Torsion

CHAPTER OVERVIEW

Chapter 3 is concerned with the twisting of circular bars and hollow shafts acted upon by torsional moments. First, we consider **uniform torsion** which refers to the case in which torque is constant over the length of a prismatic shaft, while **nonuniform torsion** describes cases in which the torsional moment and/or the torsional rigidity of the cross section varies over the length. As for the case of axial deformations, we must relate stress and strain and also applied loading and deformation. For torsion, recall that Hooke's Law for shear states that shearing stresses, τ, are proportional to shearing strains, γ, with the constant of proportionality being G, the shearing modulus of elasticity. Both shearing stresses and shearing strains vary linearly with increasing radial distance in the cross section, as described by the **torsion formula**. The angle of twist, φ, is proportional to the internal torsional moment and the torsional flexibility of the circular bar. Most of the discussion in this chapter is devoted to linear elastic behavior and small rotations of statically determinate members. However, if the bar is **statically indeterminate**, we must augment the equations of statical equilibrium with compatibility equations (which rely on **torque-displacement relations**) to solve for any unknowns of interest, such as support moments or internal torsional moments in members. Stresses on inclined sections also are investigated as a first step toward a more complete consideration of plane stress states in later chapters. Finally, a number of specialized and advanced topics (such as strain energy, shear flow in thin-walled tubes, and stress concentrations in torsion) are introduced at the end of this chapter.

The topics in Chapter 3 are organized as follows:

*Specialized and/or advanced topics

Torsion الالتواء

د. هشام خالدي

Ch3

$$\tau = \frac{Tr}{J}$$

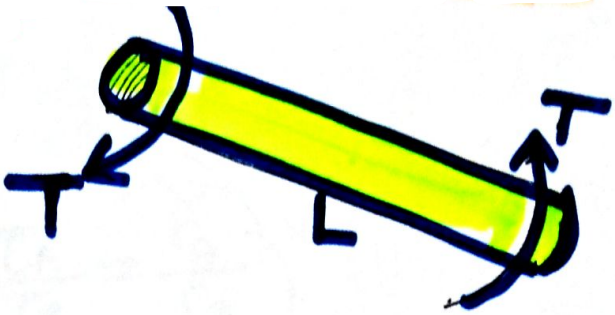
$$\phi = \frac{TL}{GJ}$$

$$r\phi = L\gamma$$

$$\tau = G\gamma$$

$$P = T\omega$$

$$\omega = 2\pi f$$



- τ الاجهاد القص = Shear stress , Pa
- γ الانفعال القص = Shear strain , rad
- T العزم = Torque . N.m
- $I_p = J$ عزم القصور القطبي = Polar moment of inertia m^4
- G نوع المعدن = modulus of rigidity Pa
- ϕ زاوية الالتواء = Angle of Twist rad
- r نصف القطر = radius , m
- L الطول = Length of shaft . m
- P القدرة = power . Watt
- ω السرعة الزاوية = Angular speed . (rad/s)
- f التردد = frequency . (Hz)
- n السرعة = Angular velocity (rpm) revolution per minute دوره في الدقيقه

$$\omega = \frac{2\pi}{60} n$$

Solid

$$J = \frac{\pi}{2} r^4$$

$$J = \frac{\pi}{32} d^4$$

Hollow

$$J = \frac{\pi}{2}(r_o^4 - r_i^4)$$

$$J = \frac{\pi}{32}(d_o^4 - d_i^4)$$

Hollow shaft $d_o = 20\,mm$, $d_i = 16\,mm$, $T = 40\ N.m$, $G = 80\ GPa$, $L = 1.6\ m$.

(1) Find τ and ϕ ? $\Rightarrow$ Hollow.

(2) Find τ and ϕ ? $\Rightarrow$ Solid

(1) $J = \frac{\pi}{32}(20^4 - 16^4) = 9273.48\ mm^4$

$$* \quad \tau = \frac{Tr}{J} = \frac{(40)(10 * 10^{-3})}{9273.48 * 10^{-12}}$$

$$\Rightarrow \tau = 34.5\ MPa.$$

$$* \quad \phi = \frac{TL}{GJ} = \frac{(40)(1.6)}{(80 * 10^9)(9273.48 * 10^{-12})}$$

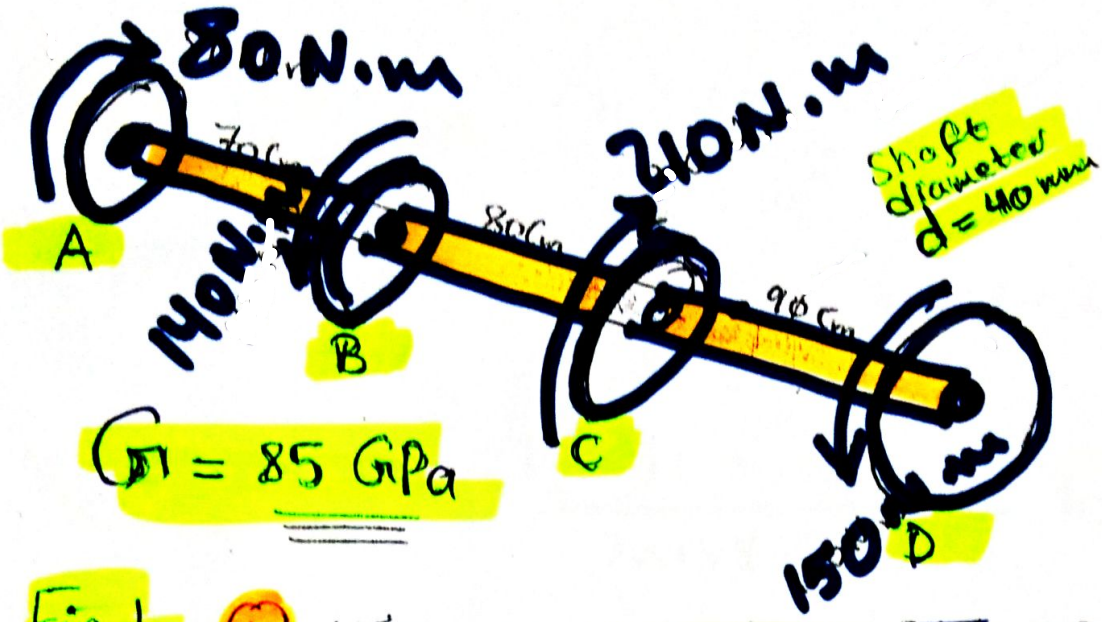
$$\phi = 0.086\ rad$$

$$\phi = 0.086 \left(\frac{180^\circ}{\pi}\right) = 4.9^\circ$$

(2) $* \quad J = \frac{\pi}{32}[20^4] = 15708\ mm^4$.

$$* \quad \tau = \frac{Tr}{J} = \frac{(40)(10 * 10^{-3})}{15708 * 10^{-12}} = 25.46\ MPa$$

$$* \quad \phi = \frac{TL}{GJ} = \frac{(40)(1.6)}{(80 * 10^9)(15708 * 10^{-12})} = \text{---} \checkmark \text{---}\ rad.$$



$G = 85\ GPa$

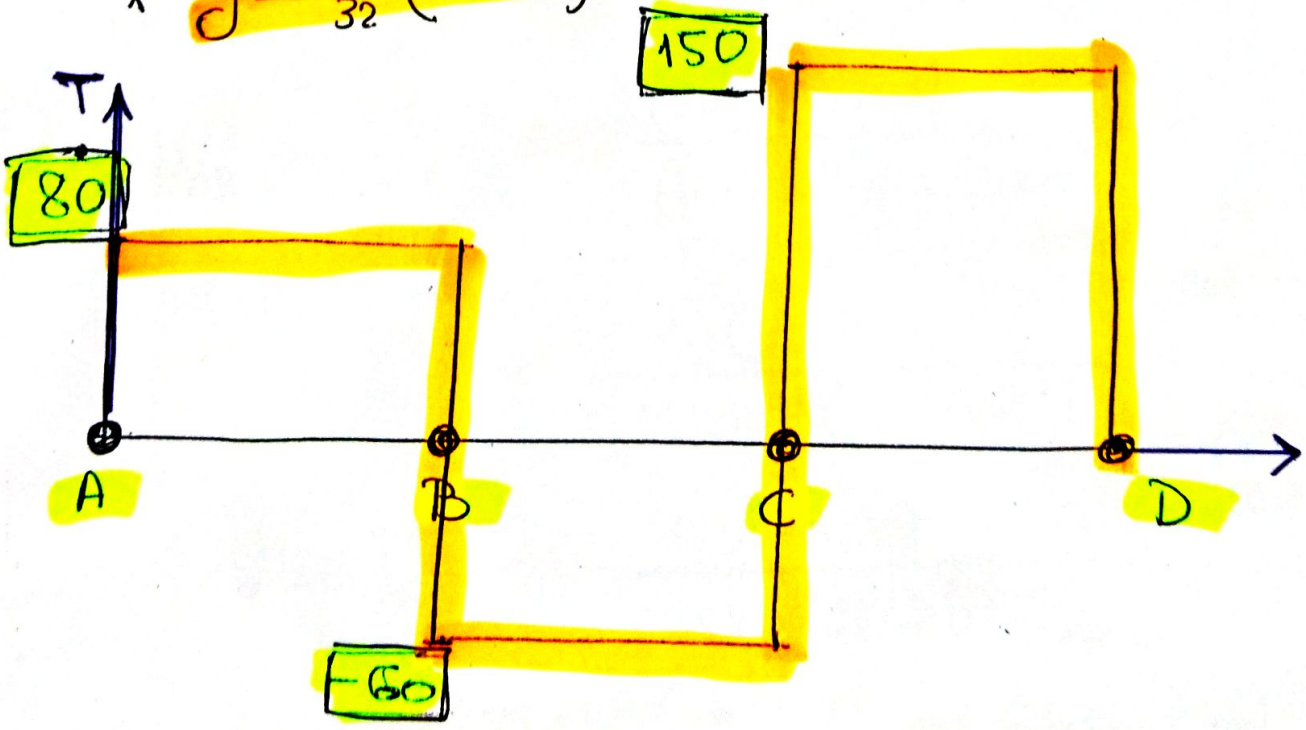
Find (1) The max. shear stress τ_{max} ?

(2) The Total angle of Twist ϕ_{AD} ?
زاوية الالتواء الكلية

(3) The angle of Twist between BD $\Rightarrow$ ϕ_{BD} ?

---

* $J = \frac{\pi}{32}(0.040)^4 = 2.513 \times 10^{-7}\ m^4$



Torque diagram.

① $\tau_{AB} = \frac{Tr}{J} = \frac{(80)(0.020)}{2.513 \times 10^{-7}} = 6.37$ MPa

$\tau_{BC} = \frac{Tr}{J} = \frac{(-60)(0.020)}{2.513 \times 10^{-7}} = -4.78$ MPa

$\tau_{CD} = \frac{Tr}{J} = \frac{(150)(0.020)}{2.513 \times 10^{-7}} = 11.94$ MPa

أخذ , الأكبر كقيمة مطلقة

$\tau_{max} = 11.94$ MPa

② $\phi_{AB} = \frac{TL}{GJ} = \frac{(80)(0.70)}{(85 \times 10^{9})(2.513 \times 10^{-7})} = 2.6 \times 10^{-3}$ rad

$\phi_{BC} = \frac{TL}{GJ} = \frac{(-60)(0.80)}{(85 \times 10^{9})(2.513 \times 10^{-7})} = 2.25 \times 10^{-3}$ rad

$\phi_{CD} = \frac{TL}{GJ} = \frac{(150)(0.90)}{(85 \times 10^{9})(2.513 \times 10^{-7})} = 6.32 \times 10^{-3}$ rad

Total $\phi_{AD} = 2.6 \times 10^{-3} - 2.25 \times 10^{-3} + 6.32 \times 10^{-3} = 6.67 \times 10^{-3}$ rad

(3) $\phi_{BD} = \phi_{BC} + \phi_{CD}$

$= -2.25 \times 10^{-3} + 6.32 \times 10^{-3}$

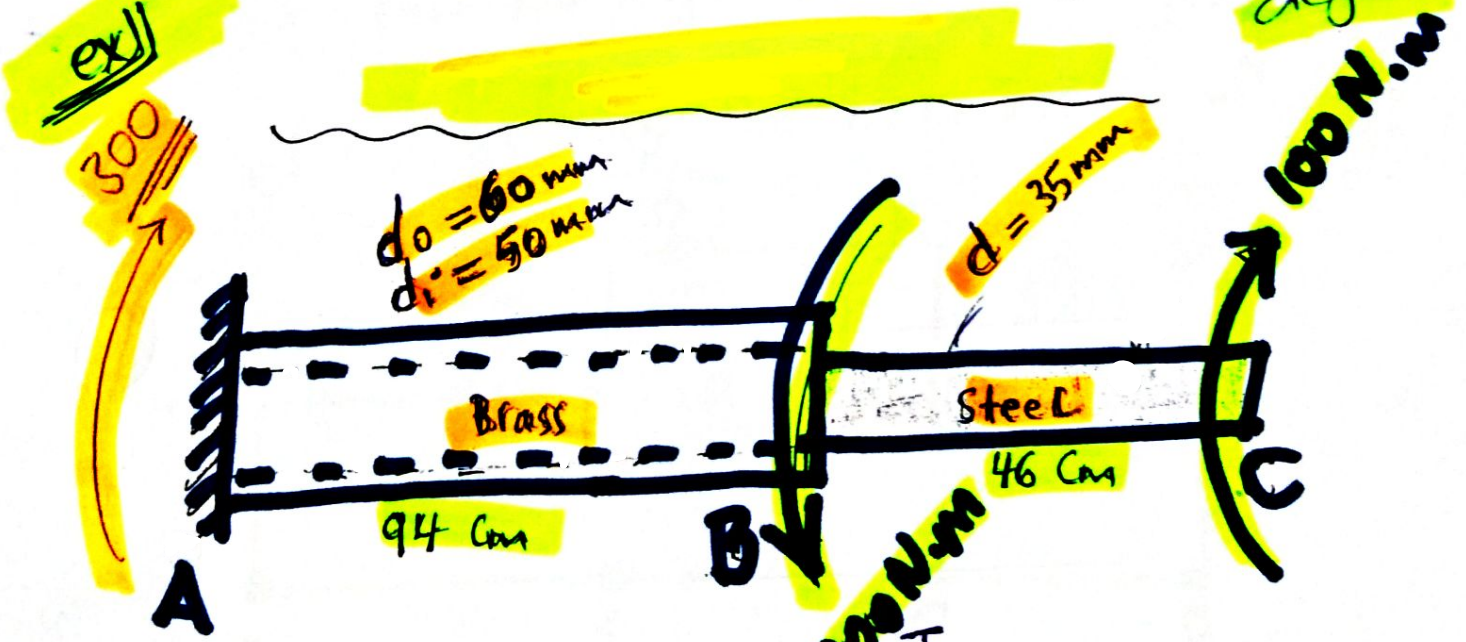
$= 4.07 \times 10^{-3}$ rad.

$= 4.07 \times 10^{-3} \frac{180}{\pi}$

$= 0.233^{\circ}$

گویل از درجه degree

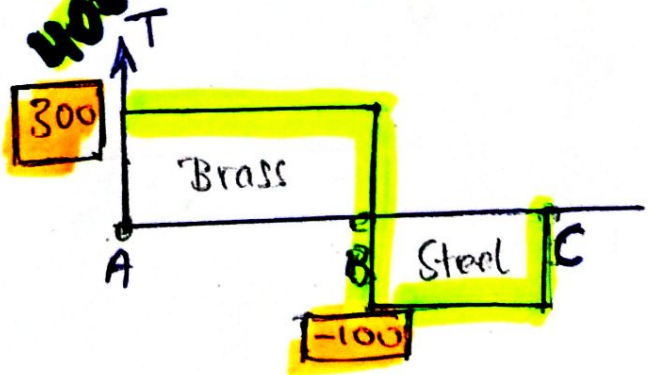
ex



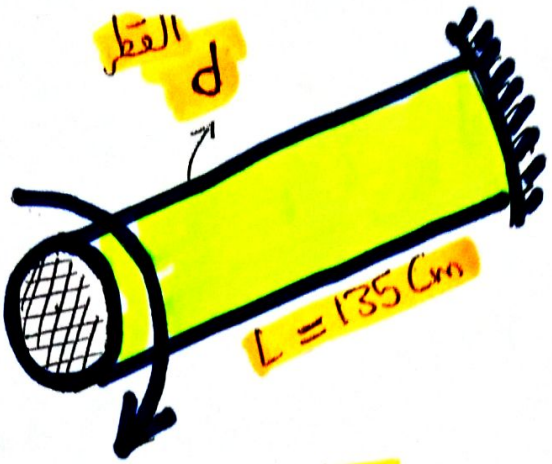
Solve alone

$G_{Brass} = 45$ GPa

$G_{steel} = 70$ GPa



Ex1



$\tau_{max} = 16$ MPa

$\phi_{max} = 1.25^\circ$

$G = 65$ GPa

Find The Shaft diameter d ?

$$* \quad \tau = \frac{Tr}{J} \Rightarrow 16 \times 10^6 = \frac{(320)\frac{d}{2}}{\frac{\pi}{32} d^3}$$

$$\Rightarrow 16 \times 10^6 = \frac{160}{\frac{\pi}{32} d^3}$$

$$\Rightarrow \text{Solve} \Rightarrow d = 0.0467 \text{ m}$$

$$d = 46.7 \text{ mm}$$

$$* \quad \phi = \frac{TL}{GJ} \Rightarrow 1.25^\circ * \frac{\pi}{180} = \frac{(320)(1.35)}{(65 \times 10^9)(\frac{\pi}{32}) d^4}$$

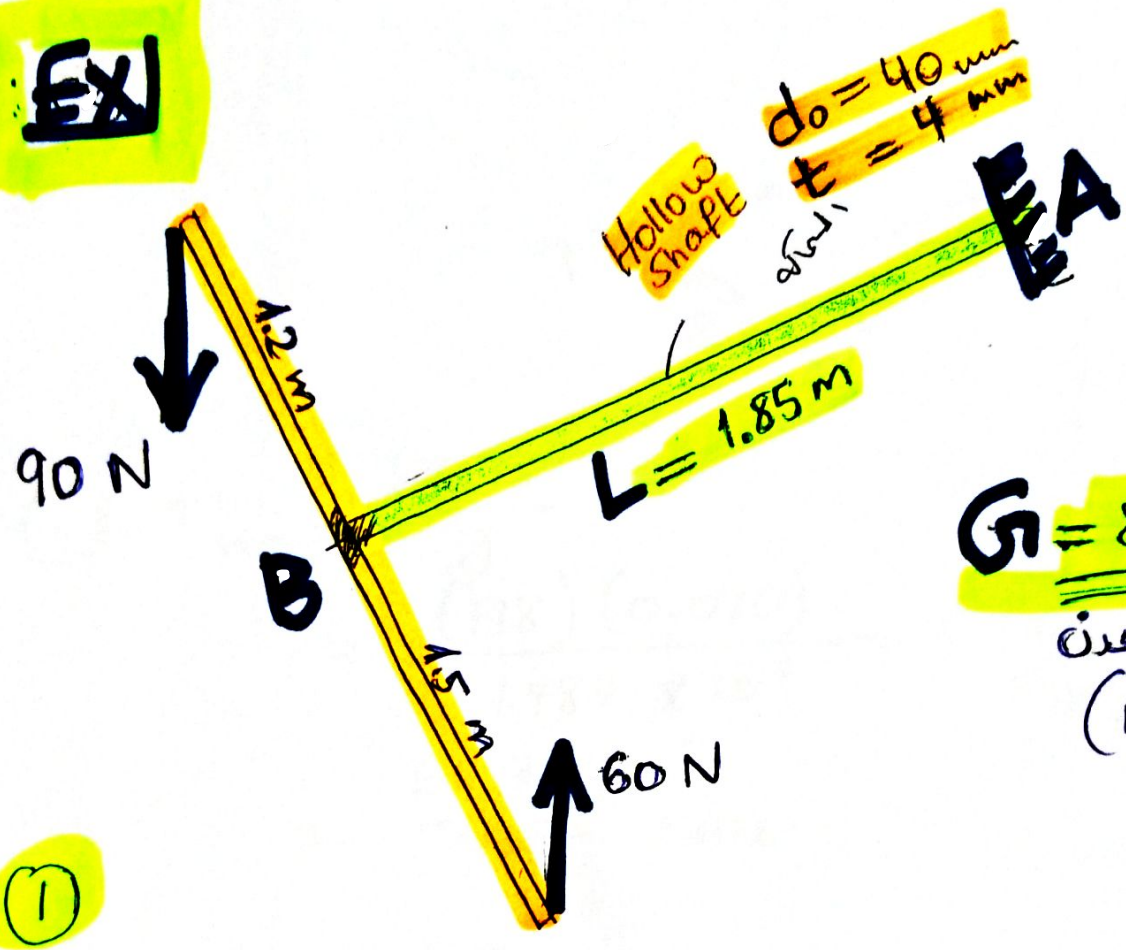
$$\Rightarrow \text{Solve} \Rightarrow d = 0.0420 \text{ m}$$

$$d = 42 \text{ mm}$$

For Safety Select $d = 46.7$ mm

Select The largest diameter

Ex



$G = 80$ GPa

معامل صلابة المادة (Rigidity)

(1) Find The maximum Shear stress in AB $\Rightarrow \tau_{max}$?

(2) Find the maximum Angle of Twist AB $\Rightarrow \phi_{max}$?

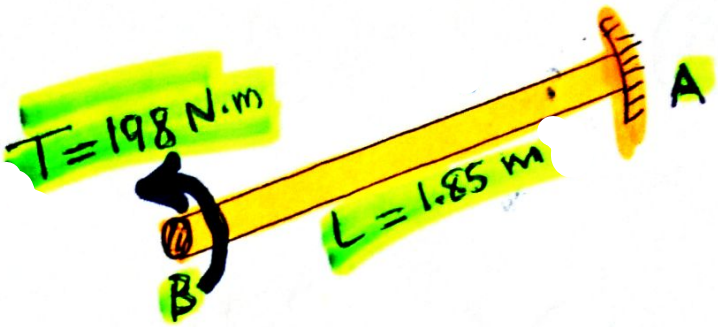
(3) Find the maximum shear Strain AB $\Rightarrow \gamma_{AB}$?

---

* Torque $T = (60)(1.5) + (90)(1.2) = 198$ N.m

* $d_i = d_o - 2t \Rightarrow d_i = 40 - 2(4) = 32$ mm

* $J = \frac{\pi}{32}(0.040^4 - 0.032^4) = 1.484 * 10^{-7}$ m^4



① $\tau_{max} = \dfrac{T r_o}{J}$

$= \dfrac{(198)(0.020)}{1.484 \times 10^{-7}}$

ENG

$= 26.7$ MPa

② $\phi_{max} = \dfrac{TL}{GJ}$

$= \dfrac{(198)(1.85)}{(80 \times 10^9)(1.484 \times 10^{-7})}$

$= 0.031$ rad

$= 0.031 \dfrac{180^\circ}{\pi} = 1.77^\circ$

③ $\tau = G\gamma \Rightarrow \gamma = \dfrac{\tau}{G}$

$\Rightarrow \gamma = \dfrac{26.7 \times 10^6}{80 \times 10^9} = 3.338 \times 10^{-4}$ rad.

Power Transmission

Power $\Rightarrow$ $P = T\omega$

$P \equiv$ Watt

$T \equiv$ N.m

$\omega \equiv$ rad/s

$\omega = 2\pi f$ $\quad f \equiv$ frequency (HZ)

$\omega = \frac{2\pi}{60} n$ $\quad n \equiv$ rpm

$\tau = \frac{Tr}{J}$ $\quad \phi = \frac{TL}{GJ}$ $\quad r\phi = \gamma L$

* $J = \frac{\pi}{2} r^4 = \frac{\pi}{32} d^4 \rightarrow$ Solid Shaft
* $J = \frac{\pi}{2}(r_o^4 - r_i^4) = \frac{\pi}{32}(d_o^4 - d_i^4) \rightarrow$ Hollow

Ex Hollow shaft transmit power 60 kW,

$d_o = 45$ mm, $t = 8$ mm, $G = 80$ GPa

$L = 2$ m, Find (1) τ_{max} (2) ϕ_{max}.

$n = 800$ rpm

* $P = T\omega \Rightarrow \omega = \frac{2\pi}{60}(800)$

$\omega = 83.8$ rad/s

* $T = \frac{60 \times 10^3}{83.8} \Rightarrow T = 716$ N.m

$d_i = d_o - 2t$

$d_i = 29$ mm

$J = \frac{\pi}{32}(d_o^4 - d_i^4)$

$J = 333 \times 10^3$ mm^4

* (1) $\tau_{max} = \frac{(716)(22.5 \times 10^{-3})}{333 \times 10^3 \times 10^{-12}} = 48$ MPa
* (2) $\phi_{max} = \frac{(716)(2)}{(80 \times 10^9)(333 \times 10^3 * 10^{-12})} = 0.0538$ rad $= 3.08°$

EX Find shaft diameter?

(power) $P = 8\ KW$

التردد frequency $f = 30\ HZ$

$\tau_{max} = 55000\ KN/m^2 = 55000\ KPa$

$= 55\ MPa$

* $\omega = 2\pi f$

$= 2\pi(30) = 188.5\ rad/s.$ السرعة الزاوية ω

* $P = T\omega$

$8*10^3 = T(188.5)$

$\Rightarrow T = 42.44\ N.m$

* $\tau = \frac{Tr}{J} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3}$

$\Rightarrow 55*10^6 = \frac{(2)(42.44)}{\pi r^3}$ $\Rightarrow d = 2r$

$\Rightarrow r = 7.89\ mm \Rightarrow d = 15.8\ mm$

Ex]

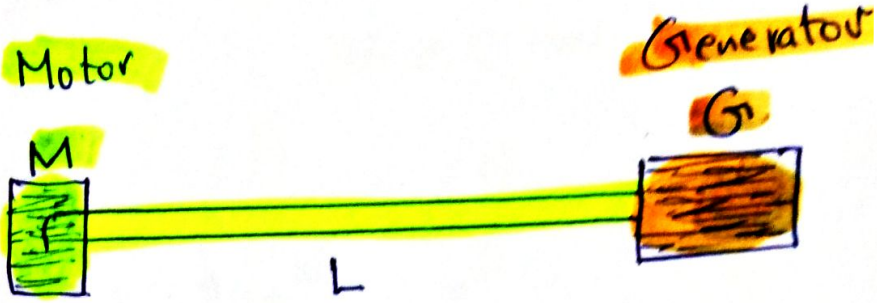
Hollow shaft

$L = 2\ m$

$d_o = 40\ mm$

when $\omega = 80\ rad/s$

$\Rightarrow$ power $P = 32\ KW$



$\tau_{all} = 140\ MPa$

$\phi_{max} = 0.05\ rad$

$G = 75\ GPa$

determine the smallest thickness (t) of the shaft ?

* $P = T\omega \Rightarrow 32 \times 10^3 = T(80)$

$\Rightarrow T = 400\ N.m$

* $\tau_{all} = \dfrac{Tr_o}{J} \Rightarrow 140 \times 10^6 = \dfrac{(400)\left(\frac{40}{2}\right)\times 10^{-3}}{\frac{\pi}{2}\left[0.02^4 - r_i^4\right]}$

$\Rightarrow r_i = 18.75\ mm$

* $\phi_{max} = \dfrac{TL}{GJ} \Rightarrow 0.05 = \dfrac{(400)(2)}{(75\times 10^9)\frac{\pi}{2}\left[0.02^4 - r_i^4\right]}$

$\Rightarrow r_i = 12.47\ mm$

* For Safety select the smallest inner diameter (or radius):

$\Rightarrow r_i = 12.47\ mm$

$\Rightarrow$ The Thickness is then:

$t = \frac{1}{2}(d_o - d_i) = r_o - r_i$

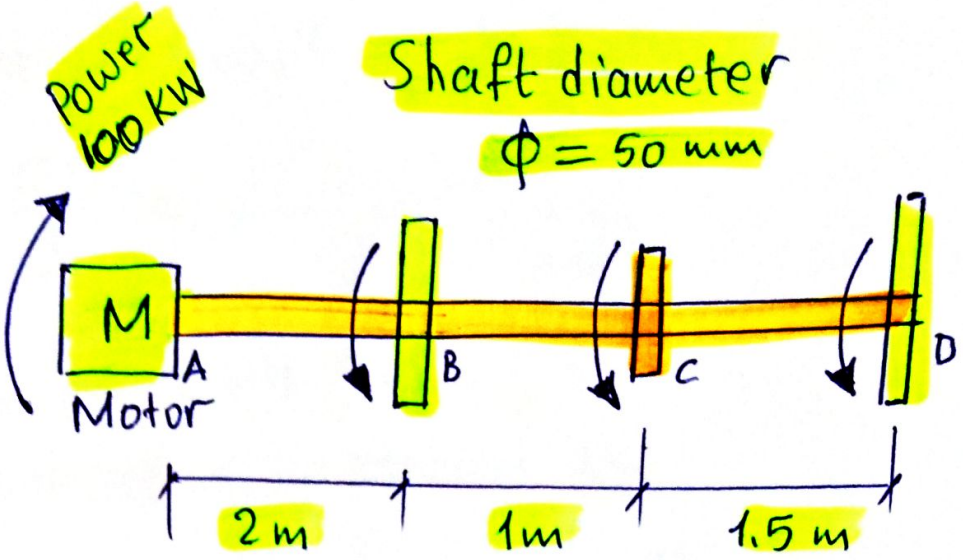
$t = (20 - 12.47)$

$t = 7.53\ mm$

Ex

Motor transmits Power of 100 KW to a shaft at 10 Hz. The gears B, C, and D drive machinery requiring power in a ratio of 1:3:4 respectively.

use $G = 80\ GPa$



Find (1) Max. Shear Stress τ_{max} ?

(2) Total angle of Twist ϕ_{AD} ?

(3) Shear Strain in portion BC $\Rightarrow \gamma_{BC}$?

---

* $\omega = 2\pi f = 2\pi(10)$

$\omega = 62.8$ rad/s

* $P = T\omega$

$100 \times 10^3 = T(62.8)$

$\Rightarrow T = 1592$ N.m

B : C : D

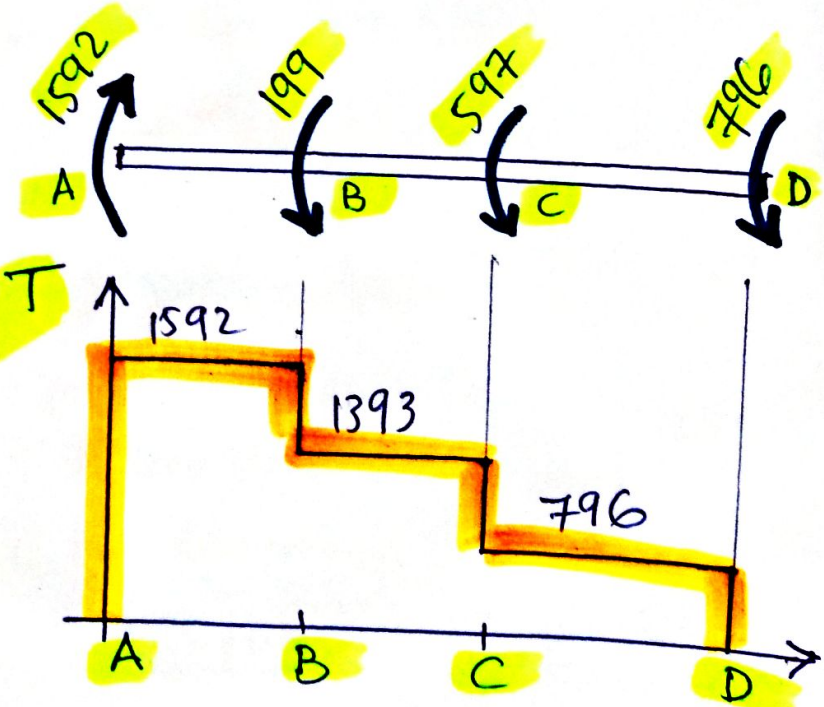
1 : 3 : 4

$\frac{1}{8}$: $\frac{3}{8}$: $\frac{4}{8}$

$\Rightarrow T_B = \frac{1}{8}(1592) = 199$ N.m

$\Rightarrow T_C = \frac{3}{8}(1592) = 597$ N.m

$\Rightarrow T_D = \frac{4}{8}(1592) = 796$ N.m



Torque diagram

* $J = \frac{\pi}{32} d^4 = \frac{\pi}{32} (50)^4 = 613 \times 10^3 \text{ mm}^4$

* $\tau_{max} = \frac{Tr}{J} = \frac{(1592)\left(\frac{50}{2} \times 10^{-3}\right)}{613 \times 10^3 \times 10^{-12}}$

(1)

$\tau_{max} = 64.9 \text{ MPa}$

This occurs in portion AB

(2) $\phi_{AB} = \frac{T_{AB} L_{AB}}{GJ} = 0.0649 \text{ rad.}$

$\phi_{BC} = \frac{T_{BC} L_{BC}}{GJ} = 0.0284 \text{ rad.}$

$\phi_{CD} = \frac{T_{CD} L_{CD}}{GJ} = 0.0243 \text{ rad.}$

$\phi_{AD} = \phi_{AB} + \phi_{BC} + \phi_{CD} = 0.1176 \text{ rad.}$

$= 6.74°$

(3) $r\phi = \gamma L \Rightarrow$ For portion BC

$\Rightarrow r = 25 \text{ mm}$

$\Rightarrow \phi = 0.0284 \text{ rad.}$

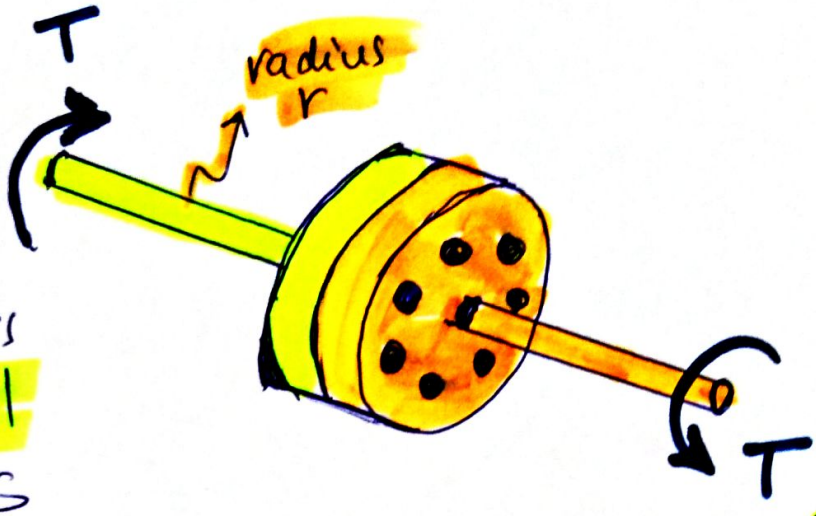
$\Rightarrow L = 1 \text{ m}$

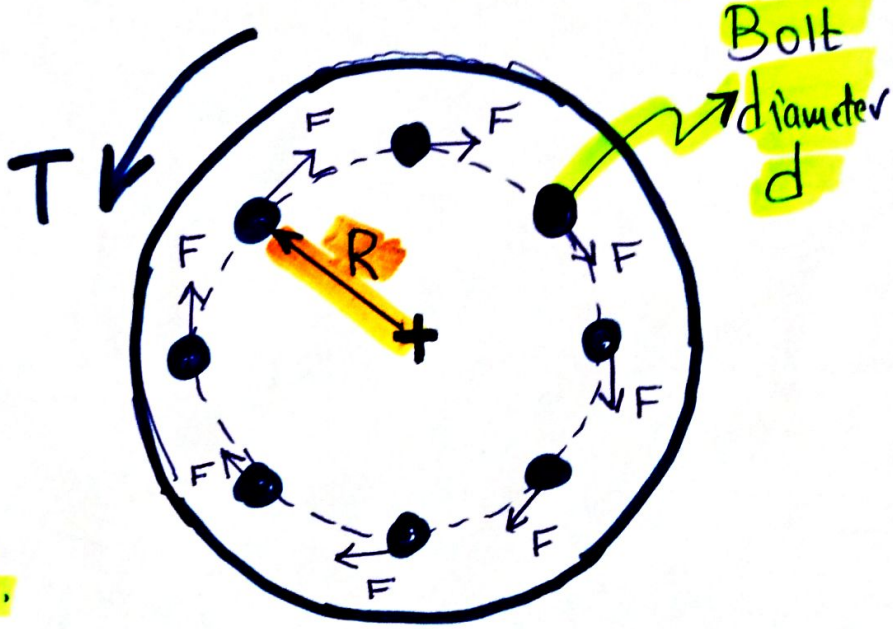
$\Rightarrow \gamma = \frac{r\phi}{L} = \frac{(25 \times 10^{-3})(0.0284)}{1}$

$\Rightarrow \gamma_{BC} = 7.10 \times 10^{-4} \text{ rad.}$

Ex1

Find the number of Bolts (n) necessary to make the max. shear stress in the shaft equal to the shear stress in the Bolts ?





$R \equiv$ Radius of Bolt's Circle.

$d \equiv$ Bolt diameter

$r \equiv$ Shaft radius

$n \equiv$ Number of Bolts.

$$* \quad T = n(FR) \Rightarrow F = \frac{T}{nR}$$

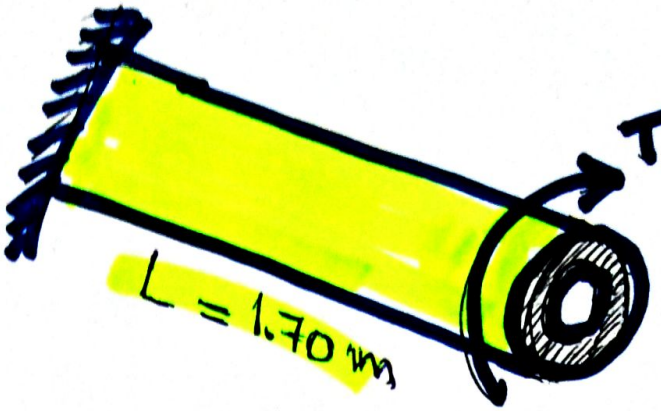
where **F** is the shear force on one Bolt.

$$* \quad \tau_{av} = \frac{\text{Force}}{\text{Area}} = \frac{T/nR}{\frac{\pi}{4}d^2} = \frac{4T}{nR\pi d^2}$$

$$* \quad \tau_{max} = \frac{Tr}{J} = \frac{Tr}{\frac{\pi}{2}r^2} = \frac{2T}{\pi r^3}$$

$$* \quad \text{But } \tau_{av} = \tau_{max} \Rightarrow n = \frac{2r^3}{Rd^2}$$

Ex



$d_o = 40\ mm$

$d_i = 28\ mm$

$G = 75\ GPa$

الصلب
steel

Find:

1. Max. shear stress τ_{max} ?
2. min shear stress τ_{min} ?
3. Angle of Twist ϕ ?
4. Find the power if the speed is 240 rpm ?

* $J = \frac{\pi}{32}\left[0.040^4 - 0.028^4\right] = 1.91 \times 10^{-7}\ m^4$

(1) $\tau_{max} = \frac{T r_o}{J} = \frac{(90)(0.020)}{1.91 \times 10^{-7}}$

$= 9.42\ MPa$

(2) $\tau_{min} = \frac{T r_i}{J} = \frac{(90)(0.014)}{1.91 \times 10^{-7}}$

$= 6.60\ MPa$

(3) $\phi = \frac{TL}{GJ} = \frac{(90)(1.70)}{(75 \times 10^9)(1.91 \times 10^{-7})}$

$= 0.01068$ radian

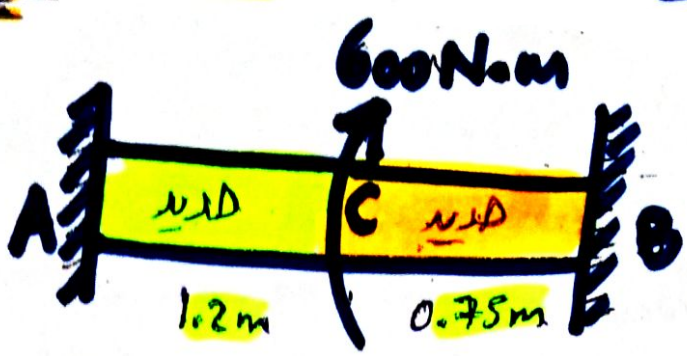
$= 0.01068 \left(\frac{180^\circ}{\pi}\right) = 0.612^\circ$

Ind. Torsion

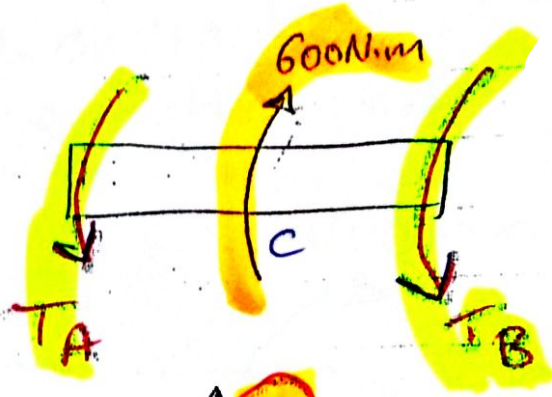
Indeterminate Torsion

$G_1 = G_2$ نفس المادة

$J_1 = J_2$ نفس المقطع



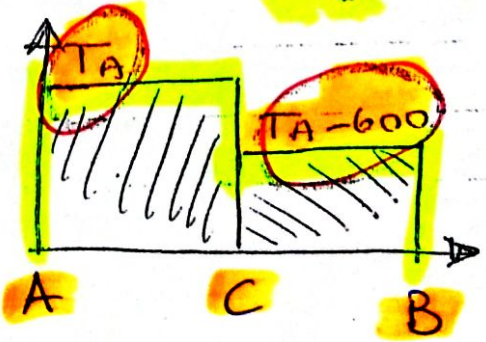
* Find the reactions at A and B ?



بما أن العامود مثبت من الطرفين

$\Leftarrow$ لا يوجد دوران $(\phi = 0)$

$$\Rightarrow \phi_1 + \phi_2 = 0$$



$$\Rightarrow \frac{T_1 L_1}{G_1 J_1} + \frac{T_2 L_2}{G_2 J_2} = 0$$

$$T_A(1.2) + (T_A - 600)(0.75) = 0$$

$$T_A = 230.8 \text{ N.m}$$

من المعادلة $T_A + T_B = 600 \Rightarrow T_B = 369.2 \text{ N.m}$

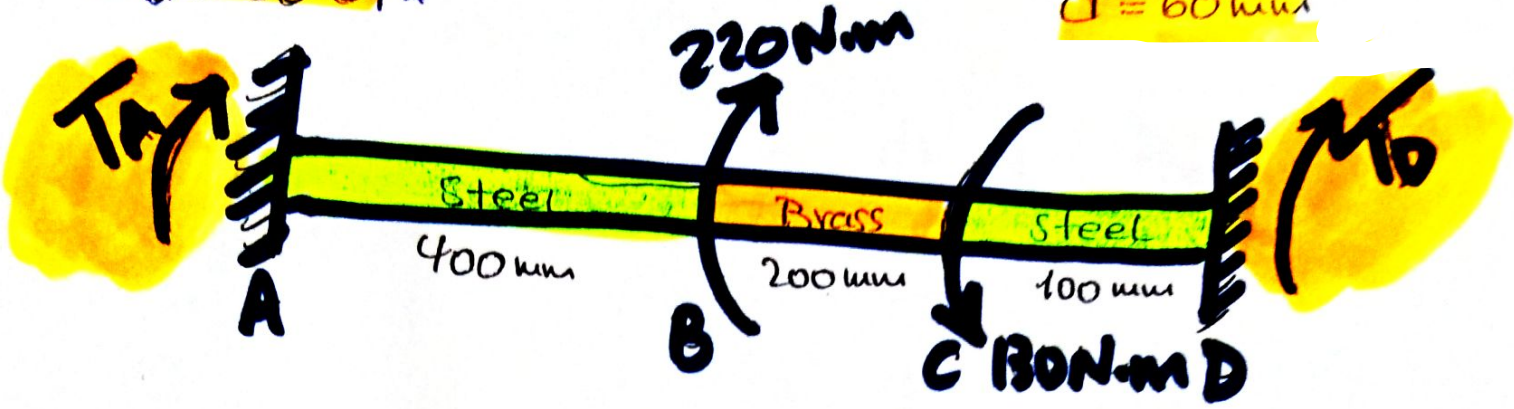
Ex1 Indeterminate Torsion

$G_{St} = 70\ GPa$ Indeterminate Torsion

$G_{Br} = 50\ GPa$

الدائرة ثابتة ⇐ مثبت من طرفين

$d = 60\ mm$

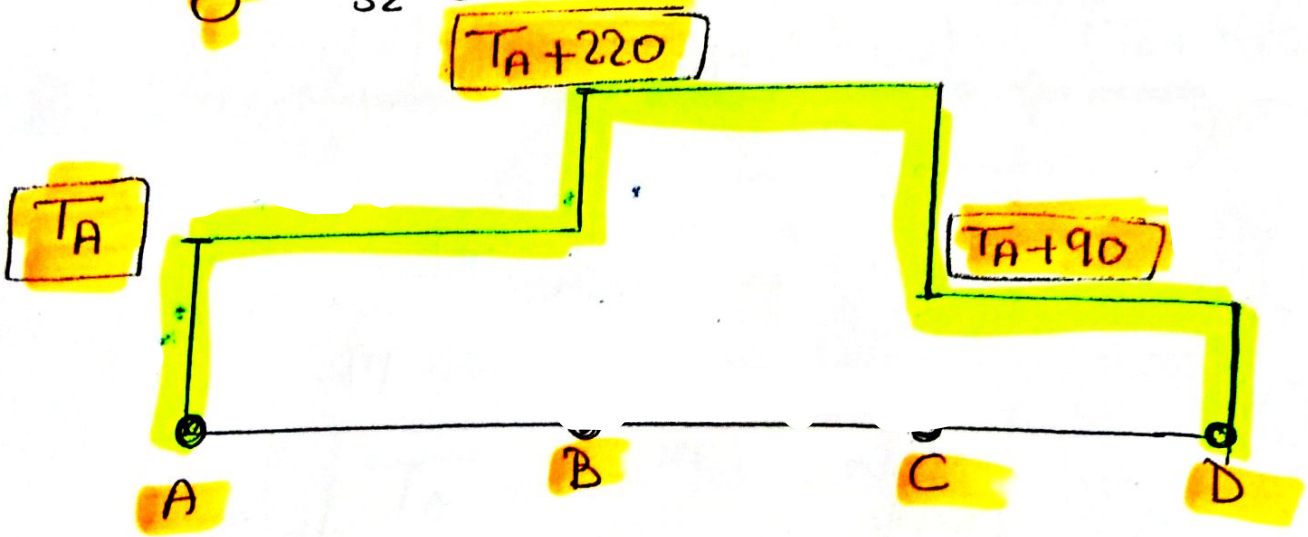


Find: ① The Reactions at A and D ? ردود الأفعال T_A T_D

② Find The shear stress in Brass ⇒ τ_{BC} ?

③ Find The Angle of Twist of AB ⇒ ϕ_{AB}

* $J = \frac{\pi}{32}(0.060)^4 = 1.272 \times 10^{-6}\ m^4$



"Torque Diagram"

(لا يوجد دوران) ← بسبب أنها مثبتة من الطرفين ✓

$$\Rightarrow \quad \phi_{AD} = 0$$

$$\Rightarrow \quad \phi_{AB} + \phi_{BC} + \phi_{CD} = 0$$

$$\frac{T_A\ (0.4)}{(70\times10^9)(1.272\times10^{-6})} + \frac{(T_A+220)\ (0.2)}{(50\times10^9)(1.272\times10^{-6})}$$

$$+ \frac{(T_A+90)\ (0.1)}{(70\times10^9)(1.272\times10^{-6})} = 0$$

$$\Rightarrow \frac{(T_A)(0.4)}{70} + \frac{(T_A+220)(0.2)}{50} + \frac{(T_A+90)(0.1)}{70} = 0$$

Solve

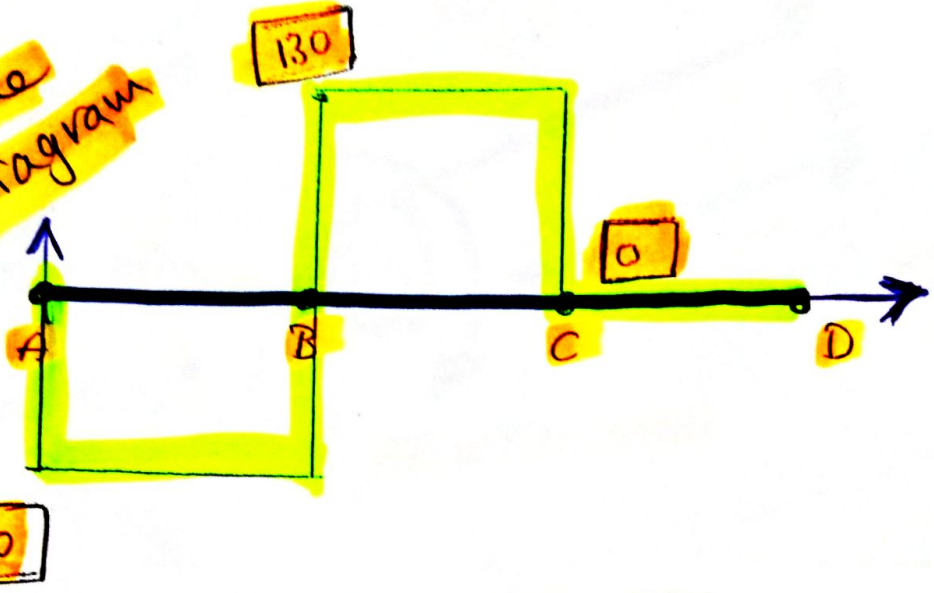
$$\Rightarrow \boxed{T_A = -90 \ \text{N.m}}$$

$$T_D = 90 + 130 - 220 = 0$$

$$\boxed{T_D = 0}$$

After find the right reactions T_A & T_B:

Redraw the Torque diagram



(2)

$$\tau_{BC} = \frac{Tr}{J} = \frac{(130)(0.030)}{1.272 \times 10^{-6}}$$

$$= 3.07 \text{ MPa}$$

(3)

$$\phi_{AB} = \frac{TL}{GJ}$$

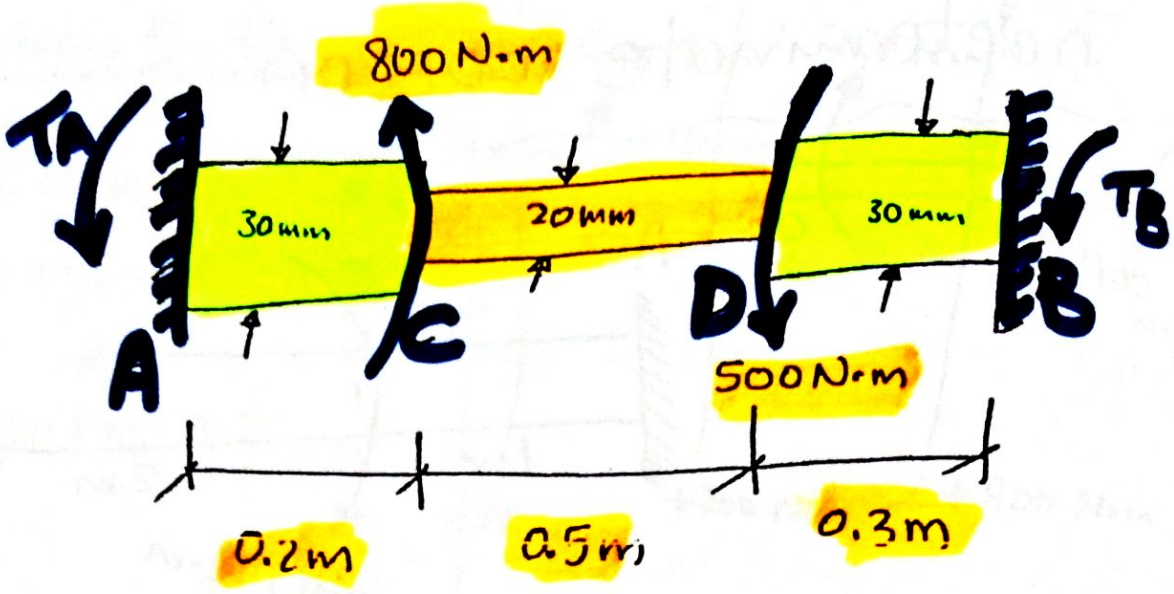
$$= \frac{(-90)(0.4)}{(70 \times 10^{9})(1.272 \times 10^{-6})}$$

$$= -4.043 \times 10^{-4} \text{ rad}$$

$$= -4.043 \times 10^{-4} \frac{180}{\pi} = -0.0232^{0}$$

in degree

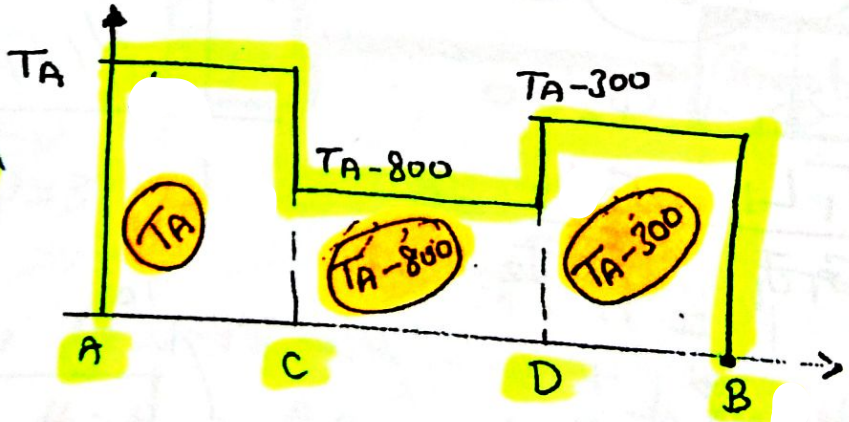
Ex

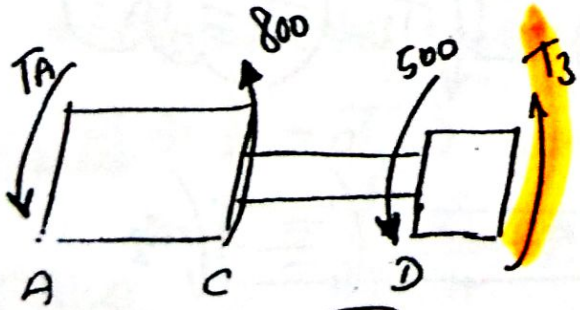


From steel $G = 70\ GPa$.

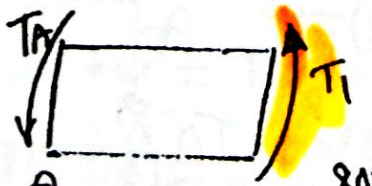
Find
1. The reactions at A & B ?
2. The max. shear stress in CD ?
3. The max shear strain in CD ?

Torque diagram

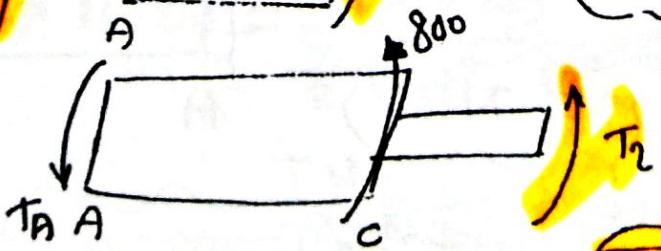




$T_3 = T_A - 300$



$T_1 = T_A$



$T_2 = T_A - 800$

① Compatability:

$\Phi = 0$

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 = 0$$

$$\Rightarrow \frac{T_1 L_1}{G_1 J_1} + \frac{T_2 L_2}{G_2 J_2} + \frac{T_3 L_3}{G_3 J_3} = 0$$

$$\Rightarrow \frac{T_A (0.2)}{\frac{\pi}{2}(15)^4 \times 10^{-12}} + \frac{(T_A - 800)(0.5)}{\frac{\pi}{2}(10)^4 \times 10^{-12}} + \frac{(T_A - 300)(0.3)}{\frac{\pi}{2}(15)^4 \times 10^{-12}} = 0$$

$\Rightarrow$ solve

$* \Rightarrow$ $T_A = 698.\ N.m$

* توازن $\sum T = 0 \Rightarrow T_A - 800 + 500 + T_B = 0$

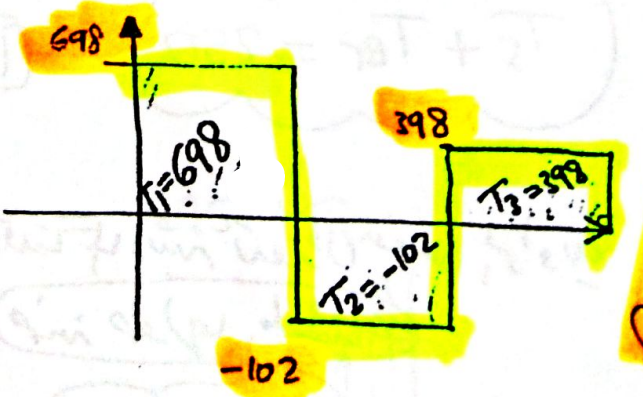
$$\Rightarrow 698 - 800 + 500 + T_B = 0$$

$\Rightarrow$ عكس الاتجاه لتوازن $T_B = -398\ N.m$

② $\tau = \frac{Tr}{J}$

$$\tau_{CD} = \frac{(-102)(10 \times 10^{-3})}{\frac{\pi}{2}(10)^4 \times 10^{-12}}$$

$\Rightarrow \tau_{CD} = -65\ MPa.$



Redraw again

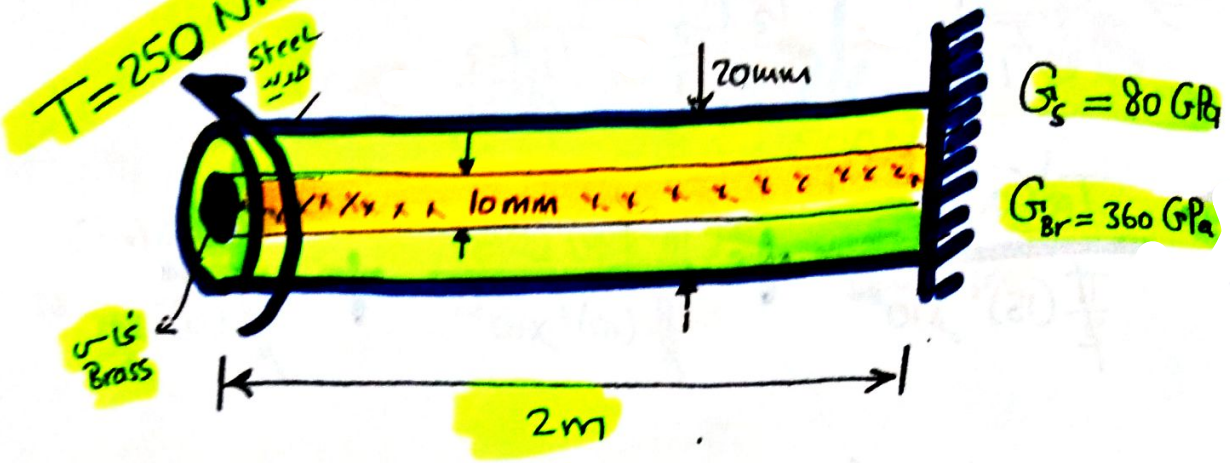
③ $\tau = G\gamma \Rightarrow \gamma_{CD} = \frac{\tau}{G} = \frac{-65 \times 10^6}{70 \times 10^9}$

$\gamma_{CD} = -9.3 \times 10^{-4}\ rad.$

Indet. Composite Materials

المواد المركبة
Composite Materials

عمود مركب بداخل بعضه

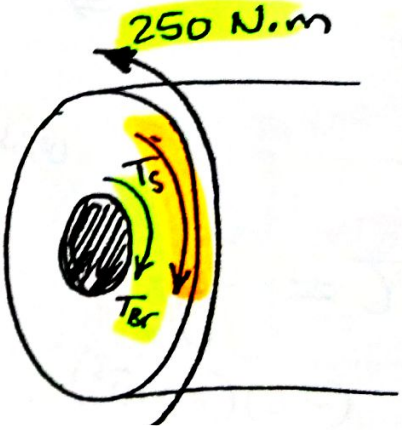


(1) Find the Torque on Steel and Brass (T_S, T_{Br})

(2) Find the shear stress (sketch) (ارسم!)

(3) Find the total Angle of Twist

(1)



من الاتزان

$$T_S + T_{Br} = 250 \quad \text{---(1)}$$

بكتبا بالنسبة لدوران نفس الزاوية
(نفس طول العمود، ملتصقين)

$$\Rightarrow \quad \phi_S = \phi_{Br}$$

$$\Rightarrow \quad \frac{T_S L_S}{G_S J_S} = \frac{T_{Br} L_{Br}}{G_{Br} J_{Br}}$$

* Note: $J_{Br} = \frac{\pi}{2} r_{Br}^4$ → Brass

$J_s = \frac{\pi}{2}\left(r_s^4 - r_{Br}^4\right)$ → Steel

$$\frac{T_s\,(2)}{[80\times10^9]\left[\frac{\pi}{2}(10)^4-(5)^4\right]\times10^{-12}} = \frac{(250-T_s)\,(2)}{(360\times10^9)\left[\frac{\pi}{2}(5)^4\right]\times10^{-12}}$$

$$\Rightarrow \frac{T_s}{80\,(10^4-5^4)} = \frac{250-T_s}{360\,(5)^4}$$

$$\Rightarrow T_s = 192 \text{ N.m}$$

* $T_{Br} = 250 - 192 \Rightarrow T_{Br} = 58$ N.m

Total angle of Twist:

$$\phi_{Total} = \phi_{Br} = \phi_s = \frac{T_s L}{G_s J_s} = \text{---- radians}$$

② Brass

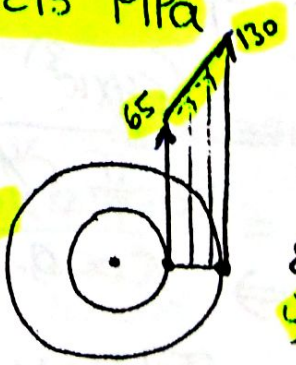
$$\tau = \frac{T_{Br}\, r}{J}$$

$$\tau_{Br} = \frac{(58)(5\times10^{-3})}{\frac{\pi}{2}(5)^4\times10^{-12}} = 295 \text{ MPa}$$

Steel

* $\tau_{min} = \frac{T_s r_i}{J} = \frac{192(5\times10^{-3})}{\frac{\pi}{2}[10^4-5^4]\times10^{-12}} = 65$ MPa

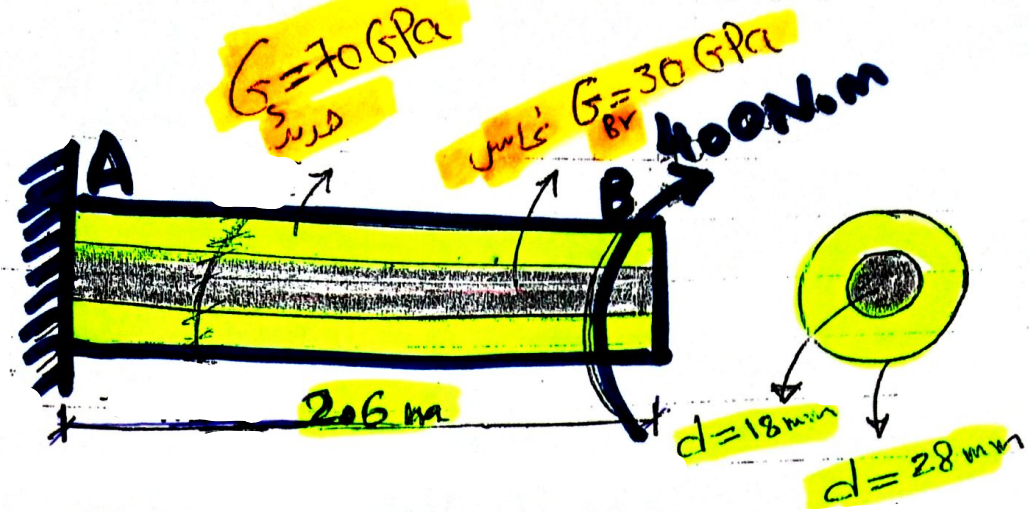
* $\tau_{max} = \frac{T_s r_o}{J} = 130$ MPa



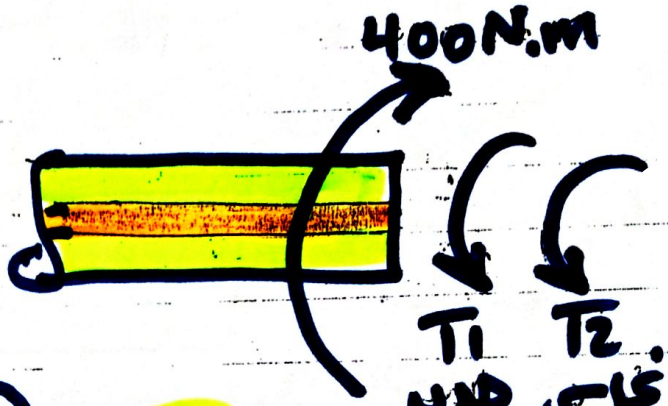
رسم sketch

③

Ex)



Find the Torque on the brass and the Torque on the steel ?



الاتزان
From Equilibrium

$$T_1 + T_2 = 400 \quad -(1)$$

و أيضاً

$$\phi_1 = \phi_2 \quad -(2)$$

$$\frac{T_1 L_1}{G_1 J_1} = \frac{T_2 L_2}{G_2 J_2}$$

$$\Rightarrow \frac{T_1 (2.6)}{(70\times10^9)\left(\frac{\pi}{2}\right)\left((14)^4-(9)^4\right)\times10^{-12}}$$

$$\frac{T_1}{222985} = \frac{T_2}{196830} = \frac{T_2 (2.6)}{(30\times10^9)\frac{\pi}{2}\left[(9)^4\right]\times10^{-12}}$$

$T_1 = 367.6\ N.m$

$T_2 = 32.4\ N.m$