.17} = 548.53 \text{ rad/s}$

- a) $P_{in} = 151.8 \text{ W}$ and the efficiency is: $\text{eff} = \frac{93.25}{151.8} = 61.43\%$
 b) The speed is 5238.1 rev/min .
 c) The copper loss is: $1.5^2 \times 10 = 22.5 \text{ W}$
 d) Other loss will be: $151.8 - 22.5 - 93.25 = 36.05 \text{ W}$
 e) $P_{in} = (115)(0.5 + (0.5^2 \times 10) + P_{out})(0.88)$
 $= 12.05 \text{ W}$

Assume T is proportional to I^2 .

$$T_{new} = \left(\frac{0.5}{1.5}\right)^2 (0.17) = 0.019 \text{ N} \cdot \text{m}$$

$$\omega = \frac{12.05}{0.019} = 637.9 \text{ rad/s}$$

$$n = 6091.9 \text{ rpm}$$

Problem 18.26**Solution:****Known quantities:**

A 240 V , 60 Hz , two-pole universal motor operates at $12,000 \text{ rev/min}$ on full load and draws a current of 6.5 A at a power factor of 0.94 lagging. The series field-winding impedance is $4.55 + j3.2 \Omega$ and the armature circuit impedance is $6.15 + j9.4 \Omega$

Find:

- a) The back emf of the motor.
 b) The mechanical power developed by the motor.
 c) The power output if the rotational loss is 65 W .
 d) The efficiency of the motor.

Assumptions:

None.

Analysis:

From $P_f = 0.94$ (lagging), we have

$$\theta = -19.95^\circ \text{ and } I_S = 6.5 \angle -19.95^\circ \text{ A}.$$

- a)
 $E_b = 146.68 - j53.25$
 $= 156.05 \angle -19.95^\circ \text{ V}$
 b)
 $P_{dev} = I_S E_b \cos 0^\circ = 1014.3 \text{ W}$
 c)
 $P_{out} = 1014.3 - 65 = 949.3 \text{ W}$
 d)

$$P_{in} = 1466.4W$$

$$eff = 64.7\%$$

Problem 18.27

Solution:

Known quantities:

A single-phase motor is drawing 20 A from a 400V, 50 Hz supply. The power factor is 0.8 lagging.

Find:

The value of capacitor connected across the circuit to raise the power factor to unity.

Assumptions:

None.

Analysis:

$V_S = 400\angle 0^\circ$, $\theta = -36.9^\circ$, therefore,

$$I_S = 20\angle 36.9^\circ = 16 - j12$$

For a unity power factor, $I_C = j12$.

We have $12 = 400\omega C$, $\omega = 314.16 \text{ rad/s}$.

Therefore, $C = 95.5 \mu F$

Problem 18.28

Solution:

Known quantities:

A way of operating a three-phase induction motor as a single-phase source is shown in Figure P18.28.

Find:

Whether the motor will work. Explain why or why not.

Assumptions:

None.

Analysis:

It will work. The b and c windings will produce a magnetic field similar to a single phase machine, that is, two components rotating in opposite directions and the a winding would act as a starting winding. The phase shift provided by the capacitor is needed to provide a nonzero starting torque.

Problem 18.29

Solution:

Known quantities:

A $\frac{1}{4} \text{ hp}$ capacitor-start motor with its output adjusted to rated value.

$$E = 115 \text{ volts}; \quad I = 3.8 \text{ A}; \quad P = 310 \text{ W}; \quad \text{rev/min} = 1725.$$

Find:

- Efficiency.
- Power factor.
- Torque in pound-inches.

Assumptions:

None.

Analysis:

a)

$$P_{out} = \frac{1}{4} \times 746 = 186.5 \text{ W}$$

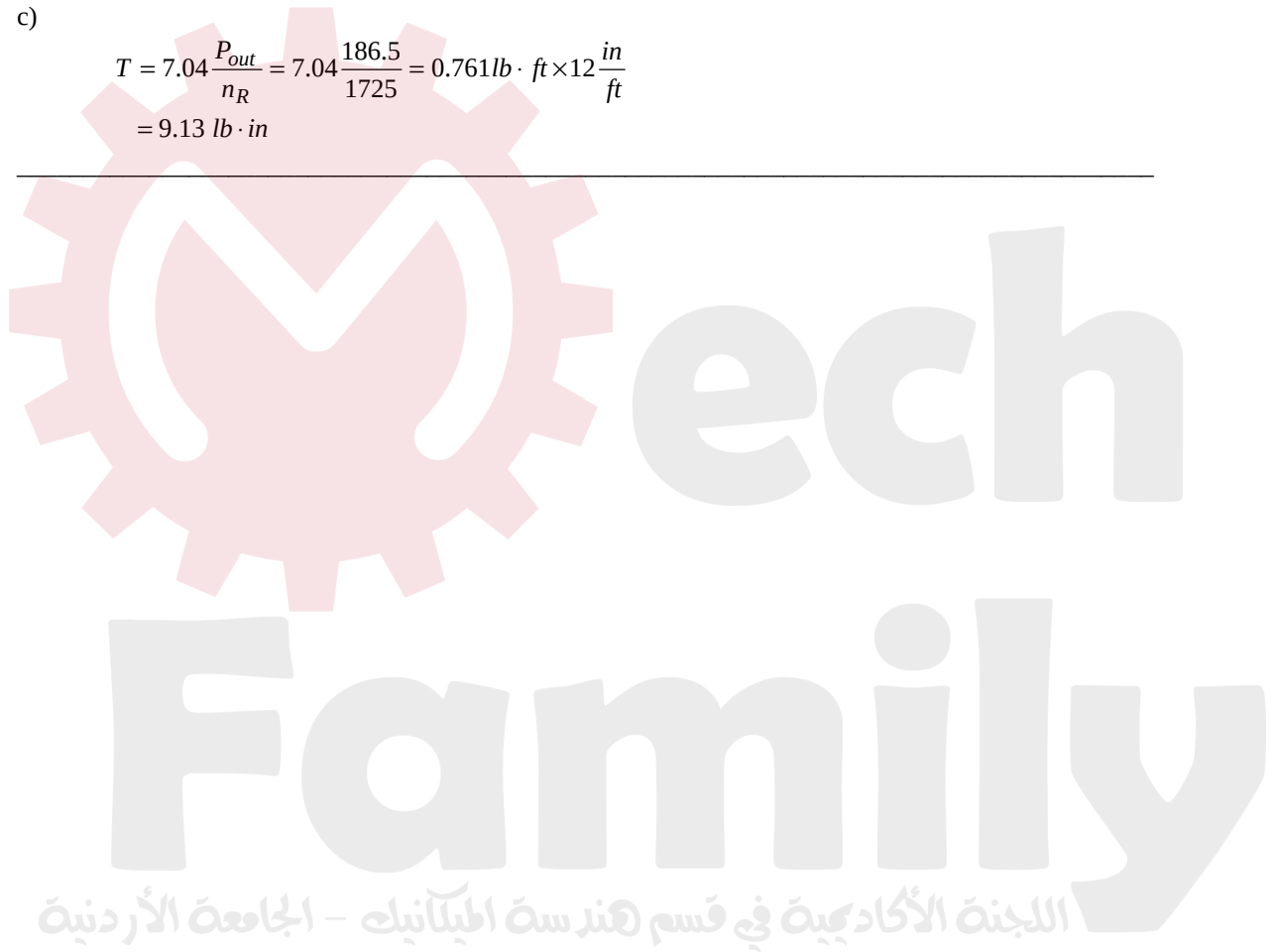
$$eff = \frac{P_{out}}{P_{in}} = 0.602 = 60.2\%$$

b)

$$pf = \frac{P}{VI} = \frac{310}{(115)(3.8)} = 0.709 \text{ lagging}$$

c)

$$T = 7.04 \frac{P_{out}}{n_R} = 7.04 \frac{186.5}{1725} = 0.761 \text{ lb} \cdot \text{ft} \times 12 \frac{\text{in}}{\text{ft}} \\ = 9.13 \text{ lb} \cdot \text{in}$$



Section 18.5: Motor Selection and Applications

Problem 18.30

Solution:

Known quantities:

The tasks are the following:

- a) Vacuum cleaner
- b) Refrigerator.
- c) Air conditioner compressor.
- d) Air conditioner fan.
- e) Variable-speed sewing machine.
- f) Clock.
- g) Electric drill.
- h) Tape drive.
- i) X-Y plotter.

Find:

The type of motor that can perform the above tasks.

Assumptions:

None.

Analysis:

The universal motor speed is easily controlled and thus it would be used for variable speed, that is, (e) and (g). The vacuum cleaner motors are often universal motors. This motor could also be used for the fan motors.

A single-phase induction motor is used for (b) and (c).

The clock should use a single-phase synchronous motor.

The tape drive would be a single-phase synchronous motor also.

An X-Y plotter uses a stepper motor.

Problem 18.31

Solution:

Known quantities:

A 5hp, 1150rev/min shunt motor. The speed control by means of a tapped field resistor is shown Figure P18.31.

Find:

The speed of the motor and the torque available at the maximum permissible load with the tap at position 3.

Assumptions:

None.

Analysis:

$$n = 230\% \times 1150 = 2645 \text{ rpm}$$

$$T = 40\% \times T_{\text{rated}}$$

$$= (0.4) \frac{(33,000)(5)}{(2\pi)(1150)} = 9.13 \text{ lb} \cdot \text{ft}$$

Problem 18.32**Solution:****Known quantities:**

The applications are the following:

- a) Inexpensive analog electric clock.
- b) Bathroom ventilator fan.
- c) Escalator which must start under all load conditions.
- d) Kitchen blender.
- e) Table model circular saw operating at about 3,500 *rev/min*.
- f) Hand-held circular saw operating at 15,000 *rev/min*.
- g) Water pump.

Find:

The single-phase motor that can apply to the above cases.

Assumptions:

None.

Analysis:

- a) reluctance
- b) shaded-pole
- c) capacitor-start
- d) universal
- e) permanent split capacitor
- f) universal
- g) permanent split capacitor

Problem 18.33**Solution:****Known quantities:**

The power required to drive a fan varies as a cube of the speed. The motor driving a shaft-mounted fan is loaded to 100 percent of its horsepower rating on the top speed connection.

Find:

The horsepower output in percent of rating at the following speed reduction.

- a) 20 percent.
- b) 30 percent.
- c) 50 percent.

Assumptions:

None.

Analysis:

a)
$$HP = (0.8)^3 = 0.512 = 51.2\% \text{ of rated}$$

b)
$$HP = (0.7)^3 = 0.343 = 34.3\% \text{ of rated}$$

c)
$$HP = (0.5)^3 = 0.125 = 12.5\% \text{ of rated}$$

Problem 18.34**Solution:****Known quantities:**

An industrial plant has a load of 800 kW at a power factor of 0.8 lagging. A synchronous motor will be used to change them to 200 kW and 0.92 .

Find:

The KVA input rating and the power factor at which the motor operates.

Assumptions:

The motor has an efficiency of 91 percent.

Analysis:

$$P_m = \frac{200\text{ K}}{0.91} = 219.8\text{ KW}$$

$$P_{new} = 800\text{ K} + 219.8\text{ K} = 1019.8\text{ KW}$$

$$Q_{new} = P_{new} \tan(\cos^{-1} 0.92) = 434.4\text{ KVAR}$$

$$Q_{old} = 800\text{ K} \tan(\cos^{-1} 0.8) = 600\text{ KVAR}$$

$$Q_m = Q_{old} - Q_{new} = 165.6\text{ KVAR}$$

$$S_m = \sqrt{P_m^2 + Q_m^2} = 274.8\text{ KVA}$$

$$pf_m = \cos \tan^{-1} \left(\frac{Q_m}{P_m} \right) = 0.8 \text{ leading}$$

Problem 18.35**Solution:****Known quantities:**

An electric machine is controlled so that its torque-speed characteristics exhibit a constant-torque region and a constant-power region as shown in Figure P18.35.

Average efficiency of the electric drive is 87%.

Machine torque is constant at 150 N-m from 0 to 2500 rpm .

The constant power region is from 2500 to 6000 rpm .

Machine drives a constant torque load requiring 75 N-m .

Find:

- Operating speed of the machine
- Electric power needed to operate the machine

Assumptions:

None

Analysis:

- Since the load requires less than the starting torque, the motor should be operating in the constant power regime. The constant power is determined from the starting torque and the base speed:

$$P = \omega_{base} T_{start} = (2500\text{ r/min}) \frac{(2\pi\text{ rad/rev})}{(60\text{ s/min})} (150\text{ N-m}) = 39.3\text{ kW}$$

The operating speed is determined from the load torque:

$$\omega = \frac{P}{T_L} = \frac{(39300W)}{(75N - m)} = 523.6 \text{ rad / s}$$

$$n = (523.6 \text{ rad / s}) \frac{(60 \text{ s / min})}{(2\pi \text{ rad / rev})} = 5000 \text{ r / min}$$

b) The electric power needed to operate the machine:

$$P_e = \frac{P}{\eta} = \frac{39.3 \text{ kW}}{0.87} = 45.1 \text{ kW}$$

Problem 18.36

Solution:

Known quantities:

The PM synchronous (brushless DC) motor in Figure P18.36

Electrical subsystem parameters: R_s , L_s , k . (motor constant), $V_s(t)$, $I_s(t)$.

Mechanical subsystem parameters: inertia and damping coefficient, J , b .

Thermal subsystem parameters: thermal resistance, specific heat, mass, R_t , c , m .

Find:

Write the differential equations describing the electrothermomechanical dynamics of the systems

Assumptions:

All heat is generated in the stator by the stator current (i.e. the heat generated in the rotor is negligible)

The rotor and stator are at the same temperature and specific heat c

The stator is highly thermally conductive

The dominant heat-transfer term is convection

Overall thermal resistance R_t from stator to air

The motor generates torque according to the equation $T_m = kI_s$

The back emf is equal to $E_b = k\omega$

Analysis:

Mechanical subsystem

$$J \frac{d\omega(t)}{dt} = T_m - b\omega(t) = kI_s(t) - b\omega(t)$$

Electrical subsystem

$$V_s(t) = L_s \frac{dI_s(t)}{dt} + R_s I_s(t) + E_b = L_s \frac{dI_s(t)}{dt} + R_s I_s(t) + k\omega(t)$$

Thermal subsystem

$$R_s I_s^2(t) - \frac{s(t)}{R_t} \text{ air} = mc \frac{ds(t)}{dt}$$

Problem 18.37

Solution:

Known quantities:

The wound separately excited motor in Figure P18.37

Electrical subsystem parameters: R_f, L_f, R_a, L_a , (motor field and armature electrical parameters), k_f, k_a, k_T (motor field and armature constants), $V_s(t), V_f(t), I_a(t), I_f(t)$.

Mechanical subsystem parameters: load inertia, damping coefficient, load torque, J, b, T_L .

Thermal subsystem parameters: $C_{t-rotor}, h_{t-rotor}, A_{rotor}$ (rotor thermal capacitance, film coefficient of heat transfer from rotor surface to air and from air to stator inner surface, rotor and inner stator surface area (assumed equal). $C_{t-stator}, h_{t-stator}, A_{stator}$ (stator thermal capacitance, film coefficient of heat transfer from stator outer surface to air, stator outer surface area).

Find:

Write the differential equations describing the electrothermomechanical dynamics of the systems

Assumptions:

Heat is generated in the stator and the rotor by the respective currents

The stator and rotor are highly thermally conductive

The dominant heat-transfer term is convection through the air gap and to ambient

Heat storage in the air gap is negligible, and the air gap is infinitely thin

The motor generates torque according to the equation $T_m = k_T I_a$

The back emf is equal to $E_b = k_a \omega$

The stator and rotor each act as a lumped thermal mass

Analysis:

Mechanical subsystem

$$J \frac{d\omega(t)}{dt} = T_m - b\omega(t) - T_L = k_T I_a(t) - b\omega(t) - T_L$$

$$k_T = k_a \phi, \phi = k_f I_f(t), k_T = k_a k_f I_f(t)$$

$$J \frac{d\omega(t)}{dt} = k_a k_f I_f(t) I_a(t) - b\omega(t) - T_L$$

Electrical subsystem

Armature Circuit:

$$V_a(t) = L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + E_b = L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_a \omega(t)$$

Field Circuit:

$$V_f(t) = L_f \frac{dI_f(t)}{dt} + R_f I_f(t)$$

Thermal subsystem

Rotor:

$$R_a I_a^2(t) - h_{rotor} A_{rotor} (\theta_R(t) - \theta_S(t)) = mc_{rotor} \frac{d\theta_R(t)}{dt}$$

Stator:

$$R_f I_f^2(t) - h_{stator} A_{stator} (\theta_S(t) - \theta_{air}) + h_{rotor} A_{rotor} (\theta_R(t) - \theta_S(t)) = mc_{stator} \frac{d\theta_S(t)}{dt}$$

Problem 19.1

Solution:

Known quantities:

The square wave signal: $x(t) = \begin{cases} 1 & \text{for } |t| \leq \eta\tau \\ -1 & \text{for } |t| < \tau \end{cases}$, $\tau = \frac{1}{300} \text{ sec}$, $\eta = 30\%$

Find:

- The Fourier series coefficients for the square wave signal.
- Frequency spectrum of the signal for the numerical values.

Analysis:

- Fourier series coefficients:

From Fourier series theory we know that the Fourier series representation of any signal is given as:

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$a_0 = \frac{1}{T} \int_T x(t) dt$$

$$a_n = \frac{2}{T} \int_T x(t) \cos(n\omega_0 t) dt$$

$$b_n = \frac{2}{T} \int_T x(t) \sin(n\omega_0 t) dt$$

where $x(t)$ is the signal, T is the period of the signal, and ω_0 is the natural frequency.

The square wave signal is an odd function, so we need to compute only the b_n Fourier coefficients. The interval of integration $0 \leq t \leq \tau$ would be convenient.

$$a_0 = \frac{1}{\tau} \left[\int_0^{\frac{\tau}{2}} dt - \int_{\frac{\tau}{2}}^{\tau} dt \right] = 0$$

$$b_n = \frac{2}{\tau} \left[\int_0^{\frac{\tau}{2}} \sin n \frac{2\pi}{\tau} t dt - \int_{\frac{\tau}{2}}^{\tau} \sin n \frac{2\pi}{\tau} t dt \right]$$

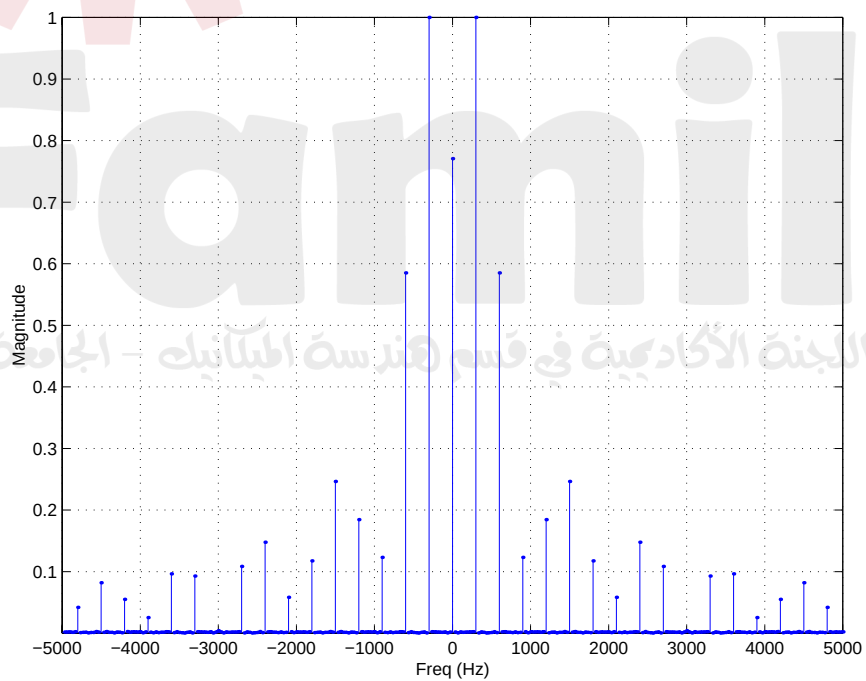
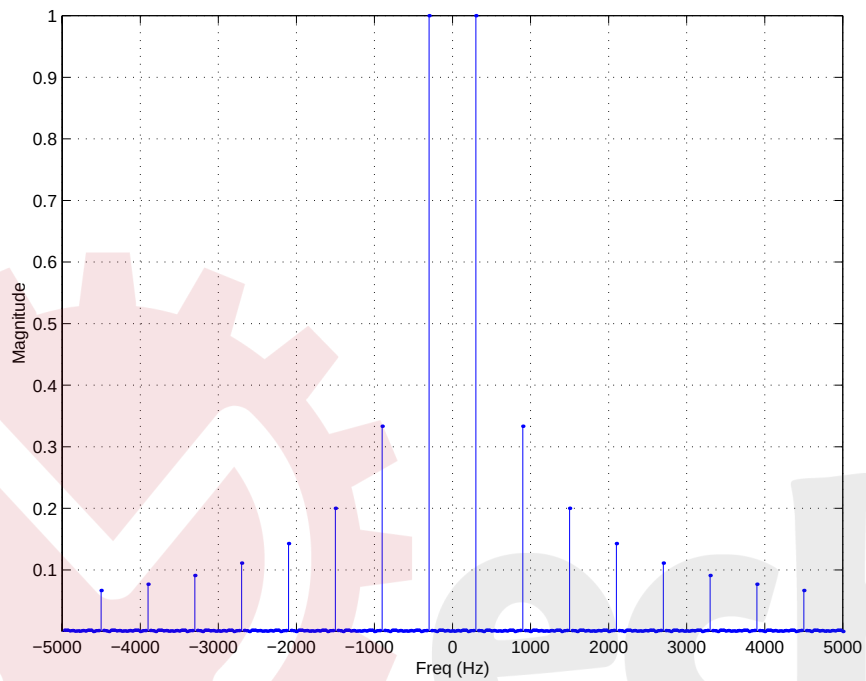
$$b_n = \frac{2}{\tau} \left[\left. \frac{-\cos \frac{n2\pi}{\tau} t}{\frac{n2\pi}{\tau}} \right|_0^{\tau/2} + \left. \frac{\cos \frac{n2\pi}{\tau} t}{\frac{n2\pi}{\tau}} \right|_{\tau/2}^{\tau} \right]$$

$$b_n = \frac{1}{n\pi} - \frac{2}{n\pi} \cos n\pi + \frac{1}{n\pi} \cos n2\pi = \frac{2}{n\pi} [1 - 2 \cos n\pi] \quad n \neq 0$$

$$b_n = \begin{cases} \frac{4}{n\pi} & n : \text{odd} \\ 0 & n : \text{even} \end{cases}$$

$$\therefore x(t) = \frac{4}{\pi} \left[\sin \frac{2\pi t}{\tau} + \frac{1}{3} \sin \frac{3\pi t}{\tau} + \dots \right]$$

- b. Frequency spectrum of the signal for $\eta = 50\%$ is shown in Figure 19.xx and for $\eta = 30\%$ is shown in Figure 19.xx



Problem 19.2

Solution:

Known quantities:

Functional form of a full-wave rectified sinusoidal signal of time period T sec, $x(t) = |\sin(\omega_0 t)|$, and natural frequency $\omega_0 = 200\pi \frac{\text{rad}}{\text{s}}$.

Find:

- The Fourier series coefficients.
- Frequency spectrum of the signal.

Analysis:

The rectified sine wave signal is an even function. Hence, we need to compute only the a_n coefficients of the Fourier series.

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} |\sin(\omega_0 t)| d(\omega_0 t) = \frac{2}{\pi} \int_0^{\pi} \sin(\omega_0 t) d(\omega_0 t)$$

$$a_0 = \frac{4}{\pi}$$

$$a_1 = \frac{2}{\pi} \int_0^{2\pi} |\sin(\omega_0 t)| \cos(\omega_0 t) d(\omega_0 t) = \frac{4}{\pi} \int_0^{\pi} \sin(\omega_0 t) \cos(\omega_0 t) d(\omega_0 t) = \frac{4}{\pi} \int_0^{\pi} \frac{1}{2} \sin(2\omega_0 t) d(\omega_0 t)$$

$$a_1 = 0$$

$$a_2 = \frac{2}{\pi} \int_0^{2\pi} |\sin(\omega_0 t)| \cos(2\omega_0 t) d(\omega_0 t) = \frac{4}{\pi} \int_0^{\pi} \sin(\omega_0 t) \cos(2\omega_0 t) d(\omega_0 t)$$

$$= \frac{4}{\pi} \int_0^{\pi} \frac{1}{2} [\sin(3\omega_0 t) - \sin(\omega_0 t)] d(\omega_0 t)$$

$$a_2 = \frac{4}{\pi} \frac{2}{3}$$

$$a_3 = 0$$

$$a_4 = -\frac{4}{\pi} \frac{2}{15}$$

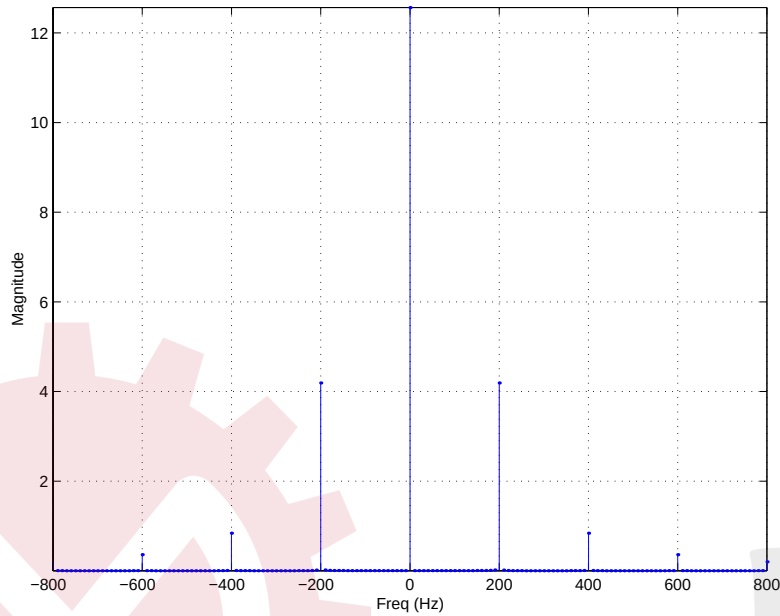
$$a_n = \begin{cases} \frac{4}{\pi} \frac{2}{(n-1)(n+1)} & n : \text{even} \\ 0 & n : \text{odd} \end{cases}$$

$$b_n = 0$$

$$x(t) = \frac{4}{\pi} \left[1 + \frac{2}{3} \cos(2\omega_0 t) - \frac{2}{15} \cos(4\omega_0 t) + \frac{2}{65} \cos(6\omega_0 t) \dots \right]$$

- The Frequency spectrum for the full wave rectified sinusoid of frequency $\omega_0 = 200\pi \frac{\text{rad}}{\text{s}}$ is shown in

Figure 19.xx.



Problem 19.3

Solution:

Known quantities:

Functional form of a full-wave rectified cosine wave of time period T sec, $x(t) = |\sin(\omega_o t)|$, and natural frequency $\omega_o = 150\pi \frac{\text{rad}}{\text{s}}$.

Find:

- The Fourier series coefficients.
- Frequency spectrum of the signal.

Analysis:

The rectified cosine wave signal is same as the rectified sine wave with a phase shift of $\frac{\pi}{2}$ rad. The Fourier series coefficients are to be found over a period of π rad. If we consider the period from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$ rad, the analysis is same as a full wave rectified sinusoid. Hence, the Fourier series coefficients are the same as computed in Problem 19.2.

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} |\cos(\omega_o t)| d(\omega_o t) = \frac{2}{\pi} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin(\omega_o t) d(\omega_o t)$$

$$a_0 = \frac{4}{\pi}$$

$$a_1 = \frac{2}{\pi} \int_0^{2\pi} |\cos(\omega_o t)| \cos(\omega_o t) d(\omega_o t) = \frac{4}{\pi} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin(\omega_o t) \cos(\omega_o t) d(\omega_o t) = \frac{4}{\pi} \int_0^{\pi} \frac{1}{2} \sin(\omega_o t) d(\omega_o t)$$

$$a_1 = 0$$

$$a_2 = \frac{2}{\pi} \int_0^{2\pi} |\cos(\omega_o t)| \cos(2\omega_o t) d(\omega_o t) = \frac{4}{\pi} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin(\omega_o t) \cos(2\omega_o t) d(\omega_o t)$$

$$= \frac{4}{\pi} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2} [\sin(3\omega_o t) - \sin(\omega_o t)] d(\omega_o t)$$

$$a_2 = \frac{4}{\pi} \frac{2}{3}$$

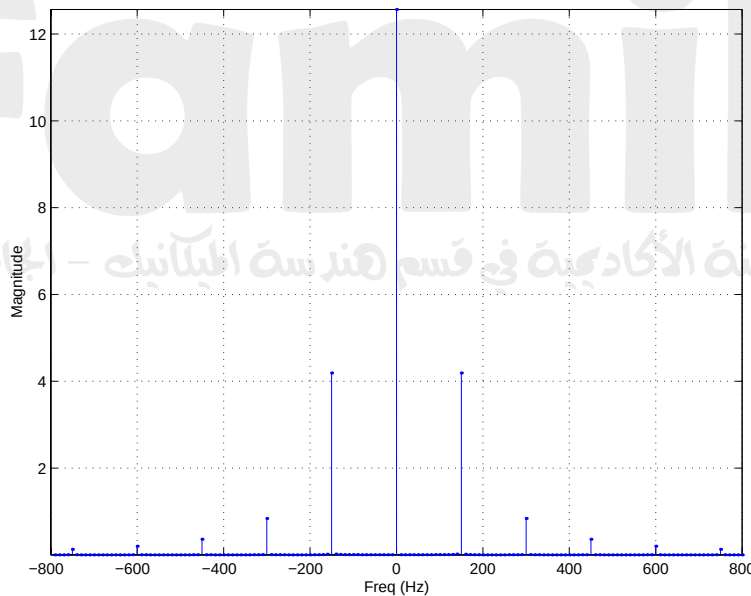
$$a_n = \begin{cases} \frac{4}{\pi} \frac{2}{(n-1)(n+1)} & n : \text{even} \\ 0 & n : \text{odd} \end{cases}$$

$$b_n = 0$$

$$x(t) = \frac{4}{\pi} \left[1 + \frac{2}{3} \cos(2\omega_o t) - \frac{2}{15} \cos(4\omega_o t) + \frac{2}{65} \cos(6\omega_o t) \cdots \right]$$

- b. The Frequency spectrum for the full wave rectified cosine wave of frequency $\omega_o = 150\pi \frac{\text{rad}}{\text{s}}$ is shown in

Figure 19.xx.



Problem 19.4

Solution:

Known quantities:

Functional form of a cosine burst as shown in Fig. 19.xx and mathematically defined as:

$$x(t) = \cos\left(\frac{\pi}{T}t\right)$$

Find:

Fourier series coefficients for the cosine burst.

Analysis:

The Fourier series coefficients can be calculated as follows:

$$a_0 = \frac{1}{T} \int_T x(t) dt = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos\left(\frac{\pi}{T}t\right) dt = \frac{1}{T} \left[\frac{T}{\pi} \sin\left(\frac{\pi}{T}t\right) \right]_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$a_0 = \frac{2}{\pi}$$

$$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos\left(\frac{\pi}{T}t\right) \cos\left(\frac{n2\pi}{T}t\right) dt$$

$$= \frac{2}{T} \frac{1}{2} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left[\cos\left(\frac{(2n+1)\pi}{T}t\right) + \cos\left(\frac{(2n-1)\pi}{T}t\right) \right] dt$$

$$= \frac{1}{T} \left[\frac{T}{(2n+1)} \sin\left(\frac{(2n+1)\pi}{T}t\right) + \frac{T}{(2n-1)} \sin\left(\frac{(2n-1)\pi}{T}t\right) \right]_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$a_n = \frac{1}{\pi} \left[\frac{1}{(2n+1)} \sin\left((2n+1)\frac{\pi}{2}\right) + \frac{1}{(2n-1)} \sin\left((2n-1)\frac{\pi}{2}\right) \right]$$

$$a_n = (-1)^{n+1} \frac{2}{(2n+1)(2n-1)\pi}$$

Since the cosine burst is an even signal the b_n coefficients are all zero. Hence $b_n = 0$.

The cosine burst signal can be written as:

$$x(t) = \frac{2}{\pi} + \frac{2}{3\pi} \cos\left(\frac{2\pi}{T}t\right) - \frac{2}{15\pi} \cos\left(\frac{4\pi}{T}t\right) + \dots$$

Problem 19.5

Solution:

Known quantities:

Functional form of a triangular pulse signal as shown in Fig. 19.xx and mathematically defined as:

$$x(t) = A \left[1 - \frac{|t|}{T} \right] (u(t+T) - u(t-T))$$

Find:

- Fourier transform of the function.
- Plot the frequency spectrum of the triangular pulse of period, $T = 0.01$ sec and amplitude, $A = 0.5$.

Analysis:

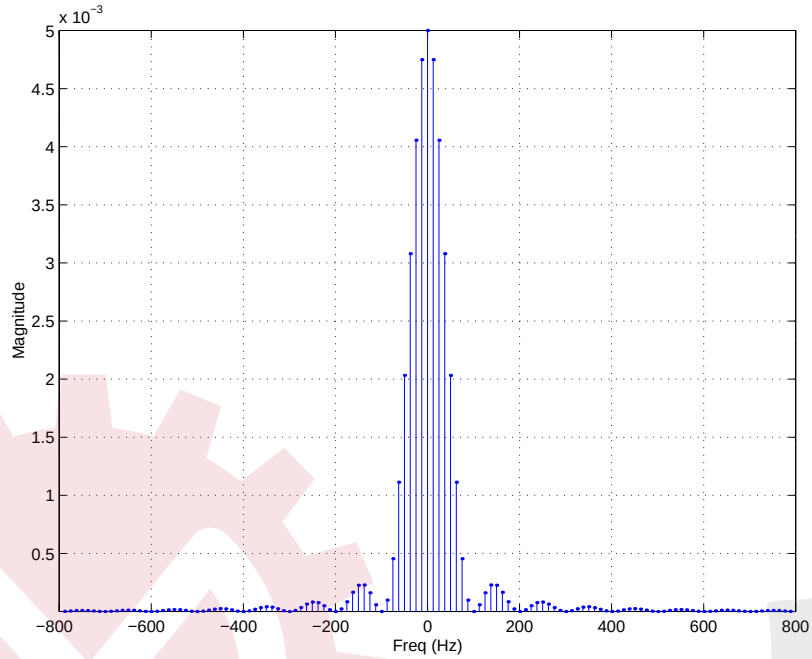
The mathematical equation for the triangular pulse can be split into a function defined over different periods as follows:

$$x(t) = \begin{cases} A \left[1 + \frac{t}{T} \right] & \text{for } -T \leq t < 0 \\ A \left[1 - \frac{t}{T} \right] & \text{for } 0 \leq t \leq T \\ 0 & \text{elsewhere} \end{cases}$$

The Fourier transform is defined as: $X(f) = \int_{-\infty}^{\infty} x(t) \exp(-j2\pi ft) dt$ and can be computed for the triangular pulse as follows:

$$\begin{aligned} X(f) &= \int_{-T}^T A \left[1 - \frac{|t|}{T} \right] \exp(-j2\pi ft) dt \\ &= \int_{-T}^0 A \left[1 + \frac{t}{T} \right] \exp(-j2\pi ft) dt + \int_0^T A \left[1 - \frac{t}{T} \right] \exp(-j2\pi ft) dt \\ &= A \left\{ \frac{j \exp(j\pi fT)}{2\pi^2 f^2 T} \left[\frac{\exp(-j\pi fT) - \exp(j\pi fT)}{2j} \right] + \frac{j \exp(-j\pi fT)}{2\pi^2 f^2 T} \left[\frac{\exp(j\pi fT) - \exp(-j\pi fT)}{2j} \right] \right\} \\ &= A \left\{ \frac{j \exp(j\pi fT)}{2\pi^2 f^2 T} [-\sin(\pi fT)] + \frac{j \exp(-j\pi fT)}{2\pi^2 f^2 T} [\sin(\pi fT)] \right\} \\ &= \frac{A}{\pi^2 f^2 T} \sin(\pi fT) \left[\frac{-\exp(-j\pi fT) + \exp(j\pi fT)}{2j} \right] \\ &= \frac{AT}{(\pi fT)^2} \sin(\pi fT) \sin(\pi fT) \\ X(f) &= AT \text{sinc}^2(fT) \quad \text{where } \text{sinc}(fT) = \frac{\sin(\pi fT)}{\pi fT} \end{aligned}$$

- The Frequency spectrum for the signal as computed from Matlab is shown in Figure 19.xx.



Problem 19.6

Solution:

Known quantities:

Functional form of a exponential pulse signal as shown in Fig. 19.xx and mathematically defined as:

$$x(t) = \begin{cases} (\exp)^{-at}, & \text{for } t > 0 \\ 0, & \text{for } t = 0 \\ -(\exp)^{at}, & \text{for } t < 0 \end{cases}$$

Find:

Fourier transform for the exponential pulse.

Analysis:

We can formulate a compact notation for the pulse signal by using “signum function” which equals +1 for positive time and -1 for negative time. This function is defined as:

$$\text{sgn}(t) = \begin{cases} 1, & \text{for } t > 0 \\ 0, & \text{for } t = 0 \\ -1, & \text{for } t < 0 \end{cases}$$

The signal $x(t)$ can be written as:

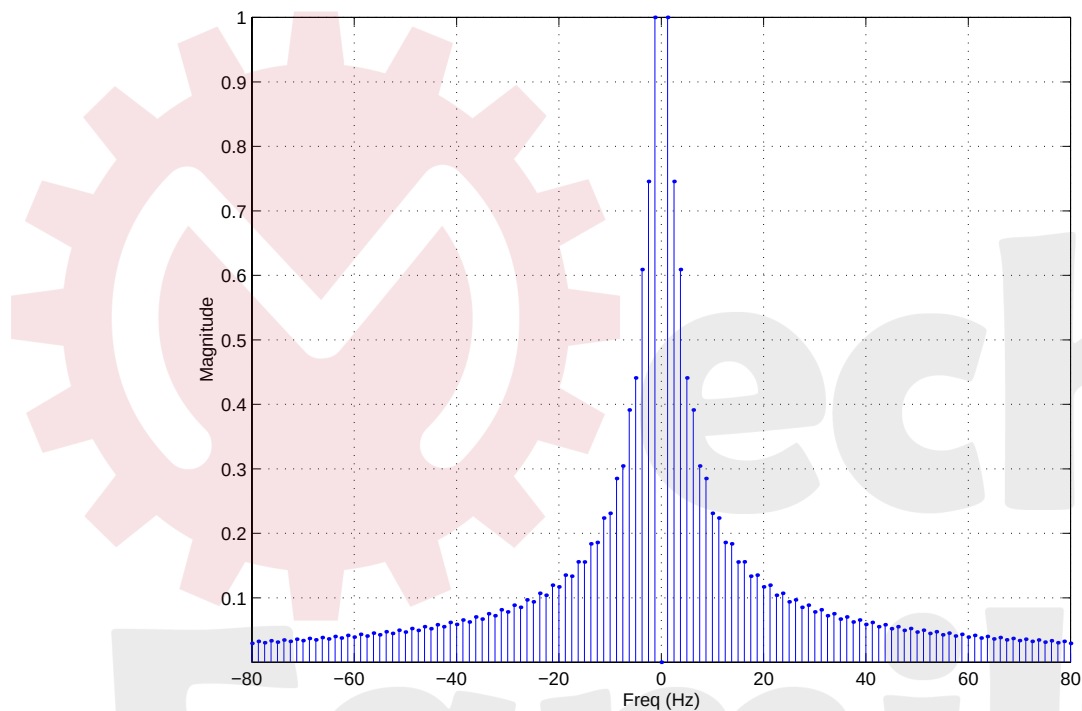
$$x(t) = \exp(-a |t|) \text{sgn}(t)$$

The Fourier transform is now calculated as follows:

$$X(f) = \int_{-\infty}^{\infty} \exp(-a |t|) \text{sgn}(t) \exp(-j2\pi ft) dt$$

$$\begin{aligned}
 &= \int_{-\infty}^0 -\exp((a - j2\pi f)t)dt + \int_0^{\infty} \exp(-(a + j2\pi f)t)dt \\
 &= -\frac{1}{a - j2\pi f} + \frac{1}{a + j2\pi f} \\
 X(f) &= \frac{-j4\pi f}{a^2 + 4\pi^2 f^2}
 \end{aligned}$$

b. The Frequency spectrum for the signal is shown in Figure 19.xx.



Problem 19.7

Solution:

Known quantities:

$$x(t) = \exp(-at) \cos(2\pi f_c t) u(t)$$

Functional form of a damped sinusoid signal as shown in Fig. 19.xx and mathematically defined as:

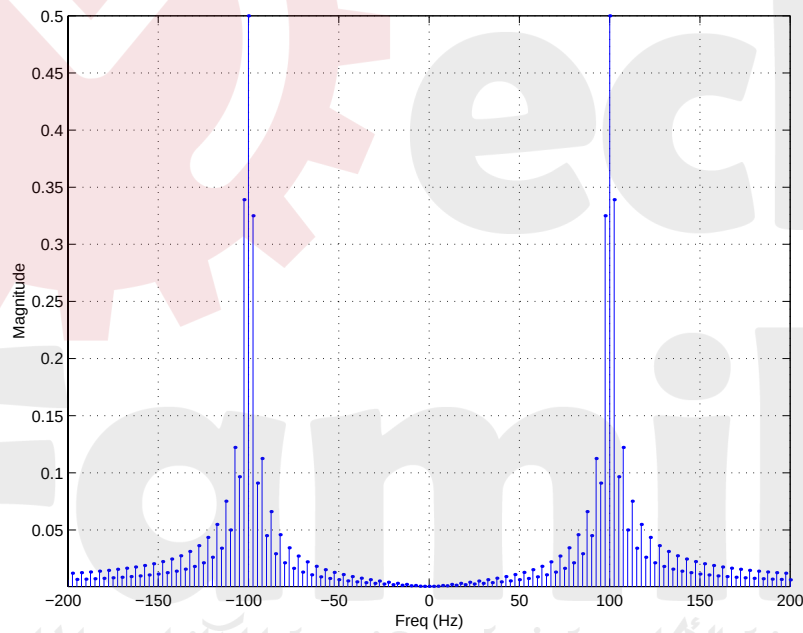
Find:

Fourier transform for the damped sinusoid.

Analysis:

$$\begin{aligned}
 X(f) &= \int_{-\infty}^{\infty} x(t) \exp(-j2\pi ft) dt \\
 &= \int_{-\infty}^{\infty} \exp(-at) \cos(2\pi f_c t) u(t) \exp(-j2\pi ft) dt \\
 &= \int_0^{\infty} \exp(-at) \frac{1}{2} [\exp(j2\pi f_c t) + \exp(-j2\pi f_c t)] \exp(-j2\pi ft) dt \\
 &= \frac{1}{2} \int_0^{\infty} \{ \exp(-[a + j2\pi(f - f_c)]t) + \exp(-[a + j2\pi(f + f_c)]t) \} dt \\
 X(f) &= \frac{1}{2} \left[\frac{1}{a + j2\pi(f - f_c)} + \frac{1}{a + j2\pi(f + f_c)} \right]
 \end{aligned}$$

b. The Frequency spectrum from Matlab is shown in Figure 19.xx.



Problem 19.8

Solution:

Known quantities:

Functional form of an ideal sampling function of frequency $\frac{1}{T_0}$ Hz as shown in Figure 19.xx and having

mathematical equation: $\delta_{T_0} = \sum_{m=-\infty}^{\infty} \delta(t - mT_0)$

Find:

- The Fourier transform for the periodic signal.
- Frequency spectrum of the signal for $T_0 = 0.01 \text{ sec}$.

Analysis:

In a limiting sense, Fourier transforms can be defined for periodic signals. Therefore, it is reasonable to represent that a periodic signal can be represented in terms of a Fourier transform, provided that this transform is permitted to include delta functions. An ideal sampling function consists of an infinite sequence of uniformly spaced delta functions. We observe that the generating function for the ideal sampling function is simply a delta function $\delta(t)$. The periodic signal can be represented in terms of the complex exponential Fourier series:

$$\delta_{T_0} = \sum_{m=-\infty}^{\infty} c_n \exp(j2\pi f_0 t)$$

where c_n is the complex Fourier series coefficients defined as:

$$\begin{aligned} c_n &= \frac{1}{T_0} \int_{-\infty}^{\infty} \delta(t) \exp(-j2\pi f_0 t) dt \\ &= f_0 G(nf_0) \end{aligned}$$

where $G(nf_0)$ is the Fourier transform of $\delta(t)$ evaluated at the frequency nf_0 . For the delta function:

$$G(nf_0) = 1 \quad \text{for all } n$$

Therefore, using the relation for Fourier transform pair for a periodic signal $g_{T_0}(t)$ with a generating function $g(t)$ and period T_0 :

$$\sum_{m=-\infty}^{\infty} g(t - mT_0) \Leftrightarrow f_0 \sum_{n=-\infty}^{\infty} G(nf_0) \delta(f - nf_0)$$

We get the Fourier transform pair for the ideal sampling function as:

$$\sum_{m=-\infty}^{\infty} \delta(t - mT_0) \Leftrightarrow f_0 \sum_{n=-\infty}^{\infty} \delta(f - nf_0)$$

We can see that the Fourier transform of a periodic train of delta functions, spaced T_0 seconds apart, consists of another set of delta functions weighted by a factor $f_0 = \frac{1}{T_0}$ and regularly spaced f_0 Hz apart along the frequency axis.

Problem 19.9**Solution:****Known quantities:**

Functional form of the modulating signal $m(t)$, the carrier signal $c(t)$, and the modulation index μ .

$$c(t) = A_c \cos(2\pi f_c t)$$

$$m(t) = A_m \cos(2\pi f_m t)$$

Find:

The average power delivered to a 1-ohm resistor.

Analysis:

The AM signal is given by:

$$s(t) = A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

Expressing the product of two cosines as the sum of sinusoidal waves, we get:

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} \mu A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2} \mu A_c \cos[2\pi(f_c - f_m)t]$$

The Fourier transform of $s(t)$ is therefore:

$$\begin{aligned} S(f) &= \frac{1}{2} A_c [\delta(f - f_c) + \delta(f + f_c)] \\ &+ \frac{1}{4} \mu A_c [\delta(f - f_c - f_m) + \delta(f + f_c + f_m)] \\ &+ \frac{1}{4} \mu A_c [\delta(f - f_c + f_m) + \delta(f + f_c - f_m)] \end{aligned}$$

Thus the spectrum of an AM wave, for sinusoidal modulation, consists of delta functions at $\pm f_c, f_c \pm f_m, -f_c \pm f_m$ as seen from its Fourier transform.

In practice, the AM wave $s(t)$ is a voltage or current wave. In either case, the average power delivered to a 1-ohm resistor by $s(t)$ is comprised of three components:

$$\text{Average power} = \frac{1}{T} \int_T x^2(t) dt$$

Using Parseval's energy relation we can find average power in frequency domain as:

$$\text{Average power} = |X(f)|^2 \quad \text{at } f = 0$$

Hence the carrier frequency, upper side-frequency and lower side-frequency power is:

$$\begin{aligned} \text{Carrier Power} &= \left| \frac{1}{2} A_c [\delta(-f_c) + \delta(f_c)] \right|^2 \\ &= \frac{1}{2} A_c^2 \end{aligned}$$

$$\begin{aligned} \text{Upper side - frequency power} &= \left| \frac{1}{4} A_c [\delta(-f_c - f_m) + \delta(f_c + f_m)] \right|^2 \\ &= \frac{1}{8} \mu^2 A_c^2 \end{aligned}$$

$$\begin{aligned} \text{Lower side - frequency power} &= \left| \frac{1}{4} A_c [\delta(-f_c + f_m) + \delta(f_c - f_m)] \right|^2 \\ &= \frac{1}{8} \mu^2 A_c^2 \end{aligned}$$

For a load resistor R different from 1-ohm, which is usually the case in practice, the expression for carrier power, upper side-frequency power, and lower side-frequency power are merely scaled by the factor $1/R$ or R , depending on whether the modulated wave $s(t)$ is a voltage or current, respectively.

Problem 19.10

Solution:

Known quantities:

Carrier signal frequency, $f_c = 0.82$ MHz, upper side-band frequency components at frequencies

$f_{s1} = 0.825$ MHz, $f_{s2} = 0.83$ MHz, $f_{s3} = 0.84$ MHz, their amplitudes and the modulation index $\mu = 1$.

Find:

- Modulating signal equation.
- Plot spectrum of the modulating signal.
- Plot the spectrum of the AM signal including the lower side-band.

Analysis:

- We know from the theory for AM that the upper side-band frequency has frequency components at frequencies:

$f_{sn} = f_c + f_{mn}$ where n is the number of frequency components in the modulating signal

and, their amplitudes in the AM signal are $\frac{1}{2}$ times the original amplitude of the modulating signal for a

modulation index $\mu = 1$. Hence, we can find the modulating signal components to be:

$$m_1(t) = 0.8 \sin(2\pi 5000t)$$

$$m_2(t) = 0.4 \sin(2\pi 10000t)$$

$$m_3(t) = 0.5 \sin(2\pi 20000t)$$

Hence the modulating signal is:

$$m(t) = 0.8 \sin(2\pi 5000t) + 0.4 \sin(2\pi 10000t) + 0.5 \sin(2\pi 20000t)$$

- The spectrum for the modulating signal is shown in Figure 19.xx
- The spectrum for the AM modulated signal with the lower side-band is shown in Figure 19.xx

Problem 19.11

Known quantities:

AM frequency spectrum from 525 kHz to 1.7 MHz, bandwidth for each channel is 10 kHz

Find:

- Number of channels that can be transmitted in the given frequency range
- The maximum modulating frequency that can be transmitted without overlap.

Analysis:

Assume: No guardband between channels.

- The frequency range allocated for AM broadcast is

$$f_R = 1700 - 525 = 1175 \text{ kHz}$$

This range is partitioned to allow 10 kHz of separation between each channel; therefore, the total number of channels, N is

$$N = \frac{1175}{10} \approx 118 \text{ channels}$$

- The carriers of two separate channels are separated by 10 kHz. If we let the maximum frequency of the message signal increase, the outer edges of both sidebands move away from the carrier frequency and into each other, thereby increasing the bandwidth of each AM channel. The maximum allowable message frequency will occur at the midpoint of the spacing between the carriers. Hence, the maximum message frequency is half the frequency spacing between the carriers.

$$f_{m(\max)} = 5 \text{ kHz}$$

