2

Axially Loaded Members

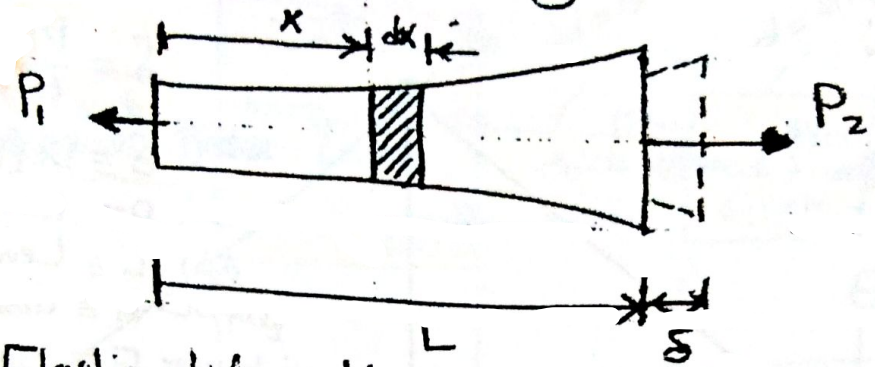
CHAPTER OVERVIEW

In Chapter 2, we consider several other aspects of axially loaded members, beginning with the determination of changes in lengths caused by loads (Sections 2.2 and 2.3). The calculation of changes in lengths is an essential ingredient in the analysis of statically indeterminate structures, a topic we introduce in Section 2.4. If the member is statically indeterminate, we must augment the equations of statical equilibrium with compatibility equations (which rely on force-displacement relations) to solve for any unknowns of interest, such as support reactions or internal axial forces in members. Changes in lengths also must be calculated whenever it is necessary to control the displacements of a structure, whether for aesthetic or functional reasons. In Section 2.5, we discuss the effects of temperature on the length of a bar, and we introduce the concepts of thermal stress and thermal strain. Also included in this section is a discussion of the effects of misfits and prestrains. A generalized view of the stresses in axially loaded bars is presented in Section 2.6, where we discuss the stresses on *inclined sections* (as distinct from *cross sections*) of bars. Although only normal stresses act on cross sections of axially loaded bars, both normal and shear stresses act on inclined sections. Stresses on inclined sections of axially loaded members are investigated as a first step toward a more complete consideration of plane stress states in later chapters. We then introduce several additional topics of importance in mechanics of materials, namely, strain energy (Section 2.7), impact loading (Section 2.8), fatigue (Section 2.9), stress concentrations (Section 2.10), and nonlinear behavior (Sections 2.11 and 2.12). Although these subjects are discussed in the context of members with axial loads, the discussions provide the foundation for applying the same concepts to other structural elements, such as bars in torsion and beams in bending.

Chapter 2 is organized as follows:

د. هاشم الخالدي

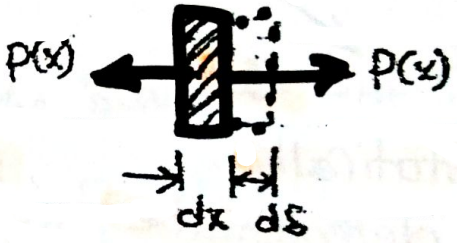
"Axial Loading"



* For Elastic deformation:-

$$\sigma = E\,\varepsilon$$

$$\frac{P(x)}{A(x)} = E\frac{d\delta}{dx}$$



$$d\delta = \frac{P(x)}{A(x)\,E}\,dx$$

$$\delta = \int_0^L \frac{P(x)}{A(x)\,E}\,dx$$

For variable P(x) and/or A(x)

* For Constant load and Constant Cross Sectional Area then $P(x) = P$ and $A(x) = A$

$$\Rightarrow \quad \delta = \frac{PL}{AE}$$

For Constant P and A

* If the bar is subjected to different axial forces and/or different Cross sections and Lengths then:

$$\delta = \sum \frac{PL}{AE}$$

Stepped Bar

Axial Deflection

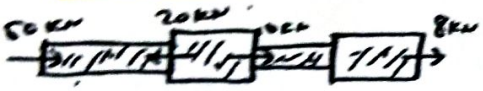
الإستطالة (الزيادة في الطول) أو الإنضغاط (النقصان في الطول)

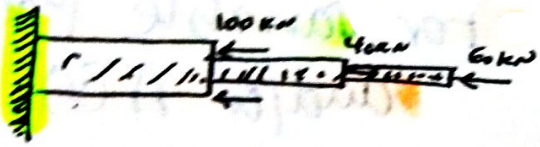
$$\delta = \frac{PL}{EA}$$

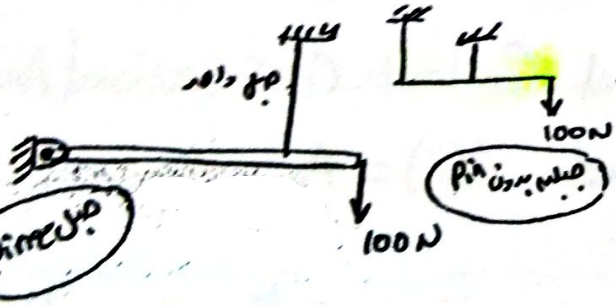
- $\delta \equiv$ Deflection الإستطالة
- $P \equiv$ Force القوة
- $L \equiv$ Length الطول
- $A \equiv$ Cross-Section Area مساحة المقطع
- $E \equiv$ Modulus of Elasticity معامل المرونة للمعدن

Statically determinate

المحدّد

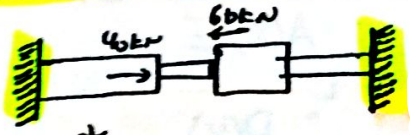


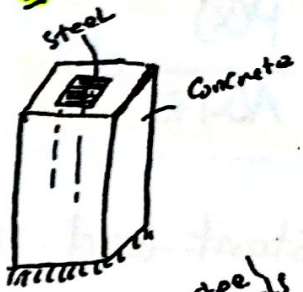




Statically indeterminate

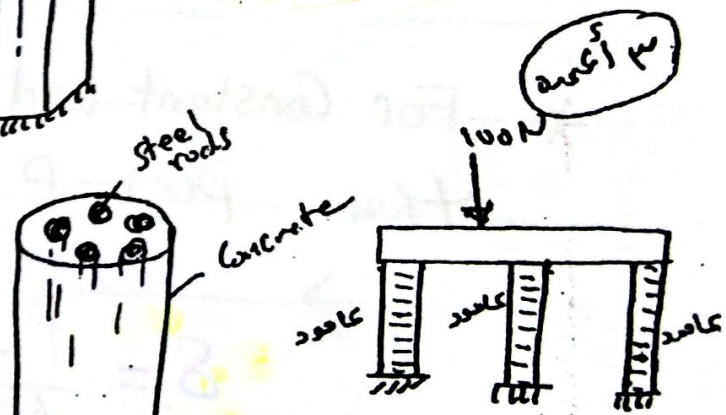
غير محدّد





Temprature

الحرارة

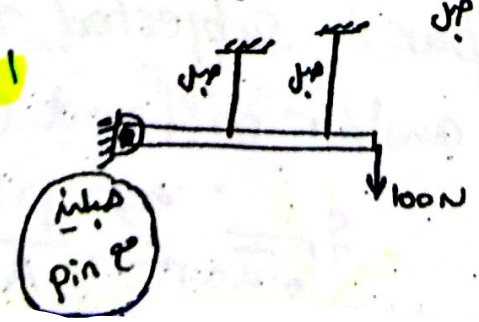


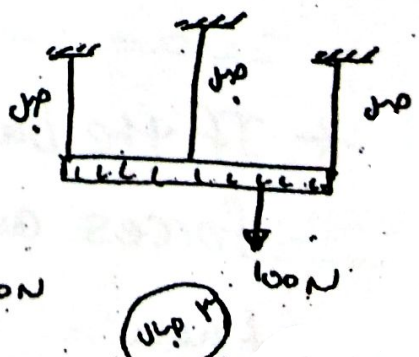
$$\delta = \frac{PL}{EA}$$

التمدد بالقوة

$$\delta = \alpha L \Delta T$$

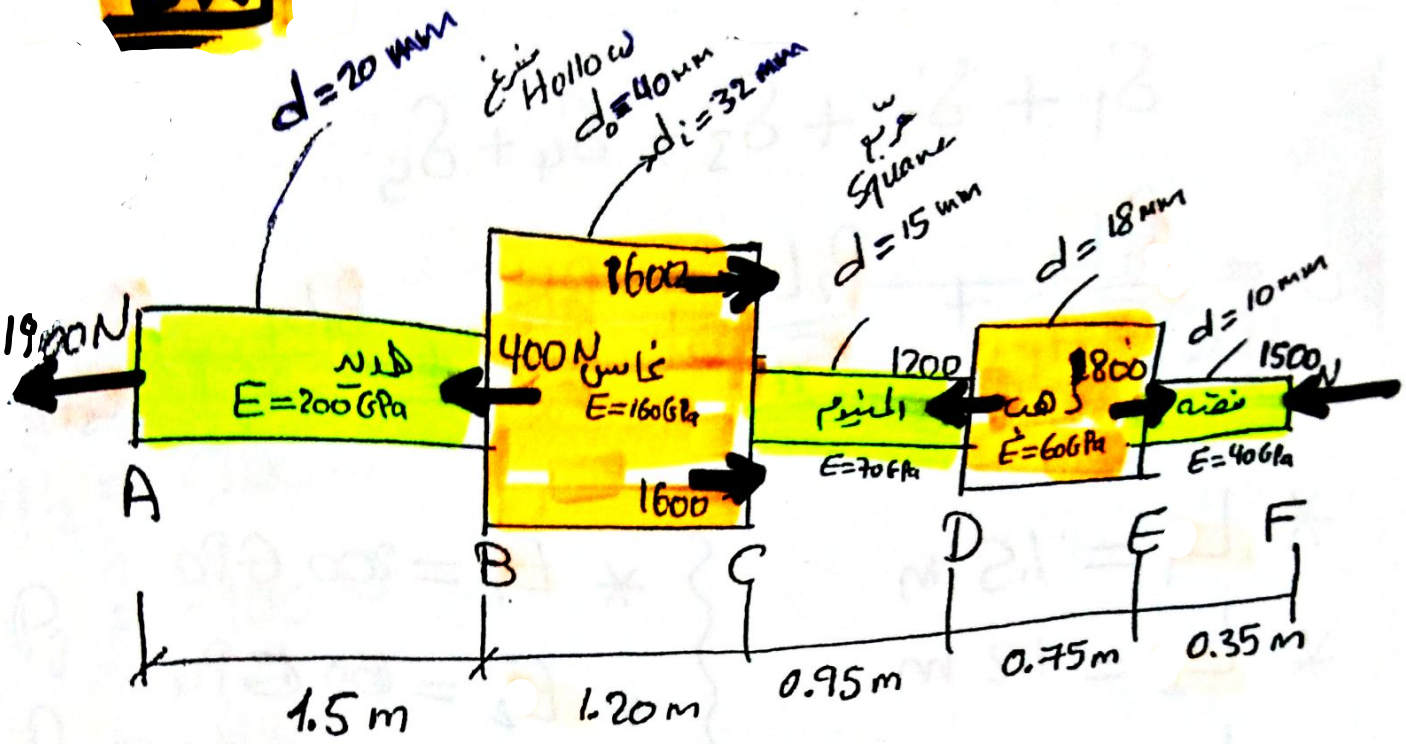
التمدد بالحرارة





الدكتور هاشم الخالدي - الجامعة الأردنية

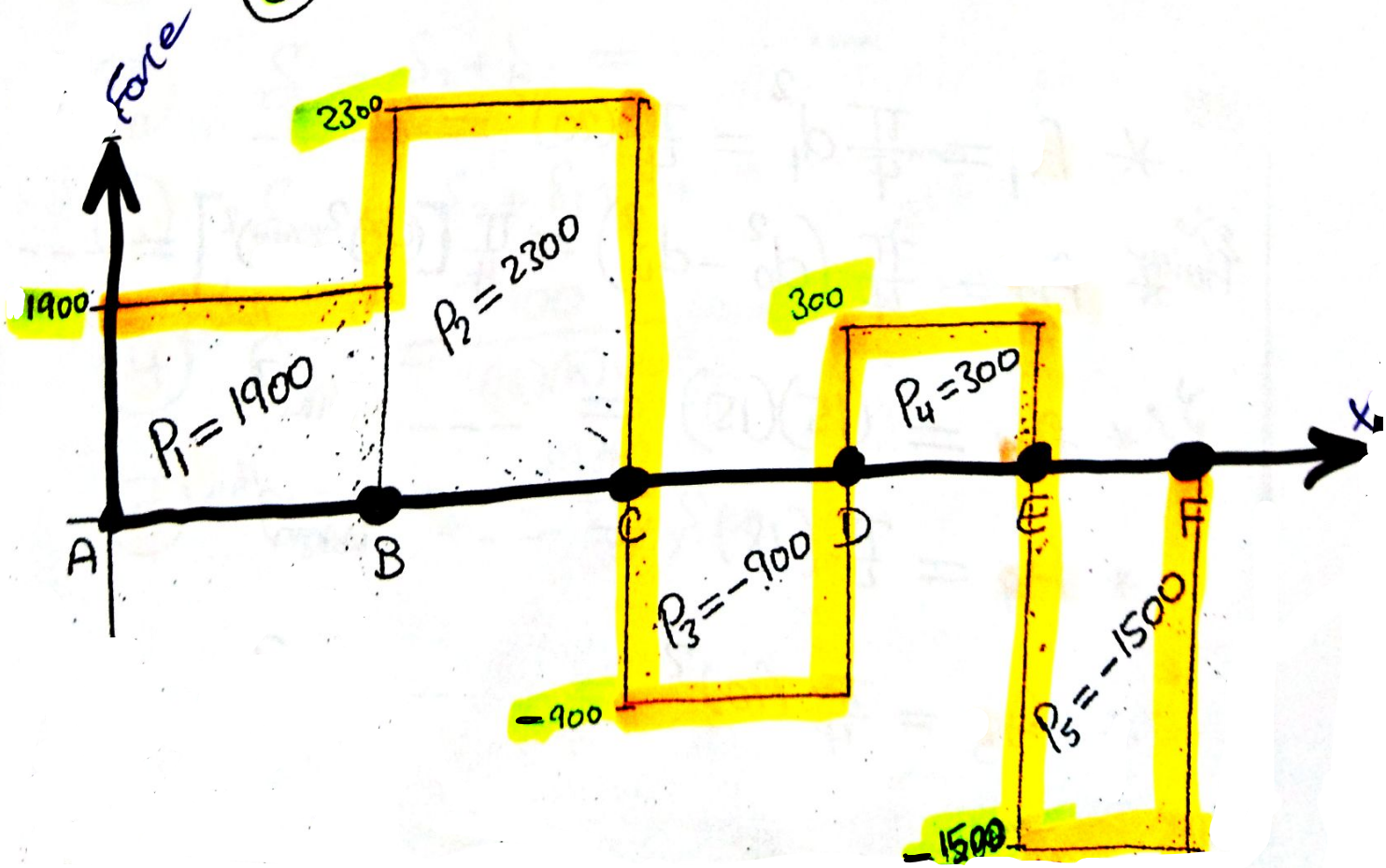
Ex



Find (1) the deflection of the Beam? δ_{AF}?

(2) δ_{DB}? (3) δ_{EB}? (4) σ_{Al}

(5) σ_{Steel}



$$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5$$

$$\delta = \frac{P_1 L_1}{E_1 A_1} + \frac{P_2 L_2}{E_2 A_2} + \frac{P_3 L_3}{E_3 A_3} + \frac{P_4 L_4}{E_4 A_4} + \frac{P_5 L_5}{E_5 A_5}$$

* $L_1 = 1.5$ m
* $L_2 = 1.2$ m
* $L_3 = 0.95$ m
* $L_4 = 0.75$ m
* $L_5 = 0.35$ m

* $E_1 = 200$ GPa
* $E_2 = 160$ GPa
* $E_3 = 70$ GPa
* $E_4 = 60$ GPa
* $E_5 = 40$ GPa

* $A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(20)^2 = \text{---}$ mm^2

Hollow * $A_2 = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}[(40)^2 - (32)^2] = \text{---}$

* $A_3 = (15)(15) = \text{---}$ mm^2

* $A_4 = \frac{\pi}{4}(18)^2 = \text{---}$ mm^2

* $A_5 = \frac{\pi}{4}(10)^2 = \text{---}$ mm^2

$P_1 = 1900\ N$

$P_2 = 2300\ N$

$P_3 = -900\ N$

$P_4 = 300\ N$

$P_5 = -1500\ N$

(1) الإزاحة الكلية $\delta_{AF} = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 = \dots\dots$ mm

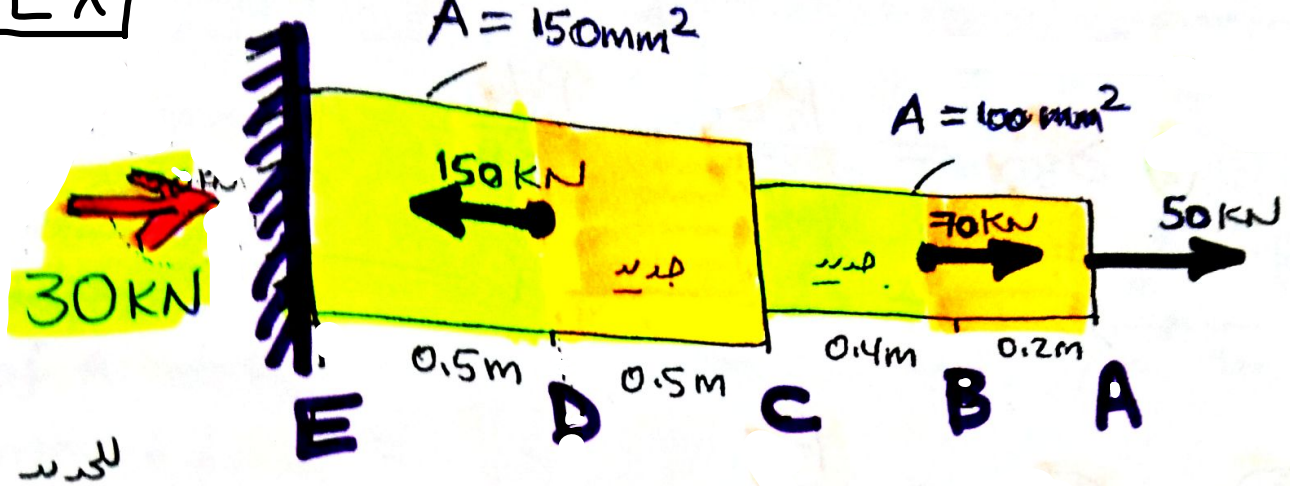
(2) $\delta_{BD} = \delta_2 + \delta_3 = \dots\dots$ mm

(3) $\delta_{EB} = \delta_2 + \delta_3 + \delta_4 = \dots\dots$ mm

(4) الإجهاد في الألمنيوم $\sigma_{Al} = \dfrac{-900}{(15)(15)\times 10^{-6}} = \dots\dots$ MPa.

(5) الإجهاد في الحديد $\sigma_{Steel} = \dfrac{1900}{\frac{\pi}{4}(20)^2 \times 10^{-6}} = \dots\dots$ MPa.

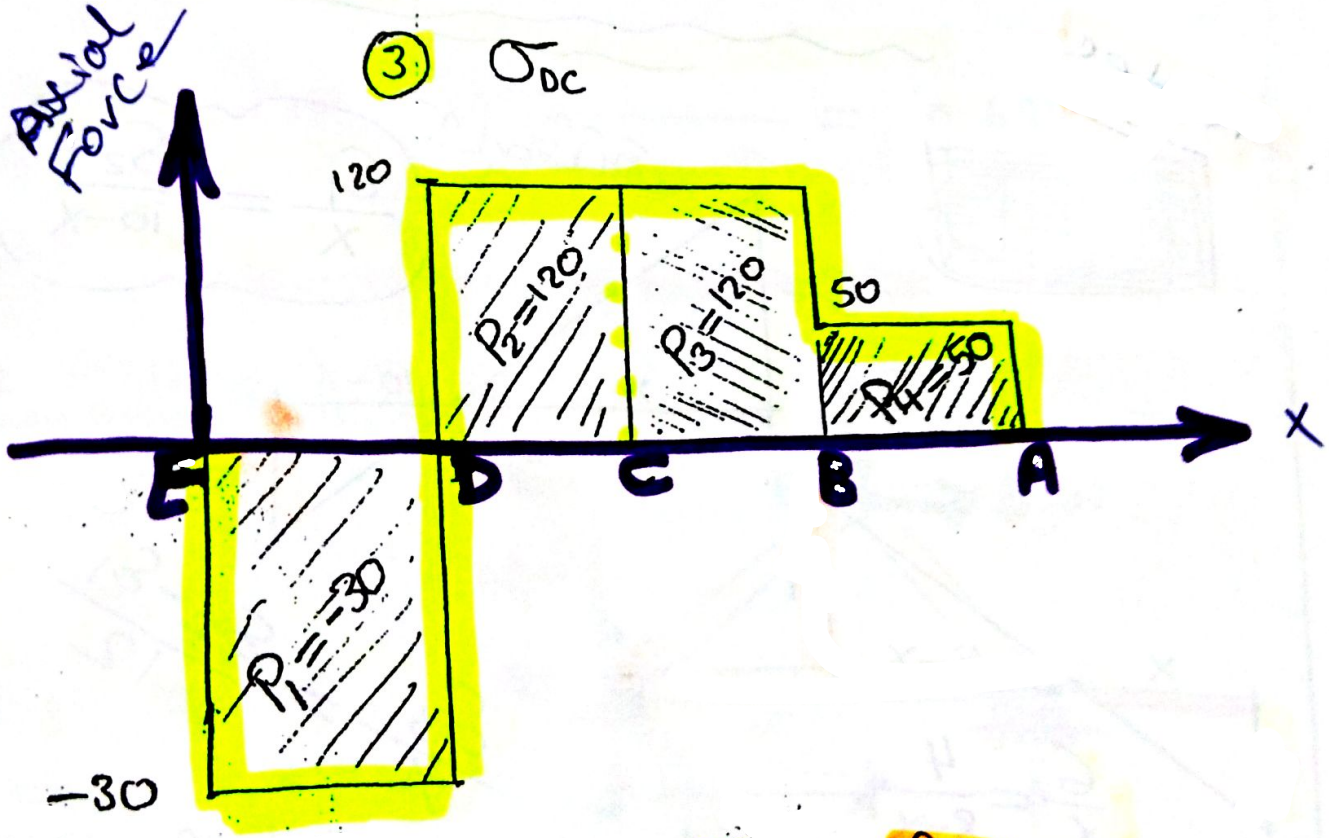
EX/



للحديد

$E = 200\ GPa$

Find ① δ_A ② δ_{BD} ③ σ_{DC}



① $$\delta_A = \frac{P_1L_1}{E_1A} + \frac{P_2L_2}{E_2A_2} + \frac{P_3L_3}{E_3A_3} + \frac{P_4L_4}{E_4A_4}$$

$$= \frac{(-30\times10^3)(0.5)}{(200\times10^9)(150\times10^{-6})} + \frac{(120\times10^3)(0.5)}{(200\times10^9)(150\times10^{-6})}$$

$$+ \frac{(120\times10^3)(0.4)}{(200\times10^9)(100\times10^{-6})} + \frac{(50\times10^3)(0.2)}{(200\times10^9)(100\times10^{-6})} = \cdots$$

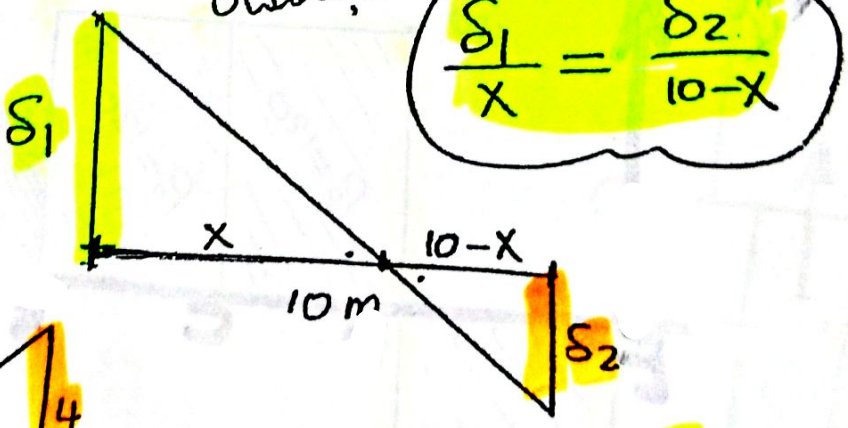
② $\delta_{BD} = \frac{P_2 L_2}{E_2 A} + \frac{P_3 L_3}{E_3 A_3}$

$= \frac{\quad}{\quad} + \frac{\quad}{\quad} = \cdots$

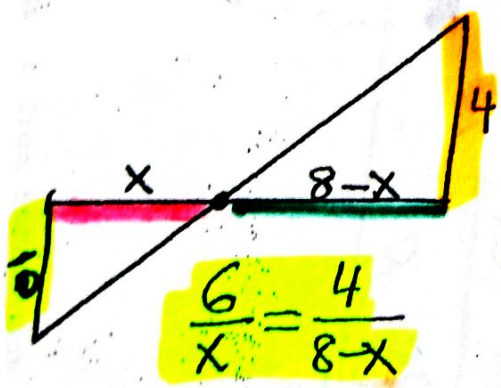
③ $\sigma_{DC} = \frac{P_2}{A}$

$= \frac{120 \times 10^3}{150 \times 10^{-6}} = 800\ MPa$

Notes



$\frac{\delta_1}{x} = \frac{\delta_2}{10-x}$



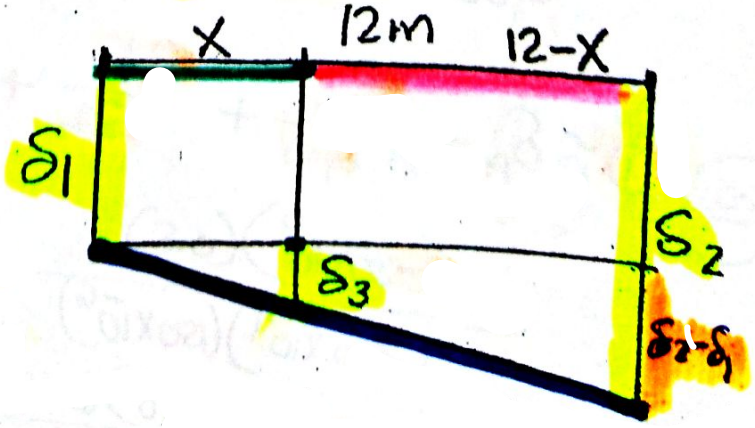
$\frac{6}{x} = \frac{4}{8-x}$

$\Rightarrow 4x = 48 - 6x$

$10x = 48$

$x = 4.8$

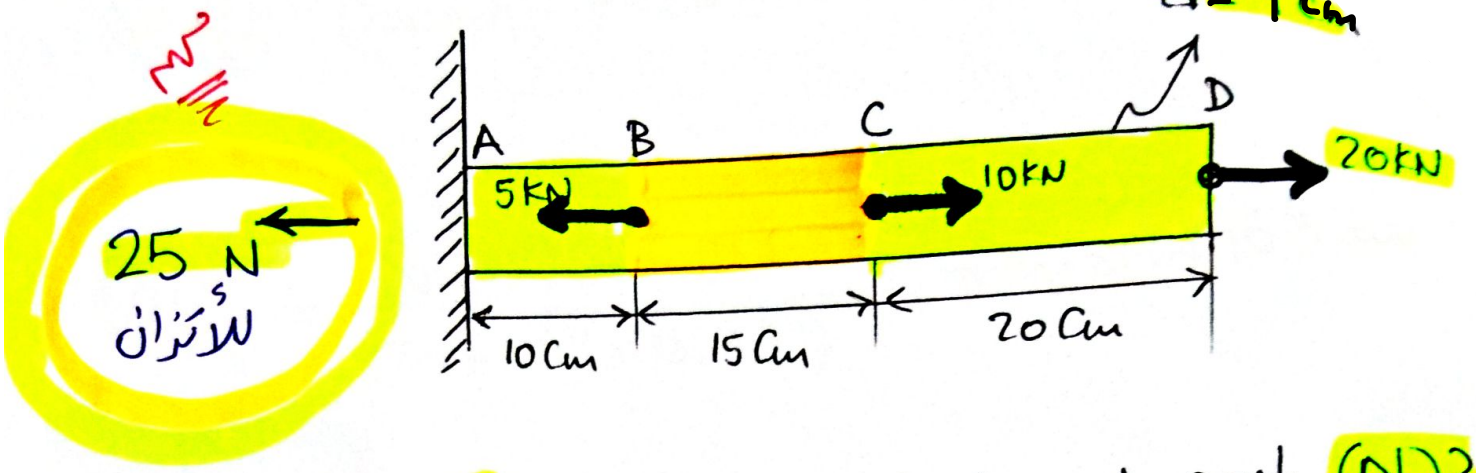
$\frac{\delta_3}{x} = \frac{\delta_2 - \delta_1}{12}$



Ex)

$E = 200$ GPa

القطر $d = 1$ cm

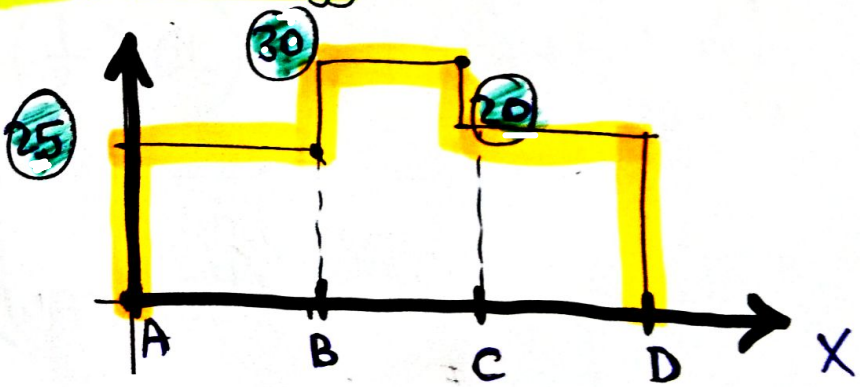


Find (1) Change in length in each part (ΔL)?

(2) the Normal stress (σ) in each part ?

(3) Total Strain energy in the bar ?

الحل



$$A = \frac{\pi}{4}(1)^2 = 0.785 \text{ Cm}^2$$

$$\sigma_{AB} = \frac{F}{A} = \frac{25 * 10^3}{0.785 * 10^{-4}} = 318.5 \text{ MPa}$$

$$\sigma_{BC} = \frac{F}{A} = \frac{30 * 10^3}{0.785 * 10^{-4}} = 382.2 \text{ MPa}$$

$$\sigma_{CD} = \frac{F}{A} = \frac{20 * 10^3}{0.785 * 10^{-4}} = 254.8 \text{ MPa}$$

$$* \ \sigma = E\varepsilon \Rightarrow \varepsilon_{AB} = \frac{\sigma}{E} \Rightarrow \varepsilon_{AB} = \frac{\Delta L}{L}$$

$$\Rightarrow \text{use } \Delta L = \frac{PL}{EA}$$

* $(\Delta L)_{AB} = \frac{(25*10^3)(10*10^{-2})}{(200*10^9)(0.785*10^{-4})} = 1.592*10^{-4}$ m

* $(\Delta L)_{BC} = \frac{(30*10^3)(15*10^{-2})}{(200*10^9)(0.785*10^{-4})} = 2.866*10^{-4}$ m

* $(\Delta L)_{CD} = \frac{(20*10^3)(20*10^{-2}}{(200*10^9)(0.785*10^{-4}} = 2.55*10^{-4}$ m

* Strain energy $U = \left(\frac{1}{2} P \Delta L\right)_{AB} + \left(\frac{1}{2} P \Delta L\right)_{BC} + \left(\frac{1}{2} P \Delta L\right)_{CD}$

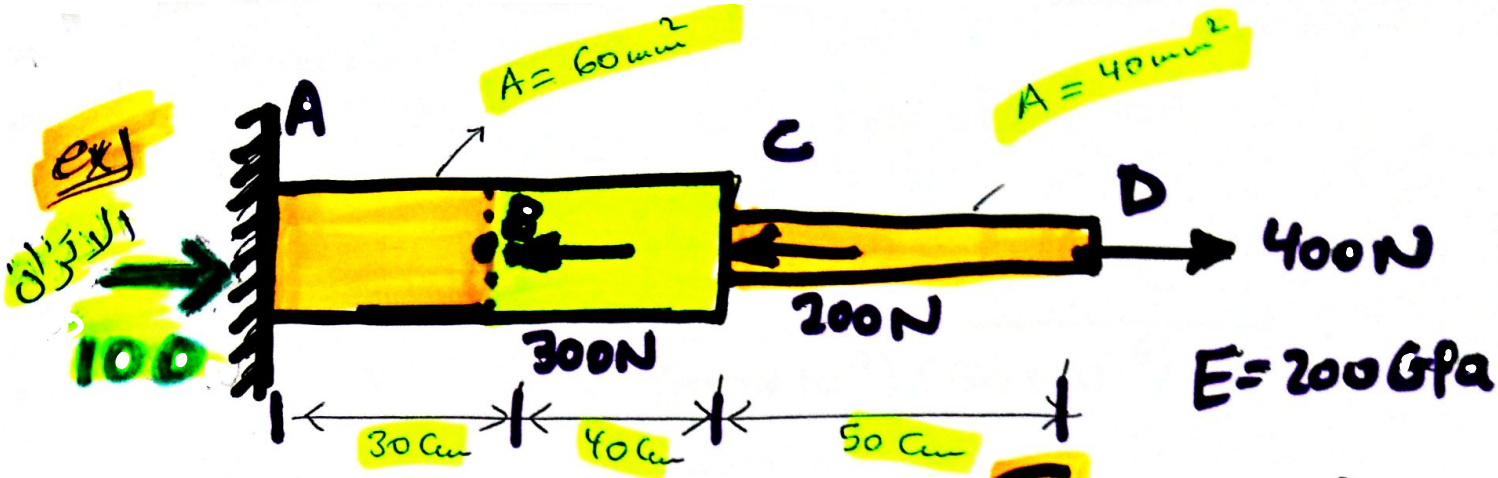
$\Rightarrow U = \frac{1}{2}(25*10^3)(1.592*10^{-4})$

$+ \frac{1}{2}(30*10^3)(2.866*10^{-4})$

$+ \frac{1}{2}(20*10^3)(2.55*10^{-4})$

$\Rightarrow U \cong 7$ joule

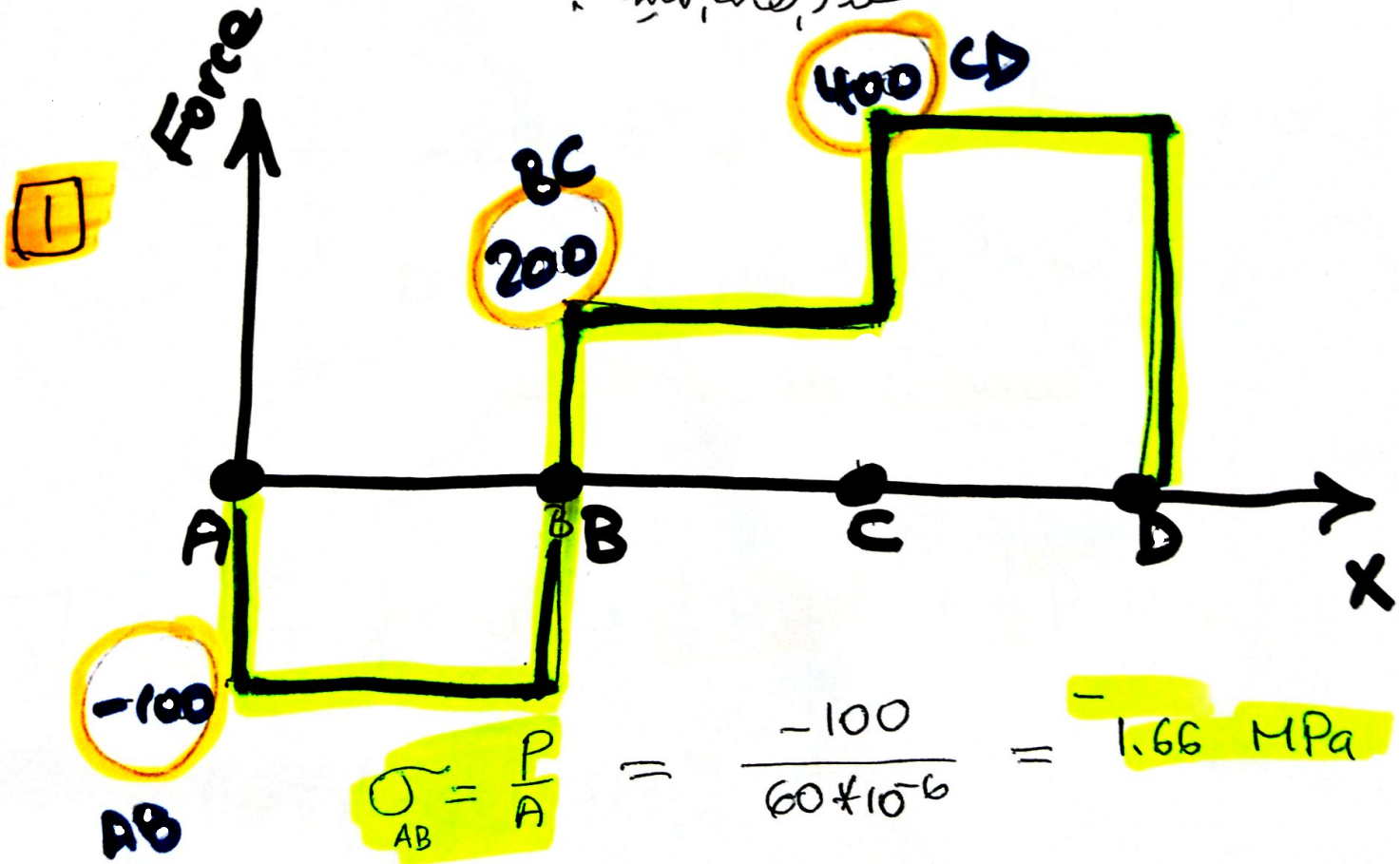
ex1



Find (1) The maximum stress σ in the Beam

(2) The Total elongation Δ ?

(3) Total Strain energy ? U



$$\sigma_{AB} = \frac{P}{A} = \frac{-100}{60*10^{-6}} = -1.66 \text{ MPa}$$

$$\sigma_{BC} = \frac{P}{A} = \frac{200}{60*10^{-6}} = 3.33 \text{ MPa}$$

$$\sigma_{CD} = \frac{P}{A} = \frac{400}{40*10^{-6}} = 10 \text{ MPa}$$

$$\Delta L = \frac{PL}{EA} + \frac{PL}{EA} + \frac{PL}{EA}$$

(2)

$$= \frac{(-100)(0.3)}{(200*10^9)(60*10^{-6})}$$

$$+ \frac{(200)(0.4)}{(200*10^9)(60*10^{-6})}$$

$$+ \frac{(400)(0.5)}{(200*10^9)(40*10^{-6})}$$

$$\Rightarrow \Delta L = -2.5*10^{-6} + 6.67*10^{-6} + 2.5*10^{-5}$$

$$\Delta L = 2.916*10^{-5} \text{ m}$$

$$\Delta L = 0.02916 \text{ mm}$$

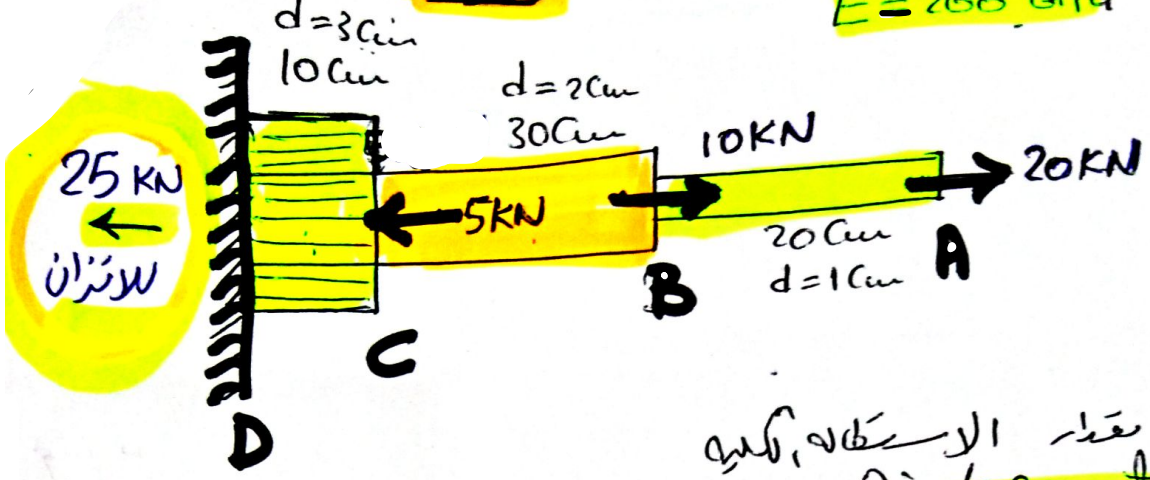
(3) $U = \frac{1}{2} P \Delta L + \frac{1}{2} P \Delta L + \frac{1}{2} P \Delta L$

$$U = \frac{1}{2}(100)(2.5*10^{-6}) + \frac{1}{2}(200)(6.67*10^{-6}) + \frac{1}{2}(400)(2.5*10^{-5})$$

$$U = 5.79*10^{-3} \text{ Joule}$$

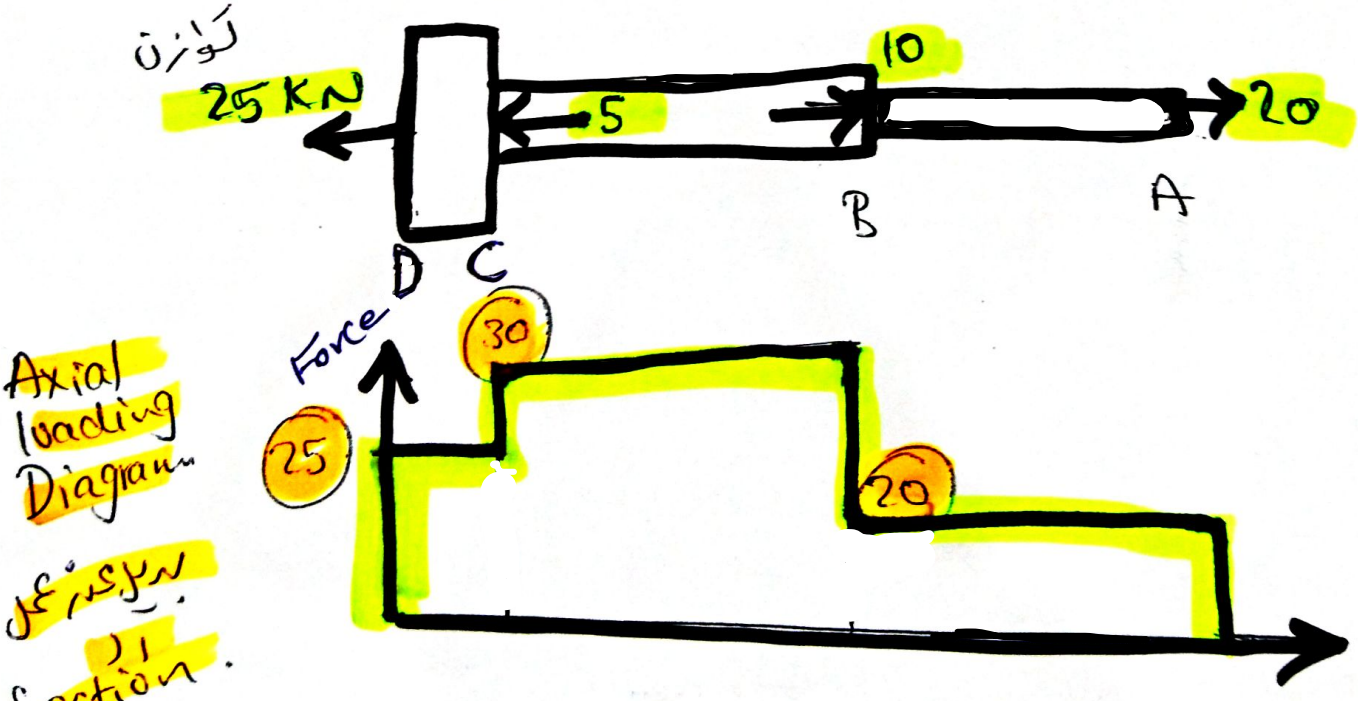
Ex1

$E = 200\ GPa$



Find (1) the relative Displacement of points A and D?

(2) Total strain energy in the Bar?



Axial loading Diagram

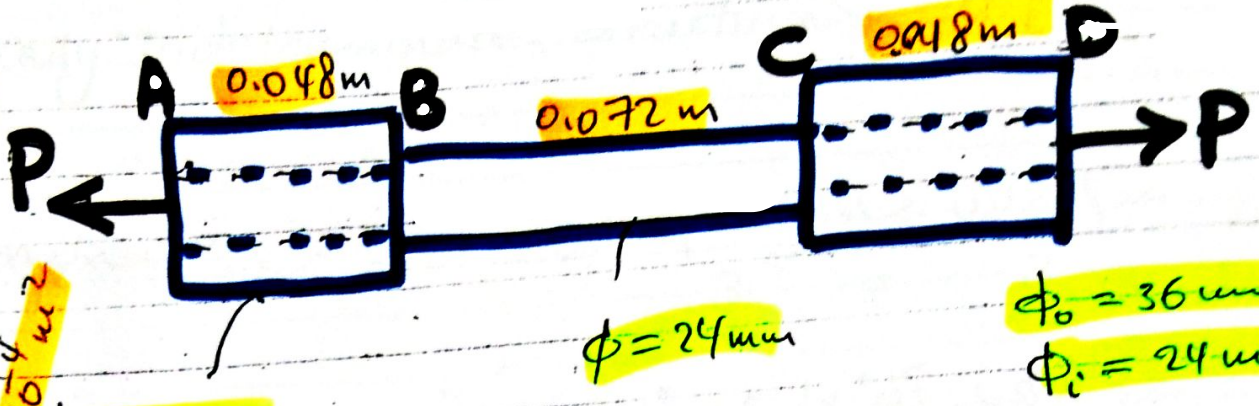
لرسم محور الـ Section

$$\delta_{DA} = \delta_1 + \delta_2 + \delta_3$$

$$= \left(\frac{PL}{EA}\right)_1 + \left(\frac{PL}{EA}\right)_2 + \left(\frac{PL}{EA}\right)_3$$

$E = 200\ GPa$

Ex



$\phi = 24\ mm$

$\phi_o = 36\ mm$

$\phi_i = 24\ mm$

$\phi_o = 36\ mm$

$\phi_i = 24\ mm$

$A_1 = \frac{\pi}{4}\left[36^2 - 24^2\right] = 5.655 \times 10^{-4}\ m^2$

$A_2 = \frac{\pi}{4}(24)^2 \times 10^{-6} = 4.524 \times 10^{-4}\ m^2$

(1) Find P so that the total deformation $= 0.05\ mm$

$$\delta_{AD} = \delta_1 + \delta_2 + \delta_3 = \frac{P_1 L_1}{E_1 A_1} + \frac{P_2 L_2}{E_2 A_2} + \frac{P_3 L_3}{E_3 A_3}$$

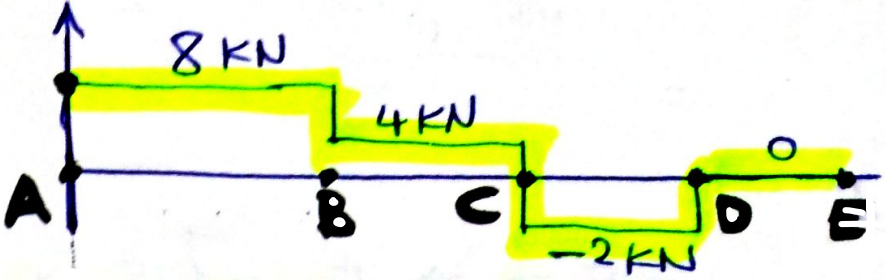
$$\Rightarrow 0.05 \times 10^{-3} = 2\left[\frac{P(0.048)}{(200\times10^9)(5.655\times10^{-4})}\right] + \left\{\frac{P(0.072)}{(200\times10^9)(4.524\times10^{-4})}\right\}$$

$$\Rightarrow 0.05 \times 10^{-3} = 8.488\times10^{-10} P + 7.958\times10^{-10} P$$

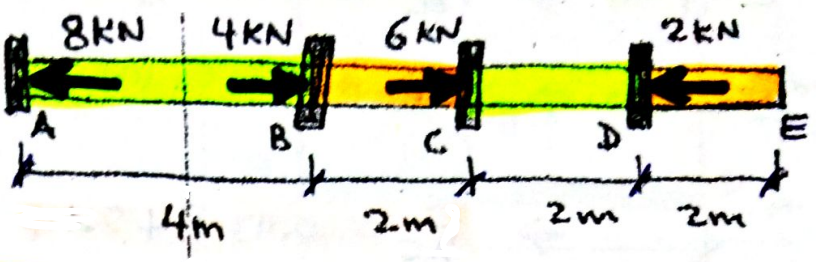
(2) Find δ_{BC} ?

$$\delta_{BC} = \frac{P_2 L_2}{E_2 A_2}$$

Ex1



Given
$d = 30\,mm$
$E = 73.1\ GPa$
Determine the displacement of A with respect to E ? $\delta_{A/E}$?



Sol.

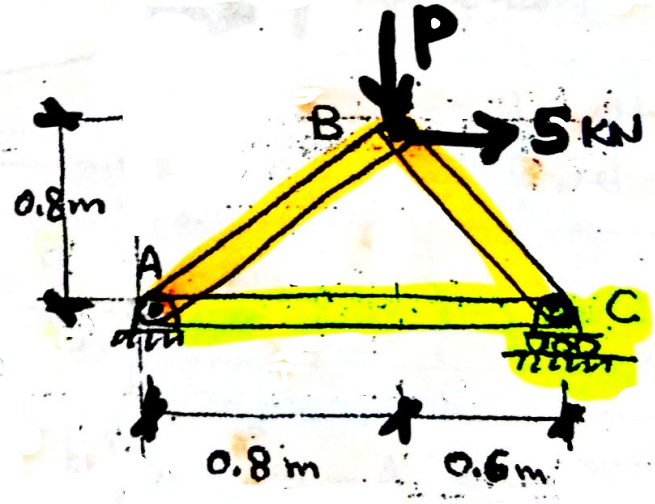
$$\delta_{A/E} = \sum \frac{PL}{AE} = \frac{1}{AE}\left[(8)(4) + (4)(2) - (6)(2)\right] \times 10^3$$

$$= \frac{36 \times 10^3}{\frac{\pi}{4}(0.03)^2 (73.1 \times 10^9)} = 0.697\ mm$$

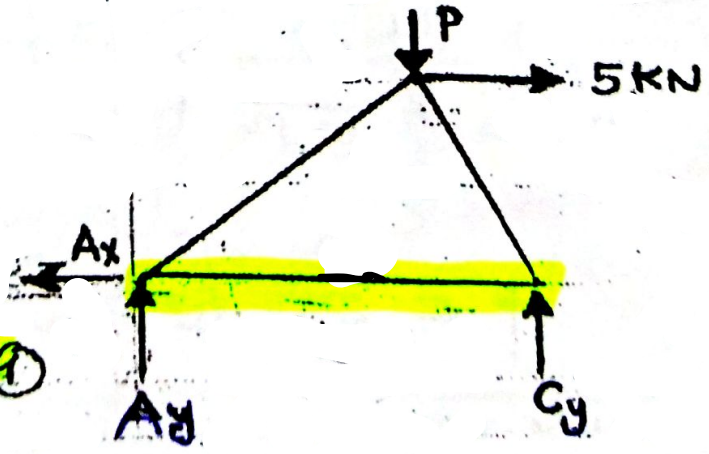
Ex1

"Truss"

$A = 400\ mm^2$
$E = 200\ GPa$.
Find P required to displace the roller 0.2 mm to the right ?



Sol.



$\Sigma M_A = 0$

$\Rightarrow C_y = 0.57P + 2.86$ ——— (1)

$\sum F_y = 0 \Rightarrow F_{BC} = 1.25\, C_y$ ——— (1)

$\sum F_x = 0 \Rightarrow -F_{AC} + 1.25\, C_y\,(0.6) = 0$ ——— (2)

$\Rightarrow F_{AC} = 0.43\,P + 2.14$

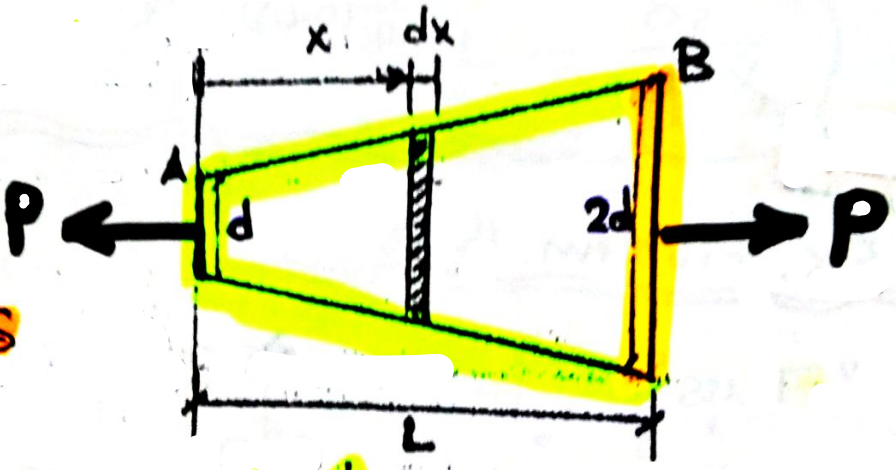
$\varepsilon_c = 0.0002 = \dfrac{(0.43P + 2.14)(10)^3(1.4)}{(400\times10^6)(200\times10^9)}$

$\Rightarrow P = 21.7\ KN$

Ex

- Tapered Bar AB
- Square Cross-Section

Find the elongation δ of the bar?



Sol.

$$\delta = \int_0^L \frac{P\,dx}{E A_x} = \frac{P}{E}\int_0^L \frac{1}{A_x}\,dx$$

where $A_x = d_x^2 = \left(\frac{d}{L}x + d\right)^2 = d^2\left(1 + \frac{x}{L}\right)^2$

$$\delta = \frac{P}{Ed^2}\int_0^L \frac{dx}{(1+\frac{x}{L})^2}$$

$$= \frac{P}{Ed^2}\left[\frac{L}{(1+\frac{x}{L})}\right]_0^L = \frac{PL}{2Ed^2}$$

Example 2-4

A tapered bar AB of solid circular cross section and length L (Fig. 2-13a) is supported at end B and subjected to a tensile load P at the free end A. The diameters of the bar at ends A and B are d_A and d_B, respectively.

Determine the elongation of the bar due to the load P, assuming that the angle of taper is small.

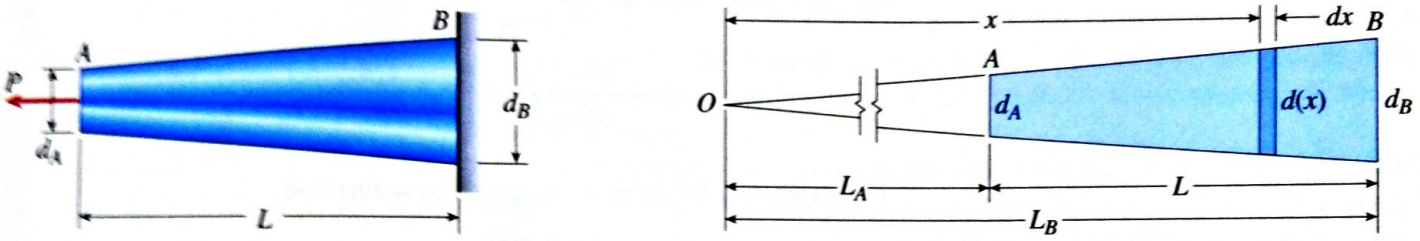


FIG. 2-13 Example 2-4. Change in length of a tapered bar of solid circular cross section

Solution

The bar being analyzed in this example has a constant axial force (equal to the load P) throughout its length. However, the cross-sectional area varies continuously from one end to the other. Therefore, we must use integration (see Eq. 2-7) to determine the change in length.

Cross-sectional area. The first step in the solution is to obtain an expression for the cross-sectional area $A(x)$ at any cross section of the bar. For this purpose, we must establish an origin for the coordinate x. One possibility is to place the origin of coordinates at the free end A of the bar. However, the integrations to be performed will be slightly simplified if we locate the origin of coordinates by extending the sides of the tapered bar until they meet at point O, as shown in Fig. 2-13b.

The distances L_A and L_B from the origin O to ends A and B, respectively, are in the ratio

$$\frac{L_A}{L_B} = \frac{d_A}{d_B} \tag{a}$$

as obtained from similar triangles in Fig. 2-13b. From similar triangles we also get the ratio of the diameter $d(x)$ at distance x from the origin to the diameter d_A at the small end of the bar:

$$\frac{d(x)}{d_A} = \frac{x}{L_A} \quad \text{or} \quad d(x) = \frac{d_A x}{L_A} \tag{b}$$

Therefore, the cross-sectional area at distance x from the origin is

$$A(x) = \frac{\pi[d(x)]^2}{4} = \frac{\pi d_A^2 x^2}{4L_A^2} \tag{c}$$

continued

Change in length. We now substitute the expression for $A(x)$ into Eq. (2-7) and obtain the elongation δ:

$$\delta = \int \frac{N(x)dx}{EA(x)} = \int_{L_A}^{L_B} \frac{Pdx(4L_A^2)}{E(\pi d_A^2 x^2)} = \frac{4PL_A^2}{\pi E d_A^2} \int_{L_A}^{L_B} \frac{dx}{x^2} \tag{d}$$

By performing the integration (see Appendix C for integration formulas) and substituting the limits, we get

$$\delta = \frac{4PL_A^2}{\pi E d_A^2} \left[-\frac{1}{x} \right]_{L_A}^{L_B} = \frac{4PL_A^2}{\pi E d_A^2} \left(\frac{1}{L_A} - \frac{1}{L_B} \right) \tag{e}$$

This expression for δ can be simplified by noting that

$$\frac{1}{L_A} - \frac{1}{L_B} = \frac{L_B - L_A}{L_A L_B} = \frac{L}{L_A L_B} \tag{f}$$

Thus, the equation for δ becomes

$$\delta = \frac{4PL}{\pi E d_A^2} \left(\frac{L_A}{L_B} \right) \tag{g}$$

Finally, we substitute $L_A/L_B = d_A/d_B$ (see Eq. a) and obtain

$$\delta = \frac{4PL}{\pi E d_A d_B} \tag{2-8}$$

This formula gives the elongation of a tapered bar of solid circular cross section. By substituting numerical values, we can determine the change in length for any particular bar.

Note 1: A common mistake is to assume that the elongation of a tapered bar can be determined by calculating the elongation of a prismatic bar that has the same cross-sectional area as the midsection of the tapered bar. Examination of Eq. (2-8) shows that this idea is not valid.

Note 2: The preceding formula for a tapered bar (Eq. 2-8) can be reduced to the special case of a prismatic bar by substituting $d_A = d_B = d$. The result is

$$\delta = \frac{4PL}{\pi E d^2} = \frac{PL}{EA}$$

which we know to be correct.

A general formula such as Eq. (2-8) should be checked whenever possible by verifying that it reduces to known results for *special cases*. If the reduction does not produce a correct result, the original formula is in error. If a correct result is obtained, the original formula may still be incorrect but our confidence in it increases. In other words, this type of check is a necessary but not sufficient condition for the correctness of the original formula.

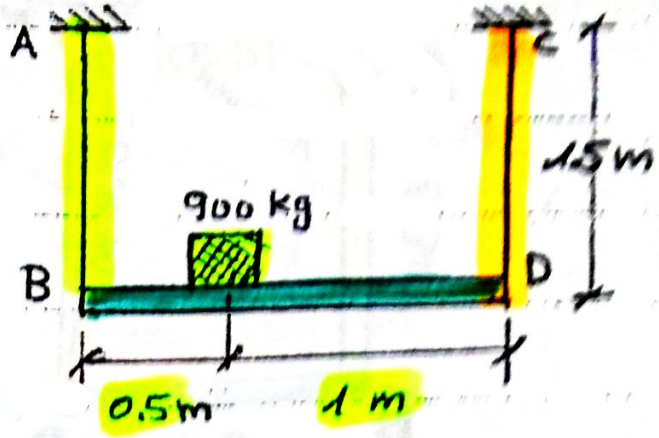
Problem

$A_{AB} = 60\ mm^2$

$A_{CD} = 120\ mm^2$

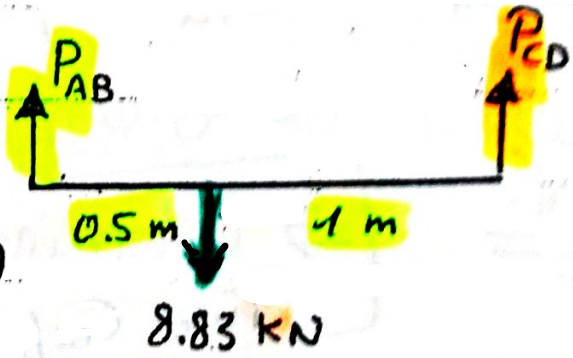
$E_{AB} = 200\ GPa$

$E_{CD} = 70\ GPa$



Calculate the vertical displacement of the weight?

Sol.



$W = (900)(9.81) = 8.83\ KN$

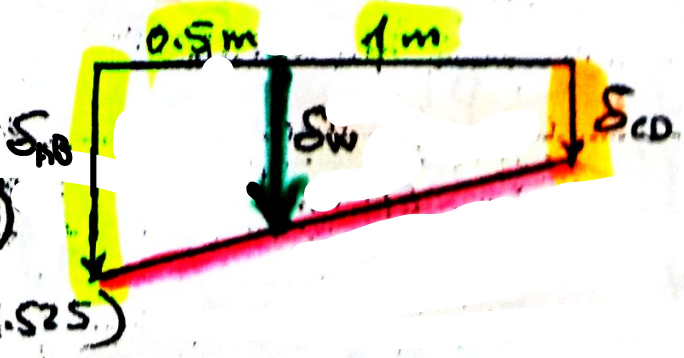
$\sum M_B = 0 \Rightarrow P_{CD} = 2.94\ KN.$

$\sum F_y = 0 \Rightarrow P_{AB} = 5.89\ KN.$

$$\delta_{AB} = \frac{P_{AB} L}{A_{AB} E_{AB}} = \frac{(5.89 \times 10^3)(1.5)}{(60 \times 10^{-6})(200 \times 10^9)} = 0.736\ mm$$

and $$\delta_{CD} = \frac{P_{CD} L}{A_{CD} E_{CD}} = \frac{(2.94 \times 10^3)(1.5)}{(120 \times 10^{-6})(70 \times 10^9)} = 0.525\ mm$$

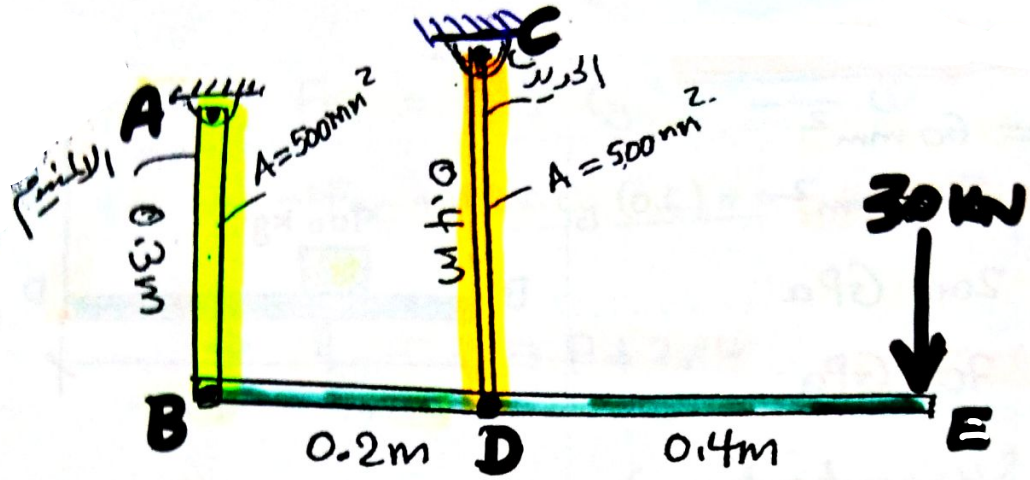
$$\frac{\delta_{AB} - \delta_{CD}}{1.5} = \frac{\delta_W - \delta_{CD}}{1}$$



$$\Rightarrow \delta_W = \delta_{CD} + \frac{1}{1.5}(\delta_{AB} - \delta_{CD})$$

$$= 0.525 + \frac{1}{1.5}(0.736 - 0.525)$$

$$= 0.666\ mm$$



$E_{Al} = 70\ GPa$ ، $E_{Steel} = 200\ GPa$

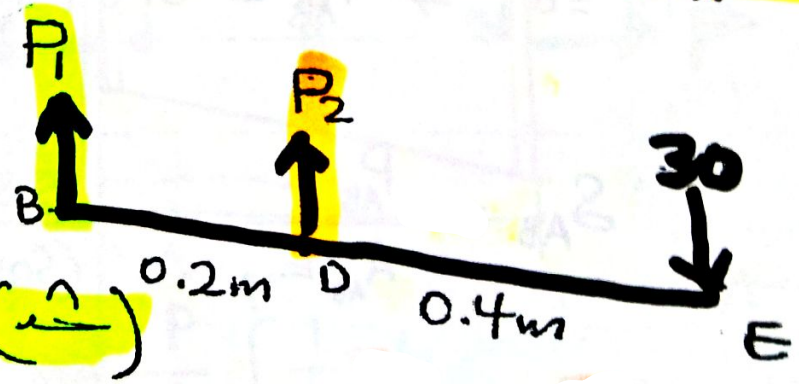
⇒ Determinante (statically) Two Cables فقط

$\Sigma M = 0$

$\Sigma F_y = 0$

$\Sigma F_x = 0$

* $\Sigma M_B = 0$



$\Rightarrow -0.2P_2 + 30(0.6) = 0$

$\Rightarrow P_2 = 90\ kN$ (↑)

* $\Sigma F_y = 0 \Rightarrow P_1 + 90 - 30 = 0$

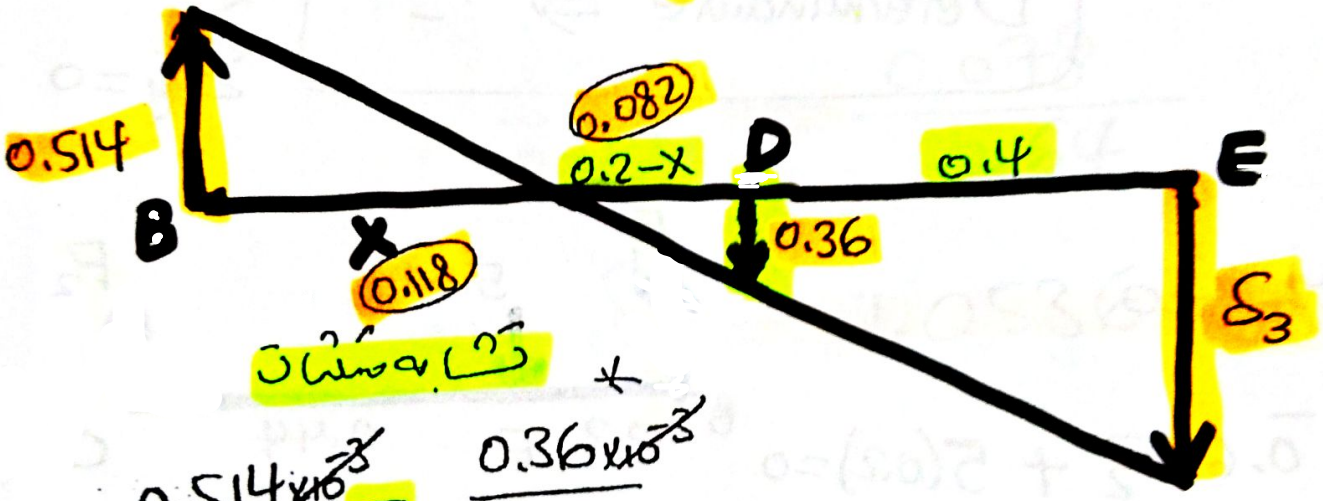
$\Rightarrow P_1 = -60\ kN$ (عكس)

$$\delta_1 = \frac{P_1 L_1}{E_1 A_1}$$

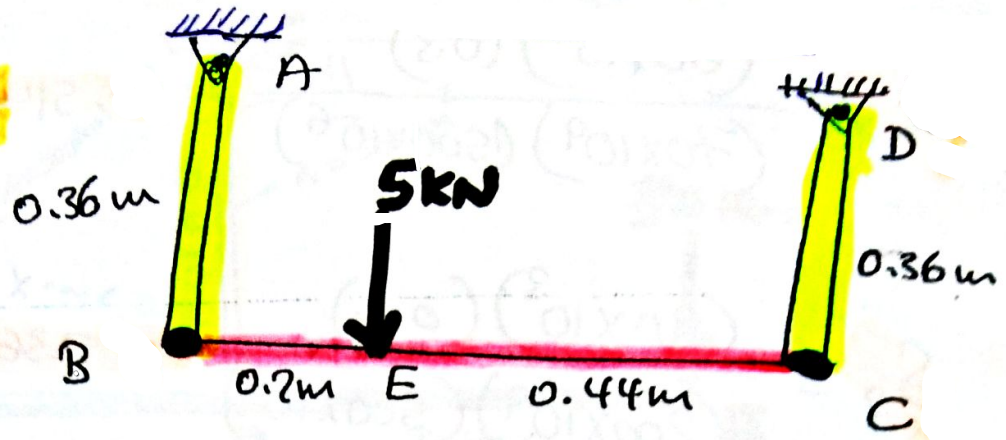
$$\delta_2 = \frac{P_2 L_2}{E_2 A_2}$$

$$\delta_1 = \frac{(60\times10^3)(0.3)}{(70\times10^9)(1500\times10^{-6})} = 0.514 \text{ mm}$$

$$\delta_2 = \frac{(90\times10^3)(0.4)}{(200\times10^9)(500\times10^{-6})} = 0.36 \text{ mm}$$

~~~

من أجل إيجاد δ_3 (تشابه مثلثان) ⟸



* تشابه مثلثان

$$\frac{0.514\times10^{-3}}{X} = \frac{0.36\times10^{-3}}{0.2-X}$$

بالاكلاوية $\Rightarrow$ $X = 0.118$ m

* تشابه مثلثان مرة أخرى

$$\frac{\delta_3}{0.482} = \frac{0.36}{0.082} \Rightarrow$$

$$\delta_3 = 2.11 \text{ mm}$$
~~~

Ex1



$A = 125\ mm^2$

$E = 70\ GPa$

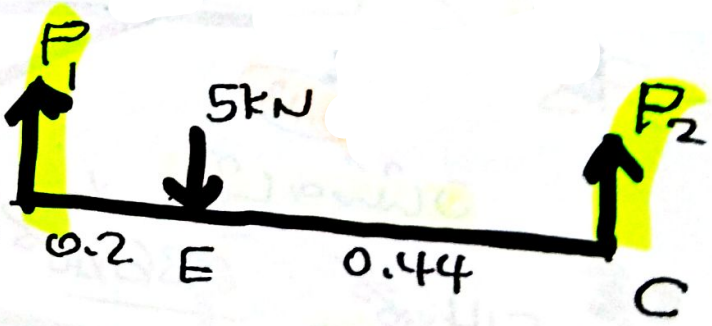
Find δ_E ?

Sol

Determinacute $\Rightarrow$ indet

$\Sigma M = 0$

$\Sigma F_y = 0$



* $\Sigma M_B = 0$

$\Rightarrow -0.64 P_2 + 5(0.2) = 0$

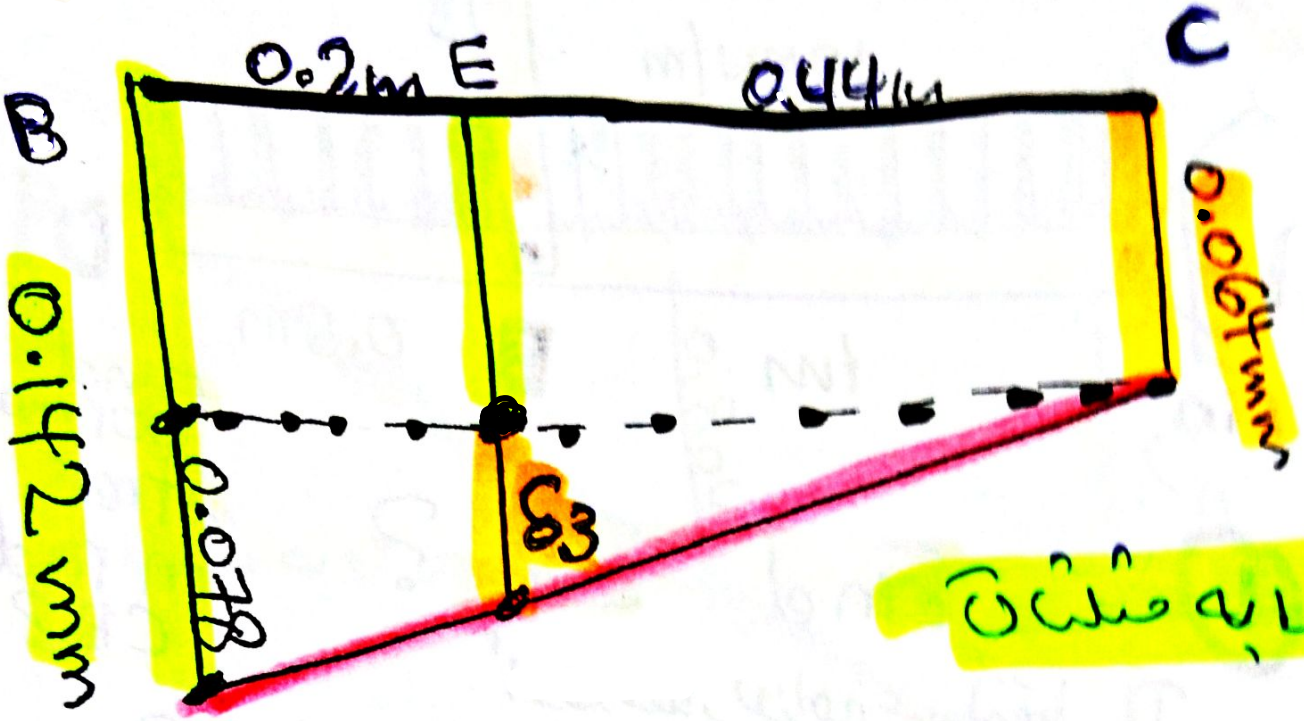
$P_2 = 1.56\ KN$

* $\Sigma F_y = 0 \Rightarrow P_1 - 5 + 1.56 = 0$

$\Rightarrow P_1 = 3.44\ KN$

* $\delta_1 = \frac{P_1 L_1}{E_1 A_1} = \frac{(3.44 \times 10^3)(0.36)}{(70 \times 10^9)(125 \times 10^{-6})} \Rightarrow \delta_1 = 0.142\ mm$

* $\delta_2 = \frac{P_2 L_2}{E_2 A_2} = \frac{(1.56 \times 10^3)(0.36)}{(70 \times 10^9)(125 \times 10^{-6})} \Rightarrow \delta_2 = 0.064\ mm$



تشابه مثلث

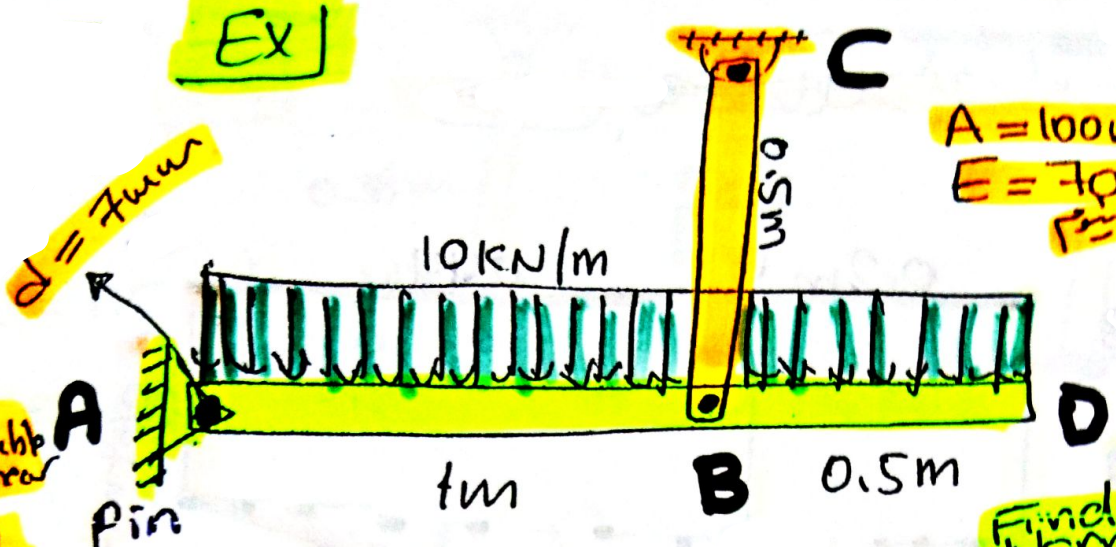
$$\frac{\delta_3}{0.44} = \frac{0.078}{0.64}$$

$$\Rightarrow \delta_3 = 0.0536 \text{ mm}$$

$$\delta_E = 0.0536 + 0.064$$

$$\delta_E = 0.1176 \text{ mm}$$

Ex



Find Normal Stress in rod CB?

1) * Find δ_D ?

D أوجد مقدار إزاحة عند النقطة

2) * Find the shear stress in pin A?

Determinate ⇒ الـ pin؟

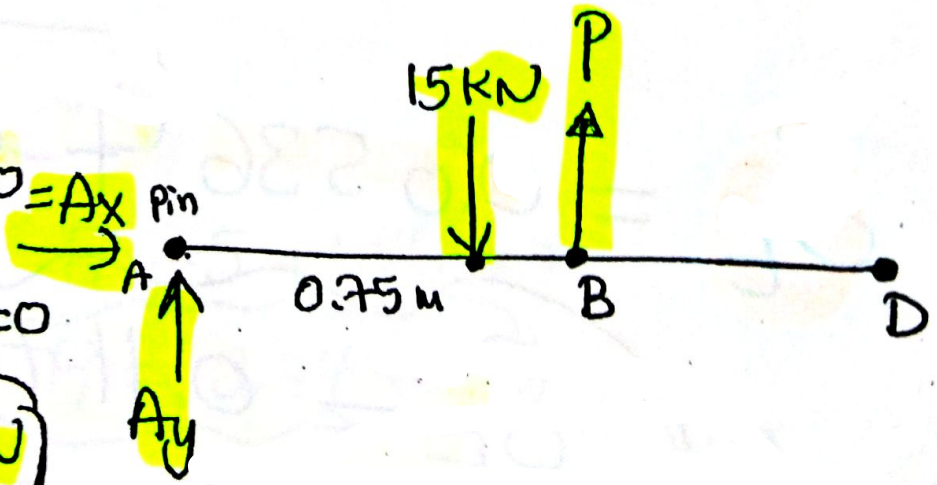
⇒ 3 unknowns

$\Sigma M = 0$

$\Sigma F_x = 0$

$\Sigma F_y = 0$

* $\Sigma M_A = 0$



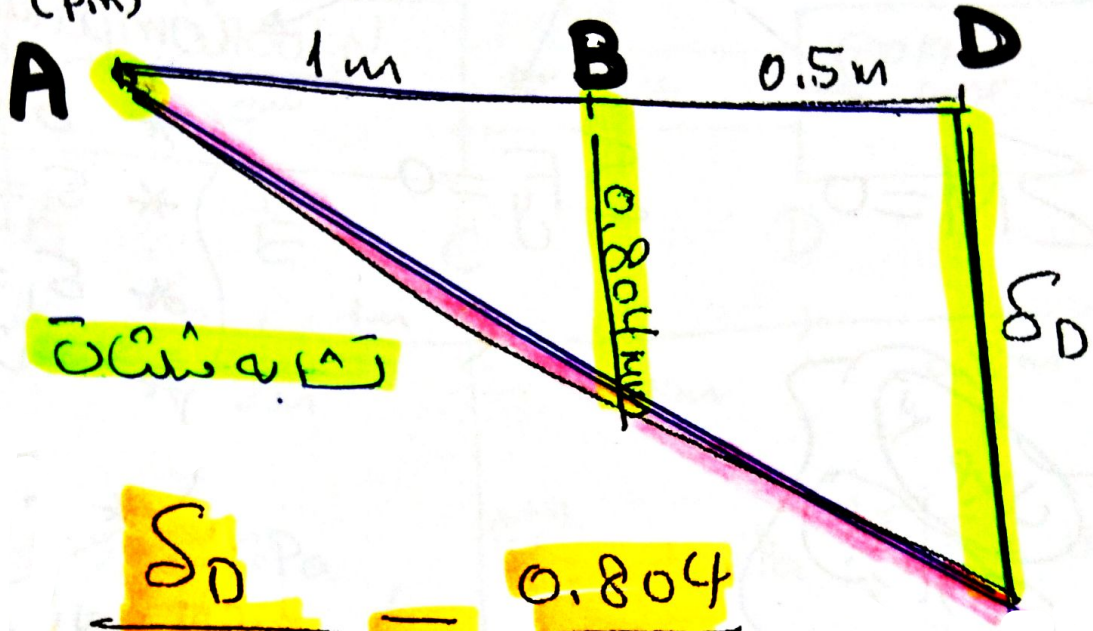
$\Rightarrow -(1)(P) + 15(0.75) = 0$

$\Rightarrow P = 11.25$ KN

* $\Sigma F_y = 0 \Rightarrow$ $A_y - 15 + 11.25 = 0 \Rightarrow$ $A_y = 3.75$ KN

* $\delta = \frac{PL}{EA} = \frac{(11.25 \times 10^3)(0.5)}{(70 \times 10^9)(100 \times 10^{-6})} \Rightarrow \delta = 0.804$

(Pin) للتثبيت



(1) تشابه مثلثات

$$\frac{\delta_D}{1.5} = \frac{0.804}{1}$$

$$\Rightarrow \delta_D = 1.206 \text{ mm}$$

(2)

* $\sigma_{CB} = \frac{P}{A} = \frac{11.25 \times 10^3}{100 \times 10^{-6}} = 112.5 \text{ MPa}$

* $\tau_A = \frac{\frac{3.75}{2} \times 10^3}{\frac{\pi}{4}(7)^2 \times 10^{-6}} = 45.5 \text{ MPa}$

Double shear

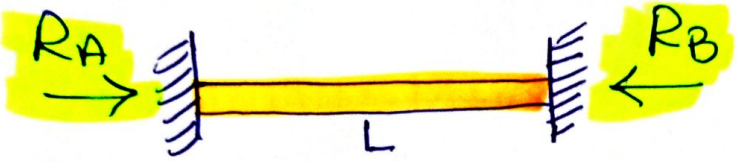
$\alpha \equiv$ thermal Expansion Coeff.

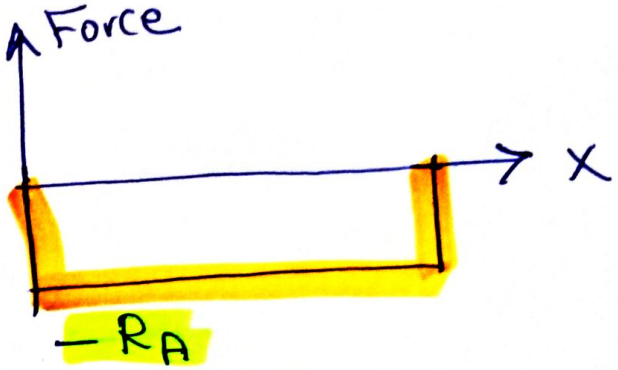
Statically indeterminate Problems (Axial loading)

$\delta_p + \delta_T = gap$

- Axial load $\delta_P = \frac{PL}{EA}$

- Temperature $\delta_T = \alpha L \Delta T$





$\delta_P = \frac{-R_A L}{EA}$

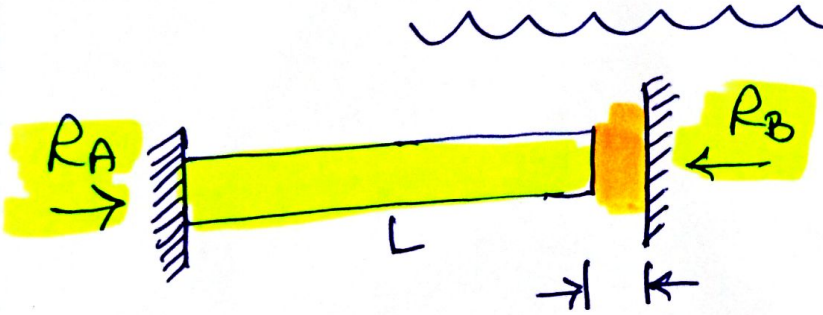
$\delta_T = \alpha L \Delta T$

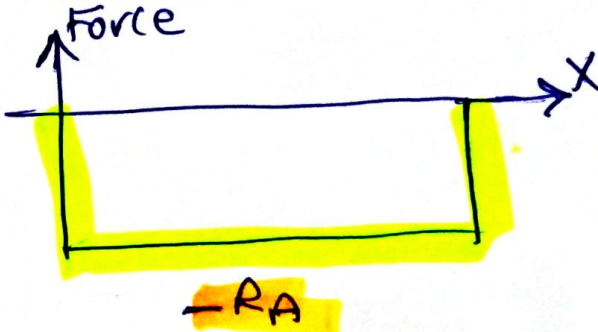
$gap = 0$

$\Rightarrow \delta_P + \delta_T = gap = 0$

$\Rightarrow$ Solve for R_A

$\Rightarrow$ then Find $R_B = R_A$





$\delta_P = \frac{-R_A L}{EA}$

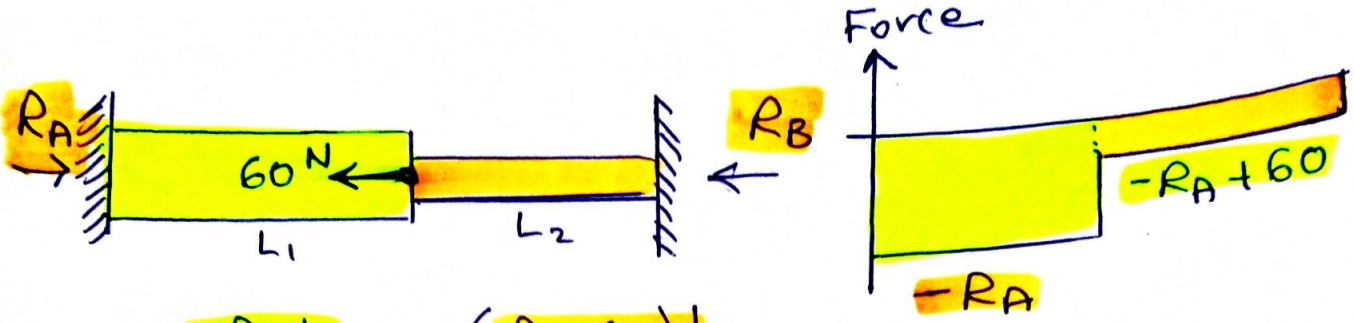
$\delta_T = \alpha L \Delta T$

$gap = 0.15 \times 10^{-3}\ m$

$\Rightarrow \delta_P + \delta_T = gap$

$\Rightarrow$ Solve for R_A

$\Rightarrow$ then Find $R_B = -R_A$



$$\delta_P = \frac{-R_A L_1}{E_1 A_1} + \frac{(-R_A+60) L_2}{E_2 A_2}$$

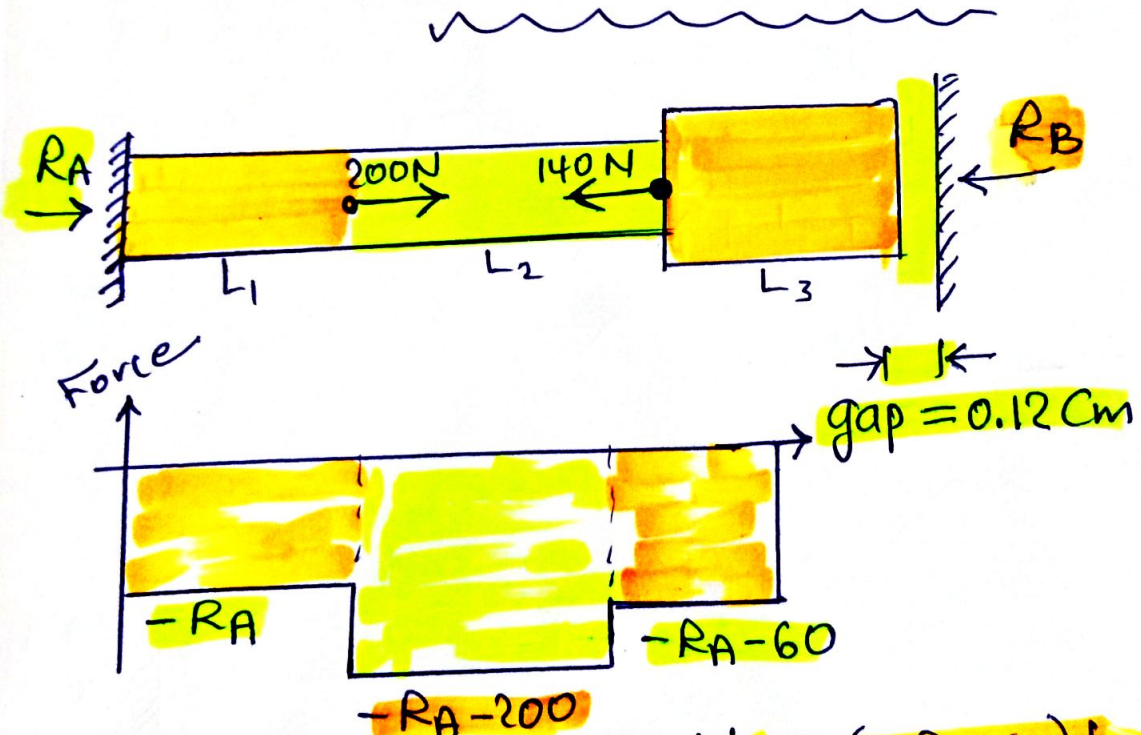
$$\delta_T = \alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T$$

$gap = 0$

$\Rightarrow \delta_P + \delta_T = gap = 0$

$\Rightarrow$ Solve for R_A

$\Rightarrow$ Find R_B : $R_B = R_A - 60$



$$\delta_P = \frac{-R_A L_1}{E_1 A_1} + \frac{(-R_A-200) L_2}{E_2 A_2} + \frac{(-R_A-60) L_3}{E_3 A_3}$$

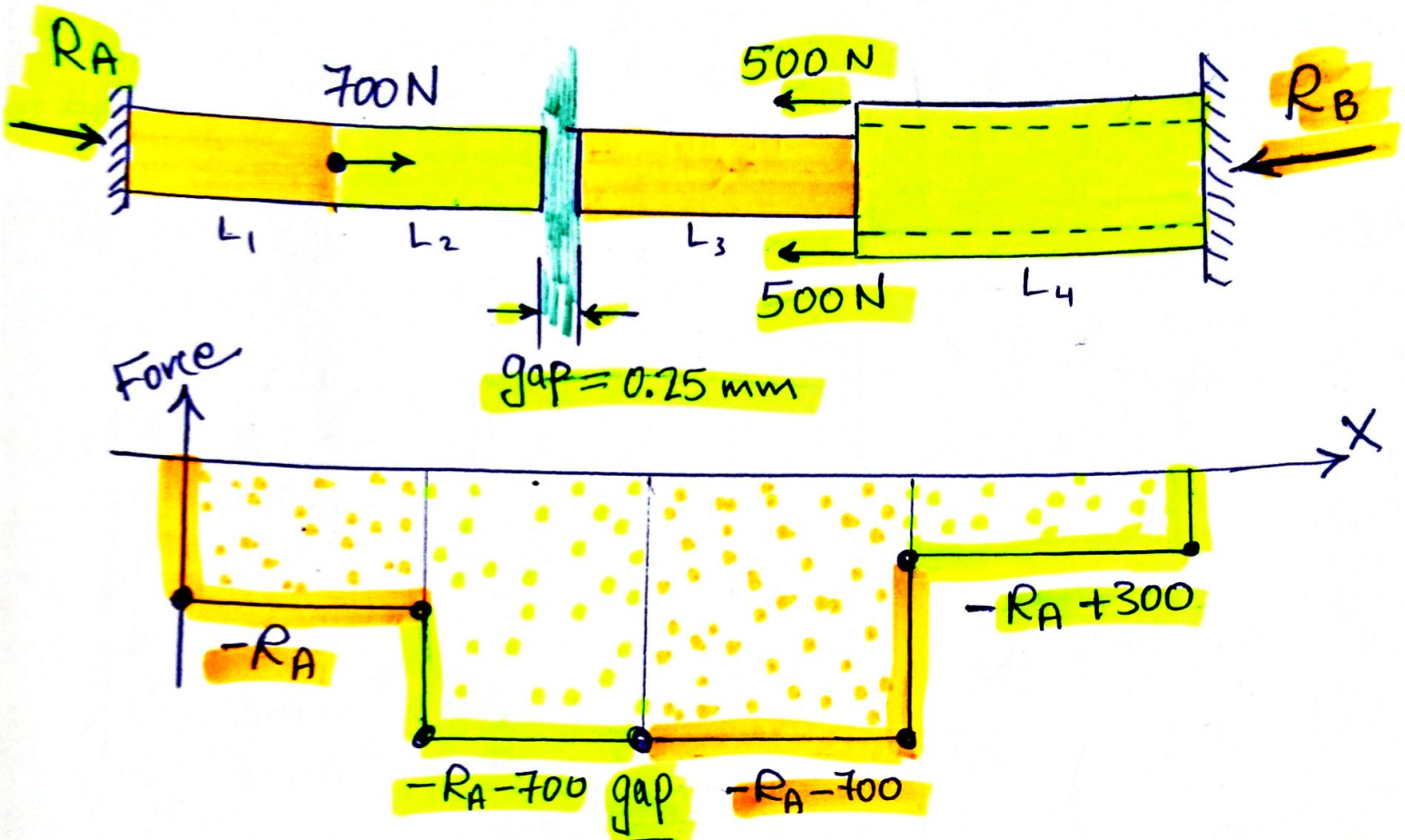
$$\delta_T = \alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T + \alpha_3 L_3 \Delta T$$

$gap = 0.12 \times 10^{-2}\ m$

$\Rightarrow \delta_P + \delta_T = gap$

$\Rightarrow$ Solve for R_A

$\Rightarrow$ Find $R_B = R_A + 200 - 140$



$$\delta_P = \frac{-R_A L_1}{E_1 A_1} + \frac{(-R_A-700)L_2}{E_2 A_2} + \frac{(-R_A-700)L_3}{E_3 A_3} + \frac{(-R_A+300)L}{E_4 A_4}$$

$$\delta_T = \alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T + \alpha_3 L_3 \Delta T + \alpha_4 L_4 \Delta T$$

$$gap = 0.25 \times 10^{-3} \text{ m}$$

$$\Rightarrow \delta_P + \delta_T = gap$$

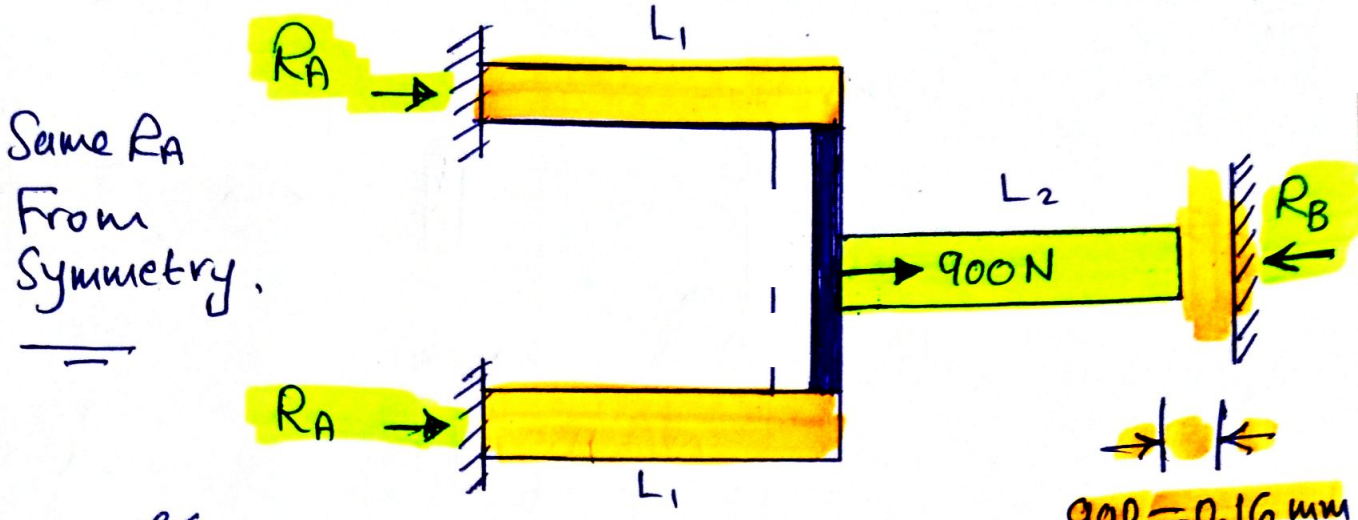
$$\Rightarrow \text{Solve for } R_A$$

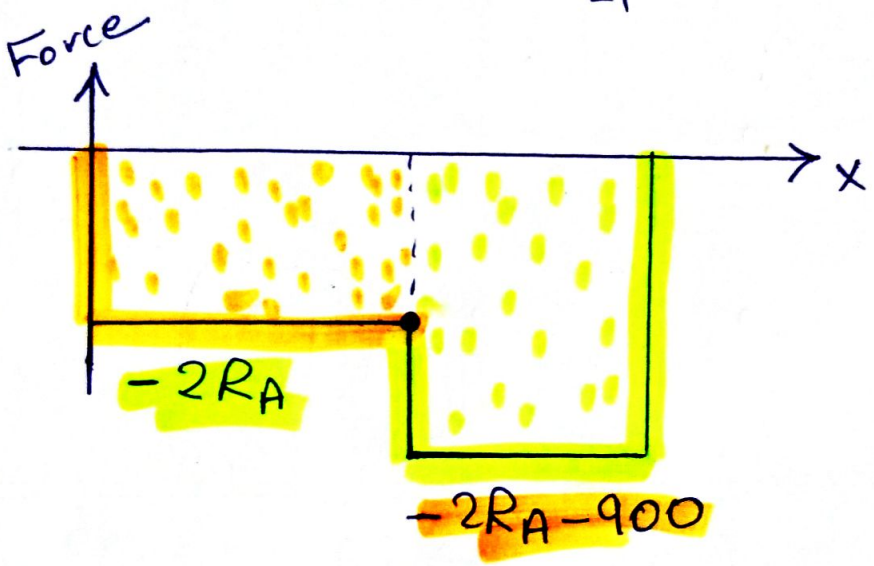
$$\Rightarrow \text{Find } R_B = R_A + 700 - 1000$$

* **Note:**

ΔT = Final Temp. − Initial Temp.

$$\Rightarrow \Delta T = T_f - T_i$$





$$\delta_P = \frac{-R_A L_1}{E_1 A_1} + \frac{(-2R_A - 900) L_2}{E_2 A_2}$$

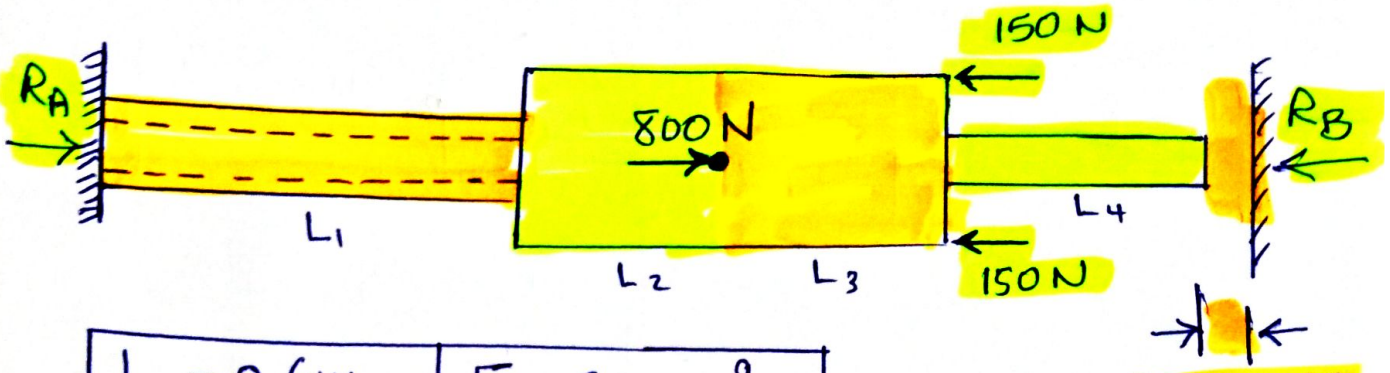
$$\delta_T = \alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T$$

$$gap = 0.16 \times 10^{-3} \text{ m}$$

$\Rightarrow \delta_P + \delta_T = gap$

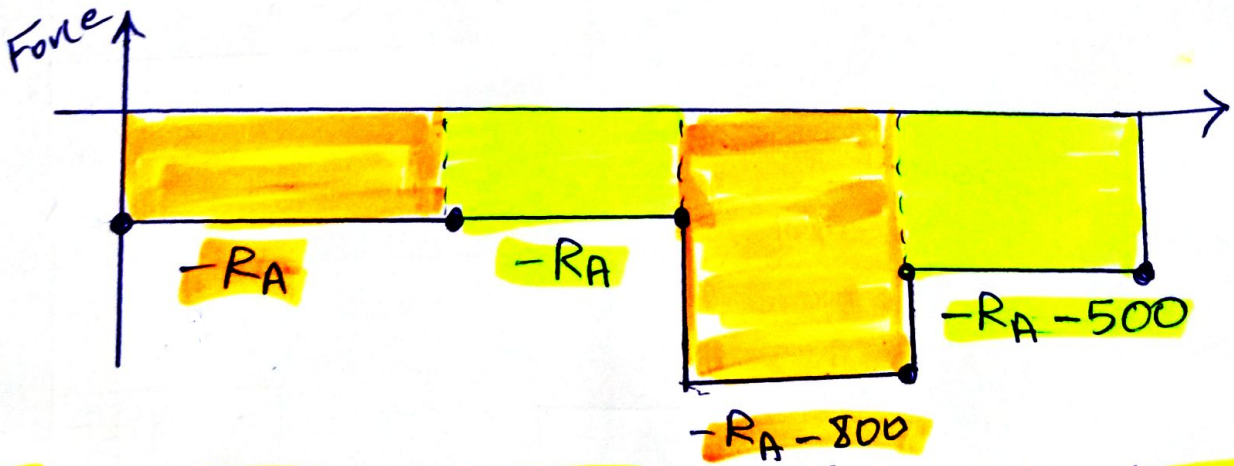
$\Rightarrow$ Solve for R_A

$\Rightarrow$ Find $R_B = 2R_A + 900$



$L_1 = 0.6m$	$E_1 = 200 \times 10^9$
$L_2 = 0.4m$	$E_2 = 140 \times 10^9$
$L_3 = 0.8m$	$E_3 = 500 \times 10^9$
$L_4 = 0.7m$	$E_4 = 70 \times 10^9$

$\alpha_1 = 24 \times 10^{-6}$	$d_o = 20mm$, $d_i = 12mm \Rightarrow A_1 = \frac{\pi}{4}(d_o^2 - d_i^2)$
$\alpha_2 = 20 \times 10^{-6}$	$d_2 = 36mm \Rightarrow A_2 = \frac{\pi}{4}d^2$
$\alpha_3 = 18 \times 10^{-6}$	$d_3 = 36mm \Rightarrow A_3 = \frac{\pi}{4}d^2$
$\alpha_4 = 12 \times 10^{-6}$	$d_4 = 8mm \Rightarrow A_4 = \frac{\pi}{4}d^2$



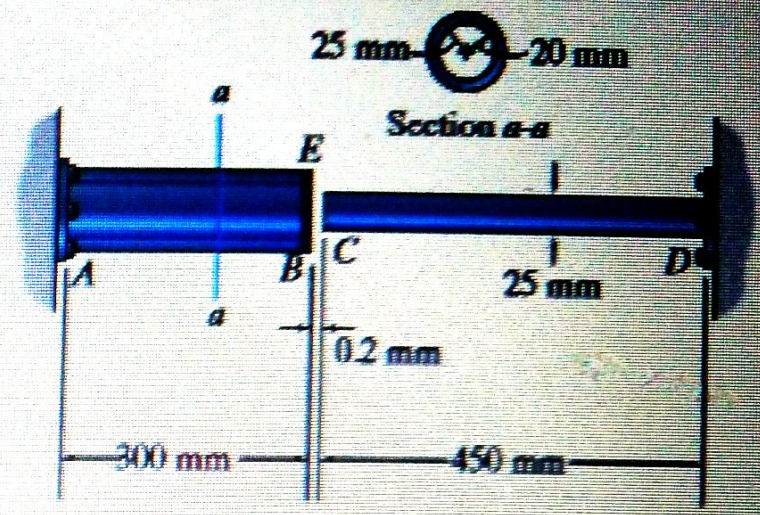
$$\delta_P = \frac{-R_A L_1}{E_1 A_1} + \frac{-R_A L_2}{E_2 A_2} + \frac{(-R_A - 800) L_3}{E_3 A_3} + \frac{(-R_A - 500) L_4}{E_4 A_4}$$

$$\delta_T = \alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T + \alpha_3 L_3 \Delta T + \alpha_4 L_4 \Delta T$$

$gap = 0.18 \times 10^{-3}$ m

* $\delta_P + \delta_T = gap$
* Solve for R_A
* $R_B = R_A + 800 - 300$

. The AM1004-T61 magnesium alloy tube AB is capped with a rigid plate E. The gap between E and end C of the 6061-T6 aluminum alloy solid circular rod CD is 0.2 mm when the temperature is at 30° C. Determine the normal stress developed in the tube and the rod if the temperature rises to 80° C. Neglect the thickness of the rigid cap.



Compatibility Equation: If tube AB and rod CD are unconstrained, they will have a free expansion of $(\delta_T)_{AB} = \alpha_{mg}\Delta T L_{AB} = 26(10^{-6})(80 - 30)(300) = 0.39$ mm and $(\delta_T)_{CD} = \alpha_{al}\Delta T L_{CD} = 24(10^{-6})(80 - 30)(450) = 0.54$ mm. Referring to the deformation diagram of the tube and the rod shown in Fig. *a*,

$$\delta = [(\delta_T)_{AB} - (\delta_F)_{AB}] + [(\delta_T)_{CD} - (\delta_F)_{CD}]$$

$$0.2 = \left[0.39 - \frac{F(300)}{\pi(0.025^2 - 0.02^2)(44.7)(10^9)}\right] + \left[0.54 - \frac{F(450)}{\frac{\pi}{4}(0.025^2)(68.9)(10^9)}\right]$$

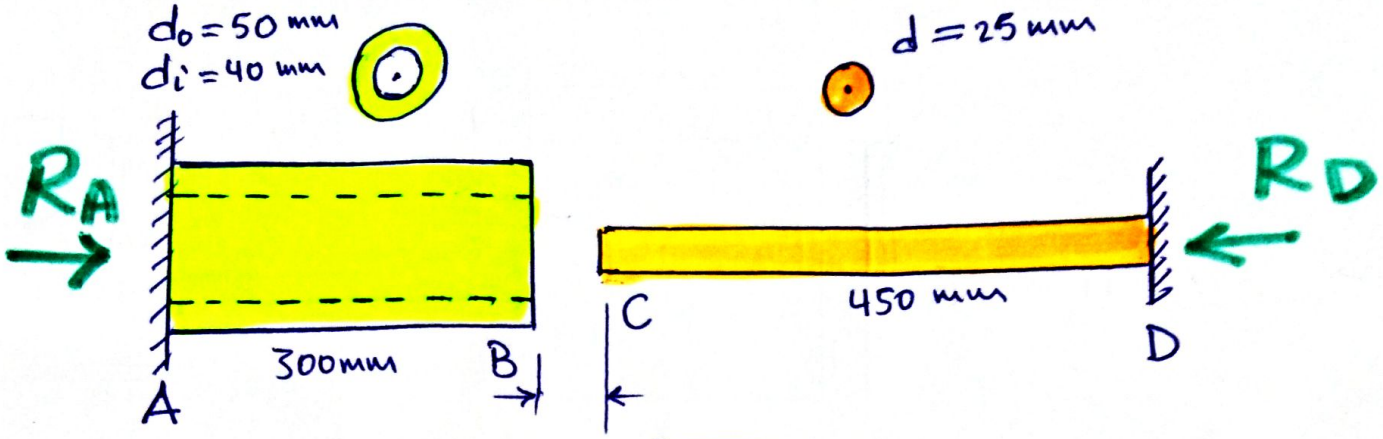
$$F = 32\,017.60\ \text{N}$$

Normal Stress:

$$\sigma_{AB} = \frac{F}{A_{AB}} = \frac{32\,017.60}{\pi(0.025^2 - 0.02^2)} = 45.3\ \text{MPa}$$ **Ans.**

$$\sigma_{CD} = \frac{F}{A_{CD}} = \frac{32\,017.60}{\frac{\pi}{4}(0.025^2)} = 65.2\ \text{MPa}$$ **Ans.**

$$F = 107\,442.47\ \text{N}$$



gap = 0.2 mm

$E_1 = 44.7$ GPa $\quad \alpha_1 = 26 \times 10^{-6}$

$E_2 = 68.9$ GPa $\quad \alpha_2 = 24 \times 10^{-6}$

(1) Find Reactions R_A and R_D ?

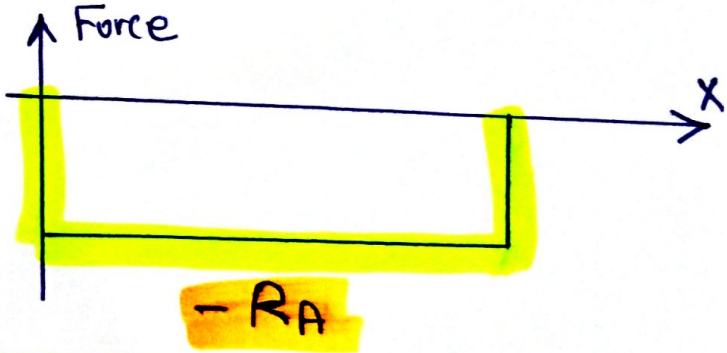
(2) Find Normal Stress in tube AB & in rod CD ?

Note: the Temp. rises from 30°c up to 80°c.

$\Delta T = T_f - T_i$

$\Delta T = 80^\circ - 30^\circ$

$\Rightarrow \Delta T = 50^\circ C$ (positive)



$\delta_T = \alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T = 0.93$ mm

$$\delta_P = \frac{-R_A L_1}{E_1 A_1} + \frac{-R_A L_2}{E_2 A_2} = -2.28 \times 10^{-5} R_A \text{ , mm}$$

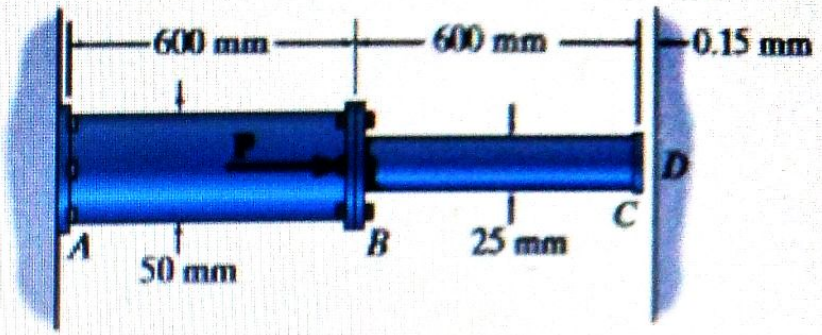
But $\delta_P + \delta_T = 0.2 \Rightarrow R_A = 32.02$ KN

$\Rightarrow R_D = R_A = 32.02$ KN

* $\sigma_{AB} = \frac{R_A}{A_{AB}} = 45.3$ MPa

* $\sigma_{CD} = \frac{R_A}{A_{CD}} = 65.2$ MPa

If the gap between C and the rigid wall at D is initially 0.15 mm, determine the support reactions at A and D when the force $\mathbf{P} = 200$ kN is applied. The assembly is made of A36 steel.



Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 200(10^3) - F_D - F_A = 0 \qquad (1)$$

Compatibility Equation: Using the method of superposition, Fig. *b*,

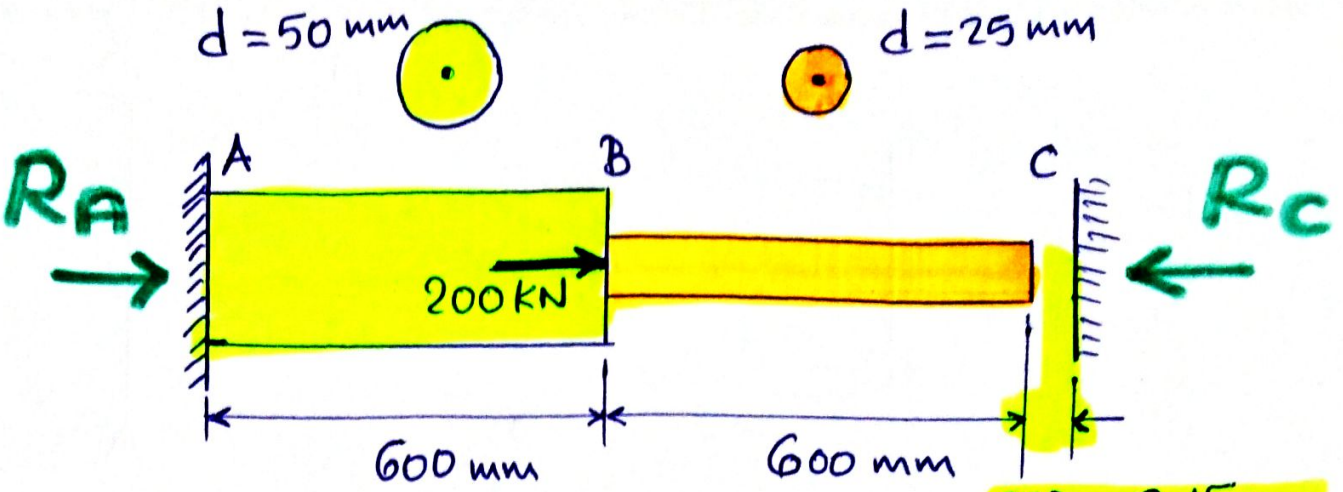
$$(\stackrel{+}{\rightarrow}) \qquad \delta = \delta_P - \delta_{F_D}$$

$$0.15 = \frac{200(10^3)(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} - \left[\frac{F_D(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} + \frac{F_D(600)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}\right]$$

$$F_D = 20\,365.05\ \text{N} = 20.4\ \text{kN} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_A = 179\,634.95\ \text{N} = 180\ \text{kN} \qquad \textbf{Ans.}$$



(1) Find Support Reactions R_A and R_C ?

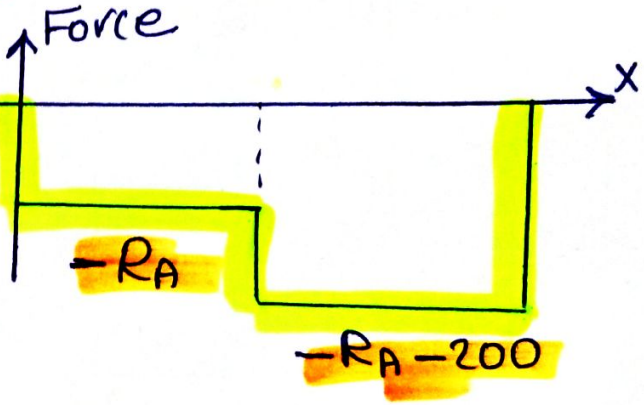
(2) Find Normal Stresses in AB and in BC ?

(3) the increase in diameter of BC assume the Poisson's Ratio $\upsilon = 0.35$ and $E = 200$ GPa ?

(No Temp. effect)

$$\delta_P = \frac{-R_A L_1}{E_1 A_1} + \frac{(-R_A - 200) L_2}{E_2 A_2}$$

But $E_1 = E_2 = 200$ GPa



$\delta_T = 0$ #

So: $\delta_P + \delta_T = gap$.

Be Careful about units $\Rightarrow R_A = -179.6$ KN

$\Rightarrow$ and $R_C = R_A + 200$

$\Rightarrow R_C = 20.4$ KN

Stresses:

$$\sigma_{AB} = \frac{-R_A}{Area} = \frac{+179.6 \times 10^3}{\frac{\pi}{4}(50)^2 \times 10^{-6}} = 365.9 \text{ MPa}$$ Tension

$$\sigma_{BC} = \frac{-R_A - 200}{Area} = \frac{-20.4 \times 10^3}{\frac{\pi}{4}(75)^2 \times 10^{-6}} = -41.6 \text{ MPa}$$ Comp.

Now: the increase in diameter Δd ?

$$\sigma_{BC} = E\,\varepsilon_{BC}$$

$$\varepsilon_{BC} = \frac{41.6 \times 10^{6}}{200 \times 10^{9}} = 2.0779 \times 10^{-4}$$

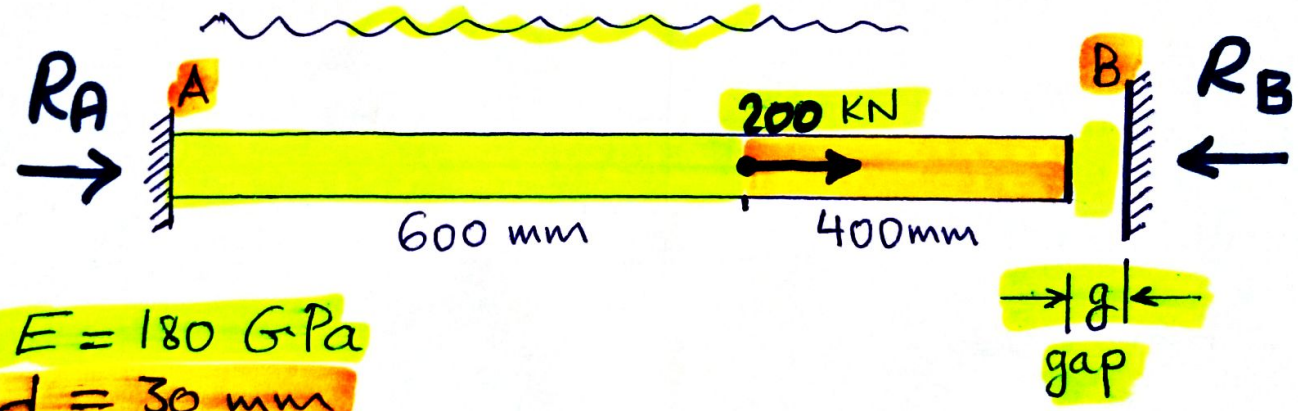
$$\varepsilon_{lat} = \upsilon\,\varepsilon_{BC} = 7.2727 \times 10^{-5}$$

$$\# \quad \varepsilon_{lat} = \frac{\Delta d}{d}$$

$$\Rightarrow \Delta d = d\,\varepsilon_{lat} = (25)(7.2727 \times 10^{-5})$$

$$\Rightarrow \Delta d = 1.818 \times 10^{-3} \text{ mm}$$

Ex



$E = 180$ GPa

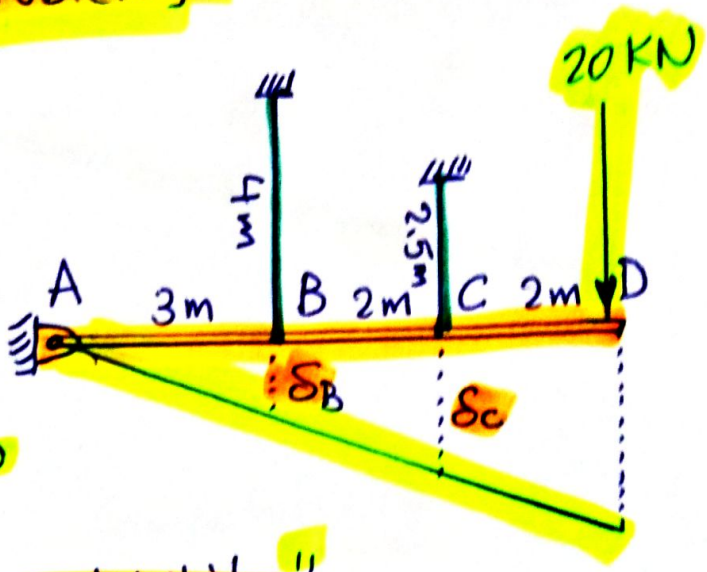
$d = 30$ mm

1) Find the gap distance g so that the reaction at B is three times the reaction at A ?

2) If $g = 1.25$ mm, find the support reactions at A and B ?

Axially loaded Members

Statically Indeterminate problems



$\sum F_y = 0$

$\sum F_x = 0$

$\sum M_A = 0$

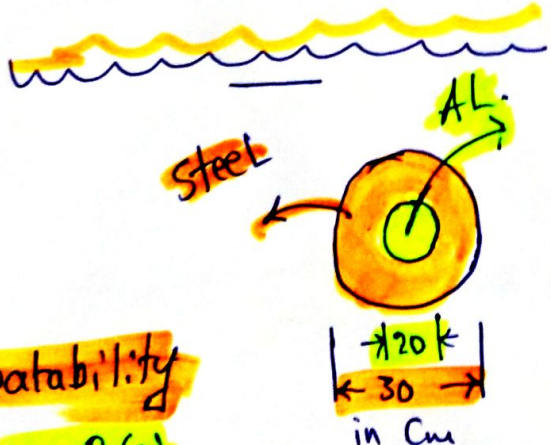
$* \rightarrow \quad 3T_B + 5T_C - 20(7) = 0$

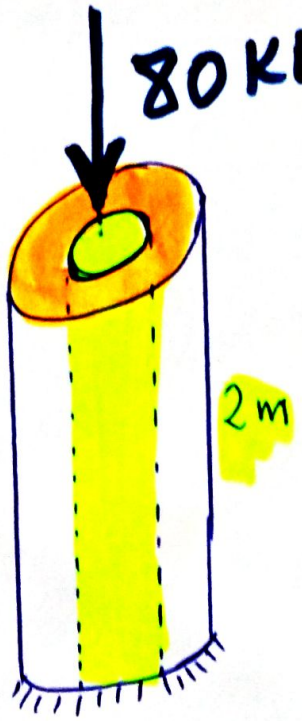
$* \quad \frac{\delta_C}{5} = \frac{\delta_B}{3}$ "Compatability"

where $\delta_B = \frac{PL}{EA} = \frac{T_B(4)}{E_1 A_1}$

$\delta_C = \frac{PL}{EA} = \frac{T_C(2.5)}{E_2 A_2}$

$\Rightarrow$ Solve For T_B and T_C.



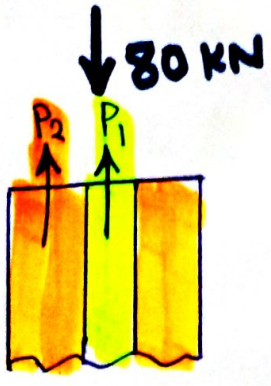


$\sum F_y = 0$

$P_1 + P_2 = 80$

$\delta_{AL} = \delta_{St}$, Compatability

$\delta_{AL} = \frac{P_1(2)}{E_1 A_1}$, $\delta_{St} = \frac{P_2(2)}{E_2 A_2}$



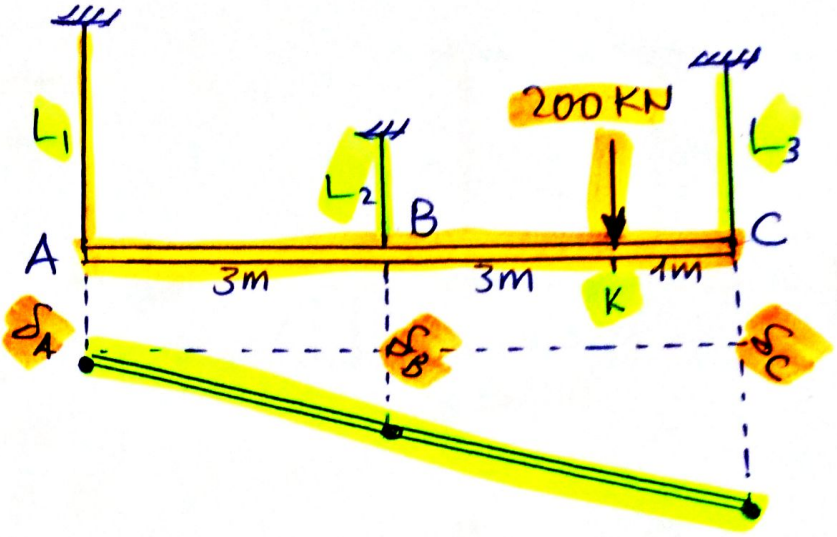
$A_1 = \frac{\pi}{4}(20)^2$

$A_2 = \frac{\pi}{4}(30^2 - 20^2)$

Solve to find P_1 and P_2

$$\Sigma M_A = 0$$

$$7T_C + 3T_B - 200(6) = 0$$



$$\Sigma F_y = 0$$

$$T_A + T_B + T_C = 200$$

$$\frac{\delta_C - \delta_A}{7} = \frac{\delta_B - \delta_A}{3}$$ Compatability

Where: $\delta_A = \frac{T_A L_1}{E_1 A_1}$, $\delta_B = \frac{T_B L_2}{E_2 A_2}$, $\delta_C = \frac{T_C L_3}{E_3 A_3}$

* Solve For Tension T_A, T_B and T_C

* you can find the deflection under the load of point K as (δ_K):

$$\frac{\delta_K - \delta_A}{6} = \frac{\delta_B - \delta_A}{3} \rightarrow \text{Solve for } \delta_K.$$

* The Normal Stress σ_C in Cable at C is

$$\sigma_C = \frac{\text{Force}}{\text{Area}} = \frac{T_C}{A_3}.$$

* The Strain in the Cable at B is ε_B:

$$\varepsilon_B = \frac{\Delta L}{L_o} = \frac{\delta_B}{L_2}.$$

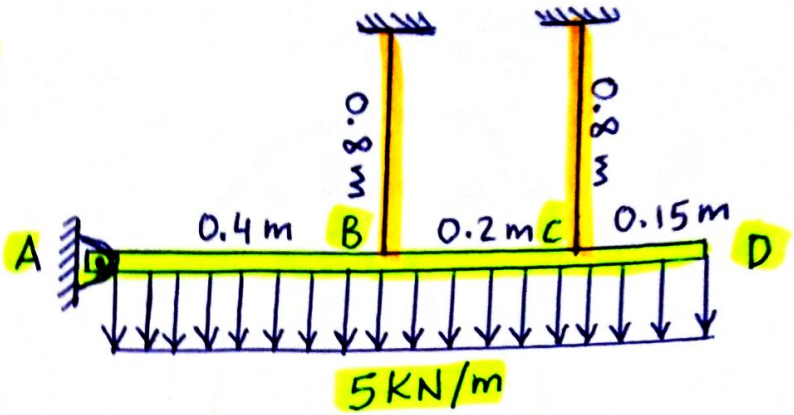
Ex)

6 mm = ϕ_C

Cable diameters ⇒ 4 mm = ϕ_B

Same material

$E = 200\ GPa$



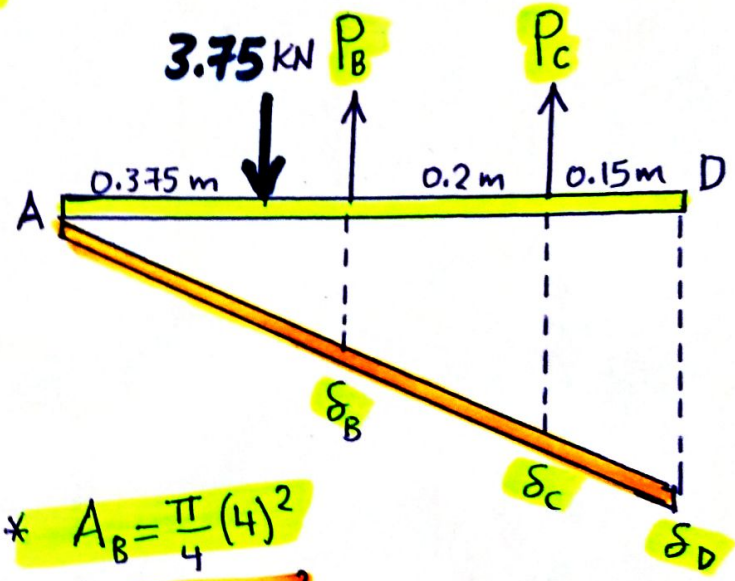
Find ① Forces in Cables ?

② deflection at D ?

③ Stress in Cable C ?

$\sum M_A = 0$

$\rightarrow 0.4 P_B + 0.6 P_C = 1.41$ — ①



$$\frac{\delta_C}{0.6} = \frac{\delta_B}{0.4}$$

$$\rightarrow \left(\frac{PL}{EA}\right)_C = 1.5 \left(\frac{PL}{EA}\right)_B$$

$P_C = 3.375\ P_B$ — ②

* $A_B = \frac{\pi}{4}(4)^2$
* $A_C = \frac{\pi}{4}(6)^2$

Solve:

$P_B = 0.58\ KN$

$P_C = 1.96\ KN$

$$\Rightarrow \delta_B = \frac{PL}{EA}$$

$$\rightarrow \delta_B = \frac{(0.58 \times 10^3)(0.8)}{(200 \times 10^9)\frac{\pi}{4}(4)^2 \times 10^{-6}}$$

$\rightarrow \delta_B = 0.185\ mm$

$\rightarrow \delta_C = 0.278\ mm$

$$\frac{\delta_D}{0.75} = \frac{\delta_B}{0.4}$$

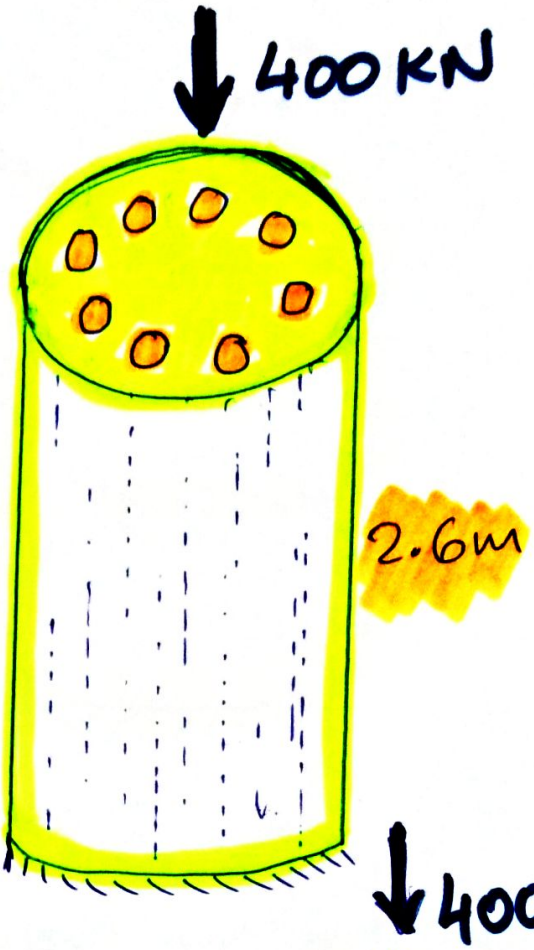
$$\Rightarrow \delta_D = \frac{0.75}{0.4}(0.185)$$

$\rightarrow \delta_D = 0.347\ mm$

$$\sigma_C = \frac{P_C}{A_C}$$

$\rightarrow \sigma_C = 69.3\ MPa$

Ex



8 rods from steel
diameter $d = 20\ mm$

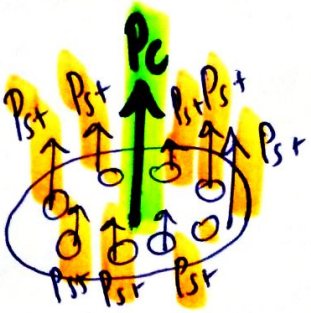
Columns from Concrete
diameter $d = 200\ mm$

$E_{St} = 200\ GPa$

$E_C = 18\ GPa$

1) Find the force in the steel rod and in Concrete?

2) Find the stress in Steel and in Concrete?



$$\Sigma F_y = 0 \Rightarrow P_C + 8P_{St} = 400$$

$$\delta_C = \delta_{St} \Rightarrow \left(\frac{PL}{EA}\right)_C = \left(\frac{PL}{EA}\right)_{St}$$ Compatability

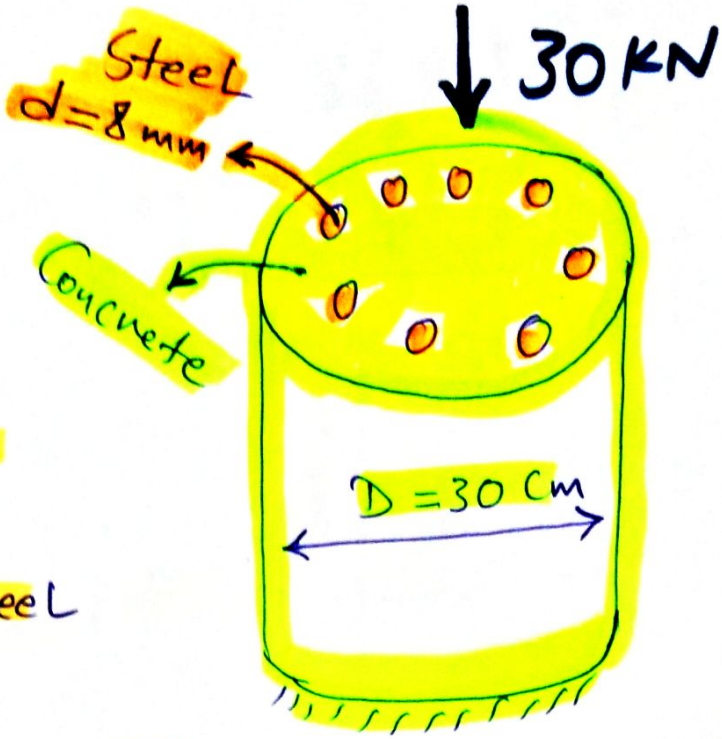
* $A_{St} = \frac{\pi}{4}(20)^2$ for each rod.

* $A_C = \frac{\pi}{4}(200)^2 - 8A_{St}$ for Concrete.

$\Rightarrow$ Solve Now for P_C and P_{St}.

* $\sigma_C = \frac{P_C}{A_C}$ and $\sigma_{St} = \frac{P_{St}}{A_{St}}$.

Ex1



$E_{St} = 200\ GPa$

$E_{cr} = 25\ GPa$

(1) Find Force in Steel and Concrete ?

(2) Find Stresses in Steel and Concrete ?

$$P_{cr} + P_{St} = 30 \quad \text{---(1)}$$

$$A_{St} = 8\left(\frac{\pi}{4}\right)d^2 = 8\left(\frac{\pi}{4}\right)(8) = 402\ mm^2 = 402\times10^{-6}\ m^2$$

$$A_{cr} = \frac{\pi}{4}D^2 - A_{St} = \frac{\pi}{4}(0.3)^2 - 402\times10^{-6} = 0.0703\ m^2$$

But $\delta_{St} = \delta_{cr}$ Compatability

$$\Rightarrow \left(\frac{PL}{EA}\right)_{St} = \left(\frac{PL}{EA}\right)_{cr} \quad \text{---(2)}$$

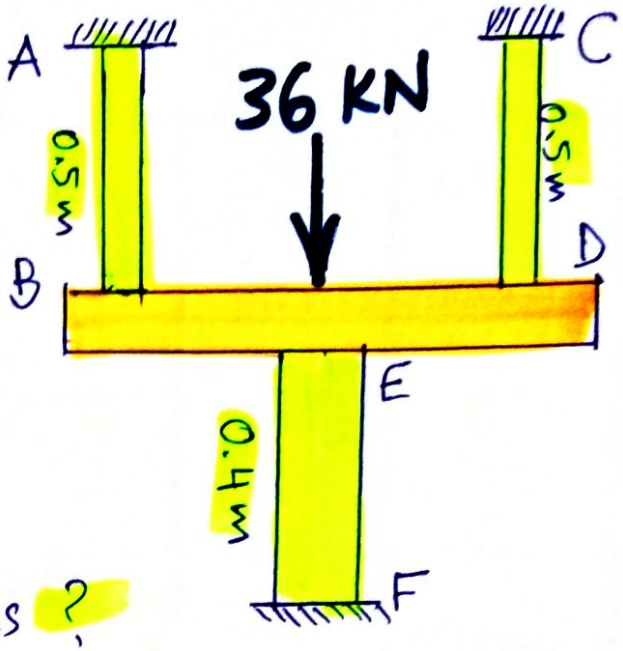
$$\Rightarrow \frac{P_{St}\ L}{(200\times10^9)(402\times10^{-6})} = \frac{(30-P_{St})\ L}{(25\times10^9)(0.0703)}$$

$$\Rightarrow P_{St} = 1.30\ KN \ \& \ P_{cr} = 28.7\ KN$$

* $\sigma_{cr} = \frac{P_{cr}}{A_{cr}} = \text{----}$

* $\sigma_{St} = \frac{P_{St}}{A_{St}} = \text{----}$

Ex)



for columns:

$E = 200\ GPa$

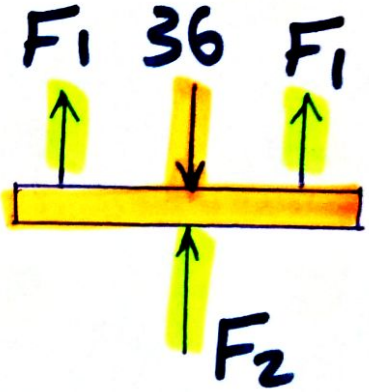
$A_{AB} = A_{CD} = 200\ mm^2$

$A_{EF} = 625\ mm^2$

Find (1) Forces in Columns ?

(2) δ_{EF} ?

(3) Stresses in Columns?



$\sum F_y = 0$:

$2F_1 + F_2 = 36 \times 10^3$ — (1)

$\delta_{AB} = \delta_{EF}$ Compatability

$$\frac{F_1(0.5)}{E(200)} = \frac{F_2(0.4)}{E(625)}$$

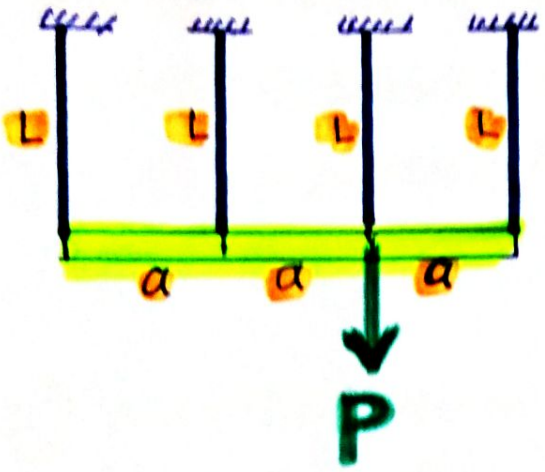
$\Rightarrow$ $312.5\,F_1 - 80\,F_2 = 0$ — (2)

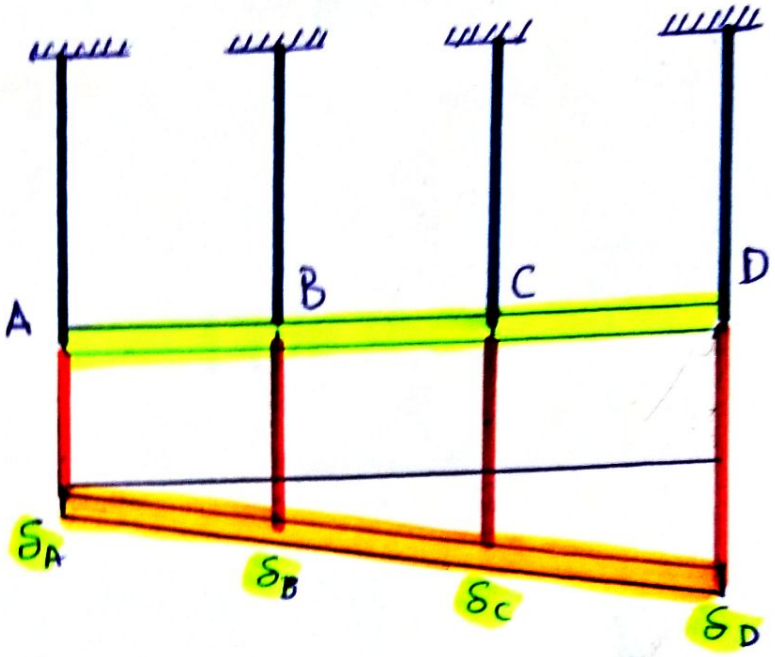
Solve : $F_1 = 6095\ N = 6.1\ kN$ & $F_2 = 23810\ N = 23.8\ kN$

* $\delta_{EF} = \frac{PL}{EA} = 0.0762\ mm$

* $\sigma_{AB} = \frac{F_1}{A_{AB}} = 30.5\ MPa$ Tension & $\sigma_{EF} = \frac{F_2}{A_{EF}} = 38.1\ MPa$ Compression

Ex





Find The Tension fore in the four Cables, F_A , F_B , F_C and F_D ?

$$\sum F_y = 0 \rightarrow F_A + F_B + F_C + F_D = P \quad \text{— (1)}$$

$$\sum M_C = 0 \rightarrow F_D - F_B - 2F_A = 0 \quad \text{— (2)}$$

$$\frac{\delta_D - \delta_A}{3a} = \frac{\delta_B - \delta_A}{a} \rightarrow F_D = 3F_B - 2F_A \quad \text{— (3)}$$

$$\frac{\delta_C - \delta_A}{2a} = \frac{\delta_B - \delta_A}{a} \rightarrow F_C = 2F_B - F_A \quad \text{— (4)}$$

Solve these 4 equations :

$F_A = \frac{1}{10} P$, $F_B = \frac{1}{5} P$

$F_C = \frac{3}{10} P$, $F_D = \frac{2}{5} P$

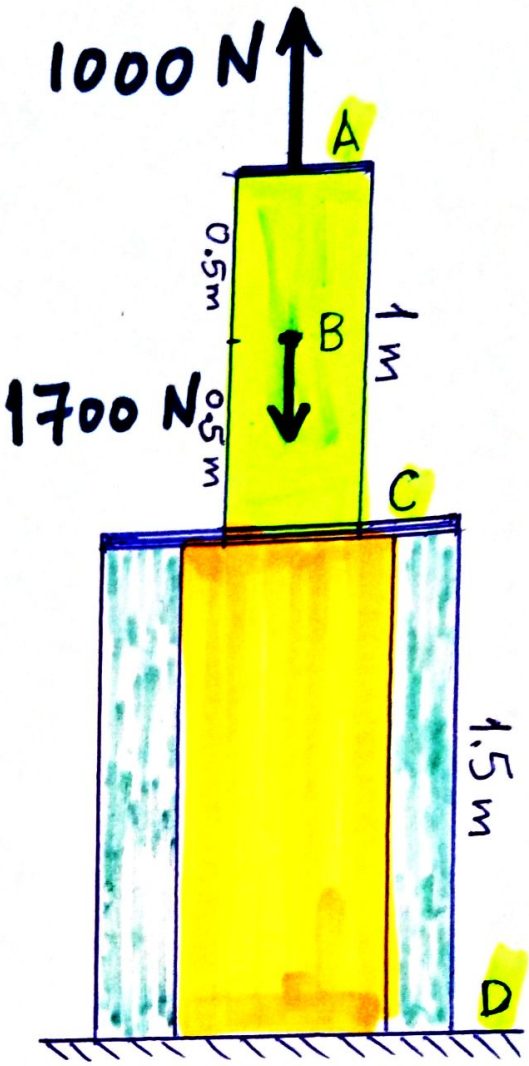
Ex1

Materials according to Colors:

$E_1 = 160$ GPa

$E_2 = 80$ GPa

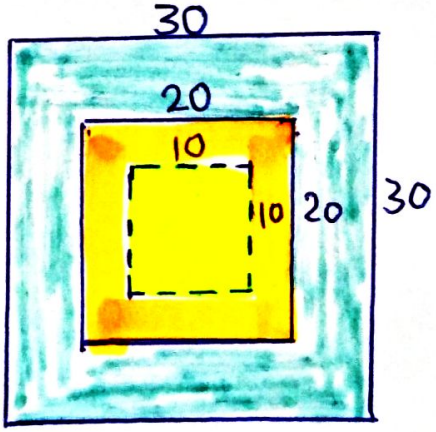
$E_3 = 120$ GPa



1. Find the Force in Material (2) and (3) ?
2. Deflection of point A ?
3. Normal Stress in Material (3) ?

- $F_{AB} = 1000$ N
- $F_{BC} = -700$ N
- $F_C = 700$ N Compression
- $P_2 + P_3 = 700$ ——— (1)
- $\delta_2 = \delta_3$ ——— (2)

Solve to find P_2 & P_3 .



Square Cross-section dimensions in Cm

- $\delta_A = \delta_{AB} + \delta_{BC} + \delta_2$
- $\sigma_3 = \frac{P_3}{A_3}$

* $A_1 = (10)^2 = 100$ Cm^2
* $A_2 = (20)^2 = 400$ Cm^2
* $A_3 = (30)^2 - (20)^2 = 500$ Cm^2