Vibrations

Dr. Basem Bdoor

1st Semester 2017

Mech Family

Notebooks

No.

Vibrations

Dr. Basem Al. bdour

1st Semester (2017-2018)

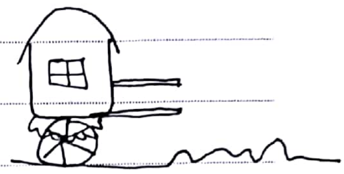
25/9/2017

No. Introduction

Physics Fundementals and science
+
Mathematics (tools) → Application
- → Civil
- → *Mechanical
- → Architecture

***
هاي الصفحة
عبارة عن
وين ما اتلف
وجهك بتلاقي
Vibrations

→ first Application was at horse Cart as the Cart was held by leaf Spring as it become flexible element and absorb the effect of road irregudarities



* Tire (Rubber with Air inside) → flexible elements

*Mechanical
- → Thermal / Fluids
- → Applied Mechanics → strength
  - → Statics
  - → Dynamics
  - → Design
  - → Machines

↓ all of them are related to Mech. Vibrations!

* FYI : electric Motor efficiency $= \dfrac{\text{Output (Mech)}}{\text{input (elect)}} = \dfrac{\tau * w}{I * V}$

→ since efficiency is not 100% that means there is energy losses Like the Vibration energy

→ Tacoma bridge Phenomena!
↳ big failure due to bibration created by wind at certain Speed.

*Vibration needs to be Considered else ⇒ no Comfort
⇒ Health Problem

Hearing is Vibration, microPhone is Vibration!, Sound isolation!,

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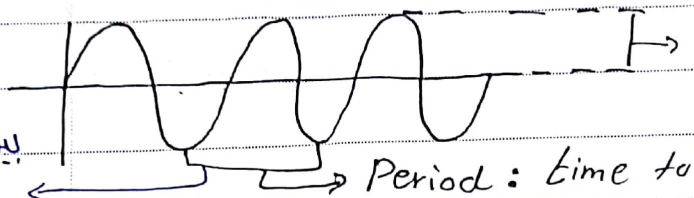
Vibration: repeated motion (positional) at period of time!

⇒ it can be represented by sinusoidal wave!

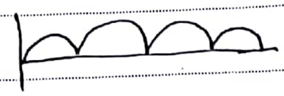


→ Amplitude

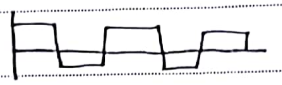
(T)

بين قمة وقمة
أو
قاع وقاع

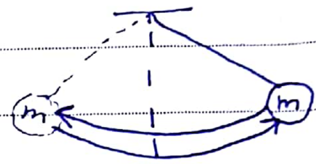
→ Period: time to complete one cycle (sec)



or



example: The Pendolume Motion



in electric field → Period is $\frac{1}{50\ 000}$ sec

(hertz)

* frequency (HZ) $= \frac{1}{T}$

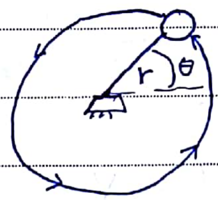
or

cycle per sec (cps)

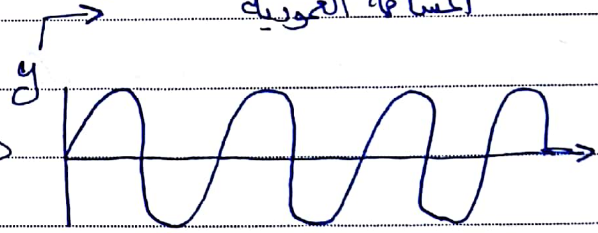
$y = r \sin \theta$

المسافة العمودية

* example:



⇒



$\phi = wt = 2\pi f t$, $\boxed{w = 2\pi f}$

No.

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* The material is available on website (Slide Share.net)
  ↳ Search in mechanical Vibration Slide Share

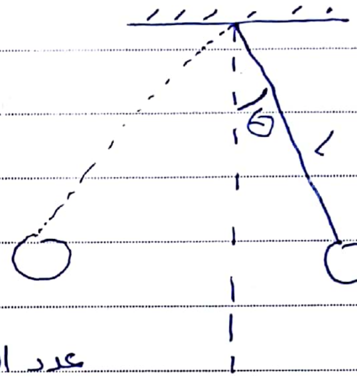
* Storing Potential energy Types : [1] gravity (height)
  [2] ~~elasticity~~ (Springs) ✓ ∝

elasticity مش Stiffness ✓

* Simplest Type of Free vibration → The pendulum.

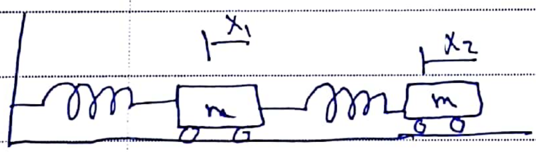
KE is Zero at Maximum height

mgL (   )



* The 3 elements of Vib system : [1] mass
  [2] spring
  [3] damper

* degree of freedom → عدد ال Parameter اللي بحتاجها علشان أوصف حركة الجسم



* 2 degree of freedom
  (2 Single and independent degree of freedom for each part)

Vibration Classification

* Free or Forced | Damped & undamped

Linear and Non-Linear | Deterministic or Random (Predictable/not Predictable)

↓ ↓

Differential Equation!

↳ واحنا ح احل هون دا

No.

* There is Assumption must be taken, then formulate and Analysing and Differentiation in order to find the Vibration Results (System Characteristic)

* Spring Element:

* for a Cantiliver beam:

the deflection at free end is

⟶ take the Force and treat like a spring so you can find the spring constant K

and

Springs in series ⟶ $\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} \dots$

Springs in parallel ⟶ $K_{eq} = K_1 + K_2 + K_3$

$F = K * \delta$ من معادلة ال
بنقدر نفهم لبيش

(Check the Slides)

2/10/2017

No.

- Vibration : Motion that repeats it self at certain pattern

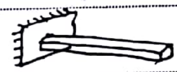
- Free vibration : Initial disturbance (excitation)
Vibrates freely, Continuous exchange between (KE & PE)
* for the Same material, Area, Cross-section
the speed of Vibration doesn't Change.

* Change in Length ⟶ Change in Stiffness (K)

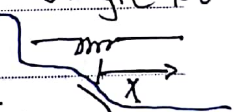
- Forced Vibration : under Continuous input from the outside of the System

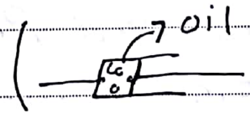
- Degree of freedom (DoF) :

- Continuous System (∞ DoF's) ex: Beam

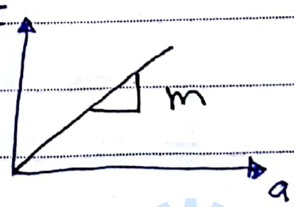


* DoF يعني عدد الـCoordinates اللي يحتاجها عشان
أقدر أوصف حركة جسم (X, Y, Z...) مثال...
- Discrete System (finite DoF's) ex: Single DoF



- Damping : - Damped ( oil or Friction)
- Undamped ⟶ No Energy dissipation

* The System that we deal with is :



[I] Mass element ⟶ Store KE

* Inertia is Body Resistance to accelerate. $F = ma$ Linear acceleration

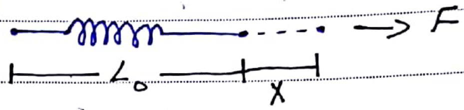
No.

- for Angular acceleration $\rightsquigarrow M = J\ddot{\theta} = J\alpha$ (rad/s²)

*Ideal point mass, no dimensions, No spring effect, No damping $\Big\}$ $m = m_1 + m_2 + \dots + m_n$

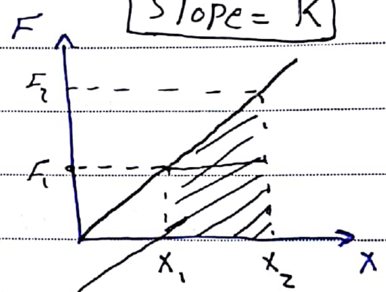
2 Spring element : (Store PE)

↳ transmits all forces



- gives flexibility for suspension system and resist the system inputs like earthquakes.

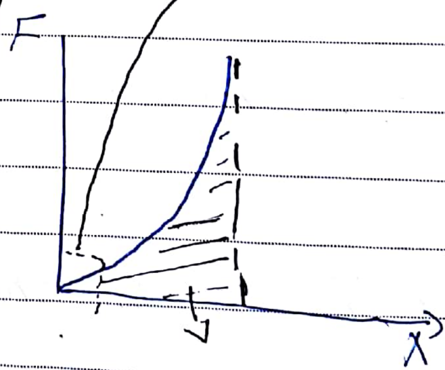
- Slope = K = Stiffness $\Big\{$ - massless, - linear, - No damping



* بس أول هاد ال Curve بتكون Linear ولما بصير في (deflection كبير (ΔX)) بصير في hardening وبطل Linear

$F = Kx$ (Linear Region)

- So in this Coarse we will deal with the linear region only for development and Predict system behavior.



- The area is Energy Stored in spring

$$\boxed{E = \frac{1}{2} K x^2}$$

$\boxed{F = Kx}$ ← integrate from force to energy

1

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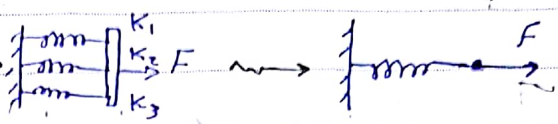
*Spring Connections:

- Spring in Parallel ⟶



$$F = F_1 + F_2 + \dots + F_n \leadsto K_{eq} * \cancel{X} = K_1 \cancel{X_1} + K_2 \cancel{X_2} + \dots + K_n \cancel{X_n} \quad \{ X = X_1 = X_2 \dots$$

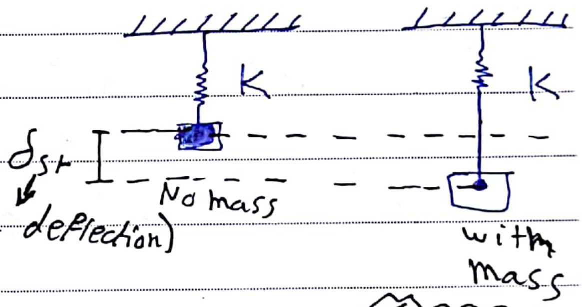
$$\boxed{K_{eq} = \sum K_n}$$

- Spring in Series:

$$X_{tot} = X_1 + X_2 + X_3 + \dots + X_n \leadsto \frac{F}{K_{eq}} = \frac{F_1}{K_1} + \frac{F_2}{K_2} + \dots + \frac{F}{K_n}$$

$$\boxed{\frac{1}{K_{eq}} = \sum \frac{1}{K_n}}$$

* يعني عكس المقاومات اللي بالكهرباء.

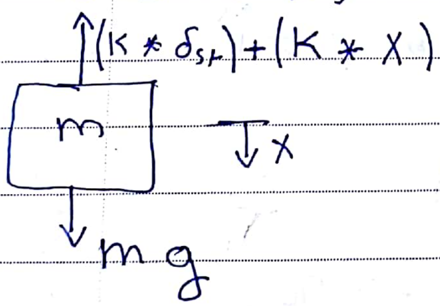


*هاي الحالة اسمها "Static equilibrium Position"

$mg = K\delta_{st}$ اللي عندها (Static deflection)

$\{F = W = m*g\}$

* F.B.D & apply Newton's 2^{nd} Law:



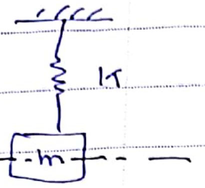
$$\leadsto \sum F_x = m\ddot{X} = \cancel{mg} - \cancel{K\delta_{st}} - KX$$

$$m\ddot{X} + KX = 0$$

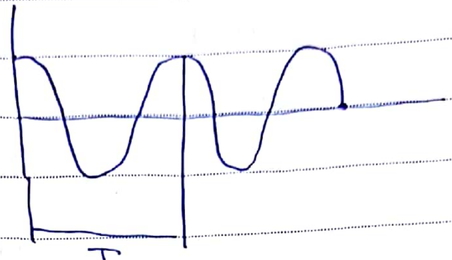
(2^{nd} ODE, homogenous, Constant Coefficient)

Initial Condition are needed $(X_0, \dot{X}_0)$

No. 4/10/2017



"Static equilibrium Position"



** $K\,\delta_{st} = mg$

$m\ddot{x} + kx = 0 \rightsquigarrow x(0) = x_0 \,,\ \dot{x}(0) = v_0$

→ Assume $x(t) = Ae^{\lambda t}$ ⇝ from the solution of 2^{st} ODE, homogenous, Linear

$\dot{x}(t) = A\lambda e^{\lambda t}$

$\ddot{x}(t) = A\lambda^2 e^{\lambda t}$

substitute in equation

$m\lambda^2 A e^{\lambda t} + kAe^{\lambda t} = 0$

$(m\lambda^2 + k)Ae^{\lambda t} = 0 \longrightarrow m\lambda^2 + k = 0$

$\lambda = \pm\sqrt{\frac{-k}{m}} = \pm i\sqrt{\frac{k}{m}}$

$\Rightarrow x(t) = B_1 \cos\left(\sqrt{\frac{k}{m}}\,t\right) + B_2 \sin\sqrt{\frac{k}{m}}\,t$

+ The Natural frequency $(w_n) = \frac{2\pi}{T_n} = 2\pi f$

$\boxed{f = \frac{1}{T_n}}$

f → (H_z)

T_n → natural period

رح اتصير معادلة با Cos & Sin

$\boxed{e^{i\alpha} = \cos\alpha t + i\sin\alpha t}$ ⟵ Euler's Identity باستخدام

No.

$X_c(t) = B_1 \cos(w_n t) + B_2 \sin(w_n t)$

$= B_1 \cos\left(\frac{2\pi}{T} t\right) + B_2 \sin\left(\frac{2\pi}{T} t\right)$

$= B_1 \cos 2\pi f t + B_2 \sin(2\pi f t)$

* B_1 and B_2 are found by The initial Condition ~~$X(0)$ & $\dot{X}$.~~

$\dot{X}(0) = V_0$

$X(0) = X_0$

$\rightarrow X(0) = B_1 + 0 = X_0$

$\rightarrow \boxed{B_1 = X_0}$

$\rightarrow \dot{X}(0) = 0 + B_2 w_n = V_0$

$\boxed{B_2 = \frac{V_0}{w_n}}$

* كل هاد الحكي اسمو Complementary Solution

$\rightarrow$ So The general Solution is

$X_{gs} = X_{cs} + \cancel{X_{PI}}^{\,0}$

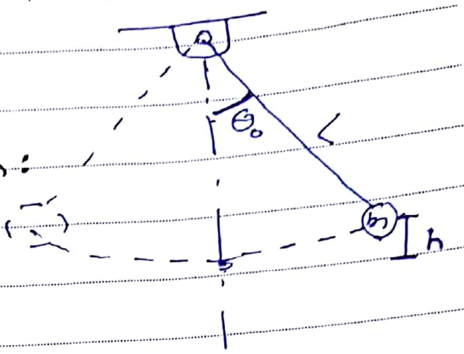
$\rightarrow$ Particular Integral

هاد بطلع لما يكون الطرف الثاني (اليمين) ما بساوي صفر

$$\boxed{X_{gs} = X_{cs} = X_0 \cos w_n t + \frac{V_0}{w_n} \sin w_n t}$$

No.

Simple Pendulum Example:



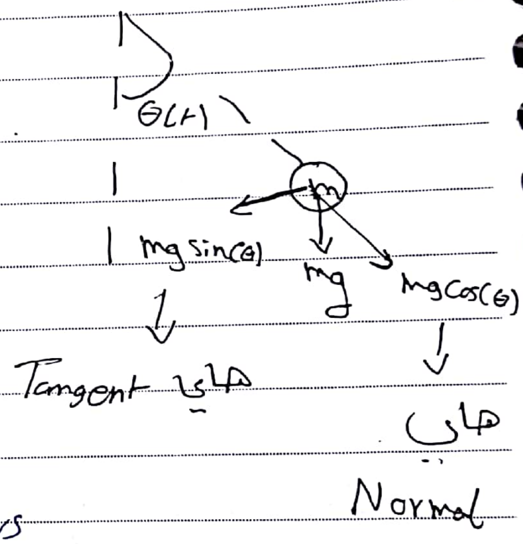
let's take a Free body diagram:

* we need to use Equation with The Variable θ

* by parallel axis Theorem
J_o (mass moment of inertia)

here (for simple Pendulum) $\boxed{J_o = m L^2}$

F.B.D



⇒ The Equation of Motion:

$$m L^2 \ddot{\theta} + m g L \sin(\theta) = 0$$

2^{nd} ODE, non linear (sin), homogenous

⇝ By Taylor Expansion and Assuming $\sin\theta \cong \theta$ (Small Angle)

$$\rightarrow m L^2 \ddot{\theta} + m g L \theta = 0$$

⇓

$$L \ddot{\theta} + g \theta = 0 \quad ; \quad g = 9.81 \text{ m/s}^2$$

Recall the Equation of the Previous Example:

$$m\ddot{x} + kx = 0 \rightsquigarrow \ddot{x} + \frac{k}{m}x = 0 \rightsquigarrow \ddot{x} + w_n^2 x = 0$$

بدنا نحل نفس الإشي بمعادلة ال Pendulum

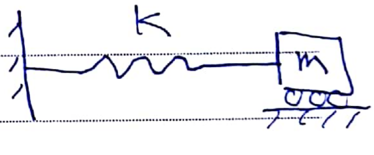
$$\ddot{\theta} + \frac{g}{L}\theta = 0$$

$$\hookrightarrow w_n^2 \rightsquigarrow w_n = \sqrt{\frac{g}{L}}$$

* انا الإشي الوحيد اللي بتحكم بالمعادلة هو الطول (L)
* بما انها نفس المعادلة فبنحل بنفس طريقة سؤال ال spring

$$\Rightarrow \theta(t) = \theta_0 \cos(w_n t) + \frac{\dot{\theta}_0}{w_n}\sin(w_n t) \qquad w_n = \sqrt{\frac{g}{L}}$$

* Energy Conservation of Energy (طريقة ثانية للحل)



$P.E = \frac{1}{2}kx^2$, $K.E = \frac{1}{2}m\dot{x}^2$

So $\rightarrow$ $\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = \text{Constant}\Big|_0$

↓ taking The derivatives

$$m\cancel{\dot{x}}\ddot{x} + kx\cancel{\dot{x}} = c \rightsquigarrow \boxed{m\ddot{x} + kx = 0}$$

Some Procedure for simple pendulum:

$KE = \frac{1}{2}I_o\dot{\theta}^2$, $PE = mg(\text{height}) = mg(L - L\cos\theta)$

$\rightarrow \frac{d}{dt}\left(\frac{1}{2}I\dot{\theta}^2 + mg(L - L\cos\theta)\right) = \text{Constant}$

↓

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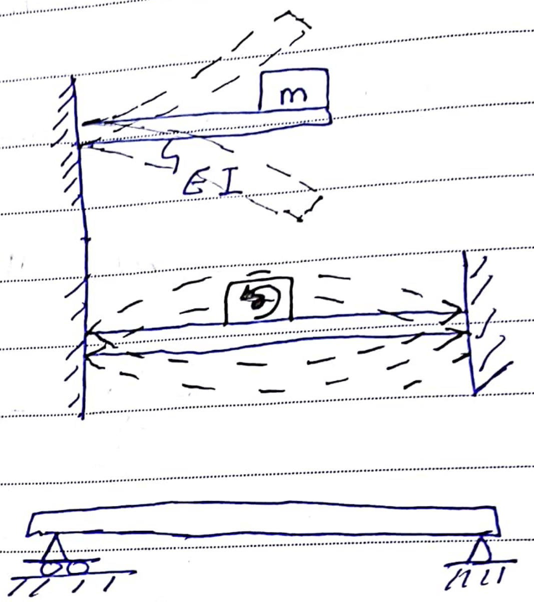
Example ⟶ Cantilever Beam



* Quiz → next week, read the first Chapter 1

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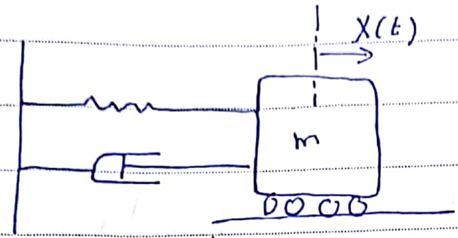
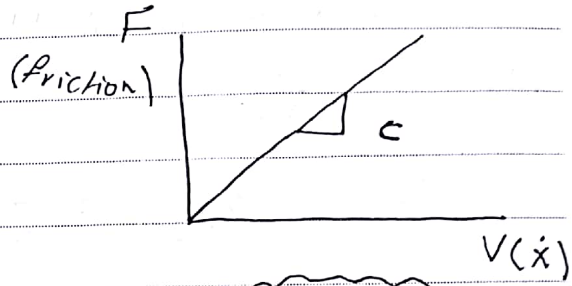
9/10/2017

Free Vibration of SDoF "Viscous Damping"

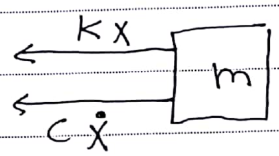


بكون في عنا Damper بتأثر بقوة احتكاك بتعتمد فقط على السرعة $\dot{x}$



$$\boxed{C = \frac{F_d}{\dot{x}}}$$

drawing The F.B.D



$$\sum F_x = m\ddot{x}$$

$$m\ddot{x} = -c\dot{x} - kx \qquad \boxed{x(0) = X_0 \,,\quad \dot{x}(0) = \dot{x}_0} \rightarrow \text{IC's}$$

→ 2^{nd} O.D.E with Constant Coefficients, ~~homogeneous~~ homogeneous

assume
$$x(t) = Ae^{\lambda t}$$
$$\dot{x}(t) = A\lambda e^{\lambda t}$$
$$\ddot{x}(t) = A\lambda^2 e^{\lambda t}$$
→ Substitute in the previous Equation

→ The Characteristic equation → $m\lambda^2 + c\lambda + k = 0$

The roots are $\lambda_{1,2} = \dfrac{-c \pm \sqrt{c^2 - 4mk}}{2m}$

$$= \frac{-c}{2m} \pm \sqrt{\frac{c^2 - 4mk}{4m^2}}$$

No.

So the roots are:

$$\frac{-c}{2m} \pm \sqrt{\frac{c^2}{2m} - \frac{k}{m}}$$

* حسب هاي المعادلة في عنا ٣ حالات (real, Zero, imaginary)

ما داخل الجذر (>0 , =0 , <0)

$\boxed{1}$ $\frac{c^2}{2m} - \frac{k}{m} = 0$, (Critically Damped)

So $\lambda_{1,2} = \frac{-c}{2m}$

$$\frac{-C_c^2}{4m} - K = 0 \rightsquigarrow C_{cr}^2 = 4mK$$

$$\boxed{C_{cr} = 2\sqrt{mK}}$$

Critical Damping (C_{cr}) هاد اسمو

* Now we will Define a new paramater [ζ]

Damping Ratio اسمو

$$\zeta = \frac{c}{C_{cr}} \rightsquigarrow \text{dimensionless !}$$

Based on ζ we have 4 Cases

if

$c > C_{cr} \longrightarrow \zeta > 1 \rightarrow$ Over damped

$c = C_{cr} \longrightarrow \zeta = 1 \rightarrow$ Critically damped

$c < C_{cr} \longrightarrow \zeta < 1 \rightarrow$ Under damped

$c = 0 \longrightarrow \zeta = 0 \rightarrow$ Undamped

No.

$\rightsquigarrow$ $C = \zeta \; C_{cr} = \zeta * 2 * \sqrt{mk}$

$\lambda_{1,2} = \quad , \quad = -\zeta \sqrt{\frac{mk}{m^2}}$

هاي اذا كانت $\sqrt{\frac{c^2}{2m} - 4mk} = 0$

So $\lambda_{1,2} = -\zeta \sqrt{\frac{k}{m}} \pm \sqrt{\frac{\zeta^2 k}{m} - \frac{k}{m}}$

$= -\zeta \, w_n \pm w_n \sqrt{\zeta^2 - 1}$

وهاي لما يكون ما داخل الجزر ما بساوي صفر

So Baseed on this equation and the 4 cases of ζ:

1. $\rightarrow$ for undamped case: $\zeta = 0$, $\lambda_{1,2} = \pm i w_n$

$x(t) = A\cos(w_n t) + B\sin(w_n t)$

2. $\rightarrow$ for under damped case: $0 < \zeta < 1$, $\lambda_{1,2} = -\zeta w_n \pm i w_n \sqrt{1-\zeta^2}$

$\rightsquigarrow$ here we define new parameter:

Damping frequency $(w_d) = w_n \sqrt{1-\zeta^2}$

(damped natural frequency)

$\lambda_{1,2} = -\zeta w_n \pm i w_d$

$$X(t) = A_1 e^{(-w_n + i w_d)t} + A_2 e^{(-\zeta w_n - i w_d)t}$$

$$X(t) = e^{-\zeta w_n t}\left(A_1 e^{i w_d t} + A_2 e^{-i w_d t}\right) \rightsquigarrow$$

No.

⇝ Using Euler's identity :

$$X(t) = e^{-\zeta w_n t}\left(B_1 \cos(w_d t) + B_2 \sin(w_d t)\right)$$

→ Recalling The I.C's → $X(0) = X_0$, $\dot{X}(0) = \dot{X}_0$
to find B_1 and B_2

① $X_0 = B_1$ ⇝ $\boxed{B_1 = X_0}$

② $\dot{x}(t) = -\zeta w_n e^{-\zeta w_n t}\left(B_1 \cos(w_d t) + B_2 \sin(w_d t)\right)$
$+ e^{-\zeta w_n t}\left(-B_1 w_d \sin(w_d t) + B_2 w_d \cos(w_d t)\right)$

هون اشتقيت معادلة
الـ $x(t)$ عشان أعوض
الـ $\dot{x}_0$ وأعرف B_2

$$\dot{X}_0 = V_0 = -\zeta w_n B_1 + B_2 w_d$$

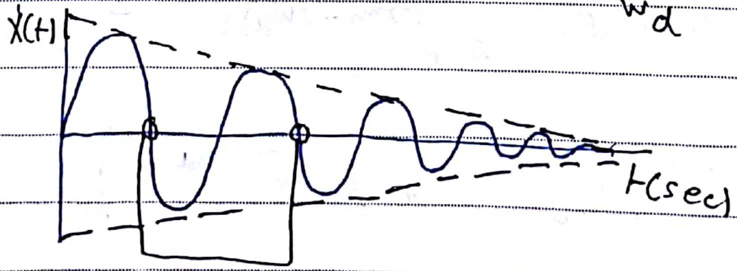
$$V_0 = -\zeta w_n X_0 + B_2 w_d$$

$$\boxed{B_2 = \frac{V_0 + \zeta w_n X_0}{w_d}}$$

عوض بالمعادلة
الأصلية

$$\Rightarrow X(t) = e^{-\zeta w_n t}\left(X_0 \cos w_d t + \frac{V_0 + \zeta w_n X_0}{w_d} \sin(w_d t)\right)$$

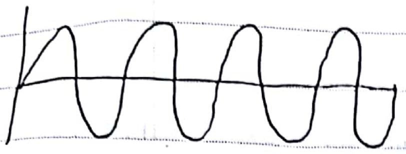
هاد الجزء
هو المسؤول
عن الـ
Decay
(exponential Decay)



T_d (damped Period) $= \dfrac{2\pi}{w_d} = \dfrac{2\pi}{w_n\sqrt{1-\zeta^2}}$

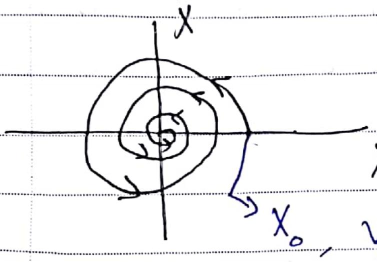
هاد تأثير الـ Damper

* بدون Damper ما في Friction Force ←



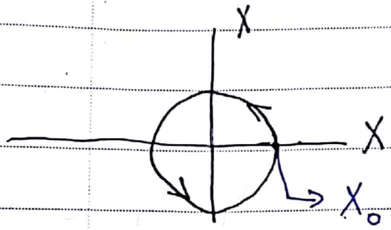
→ Energy is Conseved

* لو بدي ارسمها على $\dot{X}$ vs X | Phase Plane Plot)



→ Damping (No Energy Conservation)

X_0 , $V_0 = 0$ مثلاً



X → No Dampin (Energy is Conserved)

X_0 , $V_0 = 0$, مثلاً

* اذا كان في Decay يعني بروح 2 للصفر..

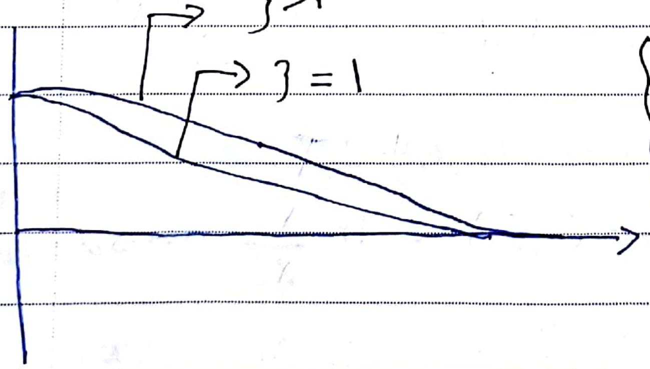
معناتو ζ أعلى , c , أعلى

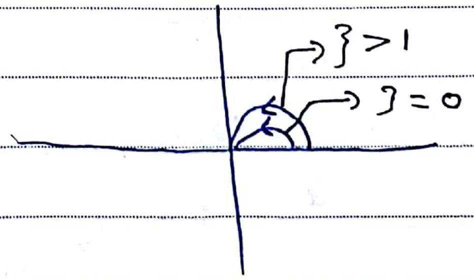
واذا ضل ثابت معناتو $\zeta = 0$, $c = 0$ ..

$\boxed{3}$ + $\boxed{4}$ Overdamped and Critically damp Case ($\zeta = 1$, $\lambda_{1,2} = -\omega_n$)

↳ repeated roots

(No Vibration)!

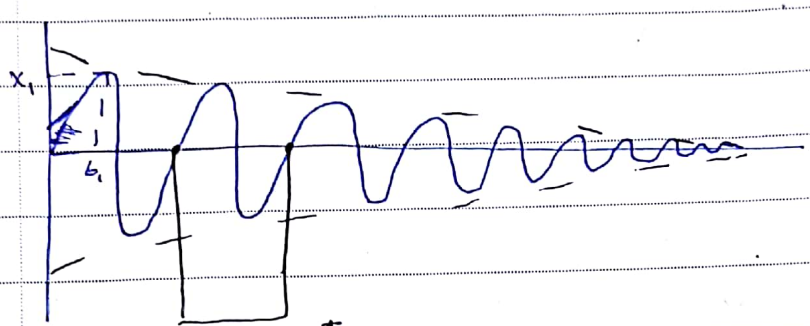




* Logarithmic Decrement:

→ Used to find experimentally the amount of damping in the system.

* مثلاً لو كان في عندي Spring أثر عليه بقوة معينة وعند الـ X_o
ومعيا Stop Watch بقدر أحسب ؟



$T_d = \frac{2\pi}{w_d}$

عندي X_1 و t_1 بس عوض بالمعادلة

بيطلع الـ Sin

$$x(t) = e^{-\zeta w_n t}\left(A\cos(w_d t + \phi)\right)$$

$$X_1 = e^{-\zeta w_n t_1}\left(A\cos(w_d t_1 + \phi)\right)$$!

* But $\boxed{t_2 = t_1 + T_d}$

$$X_2 = e^{-\zeta w_n t_2}\left(A\cos(w_d t_2 + \phi)\right)$$

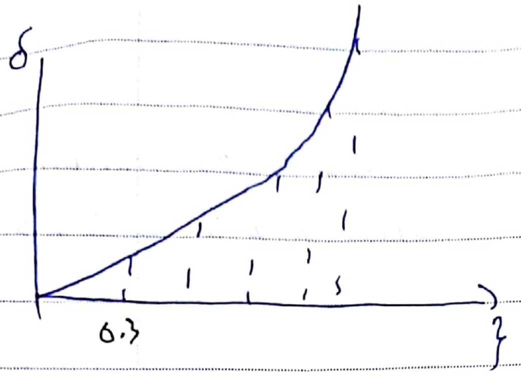
$\frac{2\pi}{w_d} * w_d = 2\pi$

$$= e^{-\zeta w_n t_1 - \zeta w_n T_d}\left(A\cos(w_d(t_1)) + 2\pi + \phi\right)$$

→ define New Parameter $(\delta) = \ln \frac{X_1}{X_2}$ → logarithmic decrement

$$\delta = \ln e^{\zeta w_n T_d} = \zeta w_n T_d = \frac{\zeta w_n 2\pi}{\sqrt{1-\zeta^2}\, w_n}, \quad \delta = \ln \frac{X_1}{X_2} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

No.



$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

→ for Lightly damped System $\zeta \ll 1$

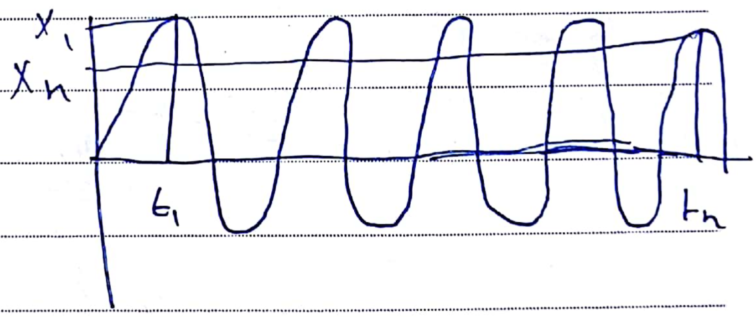
$$\ln\frac{X_1}{X_2} = \delta \cong 2\pi\zeta \Rightarrow \zeta = \frac{1}{2\pi}\ln\frac{X_1}{X_2}$$

→ for very Lightly Damped (يعني كثير قليل تأثير الـ Damper)

فبصير بدي n cycles لأقدر ألاقي

$\Rightarrow X_1 , X_n$ X_2 بتفرقا عن X_1 !

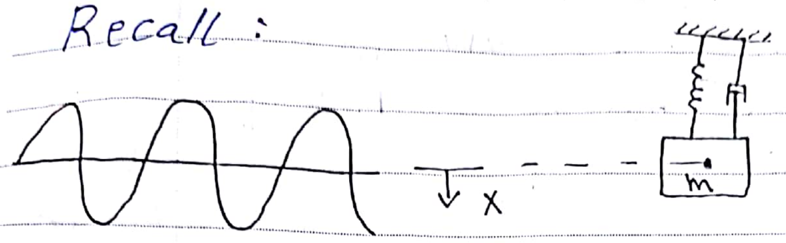
$\Rightarrow t_n = t_1 + n T_d$



11/10/2017

No.

Recall :

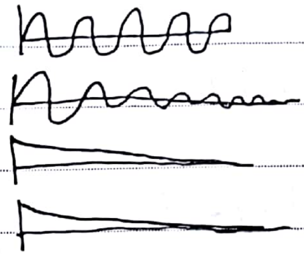


Static equilibrium with x_0, $\dot{x}_0$ as IC's

$$m\ddot{x} + Kx + c\dot{x} = 0 \quad --- \quad (1)$$

$$w_n = \sqrt{\frac{k}{m}} \;,\; C_{cr} = \sqrt{4\,Km} \;,\; \zeta = \frac{c}{C_{cr}}$$

Cases:
1. Undamped $(c=0, \zeta=0)$
2. Underdamped $(0<\zeta<1)$
3. Critically damped $(\zeta=1)$
4. Overdamped $(\zeta>1)$



$$\delta = \ln \frac{x_1}{x_2} \rightsquigarrow \delta = 2\pi\zeta \text{ (Lightly damped)}$$

devide eq (1) over m to get

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0 \;,\quad \frac{c}{m} = 2\zeta w_n \;,\quad \frac{k}{m} = w_n^2$$

$$\Rightarrow \ddot{x} + 2\zeta w_n \dot{x} + w_n^2 x = 0$$ انتبه انه معامل $\ddot{q}$ لازم يكون 1!

↳ Generalized Undamped SDoF System

ex: if the system Equation of Motion is $\underbrace{\ddot{q}}\; + \underbrace{3}_{2\zeta w_n}\dot{q} + \underbrace{25}_{w_n^2} q = 0$

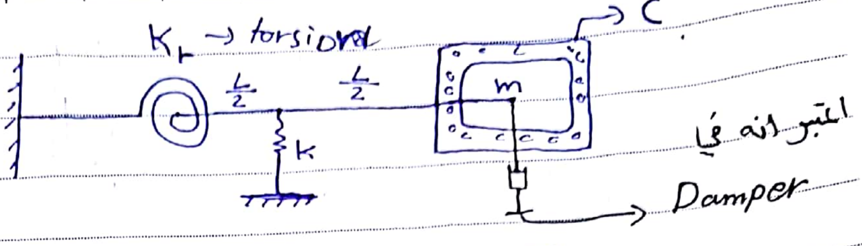
we can find w_n and ζ !

$\rightarrow w_n^2 = 25 \rightsquigarrow w_n = 5$ rad/s / $2\zeta w_n = 3 \rightarrow \zeta = \frac{3}{10} \rightsquigarrow$ أقل من 1 وأكبر من صفر

(Underdamped!)

No.

Example:



- Find: [1] Equation of motion, [2] C_{eq}, [3] K_{eq}, [4] m_{eq}
[5] ω_n, [6] ζ

- Showing: [1] Applying Newton's 2^{nd} Law
[2] Free body diagram
[3] The Coordinate System

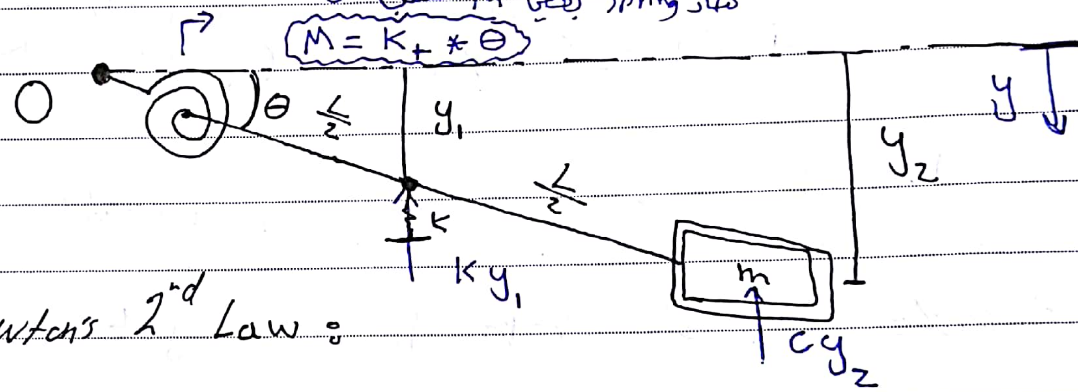
* the effect of mg is Cancelled by $k\delta_{st}$ (Static equilibrium Position)

هاد ال Spring بيعطي M حسب θ

$M = K_t * \theta$

F.B.D:

* $y_1 =$



Applying Newton's 2^{nd} Law:

$$\Sigma M_o = J_o \ddot{\theta}$$

* J_o: mass moment of inertia about O

$$ML^2\ddot{\theta} = -K_t\theta - \underbrace{ky_1}_{\text{قوة}}\underbrace{\frac{L}{2}\cos\theta}_{\text{مسافة}} - Cy_2 L\cos\theta$$

→ $J_o = mL^2$ بس على فرض انه ال rod ما الو mass فقط !!

* for Small θ, $\cos\theta = 1$ & $\sin\theta = \theta$

so $y_1 = \frac{L}{2} * \theta$, $y_2 = L * \theta$

معادلة ال Moment كلها عبارة عن القوة * المسافة بس بدي أكتبها بدلالة θ.

$$\boxed{mL^2\ddot{\theta} + K_t\theta + \frac{kL^2}{4}\theta + cL^2\dot{\theta} = 0}$$

No. $\rightarrow m_{eq} = mL^2$

$$\underbrace{mL^2}_{m_{eq}}\,\ddot{\theta} + \underbrace{\left(K_t + \frac{KL^2}{4}\right)}_{K_{eq}}\theta + \underbrace{CL^2}_{C_{eq}}\,\dot{\theta} = 0$$

بعدها تقسم المعادلة على m_{eq} عشان بصير معامل $\ddot{\theta}$ هو 1

$$m_{eq}\,\ddot{y} + C_{eq}\,\dot{x} + K\,x = 0$$

$$\Rightarrow \omega_n^2 = \frac{1}{m_{eq}}\left(K_t + \frac{KL^2}{4}\right)$$

هاد بس عشان أطلع ω_n & ζ

$$\omega_n = \sqrt{\frac{K_t}{m_{eq}} + \frac{KL^2}{4\,m_{eq}}}$$

$$\rightarrow 2\zeta\omega_n = \frac{CL^2}{m_{eq}} \rightsquigarrow \zeta = \frac{CL^2}{2\,\omega_n\,m_{eq}}$$

$$\Rightarrow \ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0$$

I.C's $= \theta(0)$, $\dot{\theta}(0)$

* Notice that we can write the equation in terms of y_2. $\theta = \frac{y_2}{L}$, $\dot{\theta} = \frac{\dot{y}_2}{L}$, $\ddot{\theta} = \frac{\ddot{y}_2}{L}$

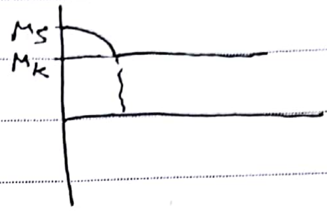
$$\ddot{y}_2 + 2\zeta\omega_n\dot{y}_2 + \omega_n^2 y_2 = 0$$

* Notice that ω_n & ζ are independent from the Coordinate System, and will always have the same Value.

No.

Dry friction Damping : "Columb damping"

$\mu_s \rightsquigarrow$ Static Coefficient of friction

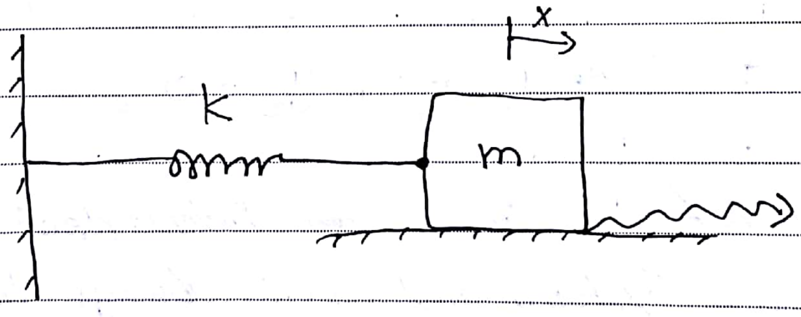


$\mu_k \rightsquigarrow$ Kinetic coeffizien of friction

μ depends on :
1) material properties of mating surfaces
2) Surface finish
3) Lubricants
4) Temperature

$$F_{friction} = \mu * N \quad N \rightarrow \text{normal Force}$$

$\hookrightarrow$ irreversible, non-Conservative, produce heat

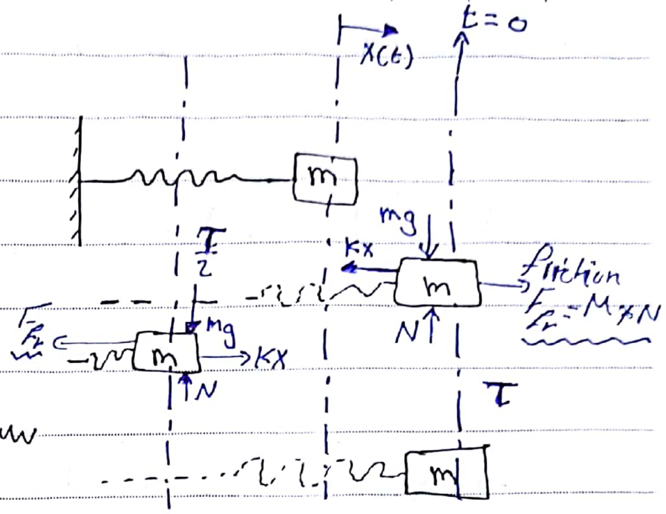


$\rightsquigarrow$ Contact means there is Friction Force!

No. 16/10/2017

- Free Vibration :

- Dry friction



- Period $= \frac{2\pi}{\omega_n} = T$

⟶ Applying Newton's 2^{nd} law

at $0 \le t \le \frac{\pi}{\omega_n}$ (half cycle)

$$\sum F_x = m\ddot{x} \rightsquigarrow m\ddot{x} + Kx = \mu * mg$$

$$\begin{cases} x(0) = x_0 \\ \dot{x}(0) = \text{Constant} = \text{zero} \end{cases}$$

⟹ General Solution: $X_g = X_{c.s} + X_{P.I}$

$\rightarrow X_{c.s} = A\cos(\omega_n t) + B\sin(\omega_n t)$

$\rightarrow X_{P.I} = \text{Constant} \longrightarrow Kc = \mu mg$

$$c = \frac{\mu mg}{K}$$

$$X_{g.s} = A\cos(\omega_n t) + B\sin(\omega_n t) + \frac{\mu mg}{K}$$

$\rightarrow X(0) = X_0 = A + \frac{\mu mg}{K} \rightsquigarrow \boxed{A = X_0 - \frac{\mu mg}{K}}$

$\rightarrow \dot{X}(0) = \dot{X}_0 = -A\sin(0) + B\cos(0) + 0 \rightarrow \boxed{B = \dot{X}_0} = \text{zero}$

(zero)

$$\rightsquigarrow X_{g.s}(t) = \left(X_0 - \frac{\mu mg}{K}\right)\cos(\omega_n t) + \frac{\mu mg}{K}$$

$$\rightsquigarrow \dot{X}_{g.s}(t) = -\omega_n * \left(X_0 - \frac{\mu mg}{K}\right)\sin(\omega_n t) \quad \boxed{* \; 0 \le t \le \frac{\pi}{\omega_n}}$$

No.

⇝ now we will find $x(\frac{\pi}{w_n})$ & $\dot{x}(\frac{\pi}{w_n})$ which will be our new IC for the 2^{nd} half $(\frac{\pi}{w_n} < t < \frac{2\pi}{w_n})$

$$\leadsto x(\frac{\pi}{w_n}) = (x_0 - \frac{\mu mg}{k}) * (-1) + \frac{\mu mg}{k}$$

$$= -x_0 + 2\frac{\mu mg}{k}$$

$\leadsto \dot{x}(\frac{\pi}{w_n}) =$ Zero ! ($V =$ zero ⇝ Switch direction!)

→ Apply Newton's 2^{nd} law $(\frac{\pi}{w_n} < t < \frac{2\pi}{w_n}$

$$\sum F = m\ddot{x} \leadsto m\ddot{x} + kx = -\mu mg$$

↳ negative sign
بنفس عكس الإتجاه !

$$x_{g.s} = x_{c.s} + x_{PI}$$

Same as before ← $x_{c.s}$

x_{PI} → $x_{PI} = C$

$kC = -\mu mg$

$C = \frac{-\mu mg}{k}$

$$@\ t = \frac{\pi}{w_n},\ \left.\begin{aligned} x &= -x_0 + \frac{2\mu mg}{k} \\ \dot{x} &= 0 \end{aligned}\right\} I.C$$

$$\leadsto x(\frac{\pi}{w_n}) = -x_0 + 2\frac{\mu mg}{k} = -A + 0 - \frac{\mu mg}{k}$$

$$A = x_0 - \frac{2\mu mg}{k} - \frac{\mu mg}{k}$$

$$A = x_0 - \frac{3\mu mg}{k}$$

No.

$$\dot{x}(t) = -A\,\omega_n \sin(\omega_n t) + B\,\omega_n \cos(\omega_n t)$$

$$\dot{x}\left(\frac{\pi}{\omega_n}\right) = 0 \;, \quad \boxed{B = 0}$$

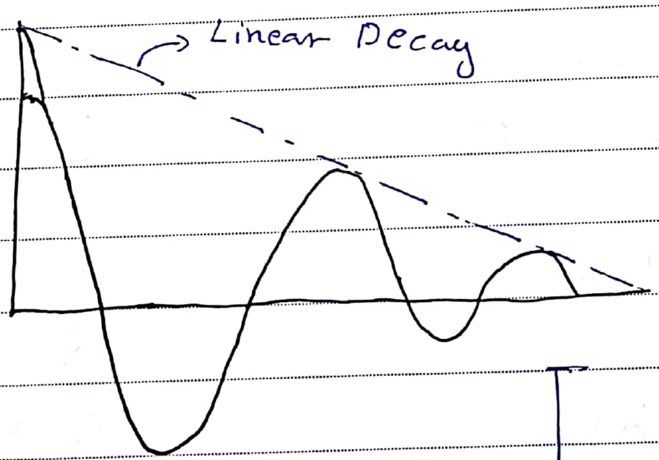
$$\rightarrow X_{g.s} = \left(X_0 - \frac{3\mu mg}{k}\right)\cos(\omega_n t) - \frac{\mu mg}{k}$$

@ end of 2nd half cycle $\left(\frac{2\pi}{\omega_n}\right)$ @ $\frac{\pi}{\omega_n} < t < \frac{2\pi}{\omega_n}$ period

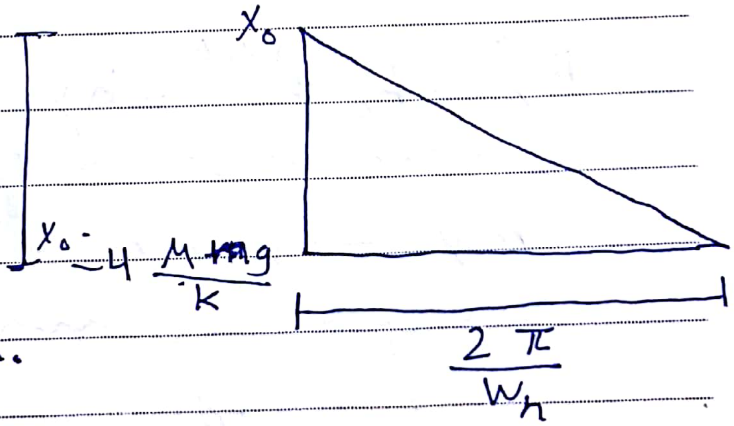
$$X\left(\frac{2\pi}{\omega_n}\right) = \left(X_0 - \frac{3\mu mg}{k}\right)(1) - \frac{\mu mg}{k}$$

$$= X_0 - 4\frac{\mu mg}{k} \quad !$$

* لاحظ انه كل نص Cycle قيمه x بتقل بقيمه ثابته مقدارها $\frac{2\mu mg}{k}$ ، يعني رح يكون عنا Linear Decay !



* Dry friction doesn't effect the natural frequency



$$m = \frac{-4\frac{\mu mg}{k}}{\left(\frac{2\pi}{\omega_n}\right)} = \frac{\Delta y}{\Delta x} \ldots$$

Chapter 3 : Forced Vibration of SDoF "Harmonic" excitation

$F(t) = F_o \sin(\Omega t)$ ✓ يعني قوة الها f ثابتة

Amplitude ↲ ↳ excitation frequency (rad/s)

$f = \frac{2\pi}{\Omega}$

$= F_o \cos(\Omega t)$ ✓

$= F_o e^{j\Omega t}$ ✓

$\underbrace{\sin, \cos, e^{j\Omega t}}$

هدول كلهم Harmonic

↓

* Undamped System $\left(\begin{matrix} c = 0 \\ \zeta = 0 \end{matrix}\right)$

بيوفرهم من خلال :

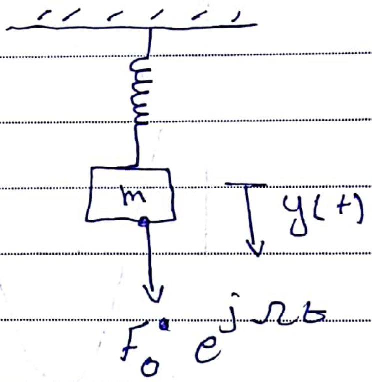
Actuator 1

Rotating 2
Unbalanced
Machine

example :

a spring with Static position under Harmonic excitation :

1 First we find the Equation of motion



→ Newton's 2^{nd} law :

$m\ddot{y} = F_o e^{j\Omega t} - ky$

$y(0)$ & $\dot{y}(0)$ are I.C's

$$m\ddot{y} + ky = F_o e^{j\Omega t}$$

↳ 2^{nd} order O.D.E BUT non-homogenous

No.

as previous Solution..

$$y_{g.s} = y_{c.s} + y_{PI}$$

$$y_{c.s} = A\cos(w_n t) + B\sin(w_n t)$$

→ Linear System Output follows input..

So we assume:

$$\left.\begin{aligned} y(t)_{PI} &= \overset{?}{Y} * e^{j\Omega t} \\ \dot{y}(t) &= Y j\Omega\, e^{j\Omega t} \\ \ddot{y}(t) &= -Y\Omega^2 e^{j\Omega t} \end{aligned}\right\}$$

Subtitute in E.o.m

$$m(-Y\Omega^2 e^{j\Omega t}) + K(Y e^{j\Omega t}) = F_o e^{j\Omega t}$$

$$(K - m\Omega^2)\, Y \cancel{e^{j\Omega t}} = F_o \cancel{e^{j\Omega t}}$$

$$\boxed{Y = \frac{F_o}{K - m\Omega^2}}$$

$$X_{PI} = \frac{F_o}{K - m\Omega^2} * e^{j\Omega t}$$

$$= \frac{F_o}{K - m\Omega^2} * \sin(\Omega t)$$

$$= \frac{F_o}{K - m\Omega^2} * \cos(\Omega t)$$

مدخول الس من نفس العيله فالحل نفس الحل

No.

let's study $Y = \dfrac{F_o}{K - m\Omega^2}$

→ devide by K ! $\quad w_n^2 = \dfrac{k}{m}$

$$Y = \frac{F_o/K}{1 - \left(\frac{m}{K}\right)\Omega^2}$$

$$Y = \frac{Y_{static}\ \boxed{1}}{1 - \frac{\Omega^2}{w^2}\ \boxed{2}}$$

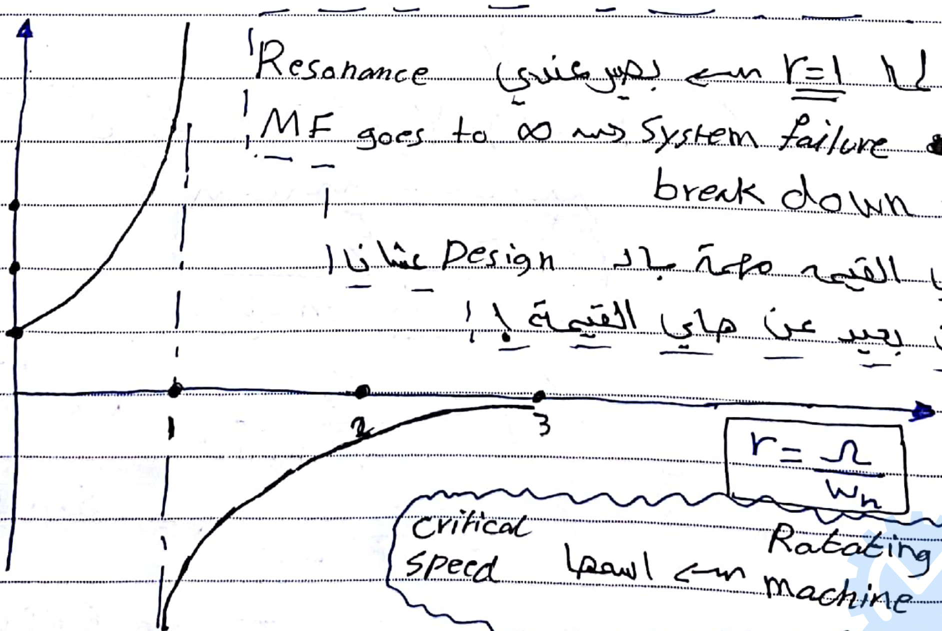
$$Y = \frac{Y_{static}}{1 - r^2}$$

$\boxed{1}$ F_o/k : Static deflection $y_{static} = \dfrac{F_o}{k}$

$\boxed{2}$ $r = \dfrac{\Omega}{w_n}$ → frequency ratio → the ratio of external excitation frequency to the System natural frequency

$\dfrac{Y}{Y_{static}}$ → magnification factor

↳ draw r Vs MF :



$\dfrac{Y}{Y_{static}}$

$\dfrac{Y}{y_{st}} = \dfrac{1}{1 - r^2}$

$r = \dfrac{\Omega}{w_n}$

* لما $\underline{r=1}$ ← بصير عندي Resonance

MF goes to ∞ → System failure / break down !

* هاي القيمة مهمة بالـ Design عشانا !!
أكون بعيد عن هاي القيمة !!

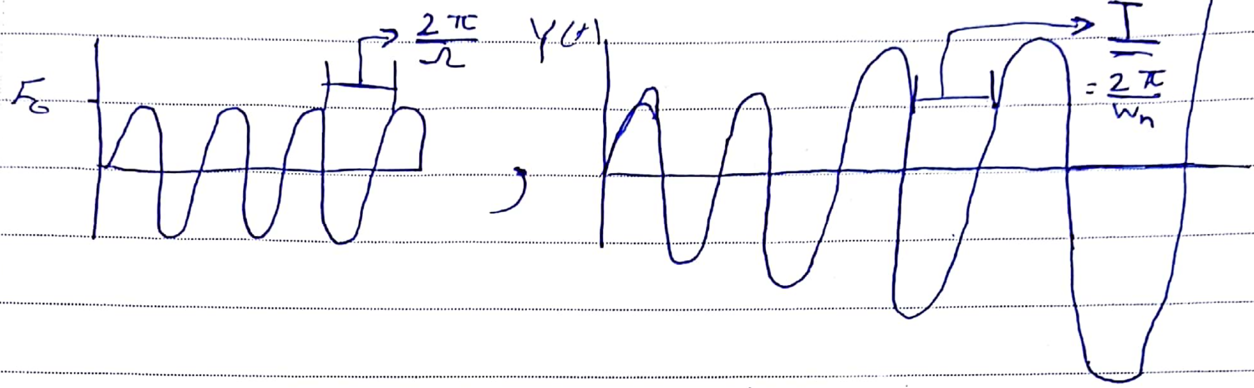
$r = 1$ ⟹ $w_n = \Omega$ → Resonance

Critical speed ← اسمها بالـ Rotating machine

Original Equipment Manufacturer (OEM) → Provide 5 critical speed and How to pass

No.

Resonance occure when $w_n = \Omega$ (Same phase)

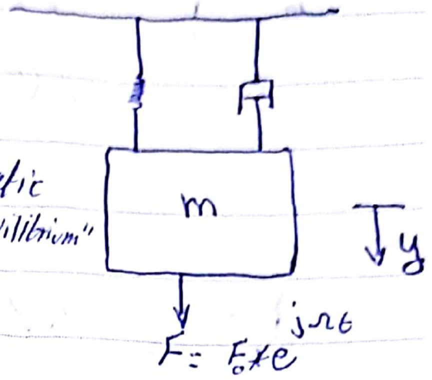


* عشان هيك بلش يكبر و بروح الى ∞ !

23/10/2017

No.

For underdamped $(0 < \zeta < 1)$



Equation of motion :

$$m\ddot{y} + c\dot{y} + Ky = F_0 e^{j\Omega t}$$

while $y(0)$ & $\dot{y}(0)$ are I.C's

$$y_{g.s} = y_{c.s} + y_{PI}$$

* for Linear System ⟿ Output follows the input BUT with Lag!

For the Particular Solution ⟿ $Y_{PI} = Y * e^{j\Omega t}$

$$\dot{Y}_{PI} = Y j \Omega e^{j\Omega t}$$

$$\ddot{Y}_{PI} = -Y \Omega^2 e^{j\Omega t}$$

Subtitute in EoM

⟿

$$-mY\Omega^2 \cancel{e^{j\Omega t}} + cYj\Omega \cancel{e^{j\Omega t}} + KY\cancel{e^{j\Omega t}} = F_0 \cancel{e^{j\Omega t}}$$

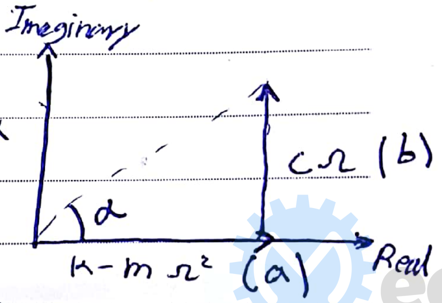
$$⟿ \quad Y = \frac{F_0}{(\underbrace{K - m\Omega^2}_{\text{real}} + \underbrace{cj\Omega}_{\text{imaginary}})}$$

Complex Dynamic Stiffness

$$a + jb = \sqrt{a^2 + b^2} * e^{j\alpha}$$

$$\alpha = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\text{So} ⟿ K - m\Omega^2 + c\Omega j = \sqrt{(K - m\Omega^2)^2 + (c\Omega)^2} * e^{j\alpha}$$



ثابتة مع تغير α

and $\alpha = \tan^{-1}\left(\frac{c\Omega}{K - m\Omega^2}\right)$

بس الاتجاه

* ال $e^{j\alpha}$ خارج الجذر لأنه أنا بس محتاج القيمة $\sqrt{(\ )^2 + (\ )^2}$

No.

$$\rightsquigarrow Y = \frac{F_0 \, e^{-j\alpha}}{\sqrt{(k-m\Omega^2)+(c\Omega)^2}} \rightarrow \text{devide by } (K)$$

$$Y = \frac{F_0/K}{\sqrt{\left(1-\frac{\Omega^2}{w_n^2}\right)^2+(2\zeta r)^2}}$$

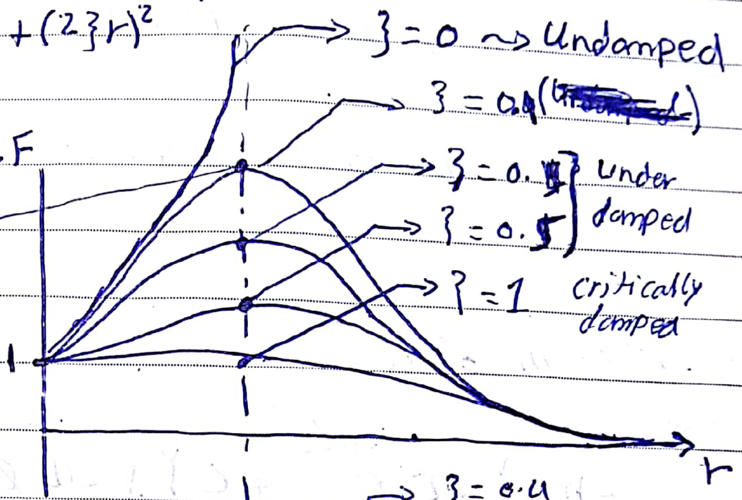
$\rightarrow y_{static}!$

and $\boxed{r=\frac{\Omega}{w_n}}$

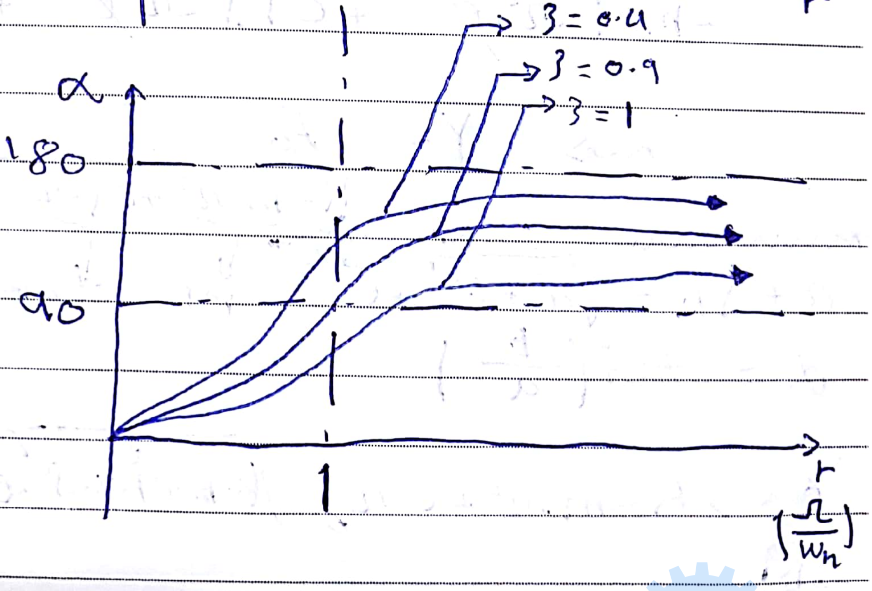
$$\frac{Y}{y_{st}} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = M.F$$

$$\left|\frac{Y}{y_{st}}\right| = M.F$$

(near resonance)



* α بتحدد قيمة
ال Phase shift
بين input & output



Plot the Curve : $r = (0.5 - 5)$, increment (0.1)

for $\zeta = 0.1, 0.8, 1, 1.2$

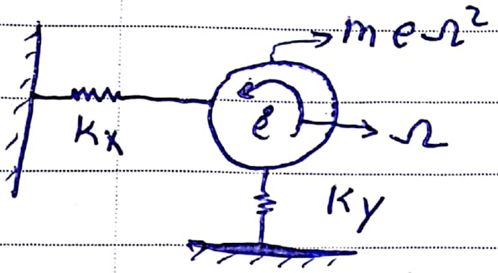
X-axis $\rightarrow r$

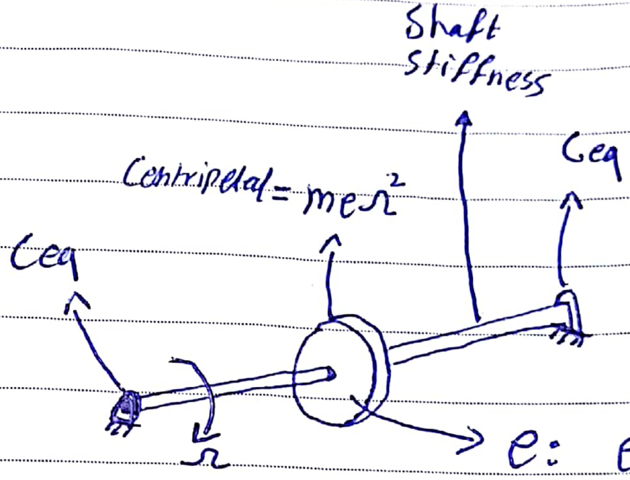
Y-axis $\rightarrow \dfrac{Y}{Y_{st}}$

$$y_{c.s} = e^{-\zeta\omega_n t}\left(A\cos(\omega_n t) + B\sin(\omega_n t)\right)$$

$$y_{P.I} = \frac{F_0/k}{\sqrt{(1-r^2)+(2\zeta r)^2}} * e^{-j\omega t}$$

* Jefcot rotor :





e: eccentricity "difference between mass center and the geometrically rotation center

Homework: $\boxed{1}$ Plot (the above)

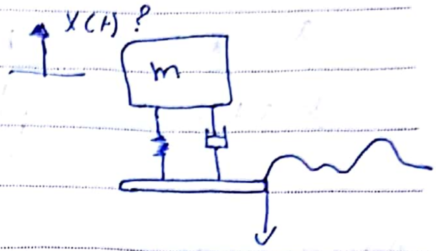
$\boxed{2}$ find the equation of motion for Jefcot rotor (موجودة بالكتاب)

30/10/2017

No.

Response of SDOF System under Base Harmonic excitation

The equation of motion

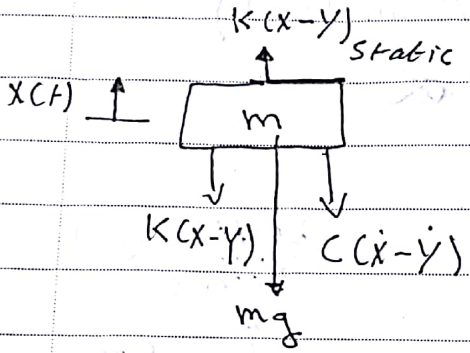


road irregulaty

$y = Y e^{j\Omega t}$

⟿ Free body diagram

assoming $x > y$.



* Applying Newton's 2^{nd} law:

$$\sum F_x = m\ddot{x} = -K(x-y) - c(\dot{x}-\dot{y})$$

$$\Rightarrow \quad m\ddot{x} + c\dot{x} + Kx = c\dot{y} + Ky$$

* as $y(t)$ is Known displacement

$x, \dot{x}$ @ Zero are oor Initial Conditions

$$y = Y e^{j\Omega t}, \quad \dot{y} = Y j \Omega e^{j\Omega t}, \quad \ddot{Y} = -Y\Omega^2 e^{j\Omega t}$$

Sobtitute in the orginal Equation:

$$m\ddot{x} + c\dot{x} + Kx = c(Y j \Omega e^{j\Omega t}) + K(Y e^{j\Omega t})$$

No.

So we have Linear system Output that is following the input.

$$X_{g.s} = X_{c.s} + X_{PI}$$

Homogeneous نفس الحل بال

For the particular Integral (X_{PI})

$$x = \overset{?}{X} e^{j\Omega t} \rightarrow \dot{x} = X j \Omega e^{j\Omega t} \rightarrow \ddot{x} = -X\Omega^2 e^{j\Omega t}$$

Subtitute in the original Equation:

$$(K - m\Omega^2) X e^{j\Omega t} + j c\Omega X e^{j\Omega t} = (K + jc\Omega) Y e^{j\Omega t}$$

Output هاي ال

$$\frac{X}{Y} = \frac{K + jc\Omega}{(K - m\Omega^2) + jc\Omega}$$

input هاي ال

هاي النسبة dimensionless واسمها Displacement Transmibility Ratio

and, $K + jc\Omega = \sqrt{K^2 + c^2\Omega^2}\, e^{j\phi}$

$$\phi = \tan^{-1}\left(\frac{c\Omega}{K}\right)$$

and $(K - m\Omega^2) + jc\Omega = \sqrt{(K - m\Omega^2)^2 + c^2\Omega^2} * e^{j\beta}$

$$\beta = \tan^{-1}\left(\frac{c\Omega}{(K - m\Omega^2)}\right)$$

$$\frac{X}{Y} = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

devide by K to get

$$* \; r = \frac{\Omega}{\omega_n}, \quad \zeta = \frac{C}{C_{cr}}$$

* من المعادلة هاي اذا ما كان عندي Damping ⟵ $C=0$

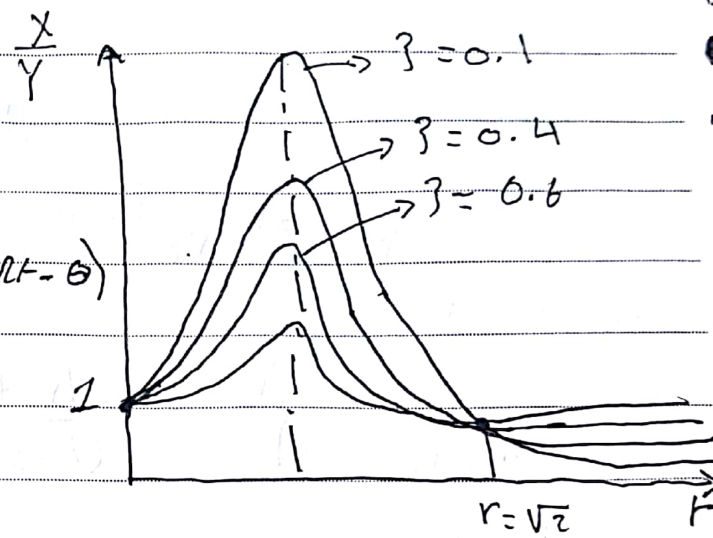
بتصير المعادلة

$$\frac{X}{Y} = \frac{1}{1-r^2}$$

* to find The Maximum Value ⟶ derrive $\left(\frac{d\left(\frac{X}{Y}\right)}{dr}\right)$

$$X_p(t) = \frac{Y\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}\, e^{j(\Omega t-\theta)}$$

$$X_p(t) = \frac{Y\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} * \sin(\Omega t-\theta)$$

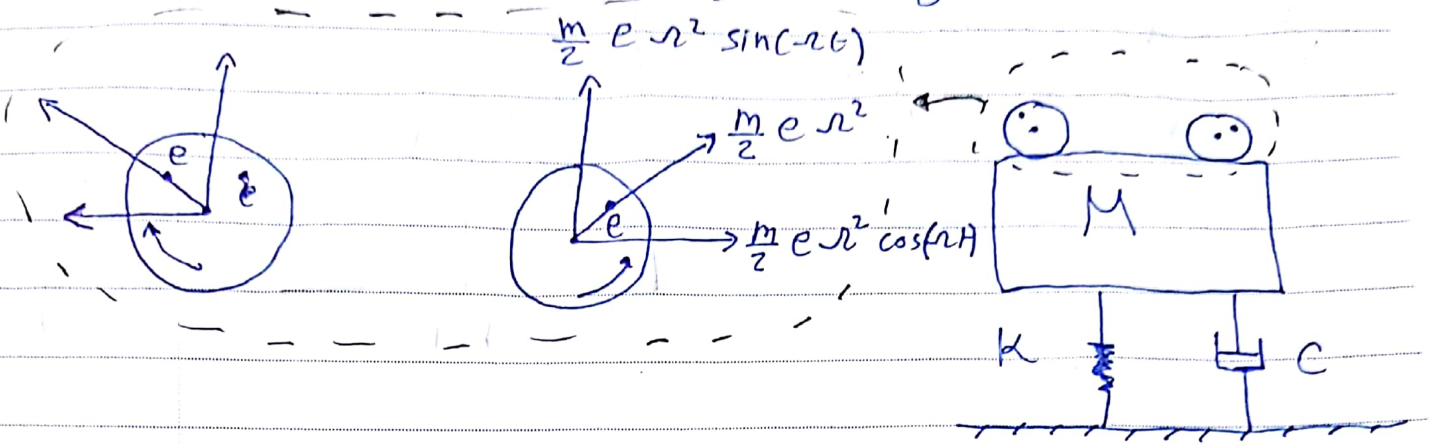


$$X_{g.s} = e^{-\zeta w_n t}\left(A\cos w_d t + B\sin w_d t\right) +$$

$$+ \frac{Y\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} * \sin(\Omega t-\theta)$$

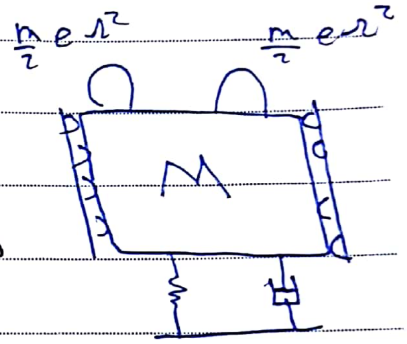
⟶ Apply The I.C'S to find A and B.

Response of SDOF due to rotating unbalance :



الـ e هي الـ eccentricity وهي بعد الـ Center of mass
عن الـ Center of rotation .
وهي السبب الرئيسي للـ Vibration.

For a SDOF ⟶ وأحلا اذا كانو نفس
بعض فالمحصلة
لليمين و اليسار بلغوا بعض!



⇒ Replace $F_0 = m e \Omega^2$

$X(t) = X e^{j\Omega t}$, $F(t) = m e \Omega^2 e^{j\Omega t}$

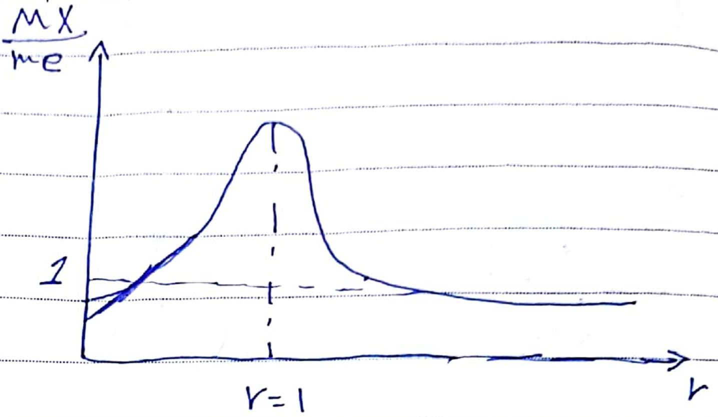
$$X = \frac{F_0}{\sqrt{(K - M\Omega^2)^2 + (C\Omega)^2}}$$ ⟶ if we devide by K

and $r = \Omega$, $\zeta = \frac{C}{C_{cr}}$, $K = M\omega_n^2$

$\frac{\Omega^2}{\omega_n} = M$

$$\frac{MX}{me} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$\cancel{\frac{X}{Y} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}}$$



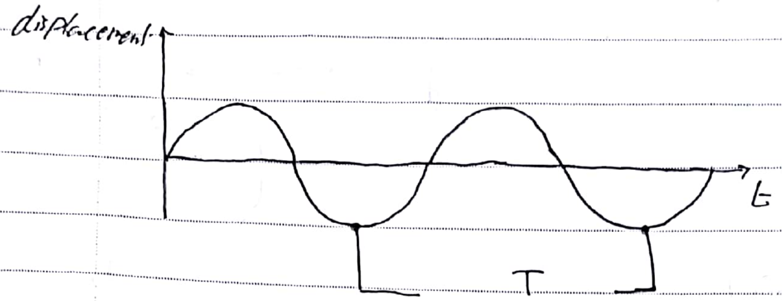
End of first exam material

1/11/2017

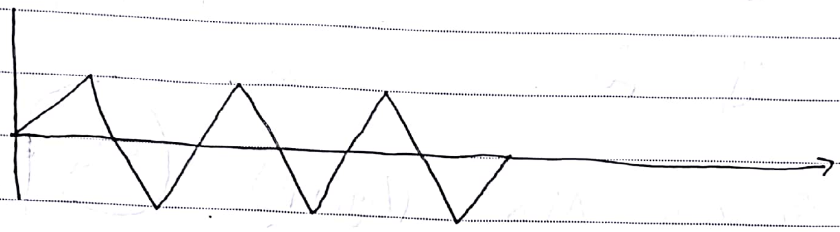
No.

Revision lecture

* Vibration: Dynamic Motion that repeats it self.



↳ period (seconds) (time) to Complete one Cycle



frequency (f): no. of cycle Completed in one Second (CPS) (H_z)

Mathimatically: $\omega = 2\pi f = \frac{2\pi}{T}$ (rad/s)

* f = rps = $\frac{rpm}{60}$!

* Classification
  - → Free Vibration → Mathematical Model by Physical rules (Newton)
  - → Forced Vibration

→ F.B.D with Assumption

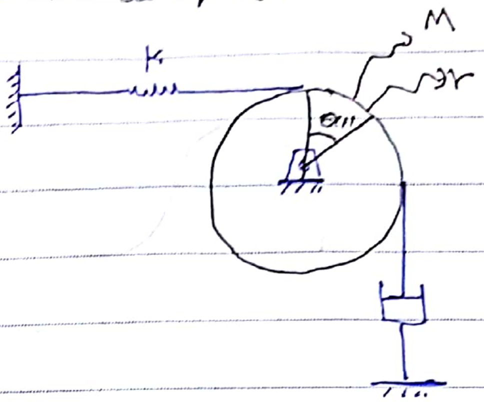
Applied to each DoF ← No. of independent Position Co-ordinate

No.

For Free vibration : $x(t)$, $Y(t)$, $\theta(t)$, $q(t)$

Classification : - Undamped - Under damped

- Critically damped - Over damped

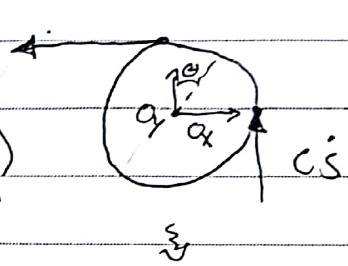


Assumptions :

1. Small motions
2. Linear Spring
3. Viscous damper

applying Newton's 2nd law:

*The F.B.D is drawn with deflections



ccw $\sum M_o = J_o \ddot{\theta}$

$J_o \rightarrow$ for disk $\frac{1}{2} M R^2$ $(Kg.m^2)$

$\frac{1}{2} M R^2 \ddot{\theta} = -K S r - c \dot{S} r$

$S = r\theta$

$\dot{S} = r\dot{\theta}$

replace S by θ

$\Rightarrow \frac{1}{2} M R^2 \ddot{\theta} + K r^2 \theta + c r^2 \dot{\theta} = 0$

$M \ddot{\theta} + 2c \dot{\theta} + 2K\theta = 0$ $\rightsquigarrow$ devide by M

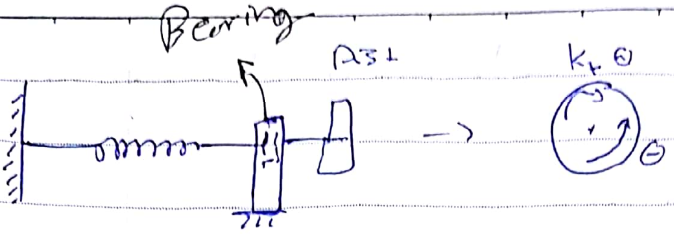
$2\zeta w_n$

$\ddot{\theta} + \frac{2c}{M} \dot{\theta} + \frac{2K}{M} \theta = 0$

$w_n = \sqrt{\frac{2K}{m}}$ (rad/s)

$\zeta = \frac{c}{M w_n}$

No.



$$\Sigma M_o = J_o \ddot{\theta}$$

$$J \ddot{\theta} + k_t \theta = 0$$

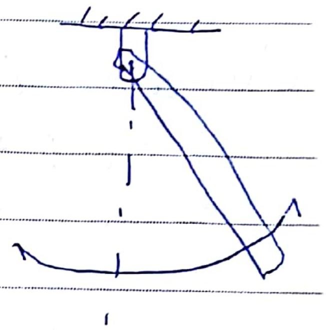
$$\ddot{\theta} + \frac{k_t}{J_o}\theta = 0$$

$$w_n^2 = \frac{k}{J_o}$$

$$w_n = \sqrt{\frac{k}{J_o}}$$

For Compound Pendulum (like Normal Pendulum)

$w_n =$ [illegible]



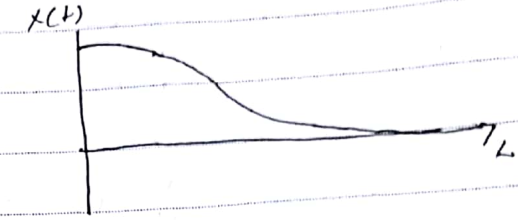
No.

تكملة اشتقاق المعادلات لآخر حالتين

Critically damped ($\zeta = 1$, $C = C_{cr}$)

$$\lambda_{1,2} = \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right) \omega_n$$

$$= -\omega_n$$



$$\rightsquigarrow \quad x(t) = C_1 e^{-\omega_n t} + C_2 \, t \, e^{-\omega_n t}$$

$$= e^{-\omega_n t} (C_1 + C_2 t)$$

if $x(0) = x_0$, $\dot{x}(0) = \dot{x}_0$ $\rightsquigarrow$ $C_1 = X_0$, $C_2 = \dot{X}_0 + X_0 \omega_n$

$$x(t) = \left(X_0 + (\dot{X}_0 + X_0 \omega_n) t\right) e^{-\omega_n t}$$

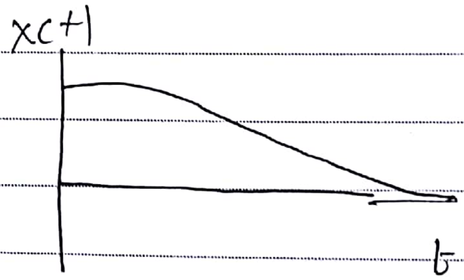
Over damped

$\sqrt{\zeta^2 - 1} \rightsquigarrow$ Positive

$$\left.\begin{aligned} \lambda_1 &= \left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n \\ \lambda_2 &= \left(-\zeta - \sqrt{\zeta^2-1}\right)\omega_n \end{aligned}\right\} \rightsquigarrow$$ Both will always be negative

$$x(t) = C_1 e^{\left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n t} + C_2 e^{\left(-\zeta - \sqrt{\zeta^2-1}\right)\omega_n t}$$

For $x(0) = x_0$, $\dot{x}(0) = \dot{X}_0 \rightsquigarrow$



$$C_1 = \frac{X_0 \omega_n \left(\zeta + \sqrt{\zeta^2-1}\right) + \dot{X}_0}{2\omega_n \sqrt{\zeta^2 - 1}}$$

$$C_2 = \frac{-X_0 \omega_n \left(\zeta - \sqrt{\zeta^2-1}\right) - \dot{X}_0}{2\omega_n \sqrt{\zeta^2 - 1}}$$

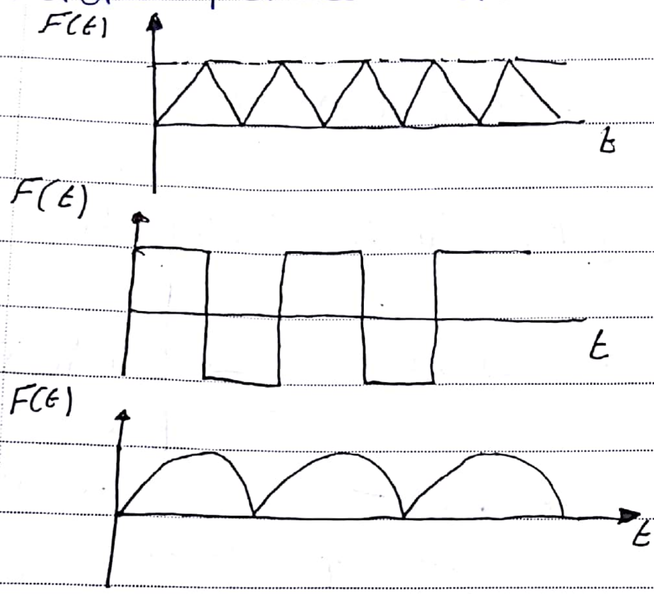
6-11-2017

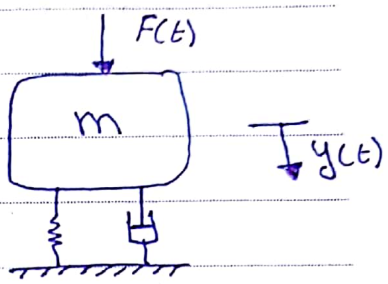
No.

Vibration

Chapter 4 : Response under General loading

[1] General periodic force





* Any periodic function Can be represented as a series Summation using Fourier Series.

$$F(t) = \frac{a_0}{2} + \sum_{i=1}^{\infty} a_i \cos(i\omega t) + b_i \sin(i\omega t)$$

where $\omega = \frac{2\pi}{T} = 2\pi f$

Fundemental frequency

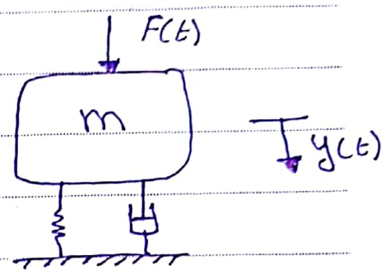
6-11-2017

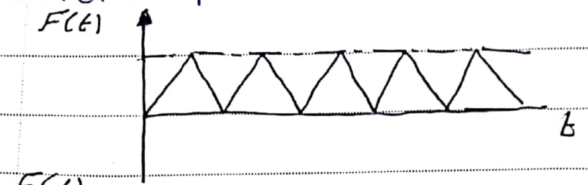
No.

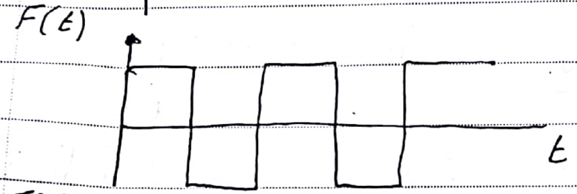
Vibration

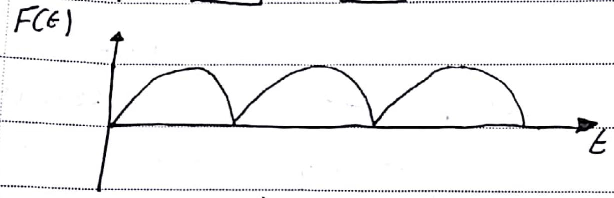
Chapter 4: Response under General loading

[1] General periodic force









* Any periodic function can be represented as a series summation using Fourier series.

$$F(t) = \frac{a_0}{2} + \sum_{i=1}^{\infty} a_i \cos(i\omega t) + b_i \sin(i\omega t)$$

where $\omega = \frac{2\pi}{T} = 2\pi f$

Fundemental frequency

No.

Multiply both Sides of the equation by $\cos(i\omega t)$ and integrate over One period

$$\int_0^T F(t)\cos(i\omega t)\,dt = \int_0^T \frac{a_0}{2}\cos(i\omega t)\,dt$$

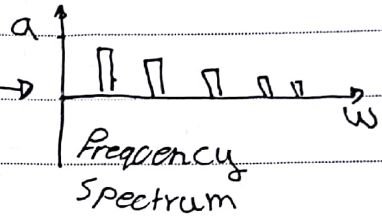
$$+ \sum \int_0^T a_i \cos^2(i\omega t)\,dt$$

$0 \to \infty$

$$+ \sum \int_0^T b_i \sin(i\omega t)\cos(i\omega t)\,dt$$

$$* \quad a_i = \frac{2}{T}\int_0^T F(t)\cos(i\omega t)\,dt$$

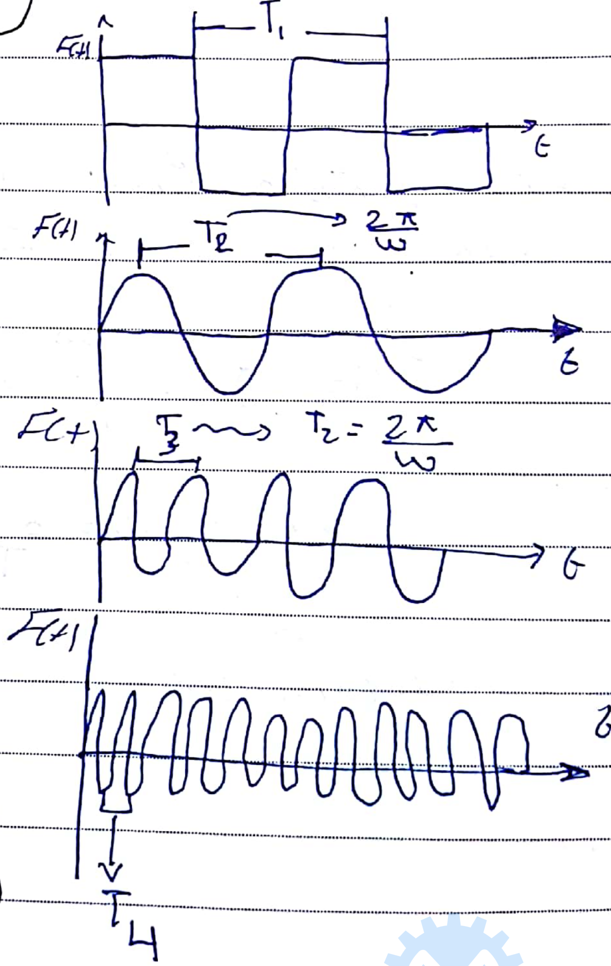
$$* \quad b_i = \frac{2}{T}\int_0^T F(t) * \sin(i\omega t)\,dt$$



The Fourier Coefficients:

$* \; m\ddot{y} + c\dot{y} + Ky = F(t)$

$$\Rightarrow m\ddot{y} + c\dot{y} + Ky = \frac{a_0}{2} + \sum_{i=1}^{\infty} a_i\cos(i\omega t) + \sum_{i=1}^{\infty}\sin(i\omega t)$$



after Solving the O.D.E for the given forcing function $y_{g.s} = y_{c.s} + y_{PI}$

Since we are dealing with linear System the methed of Superposition applies, then:

$$m\ddot{y}_i + c\dot{y}_i + Ky_i = a_i\cos(i\omega t)$$

$$m\ddot{y}_i + c\dot{y}_i + Ky_i = b_i\sin(i\omega t)$$

$$m\ddot{y}_i + c\dot{y}_i + Ky_i = \frac{a_0}{2}$$

↳ See next Page for the General Eq.

13/11/2017

No.

*Harmonic motion:

$y(t) = \sum_{i=1}^{\infty} y_i(t)$, for all the harmonics.

the particular Integral Solution is:

$$y_{PI} = \sum_{i=0}^{\infty} \frac{iA_i/k \quad \cos(i\omega t + \phi_i)}{\sqrt{(1-(ir)^2)^2 + (2\zeta ir)^2}}$$

$$+ \frac{iB_i/k \quad \sin(i\omega t + \phi_i)}{\sqrt{(1-(ir)^2)^2 + (2\zeta ir)^2}}$$

*Resonance:

$\Omega = \omega_n \Longrightarrow r = 1$

$i\Omega = \omega_n \Longrightarrow r = i$

$\rightarrow$ 2 points!

$\rightarrow$ Possibilities of resonance increase!

the general equation for previous page

$$\rightsquigarrow y_i(t) = \frac{a_i/k}{\sqrt{(1-(ir)^2)^2 + (2\zeta ir)^2}}, \cos(i\omega t - \phi)$$

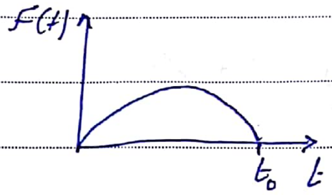
No.

So if the General loading is :

1. Periodic ⟶ Fourier analysis ⟶ Cos & Sine
   ⟶ exponential form

2. Non-periodic input :
   - doesn't repeat it self, act for sometime and disappears.

example :



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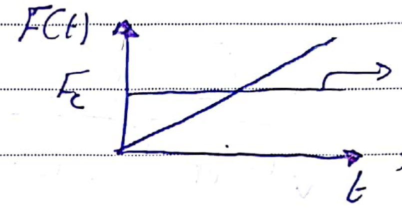
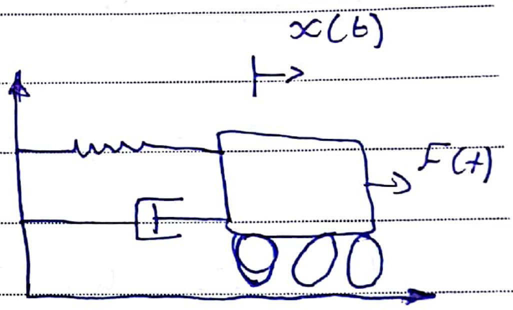


⟶ we assume a Constant Force

,

So for this system :



E.O.M :

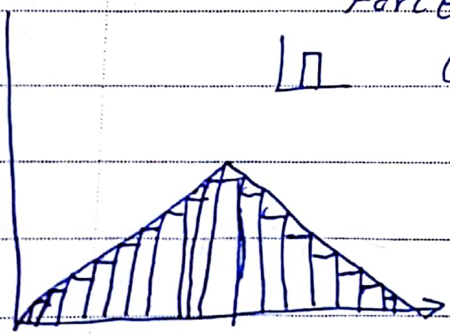
$$m\ddot{x} + c\dot{x} + kx = F_c$$

F_c ⟶ F(t) بدل الـ

2^{nd} order O.D.E !

مثال سس لو كان عندي هاي الـ Force

وقسمتها لمجموعة من ⊓ Constant Force..



وعشان الـ Vibration بقدر أطبق

عليها الـ (Super position)

الـ Output بساوي مجموع الـ Output لكل input ...

So we get : [1] Response to each Pulse
[2] Summation
[3] Duhamel's Integral (Convolution)

* Alternative methods : (1) Laplace transformation
[2] Numerical integration

* What actually happens in Laplace domain
we freeze the time and study the behavior in S-domain , S is $(i\omega)$.

* it's more Like Conversion

$$\mathcal{L}(f(t)) = \int_{0}^{\infty} f(t) * e^{-st} = F(s)$$

time-domain   S-domain

* Laplace function Conditions:

* anything physical can be expressed in Laplace!

* it Converts from differential to polynomial expressions.

* Valid method for Solving Linear System differential equation

No.

* it takes Case of the initial Condition automatically

* to go to time domain we do inverse Laplace transformation.

Check Laplace transformation table for Solving

Section 4.7 → Very important !

The SDOF System :

$$m\ddot{x} + c\dot{x} + Kx = f(t)$$

taking the laplace

$$\mathcal{L}(m\ddot{x} + c\dot{x} + Kx) = \mathcal{L}(f(t))$$

* for I'c = zero

15/11/2017

No.

Recall ⇝ $X(t) = \int_0^\infty x(t)\, e^{-st}\, dt$ Linear ODE

Our equation is $m \frac{d^2x}{d^2t} + c \frac{dx}{dt} + kx = f(t)$

$\mathcal{L}(x(t)) = X(s)$

For $\dot{x}$ ⇝ $\mathcal{L}\left(\frac{dx}{dt}\right) = \int_0^\infty \frac{dx}{dt} e^{-st} dt$ → integration by Parts

$u = e^{-st} \longrightarrow du = -s e^{-st}$

$dv = \frac{dx}{dt} \longrightarrow v = x(t)$

$\mathcal{L}\left(\frac{dx}{dt}\right) = s\, X(s) - X(0)$ → Initial Condition

$\mathcal{L}\left(\frac{d^2x}{d^2t}\right) = s^2 X(s) - s\,X(0) - \dot{X}(0)$

* From Laplace Transform table (b'ap) ⇝ $\mathcal{L}(c) = \frac{c}{s}$ → Step input

$\mathcal{L}(ct) = \frac{c}{s^2}$ → Ramp input

$\mathcal{L}(m\ddot{x} + c\dot{x} + kx) = \mathcal{L}(f(t))$

$m(s^2 X(s) - sX(0) - \dot{X}(0)) + c(sX(s) - X(0)) + k(X(s)) = F(s)$

$X(s)\,(ms^2 + cs + k) = F(s) + x(0)\,(ms + c) + m\,\dot{X}(0)$

Laplace take care of Initial Conditions automatically!

No.

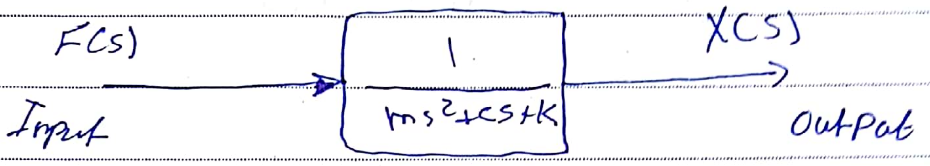
For Zero I'Cs $\rightarrow$ $X(0) = 0$, $\dot{X}(0) = 0$

$$X(s) = \frac{F(s)}{ms^2 + cs + k}$$

Transfer function $\rightarrow$ $\frac{\text{Output}}{\text{input}}$

$$= \frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$$

drawing the Block diagram :



after Solving for the Output X(s) back to time domain (Laplace Transform ~~...~~(inverse))

$x(t) = \mathcal{L}^{-1}(X(s))$

* Final and Initial Val
Theorem. (Steady Stat
and

this involves Complex integral
Usually we use tables and for Linear System

→ any problem Can be Solved by Partial fraction expansion

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No.

Partial fraction example:

$$X(s) = \frac{s}{s^2+3s+2}$$

هاد الشكل ما الو Laplace inverse
فبعمل partial fraction expansion

$$X(s) = \frac{s}{\underbrace{s^2+3s+2}_{=(s+1)(s+2)}} = \frac{a}{(s+1)} + \frac{b}{(s+2)}$$

وحد مقامات
عشان البسط
يساوي البسط

$\rightsquigarrow$ $s = a(s+2) + b(s+1)$

عوض -1 و -2
عشان توجد
قيم a & b

$s=-1$

$-1 = a + 0$

$\rightsquigarrow$ $a = -1$

$s=-2$

$-2 = 0 + -b$

$b = 2$

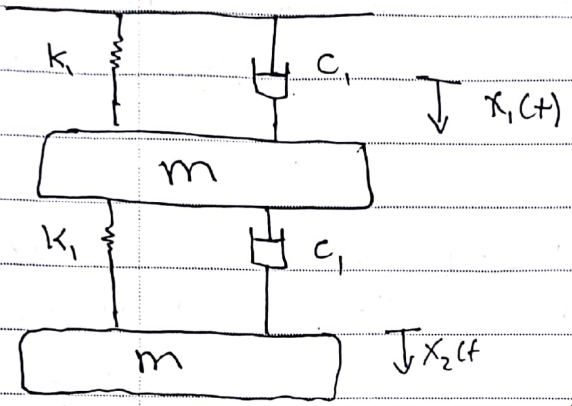
So $\rightsquigarrow$ $\mathcal{L}^{-1}\left(\frac{-1}{(s+1)} + \frac{2}{s+2}\right) = -e^{-t} + 2e^{-2t}$!

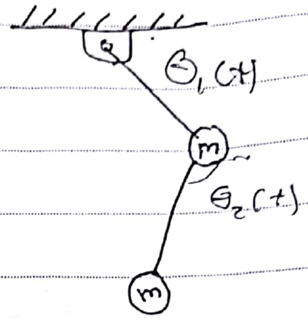
"Solved easily"

No.

Ch : Vibration of 2 DoF systems

Examples





→ The minimum number of Position Coordinates needed to describe the motion is 2.

→ the 2 Coordinates are Completely indepedent from each other, can't be related in an equation to Convert it into a S'DOF.

*For Free vibration we are after "Natural Frequencies" and "modes" of free vibration

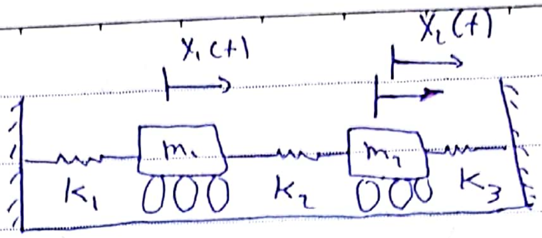
→ (العلاقة بين الحركتين سببتحركوا مع بعض أو عكس بعض)

Steps:

1. First we should (idealize) our system "Choose the Cordinates"
2. apply physical laws to develope the E.O.M
3. Perform to the resulting equations

→ Natural Frequency and modes

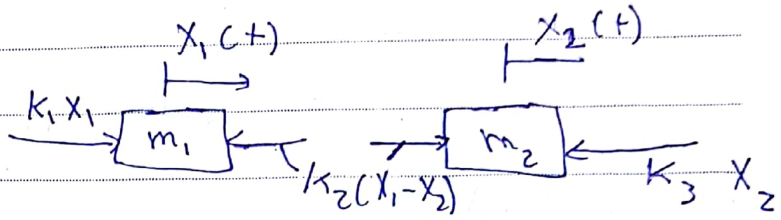
Example:



Apply Newton's 2^{nd} law for each of the mass at a time

F.B.D

• $x_1(t) > x_2(t)$



* for m_1

$$\sum F_{x_1} = m_1 \ddot{x}_1 = -k_1 x_1 - k_2 (x_1 - x_2)$$

$$m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 = 0 \quad ------ \quad (1)$$

* for m_2 :

$$\sum F_{x_2} = m_2 \ddot{x}_2 = k_2 (x_1 - x_2) - k_3 x_2$$

$$m_2 \ddot{x}_2 = +(k_2 + k_3) x_2 - k_2 x_1 \quad --- \quad (2)$$

These 2 equations

Should be used together

to be Solved

Re-arrange in matrix form

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{\boxed{\text{Mass matrix or Inertia matrix}}} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}}_{\boxed{\text{Stiffness matrix}}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

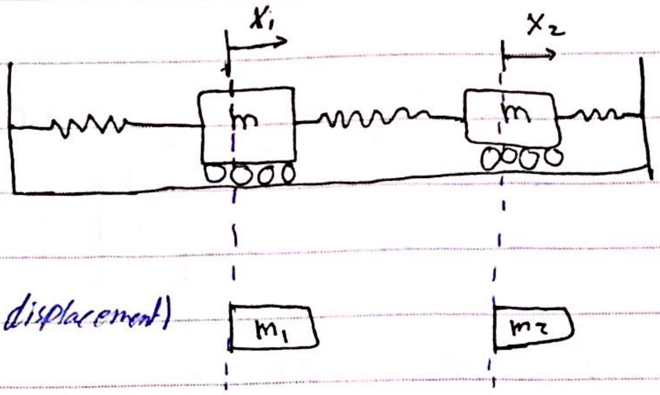
هدول بعبيهم بحيث لما أضربهم يطلع
هي نفس المعادلتين

22/11/2017

No.

2 DoF Systems

1- undamped

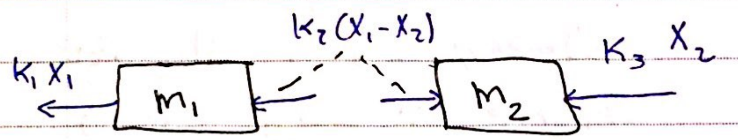


* Free vibration (initial displacement)

* Free body diagram

Assumption $x_1 > x_2$

(يعني الـ Spring اللي بينهم رح ينضغط)



Applying Newton's 2nd law :

m_1 $\quad m_1 \ddot{x}_1 = -k_1 x_1 - k_2(x_1 - x_2)$

$\longrightarrow m_1 \ddot{x}_1 + k_1 x_1 + k_2(x_1 - x_2) = 0$

m_2 $\quad m_2 \ddot{x}_2 = +k_2(x_1 - x_2) - k_3 x_2$

$\longrightarrow m_2 \ddot{x}_2 - k_2(x_1 - x_2) + k_3 x_2 = 0$

Matrix بقدر أمثلها بـ

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ ***

No.

* In general, any two degree of freedom System

$$\begin{bmatrix} m_{ii} & m_{ij} \\ m_{ji} & m_{jj} \end{bmatrix} \begin{bmatrix} \ddot{x}_i \\ \ddot{x}_j \end{bmatrix} + \begin{bmatrix} k_{ii} & k_{ij} \\ k_{ji} & k_{jj} \end{bmatrix} \begin{bmatrix} x_i \\ x_j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

if $m_{ij} = m_{ji}$ & $k_{ij} = k_{ji}$ → Symmetric → Positive Definite

يعني الـ Determinate $\neq 0$
وبقدر أوجد الـ Inverse

if $m_{ij} = 0$ → Dynamically unCoupled

$m_{ij} \neq 0$ → Dynamically Coupled

* Coupled → معناها انه الحركتين مربوطين ببعض
يعني في علاقة بينهم !

if $k_{ij} = 0$ → Statically unCoupled

$k_{ij} \neq 0$ → Statically Coupled

* For the previous example **

it is Dynamically UnCoupled and Statically Coupled

No.

Free Vibration Analysis:

* Two Natural frequencies
* Two Mode Shapes (Natural Coordinate)

→ Eigen value Problem

* Assume motion to be harmonic.

$$x_1 = \underset{\sim}{X_1} \sin(\omega t) \rightsquigarrow \ddot{x}_1 = -\underset{\sim}{X_1}\, \omega^2 \sin(\omega t)$$

$$x_2 = \underset{\sim}{X_2} \sin(\omega t) \rightsquigarrow \ddot{x}_2 = -\underset{\sim}{X_2}\, \omega^2 \sin(\omega t)$$

Subtitute in the Equation of Motion ...

عوض هدول وخد عامل مشترك X_1 و X_2

$$\left[-\omega^2 \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\right] \underbrace{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}}_{\text{doesn't equal Zero}} \sin(\omega t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

* اذا كان X_1 و X_2 ما بساوي صفر، معناتو القوس التاني لازم يساوي صفر

$$\begin{bmatrix} k_1 + k_2 - \omega^2 m_1 & -k_2 \\ -k_2 & k_2 + k_3 - \omega^2 m_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Eigen Value problem, Determinate $= 0$

$$\begin{vmatrix} (k_1+k_2) - w^2 m_1 & -k_2 \\ -k_2 & k_2 + k_3 - w^2 m_2 \end{vmatrix} = 0$$

$$\rightsquigarrow \left((k_1+k_2) - w^2 m_1\right)\left((k_2+k_3 - w^2 m_2\right) - k_2^2 = 0$$

$$\rightsquigarrow (k_1+k_2)(k_2+k_3) - (k_1+k_2) w^2 m_2$$

$$- m_1 w^2 (k_2+k_3) + w^4 m_2 m_1 - k_2^2 = 0$$

$$\Downarrow$$

$$m_1 m_2 w^4 - w^2 \left(m_1 (k_2+k_3) + m_2 (k_1+k_2)\right)$$

$$+ (k_1 k_2 + k_1 k_3 + k_2 k_3) = 0$$

Assume $\lambda = w^2 \rightsquigarrow$ يعني بطلع عندي معادلة تربيعية

$$\rightarrow m_1 m_2 \lambda^2 - \left(m_1 (k_2+k_3) + m_2 (k_1+k_2)\right) \lambda$$

$$+ (k_1 k_2 + k_1 k_3 + k_2 k_3) = 0$$

$\rightsquigarrow$

No.

$$\lambda_{1,2} = \frac{\left(m_1(k_2+k_3) + m_2(k_1+k_2) \pm \sqrt{\left(m_1(k_2+k_3) + m_2(k_1+k_2)\right)^2 - 4 m_1 m_2 (k_1 k_2 + k_1 k_3 + k_2 k_1)} \right)}{2 m_1 m_2}$$

القانون العام ← كثير طويل XD.

$\lambda_1 = w_{n_1}^2 \longrightarrow w_{n_1} = \sqrt{\lambda_1}$ rad/s

$\lambda_2 = w_{n_2}^2 \longrightarrow w_{n_2} = \sqrt{\lambda_2}$ rad/s

Which are the two natural frequencies of the System

→ Now go back to eqn (*) solve for

Vector which is the eigen vector $\begin{Bmatrix} X_1^{(1)} \\ X_2^{(1)} \end{Bmatrix}$

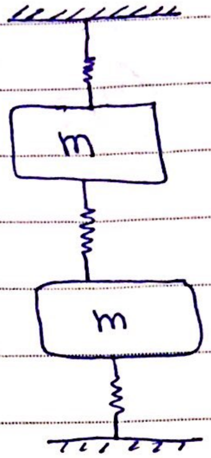
(mode Shape)

Similerly, Subtitute for w_{n_2} to Solve for the eigen Value which is the 2nd mode Shape

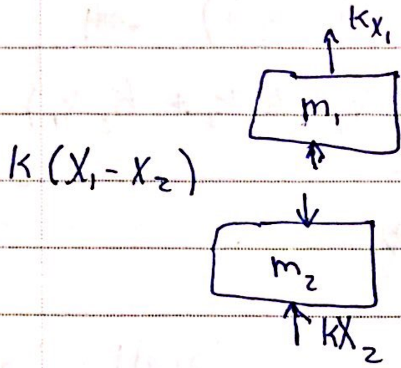
27/11/2017

No.

Example



first we take the diflicted Shape ..
and define the forces acting on the 2 masses.



for the first mass : $\sum F_{x_1} = m\ddot{X}_1 = -kX_1 - k(X_1 - X_2)$

$$\rightsquigarrow \; m\ddot{X}_1 + kX_1 + k(X_1 - X_2) = 0$$

for the Second mass : $\sum F_{x_2} = m\ddot{X}_2 = -kX_2 + k(X_1 - X_2)$

$$m\ddot{X}_2 + 2kX_2 + k(X_1 - X_2) = 0$$

$$\boxed{m\ddot{X}_1 + 2kX_1 - kX_2 = 0} \qquad \boxed{m\ddot{X}_2 + 2kX_2 - kX_1 = 0}$$

$$\rightsquigarrow \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{bmatrix} + \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

it is Eigen Value Problem ..

$$\left[[K] - w^2 [m] \right] [X] = 0$$

$$\begin{bmatrix} 2K - m w^2 & -K \\ -K & 2K - m w^2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = 0$$

$$\Delta (w^2) = \begin{vmatrix} 2K - m w^2 & -K \\ -K & 2K - m w^2 \end{vmatrix} = 0$$

$$(2K - m w^2)^2 - K^2 = 0$$

$$4K^2 - 4 K m w^2 + m^2 w^4 - K^2 = 0$$

$$m^2 w^4 - 4 K m w^2 + 3K^2$$

assume $w^4 = \lambda^2$

$$m^2 \lambda^2 - 4 K m \lambda + 3K^2 = 0$$

ال roots لهاي المعادلة بطلعلي w_{n1} & w_{n2}

No.

$$\omega^2 = \frac{km \pm \sqrt{16k^2m^2 - 12m^2k^2}}{2m^2}$$

$$= \frac{4km \pm 2mk}{2m^2}$$

$$\omega_2^2 = \frac{6k}{2m} = \frac{3k}{2m} \rightsquigarrow \omega_2 = \sqrt{\frac{3}{2}\frac{k}{m}}$$

$$\omega_1^2 = \frac{2km}{2m^2} = \frac{k}{m} \rightsquigarrow \omega_1 = \sqrt{\frac{k}{m}}$$

Recall $\quad (2k - m\omega^2)x_1 - kx_2 = 0$

Substitute By ω_1, and define $r_1 = \frac{X_1}{X_2}$

$$r_1 = \frac{X_1}{X_2} = \frac{k}{2k - m\omega_1^2} = \frac{k}{2k - \cancel{m}\frac{k}{\cancel{m}}} = 1$$

هاد الرمز يعني يتعامل مع ω_1

$$\rightsquigarrow \begin{Bmatrix} X_1^{(1)} \\ X_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} X_1^{(1)} \\ \frac{X_1^{(1)}}{r_1} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$m_1 \uparrow$ هيك يعني بتحركوا مع بعض

$m_2 \uparrow$

For the 2^{nd} natural frequency.

$$(2K - m\omega_2^2)\, X_1^{(2)} - K\, X_2^{(2)} = 0$$

$$r_2 = \frac{X_2^{(2)}}{X_1^{(2)}} = \frac{2K - m\omega_2^2}{K}$$

$$= \frac{2K - m\,3\frac{K}{m}}{m} = \frac{-K}{K} = -1$$

$$\rightsquigarrow \begin{bmatrix} X_1^{(2)} \\ X_2^{(2)} \end{bmatrix} = \begin{bmatrix} X_1 \\ r_2 X_1 \end{bmatrix} = X_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

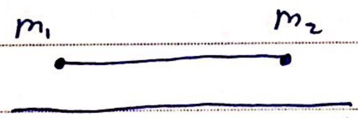
m_1 $\downarrow$
m_2 $\uparrow$

يعني بتحركوا عكس بعض

الخط مستقيم عشان $m_1 = m_2$

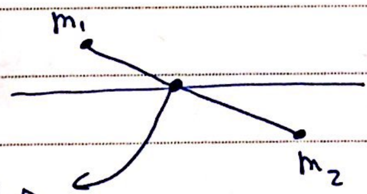
فبكون مايل بدون ما يعفج $m_1 \neq$

First mode



$$\omega_1 = \sqrt{\frac{K}{m}}$$

Second mode



$$\omega_2 = \sqrt{\frac{3K}{m}}$$

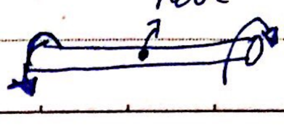
Node point هاي اسمها

(No motion)

(No Deflection)

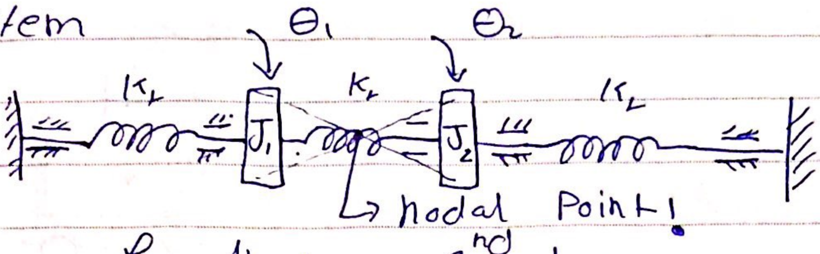
* Role: No.
No. of nodes = Mode − 1
For each mode

Torsion example



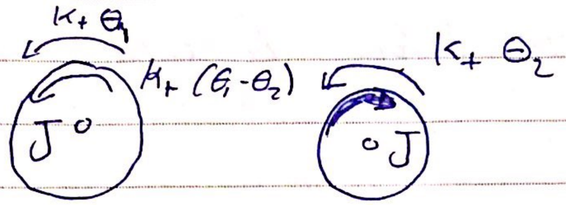
No.

Torsional System



الـ node انفصلها
بتقرب على اللي الـ
J أكبر

for Newtons 2^{nd} law



$$\sum M_{J_1} = J_1\ddot{\theta}_1 = -K_t\theta_1 - K_t(\theta_1-\theta_2) = 0$$

$$\boxed{J\ddot{\theta}_1 + 2K_1\theta_1 - K_t\theta_2 = 0}$$

$$\sum M_{J_2} = J_2\ddot{\theta}_2 = -K_t\theta_2 + K_t(\theta_1-\theta_2)$$

$$J_2\ddot{\theta}_2 = -K_t\theta_2 + K_t(\theta_1-\theta_2)$$

$$\boxed{J_2\ddot{\theta}_2 + 2K_t\theta_2 - K_t\theta_1 = 0}$$

so

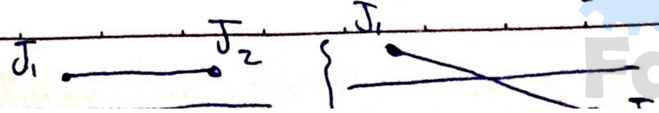
$$\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 2K_t & -K_t \\ -K_t & 2K_t \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Same as Previous example!

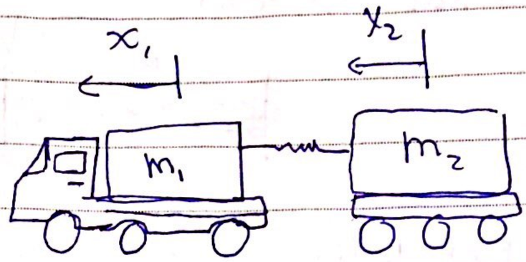
$$W_1 = \sqrt{\frac{K_t}{J}}, \quad W_2 = \sqrt{\frac{3K_t}{J}}$$

mode 1 = $\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$ mode 2 = $\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$

and Same nodes



Example..



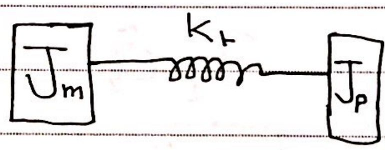
homework

Find the natural frequencies and mode shapes.

* From the Solution, we can see that $\omega_n = 0$, $T_{period} = \infty$

which means that there is no vibration..

and this happen in torsional systems as in pumps and motor.



$$\begin{bmatrix} J_m & 0 \\ 0 & J_p \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k_t & -k_t \\ -k_t & k_t \end{bmatrix}}_{} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Delta K = 0$!!

Due to rigid body mode...

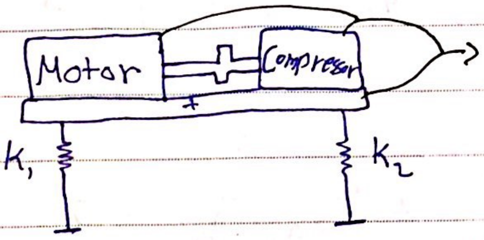
29/11/2017

No.

any 2DoF can be represented in a matrix

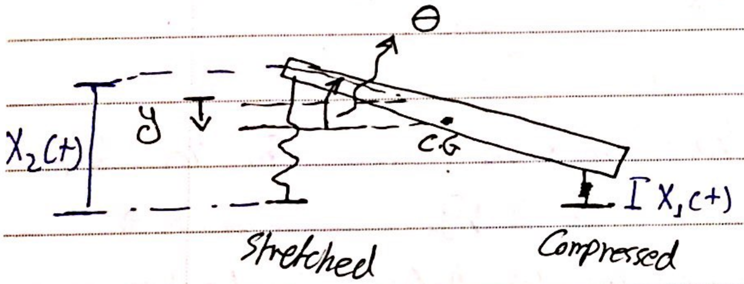
$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{21} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Example :



→ assuming the bodies are Rigid body

First ⇝ What are the degree of Freedom ? ⇝ $\underline{\underline{2}}$



* بقدر أوصف الحركة بـ

1) y (حركة الـ C.G يشكل عمودي)

② Rotational Motion ⟸ θ

حولين الـ C.G

* وبنفس الوقت بقدر أوصف هاي الحركة بـ $x_1(t)$ & $x_2(t)$

وهيك برضو 2DoF ..

بمعنى آخر

There must be some Coordinate system $q_1(t)$ & $q_2(t)$ that we can use and result a new system of equation (even if there is no Physical meaning) that will lead to unCoupled system which mean the two equation can be solved independently. These Coordinates are the Natural (Principle) Coordinates

No.

* Un Coupled ⇝ يعني مش متصلين ببعض
ولا إلهم دخل ببعض ، وبتصير الـ matrix

$$\begin{bmatrix} m_{qq_1} & 0 \\ 0 & m_{qq_2} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} k_{qq_1} & 0 \\ 0 & k_{qq_2} \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$m_{qq_1}\ddot{q}_1 + k_{qq_1} q_1 = 0 \qquad q_1(t) = ?$$

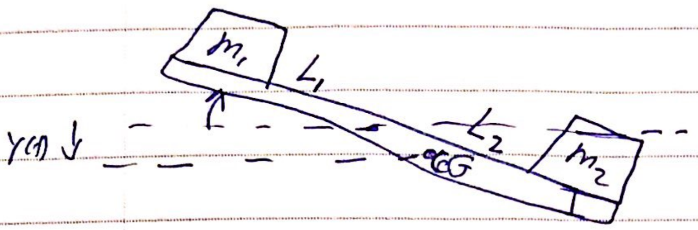
$\boxed{\text{Independent}}$

$$m_{qq_2}\ddot{q}_2 + k_{qq_2} q_2 = 0 \qquad q_2(t) = ?$$

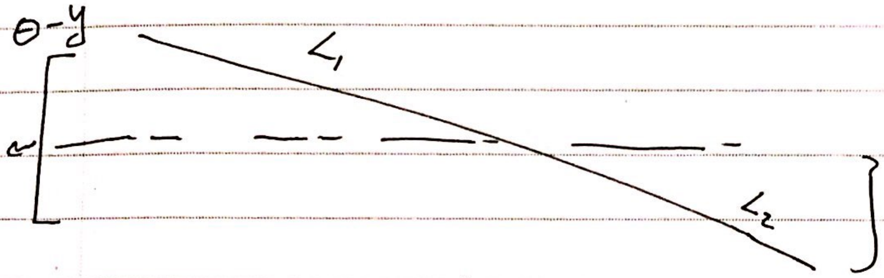
⇝ There must be a transformation that can transform [m] and [k] from [x] to [q]
i.e from physical coordinates to natural coordinates ...

No.

Example : recall the motor and compressor and solve it with $y(t)$ and $\theta(t)$



$x_1 = L_1\theta - y$



$\rightsquigarrow x_2 = y + L_2\theta$

Apply Newton's 2^{nd} law

homework $\rightsquigarrow$ find the Equation of motion

11/12/2017

No.

Recall the 2 DoF Free vibration Matrix :

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$\rightsquigarrow$ Eigen Value problem $\rightsquigarrow$ ω_1, ω_2

$$\begin{bmatrix} x_1^{(1)} & x_1^{(2)} \\ x_2^{(1)} & x_2^{(2)} \end{bmatrix} \rightsquigarrow \text{Modal Matrix } [V]$$

transposed

$$\Rightarrow [m_{new}] = [V]^T [m] [V]$$

$$= \begin{bmatrix} m_{1new} & 0 \\ 0 & m_{2new} \end{bmatrix}$$

لما أضرب ب V و $[V]^T$ بعطيني Diagonal Matrix وما معناه إنه صار عندي Uncoupled equations.

-- Similarly for K.

Swap between rows and Columns

$$[K_{new}] = [V]^T [\quad][\quad]$$

$$= \begin{bmatrix} K_{1new} & 0 \\ 0 & K_{2new} \end{bmatrix}$$

that will give Modal equations that are uncopled where each of them can be solved independently from the other with new Coordinate $\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$ and are called the natural Coordination

No.

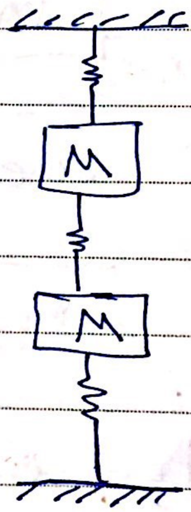
$$\begin{bmatrix} m_{1new} & 0 \\ 0 & m_{2\,new} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} K_{1\,new} & 0 \\ 0 & K_{2\,new} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ UnCoupled $\Rightarrow$ $m_{1\,new}\,\ddot{q}_1 + K_{new}\,q_1 = 0$

$m_{2\,new}\,\ddot{q}_2 + K_{new}\,q_2 = 0$

* this can be applied to any undamped multi degree of freedom systems (Check)

H.w on Wednesday (Very Important to be Studied and Solved)



1. Formulate Equation of motion
2. Find natural frequencies
3. Find modal matrix
4. uncouple equations
5. get final uncoupled equations

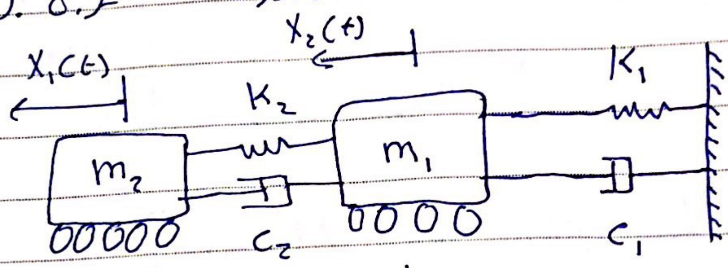
$K = 100\ N/m$
$m = 1\ kg$

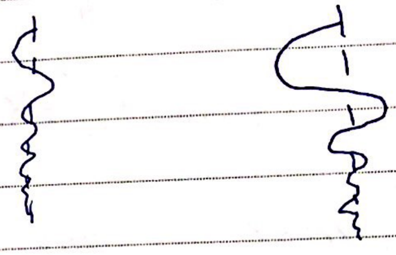
No.

* Damped 2 D.o.f System

→ Formulate the modal Equation of motion

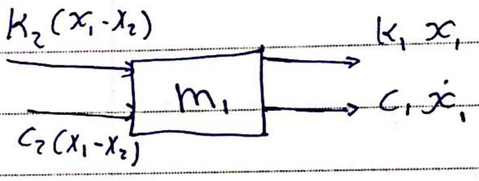
→ Newton's Laws





motion for Light damping and with initial disturbance

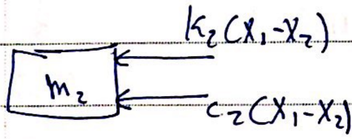
F.B.D



* Assumptions:

- Linear Springs
- Linear dampers
- Small motion

and



So the Equation of motion

→ Apply Newton's 2nd law

$$\sum F_{x_1} = m_1\ddot{x}_1 = -Kx_1 - c_1\dot{x}_1 - K_2(x_1-x_2) - c_2(\dot{x}_1-\dot{x}_2)$$

$$\rightsquigarrow m_1\ddot{x}_1 + (c_1+c_2)\dot{x}_1 - c_2\dot{x}_2 + (K_1+K_2)x_1 - K_2x_2 = 0$$

$$\sum F_{x_2} = m_2\ddot{x}_2 = c_2(\dot{x}_1-\dot{x}_2) + k_2(x_1-x_2)$$

$$\rightsquigarrow m_2\ddot{x}_2 + c_2\dot{x}_2 - c_2\dot{x}_1 + k_2x_2 - k_2x_1 = 0$$

the matrix for these 2 equation is

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{\text{mass matrix}} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}}_{\text{damping matrix}} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k_1+k_2 & -k_1 \\ -k_2 & k_2 \end{bmatrix}}_{\text{stiffness matrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Now what is the Eigen Value problem for this System?

Recall → Eigen Value → Natural frequencies
Eigen Vector → Mode Shapes..

$*j e^{j\omega t}$ $= \dot{X} \Rightarrow$ complex عن Damping من الـ $\dot{x}$ ولما يكون عندي
imaginary مش Real number هو w_n والـ
first order لـ 2nd order فالحل انه أحول الـ

⇒ to find eigen Values / eigen Vectors ⇒ 1st order System

$$[m][\ddot{x}] + [c][\dot{x}] + [k][x] = [0] \; ; \quad x = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

y_1, y_2, y_3, y_4 بدي أحول ويطلع عندي *

$\begin{Bmatrix} y_{1\,new} \\ y_{2\,new} \end{Bmatrix}$ → new Coordination System such that $y_1 = x_1$

$\dot{y}_2 = \ddot{x}_1 \leftarrow y_2 = \dot{x}_1$

$y_3 = x_2$

$\dot{y}_4 = \ddot{x}_2 \leftarrow y_4 = \dot{x}_2$

No.

* $[m^{-1}] * [m] \rightsquigarrow$ Identity $[I]$ ١.

$[m]^{-1}$ اضرب ب *

$$\rightsquigarrow [\ddot{x}] + [m]^{-1}[c]\{\dot{x}\} + [m]^{-1}[k]\{x\} = \{0\}$$

* $\dot{y}_1 = y_2$

$\dot{y}_3 = y_4$

$$\begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{Bmatrix} = \underbrace{\left[\begin{array}{c|c} I & 0 \\ \hline -[m]^{-1}[k] & -[m]^{-1}[c] \end{array}\right]}_{[A]} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

$[A]$

Check the numerical Example *

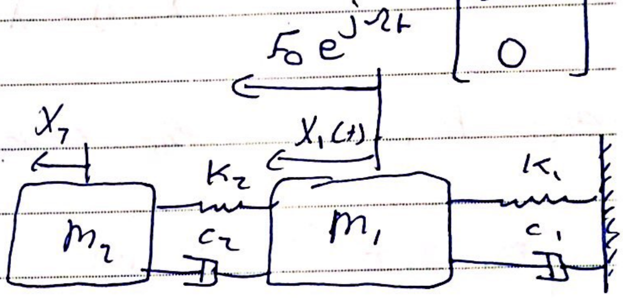
No.

Forced Vibration (Harmonic excitation)

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}+\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} F_0 e^{j\Omega t} \\ 0 \end{bmatrix}$$

Ω is the forcing Frequency

and output follows the input



$$x_1(t) = X_1 e^{j\Omega t}$$

$$x_2(t) = X_2 e^{j\Omega t}$$

and $\dot{x}_1 = X_1 j\Omega e^{j\Omega t}$ and $\ddot{x}_1 = -X_1\Omega^2 e^{j\Omega t}$

$\dot{x}_2 = X_2 j\Omega e^{j\Omega t}$ $\ddot{x}_2 = -X_2\Omega^2 e^{j\Omega t}$

* خد $e^{j\Omega t}$ عامل مشترك واشطبه
* خد X_1 و X_2 عامل مشترك

$\Downarrow$

$$\begin{bmatrix} k_{11}-m_{11}\Omega^2+jc_{11}\Omega & k_{12}-m_{12}\Omega^2+jc_{12}\Omega \\ k_{21}-m_{21}\Omega^2+jc_{21}\Omega & k_{22}-m_{22}\Omega^2+jc_{22}\Omega \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}=\begin{Bmatrix} F_0 \\ 0 \end{Bmatrix}$$

عشان نسهل الشغل بنعوض عن $Z_{rs} = k_{rs} - m_{rs}\Omega^2 + jc_{rs}\Omega$

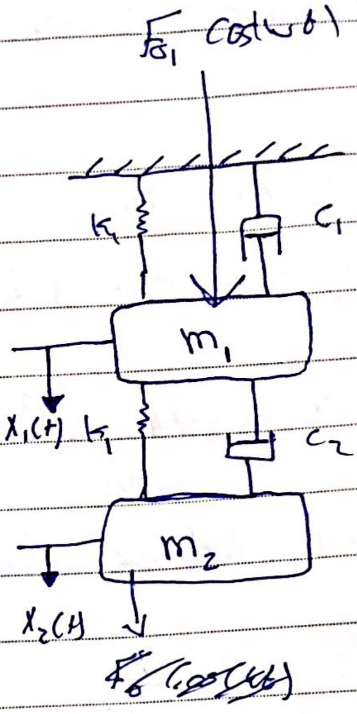
$$\underbrace{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}}_{\text{Mechanical impedence}}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}=\begin{Bmatrix} F_0 \\ 0 \end{Bmatrix} \xrightarrow{\text{ومنه}} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}=[Z]^{-1}\begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

وهيك بوجد X_2 & X_1

13/12/2017

No.

Harmanic Excitation (Steady state)



Anather example:

and we assomed that the Output follows the Input ⟶ $x_1(t) = X_1(i\omega)\cos(\omega t)$

$x_2(t) = X_2(i\omega)\cos(\omega t)$

and reached for the impedence matrix

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} X_1(i\omega) \\ X_2(i\omega) \end{bmatrix} = \begin{bmatrix} F_{o_1} \\ 0 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} X_1(i\omega) \\ X_2(i\omega) \end{bmatrix} = \left[Z^{-1}\right]\left[F\right]$$

$$\rightsquigarrow Z_{rs} = k_{rs} - m_{rs}\omega^2 + i\,c_{rs}\,\omega$$

$$\longrightarrow [Z]^{-1} = \frac{1}{\underbrace{Z_{12}^2 - Z_{11}Z_{22}}}\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{12} & Z_{11} \end{bmatrix}$$

$\dfrac{1}{\text{Determinant}}$ هاي اللي فيه

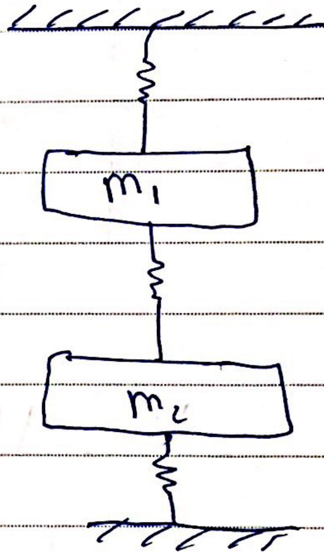
$$X_1(i\omega) = \frac{Z_{22}\,F_{01}}{Z_{12}^2 - Z_{11}Z_{22}}$$

and

$$X_2(i\omega) = \frac{-Z_{12}\,F_{01}}{Z_{12}^2 - Z_{11}Z_{22}}$$

Example 8 (مثال محلول بالكتاب)
(نفس ال homework)

Solution:



$$m_{12} = m_{21} = 0 \qquad C_{ij} = 0$$

$$m_{11} = m_{22} = m \qquad K_{11} = K_{22} = 2K$$

$$K_{12} = K_{21} = -K$$

the impedence is

$$Z_{11} = 2K - m\omega^2$$

$$Z_{12} = Z_{21} = -K$$

$$Z_{22} = 2K - m\omega^2$$

So $\rightsquigarrow$

$$X_1(i\omega) = \frac{(2K - m\omega^2)\,F_{01}}{K^2 - (2K - m\omega^2)^2}$$

$$X_2(i\omega) = \frac{K\,F_{01}}{K^2 - (2K - m\omega^2)^2}$$

$$\omega_1 = \sqrt{\frac{K}{m}}$$

$$\omega_2 = \sqrt{\frac{3K}{m}}$$

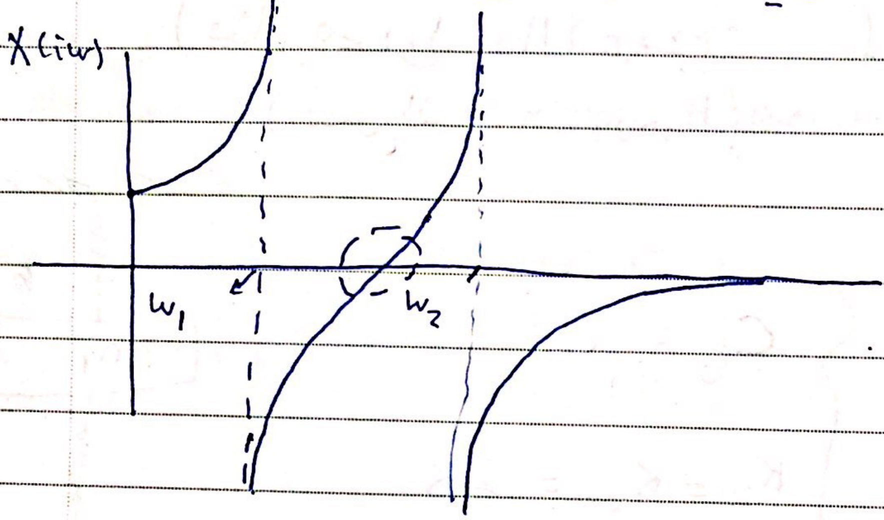
بدنا نرسمها ... خلينا انشوف المقام

$$K^2-(2k-m\omega^2)=0 \Rightarrow \omega=\sqrt{\frac{k}{m}} \rightarrow \infty \text{ بروح للـ}$$

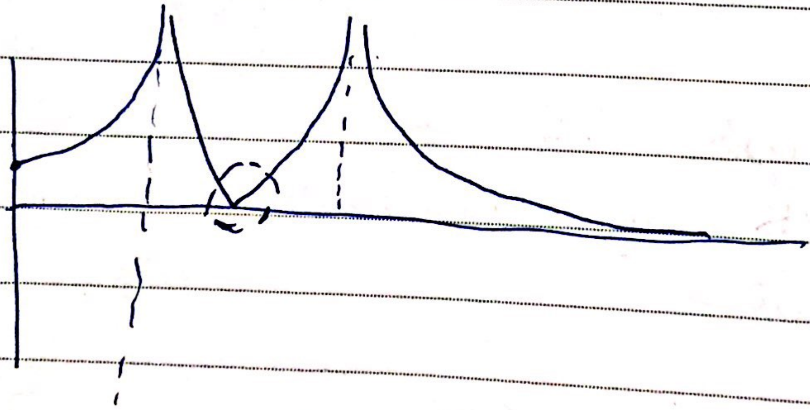
and

$$\omega=\sqrt{\frac{3k}{m}}$$

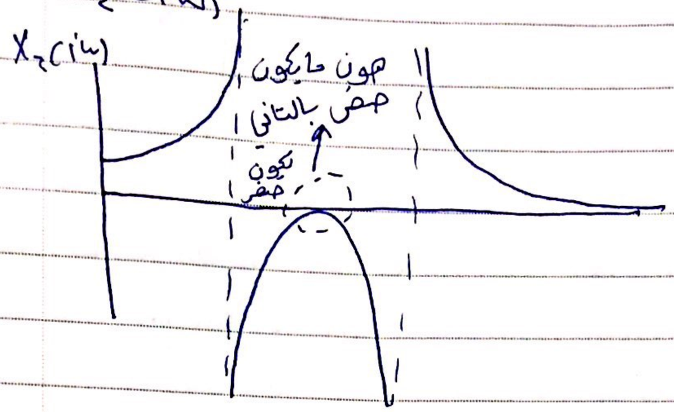
فلما الـ Ω بساوي $\sqrt{\frac{k}{m}}$ أو $\sqrt{\frac{3k}{m}}$ ← بروح للـ ∞



أو اذا كانت $|X_1|$ (مطلق)



For $x_2(i\omega)$



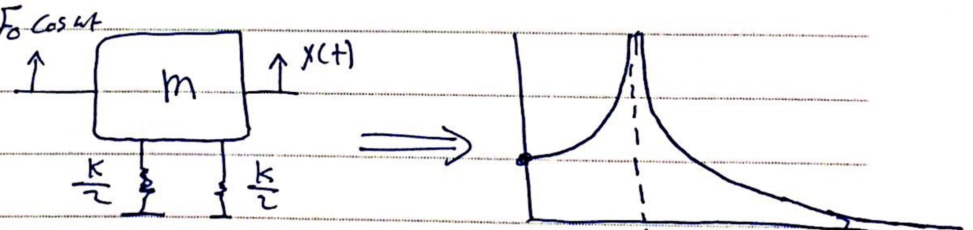
*أهم ملاحظة بجدول الرسمتين انه في منطقة بينها قيمة ال Vibration
بتساوي صفر سـ ومن هون صارعنا اشي بنستعمله اسمو

Dynamic Vibration Absorber idea

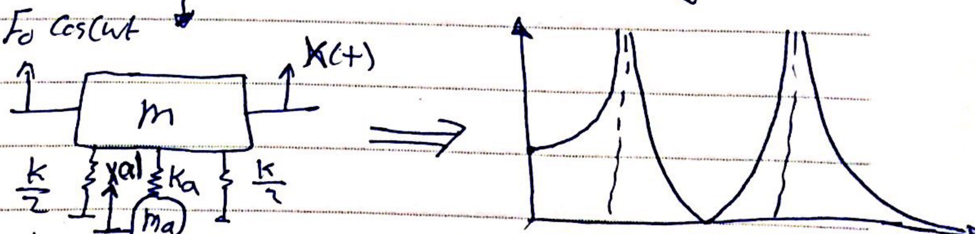
* من خلال اني أعلق mass معينة عند ال Ω اللي
بشتغل عليها وبتخفيلي اياه ...

* هاد الحكي موجود ب Chapter 9: Vibration Control

* Example



بدي أعلق ال mass وهيك
2 DoF صار عندي



بكون بس ma بتحرك

هيك صار عندي 2 ω_n وبطل عند اللي بدي اياه + الصفر اللي أنا شغال عليه...

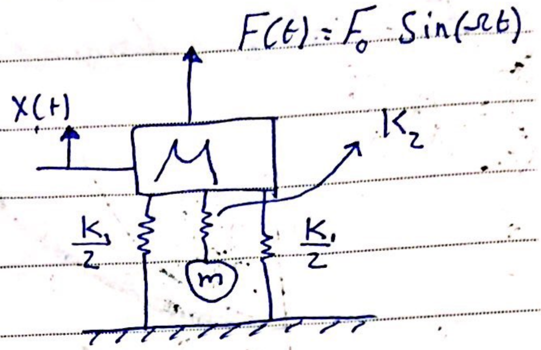
* if Absorber frequency is tuned to be equal to the external excitation frequency lo amplitude will remain primary

Check the Doctor Paper (Dual dynamic absorber 1999)

18/12/2017

No.

Dynamic absorber



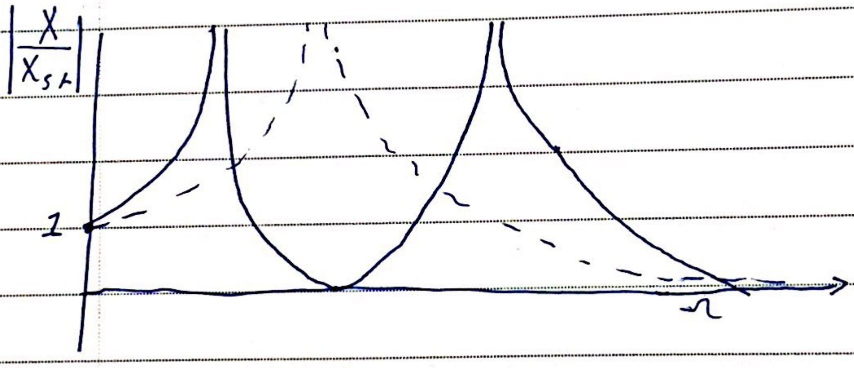
→ For undamped System

$m_1 \ddot{x}_1 + K x_1 = F_0 \sin(\Omega t)$

where Ω is the forcing frequency

* if we add this mass, the System become 2 DoF

$\Omega \to \omega_1 \longrightarrow \omega_1 = \sqrt{\frac{k}{m}}$



$m_1 \ddot{x}_1 + K_1 x_1 + K_2 (X_1 - X_2) = F_0 \sin(\Omega t)$

$m_2 \ddot{x}_2 + K_2 (x_2 - X_1) = 0$

the Steady state response:

$x_1(t) = X_1 \sin \Omega t$

$X_2(t) = X_2 \sin \Omega t$

$\ddot{x}_1 = -\Omega^2 X_1 \sin \Omega t$

$\ddot{x}_2 = -\Omega^2 X_2 \sin \Omega t$

نعوض بـ المعادلة الأصلية

$m_1(-\Omega X_1 \sin(\Omega t)) + K_1 X_1 \sin(\Omega t)$
$+ K_2 (X_1 - X_2) \sin(\Omega t) = F_0 \sin \Omega t$

So:

$X_1 = \frac{(K_2 - m\Omega^2) F_0}{\Delta}$

&

$X_2 = \frac{K F_0}{\Delta}$

Dual Dynamic absorber

No.

* after adding (m_{a2}, k_{a}) System become 2 DoF System

→ Vibration is a form of Energy...
So it will go to the System even if it is Close to resonance (by the easiest way which mean to the added mass)

* if absorber frequency is tuned to be equal to the external excitation frequency, so the amplitude will remain at primary.