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DATE Monday

* Engineering mechanics:-

may be defined as the science which describes and the conditions of rest or motion of bodies under the area of forces.

* The Parallelogram law for the addition of forces:

Any 2 forces acting on a particle may be replaced by a single force called their resultant.

Particle → جزيء

* The principle of Transmissibility:

Conditions of equilibrium will remain unchanged if the force acting at a given point of the rigid body is replaced by a force of the same magnitude and direction at a different point provided that the two forces have the same line of action.

* Newton's laws*

* Force systems:- 2D

Force is the action of one body on another

types of force: ① actual contact ② remote.

Force is characterized by :-

* point of application
* magnitude and
* direction.

Units → Newtons → N

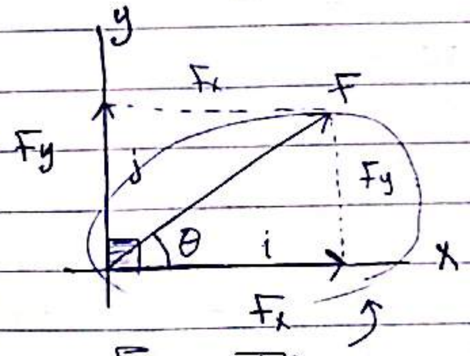
* إذا كانت الوحدة مُشتقة من اسم
نرمز لها بحرف capital
مثل N
* إذا كانت الوحدة ليست
مشتقة من اسم ← حرف Small ← مثل k

* kg * kN ←

** Components of force:-

* Rectangular Components

يجب أن يكونوا متعامدات



Force Triangle

$* F_x = F \cos\theta$

$* F_y = F \cos(90-\theta)$

$= F \sin\theta$

$* F = \sqrt{F_x^2 + F_y^2}$

$* \theta = \tan^{-1} \frac{F_y}{F_x}$

*vector expression of the force

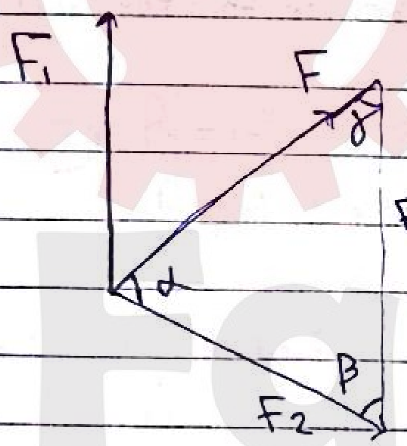
$$\vec{F} = \vec{F}_x + \vec{F}_y$$

→ $\vec{F}_x = F_x i$

→ $\vec{F}_y = F_y j$

→ $\cancel{\vec{F}}$ $\vec{F} = F_x i + F_y j = F(\cos\theta i + \sin\theta)$

* Non-Rectangular Components:- ليس هناك تعامد



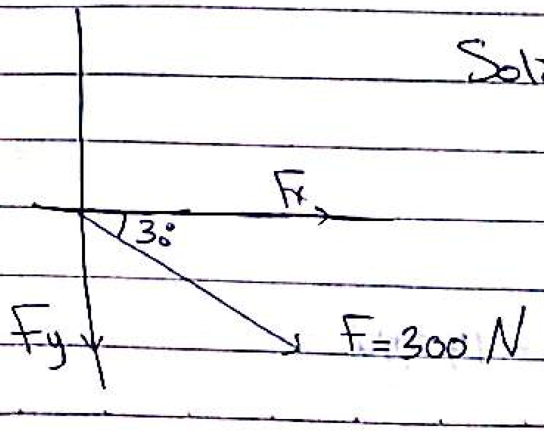
*law of sines:

→ $$\frac{F}{\sin\beta} = \frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\gamma}$$

*law of cosines:

→ $$F^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\beta$$

* Example:- calculate Fx and Fy and express F as a vector



Sol: $F_x = F\cos 30°$

$= 300 * 0.78 = 260$ N

$F_y = F\sin 30°$ لتحديد الإتجاه

$= 300 * 0.5 = -150$ N

ملاحظة

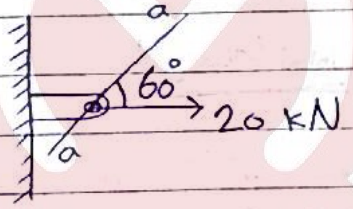
* $\sin(180-\theta)=\sin\theta$ * $\sin(90-\theta)=\cos\theta$

* F as a vector :-

$F = F_x + F_y$
$= F\cos\theta\, i + F\sin\theta\, j$

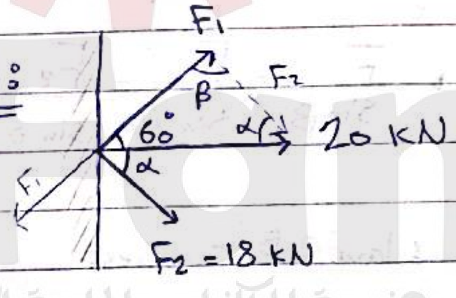
$F = (260i - 150j)\, N$

* Example :-



Replaced the 20kN force by:
F_1 along axis a-a
$F_2 = 18\,kN$
Determain F_1 and α which F_2 makes with the horizantal axis

* Sol:



* Law of sines :-

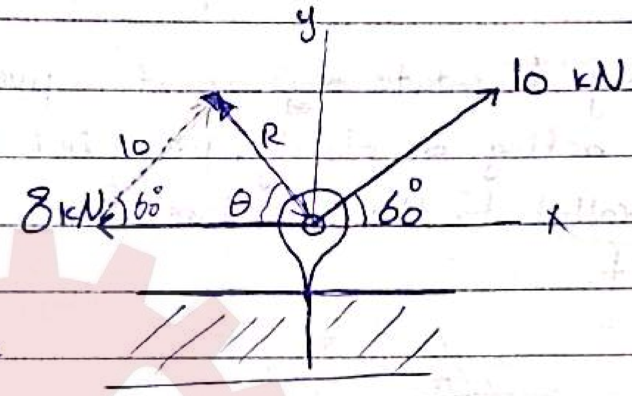
$$\boxed{\frac{20}{\sin\beta} = \frac{18}{\sin 60^\circ}} = \frac{F_1}{\sin\alpha} \longrightarrow \beta = 74.2^\circ \text{ or } 105.8^\circ$$

$\rightarrow 60^\circ + \alpha + \beta = 180^\circ$
So $\alpha = 14.2^\circ$ or 45.8°

$\rightarrow$ For $\alpha = 45.8^\circ \rightarrow \dfrac{F_1}{\sin 45.8^\circ} = \dfrac{20}{\sin 74.2^\circ} \rightarrow F_1 = 14.9\,kN$

→ For $\alpha = 14.2°$: $\frac{F_1}{\sin 14.2°} = \frac{20}{\sin 105.8°} \Rightarrow F_1 = 5.10 \text{ kN}$

* **Example:**



Determine the resultant R

*a Parallelogram rule for vector addition

*b summing scalar components

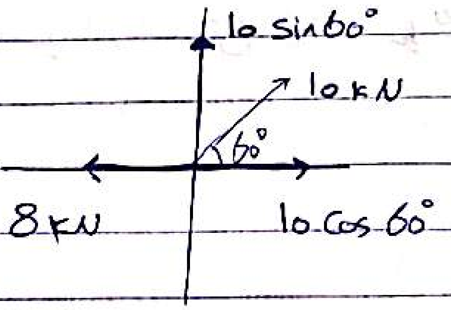
→ Sol: ⓐ $R^2 = 10^2 + 8^2 - 2(8)(10)\cos 60°$

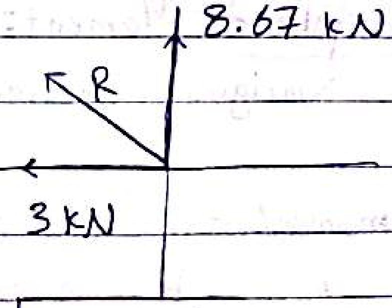
$R = 9.17 \text{ kN}$

→ law of sines:

$\frac{R}{\sin 60°} = \frac{10}{\sin\theta} \rightarrow \theta = 70.8°$

ⓑ



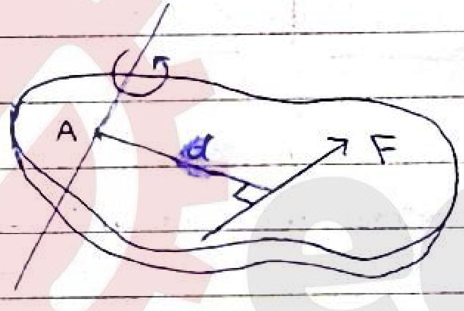


$R = \sqrt{3^2 + 8.67^2}$

$R = 9.17 \text{ kN}$

* Moment العُزوم

→ the tendency to rotate about agiven axis caused by the force acting on abody, provided that this axis is not parallel to the force line of action and doesn't intersect it.



* Magnitude:

* $M = F.d$
* Units: N.m
* Signs: ↺+ , ↻−

* نستخدم قاعدة اليد اليمنى
← الأصابع مع اتجاه الحركة
والإبهام اتجاه **M**

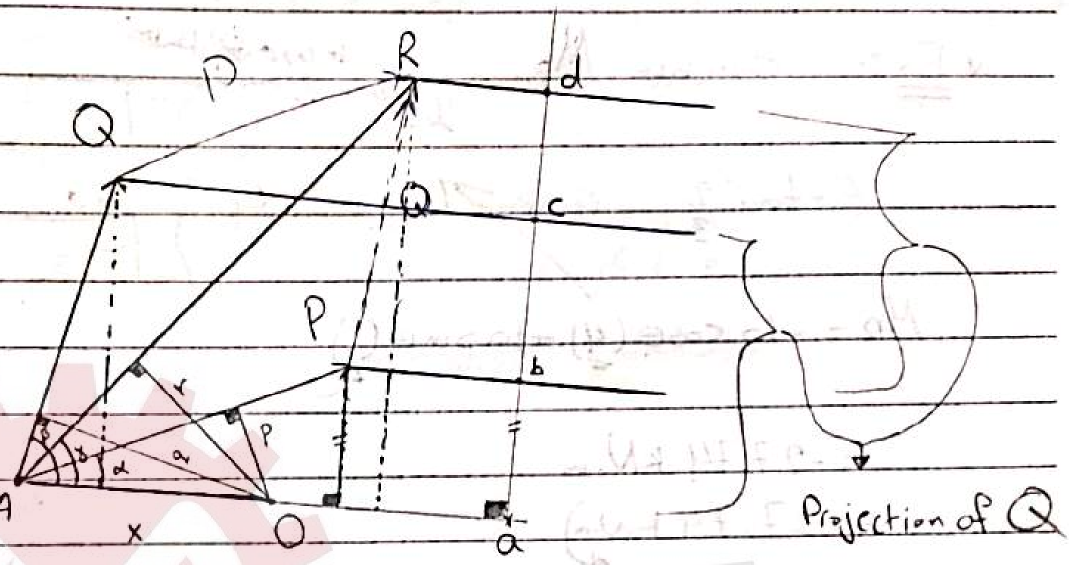
* Principle of Moment:

Varignon's Theorem

يعني ← المومنت للقوة يساوي المومنت لمركباتها

moment of R about O = R.r
" " P " O = P.p
" " Q " O = Q.q

*Projection for any force means rectangular components



*ac = bd → projection of Q

*من الرسمة :-

* ad = ab + bd

= ab + ac

$$\to X = \frac{p}{\sin\alpha}$$

$$\to X = \frac{r}{\sin\gamma}$$

$$\to X = \frac{q}{\sin\beta}$$

⊛ $ab = P\sin\alpha$

⊛ $ac = Q\sin\beta$

⊛ $ad = R\sin\gamma$

$$\underset{ad}{R\sin\gamma} = \underset{ab}{P\sin\alpha} + \underset{ac}{Q\sin\beta}$$

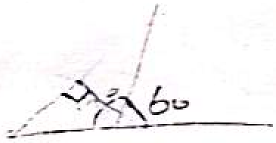
*نضرب الطرفين بـ X

*AO = X

$$\to xR\sin\gamma = xP\sin\alpha + xQ\sin\beta$$

$$\to \frac{r}{\sin\gamma}R\sin\gamma = \frac{p}{\sin\alpha}P\sin\alpha + \frac{q}{\sin\beta}Q\sin\beta$$

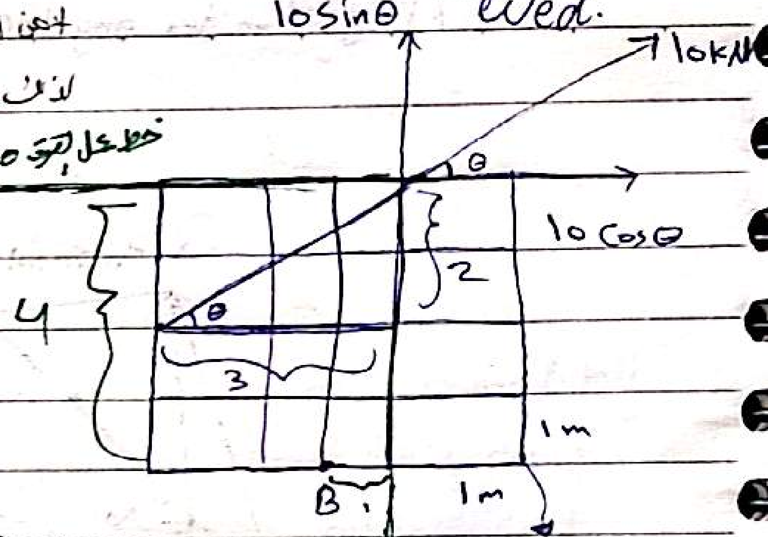
$$\to rR = pP + qQ$$



25/6/2014
Wed.

*من الصعب إيجاد المسافة العمودية بين B وخط عمل القوة ، لذلك نجد M لمركبات القوة لأن إيجاد المسافات العمودية للمركبات

*<u>Ex</u>:- Calculate M_B.



خط عمل القوة 10 cos θ يبعد عن B 4

$\theta = \tan^{-1}\frac{2}{3}$

$M_B = -10\cos\theta(\underline{4}) + 10\sin\theta(\underline{1})$

خط عمل القوة 10 sinθ يبعد عن B 1

$= -27.74$ kN.m

$= 27.74$ kN.m ↻

*<u>Ex</u>:- calculate the Tension in the cable to cause $M_o = 72$ kN.m

given $M_o = T.r = 72$ kN.m

$\theta = \tan^{-1}\frac{26}{27}$

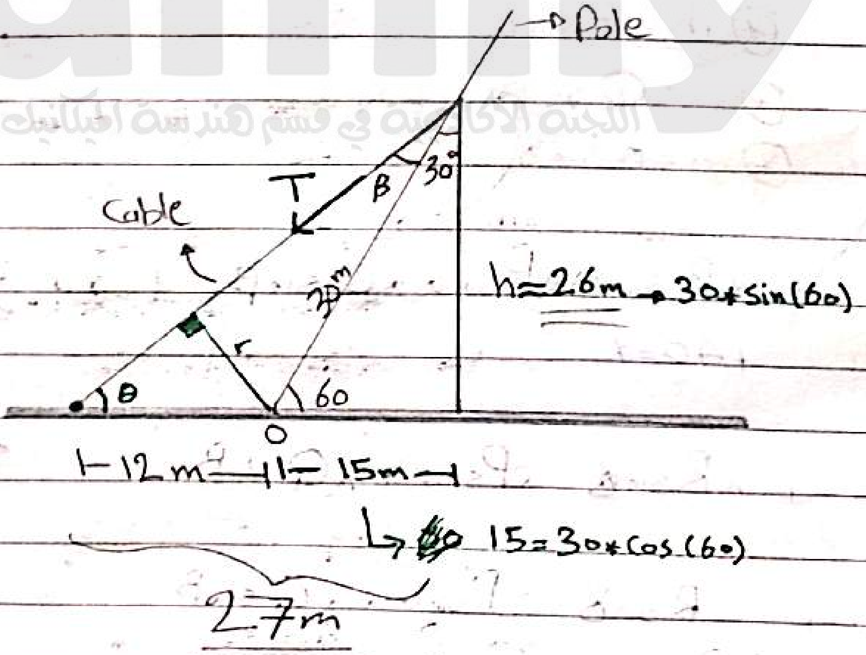
$(\beta + 30°) = \tan^{-1}\frac{27}{26}$ ← مقابل ← مجاور

$\beta = 16.1°$

$T = \frac{M_o}{r}$

$= \frac{72}{30(\sin\beta)}$

$T = 8.65$ kN.



h=26m → 30*sin(60)

↳ 15=30*cos(60)

27m

$M_A = 15.8\ N.m \circlearrowright$
$= R.\ 40 \cos\alpha$

$x = 49mm\ ,\ y = 63.2mm$

*Ex:-

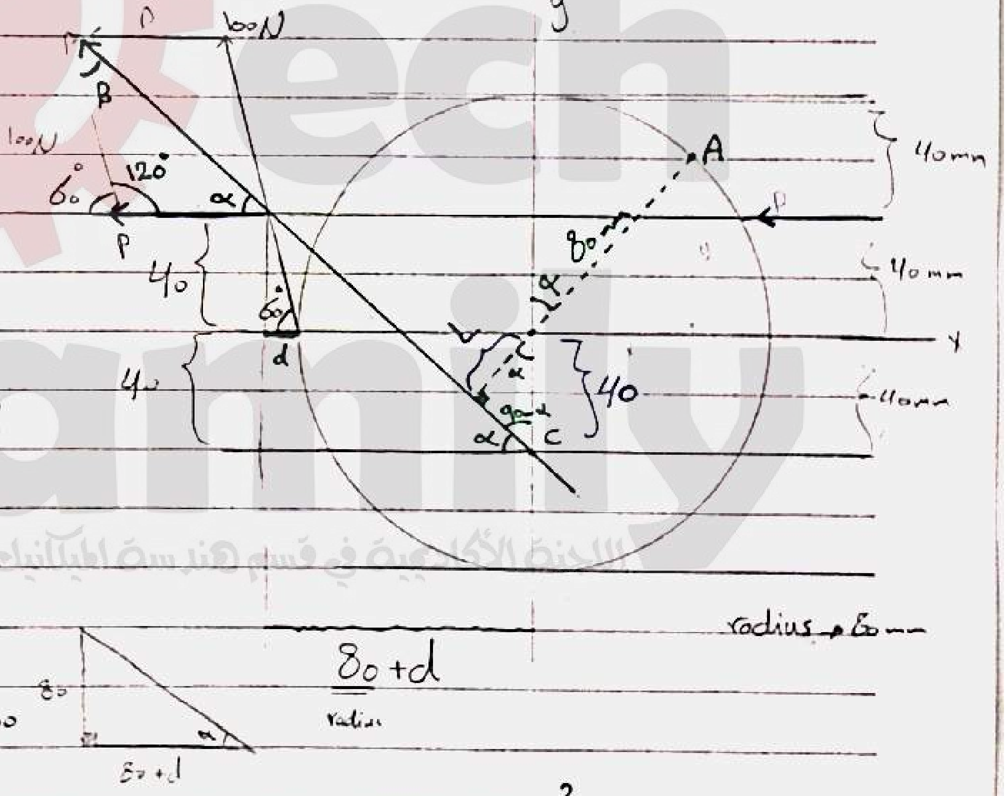
النقطة A ـــ أبعد نقطة عن R وذلك حتى يكون M أكبر شي
وحتى تكون أبعد شي يجب أن تكون
عمودية على R وتمر في المركز.

$M_C = 0$

moment of the two forces shown about C is zero.

Determine:-

P, $\underline{\underline{R}}$, x and y of point A
resultant about which M is $\underline{max}$



$$*\ \alpha = \tan^{-1}\frac{80}{80+d}$$

$$*\ \tan 60^\circ = \frac{40}{d}$$

$$\rightarrow d = \frac{40}{\tan 60^\circ} = 23.1\ mm$$

$$\rightarrow \alpha = \tan^{-1}\frac{80}{80+23.1} = 37.8^\circ$$

$$\rightarrow \beta = 22.2^\circ$$

3

$$\boxed{\begin{aligned} x &= 80 \sin\alpha = 49 \\ y &= 80 \cos\alpha = 63.2 \end{aligned}}$$

*Law of sines:-

1 2

$$\frac{P}{\sin\beta} = \frac{100}{\sin\alpha} = \frac{R}{\sin 120} \rightarrow \boxed{P = 61.6N}\ ,\ \boxed{R = 141N}$$

force triangle

حوّل من mm ← m
بالقسمة على 1000

$$* \text{Now} \rightarrow M_A = R \cdot (\underbrace{80 + \underline{40\cos\alpha}}_{\rightarrow})$$

↳ $L = 40\cos\alpha$

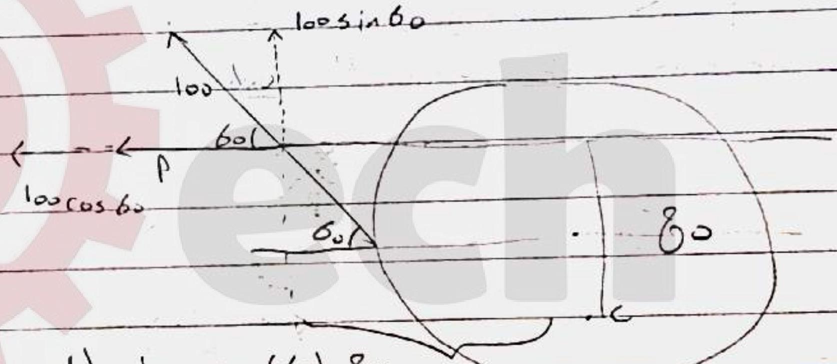
$= 141\,(80 + 40\cos\alpha)$

$= 15.7$ N.m

$*80 + 40\cos\alpha$

→ distance between A and R.

* طريقة أخرى لإيجاد P, R دون استخدام Law of sines.



$M_C = \text{Zero}$

$P \cdot 80 = 100\sin(60) \cdot (80 + \underset{23.1}{\underline{d}}) - 100\cos(60) \cdot 80$

↳ $80 + d$

$P = 61.6$ N

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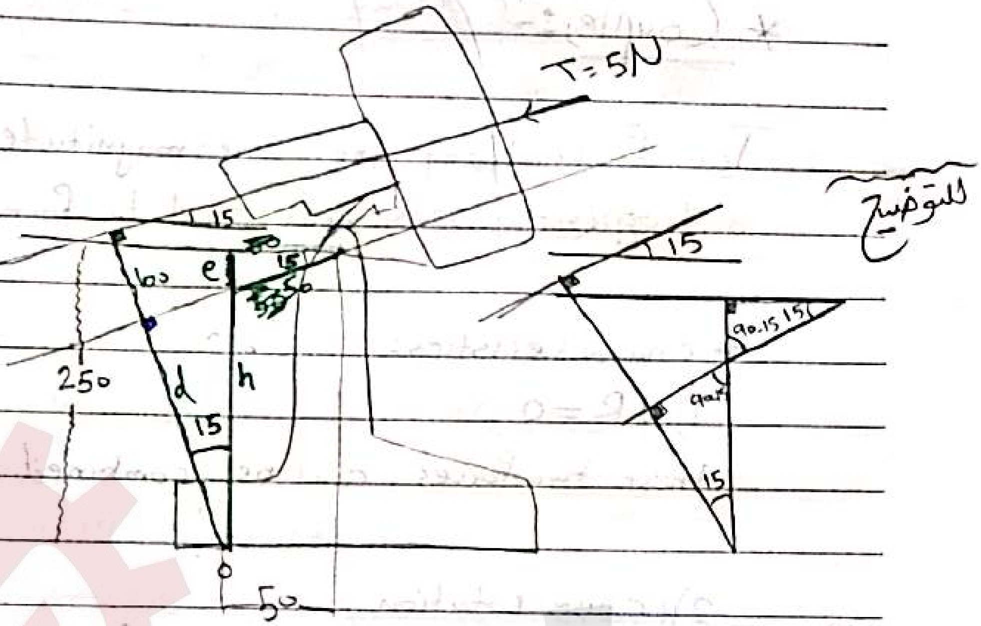
$M_o = T(d+100)$

$d = h\cos 15°$

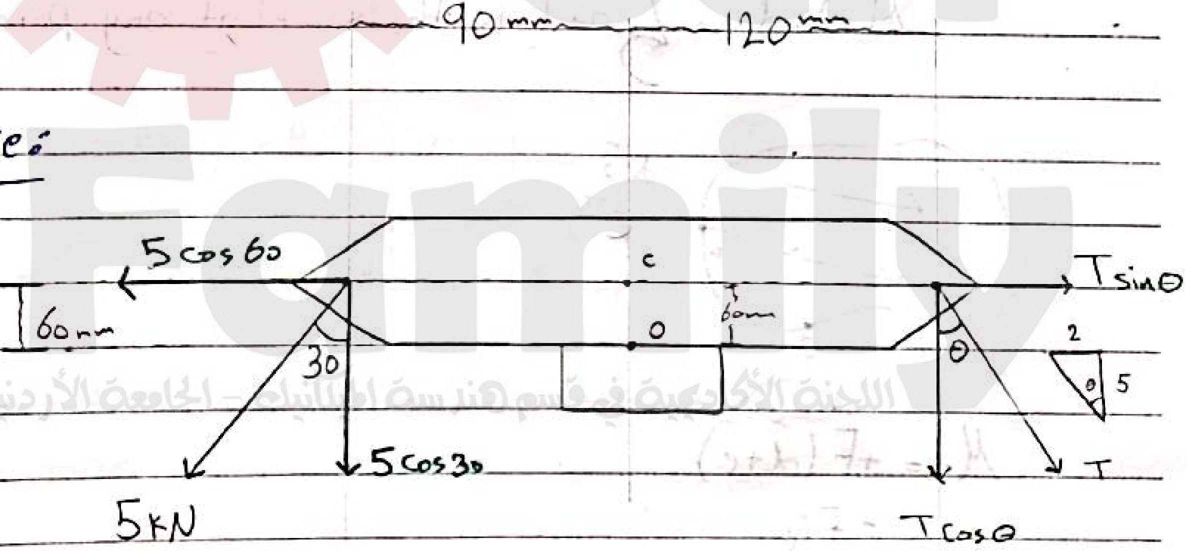
$h = 250 - e$

$h = 250 - 50\sin 15$
$= 237.5$ mm



$M_o = 5(237.5\cos 15° + 100)$
$= 1.64$ N.m ↺

*Example:



$\theta = \tan^{-1}\frac{2}{5} = 0.4 = 21.8°$

$M_o =$ Zero
$= -T\sin\theta(60)$
$- T\cos\theta(120)$
$+ 5\cos 60(60)$
$+ 5\cos 30(90) \Rightarrow T = 4.04$ kN

* قوتين لهما نفس المقدار + خطي عملهما متوازيان + متعاكستان بالإتجاه.

* Couple :-

Two forces having the same magnitude, Padlel lines of action and opposite sense are said to form a couple.

→ Characteristics:-

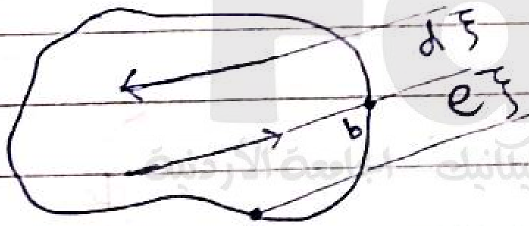
1) * $R=0$

these two forces can't be combined Into a single force

2) * cause rotation.

3) * the moment of the couple is given by

$M = F.d \circlearrowleft$ and is the same at any point

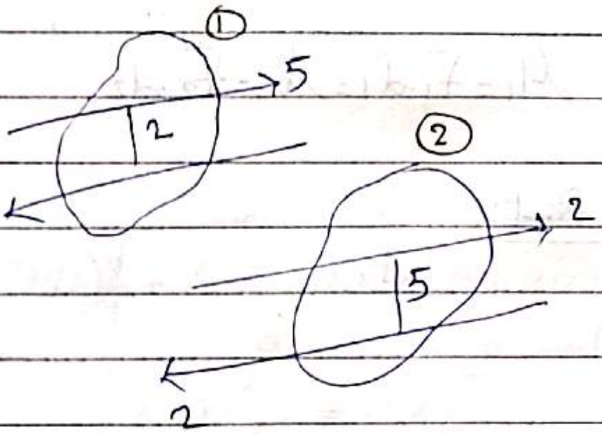


$$M_a = +F(d+e)$$
$$-F(e)$$
$$= +F.d$$

$M_b = +F d$

* Equivalent couple:-

→ couples are said to be equivalent if they produce the same moment



$M_1 = 5 \cdot 2 = 10$ $M_2 = 2 \cdot 5 = 10$

29/6/2014 Sunday

* Two Systems of forces are equivalent if we can transform on of them into the other by means of one or several of the following operations:-

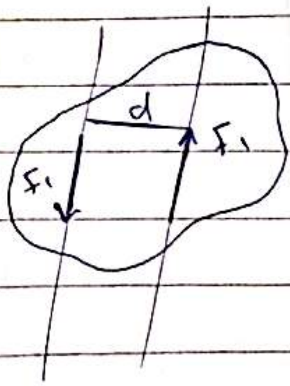
⊛ Replacing two forces by their resultant.

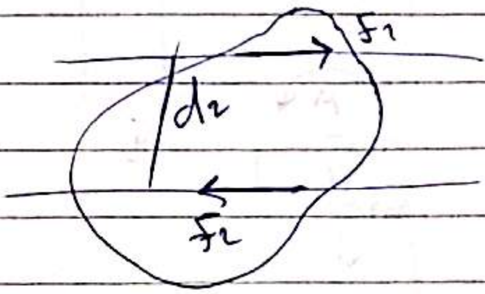
⊛ Resolving a force into two components

⊛ Canceling two equal and opposite forces acting on a particle.

⊛ Attaching to the same particle two equal and opposite forces

⊛ Moving a force along it's line of action.





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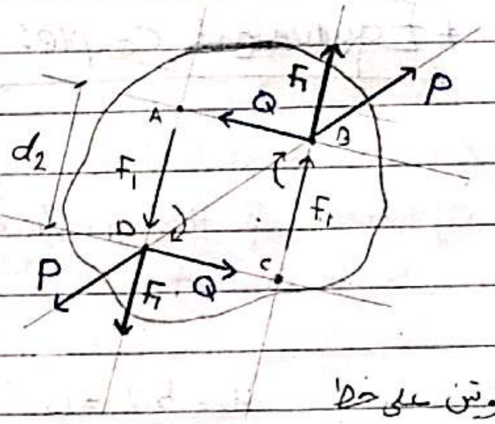
$M_1 = F_1 \cdot d_1 = M_2 = F_2 \cdot d_2$



*** Proof:-**

$$F_1 d_1 = \overset{\text{صفر}}{M_{P_s}} + M_{Q_s}$$

$$= Q d_2$$

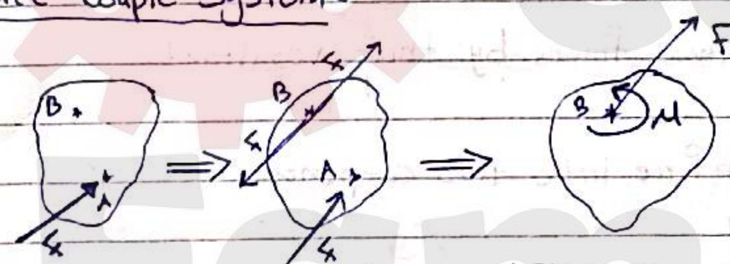
$$= F_2 d_2$$

$\Longrightarrow Q = F_2$

* ننقل القوتين على خطا عملها.

* تُختصر القوتين P لأنهما متساويتان ومتعاكستان وعلى نفس خط عمل القوى.

*** Force couple System:-**

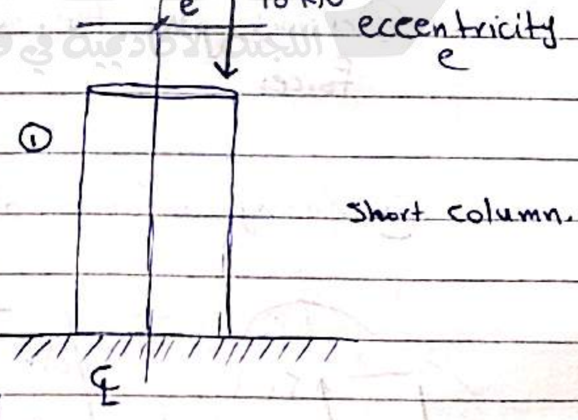


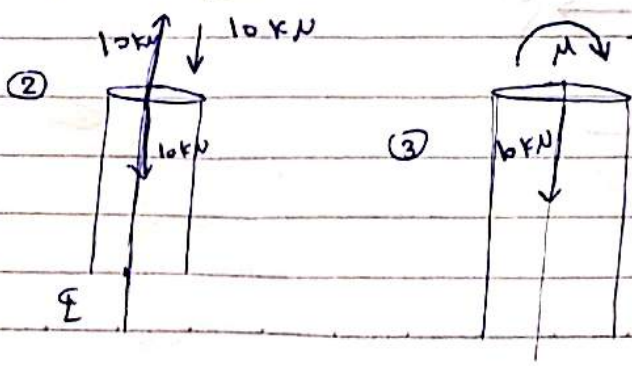
* ننقل قوة من A ← B: ① احسب M عند النقطة B ② انقل القوة.

*** Example:**

If $M_{℄} = 800$ N.m

determine e.



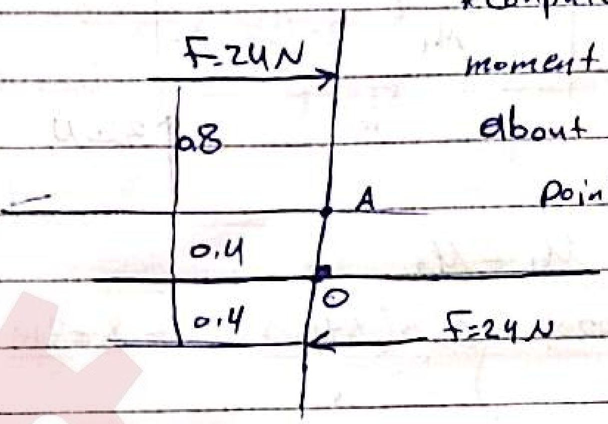


$10.000\,e = 800\ N.m$

$e = 80\ mm.$

* Example:

*Compute the combined moment of two 24N forces about point o, and point A

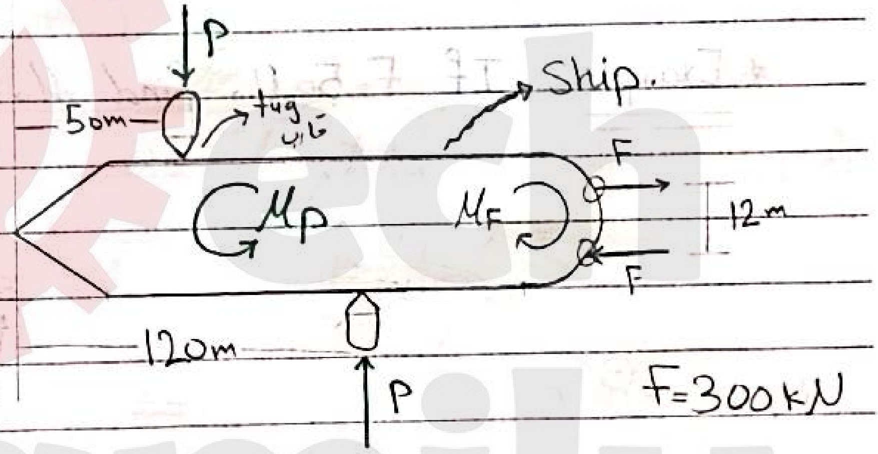


$M_A = M_O = F.d$

$= 24\,(1.6)$

$= 38.4\ N.m$

* Example:



*Sol:-

$P\,(70) = F\,(12)$

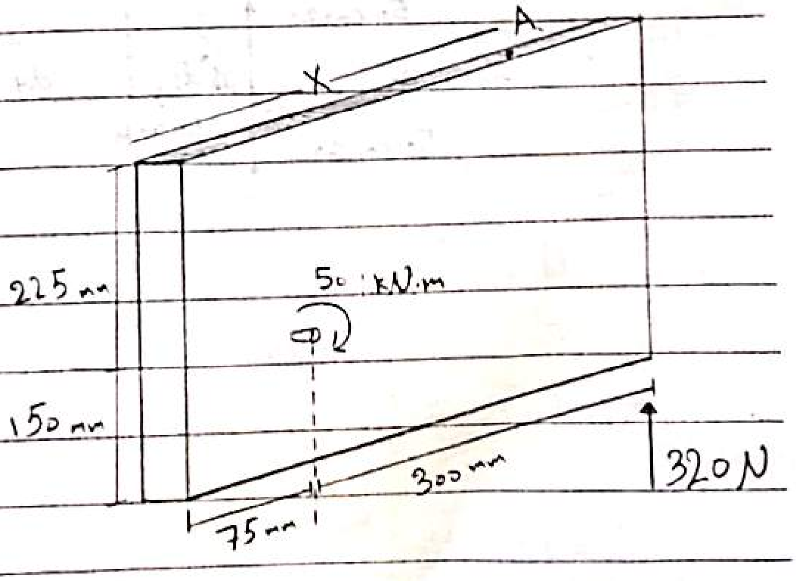
$$P = \frac{300(12)}{70}$$

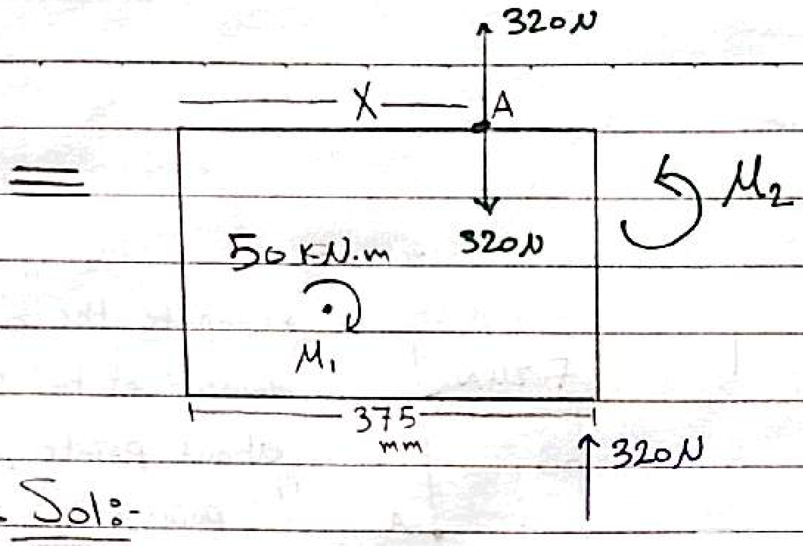
$\rightarrow P = 51.4\ kN$

* Example:-

The (50N.m) couple and the 320N force are replaced by a single force at A

* determine X



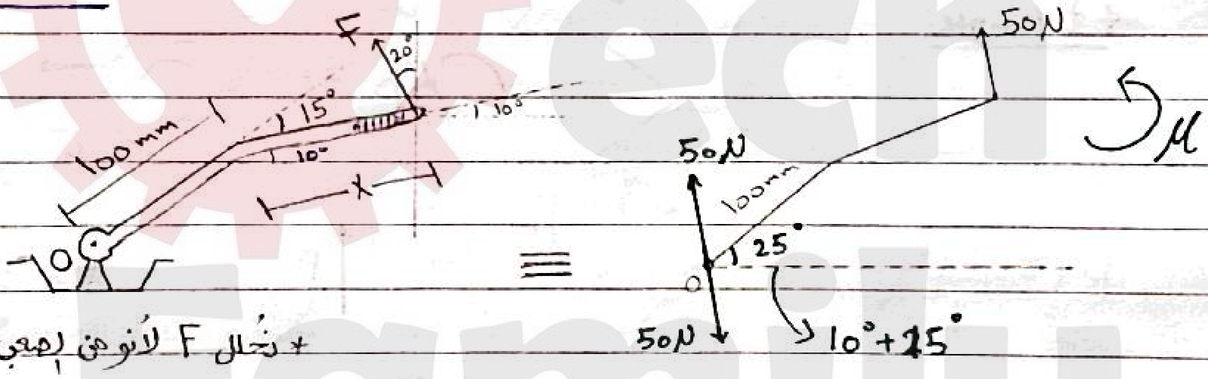


→ Sol:-

$M_1 = M_2$

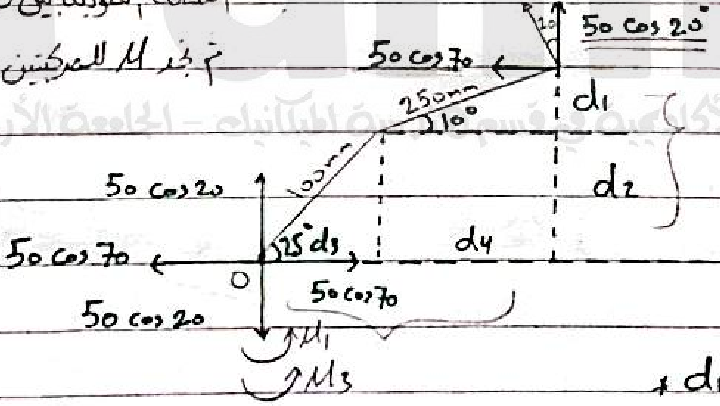
$50(1000) = 320(375-x) \Rightarrow X = 218.75 \Rightarrow X = 219\ mm$

* Example:- If $F = 50N$, find M_o $(X = 250\ mm)$



* نحلل F لأنو هن الصعب إيجاد المسافة العمودية بين O وخط عمل F

ثم نجد M للمركبتين.



M_1 ← ناتج عن المركبة الأولى

M_2 ← ناتج عن المركبة الثانية

$* d_1 = 250 \sin 10$

$* d_2 = 100 \sin 25$

$* d_3 = 100 \cos 25$

$* d_4 = 250 \cos 10$

$M_0 = M_1 + M_2$

$M_1 = 50 \cos 20 (d_3 + d_4)$

$M_2 = 50 \cos 70 (d_1 + d_2)$

$M_0 = 17.3\ N.m$ ↺

* لا تنسى تحويل الأطوال إلى m

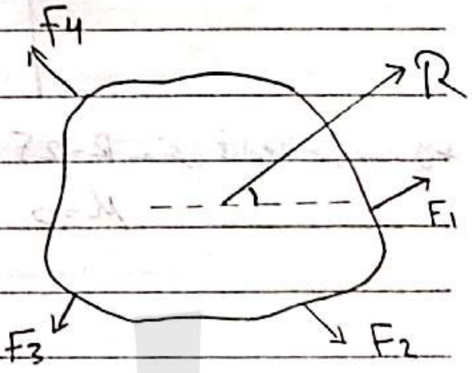
* Resultant :

The resultant may be defined as the single force or (couple) which replace the orginal forces with no change to the external effect.

* $R_x = \Sigma F_x = f_{1x} + F_{2x} + \dots$

* $R_y = \Sigma F_y = F_{1y} + F_{2y} + \dots$

$\rightarrow R = \sqrt{R_x^2 + R_y^2}$, $\theta = \tan^{-1} \frac{R_y}{R_x}$



** The principle of moment :- (to determine point of apply)

$Rd = F_1d_1 + F_2d_2 + F_3d_3$

$\rightarrow d = \frac{\Sigma Fd}{R}$

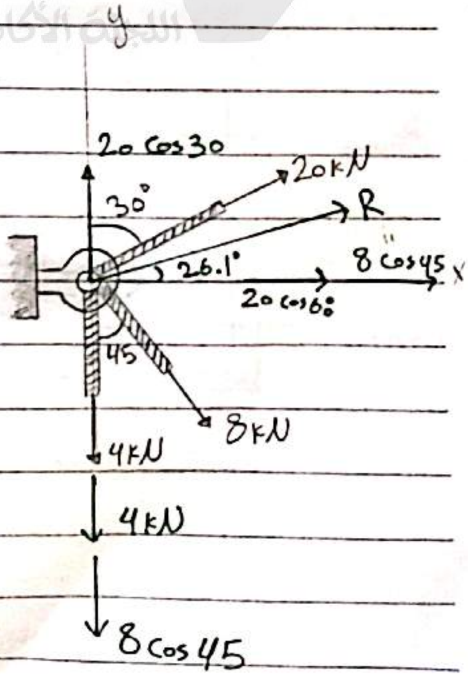
* Example:- Find R!?

$R_x = \Sigma F_x = 80 \cos 45 + 20 \cos 60^\circ$
$= 15.65$ kN

$R_y = \Sigma F_y = 20 \cos 30 - 4 - 8 \cos 45$
$= 7.66$ kN

$R = \sqrt{15.65^2 + 7.66^2}$
$= 17.42$ kN

$\theta = \tan^{-1} \frac{R_y}{R_x} = 26.1^\circ$



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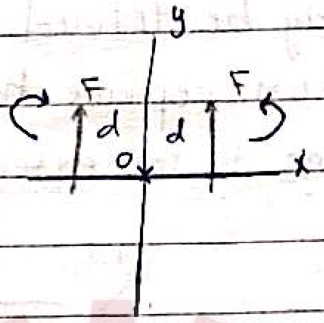
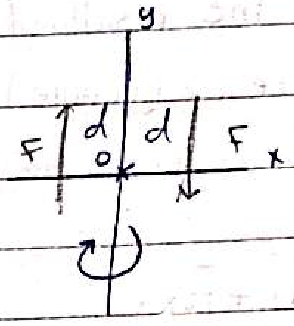
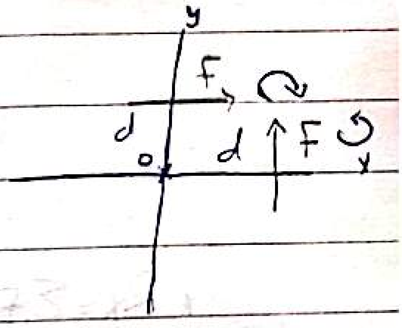
* Example:-







$R = 2F$ وتكون في الوسط على امتداد +y

$M = 0$

$R = 0$

$M = F \cdot 2d$

$R = F\sqrt{2}$

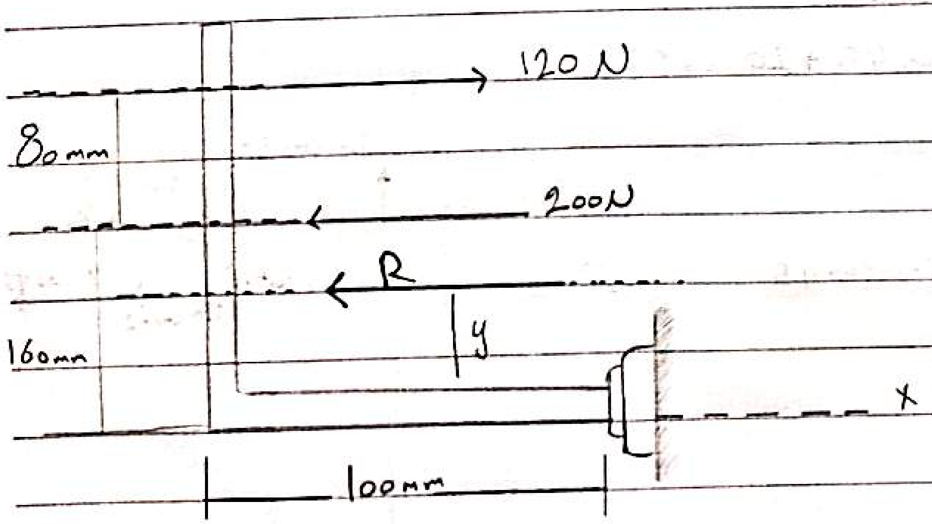
$M = 0$

* Example:-

Replace the two forces by a single force R, ① and determine y ②

① $R = \sum F_x = 120 - 200 = -80\ N$

② $R \cdot y = 200(160) - 120(240) \Rightarrow y = 40\ mm$



*** Example:-**

Four Slender rods subjected to force as shown. Determine F and θ if the resultant R is equal to 100N vertically upward.

$* R_x = \sum F_x = 0$

$1000 - 750 \cos 30^\circ + F \cos\theta + 600 \cos 50^\circ = 0$

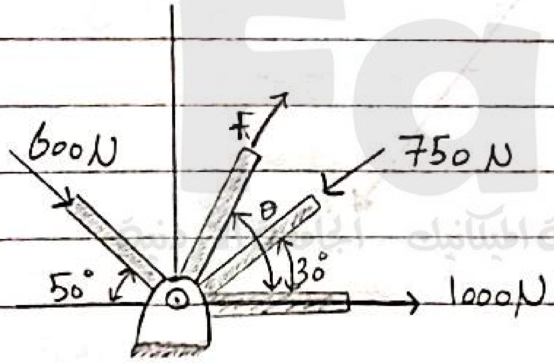
$\Rightarrow F \cos\theta = -736 N$

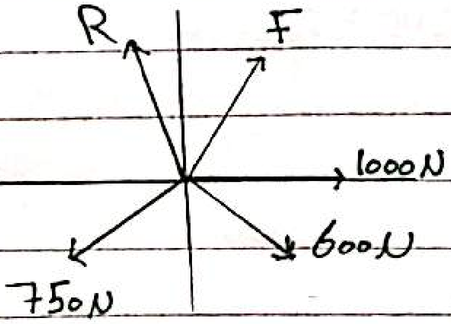
$* R_y = \sum F_y = 100$

$-750 \sin 30 + F \sin\theta - 600 \sin 50 = 100$

$\Rightarrow F \sin\theta = 935 N$

$\Rightarrow \theta = 128.2 \Rightarrow F = 1190 N.$





* Force Systems 3D

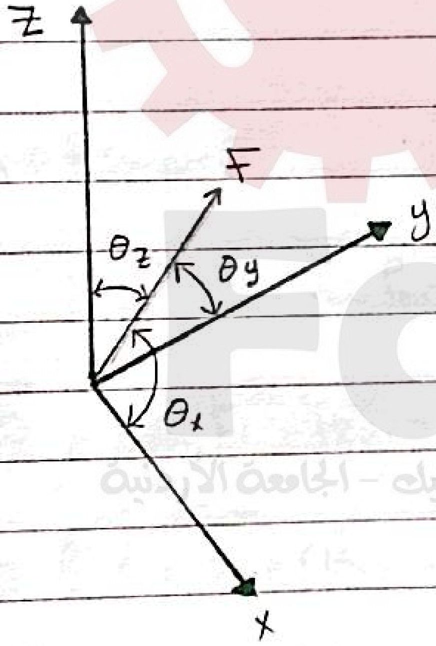
* اليد اليمنى :-

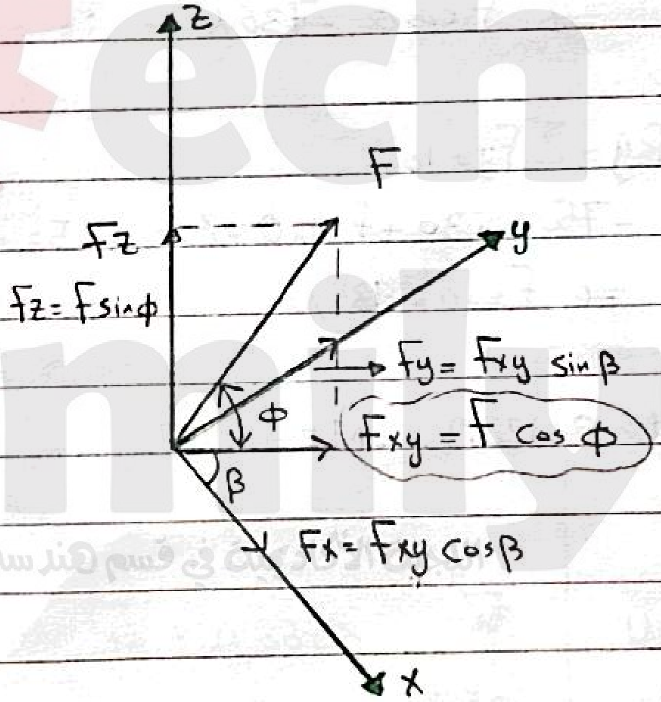
← يكون الدوران من X ← Y بأصابع اليد

والإبهام يكون +Z

The rectangular components of the force F in ~~the 3 dimi~~ the 3 directions are

* $F_x = F \cos \theta_x$
* $F_y = F \cos \theta_y$
* $F_z = F \cos \theta_z$





→ $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

→ $F = \sqrt{F_z^2 + \underbrace{F_{xy}^2}_{= F_x^2 + F_y^2}}$

→ $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

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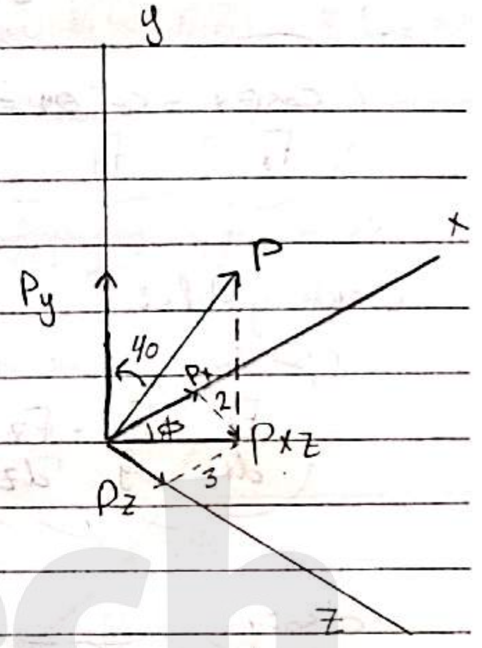
DATE Wed.

* Example:- ①

If $P_x = 60 kN$, find P_y and P



*Sol:- $P_y = P\cos 40$

$P_{xz} = P\sin 40$

$\Phi = \tan^{-1}\frac{2}{3} = 33.69°$

$P_x = P_{xz}\cos\phi$

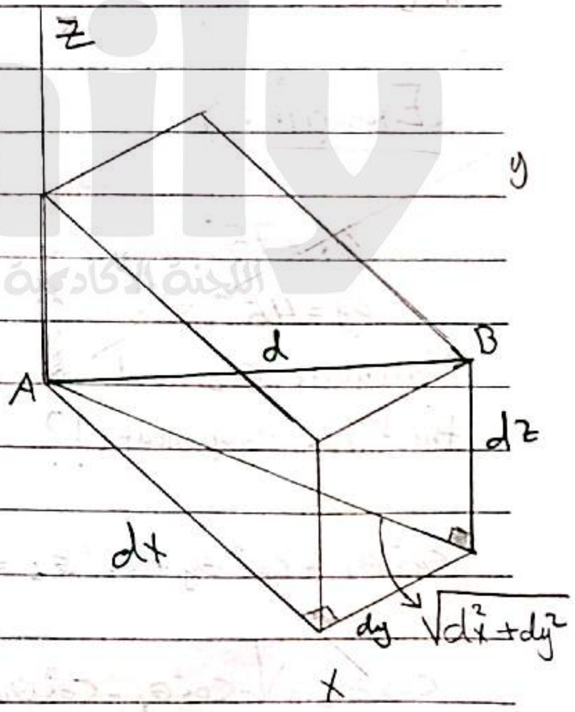
$= P\sin 40°\cos\phi$

$= 60\ kN$

$\Rightarrow P = 112.2\ kN$

$P_y = P\cos 40°$

$= 85.94\ kN$



$AB = d$

$dx = x_2 - x_1$

$dy = y_2 - y_1$

$dz = z_2 - z_1$

$d = \sqrt{dx^2 + dy^2 + dz^2}$

← حاوِل تتخيلها صح Pp:

→ let $\cos\theta_x = L$

$\cos\theta_y = m$, we now that $L^2 + m^2 + n^2 = 1$

$\cos\theta_z = n$

also we can say that:-

$$\boxed{\frac{\cos\theta_x}{F_x}=\frac{\cos\theta_y}{F_y}=\frac{\cos\theta_z}{F_z}=\frac{1}{F}}$$

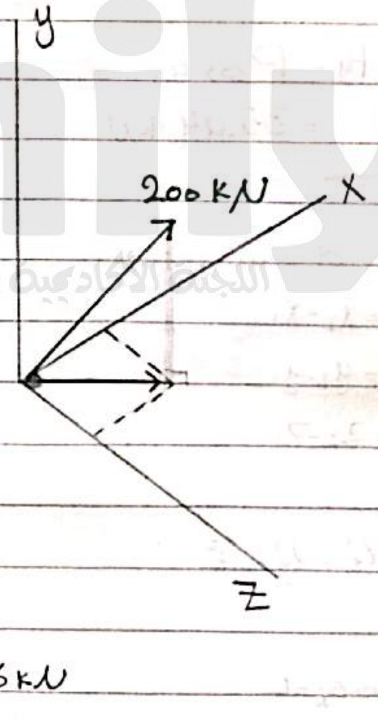
which yields:

$$\boxed{\frac{F_x}{d_x}=\frac{F_y}{d_y}=\frac{F_z}{d_z}=\frac{F}{d}}$$

also:

$$\boxed{\frac{\cos\theta_x}{d_x}+\frac{\cos\theta_y}{d_y}+\frac{\cos\theta_z}{d_z}=\frac{1}{d}}$$

Example^3:



$\theta_x=75^\circ$

$\theta_y=45^\circ$

Calculate θ_z and the force components!?

$$\cos^2\theta_x+\cos^2\theta_y+\cos^2\theta_z=1$$

$$\cos\theta_z=\sqrt{-\cos^2\theta_x-\cos^2\theta_y}$$

$\theta_z=48.85^\circ$

$F_x=F\cos\theta_x=51.76\,kN$

$F_y=F\cos\theta_y=141.42\,kN$

$F_z=F\cos\theta_z=131.6\,kN$

$$F=(51.76i+141.42j+131.6k)\,kN$$

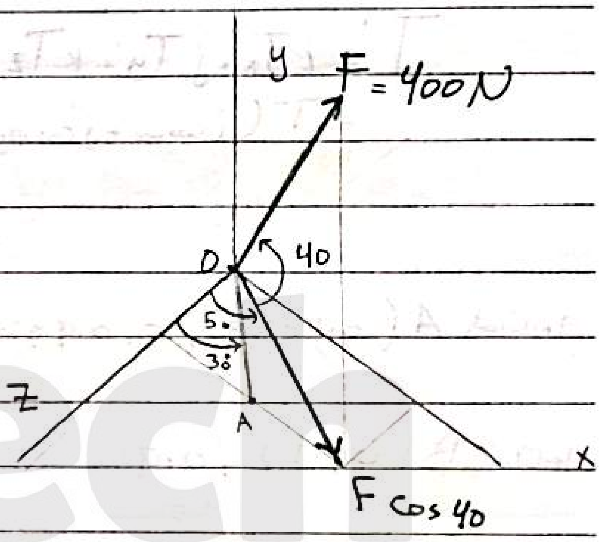
*Example: ^{4} What is the projection of F onto Line OA?

↳ Rectangular component in the direction of Line OA.

Sol: $F = iF_x + jF_y + kF_z$

$F_x \to F\cos\theta_x$, $F_y \to F\cos\theta_y$, $F_z \to F\cos\theta_z$

$F_y = 400\cos 50^\circ = 257\ N.$



$F_x = 400\cos 40^\circ \cos 40^\circ$
$= 234.7\ N$

$F_z = 400\cos 40^\circ \cos 50^\circ$
$= 197\ N$

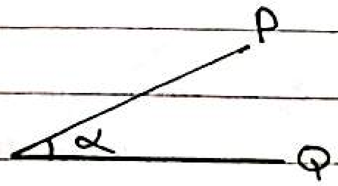
$\to F = (234.7\,i + 257\,j + 197\,k)\ N$

*Remark: (projection → Rectangular Comps)

So: $F_{OA} = F\cos 40^\circ \cos 20^\circ = 288\ N.$

*Dot Product:-

$P.Q = P.Q\cos\alpha$



* هو عبارة عن مركبة أحد المتجهين في اتجاه الآخر مضروبة في المتجه الآخر.

$P.\underline{Q\cos\alpha}$

↳ مركبة Q في اتجاه P

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* Example:- (2/107 / Page 69)

T_{CD} ?!

$$T_{CD} = T . \lambda$$

$$T = iT_x + jT_y + kT_z$$
$$= T(i\cos\theta_x + j\cos\theta_y + k\cos\theta_z)$$

احداثياتها $A(0.9\cos 30°, 0, 0.9\sin 30°)$

احداثياتها $B(0, 1.2, 0.9)$

$d_x = -0.9\cos 30° = -0.779$ m
$d_y = 1.2$ m
$d_z = 0.45$ m

$\overline{AB} = d = \sqrt{d_x^2 + d_y^2 + d_z^2} = 1.5$ m.

$$\cos\theta_x = \frac{d_x}{d}$$

$$\cos\theta_y = \frac{d_y}{d}$$

* قيمة λ دائماً $\boxed{1}$ Unit vector.

$$\cos\theta_z = \frac{d_z}{d}$$

$$\rightarrow T = \frac{\overset{T}{100}}{\underset{d}{1.5}}(\overset{d_x}{-0.779i} + \overset{d_y}{1.2j} + \overset{d_z}{0.45K})$$

$$\rightarrow \lambda = \frac{1}{1.5}(0.779i + 1.2j + 0.45K)$$

ارجع احسبهم في البيت

← واضرب المركبات في بعض

$T_{CD} = T \cdot \lambda = 46N$

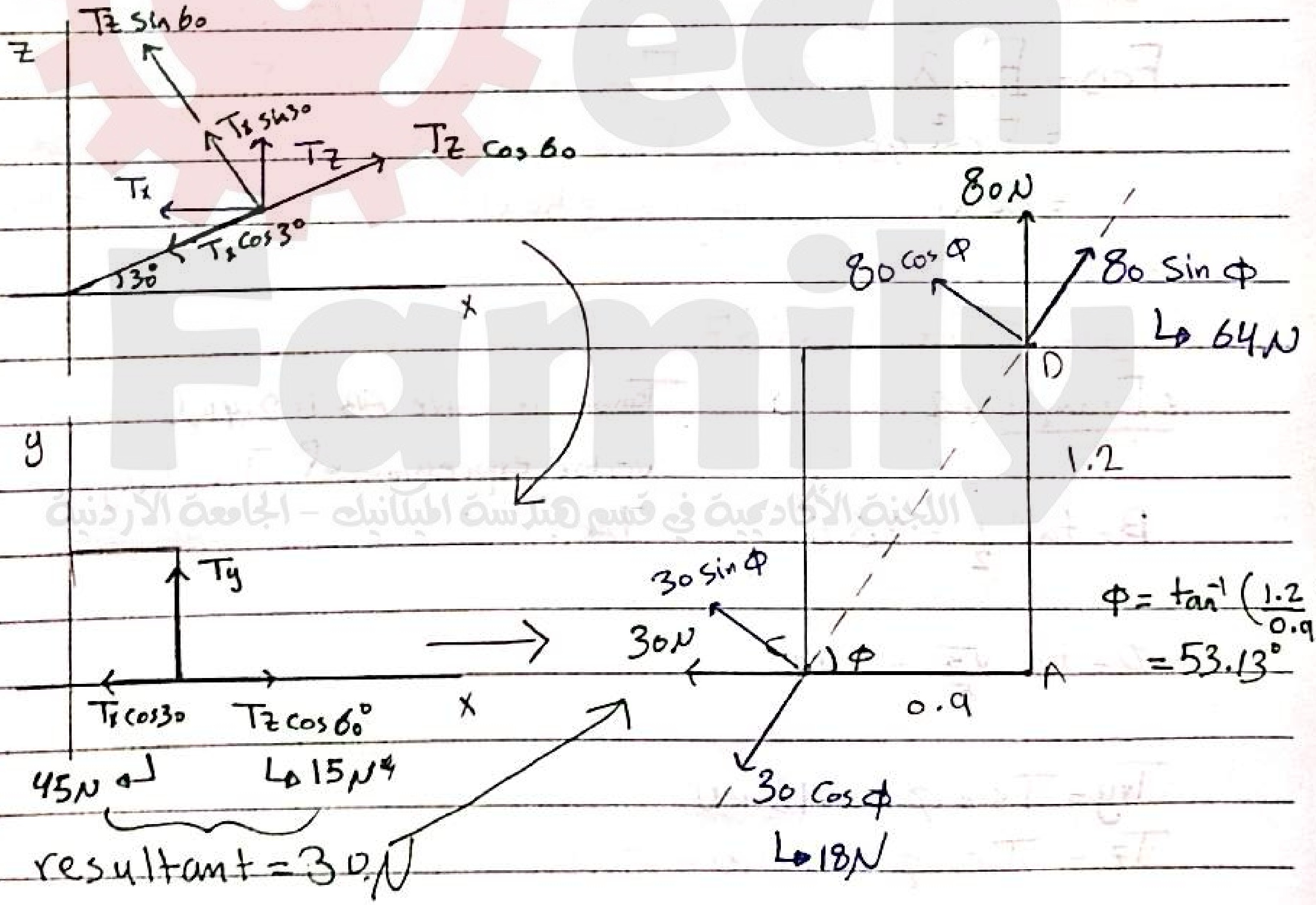
$T_{CD}\left(\frac{100 * -0.779}{1.5} * \frac{0.779}{1.5}\right) + \left(\frac{1.2 * 100}{1.5} * \frac{1.2}{1.5}\right) + \left(\frac{0.45 * 100}{1.5} * \frac{0.45}{1.5}\right)$

$= -27 + 64 + 9 = 46$

\~\~\~\~\~

$* T_x = T\cos\theta_x = T\frac{d_x}{d} = 52N$ $\quad * T_y = T\cos\theta_y = T\frac{d_y}{d} = 80N$

$* T_z = T\cos\theta_z = T\frac{d_z}{d} = 30N$



$\Phi = \tan^{-1}\left(\frac{1.2}{0.9}\right)$
$= 53.13°$

resultant = 30 N

$\therefore 64 - 18 = 46$ ✓ الحل صحيح

* Exam : 17/7/2014 _ Thursday *

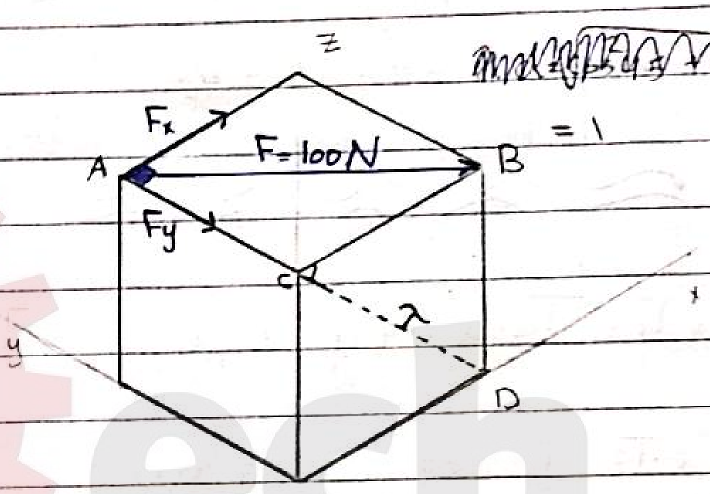
PAGE 6/7/2014
DATE Sunday.

*** Example :** ① Calculate F_{CD}?!

$F_x = F\cos 45°$

$F_y = F\cos 45°$

$F_Z = 0$



= 1

→ $F' = F(i\cos\theta_x + j\cos\theta_y)$

$= F(i\cos 45 - j\cos 45)$

→ $\lambda = 1(i\cos 45° - K\cos 45)$

$F_{CD} = F' \cdot \lambda$

$= F\cos^2 45$

$= 50N$

Zero Zero

$F' \cdot \lambda = F\cos 45°(\cos 45) + F\cos 45(Zero) + Zero(\cos 45°)$

$F' \cdot \lambda = F\cos^2(45)$

$= 50N$

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*** Example : 2** Tension in Cable AB is 2.4kN
vector expression of T acting on AC
T_{AC}.

$\beta = \tan^{-1}\frac{1}{2} = 26.6°$

$\phi = \tan^{-1}\frac{\sqrt{5}}{5} = 24.1°$

$T_{xy} = T\sin\phi = 0.98\,kN$

$T_Z = T\cos\phi = 2.19\,kN = -2.19\,kN$ (downward) ↓

$T_x = T_{xy}\cos\beta = 0.876\,kN$

$T_y = T_{xy}\sin\beta = 0.438\,kN$

$T_{AC} = T \cdot \lambda$

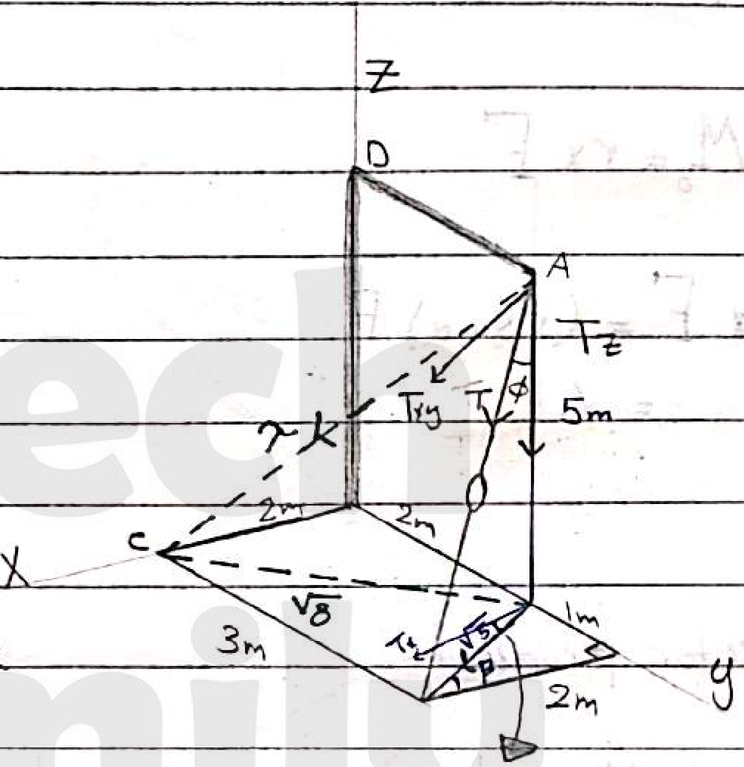
$$\lambda = \frac{1}{5.74}(\overset{dx}{2i} - \overset{dy}{2j} - \overset{dz}{5k})$$

d

$$\boxed{d = dx^2 + dy^2 + dz^2}$$

$$d = \sqrt{(\sqrt{8})^2 + 5^2}$$

$= 5.74$



T_{xy} Parallel to $\sqrt{5}$

$T = 0.876i + 0.438j - 2.19k$

$\lambda = \frac{1}{5.74}(2i - 2j - 5k)$

$T \cdot \lambda = 2.06\,kN$

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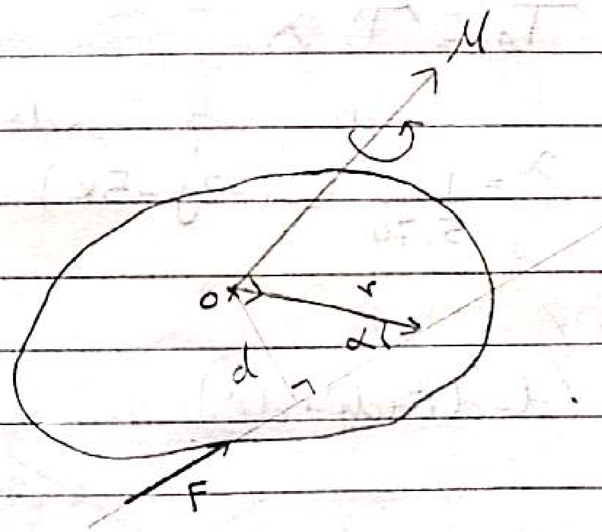
DATE Mon.

*Moment 3D



$M_o = F.d$

$M_o = r \times F$

$r \times F = (r\sin\alpha) F$
$= d.F$

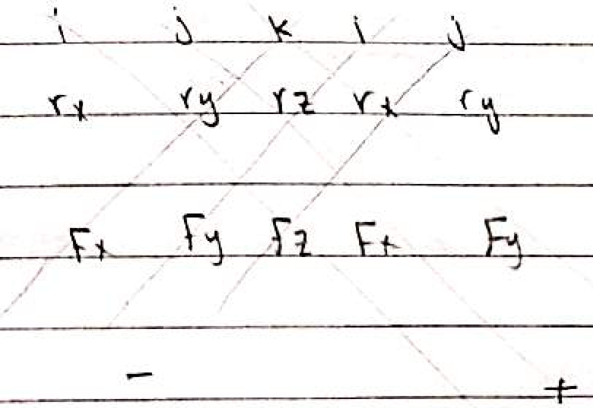
*vector product

$$M_o = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$M_o = i\overset{M_x}{(r_yF_z - r_zF_y)} + j\overset{M_y}{(r_zF_x - r_xF_z)} + k\overset{M_z}{(r_xF_y - r_yF_x)}$$

أو $\ominus$ $j(r_xF_z - r_zF_x)$ *إنتبه للترتيب

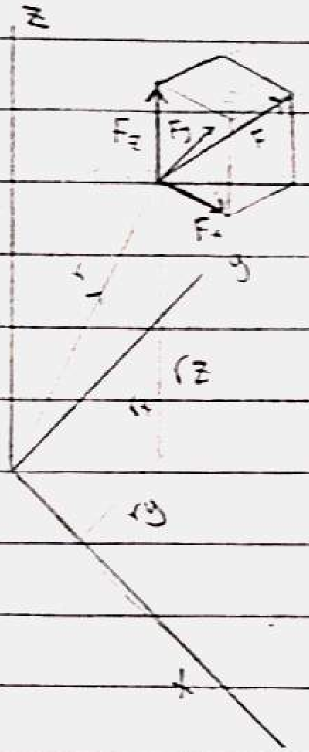
$$M_o = \sqrt{M_x^2 + M_y^2 + M_z^2}$$



$M_x = -F_y\, r_z + F_z \cdot r_y$

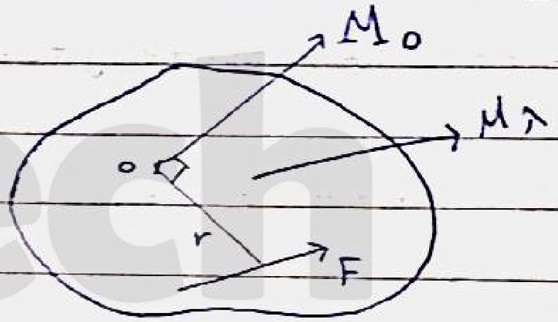
$M_y = +F_x \cdot r_z - F_z\, r_x$

$M_z = +F_y \cdot r_x - F_x \cdot r_y$



$M_\lambda = M_o \cdot \lambda$

$= r \times F \cdot \underline{n} \rightarrow$ unit vector in the direction of λ



$\underline{M}_\lambda = (r \times F \cdot n)\, n$

* Example:

Calculate M_o=?!

T = 1.2 kN

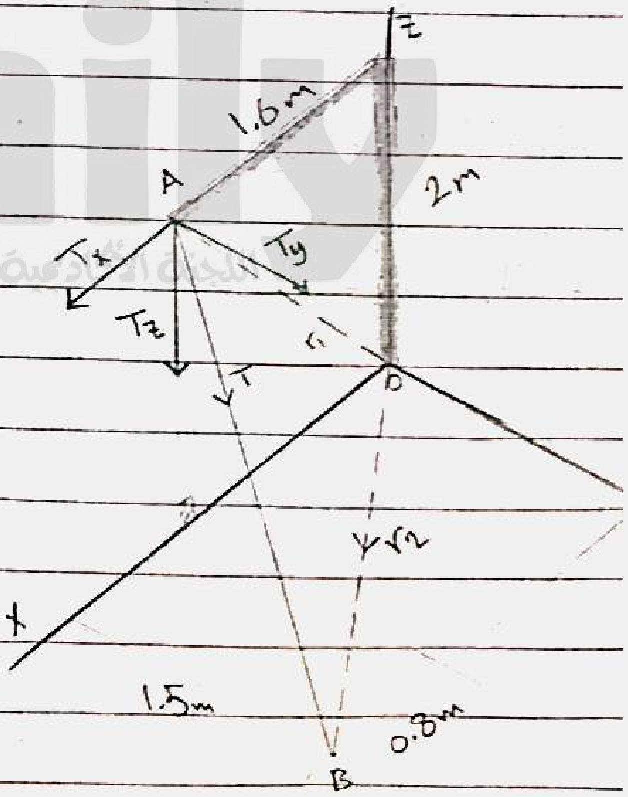
$d_x = 0.8m$

$d_y = 1.5m$

$d_z = -2m$

~~$d = 2.65$~~

$d = 2.625m$



$T_x = \frac{d_x}{d} . T$

~~d_y~~ $T_y = \frac{d_y}{d} . T$

$T_z = \frac{d_z}{d} T$

$$T = iT_x + jT_y + kT_z$$

$$= \frac{1.2}{2.825}(0.8i + 1.5j - 2k)$$

$M_o = r_2 \times T$

r ← أي متجه من o إلى خط عمل T

إمّا r_1 ، r_2

$r = 2.4i + 1.5j$

✓ $M_o = 0.457(-3i + 4.8j + 2.4k)$

$M_o = \sqrt{M_x^2 + M_y^2 + M_z^2} = 2.81$ kN.m.

═

✓ $M_x = -T_y(2) = 1.37$ kN.m $\Rightarrow$ ($T_x \rightarrow$ لا تعمل moment لأنها موازية

$T_z \rightarrow$ لا تعمل moment لأنها تقطع x-axis

$M_y = T_x(2) + T_{\cancel{z}}(1.6) = 2.194$ kN.m

$M_z = T_y(1.6) = 1.097$ kN.m $\Rightarrow$ $T_x \rightarrow$ z-axis تقطع

$M_o = \sqrt{M_x^2 + M_y^2 + M_z^2} = 2.81$ kN.m ✓

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DATE Week.

* Equilibrium :-

A rigid body is said to be in equilibrium when the exerted force acting on it form a system of forces equivalent to zero:

$$R = \Sigma F = 0 \quad \text{and}$$
$$M = \Sigma M = 0$$

* Mechanical System Isolation :-

Free Body Diagram FBD

diagramatic representation of the body or bodies under Consideration showing all forces applied to it by other bodies that imagined to be removed.

* Construction of the F.B.D. :-

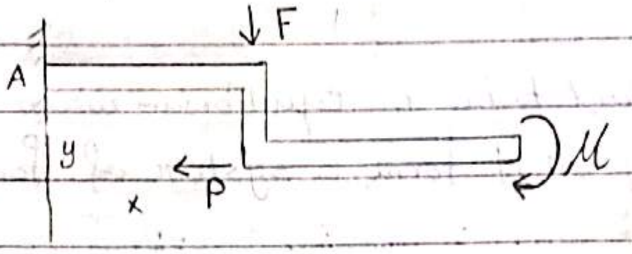
→ step 1 : choice of the Free body to be isolated. A body which gives all unknowns

→ step 2 : Detaching the body from the ground and separating the body by a diagram representing its complete external boundaries.

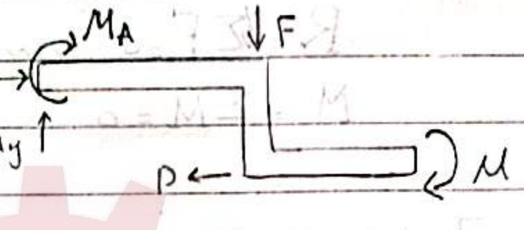
→ step 3 : Indicating all external forces.
→ diagram should include axes, dimentions, directions

* Example:-

*



⟹ F.B.D.



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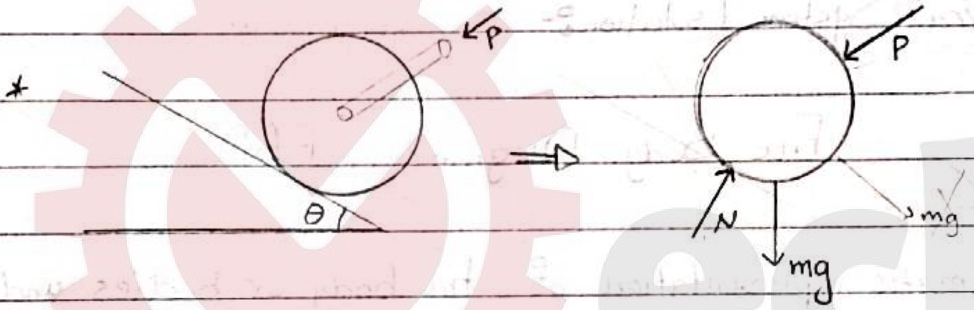

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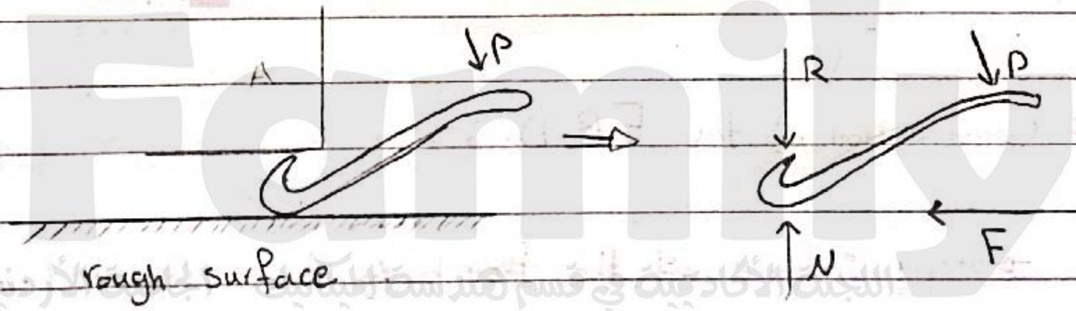

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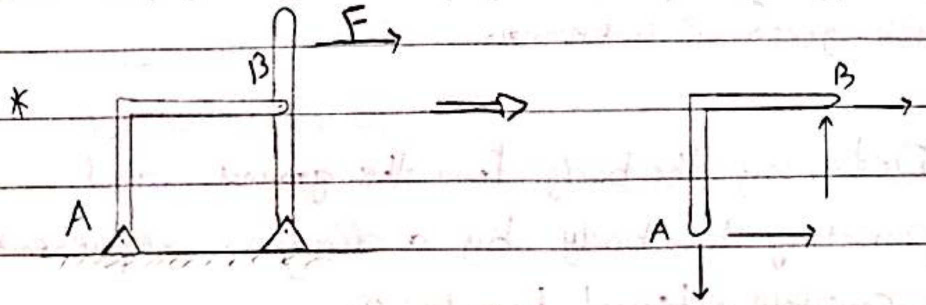
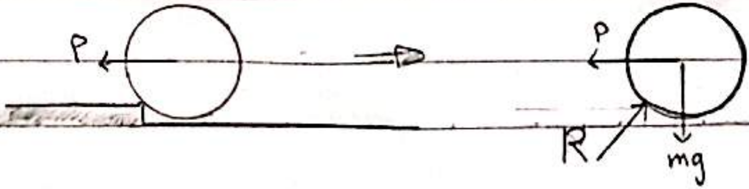
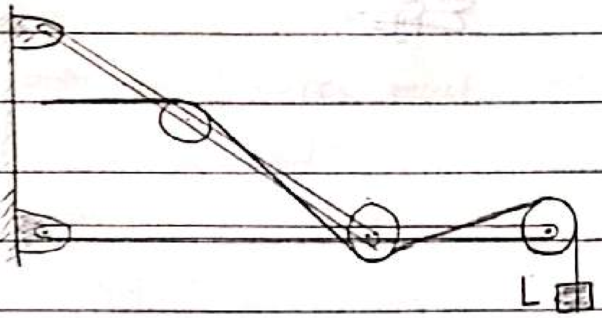


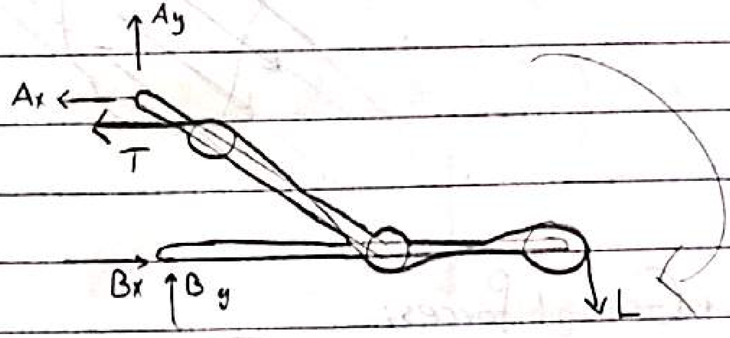
* Example: wheel of mass m on verge of being rolled over curb by Pull P?

* على وشك إنها تنقلب



* Example :-





* Equilibrium Conditions :

The necessary and $\overset{\text{كافية}}{\text{sufficient}}$ conditions for complete equilibrium in 2D are:

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0 \quad \text{and} \quad \Sigma M = 0$$

* Categories of equilibrium:

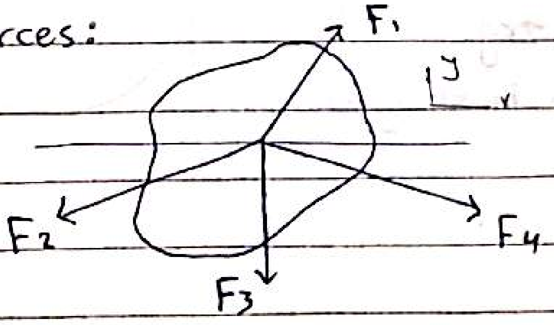
* Collinear forces:

$\Sigma F_x = 0$

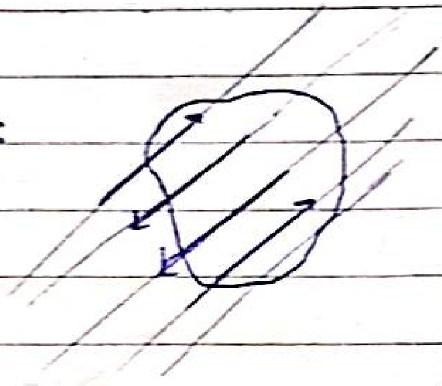
(من نفس النقطة) * Concurrent forces:

$\Sigma F_x = 0$

$\Sigma F_y = 0$



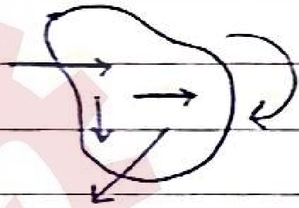
+ Parallel forces:



$\Sigma F_x = 0$, $\Sigma M = 0$

~~$\Sigma F_y =$~~

+ General forces:



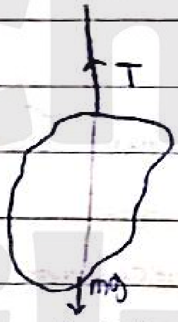
$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$

13/7/2014: Sunday

*Special Cases of Equilibrium:-

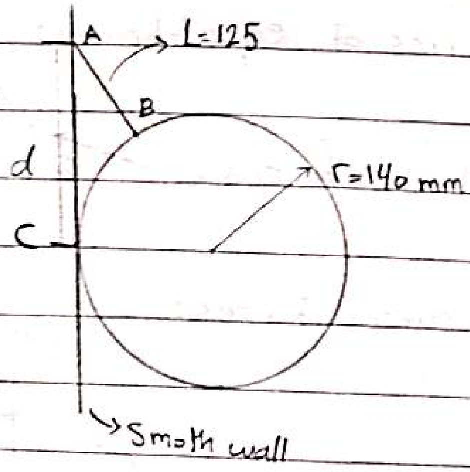


1] Equilibrium of a two-force body.

the two forces must be equal, opposite and collinear.

2] Equilibrium of 3-Force body.

Multi ٣ أختر من the 3 forces must be either concurrent or Parallel



+ Example:

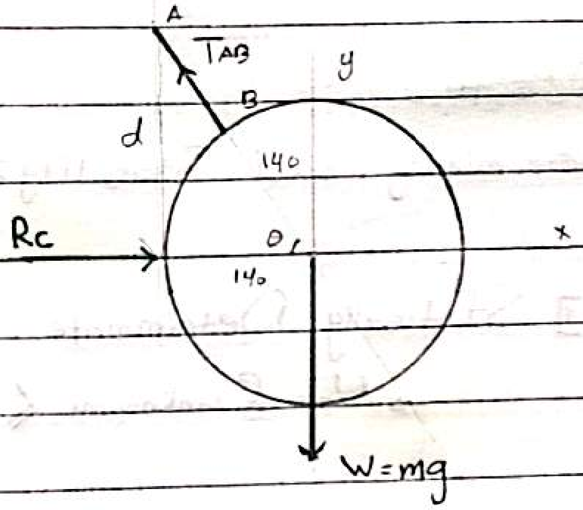
Thin ring of mass 2kg.

Determine d, Tinsino in string AB

R_c

The ring is 3-Force body

⇒ the 3 Forces must be concurrent.



Sol:

$d = \sqrt{265^2 + 140^2} = 225 \text{ mm}$

$+ \Sigma F_x = 0$

$T_{AB} \cos\theta = R_C$

$+ \Sigma F_y = 0$

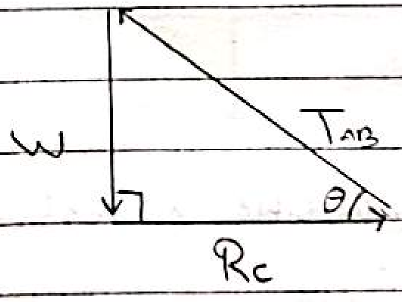
$T_{AB} \sin\theta = W$

$T_{AB} = \frac{2(9.81)}{\sin\theta} = 23.11 N$

⇒ From $\Sigma F_x = 0$

$R_C = 12.21 N$

From the force triangle:



$\frac{T_{AB}}{\sin 90} = \frac{R_C}{\cos\theta} = \frac{W}{\sin\theta}$

14/7/201

~~* Determiac~~

+ Determinacy and Stability:

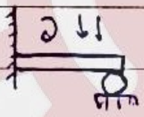
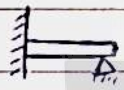
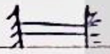
1] Statically Determinate

→ No. of unknowns ≤ Equations of equi.

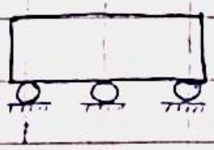
2] Statically Indeterminate

→ No. of unknowns > Equations of equi.

* Examples:-

	Unknowns	Equ.	
	4	3	⇒ 1st degree Indeterminate.
	5	3	⇒ 2nd degree indeterminate
	6	3	⇒ 3rd degree indeterminate.

* Stability:

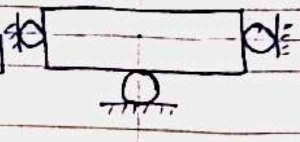
1]  , unstable no restaint against horizontal movement

all constraint parallel

كوابح

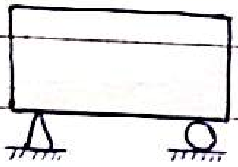
حتى يكون Stable القوى يجب أن لاتكون متوازية ولامتقاطعة

له الناتجة من الدعامات أو الكوابح

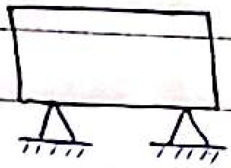
2]  , unstable no restaint against rotation.

contraint concurrent

كوابح



Stable determinate adequate constraints.



Stable Indet.

↳ Redundant constraints (causes extra fixibility)

① 14/7/2014

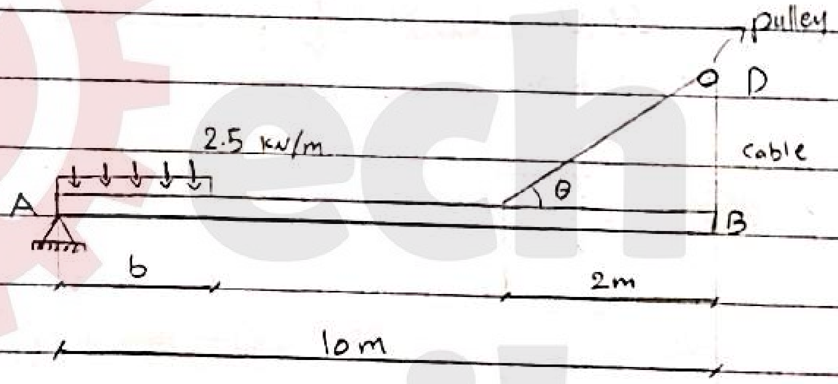
* Example:

$\theta = 60^\circ$

Beam AB of 100kg

If T_{max} in the cable is 800 N

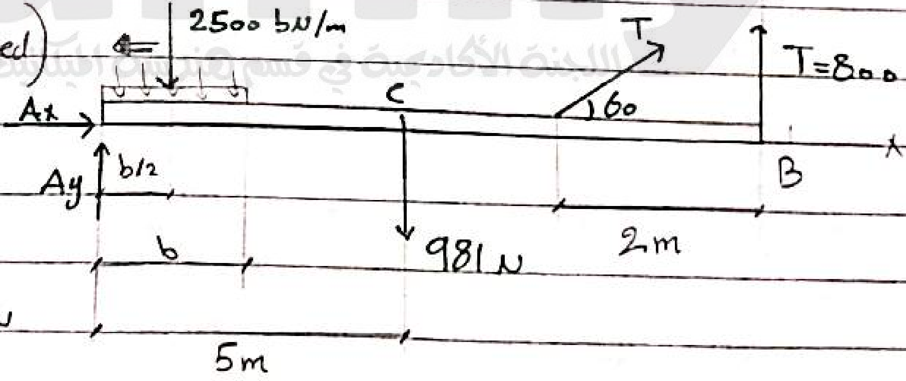
determine b, Ax, and Ay



* Sol:- F.B.D. for the Beam.

2500b : (equivelant concetrated force)

* 2500 N/m → each 1m make force equal to 2500 N on the beam



$$\rightarrow \Sigma M_A = 0 \quad -2500b\left(\frac{b}{2}\right) - 981(5) + T\sin(60) + T(10) = 0$$

$$\Rightarrow b = 2.63\ m$$

~~$\Sigma F_y = 0$~~

$*\Sigma F_x = 0$

$A_x + T\cos 60 = 0$

$A_x = 400\ N \leftarrow$

$\Sigma F_y = 0$

$A_y - 2500(2.63) - 981 + T\sin 60^\circ + T = 0$

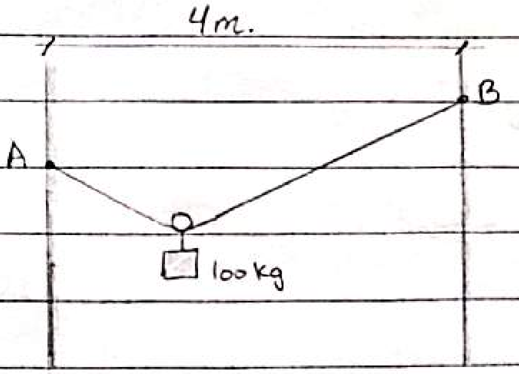
$A_y = 6063\ N$

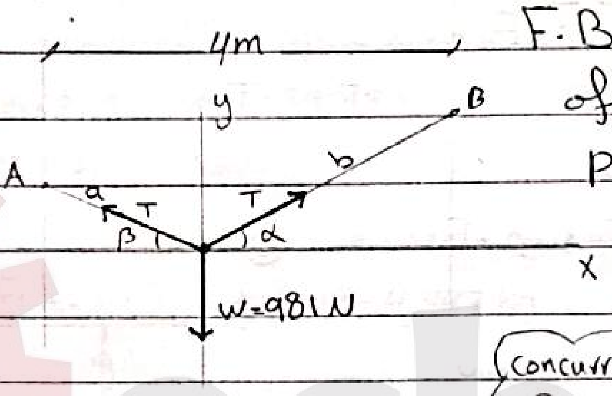
$= 6.06\ kN.$

15/7/2014 Tuesday.

① *Example:

length of cable AB = 6m
Calculate the Tinsion in the cable.





F.B.D of the Pulley.

concurrent forces

*$\Sigma F_x=0$

$T\cos\alpha = T\cos\beta$

$\Rightarrow \alpha=\beta$

from geometry:

$a\cos\beta + b\cos\alpha = 4$

$(a+b)\cos\beta = 4$

$a+b=6 \Rightarrow \beta=\cos^{-1}\frac{4}{6}$, $\beta=48^\circ$

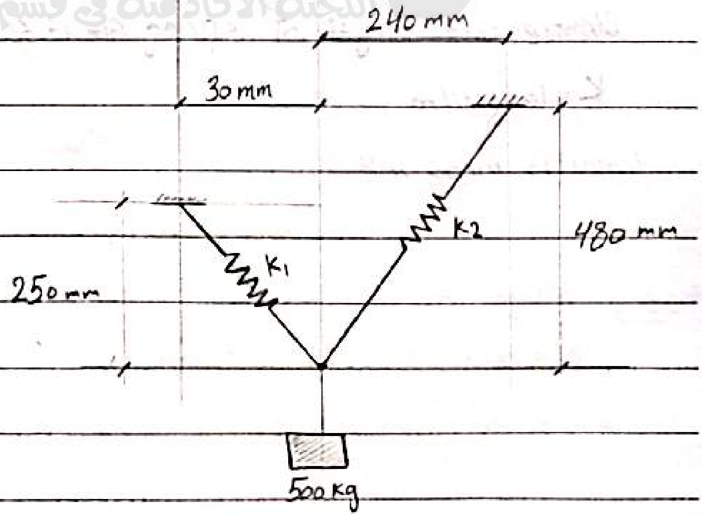
*$\Sigma F_y=0$

$2T\sin\beta = 981N$

$T=658N.$

*Example:

Force in the spring
$F=K.x$
x: elongation or contraction

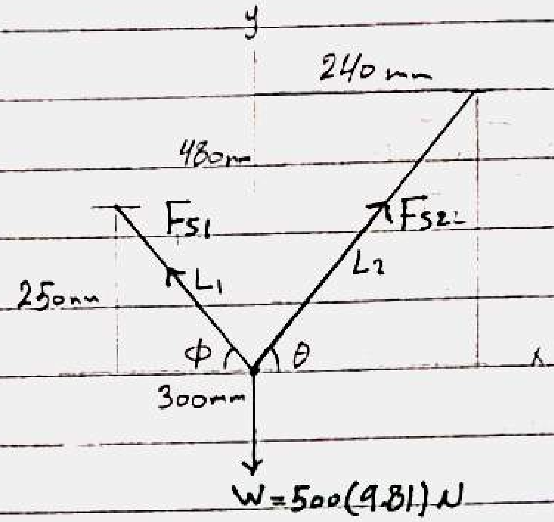


*Springs stifness
$K_1 = 20$ KN/m
$K_2 = 10$ KN/m

Determine the unstretched length of the springs.

$\theta = \tan^{-1} \frac{480}{240} = 63.43^\circ$

$\phi = \tan^{-1} \frac{250}{300} = 39.8^\circ$



$* \sum F_x = 0$

$F_{s2} \cos\theta = F_{s1} \cos\phi \quad \ldots ①$

$+\sum F_y = 0$

$W = F_{s1} \sin\phi + F_{s2} \cos\theta \quad \ldots ②$

$L_1 = 390.5$ mm

$L_2 = 536.7$ mm

$\Rightarrow F_{s1} = 2.253$ kN

$\Rightarrow F_{s2} = 3.871$ kN

$\rightarrow F_{s1} = K_1 . X_1 \Rightarrow L_1 - L_{01} = \frac{F_{s1}}{K_1}$

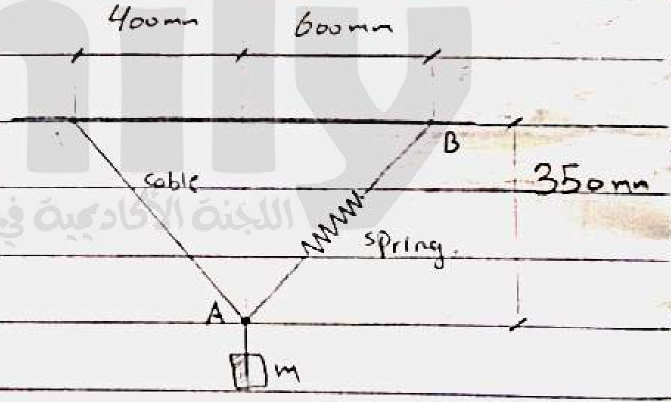
$L_{01} = 278$ mm

الطول الأصلي غير المشدود

$\rightarrow F_{s2} = K_2 X_2 \Rightarrow L_2 - L_{02} = \frac{F_{s2}}{K_2}$

$L_{02} = 150$ mm.

* Example:



Unstretched length of spring AB is 660mm

K=1000 N/m

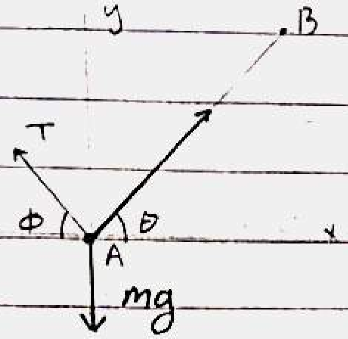
What is mass m?

+Sol: length AB = 694.62 mm

$e = 694.62 - 660 = 34.62$ mm

$\theta = 30.26^\circ$ $(\tan^{-1} \frac{350}{600})$

$\phi = 41.2^\circ$ $(\tan^{-1} \frac{350}{400})$



$F_s = k \cdot e$

$= 1000 \cdot \frac{(34.62)}{1000} \rightarrow$ حوّل لـ m

$= 34.62\ N$

$+ \sum F_x = 0$

$F_s \cos\theta = T\cos\phi \rightarrow T = \frac{\overset{F_s}{(34.62)} * \cos 30.26^\circ}{\cos 41.2^\circ} \Rightarrow T = 39.74\ N$

$+ \sum F_y = 0$

$F_s \sin\theta + T\sin\phi = mg \rightarrow m = \frac{F_s \sin\theta + T\sin\phi}{g} \rightarrow m = 4.447\ kg$

16/7/2014

* Example:

rod AB $m = 40$ kg

wheels 80 kg each

→ Show that the 3 bodies can not be in equi. as shown.