Noise

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Vibration & Noise Control

26 / 9 /2019 Thurs

→ **Acoustics**: the science that deals with sound generation, propagation, & transmission.

→ **Sound**: Vibration or disturbance of an elastic medium that propagates with speed which depends on the characteristics of the medium.

→ **Noise**: any unwanted sound

→ **pure tone**: sound with single frequency

→ **Complex Sound**: multi frequency sound

→ Discussion:

- Recall: vibration means oscillation of a particle back and forth about its equilibrium position
- Sound is created by a vibrating object. It is easy to detect the vibrations of many "Sources of sound". e.g:

– By lightly touching the speakers of your phone while your are listening to a music, you can feel its vibrations as a tingling sensation in your finger-tips. Note: in this example, the source of sound is the phone speaker.

– If you touch your throat while singing or speaking, you can feel the vibrations of your vocal cords, which represents the source of sound.

- For such vibrations to be heard as sound, there must be a medium through which they can travel from the vibrating source to the ear. In otherwords, you can't hear sounds if you were in a vacuum space. Sound can travel through liquids, solids & gases like air.
- The vibrations of a sound source (example: speaker) cause the neighbouring air molecules to be alternately squeezed together & pulled apart. These air molecules then push and pull against their neighbours, which in turn, push & pull against their neighbours.
- In this way, a series of compressions (regions of high pressure) & rarefactions (regions of low pressure) is generated. This series of pressure fluctuation is called sound wave

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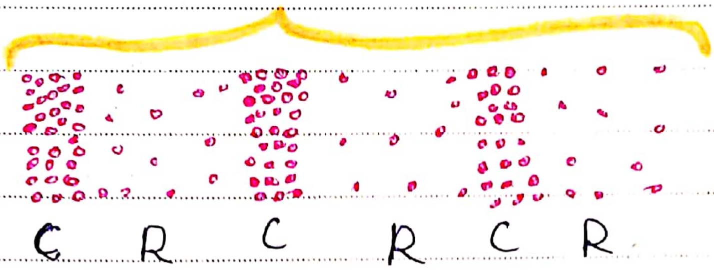
Sound Wave



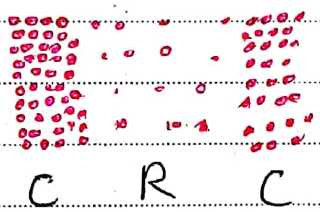
• = air particle

C = Compression

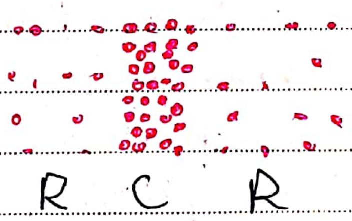
R = Rarefaction

• The particles do not move down the way with the wave, but oscillate back & forth about their individual ~~posit~~ equilibrium position

• The pressure at a certain region in the medium alternates between compressions & rarefactions. Thus if at one instant, a region in the medium experiences compression, the region adjacent to it along the line of propagation is expected to be experiencing rarefaction. Then as time progresses, the region which was experiencing compression, will undergo ~~#~~ a rarefaction; while the region adjacent to it undergoes compression



⇒ at t_1



⇒ at $t_1 + \Delta t$

C R C ⇒ at $t_1 + 2\Delta t$

• If we plot the pressure variation at a certain region against time, we will obtain the following:

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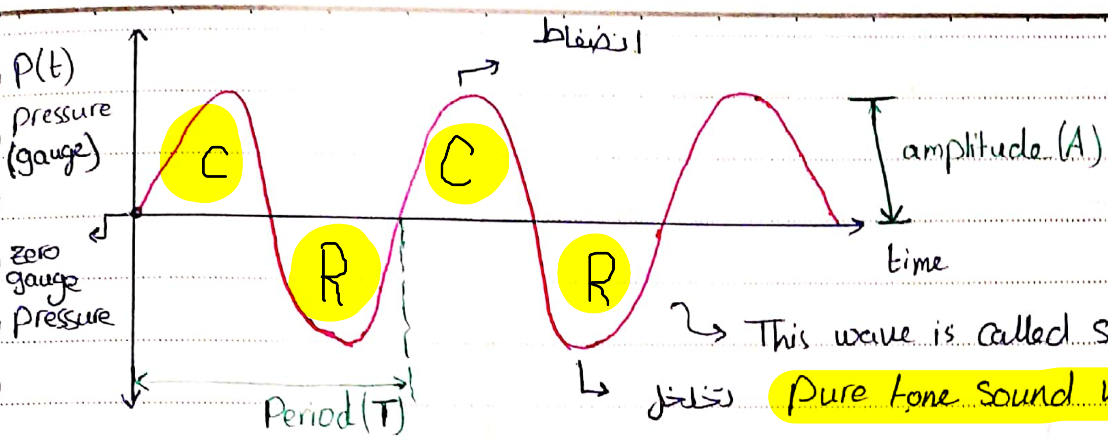


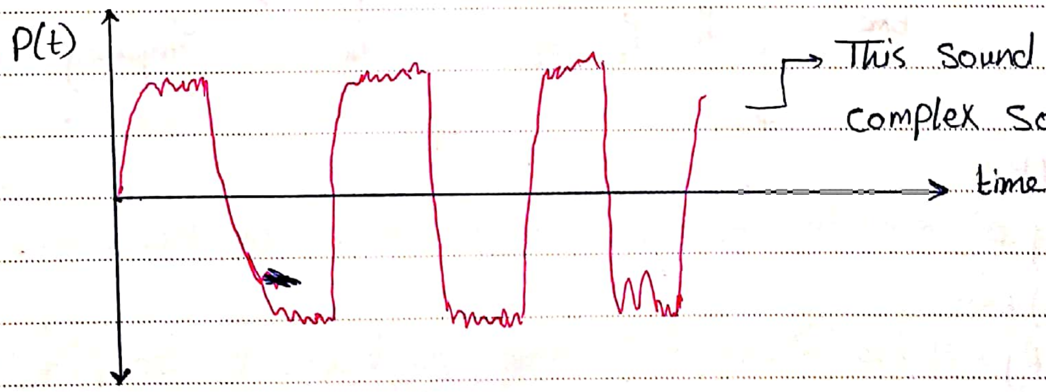
This wave is called sound wave. It is called Pure tone sound wave

• Characteristics of sound wave:

1. Amplitude (A)
2. Period (T); Frequency $f = \frac{1}{T}$

• The sound wave shown above is called pure tone, since it has a single frequency (or it can be described by a single sinusoidal function)

• Most sounds that we hear are complex sounds or random sounds. i.e they are multifrequency sounds (They can be described by a series of sinusoidal functions "Fourier series")



This sound wave is called complex sound wave

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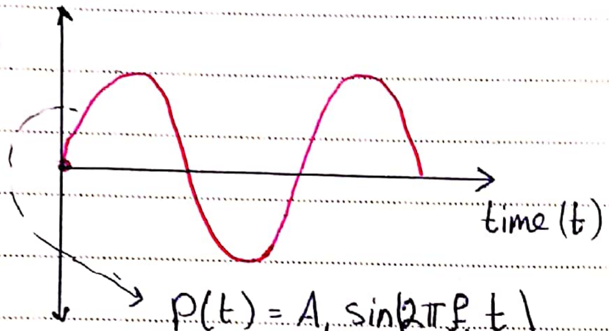
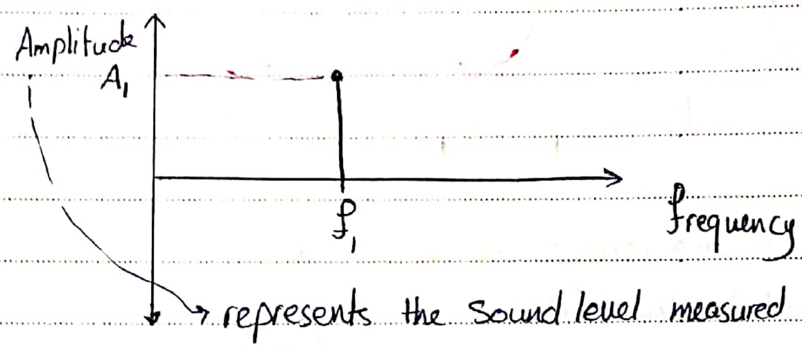
- Comparison between pure tone & complex sound

- Pure tone:

Time domain $\xrightarrow{\text{Fourier transform}}$ Frequency domain



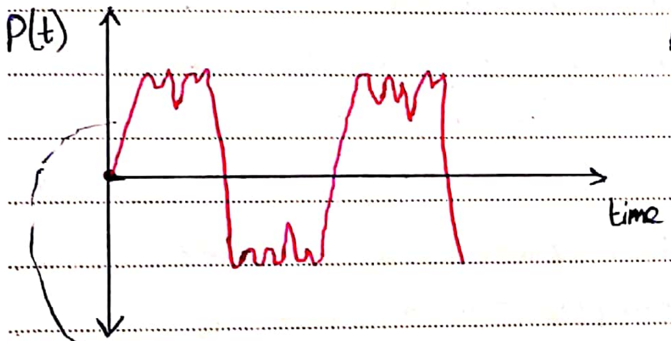


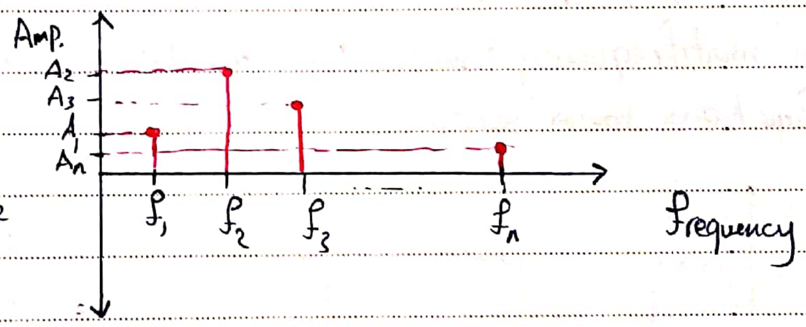
$P(t) = A_1 \sin(2\pi f_1 t)$

→ represents the sound level measured in dB

- Complex tone:

Time domain $\xrightarrow{\text{Fourier transform}}$ Frequency domain





$$P(t) = A_1 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t) + A_3 \sin(2\pi f_3 t) + \cdots + A_n \sin(2\pi f_n t)$$

- The time domain curve is called time history
- The frequency domain curve is called spectrum

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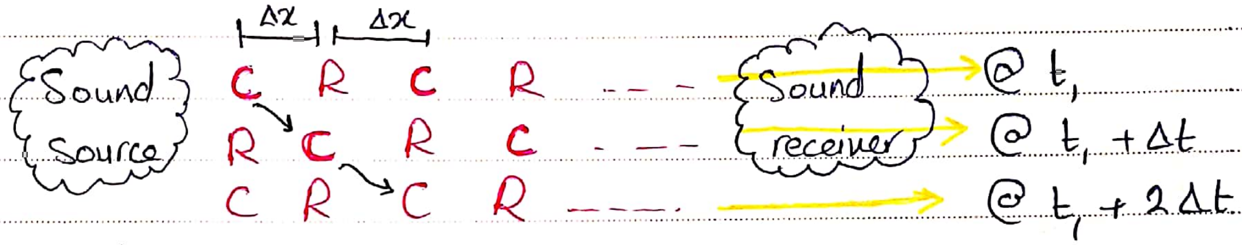
• Actually, sound waves are function of space & time. i.e $P = P(x, y, z, t)$

مثلاً القريب مني بيسمعني أحسن من البعيد عني

• Pure tone is an assumption , however, we can generate them in labs using sound generators

يعني ~~...~~ الأصوات ما بتكون single frequency

• As we said before, the particles do not move down with the wave, but oscillate back & forth about their individual equilibrium position. Also, the pressure at a certain region oscillates between compression and rarefaction



لـ جزيئات الهوا بتهتز لكنها لا تتحرك
حركة خطية. لكن مثلاً حط مع
مرور الزمن كأنه الانضغاط ع يتحرك
Δx خلال Δt وهاد بمثل سرعة
الصوت

→ Sound speed $= \frac{dx}{dt} = C$

→ Speed of sound in air = 341 m/s at room temp

• Speed of sound in gases < liquids < solids , because, in solids, the atoms are very close to each other compared to liquids & gases, hence, the transmission of vibration (sound) will be easier.

Bulk modulus

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→ Goals of designing a noise environment:

- Prevent annoyance.
- prevent speech interference
- prevent hearing loss.

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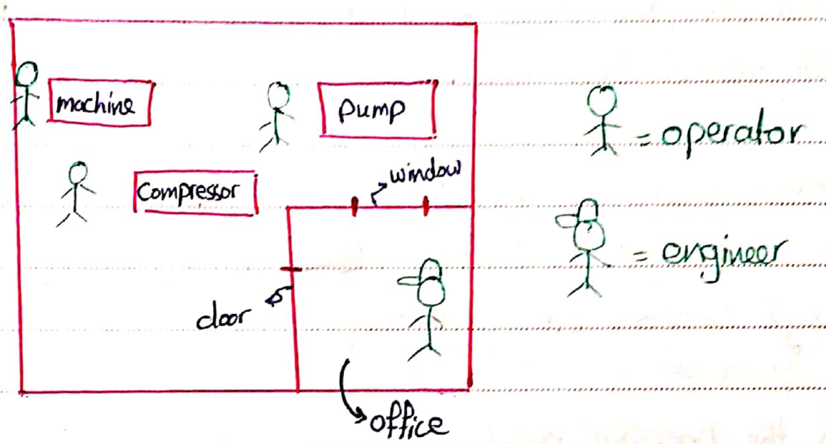
→ Major tasks for a noise control engineer:

[1] Identification of the source, Path & receiver
[2] Selection of the instruments needed for the evaluation
[3] Measurement of sound levels
[4] Evaluation within the standards
[5] Noise Control initiation

→ To understand the previous steps, let's take an example:

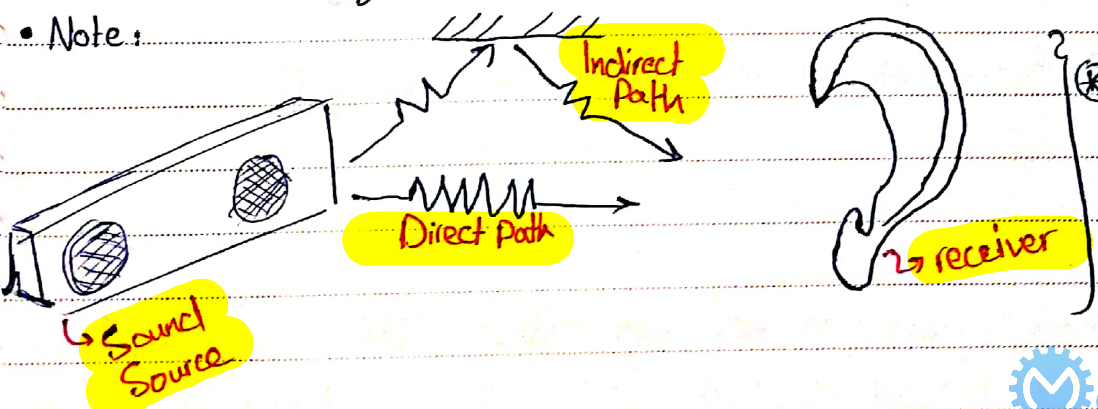
• Consider a workshop as a noise environment to be set under study:



[1] Source: machine, compressor, pump
Path: air within the space, door, window.
receiver: operator, engineer
} Generally speaking.

• Actually, noise measurements may be needed to locate the "source" of noise in a piece of machinery. In addition, the transmission "path" for noise in a system may ~~by~~ be identified through acoustic measurements. } More accurately

• Note:



} (*) New termenologies:

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• There are 3 components of a general noise system: source of noise, path of the noise, and the receiver. The path may be direct from the source to the receiver, or the path may be indirect

[2] It is important that the measuring equipment be properly selected to measure sound properties. This selection is based on our target of measurement.

e.g.:

• When we have a basic situation in which we need to assess the severity of environmental noise, we may need to measure "only" the overall level of sound using simple sound level meter.

• There are cases where we require a more detailed analysis of the noise. In these cases, a sound level meter + ~~Sound~~ filter + acoustic spectrum analyzer + Data storage system are required

↳ let's explain the previous points:

* Sound level is measured in decibels (dB).

* ~~We can either measure:~~

~~[•] The overall sound level of the noise~~

~~[•] or~~

* Noise is a multifrequency sound, hence, we can either measure:

[•] The overall sound level of the noise

[•] The ~~sound level~~ corresponding sound level for each frequency

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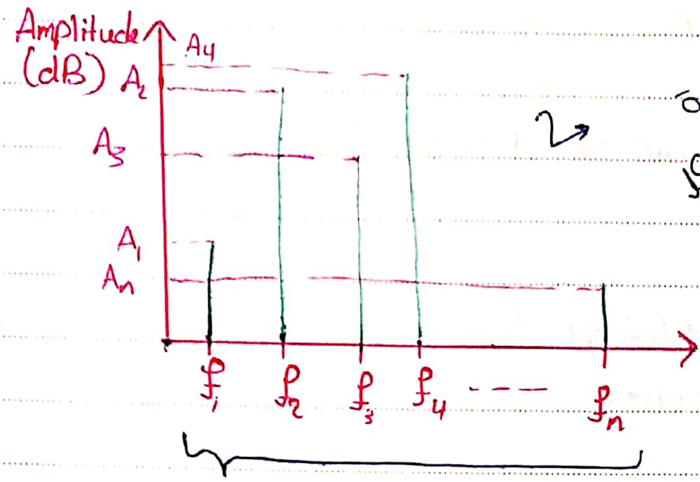
***The following spectrum is a spectrum of a noise**



* ملاحظة : ع أرض الواقع الرسمة
ما بتكون هيك ع شكل spikes
الرسمة بتكون ع شكل
continuous function

[1] The range of frequency is controlled using a $\boxed{\text{filter}}$
There are 3 types of filters:

- **low pass filter**: this filter will consider all the frequencies below a predefind value f_{Low} (i.e frequency range $= [0 - f_{Low}]$)
- **high pass filter**: this filter will consider all the frequencies above a predefined value f_{high} (i.e range frequency $= [f_{high} - \infty]$)
- **band pass filter**: will consider the frequencies **the fall between 2 predefined values** (i.e range frequency $= [f_{low} - f_{high}]$

[2] **The amplitude or level of sound** is measured for each frequency using $\boxed{\text{Sound level meter}}$

[3] The above data (frequency + sound level) is manipulated and plotted using $\boxed{\text{acoustic spectrum analyzer}}$

[4] The above data can be stored with the aid of a $\boxed{\text{data storage system}}$

Note:
-Referring to point [2], sound level can be measured for each frequency. Also, an overall sound level (i.e a single value) can be measured for the above noise spectrum

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- Humans can hear from about 20Hz to 20KHz, unless you are old and can hear from 30HZ to 15KHZ!

[3] Once the proper instruments are selected, one can perform sound level measurements.

[4] We compare the results obtained from measurements with the standard values given by famous societies

[5] Noise control initiation → next lecture will cover this point

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→ **Noise Control initiation:**

• Noise Control engineer has to carry out the procedure discussed in the previous lecture to control the noise either

1] at the source

2] along the path

3] or at the receiver

• The most effective noise control method is to eliminate noise at the source.

[*] Noise control at the source

[1] Cover the source with sound enclosure that is lined with acustical material to absorb the unwanted sound.

[2] Solve vibration problem

[3] Vibration isolation

[4] Change the mode of operation if it is possible

[5] Regulating pressure & velocity in fluid systems [e.g piping system, duct system]

[6] Source relocation

→ Let's consider each of the above methods in more detail:

[1] **Sound enclosures:**

• All hospitals, hotels must be provided with standby electrical generators that will be utilized in electrical power interruption situations. These generators are covered with sound enclosures to eliminate their noise. & provide a better environment for the occupants

• Later, we will learn how to design sound enclosures.

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2 Solving vibration problem:

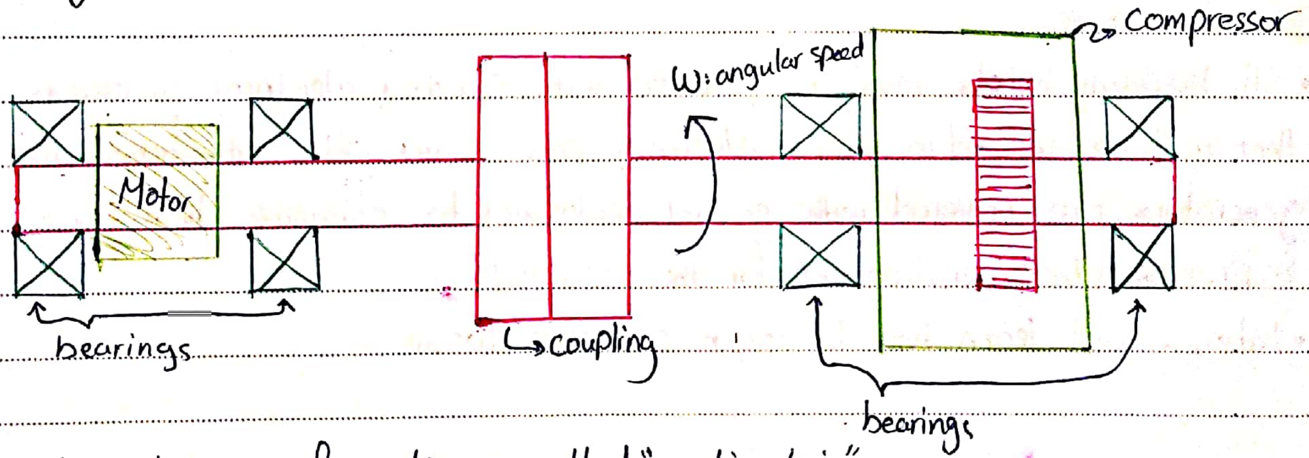
• In "most" cases, noise problems occur due to vibration problems. In other words if ~~solve~~ these vibration problems are solved, noise problems will disappear spontaneously.

• Recall [from mechanical vibrations course]: Steps of solving a vibration problem:

1. Constructing the mathematical model (Equation of motion) of the physical model by applying physics laws, such as: Newton's 2nd law.
2. Solving the equation of motion to obtain the response of the model
3. Analyze the response curve of the model to get rid of the vibration problem.

• When we want to solve a vibration problem, we have to find out the component which has excessive level of vibration (i.e large amplitude of vibration. In other words, ~~we have to locate~~ if we have a noisy machine that consists of many ~~components~~ components, we have to locate ~~the~~ the component which has the greatest level of vibration, since it contributes the most to the overall noise level

• Example: Suppose that we have a noisy machine that consists of: a motor, a coupling, & a compressor,



Note: the above configuration is called "machine train"

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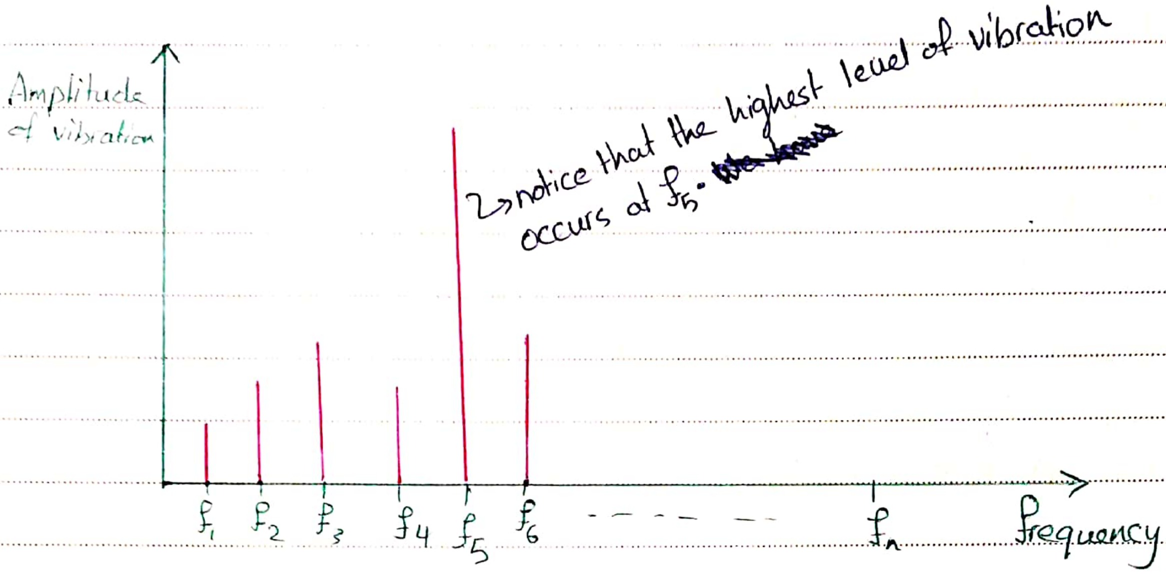
• This machine is noisy due to the existance of excessive vibration levels.
• To solve this vibration problem, we have to perform sound level measurements to obtain the spectrum of the noise:



• As we said before, the previous machine train consists of many components. Each component has a certain frequency. We have to identify the frequencies of all components, so that we can discover which component has the frequency f_5, since it has the highest level of vibration as indicated in the above figure. We have to solve the vibration problem of this component since it has the most contribution to the overall sound level.

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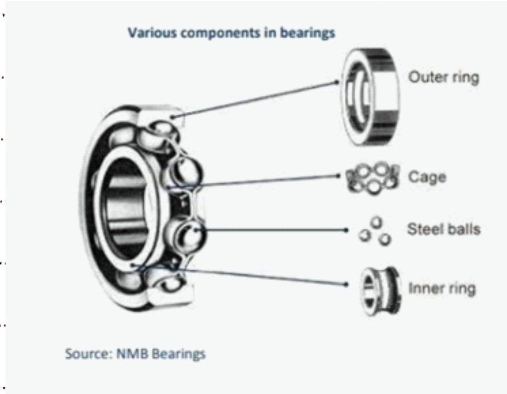
• Based on the previous discussion, we have to identify the frequencies of all the components that form the machine train:

(*) Bearing frequencies: the adjacent figure shows the components of a bearing. When a bearing spins, any defect or irregularities in the raceway surfaces or the rolling elements excites periodic frequencies. A machine with defective bearing can generate 4 frequencies. These frequencies are:



① Inner race frequency: It is the rate at which a ball passes a defect in the inner race.

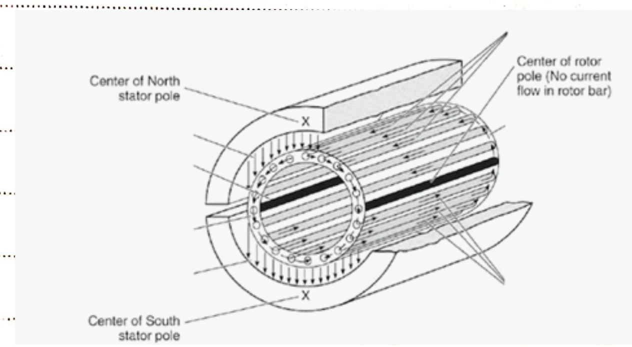
② Outer race frequency: It is the rate at which a ball passes a defect in the outer race. [Note: $f_{\text{Inner race}}$ is often slightly lower than $f_{\text{outer race}}$ as the vibration is generated further away from the transducer
↳ the device which detects the vibration

③ Ball spin frequency: the rate at which a defect on the ball contacts the inner race or outer race.

④ Cage frequency: the rate at which the bearing's cage rotates.

• These frequencies depend on the rpm of the shaft & the geometry of the bearing

(*) Rotor's bars pass frequency: the adjacent figure shows the components of an induction motor. Any motor consists mainly of a rotor & stator. Notice that the rotor of an induction motor has bars. Rotor's bars pass frequency is defined as the rate at which the bars pass a fixed point.



∴ Rotor's bars pass frequency = RPM of the shaft x number of bars.

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(*) Fan blades pass frequency: Some motors are equipped with a fan for cooling purposes. Fan blades pass frequency is defined as the rate at which the blades of the fan pass a fixed point.

∴ Fan blades pass frequency = RPH of the shaft x number of fan blades.

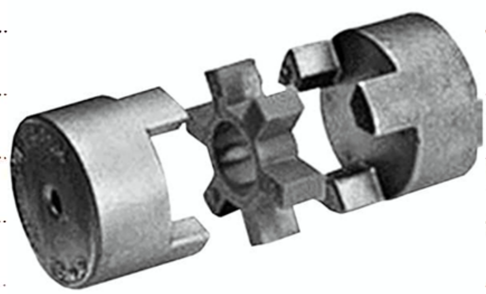
• Check the figure of the machine train & notice that the motor has 2 bearings. Assuming that the 2 bearings are identical, one can conclude that the motor has 6 frequencies as explained below:

4 bearing frequencies
\+ 1 Rotor's bars frequency
1 fan blades frequency

---

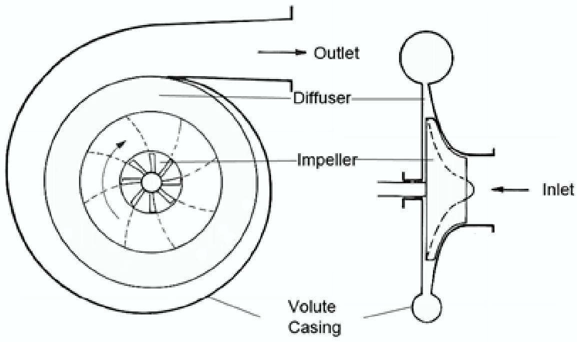
6 frequencies

(*) Coupling frequency: The adjacent figure shows a jaw type coupling, which connects the shaft of the motor to the shaft of the compressor, so that they run with the same RPH.



Coupling frequency = RPH x number of jaws

(*) Vane pass frequency: The adjacent figure shows the components of a centrifugal compressor. The impeller of the compressor has vanes. Vane pass frequency is the rate at which the vanes of the impeller pass a fixed point.



Centrifugal compressor schematic diagram

∴ Vane pass frequency = RPH of the shaft x number of vanes

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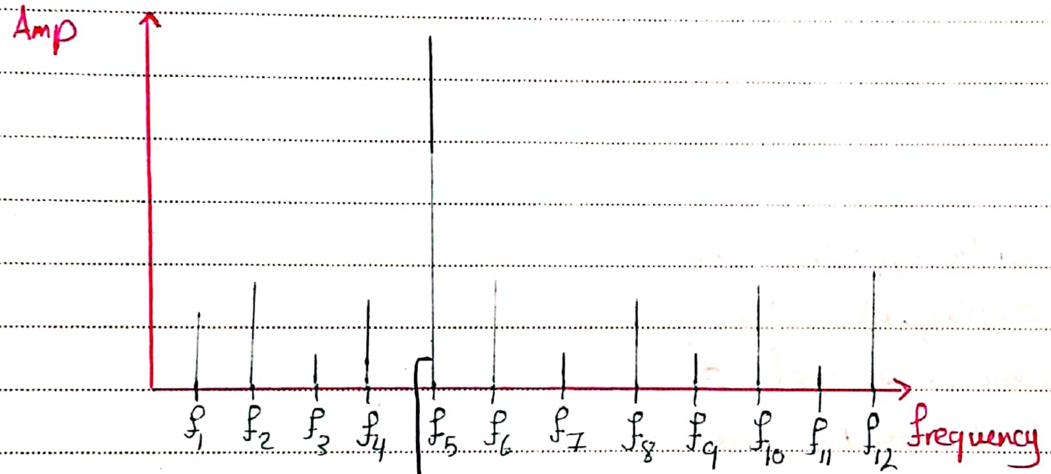
- Check the machine train figure & notice that the compressor has 2 bearings. Assuming that the bearings are identical, one can conclude that the compressor has 5 frequencies as explained below

4 → bearing frequencies
+ 1 → Vane pass frequency

5 - frequencies

- The entire machine train has 12 frequencies = 6 frequencies of the motor + coupling frequency + 5 frequencies of the compressor.



↳ Since all frequencies are now identified, we can recognize the source of this frequency to solve the vibration problem.

If f_5 represents the fan blades pass frequency, we can reduce the amplitude of vibration by redesigning the fan blades; We can either $\boxed{1}$ increase the thickness of the blades $\boxed{2}$ Stiffen the blades

أحيانًا مرات ريش المروحة بتكون كثير flexible فبتسبب high level of vibration عشان
أحل المشكلة ← بقلل ال flexibility عن طريق اني أزيد ال thickness تبع
ال blades أو استخدم مادة تكون stiff أكثر

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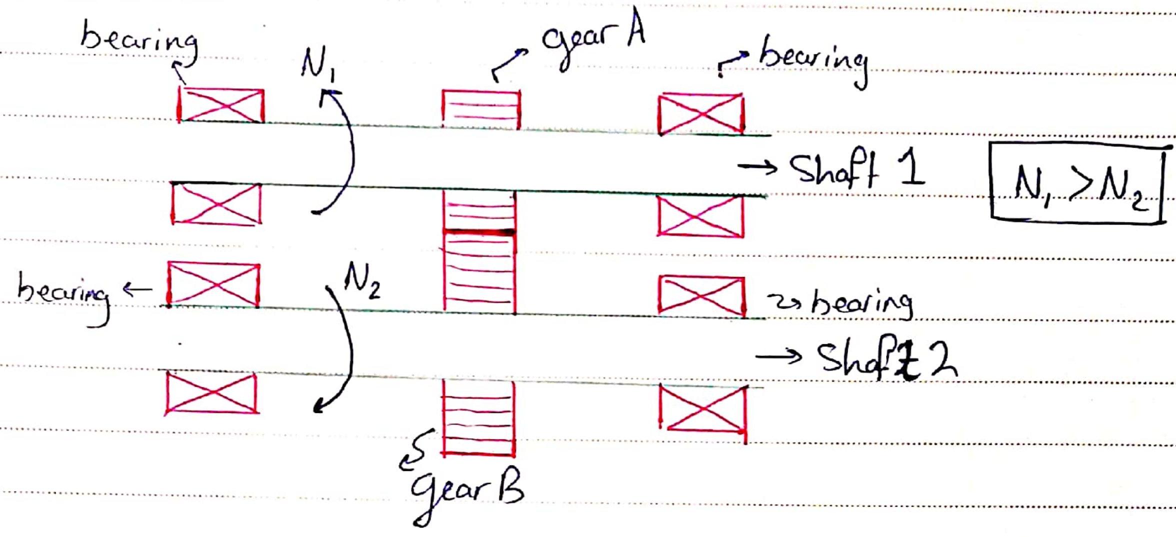
Example 2: Identify all the frequencies of the following gear box

* All bearings are identical



gear tooth pass frequency of gear A + gear tooth frequency of gear B + 4 bearing frequencies of shaft 1 + 4 bearing frequencies of shaft 2 = 10 freq.

↳ Although the bearings are identical, we calculated the bearing frequencies twice because $N_1 \neq N_2$ (Recall: bearing frequencies depend on the RPM of the shaft)

[3] Source relocation

It is a low-cost solution to eliminate the noise. If the source of noise in a workshop is the compressor, we can install the compressor outside the workshop - if possible - to reduce the noise level inside the workshop.

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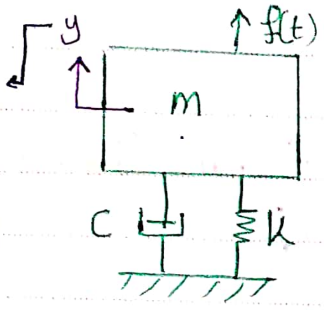
[*] Continue: Noise Control at the source

[4] Vibration Isolation

Recall from mechanial vibrations Course:

represents the displacement experienced by the system



→ Consider a forced vibration problem.

- Assume:
- f(t) is a harmonic force
- The system represents a rotating shaft.
  i.e. m = mass of the shaft
  C & K ≡ describe the support structure of the shaft (bearings)

→ ~~Since~~ Our goal is to reduce noise caused by vibration. To protect the system from this undesirable vibration, we have to study the effect of f(t) on our system.

→ The most important parameter that we can identify which shows the effect of f(t) on the system is TR = Transmissibility Ratio.

→ Hence, by understanding this parameter we can protect our system from undesirable vibration.

→ TR = is a quantity that measures the impact of a force on a system.

→ If a harmonic force is acting on the system, obviously, the force will cause a displacement in the system (i.e the mass m will move back & forth about its equilibrium position.

→ As a result of this displacement, the support structure (which is represented by the stiffness & damping elements) will be subjected to a force. This transmitted force to the support structure is given by

$$f_T = ky + c\dot{y} \quad (f_T = \text{transmitted force})$$

↳ The force experienced by the support structure (bearings) is designated by f_T. You have to know what force the support structure has to handle.

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→ Mathematically, TR is defined as follows:

$$TR = \frac{\text{Maximum force transmitted}}{\text{Maximum force applied}}$$

→ We are going to find these values

- The force applied on the system is $f(t) = F_o \sin\omega t$ (harmonic force)

  $\therefore$ Max. force applied = Amplitude of $f(t) = F_o$

- Recall from vibrations course:

  When a mass-spring-damper system is subjected to a harmonic force $f(t) = F_o \sin\omega t$, its displacement $y(t)$ is given by

$$y(t) = \frac{F_o}{K} * \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \sin(\omega t - \phi)$$

Substitute

$$\dot{y}(t) = \frac{F_o}{K} * \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} * \omega \cos(\omega t - \phi)$$

$$F_T = Ky + C\dot{y}$$

$$F_T = \underbrace{\cancel{K} * \frac{F_o}{\cancel{K}} * \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}}_{\text{Call it } A_1} \sin(\omega t - \phi) + \underbrace{C * \frac{F_o}{K} * \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} * \omega}_{\text{Call it } A_2} \cos(\omega t - \phi)$$

$$F_T = A_1 \sin(\omega t - \phi) + A_2 \cos(\omega t - \phi)$$

To find the maximum F_T, we will do the following trick

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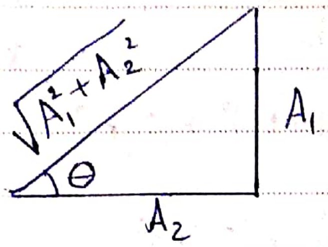
$$F_T = A_1 \sin(\omega t - \phi) + A_2 \cos(\omega t - \phi)$$

Define an angle θ

where

$$\sin\theta = \frac{A_1}{\sqrt{A_1^2 + A_2^2}}$$



$$\cos\theta = \frac{A_2}{\sqrt{A_1^2 + A_2^2}}$$

do this multiplication

$$F_T = \left[A_1 \sin(\omega t - \phi) + A_2 \cos(\omega t - \phi)\right] * \frac{\sqrt{A_1^2 + A_2^2}}{\sqrt{A_1^2 + A_2^2}}$$

$$= \sqrt{A_1^2 + A_2} \left[\frac{A_1}{\sqrt{A_1^2 + A_2^2}} \sin(\omega t - \phi) + \frac{A_2}{\sqrt{A_1^2 + A_2^2}} \cos(\omega t - \phi)\right]$$

$$= \sqrt{A_1^2 + A_2^2} \left[\sin\theta \sin(\omega t - \phi) + \cos\theta \cos(\omega t - \phi)\right]$$

Recall from Calculus : $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

$$\therefore F_T = \sqrt{A_1^2 + A_2^2} \quad \cos(\omega t - \phi - \theta)$$

Therefore, the max. transmitted force = amplitude of $F_T = \sqrt{A_1^2 + A_2^2}$

$$\therefore F_{T\max} = \sqrt{A_1^2 + A_2^2} = \sqrt{\left(\frac{F_0}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}\right)^2 + \left(\frac{C * F_0 * \omega}{K\sqrt{(1-r^2)^2 + (2\zeta r)^2}}\right)^2}$$

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$$F_{T_{max}} = \sqrt{\left[\frac{F_o}{\sqrt{(1-r^2)^2+(2\xi r)^2}}\right]^2 * \left(1+\left(\frac{CW}{K}\right)^2\right)}$$

$$= \frac{F_o}{\sqrt{(1-r^2)^2+(2\xi r)^2}} * \sqrt{1+\left(\frac{CW}{K}\right)^2}$$

$$\frac{C^2 W^2}{K^2} = \frac{C^2}{K^2} W^2 * \frac{W_n^2}{W_n^2}$$

r^2, where r = frequency ratio

$$= \frac{C^2}{K^2} * r^2 * W_n^2$$

$$= \frac{C^2}{K^2} * r^2 * \frac{K}{m} * \frac{4}{4}$$

$$= \frac{C^2}{4Km} * r^2 * 4$$

$$= 4r^2\xi^2 = (2\xi r)^2$$

$$= \frac{F_o \sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\therefore TR = \frac{\text{max. transmitted force}}{\text{max. applied force}}$$

$$= \frac{\dfrac{\cancel{F_o} * \sqrt{(1+(2\xi r)^2)}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}}{\cancel{F_o}} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

We will plot it on the next page

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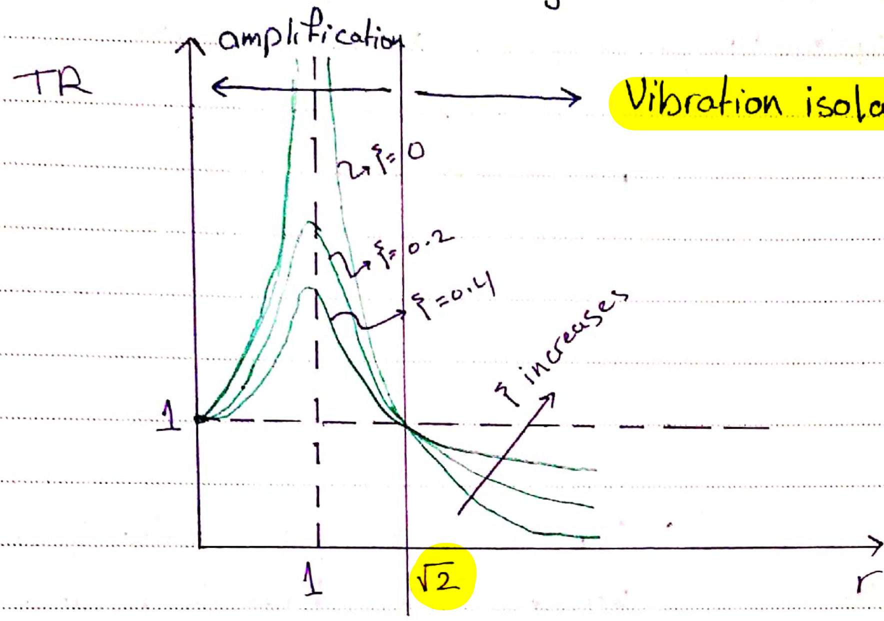
→ TR must be minimized to reduce the unpleasant effect of the forcing function $f(t)$ on the support structure of the system.

→ If TR is plotted against r for various values of ξ



→ To reduce TR : $r = \frac{\omega}{\omega_n}$ must be greater than $\sqrt{2}$ (Vibration isolation Zone)

$$r = \frac{\omega}{\omega_n} = \frac{\omega}{\sqrt{\frac{K}{m}}}$$

, We can increase r by:

1. Increasing ω ⇝ Sometimes We can't change the operating conditions of the System
2. Reducing K
3. Increasing m ⇝ Sometimes, adding additional mass to the system is not an effective solution, since the original System itself is heavy

→ Notice that for $r > \sqrt{2}$, as $\xi \uparrow$ TR gets larger, we can Control C (damping coefficient) to reduce TR.

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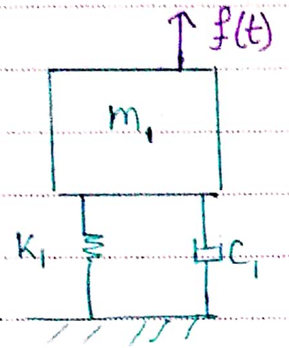
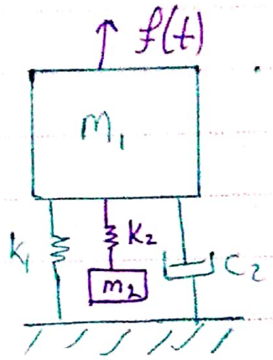
[5] Vibration absorber

→ If the system must operate at $w = w_n$, it will be subjected to an extreme level of vibration. To solve this problem, a mass is introduced to the original system through a spring as shown below

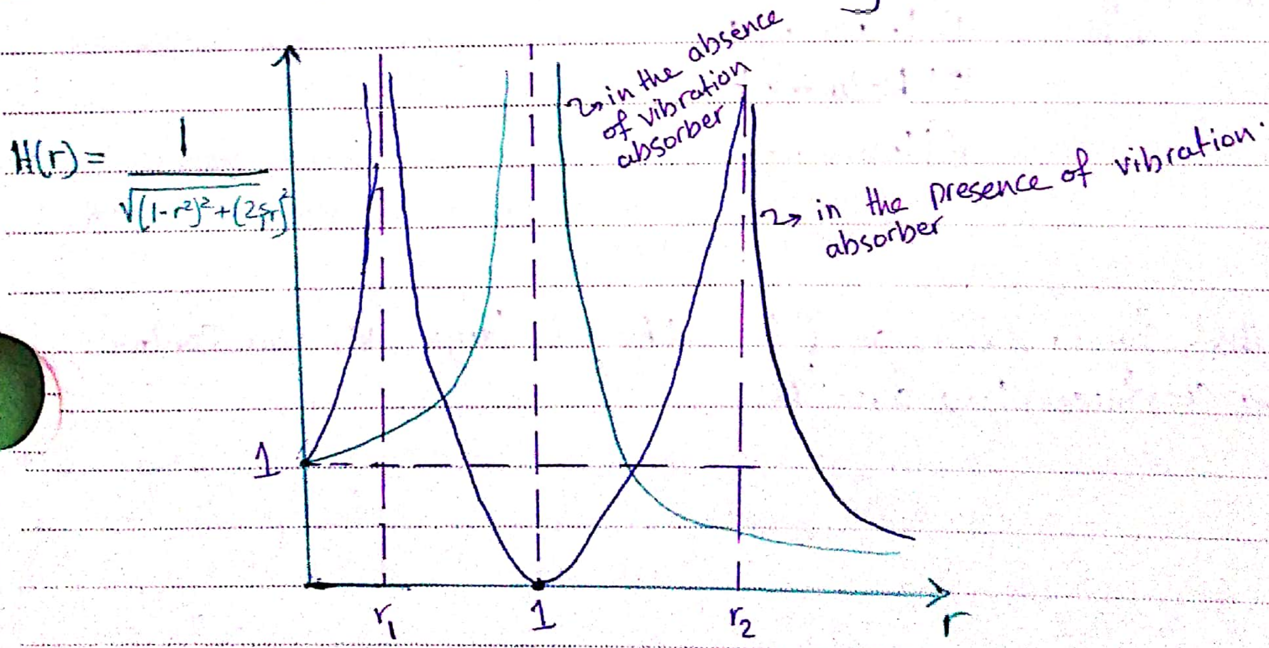




→ This system is a single degree of freedom system. It has a single natural frequency

→ The system became a 2 DOF system, therefore, it will have 2 natural frequencies

→ If the frequency response function is plotted for both systems shown above, we will obtain the following



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→ later we will discuss how to find m_2 & K_2 of the vibration absorber.

→ Note: What is $H(r)$ = Frequency response function?

Recall: When a mass spring damper system is subjected to harmonic force, its displacement is given by

$$y(t) = \frac{F_o}{K} * \underbrace{\frac{1}{\sqrt{(1-r^2)^2+(2\xi r)^2}}}_{H(r)} \sin(\omega t - \phi)$$

Notice that the maximum displacement experienced by the system is

$$y_{max} = \frac{F_o}{K} * H(r)$$

∴ $H(r)$ indicates the level of vibration experienced by the system.
(i.e as $H(r)$ gets larger, the amplitude of vibration gets larger)

"undesirable"

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→ Now, we will discuss some characteristics of sound wave

[*] Sound speed:

The speed of sound depends on the elasticity & density of the medium. Check the following video to understand the effect of the Bulk modulus & density of the medium on the sound speed.
Note: Bulk modulus: is defined as the pressure required Δp to compress a unit volume $1 m^3$ to decrease its volume by an amount of ΔV (i.e it indicates how hard to compress a material)

• Link : https://www.youtube.com/watch?v=yF4cvbAYjwI&feature=share&fbclid=IwAR0ksdYn6QZZcwt7J_9o_ypBmwl2UcEszp85SQkCtLxK9KK4BXvktdEpd90

[*] How obstacles affect sound wave?

When sound hits a material, three things can happen:

[1] Reflection

Sound is bounced off a surface. This usually occurs on flat rigid surfaces with a lot of mass like concrete or brick walls. Because the sound wave can't penetrate very far into the surface, the wave is turned back on itself. The sound bouncing back off the surface creates an echo!

[2] Transmission:

Some of the sound can penetrate thin walls

[3] absorption:

As sound travels through a surface, absorption may occur. Since sound is a regular vibration of atoms or molecules, some of the kinetic energy of the molecules is lost & turned into heat. (i.e absorption is the process of converting the kinetic energy of the molecules into heat). The best absorptive material is full of holes that the vibrating molecules can bounce around in & loss their kinetic energy. The energy lost as heat is too ~~small~~ small to be felt

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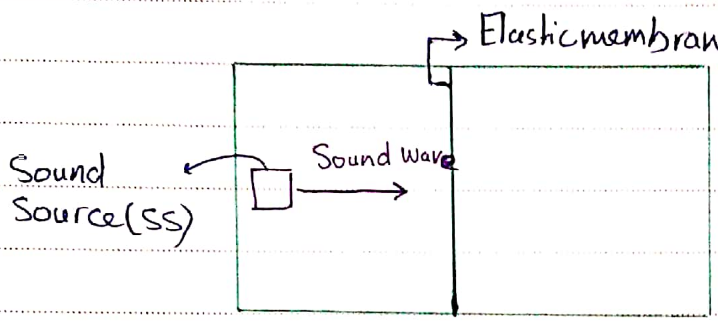
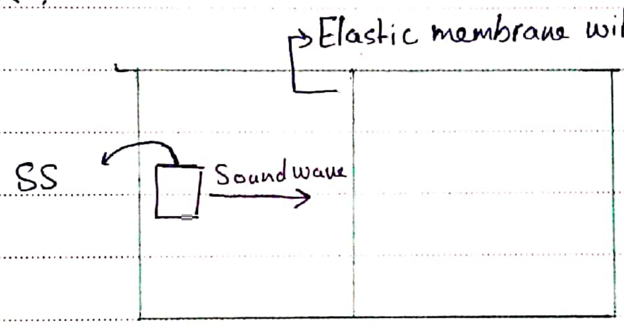
→ Based on the previous discussion, consider the following cases

• Case 1:



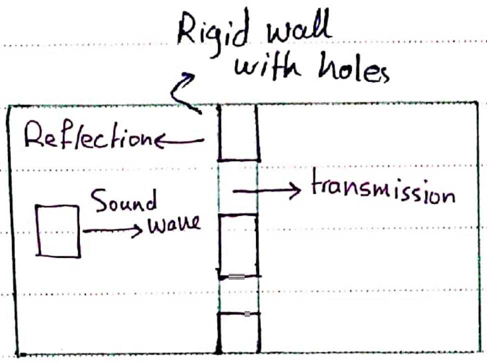
- Complete transmission will occur
- Reflection will not occur, since the membrane have negligible mass.
- Absorption will not occur.

• Case 2:



- Transmission will occur since it is like a thin wall.
- A little reflection will occur, because the membrane has a mass

• Case 3:



- Transmission will occur through the holes.
- More reflection will occur compared to case 2, since it has more mass. (the rigid wall is massive)
- Absorption will occur, since the vibrating molecules will hit the edges of the wall & lose their kinetic energy

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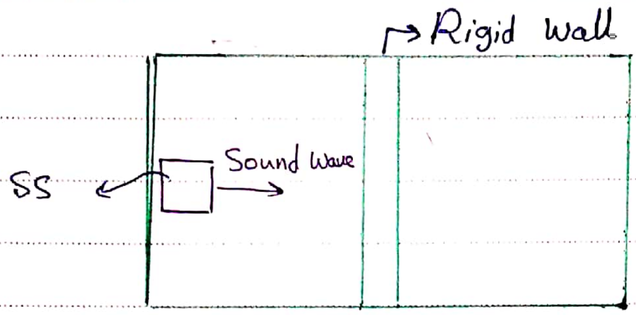
• Case 4:



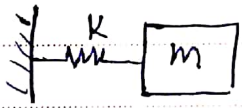
↝ Complete reflection will occur, since the wall is thick

↝ Transmission will not occur, since the wall is thick

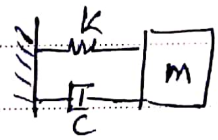
→ The previous cases can be modeled as mass spring ~~system~~ damper system:

• Case 1: no model, because the membrane is massless.

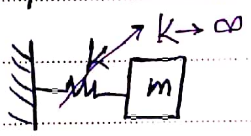
• Case 2:



• Case 3:



• Case 4:



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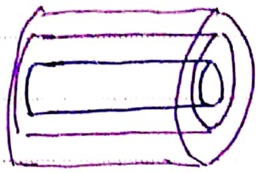
[*] Types of sound waves based on the shape of the wave

1- Spherical

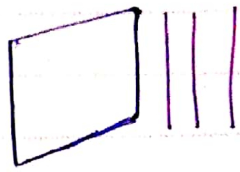
e.g. microphone

2 - cylindrical



e.g: pipe
a row of cars

3- plane



e.g: ducts of air conditioning systems.

- In this course, we will only deal with point sound sources (i.e they emit spherical wave)

[*] Acoustic power (Sound power), Sound intensity, and Sound pressure. What is the difference?

→ These 3 terms measure different aspects of sound, but can all be expressed in decibels. A decibel is not unit of measure, but rather a logarithmic ratio between 2 numbers (a measured quantity & a reference number).

Measurement	Measurement unit	Commonly reported in
Sound power	Watts	dB (ref = 1×10^{-12} W)
Sound Intensity	Watts/m^2	dB (ref = 1×10^{-12} W/m^2)
Sound pressure	Pa	dB (ref = 2×10^{-5} Pa)

ref → means reference

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→ An analogy between a heater placed in a cold room, versus a sound emitting object in quiet room, can be used to illustrate the differences between pressure, power & intensity.

Heater power (Watts) = Sound power (Watts)
Heat flow (Watts/m^2) = Sound intensity (Watts/m^2)
Temperature (degrees) = Sound pressure (Pa)

- Temperature & Sound pressure

At every position in the room, there is a specific temperature level, which is measured in degrees. Likewise, at every position in the room with the sound source, there is a particular sound pressure level, which is measured in Pascals. As heat is produced, the temperature level is higher closer to the heater. like temperature, the sound pressure is higher closer to the noise emitting object. Both sound pressure & temperature depend on the location & distance away from the object.

- Sound power & heater power

The heater generates a particular amount of heat per hour. The power required to generate this heat is the same, no matter what the temperature of the room. Heater power is measured in Watts. Sound power operates on the same principal. The sound power of a source is "solely" a property of that source, and is independent of the sound pressure levels in the room. Sound power is the rate at which sound energy is emitted per unit time. It is measured in Watts.

- Sound intensity & Heat flow

Heat travels & flows throughout the room. Sound intensity is the measure of flow of sound. This flow is observed over a specific area, hence the units of sound intensity are W/m^2

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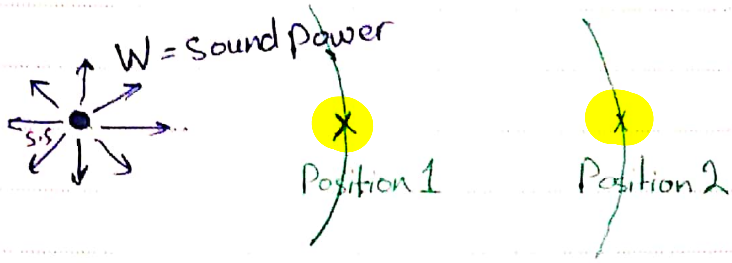
→ Consider the following point sound source



- Assume that the point source represents a machine running at steady state (i.e it is emitting ~~sou~~ constant amount of sound energy per unit time) W = constant).
- Assume a free field (i.e there are no barriers that may cause absorption & reduce W)

- Sound Intensity $I = \dfrac{W \text{ (acoustic Power)}}{A \text{ (Area)}}$

$$I_1 (\text{@ Position } 1) = \frac{W}{4\pi r_1^2} \quad , \quad I_2 = \frac{W}{4\pi r_2^2}$$

- $I_1 > I_2$, but W = constant, because the machine is running at steady state + No barriers are found.

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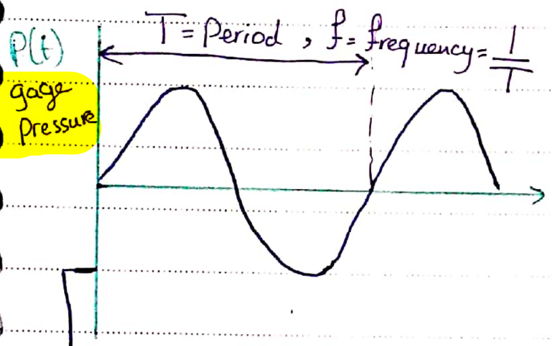
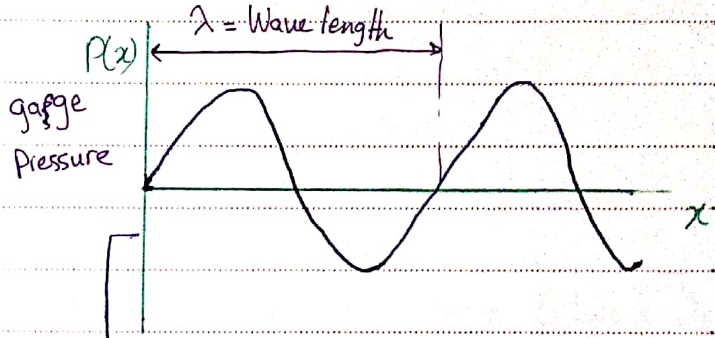
→ Sound waves can be ~~[illegible]~~ represented in different ways. Check the following video to understand how can we represent sound waves & learn new parameters Such as: Period, Frequency & wave length.

• Link https://www.youtube.com/watch?v=-_xZZt99MzY&feature=share&fbclid=IwAR25-7MzFfXwadRSY4di2UcD83b964TQO6baSwYlur5O23jeBtWBCy8iBI4



↳ represents the pressure Variation "at a certain Position" with time



↳ represents the pressure Variation "at a certain instant" with position.

$P(x) = P_o \sin(Kx + \phi)$ ↳ in the above figure $\phi = zero$

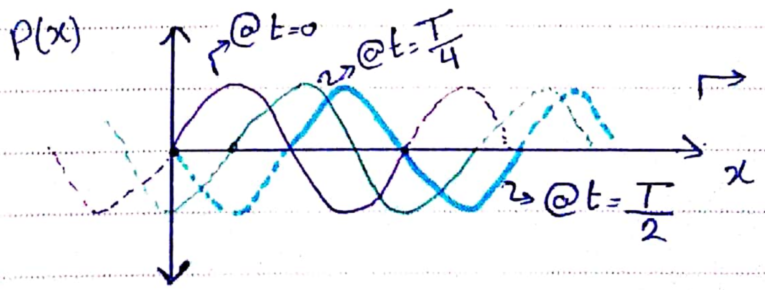
يعني هاي هي الصورة العامة لـ P(x)

→ Check also the following video, which explains how to find the sound speed.

• Link https://www.youtube.com/watch?v=UgE2GIQwUCw&feature=share&fbclid=IwAR3wJpiLZu4Na9rrHmau94ZpcXq-eONthLz4GVmOplf1tNgWc6Czgm7DC88

→ Can We find $P(x,t)$?

Based on the previous video, we can conclude that the sound wave travels a complete wave length λ after T seconds
↳ Period



→ لو رسمنا رسمة P(x) عند
أزمنة مختلفة رح نلاحظ
انها رح تتحرك طول مقداره
λ بعد زمن مقداره T

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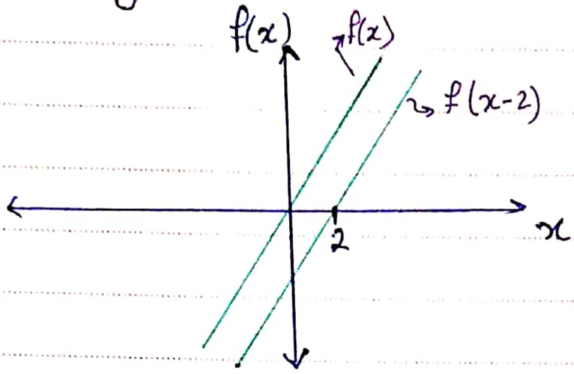
Recall from Calculus

if $f(x) = x$, $f(x-2)$ has the same plot as $f(x)$ but shifted 2 units to the right



→ Notice that $P(x)$ is shifted to the right with time (check the figure in the previous page)

assume $\underline{h}$ represents the amount of shift after $\underline{t}$ seconds

$$P(x) = P_o \sin(kx + \phi)$$

$$P(x-h) = P_o \sin(K(x-h) + \phi)$$

As we said before, the sound wave travells λ after T seconds

$$\left.\begin{array}{l} \lambda \longrightarrow T \\ h \longrightarrow t \end{array}\right\} \quad h = t * \frac{\lambda}{T} = t * \underset{\text{speed of sound}}{C}$$

$$\therefore P(x-h) = P_o \sin(K(x - ct) + \phi)$$

$$= P_o \sin(K\underline{x} - Kc\underline{t} + \phi) = P(x, t) \;!$$

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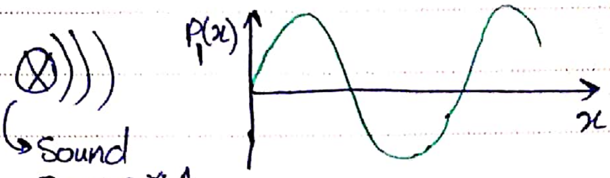
[*] Notes:

- To reduce sound or noise level we can use sound cancellation technique

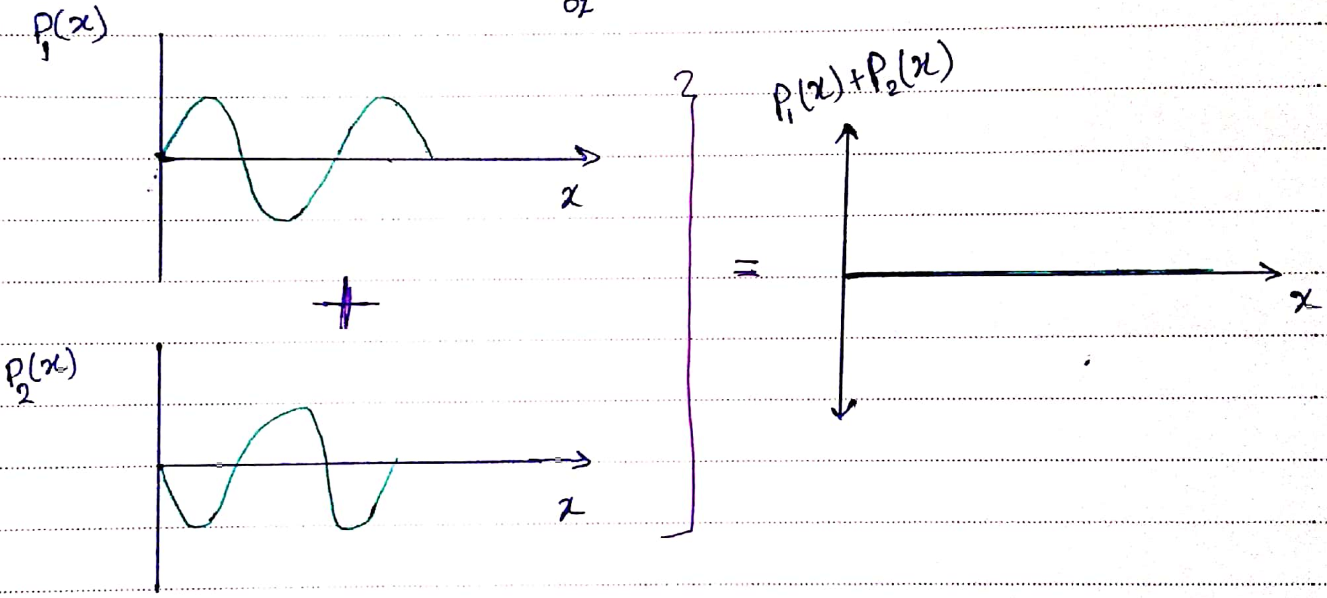


Sound source ※ 1 (it causes noise)

⇓

Sound wave of the sound source at a certain instant $\underline{\underline{t}}$

We can bring another sound source, so that both sources emit identical waves that are shifted out of phase.



- $C = \frac{\lambda}{T} = \lambda f$: for a certain medium (C = constant), low frequency waves means long wave lengths & vice versa

- To reduce noise level, we can use barriers that prevent the transmission of the sound. However, barriers can't prevent low frequency waves from being transmitted through them, why?

Think about what happens when a soundwave "hits" a wall. Really what that means is that there's a high pressure area on one side of the wall (normal pressure on the other) followed by that high pressure area becoming a low pressure area.

So while the sound wave's pressure is high, the air is pushing the wall, causing it to move a bit. This stretches the elastic mediums within the wall (like pushing on a block of jello). Eventually these elastic forces cause the far side of the wall to move, which pushes on the air on the other side, transmitting the sound. When the low pressure region hits, the elastic energy pushes the wall back towards this low pressure area. Once again, this transmits the sound to the other side.

For low frequency sounds this is most of the story. The movement of the wall is relatively fast compared to the period of the sound wave. For high frequency sounds, however, it gets more interesting. With high frequency sounds, the low pressure trough might occur while much of the energy of the sound wave is still propagating through the wall (the jello is still squished, and hasn't had a chance to release outwards towards the other side). Now the elastic energy in this wall is "happy" to go in any direction, so when the low pressure wave starts to form in the air, some of the energy put into the wall in the high pressure phase doesn't ever get through the wall. It is instead "put to work" pulling the wall back to its original shape.

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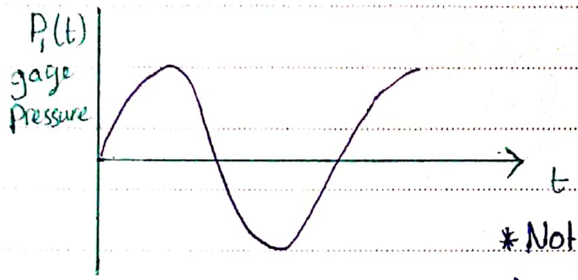
[*] Vector Analysis of Sound

• Suppose that we have 2 sound sources that have the following Sound waves

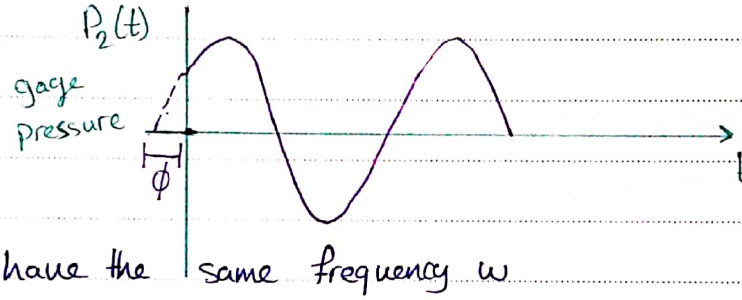
• S.S # 1



$P_1(t) = P_1 \sin(\underline{\underline{\omega}} t)$

*Note: The variation in pressure of a sound wave is in the order of μPa!

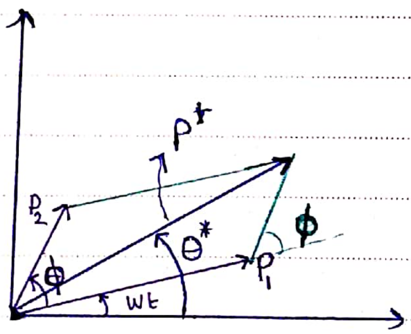
• S.S # 2



$P_2(t) = P_2 \sin(\underline{\omega} t + \phi)$

- both waves have the same frequency ω

• To find $P_{tot}(t) = P_1(t) + P_2(t) = P^* \sin\theta^*$, We will use Vector (or phasor) analysis as explained below (زي الاكثركال)



• Based on the law of Cosines $P^{*2} = P_1^2 + P_2^2 - 2P_1P_2 \cos(180 - \phi)$
$= P_1^2 + P_2^2 + 2P_1P_2 \cos\phi$

• if $P_1 = P_2$ (Same amplitude of the ~~sin~~ Sound waves) & $\phi = 0$ (in Phase Sound waves) $\Rightarrow P^{*2} = 4P_1^2 \Rightarrow P^* = 2P_1$

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• If $P_1 = P_2$ & $\phi = 180°$ (out of phase sound waves)

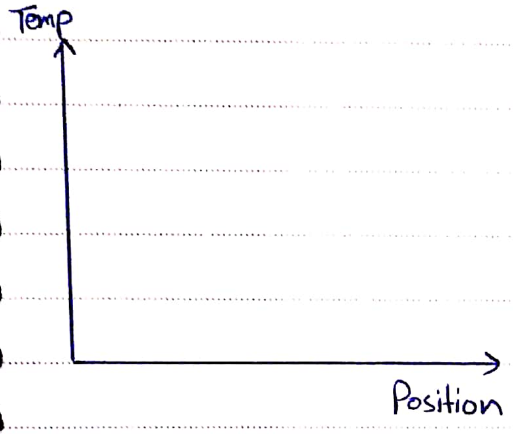
$$P_*^2 = P_1^2 + P_1^2 - 2P_1^2 = \text{Zero}$$ "Sound Cancellation"

[*] Sound power level, Sound intensity level, Sound pressure level

→ As we said before: Sound power, sound intensity, sound pressure are different quantities that can be expressed in decibels.

→ A decibel is not a unit of measure, but rather a logarithmic ratio between 2 numbers (measured quantity & a reference number)

→ Logrithmic scale is used when the variation in a quantity is large



* لو مثلاً كنت بدي أمثل Temperature-position Data
وكانت معظم درجات الحرارة ضمن ال 1000-1500°C = range
بس كان عندي نقطة درجة الحرارة عندها 50°C.
هاي النقطة ما رح تبين (أو تكون واضحة) إذا
مثلت ال data باستخدام linear scale
لأنه 50°C أصغر بكثير من 1000°C
⇐ حتى تبين النقطة بستخدم logrithmic scale

* Note: 1 meter ≡ 100 centi-meter = 10 deci-meter

• Centi & deci are prefixes (بالعربي بادئة):

Centi $= 10^{-2}$ ↝ 100 centimeter $= 100 \times 10^{-2}$ meter $= 1$ m

deci $= 10^{-1}$ ↝ 10 decimeter $= 10 \times 10^{-1}$ meter $= 1$ m

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Sound power level L_w

- L_w is defined as follows

$$L_W = \log\left[\frac{W}{W_r}\right]$$

W = measured sound power (Watts)

W_r = reference Value = 10^{-12} Watts

This logarithmic ratio has fictitious unit called Bel. When you multiply this ratio by 10, it becomes decibel → ديسبل

$$\therefore L_W = 10\log\left[\frac{W}{W_r}\right] \quad , (dB) = \text{decibel}$$

* Consider 2 identical sound sources, that are emitting the same sound wave, & both have the same sound Power $W_1 = W_2$

$$L_{W_1} (\text{for a single sound source}) = 10\log\left[\frac{W_1}{W_R}\right]$$

$$L_{W_{tot}} (\text{for both sound sources}) = 10\log\left[\frac{W_1 + W_2}{W_R}\right] = 10\log\left[\frac{2W_1}{W_R}\right]$$

$$= 10\left[\log(2) + \log\frac{W_1}{W_R}\right]$$

$$L_{W_{tot}} = 10\log 2 + 10\log\frac{W_1}{W_R} = 3\text{ dB} + L_{W_1}$$

Conclusion: Doubling the acoustic Power increases L_w by 3 dB

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8 / 10 / 2019 Tues.

→ Sound Intensity level L_I

- L_I is defined as

$$L_I = 10 \log\left(\frac{I}{I_r}\right) \quad , [dB]$$

where I = measured intensity level (W/m^2)
I_r = reference value $= 10^{-12}\ W/m^2$

- Recall:

$$W = I * A$$

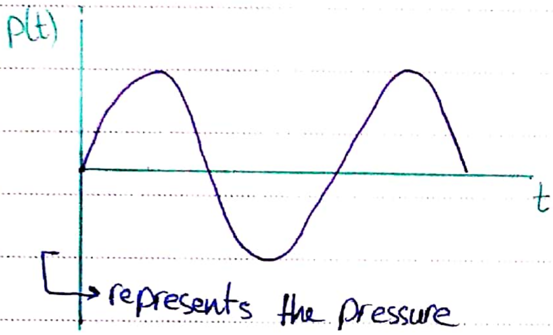
$$W = \frac{d(\text{Energy})}{dt} = \text{Force} \times \text{Velocity}$$

Force → Force exerted by the vibrating molecules of the sound wave

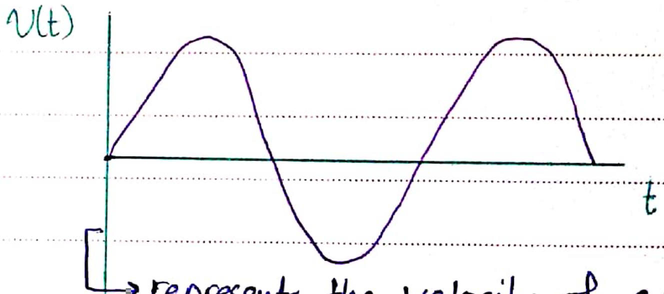
Velocity → Velocity of the vibrating molecules of the sound wave

* note:

- Work = Force x distance , [J]
  $= F \times d$
- $\frac{d}{dt}(F \times d) = F * V$



→ represents the pressure exerted by a vibrating molecule with time



→ represents the velocity of a vibrating molecule with time

- For a free wave, the following equation is satisfied

$$\rho_0 C = \frac{P_{rms}}{V_{rms}}$$

P_{rms} ⇝ rms value of $p(t)$

V_{rms} ⇝ rms value of V

C → Speed of Sound in the medium

ρ_0 → density of the medium at room temp

- **Free wave means**: A wave that doesn't ~~experence~~ experience absorption or reflection

$P(t)$ & $V(t)$ of a freewave are synchronous (i.e. P_{max} & V_{max} are attained at the same instant)

* يعني $P(t)$, $V(t)$ إلهم نفس الـ frequency وما في Phase shift بينهم

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- $\rho_o C$ = is called acoustic impedance of the medium

acoustic impedance of air = $1.21 \frac{Kg}{m^3} * 341\ m/s = 412.61 \frac{Kg}{m^2 \cdot s}$

- P_{rms} & V_{rms} are found as follows

$$P_{rms} = \underbrace{\sqrt{\underbrace{\frac{1}{T}\int_0^T \overbrace{P^2}^{\text{square}}(t)\,dt}_{\text{mean}}}}_{\text{root}}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

* Notes:
- rms = root mean square
- The mean of a value $\bar{X}$ is given by

$$\bar{X} * T = \int_0^T x(t)\,dt$$

$$\bar{X} = \frac{1}{T}\int_0^T x(t)\,dt$$

→ Now,

$W = I * A$ = Force * Velocity

$$I * \cancel{A} = P_{rms} * \cancel{A} * V_{rms} \Rightarrow I = P_{rms} * V_{rms} \quad , \quad \rho_o C = \frac{P_{rms}}{V_{rms}}$$

$$I = P_{rms} * \frac{P_{rms}}{\rho_o C}$$

$$\therefore I = \frac{P_{rms}^2}{\rho_o C}$$

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$\rightarrow$ **Sound pressure level L_p**

- L_p is defined as

will be omitted from now on

$$L_p = 10 \log\left[\frac{P_{rms}}{P_{rms,ref}}\right]^2 = 20 \log\left[\frac{P}{P_{ref}}\right], [dB], \text{ where}$$

P = Sound pressure (Pa)

P_{ref} = ref value = 20×10^{-6} Pa

- Recall

$$L_I = 10 \log\left[\frac{I}{I_r}\right], \quad I = \frac{P^2}{\rho_o C}, \quad I_r = \frac{P_r^2}{\rho_o C}$$

$$L_I = 10 \log\left[\frac{P^2}{\cancel{\rho_o C}} * \frac{\cancel{\rho_o C}}{P_r^2}\right] = 10 \log\left[\frac{P^2}{P_r^2}\right] = 20 \log\left[\frac{P}{P_r}\right]$$

$$\therefore L_I = L_p!$$

$\rightarrow$ Consider a free field that contains a sound source which emits acoustic power at a constant rate

$\otimes \xrightarrow{W=const}$ $\quad\quad$ x r_1 $\quad\quad$ x $r_2 = 2r_1$

$$L_W = 10 \log\left[\frac{W}{W_r}\right] \equiv \text{Constant}$$

$$L_{I_1} = 10 \log\left[\frac{I_1}{I_r}\right] = 10 \log\left[\frac{W}{(4\pi r_1^2) I_r}\right]$$

$$L_{I_2} = 10 \log\left[\frac{I_2}{I_r}\right] = 10 \log\left[\frac{W}{(4\pi r_2^2) I_r}\right] = 10 \log\left[\frac{W}{4(4\pi r_1^2) I_r}\right]$$

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$$L_{I_2} = 10\left(\log\left(\frac{W}{(4\pi r_1^2)I_r}\right) + \log\left(\frac{1}{4}\right)\right)$$

$$= 10\log\left(\frac{W}{(4\pi r_1^2)I_r}\right) + 10\log\frac{1}{4} = L_{I_1} - 6\ dB$$

∴ Conclusion: If the distance ~~is doubled~~ from the sound source is doubled, the Sound intensity level will be reduced by 6 dB

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$$L_{I_2} = 10\left(\log\left(\frac{W}{(4\pi r_1^2)I_r}\right) + \log\left(\frac{1}{4}\right)\right)$$

$$= 10\log\left(\frac{W}{(4\pi r_1^2)I_r}\right) + 10\log\frac{1}{4} = L_{I_1} - 6\ dB$$

∴ Conclusion: If the distance ~~is doubled~~ from the sound source is doubled, the sound intensity level will be reduced by 6 dB

* Notes:

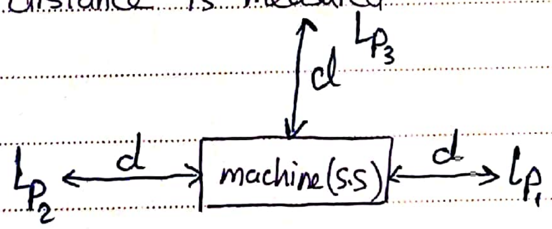
- Recall: Types of sound sources based on the shape of the sound wave they emit: [1] Spherical [2] Cylindrical [3] plane

~~Types of~~

- Sound Sources can also be classified as

[1] Directional

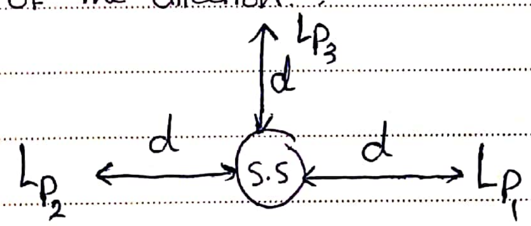
The value of L_p depends on the distance from the sound source and the direction along which the distance is measured



$$L_{P_1} \neq L_{P_2} \neq L_{P_3}$$

[2] Non-directional

The value of L_p depends only on the distance from the sound source regardless of the direction.



$$L_{P_1} = L_{P_2} = L_{P_3}$$

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→ The following figure gives the sound pressure level of various application measured a meter from the sound source

$$L_p = 20 \log\left[\frac{P_{rms}}{P_{ref}}\right] \quad , P_{ref} = 20\ \mu Pa = 20 \times 10^{-6}\ Pa$$

Application	P_{rms} (Pa)	L_p (dB)
Threshold of hearing	20μ → أقل من هيك ما منسمع	0
Residential Area	2×10^{-3}	40
Normal speech	0.02	60
Vacuum cleaner	0.2	80
pneumatic hammer	2	100
Jet engine	200	140

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☒ How to add decibels?

→ We can't add decibels like this: $60\,dB + 60\,dB = 120\,dB$ ✗ wrong

→ However, we can add sound power $0.25W + 0.03W = 0.28W$

We can add sound intensity $0.25\frac{W}{m^2} + 0.03\frac{W}{m^2} = 0.28\,W$

→ Consider 2 identical sound sources that emit the same sound power, & we want to measure L_p (or L_I) at a certain location:

$$L_p = L_I = 10\log\left[\frac{I}{I_r}\right]$$

$$L_I \text{ (for a single S.S)} = 10\log\left[\frac{W}{A\,I_r}\right]$$

$$L_I \text{ (for both S.S)} = 10\log\left[\frac{2W}{A\,I_r}\right] = 10\left(\log 2 + \log\left[\frac{W}{A\,I_r}\right]\right)$$

$$= 10\log 2 + 10\log\left(\frac{W}{A\,I_r}\right)$$

$$L_I = 3\,dB + L_{I,\text{single ss.}}$$

∴ Conclusion: if W is doubled, L_I at a certain position will be increased by 3 dB.

Hence, if $L_{I_1} = 80\,dB$, $L_{I_2} = 80\,dB \Rightarrow L_{I,tot} = 83\,dB$

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→ Rule:

- X = represents the difference between 2 pressure levels $= L_{P_1} - L_{P_2}$

if $X = 0$ (i.e. $L_{P_1} = L_{P_2} \Rightarrow L_{P_{tot}} = L_{P_1} + 3$ dB)

→ What if $X \neq 0$?

Example: Consider

$L_{I_1} = 80$ dB , $L_{I_2} = 75$ dB , find $L_{I_{tot}}$?

Here $X = 80 - 75 = 5$ dB ∴ We can't use the above rule.

$$L_{I_1} = 10 \log\left[\frac{I_1}{I_r}\right] \Rightarrow \frac{L_{I_1}}{10} = \log\left[\frac{I_1}{I_r}\right] \Rightarrow \boxed{I_1 = I_r * 10^{\frac{L_{I_1}}{10}}}$$

$$L_{I_2} = 10 \log\left[\frac{I_2}{I_r}\right] \Rightarrow \frac{L_{I_2}}{10} = \log\left[\frac{I_2}{I_r}\right] \Rightarrow \boxed{I_2 = I_r * 10^{\frac{L_{I_2}}{10}}}$$

$$I_{tot} = I_1 + I_2 = I_r * 10^{L_{I_1}/10} + I_r * 10^{L_{I_2}/10}$$

$$= I_r\left[10^{L_{I_1}/10} + 10^{L_{I_2}/10}\right]$$

$$L_{I_{tot}} = 10 \log\left[\frac{I_{tot}}{I_r}\right] = 10 \log\left[\frac{\cancel{I_r}\left[10^{L_{I_1}/10} + 10^{L_{I_2}/10}\right]}{\cancel{I_r}}\right]$$

$$L_{I_{tot}} = 10 \log\left[10^{L_{I_1}/10} + 10^{L_{I_2}/10}\right]$$

$$= 10 \log\left[10^{80/10} + 10^{75/10}\right] = 81.19 \text{ dB}$$

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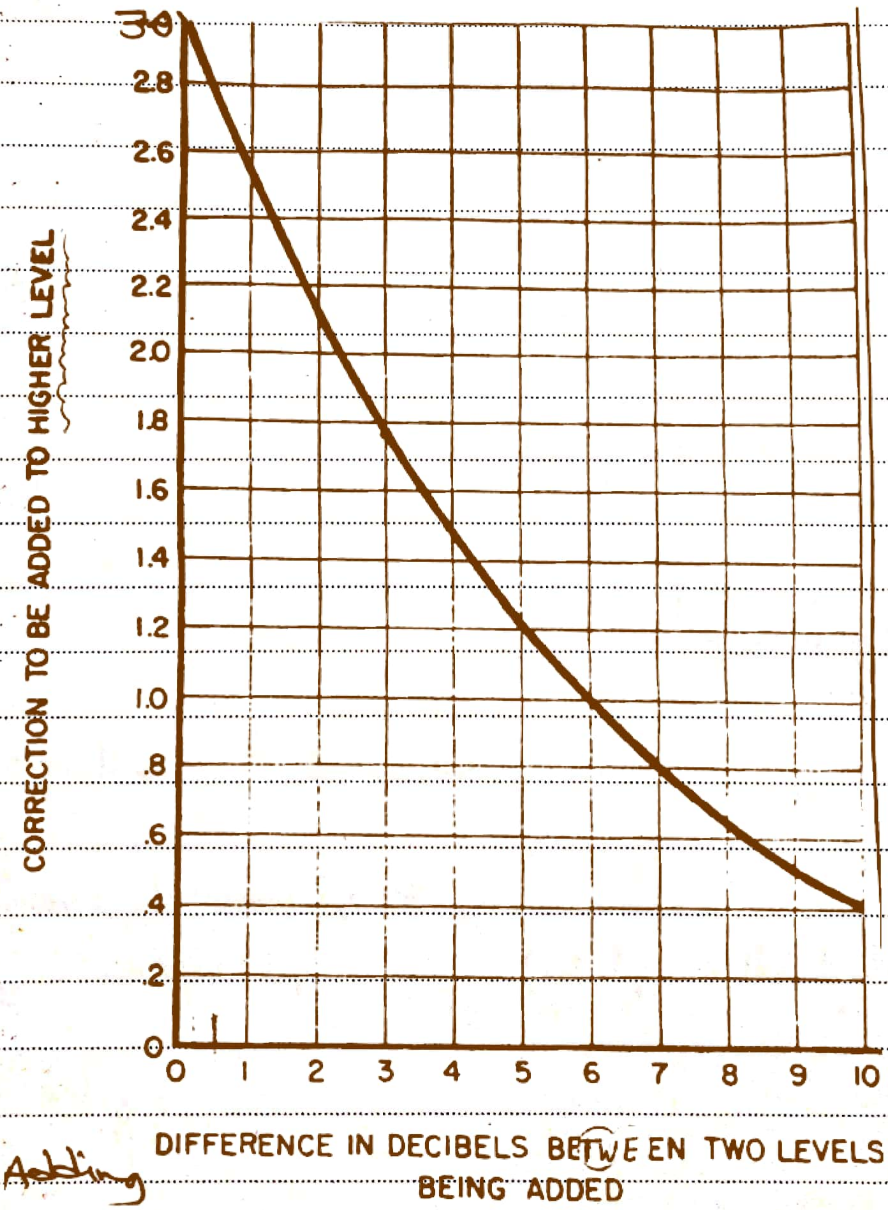
→ In general If we want to add:

$L_{I_1}, L_{I_2}, L_{I_3}, \ldots\ldots, L_{I_n}$

$$L_{I_{tot}} = 10 \log\left[10^{L_{I_1}/10} + 10^{L_{I_2}/10} + 10^{L_{I_3}/10} + \cdots + 10^{L_{I_n}/10}\right]$$

→ However, in the field, we don't use the previous formula, since we don't use calculators in the field! Instead, we use a chart like the one shown below.



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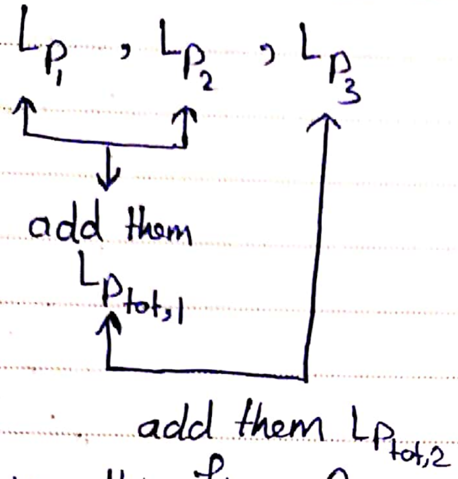
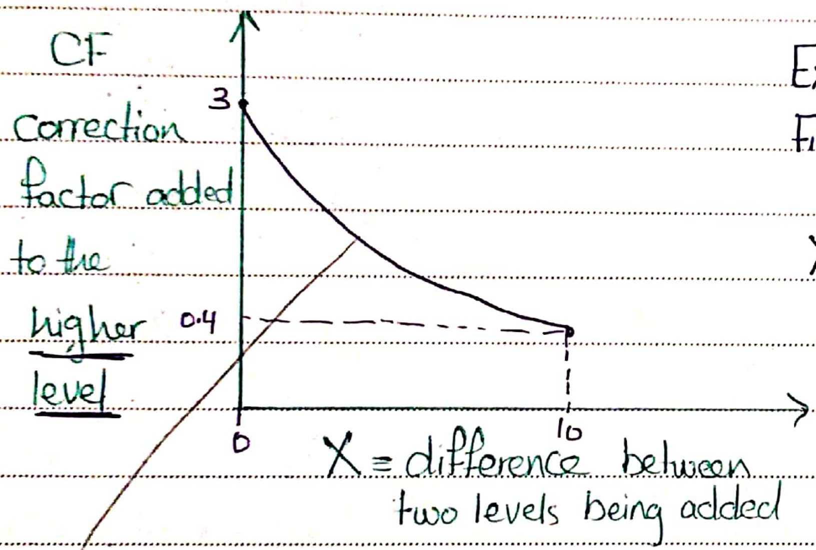
10 / 10 /2019 Thurs.

→ This figure is used when we want to add ~~to~~ 2 sound pressure levels. If we want to add more than 2 sound levels, we have to do the following



→ How to use this figure?



Example: if $L_{p_1} = 30$ dB, $L_{p_2} = 40$ dB

Find $L_{p_{tot}}$

$X = 40 - 30 = 10$ dB

$X = 10$ dB $\Rightarrow CF = 0.4$

$L_{p_{tot}} = 0.4 + 40 = 40.4$

↳ متجمعها الـ 40 dB

مش الـ 30dB (متجمعها الـ

(higher level

↳ To find the equation of the curve

let

X = the difference between L_{p_1} & L_{p_2}

$X = L_{p_1} - L_{p_2}$, where $L_{p_1} \geq L_{p_2}$ (i.e L_{p_1} is the higher level)

$L_{p_{tot}} = CF + L_{p_1}$

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$$L_{P_{tot}} = CF + L_{P_1}$$

$$CF = L_{P_{tot}} - L_{P_1}$$

* Note:

- $I_t = I_1 + I_2$
- $L_{P_{tot}} = L_{I_{tot}} = 10 \log\left[\frac{I_1 + I_2}{I_r}\right] = 10 \log\left[\frac{I_1}{I_r} + \frac{I_2}{I_r}\right]$
- $L_{P_1} = L_{I_1} = 10 \log\left[\frac{I_1}{I_r}\right] \rightarrow \frac{I_1}{I_r} = 10^{L_{P_1}/10}$
- $L_{P_2} = 10 \log\left[\frac{I_2}{I_r}\right] \rightarrow \frac{I_2}{I_r} = 10^{L_{P_2}/10}$

$$CF = L_{P_{tot}} - L_{P_1}$$

$$= 10 \log\left[10^{L_{P_1}/10} + 10^{L_{P_2}/10}\right] - L_{P_1} \quad , \quad X = L_{P_1} - L_{P_2}$$

$$\therefore L_{P_2} = L_{P_1} - X$$

$$CF = 10 \log\left[10^{L_{P_1}/10} + 10^{(L_{P_1} - x)/10}\right] - L_{P_1}$$

$$10^{L_{P_1}/10} * 10^{-x/10}$$

$$CF = 10 \log\left[10^{L_{P_1}/10}\left(1 + 10^{-x/10}\right)\right] - L_{P_1}$$

$$= 10 \log 10^{L_{P_1}/10} + 10 \log\left(1 + 10^{-x/10}\right) - L_{P_1}$$

$$= \cancel{10} * \frac{\cancel{L_{P_1}}}{\cancel{10}} + 10 \log\left(1 + 10^{-x/10}\right) - \cancel{L_{P_1}}$$

$\therefore CF = 10 \log\left(1 + 10^{-x/10}\right)$ ⇝ This is the equation of the curve.

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Example: Add the following decibels:

65 dB, 70 dB, 72 dB, 73 dB, 85 dB, 90 dB, 95 dB.

Solution:

Method I:

No.	L_p(dB)	$\frac{I_i}{I_r} = 10^{L_{p_i}/10}$
1	65	3162277.66
2	70	10^7
3	72	15848931.92
4	73	19952623.15
5	85	316227766
6	90	10^9
7	95	3162277660

$$\sum_{i=1}^{7} 10^{L_{p_i}/10} = 4527469259$$

$$L_{P_{tot}} = L_{I_{tot}} = 10 \log\left(\sum_{i=1}^{7} 10^{L_{P_i}/10}\right) = 96.6 \text{ dB}$$

Method 2: (we will use the Chart) , $CF = 10 \log(1 + 10^{-x/10})$

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95, 90 → $X = 95 - 90 = 5\text{dB}$, $CF = 1.2$ } $L_{P_{tot_1}} = 95 + 1.2 = 96.2$ dB

85 → $X = 96.2 - 85 = 11.2$, $CF = 0.3$ ⇒ $L_{P_{tot_2}} = 96.2 + 0.3 = 96.5$ dB

73 → $X = 96.5 - 73 = 23.5$, $CF = 0.019$ ⇒ $L_{P_{tot_3}} = 96.5 + 0.019 = 96.51$ dB

72

70

65

Continue

*Note: I used the ~~value~~ formula of $CF = 10 \log(1 + 10^{-x/10})$ to find CF

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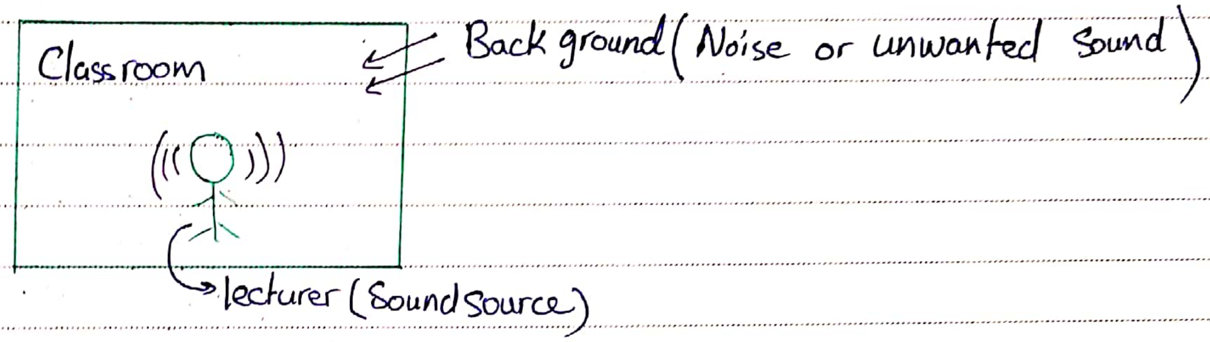
⊠ Subtracting Decibels

→ Consider the following noise environment



→ The total sound intensity "at a certain position" in the space of the class room is

$$I_{tot} = I_S + I_B ,$$

I_S ↓ Sound Source, I_B ↓ Background

→ To find I_S, we have to do subtraction

$$I_S = I_{tot} - I_B$$

(Recall: We can't subtract decibels in the following way: $L_{P_S} = L_{P_{tot}} - L_{P_B}$ Wrong X)

→ Note: $L_{P_{tot}}$ can be measured experimentally using a sound level meter. L_{P_B} can be also measured by turning off the sound source. (We can turn off the sound source, but we can't control the noise coming from outside the classroom). Since $L_{P_{tot}}$ & L_{P_B} can be measured, we can calculate L_{P_S} which can't be measured, because we can't eliminate L_{P_B}. This is why we need to learn how to subtract decibels.

→ We will use a chart to do subtracting & find L_{P_S}

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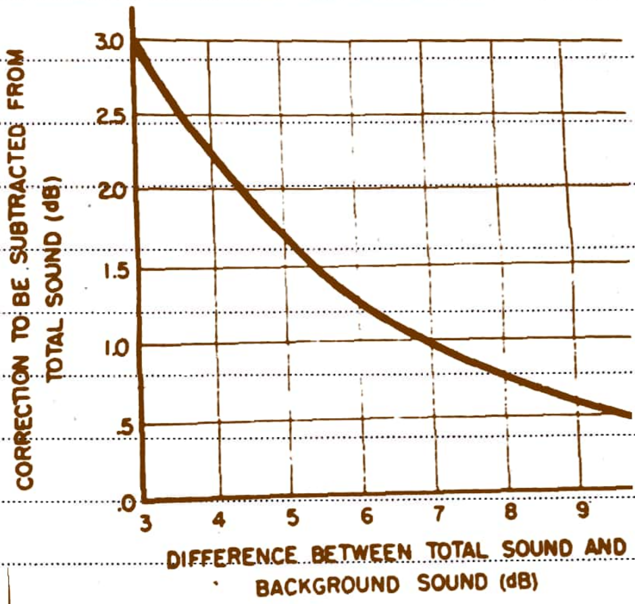
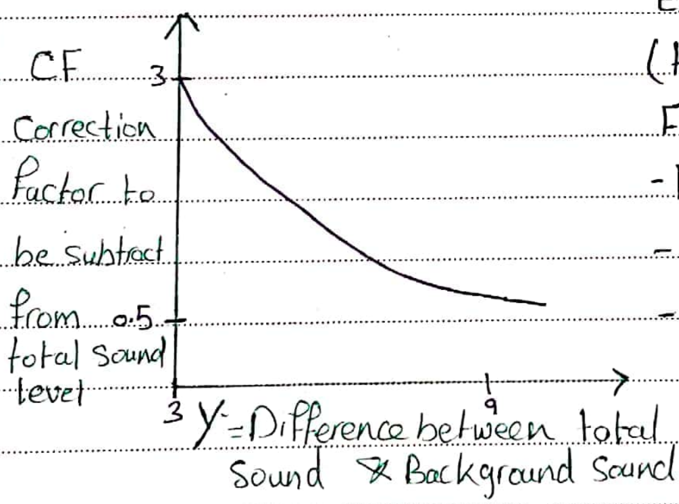


subtracting

→ How to use this Chart?



Example if $L_{P_{tot}} = 77$ dB , $L_{P_B} = 70$ dB
(these are measured experimentally)

Find L_{P_S}?

- Find $Y = L_{P_{tot}} - L_{P_B} = 77 - 70 = 7$ dB
- when $Y = 7$ dB $\longrightarrow$ CF = 1
- $\therefore L_{P_S} = L_{P_{tot}} - CF = 77 - 1 = 76$ dB

→ Find the equation of the curve shown above

$$L_{P_S} = L_{P_{tot}} - CF \Rightarrow CF = L_{P_{tot}} - L_{P_S}$$

$$I_{tot} = I_S + I_B$$
$$I_S = I_{tot} - I_B$$

$$= L_{P_{tot}} - 10 \log \left[\frac{I_S}{I_r} \right]$$

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$$CF = L_{P_{tot}} - 10\log\left[\frac{I_{tot}}{I_r} - \frac{I_B}{I_r}\right] , \quad L_{P_{tot}} = 10\log\left[\frac{I_{tot}}{I_r}\right]$$

$$L_{PB} = 10\log\left[\frac{I_B}{I_r}\right]$$

$$CF = L_{P_{tot}} - 10\log\left[10^{L_{P_{tot}}/10} - 10^{L_{P_B}/10}\right] , \quad Y = L_{P_{tot}} - L_{P_B}$$

$$CF = L_{P_{tot}} - 10\log\left[10^{L_{P_{tot}}/10} - \underbrace{10^{(L_{P_{tot}}-Y)/10}}_{10^{L_{P_{tot}}/10} * 10^{-Y/10}}\right]$$

$$CF = L_{P_{tot}} - 10\log\left[10^{L_{P_{tot}}/10}\left(1 - 10^{-Y/10}\right)\right]$$

$$= L_{P_{tot}} - 10\log 10^{L_{P_{tot}}/10} - 10\log\left(1 - 10^{-Y/10}\right)$$

$$CF = \cancel{L_{P_{tot}}} - \cancel{10} * \frac{L_{P_{tot}}}{\cancel{10}} - 10\log\left(1 - 10^{-Y/10}\right)$$

$$\therefore CF = -10\log\left(1 - 10^{-Y/10}\right)$$

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[*] Averaging levels

→ Till now, we learnt how to add & subtract sound pressure levels. Now, we will learn how to find the average of sound pressure levels. But before doing that, let's find a relation between L_W & L_I (or L_P):

$W = I * A$

$$\frac{W}{W_r} = \frac{I * A}{I_r}$$

although these quantities have different units $[W_r] = Watt$, $[I_r] = Watt/m^2$, they have the same numerical value $W_r = 10^{-12}$, $I_r = 10^{-12}$

→ In other words, the numerical value of $\frac{W}{W_r}$ is equal to the numerical value of $\frac{I * A}{I_r}$

→ Take the logarithm of both sides of the above equation & multiply both sides by 10:

$$10 * \log\left(\frac{W}{W_r}\right) = 10 * \log\left(\frac{I * A}{I_r}\right)$$

$$10 \log\left(\frac{W}{W_r}\right) = 10\left(\log\left(\frac{I}{I_r}\right) + \log(A)\right)$$

$$10 \log\left(\frac{W}{W_r}\right) = 10 \log\left(\frac{I}{I_r}\right) + 10 \log(A)$$

$$L_W = L_P + 10 \log(A)$$

→ L_P (or L_I) = Sound pressure level → can be measured using sound level meter. However, L_W can't be measured, therefore, it is calculated using the previous relation

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→ Now, we will learn how to find the average of sound pressure levels.
Consider the following sound pressure levels:

$L_{P_1}, L_{P_2}, L_{P_3}, \ldots, L_{P_n}$, We want to find their average $\overline{L}_P$ (or $\overline{L}_I$)

$$\frac{I_1}{I_r} = 10^{L_{P_1}/10}$$

$$\frac{I_2}{I_r} = 10^{L_{P_2}/10}$$

$$\frac{I_3}{I_r} = 10^{L_{P_3}/10}$$

$$\vdots$$

$$\frac{I_n}{I_r} = 10^{L_{P_n}/10}$$

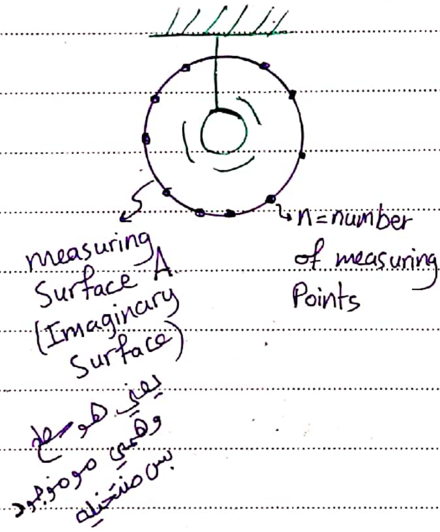
$$\frac{\overline{I}}{I_r} = \frac{\sum_{i=1}^{n} \frac{I_i}{I_r}}{n} = \frac{\sum_{i=1}^{n} 10^{L_{P_i}/10}}{n}$$

$$\overline{L}_I = 10 \log\left(\frac{\overline{I}}{I_r}\right) = 10 \log\left(\frac{\sum_{i=1}^{n} 10^{L_{P_i}/10}}{n}\right)$$

$$\overline{L}_I = 10 \log\left(\sum_{i=1}^{n} 10^{L_{P_i}/10}\right) + 10 \log\left(\frac{1}{n}\right)$$

$$\overline{L}_I = 10 \log\left(\sum_{i=1}^{n} 10^{L_{P_i}/10}\right) - 10 \log(n) \quad ※$$

→ Why is it important to know how to find the average of levels?
Consider the following problem:



- We have a suspended speaker. It acts as a point sound source. We want to find the sound power level of this source L_W. We will use the formula $L_W = \overline{L}_P + 10 \log A$
- First of all, we have to determine the surface A, since we will measure the sound pressure level along this surface
- We will take a number of sound ~~along~~ pressure level measurements along this surface (i.e we will take more than one reading)
- Find $\overline{L}_P$
- Find $L_W = \overline{L}_P + 10 \log A$

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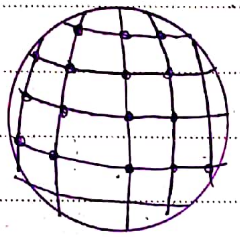
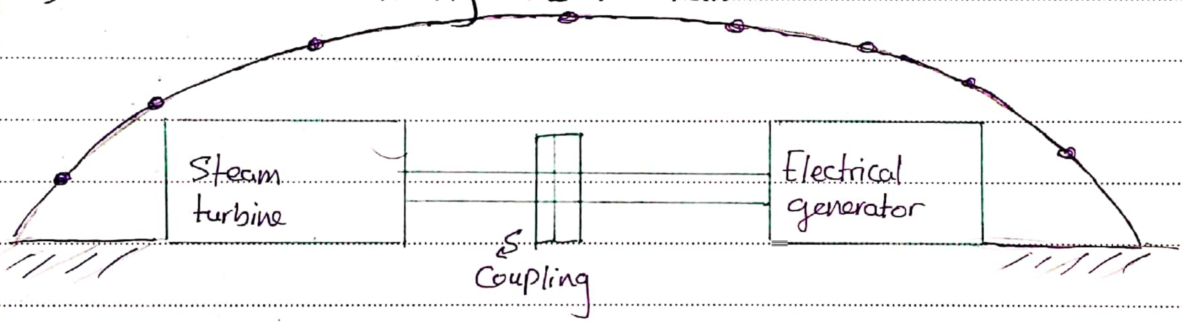


→ This is a 3-D view of the measuring surface A & measuring points n

→ There is a recommended number of measuring points (n) [will be discussed in the next lecture]

→ For point sound sources (like the speaker shown in the previous page), the values of Lp measured along the spherical surface must be equal (theoretically speaking), ~~beca~~ because point sound sources are non-directional sound sources. However, in reality, they are not equal, due to the existance of barriers which cause reflection & absorption. This is why we need to do more than a single sound pressure level measurement & find their average

→ Consider the following machine train:



→ We can't suspend this machine train! Hence, we have to select a measuring surface other than the spherical surface, like hemispherical surface (shown above) or conformal surface (discussed in the next lecture

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[*] Evaluation of sound power level L_W

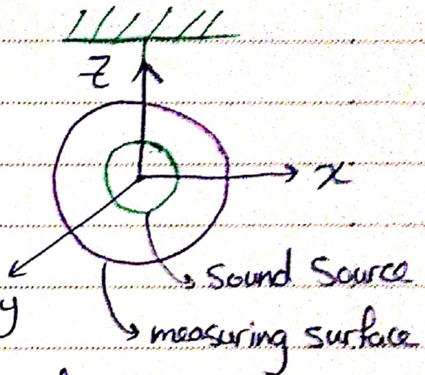
→ As discussed in the previous lecture, L_W is calculated as follows:

$$L_W = \bar{L}_p + 10 \log A$$

A → area of the measuring surface

→ In this lecture we will recognize the types of measuring surfaces, the recommended number of measuring points & their positions along the measuring surface.

[I] Spherical measuring surface



→ The table shown below gives the recommended array of microphone positions in a free field for a spherical measuring surface.

→ Microphone means sound pressure level meter

→ Notice that the recommended number of measuring points is 20.

→ The table gives the Cartezian coordinates (x, y, z) of each measuring point with the origin located at the center of the sound source.

→ "r" = radius of the measuring surface. "r" must be $\geq 1.5 *$ largest dimension of the sound source

spherical

TABLE [redacted] Recommended array of microphone positions in a free field[3.16]*

No.	$\frac{x}{r}$	$\frac{y}{r}$	$\frac{z}{r}$
1	−0.99	0	0.15
2	0.50	−0.86	0.15
3	0.50	0.86	0.15
4	−0.45	0.77	0.45
5	−0.45	−0.77	0.45
6	0.89	0	0.45
7	0.33	0.57	0.75
8	−0.66	0	0.75
9	0.33	−0.57	0.75
10	0	0	1.0
11	0.99	0	−0.15
12	−0.50	0.86	−0.15
13	−0.50	−0.86	−0.15
14	0.45	−0.77	−0.45
15	0.45	0.77	−0.45
16	−0.89	0	−0.45
17	−0.33	−0.57	−0.75
18	0.66	0	−0.75
19	−0.33	0.57	−0.75
20	0	0	−1.0

The locations of 20 points associated with equal areas on the surface of a sphere of radius r are shown in this table which gives the Cartesian coordinates (x, y, z) with origin at the centre of the source. The z-axis is chosen perpendicularly upward from a horizontal plane $(z = 0)$.

* (Reproduced by permission of the British Standards Institution, London.)

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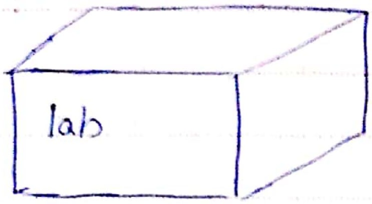
→ Notice that the caption of the table is: "Recommended array of microphone positions in a free field"

↓

i.e. barriers mustnot exist

Therefore, this type of measurement can't be performed outdoors. It must be performed in labs.



⇝ Walls, floor & ceiling of the lab room must be lined with acoustical material to absorb the sound & prevent reflection.

↳ will affect the values of L_p.

$\boxed{2}$ Hemispherical measuring surface.

→ Check the next page which shows:

1) A figure that depicts the location of microphones for a hemispherical measurement.

2) Two tables that give the coordinates of the measuring points:

(a) This table is used when the nature of sound wave emitted from the sound source is unkown or when the sound wave is complex (not pure tone).

(b) This table is used if the sound source emits pure tone or predominant pure tone

→ Notice that the recommended number of the measuring points is 10.

→ The cartesian coordinates (x, y, z) of each measuring point are obtained from the table with the origin located at the center of the hemisphere

→ "r" = radius of the hemisphere. "r" must be $\geq 1.5 *$ largest dimension of the sound source

→ What does "Predominant pure tone" mean?

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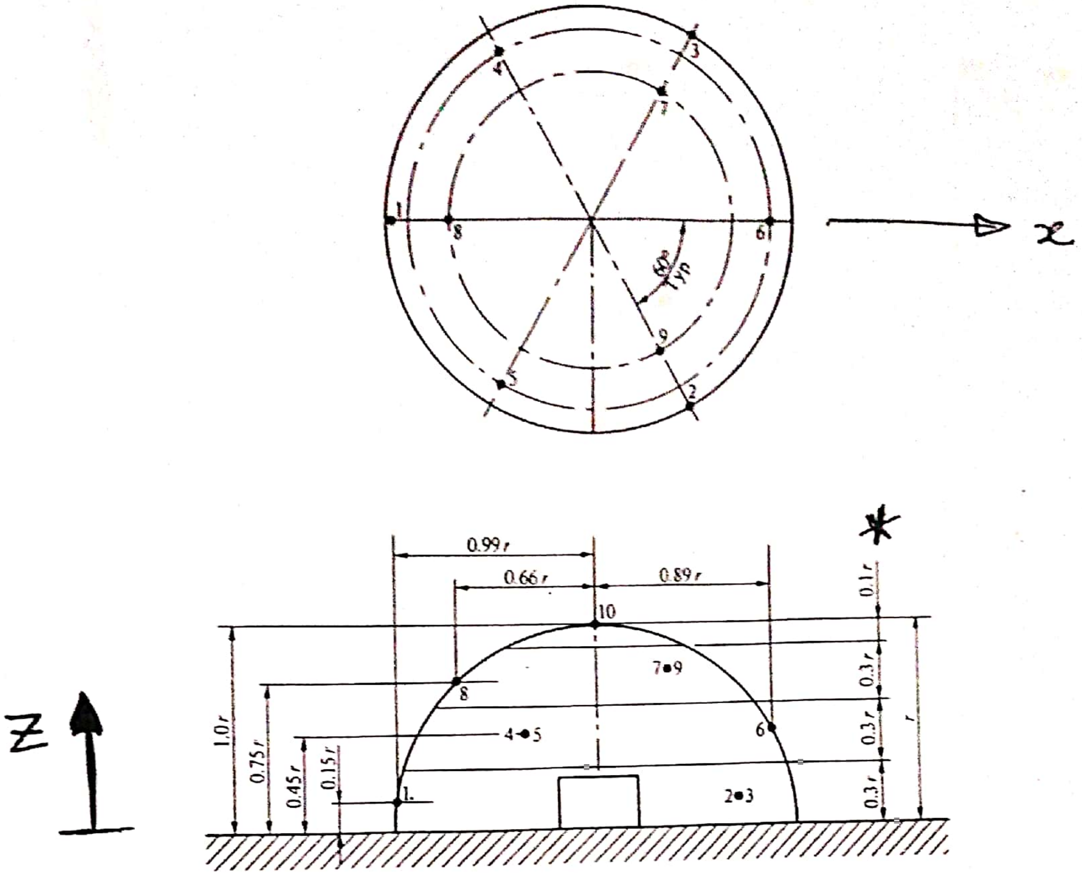

Fig. [redacted] Location of microphones for a 10-point hemispherical measurement (Reproduced by permission of the British Standards Institution, London).[3.16]

TABLE [redacted] Coordinates of measuring points on a hemispherical surface[3.16]†

(a) Coordinates of key measurement points

No.	$\frac{x}{r}$	$\frac{y}{r}$	$\frac{z}{r}$
1	−0.99	0	0.15
2	0.50	−0.86	0.15
3	0.50	0.86	0.15
4	−0.45	0.77	0.45
5	−0.45	−0.77	0.45
6	0.89	0	0.45
7	0.33	0.57	0.75
8	−0.66	0	0.75
9	0.33	−0.57	0.75
10	0	0	1.0

(b) Recommended microphone positions when the source emits predominant pure tones*

No.	$\frac{x}{r}$	$\frac{y}{r}$	$\frac{z}{r}$
1	0.16	−0.96	0.22
2	0.78	−0.60	0.20
3	0.78	0.55	0.31
4	0.16	0.90	0.41
5	−0.83	0.32	0.45
6	0.83	−0.40	0.38
7	−0.26	−0.65	0.71
8	0.74	−0.07	0.67
9	−0.26	0.50	0.83
10	0.10	−0.10	0.99

* If the source emits predominant pure tones, strong interference effects may occur if several microphone positions are placed at the same height above the reflecting plane. In such cases the use of a microphone array with the coordinates given in (b) is recommended.
† (Reproduced by permission of the British Standards Institution, London.)

$r \geq 1.5 * $ Largest dimension

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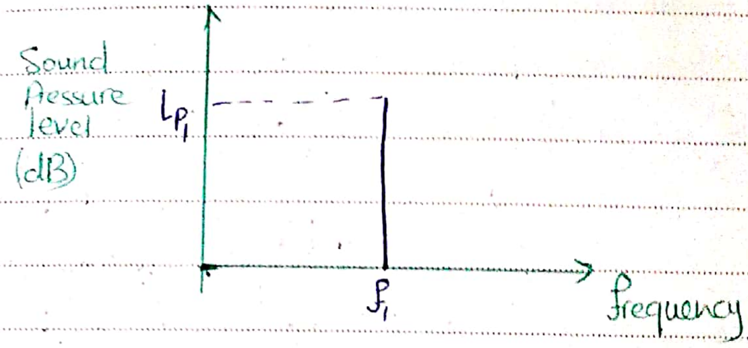
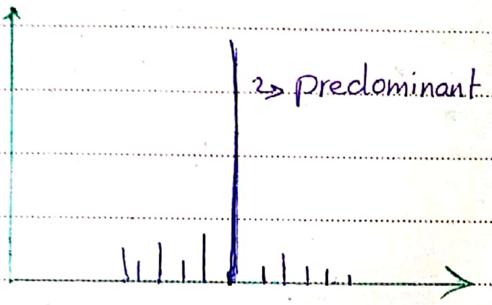
→ pure tone sound means single frequency sound wave



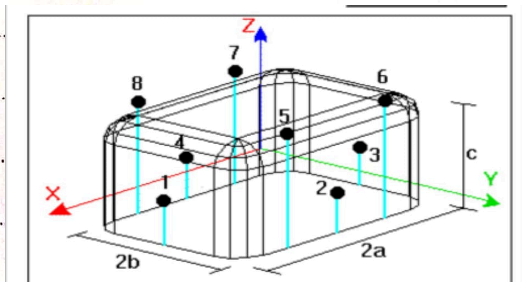
→ Predominant pure tone has the following spectrum



* يعني كأنه pure tone

3| Conformal surface.

→ The following figure shows a 3D view of the conformal surface.



Conformal Surface (8 mics)

→ The recommended number of the measuring points for a conformal surface is 16.

→ The location (x,y,z) of each measuring point is not tabulated. However, the Cartesian coordinates of each point are found from the figure shown in the next page.

→ The figure shows top & front views of the conformal surface. Based on the various parameters: $d, a, b, L_1, L_2, L_3, h_1, h_2$, one can find the location of the measuring points which are designated by the numbers from 1 to 16.

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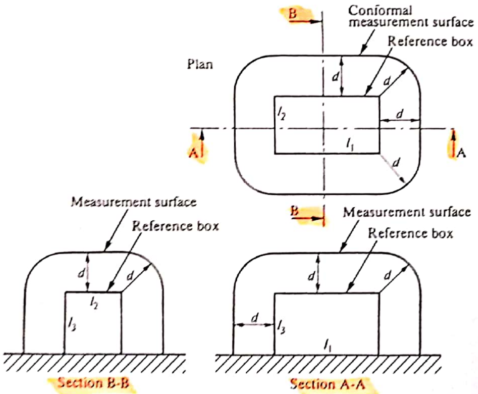
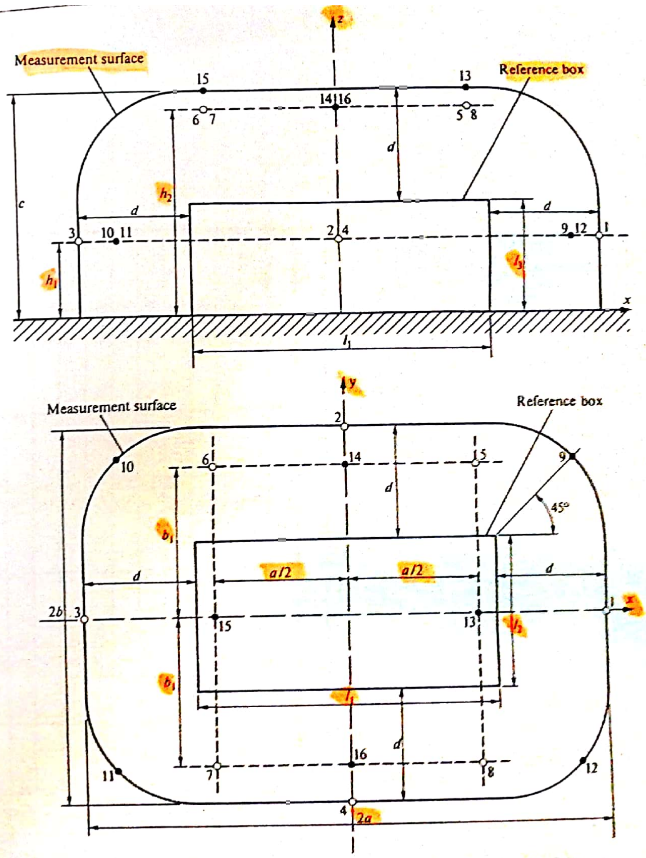


Fig. [redacted] (c) Reference box and conformal surface (reproduced by permission of the British Standards Institution, London).[3,18]

ISO
Ref. Box



$h_1 = 0.25\,(b+c-d)$	$a = 0.5\,l_1 + d$	○ Key measurement points
$h_2 = 0.75\,(b+c-d) < c$	$b = 0.5\,l_2 + d$	● Additional measurement points
$b_1 = 0.5\,(b+c-d) < b$	$c = l_3 + d$	

where d is the measurement distance, normally 1 m.

The minimum height of the microphone above the ground shall be 0.15 m.

Fig. [redacted] (d) Microphone positions on the conformal surface (reproduced by permission of the British Standards Institution, London).[3,18]

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4 Measuring Surface for horizontal machine train

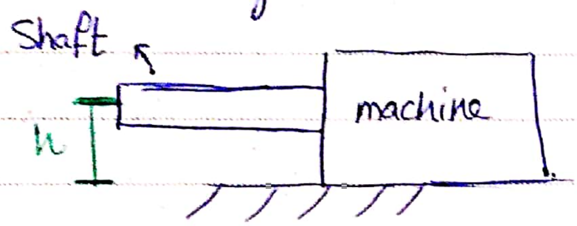
→ The following figure Shows the location of the measuring points along a measuring surface that can be used for horizontal machine train

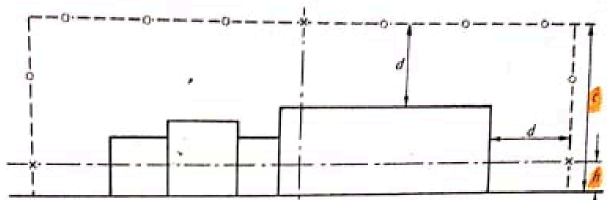
→ d is a dimension that depends on the value of ℓ = longest dimension of the machine.

if $l \geq 0.25$ $\quad d = 1$

if $l < 0.25$ $\quad$ both inequalities must be satisfied $4\ell \leq d \leq 1$ & $d > 0.25$

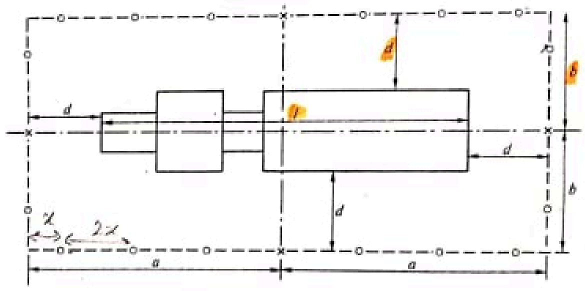
→ h = Shaft height or 0.25 m whichever is greater





Prescribed path in vertical plane

l (m)	d (m)
≥ 0.25	1
< 0.25	$4l \leq d \leq 1$ $d > 0.25$



Prescribed path in horizontal plane (at height h above reflecting plane)

h – shaft height or 0.25 metre wichever is greater

o – key measuring point

× – other measuring points marked off at intervals of 1 m from key points

Fig. [redacted] (a) Location of measuring points and prescribed paths for horizontal machines.[3,5]

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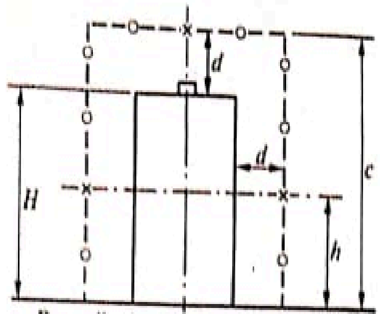
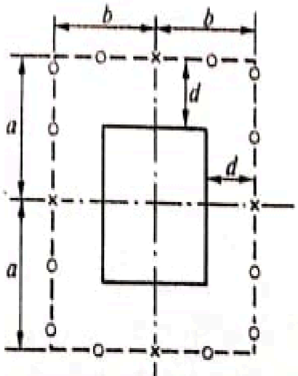
5 Measuring surface for vertical machines

→ The following figure shows the location of the measuring points ~~that~~ along a measuring surface that can be used for vertical machine train



Prescribed path in vertical plane

H (m)	d (m)
≥ 0.25	1
< 0.25	$4l \le d < 1$ $d > 0.25$



$h = \frac{H}{2}$ but not less than 0.25 m

× key measuring points

o other measuring points marked off at intervals of 1 m from key points

Prescribed path in horizontal plane (at height h above reflecting plane)

Fig. [redacted] (b) Location of measuring points and prescribed paths for vertical machines.[3,5]

3

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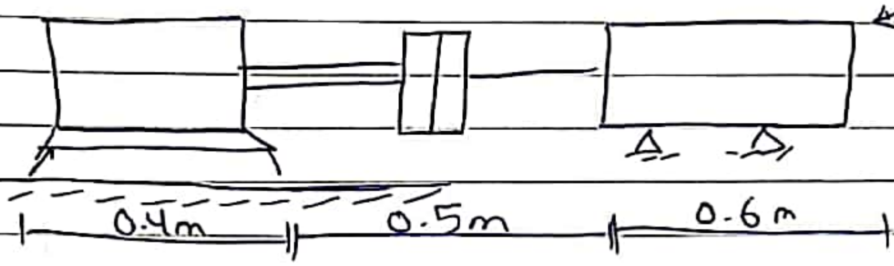
⇒ 17/10/2019:

$\phi = 0.75m$

$\phi = 0.4m$



	Lp (dB)	$10^{Lp/10}$		Lp (dB)	$10^{Lp/10}$
1	80	10^{8}	9	70	10^{7}
2	70	$10^{7.0}$	10	73	$10^{7.3}$
3	85	$10^{8.5}$	11	90	10^{9}
4	75	$10^{7.5}$	12	100	$10^{10.0}$
5	70	10^{7}	13	105	$10^{10.5}$
6	90	10^{9}	14	95	$10^{9.5}$
7	65	$10^{6.5}$	15	85	$10^{8.5}$
8	70	10^{7}	16	90	10^{9}
total					

$L_w = ?$

① $L_w = \bar{L}_p + 10 \log A$

② $L_p = 10 \log \left(\sum_{i=1}^{16} 10^{Lp/10} \right) - 10 \log(n)$

Sol → $10 \log (4.861 \times 10^{10}) - 10 \log (16)$

$\bar{L}_p = 106.867 - 10 \log(16) = 94.825 \text{ dB}.$

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⇒ hemispherical surface to find our the Area:-

1- assume the acoustical center with r as a radius.

2- $r \geq 1.5 * \text{largest dimension}$.

⇒ the largest dimension $= 1.5\,m = 0.4 + 0.5 + 0.6$

$r \geq 1.5 * 1.5 \Rightarrow \boxed{r \geq 2.25\,m}$ #

$A_{h.s} = \frac{1}{2} * 4 * \pi * r^2 = 31,8\,m^2$

$\boxed{L_W = 94,8 + 10 \log(31,8) = 109,85\ dB.}$ #

→ Sound power level.

⇒ **Conformal Surface**:

المساحة عالكونفورمل تعبر

finding the area :-

(1) $l_1 =$ overall length $= 1.5\,m$ → Reference box.

$l_2 =$ overall width $= 0.75\,m$

$l_3 =$ overall hight $= 0.75 + 0.15$

$l_3 = 0.9\,m$ → steel frame (foundation)

$d =$ normally $1\,m$ → (radius of the cylinder corner)

⇒ $a = 0.5\,l_1 + d = 0.5 * 1.5 + 1 = \frac{7}{4} = 1.75\,m$

(2) $b = 0.5\,l_2 + d = 0.5 * 0.75 + 1 = 1.375\,m$

$c = l_3 + d = 0.9 + 1 = 1.9$

(3) $h_1 = 0.25(b + c - d) = 0.25(1.375 + 1.9 - 1) = 0.56875\,m$

$h_2 = 0.75(b + c - d) < c = 1.706 < 1.9$ ✓

$b_1 = 0.5(b + c - d) < b = 1.1375 < 1.375$ ✓

location # 9, #12 :-

x-coordinate
Y-coordinate
Z-coordinate $\Big\}$ of point 9 :-

كل النقاط اللي على الـ سلندر الهوسنس
المسافات والحركات بالعينات

$x = l_1/2 + d\cos(45°) = \frac{1.5}{2} + 1 \times \frac{1}{\sqrt{2}} = 1.457\ m$

$y = l_2/2 + d\sin(45°) = \frac{0.75}{2} + \sin(45°) = 1.082\ m$

$z = h_1 = 0.56875\ m$

* Point 12 : $x = l_1/2 + 1\cos(-45) = 1.457\ m$
$y = -l_2/2 + 1\sin(-45) = -1.082\ m$
$z = h_1 = 0.56875\ m$

=> point 11 : $x = -[l_1/2 + \cos(45)] = -1.457$
$y = -[l_2/2 + \sin(45)] = -1.082\ m$
$z = h_1 = 0.56875$

=> point 14 : $x = 0$
$y = b = 1.1375\ m$
$z = h_2 = 1.70625\ m$

point 7 : $x = -a/2 = 1.75/2 = -0.875$
$y = -[h_1 - (l_2/2)] = -[1.1375 - (\frac{0.75}{2})] = -0.7625\ m$
$z = h_2 = 1.70625\ m$

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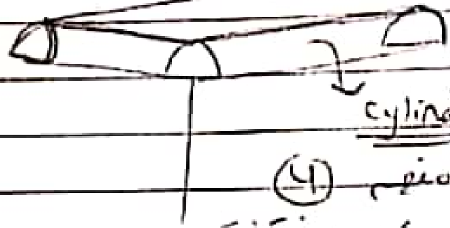
⑥ $2 * \frac{1}{4}$ (cylinder) $= \frac{1}{2}$ cylinder

Top cylinders :



cylinders

في منهم (4) بس بأبعاد مختلفة

بحتوي $\frac{1}{2}$ cylinder

أبعادها متطابقة

$\frac{1}{2}$ cylinder ثابتة

$= A_6 = \frac{2\pi L r}{2} \rightarrow$ (للضرب بوحدة)

$A_6 = \frac{2\pi}{2} * l_1 * d *$

$= \pi * 1.5 * 1$

$\boxed{A_6 = 4.7123 \ m^2}$

للضرب بالثانية $A_7 = \frac{2\pi}{2} * l_2 * d$ → radius

$= \pi * 0.75 * 1$

$\boxed{A_7 = 2.3562 \ m^2}$

$\Rightarrow$ total Area $= A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7$

$\boxed{A_{total} = 25.3 \ m^2}$ #

$\Rightarrow L_w = 94,8 + 10 * \log(25.3) = 108,83$ dB

$\rightarrow L_{w_{spherical}} - L_{w_{conformal}} = 109,35 - 108,83 = 0.5187$

(can be neglected!).

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22 /10 /2019 Tues

[*] Sound measuring devices:

→ Many types of measuring devices can be used for the measurement of sound depending on the purpose of the study, the extent of information that is desired about the sound, and the characteristics of sound. In this lecture we will consider 2 types of sound measuring devices:

[1] Sound level meter

[2] Sound Spectrum Analyzer

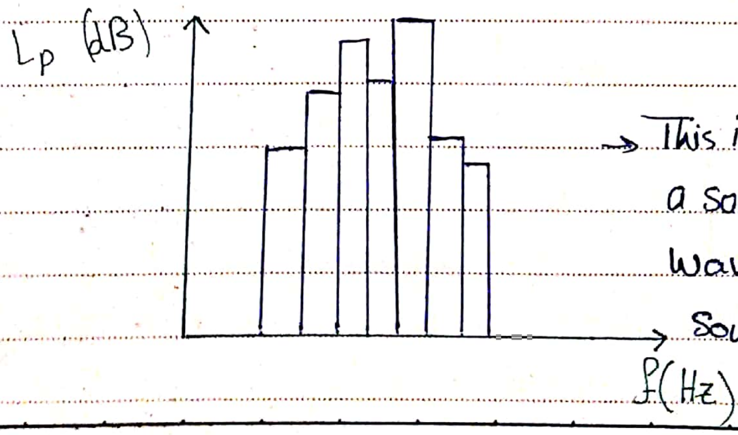
[1] Sound level meter (measures the sound pressure level of a sound wave)

→ It is an instrument with a microphone. The diaphragm of the microphone responds to changes in air pressure caused by sound waves (the sound wave might be a pure tone or complex wave). This movement of the diaphragm, i.e. the sound pressure deviation, is converted to an electrical signal. The total sound pressure level is displayed on a screen in decibels.

→ This is a Sound level meter

[2] Sound Spectrum Analyzer

→ it is a device that depicts the spectrum (plot of sound pressure level against frequency) of a sound wave



→ This is a spectrum of a sound wave. It is a complex sound wave since it is a multifrequency sound.

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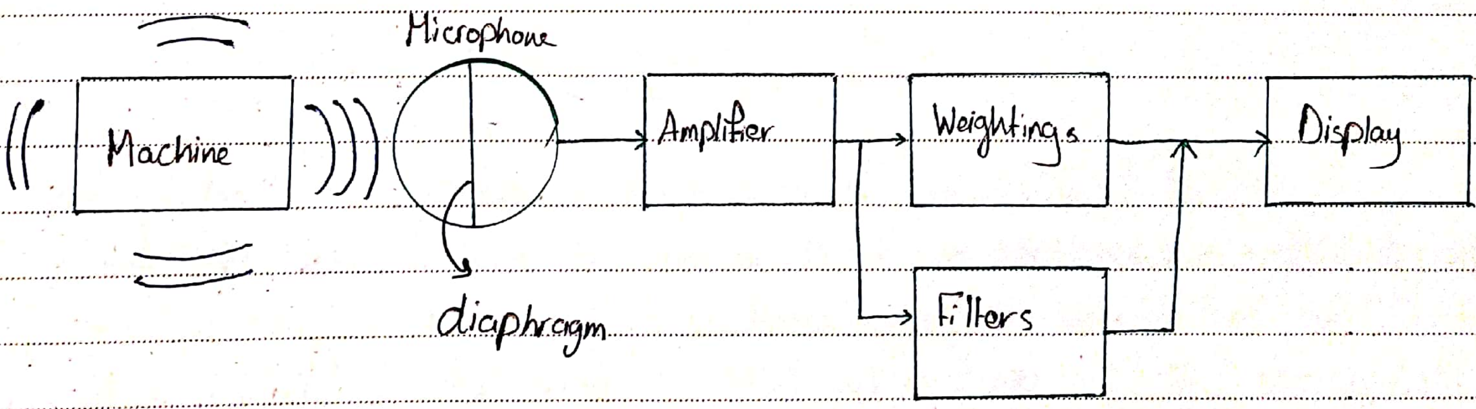
→ This device is provided with filters. Filters are circuits that allow measuring the sound pressure level for a certain range of frequency. There are 3 types of ~~sound~~ filters:

1. Low pass filter: Considers all the frequencies below a predefined value f_{low} (i.e frequency range $[0 - f_{low}]$)
2. High pass filter: Considers all the frequencies above a predefined value (i.e frequency range $[f_{high} - \infty]$)
3. Band pass filter (most common): Considers all the frequencies that fall between 2 predefined values (i.e frequency range = $[f_{low} - f_{high}]$)

→ This is a sound spectrum analyzer.

→ Modern sound level meters are embedded with sound spectrum analyzers, so that they are able to display: ① The overall sound pressure level
② The sound spectrum

→ let's take a look at the components included inside a modern sound level meter:



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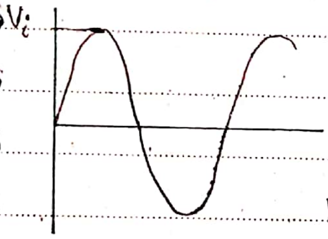
→ The membrane or diaphragm of the microphone is made of a piezoelectric material. Piezoelectric materials are smart materials that can convert the pressure applied on them into a voltage signal.

Pressure Signal → Piezo material → Voltage signal

→ This voltage signal is delivered to an amplifier, to amplify the amplitude of the voltage signal to maintain the signal

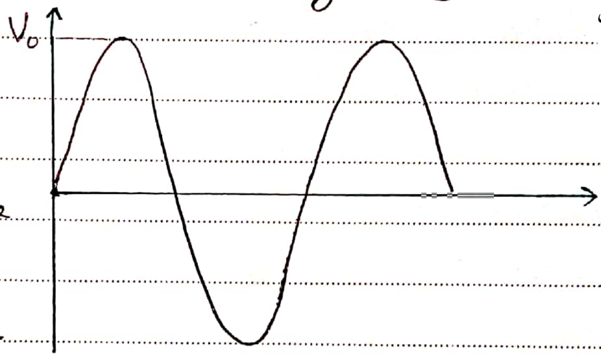
Input Voltage signal → amplifier → Output Voltage signal

وظيفة ال(Amplifier) هي
تكبير الأشارة دون تغريها حتى
لا تضيع في شبكة الأسلاك



▣ $V_o = a V_i$

▣ a is a positive constant that has a value $a > 1$



→ Filters are used to analyze the sound spectrum. As we said before, there are 3 types of filters. The most common one is the band pass filter. It is often used because the range of human hearing is 20Hz ÷ 20kHz.

↳ This is our interest ∴ band pass filters are used.

There are 3 types of band pass filters:

1. 1-Octave band filter
2. 1/3-Octave band filter $(1/3 = \frac{1}{3})$
3. 1/10-Octave band filter $(1/10 = \frac{1}{10})$

What is the story behined these types of band pass filters?

Sometimes, we want to know the frequencies & the corresponding amplitudes of a noise. But, we are not intrested in knowing all of them (it will be quite complicated and with no real advantage all the frequencies from 20Hz to 20KHz, So we divide the entire spectrum of the noise

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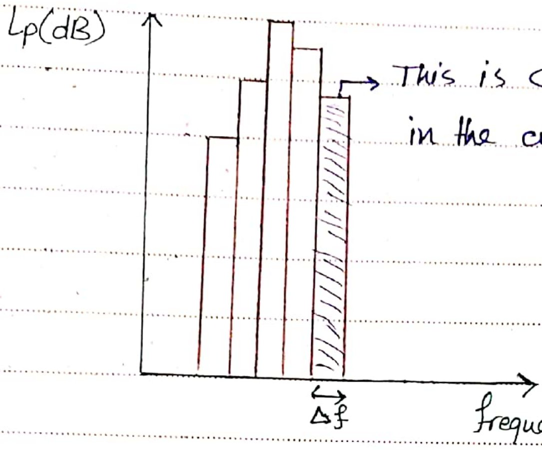
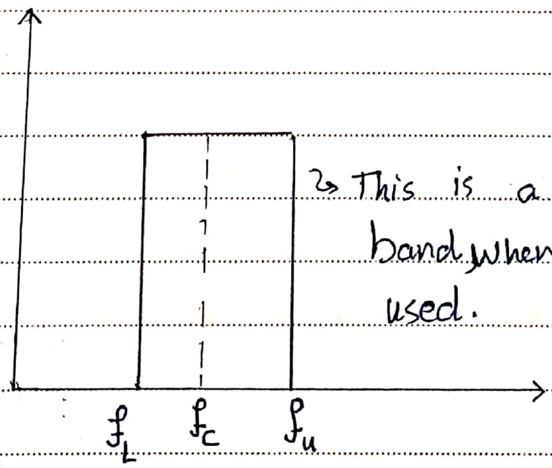
in frequency bands as shown below



→ This is called a band. There are 5 bands shown in the adjacent spectrum.

→ 1-Octave, $\frac{1}{3}$-Octave, and $\frac{1}{10}$-octave bands. What is the difference?

- An octave refers to the interval between one frequency and its double. There is one octave between frequencies 1000 Hz and 2000 Hz. There is another one octave between 1000 Hz & 500 Hz
- In 1-Octave band filter, the spectrum is splited into bands, such that the ~~band~~ frequency interval of a single band = 1-Octave (i.e $f_{upper} = 2 f_{lower}$). Each band has a central frequency, which is defined as $f_{central} = \sqrt{f_{upper} * f_{lower}}$



→ This is a representation of a single band when 1-Octave band filter is used.

$$f_u = 2 f_L$$

$$f_c = \sqrt{f_L f_u}$$

f_L=f_c/(2^0.5)
f_H=f_c*(2^0.5)

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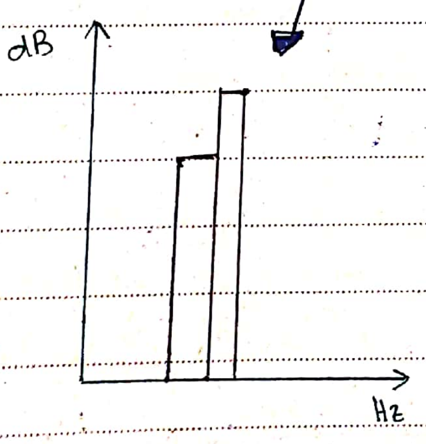
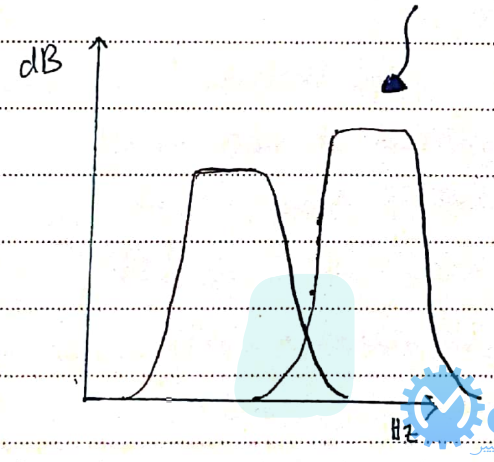
→ If we consider the human hearing range ; the range will be splited into Octaves if 1-Octave band filter is used, as shown below

f_L (Hz)	f_c (Hz)	f_u (Hz)	
22	31.5	44	band #1
44	63	88	band #2
88	125	177	band #3
177	250	355	band #4
355	500	710	band #5
710	1000	1420	band #6
1420	2000	2840	band #7
2840	4000	5680	band #8
5680	8000	1360	band #9
11360	16000	22720	band #10

- Notice that when 1-Octave band filter is used, the human hearing frequency range is splited into 10 bands.
- $f_u \approx 2 f_L$, $f_c \approx \sqrt{f_L f_u}$
- In 1/3-octave band filter, the spectrum is splited into smaller bands, such that $f_u = 2^{(1/3)} f_L$
- In 1/10-octave band filter, the spectrum is splited further, such that $f_u = 2^{(1/10)} f_L$
- Bands are often drawn like this / However, actually, they appear like this





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→ As we said before, Sound level meters measure the overall sound pressure level $L_{P_{tot}}$ of a sound wave. This value $L_{P_{tot}}$ can be measured in different ways:

[1] Flat network (or Z-weighting)
[2] A-network (or A-weighting)
[3] B-network (or B-weighting)
[4] C-network (or C-weighting)

These are called weightings

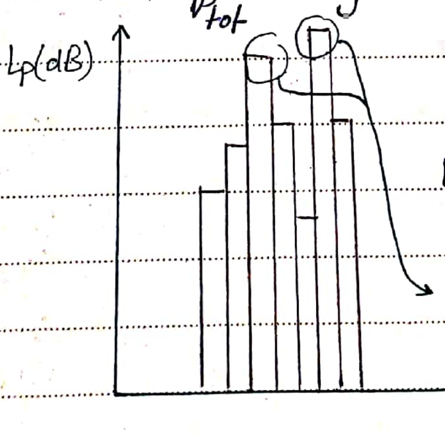
- Each weighting gives a different $L_{P_{tot}}$ value!
- A flat network or (Z-weighting) calculates $L_{P_{tot}}$ using the following formula:

$$L_{P_{tot}} = 20 \log \left(\frac{P_{tot,rms}}{P_{ref}} \right) , \quad [dB]$$

- The other weightings (i.e. A, B, C-weightings) adjust the previous value in a certain way, such that another 3 values of $L_{P_{tot}}$ are obtained.
- Measurements made using A-weighting are usually shown with dB(A) to show that A-weighting was utilized to measure $L_{P_{tot}}$. ~~The~~ The same thing is applied to Z, B, and C weightings. i.e if $L_{P_{tot}}$ is measured in dB(C), this indicates the C-weighting was utilized to measure $L_{P_{tot}}$, and so on.
- Later, we will discuss the difference between these weightings.

→ Finally, both $L_{P_{tot}}$ & the sound spectrum are displayed on a screen.

If the measured $L_{P_{tot}}$ is greater than $L_{P_{Standard}}$, $L_{P_{tot}}$ must be reduced.



* To reduce $L_{P_{tot}}$:

[1] We have to identify the frequencies of the components of the machine

[2] Based on the sound spectrum, we can define the components which ~~have~~ contribute the most to the overall sound pressure level

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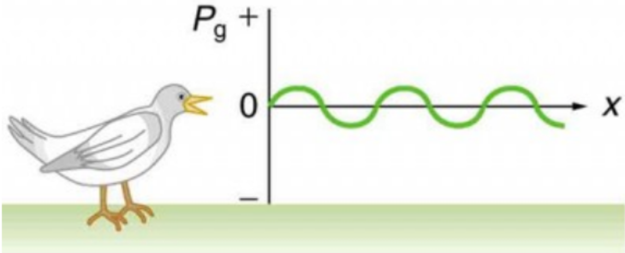
[*] Loudness level: is the human impression of the strength of a sound.

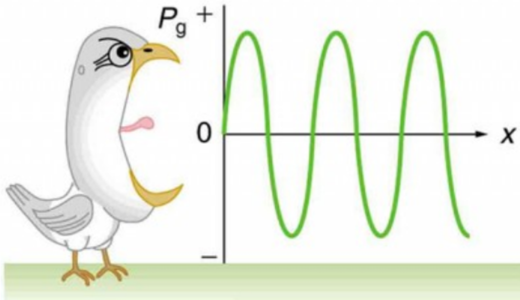
→ If two sound sources emit pure tone that have the following characteristics:

Sound source ※1: frequency: f_1, amplitude: P_1

Sound source ※2: frequency: f_2, amplitude: P_2

Where $f_1 = f_2$ but $P_2 > P_1$ (i.e. $L_{P_2} > L_{P_1}$)





We will find that the sound emitted by the 2nd sound source will be louder than the sound emitted by the 1st sound source.

→ In the early 1930s, it was found that the frequency of a sound affects our perception (حس، إدراك) of the sound loudness. In other words, two different frequencies at the same pressure level (i.e. $P_1 = P_2$ but $f_1 \neq f_2$) may not sound the same loudness level to us because of the way our hearing mechanism processes them.

→ Loudness level is a quantity that depends on both the sound pressure level L_P & the frequency of the sound. It ~~indicates~~ indicates our perception of loudness.

→ Loudness level is measured in phones. By definition:

1 phon = 1 dB @ 1000 Hz

40 phones = 40 dB @ 1000 Hz

x phones = x dB @ 1000 Hz

- Notice that loudness level depends on both L_P & f of the sound
- As the number of phones increases, the sound gets louder (i.e. 40 phones-sound is louder than a sound that has a loudness level of 1 phon)

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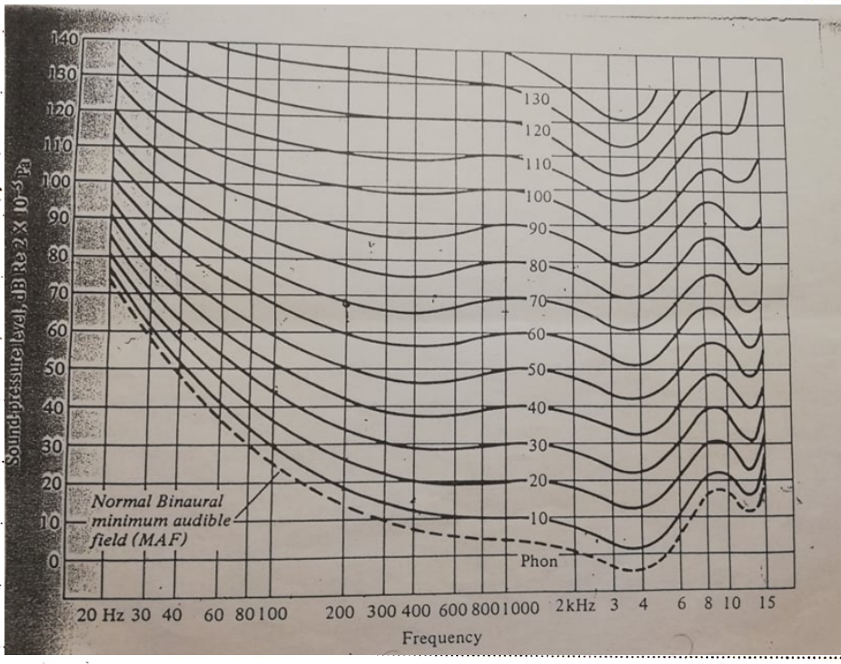
→ The following figure shows equal loudness curves (i.e. the points along a certain curve have the same loudness level



→ The curves were determined experimentally. To generate the entire 60-phon curve, volunteers were subjected to a 1 KHZ sound at 60 dB (This is a loudness of 60phones). Sounds with different frequencies were then played; the volunteer adjusted the decibel level until it was perceived to have the same loudness level as it had at 1KHz & 60 dB. To generate the other curves, the same procedure was followed.

→ From the above figure, notice the following:

• @ 1000 HZ ; loudness level = sound pressure level

• The dashed curve ~~shown~~ found at the bottom represents the curve for the threshold of hearing

• Notice that at a certain frequency, as the sound pressure level increases, the loudness level also increases.

• ~~At~~ The human error is less sensitive at low frequencies (i.e at low frequencies, the sound pressure level must be increased greatly to reach the threshold of hearing curve.

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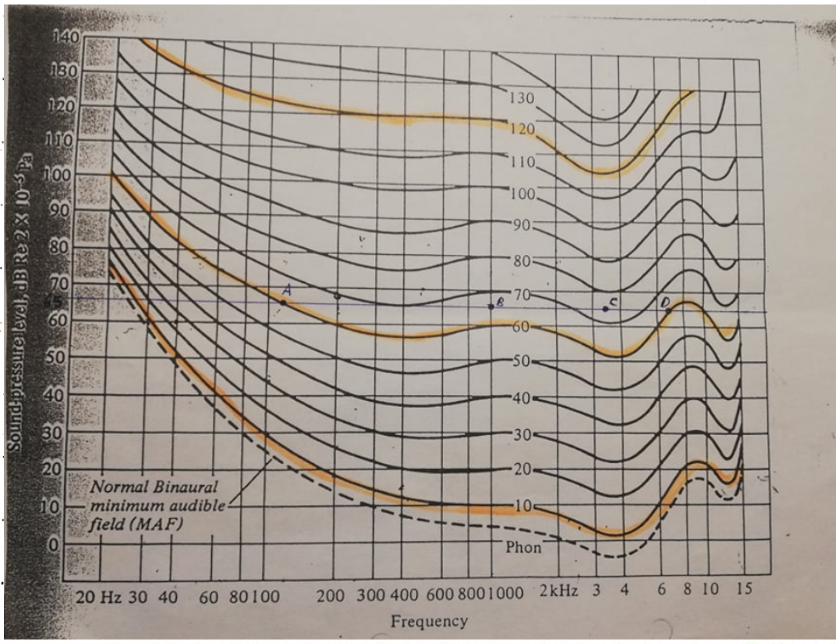
→ Three examples curves from the equal loudness curves are shown below corresponding to very soft, mid range, and very loud sounds

صوت منخفض　　　صوت معتدل　　　صوت عالي



→ Examination of these 3 curves makes it evident that there is considerable difference between the ear's response at different sound levels

يعني استجابة الإنسان للأصوات المنخفضة تختلف عن
استجابته للأصوات العالية أو العالية

→ Notice that the curve of very loud sound is much ~~flater~~ flatter than the curve of very soft sounds. i.e

~~If we consider 120 phon curve~~:

If we consider 120 phon-curve: at $f = 40\,Hz$, $L_p = 131\,dB$
$f = 200\,Hz$, $L_p = 120\,dB$ } less reduction "flatter curve"

If we consider 10 phon-curve: at $f = 40\,Hz$, $L_p = 50\,dB$
$f = 200\,Hz$, $L_p = 19\,dB$ } more reduction

Based on the previous fact, one can conclude that at high sound pressure levels, varying the frequency doesn't affect loudness level very much
e.g @ $L_p = 130\,dB$, if the frequency is varied from $f = 40\,Hz$ to $f = 800\,Hz$ loudness level will increase from ≈ 120 phon to 130 phon.
however @ $L_p = 60\,dB$, if the frequency is varied from $f = 40\,Hz$ to

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$f = 800\,Hz$, loudness level will increase from 25 phon to 61 phons "صوت زيادة أكبر"

→ The example sounds A, B, C, and D all have the same sound pressure level of 65 dB. However, this doesn't imply that they have the same loudness to the human ear. We can say that sounds A & D have the same loudness since both are on the same equal loudness curve. Sound B is above the 60 phon curve, so that implies it would be perceived as louder than A or D. In fact, since sound B is at 1000 Hz and has $L_p = 65\,dB$, we can say that its loudness is 65 phons. Finally, Sound C at 65 dB is the loudest of the four sounds, since it shows the greatest distance above the 60-phone curve.

→ To sum up, loudness level of a sound depends on both L_p & f. ~~Hence~~

Hence, when we want to reduce the noise in an environment, it is meaningless to ~~state~~ mention the sound pressure level of the noise only, we have to mention the frequency as well. For instance if $L_p = 80\,dB$, $f = 20\,Hz \Rightarrow$ loudness level $= 20$ phons

soft sound ← على الرغم من انه ال $L_p = 80dB$ بس هاد الصوت يعتبر منخفض جداً

لأنه هو موجود على frequency صغيرة

→ For that purpose, sound level meters are provided with A, B, C weightings which are used to take the effect of frequency on ~~the~~ our perception of loudness into consideration!

→ In flat network (Z-weighting): no correction is made ~~for~~ & on the recorded value of L_p $\left[\text{i.e. } L_p = 20\log\left(\frac{P_{rms}}{P_{ref}}\right)\right]$. A, B, C weightings adjust the measurement to account for the way in which the ear responds to different frequencies of sound.

→ If the uncorrected value of L_p is low → A-weighting is used

→ If the uncorrected value of L_p is high → C-weighting is used

→ If the uncorrected value of L_p is moderate → B-weighting is used.

? ← والسبب في ذلك انه احنا حكينا انه استجابة أذن الانسان للأصوات المنخفضة

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تختلف عن استجابتها للأصوات المعتدلة والعالية. عشان هيك

كل weighting بستخدم لـ range معين من ال L_p لأنه ال correction ع يختلف

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أمنو الحضارة

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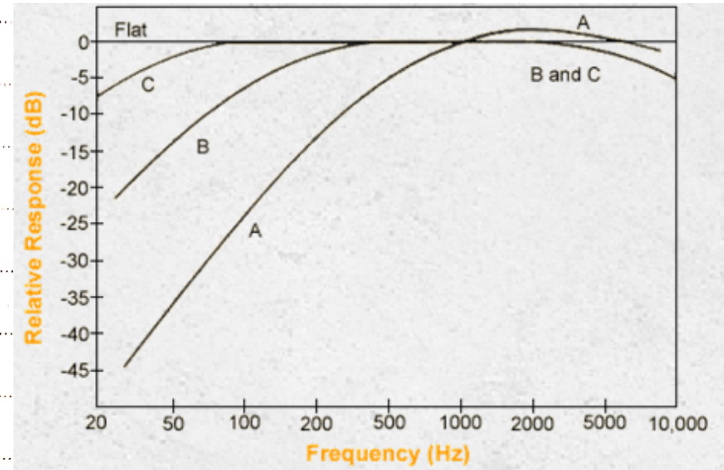
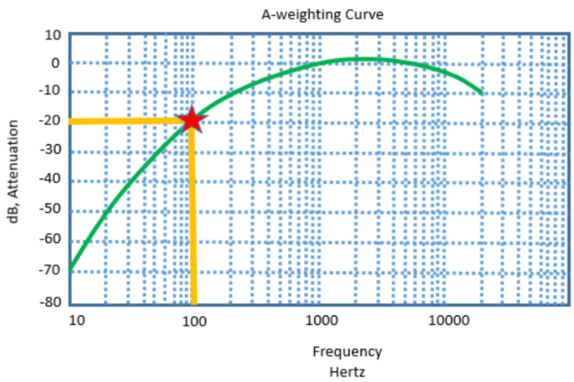
→ Each weighting has a curve as shown below. The curve shows decibels of attenuation or gain at every frequency over the range of human hearing. This gain/attenuation is applied to the uncorrected measurement of Lp.

→ Note: [Flat means Z-weighting] ⇒ No correction



→ How do we use the curves?

Suppose you measure 60 dB with a sound level meter at 100 Hz (uncorrected value of Lp). If you look at the A-weighting curve at 100 Hz, an attenuation value of -20 dB is indicated.



Subtracting 20 dB from 60 dB yields 40 dB. To indicate that A-weighting was used, the recorded value is written as Lp = 40 dB(A) or Lp = 40 dBA

للتوضيح :

هلأ ال flat network بتقيس ال P, rms و بتحسبلك ال Lp, tot بناء ع القانون.. باقي الأنواع بياخدوا قيمة ال Lp,tot و بعدلوا عليها بناء عال frequency .. لأنه اذن الإنسان لما تسمع الصوت، علو الصوت مش بس بيعتمد عال Lp tot.. و بيعتمد كمان عال frequency.. يعني مثال : لو حكيتلك انه ال Lp, tot = 90 dB ح تفكر انه هاد الصوت عالي بس لو حكيتلك انه Lp, tot=90 dB و f=20 Hz فتحت ال loudness curves رح تلاقي انه ال loudness لهاد الصوت 40 phon.. يعني مش كتير عالي هاد الصوت.. فهاد معناه انه احنا البشر لما نيجي نستخدم ال sound level meter ما بهمنا فقط قيمة ال Lp, tot لأنه اذن الإنسان ما بتعتمد فقط عال sound pressure level, بتعتمد كمان عال frequency.. فهلأ ال Flat network بتحسبلك ال Lp, tot بناء ع القانون.. أما ال A, B, C- networks بياخدوا قيمة ال Lp, tot اللي انحسبت من خلال ال flat network و بعدلوا عليها بناء عال frequency. فمثلا ال A network لو شافت انه ال f=20 Hz Lp, tot=90dB فهي لما تيجي تعرضلك ال Lp, tot ما بتعرضها 90 dB.. بتعرضها أقل من هيك عشان ما تحس انه الصوت عالي

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24 /10 /2019 Thursday

[*] Loudness level & Loudness

Question: Does loudness level completely define the acoustic sensation?
Answer: 2 tones with the same loudness level (i.e corresponding to the same curve) produce the same acoustic level feeling. However, a relative change in perception is not proportional to loudness level. In other words: a tone of 60 phons doesn't produce 2 times the feeling than a tone of 30 phons!
Consequently, loudness level doesn't define completely the relative change in acoustic sensation.
∴ A new parameter called LOUDNESS is defined that corrects this limitation
→ We will discuss: loudness of pure tones and loudness of complex sounds.

[*] Loudness of pure tone

→ The relationship between loudness (sone) and loudness level (phon) is given by the following formula

$$L = 2^{\frac{LL-40}{10}}$$

L: loudness (sone)
LL: loudness level (Phon)

→ Based on the relation:

- $L = 1$ sone $\Rightarrow$ $LL = 40$ phon
- If the LL is increased by 10 phons, the number of ~~soon~~ sones will be doubled

LL (Phone)	L (Sone)
40	1
50	2
60	4
70	8
80	16

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Example: If $L_p = 20\,dB$, $f = 1000\,Hz$, find the loudness

$L_p = 20\,dB$, $f = 1000\,Hz \longrightarrow LL = 20$ Phons

$$L = 2^{\frac{20-40}{10}} = 2^{-2} = \frac{1}{2^2} = 0.25 \text{ sone}$$

Example: If $L_p = 50\,dB$, $f = 400\,HZ$, find the loudness

$L_p = 50\,dB$, $f = 400\,HZ \Longrightarrow LL = 53$ Phon

$$L = 2^{\frac{53-20}{10}} = 2.46 \text{ sone}$$

Example: How many times louder would a typical person perceive 70 phon sound to be compared to 40 phon sound?

$LL = 40$ phone $\Longrightarrow L = 1$ sone

$LL = 70$ phone $\Longrightarrow L = 8$ sone $\quad \therefore$ 8 times louder

[*] Loudness of complex Sound:

→ To calculate the loudness of complex sound; you have to remember that the spectrum of complex sound is made of bands. Each band has a central frequency f_c & sound pressure level L_p.

→ Given the f_c & L_p of each band, one can find the loudness index of the band from the tables found in the next 2 pages

→ The loudness of the complex Sound is given by

$$L_t = L_{max} + a\left(\left(\sum_{i=1}^{n} L_i\right) - L_{max}\right)$$

L_t = total loudness of the complex sound (in Sones)

L_{max} = maximum loudness index value

L_i = Individual loudness index value of each band

$\sum L_i$ = Sum of loudness index values of all bands

$a = 0.3$ for 1-octave bands

$a = 0.15$ for 1/3 octave bands.

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كرأيت التغيير

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Band Level (dB)	Band Loudness Index (~~dB~~)									Loudness (sones)	Loudness Level (phones)
	Central Frequency (Hz)										
	31.5	63	125	250	500	1000	2000	4000	8000		
20						0.18	0.3	0.45	0.61	0.25	20
21						0.22	0.35	0.5	0.67	0.27	21
22					0.07	0.26	0.4	0.55	0.73	0.29	22
23					0.12	0.3	0.45	0.61	0.8	0.31	23
24					0.16	0.35	0.5	0.67	0.87	0.33	24
25					0.21	0.4	0.55	0.73	0.94	0.35	25
26					0.26	0.45	0.61	0.8	1.02	0.38	26
27					0.31	0.5	0.67	0.87	1.1	0.41	27
28				0.07	0.37	0.55	0.73	0.94	1.18	0.44	28
29				0.12	0.43	0.61	0.8	1.02	1.27	0.47	29
30				0.16	0.49	0.67	0.87	1.1	1.35	0.5	30
31				0.21	0.55	0.73	0.94	1.18	1.44	0.54	31
32				0.26	0.61	0.8	1.02	1.27	1.54	0.57	32
33				0.31	0.67	0.87	1.1	1.35	1.64	0.62	33
34			0.07	0.37	0.73	0.94	1.18	1.44	1.75	0.66	34
35			0.12	0.43	0.8	1.02	1.27	1.54	1.87	0.71	35
36			0.16	0.49	0.87	1.1	1.35	1.64	1.99	0.76	36
37			0.21	0.55	0.94	1.18	1.44	1.75	2.11	0.81	37
38			0.26	0.62	1.02	1.27	1.54	1.87	2.24	0.87	38
39			0.31	0.69	1.1	1.35	1.64	1.99	2.38	0.93	39
40		0.07	0.37	0.77	1.18	1.44	1.75	2.11	2.53	1.0	40
41		0.12	0.43	0.85	1.27	1.54	1.87	2.24	2.68	1.07	41
42		0.16	0.49	0.94	1.35	1.64	1.99	2.38	2.84	1.15	42
43		0.21	0.55	1.04	1.44	1.75	2.11	2.53	3.0	1.23	43
44		0.26	0.62	1.13	1.54	1.87	2.24	2.68	3.2	1.32	44
45		0.31	0.69	1.23	1.64	1.99	2.38	2.84	3.4	1.41	45
46	0.07	0.37	0.77	1.33	1.75	2.11	2.53	3.0	3.6	1.52	46
47	0.12	0.43	0.85	1.44	1.87	2.24	2.68	3.2	3.8	1.62	47
48	0.16	0.49	0.94	1.56	1.99	2.38	2.84	3.4	4.1	1.74	48
49	0.21	0.55	1.04	1.69	2.11	2.53	3.0	3.6	4.3	1.87	49
50	0.26	0.62	1.13	1.82	2.24	2.68	3.2	3.8	4.6	2.0	50
51	0.31	0.69	1.23	1.96	2.38	2.84	3.4	4.1	4.9	2.14	51
52	0.37	0.77	1.33	2.11	2.53	3.0	3.6	4.3	5.2	2.3	52
53	0.43	0.85	1.44	2.24	2.68	3.2	3.8	4.6	5.5	2.46	53
54	0.49	0.94	1.56	2.38	2.84	3.4	4.1	4.9	5.8	2.64	54
55	0.55	1.04	1.69	2.53	3.0	3.6	4.3	5.2	6.2	2.83	55
56	0.62	1.13	1.82	2.68	3.2	3.8	4.6	5.5	6.6	3.03	56
57	0.69	1.23	1.96	2.84	3.4	4.1	4.9	5.8	7.0	3.25	57
58	0.77	1.33	2.11	3.0	3.6	4.3	5.2	6.2	7.4	3.48	58
59	0.85	1.44	2.27	3.2	3.8	4.6	5.5	6.6	7.8	3.73	59
60	0.94	1.56	2.44	3.4	4.1	4.9	5.8	7.0	8.3	4.0	60
61	1.04	1.69	2.62	3.6	4.3	5.2	6.2	7.4	8.8	4.29	61
62	1.13	1.82	2.81	3.8	4.6	5.5	6.6	7.8	9.3	4.59	62
63	1.23	1.96	3.0	4.1	4.9	5.8	7.0	8.3	9.9	4.92	63
64	1.33	2.11	3.2	4.3	5.2	6.2	7.4	8.8	10.5	5.28	64
65	1.44	2.27	3.5	4.6	5.5	6.6	7.8	9.3	11.1	5.66	65
66	1.56	2.44	3.7	4.9	5.8	7.0	8.3	9.9	11.8	6.06	66
67	1.69	2.62	4.0	5.2	6.2	7.4	8.8	10.5	12.6	6.5	67
68	1.82	2.81	4.3	5.5	6.6	7.8	9.3	11.1	13.5	6.96	68
69	1.96	3.0	4.7	5.8	7.0	8.3	9.9	11.8	14.4	7.46	69
70	2.11	3.2	5.0	6.2	7.4	8.8	10.5	12.6	15.3	8.0	70
71	2.27	3.5	5.4	6.6	7.8	9.3	11.1	13.5	16.4	8.6	71
72	2.44	3.7	5.8	7.0	8.3	9.9	11.8	14.4	17.5	9.2	72
73	2.62	4.0	6.2	7.4	8.8	10.5	12.6	15.3	18.7	9.8	73
74	2.81	4.3	6.6	7.8	9.3	11.1	13.5	16.4	20.0	10.6	74

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Continue:

Band Level (dB)	Band Loudness Index (~~dB~~)									Loudness (sones)	Loudness Level (phones)
	Central Frequency (Hz)										
	31.5	63	125	250	500	1000	2000	4000	8000		
75	3.0	4.7	7.0	8.3	9.9	11.8	14.4	17.5	21.4	11.3	75
76	3.2	5.0	7.4	8.8	10.5	12.6	15.3	18.7	23.0	12.1	76
77	3.5	5.4	7.8	9.3	11.1	13.5	16.4	20.0	24.7	13.0	77
78	3.7	5.8	8.3	9.9	11.8	14.4	17.5	21.4	26.5	13.9	78
79	4.0	6.2	8.8	10.5	12.6	15.3	18.7	23.0	28.5	14.9	79
80	4.3	6.7	9.3	11.1	13.5	16.4	20.0	24.7	30.5	16.0	80
81	4.7	7.2	9.9	11.8	14.4	17.5	21.4	26.5	32.9	17.1	81
82	5.0	7.7	10.5	12.6	15.3	18.7	23.0	28.5	35.3	18.4	82
83	5.4	8.2	11.1	13.5	16.4	20.0	24.7	30.5	38.0	19.7	83
84	5.8	8.8	11.8	14.4	17.5	21.4	26.5	32.9	41	21.1	84
85	6.2	9.4	12.6	15.3	18.7	23.0	28.5	35.3	44	22.6	85
86	6.7	10.1	13.5	16.4	20.0	24.7	30.5	38.0	48	24.3	86
87	7.2	10.9	14.4	17.5	21.4	26.5	32.9	41	52	26.0	87
88	7.7	11.7	15.3	18.7	23.0	28.5	35.3	44	56	27.9	88
89	8.2	12.6	16.4	20.0	24.7	30.5	38.0	48	61	29.9	89
90	8.8	13.6	17.5	21.4	26.5	32.9	41	52	66	32.0	90
91	9.4	14.8	18.7	23.0	28.5	35.3	44	56	71	34.3	91
92	10.1	16.0	20.0	24.7	30.5	38.0	48	61	77	36.8	92
93	10.9	17.3	21.4	26.5	32.9	41	52	66	83	39.4	93
94	11.7	18.7	23.0	28.5	35.3	44	56	71	90	42.2	94
95	12.6	20.0	24.7	30.5	38.0	48	61	77	97	45.3	95
96	13.6	21.4	26.5	32.9	41	52	66	83	105	48.5	96
97	14.8	23.0	28.5	35.3	44	56	71	90	113	52.0	97
98	16.0	24.7	30.5	38.0	48	61	77	97	121	55.7	98
99	17.3	26.5	32.9	41	52	66	83	105	130	59.7	99
100	18.7	28.5	35.3	44	56	71	90	113	139	64.0	100
101	20.3	30.5	38.0	48	61	77	97	121	149	68.6	101
102	22.1	32.9	41	52	66	83	105	130	160	73.5	102
103	24.0	35.3	44	56	71	90	113	139	171	78.8	103
104	26.1	38.0	48	61	77	97	121	149	184	84.4	104
105	28.5	41	52	66	83	105	130	160	197	90.5	105
106	31.0	44	56	71	90	113	139	171	211	97.0	106
107	33.9	48	61	77	97	121	149	184	226	104	107
108	36.9	52	66	83	105	130	160	197	242	111	108
109	40.3	56	71	90	113	139	171	211	260	119	109
110	44	61	77	97	121	149	184	226	278	128	110
111	49	66	83	105	130	160	197	242	298	137	111
112	54	71	90	113	139	171	211	260	320	147	112
113	59	77	97	121	149	184	226	278	343	158	113
114	65	83	105	130	160	197	242	298	367	169	114
115	71	90	113	139	171	211	260	320		181	115
116	77	97	121	149	184	226	278	343		194	116
117	83	105	130	160	197	242	298	367		208	117
118	90	113	139	171	211	260	320			233	118
119	97	121	149	184	226	278	343			239	119
120	105	130	160	197	242	298	367			256	120
121	113	139	171	211	260	320				274	121
122	121	149	184	226	278	343				294	122
123	130	160	197	242	298	367				315	123
124	139	171	211	260	320					338	124
125	149	184	226	278	343					362	125

Table-2.2

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→ The previous table has 4 major columns:

- Band level L_p (dB)
- Band level index, given the central frequency f_c (Hz)

} related to complex sound. (i.e. to calculate the loudness of complex sound we will use these 2 columns)

- The last 2 columns are related to pure tones:

if the loudness level of a pure tone is given, one can find the loudness from the table instead of using the formula: $L = 2^{\frac{LL-40}{10}}$

Example: Consider 2 machines A and B that have the following spectrum

No	f_c (Hz)	$L_{P,A}$ (dB)	$L_{P,B}$ (dB)
1	31.5	50	45
2	63	60	70
3	125	70	50
4	250	90	80
5	500	85	70
6	1 K	100	90
7	2 K	105	110
8	4 K	70	65
9	8 K	80	75

We want to compare between them based on the total sound pressure level and loudness

(*) $L_{P_{tot}}$ can be found using the following mathematical formula:

$$L_{P_{tot}} = 10 \log \left[\sum_{i=1}^{9} 10^{L_{P_i}/10} \right]$$

or using the figure that was used previously for adding decibels

$$L_{P_{tot,A}} = 106.34 \text{ dB}$$

$$L_{P_{tot,B}} = 110.04 \text{ dB}$$

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Discussion: We have found that $L_{P,totA}$ = 106.34 dB. We can represent machine A by 9 small machines, such that each machine emits a pure tone that has a certain Sound pressure level and frequency as explained below

machine ※1 $L_p = 105$ dB @ 2 KHz
machine ※2 $L_p = 100$ dB @ 1 kHz
machine ※3 $L_p = 90$ dB @ 250 Hz
machine ※4 $L_p = 85$ dB @ 500 Hz
⋮
machine ※9 $L_p = 50$ dB @ 31.5 Hz

2. if $L_{P,standard} = 90$ dB, $L_{P,totA}$ must be reduced

• The first 3 machines contributes the most to the total sound pressure level. They must be reduced such that $L_{P,totA}$ becomes less than $L_{P,standard}$

(*) To find the loudness of both machines, We have to find the loudness index of each band in the spectrum

No	f_c	$L_{P,A}$	Loudness index (A)	$L_{P,B}$	Loudness index (B)
1	31.5	50	0.26	45	—
2	63	60	1.56	70	3.2
3	125	70	5	50	1.13
4	250	90	21.4	80	11.1
5	500	85	18.7	70	7.4
6	1 K	100	71	90	32.9
7	2 K	105	130 → $L_{max,A}$	110	184 → $L_{max,B}$
8	4 K	70	12.6	65	9.3
9	8 K	80	30.5	75	21.4

$\Sigma L_{i,A} = 291.02$ $\Sigma L_{i,B} = 270.96$

$$L_{tot} = L_{max} + 0.3\left((\Sigma L_i) - L_{max}\right)$$

$$= L_{max} + 0.3 \Sigma L_i - 0.3 L_{max} = 0.7 L_{max} + 0.3$$

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$\therefore L_{tot,A} = 178.3$ sone , $L_{tot,B} = 210$ sone

[*] Room Acoustics

→ Room acoustics describes how sound behaves in an enclosed space.

→ Sound field: is a region of an elastic medium containing sound waves

→ We will investigate 3 types of sound fields:

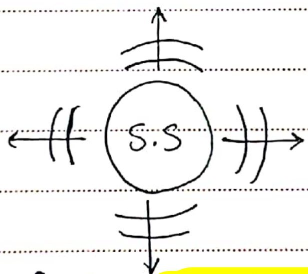
[1] Free field

[2] Reverberant field

[3] Semi-reverberant field

[1] Free field

→ Free field is a sound field with no reflecting surfaces (i.e. there are no reflections). In this field, sound waves reach an observer directly from a sound source. The sound wave passes the observer exactly once and never returns → no reflection



→ There are no barriers

∴ Reflection won't occur (Free field)

→ An example of a free field is: An anechoic room.

- anechoic means without echo (no echo means no reflection)
- An anechoic room is a special facility constructed to approximate a free field by using materials to absorb sound waves before they can be reflected. (i.e the walls, floor, ceiling of the room have absorption coefficient $\alpha = 1$" it absorbs the incident wave entirely")

→ This is an anechoic room.

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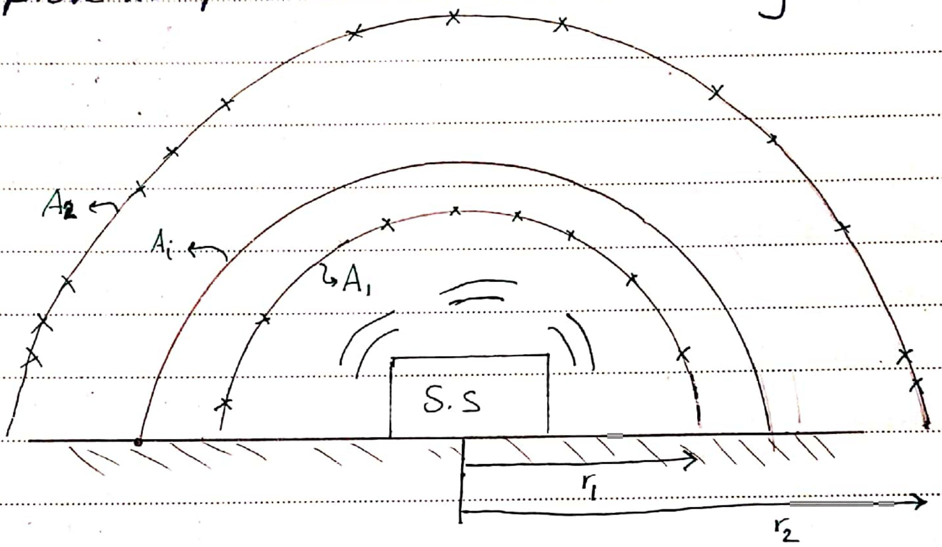
→ How to know?

- It is difficult to determine if the field is a free field or not by visual inspection. However, acoustic measurements must be done to recognize if this is a free field or not
- If a 6dB decrease in the measured sound pressure level is observed when doubling the distance from the sound source, then, this indicates the field is a free field.
- We will prove the previous statement mathematically:



- assume non-directional sound source (i.e $\overline{L_p} = L_i$)
- Recall $L_w = \overline{L_p} + 10 \log A$

Hence,

$$L_w = L_{p_1} + 10 \log A_1$$

$-$ "do subtraction"

$$L_w = L_{p_2} + 10 \log A_2$$

$$0 = L_{p_1} - L_{p_2} + 10(\log A_1 - \log A_2)$$

$$L_{p_2} - L_{p_1} = 10 \log\left(\frac{A_1}{A_2}\right) = 10 \log\left(\frac{2\pi r_1^2}{2\pi r_2^2}\right) = 20 \log\left(\frac{r_1}{r_2}\right)$$

if $r_2 = 2r_1$; $L_{p_2} - L_{p_1} = 20 \log\left(\frac{r_1}{2r_1}\right) = 20 \log\left(\frac{1}{2}\right) = -6.02\ dB$

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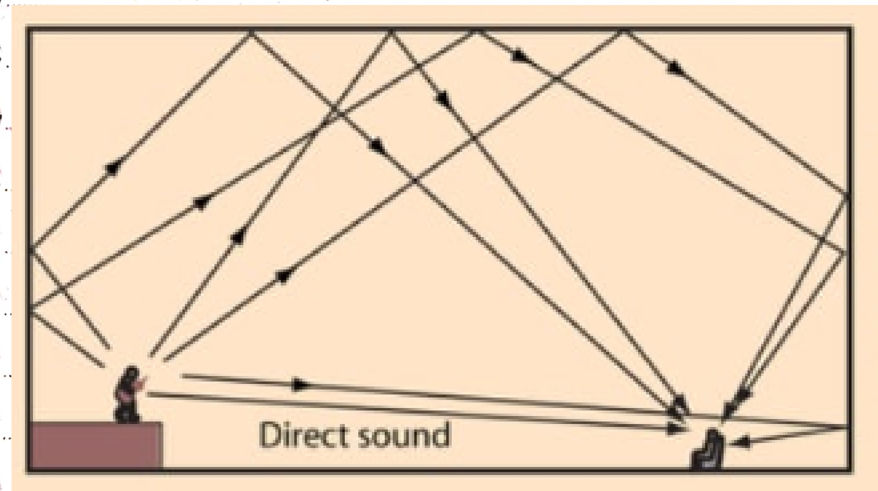
[2] Reverberant field (Diffuse field)

- Reverberant means reflective
- Sound diffusion means random reflection of Sound waves.
- Reverberation is the collection of reflected sounds from the surfaces in an enclosure like an auditorium. It is a desirable property of auditoriums to the extent that it helps to overcome the reduction in decibels caused by ~~distance~~ moving far a way from the sound source.



→ The absorption coefficient of the walls, cieling, floor of this room ~ 0 (i.e They reflect the incident sound wave entirely)

- The total sound intensity at the reciever: $I_{tot} = I_{Direct} + I_{reflected}$

قيمة ال I_{tot} رح تكون أكبر من لو كان عنا free field لأنه بحالة

ال free field رح تكون $I_{tot} = I_{Direct}$

- We can define reverberant sound field as: the region where the reflected sound ~~has~~ dominates (i.e reflected sound has a great effect on the total sound pressure level)
- How to know?

In a diffuse field, the sound level at any position is approximately the same no mater where the microphone measurement recording is made.

$I_{tot} = I_{Direct} + I_{reflected}$ السبب :

لما نكون قريبين من ال S.S ⇐ I_{Direct} كبيرة و $I_{reflected}$ صغيرة

ولما نبعد ونصير أقرب للحائط ⇐ I_{Direct} بتكون صغيرة و $I_{reflected}$ كبيرة

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[3] Semi-reverberant field

- Real noise environment are semi-reverberant (e.g: class rooms, workshops)
- In Semi-reverberant field, both reflection & absorption occur. ($0 < \alpha < 1$)
- The higher the proportion of the reflected sound, the higher the contribution of the reflected sound to the total sound level.
- Even if the sound source is turned off, the total sound level will decrease gradually (not ~~instantaneously~~ instantaneously) because of the effect of the reflected sound.
- As the surfaces become less reflective, and more absorbing of noise, the reflected sound becomes less & the situation tends to a "free field" condition, where the only significant sound is the ~~ref~~ direct sound.
- The phenomenon of reverberation has a little effect in the area very close to the sound source (i.e $I_{reflected}$ is small near the source). However, far from the source, and unless the walls are very absorbing, the sound pressure level is greatly affected by the reflected sound.
- Conclusion: Sound pressure level at a certain position depends on: [1] Acoustic power of the sound source [2] Size of the room [3] Acoustic properties of the boundaries.

→ Check this link & listen to the difference between the sound of a balloon pop in a reverberation room versus anechoic room.

https://www.youtube.com/watch?v=zq07ZFMvo-c

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[*] Absorption coefficient α: a quantity used for measuring the sound absorption of a material, and is known to be function of the frequency of incident wave

$$\alpha\ (\equiv \text{absorption coefficient}) = \frac{I_{absorbed}}{I_{incident}} = \frac{I_{incident} - I_{reflected}}{I_{incident}}$$

→ If:

- $\alpha = 1$ → means that the material absorbs the incident sound wave entirely
- $\alpha = 0$ → the material reflects the incident wave entirely

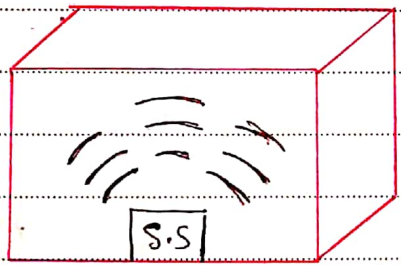
[*] Sound intensity in a reverberant field (reflective field):

→ We know that sound is a form of energy

→ We want to establish a relationship between sound intensity (which represents energy flow) in a reverberant field and energy density.

→ Energy density $\mathcal{E}$ $[J/m^3]$ = is the sound energy present in $1m^3$ of the room.

Consider the following room, which contains a sound source



$$\mathcal{E} = \frac{\text{Total sound energy present in the room}}{\text{Volume of the room}}$$

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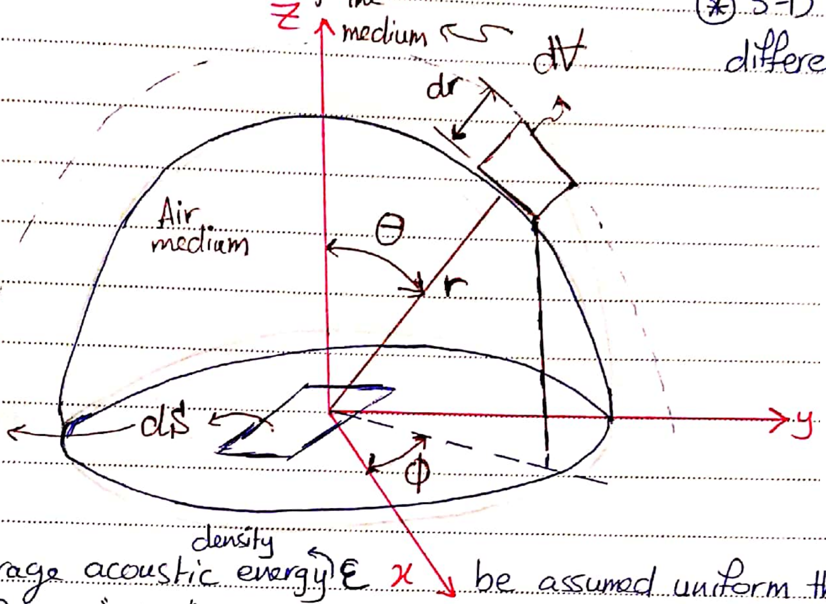
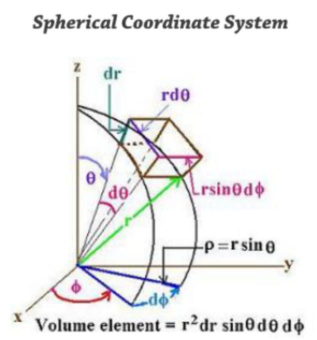
→ Now, we will find the relation between the sound intensity & the energy density:

(*) differential volume of the medium

(*) 3-D view of the differential element $d\forall$





(*) differential element of wall surface

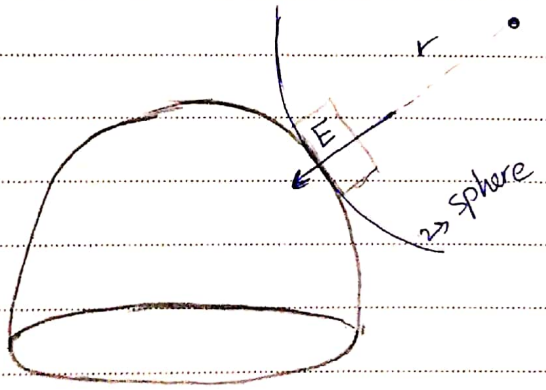
→ Let the average acoustic energy density $\mathcal{E}$ be assumed uniform throughout the region.

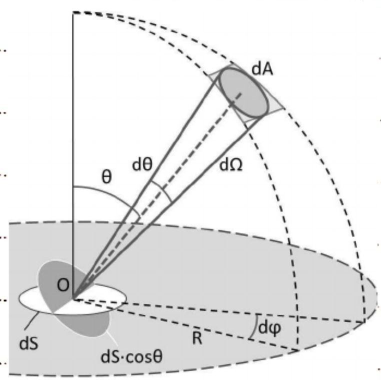
→ In the figure, "dS" represents an element of the wall surface, "$d\forall$" is a volume element in the medium at a distance "r" from "dS". The distance "r" makes an angle "θ" with the normal to "dS".

→ The acoustic energy in incremental volume $d\forall$ is $\mathcal{E}\, d\forall$.

→ As shown below, the surface area of the sphere which contains the differential volume $d\forall$ is $4\pi r^2$. In addition, the projected area of dS on the sphere is $dS \cos\theta$

→ The "ratio of the" portion of the energy which strikes the element dS to the total energy is $\dfrac{dS \cos\theta}{4\pi r^2}$





* $dS \cos\theta$ ← يمثل جزء من كرة *

$\mathcal{E} d\forall \neq$ الطاقة الي ع توصلها

$\mathcal{E} d\forall$ ع يوصلها جزء من

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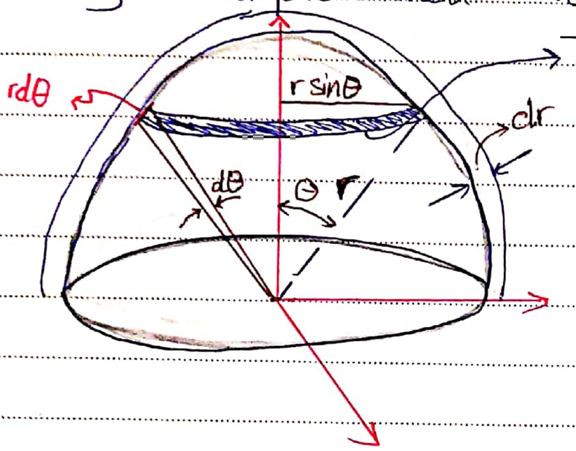
→ Hence, the energy which reaches the element dS' is given by

$$dE = \mathcal{E} * dV \left(\frac{dS' \cos\theta}{4\pi r^2} \right)$$

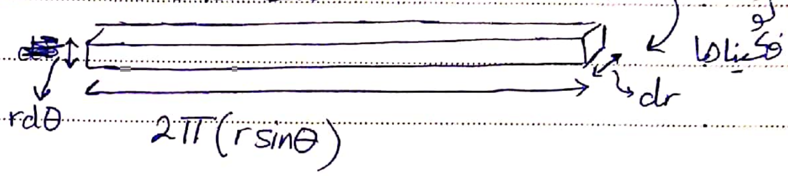
⇝ this represents the energy from dV that reaches dS'

↳ has units of Joules

→ Now consider the volume element as being part of a hemisphere shell of radius r and thickness dr. The acoustic energy reached to dS' by the complete shell is found as follows:



→ This is a circular zone from the shell



The volume of this circular zone is

$$dV = r\,d\theta * 2\pi r \sin\theta * dr$$

$$dV = 2\pi r^2 \sin\theta \, d\theta \, dr$$

∴ The energy which reaches the element dS' from the circular zone is

$$dE = \mathcal{E}\, dV \left(\frac{dS \cos\theta}{4\pi r^2} \right)$$

$$dE = \mathcal{E}\left(\cancel{2\pi r^2} \sin\theta \, d\theta \, dr\right)\left(\frac{dS \cos\theta}{\underset{2}{\cancel{4\pi r^2}}} \right)$$

$$dE = \mathcal{E}\, \frac{\sin\theta \cos\theta}{2}\, dS' \, dr \, d\theta$$

⇝ this represents the energy from the circular zone that reaches dS'

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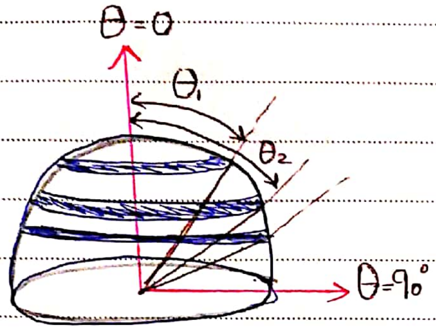
→ The hemispherical shell consists of circular Zones
∴ To find the energy that reaches dS' from the hemispherical shell, we will integrate the previous formula from $\theta=0$ to $\theta=\frac{\pi}{2}$



$$\Delta E = \frac{\mathcal{E}\, dS'\, dr}{2} \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta$$

Recall:

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\therefore \Delta E = \frac{\mathcal{E}\, dS'\, dr}{2} \int_0^{\pi/2} \frac{1}{2} \sin 2\theta$$

$$= \frac{\mathcal{E}\, dS'\, dr}{4} \left(-\frac{\cos 2\theta}{2} \right) \Big]_0^{\pi/2}$$

$$\Delta E = \frac{\mathcal{E}\, dS'\, dr}{4*2} \left(-(-1) - (-1) \right) = \frac{\mathcal{E}\, dS'\, dr}{4*\cancel{2}} * \cancel{2}$$

$$\Delta E = \frac{\mathcal{E}\, dS'\, dr}{4}$$; Recall that the sound has a speed $C = \frac{dr}{dt}$

$\therefore dr = C\,dt$

$$\Delta E = \frac{\mathcal{E}\, dS' * C\, dt}{4}$$

$$\frac{\overset{J}{\Delta E}}{\underset{m^2}{dS'} * \underset{s}{dt}} = \boxed{\frac{\mathcal{E} C}{4} = I}$$ ※ ⇝ Sound intensity in reverberant field

*note: in free field $I = \mathcal{E} C$ (لما خدنا اشتقاقها)

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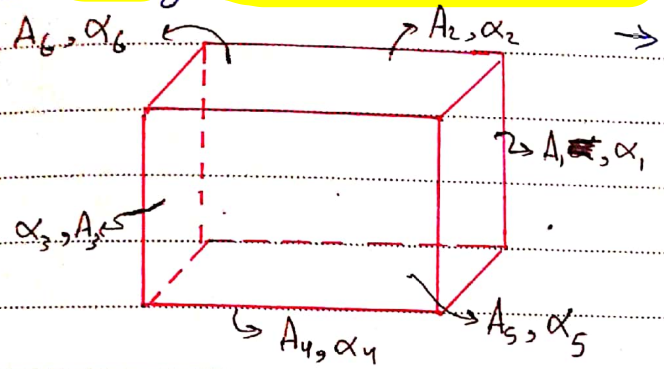
[*] Average absorption coefficient:



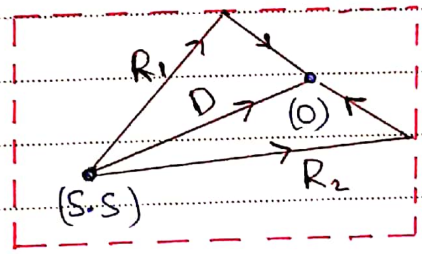
→ if the floors, walls, & ceiling of the room have different values of absorption coefficients, We can define an average absorption coefficient as follows:

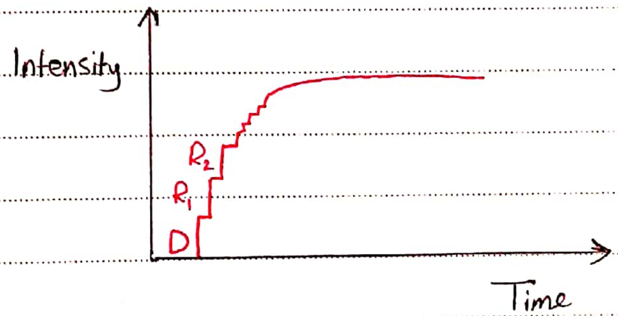
$$\bar{\alpha}\left(\Sigma A_i\right) = A_1\alpha_1 + \alpha_2 A_2 + A_3\alpha_3 + A_4\alpha_4 + A_5\alpha_5 + A_6\alpha_6$$

$$\bar{\alpha} = \frac{\Sigma(A_i\alpha_i)}{\Sigma A_i}$$

[*] Sound build up in semi reverberant field:

The following figures show what happens when a continuous sound source suddenly starts emitting sound. The direct sound is heard first, and the level gradually builds up as successive reflections (R_1 and R_2 etc.) reach the observer (O). After a short time, the rate of energy production balances the rate of absorbtion and the overall sound energy remains constant





→ We will construct a differential ~~eq~~ equation that governs this dynamical system.

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$\rightarrow \quad W = W_{absorbed} + W_{reflected}$

$\hookrightarrow$ acoustical power emitted by the sound source, W = constant

$\circledast$ $W_{absorbed} = I * A_{tot} * \bar{\alpha} = \frac{\varepsilon C}{4} \left(\sum A_i\right) * \bar{\alpha}$

$\circledast$ $W_{reflected} = \underbrace{\forall}_{m^3} * \frac{d\overbrace{\varepsilon}^{J}}{\underbrace{dt}_{s}}$ $\rightsquigarrow$ Recall: reflected power causes sound build up as shown in the previous page (i.e. ε increases)

$\rightsquigarrow \forall$ = Volume of the room

$$\therefore W = \frac{\varepsilon C}{4} \left(\sum A_i\right) \bar{\alpha} + \forall \frac{d\varepsilon}{dt}$$

$$\underbrace{\frac{W}{\forall}}_{b} = \underbrace{\left[\frac{C\left(\sum A_i\right)\bar{\alpha}}{4 * \forall}\right]}_{a} \varepsilon + \dot{\varepsilon}$$

$\rightarrow$ This is a first order ordinary differential equation that has the following form

$\dot{y} + ay = b$

$\hookrightarrow$ The solution of this D.E is given by

$$\varepsilon(t) = \frac{b}{a}\left(1 - e^{-at}\right)$$

$$= \frac{W}{\cancel{\forall}} * \frac{4\cancel{\forall}}{C\left(\sum A_i\right)\bar{\alpha}}\left(1 - e^{\frac{-C\left(\sum A_i\right)\bar{\alpha}}{4\forall}t}\right)$$

$$= \frac{4W}{C\left(\sum A_i\right)\bar{\alpha}}\left(1 - e^{\frac{-C\left(\sum A_i\right)\bar{\alpha}}{4\forall}t}\right)$$

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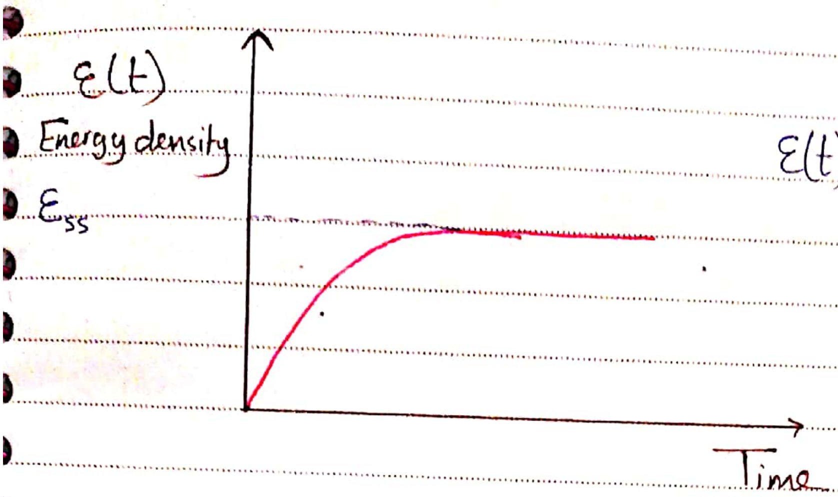
→ plot of $\mathcal{E}(t)$



$$\mathcal{E}(t) = \frac{4 * W}{C(\Sigma A_i)\bar{\alpha}} \left(1 - e^{\frac{-c(\Sigma A_i)\bar{\alpha} t}{4 \forall}}\right)$$

$\mathcal{E}_{s.s}$ = Steady state value of energy density

$$= \lim_{t \to \infty} \mathcal{E}(t)$$

$$= \frac{4 * W}{\bar{\alpha}(\Sigma A_i) c}$$

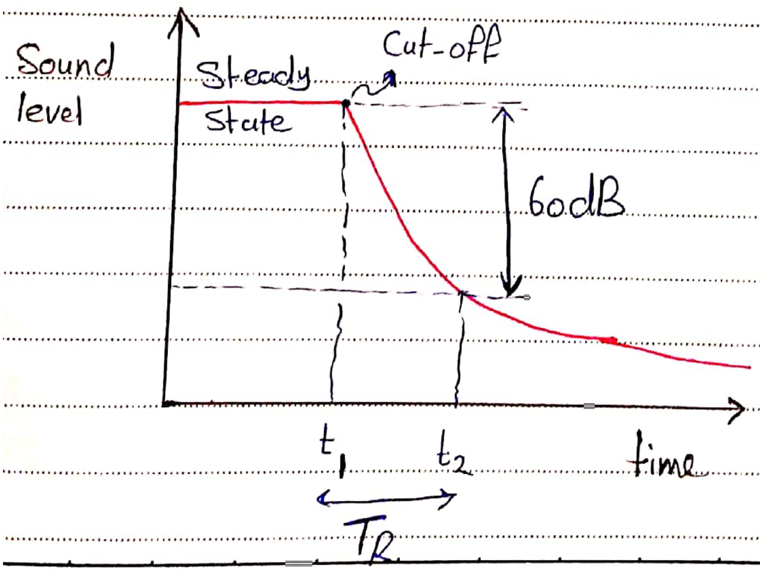
→ If $\mathcal{E}(t)$ is known, we can find $I(t)$:

$$I(t) = \frac{\mathcal{E}(t) * C}{4}$$

[*] Sound Decay

→ If now we turn off the sound source, the sound level will decay with time.

→ It is easy to understand that the more absorption coefficient the room has, the faster the sound will decay



→ The time required for the sound level to decay 60 dB is called Reverberation time T_R

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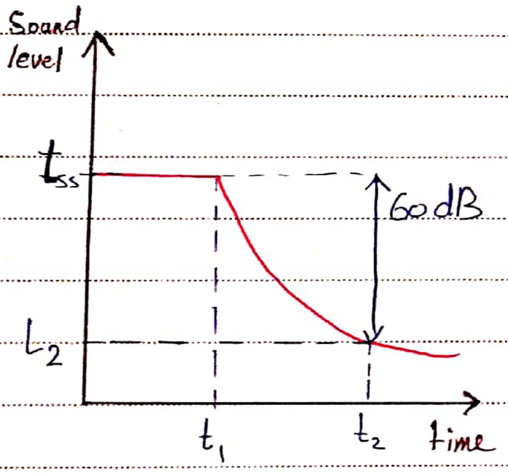
→ The differential equation which governs the sound decay is the same as the D.E which governs the sound build up, except that W (acoustic power of the sound source) is set to zero.

$$\rightarrow \underset{\text{zero}}{\cancel{W}} = \left(\frac{c(\Sigma A_i)\bar{\alpha}}{4}\right)\varepsilon + \forall \frac{d\varepsilon}{dt}$$

→ The solution of the above equation is given by

$$\varepsilon(t) = \underbrace{\frac{4W}{\bar{\alpha} c(\Sigma A_i)}}_{n} \, e^{\overbrace{\frac{-\bar{\alpha} c(\Sigma A_i)}{4\forall}}^{k} t} = n\, e^{kt}$$

→ Now, we will try to find a formula for the reverberation time "T_R"



$$* \; I(t) = \frac{\varepsilon(t) * c}{4}$$

$$* \; L_{ss} - L_2 = 60 \text{ dB}$$

$$L_{ss} - L_2 = 10 \log\left(\frac{I_{ss}}{I_r}\right) - 10 \log\left(\frac{I_2}{I_r}\right)$$

$$= 10 \log\left(\frac{I_{ss}}{\cancel{I_r}} * \frac{\cancel{I_r}}{I_2}\right) = 10 \log\left(\frac{I_{ss}}{I_2}\right)$$

$$60 \text{ dB} = 10 \log\left(\frac{\varepsilon(t_1) * \cancel{c}}{\cancel{4}} * \frac{\cancel{4}}{\cancel{c}\,\varepsilon(t_2)}\right)$$

$$60 \text{dB} = 10 \log\left(\frac{\varepsilon(t_1)}{\varepsilon(t_2)}\right) = 10 \log\left(\frac{\cancel{n}\, e^{kt_1}}{\cancel{n}\, e^{kt_2}}\right)$$

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$$60\,dB = 10\log\left(\frac{e^{kt_1}}{e^{kt_2}}\right) = 10\log\left(e^{kt_1 - kt_2}\right)$$

$$= 10\log\left(e^{-k\overbrace{(t_2 - t_1)}^{\text{reverberation time } T_R}}\right)$$

$$\frac{60}{10} = \log e^{-kT_R}$$

$$10^6 = e^{-kT_R} \Rightarrow \ln 10^6 = \ln e^{-kT_R}$$

$$-kT_R = \ln 10^6$$

$$T_R = \ln 10^6 \div (-K) = \ln 10^6 \div \left(-\left(\frac{-\bar{\alpha} * C * \Sigma A_i}{4V}\right)\right)$$

$$T_R = \frac{\ln 10^6 * 4V}{\bar{\alpha} * C * \Sigma A_i} = \frac{(\ln 10^6) * 4}{C} * \frac{V}{\bar{\alpha} * (\Sigma A_i)}$$

if $C = 343\ m/s \Rightarrow T_R = 0.1611 * \dfrac{V}{\bar{\alpha}(\Sigma A_i)}$

This theoretical formula agrees with the experimental measurements for $\bar{\alpha} \leq 0.2$

We will use another formula that is valid for any $\bar{\alpha}$

$$T_R = \frac{-0.1611 * V}{(\Sigma A_i)\ln(1 - \bar{\alpha})}$$

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* $L_{I,rev}$, not L_I

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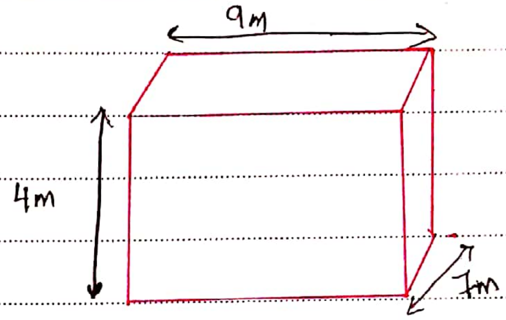
Example: a room has the following dimensions $7 \times 9 \times 4$ with $\alpha_{floor} = 0.1$, $\alpha_{wall} = 0.3$, $\alpha_{ceiling} = 0.8$. If a sound source with $L_w = 125\ dB$ is turned on. Find



① L_I after 10 ms

② Reverberation time, if the sound source is turned off

Solution:

①
$$\mathcal{E}(t) = \frac{4W}{\sum A_i \bar{\alpha} c}\left(1 - e^{-\frac{c(\sum A_i)\bar{\alpha}}{4V} * t}\right)$$

$$L_w = 10 \log\left(\frac{W}{W_R}\right) \Rightarrow \cdot\ W = W_R * 10^{L_w/10} = 10^{-12} * 10^{125/10}$$
$$= 3.16 \text{ Watt}$$

• $\sum A_i = 2 * 9 * 7 + 2 * 4 * 7 + 2 * 4 * 9 = 254\ m^2$

$$\bar{\alpha} = \frac{0.3 * 4 * 7 * 2 + 0.3 * 4 * 9 * 2 + 0.1 * 9 * 7 + 0.8 * 9 * 7}{254}$$

$$= 0.375$$

$V = 7 \times 4 \times 9 = 252\ m^3$

$c = 340\ m/s$

$\mathcal{E}(t = 0.01 \text{ sec}) = 1.07 \times 10^{-4}\ J/m^3$

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$$I = \frac{\varepsilon * C}{4} = 9.11 \times 10^{-3} \quad W/m^2$$

$$L_I = 10 \log\left(\frac{I}{I_r}\right) = 99.6 \text{ dB}$$

② $$T_R = \frac{-0.1611 * \forall}{(\Sigma A_i) \ln(1-\bar{\alpha})} = 0.34 \text{ sec}$$

* Note: notice that if $\bar{\alpha} = 1$ (Anechoin Chamber)

$$T_R = \frac{-0.1611 * \forall}{(\Sigma A_i) * \underbrace{\ln(1-1)}_{-\infty}} \Rightarrow T_R = 0 \quad (\text{no reverberation time})$$

وهذا الشي؛ صحيح لأنه لو فقعنا بالون
في Anechoic chamber رح نلاقي انه الصوت
فورًا بيختفي ، بس لو فقعنا البالون في
reverberant field ، رح نسمع صوت صدى لل
نفقع البالون ⇐ يعني في Sound decay بصير خلال أجزاء من الثانية
(الصوت ما بيختفي فجأة)

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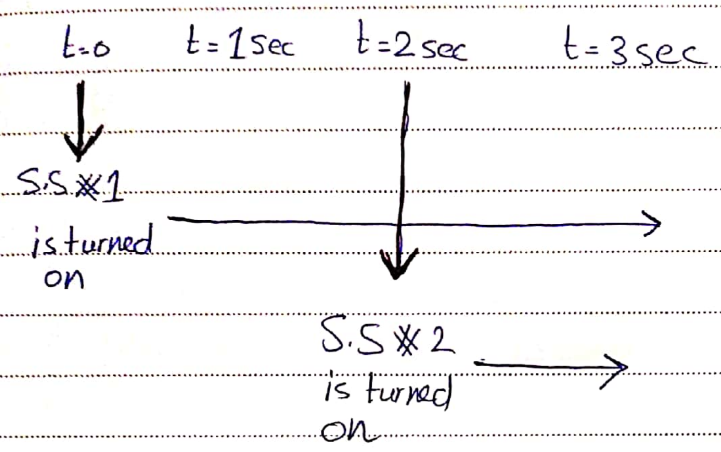
Example: if the room in the previous examples has 2 sound sources with sound power levels L_{w_1} and L_{w_2} respectively. Sound source #1 is turned on initially, and after 2 seconds, sound source #2 is turned on. Find the total sound intensity after 3 seconds.

Solution



$\therefore$ after 3 sec

$$\mathcal{E}_1 = \mathcal{E}_1(t=3\,sec)$$

$$\mathcal{E}_2 = \mathcal{E}_2(t=1\,sec)$$

$$\therefore I_1 = \frac{(\mathcal{E}_1+\mathcal{E}_2) * C}{4}$$

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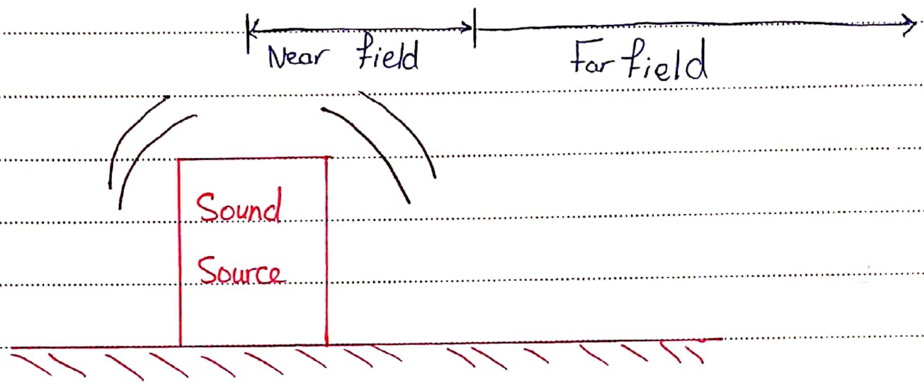
7 / 11 / Thurs

In this lecture, We will discuss 3 topics:

1. The difference between Near field and Far field in a free field
2. Definition of Directivity index in a free field
3. Find out the value of $L_{p,tot}$ in semireverberant field

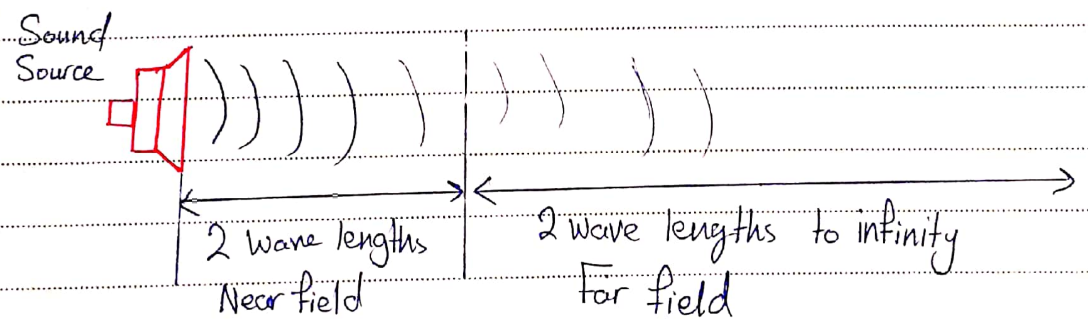
1. Near field and far field in a free field

→ Consider the following free field (i.e a field with no barriers)



* Far field

- The acoustic far field is defined as beginning at a distance of 2 wavelengths away from the sound source and extends outward to infinity as shown below. As the wave length is a function of frequency $\lambda = C/f$, the start of the far field is also a function of the frequency



- In the far field, for each doubling of distance away from the source, the sound pressure level will drop by 6dB
- In many acoustic standards, measurements are often specified at a distance of at least 1 meter from the sound source, to ensure that the measurement is taken in the far field

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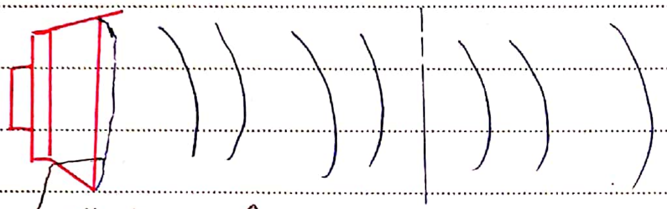
* Near field

- When close to a sound emitting object, the sound waves behave in a much more complex fashion, and there is no fixed relationship between sound pressure level and distance.
- Very close to the sound source, the sound energy moves back and forth with the vibrating ~~sound source~~ surface of the sound source, never escaping or propagating away as shown below

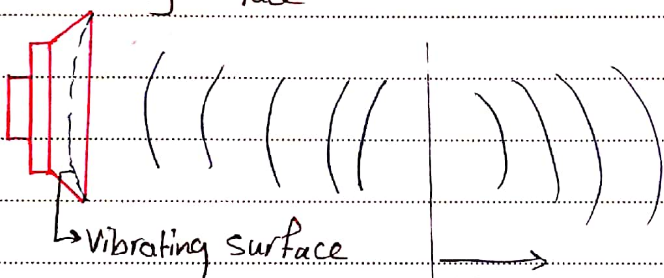
S.S

@$t = t_1$



Vibrating surface

@$t_f = t_1 + \Delta t$



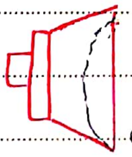
Vibrating surface

Far field

(*) هون موجات الصوت بتضل تروح وترجع مع حركة ال Vibrating Surface

(*) هون موجات الصوت بتكمل طريقها وما بترجع

(*) لما يصير شكل ال Vibrating surface هيك :



بعمل منطقة ضغط منخفض عشان هيك موجات الصوت (جزيئات الهوا) بترجع

(*) The back and forth movement of the sound ~~source~~ waves in the near field means there is no fixed relationship between distance and sound pressure level in the near field

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2) Directivity index in free field

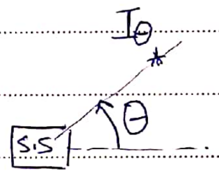
→ A simple point sound source radiates uniformly in all directions.

→ In general, the radiation of sound from a typical source is directional, being greater in some directions than in others.

→ The directional properties of a sound source may be quantified by the introduction of a directivity factor, describing the angular dependence of the sound intensity.

→ The directivity factor Q_θ is defined as

$$Q_\theta = \frac{I_\theta}{I_{point,s.s}}$$



↳ top view of a sound source (directional sound source)

Where I_θ = actual sound intensity at angle θ

$I_{Point,s.s}$ = Sound intensity of a uniform point sound source radiating the same total power W as the actual source

→ The directivity index DI is defined as

$$DI = 10 \log Q_\theta$$

→ $$I_{Point,s.s} = \frac{W}{4\pi r^2}$$

$$\therefore Q = \frac{I_\theta}{W/(4\pi r^2)} \Rightarrow I_\theta = I_{direct} = Q * \frac{W}{4\pi r^2}$$

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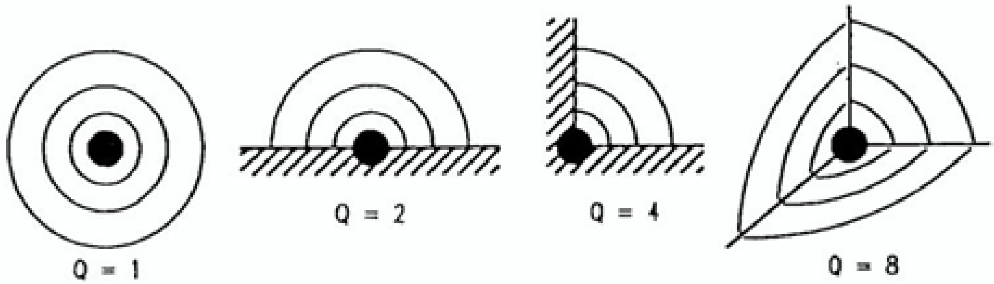
→ Special cases:

- Consider the following point sources



- Q = 1 ⇒ Omnidirectional point source
- Q = 2 ⇒ Point source over a reflecting plane
- Q = 4 ⇒ Point source in a corner
- Q = 8 ⇒ Point source in a vertex

- In the case of omnidirectional sound source

$$I_\theta = I_{Point,ss} \qquad \therefore Q = \frac{I_\theta}{I_{Point,s.s}} = 1$$

- In the case of point source over a reflecting surface, the same acoustic power is emitted over a hemisphere

$\therefore I_\theta$ is doubled

$$Q = \frac{2 I_\theta}{I_{Point,s.s}} = 2$$

- When the point source is set in a corner, the same acoustic power is emitted over a 1/4 sphere

$\therefore I_\theta$ is quadrapled

$$Q = \frac{4 I_\theta}{I_{Point,s.s}} = 4$$

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3 Sound pressure level in semireverberant field

- A real room is somewhere between a reverberant field and a free field. Therefore, the total pressure is the sum of the direct and reverberant field.
- Recall,

in reverberant field: $I_{rev} = \frac{\varepsilon c}{4}$, in free field $I_{direct} = \varepsilon c = \frac{P_{rms}^2}{\rho c}$

$\therefore$ In reverberant field $I_{rev} = \frac{P_{rms}^2}{4\rho c}$

- In semireverberant field

$$P_{rms}^2 = \left(P_{rms}^2\right)_{direct} + \left(P_{rms}^2\right)_{reverberant}$$

$$= \rho c\, I_{direct} + 4\rho c\, I_{rev} \quad (*)$$

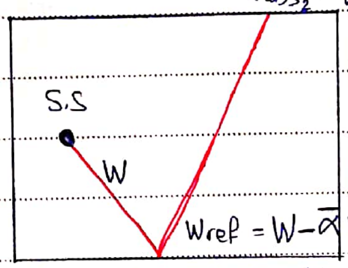
- We want to find I_{direct} and I_{rev} in terms of W (acoustic power of the sound source):

$$I_{direct}\ (\text{in free field}) = Q * \frac{W}{4\pi r^2} \quad (*)(*)$$

- To find I_{rev} , consider the following room

$W_{abs_2} = W_{ref} * \bar{\alpha} = (I_{rev} * A) * \bar{\alpha}$

recall: reverberation = reflection



$W_{ref} = W - \bar{\alpha} W = W(1-\bar{\alpha})$

first reflection

$W_{abs_1} = \bar{\alpha} W$

* approximately $W_{ref} \approx W_{abs_2}$

$$W(1-\bar{\alpha}) \approx I_{rev} * A * \bar{\alpha}$$

$$I_{rev} = \frac{W(1-\bar{\alpha})}{A\bar{\alpha}} = \frac{W}{R} \quad (*)(*)(*)$$

* R = room constant $= \frac{A\bar{\alpha}}{(1-\bar{\alpha})}$

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$\rightarrow R = \frac{A\bar{\alpha}}{1-\bar{\alpha}}$, if $\bar{\alpha} \rightarrow 1 \Rightarrow R \rightarrow \infty$

i.e. a room constant with large R indicates more absorption

$\rightarrow$ Substitute ⊛⊛ and ⊛⊛⊛ in ⊛ , the equation becomes

$$P_{rms}^2 = \rho C \left(\frac{Q\,W}{4\pi r^2} \right) + 4\rho C * \frac{W}{R}$$

$$= \rho C W \left(\frac{Q}{4\pi r^2} + \frac{4W}{R} \right)$$

$\rightarrow$ Recall:

$$L_p = 10 \log \left(\frac{P_{rms}^2}{P_{ref}^2} \right) = 10 \log \left(\frac{I}{I_{ref}} \right) \quad , I_{ref} = \frac{P_{ref}^2}{\rho C}$$

$$\frac{P_{rms}^2}{P_{ref}^2} = \frac{\rho C\ W}{P_{ref}^2} \left(\frac{Q}{4\pi r^2} + \frac{4}{R} \right)$$

$$\frac{P_{rms}^2}{P_{ref}^2} = \frac{W}{I_{ref}} \left(\frac{Q}{4\pi r^2} + \frac{4}{R} \right)$$

, recall the numerical value of I_{ref} = numerical value of W_{ref}

$$\frac{P_{rms}^2}{P_{ref}^2} = \frac{W}{W_{ref}} \left(\frac{Q}{4\pi r^2} + \frac{4}{R} \right)$$

$$10 \log \left(\frac{P_{rms}^2}{P_{ref}^2} \right) = 10 \log \left(\frac{W}{W_{ref}} * \left(\frac{Q}{4\pi r^2} + \frac{4}{R} \right) \right) \longrightarrow$$

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$$L_p = 10\log\left(\frac{W}{W_{ref}}\right) + 10\log\left[\frac{Q}{4\pi r^2} + \frac{4}{R}\right]$$

$$L_p = L_W + 10\log\left[\underbrace{\frac{Q}{4\pi r^2}}_{\text{direct field}} + \underbrace{\frac{4}{R}}_{\text{reverberant field}}\right]$$

Summary:

→ To find L_p in semireverberant field, use the previous formula

→ To find L_p in free field: (very high $\bar{\alpha}$)

$I = \varepsilon C$, $I = \frac{P_{rms}^2}{\rho C}$, $I = Q * \frac{W}{4\pi r^2}$ $\Rightarrow L_p = L_I = 10\log\left(\frac{I}{I_{ref}}\right)$

→ To find L_p in reverberant field (Very low α)

$I = \frac{\varepsilon C}{4}$, $I = \frac{P_{rms}^2}{4\rho C}$, $I = \frac{W}{R}$ $\Rightarrow L_p = L_I = 10\log\left(\frac{I}{I_{ref}}\right)$

احنا ما بنستخدمها لا approx. الا في حالة ما في

Example

if a machine emits a complex Sound, that has the following spectrum

f_c	L_W	L_p
31.5		
63		
125		
500		
1		
2		
4		
8		

* Find $L_{p,tot}$

* Find which central frequency which contributes the most to the total sound pressure level

* if at $f_c = 2000$ Hz → the sound pressure level L_p contributes the most to $L_{p,tot}$. Take the value of L_W @ 2000 Hz (assume $L_W = 120$ dB @ 2000 Hz) & analyze the sound pressure level in the room resulting from $L_W = 120$ dB @ 2000 Hz

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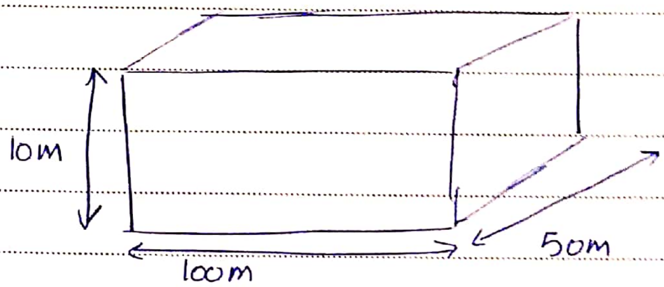
• $L_p = L_w + 10 \log\left[\frac{Q}{4\pi r^2} + \frac{4}{R}\right]$

↳ 120 dB

• assume that $Q = 2$
& the room has the following dimensions



• Given

$\alpha_{floor} = 0.02$, $\alpha_{ceiling} = 0.08$, $\alpha_{walls} = 0.39$, $\alpha_{glass} = 0.07$

These are obtained from the tables @ $f = 2000$ Hz

• No. windows 6, dimensions of the window $= 3 \times 1.5\ m^2$
• 2 doors $6 \times 5\ m^2$
• Find L_p @ 10 m, 20 m, 30 m, 40, 45 m

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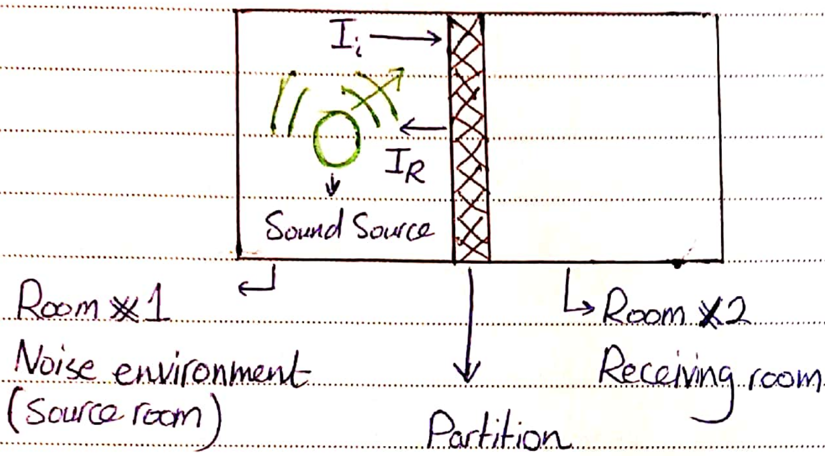
12 / 11 / Tuesday

[*] Sound Transmission

→ Consider the following two rooms which are separated by a partition.
Room #1 represents the noise environment which contains the sound source.
Room #2 represents the sound receiving room.



I_i = Incident sound intensity
I_R = Reflected sound intensity

→ $I_{absorbed} = I_i - I_R$

→ Some of the absorbed sound intensity will be dissipated in the form of "heat", while the rest will be transmitted to room #2.

→ The transmitted sound intensity is designated by I_t

→ The sound transmission coefficient τ is defined as:

$$\tau = \frac{I_t}{I_i}$$

→ the value of τ is extremely small, for that reason another quantity is defined to be utilized instead of τ. This quantity is termed as: sound transmission loss (STL)

$$STL = 10 \log\left(\frac{1}{\tau}\right)$$

→ This quantity is a function of the sound's frequency [i.e. STL = f(frequency)] and it is measured in decibels

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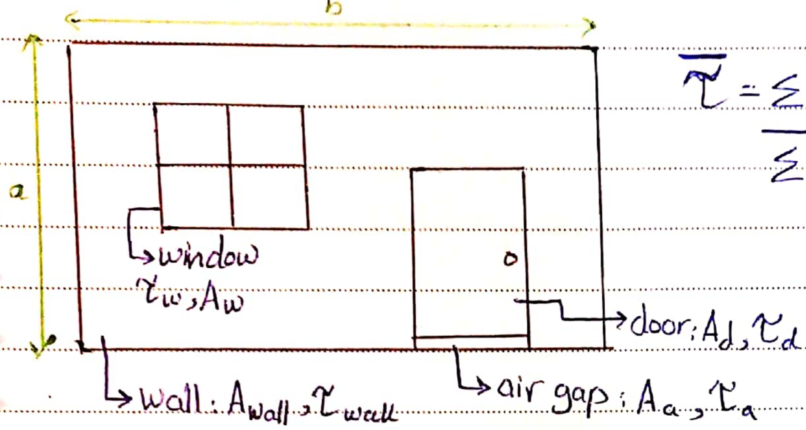
→ For a composite wall:



$$\bar{\tau} = \frac{\sum A_i \tau_i}{\sum A_i} \Rightarrow \overline{STL} = 10 \log\left(\frac{1}{\bar{\tau}}\right)$$

(Note $A_{wall} = a*b - A_{door} - A_{airgap} - A_{window}$)

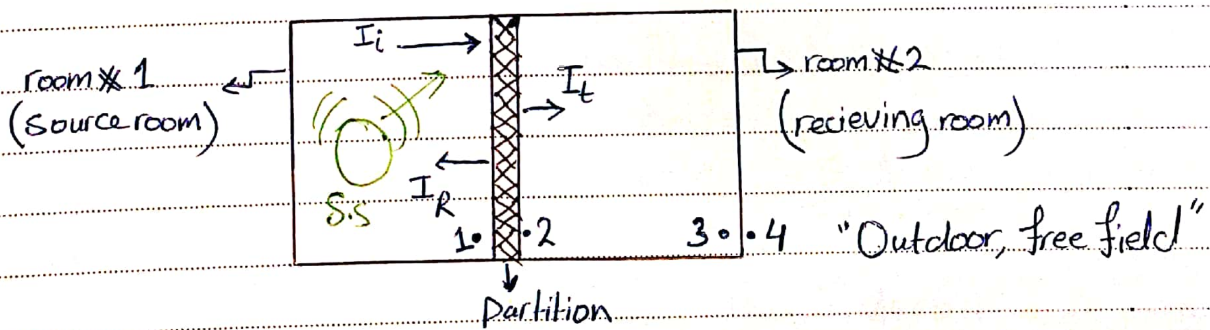
→ Note: For an air gap, the transmission coefficient $\tau = 1$, becauce air gaps transmit the incident sound intensity entirely (ie. $I_t = I_i \Rightarrow \therefore \tau = \frac{I_t}{I_i} = 1$)

Therefore

$$STL\Big|_{airgap} = 10 \log\left(\frac{1}{1}\right) = 0$$

17/11 Sunday

→ Consider the following two rooms which are separated by a partition.



→ The sound pressure level at locations 1, 2, 3, and 4 are given as follows:

- $L_{p_1} = L_w + 10 \log\left[\frac{Q}{4\pi r_1^2} + \frac{4}{R_1}\right]$

Q = directivity factor of the sound source

r_1 = the distance from the sound source to the location 1

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R_1 = room constant of room 1 $= \frac{\sum A_i * \bar{\alpha}_1}{1-\bar{\alpha}_1}$ the sum of the areas of walls, floor and ceiling of room 1

average absorption coefficient of room 1

- $L_{p_2} = L_{p_1} - STL\Big|_{Partition} + 10 \log\left(\frac{S_{partition}}{R_2} + \frac{1}{4}\right)$

the area of the Partition ~~…~~ ~~…~~ through which sound transmission is taking place

room constant of room 2

- $L_{p_3} = L_{p_2}$ (This is a conservative assumption. In reality L_{p_3} is slightly less than L_{p_2})

- $L_{p_4} = L_{p_3} - STL\Big|_{Partition} + 10 \log\left(\frac{S_{partition}}{R_{outdoor}} + \frac{1}{4}\right)$

For a free field $\alpha = 1$. Outdoors can be considered as ~~a~~ a free field

$\therefore R_{outdoor} = \frac{\sum A_i * \alpha}{1-\alpha} = \infty$

$\therefore \frac{S_{partition}}{R_{outdoor}} = zero$

$L_{p_4} = L_{p_3} - STL\Big|_{Partition} + 10 \log\left(\frac{1}{4}\right)$

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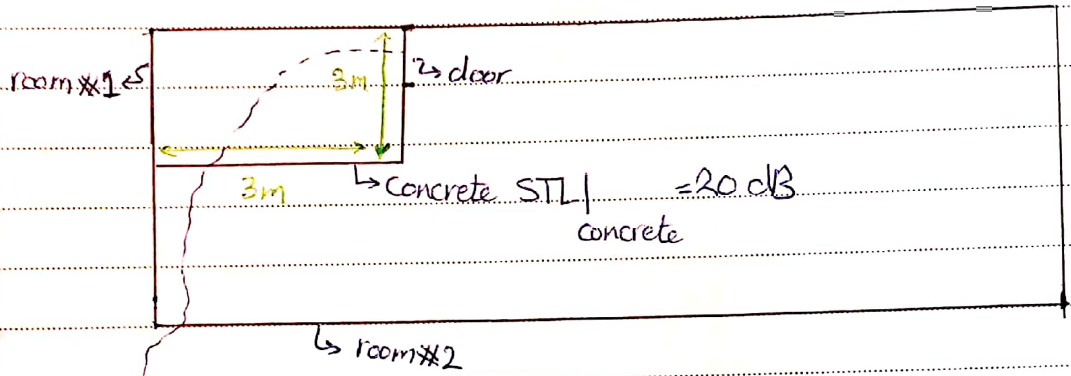
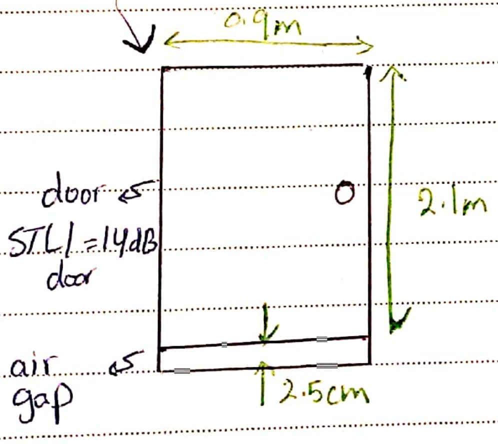
→ Note: the difference between L_{p_1} and L_{p_2} in the previous discussion is termed as: Noise Reduction

$$\text{Noise Reduction} = L_{p_1} - L_{p_2} = STL|_{\text{Partition}} - 10\log\left(\frac{S_{\text{partition}}}{R_2} + \frac{1}{4}\right)$$

→ Example: Find $\overline{STL}$ of the following partition



*height = 4.5m



• Solution: the partition consists of 2 concrete walls ~~and~~ a door, and an air gap.

$$STL|_{\text{concrete}} = 20\text{ dB} = 10\log\left[\frac{1}{\tau_{\text{conc}}}\right]$$

$$\tau_{\text{concrete}} = \frac{1}{10^{20/10}} = 0.01$$

$$\tau_{\text{door}} = \frac{1}{10^{14/10}} = 0.03981$$

$$\tau_{\text{airgap}} = 1$$

$$\bar{\tau} = \left[0.01\left(3\times4.5 + \left(3\times4.5 - 0.9*\left(2.1 + 2.5\times10^{-2}\right)\right)\right) + 0.03981*0.9*2.1 + 1*0.9*2.5\times10^{-2}\right] \Big/ \left(3\times4.5 + 3\times4.5\right) = 0.0129117$$

$$\overline{STL} = 10\log\left(\frac{1}{\bar{\tau}}\right) = 18.89\text{ dB}$$

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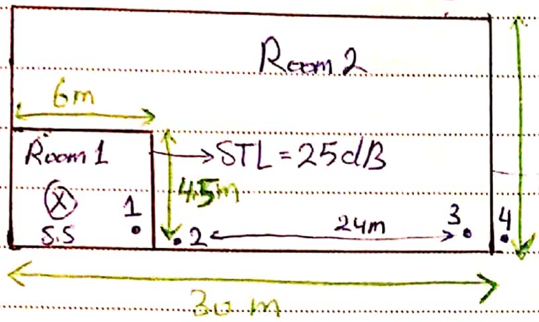
→ Example: Consider the following two rooms, which are separated by a partition. Find L_{p_1}, L_{p_2}, L_{p_3}, L_{p_4}



* height = 9 m , $W_{sound\,source}$ = 2 Watts
* $\bar{\alpha}_{room\,1} = 0.1$, $\bar{\alpha}_{room\,2} = 0.3$

↳ STL = 30dB

Solution:

$$L_w = 10 \log\left[\frac{W_{source}}{W_{ref}}\right] = 10 \log\left[\frac{2}{10^{-12}}\right] = 123 \text{ dB}$$

$$L_{p_1} = L_w + 10 \log\left[\frac{4}{R_1} + \frac{Q}{4\pi r_1^2}\right]$$

$$R_1 = \frac{\alpha_1 \sum A_i}{1-\alpha_1} = \frac{0.1 * (6\times 9\times 2 + 4.5\times 9\times 2 + 6\times 4.5\times 2)}{1-0.1} = 27 \text{ m}^2$$

Since the S.S is installed on the ground ⇒ Q = 2

r_1 = the distance between the Sound source and location 1 = 3 m

$\therefore L_{p_1} = 115.2$ dB

$$L_{p_2} = \underbrace{L_{p_1}}_{115.2} - \underbrace{STL|_{Partition}}_{25} + 10 \log\left[\frac{S_{Partition}}{R_2} + \frac{1}{4}\right]$$

↳ $S_{Partition} = 6\times 9 + 4.5\times 9 = 94.5 \text{ m}^2$

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$R_2 = \dfrac{\bar{\alpha}_2 \, \Sigma A_i}{1-\bar{\alpha}_2}$, $(\Sigma A_i) = 30\times9 + 12\times9 + (30-6)\times9 + 4.5\times9 + 6\times9 + (12-4.5)\times9$

$+ 2\times(30\times12) - 2\times6\times4.5 = 1422 \text{ m}^2$

↳ represents the sum of the walls, floor & ceiling areas of room 2

$R_2 = \dfrac{0.3\times1422}{1-0.3} = 609.4$

$\therefore L_{P_2} = 86.27 \text{ dB}$

assume $L_{P_3} = 85 \text{ dB}$

$L_{P_4} = L_{P_3} - STL + 10\log\left[\dfrac{S_{partition}}{R_{outdoor}} + \dfrac{1}{4}\right]$

$R_{outdoor}$ ↳ ∞

$= 85 - 30 + 10\log\left(\frac{1}{4}\right) = 49 \text{ dB}$

$\boxed{*}$ Vibration Control

In this part of the course, we will cover the following topics

Vibration Systems

- Single DOF
  - Forced vibration (damped / undamped)
    - Periodic
    - harmonic
      - direct
      - base excitation
      - rotating unbalance
- 2 DOF

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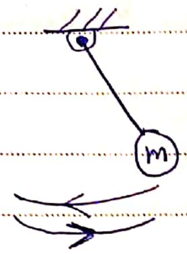
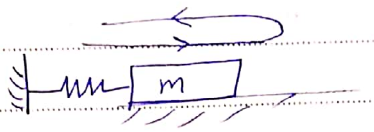
21 / 11 / Thursday

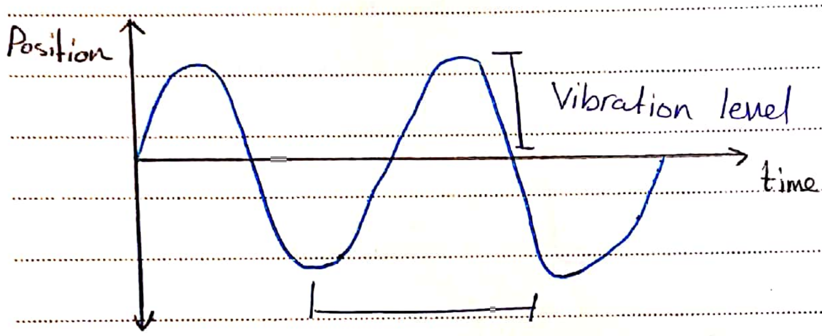
→ Vibrations: motion that repeats itself with time in a certain position.

e.g:





→ These vibrations can be represented by a plot of position against time

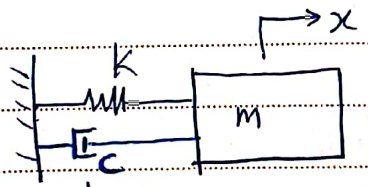


T: period: time required to complete 1 cycle

ω: frequency of vibration $= \frac{2\pi}{T}$

→ Vibrations can be classified as:

[I] Free vibration: In this type of vibration, the system is given an initial disturbance (i.e. $x(t=0)$ = non zero value) and "released" to oscillate.



↳ damping element (optional): this element dissipates energy

- In free vibration, if there is no energy dissipation (i.e. $c=0$), the system will "Continuously" oscillate.
- ω_n = natural frequency = the frequency with which an "undamped" system oscillates in the case of free vibration $\left[\omega_n = \sqrt{\frac{k}{m}} \right]$
- In free undamped vibrations, the energy of the system is conservative since there is neither energy dissipation nor energy injection. Hence, a continuous exchange between the potential energy of the spring and the kinetic

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energy of the Oscillating block occurs.

- From energy perspective, w_n can be defined as the rate of exchange between the potential energy & kinetic energy.

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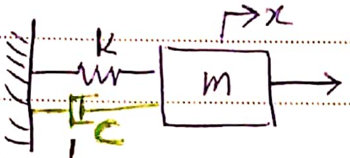
2 Forced vibration: in this type of vibration, the system oscillates due to the presence of a driving force (excitation)



$F(t) = F_0 \sin \omega t$

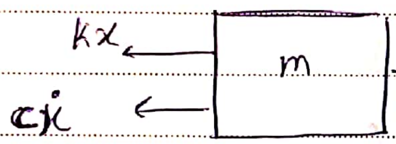
→ excitation frequency

↳ excitation level

↳ this force injects energy to the system

↳ damping element: dissipates energy

FBD



Apply Newton's 2nd law

$\xrightarrow{+} \Sigma F = m\ddot{x}$

$F_0 \sin\omega t - kx - c\dot{x} = m\ddot{x}$

$m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t$ → This equation is called: the equation of motion (EoM)

→ The maximum amplitude of vibration occurs when ω (excitation frequency) equals ω_n (natural frequency of the system. This condition is called resonance.

→ Any Vibratory System consists of:

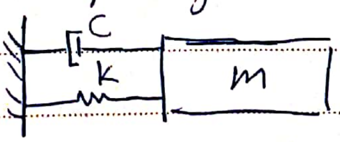
1 Inertia element (m or J) which stores kinetic energy

2 Elastic element (k or $k_{torsional}$) which stores potential energy

3 Energy dissipation element (c) [optional element]

→ To perform vibration analysis, the following procedure must be ~~exect~~ executed:

1) Modelling: converting the physical system into a mass spring damper system



≡ representation of a machine

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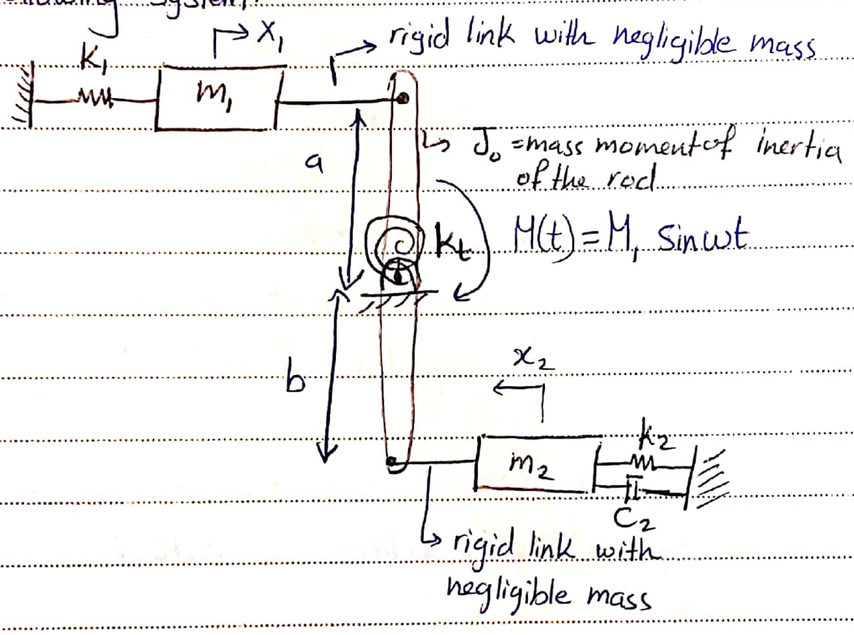
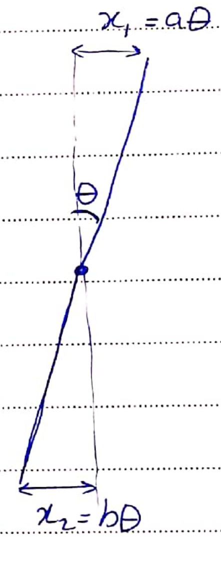
2) Application of physical laws (Newton's 2nd law or Energy method)
3) Constructing the EoM → 2nd order ODE (i.e it needs 2 initial conditions)
4) Solving the EoM to find out the system's response
5) Analysing the system's response
6) Vibration control

Example: Use the energy method to find the equation of motion of the following system:





Solution:

- This system is a single degree of freedom system, since x_1 and x_2 can be related to θ
- Note: Newton's 2nd law is often used when the system contains a single mass. energy method is easier than Newton's 2nd law if the system involves more than one mass

K.E of the system $= T = \frac{1}{2}\left[m_1\dot{x}_1^2 + m_2\dot{x}_2^2 + J_o\dot{\theta}^2\right]$

$$= \frac{1}{2}\left[m_1 a^2\dot{\theta}^2 + m_2 b^2\dot{\theta}^2 + J_o\dot{\theta}^2\right]$$

$$= \frac{1}{2}\left[m_1 a^2 + m_2 b^2 + J_o\right]\dot{\theta}^2 = \frac{1}{2} J_{eq}\dot{\theta}^2$$

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• Potential energy of the system $= V = \frac{1}{2}\left(k_t \theta^2 + k_1 x_1^2 + k_2 x_2^2\right)$

$$= \frac{1}{2}\left(k_t \theta^2 + k_1 a^2 \theta^2 + k_2 b^2 \theta^2\right)$$

$$= \frac{1}{2}\left(k_t + k_1 a^2 + k_2 b^2\right)\theta^2 = \frac{1}{2} k_{eq} \theta^2$$

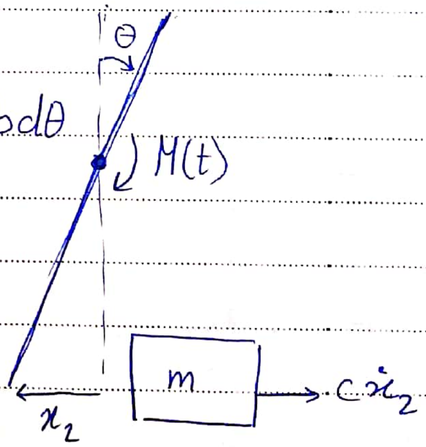
• Note:

For free undamped system $T + V = \text{Constant}$

For forced damped system $T + V =$ Work done by nonconservative forces

$$= W_{damper} + W_{\text{external force or moment}}$$

• Work done by a force $= \int \vec{F} \cdot d\vec{r}$ or Work done by a moment $= \int \vec{M} \cdot d\vec{\theta}$



→ Work done by the damping force $= -\int c\dot{x}_2\, dx_2 = -\int c(b\dot{\theta})\, b\, d\theta$

the damping force and the displacement x_2 have opposite directions. as shown in the adjacent figure

$$W_{damper} = -\int c b^2 \dot{\theta}\, d\theta$$

→ Work done by the external moment $= + \int M_0 \sin\omega t\, d\theta$

→ Both $M(t)$ and θ have the same direction

$$T + V = \int \left(M_0 \sin\omega t - c b^2 \dot{\theta}\right) d\theta = \int \left(M_0 \sin\omega t - c b^2 \dot{\theta}\right) \dot{\theta}\, dt$$

$$\frac{1}{2} J_{eq} \dot{\theta}^2 + \frac{1}{2} k_{eq} \theta^2 = \int \left(M_0 \sin\omega t - c b^2 \dot{\theta}\right) \dot{\theta}\, dt$$

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Take the time derivative of both sides of the previous equation

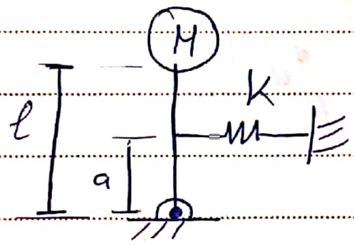
$$\frac{d}{dt}\left[T+V\right]=\frac{d}{dt}\left[\int M_0 \sin\omega t - c(b^2\dot{\theta})\,\dot{\theta}\,dt\right]$$

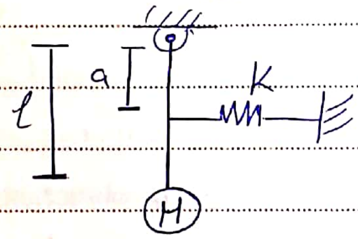
$$\frac{1}{\cancel{2}} J_{eq}\left(\cancel{2}\cancel{\dot{\theta}}\ddot{\theta}\right)+\frac{1}{\cancel{2}} k_{eq}\left(\cancel{2}\theta\cancel{\dot{\theta}}\right)=\left(M_0 \sin\omega t - cb^2\dot{\theta}\right)\cancel{\dot{\theta}}$$

$$J_{eq}\ddot{\theta}+cb^2\dot{\theta}+K_{eq}\theta=M_0 \sin\omega t$$

Homework:

- Find the EoM of the previous system using Newton's 2nd law
- Find the natural frequency of the following systems:





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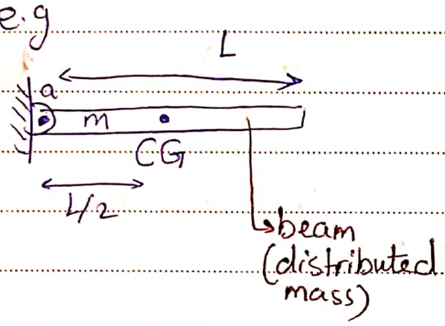
→ In this lecture, the elements of the vibratory system (Inertia element, Stiffness element, damping element) will be discussed.

☐ Inertia element;

This element stores the K.E of the system.

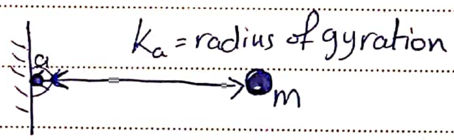
e.g



According to the Parallel axis theorem

$$J_a = \frac{1}{12} m L^2 + m \left(\frac{L}{2}\right)^2 = \frac{m L^2}{3}$$

If we want to replace the above distributed mass with a concentrated mass such that both of them would have the same moment of inertia, the concentrated mass must be set at a certain distance termed as "radius of gyration"



$$J_a = m K_a^2 = \frac{m L^2}{3} \Rightarrow K_a = \frac{L}{\sqrt{3}}$$

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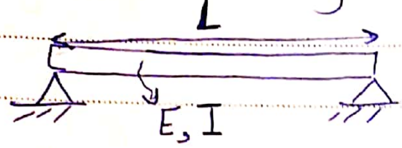
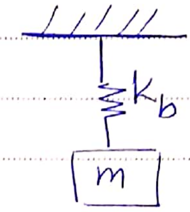
[*] Stiffness element

Consider the following beams which are supported as shown below

[1]



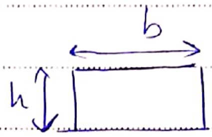
Simply supported beam =



$k_b = \frac{48EI}{L^3}$

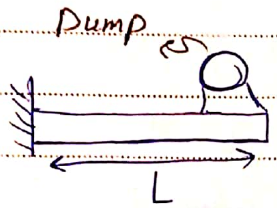
↳ stiffness at $x = \frac{L}{2}$

I = area moment of inertia of the beam $= \frac{bh^3}{12}$



↳ cross sectional area of the beam

[2]

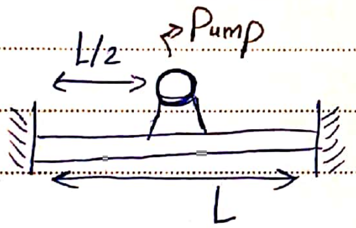


Clamped free beam =

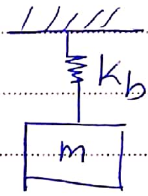
$k_b = \frac{3EI}{L^3}$

↳ stiffness at $x = L$

[3]



Clamped Clamped beam =



$k_b = \frac{192\,EI}{L^3}$

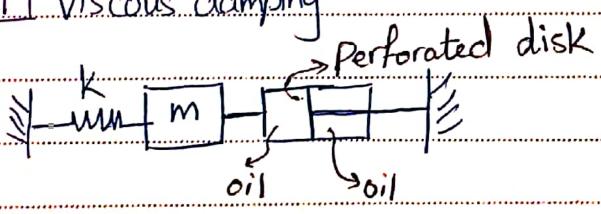
↳ stiffness at $x = \frac{L}{2}$

[*] Damping element

a mechanism by which energy is dissipated.

e.g Drag force, Viscous damping, Dry friction (coloumb damping)

[1] Viscous damping



↳ This dashpot dissipates energy

→ The damping Force F_d is ~~directly~~ directly proportional to the Velocity of the block m

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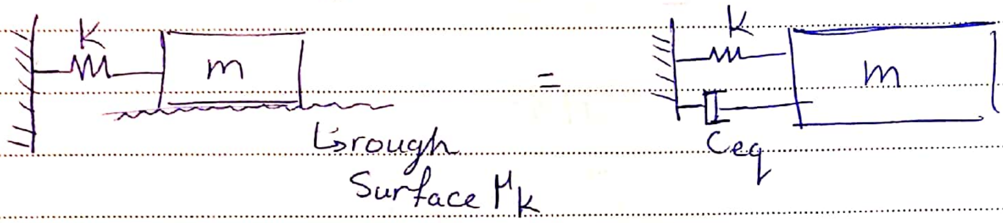
$F_d \propto \dot{x}$ (actually this is an assumption)

$F_d = C\dot{x}$
↳ damping coefficient

[2] Coloumb damping

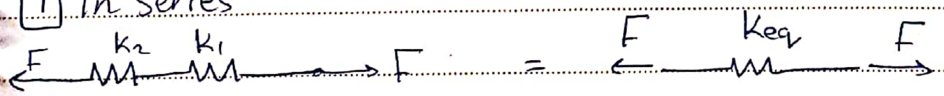


$C_{eq} = \dfrac{4\mu_k N}{\pi \omega X}$

μ_k = kinetic friction
N = Normal force
ω = Vibration frequency
X = amplitude of vibration

Note:
→The equivalent stiffness of a group of springs connected:

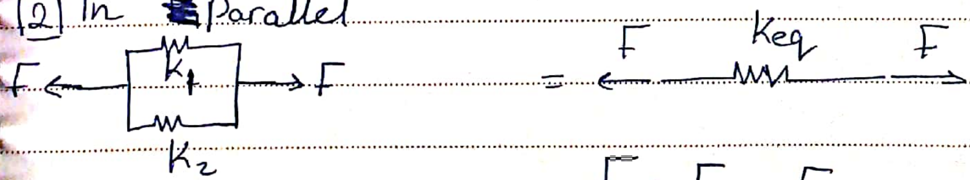
[1] In series



$\delta_{tot} = \delta_1 + \delta_2$

$\dfrac{\cancel{F}}{K_{eq}} = \dfrac{\cancel{F}}{k_1} + \dfrac{\cancel{F}}{k_2} \Rightarrow \dfrac{1}{K_{eq}} = \sum_{i=1}^{n} \dfrac{1}{k_i}$

[2] In Parallel



$F = F_1 + F_2$

$K_{eq}\cancel{\delta} = k_1\cancel{\delta} + k_2\cancel{\delta} \Rightarrow K_{eq} = \sum_{i=1}^{n} k_i$

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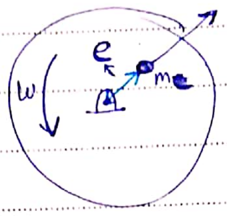
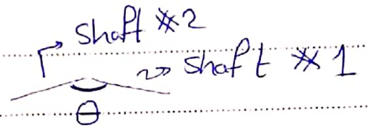
Sources of vibration:

1) Rotating unbalance

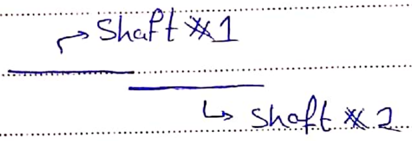


[2] Misalignment of two shafts

→ angular misalignment



→ parallel misalignment



[3] Bearings

[4] Mechanical loosness

[5] Cavitation

[6] Drag force

[7] Flow induced vibration

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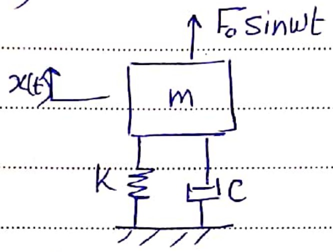
1 / 12 / Sunday

[*] Types of excitation: Mechanical vibration are caused by one of the following excitation types

1) Direct harmonic: a harmonic force is applied directly to the system (harmonic means sinusoidal)

[*] EoM:

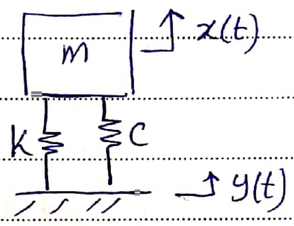
$$m\ddot{x} + c\dot{x} + kx = F_o \sin\omega t$$



2) Base excitation: the system is excited due to the oscillation of the base or the foundation of the system

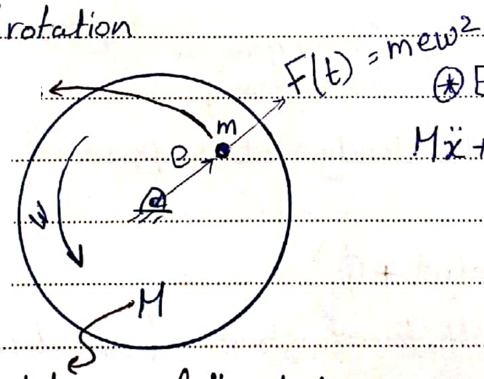
[*] EoM:

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$



3) Rotating unbalance: the system is excited due to the uneven distribution of the mass around the axis of rotation

This eccentric mass represents an irregularity in the mass distribution



total mass of the disk

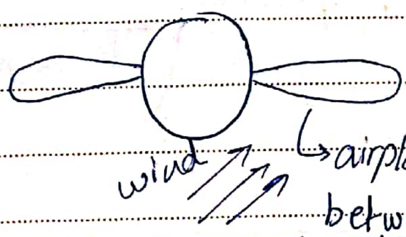
[*] EoM

$$M\ddot{x} + c\dot{x} + kx = me\omega^2 \sin\omega t$$

4) Parametric excitation (This type of excitation will not be covered in this course)

$$m\ddot{x} + c\dot{x} + (a + b\cos\Omega t)\,x = 0$$

5) Fluid structure interaction (will not be covered in this course)



airplane wing vibrates due to the interaction between the fluid and the solid

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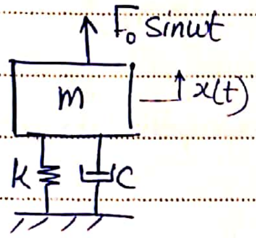
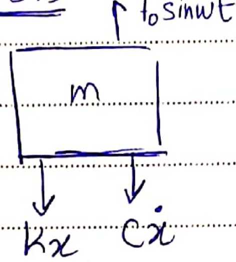
→ We will cover the first three types ~~in~~ of excitation in more detail.

$\boxed{1}$ Direct harmonic excitation



⇒

FBD



⇒ EoM

$+\uparrow \sum F = m\ddot{x}$

$F_o \sin\omega t - kx - c\dot{x} = m\ddot{x}$

$m\ddot{x} + c\dot{x} + kx = F_o \sin\omega t$

- The general solution of the EoM has two parts

$$x_{general}(t) = x_{homogeneous}(t) + x_{Particular}(t)$$

- $x_{homogeneous}(t)$ is found by equating the right hand side of the equation to zero (i.e $m\ddot{x} + c\dot{x} + kx = 0$). We are not intrested in $x_{homogeneous}(t)$ since as $t \to \infty$

$x_{homogeneous} \to$ zero (i.e. $x_{homogeneous}$ represents the transient response of the system)

- $x_{Particular}(t)$ represents the steady state response of the system,
- assume:

$$x_{Particular}(t) = X \sin(\omega t + \phi)$$

↳ amplitude of vibration (vibration level)

- Find $\dot{x}_{Particular}$ & $\ddot{x}_{Particular}$ and substitute them in the EoM

in order to find the value of X and ϕ:

$$X = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \quad ; \quad \phi = \tan^{-1}\left(\frac{c\omega}{k - \omega m^2}\right)$$

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$$\cdot X = \frac{F_0 \div K}{\sqrt{(K-m\omega^2)^2+(c\omega)^2} \div K} = \frac{F_0/K}{\sqrt{\left(1-\frac{m}{K}\omega^2\right)^2+\left(\frac{c}{K}\omega\right)^2}}$$

- $\frac{F_0}{K}$ is termed as static deflection "δ_{static}"

- $\omega_n = \sqrt{\frac{K}{m}} \Rightarrow \frac{m}{K} = \frac{1}{\omega_n^2}$

- $\frac{c}{K}\omega = \frac{c}{K}\frac{\sqrt{m}}{\sqrt{m}} * \frac{2}{2}\omega = \frac{c}{\sqrt{K}\sqrt{K}} * \frac{\sqrt{m}}{\sqrt{m}} * \frac{2}{2} * \omega = 2 * \frac{c}{2\sqrt{Km}} * \frac{\omega}{\omega_n}$

$\frac{\sqrt{m}}{\sqrt{K}} = \frac{1}{\omega_n}$ $\qquad = 2\zeta\frac{\omega}{\omega_n}$

- r = frequency ratio $= \frac{\omega}{\omega_n}$

$$\therefore X = \frac{\delta_{static}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

- MF = magnification factor $= \frac{X}{\delta_{static}} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$

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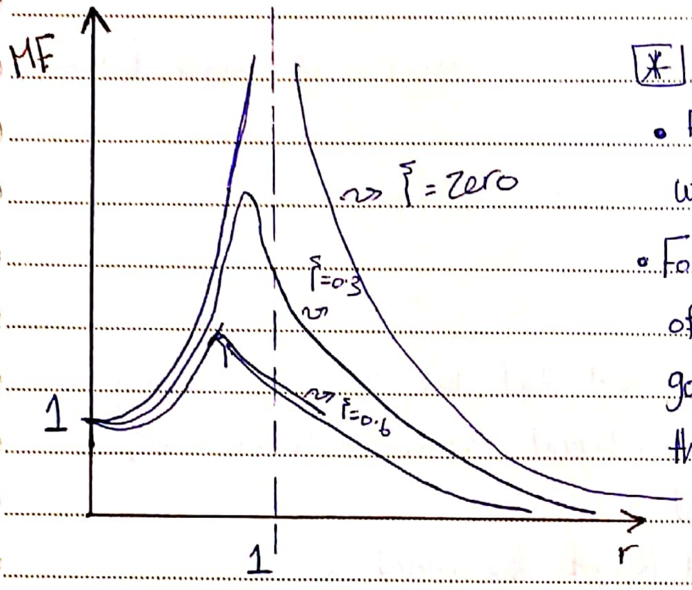
→ If the magnification factor is plotted against r for different values of ζ, the following curves would be obtained:



$\boxed{*}$ Observations :

- For $\zeta = 0$, the ~~mg~~ magnification factor $\to \infty$ when $r = 1$, i.e. $w = w_n$ (resonance condition)
- For $\zeta \neq 0$, the peak values are to the left of $r=1$ line. The higher the damping ratio goes, the further this peak value shifts to the left.

- To find the r^* value for which these curves are maximum, we have to take the derivative of HF with respect to r

$$\frac{dMF}{dr} = \frac{d}{dr}\left[\frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}\right] = 0$$

$$r^* = \frac{w^*}{w_n} = \sqrt{1-2\zeta^2}$$ ⇒ note: this relation is only valid for $\zeta < 0.707$

To find the maximum amplitude of vibration substitute r^* in MF

$$MF = \frac{X_{max}}{F_o/K} = \frac{1}{\sqrt{\left(1-(1-2\zeta^2)\right)^2 + \left(2\zeta\sqrt{1-2\zeta^2}\right)^2}} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$=$

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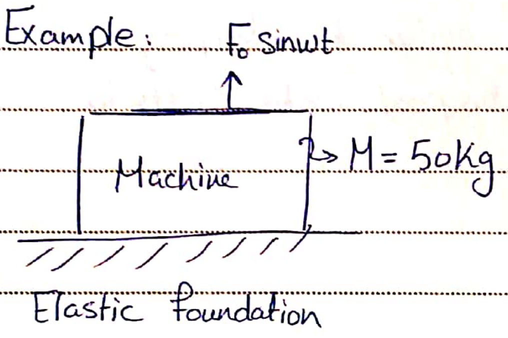
Based on the previous discussion, to control the vibration level, r must be magnified ~~s~~ beyond r^*. To increase r:

$r = \dfrac{\omega}{\omega_n \sim \sqrt{\frac{K}{m}}}$

⇝ increase ω (in most cases ω = rotational speed of the machine)

⇝ increase m or reduce K

Example:



It was observed that the maximum steady state vibration level = 3mm @ ω = 25 rad/s & F_0 = 75 N

Find C and K of the machine

Solution:

$$\frac{X_{max}}{F_0/K} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \rightsquigarrow \frac{3\times10^{-3}}{75/K} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\omega^* = \omega_n\sqrt{1-2\zeta^2}$$

$$25 = \sqrt{\frac{K}{50}}\sqrt{1-2\zeta^2} \Rightarrow K = \frac{25\times50}{(1-2\zeta^2)}$$

$$\frac{3\times10^{-3}}{75} = \frac{25\times50}{(1-2\zeta^2)} * \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$(1-2\zeta^2)(2\zeta)\sqrt{1-\zeta^2} = \frac{25\times50\times75}{3\times10^{-3}}$$

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Square both sides of the equation

$$(1-2\zeta^2)^2\, 4\zeta^2(1-\zeta^2) = \left(\frac{25 \times 50 \times 75}{3\times10^{-3}}\right)^2$$

$$(1-4\zeta^2+4\zeta^4)\,4\zeta^2(1-\zeta^2) = 9.765625 \times 10^{14}$$

$$(4\zeta^2-16\zeta^4+16\zeta^6)(1-\zeta^2) = 9.76\times10^{14}$$

$$4\zeta^2-16\zeta^4+16\zeta^6-4\zeta^4+16\zeta^6-16\zeta^8 = 9.76\times10^{14}$$

$$-16\zeta^8+32\zeta^6-20\zeta^4+4\zeta^2 = 9.76\times10^{14}$$

let $Z = \zeta^2$

$$-16Z^4+32Z^3-20Z^2+4Z = 9.76\times10^{14}$$

Find Z $\rightsquigarrow$ Find ζ $\rightsquigarrow$ find $K = \dfrac{25\times50}{1-\zeta^2}$ $\rightsquigarrow$ $\zeta = \dfrac{C}{2\sqrt{km}}$ find C

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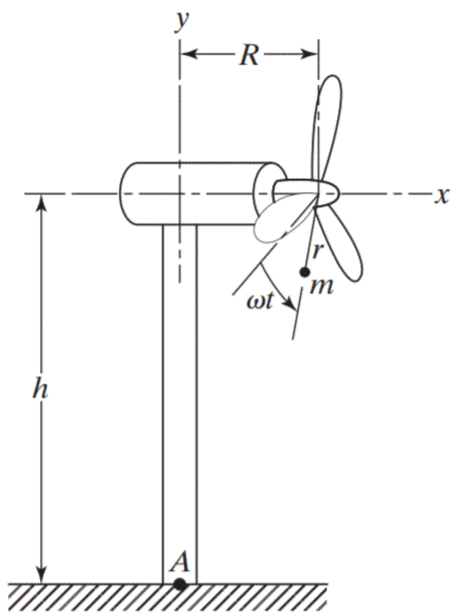
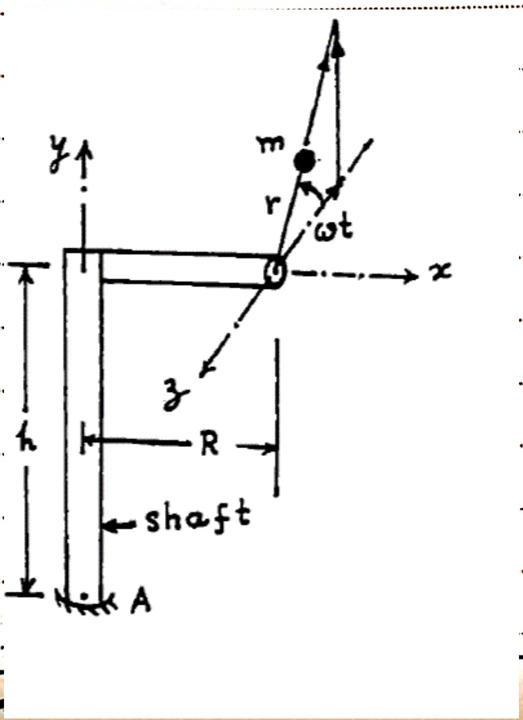
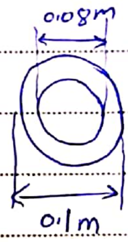
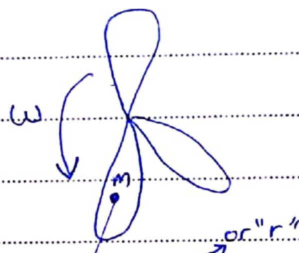
3-17 A three bladed wind turbine has a small unbalanced m located at a radius r in the plane of the blade. The blades are located from the central vertical y-axis at a distance R and rotate at an angular velocity of ω. If the supporting structure can be modeled as a hollow steel shaft of outer diameter of 0.1 m and inner diameter of 0.08 m, determine the maximum stresses developed at the base of the support (Point A). The mass moment of inertia of the turbine system about the vertical y axis is J_o. Assume $R = 0.5\ m$, $m = 0.1\ kg$, $r = 0.1\ m$, $J_o = 100\ kg.m^2$, $h = 8\ m$, $\omega = 31.416\ rad/s$.





Solution

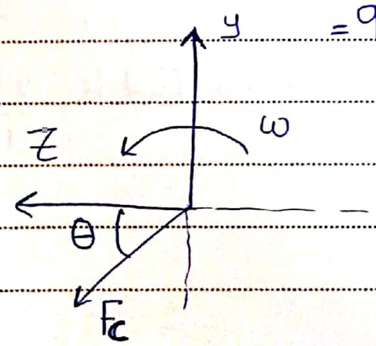




* $F_c = m e \omega^2$ (e or "r")

$= 0.1 \times 0.1 \times (31.416)^2$

$= 9.869\ N$



* $F_z = F_c \cos\theta$

$= 9.869 \cos\theta$

$= 9.869 \cos \omega t$

F_z causes torsion

$\text{Torque} = R \times F_z$

$= 0.5 \times 9.869 \cos \omega t$

$= 4.9348 \cos \omega t\ N.m$

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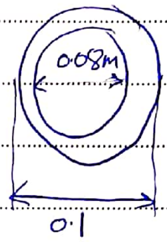
* The maximum torque occurs when θ = zero

$T_{max} = 4.93498$ N.m

* The torsional stress resulting from that torque:

$$\tau_a = \frac{T_{max} * C}{I_y} \; ; \quad C = \frac{0.1}{2} = 0.05\,m$$

$$I_y = \frac{\pi}{64}\left(0.1^4 - 0.08^4\right)$$



→ cross section of the shaft

$= 8.5138 \times 10^4\ N/m^2$

* $F_y = F_c \sin\theta = 9.869 \sin\theta = 9.869 \sin\omega t$

F_y causes bending moment

$M = F_y * R = 9.869 \times 0.5 \times \sin\omega t$

$= 4.9348 \sin\omega t$ N.m

* The maximum bending moment occurs when $\theta = 90^\circ$

$M_{max} = 4.93498$ N.m

* The resulting stress resulting from that moment

$$\sigma_a = \frac{M_{max} * C}{I_z} = \frac{4.93498 * \frac{0.1}{2}}{\frac{\pi}{32}\left(0.1^4 - 0.08^4\right)} = 4.2569 \times 10^4\ N/m^2$$

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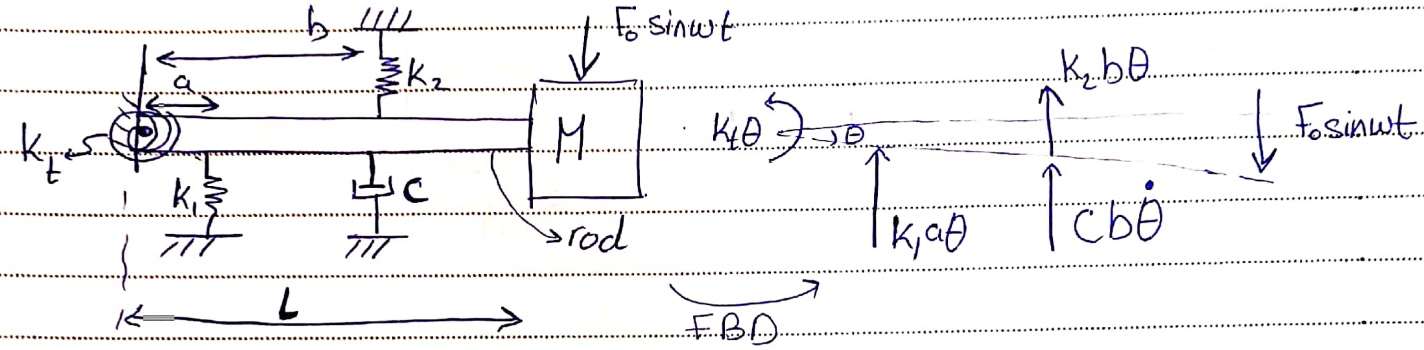
3-25 Derive the equation of motion and find the steady-state solution of the system shown below for rotational motion about the hinge O for the following data: $K = 5000\ N/m$, $L = 1\ m$, $m_{rod} = 10\ kg$, $a = 0.25\ m$, $b = 0.5\ m$, $C = 250\ N.S/m$, $M = 50\ kg$, $F_o = 500\ N$, $\omega = 1000\ rpm$, $K_t = 2\ kN/rad$



Solution:

$$\overset{+}{\circlearrowleft} \sum M_o = J\ddot{\theta}$$

$$-k_t\theta - k_1 a\theta * a - k_2 b\theta * (b) - Cb\dot{\theta}(b) + F_o L \sin\omega t = \left(\frac{1}{12} m_{rod} \ell^2 + m_{rod}\left(\frac{L}{2}\right)^2 + ML^2\right)\ddot{\theta}$$

$$\underbrace{\left(ML^2 + \frac{m_{rod} L^2}{3}\right)}_{J_{eq}}\ddot{\theta} + \underbrace{Cb^2}_{C_{eq}}\dot{\theta} + \underbrace{(k_t + k_1 a^2 + k_2 b^2)}_{k_{eq}}\theta = \underbrace{F_o L}_{F_{eq}} \sin\omega t$$

$$\omega_n = \sqrt{\frac{k_t + k_1 a^2 + k_2 b^2}{ML^2 + \frac{m_{rod} L^2}{3}}} = \sqrt{\frac{2000 + 5000(0.25^2 + 0.5^2)}{50 * 1 + \frac{10}{3}(1)^2}}$$

note: $k_1 = k_2 = K$

$$= 8.17\ rad/s$$

$$\zeta = \frac{C_{eq}}{2\sqrt{k_{eq} J_{eq}}} = \frac{250 * (0.5)^2}{2\sqrt{3562.5 * 53.33}} = 0.0717$$

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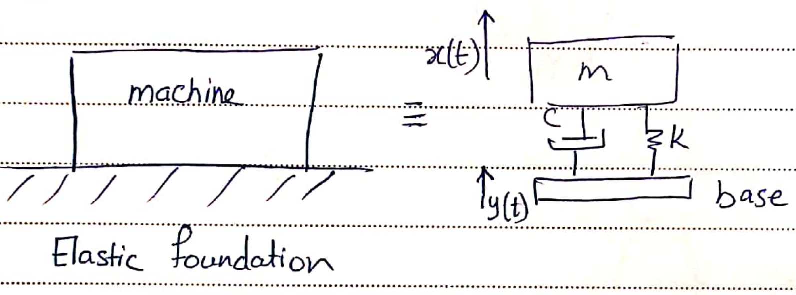
$$\theta_{ss} = \frac{F_{eq}}{\sqrt{(K_{eq} - J_{eq}\,\omega^2)^2 + (C_{eq}\,\omega)^2}} = \frac{500 * 1}{\sqrt{\left(3562.5 - 53.33 * \left(1000 * \frac{2\pi}{60}\right)^2\right)^2 + \left(250 * 0.5^2 * 1000 \times \frac{2\pi}{60}\right)^2}}$$

$= 0.86 \times 10^{-3}$ rad $= 0.049$ deg

5/12 Thursday

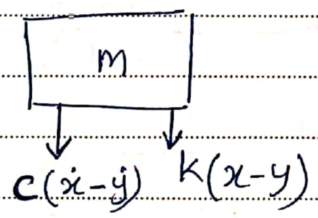
[*] Base excitation:



→ Now, Instead of having a force acting on the mass, the base that ~~this~~ the system is attached to is moving.

→ Our target is to find the response $x(t)$ of the system.

FBD:



EoM:

$+\uparrow \sum F = m\ddot{x}$

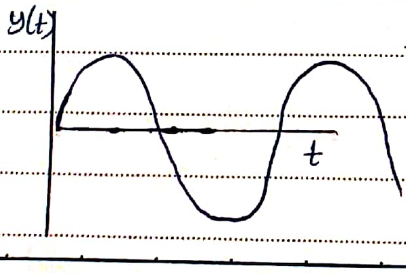
$-k(x-y) - c(\dot{x}-\dot{y}) = m\ddot{x}$

$\Downarrow$

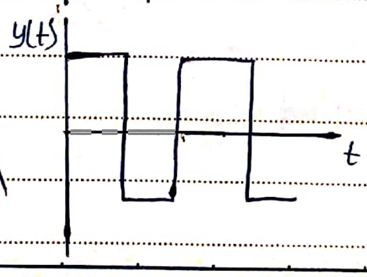
$m\ddot{x} + c\dot{x} + kx = ky + c\dot{y}$

→ $y(t)$, which represents the movement of the base, might be:

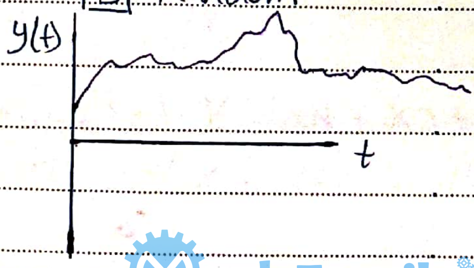
[1] harmonic



[2] Periodic



[3] Random



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→ Assuming harmonic base excitation i.e.

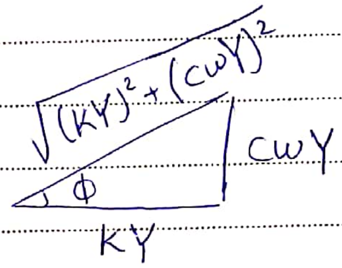
$$y(t) = Y \sin\omega t$$

$$\dot{y}(t) = Y\omega \cos\omega t$$

→ the EoM becomes:

$$m\ddot{x} + c\dot{x} + kx = kY\sin\omega t + c\omega Y\cos\omega t$$

Define angle ϕ, where



$$\cos\phi = \frac{kY}{\sqrt{(kY)^2+(c\omega Y)^2}}$$

$$\sin\phi = \frac{c\omega Y}{\sqrt{(kY)^2+(c\omega Y)^2}}$$

$$m\ddot{x} + c\dot{x} + kx = \left(kY\sin\omega t + c\omega Y\cos\omega t\right) * \frac{\sqrt{(kY)^2+(c\omega Y)^2}}{\sqrt{(kY)^2+(c\omega Y)^2}}$$

$$m\ddot{x} + c\dot{x} + kx = \left(\frac{kY}{\sqrt{(kY)^2+(c\omega Y)^2}}\sin\omega t + \frac{c\omega Y}{\sqrt{(kY)^2+(c\omega Y)^2}}\cos\omega t\right)\sqrt{(kY)^2+(c\omega Y)^2}$$

$$= \left(\cos\phi\sin\omega t + \sin\phi\cos\omega t\right)\sqrt{(kY)^2+(c\omega Y)^2}$$

$$= \sqrt{(kY)^2+(c\omega Y)^2}\,\sin(\omega t + \phi)$$

$$m\ddot{x} + c\dot{x} + kx = \underbrace{Y\sqrt{k^2+(c\omega)^2}}_{F_0}\sin(\omega t + \phi)$$

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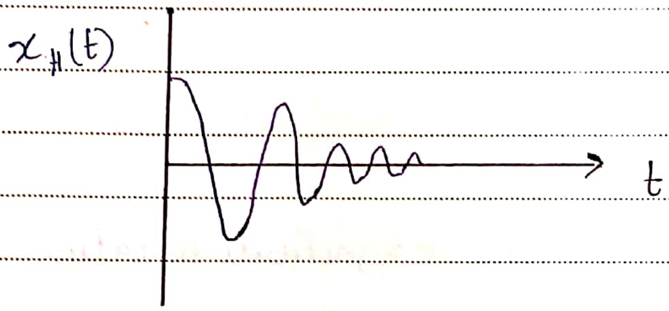
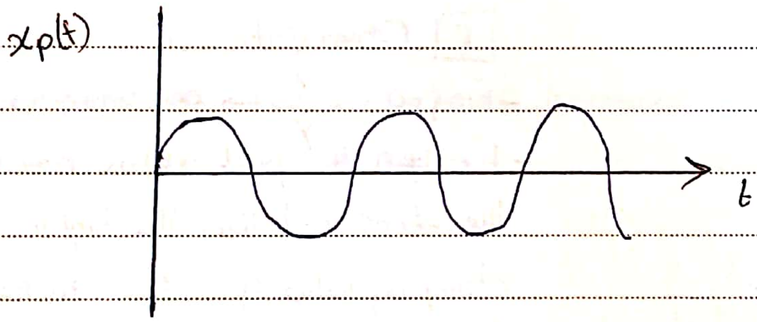
→ The system's response consists of 2 parts:

[1] Homogeneous (transient response)



⇒ We are not intrested in the transient response, since it will die after a certain period of time

[2] Particular (Steady state response)



Assume $x_p(t) = X \sin(\omega t + \beta)$

Recall from direct harmonic excitation lecture

$$X = \frac{F_0}{\sqrt{(K - m\omega^2)^2 + (c\omega)^2}}$$, Based on the EoM, $F_0 = Y\sqrt{k^2 + (c\omega)^2}$

$$\therefore X = \frac{Y\sqrt{k^2 + (c\omega)^2} \div K}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2} \div K} \Rightarrow X = \frac{Y\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

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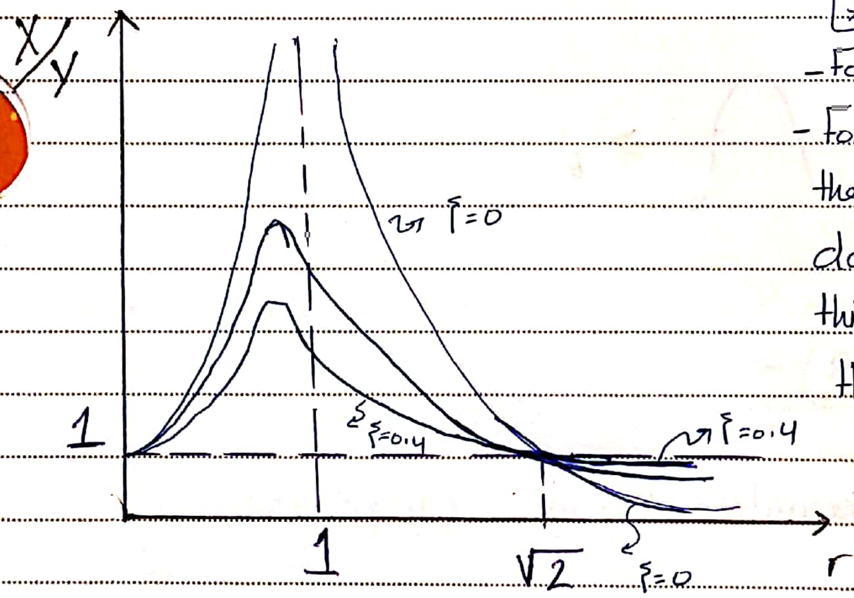
→ $\frac{X}{Y}$ = amplitude ratio = $\sqrt{\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}}$

→ X = amplitude of vibration of the mass "m"
Y = amplitude of vibration of the base

→ Sometimes, $\frac{X}{Y}$ is called transmissibility or magnification ratio

→ If $\frac{X}{Y}$ is plotted against r for different values of ξ, the following curves will be obtained :



☒ Observations:
- For $\xi=0$, $\frac{X}{Y} \rightarrow \infty$ when r = 1
- For $\xi \neq 0$, the peak values are to the left of r = 1 line. The higher the damping ratio goes, the further this peak value shifts to the left.

- To find the value r^* at which the peak value occurs, take the derivative of $\frac{X}{Y}$ with respect to r and equate it to zero

$$\frac{d}{dr}\left[\frac{X}{Y}\right]=0 \Rightarrow r^* = \frac{1}{2\xi} * \left[\sqrt{1+8\xi^2} - 1\right]^{1/2}$$

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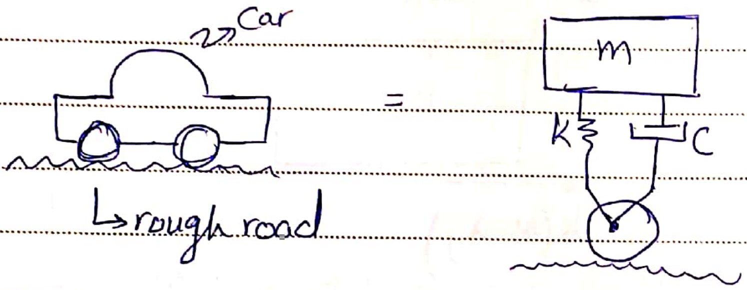
- For $r > \sqrt{2}$ (i.e. $\omega > \sqrt{2}\,\omega_n$) $\Rightarrow \frac{X}{Y} < 1$

In other words, the amplitude of vibration of the mass "m" becomes less than the amplitude of vibration of the base.

→ Real example of base excitation



[*] Transmissibility ratio in direct harmonic excitation:
Vibration isolation concept

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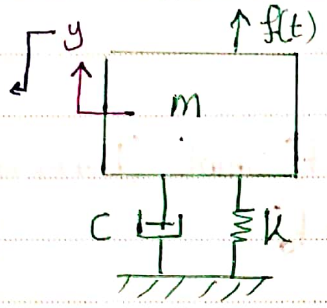
[4] Vibration Isolation

Recall from mechanical vibrations course:



represents the displacement experienced by the system

→ Consider a forced vibration problem:

• Assume:

- $f(t)$ is a harmonic force

- The system represents a rotating shaft.

i.e. m = mass of the shaft

c & k ≡ describe the support structure of the shaft (bearings)

→ Our goal is to reduce noise caused by vibration. To protect the system from this undesirable vibration, we have to study the effect of $f(t)$ on our system.

→ The most important parameter that we can identify which shows the effect of $f(t)$ on the system is TR = Transmissibility Ratio.

→ Hence, by understanding this parameter we can protect our system from undesirable vibration.

→ TR = is a quantity that measures the impact of a force on a system.

→ If a harmonic force is acting on the system, obviously, the force will cause a displacement in the system (i.e the mass m will move back & forth about its equilibrium position.

→ As a result of this displacement, the support structure (which is represented by the stiffness & damping elements) will be subjected to a force. This transmitted force to the support structure is given by

$$f_T = ky + c\dot{y} \quad (f_T = \text{transmitted force})$$

↳ The force experienced by the support structure (bearings) is designated by f_T. You have to know what force the support structure has to handle.

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→ Mathematically, TR is defined as follows:

$$TR = \frac{\text{Maximum force transmitted}}{\text{Maximum force applied}}$$

→ We are going to find these values

- The force applied on the system is $f(t) = F_o \sin \omega t$ (harmonic force)

$\therefore$ Max. force applied = Amplitude of $f(t) = F_o$

- Recall from vibrations course:

When a mass-spring-damper system is subjected to a harmonic force $f(t) = F_o \sin \omega t$, its displacement $y(t)$ is given by

$$y(t) = \frac{F_o}{K} * \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \sin(\omega t - \phi)$$

Substitute

$$\dot{y}(t) = \frac{F_o}{K} * \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} * \omega \cos(\omega t - \phi)$$

$$F_T = Ky + C\dot{y}$$

$$F_T = \underbrace{\cancel{K} * \frac{F_o}{\cancel{K}} * \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}}_{\text{Call it } A_1} \sin(\omega t - \phi) + \underbrace{C * \frac{F_o}{K} * \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} * \omega}_{\text{Call it } A_2} \cos(\omega t - \phi)$$

$$F_T = A_1 \sin(\omega t - \phi) + A_2 \cos(\omega t - \phi)$$

To find the maximum F_T, we will do the following trick

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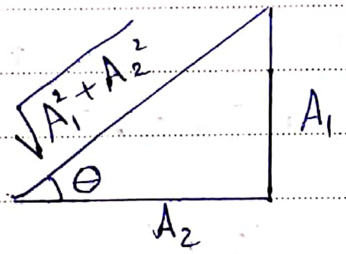
$$F_T = A_1 \sin(\omega t - \phi) + A_2 \cos(\omega t - \phi)$$

Define an angle Θ

where

$$\sin\Theta = \frac{A_1}{\sqrt{A_1^2 + A_2^2}}$$



$$\cos\Theta = \frac{A_2}{\sqrt{A_1^2 + A_2^2}}$$

do this multiplication

$$F_T = \left[A_1 \sin(\omega t - \phi) + A_2 \cos(\omega t - \phi)\right] * \frac{\sqrt{A_1^2 + A_2^2}}{\sqrt{A_1^2 + A_2^2}}$$

$$= \sqrt{A_1^2 + A_2} \left[\frac{A_1}{\sqrt{A_1^2 + A_2^2}} \sin(\omega t - \phi) + \frac{A_2}{\sqrt{A_1^2 + A_2^2}} \cos(\omega t - \phi)\right]$$

$$= \sqrt{A_1^2 + A_2^2} \left[\sin\Theta \sin(\omega t - \phi) + \cos\Theta \cos(\omega t - \phi)\right]$$

Recall from Calculus : $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

$$\therefore F_T = \sqrt{A_1^2 + A_2^2} \cos(\omega t - \phi - \theta)$$

Therefore, the max. transmitted force = amplitude of $F_T = \sqrt{A_1^2 + A_2^2}$

$$\therefore F_{T\,max} = \sqrt{A_1^2 + A_2^2} = \sqrt{\left(\frac{F_o}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}\right)^2 + \left(\frac{C * F_o * \omega}{K\sqrt{(1-r^2)^2 + (2\zeta r)^2}}\right)^2}$$

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$$F_{T\,max} = \sqrt{\left[\frac{F_o}{\sqrt{(1-r^2)^2+(2\xi r)^2}}\right]^2 * \left(1+\left(\frac{CW}{K}\right)^2\right)}$$

$$= \frac{F_o}{\sqrt{(1-r^2)^2+(2\xi r)^2}} * \sqrt{1+\left(\frac{CW}{K}\right)^2}$$

r^2, where r = frequency ratio

$$\frac{C^2W^2}{k^2} = \frac{C^2}{k^2} \frac{W^2}{W_n^2} * W_n^2$$

$$= \frac{C^2}{k^2} * r^2 * W_n^2$$

$$= \frac{C^2}{K^2} * r^2 * \frac{K}{m} * \frac{4}{4}$$

$$= \frac{C^2}{4Km} * r^2 * 4$$

$$= 4r^2\xi^2 = (2\xi r)^2$$

$$= \frac{F_o \sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\therefore TR = \frac{\text{max. transmitted force}}{\text{max. applied force}}$$

$$= \frac{\dfrac{\cancel{F_o} * \sqrt{(1+(2\xi r)^2)}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}}{\cancel{F_o}} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

We will plot it on the next page

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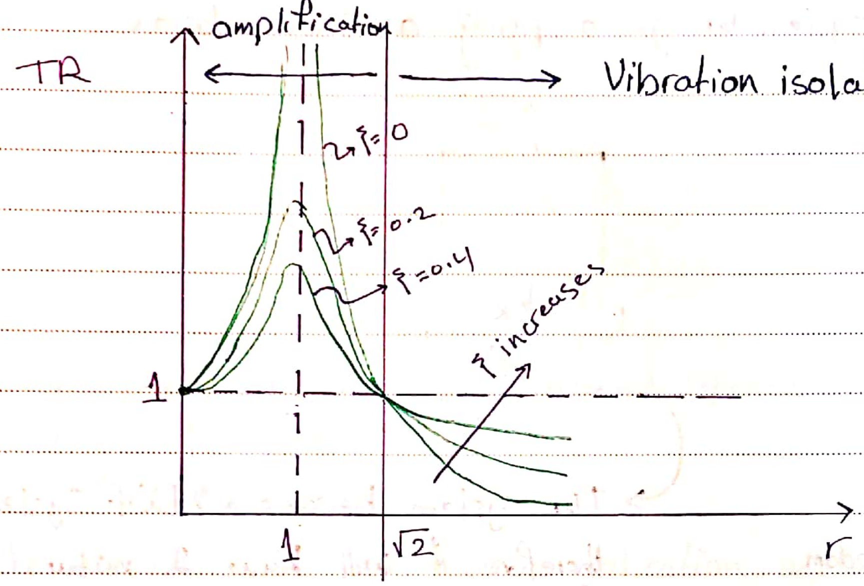
→ TR must be minimized to reduce the unpleasant effect of the forcing function $f(t)$ on the support structure of the system.

→ If TR is plotted against r for various values of ξ



→ To reduce TR : $r = \frac{\omega}{\omega_n}$ must be greater than $\sqrt{2}$ (Vibration isolation Zone)

$$r = \frac{\omega}{\omega_n} = \frac{\omega}{\sqrt{\frac{K}{m}}}$$

, We can increase r by:

1. Increasing ω ⇝ Sometimes we can't change the operating conditions of the system
2. Reducing K
3. Increasing m ⇝ Sometimes, adding additional mass to the system is not an effective solution, since the original system itself is heavy

→ Notice that for $r > \sqrt{2}$, as $\xi \uparrow$ TR gets larger, we can control C (damping coefficient) to reduce TR.

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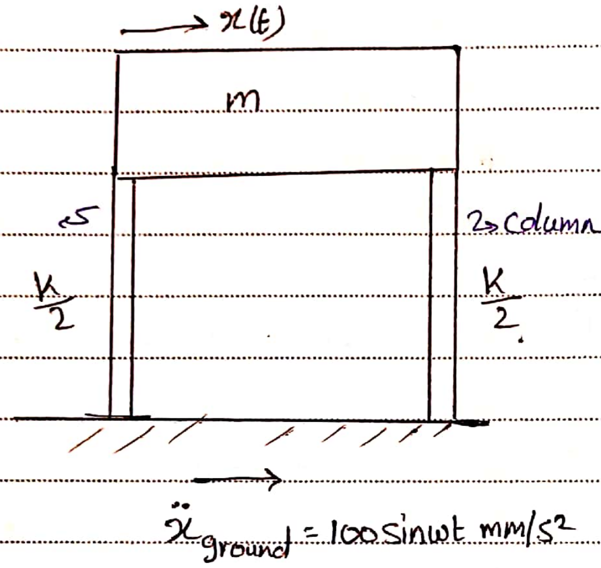
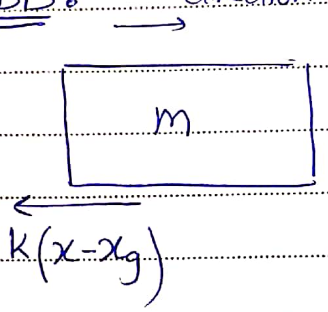
3-52 A single-story building frame is subjected to a harmonic ground acceleration, as shown below. Find the steady state motion of the floor. Given $m = 2000\,kg$, $K = 0.1\ MN/m$, $\omega = 25\,rad/s$

$\ddot{x}_g(t) = 100 \sin\omega t \ mm/s^2$



Solution:

FBD: direction of motion



EoM

$\overset{+}{\rightarrow} \sum F = m\ddot{x}$

$$-K(x - x_g) = m\ddot{x} \Rightarrow K x_g = m\ddot{x} + Kx$$

Assume

$$x_g = X_g \sin\omega t$$

$$\dot{x}_g = X_g \omega \cos\omega t$$

(ضربنا بـ 10^{-3} لأنه وحدة الـ $\ddot{x}_g$ هي $\frac{mm}{s^2}$)

$$\ddot{x}_g = -X_g \omega^2 \sin\omega t \Rightarrow X_g \omega^2 = 100 \Rightarrow X_g = \frac{100}{(25)^2} \times 10^{-3} = 1.6 \times 10^{-4}\ m$$

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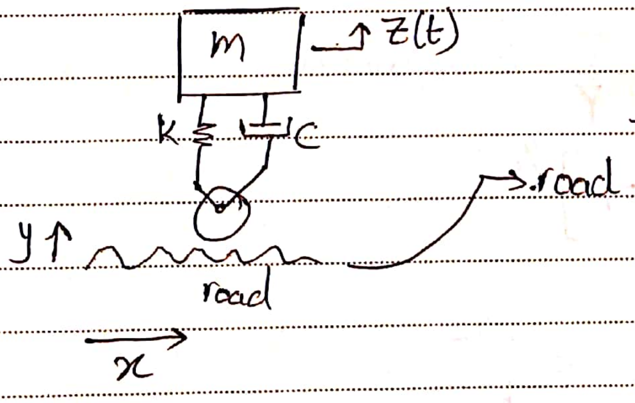
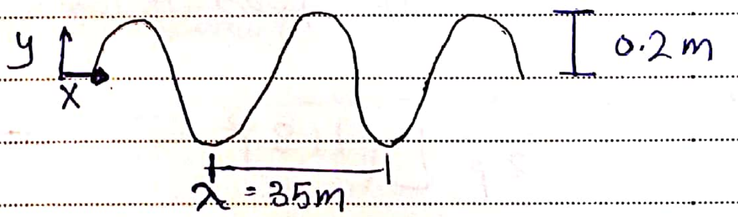
$$\frac{X}{X_g} = \sqrt{\frac{k^2 + \cancel{(c\omega)^2}^{\,zero}}{(k - m\omega^2)^2 + \cancel{(c\omega)^2}^{\,zero}}} \quad \text{"no damping"}$$

$$X = 1.6 \times 10^{-4} \sqrt{\frac{(0.1 \times 10^6)^2}{(0.1 \times 10^6 - 2000(25)^2)^2}} = 1.3913 \times 10^{-5} \text{ m}$$

3-55 - An automobile is modeled as a single degree of freedom system vibrating in the vertical direction. It is driven along a road whose elevation varies sinusoidally as shown below. If the natural frequency of the automobile is 2 Hz and the damping ratio of the shock absorbers is $\zeta = 0.15$, determine the amplitude of vibration of the automobile at a speed of 60 km/hr. If the speed of the automobile is varied Find the most unfavorable speed





$y = 0.2 \sin\theta$

1 cycle $\longrightarrow$ 35 m

2π rad $\longrightarrow$ $\lambda = 35$ m

θ rad $\longrightarrow$ X

$$\theta = \frac{2\pi}{35} * X \quad , \quad V = \frac{X}{t} \text{ (Car speed)}$$

$$\theta = \frac{2\pi}{35} * V * t$$

$$\therefore y(t) = 0.2 \sin\left(\frac{2\pi}{35} V t\right)$$

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The car speed $= 60 \frac{km}{hr} = \frac{60 \times 10^3}{3600} m/s = \frac{50}{3} m/s$

$\therefore y(t) = 0.2 \sin\left(\frac{2\pi}{35} \times \frac{50}{3} t\right) = 0.2 \sin(3t)$

$0.2 \rightarrow Y$; $\hookrightarrow w = 3 rad/s$

$f_n = 2 Hz \rightarrow w_n = 2\pi f_n = 2\pi * 2 = 12.566 \ rad/s$

$r = \frac{w}{w_n} = \frac{3}{12.566} = 0.2387$

$$\frac{Z}{Y} = \sqrt{\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}}$$

$$Z = 0.2 * \sqrt{\frac{1 + (2*0.15*0.2387)^2}{(1-0.2387^2)^2 + (2*0.15*0.2387)^2}} = 0.21 \ m$$

The most unfavorable speed is when $\frac{Z}{Y}$ is maximum

$$r^* = \frac{w^*}{w_n} = \frac{1}{2\zeta}\left[\sqrt{1+8\zeta^2} - 1\right]^{1/2}$$

$$w^* = 12.566 * \frac{1}{2*0.15}\left[\sqrt{1+8(0.15)^2} - 1\right]^{1/2}$$

$= 12.303 \ rad/s$

$w^* = \frac{2\pi}{35} * V_{unfavorable} \Rightarrow V_{unfavorable} = 68.535 \ m/s$

$= 246.72 \ Km/hr$

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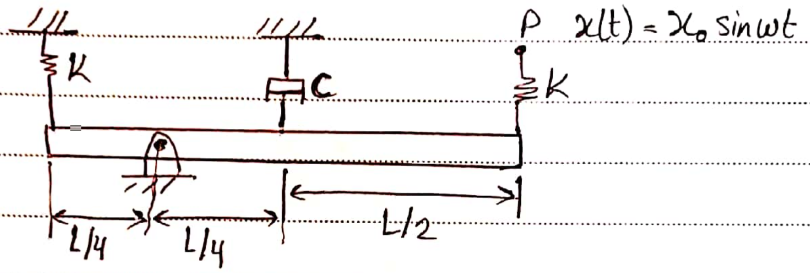
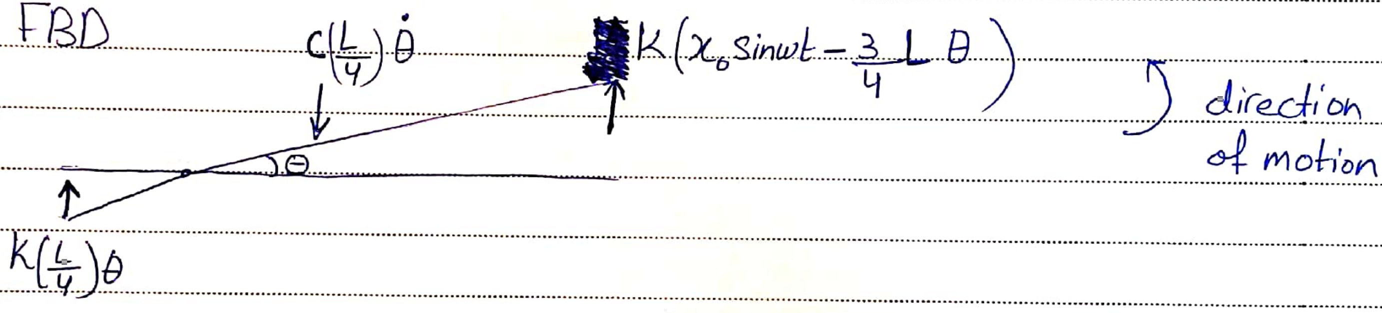
3-59 A uniform bar of mass m is pivoted at point O and supported at the ends by 2 springs. End P is subjected to a sinusoidal displacement $x(t) = x_0 \sin\omega t$. Find the steady state angular displacement of the bar



Solution:

FBD



$$\stackrel{+}{\circlearrowright} \sum M_o = J\ddot{\theta}$$

$$-K\left(\frac{L}{4}\right)\theta * \frac{L}{4} - C\left(\frac{L}{4}\right)\dot{\theta} * \frac{L}{4} + K\left(x_0 \sin\omega t - \frac{3}{4}L\theta\right) * \frac{3L}{4} =$$

$$\left(\frac{1}{12} mL^2 + m\left(\frac{L}{4}\right)^2\right)\ddot{\theta}$$

$$\left(\frac{7}{48} mL^2\right)\ddot{\theta} + \frac{CL^2}{16}\dot{\theta} + \left(\frac{KL^2}{16} + K\left(\frac{3}{4}\right)^2 L^2\right)\theta = Kx_0 * \frac{3L}{4} \sin\omega t$$

$$\underbrace{\frac{7}{48} mL^2}_{J_{eq}}\ddot{\theta} + \underbrace{\frac{CL^2}{16}}_{C_{eq}}\dot{\theta} + \underbrace{\frac{5}{8} KL^2}_{K_{eq}}\theta = \underbrace{Kx_0 * \frac{3L}{4}}_{F_{o\,eq}} \sin\omega t$$

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$$\theta_{ss} = \frac{F_{oeq}}{\sqrt{\left(K_{eq} - \cancel{m}\, J_{eq}\, w^2\right)^2 + \left(\cancel{\xi}\, C_{eq}\, w\right)^2}}$$

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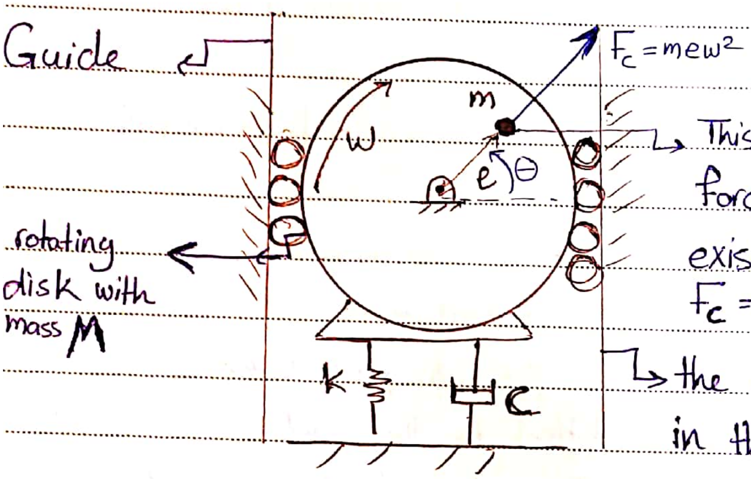
[*] Rotating unbalance

- A common Source of harmonic vibration is the uneven distribution of mass around an axis of rotation of a rotating machinery as shown below

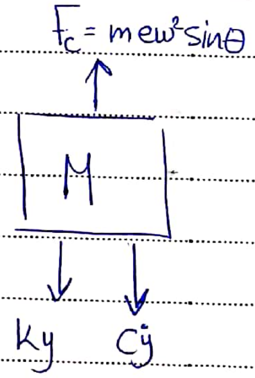


↳ This is an eccentric mass. A centrifugal force is induced ~~to the~~ due to the existance of that mass.

$F_c = m\frac{V^2}{e} = m\frac{(e\omega)^2}{e} = me\omega^2$

↳ the disk movement is blocked in the horizontal direction, It can only oscillate in the vertical direction

→ Applying Newton's 2nd law in the vertical direction



$+\uparrow \sum F_y = M\ddot{y}$; $-ky - c\dot{y} + me\omega^2\sin\theta = M\ddot{y}$

(*) $\omega = \frac{\theta}{t} \rightarrow \theta = \omega t$

$$M\ddot{y} + c\dot{y} + ky = \underbrace{me\omega^2}_{F_o} \sin \omega t$$

- The steady state Solution is $y_p(t) = Y_{ss}\sin(\omega t + \phi)$

$$\frac{M}{K} = \frac{1}{\omega_n^2} \; ; \; \frac{\omega^2}{\omega_n^2} = r^2$$

$$Y_{ss} = \frac{me\omega^2 \div K}{\sqrt{(K - M\omega^2)^2 + (c\omega)^2} \div K} = \frac{\frac{m}{K} e\,\omega^2 * \frac{M}{M}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$Y_{ss} = \frac{\frac{me}{M} r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

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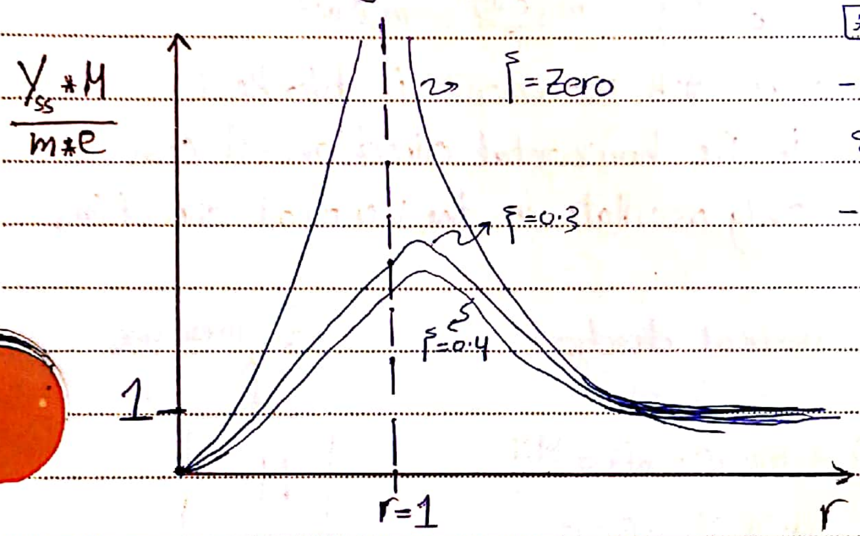
$$\frac{Y_{ss} * M}{me} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

; * Note:

M = mass of the rotating disk

m = eccentric mass

→ If we plot $\frac{Y_{ss} * M}{m * e}$ against r, for different values of ζ, we will obtain the following curves:



☐ Observations:

- For $\zeta \neq 0$, the peak value is shifted to the right.
- The value of r^* at which the peak value occurs is given by:

$$r^* = \frac{1}{\sqrt{1-2\zeta^2}}$$

- Hence $Y_{ss,max}$ is found by substituting the value of r^* in the equation shown at the top of the page

$$\frac{Y_{ss,max} * M}{m * e} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

→ Solve examples 3-6 & 3-7

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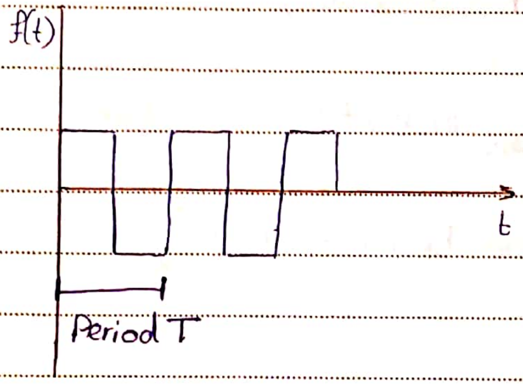
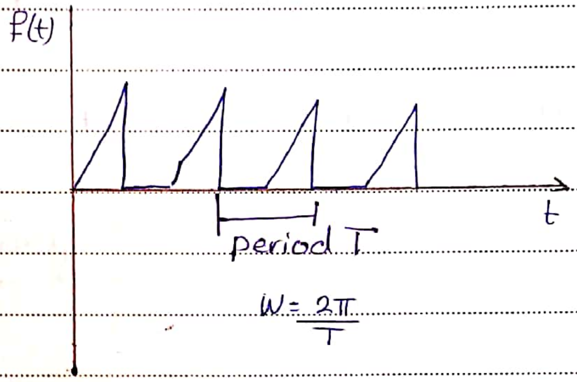
[*] Periodic excitation:

→ Sometimes, the excitation force is not harmonic (i.e. described by a single sinusoidal wave), but periodic.

e.g of periodic functions





→ In this case, we use Fourier Series to convert such functions into a series of sinusoidal functions

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

Periodic function

Converted into an infinite sum of sines and cosines

where a_0, a_n, and b_n are found as follows

$$a_0 = \frac{2}{T}\int_0^T F(t)\,dt \quad ; \; T \equiv \text{period} \; , \; F(t) = \text{periodic function}$$

$$a_n = \frac{2}{T}\int_0^T F(t)\cos(n\omega t)\,dt \quad ; \; n = 1, 2, 3, \ldots$$

$$b_n = \frac{2}{T}\int_0^T F(t)\sin(n\omega t)\,dt \quad ; \; n = 1, 2, 3, \ldots$$

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→ When we want to solve such these problems by hand, we can't make an infinite sum of sines & cosines, hence, based on the required accuracy, a certain number of sines and cosines terms will be considered:

$$F(t) \approx \frac{a_0}{2} + a_1 \cos(\omega t) + a_2 \cos(2\omega t) + \cdots + a_n \cos(n\omega t)$$
$$+ b_1 \sin(\omega t) + b_2 \sin(2\omega t) + \cdots + b_n \sin(n\omega t)$$

→ Each sine and cosine having the same frequency can be represented by a single sinusoidal function:

$$F(t) \approx \frac{a_0}{2} + \sqrt{a_1^2 + b_1^2}\, \sin(\omega t + \phi_1) + \sqrt{a_2^2 + b_2^2}\, \sin(2\omega t + \phi_2) + \cdots$$
$$+ \sqrt{a_n^2 + b_n^2}\, \sin(n\omega t + \phi_n)$$

→ The equation of motion of a mass spring damper system, subjected to a periodic excitation is given by:

$$m\ddot{x} + c\dot{x} + kx = \frac{a_0}{2} + \sqrt{a_1^2 + b_1^2}\, \sin(\omega t + \phi_1) + \sqrt{a_2^2 + b_2^2}\, \sin(2\omega t + \phi_2) + \cdots$$

→ The steady state solution of the above equation is found using the superposition principle:

P stands for Particular solution

$$m\ddot{x}_0 + c\dot{x}_0 + kx_0 = \frac{a_0}{2} \Rightarrow \text{Find } x_{o,p}(t)$$

$$m\ddot{x}_1 + c\dot{x}_1 + kx_1 = \sqrt{a_1^2 + b_1^2}\, \sin(\omega t + \phi_1) \Rightarrow \text{Find } x_{1,p}(t)$$

$$m\ddot{x}_2 + c\dot{x}_2 + kx_2 = \sqrt{a_2^2 + b_2^2}\, \sin(2\omega t + \phi_2) \Rightarrow \text{Find } x_{2,p}(t)$$

$$\vdots$$

$$m\ddot{x}_n + c\dot{x}_n + kx_n = \sqrt{a_n^2 + b_n^2}\, \sin(n\omega t + \phi_n) \Rightarrow \text{Find } x_{n,p}(t)$$

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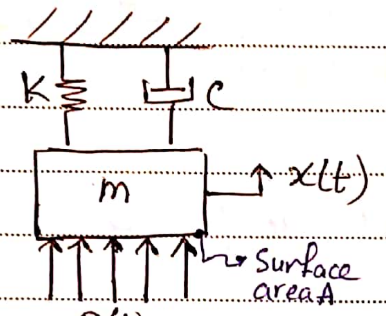
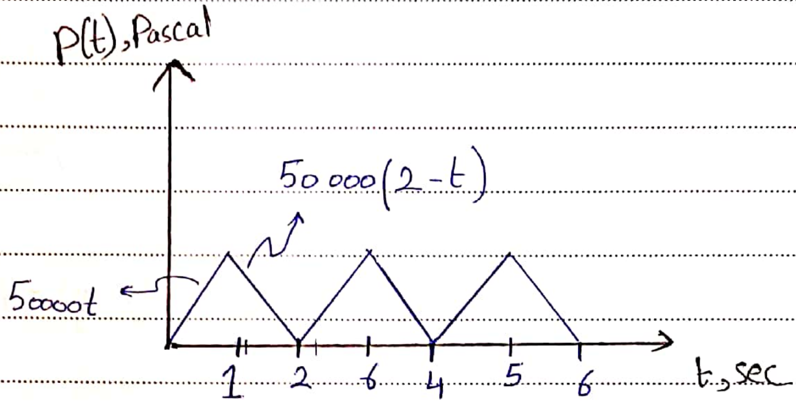
$\therefore x(t) \equiv$ Total steady state solution $= x_{0,p}(t) + x_{1,p}(t) + x_{2,p}(t) + \cdots + x_{n,p}(t)$

Example (4-4):





Period = T = 2 sec

Find the steady state response

Solution:

The acting force is found by multiplying the pressure by the area of the block A

$$F(t) = P(t) * A \quad ; \quad P(t) = \begin{cases} 50000t, & 0 \le t \le 1 \\ 50000(2-t), & 1 \le t \le 2 \end{cases}$$

$$a_0 = \frac{2}{T}\int_0^T F(t)\,dt \; ; \; a_0 = \frac{2}{2}\left[A\int_0^1 50.000t\,dt + A\int_1^2 50.000(2-t)\,dt\right]$$

$$= A\left(\frac{50.000t^2}{2}\right)\Bigg]_0^1 + \left(A\,50.000\left(2t - \frac{t^2}{2}\right)\right)\Bigg]_1^2$$

$$= A * \frac{50.000}{2} + 2 * A * 50000 - 1.5 * A * 50.000$$

$$= A * 50.000$$

$$a_1 = \frac{2}{2}\left[A\int_0^1 50000\,t\,\cos\omega t\,dt + A\int_1^2 50000(2-t)\cos\omega t\,dt\right] \quad ; \omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

This integration could by found using integration by Parts,
(In the test I will use the calculator to find the value of the integration, but don't forget to change the angle settings from degrees to radians)

$$a_1 = \frac{-2\times10^5 A}{\pi^2}$$

$$b_1 = \frac{2}{2}\left[A\int_0^1 50000\sin(\pi t)\,dt + A\int_1^2 50000(2-t)\sin(\pi t)\,dt\right] = 0$$

$$a_2 = \frac{2}{2}\left[A\int_0^1 50.000\cos(2\pi t)\,dt + A\int_1^2 50.000(2-t)\cos(2\pi t)\,dt\right] = 0$$

$$a_3 = \frac{2}{2}\left[A\int_0^1 50000\cos(3\pi t)\,dt + A\int_1^2 50.000(2-t)\cos(3\pi t)\,dt\right]$$

$$= \frac{-2\times10^5 A}{(3\pi)^2}$$

* All values of $b_1, b_2, \dots b_n$ are reduced to zero, because $F(t)$ is an even function
* If $F(t)$ is an odd function, all values of $a_1, a_2, a_3, \dots$ will be reduced to zero

$$\therefore F(t) \sim \underset{\frac{a_0}{2}}{25000A} - \underbrace{\frac{2\times10^5 A\cos\omega t}{\pi^2}}_{f_0} - \underbrace{\frac{2\times10^5 A\cos(3\omega t)}{(3\pi)^2}}_{f_0}$$

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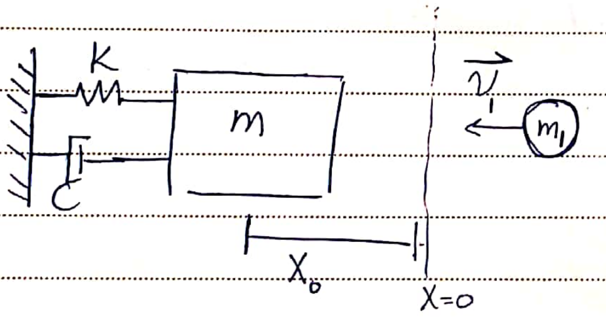
The steady state response of the system is given by:

$$x_p(t) = \frac{25000A}{k} - \frac{\left(2\times10^5 A/(k\pi^2)\right)}{\sqrt{(1-r^2)^2+(2\xi r)^2}} * \cos(\omega t - \phi_1)$$

$$- \frac{\left(2\times10^5 A/(k(3\pi)^2)\right)}{\sqrt{(1-(3r)^2)^2+(2\xi(3r))^2}} * \cos(3\omega t - \phi_1)$$

→ Consider the following free damped vibration problem



$$m\ddot{x} + c\dot{x} + kx = 0$$

- The mass m is initially displaced (i.e. $x(t=0) = x_o$), and a ball of mass m_1 travelling horizontally towards the block m is going to strike that block;

$\dot{x}(t=0) = v_o$

↳ the velocity of m after the impact; v_o is found using the conservation of momentum principle:

$$m_1 v_1 + m\underset{\text{zero}}{v} = m_1 \underset{\text{the speed of } m_1 \text{ after the impact}}{v_2} + m v_o \Rightarrow v_o = \frac{m_1(v_1 - v_2)}{m}$$

zero ↓ the block is initially at rest

$$x(t) = e^{-\xi\omega_n t}\left(A\cos\omega_d t + B\sin\omega_d t\right) \quad ; \omega_d = \omega_n\sqrt{1-\xi^2}$$

= damped natural frequency

↳ Found using the initial conditions $x(0)$, $\dot{x}(0)$

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$$x(t=0) = 1\left(A*1+0\right) = x_0 \Rightarrow A = x_0$$

$$\dot{x}(t) = e^{-\zeta\omega_n t}\left(-A\,\omega_d \sin\omega_d t + B\,\omega_d \cos\omega_d t\right) + (-\zeta\omega_n)\,e^{-\zeta\omega_n t}\left(A\cos\omega_d t + B\sin\omega_d t\right)$$

$$\dot{x}(t=0) = 1*\left(B\,\omega_d\right) + (-\zeta\omega_n)(A) = V_0$$

$$B = \frac{V_0 + \zeta\omega_n A}{\omega_d} = \frac{V_0 + \zeta\omega_n x_0}{\omega_d}$$

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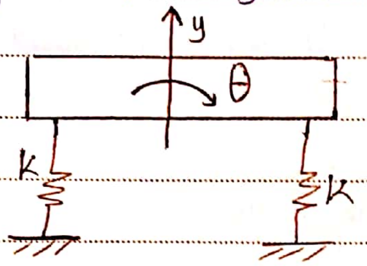
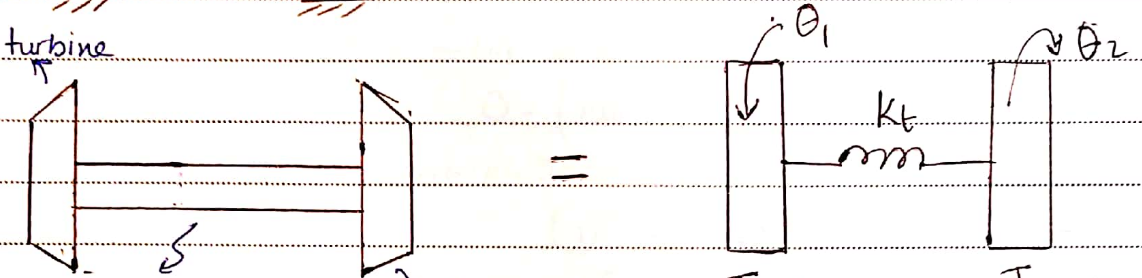
[*] 2 DOF systems

→ In this lecture, we will extend our discussion to 2 DOF systems.

e.g. of 2 DOF systems:

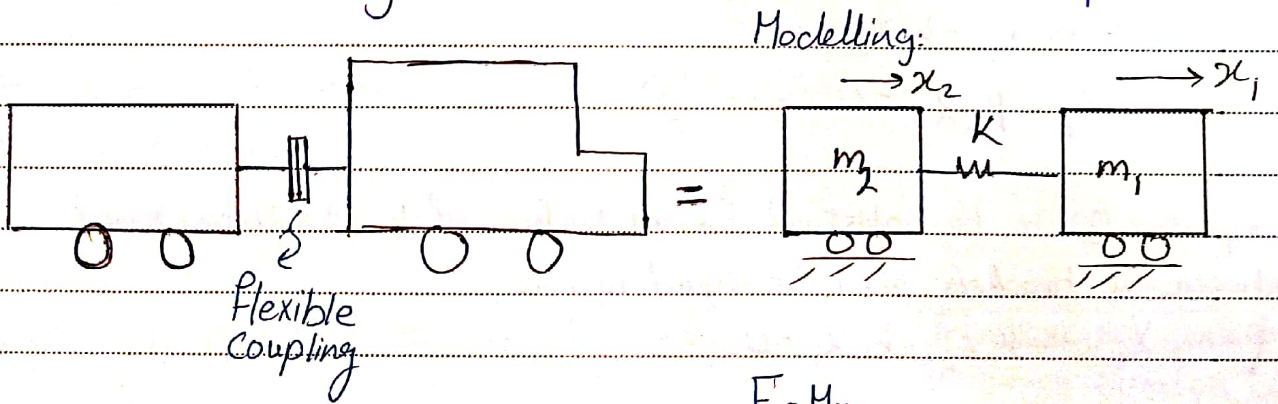




Turbocharger of a car

→ In 2 DoF systems, 2 EoM can be constructed and 2 natural ~~frequen~~ frequencies are possessed by those systems

→ Consider the following 2 DOF, and find their natural frequencies



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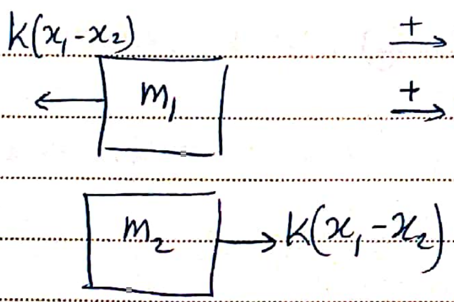

EoMs:

$\xrightarrow{+} \Sigma F = m_1 \ddot{x}_1 \; ; \; -K(x_1-x_2) = m_1 \ddot{x}_1$

$\xrightarrow{+} \Sigma F = m_2 \ddot{x}_2 \; ; \; K(x_1-x_2) = m_2 \ddot{x}_2$

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EoMs:

$$m_1 \ddot{x}_1 + k_1 x_1 - k x_2 = 0$$
$$m_2 \ddot{x}_2 + k x_2 - k x_1 = 0$$

assume

$x_1 = X_1 \sin\omega t$ $\quad$ $x_2 = X_2 \sin\omega t$

$\dot{x}_1 = X_1 \omega \cos\omega t$ $\quad$ $\dot{x}_2 = X_2 \omega \cos\omega t$

$\ddot{x}_1 = -X_1 \omega^2 \sin\omega t$ $\quad$ $\ddot{x}_2 = -X_2 \omega^2 \sin\omega t$

Substitute these in the EoMs

- $m_1(-X_1 \omega^2 \sin\omega t) + k(X_1 \sin\omega t) - k(X_2 \sin\omega t) = 0$

$(-m_1 X_1 \omega^2 + k X_1 - k X_2) \sin\omega t = 0$ --- (1)

- $m_2(-X_2 \omega^2 \sin\omega t) + k(X_2 \sin\omega t) - k(X_1 \sin\omega t) = 0$

$(-m_2 X_2 \omega^2 + k X_2 - k X_1) \sin\omega t = 0$ --- (2)

- For equation (1) to be satisfied for all values of t, the terms found between the brackets must be equal to zero

$-m_1 X_1 \omega^2 + k X_1 - k X_2 = 0$

$(k - m_1 \omega^2) X_1 - k X_2 = 0$ --- (3)

- For equation (2) to be satisfied for all values of t, the terms found between the brackets must be equal to zero

$-m_2 X_2 \omega^2 + k X_2 - k X_1 = 0$

$(k - m_2 \omega^2) X_2 - k X_1 = 0$ --- (4)

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- Eqs ③ and ④ can be set in matrix form as shown below:

$$\underbrace{\begin{bmatrix} k-m_1\omega^2 & -k \\ -k & k-m_2\omega^2 \end{bmatrix}}_{\text{call it matrix "A"}} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- Recall from Engineering mathematics 2:

- if the determinant of a matrix ≠ zero, ⇒ We can find the inverse of that matrix
- If the determinant of a matrix = zero ⇒ the matrix has no inverse matrix

- Based on the above statements, if the det(A) ≠ zero,

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} k-m_1\omega^2 & -k \\ -k & k-m_1\omega^2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This will yield: $X_1 = 0$, $X_2 = 0$ ⇒ absurd!

مش منطقة لانه كأنه عم يحكي انه مافي ولا في حركة vibration

- Hence, det(A) must be set to be equal to zero

$$\det(A) = (k - m_1\omega^2)(k - m_2\omega^2) - k^2 = 0$$

→ This equation is called frequency equation, since the values of the natural frequency are obtained from this equation

$$\cancel{k^2} - k m_2\omega^2 - k m_1\omega^2 + m_1 m_2\omega^4 \cancel{- k^2} = 0$$

$$\omega^2(-k m_2 - k m_1 + m_1 m_2\omega^2) = 0$$

$$\omega = 0 \qquad \omega = \sqrt{\frac{k(m_2+m_1)}{m_1 m_2}}$$

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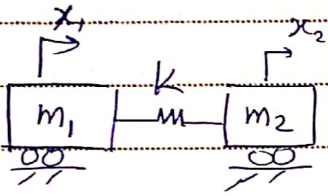
- The system has 2 natural frequencies

$$\omega_1 = 0 \quad , \quad \omega_2 = \sqrt{\frac{k(m_1+m_2)}{m_1 m_2}}$$

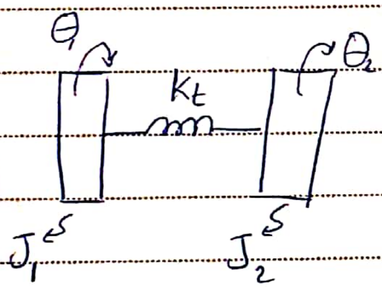
$\Downarrow$

zero frequency means that both masses m_1 and m_2 translate with no oscillation (i.e. They move as a single rigid body)

- Such systems, which have zero frequency, are called semidefinite systems

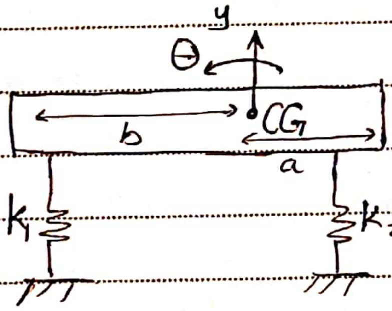
- 

$$\Rightarrow \quad \omega_1 = 0 \quad , \quad \omega_2 = \sqrt{\frac{k(m_1+m_2)}{m_1 m_2}}$$



$$\Rightarrow \quad \omega_1 = 0 \quad , \quad \omega_2 = \sqrt{\frac{k_t(J_1+J_2)}{J_1 J_2}}$$

$\rightarrow$ Consider the following 2 DOF system and Find their natural frequencies



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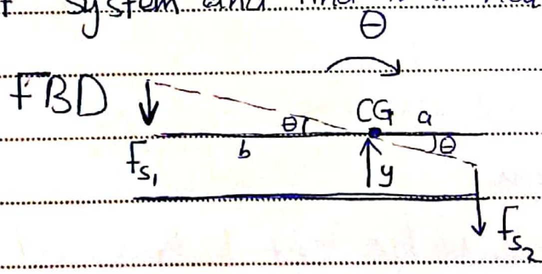

$$f_{s_1} = k_1(y+b\theta) \quad , \quad f_{s_2} = k_2(y-a\theta)$$

EoM:

$$+\uparrow \sum F_y = m\ddot{y} \quad ; \quad -k_1(y+b\theta) - k_2(y-a\theta) = m\ddot{y}$$

$$\circlearrowleft^{+} \sum M_{CG} = J\ddot{\theta} \quad ; \quad -k_1(y+b\theta)b + k_2(y-a\theta)a = J\ddot{\theta}$$

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$$-k_1 y - k_1 b\theta - k_2 y + k_2 a\theta = m\ddot{y}$$

$$m\ddot{y} + (k_1+k_2)y - (k_2 a - k_1 b)\theta = 0 \quad --- (1)$$

$$-bk_1 y - k_1 b^2\theta + k_2 a y - k_2 a^2\theta = J\ddot{\theta}$$

$$J\ddot{\theta} + (k_1 b^2 + k_2 a^2)\theta - (k_2 a - bk_1)y = 0 \quad --- (2)$$

• assume

$y = Y \sin\omega t \quad , \theta = \Theta \sin\omega t$

• Eqs 1 and 2 becomes

$$-mY\omega^2 + (k_1+k_2)Y - (k_2 a - k_1 b)\Theta = 0$$

$$-J\Theta\omega^2 + (k_1 b^2 + k_2 a^2)\Theta - (k_2 a - k_1 b)Y = 0$$

$\Downarrow$ matrix form

$$\underbrace{\begin{bmatrix} -m\omega^2 + (k_1+k_2) & -(k_2 a - k_1 b) \\ -(k_2 a - k_1 b) & -J\omega^2 + (k_1 b^2 + k_2 a^2) \end{bmatrix}}_{\text{call it matrix "A"}} \begin{bmatrix} Y \\ \Theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

• assume $m = 50\,kg$, $J = 25\,kg\cdot m^2$, $a = 0.25$, $b = 0.75$, $k_1 = 2\times10^4\,N/m$
$k_2 = 3\times10^4\,N/m$

$$\det(A) = (-50\omega^2 + 5\times10^4)(-25\omega^2 + 13125) - 5625\times10^4 = 0$$

$$1250\omega^4 - 656250\omega^2 - 1250000\omega^2 + 65625\times10^4 - 5625\times10^4 = 0$$

$$1250\omega^4 - 1906250\omega^2 + 6\times10^8 = 0$$

$\omega_2^2 = 1080.94 \qquad \omega_1^2 = 444.05 \Rightarrow \omega_1 = 21.07\ rad/s \ , \omega_2 = 32.87\ rad/s$

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The solution becomes

$y(t) = Y_1 \sin\omega_1 t + Y_2 \sin\omega_2 t$

$\theta(t) = \Theta_1 \sin\omega_1 t + \Theta_2 \sin\omega_2 t$

$Y_1, Y_2, \Theta_1, \Theta_2$ are found using the given initial conditions

$y(0), \dot{y}(0), \theta(0), \dot{\theta}(0)$

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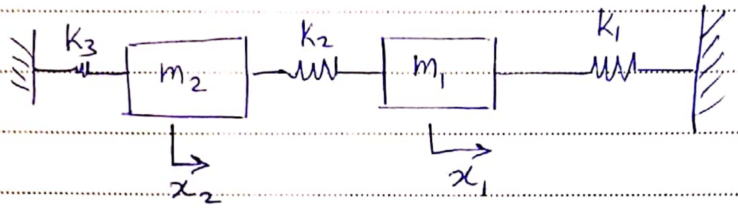
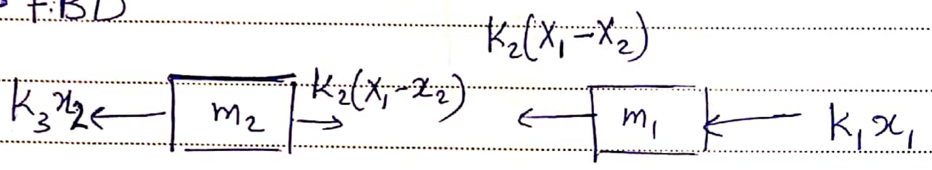
[*] Continue 2 DOF systems + Mode shapes

$m_1 = m_2 = m$
$k_1 = k_2 = k_3 = k$



→ Consider the 2 DOF system shown above; Find the EoMs + natural frequencies + mode shapes.

- FBD



- EoM for m_1

$\xrightarrow{+} \Sigma F = m_1 \ddot{x}_1$; $-k_1 x_1 - k_2(x_1 - x_2) = m_1 \ddot{x}_1$; $k_1 = k_2 = k_3 = k$

$-k x_1 - k x_1 + k x_2 = m_1 \ddot{x}_1$; $m_1 = m_2$

$m\ddot{x}_1 + 2k x_1 - k x_2 = 0$ --- (1)

- EoM for m_2

$\xrightarrow{+} \Sigma F = m_2 \ddot{x}_2$; $k_2(x_1 - x_2) - k_3 x_2 = m_2 \ddot{x}_2$; $k_1 = k_2 = k_3 = k$
$m_1 = m_2$

$m\ddot{x}_2 + 2k x_2 - k x_1 = 0$ — (2)

- Assume

$x_1(t) = X_1 \sin(\omega t + \phi)$
$x_2(t) = X_2 \sin(\omega t + \phi)$

- Eqs (1) and (2) becomes as follows

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$-m\omega^2 X_1 + 2kX_1 - kX_2 = 0 \longrightarrow (2k - m\omega^2)X_1 - kX_2 = 0$

$-m\omega^2 X_2 + 2kX_2 - kX_1 = 0 \longrightarrow (2k - m\omega^2)X_2 - kX_1 = 0$

→ • Matrix form:

$$\begin{bmatrix} 2k - m\omega^2 & -k \\ -k & 2k - m\omega^2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

• Construct the frequency equation

$(2k - m\omega^2)^2 - k^2 = 0$

$4k^2 - 4km\omega^2 + m^2\omega^4 - k^2 = 0$

$m^2\omega^4 - 4km\omega^2 + 3k^2 = 0$

$$\omega^2 = \frac{-(-4km) \pm \sqrt{(-4km)^2 - 4 * m^2 * 3k^2}}{2m^2}$$

$$= \frac{4km \pm \sqrt{16k^2m^2 - 12k^2m^2}}{2m^2} = \frac{4km \pm 2km}{2m^2}$$

$\therefore \omega_1^2 = \frac{4km - 2km}{2m^2} = \frac{k}{m}$ or $\omega_2^2 = \frac{4km + 2km}{2m^2} = \frac{3k}{m}$

$\omega_1 = \sqrt{\frac{k}{m}}$ $\qquad \omega_2 = \sqrt{\frac{3k}{m}}$ ⇒ These are the natural freq. of the system

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- A mode shape is a specific pattern of vibration ~~exect~~ executed by the system.
- To find the mode shape associated with the first natural frequency of the system $\omega_1 = \sqrt{\frac{K}{m}}$, take the 1st equation in the matrix and find the ratio of $\frac{X_1}{X_2}$:

$$(2k - m\omega^2) X_1 - K X_2 = 0 \Rightarrow \left(2K - m * \frac{K}{m}\right) X_1 - K X_2 = 0$$

$\sqrt{\frac{K}{m}}$

indicates 1st mode shape $\quad \frac{X_1^{(1)}}{X_2^{(1)}} = 1 \Rightarrow$ This indicates that both masses oscillate with the same amplitude when they oscillate at $\omega = \sqrt{K/m}$

- The mode shape associated with the 2nd natural frequency of the system $\omega_2 = \sqrt{\frac{3K}{m}}$ is found in the same manner

$$(2k - m\omega^2) X_1 - k X_2 = 0 \Rightarrow \left(2k - m\left(\frac{3K}{m}\right)\right) X_1 - k X_2 = 0$$

$\sqrt{\frac{3k}{M}}$

$\frac{X_1^{(2)}}{X_2^{(2)}} = -1 \Rightarrow$ This indicates that both masses will oscillate with the same amplitude but in opposite directions when they vibrate at $\omega = \sqrt{\frac{3k}{m}}$

- The general solution of $x_1(t)$ and $x_2(t)$:

$$* \; x_1(t) = X_1^{(1)} \sin\left(\sqrt{\frac{K}{m}}\, t + \phi_1\right) + X_1^{(2)} \sin\left(\sqrt{\frac{3k}{m}}\, t + \phi_2\right)$$

$$* \; x_2(t) = X_2^{(1)} \sin\left(\sqrt{\frac{K}{m}}\, t + \phi_1\right) + X_2^{(2)} \sin\left(\sqrt{\frac{3k}{m}}\, t + \phi_2\right)$$

$$= -X_1^{(1)} \sin\left(\sqrt{\frac{K}{m}}\, t + \phi_1\right) - X_1^{(2)} \sin\left(\sqrt{\frac{3K}{m}}\, t + \phi_2\right)$$

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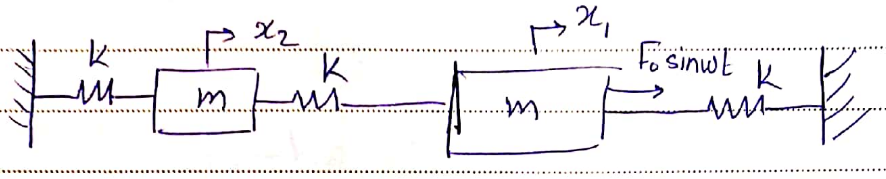
ϕ_1, ϕ_2, $X_1^{(1)}$, $X_1^{(2)}$ are found using the initial conditions.

[*] Forced vibration in 2DOF systems

→ Consider the previous 2DOF system with a harmonic force applied on one of the masses:



• EoMs:

$$m\ddot{x}_1 + 2kx_1 - kx_2 = F_0 \sin\omega t$$
$$m\ddot{x}_2 + 2kx_2 - kx_1 = 0$$

• Assume (ω = excitation frequency)

$$x_1(t) = X_1 \sin\omega t$$
$$x_2(t) = X_2 \sin\omega t$$

• Our target is to find X_1 & X_2

• The EoM becomes:

$$-mX_1\omega^2 + 2kX_1 - kX_2 = F_0$$
$$-m\omega^2 X_2 + 2kX_2 - kX_1 = 0$$

⇓ matrix form

$$\begin{bmatrix} 2k-m\omega^2 & -k \\ -k & 2k-m\omega^2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 2k-m\omega^2 & -k \\ -k & 2k-m\omega^2 \end{bmatrix}^{-1} \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

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The inverse of a matrix is found as follows

if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{\det(A)} * \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\det(A) = ad - cb$

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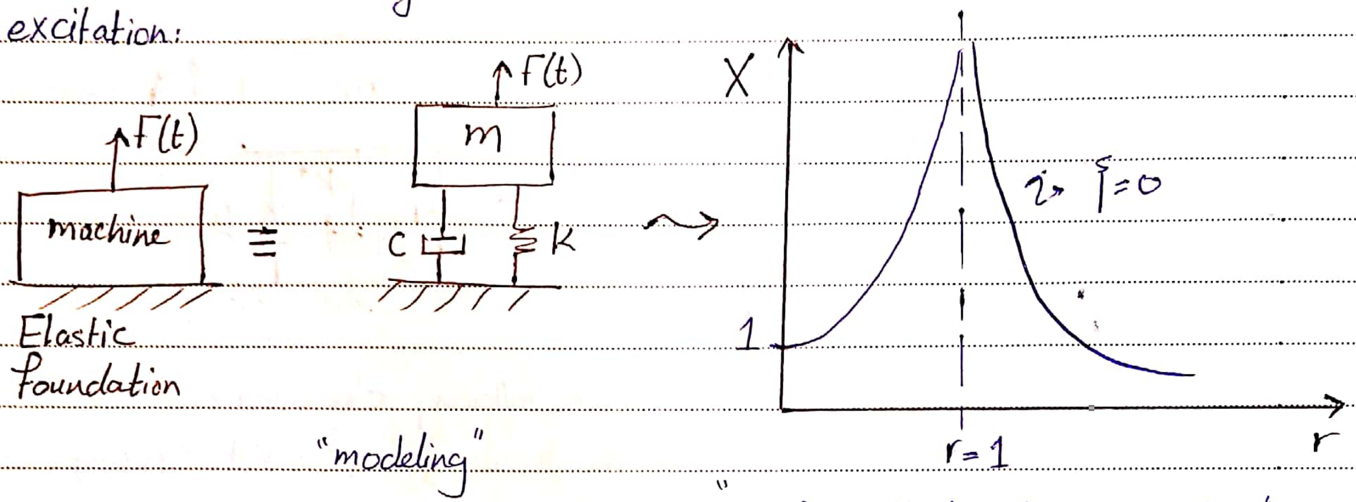
[*] Vibration Absorber

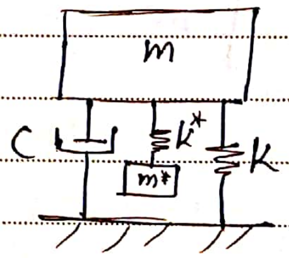
→ Consider the following 1 DOF system subjected to a direct harmonic excitation:



"plot of amplitude ratio against r"

→ If the system is operating near r = 1 the vibration level will be very high.

→ One of the most effective inexpensive techniques to reduce the vibration level is the vibration absorber. An auxiliary mass is introduced to the original system through a spring as shown below



→ The combination of k^* and m^* forms the vibration absorber

→ Notice that the original system is a 1 DOF system. The new system is 2 DOF system i.e. it has 2 natural frequencies.

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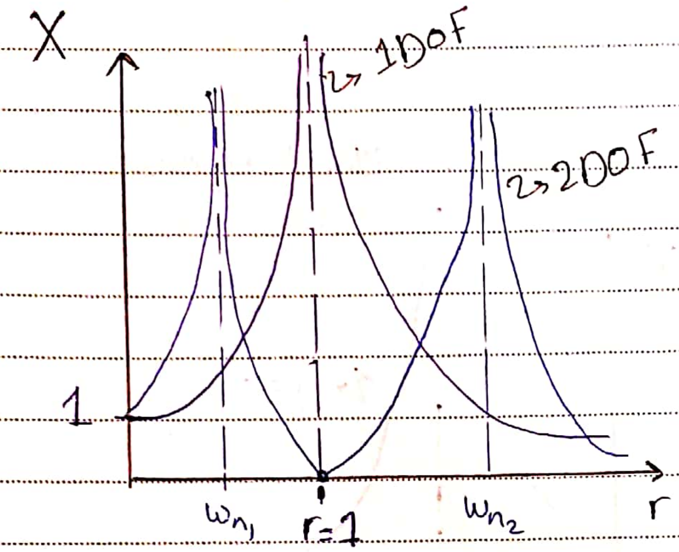
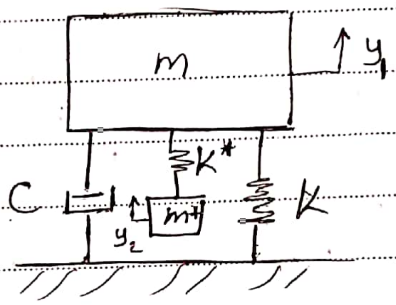


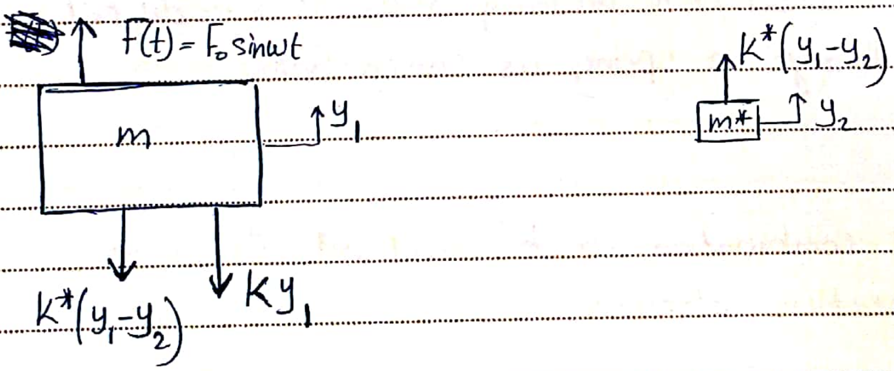


→ Note: We have to distinguish between the following designations:

- $\omega_{n_1}, \omega_{n_2}$ ≡ the natural frequencies of the new system (2DOF system)
- ω = excitation frequency of the forcing function
- ω_1 = natural frequency of the original System (1DOF System), $\omega_1 = \sqrt{k/m}$
- ω_2 = natural frequency of the vibration absorber alone, $\omega_2 = \sqrt{k^*/m^*}$

→ Our goal is to design the vibration absorber (ie. to find k^* & m^*)

→ First, we will construct the EoMs of the 2DOF system.



FBD of both m & m^*

"assume C = zero, worst case scenario"

$$+\uparrow \sum F = m\ddot{y}_1 \; ; \; F_o \sin\omega t - k y_1 - k^*(y_1 - y_2) = m\ddot{y}_1$$

$$m\ddot{y}_1 + (k + k^*) y_1 - k^* y_2 = F_o \sin\omega t \quad ---(1)$$

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$+\uparrow \sum F = m^* \ddot{y}_2 \; ; \quad K^*(y_1 - y_2) = m^* \ddot{y}_2$

$$m^* \ddot{y}_2 + K^* y_2 - K^* y_1 = 0 \quad --- (2)$$

- Assume

$y_1(t) = Y_1 \sin wt \quad ; \; Y_1$ = Vibration amplitude of m

$y_2(t) = Y_2 \sin wt \quad ; \; Y_2$ = Vibration amplitude of m^*

- The EoMs become:

$$-m w^2 Y_1 + (K + K^*) Y_1 - K^* Y_2 = F_0$$

$$-m^* w^2 Y_2 + K^* Y_2 - K^* Y_1 = 0$$

$\Downarrow$ matrix form

$$\underbrace{\begin{bmatrix} K + K^* - m w^2 & -K^* \\ -K^* & K^* - m^* w^2 \end{bmatrix}}_{\text{call it "A"}} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

- To find the natural frequencies of the 2DOF system, we have to construct the frequency equation

$$\det(A) = (K + K^* - m w^2)(K^* - m^* w^2) - K^* = 0 \quad --- (3)$$

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• To Find the amplitudes of vibration Y_1 & Y_2 :

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} K+K^*-m\omega^2 & -K^* \\ -K^* & K^*-m^*\omega^2 \end{bmatrix}^{-1} \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{det(A)} \begin{bmatrix} K^*-m^*\omega^2 & K^* \\ K^* & K+K^*-m\omega^2 \end{bmatrix} \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$Y_1 = \frac{1}{det(A)} * (K^*-m^*\omega^2) F_0$$; To eliminate Y_1 :

i.e. $Y_1 = 0$

$$K^*-m^*\omega^2 = 0$$

$$Y_2 = \frac{1}{det(A)} * K^* * F_0$$

$$\omega = \sqrt{\frac{K^*}{m^*}}$$

$\therefore K^*$ & m^* are selected such that $\sqrt{\frac{K^*}{m^*}}$ = excitation frequency

→ Y_1 is often represented in terms of ω_1 & ω_2 :

$$Y_1 = \frac{(K^*-m^*\omega^2) F_0 \quad \div (K^*K)}{(K+K^*-m\omega^2)(K^*-m^*\omega^2) - K^{*2} \div (K^*K)}$$

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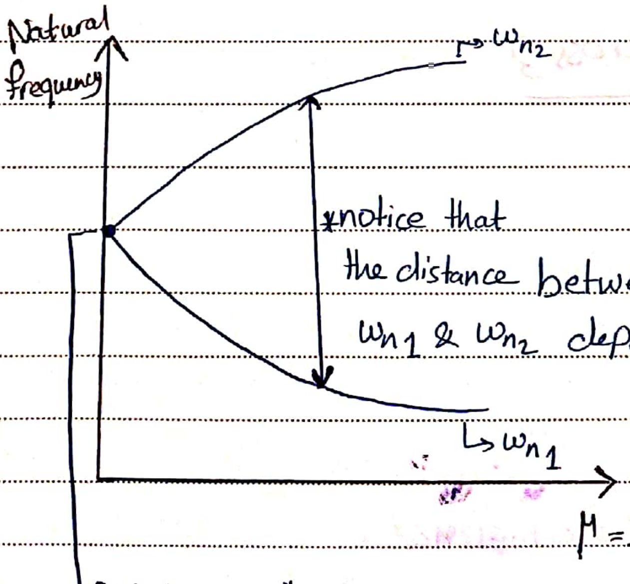
$$Y_1 = \frac{\left(1 - \frac{m^*}{k^*}\omega^2\right)\frac{F_o}{k}}{\left(1 + \frac{k^*}{k} - \frac{m}{k}\omega^2\right)\left(1 - \frac{m^*}{k^*}\omega^2\right) - \frac{k^*}{k}}$$

$$\frac{k^*}{k} = \frac{k^*}{m^*} * \frac{m^*}{m} * \frac{m}{k} = \frac{m^*}{m} * \frac{\omega_2^2}{\omega_1^2} = \mu\left(\frac{\omega_2}{\omega_1}\right)^2 \quad ; \mu = \text{mass ratio}$$

$$\therefore Y_1 = \frac{\left(1 - \left(\frac{\omega}{\omega_2}\right)^2\right)\frac{F_o}{k}}{\left(1 + \mu\left(\frac{\omega_2}{\omega_1}\right)^2 - \left(\frac{\omega}{\omega_1}\right)^2\right)\left(1 - \left(\frac{\omega}{\omega_2}\right)^2\right) - \mu\left(\frac{\omega_2}{\omega_1}\right)^2}$$

→ The following curve shows the plot of the natural frequencies ω_{n_1} and ω_{n_2} of the 2DOF against μ for $\frac{\omega_2}{\omega_1} = 1$

*لأ احنا بس
رح نتعامل
مع هاي الحالة



*notice that the distance between ω_{n1} & ω_{n2} depends on μ →

* بتهمنا عشان كا نعمل design لل Vibration absorber تكون ال operating speed range بين ال ω_{n1} و ω_{n2}

*When $m^* = 0$
The system is 1DOF (i.e. it has single natural frequency $= \sqrt{\frac{k}{m}}$)

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→ The relation between ω_{n_1} , ω_{n_2} & μ are given as follows

$$\left(\frac{\omega_{1n}}{\omega_2}\right)^2 = \frac{\left(1+(1+\mu)\left(\frac{\omega_2}{\omega_1}\right)^2\right) - \left[\left[1+(1+\mu)\frac{\omega_2}{\omega_1}\right]^2 - 4\left(\frac{\omega_2}{\omega_1}\right)^2\right]^{1/2}}{2\left(\frac{\omega_2}{\omega_1}\right)^2}$$

$$\left(\frac{\omega_{2n}}{\omega_2}\right)^2 = \frac{\left(1+(1+\mu)\left(\frac{\omega_2}{\omega_1}\right)^2\right) + \left[\left[1+(1+\mu)\frac{\omega_2}{\omega_1}\right]^2 - 4\left(\frac{\omega_2}{\omega_1}\right)^2\right]^{1/2}}{2\left(\frac{\omega_2}{\omega_1}\right)^2}$$

هاي العلاقات اشتقيناها من المعادلة رقم 3

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EXAMPLE 9.16

Absorber for Motor-Generator Set

A motor-generator set, shown in Fig. 9.36, is designed to operate in the speed range of 2000 to 4000 rpm. However, the set is found to vibrate violently at a speed of 3000 rpm due to a slight unbalance in the rotor. It is proposed to attach a cantilever mounted lumped-mass absorber system to eliminate the problem. When a cantilever carrying a trial mass of 2 kg tuned to 3000 rpm is attached to the set, the resulting natural frequencies of the system are found to be 2500 rpm and 3500 rpm. Design the absorber to be attached (by specifying its mass and stiffness) so that the natural frequencies of the total system fall outside the operating-speed range of the motor-generator set.

Solution: The natural frequencies ω_1 of the motor-generator set and ω_2 of the absorber are given by

$$\omega_1 = \sqrt{\frac{k_1}{m_1}}, \qquad \omega_2 = \sqrt{\frac{k_2}{m_2}} \tag{E.1}$$

The resonant frequencies Ω_1 and Ω_2 of the combined system are given by Eq. (9.146). Since the absorber ($m = 2$ kg) is tuned, $\omega_1 = \omega_2 = 314.16$ rad/s (corresponding to 3000 rpm). Using the notation

$$\mu = \frac{m_2}{m_1}, \qquad r_1 = \frac{\Omega_1}{\omega_2}, \qquad \text{and} \qquad r_2 = \frac{\Omega_2}{\omega_2}$$

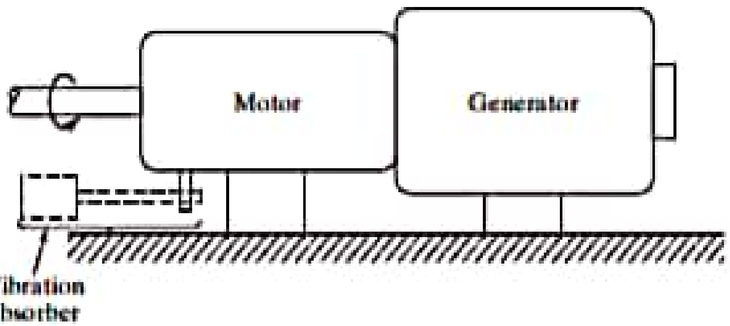

FIGURE 9.36 Motor-generator set.

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Eq. (9.146) becomes

$$r_1^2, r_2^2 = \left(1 + \frac{\mu}{2}\right) \mp \sqrt{\left(1 + \frac{\mu}{2}\right)^2 - 1} \tag{E.2}$$

Since Ω_1 and Ω_2 are known to be 261.80 rad/s (or 2500 rpm) and 366.52 rad/s (or 3500 rpm), respectively, we find that

$$r_1 = \frac{\Omega_1}{\omega_2} = \frac{261.80}{314.16} = 0.8333$$

$$r_2 = \frac{\Omega_2}{\omega_2} = \frac{366.52}{314.16} = 1.1667$$

Hence

$$r_1^2 = \left(1 + \frac{\mu}{2}\right) - \sqrt{\left(1 + \frac{\mu}{2}\right)^2 - 1}$$

or

$$\mu = \left(\frac{r_1^4 + 1}{r_1^2}\right) - 2 \tag{E.3}$$

Since $r_1 = 0.8333$, Eq. (E.3) gives $\mu = m_2/m_1 = 0.1345$ and $m_1 = m_2/0.1345 = 14.8699$ kg. The specified lower limit of Ω_1 is 2000 rpm or 209.44 rad/s, and so

$$r_1 = \frac{\Omega_1}{\omega_2} = \frac{209.44}{314.16} = 0.6667$$

With this value of r_1, Eq. (E.3) gives $\mu = m_2/m_1 = 0.6942$ and $m_2 = m_1(0.6942) = 10.3227$ kg. With these values, the second resonant frequency can be found from

$$r_2^2 = \left(1 + \frac{\mu}{2}\right) + \sqrt{\left(1 + \frac{\mu}{2}\right)^2 - 1} = 2.2497$$

which gives $\Omega_2 \simeq 4499.4$ rpm, larger than the specified upper limit of 4000 rpm. The spring stiffness of the absorber is given by

$$k_2 = \omega_2^2 m_2 = (314.16)^2(10.3227) = 1.0188 \times 10^6 \text{ N/m}$$

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$$T_r^2 = \frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2} \quad \text{with } r = \frac{\omega}{\omega_n} \qquad (E_1)$$

At $r=1$, $T_r^2 = \dfrac{1+4\zeta^2}{4\zeta^2}$; $6.25 = \dfrac{1+4\zeta^2}{4\zeta^2} \Rightarrow \zeta = 0.2182$

At operating speed, $T_r = 0.1$ and $\zeta = 0.2182$; Eq. (E_1) gives

$$(0.1)^2 = \frac{1+4(0.2182)^2 r^2}{(1-r^2)^2+4(0.2182)^2 r^2}$$

which, upon simplification, becomes

$$r^4 - 20.8595\, r^2 - 99 = 0 \quad \Rightarrow \quad r^2 = 24.8443$$

or $\quad r = \dfrac{\omega_0}{\omega_n} = 4.9844$

Since $\omega_0 = 62.832$ rad/sec, $\omega_n = 12.6057$ rad/sec $= \sqrt{\dfrac{k}{m}}$

$$k = \omega_n^2 m = (12.6057)^2 \left(\frac{800}{9.81}\right) = 12958.5054 \text{ N/m}$$

$\therefore$ Isolator is defined by

$$k = 12958.5054 \text{ N/m}$$

$$c = 2m\omega_n\zeta = 2\left(\frac{800}{9.81}\right)(12.6057)(0.2182) = 448.6139 \ \frac{N\text{-}s}{m}$$

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(9.36) $me = 1.0$ kg-m ; $\omega = 800$ to 2000 rpm $= 83.776$ to 209.44 rad/sec
$F_T = 7018$ N at 800 rpm and 43865 N at 2000 rpm
$F_T \leq 6000$ N over the speed range ; $\zeta = 0.08$

To find k.

$$\text{Relation be satisfied:}\quad \frac{F_T}{m\,e\,\omega^2} = \left\{\frac{1+(2\,\zeta\,r)^2}{(1-r^2)^2+(2\,\zeta\,r)^2}\right\}^{\frac{1}{2}} \leq \frac{6000}{m\,e\,\omega^2}$$

$$\text{or}\quad \left\{\frac{1+0.0256\,r^2}{(1-r^2)^2+0.0256\,r^2}\right\}^{\frac{1}{2}} \leq \frac{6000}{7018} = 0.8549 \text{ at } \omega = 800 \text{ rpm}$$

$$\text{and}\quad \leq \frac{6000}{43865} = 0.1368 \text{ at } \omega = 2000 \text{ rpm} \qquad (1)$$

Equating the left side of Eq. (1) to 0.85 at $\omega = 800$ rpm, we obtain

$$\frac{1+0.0256\,r_1^2}{1+r_1^4-2\,r_1^2+0.0256\,r_1^2} = 0.7225$$

$$\text{or}\quad r_1^4 - 2.00983\,r_1^2 - 0.3841 = 0 \quad \text{or}\quad r_1^2 = 2.1856 \text{ (positive root)}$$
$$\text{or}\quad r_1 = 1.4784$$

Equating the left side of Eq. (1) to 0.135 at $\omega = 2000$ rpm, we obtain

$$\frac{1+0.0256\,r_2^2}{1+r_2^4-2\,r_2^2+0.0256\,r_2^2} = 0.018225$$

$$\text{or}\quad r_2^4 - 3.3791\,r_2^2 - 53.8697 = 0 \quad \text{or}\quad r_2^2 = 9.2211 \text{ (positive root)}$$
$$\text{or}\quad r_2 = 3.0366$$

By selecting $r_2 = 3.0366$, we obtain $\omega_n = \dfrac{\omega}{r_2} = \dfrac{209.44}{3.0366} = 68.9713$ rad/sec. If $r_1 = 1.4784$ is selected, we obtain $\omega_n = \dfrac{\omega}{r_1} = \dfrac{83.776}{1.4784} = 56.6667$ rad/sec. Thus $\omega_n = 56.6667$ rad/sec satisfies the transmitted force requirement at both ends of the operating speed.

Verification:

At the speed 2000 rpm , the value of $r = \dfrac{\omega}{\omega_n}$ is: $r = \dfrac{209.44}{56.6667} = 3.6960$

This gives $r^2 = 13.6604$ and

$$\left\{\frac{1+0.3497}{(1-13.6604)^2+0.3497}\right\}^{\frac{1}{2}} = 0.09166 < 0.1368 \text{ of Eq. (1).}$$

Stiffness of the isolator:

$$k = M\,\omega_n^2 = (200)\,(56.6667^2) = 64.2223\,(10^4)\text{ N/m}$$

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* Suggested problems: 9.63 ⟶ 9.73

* Suggested examples: 9.27 ⟶ 9.40

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