5

Stresses in Beams (Basic Topics)

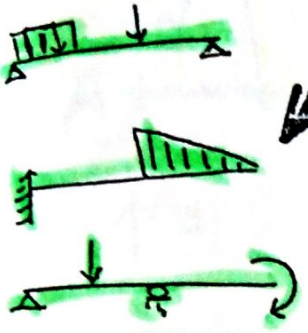
CHAPTER OVERVIEW

Chapter 5 is concerned with stresses and strains in beams which have loads applied in the xy plane, a plane of symmetry of the cross section, resulting in beam deflection in that same plane, known as the *plane of bending*. Both *pure bending* (beam flexure under constant bending moment) and *nonuniform bending* (flexure in the presence of shear forces) are discussed (Section 5.2). We will see that strains and stresses in the beam are directly related to the *curvature* κ of the deflection curve (Section 5.3). A *strain-curvature relation* will be developed from consideration of longitudinal strains developed in the beam during bending; these strains vary linearly with distance from the neutral surface of the beam (Section 5.4). When Hooke's law (which applies for linearly elastic materials) is combined with the strain-curvature relation, we find that the neutral axis passes through the centroid of the cross section. As a result, x and y axes are seen to be principal centroidal axes. By consideration of the moment resultant of the normal stresses acting over the cross section, we next derive the *moment-curvature relation* which relates curvature (κ) to moment (M) and flexural rigidity (EI). This will lead to the differential equation of the beam elastic curve, a topic for consideration in Chapter 9 when we will discuss beam deflections in detail. Of immediate interest here, however, are beam stresses, and the moment-curvature relation is next used to develop the *flexure formula* (Section 5.5). The *flexure formula* shows that normal stresses (σ_x) vary linearly with distance (y) from the neutral surface and depend on bending moment (M) and moment of inertia (I) of the cross section. Next, the section modulus (S) of the beam cross section is defined and then used in *design* of beams in Section 5.6. In beam design, we use the maximum bending moment (M_{max}) (obtained from the bending moment diagram (Section 4.5)) and the allowable normal stress for the material (σ_{allow}) to compute the required section modulus, then select an appropriate beam of steel or wood from the tables in Appendices E and F. If the beam is nonprismatic (Section 5.7), the flexure formula still applies provided that changes in cross-sectional dimensions are gradual. However, we cannot assume that the maximum stresses occur at the cross section with the largest bending moment.

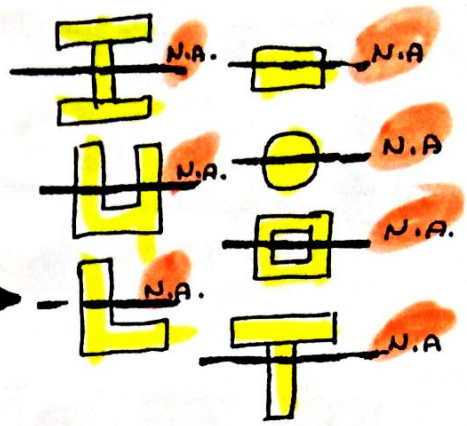
Ch 5 Bending Stress

د. هاشم الخالدي

Bending stress



$$\sigma = \frac{My}{I}$$



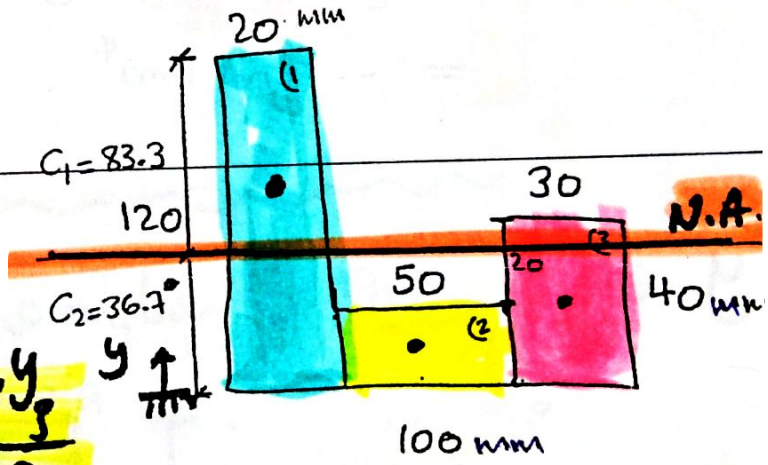
بالرسم M_{max}

بالتكامل M عند نقطة معينة

ex)

① Find N.A. ?

② Find I ?



① $$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$A_1 = 2400$, $y_1 = 60$

$A_2 = 1000$, $y_2 = 10$

$A_3 = 1200$, $y_3 = 20$

$$\Rightarrow \bar{y} = \frac{144000 + 10000 + 24000}{2400 + 1000 + 12000}$$

$$\boxed{\bar{y} = 36.7 \text{ mm}}$$

② $$I = I_1 + I_2 + I_3 = \left[\frac{1}{12}(20)(120)^3 + 2400\,(83.3 - 60)^2\right] + \left[\frac{1}{12}(50)(20)^3 + 1000\,(36.7 - 10)^2\right] + \left[\frac{1}{12}(30)(40)^3 + 1200\,(36.7 - 20)^2\right]$$

انظر خلف الصفحة

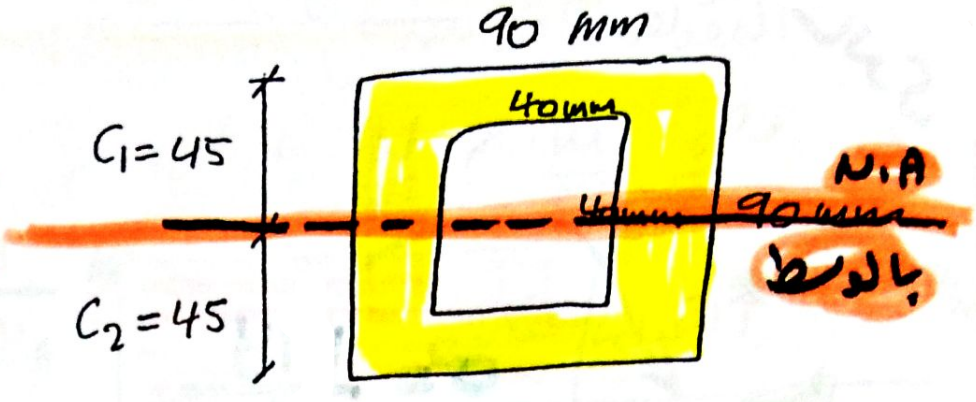
$I = \ldots\ldots mm^4$

ex

Find

① N.A. ?

② I ?



الحل

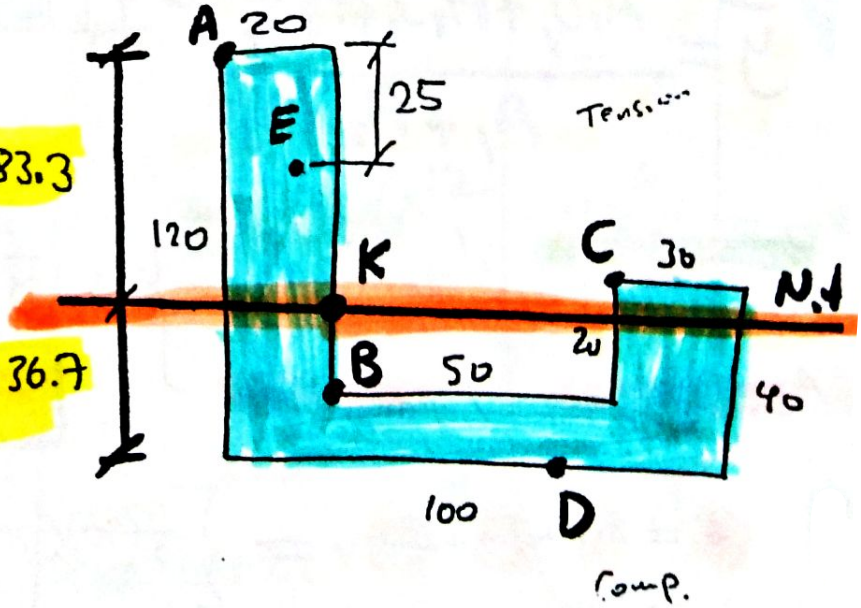
① N.A. بالوسط

② $I = I_1 - I_2$

$= \left[\frac{1}{12}(90)(90)^3\right] - \left[\frac{1}{12}(40)(40)^3\right]$

$= \text{------}\ mm^4$.

y ⇒ عبارة عن البعد عن N.A.



$C_1 = 83.3$

$C_2 = 36.7$

$y_A = 83.3\ mm$

$y_B = 36.7 - 20 =$

$y_C = 40 - 36.7 =$

$y_D = 36.7$

$y_E = 83.3 - 25$

$y_K = 0$

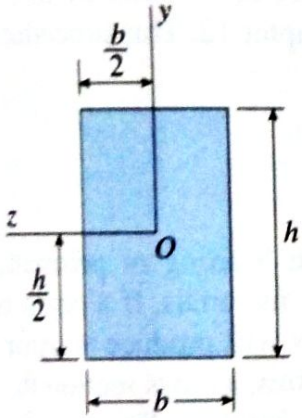
$$\sigma_1 = -\frac{Mc_1}{I} = -\frac{M}{S_1} \qquad \sigma_2 = \frac{Mc_2}{I} = \frac{M}{S_2} \tag{5-14a,b}$$

in which

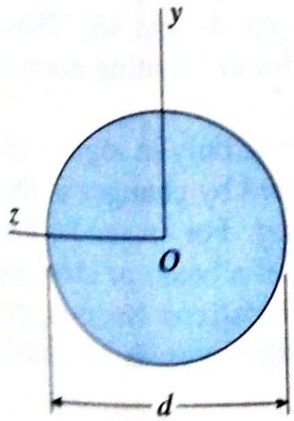
$$S_1 = \frac{I}{c_1} \qquad S_2 = \frac{I}{c_2} \tag{5-15a,b}$$

The quantities S_1 and S_2 are known as the **section moduli** of the cross-sectional area. From Eqs. (5-15a and b) we see that each section modulus has dimensions of length to the third power (for example, mm^3). Note that the distances c_1 and c_2 to the top and bottom of the beam are always taken as positive quantities.

The advantage of expressing the maximum stresses in terms of section moduli arises from the fact that each section modulus combines the beam's relevant cross-sectional properties into a single quantity. Then this quantity can be listed in tables and handbooks as a property of the beam, which is a convenience to designers. (Design of beams using section moduli is explained in the next section.)



(a)



$I = \frac{\pi d^4}{64}$ $S = \frac{\pi d^3}{32}$

(b)

FIG. 5-12 Doubly symmetric cross-sectional shapes

Doubly Symmetric Shapes

If the cross section of a beam is symmetric with respect to the z axis as well as the y axis (*doubly symmetric cross section*), then $c_1 = c_2 = c$ and the maximum tensile and compressive stresses are equal numerically:

$$\sigma_1 = -\sigma_2 = -\frac{Mc}{I} = -\frac{M}{S} \quad \text{or} \quad \sigma_{max} = \frac{M}{S} \tag{5-16a,b}$$

in which

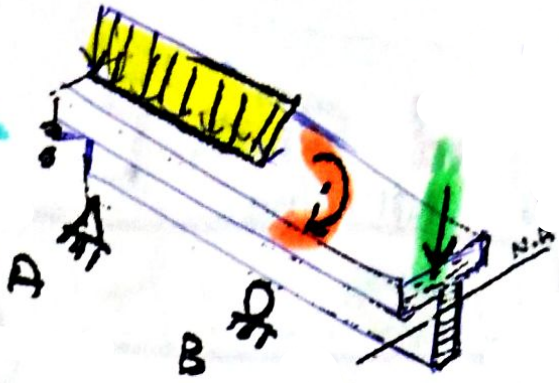
$$S = \frac{I}{c} \tag{5-17}$$

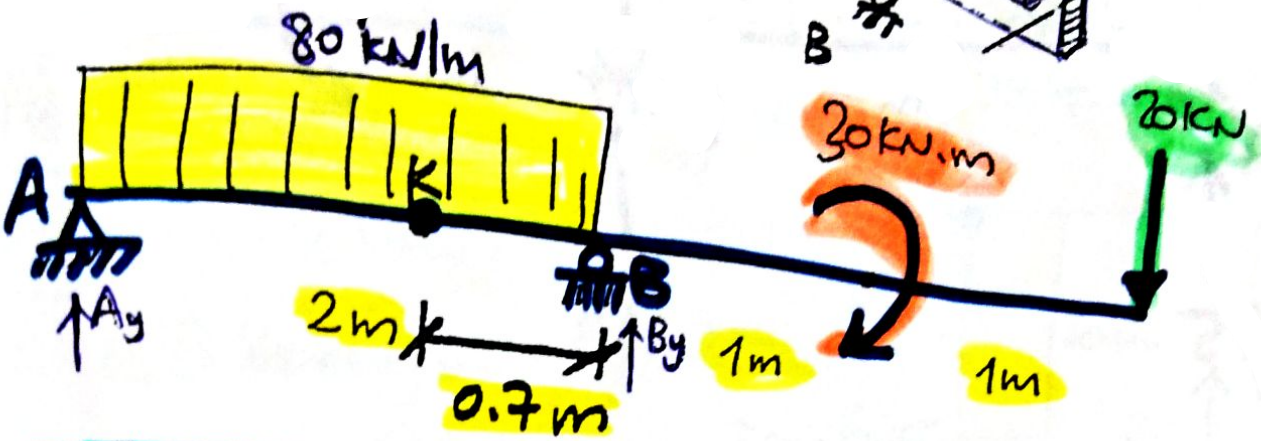
is the only section modulus for the cross section.

For a beam of rectangular cross section with width b and height h (Fig. 5-12a), the moment of inertia and section modulus are

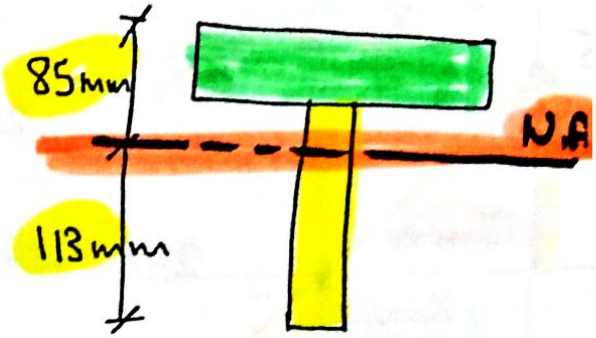
$$I = \frac{bh^3}{12} \qquad S = \frac{bh^2}{6} \tag{5-18a,b}$$

Bending Moment





* Find the $(\sigma_K)_{max}$?



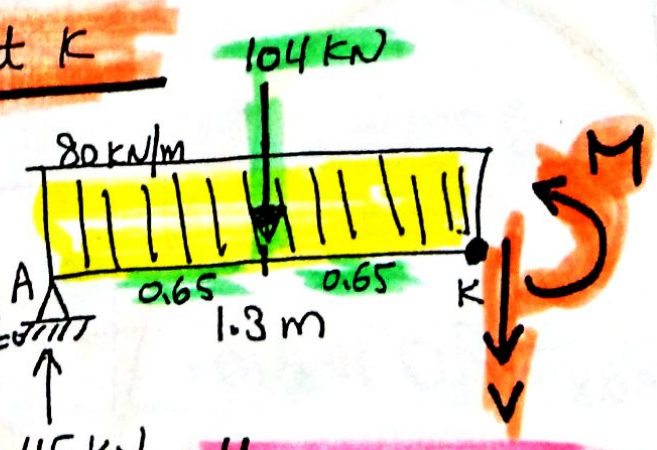
$\sum M_A = 0 \Rightarrow 30 + 20(4) + 160(1) - 2B_y = 0$

$\Rightarrow B_y = 135\ kN.$

$\sum F_y = 0 \Rightarrow A_y - 160 + 135 - 20 = 0$

$A_y = 45\ kN.$

Section at K



$\sum M_{cut} = 0$

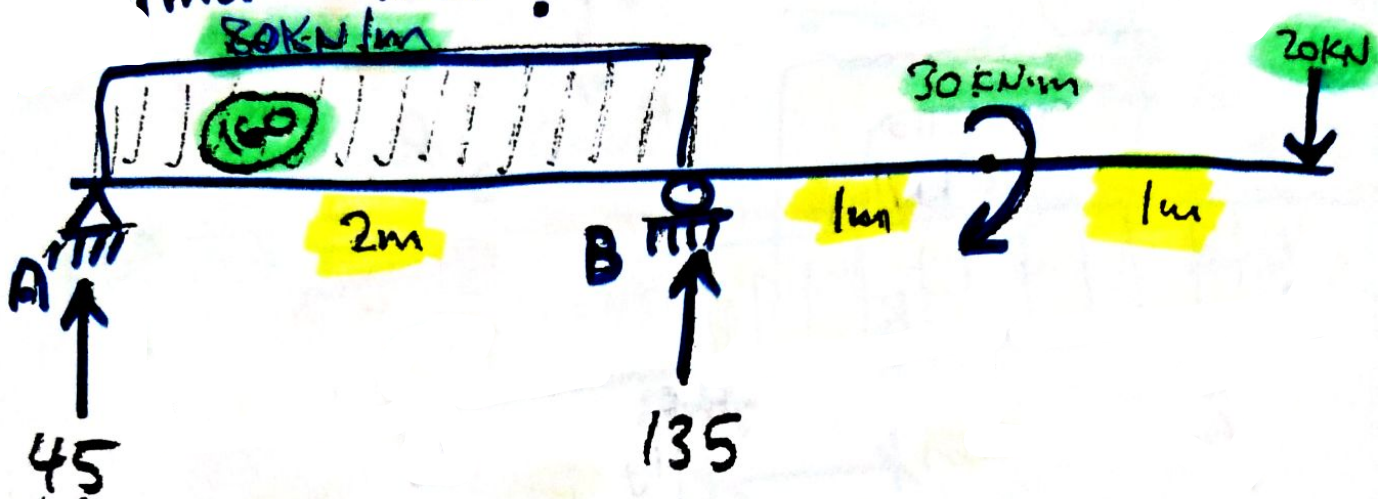
$-M - 104(0.65) + 45(1.3) = 0$

$M_K = -9.1\ kN.m$

$y_{max} = 113\ mm.$

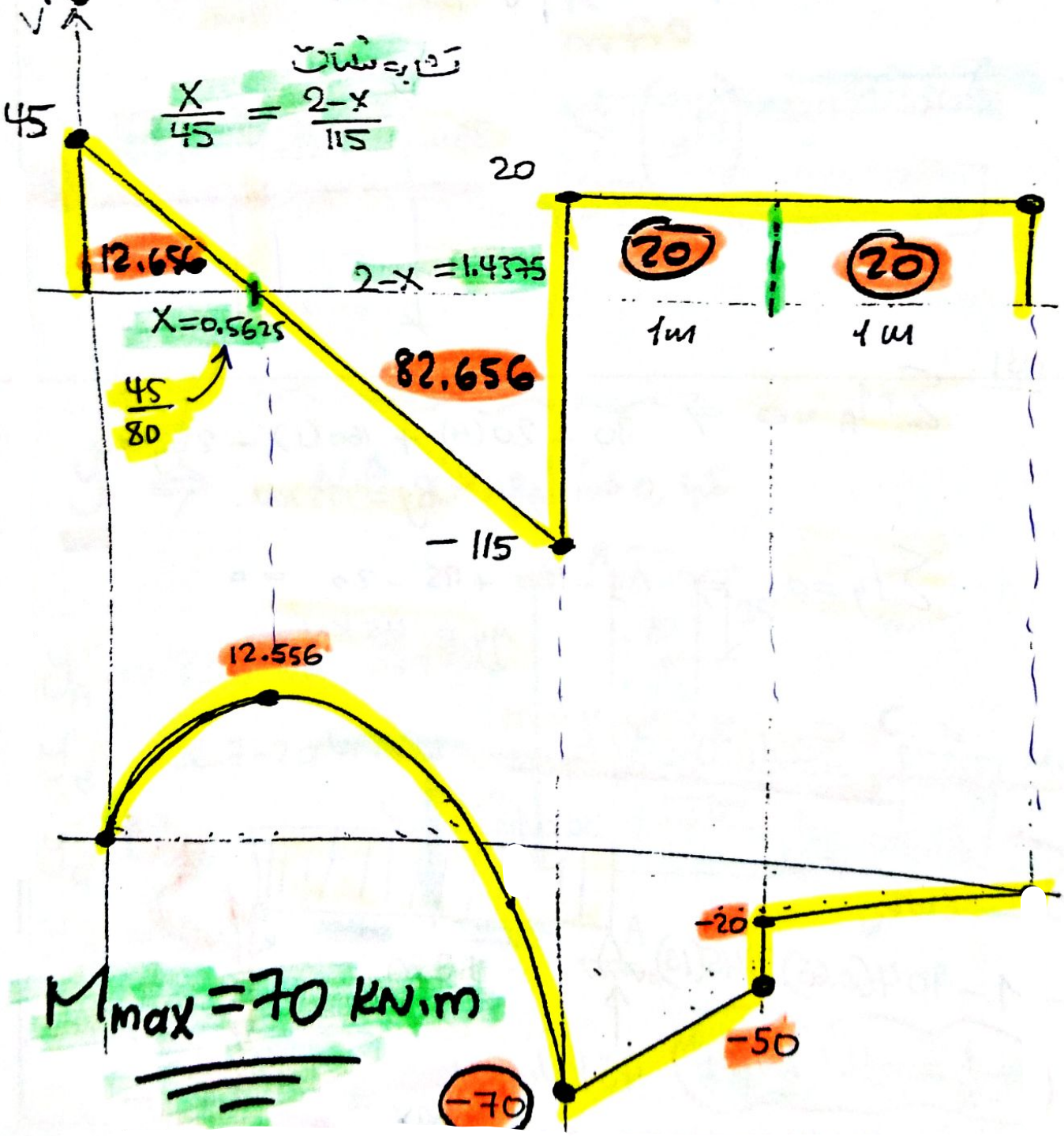
Now

Find σ_{max} ?

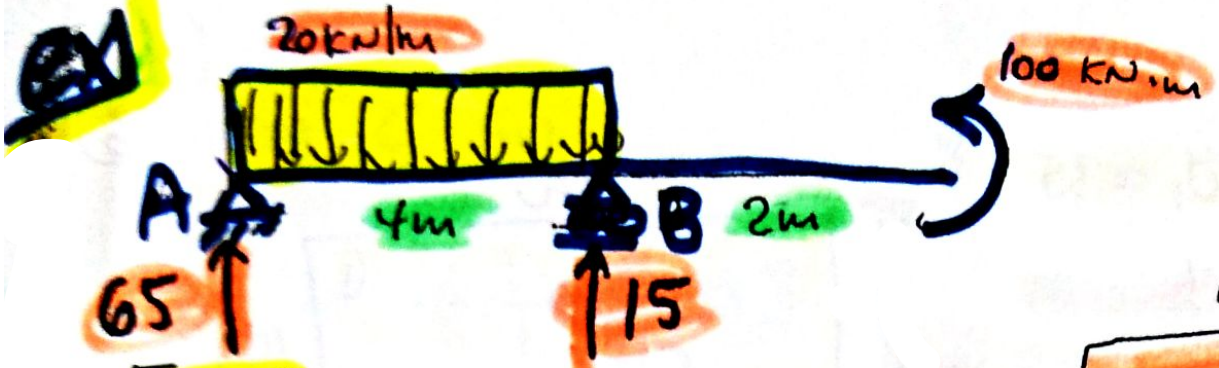


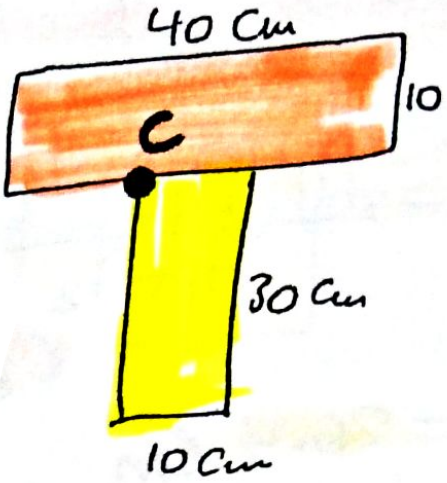
تشابه مثلثات

$$\frac{X}{45} = \frac{2-X}{115}$$



$M_{max} = 70$ KN.m





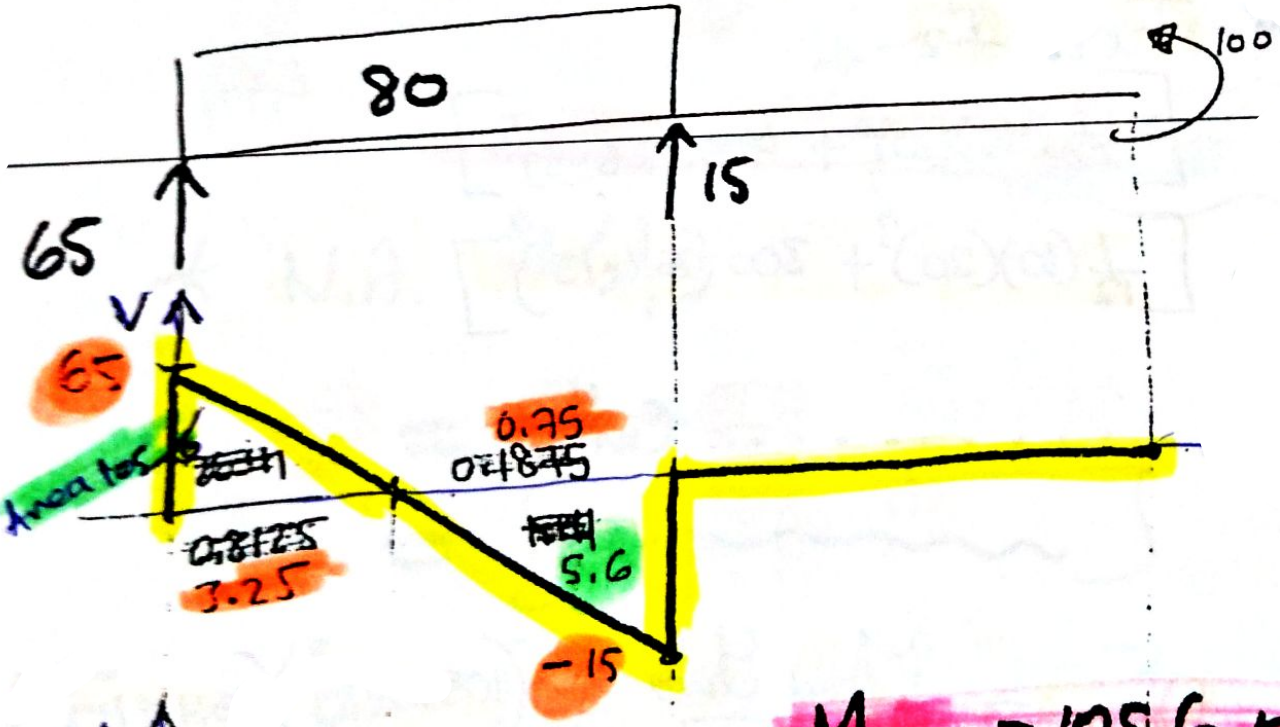
Find ① the Max. bending stress

(σ_{max})

② Beneling stess at C ?

$(\sigma_{max})_C$

* Reactions ✓



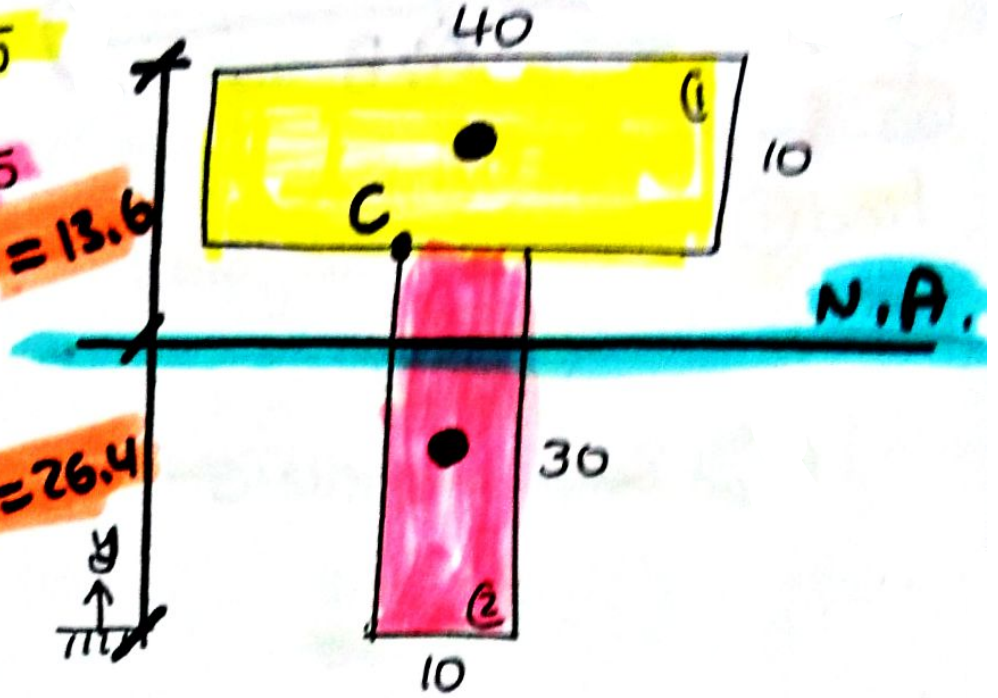
$M_{max} = 105.6$ kN.m

$A_1 = 400$, $y_1 = 35$

$A_2 = 300$, $y_2 = 15$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$\bar{y} = 26.4$ Cm



$$I = I_1 + I_2$$

$$= \left[\frac{1}{12}(40)(10)^3 + 400(13.6-5)^2\right]$$

$$+ \left[\frac{1}{12}(10)(30)^3 + 300(26.4-15)^2\right]$$

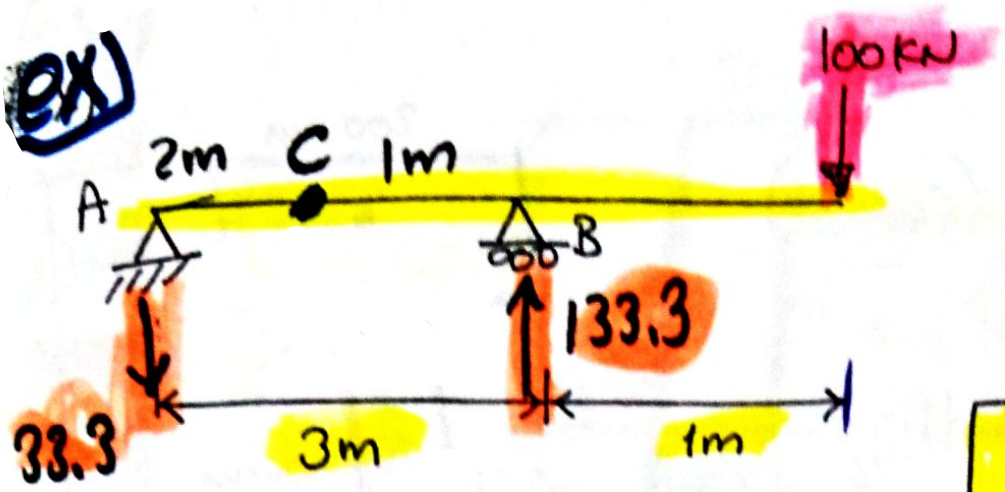
$$= \ldots\ldots\ldots\ldots \text{ Cm}^4$$

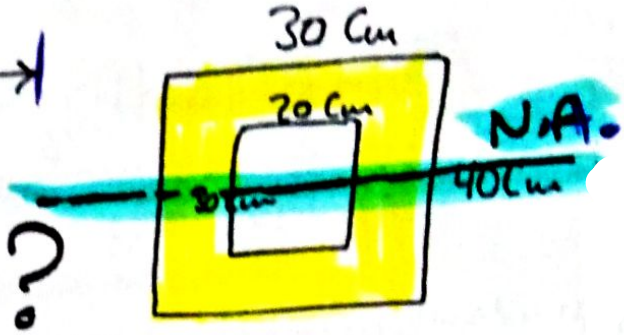
① $$\sigma_{max} = \frac{M_{max}\, y_{max}}{I} = \frac{(105.6 \times 10^3)(26.4 \times 10^{-2})}{(\ldots\ldots) \times 10^{-8}}$$

$$= \ldots\ldots\ldots \text{ Pa}$$

② $$(\sigma_{max})_C = \frac{M_{max}\, y_C}{I} = \frac{(105.6 \times 10^3)(3.6 \times 10^{-2})}{\ldots\ldots \times 10^{-8}}$$

ex)





Find σ at point C ?

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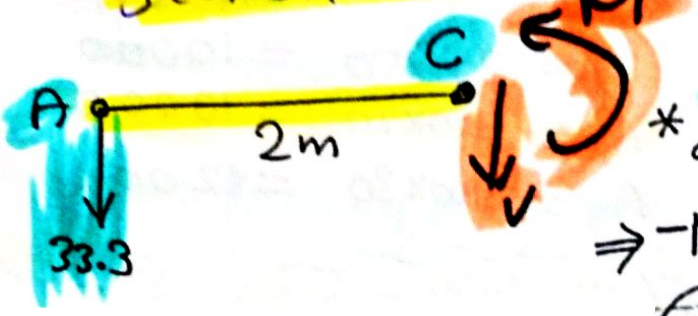
Section at C



$* \sum M_{cut} = 0$

$\Rightarrow -M - 33.3(2) = 0$

$M_c = -66.6 \text{ KN.m}$

* N.A. في الوسط

$$* \quad I = I_1 - I_2$$

$$= \left[\frac{1}{12}(30)(40)^3 \right] - \left[\frac{1}{12}(20)(30)^3 \right]$$

$$= \text{-------} \ Cm^4 .$$

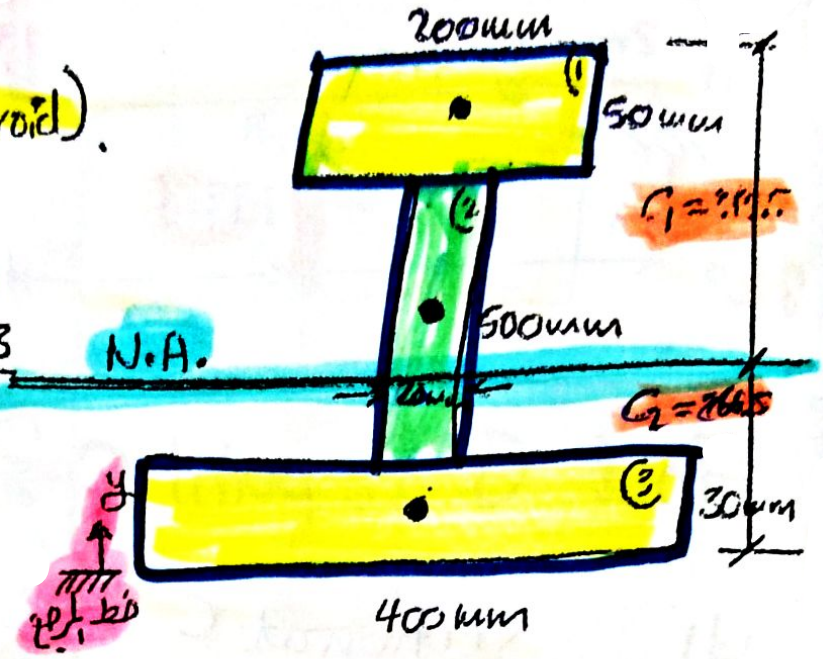
$$* \quad \sigma_c = \frac{M_c \, y_{max}}{I} = \frac{(66.6 \times 10^3)(20 \times 10^{-2})}{\text{-------} \times 10^{-8}}$$

Ex)

Find the

① Neutral Axis (Centroid) (N.A.).

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$



$y_1 = 555$

$y_2 = 280$

$y_3 = 15$

$A_1 = 200 \times 50 = 10000$

$A_2 = 500 \times 20 = 10000$

$A_3 = 400 \times 30 = 12000$

$\Rightarrow \bar{y} = 266.5 \text{ mm}$

② the moment of Inertia about the (N.A).

$$I = I_1 + I_2 + I_3$$

$$= \left[\frac{1}{12}(200)(50)^3 + 10000(313.5-25)^2\right]$$

$$+ \left[\frac{1}{12}(20)(500)^3 + 10000(313.5-300)^2\right]$$

$$+ \left[\frac{1}{12}(400)(30)^3 + 12000(266.5-15)^2\right]$$

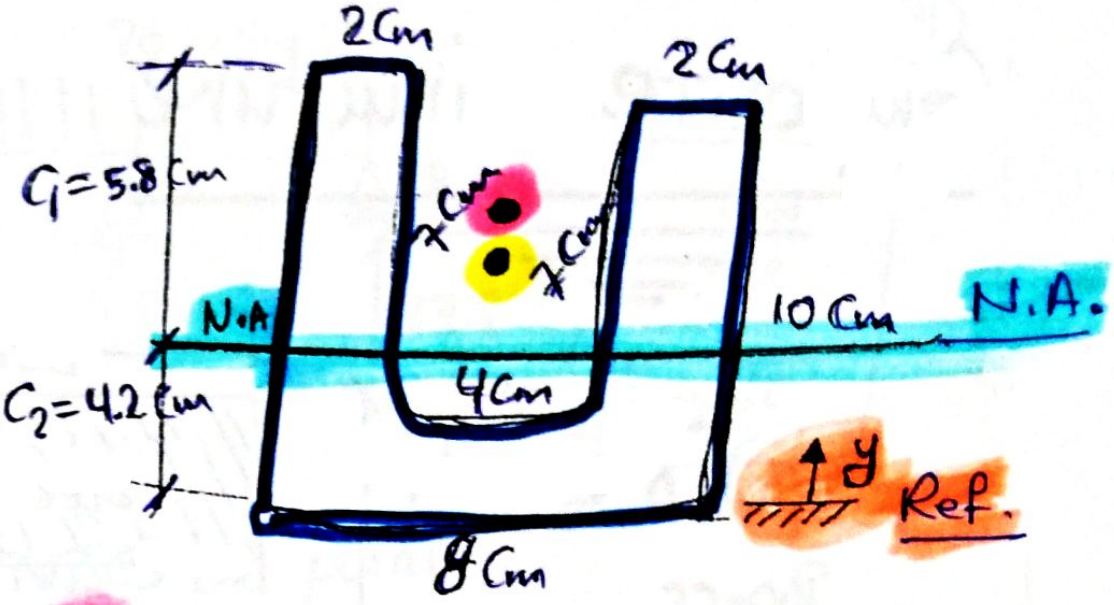
$$= \ldots\ldots\ldots \text{ mm}^4$$

Ex)

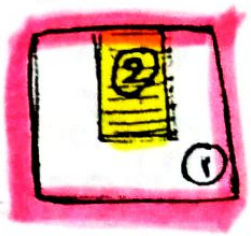
Find

1) N.A ?

2) I ?



$$\bar{y} = \frac{y_1 A_1 - y_2 A_2}{A_1 - A_2}$$



$y_1 = 5$

$y_2 = 6.5$

$A_1 = 80$

$A_2 = 28$

$\Rightarrow \bar{y} = 4.2$ Cm

$$I = I_1 - I_2$$

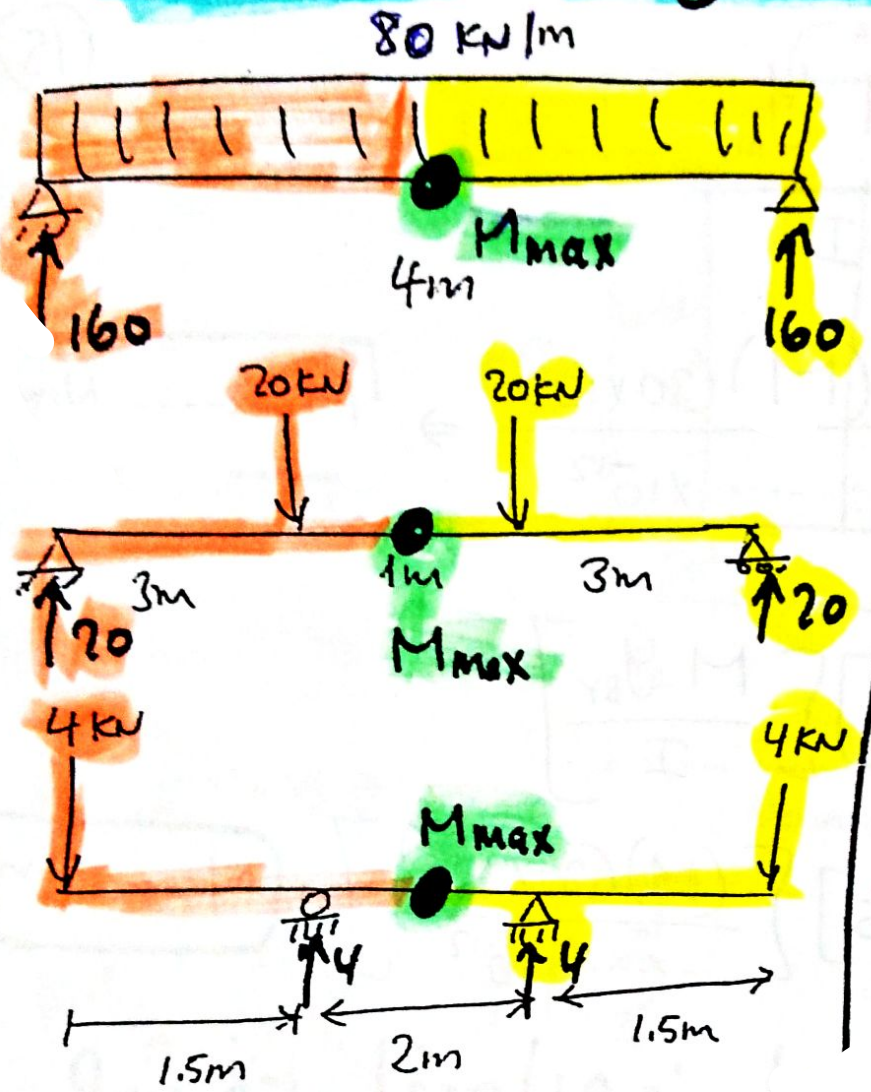
$$= \left[\frac{1}{12}(8)(10)^3 + 80\,(5.8-5)^2\right]$$

$$- \left[\frac{1}{12}(4)(7)^3 + 28\,(5.8-3.5)^2\right]$$

$$= \text{------ } Cm^4 .$$

Notes

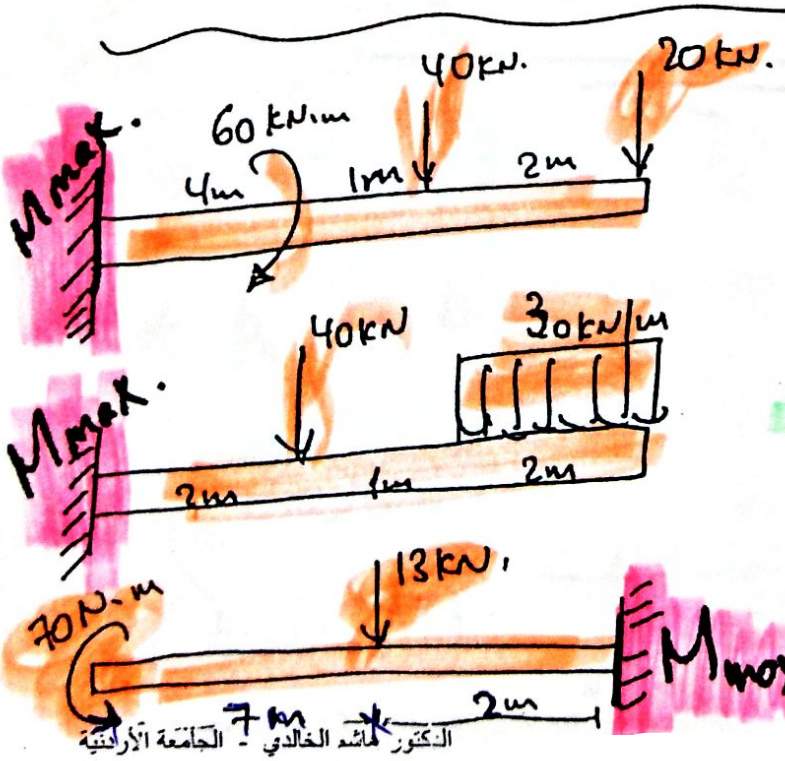
لاحظ التماثل لحساب (M_{max})



$M_{max} = 160$

$M_{max} = 60$

$M_{max} = 6$



$M_{max} = -20(7) - 40(5) - 60$

$M_{max} = -60(4) - 40(2)$

$M_{max.} = 96$ KN.m.

Bending الانثناء

Normal

$$\sigma = \frac{M y}{I_{N.A.}}$$

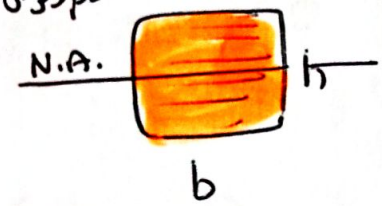
Shear

$$\tau = \frac{V Q}{I_{N.A.} t}$$

$\sigma \equiv$ Normal Bending stress (Pa)
$\tau \equiv$ shear Bending stress (Pa)
$M \equiv$ Bending Moment. (N.m)
$V \equiv$ shear force. (N)
$Q \equiv$ Moment Area. (m^3)
$I \equiv$ Moment of Inertia (m^4)
$t \equiv$ Thickness (m)
$y \equiv$ distance from N.A (m)

محور الاتزان

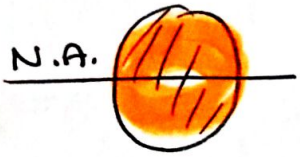
$$I = \frac{1}{12} b h^3$$



$$A = b h$$

$$I = \frac{\pi}{4} r^4$$

$$I = \frac{\pi}{64} d^4$$

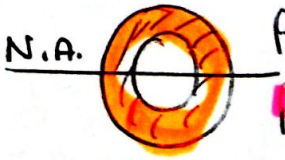


$$A = \pi r^2$$

$$A = \frac{\pi}{4} d^2$$

$$I = \frac{\pi}{4}\left(r_o^4 - r_i^4\right)$$

$$I = \frac{\pi}{64}\left(d_o^4 - d_i^4\right)$$



$$A = \pi\left(r_o^2 - r_i^2\right)$$

$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right)$$

$$I_{N.A} = I + A d^2$$

$d \Rightarrow$ البعد المركز المساحة عن (N.A)

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + \cdots}{A_1 + A_2 + A_3 + \cdots}$$

المركز المساحة (centroid) (N.A.)

Bending shear stress

$$\tau = \frac{VQ}{It}$$

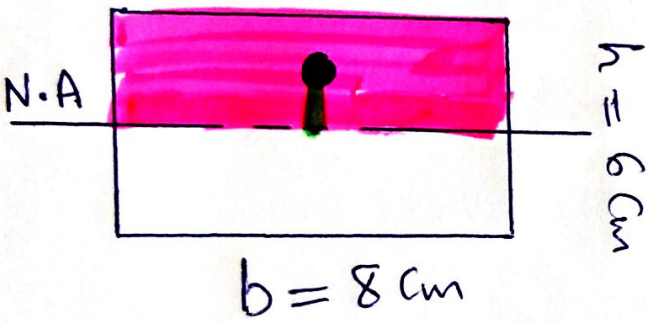
$\tau \equiv$ Bending shear stress

$V \equiv$ Shear Force

$Q =$ Moment Area

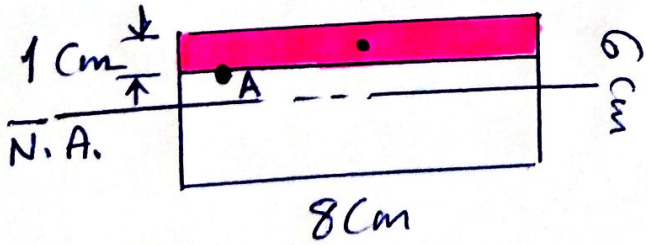
$I \equiv$ Area moment of Inertia

$t \equiv$ thickness / cross-section

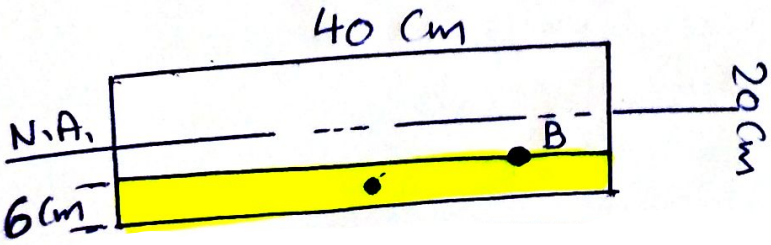


$$Q_{max} = Q_{N.A.}$$
$$= (\text{Area})(\text{Arm})$$
$$= \left(\frac{1}{2}bh\right)\left(\frac{h}{4}\right)$$
$$= \frac{1}{8}bh^2$$
$$= \frac{1}{8}(8)(6)^2 = 36\ Cm^3$$

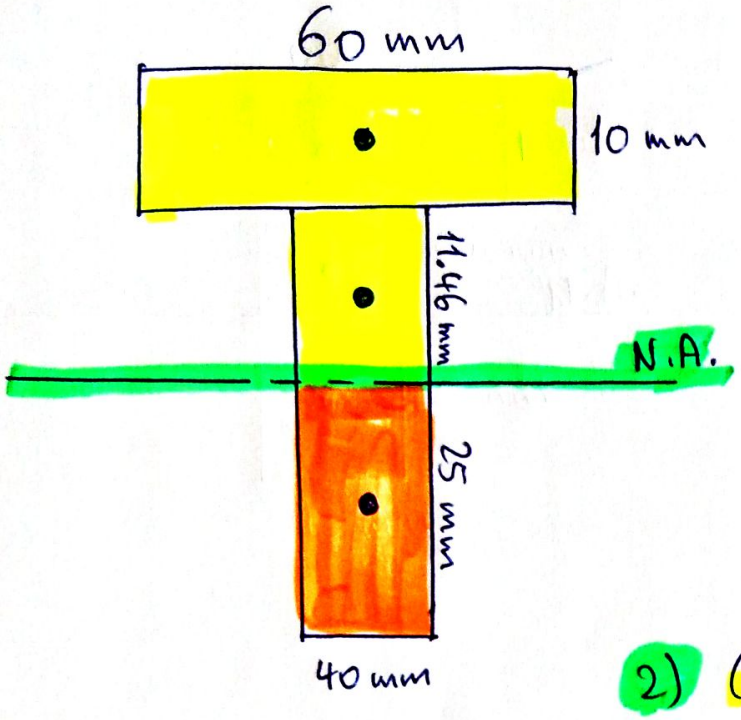
* the Arm is the distance from the Centroid point of the shaded Area to the nuetral axis(line) N.A.



$$Q_A = (\text{Area})(\text{Arm})$$
$$= (8*1)(2.5) = 20\ Cm^3$$



$$Q_B = (\text{Area})(\text{Arm})$$
$$= (40*6)(7)$$
$$= 1680\ Cm^3$$



To find $Q_{max} = Q_{N.A}$:

1) Consider the Area under the N.A :

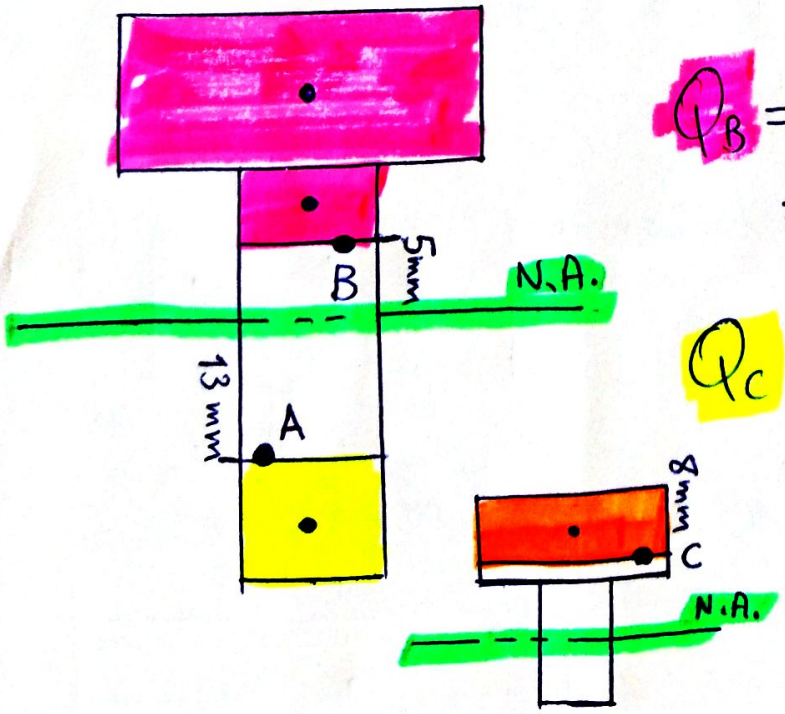
$$Q_{max} = (Area)(Arm)$$
$$= (40)(25)(12.5)$$
$$= 12500 \ mm^3$$

2) Consider Now the Area above the N.A.

$$Q_{max} = (Area)(Arm) + (Area)(Arm)$$
$$= (60)(10)(16.46) + (40)(11.46)\left(\frac{11.46}{2}\right)$$
$$= 12500 \ mm^3$$

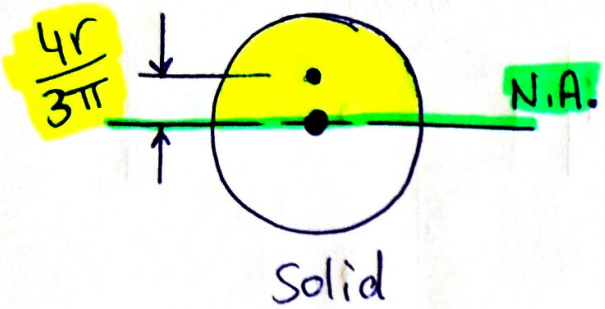
⇒ The Same answer

$$Q_A = (40)(12)(19) = 9120 \ mm^3$$



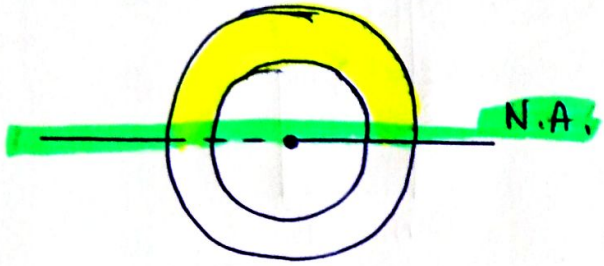
$$Q_B = (60)(10)(16.46) + (40)(6.46)(8.23)$$
$$= 12002.6 \ mm^3$$

$$Q_C = (60)(8)(17.46)$$
$$= 8380.8 \ mm^3 .$$



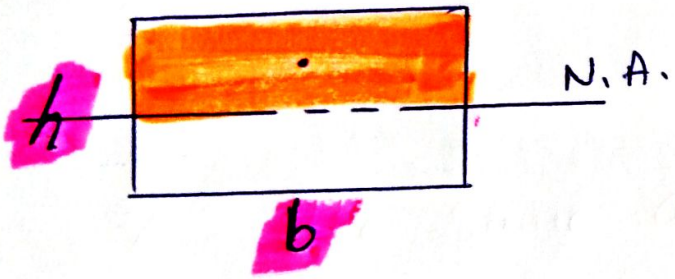
Solid

$$Q_{max} = (\text{Area})(\text{Arm})$$
$$= \left(\frac{\pi}{2} r^2\right)\left(\frac{4r}{3\pi}\right)$$
$$= \frac{2}{3} r^3$$

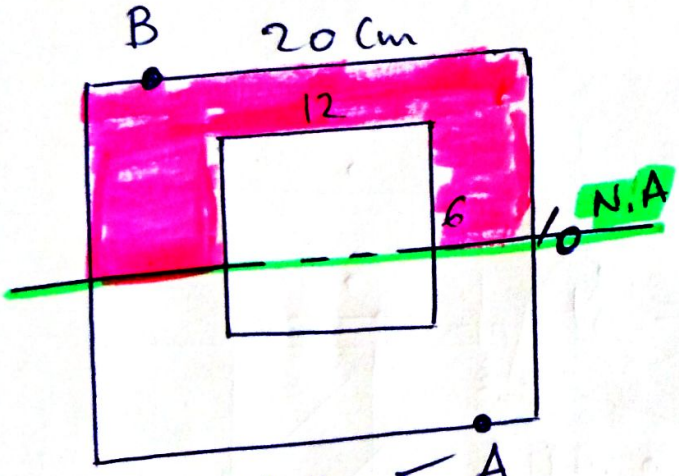


Hollow

$$Q_{max} = \frac{2}{3}\left(r_o^3 - r_i^3\right)$$



$$Q_{max} = \left(\frac{1}{2} bh\right)\left(\frac{h}{4}\right)$$
$$= \frac{1}{8} bh^2$$



Hollow

$$Q_{max} = \frac{1}{8}(20)(10)^2 - \frac{1}{8}(12)(6)^2$$
$$= 196 \text{ Cm}^3$$

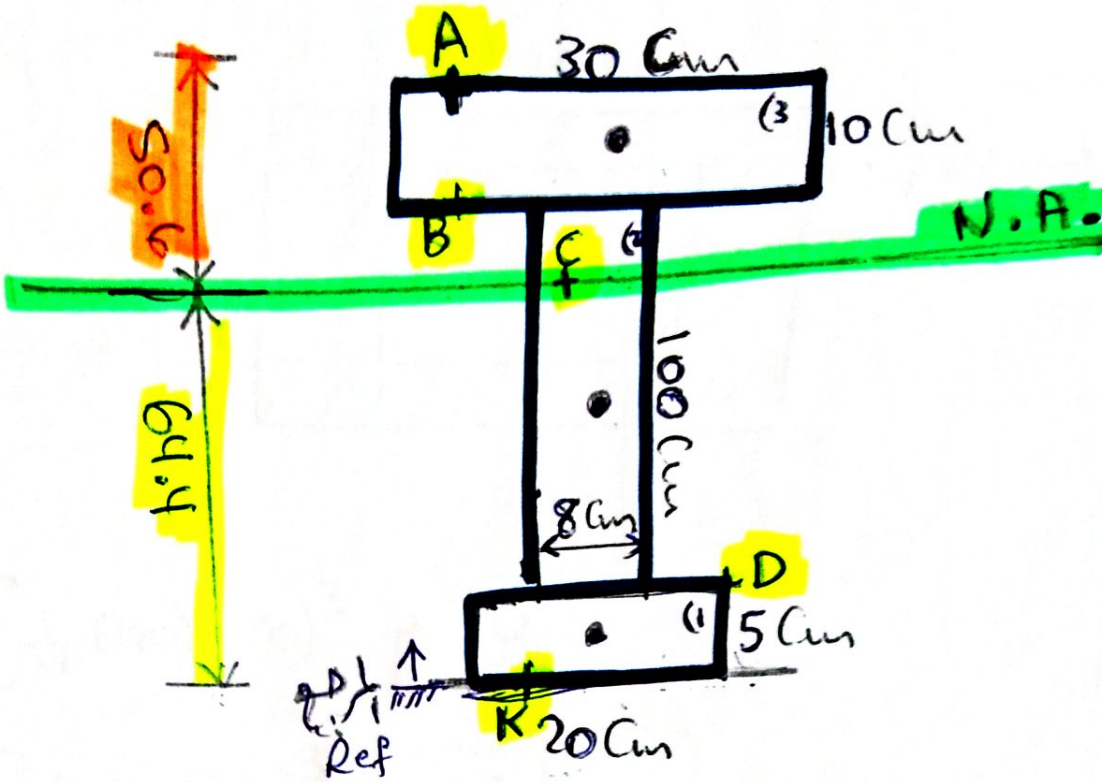
* Q_A = Zero
* Q_B = zero

Ex

Find

① N.A

② $I_{N.A.}$



① $y_1 = 2.5$ Cm (5/2) $\qquad A_1 = 100$

$y_2 = 55$ Cm (50+5) $\qquad A_2 = 800$

$y_3 = 110$ Cm (5+100+5) $\qquad A_3 = 300$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = 64.4 \text{ Cm}$$

② $$I_{N.A.} = \left[\frac{1}{12}(20)(5)^3 + 100(61.9)^2\right] + \left[\frac{1}{12}(8)(100)^3 + 800(9.4)^2\right] + \left[\frac{1}{12}(30)(10)^3 + 300(45.6)^2\right] = \text{------ Cm}^4$$

$d = 64.4 - 2.5$

distance (y)

$y_A = 50.6$

$y_B = 40.6$

$y_C = 0$

$y_D = 59.4$

$y_K = 64.4$

$y_{max} = 64.4$

السماكة

Thickness (t)

$t_A = 30$

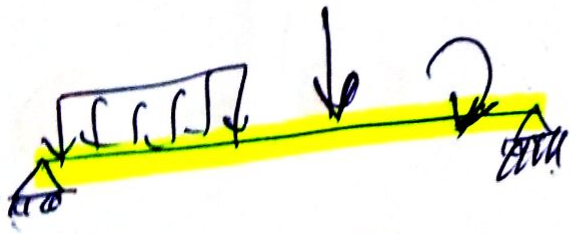
$t_B = 30$

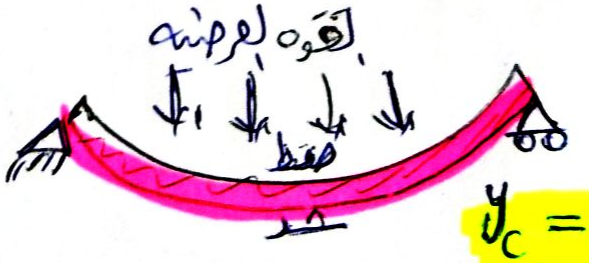
$t_C = 8$

$t_D = 20$

$t_K = 20$

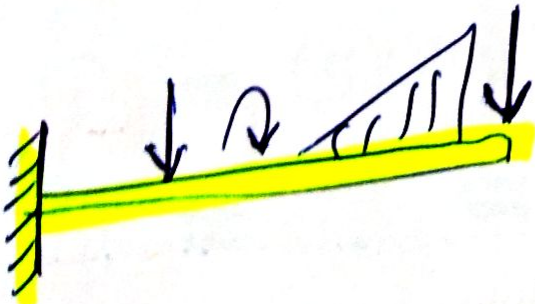
$t_{N.A} = 8$

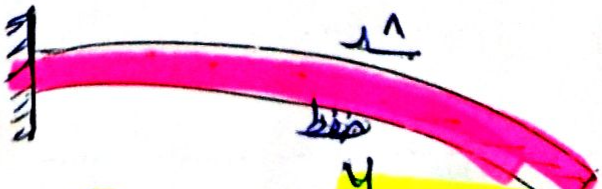




$y_c = 50.6$

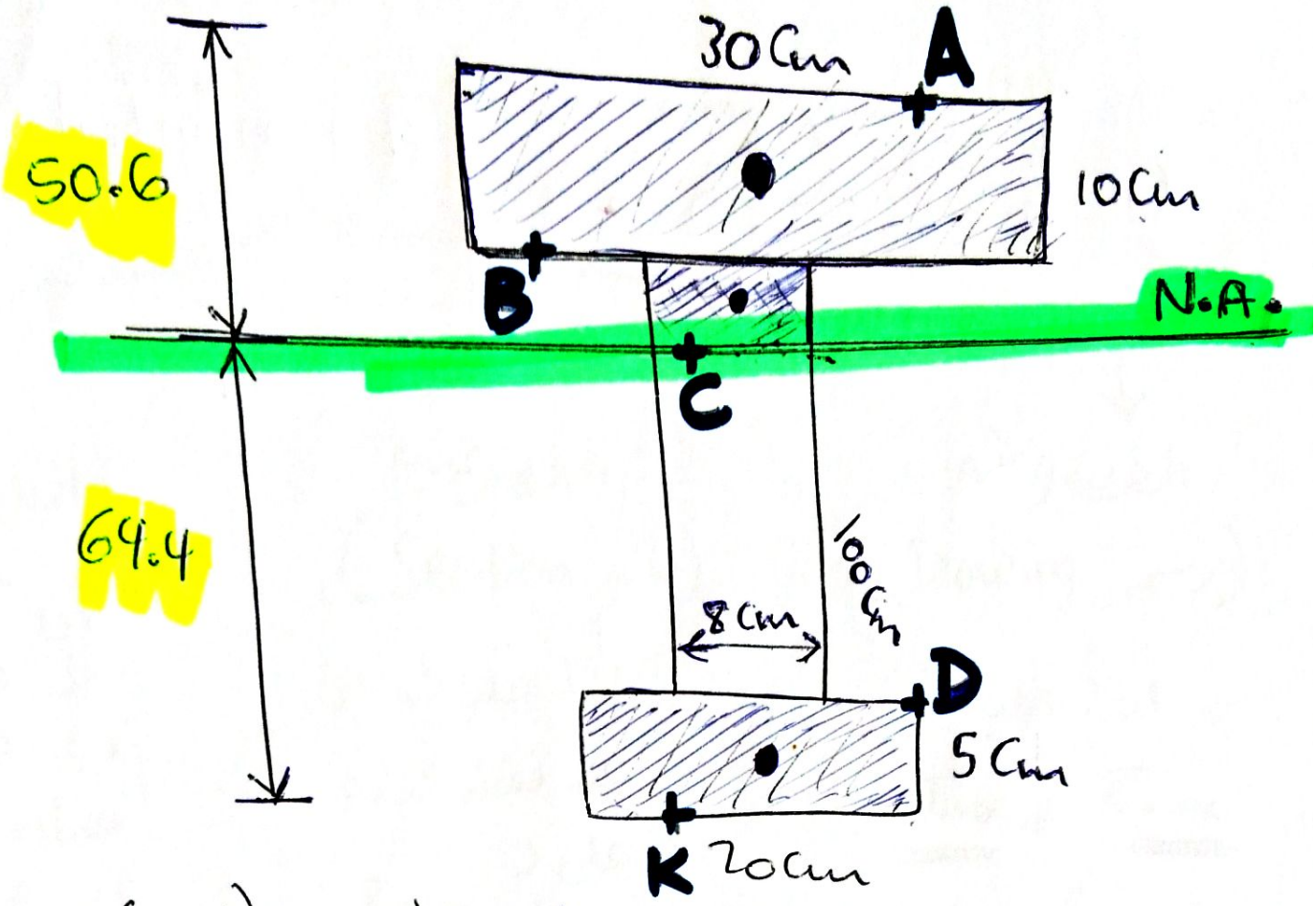
$y_t = 64.4$





$\sigma = \frac{M\,y}{I}$ → $y_c = 64.4$

$y_t = 50.6$



الابتعاد والهروب من (N.A)

* $Q_B = (\text{Area})(\text{distance})$

$= (30)(10)(45.6) = 13680 \ Cm^3$

* $Q_A = (0)$

* $Q_K = 0$

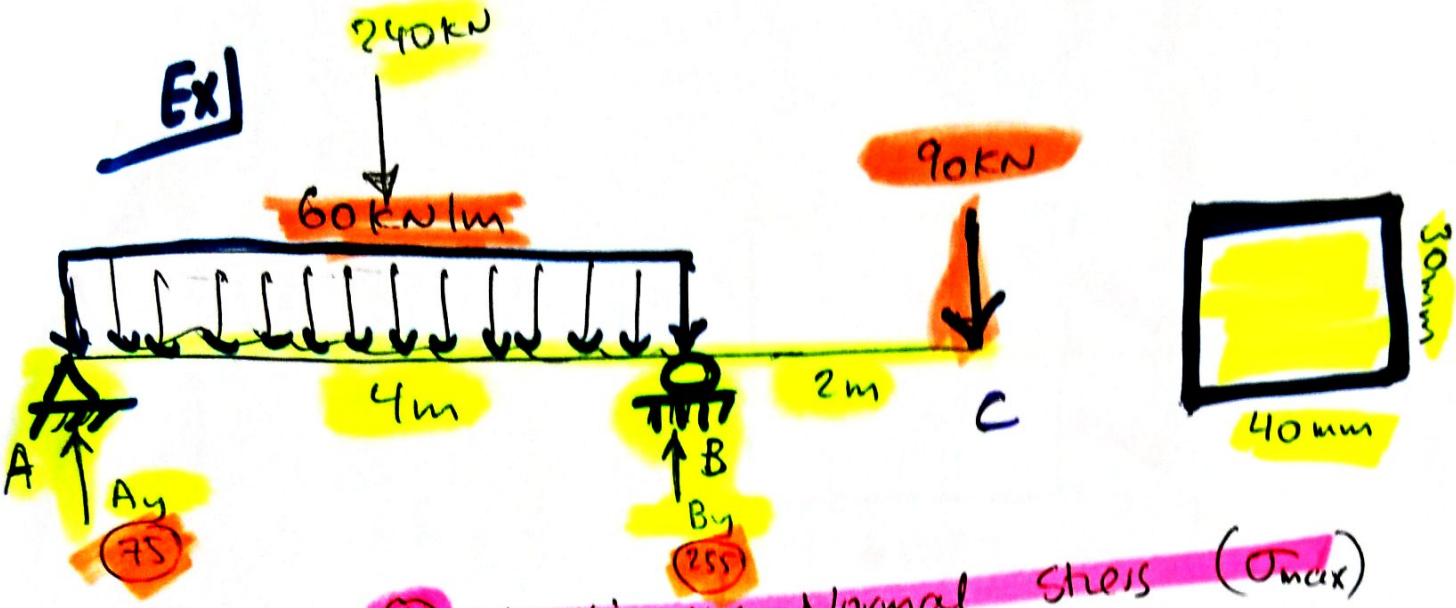
* $Q_D = (5)(20)(61.9) = 6190 \ Cm^3$.

* $Q_C = Q_{N.A} = Q_{max}$.

من الأعلى $\rightarrow = (30)(10)(45.6) + (40.6)(8)(20.3) = 20273.44 \ Cm^3$

أو من الأسفل $\rightarrow = (20)(5)(61.9) + (51.4)(8)(29.7) = 20273.44 \ Cm^3$

Ex



Find
1. maximum Normal stress (σ_{max})
2. maximum shear stress (τ_{max})

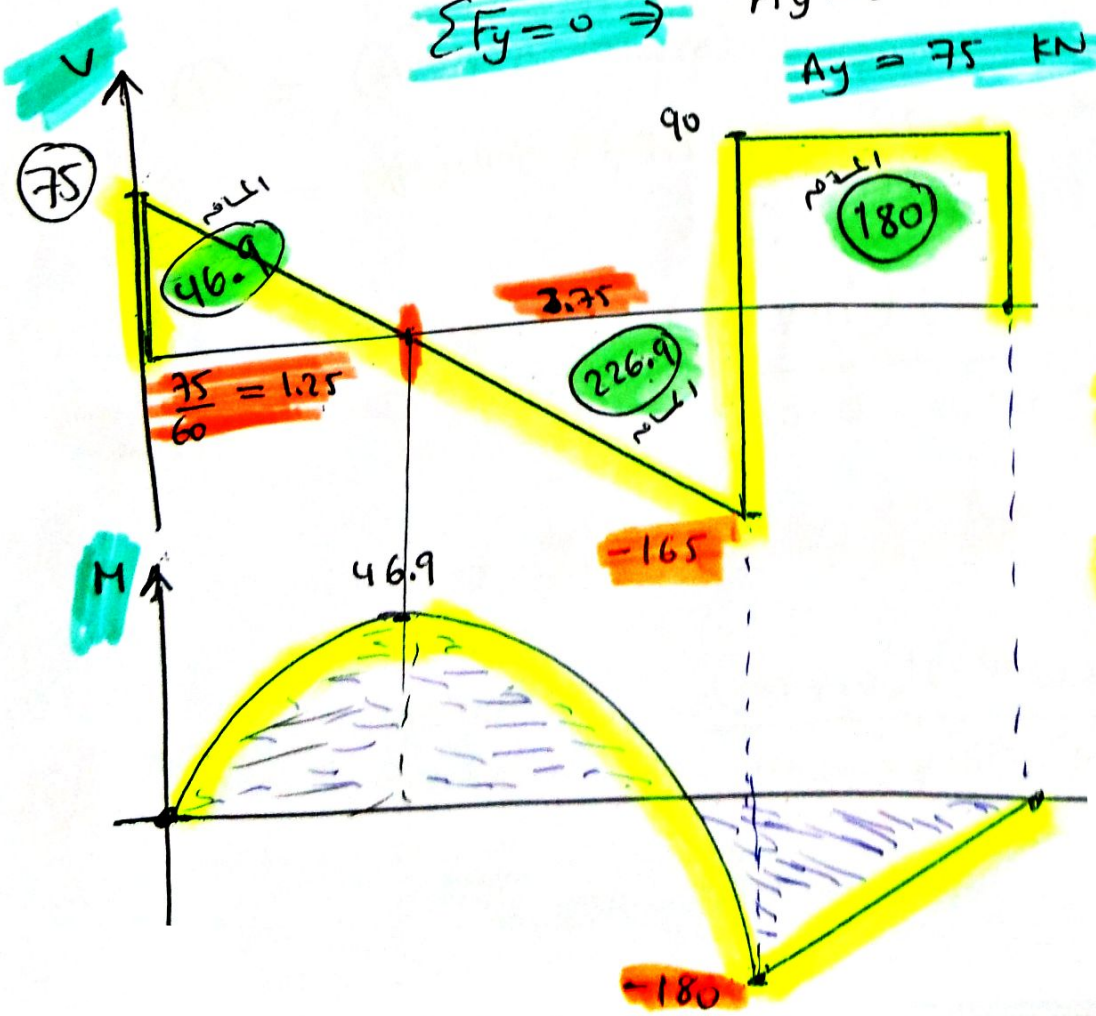
ستاتيكا $\sum M_A = 0 \Rightarrow 4B_y - 240(2) - 90(6) = 0$

$B_y = 255$ KN

$\sum F_x = 0 \Rightarrow A_x = 0$

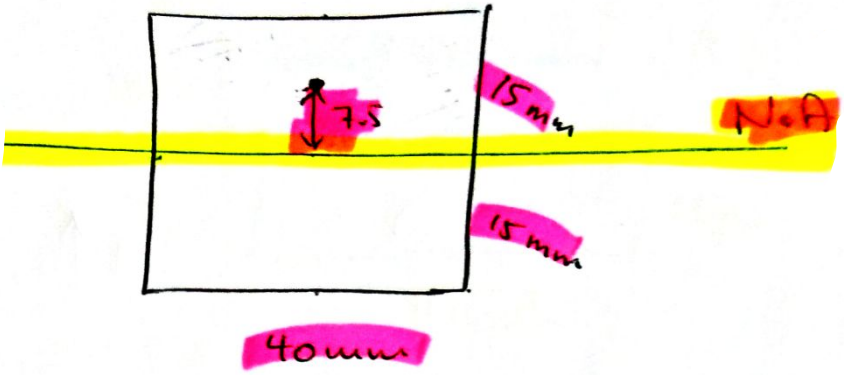
$\sum F_y = 0 \Rightarrow A_y - 240 + 255 - 90 = 0$

$A_y = 75$ KN.



$V_{max} = 165$ KN.

$M_{max} = 180$ KN.m



* $I = \frac{1}{12} b h^3 = \frac{1}{12}(40)(30)^3 = 90000 \ mm^4$.

* $y = 15 \ mm$.

* $b = 40 \ mm$ عند N.A

* $Q = (\text{Area})(\text{distance})$ (المساحة) (المسافة)

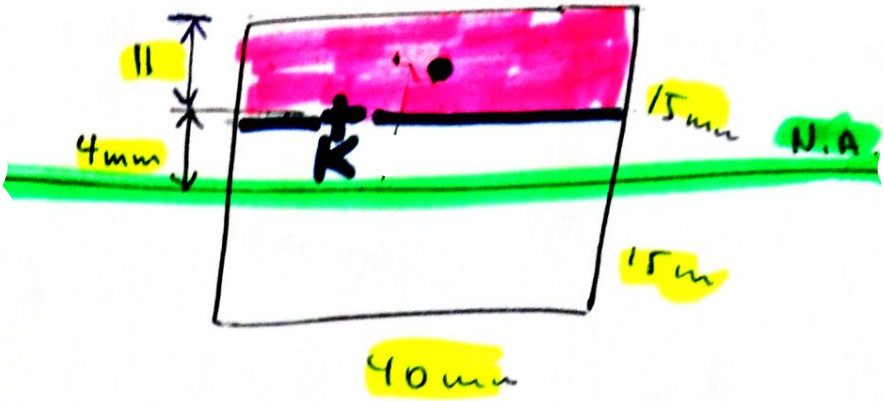
$= (15)(40)(7.5) = 4500 \ mm^3$

1) $\sigma_{max} = \frac{My}{I} = \frac{(180 * 10^3)(15 * 10^{-3})}{90000 * 10^{-12}}$

$\Rightarrow \sigma_{max} = 30 \ GPa$.

2) $\tau_{max} = \frac{VQ}{Ib} = \frac{(165 * 10^3)(4500 * 10^{-9})}{(90000 * 10^{-12})(40 * 10^{-3})}$

$\tau_{max} = 206.25 \ MPa$



3) Find Normal stress at point K ⇒ (σ_K) ?

$$\sigma_K = \frac{M y_K}{I}$$

$$= \frac{(180 * 10^{3})(4 * 10^{-3})}{90000 * 10^{-12}} = --$$

$$= 8 \text{ GPa.}$$

4) Find shear stress at point K ⇒ (τ_K) ?

$$\tau_K = \frac{V Q_K}{I b_K}$$

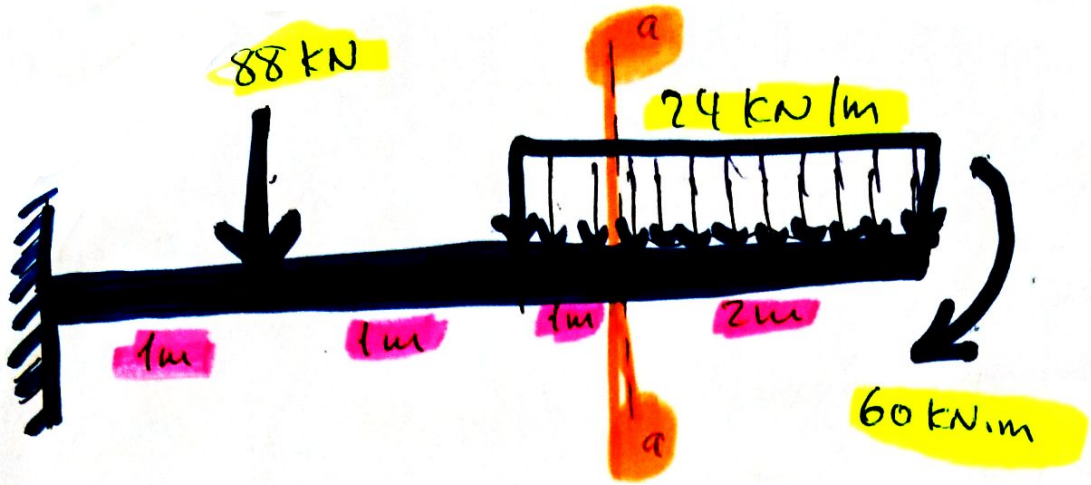
$$= \frac{(165 * 10^{3})(4480 * 10^{-9})}{(90000 * 10^{-12})(40 * 10^{-3})}$$

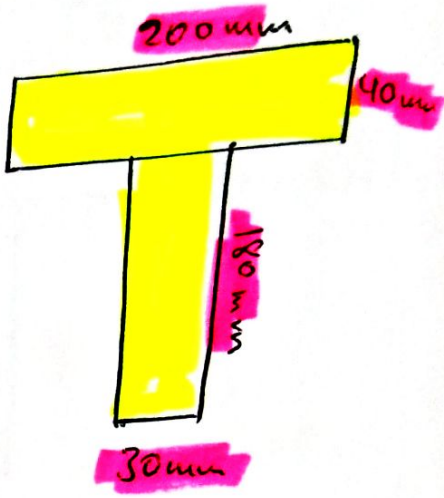
$$= ---- \checkmark$$

$Q_K =$ (المساحة)(المسافة)

$= (40)(11)(9.5)$

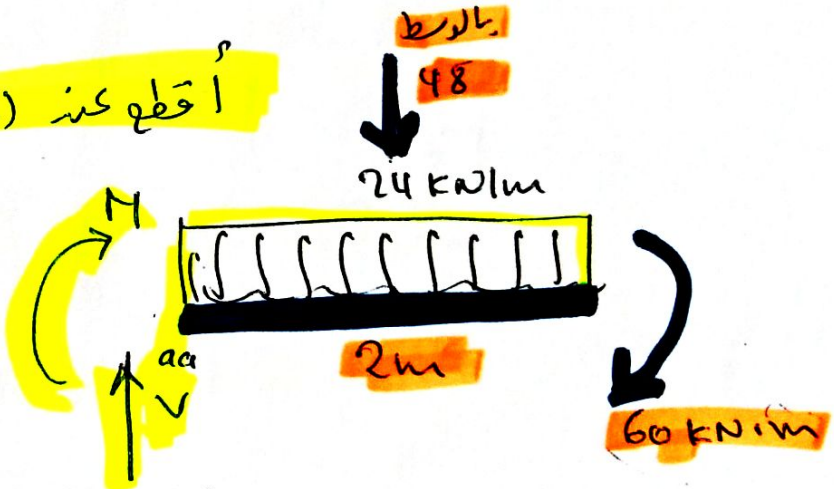
$= 4480 \text{ mm}^3$





Find 1) σ_{max} } at section (a-a).
2) τ_{max}

أقطع عند (a-a)



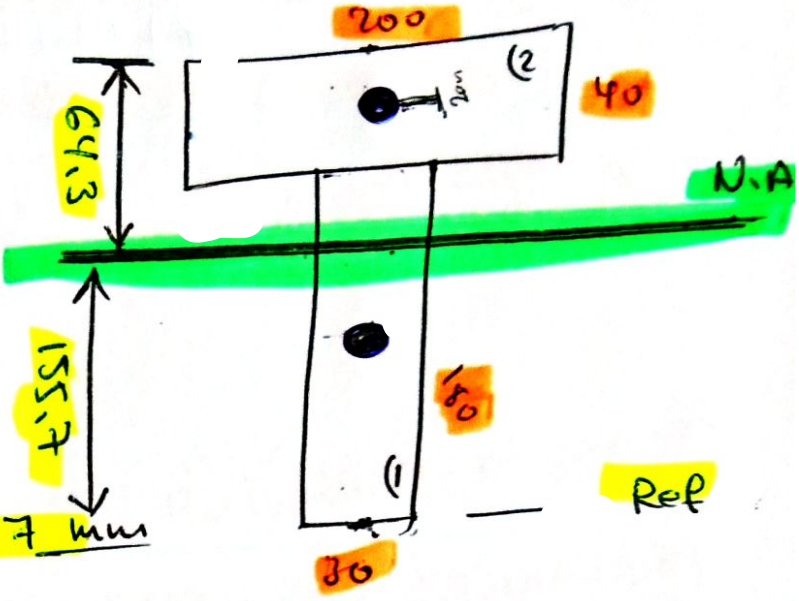
$\sum M_{aa} = 0 \Rightarrow -M - 48(1) - 60 = 0$

$M = -108$ kN.m

$\sum F_y = 0 \Rightarrow +V - 48 = 0 \Rightarrow V = 48$ kN

$y_1 = 90$, $A_1 = 5400$

$y_2 = 200$, $A_2 = 8000$



$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = 155.7 \text{ mm}$$

$$* \quad I = \left[\underbrace{\frac{1}{12}(30)(180)^3}_{\frac{1}{12}bh^3} + \underbrace{5400\,(\overset{y_1}{90} - \overset{\bar{y}}{155.7})^2}_{Ad^2} \right] + \left[\frac{1}{12}(200)(40)^3 + 8000(200 - 155.7)^2 \right]$$

$$\Rightarrow \quad I = 54655632.7 \text{ mm}^4 .$$

$$* \quad Q_{max.} = \underbrace{(155.7)(30)}_{\text{Area}} \underbrace{\left(\frac{155.7}{2}\right)}_{\text{المسافة}} = 363637.4 \text{ mm}^3$$

$$* \quad b_{N.A} = 30 \text{ mm} \Rightarrow \text{N.A. العرض عند}$$

$$* \quad y_{max} = 155.7 \text{ mm}$$

$$\sigma = \frac{My}{I}$$

$$\tau = \frac{VQ}{Ib}$$

1)

$$\sigma_{max} = \frac{My}{I} = \frac{(108 * 10^{3})(155.7 * 10^{-3})}{54655632.7 * 10^{-12}}$$

$$= 307.7 \text{ MPa.}$$ (ضغط Compression)

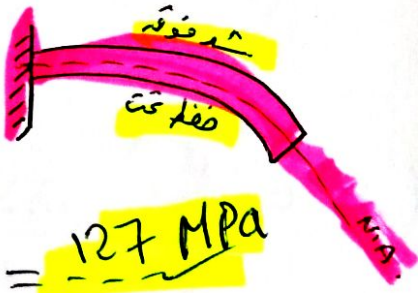
2)

$$\tau_{max} = \frac{VQ}{Ib} = \frac{(48 * 10^{3})(363637.4 * 10^{-9})}{(54655632.7 * 10^{-12})(30 * 10^{-3})}$$

$$= 10.6 \text{ MPa}$$

3)

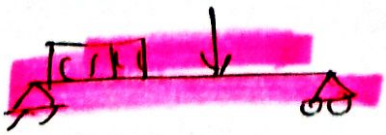
$$(\sigma_{max})_{\text{شد tension}} = \frac{My}{I} \quad (y \to 64.3)$$



$$= \frac{(108 * 10^{3})(64.3 * 10^{-3})}{54655632.7 * 10^{-12}} = 127 \text{ MPa}$$

4)

$$(\sigma_{max.})_{\text{ضغط Compression}} = \frac{My}{I} \quad (y \to 155.7)$$

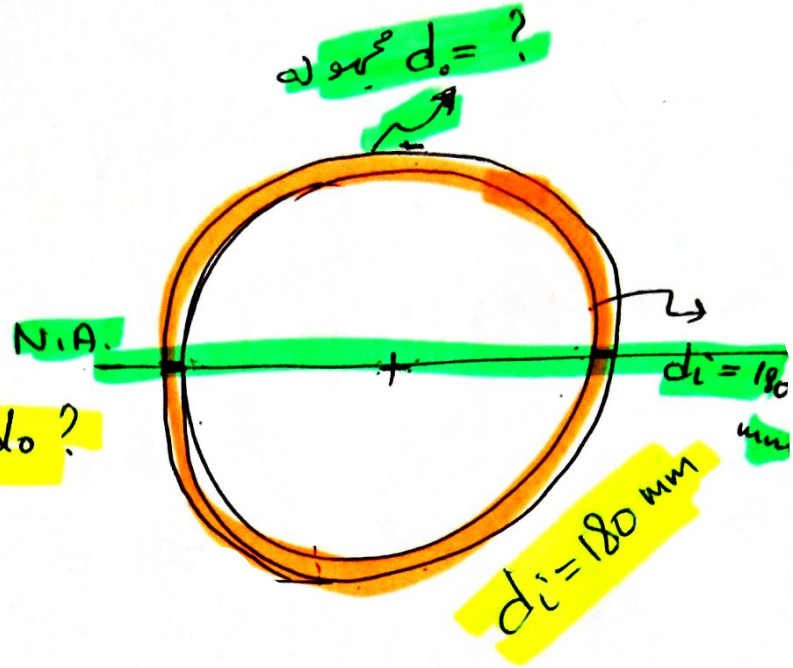


$V = 65$ kN.

$M = 100$ kN.m

ex

$\sigma_{max} = 420$ MPa.



Find ① Outer diameter d_o ?

② τ_{max} ?

$$\sigma = \frac{My}{I}$$

$$420 * 10^6 = \frac{(100 * 10^3)(r_o)}{\frac{\pi}{4}(r_o^4 - 0.09^4)}$$

1)

$$\Rightarrow \quad r_o = \frac{329867228.6}{100 * 10^3}(r_o^4 - 0.09^4)$$

$$\Rightarrow \quad r_o = 3298.6\left[r_o^4 - 0.09^4\right]$$

$$\Rightarrow \quad r_o = 0.0989 \text{ m}$$

$$r_o = 98.9 \text{ mm}$$

$$d_o = 2r_o \Rightarrow \boxed{d_o = 197.8 \text{ mm}}$$

2) * $Q_{max} = \frac{2}{3}\left(r_o^3 - r_i^3\right)$

$$= \frac{2}{3}\left(98.9^3 - 90^3\right) = 158907.8 \text{ mm}^3$$

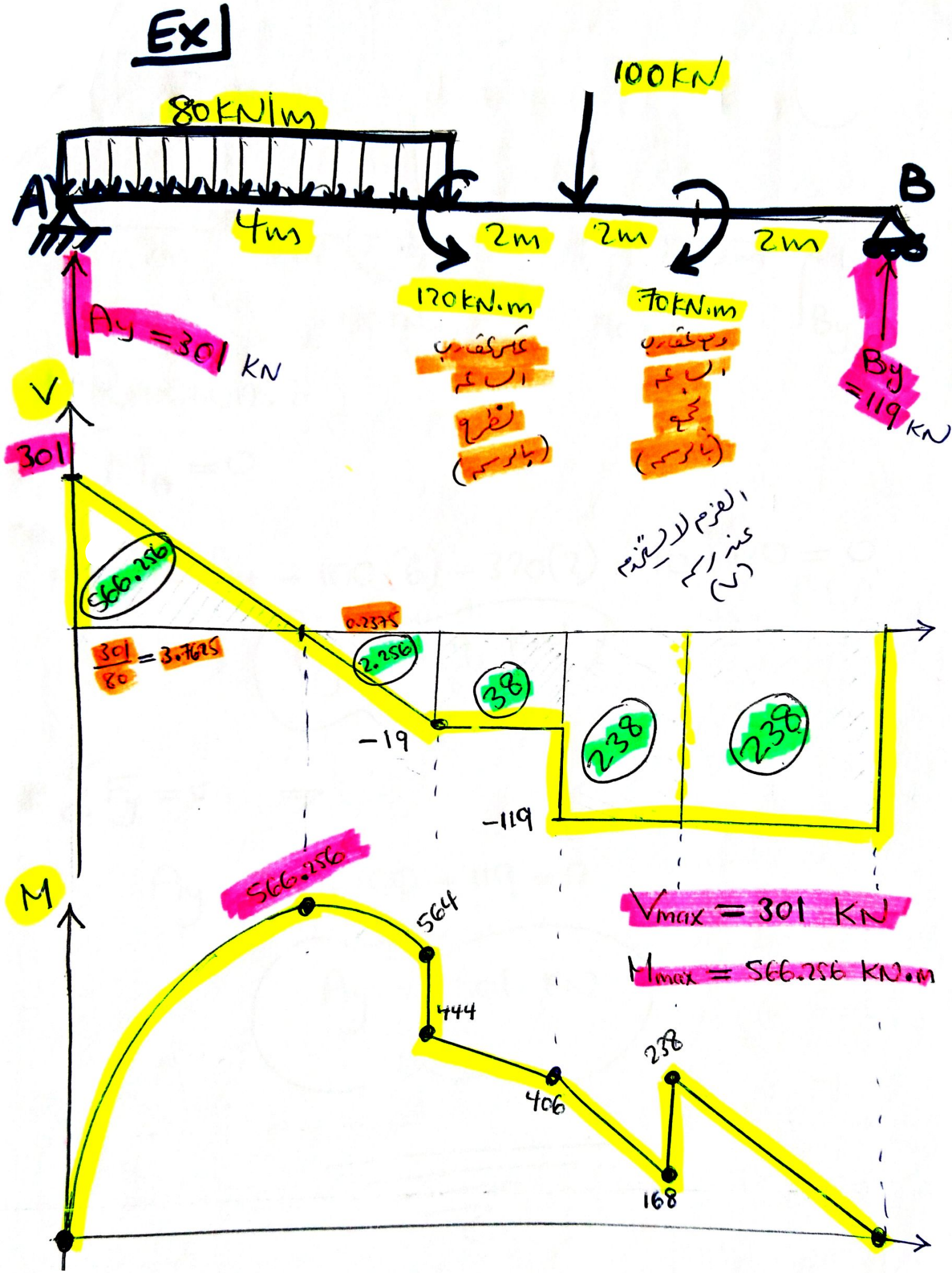
العرض * $b = d_o - d_i = 197.8 - 180 = 17.8 \text{ mm}$
N.A.

$$\Rightarrow \tau = \frac{V\,Q}{I\,b}$$

$$= \frac{(65 * 10^3)(158907.8 * 10^{-9})}{\frac{\pi}{4}\left(0.989^4 - 0.09^4\right)\left(17.8 * 10^{-3}\right)}$$

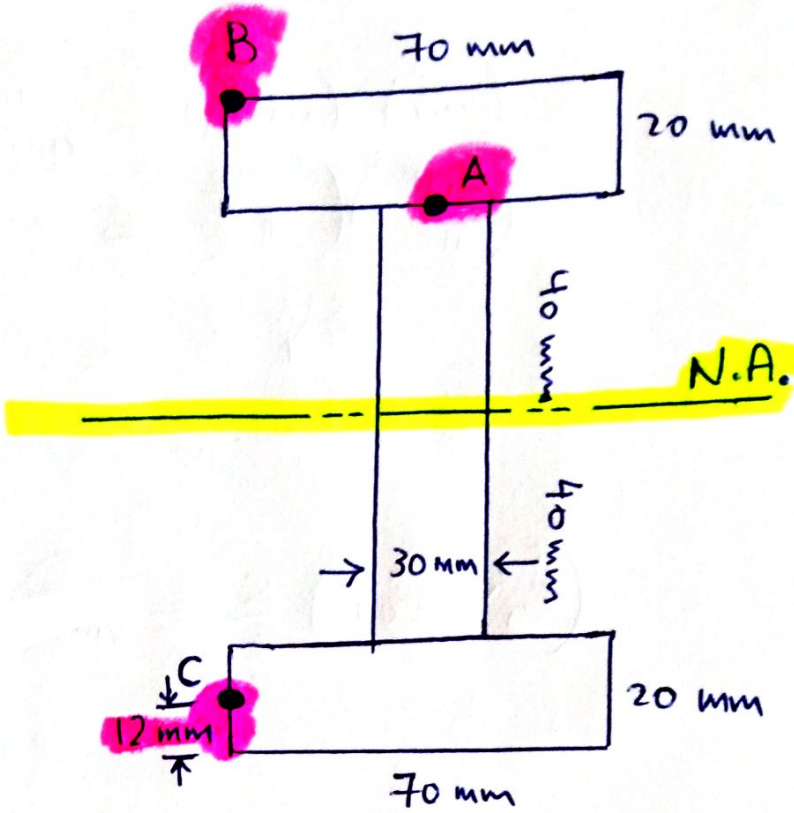
$$= 731.2 \text{ MPa.}$$

Ex


80 KN/m
100 KN
A
B
4m
2m
2m
2m
120 KN.m
70 KN.m
Ay = 301 KN
By = 119 KN
V
301
566.256
0.2375
2.256
38
238
238
-19
-119
301/80 = 3.7625
M
566.256
564
444
406
238
168
Vmax = 301 KN
Mmax = 566.256 KN.m

Find:

1. σ_{max} ?
2. τ_{max} ?
3. σ_A ?
4. τ_A ?
5. σ_B ?
6. τ_B ?
7. σ_C ?
8. τ_C ?



* $I = \frac{1}{12}(70)(120)^3 - \frac{1}{12}(40)(80)^3 = 8373333 \text{ mm}^4$
* $y_A = 40 \text{ mm}$
* $y_B = 60 \text{ mm} = y_{max}$
* $y_C = 48 \text{ mm}$
* $t_A = \begin{cases} 70 \text{ mm (flange)} \\ 30 \text{ mm (web)} \end{cases}$
* $t_B = 70 \text{ mm}$
* $t_C = 70 \text{ mm}$
* $Q_A = (70)(20)(50) = 70000 \text{ mm}^3$
* $Q_B = \text{zero}$
* $Q_C = (70)(12)(54) = 45360 \text{ mm}^3$
* $Q_{max} = Q_{N.A.} = (70)(20)(50) + (40)(30)(20) = 94000 \text{ mm}^3$

From V & M Diagrams

$V_{max} = 301 \text{ KN}$

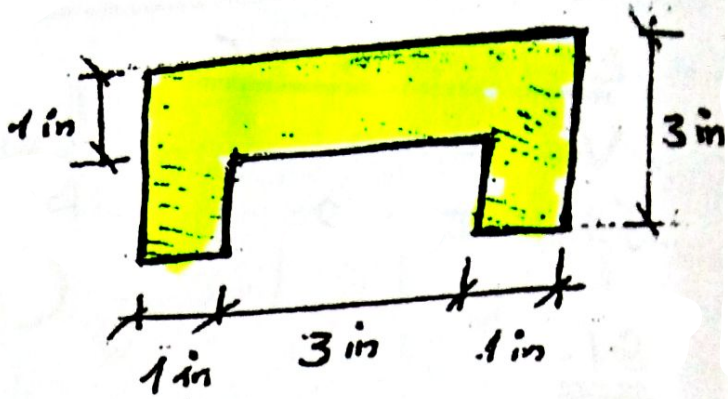
$M_{max} = 566.256 \text{ KN.m}$

$$\sigma = \frac{My}{I} \quad \& \quad \tau = \frac{VQ}{It}$$

Ex

$\tau_{max} = 8$ ksi

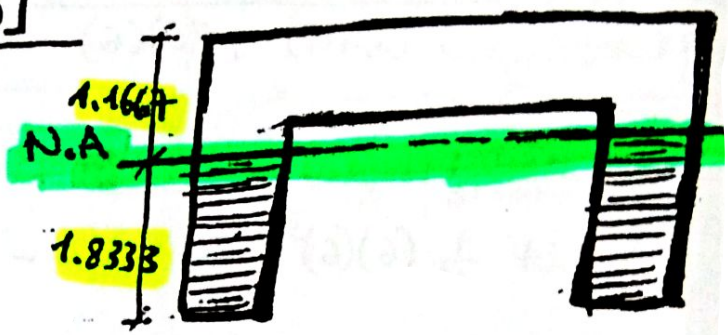
Find the shear force V



$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

$$\bar{y} = \frac{0.5(1)(5) + 2[2(1)(2)]}{(1)(5) + 2(1)(2)}$$

$$= 1.1667 \text{ in}$$



$$I = \frac{1}{12}(5)(1)^3 + (5)(1)(1.1667 - 0.5)^2 + 2\left[\frac{1}{12}(1)(2)^3 + (1)(2)(2 - 1.1667)^2\right] = 6.75 \text{ in}^4$$

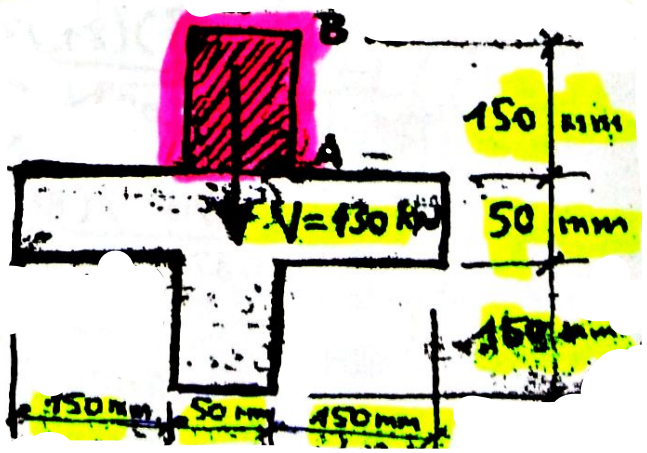
$$Q_{max} = \sum \bar{y}' A' = 2\left[\frac{1.8333}{2}(1.8333)(1)\right] = 3.3611 \text{ in}^3$$

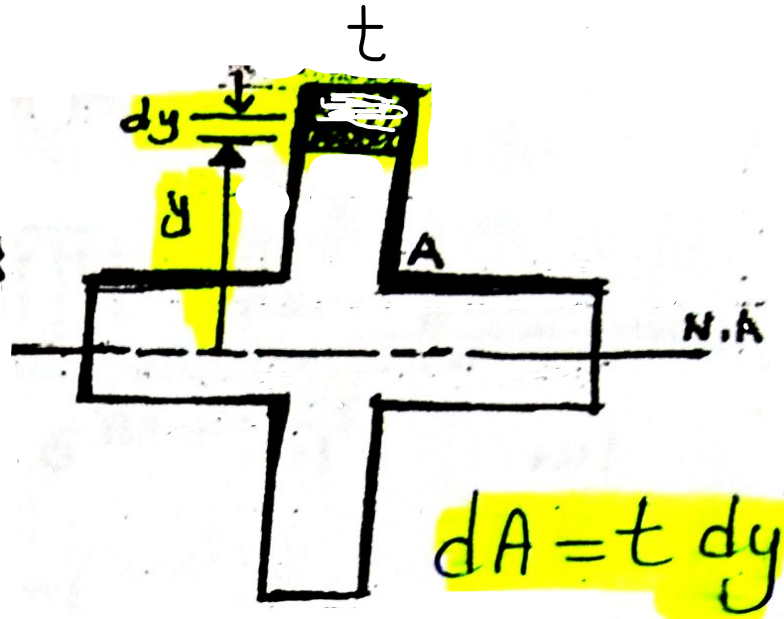
$$\tau_{max} = \frac{V Q_{max}}{I t} \Rightarrow 8 \times 10^3 = \frac{V(3.3611)}{6.75(2)(1)}$$

$$\Rightarrow V = 32.1 \text{ kip}$$

Ex

Given the shear force V=130 kN, compute the resultant shear force developed in the vertical segment AB ?





$$I = \frac{1}{12}(0.05)(0.35)^3 + \frac{1}{12}(0.3)(0.05)^3$$

$$= 0.18177 \times 10^{-3}\ m^4$$

For segment AB :

$$A' = 0.05(0.175 - y)$$

$$\bar{y}' = y + \frac{0.175 - y}{2} = \frac{1}{2}(0.175 + y)$$

$$Q = \bar{y}' A' = 0.025(0.030625 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{(130)(0.025)(0.030625 - y^2)}{(0.18177 \times 10^{-3})(0.05)}$$

$$\tau = 10951.3 - 357593.1\, y^2$$

But $V = \int \tau\, dA$ and $dA = 0.05\, dy$

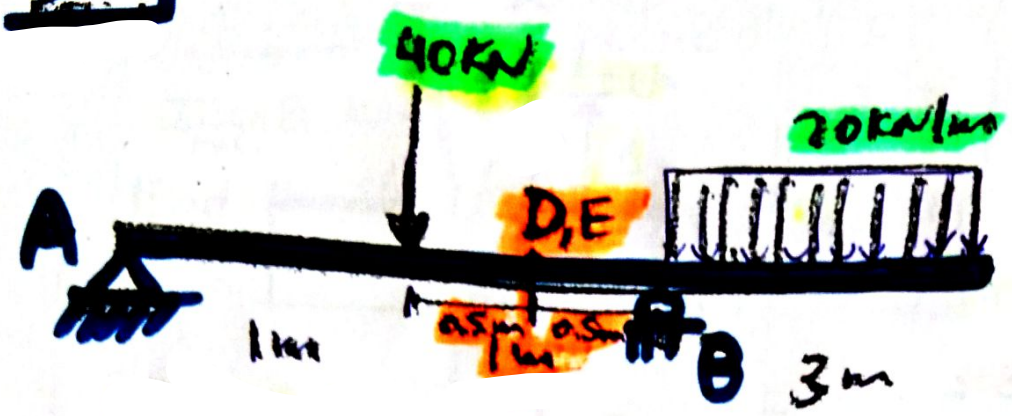
$$\Rightarrow V = \int_{0.025}^{0.175} (10951.3 - 357593.1\, y^2)(0.05)\, dy$$

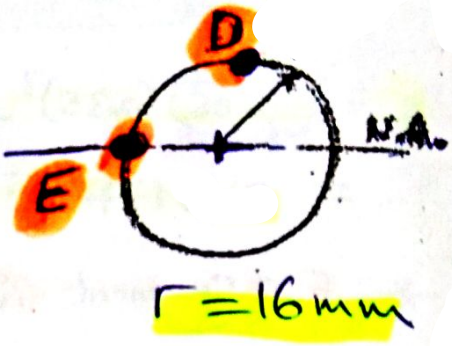
$$= \int_{0.025}^{0.175} (547.6 - 17879.7\, y^2)\, dy$$

$$= 50.3\ KN$$

$$dA = t\, dy$$

Ex



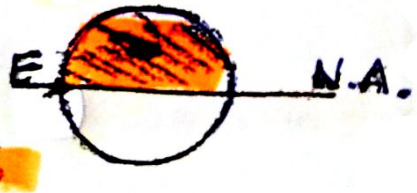


Find the ① Normal stress at point D, E

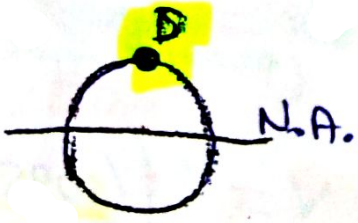
② the shear stress at point D,E

* $I = \frac{\pi}{2} r^4 = \frac{\pi}{2}(16)^4 = 102944 \ mm^4$

* $Q_E = \frac{2}{3} r^3 = \frac{2}{3}(16)^3 = 2731 \ mm^3$



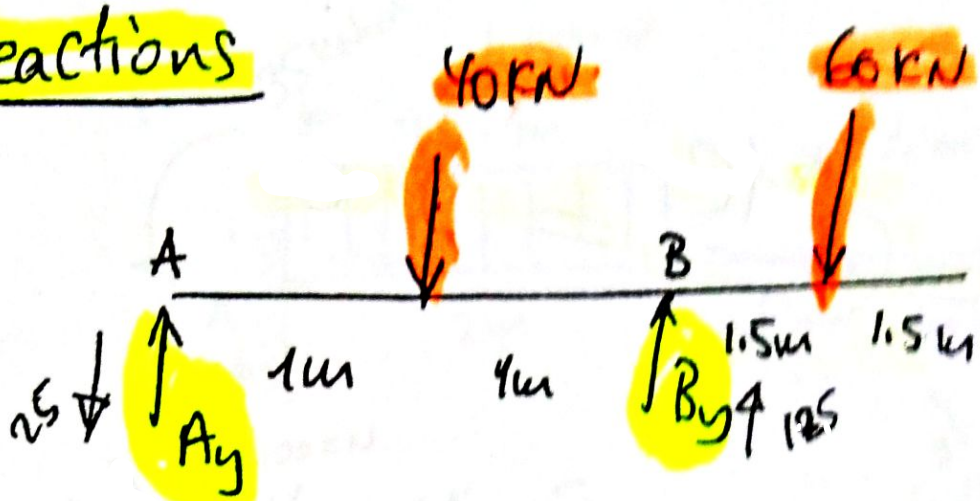
* $Q_D = 0$



* $C_E = 0$

* $C_D = r = 16 \ mm$

Reactions



+ve

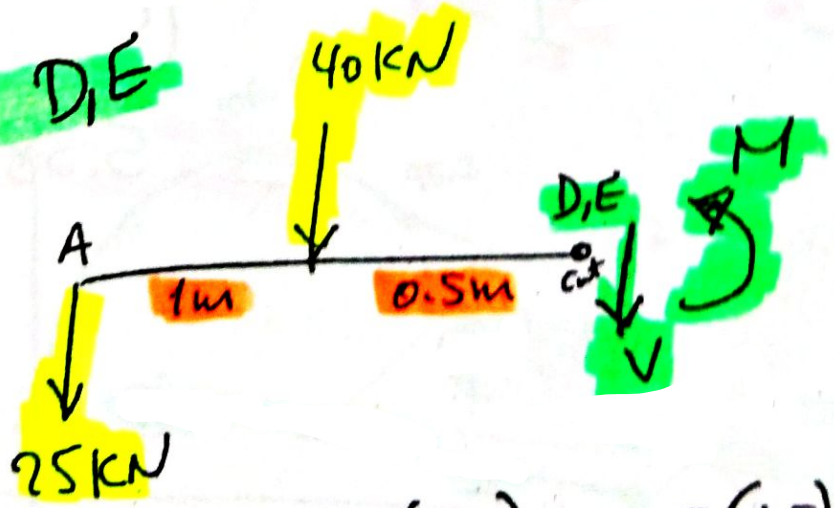
$$\sum M_A = 0 \Rightarrow 2B_y - 60(3.5) - 40(1) = 0$$

$$\Rightarrow B_y = 125 \text{ KN}$$

$$\sum F_y = 0 \Rightarrow A_y - 40 - 60 + 125 = 0$$

عكس الاتجاه $A_y = -25$ KN

قطع عند
Section at D,E



$$\sum M_{cut} = 0 \Rightarrow M + 40(0.5) + 25(1.5) = 0$$

$$M_{D,E} = -57.5 \text{ KN.m}$$

$$\sum F_y = 0 \Rightarrow 25 - 40 - V = 0 \Rightarrow V_{D,E} = -65 \text{ KN}$$

$$\sigma_E = \frac{MC}{I} = 0$$

$$\sigma_D = \frac{MC}{I} = \frac{(57.5 \times 10^{3})(16 \times 10^{-3})}{102944 \times 10^{-12}}$$

$$= 8.93 \text{ GPa}$$

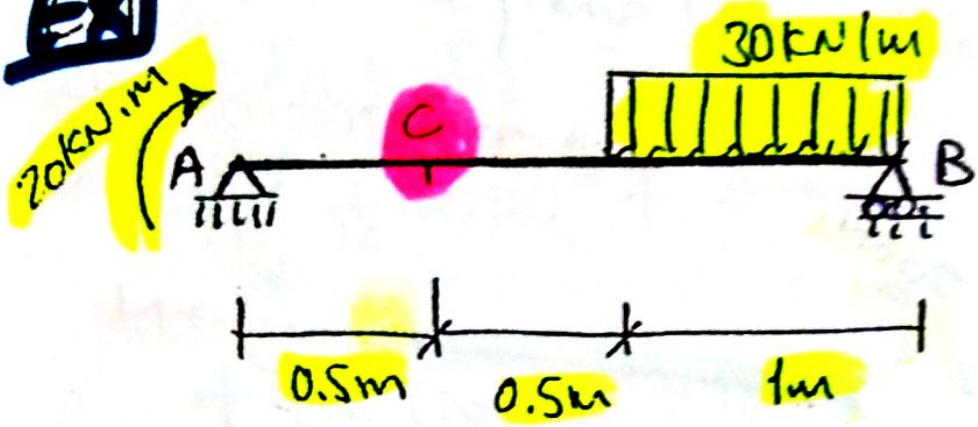
Compression bad

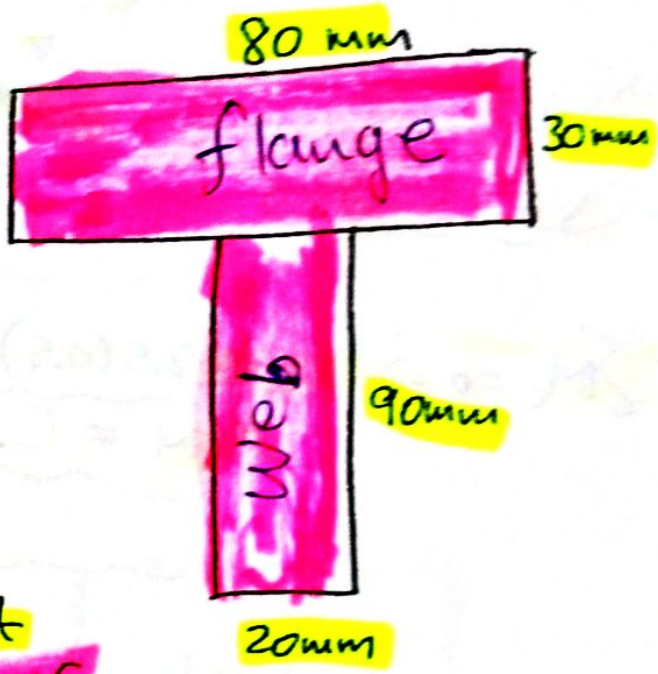
$$\tau_E = \frac{VQ}{It} = \frac{(65 \times 10^{3})(2731 \times 10^{-9})}{(102944 \times 10^{-12})(32 \times 10^{-3})}$$

$$= 5.38 \text{ MPa}$$

$$\tau_D = \frac{VQ}{It} = 0$$

Ex



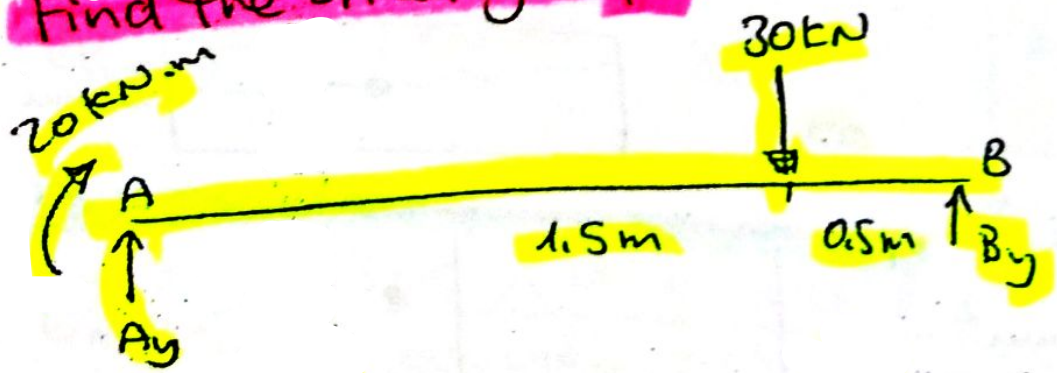


1) Find the max. shear stress at point C ?

2) Draw the shear stress (τ) profile for the section at point C ?

3) Find the Normal bending stress (σ) and draw it at C ?

4) Find the shear jump ? أوجد مقدار قفزة shear



-ve $\sum M_B = 0 \Rightarrow 20 + 2A_y - 30(0.5) = 0$

$2A_y = 15 - 20 = -5$

$A_y = -2.5$ kN.

Reactions

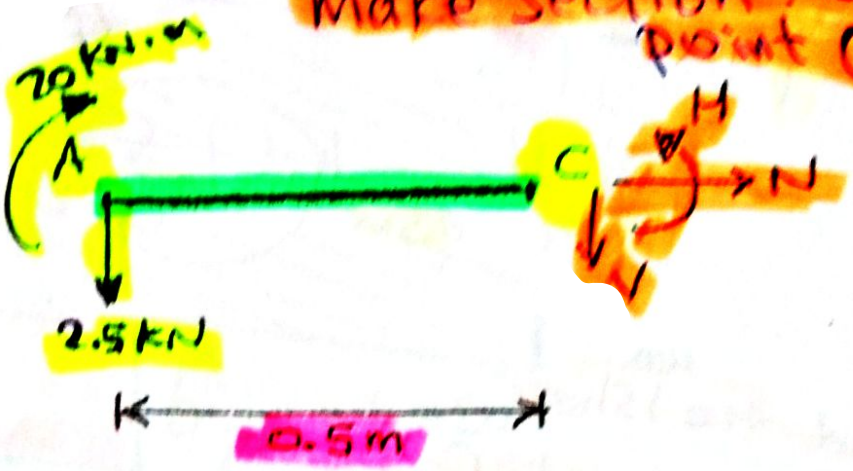
Make Section at point C

* $\Sigma F_x = 0$

$\rightarrow N = 0$

* $\Sigma F_y = 0$

$\rightarrow V = 2.5\ kN$



* $\Sigma M_C = 0 \Rightarrow 20 - 2.5(0.5) - M = 0$

$M = 18.75\ kN.m$

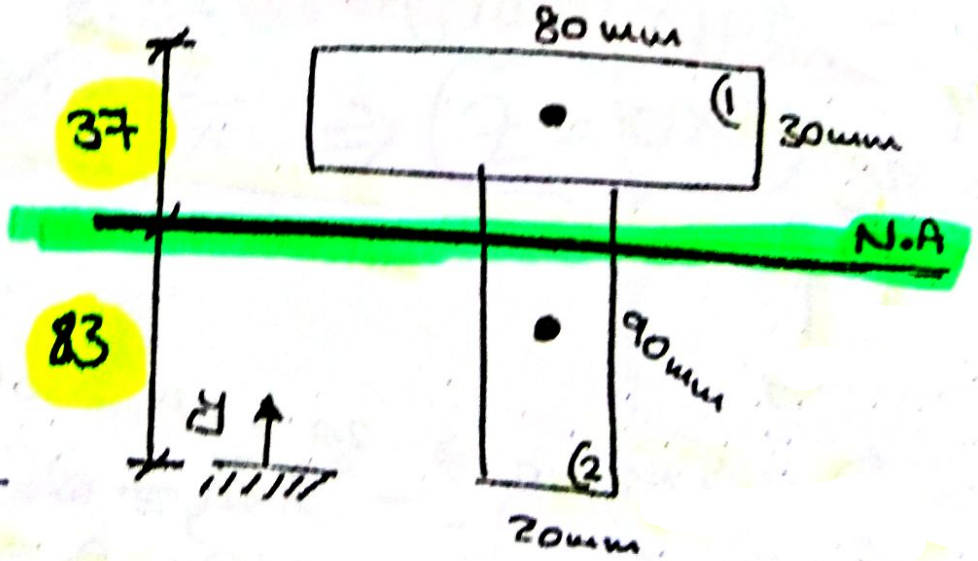
والآن يجب حساب موقع الـ Centroid

$\bar{y}_1 = 105\ mm$

$\bar{y}_2 = 45\ mm$

$A_1 = 2400\ mm^2$

$A_2 = 1800\ mm^2$



$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2}$$

$$\bar{y} = \frac{2400(105) + 1800(45)}{2400 + 1800} \cong 83\ mm$$

$$I = I_1 + I_2$$

$$= \left[\frac{1}{12}(80)(30)^3 + 2400(22)^2\right]$$

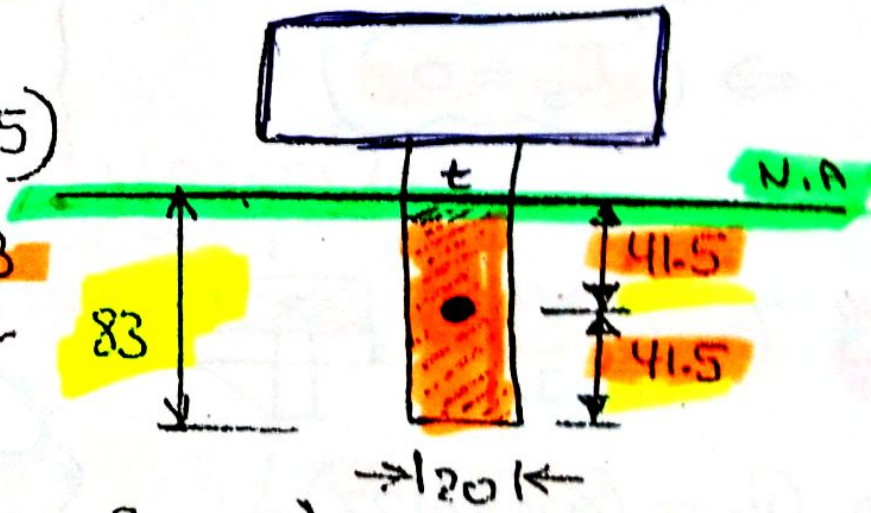
$$+ \left[\frac{1}{12}(20)(90)^3 + 1800(38)^2\right]$$

$$* \quad I = 5.155 \times 10^6 \ mm^4 .$$

① $$\tau_{max} = \frac{V Q_{max}}{I t}$$

$$\therefore \quad Q_{max} = (83)(20)(41.5)$$

$$Q_{max} = 68.9 \times 10^3 \ mm^3$$

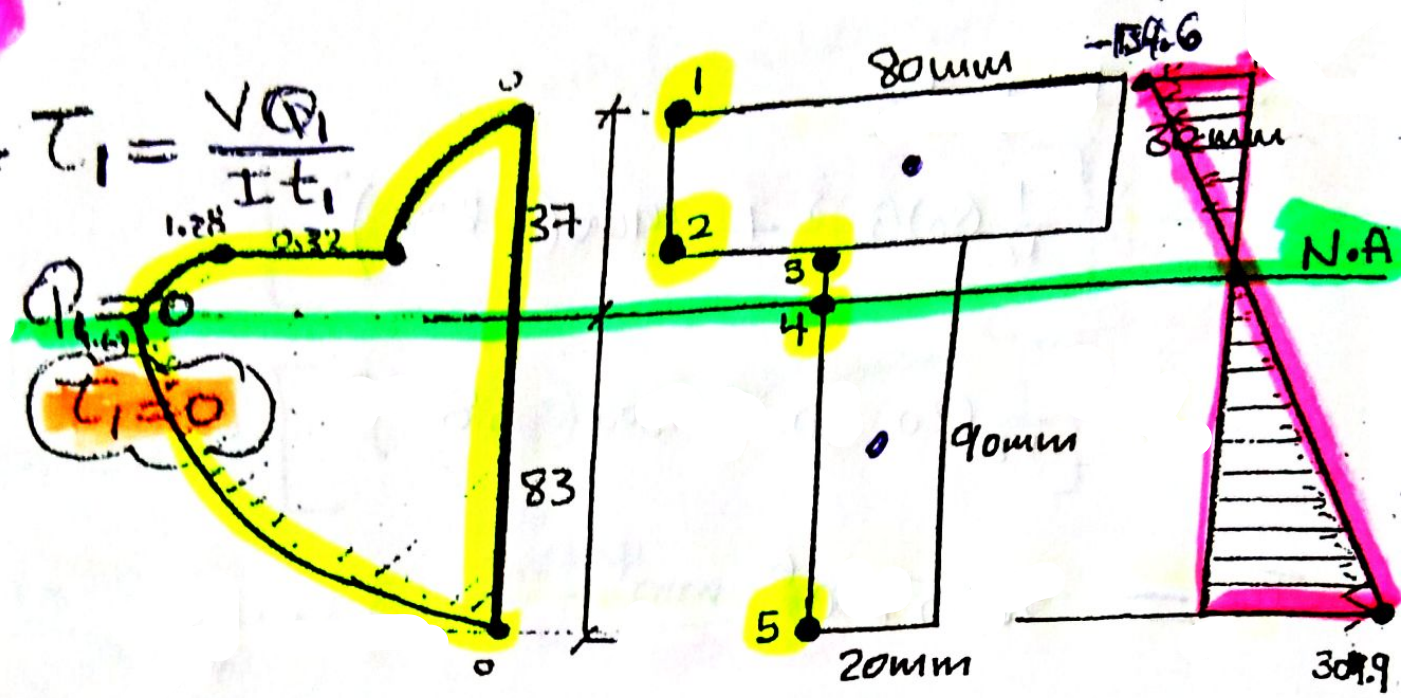


$$\tau_{max} = \frac{(2.5 \times 10^3)(68.9 \times 10^3 \times 10^{-9})}{(5.155 \times 10^6 \times 10^{-12})(20 \times 10^{-3})}$$

$$\tau_{max} = 1.67 \ MPa$$ at the N.A.

②

$$* \ \tau_1 = \frac{VQ_1}{It_1}$$



$Q_1 = 0$

$\tau_1 = 0$

$$* \ \tau_5 = \frac{VQ_5}{It_5}$$

$* \ Q_5 = 0$

$\Rightarrow \tau_5 = 0$

$\tau_4 = \tau_{N.A} = \tau_{max}$

$\tau_4 = 1.67 \ MPa$

محسوبة سابقاً

$$* \ \tau_2 = \frac{VQ_2}{It}$$

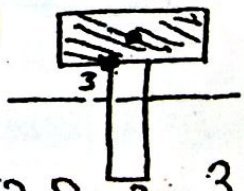
$\Rightarrow Q_2 = (80)(30)(22)$

$Q_2 = 52.8 \times 10^3 \ mm^3$

$$\tau_2 = \frac{(2.5\times10^3)(52.8\times10^3\times10^{-9})}{(5.155\times10^6\times10^{-12})(80\times10^{-3})}$$

$\tau_2 = 0.32 \ MPa$

$$* \ \tau_3 = \frac{VQ_3}{It}$$



$\Rightarrow Q_3 = Q_2 = 52.8\times10^3 \ mm^3$

$$\tau_3 = \frac{(2.5\times10^3)(52.8\times10^3\times10^{-9})}{(5.155\times10^6\times10^{-12})(20\times10^{-3})}$$

$\tau_3 = 1.28 \ MPa$

③ $$\sigma_1 = \frac{M y_1}{I}$$

$$\sigma_1 = \frac{(18.75\times10^{3})(37\times10^{-3})}{5.155\times10^{6}\times10^{-12}}$$

$$\sigma_1 = -134.6 \text{ MPa}$$ Compression

$$\sigma_4 = \frac{M y_4}{I}, \quad y_4 = 0$$

$$\sigma_4 = 0$$

$$\sigma_5 = \frac{M y_5}{I}$$

$$\sigma_5 = \frac{(18.75\times10^{3})(83\times10^{-3})}{5.155\times10^{6}\times10^{-12}}$$

$$\sigma_5 = +301.9 \text{ MPa}$$ Tension

④ Shear Jump $= \tau_3 - \tau_2$ Bet. web and flange

$$= 1.28 - 0.32$$

$$\tau_{Jump} = 0.96 \text{ MPa}$$

Shear flow (nails)

المسامير

$$q = \frac{VQ}{I}$$

(q) Shear flow

V ≡ قوة القص في المقطع

$$q^* = \frac{q}{n}$$

n ≡ no. of nails

عدد المسامير في المقطع الواحد وليس عدد المسامير الكلي

$$F = q^* S$$

F ≡ قوة القص في المسمار

Shear force in nail

S ≡ المسافة فابين المسامير

nail Spacing.

$$\tau = \frac{F}{\frac{\pi}{4} d^2}$$

→ τ ≡ Shear stress in nail

الجهد القص في المسمار

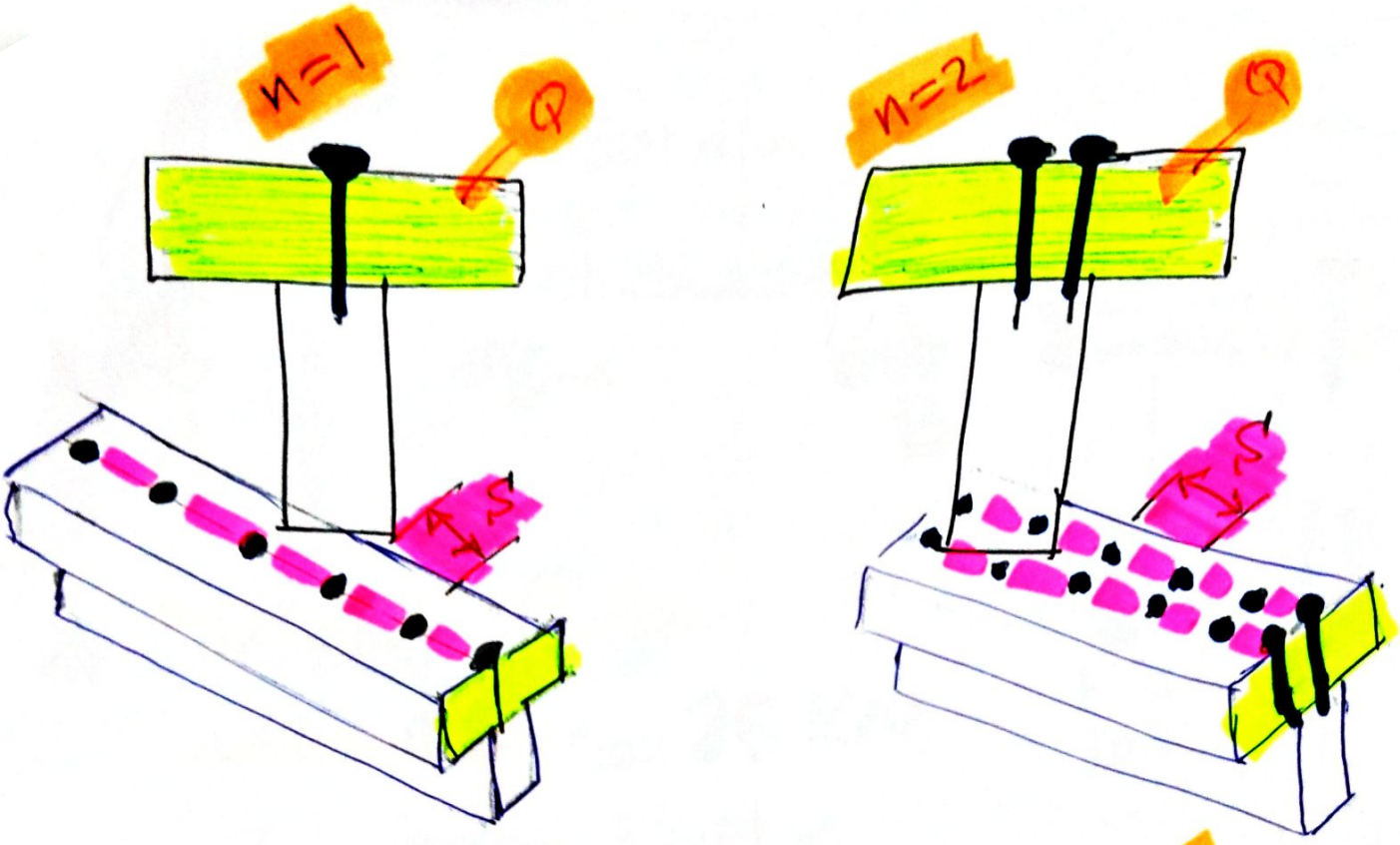
d ≡ diameter

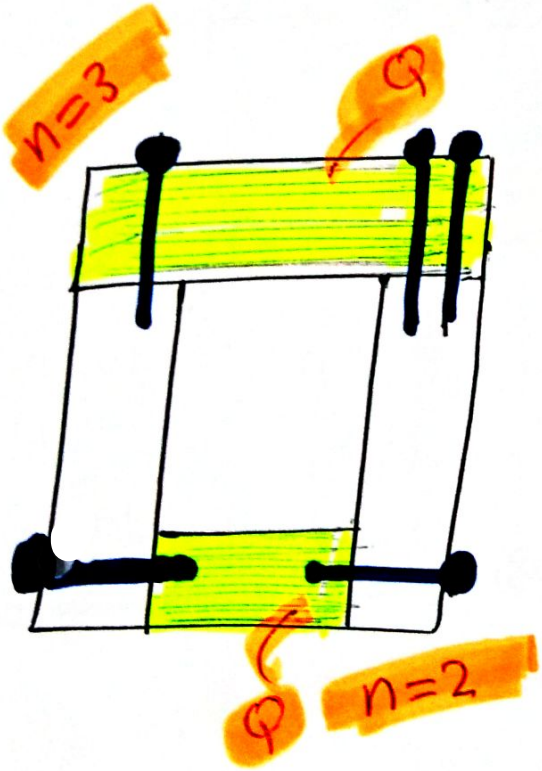
قطر المسمار

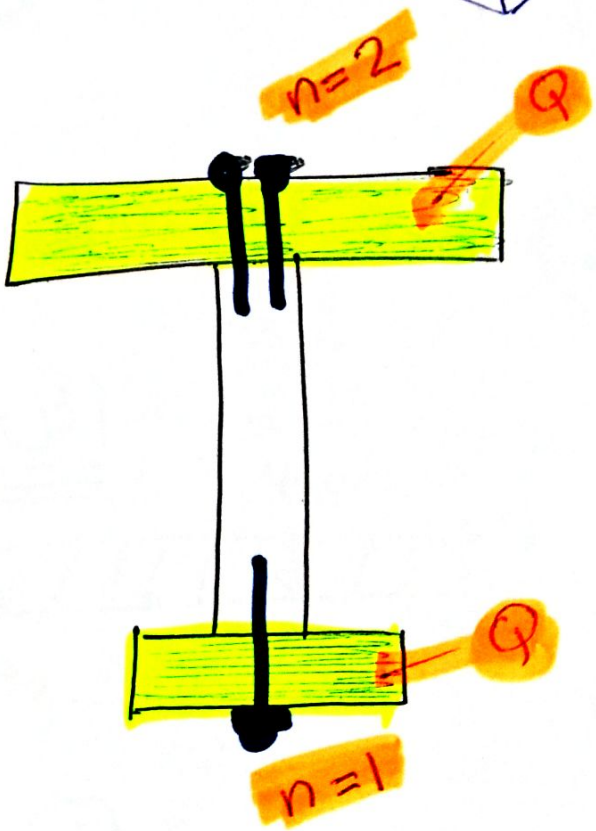
$$\tau = \frac{VQ}{Ib}$$

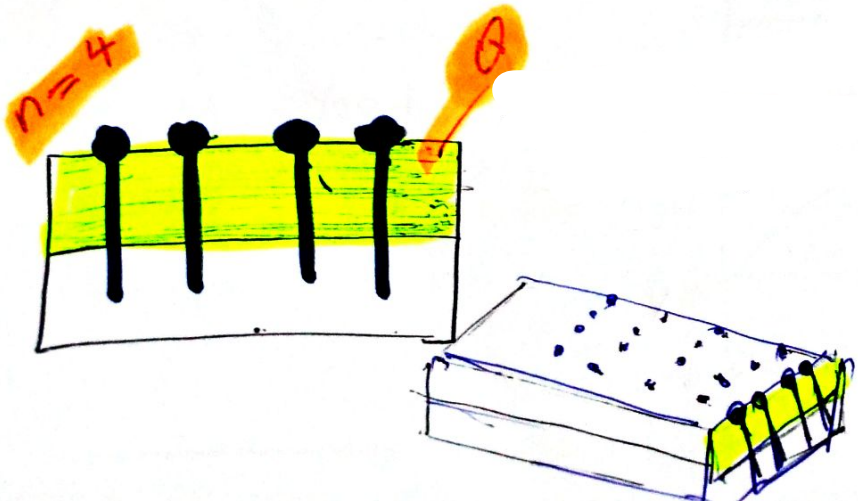
→ τ ≡ Shear stress in Section

الجهد القص في المقطع

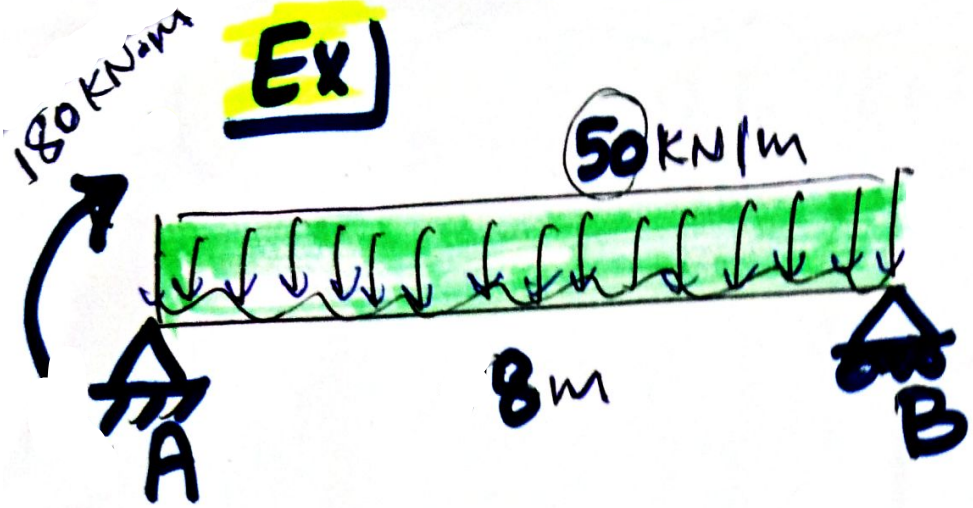

n=1
Q
S
n=2
Q
S

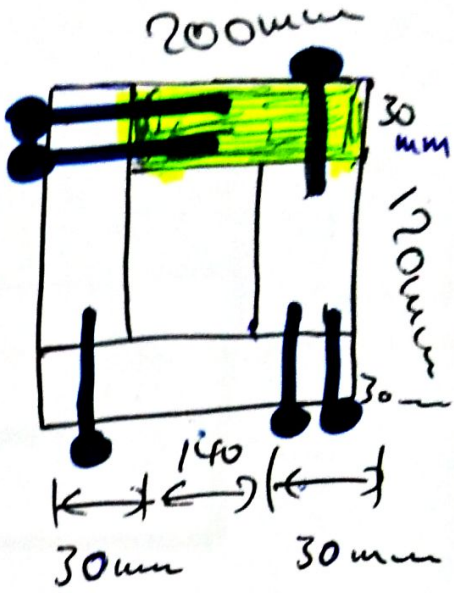

n=3
Q
Q
n=2


n=2
Q
Q
n=1


n=4
Q

Ex





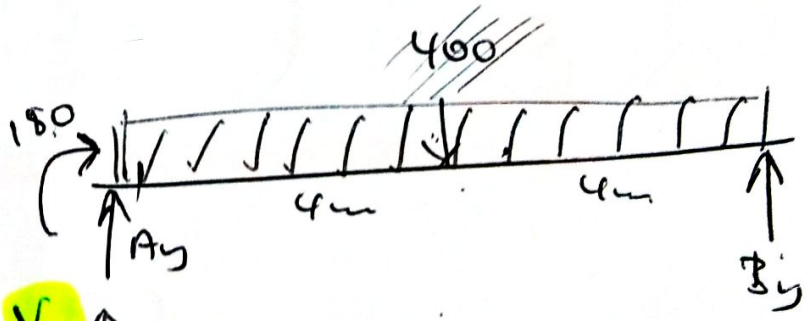
* The shear force in the nail $F = 25$ KN

⇒ Find the Nail spacing S ?

$\Sigma M_A = 0$

$\Rightarrow 8B_y - 400(4) - 180 = 0$

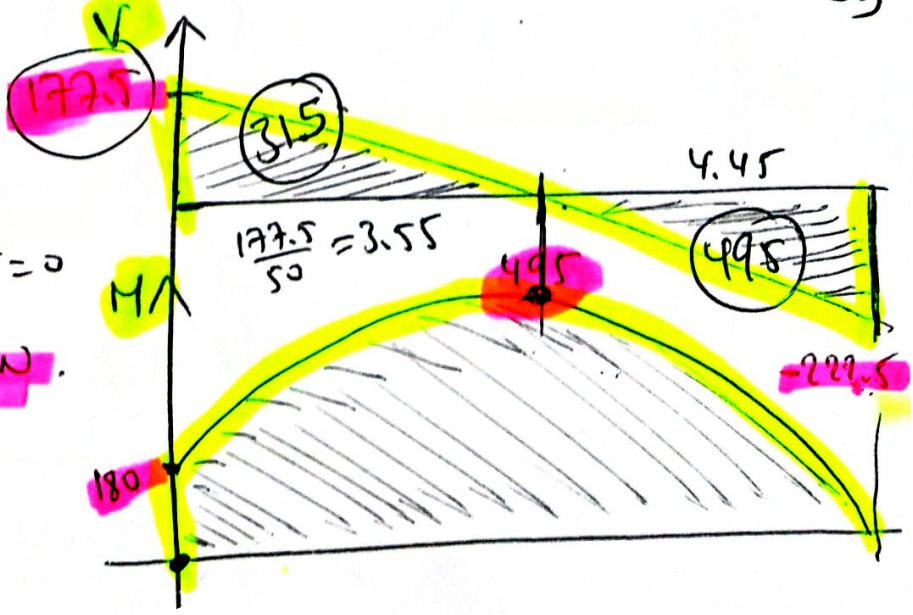
$B_y = 222.5$ KN



$\Sigma F_y = 0$

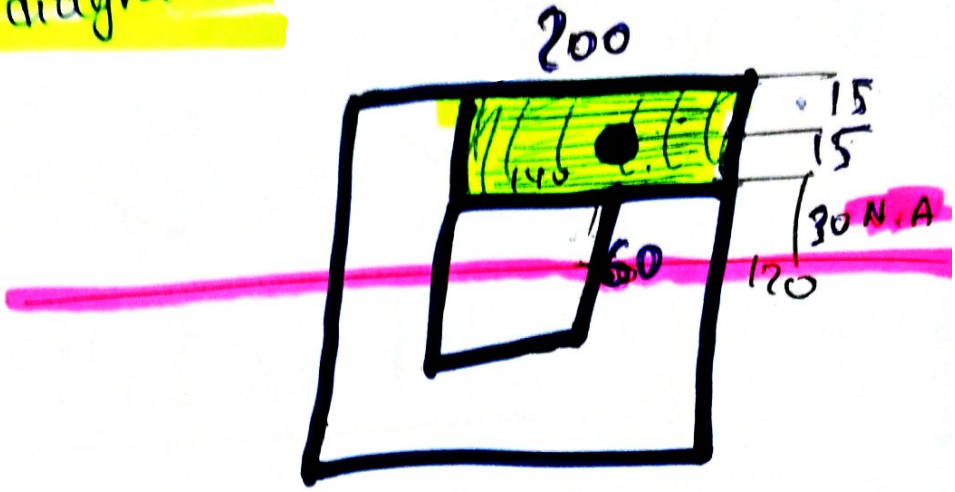
$\Rightarrow A_y - 400 + 222.5 = 0$

$A_y = 177.5$ KN.



$$V_{max} = 222.5 \text{ KN}$$

From diagram



$$* \; I = \frac{1}{12}(200)(120)^3 - \frac{1}{12}(140)(60)^3$$

$$\Rightarrow \quad I = 26280000 \text{ mm}^4$$

$$* \; Q = (170)(30)(45)$$

$$= 229500 \text{ mm}^3$$

$$q = \frac{V\,Q}{I}$$

$$q = \frac{(222.5 * 10^3)(229500 * 10^{-9})}{26280000 * 10^{-12}}$$

$$q = 194306.5 \ N/m$$

$$q^* = \frac{194306.5}{3}$$

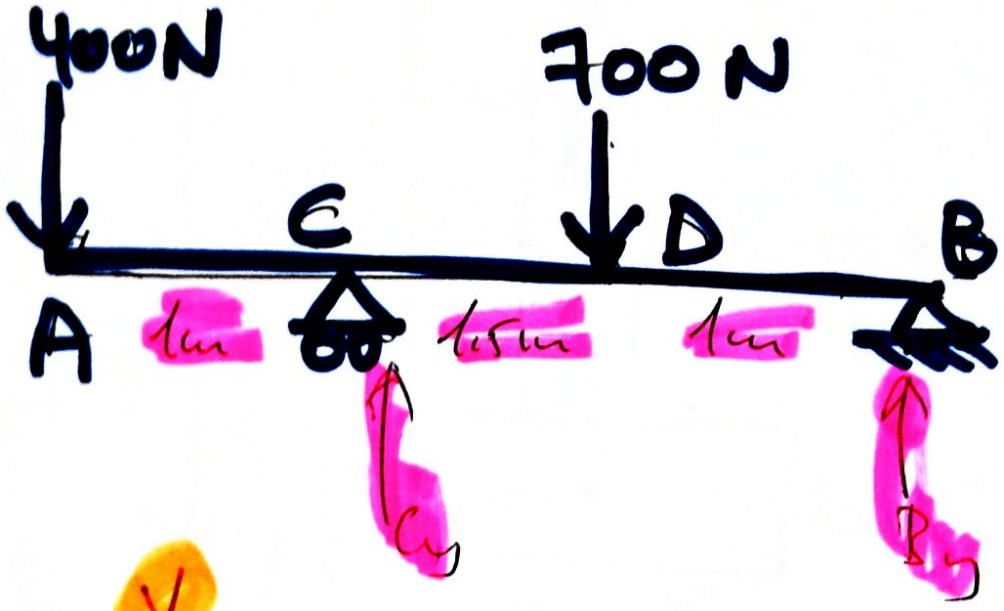
عدد المسامير

$$q^* = 64768.8 \ N/m$$

$$F = q^* \delta$$

$$\delta = \frac{25000}{64768.8} = 38.6 \ mm$$

Ex



$\sum M_C = 0$

$2.5\,B_y - 700(1.5) + 400(1) = 0$

$\boxed{B_y = 260}$

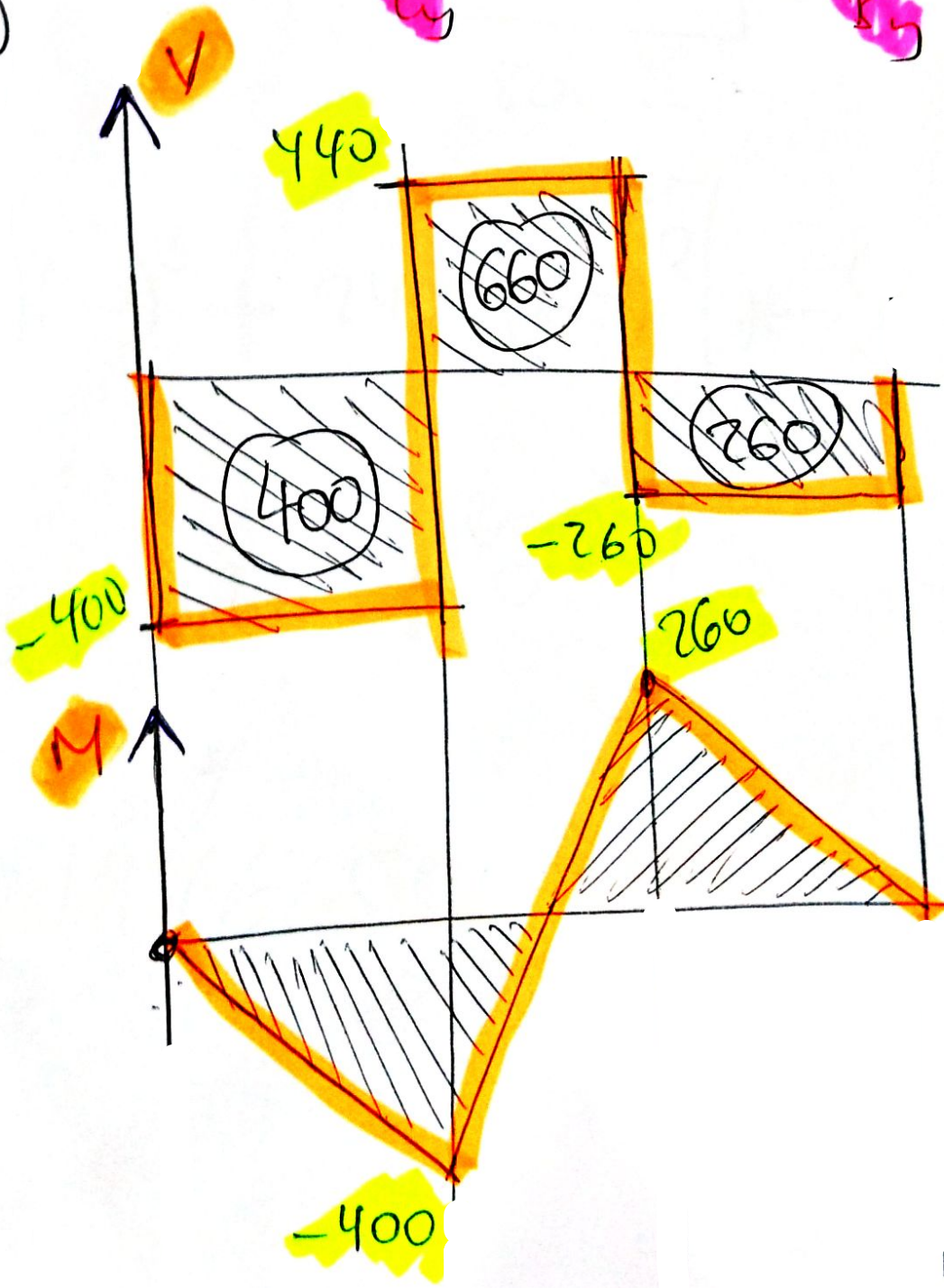
$\sum F_y = 0$

$C_y - 400 - 700 + 260 = 0$

$\boxed{C_y = 840}$

$V_{max} = 440$

$M_{max} = 400$

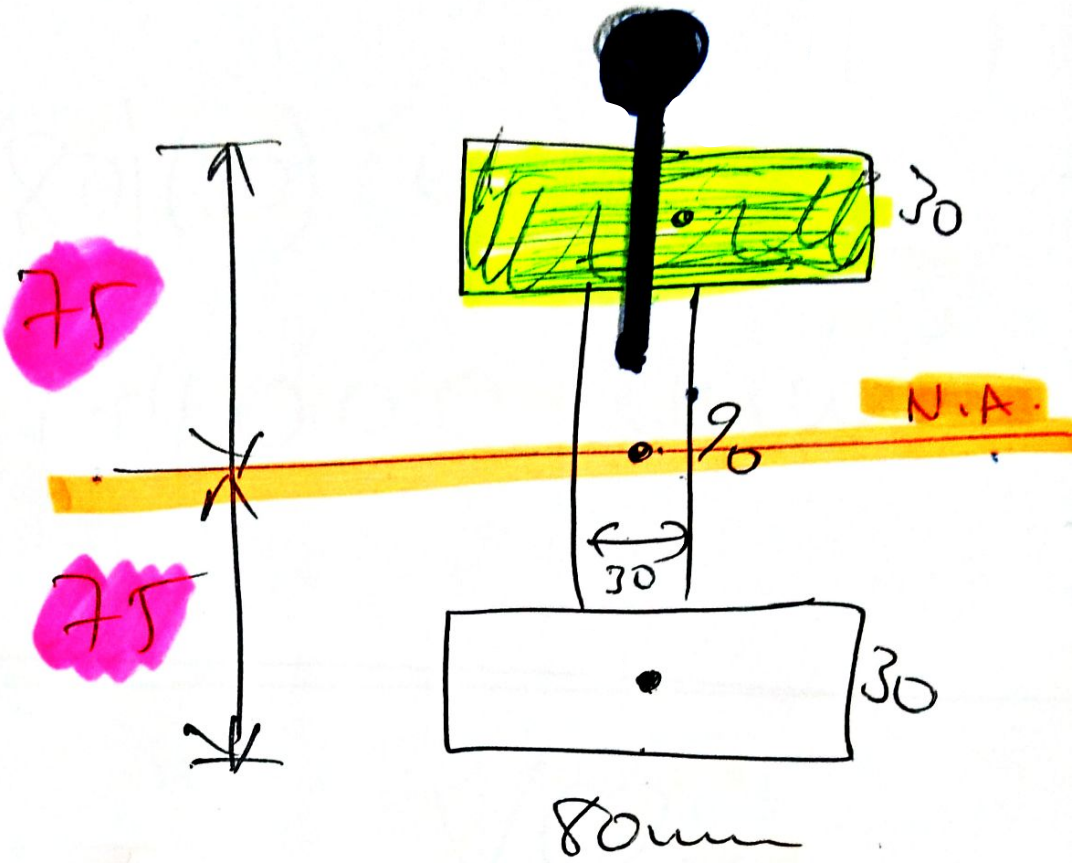


$y_1 = 15 \quad A_1 = \checkmark$

$y_2 = 75 \quad A_2 = \checkmark$

$y_3 = 135 \quad A_3 = \checkmark$

$\bar{y} = \frac{\cdots}{\cdots} = 75$



$$I = \left[\frac{1}{12}(80)(30)^3 + 2400\,(\overset{15-75}{60})^2\right] * 2$$

$$+ \left[\frac{1}{12}(30)(90)^3 + 0^{(75-75)^2}\right]$$

$$I = 19462500 \ mm^4$$

$Q_{\text{الجزء}} = (80)(30)(60)$

$Q_{\text{الجزء}} = 144000 \ mm^3$

flange

1

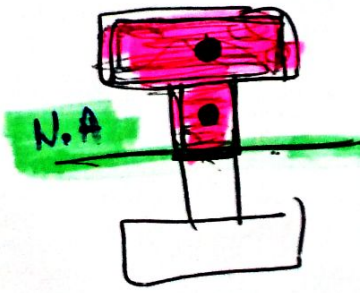
$$\tau_{max} = \frac{V Q_{max}}{I b}$$

$$\tau = \frac{(440)(17.4375 * 10^{-9})}{(19462500 * 10^{-12})(30 * 10^{-3})}$$

المسافة عن N.A

$$\tau = 13.17 \ KPa$$

$$Q_{max} = (80)(30)(60) + (30)(45)(22.5) = 17.4375 \ mm^3$$



2

$$\sigma_{max} = \frac{My}{I}$$

$$\sigma_{max} = \frac{(400)(75 * 10^{-3})}{19462500 * 10^{-12}}$$

$$\boxed{\sigma_{max} = 1.54 \text{ MPa}}$$

3

$$q_{max} = \frac{V_{max} Q_{max}}{I}$$ Shear flow

$$q_{max} = \frac{(440)(1743750 * 10^{-9})}{19462500 * 10^{-12}}$$

معطاه بالسؤال

4

$F = 1000\ N$ given ✓

$$q_{\text{البعد}} = \frac{V\,Q_{flange}}{I}$$

$$= \frac{(440)(144000 * 10^{-9})}{19462500 * 10^{-12}}$$

$$= \ldots\ N/m$$

then $F = q^* S$

$\Rightarrow$ Find S ✓

$$q^* = \frac{q}{n}$$

$$q^* = \frac{q}{1} = \ldots\ N/m$$

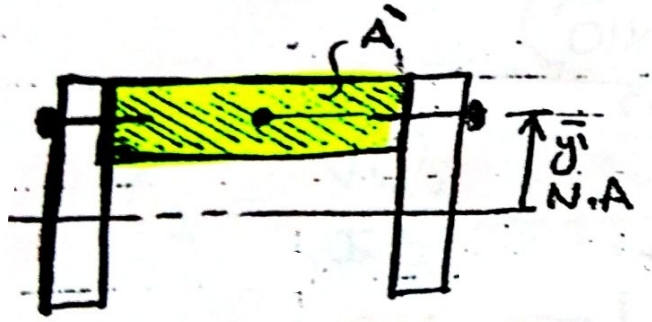
Shear Flow in Built-Up Members

Beam members consist or "built up" from several composite parts.

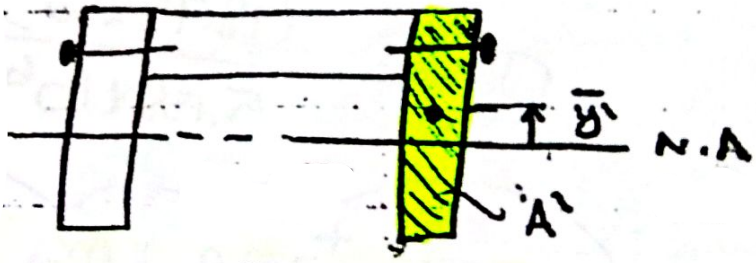
** The <u>shear flow</u> (q) is a measure of the force per unit length along a longitudinal axis of a beam.

$$q = \frac{VQ}{I} \quad , \quad Q = \bar{y}' A'$$

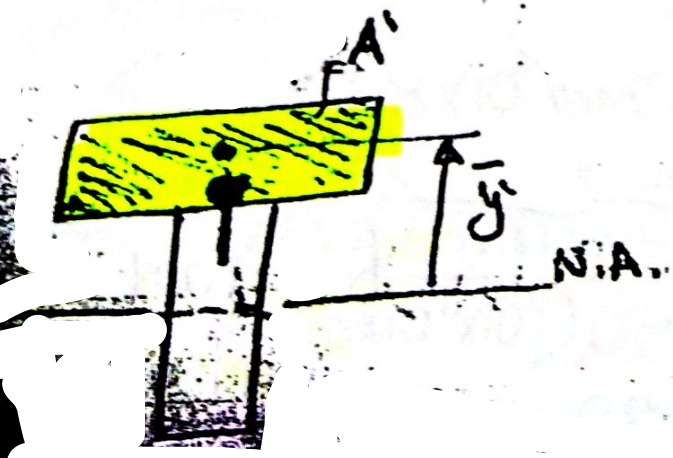
** The fasteners (such as nails, glue, weleding ...) are necessary to resist the shear flow and to keep the component parts from sliding relative to one another.



(q/2)



(q)



Shear flow (q) Shear جريان

$$q = \frac{VQ}{I}$$

$$q^* = \frac{q}{n}$$

$$\tau = \frac{F}{A}$$

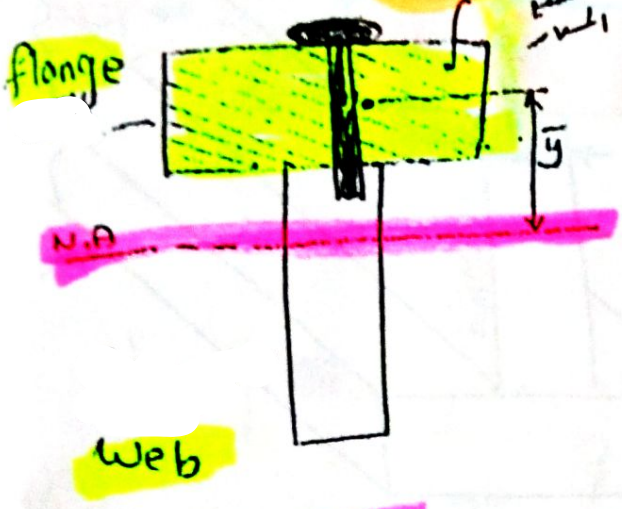
$$F = q^* S$$

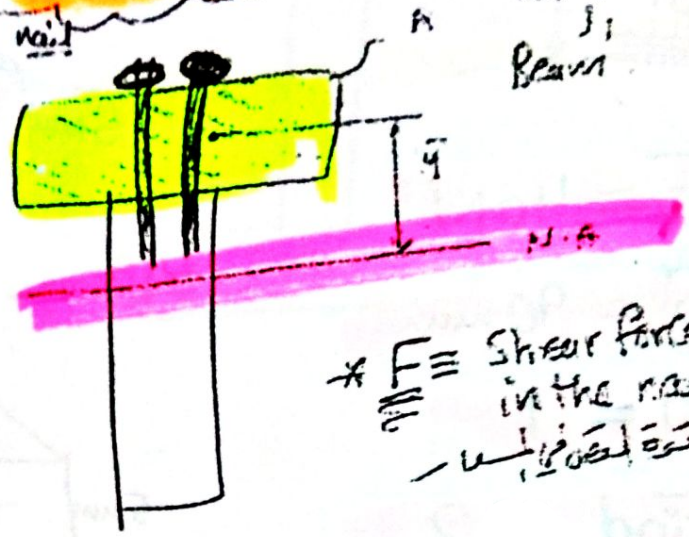
$S \equiv$ nail spacing

المسافة بين المسامير

A مساحة المقطع المسامير

A مساحة المقطع الـ Beam





$* F \equiv$ Shear force in the nail

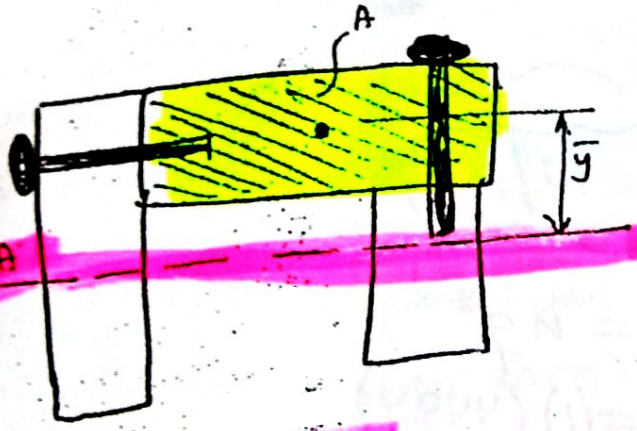
قوة القص في المسمار

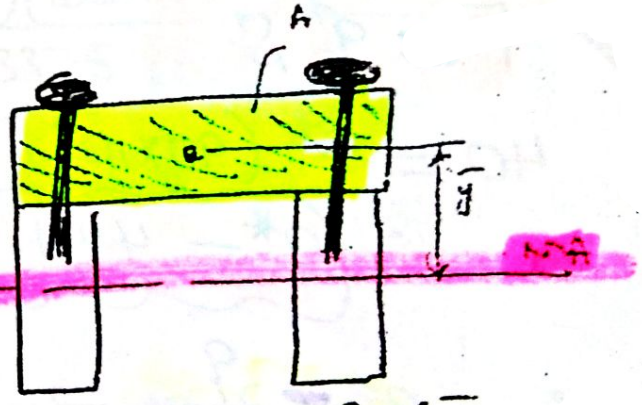
$n = 1$

$Q = A\bar{y}$

$n = 2$

$Q = A\bar{y}$



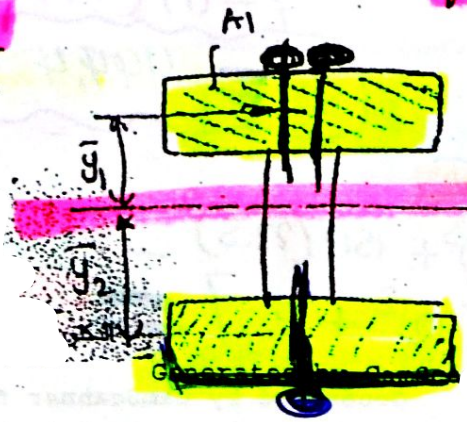


$n = 2$

$Q = A\bar{y}$

$n = 2$

$Q = A\bar{y}$



$n = 2$

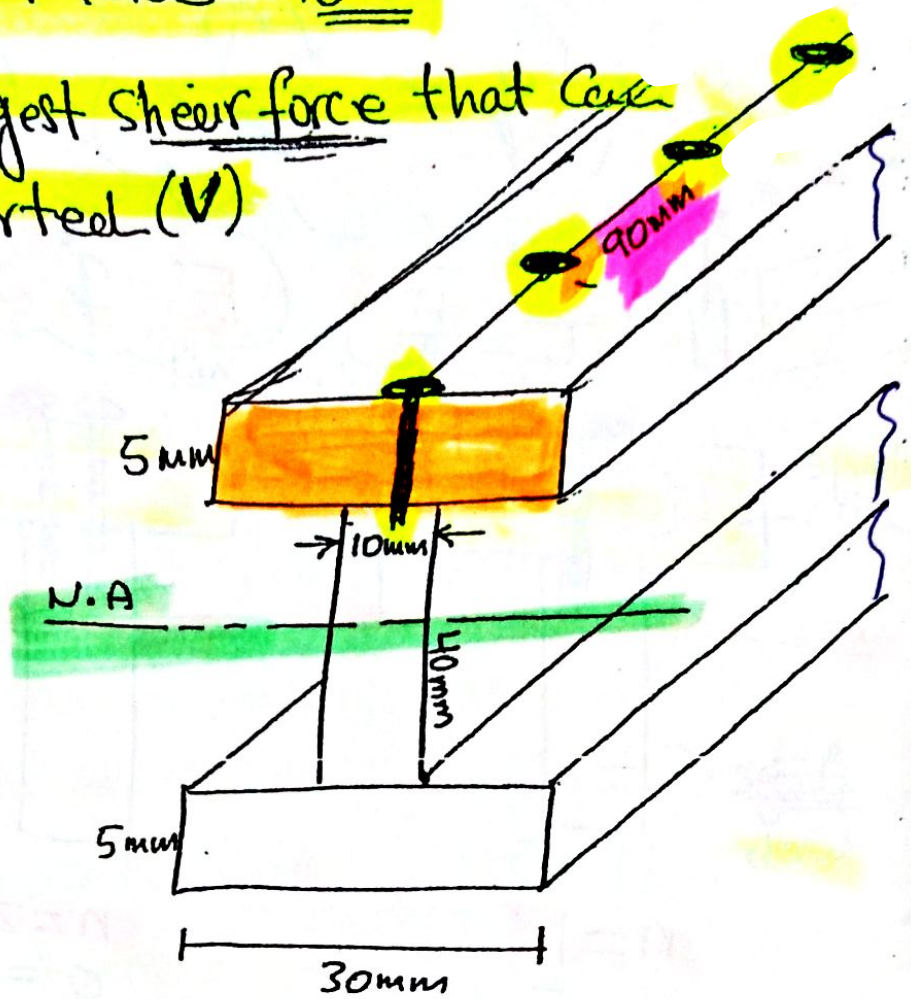
$Q_1 = A_1 \bar{y}_1$

$n = 1$

Ex)

Nails of shear force $40\,N$ (F)

Find the largest shear force that can be supported (V)



* $F = 40N$
* $S = 90\,mm$
* $n = 1$
* Find **V**?

$\Rightarrow$ $F = q^* S$

$40 = q^* (90 \times 10^{-3})$

$\Rightarrow q^* = 444.4\ N/m$

$\therefore\ q^* = \frac{q}{n} \Rightarrow q = n q^*$

$q = (1)(444.4)$

$q = 444.4\ N/m$

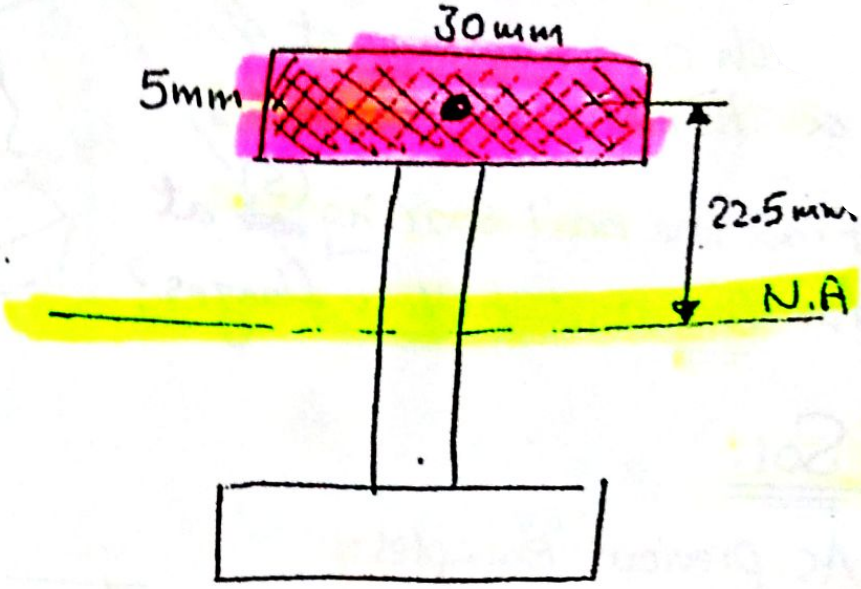
* $I = I_1 + I_2 + I_3$

$= 2\left[\frac{1}{12}(30)(5)^3 + 150(22.5)^2\right]$

$+ \left[\frac{1}{12}(10)(40)^3 + 0\right] = 205.83 \times 10^3\ mm^4$

$$Q_{flange} = (150)(22.5)$$

$$Q = 3375 \text{ mm}^3$$



$$\therefore \quad q = \frac{VQ}{I}$$

$$\Rightarrow \quad V = \frac{qI}{Q}$$

$$V = \frac{(444.4)(205.83 \times 10^{3} \times 10^{-12})}{3375 \times 10^{-9}}$$

$$V = 27.1 \text{ N}$$

مقدار قوة القص في المقطع

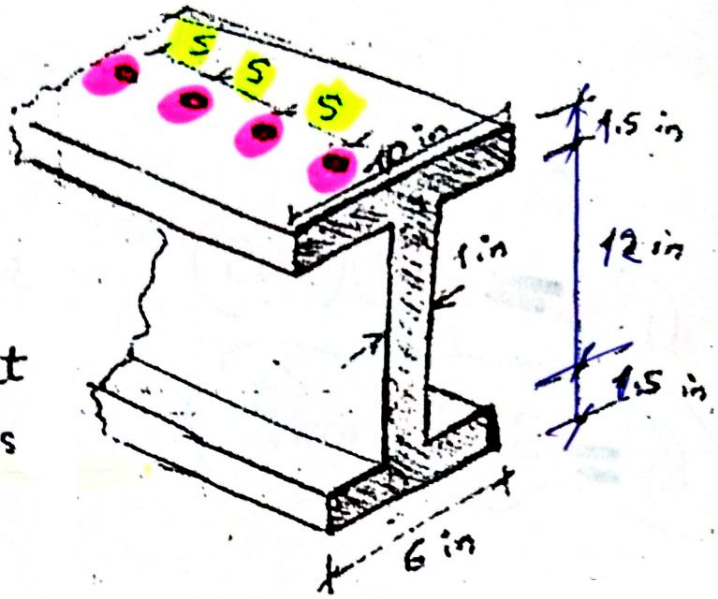
the shear force in the cross-section

Ex

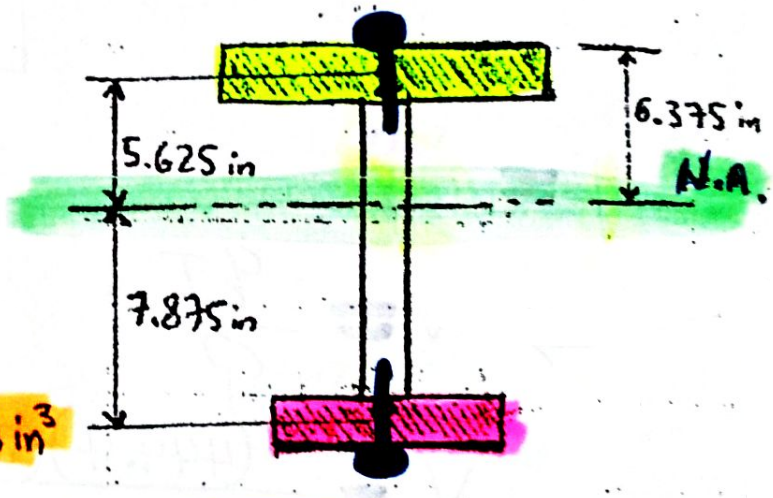
$V = 5$ Kip

each nail can support a shear force of 500 lb

Find the nail spacing (S) at the top and bottom flanges



Sol.



$\bar{y} = 6.375$ in

$I = 1196.44$ in^4

$Q_t = \bar{y}_t' A_t'$

$= 5.625(10)(1.5) = 84.38$ in^3

$Q_b = \bar{y}_b' A_b' = 7.875(6)(1.5) = 70.88$ in^3

$$q_t = \frac{VQ_t}{I} = \frac{(5\times10^3)(84.38)}{1196.44} = 352.61 \text{ lb/in}$$

$$q_b = \frac{VQ_b}{I} = \frac{(5\times10^3)(70.88)}{1196.44} = 296.19 \text{ lb/in}$$

* The shear force in the nail $\boxed{F = qS} \Rightarrow S = \frac{F}{q}$

$$\Rightarrow S_t = \frac{F}{q_t} = \frac{500}{352.61} = 1.42 \text{ in}$$

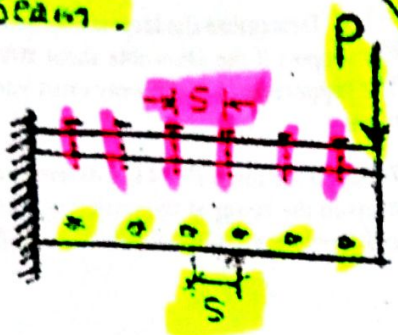
and $S_b = \frac{F}{q_b} = \frac{500}{296.19} = 1.69$ in

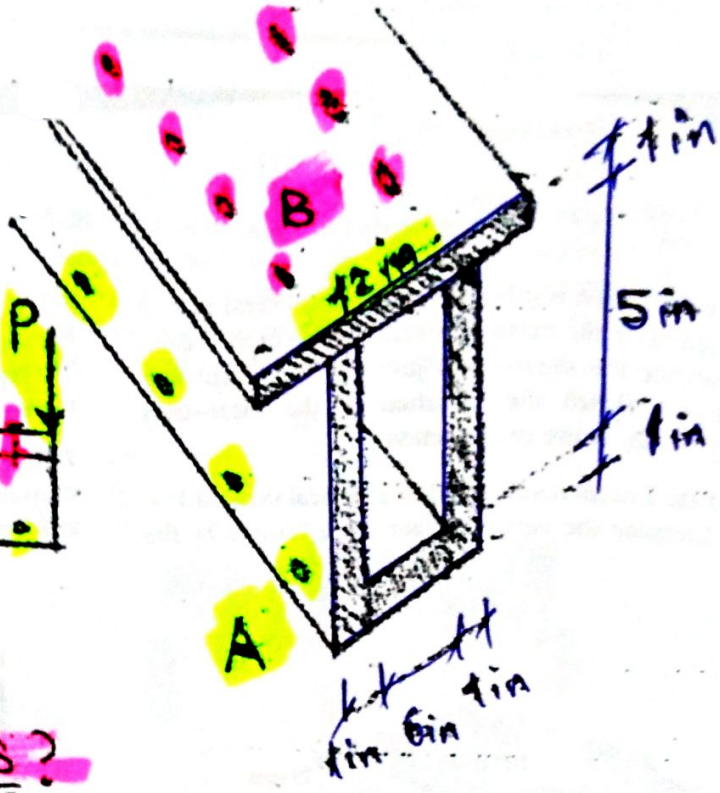
Ex

$P = 2$ kip

nail spacing $S = 2$ in

Cantilevered Beam.



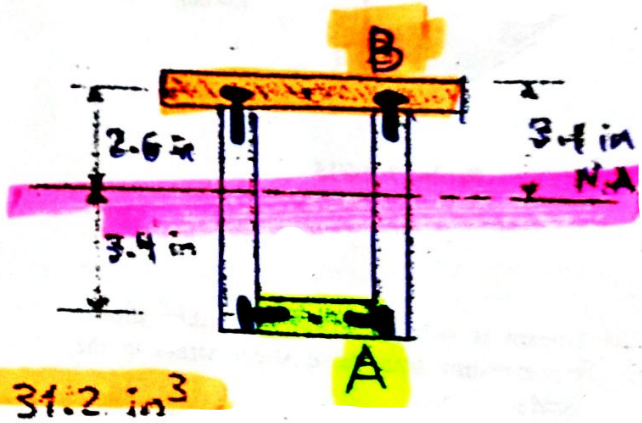


Find the shear force resisted by each nail at A and B?

Sol.

$\bar{y} = 3.10$ in

$I = 197.7$ in^4



$Q_B = \bar{y}'_B A'_B = 2.6(12)(1) = 31.2$ in^3

$Q_A = \bar{y}'_A A'_A = 3.4(6)(1) = 20.4$ in^3

But $V = P = 2$ kip

$\therefore q^*_B = \frac{1}{2}\frac{VQ_B}{I} = \frac{1}{2}\frac{2(31.2)}{197.7} = 157.8$ lb/in

and $q^*_A = \frac{1}{2}\frac{VQ_A}{I} = \frac{1}{2}\frac{2(20.4)}{197.7} = 103.2$ lb/in

* the shear force in each nail is:

$F_B = q^*_B S = (157.8)(2) = 316$ lb

and: $F_A = q^*_A S = (103.2)(2) = 206$ lb

EXAMPLE

The beam is constructed from four boards glued together as shown in Fig. 7–16*a*. If it is subjected to a shear of $V = 850$ kN, determine the shear flow at B and C that must be resisted by the glue.

SOLUTION

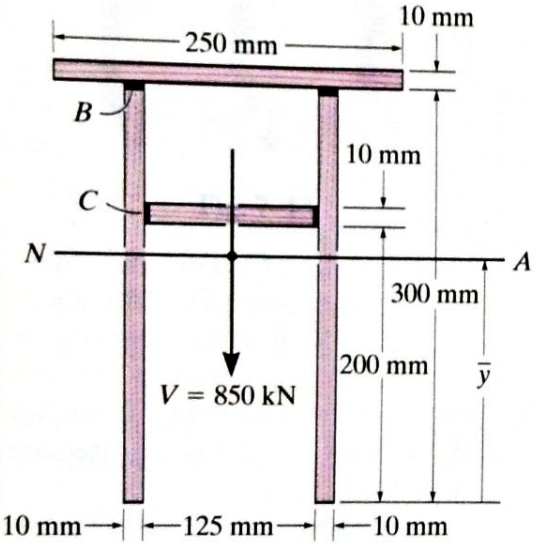
Section Properties. The neutral axis (centroid) will be located from the bottom of the beam, Fig. 7–16*a*. Working in units of meters, we have

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2[0.15\text{ m}](0.3\text{ m})(0.01\text{ m}) + [0.205\text{ m}](0.125\text{ m})(0.01\text{ m}) + [0.305\text{ m}](0.250\text{ m})(0.01\text{ m})}{2(0.3\text{ m})(0.01\text{ m}) + 0.125\text{ m}(0.01\text{ m}) + 0.250\text{ m}(0.01\text{ m})}$$

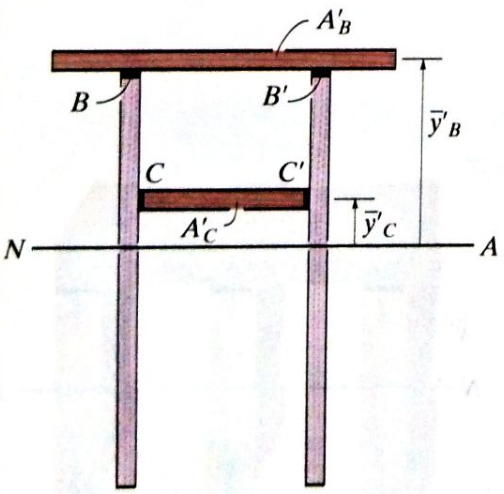
$$= 0.1968\text{ m}$$

The moment of inertia about the neutral axis is thus

$$I = 2\left[\frac{1}{12}(0.01\text{ m})(0.3\text{ m})^3 + (0.01\text{ m})(0.3\text{ m})(0.1968\text{ m} - 0.150\text{ m})^2\right]$$
$$+ \left[\frac{1}{12}(0.125\text{ m})(0.01\text{ m})^3 + (0.125\text{ m})(0.01\text{ m})(0.205\text{ m} - 0.1968\text{ m})^2\right]$$
$$+ \left[\frac{1}{12}(0.250\text{ m})(0.01\text{ m})^3 + (0.250\text{ m})(0.01\text{ m})(0.305\text{ m} - 0.1968\text{ m})^2\right]$$
$$= 87.52(10^{-6})\text{ m}^4$$



(a)



(b)

Fig. 7–16

Since the glue at B and B' in Fig. 7–16*b* "holds" the top board to the beam, we have

$$Q_B = \bar{y}'_B A'_B = [0.305\text{ m} - 0.1968\text{ m}](0.250\text{ m})(0.01\text{ m})$$
$$= 0.271(10^{-3})\text{ m}^3$$

Likewise, the glue at C and C' "holds" the inner board to the beam, Fig. 7–16*b*, and so

$$Q_C = \bar{y}'_C A'_C = [0.205\text{ m} - 0.1968\text{ m}](0.125\text{ m})(0.01\text{ m})$$
$$= 0.01026(10^{-3})\text{ m}^3$$

Shear Flow. For B and B' we have

$$q'_B = \frac{VQ_B}{I} = \frac{850(10^3)\text{ N}(0.271(10^{-3})\text{ m}^3)}{87.52(10^{-6})\text{ m}^4} = 2.63\text{ MN/m}$$

And for C and C',

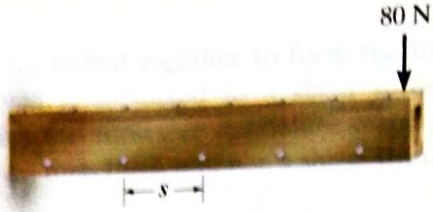
$$q'_C = \frac{VQ_C}{I} = \frac{850(10^3)\text{ N}(0.01026(10^{-3})\text{ m}^3)}{87.52(10^{-6})\text{ m}^4} = 0.0996\text{ MN/m}$$

Since *two seams* are used to secure each board, the glue per meter length of beam at each seam must be strong enough to resist *one-half* of each calculated value of q'. Thus,

$$q_B = 1.31\text{ MN/m} \quad \text{and} \quad q_C = 0.0498\text{ MN/m} \qquad \textit{Ans.}$$

EXAMPLE

A box beam is constructed from four boards nailed together as shown in Fig. 7–17*a*. If each nail can support a shear force of 30 N, determine the maximum spacing s of the nails at B and at C so that the beam will support the force of 80 N.

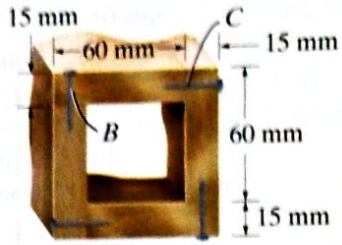


SOLUTION

Internal Shear. If the beam is sectioned at an *arbitrary point* along its length, the internal shear required for equilibrium is always $V = 80$ N and so the shear diagram is shown in Fig. 7–17*b*.

Section Properties. The moment of inertia of the cross-sectional area about the neutral axis can be determined by considering a 75-mm × 75-mm square minus a 45-mm × 45-mm square.

$$I = \frac{1}{12}(75\text{ mm})(75\text{ mm})^3 - \frac{1}{12}(45\text{ mm})(45\text{ mm})^3 = 2295(10^3)\text{ mm}^4$$



(a)

The shear flow at B is determined using Q_B found from the darker shaded area shown in Fig. 7–17*c*. It is this "symmetric" portion of the beam that is to be "held" onto the rest of the beam by nails on the left side and by the fibers of the board on the right side. Thus,

$$Q_B = \bar{y}'A' = [30\text{ mm}](75\text{ mm})(15\text{ mm}) = 33750\text{ mm}^3$$

Likewise, the shear flow at C can be determined using the "symmetric" shaded area shown in Fig. 7–17*d*. We have

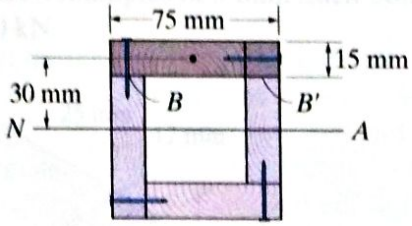
$$Q_C = \bar{y}'A' = [30\text{ mm}](45\text{ mm})(15\text{ mm}) = 20250\text{ mm}^3$$

(b)

Shear Flow.

$$q_B = \frac{VQ_B}{I} = \frac{80\text{ N}(33750\text{ mm}^3)}{2295(10)^3\text{ mm}^4} = 1.176\text{ N/mm}$$

$$q_C = \frac{VQ_C}{I} = \frac{80\text{ N}(20250\text{ mm}^3)}{2295(10)^3\text{ mm}^4} = 0.7059\text{ N/mm}$$



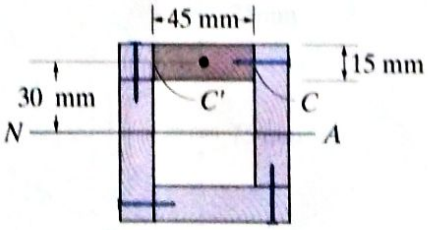
(c)

These values represent the shear force per unit length of the beam that must be resisted by the nails at B and the fibers at B', Fig. 7–17*c*, and the nails at C and the fibers at C', Fig. 7–17*d*, respectively. Since in each case the shear flow is resisted at *two* surfaces and each nail can resist 30 N, for B the spacing is

$$s_B = \frac{30\text{ N}}{(1.176/2)\text{ N/mm}} = 51.0\text{ mm} \qquad \text{Use } s_B = 55\text{ mm} \quad \textit{Ans.}$$

And for C,

$$s_C = \frac{30\text{ N}}{(0.7059/2)\text{ N/mm}} = 85.0\text{ mm} \qquad \text{Use } s_C = 85\text{ mm} \quad \textit{Ans.}$$



(d)

Fig. 7–17

EXAMPLE

Nails having a total shear strength of 40 N are used in a beam that can be constructed either as in Case I or as in Case II, Fig. 7–18. If the nails are spaced at 90 mm, determine the largest vertical shear that can be supported in each case so that the fasteners will not fail.

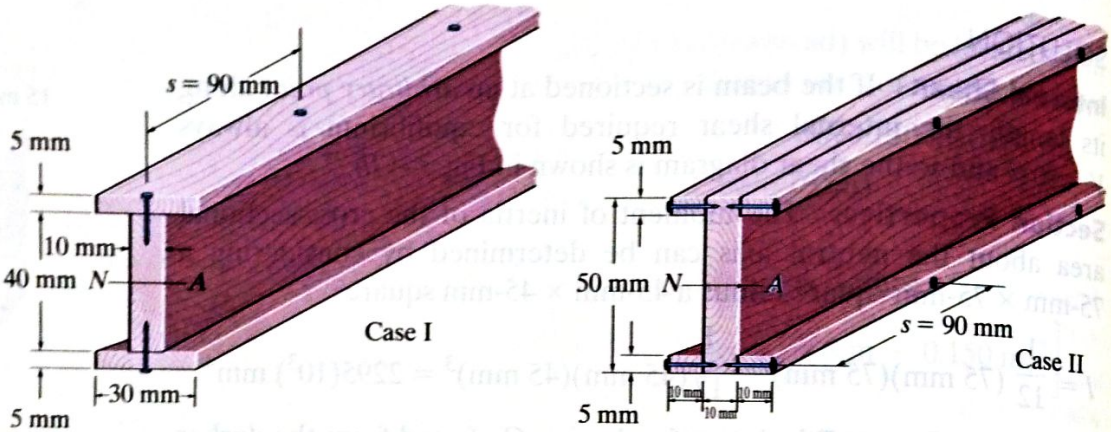


Fig. 7–18

SOLUTION

Since the cross section is the same in both cases, the moment of inertia about the neutral axis is

$$I = \frac{1}{12}(30\text{ mm})(50\text{ mm})^3 - 2\left[\frac{1}{12}(10\text{ mm})(40\text{ mm})^3\right] = 205\,833\text{ mm}^4$$

Case I. For this design a single row of nails holds the top or bottom flange onto the web. For one of these flanges,

$$Q = \bar{y}'A' = [22.5\text{ mm}](30\text{ mm}(5\text{ mm})) = 3375\text{ mm}^3$$

so that

$$q = \frac{VQ}{I}$$

$$\frac{40\text{ N}}{90\text{ mm}} = \frac{V(3375\text{ mm}^3)}{205\,833\text{ mm}^4}$$

$$V = 27.1\text{ N} \qquad \textit{Ans.}$$

Case II. Here a single row of nails holds one of the side boards onto the web. Thus,

$$Q = \bar{y}'A' = [22.5\text{ mm}](10\text{ mm}(5\text{ mm})) = 1125\text{ mm}^3$$

$$q = \frac{VQ}{I}$$

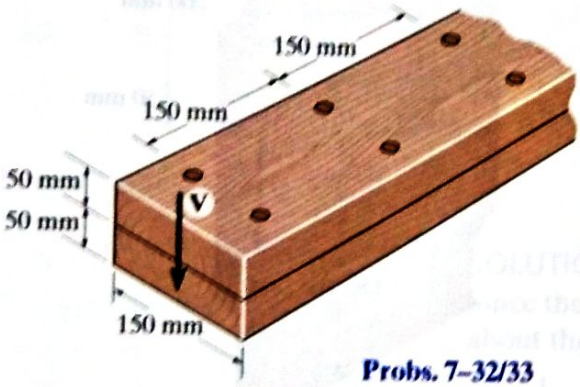
$$\frac{40\text{ N}}{90\text{ mm}} = \frac{V(1125\text{ mm}^3)}{205\,833\text{ mm}^4}$$

$$V = 81.3\text{ N} \qquad \textit{Ans.}$$

PROBLEMS

***7–32.** The beam is constructed from two boards fastened together at the top and bottom with two rows of nails spaced every 150 mm. If each nail can support a 2.5-kN shear force, determine the maximum shear force V that can be applied to the beam.

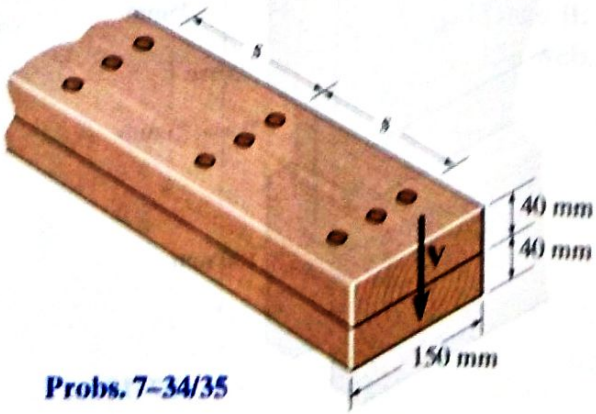
•7–33. The beam is constructed from two boards fastened together at the top and bottom with two rows of nails spaced every 150 mm. If an internal shear force of $V = 3$ kN is applied to the boards, determine the shear force resisted by each nail.



Probs. 7–32/33

7–34. The beam is constructed from two boards fastened together with three rows of nails spaced $s = 50$ mm apart. If each nail can support a 2.25-kN shear force, determine the maximum shear force V that can be applied to the beam. The allowable shear stress for the wood is $\tau_{allow} = 2.1$ MPa.

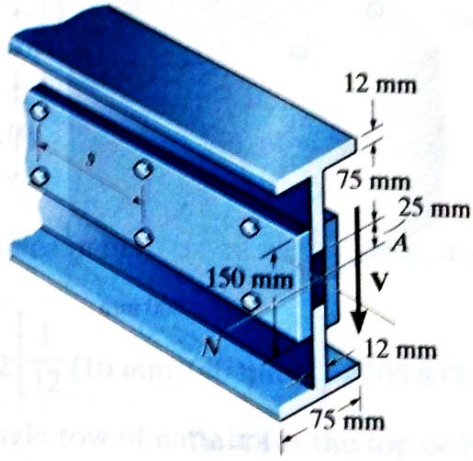
7–35. The beam is constructed from two boards fastened together with three rows of nails. If the allowable shear stress for the wood is $\tau_{allow} = 1$ MPa, determine the maximum shear force V that can be applied to the beam. Also, find the maximum spacing s of the nails if each nail can resist 3.25 kN in shear.



Probs. 7–34/35

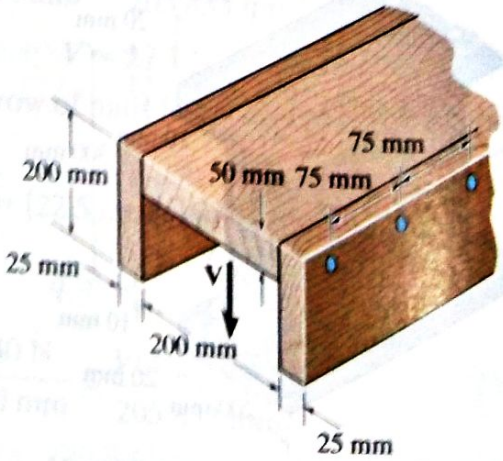
***7–36.** The beam is fabricated from two equivalent structural tees and two plates. Each plate has a height of 150 mm and a thickness of 12 mm. If a shear of $V = 250$ kN is applied to the cross section, determine the maximum spacing of the bolts. Each bolt can resist a shear force of 75 kN.

•7–37. The beam is fabricated from two equivalent structural tees and two plates. Each plate has a height of 150 mm and a thickness of 12 mm. If the bolts are spaced at $s = 200$ mm determine the maximum shear force V that can be applied to the cross section. Each bolt can resist a shear force of 75 kN.



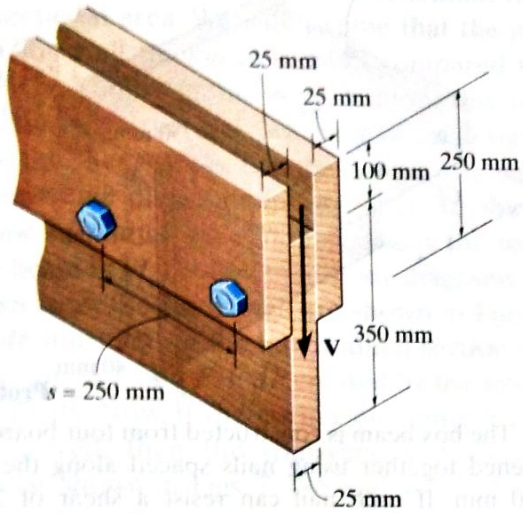
Probs. 7–36/37

7–38. The beam is subjected to a shear of $V = 2$ kN. Determine the average shear stress developed in each nail if the nails are spaced 75 mm apart on each side of the beam. Each nail has a diameter of 4 mm.



Prob. 7–38

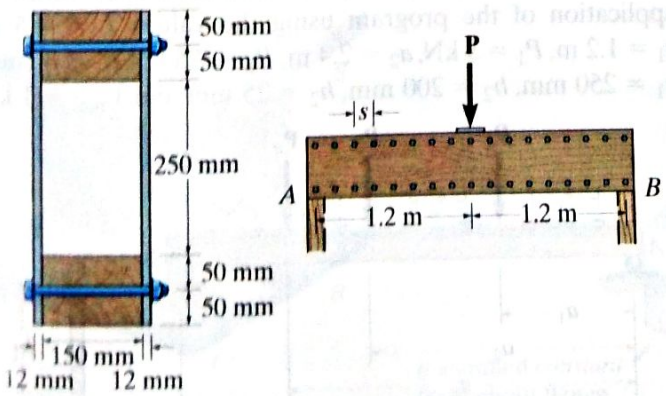
7–39. A beam is constructed from three boards bolted together as shown. Determine the shear force developed in each bolt if the bolts are spaced $s = 250$ mm apart and the applied shear is $V = 35$ kN.



Prob. 7–39

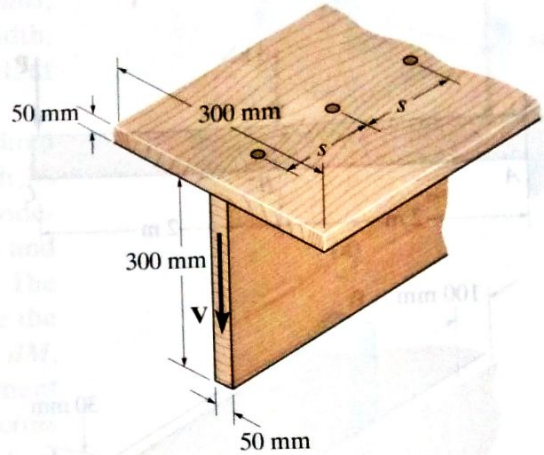
***7–40.** The double-web girder is constructed from two plywood sheets that are secured to wood members at its top and bottom. If each fastener can support 3 kN in single shear, determine the required spacing s of the fasteners needed to support the loading $P = 15$ kN. Assume A is pinned and B is a roller.

•7–41. The double-web girder is constructed from two plywood sheets that are secured to wood members at its top and bottom. The allowable bending stress for the wood is $\sigma_{allow} = 56$ MPa and the allowable shear stress is $\tau_{allow} = 21$ MPa. If the fasteners are spaced $s = 150$ mm and each fastener can support 3 kN in single shear, determine the maximum load P that can be applied to the beam.



Probs. 7–40/41

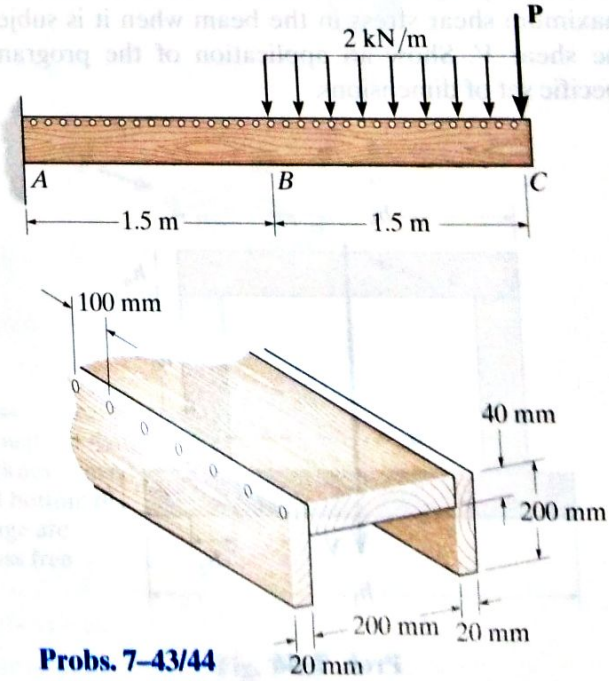
7–42. The T-beam is nailed together as shown. If the nails can each support a shear force of 4.5 kN, determine the maximum shear force V that the beam can support and the corresponding maximum nail spacing s to the nearest multiples of 5 mm. The allowable shear stress for the wood is $\tau_{allow} = 3$ MPa.



Prob. 7–42

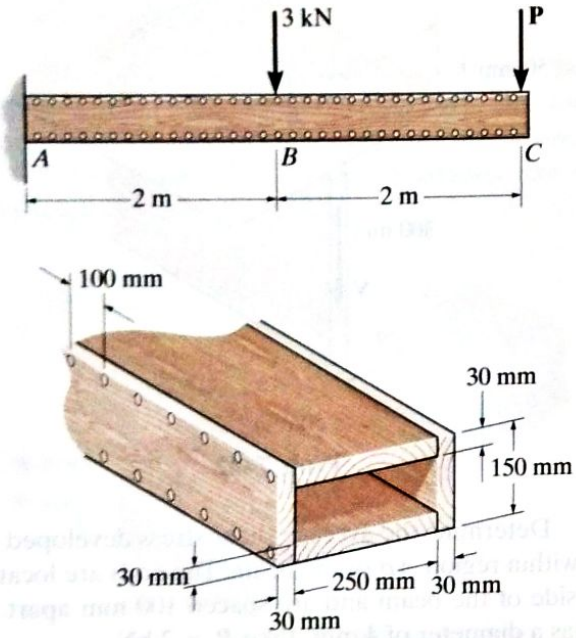
7–43. Determine the average shear stress developed in the nails within region AB of the beam. The nails are located on each side of the beam and are spaced 100 mm apart. Each nail has a diameter of 4 mm. Take $P = 2$ kN.

***7–44.** The nails are on both sides of the beam and each can resist a shear of 2 kN. In addition to the distributed loading, determine the maximum load P that can be applied to the end of the beam. The nails are spaced 100 mm apart and the allowable shear stress for the wood is $\tau_{allow} = 3$ MPa.



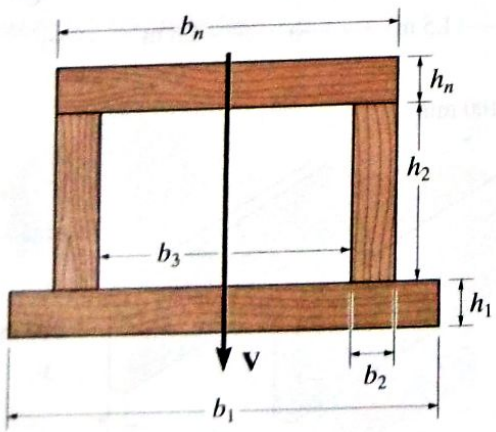
Probs. 7–43/44

•7–45. The beam is constructed from four boards which are nailed together. If the nails are on both sides of the beam and each can resist a shear of 3 kN, determine the maximum load P that can be applied to the end of the beam.



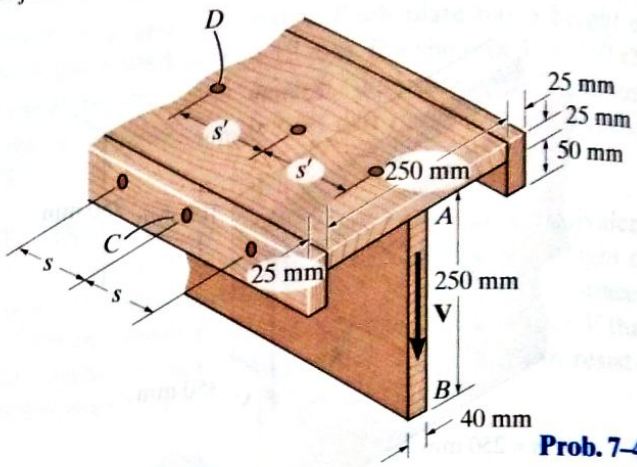
Prob. 7–45

7–46. A built-up timber beam is made from the four boards, each having a rectangular cross section. Write a computer program that can be used to determine the maximum shear stress in the beam when it is subjected to the shear V. Show an application of the program for a specific set of dimensions.



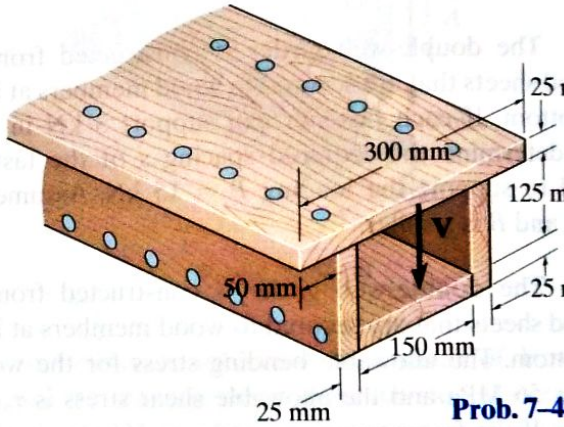
Prob. 7–46

7–47. The beam is made from four boards nailed together as shown. If the nails can each support a shear force of 500 N, determine their required spacing s' and s if the beam is subjected to a shear of $V = 3.5$ kN.



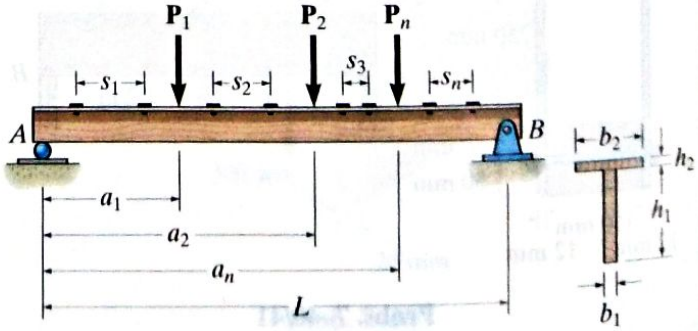
Prob. 7–47

*7–48. The box beam is constructed from four boards that are fastened together using nails spaced along the beam every 50 mm. If each nail can resist a shear of 250 N, determine the greatest shear V that can be applied to the beam without causing failure of the nails.



Prob. 7–48

7–49. The timber T-beam is subjected to a load consisting of n concentrated forces, P_n. If the allowable shear V_{nail} for each of the nails is known, write a computer program that will specify the nail spacing between each load. Show an application of the program using the values $L = 4.5$ m, $a_1 = 1.2$ m, $P_1 = 3$ kN, $a_2 = 2.4$ m, $P_2 = 7.5$ kN, $b_1 = 40$ mm, $h_1 = 250$ mm, $b_2 = 200$ mm, $h_2 = 25$ mm and $V_{nail} = 1$ kN.

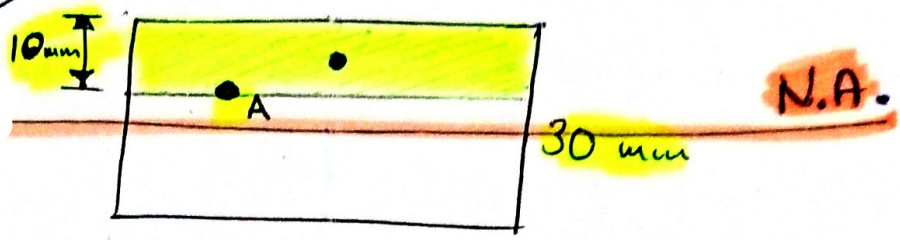


Prob. 7–49

Ex

$I = \frac{1}{12}(40)(30)^3 =$ ✓

$b_A = 40$ mm

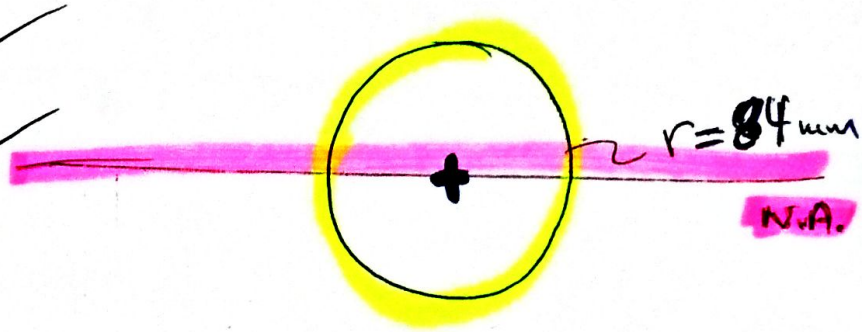


$Q_A = (10)(40)(10) = 4000$ mm^3

$Q_{max} = (15)(40)(7.5) = 4500$ mm^3

$I = \frac{\pi}{4} r^4 = \frac{\pi}{4}(84)^4 =$ ✓

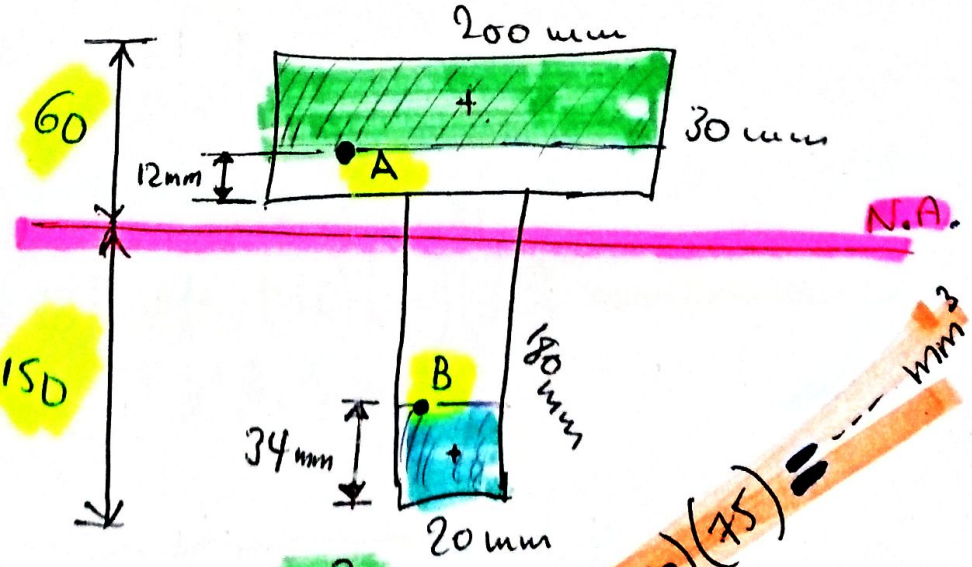
$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(168)^4 =$ ✓



$b_{max} = 168$ mm

$\boxed{Q_{max} = \frac{2}{3} r^3}$ قانون هفظ للدائرة $= \frac{2}{3}(84)^3 =$ ---- mm^3

$I =$ قطعتين كما تعلمنا سابقاً



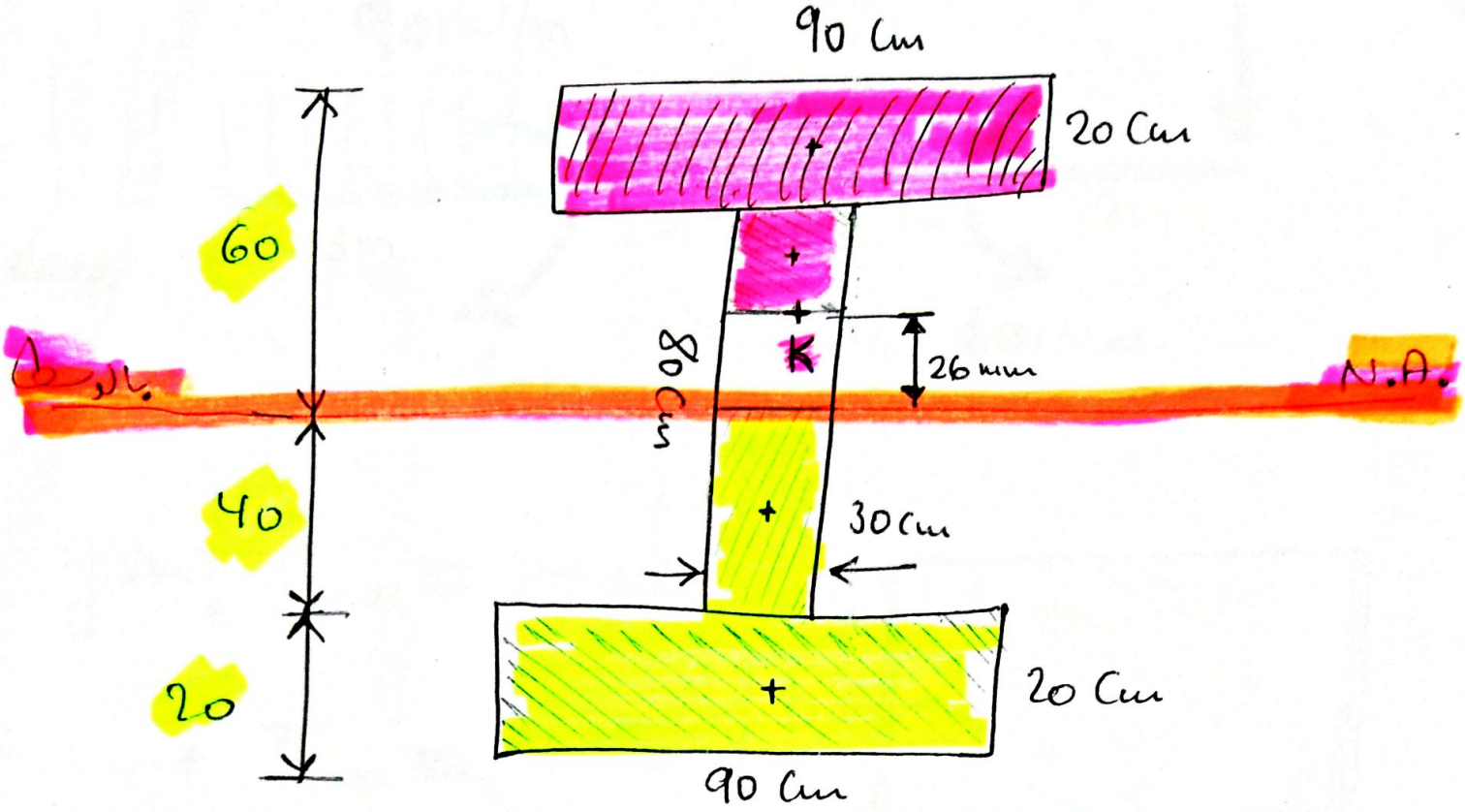
$b_A = 200$ mm

$b_B = 20$ mm

$Q_A = (200)(18)(51) =$ --- mm^3

$Q_B = (20)(34)(133) =$ --- mm^3

$Q_{max} = (20)(150)(75) =$ --- mm^3



* Find Q_{max} ?

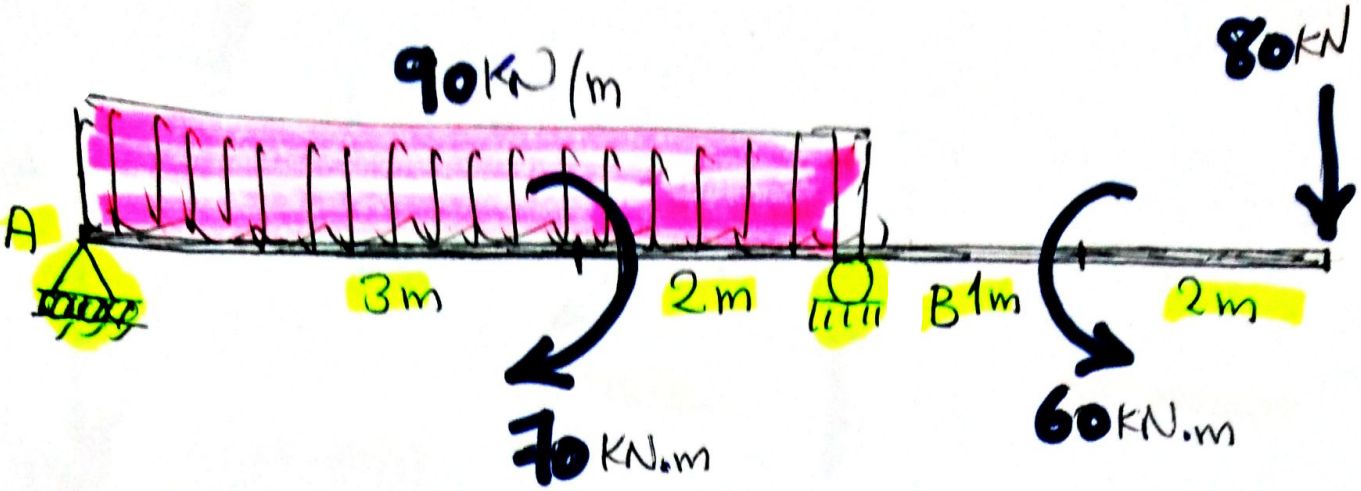
$$Q_{max} = (30)(40)(20) + (90)(20)(50) = \text{----} \ mm^3$$

المساحة الصفراء

* Find Q_K ?

$$Q_K = (30)(14)(33) + (90)(20)(50) = \text{----} \ mm^3$$

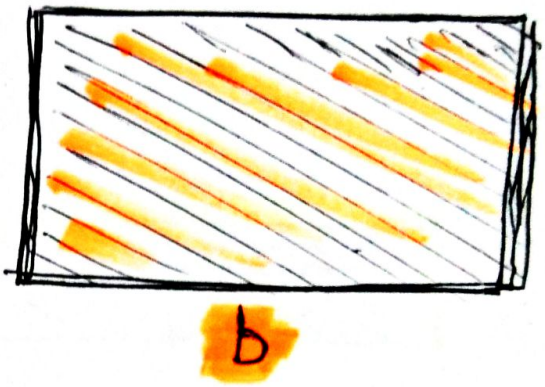
المساحة الحمراء



given

* $\sigma_{max} = 300$ MPa

* $\tau_{max} = 120$ MPa



Find The width b ?

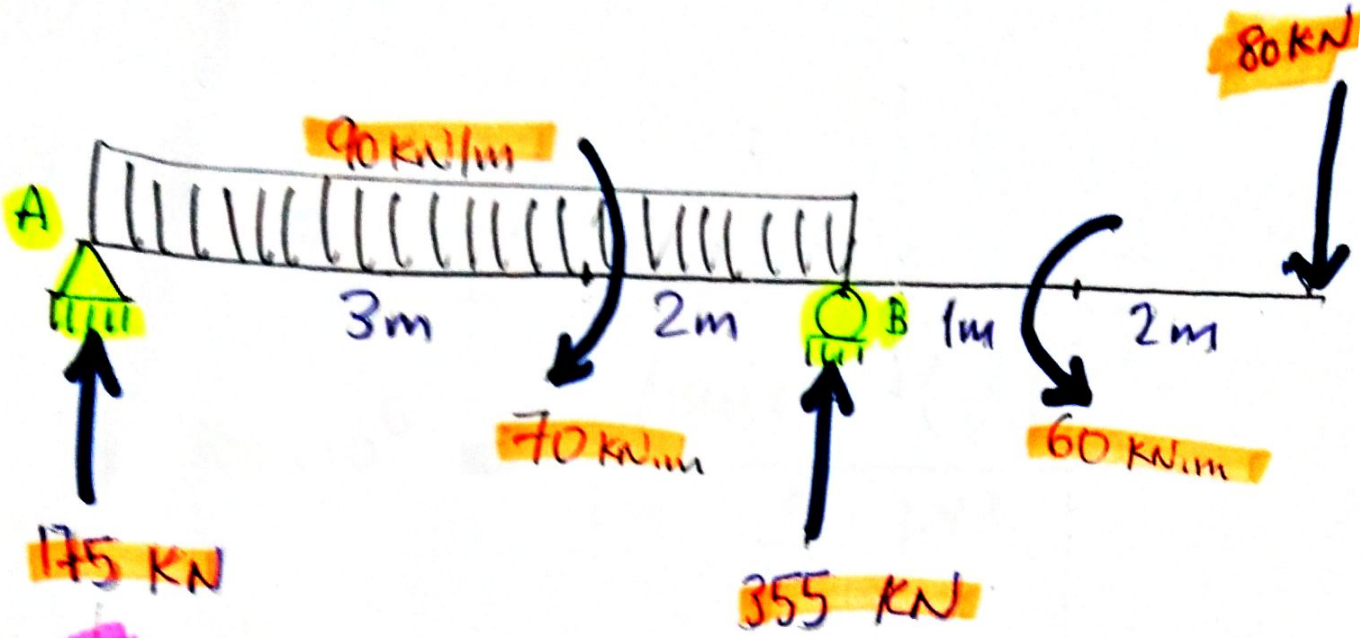
* $y_{max} = \frac{\frac{3}{4}b}{2} = \frac{3}{8}b$

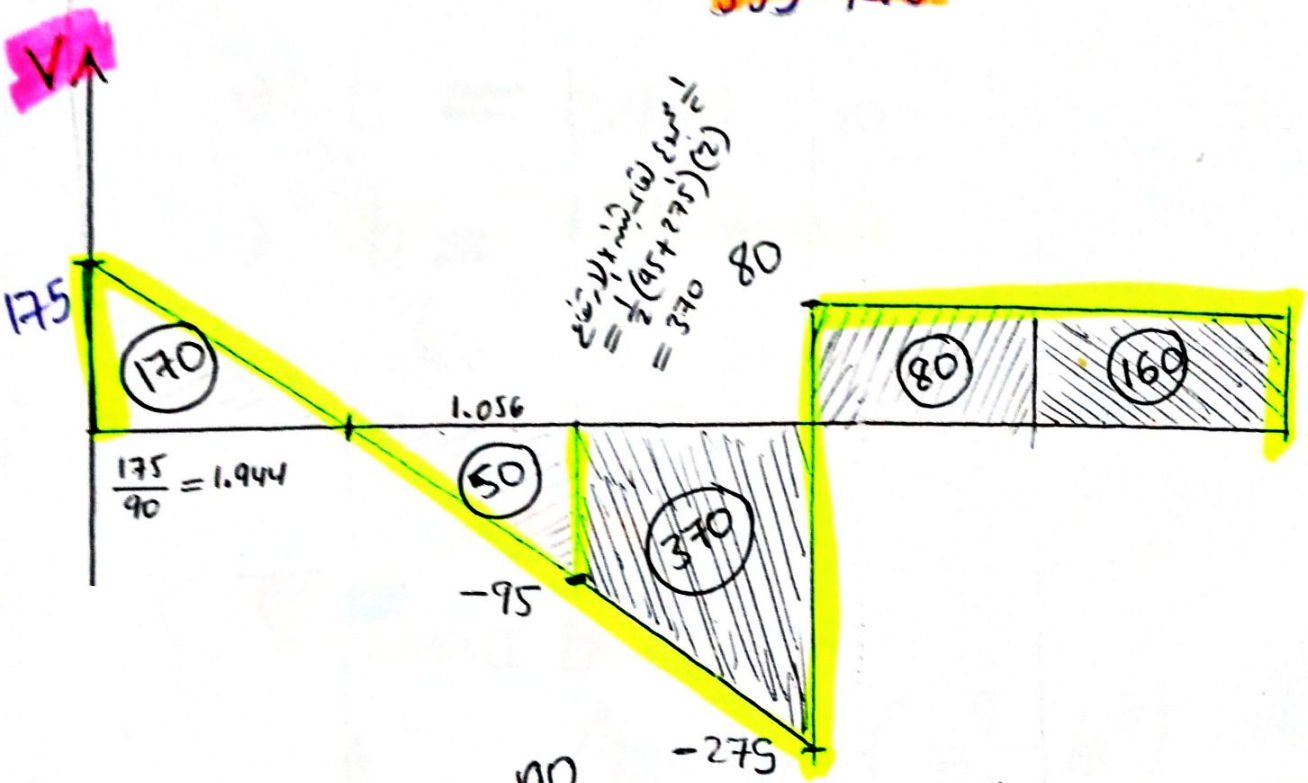
* $b_{max.} = b$

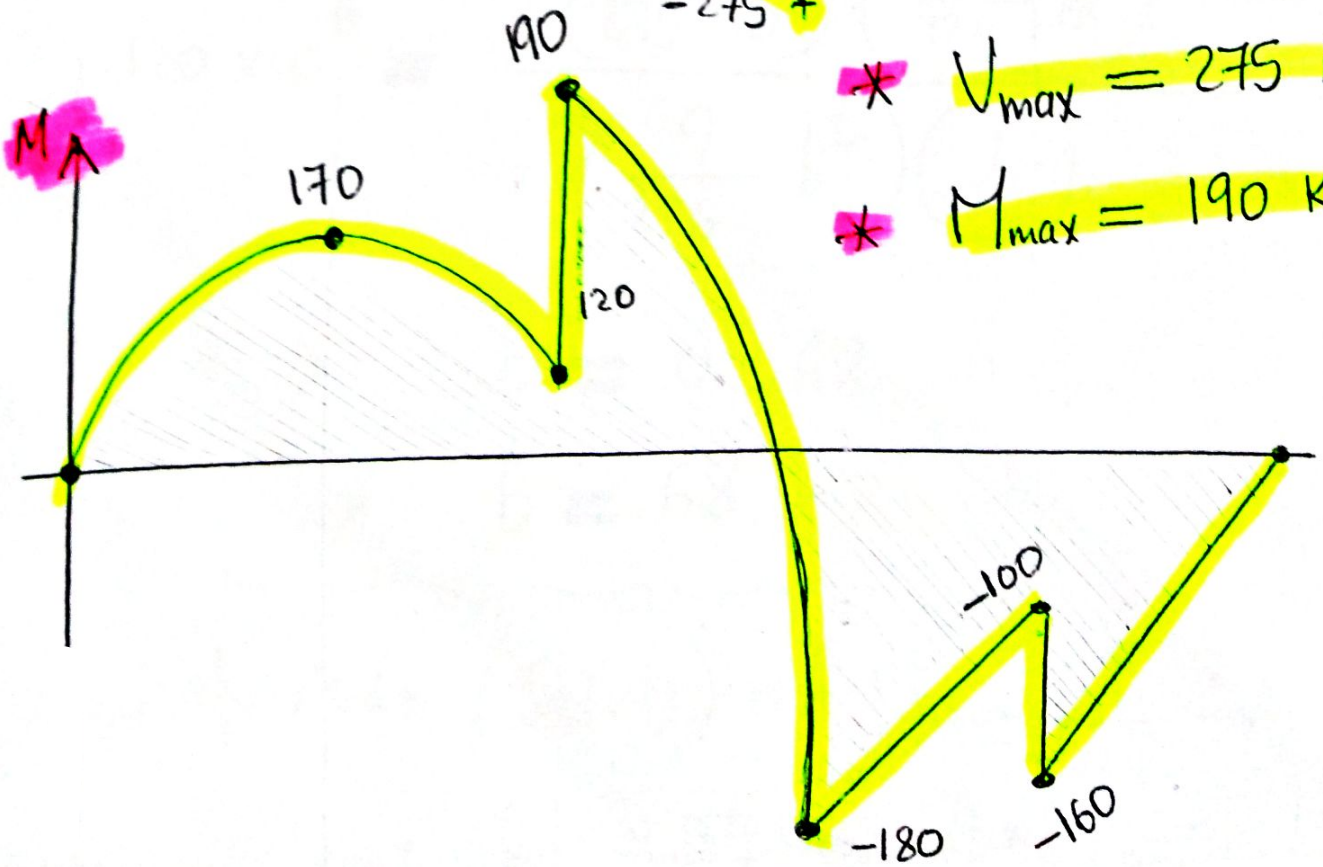
* $I = \frac{1}{12}bh^3 = \frac{1}{12}b\left(\frac{3}{4}b\right)^3$

$$I = \frac{9}{256}b^4$$

* $Q_{max} = (b)\left(\frac{3}{8}b\right)\left(\frac{3}{16}b\right) = \frac{9}{128}b^3$







* $V_{max} = 275$ KN
* $M_{max} = 190$ KN.m

$$\sigma = \frac{My}{I}$$

$$\Rightarrow 300 \times 10^6 = \frac{(190 \times 10^3)(\frac{3}{8} b)}{\frac{9}{256} b^4}$$

$$\Rightarrow b = 0.189 \quad m$$

$$\Rightarrow b = 189 \quad mm$$

$$\tau = \frac{VQ}{Ib}$$

$$120 \times 10^6 = \frac{(275 \times 10^3)(\frac{9}{128} b^3)}{(\frac{9}{256} b^4)(b)}$$

$$\Rightarrow b = 0.068 \quad m$$

$$\Rightarrow b = 68 \quad mm$$

** حيث نأخذ بمبدأ الأمان (Safety) ونختار العرض الأكبر

وهو ($b = 189$ mm) .

لنفس السؤال السابق ولكن
المعتبر أن المقطع دائري

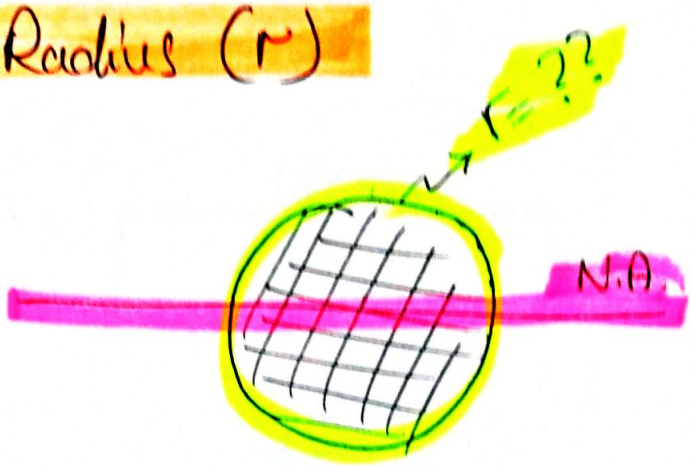
$\Rightarrow$ Find The Radius (r)

$b = 2r$

$y = r$

$I = \frac{\pi}{4} r^4$

$Q = \frac{2}{3} r^3$



$$\sigma = \frac{My}{I}$$

$$\Rightarrow 300 \times 10^6 = \frac{(190 \times 10^3)(r)}{\frac{\pi}{4} r^4}$$

$$\Rightarrow r = 0.093 \text{ m}$$

$$\Rightarrow r = 93 \text{ mm}$$

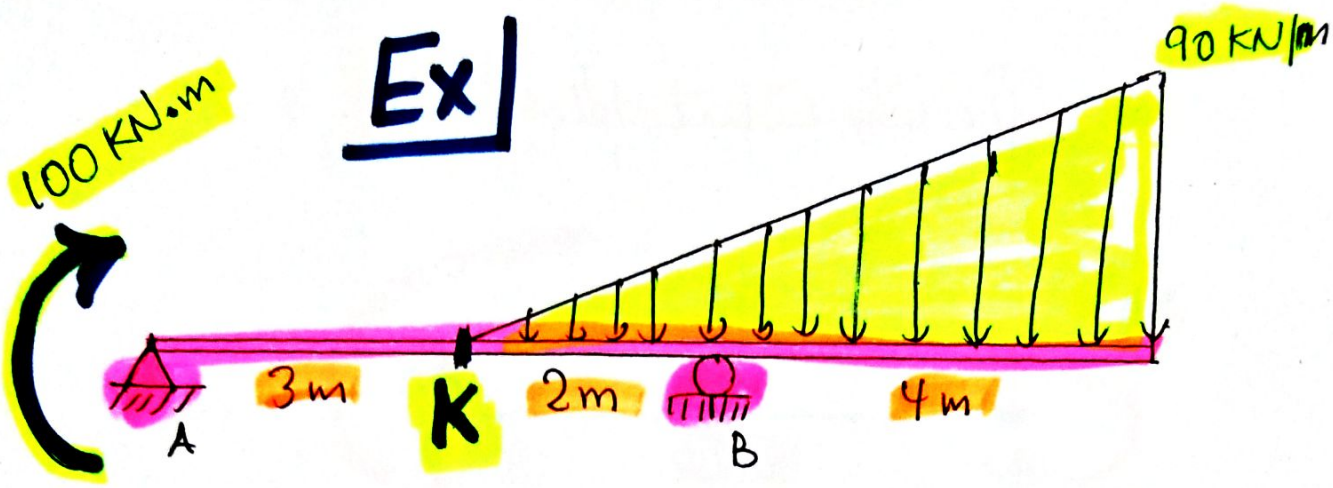
$$\tau = \frac{VQ}{Ib}$$

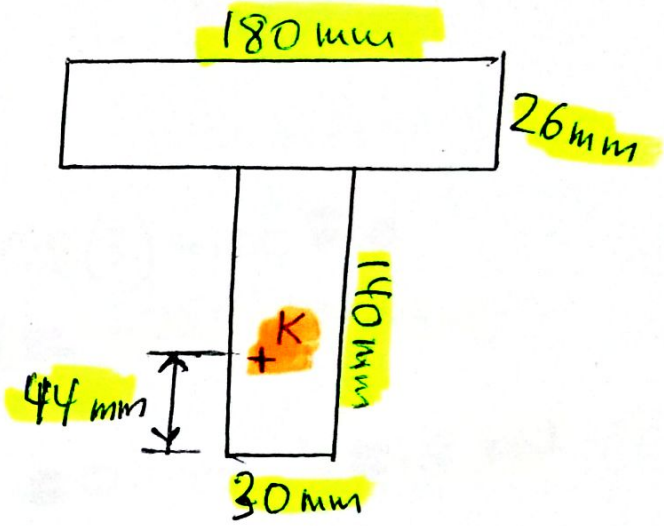
$$\Rightarrow 120 * 10^6 = \frac{(275 \times 10^3)\left(\frac{2}{3} r^3\right)}{\left(\frac{\pi}{4} r^4\right)(2r)}$$

$$\Rightarrow r = 0.031 \text{ m} = 31 \text{ mm}$$

حيث نختار الأكبر ($r = 93$ mm)

Ex





Find :

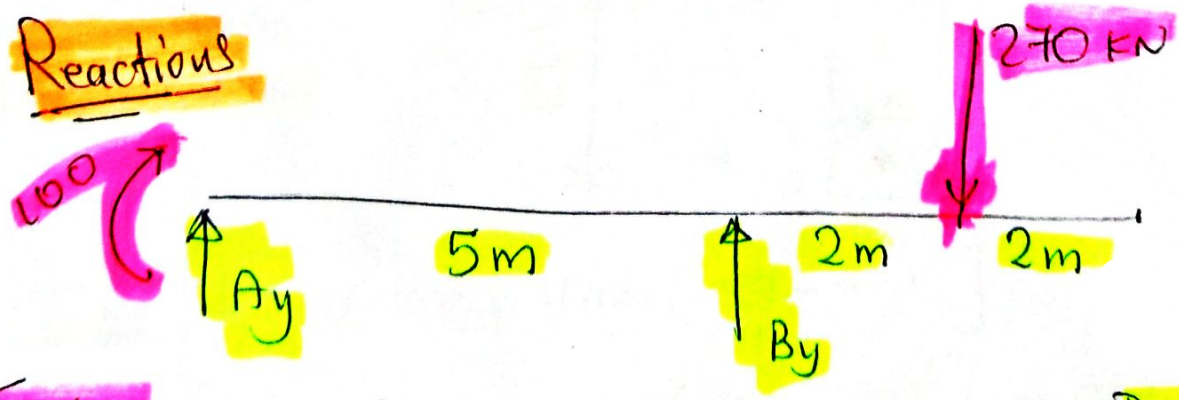
1. Normal Bending Stress at point K ?

2. Shear Bending Stress at point K ?

3. Radius of Curvature at point K given that $E = 125$ GPa ?

$\frac{1}{2}(6)(90) = 270$ KN

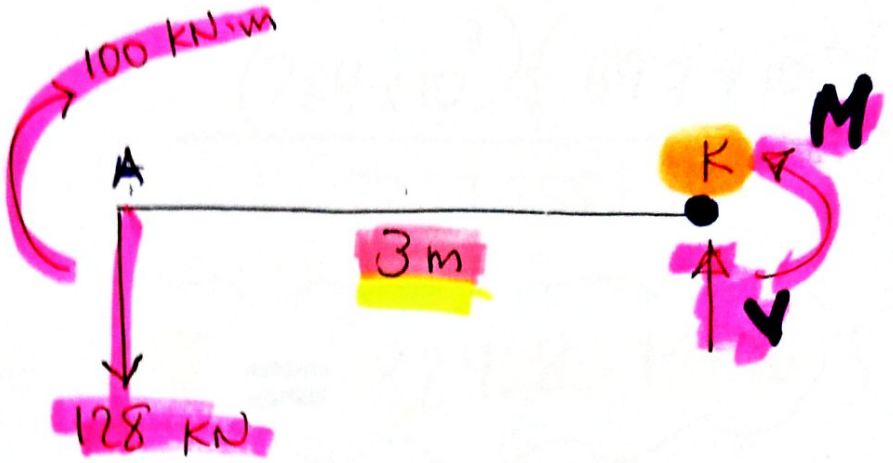
Reactions



$\sum M_A = 0 \Rightarrow 5B_y - 270(7) - 100 = 0 \rightarrow B_y = 398$ KN

$\sum F_y = 0 \Rightarrow A_y + 398 - 270 = 0 \rightarrow A_y = -128$ KN

* والآن نقطع عند النقطة K



$\sum M_K = 0 \Rightarrow M + 128(3) - 100 = 0$

$\Rightarrow M = -284 \text{ kN.m}$

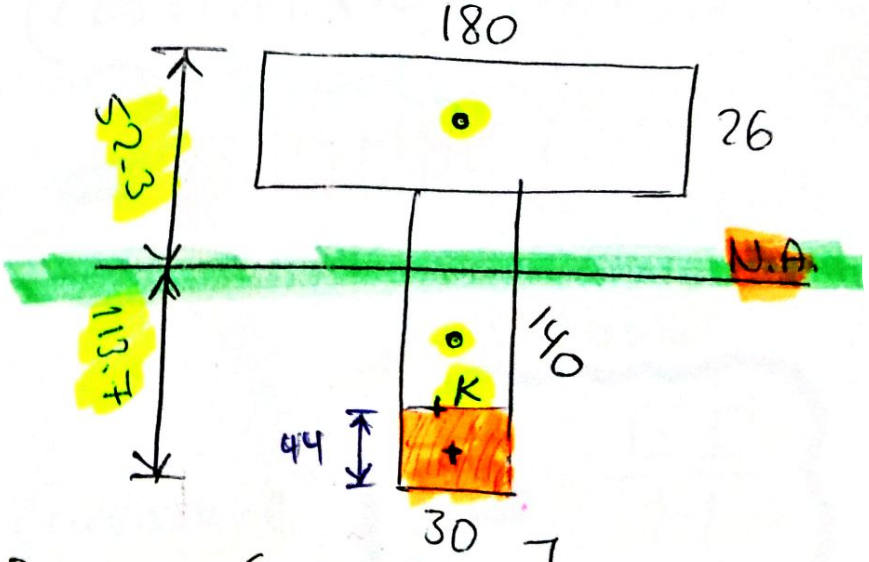
$\sum F_y = 0 \Rightarrow V - 128 = 0 \rightarrow V = 128 \text{ kN}$

$y_1 = 70 \quad A_1 = 4200$

$y_2 = 153 \quad A_2 = 4680$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$\bar{y} = 113.7 \text{ mm}$



* $I = \left[\frac{1}{12}(30)(140)^3 + 4200(113.7-70)^2\right] + \left[\frac{1}{12}(180)(26)^3 + 4680(113.7-153)^2\right] = 22372551 \text{ mm}^4$

* $Q_K = (44)(30)(91.7) = 121044 \text{ mm}^3$

* $y_K = 69.7 \text{ mm}$, $b_K = 30 \text{ mm}$

1

$$\sigma_K = \frac{M_K y_K}{I}$$

$$\sigma_K = \frac{(284 \times 10^{3})(69.7 \times 10^{-3})}{2237255 \times 10^{-12}}$$

$$\sigma_K = 884.8 \text{ MPa}$$

2

$$\tau_K = \frac{V_K Q_K}{I b_K}$$

$$\tau_K = \frac{(128 \times 10^{3})(121044 * 10^{-9})}{(2237255 * 10^{-12})(30 \times 10^{-3})}$$

$$\tau_K = 23.1 \text{ MPa}$$

قانون الحفظ / مهم

3

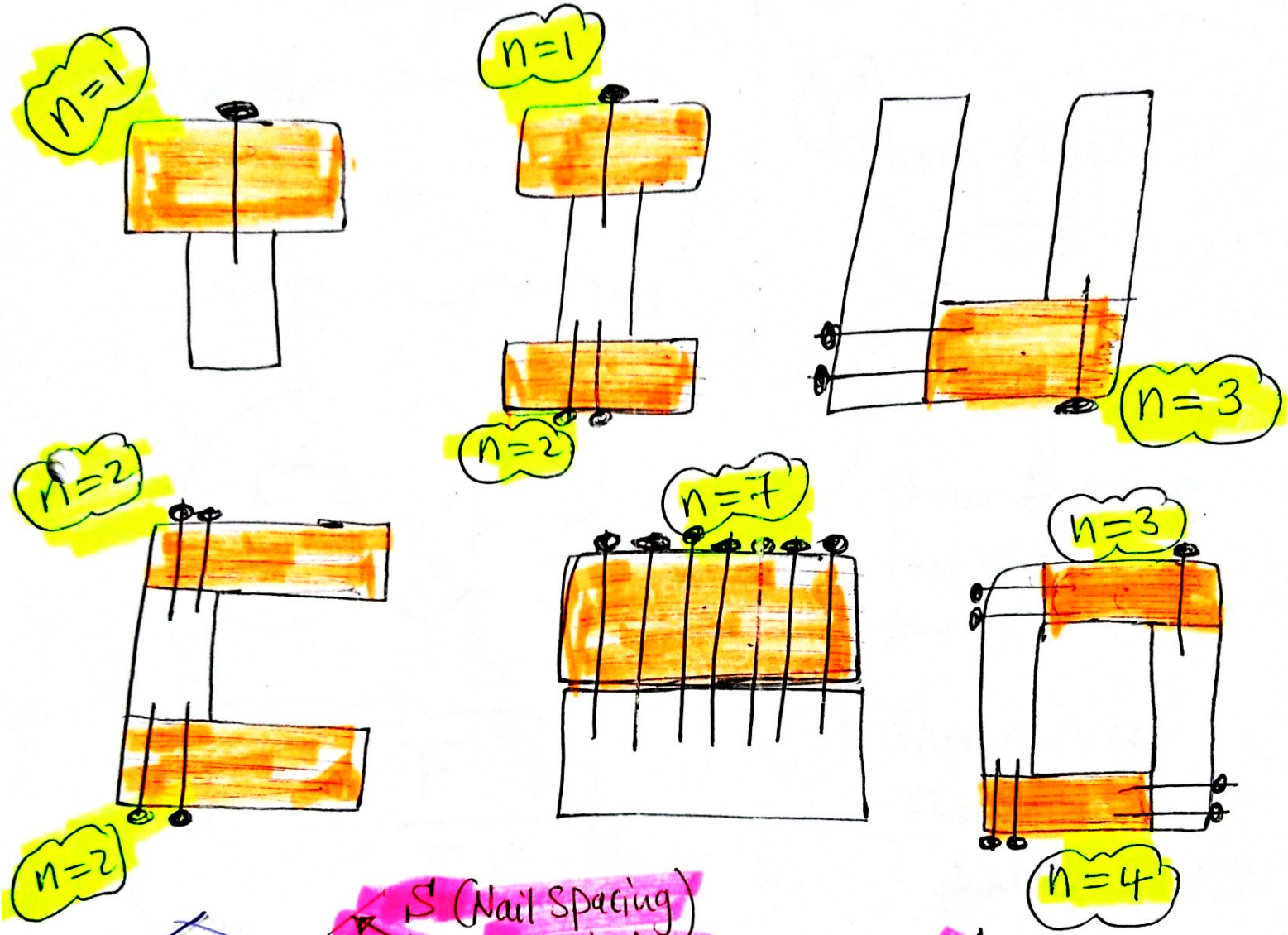
Radius of Curvature

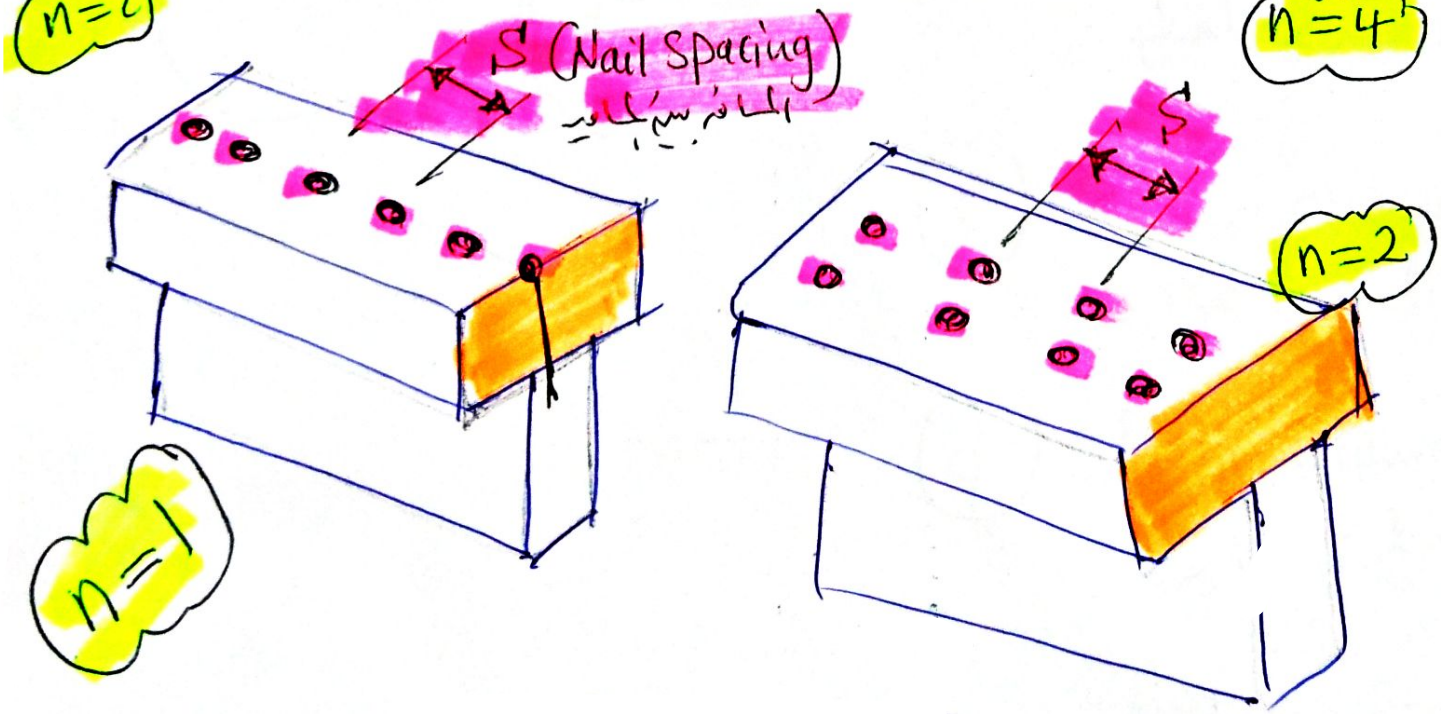
$$\rho = \frac{EI}{M}$$

$$\rho_K = \frac{(125 * 10^{9})(2237255 * 10^{-12})}{284 * 10^{3}}$$

$$\rho_K = 9.85 \text{ m}$$

Shear flow (q) (Nails) المسامير

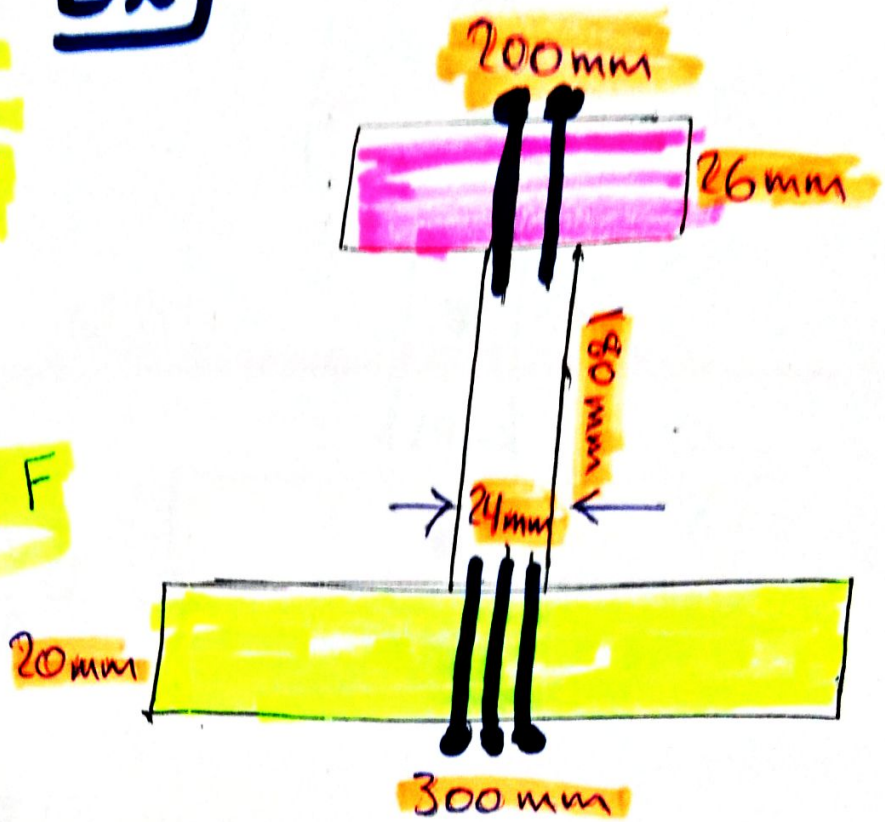




Ex)

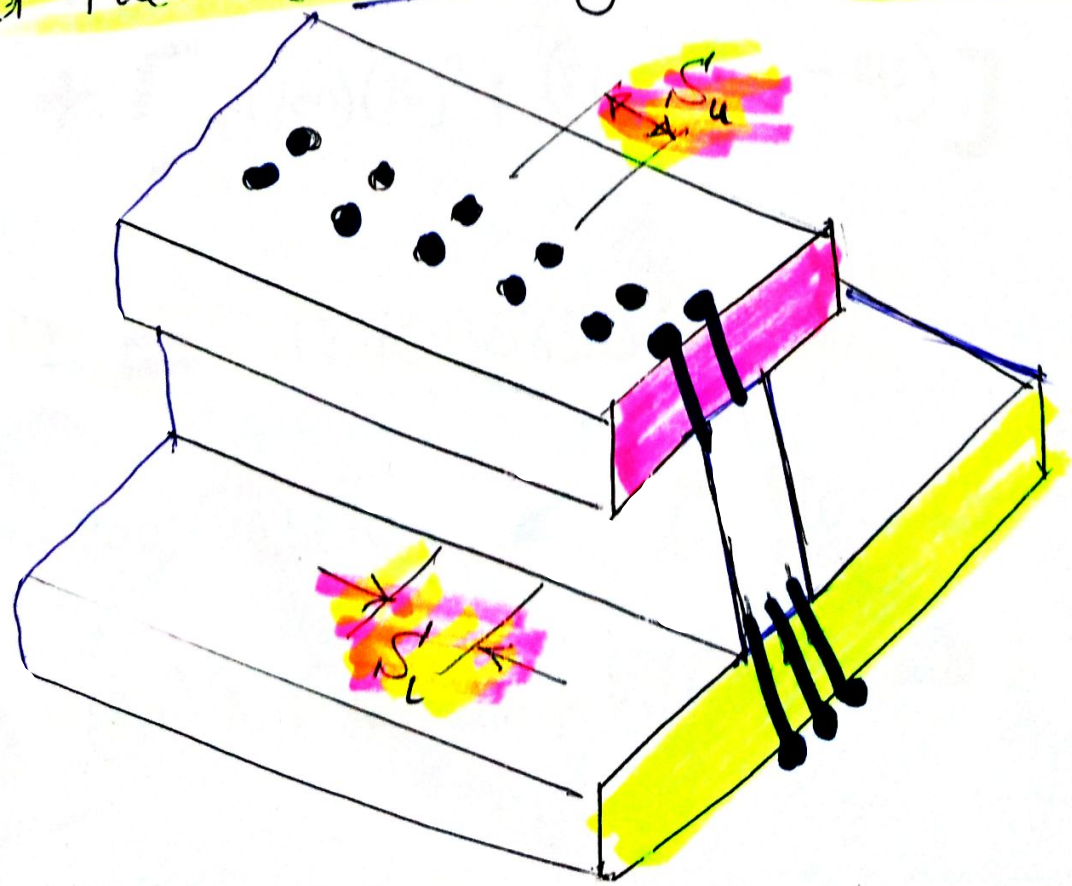
The shear Force in the section is (60 KN) ⇒ V

and the shear force in each Bolt is (6 KN) ⇒ F



1) Find The Nail Spacing (S_u) in the upper (العلوي) flange?

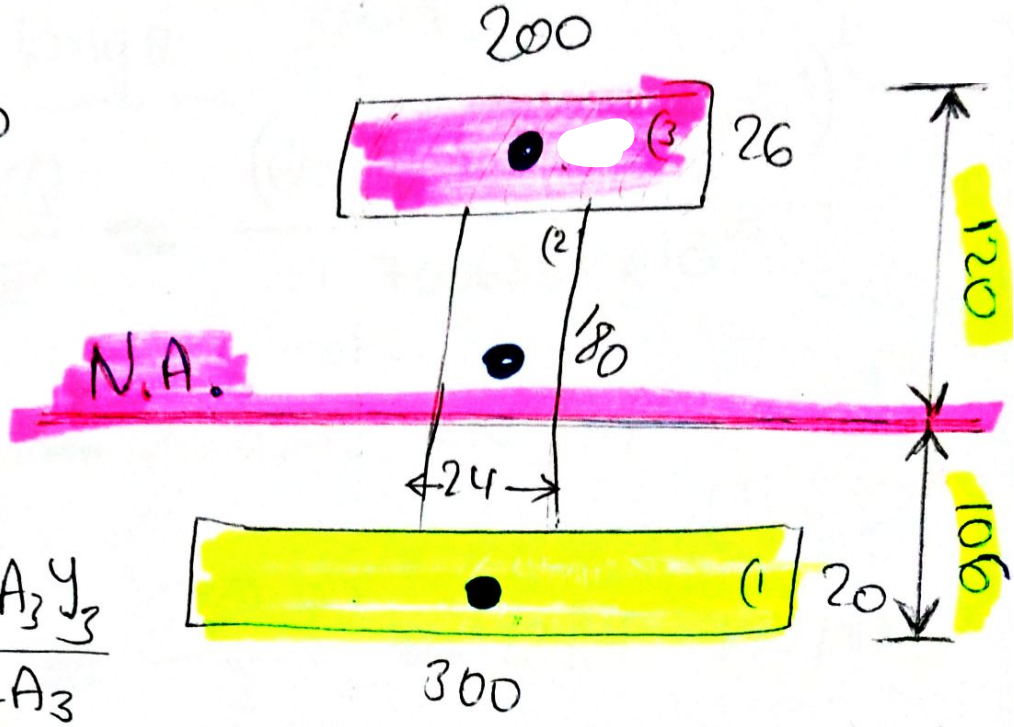
2) Find The Nail Spacing (S_L) in the lower (السفلي) flange?



$y_1 = 10 \quad A_1 = 6000$

$y_2 = 110 \quad A_2 = 4320$

$y_3 = 213 \quad A_3 = 5200$



$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\Rightarrow \quad \bar{y} = 106 \text{ mm}$$

$$I = \left[\frac{1}{12}(300)(20)^3 + (6000)(106-10)^2\right] + \left[\frac{1}{12}(24)(180)^3 + (4320)(106-110)^2\right] + \left[\frac{1}{12}(200)(26)^3 + (5200)(106-213)^2\right]$$

$$I = 127056853 \cdot \text{mm}^4$$

العلوي

$$Q_u = (200)(26)(107) = 556400 \text{ mm}^3$$

السفلي

$$Q_L = (300)(20)(96) = 576000 \text{ mm}^3$$

upper flange العلوية

* $q = \frac{VQ}{I} = \frac{(60 \times 10^3)(5564000 \times 10^{-9})}{127056853 \times 10^{-12}}$

$\Rightarrow q = 262.7 \text{ KN/m}$

العلوية $n = 2$

* $q^* = \frac{q}{n} = \frac{262.7}{2} = 131.4 \text{ KN/m}$

* $F = q^* S \Rightarrow S = \frac{6 \times 10^3}{131.4 \times 10^3}$

$\Rightarrow S = 0.0457 \text{ m}$

$\Rightarrow S_u = 45.7 \text{ mm}$

lower flange السفلية

* $q = \frac{VQ}{I} = \frac{(60 \times 10^3)(5760000 \times 10^{-9})}{127056853 \times 10^{-12}}$

$\Rightarrow q = 272 \text{ KN/m}$

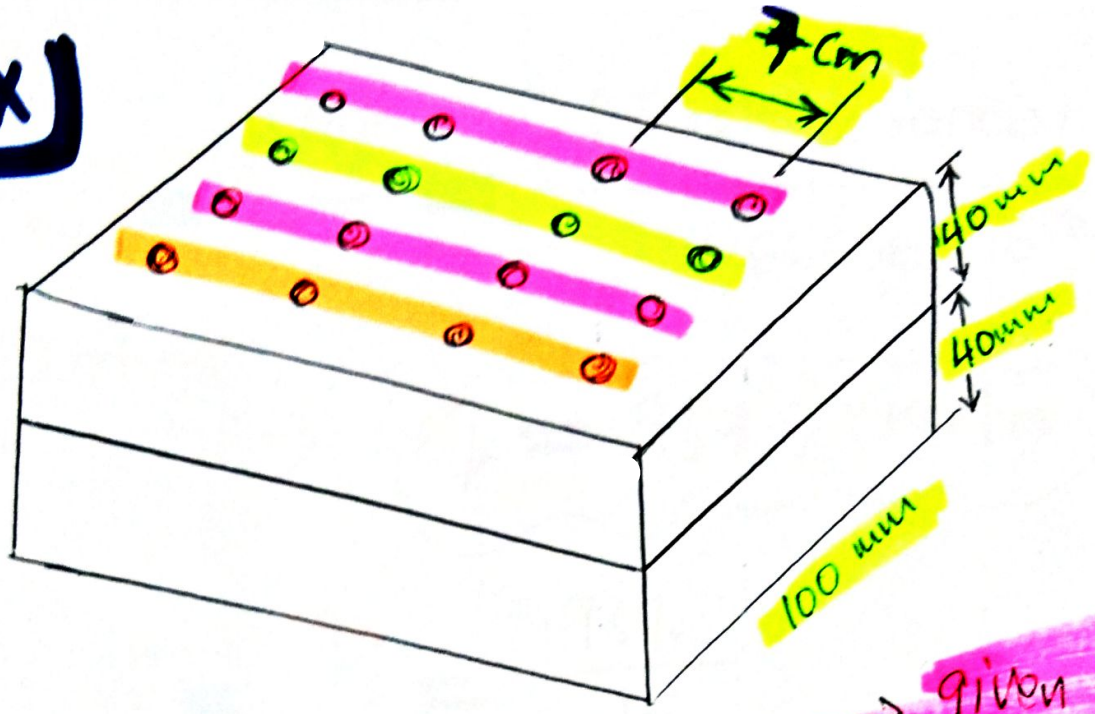
السفلية $n = 3$

* $q^* = \frac{q}{n} = \frac{272}{3} = 90.7 \text{ KN/m}$

* $F = q^* S \Rightarrow S = \frac{6 \times 10^3}{90.7 \times 10^3} = 0.0662 \text{ m}$

$\Rightarrow S_L = 66.2 \text{ mm}$

Ex



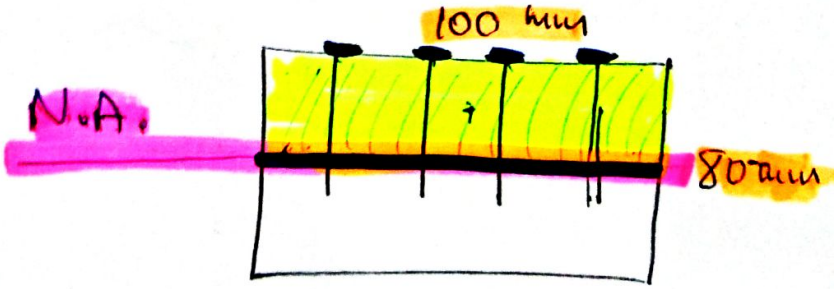
⇒ given $V = 50\ kN$

* The allowable shear stress in each Bolt (Nail) is [redacted] 22 GPa.

Find

* The Nail diameter?
* Nail spacing $S = 7\ cm$

أوجد قطر المسمار؟



$I = \frac{1}{12} b h^3$

$I = \frac{1}{12}(100)(80)^3$

$I = 4266666\ mm^4$

$n = 4$

$Q = (100)(40)(20) = 80000\ mm^3$

$$q = \frac{VQ}{I} = \frac{(50\times10^{3})(80000\times10^{-9})}{4266666\times10^{-12}}$$

$$\Rightarrow \quad q = 937.5 \ \text{KN/m}$$

$$q^{*} = \frac{q}{n} = \frac{937.5}{4}$$

$$\Rightarrow \quad q^{*} = 234.4 \ \text{KN/m}$$

$$F = q^{*} S$$

$$\Rightarrow \quad F = (234.4 * 10^{3})(7 * 10^{-2})$$

$$\Rightarrow \quad F = 3349 \ \text{KN}$$

$$\Rightarrow \quad \tau = \frac{F}{\frac{\pi}{4} d^{2}}$$

$$\Rightarrow \quad 22 * 10^{9} = \frac{3349 * 10^{3}}{\frac{\pi}{4}(d^{2})}$$

$$\Rightarrow \quad d = 0.0139 \ \text{m} \Rightarrow d = 13.9 \ \text{mm}$$

قطر المسمار

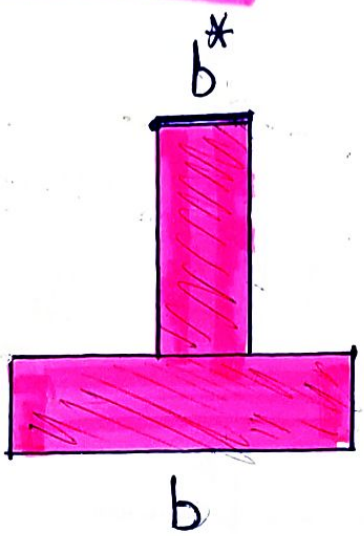
Dr. Hashem Alkhaldi

Composite Beams

* Transform material (1) to material (2)

* Transformation factor:

$$n = \frac{E_1}{E_2}$$



* $b^* = n\,b$

(1) ≡ Transformed material.

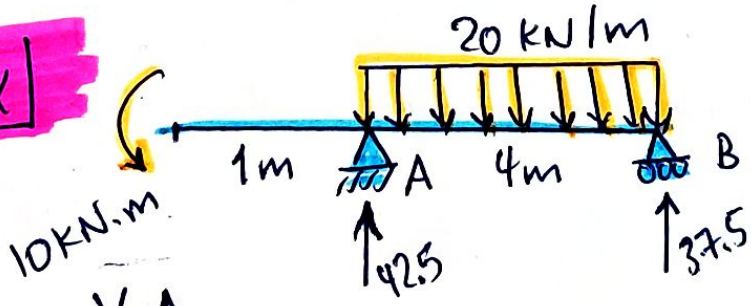
(2) ≡ Base material.

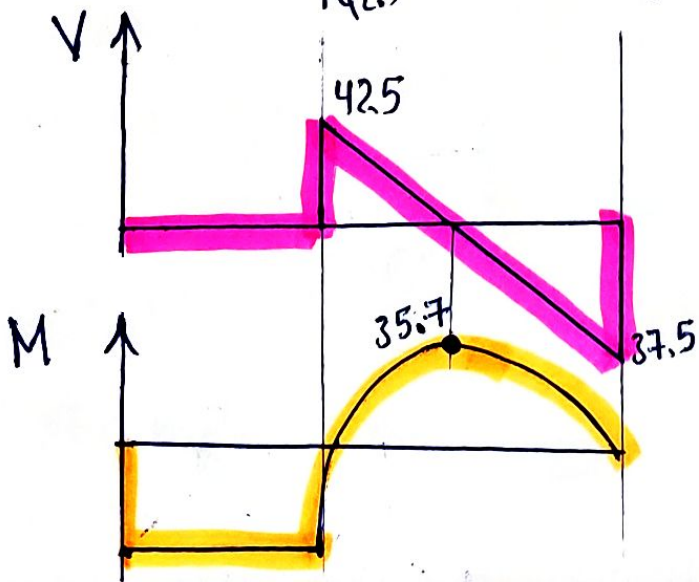
* $\sigma_A = (n)\dfrac{M y_A}{I}$

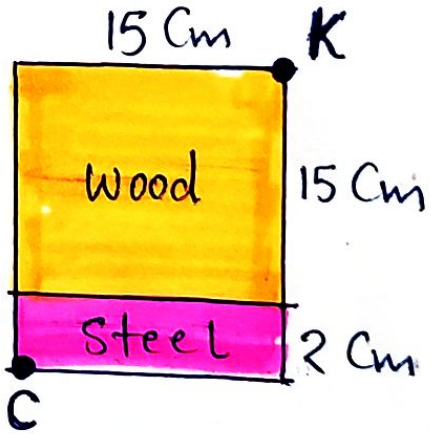
* $\sigma_C = \dfrac{M y_C}{I}$

د. هاشم الخالدي

Ex







$E_w = 12$ GPa

$E_s = 200$ GPa

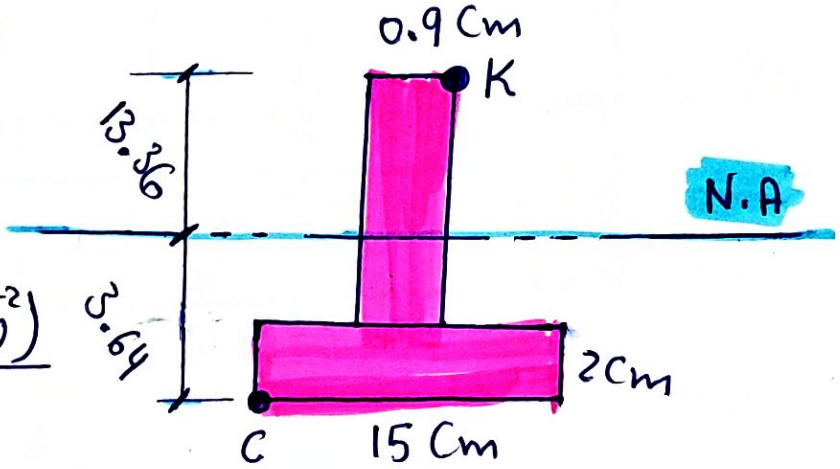
* Find Stresses @ points C & K ?

* Transform Wood to Steel :

$$n = \frac{E_w}{E_s} = 0.06$$

* $b^* = nb = (0.06)(15) = 0.9 \text{ Cm}$

* $I = 935.8 \text{ Cm}^4$



* $\sigma_c = \dfrac{M y_c}{I}$

$$\Rightarrow \quad = \frac{(35.7\times10^3)(3.64\times10^{-2})}{935.8\times10^{-8}}$$

$$= 136.2 \text{ MPa, Tension}$$

* $\sigma_K = (n)\dfrac{M y_K}{I}$

$$= (0.06)\frac{(35.7\times10^3)(13.36\times10^{-2})}{935.8\times10^{-8}} = 30.12 \text{ MPa, Compression.}$$

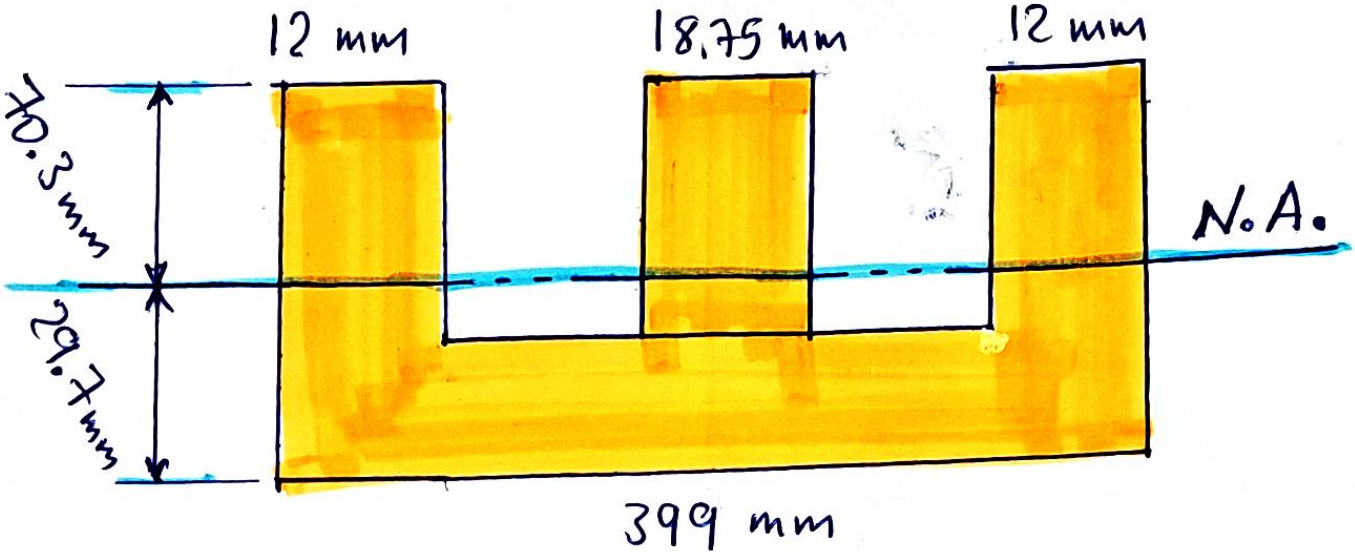
Ex

* $E_w = 10 \text{ GPa}$
* $E_s = 200 \text{ GPa}$
* $\sigma_s = 150 \text{ MPa}$
* $\sigma_w = 14 \text{ MPa}$

* Find the maximum Bending Moment M ?

* Transform Wood to Steel

$$n = \frac{E_w}{E_s} = 0.05 \quad , \quad b^* = 0.05(375) = 18.75 \text{ mm}$$



* $I = 7.78 \times 10^6 \text{ mm}^4$

* $\sigma_s = \frac{M y_s}{I} \Rightarrow 150 \times 10^6 = \frac{(M)(70.3 \times 10^{-3})}{7.78 \times 10^6 \times 10^{-12}}$

$$\Rightarrow M = 16.6 \text{ KN.m}$$

* $\sigma_w = (n) \frac{M y_w}{I} \Rightarrow 14 \times 10^6 = 0.05 \frac{(M)(70.3 \times 10^{-3})}{7.78 \times 10^6 \times 10^{-12}}$

$$\Rightarrow M = 30.9 \text{ KN.m}$$

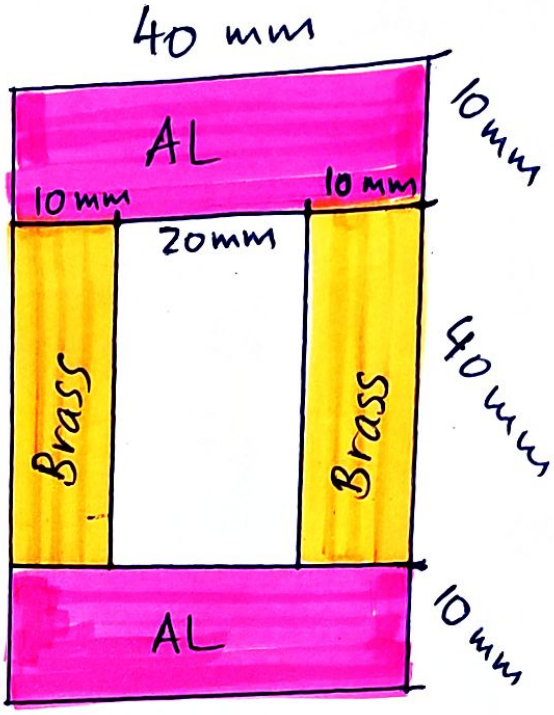
⇒ the max. Bending moment which Controls is:

$$M = 16.6 \text{ KN.m}$$

Ex

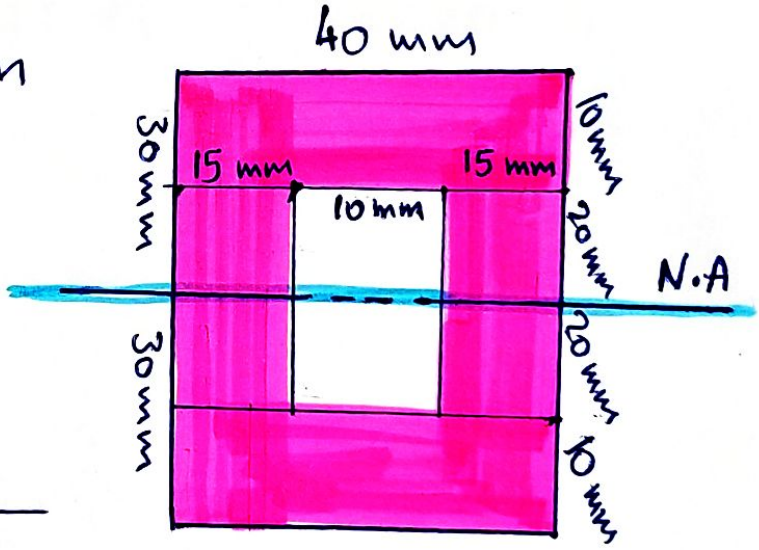
* $E_{AL} = 70\ GPa$
* $E_{Br} = 105\ GPa$
* $\sigma_{AL} = 100\ MPa$
* $\sigma_{Br} = 160\ MPa$

⇒ Find the maximum Bending Moment M ?



⇒ Transform Brass to AL :

* $n = \dfrac{E_{Br}}{E_{AL}} = 1.5$
* $b^* = (1.5)(10) = 15\ mm$
* $I = 667 \times 10^3\ mm^4$



* $\sigma_{AL} = \dfrac{M\, y_{AL}}{I}$

$$\Rightarrow 100 \times 10^6 = \frac{(M)(30 \times 10^{-3})}{667 \times 10^3 \times 10^{-12}}$$

$$\Rightarrow M = 2.22\ KN.m$$

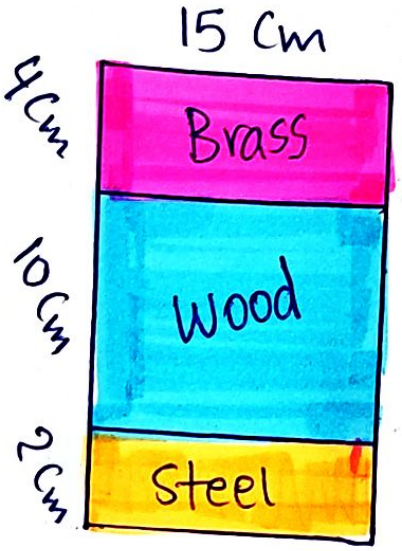
* $\sigma_{Br} = (n)\dfrac{M\, y_{Br}}{I} \Rightarrow 160 \times 10^6 = \dfrac{(1.5)(M)(20 \times 10^{-3})}{667 \times 10^3 \times 10^{-12}}$

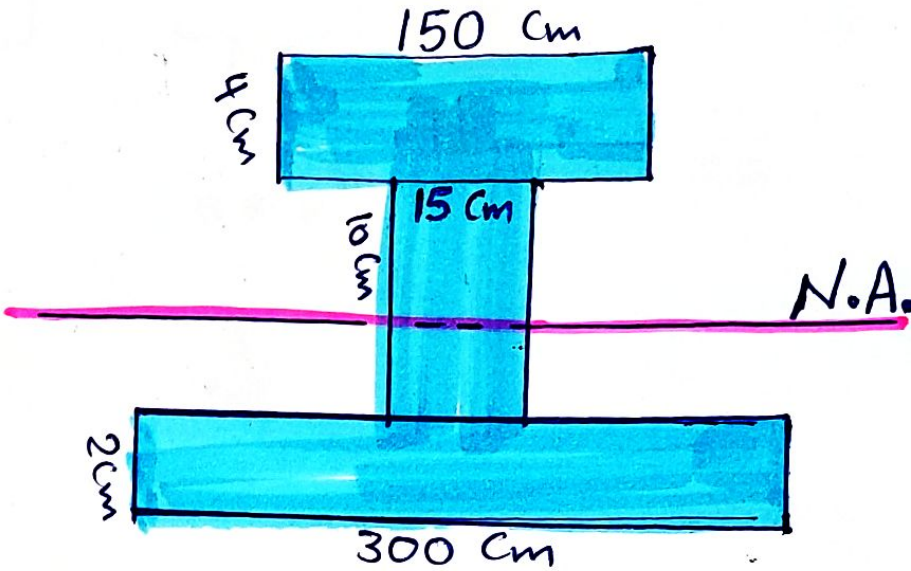
$$\Rightarrow M = 3.56\ KN.m$$

* The max. Bending Moment $M_{max} = 2.22\ KN.m$

$E_{st} = 200$ GPa
$E_{br} = 100$ GPa
$E_w = 10$ GPa

Ex] Three materials

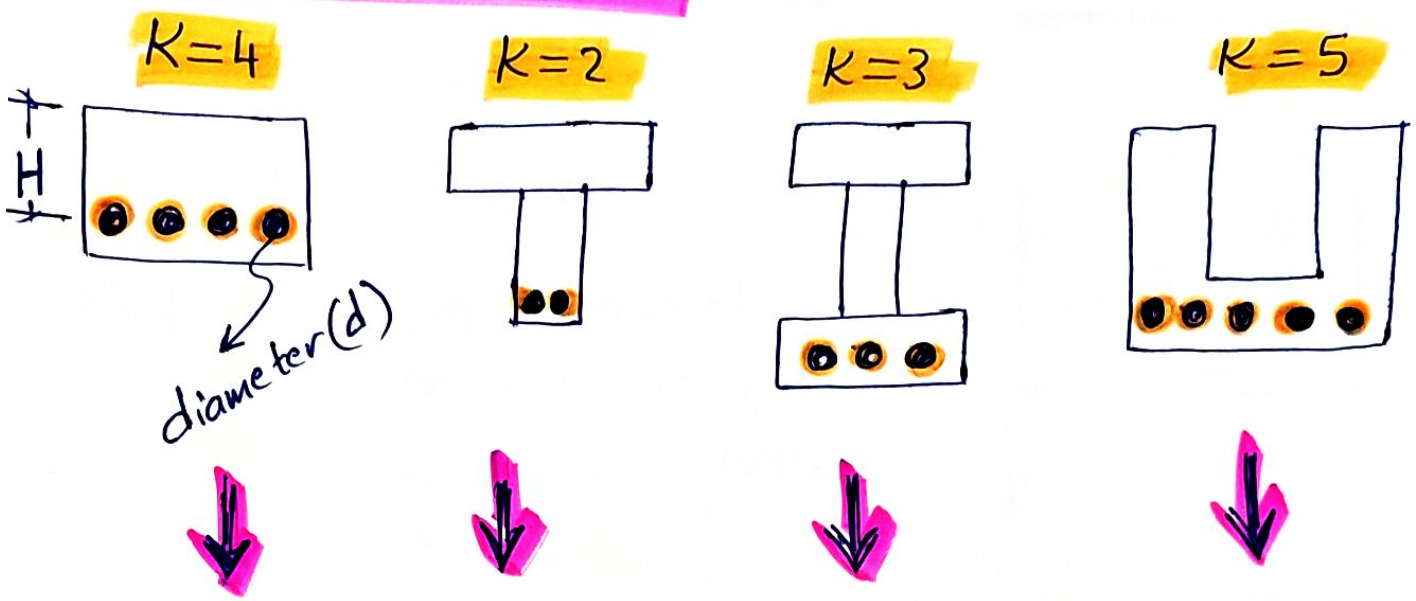


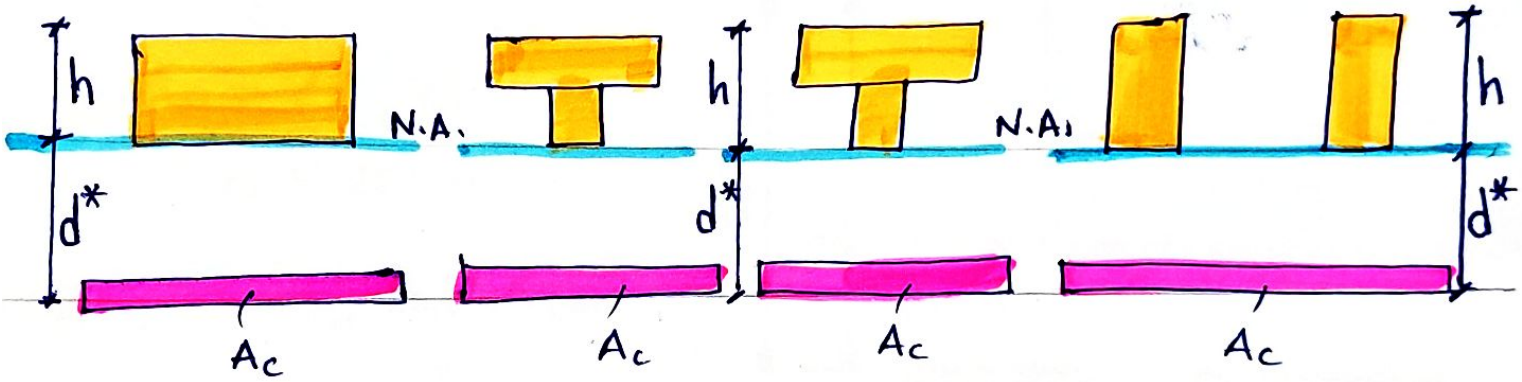


* Transform Brass to wood: $n_1 = \frac{E_{br}}{E_w} = 10$
* Transform Steel to wood: $n_2 = \frac{E_{st}}{E_w} = 20$
* $b_1^* = 10(15) = 150$ Cm .
* $b_2^* = 20(15) = 300$ Cm .
* $\sigma_w = \frac{M y_w}{I}$
* $\sigma_{Br} = (n_1) \frac{M y_{Br}}{I}$
* $\sigma_{st} = (n_2) \frac{M y_{st}}{I}$

Reinforced Concrete

Dr. Hashem Alkhaldi





1 $n = \dfrac{E_{st}}{E_c}$

2 $A_{st} = K \dfrac{\pi}{4} d^2$

$A_c = n A_{st}$

3 Find the location of the N.A ⇒ Find h ?

$\sum Q_{up} = \sum Q_{down}$

⇒ $\sum_{up.} \bar{y} A = \sum_{down} \bar{y} A$

4 Find $I = I_{up} + I_{down}$

$* \; I_{down} = A_c (d^*)^2$

5 $\sigma_c = \dfrac{M y_c}{I} = \dfrac{M h}{I}$

6 $\sigma_{st} = n \dfrac{M y_{st}}{I} = n \dfrac{M d^*}{I}$

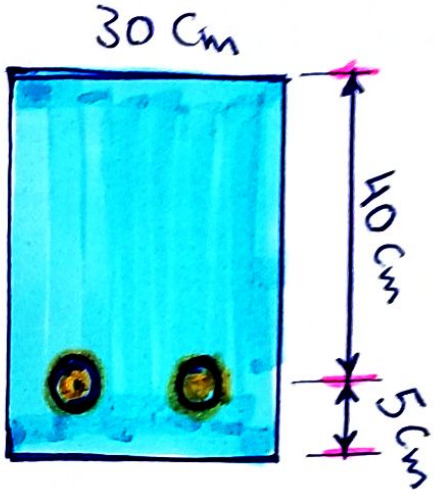
*Note : $d^* = H - h$

Ex

$E_c = 25\ GPa$

$E_{st} = 200\ GPa$

$M_{max} = 60\ kN.m$



Find: 1) The max. Bending stress in Concrete σ_c ?

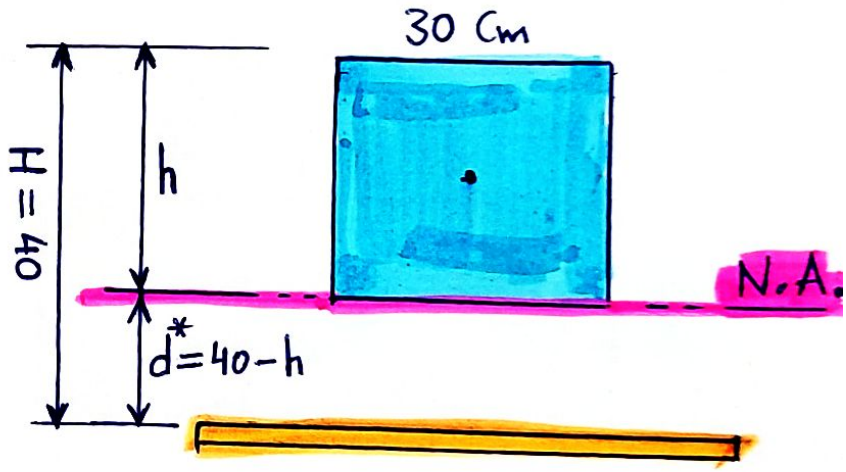
2) The max. Bending stress in steel σ_{st} ?

Rod diameter $d = 25\ mm$.

* $n = \frac{E_{st}}{E_c} = 8$

* $A_{st} = 2\left[\frac{\pi}{4}\right](2.5)^2$
$= 9.81\ Cm^2$

* $A_c = n\,A_{st} = 78.5\ Cm^2$



* $\sum_{up} \bar{y}A = \sum_{down} \bar{y}A$

$\Rightarrow 30h\left[\frac{h}{2}\right] = 78.5[40-h]$

$\Rightarrow 15h^2 + 78.5h - 3140 = 0 \Rightarrow$ $h = 12\ Cm$ & $d^* = 28\ Cm$

* $I = I_{up} + I_{down} = \left[\frac{1}{12}(30)(12)^3 + 30(12)(6)^2\right] + \left[78.5(28)^2\right]$

$\Rightarrow I = 74864\ Cm^4$.

* $\sigma_c = \frac{Mh}{I} = 9.62\ MPa$.

* $\sigma_{st} = (n)\frac{M d^*}{I} = 171\ MPa$.

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Ex

$E_c = 25\ GPa$

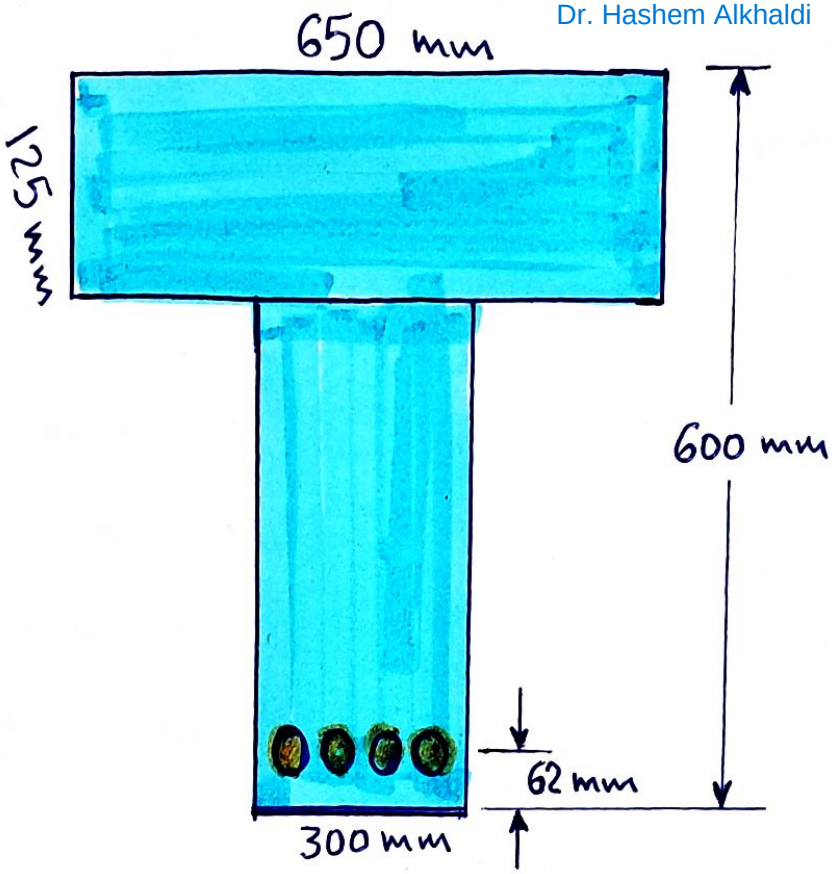
$E_{st} = 200\ GPa$

$d = 25\ mm$

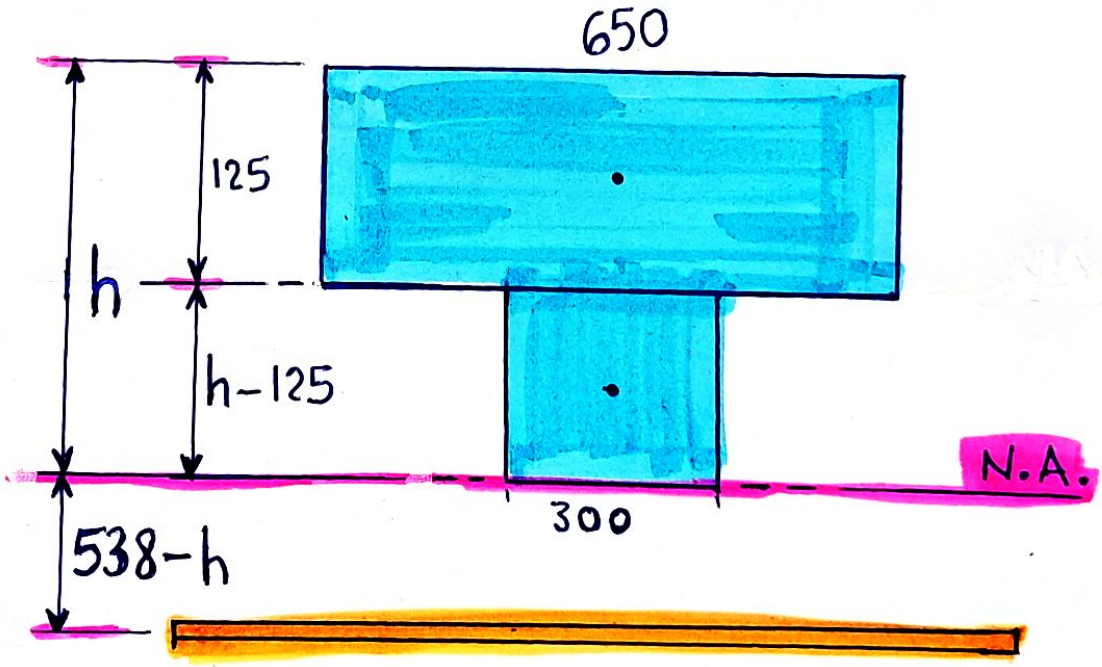
$(\sigma_{all})_{st} = 180\ MPa.$

$(\sigma_{all})_c = 12\ MPa.$

Find the maximum Bending moment on this section M_{max}?



* $n = \frac{200}{25} = 8$
* $A_s = 4\left[\frac{\pi}{4}\right](25)^2$
  $= 1963.5\ mm^2$
* $A_c = 8\ A_{st}$
  $= 15708\ mm^2$
* $H = 600 - 62$
  $= 538\ mm$



* $\sum_{up} A\bar{y} = \sum_{down} A\bar{y}$

$$\Rightarrow 300[h-125]\left[\frac{h-125}{2}\right] + (650)(125)\left(h - \frac{125}{2}\right) = 15708[538-h]$$

$$\Rightarrow h = 139.2\ mm$$

$$\Rightarrow d^* = 538 - h = 398.8\ mm$$

* $I = I_{up} + I_{down}$

* $I_{up} = \frac{1}{12}(650)(125)^3 + (650)(125)(76.7)^2$
$+ \frac{1}{12}(300)(14.2)^3 + (300)(14.2)(7.1)^2$
$= 584065412 \ mm^4$.

* $I_{down} = A_c (d^*)^2 = 15708(398.8)^2$
$= 2498222940 \ mm^4$.

$\Rightarrow \quad I = 3082288352 \ mm^4$.

* $\sigma_c = \frac{Mh}{I} \Rightarrow 12 \times 10^6 = \frac{M(139.2 \times 10^{-3})}{3082288352 \times 10^{-12}}$

$\Rightarrow M = 265.7 \ KN.m$

* $\sigma_{st} = n \frac{M d^*}{I} \Rightarrow 180 \times 10^6 = 8 \frac{M(398.8 \times 10^{-3})}{3082288352 \times 10^{-12}}$

$\Rightarrow M = 173.9 \ KN.m$

* The maximum Bending Moment M_{max} is :

$M_{max} = 173.9 \ KN.m$

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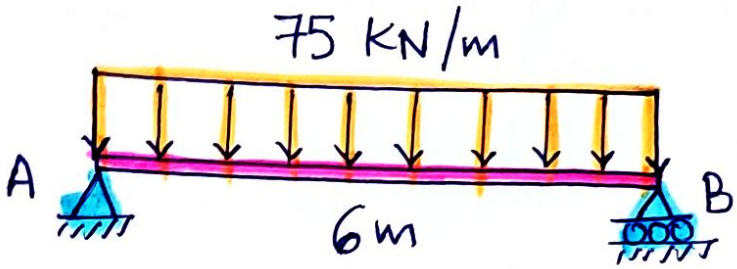
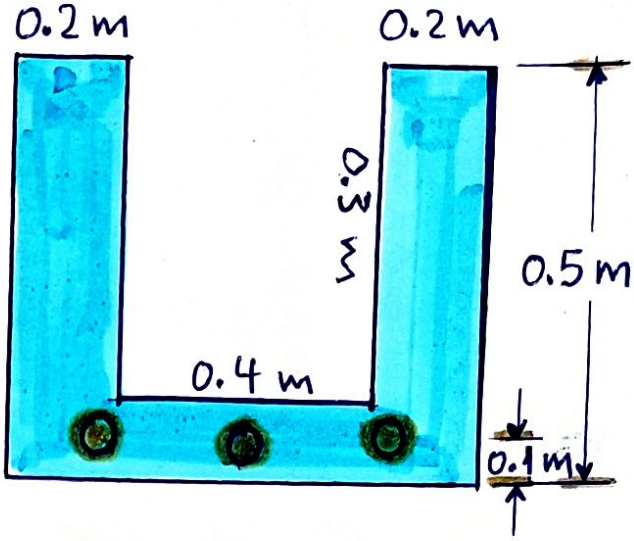
Ex

$E_c = 25\ GPa$, $E_{st} = 200\ GPa$

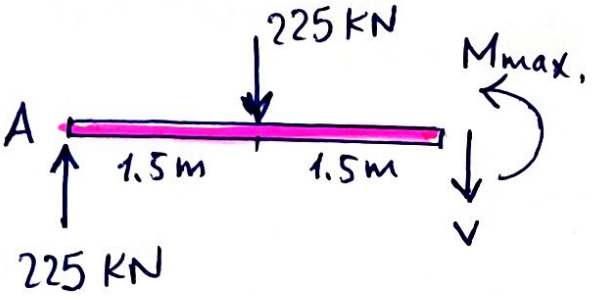


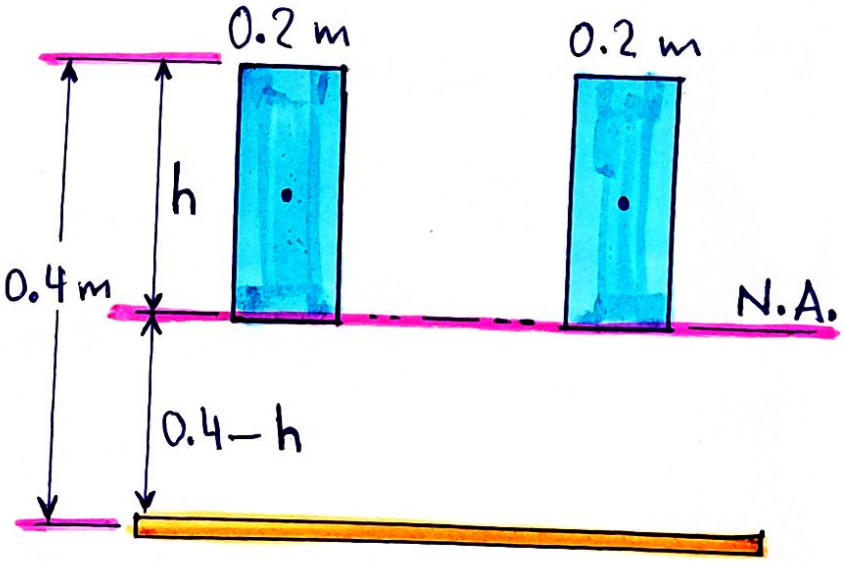


Find the maximum Bending Stresses in Steel & Concrete ?

Rod diameter $d = 20\ mm$

* Since the Beam is Symmetrical ⇒ The max. Bending moment will be in the middle.





⇒ $M_{max} = 337.5\ KN.m$

* $n = \dfrac{E_{st}}{E_c} = \dfrac{200}{25} = 8$.
* $A_{st} = 3\left[\frac{\pi}{4}\right](0.02)^2 = 9.42 \times 10^{-4}\ m^2$.
* $A_c = n\ A_{st} = 7.54 \times 10^{-3}\ m^2$.

$$* \quad \sum_{up} Q = \sum_{down} Q \Rightarrow \sum_{up} A\bar{y} = \sum_{down} A\bar{y}$$

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$$\Rightarrow 2\left[(0.2h)\left(\frac{h}{2}\right)\right] = 7.54\times10^{-3}\left[0.4-h\right]$$

$$\Rightarrow 0.2h^2 + 7.54\times10^{-3}h - 3.016\times10^{-3} = 0$$

$$\Rightarrow h = 0.1054 \text{ m} = 105.4 \text{ mm}.$$

$$* \quad I = I_{up} + I_{down}, \quad d^* = 0.4 - h, \quad d^* = 0.2946 \text{ m}$$

$$* \quad I_{down} = A_c (d^*)^2 = 6.544\times10^{-4} \text{ m}^4.$$

$$* \quad I_{up} = 2\left[\frac{1}{12}(0.2)(0.1054)^3 + (0.2)(0.1054)\left(\frac{0.1054}{2}\right)^2\right] = 1.561\times10^{-4} \text{ m}^4.$$

$$\Rightarrow I = 8.105\times10^{-4} \text{ m}^4.$$

$$* \quad \sigma_c = \frac{Mh}{I} = 43.9 \text{ MPa}.$$

$$* \quad \sigma_{st} = n\frac{Md^*}{I} = 981.4 \text{ MPa}.$$