Lecture (1) / /

Chapter (1)

* vibration:- motion about equilibrium position . الطاقة اللي بتتغير

[mass, Rotary inertia]

Vibrating system : 1) Inertial element (KE) عنصر جامد

2 basic elements : 2) Elastic element (PE) عنصر مرن

(system at rest) ↳ spring ← linear / Torsional

Beam

* 3) Dissipative element → Friction

opptional

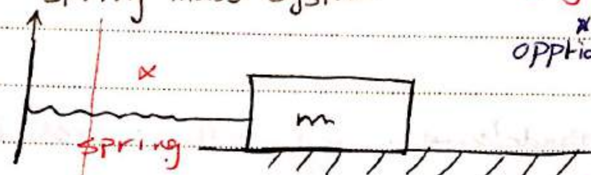
dry / Fluid: Drag force

Viscous damping

Equilibrium Motion.

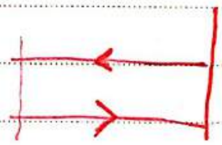
spring mass system

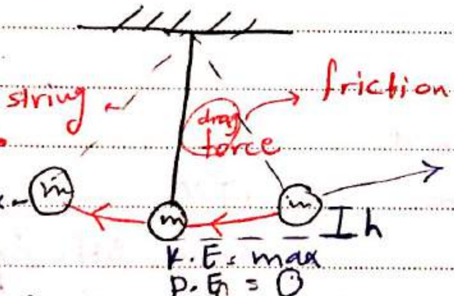
Ex.



without friction

(P.E) stored in spring.





Ex.

friction force

P.E = mgh

K.E = 0

P.E = max

K.E = 0

K.E = max

P.E = 0

* classification of vibration:-

Free vibration without force "A system is left to vibrate on its own after an initial" ↑ No External force acts on.

1) Free or Forced → we have cont. energy input on the system.

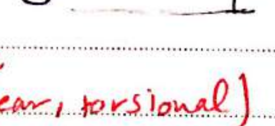
Repeating external force

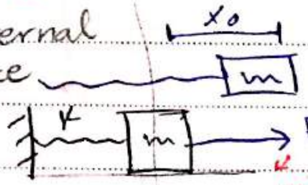
2) linear or non-linear

3) Damped or un-damped.

4) Discrete or Continuos منفصل

5) Random or Deterministic





Free

with this force it will be forced

inertia (m, J) / Elastic (Linear, torsional)

linear: all Basic components of a vibratory system behave linearly (Linear DEs)

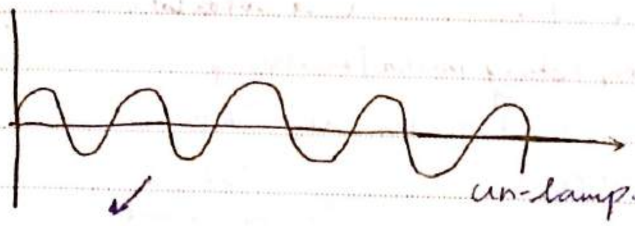
$F = kx$ $\quad$ $T = k_t\theta$ $\quad$ linear

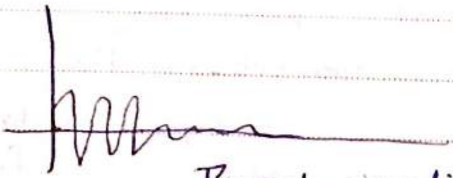
$F = k_0 + k_1x + k_2x^2 + k_3x^3 + \dots$

it's non linear

when any energy is lost in friction or other resistance.

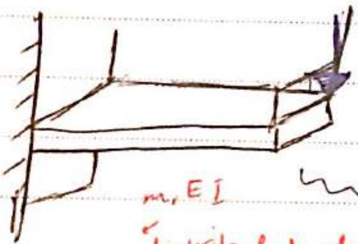
3) Damped or Un-damped.





Damped vibration or Pendulum moving with air resistance.

4) discrete or Conti.



m, E I → shaft or beam "elastic element"

"distributed elastisity" ⇓ Conti. system ⇒ with PDE

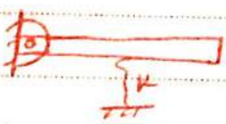
Partial diff. Equation

the diff. between beam & rod.

Discrete →

Particles.

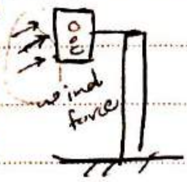
ODE



Rigid Rod.

5) Randome or deter.

↳ if the value or mag. of the (force or motion) acting on a vibratory system is known at any given time.



Random force → Random vibration.

if determined → deter. ⇝ $F(x) = F_o \sin \omega t$

$F_o \cos \omega t$

*Procedure for Vibration Analysis:-

1) Mathmatical modeling [Physical Model] ⇝ what is the diff ?

↓

2) Governing Equation [Eqn. of motion]

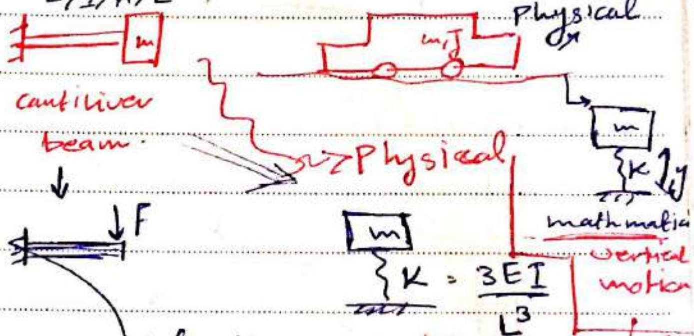
↓ EOM

3) solution of EOM

↓

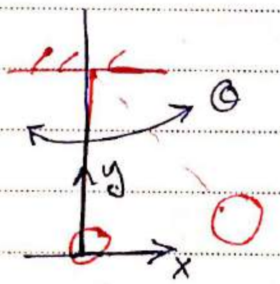
4) Analysis.

E, I, A, L



deflection

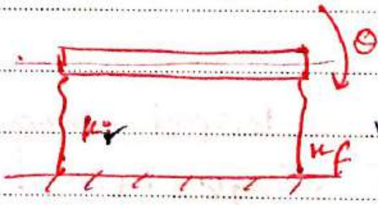
without force it will back to it's original shape.



$x^2 + y^2 = l^2$

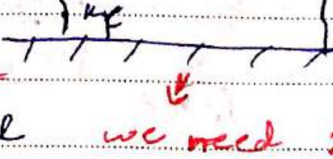
↓ Not independent coordinates, then 1 equation.

1 motion Equation of the vertical



rotational Motion.

* if we have two motions in the same system.

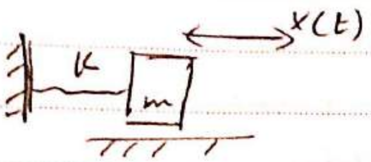


كيف بدي ال system ?
digree of accuracy

vertical & rotational

vertical & rotational with independent coordinates.

we need 2 equation of motion.



2) Governing Equation.

Newton's 2nd law

$\sum F = ma$

$\sum M = \sum M_{eff}$

T+U = Mechanical.

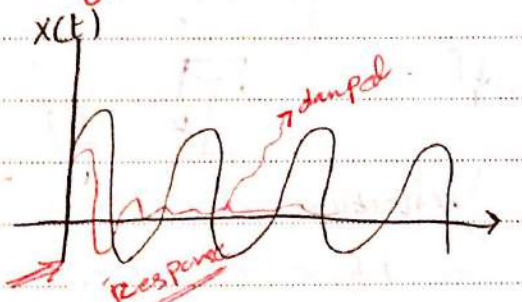
$m\ddot{x} + kx = 0$

$m\ddot{x} + kx = 0$
discreter
ODE "2nd"
$x(0)$
$\dot{x}(0)$

3) solution

$$x(t) = A\cos\sqrt{\frac{k}{m}}\,t + B\sin\sqrt{\frac{k}{m}}\,t$$

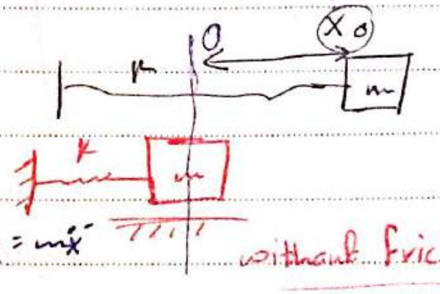
4) Analysis.



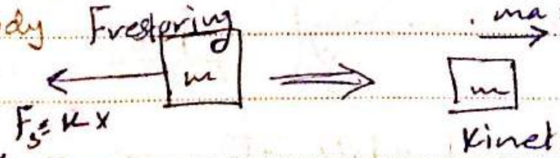
"Motion about equilibrium position"

⇒ Free, Un-damped, linear, discrete
SDOF → single degree of freedom.

$$\boxed{m\ddot{x} + c\dot{x} + kx = F(t)}$$



F.B.D "isolation of the body" ⇐
free body diagram



+ve →

$\sum F = m\ddot{x}$

$-kx = m\ddot{x} \rightarrow \boxed{m\ddot{x} + kx = 0}$ Governing Equation
2nd ODE

F.B.D ✓

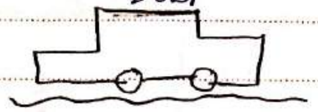
Lecture (2) / /

$x(0) = x_0$ "delta"
$\dot{x}(0) = 0$ || = initial conditions.

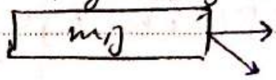
* Examples

2 dof

1)



Rigid Body

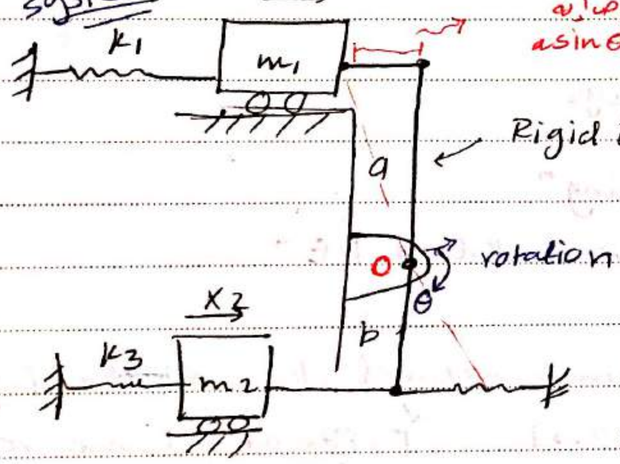


$\frac{1}{2} m \dot{x}^2$
$\frac{1}{2} J \dot{\theta}^2$

→ Vibratory system → Equivalent mass
" stifness

Vibratory system

2)



$a\sin\theta$ → المفروض انه
لو صغير $\sin\theta = \theta$

Rigid Body m_r, J_r

rotation

dof "Degrees of freedom" = 3 ??
لازم الشوف اذا dependent
or
un-dependent

$\theta = \frac{x_1}{a}$ ← $x_1 = a\theta$] الشين بيتعدوا
← $x_2 = b\theta$] عالزاوية

$x_2 = \frac{b x_1}{a}$ اذن معتمدات

$\not{d}$ dof = 3 بعضها اذن

dof = 1 , $\underbrace{x_1, x_2}, \theta$ } coordinates
3 coordinates
but sdof

$K.E|_{system} = T_{system}$

$$T_{system} = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} J_o \dot{\theta}^2$$

بالنسبة لنقطة O لانه لها rotation المسار

حساب الـ K.E

$$T_{sys.} = \frac{1}{2} \left\{ m_1 \dot{x}_1^2 + m_2 \left(\frac{b}{a}\right)^2 \dot{x}_1^2 + J_o \frac{\dot{x}_1^2}{a^2} \right\}$$

$$T_{sys.} = \frac{1}{2} \left(\underbrace{m_1 + \frac{b^2}{a^2} m_2}_{m_{equivalent}} + \frac{J_o}{a^2} \right) \dot{x}_1^2$$

meq.

حساب ال P.E

$$V_{system} = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2 + \frac{1}{2}k_3 x_2^2$$

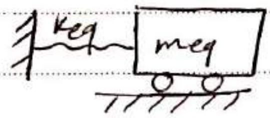
$$= \frac{1}{2}\left[k_1 x_1^2 + k_2\left(\frac{b}{a}\right)^2 x_1^2 + k_3\left(\frac{b}{a}\right) x_1^2\right]$$

$$= \frac{1}{2}\left\{\underbrace{k_1 + k_2\left(\frac{b}{a}\right)^2 + k_3\left(\frac{b}{a}\right)^2}_{k_{eq.}}\right\} x_1^2$$

* To find meq
  K.E كبب
* To find keq
  P.E كبب

هلا لقدر الأشكله

Reduction for the system

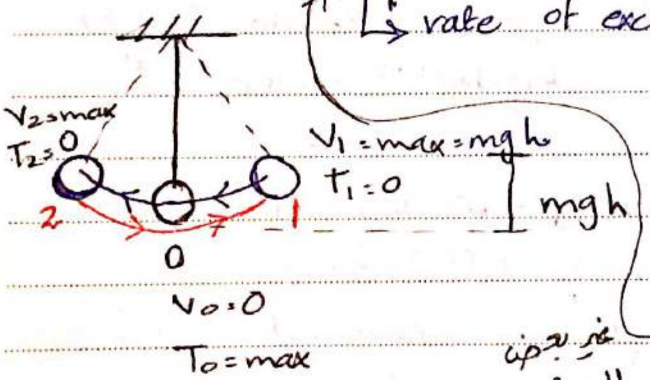


sdof زي

Phy. parameters for the sys.
ث ال

m, l ←

* Natural Frequancy :- "Physical meaning"
  ↳ rate of exchange between K.E & P.E"



. Cycle: time required to complete 1 cycle
1 → 2 → 1
يرجع لنفس النقطة
τ = Period = time req.
(S)

f = frequency = no. of cycles per second.

$$= \frac{1}{\tau} = \left(\frac{1}{s}\right); (Hz)$$

غير بعرف ال Natural function in physical parameters

system اذا كان ال

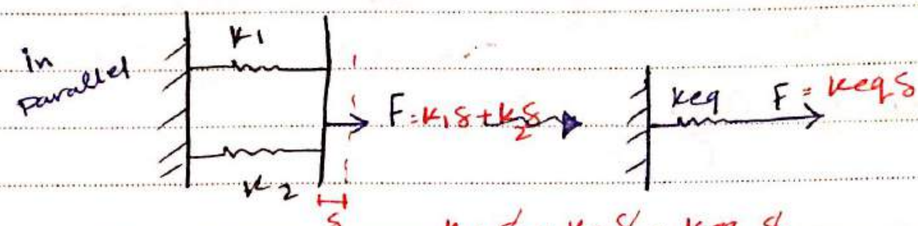
free frequency = Natural frequency

& forced اما

* Elastic elements:-

spring Linear ; $F_s = k\delta$

$F = kx$ [Extension / compression δ].



in parallel

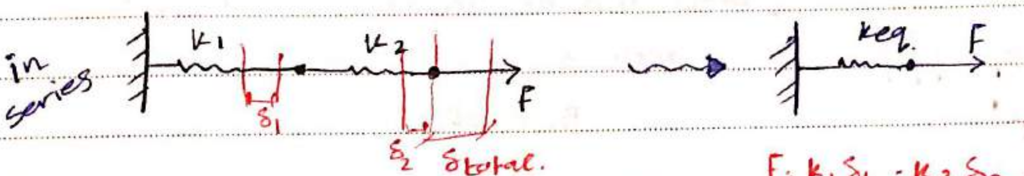
$F = k_1\delta + k_2\delta$

$F = k_{eq}\delta$

$k_1\delta + k_2\delta = k_{eq}\delta$

$\boxed{k_1 + k_2 = k_{eq}}$

Linear Spring



in series

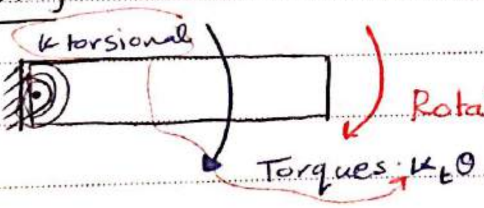
δ_1, δ_2, δ_{total}

$F = k_1\delta_1 = k_2\delta_2 = k_{eq}\delta_{total}$

$\delta_{total} = \delta_1 + \delta_2$

$\boxed{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}}$

Torsional spring

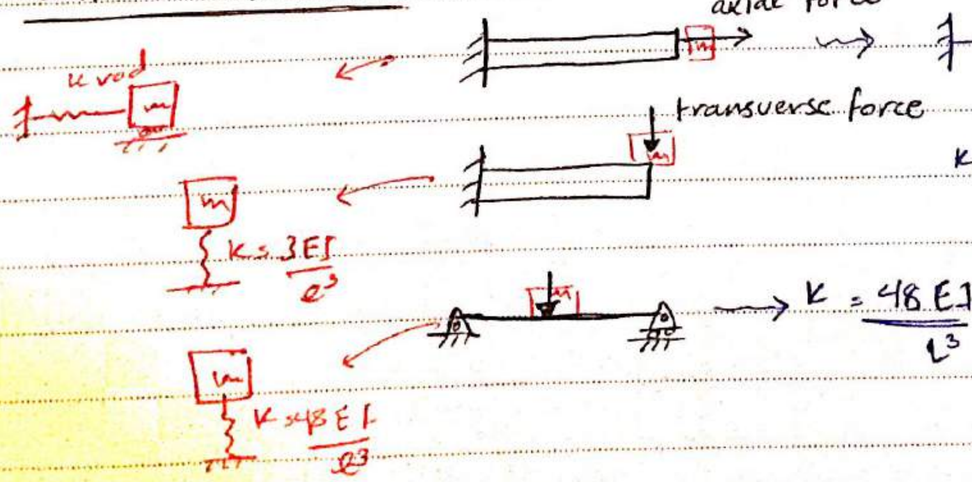


k torsional

Rotation

Torques $= k_t\theta$

Beam [Elastic Beam]



axial force

$k_{rod} = \frac{EA}{L}$

k rod

transverse force

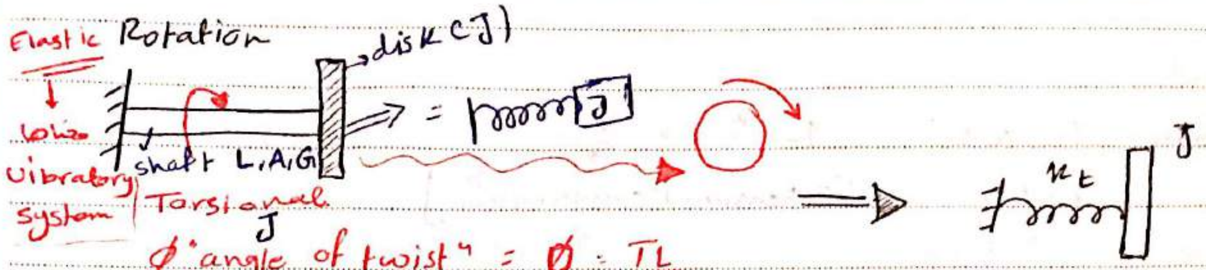
$k = \frac{3EI}{L^3}$

$k = \frac{3EI}{\ell^3}$

$\rightarrow k = \frac{48EI}{L^3}$?

$k = \frac{48EI}{\ell^3}$

/ /



ϕ "angle of twist" $= \phi = \frac{TL}{G I_{p(polar)}}$

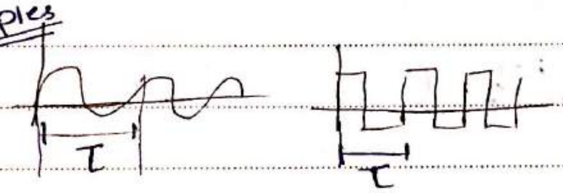
$$k_t = \frac{T}{\phi} = \frac{G I_p}{l}$$

*simple Harmonic Motion :-

↳ simplest form of Periodic motion.

↳ direct relation between cos & sin

Examples



$y(t) = A \sin \omega t$

$= B \cos \omega t$.

suggested

problem 1.9

problem 1.18

problem 1.20

problem 1.24

problem 1.31

problem 1.53

* ملاحظات مهمة :-

١- خذ rigid لو بدك تحل Moment حول نقطة الـ bar الموجودة F_1, F_2 - - - بجيب ع ضرب قوة بشرط x المسافة العمودية.

٢- $J_s = \frac{2}{5} m_s r_s^2$

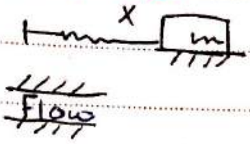
٣- عشان اعرف كم عدد DOF = عدد الـ masses بالـ system

عدد الـ n بعدد الـ springs بالـ system

/ /

* Damping Elements

⇒ Friction → Solid
↳ dissipation → Fluid
of Energy → Material.



$\tau = \mu \frac{dv}{dy}$

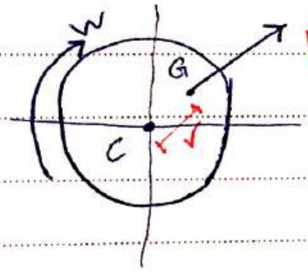
→ vibrationg sys.
→ Free / Forced
→ dof
→ Frequency
→ Natural Frequency

damping force = (const.)(velocity) $\dot{x}$
↓ viscous damping

Chapter (2)

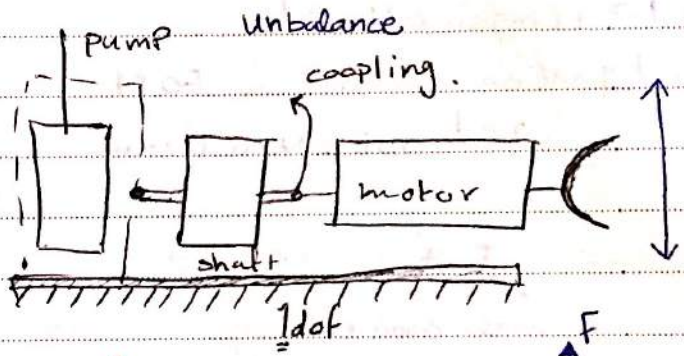
Vibrations of single degree of freedom system (undamped)

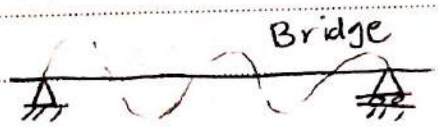
* Sources of vibration:-

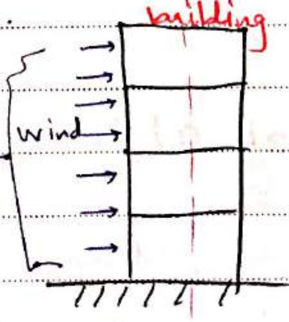


$F_c = \frac{mv^2}{r} = m\omega^2 r$

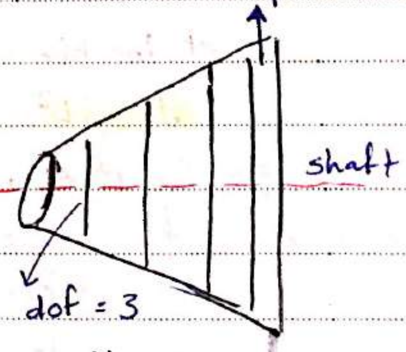
source الأهم
[un-balance].



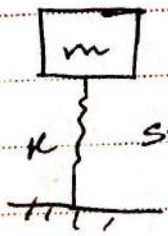




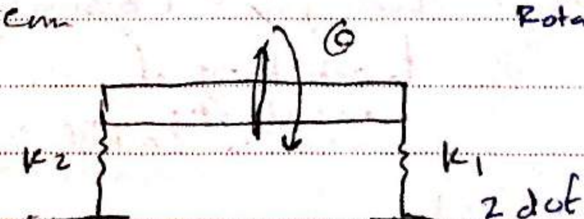
Random
$\sum A_i e^{i\omega_i t}$



لفرجم
↑
horizontal
↑
Rotation

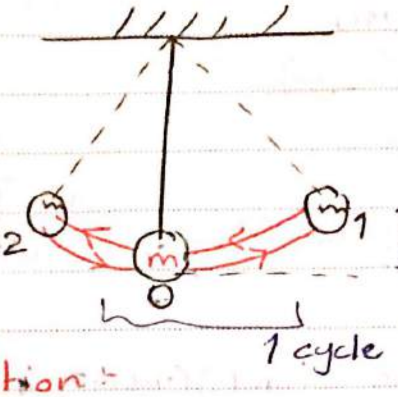


spring Mass system
single dof.



2 dof

No- Damping → Air Resistance = 0



* Pendulum

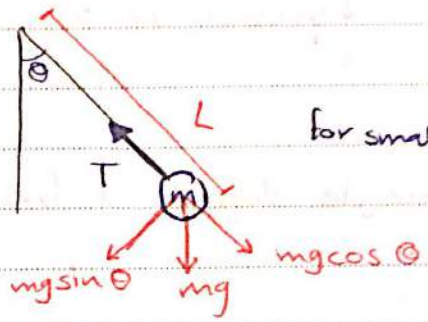
$\underline{\Sigma M}$ الأسهل دائماً

* Equation of Motion :-

+ve) $\Sigma M_o = \Sigma M_{eff}$

$= J\ddot{\theta}$

$(-mg\sin\theta)L = J\ddot{\theta}$



for small vibration level

$\sin\theta \approx \theta$

$[J\ddot{\theta} + (mg\sin\theta)L = 0]$

↳ Equation of Motion EOM

2^{nd} ODE : Non linear.

⟹ $J\ddot{\theta} + mg\theta L = 0$

mass moment of Inertia.

$J = mL^2$

So, $(mL^2\ddot{\theta} + mg\theta L = 0) / L^2 m$

↓

$\ddot{\theta} + \underbrace{\frac{g}{L}}_{\text{Nat. frequency}}^{\omega^2_n}\theta = 0$; 2^{nd} ODE → linear.

Solu

$\theta(t) = A\cos\sqrt{\frac{g}{L}}\,t + B\sin\sqrt{\frac{g}{L}}\,t$

↳ arbitary constants ↳ Initial time conditions

$\theta(0) = \theta_0$

$\dot{\theta}(0) = \dot{\theta}_0$

$\theta(t) = e^{st}$ = solu

$s^2 e^{st} + \frac{g}{L}e^{st} = 0$

$s^2 + \frac{g}{L} = 0$ → $\sqrt{-1} = i$

$s^2 = -\frac{g}{L}$

$s_{1,2} = \pm i\sqrt{\frac{g}{L}}$ → $\theta_1(t) = e^{i\sqrt{g/L}\,t}$

$\theta_1(t) = \cos\sqrt{\frac{g}{L}}\,t + i\sin\sqrt{\frac{g}{L}}\,t$

$\theta_2(t) = \cos\sqrt{\frac{g}{L}}\,t - i\sin\sqrt{\frac{g}{L}}\,t$

$\dot{\theta}(t) = -\sqrt{\frac{g}{L}}\,A\sin\sqrt{\frac{g}{L}}\,t + B\sqrt{\frac{g}{L}}\cos\sqrt{\frac{g}{L}}\,t$

$\theta_0 = A(1)$ to $A = \theta_0$

$\dot{\theta}_0 = 0 + B\sqrt{\frac{g}{L}}\,(1) \rightarrow B = \frac{\dot{\theta}_0}{\sqrt{g/L}}$

solu of EOM

$$\boxed{\theta(t) = \theta_0\cos\sqrt{\frac{g}{L}}\,t + \frac{\dot{\theta}_0}{\sqrt{g/L}}\sin\sqrt{\frac{g}{L}}\,t}$$

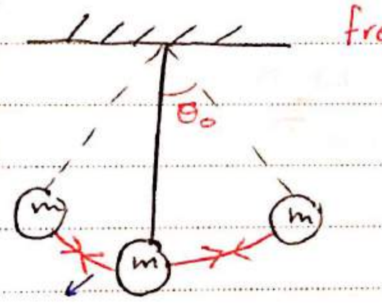
assume $\theta_0 =$ some value

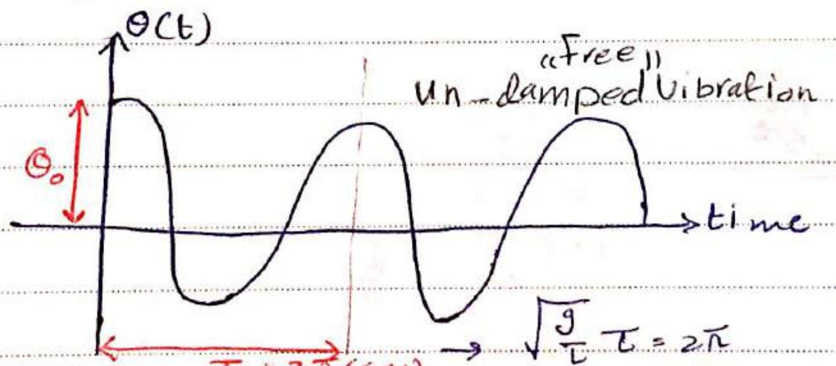
$\dot{\theta}_0 = 0$

$\theta(t) = \theta_0\cos\sqrt{\frac{g}{L}}\,t$ → Response 1st cycle

free vibration

for this Example





Periodic + simple harmonic Motion

لأن نها على شكل

sin & cos

· 1 cycle = 360°

= 2π (rad)

→ $\sqrt{\frac{g}{L}}\,T = 2\pi$

T ↓ time required to complete one cycle

$T = \frac{2\pi}{\sqrt{g/L}}$

$F = \frac{1}{T}$ [HZ] ← $\frac{1}{s}$

$= \sqrt{g/L}\,/\,2\pi$

→ $\sqrt{\frac{g}{L}} = \omega$ = freq. (rad/s) ← ω_{nat} impo!!

→ $\omega = 2\pi f = \frac{2\pi}{T}$

No. of cycles presented.

circular freq. $\boxed{\omega} = \sqrt{\frac{g}{L}}$ = Nat. frequency (ω_n)

$\omega_n = \sqrt{\frac{g}{L}}$ $\quad [\ddot{\theta} + \omega_n^2\,\theta = 0]$ → EOM

$\theta(t) = \theta_0 \cos \omega t$] → Response of the pendulum

• Natural frequency:-

$\omega_n = \sqrt{\frac{g}{L}} \Rightarrow \theta(t) = \theta_0 \cos \omega_n t$

↳ $\ddot{\theta} + \frac{mgL}{mL^2}\theta = 0$

$[\ddot{\theta} + \omega_n^2 \theta = 0$

→ $\ddot{x} + (\omega_n^2) x = 0$

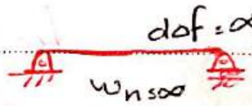
$\ddot{y} + (\omega_n^2) y = 0$] → حسب كيف يتحرك ؟

*Note:-

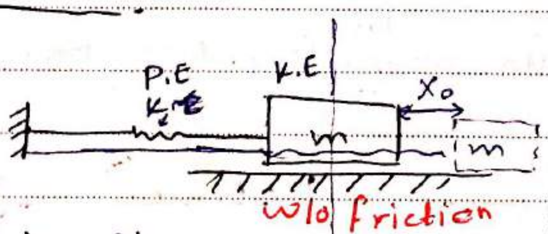
- infinit No. of degrees of freedom
  ↳ infinite Natural freq.
- 2-deg of freedom → 2 Natural freq.
- 1-deg of freedom → 1 Natural freq.

No. of Natural frequency = No. of degree of freedom = No. Equ. of Motions.

Solution = Response.



*Exp:-



→ Free Vibration

→ SDOF

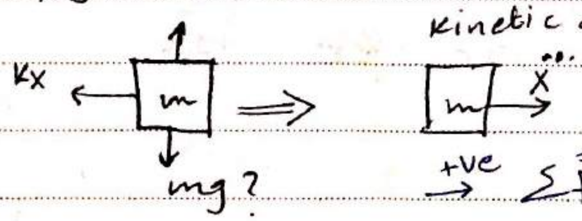
$\omega_n = \sqrt{\frac{k}{m}}$

O Equilibrium position

*Spring Mass system

$\sum F = m\vec{a}$

F.B.D



$\xrightarrow{+ve}$ $\sum \vec{F} = m\vec{a} = m\ddot{x} = \sum$ effec.

$-kx = m\ddot{x}$

$-m\ddot{x} + kx = 0$; EOM

$\ddot{x} + \frac{k}{m} x = 0$

$\boxed{\ddot{x} + \omega_n^2 x = 0}$

/ /

$\rightarrow x(t) = A\cos\sqrt{\frac{k}{m}}\,t + B\sin\sqrt{\frac{k}{m}}\,t$

$x(t) = A\cos w_n t + B\sin w_n t$

$x_0 = A$

$B = \frac{\dot{x}_0}{w_n}$ $\rightarrow x(0) = x_0 = A + 0 \rightarrow A = x_0$

$\dot{x}(t) = -w_n A\sin w_n t + w_n B\cos w_n t$

$\dot{x}(0) = 0 + w_n B(1) = \dot{x}_0$

$B = \frac{\dot{x}_0}{w_n}$

$$\boxed{x(t) = x_0\cos w_n t + \frac{\dot{x}_0}{w_n}\sin w_n t}$$

Response of the spring mass system.

cos بقدر احطها بدلالة $\Leftarrow = C\sin(w_n t + \phi)$

single الام $= C[\sin w_n t\cos\phi + \cos w_n t\sin\phi]$

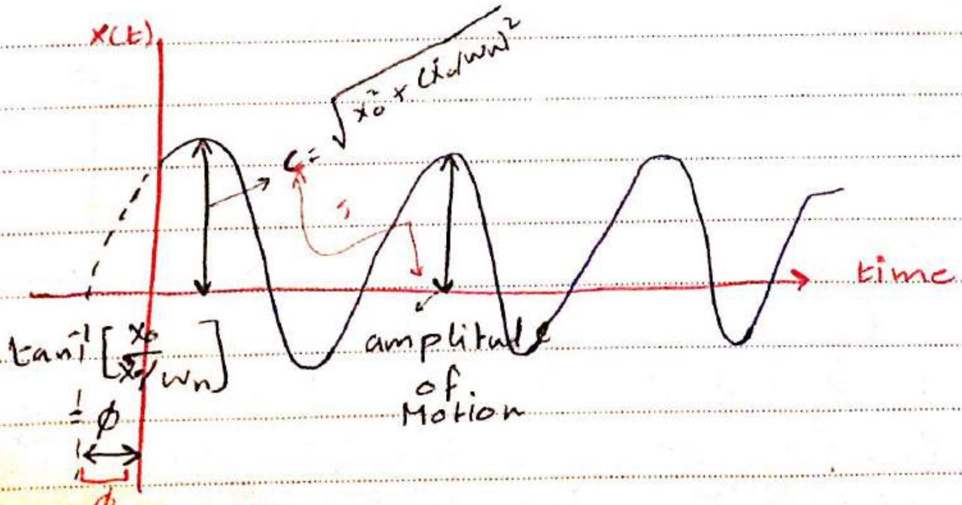
function $\rightarrow \frac{\dot{x}_0}{w_n} = C\cos\phi$] sine coefficients

$\rightarrow x_0 = C\sin\phi$] cosine "

$\hookrightarrow \tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{x_0}{(\dot{x}_0/w_n)} \longrightarrow \phi = \tan^{-1}\left[\frac{x_0}{\dot{x}_0/w_n}\right]$

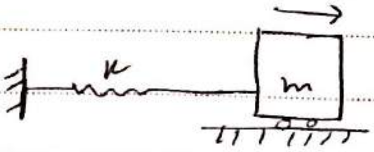
$\sin^2\phi + \cos^2\phi = 1$

$C^2 = x_0^2 + (\dot{x}_0/w_n)^2 \rightarrow C = \sqrt{x_0^2 + (\dot{x}_0/w_n)^2}$



$\hookrightarrow$ الرئيسية sin. func. + cos fun.

/ /



$T = \frac{1}{2} m \dot{x}^2$

$V = \frac{1}{2} K x^2$

free , Un-damped

No-dissipation of Energy.

T = Kinetic Energy

V = Potential "

T + V = constant = Mechanical Energy

Spring-Mass System

Energy بطريقة ال Method.

↳ نفس جواب $\sum F = m\vec{a}$

Valid only for Un-damped system. ← $T + V = E$

$\frac{d}{dt}(T+V) = 0$;

$\frac{d}{dt}\left[\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2\right] = 0$

$\frac{1}{2} m . 2 . \dot{x} \ddot{x} + \frac{1}{2} K . 2 . x \dot{x} = 0$

↳ $[m\ddot{x}(\dot{x}) + Kx(\dot{x}) = 0$

*divided by $\dot{x} \neq 0$

$[m\ddot{x} + Kx = 0]$

⇓

$$\boxed{\ddot{x} + W_n^2 x = 0}$$

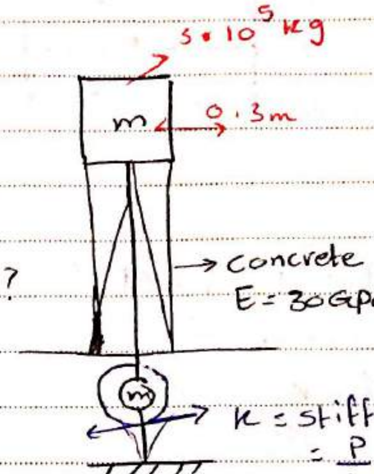
/ /

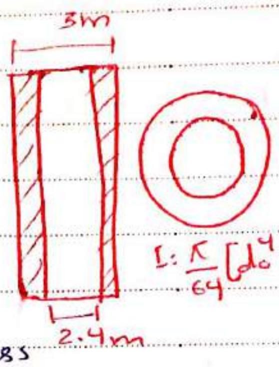
Exp 2-1:-

1) $w_n = \sqrt{\frac{k}{m}} = ?$

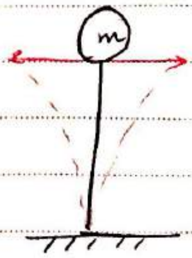
2) Vibration Response if $x_o = 0.3$, $\dot{x}_o = 0$.

3) max. values of the velocity and acc.?



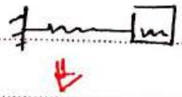


$I = \frac{\pi}{64}[d_o^4 - d_i^4]$



k = stiffness

k = stiffness $= \frac{P}{\text{deflection}}$



"Mathmatical Model"

$k = \frac{3E\,I}{L^3}$ → 2nd moment of Inertia

EOM:

$m\ddot{x} + kx = 0$

$\ddot{x} + w_n^2 x = 0$

$w_n = \sqrt{\frac{k}{m}}$ = rad/s

EOM من الأساس

2) $x(t) = x_o \cos w_n t + \frac{\dot{x}_o}{w_n} \sin w_n t$

$x(t) = \sqrt{x_o^2 + \left(\frac{\dot{x}_o}{w_n}\right)^2} \rightarrow [\sin(w_n t + \phi)]$

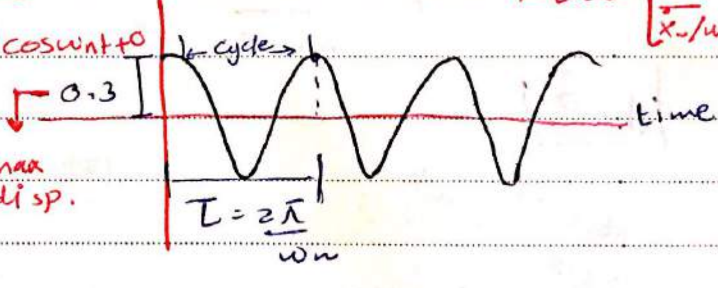
$\tan^{-1}\left[\frac{x_o}{\dot{x}_o/w_n}\right] = \phi$

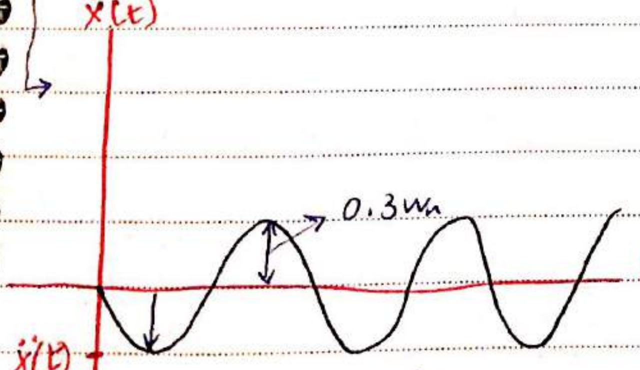
$x(t) = 0.3 \cos w_n t + 0$

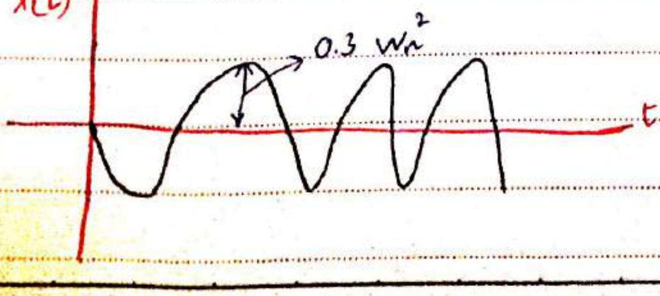
$\dot{x}(t) = -w_n\, 0.3 \sin w_n t$

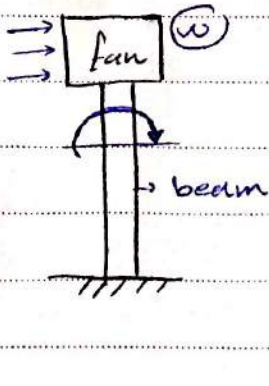
$\ddot{x}(t) = -w_n^2\, 0.3 \cos w_n t$

$x(t) = (0.3)\cos w_n t + 0$





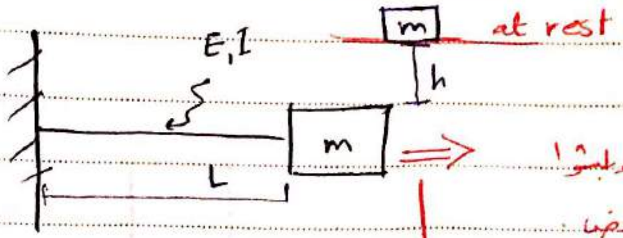




/ /

Exp 2.2:-

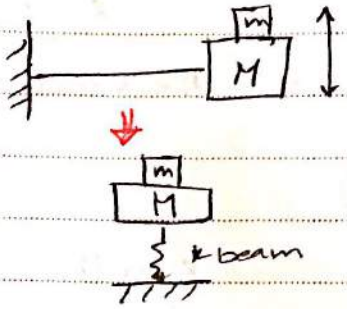
Response of the beam ?



حيصير فيه impact وبيشوا يتحركوا مع بعض

$$k_{beam} = \frac{3EI}{L^3}$$

$$\underbrace{[m+M]}_{\text{total mass}}\ddot{y} + ky = 0$$



. يتحرك عال y

$$\ddot{y} + \underbrace{\frac{k}{(M+m)}}_{\omega_n^2} y = 0$$

$$\rightarrow y(t) = y_0 \cos \omega_n t + \frac{\dot{y}_0}{\omega_n} \sin \omega_n t$$

$$= \sqrt{y_0^2 + \left(\frac{\dot{y}_0}{\omega_n}\right)^2} \, , \, \sin\left[\omega_n t + \tan^{-1}\left[\frac{y_0}{\dot{y}_0/\omega_n}\right]\right]$$

$$\underbrace{M(0)}_{v_i} + m \underbrace{\sqrt{2gh}}_{v_i} = [m+M]v_f$$

$$v_f = \frac{m\sqrt{2gh}}{m+M} = \dot{y}_0$$

$\boxed{y_0 = 0}$ at rest in impact

نعوض فيهم

بمعادلة ال response

الحركة

$\downarrow \sum F_y = m\ddot{y}$

$-k\Delta y = m\ddot{y}$

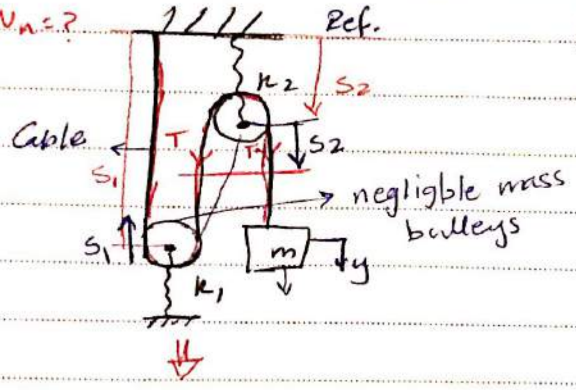
$-k_{eq}\Delta y = m\ddot{y}$

$m\ddot{y} + k_{eq}\Delta y = 0$

$\ddot{y} + \underbrace{\frac{k_{eq}}{m}}_{w_n^2}\Delta y = 0$

Exp 2-5:-

$W_n = ?$



$\rightarrow \sum F_y = m\ddot{y}$ لقدر الاطهاء

* $T_{system} = \frac{1}{2} m v^2 = \frac{1}{2} m\dot{y}^2$

* $V_{system} = \frac{1}{2} k_1 S_1^2 + \frac{1}{2} k_2 S^2$

* $S_1 + (S_1 - S_2) + (S - S_2) = L \rightsquigarrow$ Dynamics

$\hookrightarrow 2S_1 - 2S_2 + S = L$

* $[2S_1 - 2S_2 + S = L]\ t_1$

$[2S_1 - 2S_2 + S = L]\ t_2$

$\hookrightarrow 2\Delta S_1 - 2\Delta S_2 - \Delta S = 0$

Equivelent Mathmatecal Model

$W_n = \sqrt{\frac{k_{eq}}{m}}$

$(\Delta S = 2\Delta S_2 - 2\Delta S_1) \times -1$

$= 2\Delta S_2 + 2\Delta S_1$

$\Delta S = 2[\Delta S_2 + \Delta S_1]$

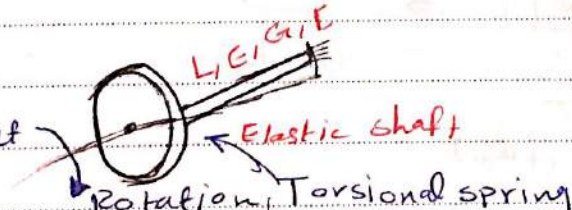
$\Delta S = \Delta y = 2\left[\frac{2\cancel{T}}{k_2} + \frac{2\cancel{T}}{k_1}\right] = \frac{\cancel{T}}{k_{eq}}$

$= 4\left[\frac{1}{k_2} + \frac{1}{k_1}\right] = \frac{1}{k_{eq}} \Longrightarrow W_n = \sqrt{\frac{k_{eq}}{m}}$

Translitional Motion

* Torsional Motion:-

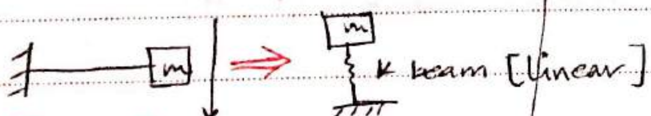
J = mass moment of inertia.



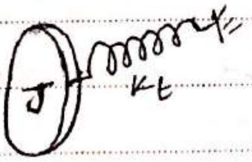
* angle of twist $= \frac{TL}{GI_p}$

k_t = torsional spring const.

$= \frac{T}{\phi} = \frac{GI_p}{L}$



$J\ddot{\theta} + k_t\theta = 0$



/ /

$\sum M_o = J\ddot{\theta}$

$-k_t\theta = J\ddot{\theta}$

$$\boxed{J\ddot{\theta} + k_t\theta = 0}$$

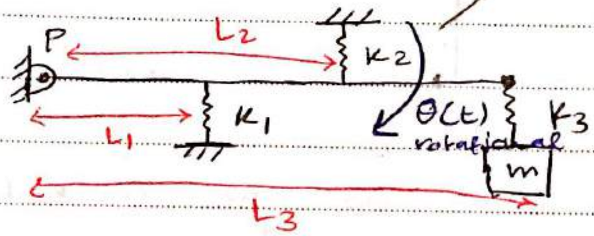
$$\ddot{\theta} + \underbrace{\left(\frac{k_t}{J}\right)}_{\omega_n^2}\theta = 0$$

$$\theta(t) = \theta_o \cos\omega_n t + \frac{\dot{\theta}_o}{\omega_n}\sin\omega_n t$$

* Monday 18/6/2015

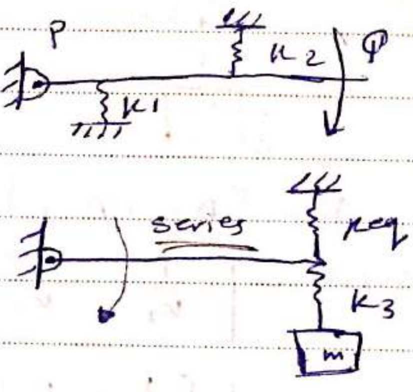
2-7

ω_n ?



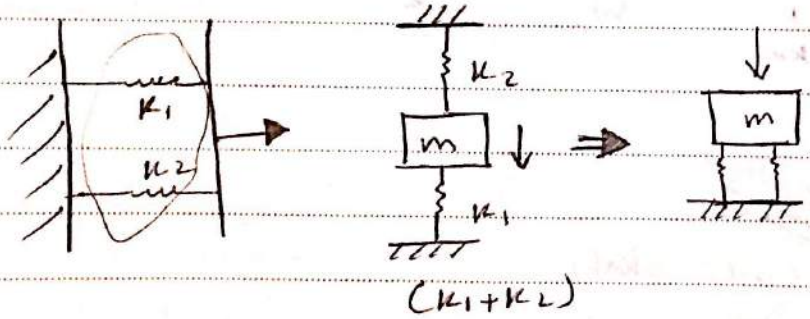
k_1, k_2 not the same δ

one comp., second extension



* EOM:

$\sum M_p = \sum M_{effective} = J\ddot{\theta}$



$$V_{potential} = \tfrac{1}{2}k_1[L_1\theta]^2 + \tfrac{1}{2}k_2[L_2\theta]^2 = \tfrac{1}{2}k_{eq}[L_3\theta]^2$$

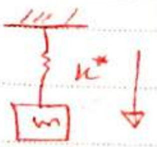
$$k_1L_1^2 + k_2L_2^2 = k_{eq}L_3^2 \ ;$$

k^* = equivalent stiffness of the system $\Rightarrow \frac{1}{k^*} = \frac{1}{k_3} + \frac{1}{k_{eq}}$

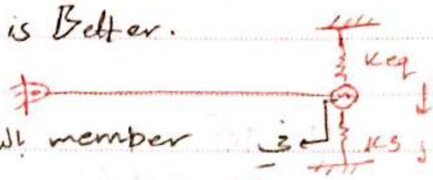
k_3, k_{eq} = in series

/ /

* If there is Rotation → Energy Method is Better.



$m\ddot{y} + K^* y = 0$



بالعرض member → in parallel.

Equivalent Mathmatical Model.

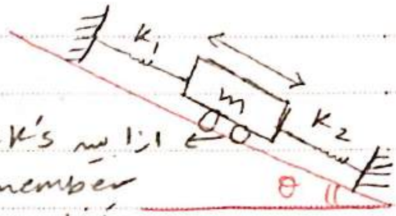
$\omega_n = \sqrt{\frac{K^*}{m}}$

$K^* = K_{eq} + K_s$

2-9

W_n ?

in parallel.



2K's اذا بنفس member بكونوا in parallel → same δ

* by Energy Method:

$T = \frac{1}{2} m \dot{\theta}^2$, $V = \frac{1}{2} K_1 \theta^2 + \frac{1}{2} K_2 \theta^2$

$T + V = E$

$\frac{d}{dt}(T+V) = 0 \rightarrow \frac{d}{dt}(\frac{1}{2} K_1 \theta^2 - - -)$

$\frac{1}{2} K_1 2\theta\dot{\theta} + \frac{1}{2} K_2 2\theta\dot{\theta} + \frac{1}{2} m 2\dot{\theta}\ddot{\theta} = 0$

$K_1 \theta + K_2 \theta + m\ddot{\theta} = 0$

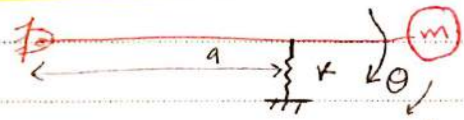
$\ddot{\theta} + (\underbrace{\frac{K_1 + K_2}{m}}_{W_n^2}) \theta = 0$

$W_n = \sqrt{\frac{K_1 + K_2}{m}}$

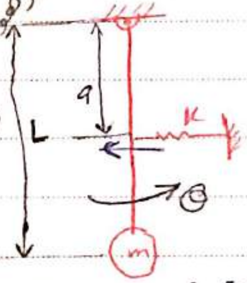
ما بيفرق

شرط هون $L/2$

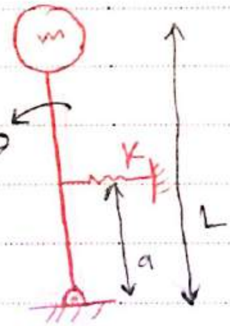
*Exp:



(1) equilibrium



(2)



(3)

$F_s = (a \sin\theta) K$
$\simeq a\theta K$

فالوزن ما دخل

$\underbrace{-K a \theta}_{F_s}[a] = mL^2 \ddot{\theta}$

خطأ

$mL\ddot{\theta} + Ka^2 \theta = 0$

$W_n = \sqrt{\frac{Ka^2}{mL^2}}$

$\Sigma M_o = [mL^2] \ddot{\theta}$

$\underbrace{-[a\theta] K}_{F_s}[a] = mL^2 \ddot{\theta}$

$mL^2 \ddot{\theta} + Ka^2 \theta = 0$

$W_n = \sqrt{\frac{Ka^2}{mL^2}}$

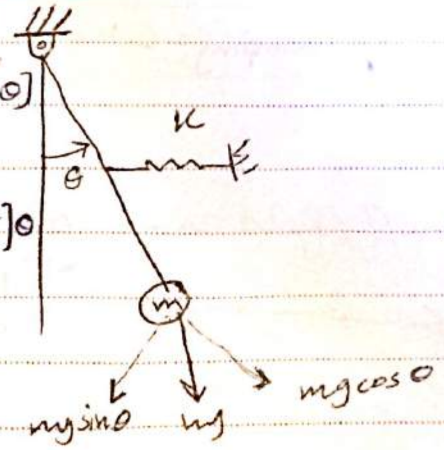
$\Sigma M_o = mL^2 \ddot{\theta}$

$-K_1 \theta(a) - mg[\sin\theta]$

$* L = mL^2 \ddot{\theta}$

$mL^2 \ddot{\theta} + [mgL + Ka^2]\theta = 0$

$W_n = \sqrt{\frac{mgL + Ka^2}{mL^2}}$

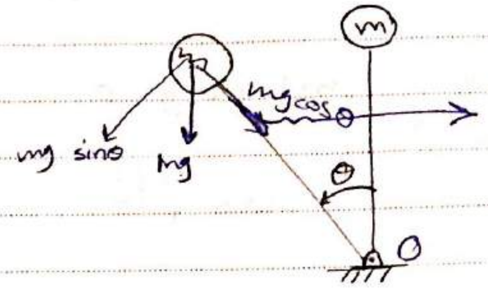


* هون عملنا اعتبار للوزن لأنه في Tension بساوية.

(3)

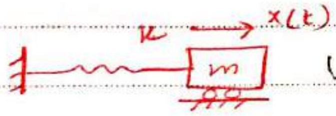
$$\sum M_o = J\ddot{\theta}$$

$$mg\theta L - Ka^2\theta = mL^2\ddot{\theta}$$



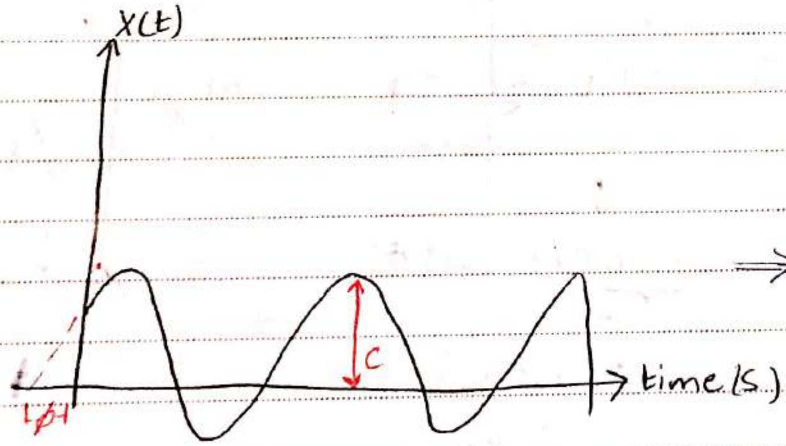
$$mL^2\ddot{\theta} + [Ka^2 - mgL]\theta = 0$$

$$W_n = \sqrt{\frac{Ka^2 - mgL}{mL^2}}$$

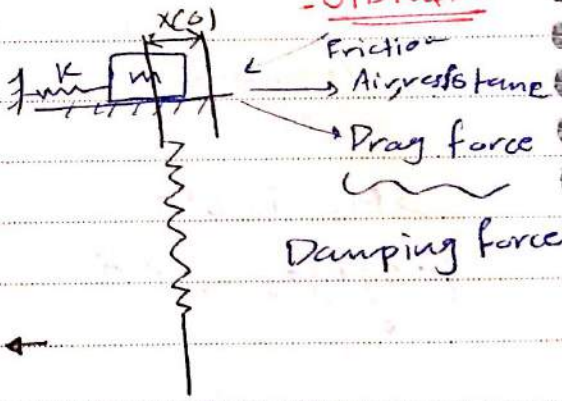


Un-damped → Vibration till infinity

No-Dissipation (No losses)



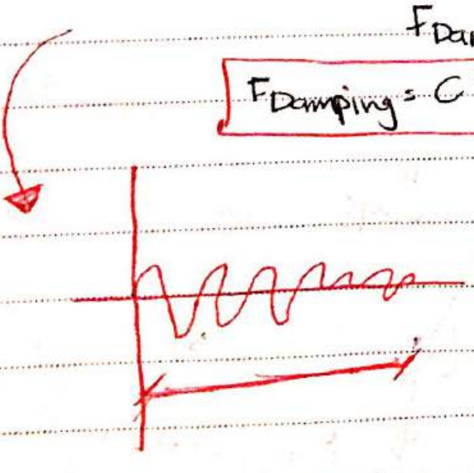
damped -Vibration



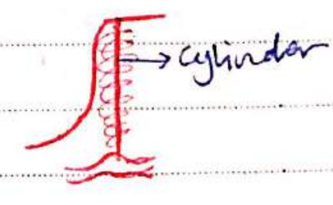
$F_{Damping} \propto$ Velocity

$$\boxed{F_{Damping} = CV = C\dot{x}}$$

C = damping coefficient $\left[\frac{N \cdot s}{m}\right]$; Experimental Calculated

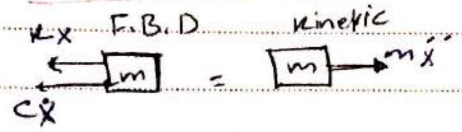


response of the system



Scanned by CamScanner

* Damped - Vibration



$-kx - c\dot{x} = m\ddot{x}$

$\boxed{m\ddot{x} + c\dot{x} + kx = 0}$ EOM for spring - Mass damper system

عالقانون العام :- $\quad s_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

assume $x(t) = e^{st}$

$\to ms^2 + cs + k = 0$

$s_{1,2} = \dfrac{-c \pm \sqrt{(c^2 - 4km)}}{2m}$

+ve → No - Vibration.

0 → $c^2 = 4km$ → No - Vibration.

$c = 2\sqrt{km}$

-ve → Vibration

$i\sqrt{4km - c^2}$, complex No. $= \sqrt{-1}$

$c = 2\sqrt{km}$; critical value of damping.

if $c > c_{cr}$; over - damped
$c = c_{cr}$; critically damped. } No - Vibration
$c < c_{cr}$; Under damped.] vibration

$\to s^2 + \frac{c}{m}s + \frac{k}{m}$

$s_{1,2} = \left(\frac{-c}{m} \mp \sqrt{\frac{c^2}{m} - \frac{4k}{m}}\right) / 2$

$\boxed{\frac{k}{m} = w_n^2}$ $\qquad \boxed{\frac{c}{m} = \left(\frac{c}{c_{r.}}\right) \cdot \left(\frac{c_{rr.}}{m}\right)}$

* damping ratio $= \zeta = \dfrac{c}{c_{cr}}$

$s_{1,2} = \dfrac{-c}{2m} \mp \sqrt{\left(\dfrac{c}{2m}\right)^2 - \dfrac{4km}{4m^2}}$

$\to s_{1,2} = \dfrac{-c}{c_{cr}} \cdot \dfrac{c_{cr}}{m} \pm \sqrt{\left[\dfrac{c}{c_{r}} \cdot \dfrac{c_{r}}{m} \cdot \dfrac{1}{2}\right]^2 - w_n^2}$

$\frac{C}{C_{cr}} = \zeta$; $\frac{C_{cr}}{2m} = \frac{2\sqrt{K}\cancel{\sqrt{m}}}{2\sqrt{m}\cancel{\sqrt{m}}}$

zeta

$$\frac{C_{cr}}{2m} = \frac{\sqrt{K}}{\sqrt{m}} = \omega_n$$

* Then:-

$$S_{1,2} = -\zeta\omega_n \mp \sqrt{\zeta^2\omega_n^2 - \omega_n^2}$$

$$= \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n \quad ;$$

1- If $\zeta > 1$, +ve → over damping system

2- If $\zeta = 1$; 0 → critacally

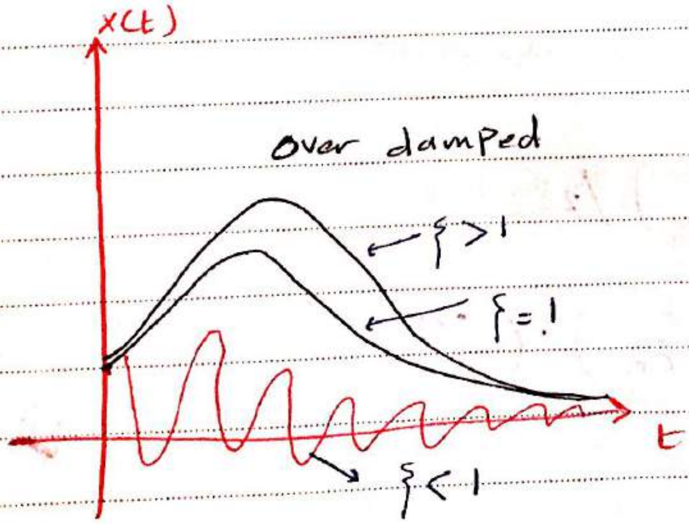
Vibration. ← 3- If $\zeta < 1$; -ve → under damped

* case 1:- $\zeta > 1$

$S_1 = \left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n$; $S_2 = \left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n$.

⇝ soln:- $x(t) = C_1 e^{S_1 t} + C_2 e^{S_2 t}$

$x(t) = C_1 e^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t} + C_2 e^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t}$



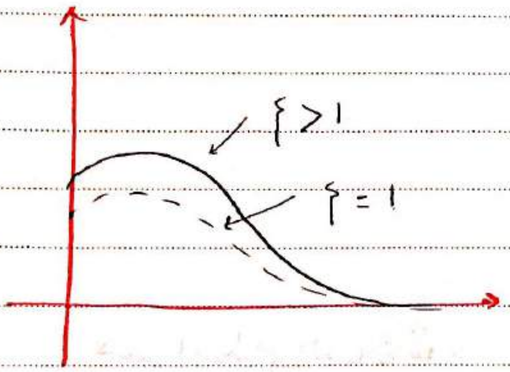
* Case 2 :- $\xi = 1$

$s_1 = -\xi w_n \quad ; \quad s_2 = -\xi w_n$

$x(t) = D_1 e^{s_1 t} + D_2 (t) e^{s_2 t}$

$$\rightarrow \boxed{x(t) = D_1 e^{-\xi w_n t} + (D_2)\, t\, e^{-\xi w_n t}}$$

$x(t) = (D_1 + t D_2)\, e^{-\xi w_n t}$



*case 3 :- $\xi < 1$

$s_1 = (-\xi - \underbrace{\sqrt{\xi^2 - 1}}) \, w_n = (-\xi - i\sqrt{1-\xi^2})\, w_n$

حيطلع سالب

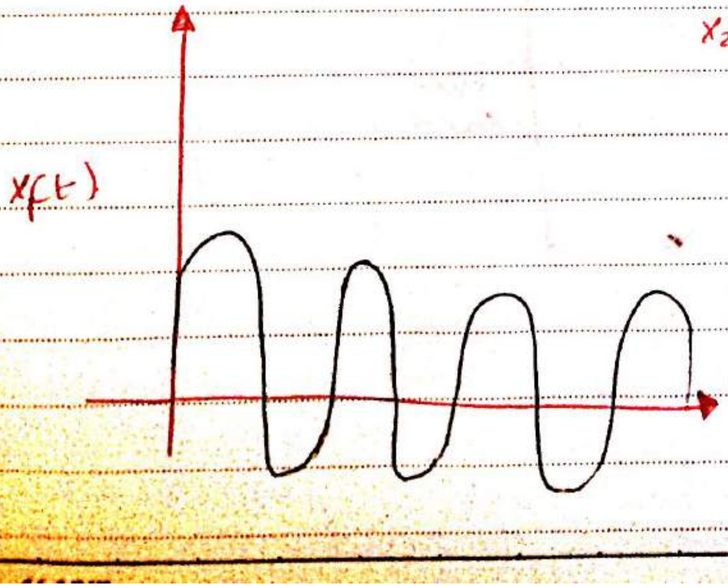
$s_2 = (-\xi + i\sqrt{1-\xi^2})\, w_n$

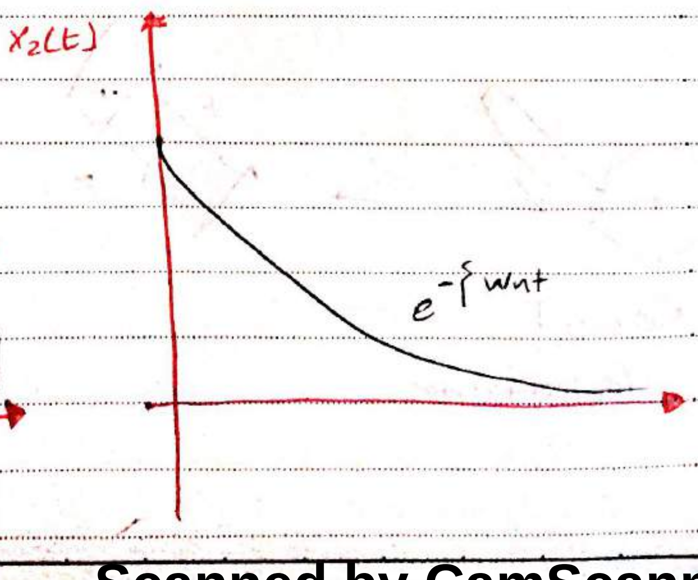
$x(t) = A^* e^{(-\xi - i\sqrt{1-\xi^2}) w_n t}$
$\quad + B^* e^{(-\xi + i\sqrt{1-\xi^2}) w_n t}$

*Note that :- $e^{i\theta} = \cos\theta + i\sin\theta$
$e^{-i\theta} = \cos\theta - i\sin\theta$

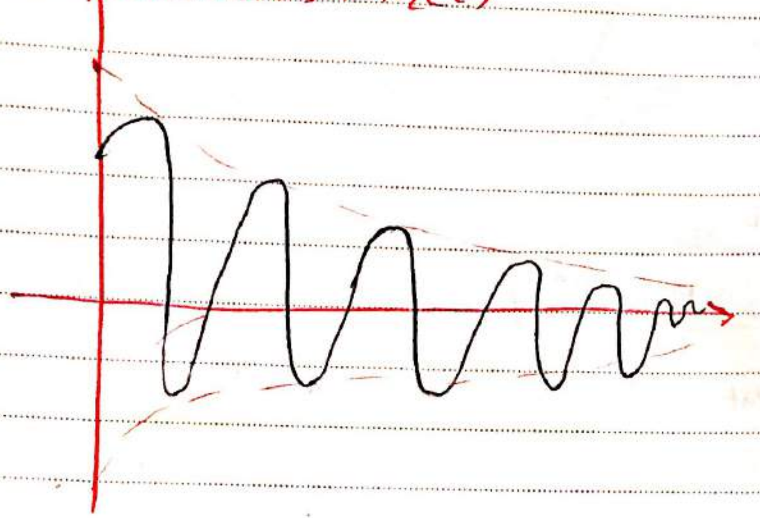
$A^* + iB^*$
$A^* - iB^*$

$$x(t) = \underbrace{\left[C \sin(\sqrt{1-\xi^2}\, w_n t + \phi) \right]}_{x_1 t} \underbrace{e^{-\xi w_n t}}_{x_2 t}$$



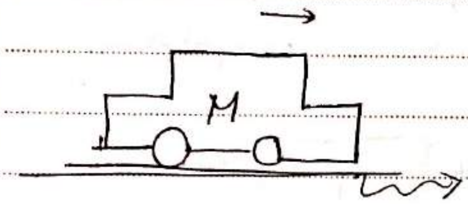


$x(t) = x_1(t) + x_2(t)$

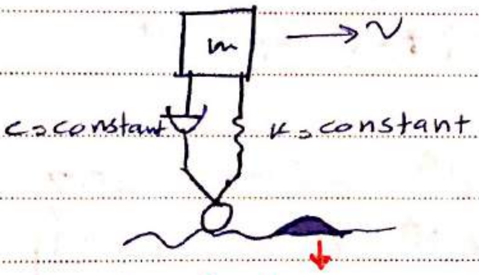


Oscillations + Decaying

* Ex 1 :



*عند احجام السيارة بتكون
Over Damped.

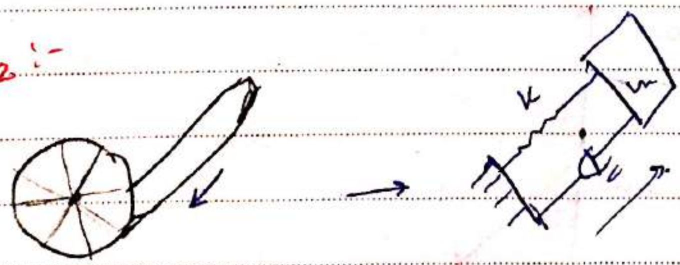


بدي مطب عشان نغير
ال Oscillation

$\underset{\text{بتغير ال mass}}{C_{cr}} = \sqrt{4Km} = 2\sqrt{K}\sqrt{m}$; $\zeta = \frac{C}{C_{cr}} = \frac{C}{2\sqrt{K}\sqrt{m}}$

*لما تزيد m ← بتقل ← بتصير Under damped

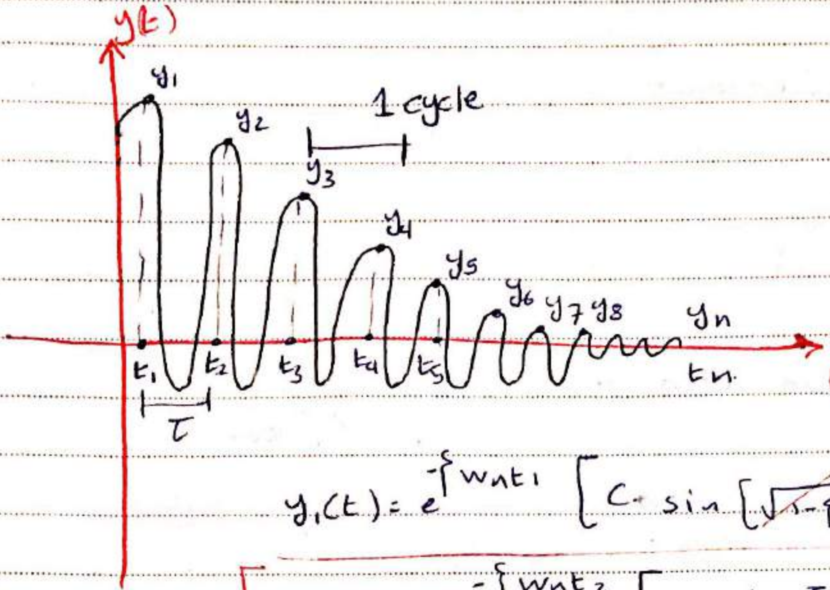
* Ex 2 :-



the same

Recoil
Mechanisim

/ /



* τ_d = damped Period

$\longrightarrow t_2 = t_1 + \tau$

* w_d : damped Nat. frequency.

$$y_1(t) = e^{-\xi w_n t_1}\left[C \sin\left[\cancel{\sqrt{1-\xi^2}\, w_n t_1 + \phi}\right]\right]$$

$$y_2(t) = e^{-\xi w_n t_2}\left[C \sin\left[\cancel{\sqrt{1-\xi^2}\, w_n t_2 + \phi}\right]\right]$$

divide y_1/y_2

$$\frac{y_1(t)}{y_2(t)} = \left(e^{-\xi w_n t_1}\right)\cdot\left(e^{+\xi w_n t_2}\right)$$

$$\longrightarrow \frac{y_1(t)}{y_2(t)} = e^{(-\xi w_n t_1 + \xi w_n t_2)}$$

$$\rightarrow \frac{y_1(t)}{y_2(t)} = e^{-\xi \cancel{w_n t_1} + \xi \cancel{w_n t_1} + \xi w_n \tau}$$

$$\frac{y_1(t)}{y_2(t)} = e^{\xi w_n \tau_d}$$

$\Rightarrow \sqrt{1-\xi^2}\, w_n$ = damped Nat. frequency.

$$\sqrt{1-\xi^2}\, w_n = w_d$$

* $\tau_d = \dfrac{2\pi}{w_d}$

$$\tau_d = \frac{2\pi}{w_n\sqrt{1-\xi^2}}$$

$$\frac{y_1(t)}{y_2(t)} = e^{\xi w_n\left[\frac{2\pi}{w_n(\sqrt{1-\xi^2})}\right]}$$

$$\xi = \frac{C}{2\sqrt{Km}}$$

$$\boxed{\frac{y_1(t)}{y_2(t)} = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}}}$$

/ /

* δ = logarithmic decremant

$$\delta = \ln\left[\frac{y_1(t)}{y_2(t)}\right] = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

* If we find y_1 & y_2 we can measure $\zeta = \frac{C}{2\sqrt{km}}$

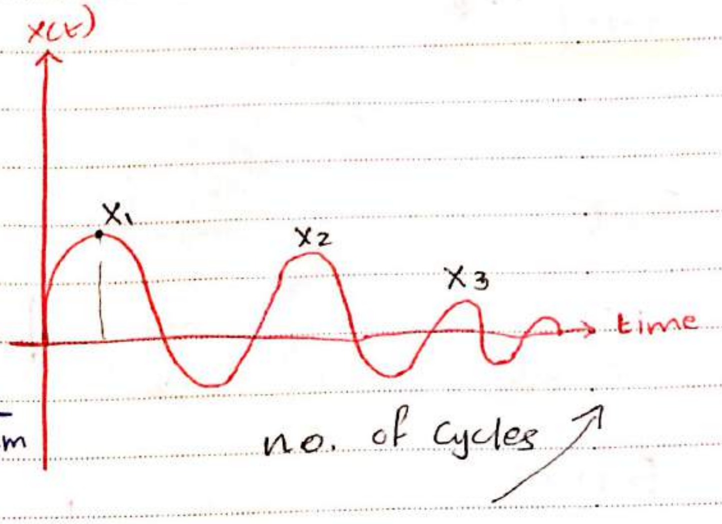
* $w_n^2 = \frac{k}{m}$ $\rightarrow w_d = w_n\sqrt{1-\zeta^2}$

* $\tau_d = \frac{2\pi}{w_d} = \frac{2\pi}{w_n\sqrt{1-\zeta^2}}$ → ملخص القوانين

* $\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\ln\left(\frac{x_1}{x_n}\right) = \ln\underbrace{\left[\frac{x_1}{x_2}\cdot\frac{x_2}{x_3}\ \cdots\ \frac{x_{n-1}}{x_n}\right]}_{\text{for n cycles}}$$



$$\delta = \frac{1}{n}\ln\left[\frac{x_1}{x_{n+1}}\right] = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \ ; \ \xi = \frac{c}{\sqrt{4km}}$$

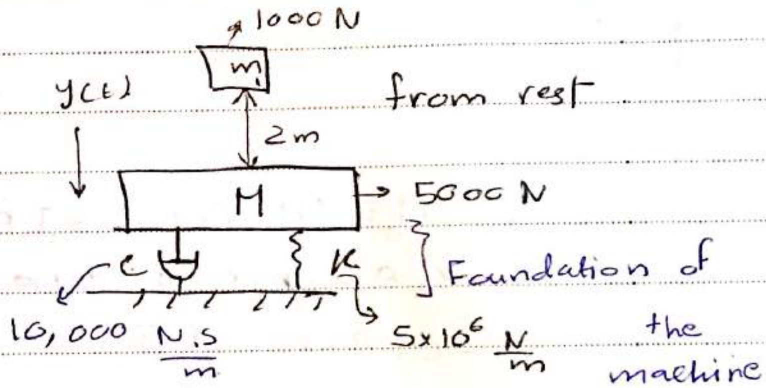
n = no. of cycles

Exp (2-6) Forcing process

impact [e ≡ coefficient of restitution = 0.4]

عماتجب بعد ال impact للجسمين نفس السرعة

الإزاحات Response ?



$e = 0$

$$y(t) = e^{-\xi w_n t}\left[y_o \cos w_d t + \frac{\dot{y}_o}{w_d}\sin w_d t\right] \ ; \ w_d = w_n\sqrt{1-\xi^2}$$

y_o = 0, $\dot{y}_o$ = ??

نفس صيغة الي مع damping

velocity of M

بدل w_n بالـ w_d ضرب الإزاحة $e^{-\xi w_n t}$

from rest

$$* \ V_{m_1} = \sqrt{2gh} = \sqrt{2\times 9\times 2}$$

→ conservation of momentum

$$mV_{m_1} + M(0) = mV_{m_2} + MV_{M_2} \quad \dot{y}_o ? \ \text{---} \ ①$$

$$e = \frac{V_{M2} - V_{m2}}{V_{m_1} - 0} \ ; \ eV_{m_1} = \dot{y}_o - V_{m_2} \ \text{---} \ ② \ ; \ \dot{y}_o = 1.4616 \ m/s$$

eV_{m_1} = 2.5056, V_{m_2} = −1.044

= after impact / before impact

$$w_n = \sqrt{\frac{K}{M}} = 99.04 \ ; \ w_d = w_n\sqrt{1-\xi^2} = 98.553 \ ; \ \xi = \frac{C}{\sqrt{4KM}} = 0.099$$

المربوط فيه C

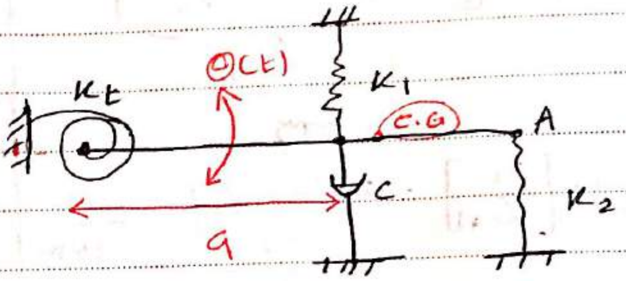
المربوط فيه

* نعوض في الأرقام بمعادلة الـ y(t) Response

2-11 } impo.
2-12 } EXP.

(Torsional Motion)

2-76



$\overline{OA} = (m) \rightsquigarrow$ "rigid" rod with mass

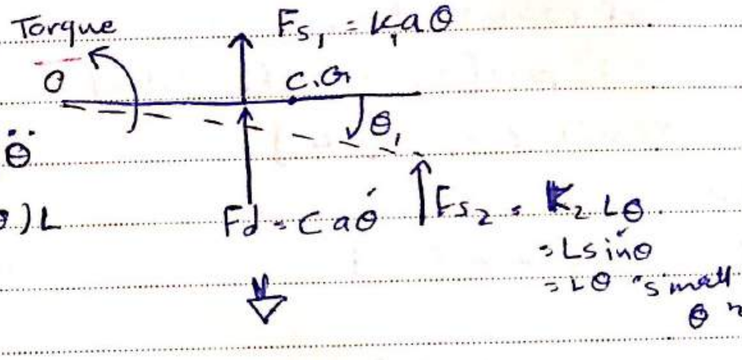
C.G @ $(\frac{L}{2}) \rightsquigarrow$ في منتصف الجسم

EOM ?

$W_n = ?$

$\zeta = ??$

F.B.D



+ve) $\sum M_o = \sum M\ \text{effective} = J_o \ddot{\theta}$

$-K_t\theta - (K_1 a\theta)\, a - (K_2 L\theta) L$

$- Ca\dot{\theta}(a) = J_o\ddot{\theta}$

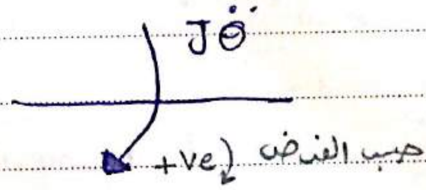
$\bar{J} = \frac{mL^2}{12}$

$J_o = \bar{J} + m(\frac{L}{2})^2$

$= (\frac{mL^2}{3})$

$$\left(\frac{mL^2}{3}\right)\ddot{\theta} + Ca^2\dot{\theta} + (K_t + K_1 a^2 + K_2 L^2)\,\theta = 0$$

EOM



$$\omega_n = \sqrt{\frac{K_t + K_1 a^2 + K_2 L^2}{\frac{mL^2}{3}}} \quad ; \quad \zeta = \frac{Ca^2}{2\sqrt{(K_t + K_1 a^2 + K_2 L^2)\cdot(mL^2/3)}}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

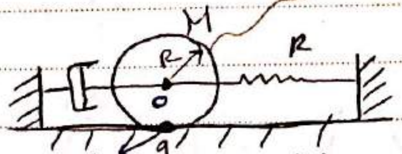
$\zeta > 1$

$\zeta = 1$

$\zeta < 1$

sugg.
2.74
2.78
2.102
2.113
2.114 | 2-112

General plane Motion



IC pure - Rolling → Type of Motion
without slipping

EOM?
Wn?
Wd
ζ

at Ic.

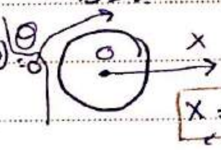
$\underline{\sum M_{@}} = \sum M_{effective}$

$J_o = \frac{MR^2}{2}$ ↓ for disk

$-CR\dot{x} - KRx = J\ddot{\theta} + (m\ddot{x})R$

F.B.D

$(J\ddot{\theta} + mR(R\ddot{\theta})) + CR(R\dot{\theta}) + KR(R\theta)$

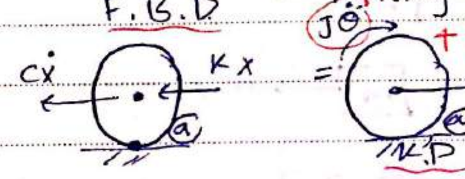


$$\boxed{(mR^2 + J)\,\ddot{\theta} + CR^2\dot{\theta} + KR^2\theta = 0}$$; EOM

$\boxed{x = R\theta}$ because of pure Rolling

$$w_n = \sqrt{\frac{KR^2}{(J + mR^2)}}$$ ⇝ مابتقدر تكتبها بدلالة x لأنها Physical Parameter

F.B.D



General plane Motion
rotational + Translational

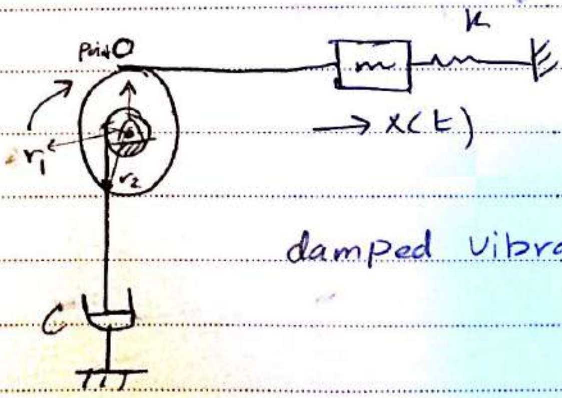
$$\zeta = \frac{CR^2}{2\sqrt{KR^2(mR^2 + J)}}$$ → معامل $\dot{\theta}$; → $\sqrt{\text{معامل } \ddot{\theta} \cdot \text{معامل } \theta}$

$\dot{x} = R\dot{\theta}$
$\ddot{x} = R\ddot{\theta}$

2-119

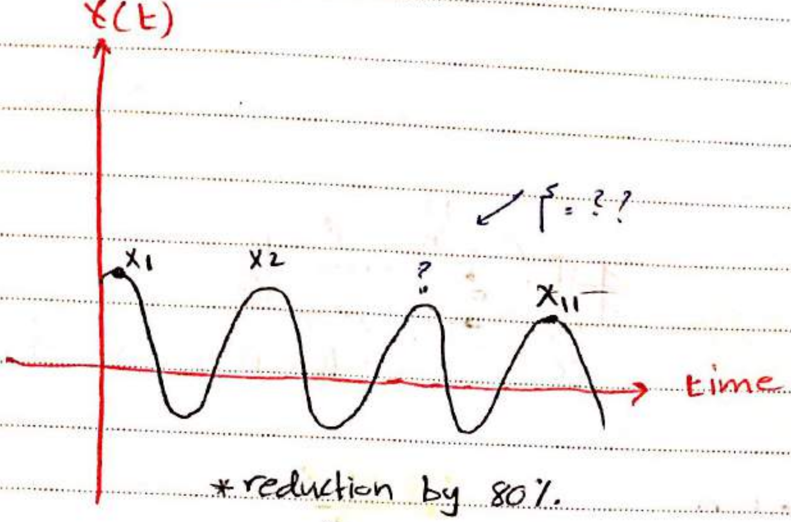
$w_n = 5\,Hz$
$m = 10\,kg$ K?
$J_o = 5\,kg.m^2$ amount of damping?
$r_1 = 10\,cm$
$r_2 = 25\,cm$

J_o
$\theta(t)$



damped Vibration

/ /



$$\delta = \frac{1}{10}\ln\left[\frac{x_1}{x_{11}}\right] = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

step $\boxed{1}$

* reduction by 80%.

in 10 cycles

ذكر عدد ال cycles و δ.

$$\delta = \frac{1}{10}\ln\left[\frac{1}{0.2}\right] = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

1-0.8

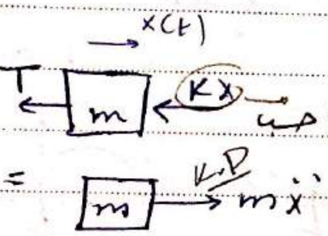
* F.B.D.

step $\boxed{2}$

$\xrightarrow{+ve} \Sigma F = m\ddot{x}$; $-T - Kx = m\ddot{x}$ --- (1) for the traslation

$\curvearrowleft_{+ve} \Sigma M_o = J_o\ddot{\theta}$; $T(r_2) - r_1 c r_1\dot{\theta} = J_o\ddot{\theta}$ --- (2) for rotation

1-



بنفرض اتجاه الحركة

حركة ال mass

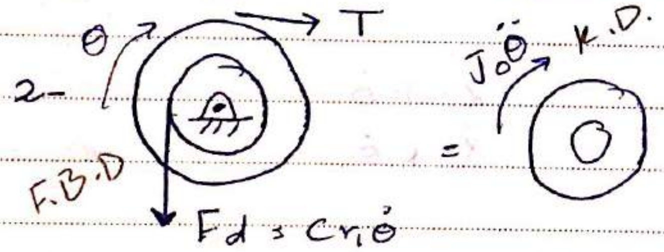
$x = r_2\theta$

$\dot{x} = r_2\dot{\theta}$

$\ddot{x} = r_2\ddot{\theta}$

2 masses, 2 coordinates but s.d.o dependent

2-



* from equation (1) ⟹

$T = -Kx - m\ddot{x}$

$$r_2\left[-Kx - m\ddot{x}\right] - c r_1^2\left[\frac{\dot{x}}{r_2}\right] = J_o\left[\frac{\ddot{x}}{r_2}\right] \quad --- (3)$$

$$\boxed{\rightarrow\left[\frac{J_o}{r_2} + m r_2\right]\ddot{x} + c\frac{r_1^2}{r_2}\dot{x} + K r_2 x = 0}$$

EOM

δ بطلع من ← $\xi = \dfrac{c(r_1^2/r_2)}{2\sqrt{K r_2\left(\frac{J_o}{r_2} + m r_2\right)}}$

$\xi = 0.02561$

$\omega_n = 5\,Hz = 2\pi(5)\ Rad/s = \sqrt{\dfrac{k r_2 \to k_{eq}}{(\frac{J_o}{r_2} + m r_2) \to m_{eq}}} \Rightarrow k \Rightarrow N/m$

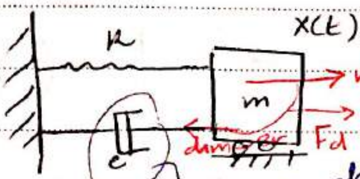
$\Downarrow$

$C \Rightarrow \dfrac{N.s}{m}$

$k = 8.8827 \times 10^4$

$C = 96.5134\ N.s/m$

* EXP:-



→ motion

→ $\theta = 180°$ ⇝ why negative sign(−)?!

→ damper [work done by the damper]

$$\boxed{m\ddot{x} + C\dot{x} + kx = 0}$$

$W_d \equiv$ work done by the damper

$$= \int_{r_1}^{r_2} \underbrace{\vec{F}_d}_{\text{damper force}} \cdot d\vec{r} = \int_{x_1}^{x_2} C(\underbrace{\vec{\dot{x}}}_{\text{velocity}}) \cdot dx$$

$$= -\int_{x_1}^{x_2} c(\dot{x})(dx) \qquad \boxed{\dot{x} = \frac{dx}{dt}}$$

$$W_d = -\int_{t_1}^{t_2} c\dot{x}\,\dot{x}\,dt = -c\int_{t_1}^{t_2} \dot{x}^2\,dt$$

2nd Law

using the Energy Method:-

$T = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{x}^2$

$V = \frac{1}{2}kx^2$

$(T+V)$ = work done by the damper

$$\frac{d}{dt}\left[(T+V)\right] = \frac{d}{dt}\left[W_d\right]$$

$$\frac{d}{dt}\left[\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right] = \frac{d}{dt}\left[-c\int_{t_1}^{t_2}\dot{x}^2\,dt\right]$$

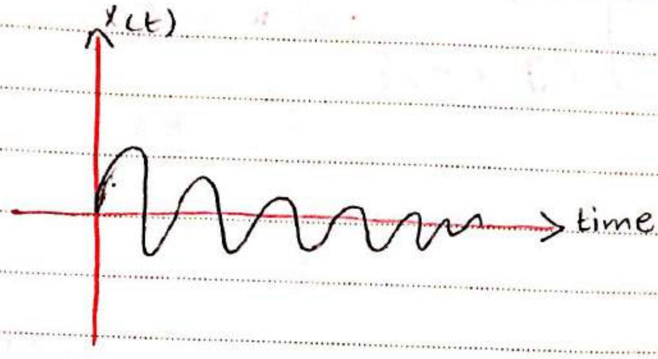
$$\left(\frac{1}{2}m\,\cancel{2}\dot{x}\ddot{x} + \frac{1}{2}k\,\cancel{2}x\dot{x} = -c\dot{x}^2\right)$$

$$\{m\ddot{x} + kx + c\dot{x}\}\dot{x} = 0$$

$\boxed{m\ddot{x} + C\dot{x} + kx = 0}$; EOM "spring mass-damper system"

Using the Energy Method.

* Damped vibration :



$* \; \delta = \dfrac{2\pi\xi}{\sqrt{1-\xi^2}}$

$* \; \delta = \dfrac{1}{n} \ln \left(\dfrac{x_1}{x_{n+1}} \right)$

Over 1 cycle ; assume simple Harmonic Motion "SHM"

$* \; \underline{V_0} = \frac{1}{2} k x_0^2 \rightarrow$ initial Energy

$\rightarrow \; x(t) = X \sin \omega_d t$

ΔW = work done by the damper over 1 cycle.

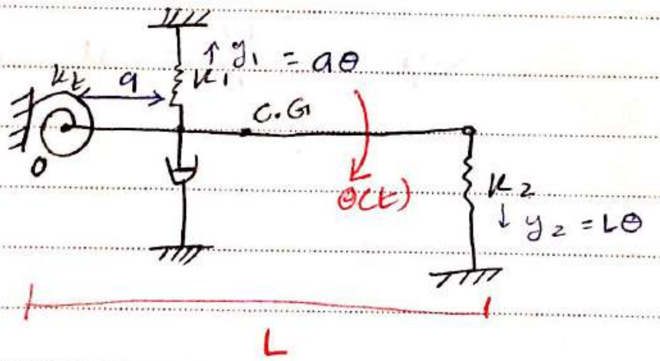
$\Longrightarrow \Delta W = \int_{t_1}^{t_2} c(\dot{x})^2 dt = \pi c X^2 \omega_d$

$\Delta W_{cycle})_{1cycle} = \pi c X^2 \omega_d$

$T_d = \dfrac{2\pi}{\omega_d}$ $\qquad \omega_n \sqrt{1-\xi^2}$

2-76

EOM



$T = \frac{1}{2} J_o \dot{\theta}^2 \quad ; \quad V = \frac{1}{2} k_1 y_1^2 + \frac{1}{2} k_2 y_2^2 + \frac{1}{2} k_t \theta^2$

"No masses"

$V = \frac{1}{2} \left[k_1 a^2 \theta^2 + k_2 L^2 \theta^2 + k_t \theta^2 \right]$

$\boxed{\begin{array}{l} dr = dy_1 \\ \dot{y}_1 = \dfrac{dy_1}{dt} \end{array}}$

displacement

$W_d = -\int \underbrace{c\dot{y}_1 (\dot{y}_1 dt)}_{F_d \cdot \vec{dr}} = -c \int_{t_1}^{t_2} a^2 \dot{\theta}^2 \, dt$

$\rightarrow \; \dfrac{d}{dt}(T+V) = \dfrac{d}{dt}(W_d)$

$$\rightarrow J_o\ddot{\theta}\dot{\theta} + [k_1a^2 + k_2l^2 + k_t]\theta\dot{\theta} + Ca^2\dot{\theta}^2 = 0$$

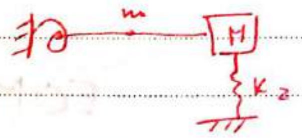
$$\rightarrow \dot{\theta}\underbrace{\left[J_o\ddot{\theta} + Ca^2\dot{\theta} + (k_1a^2 + k_2l^2 + k_t)\theta\right\}}_{EOM} = 0$$

$$J_o = \bar{J} + m\left(\frac{L}{2}\right)^2$$

$$J_o = \frac{mL^2}{12} + \frac{mL^2}{4} = \frac{mL^2}{3}$$

* Note :-

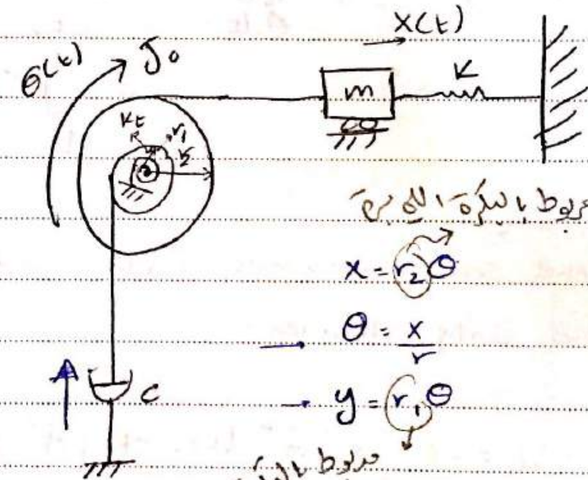
If there is a mass.



$$T = \frac{1}{2}J_o\dot{\theta}^2 + \frac{1}{2}M(L\dot{\theta})^2$$

* 2-119

EOM ??



$$T = \frac{1}{2}J_o\dot{\theta}^2 + \frac{1}{2}m\dot{x}^2$$

$$V = \frac{1}{2}kx^2 + \frac{1}{2}k_t\theta^2$$

$$W_d = -\int (c\dot{y})\,dy = -\int c\dot{y}(\dot{y}\,dt)$$

$$= -c\int_{t_1}^{t_2}\dot{y}^2\,dt$$

مربوط بالبكرة اللي برة

$$x = r_2\theta$$

$$\rightarrow \theta = \frac{x}{r}$$

$$\rightarrow y = r_1\theta$$

مربوط بالبكرة اللي جوا

$$\rightarrow T = \frac{1}{2}\left[J_o\dot{\theta}^2 + m r_2^2\dot{\theta}^2\right] ; V = \frac{1}{2}\left[k_t\theta^2 + kr_2^2\theta^2\right]$$

/ /

$$W_d = -c\int_{t_1}^{t_2} r_1^2\,\dot{\theta}^2\,dt$$

$$\rightsquigarrow \frac{d}{dt}(T+V) = \frac{d}{dt}[W_d].$$

$$\underbrace{\dot{\theta}\left\{(J_o + m r_2^2)\ddot{\theta} + c r_1^2\dot{\theta} + (k_t + r_2^2 k)\theta\right\}}_{EOM} = 0$$

* for torsional damper :-

$T_d = C_t\dot{\theta}$

same thing with torsional constant.

* Torsional Damper :-

$$W_d = \int(\vec{T_d})\cdot(d\vec{\theta}) = -\int c_t\,\dot{\theta}[\dot{\theta}\,dt]$$

Linear الـ $\dot{x}$ $\quad \dot{\theta}\,dt$

damper $\rightarrow F_d$

$$= -\int_{t_1}^{t_2} C_t\,\dot{\theta}^2\,dt$$

* for the same previous Exp. with torsional damper :-

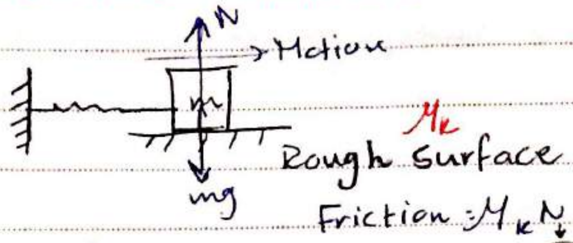
→ the only change :-

$$W_d = \underbrace{(-)c\int_{t_1}^{t_2} r_1^2\,\dot{\theta}^2\,dt}_{\text{linear}}\ \underbrace{(-)c_t\int_{t_1}^{t_2}\dot{\theta}^2\,dt}_{\text{Torsional}}.$$

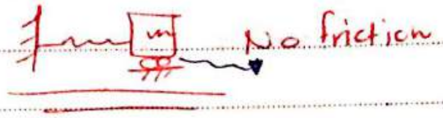
|
|
|

Wed. 28/10/2015

* Motion with friction force :-



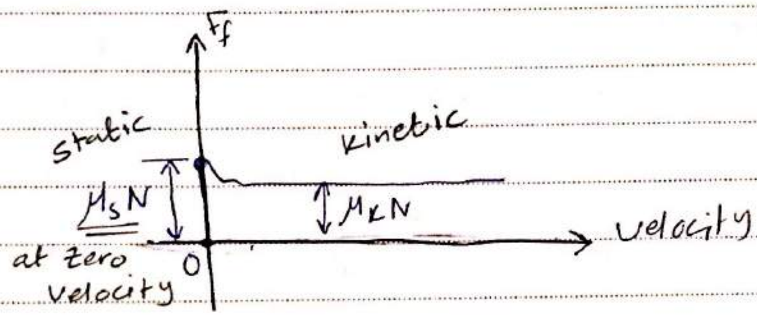
Friction $= \mu_k N$



Dry-friction $\qquad F_f = \mu_k\, mg = \text{constant} \rightarrow$ هنا كانت متغيرة

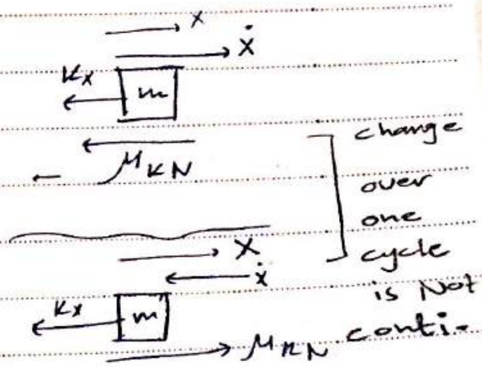
⇓ $\qquad$ ⇓ no viscous friction.

columb friction

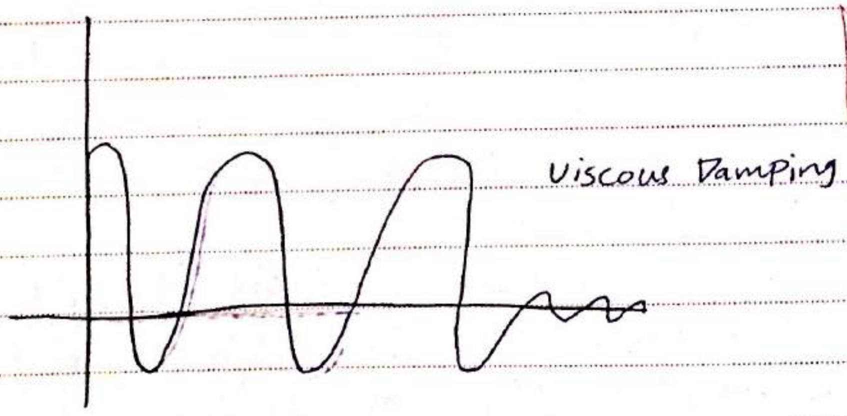


* Not continuous response like Viscouse friction & we calculate restoring the response every $\frac{1}{2}$ cycle.

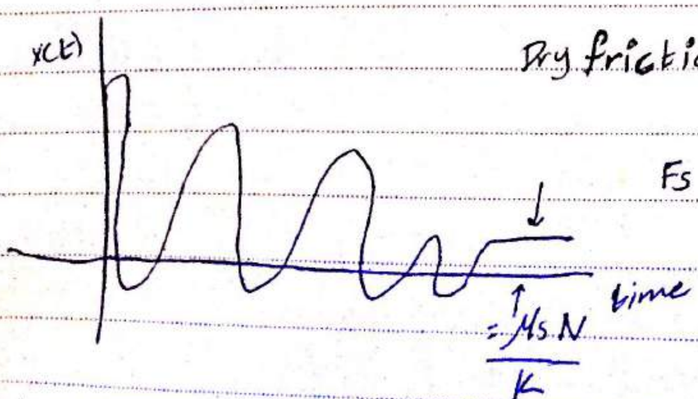
$\dot{x}$ of 1/2 cycle will be initial condition for the second cycle.

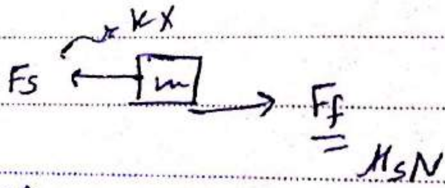


$$\boxed{\begin{aligned} m\ddot{x} + kx &= \mu_k N \\ &= -\mu_k N \end{aligned}}$$



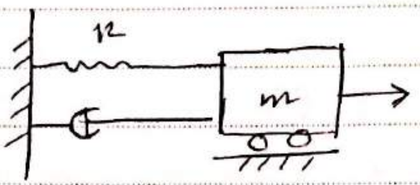
free Vibration





→ Vibratory systems subjected to Harmonic Excitation.

$\sin \omega t, \cos \omega t$

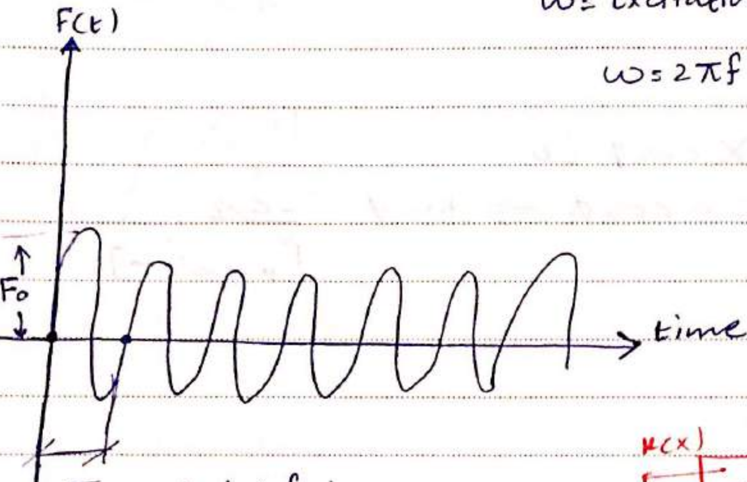


$F(t) = F_o \sin \omega t$

↗ Excitation level

ω = Excitation frequency (rad/s)

$\omega = 2\pi f = \frac{2\pi}{\tau}$



τ = period ; $f = \frac{1}{\tau}$

to complete 1 cycle

$F(t) - c\dot{x} - kx = m\ddot{x}$

EOM $\boxed{m\ddot{x} + c\dot{x} + kx = F_o \sin \omega t}$

Response ⟹ x(t)

homo. solution, particular Integral

$(X_h(t)) + (X_p(t))$

2nd, ODE, linear, Non-Homogeneous

$X_h(t)$ = free vibration $= e^{-\zeta \omega_n t} \left[\underset{C_1}{A_o} \cos \omega_n \sqrt{1-\zeta^2}\, t + \underset{C_2}{B_o} \sin \underbrace{\omega_n \sqrt{1-\zeta^2}}_{\omega_d}\, t \right]$

$X_p(t)$ ≡ particular Integral   1 function, phase shift

$X_p(t) = A \sin \omega t + B \cos \omega t = X \sin(\omega t + \phi)$

$= X \sin \omega t \cos\phi + X \cos \omega t \sin\phi$

$\dot{X}_p(t) = \omega A \cos \omega t - \omega B \sin \omega t = \omega X \cos \omega t \cos\phi - \omega X \sin \omega t \sin\phi$

$\ddot{X}_p(t) = -\omega^2 A \sin \omega t - \omega^2 B \cos \omega t = -\omega^2 X \sin \omega t \cos\phi - \omega^2 X \cos \omega t \sin\phi$

$-\omega^2 m X \{ \sin \omega t \cos\phi + \cos \omega t \sin\phi \} + c\omega X \{ \cos \omega t \cos\phi - \sin \omega t \sin\phi \}$
$+ kX \{ \sin \omega t \cos\phi + \cos \omega t \sin\phi \}$
$= F_o \sin \omega t$

* sin wt , coefficient

$$-w^2 m X \cos\phi - cwX\sin\phi + kX\cos\phi = F_o \quad (1)$$

* cos wt

$$-w^2 m X \sin\phi + cwX\cos\phi + kX\sin\phi = 0 \quad (2)$$

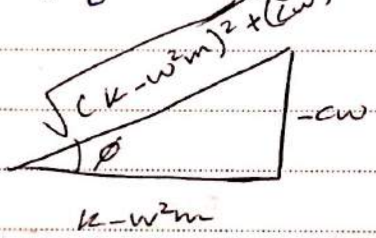
مافي عندي cosine term عشانهيك = 0

* from equ.(2)

$$X[k - w^2 m]\sin\phi + cwX\cos\phi = 0$$

$$\sin\phi\,[k - w^2 m] = -cw\cos\phi \Rightarrow \tan\phi = \frac{-cw}{[k - w^2 m]}$$

;



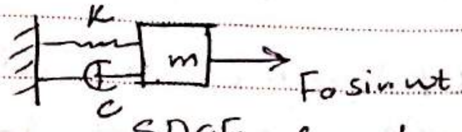
* substitute into eq(1) :-

$$X[k - w^2 m]\frac{(k - w^2 m)}{\sqrt{--}} + cwX\frac{(cw)}{\sqrt{}} = F_o$$

$$\frac{X(k - w^2 m)^2}{\sqrt{}} + \frac{X(cw)^2}{\sqrt{}} = F_o \; ; \; X\underbrace{\left[(k - w^2 m)^2 + (cw)^2\right]^{1/2}} = F_o$$

$$X = \frac{F_o}{\sqrt{(k - mw^2)^2 + (cw)^2}} \; ; \; \phi = \tan^{-1}\left(\frac{-cw}{k - mw^2}\right)$$

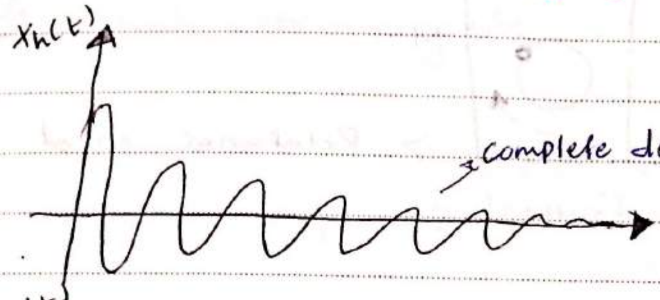
$x_p(t) = X\sin(wt + \phi)$



SDOF , forced

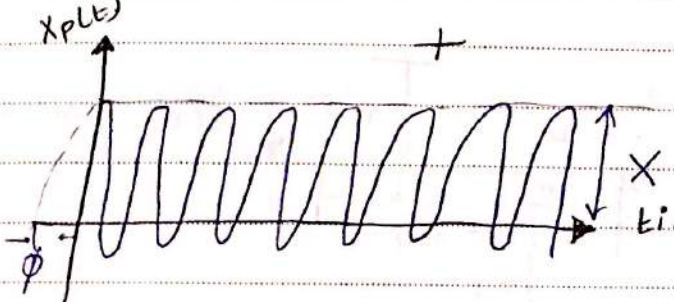
/ /

→ Response of the vibratry system:- $x(t) = x_h(t) + x_p(t)$



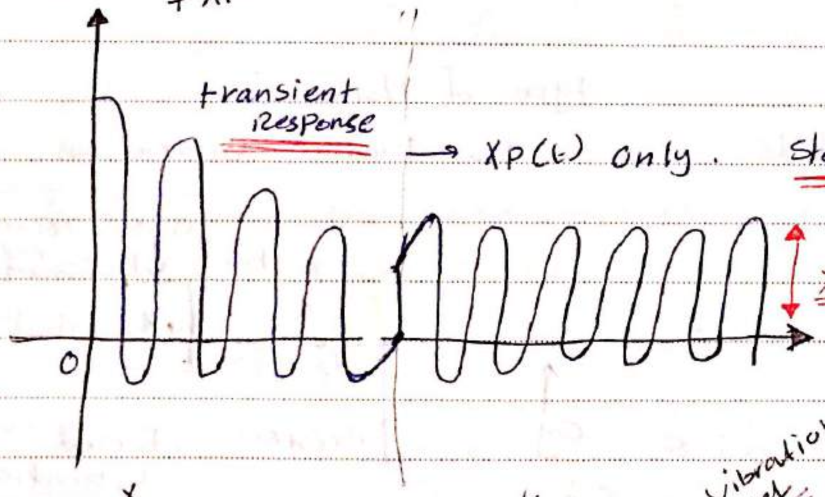
$\Rightarrow \omega_d = \omega_n \sqrt{1-\zeta^2}$

+



$\Rightarrow$ freq. $= \omega$.

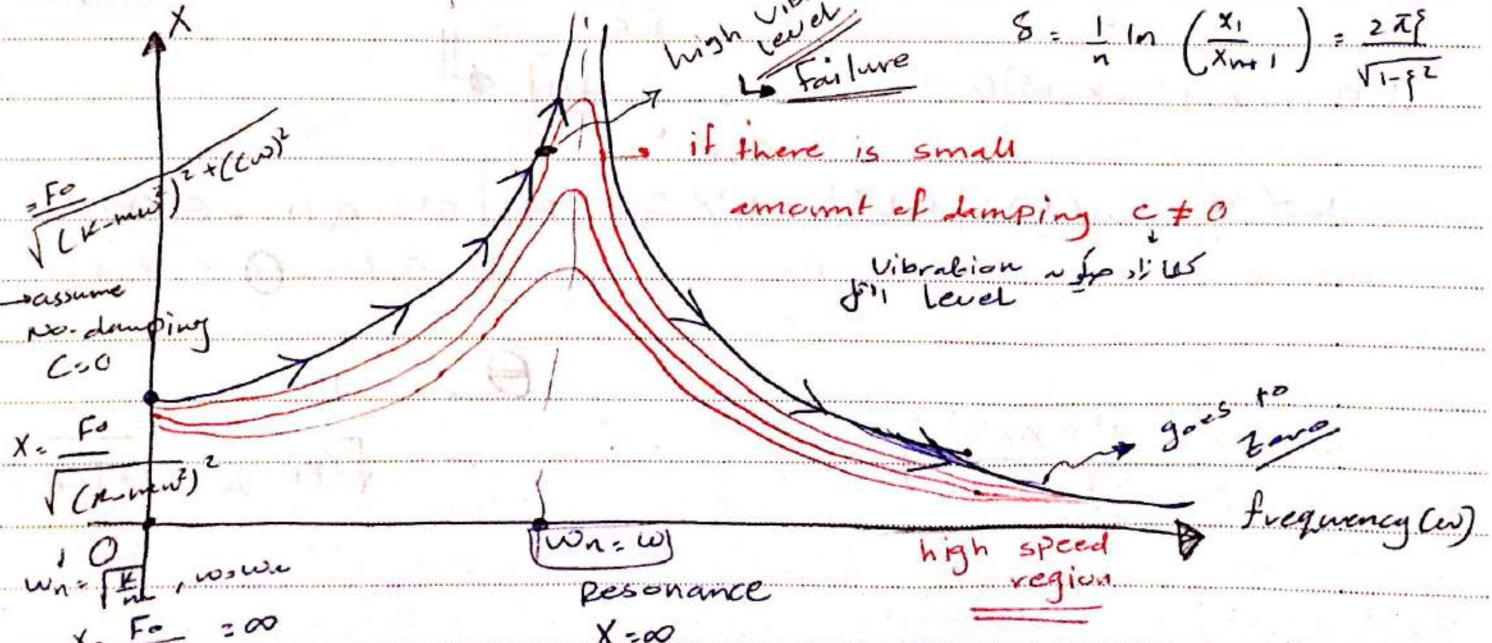
$x(t) = x_h(t) + x_p(t)$ =



* Free Vibration :- ببتو منها ؟؟؟

ω_n, c

$\delta = \frac{1}{n} \ln \left(\frac{x_1}{x_{n+1}}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$



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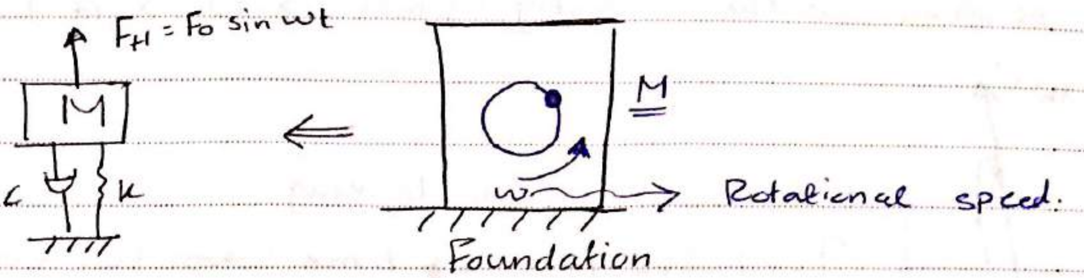


3-24

C.G الإزاحة دي

$m = 10\ kg,\ L = 1m$

$M = 50\ kg$

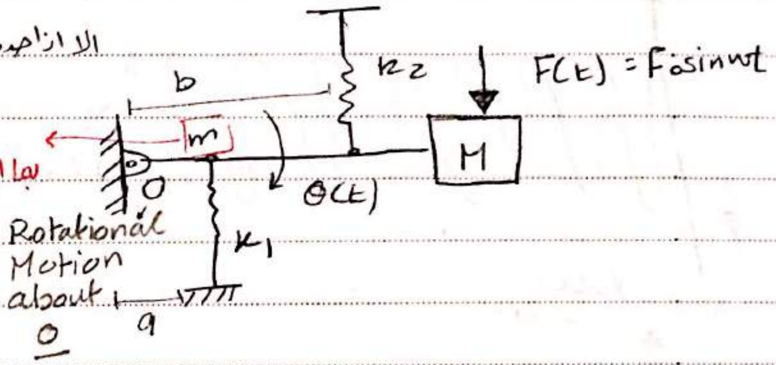
$a = 0.25,\ b = 0.5$

$F_0 = 500\ N;\ w = 1000\ rpm$

$k_1 = k_2 = 5000\ N/m$

→ EOM, steady state amplitude.

uniform rod mass بما إنه ... خال ال C.G بالنص

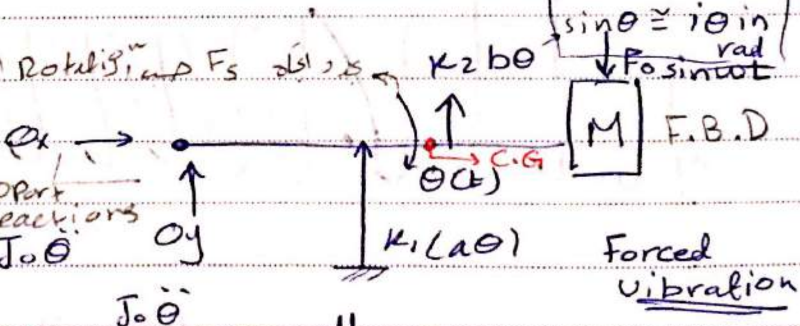


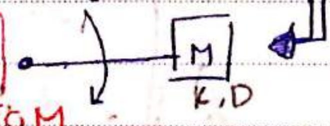
type of Motion:
Rotational Vibration.

→ Rotational Motion

$+ve) \sum M_o = J_o \ddot{\theta}$

$L F_0 \sin wt - \underbrace{(k_1 a\theta)a}_{F_{s_1}} - (k_2 b\theta)b = J_o \ddot{\theta}$





$$\boxed{J_o \ddot{\theta} + (k_1 a^2 + k_2 b^2)\theta = L F_0 \sin wt}$$

EOM

$J_o = \left(\frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2 + ML^2\right) = L^2\left(\frac{m}{3} + M\right)$

rod لل — parallel axis theorem

$\theta(t) = \theta_h(t) + \theta_p(t)$

$\theta_p(t) = \Theta_{s.s} \sin(\omega t + \phi)$

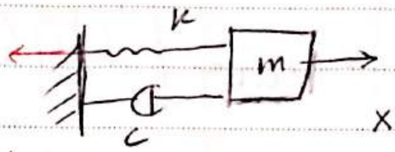
$\Theta_{s.s} = \dfrac{M_o}{\sqrt{(k_t - J_o w^2)^2 + [c_t w]^2}}$

$\omega_n^2 = \left(\dfrac{k_1 a^2 + k_2 b^2}{J_o}\right);$

/ /

مقارنة مع المحاضرة الماضية

$\Theta_{s.s}$ = steady state amplitude



$X_p(t) = X\sin(wt+\phi)$

$$= \frac{LF_0}{\sqrt{((k_1a^2+k_2b^2)-J_0w^2)^2+(0)}}$$ undamped forced vibration

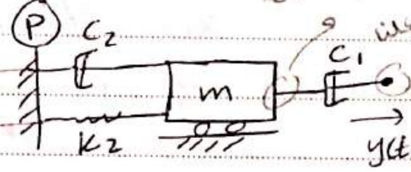
$$X = \frac{F_0}{\sqrt{(k-mw^2)^2+(cw)^2}}$$

$w = 1000 * \frac{2\pi}{60}$ [rad/s]

$m\ddot{x} + c\dot{x} + kx = F_0 \sin wt$

$J_0\ddot{\theta} + C_t\dot{\theta} + k_t\theta = M_0 \sin wt$

3-35



$y(t) = Y\cos wt$

x(t) تابع

$(\dot{x}-\dot{y})$ عشان هو بين

تابع للزمن y(t)

Sugg.
3-25
3-35

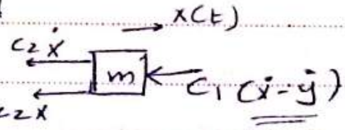
a) EOM

b) X_{ss}

c) Force → transmitted to the support at P.

translational Motion

f.B.d

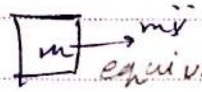


net velocity

$\overset{+ve}{\rightarrow} \sum F = m\ddot{x}$; $-c_2\dot{x} - k_2x - c_1(\dot{x}-\dot{y}) = m\ddot{x}$

$\underbrace{m\ddot{x}}_{mass} + \underbrace{\{c_1+c_2\}}_{damping}\dot{x} + \underbrace{k_2x}_{stiffness} = c_1\dot{y} = c_1[-wY\sin wt]$

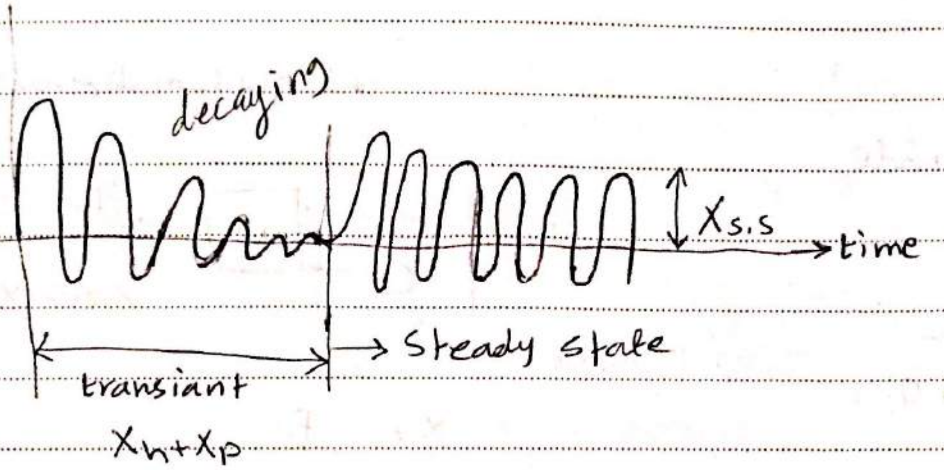
$= \underbrace{-wc_1Y}_{F_0}\sin wt$ | k.d

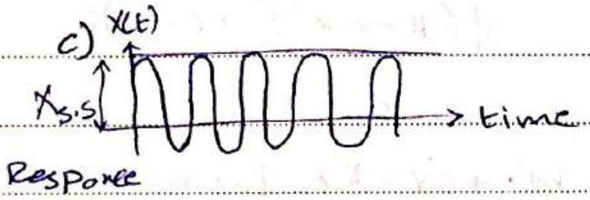


equivalent Kinetic diagram

$$X = \frac{(-wc_1Y)}{\sqrt{(k_2-mw^2)^2+((c_1+c_2)w)^2}}$$; steady state amplitude

sugg.
3-47
3-46
3-49





at the support $[F = F_c + F_s]$

$x_p(t) = X\sin(\omega t + \phi)$

$F_c = C_2\,\dot{x}_p(t) = C_2\omega X\cos(\omega t + \phi)$

$\phi = \tan^{-1}\dfrac{-c\omega}{(k - m\omega^2)}$

$F_s = K_2 X_p(t) = K_2 X\sin(\omega t + \phi)$

$\dot{x}_p(t) = \omega X\cos(\omega t + \phi)$

[illegible] Phase shift

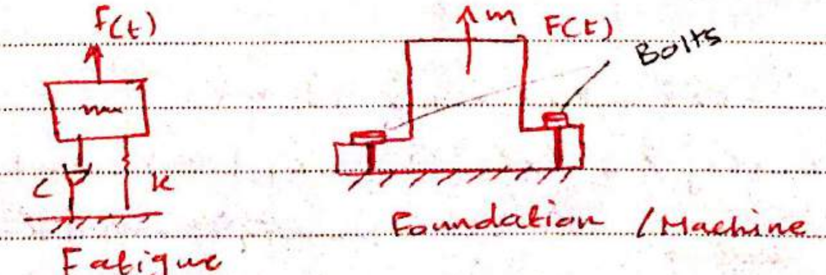
$*F_{support} =$

$-C_2\omega X\cos(\omega t + \phi) + K_2 X\sin(\omega t + \phi)$

$= (C_2\omega X + K_2 X)^{1/2}\sin(\omega t + \underline{\underline{\Psi}})$

Phase shift

Mathmateecal Modeling ←



tension [illegible] Comp.

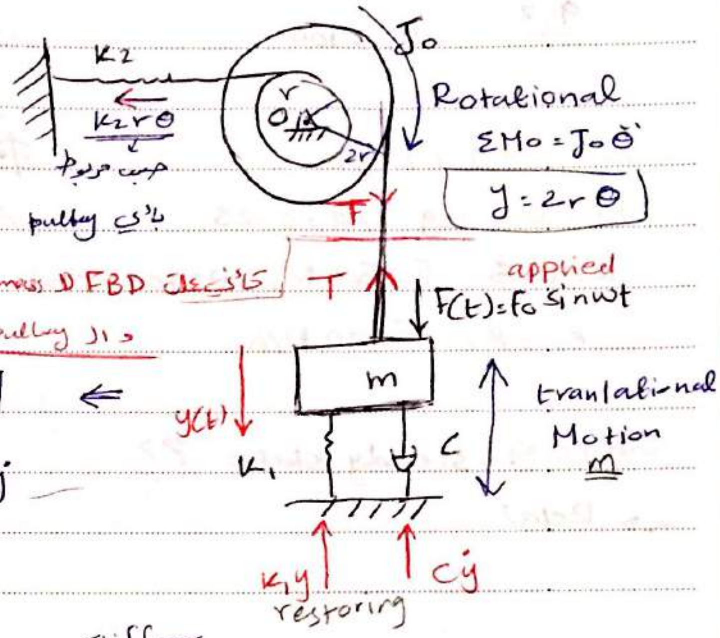
3-48

Steady state Response ?

$$-k_1 y - c\dot{y} + F_o \sin\omega t - T = m\ddot{y} \; ; \quad (1)$$

$$T = F_o \sin\omega t - k_1 y - c\dot{y} - m\ddot{y}$$

$$T(2r) - k_2 (r\theta) r = J_o \ddot{\theta}$$

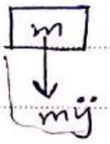


كأني عملت FBD لل mass لحال

و ال pulley لحال وعوضت عن T من المعادلة الأولى

$$J_o\ddot{\theta} + k_2 r^2 \theta - 2r T =$$

بتأثير



$$= J_o\ddot{\theta} + k_2 r^2 \theta - 2r \begin{bmatrix} F_o \sin\omega t \\ -k_1 y - c\dot{y} \\ -m\ddot{y} \end{bmatrix} = 0$$

$$\left(\underbrace{J_o \frac{\ddot{y}}{2r} + 2rm\ddot{y}}_{\text{Inertia terms}}\right) + \underbrace{2rc\dot{y}}_{\text{damping terms}} + \left(\underbrace{2rk_1 y + k_2 r^2 \cdot \frac{y}{2r}}_{\text{stiffness}}\right) = 2rF_o \sin\omega t \quad \boxed{EOM}$$

$$\rightarrow (m_{eq})\ddot{y} + c_{eq}\dot{y} + k_{eq} y = (2r F_o)\sin\omega t$$

$$Y = \frac{2rF_o}{\sqrt{(k_{eq} - m_{eq}\omega^2)^2 + (c_{eq}\omega)^2}}$$

هاي تحويشة لأنه مجمعة السبرنج والمخمد / ما بتغير الحالة

$$X = \frac{F_o / k}{\dfrac{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}{k}}$$

$$X = \frac{(\delta_{st})}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{c\omega}{k}\right)^2}}$$

$$2\zeta\frac{\omega}{\omega_n}$$

$$X = \frac{\delta_{st}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \quad ; \; r = \text{ratio of } \left(\frac{\omega}{\omega_n}\right)$$

amplitude ratio or Magnification ratio

$$\frac{c\omega}{k} = \frac{c}{c_{cr}} \cdot \frac{c_{cr} \cdot \omega}{k}$$

$$M \quad \left(\frac{X}{\delta_{st}}\right) = \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \quad \text{dimension less form}$$

$$= \zeta \frac{2\sqrt{k}\sqrt{m}\,\omega}{\sqrt{k}\sqrt{k}}$$

Steady state amplitude

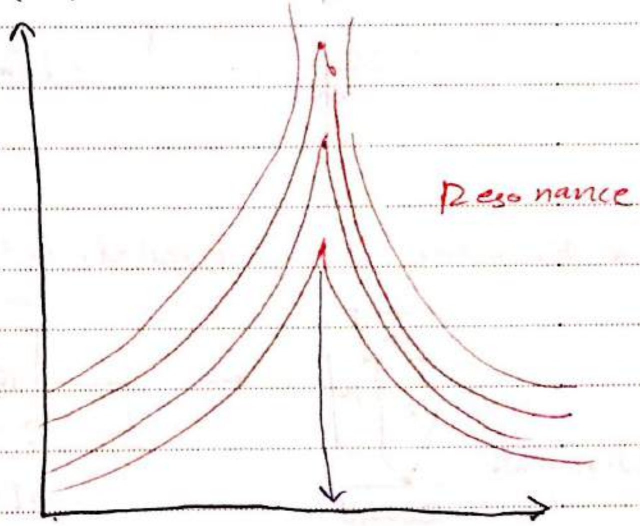
$$X = \frac{\delta_{st}}{\sqrt{(1-r^2) + (2\xi r)^2}}$$; r = ratio of $\left(\frac{w}{w_n}\right)$

$$M = \left(\frac{X}{\delta_{st}}\right) = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

amplitude ratio,

Magnification Ratio → steady state amplitude

$M = \left(\frac{X}{\delta_{st}}\right)$



Frequency -Response -Curves. $r = \frac{\omega}{\omega_n}$

$$X = \frac{F_o/k}{\sqrt{((k-m\omega^2)^2 + (c\omega)^2)}/k}$$

$$\frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$\delta_{st} = F_o/k$

$\xi = \frac{c}{C_{cr}}$, $C_{cr} = 2\sqrt{km}$

$r = \frac{\omega}{\omega_n}$, $\omega_n = \sqrt{\frac{k}{m}}$

for $\xi = 0$

$$\frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1-r)^2}} = \infty$$

* max $\left(\frac{X}{\delta_{st}}\right)$ occurs at $r = \sqrt{1-2\xi^2}$, just valid for $0 < r < \frac{1}{\sqrt{2}}$

$$\frac{dM}{dr} = 0 \rightarrow \frac{d}{dr}\left[\frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}\right] = 0 \quad ?$$

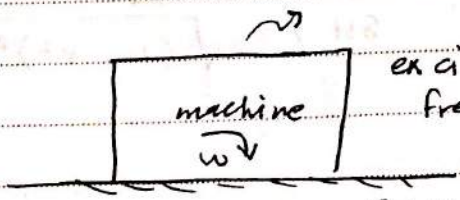
or $\sqrt{1-2\xi^2}$

$$\frac{X}{\delta_{st}}\Big|_{max} = M\Big|_{max} = \frac{1}{\sqrt{\left(1-(1-2\zeta^2)^2\right)^2 + \left(2\zeta\sqrt{1-2\zeta^2}\right)^2}}$$

$$M|_{max} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

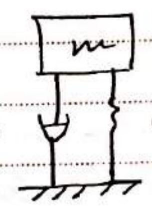
$$\hookrightarrow \frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$r = \frac{\omega}{\omega_n}$, $\zeta = \frac{c}{2\sqrt{km}}$



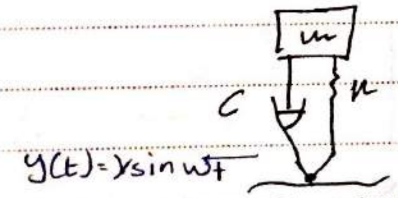
steady state [Full speed]

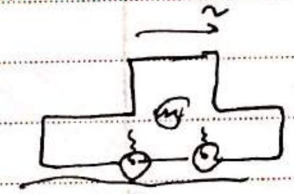
* Harmonic Basic Excitation:



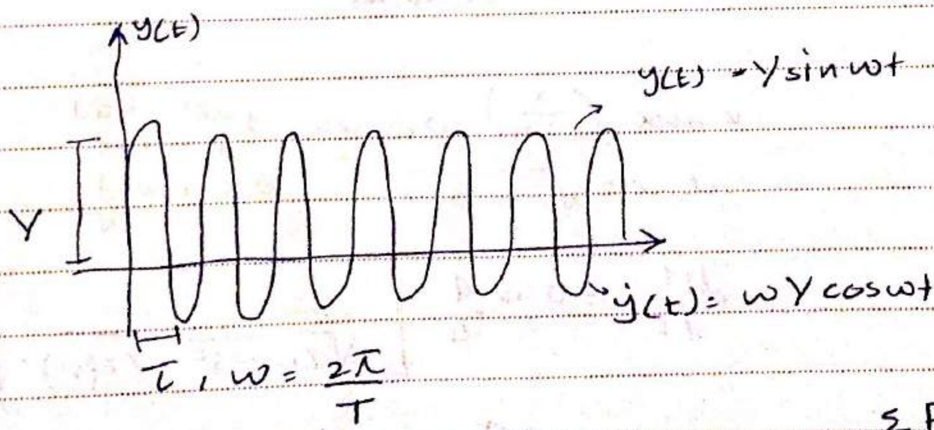
spring-damper

$F(t) = F_o \cos \omega t$

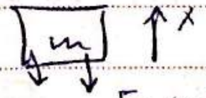




SDOF



F.B.D



$F_D = c(\dot{x}-\dot{y})$, $F_s = k(x-y)$

$\Sigma F = m\ddot{x}$;

$-k(x-y) - c(\dot{x}-\dot{y}) = m\ddot{x}$

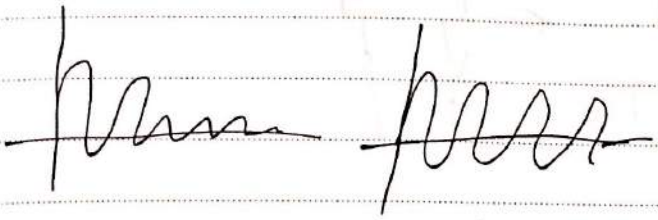
$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$

$\rightarrow m\ddot{x} + c\dot{x} + kx = c\omega Y \cos\omega t + kY \sin\omega t$

$\rightarrow m\ddot{x} + c\dot{x} + kx = Y\left(\underbrace{\sqrt{k^2 + (c\omega)^2}}_{F_o}\right)\sin(\omega t + \psi)$

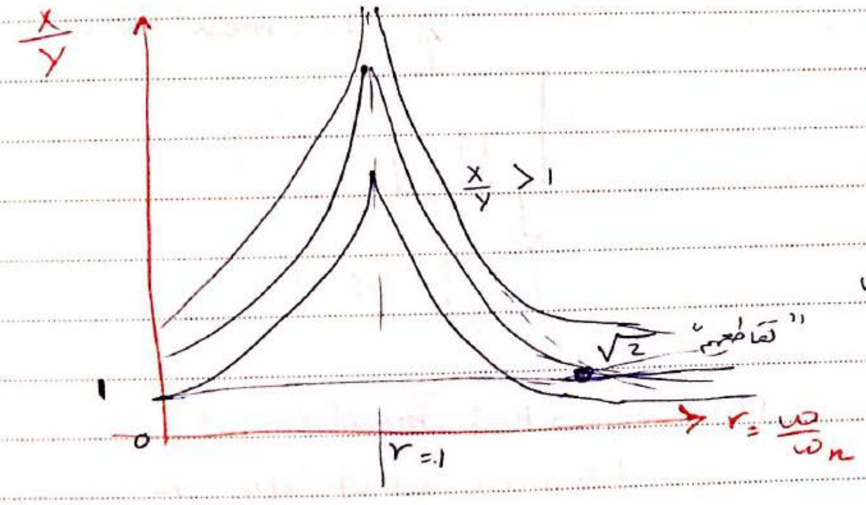
$x(t) = \text{Response}$

$= x_h(t) + x_p(t)$



$x_p(t) = X \sin(\omega t + \phi)$

$$X = \frac{Y\sqrt{k^2 + (c\omega)^2}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \qquad \frac{X}{Y} = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}$$

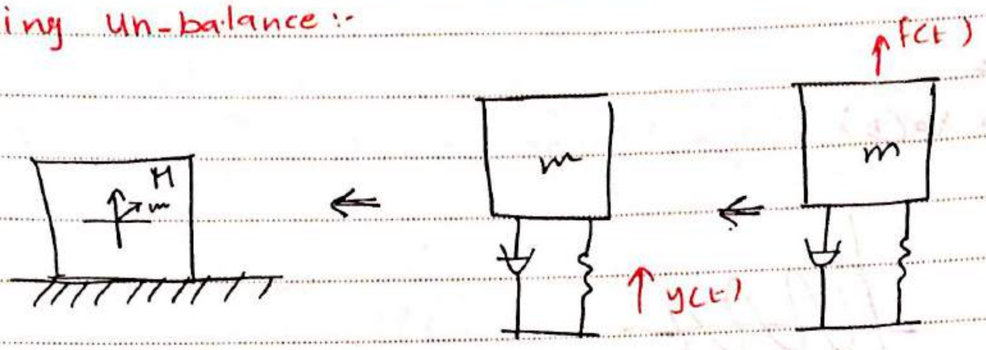


$r = \frac{\omega}{\omega_n} = \sqrt{2}$

$\omega > \sqrt{2}\,\omega_n$, $\frac{X}{Y} < 1$, $\Rightarrow X < Y$

/ /

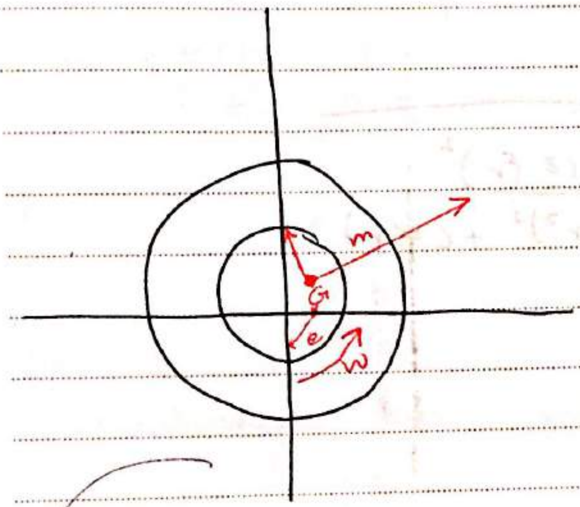
* Rotating Un-balance :-



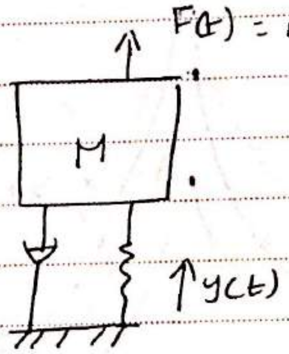
$$F_{\text{centrifugal force}} = m \frac{v^2}{\rho} = m a_n$$

$$= m \frac{(e\omega)^2}{e}$$

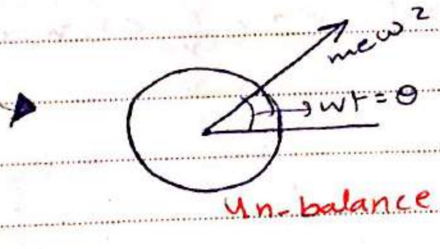
$$= m e \omega^2$$



$$F(t) = m e \omega^2 \sin \omega t$$



$$F(t) = (m e \omega^2) \sin \theta$$
$$= (m e \omega^2) \sin \omega t$$



Un-balance

$$M\ddot{y} + C\dot{y} + Ky = m e \omega^2 \sin \omega t$$

harmonic + particular solution.

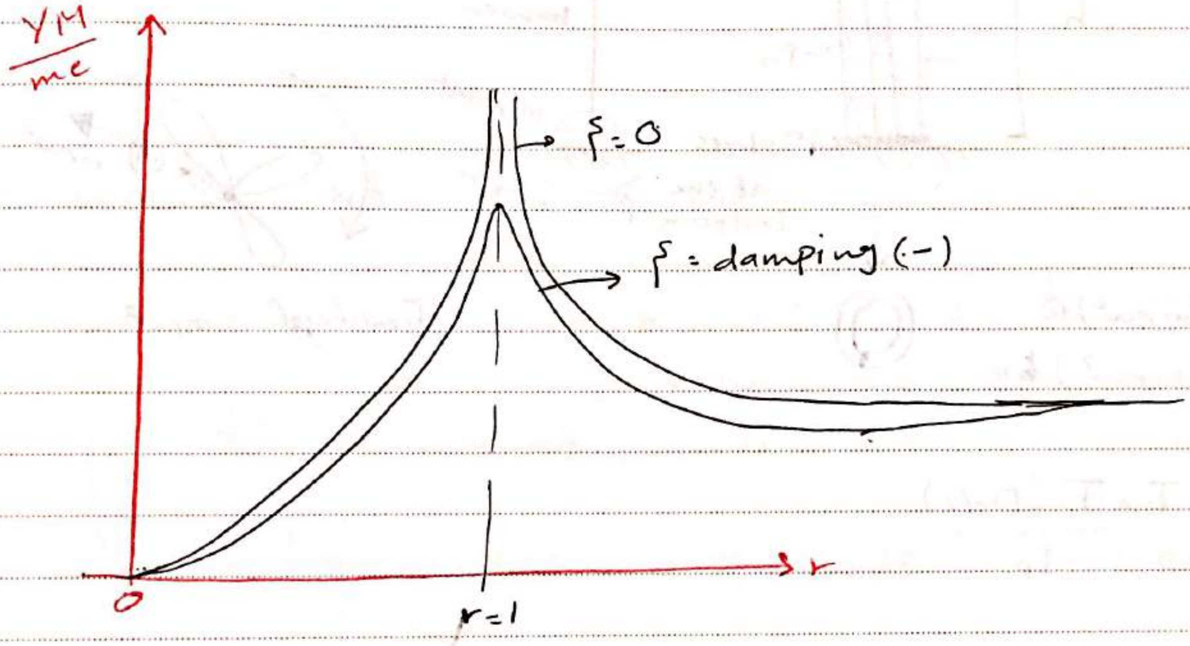
$$y_h(t) + y_p(t) = y(t)$$

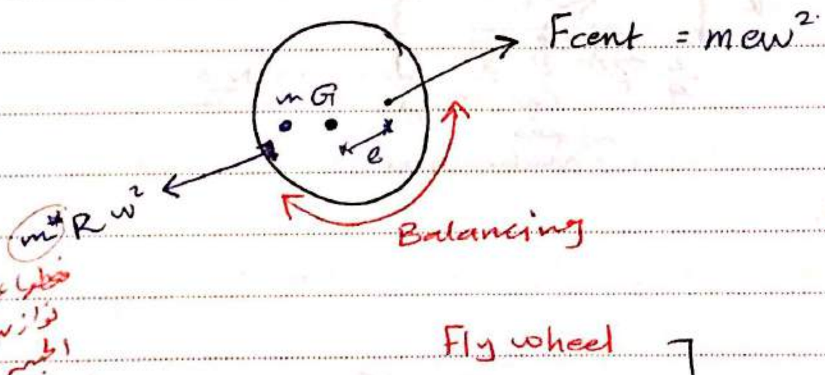
$$Y \sin(\omega t + \phi)$$

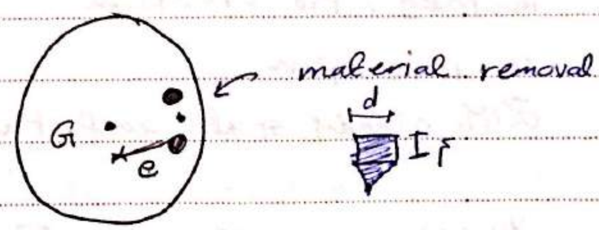
$$Y = \frac{(m e \omega^2)/K}{\left(\sqrt{(K - M\omega^2)^2 + (C\omega)^2}\right)/K}$$, steady state amplitude

$\therefore K \omega_n^2 M$

$$\rightarrow \quad Y = \frac{\frac{m e \omega^2}{M \omega_n^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\left(\zeta \frac{\omega}{\omega_n}\right)\right)^2}} \Rightarrow \frac{MY}{me} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$



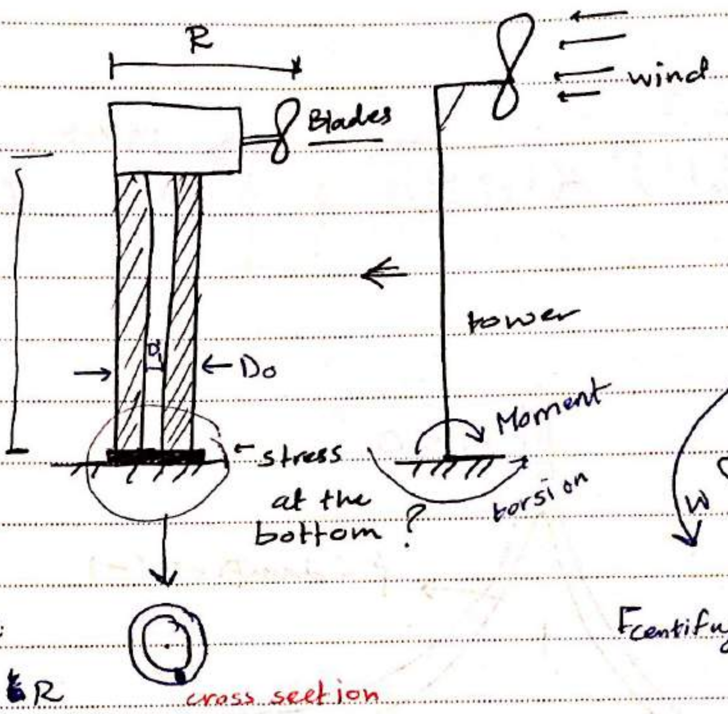


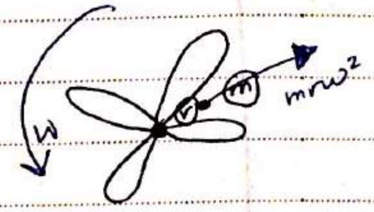


Fly wheel
Impeller
Disk
$\rightarrow$ نقدر نحط
m^* زي mass
وبقدر اشيل mass
من احد الأجسام جدول.

3-1
3-2] Ex.
3-3
3-7 idea.

3-17





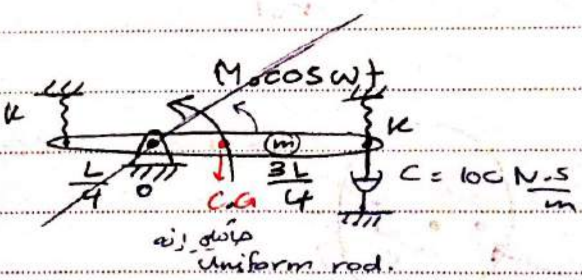
$F_{centrifugal} = mr\omega^2$

Torsion $= (mr\omega^2)R$

Moment $= (mr\omega^2)R$

$\sigma = \frac{Mc}{I}$; $\tau = \frac{T(D_o/2)}{I_p}$

3-25



$L = 1m$, $k = 5000\ N/m$

$m = 10kg$; $M_o = 100\ N.m$

$\omega = 1000$ rpm

EOM, steady state amplitude.

$\circlearrowleft +ve\ \sum M_o = \sum M_{effective} = J_o\ddot{\theta}$

$$M_o\cos\omega t - k\left(\frac{L}{4}\right)^2\theta - k\left(\frac{3L}{4}\right)^2\theta - C\left(\frac{3L}{4}\right)^2\dot{\theta} = \left(\frac{mL^2}{12} + m\left(\frac{L}{4}\right)^2\right)\ddot{\theta}$$

$$\boxed{\underbrace{mL^2\left(\frac{1}{12}+\frac{1}{16}\right)}_{m_{eq}}\ddot{\theta} + \underbrace{C\left(\frac{3L}{4}\right)^2}_{C_{eq}}\dot{\theta} + \underbrace{k\left(\frac{L^2}{16}+\frac{9L^2}{16}\right)}_{k_{eq}}\theta = M_o\cos\omega t} \quad \text{EOM}$$

/ /

$$\omega_n^2 = \frac{kL^2\left(\frac{10}{16}\right)}{mL^2\left(\frac{7}{48}\right)} \quad ; \quad 2\sqrt{km}.$$

$$\theta_p(t) = \Theta \cos(\omega t + \phi)$$

$$\Theta = \frac{M_o}{\sqrt{(k_{eq} - m_{eq}\omega^2)^2 + (C_{eq}\omega)^2}}$$

* By Energy Method:-

$$T_{system} = \frac{1}{2} J_o \dot{\theta}^2$$

$$V_{system} = \frac{1}{2}\left[k\left(\frac{L}{4}\theta\right)^2 + k\left(\frac{3L}{4}\theta\right)^2\right]$$

$$W_{system} = \left(\int \vec{F}.d\vec{r}\right)_{damping} \Longrightarrow \int -c\left(\frac{3L}{4}\right)\dot{\theta}\, dr = \int -c\left(\frac{3L}{4}\right)\dot{\theta}\left(\frac{3L}{4}\right) d\theta$$

$$= \int (\vec{M}.\vec{d\theta})_{Moment} \longrightarrow \int (M_o \cos \omega t)\, d\theta$$

$$\frac{d\theta}{dt} = \dot{\theta}$$

$$= \int M_o \cos \omega t\, d\theta - \int c\left(\frac{3L}{4}\right)^2 \dot{\theta}\, d\theta$$

$$d\theta = \dot{\theta}\, dt$$

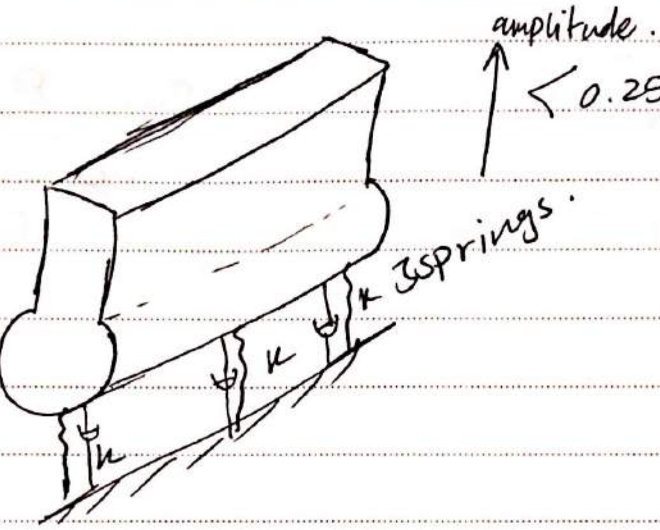
$$= \int [M_o \cos \omega t\, \dot{\theta}\, dt - \int c\left(\frac{3L}{4}\right)^2 \dot{\theta}\, \dot{\theta}\, dt$$

$$* \frac{d}{dt}(T+V) = \frac{d}{dt}(W_{system})$$

$$J_o \ddot{\theta}\dot{\theta} + \left(k\left(\frac{L}{4}\right)^2 + k\left(\frac{3L}{4}\right)^2\right)\theta \cancel{\dot{\theta}} = M_o \cos \omega t\, \cancel{\dot{\theta}} - C\left(\frac{3L}{4}\right)^2 \dot{\theta}\dot{\theta} dt$$

3-30

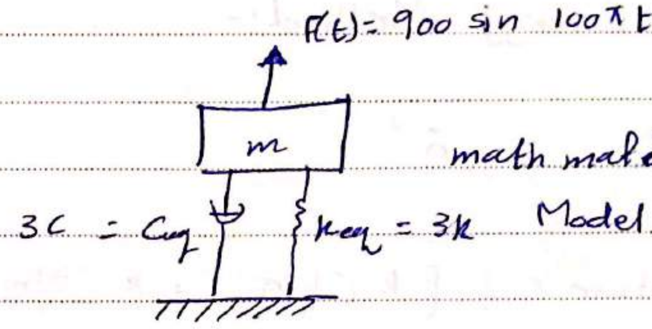
c, k ??



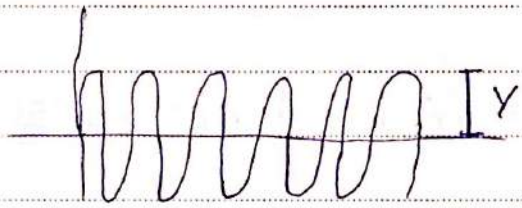
225 kg

$F(t) = 900 \sin 100\pi\, t$

F_o $\quad$ W



math malical Model



$m\ddot{y} + c_{eq}\dot{y} + k_{eq}\, y = F_o \sin \omega t$

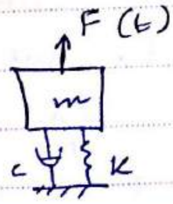
$\rightarrow y_p(t) = \underline{\underline{Y}} \sin(\omega t + \phi)$

$$Y = \frac{F_o}{\sqrt{(k_{eq} - m\omega) + (c_{eq}\omega)^2}}$$

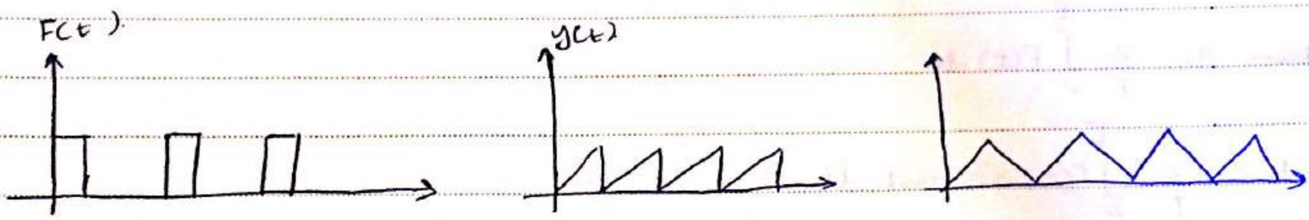
assume $k_{eq} = 1$ kN.m

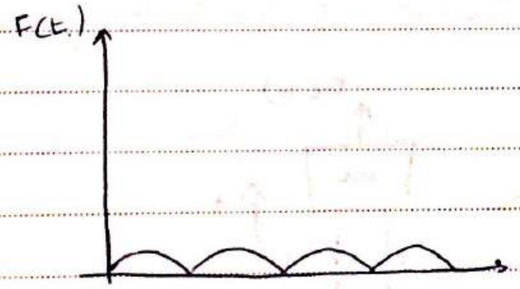
"General forcing Conditions"

F(t)
- → periodic Excitation "Not harmonic" ← simplest form for periodic Motion
- → Pulse.
- → arbitrary shape.



1)





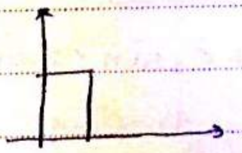
→ Fourier Transform

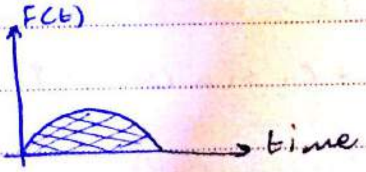
$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\underset{\substack{\downarrow \\ \text{frequency}}}{\omega} t + \sum_{n=1}^{\infty} b_n \sin n\omega t \Big]$$ حوله لـ Harmonic

$$\omega = \frac{2\pi}{\tau}$$

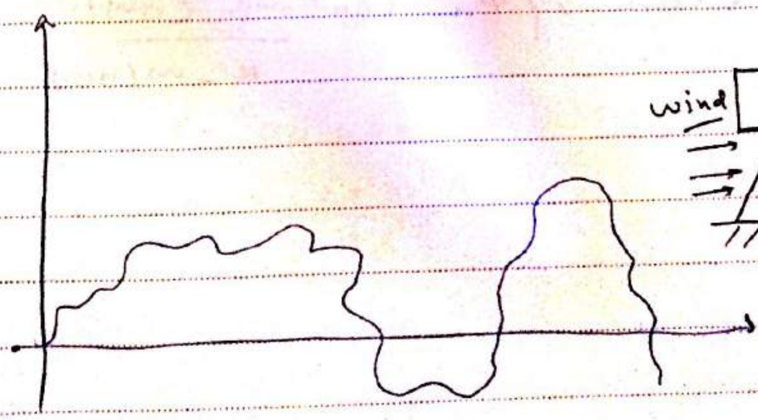
2) Pulse , single Pulse

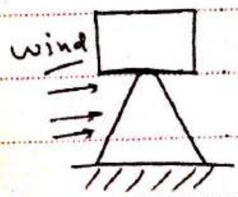
سيارة ماشية عطريق وطلع مطب ! eg.





3) arbitrary → Random



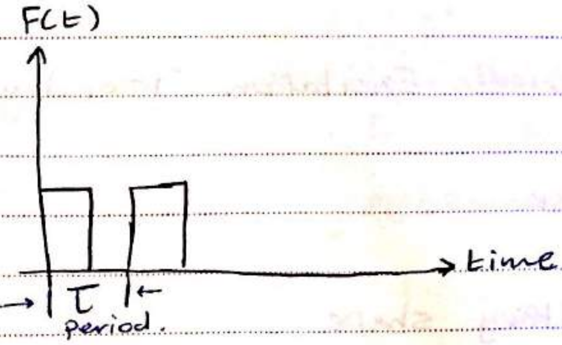


/ /

* Fourier Transform:-

ω = Excitation frequency

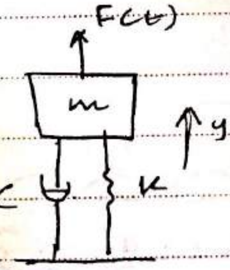
$= \frac{2\pi}{\tau}$



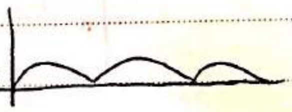
$a_n = \frac{2}{\tau} \int_0^{\tau} F(t) \cos n\omega t \, dt$, $n = 1, \dots \infty$

$a_0 = \frac{2}{\tau} \int_0^{\tau} F(t) \, dt$

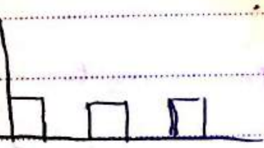
$b_n = \frac{2}{\tau} \int_0^{\tau} F(t) \sin n\omega t \, dt$, $n = 1, \dots \infty$



$m\ddot{y} + c\dot{y} + ky = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t$



$+ \sum_{n=1}^{\infty} b_n \sin n\omega t$



F(t) عبر عنها ك Fourier series

$\hookrightarrow m\ddot{y} + c\dot{y} + ky = \frac{a_0}{2} + a_1 \cos \omega t + b_1 \sin \omega t + a_2 \cos 2\omega t + b_2 \sin 2\omega t + \dots$

$m\ddot{y} + c\dot{y} + ky = \frac{a_0}{2} + c_1 \sin(\omega t + \phi_1) + c_2 \sin(2\omega t + \phi_2) + c_3 \sin(3\omega t + \phi_3) + \dots$

EOM

damping

$c_n = \sqrt{a_n^2 + b_n^2}$; $\left(\phi_n = \tan^{-1} \frac{-c^{*}(n\omega)}{k - m(n\omega)^2} \right)$

Scanned by CamScanner

Exp. $Y_{P_3} \sin(3\omega t + \Upsilon_3)$

. Response $y_p(t) \equiv$ particular solution.

$\hookrightarrow y_p(t) = y_{P_0}(t) + y_{P_1}(t) + y_{P_2}(t) + y_{P_3}(t) + \cdots$

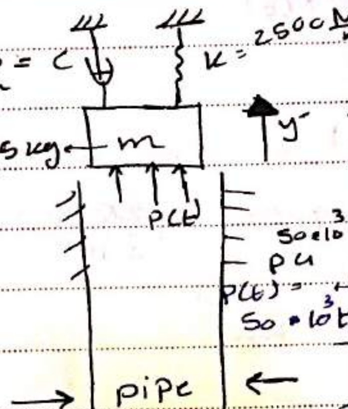
$\Rightarrow y_{P_n}(t) = Y_{P_n} \sin(n\omega t + \Upsilon_n)$; $Y = \dfrac{F_0}{\sqrt{(K-m\omega^2)^2 + (c^*\omega)^2}}$

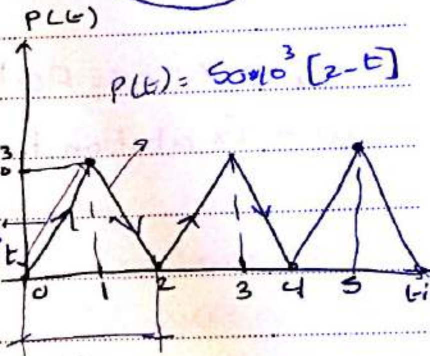
$$Y_{P_n} = \frac{C_n}{\sqrt{(K - m(n\omega)^2)^2 + [c^*(n\omega)]^2}} \quad ; \Upsilon_n = \cdots$$

Response -amplitude

Exp. 4-1
4-2

Ex. 4-4





$\tau = 2s$

$\omega = \frac{2\pi}{\tau} = \frac{2\pi}{2} = \pi$ rad

$A = \frac{\pi}{4}(50)^2 = 0.000625\pi\ m^2$

$m\ddot{y} + c\dot{y} + Ky = A(P(t))$

* $a_n = \frac{2}{\tau}\left[\int_0^1 (50*10^3 t)\cos n\omega t\, dt + \int_1^2 (50*10^3[2-t])\cos n\omega t\, dt\right]$ → تعويض

0 to 1 ← function الـ اول
ثاني 1 to 2

* $b_n = \frac{2}{2}\left[\int_0^1 50*10^3 t \sin n\omega t\, dt + \int_1^2 50*10^3[2-t]\sin n\omega t\, dt\right]$

* $a_0 = \frac{2}{2}\left[\int_0^1 50*10^3 t\, dt + \int_1^2 50*10^3(2-t)\, dt\right]$

$a_0 = 50*10^3 A$

$a_1 = -\dfrac{2*10^5}{\pi^2} A$

$a_2 = a_4 = a_6 = 0$

$a_3 = \dfrac{-2*10^5 A}{9\pi^2}$

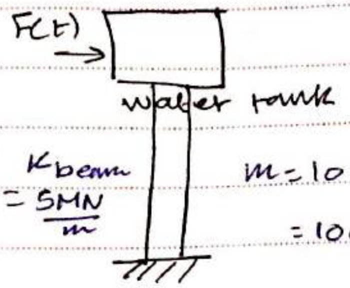
$b_1 = b_2 = b_3 = \cdots = 0$

4.10, 4.6 sugg.

/ /

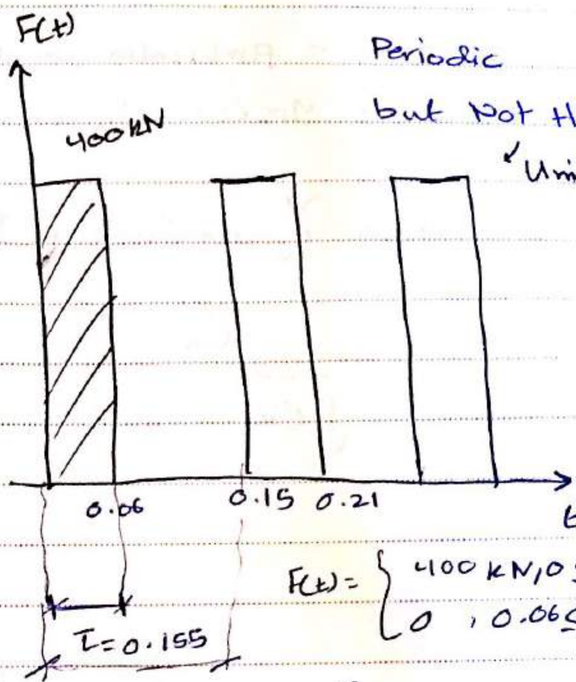
4.15

periodic force



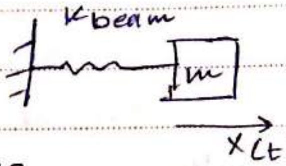
$K_{beam} = 5\frac{MN}{m}$

$m = 10\,Mg = 10000\,kg$



Periodic but Not Harmonic, Uniform

sdof



x(t) ??

$$F(t) = \begin{cases} 400\,kN, & 0 \le t < 0.06 \\ 0, & 0.06 \le t < 0.15 \end{cases}$$

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

$m\ddot{x} + kx = F(t)$

ω = Excitation frequency $= \left(\frac{2\pi}{0.15}\right)$

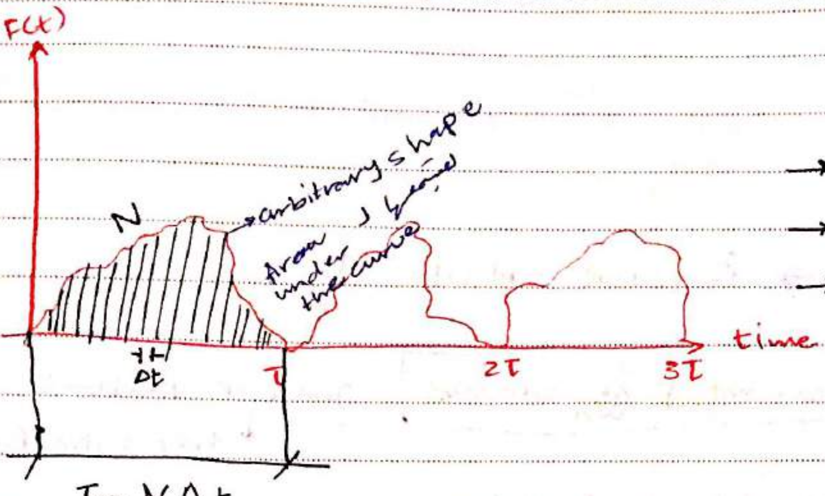
$\omega, 2\omega, 3\omega, --- n\omega$

$$a_n = \frac{2}{\tau}\int_0^{\tau} F(t)\cos n\omega t\,dt \; ; \; b_n = \frac{2}{\tau}\int_0^{\tau} F(t)\sin n\omega t\,dt$$

$$a_n = \frac{2}{\tau}\left[\int_0^{0.06} 400\cos n\omega t\,dt + \int_{0.06}^{0.15} 0\,dt\right] ; \; b_n = \frac{2}{\tau}\left[\int_0^{0.06} 400\sin n\omega t\,dt + 0\right]$$

$a_n = \checkmark$, $b_n = \checkmark$

$$a_0 = \frac{2}{\tau}\int_0^{0.06} 400\,dt$$



→ Periodic

→ Not Harmonic

→ Not uniform

$$a_n = \frac{2}{\tau}\left[\int_0^{\tau} F(t)\cos n\omega t \, dt\right]$$

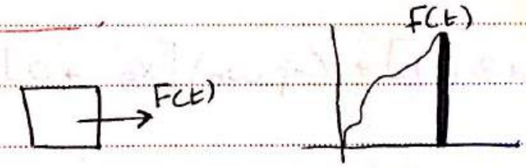
$$a_n = \frac{2}{N(\Delta t)} \sum_{i=1}^{N} F_i \cos n\omega t_i \, (\cancel{\Delta t})$$

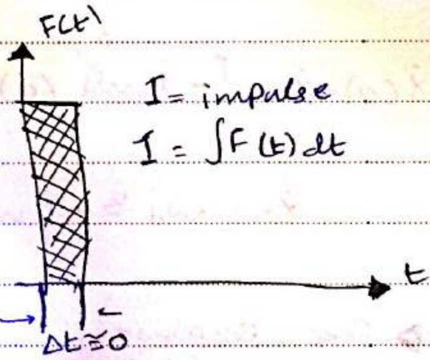
$$b_n = \frac{2}{N} \sum_{i=1}^{N} F_i \sin n \, n\omega t_i$$

Numerical Integration → If Not Uniform

---

* Impulse *





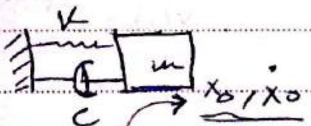
I = impulse

$I = \int F(t)\,dt$

$$\vec{L}_1 + \int_0^t F(t)\,dt = \vec{L}_2 \; ; \; \vec{L} = m\vec{v}$$

→ $\boxed{m\vec{v}_1 + \text{Impulse} = m\vec{v}_2}$

→ $\boxed{\vec{H}_{o1} + \int M(t)\,dt = \vec{H}_{o2}}$ angular Momentum



$X(0) = X_o$ → Free vibration

$\dot{X}(0) = \dot{X}_o$ → Force for a short period of time

impulse

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$X(t) = e^{-f\omega_n t}\left[A\cos\omega_d t + B\sin\omega_d t\right]$$

$x_0 = 0$

$\dot{x}(0) = \dot{x}_0$

$$\underset{L_1}{0} + \int_0^t F(t)\,dt = m\dot{x}_0 \Rightarrow \dot{x}_0 \text{ initial velocity} = \frac{I}{m}$$

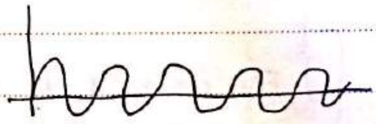
→ No-damping , $\boxed{x(t) = (x_0)\cos\omega_n t + \frac{\dot{x}_0}{\omega_n}\sin\omega_n t}$ general Response

↓ Free Vibration

$\cancel{(x_0)}$ 60

→ $x(t)$ = response due to Impulse

$$= \boxed{\frac{I}{m\,\omega_n}\sin\omega_n t}$$



→ Damping

$x(0) = x_0 = A + 0 \rightarrow A = x_0$

$\dot{x}(t) = e^{-\zeta\omega_n t}\left[-\omega_d A\sin\omega_d t + \omega_d B\cos\omega_d t\right] + \left[A\cos\omega_d t + B\sin\omega_d t\right]e^{-\zeta\omega_n t}(-\zeta\omega_n)$

$\dot{x}(0) = \dot{x}_0 = \left[-x_0\cancel{\omega_d}\,(0) + \omega_d B(1)\right] + (-\zeta\omega_n)\left[x_0 + 0\right]$

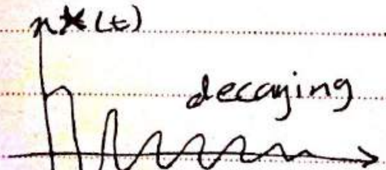
$$\dot{x}_0 = \omega_d B - \zeta\omega_n x_0 \Rightarrow B = \frac{\dot{x}_0 + \zeta\omega_n x_0}{\omega_d}$$

⇒ Free - Response

$$x(t) = e^{-\zeta\omega_n t}\left[x_0\cos\omega_d t + \frac{\dot{x}_0 + \zeta\omega_n x_0}{\omega_d}\sin\omega_d t\right]$$

→ for the impulse → $x_0 = 0$; $\dot{x}_0 = \frac{I}{m}$

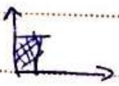
- damped case -

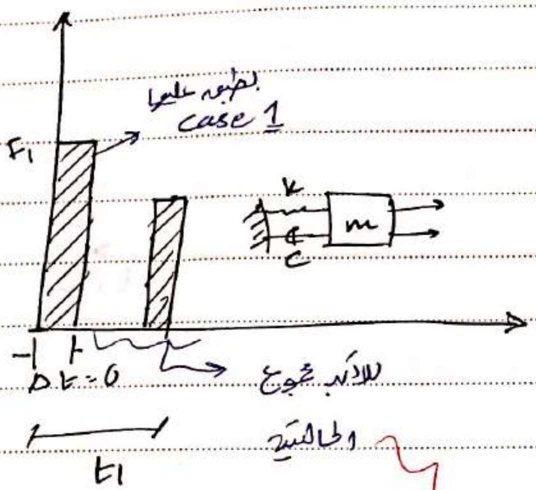


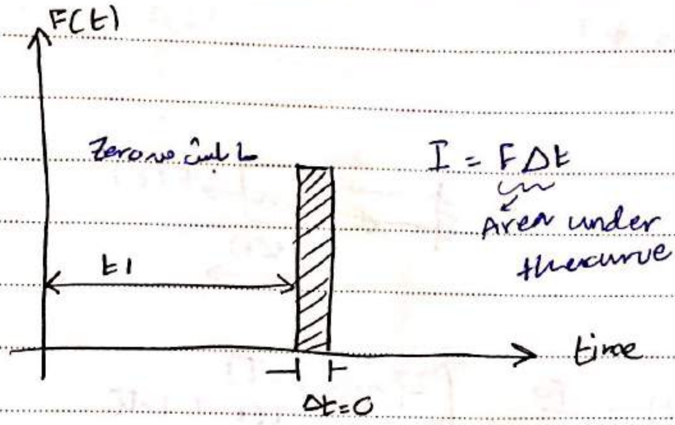
$$x(t) = e^{-\zeta\omega_n t}\frac{\dot{x}_0}{\omega_d}\sin\omega_d t = e^{-\zeta\omega_n t}\cdot\frac{I}{m\omega_d}\sin\omega_d t$$

Cases

[1] $x_1(t) = \frac{I}{m\omega_d} g(t)$; $g(t) = e^{-\zeta\omega_n t} \sin \omega_d t$







[2] $x_2(t) = \frac{I}{m\omega_d} g(t - t_1)$

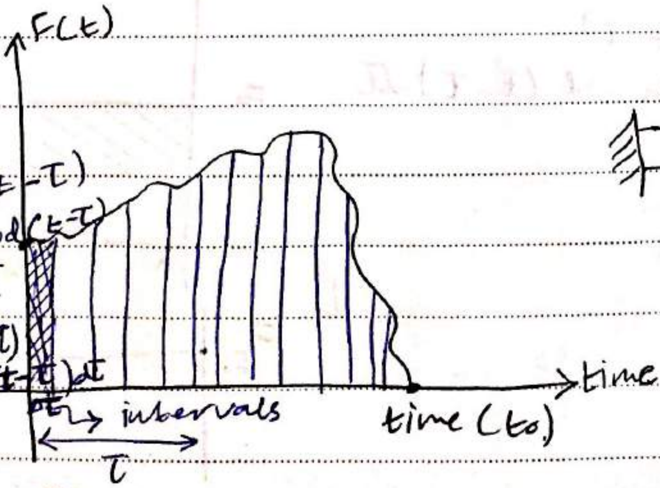
case [3] → بين الحالتين

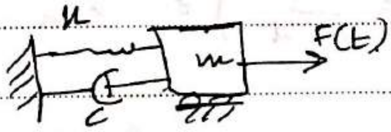
* when $0 < t < t_1$; $x(t) = x_1(t)$

$t_1 \geq t$, $x(t) = x_1(t) + x_2(t)$ → super position

$$x(t) = \begin{cases} \frac{I_1}{m\omega_d} e^{-\zeta\omega_n t} \sin\omega_d t , & t < \tau \\ \frac{I_1}{m\omega_d} e^{-\zeta\omega_n t} \sin\omega_d t + \frac{I_2}{m\omega_d} e^{-\zeta\omega_n (t-\tau)} \sin\omega_d (t-\tau) , & t \geq \tau \end{cases}$$

*assume we have





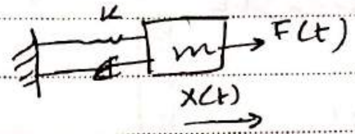
$$x(t) = \sum_i \frac{\underbrace{f_i(\Delta\tau)}_{\text{Impulse}} e^{-\zeta\omega_n(t-\tau)} \sin\omega_d(t-\tau)}{m\omega_d}$$

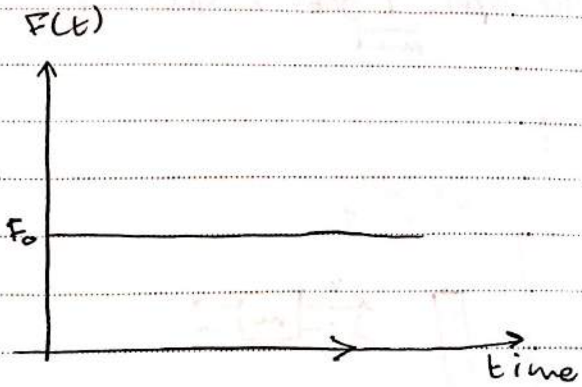
$$x(t) = \frac{1}{m\omega_d} \int_0^{t_0} F(\tau) e^{-\zeta\omega_n(t-\tau)} \sin\omega_d(t-\tau)\, d\tau$$

Intervals لا نهائية

Convolution Integral

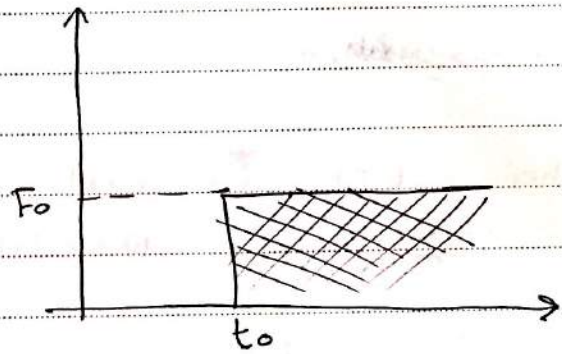
Exp # 1





$$x(t) = \frac{F_o}{m\omega_d} \int_0^{\infty} e^{-\zeta \omega_n (t-\tau)} \sin \omega_d (t-\tau)\, d\tau$$

$\omega_d = \omega_n \sqrt{1-\zeta^2}$



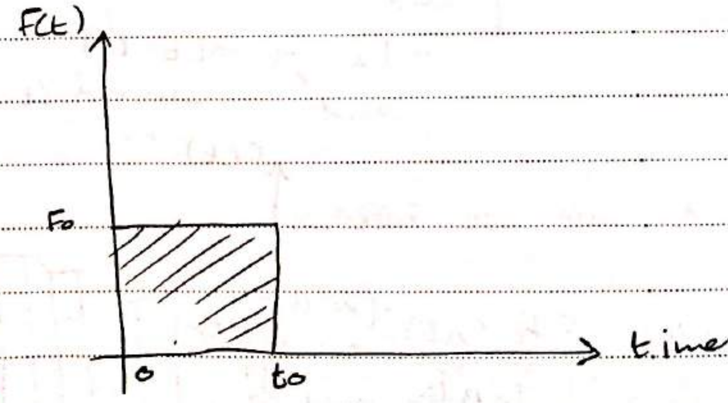
Response, $x(t - t_o)$

$$x(t) = \frac{F_o}{m\omega_d} \int_{t_o}^{\infty} e^{-\zeta \omega_n (t-\tau)} \sin \omega_d (t-\tau)\, d\tau$$

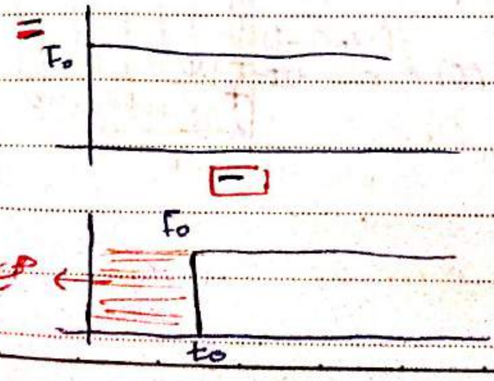
انتب

or 2

$$x(t) = \frac{F_o}{m\omega_d} \int_0^{t_o} e^{-\zeta \omega_n (t-\tau)} \sin \omega_d (t-\tau)\, d\tau$$

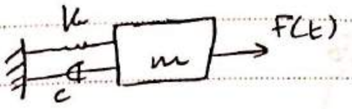


1 =

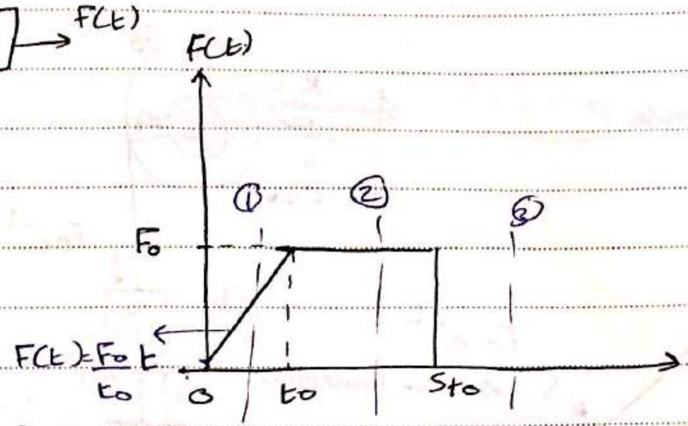


هي الربح

Exp:-



1) response for $t < t_0$
2) response for $t < 5t_0$
3) Response for $t > 5t_0$

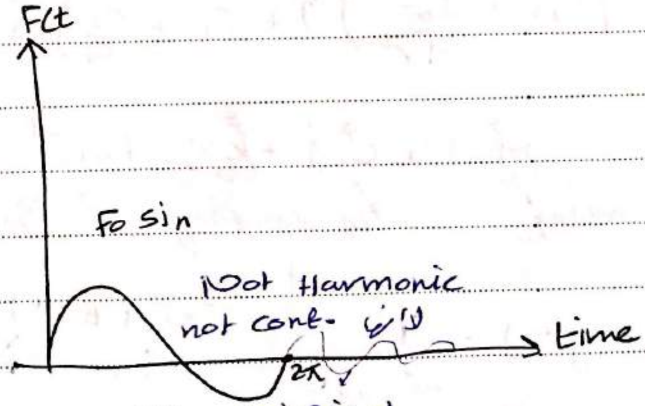


$$x_1(t) = \frac{1}{m\omega_d} \int_0^t \frac{F_0}{t_0}(\tau)\, e^{-\zeta\omega_n(t-\tau)} \sin\omega_d (t-\tau)\, d\tau \; ; \; t \le t_0$$

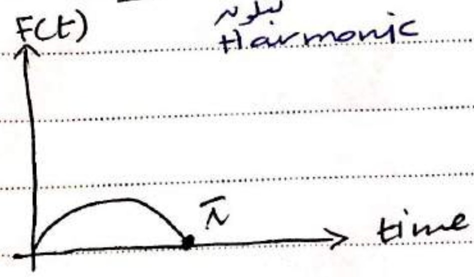
$$x_2(t) = \frac{1}{m\omega_d} \left[\int_0^{t_0} \frac{F_0 \tau}{t_0} e^{-\zeta\omega_n(t-\tau)} \sin\omega_d (t-\tau)\, d\tau + \int_{t_0}^{t} F_0 e^{-\zeta\omega_n(t-\tau)} \sin\omega_d(t-\tau)\, d\tau \right]$$

$$x_3(t) = \frac{1}{m\omega_d} \left\{ x_2 + \int_{t_0}^{5t_0} F_0 e^{-\zeta\omega_n(t-\tau)} \sin\omega_d (t-\tau)\, d\tau + 0 \right.$$

Exp:-



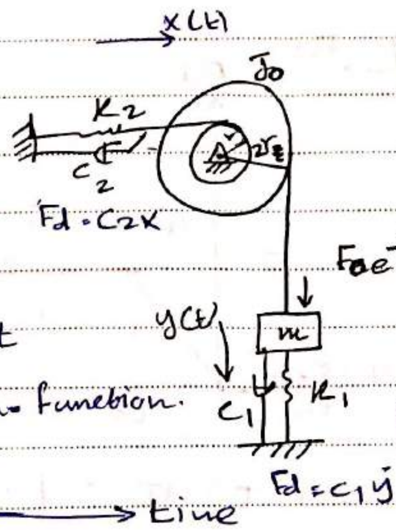
or Periodic لو تكون Harmonic

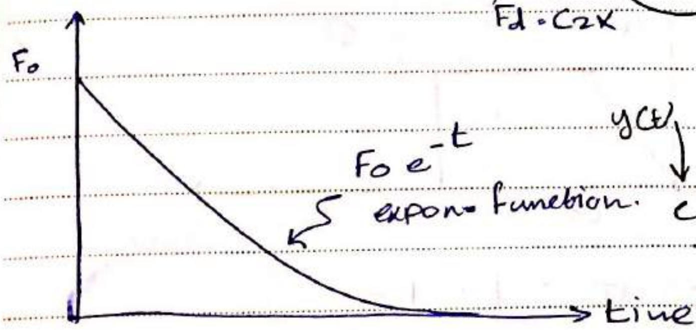


Exp: 4-13

4-35

Response?





$y = 2r\theta$

$x = r\theta$

$\boxed{x = \frac{y}{2}}$

4-12
4-15
4-16
4-18
4-20
4-19
4-21
4-23
4-24
4-25
4-32
4-33
4-34
4-35

$$T = \frac{1}{2}\left[m\dot{y}^2 + J_o \frac{\dot{y}^2}{4r^2}\right] = \frac{1}{2}\left[m + \frac{J_o}{4r^2}\right]\dot{y}^2$$

$$V = \frac{1}{2}\left[k_1 + k_2\left(\frac{1}{2}\right)^2\right]y^2$$

$$W_{damper} = -\int \left(c_1 \dot{y}\dot{y}\,dt + c_2 \frac{\dot{y}}{2}\frac{\dot{y}}{2}\,dt\right)$$

$\frac{d}{dt}(T+V) = \frac{d}{dt}(W_{dampers}) + \frac{d}{dt}$(work done by the force)

$$\left(m + \frac{J_o}{4r^2}\right)\ddot{y} + \left(c_1 + \frac{c_2}{4}\right)\dot{y} + \left(k_1 + \frac{k_2}{4}\right)y = F(t) = F_o e^{-t}$$

$$m^* \ddot{y} + c^* \dot{y} + k^* y = F_o e^{-t}$$

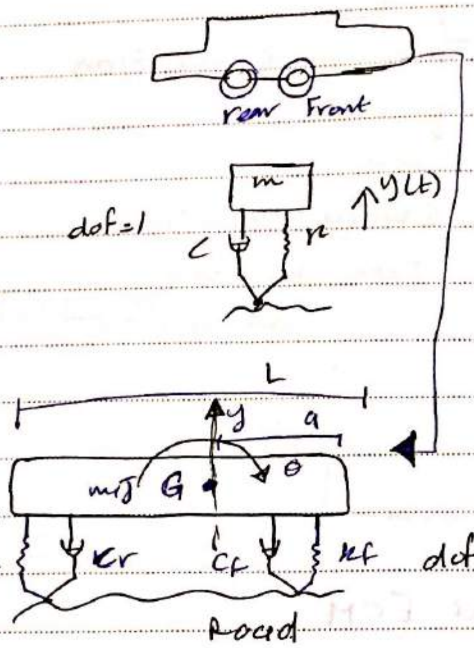
mass eq. — eq. damping — eq. stiffness.

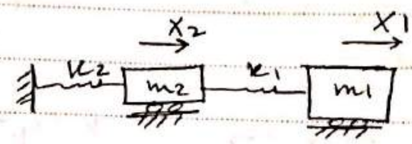
بتغير integ. لل τ

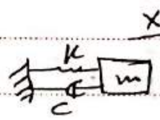
$$y(t) = \frac{1}{m^* \omega_d}\int_0^t \left(e^{-\zeta\omega_n(t-\tau)} \sin \omega_d (t-\tau)\right) F_o e^{-\tau}\, d\tau$$

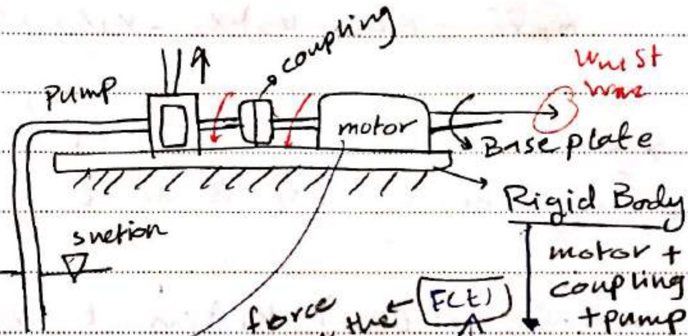
Chapter (5) / /

↳ Two-Degrees-of Freedom



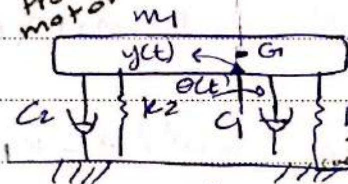






motor + coupling + pump

force from the motor → F(t)



اعتبرتم rigid بالنسبة لبعض

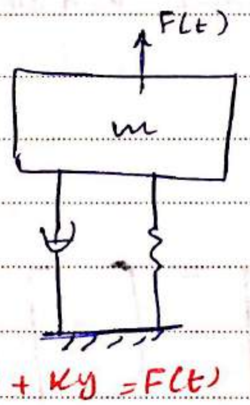
angular velocity

dof = 2

2 Equ. $y(t)$

→ Coupled $\theta(t)$

→ 2 Natural frequency



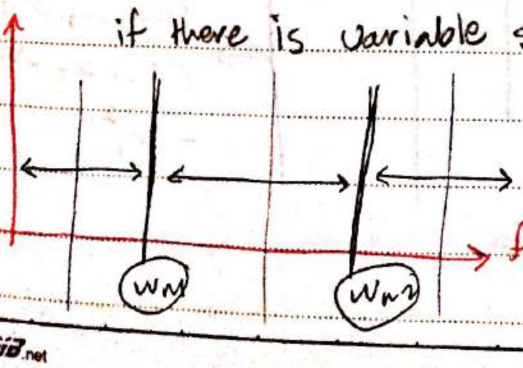
sdof

$$m\ddot{y} + c\dot{y} + ky = F(t)$$

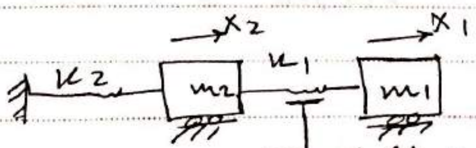
→ ω_n

No. of Natural frequency = No. of degrees of freedom.

if there is variable speed



خلي بالك عشان الـ Resonance

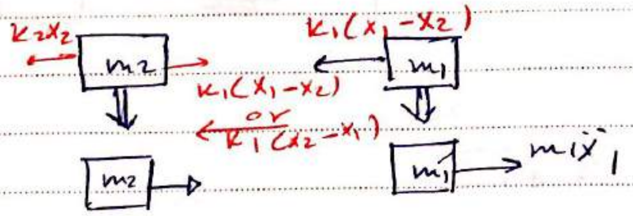


Translational Motion

مادام في
Elastic element
then it's 2 deg.

F.B.D

K.D



$\Sigma F = \Sigma$effective $= ma = m\ddot{x}$

$\rightarrow -k_1(x_1-x_2) = m_1\ddot{x}_1$

$m_1\ddot{x}_1 + k_1x_1 - k_1x_2 = 0$; --- ① 1st EOM.

coupling Term

$\rightarrow -k_2x_2 - k_1(x_2-x_1) = m_2\ddot{x}_2$

$m_2\ddot{x}_2 + (k_1+k_2)x_2 - k_1x_1 = 0$ --- ② 2nd EOM

Harmonic Excitation

$\rightarrow$ solution, assume $\begin{cases} x_1 = A\sin\omega t \\ x_2 = B\sin\omega t \end{cases}$ certain Amplitude

$\rightarrow$ freevibration

$-A\omega^2 \cancel{\sin\omega t}(m_1) - k_1A\cancel{\sin\omega t} - k_1B\cancel{\sin\omega t} = 0$

$-B\omega^2 \cancel{\sin\omega t}(m_2) + (k_1+k_2)B\cancel{\sin\omega t} - k_1A\cancel{\sin\omega t} = 0$

$(k_1 - m_1\omega^2)A - k_1B = 0$

$((k_1+k_2) - m_2\omega^2)B - k_1A = 0$

$$\begin{bmatrix} k_1 - m_1\omega^2 & -k_1 \\ -k_1 & k_1+k_2-m_2\omega^2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$ no-vibra trivial solution

; $|\quad| \Rightarrow (k_1 - m_1\omega^2) * (k_1+k_2-\omega^2m_2) - k_1^2$

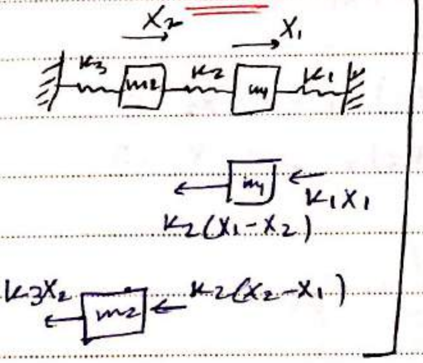
determinate $= 0$ or $\begin{bmatrix} A \\ B \end{bmatrix} = 0$

✗ trivial solution

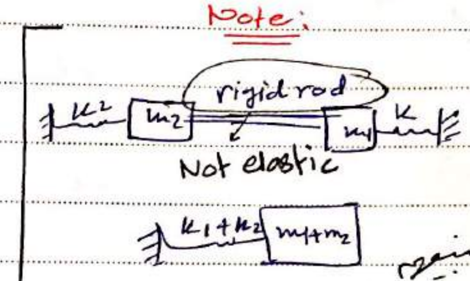
$(k_1 - m_1\omega^2)(k_1 + k_2 - m_2\omega^2) - k_1^2 = 0$, frequency Equation Or Characteristic Equation.

contains all the Physical parameters $(k_1, k_2, m_1, m_2, \omega)$

Note:-



Note:



هون ما في بينهم Elastic element

بيتحركوا بنفس الـ deflection مع بعض

فيعتبر نظام SDOF

$\boxed{\begin{matrix} m_1 = m_2 = m \\ k_1 = k_2 = k \end{matrix}}$ assumbtion for simplicity (Just) → Not general case

$(k - \omega^2 m)(2k - \omega^2 m) - k^2 = 0$

$2k^2 - \omega^2 mk - 2mk\omega^2 + (\omega^2)^2 m^2 - k^2 = 0$

If $k_1 = k_2$

$m_1 = m_2$

quadratic Equation $\boxed{m^2(\omega^2)^2 - 3mk(\omega^2) + k^2 = 0}$ → 2 solutions

$\Rightarrow (\omega^2)_{1st, 2nd} = \dfrac{3mk \pm \sqrt{9m^2k^2 - 4m^2k^2}}{2m^2}$

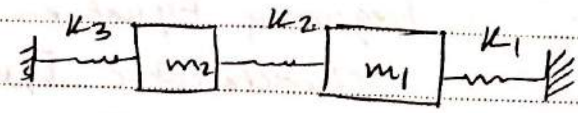
ω_{n_1} = First Nat. Freq. $= \sqrt{\dfrac{k}{m}\left(\dfrac{3 - \sqrt{5}}{2}\right)}$

$\omega^2\Big|_{1st} = \dfrac{3mk - mk\sqrt{5}}{2m^2} = \dfrac{k}{m}\left(\dfrac{3 - \sqrt{5}}{2}\right)$

ω_{n_2} = Second Nat. Freq. $= \sqrt{\dfrac{k}{m}\left(\dfrac{3 + \sqrt{5}}{2}\right)}$

$\omega^2\Big|_{2nd} = \dfrac{3mk + mk\sqrt{5}}{2m^2} = \dfrac{k}{m}\left(\dfrac{3 + \sqrt{5}}{2}\right)$

H.W



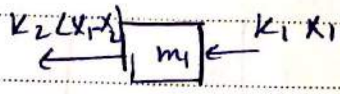
$m_1 = 5\,kg$; $k_1 = 50\ kN/m$

$m_2 = 10\,kg$; $k_2 = 75\ kN/m$

$k_3 = 100\,kN/m$

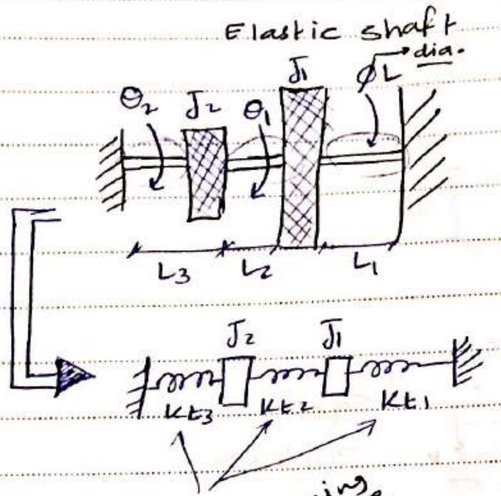
$\omega_{n_1} = ??$

$\omega_{n_2} = ??$



$$m_1 \ddot{x}_1 + (k_1 + k_2)\, x_1 - k_2 x_2 = 0$$

$$m_2 \ddot{x}_2 + (k_2 + k_3)\, x_2 - k_2 x_1 = 0$$



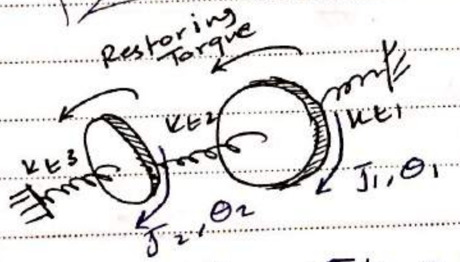
k_t = torsional spring

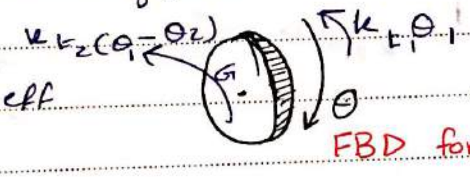
$= \frac{GI_p}{L}$ → constant polar moment of area.

$k_{t1} = \frac{GI_{P_1}}{L_1}$ (G → shear modulus; I_{P_1} → Polar moment of area.)

$k_{t2} = \frac{GI_{P_2}}{L_2}$

$k_{t3} = \frac{GI_{P_3}}{L_3}$



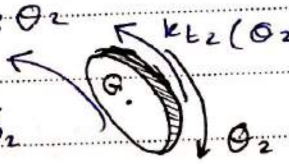


$\circlearrowleft \Sigma M_G = \Sigma M_{eff}$

FBD for Disk ①

$$-k_{t1}\theta_1 - k_{t2}(\theta_1-\theta_2) = J_1\ddot{\theta}_1$$

$$J_1\ddot{\theta}_1 + (k_{t1}+k_{t2})\theta_1 - k_{t2}\theta_2 = 0 \quad \text{---①}$$



$$-k_{t2}(\theta_2-\theta_1) - k_{t3}\theta_2 = J_2\ddot{\theta}_2$$

FBD for Disk ②

$$J_2\ddot{\theta}_2 + (k_{t2}+k_{t3})\theta_2 - k_{t2}\theta_1 = 0 \quad \text{---②}$$

$\theta_1(t) = A\sin(\omega t)$, $\theta_2(t) = B\sin(\omega t)$.

$((k_{t1}+k_{t2}) - \omega^2 J_1)A - k_{t2}B = 0$;

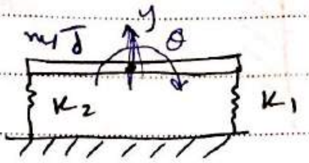
$((k_{t2}+k_{t3}) - \omega^2 J_2)B - k_{t2}A = 0$;

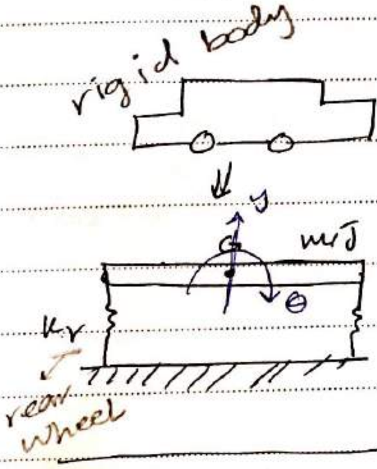
$$\begin{bmatrix} k_{t1}+k_{t2}-\omega^2 J_1 & -k_{t2} \\ -k_{t2} & k_{t2}+k_{t3}-\omega^2 J_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Delta|\,| = 0 ; \; (k_{t1}+k_{t2}-\omega^2 J_1)(k_{t2}+k_{t3}-\omega^2 J_2) - k_{t2}^2 = 0$$

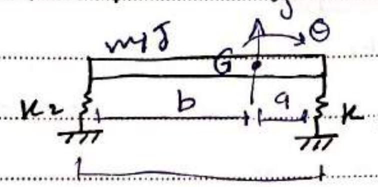
* Translation + Rotation [G P M] → General plane Motion.

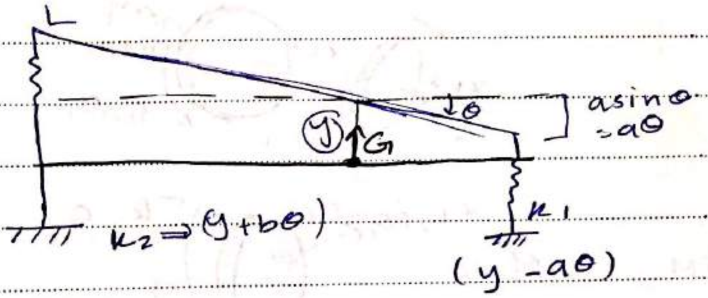
Exp's:-

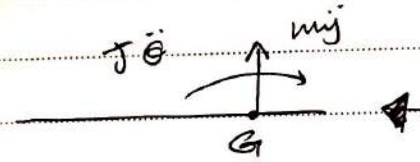




* Exp:-

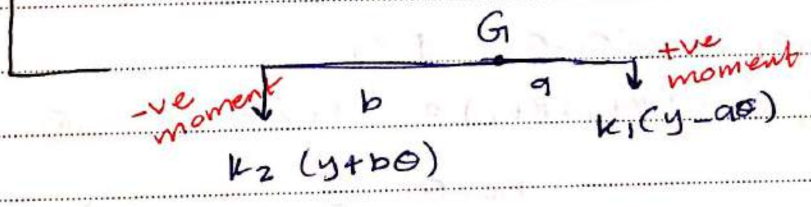






Equivalent FBD

Translation + Rotation



$\Sigma F = m\ddot{y}$ --- (1)

$\Sigma M_G = J\ddot{\theta}$

$-k_1(y-a\theta) - k_2(y+b\theta) = m\ddot{y}$; $m\ddot{y} + (k_1+k_2)y - (k_1 a\theta - k_2 b\theta) = 0$

$-k_2 b\theta$

(1) ——— $-(k_1 a - k_2 b)\theta = 0$

Coupling Term

$\circlearrowright + \Sigma M_G = J\ddot{\theta}$

$k_1(y-a\theta)a - k_2(y+b\theta)b = J\ddot{\theta}$; $J\ddot{\theta} + (k_1 a^2 + k_2 b^2)\theta$

$- (k_1 a y - k_2 b y) = 0$

$-(k_1 a - k_2 b)y = 0$ --- (2)

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assume; $y(t) = A\sin\omega t$

$\theta(t) = B\sin\omega t$

$$\begin{bmatrix} k_1+k_2-\omega^2 m & -(k_1a-k_2b) \\ -(k_1a-k_2b) & (k_1a^2+k_2b^2-\omega^2 J) \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

* frequency equation; $(k_1+k_2-\omega^2 m)(k_1a^2+k_2b^2-\omega^2 J)-(k_1a-k_2b)^2=0$

$(A^*)(\omega^2)^2 + (B^*)(\omega^2) + (C^*) = 0$

- W_{n1} = 1st Natural frequency $= \sqrt{W_1^2}$ $\sqrt{\dfrac{-B^* - \sqrt{B^{*2}-4A^*C}}{2A^*C}}$
- W_{n2} = 2nd Natural frequency $= \sqrt{W_2^2}$ =

W_{n1} ; W_{n2}

$y(t) = A_1 \sin\omega_{n1}t + A_2 \sin\omega_{n2}t$

$\theta(t) = B_1 \sin\omega_{n1}t + B_2 \sin\omega_{n2}t$

Arbitrary Constants.

$y(0)$; $\dot{y}(0)$, $\theta(0)$, $\dot{\theta}(0)$

4 initial Conditions.

$(k_1+k_2-\omega^2 m)A - (k_1a-k_2b)B =$

$((k_1+k_2)-\omega_{n1}^2 m)A_1 - (k_1a-k_2b)B_1 = 0$

$A_1 = \dfrac{k_1a-k_2b}{(k_1+k_2-\omega_{n1}^2 m)} B_1$

الاجابة الثانية $A_2 = \dfrac{k_1a-k_2b}{(k_1+k_2-\omega_{n2}^2 m)} B_2$

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$A_2 = r_2 B_2$

$A_1 = r_1 B_1$

$m = 50\ kg$, $J = 20\ kg.m^2$

$k_1 = 50\ kN/m$; $k_2 = \cancel{75}\ kN/m$ 100

$a = 1.5\ m$

$b = 1.0\ m$

$\left.\begin{array}{l} \omega_{n1} \\ \omega_{n2} \\ r_1 \\ r_2 \end{array}\right]$?? → ω_n اللي بتطلع اصغر هي الي بحسبها ω_{n1}

* If the ratios: $r_1, r_2 = 0$, then the coupling term $= 0$
that means we have 2EOM → SDOF

$$\omega_{n1} = \sqrt{\frac{k_1 + k_2}{m}} \rightarrow SDOF$$

$$\omega_{n2} = \sqrt{\frac{k_1 a^2 + k_2 b^2}{J}} \rightarrow SDOF$$

No coupling during the motion.

$\omega_{n1} = 54$ rad/s

$\omega_{n2} = 103$ rad/s

اصغر هي ω_{n1}

$y = A \sin[\omega t + \phi] \rightarrow \ddot{y} = -\omega^2 A \sin(\omega t + \phi)$

$\theta = B \sin[\omega t + \phi] \rightarrow \ddot{\theta} = -\omega^2 B \sin(\omega t + \phi)$

↳ $y(t) \equiv$ response →

→ $y(t) = A_1 \sin(\omega_1 t + \phi) + A_2 \sin(\omega_2 t + \phi)$

→ $\theta(t) = \text{response} = B_1 \sin(\omega_1 t + \phi) + B_2 \sin(\omega_2 t + \phi)$

$\dot{y}(t) = \omega_1 A_1 \cos(\omega_1 t + \phi) + A_2 \omega_2 \cos(\omega_2 t + \phi)$

$\dot{y}(0) = \dot{y}_0$

→ $y(0) = y_0$

$\theta(0) = \theta_0$

$\dot{\theta}(0) = \dot{\theta}_0$

$\dot{\theta}(t) = ---$

$\frac{A}{B} = \frac{k_1 a - k_2 b}{(k_1+k_2) - \omega^2 m}$, $\left.\frac{A}{B}\right|_{\omega=\omega_{n_1}} \overset{(-ve)}{=} -6.157$; $\left.\frac{A}{B}\right|_{\omega=\omega_{n_2}} = 0.0649$.

1^{st} Mode-shape $\Leftarrow$ [1^{st} ratio ; $A = -6.157\,B$

2^{nd} Mode-shape $\Leftarrow$ [2^{nd} ratio ; $A = 0.0649\,B$.

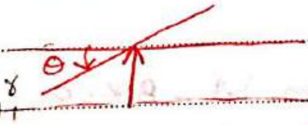
→ assume $\dot{y}_o = \dot{\theta}_o = 0$ → for simplicity

إذا بتتحرك على الحالة 1^{st} ratio ; $\underset{+ve}{y(0)} = -6.157\,\underset{-ve}{\theta(0)}$ → out-of-phase

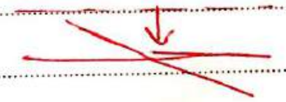
بتحرك على 2^{nd} ratio ; $\underbrace{y(0)}_{+ve} = \underbrace{0.0649\,\theta(0)}_{+ve}$ → In-phase

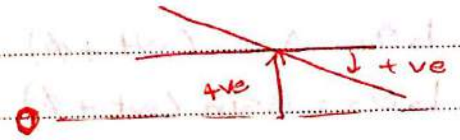
1^{st} Wn only

لأنه اعتبرنا ال clockwise positive



or, if y_o -ve





* If $k_2 = 75$

$\omega_{n_1} = \sqrt{\frac{(50+75)*10^3}{50}} = 50$ rad/sec

$\omega_{n_2} = \sqrt{\frac{k_1 a^2 + k_2 b^2}{20}} = 96.8$ rad/sec

بتحرك لحال y
لحال θ لحال
لأنه ما في بينهم ratio
↳ No-coupling

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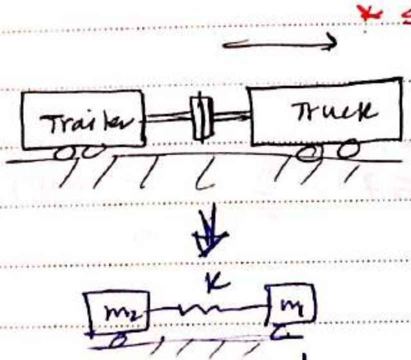
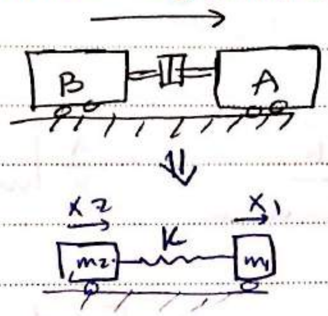
* Semi - Definite system *





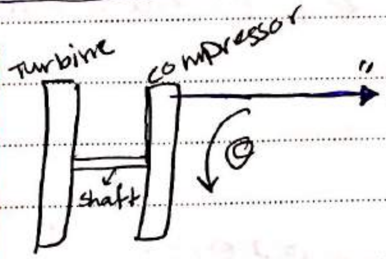
(Traslational)

$$m_1\ddot{x}_1 + kx_1 - kx_2 = 0$$

$$m_2\ddot{x}_2 + kx_2 - kx_1 = 0$$

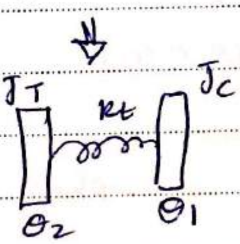
→ assume : $x_1 = A\sin(\omega t + \phi)$

$x_2 = B\sin(\omega t + \phi)$



Turbo charger

(Rotational)



$$(k - \omega^2 m_1)A - Bk = 0$$

$$(k - \omega^2 m_2)B - A\cdot k = 0$$

$$J_C\ddot{\theta}_1 + K_t\theta_1 - K_t\theta_2 = 0$$

$$J_T\ddot{\theta}_2 + K_t\theta_2 - k_t\theta_1 = 0$$

↳ $\theta_1 = A\sin(\omega t + \phi)$

↳ $\theta_2 = B\sin(\omega t + \phi)$

$$\begin{bmatrix} k - \omega^2 m_1 & -k \\ -k & k - \omega^2 m_2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(k - \omega^2 m_1)(k - \omega^2 m_2) - k^2 = 0$$

$$\cancel{k^2} - \omega^2 m_2 k - \omega^2 m_1 k + (\omega^2)^2 m_1 m_2 - \cancel{k^2} = 0$$

$$(\omega^2)^2\left[m_1 m_2\right] - \left[m_2 k + m_1 k\right]\omega^2 = 0 \rightarrow \omega^2\left[\omega^2(m_1 m_2) - (m_1 + m_2)k\right] = 0$$

↳ 1st ; $\omega^2 = 0 \Rightarrow \omega_{n_1} = 0$

↳ 2nd ; $\omega^2 = \frac{k(m_1 + m_2)}{m_1 m_2} \Rightarrow \omega_{n_2} = \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$

$w_{n_1} = 0 \longrightarrow$ that means:- m_1 & m_2 move as arigid body with the same w_n. no- vibration

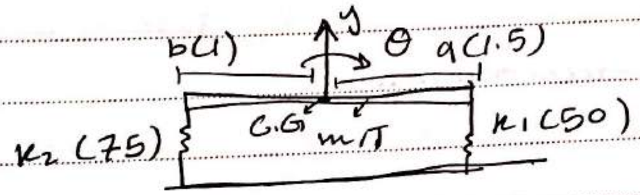
بصيرش عربات القطار، بيحركوا كواحدة ب W_n.

* Generalized coordinates

$(y, \theta, x) \,/\, (y_1, y_2) \rightarrow$ you can define them

5-1 sugg.
5-2 Examples
5-3
5-4
5-5

1)



$$m\ddot{y} + (k_1 - k_2)y - (k_1 a - k_2 b)\theta = 0 \quad \text{--- (1)}$$

$$J\ddot{\theta} + (k_1 a^2 + k_2 b^2)\theta - (k_1 a - k_2 b)y = 0$$

$(k_1 a - k_2 b) \equiv$ coupling term (elastic static coupling term) stiffness matrix

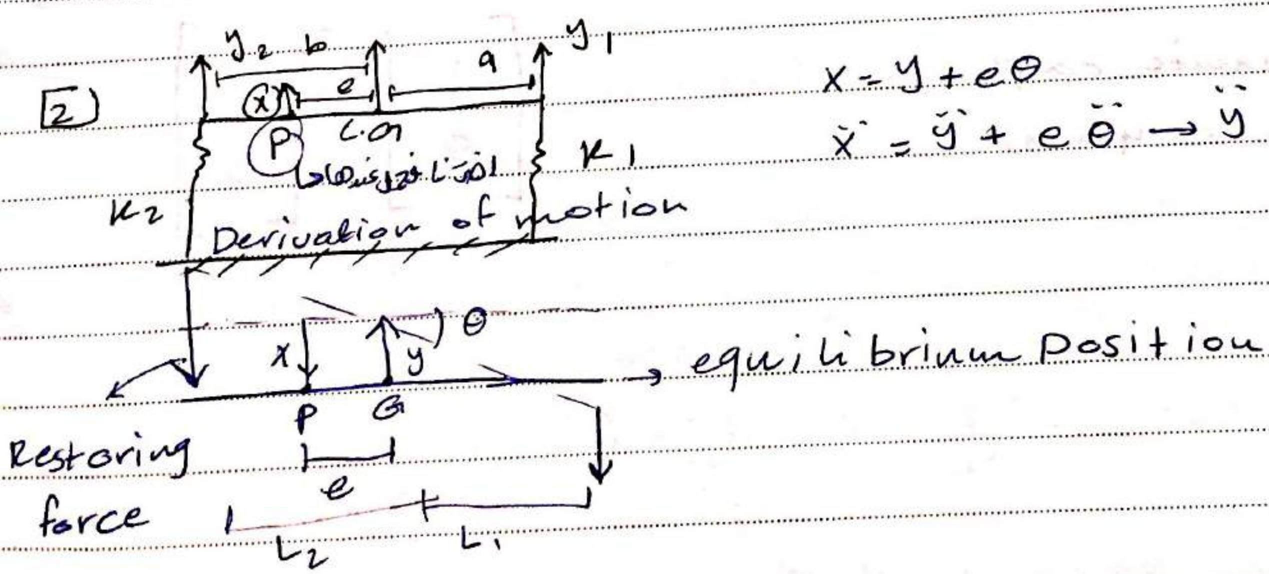
$50(1.5) - 75(1) = 0$

↳ causes reduction

$\hookrightarrow m\ddot{y} + (k_1 + k_2)y = 0 \quad \text{--- } 1^*$

$J\ddot{\theta} + (k_1 a^2 + k_2 b^2) = 0 \quad \text{--- } 2^*$

2)



$x = y + e\theta$

$\ddot{x} = \ddot{y} + e\ddot{\theta} \rightarrow \ddot{y} = \ddot{x} - e\ddot{\theta}$

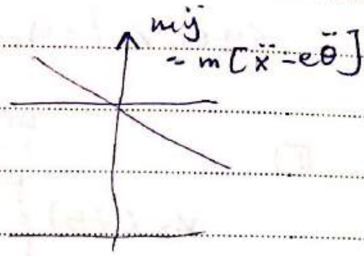
/ /

$\Sigma F = \Sigma F_{eff}$

$-k_1(x - L_1\theta) - k_2(x + L_2\theta) = m(\ddot{x} - \ddot{\theta})$ — ③

$\boxed{\hookrightarrow m\ddot{x} - m\ddot{\theta} + (k_1 + k_2)x - (k_2L_2 - k_1L_1)\theta}$ — ③

$\Sigma M_p = \Sigma M_{eff}$.



$J\ddot{\theta}$ + moment of mass acceleration

4) $\Sigma M_p = J\ddot{\theta} - em(\ddot{x} - e\ddot{\theta})$

$L_1k_1(x + L_1\theta) - L_2k_2(x - L_2\theta) = \bar{J}\ddot{\theta} - em(\ddot{x} - e\ddot{\theta})$

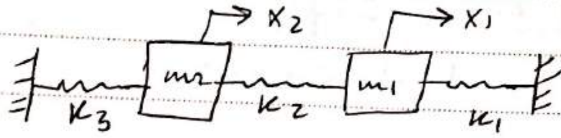
$(\bar{J} + me^2)\ddot{\theta} - me\ddot{x} + (k_2L_2^2 + k_1L_1^2)\theta - (k_2L_2 - k_1L_1)x = 0$

$$\begin{bmatrix} m & -me \\ -me & \bar{J} + me^2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -(k_2L_2 - k_1L_1) \\ -(k_2L_2 - k_1L_1) & k_1L_1^2 + k_2L_2^2 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

↳ dynamic coupling

(Inertia coupling)

* Energy Method to derive the equation of Motion



* by Newton's second Law :-

$$m_1\ddot{x}_1 + (k_1 + k_2)x_1 + k_2 x_2 = 0 \quad \text{--- } ①$$

$$m_2\ddot{x}_2 + (k_2 + k_3 \; x_2) - k_2 x_1 = 0 \quad \text{--- } ②$$

* by Energy Method :

$T_{sys.}$ = Kinetic Energy of the system $= \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$

V_{sys} = Potential Energy of the system $= \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 (x_1 - x_2)^2 + \frac{1}{2} k_3 x_2^2$.

↳ Lagrangian Method ; $(\mathcal{L}) = (T - V)$

↳ for multiple degrees of freedom

$$(\mathcal{L})_{sys.} = \frac{1}{2}\left[m_1\dot{x}_1^2 + m_2\dot{x}_1^2 - k_1 x_1^2 - k_2(x_1 - x_2)^2 - k_3 x_2^2\right]$$

↳ system's Lagrangian

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_n}\right) - \left(\frac{\partial \mathcal{L}}{\partial x_n}\right) = 0 \Rightarrow n = \text{degrees of freedom}$$

x_1 ; $\frac{\partial \mathcal{L}}{\partial \dot{x}_1} = \frac{1}{2} m_1 (2\dot{x}_1) = m_1 \dot{x}_1$; $\frac{\partial L}{\partial x_1} = -\frac{1}{\cancel{2}} k_1 (\cancel{2} x_1)$ $- \frac{1}{\cancel{2}} (\cancel{2} k_2)(x_1 - x_2)$

x_2 ; $\frac{\partial \mathcal{L}}{\partial \dot{x}_2} = \frac{1}{2} m_2 (2\dot{x}_2) = m_2 \dot{x}_2$

; $\frac{\partial \mathcal{L}}{\partial x_2} = -\frac{1}{\cancel{2}} k_3 (\cancel{2} x_2) - \frac{1}{\cancel{2}} (\cancel{2} k_2)(x_1 - x_2)$

EOM (1) ; $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_1}\right) - \frac{\partial \mathcal{L}}{\partial x_1} = 0$; $\frac{d}{dt}[m_1\dot{x}_1] - (-k_1 x_1 - k_2(x_1 - x_2)) = 0$

$$\boxed{m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = 0} \quad \text{--- } ①$$

/ /

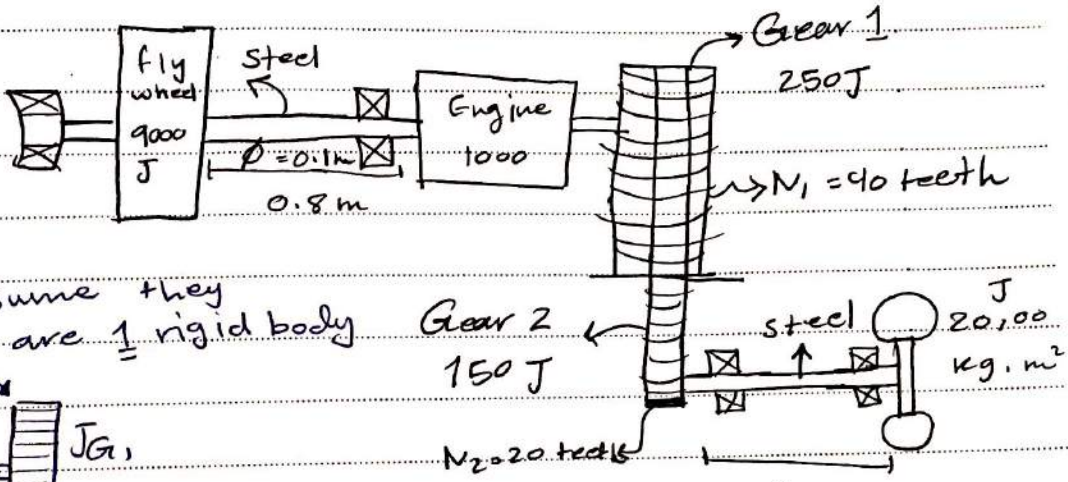
$$EOM(2);\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} = 0;\ \frac{d}{dt}[m_2 \dot{x}_2] + (k_3+k_2)x_2 - k_2 x_1 = 0$$

$$m_2\ddot{x}_2 + (k_3+k_2)x_2 - k_2 x_1 = 0 \quad --- (2)$$

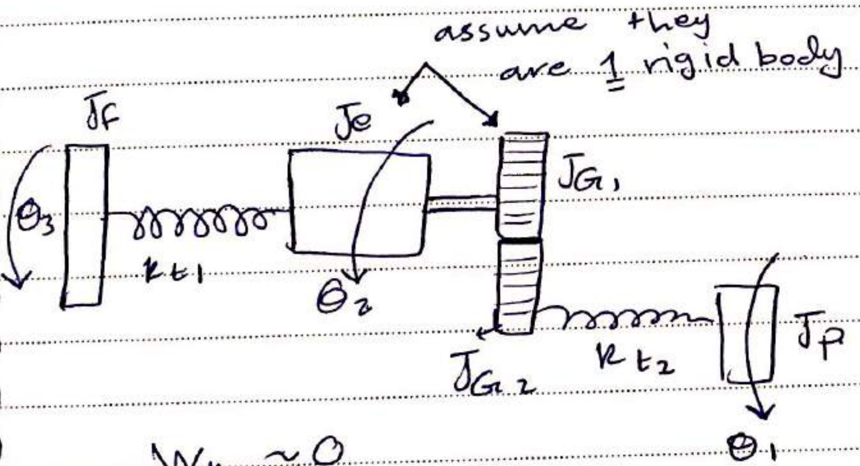
Ex. 5-5, Engine,

mass moment of Inertia

are given



$$k_t = \frac{G I_p}{L}$$



$W_{n1} \simeq 0$

$W_{n2} = \checkmark$

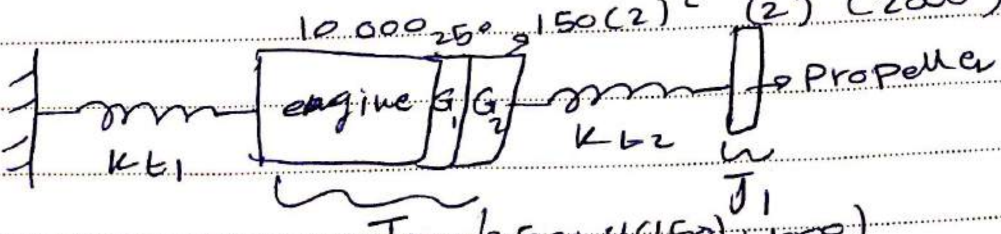
$W_{n3} = \checkmark$

20 : 40 — ط G_1

$N_1 : N_2$ — G_2، قد طه عكس

$$T_p = \frac{1}{2} J_p \dot{\theta}_1^2 = \frac{1}{2} J_p (2\dot{\theta}_2)^2 = \frac{1}{2} J_p (2)^2 \dot{\theta}_2^2$$

Reduction to (2DOF)



$J_2 = (250 + 4(150) + 1000)$

$$J_1 \ddot{\theta}_1 + K_{t2}\theta_1 - K_{t2}\theta_2 = 0 \quad \text{---} (1)$$

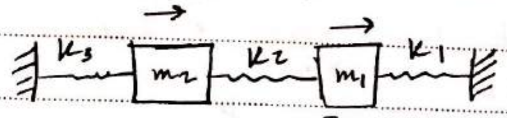
$$J_2 \ddot{\theta}_2 + (K_{t1} + K_{t2})\theta_2 - K_{t2}\theta_1 = 0 \quad \text{---} (2)$$

] EOM'S

forced vibration.   / /

$\omega_{n1} = \checkmark \quad \omega_{n2} = \checkmark$

$F_2(t) = F_{o2} \sin \omega t$



$F_1(t) = F_{o1} \sin \omega t$

$$\left.\begin{aligned} m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 &= F_{o1}\sin\omega t \\ m_2\ddot{x}_2 + (k_2+k_3)x_2 - k_2x_1 &= F_{o2}\sin\omega t \end{aligned}\right\} ; \quad \begin{aligned} x_1(t) &= X_1 \sin\omega t \\ x_2(t) &= X_2 \sin\omega t \end{aligned}$$

$$\left[(k_1+k_2) - \omega^2 m_1\right] X_1 \cancel{\sin\omega t} - k_2 X_2 \cancel{\sin\omega t} = F_{o1} \cancel{\sin\omega t}$$

$$\left[(k_2+k_3) - \omega^2 m_2\right] X_2 \cancel{\sin\omega t} - k_2 X_1 \cancel{\sin\omega t} = F_{o2} \cancel{\sin\omega t}$$

$$\begin{bmatrix} k_1+k_2-\omega^2 m_1 & -k_2 \\ -k_2 & k_2+k_3-\omega^2 m_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_{o1} \\ F_{o2} \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{1}{D} \begin{bmatrix} (k_2+k_3-\omega^2 m_2) & k_2 \\ k_2 & (k_1+k_2-\omega^2 m_1) \end{bmatrix} * \begin{bmatrix} F_{o1} \\ F_{o2} \end{bmatrix}$$

$$\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{b}$$

$$\underset{\sim}{x} = \underset{\sim}{A}^{-1} (\underset{\sim}{b})$$

$$= \frac{1}{D} \begin{bmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix}$$

$$D = (a_{11}a_{22} - a_{21}a_{12})$$

/ /

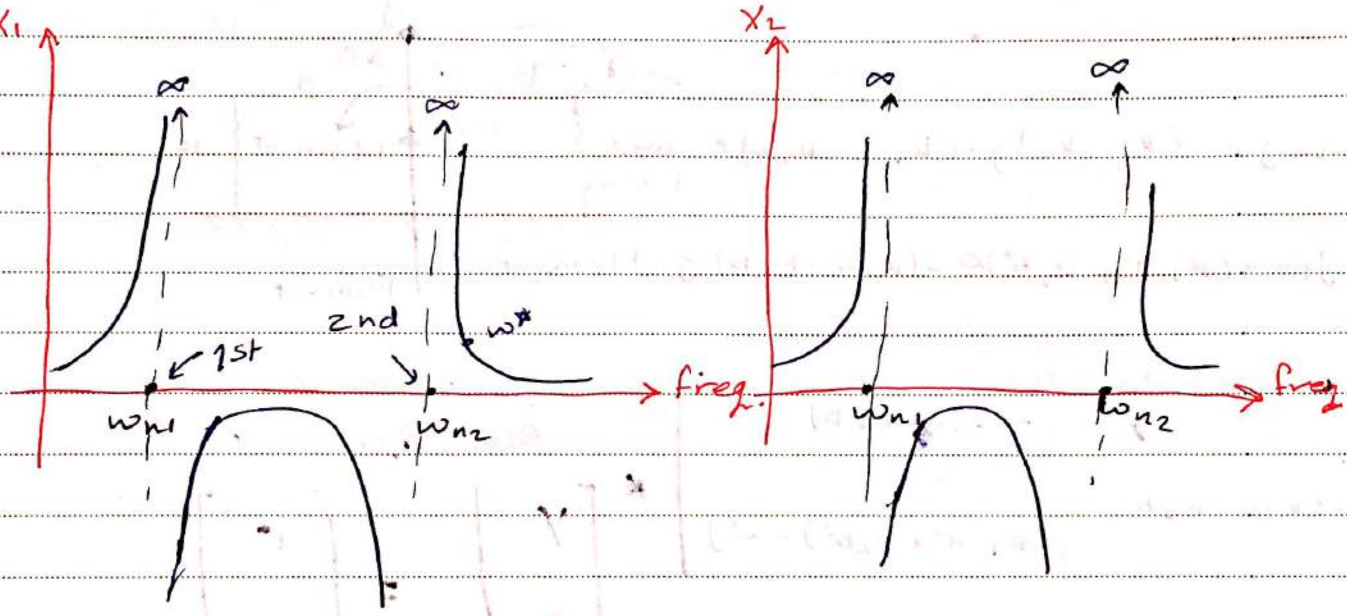
$$X_1 = \frac{(k_2+k_3-\omega^2 m_2)F_{01} + k_2 F_{02}}{\Delta}$$

$$\Delta \Leftarrow [(k_1+k_2)-\omega^2 m_1][(k_2+k_3)-\omega^2 m_2] - k_2^2$$

$$X_2 = \frac{k_2 F_{01} + ((k_1+k_2)-\omega^2 m_1) F_{02}}{\Delta}$$

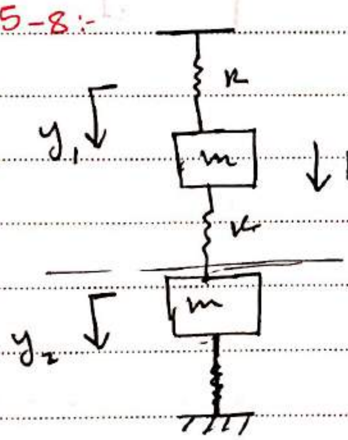
$\Delta = 0 \Rightarrow \omega_{n_1} = ?$

$\omega_{n2} = ?$



*Example 5-8 :-

Example 5.8



$$m\ddot{y}_1 + 2ky_1 - ky_2 = F_{01}\sin\omega t$$

$$m\ddot{y}_2 + 2ky_2 - ky_1 = 0$$

$$\begin{bmatrix} 2k-\omega^2 m & -k \\ -k & 2k-\omega^2 m \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

D. freq. equ. $= (2k-\omega^2 m)^2 - k^2 = 0$

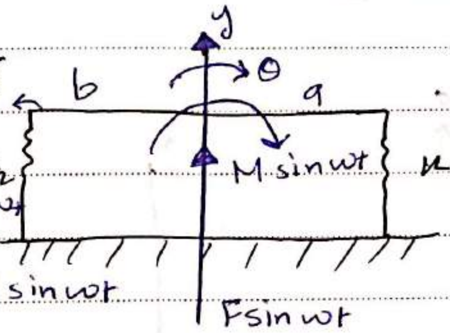
$\omega_{n1} = \checkmark$

$\omega_{n2} = \checkmark$

Free-vibration analysis:-

$$Y_1 = \frac{(2k - \omega^2 m_2) F_{01} + 0}{\Delta} \; ; \; y_1(t) = Y_1 \sin \omega t$$

$$Y_2 = \frac{k F_{01} + 0}{\Delta} \; ; \; y_2(t) = Y_2 \sin \omega t$$



m, J

$\rightarrow m\ddot{y} + (k_1 + k_2) y - (k_1 a - k_2 b)\theta = F \sin \omega t$

$\rightarrow J\ddot{\theta} + (k_1 a^2 + k_2 b^2)\theta - (k_1 a - k_2 b) y = M \sin \omega t$

$y(t) = Y \sin \omega t$

$\theta(t) = \Theta \sin \omega t$

$$\begin{bmatrix} k_1 + k_2 - \omega^2 m & -(k_1 a - k_2 b) \\ -(k_1 a - k_2 b) & (k_1 a^2 + k_2 b^2) - \omega^2 J \end{bmatrix} * \begin{bmatrix} Y \\ \Theta \end{bmatrix} = \begin{bmatrix} F \\ M \end{bmatrix}$$

$$\begin{bmatrix} Y \\ \Theta \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}^{-1} \begin{bmatrix} F \\ M \end{bmatrix}$$

5-1 , 5-32
5-2 , 5-34
5-3 , 5-37
5-4 , 5-43
5-5 , 5-50
5-6 , 5-78
5-7
5-8
5-9