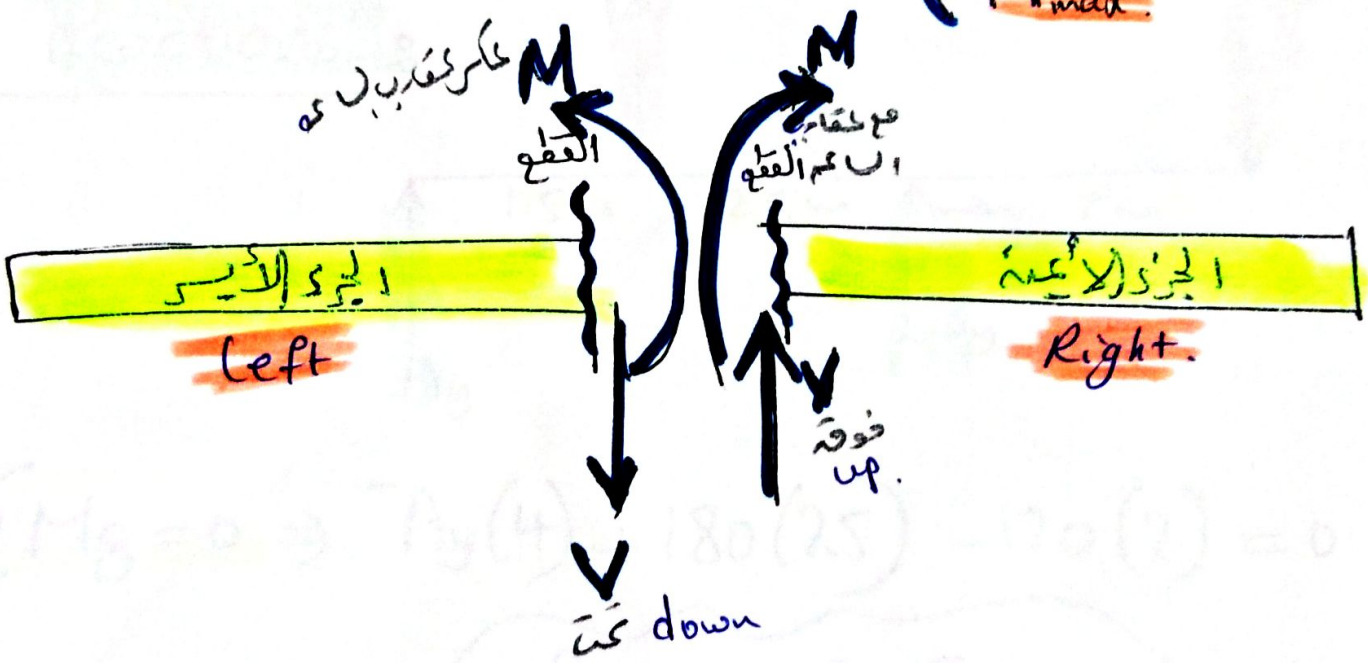
Ch4

Shear force (V) & Bending Moment (M)

قوة القص — عزم الانحناء

لحساب طريقتان

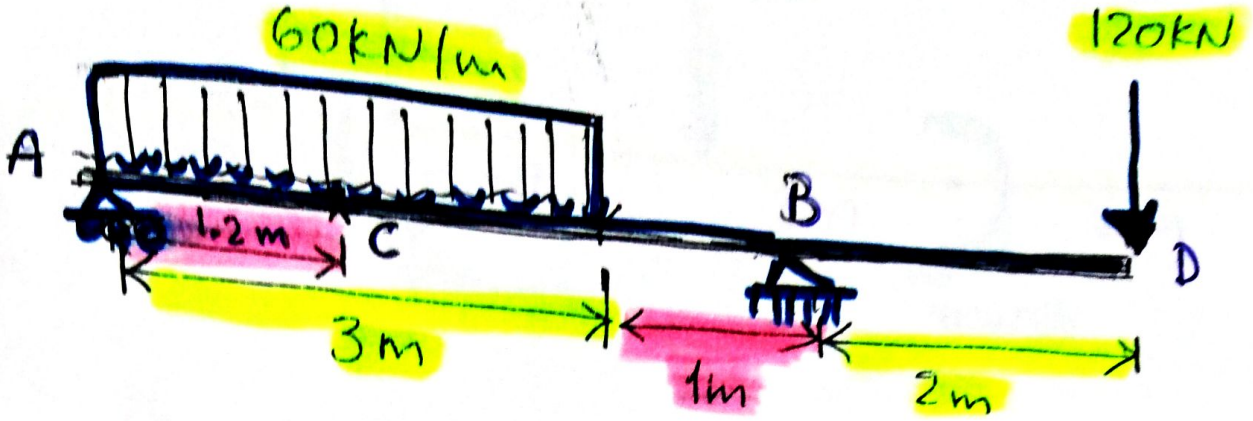
- **Section** — طريقة القطع
  - لإيجاد V و M
  - عند نقطة محددة
- **Drawing** — طريقة الرسم
  - لإيجاد
  - V_{max}
  - M_{max}



then $\sum M = 0$

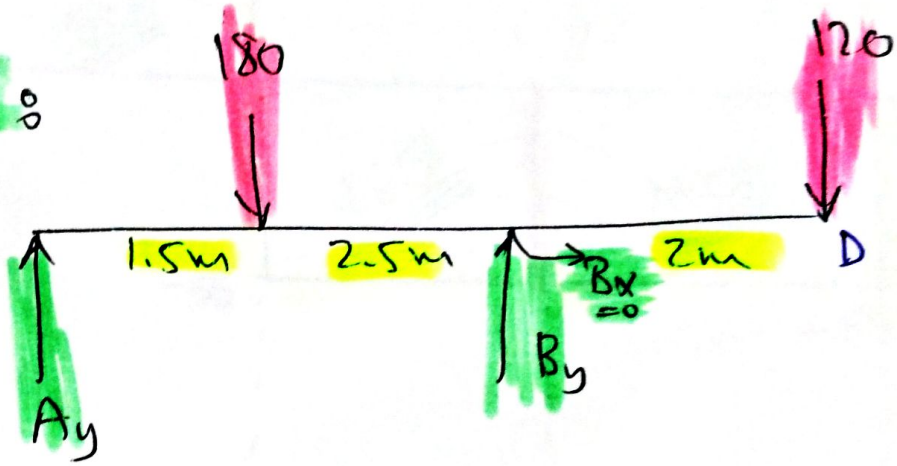
$\sum F_x = 0$

$\sum F_y = 0$



Find the shear force V_c and Bending Moment M_c for point C ?

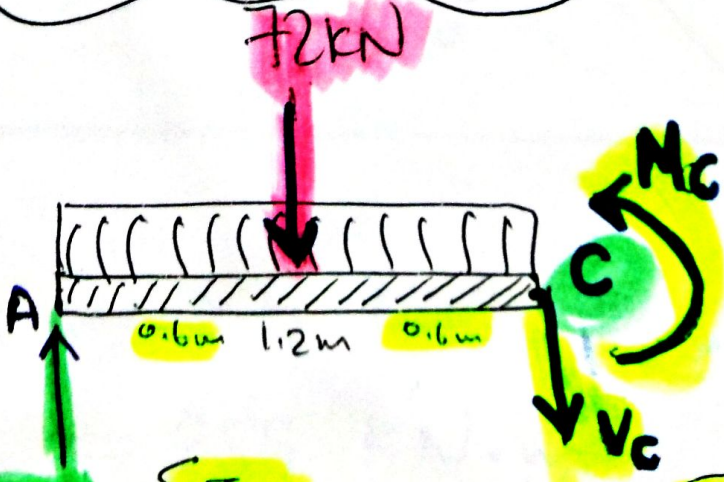
Reactions :



$$\sum M_B = 0 \Rightarrow -Ay(4) + 180(2.5) - 120(2) = 0$$

$$Ay = 52.5 \text{ KN}$$

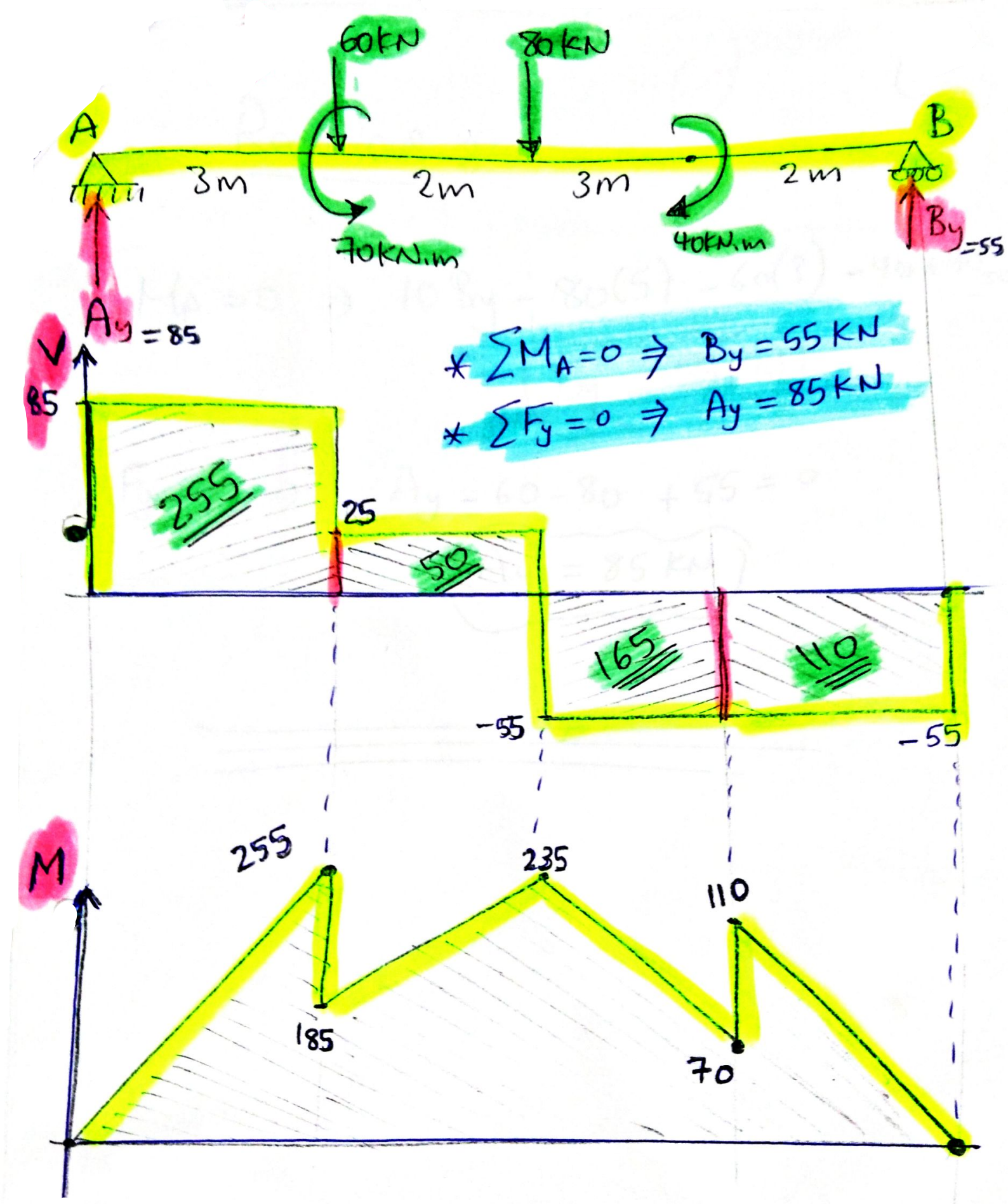
* **Section at C**



$$\sum M_c = 0$$
$$\Rightarrow M_c + 72(0.6) - 52.5(1.2) = 0$$

$$M_c = 19.8 \text{ kN.m}$$

$$\sum F_y = 0 \Rightarrow 52.5 - 72 - V_c = 0 \quad V_c = -19.5$$

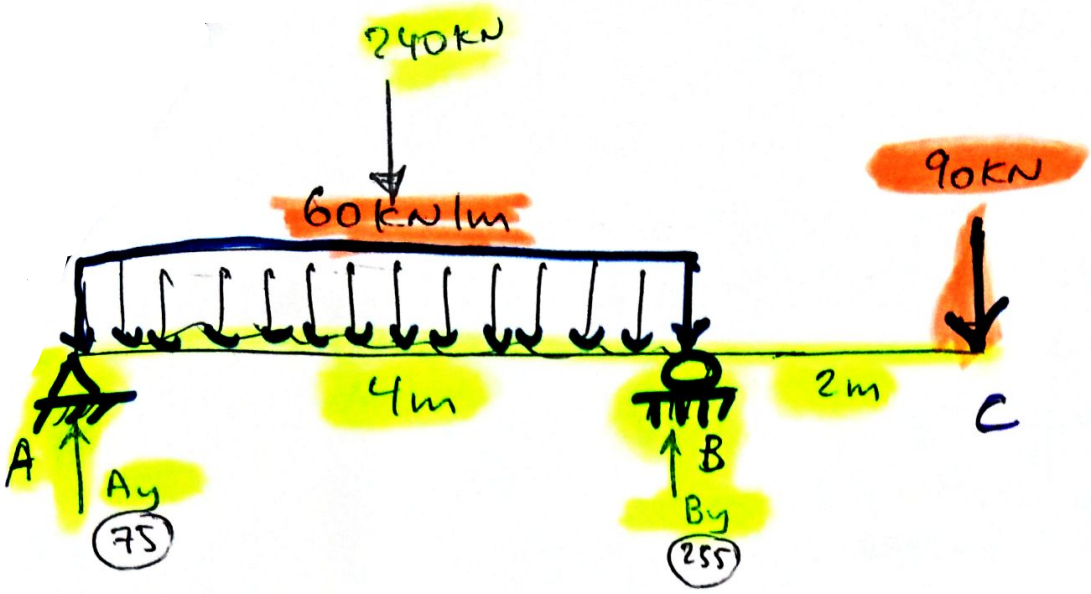


$* \sum M_A = 0 \Rightarrow B_y = 55\ KN$

$* \sum F_y = 0 \Rightarrow A_y = 85\ KN$

أعظم قيمة $V_{max} = 85\ KN$

أعظم قيمة $M_{max} = 255\ KN.m$



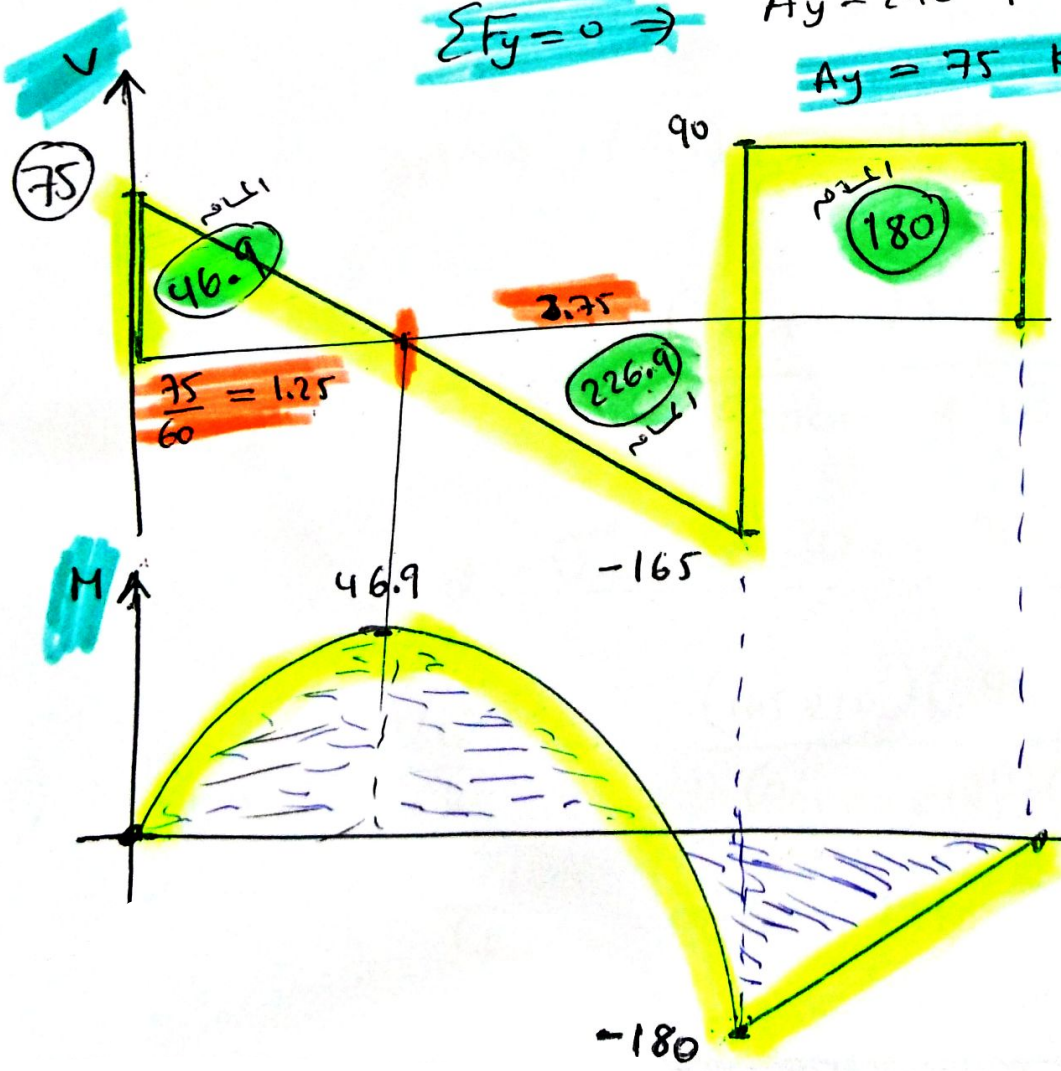
بتطبيق $\sum M_A = 0 \Rightarrow 4B_y - 240(2) - 90(6) = 0$

$B_y = 255\ KN$

$\sum F_x = 0 \Rightarrow A_x = 0$

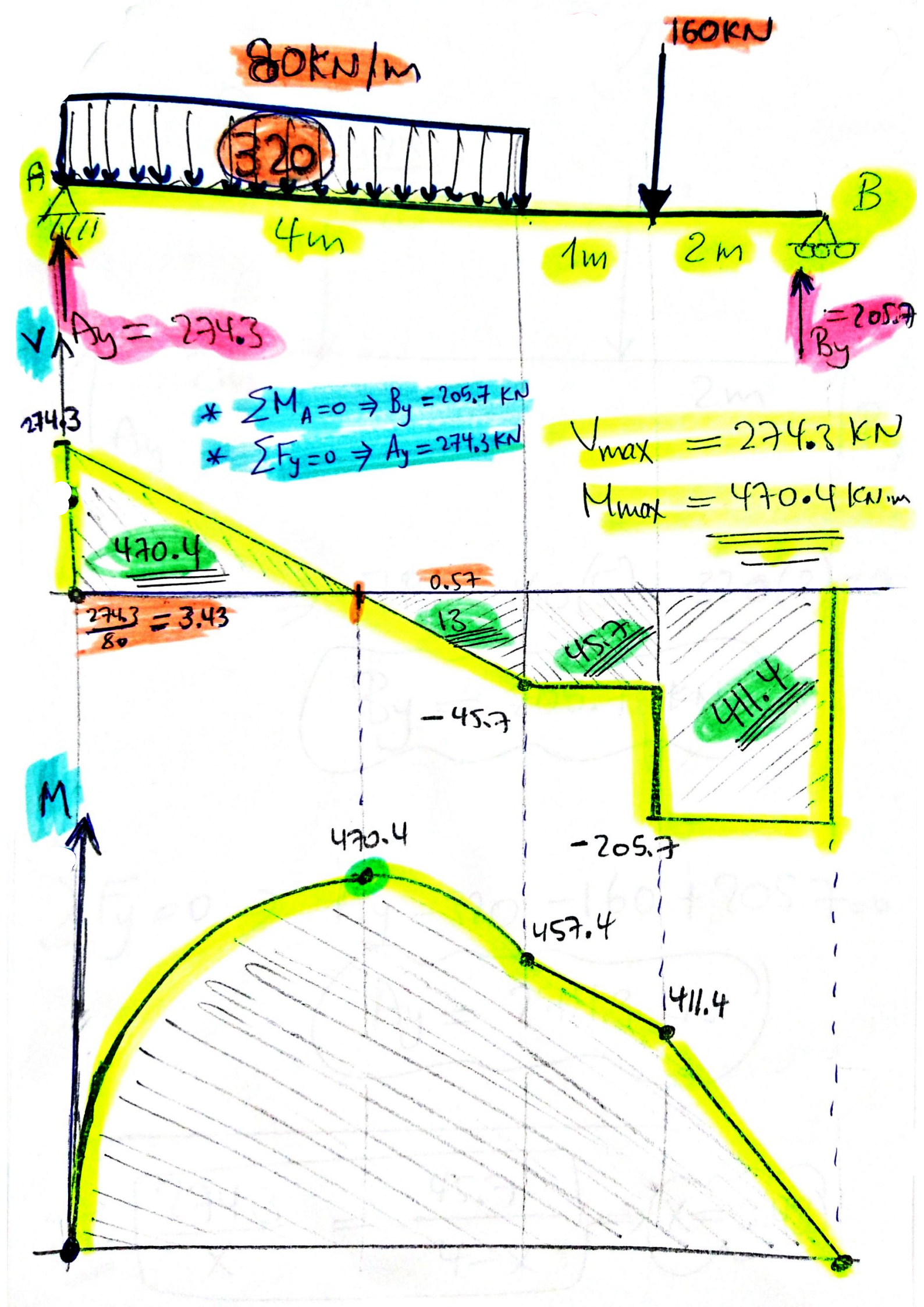
$\sum F_y = 0 \Rightarrow A_y - 240 + 255 - 90 = 0$

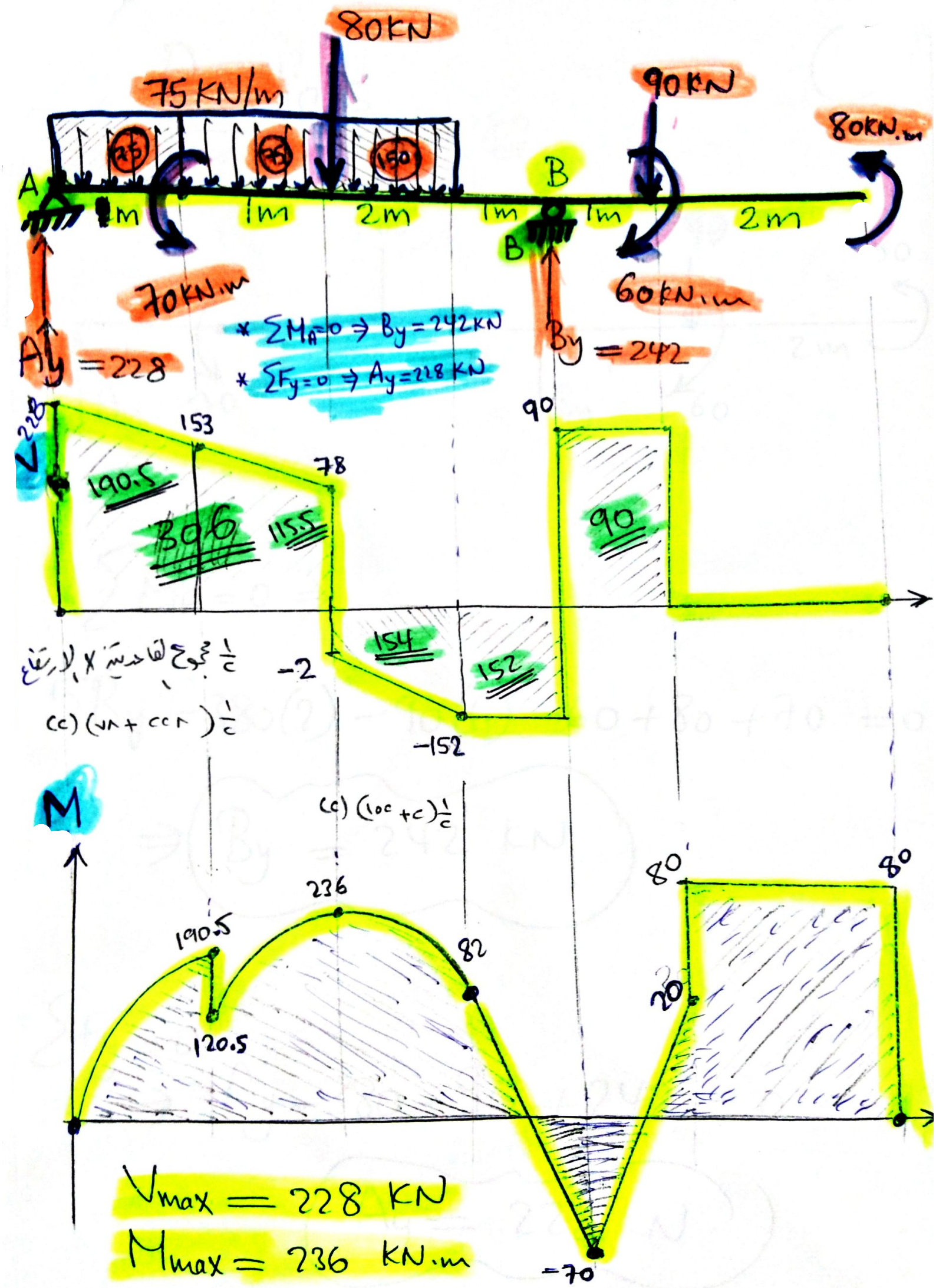
$A_y = 75\ KN$

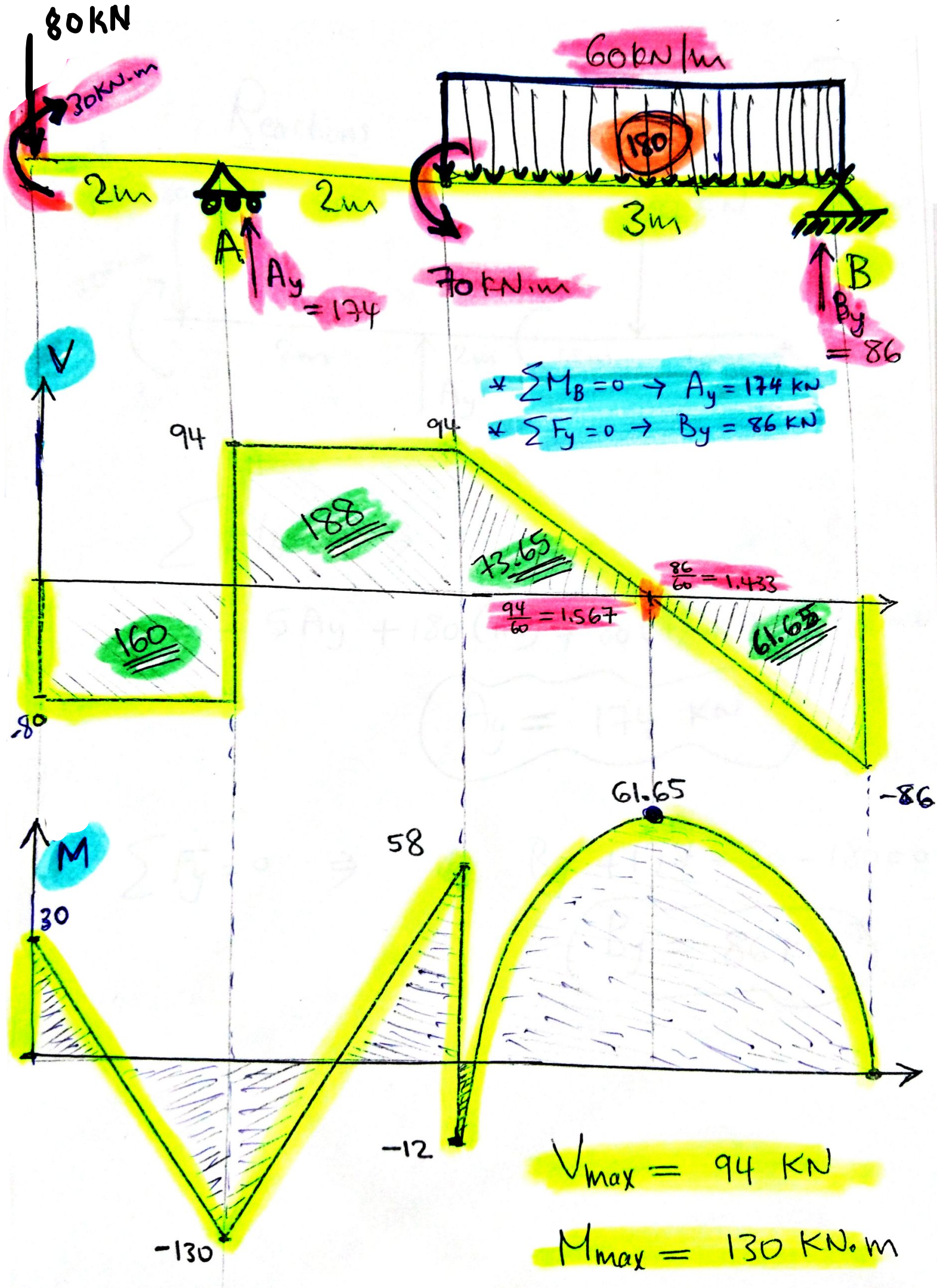


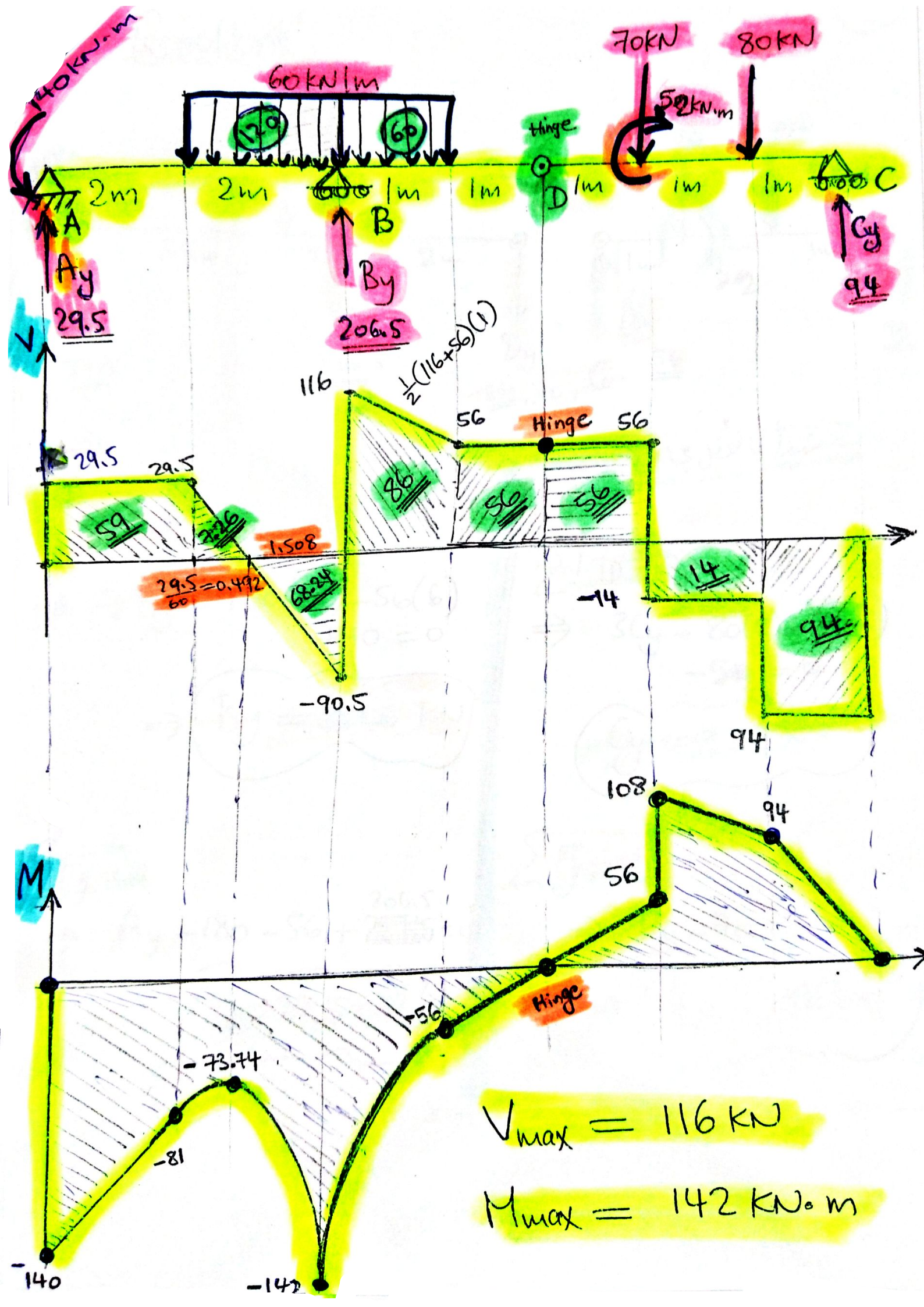
$V_{max} = 165\ KN.$

$M_{max} = 180\ KN.m$


80KN/m
160KN
320
A
B
4m
1m
2m
Ay = 274.3
By = 205.7
* $\sum M_A = 0 \Rightarrow B_y = 205.7$ KN
* $\sum F_y = 0 \Rightarrow A_y = 274.3$ KN
$V_{max} = 274.3$ KN
$M_{max} = 470.4$ KN.m
V
274.3
470.4
$\frac{274.3}{80} = 3.43$
0.57
13
45.7
411.4
-45.7
-205.7
M
470.4
457.4
411.4


80KN
75 KN/m
90KN
80KN.m
A
B
B
1m
1m
2m
1m
1m
2m
70KN.m
60KN.m
* ΣMA=0 ⇒ By=242KN
* ΣFy=0 ⇒ Ay=228 KN
Ay =228
By =242
228
153
78
90
190.5
306
115.5
90
154
152
-2
-152
M
190.5
236
120.5
82
80
80
20
-70
Vmax = 228 KN
Mmax = 236 KN.m

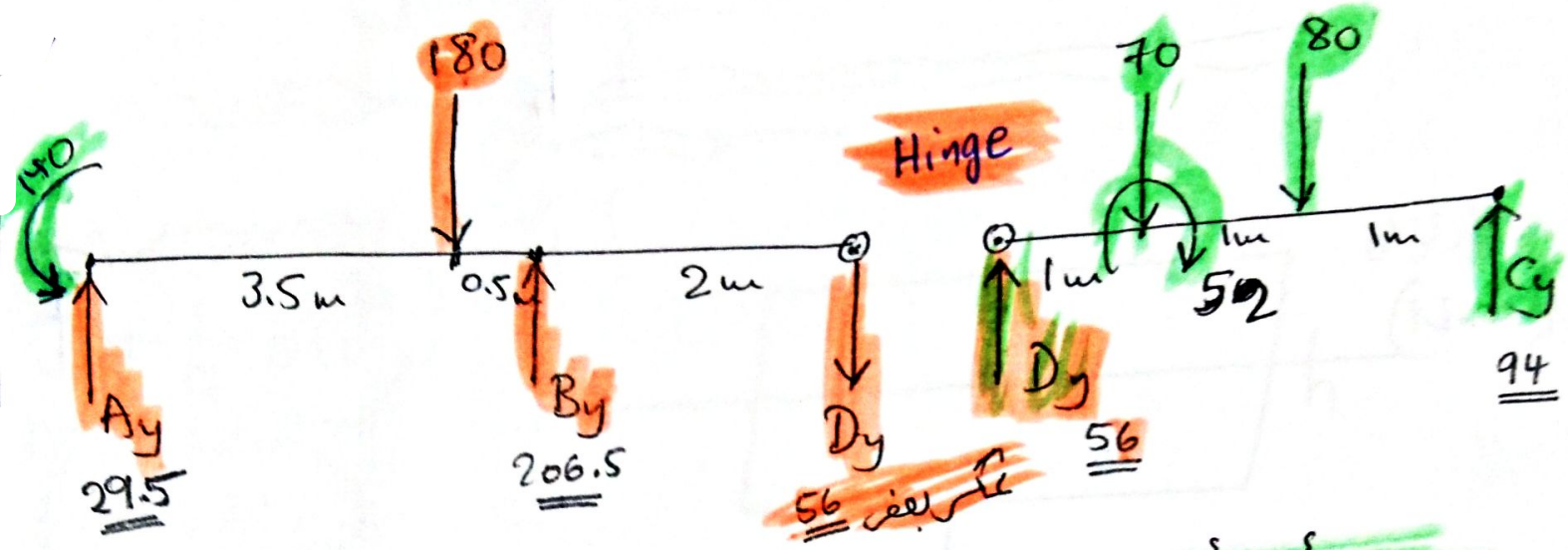

80kN
30kN.m
60kN/m
180
2m
2m
3m
A
A_y = 174
70kN.m
B
B_y = 86
V
94
94
188
73.65
160
61.65
-80
-86
$\frac{86}{60}$ = 1.433
$\frac{94}{60}$ = 1.567
M
30
58
61.65
-12
-130
* $\sum M_B = 0 \rightarrow A_y = 174$ kN
* $\sum F_y = 0 \rightarrow B_y = 86$ kN
$V_{max} = 94$ kN
$M_{max} = 130$ kN.m



$V_{max} = 116\ KN$

$M_{max} = 142\ KN \cdot m$

Reactions



نبدأ بالأقل عدد لهيل دائماً

$\sum M_A = 0$

$\Rightarrow 4By - 180(3.5) - 56(6) + 140 = 0$

$\Rightarrow By = \frac{206.5}{\cancel{29.5}}$ KN

$\sum M_D = 0$

$\Rightarrow 3Cy - 80(2) - 70(1) - 52 = 0$

$Cy = 94$ KN

$\sum F_y = 0$

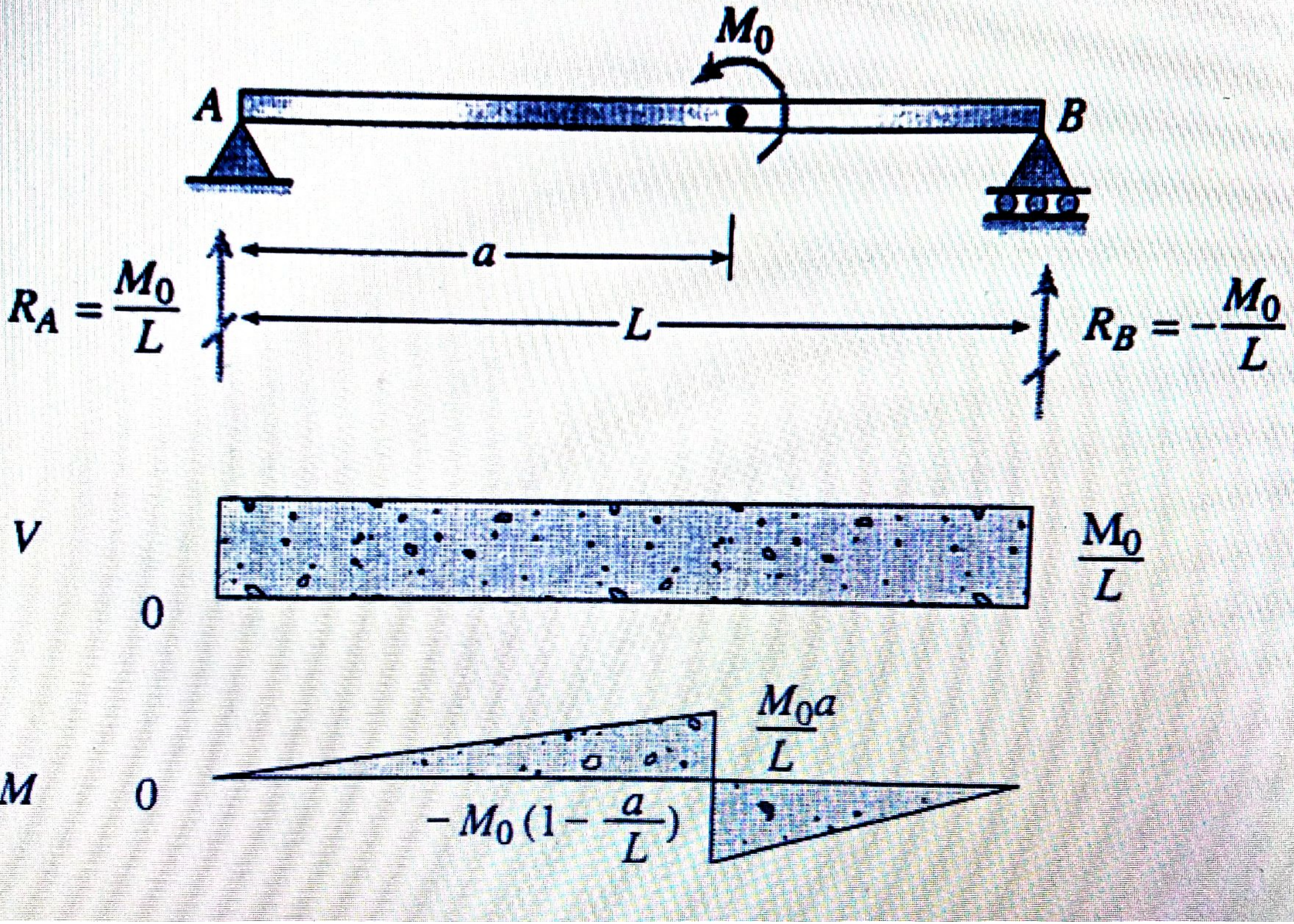
$\Rightarrow Ay - 180 - 56 + \frac{206.5}{\cancel{29.5}} = 0$

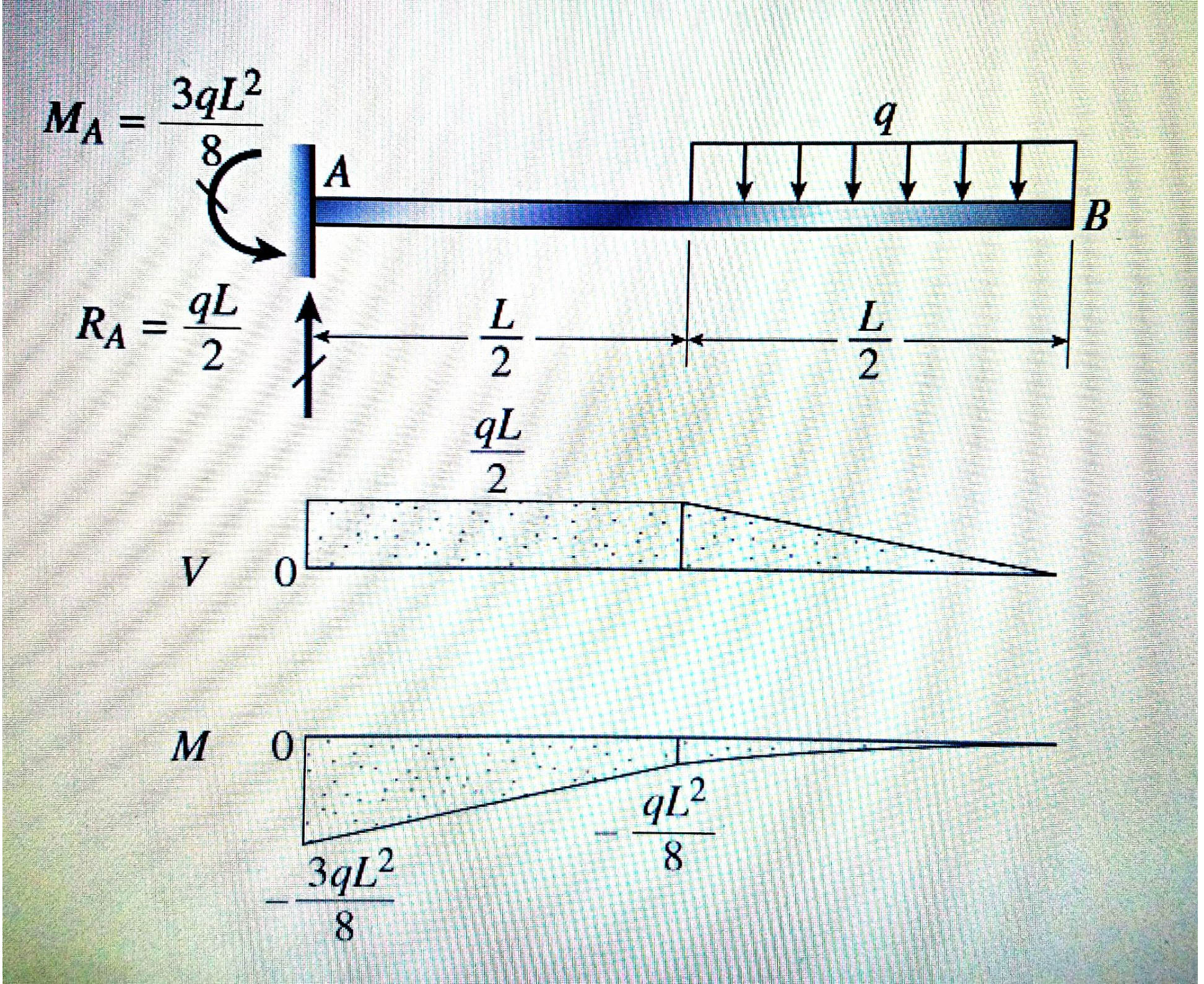
$Ay = 29.5$ KN

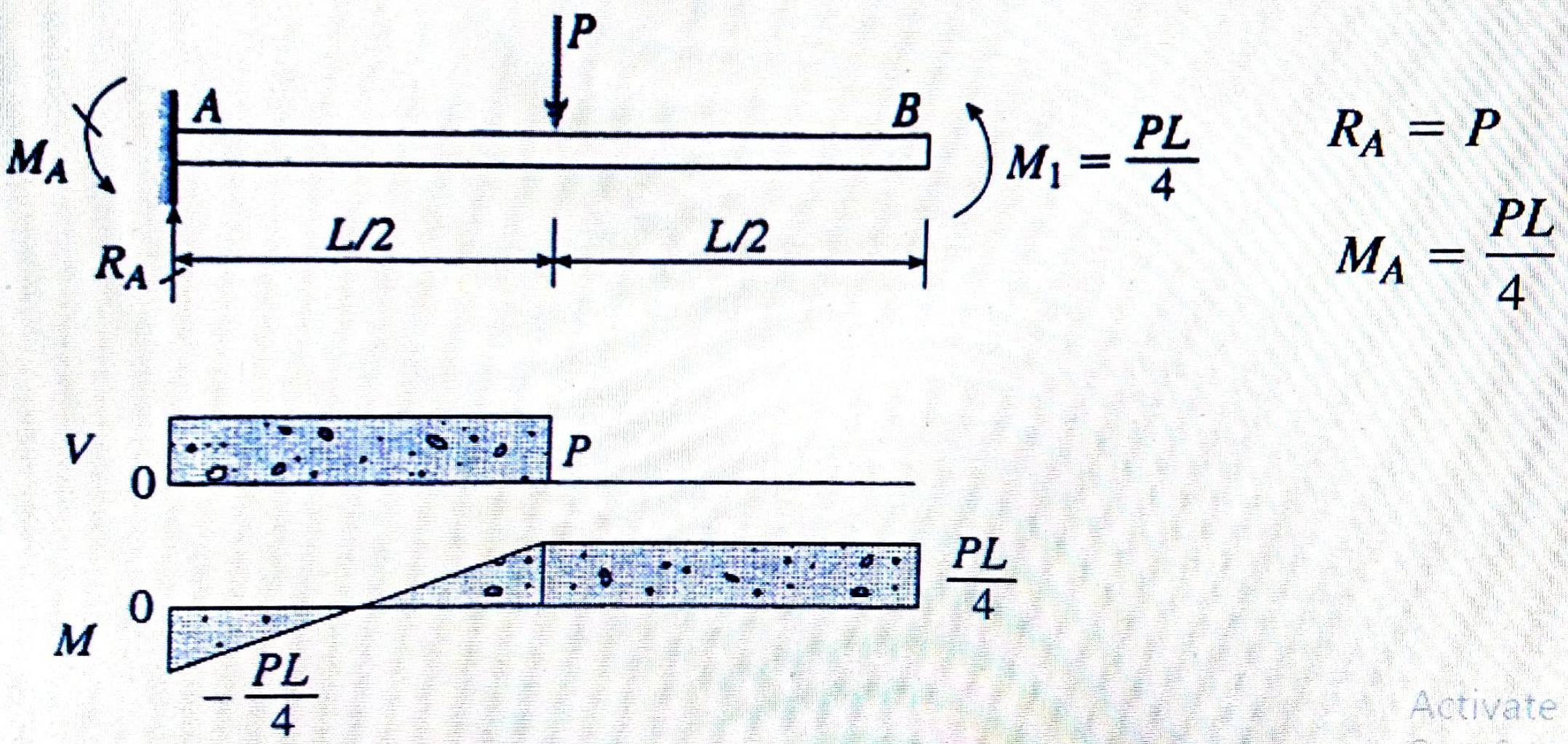
$\sum F_y = 0$

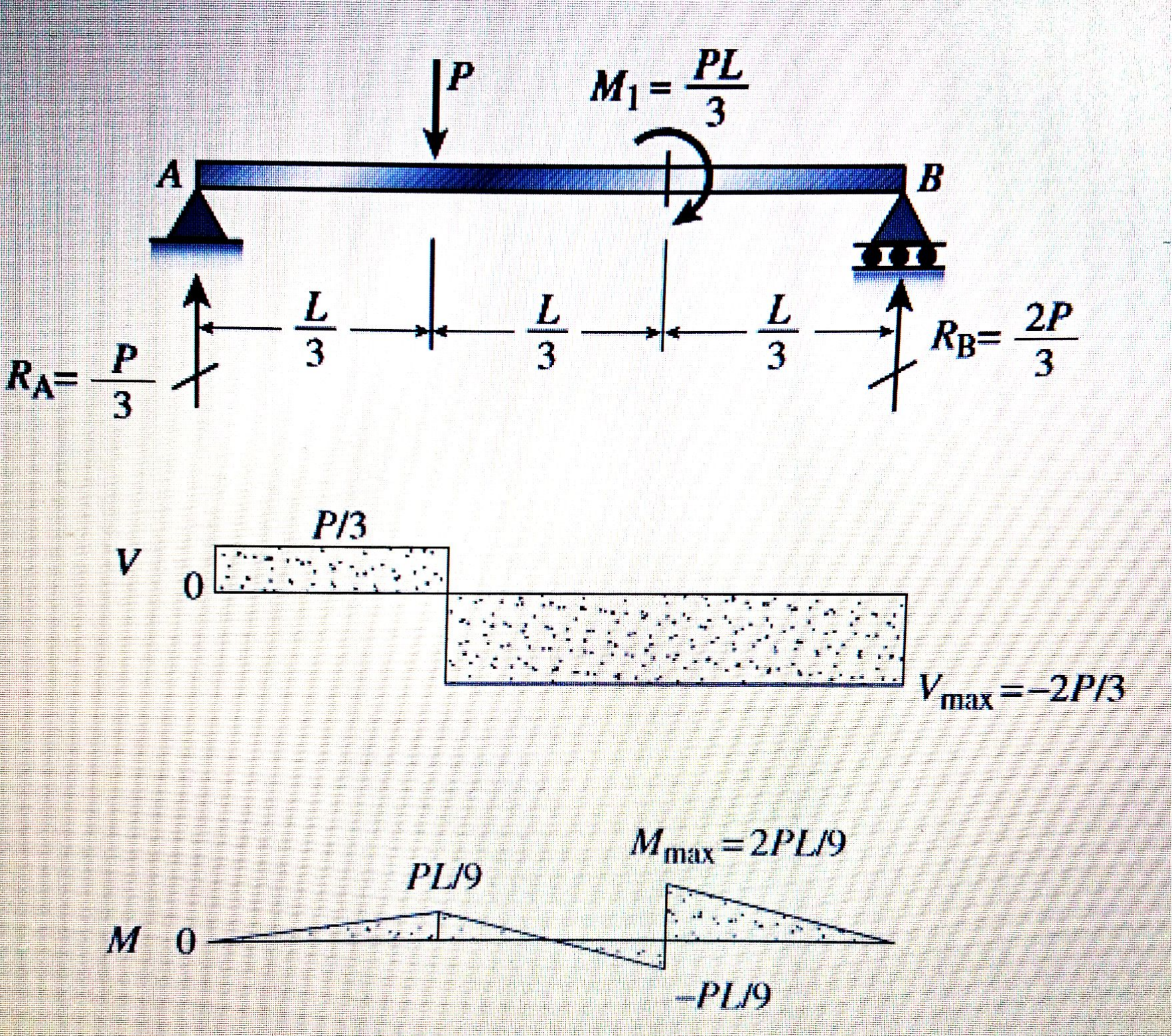
$\Rightarrow Dy - 70 - 80 + 94 = 0$

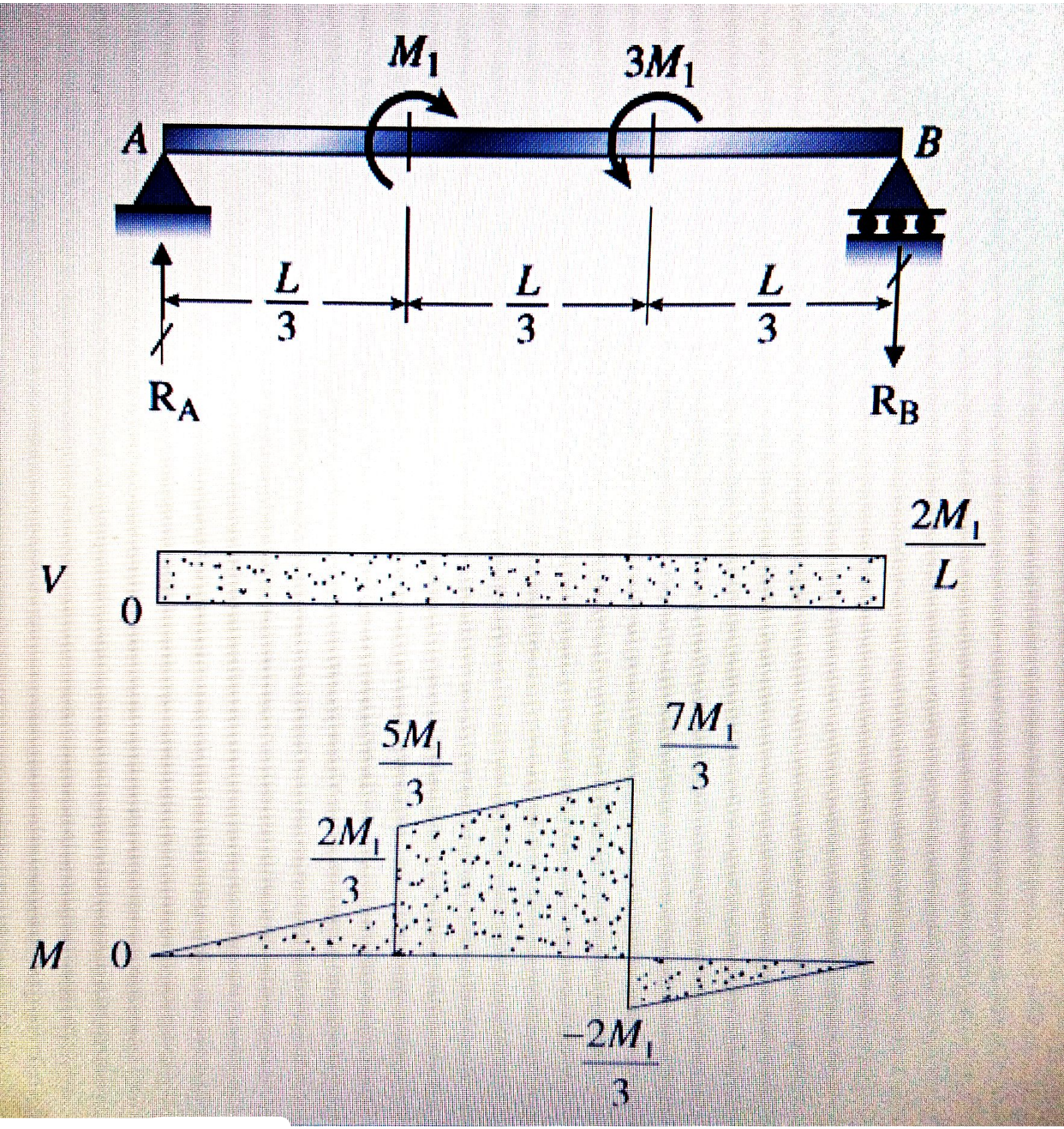
$\Rightarrow Dy = 56$ KN

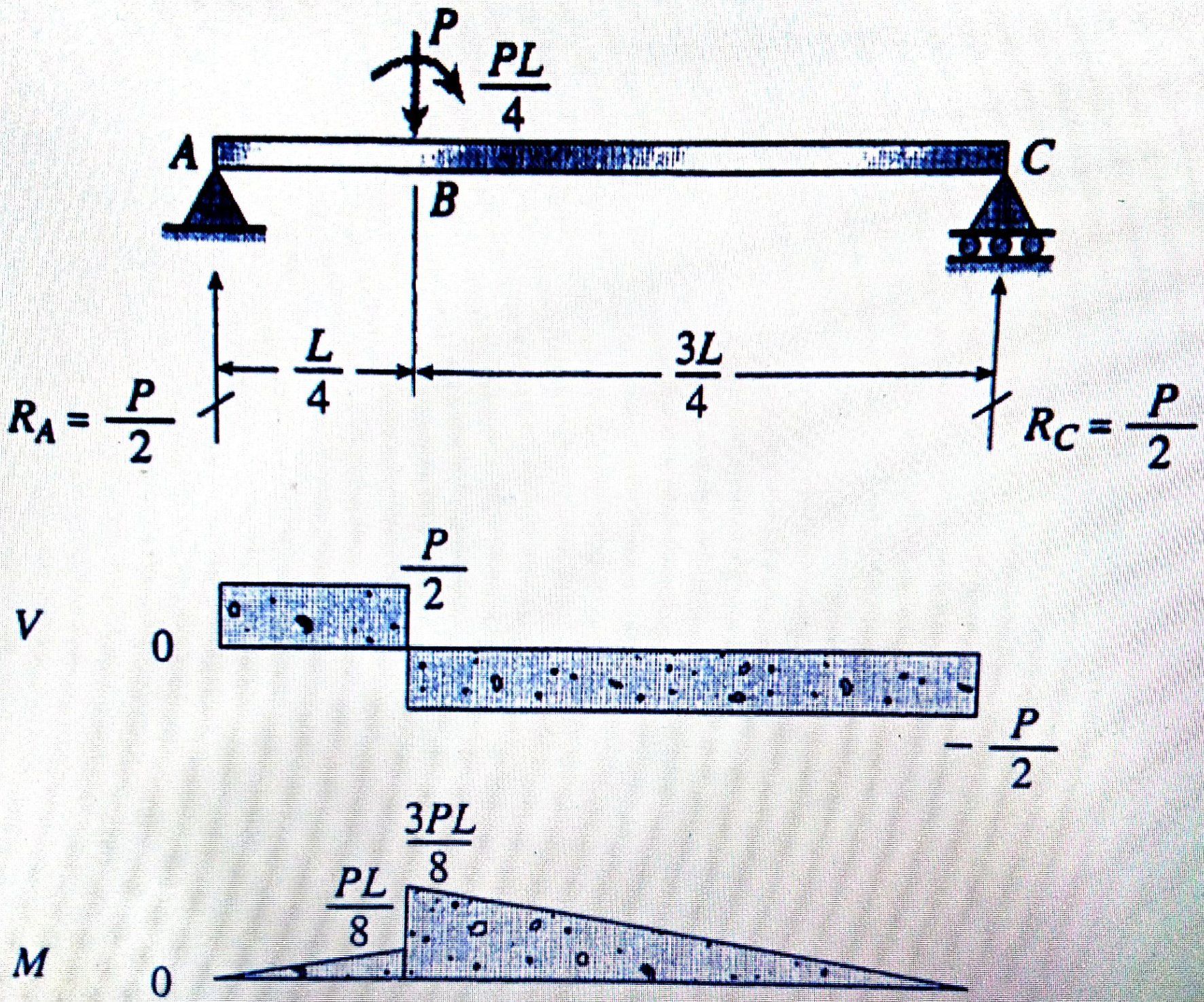

M_0
A
B
a
L
$R_A = \frac{M_0}{L}$
$R_B = -\frac{M_0}{L}$
V
0
$\frac{M_0}{L}$
M
0
$\frac{M_0 a}{L}$
$-M_0\left(1-\frac{a}{L}\right)$

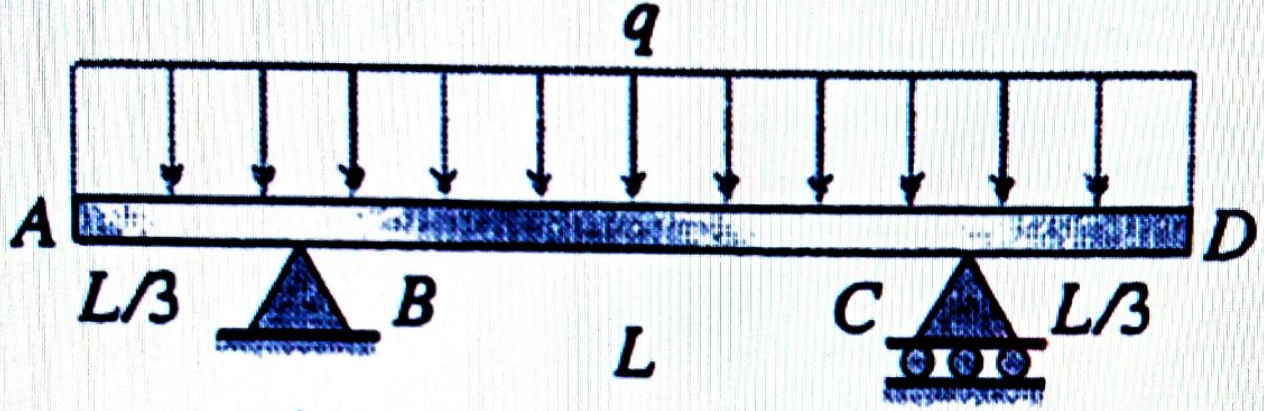

$M_A = \frac{3qL^2}{8}$
$R_A = \frac{qL}{2}$
A
B
q
$\frac{L}{2}$
$\frac{L}{2}$
$\frac{qL}{2}$
V
0
M
0
$-\frac{qL^2}{8}$
$-\frac{3qL^2}{8}$


P
A
B
M_A
$M_1 = \frac{PL}{4}$
$R_A = P$
$M_A = \frac{PL}{4}$
L/2
L/2
R_A
V
0
P
M
0
$\frac{PL}{4}$
$-\frac{PL}{4}$

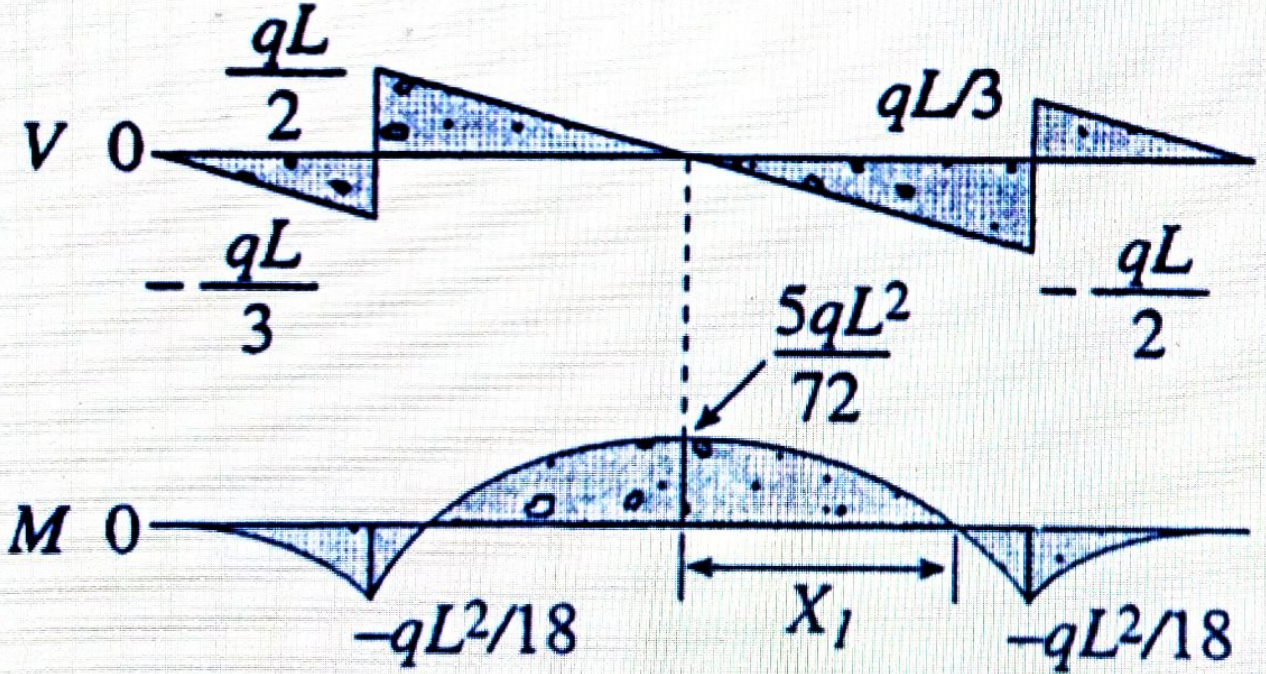

P
$M_1 = \frac{PL}{3}$
A
B
$\frac{L}{3}$
$\frac{L}{3}$
$\frac{L}{3}$
$R_A = \frac{P}{3}$
$R_B = \frac{2P}{3}$
V
P/3
0
$V_{max} = -2P/3$
M
0
PL/9
$M_{max} = 2PL/9$
−PL/9


M1
3M1
A
B
L/3
L/3
L/3
RA
RB
V
0
2M1/L
5M1/3
7M1/3
2M1/3
M
0
−2M1/3

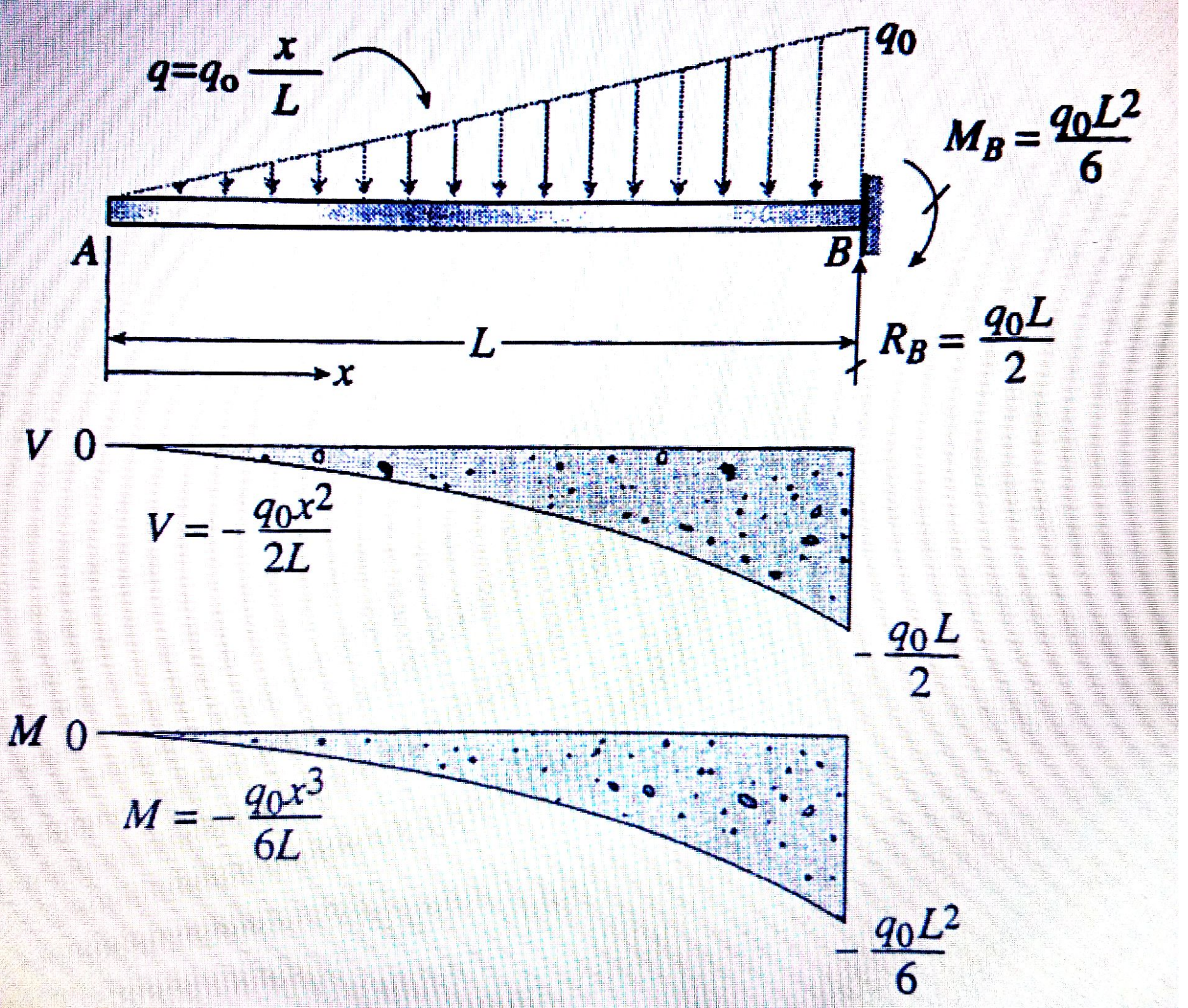

P
PL/4
A
B
C
L/4
3L/4
$R_A = \frac{P}{2}$
$R_C = \frac{P}{2}$
V
P/2
0
−P/2
M
3PL/8
PL/8
0

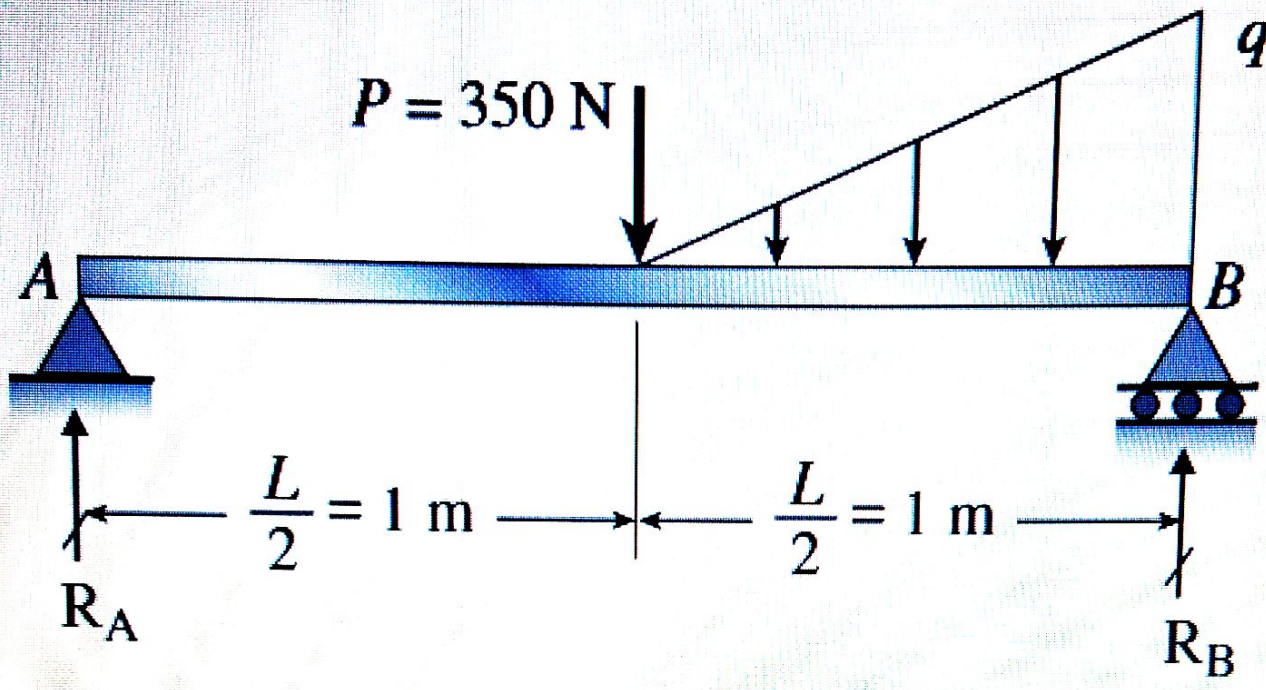


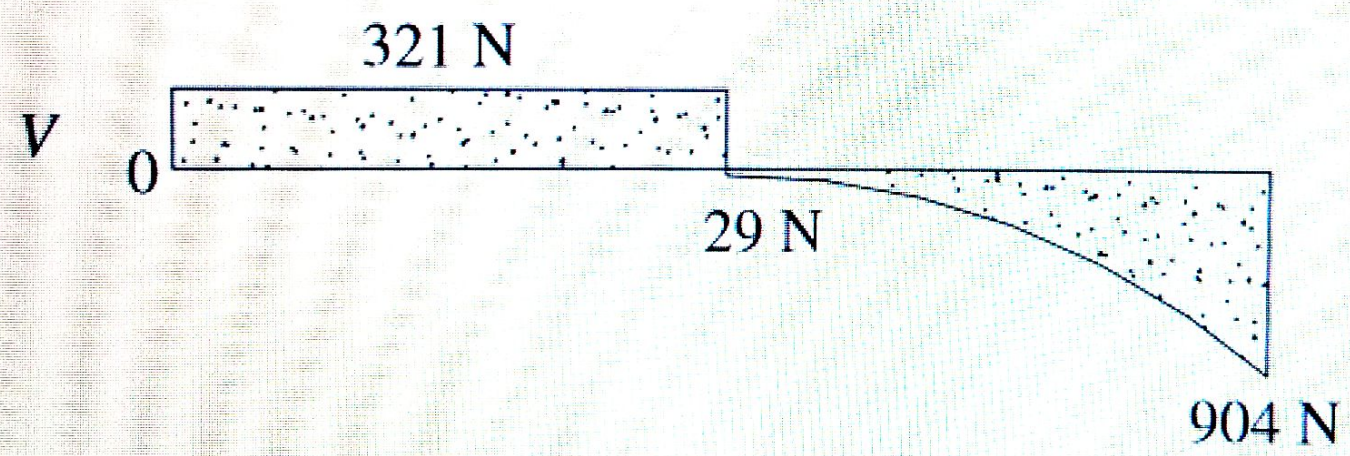
$$R_B = \frac{5qL}{6} \uparrow \qquad \uparrow R_C = \frac{5qL}{6}$$

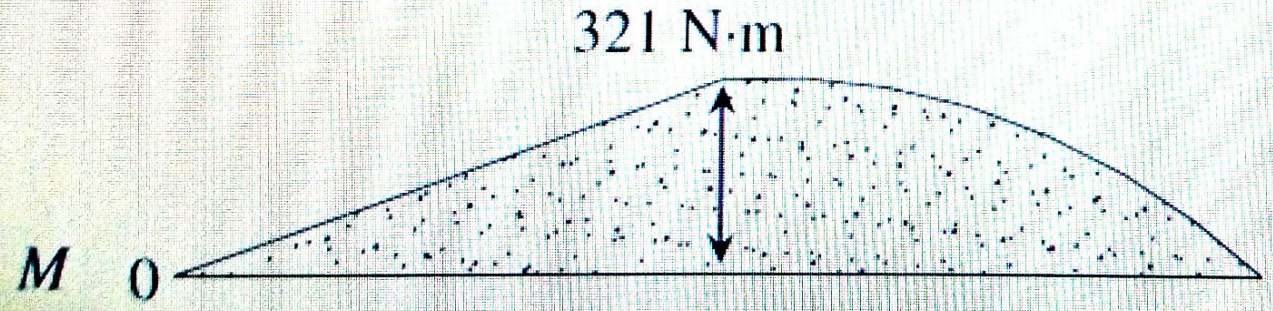


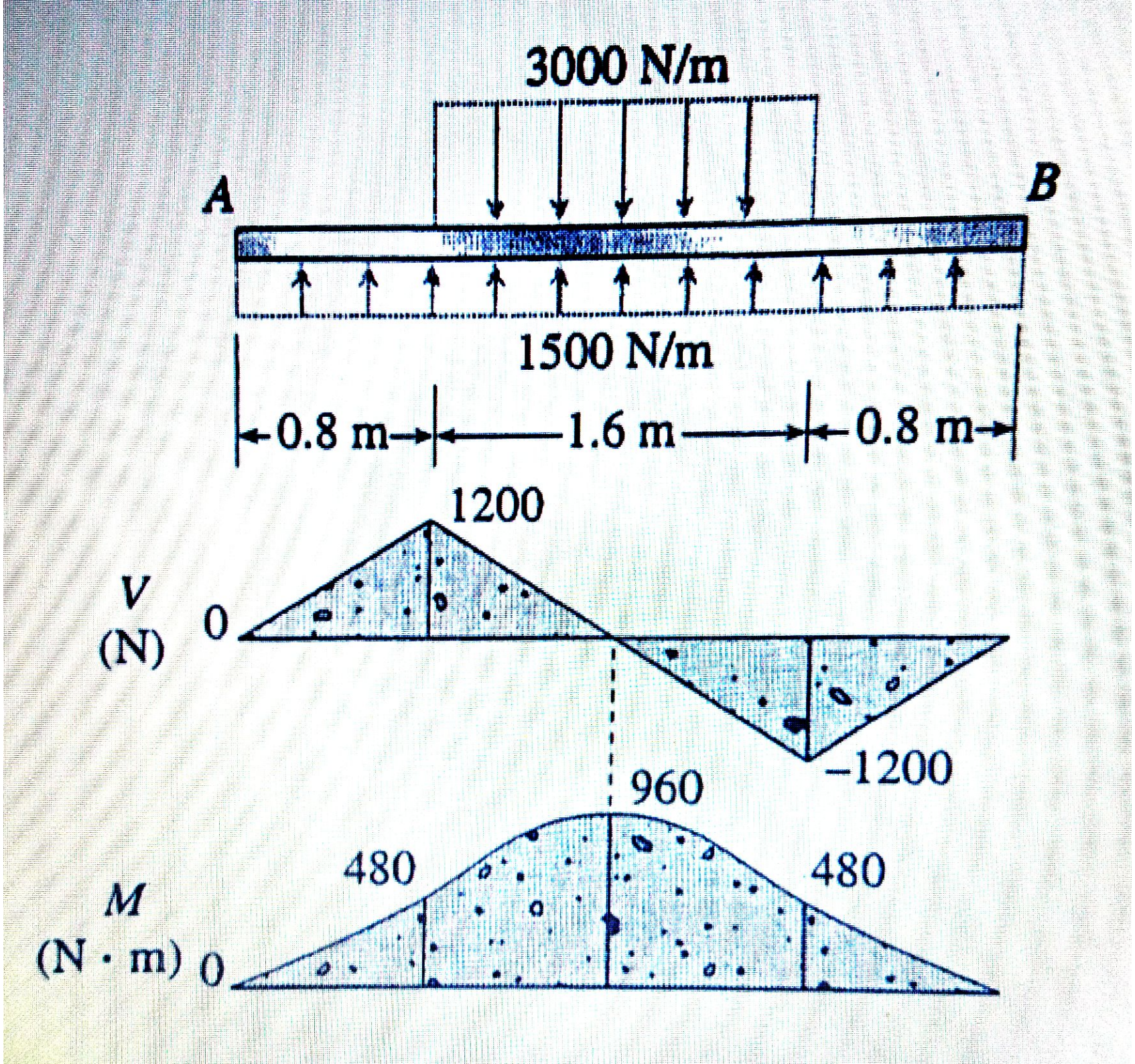
$$x_1 = L\frac{\sqrt{5}}{6} = 0.3727L$$

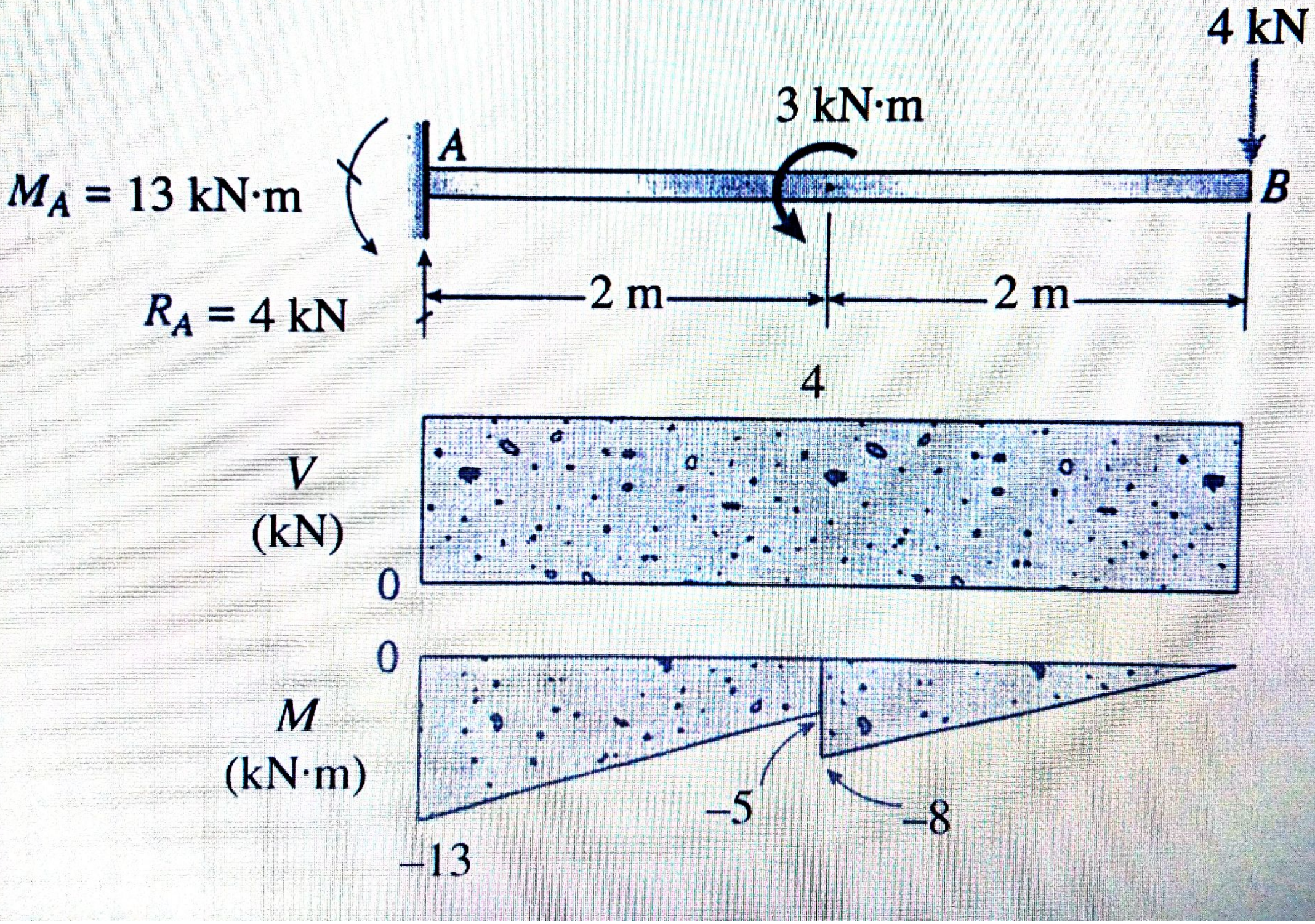

$q = q_0 \frac{x}{L}$
q_0
$M_B = \frac{q_0 L^2}{6}$
A
B
L
x
$R_B = \frac{q_0 L}{2}$
V 0
$V = -\frac{q_0 x^2}{2L}$
$-\frac{q_0 L}{2}$
M 0
$M = -\frac{q_0 x^3}{6L}$
$-\frac{q_0 L^2}{6}$

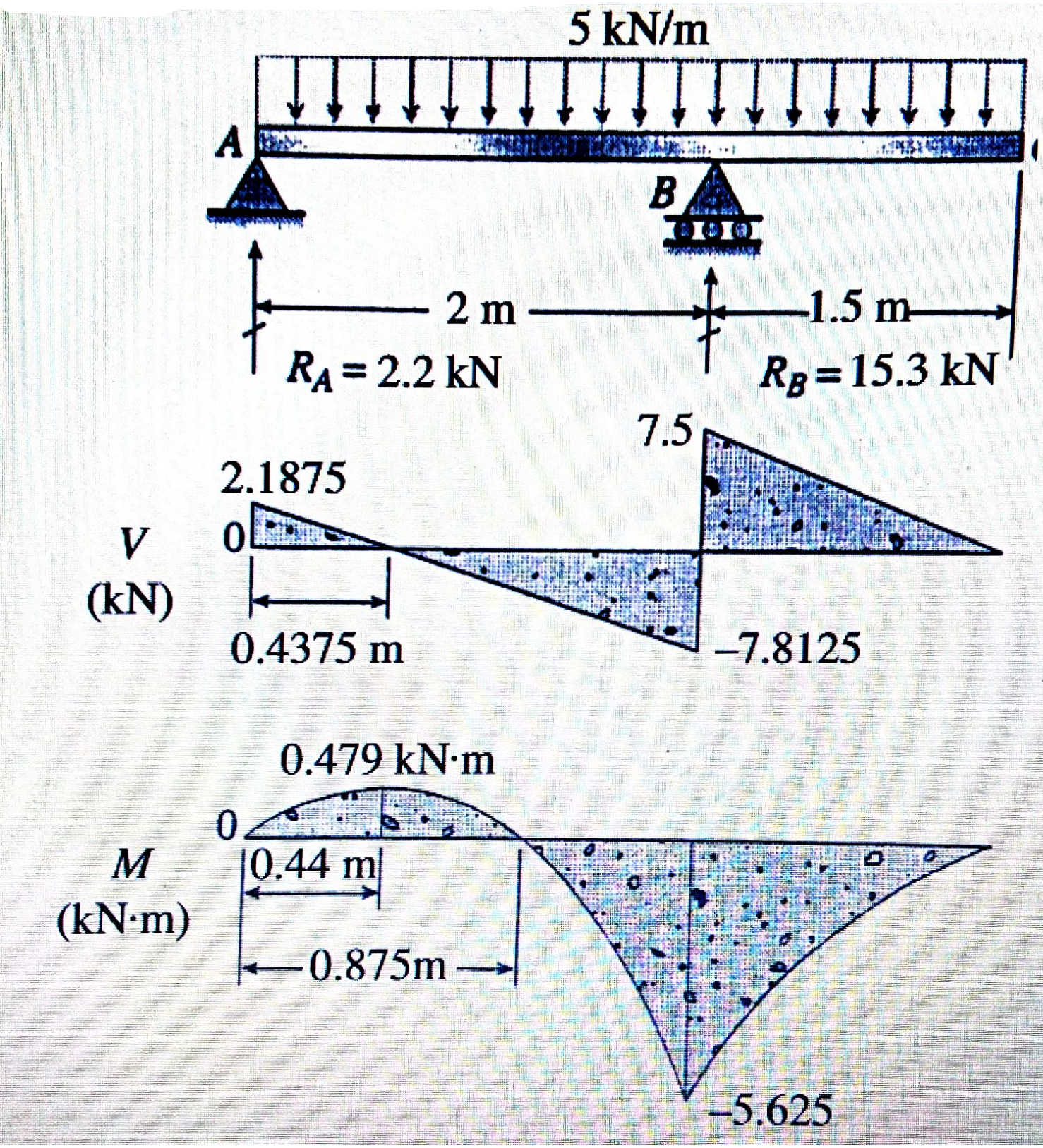

$q_0 = 1750$ N/m
$P = 350$ N
A
B
$\frac{L}{2} = 1$ m
$\frac{L}{2} = 1$ m
R_A
R_B

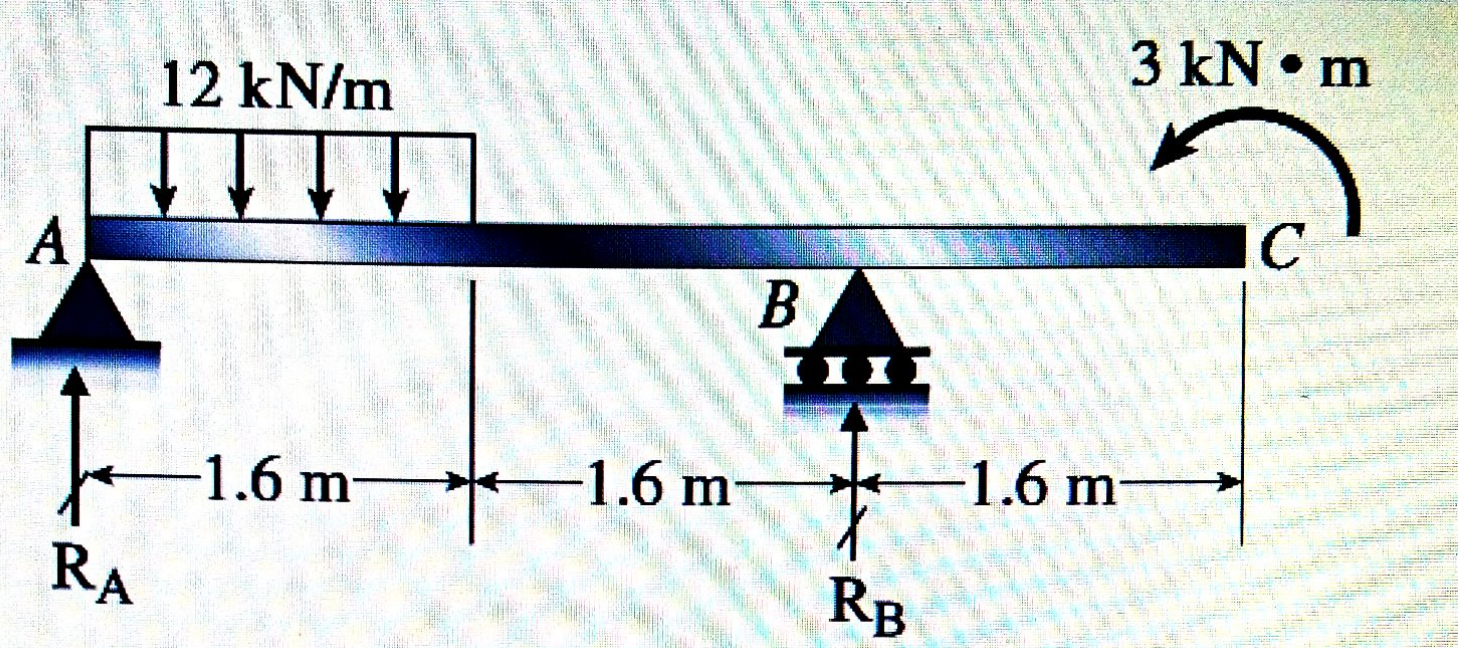

V
321 N
0
29 N
904 N

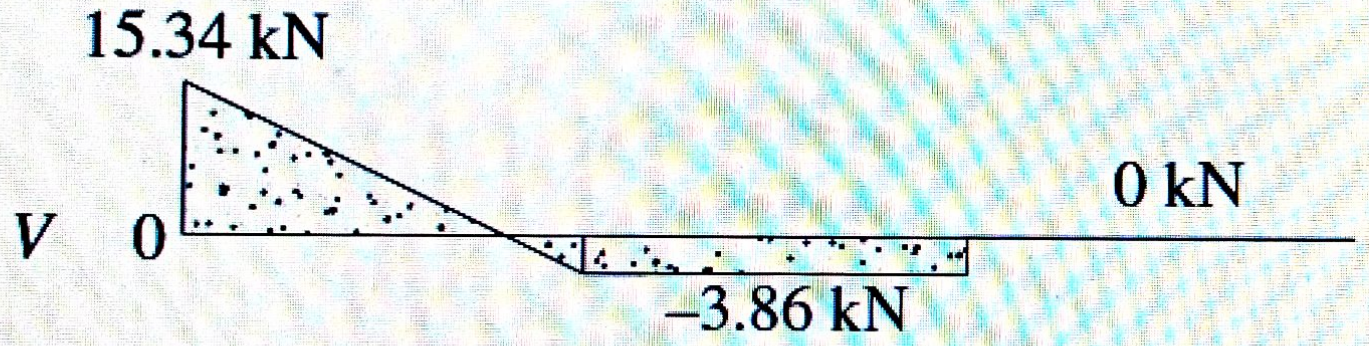

321 N·m
M
0

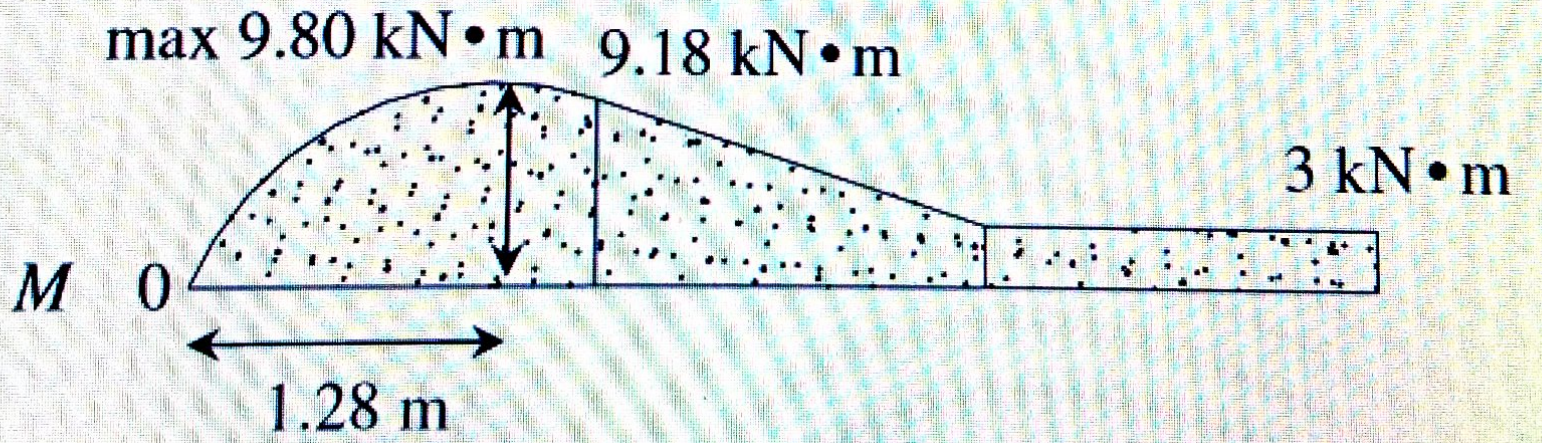

3000 N/m
A
B
1500 N/m
0.8 m
1.6 m
0.8 m
1200
V
(N)
0
−1200
960
480
480
M
(N · m)
0


4 kN
3 kN·m
A
B
M_A = 13 kN·m
R_A = 4 kN
2 m
2 m
4
V
(kN)
0
0
M
(kN·m)
−5
−8
−13


5 kN/m
A
B
2 m
1.5 m
$R_A = 2.2$ kN
$R_B = 15.3$ kN
7.5
2.1875
V
(kN)
0
0.4375 m
−7.8125
0.479 kN·m
0
M
(kN·m)
0.44 m
0.875m
−5.625

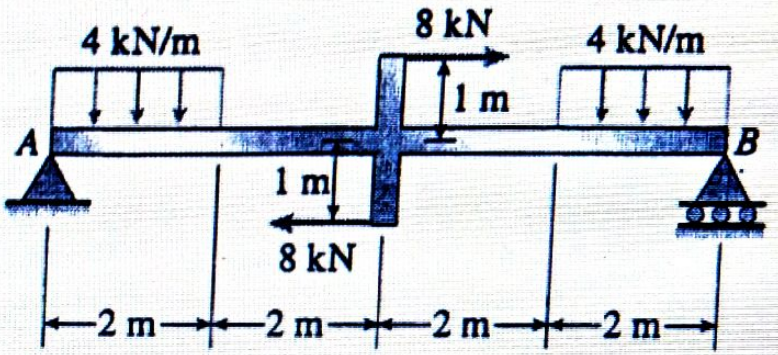

12 kN/m
3 kN • m
A
B
C
1.6 m
1.6 m
1.6 m
R_A
R_B

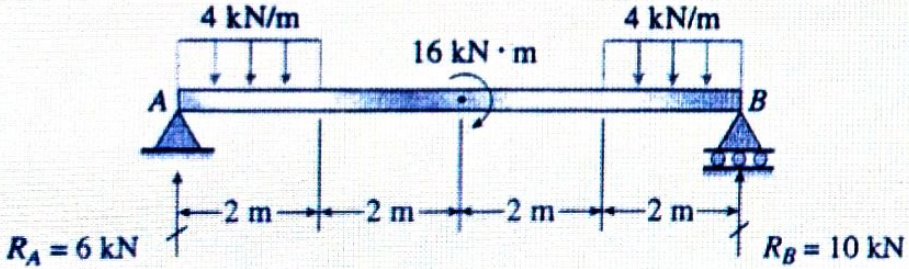

15.34 kN
V
0
0 kN
−3.86 kN

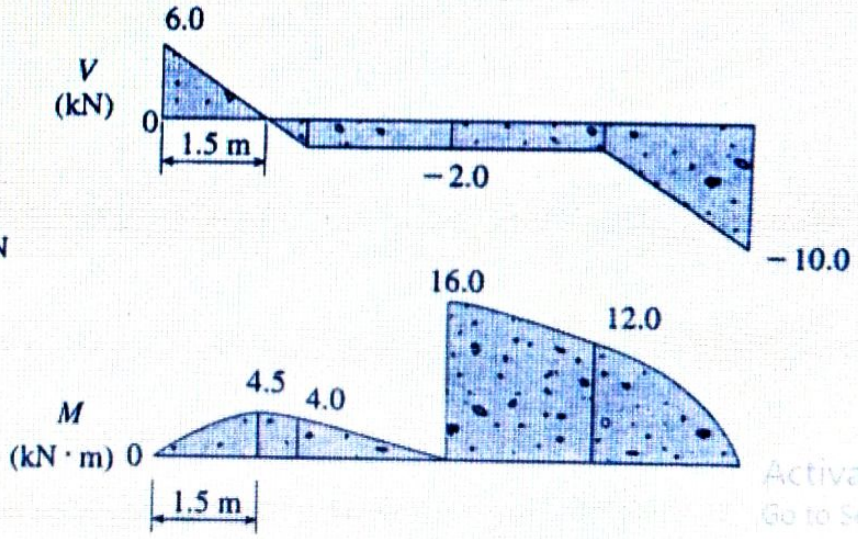

max 9.80 kN • m
9.18 kN • m
3 kN • m
M
0
1.28 m

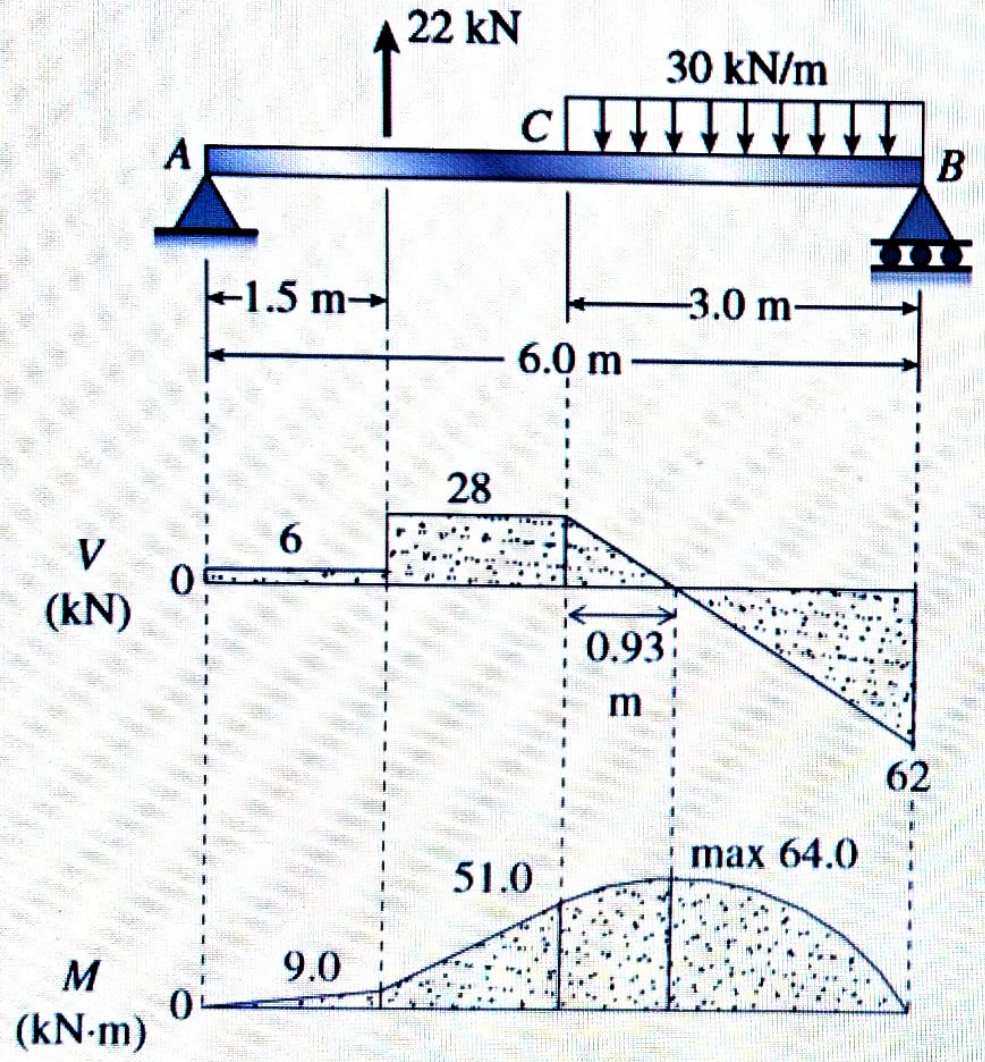
A simple beam AB is loaded by two segments of uniform load and two horizontal forces acting at the ends of a vertical arm (see figure).

Draw the shear-force and bending-moment diagrams for this beam.









$$\Sigma M_A = 0$$

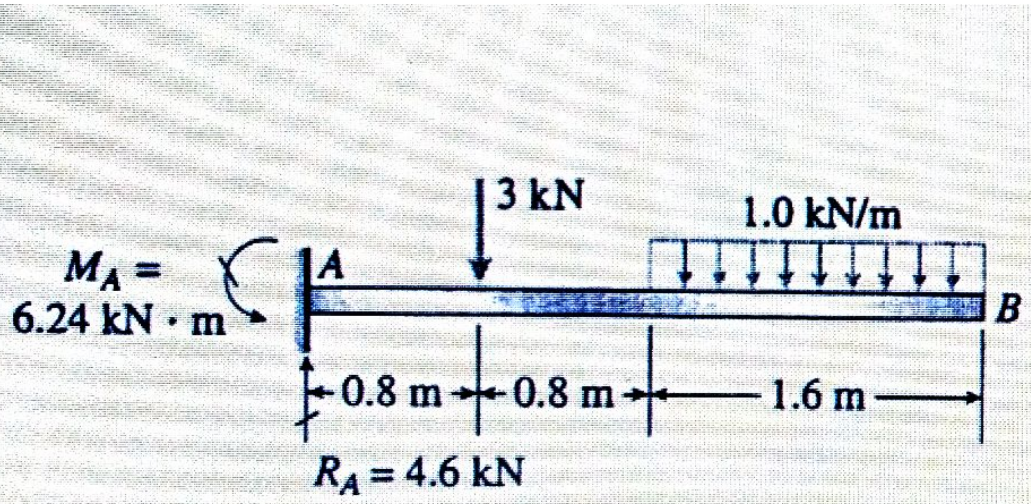
$$R_B = \frac{1}{6.0\text{m}}\left[(-22.0\text{ kN})(1.5\text{ m}) + \left(30\frac{\text{kN}}{\text{m}}\right)(3.0\text{ m})(4.5\text{ m})\right]$$

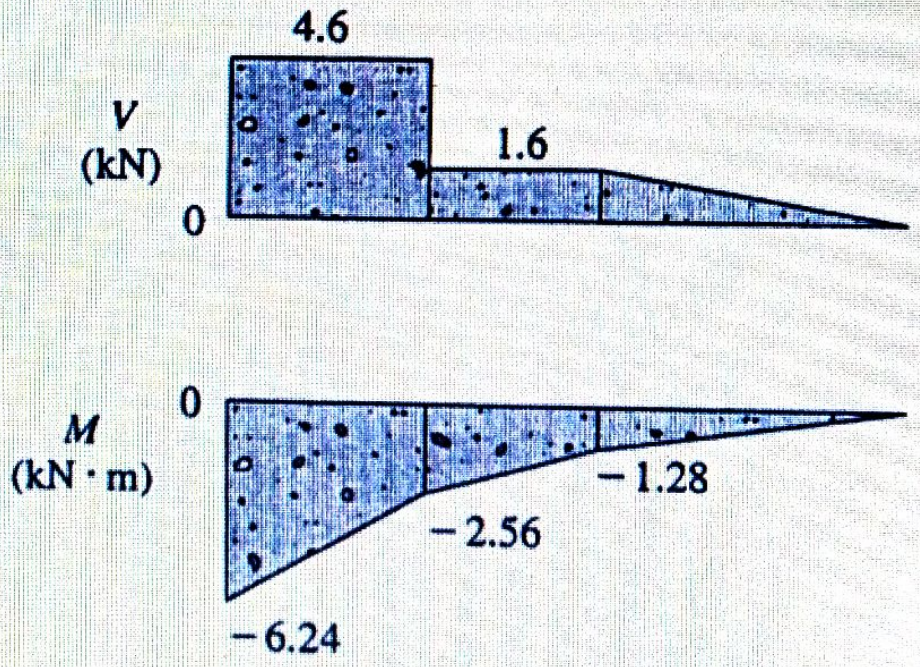
$$R_B = 62.0\text{ kN}$$

$$\Sigma F_y = 0$$

$$R_A = -22.0\text{ kN} + \left(30.0\frac{\text{kN}}{\text{m}}\right)(3.0\text{ m}) - R_B$$

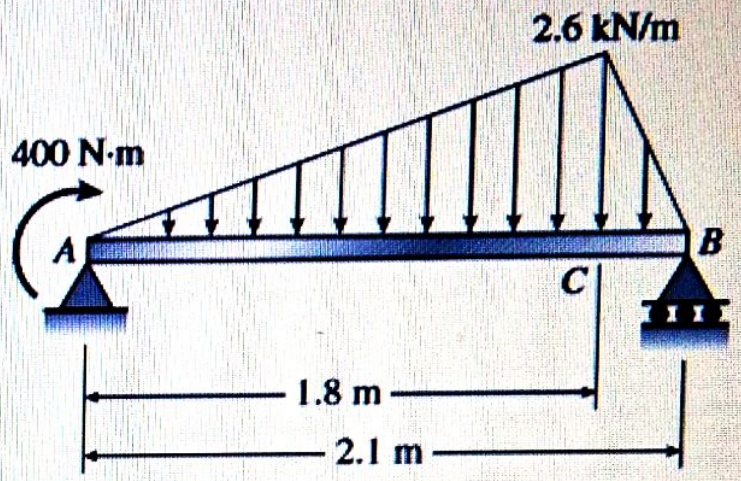
$$R_A = 6.0\text{ kN}$$


3 kN
1.0 kN/m
M_A =
6.24 kN · m
A
B
0.8 m
0.8 m
1.6 m
R_A = 4.6 kN


4.6
V
(kN)
1.6
0
0
M
(kN · m)
−1.28
−2.56
−6.24

The simple beam *ACB* shown in the figure is subjected to a triangular load of maximum intensity 2.6 kN/m and a concentrated moment of 400 Nm at *A*.

Draw the shear-force and bending-moment diagrams for this beam.



$$\Sigma M_A = 0$$

$$R_B = \frac{1}{2.1\text{m}}\left[400\ \text{N·m} + \frac{1}{2}\left(2.6\frac{\text{kN}}{\text{m}}\right)(1.8\ \text{m}) \times \left(\frac{2}{3}1.8\,\text{m}\right) + \frac{1}{2}\left(2.6\frac{\text{kN}}{\text{m}}\right)(0.3\text{m}) \times \left(1.8\,\text{m} + \frac{1}{3}0.3\ \text{m}\right)\right]$$

$$R_B = 1880\ \text{N}$$

$$\Sigma F_y = 0$$

$$R_A = \frac{1}{2}\left(2.6\frac{\text{kN}}{\text{m}}\right)(1.8\text{m}) + \frac{1}{2}\left(2.6\frac{\text{kN}}{\text{m}}\right)(0.3\ \text{m}) - R_B$$

$$R_A = 850\ \text{N}$$