

Thermodynamics: An Engineering Approach, 7th Edition
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Chapter 7

ENTROPY

Objectives

- Apply the second law of thermodynamics to processes.
- Define a new property called *entropy* to quantify the second-law effects.
- Establish the *increase of entropy principle*.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

Key Concepts of Entropy:

1. Disorder and Randomness:

1. Entropy is often described as a measure of the disorder or randomness of a system. A system with higher entropy has more possible microstates (ways the components can be arranged), leading to greater disorder.
2. For example, a gas has higher entropy than a solid because the molecules in a gas are more disordered and can occupy a greater number of positions and velocities.

2. Second Law of Thermodynamics:

1. The Second Law of Thermodynamics states that in an isolated system, the total entropy will either increase or remain constant over time; it never decreases. This law implies that natural processes tend to move towards a state of maximum entropy, or maximum disorder.
2. This is why, for example, heat naturally flows from a hot object to a cold one, leading to a more uniform distribution of energy (higher entropy).

3. Irreversibility:

1. Entropy is closely related to the concept of irreversibility in thermodynamic processes. Processes that increase entropy are irreversible because they result in a greater spread of energy that cannot be fully recovered or reversed.

4. Units of Entropy:

1. The unit of entropy in the International System of Units (SI) is the joule per kelvin (J/K).

5. Mathematical Definition:

- For a reversible process, the change in entropy ΔS can be defined as:

$$\Delta S = \frac{Q_{\text{rev}}}{T}$$

where:

- Q_{rev} is the heat absorbed or released by the system in a reversible process.
- T is the absolute temperature at which the process occurs.
- In statistical mechanics, entropy S is related to the number of microstates Ω by the Boltzmann equation:

$$S = k_B \ln \Omega$$

where:

- k_B is the Boltzmann constant.
- Ω is the number of possible microstates of the system.

6. Entropy and the Universe:

- The concept of entropy is not limited to individual systems; it applies to the entire universe.
- The Second Law of Thermodynamics suggests that the total entropy of the universe is constantly increasing, leading to the idea that the universe is gradually moving towards a state of maximum entropy, often referred to as "heat death," where no more useful work can be extracted from energy differences.

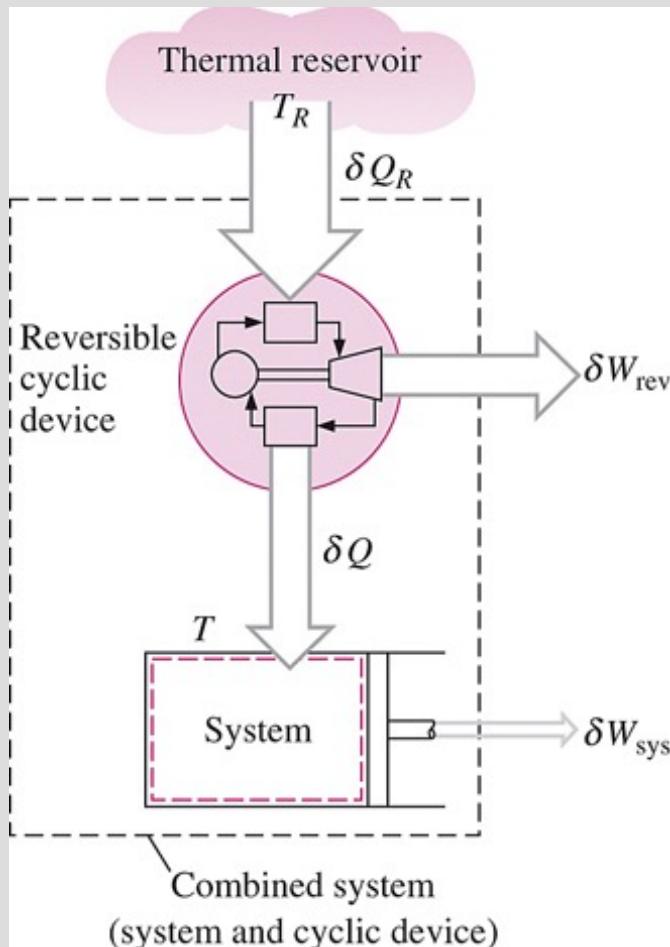
Practical Implications:

- **Heat Engines:** In heat engines, some energy is always lost as waste heat due to the increase in entropy, limiting the engine's efficiency.
- **Refrigeration:** In refrigeration cycles, work is required to decrease entropy in one part of the system, resulting in an increase in entropy elsewhere.
- **Chemical Reactions:** Entropy plays a significant role in determining the spontaneity of chemical reactions. A reaction tends to occur spontaneously if it leads to an increase in the entropy of the universe.

Summary:

Entropy is a measure of disorder and the unavailability of energy to perform work, and it plays a central role in understanding the natural direction of processes and the limitations of energy conversions in thermodynamics.

ENTROPY



The system considered in the development of the Clausius inequality.

$$\oint \frac{\delta Q}{T} \leq 0 \quad \delta W_C = \delta Q_R - dE_C$$

$$\frac{\delta Q_R}{T_R} = \frac{\delta Q}{T} \quad \delta W_C = T_R \frac{\delta Q}{T} - dE_C$$

$$W_C = T_R \oint \frac{\delta Q}{T}$$

$$\oint \frac{\delta Q}{T} \leq 0 \quad \text{Clausius inequality}$$

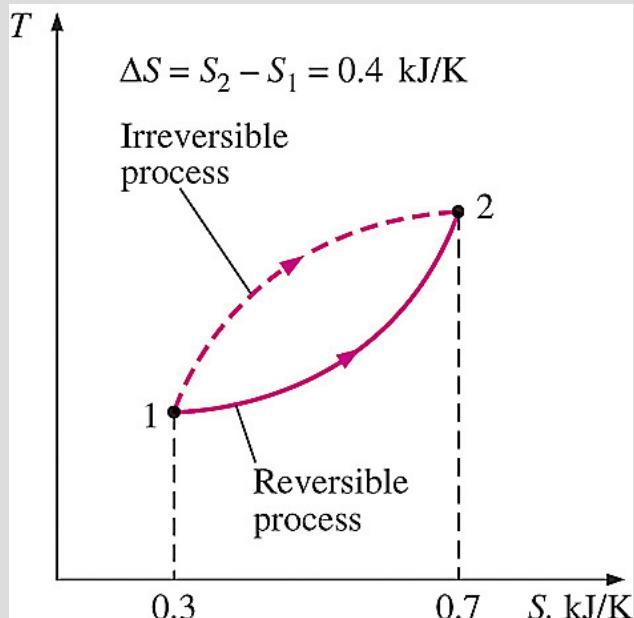
$$\oint \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = 0$$

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \quad (\text{kJ/K})$$

Formal definition of entropy

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}}$$

The equality in the Clausius inequality holds for totally or just internally reversible cycles and the inequality for the irreversible ones.



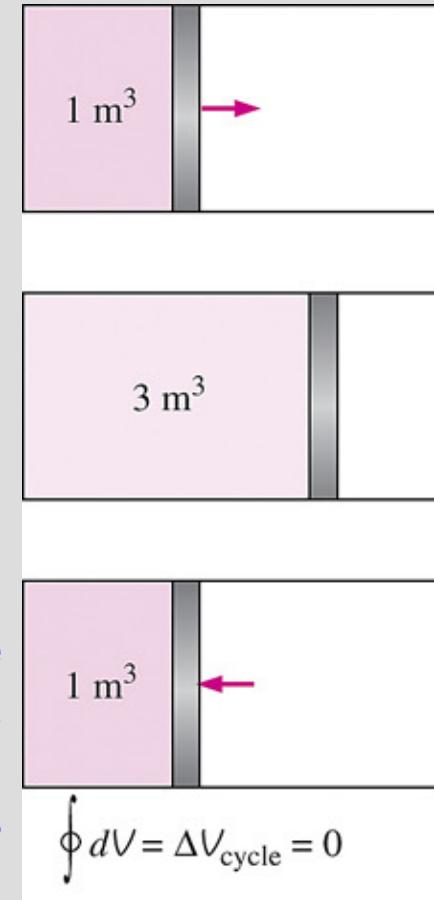
$$\oint \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = 0$$

A quantity whose cyclic integral is zero (i.e., a property like volume)

Entropy is an extensive property of a system.

The entropy change between two specified states is the same whether the process is reversible or irreversible.

The net change in volume (a property) during a cycle is always zero.



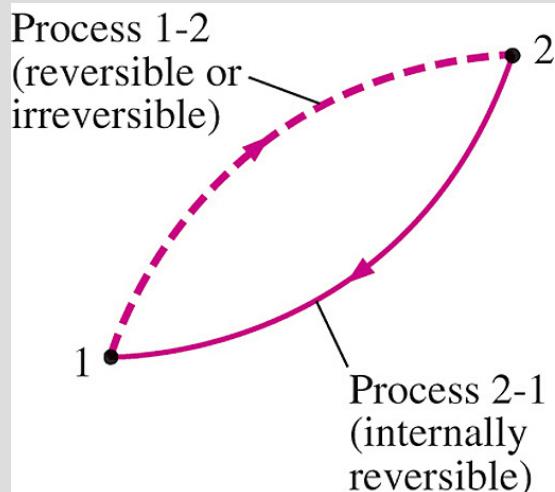
A Special Case: Internally Reversible Isothermal Heat Transfer Processes

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = \int_1^2 \left(\frac{\delta Q}{T_0} \right)_{\text{int rev}} = \frac{1}{T_0} \int_1^2 (\delta Q)_{\text{int rev}}$$

$$\Delta S = \frac{Q}{T_0}$$

This equation is particularly useful for determining the entropy changes of thermal energy reservoirs.

THE INCREASE OF ENTROPY PRINCIPLE



A cycle composed of a reversible and an irreversible process.

$$\oint \frac{\delta Q}{T} \leq 0 \quad \int_1^2 \frac{\delta Q}{T} + \int_2^1 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \leq 0$$

$$\int_1^2 \frac{\delta Q}{T} + S_1 - S_2 \leq 0 \quad S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

$$dS \geq \frac{\delta Q}{T}$$

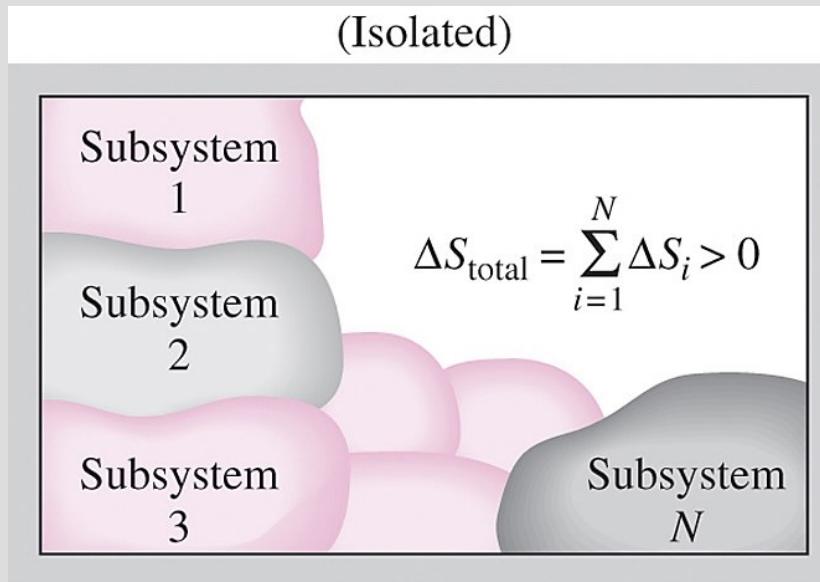
The equality holds for an internally reversible process and the inequality for an irreversible process.

$$\Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$

Some entropy is *generated* or *created* during an irreversible process, and this generation is due entirely to the presence of irreversibilities.

The entropy generation S_{gen} is always a *positive* quantity or zero.
Can the entropy of a system during a process decrease?



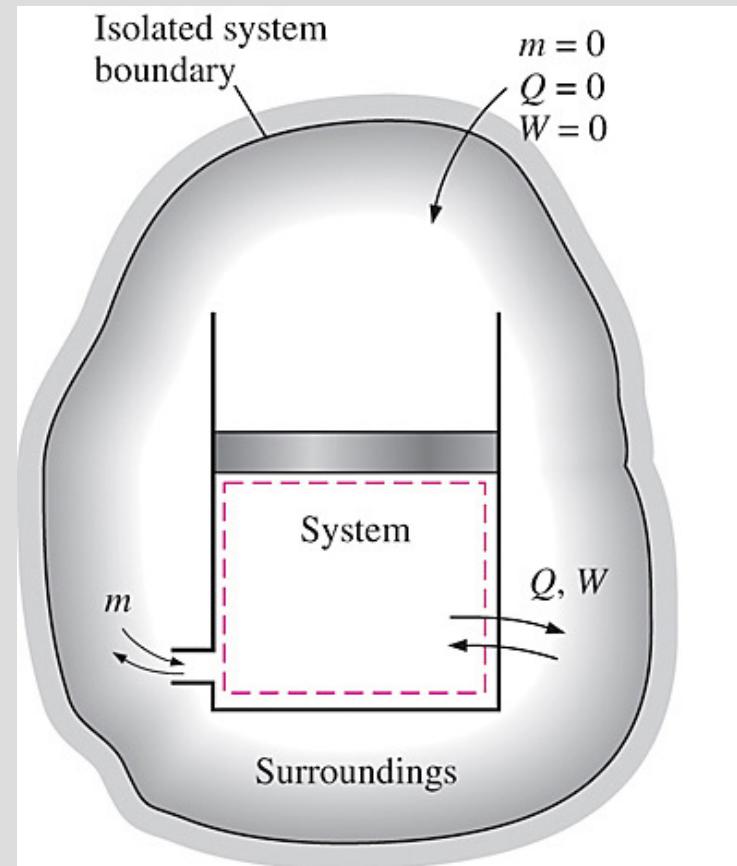
The entropy change of an isolated system is the sum of the entropy changes of its components, and is never less than zero.

$$\Delta S_{\text{isolated}} \geq 0$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$

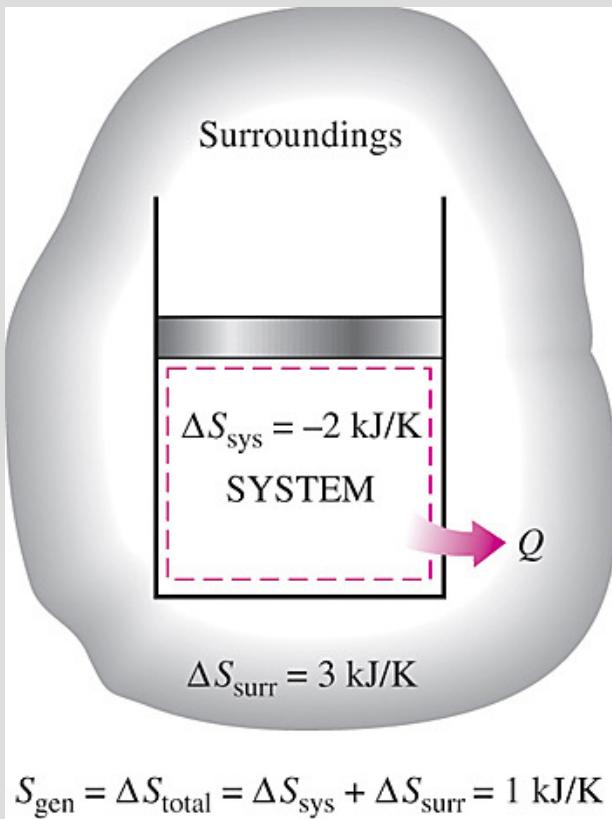
$$S_{\text{gen}} \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases}$$

The increase of entropy principle



A system and its surroundings form an isolated system.

Some Remarks about Entropy

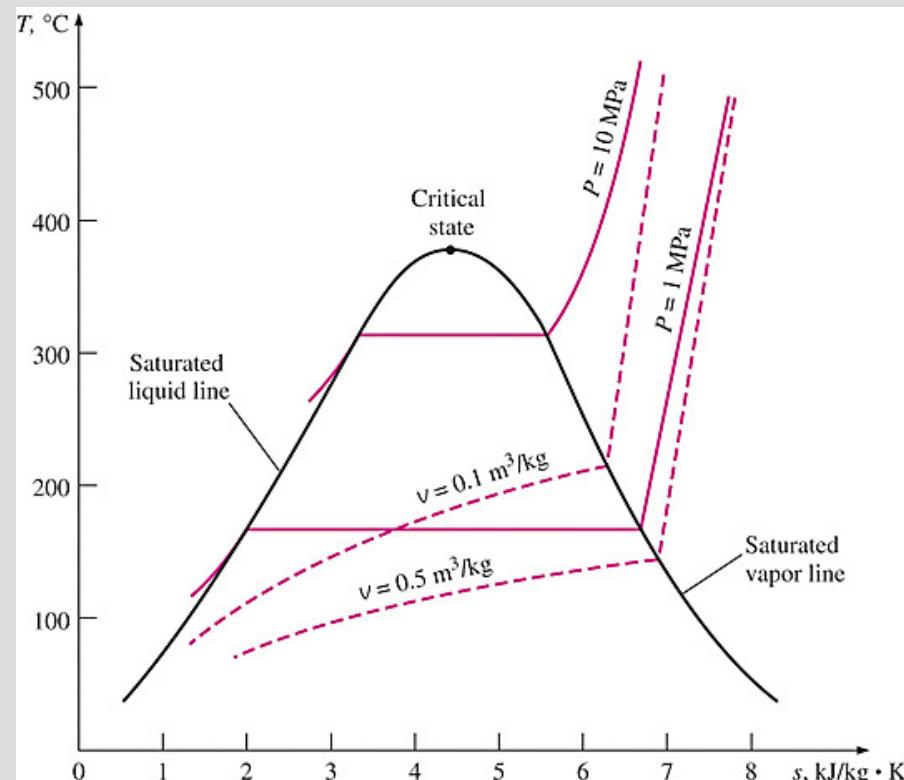
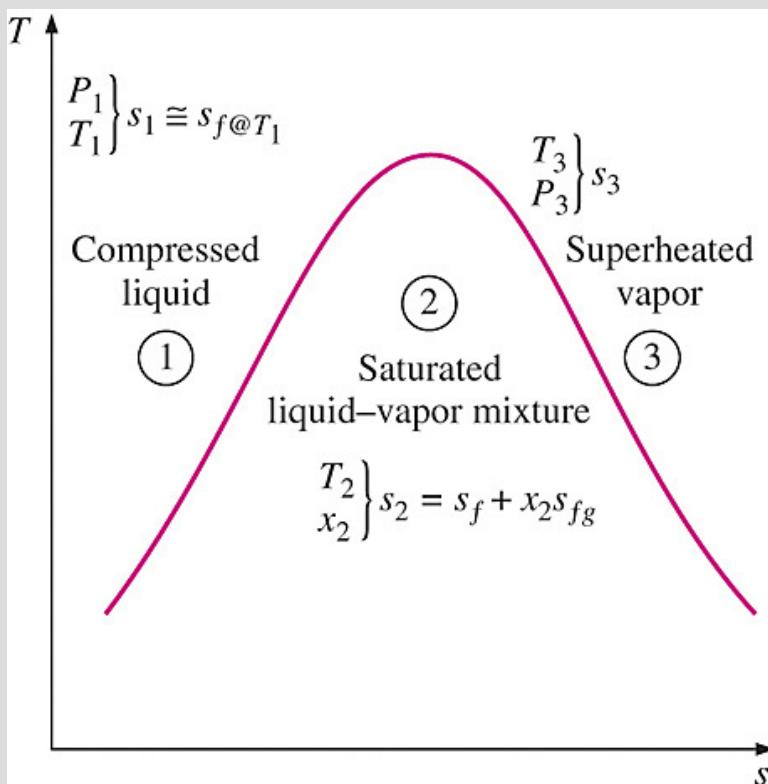


The entropy change of a system can be negative, but the entropy generation cannot.

1. Processes can occur in a *certain* direction only, not in *any* direction. A process must proceed in the direction that complies with the increase of entropy principle, that is, $S_{\text{gen}} \geq 0$. A process that violates this principle is impossible.
2. Entropy is a *nonconserved property*, and there is *no* such thing as the *conservation of entropy principle*. Entropy is conserved during the idealized reversible processes only and increases during *all* actual processes.
3. The performance of engineering systems is degraded by the presence of irreversibilities, and *entropy generation* is a measure of the magnitudes of the irreversibilities during that process. It is also used to establish criteria for the performance of engineering devices.

ENTROPY CHANGE OF PURE SUBSTANCES

Entropy is a property, and thus the value of entropy of a system is fixed once the state of the system is fixed.



Schematic of the T - s diagram for water.

The entropy of a pure substance is determined from the tables (like other properties).

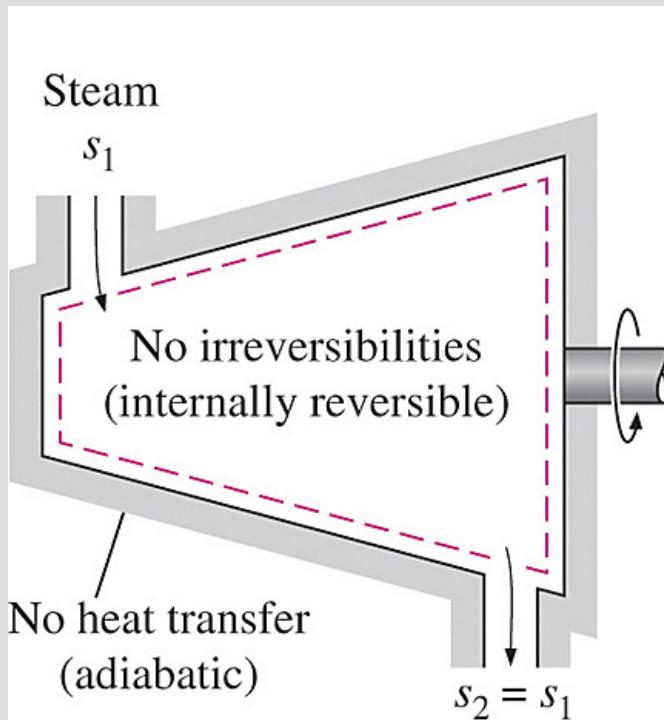
Entropy change

$$\Delta S = m\Delta s = m(s_2 - s_1) \quad (\text{kJ/K})$$

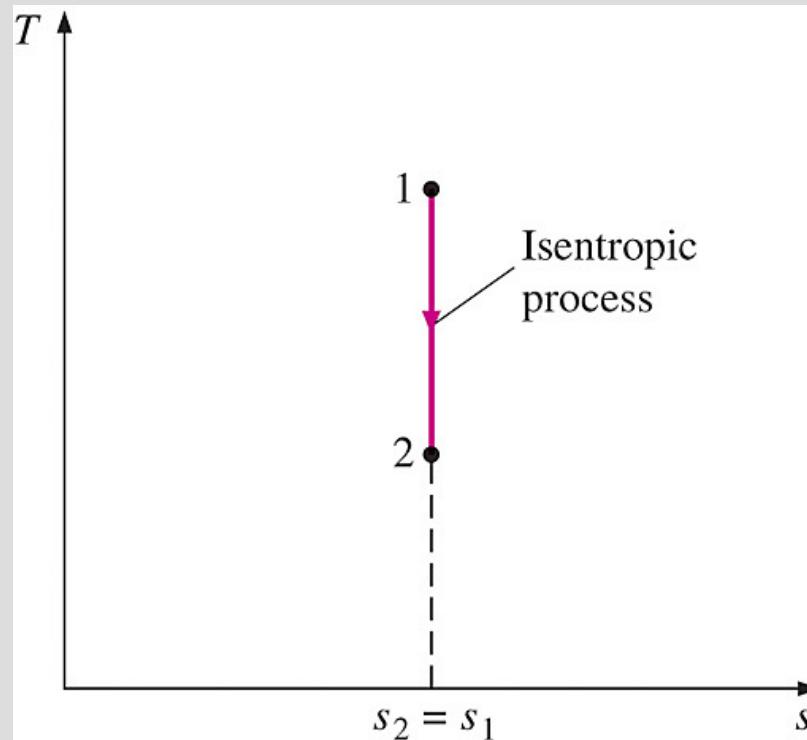
ISENTROPIC PROCESSES

A process during which the entropy remains constant is called an **isentropic process**.

$$\Delta s = 0 \quad \text{or} \quad s_2 = s_1 \quad (\text{kJ/kg} \cdot \text{K})$$

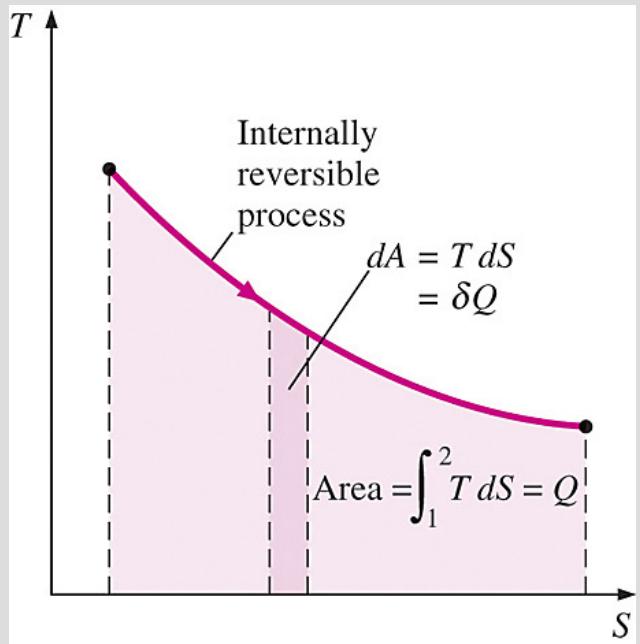


During an internally reversible, adiabatic (isentropic) process, the entropy remains constant.



The isentropic process appears as a *vertical* line segment on a *T-s* diagram.

PROPERTY DIAGRAMS INVOLVING ENTROPY



$$\delta Q_{\text{int rev}} = T \, dS$$

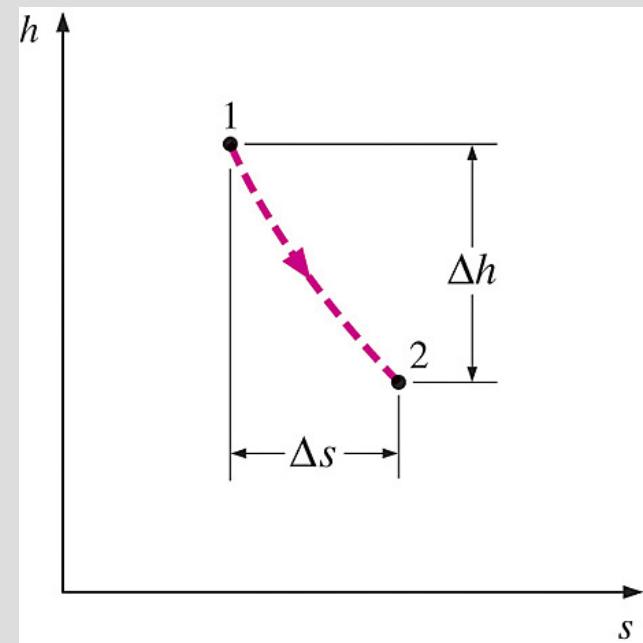
$$Q_{\text{int rev}} = \int_1^2 T \, dS$$

$$\delta q_{\text{int rev}} = T \, ds$$

$$q_{\text{int rev}} = \int_1^2 T \, ds$$

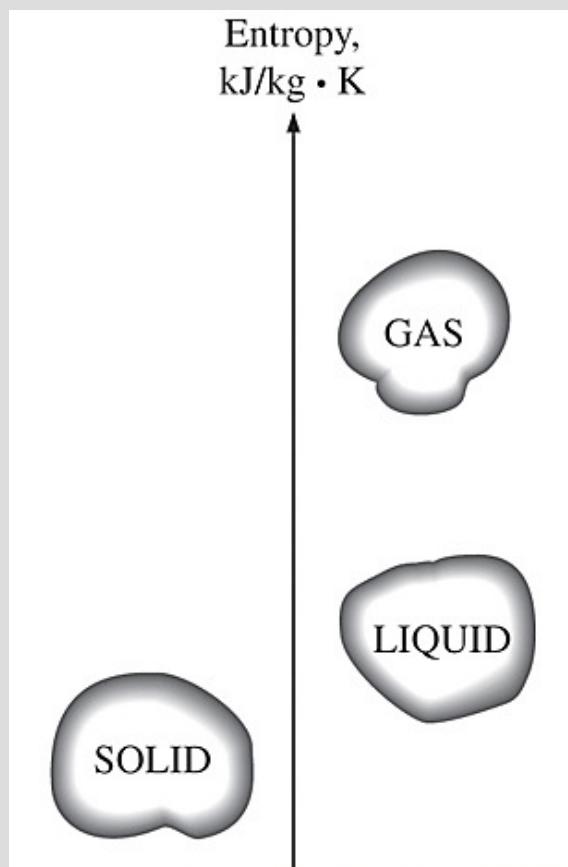
$$Q_{\text{int rev}} = T_0 \, \Delta S$$

$$q_{\text{int rev}} = T_0 \, \Delta s$$



Mollier diagram: The h - s diagram

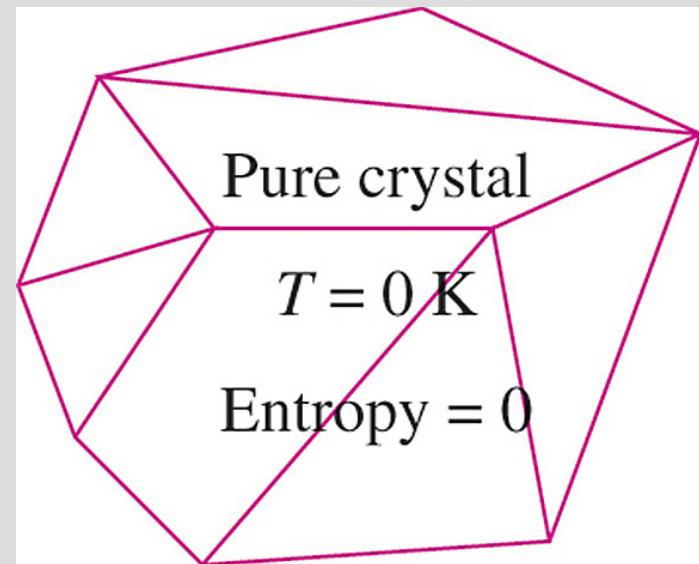
WHAT IS ENTROPY?



Boltzmann relation

$$S = k \ln p$$

$$k = 1.3806 \times 10^{-23} \text{ J/K}$$

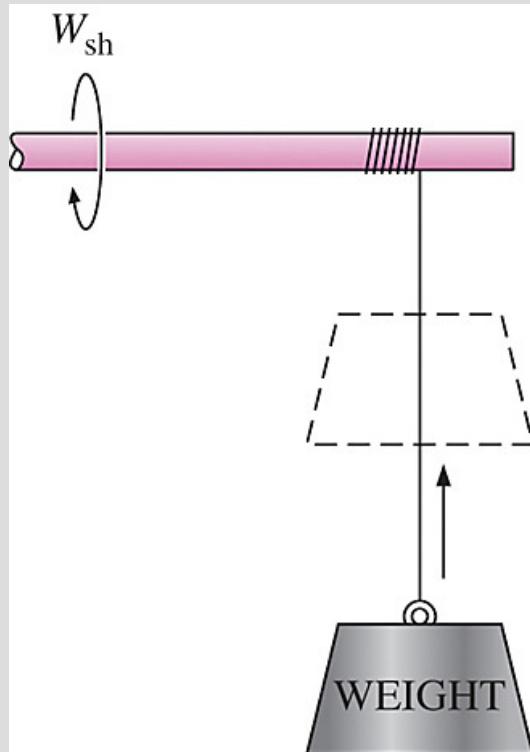


A pure crystalline substance at absolute zero temperature is in perfect order, and its entropy is zero (**the third law of thermodynamics**).

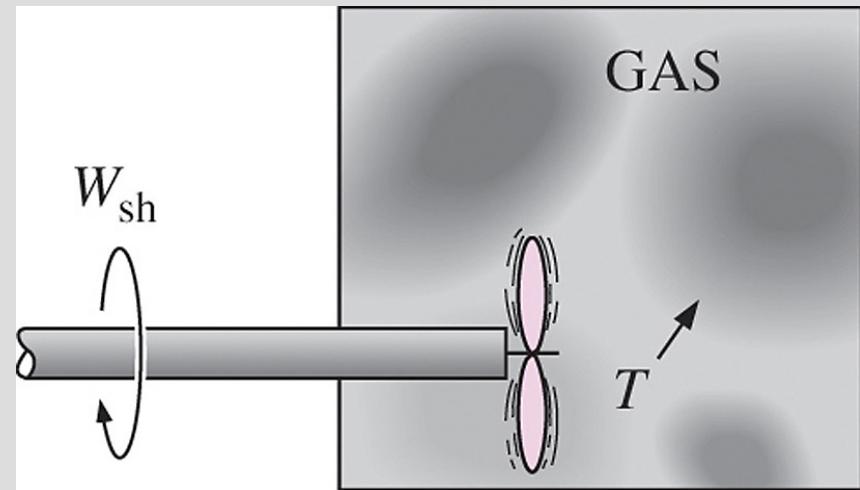
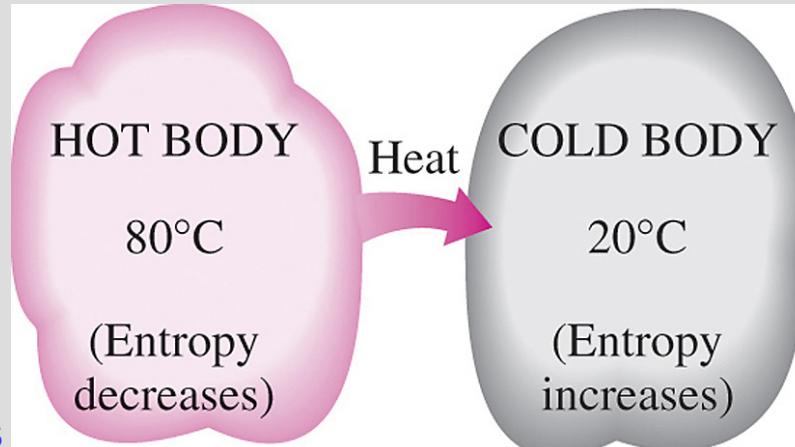


The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.

Disorganized energy does not create much useful effect, no matter how large it is.



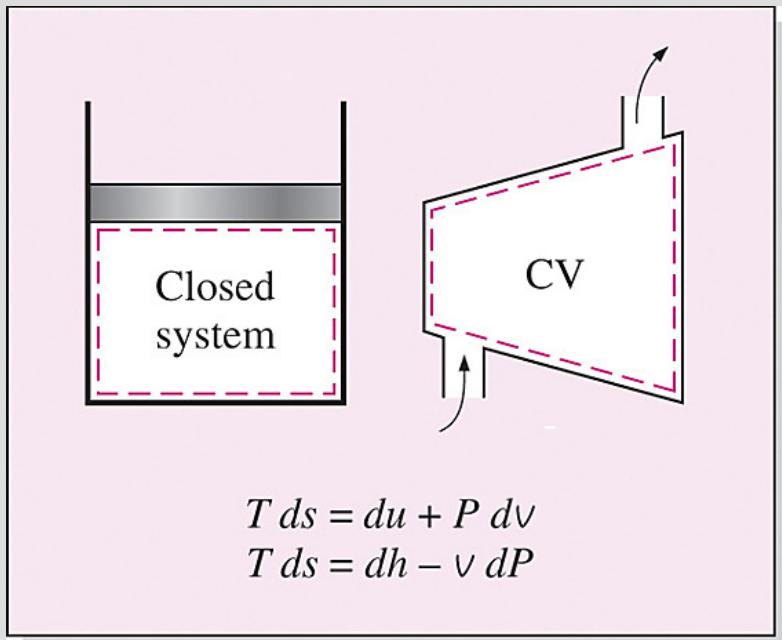
In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.



The paddle-wheel work done on a gas increases the level of disorder (entropy) of the gas, and thus energy is degraded during this process.

During a heat transfer process, the net entropy increases. (The increase in the entropy of the cold body more than offsets the decrease in the entropy of the hot body.)

THE $T ds$ RELATIONS



The $T ds$ relations are valid for both reversible and irreversible processes and for both closed and open systems.

$$\delta Q_{\text{int rev}} - \delta W_{\text{int rev,out}} = dU$$

$$\delta Q_{\text{int rev}} = T ds$$

$$\delta W_{\text{int rev,out}} = P dV$$

$$T ds = dU + P dV \quad (\text{kJ})$$

$$T ds = du + P dv \quad (\text{kJ/kg})$$

the first $T ds$, or *Gibbs equation*

$$h = u + Pv$$

$$dh = du + P dv + v dP \quad \left. \begin{array}{l} \\ T ds = du + P dv \end{array} \right\} T ds = dh - v dP$$

the second $T ds$ equation

$$ds = \frac{du}{T} + \frac{P dv}{T}$$

$$ds = \frac{dh}{T} - \frac{v dP}{T}$$

Differential changes
in entropy in terms
of other properties

ENTROPY CHANGE OF LIQUIDS AND SOLIDS

$$ds = \frac{du}{T} + \frac{P dv}{T}$$

Since $dv \cong 0$ for liquids and solids

$$ds = \frac{du}{T} = \frac{c dT}{T}$$

since $c_p = c_v = c$ and $du = c dT$

Liquids and solids can be approximated as *incompressible substances* since their specific volumes remain nearly constant during a process.

Liquids, solids: $s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \cong c_{\text{avg}} \ln \frac{T_2}{T_1}$ (kJ/kg · K)

For an isentropic process of an incompressible substance

Isentropic: $s_2 - s_1 = c_{\text{avg}} \ln \frac{T_2}{T_1} = 0 \rightarrow T_2 = T_1$

THE ENTROPY CHANGE OF IDEAL GASES

From the first $T \, ds$ relation

$$ds = \frac{du}{T} + \frac{P \, dv}{T} \quad du = c_v \, dT$$
$$P = RT/v$$

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

From the second $T \, ds$ relation

$$ds = \frac{dh}{T} - \frac{v \, dP}{T}$$

$$dh = c_p \, dT \quad v = RT/P$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$Pv = RT$$
$$du = C_v \, dT$$
$$dh = C_p \, dT$$

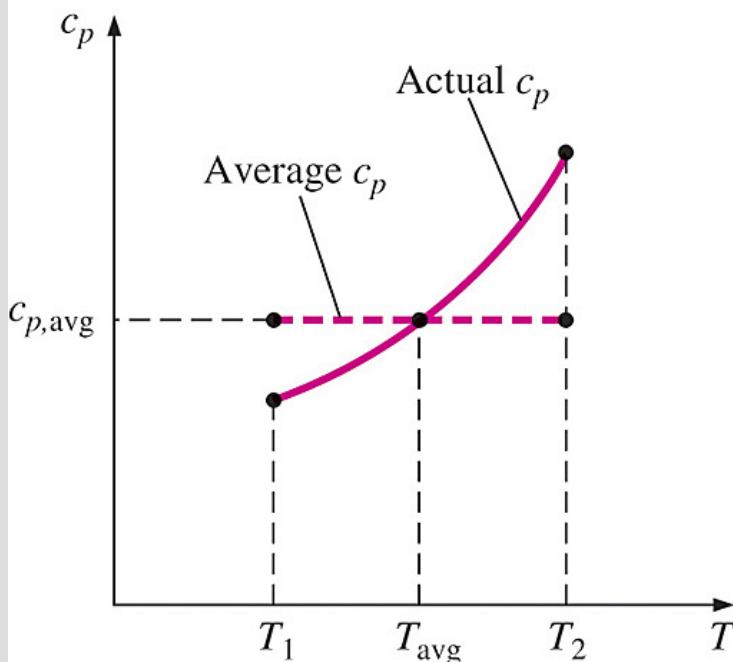
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Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1} \rightarrow s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1} \rightarrow s_2 - s_1 = c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

(kJ/kg · K)



Entropy change of an ideal gas on a unit-mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{v,\text{avg}} \ln \frac{T_2}{T_1} + R_u \ln \frac{v_2}{v_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{p,\text{avg}} \ln \frac{T_2}{T_1} - R_u \ln \frac{P_2}{P_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

Under the constant-specific-heat assumption, the specific heat is assumed to be constant at some average value.

Variable Specific Heats (Exact Analysis)

We choose absolute zero as the reference temperature and define a function s° as

$$s^\circ = \int_0^T c_p(T) \frac{dT}{T}$$

$$\int_1^2 c_p(T) \frac{dT}{T} = s_2^\circ - s_1^\circ$$

On a unit-mass basis

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \quad (\text{kJ/kg} \cdot \text{K})$$

On a unit-mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{s}_2^\circ - \bar{s}_1^\circ - R_u \ln \frac{P_2}{P_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

T, K	$s^\circ, \text{kJ/kg} \cdot \text{K}$
.	.
.	.
300	1.70203
310	1.73498
320	1.76690
.	.
.	.
.	.
(Table A-17)	

The entropy of an ideal gas depends on both T and P . The function s represents only the temperature-dependent part of entropy.

Isentropic Processes of Ideal Gases

Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Setting this eq. equal to zero, we get

$$\ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_2}{v_1}$$

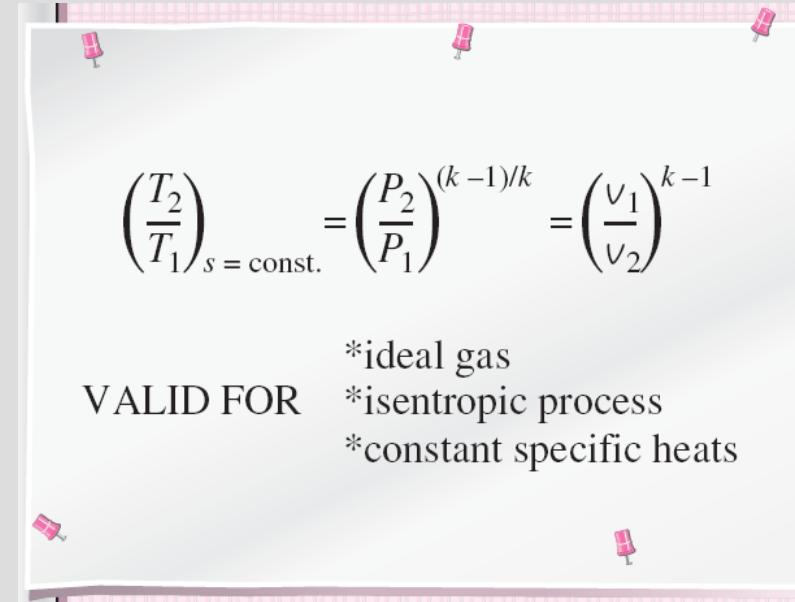
$$\ln \frac{T_2}{T_1} = \ln \left(\frac{v_1}{v_2} \right)^{R/c_v}$$

$$R = c_p - c_v, k = c_p/c_v$$

and thus $R/c_v = k - 1$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const.}} = \left(\frac{v_1}{v_2} \right)^{k-1}$$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const.}} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k} \quad \left(\frac{P_2}{P_1} \right)_{s=\text{const.}} = \left(\frac{v_1}{v_2} \right)^k$$



The isentropic relations of ideal gases are valid for the isentropic processes of ideal gases only.

$$Tv^{k-1} = \text{constant}$$

$$TP^{(1-k)/k} = \text{constant}$$

$$Pv^k = \text{constant}$$

Isentropic Processes of Ideal Gases

Variable Specific Heats (Exact Analysis)

$$0 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \rightarrow s_2^\circ = s_1^\circ + R \ln \frac{P_2}{P_1}$$

Relative Pressure and Relative Specific Volume

$$\frac{P_2}{P_1} = \exp \frac{s_2^\circ - s_1^\circ}{R}$$

$\exp(s^\circ/R)$ is the relative pressure P_r .

$$\frac{P_2}{P_1} = \frac{\exp(s_2^\circ/R)}{\exp(s_1^\circ/R)}$$

$$\left(\frac{P_2}{P_1} \right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \rightarrow \frac{v_2}{v_1} = \frac{T_2}{T_1} \frac{P_1}{P_2} = \frac{T_2}{T_1} \frac{P_{r1}}{P_{r2}} = \frac{T_2/P_{r2}}{T_1/P_{r1}}$$

$$\left(\frac{v_2}{v_1} \right)_{s=\text{const.}} = \frac{v_{r2}}{v_{r1}}$$

T/P_r is the relative specific volume v_r .

The use of P_r data for calculating the final temperature during an isentropic process.

Process: isentropic
Given: P_1 , T_1 , and P_2
Find: T_2

$$\begin{array}{c} \frac{T}{T_1} \\ \vdots \\ T_2 \end{array} \quad \begin{array}{c} \frac{P_r}{P_{r1}} \\ \vdots \\ P_{r2} \end{array}$$

$\xrightarrow{\text{read}} \quad \xleftarrow{\text{read}}$

$$P_{r2} = \frac{P_2}{P_1} P_{r1}$$

Process: isentropic
Given: v_1 , T_1 , and v_2
Find: T_2

$$\begin{array}{c} \frac{T}{T_1} \\ \vdots \\ T_2 \end{array} \quad \begin{array}{c} \frac{v_r}{v_{r1}} \\ \vdots \\ v_{r2} \end{array}$$

$\xrightarrow{\text{read}} \quad \xleftarrow{\text{read}}$

$$v_{r2} = \frac{v_2}{v_1} v_{r1}$$

The use of v_r data for calculating the final temperature during an isentropic process

REVERSIBLE STEADY-FLOW WORK

$$\delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + dke + dpe$$

$$\left. \begin{aligned} \delta q_{\text{rev}} &= T ds & (\text{Eq. 7-16}) \\ T ds &= dh - v dP & (\text{Eq. 7-24}) \end{aligned} \right\} \quad \delta q_{\text{rev}} = dh - v dP$$

$$-\delta w_{\text{rev}} = v dP + dke + dpe$$

$$w_{\text{rev}} = - \int_1^2 v dP - \Delta \text{ke} - \Delta \text{pe}$$

$$w_{\text{rev}} = - \int_1^2 v dP$$

When kinetic and potential energies are negligible

$$w_{\text{rev,in}} = \int_1^2 v dP + \Delta \text{ke} + \Delta \text{pe}$$

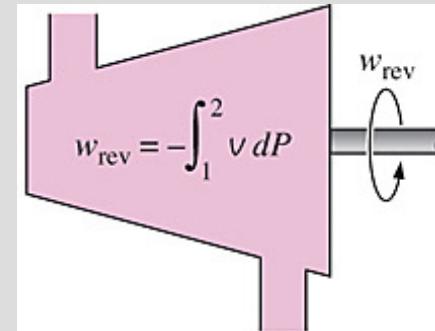
$$w_{\text{rev}} = -v(P_2 - P_1) - \Delta \text{ke} - \Delta \text{pe}$$

For the steady flow of a liquid through a device that involves no work interactions (such as a pipe section), the work term is zero (**Bernoulli equation**):

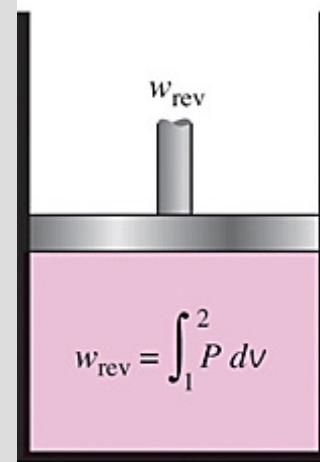
$$v(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

$$\begin{aligned} w &= - \int_1^2 v dP \\ w &= - \int_1^2 V dP \\ \mathcal{W} &= - \int_1^2 V dP \end{aligned}$$

The larger the specific volume, the greater the work produced (or consumed) by a steady-flow device.



(a) Steady-flow system



(b) Closed system

Reversible work relations for steady-flow and closed systems.

Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work when the Process Is Reversible

Taking heat input and work output positive:

$$\delta q_{\text{act}} - \delta w_{\text{act}} = dh + dke + dpe \quad \text{Actual}$$

$$\delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + dke + dpe \quad \text{Reversible}$$

$$\delta q_{\text{act}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta w_{\text{rev}}$$

$$\delta w_{\text{rev}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta q_{\text{act}}$$

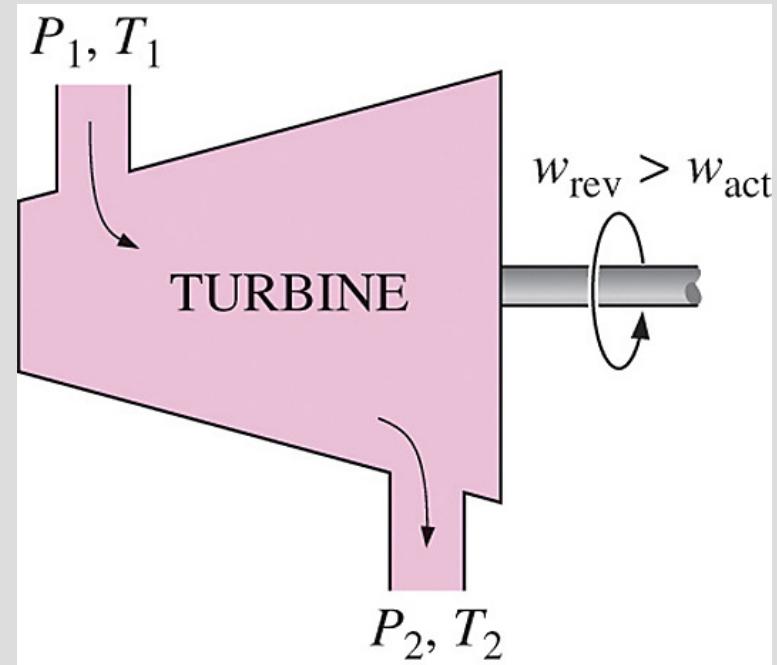
$$\delta q_{\text{rev}} = T \, ds \quad ds \geq \frac{\delta q_{\text{act}}}{T}$$

$$\frac{\delta w_{\text{rev}} - \delta w_{\text{act}}}{T} = ds - \frac{\delta q_{\text{act}}}{T} \geq 0$$

$$\delta w_{\text{rev}} \geq \delta w_{\text{act}}$$

$$w_{\text{rev}} \geq w_{\text{act}}$$

Work-producing devices such as turbines deliver more work, and work-consuming devices such as pumps and compressors require less work when they operate reversibly.



A reversible turbine delivers more work than an irreversible one if both operate between the same end states.

MINIMIZING THE COMPRESSOR WORK

$$w_{\text{rev,in}} = \int_1^2 v \, dP$$

When kinetic and potential energies are negligible

Isentropic ($Pv^k = \text{constant}$):

$$w_{\text{comp,in}} = \frac{kR(T_2 - T_1)}{k - 1} = \frac{kRT_1}{k - 1} \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right]$$

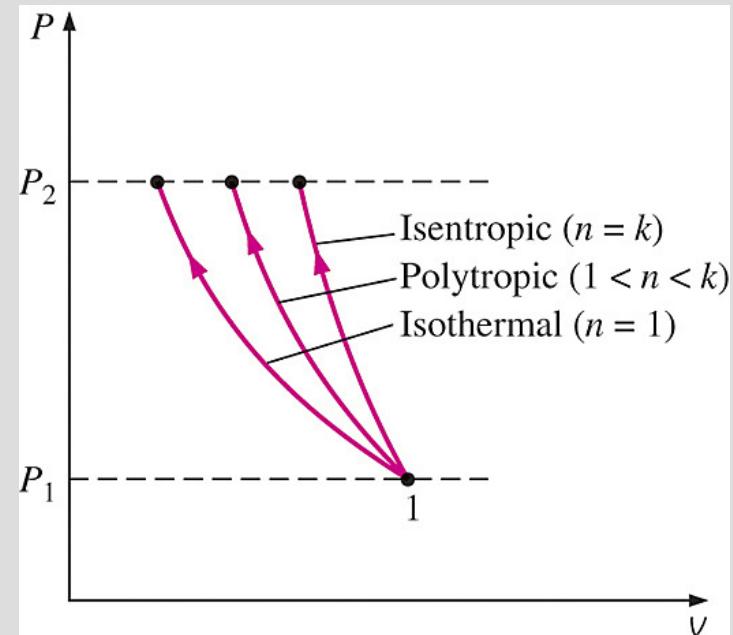
Polytropic ($Pv^n = \text{constant}$):

$$w_{\text{comp,in}} = \frac{nR(T_2 - T_1)}{n - 1} = \frac{nRT_1}{n - 1} \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]$$

Isothermal ($Pv = \text{constant}$):

$$w_{\text{comp,in}} = RT \ln \frac{P_2}{P_1}$$

The adiabatic compression ($Pv^k = \text{constant}$) requires the maximum work and the isothermal compression ($T = \text{constant}$) requires the minimum. **Why?**

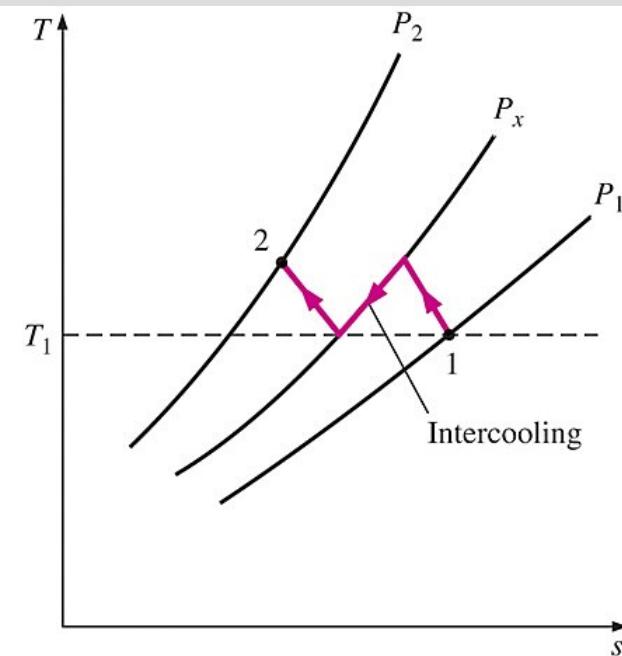
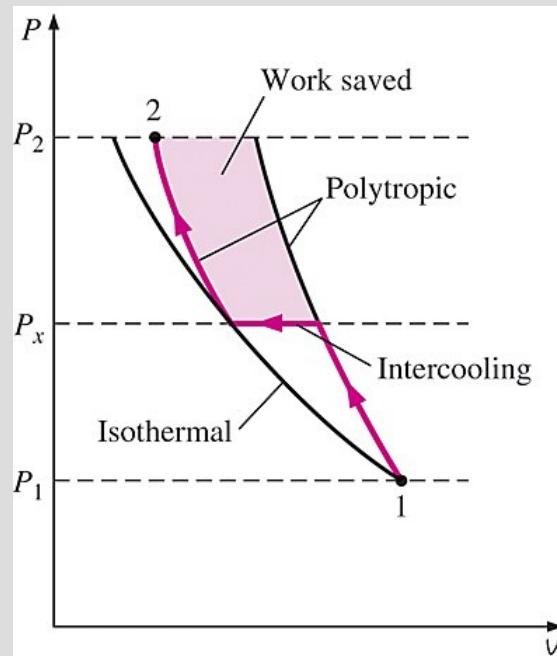


P-v diagrams of isentropic, polytropic, and isothermal compression processes between the same pressure limits.

Multistage Compression with Intercooling

The gas is compressed in stages and cooled between each stage by passing it through a heat exchanger called an *intercooler*.

P-v and T-s
diagrams for a two-stage steady-flow compression process.



$$w_{\text{comp,in}} = w_{\text{comp I,in}} + w_{\text{comp II,in}}$$

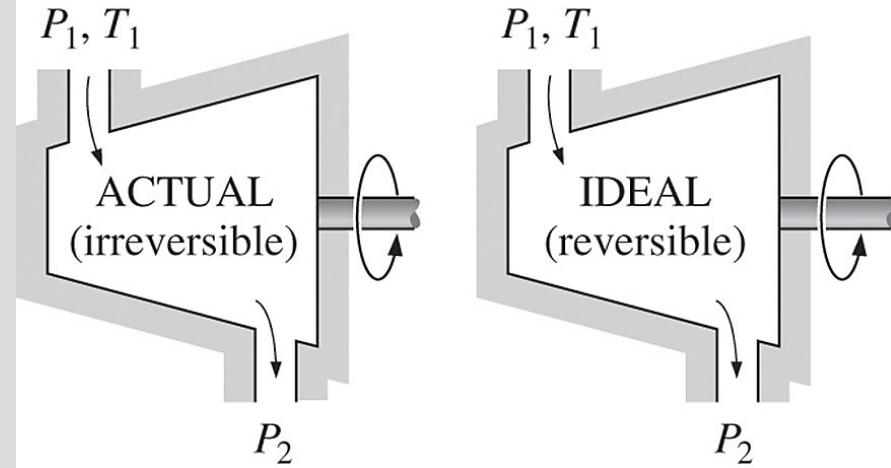
$$= \frac{nRT_1}{n-1} \left[\left(\frac{P_x}{P_1} \right)^{(n-1)/n} - 1 \right] + \frac{nRT_1}{n-1} \left[\left(\frac{P_2}{P_x} \right)^{(n-1)/n} - 1 \right]$$

$$P_x = (P_1 P_2)^{1/2} \quad \text{or} \quad \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

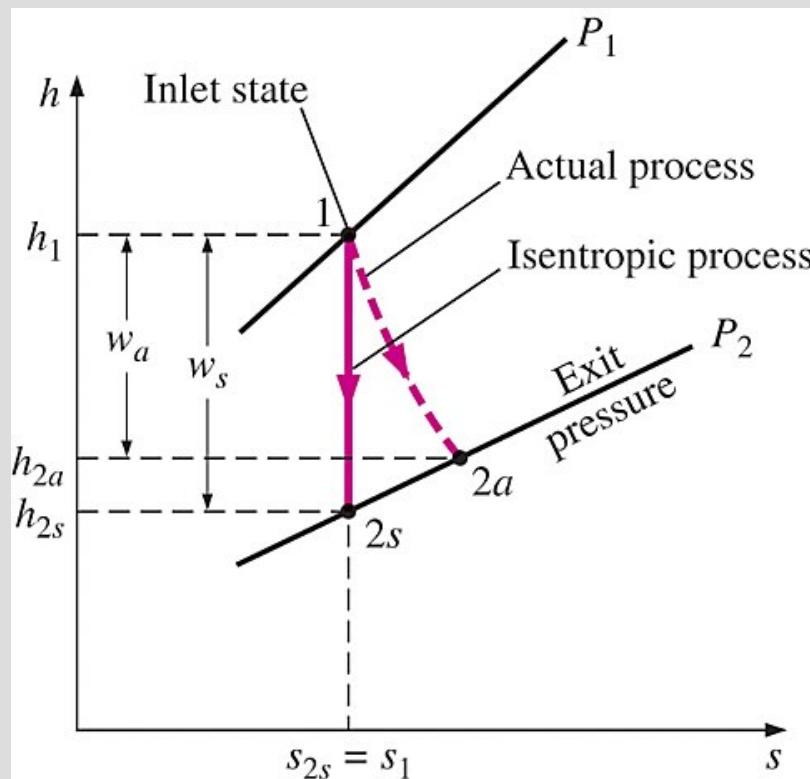
To minimize compression work during two-stage compression, the pressure ratio across each stage of the compressor must be the same.

ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

The isentropic process involves no irreversibilities and serves as the ideal process for **adiabatic devices**.



Isentropic Efficiency of Turbines



$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

The h - s diagram for the actual and isentropic processes of an adiabatic turbine.

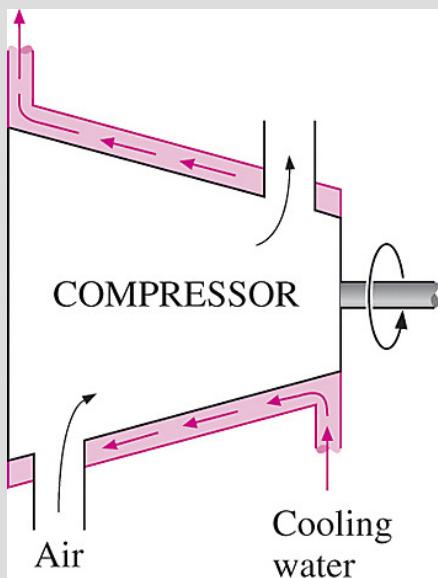
Isentropic Efficiencies of Compressors and Pumps

$$\eta_c = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

$$\eta_c \approx \frac{h_{2s} - h_1}{h_{2a} - h_1} \quad \text{When kinetic and potential energies are negligible}$$

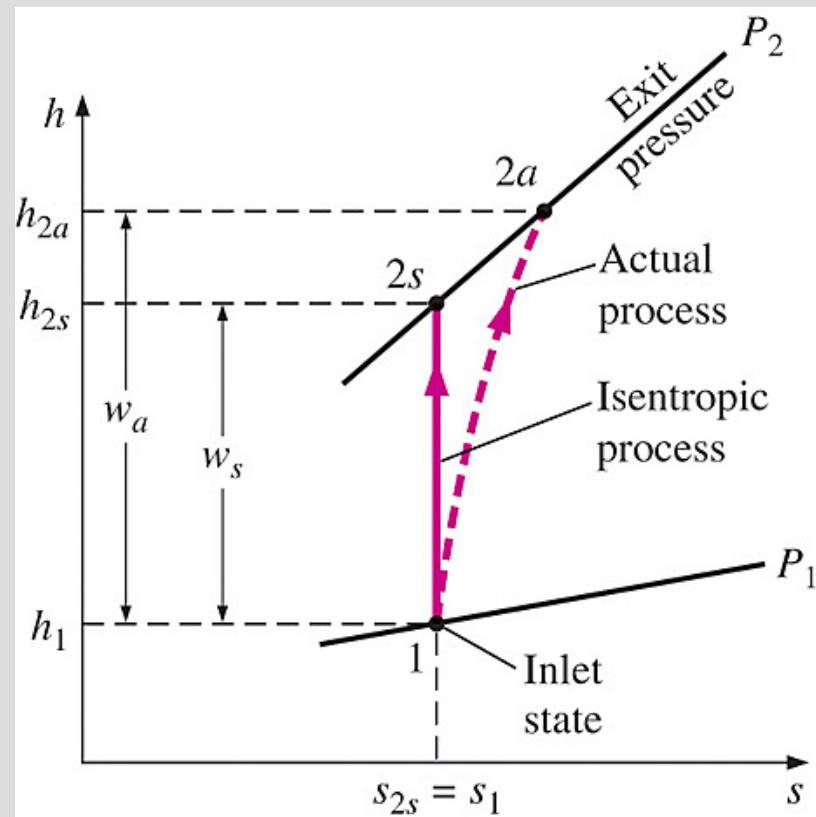
$$\eta_p = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1} \quad \text{For a pump}$$

$$\eta_c = \frac{w_t}{w_a} \quad \text{Isothermal efficiency}$$



Compressors are sometimes intentionally cooled to minimize the work input.

The *h-s* diagram of the actual and isentropic processes of an adiabatic compressor.



Can you use isentropic efficiency for a non-adiabatic compressor?

Can you use isothermal efficiency for an adiabatic compressor?

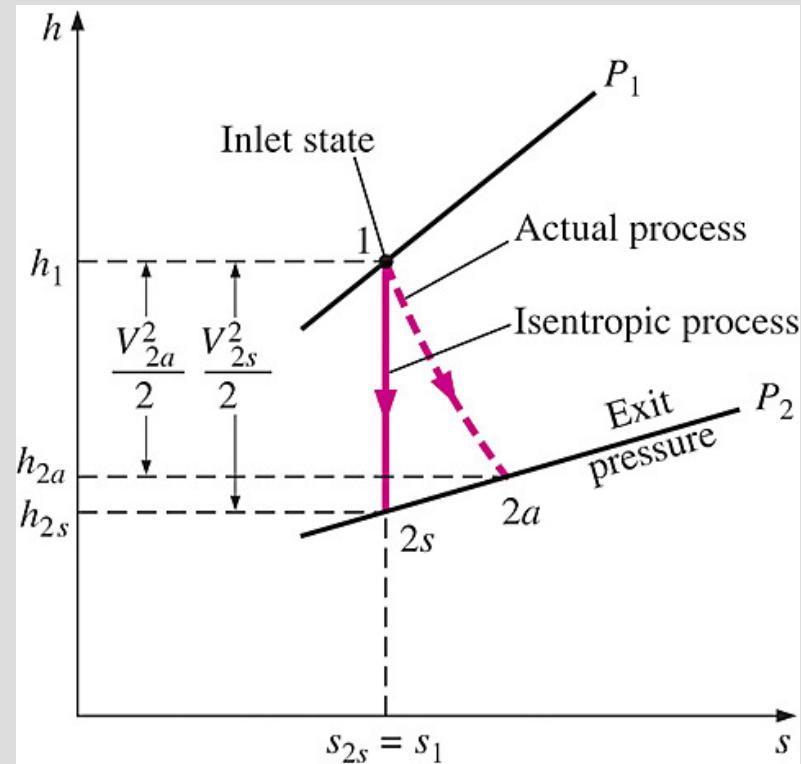
Isentropic Efficiency of Nozzles

$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2}$$

If the inlet velocity of the fluid is small relative to the exit velocity, the energy balance is

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

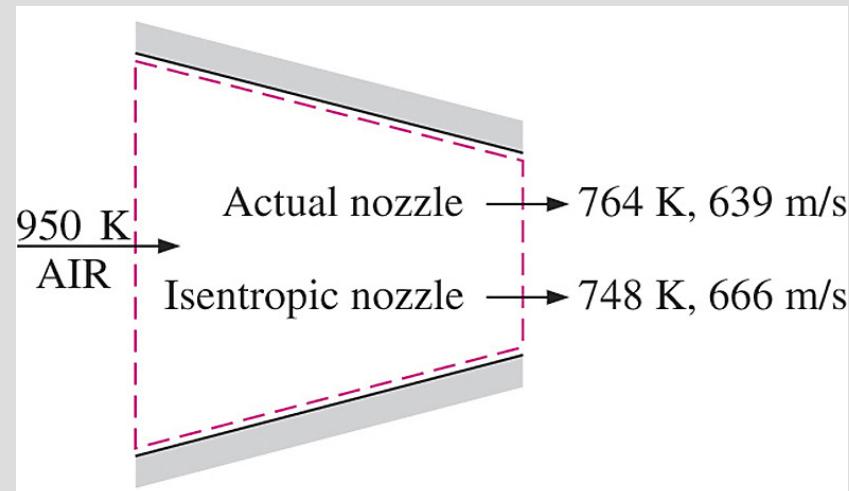
The *h-s* diagram of the actual and isentropic processes of an adiabatic nozzle.



Then,

$$\eta_T \equiv \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

A substance leaves actual nozzles at a higher temperature (thus a lower velocity) as a result of friction.



ENTROPY BALANCE

$$\left(\begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{entering} \end{array} \right) - \left(\begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{leaving} \end{array} \right) + \left(\begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{generated} \end{array} \right) = \left(\begin{array}{c} \text{Change in the} \\ \text{total entropy} \\ \text{of the system} \end{array} \right)$$

$$S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{system}}$$

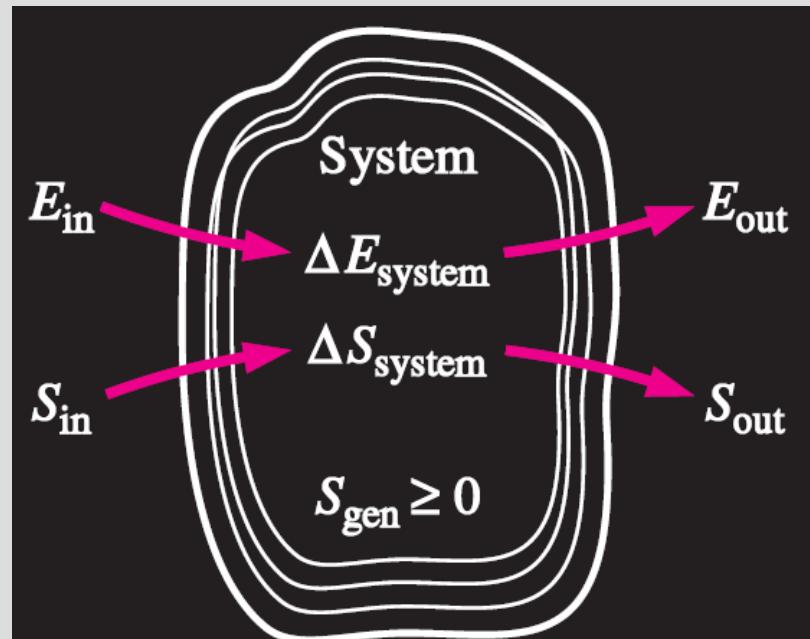
Entropy Change of a System, ΔS_{system}

$$\Delta S_{\text{system}} = S_{\text{final}} - S_{\text{initial}} = S_2 - S_1$$

When the properties of the system are not uniform

$$S_{\text{system}} = \int s \, \delta m = \int_V s \rho \, dV$$

Energy and entropy balances for a system.



$$\Delta E_{\text{system}} = E_{\text{in}} - E_{\text{out}}$$
$$\Delta S_{\text{system}} = S_{\text{in}} - S_{\text{out}} + S_{\text{gen}}$$

Mechanisms of Entropy Transfer, S_{in} and S_{out}

1 Heat Transfer

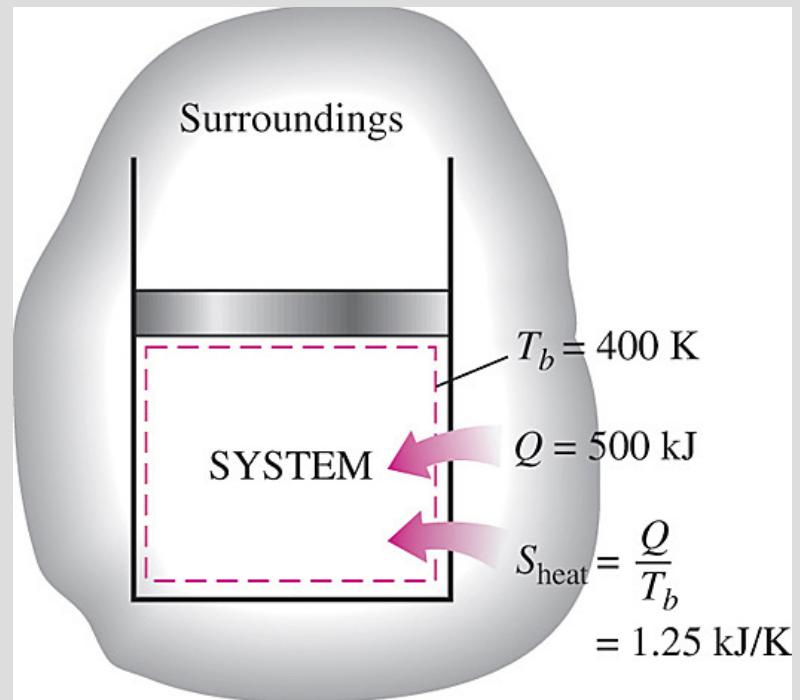
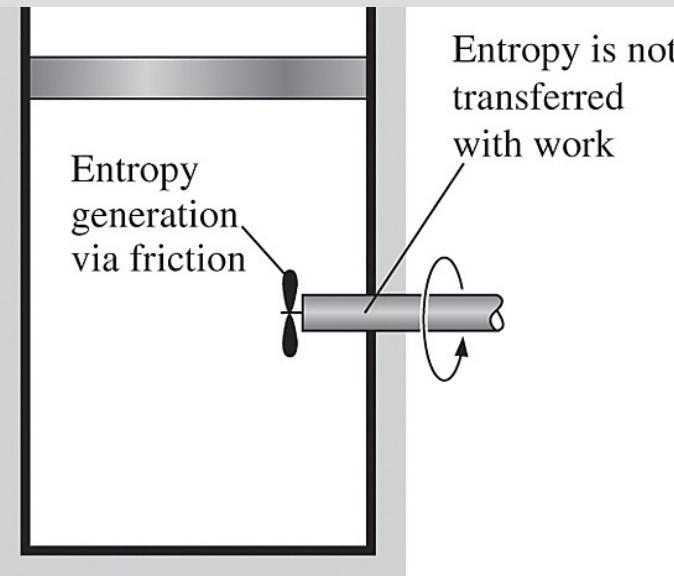
Entropy transfer by heat transfer:

$$S_{heat} = \frac{Q}{T} \quad (T = \text{constant})$$

$$S_{heat} = \int_1^2 \frac{\delta Q}{T} \cong \sum \frac{Q_k}{T_k}$$

Entropy transfer by work:

$$S_{work} = 0$$



Heat transfer is always accompanied by entropy transfer in the amount of Q/T , where T is the boundary temperature.

No entropy accompanies work as it crosses the system boundary. But entropy may be generated within the system as work is dissipated into a less useful form of energy.

Mechanisms of Entropy Transfer, S_{in} and S_{out}

2 Mass Flow

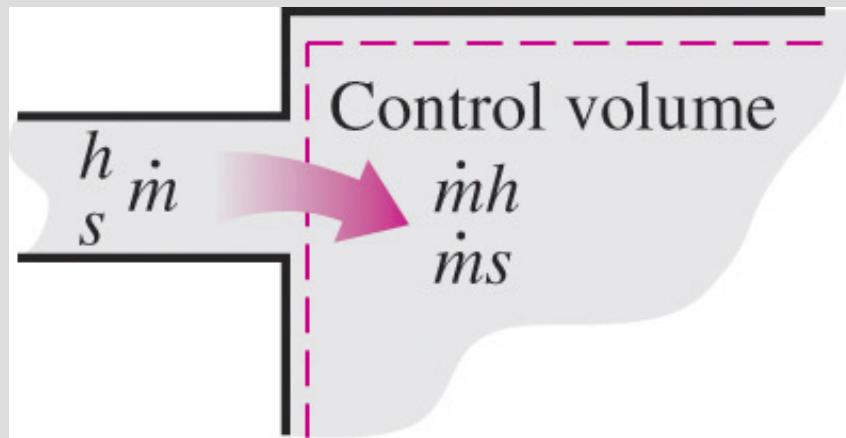
Entropy transfer by mass:

$$S_{\text{mass}} = ms$$

When the properties of the mass change during the process

$$\dot{S}_{\text{mass}} = \int_{A_c} s \rho V_n dA_c$$

$$S_{\text{mass}} = \int s \delta m = \int_{\Delta t} \dot{S}_{\text{mass}} dt$$



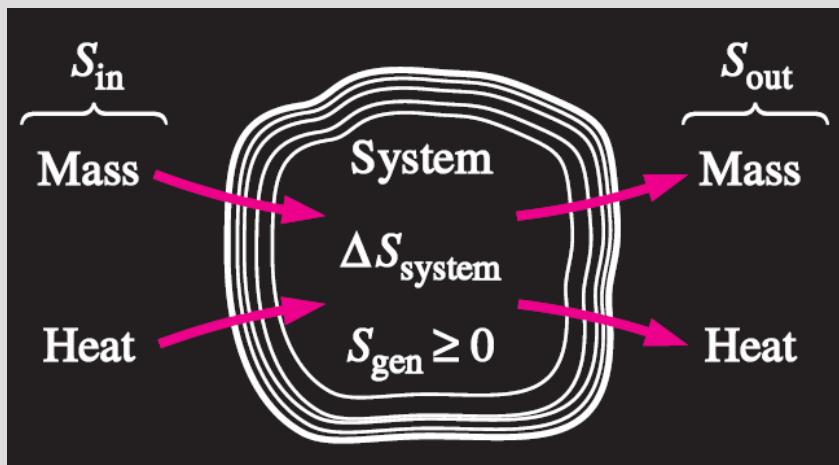
Mass contains entropy as well as energy, and thus mass flow into or out of system is always accompanied by energy and entropy transfer.

Entropy Generation, S_{gen}

$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{\text{gen}}}_{\text{Entropy generation}} = \underbrace{\Delta S_{\text{system}}}_{\text{Change in entropy}} \quad (\text{kJ/K})$$

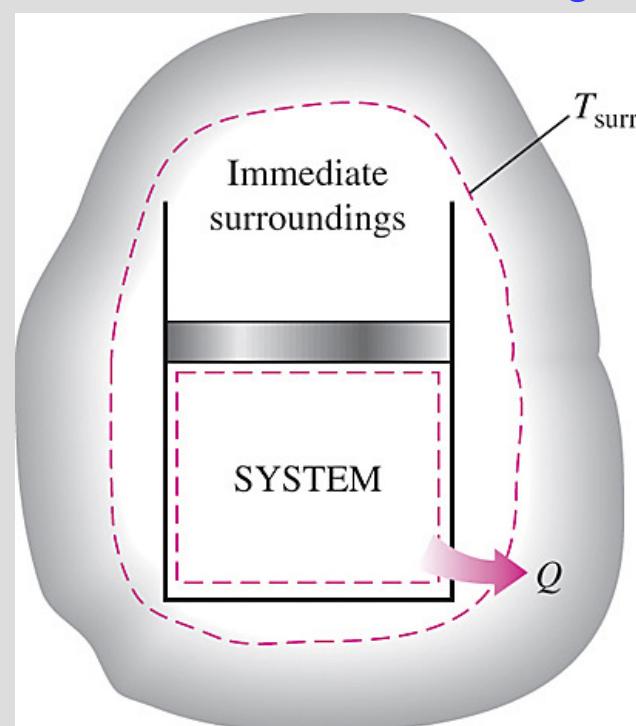
$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{dS_{\text{system}}/dt}_{\text{Rate of change in entropy}} \quad (\text{kW/K})$$

$$(S_{\text{in}} - S_{\text{out}}) + S_{\text{gen}} = \Delta S_{\text{system}} \quad (\text{kJ/kg} \cdot \text{K})$$



Mechanisms of entropy transfer for a general system.

Entropy generation outside system boundaries can be accounted for by writing an entropy balance on an extended system that includes the system and its immediate surroundings.



Closed Systems

Closed system:
$$\sum \frac{Q_k}{T_k} + S_{\text{gen}} = \Delta S_{\text{system}} = S_2 - S_1 \quad (\text{kJ/K})$$

The entropy change of a closed system during a process is equal to the sum of the net entropy transferred through the system boundary by heat transfer and the entropy generated within the system boundaries.

Adiabatic closed system:
$$S_{\text{gen}} = \Delta S_{\text{adiabatic system}}$$

System + Surroundings:
$$S_{\text{gen}} = \sum \Delta S = \Delta S_{\text{system}} + \Delta S_{\text{surroundings}}$$

$$\Delta S_{\text{system}} = m(s_2 - s_1) \quad \Delta S_{\text{surr}} = Q_{\text{surr}}/T_{\text{surr}}$$

$$\sum \frac{\dot{Q}_k}{T_k} + \sum m_i s_i - \sum m_e s_e + S_{\text{gen}} = (S_2 - S_1)_{\text{CV}} \quad (\text{kJ/K})$$

$$\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{\text{gen}} = dS_{\text{CV}}/dt \quad (\text{kW/K})$$

Steady-flow:

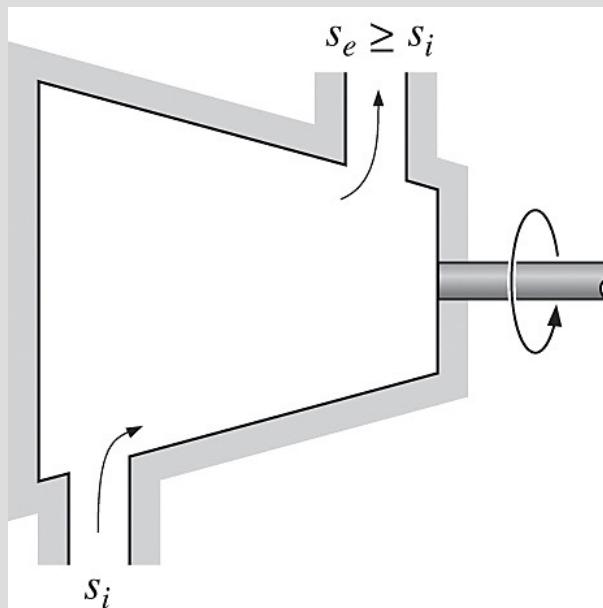
$$\dot{S}_{\text{gen}} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$$

Steady-flow, single-stream:

$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i) - \sum \frac{\dot{Q}_k}{T_k}$$

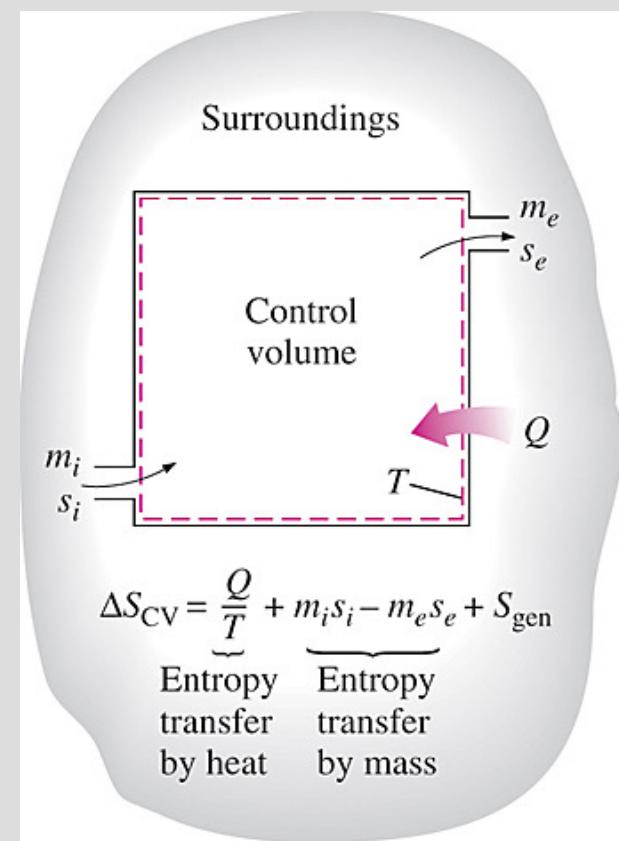
Steady-flow, single-stream, adiabatic:

$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i)$$



The entropy of a substance always increases (or remains constant in the case of a reversible process) as it flows through a single-stream, adiabatic, steady-flow device.

Control Volumes



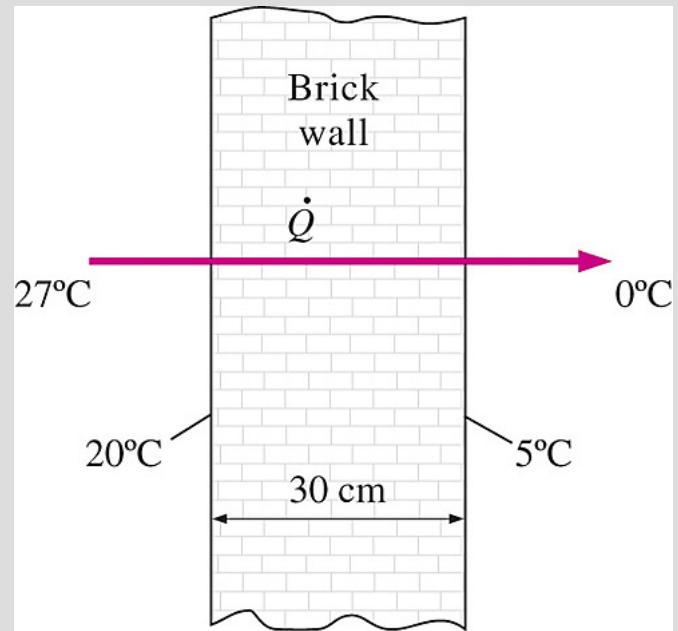
The entropy of a control volume changes as a result of mass flow as well as heat transfer.

EXAMPLES

Entropy balance for heat transfer through a wall

$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{gen}}_{\text{Rate of entropy generation}} = \underbrace{\frac{dS_{system}}{dt}}_{\substack{0 \text{ (steady)} \\ \text{Rate of change in entropy}}}$$

$$\left(\frac{\dot{Q}}{T}\right)_{in} - \left(\frac{\dot{Q}}{T}\right)_{out} + \dot{S}_{gen} = 0$$

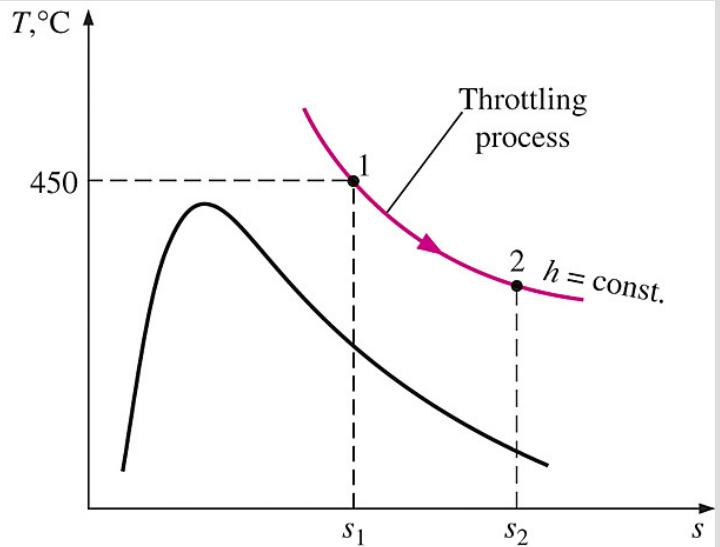
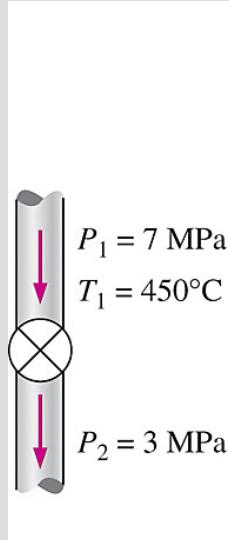


Entropy balance for a throttling process

$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{gen}}_{\text{Rate of entropy generation}} = \underbrace{\frac{dS_{system}}{dt}}_{\substack{0 \text{ (steady)} \\ \text{Rate of change in entropy}}}$$

$$\dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen} = 0$$

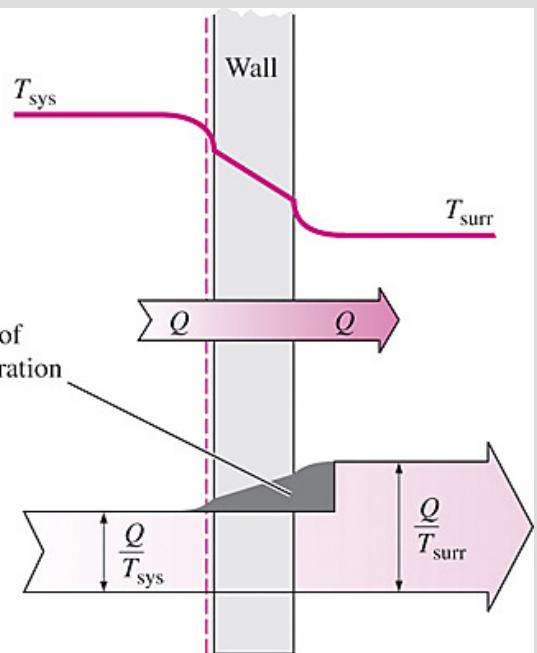
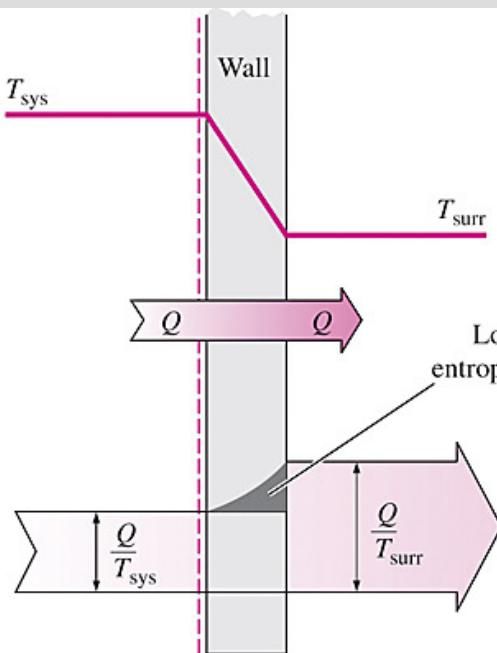
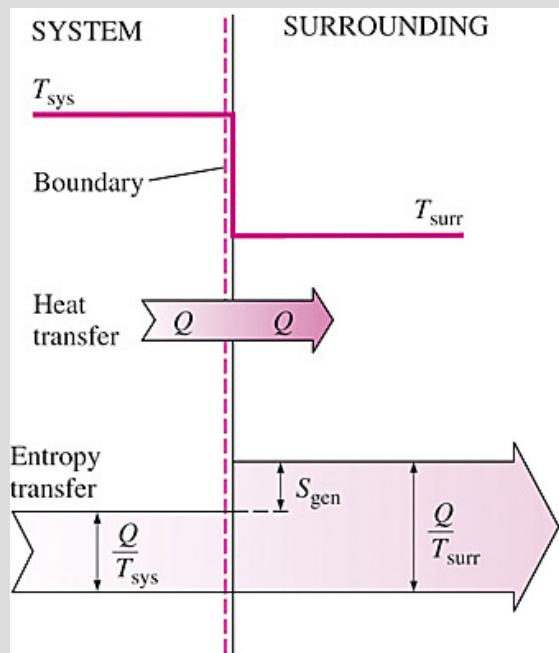
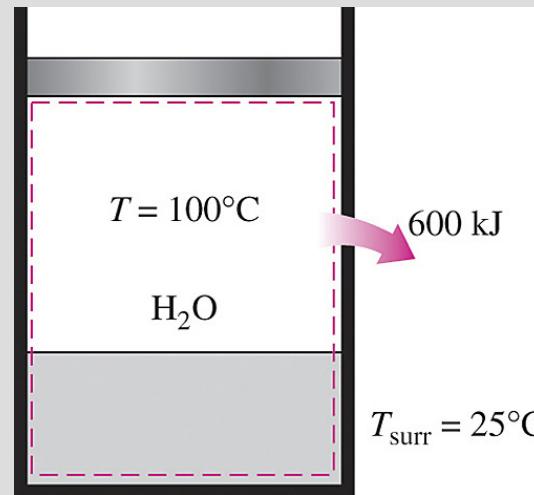
$$\dot{S}_{gen} = \dot{m}(s_2 - s_1)$$



Entropy generation associated with a heat transfer process

$$\underbrace{S_{in} - S_{out}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{gen}}_{\text{Entropy generation}} = \underbrace{\Delta S_{system}}_{\text{Change in entropy}}$$

$$-\frac{Q_{out}}{T_b} + S_{gen} = \Delta S_{system}$$



Graphical representation of entropy generation during a heat transfer process through a finite temperature difference.

Summary

- Entropy
- The Increase of entropy principle
- Some remarks about entropy
- Entropy change of pure substances
- Isentropic processes
- Property diagrams involving entropy
- What is entropy?
- The $T ds$ relations
- Entropy change of liquids and solids
- The entropy change of ideal gases
- Reversible steady-flow work
- Minimizing the compressor work
- Isentropic efficiencies of steady-flow devices
- Entropy balance