

PROBLEM 2-1

Statement: Find three (or other number as assigned) of the following common devices. Sketch careful kinematic diagrams and find their total degrees of freedom.

- An automobile hood hinge mechanism
- An automobile hatchback lift mechanism
- An electric can opener
- A folding ironing board
- A folding card table
- A folding beach chair
- A baby swing
- A folding baby walker
- A fancy corkscrew as shown in Figure P2-9
- A windshield wiper mechanism
- A dump-truck dump mechanism
- A trash truck dumpster mechanism
- A pickup tailgate mechanism
- An automobile jack
- A collapsible auto radio antenna

Solution: See Mathcad file P0201.

Equation 2.1c is used to calculate the mobility (*DOF*) of each of the models below.

- a. An automobile hood hinge mechanism.

The hood (3) is linked to the body (1) through two rocker links (2 and 4).

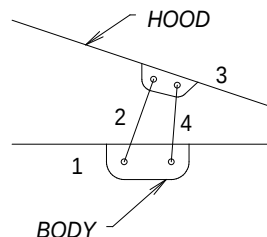
$$\text{Number of links} \quad L := 4$$

$$\text{Number of full joints} \quad J_1 := 4$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



- b. An automobile hatchback lift mechanism.

The hatch (2) is pivoted on the body (1) and is linked to the body by the lift arm, which can be modeled as two links (3 and 4) connected through a translating slider joint.

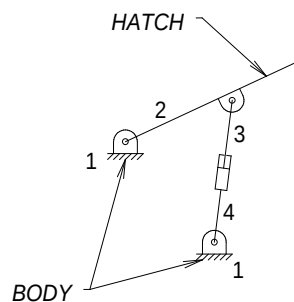
$$\text{Number of links} \quad L := 4$$

$$\text{Number of full joints} \quad J_1 := 4$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



- c. An electric can opener has 2 *DOF*.

- d. A folding ironing board.

The board (1) itself has one pivot (full) joint and one pin-in-slot sliding (half) joint. The two legs (2 and 3) have a common pivot. One leg connects to the pivot joint on the board and the other to the slider joint.

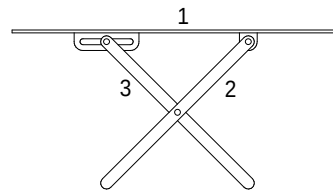
Number of links $L := 3$

Number of full joints $J_1 := 2$

Number of half joints $J_2 := 1$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



e. A folding card table has 7 *DOF*: One for each leg, 2 for location in *xy* space, and one for angular orientation.

f. A folding beach chair.

The seat (3) and the arms (6) are ternary links. The seat is linked to the front leg (2), the back (5) and a coupler link (4). The arms are linked to the front leg (2), the rear leg (1), and the back (5). Links 1, 2, 4, and 5 are binary links. The analysis below is appropriate when the chair is not fully opened. When fully opened, one or more links are prevented from moving by a stop. Subtract 1 *DOF* when forced against the stop.

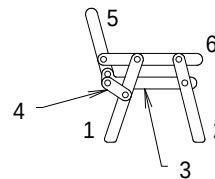
Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



g. A baby swing has 4 *DOF*: One for the angular orientation of the swing with respect to the frame, and 3 for the location and orientation of the frame with respect to a 2-D frame.

h. A folding baby walker has 4 *DOF*: One for the degree to which it is unfolded, and 3 for the location and orientation of the walker with respect to a 2-D frame.

i. A fancy corkscrew has 2 *DOF*: The screw can be rotated and the arms rotate to translate the screw.

j. A windshield wiper mechanism has 1 *DOF*: The position of the wiper blades is defined by a single input.

k. A dump-truck dump mechanism has 1 *DOF*: The angle of the dump body is determined by the length of the hydraulic cylinder that links it to the body of the truck.

l. A trash truck dumpster mechanism has 2 *DOF*: These are generally a rotation and a translation.

m. A pickup tailgate mechanism has 1 *DOF*:

n. An automobile jack has 4 *DOF*: One is the height of the jack and the other 3 are the position and orientation of the jack with respect to a 2-D frame.

o. A collapsible auto radio antenna has as many *DOF* as there are sections, less one.

PROBLEM 2-2

Statement: How many DOF do you have in your wrist and hand combined?

Solution: See Mathcad file P0202.

1. Holding the palm of the hand level and facing toward the floor, the hand can be rotated about an axis through the wrist that is parallel to the floor (and perpendicular to the forearm axis) and one perpendicular to the floor (2 *DOF*). The wrist can rotate about the forearm axis (1 *DOF*).
2. Each finger (and thumb) can rotate up and down and side-to-side about the first joint. Additionally, each finger can rotate about each of the two remaining joints for a total of 4 *DOF* for each finger (and thumb).
3. Adding all *DOF*, the total is

Wrist	1
Hand	2
Thumb	4
Fingers 4x4	<u>16</u>
TOTAL	23

PROBLEM 2-3

Statement: How many *DOF* do the following joints have?

- a. Your knee
- b. Your ankle
- c. Your shoulder
- d. Your hip
- e. Your knuckle

Solution: See Mathcad file P0203.

- a. Your knee.
1 *DOF*: A rotation about an axis parallel to the ground.
- b. Your ankle.
3 *DOF*: Three rotations about mutually perpendicular axes.
- c. Your shoulder.
3 *DOF*: Three rotations about mutually perpendicular axes.
- d. Your hip.
3 *DOF*: Three rotations about mutually perpendicular axes.
- e. Your knuckle.
2 *DOF*: Two rotations about mutually perpendicular axes.

PROBLEM 2-4

Statement: How many DOF do the following have in their normal environment?

- a. A submerged submarine
- b. An earth-orbit satellite
- c. A surface ship
- d. A motorcycle (road bike)
- e. A two-button mouse
- f. A computer joy stick.

Solution: See Mathcad file P0204.

- a. A submerged submarine.
Using a coordinate frame attached to earth, or an inertial coordinate frame, a submarine has 6 *DOF*: 3 linear coordinates and 3 angles.
- b. An earth-orbit satellite.
If the satellite was just a particle it would have 3 *DOF*. But, since it probably needs to be oriented with respect to the earth, sun, etc., it has 6 *DOF*.
- c. A surface ship.
There is no difference between a submerged submarine and a surface ship, both have 6 *DOF*. One might argue that, for an earth-centered frame, the depth of the ship with respect to mean sea level is constant, however that is not strictly true. A ship's position is generally given by two coordinates (longitude and latitude). For a given position, a ship can also have pitch, yaw, and roll angles. Thus, for all practical purposes, *a surface ship has 5 DOF*.
- d. A motorcycle.
At an intersection, the motorcycle's position is given by two coordinates. In addition, it will have some heading angle (turning a corner) and roll angle (if turning). Thus, there are 4 *DOF*.
- e. A two-button mouse.
A two-button mouse has 4 *DOF*. It can move in the x and y directions and each button has 1 *DOF*.
- f. A computer joy stick.
The joy stick has 2 *DOF* (x and y) and orientation, for a total of 3 *DOF*.

PROBLEM 2-5

Statement: Are the joints in Problem 2-3 force closed or form closed?

Solution: See Mathcad file P0205.

They are force closed by ligaments that hold them together. None are geometrically closed.

PROBLEM 2-6

Statement: Describe the motion of the following items as pure rotation, pure translation, or complex planar motion.

- a. A windmill
- b. A bicycle (in the vertical plane, not turning)
- c. A conventional "double-hung" window
- d. The keys on a computer keyboard
- e. The hand of a clock
- f. A hockey puck on the ice
- g. A "casement" window

Solution: See Mathcad file P0206.

- a. A windmill.
Pure rotation.
- b. A bicycle (in the vertical plane, not turning).
Pure translation for the frame, complex planar motion for the wheels.
- c. A conventional "double-hung" window.
Pure translation.
- d. The keys on a computer keyboard.
Pure translation.
- e. The hand of a clock.
Pure rotation.
- f. A hockey puck on the ice.
Complex planar motion.
- g. A "casement" window.
Pure rotation.

PROBLEM 2-7

Statement: Calculate the mobility of the linkages assigned from Figure P2-1 part 1 and part 2.

Solution: See Figure P2-1 and Mathcad file P0207.

1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

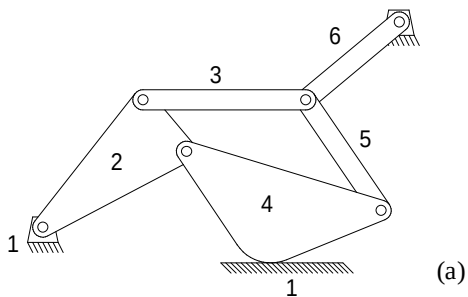
a. Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 1$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 0$$



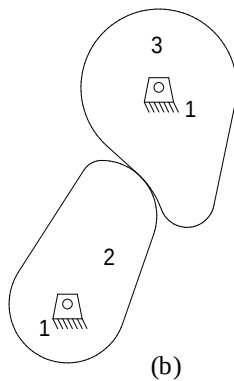
b. Number of links $L := 3$

Number of full joints $J_1 := 2$

Number of half joints $J_2 := 1$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



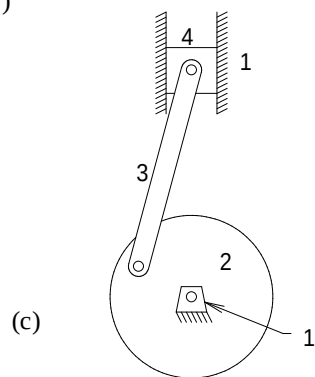
c. Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



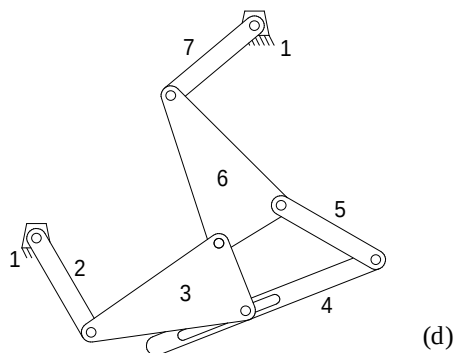
d. Number of links $L := 7$

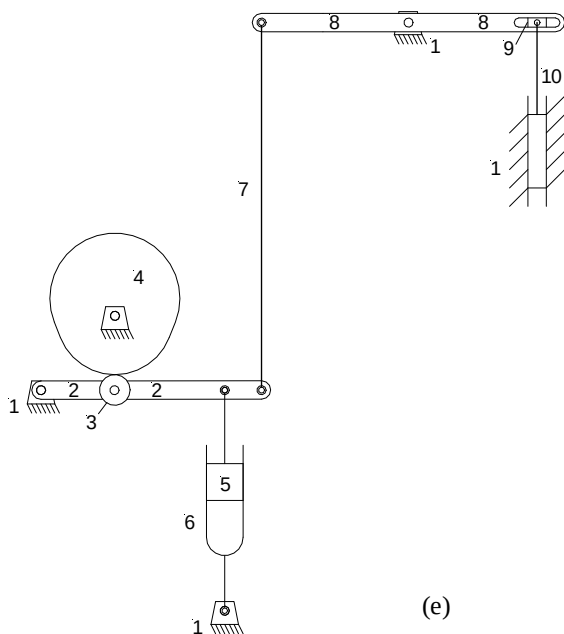
Number of full joints $J_1 := 7$

Number of half joints $J_2 := 1$

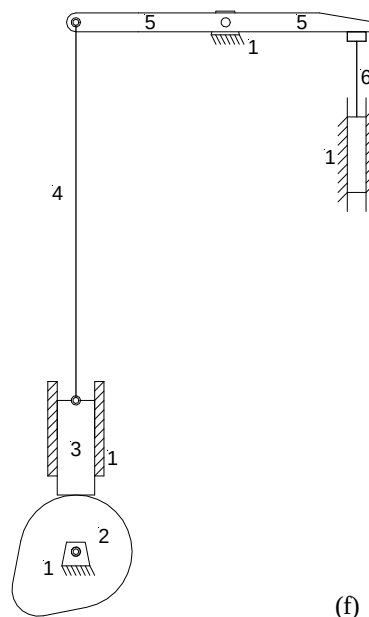
$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 3$$





(e)



(f)

- e. Number of links $L := 10$
 Number of full joints $J_1 := 13$
 Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$

- f. Number of links $L := 6$
 Number of full joints $J_1 := 6$
 Number of half joints $J_2 := 2$

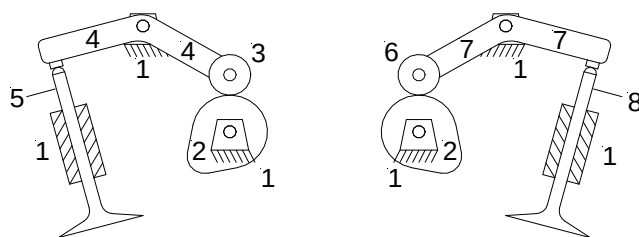
$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$

- g. Number of links $L := 8$
 Number of full joints $J_1 := 9$
 Number of half joints $J_2 := 2$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$

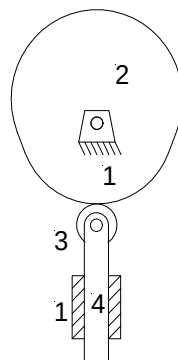


(g)

- h. Number of links $L := 4$
 Number of full joints $J_1 := 4$
 Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



(h)

PROBLEM 2-8

Statement: Identify the items in Figure P2-1 as mechanisms, structures, or preloaded structures.

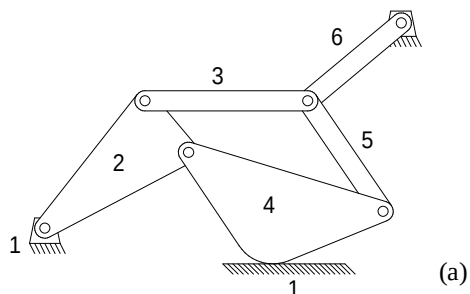
Solution: See Figure P2-1 and Mathcad file P0208.

1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility and the definitions in Section 2.5 of the text to classify the linkages.

a. Number of links $L := 6$
 Number of full joints $J_1 := 7$
 Number of half joints $J_2 := 1$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

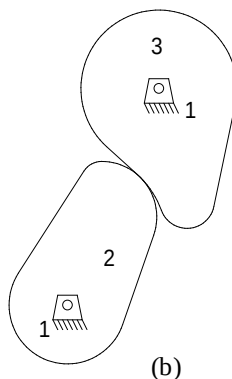
$$M = 0 \quad \text{Structure}$$



b. Number of links $L := 3$
 Number of full joints $J_1 := 2$
 Number of half joints $J_2 := 1$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

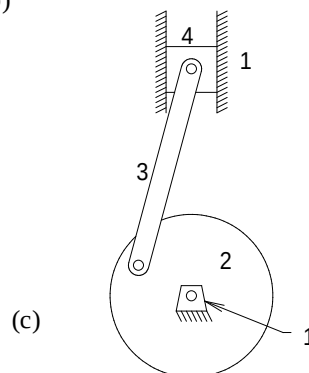
$$M = 1 \quad \text{Mechanism}$$



c. Number of links $L := 4$
 Number of full joints $J_1 := 4$
 Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

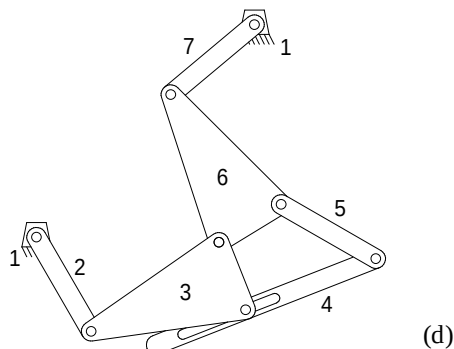
$$M = 1 \quad \text{Mechanism}$$



d. Number of links $L := 7$
 Number of full joints $J_1 := 7$
 Number of half joints $J_2 := 1$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 3 \quad \text{Mechanism}$$



PROBLEM 2-9

Statement: Use linkage transformation on the linkage of Figure P2-1a to make it a 1-DOF mechanism.

Solution: See Figure P2-1a and Mathcad file P0209.

1. The mechanism in Figure P2-1a has mobility:

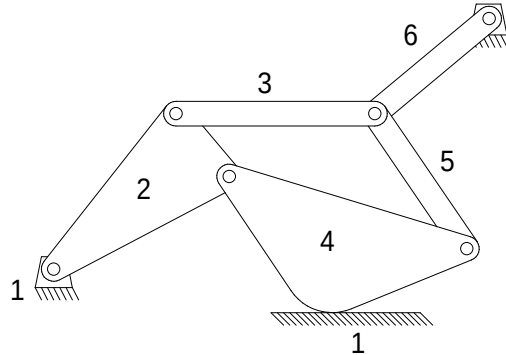
$$\text{Number of links} \quad L := 6$$

$$\text{Number of full joints} \quad J_1 := 7$$

$$\text{Number of half joints} \quad J_2 := 1$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 0$$



2. Use rule 2, which states: "Any full joint can be replaced by a half joint, but this will increase the *DOF* by one." One way to do this is to replace one of the pin joints with a pin-in-slot joint such as that shown in Figure 2-3c. Choosing the joint between links 2 and 4, we now have mobility:

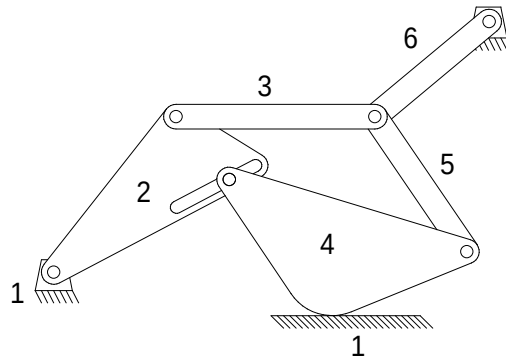
$$\text{Number of links} \quad L := 6$$

$$\text{Number of full joints} \quad J_1 := 6$$

$$\text{Number of half joints} \quad J_2 := 2$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



PROBLEM 2-10

Statement: Use linkage transformation on the linkage of Figure P2-1d to make it a 2-DOF mechanism.

Solution: See Figure P2-1d and Mathcad file P0210.

1. The mechanism in Figure P2-1d has mobility:

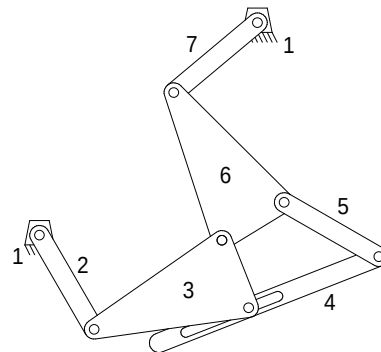
$$\text{Number of links} \quad L := 7$$

$$\text{Number of full joints} \quad J_1 := 7$$

$$\text{Number of half joints} \quad J_2 := 1$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 3$$



2. Use rule 3, which states: "Removal of a link will reduce the *DOF* by one." One way to do this is to remove link 7 such that link 6 pivots on the fixed pin attached to the ground link (1). We now have mobility:

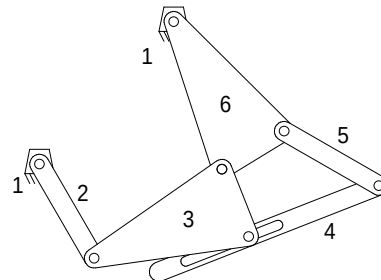
$$\text{Number of links} \quad L := 6$$

$$\text{Number of full joints} \quad J_1 := 6$$

$$\text{Number of half joints} \quad J_2 := 1$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 2$$



PROBLEM 2-11

Statement: Use number synthesis to find all the possible link combinations for 2-DOF, up to 9 links, to hexagonal order, using only revolute joints.

Solution: See Mathcad file P0211.

1. Use equations 2.4a and 2.6 with $DOF = 2$ and iterate the solution for valid combinations. Note that the number of links must be odd to have an even DOF (see Eq. 2.4). The smallest possible 2-DOF mechanism is then 5 links since three will give a structure (the delta triplet, see Figure 2-7).

$$L := B + T + Q + P + H \quad L - 3 - M := T + 2 \cdot Q + 3 \cdot P + 4 \cdot H \quad M := 2$$

$$L - 5 := T + 2 \cdot Q + 3 \cdot P + 4 \cdot H$$

2. For $L := 5$

$$0 := T + 2 \cdot Q + 3 \cdot P + 4 \cdot H \quad 0 = T = Q = P = H \quad B := 5$$

3. For $L := 7$

$$2 := T + 2 \cdot Q + 3 \cdot P + 4 \cdot H \quad H := 0 \quad P := 0$$

Case 1:

$$Q := 0 \quad T := 2 - 2 \cdot Q - 3 \cdot P - 4 \cdot H \quad T = 2$$

$$B := L - T - Q - P - H \quad B = 5$$

Case 2:

$$Q := 1 \quad T := 2 - 2 \cdot Q - 3 \cdot P - 4 \cdot H \quad T = 0$$

$$B := L - T - Q - P - H \quad B = 6$$

4. For $L := 9$

$$4 := T + 2 \cdot Q + 3 \cdot P + 4 \cdot H$$

Case 1:

$$H := 1 \quad T := 0 \quad Q := 0 \quad P := 0$$

$$B := L - T - Q - P - H \quad B = 8$$

Case 2a:

$$H := 0 \quad 4 := T + 2 \cdot Q + 3 \cdot P$$

$$9 := B + T + Q + P$$

Case 2b:

$$P := 1 \quad 1 := T + 2 \cdot Q \quad Q := 0 \quad T := 1$$

$$B := L - T - Q - P - H \quad B = 7$$

Case 2c:

$$P := 0 \quad 4 := T + 2 \cdot Q$$

$$9 := B + T + Q$$

Case 2c1:

$$Q := 2 \quad T := 4 - 2 \cdot Q \quad T = 0$$

$$B := 9 - T - Q \quad B = 7$$

Case 2c2:

$$Q := 1 \quad T := 4 - 2 \cdot Q \quad T = 2$$

$$B := 9 - T - Q \quad B = 6$$

Case 2c3:

$$Q := 0 \quad T := 4 - 2 \cdot Q \quad T = 4$$

$$B := 9 - T - Q \quad B = 5$$

PROBLEM 2-12

Statement: Find all of the valid isomers of the eightbar 1-DOF link combinations in Table 2-2 (p. 38) having

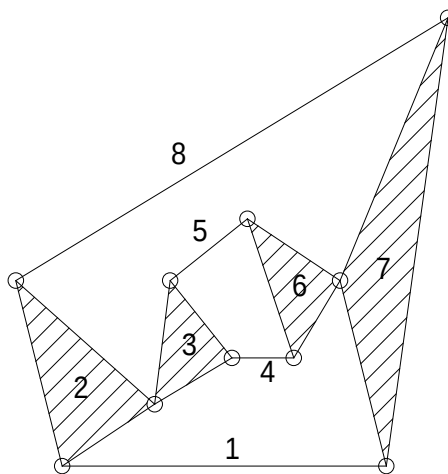
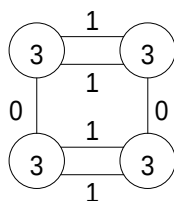
- Four binary and four ternary links.
- Five binaries, two ternaries, and one quaternary link.
- Six binaries and two quaternary links.
- Six binaries, one ternary, and one pentagonal link.

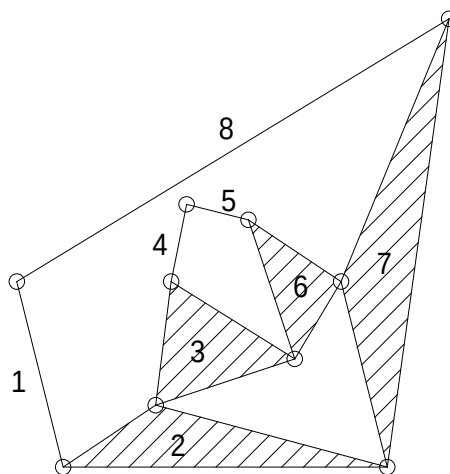
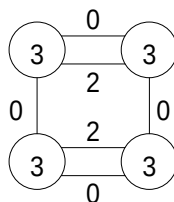
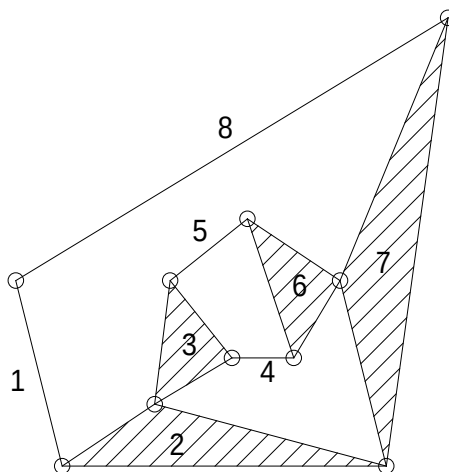
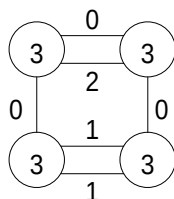
Solution: See Mathcad file P0212.

- Table 2-3 lists 16 possible isomers for an eightbar chain. However, Table 2-2 shows that there are five possible link sets, four of which are listed above. Therefore, we expect that the 16 valid isomers are distributed among the five link sets and that there will be fewer than 16 isomers among the four link sets listed above.
- One method that is helpful in finding isomers is to represent the linkage in terms of molecules as defined in Franke's Condensed Notations for Structural Synthesis. A summary of the rules for obtaining Franke's molecules follows:
 - The links of order greater than 2 are represented by circles.
 - A number is placed within each circle (the "valence" number) to describe the type (ternary, quaternary, etc.) of link.
 - The circles are connected using straight lines. The number of straight lines emanating from a circle must be equal to its valence number.
 - Numbers (0, 1, 2, etc.) are placed on the straight lines to correspond to the number of binary links used in connecting the higher order links.
 - There is one-to-one correspondence between the molecule and the kinematic chain that it represents.

a. Four binary and four ternary links.

Draw 4 circles with valence numbers of 3 in each. Then find all unique combinations of straight lines that can be drawn that connect the circles such that there are exactly three lines emanating from each circle and the total of the numbers written on the lines is exactly equal to 4. In this case, there are three valid isomers as depicted by Franke's molecules and kinematic chains below.

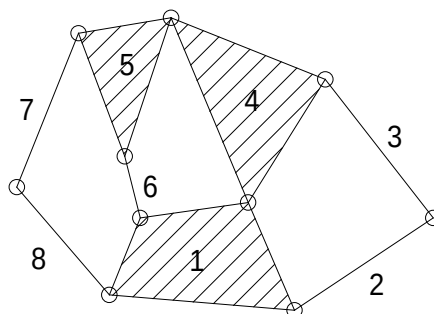
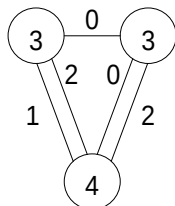


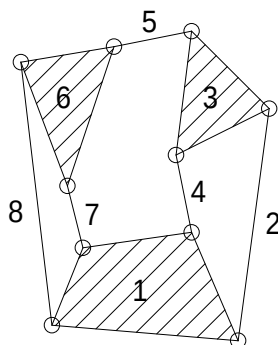
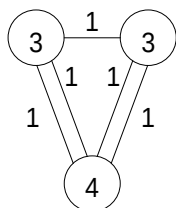
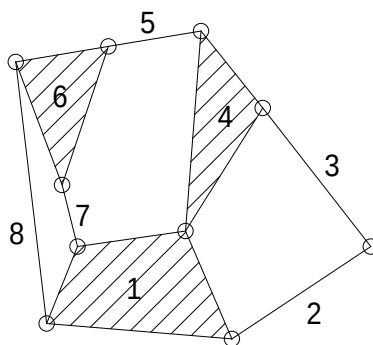
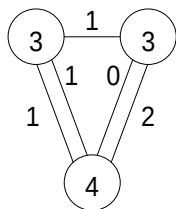
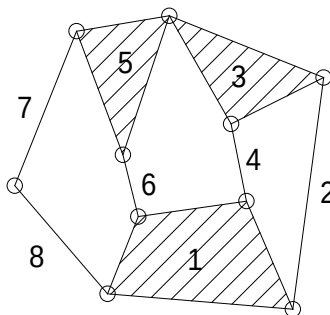
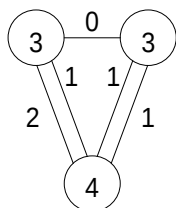
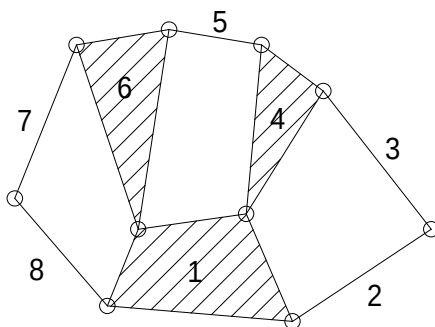
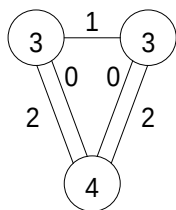


The mechanism shown in Figure P2-5b is the same eightbar isomer as that depicted schematically above.

b. Five binaries, two ternaries, and one quaternary link.

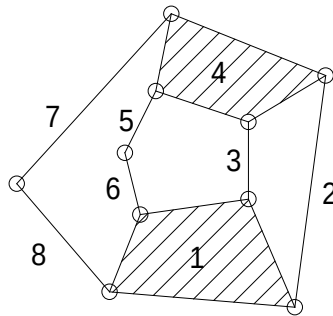
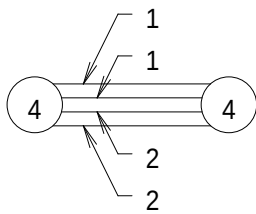
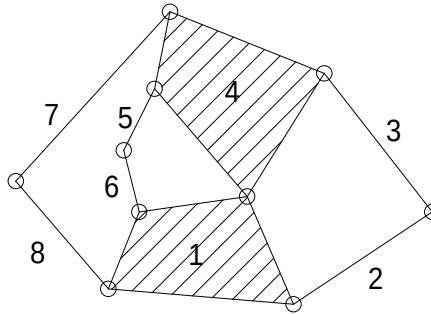
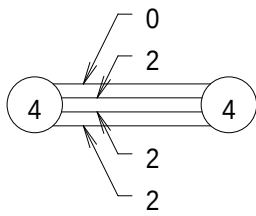
Draw 2 circles with valence numbers of 3 in each and one with a valence number of 4. Then find all unique combinations of straight lines that can be drawn that connect the circles such that there are exactly three lines emanating from each circle with valence of three and four lines from the circle with valence of four; and the total of the numbers written on the lines is exactly equal to 5. In this case, there are five valid isomers as depicted by Franke's molecules and kinematic chains below.





c. Six binaries and two quaternary links.

Draw 2 circles with valence numbers of 4 in each. Then find all unique combinations of straight lines that can be drawn that connect the circles such that there are exactly four lines emanating from each circle and the total of the numbers written on the lines is exactly equal to 6. In this case, there are two valid isomers as depicted by Franke's molecules and kinematic chains below.



d. Six binaries, one ternary, and one pentagonal link.

There are no valid implementations of 6 binary links with 1 pentagonal link.

PROBLEM 2-13

Statement: Use linkage transformation to create a 1-DOF mechanism with two sliding full joints from a Stephenson's sixbar linkage as shown in Figure 2-14a (p. 47).

Solution: See Figure 2-14a and Mathcad file P0213.

1. The mechanism in Figure 2-14a has mobility:

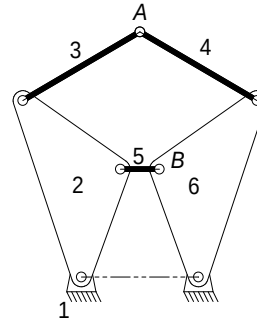
$$\text{Number of links} \quad L := 6$$

$$\text{Number of full joints} \quad J_1 := 7$$

$$\text{Number of half joints} \quad J_2 := 0$$

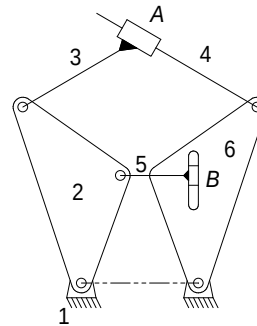
$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



2. Use rule 1, which states: "Revolute joints in any loop can be replaced by prismatic joints with no change in DOF of the mechanism, provided that at least two revolute joints remain in the loop." One way to do this is to replace pin joints at A and B with translating full slider joints such as that shown in Figure 2-3b.

Note that the sliders are attached to links 3 and 5 in such a way that they can not rotate relative to the links. The number of links and 1-DOF joints remains the same. There are no 2-DOF joints in either mechanism.



PROBLEM 2-14

Statement: Use linkage transformation to create a 1-DOF mechanism with one sliding full joint and a half joint from a Stephenson's sixbar linkage as shown in Figure 2-14b (p. 48).

Solution: See Figure 2-14a and Mathcad file P0213.

1. The mechanism in Figure 2-14b has mobility:

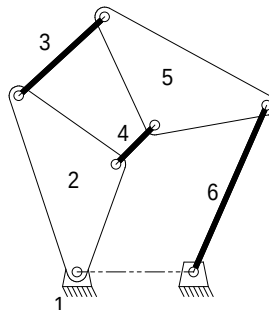
$$\text{Number of links} \quad L := 6$$

$$\text{Number of full joints} \quad J_1 := 7$$

$$\text{Number of half joints} \quad J_2 := 0$$

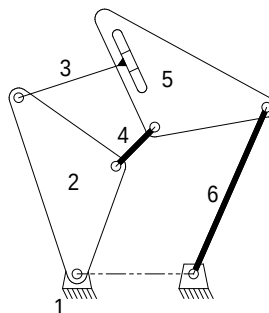
$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



2. To get the sliding full joint, use rule 1, which states: "Revolute joints in any loop can be replaced by prismatic joints with no change in *DOF* of the mechanism, provided that at least two revolute joints remain in the loop." One way to do this is to replace pin joint links 3 and 5 with a translating full slider joint such as that shown in Figure 2-3b.

Note that the slider is attached to link 3 in such a way that it can not rotate relative to the link. The number of links and 1-DOF joints remains the same.



3. To get the half joint, use rule 4 on page 42, which states: "The combination of rules 2 and 3 above will keep the original *DOF* unchanged." One way to do this is to remove link 6 (and its two nodes) and insert a half joint between links 5 and 1.

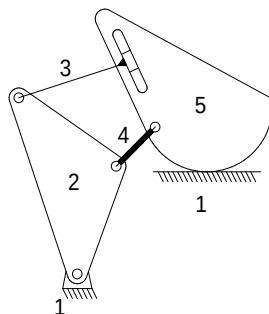
$$\text{Number of links} \quad L := 5$$

$$\text{Number of full joints} \quad J_1 := 5$$

$$\text{Number of half joints} \quad J_2 := 1$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



PROBLEM 2-15

Statement: Calculate the Grashof condition of the fourbar mechanisms defined below. Build cardboard models of the linkages and describe the motions of each inversion. Link lengths are in inches (or double given numbers for centimeters).

Part 1.

a.	2	4.5	7	9
b.	2	3.5	7	9
c.	2	4.0	6	8

Part 2.

d.	2	4.5	7	9
e.	2	4.0	7	9
f.	2	3.5	7	9

Solution: See Mathcad file P0215

1. Use inequality 2.8 to determine the Grashof condition.

$Condition(a, b, c, d) :=$	$S \leftarrow \min(a, b, c, d)$ $L \leftarrow \max(a, b, c, d)$ $SL \leftarrow S + L$ $PQ \leftarrow a + b + c + d - SL$ $return \text{ "Grashof" if } SL < PQ$ $return \text{ "Special Grashof" if } SL = PQ$ $return \text{ "non-Grashof" otherwise}$

- a. $Condition(2, 4.5, 7, 9) = \text{"Grashof"}$
- b. $Condition(2, 3.5, 7, 9) = \text{"non-Grashof"}$
- c. $Condition(2, 4.0, 6, 8) = \text{"Special Grashof"}$

This is a special case Grashof since the sum of the shortest and longest is equal to the sum of the other two link lengths.

- d. $Condition(2, 4.5, 7, 9) = \text{"Grashof"}$
- e. $Condition(2, 4.9, 7, 9) = \text{"Grashof"}$
- f. $Condition(2, 3.5, 7, 9) = \text{"non-Grashof"}$

PROBLEM 2-16

Statement: Which type(s) of electric motor would you specify

- a. To drive a load with large inertia.
- b. To minimize variation of speed with load variation.
- c. To maintain accurate constant speed regardless of load variations.

Solution: See Mathcad file P0216.

- a. Motors with high starting torque are suited to drive large inertia loads. Those with this characteristic include series-wound, compound-wound, and shunt-wound DC motors, and capacitor-start AC motors.
- b. Motors with flat torque-speed curves (in the operating range) will minimize variation of speed with load variation. Those with this characteristic include shunt-wound DC motors, and synchronous and capacitor-start AC motors.
- b. Speed-controlled DC motors will maintain accurate constant speed regardless of load variations.

PROBLEM 2-17

Statement: Describe the difference between a cam-follower (half) joint and a pin joint.

Solution: See Mathcad file P0217.

1. A pin joint has one rotational *DOF*. A cam-follower joint has 2 *DOF*, rotation and translation. The pin joint also captures its lubricant in the annulus between pin and bushing while the cam-follower joint squeezes its lubricant out of the joint.

PROBLEM 2-18

Statement: Examine an automobile hood hinge mechanism of the type described in Section 2.14. Sketch it carefully. Calculate its *DOF* and Grashof condition. Make a cardboard model. Analyze it with a free-body diagram. Describe how it keeps the hood up.

Solution: Solution of this problem will depend upon the specific mechanism modeled by the student.

PROBLEM 2-19

Statement: Find an adjustable arm desk lamp of the type shown in Figure P2-2. Sketch it carefully. Measure it and sketch it to scale. Calculate its *DOF* and Grashof condition. Make a cardboard model. Analyze it with a free-body diagram. Describe how it keeps itself stable. Are there any positions in which it loses stability? Why?

Solution: Solution of this problem will depend upon the specific mechanism modeled by the student.

PROBLEM 2-20

Statement: The torque-speed curve for a 1/8 hp permanent magnet (PM) DC motor is shown in Figure P2-3. The rated speed for this fractional horsepower motor is 2500 rpm at a rated voltage of 130V. Determine:

- The rated torque in oz-in (ounce-inches, the industry standard for fractional hp motors)
- The no-load speed
- Plot the power-torque curve and determine the maximum power that the motor can deliver.

Given: Rated speed, N_R $N_R := 2500 \cdot \text{rpm}$ Rated power, H_R $H_R := \frac{1}{8} \cdot \text{hp}$

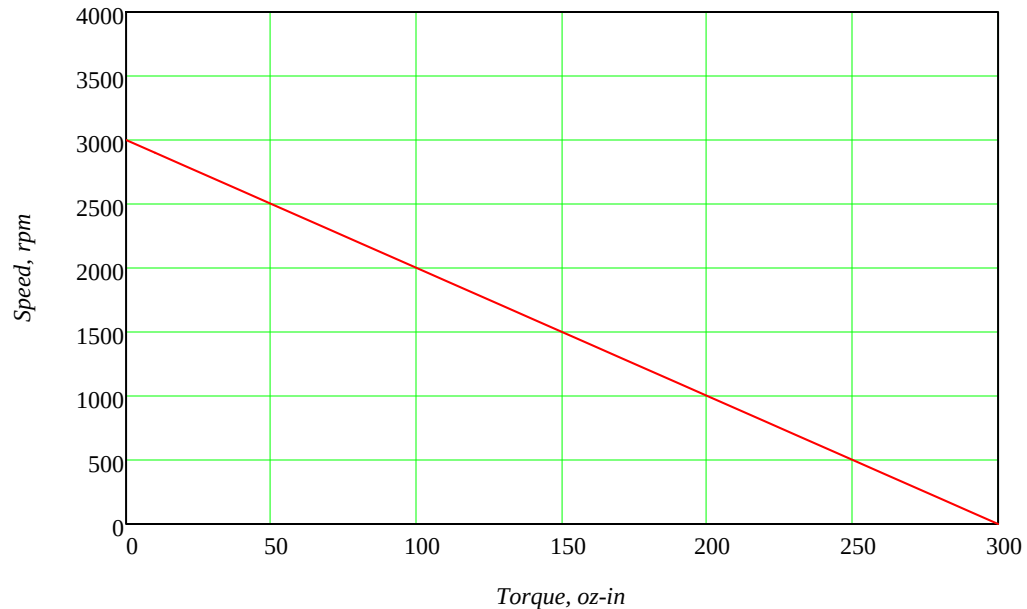


Figure P2-3 Torque-speed Characteristic of a 1/8 HP, 2500 rpm PM DC Motor

Solution: See Figure P2-3 and Mathcad file P0220.

- a. The rated torque is found by dividing the rated power by the rated speed:

$$\text{Rated torque, } T_R \quad T_R := \frac{H_R}{N_R} \quad T_R = 50 \cdot \text{ozf} \cdot \text{in}$$

- b. The no-load speed occurs at $T = 0$. From the graph this is 3000 rpm.

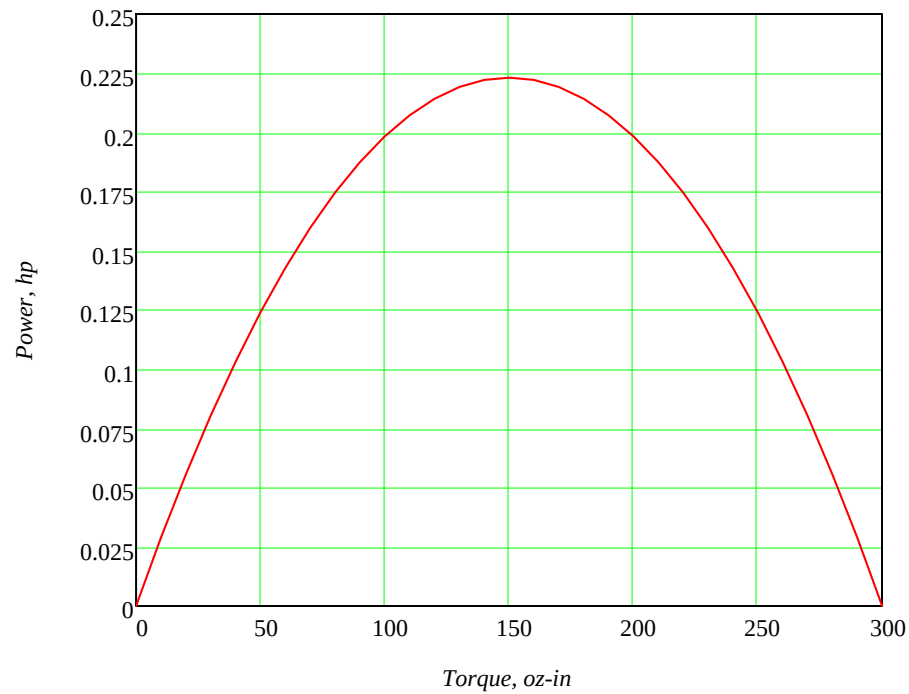
- c. The power is the product of the speed and the torque. From the graph the equation for the torque-speed curve is:

$$N(T) := -\frac{3000 \cdot \text{rpm}}{300 \cdot \text{ozf} \cdot \text{in}} \cdot T + 3000 \cdot \text{rpm}$$

and the power, therefore, is:

$$H(T) := -10 \cdot \frac{\text{rpm}}{\text{ozf} \cdot \text{in}} \cdot T^2 + 3000 \cdot \text{rpm} \cdot T$$

Plotting the power as a function of torque over the range $T := 0 \cdot \text{ozf} \cdot \text{in}, 10 \cdot \text{ozf} \cdot \text{in} .. 300 \cdot \text{ozf} \cdot \text{in}$



Maximum power occurs when $dH/dT = 0$. The value of T at maximum power is:

$$\text{Value of } T \text{ at } H_{\max} \quad T_{H\max} := 3000 \cdot \text{rpm} \cdot \frac{\text{ozf} \cdot \text{in}}{2 \cdot 10 \cdot \text{rpm}} \quad T_{H\max} = 150 \cdot \text{ozf} \cdot \text{in}$$

$$\text{Maximum power} \quad H_{\max} := H(T_{H\max}) \quad H_{\max} = 0.223 \cdot \text{hp}$$

$$\text{Speed at max power} \quad N_{H\max} := N(T_{H\max}) \quad N_{H\max} = 1500 \cdot \text{rpm}$$

Note that the curve goes through the rated power point of 0.125 hp at the rated torque of 50 oz-in.

PROBLEM 2-21

Statement: Find the mobility of the mechanisms in Figure P2-4.

Solution: See Figure P2-4 and Mathcad file P0221.

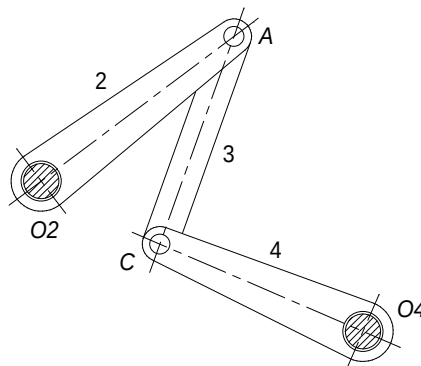
1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

- a. This is a basic fourbar linkage. The input is link 2 and the output is link 4. The cross-hatched pivot pins at O_2 and O_4 are attached to the ground link (1).

Number of links $L := 4$
 Number of full joints $J_1 := 4$
 Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$

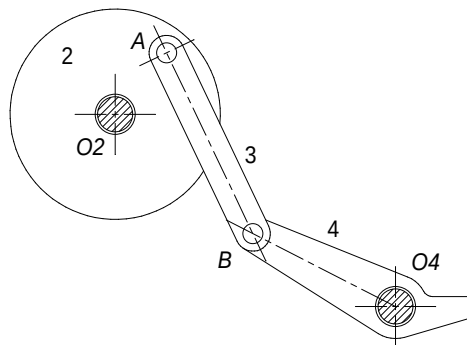


- b. This is a fourbar linkage. The input is link 2, which in this case is the wheel 2 with a pin at A, and the output is link 4. The cross-hatched pivot pins at O_2 and O_4 are attached to the ground link (1).

Number of links $L := 4$
 Number of full joints $J_1 := 4$
 Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$

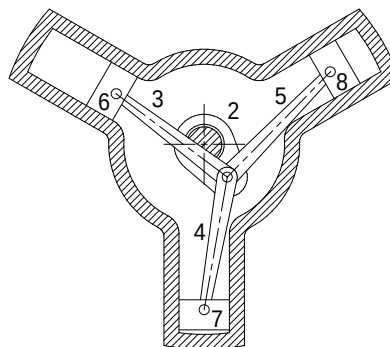


- c. This is a 3-cylinder, rotary, internal combustion engine. The pistons (sliders) 6, 7, and 8 drive the output crank (2) through piston rods (couplers 3, 4, and 5). There are 3 full joints at the crank where rods 3, 4, and 5 are pinned to crank 2. The cross-hatched crank-shaft at O_2 is supported by the ground link (1) through bearings.

Number of links $L := 8$
 Number of full joints $J_1 := 10$
 Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



- d. This is a fourbar linkage. The input is link 2, which in this case is a wheel with a pin at A, and the output is the vertical member on the coupler, link 3. Since the lengths of links 2 and 4 (O_2A and O_4B) are the same, the coupler link (3) has curvilinear motion and AB remains parallel to O_2O_4 throughout the cycle. The cross-hatched pivot pins at O_2 and O_4 are attached to the ground link (1).

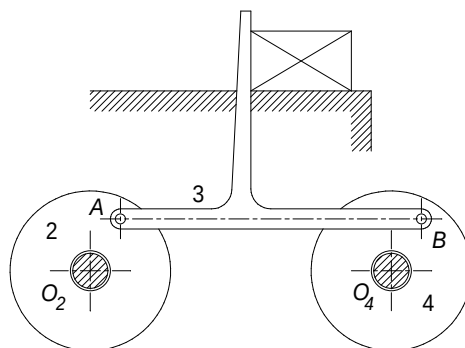
$$\text{Number of links} \quad L := 4$$

$$\text{Number of full joints} \quad J_1 := 4$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



- e. This is a fourbar linkage with an output dyad. The input (rocker) is link 2 and the output (rocker) is link 8. Links 5 and 6 are redundant, i.e. the mechanism will have the same motion if they are removed. The input fourbar consists of links 1, 2, 3, and 4. The output dyad consists of links 7 and 8. The cross-hatched pivot pins at O_2 , O_4 and O_8 are attached to the ground link (1). In the calculation below, the redundant links and their joints are not counted (subtract 2 links and 4 joints from the totals).

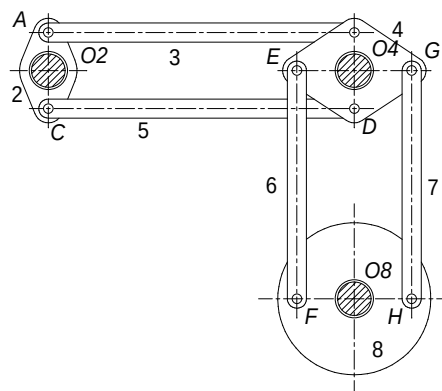
$$\text{Number of links} \quad L := 6$$

$$\text{Number of full joints} \quad J_1 := 7$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



- f. This is a fourbar offset slider-crank linkage. The input is link 2 (crank) and the output is link 4 (slider block). The cross-hatched pivot pin at O_2 is attached to the ground link (1).

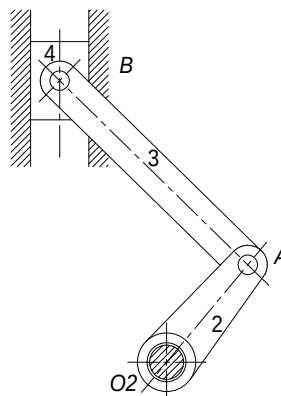
$$\text{Number of links} \quad L := 4$$

$$\text{Number of full joints} \quad J_1 := 4$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



- g. This is a fourbar linkage with an alternate output dyad. The input (rocker) is link 2 and the outputs (rockers) are links 4 and 6. The input fourbar consists of links 1, 2, 3, and 4. The alternate output dyad consists of links 5 and 6. The cross-hatched pivot pins at O_2 , O_4 and O_6 are attached to the ground link (1).

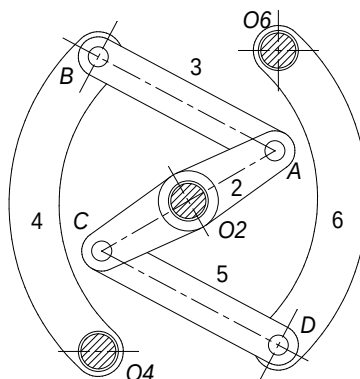
$$\text{Number of links} \quad L := 6$$

$$\text{Number of full joints} \quad J_1 := 7$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



- h. This is a ninebar mechanism with three redundant links, which reduces it to a sixbar. Since this mechanism is symmetrical about a vertical centerline, we can split it into two mirrored mechanisms to analyze it. Either links 2, 3 and 5 or links 7, 8 and 9 are redundant. To analyze it, consider 7, 8 and 9 as the redundant links. Analyzing the ninebar, there are two full joints at the pins A, B and C for a total of 12 joints.

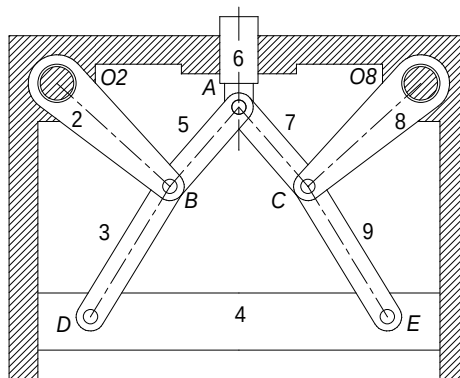
$$\text{Number of links} \quad L := 9$$

$$\text{Number of full joints} \quad J_1 := 12$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 0$$



The result is that this mechanism seems to be a structure. By splitting it into mirror halves about the vertical centerline the mobility is found to be 1. Subtract the 3 redundant links and their 5 (6 minus the joint at A) associated joints to determine the mobility of the mechanism.

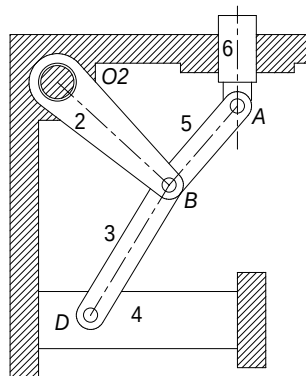
$$\text{Number of links} \quad L := 9 - 3$$

$$\text{Number of full joints} \quad J_1 := 12 - 5$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



PROBLEM 2-22

Statement: Find the Grashof condition and Barker classifications of the mechanisms in Figure P2-4a, b, and d.

Solution: See Figure P2-4 and Mathcad file P0222.

1. Use inequality 2.8 to determine the Grashof condition and Table 2-4 to determine the Barker classification.

```

Condition(a,b,c,d) :=
    S ← min(a,b,c,d)
    L ← max(a,b,c,d)
    SL ← S + L
    PQ ← a + b + c + d - SL
    return "Grashof"    if SL < PQ
    return "Special Grashof" if SL = PQ
    return "non-Grashof" otherwise
  
```

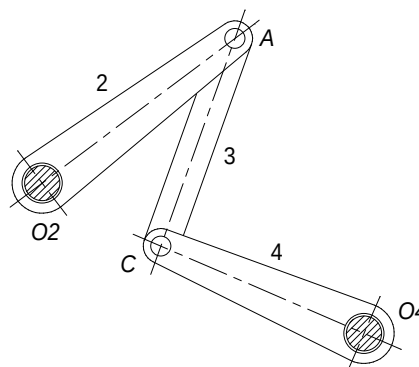
- a. This is a basic fourbar linkage. The input is link 2 and the output is link 4. The cross-hatched pivot pins at O_2 and O_4 are attached to the ground link (1).

$$L_1 := 174 \quad L_2 := 116$$

$$L_3 := 108 \quad L_4 := 110$$

$$\text{Condition}(L_1, L_2, L_3, L_4) = \text{"non-Grashof"}$$

This is a Barker Type 5 RRR1 (non-Grashof, longest link grounded).



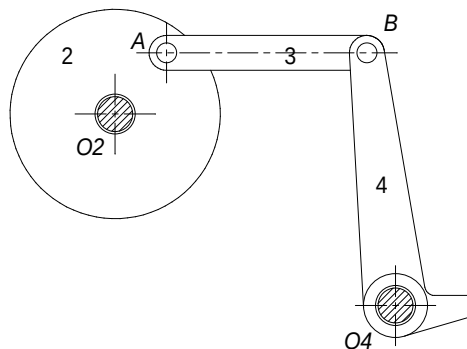
- b. This is a fourbar linkage. The input is link 2, which in this case is the wheel with a pin at A, and the output is link 4. The cross-hatched pivot pins at O_2 and O_4 are attached to the ground link (1).

$$L_1 := 162 \quad L_2 := 40$$

$$L_3 := 96 \quad L_4 := 122$$

$$\text{Condition}(L_1, L_2, L_3, L_4) = \text{"Grashof"}$$

This is a Barker Type 2 GCRR (Grashof, shortest link is input).



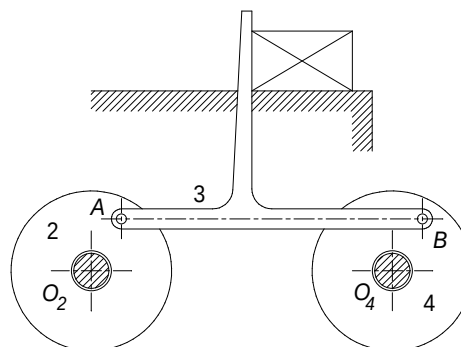
- d. This is a fourbar linkage. The input is link 2, which in this case is a wheel with a pin at A, and the output is the vertical member on the coupler, link 3. Since the lengths of links 2 and 4 (O_2A and O_4B) are the same, the coupler link (3) has curvilinear motion and AB remains parallel to O_2O_4 throughout the cycle. The cross-hatched pivot pins at O_2 and O_4 are attached to the ground link (1).

$$L_1 := 150 \quad L_2 := 30$$

$$L_3 := 150 \quad L_4 := 30$$

$$\text{Condition}(L_1, L_2, L_3, L_4) = \text{"Special Grashof"}$$

This is a Barker Type 13 S2X (special case Grashof, two equal pairs, parallelogram).



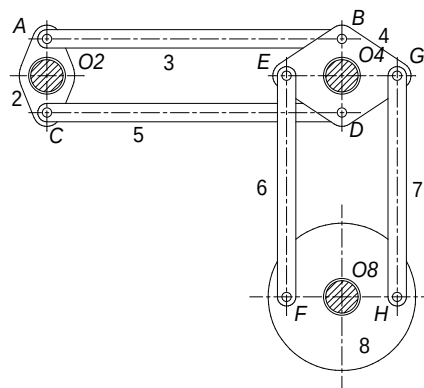
PROBLEM 2-23

Statement: Find the rotability of each loop of the mechanisms in Figure P2-4e, f, and g.

Solution: See Figure P2-4 and Mathcad file P0223.

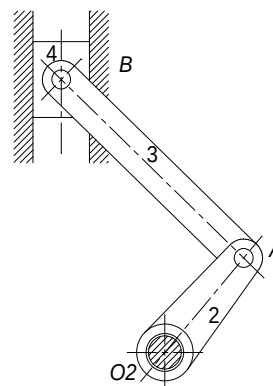
1. Use inequality 2.15 to determine the rotability of each loop in the given mechanisms.

- e. This is a fourbar linkage with an output dyad. The input (rocker) is link 2 and the output (rocker) is link 8. Links 5 and 6 are redundant, i.e. the mechanism will have the same motion if they are removed. The input fourbar consists of links 1, 2, 3, and 4. The output dyad consists of links 7 and 8. The cross-hatched pivot pins at O_2 , O_4 and O_8 are attached to the ground link (1). In the calculation below, the redundant links and their joints are not counted (subtract 2 links and 4 joints from the totals).



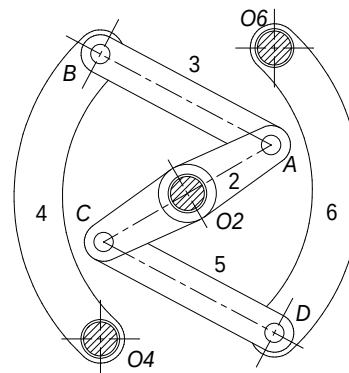
There are two loops in this mechanism. The first loop consists of links 1, 2, 3 (or 5), and 4. The second consists of links 1, 4, 7 (or 6), and 8. By inspection, we see that the sum of the shortest and longest in each loop is equal to the sum of the other two. Thus, both loops are Class III.

- f. This is a fourbar offset slider-crank linkage. The input is link 2 (crank) and the output is link 4 (slider block). The cross-hatched pivot pin at O_2 is attached to the ground link (1).



We can analyze this linkage if we replace the slider (4) with an infinitely long binary link that is pinned at B to link 3 and pinned to ground (1). Then links 1 and 4 are both infinitely long. Since these two links are equal in length and, if we say they are finite in length but very long, the rotability of the mechanism will be determined by the relative lengths of 2 and 3. Thus, this is a Class I linkage since link 2 is shorter than link 3.

- g. This is a fourbar linkage with an alternate output dyad. The input (rocker) is link 2 and the outputs (rockers) are links 4 and 6. The input fourbar consists of links 1, 2, 3, and 4. The alternate output dyad consists of links 5 and 6. The cross-hatched pivot pins at O_2 , O_4 and O_6 are attached to the ground link (1).



$$\begin{aligned} r_1 &:= 87 & r_2 &:= 49 \\ r_3 &:= 100 & r_4 &:= 153 \end{aligned}$$

Using the notation of inequality 2.15, $N := 4$

$$\begin{aligned} L_N &:= r_4 & L_1 &:= r_2 \\ L_2 &:= r_1 & L_3 &:= r_3 \end{aligned}$$

$$L_N + L_1 = 202 \qquad L_2 + L_3 = 187$$

Since $L_N + L_1 > L_2 + L_3$ this is a class II mechanism.

PROBLEM 2-24

Statement: Find the mobility of the mechanisms in Figure P2-5.

Solution: See Figure P2-5 and Mathcad file P0224.

1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility. In the kinematic representations of the linkages below, binary links are depicted as single lines with nodes at their end points whereas higher order links are depicted as 2-D bars.
- a. This is a sixbar linkage with 4 binary (1, 2, 5, and 6) and 2 ternary (3 and 4) links. The inverted U-shaped link at the top of Figure P2-5a is represented here as the binary link 6.

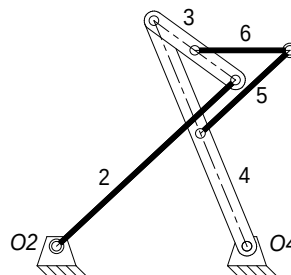
$$\text{Number of links} \quad L := 6$$

$$\text{Number of full joints} \quad J_1 := 7$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



- b. This is an eightbar linkage with 4 binary (1, 4, 7, and 8) and 4 ternary (2, 3, 5, and 6) links. The inverted U-shaped link at the top of Figure P2-5b is represented here as the binary link 8.

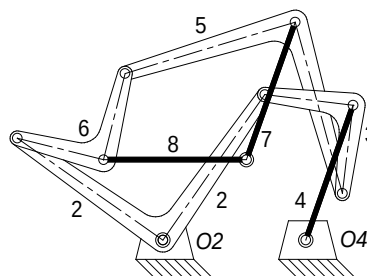
$$\text{Number of links} \quad L := 8$$

$$\text{Number of full joints} \quad J_1 := 10$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



PROBLEM 2-25

- Statement:** Find the mobility of the ice tongs in Figure P2-6.
- When operating them to grab the ice block.
 - When clamped to the ice block but before it is picked up (ice grounded).
 - When the person is carrying the ice block with the tongs.

Solution: See Figure P2-6 and Mathcad file P0225.

- Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

- In this case there are two links and one full joint and 1 *DOF*.

$$\text{Number of links} \quad L := 2$$

$$\text{Number of full joints} \quad J_1 := 1$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

- When the block is clamped in the tongs another link and two more full joints are added reducing the *DOF* to zero (the tongs and ice block form a structure).

$$\text{Number of links} \quad L := 2 + 1$$

$$\text{Number of full joints} \quad J_1 := 1 + 2$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 0$$

- When the block is being carried the system has at least 4 *DOF*: *x*, *y*, and *z* position and orientation about a vertical axis.

PROBLEM 2-26

Statement: Find the mobility of the automotive throttle mechanism shown in Figure P2-7.

Solution: See Figure P2-7 and Mathcad file P0226.

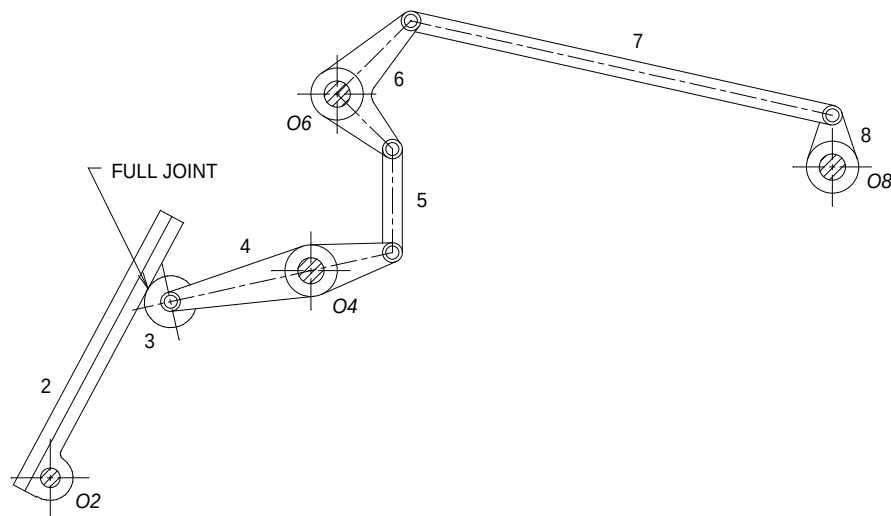
1. This is an eightbar linkage with 8 binary links. It is assumed that the joint between the gas pedal (2) and the roller (3) that pivots on link 4 is a full joint, i.e. the roller rolls without slipping. The pivot pins at O_2 , O_4 , O_6 , and O_8 are attached to the ground link (1). Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

Number of links $L := 8$

Number of full joints $J_1 := 10$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

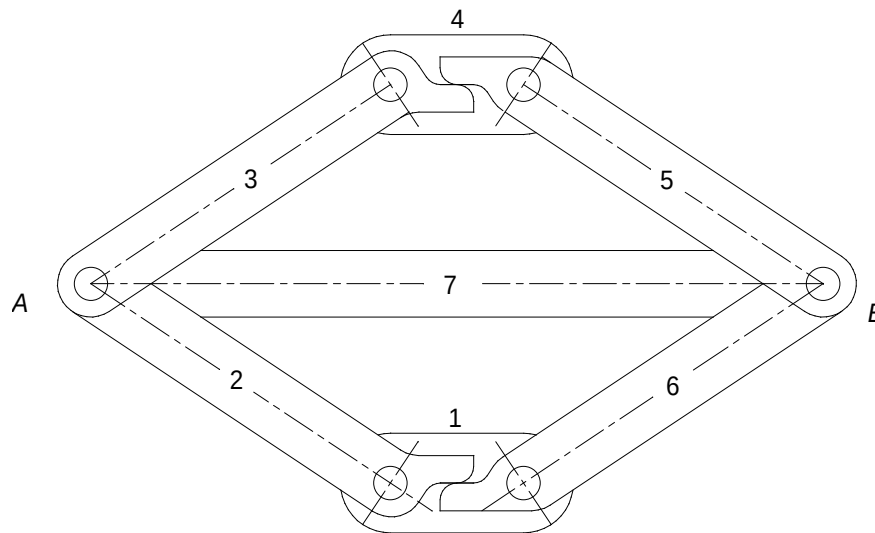


PROBLEM 2-27

Statement: Sketch a kinematic diagram of the scissors jack shown in Figure P2-8 and determine its mobility. Describe how it works.

Solution: See Figure P2-8 and Mathcad file P0227.

1. The scissors jack depicted is a seven link mechanism with eight full and two half joints (see kinematic diagram below). Link 7 is a variable length link. Its length is changed by rotating the screw with the jack handle (not shown). The two blocks at either end of link 7 are an integral part of the link. The block on the left is threaded and acts like a nut. The block on the right is not threaded and acts as a bearing. Both blocks have pins that engage the holes in links 2, 3, 5, and 6. Joints *A* and *B* have 2 full joints apiece. For any given length of link 7 the jack is a structure ($DOF = 0$). When the screw is turned to give the jack a different height the jack has 1 DOF .



Number of links $L := 7$

Number of full joints $J_1 := 8$

Number of half joints $J_2 := 2$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 0$$

PROBLEM 2-28

Statement: Find the mobility of the corkscrew in Figure P2-9.

Solution: See Figure P2-9 and Mathcad file P0228.

1. The corkscrew is made from 4 pieces: the body (1), the screw (2), and two arms with teeth (3), one of which is redundant. The second arm is present to balance the forces on the assembly but is not necessary from a kinematic standpoint. So, kinematically, there are 3 links (body, screw, and arm), 2 full joints (sliding joint between the screw and the body, and pin joint where the arm rotates on the body), and 1 half joint where the arm teeth engage the screw "teeth". Using equation 2.1c, the *DOF* (mobility) is

$$\text{Number of links} \quad L := 3$$

$$\text{Number of full joints} \quad J_1 := 2$$

$$\text{Number of half joints} \quad J_2 := 1$$

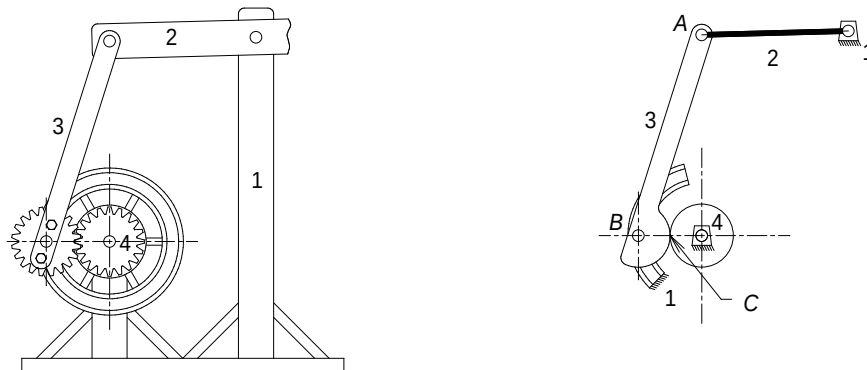
$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

PROBLEM 2-29

Statement: Figure P2-10 shows Watt's sun and planet drive that he used in his steam engine. The beam 2 is driven in oscillation by the piston of the engine. The planet gear is fixed rigidly to link 3 and its center is guided in the fixed track 1. The output rotation is taken from the sun gear 4. Sketch a kinematic diagram of this mechanism and determine its *DOF*. Can it be classified by the Barker scheme? If so, what Barker class and subclass is it?

Solution: See Figure P2-10 and Mathcad file P0229.

1. Sketch a kinematic diagram of the mechanism. The mechanism is shown on the left and a kinematic model of it is sketched on the right. It is a fourbar linkage with 1 *DOF* (see below).



2. Use equation 2.1c to determine the *DOF* (mobility). There are 4 links, 3 full pin joints, 1 half pin-in-slot joint (at *B*), and 1 half joint (at the interface *C* between the two gears, shown above by their pitch circles). Links 1 and 3 are ternary.

Kutzbach's mobility equation (2.1c)

$$\text{Number of links} \quad L := 4$$

$$\text{Number of full joints} \quad J_1 := 3$$

$$\text{Number of half joints} \quad J_2 := 2$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

3. The Barker classification scheme requires that we have 4 link lengths. The motion of link 3 can be modeled by a basic fourbar if the half joint at *B* is replaced with a full pin joint and a link is added to connect *B* and the fixed pivot that is coincident with the center of curvature of the slot that guides pin *B*.

$$L_1 := 2.15 \quad L_2 := 1.25$$

$$L_3 := 1.80 \quad L_4 := 0.54$$

This is a Grashof linkage and the Barker classification is I-4 (type 4) because the shortest link is the output.

PROBLEM 2-30

Statement: Figure P2-11 shows a bicycle hand brake lever assembly. Sketch a kinematic diagram of this device and draw its equivalent linkage. Determine its mobility. Hint: Consider the flexible cable to be a link.

Solution: See Figure P2-11 and Mathcad file P0230.

1. The motion of the flexible cable is along a straight line as it leaves the guide provided by the handle bar so it can be modeled as a translating full slider that is supported by the handlebar (link 1). The brake lever is a binary link that pivots on the ground link. Its other node is attached through a full pin joint to a third link, which drives the slider (link 4).

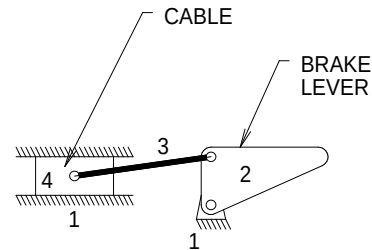
Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



PROBLEM 2-31

Statement: Figure P2-12 shows a bicycle brake caliper assembly. Sketch a kinematic diagram of this device and draw its equivalent linkage. Determine its mobility under two conditions.

- Brake pads not contacting the wheel rim.
- Brake pads contacting the wheel rim.

Hint: Consider the flexible cable to be replaced by forces in this case.

Solution: See Figure P2-12 and Mathcad file P0231.

- The rigging of the cable requires that there be two brake arms. However, kinematically they operate independently and can be analyzed that way. Therefore, we only need to look at one brake arm. When the brake pads are not contacting the wheel rim there is a single lever (link 2) that is pivoted on a full pin joint that is attached to the ground link (1). Thus, there are two links (frame and brake arm) and one full pin joint.

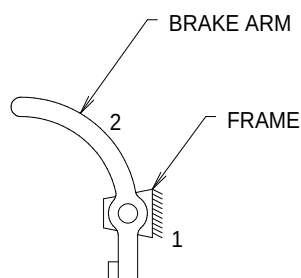
Number of links $L := 2$

Number of full joints $J_1 := 1$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



- When the brake pad contacts the wheel rim we could consider the joint between the pad, which is rigidly attached to the brake arm and is, therefore, a part of link 2, to be a half joint. The brake arm (with pad), wheel (which is constrained from moving laterally by the frame), and the frame constitute a structure.

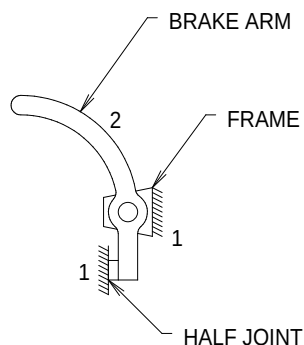
Number of links $L := 2$

Number of full joints $J_1 := 1$

Number of half joints $J_2 := 1$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 0$$



PROBLEM 2-32

Statement: Find the mobility, the Grashof condition, and the Barker classifications of the mechanism in Figure P2-13.

Solution: See Figure P2-13 and Mathcad file P0232.

1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

When there is no cable in the jaw or before the cable is crimped this is a basic fourbar mechanism with with 4 full pin joints:

$$\text{Number of links} \quad L := 4$$

$$\text{Number of full joints} \quad J_1 := 4$$

$$\text{Number of half joints} \quad J_2 := 0 \quad M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

When there is a cable in the jaw this is a threebar mechanism with with 3 full pin joints. While the cable is clamped the jaws are stationary with respect to each other so that link 4 is grounded along with link 1, leaving only three operational links.

$$\text{Number of links} \quad L := 3$$

$$\text{Number of full joints} \quad J_1 := 3$$

$$\text{Number of half joints} \quad J_2 := 0 \quad M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 0$$

2. Use inequality 2.8 to determine the Grashof condition and Table 2-4 to determine the Barker classification.

$$\begin{aligned} \text{Condition}(a, b, c, d) := & \left| \begin{array}{l} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} \quad \text{if } SL < PQ \\ \text{return "Special Grashof"} \quad \text{if } SL = PQ \\ \text{return "non-Grashof"} \quad \text{otherwise} \end{array} \right. \end{aligned}$$

$$L_1 := 0.92 \quad L_2 := 0.27$$

$$L_3 := 0.50 \quad L_4 := 0.60$$

$$\text{Condition}(L_1, L_2, L_3, L_4) = \text{"non-Grashof"}$$

The Barker classification is II-1 (Type 5) RRR1 (non-Grashof, longest link grounded).

PROBLEM 2-33

- Statement:** The approximate torque-speed curve and its equation for a 1/4 hp shunt-wound DC motor are shown in Figure P2-14. The rated speed for this fractional horsepower motor is 10000 rpm at a rated voltage of 130V. Determine:
- The rated torque in oz-in (ounce-inches, the industry standard for fractional hp motors)
 - The no-load speed
 - The operating speed range
 - Plot the power-torque curve in the operating range and determine the maximum power that the motor can deliver in the that range.

Given: Rated speed, N_R $N_R := 10000 \cdot \text{rpm}$ Rated power, H_R $H_R := \frac{1}{4} \cdot \text{hp}$

$$N(T) := \begin{cases} -0.1 \cdot \frac{N_R}{T_R} \cdot T + 1.1 \cdot N_R & \text{if } T \leq 62.5 \cdot \text{ozf} \cdot \text{in} \\ -1.7 \cdot \frac{N_R}{T_R} \cdot T + 5.1 \cdot N_R & \text{otherwise} \end{cases} \quad T := 0 \cdot \text{ozf} \cdot \text{in}, 2.5 \cdot \text{ozf} \cdot \text{in} \dots 75 \cdot \text{ozf} \cdot \text{in}$$

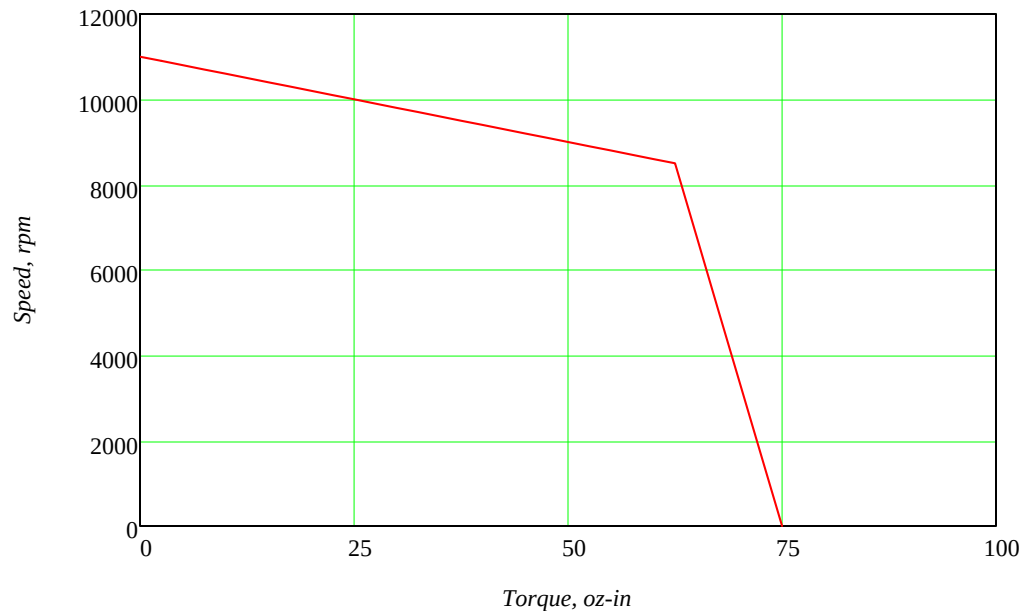


Figure P2-14 Torque-speed Characteristic of a 1/4 HP, 10000 rpm DC Motor

Solution: See Figure P2-3 and Mathcad file P0220.

- a. The rated torque is found by dividing the rated power by the rated speed:

$$\text{Rated torque, } T_R \quad T_R := \frac{H_R}{N_R} \quad T_R := 25 \cdot \text{ozf} \cdot \text{in}$$

- b. The no-load speed occurs at $T = 0$. From the graph this is 11000 rpm.
- c. The operating speed range for a shunt-wound DC motor is the speed at which the motor begins to stall up to the no-load speed. For the approximate torque-speed curve given in this problem the minimum speed is defined as the speed at the knee of the curve.

$$N_{op_{min}} := N(62.5 \cdot \text{ozf} \cdot \text{in}) \quad N_{op_{min}} = 8500 \text{ rpm}$$

$$Nop_{max} := N(0 \cdot \text{ozf} \cdot \text{in})$$

$$Nop_{max} = 11000 \text{ rpm}$$

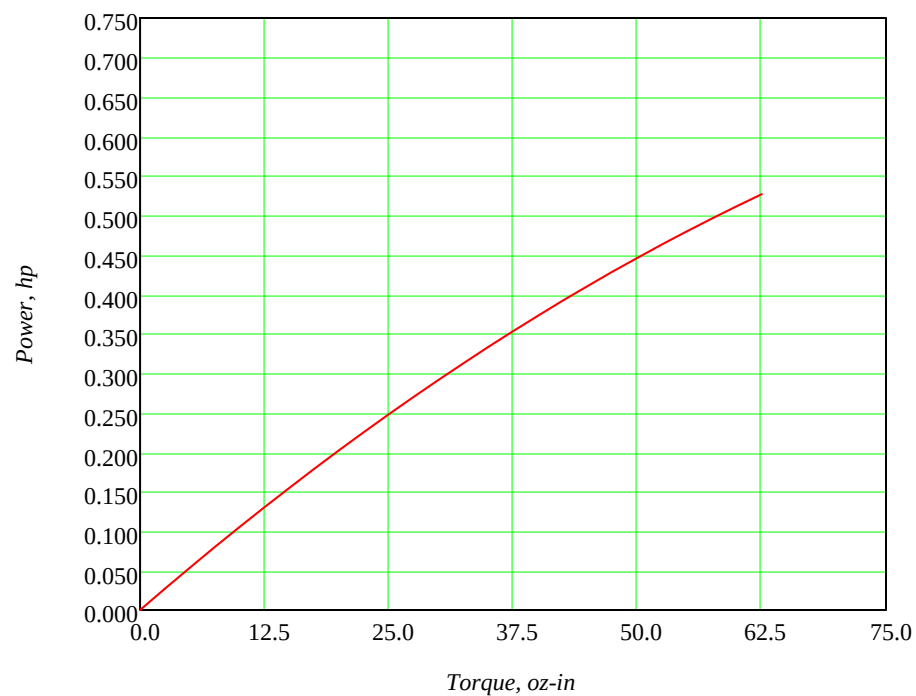
- d. The power is the product of the speed and the torque. From the graph the equation for the torque-speed curve over the operating range is:

$$N(T) := -\frac{40 \cdot \text{rpm}}{\text{ozf} \cdot \text{in}} \cdot T + 11000 \cdot \text{rpm}$$

and the power, therefore, is:

$$H(T) := N(T) \cdot T$$

Plotting the power as a function of torque over the range $T := 0 \cdot \text{ozf} \cdot \text{in}, 2.5 \cdot \text{ozf} \cdot \text{in} \dots 62.5 \cdot \text{ozf} \cdot \text{in}$



Maximum power occurs at the maximum torque in the operating range. The value of T at maximum power is:

Value of T at H_{max} $T_{Hmax} := 62.5 \cdot \text{ozf} \cdot \text{in}$

$$T_{Hmax} = 62.5 \cdot \text{ozf} \cdot \text{in}$$

Maximum power $H_{max} := H(T_{Hmax})$

$$H_{max} = 0.527 \cdot \text{hp}$$

Speed at max power $N_{Hmax} := N(T_{Hmax})$

$$N_{Hmax} = 8500 \cdot \text{rpm}$$

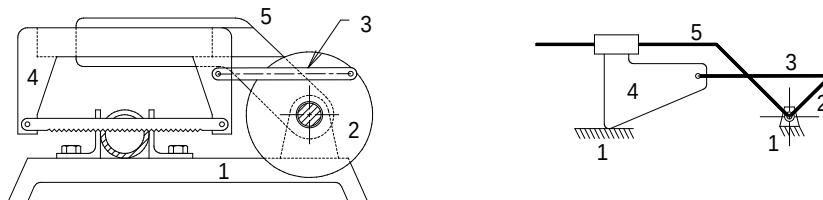
Note that the curve goes through the rated power point of 0.25 hp at the rated torque of 25 oz-in.

PROBLEM 2-34

Statement: Figure P2-15 shows a power hacksaw, used to cut metal. Link 5 pivots at O_5 and its weight forces the sawblade against the workpiece while the linkage moves the blade (link 4) back and forth within link 5 to cut the part. Sketch its kinematic diagram, determine its mobility and its type (i.e., is it a fourbar, a Watt's sixbar, a Stephenson's sixbar, an eightbar, or what?) Use reverse linkage transformation to determine its pure revolute-jointed equivalent linkage.

Solution: See Figure P2-15 and Mathcad file P0234.

1. Sketch a kinematic diagram of the mechanism. The mechanism is shown on the left and a kinematic model of it is sketched on the right. It is a fivebar linkage with 1 *DOF* (see below).



2. Use equation 2.1c to determine the *DOF* (mobility). There are 5 links, 4 full pin joints, 1 full sliding joint, and 1 half joint (at the interface between the hacksaw blade and the pipe being cut).

Kutzbach's mobility equation (2.1c)

$$\text{Number of links} \quad L := 5$$

$$\text{Number of full joints} \quad J_1 := 5$$

$$\text{Number of half joints} \quad J_2 := 1$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

3. Use rule 1 to transform the full sliding joint to a full pin joint for no change in *DOF*. Then use rules 2 and 3 by changing the half joint to a full pin joint and adding a link for no change in *DOF*. The resulting kinematically equivalent linkage has 6 links, 7 full pin joints, no half joints, and is shown below.

Kutzbach's mobility equation (2.1c)

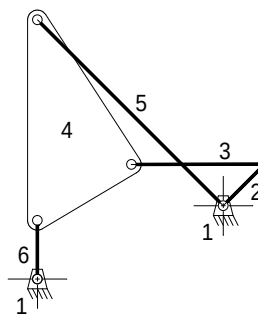
$$\text{Number of links} \quad L := 6$$

$$\text{Number of full joints} \quad J_1 := 7$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$

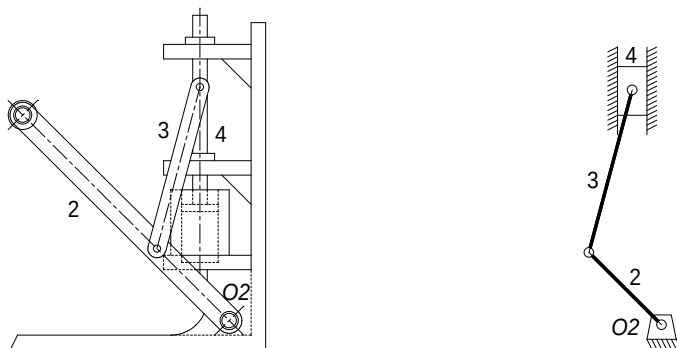


PROBLEM 2-35

Statement: Figure P2-16 shows a manual press used to compact powdered materials. Sketch its kinematic diagram, determine its mobility and its type (i.e., is it a fourbar, a Watt's sixbar, a Stephenson's sixbar, an eightbar, or what?) Use reverse linkage transformation to determine its pure revolute-jointed equivalent linkage.

Solution: See Figure P2-16 and Mathcad file P0235.

1. Sketch a kinematic diagram of the mechanism. The mechanism is shown on the left and a kinematic model of it is sketched on the right. It is a fourbar linkage with 1 *DOF* (see below).



2. Use equation 2.1c to determine the *DOF* (mobility). There are 4 links, 3 full pin joints, 1 full sliding joint, and 0 half joints. This is a fourbar slider-crank.

Kutzbach's mobility equation (2.1c)

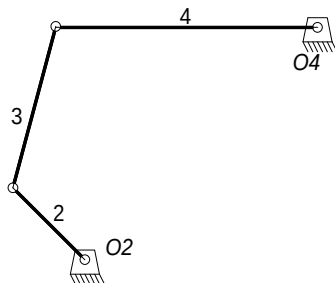
$$\text{Number of links} \quad L := 4$$

$$\text{Number of full joints} \quad J_1 := 4$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

3. Use rule 1 to transform the full sliding joint to a full pin joint for no change in *DOF*. The resulting kinematically equivalent linkage has 4 links, 4 full pin joints, no half joints, and is shown below.

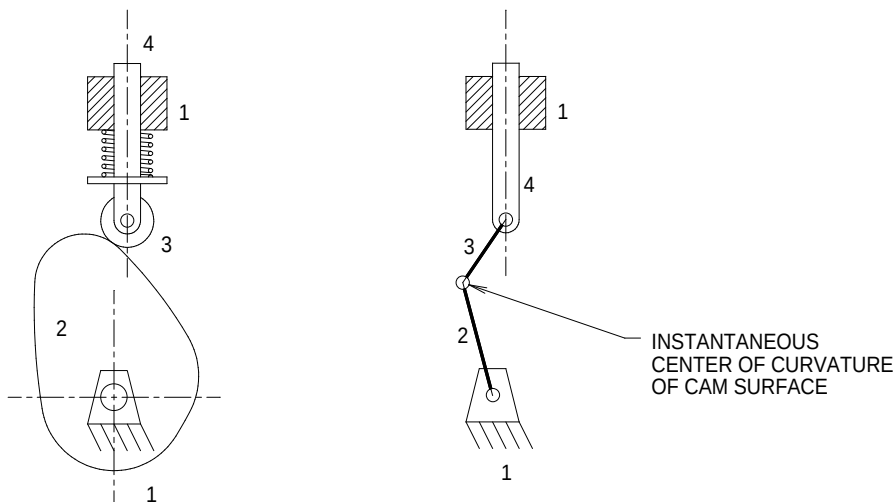


PROBLEM 2-36

Statement: Sketch the equivalent linkage for the cam and follower mechanism in Figure P2-17 in the position shown. Show that it has the same *DOF* as the original mechanism.

Solution: See Figure P2-17 and Mathcad file P0236.

1. The cam follower mechanism is shown on the left and a kinematically equivalent model of it is sketched on the right.



2. Use equation 2.1c to determine the *DOF* (mobility) of the original mechanism. There are 4 links, 2 full pin joints, 1 full sliding joint, 1 pure rolling joint and 0 half joints.

Kutzbach's mobility equation (2.1c)

$$\text{Number of links} \quad L := 4$$

$$\text{Number of full joints} \quad J_1 := 4$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

3. Use equation 2.1c to determine the *DOF* (mobility) of the equivalent mechanism. There are 4 links, 3 full pin joints, 1 full sliding joint, and 0 half joints. This is a fourbar slider-crank.

Kutzbach's mobility equation (2.1c)

$$\text{Number of links} \quad L := 4$$

$$\text{Number of full joints} \quad J_1 := 4$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

PROBLEM 2-37

Statement: Describe the motion of the following rides, commonly found at an amusement park, as pure rotation, pure translation, or complex planar motion.

- a. A Ferris wheel
- b. A "bumper" car
- c. A drag racer ride
- d. A roller coaster whose foundation is laid out in a straight line
- e. A boat ride through a maze
- f. A pendulum ride
- g. A train ride

Solution: See Mathcad file P0211.

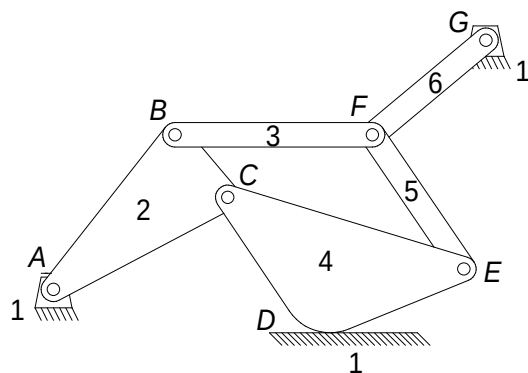
- a. A Ferris wheel
Pure rotation.
- b. A "bumper car"
Complex planar motion.
- c. A drag racer ride
Pure translation.
- d. A roller coaster whose foundation is laid out in a straight line
Complex planar motion.
- e. A boat ride through a maze
Complex planar motion.
- f. A pendulum ride
Pure rotation.
- g. A train ride
Complex planar motion.

PROBLEM 2-38

- Statement:** Figure P2-1a is an example of a mechanism. Number the links, starting with 1. (Hint: Don't forget the "ground" link.) Letter the joints alphabetically, starting with A.
- Using the link numbers, describe each link as binary, ternary, etc.
 - Using the joint letters, determine each joint's order.
 - Using the joint letters, determine whether each is a half or full joint.

Solution: See Figure P2-1a and Mathcad file P0238.

- Label the link numbers and joint letters for Figure P2-1a.



- Using the link numbers, describe each link as binary, ternary, etc.

<u>Link No.</u>	<u>Link Order</u>
1	Ternary
2	Ternary
3	Binary
4	Ternary
5	Binary
6	Binary

- Using the joint letters, determine each joint's order and whether each is a half or full joint.

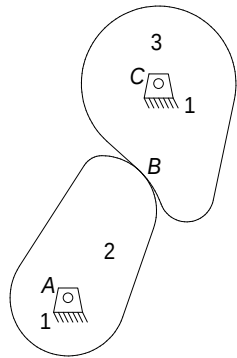
<u>Joint Letter</u>	<u>Joint Order</u>	<u>Half/Full</u>
A	1	Full
B	1	Full
C	1	Full
D	1	Half
E	1	Full
F	2	Full
G	1	Full

PROBLEM 2-39

- Statement:** Figure P2-1b is an example of a mechanism. Number the links, starting with 1. (Hint: Don't forget the "ground" link.) Letter the joints alphabetically, starting with A.
- Using the link numbers, describe each link as binary, ternary, etc.
 - Using the joint letters, determine each joint's order.
 - Using the joint letters, determine whether each is a half or full joint.

Solution: See Figure P2-1b and Mathcad file P0239.

- Label the link numbers and joint letters for Figure P2-1b.



- Using the link numbers, describe each link as binary, ternary, etc.

<u>Link No.</u>	<u>Link Order</u>
1	Binary
2	Binary
3	Binary

- Using the joint letters, determine each joint's order and whether each is a half or full joint.

<u>Joint Letter</u>	<u>Joint Order</u>	<u>Half/Full</u>
A	1	Full
B	1	Half
C	1	Full

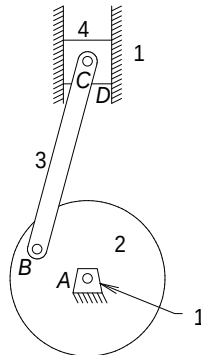
PROBLEM 2-40

Statement: Figure P2-1c is an example of a mechanism. Number the links, starting with 1. (Hint: Don't forget the "ground" link.) Letter the joints alphabetically, starting with A.

- Using the link numbers, describe each link as binary, ternary, etc.
- Using the joint letters, determine each joint's order.
- Using the joint letters, determine whether each is a half or full joint.

Solution: See Figure P2-1c and Mathcad file P0240.

- Label the link numbers and joint letters for Figure P2-1c.



- Using the link numbers, describe each link as binary, ternary, etc.

<u>Link No.</u>	<u>Link Order</u>
1	Binary
2	Binary
3	Binary
4	Binary

- Using the joint letters, determine each joint's order and whether each is a half or full joint.

<u>Joint Letter</u>	<u>Joint Order</u>	<u>Half/Full</u>
A	1	Full
B	1	Full
C	1	Full
D	1	Full

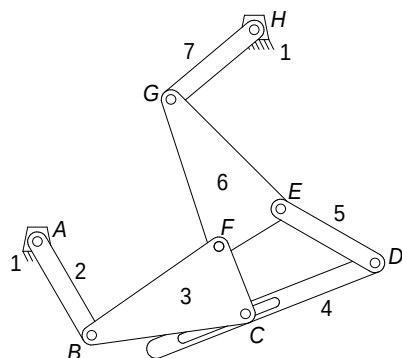
PROBLEM 2-41

Statement: Figure P2-1d is an example of a mechanism. Number the links, starting with 1. (Hint: Don't forget the "ground" link.) Letter the joints alphabetically, starting with A.

- Using the link numbers, describe each link as binary, ternary, etc.
- Using the joint letters, determine each joint's order.
- Using the joint letters, determine whether each is a half or full joint.

Solution: See Figure P2-1d and Mathcad file P0241.

- Label the link numbers and joint letters for Figure P2-1d.



- Using the link numbers, describe each link as binary, ternary, etc.

<u>Link No.</u>	<u>Link Order</u>
1	Binary
2	Binary
3	Ternary
4	Binary
5	Binary
6	Ternary
7	Binary

- Using the joint letters, determine each joint's order and whether each is a half or full joint.

<u>Joint Letter</u>	<u>Joint Order</u>	<u>Half/Full</u>
A	1	Full
B	1	Full
C	1	Half
D	1	Full
E	1	Full
F	1	Full
G	1	Full
H	1	Full

PROBLEM 2-42

Statement: Find the mobility, Grashof condition and Barker classification of the oil field pump shown in Figure P2-18.

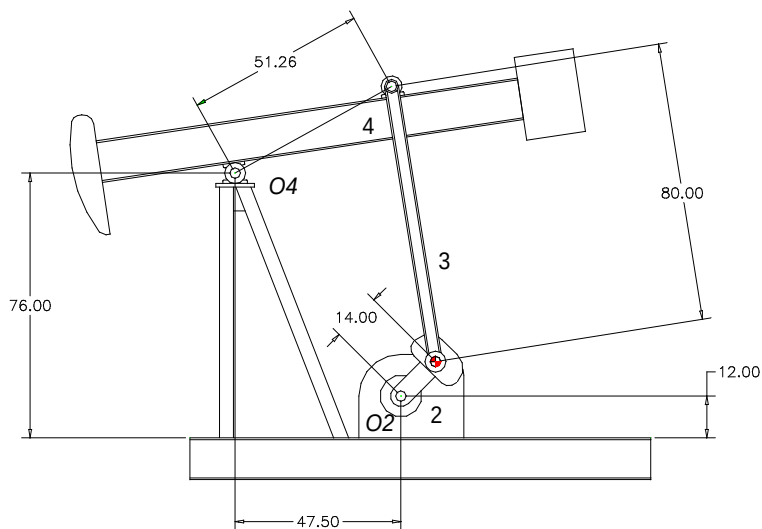
Solution: See Figure P2-18 and Mathcad file P0242.

1. Use inequality 2.8 to determine the Grashof condition and Table 2-4 to determine the Barker classification.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```



This is a basic fourbar linkage. The input is the 14-in-long crank (link 2) and the output is the top beam (link 4). The mobility (*DOF*) is found using equation 2.1c (Kutzbach's modification):

$$\text{Number of links} \quad L := 4$$

$$\text{Number of full joints} \quad J_1 := 4$$

$$\text{Number of half joints} \quad J_2 := 0 \quad M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

The link lengths and Grashof condition are

$$L_1 := \sqrt{(76 - 12)^2 + 47.5^2} \quad L_1 = 79.701 \quad L_2 := 14 \quad L_3 := 80 \quad L_4 := 51.26$$

$$\text{Condition}(L_1, L_2, L_3, L_4) = \text{"Grashof"}$$

This is a Barker Type 2 GCRR (Grashof, shortest link is input).

PROBLEM 2-43

Statement: Find the mobility, Grashof condition and Barker classification of the aircraft overhead bin shown in Figure P2-19.

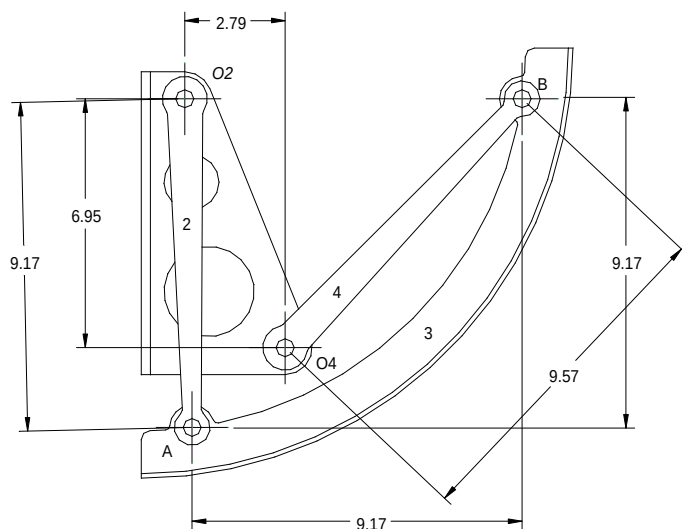
Solution: See Figure P2-19 and Mathcad file P0243.

1. Use inequality 2.8 to determine the Grashof condition and Table 2-4 to determine the Barker classification.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```



This is a basic fourbar linkage. The input is the link 2 and the output is link 4. The mobility (*DOF*) is found using equation 2.1c (Kutzbach's modification):

$$\text{Number of links} \quad L := 4$$

$$\text{Number of full joints} \quad J_1 := 4$$

$$\text{Number of half joints} \quad J_2 := 0 \quad M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

The link lengths and Grashof condition are

$$L_1 := \sqrt{2.79^2 + 6.95^2} \quad L_1 = 7.489 \quad L_2 := 9.17$$

$$L_3 := \sqrt{9.17^2 + 9.17^2} \quad L_3 = 12.968 \quad L_4 := 9.57$$

$$\text{Condition}(L_1, L_2, L_3, L_4) = \text{"non-Grashof"}$$

This is a Barker Type 7 RRR3 (non-Grashof, longest link is coupler).

PROBLEM 2-44

Statement: Figure P2-20 shows a "Rube Goldberg" mechanism that turns a light switch on when a room door is opened and off when the door is closed. The pivot at O_2 goes through the wall. There are two spring-loaded piston-in cylinder devices in the assembly. An arrangement of ropes and pulleys inside the room transfers the door swing into a rotation of link 2. Door opening rotates link 2 CW, pushing the switch up as shown in the figure, and door closing rotates link 2 CCW, pulling the switch down. Find the mobility of the linkage.

Solution: See Figure P2-20 and Mathcad file P0244.

1. Examination of the figure shows 20 links (including the the switch) and 28 full joints. The second piston-in cylinder that actuates the switch is counted as a single binary link of variable length with joints at its ends. The other cylinder consists of two binary links, each link having one pin joint and one slider joint. There are no half joints.
2. Use equation 2.1c to determine the *DOF* (mobility).

Kutzbach's mobility equation (2.1c)

Number of links $L := 20$

Number of full joints $J_1 := 28$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

3. An alternative is to ignore the the first piston-in cylinder that acts on the third bellcrank from O_2 since it does not affect the the motion of the linkage (it acts only as a damper.) In that case, subtract two links and three full joints, giving $L = 18$, $J_1 = 25$ and $M = 1$.

PROBLEM 2-45

Statement: All of the eightbar linkages in Figure 2-11 part 2 have eight possible inversions. Some of these will give motions similar to others. Those that have distinct motions are called *distinct inversions*. How many distinct inversions does the linkage in row 4, column 1 have?

Solution: See Figure 2-11, part 2 and Mathcad file P0245.

1. This isomer has one quaternary, two ternary, and five binary links arranged in a symmetrical fashion. Due to this symmetry, grounding link 2 or 7 gives the same inversion, as do grounding 3 or 6 and 4 or 5. This makes 3 of the possible 8 inversions the same leaving 5 distinct inversions. Distinct inversions are obtained by grounding link 1, 2, 3, 4, or 8 (or 1, 5, 6, 7, or 8) for a total of 5 distinct inversions.

PROBLEM 2-46

Statement: All of the eightbar linkages in Figure 2-11 part 2 have eight possible inversions. Some of these will give motions similar to others. Those that have distinct motions are called *distinct inversions*. How many distinct inversions does the linkage in row 4, column 2 have?

Solution: See Figure 2-11, part 2 and Mathcad file P0246.

1. This isomer has four ternary, and four binary links arranged in a symmetrical fashion. Due to this symmetry, grounding link 1 or 5 gives the same inversion, as do grounding 2 or 8, 4 or 6, and 3 or 7. This makes 4 of the possible 8 inversions the same leaving 4 distinct inversions. Distinct inversions are obtained by grounding link 1, 2, 3, or 4 (or 5, 6, 7, or 8) for a total of 4 distinct inversions.

PROBLEM 2-47

Statement: All of the eightbar linkages in Figure 2-11 part 2 have eight possible inversions. Some of these will give motions similar to others. Those that have distinct motions are called *distinct inversions*. How many distinct inversions does the linkage in row 4, column 3 have?

Solution: See Figure 2-11, part 2 and Mathcad file P0247.

1. This isomer has four ternary, and four binary links arranged in a symmetrical fashion. Due to this symmetry, grounding link 2 or 4 gives the same inversion, as does grounding 5 or 7. This makes 2 of the possible 8 inversions the same leaving 6 distinct inversions. Distinct inversions are obtained by grounding link 1, 2, 3, 5, 6 or 8 (or 1, 3, 4, 6, 7, or 8) for a total of 6 distinct inversions.

PROBLEM 2-48

Statement: Find the mobility of the mechanism shown in Figure 3-33.

Solution: See Figure 3-33 and Mathcad file P0248.

1. Use equation 2.1c to determine the *DOF* (mobility). There are 6 links, 7 full pin joints (two at *B*), and no half-joints.

Kutzbach's mobility equation (2.1c)

Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

PROBLEM 2-49

Statement: Find the mobility of the mechanism shown in Figure 3-34.

Solution: See Figure 3-34 and Mathcad file P0249.

1. Use equation 2.1c to determine the *DOF* (mobility). There are 6 links, 7 full pin joints, and no half-joints.

Kutzbach's mobility equation (2.1c)

Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

PROBLEM 2-50

Statement: Find the mobility of the mechanism shown in Figure 3-35.

Solution: See Figure 3-35 and Mathcad file P0250.

1. Use equation 2.1c to determine the *DOF* (mobility). There are 6 links, 7 full pin joints, and no half-joints.

Kutzbach's mobility equation (2.1c)

Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

PROBLEM 2-51

Statement: Find the mobility of the mechanism shown in Figure 3-36.

Solution: See Figure 3-36 and Mathcad file P0251.

1. Use equation 2.1c to determine the *DOF* (mobility). There are 8 links, 10 full pin joints (two at O_4), and no half-joints.

Kutzbach's mobility equation (2.1c)

Number of links $L := 8$

Number of full joints $J_1 := 10$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

PROBLEM 2-52

Statement: Find the mobility of the mechanism shown in Figure 3-37.

Solution: See Figure 3-37 and Mathcad file P0252.

1. Use equation 2.1c to determine the *DOF* (mobility). There are 6 links, 7 full pin joints (two at O_4), and no half-joints.

Kutzbach's mobility equation (2.1c)

Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

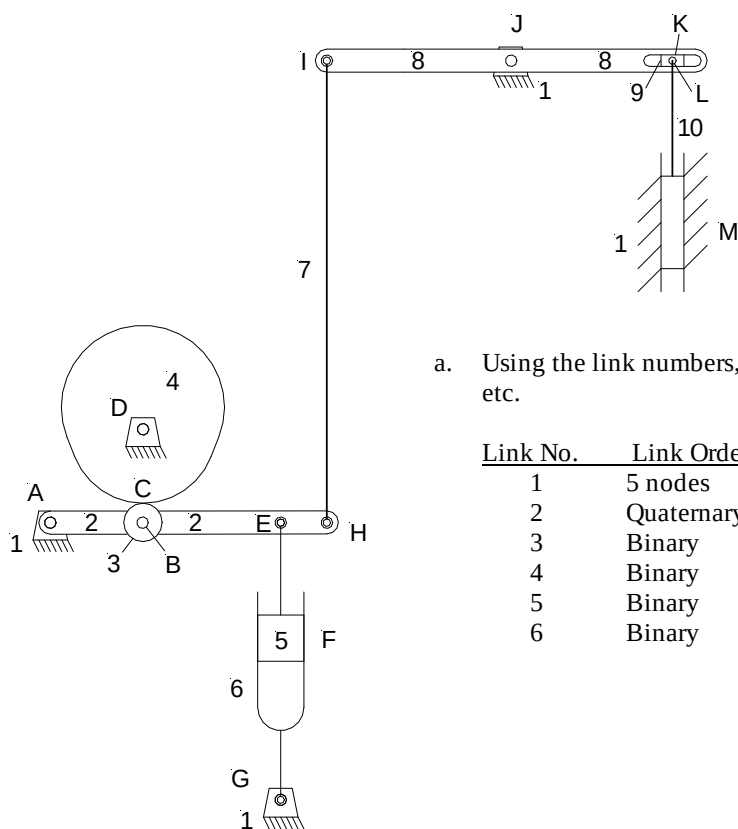
$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

PROBLEM 2-53

- Statement:** Figure P2-1e is an example of a mechanism. Number the links, starting with 1. (Hint: Don't forget the "ground" link.) Letter the joints alphabetically, starting with A.
- Using the link numbers, describe each link as binary, ternary, etc.
 - Using the joint letters, determine each joint's order.
 - Using the joint letters, determine whether each is a half or full joint.

Solution: See Figure P2-1e and Mathcad file P0253.

- Label the link numbers and joint letters for Figure P2-1e.



- Using the link numbers, describe each link as binary, ternary, etc.

Link No.	Link Order	Link No.	Link Order
1	5 nodes	7	Binary
2	Quaternary	8	Ternary
3	Binary	9	Binary
4	Binary	10	Binary
5	Binary		
6	Binary		

- Using the joint letters, determine each joint's order and whether each is a half or full joint.

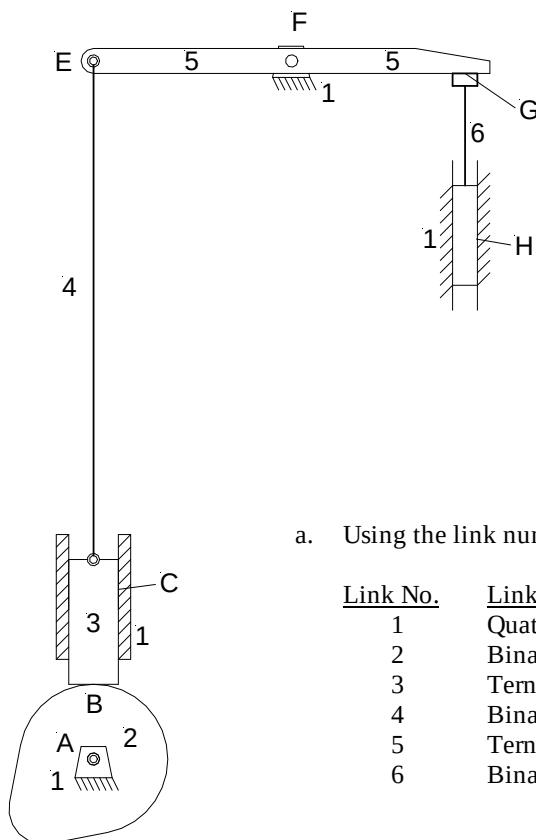
Joint Letter	Joint Order	Half/Full	Joint Classification
A	1	Full	Grounded rotating joint
B	1	Full	Moving rotating joint
C	1	Full	Pure rolling joint
D	1	Full	Grounded rotating joint
E	1	Full	Moving rotating joint
F	1	Full	Moving translating joint
G	1	Full	Grounded rotating joint
H	1	Full	Moving rotating joint
I	1	Full	Moving rotating joint
J	1	Full	Grounded rotating joint
K	1	Full	Moving translating joint
L	1	Full	Moving rotating joint
M	1	Full	Grounded translating joint

PROBLEM 2-54

- Statement:** Figure P2-1f is an example of a mechanism. Number the links, starting with 1. (Hint: Don't forget the "ground" link.) Letter the joints alphabetically, starting with A.
- Using the link numbers, describe each link as binary, ternary, etc.
 - Using the joint letters, determine each joint's order.
 - Using the joint letters, determine whether each is a half or full joint.

Solution: See Figure P2-1f and Mathcad file P0254.

- Label the link numbers and joint letters for Figure P2-1f.



- Using the link numbers, describe each link as binary, ternary, etc.

<u>Link No.</u>	<u>Link Order</u>
1	Quaternary
2	Binary
3	Ternary
4	Binary
5	Ternary
6	Binary

- Using the joint letters, determine each joint's order and whether each is a half or full joint.

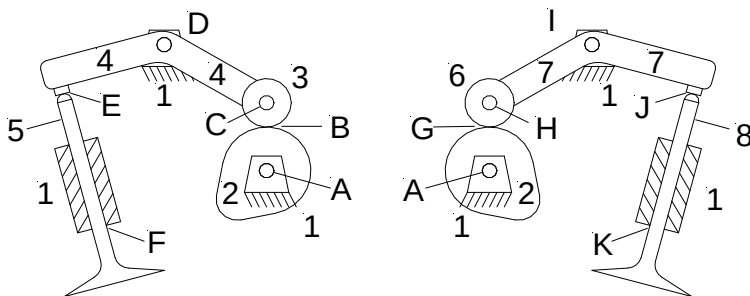
<u>Joint Letter</u>	<u>Joint Order</u>	<u>Half/Full</u>	<u>Joint Classification</u>
A	1	Full	Grounded rotating joint
B	1	Full	Moving half joint
C	1	Full	Grounded translating joint
D	1	Full	Moving rotating joint
E	1	Full	Moving rotating joint
F	1	Full	Grounded rotating joint
G	1	Full	Moving half joint
H	1	Full	Grounded translating joint

PROBLEM 2-55

- Statement:** Figure P2-1g is an example of a mechanism. Number the links, starting with 1. (Hint: Don't forget the "ground" link.) Letter the joints alphabetically, starting with A.
- Using the link numbers, describe each link as binary, ternary, etc.
 - Using the joint letters, determine each joint's order.
 - Using the joint letters, determine whether each is a half or full joint.

Solution: See Figure P2-1g and Mathcad file P0255.

- Label the link numbers and joint letters for Figure P2-1g.



- Using the link numbers, describe each link as binary, ternary, etc.

<u>Link No.</u>	<u>Link Order</u>	<u>Link No.</u>	<u>Link Order</u>
1	5 nodes	5	Binary
2	Binary	6	Binary
3	Binary	7	Ternary
4	Ternary	8	Binary

- Using the joint letters, determine each joint's order and whether each is a half or full joint.

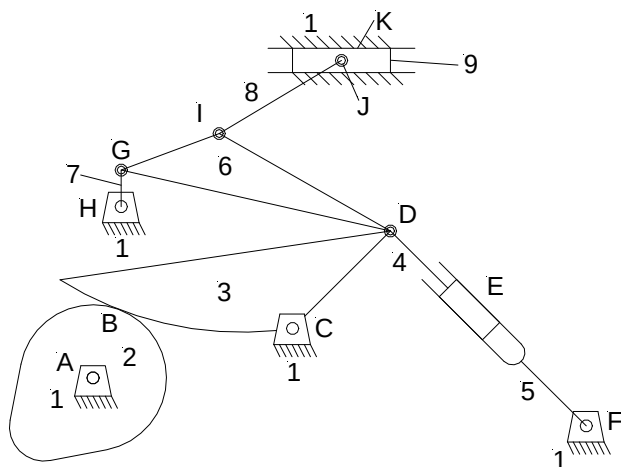
<u>Joint Letter</u>	<u>Joint Order</u>	<u>Half/Full</u>	<u>Joint Classification</u>
A	1	Full	Grounded rotating joint
B	1	Full	Moving rolling joint
C	1	Full	Moving rotating joint
D	1	Full	Grounded rotating joint
E	1	Half	Moving sliding joint
F	1	Full	Grounded translating joint
G	1	Full	Moving rolling joint
H	1	Full	Moving rotating joint
I	1	Full	Grounded rotating joint
J	1	Half	Moving sliding joint
K	1	Full	Grounded translating joint

PROBLEM 2-56

Statement: For the example linkage shown in Figure 2-4 find the number of links and their respective link orders, the number of joints and their respective orders, and the mobility of the linkage.

Solution: See Figure 2-4 and Mathcad file P0256.

1. Label the link numbers and joint letters for Figure 2-4 example.



2. Using the link numbers, describe each link as binary, ternary, etc.

<u>Link No.</u>	<u>Link Order</u>	<u>Link No.</u>	<u>Link Order</u>
1	5 nodes	6	Ternary
2	Binary	7	Binary
3	Ternary	8	Binary
4	Binary	9	Binary
5	Binary		

3. Using the joint letters, determine each joint's order and whether each is a half or full joint.

<u>Joint Letter</u>	<u>Joint Order</u>	<u>Half/Full</u>	<u>Joint Classification</u>
A	1	Full	Grounded rotating joint
B	1	Half	Moving sliding joint
C	1	Full	Grounded rotating joint
D	2	Full	Moving rotating joint
E	1	Full	Moving translating joint
F	1	Full	Grounded rotating joint
G	1	Full	Moving rotating joint
H	1	Full	Grounded rotating joint
I	1	Full	Moving rotating joint
J	1	Full	Moving rotating joint
K	1	Full	Grounded translating joint

4. Use equation 2.1c to calculate the *DOF* (mobility).

Kutzbach's mobility equation (2.1c)

$$\begin{aligned}
 \text{Number of links } L &:= 9 & \text{Number of full joints } J_1 &:= 11 & \text{Number of half joints } J_2 &:= 1 \\
 M &:= 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 & M &= 1
 \end{aligned}$$

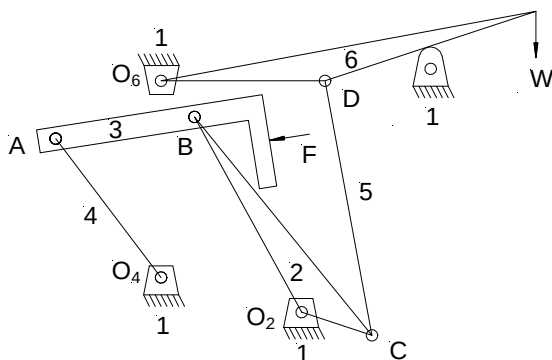
PROBLEM 2-57

Statement: For the linkage shown in Figure 2-5b find the number of joints and their respective orders, and mobility for:

- The condition of a finite load W in the direction shown and a zero F
- The condition of a finite load W and a finite load F both in the directions shown after link 6 is off the stop.

Solution: See Figure 2-5b and Mathcad file P0257.

1. Label the link numbers and joint letters for Figure 2-5b.



a) The condition of a finite load W in the direction shown and a zero F : Using the joint letters, determine each joint's order and whether each is a half or full joint. Link 6 is grounded so joint D is a grounded rotating joint and O_6 is not a joint. For this condition there is a total of 6 full joints and no half joints.

<u>Joint Letter</u>	<u>Joint Order</u>	<u>Half/Full</u>	<u>Joint Classification</u>
O_2	1	Full	Grounded rotating joint
B	1	Full	Moving rotating joint
C	1	Full	Moving rotating joint
D	1	Full	Grounded rotating joint
O_4	1	Full	Grounded rotating joint
A	1	Full	Moving rotating joint

Use equation 2.1c to calculate the DOF (mobility).

Kutzbach's mobility equation (2.1c)

$$\text{Number of links } L := 5 \quad \text{Number of full joints } J_1 := 6 \quad \text{Number of half joints } J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 0$$

b) The condition of a finite load W and a finite load F both in the directions shown after link 6 is off the stop. Using the joint letters, determine each joint's order and whether each is a half or full joint.

<u>Joint Letter</u>	<u>Joint Order</u>	<u>Half/Full</u>	<u>Joint Classification</u>
A	1	Full	Moving rotating joint
B	1	Full	Moving rotating joint
C	1	Full	Moving rotating joint
D	1	Full	Moving rotating joint
O_2	1	Full	Grounded rotating joint
O_4	1	Full	Grounded rotating joint
O_6	1	Full	Grounded rotating joint

Use equation 2.1c to calculate the DOF (mobility).

Kutzbach's mobility equation (2.1c)

$$\text{Number of links } L := 6 \quad \text{Number of full joints } J_1 := 7 \quad \text{Number of half joints } J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

PROBLEM 2-58

Statement: Figure P2-21a shows a "Nuremberg scissors" mechanism. Find its mobility.

Solution: See Figure P2-21a and Mathcad file P0258.

1. Use equation 2.1c to calculate the *DOF* (mobility).

Kutzbach's mobility equation (2.1c)

$$\text{Number of links} \quad L := 10$$

$$\text{Number of full joints} \quad J_1 := 13$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

PROBLEM 2-59

Statement: Figure P2-21b shows a mechanism. Find its mobility and classify its isomer type.

Solution: See Figure P2-21b and Mathcad file P0259.

1. Use equation 2.1c to calculate the *DOF* (mobility).

Kutzbach's mobility equation (2.1c)

$$\text{Number of links} \quad L := 6$$

$$\text{Number of full joints} \quad J_1 := 7$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

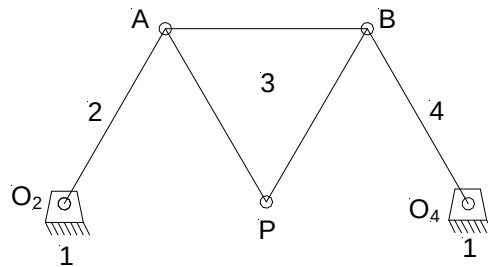
2. Using Figure 2-9, we see that the mechanism is a **Stephenson's sixbar isomer** (the two ternary links are connected with two binary links and one dyad).

PROBLEM 2-60

Statement: Figure P2-21c shows a circular saw mounted on the coupler of a fourbar linkage. The centerline of the saw blade is at a coupler point that moves in an approximate straight line. Draw its kinematic diagram and determine its mobility.

Solution: See Figure P2-21c and Mathcad file P0260.

1. Draw a kinematic diagram of the mechanism. The saw's rotation axis is at point P and the saw is attached to link 3.



2. Use equation 2.1c to calculate the *DOF* (mobility).

Kutzbach's mobility equation (2.1c)

$$\text{Number of links} \quad L := 4$$

$$\text{Number of full joints} \quad J_1 := 4$$

$$\text{Number of half joints} \quad J_2 := 0$$

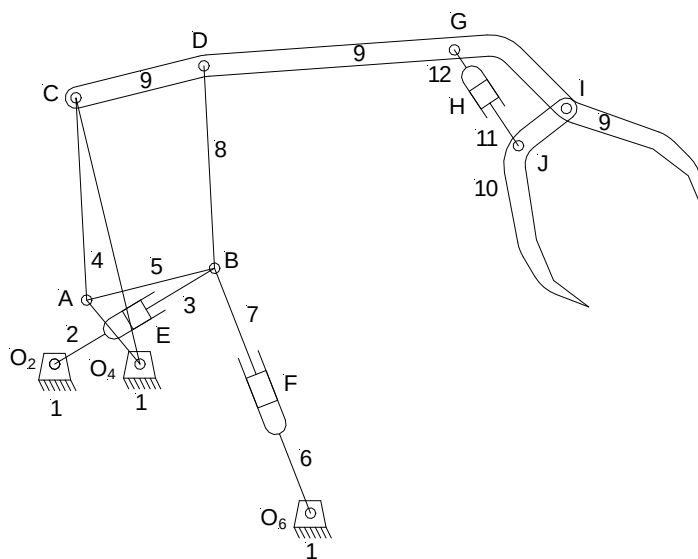
$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

PROBLEM 2-61

- Statement:** Figure P2-21d shows a log transporter. Draw a kinematic diagram of the mechanism, specify the number of links and joints, and then determine its mobility:
- For the transporter wheels locked and no log in the "claw" of the mechanism
 - For the transporter wheels locked with it lifting a log
 - For the transporter moving a log to a destination in a straight line.

Solution: See Figure P2-21d and Mathcad file P0261.

- Draw a kinematic diagram of the mechanism. Link 1 is the frame of the transporter. Joint B is of order 3. Actuators E and F provide two inputs (to get x-y motion) and actuator H provides an additional input for clamping logs.



- Wheels locked, no log in "claw."

Kutzbach's mobility equation (2.1c)

$$\text{Number of links} \quad L := 12$$

$$\text{Number of full joints} \quad J_1 := 15$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 3$$

- Wheels locked, log held tightly in the "claw." With a log held tightly between links 9 and 10 a structure will be formed by links 9 through 12 and the log so that there will only be 9 links and 11 joints active.

$$\text{Number of links} \quad L := 9$$

$$\text{Number of full joints} \quad J_1 := 11$$

$$\text{Number of half joints} \quad J_2 := 0$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 2$$

- Transporter moving in a straight line with the log holding mechanism inactive. There are two tires, the transporter frame, and the ground, making 4 links and two points of contact with the ground and two axels, making 4 joints.

Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

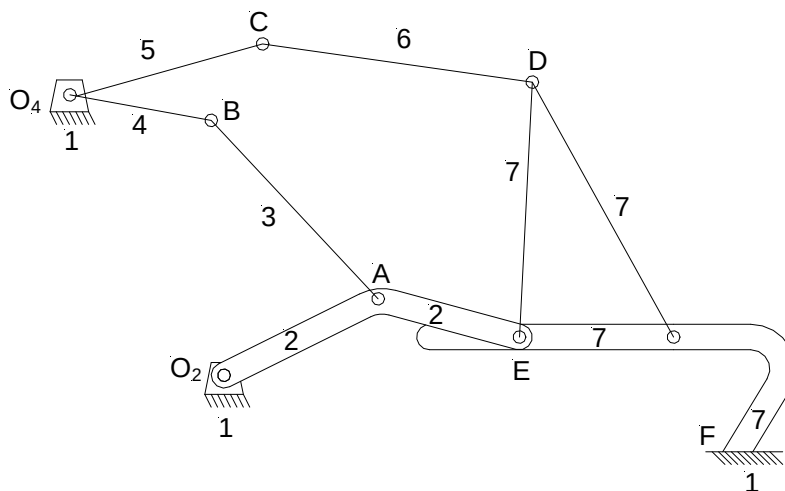
PROBLEM 2-62

Statement: Figure P2-21d shows a plow mechanism attached to a tractor. Draw its kinematic diagram and find its mobility including the earth as a "link."

- When the tractor is stopped and the turnbuckle is fixed. (Hint: Consider the tractor and the wheel to be one with the earth.)
- When the tractor is stopped and the turnbuckle is being adjusted. (Same hint.)
- When the tractor is moving and the turnbuckle is fixed. (Hint: Add the moving tractor's DOF to those found in part a.)

Solution: See Figure P2-21e and Mathcad file P0262.

- Draw a kinematic diagram of the mechanism with the ground, tractor wheels, and tractor frame as link 1. Joint O_4 is of order 2 and joint F is a half joint. The plow and its truss structure attach at joints D and E. Since the turnbuckle is fixed it can be modeled as a single binary link (6).



- Tractor stopped and turnbuckle fixed.

Kutzbach's mobility equation (2.1c)

$$\text{Number of links} \quad L := 7$$

$$\text{Number of full joints} \quad J_1 := 8$$

$$\text{Number of half joints} \quad J_2 := 1$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

- When the tractor is stopped and the turnbuckle is being adjusted. Between joints C and D we now have 2 net links (2 links threaded LH and RH on one end and the turnbuckle body) and 1 additional helical full joint.

$$\text{Number of links} \quad L := 8$$

$$\text{Number of full joints} \quad J_1 := 9$$

$$\text{Number of half joints} \quad J_2 := 1$$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 2$$

- When the tractor is moving and the turnbuckle is fixed. If the tractor moved only in a straight line we would add 1 DOF to the 1 DOF that we got in part a for a total of $M = 2$. More realistically, the tractor can turn and move up and down hills so that we would add 3 DOF to the 1 DOF of part a to get a total of 4 DOF.

PROBLEM 2-63

Statement: Figure P2-22 shows a Hart's inversor sixbar linkage. a) Is it a Watt or Stephenson linkage? b) Determine its inversion, i.e. is it a type I, II, or III?

Solution: See Figure P2-22, Figure 2-14, and Mathcad file P0263.

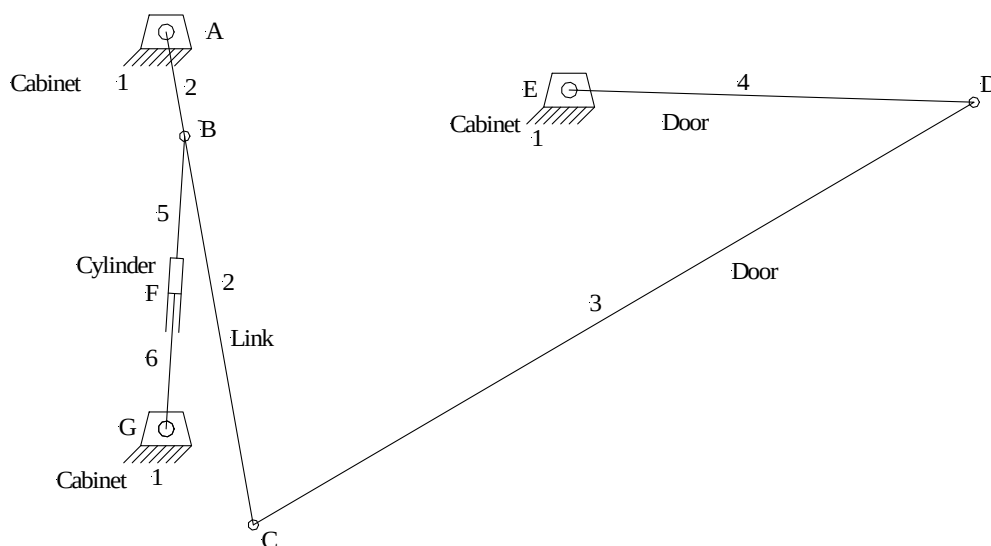
1. From Figure 2-14 we see that the Watt's sixbar has the two ternary links connected with a common joint while the Stephenson's sixbar has the two ternary links connected by binary links. Thus, Hart's inversor is a Watt's sixbar (links 1 and 2, the ternary links, are connected at a common joint). Further, the **Hart's linkage is a Watt's sixbar inversion I** since neither of the ternary links is grounded.

PROBLEM 2-64

Statement: Figure P2-23 shows the top view of the partially open doors on one side of an entertainment center cabinet. The wooden doors are hinged to each other and one door is hinged to the cabinet. There is also a ternary, metal link attached to the cabinet and door through pin joints. A spring-loaded piston-in cylinder device attaches to the ternary link and the cabinet through pin joints. Draw a kinematic diagram of the door system and find the mobility of this mechanism.

Solution: See Figure P2-23 and Mathcad file P0264.

1. Draw the kinematic diagram of this sixbar mechanism. The spring-loaded piston is just an in-line sliding joint (links 5 and 6, and joint F). The doors are binary links (3 and 4), and the metal ternary link (2) has nodes at A, B, and C. Link 1 is the cabinet.



2. Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

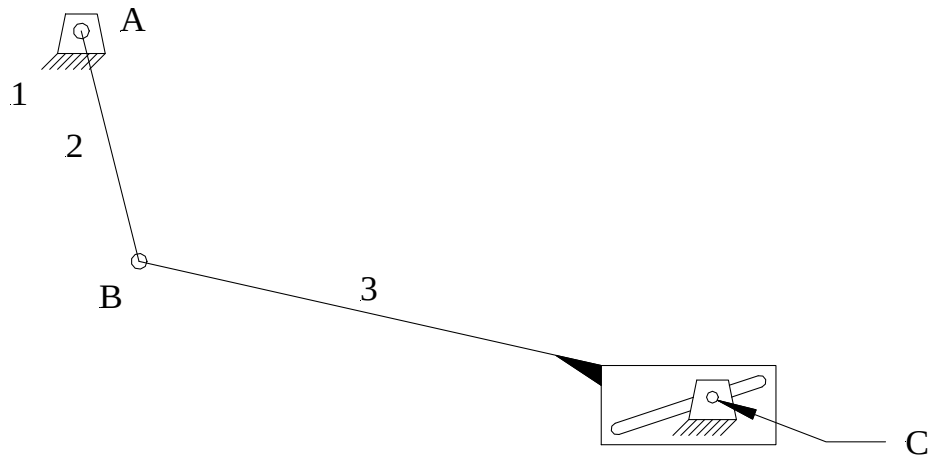
$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

PROBLEM 2-65

Statement: Figure P2-24a shows the seat and seat-back of a reclining chair with the linkage that connects them to the chair frame. Draw its kinematic diagram and determine its mobility with respect to the frame of the chair.

Solution: See Figure P2-24a and Mathcad file P0265.

1. Draw a kinematic diagram of the mechanism. The chair-back attaches to link 2 and the seat with the attached slider slot is link 3. The node at C is a half-joint as it allows two degrees of freedom.



2. Determine the mobility of the mechanism.

Kutzbach's mobility equation (2.1c)

Number of links $L := 3$

Number of full joints $J_1 := 2$

Number of half joints $J_2 := 1$

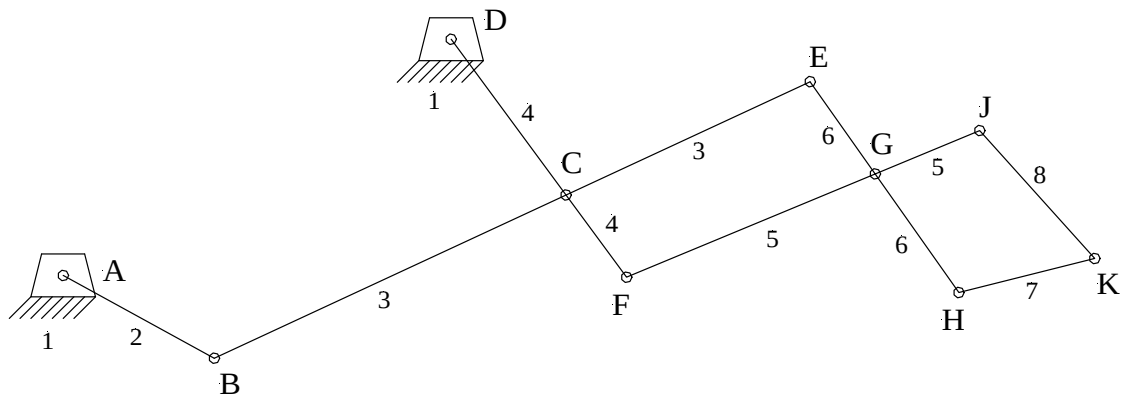
$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

PROBLEM 2-66

Statement: Figure P2-24b shows the mechanism used to extend the foot support on a reclining chair. Draw its kinematic diagram and determine its mobility with respect to the frame of the chair.

Solution: See Figure P2-24b and Mathcad file P0266.

1. Draw a kinematic diagram of the mechanism. Link 1 is the frame.



2. Determine the mobility of the mechanism.

Kutzbach's mobility equation (2.1c)

Number of links $L := 8$

Number of full joints $J_1 := 10$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \quad M = 1$$

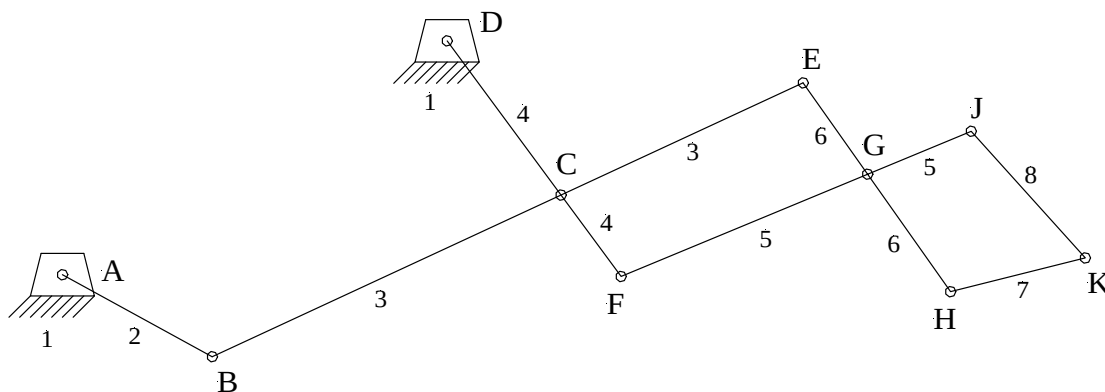
PROBLEM 2-67

Statement: Figure P2-24b shows the mechanism used to extend the foot support on a reclining chair. Number the links, starting with 1. (Hint: Don't forget the "ground" link.) Letter the joints alphabetically, starting with A.

- Using the link numbers, describe each link as binary, ternary, etc.
- Using the joint letters, determine each joint's order.
- Using the joint letters, determine whether each is a half or full joint.

Solution: See Figure P2-24b and Mathcad file P0267.

- Label the link numbers and joint letters for Figure P2-24b.



- Using the link numbers, describe each link as binary, ternary, etc.

<u>Link No.</u>	<u>Link Order</u>
1	Binary
2	Binary
3	Ternary
4	Ternary
5	Ternary
6	Ternary
7	Binary
8	Binary

- Using the joint letters, determine each joint's order and whether each is a half or full joint.

<u>Joint Letter</u>	<u>Joint Order</u>	<u>Half/Full</u>
A	1	Full
B	1	Full
C	1	Full
D	1	Half
E	1	Full
F	1	Full
G	1	Full
H	1	Full
J	1	Full
K	1	Full

PROBLEM 2-68

Statement: Figure P2-24 shows a sixbar linkage. a) Is it a Watt or Stephenson linkage? b) Determine its inversion, i.e. is it a type I, II, or III?

Solution: See Figure P2-24, Figure 2-14, and Mathcad file P0268.

1. From Figure 2-14 we see that the Watt's sixbar has the two ternary links connected with a common joint while the Stephenson's sixbar has the two ternary links connected by binary links. Thus, the sixbar linkage shown is a Watt's sixbar (links 3 and 4, the ternary links, are connected at a common joint). Further, the **linkage shown is a Watt's sixbar inversion I** since neither of the ternary links is grounded.

PROBLEM 3-1

Statement: Define the following examples as path, motion, or function generation cases.

- a. A telescope aiming (star tracking) mechanism
- b. A backhoe bucket control mechanism
- c. A thermostat adjusting mechanism
- d. A computer printing head moving mechanism
- e. An XY plotter pen control mechanism

Solution: See Mathcad file P0301.

- a. **Path generation.** A star follows a 2D path in the sky.
- b. **Motion generation.** To dig a trench, say, the position and orientation of the bucket must be controlled.
- c. **Function generation.** The output is some desired function of the input over some range of the input.
- d. **Path generation.** The head must be at some point on a path.
- e. **Path generation.** The pen follows a straight line from point to point.

PROBLEM 3-2

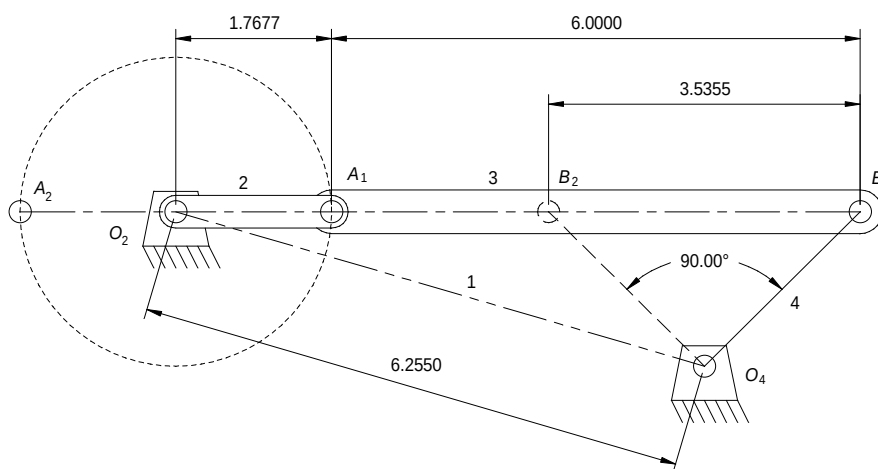
Statement: Design a fourbar Grashof crank-rocker for 90 deg of output rocker motion with no quick return. (See Example 3-1.) Build a cardboard model and determine the toggle positions and the minimum transmission angle.

Given: Output angle $\theta_4 := 90\text{-deg}$

Solution: See Example 3-1 and Mathcad file P0302.

Design choices: Link lengths: Link 3 $L_3 := 6.000$ Link 4 $L_4 := 2.500$

1. Draw the output link O_4B in both extreme positions, B_1 and B_2 , in any convenient location such that the desired angle of motion θ_4 is subtended. In this solution, link 4 is drawn such that the two extreme positions each make an angle of 45 deg to the vertical.
2. Draw the chord B_1B_2 and extend it in any convenient direction. In this solution it was extended to the left.
3. Layout the distance A_1B_1 along extended line B_1B_2 equal to the length of link 3. Mark the point A_1 .
4. Bisect the line segment B_1B_2 and layout the length of that radius from point A_1 along extended line B_1B_2 . Mark the resulting point O_2 and draw a circle of radius O_2A_1 with center at O_2 .
5. Label the other intersection of the circle and extended line B_1B_2 , A_2 .
6. Measure the length of the crank (link 2) as O_2A_1 or O_2A_2 . From the graphical solution, $L_2 := 1.76775$
7. Measure the length of the ground link (link 1) as O_2O_4 . From the graphical solution, $L_1 := 6.2550$



8. Find the Grashof condition.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Condition(L_1, L_2, L_3, L_4) = "Grashof"

PROBLEM 3-3

Statement: Design a fourbar mechanism to give the two positions shown in Figure P3-1 of output rocker motion with no quick-return. (See Example 3-2.) Build a cardboard model and determine the toggle positions and the minimum transmission angle.

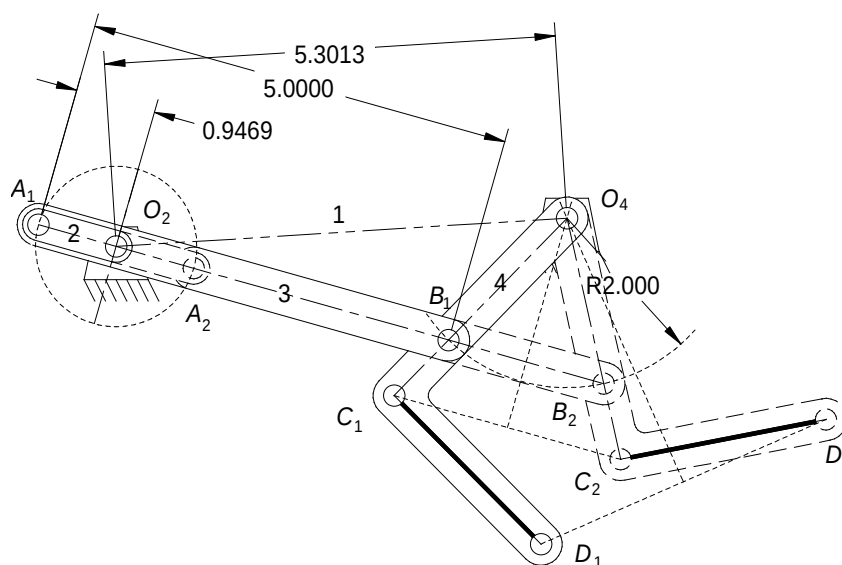
Given: Coordinates of A_1, B_1, A_2 , and B_2 (with respect to A_1):

$$\begin{array}{llll} x_{A1} := 0.00 & x_{B1} := 1.721 & x_{A2} := 2.656 & x_{B2} := 5.065 \\ y_{A1} := 0.00 & y_{B1} := -1.750 & y_{A2} := -0.751 & y_{B2} := -0.281 \end{array}$$

Solution: See Figure P3-1 and Mathcad file P0303.

Design choices: Link length: Link 3 $L_3 := 5.000$ Link 4 $L_4 := 2.000$

- Following the notation used in Example 3-2 and Figure 3-5, change the labels on points A and B in Figure P3-1 to C and D , respectively. Draw the link CD in its two desired positions, C_1D_1 and C_2D_2 , using the given coordinates.
- Draw construction lines from C_1 to C_2 and D_1 to D_2 .
- Bisect line C_1C_2 and line D_1D_2 and extend their perpendicular bisectors to intersect at O_4 .
- Using the length of link 4 (design choice) as a radius, draw an arc about O_4 to intersect both lines O_4C_1 and O_4C_2 . Label the intersections B_1 and B_2 .
- Draw the chord B_1B_2 and extend it in any convenient direction. In this solution it was extended to the left.
- Layout the distance A_1B_1 along extended line B_1B_2 equal to the length of link 3. Mark the point A_1 .
- Bisect the line segment B_1B_2 and layout the length of that radius from point A_1 along extended line B_1B_2 . Mark the resulting point O_2 and draw a circle of radius O_2A_1 with center at O_2 .
- Label the other intersection of the circle and extended line B_1B_2 , A_2 .
- Measure the length of the crank (link 2) as O_2A_1 or O_2A_2 . From the graphical solution, $L_2 := 0.9469$
- Measure the length of the ground link (link 1) as O_2O_4 . From the graphical solution, $L_1 := 5.3013$



11. Find the Grashof condition.

$$\begin{array}{l} \text{Condition}(a,b,c,d) := \left\{ \begin{array}{l} S \leftarrow \min(a,b,c,d) \\ L \leftarrow \max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array} \right. \\ \text{Condition}(L_1, L_2, L_3, L_4) = \text{"Grashof"} \end{array}$$

PROBLEM 3-4

Statement: Design a fourbar mechanism to give the two positions shown in Figure P3-1 of coupler motion. (See Example 3-3.) Build a cardboard model and determine the toggle positions and the minimum transmission angle. Add a driver dyad. (See Example 3-4.)

Given: Position 1 offsets: $x_{A1B1} := 1.721 \cdot in$ $y_{A1B1} := 1.750 \cdot in$

Solution: See figure below for one possible solution. Input file P0304.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-04.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-04.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line AB with construction lines, i.e., lines from A_1 to A_2 and B_1 to B_2 .
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of A_1A_2 was extended downward and the bisector of B_1B_2 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6 , respectively. In the solution below the distances O_4A and O_6B were each selected to be 4.000 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 6.457 in.
4. The fourbar stage is now defined as O_4ABO_6 with link lengths

$$\text{Link 5 (coupler)} \quad L_5 := \sqrt{x_{A1B1}^2 + y_{A1B1}^2} \quad L_5 = 2.454 \cdot in$$

$$\text{Link 4 (input)} \quad L_4 := 4.000 \cdot in \quad \text{Link 6 (output)} \quad L_6 := 4.000 \cdot in$$

$$\text{Ground link 1b} \quad L_{1b} := 6.457 \cdot in$$

5. Select a point on link 4 (O_4A) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it D . (Note that link 4 is now a ternary link with nodes at O_4 , D , and A .) In the solution below the distance O_4D was selected to be 2.000 in.
6. Draw a construction line through D_1D_2 and extend it to the left.
7. Select a point on this line and call it O_2 . In the solution below the distance CD was selected to be 4.000 in.
8. Draw a circle about O_2 with a radius of one-half the length D_1D_2 and label the intersections of the circle with the extended line as C_1 and C_2 . In the solution below the radius was measured as 0.6895 in.
9. The driver fourbar is now defined as O_2CDO_4 with link lengths

$$\text{Link 2 (crank)} \quad L_2 := 0.6895 \cdot in \quad \text{Link 3 (coupler)} \quad L_3 := 4.000 \cdot in$$

$$\text{Link 4a (rocker)} \quad L_{4a} := 2.000 \cdot in \quad \text{Link 1a (ground)} \quad L_{1a} := 4.418 \cdot in$$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$\text{Shortest link} \quad S := L_2 \quad S = 0.6895 \cdot in$$

$$\text{Longest link} \quad L := L_{1a} \quad L = 4.4180 \cdot in$$

$$\text{Other links} \quad P := L_3 \quad P = 4.0000 \cdot in$$

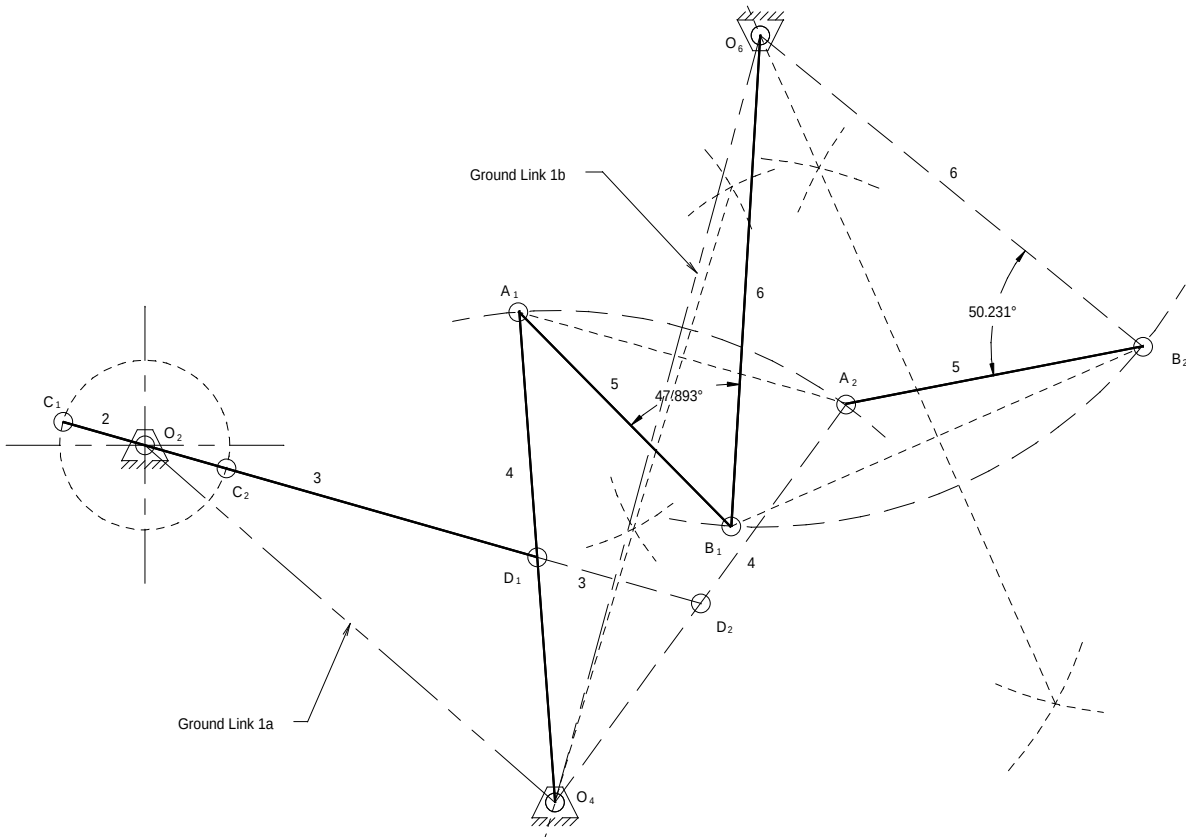
$$Q := L_{4a} \quad Q = 2.0000 \cdot in$$

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Condition(S,L,P,Q) = "Grashof"



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4ABO_6 is non-Grashoff with toggle positions at $\theta_2 = -71.9$ deg and $+71.9$ deg. The minimum transmission angle is 35.5 deg. The fourbar operates between $\theta_2 = +21.106$ deg and -19.297 deg.

PROBLEM 3-5

Statement: Design a fourbar mechanism to give the three positions of coupler motion with no quick return shown in Figure P3-2. (See also Example 3-5.) Ignore the points O_2 and O_4 shown. Build a cardboard model and determine the toggle positions and the minimum transmission angle. Add a driver dyad.

Solution: See Figure P3-2 and Mathcad file P0305.

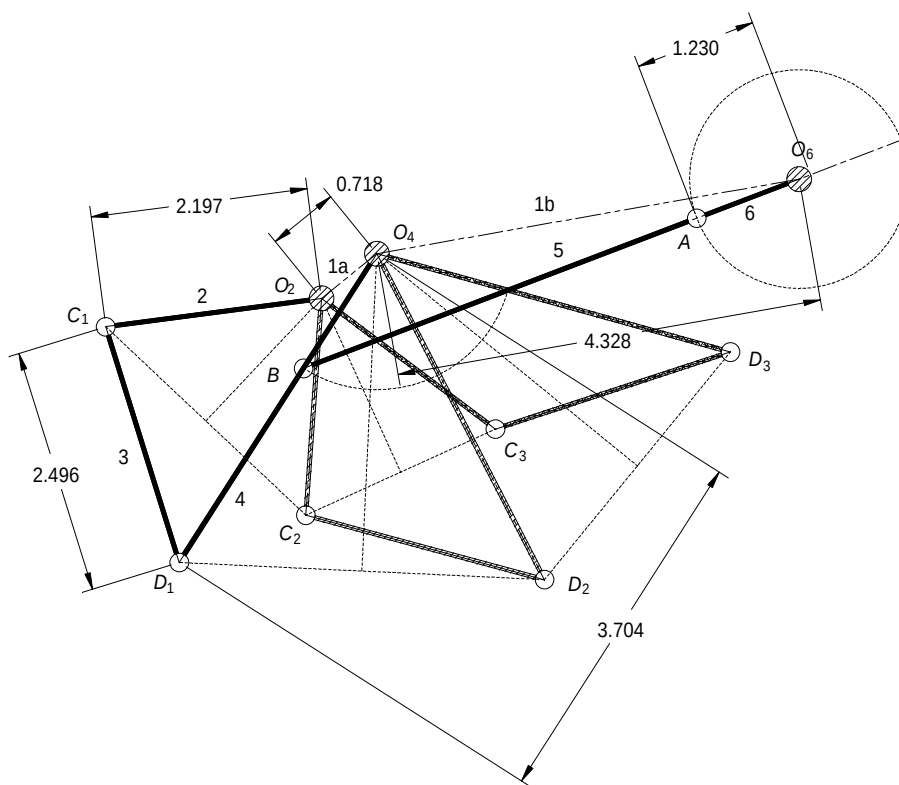
Design choices:

$$\text{Length of link 5: } L_5 := 4.250 \quad \text{Length of link 4b: } L_{4b} := 1.375$$

1. Draw link CD in its three design positions C_1D_1 , C_2D_2 , C_3D_3 in the plane as shown.
2. Draw construction lines from point C_1 to C_2 and from point C_2 to C_3 .
3. Bisect line C_1C_2 and line C_2C_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_2 .
4. Repeat steps 2 and 3 for lines D_1D_2 and D_2D_3 . Label the intersection O_4 .
5. Connect O_2 with C_1 and call it link 2. Connect O_4 with D_1 and call it link 4.
6. Line C_1D_1 is link 3. Line O_2O_4 is link 1 (ground link for the fourbar). The fourbar is now defined as O_2CDO_4 and has link lengths of

$$\text{Ground link 1a } L_{1a} := 0.718 \quad \text{Link 2 } L_2 := 2.197$$

$$\text{Link 3 } L_3 := 2.496 \quad \text{Link 4 } L_4 := 3.704$$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{cases}$$

$$\text{Condition}(L_{1a}, L_2, L_3, L_4) = \text{"Grashof"}$$

8. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 4 is now a ternary link with nodes at O_4 , C , and B .) In the solution above the distance O_4B was selected to be $L_{4b} = 1.375$.
9. Draw a construction line through B_1B_3 and extend it up to the right.
10. Layout the length of link 5 (design choice) along the extended line. Label the other end A .
11. Draw a circle about O_6 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3 . In the solution below the radius was measured as $L_6 := 1.230$.
12. The driver fourbar is now defined as O_4BAO_6 with link lengths

Link 6 (crank) $L_6 = 1.230$

Link 5 (coupler) $L_5 = 4.250$

Link 1b (ground) $L_{1b} := 4.328$

Link 4b (rocker) $L_{4b} = 1.375$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$\text{Condition}(L_6, L_{1b}, L_{4b}, L_5) = \text{"Grashof"}$$

PROBLEM 3-6

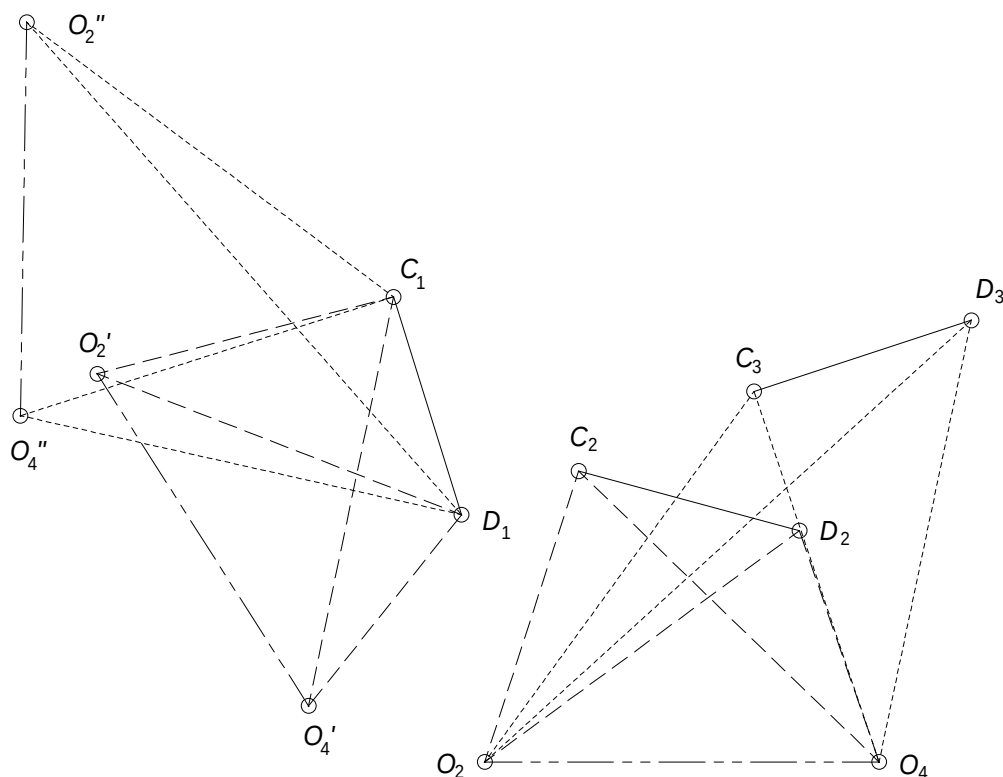
Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-2 using the fixed pivots O_2 and O_4 shown. Build a cardboard model and determine the toggle positions and the minimum transmission angle. Add a driver dyad.

Solution: See Figure P3-2 and Mathcad file P0306.

Design choices:

$$\text{Length of link 5: } L_5 := 5.000 \quad \text{Length of link 2b: } L_{2b} := 2.000$$

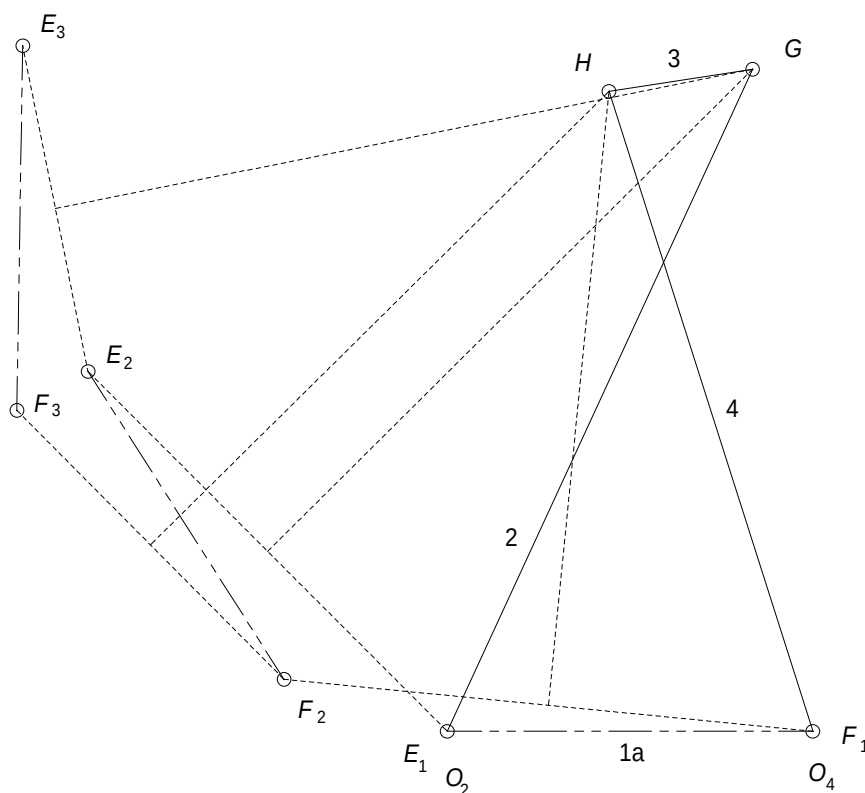
1. Draw link CD in its three design positions C_1D_1 , C_2D_2 , C_3D_3 in the plane as shown.
2. Draw the ground link O_2O_4 in its desired position in the plane with respect to the first coupler position C_1D_1 .
3. Draw construction arcs from point C_2 to O_2 and from point D_2 to O_2 whose radii define the sides of triangle $C_2O_2D_2$. This defines the relationship of the fixed pivot O_2 to the coupler line CD in the second coupler position.
4. Draw construction arcs from point C_2 to O_4 and from point D_2 to O_4 whose radii define the sides of triangle $C_2O_4D_2$. This defines the relationship of the fixed pivot O_4 to the coupler line CD in the second coupler position.
5. Transfer this relationship back to the first coupler position C_1D_1 so that the ground plane position $O_2'O_4'$ bears the same relationship to C_1D_1 as O_2O_4 bore to the second coupler position C_2D_2 .
6. Repeat the process for the third coupler position and transfer the third relative ground link position to the first, or reference, position.
7. The three inverted positions of the ground link that correspond to the three desired coupler positions are labeled O_2O_4 , $O_2'O_4'$, and $O_2''O_4''$ in the first layout below and are renamed E_1F_1 , E_2F_2 , and E_3F_3 , respectively, in the second layout, which is used to find the points G and H .



8. Draw construction lines from point E_1 to E_2 and from point E_2 to E_3 .

9. Bisect line E_1E_2 and line E_2E_3 and extend their perpendicular bisectors until they intersect. Label their intersection G .
10. Repeat steps 2 and 3 for lines F_1F_2 and F_2F_3 . Label the intersection H .
11. Connect E_1 with G and label it link 2. Connect F_1 with H and label it link 4. Reverting, E_1 and F_1 are the original fixed pivots O_2 and O_4 , respectively.
12. Line GH is link 3. Line O_2O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_2GHO_4 and has link lengths of

Ground link 1a	$L_{1a} := 4.303$	Link 2	$L_2 := 8.597$
Link 3	$L_3 := 1.711$	Link 4	$L_4 := 7.921$



13. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof"   if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$Condition(L_{1a}, L_2, L_3, L_4) = \text{"Grashof"}$

The fourbar that will provide the desired motion is now defined as a Grashof double crank in the crossed configuration. It now remains to add the original points C_1 and D_1 to the coupler GH and to define the driving dyad.

14. Select a point on link 2 (O_2G) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 2 is now a ternary link with nodes at O_2 , B , and G .) In the solution below, the distance O_2B was selected to be $L_{2b} = 2.000$.
15. Draw a construction line through B_1B_3 and extend it up to the right.
16. Layout the length of link 5 (design choice) along the extended line. Label the other end A .
17. Draw a circle about O_6 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3 . In the solution below the radius was measured as $L_6 := 0.412$.
18. The driver fourbar is now defined as O_2BAO_6 with link lengths

Link 6 (crank) $L_6 = 0.412$

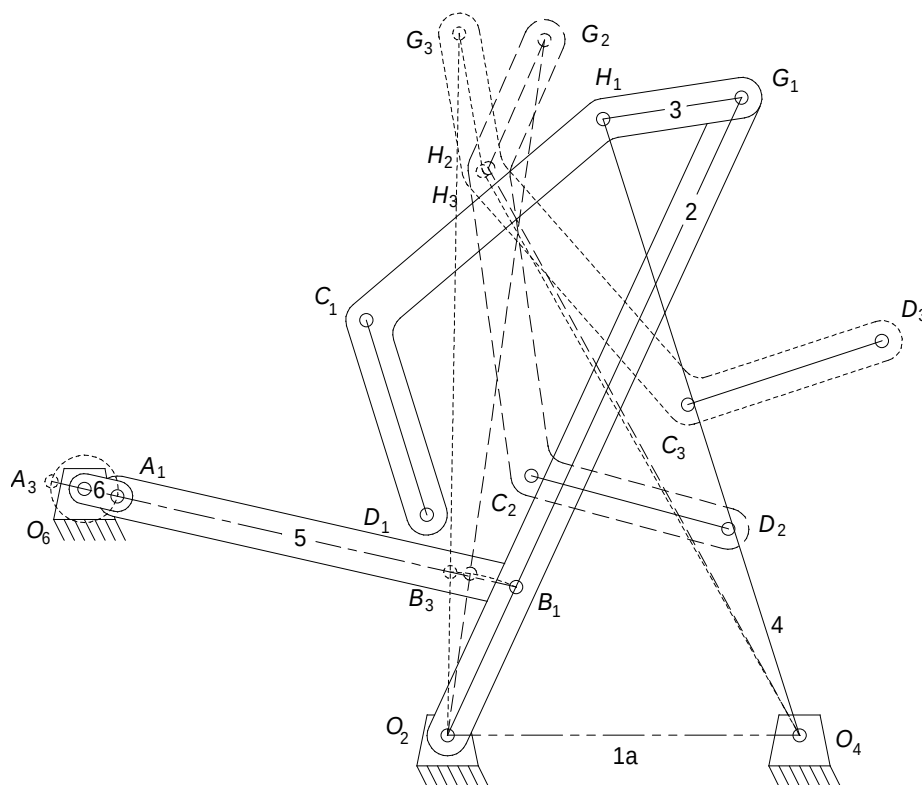
Link 5 (coupler) $L_5 = 5.000$

Link 1b (ground) $L_{1b} := 5.369$

Link 2b (rocker) $L_{2b} = 2.000$

19. Use the link lengths in step 18 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$\text{Condition}(L_6, L_{1b}, L_{2b}, L_5) = \text{"Grashof"}$$



PROBLEM 3-7

Statement: Repeat Problem 3-2 with a quick-return time ratio of 1:1.4. (See Example 3.9). Design a fourbar Grashof crank-rocker for 90 degrees of output rocker motion with a quick-return time ratio of 1:1.4.

Given: Time ratio $T_r := \frac{1}{1.4}$

Solution: See figure below for one possible solution. Also see Mathcad file P0307.

- Determine the crank rotation angles α and β , and the construction angle δ from equations 3.1 and 3.2.

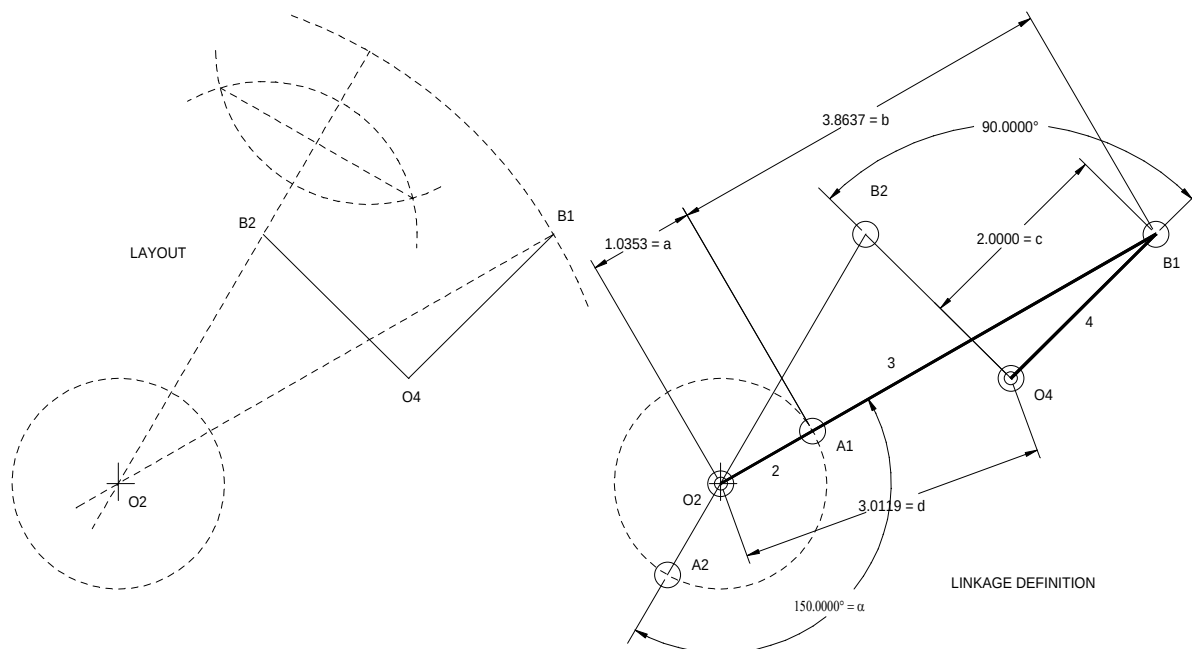
$$T_r = \frac{\alpha}{\beta} \quad \alpha + \beta = 360 \cdot \text{deg}$$

$$\text{Solving for } \beta, \alpha, \text{ and } \delta \quad \beta := \frac{360 \cdot \text{deg}}{1 + T_r} \quad \beta = 210 \text{ deg}$$

$$\alpha := 360 \cdot \text{deg} - \beta \quad \alpha = 150 \text{ deg}$$

$$\delta := \beta - 180 \cdot \text{deg} \quad \delta = 30 \text{ deg}$$

- Start the layout by arbitrarily establishing the point O_4 and from it layoff two lines of equal length, 90 deg apart. Label one B_1 and the other B_2 . In the solution below, each line makes an angle of 45 deg with the horizontal and has a length of 2.000 in.
- Layoff a line through B_1 at an arbitrary angle (but not zero deg). In the solution below, the line is 30 deg to the horizontal.
- Layoff a line through B_2 that makes an angle δ with the line in step 3 (60 deg to the horizontal in this case). The intersection of these two lines establishes the point O_2 .
- From O_2 draw an arc that goes through B_1 . Extend O_2B_2 to meet this arc. Erect a perpendicular bisector to the extended portion of the line and transfer one half of the line to O_2 as the length of the input crank.



6. For this solution, the link lengths are:

Ground link (1) $d := 3.0119 \cdot in$

Crank (2) $a := 1.0353 \cdot in$

Coupler (3) $b := 3.8637 \cdot in$

Rocker (4) $c := 2.000 \cdot in$

PROBLEM 3-8

Statement: Design a sixbar drag link quick-return linkage for a time ratio of 1:2, and output rocker motion of 60 degrees. (See Example 3-10.)

Given: Time ratio $T_r := \frac{1}{2}$

Solution: See figure below for one possible solution. Also see Mathcad file P0308.

1. Determine the crank rotation angles α and β from equation 3.1.

$$T_r = \frac{\alpha}{\beta} \quad \alpha + \beta = 360 \cdot \text{deg}$$

$$\text{Solving for } \beta \text{ and } \alpha \quad \beta := \frac{360 \cdot \text{deg}}{1 + T_r} \quad \beta = 240 \text{ deg}$$

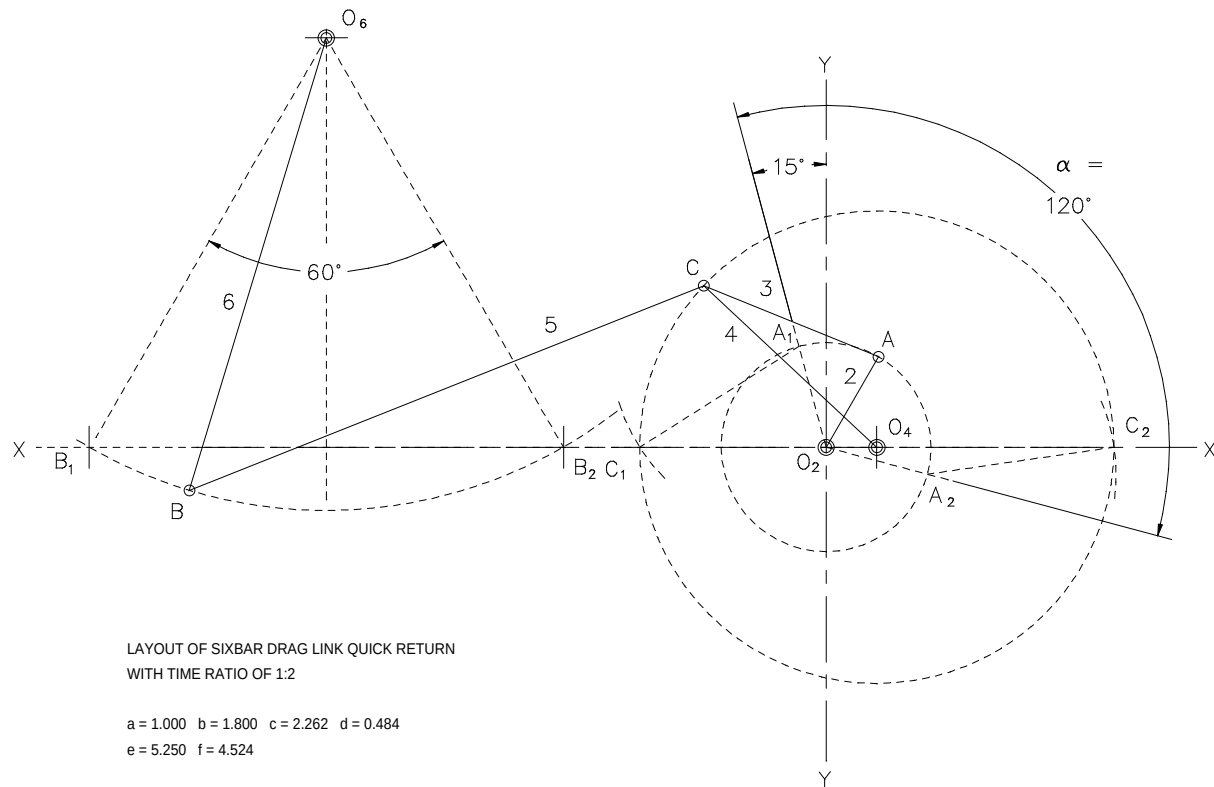
$$\alpha := 360 \cdot \text{deg} - \beta \quad \alpha = 120 \text{ deg}$$

2. Draw a line of centers XX at any convenient location.
3. Choose a crank pivot location O_2 on line XX and draw an axis YY perpendicular to XX through O_2 .
4. Draw a circle of convenient radius O_2A about center O_2 . In the solution below, the length of O_2A is $a := 1.000 \cdot \text{in}$.
5. Lay out angle α with vertex at O_2 , symmetrical about quadrant one.
6. Label points A_1 and A_2 at the intersections of the lines subtending angle α and the circle of radius O_2A .
7. Set the compass to a convenient radius AC long enough to cut XX in two places on either side of O_2 when swung from both A_1 and A_2 . Label the intersections C_1 and C_2 . In the solution below, the length of AC is $b := 1.800 \cdot \text{in}$.
8. The line O_2A is the driver crank, link 2, and the line AC is the coupler, link 3.
9. The distance C_1C_2 is twice the driven (dragged) crank length. Bisect it to locate the fixed pivot O_4 .
10. The line O_2O_4 now defines the ground link. Line O_4C is the driven crank, link 4. In the solution below, O_4C measures $c := 2.262 \cdot \text{in}$ and O_2O_4 measures $d := 0.484 \cdot \text{in}$.
11. Calculate the Grashoff condition. If non-Grashoff, repeat steps 7 through 11 with a shorter radius in step 7.

$$\begin{aligned} \text{Condition}(a, b, c, d) := & \left| \begin{array}{l} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} \quad \text{if } SL < PQ \\ \text{return "Special Grashof"} \quad \text{if } SL = PQ \\ \text{return "non-Grashof"} \quad \text{otherwise} \end{array} \right. \end{aligned}$$

$$\text{Condition}(a, b, c, d) = \text{"Grashof"}$$

12. Invert the method of Example 3-1 to create the output dyad using XX as the chord and O_4C_1 as the driving crank. The points B_1 and B_2 will lie on line XX and be spaced apart a distance that is twice the length of O_4C (link 4). The pivot point O_6 will lie on the perpendicular bisector of B_1B_2 at a distance from XX which subtends the specified output rocker angle, which is 60 degrees in this problem. In the solution below, the length BC was chosen to be $e := 5.250 \cdot \text{in}$.



13. For the design choices made (lengths of links 2, 3 and 5), the length of the output rocker (link 6) was measured as $f := 4.524 \text{ in.}$

PROBLEM 3-9

Statement: Design a crank-shaper quick-return mechanism for a time ratio of 1:3 (Figure 3-14, p. 112).

Given: Time ratio $T_R := \frac{1}{3}$

Solution: See Figure 3-14 and Mathcad file P0309.

Design choices:

Length of link 2 (crank) $L_2 := 1.000$

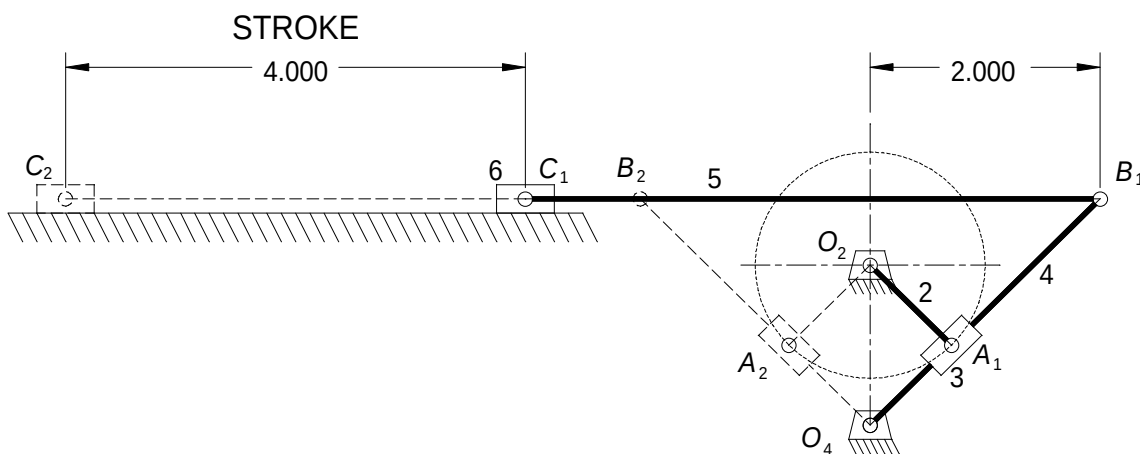
Length of stroke $S := 4.000$

Length of link 5 (coupler) $L_5 := 5.000$

1. Calculate α from equations 3.1.

$$T_R := \frac{\alpha}{\beta} \quad \alpha + \beta := 360 \cdot \text{deg} \quad \alpha := \frac{360 \cdot \text{deg}}{1 + \frac{1}{T_R}} \quad \alpha = 90.000 \text{ deg}$$

2. Draw a vertical line and mark the center of rotation of the crank, O_2 , on it.
3. Layout two construction lines from O_2 , each making an angle $\alpha/2$ to the vertical line through O_2 .
4. Using the chosen crank length (see Design Choices), draw a circle with center at O_2 and radius equal to the crank length. Label the intersections of the circle and the two lines drawn in step 3 as A_1 and A_2 .
5. Draw lines through points A_1 and A_2 that are also tangent to the crank circle (step 2). These two lines will simultaneously intersect the vertical line drawn in step 2. Label the point of intersection as the fixed pivot center O_4 .
6. Draw a vertical construction line, parallel and to the right of O_2O_4 , a distance $S/2$ (one-half of the output stroke length) from the line O_2O_4 .
7. Extend line O_4A_1 until it intersects the construction line drawn in step 6. Label the intersection B_1 .
8. Draw a horizontal construction line from point B_1 , either to the left or right. Using point B_1 as center, draw an arc of radius equal to the length of link 5 (see Design Choices) to intersect the horizontal construction line. Label the intersection as C_1 .
9. Draw the slider blocks at points A_1 and C_1 and finish by drawing the mechanism in its other extreme position.



PROBLEM 3-10

Statement: Find the two cognates of the linkage in Figure 3-17 (p. 116). Draw the Cayley and Roberts diagrams. Check your results with program FOURBAR.

Given: Link lengths:

Ground link $L_1 := 2$

Crank $L_2 := 1$

Coupler $L_3 := 3$

Rocker $L_4 := 3.5$

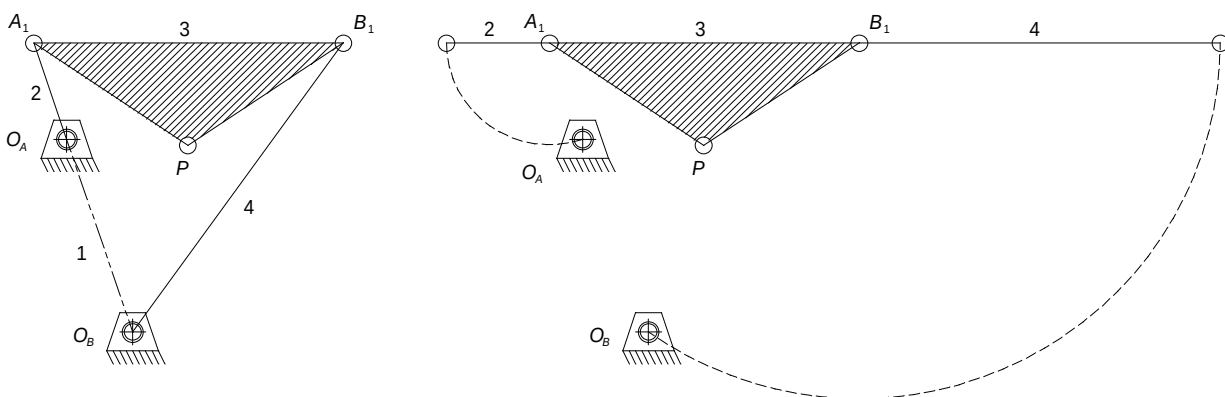
Coupler point data:

$A_1P := 1.800$ $\delta_1 := -34.000 \cdot \text{deg}$

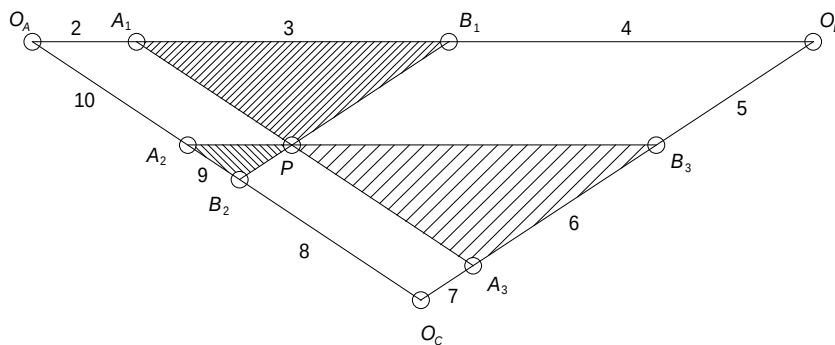
$B_1P := 1.813$ $\gamma_1 := -33.727 \cdot \text{deg}$

Solution: See Figure 3-17 and Mathcad file P0310.

1. Draw the original fourbar linkage, which will be cognate #1, and align links 2 and 4 with the coupler.



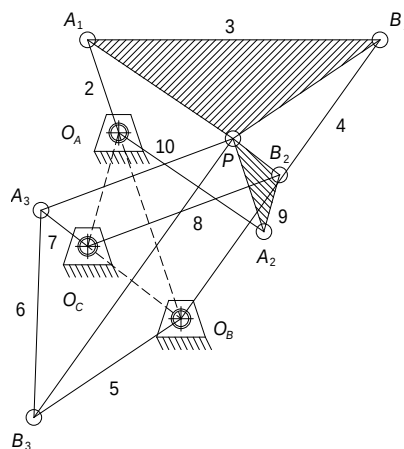
2. Construct lines parallel to all sides of the aligned fourbar linkage to create the Cayley diagram (see Figure 3-24).

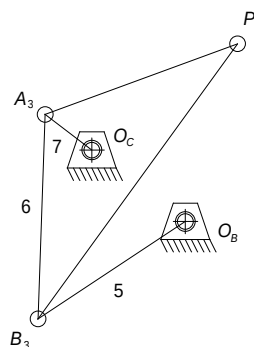


3. Return links 2 and 4 to their fixed pivots O_A and O_B and establish O_C as a fixed pivot by making triangle $O_A O_B O_C$ similar to $A_1 B_1 P$.
4. Separate the three cognates. Point P has the same path motion in each cognate.
5. Calculate the cognate link lengths based on the geometry of the Cayley diagram (Figure 3-24c, p. 114).

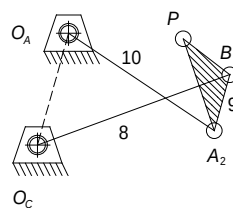
$$L_5 := B_1P \quad L_5 = 1.813$$

$$L_6 := \frac{L_4}{L_3} \cdot B_1P \quad L_6 = 2.115$$





Cognate #3



Cognate #2

$$L_{10} := A_1P \quad L_{10} = 1.800 \quad L_9 := \frac{L_2}{L_3} \cdot A_1P \quad L_9 = 0.600$$

$$L_7 := L_9 \cdot \frac{B_1P}{A_1P} \quad L_7 = 0.604$$

$$L_8 := L_6 \cdot \frac{A_1P}{B_1P} \quad L_8 = 2.100$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

$$L_{1BC} := \frac{L_1}{L_3} \cdot B_1P \quad L_{1BC} = 1.209$$

$$L_{1AC} := \frac{L_1}{L_3} \cdot A_1P \quad L_{1AC} = 1.200$$

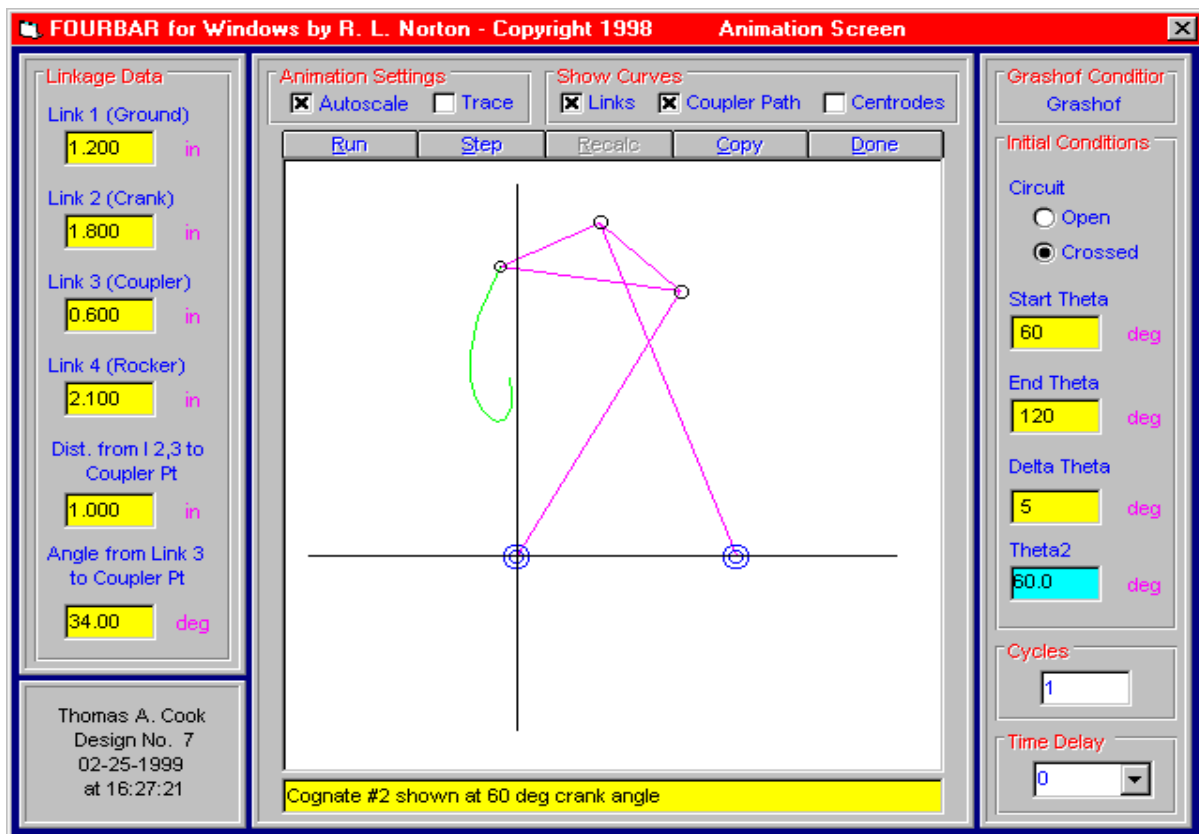
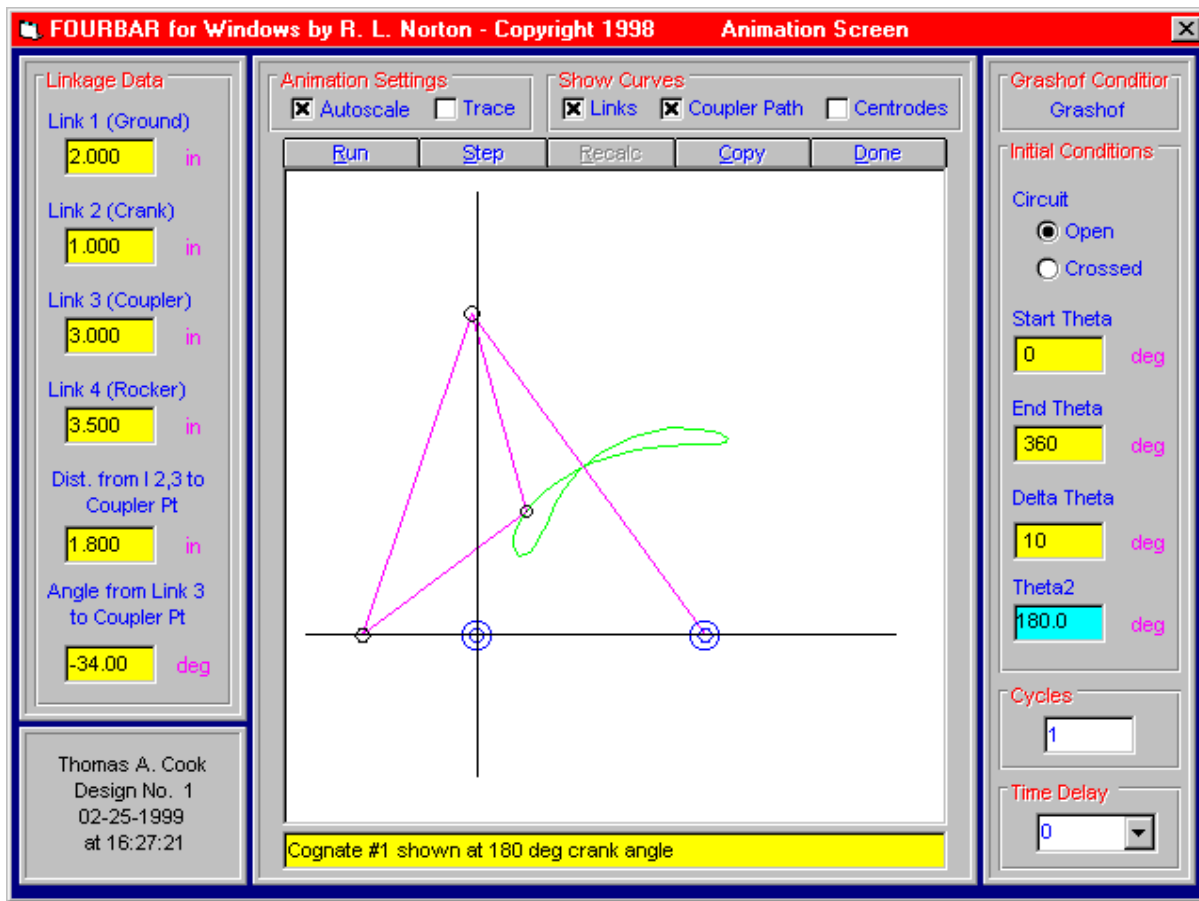
Calculate the coupler point data for cognates #2 and #3

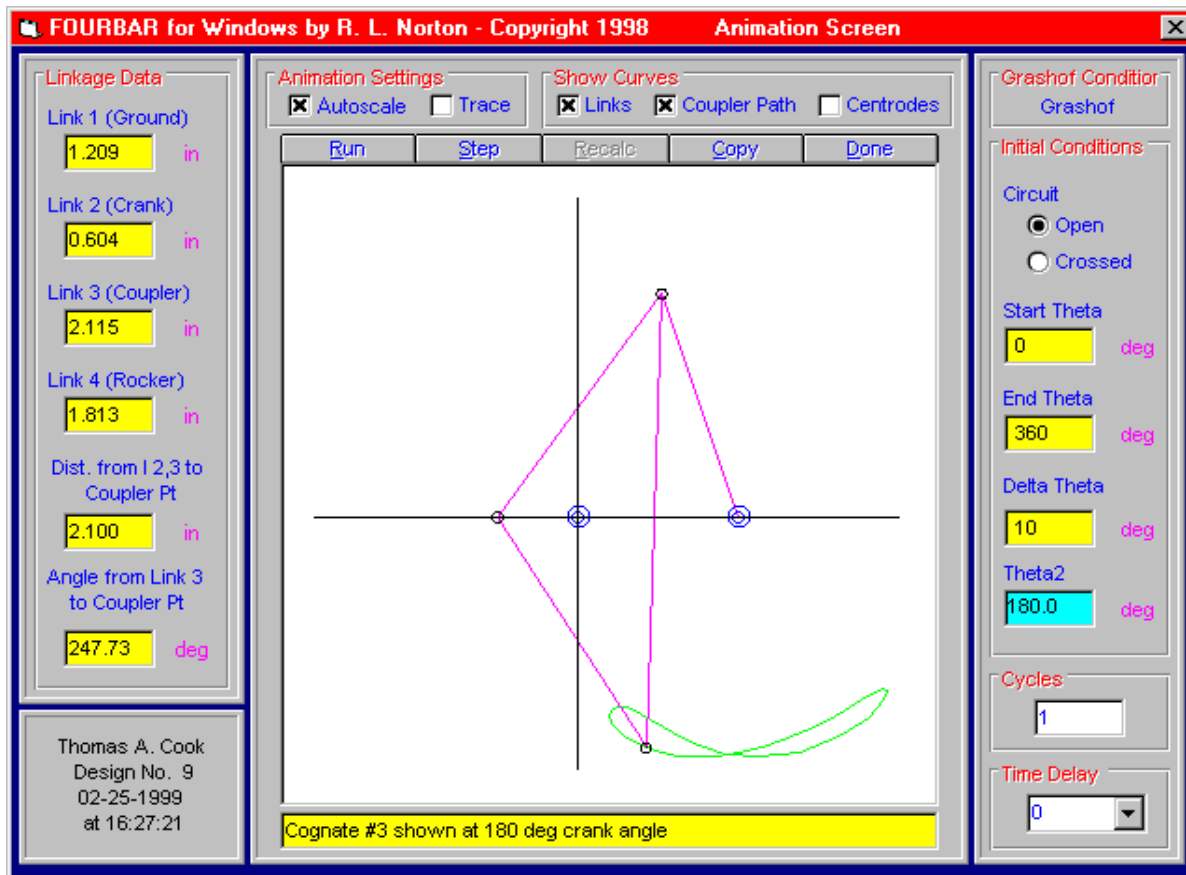
$$\begin{aligned} A_3P &:= L_8 & A_3P &= 2.100 & A_2P &:= L_2 & A_2P &= 1.000 \\ \delta_3 &:= 180 \cdot \text{deg} - (\delta_1 + \gamma_1) & \delta_3 &= 247.727 \text{ deg} & \delta_2 &:= -\delta_1 & \delta_2 &= 34.000 \text{ deg} \end{aligned}$$

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.000$	$L_{1AC} = 1.200$	$L_{1BC} = 1.209$
Crank length	$L_2 = 1.000$	$L_{10} = 1.800$	$L_7 = 0.604$
Coupler length	$L_3 = 3.000$	$L_9 = 0.600$	$L_6 = 2.115$
Rocker length	$L_4 = 3.500$	$L_8 = 2.100$	$L_5 = 1.813$
Coupler point	$A_1P = 1.800$	$A_2P = 1.000$	$A_3P = 2.100$
Coupler angle	$\delta_1 = -34.000 \text{ deg}$	$\delta_2 = 34.000 \text{ deg}$	$\delta_3 = 247.727 \text{ deg}$

6. Verify that the three cognates yield the same coupler curve by entering the original link lengths in program FOURBAR and letting it calculate the cognates.





Note that cognate #2 is a Grashof double rocker and, therefore, cannot trace out the entire coupler curve.

PROBLEM 3-11

Statement: Find the three equivalent geared fivebar linkages for the three fourbar cognates in Figure 3-25a (p. 125). Check your results by comparing the coupler curves with programs FOURBAR and FIVEBAR.

Given: Link lengths: Coupler point data:
 Ground link $L_1 := 39.5$ Crank $L_2 := 15.5$ $A1P := 26.0$ $\delta_1 := 63.000 \cdot \text{deg}$
 Coupler $L_3 := 14.0$ Rocker $L_4 := 20.0$

Solution: See Figure 3-25a and Mathcad file P0311.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$B1P := \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} \quad B1P = 23.270$$

$$\gamma_1 := \arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) \quad \gamma_1 = 84.5843 \text{ deg}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \quad L_5 = 23.270 \quad L_6 := \frac{L_4}{L_3} \cdot B1P \quad L_6 = 33.243$$

$$L_{10} := A1P \quad L_{10} = 26.000 \quad L_9 := \frac{L_2}{L_3} \cdot A1P \quad L_9 = 28.786$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \quad L_7 = 25.763 \quad L_8 := L_6 \cdot \frac{A1P}{B1P} \quad L_8 = 37.143$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_4 \quad A3P = 20.000 \quad A2P := L_2 \quad A2P = 15.500$$

$$\delta_3 := \gamma_1 \quad \delta_3 = 84.584 \text{ deg} \quad \delta_2 := -\delta_1 \quad \delta_2 = -63.000 \text{ deg}$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

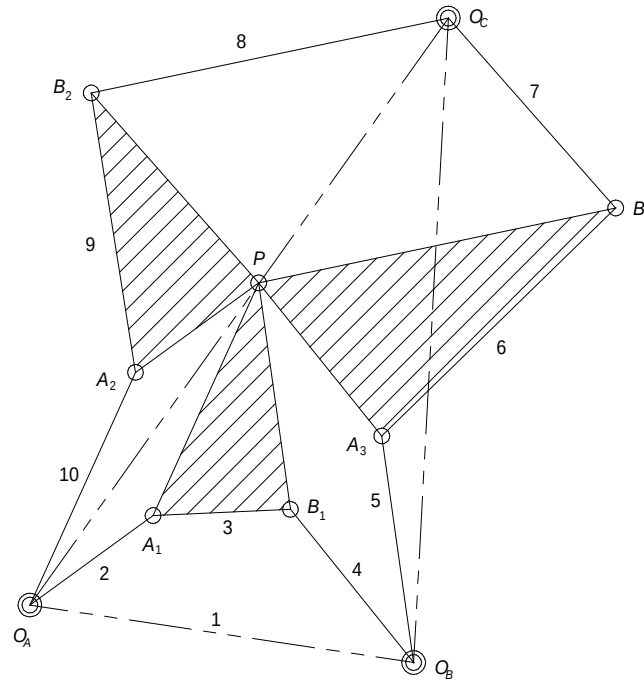
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \quad L_{1BC} = 65.6548 \quad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \quad L_{1AC} = 73.3571$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

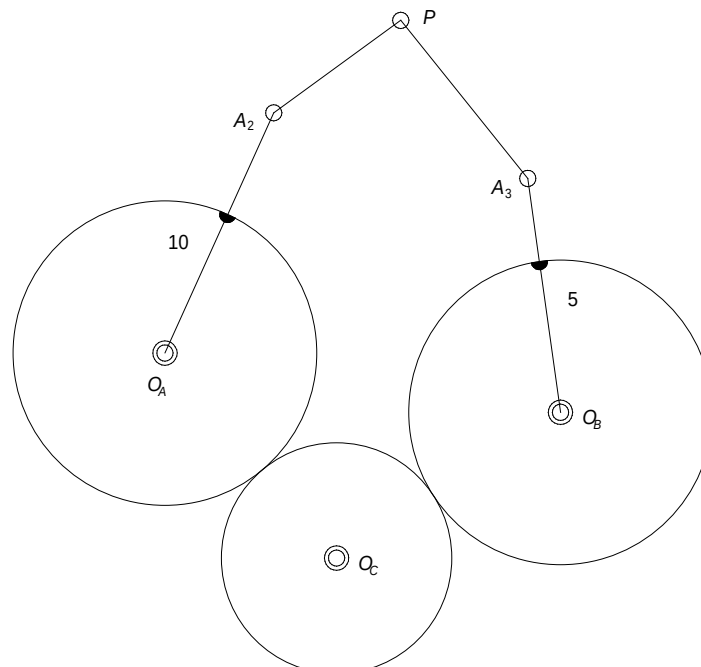
SUMMARY OF COGNATE SPECIFICATIONS:

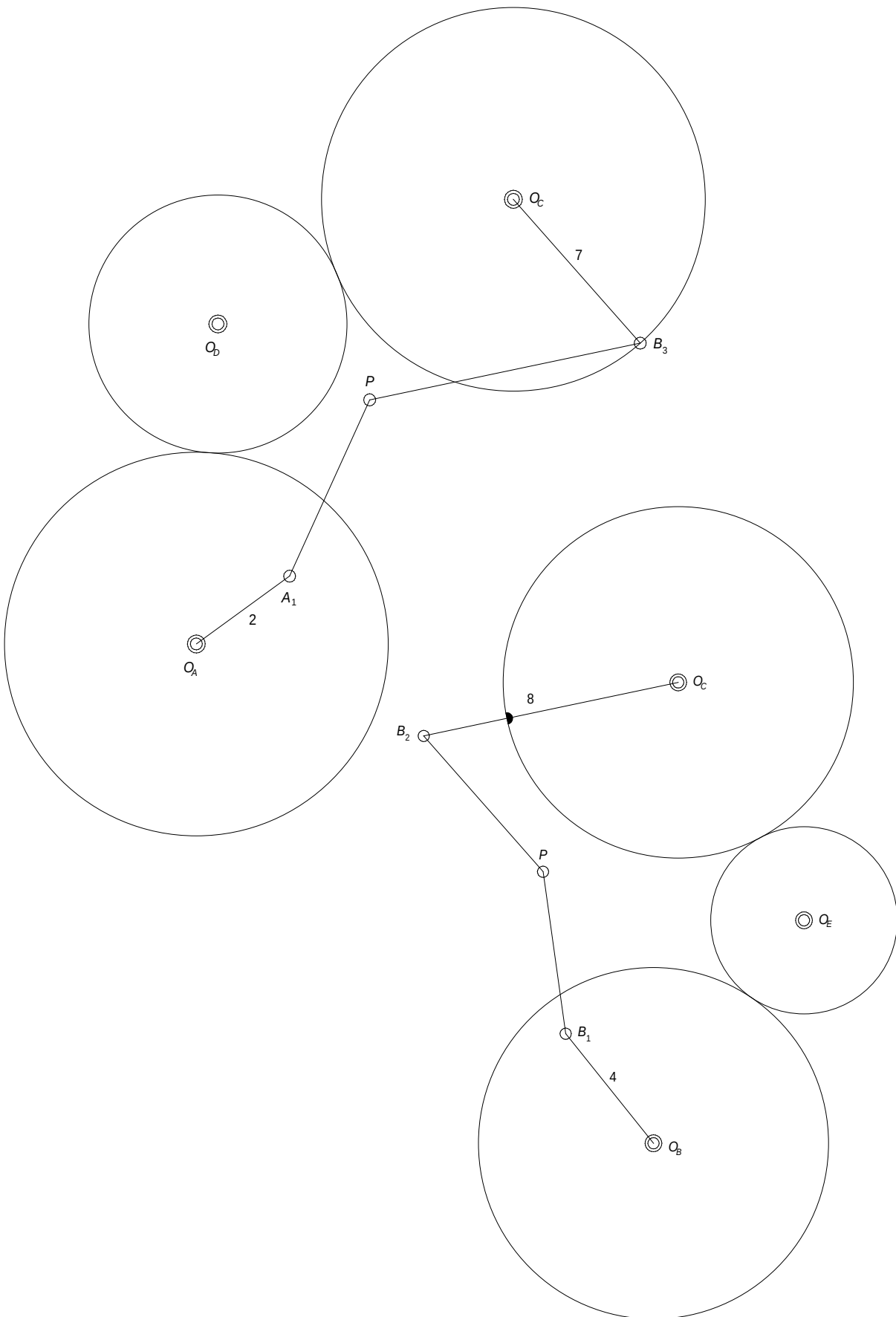
	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 39.500$	$L_{1AC} = 73.357$	$L_{1BC} = 65.655$
Crank length	$L_2 = 15.500$	$L_{10} = 26.000$	$L_7 = 25.763$
Coupler length	$L_3 = 14.000$	$L_9 = 28.786$	$L_6 = 33.243$

Rocker length	$L_4 = 20.000$	$L_8 = 37.143$	$L_5 = 23.270$
Coupler point	$A_1P = 26.000$	$A_2P = 15.500$	$A_3P = 20.000$
Coupler angle	$\delta_1 = 63.000 \text{ deg}$	$\delta_2 = -63.000 \text{ deg}$	$\delta_3 = 84.584 \text{ deg}$



4. The three geared fivebar cognates can be seen in the Roberts diagram. They are: $O_A A_2 P A_3 O_B$, $O_A A_1 P B_3 O_C$, and $O_B B_1 P B_2 O_C$. They are shown individually below with their associated gears.

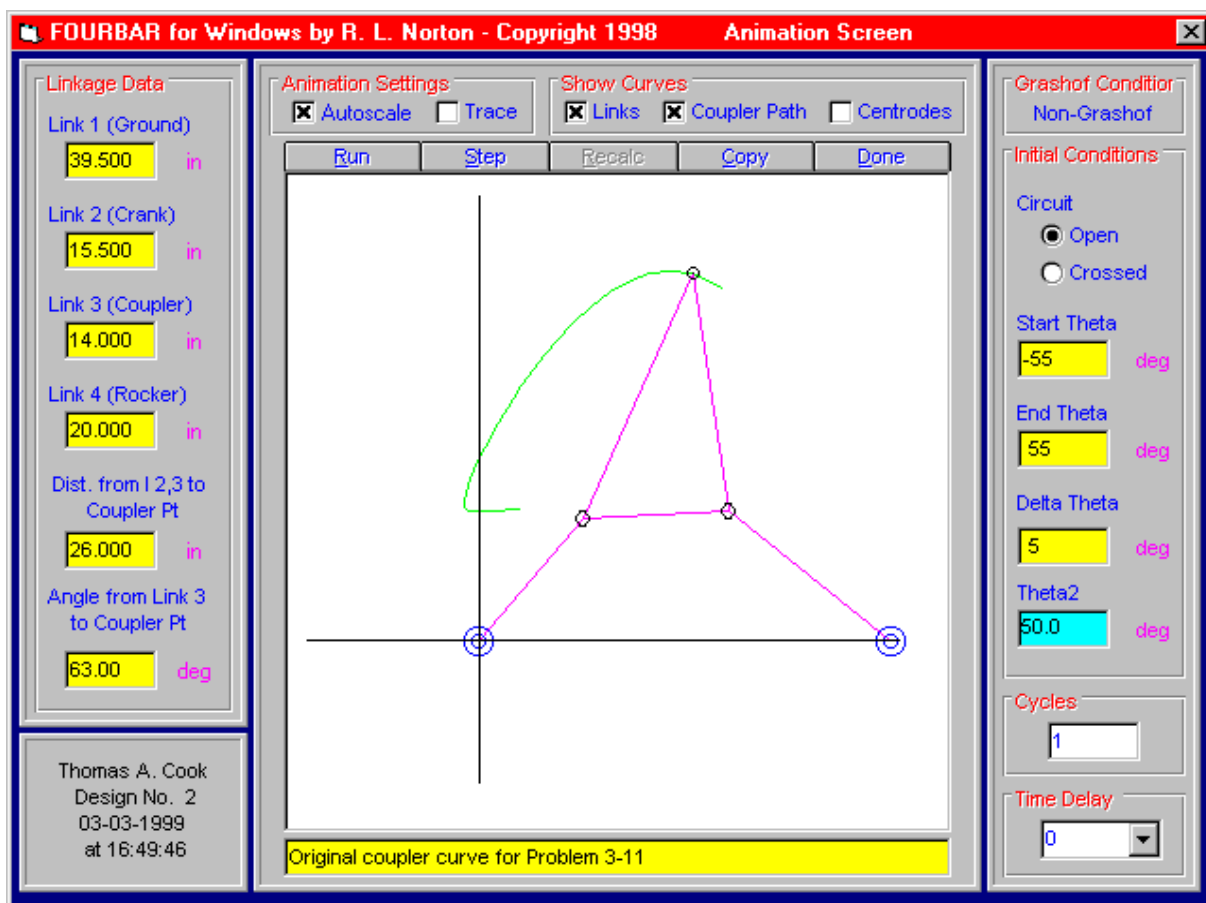




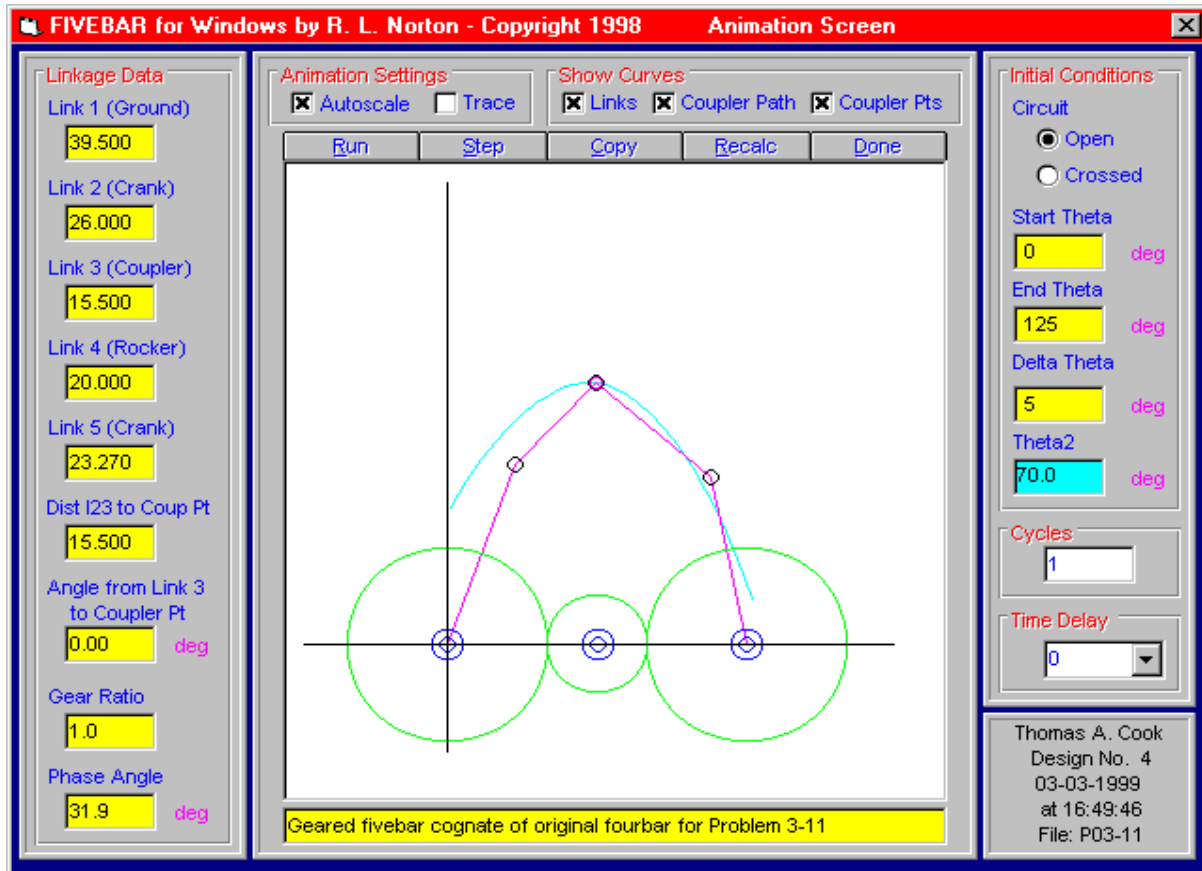
SUMMARY OF GEARED FIVEBAR COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 39.500$	$L_{1AC} = 73.357$	$L_{1BC} = 65.655$
Crank length	$L_{10} = 26.000$	$L_2 = 15.500$	$L_4 = 20.000$
Coupler length	$A2P = 15.500$	$A1P = 26.000$	$L_5 = 23.270$
Rocker length	$A3P = 20.000$	$L_8 = 37.143$	$L_7 = 25.763$
Crank length	$L_5 = 23.270$	$L_7 = 25.763$	$L_8 = 37.143$
Coupler point	$A2P = 15.500$	$A1P = 26.000$	$B1P = 23.270$
Coupler angle	$\delta_1 := 0.00 \cdot \text{deg}$	$\delta_2 := 0.00 \cdot \text{deg}$	$\delta_3 := 0.00 \cdot \text{deg}$

5. Enter the cognate #1 specifications into program FOURBAR to get a trace of the coupler path.



6. Enter the geared fivebar cognate #1 specifications into program FIVEBAR to get a trace of the coupler path for the geared fivebar (see next page).



PROBLEM 3-12

Statement: Design a sixbar, single-dwell linkage for a dwell of 90 deg of crank motion, with an output rocker motion of 45 deg.

Given: Crank dwell period: 90 deg.
Output rocker motion: 45 deg.

Solution: See Figures 3-20, 3-21, and Mathcad file P0312.

Design choices:

Ground link ratio, $L_1/L_2 = 2.0$: $GLR := 2.0$

Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.5$: $CLR := 2.5$

Coupler angle, $\gamma := 72 \cdot \text{deg}$

Crank length, $L_2 := 2.000$

1. For the given design choices, determine the remaining link lengths and coupler point specification.

$$\text{Coupler link (3) length} \quad L_3 := CLR \cdot L_2 \quad L_3 = 5.000$$

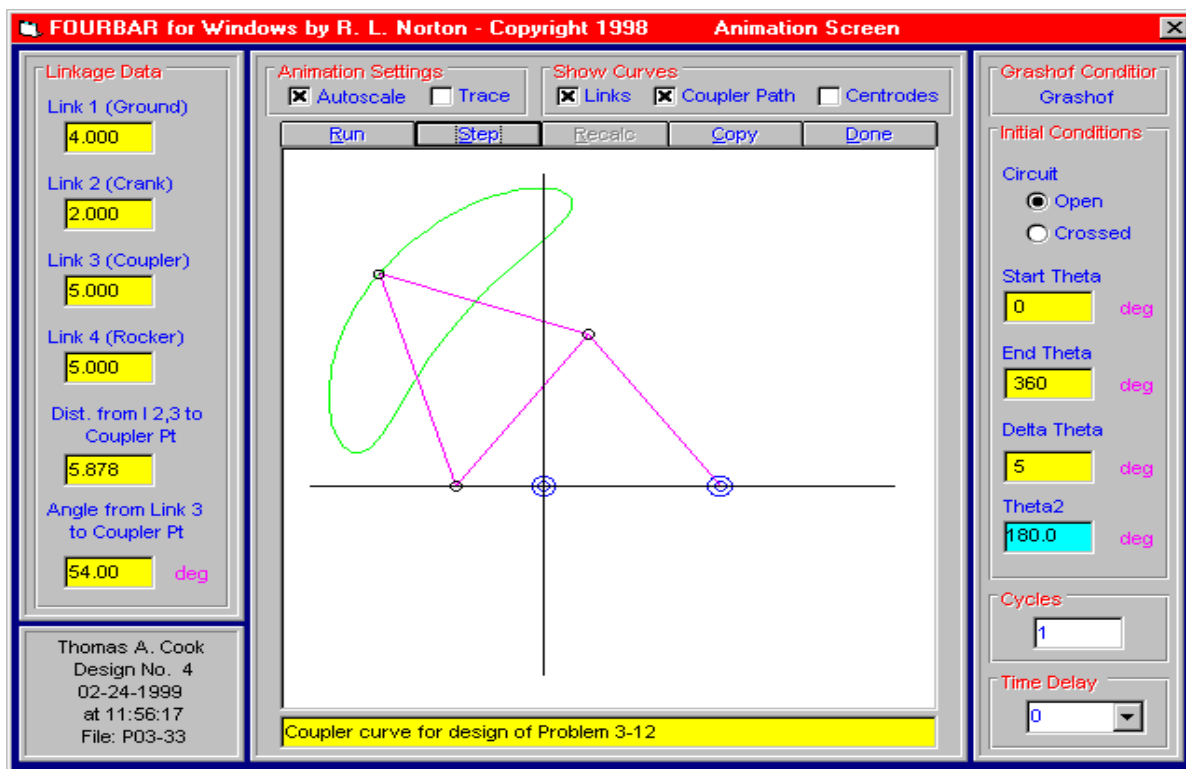
$$\text{Rocker link (4) length} \quad L_4 := CLR \cdot L_2 \quad L_4 = 5.000$$

$$\text{Ground link (1) length} \quad L_1 := GLR \cdot L_2 \quad L_1 = 4.000$$

$$\text{Angle PAB} \quad \delta := \frac{180 \cdot \text{deg} - \gamma}{2} \quad \delta = 54.000 \text{ deg}$$

$$\text{Length AP on coupler} \quad AP := 2 \cdot L_3 \cdot \cos(\delta) \quad AP = 5.878$$

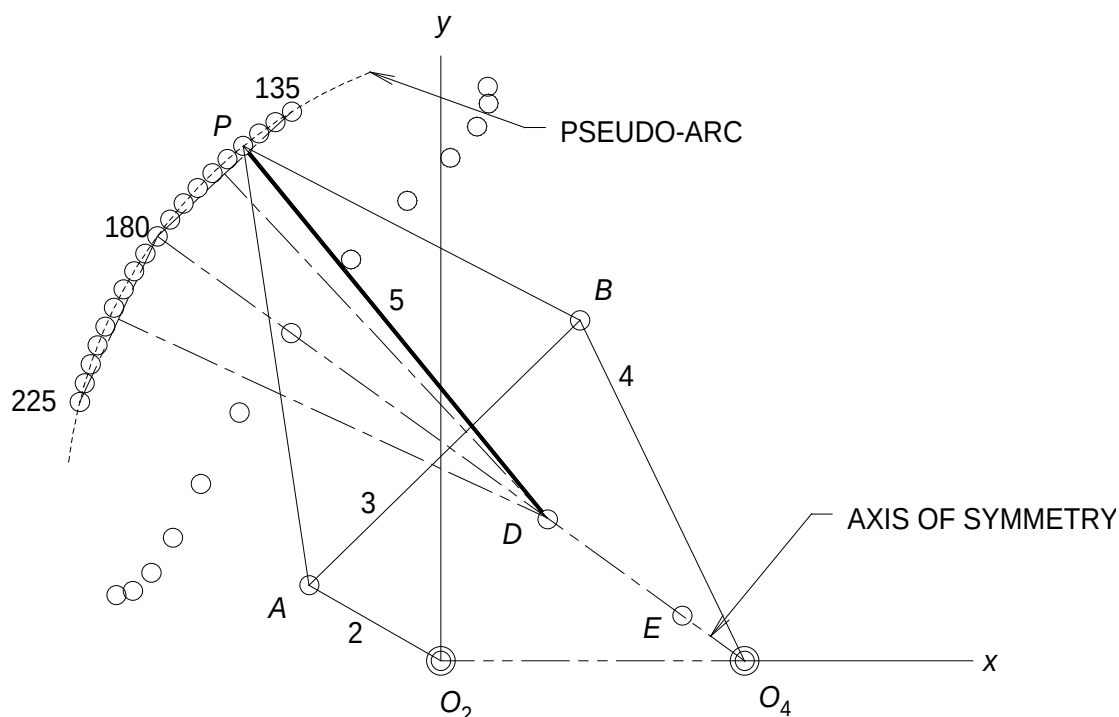
2. Enter the above data into program FOURBAR, plot the coupler curve, and determine the coordinates of the coupler curve in the selected range of crank motion, which in this case will be from 135 to 225 deg..



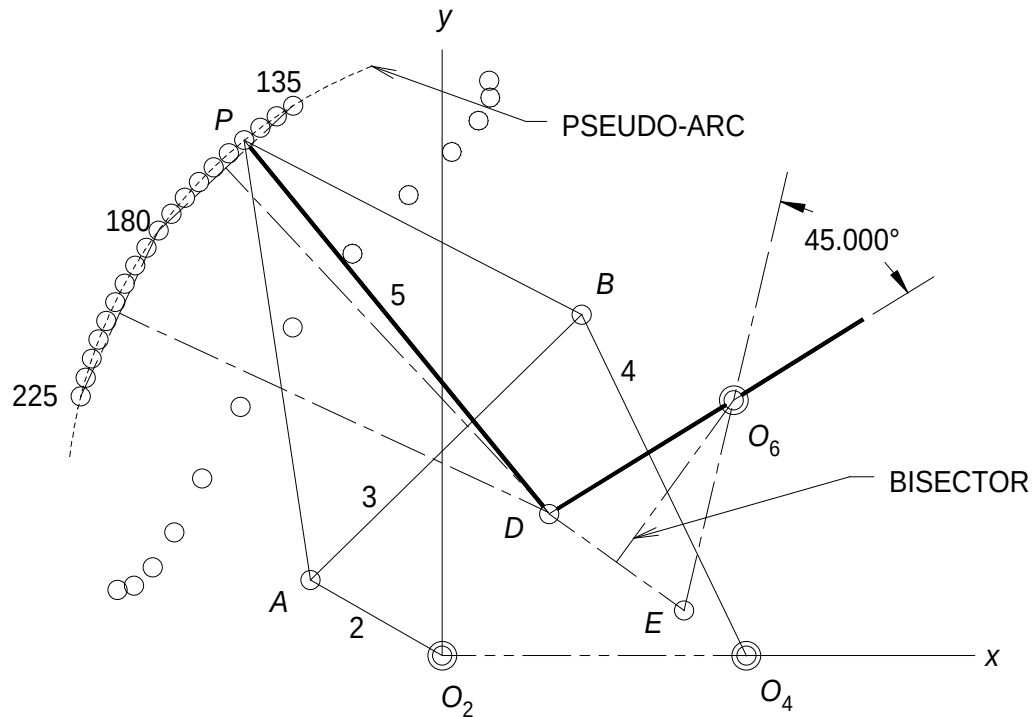
4. The position of the end of link 5 at point D will remain nearly stationary while the crank moves from 135 to 225° deg. As the crank motion causes the coupler point to move around the coupler curve there will be another extreme position of the end of link 5 that was originally at D . Since a symmetrical linkage was chosen, the other extreme position will be located along a line through the axis of symmetry (see Figure 3-20) a distance equal to the length of link 5 measured from the point where the axis of symmetry intersects the coupler curve near the 0° deg coupler point. Establish this point and label it E .

FOURBAR for Windows File P03-12.DAT

Angle Step Deg	Coupler Pt X	Coupler Pt Y	Coupler Pt Mag	Coupler Pt Ang
300	-4.271	0.869	4.359	168.495
310	-4.054	0.926	4.158	167.133
320	-3.811	1.165	3.985	162.998
330	-3.526	1.628	3.883	155.215
340	-3.159	2.343	3.933	143.437
350	-2.651	3.286	4.222	128.892
0	-1.968	4.336	4.762	114.414
10	-1.181	5.310	5.440	102.534
20	-0.441	6.085	6.101	94.142
30	0.126	6.654	6.656	88.914
40	0.478	7.068	7.085	86.129
50	0.631	7.373	7.400	85.111
60	0.617	7.598	7.623	85.354



5. The line segment DE represents the maximum displacement that a link of the length equal to link 5, attached at P , will reach along the axis of symmetry. Construct a perpendicular bisector of the line segment DE and extend it to the right (or left, which ever is convenient). Locate fixed pivot O_6 on the bisector of DE such that the lines O_6D and O_6E subtend the desired output angle, in this case 45° . Draw link 6 from D through O_6 and extend it to any convenient length. This is the output link that will dwell during the specified motion of the crank. See next page for the completed layout and further linkage specifications.



SUMMARY OF LINKAGE SPECIFICATIONS

Original fourbar:

Ground link	$L_1 = 4.000$	
Crank	$L_2 = 2.000$	
Coupler	$L_3 = 5.000$	
Rocker	$L_4 = 5.000$	
Coupler point	$AP = 5.878$	$\delta = 54.000 \text{ deg}$

Added dyad:

Coupler	$L_5 := 6.363$	
Output	$L_6 := 2.855$	
Pivot O_6	$x := 3.833$	$y := 3.375$

PROBLEM 3-13

Statement: Design a sixbar double-dwell linkage for a dwell of 90 deg of crank motion, with an output of rocker motion of 60 deg, followed by a second dwell of about 60 deg of crank motion.

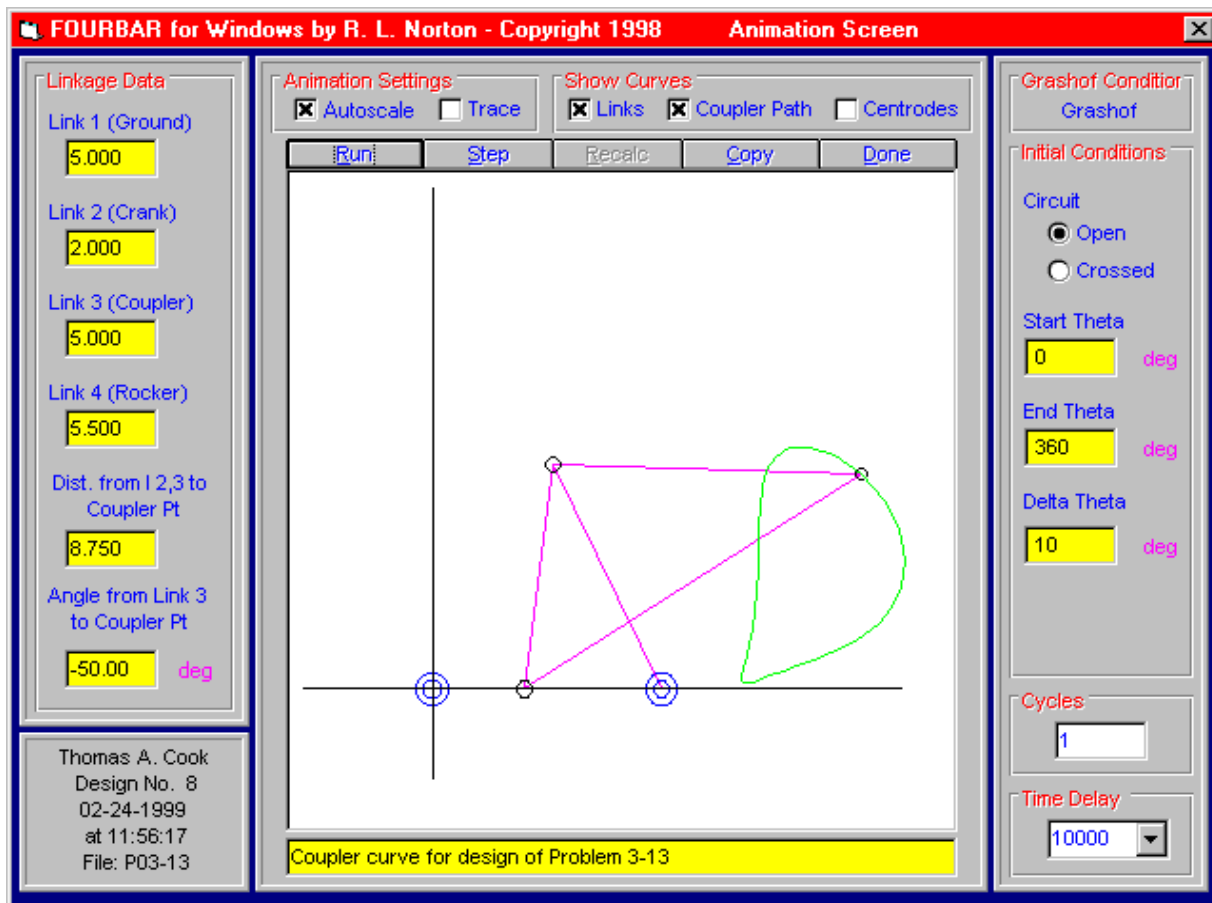
Given: Initial crank dwell period: 90 deg
 Final crank dwell period: 60 deg (approx.)
 Output rocker motion between dwells: 60 deg

Solution: See Mathcad file P0313.

Design choices:

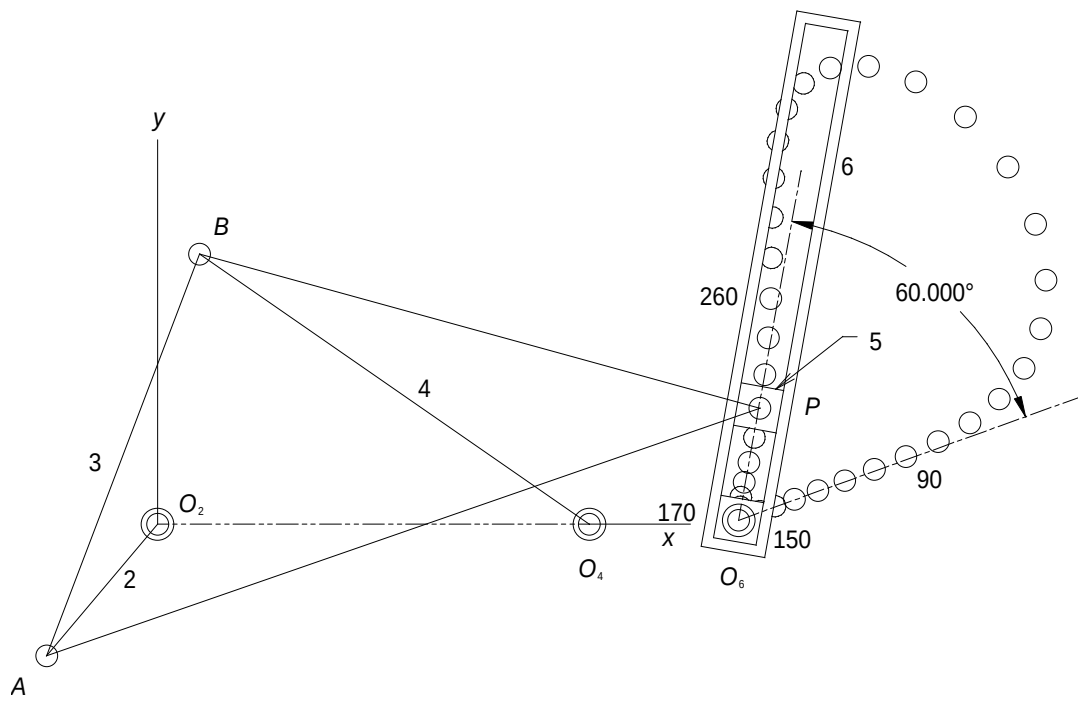
Ground link length	$L_1 := 5.000$	Crank length	$L_2 := 2.000$
Coupler link length	$L_3 := 5.000$	Rocker length	$L_4 := 5.500$
Coupler point data:	$AP := 8.750$	$\delta := -50 \cdot \text{deg}$	

1. In the absence of a linkage atlas it is difficult to find a coupler curve that meets the specifications. One approach is to start with a symmetrical linkage, using the data in Figure 3-21. Then, using program FOURBAR and by trial-and-error, adjust the link lengths and coupler point data until a satisfactory coupler curve is found. The link lengths and coupler point data given above were found this way. The resulting coupler curve is shown below and a printout of the coupler curve coordinates taken from FOURBAR is also printed below.



FOURBAR for Windows		File		P03-13.DAT	
Angle Step Deg	Cpler Pt X	Cpler Pt Y	Cpler Pt Mag	Cpler Pt Ang	
0.000	9.353	4.742	10.487	26.886	
10.000	9.846	4.159	10.688	22.900	
20.000	10.167	3.491	10.750	18.951	
30.000	10.286	2.840	10.671	15.437	
40.000	10.226	2.274	10.476	12.537	
50.000	10.031	1.815	10.194	10.257	
60.000	9.746	1.457	9.854	8.503	
70.000	9.406	1.180	9.480	7.152	
80.000	9.039	0.963	9.090	6.081	
90.000	8.665	0.787	8.701	5.187	
100.000	8.301	0.637	8.325	4.391	
110.000	7.958	0.507	7.974	3.644	
120.000	7.647	0.391	7.657	2.928	
130.000	7.376	0.291	7.382	2.256	
140.000	7.151	0.209	7.154	1.671	
150.000	6.977	0.151	6.978	1.242	
160.000	6.853	0.126	6.854	1.051	
170.000	6.778	0.140	6.779	1.182	
180.000	6.748	0.201	6.751	1.708	
190.000	6.755	0.316	6.763	2.678	
200.000	6.792	0.488	6.809	4.110	
210.000	6.847	0.719	6.885	5.996	
220.000	6.912	1.008	6.985	8.300	
230.000	6.976	1.351	7.105	10.963	
240.000	7.031	1.741	7.243	13.911	
250.000	7.073	2.170	7.398	17.057	
260.000	7.099	2.626	7.569	20.302	
270.000	7.112	3.098	7.757	23.536	
280.000	7.120	3.570	7.965	26.632	
290.000	7.137	4.030	8.196	29.448	
300.000	7.184	4.458	8.455	31.819	
310.000	7.288	4.834	8.746	33.555	
320.000	7.481	5.131	9.072	34.446	
330.000	7.792	5.312	9.430	34.286	
340.000	8.233	5.332	9.809	32.931	
350.000	8.779	5.147	10.177	30.384	
360.000	9.353	4.742	10.487	26.886	

- Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Fit tangent lines to the nearly straight portions of the curve. Label their intersection O_6 . The coordinates of O_6 are (6.729, 0.046).
- Design link 6 to lie along these straight tangents, pivoted at O_6 . Provide a slot in link 6 to accommodate slider block 5, which pivots on the coupler point P . (See next page).
- The beginning and ending crank angles for the dwell portions of the motion are indicated on the layout and in the table above by boldface entries.



PROBLEM 3-14

Statement: Figure P3-3 shows a treadle-operated grinding wheel driven by a fourbar linkage. Make a cardboard model of the linkage to any convenient scale. Determine its minimum transmission angles. Comment on its operation. Will it work? If so, explain how it does.

Given: Link lengths: Link 2 $L_2 := 0.60 \cdot m$ Link 3 $L_3 := 0.75 \cdot m$
 Link 4 $L_4 := 0.13 \cdot m$ Link 1 $L_1 := 0.90 \cdot m$

Grashof condition function:

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Solution: See Mathcad file P0314.

1. Determine the Grashof condition of the mechanism from inequality 2.8 and its Barker classification from Table 2-4.

Grashof condition: $Condition(L_1, L_2, L_3, L_4) = \text{"Grashof"}$

Barker classification: Class I-4, Grashof rocker-rocker-crank, GRRC, since the shortest link is the output link.

2. As a Grashof rocker-crank, the minimum transmission angle will be 0 deg, twice per revolution of the output (link 4) crank.
3. Despite having transmission angles of 0 deg twice per revolution, the mechanism will work. That is, one will be able to drive the grinding wheel from the treadle (link 2). The reason is that the grinding wheel will act as a flywheel and will carry the linkage through the periods when the transmission angle is low. Typically, the operator will start the motion by rotating the wheel by hand.

PROBLEM 3-15

Statement: Figure P3-4 shows a non-Grashof fourbar linkage that is driven from link O_2A . All dimensions are in centimeters (cm).

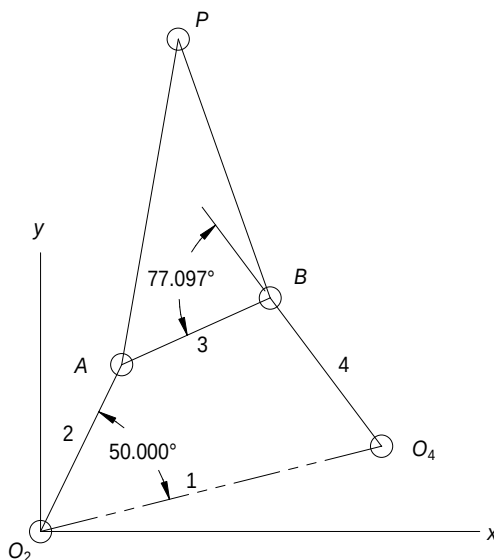
- Find the transmission angle at the position shown.
- Find the toggle positions in terms of angle AO_2O_4 .
- Find the maximum and minimum transmission angles over its range of motion.
- Draw the coupler curve of point P over its range of motion.

Given: Link lengths:

Link 1 (ground)	$L_1 := 95\text{ mm}$	Link 2 (driver)	$L_2 := 50\text{ mm}$
Link 3 (coupler)	$L_3 := 44\text{ mm}$	Link 4 (driven)	$L_4 := 50\text{ mm}$

Solution: See Figure P3-4 and Mathcad file P0315.

- To find the transmission angle at the position shown, draw the linkage to scale in the position shown and measure the transmission angle ABO_4 .



The measured transmission angle at the position shown is 77.097 deg.

- The toggle positions will be symmetric with respect to the O_2O_4 axis and will occur when links 3 and 4 are colinear. Use the law of cosines to calculate the angle of link 2 when links 3 and 4 are in toggle.

$$(L_3 + L_4)^2 := L_1^2 + L_2^2 - 2 \cdot L_1 \cdot L_2 \cdot \cos(\theta_2)$$

where θ_2 is the angle AO_2O_4 . Solving for θ_2 ,

$$\theta_2 := \arccos \left[\frac{L_1^2 + L_2^2 - (L_3 + L_4)^2}{2 \cdot L_1 \cdot L_2} \right] \quad \theta_2 = 73.558 \text{ deg}$$

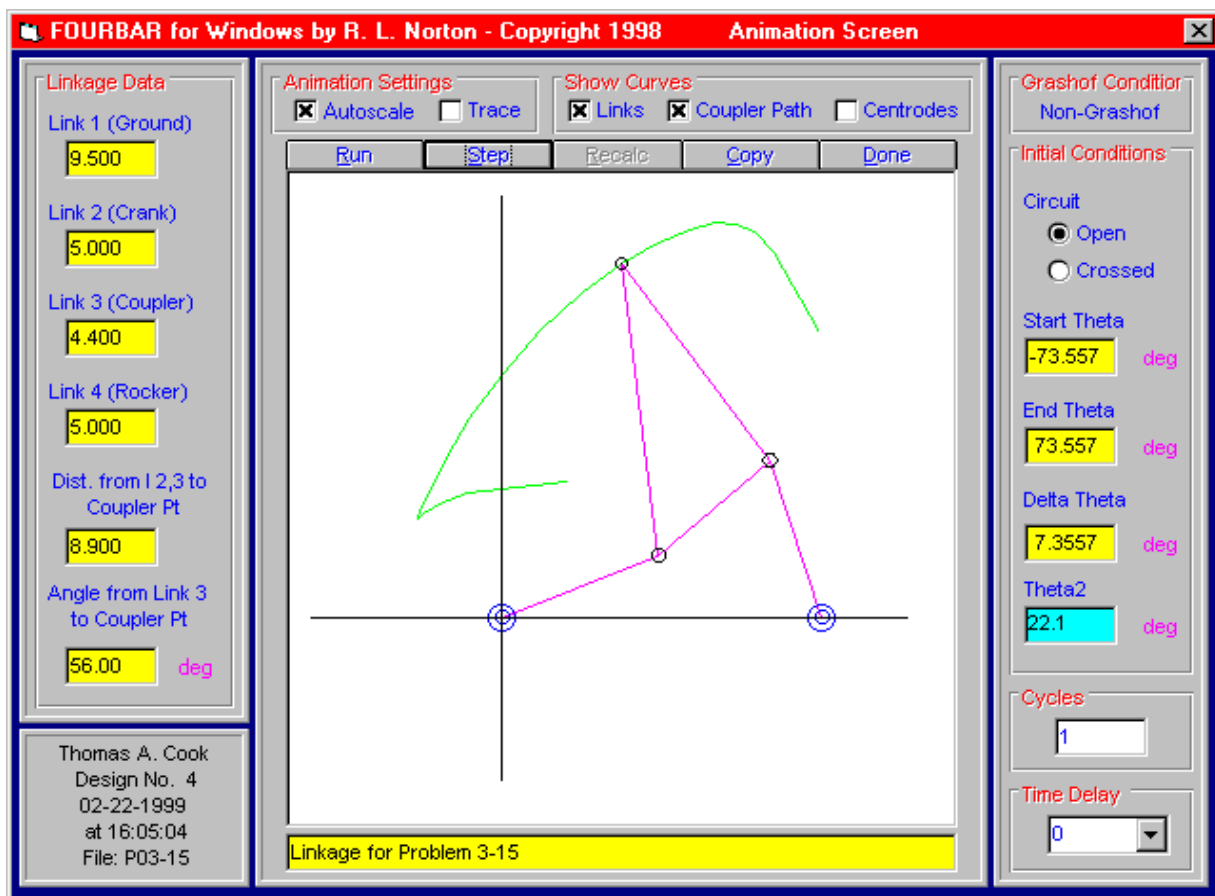
The other toggle position occurs at $-\theta_2 = -73.558 \text{ deg}$

3. Use the program FOURBAR to find the maximum and minimum transmission angles.

Angle Step Deg	Theta2 Mag degrees	Theta3 Mag degrees	Theta4 Mag degrees	Trans Ang Mag degrees
-73.557	-73.557	30.861	-149.490	0.352
-58.846	-58.846	64.075	-176.312	60.387
-44.134	-44.134	77.168	170.696	86.472
-29.423	-29.423	83.147	157.514	74.367
-14.711	-14.711	80.604	142.103	61.499
0.000	0.000	68.350	125.123	56.773
14.711	14.711	50.145	111.644	61.499
29.423	29.423	32.106	106.473	74.367
44.134	44.134	16.173	109.701	86.472
58.846	58.846	0.566	120.179	60.387
73.557	73.557	-30.486	149.159	0.355

A partial output from FOURBAR is shown above. From it, we see that the maximum transmission angle is approximately 86.5 deg and the minimum is zero deg.

4. Use program FOURBAR to draw the coupler curve with respect to a coordinate frame through O_2O_4 .



PROBLEM 3-16

Statement: Draw the Roberts diagram for the linkage in Figure P3-4 and find its two cognates. Are they Grashof or non-Grashof?

Given: Link lengths: Coupler point data:
 Ground link $L_1 := 9.5$ Crank $L_2 := 5$ $A1P := 8.90$ $\delta_1 := 56.000 \cdot \text{deg}$
 Coupler $L_3 := 4.4$ Rocker $L_4 := 5$

Solution: See Figure P3-4 and Mathcad file P0316.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$B1P := \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} \quad B1P = 7.401$$

$$\gamma_1 := \arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) \quad \gamma_1 = 94.4701 \text{ deg}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \quad L_5 = 7.401 \quad L_6 := \frac{L_4}{L_3} \cdot B1P \quad L_6 = 8.410$$

$$L_{10} := A1P \quad L_{10} = 8.900 \quad L_9 := \frac{L_2}{L_3} \cdot A1P \quad L_9 = 10.114$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \quad L_7 = 8.410 \quad L_8 := L_6 \cdot \frac{A1P}{B1P} \quad L_8 = 10.114$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_4 \quad A3P = 5.000 \quad A2P := L_2 \quad A2P = 5.000$$

$$\delta_3 := \gamma_1 \quad \delta_3 = 94.470 \text{ deg} \quad \delta_2 := -\delta_1 \quad \delta_2 = -56.000 \text{ deg}$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

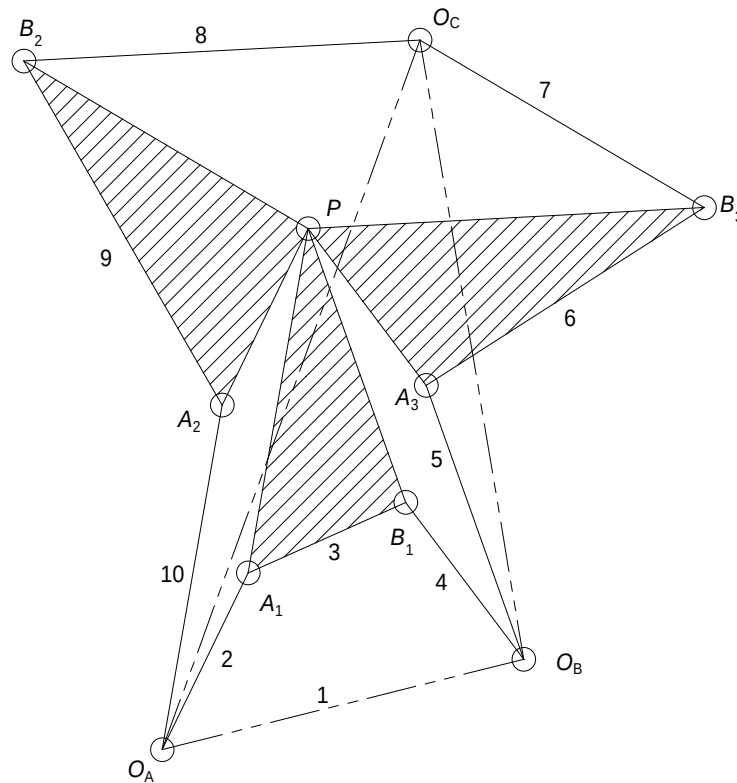
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \quad L_{1BC} = 15.9793 \quad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \quad L_{1AC} = 19.2159$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 9.500$	$L_{1AC} = 19.216$	$L_{1BC} = 15.979$
Crank length	$L_2 = 5.000$	$L_{10} = 8.900$	$L_7 = 8.410$
Coupler length	$L_3 = 4.400$	$L_9 = 10.114$	$L_6 = 8.410$

Rocker length	$L_4 = 5.000$	$L_8 = 10.114$	$L_5 = 7.401$
Coupler point	$A_1P = 8.900$	$A_2P = 5.000$	$A_3P = 5.000$
Coupler angle	$\delta_1 = 56.000 \text{ deg}$	$\delta_2 = -56.000 \text{ deg}$	$\delta_3 = 94.470 \text{ deg}$



6. Determine the Grashof condition of each of the two additional cognates.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Cognate #2: $Condition(L_{10}, L_{1AC}, L_8, L_9) = \text{"non-Grashof"}$

Cognate #3: $Condition(L_5, L_{1BC}, L_6, L_7) = \text{"non-Grashof"}$

PROBLEM 3-17

Statement: Design a Watt-I sixbar to give parallel motion that follows the coupler path of point P of the linkage in Figure P3-4.

Given: Link lengths: Coupler point data:

Ground link	$L_1 := 9.5$	Crank	$L_2 := 5$	$A1P := 8.90$	$\delta_1 := 56.000 \cdot deg$
Coupler	$L_3 := 4.4$	Rocker	$L_4 := 5$		

Solution: See Figure P3-4 and Mathcad file P0317.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB .

$$B1P := \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} \quad B1P = 7.401$$

$$\gamma_1 := \arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) \quad \gamma_1 = 94.4701 \text{ deg}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \quad L_5 = 7.401 \quad L_6 := \frac{L_4}{L_3} \cdot B1P \quad L_6 = 8.410$$

$$L_{10} := A1P \quad L_{10} = 8.900 \quad L_9 := \frac{L_2}{L_3} \cdot A1P \quad L_9 = 10.114$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \quad L_7 = 8.410 \quad L_8 := L_6 \cdot \frac{A1P}{B1P} \quad L_8 = 10.114$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_4 \quad A3P = 5.000 \quad A2P := L_2 \quad A2P = 5.000$$

$$\delta_3 := \gamma_1 \quad \delta_3 = 94.470 \text{ deg} \quad \delta_2 := -\delta_1 \quad \delta_2 = -56.000 \text{ deg}$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

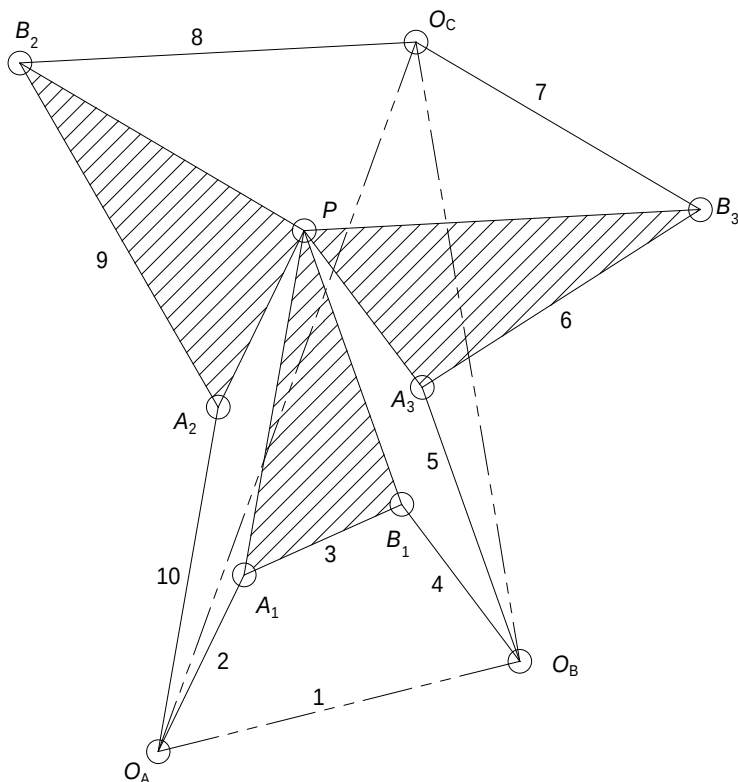
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \quad L_{1BC} = 15.9793 \quad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \quad L_{1AC} = 19.2159$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

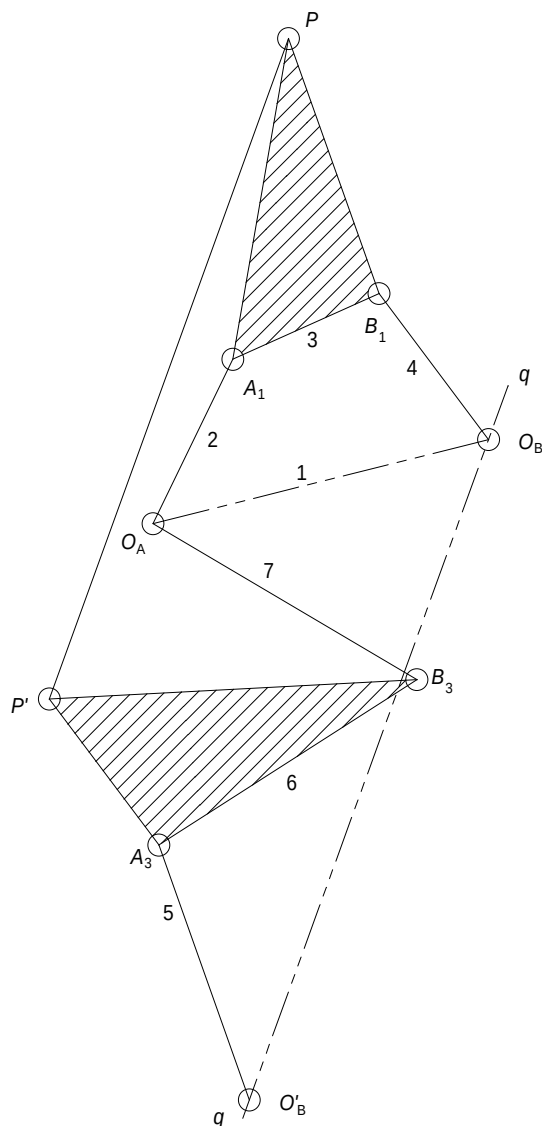
SUMMARY OF COGNATE SPECIFICATIONS:

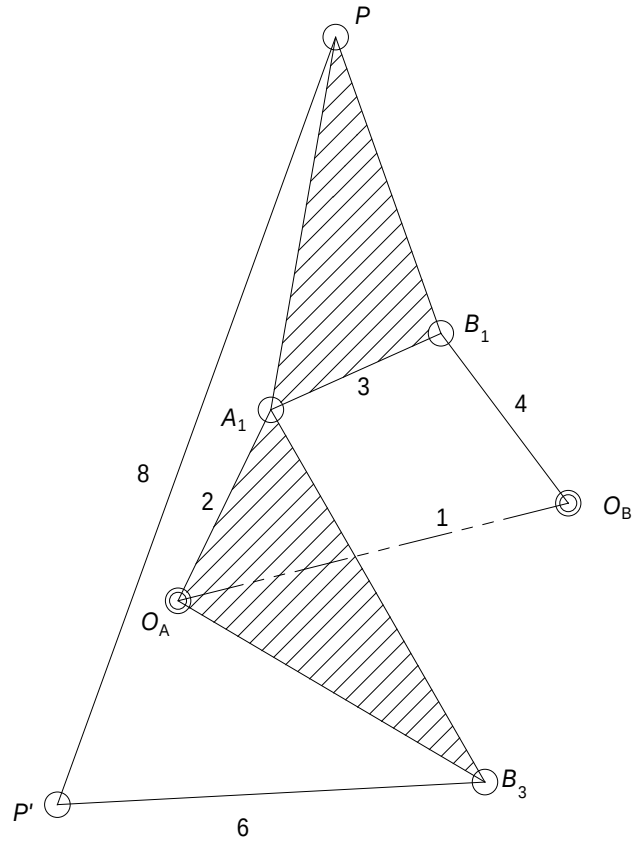
	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 9.500$	$L_{1AC} = 19.216$	$L_{1BC} = 15.979$
Crank length	$L_2 = 5.000$	$L_{10} = 8.900$	$L_7 = 8.410$
Coupler length	$L_3 = 4.400$	$L_9 = 10.114$	$L_6 = 8.410$

Rocker length	$L_4 = 5.000$	$L_8 = 10.114$	$L_5 = 7.401$
Coupler point	$A_1P = 8.900$	$A_2P = 5.000$	$A_3P = 5.000$
Coupler angle	$\delta_1 = 56.000 \text{ deg}$	$\delta_2 = -56.000 \text{ deg}$	$\delta_3 = 94.470 \text{ deg}$



- All three of these cognates are non-Grashof and will, therefore, have limited motion. However, following Example 3-11, discard cognate #2 and retain cognates #1 and #3. Draw line qq parallel to line O_AO_C and through point O_B . Without allowing links 5, 6, and 7 to rotate, slide them as an assembly along lines O_AO_C and qq until the free end of link 7 is at O_A . The free end of link 5 will then be at point O'_B and point P on link 6 will be at P' . Add a new link of length O_AO_C between P and P' . This is the new output link 8 and all points on it describe the original coupler curve.
- Join links 2 and 7, making one ternary link. Remove link 5 and reduce link 6 to a binary link. The result is a Watt-I sixbar with links numbered 1, 2, 3, 4, 6, and 8 (see next page). Link 8 is in curvilinear translation and follows the coupler path of the original point P .





PROBLEM 3-18

Statement: Design a Watt-I sixbar to give parallel motion that follows the coupler path of point P of the linkage in Figure P3-4 and add a driver dyad to drive it over its possible range of motion with no quick return. (The result will be an 8-bar linkage).

Given: Link lengths: Coupler point data:

Ground link	$L_1 := 9.5$	Crank	$L_2 := 5$	A1P := 8.90	$\delta_1 := 56.000 \cdot \text{deg}$
Coupler	$L_3 := 4.4$	Rocker	$L_4 := 5$		

Solution: See Figure P3-4 and Mathcad file P0318.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$B1P := \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} \quad B1P = 7.401$$

$$\gamma_1 := \arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) \quad \gamma_1 = 94.4701 \text{ deg}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \quad L_5 = 7.401 \quad L_6 := \frac{L_4}{L_3} \cdot B1P \quad L_6 = 8.410$$

$$L_{10} := A1P \quad L_{10} = 8.900 \quad L_9 := \frac{L_2}{L_3} \cdot A1P \quad L_9 = 10.114$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \quad L_7 = 8.410 \quad L_8 := L_6 \cdot \frac{A1P}{B1P} \quad L_8 = 10.114$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_4 \quad A3P = 5.000 \quad A2P := L_2 \quad A2P = 5.000$$

$$\delta_3 := \gamma_1 \quad \delta_3 = 94.470 \text{ deg} \quad \delta_2 := -\delta_1 \quad \delta_2 = -56.000 \text{ deg}$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

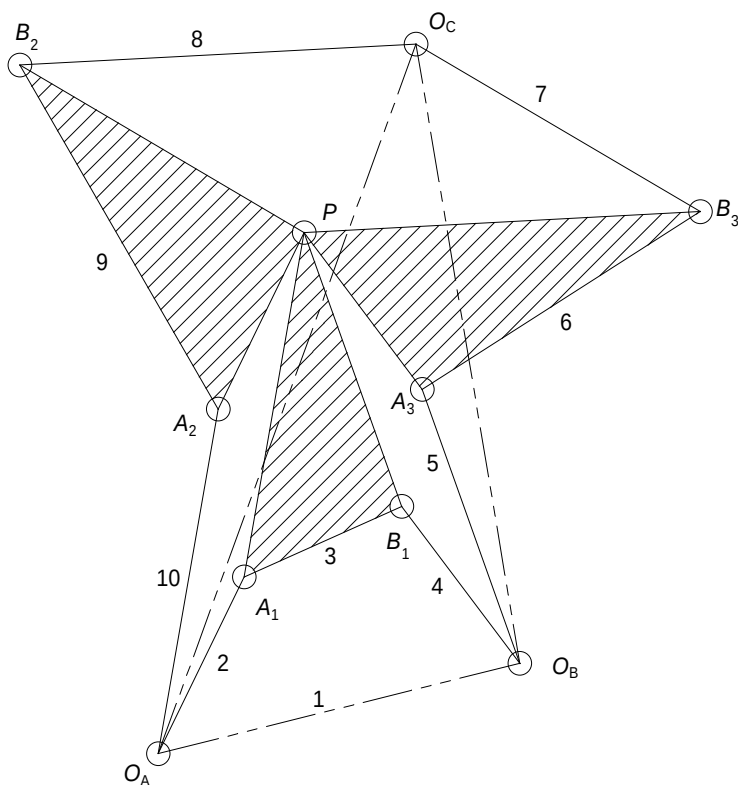
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \quad L_{1BC} = 15.9793 \quad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \quad L_{1AC} = 19.2159$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

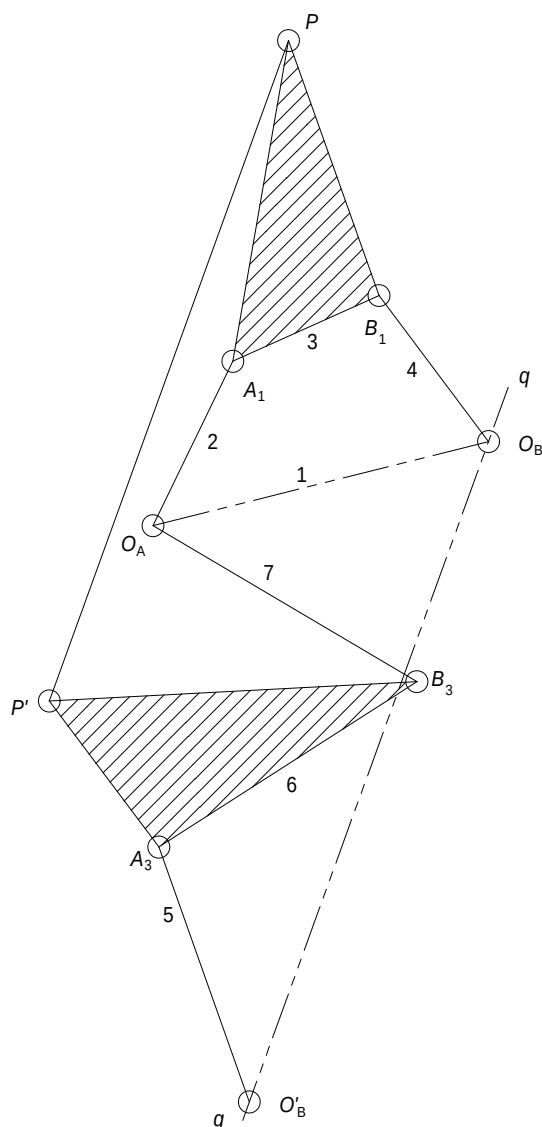
SUMMARY OF COGNATE SPECIFICATIONS:

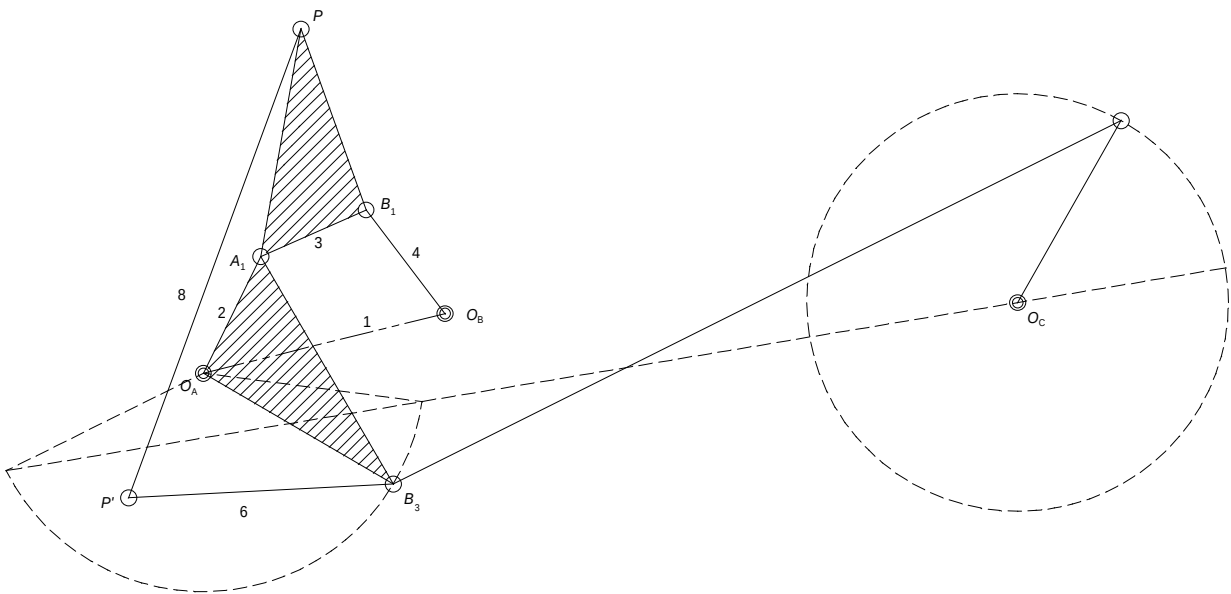
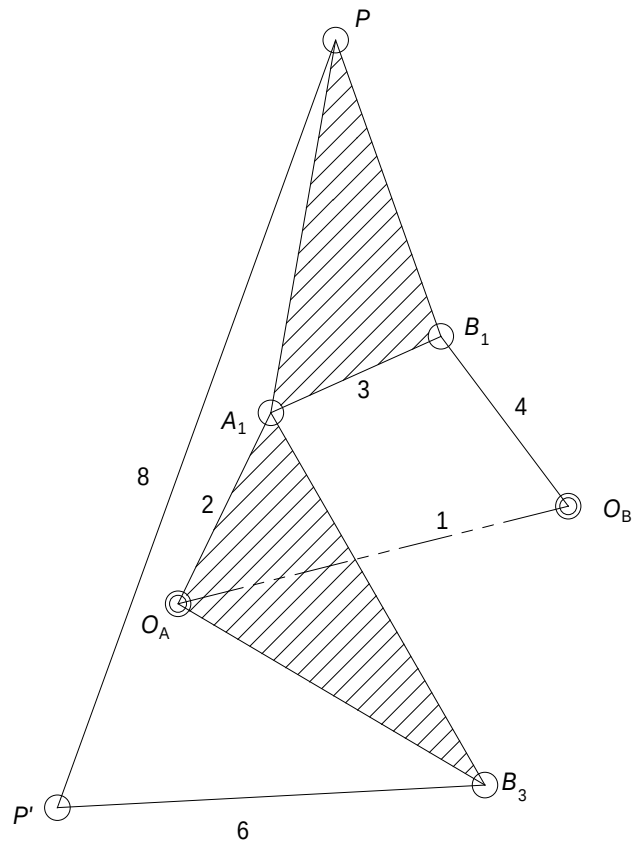
	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 9.500$	$L_{1AC} = 19.216$	$L_{1BC} = 15.979$
Crank length	$L_2 = 5.000$	$L_{10} = 8.900$	$L_7 = 8.410$
Coupler length	$L_3 = 4.400$	$L_9 = 10.114$	$L_6 = 8.410$

Rocker length	$L_4 = 5.000$	$L_8 = 10.114$	$L_5 = 7.401$
Coupler point	$A_1P = 8.900$	$A_2P = 5.000$	$A_3P = 5.000$
Coupler angle	$\delta_1 = 56.000 \text{ deg}$	$\delta_2 = -56.000 \text{ deg}$	$\delta_3 = 94.470 \text{ deg}$



- All three of these cognates are non-Grashof and will, therefore, have limited motion. However, following Example 3-11, discard cognate #2 and retain cognates #1 and #3. Draw line qq parallel to line O_AO_C and through point O_B . Without allowing links 5, 6, and 7 to rotate, slide them as an assembly along lines O_AO_C and qq until the free end of link 7 is at O_A . The free end of link 5 will then be at point O'_B and point P on link 6 will be at P' . Add a new link of length O_AO_C between P and P' . This is the new output link 8 and all points on it describe the original coupler curve.
- Join links 2 and 7, making one ternary link. Remove link 5 and reduce link 6 to a binary link. The result is a Watt-I sixbar with links numbered 1, 2, 3, 4, 6, and 8 (see next page). Link 8 is in curvilinear translation and follows the coupler path of the original point P .
- Add a driver dyad following Example 3-4.





PROBLEM 3-19

Statement: Design a pin-jointed linkage that will guide the forks of the fork lift truck in Figure P3-5 up and down in an approximate straight line over the range of motion shown. Arrange the fixed pivots so they are close to some part of the existing frame or body of the truck.

Given: Length of straight line motion of the forks: $\Delta x := 1800 \text{ mm}$

Solution: See Figure P3-5 and Mathcad file P0319.

Design choices:

Use a Hoeken-type straight line mechanism optimized for straightness.

Maximum allowable error in straightness of line: $\Delta C_y := 0.096\%$

- Using Table 3-1 and the required length of straight-line motion, determine the link lengths.

Link ratios from Table 3-1 for

$$\Delta C_y = 0.096\%$$

$$L1overL2 := 2.200 \quad L3overL2 := 2.800 \quad \Delta x over L2 := 4.181$$

Link lengths:

$$\text{Crank} \quad L_2 := \frac{\Delta x}{\Delta x over L2} \quad L_2 = 430.5 \text{ mm}$$

$$\text{Coupler} \quad L_3 := L3overL2 \cdot L_2 \quad L_3 = 1205.5 \text{ mm}$$

$$\text{Ground link} \quad L_1 := L1overL2 \cdot L_2 \quad L_1 = 947.1 \text{ mm}$$

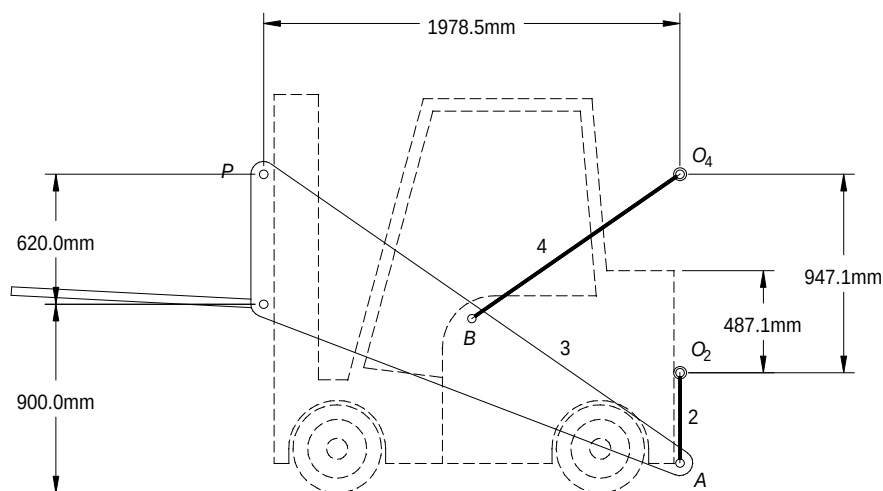
$$\text{Rocker} \quad L_4 := L_3 \quad L_4 = 1205.5 \text{ mm}$$

$$\text{Coupler point} \quad BP := L_3 \quad BP = 1205.5 \text{ mm}$$

- Calculate the distance from point P to pivot O_4 (C_y).

$$C_y := \sqrt{(2 \cdot L_3)^2 - (L_1 + L_2)^2} \quad C_y = 1978.5 \text{ mm}$$

- Draw the fork lift truck to scale with the mechanism defined in step 1 superimposed on it..



PROBLEM 3-20

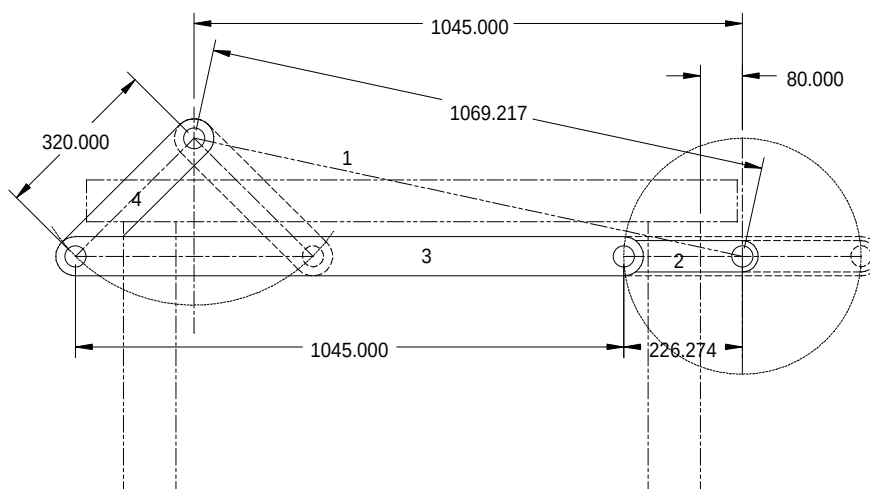
Statement: Figure P3-6 shows a "V-link" off-loading mechanism for a paper roll conveyor. Design a pin-jointed linkage to replace the air cylinder driver that will rotate the rocker arm and V-link through the 90 deg motion shown. Keep the fixed pivots as close to the existing frame as possible. Your fourbar linkage should be Grashof and be in toggle at each extreme position of the rocker arm.

Given: Dimensions scaled from Figure P3-6:
Rocker arm (link 4) distance between pin centers: $L_4 := 320 \cdot \text{mm}$

Solution: See Figure P3-6 and Mathcad file P0320.

Design choices:

1. Use the same rocker arm that was used with the air cylinder driver.
 2. Place the pivot O_2 80 mm to the right of the right leg and on a horizontal line with the center of the pin on the rocker arm.
 3. Design for two-position, 90 deg of output rocker motion with no quick return, similar to Example 3-2.
1. Draw the rocker arm (link 4) O_4B in both extreme positions, B_1 and B_2 , in any convenient location such that the desired angle of motion θ_4 is subtended. In this solution, link 4 is drawn such that the two extreme positions each make an angle of 45 deg to the vertical.
 2. Draw the chord B_1B_2 and extend it in any convenient direction. In this solution it was extended horizontally to the left.
 3. Mark the center O_2 on the extended line such that it is 80 mm to the right of the right leg. This will allow sufficient space for a supporting pillow block bearing.
 4. Bisect the line segment B_1B_2 and draw a circle of that radius about O_2 .
 5. Label the two intersections of the circle and extended line B_1B_2 , A_1 and A_2 .
 6. Measure the length of the coupler (link 3) as A_1B_1 or A_2B_2 . From the graphical solution, $L_3 := 1045 \cdot \text{mm}$
 7. Measure the length of the crank (link 2) as O_2A_1 or O_2A_2 . From the graphical solution, $L_2 := 226.274 \cdot \text{mm}$
 8. Measure the length of the ground link (link 1) as O_2O_4 . From the graphical solution, $L_1 := 1069.217 \cdot \text{mm}$



9. Find the Grashof condition.

$$\text{Condition}(a, b, c, d) := \left\{ \begin{array}{l} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} \quad \text{if } SL < PQ \\ \text{return "Special Grashof"} \quad \text{if } SL = PQ \\ \text{return "non-Grashof"} \quad \text{otherwise} \end{array} \right.$$
$$\text{Condition}(L_1, L_2, L_3, L_4) = \text{"Grashof"}$$

PROBLEM 3-21

Statement: Figure P3-7 shows a walking-beam transport mechanism that uses a fourbar coupler curve, replicated with a parallelogram linkage for parallel motion. Note the duplicate crank and coupler shown ghosted in the right half of the mechanism - they are redundant and have been removed from the duplicate fourbar linkage. Using the same fourbar driving stage (links 1, 2, 3, 4 with coupler point P), design a Watt-I sixbar linkage that will drive link 8 in the same parallel motion using two fewer links.

Given: Link lengths: Coupler point data:
 Ground link $L_1 := 2.22$ Crank $L_2 := 1$ $A1P := 3.06$ $\delta_1 := 31.000 \cdot \text{deg}$
 Coupler $L_3 := 2.06$ Rocker $L_4 := 2.33$

Solution: See Figure P3-7 and Mathcad file P0321.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$B1P := \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} \quad B1P = 1.674$$

$$\gamma_1 := \arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) \quad \gamma_1 = 109.6560 \text{ deg}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \quad L_5 = 1.674 \quad L_6 := \frac{L_4}{L_3} \cdot B1P \quad L_6 = 1.893$$

$$L_{10} := A1P \quad L_{10} = 3.060 \quad L_9 := \frac{L_2}{L_3} \cdot A1P \quad L_9 = 1.485$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \quad L_7 = 0.812 \quad L_8 := L_6 \cdot \frac{A1P}{B1P} \quad L_8 = 3.461$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_8 \quad A3P = 3.461 \quad A2P := L_2 \quad A2P = 1.000$$

$$\delta_3 := -[180 \cdot \text{deg} - (\delta_1 + \gamma_1)] \quad \delta_2 := -\delta_1 \quad \delta_2 = -31.000 \text{ deg}$$

$$\delta_3 = -39.344 \text{ deg}$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

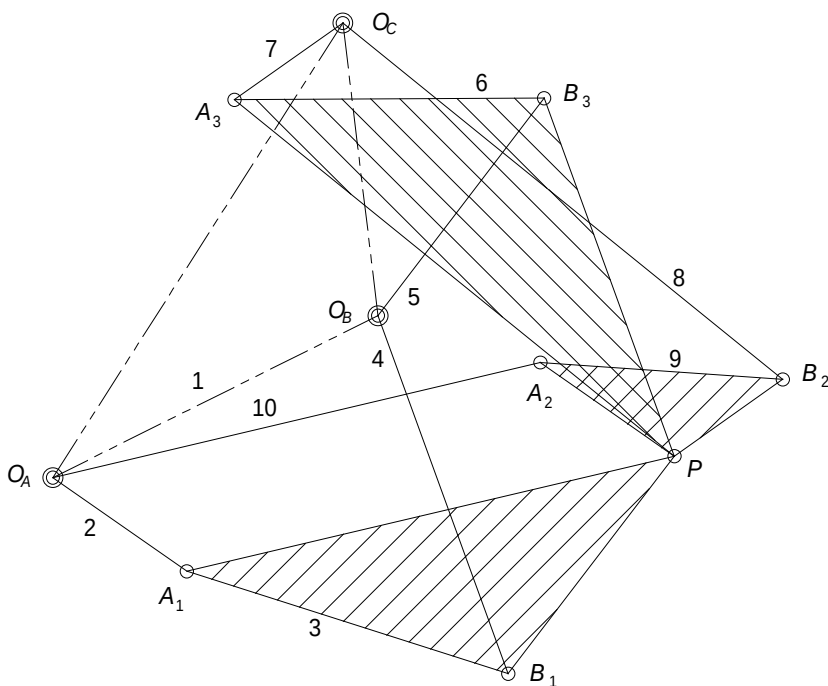
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \quad L_{1BC} = 1.8035 \quad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \quad L_{1AC} = 3.2977$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 3.298$	$L_{1BC} = 1.804$

Crank length	$L_2 = 1.000$	$L_{10} = 3.060$	$L_7 = 0.812$
Coupler length	$L_3 = 2.060$	$L_9 = 1.485$	$L_6 = 1.893$
Rocker length	$L_4 = 2.330$	$L_8 = 3.461$	$L_5 = 1.674$
Coupler point	$A_1P = 3.060$	$A_2P = 1.000$	$A_3P = 3.461$
Coupler angle	$\delta_1 = 31.000 \text{ deg}$	$\delta_2 = -31.000 \text{ deg}$	$\delta_3 = -39.344 \text{ deg}$



4. Determine the Grashof condition of each of the two additional cognates.

```

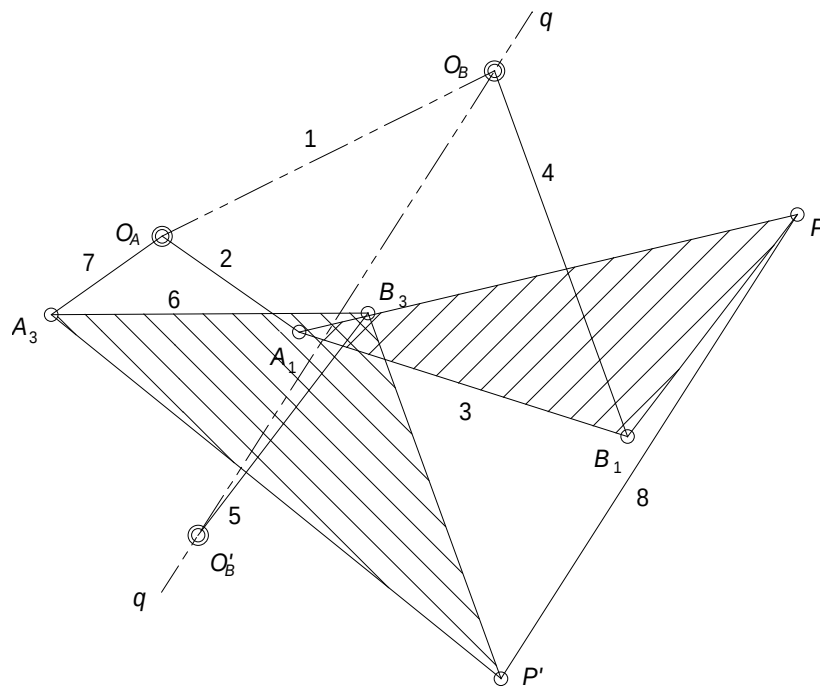
Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

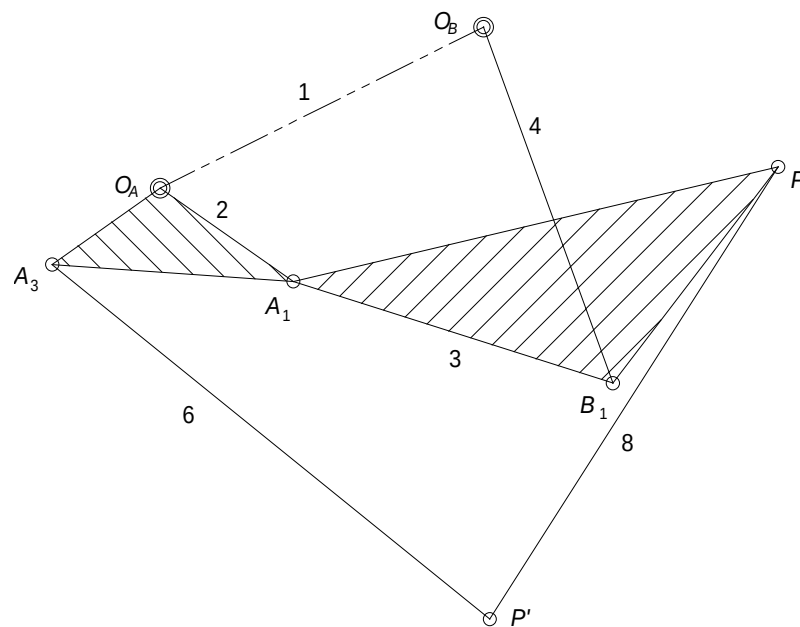
Cognate #2: $Condition(L_8, L_9, L_{10}, L_{1AC}) = \text{"Grashof"}$

Cognate #3: $Condition(L_5, L_6, L_7, L_{1BC}) = \text{"Grashof"}$

5. Both of these cognates are Grashof but cognate #3 is a crank rocker. Following Example 3-11, discard cognate #2 and retain cognates #1 and #3. Draw line qq parallel to line O_AO_C and through point O_B . Without allowing links 5, 6, and 7 to rotate, slide them as an assembly along lines O_AO_C and qq until the free end of link 7 is at O_A . The free end of link 5 will then be at point O'_B and point P on link 6 will be at P' . Add a new link of length O_AO_C between P and P' . This is the new output link 8 and all points on it describe the original coupler curve.



6. Join links 2 and 7, making one ternary link. Remove link 5 and reduce link 6 to a binary link. The result is a Watt-I sixbar with links numbered 1, 2, 3, 4, 6, and 8 (see next page). Link 8 is in curvilinear translation and follows the coupler path of the original point P . The walking-beam (link 8 in Figure P3-7) is rigidly attached to link 8 below.



PROBLEM 3-22

Statement: Find the maximum and minimum transmission angles of the fourbar driving stage (links L_1, L_2, L_3, L_4) in Figure P3-7 (to graphical accuracy).

Given: Link lengths: Link 2 $L_2 := 1.00$ Link 3 $L_3 := 2.06$
Link 4 $L_4 := 2.33$ Link 1 $L_1 := 2.22$

Grashof condition function:

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

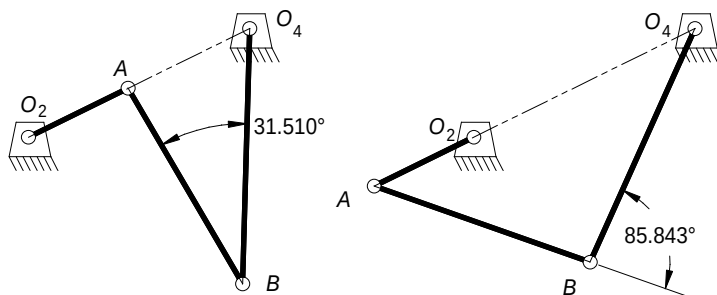
Solution: See Figure P3-7 and Mathcad file P0322.

1. Determine the Grashof condition of the mechanism from inequality 2.8 and its Barker classification from Table 2-4.

Grashof condition: $Condition(L_1, L_2, L_3, L_4) = \text{"Grashof"}$

Barker classification: Class I-2, Grashof crank-rocker-rocker, GCRR, since the shortest link is the input link.

2. It can be shown (see Section 4.10) that the minimum transmission angle for a fourbar GCRR linkage occurs when links 2 and 1 (ground link) are colinear. Draw the linkage in these two positions and measure the transmission angles.



3. As measured from the layout, the minimum transmission angle is 31.5 deg. The maximum is 90 deg.

PROBLEM 3-23

Statement: Figure P3-8 shows a fourbar linkage used in a power loom to drive a comb-like reed against the thread, "beating it up" into the cloth. Determine its Grashof condition and its minimum and maximum transmission angles to graphical accuracy.

Given: Link lengths: Link 2 $L_2 := 2.00 \cdot \text{in}$ Link 3 $L_3 := 8.375 \cdot \text{in}$
Link 4 $L_4 := 7.187 \cdot \text{in}$ Link 1 $L_1 := 9.625 \cdot \text{in}$

Grashof condition function:

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof"   if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

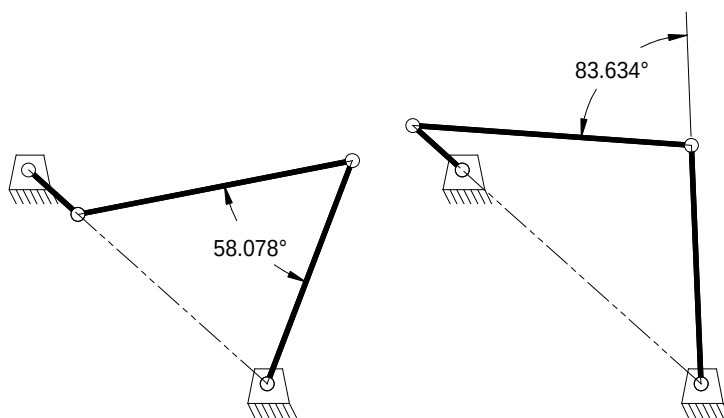
Solution: See Figure P3-8 and Mathcad file P0323.

1. Determine the Grashof condition of the mechanism from inequality 2.8 and its Barker classification from Table 2-4.

Grashof condition: $\text{Condition}(L_1, L_2, L_3, L_4) = \text{"Grashof"}$

Barker classification: Class I-2, Grashof crank-rocker-rocker, GCRR, since the shortest link is the input link.

2. It can be shown (see Section 4.10) that the minimum transmission angle for a fourbar GCRR linkage occurs when links 2 and 1 (ground link) are colinear. Draw the linkage in these two positions and measure the transmission angles.



3. As measured from the layout, the minimum transmission angle is 58.1 deg. The maximum is 90.0 deg.

PROBLEM 3-24

Statement: Draw the Roberts diagram and find the cognates for the linkage in Figure P3-9.

Given: Link lengths: Coupler point data:
 Ground link $L_1 := 2.22$ Crank $L_2 := 1.0$ $A1P := 3.06$ $\delta_1 := -31.00 \cdot \text{deg}$
 Coupler $L_3 := 2.06$ Rocker $L_4 := 2.33$

Solution: See Figure P3-9 and Mathcad file P0324.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$B1P := \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} \quad B1P = 1.674$$

$$\gamma_1 := -\arccos\left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) \quad \gamma_1 = -109.6560 \text{ deg}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \quad L_5 = 1.674 \quad L_6 := \frac{L_4}{L_3} \cdot B1P \quad L_6 = 1.893$$

$$L_{10} := A1P \quad L_{10} = 3.060 \quad L_9 := \frac{L_2}{L_3} \cdot A1P \quad L_9 = 1.485$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \quad L_7 = 0.812 \quad L_8 := L_6 \cdot \frac{A1P}{B1P} \quad L_8 = 3.461$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_8 \quad A3P = 3.461 \quad A2P := L_2 \quad A2P = 1.000$$

$$\delta_3 := 180 \cdot \text{deg} - |\delta_1 + \gamma_1| \quad \delta_3 = 39.344 \text{ deg} \quad \delta_2 := -\delta_1 \quad \delta_2 = 31.000 \text{ deg}$$

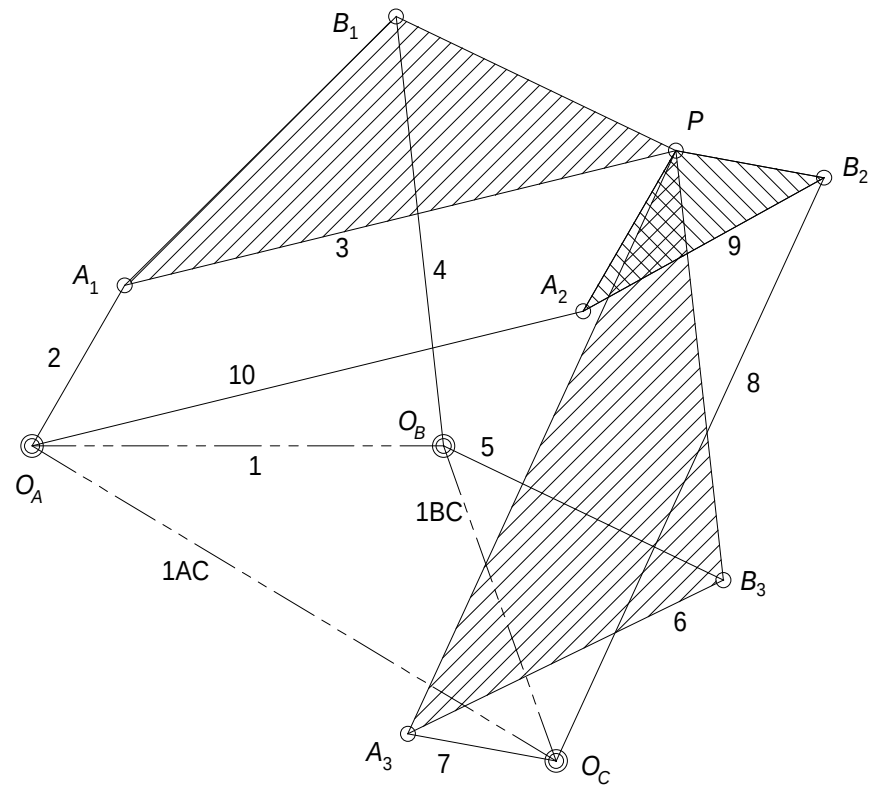
From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \quad L_{1BC} = 1.8035 \quad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \quad L_{1AC} = 3.2977$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 3.298$	$L_{1BC} = 1.804$
Crank length	$L_2 = 1.000$	$L_{10} = 3.060$	$L_7 = 0.812$
Coupler length	$L_3 = 2.060$	$L_9 = 1.485$	$L_6 = 1.893$
Rocker length	$L_4 = 2.330$	$L_8 = 3.461$	$L_5 = 1.674$
Coupler point	$A1P = 3.060$	$A2P = 1.000$	$A3P = 3.461$
Coupler angle	$\delta_1 = -31.000 \text{ deg}$	$\delta_2 = 31.000 \text{ deg}$	$\delta_3 = 39.344 \text{ deg}$



PROBLEM 3-25

Statement: Find the equivalent geared fivebar mechanism cognate of the linkage in Figure P3-9.

Given: Link lengths:

Coupler point data:

$$\begin{array}{llllll} \text{Ground link} & L_1 := 2.22 & \text{Crank} & L_2 := 1.0 & A1P := 3.06 & \delta_1 := -31.00 \cdot \text{deg} \\ \text{Coupler} & L_3 := 2.06 & \text{Rocker} & L_4 := 2.33 & & \end{array}$$

Solution: See Figure P3-9 and Mathcad file P0325.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$\begin{aligned} B1P &:= \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} & B1P &= 1.674 \\ \gamma_1 &:= -\arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) & \gamma_1 &= -109.6560 \text{ deg} \end{aligned}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$\begin{aligned} L_5 &:= B1P & L_5 &= 1.674 & L_6 &:= \frac{L_4}{L_3} \cdot B1P & L_6 &= 1.893 \\ L_{10} &:= A1P & L_{10} &= 3.060 & L_9 &:= \frac{L_2}{L_3} \cdot A1P & L_9 &= 1.485 \\ L_7 &:= L_9 \cdot \frac{B1P}{A1P} & L_7 &= 0.812 & L_8 &:= L_6 \cdot \frac{A1P}{B1P} & L_8 &= 3.461 \end{aligned}$$

Calculate the coupler point data for cognates #2 and #3

$$\begin{aligned} A3P &:= L_8 & A3P &= 3.461 & A2P &:= L_2 & A2P &= 1.000 \\ \delta_3 &:= 180 \cdot \text{deg} - |\delta_1 + \gamma_1| & \delta_3 &= 39.344 \text{ deg} & \delta_2 &:= -\delta_1 & \delta_2 &= 31.000 \text{ deg} \end{aligned}$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

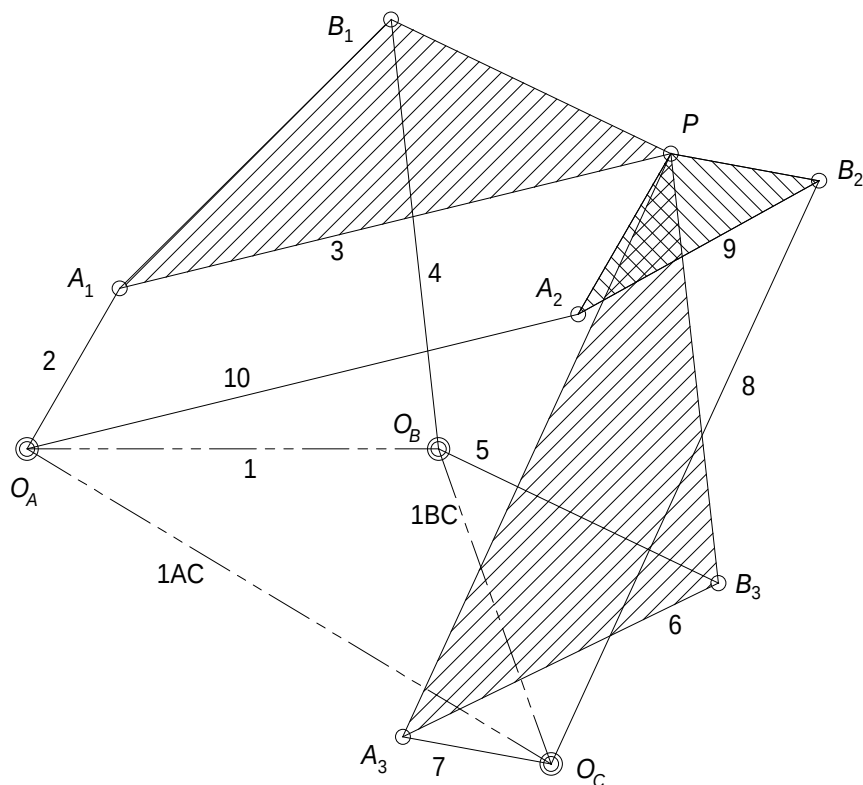
$$\begin{aligned} L_{1BC} &:= \frac{L_1}{L_3} \cdot B1P & L_{1BC} &= 1.8035 & L_{1AC} &:= \frac{L_1}{L_3} \cdot A1P & L_{1AC} &= 3.2977 \end{aligned}$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 3.298$	$L_{1BC} = 1.804$
Crank length	$L_2 = 1.000$	$L_{10} = 3.060$	$L_7 = 0.812$
Coupler length	$L_3 = 2.060$	$L_9 = 1.485$	$L_6 = 1.893$
Rocker length	$L_4 = 2.330$	$L_8 = 3.461$	$L_5 = 1.674$

Coupler point	$A_1P = 3.060$	$A_2P = 1.000$	$A_3P = 3.461$
Coupler angle	$\delta_1 = -31.000 \text{ deg}$	$\delta_2 = 31.000 \text{ deg}$	$\delta_3 = 39.344 \text{ deg}$

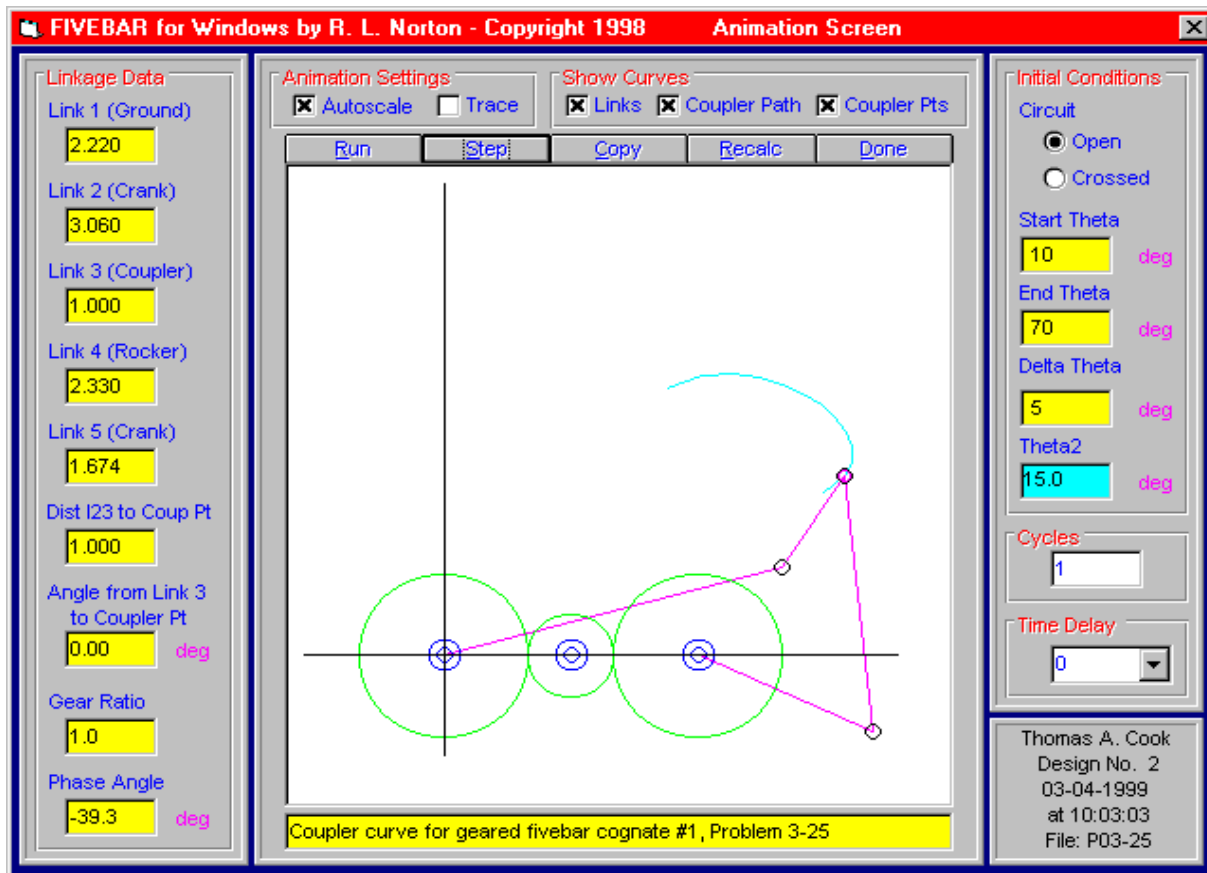
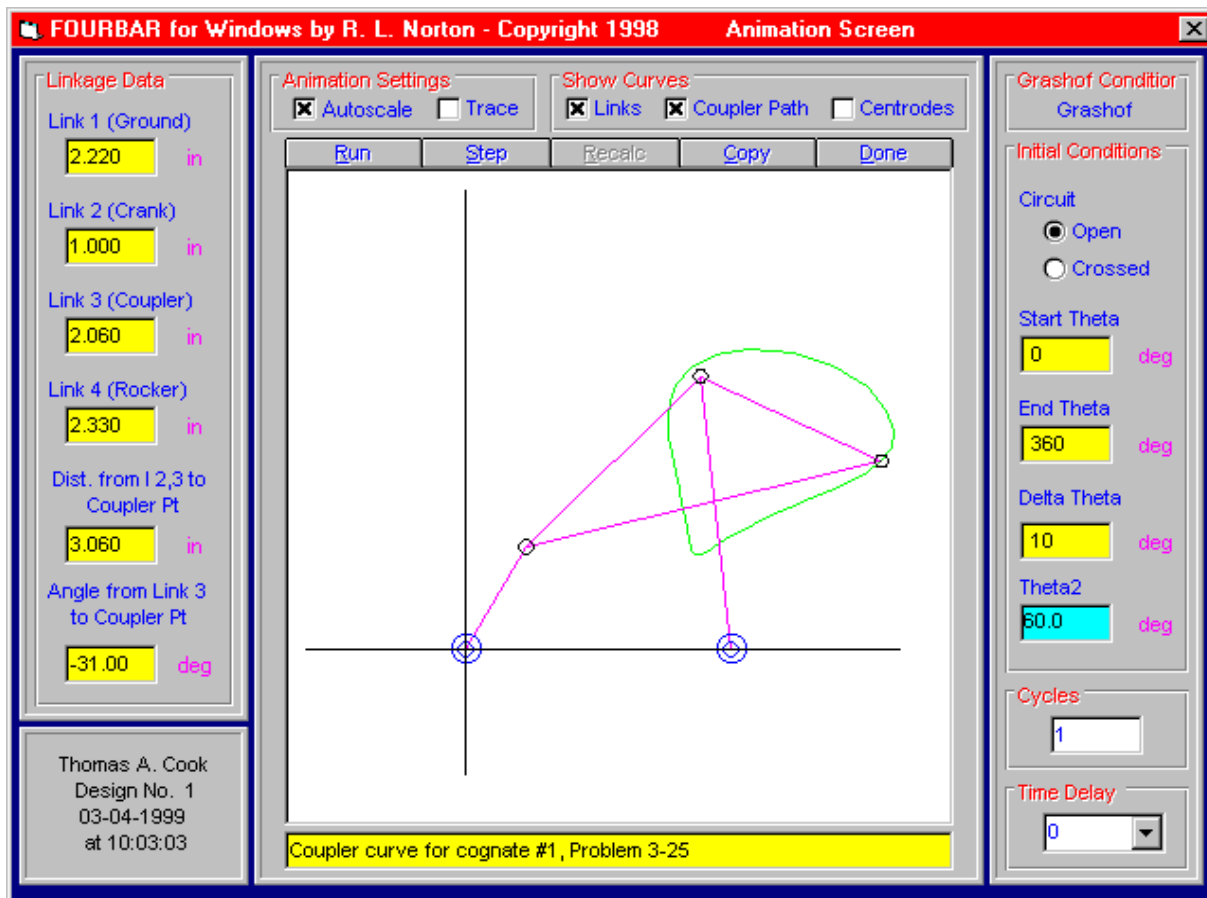


4. The three geared fivebar cognates can be seen in the Roberts diagram. They are: $O_A A_2 P B_3 O_B$, $O_A A_1 P A_3 O_C$, and $O_B B_1 P B_2 O_C$. The three geared fivebar cognates are summarized in the table below.

SUMMARY OF GEARED FIVEBAR COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 3.298$	$L_{1BC} = 1.804$
Crank length	$L_{10} = 3.060$	$L_2 = 1.000$	$L_4 = 2.330$
Coupler length	$A_2P = 1.000$	$A_1P = 3.060$	$L_5 = 1.674$
Rocker length	$L_4 = 2.330$	$L_8 = 3.461$	$L_7 = 0.812$
Crank length	$L_5 = 1.674$	$L_7 = 0.812$	$L_8 = 3.461$
Coupler point	$A_2P = 1.000$	$A_1P = 3.060$	$B_1P = 1.674$
Coupler angle	$\delta_1 := 0.00 \cdot \text{deg}$	$\delta_2 := 0.00 \cdot \text{deg}$	$\delta_3 := 0.00 \cdot \text{deg}$

5. Enter the cognate #1 specifications into program FOURBAR to get a trace of the coupler path (see next page)
6. Enter the geared fivebar cognate #1 specifications into program FIVEBAR to get a trace of the coupler path for the geared fivebar (see next page).



PROBLEM 3-26

Statement: Use the linkage in Figure P3-9 to design an eightbar double-dwell mechanism that has a rocker output through 45 deg.

Given: Link lengths: Coupler point data:

Ground link $L_1 := 2.22$ Crank $L_2 := 1.0$ $A1P := 3.06$ $\delta_1 := -31.00 \cdot \text{deg}$

Coupler $L_3 := 2.06$ Rocker $L_4 := 2.33$

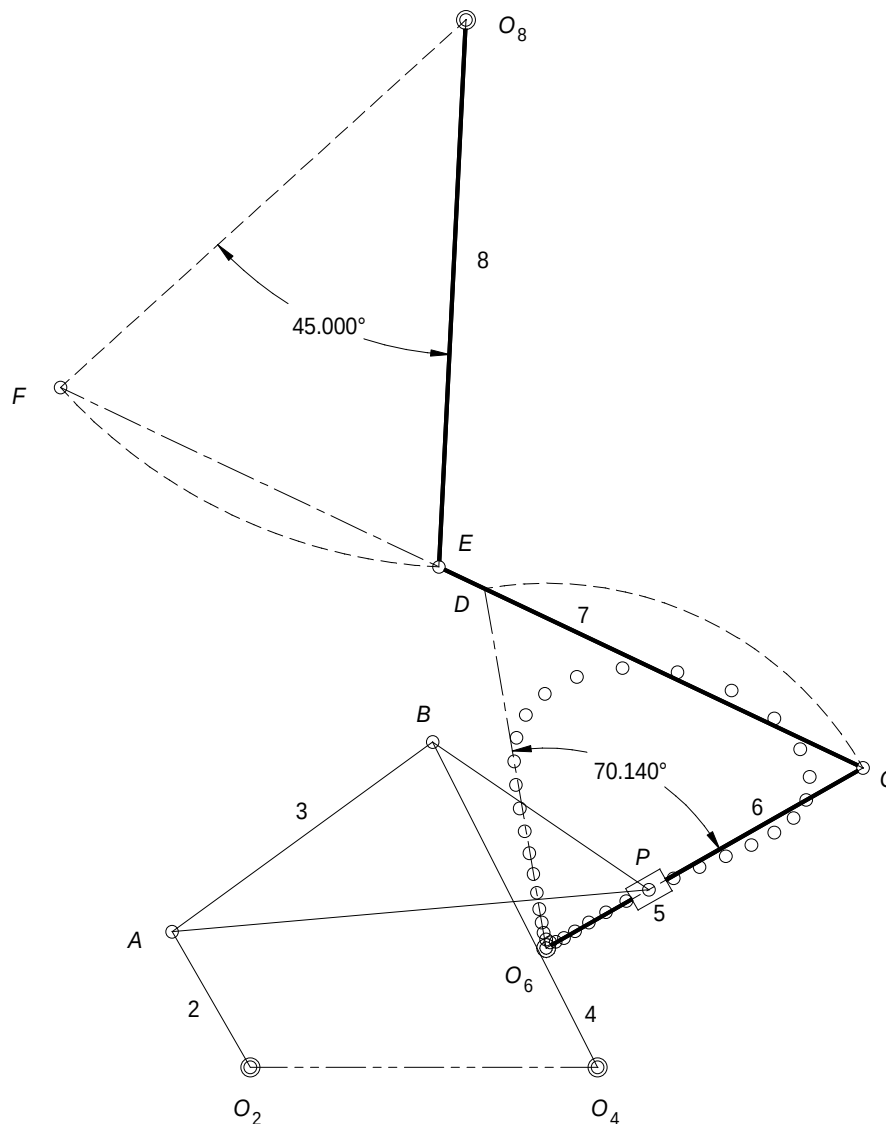
Solution: See Figure P3-9 and Mathcad file P0326.

1. Enter the given data into program FOURBAR and print out the resulting coupler point coordinates (see table below).

FOURBAR for Windows File P03-26.DAT

Angle Step Deg	Cpler Pt X	Cpler Pt Y	Cpler Pt Mag	Cpler Pt Ang
0.000	2.731	2.523	3.718	42.731
10.000	3.077	2.407	3.906	38.029
20.000	3.350	2.228	4.023	33.626
30.000	3.515	2.032	4.060	30.035
40.000	3.576	1.855	4.028	27.412
50.000	3.554	1.708	3.943	25.672
60.000	3.473	1.592	3.820	24.635
70.000	3.350	1.499	3.671	24.107
80.000	3.203	1.420	3.503	23.915
90.000	3.040	1.348	3.326	23.915
100.000	2.872	1.278	3.144	23.988
110.000	2.706	1.207	2.963	24.039
120.000	2.548	1.135	2.789	24.001
130.000	2.403	1.062	2.627	23.834
140.000	2.274	0.990	2.480	23.533
150.000	2.164	0.925	2.354	23.134
160.000	2.075	0.869	2.249	22.719
170.000	2.005	0.826	2.168	22.404
180.000	1.953	0.802	2.111	22.326
190.000	1.917	0.798	2.076	22.614
200.000	1.892	0.817	2.061	23.365
210.000	1.875	0.860	2.063	24.632
220.000	1.862	0.925	2.079	26.417
230.000	1.848	1.011	2.107	28.678
240.000	1.832	1.115	2.145	31.340
250.000	1.810	1.235	2.192	34.306
260.000	1.784	1.367	2.248	37.463
270.000	1.754	1.508	2.313	40.683
280.000	1.723	1.654	2.388	43.826
290.000	1.698	1.804	2.477	46.730
300.000	1.687	1.955	2.582	49.207
310.000	1.702	2.105	2.707	51.038
320.000	1.761	2.251	2.858	51.965
330.000	1.883	2.386	3.040	51.715
340.000	2.088	2.494	3.253	50.064
350.000	2.380	2.550	3.488	46.967
360.000	2.731	2.523	3.718	42.731

- Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Fit tangent lines to the nearly straight portions of the curve. Label their intersection O_6 .
- Design link 6 to lie along these straight tangents, pivoted at O_6 . Provide a guide on link 6 to accommodate slider block 5, which pivots on the coupler point P .



- Extend link 6 a convenient distance to point C . Draw an arc through point C with center at O_6 . Label the intersection of the arc with the other tangent line as point D . Attach link 7 to the pivot at C . The length of link 7 is CE , a design choice. Extend line CDE from point E a distance equal to CD . Label the end point F . Layout two intersecting lines through E and F such that they subtend an angle of 45° . Label their intersection O_8 . The link lengths and locations of O_6 and O_8 are:

Link 6	$L_6 := 2.330$	Link 7	$L_7 := 3.000$	Link 8	$L_8 := 3.498$
Fixed pivot O_6 :	$x := 1.892$	Fixed pivot O_8 :	$x := 1.379$		
	$y := 0.762$		$y := 6.690$		

PROBLEM 3-27

Statement: Use the linkage in Figure P3-9 to design an eightbar double-dwell mechanism that has a slider output stroke of 5 crank units.

Given: Link lengths: Crank Coupler point data:
Ground link $L_1 := 2.22$ $L_2 := 1.0$ $A1P := 3.06$ $\delta_1 := -31.00 \cdot \text{deg}$
Coupler $L_3 := 2.06$ Rocker $L_4 := 2.33$

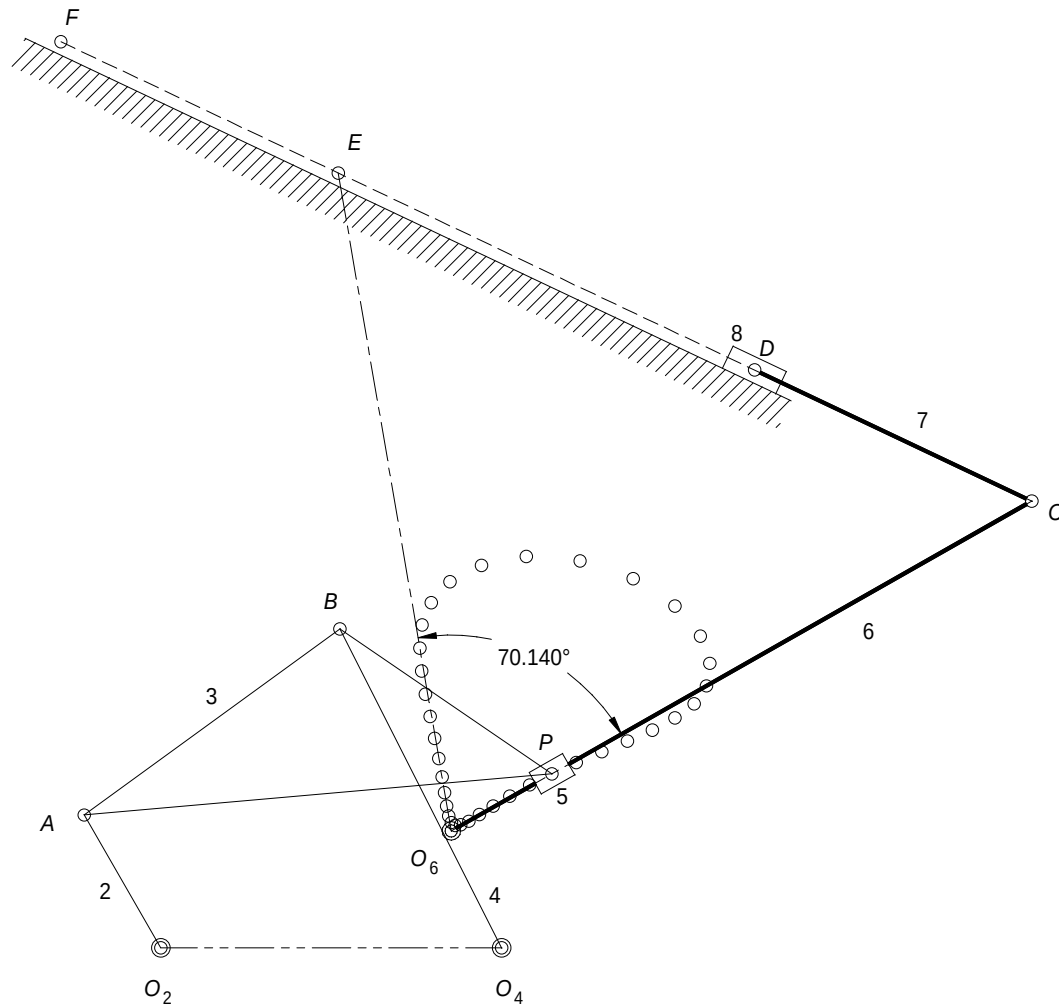
Solution: See Figure P3-9 and Mathcad file P0327.

1. Enter the given data into program FOURBAR and print out the resulting coupler point coordinates (see table below).

FOURBAR for Windows File P03-26.DAT

Angle Step Deg	Cpler Pt X	Cpler Pt Y	Cpler Pt Mag	Cpler Pt Ang
0.000	2.731	2.523	3.718	42.731
10.000	3.077	2.407	3.906	38.029
20.000	3.350	2.228	4.023	33.626
30.000	3.515	2.032	4.060	30.035
40.000	3.576	1.855	4.028	27.412
50.000	3.554	1.708	3.943	25.672
60.000	3.473	1.592	3.820	24.635
70.000	3.350	1.499	3.671	24.107
80.000	3.203	1.420	3.503	23.915
90.000	3.040	1.348	3.326	23.915
100.000	2.872	1.278	3.144	23.988
110.000	2.706	1.207	2.963	24.039
120.000	2.548	1.135	2.789	24.001
130.000	2.403	1.062	2.627	23.834
140.000	2.274	0.990	2.480	23.533
150.000	2.164	0.925	2.354	23.134
160.000	2.075	0.869	2.249	22.719
170.000	2.005	0.826	2.168	22.404
180.000	1.953	0.802	2.111	22.326
190.000	1.917	0.798	2.076	22.614
200.000	1.892	0.817	2.061	23.365
210.000	1.875	0.860	2.063	24.632
220.000	1.862	0.925	2.079	26.417
230.000	1.848	1.011	2.107	28.678
240.000	1.832	1.115	2.145	31.340
250.000	1.810	1.235	2.192	34.306
260.000	1.784	1.367	2.248	37.463
270.000	1.754	1.508	2.313	40.683
280.000	1.723	1.654	2.388	43.826
290.000	1.698	1.804	2.477	46.730
300.000	1.687	1.955	2.582	49.207
310.000	1.702	2.105	2.707	51.038
320.000	1.761	2.251	2.858	51.965
330.000	1.883	2.386	3.040	51.715
340.000	2.088	2.494	3.253	50.064
350.000	2.380	2.550	3.488	46.967
360.000	2.731	2.523	3.718	42.731

- Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Fit tangent lines to the nearly straight portions of the curve. Label their intersection O_6 .
- Design link 6 to lie along these straight tangents, pivoted at O_6 . Provide a guide on link 6 to accommodate slider block 5, which pivots on the coupler point P .



- Extend link 6 and the other tangent line until points C and E are 5 units apart. Attach link 7 to the pivot at C . The length of link 7 is CD , a design choice. Extend line CDE from point D a distance equal to CE . Label the end point F . As link 6 travels from C to E , slider block 8 will travel from D to F , a distance of 5 units. The link lengths and location of O_6 :

$$\text{Link 6} \quad L_6 := 4.351 \quad \text{Link 7} \quad L_7 := 2.000$$

$$\text{Fixed pivot } O_6: \quad x := 1.892$$

$$y := 0.762$$

PROBLEM 3-28

Statement: Use two of the cognates in Figure 3-26 (p. 126) to design a Watt-I sixbar parallel motion mechanism that carries a link through the same coupler curve at all points. Comment on its similarities to the original Roberts diagram.

Given: Link lengths: Coupler point data:

Ground link $L_1 := 45$ Crank $L_2 := 56$ $A1P := 11.25$ $\delta_1 := 0.000 \cdot \text{deg}$

Coupler $L_3 := 22.5$ Rocker $L_4 := 56$

Solution: See Figure 3-26 and Mathcad file P0328.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$B1P := \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} \quad B1P = 11.250$$

$$\gamma_1 := \arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) \quad \gamma_1 = 0.0000 \text{ deg}$$

2. Use the Cayley diagram (see Figure 3-26) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \quad L_5 = 11.250 \quad L_6 := \frac{L_4}{L_3} \cdot B1P \quad L_6 = 28.000$$

$$L_{10} := A1P \quad L_{10} = 11.250 \quad L_9 := \frac{L_2}{L_3} \cdot A1P \quad L_9 = 28.000$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \quad L_7 = 28.000 \quad L_8 := L_6 \cdot \frac{A1P}{B1P} \quad L_8 = 28.000$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_4 \quad A3P = 56.000 \quad A2P := L_2 \quad A2P = 56.000$$

$$\delta_3 := \delta_1 \quad \delta_3 = 0.000 \text{ deg} \quad \delta_2 := \delta_1 \quad \delta_2 = 0.000 \text{ deg}$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

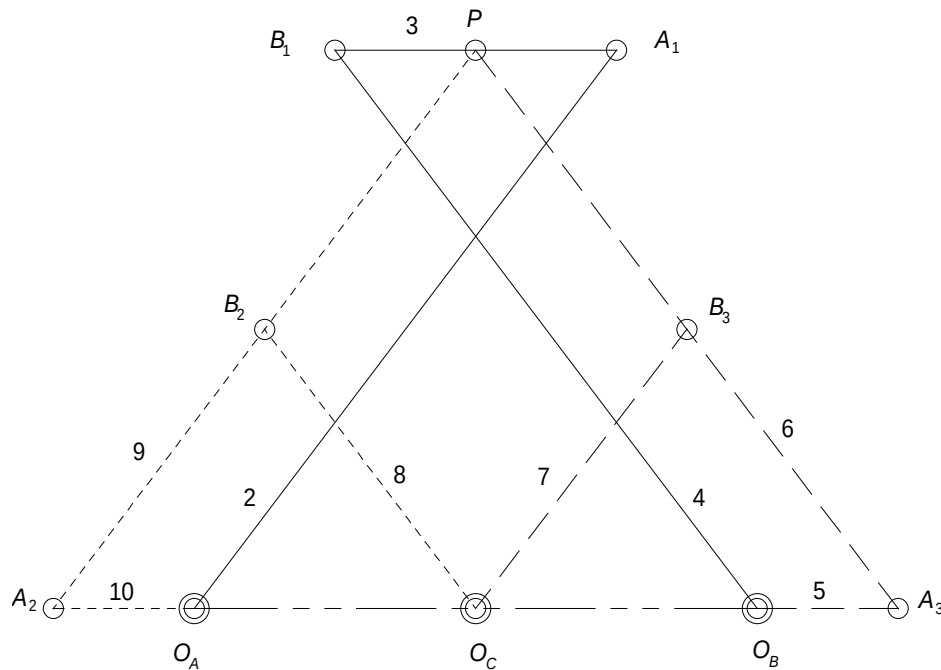
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \quad L_{1BC} = 22.5000 \quad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \quad L_{1AC} = 22.5000$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

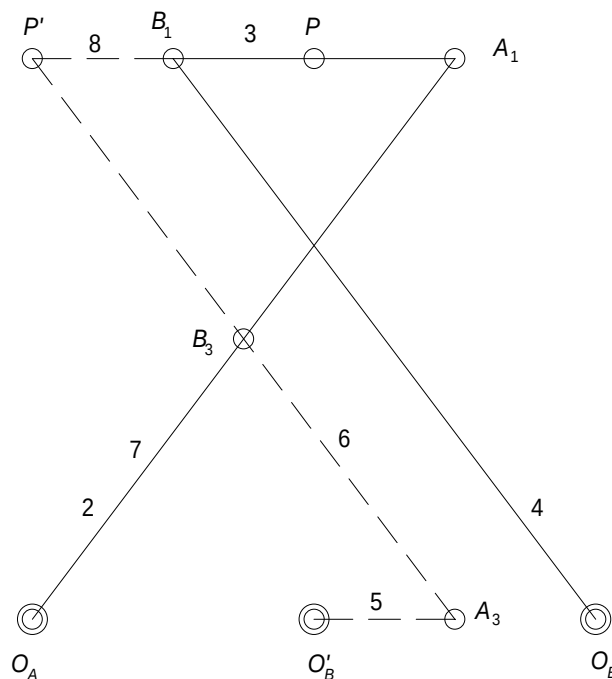
SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 45.000$	$L_{1AC} = 22.500$	$L_{1BC} = 22.500$
Crank length	$L_2 = 56.000$	$L_{10} = 11.250$	$L_7 = 28.000$
Coupler length	$L_3 = 22.500$	$L_9 = 28.000$	$L_6 = 28.000$

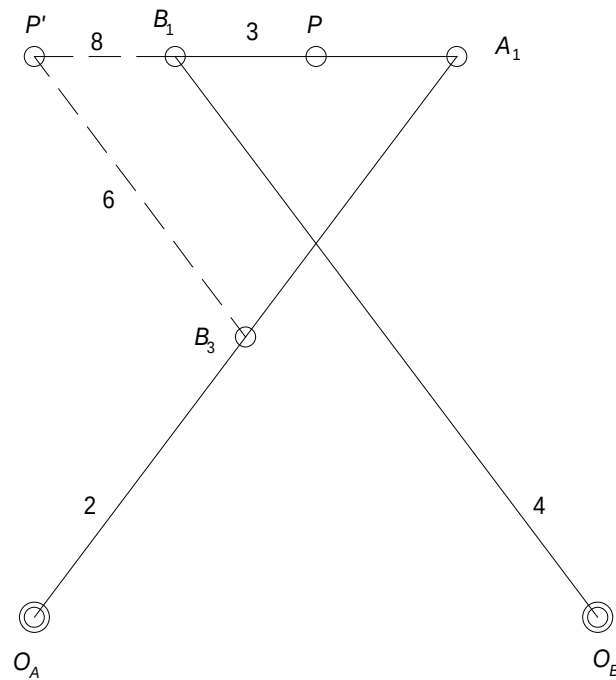
Rocker length	$L_4 = 56.000$	$L_8 = 28.000$	$L_5 = 11.250$
Coupler point	$A_1P = 11.250$	$A_2P = 56.000$	$A_3P = 56.000$
Coupler angle	$\delta_1 = 0.000 \text{ deg}$	$\delta_2 = 0.000 \text{ deg}$	$\delta_3 = 0.000 \text{ deg}$



4. Both of these cognates are identical. Following Example 3-11, discard cognate #2 and retain cognates #1 and #3. Without allowing links 5, 6, and 7 to rotate, slide them as an assembly along line O_AO_C until the free end of link 7 is at O_A . The free end of link 5 will then be at point O'_B and point P on link 6 will be at P' . Add a new link of length O_AO_C between P and P' . This is the new output link 8 and all points on it describe the original coupler curve.



5. Join links 2 and 7, making one ternary link. Remove link 5 and reduce link 6 to a binary link. The result is a Watt-I sixbar with links numbered 1, 2, 3, 4, 6, and 8. Link 8 is in curvilinear translation and follows the coupler path of the original point P . Link 8 is a binary link with nodes at P and P' . It does not attach to link 4 at B_1 .



PROBLEM 3-29

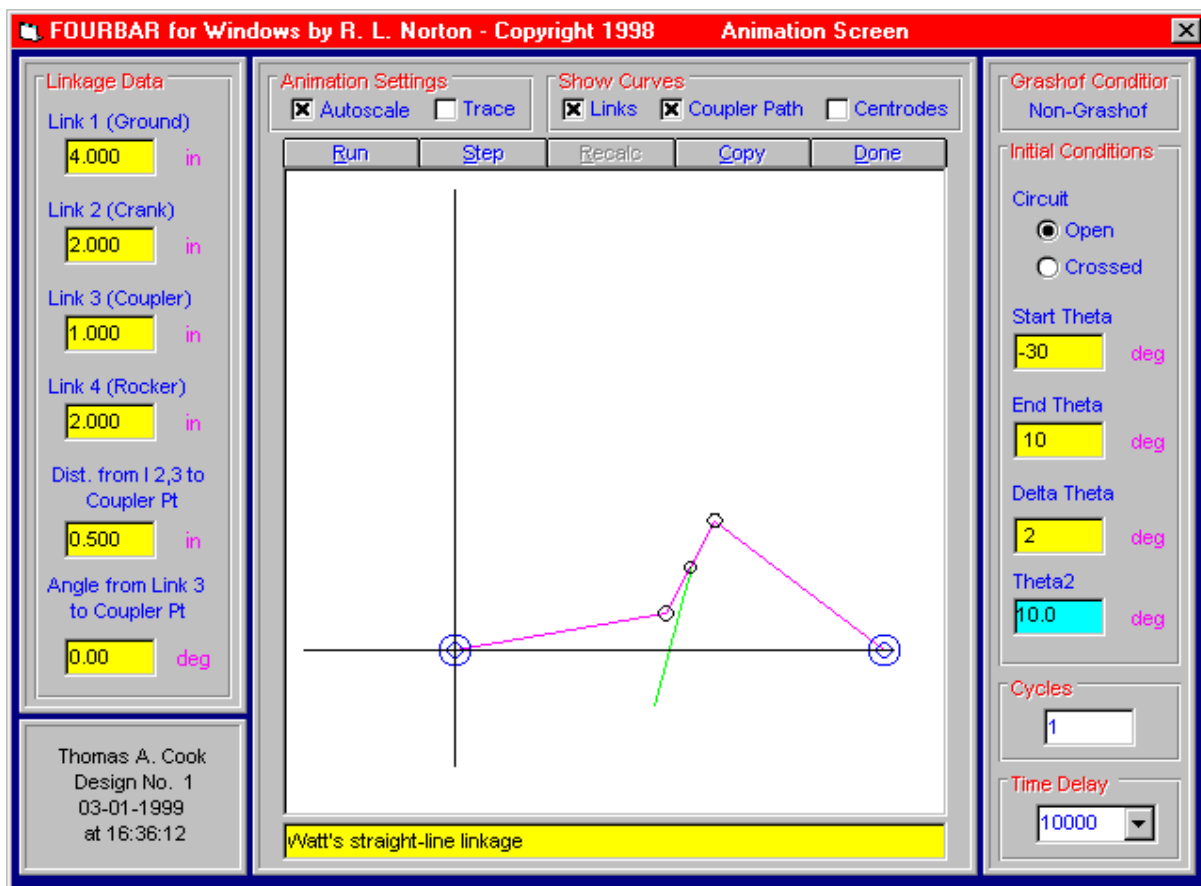
Statement: Find the cognates of the Watt straight-line mechanism in Figure 3-29a (p. 131).

Given: Link lengths: Coupler point data:

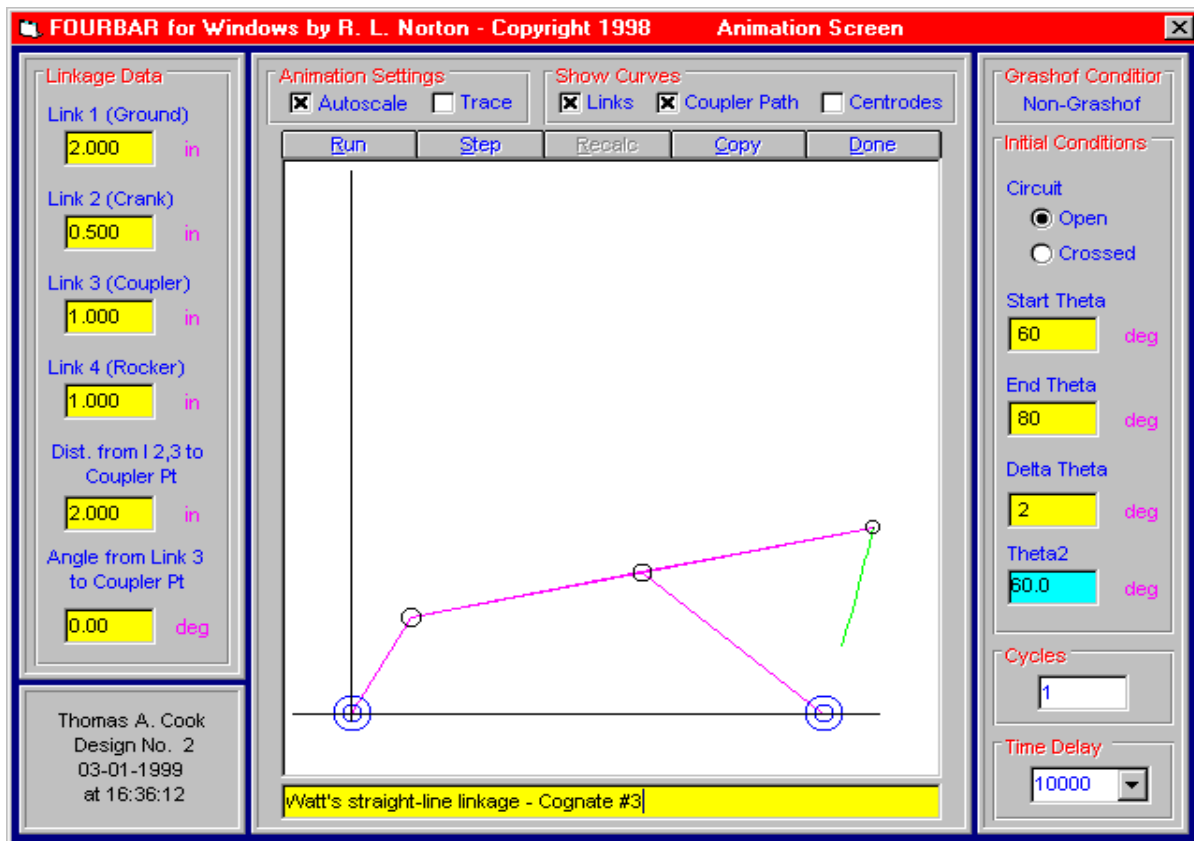
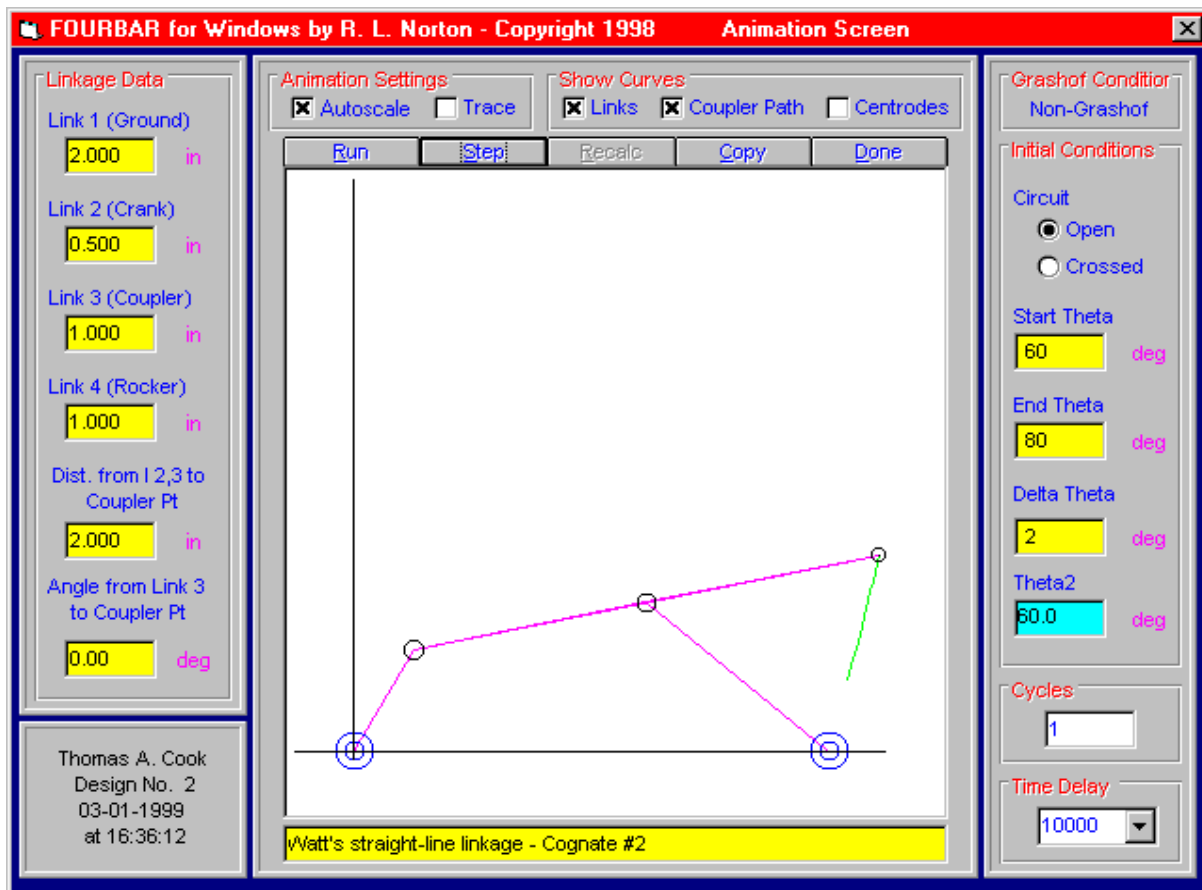
Ground link	$L_1 := 4$	Crank	$L_2 := 2$	$A1P := 0.500$	$\delta_1 := 0.00\text{-deg}$
Coupler	$L_3 := 1$	Rocker	$L_4 := 2$	$B1P := 0.500$	$\gamma_1 := 0.00\text{-deg}$

Solution: See Figure 3-29a and Mathcad file P0329.

1. Input the link dimensions and coupler point data into program FOURBAR.



2. Use the Cognate pull-down menu to get the link lengths for cognates #2 and #3 (see next page). Note that, for this mechanism, cognates #2 and #3 are identical. All three mechanisms are non-Grashof with limited crank angles.



PROBLEM 3-30

Statement: Find the cognates of the Roberts straight-line mechanism in Figure 3-29b.

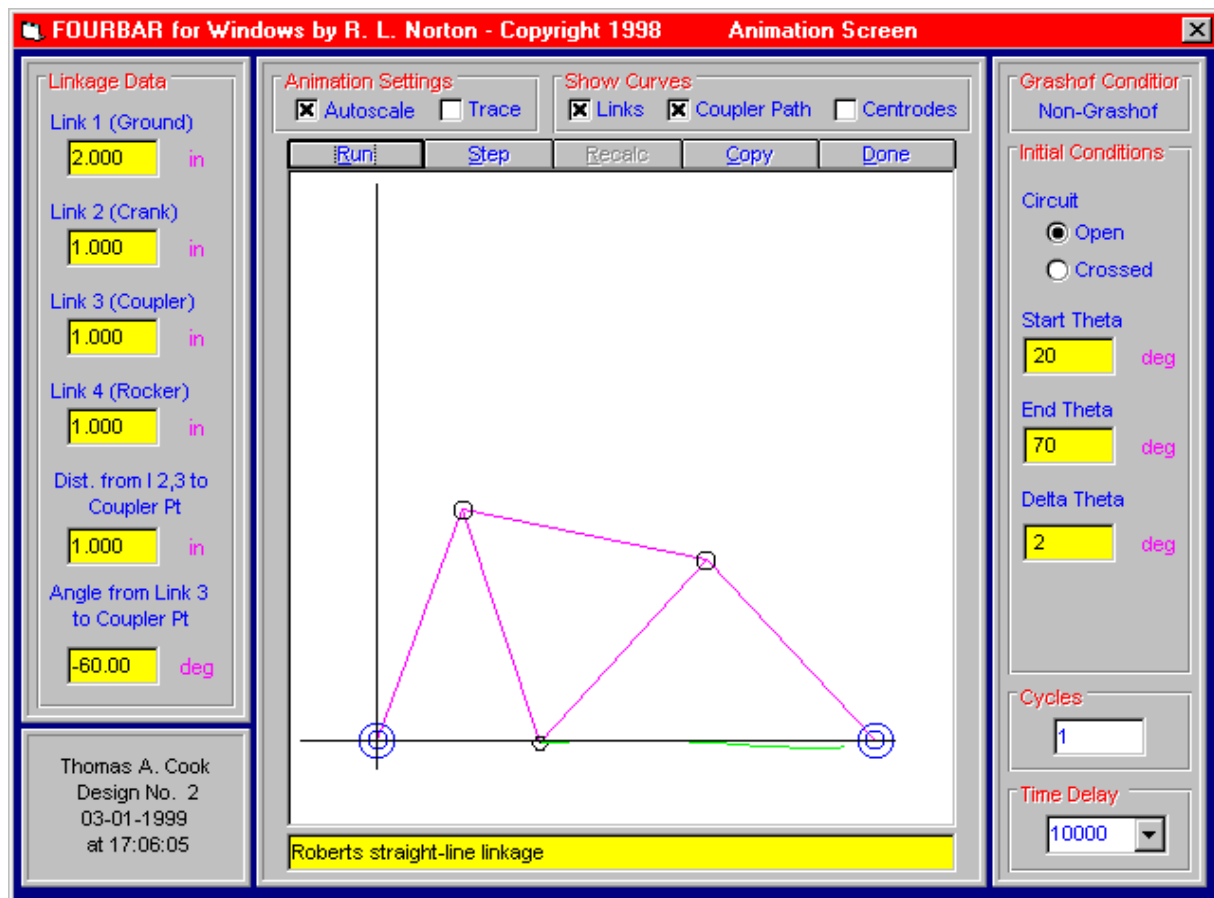
Given: Link lengths:

Ground link	$L_1 := 2$	Crank	$L_2 := 1$	$A1P := 1.000$	$\delta_1 := -60.0\text{-deg}$
Coupler	$L_3 := 1$	Rocker	$L_4 := 1$	$B1P := 1.000$	$\gamma_1 := -60.0\text{-deg}$

Coupler point data:

Solution: See Figure 3-29b and Mathcad file P0330.

1. Input the link dimensions and coupler point data into program FOURBAR.



2. Note that, for this mechanism, cognates #2 and #3 are identical with cognate #1 because of the symmetry of the linkage (draw the Cayley diagram to see this). All three mechanisms are non-Grashof with limited crank angles.

PROBLEM 3-31

Statement: Design a Hoeken straight-line linkage to give minimum error in velocity over 22% of the cycle for a 15-cm-long straight line motion. Specify all linkage parameters.

Given: Length of straight line motion: $\Delta x := 150 \cdot \text{mm}$
 Percentage of cycle over which straight line motion takes place: 22%

Solution: See Figure 3-30 and Mathcad file P0331.

1. Using Table 3-1 and the required length of straight-line motion, determine the link lengths.

Link ratios from Table 3-1 for 22% cycle:

$$L1\text{over}L2 := 1.975 \quad L3\text{over}L2 := 2.463 \quad \Delta x\text{over}L2 := 1.845$$

Link lengths:

$$\text{Crank} \quad L_2 := \frac{\Delta x}{\Delta x\text{over}L2} \quad L_2 = 81.30 \text{ mm}$$

$$\text{Coupler} \quad L_3 := L3\text{over}L2 \cdot L_2 \quad L_3 = 200.24 \text{ mm}$$

$$\text{Ground link} \quad L_1 := L1\text{over}L2 \cdot L_2 \quad L_1 = 160.57 \text{ mm}$$

$$\text{Rocker} \quad L_4 := L_3 \quad L_4 = 200.24 \text{ mm}$$

$$\text{Coupler point} \quad AP := 2 \cdot L_3 \quad AP = 400.49 \text{ mm}$$

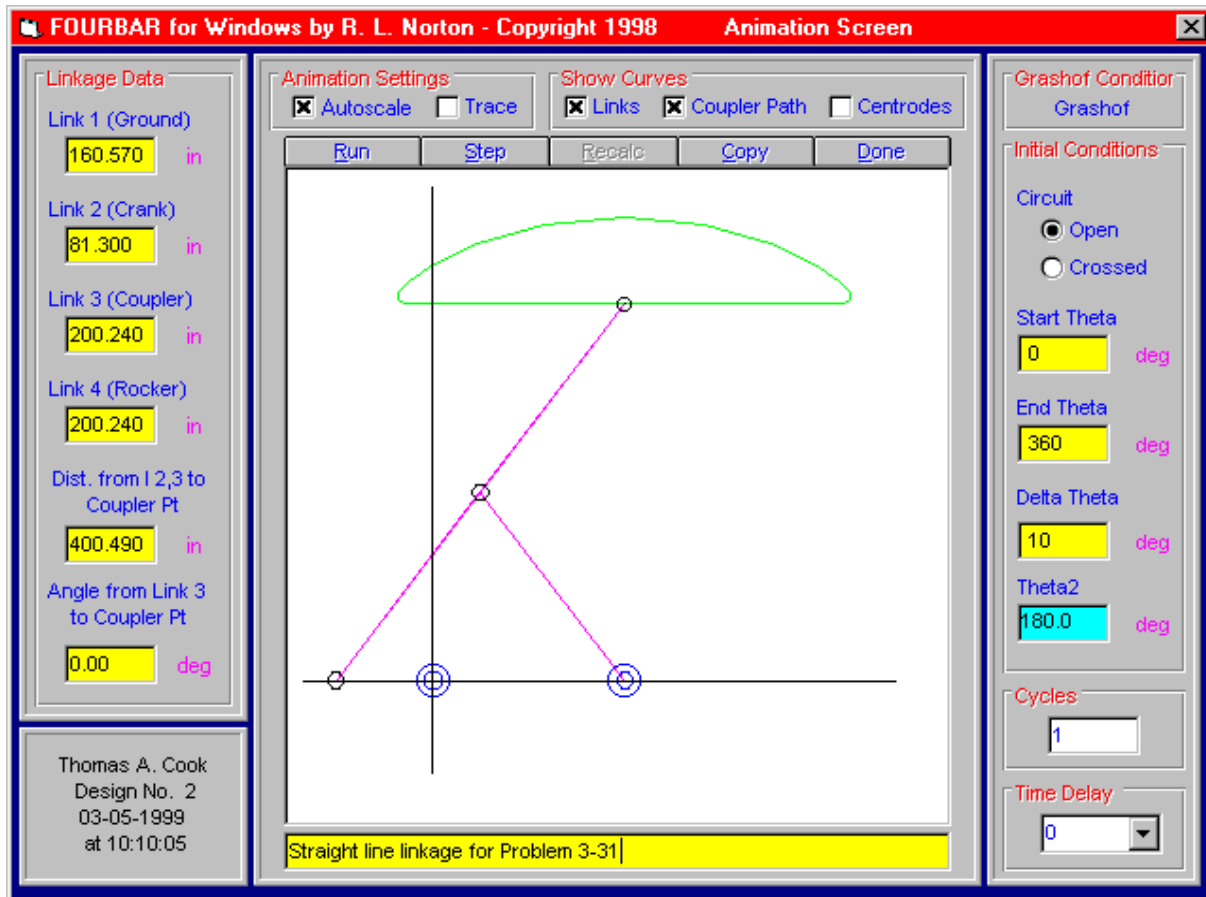
2. Calculate the distance from point P to pivot $O_4 (C_y)$ when crank angle is 180 deg.

$$C_y := \sqrt{(2 \cdot L_3)^2 - (L_1 + L_2)^2} \quad C_y = 319.20 \text{ mm}$$

3. Enter the link lengths into program FOURBAR to verify the design (see next page for coupler point curve). Using the PRINT facility, determine the x,y coordinates of the coupler curve and the x,y components of the coupler point velocity in the straight line region. A table of these values is printed below. Notice the small deviations over the range of crank angles from the y-coordinate and the x-velocity at a crank angle of 180 deg.

FOURBAR for Windows File P03-31.DOC

Angle Step Deg	Cpler Pt X mm	Cpler Pt Y mm	Veloc CP X mm/sec	Veloc CP Y mm/sec
140	235.60	319.95	-1,072.61	-10.73
150	216.84	319.72	-1,076.20	-14.74
160	198.06	319.46	-1,075.51	-13.54
170	179.31	319.27	-1,073.75	-7.99
180	160.58	319.20	-1,072.93	0.02
190	141.85	319.27	-1,073.75	8.03
200	123.09	319.47	-1,075.52	13.58
210	104.31	319.72	-1,076.22	14.78
220	85.55	319.95	-1,072.63	10.76



PROBLEM 3-32

Statement: Design a Hoeken straight-line linkage to give minimum error in straightness over 39% of the cycle for a 20-cm-long straight line motion. Specify all linkage parameters.

Given: Length of straight line motion: $\Delta x := 200 \cdot mm$
 Percentage of cycle over which straight line motion takes place: 39%

Solution: See Figure 3-30 and Mathcad file P0332.

1. Using Table 3-1 and the required length of straight-line motion, determine the link lengths.

Link ratios from Table 3-1 for 39% cycle:

$$L1overL2 := 2.500 \quad L3overL2 := 3.250 \quad \Delta x over L2 := 3.623$$

Link lengths:

$$\text{Crank} \quad L_2 := \frac{\Delta x}{\Delta x over L2} \quad L_2 = 55.20 \text{ mm}$$

$$\text{Coupler} \quad L_3 := L3overL2 \cdot L_2 \quad L_3 = 179.41 \text{ mm}$$

$$\text{Ground link} \quad L_1 := L1overL2 \cdot L_2 \quad L_1 = 138.01 \text{ mm}$$

$$\text{Rocker} \quad L_4 := L_3 \quad L_4 = 179.41 \text{ mm}$$

$$\text{Coupler point} \quad AP := 2 \cdot L_3 \quad AP = 358.82 \text{ mm}$$

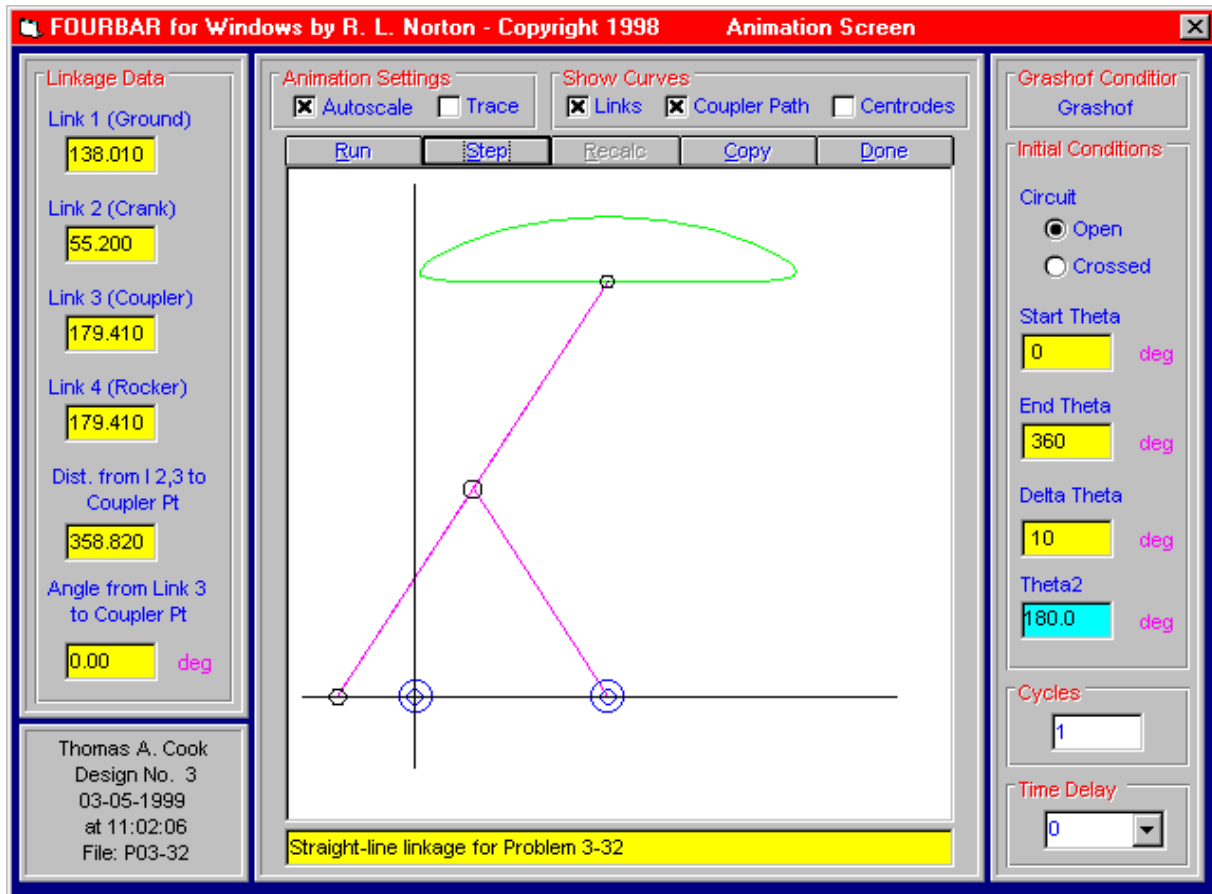
2. Calculate the distance from point P to pivot $O_4 (C_y)$ when crank angle is 180 deg.

$$C_y := \sqrt{(2 \cdot L_3)^2 - (L_1 + L_2)^2} \quad C_y = 302.36 \text{ mm}$$

3. Enter the link lengths into program FOURBAR to verify the design (see next page for coupler point curve). Using the PRINT facility, determine the x,y coordinates of the coupler curve and the x,y components of the coupler point velocity in the straight line region. A table of these values is printed below. Notice the small deviations over the range of crank angles from the y-coordinate and the x-velocity from a crank angle of 180 deg.

FOURBAR for Windows File P03-32.DAT

Angle Step Deg	Coupler Pt X mm	Coupler Pt Y mm	Veloc CP X mm/sec	Veloc CP Y mm/sec
110	237.992	302.408	-696.591	-6.416
120	225.289	302.361	-755.847	-0.019
130	211.710	302.378	-797.695	1.426
140	197.521	302.398	-826.217	0.664
150	182.927	302.399	-844.774	-0.483
160	168.076	302.385	-856.043	-1.052
170	153.076	302.368	-861.994	-0.800
180	138.010	302.360	-863.841	0.000
190	122.944	302.368	-861.994	0.800
200	107.944	302.385	-856.043	1.052
210	93.093	302.399	-844.774	0.483
220	78.499	302.398	-826.217	-0.664
230	64.311	302.378	-797.695	-1.426
240	50.731	302.361	-755.847	0.019
250	38.028	302.408	-696.591	6.416



PROBLEM 3-33

Statement: Design a linkage that will give a symmetrical "kidney bean" shaped coupler curve as shown in Figure 3-16 (p. 114 and 115). Use the data in Figure 3-21 (p. 120) to determine the required link ratios and generate the coupler curve with program FOURBAR.

Solution: See Figures 3-16, 3-21, and Mathcad file P0333.

Design choices:

Ground link ratio, $L_1/L_2 = 2.0$: $GLR := 2.0$

Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.5$: $CLR := 2.5$

Coupler angle, $\gamma := 72 \cdot \text{deg}$

Crank length, $L_2 := 2.000$

- For the given design choices, determine the remaining link lengths and coupler point specification.

Coupler link (3) length $L_3 := CLR \cdot L_2$ $L_3 = 5.000$

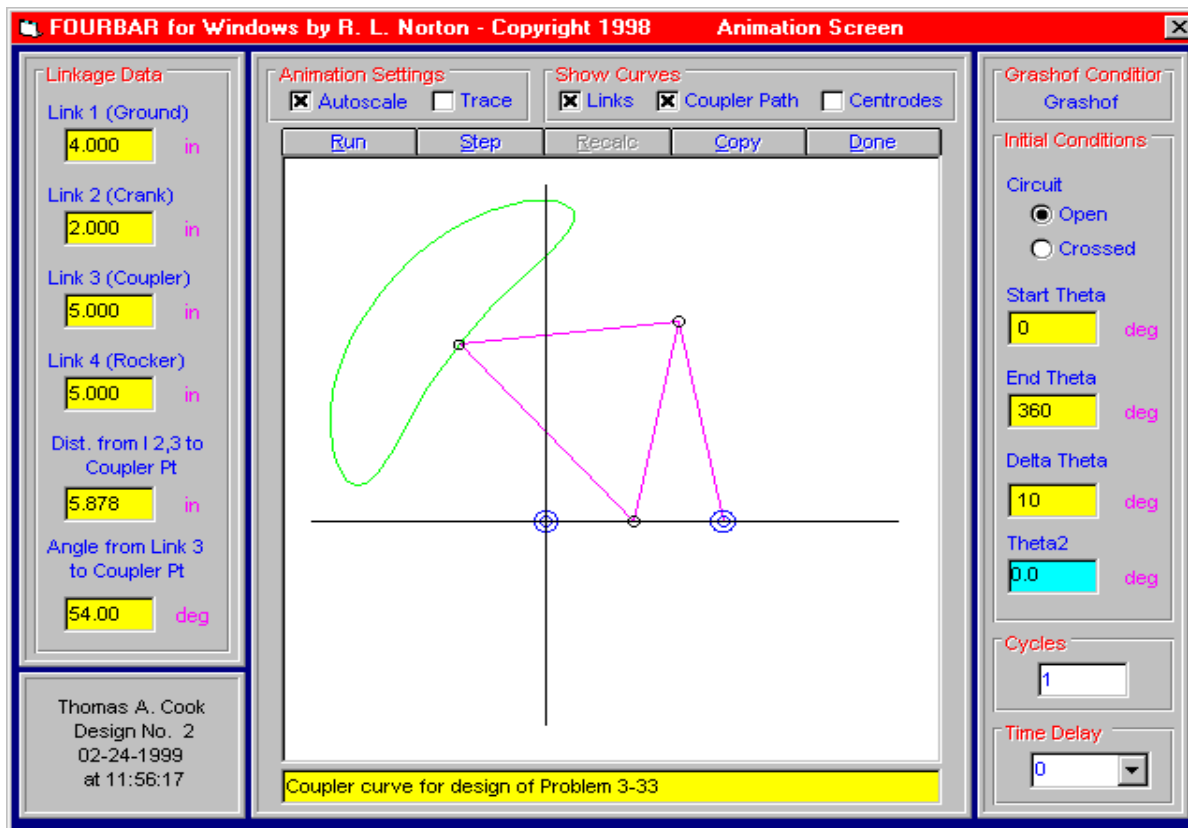
Rocker link (4) length $L_4 := CLR \cdot L_2$ $L_4 = 5.000$

Ground link (1) length $L_1 := GLR \cdot L_2$ $L_1 = 4.000$

Angle PAB $\delta := \frac{180 \cdot \text{deg} - \gamma}{2}$ $\delta = 54.000 \text{ deg}$

Length AP on coupler $AP := 2 \cdot L_3 \cdot \cos(\delta)$ $AP = 5.878$

- Enter the above data into program FOURBAR and plot the coupler curve.



PROBLEM 3-34

Statement: Design a linkage that will give a symmetrical "double straight" shaped coupler curve as shown in Figure 3-16. Use the data in Figure 3-21 to determine the required link ratios and generate the coupler curve with program FOURBAR.

Solution: See Figures 3-16, 3-21, and Mathcad file P0334.

Design choices:

Ground link ratio, $L_1/L_2 = 2.5$: $GLR := 2.5$

Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.5$: $CLR := 2.5$

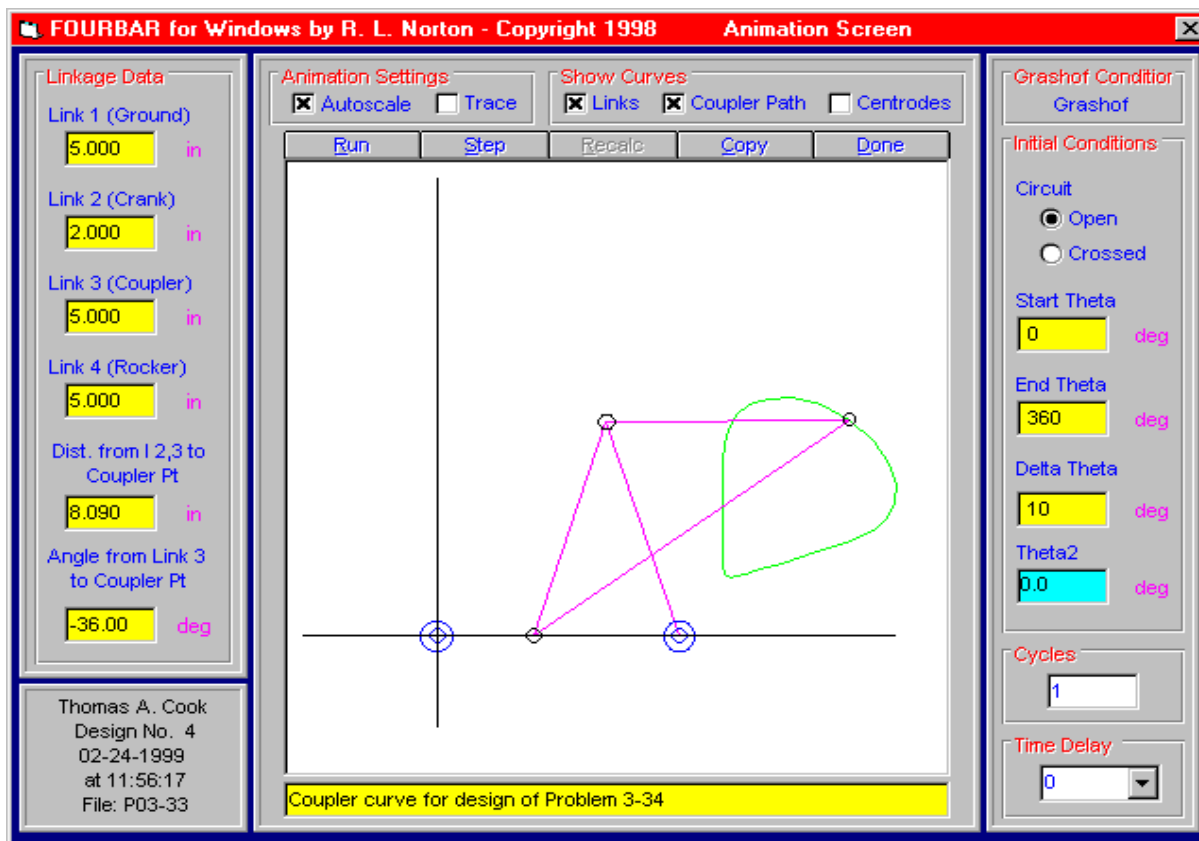
Coupler angle, $\gamma := 252 \cdot \text{deg}$

Crank length, $L_2 := 2.000$

- For the given design choices, determine the remaining link lengths and coupler point specification.

Coupler link (3) length	$L_3 := CLR \cdot L_2$	$L_3 = 5.000$
Rocker link (4) length	$L_4 := CLR \cdot L_2$	$L_4 = 5.000$
Ground link (1) length	$L_1 := GLR \cdot L_2$	$L_1 = 5.000$
Angle PAB	$\delta := \frac{180 \cdot \text{deg} - \gamma}{2}$	$\delta = -36.000 \text{ deg}$
Length AP on coupler	$AP := 2 \cdot L_3 \cdot \cos(\delta)$	$AP = 8.090$

- Enter the above data into program FOURBAR and plot the coupler curve.



PROBLEM 3-35

Statement: Design a linkage that will give a symmetrical "scimitar" shaped coupler curve as shown in Figure 3-16. Use the data in Figure 3-21 to determine the required link ratios and generate the coupler curve with program FOURBAR. Show that there are (or are not) true cusps on the curve.

Solution: See Figures 3-16, 3-21, and Mathcad file P0334.

Design choices:

Ground link ratio, $L_1/L_2 = 2.0$: $GLR := 2.0$

Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.5$: $CLR := 2.5$

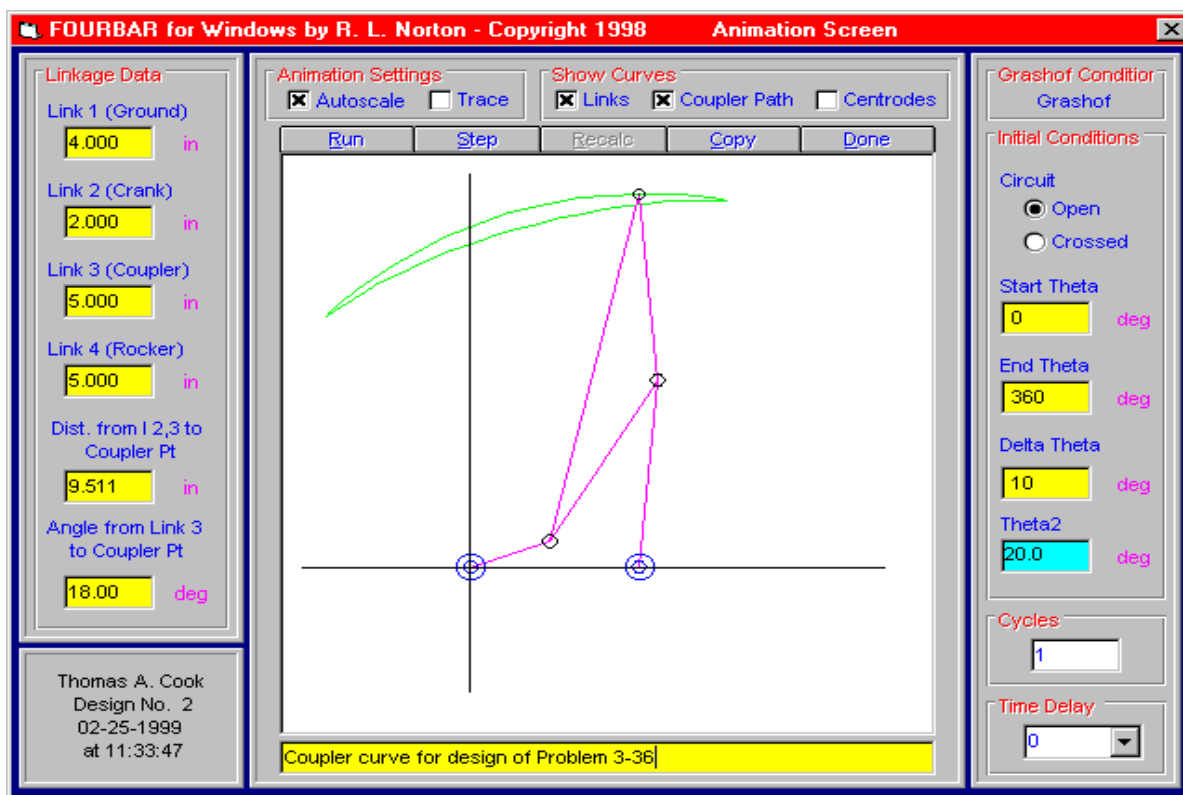
Coupler angle, $\gamma := 144 \cdot \text{deg}$

Crank length, $L_2 := 2.000$

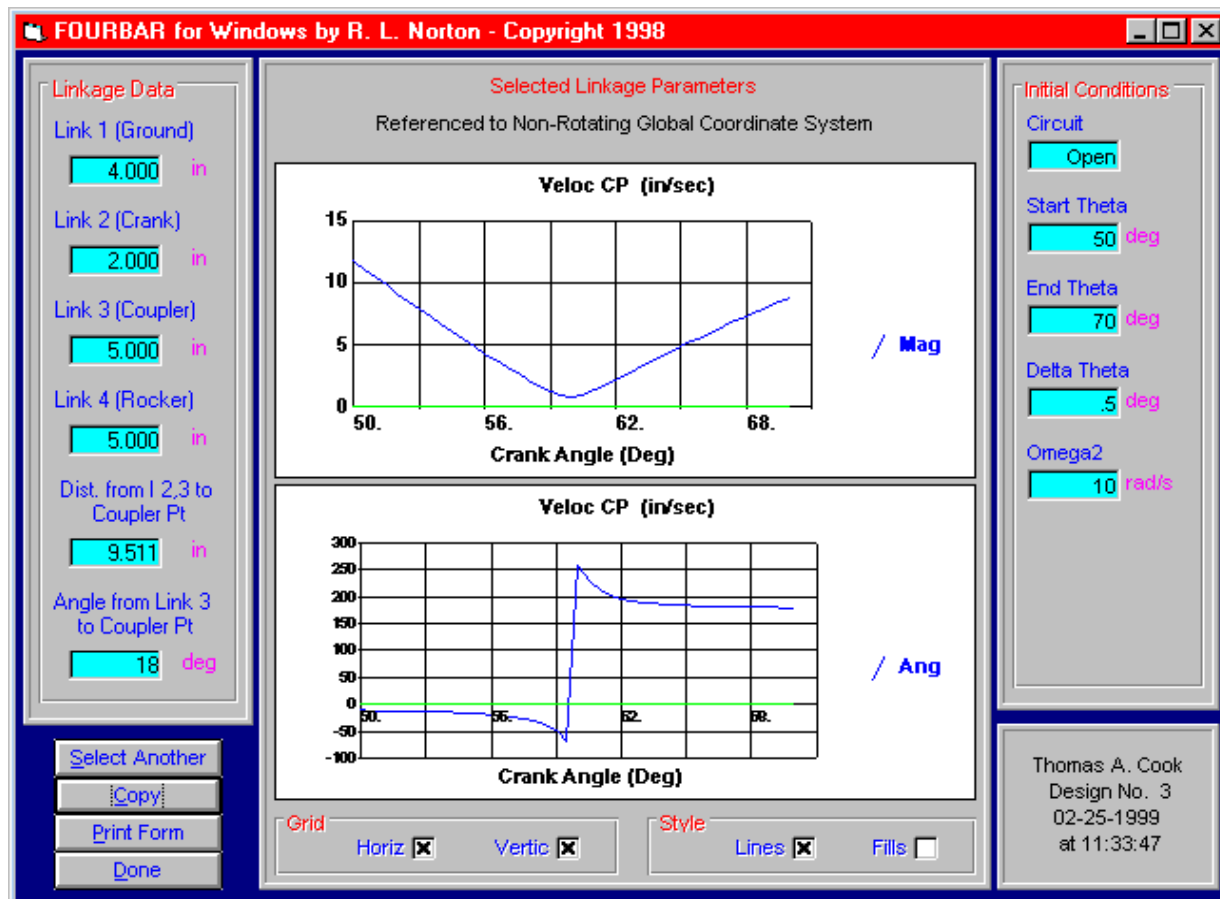
- For the given design choices, determine the remaining link lengths and coupler point specification.

Coupler link (3) length	$L_3 := CLR \cdot L_2$	$L_3 = 5.000$
Rocker link (4) length	$L_4 := CLR \cdot L_2$	$L_4 = 5.000$
Ground link (1) length	$L_1 := GLR \cdot L_2$	$L_1 = 4.000$
Angle PAB	$\delta := \frac{180 \cdot \text{deg} - \gamma}{2}$	$\delta = 18.000 \text{ deg}$
Length AP on coupler	$AP := 2 \cdot L_3 \cdot \cos(\delta)$	$AP = 9.511$

- Enter the above data into program FOURBAR and plot the coupler curve.



3. The points at the ends of the "scimitar" will be true cusps if the velocity of the coupler point is zero at these points. Using FOURBAR's plotting utility, plot the magnitude and angle of the coupler point velocity vector. As seen below for the range of crank angle from 50 to 70 degrees, the magnitude of the velocity does not quite reach zero. Therefore, these are not true cusps.



PROBLEM 3-36

Statement: Find the Grashof condition, inversion, any limit positions, and the extreme values of the transmission angle (to graphical accuracy) of the linkage in Figure P3-10.

Given: Link lengths: Link 2 $L_2 := 0.785$ Link 3 $L_3 := 0.356$
 Link 4 $L_4 := 0.950$ Link 1 $L_1 := 0.544$

Grashof condition function:

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise
  
```

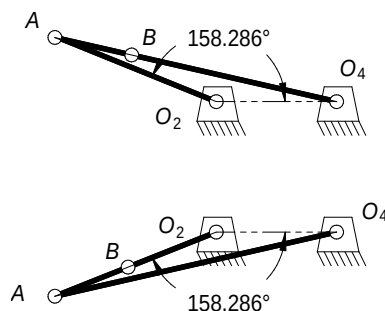
Solution: See Figure P3-10 and Mathcad file P0336.

1. Determine the Grashof condition of the mechanism from inequality 2.8 and its Barker classification from Table 2-4.

Grashof condition: $Condition(L_1, L_2, L_3, L_4) = \text{"Grashof"}$

Barker classification: Class I-3, Grashof rocker-crank-rocker, GRCR, since the shortest link is the coupler link.

2. A GRCR linkage will have two toggle positions. Draw the linkage in these two positions and measure the input link angles.



3. As measured from the layout, the input link angles at the toggle positions are: +158.3 and -158.3 deg.
4. Since the coupler link in a GRCR linkage can make a full rotation with respect to the input and output rockers, the minimum transmission angle is 0 deg and the maximum is 90 deg.

PROBLEM 3-37

Statement: Draw the Roberts diagram and find the cognates for the linkage in Figure P3-10.

Given: Link lengths: Coupler point data:
 Ground link $L_1 := 0.544$ Crank $L_2 := 0.785$ $A1P := 1.09$ $\delta_1 := 0.00 \cdot \text{deg}$
 Coupler $L_3 := 0.356$ Rocker $L_4 := 0.950$

Solution: See Figure P3-10 and Mathcad file P0337.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$B1P := \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} \quad B1P = 0.734$$

$$\gamma_1 := \arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) \quad \gamma_1 = 180.0000 \text{ deg}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \quad L_5 = 0.734 \quad L_6 := \frac{L_4}{L_3} \cdot B1P \quad L_6 = 1.959$$

$$L_{10} := A1P \quad L_{10} = 1.090 \quad L_9 := \frac{L_2}{L_3} \cdot A1P \quad L_9 = 2.404$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \quad L_7 = 1.619 \quad L_8 := L_6 \cdot \frac{A1P}{B1P} \quad L_8 = 2.909$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_4 \quad A3P = 0.950 \quad A2P := L_2 \quad A2P = 0.785$$

$$\delta_3 := 180 \cdot \text{deg} - \delta_1 \quad \delta_3 = 180.000 \text{ deg} \quad \delta_2 := -\delta_1 \quad \delta_2 = 0.000 \text{ deg}$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

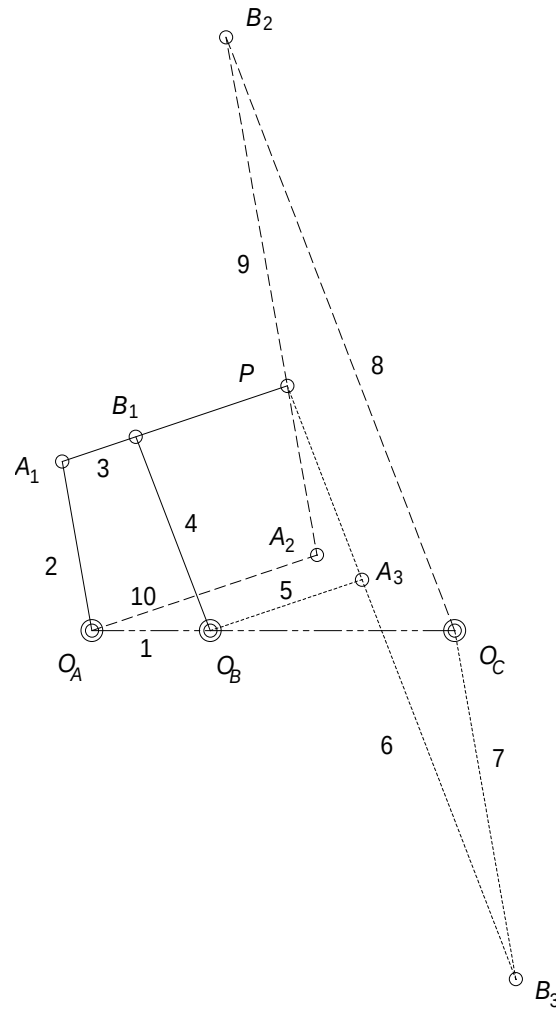
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \quad L_{1BC} = 1.1216 \quad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \quad L_{1AC} = 1.6656$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 0.544$	$L_{1AC} = 1.666$	$L_{1BC} = 1.122$
Crank length	$L_2 = 0.785$	$L_{10} = 1.090$	$L_7 = 1.619$
Coupler length	$L_3 = 0.356$	$L_9 = 2.404$	$L_6 = 1.959$

Rocker length	$L_4 = 0.950$	$L_8 = 2.909$	$L_5 = 0.734$
Coupler point	$A_1P = 1.090$	$A_2P = 0.785$	$A_3P = 0.950$
Coupler angle	$\delta_1 = 0.000 \text{ deg}$	$\delta_2 = 0.000 \text{ deg}$	$\delta_3 = 180.000 \text{ deg}$



PROBLEM 3-38

Statement: Find the three geared fivebar cognates of the linkage in Figure P3-10.

Given: Link lengths: Coupler point data:
 Ground link $L_1 := 0.544$ Crank $L_2 := 0.785$ $A1P := 1.09$ $\delta_1 := 0.00 \cdot \text{deg}$
 Coupler $L_3 := 0.356$ Rocker $L_4 := 0.950$

Solution: See Figure P3-10 and Mathcad file P0338.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$B1P := \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} \quad B1P = 0.734$$

$$\gamma_1 := \arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) \quad \gamma_1 = 180.0000 \text{ deg}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \quad L_5 = 0.734 \quad L_6 := \frac{L_4}{L_3} \cdot B1P \quad L_6 = 1.959$$

$$L_{10} := A1P \quad L_{10} = 1.090 \quad L_9 := \frac{L_2}{L_3} \cdot A1P \quad L_9 = 2.404$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \quad L_7 = 1.619 \quad L_8 := L_6 \cdot \frac{A1P}{B1P} \quad L_8 = 2.909$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_4 \quad A3P = 0.950 \quad A2P := L_2 \quad A2P = 0.785$$

$$\delta_3 := 180 \cdot \text{deg} - \delta_1 \quad \delta_3 = 180.000 \text{ deg} \quad \delta_2 := -\delta_1 \quad \delta_2 = 0.000 \text{ deg}$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \quad L_{1BC} = 1.1216 \quad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \quad L_{1AC} = 1.6656$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

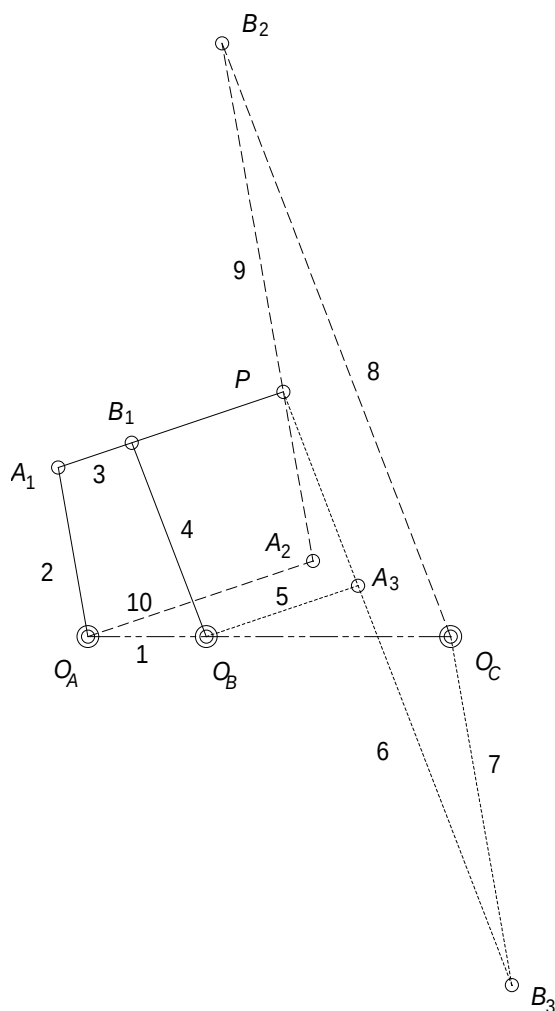
	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 0.544$	$L_{1AC} = 1.666$	$L_{1BC} = 1.122$
Crank length	$L_2 = 0.785$	$L_{10} = 1.090$	$L_7 = 1.619$
Coupler length	$L_3 = 0.356$	$L_9 = 2.404$	$L_6 = 1.959$
Rocker length	$L_4 = 0.950$	$L_8 = 2.909$	$L_5 = 0.734$
Coupler point	$A1P = 1.090$	$A2P = 0.785$	$A3P = 0.950$

Coupler angle

$\delta_1 = 0.000 \text{ deg}$

$\delta_2 = 0.000 \text{ deg}$

$\delta_3 = 180.000 \text{ deg}$

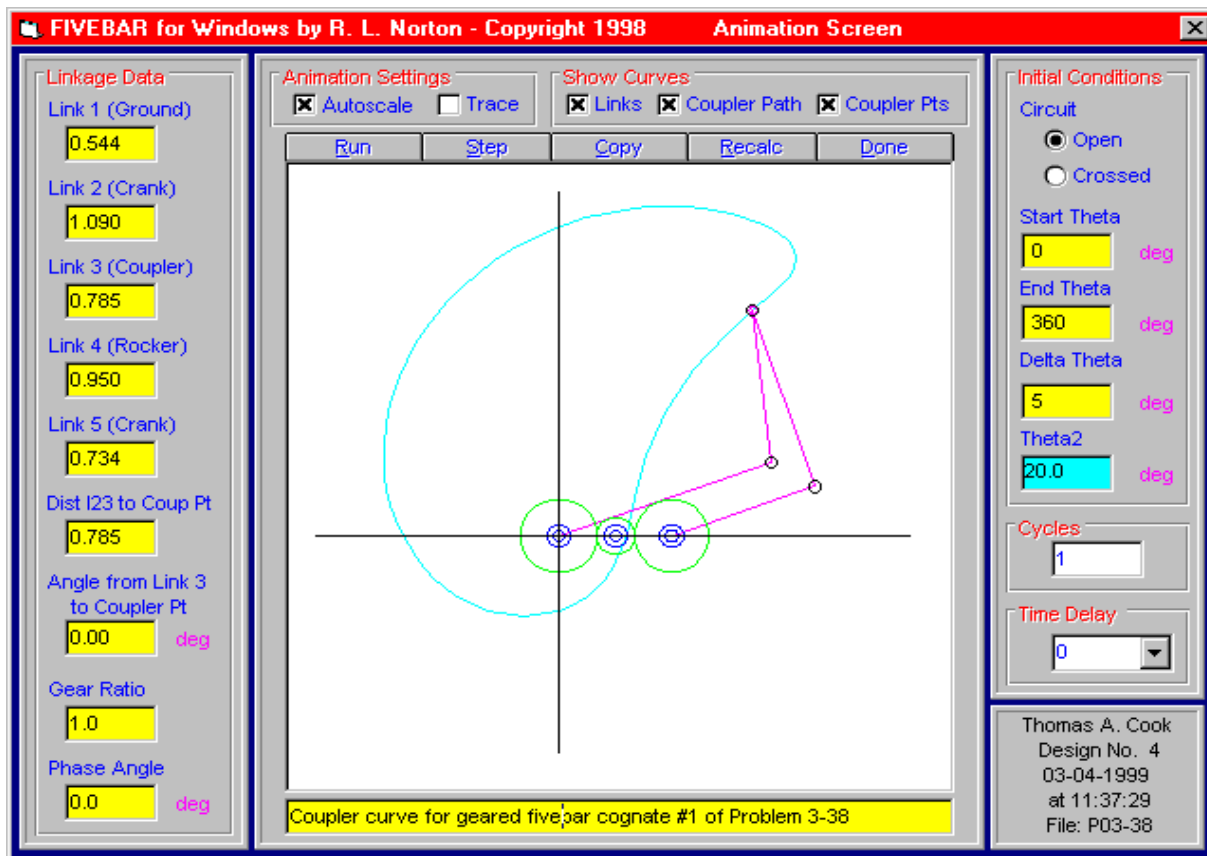
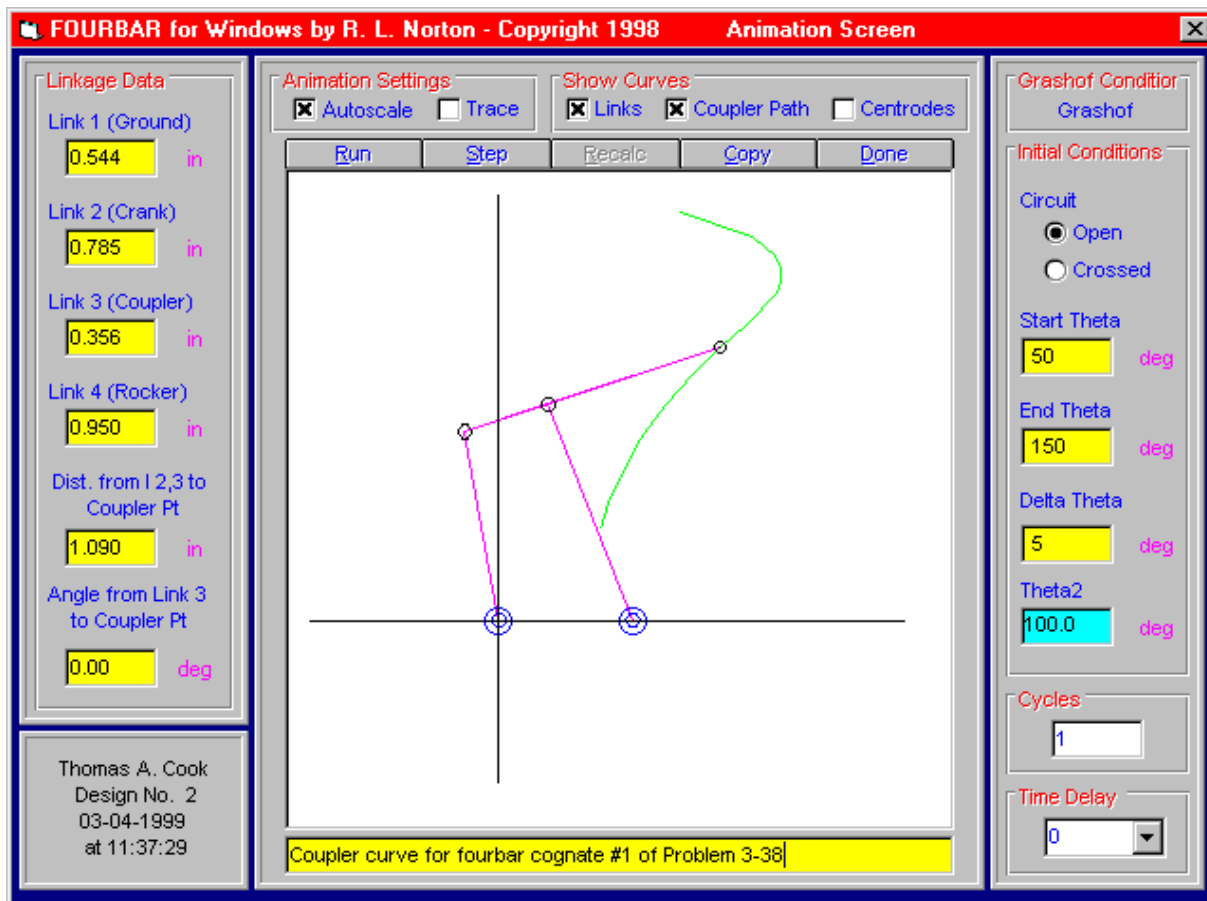


4. The three geared fivebar cognates can be seen in the Roberts diagram. They are: $O_A A_2 P A_3 O_B$, $O_A A_1 P B_3 O_C$, and $O_B B_1 P B_2 O_C$. They are specified in the summary table below.

SUMMARY OF GEARED FIVEBAR COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 0.544$	$L_{1AC} = 1.666$	$L_{1BC} = 1.122$
Crank length	$L_{10} = 1.090$	$L_2 = 0.785$	$L_4 = 0.950$
Coupler length	$A_2P = 0.785$	$A_1P = 1.090$	$L_5 = 0.734$
Rocker length	$A_3P = 0.950$	$L_8 = 2.909$	$L_7 = 1.619$
Crank length	$L_5 = 0.734$	$L_7 = 1.619$	$L_8 = 2.909$
Coupler point	$A_2P = 0.785$	$A_1P = 1.090$	$B_1P = 0.734$
Coupler angle	$\delta_1 := 0.00 \cdot \text{deg}$	$\delta_2 := 0.00 \cdot \text{deg}$	$\delta_3 := 0.00 \cdot \text{deg}$

5. Enter the cognate #1 specifications into program FOURBAR to get a trace of the coupler path (see next page).
6. Enter the geared fivebar cognate #1 specifications into program FIVEBAR to get a trace of the coupler path for the geared fivebar (see next page).



PROBLEM 3-39

Statement: Find the Grashof condition, any limit positions, and the extreme values of the transmission angle (to graphical accuracy) of the linkage in Figure P3-11.

Given: Link lengths: Link 2 $L_2 := 0.86$ Link 3 $L_3 := 1.85$
 Link 4 $L_4 := 0.86$ Link 1 $L_1 := 2.22$

Grashof condition function:

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof"   if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise
  
```

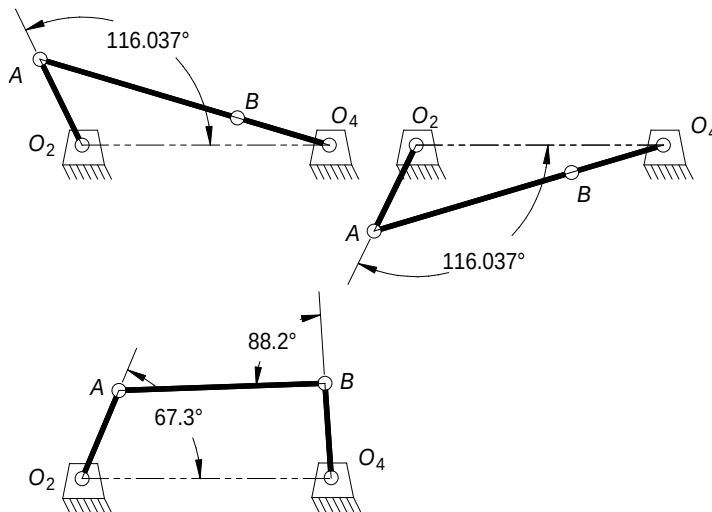
Solution: See Figure P3-11 and Mathcad file P0339.

1. Determine the Grashof condition of the mechanism from inequality 2.8 and its Barker classification from Table 2-4.

Grashof condition: $Condition(L_1, L_2, L_3, L_4) = \text{"non-Grashof"}$

Barker classification: Class II-1, non-Grashof triple rocker, RRR1, since the longest link is the ground link.

2. An RRR1 linkage will have two toggle positions. Draw the linkage in these two positions and measure the input link angles.



3. As measured from the layout, the input link angles at the toggle positions are: +116 and -116 deg.
4. Since the coupler link in an RRR1 linkage cannot make a full rotation with respect to the input and output rockers, the minimum transmission angle is 0 deg and the maximum is 88 deg.

PROBLEM 3-40

Statement: Draw the Roberts diagram and find the cognates for the linkage in Figure P3-11.

Given: Link lengths: Coupler point data:

Ground link $L_1 := 2.22$ Crank $L_2 := 0.86$ $A1P := 1.33$ $\delta_1 := 0.00 \cdot \text{deg}$

Coupler $L_3 := 1.85$ Rocker $L_4 := 0.86$

Solution: See Figure P3-11 and Mathcad file P0340.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$B1P := \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} \quad B1P = 0.520$$

$$\gamma_1 := \arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) \quad \gamma_1 = 0.0000 \text{ deg}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \quad L_5 = 0.520 \quad L_6 := \frac{L_4}{L_3} \cdot B1P \quad L_6 = 0.242$$

$$L_{10} := A1P \quad L_{10} = 1.330 \quad L_9 := \frac{L_2}{L_3} \cdot A1P \quad L_9 = 0.618$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \quad L_7 = 0.242 \quad L_8 := L_6 \cdot \frac{A1P}{B1P} \quad L_8 = 0.618$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_8 \quad A3P = 0.618 \quad A2P := L_7 \quad A2P = 0.242$$

$$\delta_3 := 180 \cdot \text{deg} \quad \delta_3 = 180.000 \text{ deg} \quad \delta_2 := 180 \cdot \text{deg} \quad \delta_2 = 180.000 \text{ deg}$$

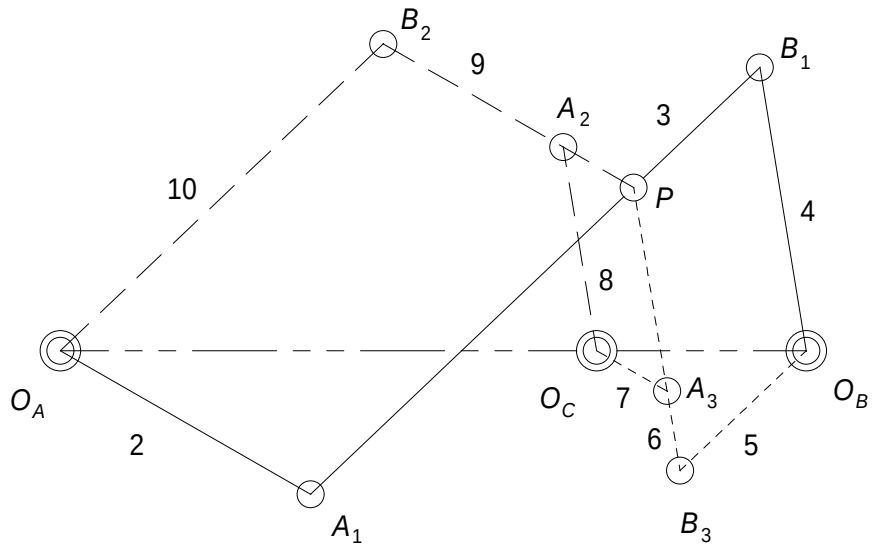
From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \quad L_{1BC} = 0.6240 \quad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \quad L_{1AC} = 1.5960$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 1.596$	$L_{1BC} = 0.624$
Crank length	$L_2 = 0.860$	$L_{10} = 1.330$	$L_7 = 0.242$
Coupler length	$L_3 = 1.850$	$L_9 = 0.618$	$L_6 = 0.242$
Rocker length	$L_4 = 0.860$	$L_8 = 0.618$	$L_5 = 0.520$
Coupler point	$A1P = 1.330$	$A2P = 0.242$	$A3P = 0.618$
Coupler angle	$\delta_1 = 0.000 \text{ deg}$	$\delta_2 = 180.000 \text{ deg}$	$\delta_3 = 180.000 \text{ deg}$



PROBLEM 3-41

Statement: Find the three geared fivebar cognates of the linkage in Figure P3-11.

Given: Link lengths:

Coupler point data:

$$\begin{array}{llllll} \text{Ground link} & L_1 := 2.22 & \text{Crank} & L_2 := 0.86 & A1P := 1.33 & \delta_1 := 0.00 \cdot \text{deg} \\ \text{Coupler} & L_3 := 1.85 & \text{Rocker} & L_4 := 0.86 & & \end{array}$$

Solution: See Figure P3-11 and Mathcad file P0341.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$\begin{aligned} B1P &:= \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} & B1P &= 0.520 \\ \gamma_1 &:= \arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) & \gamma_1 &= 0.0000 \text{ deg} \end{aligned}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \quad L_5 = 0.520 \quad L_6 := \frac{L_4}{L_3} \cdot B1P \quad L_6 = 0.242$$

$$L_{10} := A1P \quad L_{10} = 1.330 \quad L_9 := \frac{L_2}{L_3} \cdot A1P \quad L_9 = 0.618$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \quad L_7 = 0.242 \quad L_8 := L_6 \cdot \frac{A1P}{B1P} \quad L_8 = 0.618$$

Calculate the coupler point data for cognates #2 and #3

$$A3P := L_8 \quad A3P = 0.618 \quad A2P := L_7 \quad A2P = 0.242$$

$$\delta_3 := 180 \cdot \text{deg} \quad \delta_3 = 180.000 \text{ deg} \quad \delta_2 := 180 \cdot \text{deg} \quad \delta_2 = 180.000 \text{ deg}$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

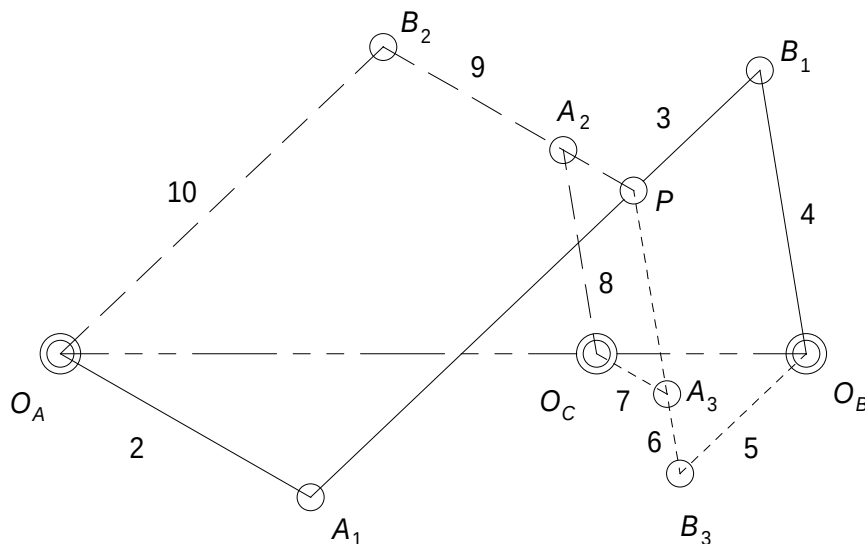
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \quad L_{1BC} = 0.6240 \quad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \quad L_{1AC} = 1.5960$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 1.596$	$L_{1BC} = 0.624$
Crank length	$L_2 = 0.860$	$L_{10} = 1.330$	$L_7 = 0.242$
Coupler length	$L_3 = 1.850$	$L_9 = 0.618$	$L_6 = 0.242$
Rocker length	$L_4 = 0.860$	$L_8 = 0.618$	$L_5 = 0.520$

Coupler point	$A_1P = 1.330$	$A_2P = 0.242$	$A_3P = 0.618$
Coupler angle	$\delta_1 = 0.000 \text{ deg}$	$\delta_2 = 180.000 \text{ deg}$	$\delta_3 = 180.000 \text{ deg}$

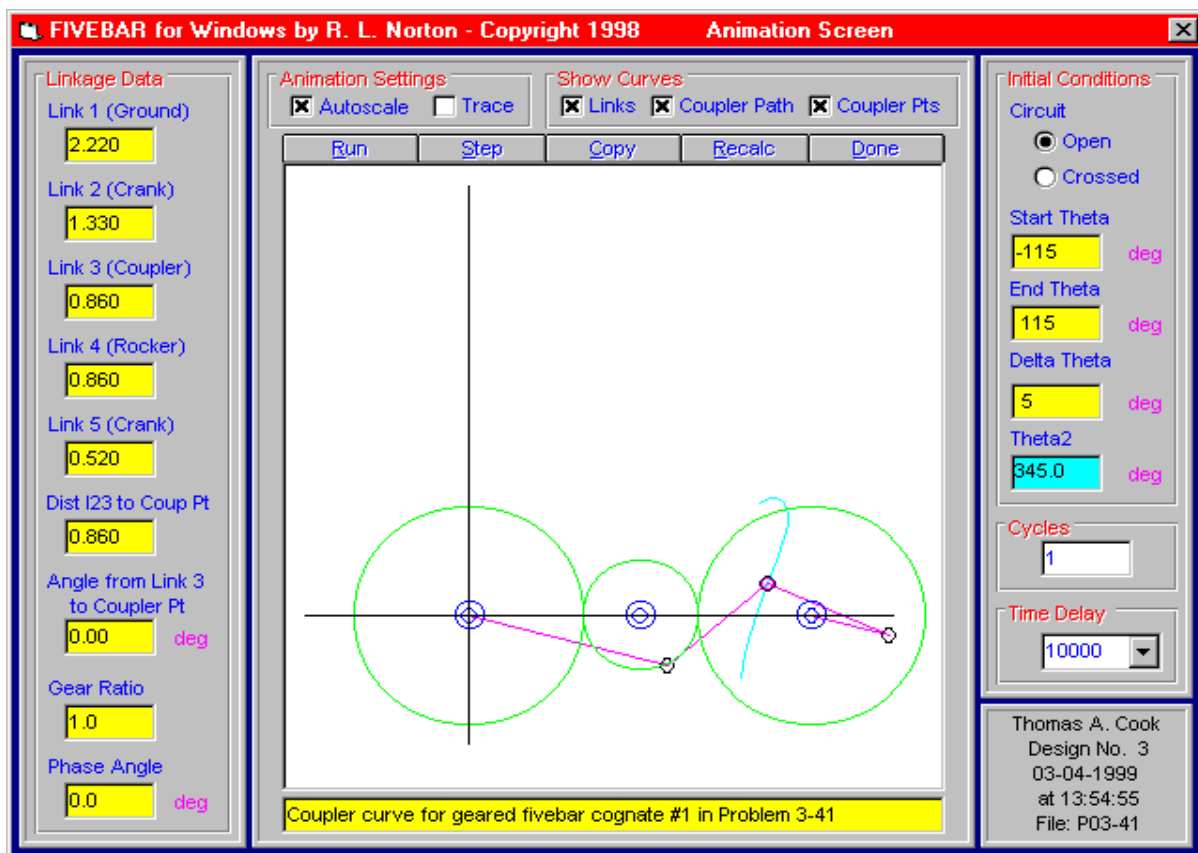
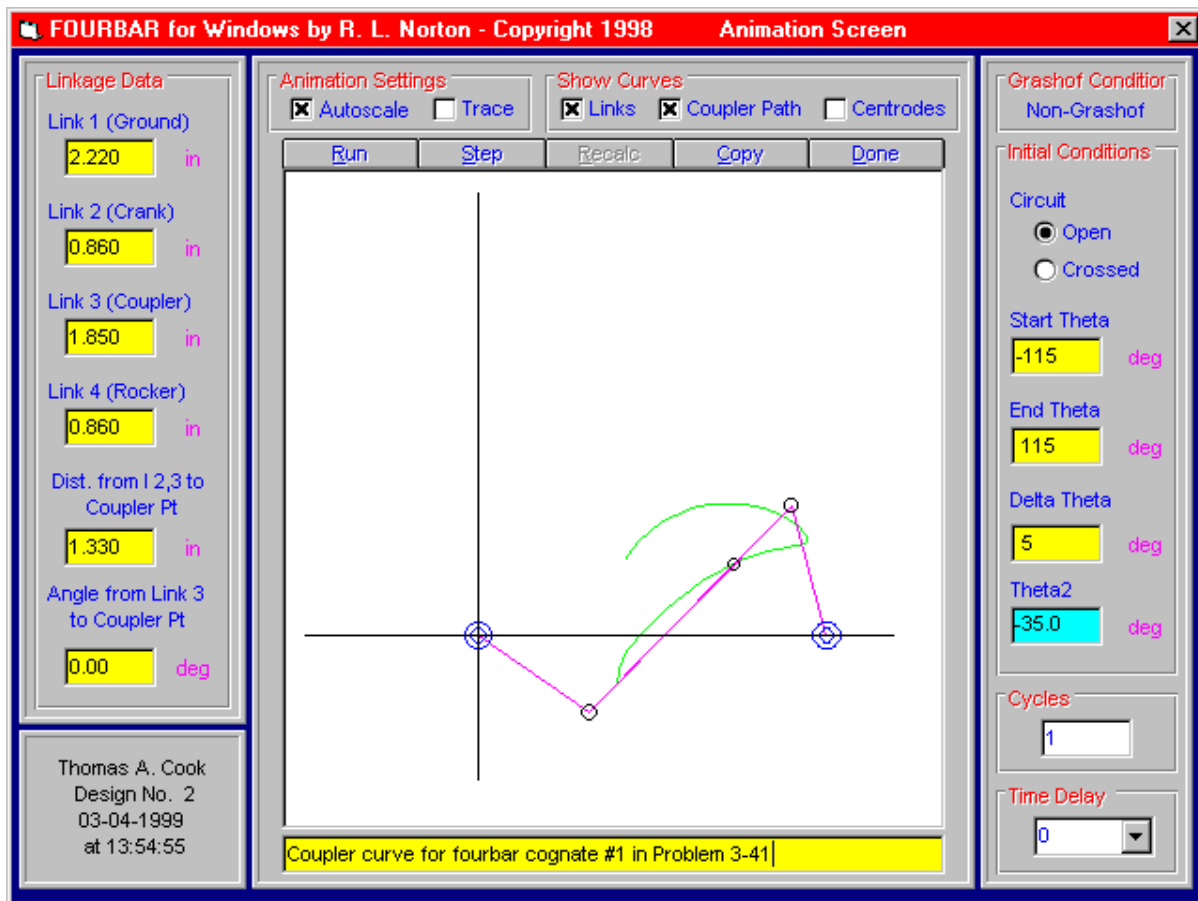


4. The three geared fivebar cognates can be seen in the Roberts diagram. They are: $O_A B_2 P B_3 O_B$, $O_A A_1 P A_3 O_C$, and $O_B B_1 P A_2 O_C$. The three geared fivebar cognates are summarized in the table below.

SUMMARY OF GEARED FIVEBAR COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 2.220$	$L_{1AC} = 1.596$	$L_{1BC} = 0.624$
Crank length	$L_{10} = 1.330$	$L_2 = 0.860$	$L_4 = 0.860$
Coupler length	$L_2 = 0.860$	$A_1P = 1.330$	$L_5 = 0.520$
Rocker length	$L_4 = 0.860$	$L_8 = 0.618$	$L_7 = 0.242$
Crank length	$L_5 = 0.520$	$L_7 = 0.242$	$L_8 = 0.618$
Coupler point	$L_2 = 0.860$	$A_1P = 1.330$	$B_1P = 0.520$
Coupler angle	$\delta_1 := 0.00 \cdot \text{deg}$	$\delta_2 := 0.00 \cdot \text{deg}$	$\delta_3 := 0.00 \cdot \text{deg}$

- Enter the cognate #1 specifications into program FOURBAR to get a trace of the coupler path (see next page)
- Enter the geared fivebar cognate #1 specifications into program FIVEBAR to get a trace of the coupler path for the geared fivebar (see next page).



PROBLEM 3-42

Statement: Find the Grashof condition, any limit positions, and the extreme values of the transmission angle (to graphical accuracy) of the linkage in Figure P3-12.

Given: Link lengths: Link 2 $L_2 := 0.72$ Link 3 $L_3 := 0.68$
Link 4 $L_4 := 0.85$ Link 1 $L_1 := 1.82$

Grashof condition function:

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

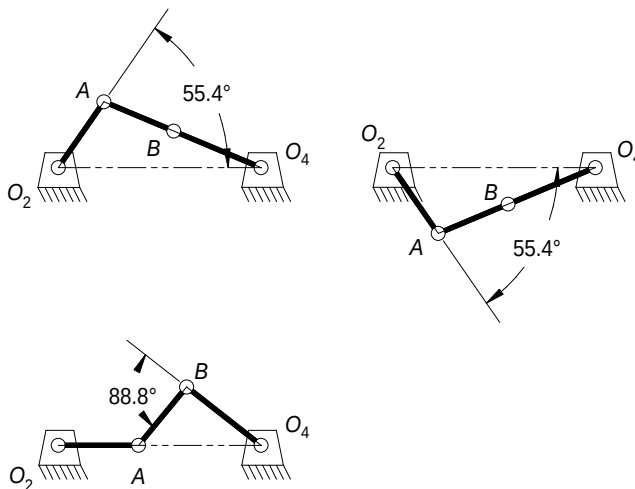
Solution: See Figure P3-12 and Mathcad file P0342.

1. Determine the Grashof condition of the mechanism from inequality 2.8 and its Barker classification from Table 2-4.

Grashof condition: $Condition(L_1, L_2, L_3, L_4) = \text{"non-Grashof"}$

Barker classification: Class II-1, non-Grashof triple rocker, RRR1, since the longest link is the ground link.

2. An RRR1 linkage will have two toggle positions. Draw the linkage in these two positions and measure the input link angles.



3. As measured from the layout, the input link angles at the toggle positions are: +55.4 and -55.4 deg.
4. Since the coupler link in an RRR1 linkage it cannot make a full rotation with respect to the input and output rockers, the minimum transmission angle is 0 deg and the maximum is 88.8 deg.

PROBLEM 3-43

Statement: Draw the Roberts diagram and find the cognates for the linkage in Figure P3-12.

Given: Link lengths:

Coupler point data:

$$\begin{array}{llll} \text{Ground link} & L_1 := 1.82 & \text{Crank} & L_2 := 0.72 \\ \text{Coupler} & L_3 := 0.68 & \text{Rocker} & L_4 := 0.85 \end{array} \quad \begin{array}{ll} A1P := 0.97 & \delta_1 := 54.0 \cdot \text{deg} \end{array}$$

Solution: See Figure P3-12 and Mathcad file P0343.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$\begin{aligned} B1P &:= \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} & B1P &= 0.792 \\ \gamma_1 &:= \arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) & \gamma_1 &= 82.0315 \text{ deg} \end{aligned}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$\begin{array}{llll} L_5 := B1P & L_5 = 0.792 & L_6 := \frac{L_4}{L_3} \cdot B1P & L_6 = 0.990 \end{array}$$

$$\begin{array}{llll} L_{10} := A1P & L_{10} = 0.970 & L_9 := \frac{L_2}{L_3} \cdot A1P & L_9 = 1.027 \end{array}$$

$$\begin{array}{llll} L_7 := L_9 \cdot \frac{B1P}{A1P} & L_7 = 0.839 & L_8 := L_6 \cdot \frac{A1P}{B1P} & L_8 = 1.212 \end{array}$$

Calculate the coupler point data for cognates #2 and #3

$$\begin{array}{llll} A3P := L_4 & A3P = 0.850 & A2P := L_2 & A2P = 0.720 \\ \delta_3 := \gamma_1 & \delta_3 = 82.032 \text{ deg} & \delta_2 := -\delta_1 & \delta_2 = -54.000 \text{ deg} \end{array}$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

$$\begin{array}{llll} L_{1BC} := \frac{L_1}{L_3} \cdot B1P & L_{1BC} = 2.1208 & L_{1AC} := \frac{L_1}{L_3} \cdot A1P & L_{1AC} = 2.5962 \end{array}$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

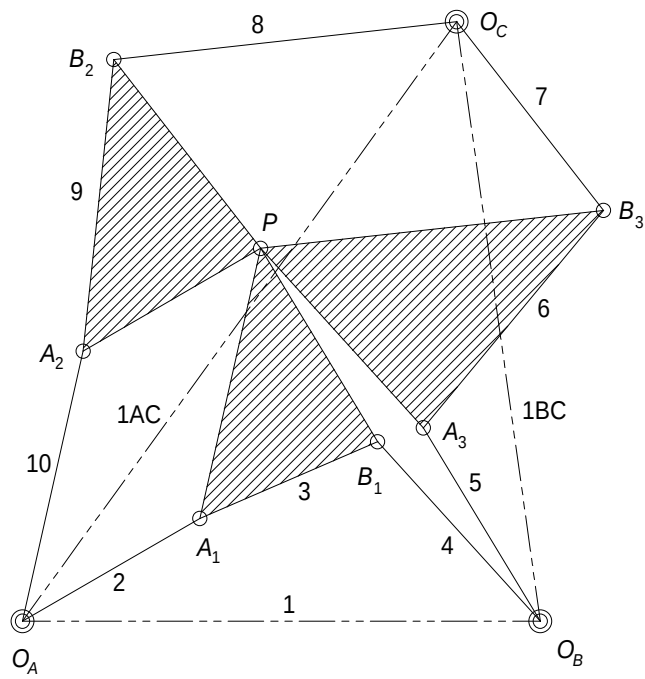
	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 1.820$	$L_{1AC} = 2.596$	$L_{1BC} = 2.121$
Crank length	$L_2 = 0.720$	$L_{10} = 0.970$	$L_7 = 0.839$
Coupler length	$L_3 = 0.680$	$L_9 = 1.027$	$L_6 = 0.990$
Rocker length	$L_4 = 0.850$	$L_8 = 1.212$	$L_5 = 0.792$
Coupler point	$A1P = 0.970$	$A2P = 0.720$	$A3P = 0.850$

Coupler angle

$\delta_1 = 54.000 \text{ deg}$

$\delta_2 = -54.000 \text{ deg}$

$\delta_3 = 82.032 \text{ deg}$



PROBLEM 3-44

Statement: Find the three geared fivebar cognates of the linkage in Figure P3-12.

Given: Link lengths:

Coupler point data:

$$\begin{array}{llllll} \text{Ground link} & L_1 := 1.82 & \text{Crank} & L_2 := 0.72 & A1P := 0.97 & \delta_1 := 54.0 \cdot \text{deg} \\ \text{Coupler} & L_3 := 0.68 & \text{Rocker} & L_4 := 0.85 & & \end{array}$$

Solution: See Figure P3-12 and Mathcad file P0344.

1. Calculate the length BP and the angle γ using the law of cosines on the triangle APB.

$$\begin{aligned} B1P &:= \left(L_3^2 + A1P^2 - 2 \cdot L_3 \cdot A1P \cdot \cos(\delta_1) \right)^{0.5} & B1P &= 0.792 \\ \gamma_1 &:= \arccos \left(\frac{L_3^2 + B1P^2 - A1P^2}{2 \cdot L_3 \cdot B1P} \right) & \gamma_1 &= 82.0315 \text{ deg} \end{aligned}$$

2. Use the Cayley diagram (see Figure 3-24) to calculate the link lengths of the two cognates. Note that the diagram is made up of three parallelograms and three similar triangles

$$L_5 := B1P \quad L_5 = 0.792 \quad L_6 := \frac{L_4}{L_3} \cdot B1P \quad L_6 = 0.990$$

$$L_{10} := A1P \quad L_{10} = 0.970 \quad L_9 := \frac{L_2}{L_3} \cdot A1P \quad L_9 = 1.027$$

$$L_7 := L_9 \cdot \frac{B1P}{A1P} \quad L_7 = 0.839 \quad L_8 := L_6 \cdot \frac{A1P}{B1P} \quad L_8 = 1.212$$

Calculate the coupler point data for cognates #2 and #3

$$\begin{aligned} A3P &:= L_4 & A3P &= 0.850 & A2P &:= L_2 & A2P &= 0.720 \\ \delta_3 &:= \gamma_1 & \delta_3 &= 82.032 \text{ deg} & \delta_2 &:= -\delta_1 & \delta_2 &= -54.000 \text{ deg} \end{aligned}$$

From the Roberts diagram, calculate the ground link lengths for cognates #2 and #3

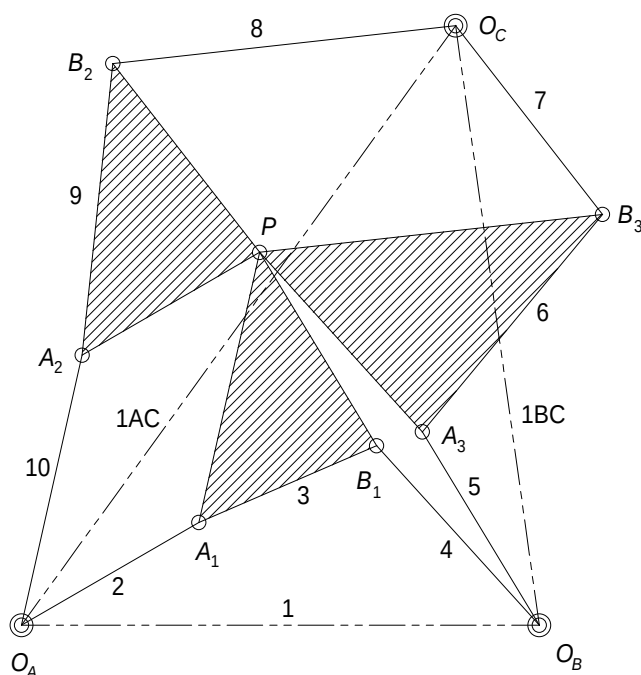
$$L_{1BC} := \frac{L_1}{L_3} \cdot B1P \quad L_{1BC} = 2.1208 \quad L_{1AC} := \frac{L_1}{L_3} \cdot A1P \quad L_{1AC} = 2.5962$$

3. Using the calculated link lengths, draw the Roberts diagram (see next page).

SUMMARY OF COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 1.820$	$L_{1AC} = 2.596$	$L_{1BC} = 2.121$
Crank length	$L_2 = 0.720$	$L_{10} = 0.970$	$L_7 = 0.839$
Coupler length	$L_3 = 0.680$	$L_9 = 1.027$	$L_6 = 0.990$
Rocker length	$L_4 = 0.850$	$L_8 = 1.212$	$L_5 = 0.792$
Coupler point	$A1P = 0.970$	$A2P = 0.720$	$A3P = 0.850$

Coupler angle $\delta_1 = 54.000 \text{ deg}$ $\delta_2 = -54.000 \text{ deg}$ $\delta_3 = 82.032 \text{ deg}$

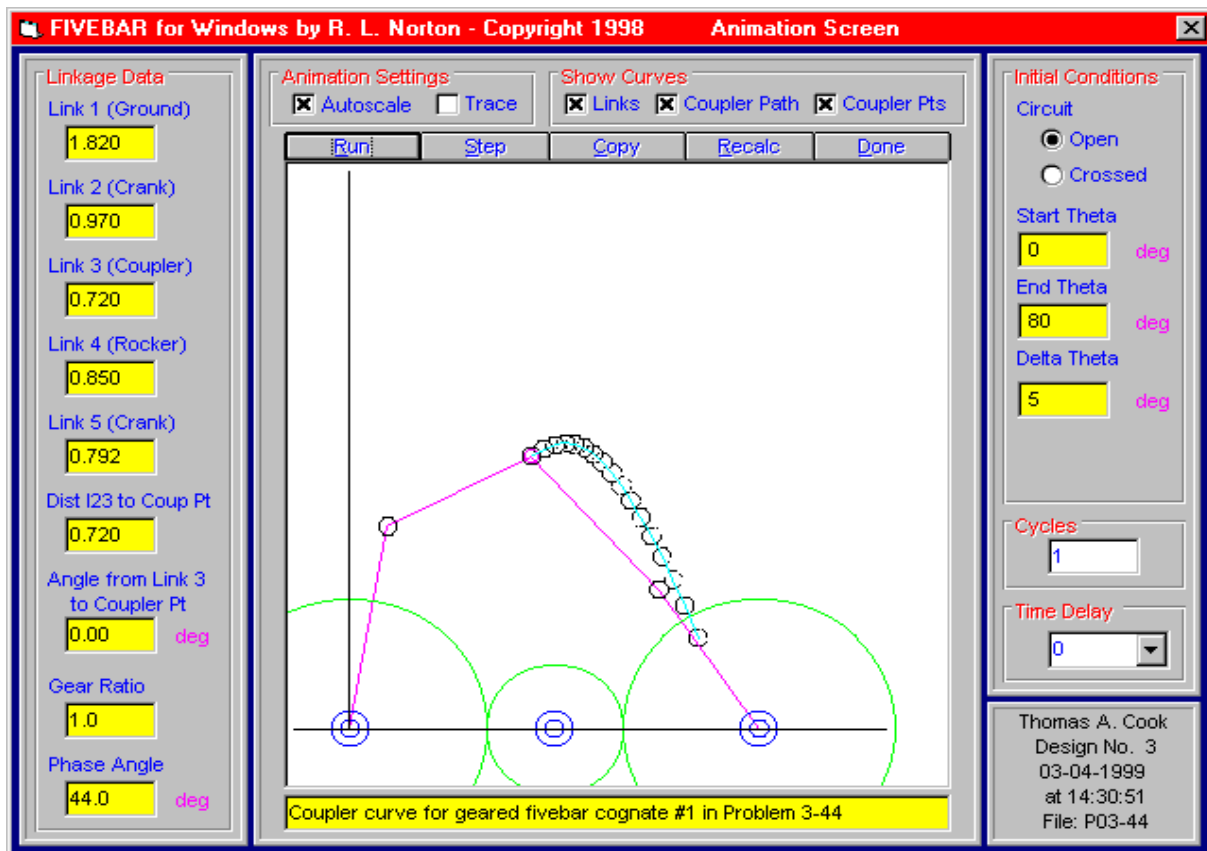
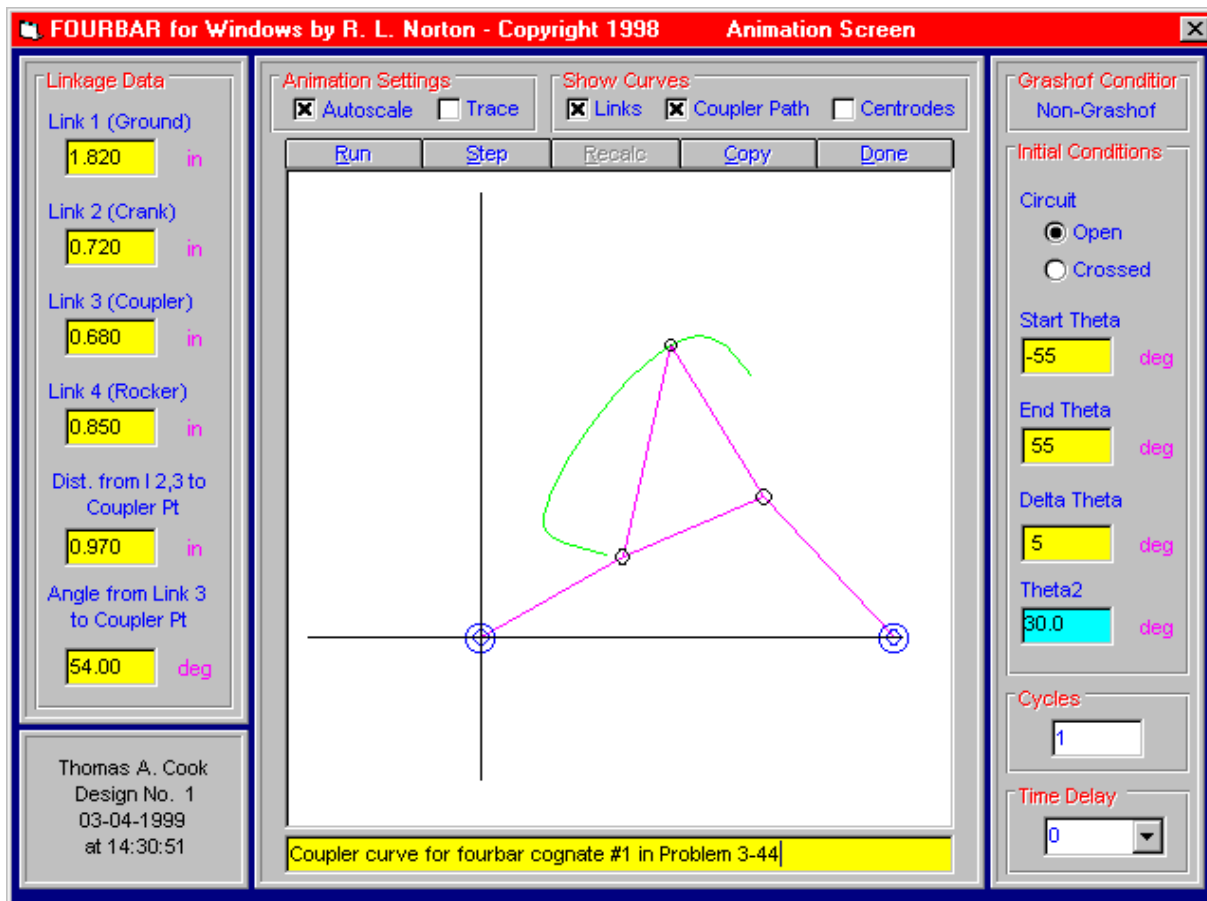


4. The three geared fivebar cognates can be seen in the Roberts diagram. They are: $O_A A_2 P A_3 O_B$, $O_A A_1 P B_3 O_C$, and $O_B B_1 P B_2 O_C$.

SUMMARY OF GEARED FIVEBAR COGNATE SPECIFICATIONS:

	Cognate #1	Cognate #2	Cognate #3
Ground link length	$L_1 = 1.820$	$L_{1AC} = 2.596$	$L_{1BC} = 2.121$
Crank length	$L_{10} = 0.970$	$L_2 = 0.720$	$L_4 = 0.850$
Coupler length	$A_2P = 0.720$	$A_1P = 0.970$	$L_5 = 0.792$
Rocker length	$A_3P = 0.850$	$L_8 = 1.212$	$L_7 = 0.839$
Crank length	$L_5 = 0.792$	$L_7 = 0.839$	$L_8 = 1.212$
Coupler point	$A_2P = 0.720$	$A_1P = 0.970$	$B_1P = 0.792$
Coupler angle	$\delta_1 := 0.00 \cdot \text{deg}$	$\delta_2 := 0.00 \cdot \text{deg}$	$\delta_3 := 0.00 \cdot \text{deg}$

5. Enter the cognate #1 specifications into program FOURBAR to get a trace of the coupler path (see next page).
6. Enter the geared fivebar cognate #1 specifications into program FIVEBAR to get a trace of the coupler path for the geared fivebar (see next page).



PROBLEM 3-45

Statement: Prove that the relationships between the angular velocities of various links in the Roberts diagram as shown in Figure 3-25 (p. 125) are true.

Given: $O_A A_1 P A_2$, $O_C B_2 P B_3$, and $O_B B_1 P A_3$ are parallelograms for any position of link 2..

Proof:

1. $O_A A_1$ and $A_2 P$ are opposite sides of a parallelogram and are, therefore, always parallel.
2. Any change in the angle of $O_A A_1$ (link 2) will result in an identical change in the angle of $A_2 P$.
3. Angular velocity is the change in angle per unit time.
4. Since $O_A A_1$ and $A_2 P$ have identical changes in angle, their angular velocities are identical.
5. $A_2 P$ is a line on link 9 and all lines on a rigid body have the same angular velocity. Therefore, link 9 has the same angular velocity as link 2.
6. $O_C B_3$ (link 7) and $B_2 P$ are opposite sides of a parallelogram and are, therefore, always parallel.
7. $B_2 P$ is a line on link 9 and all lines on a rigid body have the same angular velocity. Therefore, link 7 has the same angular velocity as links 9 and 2.
8. The same argument holds for links 3, 5, and 10; and links 4, 6, and 8.

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$Condition(L_{1b}, L_{2b}, L_5, L_6) = \text{"Grashof"}$

$min(L_{1b}, L_{2b}, L_5, L_6) = 23.003$

PROBLEM 3-47

Statement: Design a fourbar linkage to move the object in Figure P3-13 from position 2 to 3 using points A and B for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 52.000$

Solution: See Figure P3-13 and Mathcad file P0347.

Design choices:

Length of link 2 $L_2 := 130$ Length of link 4 $L_4 := 225$

Length of link 4b $L_{4b} := 40$

1. Connect the end points of the two given positions of the line AB with construction lines, i.e., lines from A_2 to A_3 and B_2 to B_3 .
2. Bisect these lines and extend their perpendicular bisectors into the base.
3. Select one point on each bisector and label them O_2 and O_4 , respectively. In the solution below the distances O_2A was selected to be $L_2 = 130.000$ and O_4B to be $L_4 = 225.000$. This resulted in a ground-link-length O_2O_4 for the fourbar of 111.758.
4. The fourbar stage is now defined as O_2ABO_4 with link lengths

Ground link 1a $L_{1a} := 111.758$ Link 2 (input) $L_2 = 130.000$

Link 3 (coupler) $L_3 = 52.000$ Link 4 (output) $L_4 = 225.000$

5. Select a point on link 4 (O_4B) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it C. (Note that link 4 is now a ternary link with nodes at O_4 , C, and B.) In the solution below the distance O_4C was selected to be $L_{4b} = 40.000$.
6. Draw a construction line through C_2C_3 and extend it downward.
7. Select a point on this line and call it O_6 . In the solution below O_6 was placed 20 units from the bottom of the base.
8. Draw a circle about O_6 with a radius of one-half the length C_1C_2 and label the intersections of the circle with the extended line as D_2 and D_3 . In the solution below the radius was measured as 10.480 units.

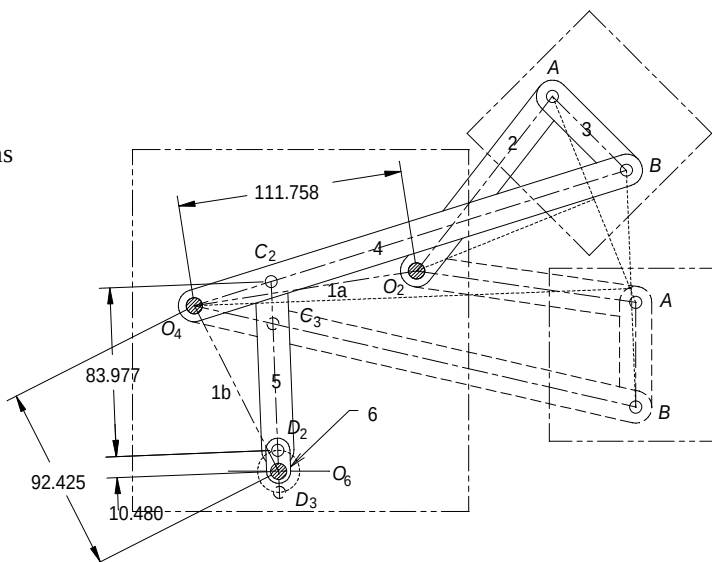
9. The driver fourbar is now defined as O_4CDO_6 with link lengths

Link 6 (crank) $L_6 := 10.480$

Link 5 (coupler) $L_5 := 83.977$

Link 1b (ground) $L_{1b} := 92.425$

Link 4b (rocker) $L_{4b} = 40.000$



10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

```

Condition( $a, b, c, d$ ) :=
    S ← min( $a, b, c, d$ )
    L ← max( $a, b, c, d$ )
    SL ← S + L
    PQ ←  $a + b + c + d - SL$ 
    return "Grashof" if  $SL < PQ$ 
    return "Special Grashof" if  $SL = PQ$ 
    return "non-Grashof" otherwise

```

$Condition(L_{1b}, L_{4b}, L_5, L_6) = \text{"Grashof"}$

$min(L_{1b}, L_{4b}, L_5, L_6) = 10.480$

PROBLEM 3-48

Statement: Design a fourbar linkage to move the object in Figure P3-13 through the three positions shown using points A and B for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 52.000$

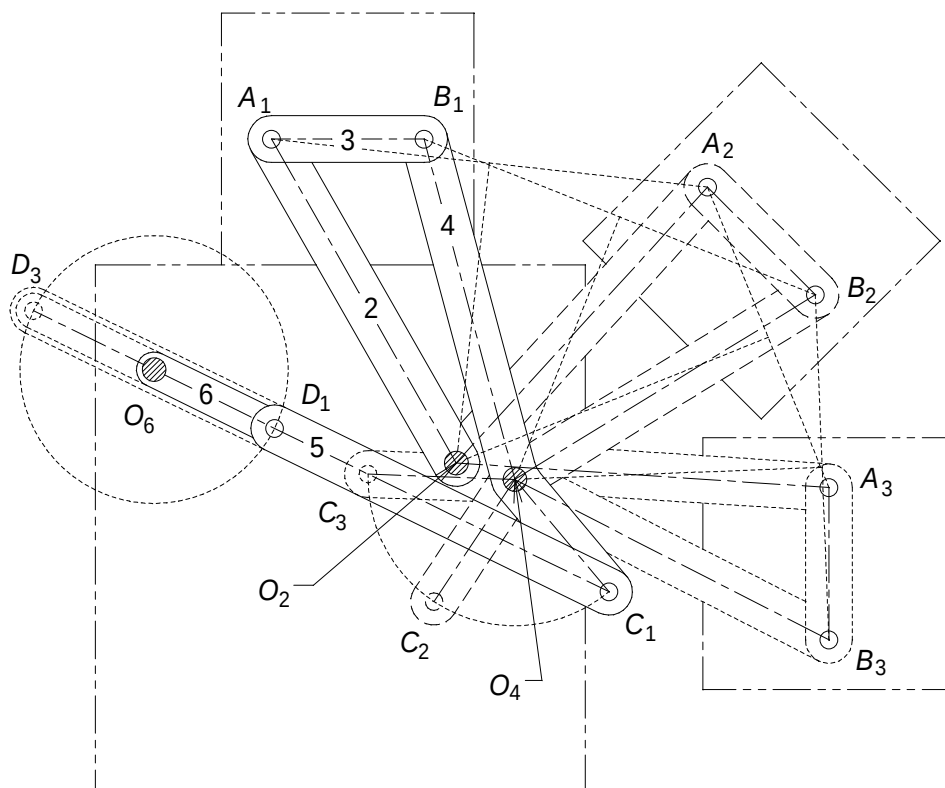
Solution: See Figure P3-13 and Mathcad file P0348.

Design choices:

Length of link 4b $L_{4b} := 50$

1. Draw link AB in its three design positions A_1B_1, A_2B_2, A_3B_3 in the plane as shown.
2. Draw construction lines from point A_1 to A_2 and from point A_2 to A_3 .
3. Bisect line A_1A_2 and line A_2A_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_2 .
4. Repeat steps 2 and 3 for lines B_1B_2 and B_2B_3 . Label the intersection O_4 .
5. Connect O_2 with A_1 and call it link 2. Connect O_4 with B_1 and call it link 4.
6. Line A_1B_1 is link 3. Line O_2O_4 is link 1 (ground link for the fourbar). The fourbar is now defined as O_2ABO_4 and has link lengths of

Ground link 1a	$L_{1a} := 20.736$	Link 2	$L_2 := 127.287$
Link 3	$L_3 = 52.000$	Link 4	$L_4 := 120.254$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$\text{Condition}(a,b,c,d) := \begin{cases} S \leftarrow \min(a,b,c,d) \\ L \leftarrow \max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(L_{1a}, L_2, L_3, L_4) = \text{"Grashof"}$$

8. Select a point on link 4 (O_4B) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it C . (Note that link 4 is now a ternary link with nodes at O_4 , C , and B .) In the solution above the distance O_4C was selected to be $L_{4b} = 50.000$.
9. Draw a construction line through C_1C_3 and extend it to the left.
10. Select a point on this line and call it O_6 . In the solution above O_6 was placed 20 units from the left edge of the base.
11. Draw a circle about O_6 with a radius of one-half the length C_1C_3 and label the intersections of the circle with the extended line as D_1 and D_3 . In the solution below the radius was measured as $L_6 := 45.719$.
12. The driver fourbar is now defined as O_4CDO_6 with link lengths

Link 6 (crank) $L_6 = 45.719$

Link 5 (coupler) $L_5 := 126.875$

Link 1b (ground) $L_{1b} := 128.545$

Link 4b (rocker) $L_{4b} = 50.000$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$\text{Condition}(L_6, L_{1b}, L_{4b}, L_5) = \text{"Grashof"}$$

$$\min(L_6, L_{1b}, L_{4b}, L_5) = 45.719$$

PROBLEM 3-49

Statement: Design a fourbar linkage to move the object in Figure P3-14 from position 1 to 2 using points A and B for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 86.000$

Solution: See Figure P3-14 and Mathcad file P0349.

Design choices:

Length of link 2 $L_2 := 125$

Length of link 4 $L_4 := 140$

Length of link 2b $L_{4b} := 50$

1. Connect the end points of the two given positions of the line AB with construction lines, i.e., lines from A_1 to A_2 and B_1 to B_2 .
2. Bisect these lines and extend their perpendicular bisectors into the base.
3. Select one point on each bisector and label them O_2 and O_4 , respectively. In the solution below the distances O_2A was selected to be $L_2 = 125.000$ and O_4B to be $L_4 = 140.000$. This resulted in a ground-link-length O_2O_4 for the fourbar of 97.195.
4. The fourbar stage is now defined as O_2ABO_4 with link lengths

Ground link 1a $L_{1a} := 97.195$

Link 2 (input) $L_2 = 125.000$

Link 3 (coupler) $L_3 = 86.000$

Link 4 (output) $L_4 = 140.000$

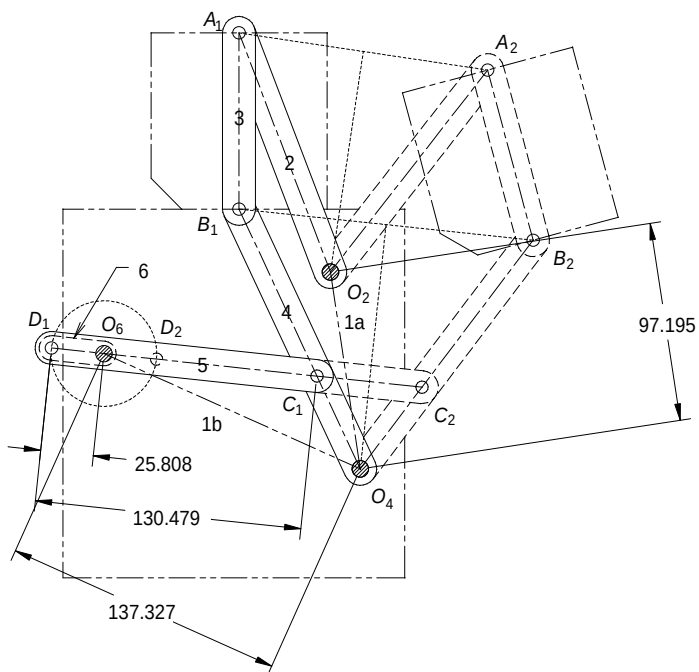
5. Select a point on link 4 (O_4B) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it C. (Note that link 4 is now a ternary link with nodes at O_4 , C, and B.) In the solution below the distance O_4C was selected to be $L_{4b} = 50.000$.
6. Draw a construction line through C_1C_2 and extend it to the left.
7. Select a point on this line and call it O_6 . In the solution below O_6 was placed 20 units from the left edge of the base.
8. Draw a circle about O_6 with a radius of one-half the length C_1C_2 and label the intersections of the circle with the extended line as D_1 and D_2 . In the solution below the radius was measured as 25.808 units.
9. The driver fourbar is now defined as O_4CDO_6 with link lengths

Link 6 (crank) $L_6 := 25.808$

Link 5 (coupler) $L_5 := 130.479$

Link 1b (ground) $L_{1b} := 137.327$

Link 4b (rocker) $L_{4b} = 50.000$



10. Use the link lengths in step 9 to find the Grashof condition of the driving fourbar (it must be Grashof and the shortest link must be link 6).

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof"   if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$Condition(L_{1b}, L_{4b}, L_5, L_6) = \text{"Grashof"}$

$min(L_{1b}, L_{4b}, L_5, L_6) = 25.808$

PROBLEM 3-50

Statement: Design a fourbar linkage to move the object in Figure P3-14 from position 2 to 3 using points A and B for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 86.000$

Solution: See Figure P3-14 and Mathcad file P0350.

Design choices:

Length of link 2 $L_2 := 130$

Length of link 4 $L_4 := 130$

Length of link 2b $L_{2b} := 50$

1. Connect the end points of the two given positions of the line AB with construction lines, i.e., lines from A_2 to A_3 and B_2 to B_3 .
2. Bisect these lines and extend their perpendicular bisectors into the base.
3. Select one point on each bisector and label them O_2 and O_4 , respectively. In the solution below the distances O_2A was selected to be $L_2 = 130.000$ and O_4B to be $L_4 = 130.000$. This resulted in a ground-link-length O_2O_4 for the fourbar of 67.395.
4. The fourbar stage is now defined as O_2ABO_4 with link lengths

Ground link 1a $L_{1a} := 67.395$

Link 2 (input) $L_2 = 130.000$

Link 3 (coupler) $L_3 = 86.000$

Link 4 (output) $L_4 = 130.000$

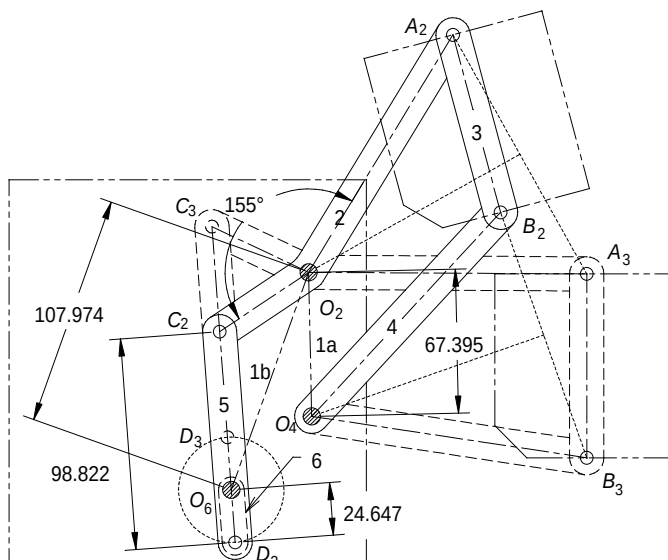
5. Select a point on link 2 (O_2A) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it C. (Note that link 4 is now a ternary link with nodes at O_2 , C, and A.) In the solution below the distance O_2C was selected to be $L_{2b} = 50.000$ and the link was extended away from A to give a better position for the driving dyad.
6. Draw a construction line through C_2C_3 and extend it downward.
7. Select a point on this line and call it O_6 . In the solution below O_6 was placed 35 units from the bottom of the base.
8. Draw a circle about O_6 with a radius of one-half the length C_1C_2 and label the intersections of the circle with the extended line as D_2 and D_3 . In the solution below the radius was measured as 24.647 units.
9. The driver fourbar is now defined as O_2CDO_6 with link lengths

Link 6 (crank) $L_6 := 24.647$

Link 5 (coupler) $L_5 := 98.822$

Link 1b (ground) $L_{1b} := 107.974$

Link 2b (rocker) $L_{2b} = 50.000$



10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$\text{Condition}(a, b, c, d) := \left\{ \begin{array}{l} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array} \right.$$

$$\text{Condition}(L_{1b}, L_{2b}, L_5, L_6) = \text{"Grashof"}$$

$$\min(L_{1b}, L_{2b}, L_5, L_6) = 24.647$$

PROBLEM 3-51

Statement: Design a fourbar linkage to move the object in Figure P3-14 through the three positions shown using points A and B for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 86.000$

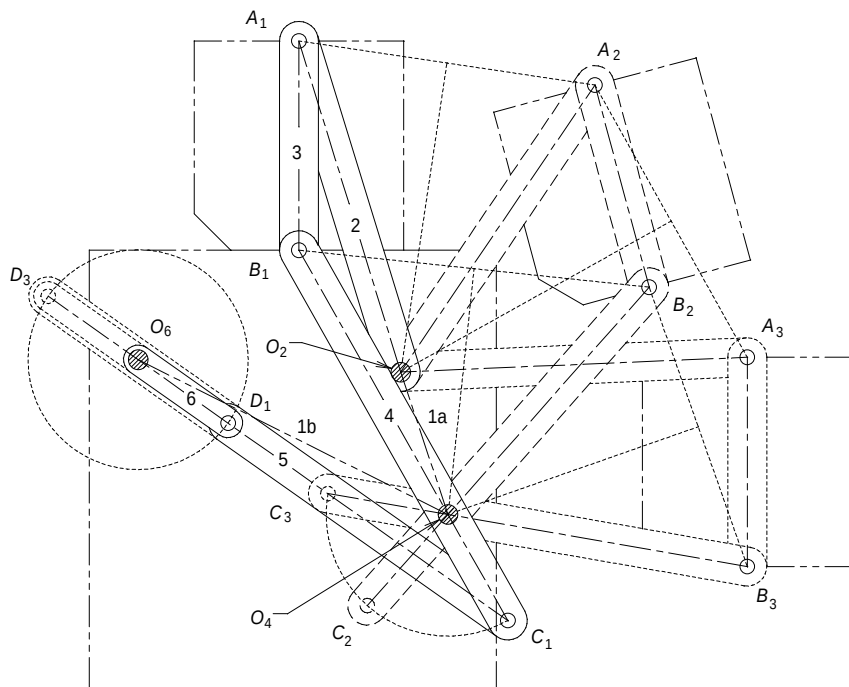
Solution: See Figure P3-14 and Mathcad file P0351.

Design choices:

Length of link 4b $L_{4b} := 50$

1. Draw link AB in its three design positions A_1B_1, A_2B_2, A_3B_3 in the plane as shown.
2. Draw construction lines from point A_1 to A_2 and from point A_2 to A_3 .
3. Bisect line A_1A_2 and line A_2A_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_2 .
4. Repeat steps 2 and 3 for lines B_1B_2 and B_2B_3 . Label the intersection O_4 .
5. Connect O_2 with A_1 and call it link 2. Connect O_4 with B_1 and call it link 4.
6. Line A_1B_1 is link 3. Line O_2O_4 is link 1 (ground link for the fourbar). The fourbar is now defined as O_2ABO_4 and has link lengths of

Ground link 1a	$L_{1a} := 61.667$	Link 2	$L_2 := 142.357$
Link 3	$L_3 = 86.000$	Link 4	$L_4 := 124.668$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

```

Condition(a,b,c,d) :=
    S ← min(a,b,c,d)
    L ← max(a,b,c,d)
    SL ← S + L
    PQ ← a + b + c + d - SL
    return "Grashof" if SL < PQ
    return "Special Grashof" if SL = PQ
    return "non-Grashof" otherwise

```

$Condition(L_{1a}, L_2, L_3, L_4) = \text{"Grashof"}$

8. Select a point on link 4 (O_4B) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it C . (Note that link 4 is now a ternary link with nodes at O_4 , C , and B .) In the solution above the distance O_4C was selected to be $L_{4b} = 50.000$.
9. Draw a construction line through C_1C_3 and extend it to the left.
10. Select a point on this line and call it O_6 . In the solution above O_6 was placed 20 units from the left edge of the base.
11. Draw a circle about O_6 with a radius of one-half the length C_1C_3 and label the intersections of the circle with the extended line as D_1 and D_3 . In the solution below the radius was measured as $L_6 := 45.178$.
12. The driver fourbar is now defined as O_4CDO_6 with link lengths

Link 6 (crank) $L_6 = 45.178$

Link 5 (coupler) $L_5 := 140.583$

Link 1b (ground) $L_{1b} := 142.205$

Link 4b (rocker) $L_{4b} = 50.000$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$Condition(L_6, L_{1b}, L_{4b}, L_5) = \text{"Grashof"}$

14. Unfortunately, although the solution presented appears to meet the design specification, a simple cardboard model will quickly demonstrate that it has a branch defect. That is, in the first position shown, the linkage is in the "open" configuration, but in the 2nd and 3rd positions it is in the "crossed" configuration. The linkage cannot get from one circuit to the other without removing a pin and reassembling after moving the linkage. The remedy is to attach the points A and B to the coupler, but not at the joints between links 2 and 3 and links 3 and 4.

PROBLEM 3-52

Statement: Design a fourbar linkage to move the object in Figure P3-15 from position 1 to 2 using points A and B for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 52.000$

Solution: See Figure P3-15 and Mathcad file P0352.

Design choices:

Length of link 2 $L_2 := 100$

Length of link 4 $L_4 := 160$

Length of link 4b $L_{4b} := 40$

1. Connect the end points of the two given positions of the line AB with construction lines, i.e., lines from A_1 to A_2 and B_1 to B_2 .
2. Bisect these lines and extend their perpendicular bisectors into the base.
3. Select one point on each bisector and label them O_2 and O_4 , respectively. In the solution below the distances O_2A was selected to be $L_2 = 100.000$ and O_4B to be $L_4 = 160.000$. This resulted in a ground-link-length O_2O_4 for the fourbar of 81.463.
4. The fourbar stage is now defined as O_2ABO_4 with link lengths

Ground link 1a $L_{1a} := 81.463$

Link 2 (input) $L_2 = 100.000$

Link 3 (coupler) $L_3 = 52.000$

Link 4 (output) $L_4 = 160.000$

5. Select a point on link 4 (O_4B) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it C. (Note that link 4 is now a ternary link with nodes at O_4 , C, and B.) In the solution below the distance O_4C was selected to be $L_{4b} = 40.000$.
6. Draw a construction line through C_1C_2 and extend it to the left.
7. Select a point on this line and call it O_6 . In the solution below O_6 was placed 20 units from the left edge of the base.
8. Draw a circle about O_6 with a radius of one-half the length C_1C_2 and label the intersections of the circle with the extended line as D_1 and D_2 . In the solution below the radius was measured as 14.351 units.

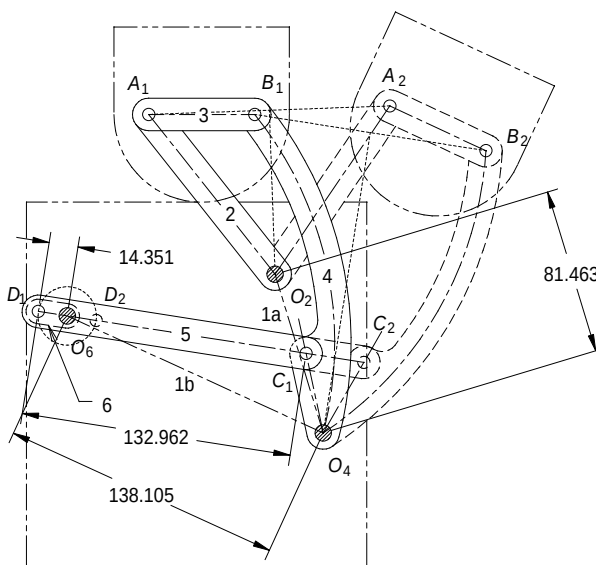
9. The driver fourbar is now defined as O_4CDO_6 with link lengths

Link 6 (crank) $L_6 := 14.351$

Link 5 (coupler) $L_5 := 132.962$

Link 1b (ground) $L_{1b} := 138.105$

Link 4b (rocker) $L_{4b} = 40.000$



10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

```

Condition( $a, b, c, d$ ) :=  $\left\{ \begin{array}{l} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array} \right.$ 

```

$\text{Condition}(L_{1b}, L_{4b}, L_5, L_6) = \text{"Grashof"}$

$\min(L_{1b}, L_{4b}, L_5, L_6) = 14.351$

PROBLEM 3-53

Statement: Design a fourbar linkage to move the object in Figure P3-15 from position 2 to 3 using points A and B for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 52.000$

Solution: See Figure P3-15 and Mathcad file P0353.

Design choices:

Length of link 2 $L_2 := 150$

Length of link 4 $L_4 := 200$

Length of link 4b $L_{4b} := 50$

1. Connect the end points of the two given positions of the line AB with construction lines, i.e., lines from A_2 to A_3 and B_2 to B_3 .
2. Bisect these lines and extend their perpendicular bisectors into the base.
3. Select one point on each bisector and label them O_2 and O_4 , respectively. In the solution below the distances O_2A was selected to be $L_2 = 150.000$ and O_4B to be $L_4 = 200.000$. This resulted in a ground-link-length O_2O_4 for the fourbar of $L_{1a} := 80.864$.
4. The fourbar stage is now defined as O_2ABO_4 with link lengths

Ground link 1a $L_{1a} = 80.864$

Link 2 (input) $L_2 = 150.000$

Link 3 (coupler) $L_3 = 52.000$

Link 4 (output) $L_4 = 200.000$

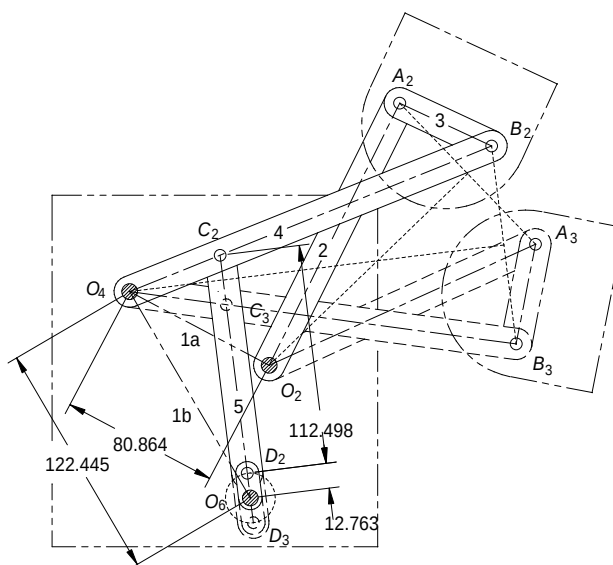
5. Select a point on link 4 (O_4B) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it C. (Note that link 4 is now a ternary link with nodes at O_4 , C, and B.) In the solution below the distance O_4C was selected to be $L_{4b} = 50.000$.
6. Draw a construction line through C_2C_3 and extend it downward.
7. Select a point on this line and call it O_6 . In the solution below O_6 was placed 25 units from the bottom of the base.
8. Draw a circle about O_6 with a radius of one-half the length C_1C_2 and label the intersections of the circle with the extended line as D_2 and D_3 . In the solution below the radius was measured as $L_6 := 12.763$.
9. The driver fourbar is now defined as O_4CDO_6 with link lengths

Link 6 (crank) $L_6 = 12.763$

Link 5 (coupler) $L_5 := 112.498$

Link 1b (ground) $L_{1b} := 122.445$

Link 4b (rocker) $L_{4b} = 50.000$



10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

```

Condition(a,b,c,d) :=
    S ← min(a,b,c,d)
    L ← max(a,b,c,d)
    SL ← S + L
    PQ ← a + b + c + d - SL
    return "Grashof"   if SL < PQ
    return "Special Grashof" if SL = PQ
    return "non-Grashof" otherwise

```

$Condition(L_{1b}, L_{4b}, L_5, L_6) = \text{"Grashof"}$

$min(L_{1b}, L_{4b}, L_5, L_6) = 12.763$

PROBLEM 3-54

Statement: Design a fourbar linkage to move the object in Figure P3-15 through the three positions shown using points A and B for attachment. Add a driver dyad to limit its motion to the range of positions shown, making it a sixbar. All fixed pivots should be on the base.

Given: Length of coupler link: $L_3 := 52.000$

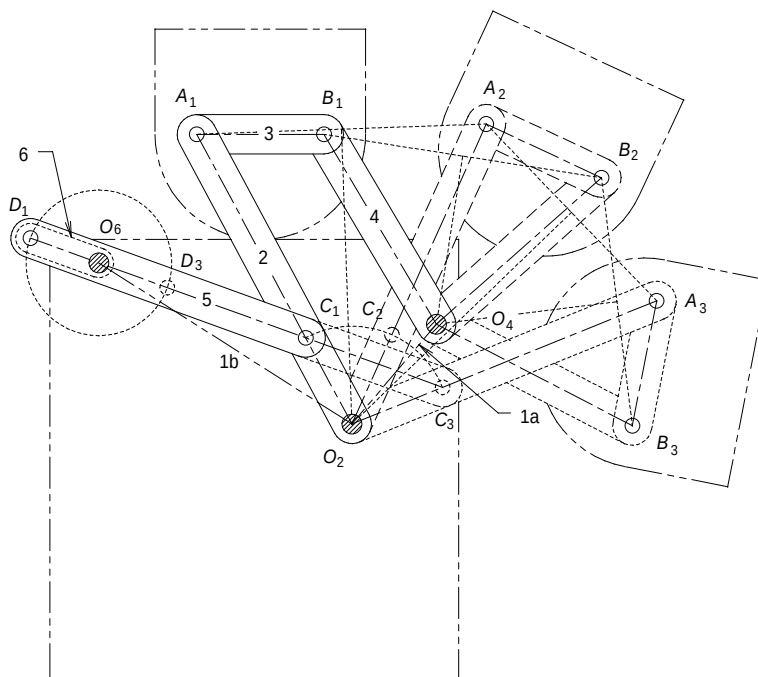
Solution: See Figure P3-15 and Mathcad file P0354.

Design choices:

Length of link 2b $L_{2b} := 40$

1. Draw link AB in its three design positions A_1B_1, A_2B_2, A_3B_3 in the plane as shown.
2. Draw construction lines from point A_1 to A_2 and from point A_2 to A_3 .
3. Bisect line A_1A_2 and line A_2A_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_2 .
4. Repeat steps 2 and 3 for lines B_1B_2 and B_2B_3 . Label the intersection O_4 .
5. Connect O_2 with A_1 and call it link 2. Connect O_4 with B_1 and call it link 4.
6. Line A_1B_1 is link 3. Line O_2O_4 is link 1 (ground link for the fourbar). The fourbar is now defined as O_2ABO_4 and has link lengths of

Ground link 1a	$L_{1a} := 53.439$	Link 2	$L_2 := 134.341$
Link 3	$L_3 = 52.000$	Link 4	$L_4 := 90.203$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.


```

Condition(a,b,c,d) :=
    S ← min(a,b,c,d)
    L ← max(a,b,c,d)
    SL ← S + L
    PQ ← a + b + c + d - SL
    return "Grashof" if SL < PQ
    return "Special Grashof" if SL = PQ
    return "non-Grashof" otherwise

```

$Condition(L_{1a}, L_2, L_3, L_4) = \text{"non-Grashof"}$

Although this fourbar is non-Grashof, there are no toggle points within the required range of motion.

8. Select a point on link 2 (O_2A) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it C . (Note that link 2 is now a ternary link with nodes at O_2 , C , and A .) In the solution above the distance O_2C was selected to be $L_{2b} = 40.000$.
9. Draw a construction line through C_1C_3 and extend it to the left.
10. Select a point on this line and call it O_6 . In the solution above O_6 was placed 20 units from the left edge of the base.
11. Draw a circle about O_6 with a radius of one-half the length C_1C_3 and label the intersections of the circle with the extended line as D_1 and D_3 . In the solution below the radius was measured as $L_6 := 29.760$.
12. The driver fourbar is now defined as O_2CDO_6 with link lengths

Link 6 (crank) $L_6 = 29.760$

Link 5 (coupler) $L_5 := 119.665$

Link 1b (ground) $L_{1b} := 122.613$

Link 2b (rocker) $L_{2b} = 40.000$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$Condition(L_6, L_{1b}, L_{2b}, L_5) = \text{"Grashof"}$

PROBLEM 3-55

Statement: Design a fourbar mechanism to move the link shown in Figure P3-16 from position 1 to position 2. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Given: Position 1 offsets: $x_{C1D1} := 3.744 \cdot \text{in}$ $y_{C1D1} := 2.497 \cdot \text{in}$

Solution: See figure below for one possible solution. Input file P0355.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-55.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-55.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line CD with construction lines, i.e., lines from C_1 to C_2 and D_1 to D_2 .
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_1C_2 was extended downward and the bisector of D_1D_2 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6 , respectively. In the solution below the distances O_4D and O_6C were each selected to be 7.500 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 15.366 in.
4. The fourbar stage is now defined as O_4CDO_6 with link lengths

$$\text{Link 5 (coupler)} \quad L_5 := \sqrt{x_{C1D1}^2 + y_{C1D1}^2} \quad L_5 = 4.500 \text{ in}$$

$$\text{Link 4 (input)} \quad L_4 := 7.500 \cdot \text{in} \quad \text{Link 6 (output)} \quad L_6 := 7.500 \cdot \text{in}$$

$$\text{Ground link 1b} \quad L_{1b} := 15.366 \cdot \text{in}$$

5. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 4 is now a ternary link with nodes at O_4 , B , and D .) In the solution below the distance O_4B was selected to be 4.000 in.
6. Draw a construction line through B_1B_2 and extend it to the right.
7. Select a point on this line and call it O_2 . In the solution below the distance AB was selected to be 6.000 in.
8. Draw a circle about O_2 with a radius of one-half the length B_1B_2 and label the intersections of the circle with the extended line as A_1 and A_2 . In the solution below the radius was measured as 1.370 in.
9. The driver fourbar is now defined as O_2ABO_4 with link lengths

$$\text{Link 2 (crank)} \quad L_2 := 1.370 \cdot \text{in} \quad \text{Link 3 (coupler)} \quad L_3 := 6.000 \cdot \text{in}$$

$$\text{Link 4a (rocker)} \quad L_{4a} := 4.000 \cdot \text{in} \quad \text{Link 1a (ground)} \quad L_{1a} := 7.080 \cdot \text{in}$$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$\begin{array}{l} \text{Condition}(a, b, c, d) := \left| \begin{array}{l} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array} \right. \end{array}$$

PROBLEM 3-56

Statement: Design a fourbar mechanism to move the link shown in Figure P3-16 from position 2 to position 3. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Given: Position 2 offsets: $x_{C2D2} := 4.355 \cdot \text{in}$ $y_{C2D2} := 1.134 \cdot \text{in}$

Solution: See figure below for one possible solution. Input file P0356.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-56.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-56.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line CD with construction lines, i.e., lines from C_2 to C_3 and D_2 to D_3 .
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_2C_3 was extended downward and the bisector of D_2D_3 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6 , respectively. In the solution below the distances O_4D and O_6C were each selected to be 6.000 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 14.200 in.
4. The fourbar stage is now defined as O_4DCO_6 with link lengths

$$\text{Link 5 (coupler)} \quad L_5 := \sqrt{x_{C2D2}^2 + y_{C2D2}^2} \quad L_5 = 4.500 \text{ in}$$

$$\text{Link 4 (input)} \quad L_4 := 6.000 \cdot \text{in} \quad \text{Link 6 (output)} \quad L_6 := 6.000 \cdot \text{in}$$

$$\text{Ground link 1b} \quad L_{1b} := 14.200 \cdot \text{in}$$

5. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 4 is now a ternary link with nodes at O_4 , B , and D .) In the solution below the distance O_4B was selected to be 4.000 in.
6. Draw a construction line through B_1B_2 and extend it to the right.
7. Select a point on this line and call it O_2 . In the solution below the distance AB was selected to be 6.000 in.
8. Draw a circle about O_2 with a radius of one-half the length B_1B_2 and label the intersections of the circle with the extended line as A_1 and A_2 . In the solution below the radius was measured as 1.271 in.
9. The driver fourbar is now defined as O_2ABO_4 with link lengths

$$\text{Link 2 (crank)} \quad L_2 := 1.271 \cdot \text{in} \quad \text{Link 3 (coupler)} \quad L_3 := 6.000 \cdot \text{in}$$

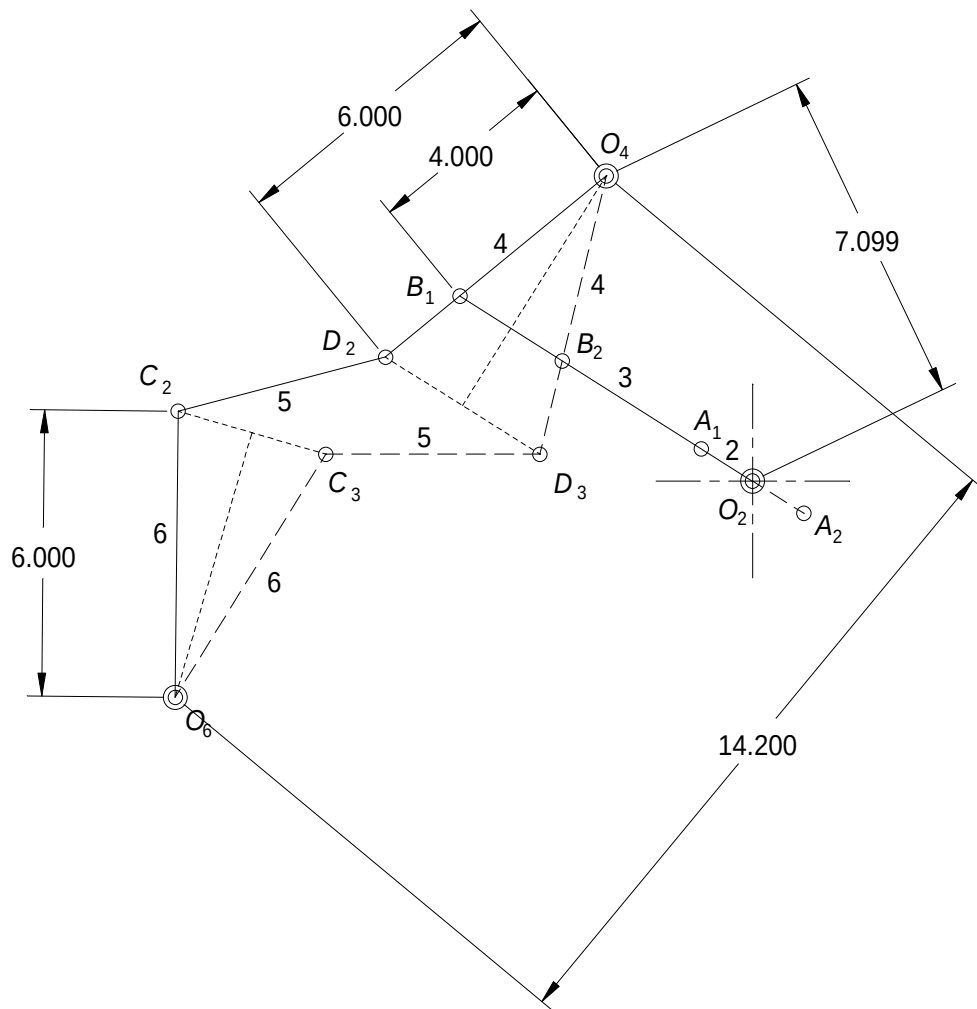
$$\text{Link 4a (rocker)} \quad L_{4a} := 4.000 \cdot \text{in} \quad \text{Link 1a (ground)} \quad L_{1a} := 7.099 \cdot \text{in}$$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$\begin{array}{l} \text{Condition}(a,b,c,d) := \left\{ \begin{array}{l} S \leftarrow \min(a,b,c,d) \\ L \leftarrow \max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array} \right. \end{array}$$

$$\text{Condition}(L_{1a}, L_2, L_3, L_{4a}) = \text{"Grashof"}$$

$$\min(L_{1a}, L_2, L_3, L_{4a}) = 1.271 \text{ in}$$



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4DCO_6 is non-Grashoff with toggle positions at $\theta_4 = -41.6$ deg and $+41.6$ deg. The fourbar operates between $\theta_4 = +26.171$ deg and -11.052 deg.

PROBLEM 3-57

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-16. Ignore the points O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Solution: See Figure P3-16 and Mathcad file P0357.

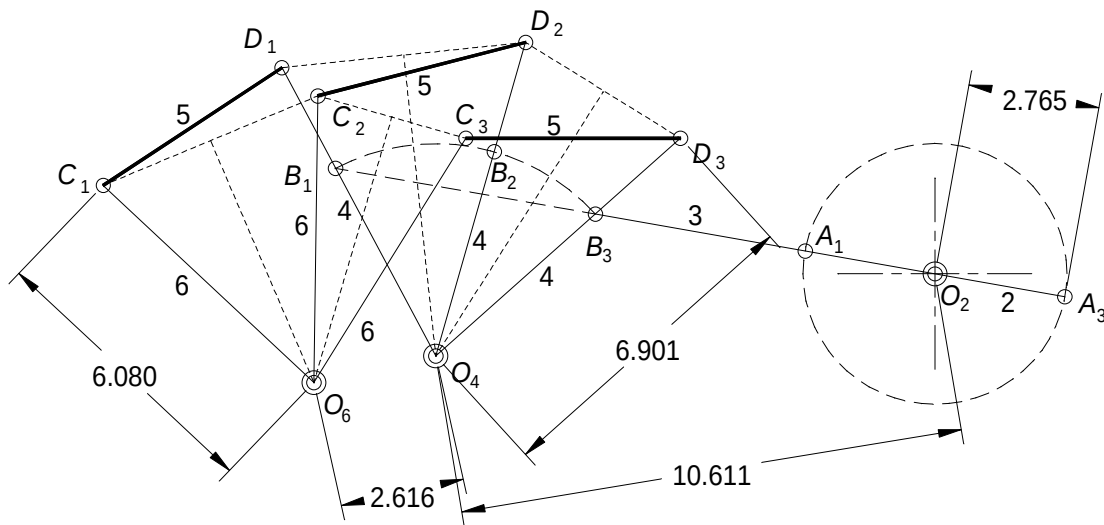
Design choices:

$$\text{Length of link 3: } L_3 := 10.000 \quad \text{Length of link 4b: } L_{4b} := 4.500$$

1. Draw link CD in its three design positions C_1D_1 , C_2D_2 , C_3D_3 in the plane as shown.
2. Draw construction lines from point C_1 to C_2 and from point C_2 to C_3 .
3. Bisect line C_1C_2 and line C_2C_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_6 .
4. Repeat steps 2 and 3 for lines D_1D_2 and D_2D_3 . Label the intersection O_4 .
5. Connect O_6 with C_1 and call it link 6. Connect O_4 with D_1 and call it link 4.
6. Line C_1D_1 is link 5. Line O_6O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_6CDO_4 and has link lengths of

$$\text{Ground link 1a } L_{1a} := 2.616 \quad \text{Link 6 } L_6 := 6.080$$

$$\text{Link 5 } L_5 := 4.500 \quad \text{Link 4 } L_4 := 6.901$$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

```

Condition(a,b,c,d) :=
    S ← min(a,b,c,d)
    L ← max(a,b,c,d)
    SL ← S + L
    PQ ← a + b + c + d - SL
    return "Grashof" if SL < PQ
    return "Special Grashof" if SL = PQ
    return "non-Grashof" otherwise
  
```

$$\text{Condition}(L_{1a}, L_4, L_5, L_6) = \text{"Grashof"}$$

8. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 4 is now a ternary link with nodes at O_4 , D , and B .) In the solution above the distance O_4B was selected to be $L_{4b} = 4.500$.
9. Draw a construction line through B_1B_3 and extend it up to the right.
10. Layout the length of link 3 (design choice) along the extended line. Label the other end A .
11. Draw a circle about O_2 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3 . In the solution below the radius was measured as $L_2 := 2.765$.
12. The driver fourbar is now defined as O_4BAO_2 with link lengths

$$\text{Link 2 (crank)} \quad L_2 = 2.765$$

$$\text{Link 3 (coupler)} \quad L_3 = 10.000$$

$$\text{Link 1b (ground)} \quad L_{1b} := 10.611$$

$$\text{Link 4b (rocker)} \quad L_{4b} = 4.500$$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$\text{Condition}(L_2, L_3, L_{1b}, L_{4b}) = \text{"Grashof"}$$

$$\min(L_2, L_3, L_{1b}, L_{4b}) = 2.765$$

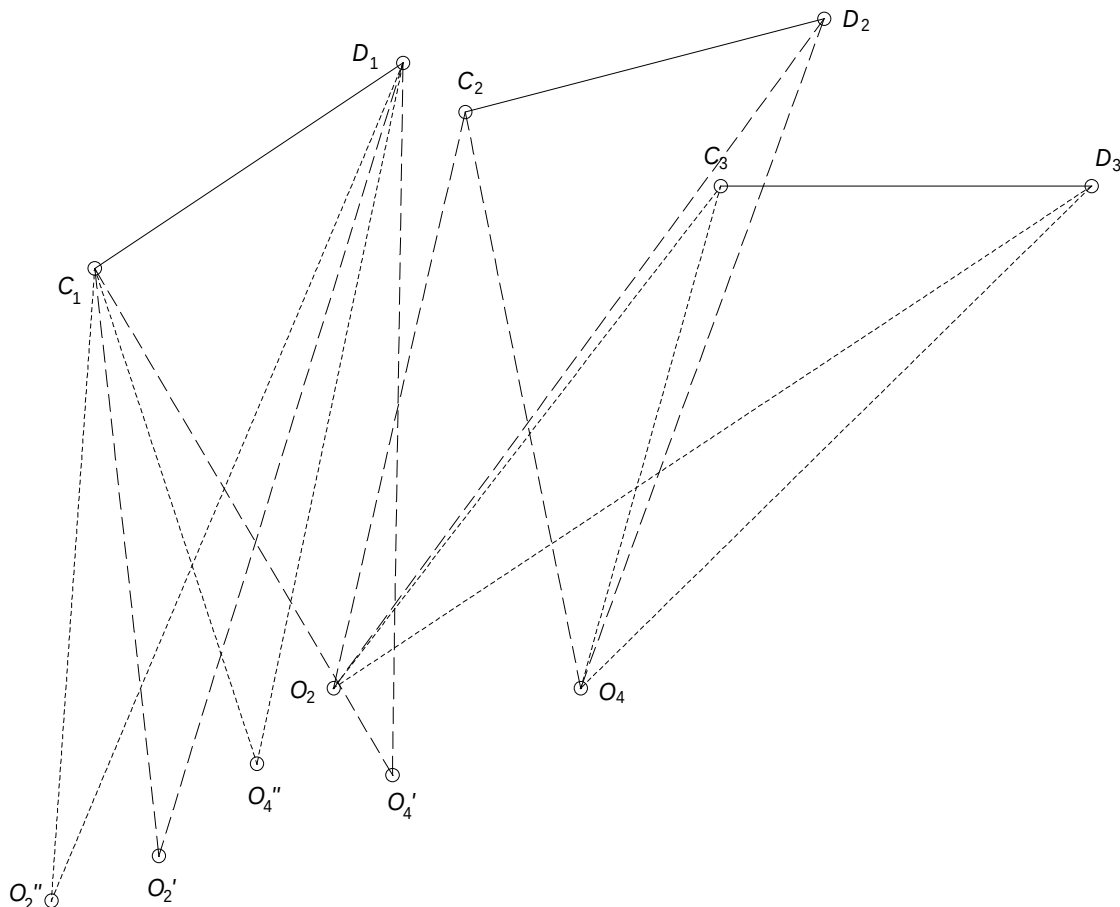
PROBLEM 3-58

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-16 using the fixed pivots O_2 and O_4 shown. (See Example 3-7.) Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Solution: See Figure P3-16 and Mathcad file P0358.

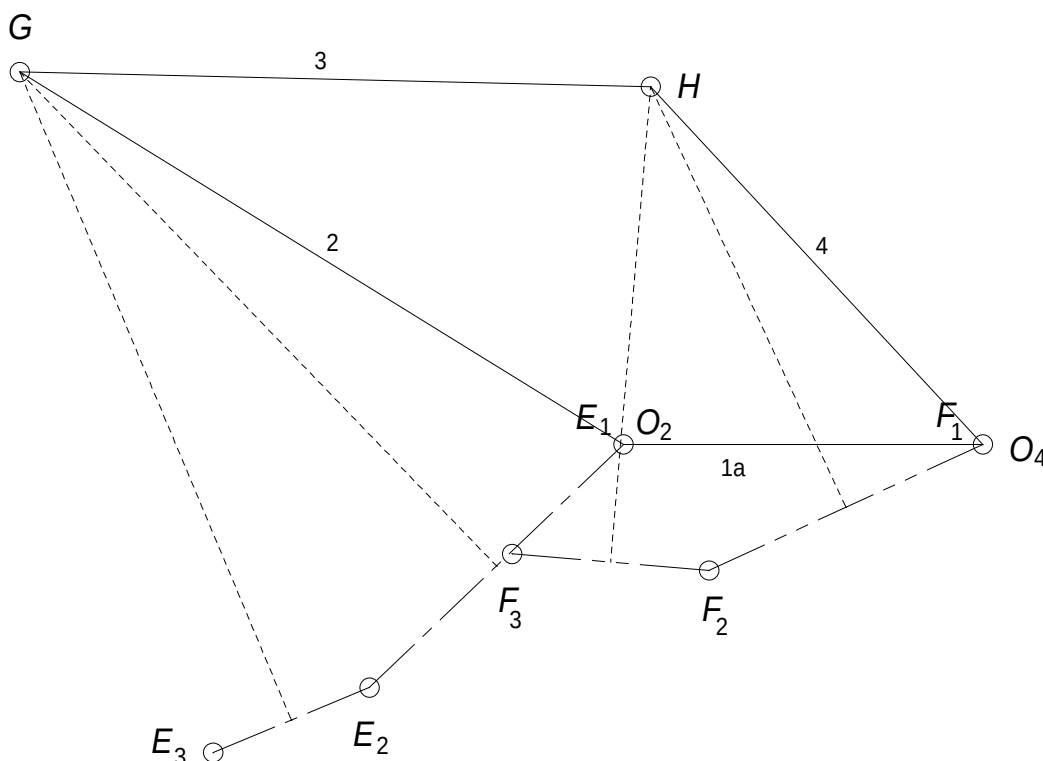
Design choices: Length of link 5: $L_5 := 5.000$ Length of link 2b: $L_{2b} := 2.500$

1. Draw link CD in its three design positions C_1D_1 , C_2D_2 , C_3D_3 in the plane as shown.
2. Draw the ground link O_2O_4 in its desired position in the plane with respect to the first coupler position C_1D_1 .
3. Draw construction arcs from point C_2 to O_2 and from point D_2 to O_2 whose radii define the sides of triangle $C_2O_2D_2$. This defines the relationship of the fixed pivot O_2 to the coupler line CD in the second coupler position.
4. Draw construction arcs from point C_2 to O_4 and from point D_2 to O_4 whose radii define the sides of triangle $C_2O_4D_2$. This defines the relationship of the fixed pivot O_4 to the coupler line CD in the second coupler position.
5. Transfer this relationship back to the first coupler position C_1D_1 so that the ground plane position $O_2'O_4'$ bears the same relationship to C_1D_1 as O_2O_4 bore to the second coupler position C_2D_2 .
6. Repeat the process for the third coupler position and transfer the third relative ground link position to the first, or reference, position.
7. The three inverted positions of the ground link that correspond to the three desired coupler positions are labeled O_2O_4 , $O_2'O_4'$, and $O_2''O_4''$ in the first layout below and are renamed E_1F_1 , E_2F_2 , and E_3F_3 , respectively, in the second layout, which is used to find the points G and H .



8. Draw construction lines from point E_1 to E_2 and from point E_2 to E_3 .
9. Bisect line E_1E_2 and line E_2E_3 and extend their perpendicular bisectors until they intersect. Label their intersection G .
10. Repeat steps 2 and 3 for lines F_1F_2 and F_2F_3 . Label the intersection H .
11. Connect E_1 with G and label it link 2. Connect F_1 with H and label it link 4. Reverting, E_1 and F_1 are the original fixed pivots O_2 and O_4 , respectively.
12. Line GH is link 3. Line O_2O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_2GHO_4 and has link lengths of

Ground link 1a	$L_{1a} := 3.000$	Link 2	$L_2 := 8.597$
Link 3	$L_3 := 1.711$	Link 4	$L_4 := 7.921$



13. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof"   if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$Condition(L_{1a}, L_2, L_3, L_4) = \text{"Grashof"}$

The fourbar that will provide the desired motion is now defined as a Grashof double crank in the crossed configuration. It now remains to add the original points C_1 and D_1 to the coupler GH and to define the driving dyad.

14. Select a point on link 2 (O_2G) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 2 is now a ternary link with nodes at O_2 , B , and G .) In the solution below, the distance O_2B was selected to be $L_{2b} = 2.500$.
15. Draw a construction line through B_1B_3 and extend it up to the left.
16. Layout the length of link 5 (design choice) along the extended line. Label the other end A .
17. Draw a circle about O_6 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3 . In the solution below the radius was measured as $L_6 := 1.541$.
18. The driver fourbar is now defined as O_2BAO_6 with link lengths

Link 6 (crank) $L_6 = 1.541$

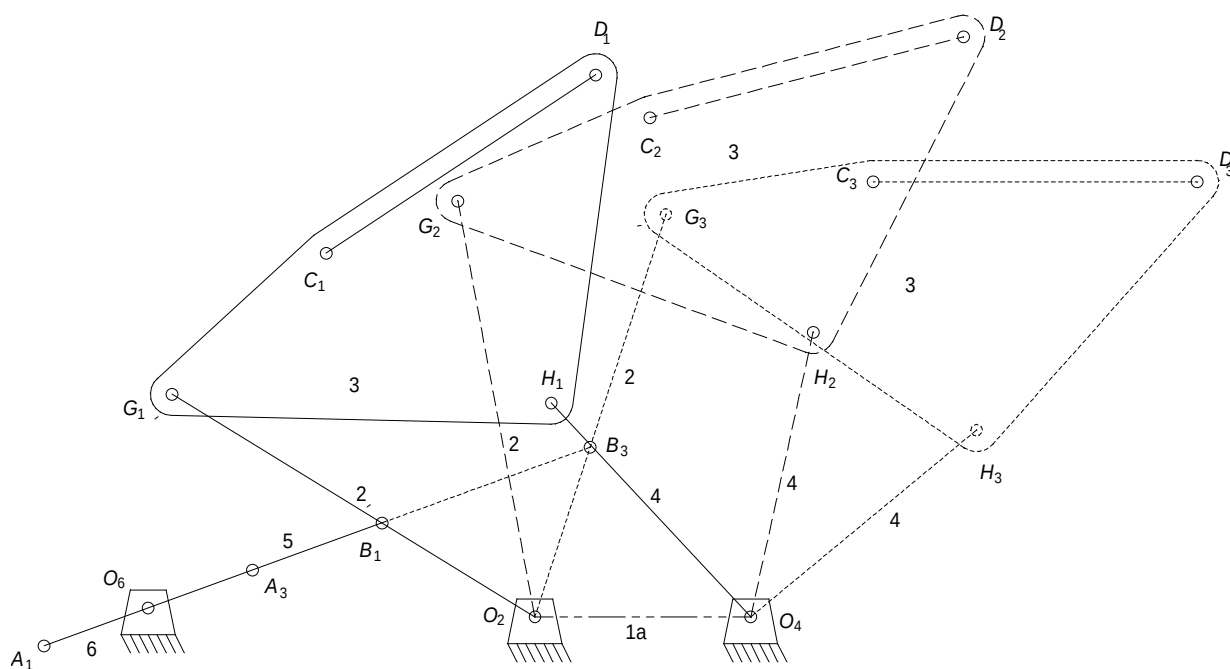
Link 5 (coupler) $L_5 = 5.000$

Link 1b (ground) $L_{1b} := 5.374$

Link 2b (rocker) $L_{2b} = 2.500$

19. Use the link lengths in step 18 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$\text{Condition}(L_6, L_5, L_{1b}, L_{2b}) = \text{"Grashof"}$$



PROBLEM 3-59

Statement: Design a fourbar mechanism to move the link shown in Figure P3-17 from position 1 to position 2. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Given: Position 1 offsets: $x_{C1D1} := 1.896 \cdot \text{in}$ $y_{C1D1} := 1.212 \cdot \text{in}$

Solution: See figure below for one possible solution. Input file P0359.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-59.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-59.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line CD with construction lines, i.e., lines from C_1 to C_2 and D_1 to D_2 .
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_1C_2 was extended downward and the bisector of D_1D_2 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6 , respectively. In the solution below the distances O_6C and O_4D were each selected to be 6.500 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 14.722 in.
4. The fourbar stage is now defined as O_4DCO_6 with link lengths

$$\text{Link 5 (coupler)} \quad L_5 := \sqrt{x_{C1D1}^2 + y_{C1D1}^2} \quad L_5 = 2.250 \text{ in}$$

$$\text{Link 4 (input)} \quad L_4 := 6.500 \cdot \text{in} \quad \text{Link 6 (output)} \quad L_6 := 6.500 \cdot \text{in}$$

$$\text{Ground link 1b} \quad L_{1b} := 14.722 \cdot \text{in}$$

5. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 4 is now a ternary link with nodes at O_4 , B , and D .) In the solution below the distance O_4B was selected to be 4.500 in.
6. Draw a construction line through B_1B_2 and extend it to the right.
7. Select a point on this line and call it O_2 . In the solution below the distance AB was selected to be 6.000 in.
8. Draw a circle about O_2 with a radius of one-half the length B_1B_2 and label the intersections of the circle with the extended line as A_1 and A_2 . In the solution below the radius was measured as 1.037 in.
9. The driver fourbar is now defined as O_2ABO_4 with link lengths

$$\text{Link 2 (crank)} \quad L_2 := 0.645 \cdot \text{in} \quad \text{Link 3 (coupler)} \quad L_3 := 6.000 \cdot \text{in}$$

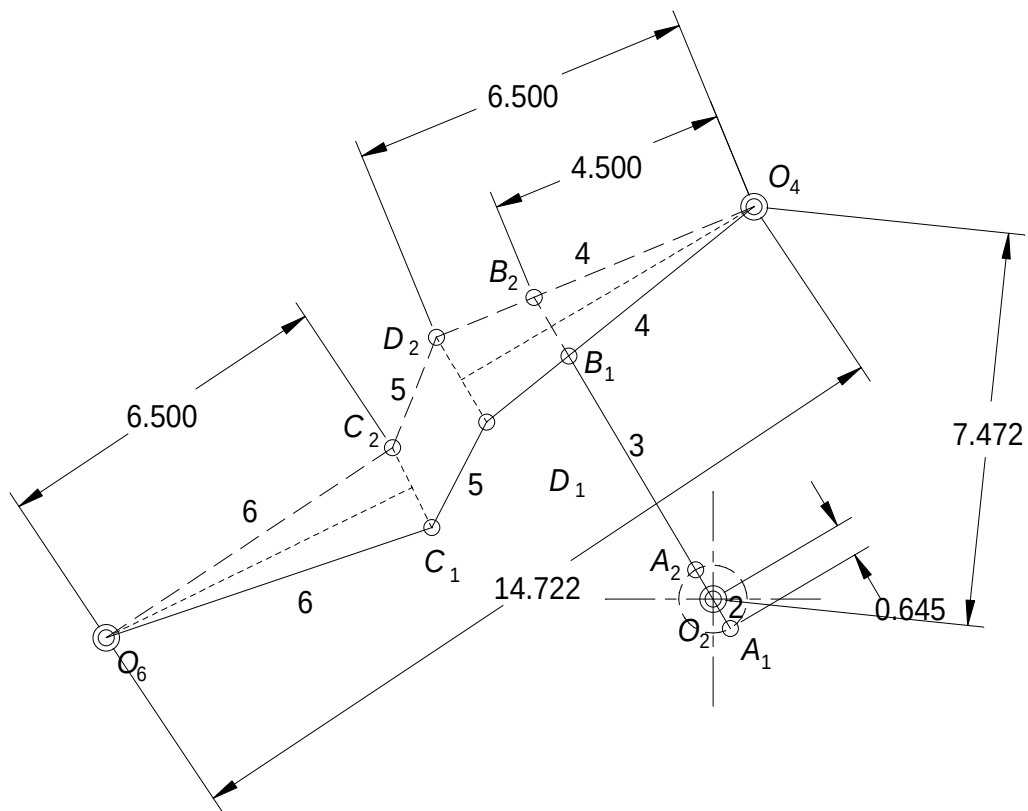
$$\text{Link 4a (rocker)} \quad L_{4a} := 4.500 \cdot \text{in} \quad \text{Link 1a (ground)} \quad L_{1a} := 7.472 \cdot \text{in}$$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$\begin{aligned} \text{Condition}(a, b, c, d) := & \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(L_{1a}, L_2, L_3, L_{4a}) = \text{"Grashof"}$$

$$\min(L_{1a}, L_2, L_3, L_{4a}) = 0.645 \text{ in}$$



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4CDO_6 is non-Grashoff with toggle positions at $\theta_4 = -17.1$ deg and $+17.1$ deg. The fourbar operates between $\theta_4 = +5.216$ deg and -11.273 deg.

PROBLEM 3-60

Statement: Design a fourbar mechanism to move the link shown in Figure P3-17 from position 2 to position 3. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Given: Position 2 offsets: $x_{C_2D_2} := 0.834 \cdot \text{in}$ $y_{C_2D_2} := 2.090 \cdot \text{in}$

Solution: See figure below for one possible solution. Input file P0360.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-60.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-60.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line CD with construction lines, i.e., lines from C_2 to C_3 and D_2 to D_3 .
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_2C_3 was extended downward and the bisector of D_2D_3 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6 , respectively. In the solution below the distances O_4D and O_6C were each selected to be 6.000 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 12.933 in.
4. The fourbar stage is now defined as O_4DCO_6 with link lengths

$$\text{Link 5 (coupler)} \quad L_5 := \sqrt{x_{C_2D_2}^2 + y_{C_2D_2}^2} \quad L_5 = 2.250 \text{ in}$$

$$\text{Link 4 (input)} \quad L_4 := 5.000 \cdot \text{in} \quad \text{Link 6 (output)} \quad L_6 := 5.000 \cdot \text{in}$$

$$\text{Ground link 1b} \quad L_{1b} := 12.933 \cdot \text{in}$$

5. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 4 is now a ternary link with nodes at O_4 , B , and D .) In the solution below the distance O_4B was selected to be 4.000 in.
6. Draw a construction line through B_1B_2 and extend it to the right.
7. Select a point on this line and call it O_2 . In the solution below the distance AB was selected to be 6.000 in.
8. Draw a circle about O_2 with a radius of one-half the length B_1B_2 and label the intersections of the circle with the extended line as A_1 and A_2 . In the solution below the radius was measured as 0.741 in.
9. The driver fourbar is now defined as O_2ABO_4 with link lengths

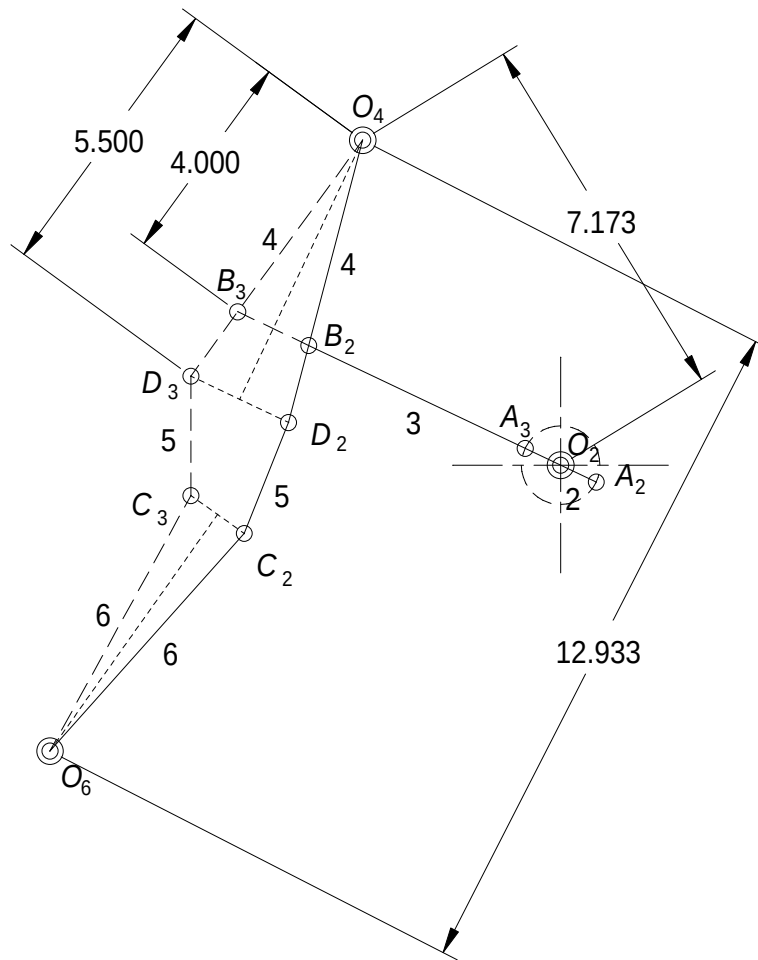
$$\text{Link 2 (crank)} \quad L_2 := 0.741 \cdot \text{in} \quad \text{Link 3 (coupler)} \quad L_3 := 6.000 \cdot \text{in}$$

$$\text{Link 4a (rocker)} \quad L_{4a} := 4.000 \cdot \text{in} \quad \text{Link 1a (ground)} \quad L_{1a} := 7.173 \cdot \text{in}$$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$\begin{aligned} \text{Condition}(a, b, c, d) := & \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(L_{1a}, L_2, L_3, L_4) = \text{"Grashof"}$$



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4DCO_6 is non-Grashoff with toggle positions at $\theta_4 = -14.9$ deg and $+14.9$ deg. The fourbar operates between $\theta_4 = +12.403$ deg and -8.950 deg.

PROBLEM 3-61

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-17. Ignore the points O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Solution: See Figure P3-17 and Mathcad file P0361.

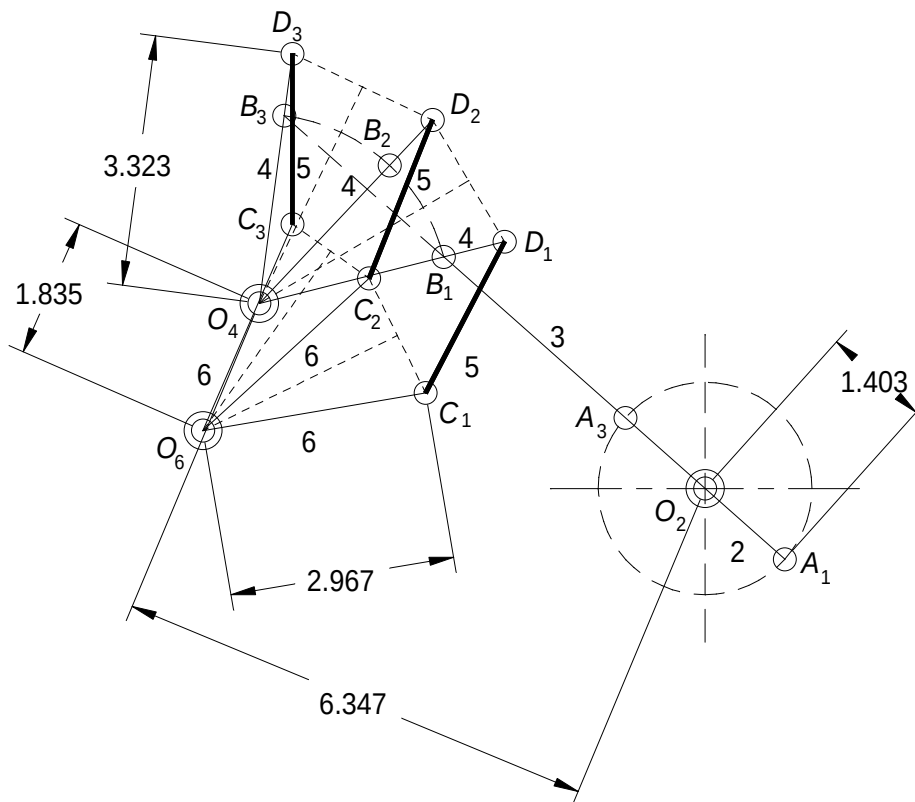
Design choices:

Length of link 3: $L_3 := 6.000$ Length of link 4b: $L_{4b} := 2.500$

1. Draw link CD in its three design positions C_1D_1 , C_2D_2 , C_3D_3 in the plane as shown.
2. Draw construction lines from point C_1 to C_2 and from point C_2 to C_3 .
3. Bisect line C_1C_2 and line C_2C_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_6 .
4. Repeat steps 2 and 3 for lines D_1D_2 and D_2D_3 . Label the intersection O_4 .
5. Connect O_2 with C_1 and call it link 2. Connect O_4 with D_1 and call it link 4.
6. Line C_1D_1 is link 5. Line O_2O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_6CDO_4 and has link lengths of

Ground link 1a	$L_{1a} := 1.835$	Link 6	$L_6 := 2.967$

Link 5 $L_5 := 2.250$ Link 4 $L_4 := 3.323$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$\text{Condition}(a, b, c, d) := \left| \begin{array}{l} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array} \right.$$

$$\text{Condition}(L_{1a}, L_4, L_5, L_6) = \text{"Grashof"}$$

8. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 4 is now a ternary link with nodes at O_4 , D , and B .) In the solution above the distance O_4B was selected to be $L_{4b} = 2.500$.
9. Draw a construction line through B_1B_3 and extend it up to the right.
10. Layout the length of link 3 (design choice) along the extended line. Label the other end A .
11. Draw a circle about O_2 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3 . In the solution below the radius was measured as $L_2 := 1.403$.
12. The driver fourbar is now defined as O_2ABO_4 with link lengths

Link 2 (crank) $L_2 = 1.403$

Link 3 (coupler) $L_3 = 6.000$

Link 1b (ground) $L_{1b} := 6.347$

Link 4b (rocker) $L_{4b} = 2.500$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$\text{Condition}(L_{1b}, L_2, L_3, L_{4b}) = \text{"Grashof"}$$

$$\min(L_{1b}, L_2, L_3, L_{4b}) = 1.403$$

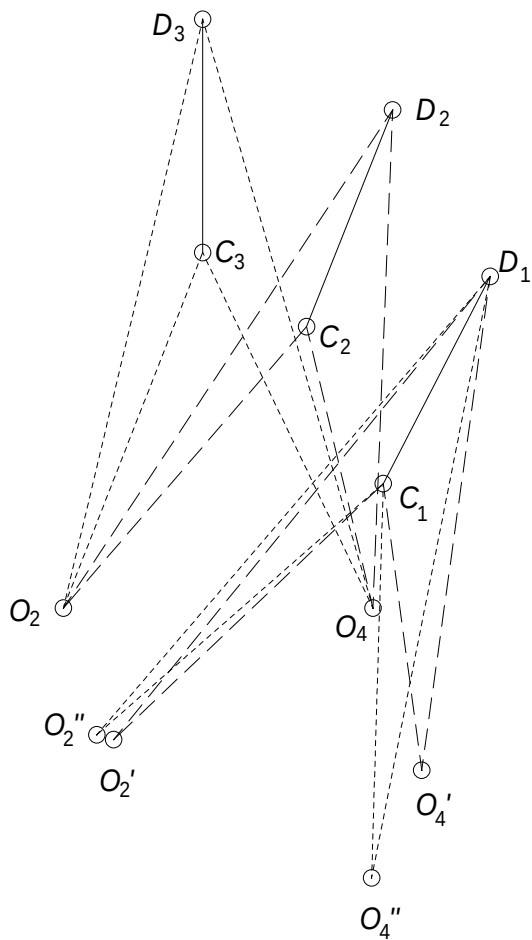
PROBLEM 3-62

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-17 using the fixed pivots O_2 and O_4 shown. (See Example 3-7.) Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Solution: See Figure P3-17 and Mathcad file P0362.

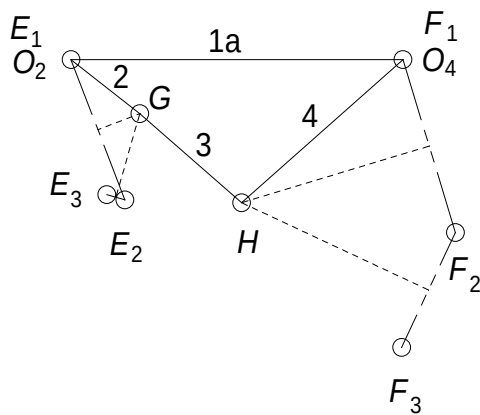
Design choices: Length of link 5: $L_5 := 4.000$ Length of link 2b: $L_{2b} := 0.791$

1. Draw link CD in its three design positions C_1D_1 , C_2D_2 , C_3D_3 in the plane as shown.
2. Draw the ground link O_2O_4 in its desired position in the plane with respect to the first coupler position C_1D_1 .
3. Draw construction arcs from point C_2 to O_2 and from point D_2 to O_2 whose radii define the sides of triangle $C_2O_2D_2$. This defines the relationship of the fixed pivot O_2 to the coupler line CD in the second coupler position.
4. Draw construction arcs from point C_2 to O_4 and from point D_2 to O_4 whose radii define the sides of triangle $C_2O_4D_2$. This defines the relationship of the fixed pivot O_4 to the coupler line CD in the second coupler position.
5. Transfer this relationship back to the first coupler position C_1D_1 so that the ground plane position $O_2'O_4'$ bears the same relationship to C_1D_1 as O_2O_4 bore to the second coupler position C_2D_2 .
6. Repeat the process for the third coupler position and transfer the third relative ground link position to the first, or reference, position.
7. The three inverted positions of the ground link that correspond to the three desired coupler positions are labeled O_2O_4 , $O_2'O_4'$, and $O_2''O_4''$ in the first layout below and are renamed E_1F_1 , E_2F_2 , and E_3F_3 , respectively, in the second layout, which is used to find the points G and H .



8. Draw construction lines from point E_1 to E_2 and from point E_2 to E_3 .
9. Bisect line E_1E_2 and line E_2E_3 and extend their perpendicular bisectors until they intersect. Label their intersection G .
10. Repeat steps 2 and 3 for lines F_1F_2 and F_2F_3 . Label the intersection H .
11. Connect E_1 with G and label it link 2. Connect F_1 with H and label it link 4. Reverting, E_1 and F_1 are the original fixed pivots O_2 and O_4 , respectively.
12. Line GH is link 3. Line O_2O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_2GHO_4 and has link lengths of

Ground link 1a	$L_{1a} := 3.000$	Link 2	$L_2 := 0.791$
Link 3	$L_3 := 1.222$	Link 4	$L_4 := 1.950$



13. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$Condition(L_{1a}, L_2, L_3, L_4) = \text{"non-Grashof"}$

The fourbar that will provide the desired motion is now defined as a non-Grashof double rocker in the crossed configuration. It now remains to add the original points C_1 and D_1 to the coupler GH and to define the driving dyad, which in this case will drive link 4 rather than link 2.

14. Select a point on link 2 (O_2G) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 2 is now a ternary link with nodes at O_2 , B , and G .) In the solution below, the distance O_2B was selected to be $L_{2b} = 0.791$. Thus, in this case B and G coincide.
15. Draw a construction line through B_1B_3 and extend it up to the left.
16. Layout the length of link 5 (design choice) along the extended line. Label the other end A .
17. Draw a circle about O_6 with a radius of one-half the length B_1B_3 and label the intersections of the circle

with the extended line as A_1 and A_3 . In the solution below the radius was measured as $L_6 := 0.727$.

18. The driver fourbar is now defined as O_2BAO_6 with link lengths

Link 6 (crank) $L_6 = 0.727$

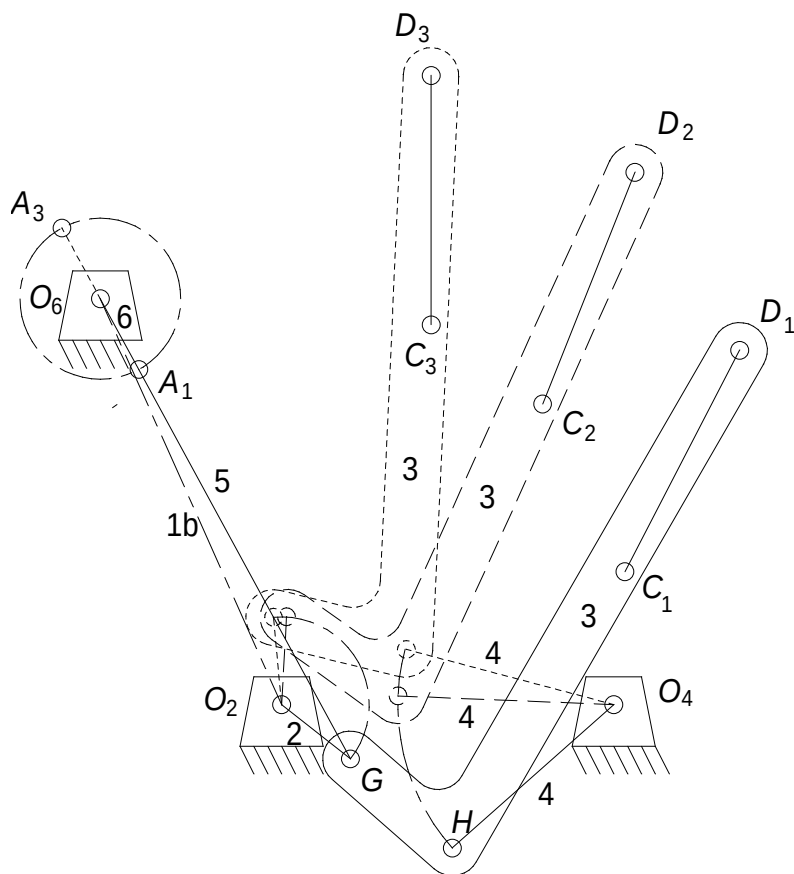
Link 5 (coupler) $L_5 = 4.000$

Link 1b (ground) $L_{1b} := 4.012$

Link 2b (rocker) $L_{2b} = 0.791$

19. Use the link lengths in step 18 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$Condition(L_6, L_5, L_{1b}, L_{2b}) = \text{"Grashof"}$



PROBLEM 3-63

Statement: Design a fourbar mechanism to move the link shown in Figure P3-18 from position 1 to position 2. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Given: Position 1 offsets: $x_{C1D1} := 1.591 \cdot \text{in}$ $y_{C1D1} := 1.591 \cdot \text{in}$

Solution: See figure below for one possible solution. Input file P0363.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-63.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-63.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line CD with construction lines, i.e., lines from C_1 to C_2 and D_1 to D_2 .
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_1C_2 was extended downward and the bisector of D_1D_2 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6 , respectively. In the solution below the distances O_4C and O_6D were each selected to be 5.000 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 10.457 in.
4. The fourbar stage is now defined as O_4CDO_6 with link lengths

$$\text{Link 5 (coupler)} \quad L_5 := \sqrt{x_{C1D1}^2 + y_{C1D1}^2} \quad L_5 = 2.250 \text{ in}$$

$$\text{Link 4 (input)} \quad L_4 := 5.000 \cdot \text{in} \quad \text{Link 6 (output)} \quad L_6 := 5.000 \cdot \text{in}$$

$$\text{Ground link 1b} \quad L_{1b} := 10.457 \cdot \text{in}$$

5. Select a point on link 4 (O_4C) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 4 is now a ternary link with nodes at O_4 , B , and C .) In the solution below the distance O_4B was selected to be 3.750 in.
6. Draw a construction line through B_1B_2 and extend it to the right.
7. Select a point on this line and call it O_2 . In the solution below the distance AB was selected to be 6.000 in.
8. Draw a circle about O_2 with a radius of one-half the length B_1B_2 and label the intersections of the circle with the extended line as A_1 and A_2 . In the solution below the radius was measured as 0.882 in.
9. The driver fourbar is now defined as O_2ABO_4 with link lengths

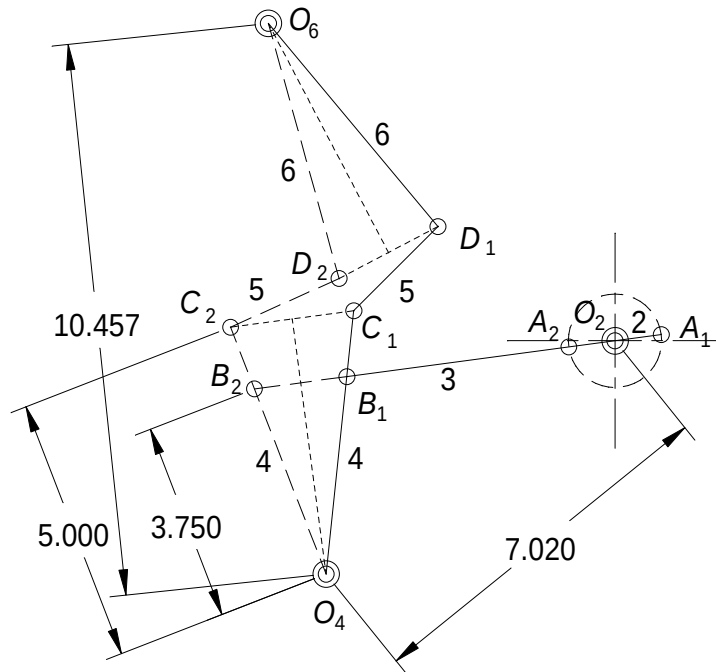
$$\text{Link 2 (crank)} \quad L_2 := 0.882 \cdot \text{in} \quad \text{Link 3 (coupler)} \quad L_3 := 6.000 \cdot \text{in}$$

$$\text{Link 4a (rocker)} \quad L_{4a} := 3.750 \cdot \text{in} \quad \text{Link 1a (ground)} \quad L_{1a} := 7.020 \cdot \text{in}$$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} \quad \text{if } SL < PQ \\ \text{return "Special Grashof"} \quad \text{if } SL = PQ \\ \text{return "non-Grashof"} \quad \text{otherwise} \end{cases}$$

$$\text{Condition}(L_{1a}, L_2, L_3, L_{4a}) = \text{"Grashof"}$$



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4CDO_6 is non-Grashoff with toggle positions at $\theta_4 = -38.5$ deg and $+38.5$ deg. The fourbar operates between $\theta_4 = +15.206$ deg and -12.009 deg.

PROBLEM 3-64

Statement: Design a fourbar mechanism to move the link shown in Figure P3-18 from position 2 to position 3. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Given: Position 2 offsets: $x_{C_2D_2} := 2.053 \cdot \text{in}$ $y_{C_2D_2} := 0.920 \cdot \text{in}$

Solution: See figure below for one possible solution. Input file P0360.mcd from the solutions manual disk to the Mathcad program for this solution, file P03-60.4br to the program FOURBAR to see the fourbar solution linkage, and file P03-60.6br into program SIXBAR to see the complete sixbar with the driver dyad included.

1. Connect the end points of the two given positions of the line CD with construction lines, i.e., lines from C_2 to C_3 and D_2 to D_3 .
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_2C_3 was extended downward and the bisector of D_2D_3 was extended upward.
3. Select one point on each bisector and label them O_4 and O_6 , respectively. In the solution below the distances O_4D and O_6C were each selected to be 5.000 in. This resulted in a ground-link-length O_4O_6 for the fourbar of 8.773 in.
4. The fourbar stage is now defined as O_4DCO_6 with link lengths

$$\text{Link 5 (coupler)} \quad L_5 := \sqrt{x_{C_2D_2}^2 + y_{C_2D_2}^2} \quad L_5 = 2.250 \text{ in}$$

$$\text{Link 4 (input)} \quad L_4 := 5.000 \cdot \text{in} \quad \text{Link 6 (output)} \quad L_6 := 5.000 \cdot \text{in}$$

$$\text{Ground link 1b} \quad L_{1b} := 8.773 \cdot \text{in}$$

5. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 4 is now a ternary link with nodes at O_4 , B , and D .) In the solution below the distance O_4B was selected to be 3.750 in.
6. Draw a construction line through B_1B_2 and extend it to the right.
7. Select a point on this line and call it O_2 . In the solution below the distance AB was selected to be 6.000 in.
8. Draw a circle about O_2 with a radius of one-half the length B_1B_2 and label the intersections of the circle with the extended line as A_1 and A_2 . In the solution below the radius was measured as 0.892 in.
9. The driver fourbar is now defined as O_2ABO_4 with link lengths

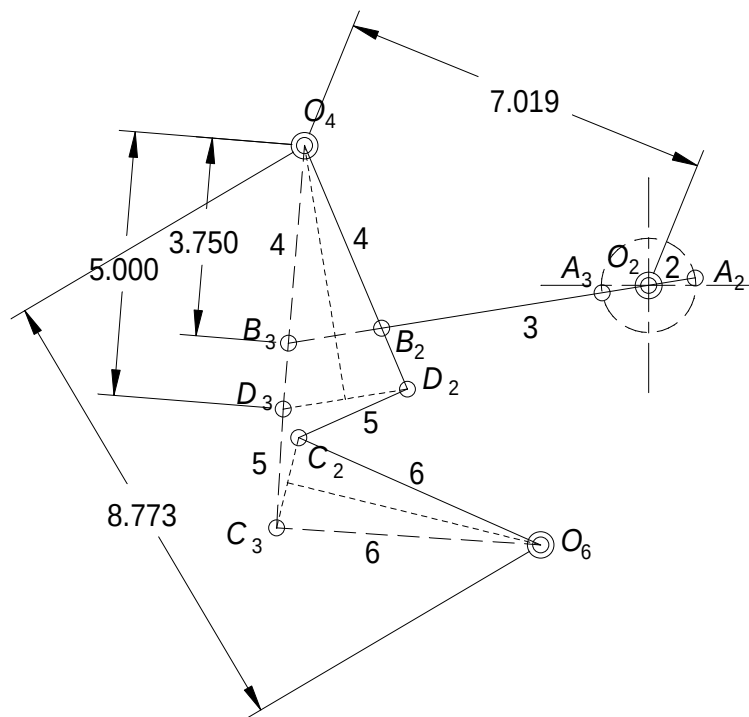
$$\text{Link 2 (crank)} \quad L_2 := 0.892 \cdot \text{in} \quad \text{Link 3 (coupler)} \quad L_3 := 6.000 \cdot \text{in}$$

$$\text{Link 4a (rocker)} \quad L_{4a} := 3.750 \cdot \text{in} \quad \text{Link 1a (ground)} \quad L_{1a} := 7.019 \cdot \text{in}$$

10. Use the link lengths in step 9 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$\begin{aligned} \text{Condition}(a, b, c, d) := & \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(L_{1a}, L_2, L_3, L_{4a}) = \text{"Grashof"}$$



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4DCO_6 is non-Grashoff with toggle positions at $\theta_4 = -55.7$ deg and $+55.7$ deg. The fourbar operates between $\theta_4 = -7.688$ deg and -35.202 deg.

PROBLEM 3-65

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-18. Ignore the points O_2 and O_4 shown. Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Solution: See Figure P3-18 and Mathcad file P0365.

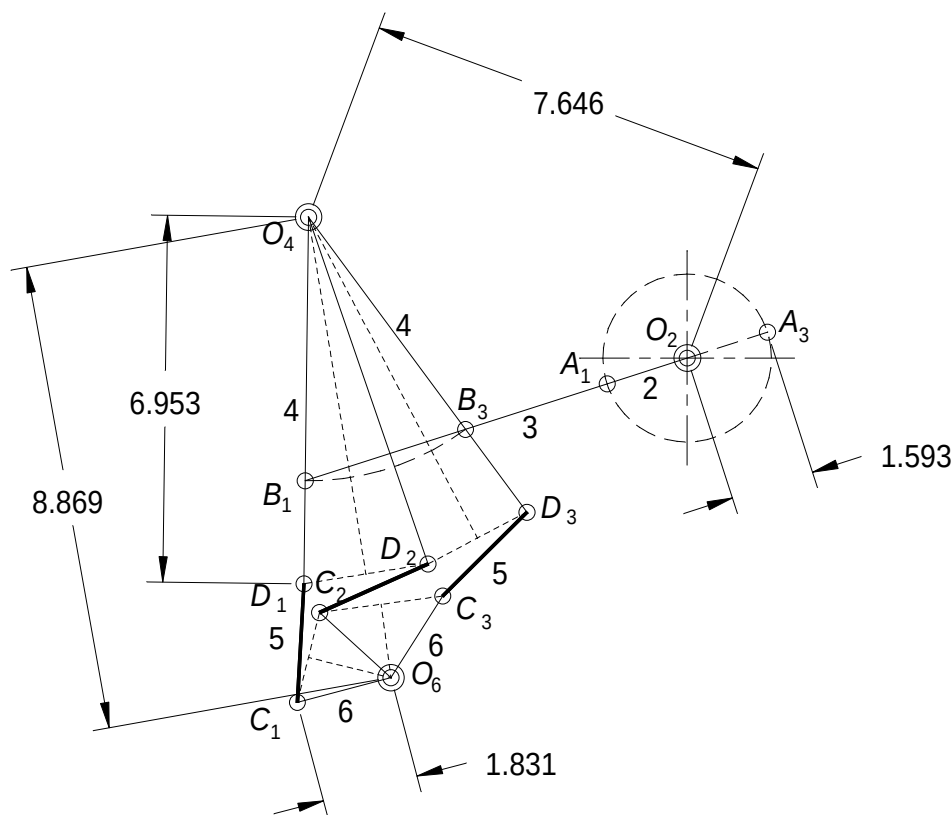
Design choices:

Length of link 3: $L_3 := 6.000$ Length of link 4b: $L_{4b} := 5.000$

1. Draw link CD in its three design positions C_1D_1 , C_2D_2 , C_3D_3 in the plane as shown.
2. Draw construction lines from point C_1 to C_2 and from point C_2 to C_3 .
3. Bisect line C_1C_2 and line C_2C_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_6 .
4. Repeat steps 2 and 3 for lines D_1D_2 and D_2D_3 . Label the intersection O_4 .
5. Connect O_6 with C_1 and call it link 6. Connect O_4 with D_1 and call it link 4.
6. Line C_1D_1 is link 5. Line O_6O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_6CDO_4 and has link lengths of

Ground link 1a $L_{1a} := 8.869$ Link 6 $L_6 := 1.831$

Link 5 $L_5 := 2.250$ Link 4 $L_4 := 6.953$



7. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} \quad \text{if } SL < PQ \\ \text{return "Special Grashof"} \quad \text{if } SL = PQ \\ \text{return "non-Grashof"} \quad \text{otherwise} \end{cases}$$

$$\text{Condition}(L_6, L_{1a}, L_4, L_5) = \text{"non-Grashof"}$$

8. Select a point on link 4 (O_4D) at a suitable distance from O_4 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 4 is now a ternary link with nodes at O_4 , D , and B .) In the solution above the distance O_4B was selected to be $L_{4b} = 5.000$.
9. Draw a construction line through B_1B_3 and extend it up to the right.
10. Layout the length of link 3 (design choice) along the extended line. Label the other end A .
11. Draw a circle about O_2 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3 . In the solution below the radius was measured as $L_2 := 1.593$.
12. The driver fourbar is now defined as O_2ABO_4 with link lengths

Link 2 (crank) $L_2 = 1.593$

Link 3 (coupler) $L_3 = 6.000$

Link 1b (ground) $L_{1b} := 7.646$

Link 4b (rocker) $L_{4b} = 5.000$

13. Use the link lengths in step 12 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 2).

$$\text{Condition}(L_{1b}, L_2, L_3, L_{4b}) = \text{"Grashof"}$$

$$\min(L_{1b}, L_2, L_3, L_{4b}) = 1.593$$

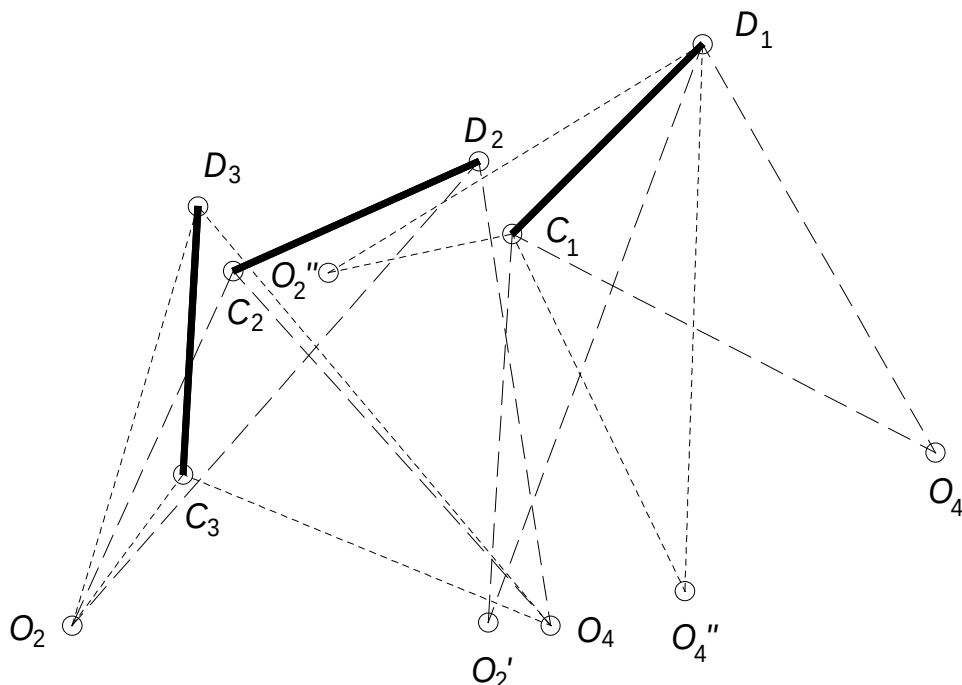
PROBLEM 3-66

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-18 using the fixed pivots O_2 and O_4 shown. (See Example 3-7.) Build a cardboard model and add a driver dyad to limit its motion to the range of positions designed, making it a sixbar.

Solution: See Figure P3-18 and Mathcad file P0366.

Design choices: Length of link 5: $L_5 := 4.000$ Length of link 2b: $L_{2b} := 2.000$

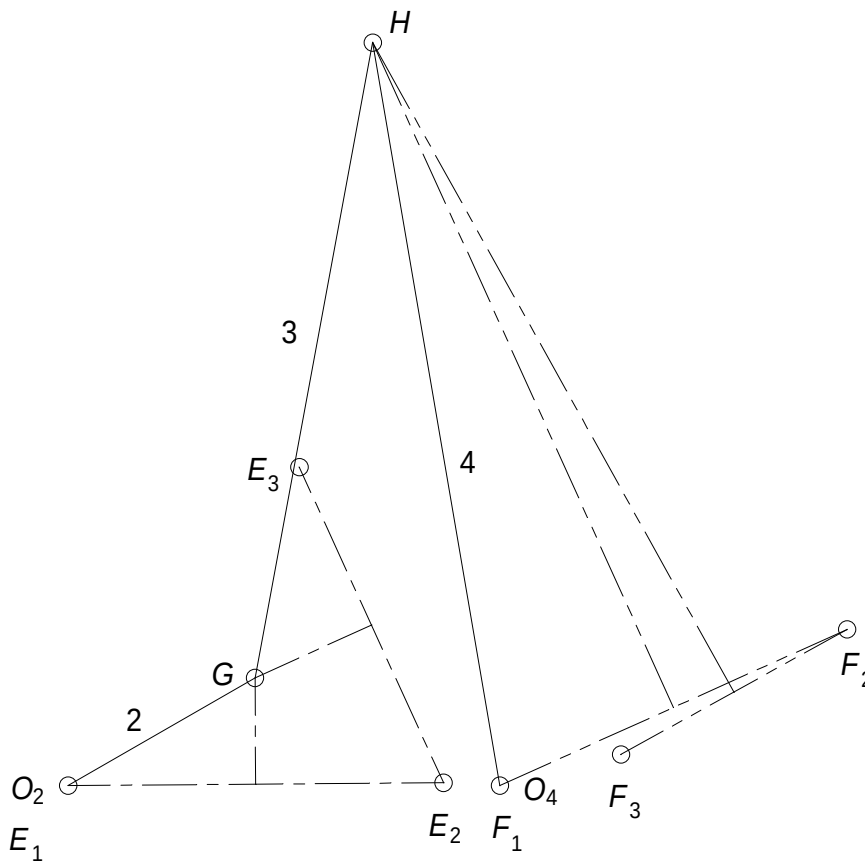
1. Draw link CD in its three design positions C_1D_1 , C_2D_2 , C_3D_3 in the plane as shown.
2. Draw the ground link O_2O_4 in its desired position in the plane with respect to the first coupler position C_1D_1 .
3. Draw construction arcs from point C_2 to O_2 and from point D_2 to O_2 whose radii define the sides of triangle $C_2O_2D_2$. This defines the relationship of the fixed pivot O_2 to the coupler line CD in the second coupler position.
4. Draw construction arcs from point C_2 to O_4 and from point D_2 to O_4 whose radii define the sides of triangle $C_2O_4D_2$. This defines the relationship of the fixed pivot O_4 to the coupler line CD in the second coupler position.
5. Transfer this relationship back to the first coupler position C_1D_1 so that the ground plane position $O_2'O_4'$ bears the same relationship to C_1D_1 as O_2O_4 bore to the second coupler position C_2D_2 .
6. Repeat the process for the third coupler position and transfer the third relative ground link position to the first, or reference, position.
7. The three inverted positions of the ground link that correspond to the three desired coupler positions are labeled O_2O_4 , $O_2'O_4'$, and $O_2''O_4''$ in the first layout below and are renamed E_1F_1 , E_2F_2 , and E_3F_3 , respectively, in the second layout, which is used to find the points G and H .



8. Draw construction lines from point E_1 to E_2 and from point E_2 to E_3 .
9. Bisect line E_1E_2 and line E_2E_3 and extend their perpendicular bisectors until they intersect. Label their intersection G .
10. Repeat steps 2 and 3 for lines F_1F_2 and F_2F_3 . Label the intersection H .

11. Connect E_1 with G and label it link 2. Connect F_1 with H and label it link 4. Reinverting, E_1 and F_1 are the original fixed pivots O_2 and O_4 , respectively.
12. Line GH is link 3. Line O_2O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_2GHO_4 and has link lengths of

Ground link 1a	$L_{1a} := 4.000$	Link 2	$L_2 := 2.000$
Link 3	$L_3 := 6.002$	Link 4	$L_4 := 7.002$



13. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

```

Condition(a,b,c,d) :=
    S ← min(a,b,c,d)
    L ← max(a,b,c,d)
    SL ← S + L
    PQ ← a + b + c + d - SL
    return "Grashof" if SL < PQ
    return "Special Grashof" if SL = PQ
    return "non-Grashof" otherwise

```

$Condition(L_{1a}, L_2, L_3, L_4) = \text{"Grashof"}$

The fourbar that will provide the desired motion is now defined as a non-Grashof crank rocker in the open configuration. It now remains to add the original points C_1 and D_1 to the coupler GH and to define the driving dyad, which in this case will drive link 4 rather than link 2.

14. Select a point on link 2 (O_2G) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it B . (Note that link 2 is now a ternary link with nodes at O_2 , B , and G .) In the solution below, the distance O_2B was selected to be $L_{2b} = 2.000$. Thus, in this case B and G coincide.
15. Draw a construction line through B_1B_3 and extend it up to the left.
16. Layout the length of link 5 (design choice) along the extended line. Label the other end A .
17. Draw a circle about O_6 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3 . In the solution below the radius was measured as $L_6 := 1.399$.
18. The driver fourbar is now defined as O_2BAO_6 with link lengths

Link 6 (crank) $L_6 = 1.399$

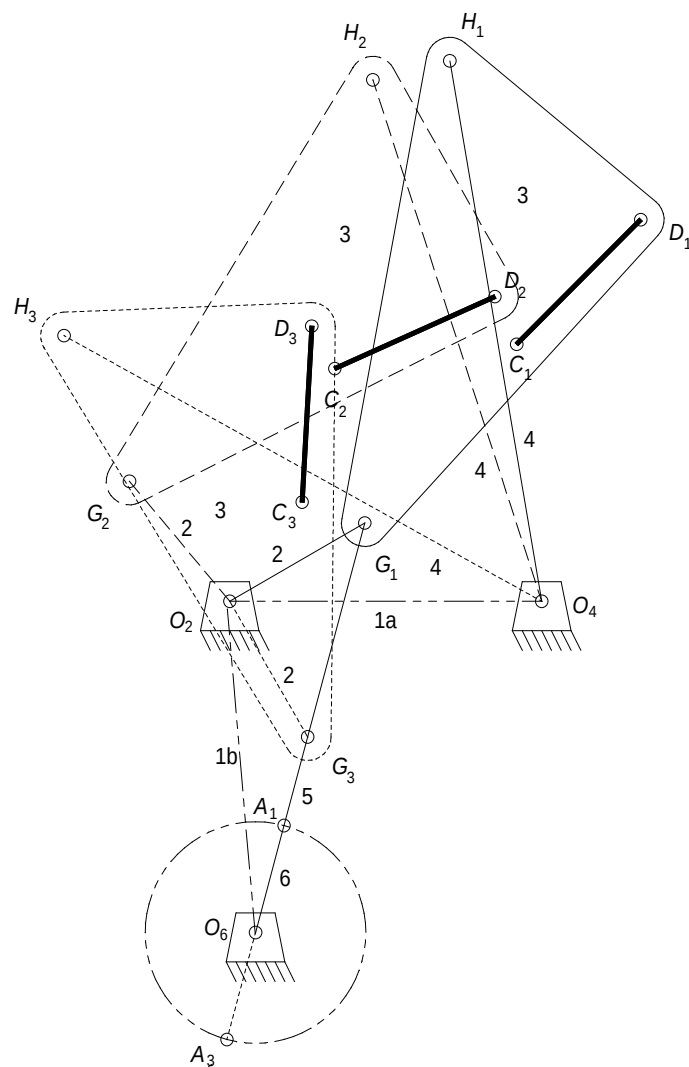
Link 5 (coupler) $L_5 = 4.000$

Link 1b (ground) $L_{1b} := 4.257$

Link 2b (rocker) $L_{2b} = 2.000$

19. Use the link lengths in step 18 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$Condition(L_6, L_{1b}, L_{2b}, L_5) = \text{"Grashof"}$



PROBLEM 3-67

Statement: Design a fourbar Grashof crank-rocker for 120 degrees of output rocker motion with a quick-return time ratio of 1:1.2. (See Example 3-9.)

Given: Time ratio $T_r := \frac{1}{1.2}$

Solution: See figure below for one possible solution. Also see Mathcad file P0367.

- Determine the crank rotation angles α and β , and the construction angle δ from equations 3.1 and 3.2.

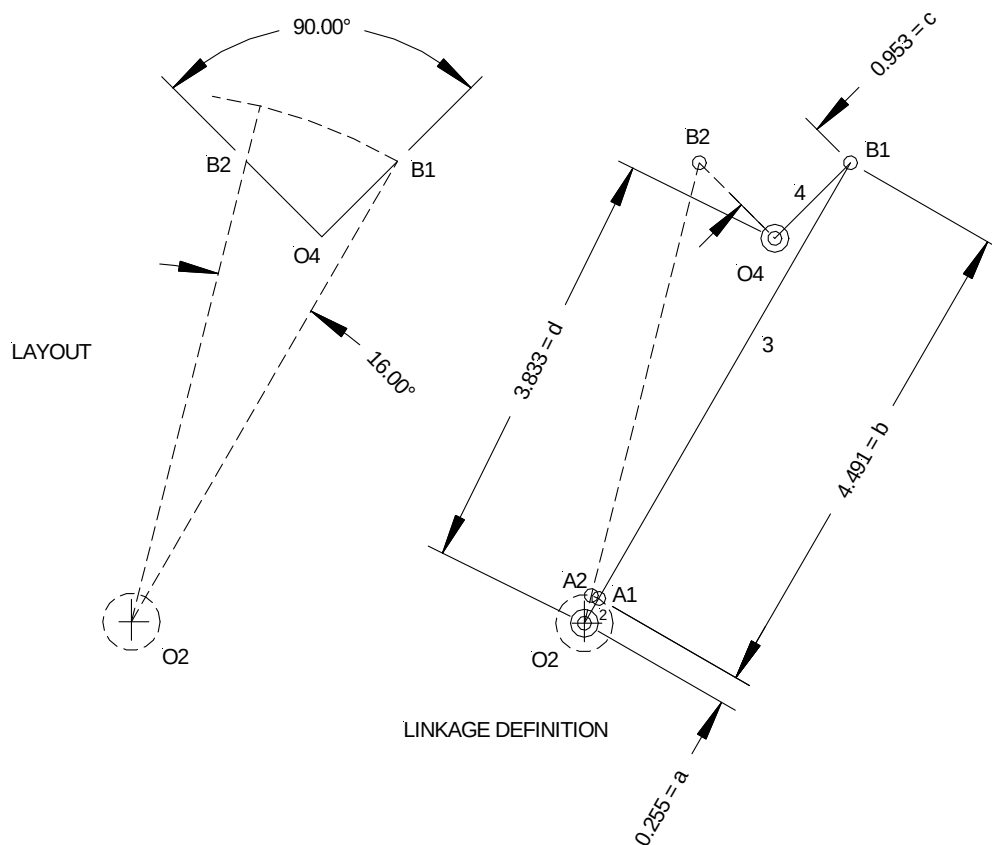
$$T_r = \frac{\alpha}{\beta} \quad \alpha + \beta = 360 \cdot \text{deg}$$

$$\text{Solving for } \beta, \alpha, \text{ and } \delta \quad \beta := \frac{360 \cdot \text{deg}}{1 + T_r} \quad \beta = 196 \cdot \text{deg}$$

$$\alpha := 360 \cdot \text{deg} - \beta \quad \alpha = 164 \cdot \text{deg}$$

$$\delta := \beta - 180 \cdot \text{deg} \quad \delta = 16 \cdot \text{deg}$$

- Start the layout by arbitrarily establishing the point O_4 and from it layoff two lines of equal length, 90 deg apart. Label one B_1 and the other B_2 . In the solution below, each line makes an angle of 45 deg with the horizontal and has a length of 1.000 in.
- Layoff a line through B_1 at an arbitrary angle (but not zero deg). In the solution below the line is 60 deg to the horizontal.



4. Layoff a line through B_2 that makes an angle δ with the line in step 3 (76 deg to the horizontal in this case). The intersection of these two lines establishes the point O_2 .
5. From O_2 draw an arc that goes through B_1 . Extend O_2B_2 to meet this arc. Erect a perpendicular bisector to the extended portion of the line and transfer one half of the line to O_2 as the length of the input crank.
6. For this solution, the link lengths are:

Ground link (1)	$d := 3.833 \cdot in$	Coupler (3)	$b := 4.491 \cdot in$
Crank (2)	$a := 0.255 \cdot in$	Rocker (4)	$c := 0.953 \cdot in$

PROBLEM 3-68

Statement: Design a fourbar Grashof crank-rocker for 100 degrees of output rocker motion with a quick-return time ratio of 1:1.5. (See Example 3-9.)

Given: Time ratio $T_r := \frac{1}{1.5}$

Solution: See figure below for one possible solution. Also see Mathcad file P0368.

- Determine the crank rotation angles α and β , and the construction angle δ from equations 3.1 and 3.2.

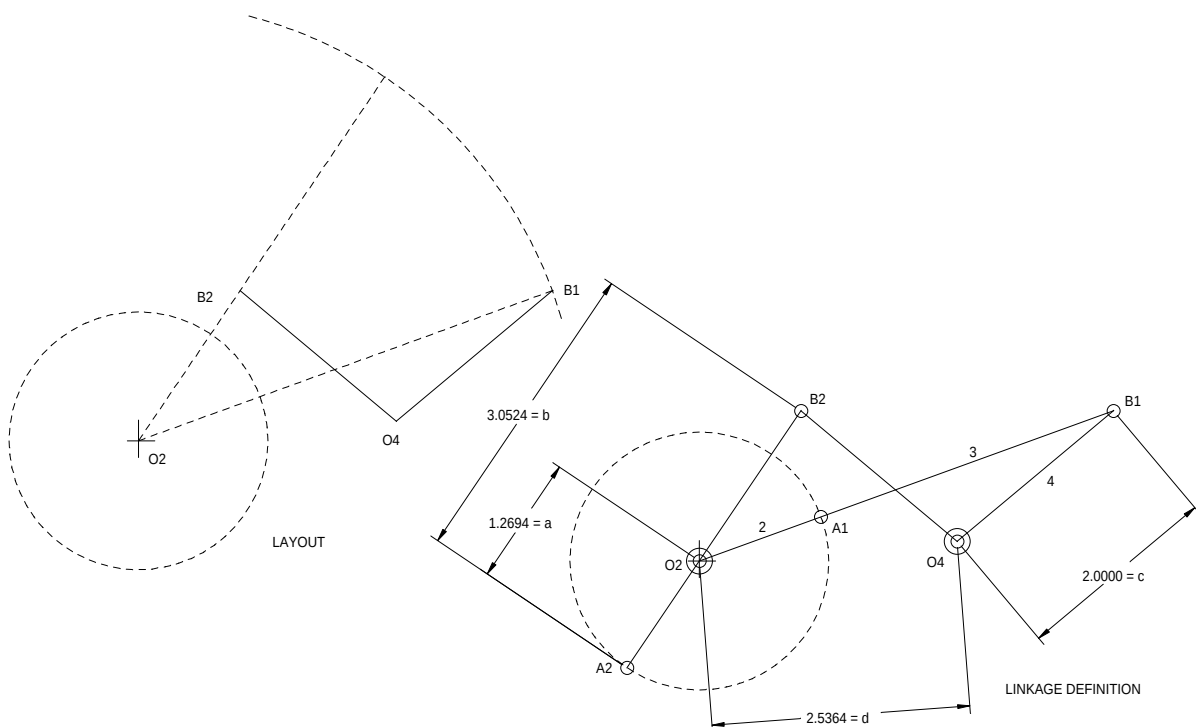
$$T_r = \frac{\alpha}{\beta} \quad \alpha + \beta = 360 \cdot \text{deg}$$

$$\text{Solving for } \beta, \alpha, \text{ and } \delta \quad \beta := \frac{360 \cdot \text{deg}}{1 + T_r} \quad \beta = 216 \text{ deg}$$

$$\alpha := 360 \cdot \text{deg} - \beta \quad \alpha = 144 \text{ deg}$$

$$\delta := \beta - 180 \cdot \text{deg} \quad \delta = 36 \text{ deg}$$

- Start the layout by arbitrarily establishing the point O_4 and from it layoff two lines of equal length, 100 deg apart. Label one B_1 and the other B_2 . In the solution below, each line makes an angle of 40 deg with the horizontal and has a length of 2.000 in.
- Layoff a line through B_1 at an arbitrary angle (but not zero deg). In the solution below the line is 20 deg to the horizontal.
- Layoff a line through B_2 that makes an angle δ with the line in step 3 (56 deg to the horizontal in this case). The intersection of these two lines establishes the point O_2 .
- From O_2 draw an arc that goes through B_1 . Extend O_2B_2 to meet this arc. Erect a perpendicular bisector to the extended portion of the line and transfer one half of the line to O_2 as the length of the input crank.



6. For this solution, the link lengths are:

Ground link (1)	$d := 2.5364 \cdot in$	Coupler (3)	$b := 3.0524 \cdot in$
Crank (2)	$a := 1.2694 \cdot in$	Rocker (4)	$c := 2.000 \cdot in$

PROBLEM 3-69

Statement: Design a fourbar Grashof crank-rocker for 80 degrees of output rocker motion with a quick-return time ratio of 1:1.33. (See Example 3-9.)

Given: Time ratio $T_r := \frac{1}{1.33}$

Solution: See figure below for one possible solution. Also see Mathcad file P0369.

- Determine the crank rotation angles α and β , and the construction angle δ from equations 3.1 and 3.2.

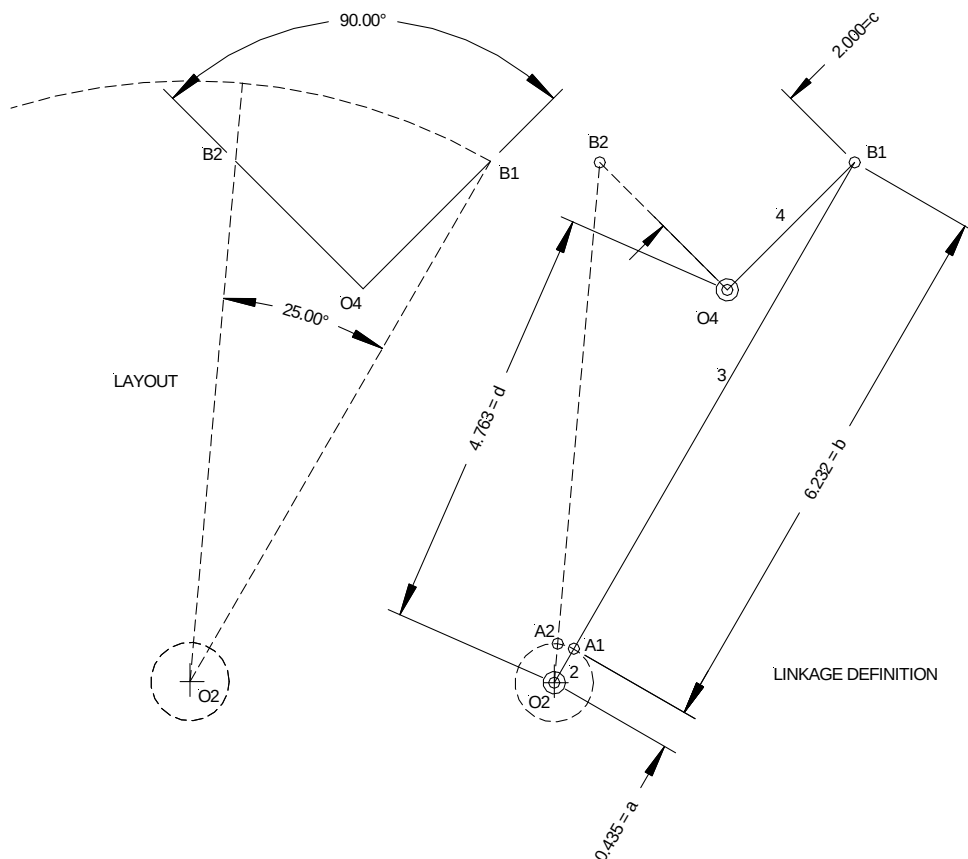
$$T_r = \frac{\alpha}{\beta} \quad \alpha + \beta = 360 \cdot \text{deg}$$

Solving for β , α , and δ $\beta := \frac{360 \cdot \text{deg}}{1 + T_r} \quad \beta = 205 \cdot \text{deg}$

$$\alpha := 360 \cdot \text{deg} - \beta \quad \alpha = 155 \cdot \text{deg}$$

$$\delta := \beta - 180 \cdot \text{deg} \quad \delta = 25 \cdot \text{deg}$$

- Start the layout by arbitrarily establishing the point O_4 and from it layoff two lines of equal length, 100 deg apart. Label one B_1 and the other B_2 . In the solution below, each line makes an angle of 40 deg with the horizontal and has a length of 2.000 in.
- Layoff a line through B_1 at an arbitrary angle (but not zero deg). In the solution below the line is 150 deg to the horizontal.



4. Layoff a line through B_2 that makes an angle δ with the line in step 3 (73 deg to the horizontal in this case). The intersection of these two lines establishes the point O_2 .
5. From O_2 draw an arc that goes through B_1 . Extend O_2B_2 to meet this arc. Erect a perpendicular bisector to the extended portion of the line and transfer one half of the line to O_2 as the length of the input crank.
6. For this solution, the link lengths are:

Ground link (1) $d := 4.763 \cdot in$

Coupler (3) $b := 6.232 \cdot in$

Crank (2) $a := 0.435 \cdot in$

Rocker (4) $c := 2.000 \cdot in$

PROBLEM 3-70

Statement: Design a sixbar drag link quick-return linkage for a time ratio of 1:4 and output rocker motion of 50 degrees. (See Example 3-10.)

Given: Time ratio $T_r := \frac{1}{4}$

Solution: See figure below for one possible solution. Also see Mathcad file P0370.

1. Determine the crank rotation angles α and β from equation 3.1.

$$T_r = \frac{\alpha}{\beta} \quad \alpha + \beta = 360 \cdot \text{deg}$$

$$\text{Solving for } \beta \text{ and } \alpha \quad \beta := \frac{360 \cdot \text{deg}}{1 + T_r} \quad \beta = 288 \text{ deg}$$

$$\alpha := 360 \cdot \text{deg} - \beta \quad \alpha = 72 \text{ deg}$$

2. Draw a line of centers XX at any convenient location.
3. Choose a crank pivot location O_2 on line XX and draw an axis YY perpendicular to XX through O_2 .
4. Draw a circle of convenient radius O_2A about center O_2 . In the solution below, the length of O_2A is $a := 1.000 \cdot \text{in}$.
5. Lay out angle α with vertex at O_2 , symmetrical about quadrant one.
6. Label points A_1 and A_2 at the intersections of the lines subtending angle α and the circle of radius O_2A .
7. Set the compass to a convenient radius AC long enough to cut XX in two places on either side of O_2 when swung from both A_1 and A_2 . Label the intersections C_1 and C_2 . In the solution below, the length of AC is $b := 2.000 \cdot \text{in}$.
8. The line O_2A is the driver crank, link 2, and the line AC is the coupler, link 3.
9. The distance C_1C_2 is twice the driven (dragged) crank length. Bisect it to locate the fixed pivot O_4 .
10. The line O_2O_4 now defines the ground link. Line O_4C is the driven crank, link 4. In the solution below, O_4C measures $c := 2.282 \cdot \text{in}$ and O_2O_4 measures $d := 0.699 \cdot \text{in}$.
11. Calculate the Grashoff condition. If non-Grashoff, repeat steps 7 through 11 with a shorter radius in step 7.

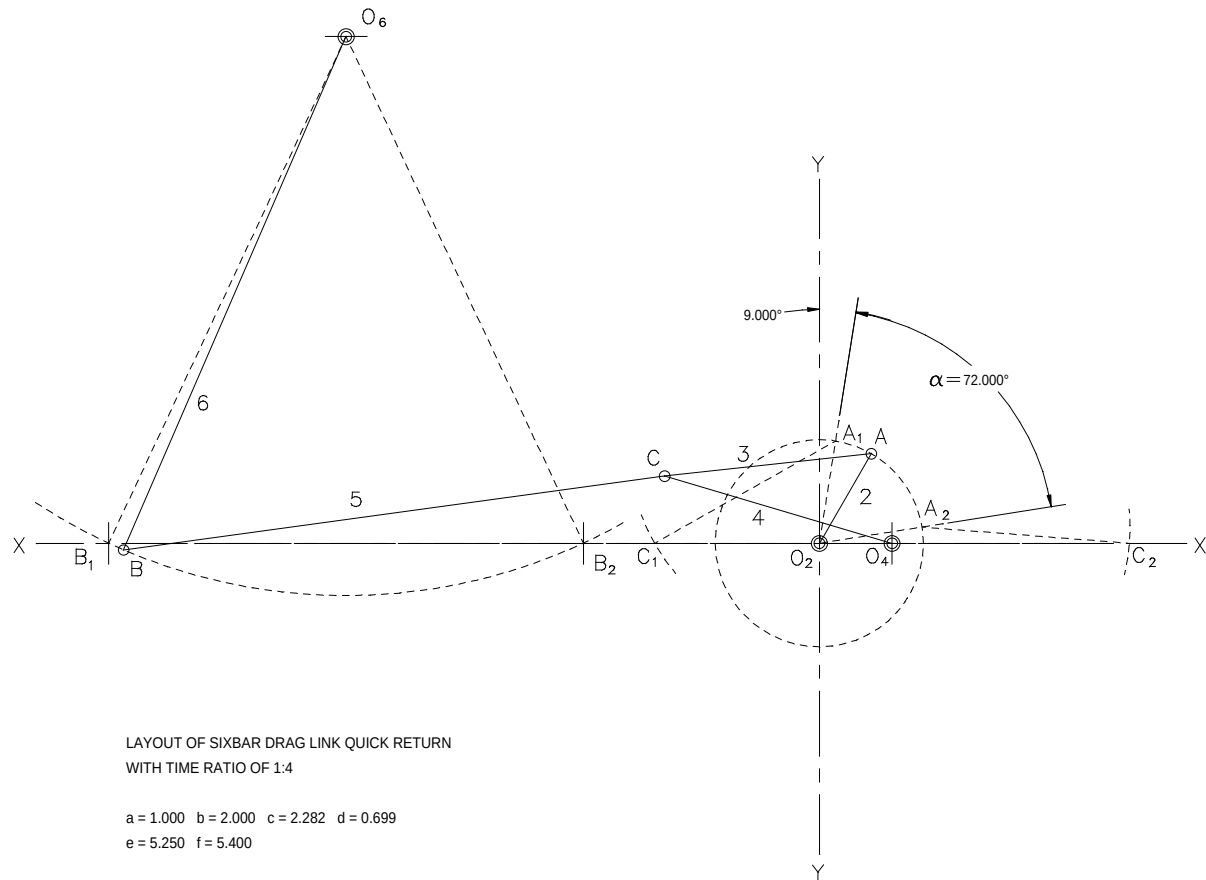
```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$\text{Condition}(a,b,c,d) = \text{"Grashof"}$

12. Invert the method of Example 3-1 to create the output dyad using XX as the chord and O_4C_1 as the driving crank. The points B_1 and B_2 will lie on line XX and be spaced apart a distance that is twice the length of O_4C (link 4). The pivot point O_6 will lie on the perpendicular bisector of B_1B_2 at a distance from XX which subtends the specified output rocker angle, which is 50 degrees in this problem. In the solution below, the length BC was chosen to be $e := 5.250 \cdot \text{in}$.



13. For the design choices made (lengths of links 2, 3 and 5), the length of the output rocker (link 6) was measured as $f := 5.400 \cdot in$.

PROBLEM 3-71

Statement: Design a crank-shaper quick-return mechanism for a time ratio of 1:2.5 (Figure 3-14, p. 112).

Given: Time ratio $T_R := \frac{1}{2.5}$

Solution: See Figure 3-14 and Mathcad file P0371.

Design choices:

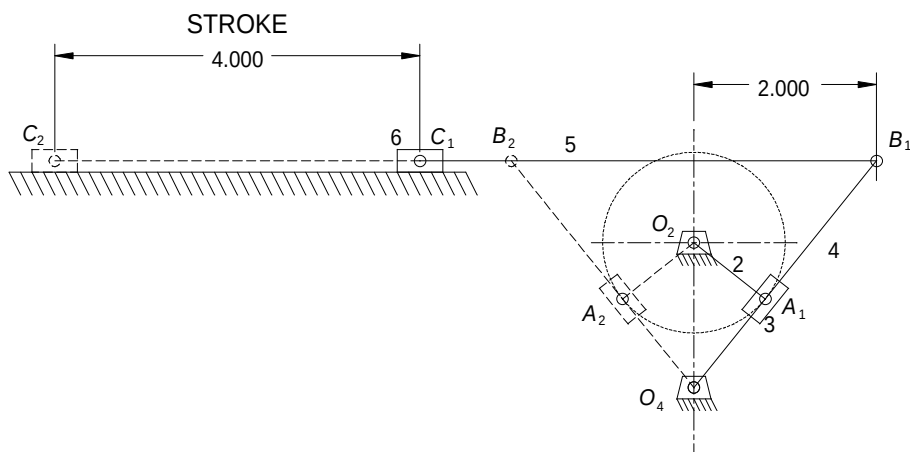
Length of link 2 (crank) $L_2 := 1.000$ Length of stroke $S := 4.000$

Length of link 5 (coupler) $L_5 := 5.000$

1. Calculate α from equations 3.1.

$$T_R := \frac{\alpha}{\beta} \quad \alpha + \beta := 360 \cdot \text{deg} \quad \alpha := \frac{360 \cdot \text{deg}}{1 + \frac{1}{T_R}} \quad \alpha = 102.86 \text{ deg}$$

2. Draw a vertical line and mark the center of rotation of the crank, O_2 , on it.
3. Layout two construction lines from O_2 , each making an angle $\alpha/2$ to the vertical line through O_2 .
4. Using the chosen crank length (see Design Choices), draw a circle with center at O_2 and radius equal to the crank length. Label the intersections of the circle and the two lines drawn in step 3 as A_1 and A_2 .
5. Draw lines through points A_1 and A_2 that are also tangent to the crank circle (step 2). These two lines will simultaneously intersect the vertical line drawn in step 2. Label the point of intersection as the fixed pivot center O_4 .
6. Draw a vertical construction line, parallel and to the right of O_2O_4 , a distance $S/2$ (one-half of the output stroke length) from the line O_2O_4 .
7. Extend line O_4A_1 until it intersects the construction line drawn in step 6. Label the intersection B_1 .
8. Draw a horizontal construction line from point B_1 , either to the left or right. Using point B_1 as center, draw an arc of radius equal to the length of link 5 (see Design Choices) to intersect the horizontal construction line. Label the intersection as C_1 .
9. Draw the slider blocks at points A_1 and C_1 and finish by drawing the mechanism in its other extreme position.



PROBLEM 3-72

Statement: Design a sixbar, single-dwell linkage for a dwell of 70 deg of crank motion, with an output rocker motion of 30 deg using a symmetrical fourbar linkage with the following parameter values: ground link ratio = 2.0, common link ratio = 2.0, and coupler angle $\gamma = 40$ deg. (See Example 3-13.)

Given: Crank dwell period: 70 deg.
 Output rocker motion: 30 deg.
 Ground link ratio, $L_1/L_2 = 2.0$: $GLR := 2.0$
 Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.0$: $CLR := 2.0$
 Coupler angle, $\gamma := 40 \cdot \text{deg}$

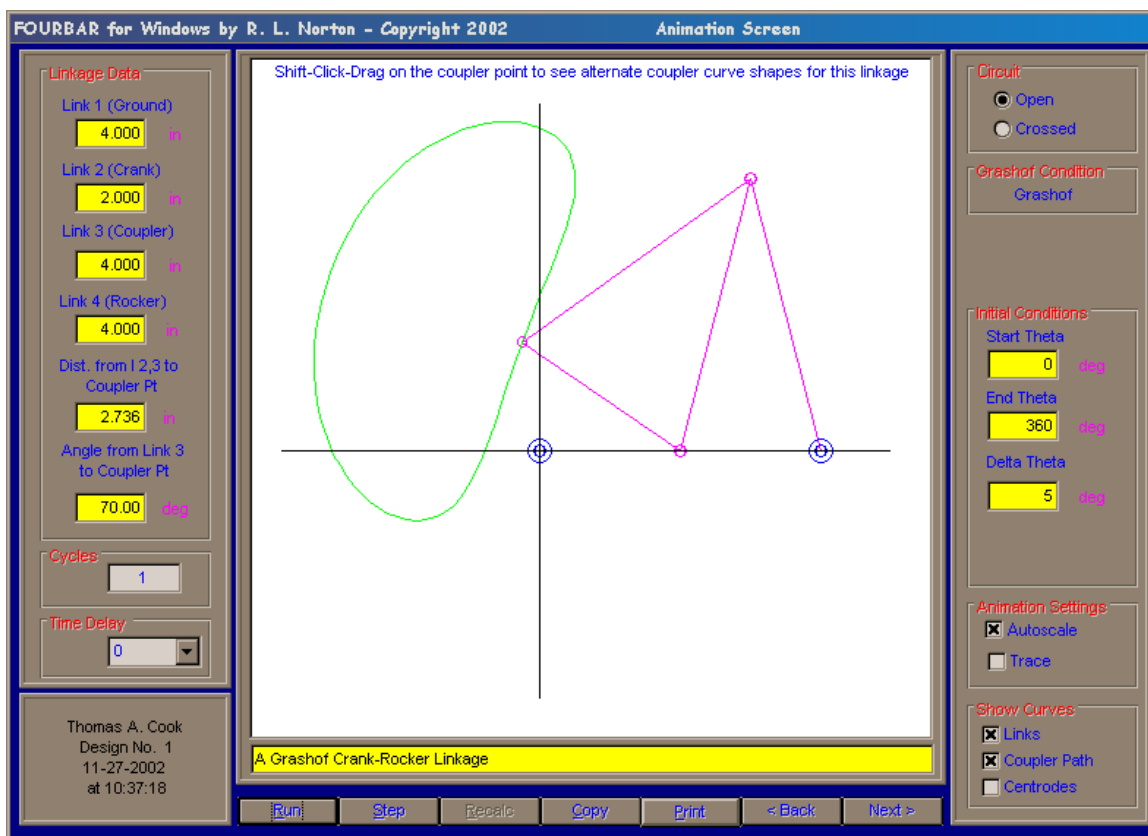
Design choice: Crank length, $L_2 := 2.000$

Solution: See Figures 3-20 and 3-21 and Mathcad file P0372.

1. For the given design choice, determine the remaining link lengths and coupler point specification.

Coupler link (3) length	$L_3 := CLR \cdot L_2$	$L_3 = 4.000$
Rocker link (4) length	$L_4 := CLR \cdot L_2$	$L_4 = 4.000$
Ground link (1) length	$L_1 := GLR \cdot L_2$	$L_1 = 4.000$
Angle PAB	$\delta := \frac{180 \cdot \text{deg} - \gamma}{2}$	$\delta = 70.000 \text{ deg}$
Length AP on coupler	$AP := 2 \cdot L_3 \cdot \cos(\delta)$	$AP = 2.736$

2. Enter the above data into program FOURBAR, plot the coupler curve, and determine the coordinates of the coupler curve in the selected range of crank motion, which in this case will be from 145 to 215 deg.

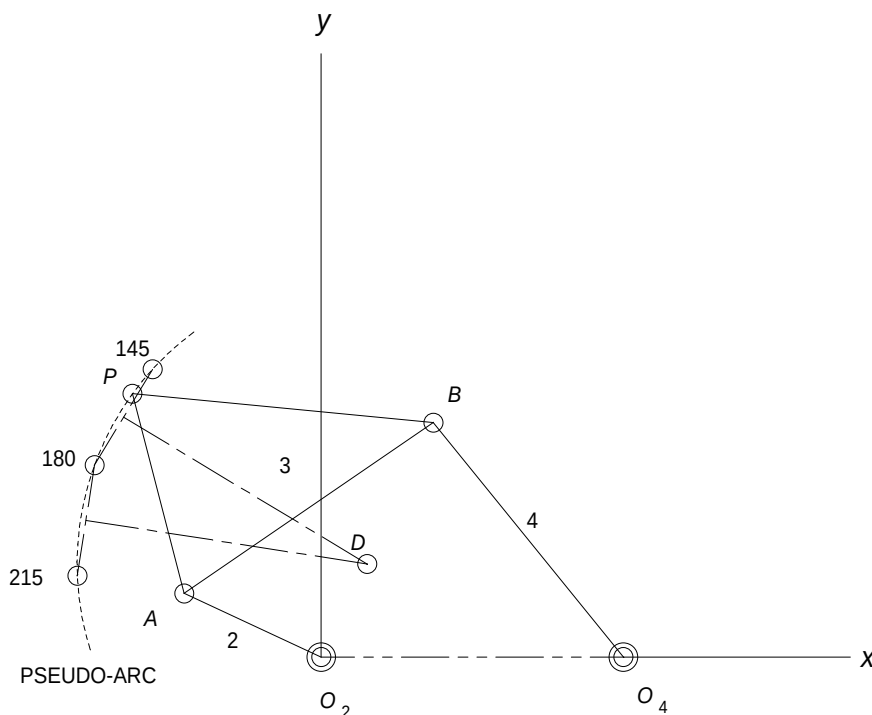


FOURBAR for Windows

File P03-72

Angle Step Deg	Coupler Pt X in	Coupler Pt Y in	Coupler Pt Mag in	Coupler Pt Ang in
145	-2.231	3.818	4.422	120.297
150	-2.368	3.661	4.360	122.895
155	-2.497	3.494	4.295	125.549
160	-2.617	3.319	4.226	128.259
165	-2.728	3.135	4.156	131.025
170	-2.829	2.945	4.083	133.846
175	-2.919	2.749	4.009	136.723
180	-2.999	2.547	3.935	139.655
185	-3.067	2.342	3.859	142.639
190	-3.124	2.133	3.783	145.674
195	-3.169	1.923	3.707	148.757
200	-3.202	1.711	3.631	151.886
205	-3.223	1.499	3.555	155.055
210	-3.232	1.289	3.479	158.261
215	-3.227	1.080	3.403	161.498

3. Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Use the points at crank angles of 145, 180, and 215 deg to define the pseudo-arc. Find the center of the pseudo-arc erecting perpendicular bisectors to the chords defined by the selected coupler curve points. The center will lie at the intersection of the perpendicular bisectors, label this point D . The radius of this circle is the length of link 5.



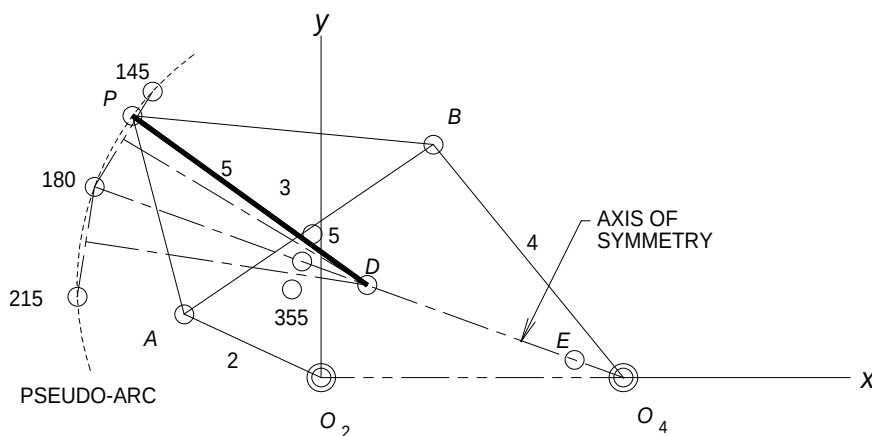
4. The position of the end of link 5 at point D will remain nearly stationary while the crank moves from 145 to 215 deg. As the crank motion causes the coupler point to move around the coupler curve there will be another extreme position of the end of link 5 that was originally at D . Since a symmetrical linkage was chosen, the other extreme position will be located along a line through the axis of symmetry (see Figure 3-20) a distance equal to the length of link 5 measured from the point where the axis of symmetry intersects the coupler curve near the 0 deg coupler point. Establish this point and label it E .

FOURBAR for Windows

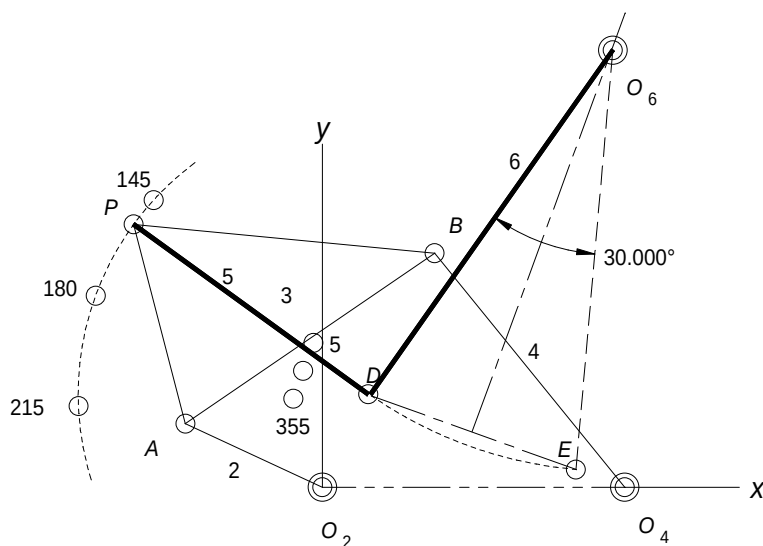
File

P03-72

Angle Step Deg	Coupler Pt X in	Coupler Pt Y in	Coupler Pt Mag n	Coupler Pt Ang in
340.000	-0.718	0.175	0.739	166.325
345.000	-0.615	0.481	0.781	142.001
350.000	-0.506	0.818	0.962	121.717
355.000	-0.386	1.178	1.240	108.135
0.000	-0.255	1.549	1.570	99.365
5.000	-0.117	1.917	1.920	93.499
10.000	0.022	2.269	2.269	89.434
15.000	0.155	2.598	2.603	86.581



5. The line segment DE represents the maximum displacement that a link of the length equal to link 5, attached at P , will reach along the axis of symmetry. Construct a perpendicular bisector of the line segment DE and extend it to the right (or left, whichever is convenient). Locate fixed pivot O_6 on the bisector of DE such that the lines O_6D and O_6E subtend the desired output angle, in this case 30 deg. Draw link 6 from D through O_6 and extend it to any convenient length. This is the output link that will dwell during the specified motion of the crank.



SUMMARY OF LINKAGE SPECIFICATIONS

Original fourbar:

Ground link $L_1 = 4.000$ Crank $L_2 = 2.000$ Coupler $L_3 = 4.000$ Rocker $L_4 = 4.000$ Coupler point $AP = 2.736$ $\delta = 70.000 \text{ deg}$

Added dyad:

Coupler $L_5 := 3.840$ Output $L_6 := 5.595$ Pivot O_6 $x := 3.841$ $y := 5.809$

PROBLEM 3-73

Statement: Design a sixbar, single-dwell linkage for a dwell of 100 deg of crank motion, with an output rocker motion of 50 deg using a symmetrical fourbar linkage with the following parameter values: ground link ratio = 2.0, common link ratio = 2.5, and coupler angle $\gamma = 60$ deg. (See Example 3-13.)

Given: Crank dwell period: 100 deg.
 Output rocker motion: 50 deg.
 Ground link ratio, $L_1/L_2 = 2.0$: $GLR := 2.0$
 Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.0$: $CLR := 2.5$
 Coupler angle, $\gamma := 60 \cdot \text{deg}$

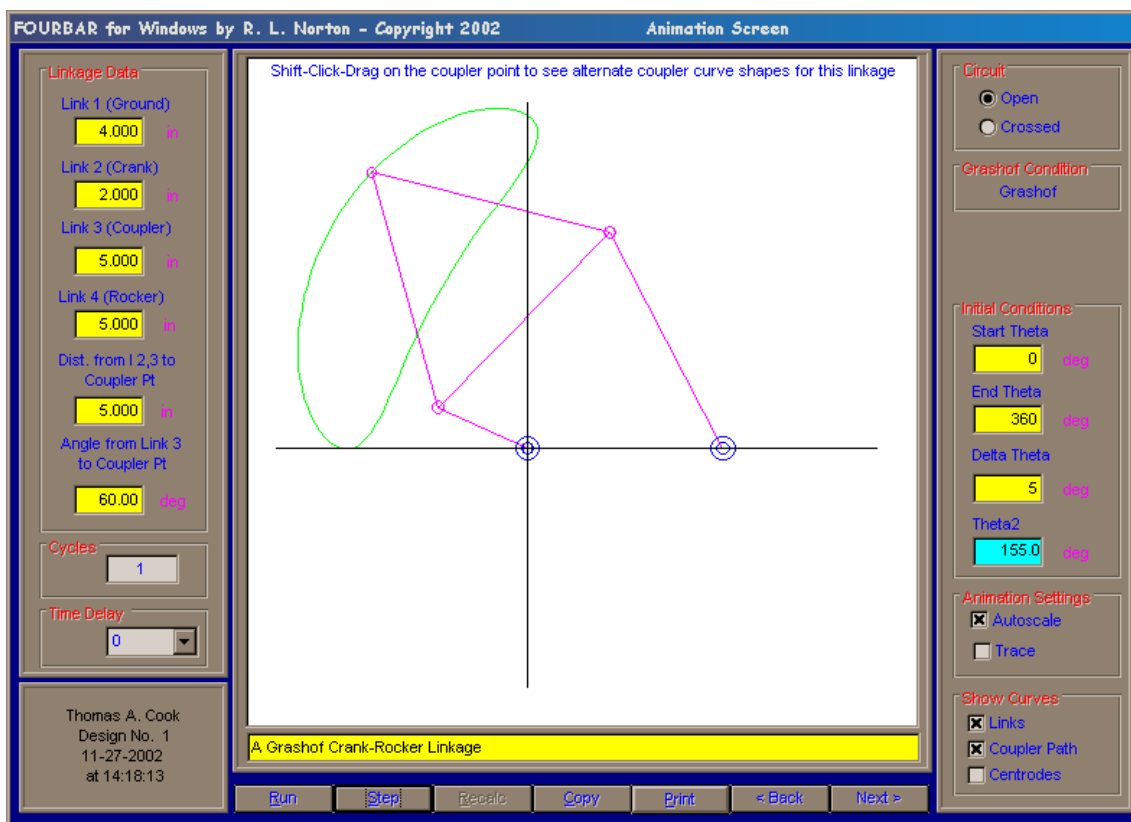
Design choice: Crank length, $L_2 := 2.000$

Solution: See Figures 3-20 and 3-21 and Mathcad file P0373.

1. For the given design choice, determine the remaining link lengths and coupler point specification.

Coupler link (3) length	$L_3 := CLR \cdot L_2$	$L_3 = 5.000$
Rocker link (4) length	$L_4 := CLR \cdot L_2$	$L_4 = 5.000$
Ground link (1) length	$L_1 := GLR \cdot L_2$	$L_1 = 4.000$
Angle PAB	$\delta := \frac{180 \cdot \text{deg} - \gamma}{2}$	$\delta = 60.000 \text{ deg}$
Length AP on coupler	$AP := 2 \cdot L_3 \cdot \cos(\delta)$	$AP = 5.000$

2. Enter the above data into program FOURBAR, plot the coupler curve, and determine the coordinates of the coupler curve in the selected range of crank motion, which in this case will be from 130 to 230 deg.

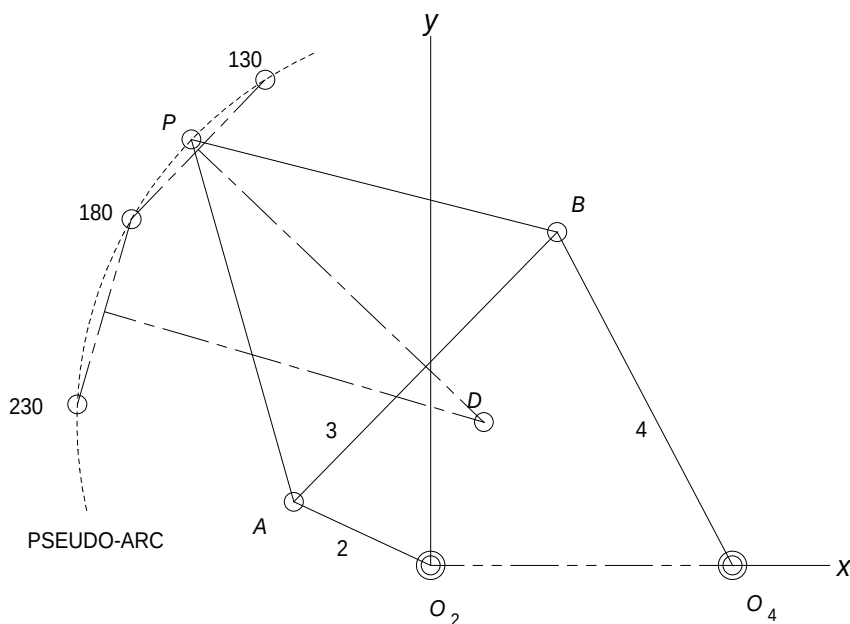


FOURBAR for Windows

File P03-73

Angle Step Deg	Coupler Pt X in	Coupler Pt Y in	Coupler Pt Mag in	Coupler Pt Ang in
130	-2.192	6.449	6.812	108.774
140	-2.598	6.171	6.695	112.833
150	-2.986	5.840	6.559	117.078
160	-3.347	5.464	6.408	121.493
170	-3.675	5.047	6.244	126.060
180	-3.964	4.598	6.071	130.765
190	-4.209	4.123	5.892	135.588
200	-4.405	3.631	5.709	140.504
210	-4.551	3.130	5.523	145.482
220	-4.643	2.629	5.336	150.482
230	-4.681	2.138	5.146	155.454

3. Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Use the points at crank angles of 130, 180, and 230 deg to define the pseudo-arc. Find the center of the pseudo-arc erecting perpendicular bisectors to the chords defined by the selected coupler curve points. The center will lie at the intersection of the perpendicular bisectors, label this point D . The radius of this circle is the length of link 5.

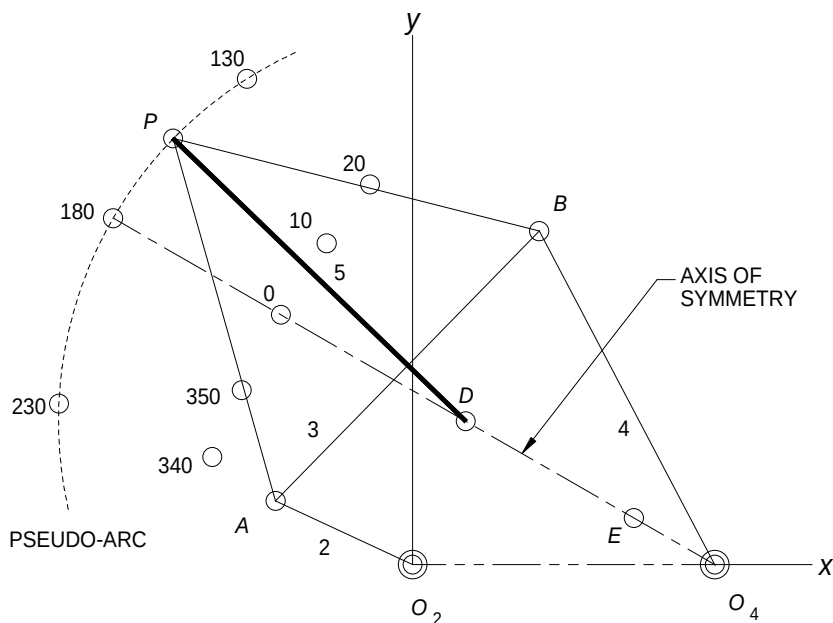


4. The position of the end of link 5 at point D will remain nearly stationary while the crank moves from 130 to 230 deg. As the crank motion causes the coupler point to move around the coupler curve there will be another extreme position of the end of link 5 that was originally at D . Since a symmetrical linkage was chosen, the other extreme position will be located along a line through the axis of symmetry (see Figure 3-20) a distance equal to the length of link 5 measured from the point where the axis of symmetry intersects the coupler curve near the 0 deg coupler point. Establish this point and label it E .

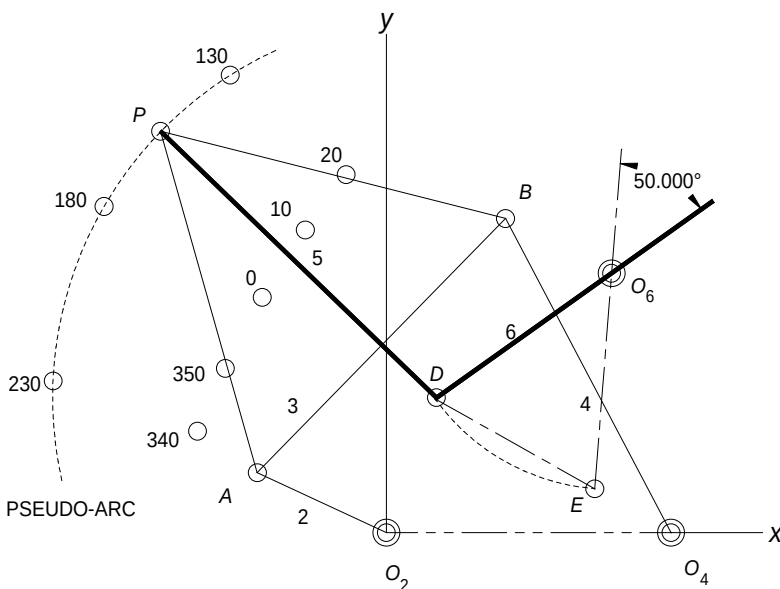
FOURBAR for Windows

File P03-73

Angle Step Deg	Coupler Pt X in	Coupler Pt Y in	Coupler Pt Mag in	Coupler Pt Ang in
340	-2.652	1.429	3.013	151.688
350	-2.262	2.316	3.237	134.332
0	-1.743	3.316	3.746	117.727
10	-1.137	4.265	4.414	104.920
20	-0.564	5.047	5.078	96.371



5. The line segment DE represents the maximum displacement that a link of the length equal to link 5, attached at P , will reach along the axis of symmetry. Construct a perpendicular bisector of the line segment DE and extend it to the right (or left, whichever is convenient). Locate fixed pivot O_6 on the bisector of DE such that the lines O_6D and O_6E subtend the desired output angle, in this case 30 deg. Draw link 6 from D through O_6 and extend it to any convenient length. This is the output link that will dwell during the specified motion of the crank.



SUMMARY OF LINKAGE SPECIFICATIONS

Original fourbar:

Ground link $L_1 = 4.000$ Crank $L_2 = 2.000$ Coupler $L_3 = 5.000$ Rocker $L_4 = 5.000$ Coupler point $AP = 5.000$ $\delta = 60.000 \text{ deg}$

Added dyad:

Coupler $L_5 := 5.395$ Output $L_6 := 2.998$ Pivot O_6 $x := 3.166$ $y := 3.656$

PROBLEM 3-74

Statement: Design a sixbar, single-dwell linkage for a dwell of 80 deg of crank motion, with an output rocker motion of 45 deg using a symmetrical fourbar linkage with the following parameter values: ground link ratio = 2.0, common link ratio = 1.75, and coupler angle $\gamma = 70$ deg. (See Example 3-13.)

Given: Crank dwell period: 80 deg.
 Output rocker motion: 45 deg.
 Ground link ratio, $L_1/L_2 = 2.0$: $GLR := 2.0$
 Common link ratio, $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.0$: $CLR := 1.75$
 Coupler angle, $\gamma := 70\text{-deg}$

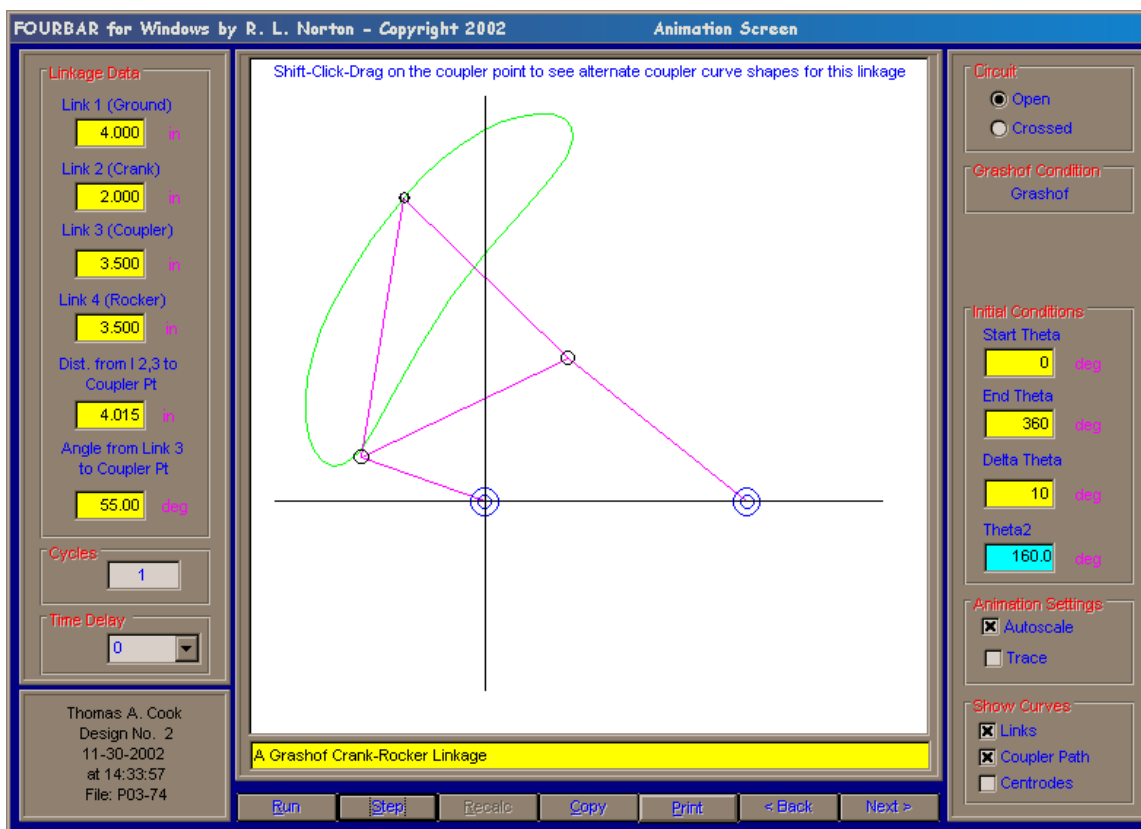
Design choice: Crank length, $L_2 := 2.000$

Solution: See Figures 3-20 and 3-21 and Mathcad file P0374.

- For the given design choice, determine the remaining link lengths and coupler point specification.

Coupler link (3) length	$L_3 := CLR \cdot L_2$	$L_3 = 3.500$
Rocker link (4) length	$L_4 := CLR \cdot L_2$	$L_4 = 3.500$
Ground link (1) length	$L_1 := GLR \cdot L_2$	$L_1 = 4.000$
Angle PAB	$\delta := \frac{180 \cdot \text{deg} - \gamma}{2}$	$\delta = 55.000 \text{ deg}$
Length AP on coupler	$AP := 2 \cdot L_3 \cdot \cos(\delta)$	$AP = 4.015$

- Enter the above data into program FOURBAR, plot the coupler curve, and determine the coordinates of the coupler curve in the selected range of crank motion, which in this case will be from 140 to 220 deg.

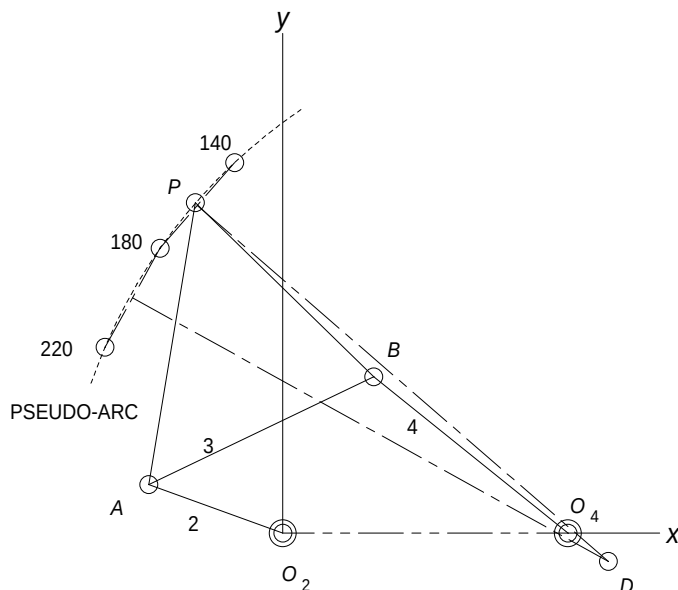


FOURBAR for Windows

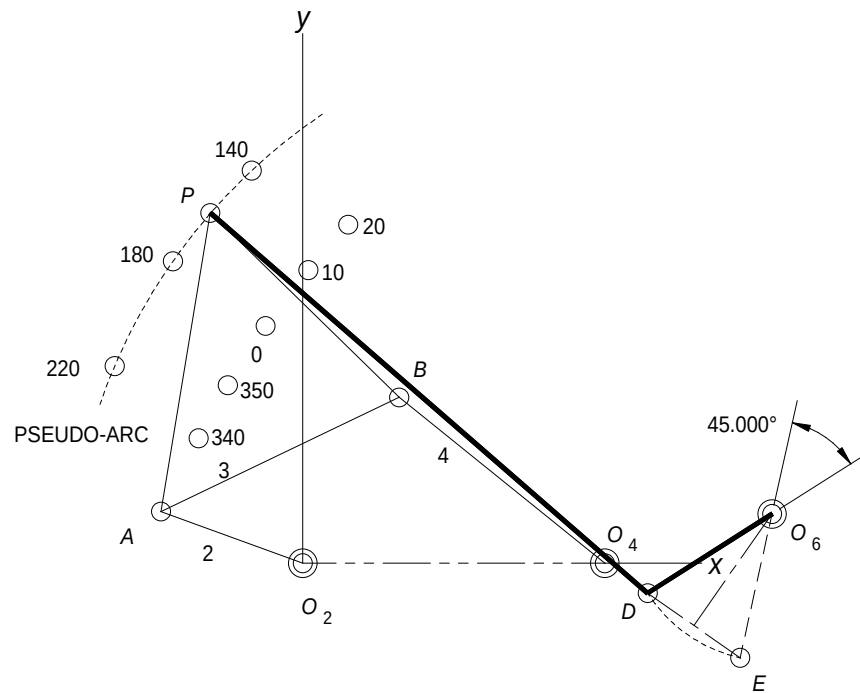
File P03-74

Angle Step Deg	Coupler Pt X in	Coupler Pt Y in	Coupler Pt Mag in	Coupler Pt Ang in
140	-0.676	5.208	5.252	97.395
150	-0.958	4.940	5.032	100.971
160	-1.226	4.645	4.804	104.781
170	-1.480	4.332	4.578	108.860
180	-1.720	4.005	4.359	113.242
190	-1.945	3.668	4.152	117.942
200	-2.153	3.322	3.958	122.946
210	-2.337	2.969	3.779	128.210
220	-2.493	2.613	3.612	133.663

3. Layout this linkage to scale, including the coupler curve whose coordinates are in the table above. Use the points at crank angles of 140, 180, and 220 deg to define the pseudo-arc. Find the center of the pseudo-arc erecting perpendicular bisectors to the chords defined by the selected coupler curve points. The center will lie at the intersection of the perpendicular bisectors, label this point D . The radius of this circle is the length of link 5.



4. The position of the end of link 5 at point D will remain nearly stationary while the crank moves from 140 to 220 deg. As the crank motion causes the coupler point to move around the coupler curve there will be another extreme position of the end of link 5 that was originally at D . Since a symmetrical linkage was chosen, the other extreme position will be located along a line through the axis of symmetry (see Figure 3-20) a distance equal to the length of link 5 measured from the point where the axis of symmetry intersects the coupler curve near the 0 deg coupler point. Establish this point and label it E .



SUMMARY OF LINKAGE SPECIFICATIONS

Original fourbar:

Ground link $L_1 = 4.000$

Crank $L_2 = 2.000$

Coupler	$L_3 = 3.500$

Rocker $L_4 = 3.500$

Coupler point $AP = 4.015$

$$\delta = 55.000 \text{ deg}$$

Added dyad:

Coupler	$L_5 := 7.676$

Output	$L_6 := 1.979$

Pivot O_6	$x := 6.217$

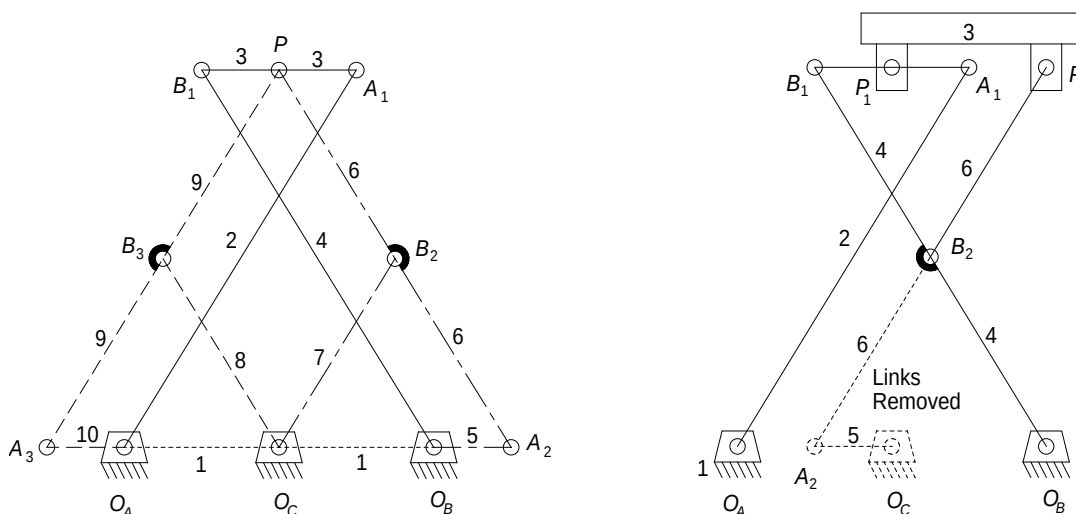
$$y := 0.653$$

PROBLEM 3-75

Statement: Using the method of Example 3-11, show that the sixbar Chebychev straight-line linkage of Figure P2-5 is a combination of the fourbar Chebychev straight-line linkage of Figure 3-29d and its Hoeken's cognate of Figure 3-29e. See also Figure 3-26 for additional information useful to this solution. Graphically construct the Chebychev sixbar parallel motion linkage of Figure P2-5a from its two fourbar linkage constituents and build a physical or computer model of the result.

Solution: See Figures P2-5, 3-29d, 3-29e, and 3-26 and Mathcad file P0375.

- Following Example 3-11 and Figure 3-26 for the Chebyshev linkage of Figure 3-29d, the fixed pivot O_C is found by laying out the triangle $O_A O_B O_C$, which is similar to $A_1 B_1 P$. In this case, $A_1 B_1 P$ is a straight line with P halfway between A_1 and B_1 and therefore $O_A O_B O_C$ is also a straight line with O_C halfway between O_A and O_B . As shown below and in Figure 3-26, cognate #1 is made up of links numbered 1, 2, 3, and 4. Cognate #2 is links numbered 1, 5, 6, and 7. Cognate #3 is links numbered 1, 8, 9, and 10.



- Discard cognate #3 and shift link 5 from the fixed pivot O_B to O_C and shift link 7 from O_C to O_B . Note that due to the symmetry of the figure above, $L_5 = 0.5 L_3$, $L_6 = L_2$, $L_7 = 0.5 L_2$ and $O_C O_B = 0.5 O_A O_B$. Thus, cognate #2 is, in fact, the Hoeken straight-line linkage. The original Chebyshev linkage with the Hoeken linkage superimposed is shown above right with the link 5 rotated to 180 deg. Links 2 and 6 will now have the same velocity as will 7 and 4. Thus, link 5 can be removed and link 6 can be reduced to a binary link supported and constrained by link 4. The resulting sixbar is the linkage shown in Figure P2-5.

PROBLEM 3-76

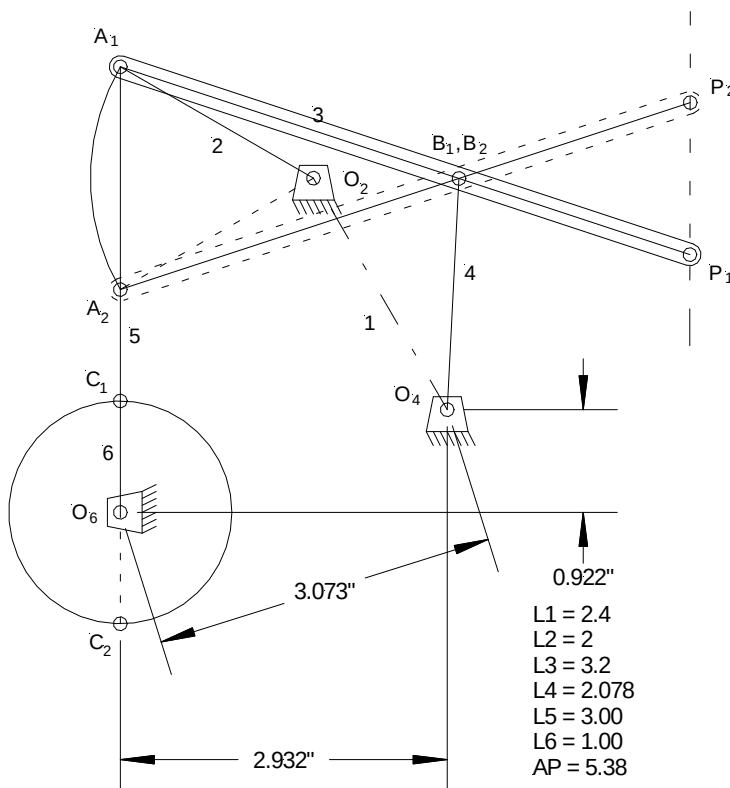
Statement: Design a driver dyad to drive link 2 of the Evans straight-line linkage in Figure 3-29f from 150 deg to 210 deg. Make a model of the resulting sixbar linkage and trace the couple curve.

Given: Output angle $\theta_2 := 60\text{-deg}$

Solution: See Figure 3-29f, Example 3-1, and Mathcad file P0376.

Design choices: Link lengths: Link 2 $L_2 := 2.000$ Link 5 $L_5 := 3.000$

1. Draw the input link O_2A in both extreme positions, A_1 and A_2 , at the specified angles such that the desired angle of motion θ_2 is subtended.
2. Draw the chord A_1A_2 and extend it in any convenient direction. In this solution it was extended downward.
3. Layout the distance A_1C_1 along extended line A_1A_2 equal to the length of link 5. Mark the point C_1 .
4. Bisect the line segment A_1A_2 and layout the length of that radius from point C_1 along extended line A_1A_2 . Mark the resulting point O_6 and draw a circle of radius O_6C_1 with center at O_6 .
5. Label the other intersection of the circle and extended line A_1A_2 , C_2 .
6. Measure the length of the crank (link 6) as O_6C_1 or O_6C_2 . From the graphical solution, $L_6 := 1.000$
7. Measure the length of the ground link (link 1) as O_2O_6 . From the graphical solution, $L_1 := 3.073$



8. Find the Grashof condition.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$Condition(L_1, L_2, L_5, L_6) = \text{"Grashof"}$

PROBLEM 3-77

Statement: Design a driver dyad to drive link 2 of the Evans straight-line linkage in Figure 3-29g from -40 deg to 40 deg. Make a model of the resulting sixbar linkage and trace the couple curve.

Given: Output angle $\theta_2 := 80\text{-deg}$

Solution: See Figure 3-29G, Example 3-1, and Mathcad file P0377.

Design choices: Link lengths: Link 2 $L_2 := 2.000$ Link 5 $L_5 := 3.000$

1. Draw the input link O_2A_1 in both extreme positions, A_1 and A_2 , at the specified angles such that the desired angle of motion θ_2 is subtended.
2. Draw the line A_1C_1 and extend it in any convenient direction. In this solution it was extended at a 30-deg angle from A_1O_2 (see note below).
3. Layout the distance A_1C_1 along extended line A_1C_1 equal to the length of link 5. Mark the point C_1 .
4. Bisect the line segment A_1A_2 and layout the length of that radius from point C_1 along extended line A_1C_1 . Mark the resulting point O_6 and draw a circle of radius O_6C_1 with center at O_6 .
5. Extend a line from A_2 through O_6 . Label the other intersection of the circle and extended line A_2O_6 , C_2 .
6. Measure the length of the crank (link 6) as O_6C_1 or O_6C_2 . From the graphical solution, $L_6 := 1.735$
7. Measure the length of the ground link (link 1) as O_2O_6 . From the graphical solution, $L_1 := 3.165$

Note: If the angle between link 2 and link 5 is zero the resulting driving fourbar will be a special Grashof. For angles greater than zero but less than 33.68 degrees it is a Grashof crank-rocker. For angles greater than 33.68 it is a non-Grashof double rocker.

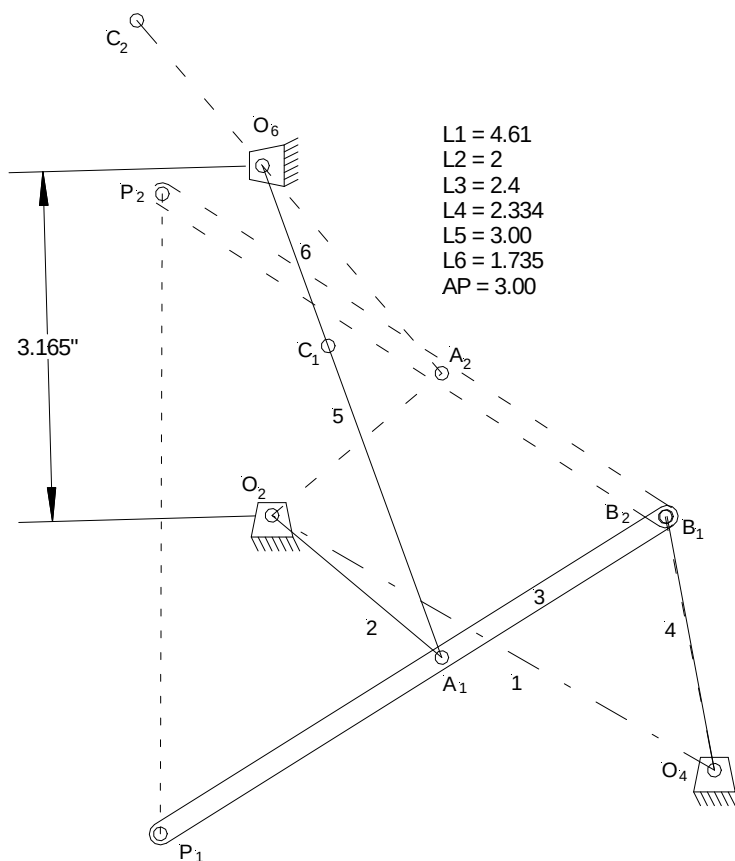
8. Find the Grashof condition.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$Condition(L_1, L_2, L_5, L_6) = \text{"Grashof"}$



PROBLEM 3-78

Statement: Figure 6 on page ix of the Hrones and Nelson atlas of fourbar coupler curves (on the book DVD) shows a 50-point coupler that was used to generate the curves in the atlas. Using the definition of the vector **R** given in Figure 3-17b of the text, determine the 10 possible pairs of values of ϕ and R for the first row of points above the horizontal axis if the gridpoint spacing is one half the length of the unit crank.

Given: Grid module $g := 0.5$

Solution: See Figure 6 H&N Atlas, Figure 3-17b, and Mathcad file P0378.

1. The moving pivot point is located on the 3rd grid line from the bottom and the third grid line from the left when the crank angle is π radians. Let the number of horizontal grid spaces from the left end of the coupler to the coupler point be $n := -2, -1 \dots 7$ and the number of vertical grid spaces from the coupler to the coupler point be $m := -2, -1 \dots 2$
2. For the first row of points above the horizontal axis shown in Figure 6, $n := -2, -1 \dots 7$ and $m := 1$.
3. The angle, ϕ , between the coupler and the line from the coupler/crank pivot to the coupler point is

$$\phi(m, n) := \text{if}\left(n \neq 0, \text{atan2}(n, m), \text{if}\left(m = 0, 0, \text{if}\left(m > 0, \frac{\pi}{2}, \frac{-\pi}{2}\right)\right)\right)$$

4. The distance, R , from the pivot to the coupler point along the same line is

$$R(m, n) := g \cdot \sqrt{m^2 + n^2}$$

$n =$	$\frac{\phi(m, n)}{\text{deg}} =$	$R(m, n) =$
-2.000	153.435	1.118
-1.000	135.000	0.707
0.000	90.000	0.500
1.000	45.000	0.707
2.000	26.565	1.118
3.000	18.435	1.581
4.000	14.036	2.062
5.000	11.310	2.550
6.000	9.462	3.041
7.000	8.130	3.536

5. The coupler point distance, R , like the link lengths A , B , and C is a ratio of the given length to the the length of the driving crank.

PROBLEM 3-79

Statement: The set of coupler curves in the Hrones and Nelson atlas of fourbar coupler curves (on the book DVD, page 16 of the PDF file) has $A = B = C = 1.5$. Model this linkage with program FOURBAR using the coupler point farthest to the left in the row shown on page 1 and plot the resulting coupler curve.

Given: $A := 1.5$ $B := 1.5$ $C := 1.5$

Solution: See Figure on page 1 H&N Atlas, Figure 3-17b, and Mathcad file P0379.

1. The moving pivot point is located on the 3rd grid line from the bottom and the third grid line from the left when the crank angle is π radians. Let the number of horizontal grid spaces from the left end of the coupler to the coupler point be $n := -2, -1 \dots 7$ and the number of vertical grid spaces from the coupler to the coupler point be $m := -2, -1 \dots 2$
2. For the second column of points to the left of the coupler pivot and the second row of points above the horizontal axis $n := -2$ and $m := 2$. The grid spacing is $g := 0.5$
3. The angle, ϕ , between the coupler and the line from the coupler/crank pivot to the coupler point is

$$\phi(m, n) := \text{if}\left(n \neq 0, \text{atan2}(n, m), \text{if}\left(m = 0, 0, \text{if}\left(m > 0, \frac{\pi}{2}, \frac{-\pi}{2}\right)\right)\right) \quad \phi(m, n) = 135.000 \text{ deg}$$

4. The distance from the pivot to the coupler point, R , along the same line is

$$R(m, n) := g \cdot \sqrt{m^2 + n^2} \quad R(m, n) = 1.414$$

5. Determine the values needed for input to FOURBAR.

Link 2 (Crank)	$a := 1$	
Link 3 (Coupler)	$b := A \cdot a$	$b = 1.500$
Link 4 (Rocker)	$c := B \cdot a$	$c = 1.500$
Link 1 (Ground)	$d := C \cdot a$	$d = 1.500$
Distance to coupler point		$R(m, n) = 1.414$
Angle from link 3 to coupler point		$\phi(m, n) = 135.000 \text{ deg}$

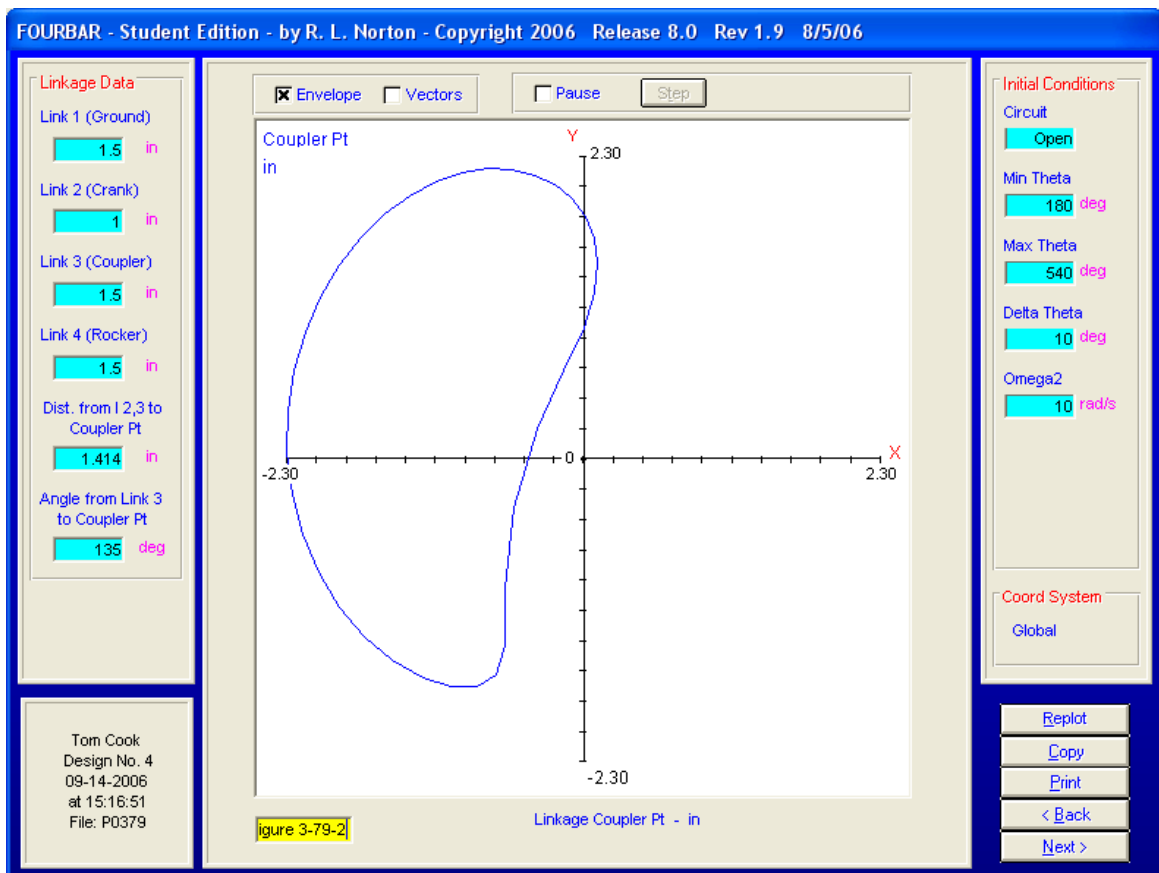
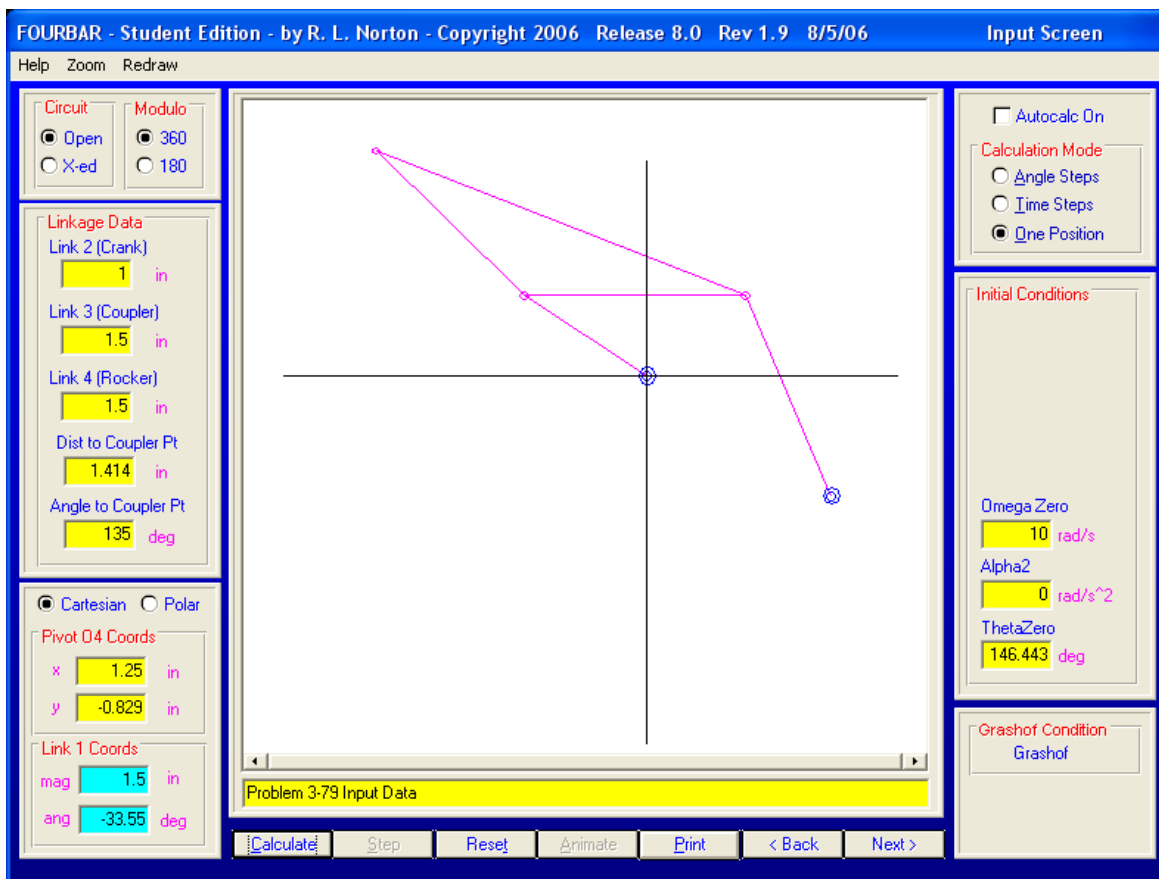
6. Calculate the coordinates of O_4 . Let the angle between links 2 and 3 be α , then

$$\alpha := \text{acos}\left[\frac{A^2 + (1 + C)^2 - B^2}{2 \cdot A \cdot (1 + C)}\right] \quad \alpha = 33.557 \text{ deg}$$

$$x_{O4} := C \cdot \cos(\alpha) \quad x_{O4} = 1.250$$

$$y_{O4} := -C \cdot \sin(\alpha) \quad y_{O4} = -0.829$$

7. Enter this data into FOURBAR and then plot the coupler curve. (See next page)



PROBLEM 3-80

Statement: The set of coupler curves on page 17 in the Hrones and Nelson atlas of fourbar coupler curves (on the book DVD, page 32 of the PDF file) has $A = 1.5$, $B = C = 3.0$. Model this linkage with program FOURBAR using the coupler point farthest to the right in the row shown and plot the resulting coupler curve.

Given: $A := 1.5$ $B := 3.0$ $C := 3.0$

Solution: See Figure on page 17 H&N Atlas, Figure 3-17b, and Mathcad file P0380.

1. The moving pivot point is located on the 3rd grid line from the bottom and the third grid line from the left when the crank angle is π radians. Let the number of horizontal grid spaces from the left end of the coupler to the coupler point be $n := -2, -1 \dots 7$ and the number of vertical grid spaces from the coupler to the coupler point be $m := -2, -1 \dots 2$
2. For the fifth column of points to the right of the coupler pivot and the first row of points above the horizontal axis $n := 5$ and $m := 1$. The grid spacing is $g := 0.5$
3. The angle, ϕ , between the coupler and the line from the coupler/crank pivot to the coupler point is

$$\phi(m, n) := \text{if}\left(n \neq 0, \text{atan2}(n, m), \text{if}\left(m = 0, 0, \text{if}\left(m > 0, \frac{\pi}{2}, \frac{-\pi}{2}\right)\right)\right) \quad \phi(m, n) = 11.310 \text{ deg}$$

4. The distance from the pivot to the coupler point, R , along the same line is

$$R(m, n) := g \cdot \sqrt{m^2 + n^2} \quad R(m, n) = 2.550$$

5. Determine the values needed for input to FOURBAR.

Link 2 (Crank)	$a := 1$	
Link 3 (Coupler)	$b := A \cdot a$	$b = 1.500$
Link 4 (Rocker)	$c := B \cdot a$	$c = 3.000$
Link 1 (Ground)	$d := C \cdot a$	$d = 3.000$
Distance to coupler point		$R(m, n) = 2.550$
Angle from link 3 to coupler point		$\phi(m, n) = 11.310 \text{ deg}$

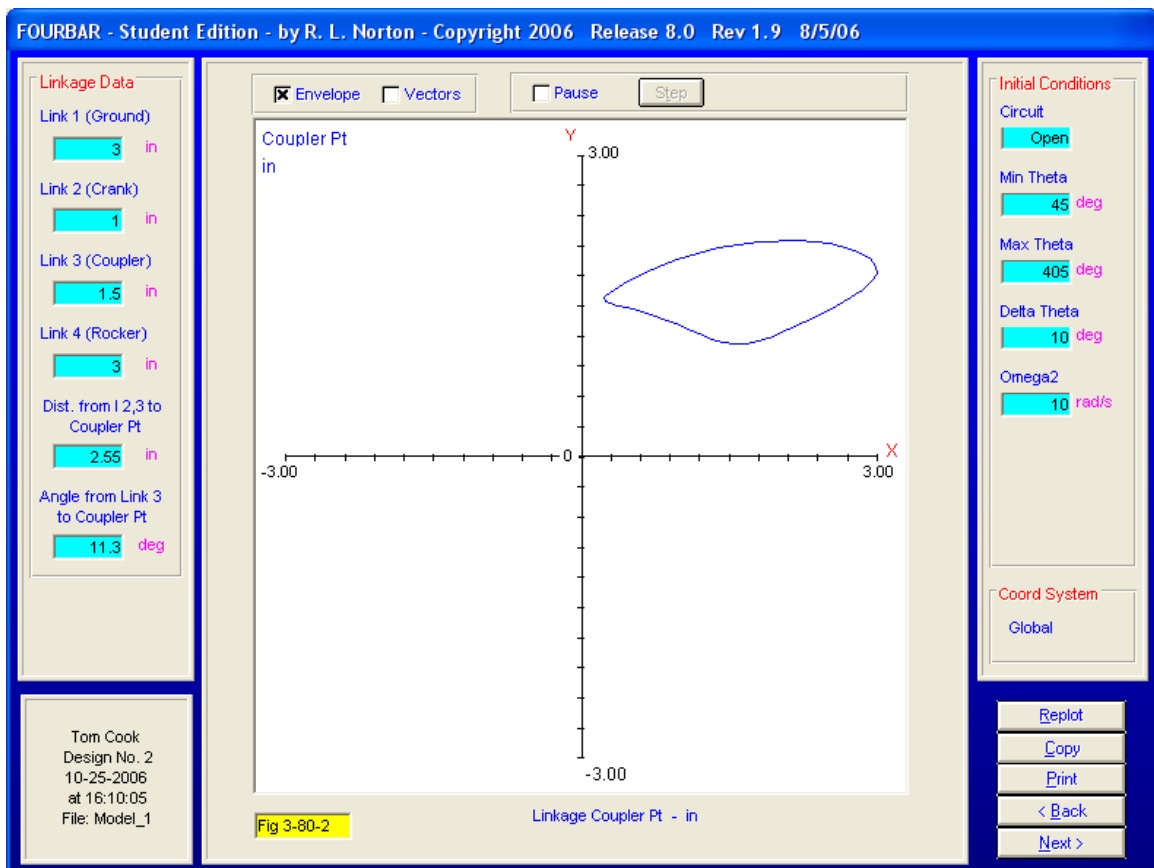
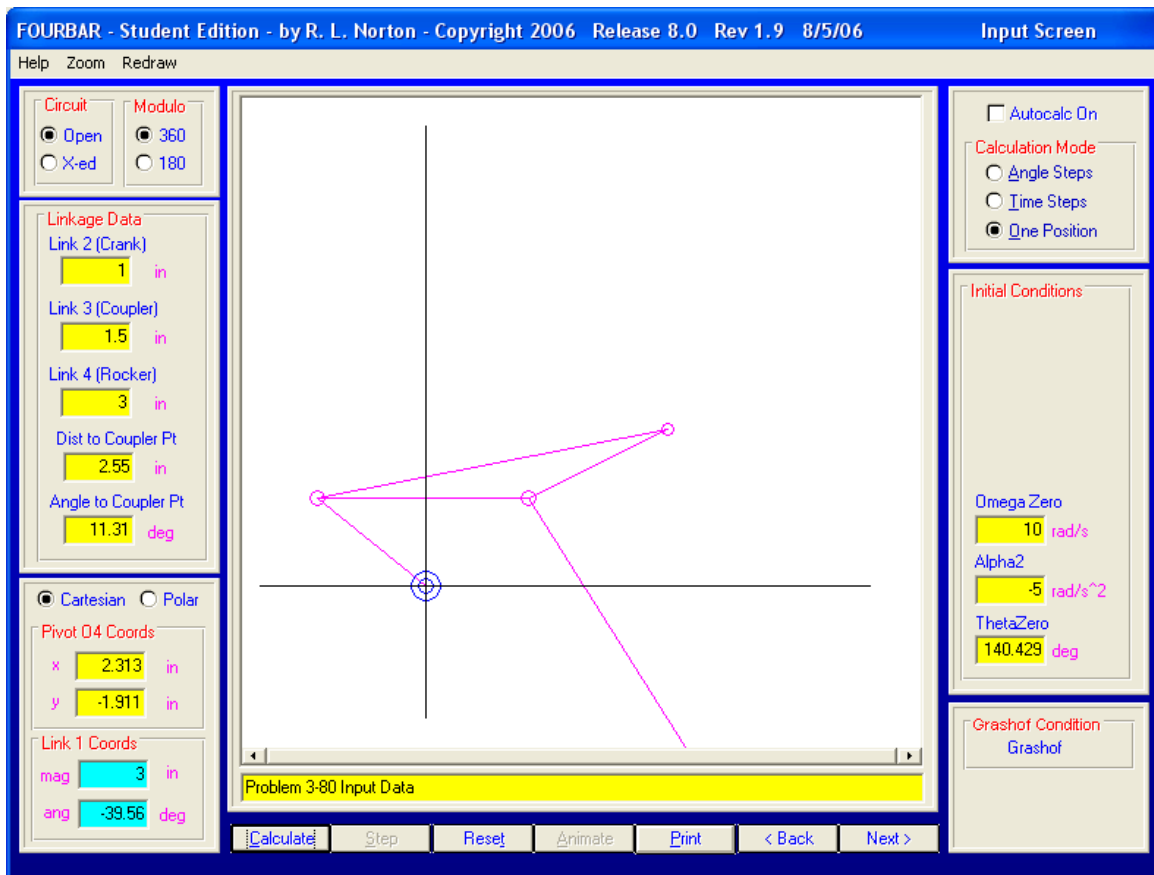
6. Calculate the coordinates of O_4 . Let the angle between links 2 and 3 be α , then

$$\alpha := \text{acos}\left[\frac{A^2 + (1 + C)^2 - B^2}{2 \cdot A \cdot (1 + C)}\right] \quad \alpha = 39.571 \text{ deg}$$

$$x_{O4} := C \cdot \cos(\alpha) \quad x_{O4} = 2.313$$

$$y_{O4} := -C \cdot \sin(\alpha) \quad y_{O4} = -1.911$$

7. Enter this data into FOURBAR and then plot the coupler curve. (See next page)



PROBLEM 3-81

Statement: The set of coupler curves on page 21 in the Hrones and Nelson atlas of fourbar coupler curves (on the book DVD, page 36 of the PDF file) has $A = 1.5$, $B = C = 3.5$. Model this linkage with program FOURBAR using the coupler point farthest to the right in the row shown and plot the resulting coupler curve.

Given: $A := 1.5$ $B := 3.5$ $C := 3.5$

Solution: See Figure on page 21 H&N Atlas, Figure 3-17b, and Mathcad file P0381.

1. The moving pivot point is located on the 3rd grid line from the bottom and the third grid line from the left when the crank angle is π radians. Let the number of horizontal grid spaces from the left end of the coupler to the coupler point be $n := -2, -1 \dots 7$ and the number of vertical grid spaces from the coupler to the coupler point be $m := -2, -1 \dots 2$
2. For the fourth column of points to the right of the coupler pivot and the second row of points above the horizontal axis $n := 4$ and $m := 2$. The grid spacing is $g := 0.5$
3. The angle, ϕ , between the coupler and the line from the coupler/crank pivot to the coupler point is

$$\phi(m, n) := \text{if}\left(n \neq 0, \text{atan2}(n, m), \text{if}\left(m = 0, 0, \text{if}\left(m > 0, \frac{\pi}{2}, \frac{-\pi}{2}\right)\right)\right) \quad \phi(m, n) = 26.565 \text{ deg}$$

4. The distance from the pivot to the coupler point, R , along the same line is

$$R(m, n) := g \cdot \sqrt{m^2 + n^2} \quad R(m, n) = 2.236$$

5. Determine the values needed for input to FOURBAR.

Link 2 (Crank)	$a := 1$	
Link 3 (Coupler)	$b := A \cdot a$	$b = 1.500$
Link 4 (Rocker)	$c := B \cdot a$	$c = 3.500$
Link 1 (Ground)	$d := C \cdot a$	$d = 3.500$
Distance to coupler point		$R(m, n) = 2.236$
Angle from link 3 to coupler point		$\phi(m, n) = 26.565 \text{ deg}$

6. Calculate the coordinates of O_4 . Let the angle between links 2 and 3 be α , then

$$\alpha := \text{acos}\left[\frac{A^2 + (1 + C)^2 - B^2}{2 \cdot A \cdot (1 + C)}\right] \quad \alpha = 40.601 \text{ deg}$$

$$x_{O4} := C \cdot \cos(\alpha) \quad x_{O4} = 2.657$$

$$y_{O4} := -C \cdot \sin(\alpha) \quad y_{O4} = -2.278$$

7. Enter this data into FOURBAR and then plot the coupler curve. (See next page)

FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06 Input Screen

Help Zoom Redraw

Circuit

☒ Open ☐ X-ed

Module

☒ 360 ☐ 180

☐ Autocalc On

Calculation Mode

☐ Angle Steps

☐ Time Steps

☒ One Position

Linkage Data

Link 2 (Crank) in

Link 3 (Coupler) in

Link 4 (Rocker) in

Dist to Coupler Pt in

Angle to Coupler Pt deg

☒ Cartesian ☐ Polar

Pivot O4 Coords

x in

y in

Initial Conditions

Omega Zero rad/s

Alpha2 rad/s^2

ThetaZero deg

Link 1 Coords

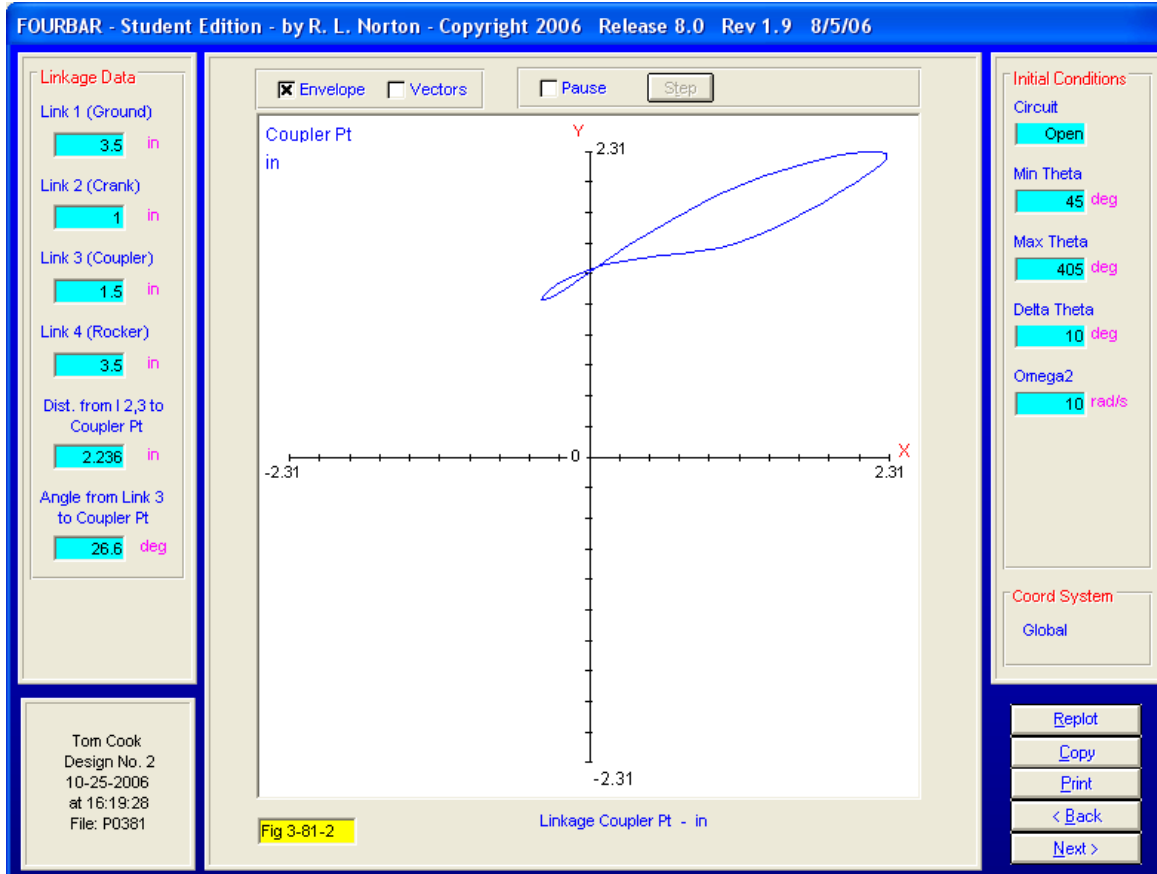
mag in

ang deg

Problem 3-81 Input Data

Grashof Condition

Grashof



PROBLEM 3-82

Statement: The set of coupler curves on page 34 in the Hrones and Nelson atlas of fourbar coupler curves (on the book DVD, page 49 of the PDF file) has $A = 2.0$, $B = 1.5$, $C = 2.0$. Model this linkage with program FOURBAR using the coupler point farthest to the right in the row shown and plot the resulting coupler curve.

Given: $A := 2.0$ $B := 1.5$ $C := 2.0$

Solution: See Figure on page 34 H&N Atlas, Figure 3-17b, and Mathcad file P0382.

1. The moving pivot point is located on the 3rd grid line from the bottom and the third grid line from the left when the crank angle is π radians. Let the number of horizontal grid spaces from the left end of the coupler to the coupler point be $n := -2, -1 \dots 7$ and the number of vertical grid spaces from the coupler to the coupler point be $m := -2, -1 \dots 2$
2. For the sixth column of points to the right of the coupler pivot and the first row of points below the horizontal axis $n := 6$ and $m := -1$. The grid spacing is $g := 0.5$
3. The angle, ϕ , between the coupler and the line from the coupler/crank pivot to the coupler point is

$$\phi(m, n) := \text{if}\left(n \neq 0, \text{atan2}(n, m), \text{if}\left(m = 0, 0, \text{if}\left(m > 0, \frac{\pi}{2}, \frac{-\pi}{2}\right)\right)\right) \quad \phi(m, n) = -9.462 \text{ deg}$$

4. The distance from the pivot to the coupler point, R , along the same line is

$$R(m, n) := g \cdot \sqrt{m^2 + n^2} \quad R(m, n) = 3.041$$

5. Determine the values needed for input to FOURBAR.

Link 2 (Crank)	$a := 1$	
Link 3 (Coupler)	$b := A \cdot a$	$b = 2.000$
Link 4 (Rocker)	$c := B \cdot a$	$c = 1.500$
Link 1 (Ground)	$d := C \cdot a$	$d = 2.000$
Distance to coupler point		$R(m, n) = 3.041$
Angle from link 3 to coupler point		$\phi(m, n) = -9.462 \text{ deg}$

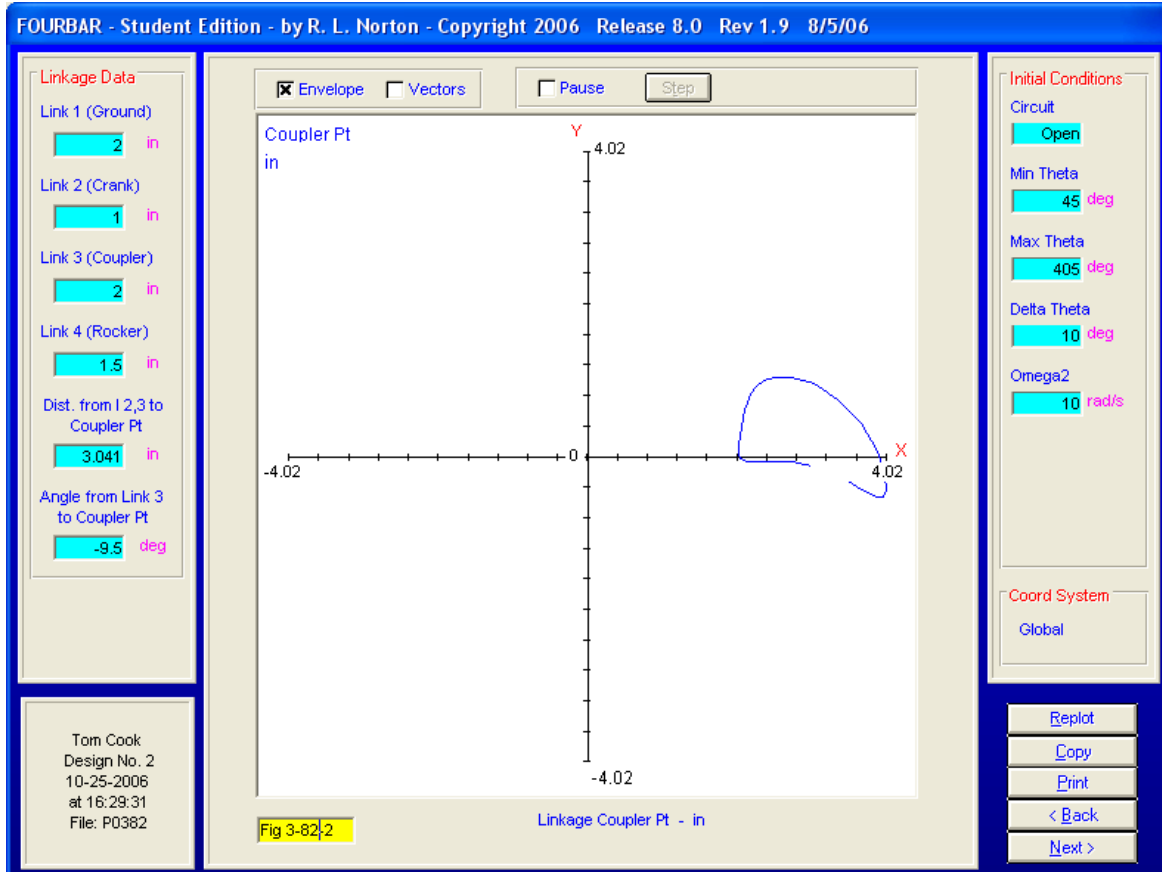
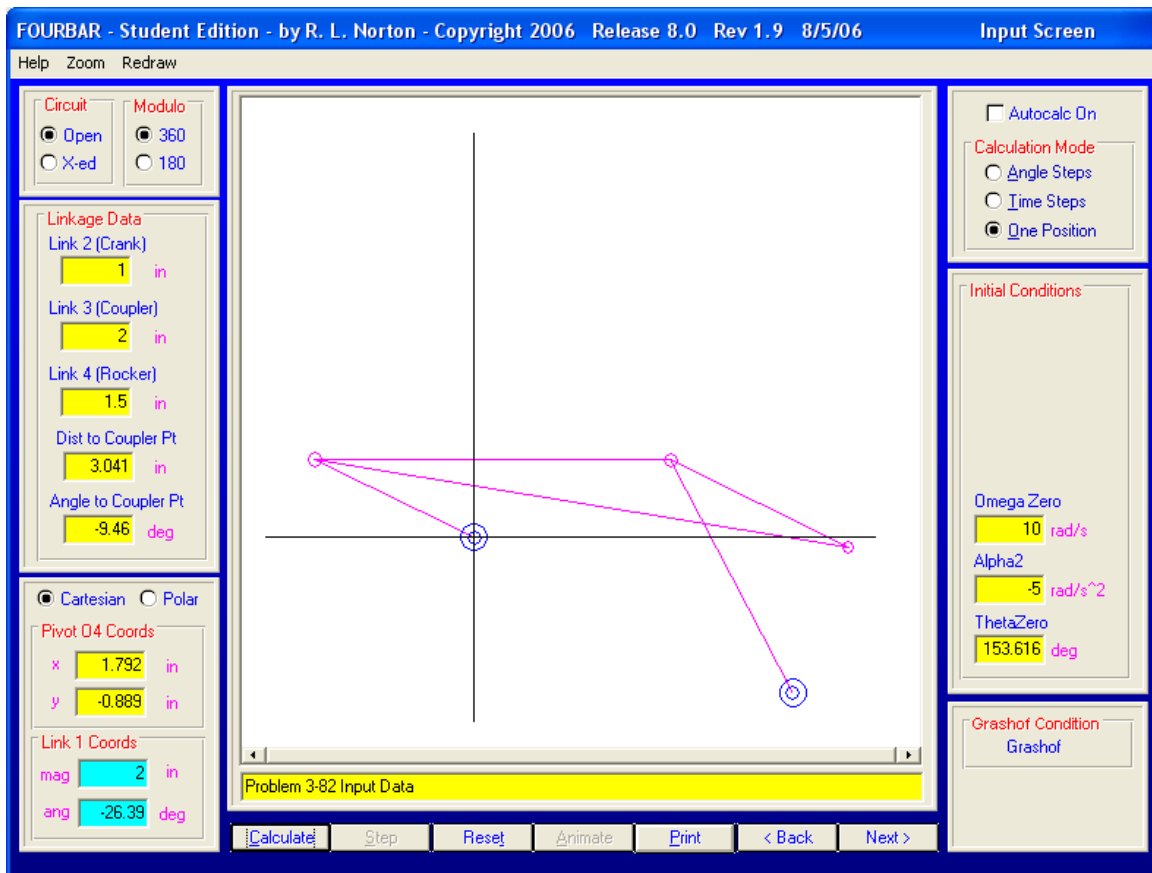
6. Calculate the coordinates of O_4 . Let the angle between links 2 and 3 be α , then

$$\alpha := \text{acos}\left[\frac{A^2 + (1 + C)^2 - B^2}{2 \cdot A \cdot (1 + C)}\right] \quad \alpha = 26.384 \text{ deg}$$

$$x_{O4} := C \cdot \cos(\alpha) \quad x_{O4} = 1.792$$

$$y_{O4} := -C \cdot \sin(\alpha) \quad y_{O4} = -0.889$$

7. Enter this data into FOURBAR and then plot the coupler curve. (See next page)



PROBLEM 3-83

Statement: The set of coupler curves on page 115 in the Hrones and Nelson atlas of fourbar coupler curves (on the book DVD, page 130 of the PDF file) has $A = 2.5$, $B = 1.5$, $C = 2.5$. Model this linkage with program FOURBAR using the coupler point farthest to the right in the row shown and plot the resulting coupler curve.

Given: $A := 2.5$ $B := 1.5$ $C := 2.5$

Solution: See Figure on page 115 H&N Atlas, Figure 3-17b, and Mathcad file P0383.

1. The moving pivot point is located on the 3rd grid line from the bottom and the third grid line from the left when the crank angle is π radians. Let the number of horizontal grid spaces from the left end of the coupler to the coupler point be $n := -2, -1 \dots 7$ and the number of vertical grid spaces from the coupler to the coupler point be $m := -2, -1 \dots 2$
2. For the second column of points to the right of the coupler pivot and the second row of points below the horizontal axis $n := 2$ and $m := -2$. The grid spacing is $g := 0.5$
3. The angle, ϕ , between the coupler and the line from the coupler/crank pivot to the coupler point is

$$\phi(m, n) := \text{if}\left(n \neq 0, \text{atan2}(n, m), \text{if}\left(m = 0, 0, \text{if}\left(m > 0, \frac{\pi}{2}, \frac{-\pi}{2}\right)\right)\right) \quad \phi(m, n) = -45.000 \text{ deg}$$

4. The distance from the pivot to the coupler point, R , along the same line is

$$R(m, n) := g \cdot \sqrt{m^2 + n^2} \quad R(m, n) = 1.414$$

5. Determine the values needed for input to FOURBAR.

Link 2 (Crank)	$a := 1$	
Link 3 (Coupler)	$b := A \cdot a$	$b = 2.500$
Link 4 (Rocker)	$c := B \cdot a$	$c = 1.500$
Link 1 (Ground)	$d := C \cdot a$	$d = 2.500$
Distance to coupler point		$R(m, n) = 1.414$
Angle from link 3 to coupler point		$\phi(m, n) = -45.000 \text{ deg}$

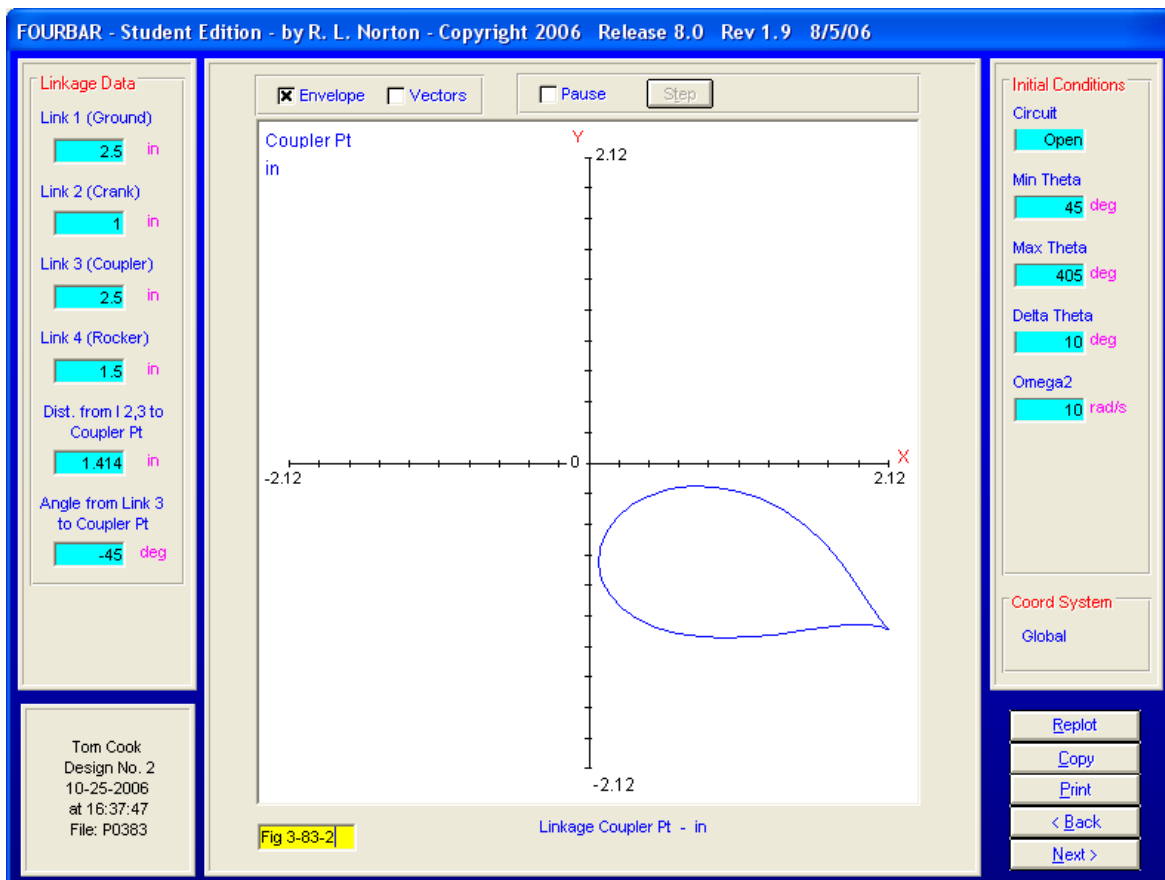
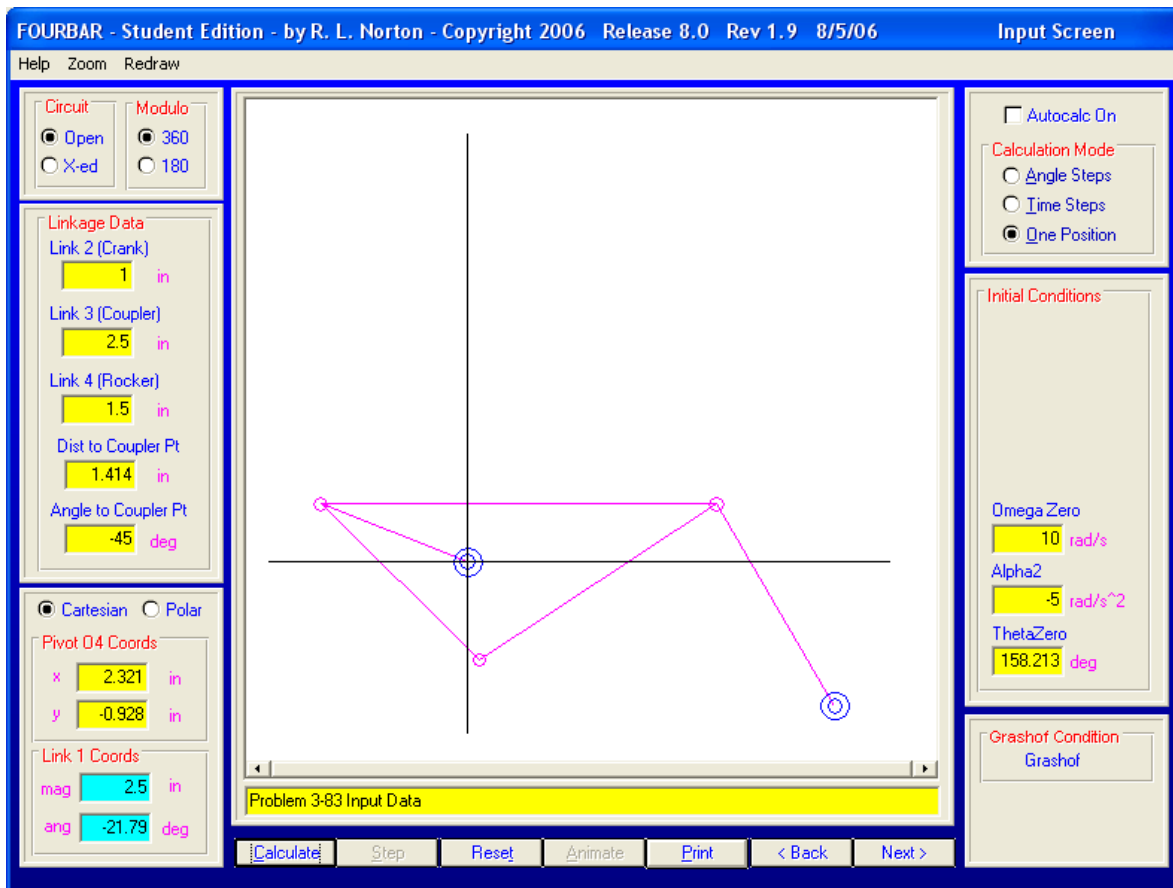
6. Calculate the coordinates of O_4 . Let the angle between links 2 and 3 be α , then

$$\alpha := \text{acos}\left[\frac{A^2 + (1 + C)^2 - B^2}{2 \cdot A \cdot (1 + C)}\right] \quad \alpha = 21.787 \text{ deg}$$

$$x_{O4} := C \cdot \cos(\alpha) \quad x_{O4} = 2.321$$

$$y_{O4} := -C \cdot \sin(\alpha) \quad y_{O4} = -0.928$$

7. Enter this data into FOURBAR and then plot the coupler curve. (See next page)



PROBLEM 3-84

Statement: Design a fourbar mechanism to move the link shown in Figure P3-19 from position 1 to position 2. Ignore the third position and the fixed pivots O_2 and O_4 shown. Build a cardboard model that demonstrates the required movement.

Given: Position 1 offsets: $x_{C1D1} := 17.186 \cdot \text{in}$ $y_{C1D1} := 0.604 \cdot \text{in}$

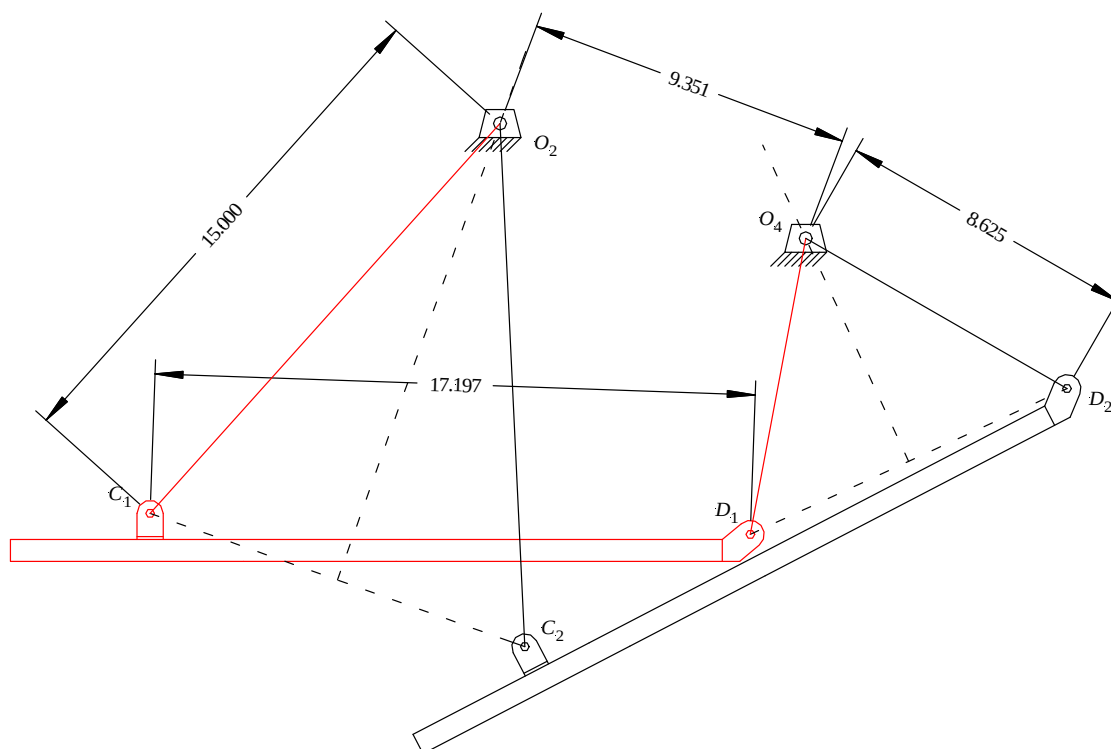
Solution: See figure below and Mathcad file P0384 for one possible solution.

1. Connect the end points of the two given positions of the line CD with construction lines, i.e., lines from C_1 to C_2 and D_1 to D_2 .
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_1C_2 was extended upward and the bisector of D_1D_2 was also extended upward.
3. Select one point on each bisector and label them O_2 and O_4 , respectively. In the solution below the distances O_2C and O_4D were selected to be 15.000 in. and 8.625 in, respectively. This resulted in a ground-link-length O_2O_4 for the fourbar of 9.351 in.
4. The fourbar is now defined as O_2CDO_4 with link lengths

$$\text{Link 3 (coupler)} \quad L_3 := \sqrt{x_{C1D1}^2 + y_{C1D1}^2} \quad L_3 = 17.197 \cdot \text{in}$$

$$\text{Link 2 (input)} \quad L_2 := 14.000 \cdot \text{in} \quad \text{Link 4 (output)} \quad L_4 := 7.000 \cdot \text{in}$$

$$\text{Ground link 1} \quad L_1 := 9.351 \cdot \text{in}$$



PROBLEM 3-85

Statement: Design a fourbar mechanism to move the link shown in Figure P3-19 from position 2 to position 3. Ignore the first position and the fixed pivots O_2 and O_4 shown. Build a cardboard model that demonstrates the required movement.

Given: Position 2 offsets: $x_{C_2D_2} := 15.524 \cdot \text{in}$ $y_{C_2D_2} := 7.397 \cdot \text{in}$

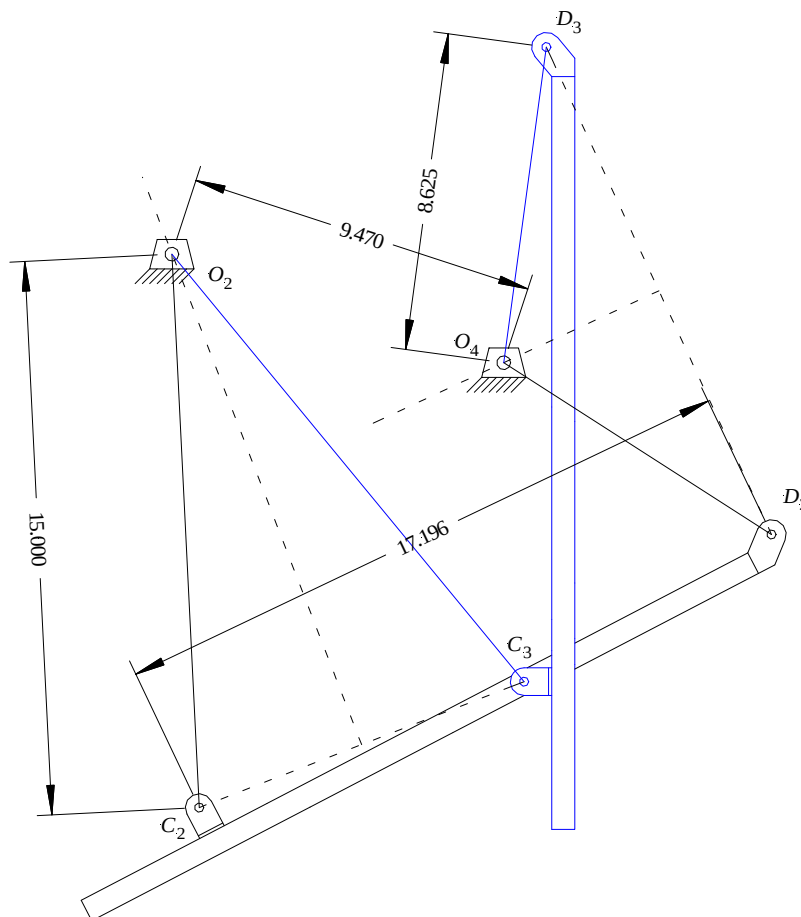
Solution: See figure below and Mathcad file P0385 for one possible solution.

1. Connect the end points of the two given positions of the line CD with construction lines, i.e., lines from C_2 to C_3 and D_2 to D_3 .
2. Bisect these lines and extend their perpendicular bisectors in any convenient direction. In the solution below the bisector of C_2C_3 was extended upward and the bisector of D_2D_3 was also extended upward.
3. Select one point on each bisector and label them O_2 and O_4 , respectively. In the solution below the distances O_2C and O_4D were selected to be 15.000 in and 8.625 in, respectively. This resulted in a ground-link-length O_2O_4 for the fourbar of 9.470 in.
4. The fourbar stage is now defined as O_2CDO_4 with link lengths

$$\text{Link 3 (coupler)} \quad L_3 := \sqrt{x_{C_2D_2}^2 + y_{C_2D_2}^2} \quad L_3 = 17.196 \cdot \text{in}$$

$$\text{Link 2 (input)} \quad L_2 := 15.000 \cdot \text{in} \quad \text{Link 4 (output)} \quad L_4 := 8.625 \cdot \text{in}$$

$$\text{Ground link 1b} \quad L_{1b} := 9.470 \cdot \text{in}$$



11. Using the program FOURBAR and the link lengths given above, it was found that the fourbar O_4DCO_6 is non-Grashoff with toggle positions at $\theta_4 = -14.9$ deg and $+14.9$ deg. The fourbar operates between $\theta_4 = +12.403$ deg and -8.950 deg.

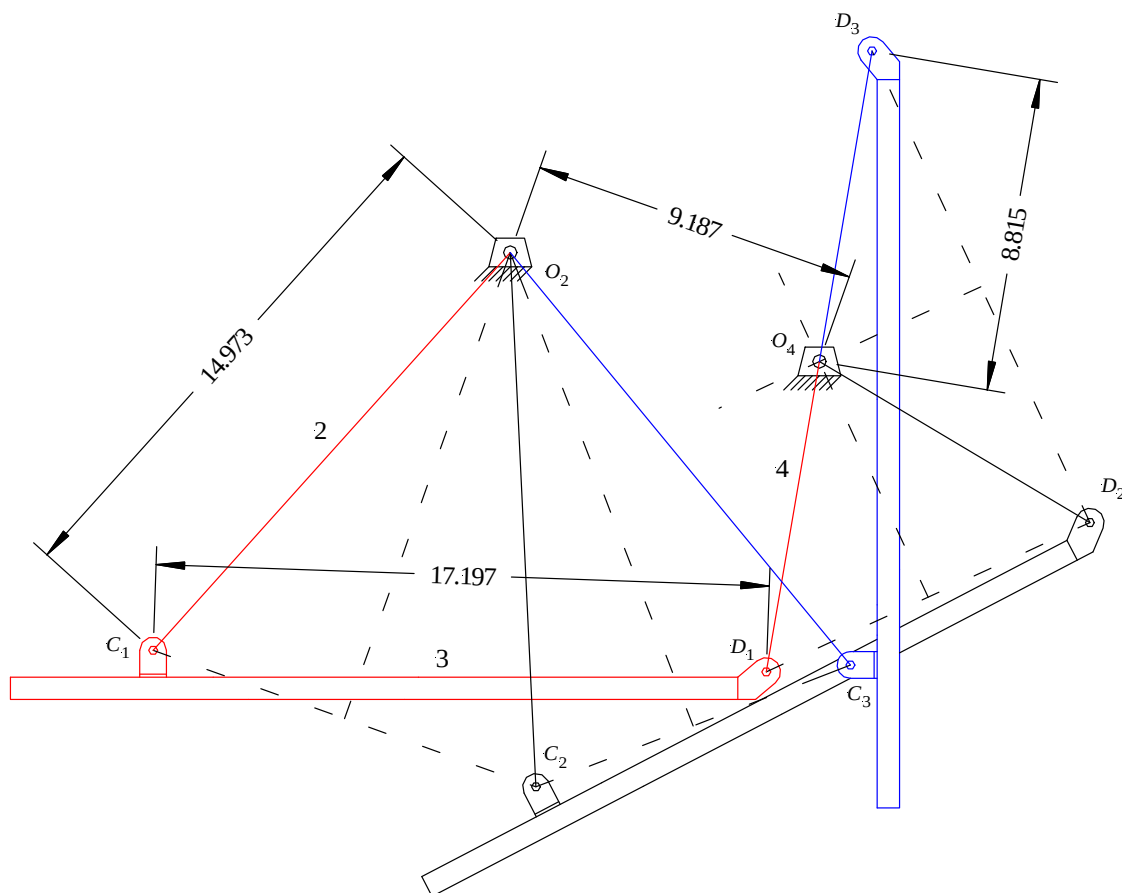
PROBLEM 3-86

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-19. Ignore the points O_2 and O_4 shown. Build a cardboard model that has stops to limit its motion to the range of positions designed.

Solution: See Figure P3-19 and Mathcad file P0386.

1. Draw link CD in its three design positions C_1D_1 , C_2D_2 , C_3D_3 in the plane as shown.
2. Draw construction lines from point C_1 to C_2 and from point C_2 to C_3 .
3. Bisect line C_1C_2 and line C_2C_3 and extend their perpendicular bisectors until they intersect. Label their intersection O_2 .
4. Repeat steps 2 and 3 for lines D_1D_2 and D_2D_3 . Label the intersection O_4 .
5. Connect O_2 with C_1 and call it link 2. Connect O_4 with D_1 and call it link 4.
6. Line C_1D_1 is link 3. Line O_2O_4 is link 1 (ground link for the fourbar). The fourbar is now defined as O_2CDO_4 and has link lengths of

Ground link 1	$L_1 := 9.187$	Link 2	$L_2 := 14.973$
Link 3	$L_3 := 17.197$	Link 4	$L_4 := 8.815$

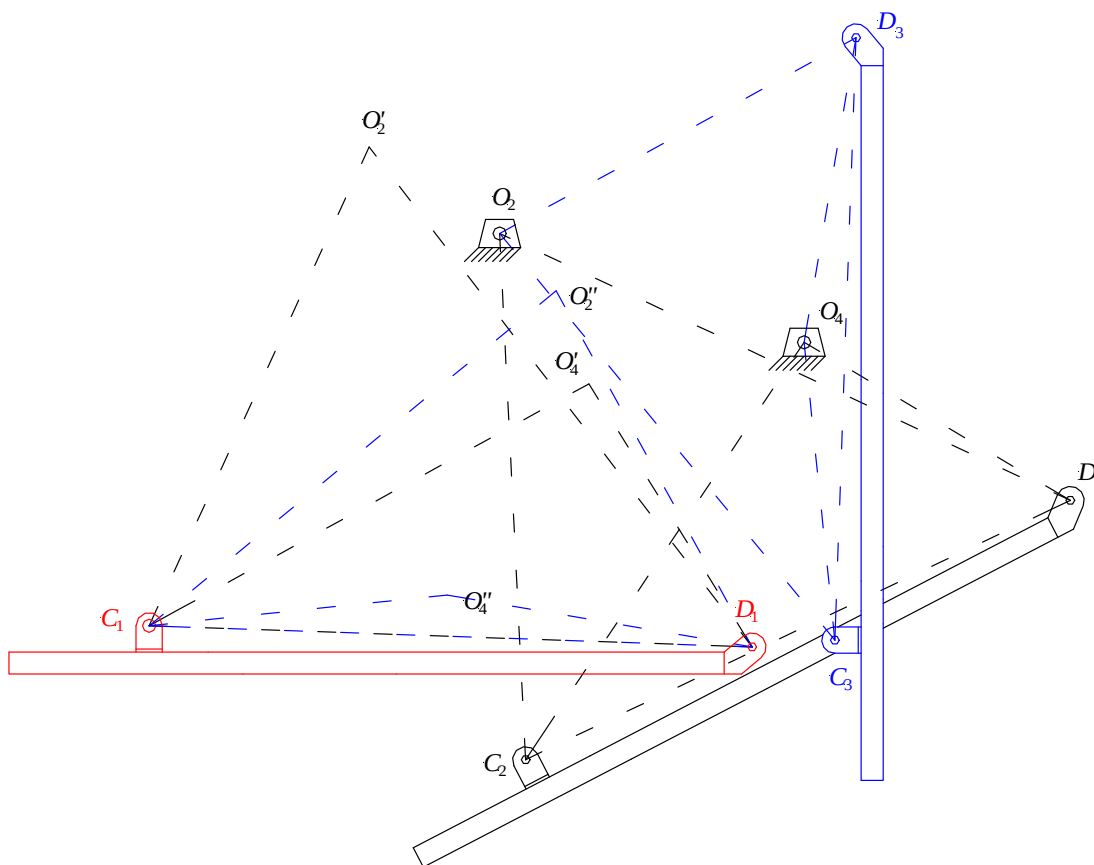


PROBLEM 3-87

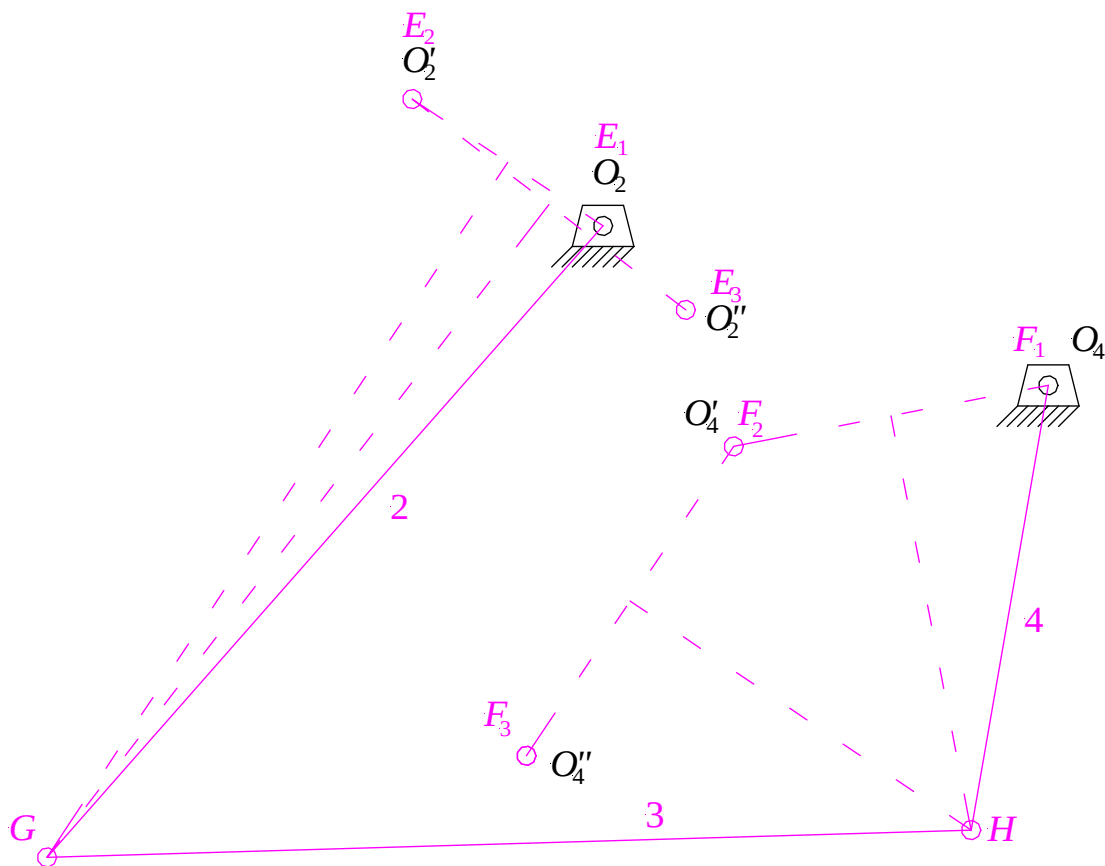
Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-17 using the fixed pivots O_2 and O_4 shown. (See Example 3-7.) Build a cardboard model that has stops to limit its motion to the range of positions designed.

Solution: See Figure P3-19 and Mathcad file P0387.

1. Draw link CD in its three design positions C_1D_1 , C_2D_2 , C_3D_3 in the plane as shown.
2. Draw the ground link O_2O_4 in its desired position in the plane with respect to the first coupler position C_1D_1 .
3. Draw construction arcs from point C_2 to O_2 and from point D_2 to O_2 whose radii define the sides of triangle $C_2O_2D_2$. This defines the relationship of the fixed pivot O_2 to the coupler line CD in the second coupler position.
4. Draw construction arcs from point C_2 to O_4 and from point D_2 to O_4 whose radii define the sides of triangle $C_2O_4D_2$. This defines the relationship of the fixed pivot O_4 to the coupler line CD in the second coupler position.
5. Transfer this relationship back to the first coupler position C_1D_1 so that the ground plane position $O_2'O_4'$ bears the same relationship to C_1D_1 as O_2O_4 bore to the second coupler position C_2D_2 .
6. Repeat the process for the third coupler position and transfer the third relative ground link position to the first, or reference, position.
7. The three inverted positions of the ground link that correspond to the three desired coupler positions are labeled O_2O_4 , $O_2'O_4'$, and $O_2''O_4''$ in the first layout below and are renamed E_1F_1 , E_2F_2 , and E_3F_3 , respectively, in the second layout, which is used to find the points G and H .



First layout for steps 1 through 7



Second layout for steps 8 through 12

8. Draw construction lines from point E_1 to E_2 and from point E_2 to E_3 .
9. Bisect line E_1E_2 and line E_2E_3 and extend their perpendicular bisectors until they intersect. Label their intersection G .
10. Repeat steps 2 and 3 for lines F_1F_2 and F_2F_3 . Label the intersection H .
11. Connect E_1 with G and label it link 2. Connect F_1 with H and label it link 4. Reverting, E_1 and F_1 are the original fixed pivots O_2 and O_4 , respectively.
12. Line GH is link 3. Line O_2O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_2GHO_4 and has link lengths of

Ground link 1a	$L_{1a} := 9.216$	Link 2	$L_2 := 16.385$
Link 3	$L_3 := 18.017$	Link 4	$L_4 := 8.786$

13. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

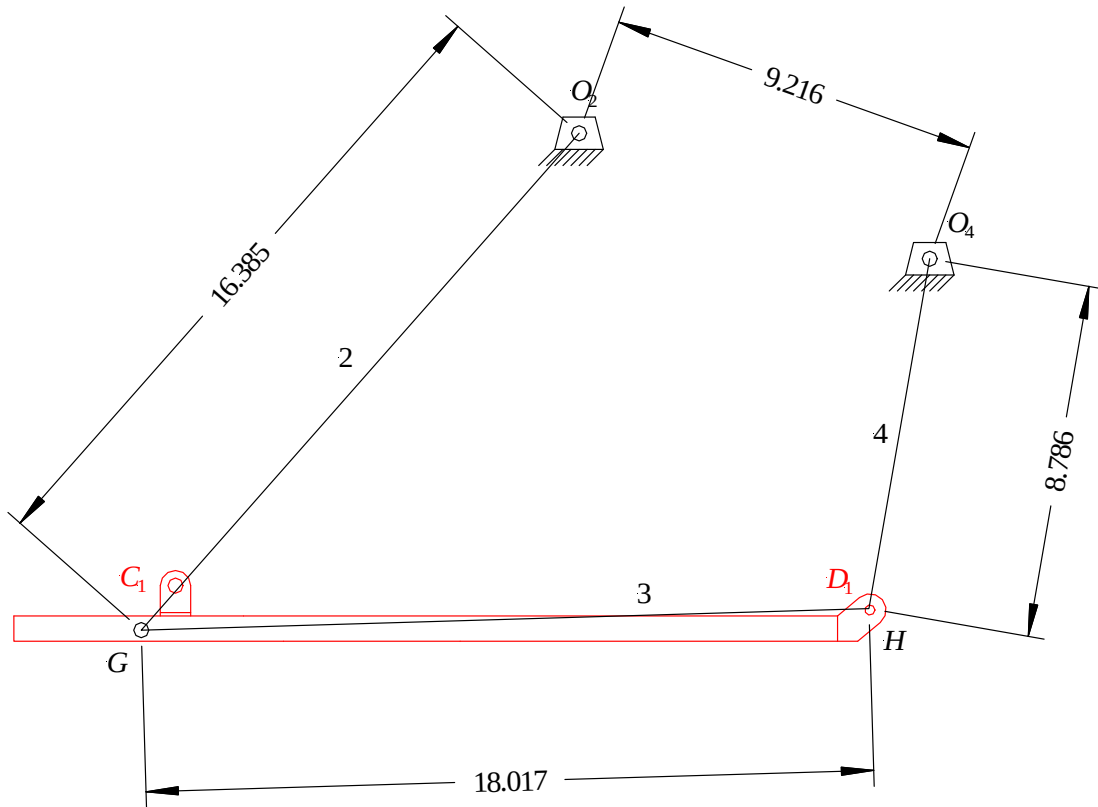
```

Condition(a,b,c,d) :=
| S ← min(a,b,c,d)
| L ← max(a,b,c,d)
| SL ← S + L
| PQ ← a + b + c + d - SL
| return "Grashof" if SL < PQ
| return "Special Grashof" if SL = PQ
| return "non-Grashof" otherwise

```

$$\text{Condition}(L_1, L_2, L_3, L_4) = \text{"non-Grashof"}$$

The fourbar that will provide the desired motion is now defined as a non-Grashof double rocker in the open configuration. It now remains to add the original points C_1 and D_1 to the coupler GH .



PROBLEM 4-7a

Statement: The link lengths and value of θ_2 for some fourbar linkages are defined in Table P4-1. The linkage configuration and terminology are shown in Figure P4-1. For row *a*, find all possible solutions (both open and crossed) for angles θ_3 and θ_4 using the vector loop method. Determine the Grashof condition.

Given: Link 1 $d := 6 \cdot \text{in}$ Link 2 $a := 2 \cdot \text{in}$
 Link 3 $b := 7 \cdot \text{in}$ Link 4 $c := 9 \cdot \text{in}$ $\theta_2 := 30 \cdot \text{deg}$

Solution: See Mathcad file P0407a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 3.0000 \qquad K_2 = 0.6667 \qquad K_3 = 2.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \qquad A = -0.7113$$

$$B := -2 \cdot \sin(\theta_2) \qquad B = -1.0000$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3 \qquad C = 3.5566$$

2. Use equation 4.10b to find values of θ_4 for the open and crossed circuits.

$$\text{Open: } \theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \qquad \theta_{41} = -242.714 \cdot \text{deg}$$

$$\theta_{41} := \theta_{41} - 360 \cdot \text{deg} \qquad \theta_{41} = -602.714 \cdot \text{deg}$$

$$\text{Crossed: } \theta_{42} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \qquad \theta_{42} = 216.340 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \qquad K_4 = 0.8571$$

$$K_5 = -0.2857$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -1.6774$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = -1.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = 2.5906$$

4. Use equation 4.13 to find values of θ_3 for the open and crossed circuits.

$$\text{Open: } \theta_{31} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \qquad \theta_{31} = -271.163 \cdot \text{deg}$$

$$\theta_{31} := \theta_{31} - 360 \cdot \text{deg} \qquad \theta_{31} = -631.163 \cdot \text{deg}$$

$$\text{Crossed: } \theta_{32} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \qquad \theta_{32} = 244.789 \cdot \text{deg}$$

5. Check the Grashof condition.

```
Condition(a,b,c,d) :=  $\left\{ \begin{array}{l} S \leftarrow \min(a,b,c,d) \\ L \leftarrow \max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array} \right.$ 
```

$Condition(a,b,c,d) = \text{"Grashof"}$

PROBLEM 4-1

Statement: A position vector is defined as having length equal to your height in inches (or centimeters). The tangent of its angle is defined as your weight in lbs (or kg) divided by your age in years. Calculate the data for this vector and:

- Draw the position vector to scale on Cartesian axes.
- Write an expression for the position vector using unit vector notation.
- Write an expression for the position vector using complex number notation, in both polar and Cartesian forms.

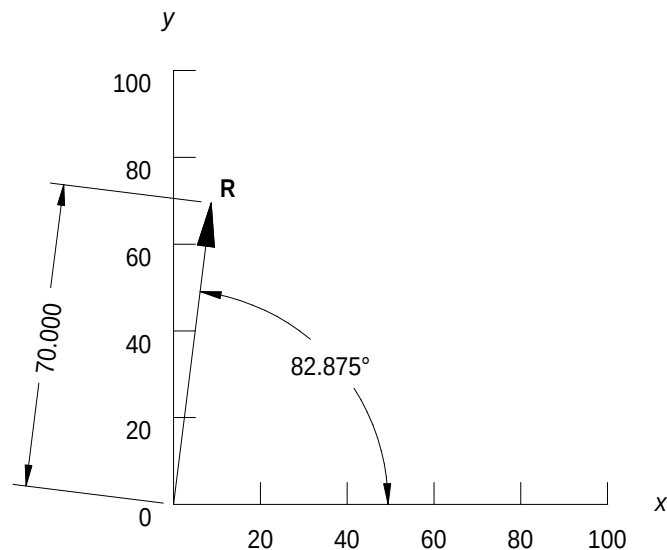
Assumptions: Height := 70, weight := 160, age := 20

Solution: See Mathcad file P0401.

- The magnitude of the vector is $R := \text{Height}$. The angle that the vector makes with the x-axis is

$$\theta := \text{atan}\left(\frac{\text{weight}}{\text{age}}\right) \quad \theta = 82.875 \text{ deg} \quad \theta = 1.446 \text{ rad}$$

- Draw the position vector to scale on Cartesian axes.



- Write an expression for the position vector using unit vector notation.

$$\mathbf{R} := R \cdot \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \quad \mathbf{R} = \begin{pmatrix} 8.682 \\ 69.459 \end{pmatrix} \quad \mathbf{R} = 8.682 \mathbf{i} + 69.459 \mathbf{j}$$

- Write an expression for the position vector using complex number notation, in both polar and Cartesian forms.

Polar form: $\mathbf{R} := 68 \cdot e^{j \cdot 1.446}$

Cartesian form: $\mathbf{R} := 8.682 + j \cdot 69.459$

PROBLEM 4-2

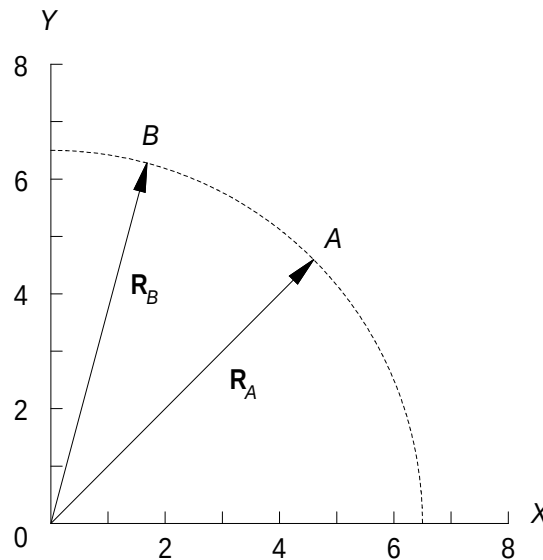
Statement: A particle is traveling along an arc of 6.5 inch radius. The arc center is at the origin of a coordinate system. When the particle is at position *A*, its position vector makes a 45-deg angle with the *X* axis. At position *B*, its vector makes a 75-deg angle with the *X* axis. Draw this system to some convenient scale and:

- Write an expression for the particle's position vector in position *A* using complex number notation, in both polar and Cartesian forms.
- Write an expression for the particle's position vector in position *B* using complex number notation, in both polar and Cartesian forms.
- Write a vector equation for the position difference between points *B* and *A*. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.
- Check the result of part *c* with a graphical method.

Given: Circle radius and vector magnitude, $R := 6.5 \text{ in}$; vector angles: $\theta_A := 45 \text{ deg}$ $\theta_B := 75 \text{ deg}$

Solution: See Mathcad file P0402.

- Establish an *X*-*Y* coordinate frame and draw a circle with center at the origin and radius *R*.
- Draw lines from the origin that make angles of 45 and 75 deg with respect to the *X* axis. Label the intersections of the lines with the circles as *A* and *B*, respectively. Make the line segment *OA* a vector by putting an arrowhead at *A*, pointing away from the origin. Label the vector $\mathbf{R}_A$. Repeat for the line segment *OB*, labeling it $\mathbf{R}_B$.



- Write an expression for the particle's position vector in position *A* using complex number notation, in both polar and Cartesian forms.

Polar form: $\mathbf{R}_A := R \cdot e^{j \cdot \theta_A}$ $\mathbf{R}_A := 6.5 \cdot e^{j \cdot \frac{\pi}{4}}$

Cartesian form: $\mathbf{R}_A := R \cdot (\cos(\theta_A) + j \cdot \sin(\theta_A))$ $\mathbf{R}_A = (4.596 + 4.596j) \text{ in}$

- Write an expression for the particle's position vector in position *B* using complex number notation, in both polar and Cartesian forms.

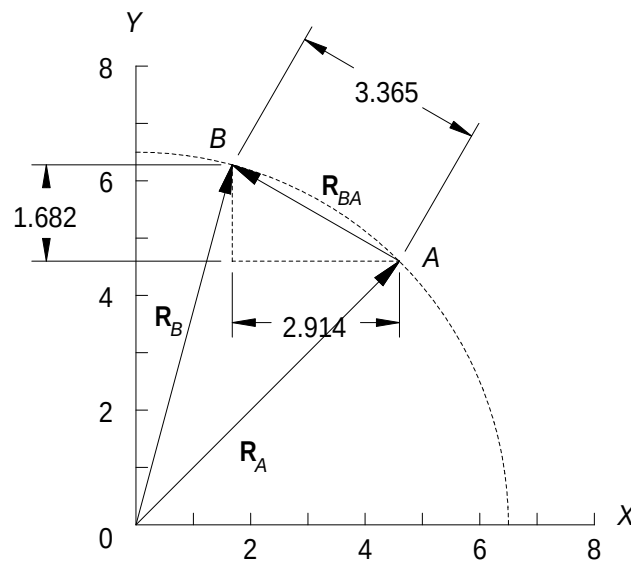
Polar form: $\mathbf{R}_B := R \cdot e^{j \cdot \theta_B}$ $\mathbf{R}_B := 6.5 \cdot e^{j \cdot \frac{75 \cdot \pi}{180}}$

Cartesian form: $\mathbf{R}_B := R \cdot (\cos(\theta_B) + j \cdot \sin(\theta_B))$ $\mathbf{R}_B = (1.682 + 6.279j) \text{ in}$

- c. Write a vector equation for the position difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.

$$\mathbf{R}_{BA} := \mathbf{R}_B - \mathbf{R}_A \qquad \mathbf{R}_{BA} = (-2.914 + 1.682j) \text{ in}$$

- d. Check the result of part c with a graphical method.



On the layout above the X and Y components of $\mathbf{R}_{BA}$ are equal to the real and imaginary components calculated, confirming that the calculation is correct.

PROBLEM 4-3

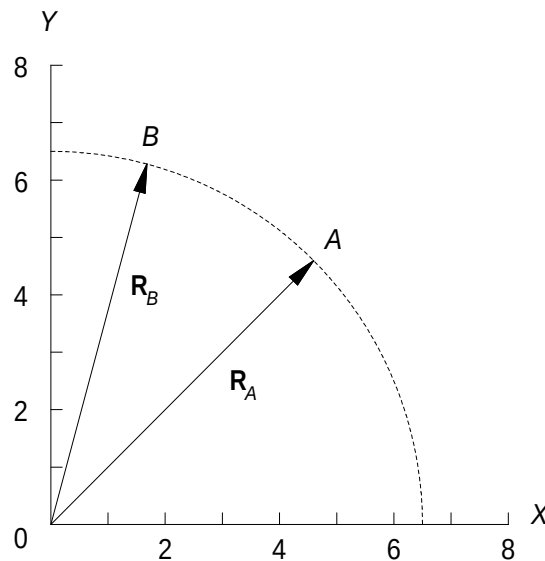
Statement: Two particles are traveling along an arc of 6.5 inch radius. The arc center is at the origin of a coordinate system. When one particle is at position *A*, its position vector makes a 45-deg angle with the *X* axis. Simultaneously, the other particle is at position *B*, where its vector makes a 75-deg angle with the *X* axis. Draw this system to some convenient scale and:

- Write an expression for the particle's position vector in position *A* using complex number notation, in both polar and Cartesian forms.
- Write an expression for the particle's position vector in position *B* using complex number notation, in both polar and Cartesian forms.
- Write a vector equation for the relative position of the particle at *B* with respect to the particle at *A*. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.
- Check the result of part *c* with a graphical method.

Given: Circle radius and vector magnitude, $R := 6.5 \text{ in}$; vector angles: $\theta_A := 45 \text{ deg}$ $\theta_B := 75 \text{ deg}$

Solution: See Mathcad file P0403.

- Establish an *X*-*Y* coordinate frame and draw a circle with center at the origin and radius *R*.
- Draw lines from the origin that make angles of 45 and 75 deg with respect to the *X* axis. Label the intersections of the lines with the circles as *A* and *B*, respectively. Make the line segment *OA* a vector by putting an arrowhead at *A*, pointing away from the origin. Label the vector $\mathbf{R}_A$. Repeat for the line segment *OB*, labeling it $\mathbf{R}_B$.



- Write an expression for the particle's position vector in position *A* using complex number notation, in both polar and Cartesian forms.

Polar form: $\mathbf{R}_A := R \cdot e^{j \cdot \theta_A}$ $\mathbf{R}_A := 6.5 \cdot e^{j \cdot \frac{\pi}{4}}$

Cartesian form: $\mathbf{R}_A := R \cdot (\cos(\theta_A) + j \cdot \sin(\theta_A))$ $\mathbf{R}_A = (4.596 + 4.596j) \text{ in}$

- Write an expression for the particle's position vector in position *B* using complex number notation, in both polar and Cartesian forms.

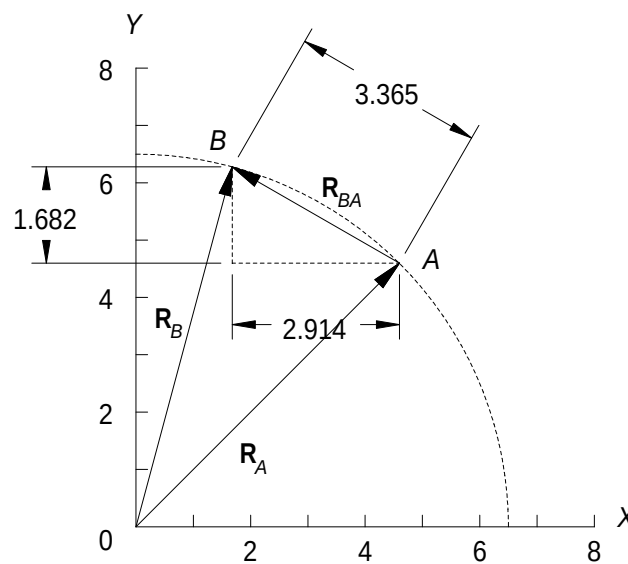
Polar form: $\mathbf{R}_B := R \cdot e^{j \cdot \theta_B}$ $\mathbf{R}_B := 6.5 \cdot e^{j \cdot \frac{75 \cdot \pi}{180}}$

Cartesian form: $\mathbf{R}_B := R \cdot (\cos(\theta_B) + j \cdot \sin(\theta_B))$ $\mathbf{R}_B = (1.682 + 6.279j) \text{ in}$

- c. Write a vector equation for the relative position of the particle at B with respect to the particle at A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.

$$\mathbf{R}_{BA} := \mathbf{R}_B - \mathbf{R}_A \quad \mathbf{R}_{BA} = (-2.914 + 1.682j) \text{ in}$$

- d. Check the result of part c with a graphical method.



On the layout above the X and Y components of $\mathbf{R}_{BA}$ are equal to the real and imaginary components calculated, confirming that the calculation is correct.

PROBLEM 4-4

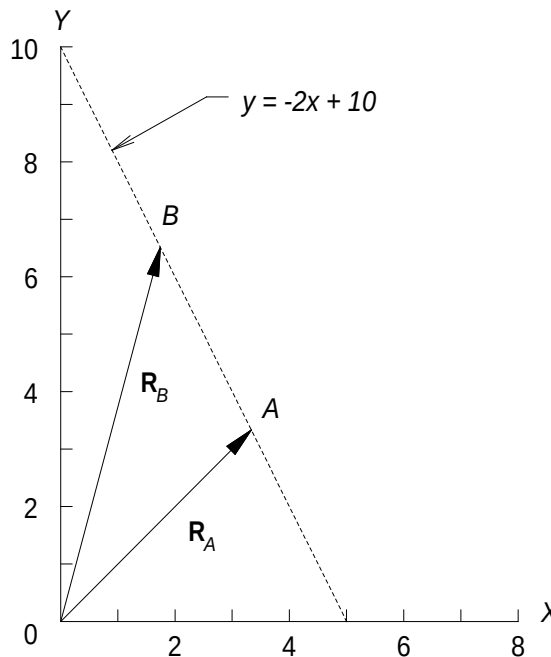
Statement: A particle is traveling along the line $y = -2x + 10$. When the particle is at position A , its position vector makes a 45-deg angle with the X axis. At position B , its vector makes a 75-deg angle with the X axis. Draw this system to some convenient scale and:

- Write an expression for the particle's position vector in position A using complex number notation, in both polar and Cartesian forms.
- Write an expression for the particle's position vector in position B using complex number notation, in both polar and Cartesian forms.
- Write a vector equation for the position difference between points B and A . Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.
- Check the result of part c with a graphical method.

Given: Vector angles: $\theta_A := 45\text{-deg}$ $\theta_B := 75\text{-deg}$

Solution: See Mathcad file P0402.

- Establish an X - Y coordinate frame and draw the line $y = -2x + 10$.
- Draw lines from the origin that make angles of 45 and 75 deg with respect to the X axis. Label the intersections of the lines with the line in step 1 as A and B , respectively. Make the line segment OA a vector by putting an arrowhead at A , pointing away from the origin. Label the vector $\mathbf{R}_A$. Repeat for the line segment OB , labeling it $\mathbf{R}_B$.



- Calculate the coordinates of points A and B .

$$x_A \cdot \tan(\theta_A) = -2 \cdot x_A + 10 \quad x_A := \frac{10}{2 + \tan(\theta_A)} \quad x_A = 3.333$$

$$y_A := x_A \cdot \tan(\theta_A) \quad y_A = 3.333$$

$$x_B \cdot \tan(\theta_B) = -2 \cdot x_B + 10 \quad x_B := \frac{10}{2 + \tan(\theta_B)} \quad x_B = 1.745$$

$$y_B := x_B \cdot \tan(\theta_B) \quad y_B = 6.511$$

4. Calculate the distances of points A and B from the origin.

$$R_A := \sqrt{x_A^2 + y_A^2} \quad R_A = 4.714$$

$$R_B := \sqrt{x_B^2 + y_B^2} \quad R_B = 6.741$$

- a. Write an expression for the particle's position vector in position A using complex number notation, in both polar and Cartesian forms.

$$\text{Polar form:} \quad \mathbf{R}_A := R_A \cdot \mathbf{e}^{j \cdot \theta_A} \quad \mathbf{R}_A := 4.714 \cdot \mathbf{e}^{j \cdot \frac{\pi}{4}}$$

$$\text{Cartesian form:} \quad \mathbf{R}_A := R_A \cdot (\cos(\theta_A) + j \cdot \sin(\theta_A)) \quad \mathbf{R}_A = 3.333 + 3.333j$$

- b. Write an expression for the particle's position vector in position B using complex number notation, in both polar and Cartesian forms.

$$\text{Polar form:} \quad \mathbf{R}_B := R_B \cdot \mathbf{e}^{j \cdot \theta_B} \quad \mathbf{R}_B := 6.741 \cdot \mathbf{e}^{j \cdot \frac{75 \cdot \pi}{180}}$$

$$\text{Cartesian form:} \quad \mathbf{R}_B := R_B \cdot (\cos(\theta_B) + j \cdot \sin(\theta_B)) \quad \mathbf{R}_B = 1.745 + 6.511j$$

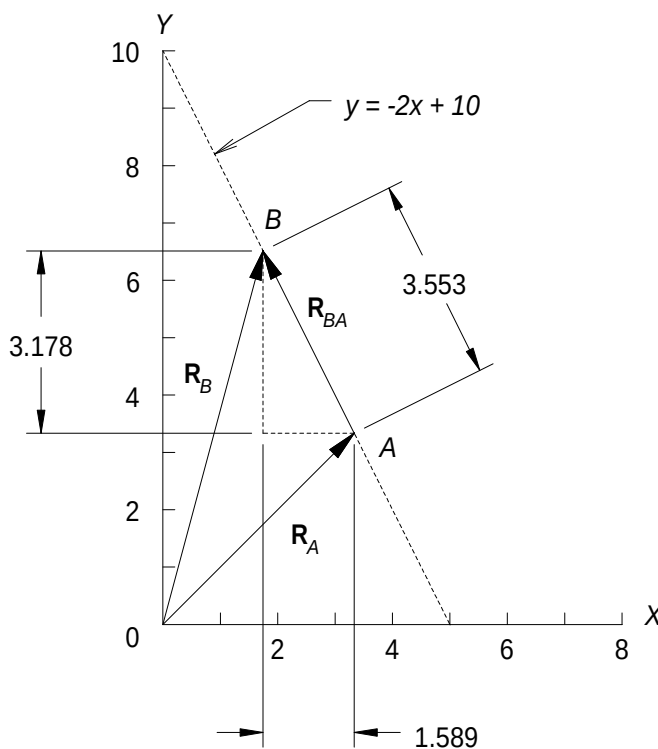
- c. Write a vector equation for the position difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.

$$\mathbf{R}_{BA} := \mathbf{R}_B - \mathbf{R}_A$$

$$\mathbf{R}_{BA} = -1.589 + 3.178j$$

- d. Check the result of part c with a graphical method.

On the layout above the X and Y components of $\mathbf{R}_{BA}$ are equal to the real and imaginary components calculated, confirming that the calculation is correct.



PROBLEM 4-5

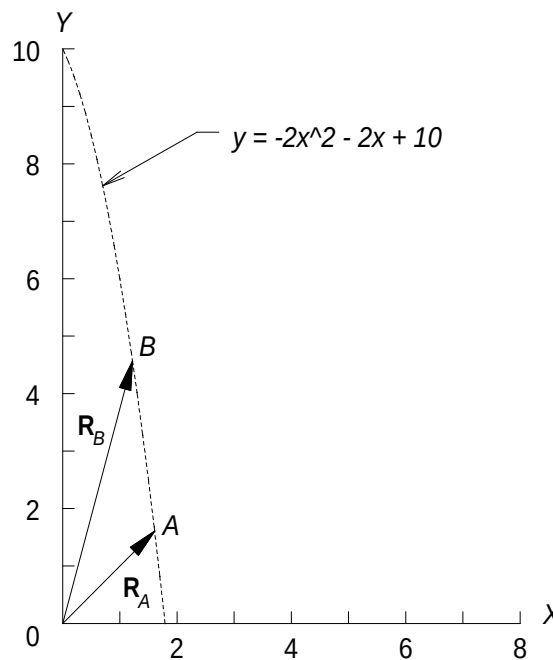
Statement: Two particles are traveling along the line $y = -2x^2 - 2x + 10$. When one particle is at position A, its position vector makes a 45-deg angle with the X axis. Simultaneously, the other particle is at position B, where its vector makes a 75-deg angle with the X axis. Draw this system to some convenient scale and:

- Write an expression for the particle's position vector in position A using complex number notation, in both polar and Cartesian forms.
- Write an expression for the particle's position vector in position B using complex number notation, in both polar and Cartesian forms.
- Write a vector equation for the relative position of the particle at B with respect to the particle at A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.
- Check the result of part c with a graphical method.

Given: Vector angles: $\theta_A := 45 \cdot \text{deg}$ $\theta_B := 75 \cdot \text{deg}$

Solution: See Mathcad file P0405.

- Establish an X-Y coordinate frame and draw the line $y = -2x^2 - 2x + 10$.
- Draw lines from the origin that make angles of 45 and 75 deg with respect to the X axis. Label the intersections of the lines with the line drawn in step 1 as A and B, respectively. Make the line segment OA a vector by putting an arrowhead at A, pointing away from the origin. Label the vector $\mathbf{R}_A$. Repeat for the line segment OB, labeling it $\mathbf{R}_B$.



- Calculate the coordinates of points A and B.

$$x_A \cdot \tan(\theta_A) = -2 \cdot x_A^2 - 2 \cdot x_A + 10$$

$$x_A := \frac{1}{2} \cdot \left[-\left(1 + \frac{\tan(\theta_A)}{2} \right) + \sqrt{\left(1 + \frac{\tan(\theta_A)}{2} \right)^2 + 20} \right] \quad x_A = 1.608$$

$$y_A := x_A \cdot \tan(\theta_A) \quad y_A = 1.608$$

$$x_B \cdot \tan(\theta_B) = -2 \cdot x_B^2 - 2 \cdot x_B + 10$$

$$x_B := \frac{1}{2} \cdot \left[-\left(1 + \frac{\tan(\theta_B)}{2}\right) + \sqrt{\left(1 + \frac{\tan(\theta_B)}{2}\right)^2 + 20} \right] \quad x_B = 1.223$$

$$y_B := x_B \cdot \tan(\theta_B) \quad y_B = 4.564$$

4. Calculate the distances of points A and B from the origin.

$$R_A := \sqrt{x_A^2 + y_A^2} \quad R_A = 2.275$$

$$R_B := \sqrt{x_B^2 + y_B^2} \quad R_B = 4.725$$

- a. Write an expression for the particle's position vector in position A using complex number notation, in both polar and Cartesian forms.

$$\text{Polar form:} \quad \mathbf{R}_A := R \cdot \mathbf{e}^{j \cdot \theta_A} \quad \mathbf{R}_A := 2.275 \cdot \mathbf{e}^{j \cdot \frac{\pi}{4}}$$

$$\text{Cartesian form:} \quad \mathbf{R}_A := R_A \cdot (\cos(\theta_A) + j \cdot \sin(\theta_A)) \quad \mathbf{R}_A = 1.608 + 1.608j$$

- b. Write an expression for the particle's position vector in position B using complex number notation, in both polar and Cartesian forms.

$$\text{Polar form:} \quad \mathbf{R}_B := R \cdot \mathbf{e}^{j \cdot \theta_B} \quad \mathbf{R}_B := 4.725 \cdot \mathbf{e}^{j \cdot \frac{75 \cdot \pi}{180}}$$

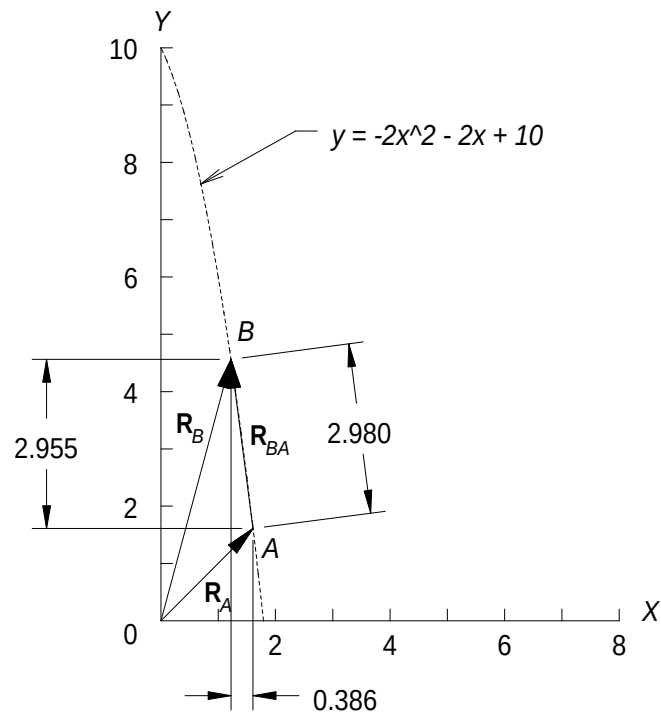
$$\text{Cartesian form:} \quad \mathbf{R}_B := R_B \cdot (\cos(\theta_B) + j \cdot \sin(\theta_B)) \quad \mathbf{R}_B = 1.223 + 4.564j$$

- c. Write a vector equation for the relative position of the particle at B with respect to the particle at A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.

$$\mathbf{R}_{BA} := \mathbf{R}_B - \mathbf{R}_A \quad \mathbf{R}_{BA} = -0.386 + 2.955j$$

- d. Check the result of part c with a graphical method.

On the layout on the next page the X and Y components of $\mathbf{R}_{BA}$ are equal to the real and imaginary components calculated, confirming that the calculation is correct.



PROBLEM 4-6a

Statement: The link lengths and value of θ_2 for some fourbar linkages are defined in Table P4-1. The linkage configuration and terminology are shown in Figure P4-1. For row *a*, draw the linkage to scale and graphically find all possible solutions (both open and crossed) for angles θ_3 and θ_4 . Determine the Grashoff condition.

Given: Link 1 $d := 6 \cdot \text{in}$ Link 2 $a := 2 \cdot \text{in}$
 Link 3 $b := 7 \cdot \text{in}$ Link 4 $c := 9 \cdot \text{in}$ $\theta_2 := 30 \cdot \text{deg}$

Solution: See figure below for one possible solution. Also see Mathcad file P0406a.

1. Lay out an xy-axis system. Its origin will be the link 2 pivot, O_2 .
2. Draw link 2 to some convenient scale at its given angle.
3. Draw a circle with center at the free end of link 2 and a radius equal to the given length of link 3.
4. Locate pivot O_4 on the x-axis at a distance from the origin equal to the given length of link 1.
5. Draw a circle with center at O_4 and a radius equal to the given length of link 4.
6. The two intersections of the circles (if any) are the two solutions to the position analysis problem, crossed and open. If the circles don't intersect, there is no solution.
7. Draw links 3 and 4 in their two possible positions (shown as solid for open and dashed for crossed in the figure) and measure their angles θ_3 and θ_4 with respect to the x-axis. From the solution below,

OPEN $\theta_{31} := 88.84 \cdot \text{deg}$

$\theta_{41} := 117.29 \cdot \text{deg}$

CROSSED $\theta_{32} := 360 \cdot \text{deg} - 115.21 \cdot \text{deg}$ $\theta_{32} = 244.790 \text{ deg}$

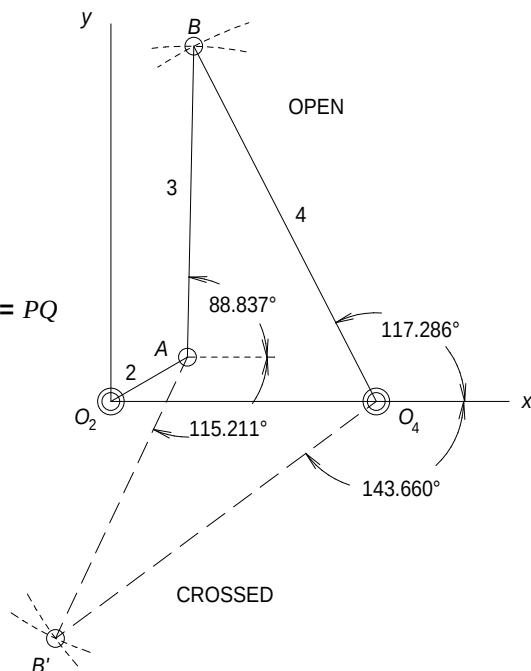
$\theta_{42} := 360 \cdot \text{deg} - 143.66 \cdot \text{deg}$ $\theta_{42} = 216.340 \text{ deg}$

8. Check the Grashof condition.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise
  
```

Condition(a,b,c,d) = "Grashof"



PROBLEM 4-7a

Statement: The link lengths and value of θ_2 for some fourbar linkages are defined in Table P4-1. The linkage configuration and terminology are shown in Figure P4-1. For row *a*, find all possible solutions (both open and crossed) for angles θ_3 and θ_4 using the vector loop method. Determine the Grashof condition.

Given: Link 1 $d := 6 \cdot \text{in}$ Link 2 $a := 2 \cdot \text{in}$
 Link 3 $b := 7 \cdot \text{in}$ Link 4 $c := 9 \cdot \text{in}$ $\theta_2 := 30 \cdot \text{deg}$
 Two argument inverse tangent

$$\text{atan2}(x,y) := \begin{cases} \text{return } 0.5 \cdot \pi & \text{if } x = 0 \wedge y > 0 \\ \text{return } 1.5 \cdot \pi & \text{if } x = 0 \wedge y < 0 \\ \text{return } \text{atan}\left(\left(\frac{y}{x}\right)\right) & \text{if } x > 0 \\ \text{atan}\left(\left(\frac{y}{x}\right)\right) + \pi & \text{otherwise} \end{cases}$$

Solution: See Mathcad file P0407a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$\begin{aligned} K_1 &:= \frac{d}{a} & K_2 &:= \frac{d}{c} & K_3 &:= \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \\ K_1 &= 3.0000 & K_2 &= 0.6667 & K_3 &= 2.0000 \\ A &:= \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 & A &= -0.7113 \\ B &:= -2 \cdot \sin(\theta_2) & B &= -1.0000 \\ C &:= K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3 & C &= 3.5566 \end{aligned}$$

2. Use equation 4.10b to find values of θ_4 for the open and crossed circuits.

$$\begin{aligned} \text{Open: } \theta_{41} &:= 2 \cdot \left(\text{atan2}\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right) \right) & \theta_{41} &= 477.286 \text{ deg} \\ \theta_{41} &:= \theta_{41} - 360 \cdot \text{deg} & \theta_{41} &= 117.286 \text{ deg} \\ \text{Crossed: } \theta_{42} &:= 2 \cdot \left(\text{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right) \right) & \theta_{42} &= 216.340 \text{ deg} \end{aligned}$$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$\begin{aligned} K_4 &:= \frac{d}{b} & K_5 &:= \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} & K_4 &= 0.8571 \\ & & & & K_5 &= -0.2857 \\ D &:= \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 & D &= -1.6774 \\ E &:= -2 \cdot \sin(\theta_2) & E &= -1.0000 \\ F &:= K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 & F &= 2.5906 \end{aligned}$$

4. Use equation 4.13 to find values of θ_3 for the open and crossed circuits.

$$\text{Open: } \theta_{31} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{31} = 448.837 \text{ deg}$$

$$\theta_{31} := \theta_{31} - 360 \cdot \text{deg} \quad \theta_{31} = 88.837 \text{ deg}$$

$$\text{Crossed: } \theta_{32} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{32} = 244.789 \text{ deg}$$

5. Check the Grashof condition.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Condition(a,b,c,d) = "Grashof"

PROBLEM 4-8

Statement: Expand equation 4.7b and prove that it reduces to equation 4.7c (p. 157).

Solution: See Mathcad file P0408.

1. Write equation 4.7b and expand the two terms that are squared.

$$b^2 := (-a \cdot \sin(\theta_2) + c \cdot \sin(\theta_4))^2 + (-a \cdot \cos(\theta_2) + c \cdot \cos(\theta_4) + d)^2 \quad (4.7b)$$

$$(-a \cdot \sin(\theta_2) + c \cdot \sin(\theta_4))^2 := a^2 \cdot \sin(\theta_2)^2 - 2 \cdot a \cdot c \cdot \sin(\theta_2) \cdot \sin(\theta_4) + c^2 \cdot \sin(\theta_4)^2 \quad (a)$$

$$(-a \cdot \cos(\theta_2) + c \cdot \cos(\theta_4) + d)^2 := a^2 \cdot \cos(\theta_2)^2 - 2 \cdot a \cdot c \cdot (\cos(\theta_2) \cdot \cos(\theta_4)) - 2 \cdot a \cdot d \cdot \cos(\theta_2) \dots \\ + 2 \cdot c \cdot d \cdot \cos(\theta_4) + c^2 \cdot \cos(\theta_4)^2 + d^2 \quad (b)$$

2. Add the two expanded terms, equations *a* and *b*, noting the identity $\sin^2 x + \cos^2 x = 1$.

$$b^2 := a^2 + c^2 + d^2 - 2 \cdot a \cdot d \cdot \cos(\theta_2) + 2 \cdot c \cdot d \cdot \cos(\theta_4) - 2 \cdot a \cdot c \cdot (\sin(\theta_2) \cdot \sin(\theta_4) + \cos(\theta_2) \cdot \cos(\theta_4))$$

This is equation 4.7c.

PROBLEM 4-9a

Statement: The link lengths, value of θ_2 , and offset for some fourbar slider-crank linkages are defined in Table P4-2. The linkage configuration and terminology are shown in Figure P4-2. For row *a*, draw the linkage to scale and graphically find all possible solutions (both open and crossed) for angles θ_3 and slider position *d*.

Given: Link 2 $a := 1.4 \cdot \text{in}$ Link 3 $b := 4 \cdot \text{in}$
Offset $c := 1 \cdot \text{in}$ $\theta_2 := 45 \cdot \text{deg}$

Solution: See figure below for one possible solution. Also see Mathcad file P0409a.

1. Lay out an xy-axis system. Its origin will be the link 2 pivot, O_2 .
2. Draw link 2 to some convenient scale at its given angle.
3. Draw a circle with center at the free end of link 2 and a radius equal to the given length of link 3.
4. Draw a horizontal line through $y = c$ (the offset).
5. The two intersections of the circle with the horizontal line (if any) are the two solutions to the position analysis problem, crossed and open. If the circle and line don't intersect, there is no solution.
6. Draw link 3 and the slider block in their two possible positions (shown as solid for open and dashed for crossed in the figure) and measure the angle θ_3 and length *d* for each circuit. From the solution below,

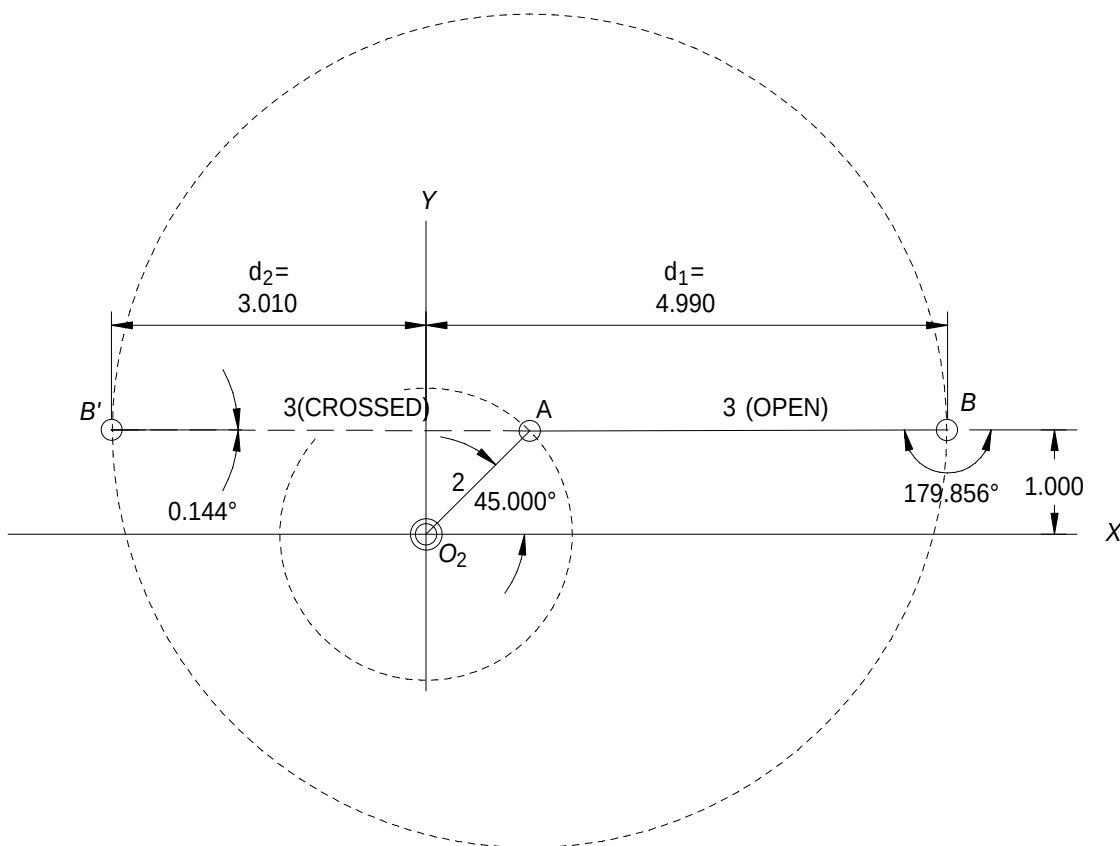
$$\theta_{31} := 360 \cdot \text{deg} - 179.856 \cdot \text{deg}$$

$$\theta_{31} = 180.144 \cdot \text{deg}$$

$$\theta_{32} := -0.144 \cdot \text{deg}$$

$$d_1 := 4.990 \cdot \text{in}$$

$$d_2 := -3.010 \cdot \text{in}$$

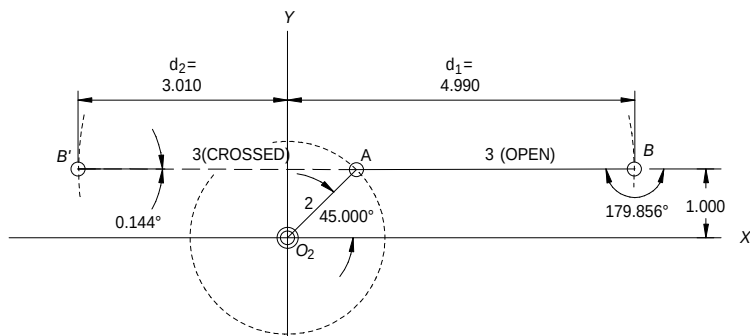


PROBLEM 4-10a

Statement: The link lengths, value of θ_2 , and offset for some fourbar slider-crank linkages are defined in Table P4-2. The linkage configuration and terminology are shown in Figure P4-2. For row *a*, using the vector loop method, find all possible solutions (both open and crossed) for angles θ_3 and slider position *d*.

Given: Link 2 $a := 1.4 \cdot \text{in}$ Link 3 $b := 4 \cdot \text{in}$
Offset $c := 1 \cdot \text{in}$ $\theta_2 := 45 \cdot \text{deg}$

Solution: See Figure P4-2 and Mathcad file P0410a.



- Determine θ_3 and *d* using equations 4.16 and 4.17.

Crossed:

$$\theta_{32} := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right) \quad \theta_{32} = -0.144 \text{ deg}$$

$$d_2 := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_{32}) \quad d_2 = -3.010 \text{ in}$$

Open:

$$\theta_{31} := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi \quad \theta_{31} = 180.144 \text{ deg}$$

$$d_1 := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_{31}) \quad d_1 = 4.990 \text{ in}$$

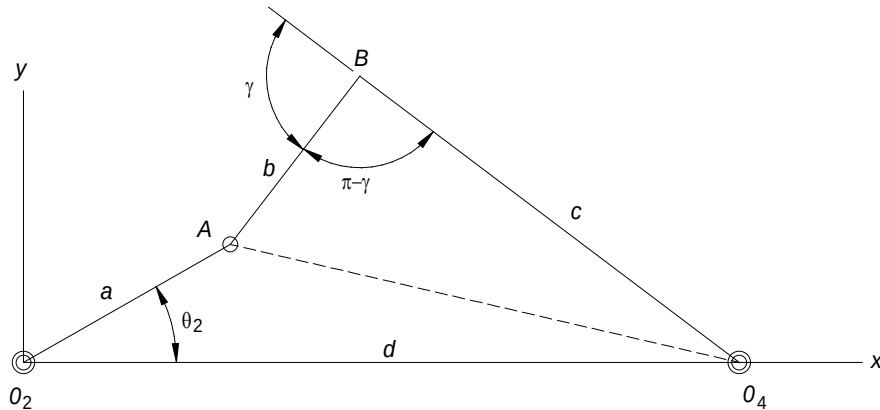
PROBLEM 4-11a

Statement: The link lengths and the value of θ_2 and γ for some inverted fourbar slider-crank linkages are defined in Table P4-3. The linkage configuration and terminology are shown in Figure P4-3. For row *a*, draw the linkage to scale and graphically find both open and closed solutions for θ_3 and θ_4 and vector $\mathbf{R}_B$.

Given: Link 1 $d := 6 \cdot \text{in}$ Link 2 $a := 2 \cdot \text{in}$
 Link 4 $c := 4 \cdot \text{in}$ $\gamma := 90 \cdot \text{deg}$ $\theta_2 := 30 \cdot \text{deg}$

Solution: See figure below for one possible solution. Also see Mathcad file P04011a.

1. Lay out an xy-axis system. Its origin will be the link 2 pivot, O_2 .
2. Draw link 2 to some convenient scale at its given angle.
- 3a. If $\gamma = 90 \text{ deg}$, locate O_4 on the x-axis at a distance equal the length of link 1 (d) from the origin. Draw a circle with center at O_4 and radius equal to the length of link 4 (c). From point A, draw two lines that are tangent to the circle. The points of tangency define the location of the points B for the open and crossed circuits.
- 3b. When γ is not 90 deg there are two approaches to a graphical solution for link 3 and the location of point B: 1) establish the position of link 4 and the angle γ by trial and error, or 2) calculate the distance from point A to point B (the instantaneous length of link 3). Using the second approach, from triangle O_2AO_4



$$AO_4^2 = a^2 + d^2 - 2 \cdot a \cdot d \cdot \cos(\theta_2)$$

and, from triangle AO_4B (for the open circuit)

$$AO_4^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos(\pi - \gamma)$$

where a , b , c , and d are the lengths of links 2, 3, 4, and 1, respectively. Eliminating AO_4 and solving for the unknown distance b for the open branch,

$$b_1 := \frac{1}{2} \cdot \left[2 \cdot c \cdot \cos(\pi - \gamma) + \sqrt{(2 \cdot c \cdot \cos(\pi - \gamma))^2 - 4 \cdot (c^2 - a^2 - d^2 + 2 \cdot a \cdot d \cdot \cos(\theta_2))} \right]$$

$$b_1 = 1.7932 \text{ in}$$

For the closed branch: $AO_4^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos(\gamma)$ and

$$b_2 := \frac{1}{2} \cdot \left[2 \cdot c \cdot \cos(\gamma) + \sqrt{(2 \cdot c \cdot \cos(\gamma))^2 - 4 \cdot (c^2 - a^2 - d^2 + 2 \cdot a \cdot d \cdot \cos(\theta_2))} \right]$$

$$b_2 = 1.7932 \text{ in}$$

Draw a circle with center at point A and radius b_1 . Draw a circle with center at O_4 and radius equal to the length of link 4 (c). The intersections of these two circles is the solution for the open and crossed locations of the point B.

4. Draw the complete linkage for the open and crossed circuits, including the slider. The results from the graphical solution below are:

OPEN $\theta_{31} := -127.333 \cdot \text{deg}$

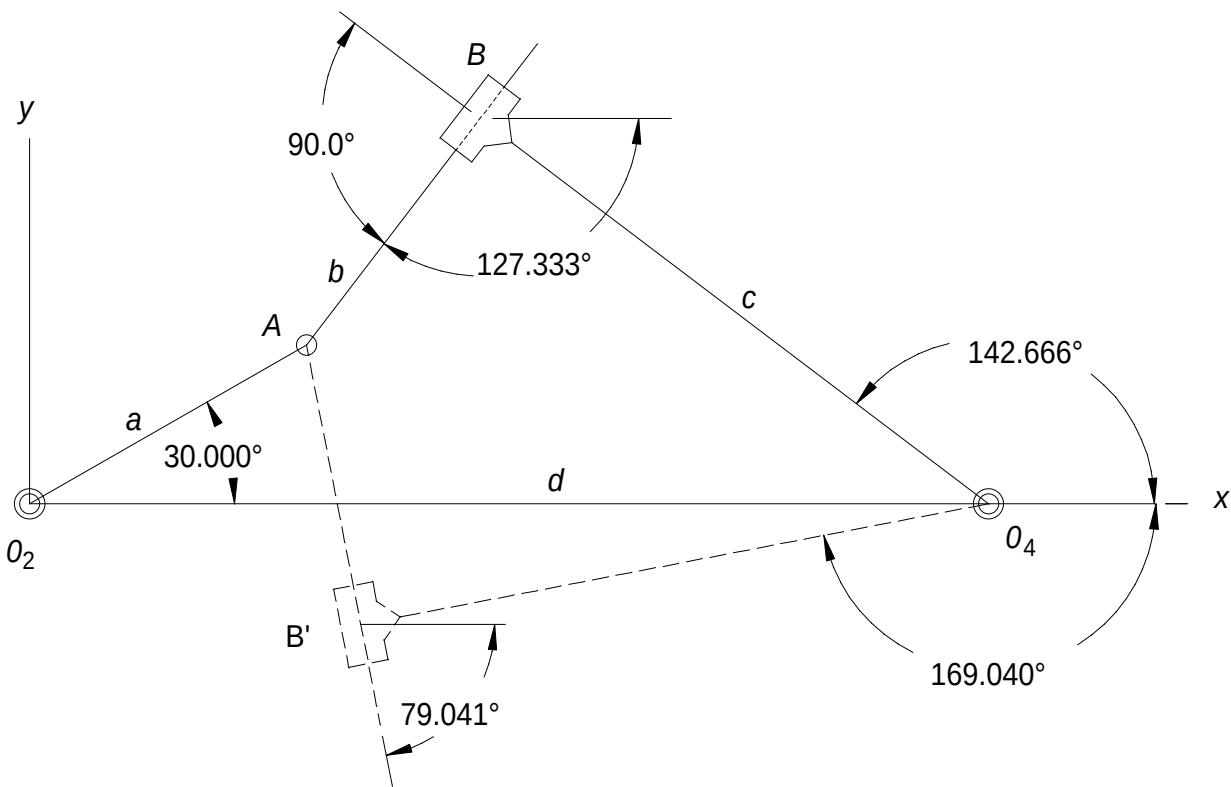
CROSSED $\theta_{32} := 100.959 \cdot \text{deg}$

$\theta_{41} := 142.666 \cdot \text{deg}$

$\theta_{42} := -169.040 \cdot \text{deg}$

$R_{B1} := 3.719$ at 40.708 deg

$R_{B2} := 2.208$ at -20.146 deg



PROBLEM 4-12a

Statement: The link lengths and the value of θ_2 and γ for some inverted fourbar slider-crank linkages are defined in Table P4-3. The linkage configuration and terminology are shown in Figure P4-3. For row *a*, using the vector loop method, find both open and closed solutions for θ_3 and θ_4 and vector $\mathbf{R}_B$.

Given: Link 1 $d := 6 \cdot \text{in}$ Link 2 $a := 2 \cdot \text{in}$
 Link 4 $c := 4 \cdot \text{in}$ $\gamma := 90 \cdot \text{deg}$ $\theta_2 := 30 \cdot \text{deg}$

Solution: See Mathcad file P0412a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.25 and 4.26.

$$\begin{aligned} P &:= a \cdot \sin(\theta_2) \cdot \sin(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \cos(\gamma) & P &= 1.000 \text{ in} \\ Q &:= -a \cdot \sin(\theta_2) \cdot \cos(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \sin(\gamma) & Q &= -4.268 \text{ in} \\ R &:= -c \cdot \sin(\gamma) & R &= -4.000 \text{ in} & T &:= 2 \cdot P & T &= 2.000 \text{ in} \\ S &:= R - Q & S &= 0.268 \text{ in} & U &:= Q + R & U &= -8.268 \text{ in} \end{aligned}$$

2. Use equation 4.26c to find values of θ_4 for the open and crossed circuits.

$$\begin{aligned} \text{OPEN} \quad \theta_{41} &:= 2 \cdot \text{atan2}\left(2 \cdot S, -T + \sqrt{T^2 - 4 \cdot S \cdot U}\right) & \theta_{41} &= 142.667 \text{ deg} \\ \text{CROSSED} \quad \theta_{42} &:= 2 \cdot \text{atan2}\left(2 \cdot S, -T - \sqrt{T^2 - 4 \cdot S \cdot U}\right) & \theta_{42} &= -169.041 \text{ deg} \end{aligned}$$

3. Use equation 4.22 to find values of θ_3 for the open and crossed circuits.

$$\begin{aligned} \text{OPEN} \quad \theta_{31} &:= \theta_{41} + \gamma & \theta_{31} &= 232.667 \text{ deg} \\ \text{CROSSED} \quad \theta_{32} &:= \theta_{42} + \gamma & \theta_{32} &= -79.041 \text{ deg} \end{aligned}$$

4. Determine the magnitude of the instantaneous "length" of link 3 from equation 4.24a.

$$\begin{aligned} \text{OPEN} \quad b_1 &:= \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_{41})}{\sin(\theta_{41} + \gamma)} & b_1 &= 1.793 \text{ in} \\ \text{CROSSED} \quad b_2 &:= \left| \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_{42})}{\sin(\theta_{42} + \gamma)} \right| & b_2 &= 1.793 \text{ in} \end{aligned}$$

5. Find the position vector $\mathbf{R}_B$ from the definition given in the text.

$$\begin{aligned} \text{OPEN} \quad \mathbf{R}_{B1} &:= a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) - b_1 \cdot (\cos(\theta_{31}) + j \cdot \sin(\theta_{31})) \\ R_{B1} &:= |\mathbf{R}_{B1}| & R_{B1} &= 3.719 \text{ in} \\ \theta_{B1} &:= \arg(\mathbf{R}_{B1}) & \theta_{B1} &= 40.707 \text{ deg} \end{aligned}$$

$$\text{CROSSED} \quad \mathbf{R}_{B2} := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) - b_2 \cdot (\cos(\theta_{32}) + j \cdot \sin(\theta_{32}))$$

$$R_{B2} := |\mathbf{R}_{B2}| \quad R_{B2} = 3.091 \text{ in}$$

$$\theta_{B2} := \arg(\mathbf{R}_{B2}) \quad \theta_{B2} = 63.254 \text{ deg}$$

PROBLEM 4-12c

Statement: The link lengths and the value of θ_2 and γ for some inverted fourbar slider-crank linkages are defined in Table P4-3. The linkage configuration and terminology are shown in Figure P4-3. For row c, using the vector loop method, find both open and closed solutions for θ_3 and θ_4 and vector $\mathbf{R}_B$.

Given: Link 1 $d := 3 \cdot \text{in}$ Link 2 $a := 10 \cdot \text{in}$
 Link 4 $c := 6 \cdot \text{in}$ $\gamma := 45 \cdot \text{deg}$ $\theta_2 := 45 \cdot \text{deg}$
 Two argument inverse tangent

$$\text{atan2}(x, y) := \begin{cases} \text{return } 0.5 \cdot \pi & \text{if } x = 0 \wedge y > 0 \\ \text{return } 1.5 \cdot \pi & \text{if } x = 0 \wedge y < 0 \\ \text{return } \text{atan}\left(\left(\frac{y}{x}\right)\right) & \text{if } x > 0 \\ \text{atan}\left(\left(\frac{y}{x}\right)\right) + \pi & \text{otherwise} \end{cases}$$

Solution: See Mathcad file P0412c.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$P := a \cdot \sin(\theta_2) \cdot \sin(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \cos(\gamma) \quad P = 7.879 \text{ in}$$

$$Q := -a \cdot \sin(\theta_2) \cdot \cos(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \sin(\gamma) \quad Q = -2.121 \text{ in}$$

$$R := -c \cdot \sin(\gamma) \quad R = -4.243 \text{ in} \quad T := 2 \cdot P \quad T = 15.757 \text{ in}$$

$$S := R - Q \quad S = -2.121 \text{ in} \quad U := Q + R \quad U = -6.364 \text{ in}$$

2. Use equation 4.22c to find values of θ_4 for the open and crossed circuits.

$$\text{OPEN} \quad \theta_{41} := 2 \cdot \text{atan2}\left(2 \cdot S, -T + \sqrt{T^2 - 4 \cdot S \cdot U}\right) - 360 \cdot \text{deg} \quad \theta_{41} = 46.400 \text{ deg}$$

$$\text{CROSSED} \quad \theta_{42} := 2 \cdot \text{atan2}\left(2 \cdot S, -T - \sqrt{T^2 - 4 \cdot S \cdot U}\right) - 360 \cdot \text{deg} \quad \theta_{42} = 163.739 \text{ deg}$$

3. Use equation 4.18 to find values of θ_3 for the open and crossed circuits.

$$\text{OPEN} \quad \theta_{31} := \theta_{41} + \gamma \quad \theta_{31} = 91.400 \text{ deg}$$

$$\text{CROSSED} \quad \theta_{32} := \theta_{42} - \gamma \quad \theta_{32} = 118.739 \text{ deg}$$

4. Determine the magnitude of the instantaneous "length" of link 3 from equation 4.20a.

$$\text{OPEN} \quad b_1 := \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_{41})}{\sin(\theta_{41} + \gamma)} \quad b_1 = 2.727 \text{ in}$$

$$\text{CROSSED} \quad b_2 := \left| \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_{42})}{\sin(\theta_{42} + \gamma)} \right| \quad b_2 = 11.212 \text{ in}$$

5. Find the position vector $\mathbf{R}_B$ from the definition given on page 162 of the text.

$$\text{OPEN} \quad \mathbf{R}_{B1} := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) - b_I \cdot (\cos(\theta_{31}) + j \cdot \sin(\theta_{31}))$$

$$R_{B1} := |\mathbf{R}_{B1}| \quad R_{B1} = 8.356 \text{ in}$$

$$\theta_{B1} := \arg(\mathbf{R}_{B1}) \quad \theta_{B1} = 31.331 \text{ deg}$$

$$\text{CROSSED} \quad \mathbf{R}_{B2} := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) - b_Z \cdot (\cos(\theta_{32}) + j \cdot \sin(\theta_{32}))$$

$$R_{B2} := |\mathbf{R}_{B2}| \quad R_{B2} = 12.764 \text{ in}$$

$$\theta_{B2} := \arg(\mathbf{R}_{B2}) \quad \theta_{B2} = -12.488 \text{ deg}$$

PROBLEM 4-13a

Statement: Find the transmission angles of the linkage in row *a* of Table P4-1.

Given: Link 1 $d := 6 \cdot \text{in}$ Link 2 $a := 2 \cdot \text{in}$
 Link 3 $b := 7 \cdot \text{in}$ Link 4 $c := 9 \cdot \text{in}$ $\theta_2 := 30 \cdot \text{deg}$

Solution: See Mathcad file P0413a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 3.0000 \quad K_2 = 0.6667 \quad K_3 = 2.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad A = -0.7113$$

$$B := -2 \cdot \sin(\theta_2) \quad B = -1.0000$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3 \quad C = 3.5566$$

2. Use equation 4.10b to find θ_4 for the open circuit.

$$\theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_{41} = -242.714 \text{ deg}$$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.8571$$

$$K_5 = -0.2857$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -1.6774$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 2.5906$$

4. Use equation 4.13 to find θ_3 for the open circuit.

$$\theta_{31} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{31} = -271.163 \text{ deg}$$

$$\theta_{31} := \theta_{31} - 360 \cdot \text{deg} \quad \theta_{31} = -631.163 \text{ deg}$$

5. Use equations 4.32 to find the transmission angle.

$$\theta_{\text{trans}}(\theta_3, \theta_4) := \begin{cases} t \leftarrow |\theta_3 - \theta_4| \\ \text{return } t \text{ if } t \leq 0.5 \cdot \pi \\ \pi - t \text{ otherwise} \end{cases} \quad \theta_{\text{trans}}(\theta_{31}, \theta_{41}) = -208.449 \text{ deg}$$

6. It can be shown that the triangle ABO_4 in Figure 4-17 is symmetric with respect to the line AO_4 for the crossed branch and, therefore, the transmission angle for the crossed branch is identical to that for the open branch.

PROBLEM 4-14

Statement: Find the minimum and maximum values of the transmission angle for all the Grashof crank-rocker linkages in Table P4-1.

Given: Table P4-1 data: $i := 1, 2 \dots 14$

$Row_i :=$	$d_i :=$	$a_i :=$	$b_i :=$	$c_i :=$
"a"	6	2	7	9
"b"	7	9	3	8
"c"	3	10	6	8
"d"	8	5	7	6
"e"	8	5	8	6
"f"	5	8	8	9
"g"	6	8	8	9
"h"	20	10	10	10
"i"	4	5	2	5
"j"	20	10	5	10
"k"	4	6	10	7
"l"	9	7	10	7
"m"	9	7	11	8
"n"	9	7	11	6

Solution: See Table P4-1 and Mathcad file P0414.

1. Determine which of the linkages in Table P4-1 are Grashof.

```

Condition(a,b,c,d) :=
    S ← min(a,b,c,d)
    L ← max(a,b,c,d)
    SL ← S + L
    PQ ← a + b + c + d - SL
    return "Grashof" if SL < PQ
    return "Special Grashof" if SL = PQ
    return "non-Grashof" otherwise

```

Row a $Condition(6, 2, 7, 9) = \text{"Grashof"}$

Row b $Condition(7, 9, 3, 8) = \text{"Grashof"}$

Row c $Condition(3, 10, 6, 8) = \text{"Grashof"}$

Row d $Condition(8, 5, 7, 6) = \text{"Special Grashof"}$

Row e $Condition(8, 5, 8, 6) = \text{"Grashof"}$

Row f $Condition(5, 8, 8, 9) = \text{"Grashof"}$

Row g $Condition(6, 8, 8, 9) = \text{"Grashof"}$

Row h $Condition(20, 10, 10, 10) = \text{"non-Grashof"}$

Row i $Condition(4, 5, 2, 5) = \text{"Grashof"}$

Row j $Condition(20, 10, 5, 10) = \text{"non-Grashof"}$

Row k $Condition(4, 6, 10, 7) = \text{"non-Grashof"}$

Row l $Condition(9, 7, 10, 7) = \text{"non-Grashof"}$

Row *m* *Condition*(9, 7, 11, 8) = "non-Grashof"

Row *n* *Condition*(9, 7, 11, 6) = "non-Grashof"

- Determine which of the Grashof linkages are crank-rockers. To be a Grashof crank-rocker, the linkage must be Grashof and the shortest link is either 2 or 4. This is true of rows *a*, *d*, and *e*.
- Use equations 4.32 and 4.33 to calculate the maximum and minimum transmission angles.

Row <i>a</i>	$i := 1$	$\mu_1 := \text{acos} \left[\frac{(b_i)^2 + (c_i)^2 - (d_i + a_i)^2}{2 \cdot b_i \cdot c_i} \right]$	
		$\mu_1 := \text{if} \left(\mu_1 > \frac{\pi}{2}, \pi - \mu_1, \mu_1 \right)$	$\mu_1 = 58.412 \text{ deg}$
		$\mu_2 := \text{acos} \left[\frac{(b_i)^2 + (c_i)^2 - (d_i - a_i)^2}{2 \cdot b_i \cdot c_i} \right]$	$\mu_2 = 25.209 \text{ deg}$
Row <i>d</i>	$i := 4$	$\mu_1 := \text{acos} \left[\frac{(b_i)^2 + (c_i)^2 - (d_i + a_i)^2}{2 \cdot b_i \cdot c_i} \right]$	
		$\mu_1 := \text{if} \left(\mu_1 > \frac{\pi}{2}, \pi - \mu_1, \mu_1 \right)$	$\mu_1 = 0.000 \text{ deg}$
		$\mu_2 := \text{acos} \left[\frac{(b_i)^2 + (c_i)^2 - (d_i - a_i)^2}{2 \cdot b_i \cdot c_i} \right]$	$\mu_2 = 25.209 \text{ deg}$
Row <i>e</i>	$i := 5$	$\mu_1 := \text{acos} \left[\frac{(b_i)^2 + (c_i)^2 - (d_i + a_i)^2}{2 \cdot b_i \cdot c_i} \right]$	
		$\mu_1 := \text{if} \left(\mu_1 > \frac{\pi}{2}, \pi - \mu_1, \mu_1 \right)$	$\mu_1 = 44.049 \text{ deg}$
		$\mu_2 := \text{acos} \left[\frac{(b_i)^2 + (c_i)^2 - (d_i - a_i)^2}{2 \cdot b_i \cdot c_i} \right]$	$\mu_2 = 18.573 \text{ deg}$

PROBLEM 4-15

Statement: Find the input angles corresponding to the toggle positions of the non-Grashof linkages in Table P4-1.

Given: Table P4-1 data: $i := 1, 2 \dots 14$

$Row_i :=$	$d_i :=$	$a_i :=$	$b_i :=$	$c_i :=$
"a"	6	2	7	9
"b"	7	9	3	8
"c"	3	10	6	8
"d"	8	5	7	6
"e"	8	5	8	6
"f"	5	8	8	9
"g"	6	8	8	9
"h"	20	10	10	10
"i"	4	5	2	5
"j"	20	10	5	10
"k"	4	6	10	7
"l"	9	7	10	7
"m"	9	7	11	8
"n"	9	7	11	6

Solution: See Table P4-1 and Mathcad file P0415.

1. Determine which of the linkages in Table P4-1 are Grashof.

```

Condition(a,b,c,d) :=
    S ← min(a,b,c,d)
    L ← max(a,b,c,d)
    SL ← S + L
    PQ ← a + b + c + d - SL
    return "Grashof" if SL < PQ
    return "Special Grashof" if SL = PQ
    return "non-Grashof" otherwise

```

Row a	Condition(6,2,7,9) = "Grashof"
Row b	Condition(7,9,3,8) = "Grashof"
Row c	Condition(3,10,6,8) = "Grashof"
Row d	Condition(8,5,7,6) = "Special Grashof"
Row e	Condition(8,5,8,6) = "Grashof"
Row f	Condition(5,8,8,9) = "Grashof"
Row g	Condition(6,8,8,9) = "Grashof"
Row h	Condition(20,10,10,10) = "non-Grashof"
Row i	Condition(4,5,2,5) = "Grashof"
Row j	Condition(20,10,5,10) = "non-Grashof"
Row k	Condition(4,6,10,7) = "non-Grashof"
Row l	Condition(9,7,10,7) = "non-Grashof"

Row m $Condition(9,7,11,8) = \text{"non-Grashof"}$

Row n $Condition(9,7,11,6) = \text{"non-Grashof"}$

2. There are six non-Grashof rows in the Table: Rows h , and j through n . For each row there are two possible arguments to the arccos function given in equation (4.37). They are:

$i := 8$ $Row_i = \text{"h"}$ $j := 1$

$$arg_{j,1} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} + \frac{b_i \cdot c_i}{a_i \cdot d_i}$$

$$arg_{j,2} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} - \frac{b_i \cdot c_i}{a_i \cdot d_i}$$

$i := 10$ $Row_i = \text{"j"}$ $j := 2$

$$arg_{j,1} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} + \frac{b_i \cdot c_i}{a_i \cdot d_i}$$

$$arg_{j,2} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} - \frac{b_i \cdot c_i}{a_i \cdot d_i}$$

$i := 11$ $Row_i = \text{"k"}$ $j := 3$

$$arg_{j,1} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} + \frac{b_i \cdot c_i}{a_i \cdot d_i}$$

$$arg_{j,2} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} - \frac{b_i \cdot c_i}{a_i \cdot d_i}$$

$i := 12$ $Row_i = \text{"l"}$ $j := 4$

$$arg_{j,1} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} + \frac{b_i \cdot c_i}{a_i \cdot d_i}$$

$$arg_{j,2} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} - \frac{b_i \cdot c_i}{a_i \cdot d_i}$$

$i := 13$ $Row_i = \text{"m"}$ $j := 5$

$$arg_{j,1} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} + \frac{b_i \cdot c_i}{a_i \cdot d_i}$$

$$arg_{j,2} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} - \frac{b_i \cdot c_i}{a_i \cdot d_i}$$

$$i := 14 \quad \text{Row}_i = "n" \quad j := 6$$

$$arg_{j,1} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} + \frac{b_i \cdot c_i}{a_i \cdot d_i}$$

$$arg_{j,2} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} - \frac{b_i \cdot c_i}{a_i \cdot d_i}$$

$$arg = \begin{pmatrix} 1.250 & 0.250 \\ 1.188 & 0.688 \\ 0.896 & -4.938 \\ 0.960 & -1.262 \\ 0.960 & -1.833 \\ 0.833 & -1.262 \end{pmatrix}$$

3. Choose the argument values that lie between plus and minus 1,

$$\theta_{2h} := \text{acos}(arg_{1,2}) \quad \theta_{2h} = 75.5 \text{ deg}$$

$$\theta_{2j} := \text{acos}(arg_{2,2}) \quad \theta_{2j} = 46.6 \text{ deg}$$

$$\theta_{2k} := \text{acos}(arg_{3,1}) \quad \theta_{2k} = 26.4 \text{ deg}$$

$$\theta_{2l} := \text{acos}(arg_{4,1}) \quad \theta_{2l} = 16.2 \text{ deg}$$

$$\theta_{2m} := \text{acos}(arg_{5,1}) \quad \theta_{2m} = 16.2 \text{ deg}$$

$$\theta_{2n} := \text{acos}(arg_{6,1}) \quad \theta_{2n} = 33.6 \text{ deg}$$

PROBLEM 4-16a

Statement: The link lengths, gear ratio, phase angle, and the value of θ_2 for some geared fivebar linkages are defined in Table P4-4. The linkage configuration and terminology are shown in Figure P4-4. For row a, draw the linkage to scale and graphically find all possible solutions for angles θ_3 and θ_4 .

Given:

Link 1	$d := 4 \cdot in$	Link 2	$a := 1 \cdot in$		
Link 3	$b := 7 \cdot in$	Link 4	$c := 9 \cdot in$	Link 5	$f := 6 \cdot in$
Gear ratio	$\lambda := 2.0$	Phase angle	$\phi := 30 \cdot deg$	Input angle	$\theta_2 := 60 \cdot deg$

Solution: See Mathcad file P0201.

1. Determine whether or not an idler is required.

$$idler := \begin{cases} \text{"required"} & \text{if } \lambda > 0 \\ \text{"not-required"} & \text{otherwise} \end{cases}$$

$$idler = \text{"required"}$$

2. Choose radii for gears 2 and 5 by making a design choice for their center distance (which must be increased if an idler is required). Let the standard center distance when no idler is required be $C := 0.5 \cdot c$ then

$$C = r_2 + r_5 \quad \text{and} \quad |\lambda| = \frac{r_2}{r_5}$$

Solving for r_2 and r_5 ,

$$r_5 := \frac{C}{|\lambda| + 1} \quad r_5 = 1.500 \, in$$

$$r_2 := r_5 \cdot |\lambda| \quad r_2 = 3.000 \, in$$

If an idler is required, increase the center distance.

$$C := \text{if}(idler = \text{"required"}, C + r_5, C) \quad C = 6.000 \, in$$

Note that the amount by which C is increased if an idler is required is a design choice that is made based on the size of the gears and the space available.

3. Using equation 4.27c, determine the angular position of link 5 corresponding to the position of link 2.

$$\theta_5 := \lambda \cdot \theta_2 + \phi \quad \theta_5 = 150 \, deg$$

4. Lay out an xy-axis system. Its origin will be the link 2 pivot, O_2 .
5. Draw link 2 to some convenient scale at its given angle.
6. Draw a circle with center at the free end of link 2 and a radius equal to the given length of link 3.
7. Locate pivot O_4 on the x-axis at a distance from the origin equal to the given length of link 1.
8. Draw link 5 to some convenient scale at its calculated angle.
9. Draw a circle with center at the free end of link 5 and a radius equal to the given length of link 4.
10. The two intersections of the circles (if any) are the two solutions to the position analysis problem, crossed and open. If the circles don't intersect, there is no solution.
11. Draw links 3 and 4 in their two possible positions (shown as solid for open and dashed for crossed in the figure) and measure their angles θ_3 and θ_4 with respect to the x-axis. From the solution below,

OPEN $\theta_{31} := 173.64 \cdot \text{deg}$

$\theta_{41} := 360 \cdot \text{deg} - 177.715 \cdot \text{deg}$

$\theta_{41} = 182.285 \text{ deg}$

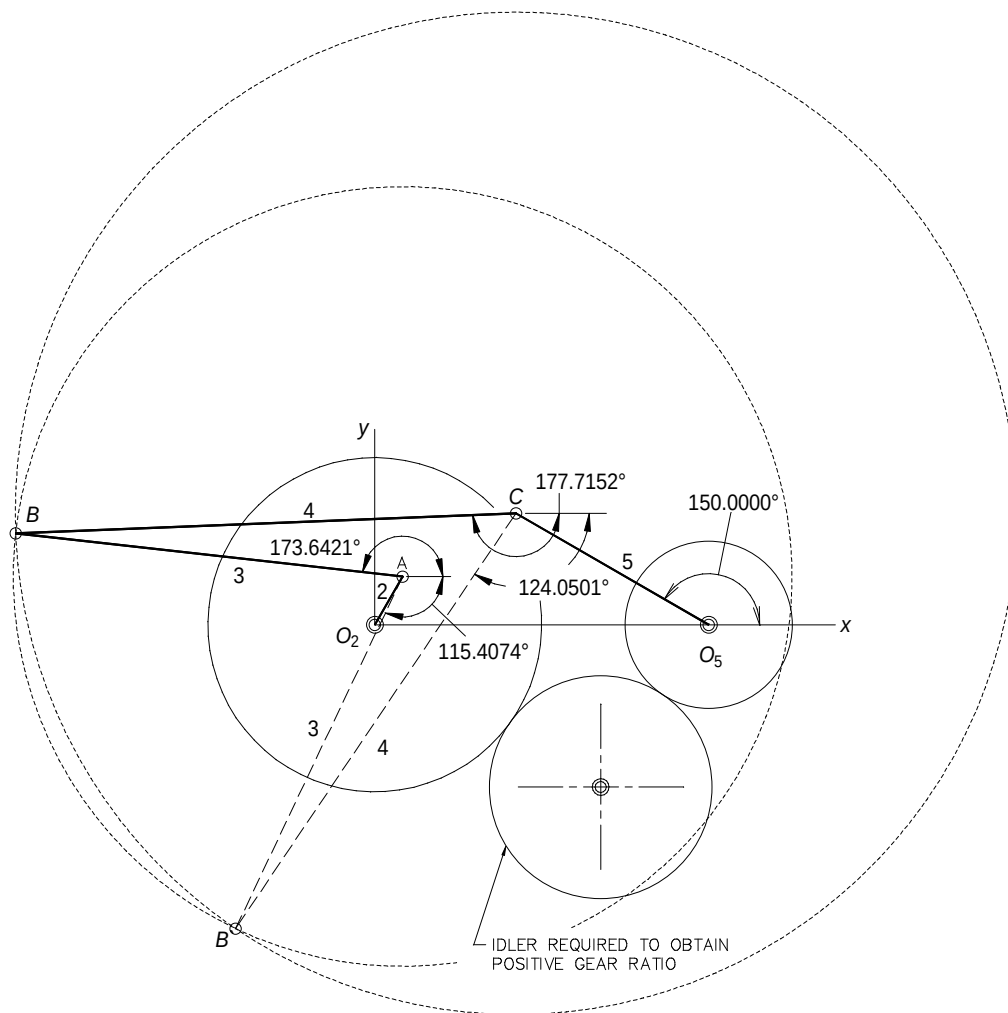
CROSSED $\theta_{32} := 360 \cdot \text{deg} - 115.407 \cdot \text{deg}$

$\theta_{41} := 360 \cdot \text{deg} - 124.050 \cdot \text{deg}$

$\theta_{32} = 244.593 \text{ deg}$

$\theta_{41} = 235.950 \text{ deg}$

12. Draw gears 2 and 5 schematically at their calculated radii. If an idler is required, draw it tangent to gears 2 and 5. Its diameter is a design choice that will be made on strength and space requirements. It does not affect the gear ratio.



PROBLEM 4-17a

Statement: The link lengths, gear ratio, phase angle, and the value of θ_2 for some geared fivebar linkages are defined in Table P4-4. The linkage configuration and terminology are shown in Figure P4-4. For row *a*, using the vector loop method, find all possible solutions for angles θ_3 and θ_4 .

Given:

Link 1	$d := 4 \cdot \text{in}$	Link 2	$a := 1 \cdot \text{in}$		
Link 3	$b := 7 \cdot \text{in}$	Link 4	$c := 9 \cdot \text{in}$	Link 5	$f := 6 \cdot \text{in}$
Gear ratio	$\lambda := 2.0$	Phase angle	$\phi := 30 \cdot \text{deg}$	Input angle	$\theta_2 := 60 \cdot \text{deg}$

Solution: See Mathcad file P0417a.

1. Determine the values of the constants needed for finding θ_3 and θ_4 from equations 4.27h and 4.27i.

$$A := 2 \cdot c \cdot (d \cdot \cos(\lambda \cdot \theta_2 + \phi) - a \cdot \cos(\theta_2) + f) \quad A = 36.6462 \text{ in}^2$$

$$B := 2 \cdot c \cdot (d \cdot \sin(\lambda \cdot \theta_2 + \phi) - a \cdot \sin(\theta_2)) \quad B = 20.412 \text{ in}^2$$

$$C := (a^2 - b^2 + c^2 + d^2 + f^2) - 2 \cdot a \cdot f \cdot \cos(\theta_2) \dots$$

$$+ [2 \cdot d \cdot (a \cdot \cos(\theta_2) - f) \cdot \cos(\lambda \cdot \theta_2 + \phi)] \dots$$

$$+ -2 \cdot a \cdot d \cdot \sin(\theta_2) \cdot \sin(\lambda \cdot \theta_2 + \phi) \quad C = 37.4308 \text{ in}^2$$

$$D := C - A \quad D = 0.78461 \text{ in}^2$$

$$E := 2 \cdot B \quad E = 40.823 \text{ in}^2$$

$$F := A + C \quad F = 74.077 \text{ in}^2$$

$$G := 2 \cdot b \cdot [- (d \cdot \cos(\lambda \cdot \theta_2 + \phi)) + a \cdot \cos(\theta_2) - f] \quad G = -28.503 \text{ in}^2$$

$$H := 2 \cdot b \cdot [- (d \cdot \sin(\lambda \cdot \theta_2 + \phi)) + a \cdot \sin(\theta_2)] \quad H = -15.876 \text{ in}^2$$

$$K := (a^2 + b^2 - c^2 + d^2 + f^2) - 2 \cdot a \cdot f \cdot \cos(\theta_2) \dots$$

$$+ [2 \cdot d \cdot (a \cdot \cos(\theta_2) - f) \cdot \cos(\lambda \cdot \theta_2 + \phi)] \dots$$

$$+ -2 \cdot a \cdot d \cdot \sin(\theta_2) \cdot \sin(\lambda \cdot \theta_2 + \phi) \quad K = -26.569 \text{ in}^2$$

$$L := K - G \quad L = 1.933 \text{ in}^2$$

$$M := 2 \cdot H \quad M = -31.751 \text{ in}^2$$

$$N := G + K \quad N = -55.072 \text{ in}^2$$

2. Use equations 4.28h and 4.28i to find values of θ_3 and θ_4 for the open and crossed circuits.

$$\text{OPEN} \quad \theta_{31} := 2 \cdot \left(\text{atan2} \left(2 \cdot L, -M + \sqrt{M^2 - 4 \cdot L \cdot N} \right) \right) \quad \theta_{31} = 173.642 \text{ deg}$$

$$\theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{41} = -177.715 \text{ deg}$$

$$\text{CROSSED} \quad \theta_{32} := 2 \cdot \left(\text{atan2} \left(2 \cdot L, -M - \sqrt{M^2 - 4 \cdot L \cdot N} \right) \right) \quad \theta_{32} = -115.407 \text{ deg}$$

$$\theta_{42} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{42} = -124.050 \text{ deg}$$

PROBLEM 4-18a

Statement: The angle between the X and x axes is 25° . Find the angular displacement of link 4 when link 2 rotates clockwise from the position shown ($+37^\circ$) to horizontal (0°). How does the transmission angle vary and what is its minimum between those two positions? Find the toggle positions of this linkage in terms of the angle of link 2.

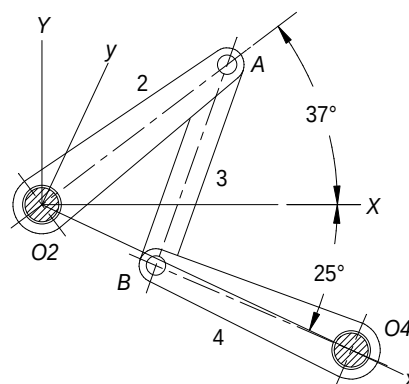
Given: Link lengths:

Crank $L_2 := 116$ Coupler $L_3 := 108$

Rocker $L_4 := 110$ Ground link $L_1 := 174$

Crank angle for position shown (relative to O_2O_4): $\theta_{21} := 62^\circ$

Crank rotation angle from position shown to horizontal: $\Delta\theta_2 := -37^\circ$



Solution: See Figure P4-5a and Mathcad file P0418a.

- Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{L_1}{L_2} \quad K_2 := \frac{L_1}{L_3} \quad K_3 := \frac{L_2^2 - L_3^2 + L_4^2 + L_1^2}{2 \cdot L_2 \cdot L_4}$$

$$K_1 = 1.5000 \quad K_2 = 1.6111 \quad K_3 = 1.7307$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

- Determine θ_4 for the position shown and after the crank has moved to the horizontal position.

$$\theta_{41} := \theta_4(\theta_{21}) \quad \theta_{41} = 183.5^\circ$$

$$\theta_{42} := \theta_4(\theta_{21} + \Delta\theta_2) \quad \theta_{42} = 212.8^\circ$$

- Subtract the two values of θ_4 to find the angular displacement of link 3 when link 2 rotates clockwise from the position shown to the horizontal.

$$\Delta\theta_4 := \theta_{42} - \theta_{41} \quad \Delta\theta_4 = 29.2^\circ$$

- Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{L_1}{L_3} \quad K_5 := \frac{L_4^2 - L_1^2 - L_2^2 - L_3^2}{2 \cdot L_2 \cdot L_3} \quad K_4 = 1.6111 \quad K_5 = -1.7280$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine θ_3 for the position shown and after the crank has moved to the horizontal position.

$$\theta_{31} := \theta_3(\theta_{21}) \quad \theta_{31} = 275.1 \text{ deg}$$

$$\theta_{32} := \theta_3(\theta_{21} + \Delta\theta_2) \quad \theta_{32} = 256.1 \text{ deg}$$

7. Use equations 4.28 to find the transmission angles.

$$\mu_1 := \pi - |\theta_{31} - \theta_{41}| \quad \mu_1 = 88.4 \text{ deg}$$

$$\mu_2 := |\theta_{32} - \theta_{42}| \quad \mu_2 = 43.4 \text{ deg}$$

The transmission angle is smaller when the crank is in the horizontal position.

8. Check the Grashof condition of the linkage.

$$\begin{aligned} \text{Condition}(a, b, c, d) &:= \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(L_1, L_2, L_3, L_4) = \text{"non-Grashof"}$$

9. Using equations 4.37, determine the crank angles (relative to the XY axes) at which links 3 and 4 are in toggle.

$$\arg_1 := \frac{L_2^2 + L_1^2 - L_3^2 - L_4^2}{2 \cdot L_2 \cdot L_1} + \frac{L_3 \cdot L_4}{L_2 \cdot L_1} \quad \arg_1 = 1.083$$

$$\arg_2 := \frac{L_2^2 + L_1^2 - L_3^2 - L_4^2}{2 \cdot L_2 \cdot L_1} - \frac{L_3 \cdot L_4}{L_2 \cdot L_1} \quad \arg_2 = -0.094$$

$$\theta_{2\text{toggle}} := \text{acos}(\arg_2) \quad \theta_{2\text{toggle}} = 95.4 \text{ deg}$$

The other toggle angle is the negative of this.

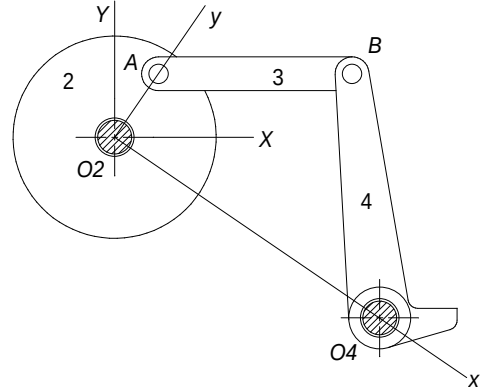
PROBLEM 4-18b

Statement: Find and plot the angular position of links 3 and 4 and the transmission angle as a function of the angle of link 2 as it rotates through one revolution.

Given: Link lengths:

Wheel (crank)	$L_2 := 40$	$a := L_2$	Coupler	$L_3 := 96$	$b := L_3$
Rocker	$L_4 := 122$	$c := L_4$	Ground link	$L_1 := 162$	$d := L_1$

Two argument inverse tangent

$$\text{atan2}(x, y) := \begin{cases} \text{return } 0.5 \cdot \pi & \text{if } x = 0 \wedge y > 0 \\ \text{return } 1.5 \cdot \pi & \text{if } x = 0 \wedge y < 0 \\ \text{return } \text{atan}\left(\left(\frac{y}{x}\right)\right) & \text{if } x > 0 \\ \text{return } \left(\text{atan}\left(\left(\frac{y}{x}\right)\right) + \pi\right) & \text{otherwise} \end{cases}$$


Solution: See Figure P4-5b and Mathcad file P0418b.

1. Check the Grashof condition of the linkage.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$\text{Condition}(a, b, c, d) = \text{"Grashof"}$

2. Define one cycle of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 4.0500 \qquad K_2 = 1.3279 \qquad K_3 = 3.4336$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2}\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)}\right) \right)$$

4. If the calculated value of θ_4 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{42}(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := \text{if}(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.6875 \quad K_5 = -2.8875$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

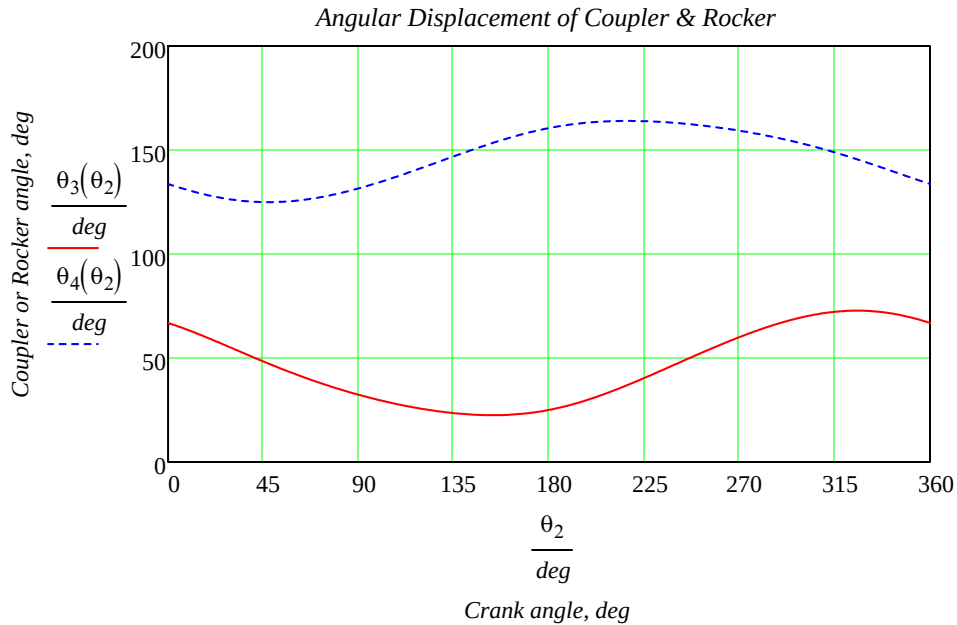
$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. If the calculated value of θ_3 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_2) := \text{if}(\theta_{31}(\theta_2) > 2 \cdot \pi, \theta_{31}(\theta_2) - 2 \cdot \pi, \theta_{31}(\theta_2))$$

$$\theta_3(\theta_2) := \text{if}(\theta_{32}(\theta_2) < 0, \theta_{32}(\theta_2) - 2 \cdot \pi, \theta_{32}(\theta_2))$$

8. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).

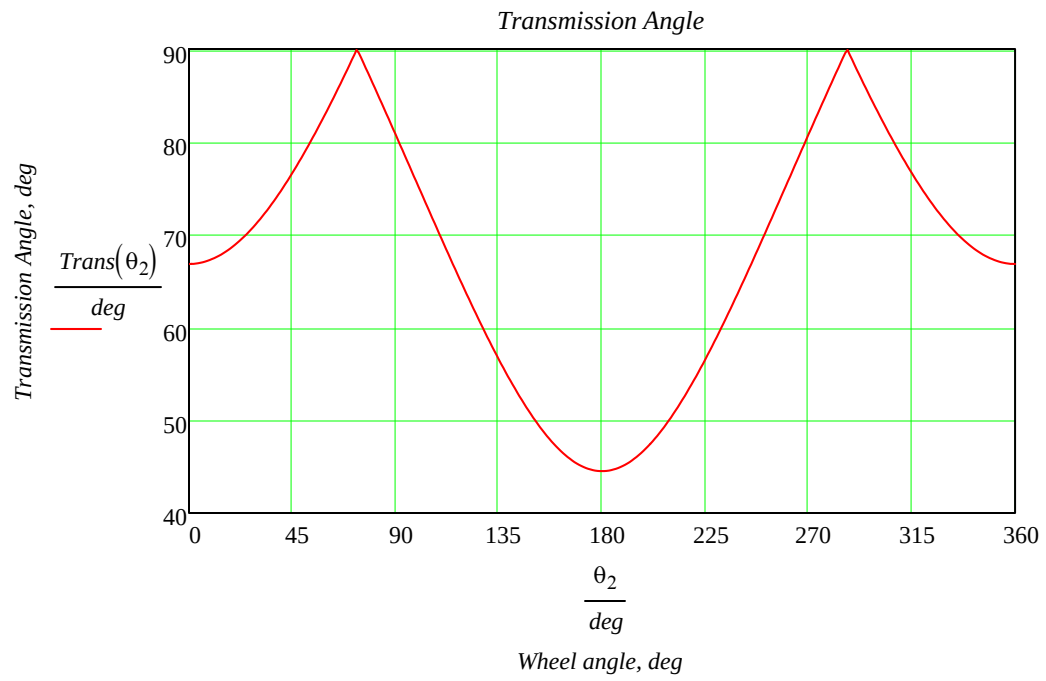


9. Use equations 4.32 to find the transmission angle.

$$\text{Tran}(\theta_2) := |\theta_3(\theta_2) - \theta_4(\theta_2)|$$

$$\text{Trans}(\theta_2) := \text{if} \left(\text{Tran}(\theta_2) > \frac{\pi}{2}, \left| \pi - \text{Tran}(\theta_2) \right|, \text{Tran}(\theta_2) \right)$$

10. Plot the transmission angle.

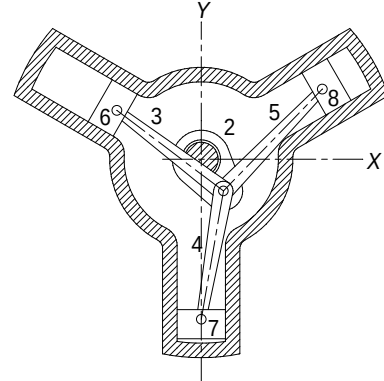


PROBLEM 4-18c

Statement: Find and plot the position of any one piston as a function of the angle of crank 2 as it rotates through one revolution. Once one piston's motion is defined, find the motions of the other two pistons and their phase relationship to the first piston.

Given:

Crank length $L_2 := 19$ $a := L_2$
 Piston-rod length $L_3 := 70$ $b := L_3$
 Offset $c := 0$



Solution: See Figure P4-5c and Mathcad file P0418c.

- Let pistons 1, 2, and 3 be links 7, 6, and 8, respectively.
- Solve first for piston 6. Establish θ_2 as a range variable: $\theta_{21} := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg}$
- Determine θ_3 and d using equations 4.16 and 4.17.

$$\theta_{31}(\theta_{21}) := \text{asin}\left(\frac{a \cdot \sin(\theta_{21}) - c}{b}\right) + \pi \quad d_1(\theta_{21}) := a \cdot \cos(\theta_{21}) - b \cdot \cos(\theta_{31}(\theta_{21}))$$

- For each piston (slider) the crank angle is measured counter-clock-wise from the centerline of the piston, which goes through the O_2 in all cases. Thus, when the crank angle for piston 1 is 0 deg, it is 120 deg for piston 2 and 240 deg for piston 3. Thus, the crank angles for pistons 2 and 3 are

$$\theta_{22}(\theta_{21}) := \theta_{21} + 120 \cdot \text{deg} \quad \theta_{23}(\theta_{21}) := \theta_{21} + 240 \cdot \text{deg}$$

- Determine θ_3 and d for pistons 2 and 3.

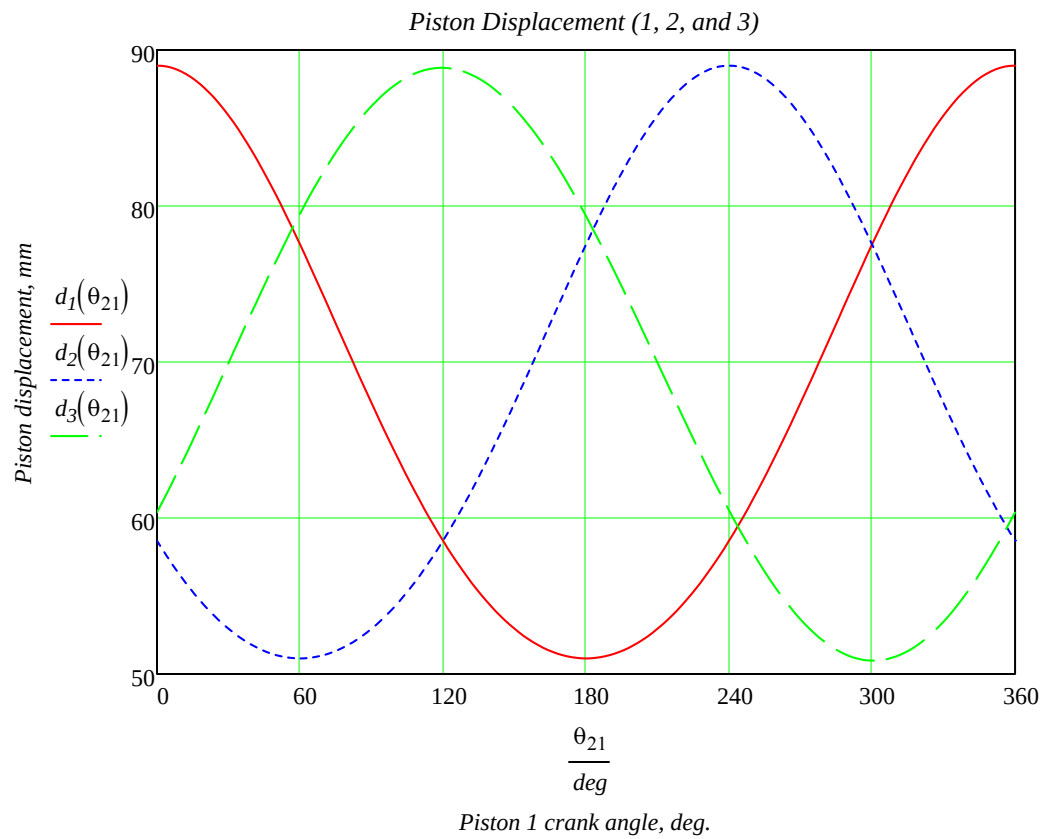
$$\theta_{32}(\theta_{21}) := \text{asin}\left(\frac{a \cdot \sin(\theta_{22}(\theta_{21})) - c}{b}\right) + \pi$$

$$d_2(\theta_{21}) := a \cdot \cos(\theta_{22}(\theta_{21})) - b \cdot \cos(\theta_{32}(\theta_{21}))$$

$$\theta_{33}(\theta_{21}) := \text{asin}\left(\frac{a \cdot \sin(\theta_{23}(\theta_{21})) - c}{b}\right) + \pi$$

$$d_3(\theta_{21}) := a \cdot \cos(\theta_{23}(\theta_{21})) - b \cdot \cos(\theta_{33}(\theta_{21}))$$

- Plot the piston displacements as a function of crank angle (referenced to line AC (see next page)).



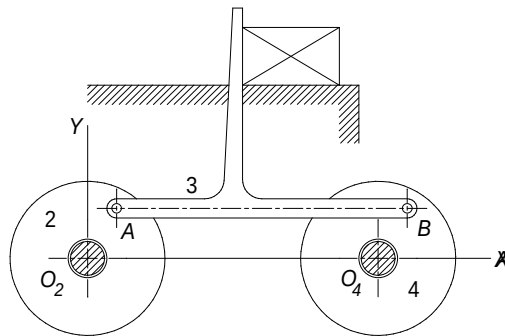
The solid line is piston 1, the dotted line is piston 2, and the dashed line is piston 3.

PROBLEM 4-18d

Statement: Find the total angular displacement of link 3 and the total stroke of the box as link 2 makes a complete revolution.

Given: Ground link $L_1 := 150$ Input crank $L_2 := 30$
Coupler link $L_3 := 150$ Output crank $L_4 := 30$

Solution: See Figure P4-5d and Mathcad file P0418d.



1. This is a special-case Grashof mechanism in the parallelogram form (see Figure 2-17 in the text). As such, the coupler link 3 executes curvilinear motion and is always parallel to the ground link 1. Thus, the total angular motion of link 3 as crank 2 makes one complete revolution is zero degrees.
2. The stroke of the box will be equal to twice the length of the crank link in one complete revolution of the crank

$$\text{stroke} := 2 \cdot L_2 \qquad \text{stroke} = 60$$

PROBLEM 4-18e

Statement: Determine the ratio of angular displacement between links 8 and 2 as a function of angular displacement of input crank 2. Plot the transmission angle at point B for one revolution of crank 2. Comment on the behavior of this linkage. Can it make a full revolution as shown?

Given: Link lengths:

Ground link (O_2O_4) $d_1 := 160$

Crank (O_2A) $a_1 := 20$

Coupler (L_3) $b_1 := 160$

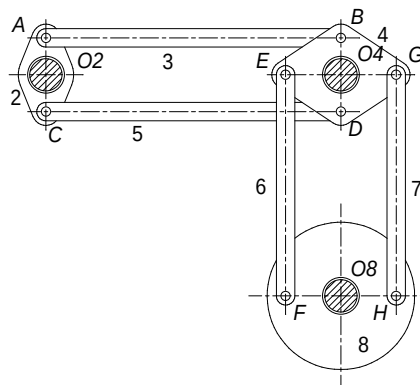
Crank (O_4B) $c_1 := 20$

Ground link (O_4O_8) $d_2 := 120$

Crank (O_4G) $a_2 := 30$

Coupler (L_6) $b_2 := 120$

Crank (O_8F) $c_2 := 30$



Solution: See Figure P4-5e and Mathcad file P0418e.

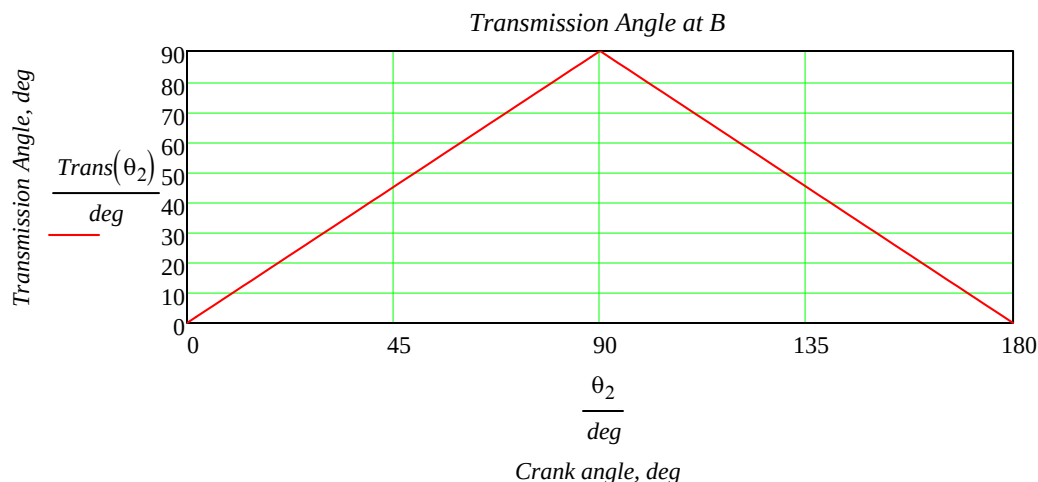
1. This is an eightbar, 1-DOF linkage with two redundant links (3 and 6 or 5 and 7) making it, effectively, a sixbar. It is composed of a fourbar (1, 2, 3, and 4) with an output dyad (7 and 8). The input fourbar is a special-case Grashof in the parallelogram configuration. Thus, the output angle is equal to the input angle and the couplers execute curvilinear motion with links 3 and 5 always parallel to the horizontal. The output dyad also behaves like a special-case Grashof with parallelogram configuration so that the angular motion of link 8 is equal to that of link 4. Therefore, the ratio of angular displacement between links 8 and 2 is unity. The mechanism is not capable of making a full revolution. The couplers 3 and 5 (also 6 and 7) cannot pass by each other near $\theta_2 = 0$ and 180 deg because of interference with the pins that connect them to their cranks.
2. Define the approximate range of motion of the input crank: $\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 180 \cdot \text{deg}$
3. Define θ_3 and θ_4 .

$$\theta_3 := 0 \cdot \text{deg} \quad \theta_4(\theta_2) := \theta_2$$

4. Use equations 4.32 to find and plot the transmission angle.

$$\text{tran}(\theta_2) := |\theta_3 - \theta_4(\theta_2)| \quad \text{Tran}(\theta_2) := \text{if}(\text{tran}(\theta_2) > \pi, \text{tran}(\theta_2) - \pi, \text{tran}(\theta_2))$$

$$\text{Trans}(\theta_2) := \text{if}\left(\text{Tran}(\theta_2) > \frac{\pi}{2}, \left|\pi - \text{Tran}(\theta_2)\right|, \text{Tran}(\theta_2)\right)$$



PROBLEM 4-18f

Statement: Find and plot the displacement of piston 4 and the angular displacement of link 3 as a function of the angular displacement of crank 2.

Given: Link lengths:
 Crank length, L_2 $a := 63$
 Piston-rod length, L_3 $b := 130$
 Offset $c := 52$

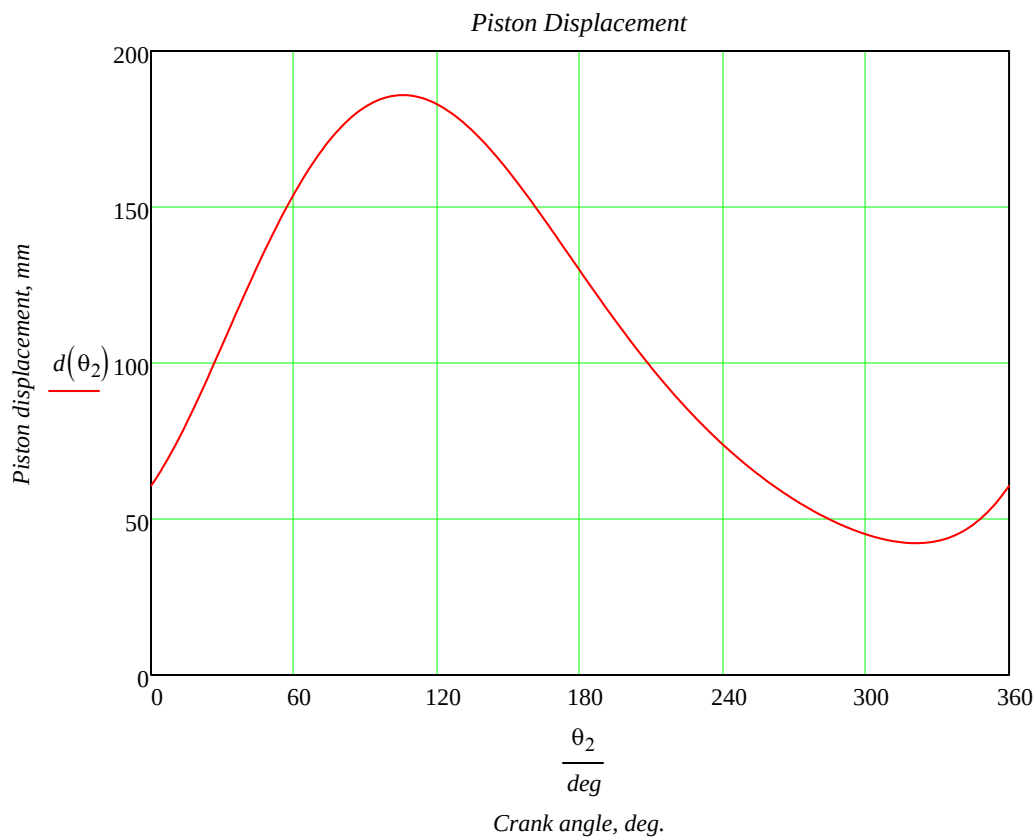
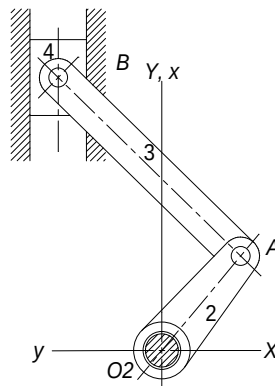
Solution: See Figure P4-5f and Mathcad file P0418f.

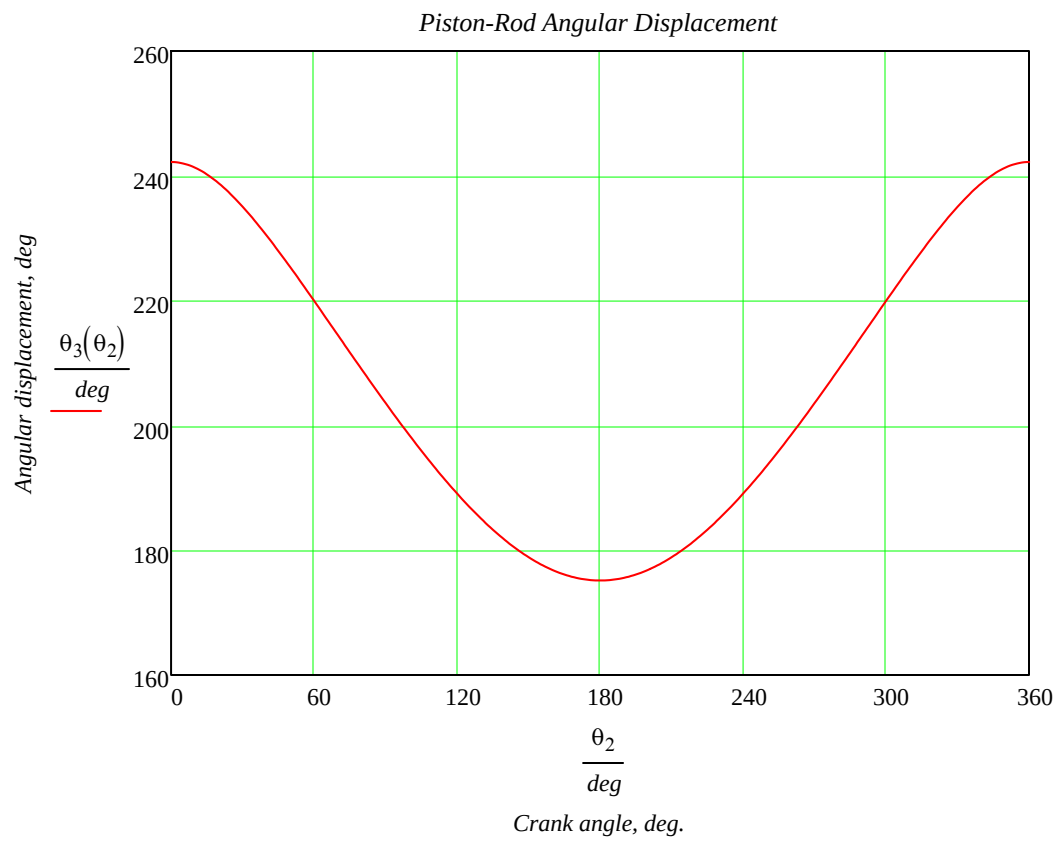
1. Establish θ_2 as a range variable: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
2. Determine θ_3 and d in global XY coord using equations 4.16 and 4.17.

$$\theta_3(\theta_2) := \text{asin}\left(-\frac{a \cdot \sin(\theta_2 - 90 \cdot \text{deg}) - c}{b}\right) + \pi$$

$$d(\theta_2) := a \cdot \cos(\theta_2 - 90 \cdot \text{deg}) - b \cdot \cos(\theta_3(\theta_2))$$

3. Plot the piston displacement (directly below) and rod angle (next page) as functions of crank angle in the global XY coordinate frame.





PROBLEM 4-18g

Statement: Find and plot the angular displacement of link 6 versus the angle of input link 2 as it is rotated from the position shown (+30 deg) to a vertical position (+90 deg). Find the toggle positions of this linkage in terms of the angle of link 2.

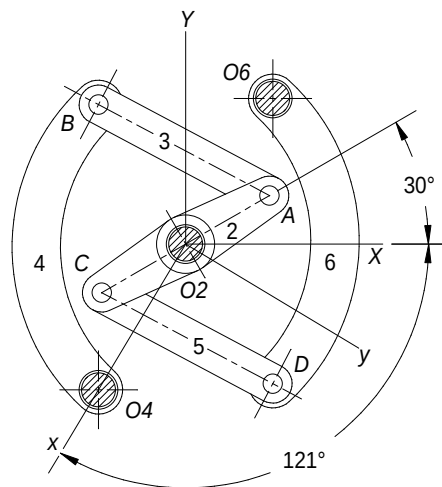
Given: Link lengths:

Input (L_2)	$a := 49$
Rocker (L_4)	$c := 153$
Coupler (L_3)	$b := 100$
Ground link (L_1)	$d := 87$

Angle from x axis to X axis: $\alpha := 121 \cdot \text{deg}$

Starting angle: $\theta_{21} := 30 \cdot \text{deg}$

Crank rotation angle from position shown to vertical: $\Delta\theta_2 := 60 \cdot \text{deg}$



Solution: See Figure P4-5g and Mathcad file P0418g.

1. Define one cycle of the input crank in global coord: $\theta_2 := \theta_{21}, \theta_{21} + 1 \cdot \text{deg} .. \theta_{21} + \Delta\theta_2$
2. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 1.7755 \quad K_2 = 0.5686 \quad K_3 = 1.5592$$

$$A(\theta_2) := \cos(\theta_2 + \alpha) - K_1 - K_2 \cdot \cos(\theta_2 + \alpha) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2 + \alpha) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2 + \alpha) + K_3$$

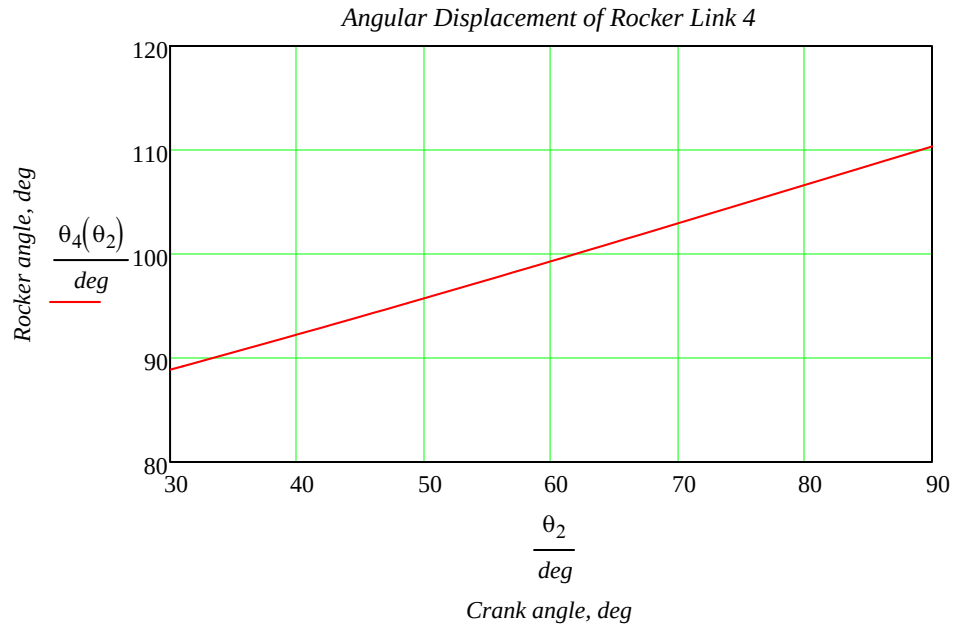
$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right) - \alpha$$

3. If the calculated value of θ_4 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{42}(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := \text{if}(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$$

4. Plot θ_4 as a function of the crank angle θ_2 (measured from the X-axis) as it rotates from the position shown to the vertical position.



4. Check the Grashof condition of the linkage.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof"   if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Condition(a,b,c,d) = "non-Grashof"

5. Using equations 4.37, determine the crank angles (relative to the x-axis) at which links 3 and 4 are in toggle.

$$arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad arg_1 = 0.840$$

$$arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad arg_2 = -6.338$$

$$\theta_{2toggle} := \cos^{-1}(arg_1) \quad \theta_{2toggle} = 32.9 \text{ deg}$$

The other toggle angle is the negative of this. Thus, in the global XY frame the toggle positions are:

$$\theta_{2XYtoggle} := \theta_{2toggle} - \alpha \quad \theta_{2XYtoggle} = -88.130 \text{ deg}$$

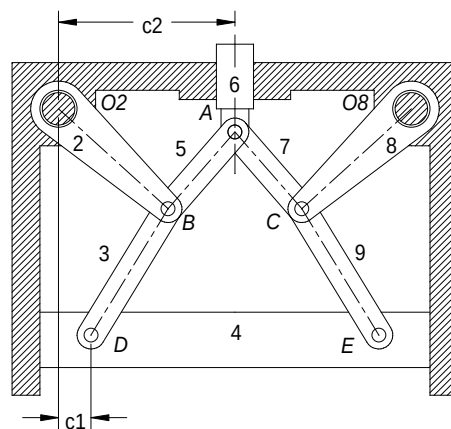
$$\theta_{2XYtoggle} := -\theta_{2toggle} - \alpha \quad \theta_{2XYtoggle} = -153.870 \text{ deg}$$

PROBLEM 4-18h

Statement: Find link 4's maximum displacement vertically downward from the position shown. What will the angle of input link 2 be at that position?

Given: Link lengths:

Crank length, L_2 or L_8	$a := 19.8 \cdot \text{mm}$
Coupler length, L_3 or L_5	$b1 := 19.4 \cdot \text{mm}$
Offset of 1, 2, 3, 4	$c1 := 4.5 \cdot \text{mm}$
Distance from O_2 to O_8	$L_1 := 45.8 \cdot \text{mm}$
Coupler length, L_5 or L_7	$b2 := 13.3 \cdot \text{mm}$
Offset of 1, 2, 5, 6	$c2 := 22.9 \cdot \text{mm}$
Angle of link 2 as shown	$\theta_{20} := 47 \cdot \text{deg}$



Solution: See Figure P4-5h and Mathcad file P0418h.

- Links 1, 2, 3, 4, 5, and 6 make up two offset slider-crank with a common crank, link 2. Links 7, 8, and 9 are kinematically redundant and contribute only to equalizing the forces in the mirror image links. Slider-crank 1, 2, 3, 4 is in the open circuit, and slider-crank 1, 2, 5, 6 is in the crossed circuit.
- Calculate the displacement of link 4 with respect to link 2 angle for the position shown in Figure P4-5h using equations 4.17 and 4.16b.

$$\theta_{310} := \text{asin}\left(\frac{a \cdot \sin(\theta_{20}) - c1}{b1}\right) + \pi \quad \theta_{310} = 149.038 \text{ deg}$$

$$d_{10} := a \cdot \cos(\theta_{20}) - b1 \cdot \cos(\theta_{310}) \quad d_{10} = 30.14 \text{ mm}$$

- Link 4 will reach its maximum downward displacement when links 8 and 9 and links 2 and 3 are in the toggle position. However, it is possible that they may not be able to reach this position because links 5 and 7 may be too short to allow links 2 and 8 to rotate far enough to reach toggle with 3 and 9, respectively.
- Using equation 4.16a, determine the angle that the crank will make with the x axis (see layout below) when links 5 and 7 are horizontal ($\theta_5 = -90 \text{ deg}$). This will be the least value of the angle θ_2 .

$$\theta_{52} := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c2}{b2}\right)$$

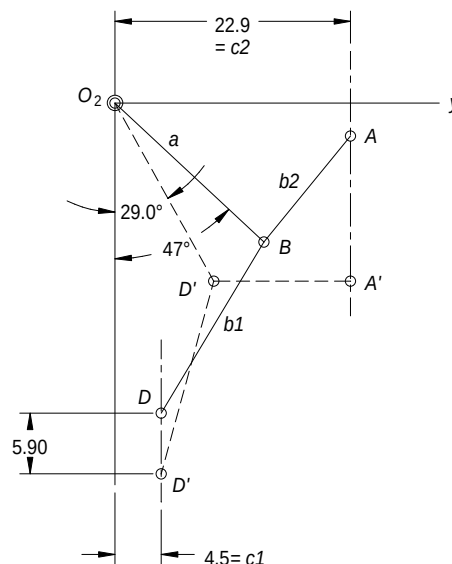
$$\theta_{52} := -90 \cdot \text{deg}$$

$$\sin(\theta_{52}) = -1.000$$

$$\frac{a \cdot \sin(\theta_2) - c2}{b2} := -1.000$$

$$\theta_2 := \text{asin}\left(\frac{c2 - b2}{a}\right)$$

$$\theta_2 = 29.00 \text{ deg}$$



5. Use equations 4.17 and 4.16b to determine the displacement of D' with respect to O_2 .

$$\theta_{31} := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c1}{b1}\right) + \pi \quad \theta_{31} = 164.759 \text{ deg}$$

$$d_1 := a \cdot \cos(\theta_2) - b1 \cdot \cos(\theta_{31}) \quad d_1 = 36.03 \text{ mm}$$

6. The maximum displacement of link 4 from the position shown in Figure P4-5h is the difference between the displacement found in step 5 and that found in step 2.

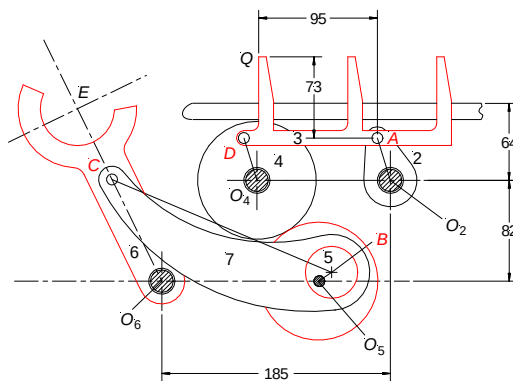
$$\Delta d_{\max} := d_1 - d_{10} \quad \Delta d_{\max} = 5.90 \text{ mm}$$

PROBLEM 4-19

Statement: For one revolution of the driving link 2 of the walking-beam indexing and pick-and-place mechanism in Figure P4-6, find the horizontal stroke of link 3 for the portion of their motion where their tips are above the platen. Express the stroke as a percentage of the crank length O_2B . What portion of a revolution of link 2 does this stroke correspond to? Also find the total angular displacement of link 6 over one revolution of link 2.

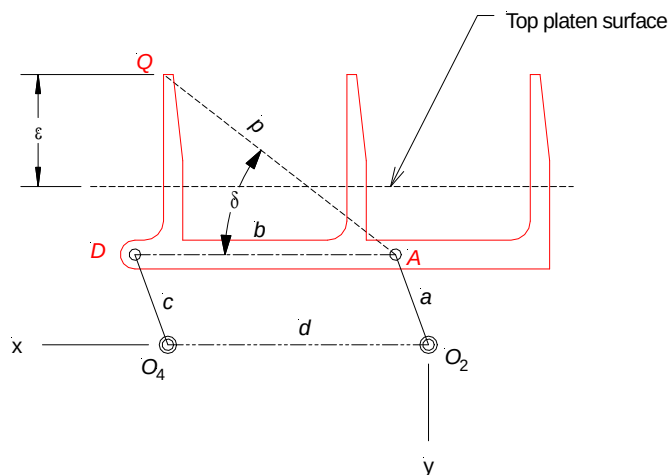
Given: Measured lengths:

Input crank length (O_2A)	$a := 40$
Coupler length (L_3)	$b := 108$
Output crank length (L_4)	$c := 40$
Ground link length (O_2O_4)	$d := 108$
Coupler data (finger at Q)	$p := 119.81$
Distance from O_2 to the platen surface	$\delta := -37.54 \cdot \text{deg}$
	$e := 64$



Solution: See Figure P4-6 and Mathcad file P0419.

- Links 1, 2, 3 and 4 are a special-case Grashof linkage in the parallelogram form. The tip of the finger at point Q (left end of the coupler) is used as the coupler point. The distance from the tip to the platen is ϵ .



- Define the crank angle as a range variable and define θ_3 , which is constant because the coupler has curvilinear motion..

$$\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

$$\theta_3 := 0 \cdot \text{deg}$$

- Use equations 4.27 to define the y-component of the vector $\mathbf{R}_p$.

$$\mathbf{R}_p := \mathbf{R}_A + \mathbf{R}_{pA}$$

$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

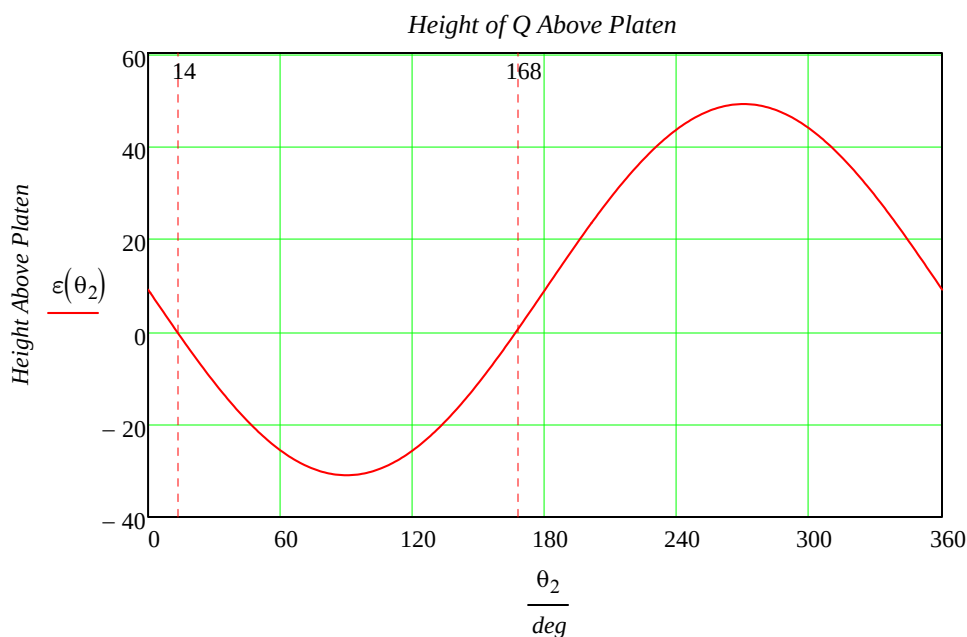
$$\mathbf{R}_{pA} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

$$R_{py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3 + \delta)$$

4. Define the distance of point Q above the platen (note the direction of the positive y axis in the figure above).

$$\varepsilon(\theta_2) := -e - R_{py}(\theta_2)$$

5. Plot ε as a function of crank angle θ_2 .



6. From the graph we see that the coupler point Q is above the platen when the crank angle is greater than 168 deg and less than 14 deg. To find the horizontal stroke during that range of θ_2 , calculate the x-components of any point on the coupler, say point A, for those two crank angles and subtract them.

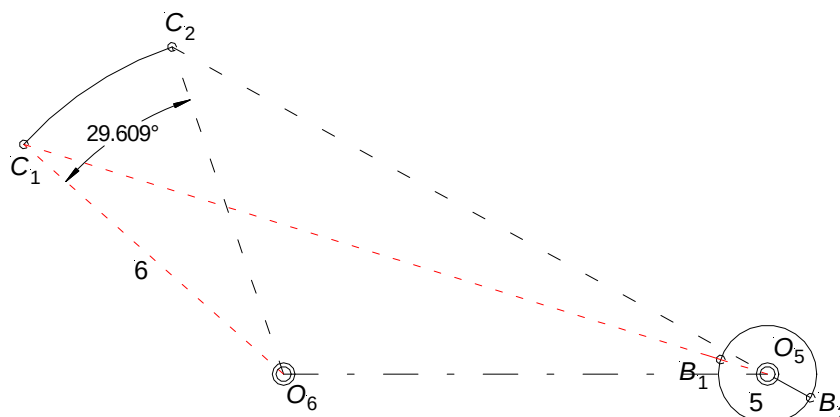
$$A_{x1} := a \cdot \cos(14 \cdot \text{deg}) \quad A_{x1} = 38.812$$

$$A_{x2} := a \cdot \cos(168 \cdot \text{deg}) \quad A_{x2} = -39.126$$

Horizontal stroke when above the platen normalized by dividing by the crank length

$$\text{Stroke} := \frac{A_{x1} - A_{x2}}{a} \quad \text{Stroke} = 1.95 \quad \text{times the crank length}$$

7. Links 1, 4, 5, and 6 constitute a Grashoff crank-rocker-rocker. The extreme positions of the output rocker (link 6) occur when links 4 and 5 are in extended and overlapping toggle positions (see Figure 3-1b in the text for example, but in this case the mechanism is in the crossed circuit).



Given link lengths:

$$L_{O5B} := 13$$

$$L_7 := 193$$

$$L_{O6C} := 92$$

$$L_{O5O6} := 128$$

In the first position (links 5 and 7 extended), the angle between link 6 and the ground link is:

$$\alpha_1 := \arccos \left[\frac{L_{O6C}^2 + L_{O5O6}^2 - (L_{O5B} + L_7)^2}{2 \cdot L_{O6C} \cdot L_{O5O6}} \right] \quad \alpha_1 = 138.312 \text{ deg}$$

In the second position (links 5 and 7 overlapping), the angle between link 6 and the ground link is:

$$\alpha_2 := \arccos \left[\frac{L_{O6C}^2 + L_{O5O6}^2 - (L_7 - L_{O5B})^2}{2 \cdot L_{O6C} \cdot L_{O5O6}} \right] \quad \alpha_2 = 108.702 \text{ deg}$$

The total angular displacement of link 6 is the difference between these two angles.

$$\Delta_{I2} := \alpha_1 - \alpha_2$$

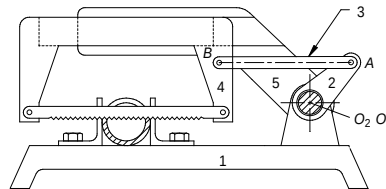
$$\Delta_{I2} = 29.609 \text{ deg}$$

PROBLEM 4-20

Statement: Figure P4-7 shows a power hacksaw, used to cut metal. Link 5 pivots at O_5 and its weight forces the saw blade against the workpiece while the linkage moves the blade (link 4) back and forth on link 5 to cut the part. It is an offset slider-crank mechanism. The dimensions are shown in the figure. For one revolution of the driving link 2 of the hacksaw mechanism on the cutting stroke, find and plot the horizontal stroke of the saw blade as a function of the angle of link 2.

Given: Link Lengths:

Crank length, L_2	$a := 75 \cdot \text{mm}$
Coupler length, L_3	$b := 170 \cdot \text{mm}$
Offset	$c := 45 \cdot \text{mm}$



Assumptions: The arm that guides the slider (hacksaw blade carrier) remains horizontal throughout the stroke.

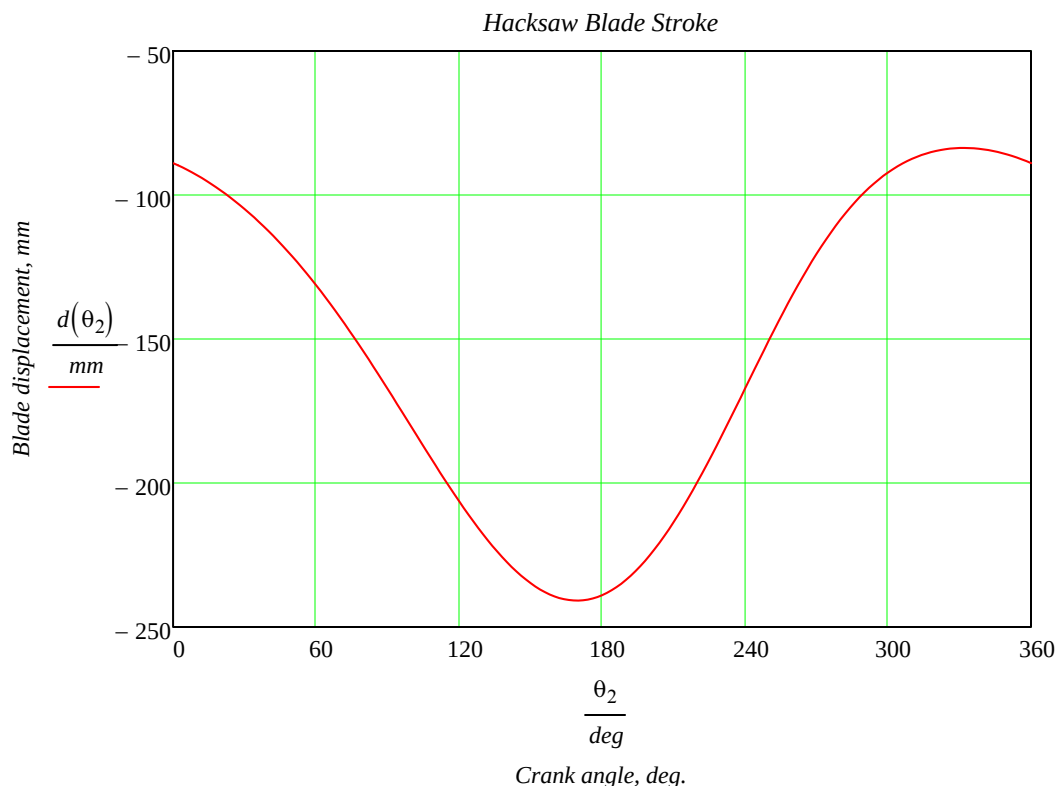
Solution: See Figure P4-7 and Mathcad file P0420.

1. This is a slider-crank mechanism in the crossed circuit. The offset is the vertical distance from the horizontal centerline through O_2 to point B.
2. Establish θ_2 as a range variable: $\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$
3. Determine θ_3 and d using equations 4.16a and 4.17.

$$\theta_3(\theta_2) := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right)$$

$$d(\theta_2) := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_3(\theta_2))$$

4. Plot the blade (point B) displacement as a function of crank angle.



PROBLEM 4-21

Statement: For the linkage in Figure P4-8, find its limit (toggle) positions in terms of the angle of link O_2A referenced to the line of centers O_2O_4 when driven from link O_2A . Then calculate and plot the xy coordinates of coupler point P between those limits, referenced to the line of centers O_2O_4 .

Given: Link lengths:

Input (O_2A) $a := 5.00 \cdot \text{in}$

Coupler (AB) $b := 4.40 \cdot \text{in}$

Rocker (O_4B) $c := 5.00 \cdot \text{in}$

Ground link $d := 9.50 \cdot \text{in}$

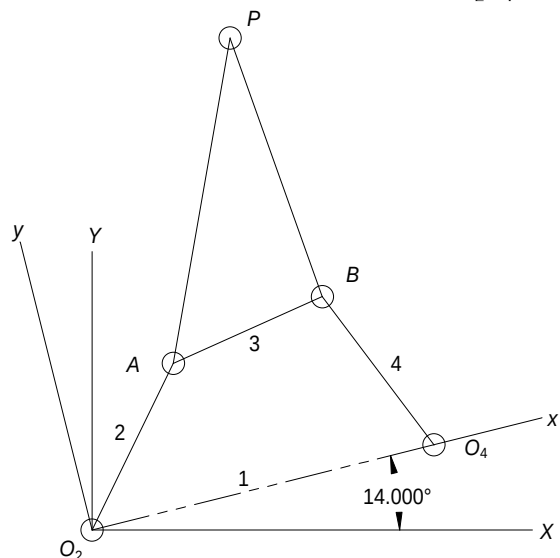
Coupler point data:

$p := 8.90 \cdot \text{in}$

$\delta := 56 \cdot \text{deg}$

Coordinate transformation angle:

$\alpha := 14 \cdot \text{deg}$



Solution: See Figure P4-8 and Mathcad file P0421.

1. Define the coordinate systems. The local frame has origin at O_2 with the positive x axis going through O_4 . Let the global frame also have its origin at O_2 with the positive X axis to the right.
2. Check the Grashof condition of the linkage.

$$\begin{aligned} \text{Condition}(a, b, c, d) &:= \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(a, b, c, d) = \text{"non-Grashof"}$$

3. Using equations 4.37, determine the crank angles (relative to the line AD) at which links 3 and 4 are in toggle.

$$\arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad \arg_1 = 1.209$$

$$\arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad \arg_2 = 0.283$$

$$\theta_{\text{toggle}} := \arccos(\arg_2) \quad \theta_{\text{toggle}} = 73.6 \cdot \text{deg}$$

The other toggle angle is the negative of this.

4. Define one cycle of the input crank between limit positions:

$$\theta_2 := -\theta_{\text{toggle}}, -\theta_{\text{toggle}} + 1 \cdot \text{deg} .. \theta_{\text{toggle}}$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \qquad K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_1 = 1.9000$$

$$K_4 = 2.1591$$

$$K_5 = -2.4911$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Use equations 4.31 to define the x- and y-components of the vector $\mathbf{R}_p$.

$$\mathbf{R}_p := \mathbf{R}_A + \mathbf{R}_{pA}$$

$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{R}_{pA} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

$$R_{px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta)$$

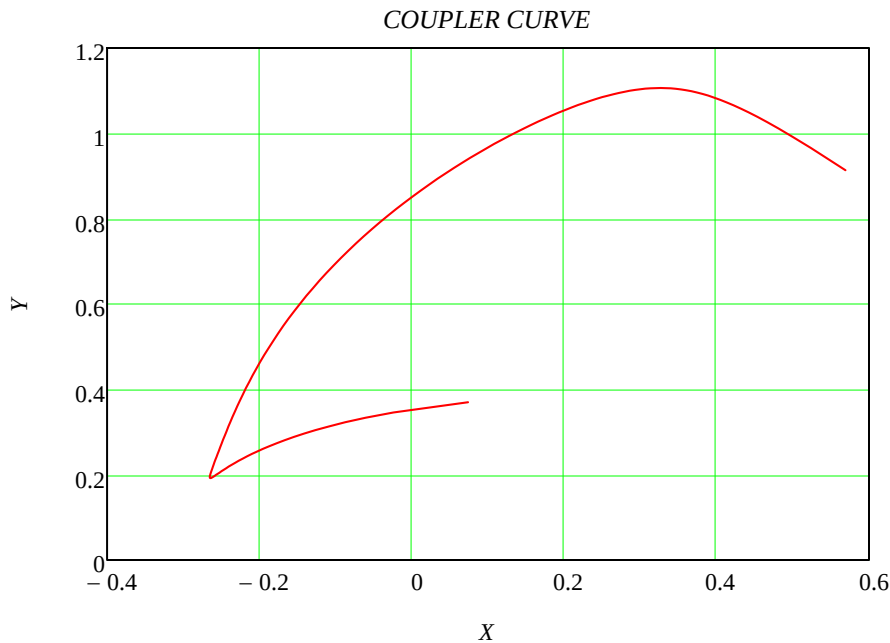
$$R_{py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

8. Transform the coupler point coordinates in the local frame to the global frame using coordinate transformation equations.

$$X_p(\theta_2) := R_{px}(\theta_2) \cdot \cos(\alpha) - R_{py}(\theta_2) \cdot \sin(\alpha)$$

$$Y_p(\theta_2) := R_{px}(\theta_2) \cdot \sin(\alpha) + R_{py}(\theta_2) \cdot \cos(\alpha)$$

9. Plot the coordinates of the coupler point in the global system.



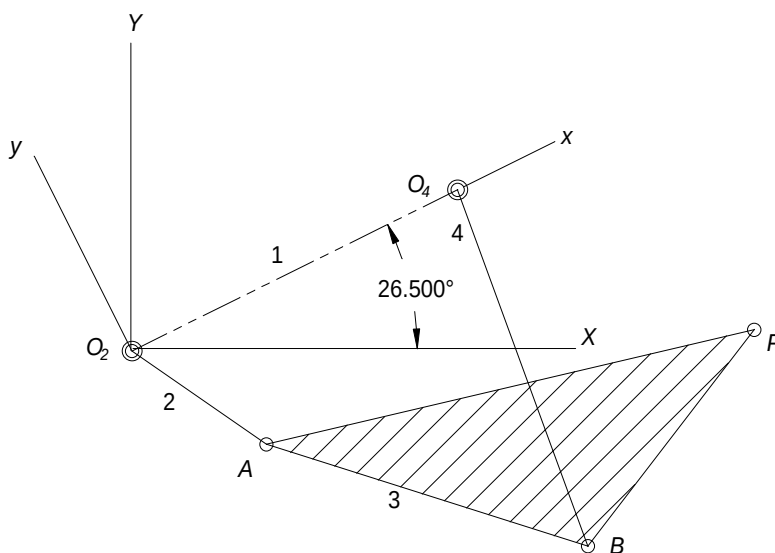
PROBLEM 4-22

Statement: For the walking beam mechanism of Figure P4-9, calculate and plot the x and y components of the position of the coupler point P for one complete revolution of the crank O_2A . Hint: Calculate them first with respect to the ground link O_2O_4 and then transform them into the global XY coordinate system (i.e., horizontal and vertical in the figure).

Given: Link lengths: Coupler point data:
 Ground link $d := 2.22$ Crank $a := 1$ $p := 3.06$ $\delta := 31.000 \cdot \text{deg}$
 Coupler $b := 2.06$ Rocker $c := 2.33$
 Coordinate transformation angle: $\alpha := 26.5 \cdot \text{deg}$

Solution: See Figure P4-9 and Mathcad file P0422.

1. Define the coordinate systems. The local frame has origin at O_2 with the positive x axis going through O_4 . Let the global frame also have its origin at O_2 with the positive X axis to the right.



2. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg}$
3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \quad K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_1 = 2.2200 \quad K_4 = 1.0777 \quad K_5 = -1.1512$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

4. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

5. Use equations 4.31 to define the x - and y -components of the vector $\mathbf{R}_P$.

$$\mathbf{R}_P := \mathbf{R}_A + \mathbf{R}_{PA}$$

$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{R}_{PA} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

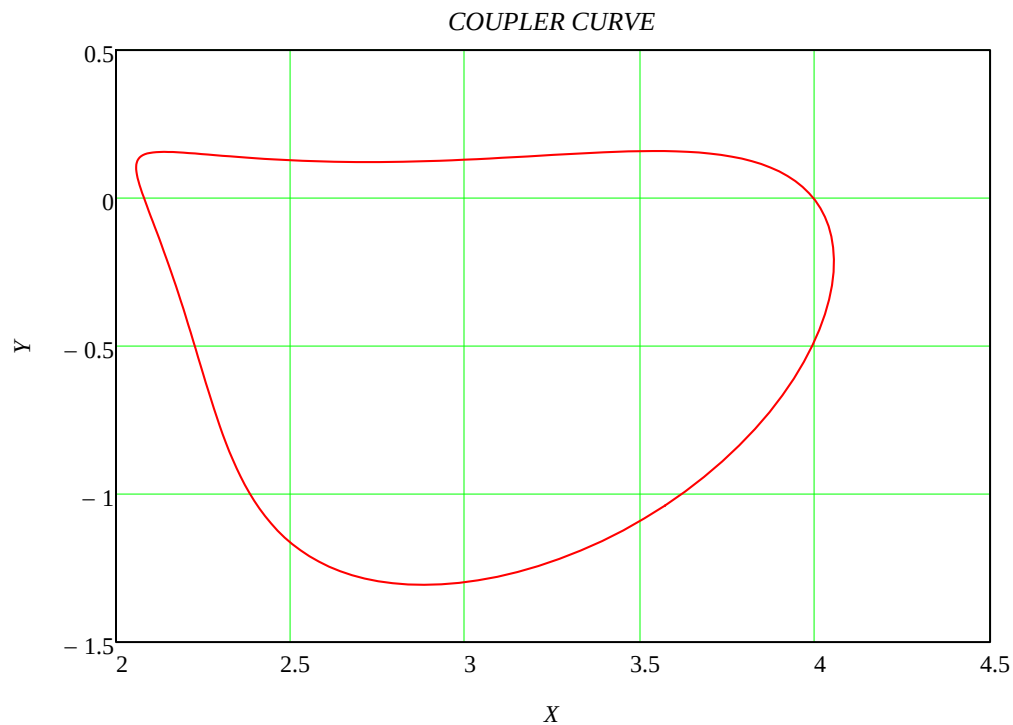
$$R_{Px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta) \quad R_{Py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

6. Transform the coupler point coordinates in the local frame to the global frame using coordinate transformation equations.

$$X_P(\theta_2) := R_{Px}(\theta_2) \cdot \cos(\alpha) - R_{Py}(\theta_2) \cdot \sin(\alpha)$$

$$Y_P(\theta_2) := R_{Px}(\theta_2) \cdot \sin(\alpha) + R_{Py}(\theta_2) \cdot \cos(\alpha)$$

7. Plot the coordinates of the coupler point in the global system.



PROBLEM 4-23

Statement: For the linkage in Figure P4-10, calculate and plot the angular displacement of links 3 and 4 and the path coordinates of point P with respect to the angle of the input crank O_2A for one revolution.

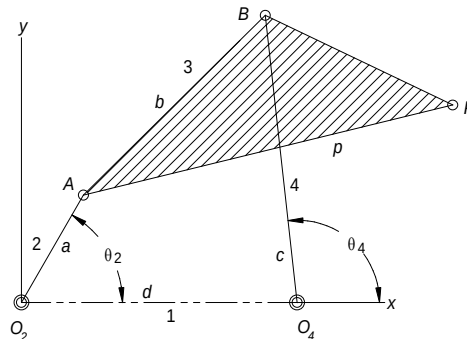
Given: Link lengths:

Ground link $d := 2.22$ Crank $a := 1.0$

Coupler $b := 2.06$ Rocker $c := 2.33$

Coupler point data:

$p := 3.06$ $\delta := -31.00 \cdot \text{deg}$



Solution: See Figure P4-10 and Mathcad file P0423.

1. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
2. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.2200 \quad K_2 = 0.9528 \quad K_3 = 1.5265$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

3. If the calculated value of θ_4 is greater than 2π , subtract 2π from it.

$$\theta_4(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0777 \quad K_5 = -1.1512$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

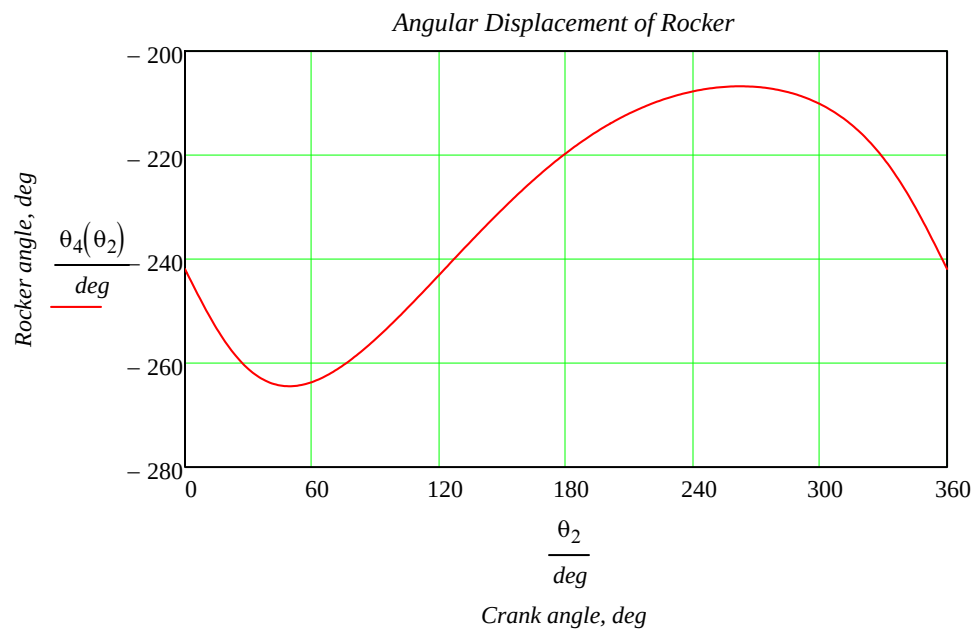
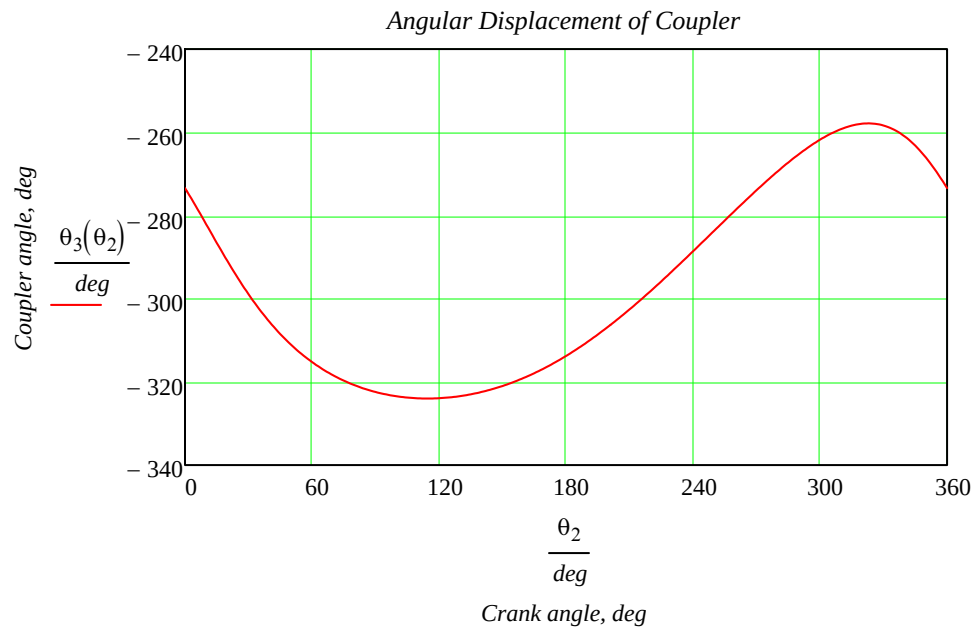
5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. If the calculated value of θ_3 is greater than 2π , subtract 2π from it.

$$\theta_3(\theta_2) := \text{if}(\theta_{31}(\theta_2) > 2 \cdot \pi, \theta_{31}(\theta_2) - 2 \cdot \pi, \theta_{31}(\theta_2))$$

7. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).



8. Use equations 4.31 to define the x- and y-components of the vector $\mathbf{R}_P$.

$$\mathbf{R}_P := \mathbf{R}_A + \mathbf{R}_{PA}$$

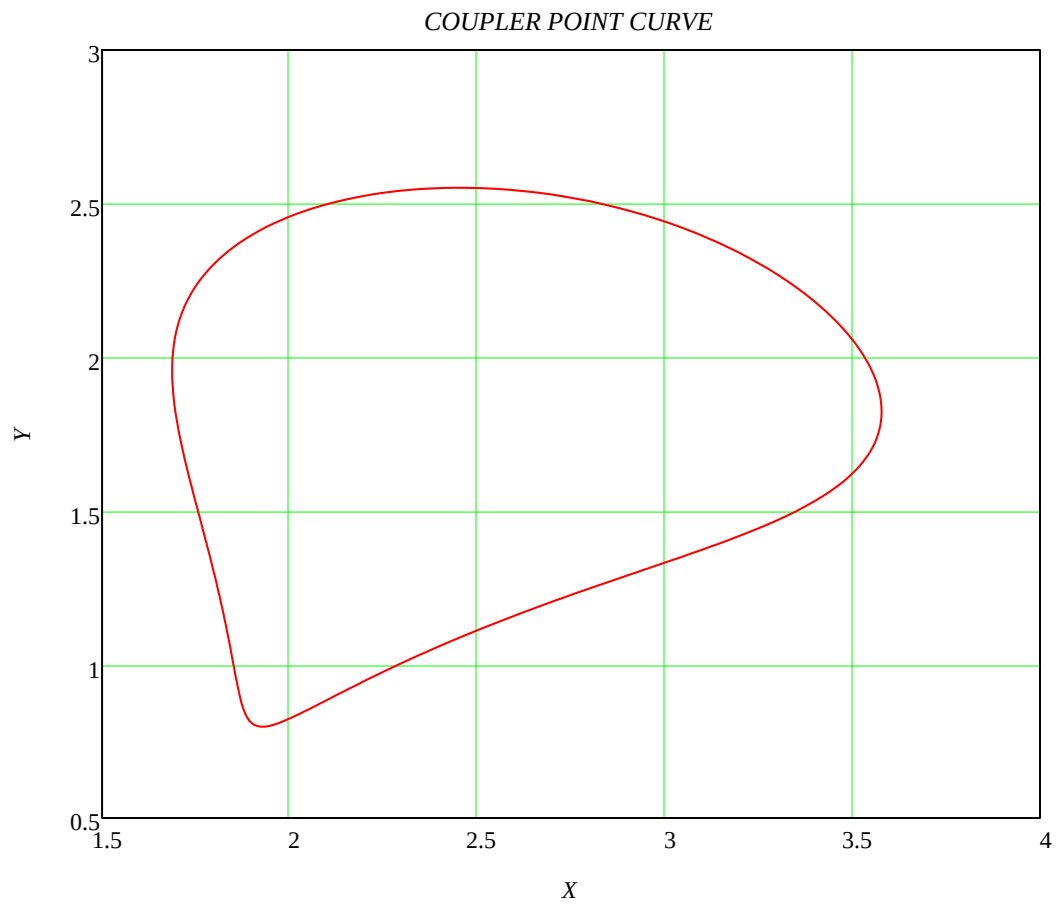
$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{R}_{PA} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

$$R_{Px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta)$$

$$R_{Py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

9. Plot the coordinates of the coupler point in the local xy coordinate system.

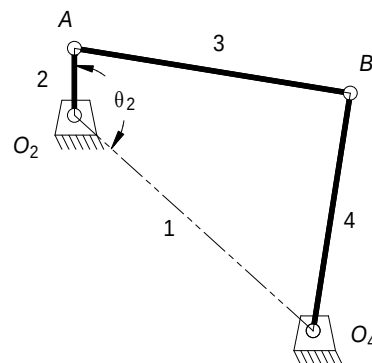


PROBLEM 4-24

Statement: For the linkage in Figure P4-11, calculate and plot the angular displacement of links 3 and 4 with respect to the angle of the input crank O_2A for one revolution.

Given: Link lengths:

- Link 2 $a := 2.00 \cdot \text{in}$
 Link 3 $b := 8.375 \cdot \text{in}$
 Link 4 $c := 7.187 \cdot \text{in}$
 Link 1 $d := 9.625 \cdot \text{in}$



Solution: See Figure P4-11 and Mathcad file P0424.

1. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg}$

2. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 4.8125 \quad K_2 = 1.3392 \quad K_3 = 2.7186$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

3. If the calculated value of θ_4 is greater than 2π , subtract 2π from it.

$$\theta_4(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.1493 \quad K_5 = -3.4367$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

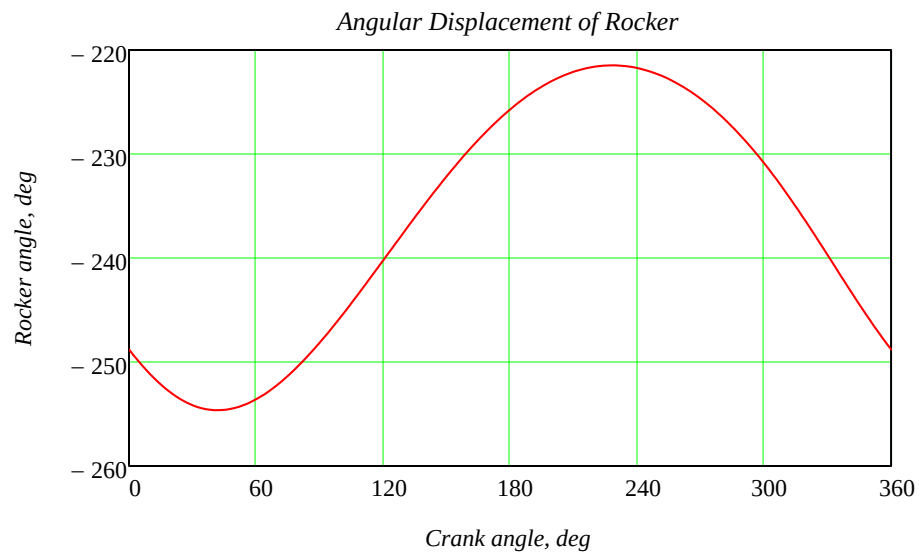
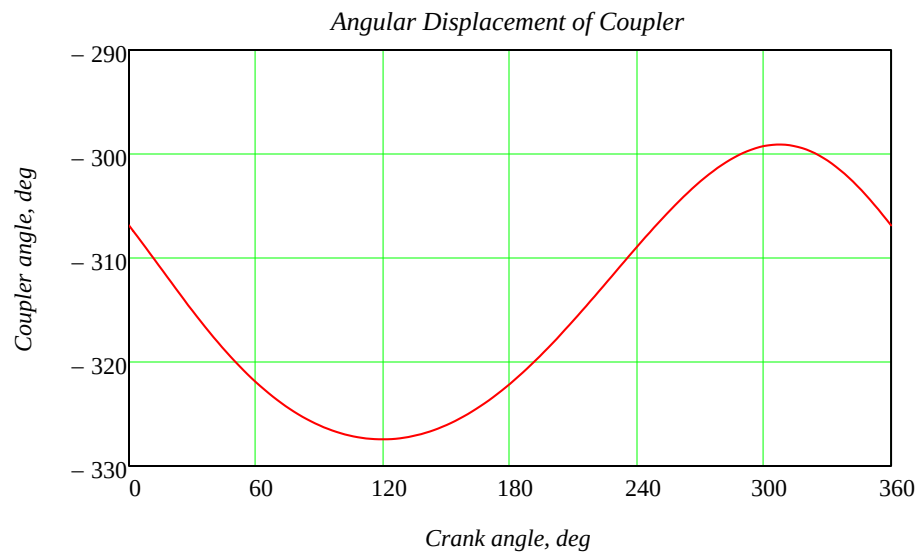
5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. If the calculated value of θ_3 is greater than 2π , subtract 2π from it.

$$\theta_3(\theta_2) := \text{if}(\theta_{31}(\theta_2) > 2 \cdot \pi, \theta_{31}(\theta_2) - 2 \cdot \pi, \theta_{31}(\theta_2))$$

7. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).



PROBLEM 4-25

Statement: For the linkage in Figure P4-12, find its limit (toggle) positions in terms of the angle of link O_2A referenced to the line of centers O_2O_4 when driven from link O_2A . Then calculate and plot the angular displacement of links 3 and 4 and the path coordinates of point P with respect to the angle of the input crank O_2A over its possible range of motion referenced to the line of centers O_2O_4 .

Given: Link lengths:

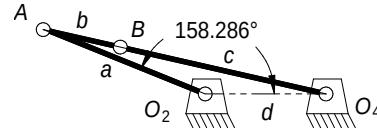
$$\text{Input } (O_2A) \quad a := 0.785$$

$$\text{Coupler } (AB) \quad b := 0.356$$

$$\text{Rocker } (O_4B) \quad c := 0.950$$

$$\text{Ground link} \quad d := 0.544$$

$$\text{Coupler point data: } p := 1.09 \quad \delta := 0 \cdot \text{deg}$$



Solution: See Figure P4-12 and Mathcad file P0425.

1. Check the Grashof condition of the linkage.

$$\begin{aligned} \text{Condition}(a, b, c, d) := & \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(a, b, c, d) = \text{"Grashof"} \quad \text{double rocker}$$

2. Using the geometry defined in Figure 3-1a in the text, determine the input crank angles (relative to the line O_2O_4) at which links 2 and 3, and 3 and 4 are in toggle.

$$\theta_{21} := \arccos \left[\frac{d^2 + (a + b)^2 - c^2}{2 \cdot d \cdot (a + b)} \right] \quad \theta_{21} = 55.937 \text{ deg}$$

$$\theta_{22} := \arccos \left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot a \cdot d} \right] \quad \theta_{22} = 158.286 \text{ deg}$$

3. Define one cycle of the input crank between limit positions:

$$\theta_2 := \theta_{21}, \theta_{21} + 1 \cdot \text{deg} .. \theta_{22}$$

4. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 0.6930$$

$$K_2 = 0.5726$$

$$K_3 = 1.1317$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. If the calculated value of θ_4 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{42}(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := \text{if}(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$$

6. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.5281 \quad K_5 = -0.2440$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

7. Use equation 4.13 to find values of θ_3 for the open circuit.

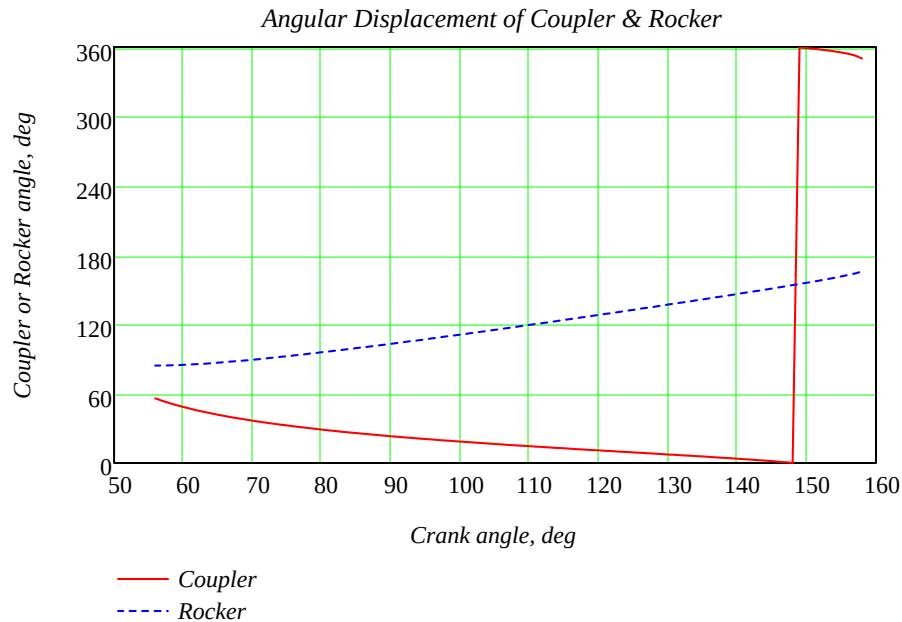
$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

8. If the calculated value of θ_3 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_2) := \text{if}(\theta_{31}(\theta_2) > 2 \cdot \pi, \theta_{31}(\theta_2) - 2 \cdot \pi, \theta_{31}(\theta_2))$$

$$\theta_3(\theta_2) := \text{if}(\theta_{32}(\theta_2) < 0, \theta_{32}(\theta_2) + 2 \cdot \pi, \theta_{32}(\theta_2))$$

9. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).



10. Use equations 4.31 to define the x - and y -components of the vector $\mathbf{R}_P$.

$$\mathbf{R}_P := \mathbf{R}_A + \mathbf{R}_{PA}$$

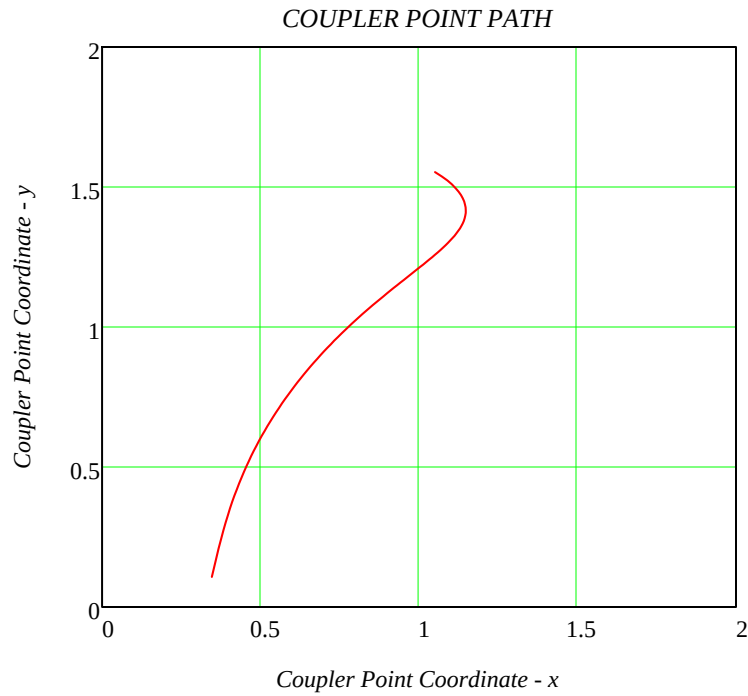
$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{R_{pA}} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

$$R_{px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta)$$

$$R_{py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

11. Plot the coordinates of the coupler point in the local xy coordinate system.



PROBLEM 4-26

Statement: For the linkage in Figure P4-13, find its limit (toggle) positions in terms of the angle of link O_2A referenced to the line of centers O_2O_4 when driven from link O_2A . Then calculate and plot the angular displacement of links 3 and 4 and the path coordinates of point P with respect to the angle of the input crank O_2A over its possible range of motion referenced to the line of centers O_2O_4 .

Given: Link lengths:

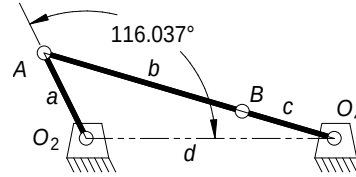
$$\text{Input } (O_2A) \quad a := 0.86$$

$$\text{Coupler } (AB) \quad b := 1.85$$

$$\text{Rocker } (O_4B) \quad c := 0.86$$

$$\text{Ground link} \quad d := 2.22$$

$$\text{Coupler point data: } p := 1.33 \quad \delta := 0 \cdot \text{deg}$$



Solution: See Figure P4-13 and Mathcad file P0426.

1. Check the Grashof condition of the linkage.

$$\begin{aligned} \text{Condition}(a, b, c, d) := & \left\{ \begin{array}{l} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array} \right. \end{aligned}$$

$$\text{Condition}(a, b, c, d) = \text{"non-Grashof"}$$

2. Using equations 4.33, determine the crank angles (relative to the line AD) at which links 3 and 4 are in toggle.

$$\arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad \arg_1 = 1.228$$

$$\arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad \arg_2 = -0.439$$

$$\theta_{2\text{toggle}} := \text{acos}(\arg_2) \quad \theta_{2\text{toggle}} = 116.037 \text{ deg}$$

The other toggle angle is the negative of this.

3. Define one cycle of the input crank between limit positions:

$$\theta_2 := -\theta_{2\text{toggle}}, -\theta_{2\text{toggle}} + 1 \cdot \text{deg} \dots \theta_{2\text{toggle}}$$

4. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.5814$$

$$K_2 = 2.5814$$

$$K_3 = 2.0181$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. If the calculated value of θ_4 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{42}(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := \text{if}(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$$

6. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.2000 \quad K_5 = -2.6244$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

7. Use equation 4.13 to find values of θ_3 for the open circuit.

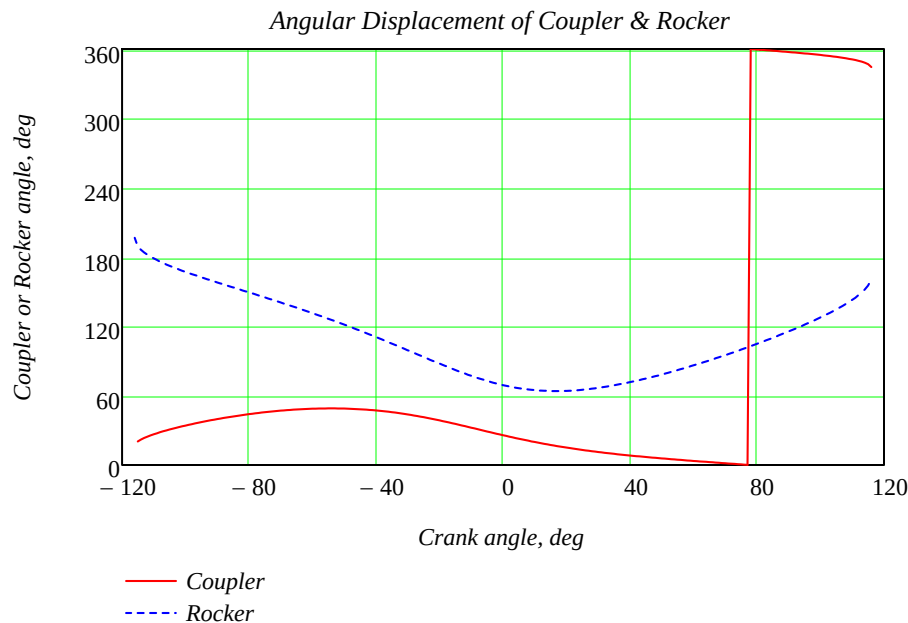
$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

8. If the calculated value of θ_3 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_2) := \text{if}(\theta_{31}(\theta_2) > 2 \cdot \pi, \theta_{31}(\theta_2) - 2 \cdot \pi, \theta_{31}(\theta_2))$$

$$\theta_3(\theta_2) := \text{if}(\theta_{32}(\theta_2) < 0, \theta_{32}(\theta_2) + 2 \cdot \pi, \theta_{32}(\theta_2))$$

9. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).



10. Use equations 4.31 to define the x- and y-components of the vector $\mathbf{R}_P$.

$$\mathbf{R}_P := \mathbf{R}_A + \mathbf{R}_{PA}$$

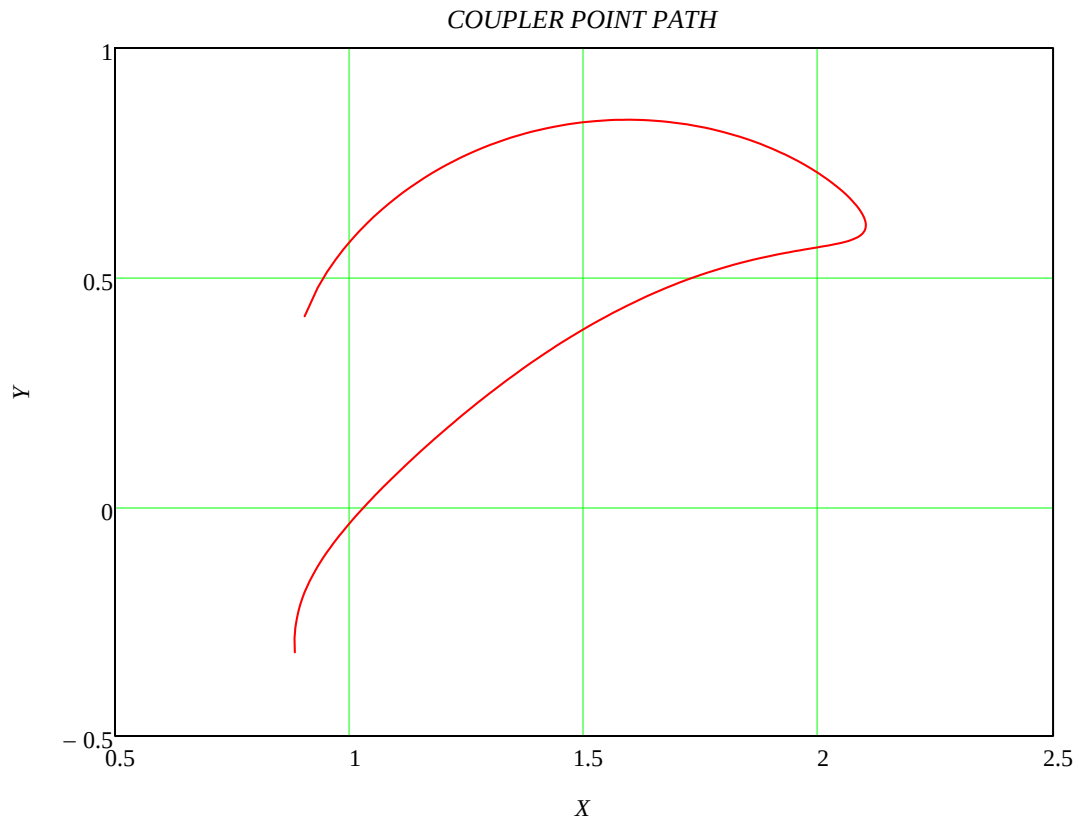
$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{R}_{PA} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

$$R_{Px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta)$$

$$R_{Py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

11. Plot the coordinates of the coupler point in the local xy coordinate system.



PROBLEM 4-27

Statement: For the linkage in Figure P4-13, find its limit (toggle) positions in terms of the angle of link O_4B referenced to the line of centers O_4O_2 when driven from link O_4B . Then calculate and plot the angular displacement of links 2 and 3 and the path coordinates of point P with respect to the angle of the input crank O_4B over its possible range of motion referenced to the line of centers O_4O_2 .

Given: Link lengths:

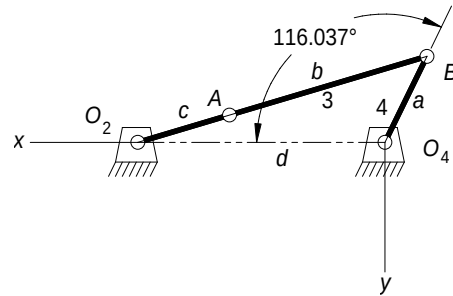
Input (O_4B) $a := 0.86$

Coupler (AB) $b := 1.85$

Rocker (O_2A) $c := 0.86$

Ground link $d := 2.22$

Coupler point data: $p := 0.52$ $\delta := 0 \cdot \text{deg}$



Solution: See Figure P4-13 and Mathcad file P0427.

1. Check the Grashof condition of the linkage.

$$\begin{aligned} \text{Condition}(a, b, c, d) &:= \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(a, b, c, d) = \text{"non-Grashof"}$$

2. Using equations 4.33, determine the crank angles (relative to the line O_4O_2) at which links 2 and 3 are in toggle.

$$\arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad \arg_1 = 1.228$$

$$\arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad \arg_2 = -0.439$$

$$\theta_{4\text{toggle}} := \arccos(\arg_2) \quad \theta_{4\text{toggle}} = 116.037 \text{ deg}$$

The other toggle angle is the negative of this.

3. Define one cycle of the input crank between limit positions:

$$\theta_4 := -\theta_{4\text{toggle}}, -\theta_{4\text{toggle}} + 1 \cdot \text{deg} .. \theta_{4\text{toggle}}$$

4. Use equations 4.8a and 4.10 to calculate θ_2 as a function of θ_4 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.5814$$

$$K_2 = 2.5814$$

$$K_3 = 2.0181$$

$$A(\theta_4) := \cos(\theta_4) - K_1 - K_2 \cdot \cos(\theta_4) + K_3$$

$$B(\theta_4) := -2 \cdot \sin(\theta_4)$$

$$C(\theta_4) := K_1 - (K_2 + 1) \cdot \cos(\theta_4) + K_3$$

$$\theta_{21}(\theta_4) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_4), -B(\theta_4) - \sqrt{B(\theta_4)^2 - 4 \cdot A(\theta_4) \cdot C(\theta_4)} \right) \right)$$

5. If the calculated value of θ_2 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{22}(\theta_4) := \text{if}(\theta_{21}(\theta_4) > 2 \cdot \pi, \theta_{21}(\theta_4) - 2 \cdot \pi, \theta_{21}(\theta_4))$$

$$\theta_2(\theta_4) := \text{if}(\theta_{22}(\theta_4) < 0, \theta_{22}(\theta_4) + 2 \cdot \pi, \theta_{22}(\theta_4))$$

6. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.2000 \quad K_5 = -2.6244$$

$$D(\theta_4) := \cos(\theta_4) - K_1 + K_4 \cdot \cos(\theta_4) + K_5$$

$$E(\theta_4) := -2 \cdot \sin(\theta_4) \quad F(\theta_4) := K_1 + (K_4 - 1) \cdot \cos(\theta_4) + K_5$$

7. Use equation 4.13 to find values of θ_3 for the open circuit.

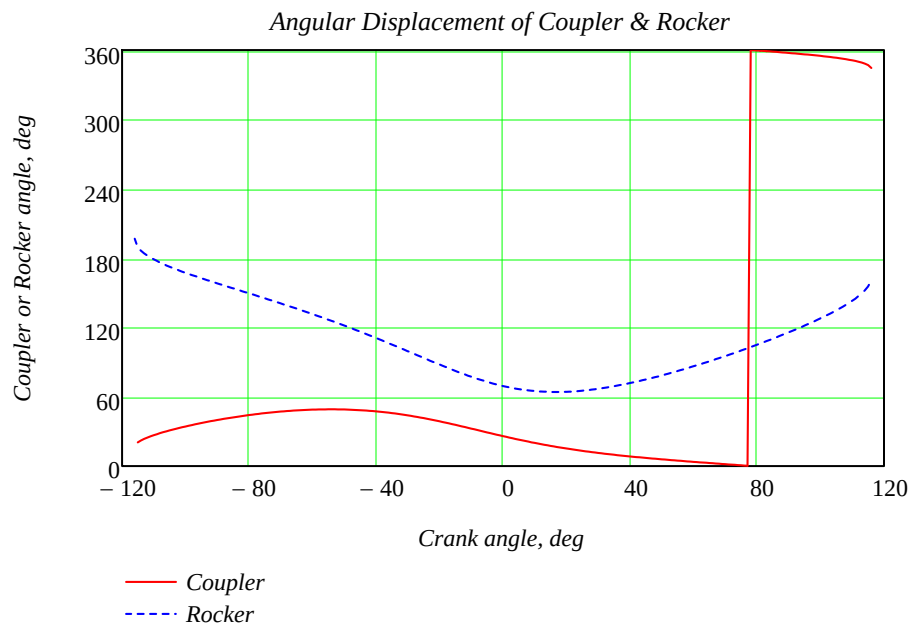
$$\theta_{31}(\theta_4) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_4), -E(\theta_4) - \sqrt{E(\theta_4)^2 - 4 \cdot D(\theta_4) \cdot F(\theta_4)} \right) \right)$$

8. If the calculated value of θ_3 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_4) := \text{if}(\theta_{31}(\theta_4) > 2 \cdot \pi, \theta_{31}(\theta_4) - 2 \cdot \pi, \theta_{31}(\theta_4))$$

$$\theta_3(\theta_4) := \text{if}(\theta_{32}(\theta_4) < 0, \theta_{32}(\theta_4) + 2 \cdot \pi, \theta_{32}(\theta_4))$$

9. Plot θ_3 and θ_2 as functions of the crank angle θ_4 (measured from the ground link).



10. Use equations 4.31 to define the x- and y-components of the vector $\mathbf{R}_P$.

$$\mathbf{R}_P := \mathbf{R}_B + \mathbf{R}_{PB}$$

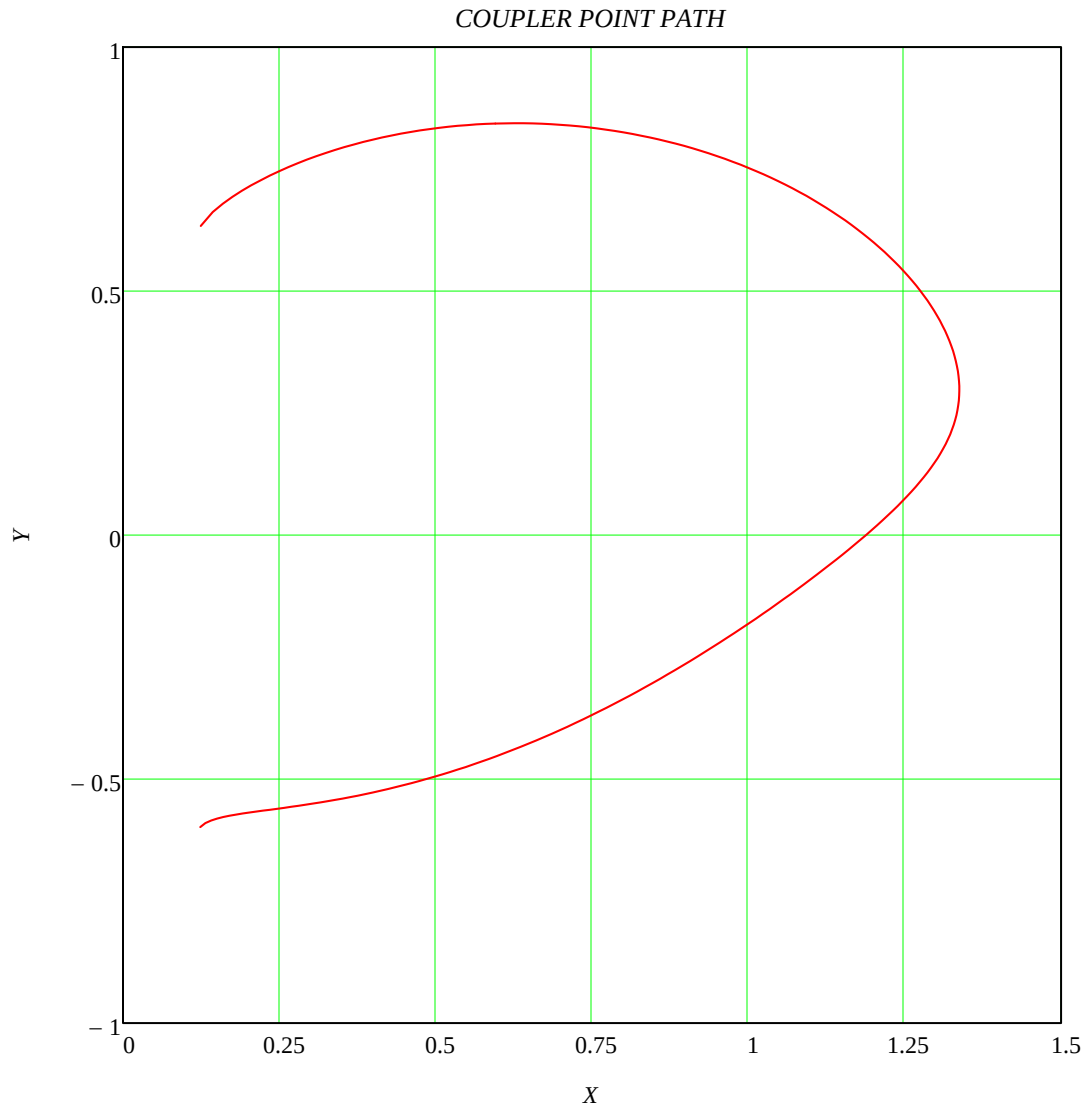
$$\mathbf{R}_B := a \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$\mathbf{R}_{PB} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

$$R_{Px}(\theta_4) := a \cdot \cos(\theta_4) + p \cdot \cos(\theta_3(\theta_4) + \delta)$$

$$R_{Py}(\theta_4) := a \cdot \sin(\theta_4) + p \cdot \sin(\theta_3(\theta_4) + \delta)$$

11. Plot the coordinates of the coupler point in the local xy coordinate system.



PROBLEM 4-28

Statement: For the rocker-crank linkage in Figure P4-14, find the maximum angular displacement possible for the treadle link (to which force F is applied). Determine the toggle positions. How does this work? Explain why the grinding wheel is able to fully rotate despite the presence of toggle positions when driven from the treadle. How would you get it started if it was in a toggle position?

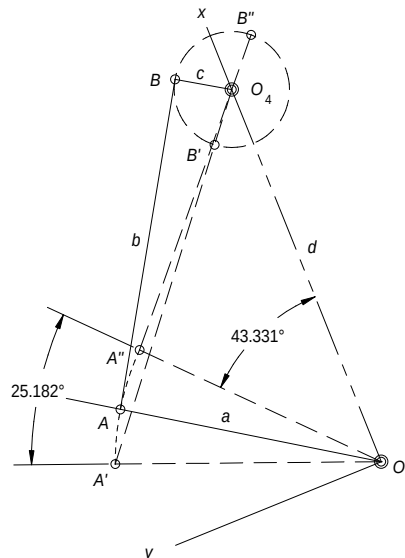
Given: Link lengths:

Input (O_2A) $a := 600 \cdot \text{mm}$

Coupler (AB) $b := 750 \cdot \text{mm}$

Rocker (O_4B) $c := 130 \cdot \text{mm}$

Ground link $d := 900 \cdot \text{mm}$



Solution: See Figure P4-14 and Mathcad file P0428.

1. Use Figure 3-1(b) in the text to calculate the angles that link O_2A makes with the ground link in the toggle positions.

$$\theta_{21} := \arccos \left[\frac{a^2 + d^2 - (b - c)^2}{2 \cdot a \cdot d} \right] \quad \theta_{21} = 43.331 \text{ deg}$$

$$\theta_{22} := \arccos \left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot a \cdot d} \right] \quad \theta_{22} = 68.513 \text{ deg}$$

2. Subtract these two angles to get the maximum angular displacement of the treadle.

$$\Delta\theta_2 := \theta_{22} - \theta_{21} \quad \Delta\theta_2 = 25.182 \text{ deg}$$

3. Despite having transmission angles of 0 deg twice per revolution, the mechanism will work. That is, one will be able to drive the grinding wheel from the treadle (link 2). The reason is that the grinding wheel will act as a flywheel and will carry the linkage through the periods when the transmission angle is low. Typically, the operator will start the motion by rotating the wheel by hand if it is in or near a toggle position.

PROBLEM 4-29

Statement: For the linkage in Figure P4-15, find its limit (toggle) positions in terms of the angle of link O_2A referenced to the line of centers O_2O_4 when driven from link O_2A . Then calculate and plot the angular displacement of links 3 and 4 and the path coordinates of point P with respect to the angle of the input crank O_2A over its possible range of motion referenced to the line of centers O_2O_4 .

Given: Link lengths:

Input (O_2A) $a := 0.72$

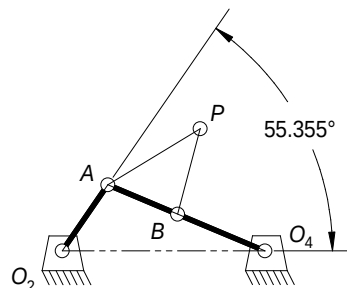
Coupler (AB) $b := 0.68$

Rocker (O_4B) $c := 0.85$

Ground link $d := 1.82$

Coupler point data: $p := 0.97$

$\delta := 54 \cdot \text{deg}$



Solution: See Figure P4-15 and Mathcad file P0429.

1. Check the Grashof condition of the linkage.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Condition(a,b,c,d) = "non-Grashof"

2. Using equations 4.37, determine the crank angles (relative to the line AD) at which links 3 and 4 are in toggle.

$$arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad arg_1 = 1.451$$

$$arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad arg_2 = 0.568$$

$$\theta_{2toggle} := \text{acos}(arg_2) \quad \theta_{2toggle} = 55.355 \text{ deg}$$

The other toggle angle is the negative of this.

3. Define one cycle of the input crank between limit positions:

$$\theta_2 := -\theta_{2toggle}, -\theta_{2toggle} + 0.5 \cdot \text{deg} .. \theta_{2toggle}$$

4. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.5278$$

$$K_2 = 2.1412$$

$$K_3 = 3.3422$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. If the calculated value of θ_4 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{42}(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := \text{if}(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$$

6. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 2.6765 \quad K_5 = -3.6465$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

7. Use equation 4.13 to find values of θ_3 for the open circuit.

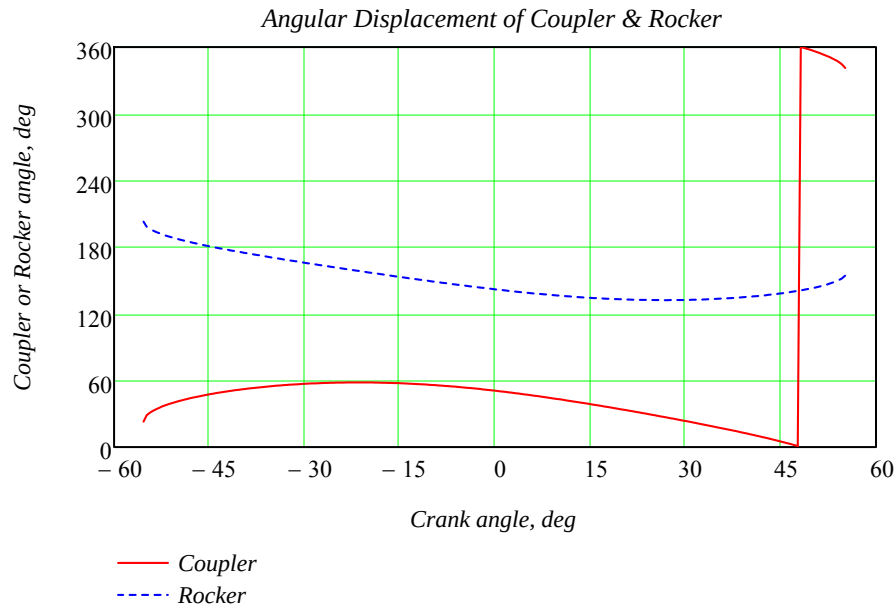
$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

8. If the calculated value of θ_3 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_2) := \text{if}(\theta_{31}(\theta_2) > 2 \cdot \pi, \theta_{31}(\theta_2) - 2 \cdot \pi, \theta_{31}(\theta_2))$$

$$\theta_3(\theta_2) := \text{if}(\theta_{32}(\theta_2) < 0, \theta_{32}(\theta_2) + 2 \cdot \pi, \theta_{32}(\theta_2))$$

9. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).



10. Use equations 4.27 to define the x- and y-components of the vector $\mathbf{R}_P$.

$$\mathbf{R}_P := \mathbf{R}_A + \mathbf{R}_{PA}$$

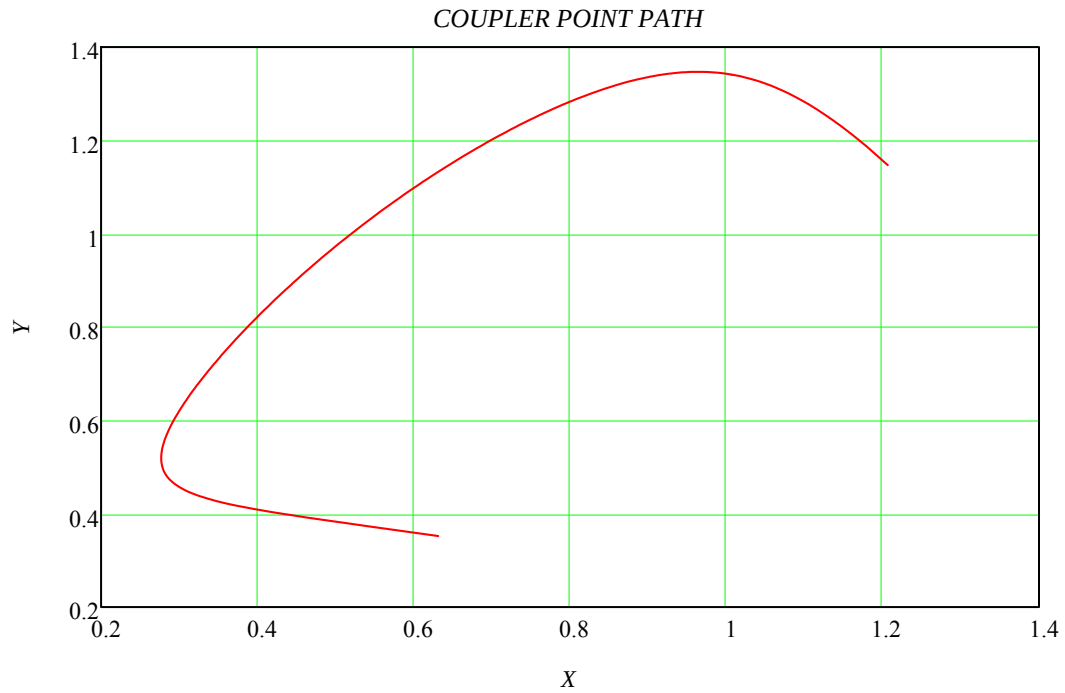
$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{R}_{PA} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

$$R_{Px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta)$$

$$R_{Py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

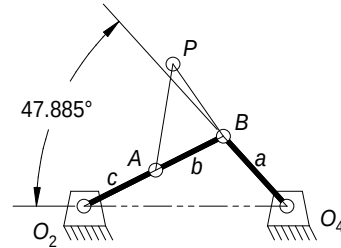
11. Plot the coordinates of the coupler point in the local xy coordinate system.



PROBLEM 4-30

Statement: For the linkage in Figure P4-15, find its limit (toggle) positions in terms of the angle of link O_4B referenced to the line of centers O_4O_2 when driven from link O_4B . Then calculate and plot the angular displacement of links 2 and 3 and the path coordinates of point P with respect to the angle of the input crank O_4B over its possible range of motion referenced to the line of centers O_4O_2 .

Given: Link lengths:
 Input (O_4B) $a := 0.85$
 Coupler (AB) $b := 0.68$
 Rocker (O_2A) $c := 0.72$
 Ground link $d := 1.82$
 Coupler point data: $p := 0.792$
 $\delta := 82.032 \cdot \text{deg}$



Solution: See Figure P4-15 and Mathcad file P0430.

1. Check the Grashof condition of the linkage.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$Condition(a,b,c,d) = \text{"non-Grashof"}$

2. Using equations 4.37, determine the crank angles (relative to the line O_4O_2) at which links 2 and 3 are in toggle.

$$arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad arg_1 = 1.304$$

$$arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad arg_2 = 0.671$$

$$\theta_{4toggle} := \text{acos}(arg_2) \quad \theta_{4toggle} = 47.885 \text{ deg}$$

The other toggle angle is the negative of this.

3. Define one cycle of the input crank between limit positions:

$$\theta_4 := -\theta_{4toggle}, -\theta_{4toggle} + 0.5 \cdot \text{deg} .. \theta_{4toggle}$$

4. Use equations 4.8a and 4.10 to calculate θ_2 as a function of θ_4 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.1412 \quad K_2 = 2.5278 \quad K_3 = 3.3422$$

$$A(\theta_4) := \cos(\theta_4) - K_1 - K_2 \cdot \cos(\theta_4) + K_3$$

$$B(\theta_4) := -2 \cdot \sin(\theta_4) \quad C(\theta_4) := K_1 - (K_2 + 1) \cdot \cos(\theta_4) + K_3$$

$$\theta_{21}(\theta_4) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_4), -B(\theta_4) - \sqrt{B(\theta_4)^2 - 4 \cdot A(\theta_4) \cdot C(\theta_4)} \right) \right)$$

5. If the calculated value of θ_2 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{22}(\theta_4) := \text{if}(\theta_{21}(\theta_4) > 2 \cdot \pi, \theta_{21}(\theta_4) - 2 \cdot \pi, \theta_{21}(\theta_4))$$

$$\theta_2(\theta_4) := \text{if}(\theta_{22}(\theta_4) < 0, \theta_{22}(\theta_4) + 2 \cdot \pi, \theta_{22}(\theta_4))$$

6. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 2.6765 \quad K_5 = -3.4420$$

$$D(\theta_4) := \cos(\theta_4) - K_1 + K_4 \cdot \cos(\theta_4) + K_5$$

$$E(\theta_4) := -2 \cdot \sin(\theta_4) \quad F(\theta_4) := K_1 + (K_4 - 1) \cdot \cos(\theta_4) + K_5$$

7. Use equation 4.13 to find values of θ_3 for the open circuit.

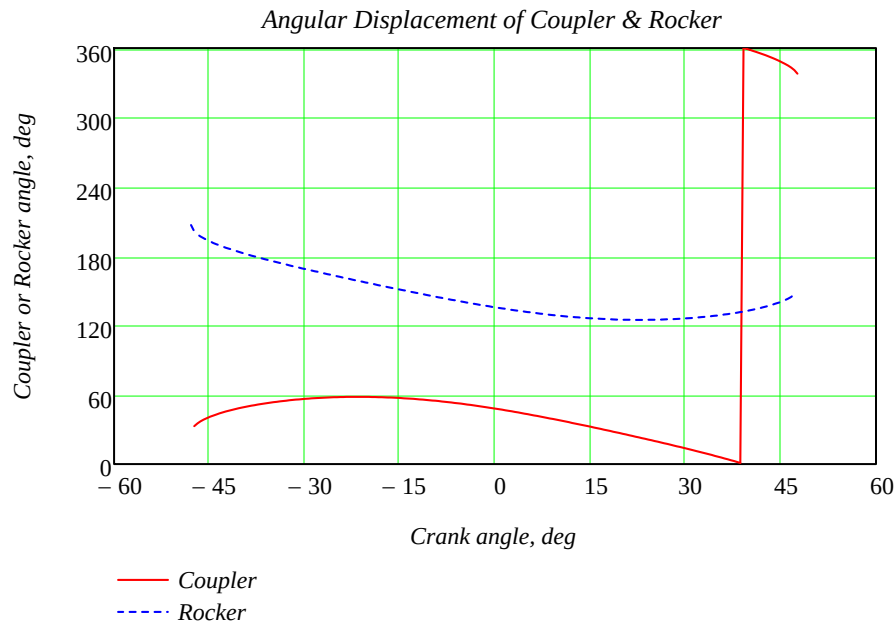
$$\theta_{31}(\theta_4) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_4), -E(\theta_4) - \sqrt{E(\theta_4)^2 - 4 \cdot D(\theta_4) \cdot F(\theta_4)} \right) \right)$$

8. If the calculated value of θ_3 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_4) := \text{if}(\theta_{31}(\theta_4) > 2 \cdot \pi, \theta_{31}(\theta_4) - 2 \cdot \pi, \theta_{31}(\theta_4))$$

$$\theta_3(\theta_4) := \text{if}(\theta_{32}(\theta_4) < 0, \theta_{32}(\theta_4) + 2 \cdot \pi, \theta_{32}(\theta_4))$$

9. Plot θ_3 and θ_2 as functions of the crank angle θ_4 (measured from the ground link).



10. Use equations 4.27 to define the x- and y-components of the vector $\mathbf{R}_P$.

$$\mathbf{R}_P := \mathbf{R}_B + \mathbf{R}_{PB}$$

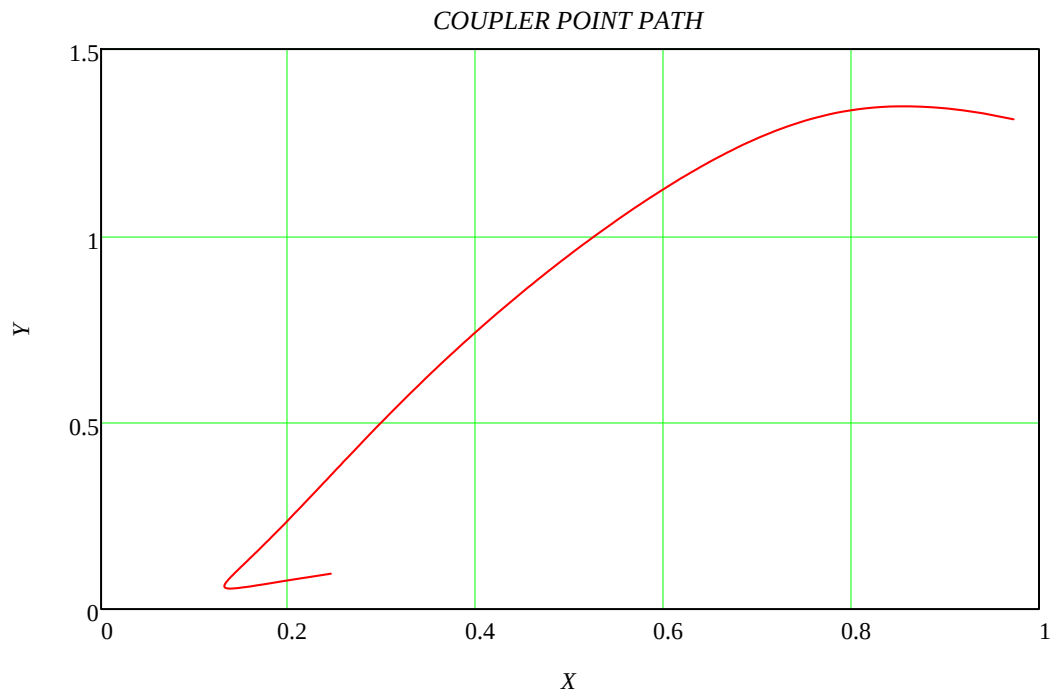
$$\mathbf{R_B} := a \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))^\mathbf{B}$$

$$\mathbf{R_{PB}} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))^\mathbf{B}$$

$$R_{Px}(\theta_4) := a \cdot \cos(\theta_4) + p \cdot \cos(\theta_3(\theta_4) + \delta)$$

$$R_{Py}(\theta_4) := a \cdot \sin(\theta_4) + p \cdot \sin(\theta_3(\theta_4) + \delta)$$

11. Plot the coordinates of the coupler point in the local xy coordinate system.



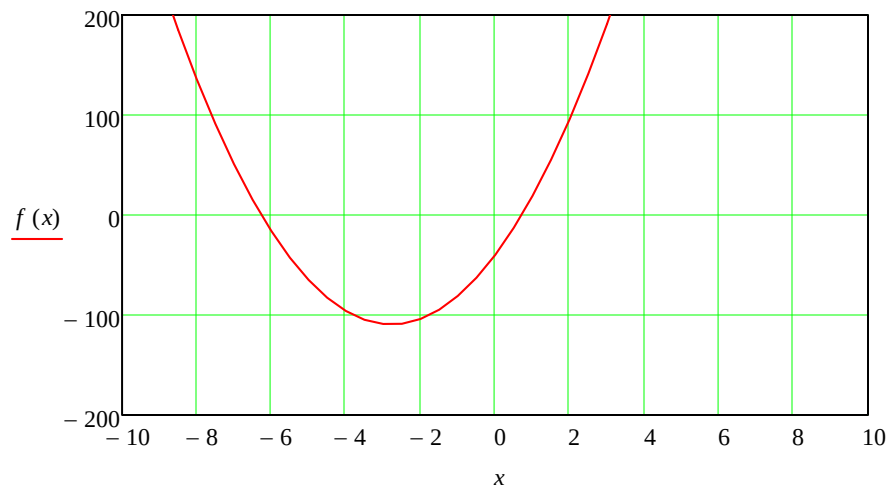
PROBLEM 4-31

Statement: Write a computer program (or use an equation solver such as *Mathcad*, *Matlab*, or *TKSolver*) to find the roots of $y = 9x^2 + 50x - 40$. Hint: Plot the function to determine good guess values.

Solution: See Mathcad file P0431.

1. Plot the function.

$$x := -10, -9.5..10 \quad f(x) := 9 \cdot x^2 + 50 \cdot x - 40$$



2. From the graph, make guesses of $x_1 := -6$, $x_2 := 1$
3. Define the program using the pseudo code in the text.

$$nroot(f, df, x) := \begin{array}{|l} y \leftarrow f(x) \\ \text{return } x \text{ if } |y| \leq TOL \\ \text{while } |y| > TOL \\ \quad \left| \begin{array}{l} x \leftarrow x - \frac{y}{df(x)} \\ y \leftarrow f(x) \end{array} \right. \end{array}$$

$$\text{where, } TOL = 1.000 \times 10^{-3}$$

4. Define the derivative of the given function. $df(x) := 18 \cdot x + 50$
5. Use the program to find the roots.

$$r_1 := nroot(f, df, x_1) \quad r_1 = -6.265$$

$$r_2 := nroot(f, df, x_2) \quad r_2 = 0.709$$

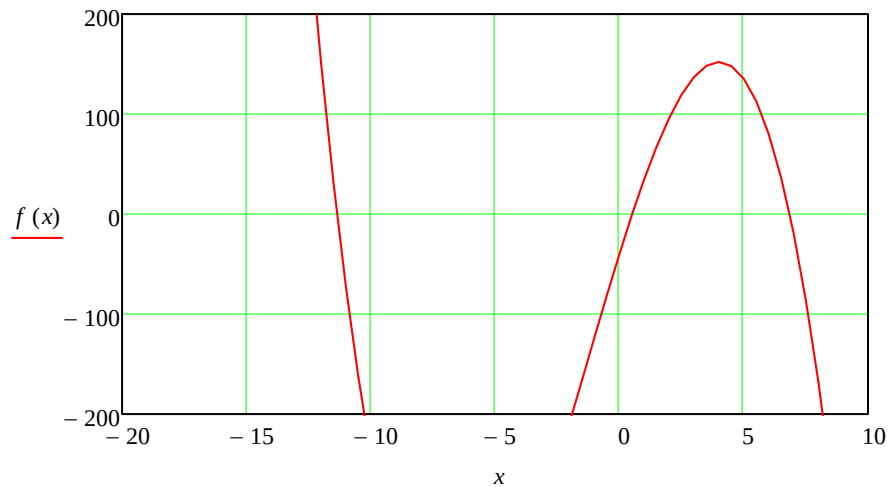
PROBLEM 4-32

Statement: Write a computer program (or use an equation solver such as *Mathcad*, *Matlab*, or *TKSolver*) to find the roots of $y = -x^3 - 4x^2 + 80x - 40$. Hint: Plot the function to determine good guess values.

Solution: See Mathcad file P0432.

1. Plot the function.

$$x := -15, -14.5 \dots 10 \quad f(x) := -x^3 - 4x^2 + 80x - 40$$



2. From the graph, make guesses of $x_1 := -11$, $x_2 := 0$, $x_3 := 7$
3. Define the program using the pseudo code in the text.

$$nroot(f, df, x) := \begin{array}{l} y \leftarrow f(x) \\ \text{return } x \text{ if } |y| \leq TOL \\ \text{while } |y| > TOL \\ \quad \left| \begin{array}{l} x \leftarrow x - \frac{y}{df(x)} \\ y \leftarrow f(x) \end{array} \right. \\ x \end{array}$$

$$\text{where, } TOL = 1.000 \times 10^{-3}$$

4. Define the derivative of the given function. $df(x) := -3x^2 - 8x + 80$
5. Use the program to find the roots.

$$r_1 := nroot(f, df, x_1) \quad r_1 = -11.355$$

$$r_2 := nroot(f, df, x_2) \quad r_2 = 0.515$$

$$r_3 := nroot(f, df, x_3) \quad r_3 = 6.840$$

PROBLEM 4-33

Statement: Figure 4-18 (p. 193) plots the cubic function from equation 4.34. Write a computer program (or use an equation solver such as *Mathcad*, *Matlab*, or *TKSolver*) to investigate the behavior of the Newton-Raphson algorithm as the initial guess value is varied from $x = 1.8$ to 2.5 in steps of 0.1 . Determine the guess value at which the convergence switches roots. Explain this root-switching phenomenon based on your observations from this exercise.

Solution: See Figure 4-18 and Mathcad file P0433.

1. Define the range of the guess value, the function, and the derivative of the function.

$$x_{\text{guess}} := 1.8, 1.9 \dots 2.5$$

$$f(x) := -x^3 - 2 \cdot x^2 + 50 \cdot x + 60 \quad df(x) := -3 \cdot x^2 - 4 \cdot x + 50$$

2. Define the root-finding program using the pseudo code in the text.

```

nroot(f, df, x) :=
| y ← f(x)
| return x if |y| ≤ TOL
| while |y| > TOL
|   | x ← x - y/df(x)
|   | y ← f(x)
| x

```

3. Find the roots that correspond to the guess values.

$$r(x_{\text{guess}}) := \text{nroot}(f, df, x_{\text{guess}})$$

$$\text{nextx}(x_{\text{guess}}) := x_{\text{guess}} - \frac{f(x_{\text{guess}})}{df(x_{\text{guess}})}$$

 $x_{\text{guess}} =$

	1
1	1.800
2	1.900
3	2.000
4	2.100
5	2.200
6	2.300
7	2.400
8	2.500

 $df(x_{\text{guess}}) =$

	1
1	33.080
2	31.570
3	30.000
4	28.370
5	26.680
6	24.930
7	23.120
8	21.250

 $\text{nextx}(x_{\text{guess}}) =$

	1
1	-2.362
2	-2.564
3	-2.800
4	-3.079
5	-3.410
6	-3.807
7	-4.289
8	-4.882

 $r(x_{\text{guess}}) =$

	1
1	-1.177
2	-1.177
3	-1.177
4	-1.177
5	-1.177
6	-1.177
7	6.740
8	-7.562

4. Find the roots of the derivative (values of x where the slope is zero).

$$ddf(x) := -6 \cdot x - 4$$

$$xz_1 := \text{nroot}(df, ddf, -5)$$

$$xz_1 = -4.803$$

$$xz_2 := \text{nroot}(df, ddf, 4)$$

$$xz_2 = 3.470$$

5. For guess values up to 2.3 , the root found is that whose slope is nearly the same as the slope of the function at the guess value. At 2.4 , the value of x that is calculated next results in a slope that throws the next x -value to the right of the extreme function value at $x = 3.470$. Subsequent estimates of x then follow down the slope to $x = 6.740$. At a guess value of 2.5 , the value of x that is calculated next is to the left of the extreme function value at $x = -4.803$. Subsequent estimates of x follow up the slope to $x = -7.562$.

PROBLEM 4-34

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular position of link 4 and the position of slider 6 in Figure 3-33 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 2.170$	Fourbar coupler (L_3)	$b := 2.067$
Output crank (L_4)	$c := 2.310$	Slider coupler (L_5)	$e := 5.40$
Fourbar ground link (L_1)	$d := 1.000$		

Solution: See Figure 3-33 and Mathcad file P0434.

1. This sixbar drag-link mechanism can be analyzed as a fourbar Grashof double crank in series with a slider-crank mechanism using the output of the fourbar, link 4, as the input to the slider-crank.
2. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit) in the global XY system.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 0.4608$$

$$K_2 = 0.4329$$

$$K_3 = 0.6755$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right) - 102 \cdot \text{deg}$$

4. If the calculated value of θ_4 is greater than 2π , subtract 2π from it and if it is negative, make it positive.

$$\theta_{42}(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

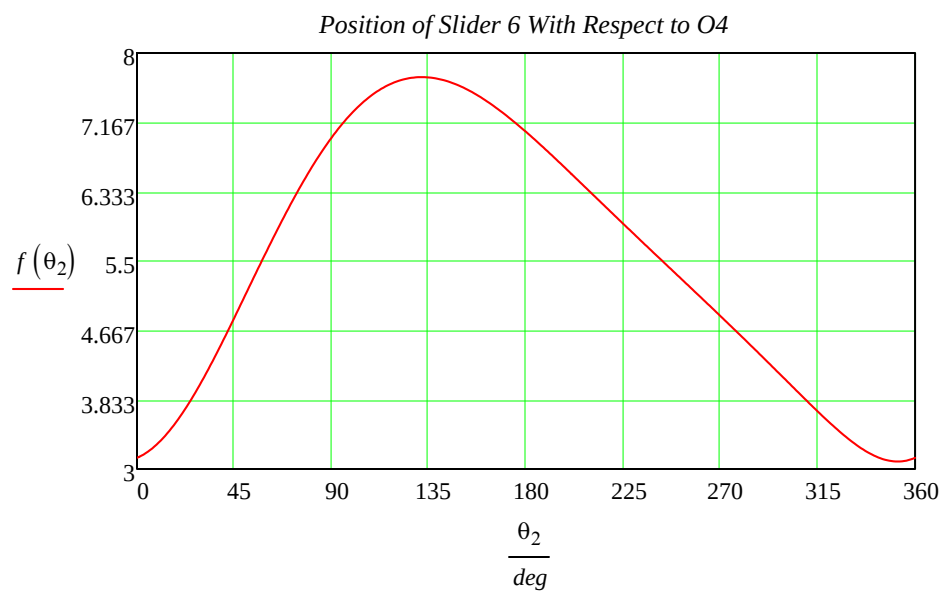
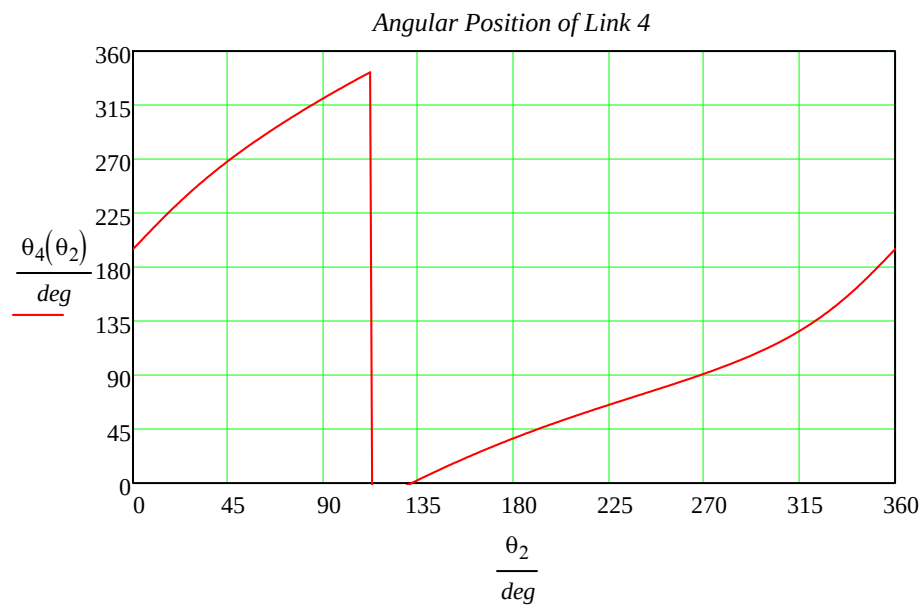
$$\theta_4(\theta_2) := \text{if}(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$$

5. Determine the slider-crank motion using equations 4.16 and 4.17 with θ_4 as the input angle.

$$\theta_5(\theta_2) := \text{asin} \left(\frac{-c \cdot \sin(\theta_4(\theta_2))}{e} \right) + \pi$$

$$f(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) - e \cdot \cos(\theta_5(\theta_2))$$

6. Plot the angular position of link 4 and the position of link 6 as functions of the angle of input link 2. See next page.



PROBLEM 4-35

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the transmission angles at points *B* and *C* of the linkage in Figure 3-33 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 2.170$	Fourbar coupler (L_3)	$b := 2.067$
Output crank (L_4)	$c := 2.310$	Slider coupler (L_5)	$e := 5.40$
Fourbar ground link (L_1)	$d := 1.000$		

Solution: See Figure 3-33 and Mathcad file P0435.

1. This sixbar drag-link mechanism can be analyzed as a fourbar Grashof double crank in series with a slider-crank mechanism using the output of the fourbar, link 4, as the input to the slider-crank.
2. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 0.4608 \quad K_2 = 0.4329 \quad K_3 = 0.6755$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. If the calculated value of θ_4 is greater than 2π , subtract 2π from it and if it is negative, make it positive.

$$\theta_{42}(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := \text{if}(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.4838 \quad K_5 = -0.5178$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. If the calculated value of θ_3 is greater than 2π , subtract 2π from it and if it is negative, make it positive.

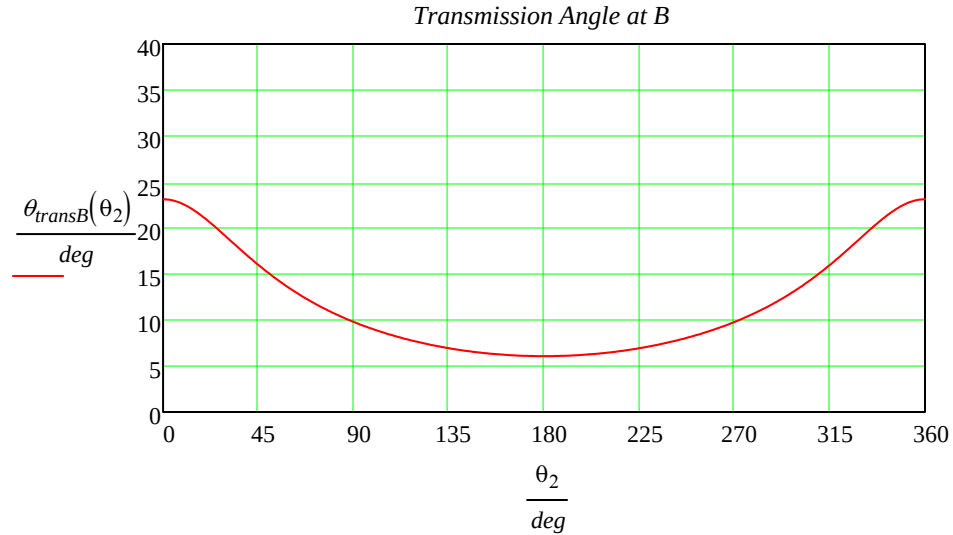
$$\theta_{32}(\theta_2) := \text{if}(\theta_{31}(\theta_2) > 2 \cdot \pi, \theta_{31}(\theta_2) - 2 \cdot \pi, \theta_{31}(\theta_2))$$

$$\theta_3(\theta_2) := \text{if}(\theta_{32}(\theta_2) < 0, \theta_{32}(\theta_2) + 2 \cdot \pi, \theta_{32}(\theta_2))$$

8. Calculate (using equations 4.32) and plot the transmission angle at B.

$$\theta_{transB1}(\theta_2) := |\theta_3(\theta_2) - \theta_4(\theta_2)|$$

$$\theta_{transB}(\theta_2) := \left| \text{if} \left(\theta_{transB1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_2), \theta_{transB1}(\theta_2) \right) \right|$$



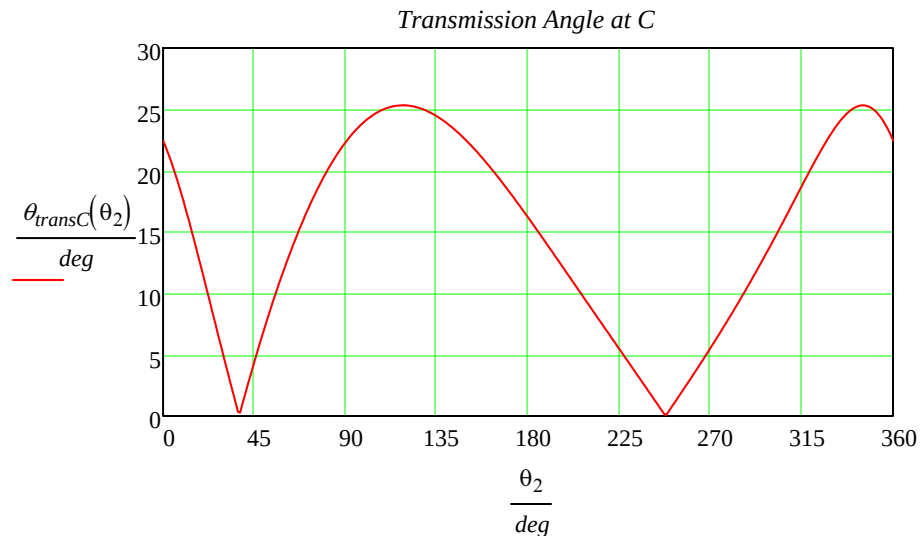
9. Determine the slider-crank motion using equations 4.16 and 4.17 with θ_4 as the input angle.

$$\theta_5(\theta_2) := \text{asin} \left(\frac{-c \cdot \sin(\theta_4(\theta_2))}{e} \right) + \pi$$

10. Calculate (using equations 4.32) and plot the transmission angle at C.

$$\theta_{transC1}(\theta_2) := |\theta_5(\theta_2)|$$

$$\theta_{transC}(\theta_2) := \left| \text{if} \left(\theta_{transC1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transC1}(\theta_2), \theta_{transC1}(\theta_2) \right) \right|$$



PROBLEM 4-36

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the path of the coupler point of the approximate straight-line linkage shown in Figure 3-29f (p. 142). Use program Fourbar to check your result.

Given: Link lengths:

Input (O_2A) $a := 1.000$ Coupler (AB) $b := 1.600$
 Rocker (O_4B) $c := 1.039$ Ground link $d := 1.200$
 Coupler point data: $p := 2.690$ $\delta := 0 \cdot \text{deg}$
 Coordinate rotation angle: $\alpha := -60 \cdot \text{deg}$

Solution: See Figure 3-29f and Mathcad file P0436.

1. Check the Grashof condition of the linkage and determine its Baker classification.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Condition(a,b,c,d) = "non-Grashof"

Since b (link 3) is the longest link and the linkage is non-Grashof, this is a Class 3 triple rocker. Using Figure 3-1a as a guide, determine the limiting values of θ_2 at the toggle positions.

For links 2 and 3 colinear:

$$\theta_{20} := \arccos\left[\frac{(b-a)^2 + d^2 - c^2}{2 \cdot d \cdot (b-a)}\right] + \pi \quad \theta_{20} = 240 \text{ deg}$$

For links 3 and 4 colinear:

$$\theta_{21} := \arccos\left[\frac{a^2 + d^2 - (b-c)^2}{2 \cdot d \cdot a}\right] \quad \theta_{21} = 27.683 \text{ deg} \quad \theta_{22} := -\theta_{21}$$

2. Define one cycle of the input crank (driving through the links 2-3 toggle position):

$$\theta_2 := \theta_{21}, \theta_{21} + 1 \cdot \text{deg} .. 360 \cdot \text{deg} + \theta_{22}$$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \quad K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_1 = 1.2000 \quad K_4 = 0.7500 \quad K_5 = -1.2251$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

4. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

5. Use equations 4.31 to define the x - and y -components of the vector $\mathbf{R}_p$.

$$\mathbf{R}_p := \mathbf{R}_A + \mathbf{R}_{pA}$$

$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{R}_{pA} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

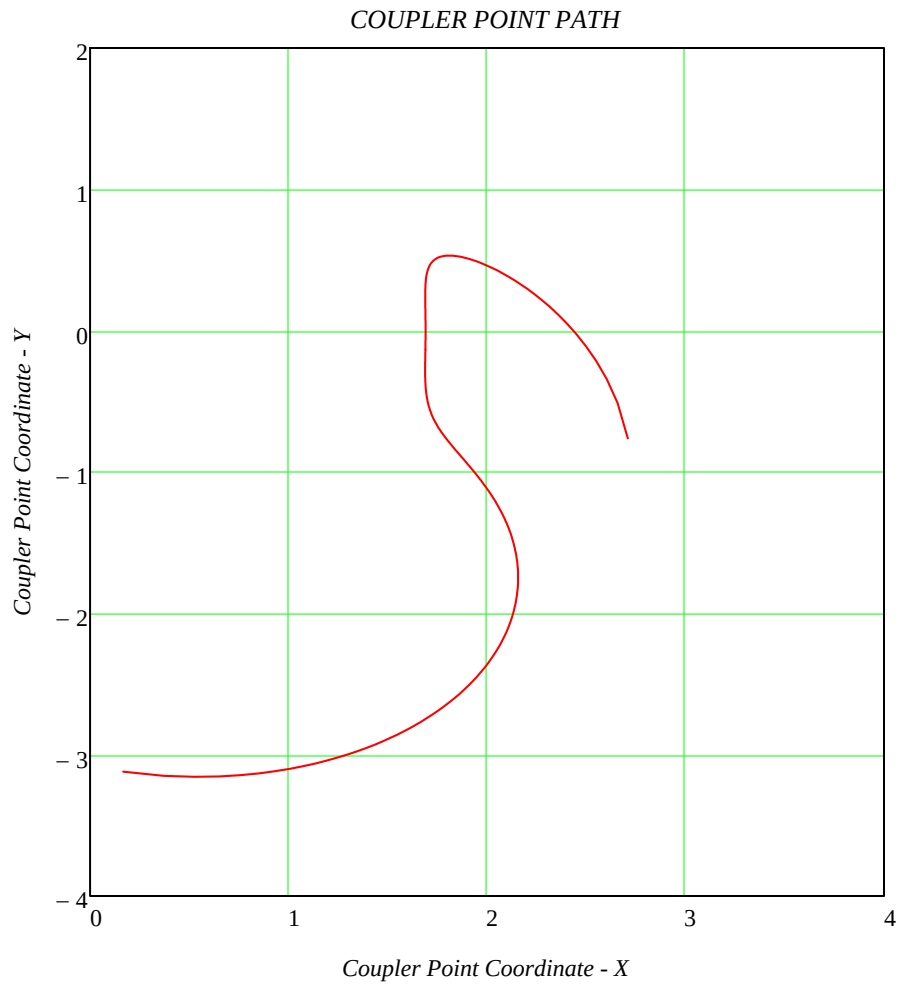
$$R_{px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta)$$

$$R_{py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

6. Plot the coordinates of the coupler point in the global X,Y coordinate system using equations 4.0b to rotate the local coordinates to a global frame.

$$P_X(\theta_2) := R_{px}(\theta_2) \cdot \cos(\alpha) - R_{py}(\theta_2) \cdot \sin(\alpha)$$

$$P_Y(\theta_2) := R_{px}(\theta_2) \cdot \sin(\alpha) + R_{py}(\theta_2) \cdot \cos(\alpha)$$



PROBLEM 4-37

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular position of link 6 in Figure 3-34 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$g := 1.556$	First coupler (L_3)	$f := 4.248$
First rocker (L_4)	$c := 2.125$	Third coupler (CD)	$b := 2.158$
Output rocker (L_6)	$a := 1.542$	Second ground link (O_4O_6)	$d := 1.000$
Angle CDB	$\delta := 36 \cdot \text{deg}$	Distance (BD)	$p := 3.274$
O_2O_4 ground link offsets:	$h_X := 3.259$	$h_Y := 2.905$	

Solution: See Figure 3-34 and Mathcad file P0437.

1. Calculate the length of the O_2O_4 ground link and the angle that it makes with the global XY system.

$$h := \sqrt{h_X^2 + h_Y^2} \quad h = 4.366 \quad \gamma := -\text{atan}\left(\frac{h_Y}{h_X}\right) \quad \gamma = -41.713 \text{ deg}$$

2. Calculate the distance BC on link 5. This is the length of vector $\mathbf{R}_{51}$. Also, calculate the angle between vectors $\mathbf{R}_{51}$ and $\mathbf{R}_{52}$

$$e := \sqrt{b^2 + p^2 - 2 \cdot b \cdot p \cdot \cos(\delta)} \quad \text{Second coupler (BC)} \quad e = 1.986$$

$$\alpha := \text{acos}\left(\frac{b^2 + e^2 - p^2}{2 \cdot b \cdot e}\right) \quad \alpha = 104.305 \text{ deg}$$

$$\beta := \pi - \alpha \quad \beta = 75.695 \text{ deg}$$

3. This is a Stephenson's sixbar linkage similar to the one shown in Figure 4-13. Since the output link 6 is known to rotate 180 deg and return for a full revolution of link 2 we can use links 6, 5, and 4 as a first-stage fourbar with known input (link 6) and then solve for vector loop equations to get the corresponding motion of link 2.
4. Define the rotation of the output crank: $\theta_6 := 90 \cdot \text{deg}, 91 \cdot \text{deg} \dots 270 \cdot \text{deg}$
5. Use equations 4.8a and 4.10 to calculate θ_4 in the local xy coordinate system as a function of θ_6 (for the crossed circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 0.6485 \quad K_2 = 0.4706 \quad K_3 = 0.4938$$

$$A(\theta_6) := \cos(\theta_6) - K_1 - K_2 \cdot \cos(\theta_6) + K_3$$

$$B(\theta_6) := -2 \cdot \sin(\theta_6) \quad C(\theta_6) := K_1 - (K_2 + 1) \cdot \cos(\theta_6) + K_3$$

$$\theta_{41}(\theta_6) := 2 \cdot \left(\text{atan2}\left(2 \cdot A(\theta_6), -B(\theta_6) + \sqrt{B(\theta_6)^2 - 4 \cdot A(\theta_6) \cdot C(\theta_6)}\right) \right)$$

6. Use equations 4.12 and 4.13 to calculate θ_5 in the local xy coordinate system as a function of θ_6 (for the crossed circuit).

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 0.463 \quad K_5 = -0.529$$

$$D(\theta_6) := \cos(\theta_6) - K_1 + K_4 \cdot \cos(\theta_6) + K_5 \quad E(\theta_6) := -2 \cdot \sin(\theta_6)$$

$$F(\theta_6) := K_1 + (K_4 - 1) \cdot \cos(\theta_6) + K_5$$

$$\theta_{521}(\theta_6) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_6), -E(\theta_6) + \sqrt{E(\theta_6)^2 - 4 \cdot D(\theta_6) \cdot F(\theta_6)} \right) \right)$$

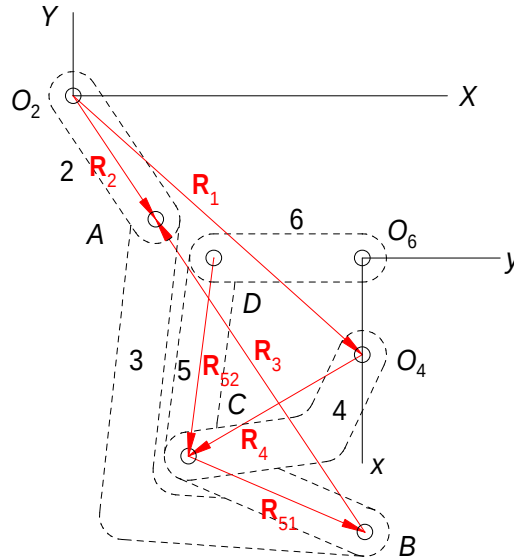
7. Transform the angles for θ_4 and θ_{52} into the global XY system and define θ_{51} in the global system.

$$\theta_4(\theta_6) := \theta_{41}(\theta_6) - 90 \cdot \text{deg}$$

$$\theta_{52}(\theta_6) := \theta_{521}(\theta_6) - 90 \cdot \text{deg}$$

$$\theta_{51}(\theta_6) := \theta_{52}(\theta_6) + \beta$$

8. Define a vector loop for the remaining links and solve the resulting vector equation by separating it into real and imaginary parts using the method of section 4.5 and the identities of equations 4.9.



$\mathbf{R}_1 + \mathbf{R}_4 + \mathbf{R}_{51} + \mathbf{R}_3 - \mathbf{R}_2 = 0$. In this equation the unknowns are θ_3 and θ_2 . Following the method of Section 4.5, substitute the complex number notation for each position vector and separate the resulting equations into real and imaginary parts:

$$f \cdot \cos(\theta_3) = g \cdot \cos(\theta_2) - G_1$$

$$f \cdot \sin(\theta_3) = g \cdot \sin(\theta_2) - G_2$$

where $G_1(\theta_6) := h \cdot \cos(\gamma) + c \cdot \cos(\theta_4(\theta_6)) + e \cdot \cos(\theta_{51}(\theta_6))$

$$G_2(\theta_6) := h \cdot \sin(\gamma) + (c \cdot \sin(\theta_4(\theta_6)) + e \cdot \sin(\theta_{51}(\theta_6)))$$

9. Solve these equations in the manner of equations 4.11 and 4.12 using the identities of equations 4.9 gives:

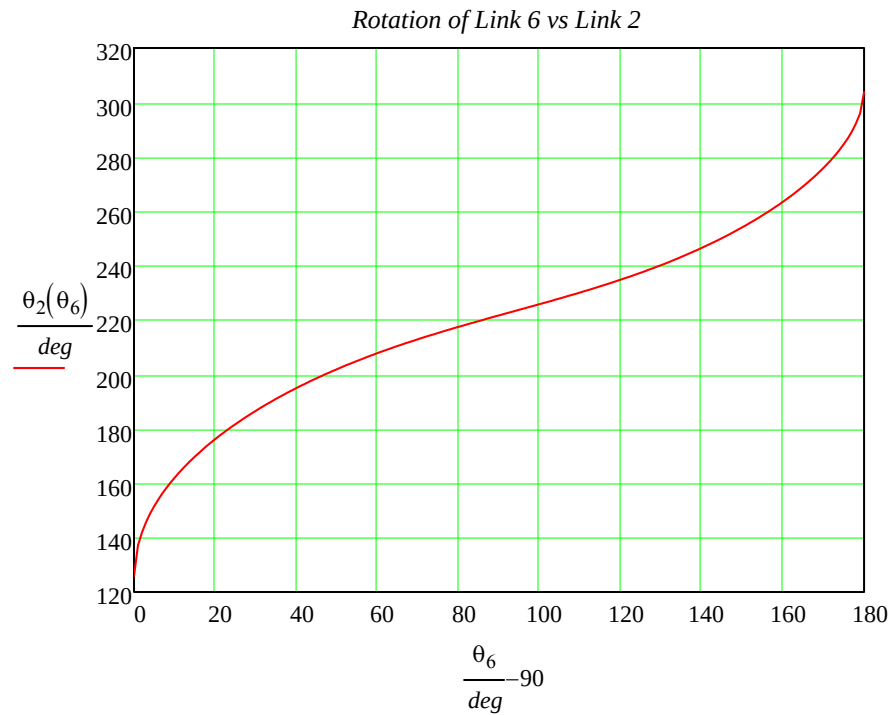
$$G_3(\theta_6) := \frac{g^2 + G_1(\theta_6)^2 + G_2(\theta_6)^2 - f^2}{2 \cdot g}$$

$$A'(\theta_6) := -G_1(\theta_6) - G_3(\theta_6) \quad B'(\theta_6) := 2 \cdot G_2(\theta_6) \quad C'(\theta_6) := G_1(\theta_6) - G_3(\theta_6)$$

$$\tan\left(\frac{\theta_2}{2}\right) = \frac{-B' + \sqrt{B'^2 - 4 \cdot A' \cdot C'}}{2 \cdot A'}$$

$$\theta_2(\theta_6) := 2 \cdot \left(\text{atan2}\left(2 \cdot A'(\theta_6), -B'(\theta_6) + \sqrt{B'(\theta_6)^2 - 4 \cdot A'(\theta_6) \cdot C'(\theta_6)}\right) \right)$$

10. Plot θ_6 vs θ_2 in global XY coordinates:



PROBLEM 4-38

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the transmission angles at points *B*, *C*, and *D* of the linkage in Figure 3-34 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$g := 1.556$	First coupler (L_3)	$f := 4.248$
First rocker (L_4)	$c := 2.125$	Third coupler (CD)	$b := 2.158$
Output rocker (L_6)	$a := 1.542$	Second ground link (O_4O_6)	$d := 1.000$
Angle CDB	$\delta := 36 \cdot \text{deg}$	Distance (BD)	$p := 3.274$
O_2O_4 ground link offsets:	$h_X := 3.259$	$h_Y := 2.905$	

Solution: See Figure 3-34 and Mathcad file P0438.

1. Calculate the length of the O_2O_4 ground link and the angle that it makes with the global XY system.

$$h := \sqrt{h_X^2 + h_Y^2} \quad h = 4.366 \quad \gamma := -\text{atan}\left(\frac{h_Y}{h_X}\right) \quad \gamma = -41.713 \text{ deg}$$

2. Calculate the distance BC on link 5. This is the length of vector $\mathbf{R}_{51}$. Also, calculate the angle between vectors $\mathbf{R}_{51}$ and $\mathbf{R}_{52}$

$$e := \sqrt{b^2 + p^2 - 2 \cdot b \cdot p \cdot \cos(\delta)} \quad \text{Second coupler (BC)} \quad e = 1.986$$

$$\alpha := \text{acos}\left(\frac{b^2 + e^2 - p^2}{2 \cdot b \cdot e}\right) \quad \alpha = 104.305 \text{ deg}$$

$$\beta := \pi - \alpha \quad \beta = 75.695 \text{ deg}$$

3. This is a Stephenson's sixbar linkage similar to the one shown in Figure 4-13. Since the output link 6 is known to rotate 180 deg and return for a full revolution of link 2 we can use links 6, 5, and 4 as a first-stage fourbar with known input (link 6) and then solve for vector loop equations to get the corresponding motion of link 2.
4. Define the rotation of the output crank: $\theta_6 := 90 \cdot \text{deg}, 91 \cdot \text{deg} \dots 270 \cdot \text{deg}$
5. Use equations 4.8a and 4.10 to calculate θ_4 in the local xy coordinate system as a function of θ_6 (for the crossed circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 0.6485 \quad K_2 = 0.4706 \quad K_3 = 0.4938$$

$$A(\theta_6) := \cos(\theta_6) - K_1 - K_2 \cdot \cos(\theta_6) + K_3$$

$$B(\theta_6) := -2 \cdot \sin(\theta_6) \quad C(\theta_6) := K_1 - (K_2 + 1) \cdot \cos(\theta_6) + K_3$$

$$\theta_{41}(\theta_6) := 2 \cdot \left(\text{atan2}\left(2 \cdot A(\theta_6), -B(\theta_6) + \sqrt{B(\theta_6)^2 - 4 \cdot A(\theta_6) \cdot C(\theta_6)}\right) \right)$$

6. Use equations 4.12 and 4.13 to calculate θ_5 in the local xy coordinate system as a function of θ_6 (for the crossed circuit).

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 0.463 \quad K_5 = -0.529$$

$$D(\theta_6) := \cos(\theta_6) - K_1 + K_4 \cdot \cos(\theta_6) + K_5 \quad E(\theta_6) := -2 \cdot \sin(\theta_6)$$

$$F(\theta_6) := K_1 + (K_4 - 1) \cdot \cos(\theta_6) + K_5$$

$$\theta_{521}(\theta_6) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_6), -E(\theta_6) + \sqrt{E(\theta_6)^2 - 4 \cdot D(\theta_6) \cdot F(\theta_6)} \right) \right)$$

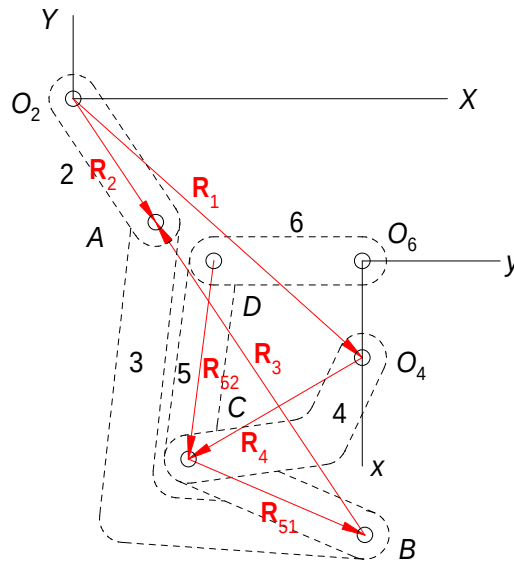
7. Transform the angles for θ_4 and θ_{52} into the global XY system and define θ_{51} in the global system.

$$\theta_4(\theta_6) := \theta_{41}(\theta_6) - 90 \cdot \text{deg}$$

$$\theta_{52}(\theta_6) := \theta_{521}(\theta_6) - 90 \cdot \text{deg}$$

$$\theta_{51}(\theta_6) := \theta_{52}(\theta_6) + \beta$$

8. Define a vector loop for the remaining links and solve the resulting vector equation by separating it into real and imaginary parts using the method of section 4.5 and the identities of equations 4.9.



$\mathbf{R}_1 + \mathbf{R}_4 + \mathbf{R}_{51} + \mathbf{R}_3 - \mathbf{R}_2 = 0$. In this equation the unknowns are θ_3 and θ_2 . Following the method of Section 4.5, substitute the complex number notation for each position vector and separate the resulting equations into real and imaginary parts:

$$g \cdot \cos(\theta_2) = f \cdot \cos(\theta_3) + G_1$$

$$g \cdot \sin(\theta_2) = f \cdot \sin(\theta_3) + G_2$$

where $G_1(\theta_6) := h \cdot \cos(\gamma) + c \cdot \cos(\theta_4(\theta_6)) + e \cdot \cos(\theta_{51}(\theta_6))$

$$G_2(\theta_6) := h \cdot \sin(\gamma) + (c \cdot \sin(\theta_4(\theta_6)) + e \cdot \sin(\theta_{51}(\theta_6)))$$

9. Solve these equations for θ_2 in the manner of equations 4.11 and 4.12 using the identities of equations 4.9 gives:

$$G_3(\theta_6) := \frac{g^2 + G_1(\theta_6)^2 + G_2(\theta_6)^2 - f^2}{2 \cdot g}$$

$$A'(\theta_6) := -G_1(\theta_6) - G_3(\theta_6) \quad B'(\theta_6) := 2 \cdot G_2(\theta_6) \quad C'(\theta_6) := G_1(\theta_6) - G_3(\theta_6)$$

$$\tan\left(\frac{\theta_2}{2}\right) = \frac{-B' + \sqrt{B'^2 - 4 \cdot A' \cdot C'}}{2 \cdot A'}$$

$$\theta_2(\theta_6) := 2 \cdot \left(\text{atan2}\left(2 \cdot A'(\theta_6), -B'(\theta_6) + \sqrt{B'(\theta_6)^2 - 4 \cdot A'(\theta_6) \cdot C'(\theta_6)}\right) \right)$$

10. Solve these equations for θ_3 in the manner of equations 4.11 and 4.12 using the identities of equations 4.9 gives:

$$G_4(\theta_6) := \frac{g^2 - G_1(\theta_6)^2 - G_2(\theta_6)^2 - f^2}{2 \cdot f}$$

$$D'(\theta_6) := -G_1(\theta_6) - G_4(\theta_6) \quad E'(\theta_6) := 2 \cdot G_2(\theta_6) \quad F'(\theta_6) := G_1(\theta_6) - G_4(\theta_6)$$

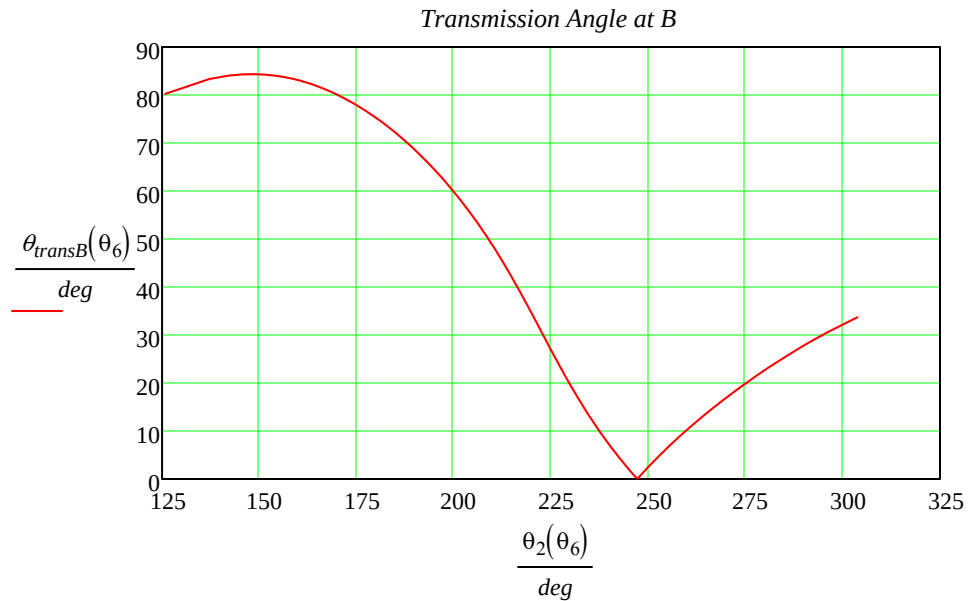
$$\tan\left(\frac{\theta_3}{2}\right) = \frac{-E' + \sqrt{E'^2 - 4 \cdot D' \cdot F'}}{2 \cdot D'}$$

$$\theta_3(\theta_6) := 2 \cdot \left(\text{atan2}\left(2 \cdot D'(\theta_6), -E'(\theta_6) + \sqrt{E'(\theta_6)^2 - 4 \cdot D'(\theta_6) \cdot F'(\theta_6)}\right) \right)$$

11. Calculate (using equations 4.32) and plot the transmission angle at B.

$$\theta_{transB1}(\theta_6) := |\theta_3(\theta_6) - \theta_{51}(\theta_6)|$$

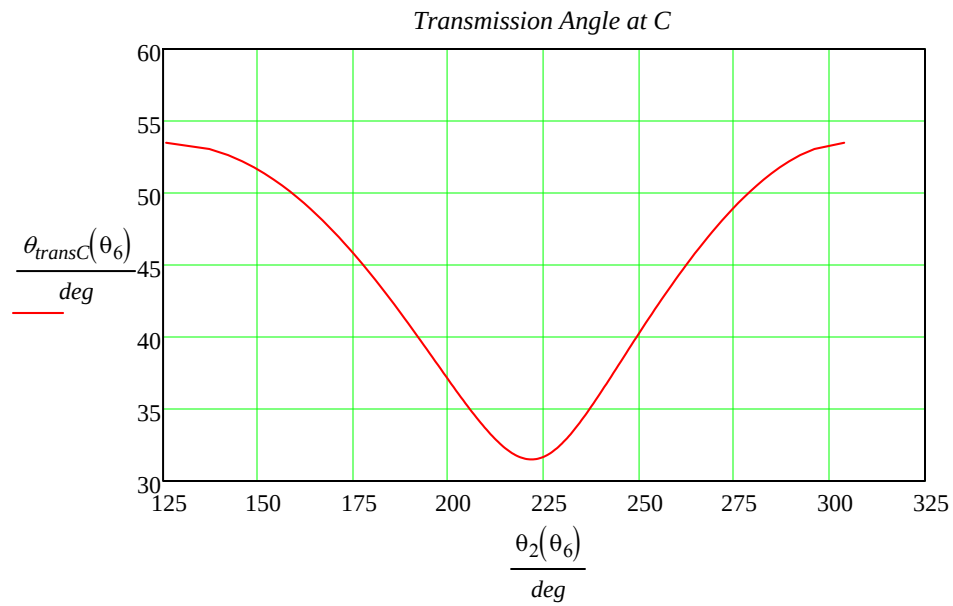
$$\theta_{transB}(\theta_6) := \left| \text{if}\left(\theta_{transB1}(\theta_6) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_6), \theta_{transB1}(\theta_6)\right) \right|$$



12. Calculate (using equations 4.32) and plot the transmission angle at C.

$$\theta_{transC1}(\theta_6) := |\theta_4(\theta_6) - \theta_{51}(\theta_6)|$$

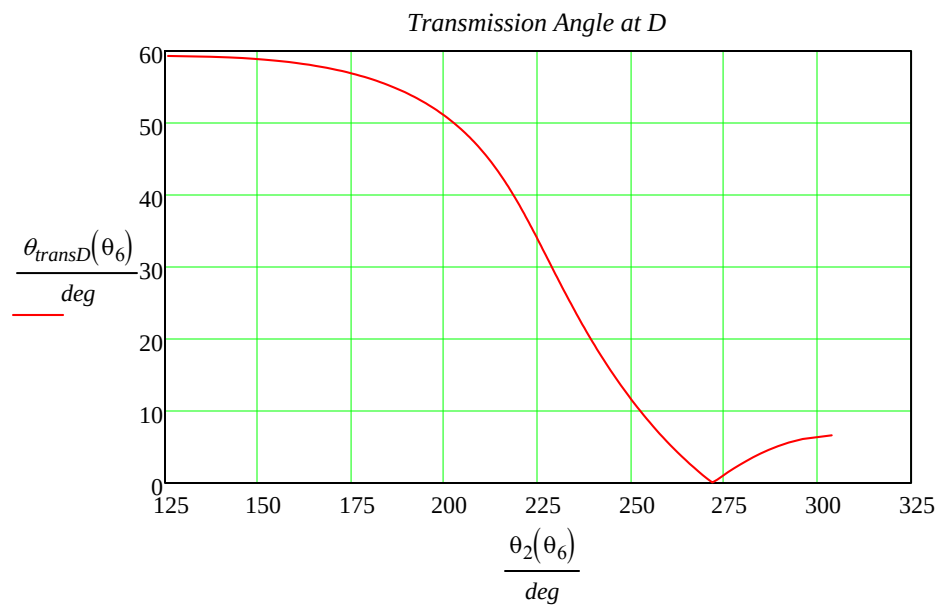
$$\theta_{transC}(\theta_6) := \left| \text{if} \left(\theta_{transC1}(\theta_6) > \frac{\pi}{2}, \pi - \theta_{transC1}(\theta_6), \theta_{transC1}(\theta_6) \right) \right|$$



13. Calculate (using equations 4.32) and plot the transmission angle at D.

$$\theta_{transD1}(\theta_6) := |\theta_6 - \theta_{s2}(\theta_6)|$$

$$\theta_{transD}(\theta_6) := \left| \text{if} \left(\theta_{transD1}(\theta_6) > \frac{\pi}{2}, \pi - \theta_{transD1}(\theta_6), \theta_{transD1}(\theta_6) \right) \right|$$



PROBLEM 4-39

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the path of the coupler point of the approximate straight-line linkage shown in Figure 3-29g (p. 142). Use program Fourbar to check your result.

Given: Link lengths:

$$\text{Input } (O_2A) \quad a := 1.000 \quad \text{Coupler } (AB) \quad b := 1.200$$

$$\text{Rocker } (O_4B) \quad c := 1.167 \quad \text{Ground link} \quad d := 2.305$$

$$\text{Coupler point data:} \quad p := 1.5 \quad \delta := 180 \cdot \text{deg}$$

$$\text{Coordinate rotation angle:} \quad \alpha := -30 \cdot \text{deg}$$

Solution: See Figure 3-29g and Mathcad file P0439.

1. Check the Grashof condition of the linkage and determine its Baker classification.

$$\begin{aligned} \text{Condition}(a, b, c, d) := & \left\{ \begin{array}{l} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array} \right. \end{aligned}$$

$$\text{Condition}(a, b, c, d) = \text{"non-Grashof"}$$

Since d (link 1) is the longest link and the linkage is non-Grashof, this is a Class 1 triple rocker. Using Figure 3-1a as a guide, determine the limiting values of θ_2 at the toggle positions.

For links 3 and 4 colinear:

$$\theta_{21} := \arccos \left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot d \cdot a} \right] \quad \theta_{21} = 81.136 \text{ deg} \quad \theta_{22} := -\theta_{21}$$

1. Define one cycle of the input crank:

$$\theta_2 := \theta_{22}, \theta_{22} + 0.5 \cdot \text{deg} .. \theta_{21}$$

2. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \quad K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_1 = 2.3050 \quad K_4 = 1.9208 \quad K_5 = -2.6630$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

3. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

4. Use equations 4.31 to define the x - and y -components of the vector $\mathbf{R}_p$.

$$\mathbf{R_P} := \mathbf{R_A} + \mathbf{R_{PA}}$$

$$\mathbf{R_A} := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{R_{PA}} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

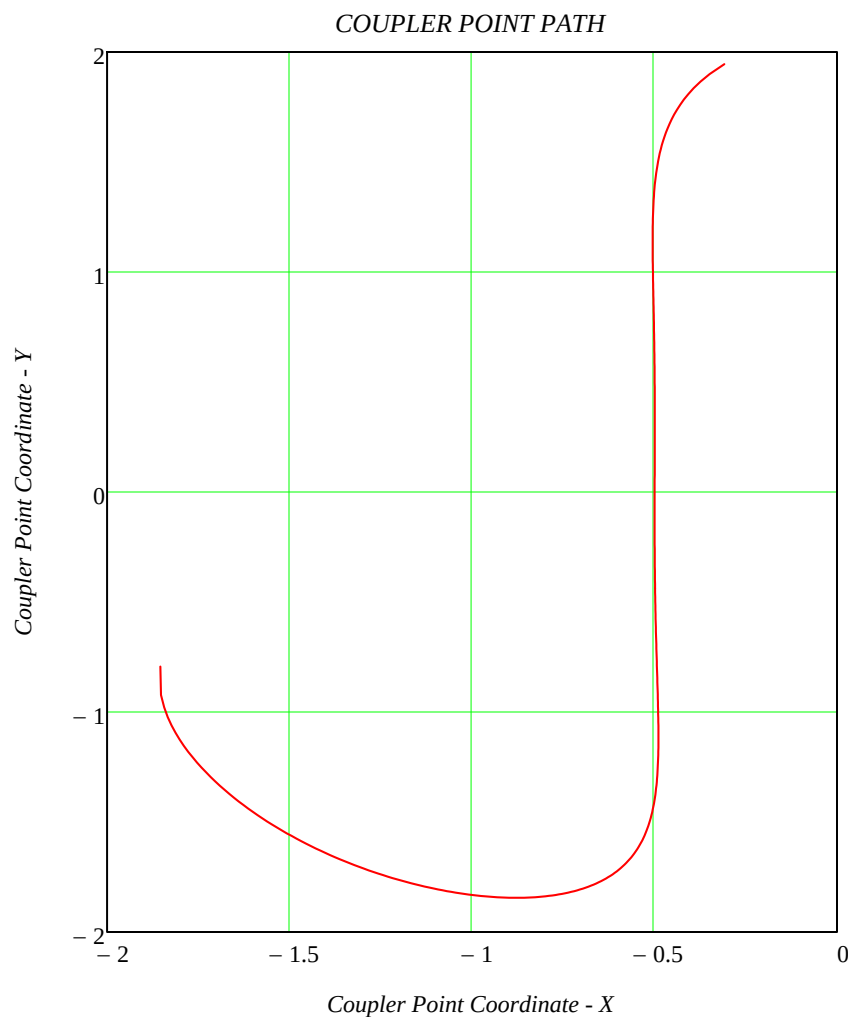
$$R_{Px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta)$$

$$R_{Py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

5. Plot the coordinates of the coupler point in the global X,Y coordinate system using equations 4.0b to rotate the local coordinates to a global frame.

$$P_X(\theta_2) := R_{Px}(\theta_2) \cdot \cos(\alpha) - R_{Py}(\theta_2) \cdot \sin(\alpha)$$

$$P_Y(\theta_2) := R_{Px}(\theta_2) \cdot \sin(\alpha) + R_{Py}(\theta_2) \cdot \cos(\alpha)$$



PROBLEM 4-40

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular position of link 6 in Figure 3-35 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 1.00$	First coupler (L_3)	$b := 3.80$
Common rocker (O_4B)	$c := 1.29$	Second coupler (L_5)	$b' := 1.29$
First ground link (O_2O_4)	$d := 3.86$	Common rocker (O_4C)	$a' := 1.43$
Output rocker (L_6)	$c' := 0.77$	Second ground link (O_4O_6)	$d' := 0.78$
Angle BO_4C	$\alpha := 157 \cdot \text{deg}$		

Solution: See Figure P3-35 and Mathcad file P0440.

1. This sixbar drag-link mechanism can be analyzed as two fourbar linkages in series that use the output of the first fourbar, link 4, as the input to the second fourbar.
2. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 3.8600 \qquad K_2 = 2.9922 \qquad K_3 = 1.2107$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Use equations 4.8a and 4.10 to calculate θ_6 as a function of θ_2 (for the open circuit).

$$\text{Input angle to second fourbar: } \theta_{42}(\theta_2) := \theta_{41}(\theta_2) + \alpha$$

$$K'_1 := \frac{d'}{a'} \qquad K'_2 := \frac{d'}{c'} \qquad K'_3 := \frac{a'^2 - b'^2 + c'^2 + d'^2}{2 \cdot a' \cdot c'}$$

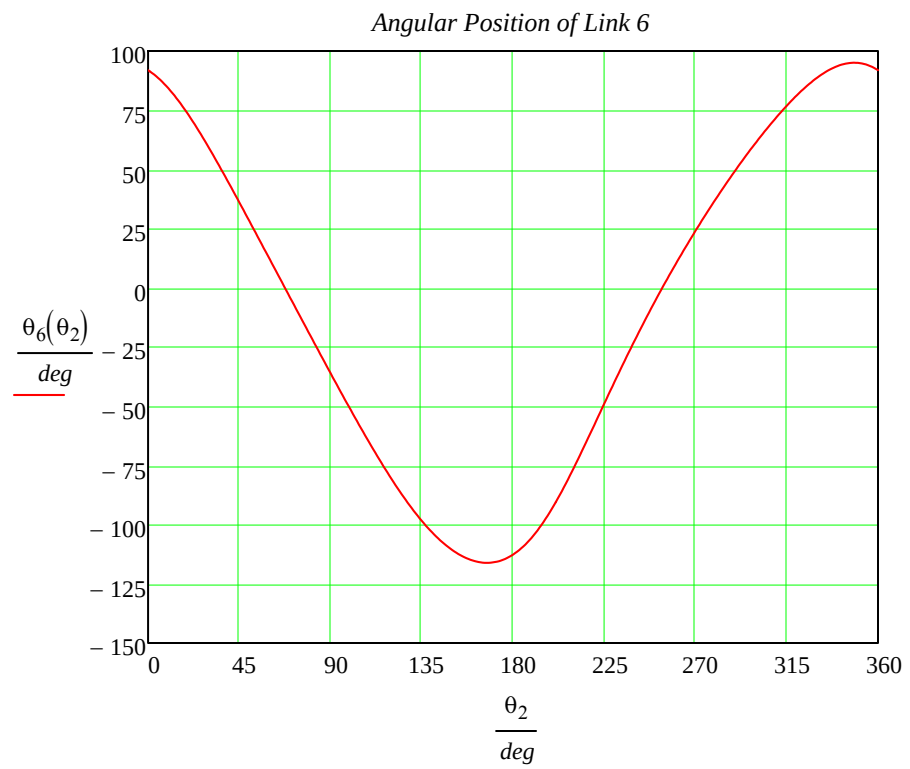
$$K'_1 = 0.5455 \qquad K'_2 = 1.0130 \qquad K'_3 = 0.7184$$

$$A'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K'_1 - K'_2 \cdot \cos(\theta_{42}(\theta_2)) + K'_3$$

$$B'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2)) \qquad C'(\theta_2) := K'_1 - (K'_2 + 1) \cdot \cos(\theta_{42}(\theta_2)) + K'_3$$

$$\theta_6(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A'(\theta_2), -B'(\theta_2) + \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)} \right) \right)$$

5. Plot the angular position of link 6 as a function of the angle of input link 2.



PROBLEM 4-41

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the transmission angles at points *B*, *C*, and *D* of the linkage in Figure 3-35 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 1.00$	First coupler (L_3)	$b := 3.80$
Common rocker (O_4B)	$c := 1.29$	Second coupler (L_5)	$b' := 1.29$
First ground link (O_2O_4)	$d := 3.86$	Common rocker (O_4C)	$a' := 1.43$
Output rocker (L_6)	$c' := 0.77$	Second ground link (O_4O_6)	$d' := 0.78$
Angle BO_4C	$\alpha := 157 \cdot \text{deg}$		

Solution: See Figure P3-35 and Mathcad file P0441.

1. This sixbar drag-link mechanism can be analyzed as two fourbar linkages in series that use the output of the first fourbar, link 4, as the input to the second fourbar.
2. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 3.8600 \qquad K_2 = 2.9922 \qquad K_3 = 1.2107$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Use equations 4.12 and 4.13 to calculate θ_3 as a function of θ_2 (for the crossed circuit).

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 1.016 \qquad K_5 = -3.773$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

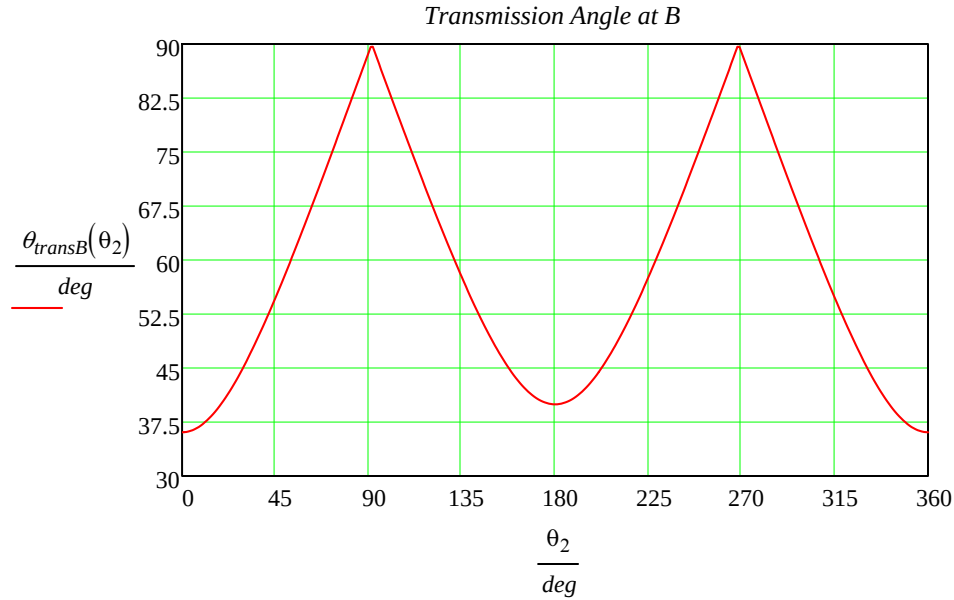
$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

5. Calculate (using equations 4.32) and plot the transmission angle at *B*.

$$\theta_{transB1}(\theta_2) := |\theta_3(\theta_2) - \theta_{41}(\theta_2)|$$

$$\theta_{transB}(\theta_2) := \left| \text{if} \left(\theta_{transB1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_2), \theta_{transB1}(\theta_2) \right) \right|$$



6. Use equations 4.8a and 4.10 to calculate θ_6 as a function of θ_2 (for the open circuit).

$$\text{Input angle to second fourbar: } \theta_{42}(\theta_2) := \theta_{41}(\theta_2) + \alpha$$

$$K'_1 := \frac{d'}{a'} \quad K'_2 := \frac{d'}{c'} \quad K'_3 := \frac{a'^2 - b'^2 + c'^2 + d'^2}{2 \cdot a' \cdot c'}$$

$$K'_1 = 0.5455$$

$$K'_2 = 1.0130$$

$$K'_3 = 0.7184$$

$$A'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K'_1 - K'_2 \cdot \cos(\theta_{42}(\theta_2)) + K'_3$$

$$B'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2)) \quad C'(\theta_2) := K'_1 - (K'_2 + 1) \cdot \cos(\theta_{42}(\theta_2)) + K'_3$$

$$\theta_6(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A'(\theta_2), -B'(\theta_2) - \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)} \right) \right)$$

7. Use equations 4.12 and 4.13 to calculate θ_5 as a function of θ_2 (for the crossed circuit).

$$K'_4 := \frac{d'}{b'} \quad K'_5 := \frac{c'^2 - d'^2 - a'^2 - b'^2}{2 \cdot a' \cdot b'}$$

$$K'_4 = 0.605$$

$$K'_5 = -1.010$$

$$D'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K'_1 + K'_4 \cdot \cos(\theta_{42}(\theta_2)) + K'_5 \quad E'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

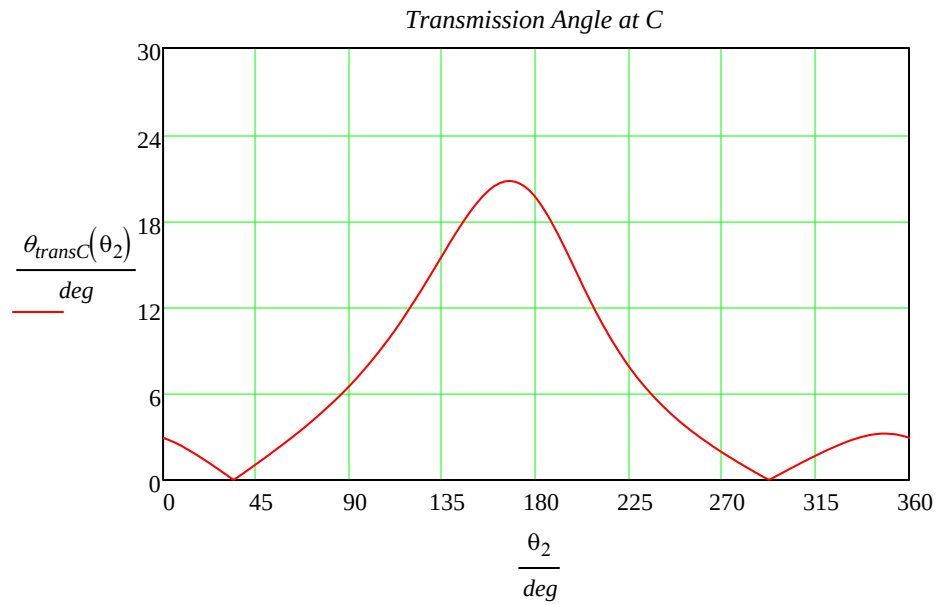
$$F'(\theta_2) := K'_1 + (K'_4 - 1) \cdot \cos(\theta_{42}(\theta_2)) + K'_5$$

$$\theta_5(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D'(\theta_2), -E'(\theta_2) + \sqrt{E'(\theta_2)^2 - 4 \cdot D'(\theta_2) \cdot F'(\theta_2)} \right) \right)$$

8. Calculate (using equations 4.32) and plot the transmission angle at C.

$$\theta_{transC1}(\theta_2) := |\theta_5(\theta_2) - \theta_{42}(\theta_2)|$$

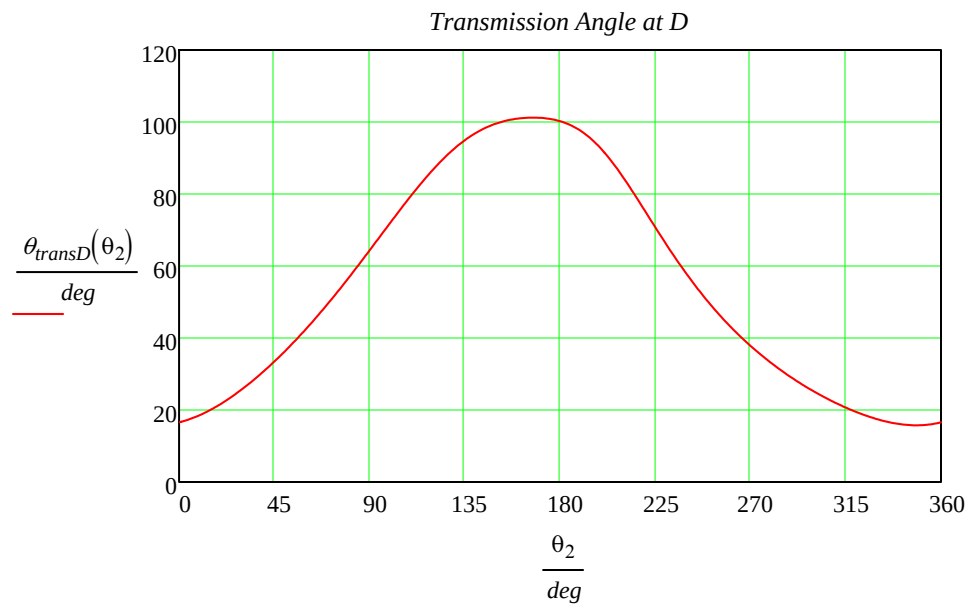
$$\theta_{transC}(\theta_2) := \left| \text{if} \left(\theta_{transC1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transC1}(\theta_2), \theta_{transC1}(\theta_2) \right) \right|$$



9. Calculate (using equations 4.32) and plot the transmission angle at D.

$$\theta_{transD1}(\theta_2) := |\theta_5(\theta_2) - \theta_6(\theta_2)|$$

$$\theta_{transD}(\theta_2) := \left| \text{if} \left(\theta_{transD1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transD1}(\theta_2), \theta_{transD1}(\theta_2) \right) \right|$$



PROBLEM 4-42

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the path of the coupler point of the approximate straight-line linkage shown in Figure 3-29h (p. 142). Use program Fourbar to check your result.

Given: Link lengths:

Input (O_2A)	$a := 1.000$	Coupler (AB)	$b := 1.000$
Rocker (O_4B)	$c := 1.000$	Ground link	$d := 2.000$
Coupler point data:	$p := 2.0$	$\delta := 0 \cdot \text{deg}$	

Solution: See Figure 3-29h and Mathcad file P0442.

1. Check the Grashof condition of the linkage and determine its Baker classification.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} \quad \text{if } SL < PQ \\ \text{return "Special Grashof"} \quad \text{if } SL = PQ \\ \text{return "non-Grashof"} \quad \text{otherwise} \end{cases}$$

$$\text{Condition}(a, b, c, d) = \text{"non-Grashof"}$$

Since d (link 1) is the longest link and the linkage is non-Grashof, this is a Class 1 triple rocker. Using Figure 3-1a as a guide, determine the limiting values of θ_2 at the toggle positions.

For links 3 and 4 colinear:

$$\theta_{21} := \arccos\left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot d \cdot a}\right] \quad \theta_{21} = 75.522 \text{ deg} \quad \theta_{22} := -\theta_{21}$$

1. Define one cycle of the input crank:

$$\theta_2 := \theta_{22}, \theta_{22} + 0.25 \cdot \text{deg} .. \theta_{21}$$

2. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \quad K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_1 = 2.0000 \quad K_4 = 2.0000 \quad K_5 = -2.5000$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

3. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2}\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)}\right) \right)$$

4. Use equations 4.31 to define the x- and y-components of the vector $\mathbf{R}_p$.

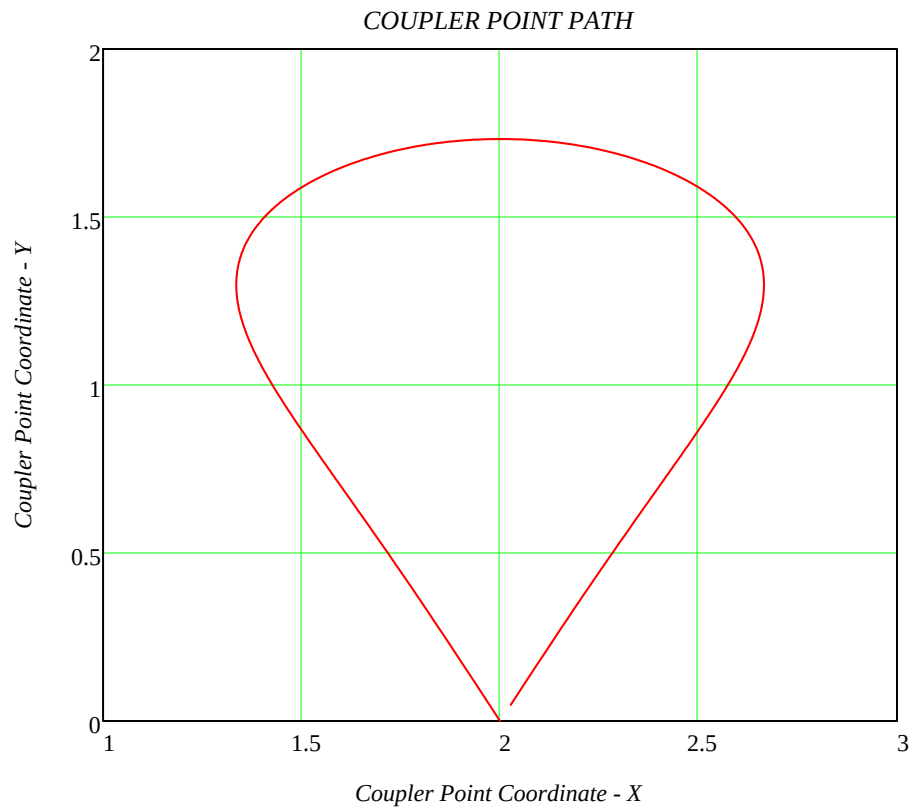
$$\mathbf{R}_p := \mathbf{R}_A + \mathbf{R}_{pA}$$

$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{R_{PA}} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

$$R_{Px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta)$$

$$R_{Py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$



PROBLEM 4-43

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular position of link 8 in Figure 3-36 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 0.450$	First coupler (L_3)	$b := 0.990$
Common rocker (O_4B)	$c := 0.590$	First ground link (O_2O_4)	$d := 1.000$
Common rocker (O_4C)	$a' := 0.590$	Second coupler (CD)	$b' := 0.325$
Output rocker (L_6)	$c' := 0.325$	Second ground link (O_4O_6)	$d' := 0.419$
Link 7 (L_7)	$e := 0.938$	Link 8 (L_8)	$f := 0.572$
Link 5 extension (DE)	$p := 0.823$	Angle DCE	$\delta := 7.0 \cdot \text{deg}$
Angle BO_4C	$\alpha := 128.6 \cdot \text{deg}$		

Solution: See Figure P3-36 and Mathcad file P0443.

1. This eightbar can be analyzed as a fourbar (links 1, 2, 3, and 4) with its output (link 4) as the input to another fourbar (links 1, 4, 5, and 6). Since links 1 and 4 are common to both, we have an eightbar linkage with links 7 & 8 included. Start by analyzing the input fourbar.
2. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.2222 \quad K_2 = 1.6949 \quad K_3 = 1.0744$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Use equations 4.8a and 4.10 to calculate θ_6 as a function of θ_2 (for the open circuit).

$$\text{Input angle to second fourbar: } \theta_{42}(\theta_2) := \theta_{41}(\theta_2) - \alpha$$

$$K'_1 := \frac{d'}{a'} \quad K'_2 := \frac{d'}{c'} \quad K'_3 := \frac{a'^2 - b'^2 + c'^2 + d'^2}{2 \cdot a' \cdot c'}$$

$$K'_1 = 0.7102 \quad K'_2 = 1.2892 \quad K'_3 = 1.3655$$

$$A'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K'_1 - K'_2 \cdot \cos(\theta_{42}(\theta_2)) + K'_3$$

$$B'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2)) \quad C'(\theta_2) := K'_1 - (K'_2 + 1) \cdot \cos(\theta_{42}(\theta_2)) + K'_3$$

$$\theta_6(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A'(\theta_2), -B'(\theta_2) - \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)} \right) \right)$$

5. Use equations 4.11b, 4.12 and 4.13 to calculate θ_5 as a function of θ_2 .

$$K'_4 := \frac{d'}{b'} \quad K'_5 := \frac{c'^2 - d'^2 - a'^2 - b'^2}{2 \cdot a' \cdot b'}$$

$$K'_4 = 1.289$$

$$K'_5 = -1.365$$

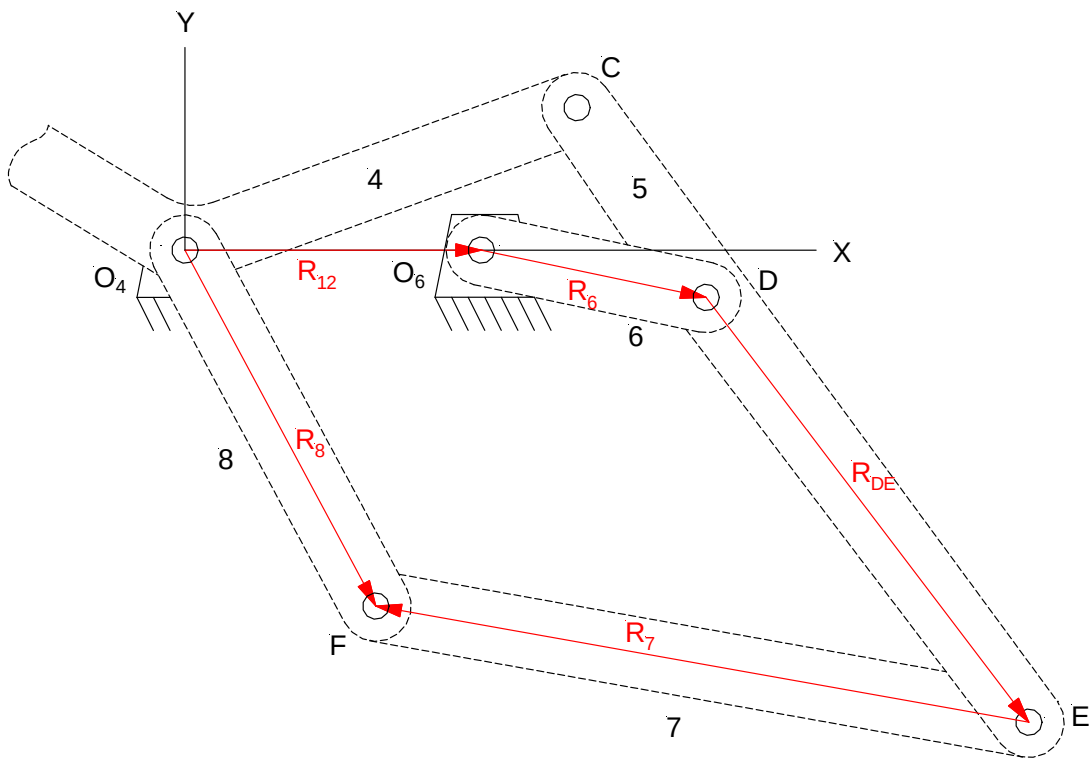
$$D'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K'_1 + K'_4 \cdot \cos(\theta_{42}(\theta_2)) + K'_5$$

$$E'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

$$F'(\theta_2) := K'_1 + (K'_4 - 1) \cdot \cos(\theta_{42}(\theta_2)) + K'_5$$

$$\theta_5(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D'(\theta_2), -E'(\theta_2) - \sqrt{E'(\theta_2)^2 - 4 \cdot D'(\theta_2) \cdot F'(\theta_2)} \right) \right)$$

6. Define a vector loop for links 1, 5, 6, 7, and 8 as shown below and write the vector loop equation.



$\mathbf{R}_{12} + \mathbf{R}_6 + \mathbf{R}_{DE} + \mathbf{R}_7 - \mathbf{R}_8 = 0$. Solving for $\mathbf{R}_7$ gives $\mathbf{R}_7 = \mathbf{R}_8 - \mathbf{R}_{12} - \mathbf{R}_6 - \mathbf{R}_{DE}$. In this equation the only unknowns are θ_7 and θ_8 . Following the method of Section 4.5, substitute the complex number notation for each position vector and separate the resulting equation into real and imaginary parts:

$$e \cdot \cos(\theta_7) = f \cdot \cos(\theta_8) - D_1$$

$$e \cdot \sin(\theta_7) = f \cdot \sin(\theta_8) - D_2$$

where $D_1(\theta_2) := d' + c' \cdot \cos(\theta_6(\theta_2)) + p \cdot \cos(\theta_5(\theta_2) + \delta)$

$$D_2(\theta_2) := c' \cdot \sin(\theta_6(\theta_2)) + p \cdot \sin(\theta_5(\theta_2) + \delta)$$

7. Solve these equations in the manner of equations 4.11 and 4.12 using the identities of equations 4.9 gives:

$$D_3(\theta_2) := \frac{f^2 + D_1(\theta_2)^2 + D_2(\theta_2)^2 - e^2}{2 \cdot f}$$

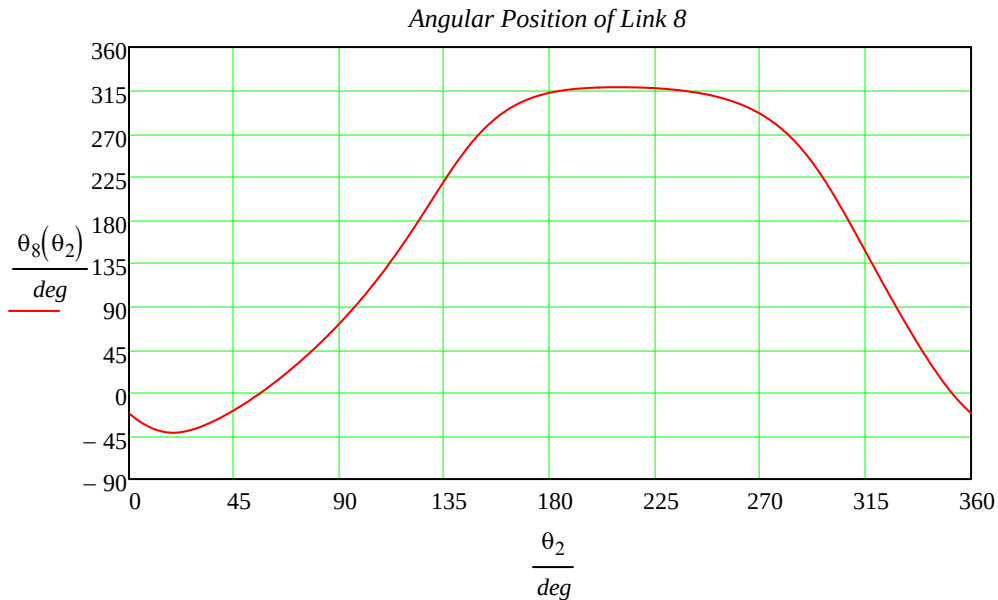
$$A'(\theta_2) := -D_1(\theta_2) - D_3(\theta_2) \quad B'(\theta_2) := 2 \cdot D_2(\theta_2) \quad C'(\theta_2) := D_1(\theta_2) - D_3(\theta_2)$$

$$\tan\left(\frac{\theta_8}{2}\right) = \frac{-B' + \sqrt{B'^2 - 4 \cdot A' \cdot C'}}{2 \cdot A'}$$

$$\theta_{81}(\theta_2) := 2 \cdot \left(\text{atan2}\left(2 \cdot A'(\theta_2), -B'(\theta_2) + \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)}\right) \right)$$

8. Plot the angular position of link 8 as a function of the angle of input link 2. If θ_{81} is greater than 360 deg, subtract 2π from it.

$$\theta_8(\theta_2) := \text{if}\left[\left(\theta_2 < \frac{\pi}{4}\right) \wedge (\theta_{81}(\theta_2) > 0), \theta_{81}(\theta_2) - 2 \cdot \pi, \theta_{81}(\theta_2)\right]$$



The graph shows that link 8 rotates 360 deg between $\theta_2 = 19$ deg and $\theta_2 = 209$ deg.

$$\theta_8(19 \cdot \text{deg}) = -42 \text{ deg} \quad \theta_8(209 \cdot \text{deg}) - 2 \cdot \pi = -42 \text{ deg}$$

$$\theta_8(209 \cdot \text{deg}) - \theta_8(19 \cdot \text{deg}) = 360.0 \text{ deg}$$

PROBLEM 4-44

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the transmission angles at points *B*, *C*, *D*, *E*, and *F* of the linkage in Figure 3-36 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 0.450$	First coupler (L_3)	$b := 0.990$
Common rocker (O_4B)	$c := 0.590$	First ground link (O_2O_4)	$d := 1.000$
Common rocker (O_4C)	$a' := 0.590$	Second coupler (CD)	$b' := 0.325$
Output rocker (L_6)	$c' := 0.325$	Second ground link (O_4O_6)	$d' := 0.419$
Link 7 (L_7)	$e := 0.938$	Link 8 (L_8)	$f := 0.572$
Link 5 extension (DE)	$p := 0.823$	Angle DCE	$\delta := 7.0 \cdot \text{deg}$
Angle BO_4C	$\alpha := 128.6 \cdot \text{deg}$		

Solution: See Figure P3-36 and Mathcad file P0444.

1. This eightbar can be analyzed as a fourbar (links 1, 2, 3, and 4) with its output (link 4) as the input to another fourbar (links 1, 4, 5, and 6). Since links 1 and 4 are common to both, we have an eightbar linkage with links 7 & 8 included. Start by analyzing the input fourbar.
2. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.2222 \quad K_2 = 1.6949 \quad K_3 = 1.0744$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Use equations 4.12 and 4.13 to calculate θ_3 as a function of θ_2 (for the open circuit).

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 1.010 \quad K_5 = -2.059$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

5. Use equations 4.8a and 4.10 to calculate θ_6 as a function of θ_2 (for the open circuit).

$$\text{Input angle to second fourbar: } \theta_{42}(\theta_2) := \theta_{41}(\theta_2) - \alpha$$

$$K'_1 := \frac{d'}{a'} \quad K'_2 := \frac{d'}{c'} \quad K'_3 := \frac{a'^2 - b'^2 + c'^2 + d'^2}{2 \cdot a' \cdot c'}$$

$$K'_1 = 0.7102$$

$$K'_2 = 1.2892$$

$$K'_3 = 1.3655$$

$$A'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K'_1 - K'_2 \cdot \cos(\theta_{42}(\theta_2)) + K'_3$$

$$B'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2)) \quad C'(\theta_2) := K'_1 - (K'_2 + 1) \cdot \cos(\theta_{42}(\theta_2)) + K'_3$$

$$\theta_6(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A'(\theta_2), -B'(\theta_2) - \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)} \right) \right)$$

6. Use equations 4.11b, 4.12 and 4.13 to calculate θ_5 as a function of θ_2 .

$$K'_4 := \frac{d'}{b'} \quad K'_5 := \frac{c'^2 - d'^2 - a'^2 - b'^2}{2 \cdot a' \cdot b'}$$

$$K'_4 = 1.289$$

$$K'_5 = -1.365$$

$$D'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K'_1 + K'_4 \cdot \cos(\theta_{42}(\theta_2)) + K'_5$$

$$E'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

$$F'(\theta_2) := K'_1 + (K'_4 - 1) \cdot \cos(\theta_{42}(\theta_2)) + K'_5$$

$$\theta_{51}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D'(\theta_2), -E'(\theta_2) - \sqrt{E'(\theta_2)^2 - 4 \cdot D'(\theta_2) \cdot F'(\theta_2)} \right) \right)$$

$$\theta_{52}(\theta_2) := \text{if}(\theta_{51}(\theta_2) > 2 \cdot \pi, \theta_{51}(\theta_2) - 2 \cdot \pi, \theta_{51}(\theta_2))$$

$$\theta_5(\theta_2) := \text{if}([105 \cdot \text{deg} < \theta_2 < 322 \cdot \text{deg}] \wedge (\theta_{52}(\theta_2) < 0), \theta_{52}(\theta_2) + 2 \cdot \pi, \theta_{52}(\theta_2))$$

7. Define a vector loop for links 1, 5, 6, 7, and 8 as shown on the next page and write the vector loop equation.

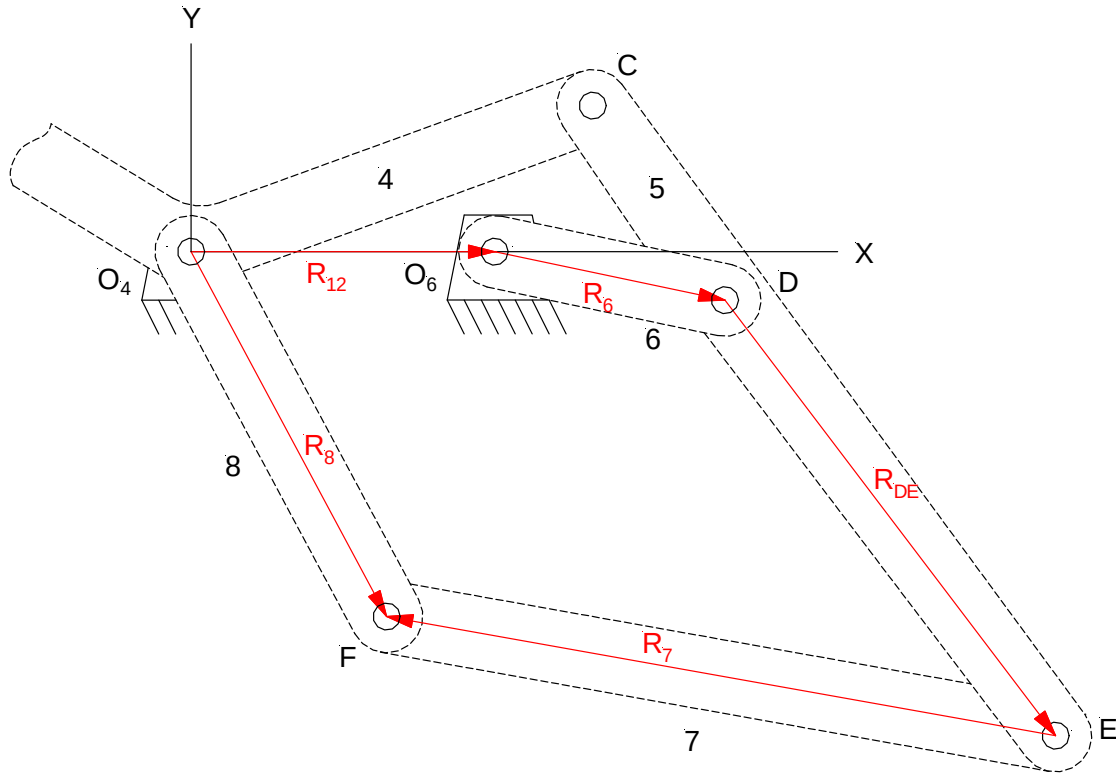
$\mathbf{R}_{12} + \mathbf{R}_6 + \mathbf{R}_{DE} + \mathbf{R}_7 - \mathbf{R}_8 = 0$. Solving for $\mathbf{R}_7$ gives $\mathbf{R}_7 = \mathbf{R}_8 - \mathbf{R}_{12} - \mathbf{R}_6 - \mathbf{R}_{DE}$. In this equation the only unknowns are θ_7 and θ_8 . Following the method of Section 4.5, substitute the complex number notation for each position vector and separate the resulting equation into real and imaginary parts:

$$e \cdot \cos(\theta_7) = f \cdot \cos(\theta_8) - D_1$$

$$e \cdot \sin(\theta_7) = f \cdot \sin(\theta_8) - D_2$$

where $D_1(\theta_2) := d' + c' \cdot \cos(\theta_6(\theta_2)) + p \cdot \cos(\theta_5(\theta_2) + \delta)$

$$D_2(\theta_2) := c' \cdot \sin(\theta_6(\theta_2)) + p \cdot \sin(\theta_5(\theta_2) + \delta)$$



8. Solve these equations in the manner of equations 4.10 using the identities of equations 4.9 gives:

$$D_3(\theta_2) := \frac{f^2 + D_1(\theta_2)^2 + D_2(\theta_2)^2 - e^2}{2 \cdot f}$$

$$A'(\theta_2) := -D_1(\theta_2) - D_3(\theta_2) \quad B'(\theta_2) := 2 \cdot D_2(\theta_2) \quad C'(\theta_2) := D_1(\theta_2) - D_3(\theta_2)$$

$$\tan\left(\frac{\theta_8}{2}\right) = \frac{-B' + \sqrt{B'^2 - 4 \cdot A' \cdot C'}}{2 \cdot A'}$$

$$\theta_{81}(\theta_2) := 2 \cdot \left(\text{atan2}\left(2 \cdot A'(\theta_2), -B'(\theta_2) + \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)}\right) \right)$$

$$\theta_8(\theta_2) := \text{if}\left[\left(\theta_2 < \frac{\pi}{4}\right) \wedge (\theta_{81}(\theta_2) > 0), \theta_{81}(\theta_2) - 2 \cdot \pi, \theta_{81}(\theta_2)\right]$$

9. Similarly, solve these equations in the manner of equations 4.11, 4.12 and 4.13 using the identities of equations 4.9 gives:

$$D_4(\theta_2) := \frac{f^2 - D_1(\theta_2)^2 - D_2(\theta_2)^2 - e^2}{2 \cdot e}$$

$$D'(\theta_2) := -D_1(\theta_2) - D_4(\theta_2) \quad E'(\theta_2) := 2 \cdot D_2(\theta_2) \quad F'(\theta_2) := D_1(\theta_2) - D_4(\theta_2)$$

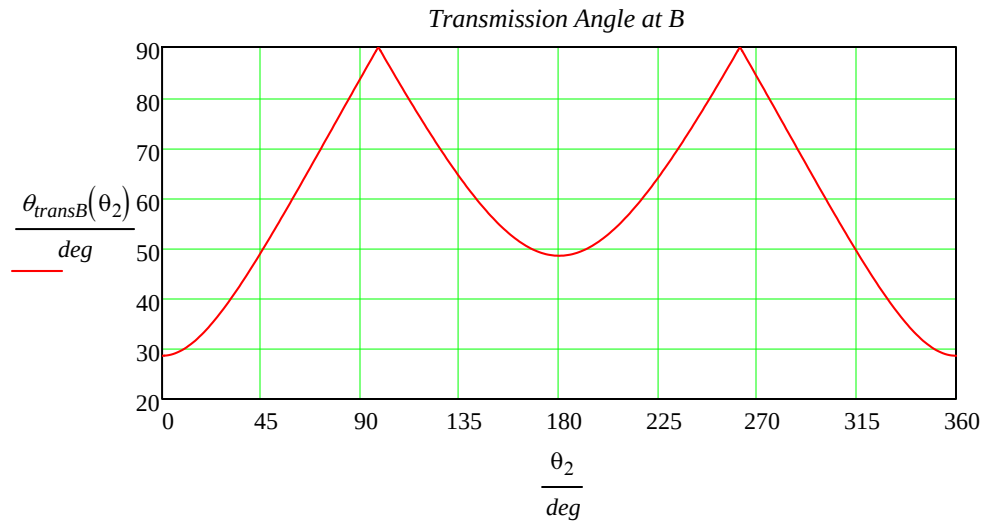
$$\tan\left(\frac{\theta_7}{2}\right) = \frac{-E' + \sqrt{E'^2 - 4 \cdot D' \cdot F'}}{2 \cdot D'}$$

$$\theta_7(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D'(\theta_2), -E'(\theta_2) + \sqrt{E'(\theta_2)^2 - 4 \cdot D'(\theta_2) \cdot F'(\theta_2)} \right) \right)$$

10. Calculate (using equations 4.32) and plot the transmission angle at B.

$$\theta_{transB1}(\theta_2) := |\theta_3(\theta_2) - \theta_{41}(\theta_2)|$$

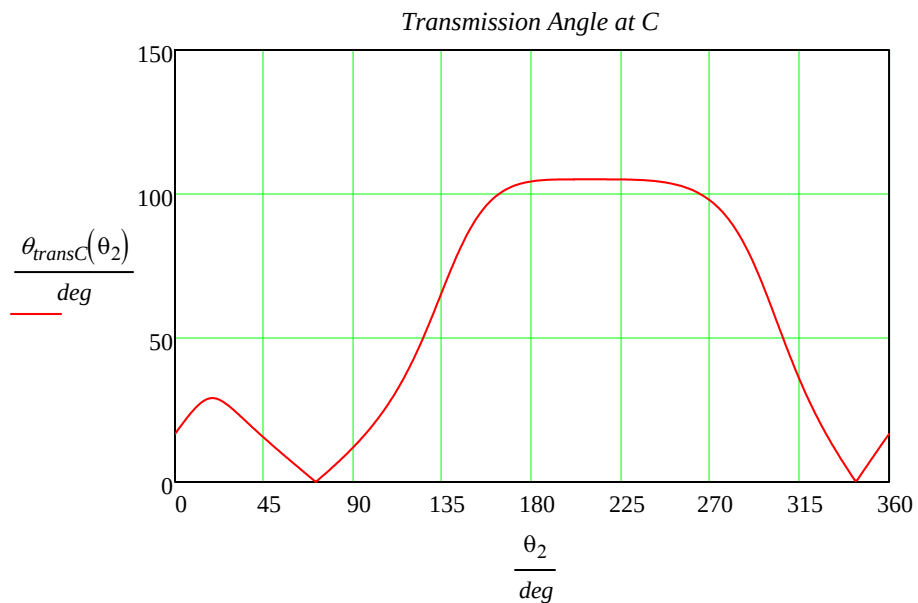
$$\theta_{transB}(\theta_2) := \left| \text{if} \left(\theta_{transB1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_2), \theta_{transB1}(\theta_2) \right) \right|$$



11. Calculate (using equations 4.32) and plot the transmission angle at C.

$$\theta_{transC1}(\theta_2) := |\theta_5(\theta_2) - \theta_{42}(\theta_2)|$$

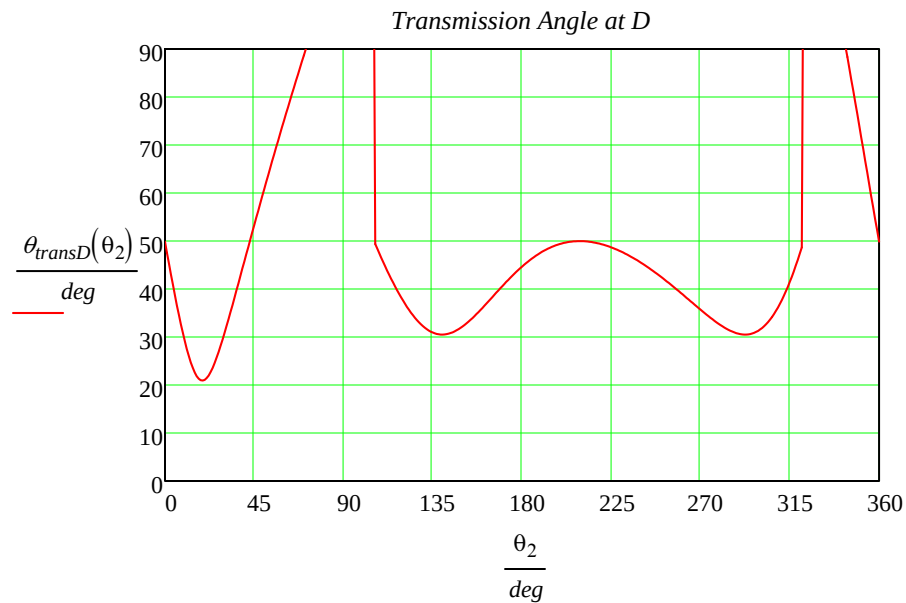
$$\theta_{transC2}(\theta_2) := \left| \text{if} \left(\frac{\pi}{2} < \theta_{transC1}(\theta_2) < 2 \cdot \pi, \pi - \theta_{transC1}(\theta_2), \theta_{transC1}(\theta_2) \right) \right|$$



12. Calculate (using equations 4.32) and plot the transmission angle at D .

$$\theta_{transD1}(\theta_2) := |\theta_5(\theta_2) - \theta_6(\theta_2)| - \pi$$

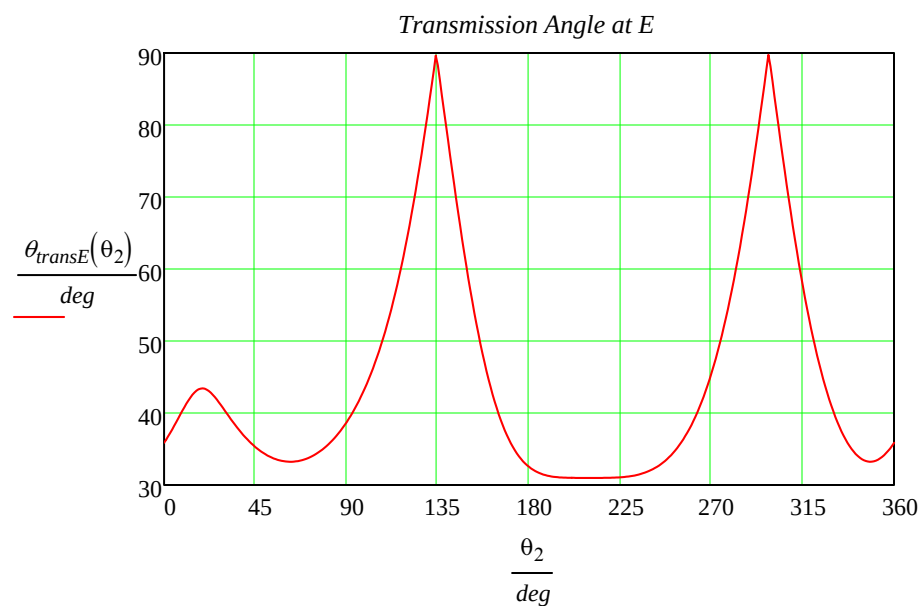
$$\theta_{transD2}(\theta_2) := \left| \text{if} \left(\frac{\pi}{2} < \theta_{transD1}(\theta_2) < \pi, \pi - \theta_{transD1}(\theta_2), \theta_{transD1}(\theta_2) \right) \right|$$



13. Calculate (using equations 4.32) and plot the transmission angle at E .

$$\theta_{transE1}(\theta_2) := |\theta_7(\theta_2) - \theta_8(\theta_2)|$$

$$\theta_{transE}(\theta_2) := \left| \text{if} \left(\theta_{transE1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transE1}(\theta_2), \theta_{transE1}(\theta_2) \right) \right|$$



PROBLEM 4-45

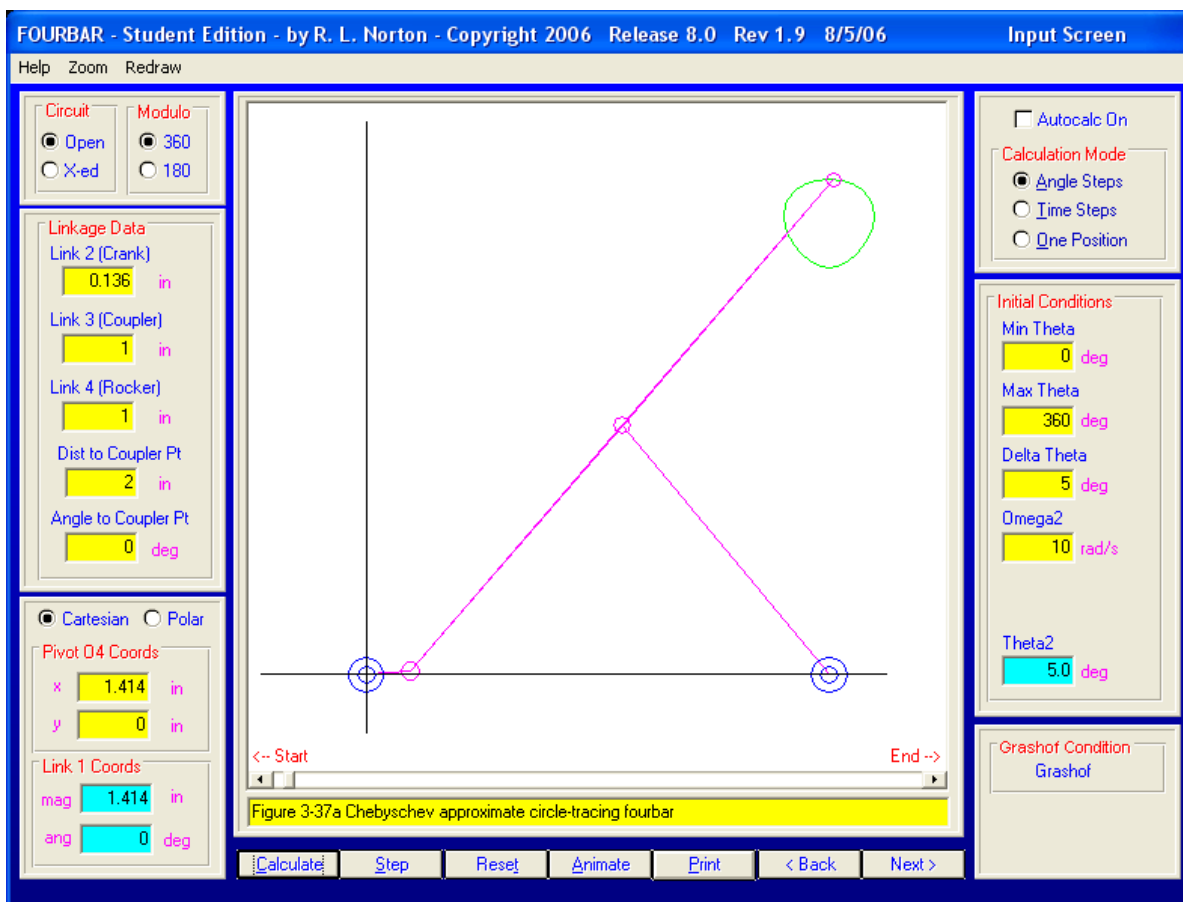
Statement: Model the linkage shown in Figure 3-37a in *Fourbar*. Export the coupler curve coordinates to *Excel* and calculate the error function versus a true circle.

Given: Link lengths:

Input (O_2A) $a := 0.136$ Coupler (AB) $b := 1.000$
 Rocker (O_4B) $c := 1.000$ Ground link $d := 1.414$
 Coupler point data: $p := 2.000$ $\delta := 0\text{-deg}$

Solution: See Figure 3-37a and Mathcad file P0445.

1. Model the linkage in *Fourbar*.



2. Write coupler point coordinates to a data file.
3. Import the data file into Excel and add columns for the true circle coordinates and radius. Analyze the coupler point radius to determine its mean, maximum deviation from mean, and average absolute deviation from its mean (See next two pages, note that the last four columns were added to the imported data).

FOURBAR P0445 Design # 2
Tom Cook 8/9/2006

Selected Linkage Parameters

a = 0.136

b = 1.000

c = 1.000

d = 1.414

Angle Step Deg	Coupler Pt X in	Coupler Pt Y in	Coupler Pt Mag in	Coupler Pt Ang in	True Circ X in	True Circ Y in	True Circ R in	Coupler Pt R in
0	1.414	1.5384	2.0895	47.413	1.4140	1.5500	0.1360	0.1244
5	1.4283	1.5379	2.0988	47.1164	1.4021	1.5495	0.1360	0.1247
10	1.4423	1.5363	2.1072	46.8058	1.3904	1.5479	0.1360	0.1255
15	1.4561	1.5336	2.1147	46.4846	1.3788	1.5454	0.1360	0.1268
20	1.4693	1.5299	2.1212	46.1562	1.3675	1.5418	0.1360	0.1284
25	1.4819	1.5251	2.1265	45.8237	1.3565	1.5373	0.1360	0.1302
30	1.4937	1.5195	2.1307	45.4901	1.3460	1.5318	0.1360	0.1322
35	1.5046	1.5129	2.1337	45.1581	1.3360	1.5254	0.1360	0.1341
40	1.5145	1.5055	2.1355	44.8303	1.3266	1.5182	0.1360	0.1359
45	1.5233	1.4974	2.136	44.5088	1.3178	1.5102	0.1360	0.1375
50	1.5309	1.4885	2.1353	44.1957	1.3098	1.5014	0.1360	0.1386
55	1.5373	1.479	2.1333	43.8925	1.3026	1.4920	0.1360	0.1394
60	1.5425	1.469	2.1301	43.6007	1.2962	1.4820	0.1360	0.1398
65	1.5465	1.4585	2.1258	43.3213	1.2907	1.4715	0.1360	0.1398
70	1.5493	1.4475	2.1203	43.0555	1.2862	1.4605	0.1360	0.1394
75	1.5508	1.4363	2.1138	42.8038	1.2826	1.4492	0.1360	0.1386
80	1.5512	1.4248	2.1063	42.567	1.2801	1.4376	0.1360	0.1376
85	1.5505	1.4131	2.0979	42.3456	1.2785	1.4259	0.1360	0.1365
90	1.5488	1.4014	2.0887	42.1401	1.2780	1.4140	0.1360	0.1354
95	1.546	1.3897	2.0788	41.9507	1.2785	1.4021	0.1360	0.1342
100	1.5424	1.378	2.0683	41.7781	1.2801	1.3904	0.1360	0.1334
105	1.5379	1.3665	2.0572	41.6225	1.2826	1.3788	0.1360	0.1327
110	1.5326	1.3552	2.0458	41.4846	1.2862	1.3675	0.1360	0.1324
115	1.5266	1.3442	2.0341	41.3647	1.2907	1.3565	0.1360	0.1325
120	1.52	1.3336	2.0221	41.2636	1.2962	1.3460	0.1360	0.1330
125	1.5128	1.3235	2.0101	41.1819	1.3026	1.3360	0.1360	0.1340
130	1.5052	1.314	1.998	41.1204	1.3098	1.3266	0.1360	0.1353
135	1.4971	1.3051	1.9861	41.0799	1.3178	1.3178	0.1360	0.1370
140	1.4887	1.2969	1.9744	41.0612	1.3266	1.3098	0.1360	0.1389
145	1.4799	1.2895	1.9629	41.0654	1.3360	1.3026	0.1360	0.1409
150	1.471	1.2829	1.9518	41.0931	1.3460	1.2962	0.1360	0.1430
155	1.4618	1.2772	1.9411	41.1453	1.3565	1.2907	0.1360	0.1449
160	1.4524	1.2725	1.931	41.2227	1.3675	1.2862	0.1360	0.1466
165	1.4429	1.2688	1.9214	41.3257	1.3788	1.2826	0.1360	0.1480
170	1.4333	1.2661	1.9124	41.455	1.3904	1.2801	0.1360	0.1492
175	1.4237	1.2645	1.9041	41.6105	1.4021	1.2785	0.1360	0.1498
180	1.414	1.2639	1.8965	41.7924	1.4140	1.2780	0.1360	0.1501
185	1.4043	1.2645	1.8897	42.0001	1.4259	1.2785	0.1360	0.1498
190	1.3947	1.2661	1.8836	42.2329	1.4376	1.2801	0.1360	0.1492
195	1.3851	1.2688	1.8784	42.49	1.4492	1.2826	0.1360	0.1480
200	1.3756	1.2725	1.8739	42.7699	1.4605	1.2862	0.1360	0.1466
205	1.3662	1.2772	1.8703	43.0708	1.4715	1.2907	0.1360	0.1449
210	1.357	1.2829	1.8674	43.3907	1.4820	1.2962	0.1360	0.1430

215	1.3481	1.2895	1.8655	43.7272	1.4920	1.3026	0.1360	0.1409
220	1.3393	1.2969	1.8643	44.0777	1.5014	1.3098	0.1360	0.1389
225	1.3309	1.3051	1.864	44.439	1.5102	1.3178	0.1360	0.1370
230	1.3228	1.314	1.8645	44.8081	1.5182	1.3266	0.1360	0.1353
235	1.3152	1.3235	1.8659	45.1815	1.5254	1.3360	0.1360	0.1340
240	1.308	1.3336	1.868	45.5556	1.5318	1.3460	0.1360	0.1330
245	1.3014	1.3442	1.871	45.9269	1.5373	1.3565	0.1360	0.1325
250	1.2954	1.3552	1.8747	46.2915	1.5418	1.3675	0.1360	0.1324
255	1.2901	1.3665	1.8793	46.6458	1.5454	1.3788	0.1360	0.1327
260	1.2856	1.378	1.8846	46.986	1.5479	1.3904	0.1360	0.1334
265	1.282	1.3897	1.8907	47.3085	1.5495	1.4021	0.1360	0.1342
270	1.2792	1.4014	1.8975	47.61	1.5500	1.4140	0.1360	0.1354
275	1.2775	1.4131	1.905	47.8871	1.5495	1.4259	0.1360	0.1365
280	1.2768	1.4248	1.9132	48.1368	1.5479	1.4376	0.1360	0.1376
285	1.2772	1.4363	1.922	48.3563	1.5454	1.4492	0.1360	0.1386
290	1.2787	1.4475	1.9314	48.5433	1.5418	1.4605	0.1360	0.1394
295	1.2815	1.4585	1.9415	48.6956	1.5373	1.4715	0.1360	0.1398
300	1.2855	1.469	1.952	48.8117	1.5318	1.4820	0.1360	0.1398
305	1.2907	1.479	1.963	48.8904	1.5254	1.4920	0.1360	0.1394
310	1.2971	1.4885	1.9744	48.9308	1.5182	1.5014	0.1360	0.1386
315	1.3047	1.4974	1.9861	48.9328	1.5102	1.5102	0.1360	0.1375
320	1.3135	1.5055	1.998	48.8966	1.5014	1.5182	0.1360	0.1359
325	1.3234	1.5129	2.0101	48.823	1.4920	1.5254	0.1360	0.1341
330	1.3343	1.5195	2.0222	48.713	1.4820	1.5318	0.1360	0.1322
335	1.3461	1.5251	2.0342	48.5685	1.4715	1.5373	0.1360	0.1302
340	1.3587	1.5299	2.0461	48.3915	1.4605	1.5418	0.1360	0.1284
345	1.3719	1.5336	2.0577	48.1846	1.4492	1.5454	0.1360	0.1268
350	1.3857	1.5363	2.0688	47.9504	1.4376	1.5479	0.1360	0.1255
355	1.3997	1.5379	2.0795	47.6921	1.4259	1.5495	0.1360	0.1247
360	1.414	1.5384	2.0895	47.413	1.4140	1.5500	0.1360	0.1244

The mean value of the coupler point radius is 0.1366

The maximum deviation from the mean is 0.0135

The average absolute deviation from the mean is 0.005127

PROBLEM 4-46

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the path of point P in Figure 3-37a as a function of the angle of input link 2. Also plot the variation (error) in the path of point P versus that of point A.

Given: Link lengths:

Input (O_2A) $a := 0.136$ Coupler (AB) $b := 1.000$
 Rocker (O_4B) $c := 1.000$ Ground link $d := 1.414$
 Coupler point data: $p := 2.000$ $\delta := 0 \cdot \text{deg}$

Solution: See Figure 3-37a and Mathcad file P0446.

1. Check the Grashof condition of the linkage.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise
Condition(a,b,c,d) = "Grashof" crank rocker

```

1. Define one cycle of the input crank:

$$\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

2. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \quad K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_1 = 10.3971 \quad K_4 = 1.4140 \quad K_5 = -7.4187$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

3. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

4. Use equations 4.31 to define the x- and y-components of the vector $\mathbf{R}_P$.

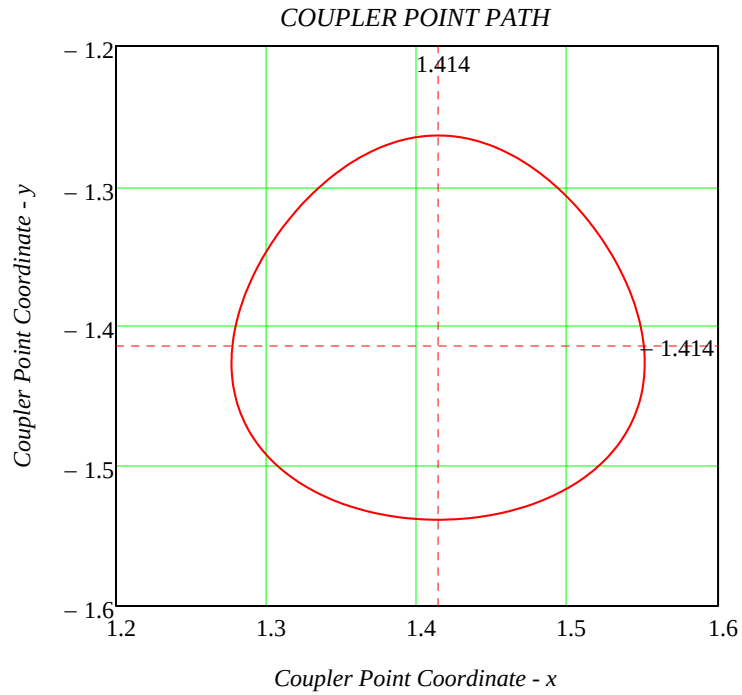
$$\mathbf{R}_P := \mathbf{R}_A + \mathbf{R}_{PA}$$

$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{R}_{PA} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

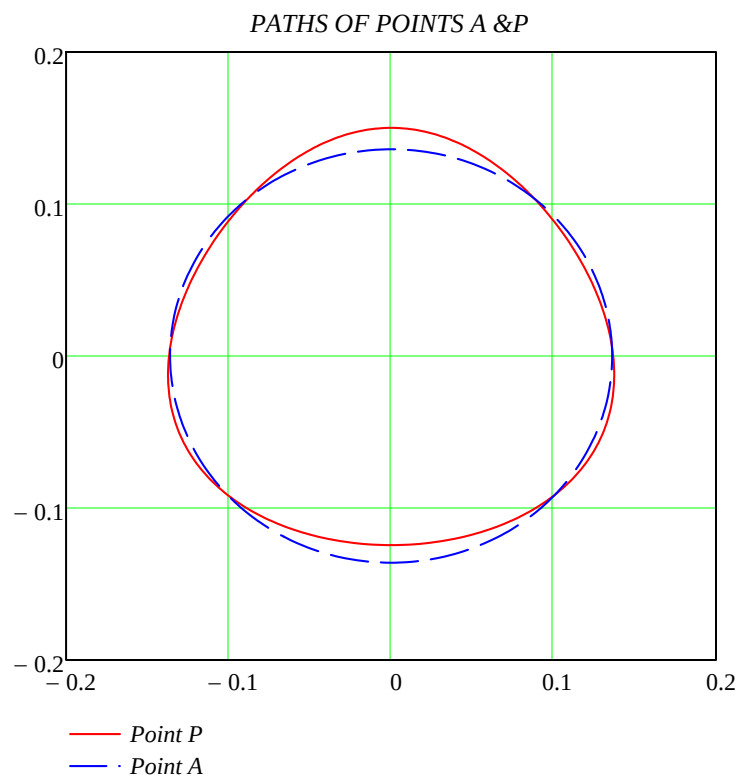
$$R_{Px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta) \quad R_{Py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

5. Plot the coordinates of the coupler point in the local xy coordinate system.



6. Replot, transforming the coupler path to 0,0 and plot the path of point A.

$$X_A(\theta_2) := a \cdot \cos(\theta_2) \quad Y_A(\theta_2) := a \cdot \sin(\theta_2)$$



PROBLEM 4-47

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the transmission angle at point B of the linkage in Figure 3-37a as a function of the angle of input link 2.

Given: Link lengths:

Input (O_2A)	$a := 0.136$	Coupler (AB)	$b := 1.000$
Rocker (O_4B)	$c := 1.000$	Ground link	$d := 1.414$
Coupler point data:	$p := 2.000$	$\delta := 0 \cdot \text{deg}$	

Solution: See Figure 3-37a and Mathcad file P0447.

1. Check the Grashof condition of the linkage.

$$\begin{aligned} \text{Condition}(a, b, c, d) := & \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(a, b, c, d) = \text{"Grashof"} \quad \text{crank rocker}$$

1. Define one cycle of the input crank:

$$\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

2. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$\begin{aligned} K_1 &:= \frac{d}{a} & K_2 &:= \frac{d}{c} & K_3 &:= \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \\ K_1 &= 10.3971 & K_2 &= 1.4140 & K_3 &= 7.4187 \\ A(\theta_2) &:= \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \\ B(\theta_2) &:= -2 \cdot \sin(\theta_2) & C(\theta_2) &:= K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3 \\ \theta_4(\theta_2) &:= 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right) \end{aligned}$$

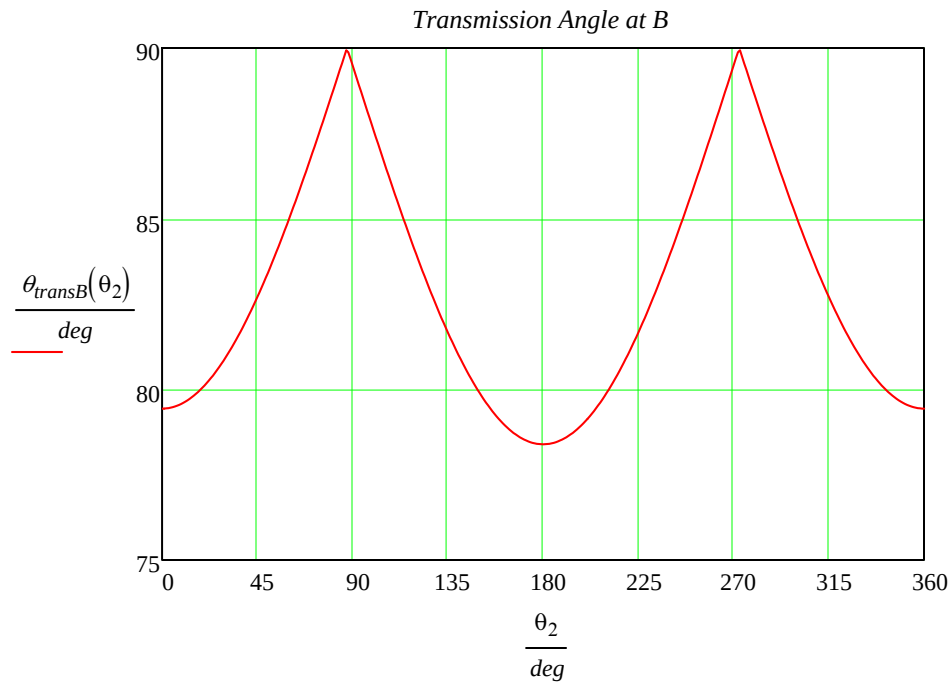
3. Use equations 4.12 and 4.13 to calculate θ_3 as a function of θ_2 (for the crossed circuit).

$$\begin{aligned} K_4 &:= \frac{d}{b} & K_5 &:= \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \\ K_4 &= 1.414 & K_5 &= -7.419 \\ D(\theta_2) &:= \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 & E(\theta_2) &:= -2 \cdot \sin(\theta_2) \\ F(\theta_2) &:= K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \\ \theta_3(\theta_2) &:= 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right) \end{aligned}$$

4. Calculate (using equations 4.32) and plot the transmission angle at B.

$$\theta_{transB1}(\theta_2) := |\theta_3(\theta_2) - \theta_4(\theta_2)|$$

$$\theta_{transB}(\theta_2) := \left| \text{if} \left(\theta_{transB1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_2), \theta_{transB1}(\theta_2) \right) \right|$$



PROBLEM 4-48

Statement: Figure 3-29f shows Evan's approximate straight-line linkage #1. Determine the range of motion of link 2 for which the point P varies no more than 0.0025 from the straight line $X = 1.690$ (assuming that O_2 is the origin of a global coordinate frame whose positive X axis is rotated 60 deg from O_2O_4).

Given:

Link lengths:

Input (O_2A) $a := 1.000$ Coupler (AB) $b := 1.600$

Rocker (O_4B) $c := 1.039$ Ground link $d := 1.200$

Coupler point data: $p := 2.690$ $\delta := 0 \cdot \text{deg}$

Coordinate rotation angle: $\alpha := -60 \cdot \text{deg}$

Solution: See Figure 3-29f and Mathcad file P0448.

1. Check the Grashof condition of the linkage and determine its Baker classification.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$Condition(a,b,c,d) = \text{"non-Grashof"}$

Since b (link 3) is the longest link and the linkage is non-Grashof, this is a Class 3 triple rocker. Using Figure 3-1a as a guide, determine the limiting values of θ_2 at the toggle positions.

For links 2 and 3 colinear:

$$\theta_{20} := \arccos\left[\frac{(b-a)^2 + d^2 - c^2}{2 \cdot d \cdot (b-a)}\right] + \pi \quad \theta_{20} = 240 \text{ deg}$$

For links 3 and 4 colinear:

$$\theta_{21} := \arccos\left[\frac{a^2 + d^2 - (b-c)^2}{2 \cdot d \cdot a}\right] \quad \theta_{21} = 27.683 \text{ deg} \quad \theta_{22} := -\theta_{21}$$

1. Define one cycle of the input crank (driving through the links 2-3 toggle position):

$$\theta_2 := \theta_{21}, \theta_{21} + 1 \cdot \text{deg} .. 360 \cdot \text{deg} + \theta_{22}$$

2. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \quad K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_1 = 1.2000 \quad K_4 = 0.7500 \quad K_5 = -1.2251$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

3. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

4. Use equations 4.31 to define the x-components of the vector $\mathbf{R}_P$.

$$\mathbf{R}_P := \mathbf{R}_A + \mathbf{R}_{PA}$$

$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

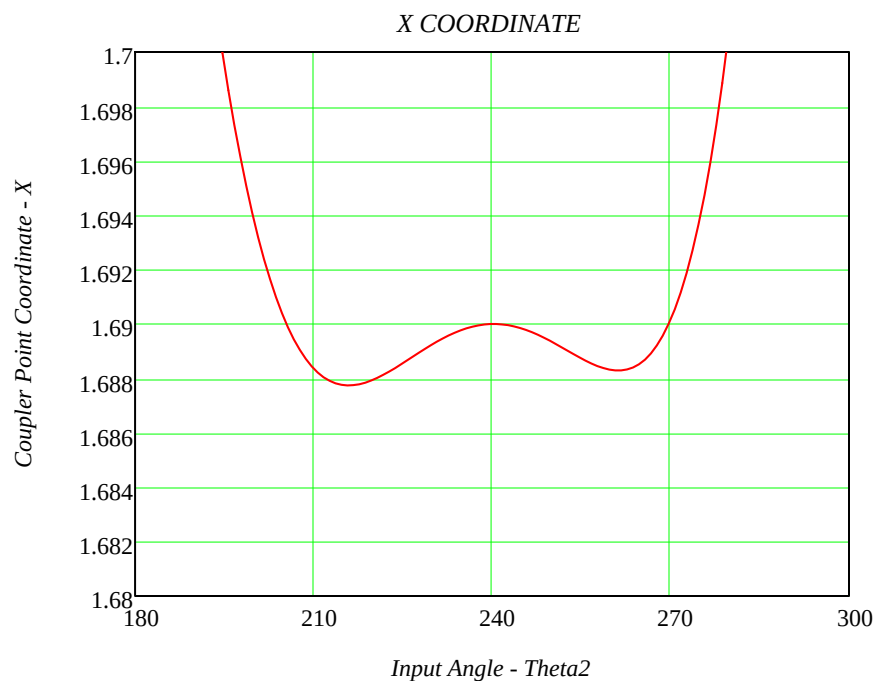
$$\mathbf{R}_{PA} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

$$R_{Px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta)$$

$$R_{Py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

5. Plot the X coordinate of the coupler point in the global X,Y coordinate system using equations 4.0b to rotate the local coordinates to a global frame.

$$P_X(\theta_2) := R_{Px}(\theta_2) \cdot \cos(\alpha) - R_{Py}(\theta_2) \cdot \sin(\alpha)$$



6. Using the graph for guess values, solve by trial and error to find θ_2 for $X = 1.6900 \pm 0.0025$.

$$P_X(201.525 \cdot \text{deg}) = 1.69250 \quad \theta_{2min} := 201.525 \cdot \text{deg}$$

$$P_X(273.450 \cdot \text{deg}) = 1.69250 \quad \theta_{2max} := 273.450 \cdot \text{deg}$$

PROBLEM 4-49

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the path of point P in Figure 3-37b as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 0.50$	First coupler (AB)	$b := 1.00$
Rocker 4 (O_4B)	$c := 1.00$	Rocker 5 (L_5)	$c' := 1.00$
Ground link (O_2O_4)	$d := 0.75$	Second coupler 6 (CD)	$b' := 1.00$
Coupler point (DP)	$p := 1.00$	Distance to OP (O_2O_p)	$d' := 1.50$

Solution: See Figure 3-37b and Mathcad file P0449.

- Links 4, 5, BC , and CD form a parallelogram whose opposite sides remain parallel throughout the motion of the fourbar 1, 2, AB , 4. Define a position vector whose tail is at point D and whose tip is at point P and another whose tail is at O_4 and whose tip is at point D . Then, since $\mathbf{R}_5 = \mathbf{R}_{AB}$ and $\mathbf{R}_{DP} = -\mathbf{R}_4$, the position vector from O_2 to P is $\mathbf{P} = \mathbf{R}_1 + \mathbf{R}_{AB} - \mathbf{R}_4$. Separating this vector equation into real and imaginary parts gives the equations for the X and Y coordinates of the coupler point P .

$$X_P = d + b \cdot \cos(\theta_3) - c \cdot \cos(\theta_4) \quad Y_P = b \cdot \sin(\theta_3) - c \cdot \sin(\theta_4)$$

- Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$
- Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 1.5000$$

$$K_2 = 0.7500$$

$$K_3 = 0.8125$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

- Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.7500 \quad K_5 = -0.8125$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

- Use equation 4.13 to find values of θ_3 for the open circuit.

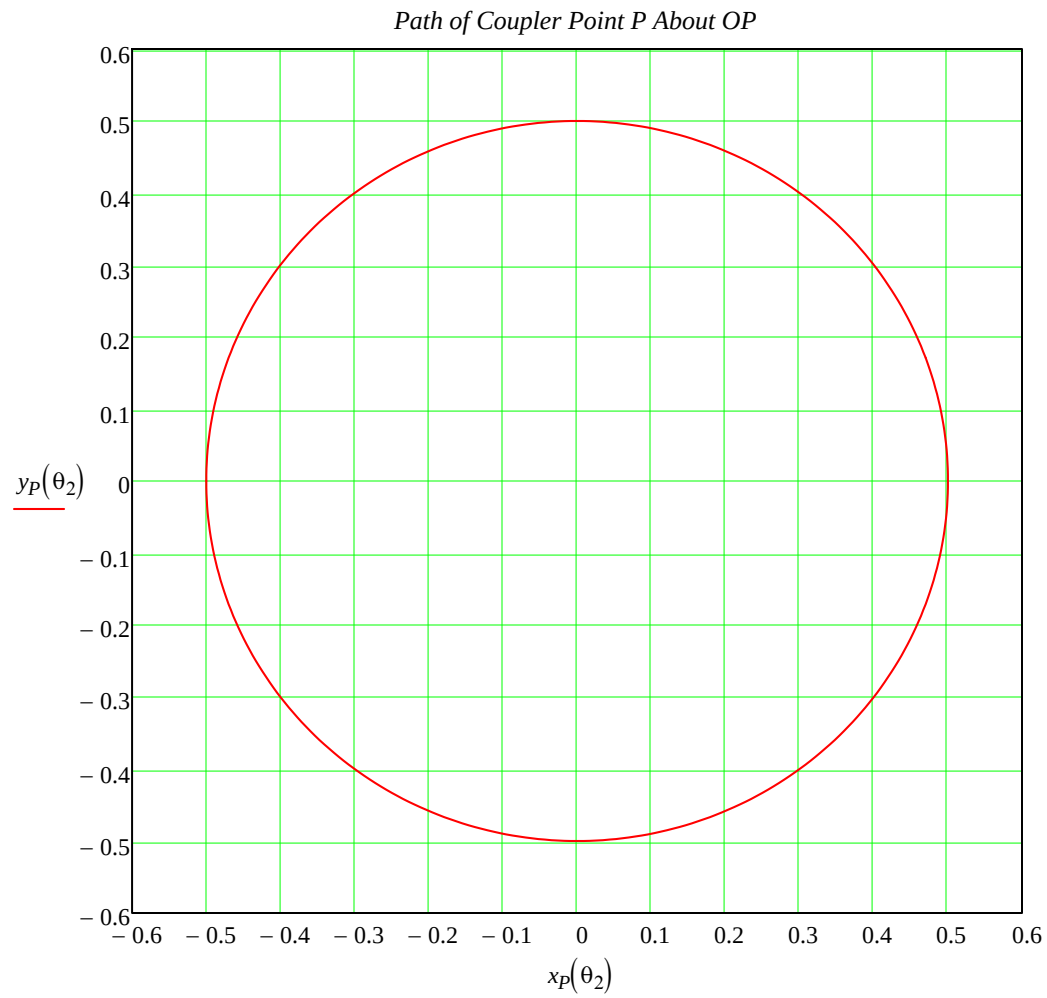
$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

- Define a local xy coordinate system with origin at O_p and with the positive x axis to the right. The coordinates of P are transformed to $x_P = X_P - d'$, $y_P = Y_P$.

$$x_P(\theta_2) := d + b \cdot \cos(\theta_3(\theta_2)) - c \cdot \cos(\theta_4(\theta_2)) - d'$$

$$y_P(\theta_2) := b \cdot \sin(\theta_3(\theta_2)) - c \cdot \sin(\theta_4(\theta_2))$$

7. Plot the path of P as a function of the angle of link 2.



PROBLEM 4-50

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the transmission angles at points *B*, *C*, and *D* of the linkage in Figure 3-37b as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 0.50$	First coupler (AB)	$b := 1.00$
Rocker 4 (O_4B)	$c := 1.00$	Rocker 5 (L_5)	$c' := 1.00$
Ground link (O_2O_4)	$d := 0.75$	Second coupler 6 (CD)	$b' := 1.00$
Coupler point (DP)	$p := 1.00$	Distance to OP (O_2O_p)	$d' := 1.50$

Solution: See Figure 3-37b and Mathcad file P0450.

- Links 4, 5, BC , and CD form a parallelogram whose opposite sides remain parallel throughout the motion of the fourbar 1, 2, AB , 4. Therefore, the transmission angles at points B and D will be the same and the transmission angle at point C will be the complement of the angle at B .
- Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
- Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 1.5000 \quad K_2 = 0.7500 \quad K_3 = 0.8125$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

- Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.7500 \quad K_5 = -0.8125$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

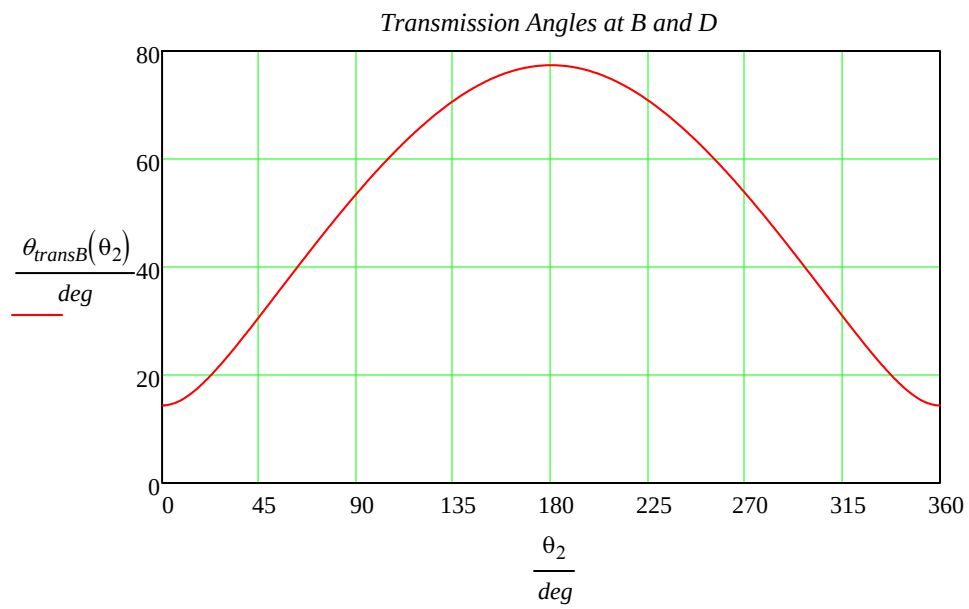
- Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

- Calculate (using equations 4.32) and plot the transmission angles at B and D .

$$\theta_{transB1}(\theta_2) := |\theta_3(\theta_2) - \theta_4(\theta_2)|$$

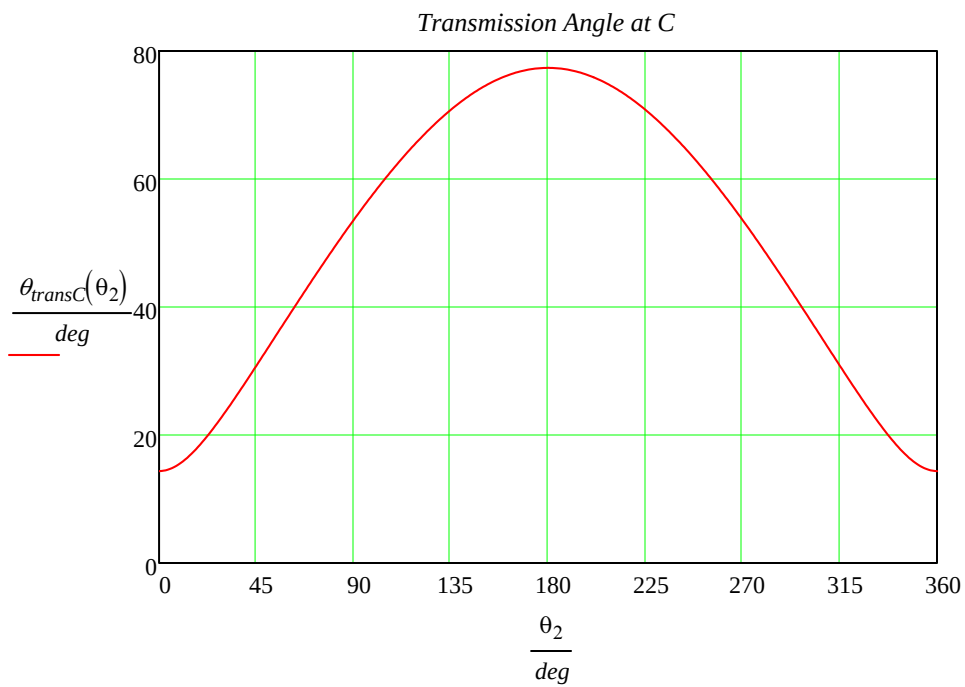
$$\theta_{transB}(\theta_2) := \left| \text{if} \left(\theta_{transB1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_2), \theta_{transB1}(\theta_2) \right) \right|$$



6. Calculate and plot the transmission angle at C.

$$\theta_{transC1}(\theta_2) := |180 \cdot \text{deg} - \theta_{transB}(\theta_2)|$$

$$\theta_{transC}(\theta_2) := \left| \text{if} \left(\theta_{transC1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transC1}(\theta_2), \theta_{transC1}(\theta_2) \right) \right|$$



PROBLEM 4-51

Statement: Figure 3-29g shows Evan's approximate straight-line linkage #2. Determine the range of motion of link 2 for which the point P varies no more than 0.005 from the straight line $X = -0.500$ (assuming that O_2 is the origin of a global coordinate frame whose positive X axis is rotated 30 deg from O_2O_4).

Given: Link lengths:

Input (O_2A) $a := 1.000$ Coupler (AB) $b := 1.200$

Rocker (O_4B) $c := 1.167$ Ground link $d := 2.305$

Coupler point data: $p := 1.50$ $\delta := 180 \cdot \text{deg}$

Coordinate rotation angle: $\alpha := -30 \cdot \text{deg}$

Solution: See Figure 3-29g and Mathcad file P0451.

1. Check the Grashof condition of the linkage and determine its Baker classification.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$Condition(a,b,c,d) = \text{"non-Grashof"}$

Since d (link 1) is the longest link and the linkage is non-Grashof, this is a Class 1 triple rocker. Using Figure 3-1a as a guide, determine the limiting values of θ_2 at the toggle positions.

For links 3 and 4 colinear:

$$\theta_{21} := \arccos\left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot d \cdot a}\right] \quad \theta_{21} = 81.136 \text{ deg} \quad \theta_{22} := -\theta_{21}$$

1. Define one cycle of the input crank (driving through the links 2-3 toggle position):

$$\theta_2 := \theta_{22}, \theta_{22} + 0.5 \cdot \text{deg} .. \theta_{21}$$

2. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \quad K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_1 = 2.3050 \quad K_4 = 1.9208 \quad K_5 = -2.6630$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

3. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2}\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)}\right) \right)$$

4. Use equations 4.31 to define the x and y-components of the vector $\mathbf{R}_P$.

$$\mathbf{R}_P := \mathbf{R}_A + \mathbf{R}_{PA}$$

$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

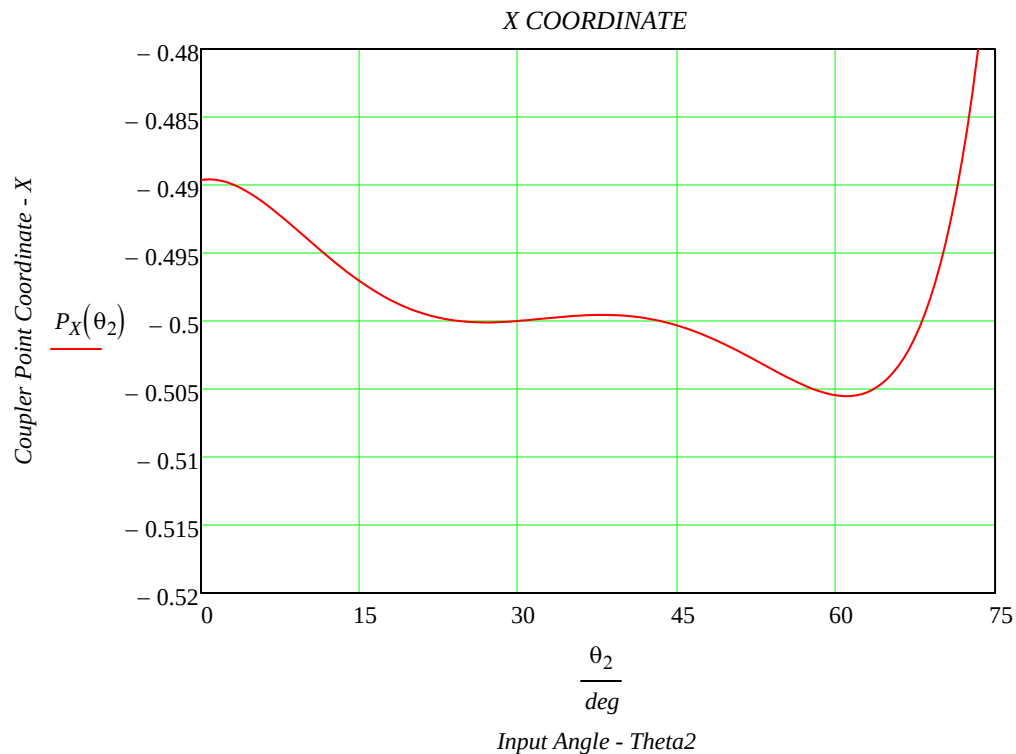
$$\mathbf{R}_{PA} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

$$R_{Px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta)$$

$$R_{Py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

5. Plot the X coordinate of the coupler point in the global X,Y coordinate system using equations 4.0b to rotate the local coordinates to a global frame.

$$P_X(\theta_2) := R_{Px}(\theta_2) \cdot \cos(\alpha) - R_{Py}(\theta_2) \cdot \sin(\alpha)$$



6. Using the graph for guess values, solve by trial and error to find θ_2 for $X = -0.500 \pm 0.005$

$$P_X(11.59 \cdot deg) = -0.49500$$

$$\theta_{2min} := 11.59 \cdot deg$$

$$P_X(57.80 \cdot deg) = -0.50500$$

$$\theta_{2max} := 57.80 \cdot deg$$

PROBLEM 4-52

Statement: For the linkage in Figure P4-16, what are the angles that link 2 makes with the positive X-axis when links 2 and 3 are in toggle positions?

Given: Link lengths:

$$\text{Input } (O_2A) \quad a := 14 \quad \text{Coupler } (AB) \quad b := 80$$

$$\text{Rocker } (O_4B) \quad c := 51.26$$

$$O_4 \text{ offset in } XY \text{ coordinates:} \quad O_{4X} := -47.5 \quad O_{4Y} := 76 - 12 \quad O_{4Y} = 64.000$$

$$\text{Ground link:} \quad d := \sqrt{O_{4X}^2 + O_{4Y}^2} \quad d = 79.701$$

Solution: See Figure P4-16 and Mathcad file P0452.

1. Check the Grashof condition of the linkage.

$$\begin{aligned} \text{Condition}(a, b, c, d) := & \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(a, b, c, d) = \text{"Grashof"} \quad \text{crank rocker}$$

2. Define the coordinate frame transformation angle:

$$\delta := \pi + \text{atan}\left(\frac{O_{4Y}}{O_{4X}}\right) \quad \delta = 126.582 \text{ deg}$$

3. Calculate the angle of link 2 in the XY system when links 2 and 3 are in the overlapped toggle position.

$$\theta_{21XY} := -\text{acos}\left[\frac{(b-a)^2 + d^2 - c^2}{2 \cdot (b-a) \cdot d}\right] + \delta \quad \theta_{21XY} = 86.765 \text{ deg}$$

4. Calculate the angle of link 2 in the XY system when links 2 and 3 are in the extended toggle position.

$$\theta_{22XY} := -\text{acos}\left[\frac{(b+a)^2 + d^2 - c^2}{2 \cdot (b+a) \cdot d}\right] + \delta \quad \theta_{22XY} = 93.542 \text{ deg}$$

PROBLEM 4-53

Statement: The coordinates of the point P_1 on link 4 in Figure P4-16 are (114.68, 33.19) with respect to the xy coordinate system when link 2 is in the position shown. When link 2 is in another position the coordinates of P_2 with respect to the xy system are (100.41, 43.78). Calculate the coordinates of P_1 and P_2 in the XY system for the two positions of link 2. What is the salient feature of the coordinates of P_1 and P_2 in the XY system?

Given: Vertical and horizontal offsets from O_2 to O_4 .

$$O_2O_4_X := 47.5 \cdot \text{in} \quad O_2O_4_Y := 64 \cdot \text{in}$$

Coordinates of P_1 and P_2 in the local system

$$P_{1X} := 114.68 \cdot \text{in} \quad P_{1Y} := 33.19 \cdot \text{in}$$

$$P_{2X} := 100.41 \cdot \text{in} \quad P_{2Y} := 43.78 \cdot \text{in}$$

Solution: See Figure P4-16 and Mathcad file P0453.

1. Calculate the angle from the global X axis to the local x axis.

$$\delta := 180 \cdot \text{deg} - \text{atan}\left(\frac{O_2O_4_Y}{O_2O_4_X}\right) \quad \delta = 126.582 \text{ deg}$$

2. Use equations 4.0b to transform the given coordinates from the local to the global system.

$$P_{1X} := P_{1x} \cdot \cos(\delta) - P_{1y} \cdot \sin(\delta) \quad P_{1X} = -95.00 \text{ in}$$

$$P_{1Y} := P_{1x} \cdot \sin(\delta) + P_{1y} \cdot \cos(\delta) \quad P_{1Y} = 72.31 \text{ in}$$

$$P_{2X} := P_{2x} \cdot \cos(\delta) - P_{2y} \cdot \sin(\delta) \quad P_{2X} = -95.00 \text{ in}$$

$$P_{2Y} := P_{2x} \cdot \sin(\delta) + P_{2y} \cdot \cos(\delta) \quad P_{2Y} = 54.54 \text{ in}$$

3. In the global XY system the X -coordinates are the same for each point, which indicates that the head on the end of the rocker beam 4 is designed such that its tangent is always parallel to the Y -axis.

PROBLEM 4-54

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular position of link 4 with respect to the XY coordinate frame and the transmission angle at point B of the linkage in Figure P4-16 as a function of the angle of input link 2 with respect to the XY frame.

Given: Link lengths:

Input (O_2A)	$a := 14$	Coupler (AB)	$b := 80$
Rocker (O_4B)	$c := 51.26$	Ground link	$d := 79.70$

Solution: See Figure P4-16 and Mathcad file P0454.

1. Check the Grashof condition of the linkage.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$Condition(a,b,c,d) = \text{"Grashof"}$ crank rocker

2. Define the coordinate frame transformation angle:

$$\delta := 90 \cdot \text{deg} + \text{atan}\left(\frac{47.5}{64}\right) \quad \delta = 126.582 \text{ deg}$$

3. Define one cycle of the input crank with respect to the XY frame:

$$\theta_{2XY} := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg} \quad \theta_2(\theta_{2XY}) := \theta_{2XY} - \delta$$

4. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$\begin{aligned}
 K_1 &:= \frac{d}{a} & K_2 &:= \frac{d}{c} & K_3 &:= \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \\
 K_1 &= 5.6929 & K_2 &= 1.5548 & K_3 &= 1.9339 \\
 A(\theta_{2XY}) &:= \cos(\theta_2(\theta_{2XY})) - K_1 - K_2 \cdot \cos(\theta_2(\theta_{2XY})) + K_3 \\
 B(\theta_{2XY}) &:= -2 \cdot \sin(\theta_2(\theta_{2XY})) & C(\theta_{2XY}) &:= K_1 - (K_2 + 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_3 \\
 \theta_{41}(\theta_{2XY}) &:= 2 \cdot \left(\text{atan2}\left(2 \cdot A(\theta_{2XY}), -B(\theta_{2XY}) + \sqrt{B(\theta_{2XY})^2 - 4 \cdot A(\theta_{2XY}) \cdot C(\theta_{2XY})} \right) \right) \\
 \theta_4(\theta_{2XY}) &:= \theta_{41}(\theta_{2XY}) + \delta - 2 \cdot \pi
 \end{aligned}$$

5. Use equations 4.12 and 4.13 to calculate θ_3 as a function of θ_2 (for the crossed circuit).

$$\begin{aligned}
 K_4 &:= \frac{d}{b} & K_5 &:= \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \\
 K_4 &= 0.996 & K_5 &= -4.607
 \end{aligned}$$

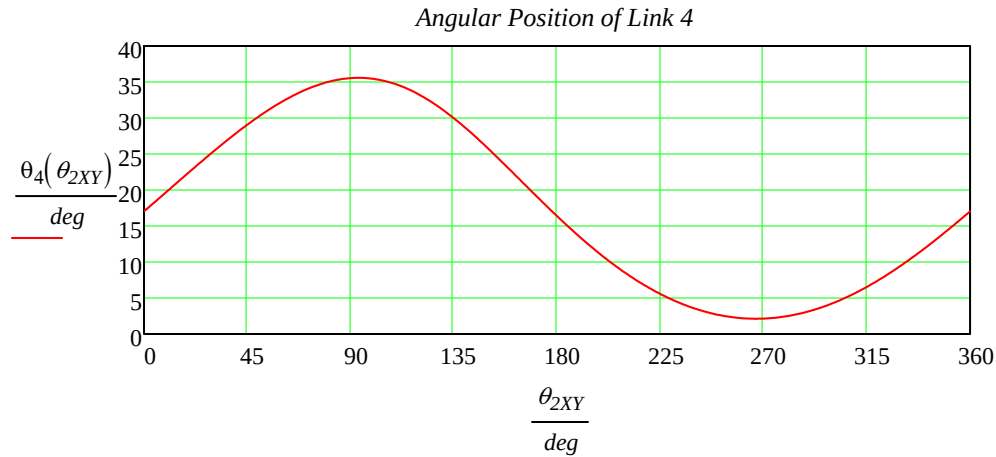
$$D(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 + K_4 \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

$$E(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$F(\theta_{2XY}) := K_1 + (K_4 - 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

$$\theta_3(\theta_{2XY}) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_{2XY}), -E(\theta_{2XY}) + \sqrt{E(\theta_{2XY})^2 - 4 \cdot D(\theta_{2XY}) \cdot F(\theta_{2XY})} \right) \right)$$

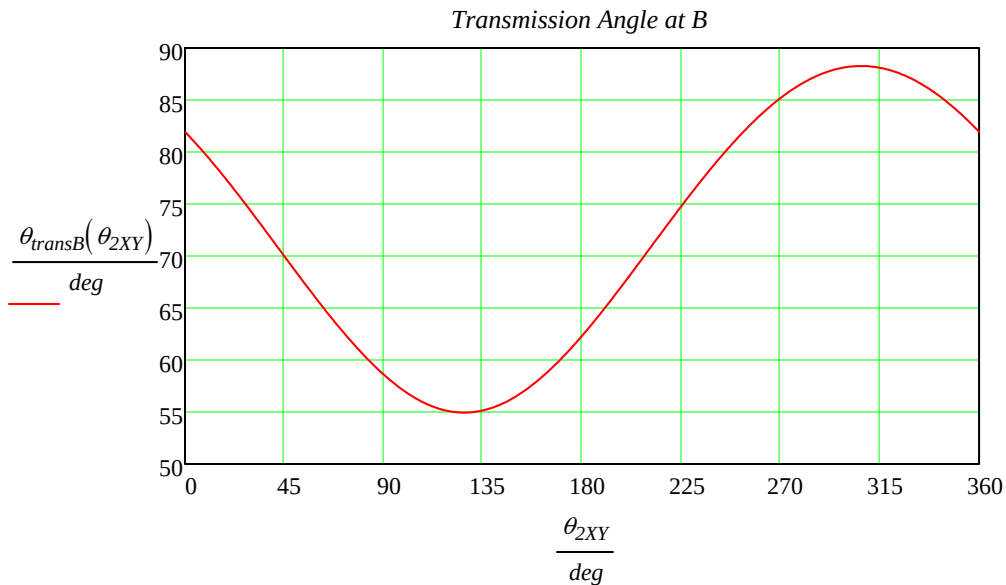
6. Plot the angular position of link 4 as a function of the input angle of link 2 with respect to the XY frame.



7. Calculate (using equations 4.32) and plot the transmission angle at B.

$$\theta_{transB1}(\theta_{2XY}) := |\theta_3(\theta_{2XY}) - \theta_{41}(\theta_{2XY})|$$

$$\theta_{transB}(\theta_{2XY}) := \left| \text{if} \left(\theta_{transB1}(\theta_{2XY}) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_{2XY}), \theta_{transB1}(\theta_{2XY}) \right) \right|$$



PROBLEM 4-55

Statement: For the linkage in Figure P4-17, calculate the maximum CW rotation of link 2 from the position shown, which is -20.60° with respect to the local xy system. What angles do link 3 and link 4 rotate through for that excursion of link 2?

Given: Link lengths:

$$\text{Input } (O_2A) \quad a := 9.17 \quad \text{Coupler } (AB) \quad b := 12.97$$

$$\text{Rocker } (O_4B) \quad c := 9.57 \quad \text{Ground link} \quad d := 7.49$$

$$\text{Initial position of link 2: } \theta_{20} := -26.00 \cdot \text{deg} + 2 \cdot \pi \quad (\text{with respect to } xy \text{ system})$$

Solution: See Figure P4-17 and Mathcad file P0455.

1. Check the Grashof condition of the linkage.

$$\begin{aligned} \text{Condition}(a, b, c, d) := & \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(d, b, a, c) = \text{"non-Grashof"}$$

2. Using equations 4.37, determine the crank angles (relative to the line O_2O_4) at which links 3 and 4 are in toggle

$$\arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad \arg_1 = 0.936$$

$$\arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad \arg_2 = -2.678$$

$$\theta_{21} := \text{acos}(\arg_1) \quad \theta_{21} = 20.55 \text{ deg}$$

The other toggle angle is the negative of this.

$$\theta_{22} := -\theta_{21} + 2 \cdot \pi \quad \theta_{22} = 339.45 \text{ deg}$$

3. Calculate the CW rotation of link 2 from the initial position to the toggle position.

$$\Delta\theta_2 := \theta_{21} - \theta_{20} \quad \Delta\theta_2 = -313.45 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$\begin{aligned} K_1 &:= \frac{d}{a} & K_4 &:= \frac{d}{b} & K_5 &:= \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \\ K_1 &= 0.8168 & K_4 &= 0.5775 & K_5 &= -0.9115 \end{aligned}$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

Initial angular position of link 3: $\theta_3(\theta_{20}) - 2 \cdot \pi = -647.755 \text{ deg}$

Final angular position of link 3: $\theta_3(\theta_{21} + 0.001 \cdot \text{deg}) = 250.764 \text{ deg}$

$$\Delta\theta_3 := \theta_3(\theta_{21} + 0.001 \cdot \text{deg}) - (\theta_3(\theta_{20}) - 2 \cdot \pi) \quad \Delta\theta_3 = 898.518 \text{ deg}$$

7. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 0.8168$$

$$K_2 = 0.7827$$

$$K_3 = 0.3621$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

Initial angular position of link 4: $\theta_4(\theta_{20}) - 2 \cdot \pi = -659.462 \text{ deg}$

Final angular position of link 4: $\theta_4(\theta_{21} + 0.001 \cdot \text{deg}) = 250.615 \text{ deg}$

$$\Delta\theta_4 := \theta_4(\theta_{21} + 0.001 \cdot \text{deg}) - (\theta_4(\theta_{20}) - 2 \cdot \pi) \quad \Delta\theta_4 = 910.077 \text{ deg}$$

PROBLEM 4-56

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the path of point P in Figure P4-17 with respect to the XY coordinate system as a function of the angle of input link 2 with respect to the XY coordinate system.

Given: Link lengths:

Input (O_2A)	$a := 9.174$	Coupler (AB)	$b := 12.971$
Rocker (O_4B)	$c := 9.573$	Ground link	$d := 7.487$
Coupler point data:	$p := 15.00$	$\delta := 0 \cdot \text{deg}$	

Solution: See Figure P4-17 and Mathcad file P0456.

1. Check the Grashof condition of the linkage.

$$\begin{aligned} \text{Condition}(a, b, c, d) := & \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(a, b, c, d) = \text{"non-Grashof"}$$

2. Using equations 4.37, determine the crank angles (relative to the line O_2O_4) at which links 3 and 4 are in toggle

$$\arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad \arg_1 = 0.937$$

$$\arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad \arg_2 = -2.679$$

$$\theta_{\text{toggle}} := \text{acos}(\arg_1) \quad \theta_{\text{toggle}} = 20.501 \text{ deg}$$

The other toggle angle is the negative of this.

3. Define the coordinate transformation angle.

$$\text{Transformation angle: } \alpha := -\text{atan}\left(\frac{6.95}{2.79}\right) \quad \alpha = -68.128 \text{ deg}$$

4. Define one cycle of the input crank between limit positions:

$$\theta_2 := \theta_{\text{toggle}}, \theta_{\text{toggle}} + 1 \cdot \text{deg} .. 2 \cdot \pi - \theta_{\text{toggle}}$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \quad K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_1 = 0.8161 \quad K_4 = 0.5772 \quad K_5 = -0.9110$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Use equations 4.31 to define the x- and y-components of the vector $\mathbf{R}_p$.

$$\mathbf{R}_p := \mathbf{R}_A + \mathbf{R}_{pA}$$

$$\mathbf{R}_A := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{R}_{pA} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

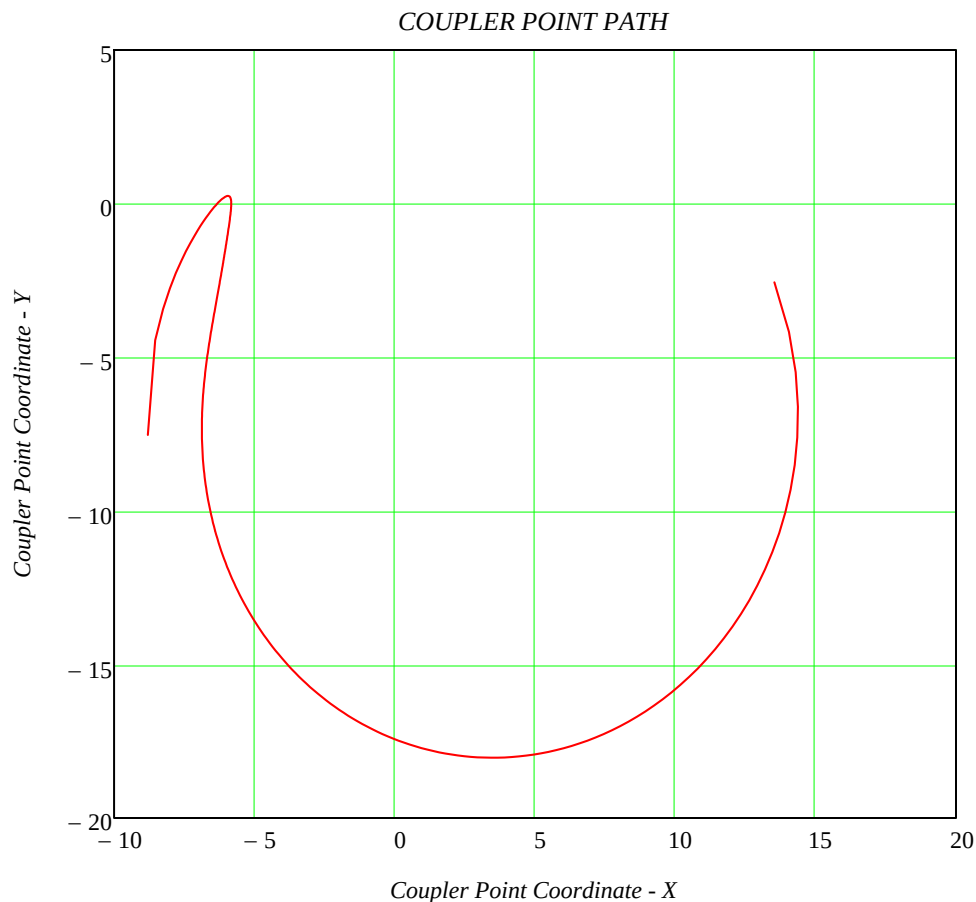
$$R_{px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta) \quad R_{py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

8. Transform these local xy coordinates to the global XY coordinate system using equations 4.0b.

$$R_{pX}(\theta_2) := R_{px}(\theta_2) \cdot \cos(\alpha) - R_{py}(\theta_2) \cdot \sin(\alpha)$$

$$R_{pY}(\theta_2) := R_{px}(\theta_2) \cdot \sin(\alpha) + R_{py}(\theta_2) \cdot \cos(\alpha)$$

9. Plot the coordinates of the coupler point in the global XY coordinate system.



PROBLEM 4-57

Statement: For the linkage in Figure P4-17, calculate the coordinates of the point P in the XY coordinate system if its coordinates in the xy system are $(2.71, 10.54)$.

Given: Vertical and horizontal offsets from O_2 to O_4 .

$$O_2O_4_X := 2.790 \cdot \text{in} \quad O_2O_4_Y := -6.948 \cdot \text{in}$$

Coordinates of P in the local system

$$P_x := 12.816 \cdot \text{in} \quad P_y := 10.234 \cdot \text{in}$$

Solution: See Figure P4-17 and Mathcad file P0457.

1. Calculate the angle from the global X axis to the local x axis.

$$\delta := \text{atan}\left(\frac{O_2O_4_Y}{O_2O_4_X}\right) \quad \delta = -68.122 \text{ deg}$$

2. Use equations 4.0b to transform the given coordinates from the local to the global system.

$$P_X := P_x \cdot \cos(\delta) - P_y \cdot \sin(\delta) \quad P_X = 14.273 \text{ in}$$

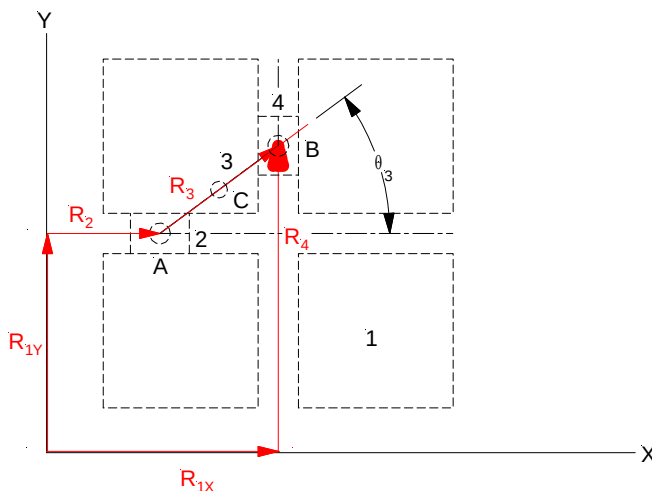
$$P_Y := P_x \cdot \sin(\delta) + P_y \cdot \cos(\delta) \quad P_Y = -8.079 \text{ in}$$

PROBLEM 4-58

Statement: The elliptical trammel in Figure P4-18 must be driven by rotating link 3 in a full circle. Derive analytical expressions for the positions of points A, B, and a point C on link 3 midway between A and B as a function of θ_3 and the length AB of link 3. Use a vector loop equation. (Hint: Place the global origin off the mechanism, preferably below and to the left and use a total of 5 vectors.) Code your solution in an equation solver such as *Mathcad*, *Matlab*, or *TKSolver* to calculate and plot the path of point C for one revolution of link 3.

Solution: See Figure P4-18 and Mathcad file P0458.

1. Establish the global XY system such that the coordinates of the intersection of the slot centerlines is at (d_X, d_Y) . Then, define position vectors $\mathbf{R}_{1X}$, $\mathbf{R}_{1Y}$, $\mathbf{R}_2$, $\mathbf{R}_3$, and $\mathbf{R}_4$ as shown below.



2. Write the vector loop equation: $\mathbf{R}_{1Y} + \mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_{1X} - \mathbf{R}_4 = 0$ then substitute the complex number notation for each position vector. The equation then becomes:

$$d_Y \cdot e^{j\left(\frac{\pi}{2}\right)} + a \cdot e^{j(0)} + c \cdot e^{j(\theta_3)} - d_X \cdot e^{j(0)} - b \cdot e^{j\left(\frac{\pi}{2}\right)} = 0$$

3. Substituting the Euler identity into this equation gives:

$$d_Y \cdot j + a + c \cdot (\cos(\theta_3) + j \cdot \sin(\theta_3)) - d_X - b \cdot j = 0$$

4. Separate this equation into its real (x component) and imaginary (y component) parts, setting each equal to zero.

$$a + c \cdot \cos(\theta_3) - d_X = 0 \quad d_Y + c \cdot \sin(\theta_3) - b = 0$$

5. Solve for the two unknowns a and b in terms of the constants d_X and d_Y and the independent variable θ_3 . Where (a, d_Y) and (d_X, b) are the coordinates of points A and B, respectively, and c is the length of link 3. With no loss of generality, let $d_X = d_Y = d$. Then,

$$a = d - c \cdot \cos(\theta_3) \quad b = d + c \cdot \sin(\theta_3)$$

6. The coordinates of the point C are:

$$C_X = d - 0.5 \cdot c \cdot \cos(\theta_3) \quad C_Y = d + 0.5 \cdot c \cdot \sin(\theta_3)$$

7. Using a local coordinate system whose origin is located at the intersection of the centerlines of the two slots and transforming the above functions to the local xy system:

$$a_x = -c \cdot \cos(\theta_3)$$

$$a_y = 0$$

$$b_x = 0$$

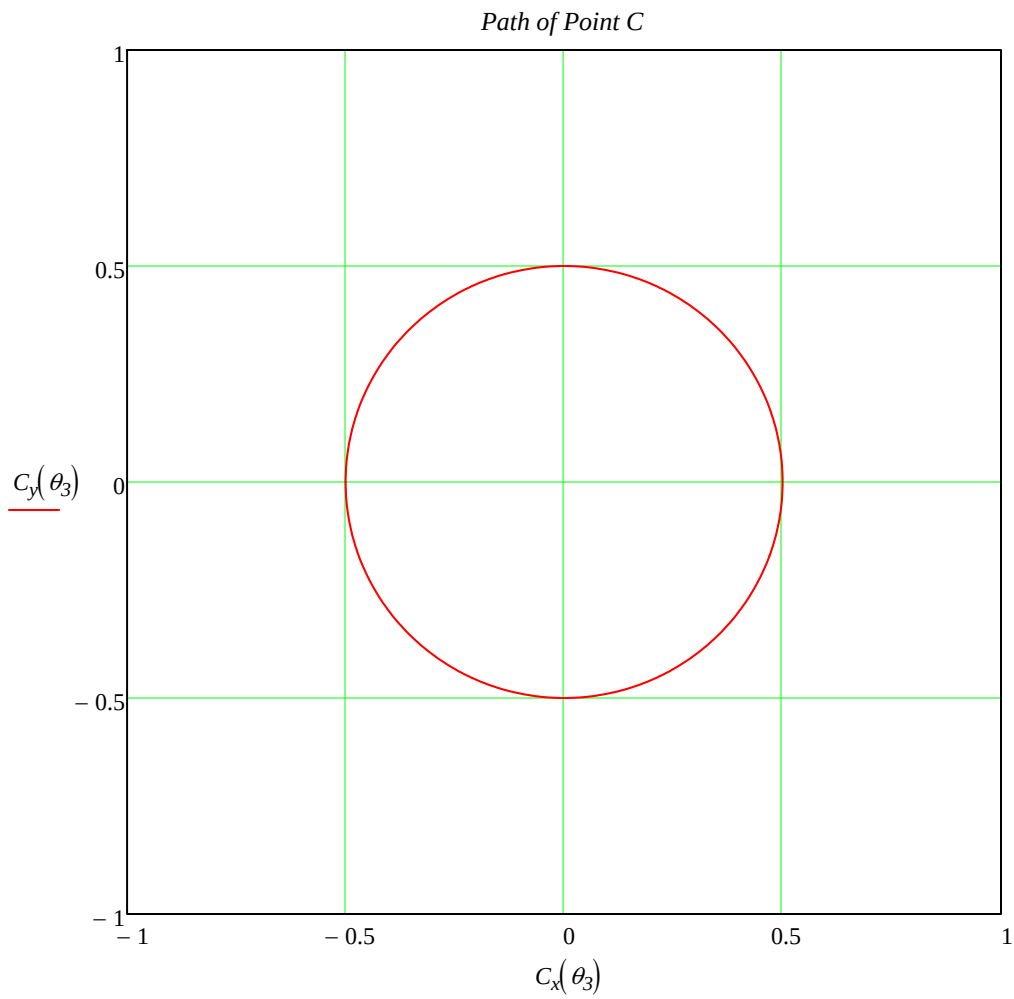
$$b_y = c \cdot \sin(\theta_3)$$

8. To plot the path of point C as a function of θ_3 , let $c := 1$ and define a range function for θ_3

$$\theta_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

$$C_x(\theta_3) := -0.5 \cdot c \cdot \cos(\theta_3)$$

$$C_y(\theta_3) := 0.5 \cdot \sin(\theta_3)$$



PROBLEM 4-59

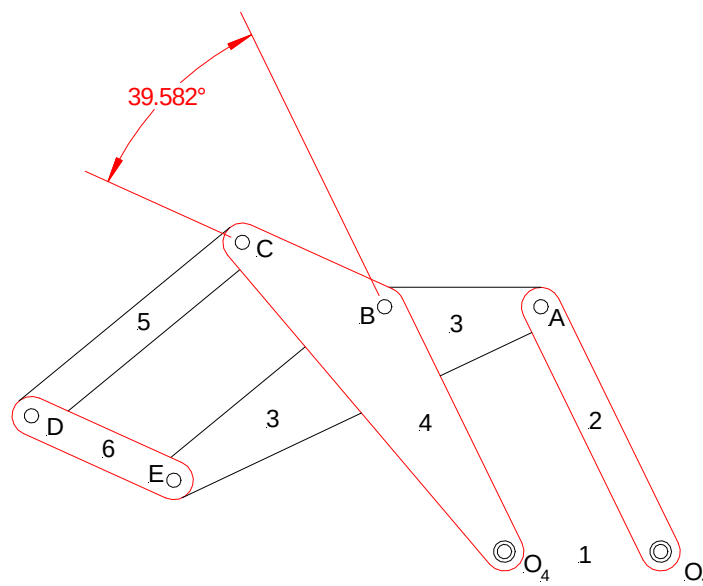
Statement: Calculate and plot the angular position of link 6 in Figure P4-19 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 1.75$	First coupler (AB)	$b := 1.00$
First rocker (O_4B)	$c := 1.75$	Ground link (O_2O_4)	$d := 1.00$
Second input (BC)	$e := 1.00$	Second coupler (L_5)	$f := 1.75$
Output rocker (L_6)	$g := 1.00$	Third coupler (BE)	$h := 1.75$
Ternary link (O_4C)	$i := 2.60$		

Solution: See Figure P4-19 and Mathcad file P0459.

1. Because the linkage is symmetrical and composed of two parallelograms the analysis can be done with simple trigonometry.



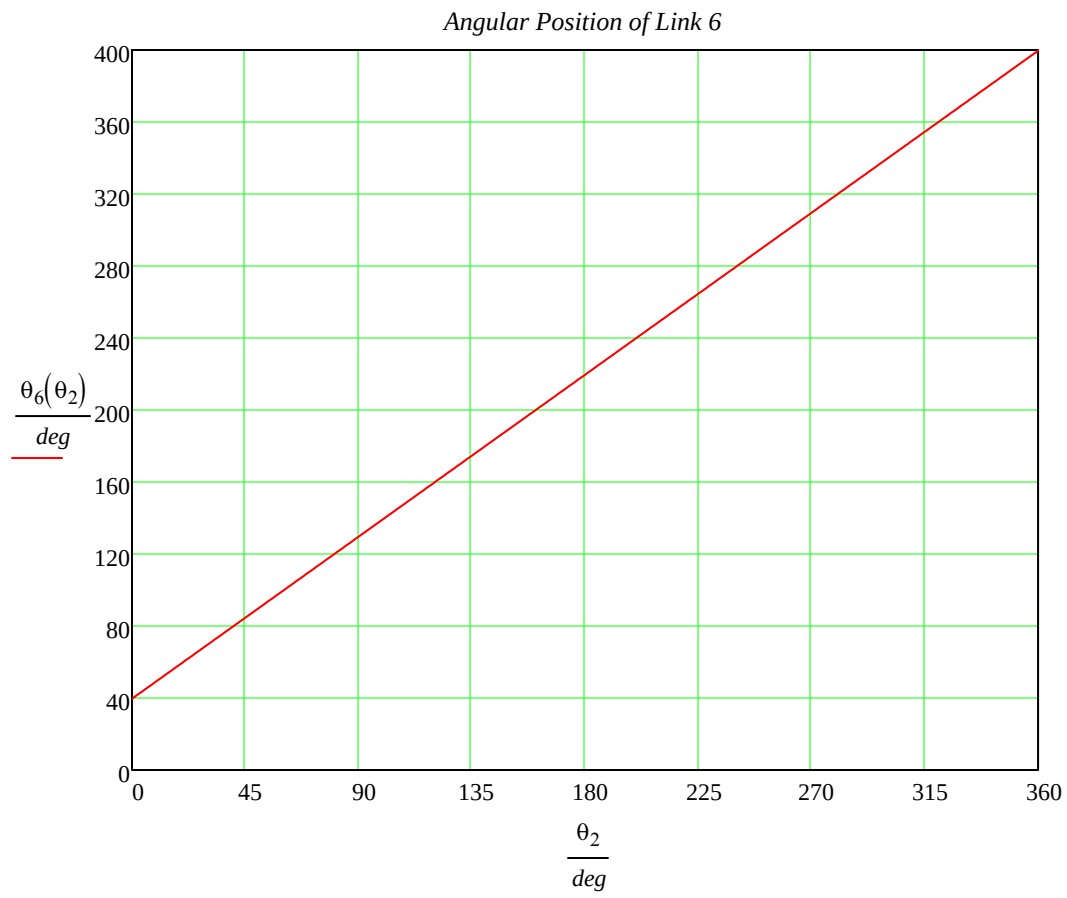
2. Calculate the fixed angle that line BC makes with the extension of line O_4B using the law of cosines.

$$\delta := \pi - \arccos\left(\frac{c^2 + e^2 - i^2}{2 \cdot c \cdot e}\right) \quad \delta = 39.582 \text{ deg}$$

3. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
4. Because links 1, 2, 3, and 4 are a parallelogram, link 4 will have the same angle as link 2 and AB will always be parallel to O_2O_4 . And because links BC , 5, 6, and BE are also a parallelogram, link 6 will have the same angle as link BC . Thus,

$$\theta_6(\theta_2) := \theta_2 + \delta$$

5. Plot the angular position of link 6 as a function of the angle of input link 2 (see next page).



PROBLEM 4-60a

Statement: The link lengths, offset, and value of d for some fourbar slider-crank linkages are defined in Table P4-5. The linkage configuration and terminology are shown in Figure P4-2. For row a , draw the linkage to scale and graphically find all possible solutions (both open and crossed) for angles θ_2 and θ_3 .

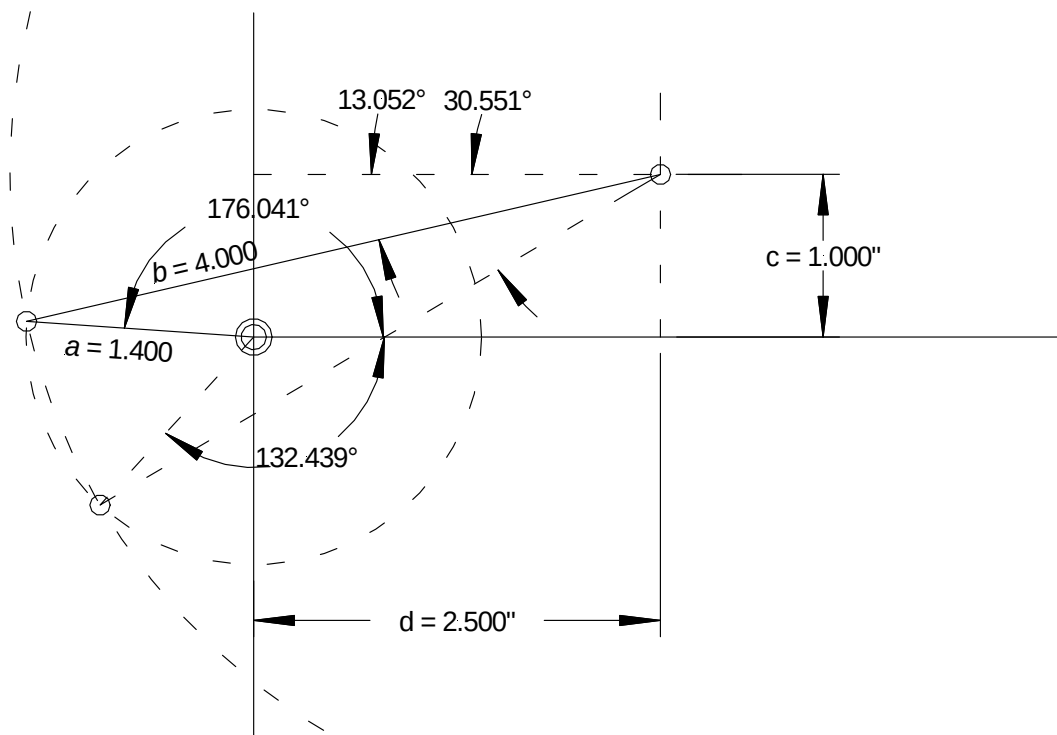
Given: Link 2 $a := 1.4 \cdot \text{in}$ Link 3 $b := 4 \cdot \text{in}$
Offset $c := 1 \cdot \text{in}$ Slider position $d := 2.5 \cdot \text{in}$

Solution: See Figure P4-2, Table P4-5, and Mathcad file P0460a.

1. Lay out an xy-axis system. Its origin will be the link 2 pivot, O_2 .
2. Draw a circle centered at the origin with radius equal to a at some convenient scale.
3. Draw construction lines to define the point (d, c) .
4. From the point (d, c) draw an arc with radius equal to b .
5. The two intersections of the circle and arc are the two solutions to the position analysis problem, crossed and open. If the circle and line don't intersect, there is no solution.
6. Draw links 2 and 3 in their two possible positions (shown as solid for branch 1 and dashed for branch 2 in the figure) and measure the angles θ_2 and θ_3 for each branch. From the solution below,

$$\text{Branch 1: } \theta_{21} := 176.041 \cdot \text{deg} \quad \theta_{31} := (180 + 13.052) \cdot \text{deg} \quad \theta_{31} = 193.052 \text{ deg}$$

$$\text{Branch 2: } \theta_{22} := -132.439 \cdot \text{deg} \quad \theta_{32} := (180 + 30.551) \cdot \text{deg} \quad \theta_{32} = 210.551 \text{ deg}$$



PROBLEM 4-61a

Statement: The link lengths, offset, and value of d for some fourbar slider-crank linkages are defined in Table P4-5. The linkage configuration and terminology are shown in Figure P4-2. For row a , using the vector loop method, find all possible solutions (both open and crossed) for angles θ_2 and θ_3 .

Given: Link 2 $a := 1.4 \cdot \text{in}$ Link 3 $b := 4 \cdot \text{in}$
Offset $c := 1 \cdot \text{in}$ Slider position $d := 2.5 \cdot \text{in}$

Solution: See Figure P4-2, Table P4-5, and Mathcad file P0461a.

1. Determine both values of θ_2 using equations 4.20 and 4.21.

$$\begin{aligned} K_1 &:= a^2 - b^2 + c^2 + d^2 & K_1 &= -6.790 \cdot \text{in}^2 \\ K_2 &:= -2 \cdot a \cdot c & K_2 &= -2.800 \cdot \text{in}^2 \\ K_3 &:= -2 \cdot a \cdot d & K_3 &= -7.000 \cdot \text{in}^2 \\ A &:= K_1 - K_3 & A &= 0.210 \cdot \text{in}^2 \\ B &:= 2 \cdot K_2 & B &= -5.600 \cdot \text{in}^2 \\ C &:= K_1 + K_3 & C &= -13.790 \cdot \text{in}^2 \\ \theta_{21} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{21} &= 176.041 \cdot \text{deg} \\ \theta_{22} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{22} &= -132.439 \cdot \text{deg} \end{aligned}$$

2. Determine both values of θ_3 using equation 4.16a or 4.17.

$$\beta(a, b, c, d, \alpha) := \begin{cases} \theta \leftarrow \text{asin}\left(\frac{a \cdot \sin(\alpha) - c}{b}\right) \\ d_1 \leftarrow a \cdot \cos(\alpha) - b \cdot \cos(\theta) \\ \text{return } \theta \text{ if } d_1 = d \\ \text{asin}\left(-\frac{a \cdot \sin(\alpha) - c}{b}\right) + \pi \text{ otherwise} \end{cases}$$

$$\begin{aligned} \theta_{31} &:= \beta(a, b, c, d, \theta_{21}) & \theta_{31} &= 193.052 \text{ deg} \\ \theta_{32} &:= \beta(a, b, c, d, \theta_{22}) & \theta_{32} &= 210.551 \text{ deg} \end{aligned}$$

PROBLEM 4-61b

Statement: The link lengths, offset, and value of d for some fourbar slider-crank linkages are defined in Table P4-5. The linkage configuration and terminology are shown in Figure P4-2. For row b , using the vector loop method, find all possible solutions (both open and crossed) for angles θ_2 and θ_3 .

Given: Link 2 $a := 2 \cdot \text{in}$ Link 3 $b := 6 \cdot \text{in}$
Offset $c := -3 \cdot \text{in}$ Slider position $d := 5 \cdot \text{in}$

Solution: See Figure P4-2, Table P4-5, and Mathcad file P0461b.

1. Determine both values of θ_2 using equations 4.20 and 4.21.

$$\begin{aligned} K_1 &:= a^2 - b^2 + c^2 + d^2 & K_1 &= 2.000 \cdot \text{in}^2 \\ K_2 &:= -2 \cdot a \cdot c & K_2 &= 12.000 \cdot \text{in}^2 \\ K_3 &:= -2 \cdot a \cdot d & K_3 &= -20.000 \cdot \text{in}^2 \\ A &:= K_1 - K_3 & A &= 22.000 \cdot \text{in}^2 \\ B &:= 2 \cdot K_2 & B &= 24.000 \cdot \text{in}^2 \\ C &:= K_1 + K_3 & C &= -18.000 \cdot \text{in}^2 \\ \theta_{21} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{21} &= 54.117 \cdot \text{deg} \\ \theta_{22} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{22} &= -116.045 \cdot \text{deg} \end{aligned}$$

2. Determine both values of θ_3 using equation 4.16a or 4.17.

$$\beta(a, b, c, d, \alpha) := \begin{cases} \theta \leftarrow \text{asin}\left(\frac{a \cdot \sin(\alpha) - c}{b}\right) \\ d_1 \leftarrow a \cdot \cos(\alpha) - b \cdot \cos(\theta) \\ \text{return } \theta \text{ if } d_1 = d \\ \text{asin}\left(-\frac{a \cdot \sin(\alpha) - c}{b}\right) + \pi \text{ otherwise} \end{cases}$$

$$\begin{aligned} \theta_{31} &:= \beta(a, b, c, d, \theta_{21}) & \theta_{31} &= 129.640 \text{ deg} \\ \theta_{32} &:= \beta(a, b, c, d, \theta_{22}) & \theta_{32} &= 168.433 \text{ deg} \end{aligned}$$

PROBLEM 4-61c

Statement: The link lengths, offset, and value of d for some fourbar slider-crank linkages are defined in Table P4-5. The linkage configuration and terminology are shown in Figure P4-2. For row c , using the vector loop method, find all possible solutions (both open and crossed) for angles θ_2 and θ_3 .

Given: Link 2 $a := 3 \cdot \text{in}$ Link 3 $b := 8 \cdot \text{in}$
Offset $c := 2 \cdot \text{in}$ Slider position $d := 8 \cdot \text{in}$

Solution: See Figure P4-2, Table P4-5, and Mathcad file P0461c.

1. Determine both values of θ_2 using equations 4.20 and 4.21.

$$\begin{aligned}
 K_1 &:= a^2 - b^2 + c^2 + d^2 & K_1 &= 13.000 \cdot \text{in}^2 \\
 K_2 &:= -2 \cdot a \cdot c & K_2 &= -12.000 \cdot \text{in}^2 \\
 K_3 &:= -2 \cdot a \cdot d & K_3 &= -48.000 \cdot \text{in}^2 \\
 A &:= K_1 - K_3 & A &= 61.000 \cdot \text{in}^2 \\
 B &:= 2 \cdot K_2 & B &= -24.000 \cdot \text{in}^2 \\
 C &:= K_1 + K_3 & C &= -35.000 \cdot \text{in}^2 \\
 \theta_{21} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{21} &= 88.803 \cdot \text{deg} \\
 \theta_{22} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{22} &= -60.731 \cdot \text{deg}
 \end{aligned}$$

2. Determine both values of θ_3 using equation 4.16a or 4.17.

$$\beta(a, b, c, d, \alpha) := \begin{cases} \theta \leftarrow \text{asin}\left(\frac{a \cdot \sin(\alpha) - c}{b}\right) \\ d_1 \leftarrow a \cdot \cos(\alpha) - b \cdot \cos(\theta) \\ \text{return } \theta \text{ if } d_1 = d \\ \text{asin}\left(-\frac{a \cdot \sin(\alpha) - c}{b}\right) + \pi \text{ otherwise} \end{cases}$$

$$\begin{aligned}
 \theta_{31} &:= \beta(a, b, c, d, \theta_{21}) & \theta_{31} &= 172.824 \cdot \text{deg} \\
 \theta_{32} &:= \beta(a, b, c, d, \theta_{22}) & \theta_{32} &= 215.249 \cdot \text{deg}
 \end{aligned}$$

PROBLEM 4-61d

Statement: The link lengths, offset, and value of d for some fourbar slider-crank linkages are defined in Table P4-5. The linkage configuration and terminology are shown in Figure P4-2. For row d , using the vector loop method, find all possible solutions (both open and crossed) for angles θ_2 and θ_3 .

Given: Link 2 $a := 3.5 \cdot \text{in}$ Link 3 $b := 10 \cdot \text{in}$
Offset $c := 1 \cdot \text{in}$ Slider position $d := -8 \cdot \text{in}$

Solution: See Figure P4-2, Table P4-5, and Mathcad file P0461d.

1. Determine both values of θ_2 using equations 4.20 and 4.21.

$$\begin{aligned}
 K_1 &:= a^2 - b^2 + c^2 + d^2 & K_1 &= -22.750 \cdot \text{in}^2 \\
 K_2 &:= -2 \cdot a \cdot c & K_2 &= -7.000 \cdot \text{in}^2 \\
 K_3 &:= -2 \cdot a \cdot d & K_3 &= 56.000 \cdot \text{in}^2 \\
 A &:= K_1 - K_3 & A &= -78.750 \cdot \text{in}^2 \\
 B &:= 2 \cdot K_2 & B &= -14.000 \cdot \text{in}^2 \\
 C &:= K_1 + K_3 & C &= 33.250 \cdot \text{in}^2 \\
 \theta_{21} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{21} &= 286.648 \cdot \text{deg} \\
 \theta_{22} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{22} &= -300.898 \cdot \text{deg}
 \end{aligned}$$

2. Determine both values of θ_3 using equation 4.16a or 4.17.

$$\beta(a, b, c, d, \alpha) := \begin{cases} \theta \leftarrow \text{asin}\left(\frac{a \cdot \sin(\alpha) - c}{b}\right) \\ d_1 \leftarrow a \cdot \cos(\alpha) - b \cdot \cos(\theta) \\ \text{return } \theta \text{ if } d_1 = d \\ \text{asin}\left(-\frac{a \cdot \sin(\alpha) - c}{b}\right) + \pi \text{ otherwise} \end{cases}$$

$$\begin{aligned}
 \theta_{31} &:= \beta(a, b, c, d, \theta_{21}) & \theta_{31} &= -25.806 \cdot \text{deg} \\
 \theta_{32} &:= \beta(a, b, c, d, \theta_{22}) & \theta_{32} &= 11.556 \cdot \text{deg}
 \end{aligned}$$

PROBLEM 4-61e

Statement: The link lengths, offset, and value of d for some fourbar slider-crank linkages are defined in Table P4-5. The linkage configuration and terminology are shown in Figure P4-2. For row e , using the vector loop method, find all possible solutions (both open and crossed) for angles θ_2 and θ_3 .

Given: Link 2 $a := 5 \cdot \text{in}$ Link 3 $b := 20 \cdot \text{in}$
Offset $c := -5 \cdot \text{in}$ Slider position $d := 15 \cdot \text{in}$

Solution: See Figure P4-2, Table P4-5, and Mathcad file P0461e.

1. Determine both values of θ_2 using equations 4.20 and 4.21.

$$\begin{aligned}
 K_1 &:= a^2 - b^2 + c^2 + d^2 & K_1 &= -125.000 \cdot \text{in}^2 \\
 K_2 &:= -2 \cdot a \cdot c & K_2 &= 50.000 \cdot \text{in}^2 \\
 K_3 &:= -2 \cdot a \cdot d & K_3 &= -150.000 \cdot \text{in}^2 \\
 A &:= K_1 - K_3 & A &= 25.000 \cdot \text{in}^2 \\
 B &:= 2 \cdot K_2 & B &= 100.000 \cdot \text{in}^2 \\
 C &:= K_1 + K_3 & C &= -275.000 \cdot \text{in}^2 \\
 \theta_{21} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{21} &= 123.804 \cdot \text{deg} \\
 \theta_{22} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{22} &= -160.674 \cdot \text{deg}
 \end{aligned}$$

2. Determine both values of θ_3 using equation 4.16a or 4.17.

$$\beta(a, b, c, d, \alpha) := \begin{cases} \theta \leftarrow \text{asin}\left(\frac{a \cdot \sin(\alpha) - c}{b}\right) \\ d_1 \leftarrow a \cdot \cos(\alpha) - b \cdot \cos(\theta) \\ \text{return } \theta \text{ if } d_1 = d \\ \text{asin}\left(-\frac{a \cdot \sin(\alpha) - c}{b}\right) + \pi \text{ otherwise} \end{cases}$$

$$\begin{aligned}
 \theta_{31} &:= \beta(a, b, c, d, \theta_{21}) & \theta_{31} &= 152.759 \cdot \text{deg} \\
 \theta_{32} &:= \beta(a, b, c, d, \theta_{22}) & \theta_{32} &= 170.371 \cdot \text{deg}
 \end{aligned}$$

PROBLEM 4-61f

Statement: The link lengths, offset, and value of d for some fourbar slider-crank linkages are defined in Table P4-5. The linkage configuration and terminology are shown in Figure P4-2. For row f , using the vector loop method, find all possible solutions (both open and crossed) for angles θ_2 and θ_3 .

Given: Link 2 $a := 3 \cdot \text{in}$ Link 3 $b := 13 \cdot \text{in}$
Offset $c := 0 \cdot \text{in}$ Slider position $d := -12 \cdot \text{in}$

Solution: See Figure P4-2, Table P4-5, and Mathcad file P0461f.

1. Determine both values of θ_2 using equations 4.20 and 4.21.

$$\begin{aligned} K_1 &:= a^2 - b^2 + c^2 + d^2 & K_1 &= -16.000 \cdot \text{in}^2 \\ K_2 &:= -2 \cdot a \cdot c & K_2 &= 0.000 \cdot \text{in}^2 \\ K_3 &:= -2 \cdot a \cdot d & K_3 &= 72.000 \cdot \text{in}^2 \\ A &:= K_1 - K_3 & A &= -88.000 \cdot \text{in}^2 \\ B &:= 2 \cdot K_2 & B &= 0.000 \cdot \text{in}^2 \\ C &:= K_1 + K_3 & C &= 56.000 \cdot \text{in}^2 \\ \theta_{21} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{21} &= 282.840 \cdot \text{deg} \\ \theta_{22} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{22} &= -282.840 \cdot \text{deg} \end{aligned}$$

2. Determine both values of θ_3 using equation 4.16a or 4.17.

$$\beta(a, b, c, d, \alpha) := \begin{cases} \theta \leftarrow \text{asin}\left(\frac{a \cdot \sin(\alpha) - c}{b}\right) \\ d_1 \leftarrow a \cdot \cos(\alpha) - b \cdot \cos(\theta) \\ \text{return } \theta \text{ if } d_1 = d \\ \text{asin}\left(-\frac{a \cdot \sin(\alpha) - c}{b}\right) + \pi \text{ otherwise} \end{cases}$$

$$\begin{aligned} \theta_{31} &:= \beta(a, b, c, d, \theta_{21}) & \theta_{31} &= -13.003 \cdot \text{deg} \\ \theta_{32} &:= \beta(a, b, c, d, \theta_{22}) & \theta_{32} &= 13.003 \cdot \text{deg} \end{aligned}$$

PROBLEM 4-61g

Statement: The link lengths, offset, and value of d for some fourbar slider-crank linkages are defined in Table P4-5. The linkage configuration and terminology are shown in Figure P4-2. For row g , using the vector loop method, find all possible solutions (both open and crossed) for angles θ_2 and θ_3 .

Given: Link 2 $a := 7 \cdot \text{in}$ Link 3 $b := 25 \cdot \text{in}$
Offset $c := 10 \cdot \text{in}$ Slider position $d := 25 \cdot \text{in}$

Solution: See Figure P4-2, Table P4-5, and Mathcad file P0461g.

1. Determine both values of θ_2 using equations 4.20 and 4.21.

$$\begin{aligned} K_1 &:= a^2 - b^2 + c^2 + d^2 & K_1 &= 149.000 \cdot \text{in}^2 \\ K_2 &:= -2 \cdot a \cdot c & K_2 &= -140.000 \cdot \text{in}^2 \\ K_3 &:= -2 \cdot a \cdot d & K_3 &= -350.000 \cdot \text{in}^2 \\ A &:= K_1 - K_3 & A &= 499.000 \cdot \text{in}^2 \\ B &:= 2 \cdot K_2 & B &= -280.000 \cdot \text{in}^2 \\ C &:= K_1 + K_3 & C &= -201.000 \cdot \text{in}^2 \\ \theta_{21} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{21} &= 88.519 \cdot \text{deg} \\ \theta_{22} &:= 2 \cdot \text{atan2}\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right) & \theta_{22} &= -44.916 \cdot \text{deg} \end{aligned}$$

2. Determine both values of θ_3 using equation 4.16a or 4.17.

$$\beta(a, b, c, d, \alpha) := \begin{cases} \theta \leftarrow \text{asin}\left(\frac{a \cdot \sin(\alpha) - c}{b}\right) \\ d_1 \leftarrow a \cdot \cos(\alpha) - b \cdot \cos(\theta) \\ \text{return } \theta \text{ if } d_1 = d \\ \text{asin}\left(-\frac{a \cdot \sin(\alpha) - c}{b}\right) + \pi \text{ otherwise} \end{cases}$$

$$\begin{aligned} \theta_{31} &:= \beta(a, b, c, d, \theta_{21}) & \theta_{31} &= 186.898 \cdot \text{deg} \\ \theta_{32} &:= \beta(a, b, c, d, \theta_{22}) & \theta_{32} &= 216.705 \cdot \text{deg} \end{aligned}$$

PROBLEM 5-1

Statement: Design a fourbar mechanism to give the two positions shown in Figure P3-1 of output rocker motion with no quick-return. (See Problem 3-3).

Given: Coordinates of the points C_1 , D_1 , C_2 , and D_2 with respect to C_1 :

$$C_{1x} := 0.0 \quad D_{1x} := 1.721 \quad C_{2x} := 2.656 \quad D_{2x} := 5.065$$

$$C_{1y} := 0.0 \quad D_{1y} := -1.750 \quad C_{2y} := -0.751 \quad D_{2y} := -0.281$$

Assumptions: Use the pivot point between links 3 and 4 (C_1 and C_2) as the precision points P_1 and P_2 . Define position vectors in the global frame whose origin is at C_1 .

Solution: See solution to Problem 3-3 and Mathcad file P0501.

- Note that this is a two-position function generation (FG) problem because the output is specified as an angular displacement of the rocker, link 4. See Section 5.13 which details the 3-position FG solution. See also Section 5.3 in which the equations for the two-position motion generation problem are derived. These are really the same problem and have the same solution. The method of Section 5.3 will be used here.
- Two solution methods are derived in Section 5.3 and are presented in equations 5.7 and 5.8 for the left dyad and equations 5.11 and 5.12 for the right dyad. The first method (equations 5.7 and 5.11) looks like the better one to use in this case since it allows us to choose the link's angular positions and excursions and solve for the lengths of links 2 and 3 (w and z). Unfortunately, this method fails in this problem because of the requirement for a non-quick-return Grashof linkage, which requires the angular displacement of link 3 in going from position 1 to position 2 to be zero ($\alpha_2 = 0$) causing a divide-by-zero error in equations 5.7d.
- Method 2 (equations 5.8 and 5.12) requires the choosing of two angles and a length for each dyad.
- To obtain the same solution as was done graphically in Problem 3-4, we need to know the location of the fixed pivot O_4 with respect to the given CD . While we could take the results from Problem 3-3 and use them here to establish the location of O_4 , that won't be done. Instead, we will use the point C as the joint between links 3 and 4 as well as the precision point P .
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$\mathbf{R}_1 := \begin{pmatrix} C_{1x} \\ C_{1y} \end{pmatrix} \quad \mathbf{R}_2 := \begin{pmatrix} C_{2x} \\ C_{2y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := \mathbf{R}_2 - \mathbf{R}_1 \quad P_{21x} = 2.656$$

$$P_{21y} = -0.751$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 2.760$$

- From the trigonometric relationships given in Figure 5-1, determine δ_2 . From the requirement for a non-quick-return, $\alpha_2 := 0$.

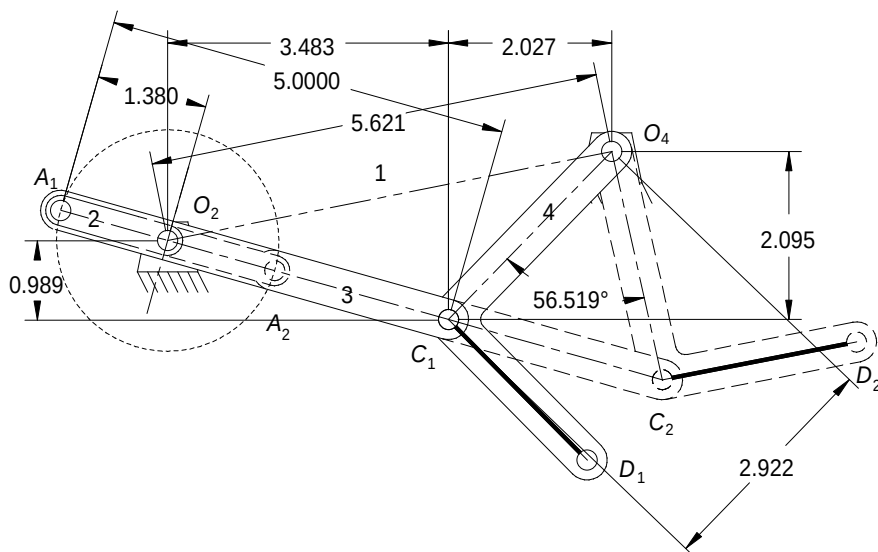
$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = -15.789 \cdot \text{deg}$$

- From a graphical solution (see figure below), determine the values necessary for input to equations 5.8.

$$z := 5.000$$

$$\beta_2 := 180 \cdot \text{deg}$$

$$\phi := \delta_2$$



8. Solve for the **WZ** dyad using equations 5.8.

$$Z_{I_X} := z \cdot \cos(\phi) \qquad Z_{I_X} = 4.811 \qquad Z_{I_Y} := z \cdot \sin(\phi) \qquad Z_{I_Y} = -1.360$$

$$A := \cos(\beta_2) - 1 \qquad A = -2.000 \qquad D := \sin(\alpha_2) \qquad D = 0.000$$

$$B := \sin(\beta_2) \qquad B = 0.000 \qquad E := p_{21} \cdot \cos(\delta_2) \qquad E = 2.656$$

$$C := \cos(\alpha_2) - 1 \qquad C = 0.000 \qquad F := p_{21} \cdot \sin(\delta_2) \qquad F = -0.751$$

$$W_{I_x} := \frac{A \cdot (-C \cdot Z_{I_x} + D \cdot Z_{I_y} + E) + B \cdot (-C \cdot Z_{I_y} - D \cdot Z_{I_x} + F)}{-2 \cdot A} \quad W_{I_x} = -1.328$$

$$W_{Iy} := \frac{A \cdot (-C \cdot Z_{Iy} - D \cdot Z_{Ix} + F) + B \cdot (C \cdot Z_{Ix} - D \cdot Z_{Iy} - E)}{-2 \cdot A} \quad W_{Iy} = 0.375$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \quad w = 1.380$$

$$\theta := atan2(W_{1x}, W_{1y}) \quad \theta = 164.211 \cdot deg$$

This is the expected value of w (one half of p_{21}) based on the design choices made in the graphical solution and the assumptions made in this problem.

9. From the graphical solution (see figure above), determine the values necessary for input to equations 5.12.

$$s := 0 \qquad \gamma_2 := 56.519 \cdot deg \qquad \psi := 0 \cdot deg$$

10. Solve for the **US** dyad using equations 5.12.

$$S_{1x} := s \cdot \cos(\psi) \qquad S_{1x} = 0.000 \qquad S_{1y} := s \cdot \sin(\psi) \qquad S_{1y} = 0.000$$

$$A := \cos(\gamma_2) - 1 \qquad A = -0.448 \qquad D := \sin(\alpha_2) \qquad D = 0.000$$

$$B := \sin(\gamma_2) \qquad B = 0.834 \qquad E := p_{21} \cdot \cos(\delta_2) \qquad E = 2.656$$

$$C := \cos(\alpha_2) - 1 \quad C = 0.000 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = -0.751$$

$$U_{1x} := \frac{A \cdot (-C \cdot S_{1x} + D \cdot S_{1y} + E) + B \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F)}{-2 \cdot A} \quad U_{1x} = -2.027$$

$$U_{1y} := \frac{A \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F) + B \cdot (C \cdot S_{1x} - D \cdot S_{1y} - E)}{-2 \cdot A} \quad U_{1y} = -2.095$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2} \quad u = 2.915$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y}) \quad \sigma = -134.048 \cdot \text{deg}$$

This is the expected value of u based on the design choices made in the graphical solution and the assumptions made in this problem.

11. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\text{Link 3: } V_{1x} := z \cdot \cos(\phi) - s \cdot \cos(\psi) \quad V_{1x} = 4.811$$

$$V_{1y} := z \cdot \sin(\phi) - s \cdot \sin(\psi) \quad V_{1y} = -1.360$$

$$\theta_3 := \text{atan2}(V_{1x}, V_{1y}) \quad \theta_3 = -15.789 \cdot \text{deg}$$

$$v := \sqrt{V_{1x}^2 + V_{1y}^2} \quad v = 5.000$$

$$\text{Link 1: } G_{1x} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma) \quad G_{1x} = 5.510$$

$$G_{1y} := w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma) \quad G_{1y} = 1.110$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y}) \quad \theta_1 = 11.391 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2} \quad g = 5.621$$

12. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \quad \theta_{2i} = 152.820 \cdot \text{deg}$$

$$\theta_{2f} := \theta_2 + \beta_2 \quad \theta_{2f} = 332.820 \cdot \text{deg}$$

13. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z \quad \delta_p := \phi - \theta_3$$

$$r_p = 5.000 \quad \delta_p = 0.000 \cdot \text{deg}$$

which is correct for the assumption that the precision point is at C.

14. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := 0 \cdot \text{deg} \quad \rho_1 = 0.000 \cdot \text{deg}$$

$$R_1 := \sqrt{C_{1x}^2 + C_{1y}^2} \quad R_1 = 0.000$$

$$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta) \quad O_{2x} = -3.483$$

$$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta) \quad O_{2y} = 0.985$$

$$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma) \quad O_{4x} = 2.027$$

$$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma) \quad O_{4y} = 2.095$$

15. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = 11.391 \cdot \text{deg}$$

16. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"Grashof"}$$

17. DESIGN SUMMARY

Link 2:	$w = 1.380$	$\theta = 164.211 \cdot \text{deg}$
Link 3:	$v = 5.000$	$\theta_3 = -15.789 \cdot \text{deg}$
Link 4:	$u = 2.915$	$\sigma = -134.048 \cdot \text{deg}$
Link 1:	$g = 5.621$	$\theta_1 = 11.391 \cdot \text{deg}$
Coupler:	$r_p = 5.000$	$\delta_p = 0.000 \cdot \text{deg}$
Crank angles:	$\theta_{2i} = 152.820 \cdot \text{deg}$	
	$\theta_{2f} = 332.820 \cdot \text{deg}$	

PROBLEM 5-2

Statement: Design a fourbar mechanism to give the two positions shown in Figure P3-1 of coupler motion.

Given: Coordinates of the points A_1 , B_1 , A_2 , and B_2 with respect to A_1 :

$$\begin{array}{llll} A_{1x} := 0.0 & B_{1x} := 1.721 & A_{2x} := 2.656 & B_{2x} := 5.065 \\ A_{1y} := 0.0 & B_{1y} := -1.750 & A_{2y} := -0.751 & B_{2y} := -0.281 \end{array}$$

Assumptions: Use the points A_1 and A_2 as the precision points P_1 and P_2 . Define position vectors in the global frame whose origin is at A_1 .

Solution: See solution to Problem 3-4 and Mathcad file P0502.

1. Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. See Section 5.3 in which the equations for the two-position motion generation problem are derived.
2. Two solution methods are derived in Section 5.3 and are presented in equations 5.7 and 5.8 for the left dyad and equations 5.11 and 5.12 for the right dyad.
3. Method 1 (equations 5.7 and 5.11) requires the choosing of three angles for each dyad. Method 2 (equations 5.8 and 5.12) requires the choosing of two angles and a length for each dyad. Method 1 is used in this solution.
4. In order to obtain the same solution as was done graphically in Problem 3-4, the necessary assumed values were taken from that solution as shown below.
5. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

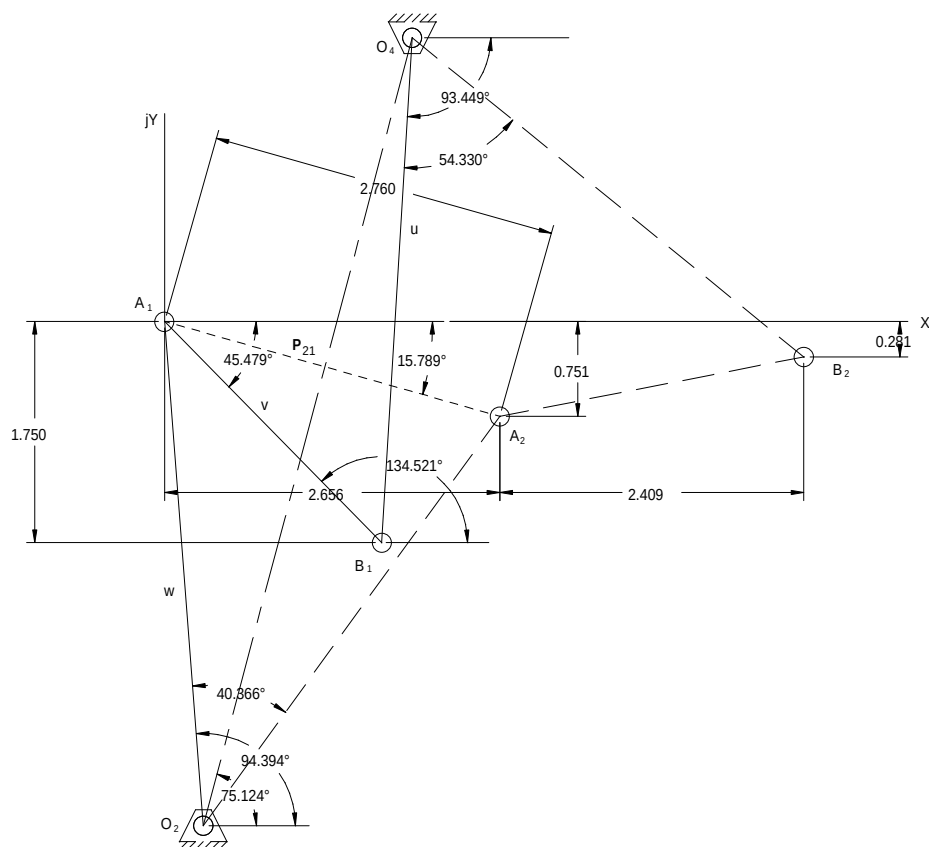
$$\begin{array}{llll} R1 := \begin{pmatrix} A_{1x} \\ A_{1y} \end{pmatrix} & R2 := \begin{pmatrix} A_{2x} \\ A_{2y} \end{pmatrix} & \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 & P_{21x} = 2.656 \\ & & & P_{21y} = -0.751 \\ p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} & & & p_{21} = 2.760 \end{array}$$

6. From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\begin{array}{ll} \alpha_2 := \text{atan2}\left[\left(A_{2x} - B_{2x}\right), \left(A_{2y} - B_{2y}\right)\right] \dots & \alpha_2 = -303.481 \cdot \text{deg} \\ \quad + -\text{atan2}\left[\left(A_{1x} - B_{1x}\right), \left(A_{1y} - B_{1y}\right)\right] & \\ \delta_2 := \text{atan2}\left(P_{21x}, P_{21y}\right) & \delta_2 = -15.789 \cdot \text{deg} \end{array}$$

7. From the graphical solution (see figure below), determine the values necessary for input to equations 5.7.

$$\theta := 94.394 \cdot \text{deg} \quad \beta_2 := -40.366 \cdot \text{deg} \quad \phi := -45.479 \cdot \text{deg}$$



8. Solve for the **WZ** dyad using equations 5.7.

$$A := \cos(\theta) \cdot (\cos(\beta_2) - 1) - \sin(\theta) \cdot \sin(\beta_2)$$

$$B := \cos(\phi) \cdot (\cos(\alpha_2) - 1) - \sin(\phi) \cdot \sin(\alpha_2)$$

$$C := p_{21} \cdot \cos(\delta_2)$$

$$D := \sin(\theta) \cdot (\cos(\beta_2) - 1) + \cos(\theta) \cdot \sin(\beta_2)$$

$$E := \sin(\phi) \cdot (\cos(\alpha_2) - 1) + \cos(\phi) \cdot \sin(\alpha_2)$$

$$F := p_{21} \cdot \sin(\delta_2)$$

$$w := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D}$$

$$z := \frac{A \cdot F - C \cdot D}{A \cdot E - B \cdot D}$$

$w = 4.000$

$$z = 0.000$$

These are the expected values of w and z based on the design choices made in the graphical solution and the assumptions made in this problem.

9. From the graphical solution (see figure above), determine the values necessary for input to equations 5.11.

$$\sigma := -93.449 \cdot \text{deg}$$

$$\gamma_2 := 54.330 \cdot \text{deg}$$

$$\psi := 134.521 \cdot deg$$

10. Solve for the **US** dyad using equations 5.11.

$$A' := \cos(\sigma) \cdot (\cos(\gamma_2) - 1) - \sin(\sigma) \cdot \sin(\gamma_2)$$

$$B' := \cos(\psi) \cdot (\cos(\alpha_2) - 1) - \sin(\psi) \cdot \sin(\alpha_2)$$

$$C := p_{21} \cdot \cos(\delta_2)$$

$$D' := \sin(\sigma) \cdot (\cos(\gamma_2) - 1) + \cos(\sigma) \cdot \sin(\gamma_2)$$

$$E' := \sin(\psi) \cdot (\cos(\alpha_2) - 1) + \cos(\psi) \cdot \sin(\alpha_2)$$

$$F := p_{21} \cdot \sin(\delta_2)$$

$$u := \frac{C \cdot E' - B' \cdot F}{A' \cdot E' - B' \cdot D'}$$

$$s := \frac{A' \cdot F - C \cdot D'}{A' \cdot E' - B' \cdot D'}$$

$$u = 4.000$$

$$s = 2.455$$

These are the expected values of u and s based on the design choices made in the graphical solution and the assumptions made in this problem.

11. Solve for the links 3 and 1 using the vector definitions of $\mathbf{V}$ and $\mathbf{G}$.

Link 3: $V_{Ix} := z \cdot \cos(\phi) - s \cdot \cos(\psi)$

$$V_{Ix} = 1.721$$

$$V_{Iy} := z \cdot \sin(\phi) - s \cdot \sin(\psi)$$

$$V_{Iy} = -1.750$$

$$\theta_3 := \text{atan2}(V_{Ix}, V_{Iy})$$

$$\theta_3 = -45.479 \cdot \text{deg}$$

$$v := \sqrt{V_{Ix}^2 + V_{Iy}^2}$$

$$v = 2.455$$

Link 1: $G_{Ix} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma)$

$$G_{Ix} = 1.655$$

$$G_{Iy} := w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma)$$

$$G_{Iy} = 6.231$$

$$\theta_1 := \text{atan2}(G_{Ix}, G_{Iy})$$

$$\theta_1 = 75.123 \cdot \text{deg}$$

$$g := \sqrt{G_{Ix}^2 + G_{Iy}^2}$$

$$g = 6.447$$

12. Determine the initial and final values of the input crank with respect to the vector $\mathbf{G}$.

$$\theta_{2i} := \theta - \theta_1$$

$$\theta_{2i} = 19.271 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_2$$

$$\theta_{2f} = -21.095 \cdot \text{deg}$$

13. Define the coupler point with respect to point A and the vector $\mathbf{V}$.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 0.000$$

$$\delta_p = 0.000 \cdot \text{deg}$$

which is correct for the assumption that the precision point is at A.

14. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := 0 \cdot \text{deg}$$

$$\rho_1 = 0.000 \cdot \text{deg}$$

$$R_1 := \sqrt{A_{Ix}^2 + A_{Iy}^2}$$

$$R_1 = 0.000$$

$$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = 0.306$$

$$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = -3.988$$

$$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma) \quad O_{4x} = 1.962$$

$$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma) \quad O_{4y} = 2.243$$

15. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = 75.123 \cdot \text{deg}$$

16. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

17. DESIGN SUMMARY

Link 2:	$w = 4.000$	$\theta = 94.394 \cdot \text{deg}$
Link 3:	$v = 2.455$	$\theta_3 = -45.479 \cdot \text{deg}$
Link 4:	$u = 4.000$	$\sigma = -93.449 \cdot \text{deg}$
Link 1:	$g = 6.447$	$\theta_1 = 75.123 \cdot \text{deg}$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000 \cdot \text{deg}$
Crank angles:	$\theta_{2i} = 19.271 \cdot \text{deg}$	
	$\theta_{2f} = -21.095 \cdot \text{deg}$	

PROBLEM 5-3

Statement: Design a fourbar mechanism to give the three positions of coupler motion with no quick return shown in Figure P3-2. (See Problem 3-5). Ignore the fixed pivot points in the Figure.

Given: Coordinates of points A and B with respect to point A_1 :

$$\begin{array}{llll} A_{1x} := 0.0 & A_{1y} := 0.0 & B_{1x} := 0.741 & B_{1y} := -2.383 \\ A_{2x} := 2.019 & A_{2y} := -1.905 & B_{2x} := 4.428 & B_{2y} := -2.557 \\ A_{3x} := 3.933 & A_{3y} := -1.035 & B_{3x} := 6.304 & B_{3y} := -0.256 \end{array}$$

Assumptions: Let points A_1, A_2 , and A_3 be the precision points P_1, P_2 , and P_3 , respectively.

Solution: See Figure P3-2 Mathcad file P0503.

1. Determine the magnitudes and orientation of the position difference vectors.

$$\begin{array}{llll} p_{21} := \sqrt{A_{2x}^2 + A_{2y}^2} & p_{21} = 2.776 & \delta_2 := \text{atan2}(A_{2x}, A_{2y}) & \delta_2 = -43.336 \cdot \text{deg} \\ p_{31} := \sqrt{A_{3x}^2 + A_{3y}^2} & p_{31} = 4.067 & \delta_3 := \text{atan2}(A_{3x}, A_{3y}) & \delta_3 = -14.744 \cdot \text{deg} \end{array}$$

2. Determine the angle changes of the coupler between precision points.

$$\begin{array}{ll} \theta_{p1} := \text{atan2}(A_{1x} - B_{1x}, A_{1y} - B_{1y}) & \theta_{p1} = 107.273 \cdot \text{deg} \\ \theta_{p2} := \text{atan2}(A_{2x} - B_{2x}, A_{2y} - B_{2y}) & \theta_{p2} = 164.856 \cdot \text{deg} \\ \theta_{p3} := \text{atan2}(A_{3x} - B_{3x}, A_{3y} - B_{3y}) & \theta_{p3} = -161.812 \cdot \text{deg} \\ \alpha_2 := \theta_{p2} - \theta_{p1} & \alpha_2 = 57.582 \cdot \text{deg} \\ \alpha_3 := \theta_{p3} - \theta_{p1} & \alpha_3 = -269.085 \cdot \text{deg} \end{array}$$

3. The free choices for this linkage are (from the graphical solution to Problem 3-5):

$$\beta_2 := 78.375 \cdot \text{deg} \quad \beta_3 := 135.560 \cdot \text{deg} \quad \gamma_2 := 59.771 \cdot \text{deg} \quad \gamma_3 := 107.023 \cdot \text{deg}$$

4. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$\begin{array}{lll} A := \cos(\beta_2) - 1 & B := \sin(\beta_2) & C := \cos(\alpha_2) - 1 \\ D := \sin(\alpha_2) & E := p_{21} \cdot \cos(\delta_2) & F := \cos(\beta_3) - 1 \\ G := \sin(\beta_3) & H := \cos(\alpha_3) - 1 & K := \sin(\alpha_3) \\ L := p_{31} \cdot \cos(\delta_3) & M := p_{21} \cdot \sin(\delta_2) & N := p_{31} \cdot \sin(\delta_3) \end{array}$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$$\begin{aligned} W_{1x} &= -2.178 & W_{1y} &= -0.286 & Z_{1x} &= 0.000 & Z_{1y} &= 0.000 \\ \theta &:= \text{atan2}(W_{1x}, W_{1y}) & \theta &= -172.523 \cdot \text{deg} & \phi &:= \text{atan2}(Z_{1x}, Z_{1y}) & \phi &= 174.344 \cdot \text{deg} \end{aligned}$$

The length of link 2 is: $w := \sqrt{(W_{1x}^2 + W_{1y}^2)}$, $w = 2.197$

The length of vector **Z** is: $z := \sqrt{(Z_{1x}^2 + Z_{1y}^2)}$, $z = 0.000$

5. Evaluate terms in the **US** coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$$\begin{aligned} A' &:= \cos(\gamma_2) - 1 & B' &:= \sin(\gamma_2) & C &:= \cos(\alpha_2) - 1 \\ D &:= \sin(\alpha_2) & E &:= p_{21} \cdot \cos(\delta_2) & F' &:= \cos(\gamma_3) - 1 \\ G' &:= \sin(\gamma_3) & H &:= \cos(\alpha_3) - 1 & K &:= \sin(\alpha_3) \\ L &:= p_{31} \cdot \cos(\delta_3) & M &:= p_{21} \cdot \sin(\delta_2) & N &:= p_{31} \cdot \sin(\delta_3) \end{aligned}$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$$\begin{aligned} U_{1x} &= -1.995 & U_{1y} &= -3.121 & S_{1x} &= -0.741 & S_{1y} &= 2.383 \\ \sigma &:= \text{atan2}(U_{1x}, U_{1y}) & \sigma &= -122.591 \cdot \text{deg} & \psi &:= \text{atan2}(S_{1x}, S_{1y}) & \psi &= 107.267 \cdot \text{deg} \end{aligned}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 3.704$

The length of vector **S** is: $s := \sqrt{(S_{1x}^2 + S_{1y}^2)}$, $s = 2.495$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 0.741$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = -2.383$

$\theta_3 := \text{atan2}(V_{1x}, V_{1y})$ $\theta_3 = -72.734 \cdot \text{deg}$

$v := \sqrt{V_{1x}^2 + V_{1y}^2}$ $v = 2.495$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 0.558$

$$G_{1y} := W_{1y} + V_{1y} - U_{1y}$$

$$G_{1y} = 0.452$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y})$$

$$\theta_1 = 39.009 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2}$$

$$g = 0.718$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1$$

$$\theta_{2i} = -211.532 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_3$$

$$\theta_{2f} = -75.972 \cdot \text{deg}$$

8. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 0.000$$

$$\delta_p = 247.078 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = 2.178$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = 0.286$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = 2.736$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = 0.738$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$$

$$\theta_{rot} = 39.009 \cdot \text{deg}$$

11. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"Grashof"}$$

12. DESIGN SUMMARY

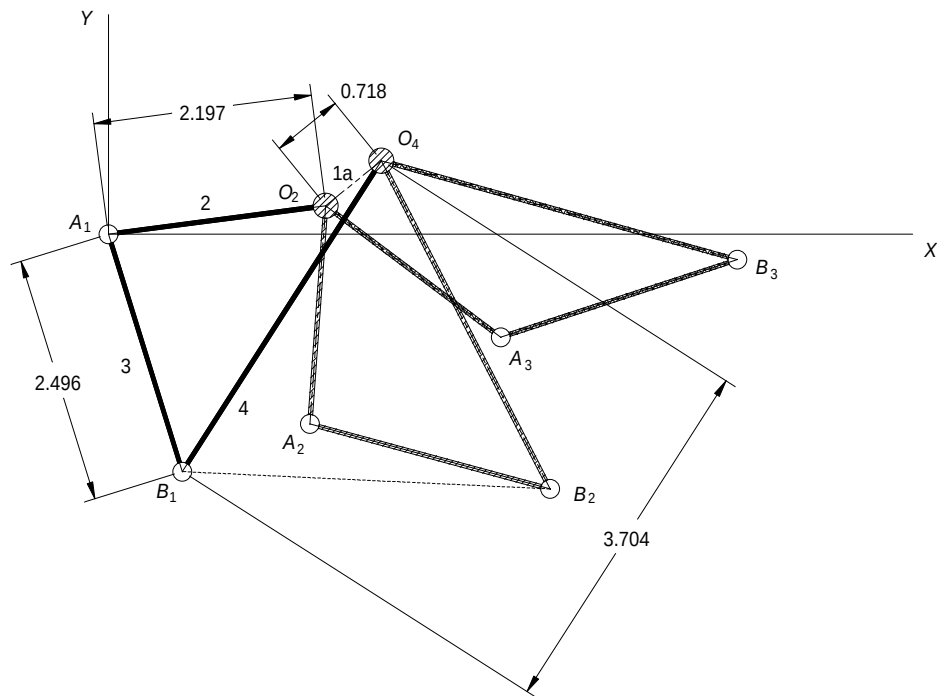
Link 2:	$w = 2.197$	$\theta = -172.523 \cdot \text{deg}$
Link 3:	$v = 2.495$	$\theta_3 = -72.734 \cdot \text{deg}$
Link 4:	$u = 3.704$	$\sigma = -122.591 \cdot \text{deg}$
Link 1:	$g = 0.718$	$\theta_1 = 39.009 \cdot \text{deg}$
Coupler:	$r_p = 0.000$	$\delta_p = 247.078 \cdot \text{deg}$

Crank angles:

$$\theta_{2i} = -211.532 \cdot \text{deg}$$

$$\theta_{2f} = -75.972 \cdot \text{deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



This is the same result as that found in Problem 3-5.

PROBLEM 5-4

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-2 (see Problem 3-6). Use analytical synthesis and design it for the fixed pivots shown.

Given: Link end points (with respect to A_1):

$$\begin{array}{llllll} A_{1x} := 0.0 & B_{1x} := 0.741 & A_{2x} := 2.019 & B_{2x} := 4.428 & A_{3x} := 3.933 & B_{3x} := 6.304 \\ A_{1y} := 0.0 & B_{1y} := -2.383 & A_{2y} := -1.905 & B_{2y} := -2.557 & A_{3y} := -1.035 & B_{3y} := -0.256 \end{array}$$

Fixed pivot points (with respect to A_1):

$$O_{2x} := 0.995 \quad O_{2y} := -5.086 \quad O_{4x} := 5.298 \quad O_{4y} := -5.086$$

Solution: See Figure P3-2 and Mathcad file P0504.

1. Determine the angle changes between precision points from the body angles given.

$$\begin{array}{ll} \theta_{p1} := \text{atan2}(A_{1x} - B_{1x}, A_{1y} - B_{1y}) & \theta_{p1} = 107.273 \cdot \text{deg} \\ \theta_{p2} := \text{atan2}(A_{2x} - B_{2x}, A_{2y} - B_{2y}) & \theta_{p2} = 164.856 \cdot \text{deg} \\ \theta_{p3} := \text{atan2}(A_{3x} - B_{3x}, A_{3y} - B_{3y}) & \theta_{p3} = -161.812 \cdot \text{deg} \\ \alpha_2 := \theta_{p2} - \theta_{p1} & \alpha_2 = 57.582 \cdot \text{deg} \\ \alpha_3 := \theta_{p3} - \theta_{p1} & \alpha_3 = -269.085 \cdot \text{deg} \end{array}$$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$\begin{array}{llll} P_{21x} := A_{2x} & P_{21x} = 2.019 & P_{31x} := A_{3x} & P_{31x} = 3.933 \\ P_{21y} := A_{2y} & P_{21y} = -1.905 & P_{31y} := A_{3y} & P_{31y} = -1.035 \\ R_{1x} := -O_{2x} & R_{1x} = -0.995 & R_{1y} := -O_{2y} & R_{1y} = 5.086 \\ R_{2x} := R_{1x} + P_{21x} & R_{2x} = 1.024 & & \\ R_{2y} := R_{1y} + P_{21y} & R_{2y} = 3.181 & & \\ R_{3x} := R_{1x} + P_{31x} & R_{3x} = 2.938 & & \\ R_{3y} := R_{1y} + P_{31y} & R_{3y} = 4.051 & & \\ R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} & R_1 = 5.182 & & \\ R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} & R_2 = 3.342 & & \\ R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} & R_3 = 5.004 & & \end{array}$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\begin{array}{ll} \zeta_1 := \text{atan2}(R_{1x}, R_{1y}) & \zeta_1 = 101.069 \cdot \text{deg} \\ \zeta_2 := \text{atan2}(R_{2x}, R_{2y}) & \zeta_2 = 72.156 \cdot \text{deg} \end{array}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = 54.048 \cdot \text{deg}$$

4. Solve for β_2 and β_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = 1.352$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = 3.679$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = -8.007$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = 5.127$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = -5.851$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = 1.294$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -90.406$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = 19.633$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = -53.487$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = -22.524$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = -19.633$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -29.689$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = 1.146 \times 10^3$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = 1.788 \times 10^3$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = 1.769 \times 10^3$$

$$\beta_{31} := 2 \cdot \text{atan}\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \quad \beta_{31} = 90.915 \cdot \text{deg}$$

$$\beta_{32} := 2 \cdot \text{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \quad \beta_{32} = 23.770 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\beta_3 := \beta_{32}$$

$$\beta_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right) \quad \beta_{21} = 16.790 \cdot \text{deg}$$

$$\beta_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right) \quad \beta_{22} = 16.790 \cdot \text{deg}$$

Since both values are the same,

$$\beta_2 := \beta_{21}$$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$\begin{aligned}
 R_{1x} &:= -O_{4x} & R_{1x} &= -5.298 & R_{1y} &:= -O_{4y} & R_{1y} &= 5.086 \\
 R_{2x} &:= R_{1x} + P_{21x} & R_{2x} &= -3.279 \\
 R_{2y} &:= R_{1y} + P_{21y} & R_{2y} &= 3.181 \\
 R_{3x} &:= R_{1x} + P_{31x} & R_{3x} &= -1.365 \\
 R_{3y} &:= R_{1y} + P_{31y} & R_{3y} &= 4.051 \\
 R_1 &:= \sqrt{R_{1x}^2 + R_{1y}^2} & R_1 &= 7.344 \\
 R_2 &:= \sqrt{R_{2x}^2 + R_{2y}^2} & R_2 &= 4.568 \\
 R_3 &:= \sqrt{R_{3x}^2 + R_{3y}^2} & R_3 &= 4.275
 \end{aligned}$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\begin{aligned}
 \zeta_1 &:= \text{atan2}(R_{1x}, R_{1y}) & \zeta_1 &= 136.170 \cdot \text{deg} \\
 \zeta_2 &:= \text{atan2}(R_{2x}, R_{2y}) & \zeta_2 &= 135.869 \cdot \text{deg} \\
 \zeta_3 &:= \text{atan2}(R_{3x}, R_{3y}) & \zeta_3 &= 108.621 \cdot \text{deg}
 \end{aligned}$$

7. Solve for γ_2 and γ_3 using equations 5.34

$$\begin{aligned}
 C_1 &:= R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) & C_1 &= -1.023 \\
 C_2 &:= R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) & C_2 &= 4.349 \\
 C_3 &:= R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) & C_3 &= -3.636 \\
 C_4 &:= -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) & C_4 &= 9.430 \\
 C_5 &:= R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) & C_5 &= -3.855 \\
 C_6 &:= -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) & C_6 &= 4.927 \\
 A_1 &:= -C_3^2 - C_4^2 & A_1 &= -102.135 \\
 A_2 &:= C_3 \cdot C_6 - C_4 \cdot C_5 & A_2 &= 18.434 \\
 A_3 &:= -C_4 \cdot C_6 - C_3 \cdot C_5 & A_3 &= -60.472 \\
 A_4 &:= C_2 \cdot C_3 + C_1 \cdot C_4 & A_4 &= -25.460 \\
 A_5 &:= C_4 \cdot C_5 - C_3 \cdot C_6 & A_5 &= -18.434 \\
 A_6 &:= C_1 \cdot C_3 - C_2 \cdot C_4 & A_6 &= -37.287 \\
 K_1 &:= A_2 \cdot A_4 + A_3 \cdot A_6 & K_1 &= 1.785 \times 10^3 \\
 K_2 &:= A_3 \cdot A_4 + A_5 \cdot A_6 & K_2 &= 2.227 \times 10^3
 \end{aligned}$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = 2.198 \times 10^3$$

$$\gamma_{31} := 2 \cdot \text{atan}\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \quad \gamma_{31} = 90.915 \cdot \text{deg}$$

$$\gamma_{32} := 2 \cdot \text{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \quad \gamma_{32} = 11.643 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\gamma_3 := \gamma_{32}$$

$$\gamma_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1}\right) \quad \gamma_{21} = 11.069 \cdot \text{deg}$$

$$\gamma_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1}\right) \quad \gamma_{22} = 11.069 \cdot \text{deg}$$

Since both angles are the same,

$$\gamma_2 := \gamma_{21}$$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 2.776$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = -43.336 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \quad p_{31} = 4.067$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y}) \quad \delta_3 = -14.744 \cdot \text{deg}$$

9. Evaluate terms in the $\mathbf{WZ}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1 \quad B := \sin(\beta_2) \quad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \quad E := p_{21} \cdot \cos(\delta_2) \quad F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3) \quad H := \cos(\alpha_3) - 1 \quad K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \quad M := p_{21} \cdot \sin(\delta_2) \quad N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the $\mathbf{W}$ and $\mathbf{Z}$ vectors are:

$$W1x = 3.594$$

$$W1y = 7.810$$

$$Z1x = -4.589$$

$$Z1y = -2.724$$

11. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2}$ $w = 8.597$

12. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$U1x = -2.400$$

$$U1y = 7.549$$

$$S1x = -2.898$$

$$S1y = -2.463$$

14. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2}$ $u = 7.921$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x$$

$$V1x = -1.691$$

$$V1y := Z1y - S1y$$

$$V1y = -0.261$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$ $v = 1.711$

$$G1x := W1x + V1x - U1x$$

$$G1x = 4.303$$

$$G1y := W1y + V1y - U1y$$

$$G1y = -5.329 \times 10^{-14}$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$ $g = 4.303$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors **W**₁, **Z**₁, **U**₁, and **S**₁.

$$O2x := -Z1x - W1x$$

$$O2x = 0.995$$

$$O2y := -Z1y - W1y$$

$$O2y = -5.086$$

$$O4x := -S1x - U1x$$

$$O4x = 5.298$$

$$O4y := -S1y - U1y$$

$$O4y = -5.086$$

These check with Figure P3-2.

17. Determine the location of the coupler point with respect to point A and line AB.

$$\text{Distance from A to P} \quad z := \sqrt{Z1x^2 + Z1y^2} \quad z = 5.336 \quad r_P := z$$

$$\text{Angle BAP } (\delta_p) \quad s := \sqrt{S1x^2 + S1y^2} \quad s = 3.803$$

$$\psi := \text{atan2}(S1x, S1y) \quad \psi = -139.639 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z1x, Z1y) \quad \phi = -149.309 \cdot \text{deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = -171.233 \cdot \text{deg}$$

$$\delta_p := \phi - \theta_3 \quad \delta_p = 21.924 \cdot \text{deg}$$

18. DESIGN SUMMARY

$$\text{Link 1:} \quad g = 4.303$$

$$\text{Link 2:} \quad w = 8.597$$

$$\text{Link 3:} \quad v = 1.711$$

$$\text{Link 4:} \quad u = 7.921$$

$$\text{Coupler point:} \quad r_P = 5.336 \quad \delta_p = 21.924 \cdot \text{deg}$$

19. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

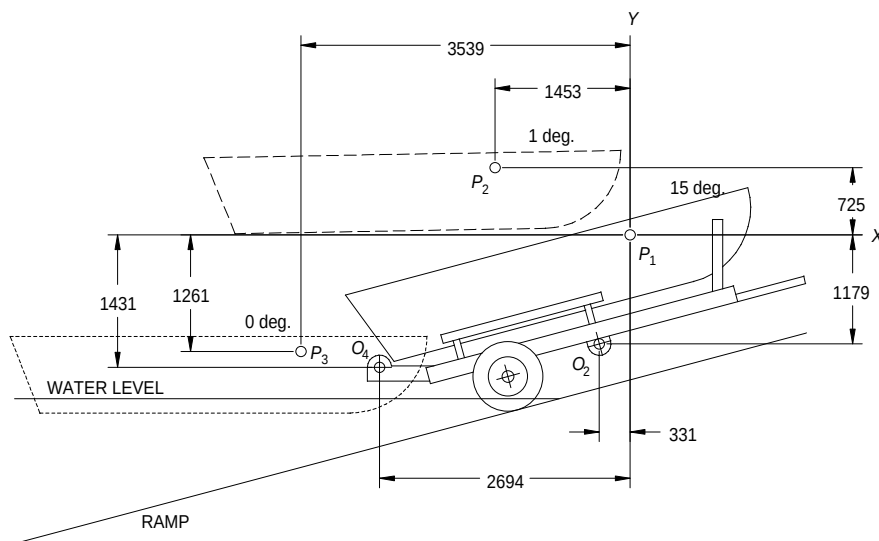
PROBLEM 5-5

Statement: See Project P3-8. Define three positions of the boat and analytically synthesize a linkage to move through them.

Assumptions: Launch ramp angle is 15 deg to the horizontal.

Solution: See Project P3-8 and Mathcad file P0505.

1. This is an open-ended design problem that has many valid solutions. First define the problem more completely than is stated by deciding on three positions for the boat to move through. The figure below shows one such set of positions (dimensions are in mm).



2. From the figure, the design choices are:

$$\begin{aligned}
 P_{21x} &:= -1453 & P_{21y} &:= 725 & P_{31x} &:= -3539 & P_{31y} &:= -1261 \\
 O_{2x} &:= -331 & O_{2y} &:= -1179 & O_{4x} &:= -2694 & O_{4y} &:= -1431 \\
 \text{Body angles:} & & \theta_{p1} &:= 15 \cdot \text{deg} & \theta_{p2} &:= 1 \cdot \text{deg} & \theta_{p3} &:= 0 \cdot \text{deg}
 \end{aligned}$$

3. The methods of Section 5.8 are used to get a solution for this problem. The solution is sensitive to small changes in the design choices so a trial-and-error approach is warranted.
4. Determine the angle changes between precision points from the body angles given.

$$\begin{aligned}
 \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -14.000 \cdot \text{deg} \\
 \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -15.000 \cdot \text{deg}
 \end{aligned}$$

5. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$\begin{aligned}
 R_{1x} &:= -O_{2x} & R_{1x} &= 331.000 & R_{1y} &:= -O_{2y} & R_{1y} &= 1.179 \times 10^3 \\
 R_{2x} &:= R_{1x} + P_{21x} & R_{2x} &= -1.122 \times 10^3 \\
 R_{2y} &:= R_{1y} + P_{21y} & R_{2y} &= 1.904 \times 10^3 \\
 R_{3x} &:= R_{1x} + P_{31x} & R_{3x} &= -3.208 \times 10^3 \\
 R_{3y} &:= R_{1y} + P_{31y} & R_{3y} &= -82.000
 \end{aligned}$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \quad R_1 = 1.225 \times 10^3$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 2.210 \times 10^3$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 3.209 \times 10^3$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 74.318 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 120.510 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = -178.536 \cdot \text{deg}$$

7. Solve for β_2 and β_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = -2.542 \times 10^3$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = -1.433 \times 10^3$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = 3.833 \times 10^3$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = -1.135 \times 10^3$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = 1.728 \times 10^3$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = 840.097$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -1.598 \times 10^7$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = 5.182 \times 10^6$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = -5.671 \times 10^6$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = -2.607 \times 10^6$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = -5.182 \times 10^6$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -1.137 \times 10^7$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = 5.096 \times 10^{13}$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = 7.370 \times 10^{13}$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = 3.015 \times 10^{13}$$

$$\beta_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \beta_{31} = 125.675 \cdot \text{deg}$$

$$\beta_{32} := 2 \cdot \operatorname{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \quad \beta_{32} = -15.000 \cdot \text{deg}$$

The second value is the same as α_3 , so use the first value

$$\beta_3 := \beta_{31}$$

$$\beta_{21} := \operatorname{acos}\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right) \quad \beta_{21} = 39.836 \cdot \text{deg}$$

$$\beta_{22} := \operatorname{asin}\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right) \quad \beta_{22} = 39.836 \cdot \text{deg}$$

Since both values are the same,

$$\beta_2 := \beta_{21}$$

8. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$R_{1x} := -O_{4x} \quad R_{1x} = 2.694 \times 10^3 \quad R_{1y} := -O_{4y} \quad R_{1y} = 1.431 \times 10^3$$

$$R_{2x} := R_{1x} + P_{21x} \quad R_{2x} = 1.241 \times 10^3$$

$$R_{2y} := R_{1y} + P_{21y} \quad R_{2y} = 2.156 \times 10^3$$

$$R_{3x} := R_{1x} + P_{31x} \quad R_{3x} = -845.000$$

$$R_{3y} := R_{1y} + P_{31y} \quad R_{3y} = 170.000$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \quad R_1 = 3.050 \times 10^3$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 2.488 \times 10^3$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 861.931$$

9. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \operatorname{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 27.976 \cdot \text{deg}$$

$$\zeta_2 := \operatorname{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 60.075 \cdot \text{deg}$$

$$\zeta_3 := \operatorname{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = 168.625 \cdot \text{deg}$$

10. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = -2.536 \times 10^3$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = -1.392 \times 10^3$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = 3.818 \times 10^3$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = -514.981$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = 1.719 \times 10^3$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = 1.419 \times 10^3$$

$$A_1 := -C_3^2 - C_4^2$$

$$A_1 = -1.484 \times 10^7$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$$

$$A_2 = 6.303 \times 10^6$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$$

$$A_3 = -5.832 \times 10^6$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$$

$$A_4 = -4.008 \times 10^6$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$$

$$A_5 = -6.303 \times 10^6$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$$

$$A_6 = -1.040 \times 10^7$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$$

$$K_1 = 3.537 \times 10^{13}$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$$

$$K_2 = 8.891 \times 10^{13}$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_5^2 - A_6^2}{2}$$

$$K_3 = 1.115 \times 10^{13}$$

$$\gamma_{31} := 2 \cdot \text{atan}\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\gamma_{31} = 151.615 \cdot \text{deg}$$

$$\gamma_{32} := 2 \cdot \text{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\gamma_{32} = -15.000 \cdot \text{deg}$$

The second value is the same as α_3 , so use the first value

$$\gamma_3 := \gamma_{31}$$

$$\gamma_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1}\right)$$

$$\gamma_{21} = 56.167 \cdot \text{deg}$$

$$\gamma_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1}\right)$$

$$\gamma_{22} = 56.167 \cdot \text{deg}$$

Since both angles are the same,

$$\gamma_2 := \gamma_{21}$$

11. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2}$$

$$p_{21} = 1.624 \times 10^3$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y})$$

$$\delta_2 = 153.482 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2}$$

$$p_{31} = 3.757 \times 10^3$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y})$$

$$\delta_3 = -160.388 \cdot \text{deg}$$

12. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$\begin{aligned}
 A &:= \cos(\beta_2) - 1 & B &:= \sin(\beta_2) & C &:= \cos(\alpha_2) - 1 \\
 D &:= \sin(\alpha_2) & E &:= p_{21} \cdot \cos(\delta_2) & F &:= \cos(\beta_3) - 1 \\
 G &:= \sin(\beta_3) & H &:= \cos(\alpha_3) - 1 & K &:= \sin(\alpha_3) \\
 L &:= p_{31} \cdot \cos(\delta_3) & M &:= p_{21} \cdot \sin(\delta_2) & N &:= p_{31} \cdot \sin(\delta_3)
 \end{aligned}$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$W1x = 1.331 \times 10^3 \quad W1y = 1.653 \times 10^3 \quad Z1x = -1.000 \times 10^3 \quad Z1y = -474.187$$

14. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2}$ $w = 2122.473$

15. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$\begin{aligned}
 A' &:= \cos(\gamma_2) - 1 & B' &:= \sin(\gamma_2) & C &:= \cos(\alpha_2) - 1 \\
 D &:= \sin(\alpha_2) & E &:= p_{21} \cdot \cos(\delta_2) & F' &:= \cos(\gamma_3) - 1 \\
 G' &:= \sin(\gamma_3) & H &:= \cos(\alpha_3) - 1 & K &:= \sin(\alpha_3) \\
 L &:= p_{31} \cdot \cos(\delta_3) & M &:= p_{21} \cdot \sin(\delta_2) & N &:= p_{31} \cdot \sin(\delta_3)
 \end{aligned}$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

16. The components of the **W** and **Z** vectors are:

$$U1x = 1.690 \times 10^3 \quad U1y = 951.273 \quad S1x = 1.004 \times 10^3 \quad S1y = 479.727$$

17. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2}$ $u = 1939.291$

18. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$\begin{aligned}
 V1x &:= Z1x - S1x & V1x &= -2.004 \times 10^3 \\
 V1y &:= Z1y - S1y & V1y &= -953.914
 \end{aligned}$$

$$\text{The length of link 3 is: } v := \sqrt{V1x^2 + V1y^2} \quad v = 2219.601$$

$$G1x := W1x + V1x - U1x \quad G1x = -2.363 \times 10^3$$

$$G1y := W1y + V1y - U1y \quad G1y = -252.000$$

$$\text{The length of link 1 is: } g := \sqrt{G1x^2 + G1y^2} \quad g = 2376.399$$

19. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$$O2x := -Z1x - W1x \quad O2x = -331.000$$

$$O2y := -Z1y - W1y \quad O2y = -1179.000$$

$$O4x := -S1x - U1x \quad O4x = -2694.000$$

$$O4y := -S1y - U1y \quad O4y = -1431.000$$

These check with the design choices shown in the figure above.

20. Determine the location of the coupler point with respect to point A and line AB.

$$\text{Distance from A to P} \quad z := \sqrt{Z1x^2 + Z1y^2} \quad z = 1106.834 \quad r_p := z$$

$$\text{Angle BAP } (\delta_p) \quad s := \sqrt{S1x^2 + S1y^2} \quad s = 1112.770$$

$$\psi := \text{atan2}(S1x, S1y) \quad \psi = 25.538 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z1x, Z1y) \quad \phi = -154.633 \cdot \text{deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = -154.547 \cdot \text{deg}$$

$$\delta_p := \phi - \theta_3 \quad \delta_p = -0.086 \cdot \text{deg}$$

21. DESIGN SUMMARY

$$\text{Link 1:} \quad g = 2376.4$$

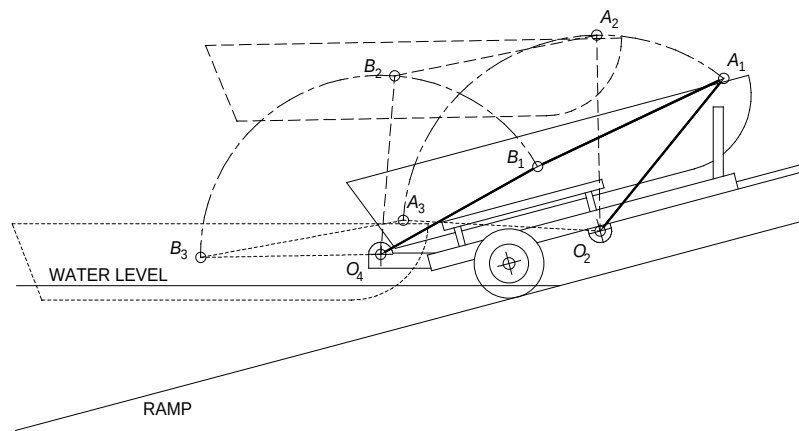
$$\text{Link 2:} \quad w = 2122.5$$

$$\text{Link 3:} \quad v = 2219.6$$

$$\text{Link 4:} \quad u = 1939.3$$

$$\text{Coupler point:} \quad r_p = 1106.8 \quad \delta_p = -0.086 \cdot \text{deg}$$

22. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct. The solution is drawn below to show the locations of the moving pivots for the three positions chosen (see next page).
23. The design needs to be checked for the presence of toggle positions within its desired range of motion. This design has none, but is close to toggle at position 1. This could be used as a locking feature.
24. The transmission angles need to be checked also. This design has poor transmission angles, especially in and near positions 1 and 3. Unfortunately, this is where a large overturning moment is created by the mass of the boat. A large mechanical advantage input device will need to be used here, such as a hydraulic cylinder or geared drive. Note that the drive mechanism must also resist being overdriven (back driven) by the load as the boat descends from its high point onto the trailer.

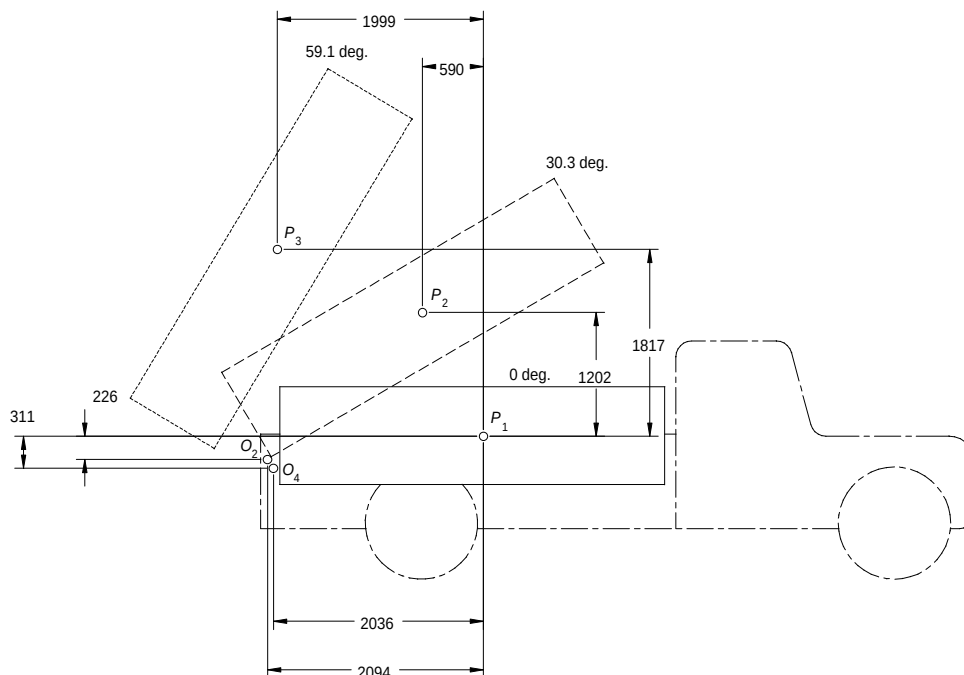


PROBLEM 5-6

Statement: See Project P3-20. Define three positions of the dumpster and analytically synthesize a linkage to move through them. The fixed pivots must be located on the existing truck.

Solution: See Project P3-20 and Mathcad file P0506.

1. This is an open-ended design problem that has many valid solutions. First define the problem more completely than it is stated by deciding on three positions for the dumpster box to move through. The figure below shows one such set of positions (dimensions are in mm).



2. From the figure, the design choices are:

$$\begin{array}{llll}
 P_{21x} := -590 & P_{21y} := 1202 & P_{31x} := -1999 & P_{31y} := 1817 \\
 O_{2x} := -2094 & O_{2y} := -226 & O_{4x} := -2036 & O_{4y} := -311 \\
 \text{Body angles:} & \theta_{p1} := 0 \cdot \text{deg} & \theta_{p2} := 30.3 \cdot \text{deg} & \theta_{p3} := 59.1 \cdot \text{deg}
 \end{array}$$

3. The methods of Section 5.8 are used to get a solution for this problem. The solution is sensitive to small changes in the design choices so a trial-and-error approach is warranted.
4. Determine the angle changes between precision points from the body angles given.

$$\begin{array}{ll}
 \alpha_2 := \theta_{p2} - \theta_{p1} & \alpha_2 = 30.300 \cdot \text{deg} \\
 \alpha_3 := \theta_{p3} - \theta_{p1} & \alpha_3 = 59.100 \cdot \text{deg}
 \end{array}$$

5. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$\begin{array}{llll}
 R_{1x} := -O_{2x} & R_{1x} = 2.094 \times 10^3 & R_{1y} := -O_{2y} & R_{1y} = 226.000 \\
 R_{2x} := R_{1x} + P_{21x} & R_{2x} = 1.504 \times 10^3 & & \\
 R_{2y} := R_{1y} + P_{21y} & R_{2y} = 1.428 \times 10^3 & & \\
 R_{3x} := R_{1x} + P_{31x} & R_{3x} = 95.000 & &
 \end{array}$$

$$\begin{aligned}
 R_{3y} &:= R_{1y} + P_{31y} & R_{3y} &= 2.043 \times 10^3 \\
 R_1 &:= \sqrt{R_{1x}^2 + R_{1y}^2} & R_1 &= 2.106 \times 10^3 \\
 R_2 &:= \sqrt{R_{2x}^2 + R_{2y}^2} & R_2 &= 2.074 \times 10^3 \\
 R_3 &:= \sqrt{R_{3x}^2 + R_{3y}^2} & R_3 &= 2.045 \times 10^3
 \end{aligned}$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\begin{aligned}
 \zeta_1 &:= \text{atan2}(R_{1x}, R_{1y}) & \zeta_1 &= 6.160 \cdot \text{deg} \\
 \zeta_2 &:= \text{atan2}(R_{2x}, R_{2y}) & \zeta_2 &= 43.515 \cdot \text{deg} \\
 \zeta_3 &:= \text{atan2}(R_{3x}, R_{3y}) & \zeta_3 &= 87.338 \cdot \text{deg}
 \end{aligned}$$

7. Solve for β_2 and β_3 using equations 5.34

$$\begin{aligned}
 C_1 &:= R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) & C_1 &= -495.777 \\
 C_2 &:= R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) & C_2 &= -212.019 \\
 C_3 &:= R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) & C_3 &= 786.433 \\
 C_4 &:= -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) & C_4 &= 130.152 \\
 C_5 &:= R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) & C_5 &= 189.927 \\
 C_6 &:= -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) & C_6 &= 176.392 \\
 A_1 &:= -C_3^2 - C_4^2 & A_1 &= -6.354 \times 10^5 \\
 A_2 &:= C_3 \cdot C_6 - C_4 \cdot C_5 & A_2 &= 1.140 \times 10^5 \\
 A_3 &:= -C_4 \cdot C_6 - C_3 \cdot C_5 & A_3 &= -1.723 \times 10^5 \\
 A_4 &:= C_2 \cdot C_3 + C_1 \cdot C_4 & A_4 &= -2.313 \times 10^5 \\
 A_5 &:= C_4 \cdot C_5 - C_3 \cdot C_6 & A_5 &= -1.140 \times 10^5 \\
 A_6 &:= C_1 \cdot C_3 - C_2 \cdot C_4 & A_6 &= -3.623 \times 10^5 \\
 K_1 &:= A_2 \cdot A_4 + A_3 \cdot A_6 & K_1 &= 3.607 \times 10^{10} \\
 K_2 &:= A_3 \cdot A_4 + A_5 \cdot A_6 & K_2 &= 8.115 \times 10^{10} \\
 K_3 &:= \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} & K_3 &= 8.816 \times 10^{10} \\
 \beta_{31} &:= 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) & \beta_{31} &= 72.976 \cdot \text{deg}
 \end{aligned}$$

$$\beta_{32} := 2 \cdot \operatorname{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \quad \beta_{32} = 59.100 \cdot \text{deg}$$

The second value is the same as α_3 , so use the first value

$$\beta_3 := \beta_{31}$$

$$\beta_{21} := \operatorname{acos}\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right) \quad \beta_{21} = 34.802 \cdot \text{deg}$$

$$\beta_{22} := \operatorname{asin}\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right) \quad \beta_{22} = 34.802 \cdot \text{deg}$$

Since both values are the same,

$$\beta_2 := \beta_{21}$$

8. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$R_{1x} := -O_{4x} \quad R_{1x} = 2.036 \times 10^3 \quad R_{1y} := -O_{4y} \quad R_{1y} = 311.000$$

$$R_{2x} := R_{1x} + P_{21x} \quad R_{2x} = 1.446 \times 10^3$$

$$R_{2y} := R_{1y} + P_{21y} \quad R_{2y} = 1.513 \times 10^3$$

$$R_{3x} := R_{1x} + P_{31x} \quad R_{3x} = 37.000$$

$$R_{3y} := R_{1y} + P_{31y} \quad R_{3y} = 2.128 \times 10^3$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \quad R_1 = 2.060 \times 10^3$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 2.093 \times 10^3$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 2.128 \times 10^3$$

9. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \operatorname{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 8.685 \cdot \text{deg}$$

$$\zeta_2 := \operatorname{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 46.297 \cdot \text{deg}$$

$$\zeta_3 := \operatorname{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = 89.004 \cdot \text{deg}$$

10. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = -486.018$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = -161.777$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = 741.712$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = 221.269$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = 154.965$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = 217.266$$

$$A_1 := -C_3^2 - C_4^2$$

$$A_1 = -5.991 \times 10^5$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$$

$$A_2 = 1.269 \times 10^5$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$$

$$A_3 = -1.630 \times 10^5$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$$

$$A_4 = -2.275 \times 10^5$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$$

$$A_5 = -1.269 \times 10^5$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$$

$$A_6 = -3.247 \times 10^5$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$$

$$K_1 = 2.406 \times 10^{10}$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$$

$$K_2 = 7.828 \times 10^{10}$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_5^2 - A_6^2}{2}$$

$$K_3 = 7.953 \times 10^{10}$$

$$\gamma_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right)$$

$$\gamma_{31} = 86.724 \cdot \text{deg}$$

$$\gamma_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right)$$

$$\gamma_{32} = 59.100 \cdot \text{deg}$$

The second value is the same as α_3 , so use the first value

$$\gamma_3 := \gamma_{31}$$

$$\gamma_{21} := \text{acos} \left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1} \right)$$

$$\gamma_{21} = 39.743 \cdot \text{deg}$$

$$\gamma_{22} := \text{asin} \left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1} \right)$$

$$\gamma_{22} = 39.743 \cdot \text{deg}$$

Since both angles are the same,

$$\gamma_2 := \gamma_{21}$$

11. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2}$$

$$p_{21} = 1338.994$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y})$$

$$\delta_2 = 116.144 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2}$$

$$p_{31} = 2701.387$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y})$$

$$\delta_3 = 137.731 \cdot \text{deg}$$

12. Evaluate terms in the $\mathbf{WZ}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1$$

$$B := \sin(\beta_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$W1x = 3.194 \times 10^3$$

$$W1y = 829.763$$

$$Z1x = -1.100 \times 10^3$$

$$Z1y = -603.763$$

14. The length of link 2 is:

$$w := \sqrt{W1x^2 + W1y^2}$$

$$w = 3299.543$$

15. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

16. The components of the **W** and **Z** vectors are:

$$U1x = 1.621 \times 10^3$$

$$U1y = 13.492$$

$$S1x = 415.016$$

$$S1y = 297.508$$

17. The length of link 4 is:

$$u := \sqrt{U1x^2 + U1y^2}$$

$$u = 1621.040$$

18. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x$$

$$V1x = -1.515 \times 10^3$$

$$V1y := Z1y - S1y$$

$$V1y = -901.272$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$ $v = 1762.404$

$$G1x := W1x + V1x - U1x \quad G1x = 58.000$$

$$G1y := W1y + V1y - U1y \quad G1y = -85.000$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$ $g = 102.903$

19. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$$O2x := -Z1x - W1x \quad O2x = -2094.000$$

$$O2y := -Z1y - W1y \quad O2y = -226.000$$

$$O4x := -S1x - U1x \quad O4x = -2036.000$$

$$O4y := -S1y - U1y \quad O4y = -311.000$$

These check with the design choices shown in the figure above.

20. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $z := \sqrt{Z1x^2 + Z1y^2}$ $z = 1254.370$ $r_p := z$

Angle BAP (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 510.637$

$$\psi := \text{atan2}(S1x, S1y) \quad \psi = 35.635 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z1x, Z1y) \quad \phi = -151.228 \cdot \text{deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = -149.244 \cdot \text{deg}$$

$$\delta_p := \phi - \theta_3 \quad \delta_p = -1.984 \cdot \text{deg}$$

21. DESIGN SUMMARY

Link 1: $g = 102.9$

Link 2: $w = 3299.5$

Link 3: $v = 1762.4$

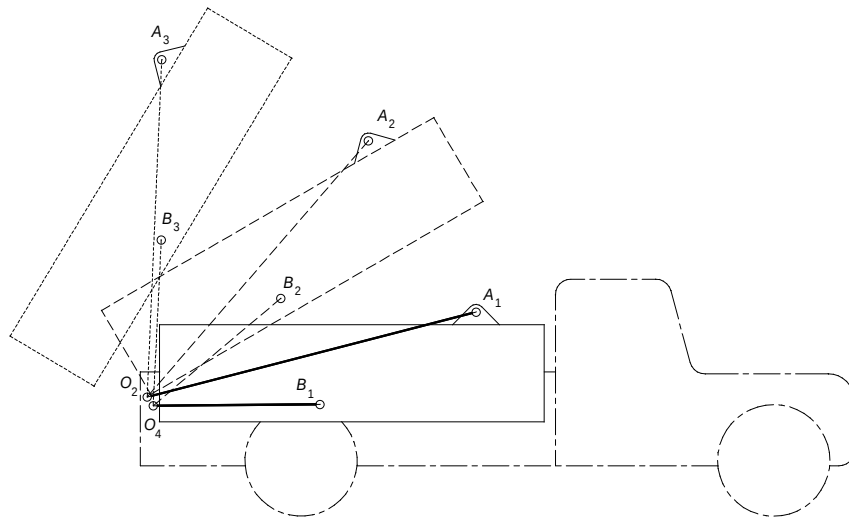
Link 4: $u = 1621.0$

Coupler point: $r_p = 1254.4$ $\delta_p = -1.984 \cdot \text{deg}$

22. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct. The solution is drawn below to show the locations of the moving pivots for the three positions chosen (see next page).

23. The design needs to be checked for the presence of toggle positions within its desired range of motion. This design has none, but is close to toggle at position 3.

24. The transmission angles need to be checked also. A large mechanical advantage input device will need to be used here, such as a hydraulic cylinder. Note that the drive mechanism must also resist being overdriven (back driven) by the load as the dumpster descends from its high point onto the truck.

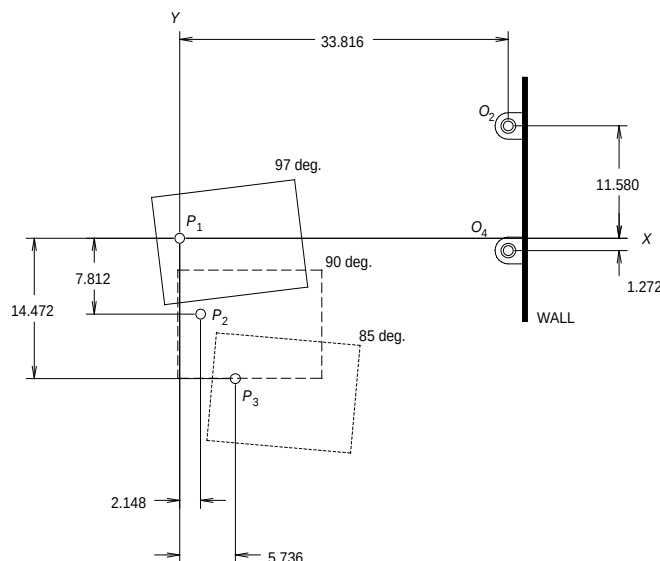


PROBLEM 5-7

Statement: See Project P3-7. Define three positions of the computer monitor and analytically synthesize a linkage to move through them. The fixed pivots must be located on the floor or wall.

Solution: See Project P3-7 and Mathcad file P0507.

1. This is an open-ended design problem that has many valid solutions. First define the problem more completely than it is stated by deciding on three positions for the computer monitor to move through. The figure below shows one such set of positions (dimensions are in inches).



2. From the figure, the design choices are:

$$\begin{array}{llll}
 P_{21x} := 2.148 & P_{21y} := -7.812 & P_{31x} := 5.736 & P_{31y} := -14.472 \\
 O_{2x} := 33.816 & O_{2y} := 11.580 & O_{4x} := 33.816 & O_{4y} := -1.272 \\
 \text{Body angles:} & \theta_{p1} := 97 \cdot \text{deg} & \theta_{p2} := 90 \cdot \text{deg} & \theta_{p3} := 85 \cdot \text{deg}
 \end{array}$$

3. The methods of Section 5.8 are used to get a solution for this problem. The solution is sensitive to small changes in the design choices so a trial-and-error approach is warranted.
4. Determine the angle changes between precision points from the body angles given.

$$\begin{array}{ll}
 \alpha_2 := \theta_{p2} - \theta_{p1} & \alpha_2 = -7.000 \cdot \text{deg} \\
 \alpha_3 := \theta_{p3} - \theta_{p1} & \alpha_3 = -12.000 \cdot \text{deg}
 \end{array}$$

5. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$\begin{array}{llll}
 R_{1x} := -O_{2x} & R_{1x} = -33.816 & R_{1y} := -O_{2y} & R_{1y} = -11.580 \\
 R_{2x} := R_{1x} + P_{21x} & R_{2x} = -31.668 & & \\
 R_{2y} := R_{1y} + P_{21y} & R_{2y} = -19.392 & & \\
 R_{3x} := R_{1x} + P_{31x} & R_{3x} = -28.080 & & \\
 R_{3y} := R_{1y} + P_{31y} & R_{3y} = -26.052 & &
 \end{array}$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \quad R_1 = 35.744$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 37.134$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 38.304$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = -161.097 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = -148.519 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = -137.146 \cdot \text{deg}$$

7. Solve for β_2 and β_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = 3.962$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = -10.052$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = -7.405$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = -21.756$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = -3.307$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = -12.019$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -528.143$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = 17.049$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = -285.981$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = -11.771$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = -17.049$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -248.020$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = 7.073 \times 10^4$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = 7.595 \times 10^3$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = 6.760 \times 10^4$$

$$\beta_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \beta_{31} = 24.258 \cdot \text{deg}$$

$$\beta_{32} := 2 \cdot \operatorname{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \quad \beta_{32} = -12.000 \cdot \text{deg}$$

The second value is the same as α_3 , so use the first value

$$\beta_3 := \beta_{31}$$

$$\beta_{21} := \operatorname{acos}\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right) \quad \beta_{21} = 12.435 \cdot \text{deg}$$

$$\beta_{22} := \operatorname{asin}\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right) \quad \beta_{22} = 12.435 \cdot \text{deg}$$

Since both values are the same,

$$\beta_2 := \beta_{21}$$

8. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$R_{1x} := -O_{4x} \quad R_{1x} = -33.816 \quad R_{1y} := -O_{4y} \quad R_{1y} = 1.272$$

$$R_{2x} := R_{1x} + P_{21x} \quad R_{2x} = -31.668$$

$$R_{2y} := R_{1y} + P_{21y} \quad R_{2y} = -6.540$$

$$R_{3x} := R_{1x} + P_{31x} \quad R_{3x} = -28.080$$

$$R_{3y} := R_{1y} + P_{31y} \quad R_{3y} = -13.200$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \quad R_1 = 33.840$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 32.336$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 31.028$$

9. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \operatorname{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 177.846 \cdot \text{deg}$$

$$\zeta_2 := \operatorname{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = -168.331 \cdot \text{deg}$$

$$\zeta_3 := \operatorname{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = -154.822 \cdot \text{deg}$$

10. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = 2.856$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = -9.867$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = -4.733$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = -21.475$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = -1.741$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = -11.924$$

$$\begin{aligned}
 A_1 &:= -C_3^2 - C_4^2 & A_1 &= -483.571 \\
 A_2 &:= C_3 \cdot C_6 - C_4 \cdot C_5 & A_2 &= 19.043 \\
 A_3 &:= -C_4 \cdot C_6 - C_3 \cdot C_5 & A_3 &= -264.299 \\
 A_4 &:= C_2 \cdot C_3 + C_1 \cdot C_4 & A_4 &= -14.646 \\
 A_5 &:= C_4 \cdot C_5 - C_3 \cdot C_6 & A_5 &= -19.043 \\
 A_6 &:= C_1 \cdot C_3 - C_2 \cdot C_4 & A_6 &= -225.402 \\
 K_1 &:= A_2 \cdot A_4 + A_3 \cdot A_6 & K_1 &= 5.929 \times 10^4 \\
 K_2 &:= A_3 \cdot A_4 + A_5 \cdot A_6 & K_2 &= 8.163 \times 10^3 \\
 K_3 &:= \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} & K_3 &= 5.630 \times 10^4 \\
 \gamma_{31} &:= 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) & \gamma_{31} &= 27.678 \cdot \text{deg} \\
 \gamma_{32} &:= 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) & \gamma_{32} &= -12.000 \cdot \text{deg}
 \end{aligned}$$

The second value is the same as α_3 , so use the first value

$$\begin{aligned}
 \gamma_{21} &:= \text{acos} \left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1} \right) & \gamma_{21} &= 14.435 \cdot \text{deg} \\
 \gamma_{22} &:= \text{asin} \left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1} \right) & \gamma_{22} &= 14.435 \cdot \text{deg}
 \end{aligned}$$

Since both angles are the same,

$$\gamma_2 := \gamma_{21}$$

11. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$\begin{aligned}
 p_{21} &:= \sqrt{P_{21x}^2 + P_{21y}^2} & p_{21} &= 8.102 \\
 \delta_2 &:= \text{atan2}(P_{21x}, P_{21y}) & \delta_2 &= -74.626 \cdot \text{deg} \\
 p_{31} &:= \sqrt{P_{31x}^2 + P_{31y}^2} & p_{31} &= 15.567 \\
 \delta_3 &:= \text{atan2}(P_{31x}, P_{31y}) & \delta_3 &= -68.379 \cdot \text{deg}
 \end{aligned}$$

12. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$\begin{array}{lll}
 A := \cos(\beta_2) - 1 & B := \sin(\beta_2) & C := \cos(\alpha_2) - 1 \\
 D := \sin(\alpha_2) & E := p_{21} \cdot \cos(\delta_2) & F := \cos(\beta_3) - 1 \\
 G := \sin(\beta_3) & H := \cos(\alpha_3) - 1 & K := \sin(\alpha_3) \\
 L := p_{31} \cdot \cos(\delta_3) & M := p_{21} \cdot \sin(\delta_2) & N := p_{31} \cdot \sin(\delta_3)
 \end{array}$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$W1x = -36.030 \quad W1y = -8.098 \quad Z1x = 2.214 \quad Z1y = -3.482$$

14. The length of link 2 is:

$$w := \sqrt{W1x^2 + W1y^2} \quad w = 36.929$$

15. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$\begin{array}{lll}
 A' := \cos(\gamma_2) - 1 & B' := \sin(\gamma_2) & C := \cos(\alpha_2) - 1 \\
 D := \sin(\alpha_2) & E := p_{21} \cdot \cos(\delta_2) & F' := \cos(\gamma_3) - 1 \\
 G' := \sin(\gamma_3) & H := \cos(\alpha_3) - 1 & K := \sin(\alpha_3) \\
 L := p_{31} \cdot \cos(\delta_3) & M := p_{21} \cdot \sin(\delta_2) & N := p_{31} \cdot \sin(\delta_3)
 \end{array}$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

16. The components of the **W** and **Z** vectors are:

$$U1x = -32.294 \quad U1y = -2.592 \quad S1x = -1.522 \quad S1y = 3.864$$

17. The length of link 4 is:

$$u := \sqrt{U1x^2 + U1y^2} \quad u = 32.398$$

18. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$\begin{array}{ll}
 V1x := Z1x - S1x & V1x = 3.736 \\
 V1y := Z1y - S1y & V1y = -7.346
 \end{array}$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$ $v = 8.241$

$$G1x := W1x + V1x - U1x \quad G1x = 0.000$$

$$G1y := W1y + V1y - U1y \quad G1y = -12.852$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$ $g = 12.852$

19. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$$O2x := -Z1x - W1x \quad O2x = 33.816$$

$$O2y := -Z1y - W1y \quad O2y = 11.580$$

$$O4x := -S1x - U1x \quad O4x = 33.816$$

$$O4y := -S1y - U1y \quad O4y = -1.272$$

These check with the design choices shown in the figure above.

20. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $z := \sqrt{Z1x^2 + Z1y^2}$ $z = 4.126$ $r_p := z$

Angle BAP (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 4.153$

$$\psi := \text{atan2}(S1x, S1y) \quad \psi = 111.494 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z1x, Z1y) \quad \phi = -57.551 \cdot \text{deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = -63.046 \cdot \text{deg}$$

$$\delta_p := \phi - \theta_3 \quad \delta_p = 5.495 \cdot \text{deg}$$

21. DESIGN SUMMARY

Link 1: $g = 12.852$

Link 2: $w = 36.929$

Link 3: $v = 8.241$

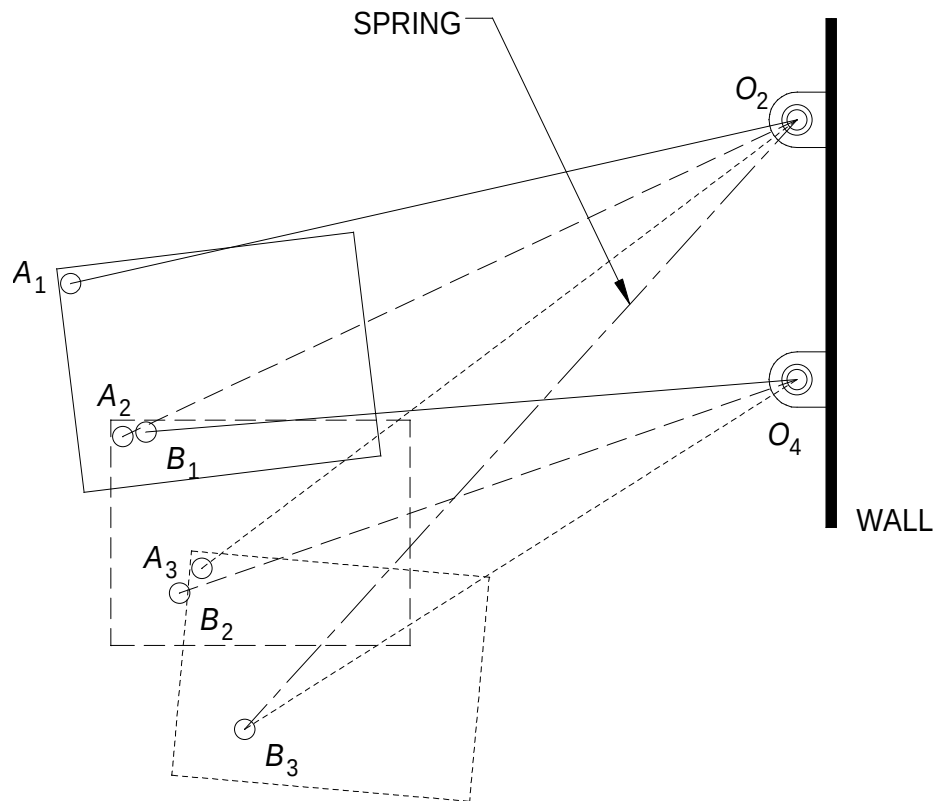
Link 4: $u = 32.398$

Coupler point: $r_p = 4.126$ $\delta_p = 5.495 \cdot \text{deg}$

22. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct. The solution is drawn below to show the locations of the moving pivots for the three positions chosen (see next page).

23. The design needs to be checked for the presence of toggle positions within its desired range of motion. This design has none.

24. The transmission angles need to be checked also. A means to support the weight of the monitor must be provided. The figure below shows a spring placed between links to provide the balancing moment. Further design and analysis needs to be done to optimize the spring placement in order to compensate for its change in force with deflection and the change in moment arm as the linkage moves.



PROBLEM 5-8

Statement: Design a linkage to carry the body in Figure P5-1 through the two positions P_1 and P_2 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown. Use the free choices given below.

Given: Coordinates of the points P_1 and P_2 with respect to P_1 :

$$P_{1x} := 0.0 \quad P_{1y} := 0.0 \quad P_{2x} := -1.236 \quad P_{2y} := 2.138$$

Angles made by the body in positions 1 and 2:

$$\theta_{p1} := 210 \cdot \text{deg} \quad \theta_{p2} := 147.5 \cdot \text{deg}$$

Free choices for the **WZ** dyad :

$$z := 1.075 \quad \beta_2 := -27.0 \cdot \text{deg} \quad \phi := 204.4 \cdot \text{deg}$$

Free choices for the **US** dyad :

$$s := 1.240 \quad \gamma_2 := -40.0 \cdot \text{deg} \quad \psi := 74.0 \cdot \text{deg}$$

Solution: See Figure P5-1 and Mathcad file P0508.

- Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. Because of the data given in the hint, the second method of Section 5.3 will be used here.
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$\mathbf{R}_1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \quad \mathbf{R}_2 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := \mathbf{R}_2 - \mathbf{R}_1 \quad P_{21x} = -1.236$$

$$P_{21y} = 2.138$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 2.470$$

- From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\alpha_2 := \theta_{p2} - \theta_{p1} \quad \alpha_2 = -62.500 \cdot \text{deg}$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = 120.033 \cdot \text{deg}$$

- Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = -0.979 \quad Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = -0.444$$

$$A := \cos(\beta_2) - 1 \quad A = -0.109 \quad D := \sin(\alpha_2) \quad D = -0.887$$

$$B := \sin(\beta_2) \quad B = -0.454 \quad E := p_{21} \cdot \cos(\delta_2) \quad E = -1.236$$

$$C := \cos(\alpha_2) - 1 \quad C = -0.538 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = 2.138$$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \quad W_{1x} = -1.462$$

$$W_{Iy} := \frac{A \cdot (-C \cdot Z_{Iy} - D \cdot Z_{Ix} + F) + B \cdot (C \cdot Z_{Ix} - D \cdot Z_{Iy} - E)}{-2 \cdot A} \quad W_{Iy} = -3.367$$

$$w := \sqrt{W_{Ix}^2 + W_{Iy}^2} \quad w = 3.670$$

$$\theta := \text{atan2}(W_{Ix}, W_{Iy}) \quad \theta = -113.472 \cdot \text{deg}$$

5. Solve for the **US** dyad using equations 5.12.

$$S_{Ix} := s \cdot \cos(\psi) \quad S_{Ix} = 0.342 \quad S_{Iy} := s \cdot \sin(\psi) \quad S_{Iy} = 1.192$$

$$A := \cos(\gamma_2) - 1 \quad A = -0.234 \quad D := \sin(\alpha_2) \quad D = -0.887$$

$$B := \sin(\gamma_2) \quad B = -0.643 \quad E := p_{21} \cdot \cos(\delta_2) \quad E = -1.236$$

$$C := \cos(\alpha_2) - 1 \quad C = -0.538 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = 2.138$$

$$U_{Ix} := \frac{A \cdot (-C \cdot S_{Ix} + D \cdot S_{Iy} + E) + B \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F)}{-2 \cdot A} \quad U_{Ix} = -3.180$$

$$U_{Iy} := \frac{A \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F) + B \cdot (C \cdot S_{Ix} - D \cdot S_{Iy} - E)}{-2 \cdot A} \quad U_{Iy} = -4.439$$

$$u := \sqrt{U_{Ix}^2 + U_{Iy}^2} \quad u = 5.461$$

$$\sigma := \text{atan2}(U_{Ix}, U_{Iy}) \quad \sigma = -125.619 \cdot \text{deg}$$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3:

$$V_{Ix} := z \cdot \cos(\phi) - s \cdot \cos(\psi) \quad V_{Ix} = -1.321$$

$$V_{Iy} := z \cdot \sin(\phi) - s \cdot \sin(\psi) \quad V_{Iy} = -1.636$$

$$\theta_3 := \text{atan2}(V_{Ix}, V_{Iy}) \quad \theta_3 = -128.914 \cdot \text{deg}$$

$$v := \sqrt{V_{Ix}^2 + V_{Iy}^2} \quad v = 2.103$$

Link 1:

$$G_{Ix} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma) \quad G_{Ix} = 0.398$$

$$G_{Iy} := w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma) \quad G_{Iy} = -0.564$$

$$\theta_1 := \text{atan2}(G_{Ix}, G_{Iy}) \quad \theta_1 = -54.796 \cdot \text{deg}$$

$$g := \sqrt{G_{Ix}^2 + G_{Iy}^2} \quad g = 0.690$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \quad \theta_{2i} = -58.677 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_2$$

$$\theta_{2f} = -85.677 \cdot \text{deg}$$

8. Define the coupler point with respect to point A and the vector $\mathbf{V}$.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 1.075$$

$$\delta_p = 333.314 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := 0 \cdot \text{deg}$$

$$\rho_1 = 0.000 \cdot \text{deg}$$

$$R_1 := \sqrt{P_{1x}^2 + P_{1y}^2}$$

$$R_1 = 0.000$$

$$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = 2.441$$

$$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = 3.811$$

$$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = 2.838$$

$$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = 3.247$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$$

$$\theta_{rot} = -54.796 \cdot \text{deg}$$

11. Determine the Grashof condition.

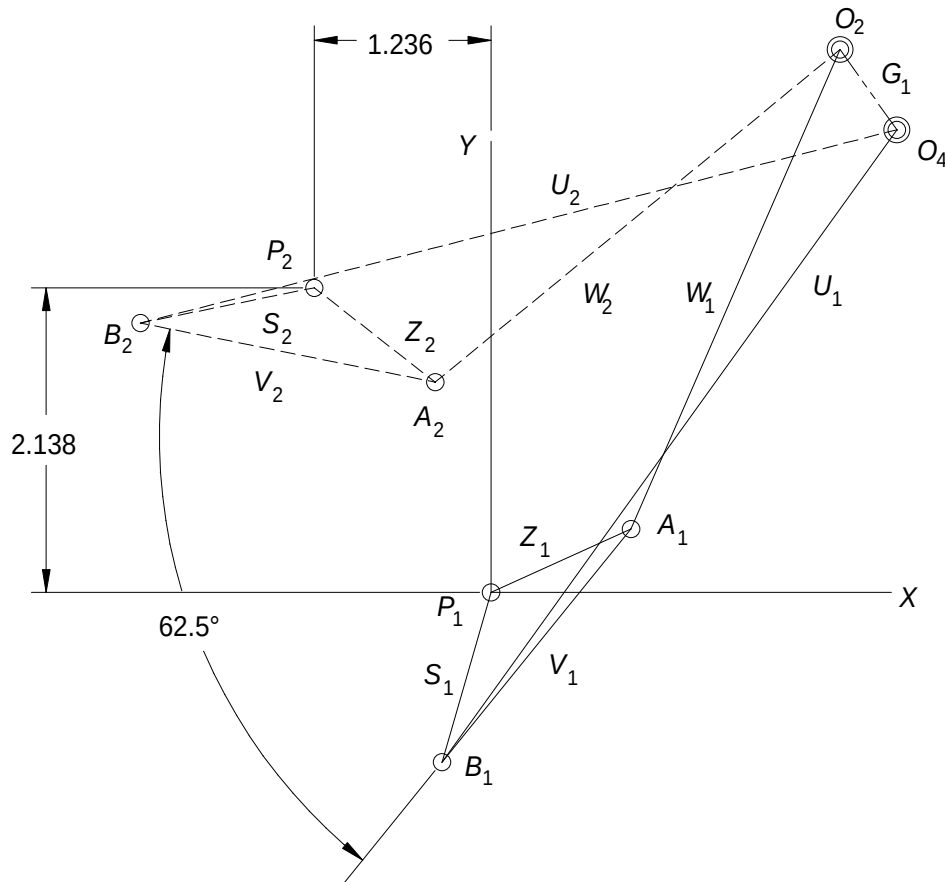
$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

12. DESIGN SUMMARY

Link 2:	$w = 3.670$	$\theta = -113.472 \cdot \text{deg}$
Link 3:	$v = 2.103$	$\theta_3 = -128.914 \cdot \text{deg}$
Link 4:	$u = 5.461$	$\sigma = -125.619 \cdot \text{deg}$
Link 1:	$g = 0.690$	$\theta_1 = -54.796 \cdot \text{deg}$
Coupler:	$r_p = 1.075$	$\delta_p = 333.314 \cdot \text{deg}$
Crank angles:	$\theta_{2i} = -58.677 \cdot \text{deg}$ $\theta_{2f} = -85.677 \cdot \text{deg}$	

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-9

Statement: Design a linkage to carry the body in Figure P5-1 through the two positions P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown. Hint: First try a rough graphical solution to create realistic values for free choices.

Given: Coordinates of the points P_2 and P_3 with respect to P_1 :

$$P_{2x} := -1.236 \quad P_{2y} := 2.138 \quad P_{3x} := -2.500 \quad P_{3y} := 2.931$$

Angles made by the body in positions 1 and 2:

$$\theta_{p2} := 147.5 \cdot \text{deg} \quad \theta_{p3} := 110.2 \cdot \text{deg}$$

Solution: See Figure P5-1 and Mathcad file P0509.

- Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$\mathbf{R}_1 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \quad \mathbf{R}_2 := \begin{pmatrix} P_{3x} \\ P_{3y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := \mathbf{R}_2 - \mathbf{R}_1 \quad P_{21x} = -1.264$$

$$P_{21y} = 0.793$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 1.492$$

- From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\alpha_2 := \theta_{p3} - \theta_{p2} \quad \alpha_2 = -37.300 \cdot \text{deg}$$

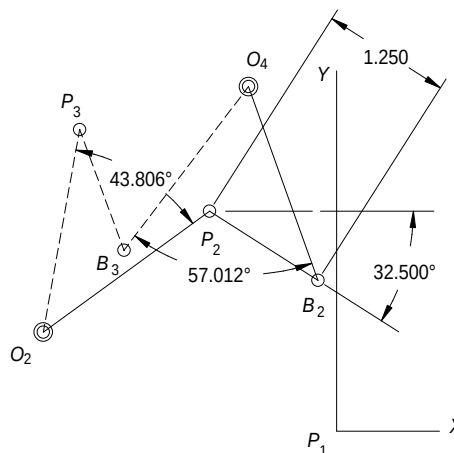
$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = 147.897 \cdot \text{deg}$$

- From a graphical solution (see figure at right), determine the values necessary for input to equations 5.8.

$$z := 0.0$$

$$\beta_2 := 43.806 \cdot \text{deg}$$

$$\phi := -32.500 \cdot \text{deg}$$



- Solve for the $\mathbf{WZ}$ dyad using equations 5.8.

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = 0.000 \quad Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = 0.000$$

$$A := \cos(\beta_2) - 1 \quad A = -0.278 \quad D := \sin(\alpha_2) \quad D = -0.606$$

$$B := \sin(\beta_2) \quad B = 0.692 \quad E := p_{21} \cdot \cos(\delta_2) \quad E = -1.264$$

$$C := \cos(\alpha_2) - 1 \quad C = -0.205 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = 0.793$$

$$W_{Ix} := \frac{A \cdot (-C \cdot Z_{Ix} + D \cdot Z_{Iy} + E) + B \cdot (-C \cdot Z_{Iy} - D \cdot Z_{Ix} + F)}{-2 \cdot A} \quad W_{Ix} = 1.618$$

$$W_{Iy} := \frac{A \cdot (-C \cdot Z_{Iy} - D \cdot Z_{Ix} + F) + B \cdot (C \cdot Z_{Ix} - D \cdot Z_{Iy} - E)}{-2 \cdot A} \quad W_{Iy} = 1.175$$

$$w := \sqrt{W_{Ix}^2 + W_{Iy}^2} \quad w = 2.000$$

$$\theta := \text{atan2}(W_{Ix}, W_{Iy}) \quad \theta = 35.994 \cdot \text{deg}$$

5. From the graphical solution (see figure above), determine the values necessary for input to equations 5.12.

$$s := 1.250 \quad \gamma_2 := -57.012 \cdot \text{deg} \quad \psi := 147.5 \cdot \text{deg}$$

6. Solve for the **US** dyad using equations 5.12.

$$S_{Ix} := s \cdot \cos(\psi) \quad S_{Ix} = -1.054 \quad S_{Iy} := s \cdot \sin(\psi) \quad S_{Iy} = 0.672$$

$$A := \cos(\gamma_2) - 1 \quad A = -0.456 \quad D := \sin(\alpha_2) \quad D = -0.606$$

$$B := \sin(\gamma_2) \quad B = -0.839 \quad E := p_{21} \cdot \cos(\delta_2) \quad E = -1.264$$

$$C := \cos(\alpha_2) - 1 \quad C = -0.205 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = 0.793$$

$$U_{Ix} := \frac{A \cdot (-C \cdot S_{Ix} + D \cdot S_{Iy} + E) + B \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F)}{-2 \cdot A} \quad U_{Ix} = 0.675$$

$$U_{Iy} := \frac{A \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F) + B \cdot (C \cdot S_{Ix} - D \cdot S_{Iy} - E)}{-2 \cdot A} \quad U_{Iy} = -1.883$$

$$u := \sqrt{U_{Ix}^2 + U_{Iy}^2} \quad u = 2.000$$

$$\sigma := \text{atan2}(U_{Ix}, U_{Iy}) \quad \sigma = -70.278 \cdot \text{deg}$$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\text{Link 3: } V_{Ix} := z \cdot \cos(\phi) - s \cdot \cos(\psi) \quad V_{Ix} = 1.054$$

$$V_{Iy} := z \cdot \sin(\phi) - s \cdot \sin(\psi) \quad V_{Iy} = -0.672$$

$$\theta_3 := \text{atan2}(V_{Ix}, V_{Iy}) \quad \theta_3 = -32.500 \cdot \text{deg}$$

$$v := \sqrt{V_{Ix}^2 + V_{Iy}^2} \quad v = 1.250$$

$$\text{Link 1: } G_{Ix} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma) \quad G_{Ix} = 1.997$$

$$G_{Iy} := w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma) \quad G_{Iy} = 2.386$$

$$\theta_1 := \text{atan2}(G_{Ix}, G_{Iy}) \quad \theta_1 = 50.070 \cdot \text{deg}$$

$$g := \sqrt{G_{Ix}^2 + G_{Iy}^2} \quad g = 3.112$$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1$$

$$\theta_{2i} = -14.077 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_2$$

$$\theta_{2f} = 29.729 \cdot \text{deg}$$

9. Define the coupler point with respect to point A and the vector $\mathbf{V}$.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 0.000$$

$$\delta_p = 0.000 \cdot \text{deg}$$

which is correct for the assumption that the precision point is at C.

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := \text{atan2}(P_{2x}, P_{2y})$$

$$\rho_1 = 120.033 \cdot \text{deg}$$

$$R_1 := \sqrt{P_{2x}^2 + P_{2y}^2}$$

$$R_1 = 2.470$$

$$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = -2.854$$

$$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = 0.963$$

$$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = -0.857$$

$$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = 3.349$$

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{\text{rot}} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$$

$$\theta_{\text{rot}} = 50.070 \cdot \text{deg}$$

12. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

13. DESIGN SUMMARY

Link 2:	$w = 2.000$	$\theta = 35.994 \cdot \text{deg}$
Link 3:	$v = 1.250$	$\theta_3 = -32.500 \cdot \text{deg}$
Link 4:	$u = 2.000$	$\sigma = -70.278 \cdot \text{deg}$
Link 1:	$g = 3.112$	$\theta_1 = 50.070 \cdot \text{deg}$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000 \cdot \text{deg}$
Crank angles:		
	$\theta_{2i} = -14.077 \cdot \text{deg}$	
	$\theta_{2f} = 29.729 \cdot \text{deg}$	

PROBLEM 5-10

Statement: Design a linkage to carry the body in Figure P5-1 through the three positions P_1 , P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown. Use the free choices given below.

Given: Coordinates of the points P_1 and P_2 with respect to P_1 :

$$P_{1x} := 0.0 \quad P_{1y} := 0.0 \quad P_{2x} := -1.236 \quad P_{2y} := 2.138$$

$$P_{3x} := -2.500 \quad P_{3y} := 2.931$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 210 \cdot \text{deg} \quad \theta_{p2} := 147.5 \cdot \text{deg} \quad \theta_{p3} := 110.2 \cdot \text{deg}$$

Free choices for the **WZ** dyad :

$$\beta_2 := 30.0 \cdot \text{deg} \quad \beta_3 := 60.0 \cdot \text{deg}$$

Free choices for the **US** dyad :

$$\gamma_2 := -10.0 \cdot \text{deg} \quad \gamma_3 := 25.0 \cdot \text{deg}$$

Solution: See Figure P5-1 and Mathcad file P0510.

- Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \quad p_{21} = 2.470 \quad \delta_2 := \text{atan2}(P_{2x}, P_{2y}) \quad \delta_2 = 120.033 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \quad p_{31} = 3.852 \quad \delta_3 := \text{atan2}(P_{3x}, P_{3y}) \quad \delta_3 = 130.463 \cdot \text{deg}$$

- Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{p2} - \theta_{p1} \quad \alpha_2 = -62.500 \cdot \text{deg}$$

$$\alpha_3 := \theta_{p3} - \theta_{p1} \quad \alpha_3 = -99.800 \cdot \text{deg}$$

- Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$A := \cos(\beta_2) - 1 \quad B := \sin(\beta_2) \quad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \quad E := p_{21} \cdot \cos(\delta_2) \quad F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3) \quad H := \cos(\alpha_3) - 1 \quad K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \quad M := p_{21} \cdot \sin(\delta_2) \quad N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$$W_{1x} = 2.920 \quad W_{1y} = 1.720 \quad Z_{1x} = -0.756 \quad Z_{1y} = -0.442$$

$$\theta := \text{atan2}(W_{1x}, W_{1y}) \quad \theta = 30.493 \cdot \text{deg} \quad \phi := \text{atan2}(Z_{1x}, Z_{1y}) \quad \phi = -149.697 \cdot \text{deg}$$

The length of link 2 is: $w := \sqrt{(W_{1x}^2 + W_{1y}^2)}$, $w = 3.389$

The length of vector **Z** is: $z := \sqrt{(Z_{1x}^2 + Z_{1y}^2)}$, $z = 0.876$

4. Evaluate terms in the **US** coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1 \quad B' := \sin(\gamma_2) \quad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \quad E := p_{21} \cdot \cos(\delta_2) \quad F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3) \quad H := \cos(\alpha_3) - 1 \quad K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \quad M := p_{21} \cdot \sin(\delta_2) \quad N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$$U_{1x} = -1.009 \quad U_{1y} = 2.693 \quad S_{1x} = -0.792 \quad S_{1y} = -2.418$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y}) \quad \sigma = 110.545 \cdot \text{deg} \quad \psi := \text{atan2}(S_{1x}, S_{1y}) \quad \psi = -108.125 \cdot \text{deg}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 2.875$

The length of vector **S** is: $s := \sqrt{(S_{1x}^2 + S_{1y}^2)}$, $s = 2.544$

5. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 0.036$

$$V_{1y} := Z_{1y} - S_{1y} \quad V_{1y} = 1.976$$

$$\theta_3 := \text{atan2}(V_{1x}, V_{1y}) \quad \theta_3 = 88.968 \cdot \text{deg}$$

$$v := \sqrt{V_{1x}^2 + V_{1y}^2} \quad v = 1.977$$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 3.965$

$$G_{1y} := W_{1y} + V_{1y} - U_{1y} \quad G_{1y} = 1.003$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y}) \quad \theta_1 = 14.202 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2} \quad g = 4.090$$

6. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \quad \theta_{2i} = 16.291 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_3 \quad \theta_{2f} = 76.291 \cdot \text{deg}$$

7. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z \quad \delta_p := \phi - \theta_3$$

$$r_p = 0.876 \quad \delta_p = -238.665 \cdot \text{deg}$$

8. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta) \quad O_{2x} = -2.164$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta) \quad O_{2y} = -1.278$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma) \quad O_{4x} = 1.801$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma) \quad O_{4y} = -0.274$$

9. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = 14.202 \cdot \text{deg}$$

10. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"Grashof"}$$

11. DESIGN SUMMARY

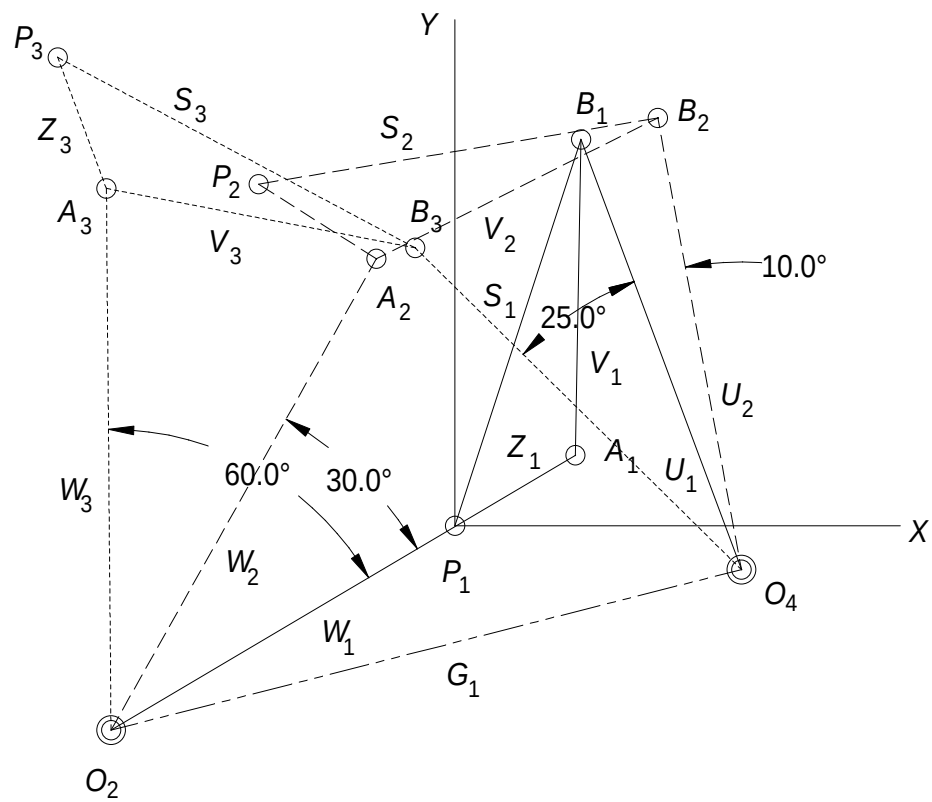
Link 2:	$w = 3.389$	$\theta = 30.493 \cdot \text{deg}$
Link 3:	$v = 1.977$	$\theta_3 = 88.968 \cdot \text{deg}$
Link 4:	$u = 2.875$	$\sigma = 110.545 \cdot \text{deg}$
Link 1:	$g = 4.090$	$\theta_1 = 14.202 \cdot \text{deg}$
Coupler:	$r_p = 0.876$	$\delta_p = -238.665 \cdot \text{deg}$

Crank angles:

$$\theta_{2i} = 16.291 \cdot \text{deg}$$

$$\theta_{2f} = 76.291 \cdot \text{deg}$$

12. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-11

Statement: Design a linkage to carry the body in Figure P5-1 through the three positions P_1 , P_2 and P_3 at the angles shown in the figure. Use analytical synthesis and design it for the fixed pivots shown.

Given:

$P_{21x} := -1.236$	$P_{21y} := 2.138$	$P_{31x} := -2.500$	$P_{31y} := 2.931$
$O_{2x} := -2.164$	$O_{2y} := -1.260$	$O_{4x} := 2.190$	$O_{4y} := -1.260$
Body angles:			
$\theta_{p1} := 210 \cdot \text{deg}$	$\theta_{p2} := 147.5 \cdot \text{deg}$	$\theta_{p3} := 110.2 \cdot \text{deg}$	

Solution: See Figure P5-1 and Mathcad file P0511.

1. Determine the angle changes between precision points from the body angles given.

$$\alpha_2 := \theta_{p2} - \theta_{p1} \quad \alpha_2 = -62.500 \cdot \text{deg}$$

$$\alpha_3 := \theta_{p3} - \theta_{p1} \quad \alpha_3 = -99.800 \cdot \text{deg}$$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$R_{1x} := -O_{2x} \quad R_{1x} = 2.164 \quad R_{1y} := -O_{2y} \quad R_{1y} = 1.260$$

$$R_{2x} := R_{1x} + P_{21x} \quad R_{2x} = 0.928$$

$$R_{2y} := R_{1y} + P_{21y} \quad R_{2y} = 3.398$$

$$R_{3x} := R_{1x} + P_{31x} \quad R_{3x} = -0.336$$

$$R_{3y} := R_{1y} + P_{31y} \quad R_{3y} = 4.191$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \quad R_1 = 2.504$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 3.522$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 4.204$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 30.210 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 74.725 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = 94.584 \cdot \text{deg}$$

4. Solve for β_2 and β_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = 0.372$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = 3.726$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = 1.209$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = 6.538$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = 1.189$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2)$$

$$C_6 = 4.736$$

$$A_1 := -C_3^2 - C_4^2$$

$$A_1 = -44.206$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$$

$$A_2 = -2.046$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$$

$$A_3 = -32.399$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$$

$$A_4 = 6.937$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$$

$$A_5 = 2.046$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$$

$$A_6 = -23.911$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$$

$$K_1 = 760.497$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$$

$$K_2 = -273.669$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_5^2 - A_6^2}{2}$$

$$K_3 = 140.232$$

$$\beta_{31} := 2 \cdot \operatorname{atan}\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\beta_{31} = 60.217 \cdot \deg$$

$$\beta_{32} := 2 \cdot \operatorname{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\beta_{32} = -99.800 \cdot \deg$$

The second value is the same as α_3 , so use the first value

$$\beta_3 := \beta_{31}$$

$$\beta_{21} := \operatorname{acos}\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right)$$

$$\beta_{21} = 30.143 \cdot \deg$$

$$\beta_{22} := \operatorname{asin}\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right)$$

$$\beta_{22} = 30.143 \cdot \deg$$

Since both values are the same,

$$\beta_2 := \beta_{21}$$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$R_{1x} := -O_{4x}$$

$$R_{1x} = -2.190$$

$$R_{1y} := -O_{4y}$$

$$R_{1y} = 1.260$$

$$R_{2x} := R_{1x} + P_{21x}$$

$$R_{2x} = -3.426$$

$$R_{2y} := R_{1y} + P_{21y}$$

$$R_{2y} = 3.398$$

$$R_{3x} := R_{1x} + P_{31x}$$

$$R_{3x} = -4.690$$

$$R_{3y} := R_{1y} + P_{31y}$$

$$R_{3y} = 4.191$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$$

$$R_1 = 2.527$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 4.825$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 6.290$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 150.086 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 135.235 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = 138.216 \cdot \text{deg}$$

7. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = -2.380$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = 3.298$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = 6.304$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = 2.247$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = 3.532$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = 0.874$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -44.796$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = -2.431$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = -24.233$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = 15.441$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = 2.431$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -22.414$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = 505.612$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = -428.679$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = 336.363$$

$$\gamma_{31} := 2 \cdot \text{atan}\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \quad \gamma_{31} = 19.215 \cdot \text{deg}$$

$$\gamma_{32} := 2 \cdot \text{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \quad \gamma_{32} = -99.800 \cdot \text{deg}$$

The second value is the same as α_3 , so use the first value

$$\gamma_3 := \gamma_{31}$$

$$\gamma_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1}\right)$$

$$\gamma_{21} = 6.628 \cdot \text{deg}$$

$$\gamma_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1}\right)$$

$$\gamma_{22} = -6.628 \cdot \text{deg}$$

Since γ_2 is not in the first quadrant ,

$$\gamma_2 := \gamma_{22}$$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2}$$

$$p_{21} = 2.470$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y})$$

$$\delta_2 = 120.033 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2}$$

$$p_{31} = 3.852$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y})$$

$$\delta_3 = 130.463 \cdot \text{deg}$$

9. Evaluate terms in the $\mathbf{WZ}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1$$

$$B := \sin(\beta_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the $\mathbf{W}$ and $\mathbf{Z}$ vectors are:

$$W1x = 2.915$$

$$W1y = 1.702$$

$$Z1x = -0.751$$

$$Z1y = -0.442$$

11. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2}$

$$w = 3.376$$

12. Evaluate terms in the $\mathbf{US}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$U1x = -1.371$$

$$U1y = 3.634$$

$$S1x = -0.819$$

$$S1y = -2.374$$

14. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2}$ $u = 3.884$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x$$

$$V1x = 0.068$$

$$V1y := Z1y - S1y$$

$$V1y = 1.932$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$ $v = 1.933$

$$G1x := W1x + V1x - U1x$$

$$G1x = 4.354$$

$$G1y := W1y + V1y - U1y$$

$$G1y = -2.220 \times 10^{-15}$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$ $g = 4.354$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors **W**₁, **Z**₁, **U**₁, and **S**₁.

$$O2x := -Z1x - W1x$$

$$O2x = -2.164$$

$$O2y := -Z1y - W1y$$

$$O2y = -1.260$$

$$O4x := -S1x - U1x$$

$$O4x = 2.190$$

$$O4y := -S1y - U1y$$

$$O4y = -1.260$$

These check with Figure P5-1.

17. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $z := \sqrt{Z1x^2 + Z1y^2}$ $z = 0.871$ $r_P := z$

Angle BAP (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 2.511$

$$\psi := \text{atan2}(S1x, S1y) \quad \psi = -109.037 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z1x, Z1y) \quad \phi = -149.555 \cdot \text{deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = 87.994 \cdot \text{deg}$$

$$\delta_p := \phi - \theta_3$$

$$\delta_p = -237.549 \cdot \text{deg}$$

18. DESIGN SUMMARY

$$\text{Link 1:} \quad g = 4.354$$

$$\text{Link 2:} \quad w = 3.376$$

$$\text{Link 3:} \quad v = 1.933$$

$$\text{Link 4:} \quad u = 3.884$$

$$\text{Coupler point:} \quad r_P = 0.871 \quad \delta_p = -237.549 \cdot \text{deg}$$

19. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-12

Statement: Design a linkage to carry the body in Figure P5-2 through the two positions P_1 and P_2 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown. Use the free choices given below.

Given: Coordinates of the points P_1 and P_2 with respect to P_1 :

$$P_{1x} := 0.0 \quad P_{1y} := 0.0 \quad P_{2x} := 1.903 \quad P_{2y} := 1.347$$

Angles made by the body in positions 1 and 2:

$$\theta_{p1} := 101.0 \cdot \text{deg} \quad \theta_{p2} := 62.0 \cdot \text{deg}$$

Free choices for the **WZ** dyad :

$$z := 2.000 \quad \beta_2 := 30.0 \cdot \text{deg} \quad \phi := 150.0 \cdot \text{deg}$$

Free choices for the **US** dyad :

$$s := 3.000 \quad \gamma_2 := 40.0 \cdot \text{deg} \quad \psi := -50.0 \cdot \text{deg}$$

Solution: See Figure P5-2 and Mathcad file P0512.

- Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. Because of the data given in the hint, the second method of Section 5.3 will be used here.
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \quad R2 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \quad P_{21x} = 1.903$$

$$P_{21y} = 1.347$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 2.331$$

- From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\alpha_2 := \theta_{p2} - \theta_{p1} \quad \alpha_2 = -39.000 \cdot \text{deg}$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = 35.292 \cdot \text{deg}$$

- Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = -1.732 \quad Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = 1.000$$

$$A := \cos(\beta_2) - 1 \quad A = -0.134 \quad D := \sin(\alpha_2) \quad D = -0.629$$

$$B := \sin(\beta_2) \quad B = 0.500 \quad E := p_{21} \cdot \cos(\delta_2) \quad E = 1.903$$

$$C := \cos(\alpha_2) - 1 \quad C = -0.223 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = 1.347$$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \quad W_{1x} = 0.452$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \quad W_{1y} = -1.896$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \quad w = 1.949$$

$$\theta := \text{atan2}(W_{1x}, W_{1y}) \quad \theta = -76.607 \cdot \text{deg}$$

5. Solve for the **US** dyad using equations 5.12.

$$S_{1x} := s \cdot \cos(\psi) \quad S_{1x} = 1.928 \quad S_{1y} := s \cdot \sin(\psi) \quad S_{1y} = -2.298$$

$$A := \cos(\gamma_2) - 1 \quad A = -0.234 \quad D := \sin(\alpha_2) \quad D = -0.629$$

$$B := \sin(\gamma_2) \quad B = 0.643 \quad E := p_{21} \cdot \cos(\delta_2) \quad E = 1.903$$

$$C := \cos(\alpha_2) - 1 \quad C = -0.223 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = 1.347$$

$$U_{1x} := \frac{A \cdot (-C \cdot S_{1x} + D \cdot S_{1y} + E) + B \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F)}{-2 \cdot A} \quad U_{1x} = 0.924$$

$$U_{1y} := \frac{A \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F) + B \cdot (C \cdot S_{1x} - D \cdot S_{1y} - E)}{-2 \cdot A} \quad U_{1y} = -6.216$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2} \quad u = 6.284$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y}) \quad \sigma = -81.540 \cdot \text{deg}$$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3:

$$V_{1x} := z \cdot \cos(\phi) - s \cdot \cos(\psi) \quad V_{1x} = -3.660$$

$$V_{1y} := z \cdot \sin(\phi) - s \cdot \sin(\psi) \quad V_{1y} = 3.298$$

$$\theta_3 := \text{atan2}(V_{1x}, V_{1y}) \quad \theta_3 = 137.980 \cdot \text{deg}$$

$$v := \sqrt{V_{1x}^2 + V_{1y}^2} \quad v = 4.927$$

Link 1:

$$G_{1x} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma) \quad G_{1x} = -4.133$$

$$G_{1y} := w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma) \quad G_{1y} = 7.617$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y}) \quad \theta_1 = 118.485 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2} \quad g = 8.667$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \quad \theta_{2i} = -195.092 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_2$$

$$\theta_{2f} = -165.092 \cdot \text{deg}$$

8. Define the coupler point with respect to point A and the vector $\mathbf{V}$.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 2.000$$

$$\delta_p = 12.020 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := 0 \cdot \text{deg}$$

$$\rho_1 = 0.000 \cdot \text{deg}$$

$$R_1 := \sqrt{P_{1x}^2 + P_{1y}^2}$$

$$R_1 = 0.000$$

$$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = 1.281$$

$$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = 0.896$$

$$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = -2.853$$

$$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = 8.514$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$$

$$\theta_{rot} = 118.485 \cdot \text{deg}$$

11. Determine the Grashof condition.

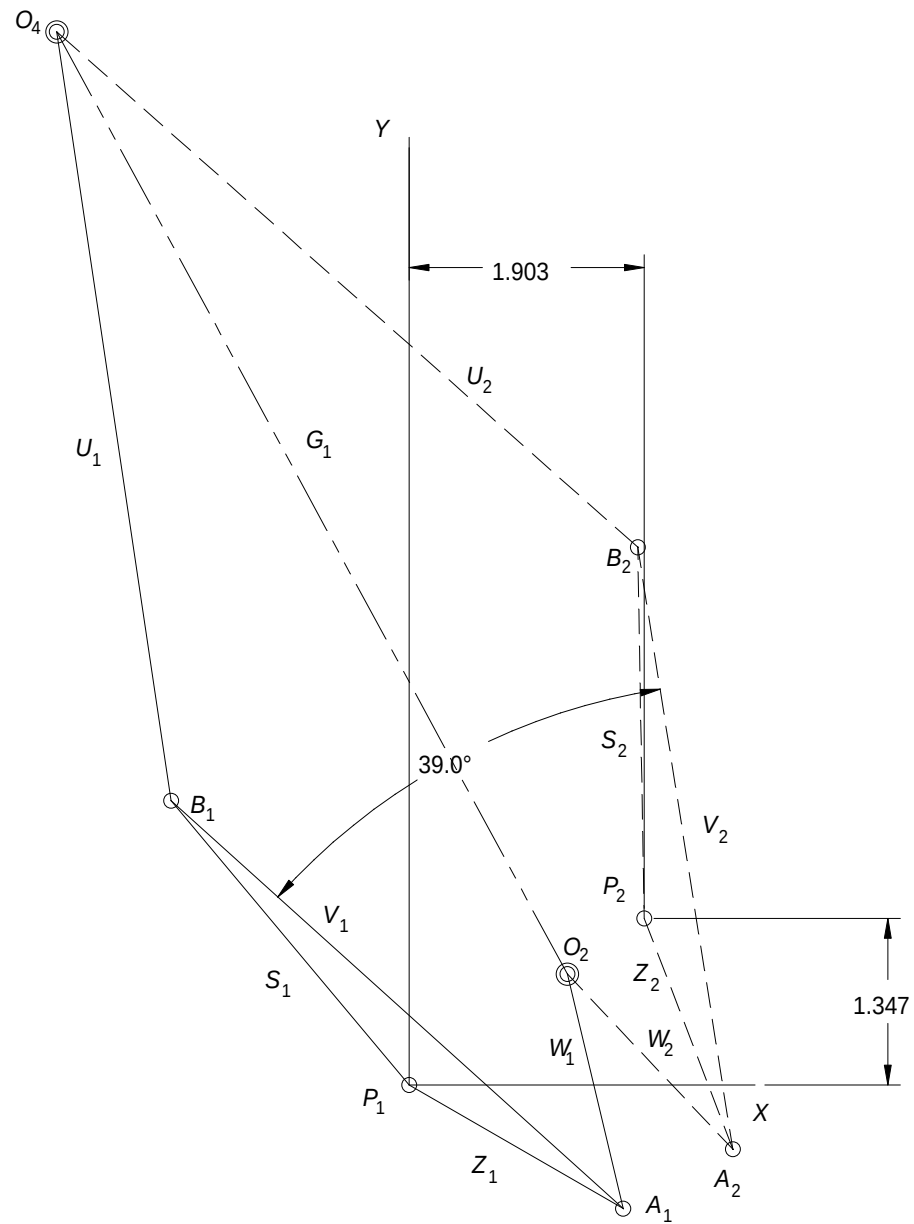
$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"Grashof"}$$

12. DESIGN SUMMARY

Link 2:	$w = 1.949$	$\theta = -76.607 \cdot \text{deg}$
Link 3:	$v = 4.927$	$\theta_3 = 137.980 \cdot \text{deg}$
Link 4:	$u = 6.284$	$\sigma = -81.540 \cdot \text{deg}$
Link 1:	$g = 8.667$	$\theta_1 = 118.485 \cdot \text{deg}$
Coupler:	$r_p = 2.000$	$\delta_p = 12.020 \cdot \text{deg}$
Crank angles:	$\theta_{2i} = -195.092 \cdot \text{deg}$ $\theta_{2f} = -165.092 \cdot \text{deg}$	

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-13

Statement: Design a linkage to carry the body in Figure P5-2 through the two positions P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown. Hint: First try a rough graphical solution to create realistic values for free choices.

Given: Coordinates of the points P_2 and P_3 with respect to P_1 :

$$P_{2x} := 1.903 \quad P_{2y} := 1.347 \quad P_{3x} := 1.389 \quad P_{3y} := 1.830$$

Angles made by the body in positions 1 and 2:

$$\theta_{p2} := 62.0 \cdot \text{deg} \quad \theta_{p3} := 39.0 \cdot \text{deg}$$

Solution: See Figure P5-2 and Mathcad file P0513.

- Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$\mathbf{R}_1 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \quad \mathbf{R}_2 := \begin{pmatrix} P_{3x} \\ P_{3y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := \mathbf{R}_2 - \mathbf{R}_1 \quad P_{21x} = -0.514$$

$$P_{21y} = 0.483$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 0.705$$

- From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\alpha_2 := \theta_{p3} - \theta_{p2} \quad \alpha_2 = -23.000 \cdot \text{deg}$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = 136.781 \cdot \text{deg}$$

- From a graphical solution (see figure next page), determine the values necessary for input to equations 5.8.

$$z := 3 \quad \beta_2 := 45.0 \cdot \text{deg} \quad \phi := 100 \cdot \text{deg}$$

- Solve for the $\mathbf{WZ}$ dyad using equations 5.8.

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = -0.521 \quad Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = 2.954$$

$$A := \cos(\beta_2) - 1 \quad A = -0.293 \quad D := \sin(\alpha_2) \quad D = -0.391$$

$$B := \sin(\beta_2) \quad B = 0.707 \quad E := p_{21} \cdot \cos(\delta_2) \quad E = -0.514$$

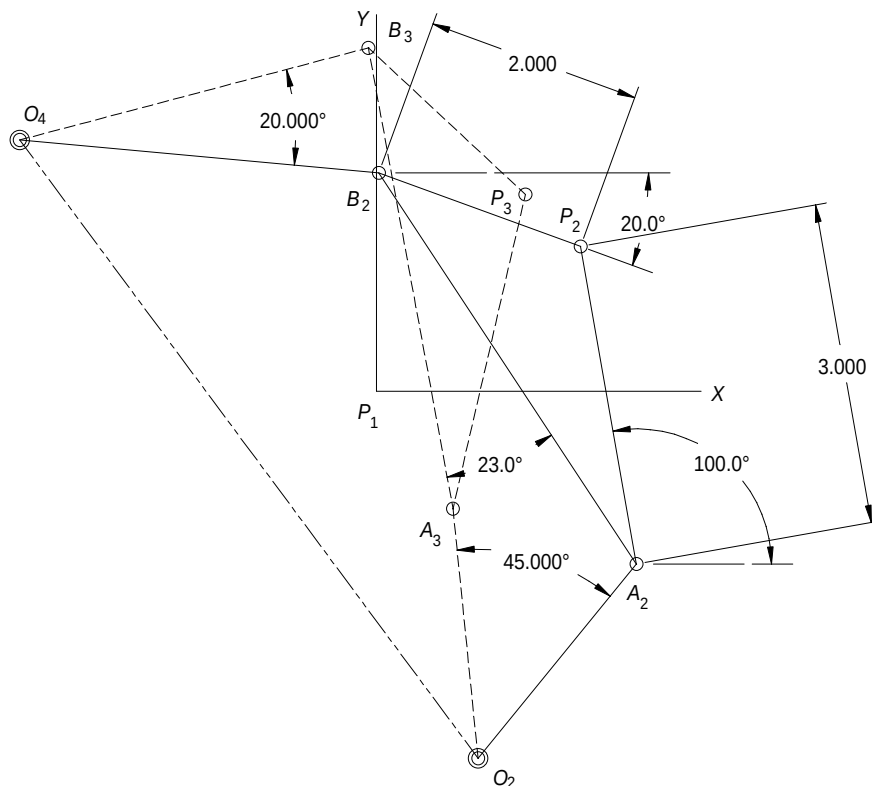
$$C := \cos(\alpha_2) - 1 \quad C = -0.079 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = 0.483$$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \quad W_{1x} = 1.476$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \quad W_{1y} = 1.807$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \quad w = 2.333$$

$$\theta := \text{atan2}(W_{1x}, W_{1y}) \quad \theta = 50.759 \cdot \text{deg}$$



5. From the graphical solution (see figure above), determine the values necessary for input to equations 5.12.

$$s := 2.000$$

$$\gamma_2 := 20.0 \cdot \text{deg}$$

$$\psi := -20.0 \cdot \text{deg}$$

6. Solve for the **US** dyad using equations 5.12.

$$S_{1x} := s \cdot \cos(\psi)$$

$$S_{1x} = 1.879$$

$$S_{1y} := s \cdot \sin(\psi)$$

$$S_{1y} = -0.684$$

$$A := \cos(\gamma_2) - 1$$

$$A = -0.060$$

$$D := \sin(\alpha_2)$$

$$D = -0.391$$

$$B := \sin(\gamma_2)$$

$$B = 0.342$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$E = -0.514$$

$$C := \cos(\alpha_2) - 1$$

$$C = -0.079$$

$$F := p_{21} \cdot \sin(\delta_2)$$

$$F = 0.483$$

$$U_{1x} := \frac{A \cdot (-C \cdot S_{1x} + D \cdot S_{1y} + E) + B \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F)}{-2 \cdot A}$$

$$U_{1x} = 3.346$$

$$U_{1y} := \frac{A \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F) + B \cdot (C \cdot S_{1x} - D \cdot S_{1y} - E)}{-2 \cdot A}$$

$$U_{1y} = -0.306$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2}$$

$$u = 3.360$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y})$$

$$\sigma = -5.216 \cdot \text{deg}$$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := z \cdot \cos(\phi) - s \cdot \cos(\psi)$

$$V_{1x} = -2.400$$

$$V_{Iy} := z \cdot \sin(\phi) - s \cdot \sin(\psi)$$

$$V_{Iy} = 3.638$$

$$\theta_3 := \text{atan2}(V_{Ix}, V_{Iy})$$

$$\theta_3 = 123.413 \cdot \text{deg}$$

$$v := \sqrt{V_{Ix}^2 + V_{Iy}^2}$$

$$v = 4.359$$

Link 1: $G_{Ix} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma)$

$$G_{Ix} = -4.271$$

$$G_{Iy} := w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma)$$

$$G_{Iy} = 5.751$$

$$\theta_1 := \text{atan2}(G_{Ix}, G_{Iy})$$

$$\theta_1 = 126.601 \cdot \text{deg}$$

$$g := \sqrt{G_{Ix}^2 + G_{Iy}^2}$$

$$g = 7.163$$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1$$

$$\theta_{2i} = -75.842 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_2$$

$$\theta_{2f} = -30.842 \cdot \text{deg}$$

9. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 3.000$$

$$\delta_p = -23.413 \cdot \text{deg}$$

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := \text{atan2}(P_{2x}, P_{2y})$$

$$\rho_1 = 35.292 \cdot \text{deg}$$

$$R_1 := \sqrt{P_{2x}^2 + P_{2y}^2}$$

$$R_1 = 2.331$$

$$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = 0.948$$

$$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = -3.414$$

$$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = -3.323$$

$$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = 2.337$$

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$$

$$\theta_{rot} = 126.601 \cdot \text{deg}$$

12. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

13. DESIGN SUMMARY

Link 2:	$w = 2.333$	$\theta = 50.759 \cdot deg$
Link 3:	$v = 4.359$	$\theta_3 = 123.413 \cdot deg$
Link 4:	$u = 3.360$	$\sigma = -5.216 \cdot deg$
Link 1:	$g = 7.163$	$\theta_1 = 126.601 \cdot deg$
Coupler:	$r_p = 3.000$	$\delta_p = -23.413 \cdot deg$
Crank angles:	$\theta_{2i} = -75.842 \cdot deg$	
	$\theta_{2f} = -30.842 \cdot deg$	

PROBLEM 5-14

Statement: Design a linkage to carry the body in Figure P5-2 through the three positions P_1 , P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown.

Given: Coordinates of the points P_1 and P_2 with respect to P_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= 1.903 & P_{2y} &:= 1.347 \\ P_{3x} &:= 1.389 & P_{3y} &:= 1.830 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 101 \cdot \text{deg} \quad \theta_{p2} := 62.0 \cdot \text{deg} \quad \theta_{p3} := 39.0 \cdot \text{deg}$$

Free choices for the **WZ** dyad :

$$\beta_2 := 40.0 \cdot \text{deg} \quad \beta_3 := 75.0 \cdot \text{deg}$$

Free choices for the **US** dyad :

$$\gamma_2 := 0.0 \cdot \text{deg} \quad \gamma_3 := 30.0 \cdot \text{deg}$$

Solution: See Figure P5-2 and Mathcad file P0514.

- Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 2.331 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= 35.292 \cdot \text{deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 2.297 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= 52.801 \cdot \text{deg} \end{aligned}$$

- Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -39.000 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -62.000 \cdot \text{deg} \end{aligned}$$

- Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$\begin{aligned} A &:= \cos(\beta_2) - 1 & B &:= \sin(\beta_2) & C &:= \cos(\alpha_2) - 1 \\ D &:= \sin(\alpha_2) & E &:= p_{21} \cdot \cos(\delta_2) & F &:= \cos(\beta_3) - 1 \\ G &:= \sin(\beta_3) & H &:= \cos(\alpha_3) - 1 & K &:= \sin(\alpha_3) \\ L &:= p_{31} \cdot \cos(\delta_3) & M &:= p_{21} \cdot \sin(\delta_2) & N &:= p_{31} \cdot \sin(\delta_3) \end{aligned}$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$$W_{1x} = 3.110$$

$$W_{1y} = -1.061$$

$$Z_{1x} = 0.297$$

$$Z_{1y} = 3.201$$

$$\theta := \text{atan2}(W_{1x}, W_{1y})$$

$$\theta = -18.843 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z_{1x}, Z_{1y})$$

$$\phi = 84.698 \cdot \text{deg}$$

The length of link 2 is: $w := \sqrt{(W_{1x}^2 + W_{1y}^2)}$, $w = 3.286$

The length of vector **Z** is: $z := \sqrt{(Z_{1x}^2 + Z_{1y}^2)}$, $z = 3.215$

4. Evaluate terms in the **US** coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$$U_{1x} = 1.658$$

$$U_{1y} = 3.361$$

$$S_{1x} = -2.853$$

$$S_{1y} = 2.013$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y})$$

$$\sigma = 63.740 \cdot \text{deg}$$

$$\psi := \text{atan2}(S_{1x}, S_{1y})$$

$$\psi = 144.792 \cdot \text{deg}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 3.748$

The length of vector **S** is: $s := \sqrt{(S_{1x}^2 + S_{1y}^2)}$, $s = 3.492$

5. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$

$$V_{1x} = 3.150$$

$$V_{1y} := Z_{1y} - S_{1y}$$

$$V_{1y} = 1.188$$

$$\theta_3 := \text{atan2}(V_{1x}, V_{1y})$$

$$\theta_3 = 20.657 \cdot \text{deg}$$

$$v := \sqrt{V_{1x}^2 + V_{1y}^2}$$

$$v = 3.367$$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$

$$G_{1x} = 4.602$$

$$G_{1y} := W_{1y} + V_{1y} - U_{1y}$$

$$G_{1y} = -3.234$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y})$$

$$\theta_1 = -35.099 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2}$$

$$g = 5.625$$

6. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1$$

$$\theta_{2i} = 16.256 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_3$$

$$\theta_{2f} = 91.256 \cdot \text{deg}$$

7. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 3.215$$

$$\delta_p = 64.041 \cdot \text{deg}$$

8. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = -3.407$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = -2.140$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = 1.195$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = -5.374$$

9. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$$

$$\theta_{rot} = -35.099 \cdot \text{deg}$$

10. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

11. DESIGN SUMMARY

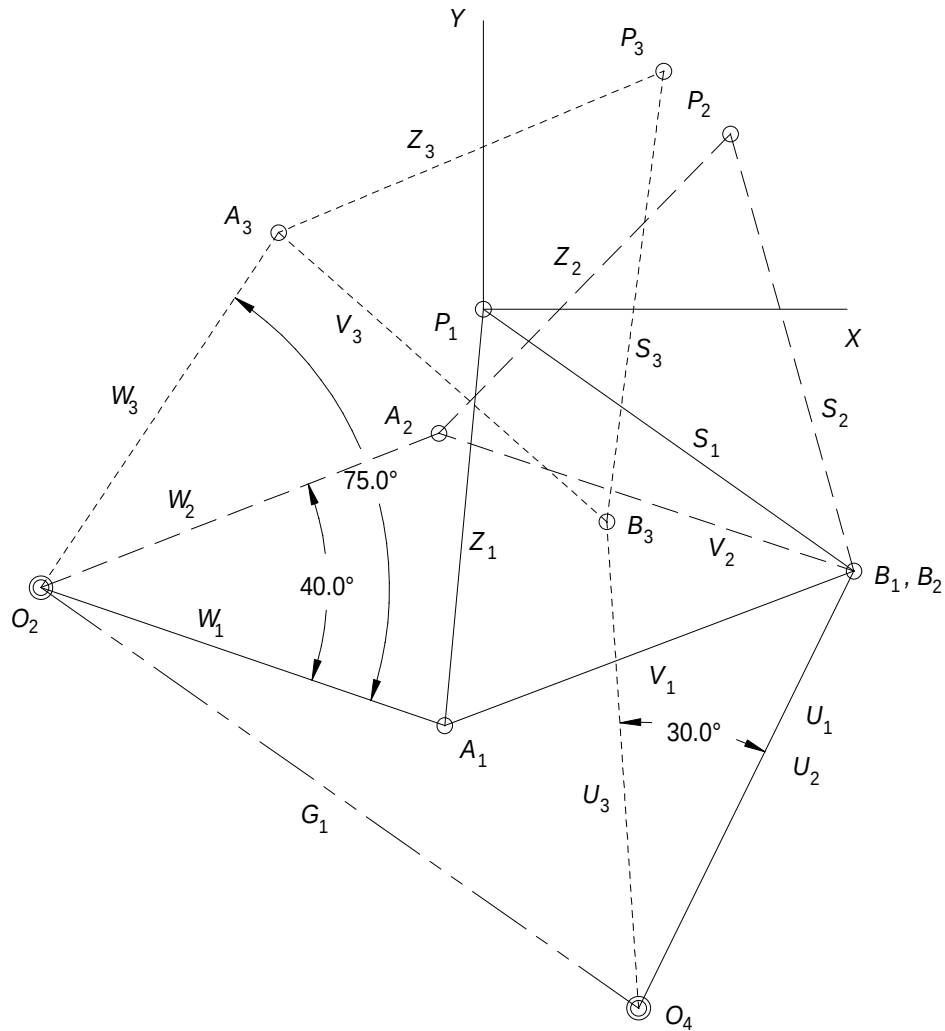
Link 2:	$w = 3.286$	$\theta = -18.843 \cdot \text{deg}$
Link 3:	$v = 3.367$	$\theta_3 = 20.657 \cdot \text{deg}$
Link 4:	$u = 3.748$	$\sigma = 63.740 \cdot \text{deg}$
Link 1:	$g = 5.625$	$\theta_1 = -35.099 \cdot \text{deg}$
Coupler:	$r_p = 3.215$	$\delta_p = 64.041 \cdot \text{deg}$

Crank angles:

$$\theta_{2i} = 16.256 \cdot \text{deg}$$

$$\theta_{2f} = 91.256 \cdot \text{deg}$$

12. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-15

Statement: Design a linkage to carry the body in Figure P5-2 through the three positions P_1 , P_2 and P_3 at the angles shown in the figure. Use analytical synthesis and design it for the fixed pivots shown.

Given:

$P_{21x} := 1.903$	$P_{21y} := 1.347$	$P_{31x} := 1.389$	$P_{31y} := 1.830$
$O_{2x} := -0.884$	$O_{2y} := -1.251$	$O_{4x} := 3.062$	$O_{4y} := -1.251$
Body angles:	$\theta_{p1} := 101 \cdot \text{deg}$	$\theta_{p2} := 62 \cdot \text{deg}$	$\theta_{p3} := 39 \cdot \text{deg}$

Solution: See Figure P5-2 and Mathcad file P0515.

1. Determine the angle changes between precision points from the body angles given.

$$\alpha_2 := \theta_{p2} - \theta_{p1} \quad \alpha_2 = -39.000 \cdot \text{deg}$$

$$\alpha_3 := \theta_{p3} - \theta_{p1} \quad \alpha_3 = -62.000 \cdot \text{deg}$$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$R_{1x} := -O_{2x} \quad R_{1x} = 0.884 \quad R_{1y} := -O_{2y} \quad R_{1y} = 1.251$$

$$R_{2x} := R_{1x} + P_{21x} \quad R_{2x} = 2.787$$

$$R_{2y} := R_{1y} + P_{21y} \quad R_{2y} = 2.598$$

$$R_{3x} := R_{1x} + P_{31x} \quad R_{3x} = 2.273$$

$$R_{3y} := R_{1y} + P_{31y} \quad R_{3y} = 3.081$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \quad R_1 = 1.532$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 3.810$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 3.829$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 54.754 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 42.990 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = 53.582 \cdot \text{deg}$$

4. Solve for β_2 and β_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = 0.103$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = 2.205$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = -0.753$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = 3.274$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = -1.313$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2)$$

$$C_6 = 2.182$$

$$A_1 := -C_3^2 - C_4^2$$

$$A_1 = -11.288$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$$

$$A_2 = 2.654$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$$

$$A_3 = -8.134$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$$

$$A_4 = -1.324$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$$

$$A_5 = -2.654$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$$

$$A_6 = -7.297$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$$

$$K_1 = 55.842$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$$

$$K_2 = 30.136$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$$

$$K_3 = -0.392$$

$$\beta_{31} := 2 \cdot \operatorname{atan}\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\beta_{31} = 118.708 \cdot \text{deg}$$

$$\beta_{32} := 2 \cdot \operatorname{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\beta_{32} = -62.000 \cdot \text{deg}$$

The second value is the same as α_3 , so use the first value

$$\beta_3 := \beta_{31}$$

$$\beta_{21} := \operatorname{acos}\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right)$$

$$\beta_{21} = 59.564 \cdot \text{deg}$$

$$\beta_{22} := \operatorname{asin}\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right)$$

$$\beta_{22} = 59.564 \cdot \text{deg}$$

Since both values are the same,

$$\beta_2 := \beta_{21}$$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$R_{1x} := -O_{4x}$$

$$R_{1x} = -3.062$$

$$R_{1y} := -O_{4y}$$

$$R_{1y} = 1.251$$

$$R_{2x} := R_{1x} + P_{21x}$$

$$R_{2x} = -1.159$$

$$R_{2y} := R_{1y} + P_{21y}$$

$$R_{2y} = 2.598$$

$$R_{3x} := R_{1x} + P_{31x}$$

$$R_{3x} = -1.673$$

$$R_{3y} := R_{1y} + P_{31y}$$

$$R_{3y} = 3.081$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$$

$$R_1 = 3.308$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$$

$$R_2 = 2.845$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 3.506$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 157.777 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 114.042 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = 118.502 \cdot \text{deg}$$

7. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = -1.111$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = 1.204$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = 1.340$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = -0.210$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = -0.433$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = -0.301$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -1.840$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = -0.495$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = 0.517$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = 1.847$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = 0.495$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -1.236$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = -1.553$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = 0.344$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = -1.033$$

$$\gamma_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{31} = -62.000 \cdot \text{deg}$$

$$\gamma_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{32} = 36.991 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\gamma_3 := \gamma_{32}$$

$$\gamma_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1}\right) \quad \gamma_{21} = 73.415 \cdot \text{deg}$$

$$\gamma_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1}\right) \quad \gamma_{22} = -73.415 \cdot \text{deg}$$

Since γ_2 is not in the first quadrant ,

$$\gamma_2 := \gamma_{22}$$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 2.331$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = 35.292 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \quad p_{31} = 2.297$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y}) \quad \delta_3 = 52.801 \cdot \text{deg}$$

9. Evaluate terms in the $\mathbf{WZ}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1$$

$$B := \sin(\beta_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the $\mathbf{W}$ and $\mathbf{Z}$ vectors are:

$$W1x = 1.262$$

$$W1y = -1.109$$

$$Z1x = -0.378$$

$$Z1y = 2.360$$

11. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2} \quad w = 1.680$

12. Evaluate terms in the $\mathbf{US}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$U1x = -0.326 \quad U1y = 0.830 \quad S1x = -2.736 \quad S1y = 0.421$$

14. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2} \quad u = 0.892$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x \quad V1x = 2.359$$

$$V1y := Z1y - S1y \quad V1y = 1.939$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2} \quad v = 3.054$

$$G1x := W1x + V1x - U1x \quad G1x = 3.946$$

$$G1y := W1y + V1y - U1y \quad G1y = -1.110 \times 10^{-15}$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2} \quad g = 3.946$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors **W**₁, **Z**₁, **U**₁, and **S**₁.

$$O2x := -Z1x - W1x \quad O2x = -0.884$$

$$O2y := -Z1y - W1y \quad O2y = -1.251$$

$$O4x := -S1x - U1x \quad O4x = 3.062$$

$$O4y := -S1y - U1y \quad O4y = -1.251$$

These check with Figure P5-2.

17. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $z := \sqrt{Z1x^2 + Z1y^2} \quad z = 2.390 \quad r_P := z$

Angle BAP (δ_p) $s := \sqrt{S1x^2 + S1y^2} \quad s = 2.769$

$$\psi := \text{atan2}(S1x, S1y) \quad \psi = 171.262 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z1x, Z1y) \quad \phi = 99.095 \cdot \text{deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = 39.430 \cdot \text{deg}$$

$$\delta_p := \phi - \theta_3$$

$$\delta_p = 59.666 \cdot \text{deg}$$

18. DESIGN SUMMARY

$$\text{Link 1:} \quad g = 3.946$$

$$\text{Link 2:} \quad w = 1.680$$

$$\text{Link 3:} \quad v = 3.054$$

$$\text{Link 4:} \quad u = 0.892$$

$$\text{Coupler point:} \quad r_P = 2.390 \quad \delta_p = 59.666 \cdot \text{deg}$$

19. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-16

Statement: Design a linkage to carry the body in Figure P5-3 through the two positions P_1 and P_2 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown.

Given: Coordinates of the points P_1 and P_2 with respect to P_1 :

$$P_{1x} := 0.0 \quad P_{1y} := 0.0 \quad P_{2x} := -0.907 \quad P_{2y} := 0.0$$

Angles made by the body in positions 1 and 2:

$$\theta_{p1} := 111.8 \cdot \text{deg} \quad \theta_{p2} := 191.1 \cdot \text{deg}$$

Free choices for the **WZ** dyad :

$$z := 1.500 \quad \beta_2 := 44.0 \cdot \text{deg} \quad \phi := -50.0 \cdot \text{deg}$$

Free choices for the **US** dyad :

$$s := 2.500 \quad \gamma_2 := -55.0 \cdot \text{deg} \quad \psi := 20.0 \cdot \text{deg}$$

Solution: See Figure P5-3 and Mathcad file P0516.

- Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$\mathbf{R}_1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \quad \mathbf{R}_2 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := \mathbf{R}_2 - \mathbf{R}_1 \quad P_{21x} = -0.907$$

$$P_{21y} = 0.000$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 0.907$$

- From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\alpha_2 := \theta_{p2} - \theta_{p1} \quad \alpha_2 = 79.300 \cdot \text{deg}$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = 180.000 \cdot \text{deg}$$

- Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = 0.964 \quad Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = -1.149$$

$$A := \cos(\beta_2) - 1 \quad A = -0.281 \quad D := \sin(\alpha_2) \quad D = 0.983$$

$$B := \sin(\beta_2) \quad B = 0.695 \quad E := p_{21} \cdot \cos(\delta_2) \quad E = -0.907$$

$$C := \cos(\alpha_2) - 1 \quad C = -0.814 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = 0.000$$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \quad W_{1x} = -1.705$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \quad W_{1y} = 2.490$$

$$w := \sqrt{W_{Ix}^2 + W_{Iy}^2}$$

$$w = 3.018$$

$$\theta := \text{atan2}(W_{Ix}, W_{Iy})$$

$$\theta = 124.405 \cdot \text{deg}$$

5. Solve for the **US** dyad using equations 5.12.

$$S_{Ix} := s \cdot \cos(\psi)$$

$$S_{Ix} = 2.349$$

$$S_{Iy} := s \cdot \sin(\psi)$$

$$S_{Iy} = 0.855$$

$$A := \cos(\gamma_2) - 1$$

$$A = -0.426$$

$$D := \sin(\alpha_2)$$

$$D = 0.983$$

$$B := \sin(\gamma_2)$$

$$B = -0.819$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$E = -0.907$$

$$C := \cos(\alpha_2) - 1$$

$$C = -0.814$$

$$F := p_{21} \cdot \sin(\delta_2)$$

$$F = 0.000$$

$$U_{Ix} := \frac{A \cdot (-C \cdot S_{Ix} + D \cdot S_{Iy} + E) + B \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F)}{-2 \cdot A}$$

$$U_{Ix} = 0.625$$

$$U_{Iy} := \frac{A \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F) + B \cdot (C \cdot S_{Ix} - D \cdot S_{Iy} - E)}{-2 \cdot A}$$

$$U_{Iy} = 2.579$$

$$u := \sqrt{U_{Ix}^2 + U_{Iy}^2}$$

$$u = 2.654$$

$$\sigma := \text{atan2}(U_{Ix}, U_{Iy})$$

$$\sigma = 76.373 \cdot \text{deg}$$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{Ix} := z \cdot \cos(\phi) - s \cdot \cos(\psi)$

$$V_{Ix} = -1.385$$

$$V_{Iy} := z \cdot \sin(\phi) - s \cdot \sin(\psi)$$

$$V_{Iy} = -2.004$$

$$\theta_3 := \text{atan2}(V_{Ix}, V_{Iy})$$

$$\theta_3 = -124.648 \cdot \text{deg}$$

$$v := \sqrt{V_{Ix}^2 + V_{Iy}^2}$$

$$v = 2.436$$

Link 1: $G_{Ix} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma)$

$$G_{Ix} = -3.715$$

$$G_{Iy} := w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma)$$

$$G_{Iy} = -2.094$$

$$\theta_1 := \text{atan2}(G_{Ix}, G_{Iy})$$

$$\theta_1 = -150.596 \cdot \text{deg}$$

$$g := \sqrt{G_{Ix}^2 + G_{Iy}^2}$$

$$g = 4.265$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1$$

$$\theta_{2i} = 275.001 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_2$$

$$\theta_{2f} = 319.001 \cdot \text{deg}$$

8. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 1.500$$

$$\delta_p = 74.648 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\begin{aligned}\rho_1 &:= 0 \cdot \text{deg} & \rho_1 &= 0.000 \cdot \text{deg} \\ R_1 &:= \sqrt{P_{1x}^2 + P_{1y}^2} & R_1 &= 0.000 \\ O_{2x} &:= R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta) & O_{2x} &= 0.741 \\ O_{2y} &:= R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta) & O_{2y} &= -1.341 \\ O_{4x} &:= R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma) & O_{4x} &= -2.975 \\ O_{4y} &:= R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma) & O_{4y} &= -3.434\end{aligned}$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{\text{rot}} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{\text{rot}} = -150.596 \cdot \text{deg}$$

11. Determine the Grashof condition.

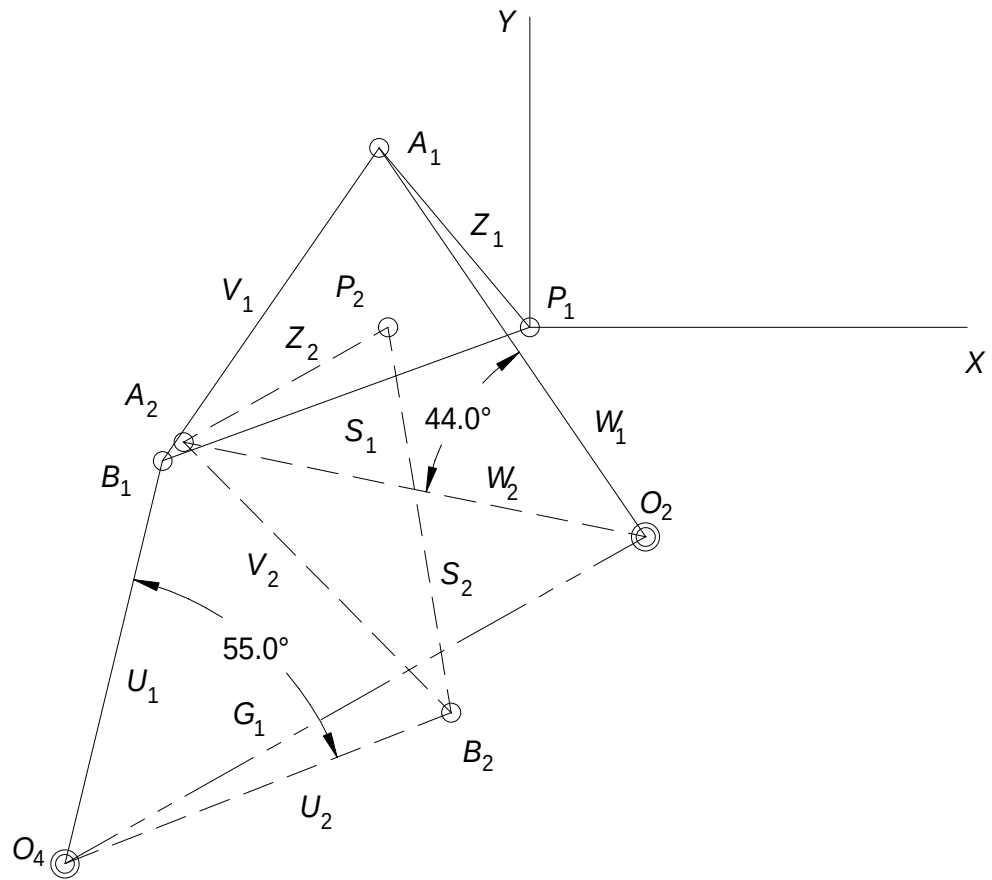
$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

12. DESIGN SUMMARY

Link 2:	$w = 3.018$	$\theta = 124.405 \cdot \text{deg}$
Link 3:	$v = 2.436$	$\theta_3 = -124.648 \cdot \text{deg}$
Link 4:	$u = 2.654$	$\sigma = 76.373 \cdot \text{deg}$
Link 1:	$g = 4.265$	$\theta_1 = -150.596 \cdot \text{deg}$
Coupler:	$r_p = 1.500$	$\delta_p = 74.648 \cdot \text{deg}$
Crank angles:	$\theta_{2i} = 275.001 \cdot \text{deg}$	$\theta_{2f} = 319.001 \cdot \text{deg}$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-17

Statement: Design a linkage to carry the body in Figure P5-3 through the two positions P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown.

Given: Coordinates of the points P_2 and P_3 with respect to P_1 :

$$P_{2x} := -0.907 \quad P_{2y} := 0.0 \quad P_{3x} := -1.447 \quad P_{3y} := 0.0$$

Angles made by the body in positions 1 and 2:

$$\theta_{p2} := 191.1 \cdot \text{deg} \quad \theta_{p3} := 237.4 \cdot \text{deg}$$

Free choices for the **WZ** dyad :

$$z := 2.000 \quad \beta_2 := 66.0 \cdot \text{deg} \quad \phi := -60.0 \cdot \text{deg}$$

Free choices for the **US** dyad :

$$s := 3.000 \quad \gamma_2 := -44.0 \cdot \text{deg} \quad \psi := 30.0 \cdot \text{deg}$$

Solution: See Figure P5-3 and Mathcad file P0517.

- Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \quad R2 := \begin{pmatrix} P_{3x} \\ P_{3y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \quad P_{21x} = -0.540$$

$$P_{21y} = 0.000$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 0.540$$

- From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\alpha_2 := \theta_{p3} - \theta_{p2} \quad \alpha_2 = 46.300 \cdot \text{deg}$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = 180.000 \cdot \text{deg}$$

- Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = 1.000 \quad Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = -1.732$$

$$A := \cos(\beta_2) - 1 \quad A = -0.593 \quad D := \sin(\alpha_2) \quad D = 0.723$$

$$B := \sin(\beta_2) \quad B = 0.914 \quad E := p_{21} \cdot \cos(\delta_2) \quad E = -0.540$$

$$C := \cos(\alpha_2) - 1 \quad C = -0.309 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = 0.000$$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \quad W_{1x} = -0.227$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \quad W_{1y} = 1.771$$

$$w := \sqrt{W_{Ix}^2 + W_{Iy}^2}$$

$$w = 1.786$$

$$\theta := \text{atan2}(W_{Ix}, W_{Iy})$$

$$\theta = 97.314 \cdot \text{deg}$$

5. Solve for the **US** dyad using equations 5.12.

$$S_{Ix} := s \cdot \cos(\psi)$$

$$S_{Ix} = 2.598$$

$$S_{Iy} := s \cdot \sin(\psi)$$

$$S_{Iy} = 1.500$$

$$A := \cos(\gamma_2) - 1$$

$$A = -0.281$$

$$D := \sin(\alpha_2)$$

$$D = 0.723$$

$$B := \sin(\gamma_2)$$

$$B = -0.695$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$E = -0.540$$

$$C := \cos(\alpha_2) - 1$$

$$C = -0.309$$

$$F := p_{21} \cdot \sin(\delta_2)$$

$$F = 0.000$$

$$U_{Ix} := \frac{A \cdot (-C \cdot S_{Ix} + D \cdot S_{Iy} + E) + B \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F)}{-2 \cdot A}$$

$$U_{Ix} = 1.077$$

$$U_{Iy} := \frac{A \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F) + B \cdot (C \cdot S_{Ix} - D \cdot S_{Iy} - E)}{-2 \cdot A}$$

$$U_{Iy} = 2.375$$

$$u := \sqrt{U_{Ix}^2 + U_{Iy}^2}$$

$$u = 2.608$$

$$\sigma := \text{atan2}(U_{Ix}, U_{Iy})$$

$$\sigma = 65.609 \cdot \text{deg}$$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{Ix} := z \cdot \cos(\phi) - s \cdot \cos(\psi)$

$$V_{Ix} = -1.598$$

$$V_{Iy} := z \cdot \sin(\phi) - s \cdot \sin(\psi)$$

$$V_{Iy} = -3.232$$

$$\theta_3 := \text{atan2}(V_{Ix}, V_{Iy})$$

$$\theta_3 = -116.310 \cdot \text{deg}$$

$$v := \sqrt{V_{Ix}^2 + V_{Iy}^2}$$

$$v = 3.606$$

Link 1: $G_{Ix} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma)$

$$G_{Ix} = -2.902$$

$$G_{Iy} := w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma)$$

$$G_{Iy} = -3.836$$

$$\theta_1 := \text{atan2}(G_{Ix}, G_{Iy})$$

$$\theta_1 = -127.111 \cdot \text{deg}$$

$$g := \sqrt{G_{Ix}^2 + G_{Iy}^2}$$

$$g = 4.810$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1$$

$$\theta_{2i} = 224.425 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_2$$

$$\theta_{2f} = 290.425 \cdot \text{deg}$$

8. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 2.000$$

$$\delta_p = 56.310 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := \text{atan2}(P_{2x}, P_{2y})$$

$$\rho_1 = 180.000 \cdot \text{deg}$$

$$R_1 := \sqrt{P_{2x}^2 + P_{2y}^2}$$

$$R_1 = 0.907$$

$$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = -1.680$$

$$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = -0.039$$

$$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = -4.582$$

$$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = -3.875$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{\text{rot}} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$$

$$\theta_{\text{rot}} = -127.111 \cdot \text{deg}$$

11. Determine the Grashof condition.

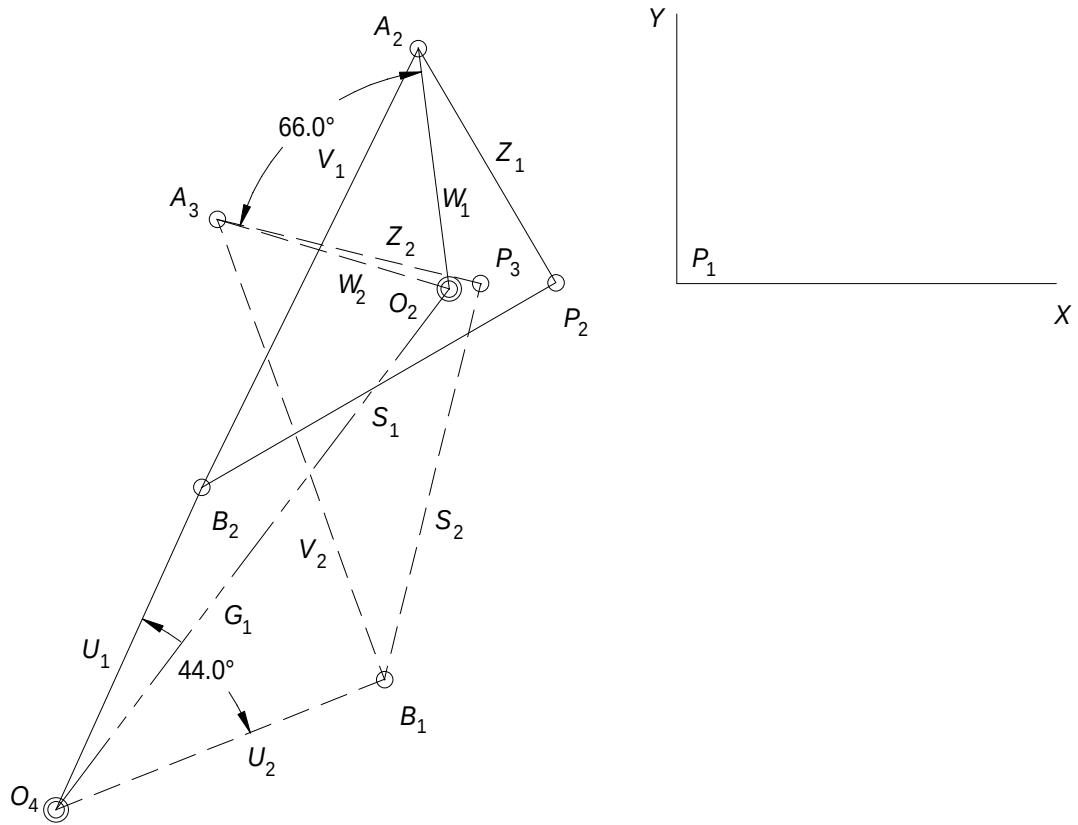
$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

12. DESIGN SUMMARY

Link 2:	$w = 1.786$	$\theta = 97.314 \cdot \text{deg}$
Link 3:	$v = 3.606$	$\theta_3 = -116.310 \cdot \text{deg}$
Link 4:	$u = 2.608$	$\sigma = 65.609 \cdot \text{deg}$
Link 1:	$g = 4.810$	$\theta_1 = -127.111 \cdot \text{deg}$
Coupler:	$r_p = 2.000$	$\delta_p = 56.310 \cdot \text{deg}$
Crank angles:	$\theta_{2i} = 224.425 \cdot \text{deg}$	$\theta_{2f} = 290.425 \cdot \text{deg}$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-18

Statement: Design a linkage to carry the body in Figure P5-3 through the three positions P_1 , P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown.

Given: Coordinates of the points P_1 and P_2 with respect to P_1 :

$$P_{1x} := 0.0 \quad P_{1y} := 0.0 \quad P_{2x} := -0.907 \quad P_{2y} := 0.0$$

$$P_{3x} := -1.447 \quad P_{3y} := 0.0$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 111.8 \cdot \text{deg} \quad \theta_{p2} := 191.1 \cdot \text{deg} \quad \theta_{p3} := 237.4 \cdot \text{deg}$$

Free choices for the **WZ** dyad :

$$\beta_2 := 40.0 \cdot \text{deg} \quad \beta_3 := 80.0 \cdot \text{deg}$$

Free choices for the **US** dyad :

$$\gamma_2 := 20.0 \cdot \text{deg} \quad \gamma_3 := 50.0 \cdot \text{deg}$$

Solution: See Figure P5-3 and Mathcad file P0518.

- Determine the magnitudes and orientation of the position difference vectors.

$$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2} \quad p_{21} = 0.907 \quad \delta_2 := \text{atan2}(P_{2x}, P_{2y}) \quad \delta_2 = 180.000 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2} \quad p_{31} = 1.447 \quad \delta_3 := \text{atan2}(P_{3x}, P_{3y}) \quad \delta_3 = 180.000 \cdot \text{deg}$$

- Determine the angle changes of the coupler between precision points.

$$\alpha_2 := \theta_{p2} - \theta_{p1} \quad \alpha_2 = 79.300 \cdot \text{deg}$$

$$\alpha_3 := \theta_{p3} - \theta_{p1} \quad \alpha_3 = 125.600 \cdot \text{deg}$$

- Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$A := \cos(\beta_2) - 1 \quad B := \sin(\beta_2) \quad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \quad E := p_{21} \cdot \cos(\delta_2) \quad F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3) \quad H := \cos(\alpha_3) - 1 \quad K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \quad M := p_{21} \cdot \sin(\delta_2) \quad N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$$W_{1x} = 1.696 \quad W_{1y} = -0.038 \quad Z_{1x} = -0.396 \quad Z_{1y} = 0.872$$

$$\theta := \text{atan2}(W_{1x}, W_{1y}) \quad \theta = -1.280 \cdot \text{deg} \quad \phi := \text{atan2}(Z_{1x}, Z_{1y}) \quad \phi = 114.406 \cdot \text{deg}$$

The length of link 2 is: $w := \sqrt{(W_{1x}^2 + W_{1y}^2)}$, $w = 1.696$

The length of vector **Z** is: $z := \sqrt{(Z_{1x}^2 + Z_{1y}^2)}$, $z = 0.958$

4. Evaluate terms in the **US** coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1 \quad B' := \sin(\gamma_2) \quad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \quad E := p_{21} \cdot \cos(\delta_2) \quad F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3) \quad H := \cos(\alpha_3) - 1 \quad K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \quad M := p_{21} \cdot \sin(\delta_2) \quad N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$$U_{1x} = 1.483 \quad U_{1y} = 0.409 \quad S_{1x} = 0.048 \quad S_{1y} = 0.650$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y}) \quad \sigma = 15.412 \cdot \text{deg} \quad \psi := \text{atan2}(S_{1x}, S_{1y}) \quad \psi = 85.790 \cdot \text{deg}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 1.538$

The length of vector **S** is: $s := \sqrt{(S_{1x}^2 + S_{1y}^2)}$, $s = 0.652$

5. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x} \quad V_{1x} = -0.444$

$$V_{1y} := Z_{1y} - S_{1y} \quad V_{1y} = 0.222$$

$$\theta_3 := \text{atan2}(V_{1x}, V_{1y}) \quad \theta_3 = 153.426 \cdot \text{deg}$$

$$v := \sqrt{V_{1x}^2 + V_{1y}^2} \quad v = 0.496$$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x} \quad G_{1x} = -0.230$

$$G_{1y} := W_{1y} + V_{1y} - U_{1y} \quad G_{1y} = -0.225$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y}) \quad \theta_1 = -135.700 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2} \quad g = 0.322$$

6. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \quad \theta_{2i} = 134.419 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_3 \quad \theta_{2f} = 214.419 \cdot \text{deg}$$

7. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z \quad \delta_p := \phi - \theta_3$$

$$r_p = 0.958 \quad \delta_p = -39.020 \cdot \text{deg}$$

8. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta) \quad O_{2x} = -1.300$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta) \quad O_{2y} = -0.834$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma) \quad O_{4x} = -1.530$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma) \quad O_{4y} = -1.059$$

9. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = -135.700 \cdot \text{deg}$$

10. Determine the Grashof condition.

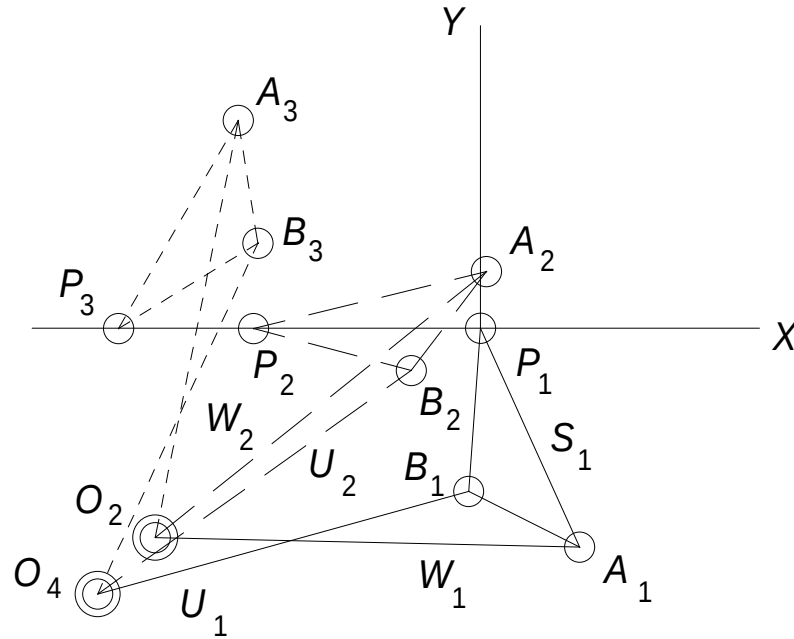
$$\begin{aligned} \text{Condition}(a, b, c, d) := & \left\{ \begin{array}{l} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} \quad \text{if } SL < PQ \\ \text{return "Special Grashof"} \quad \text{if } SL = PQ \\ \text{return "non-Grashof"} \quad \text{otherwise} \end{array} \right. \end{aligned}$$

$$\text{Condition}(g, u, v, w) = \text{"Grashof"}$$

11. DESIGN SUMMARY

Link 2:	$w = 1.696$	$\theta = -1.280 \cdot \text{deg}$
Link 3:	$v = 0.496$	$\theta_3 = 153.426 \cdot \text{deg}$
Link 4:	$u = 1.538$	$\sigma = 15.412 \cdot \text{deg}$
Link 1:	$g = 0.322$	$\theta_1 = -135.700 \cdot \text{deg}$
Coupler:	$r_p = 0.958$	$\delta_p = -39.020 \cdot \text{deg}$
Crank angles:	$\theta_{2i} = 134.419 \cdot \text{deg}$	
	$\theta_{2f} = 214.419 \cdot \text{deg}$	

12. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-19

Statement: Design a linkage to carry the body in Figure P5-3 through the three positions P_1 , P_2 and P_3 at the angles shown in the figure. Use analytical synthesis and design it for the fixed pivots shown.

Given:

$$\begin{array}{llll}
 P_{21x} := -0.907 & P_{21y} := 0.0 & P_{31x} := -1.447 & P_{31y} := 0.0 \\
 O_{2x} := -1.788 & O_{2y} := -1.994 & O_{4x} := 0.212 & O_{4y} := -1.994 \\
 \text{Body angles:} & \theta_{p1} := 111.8 \cdot \text{deg} & \theta_{p2} := 191.1 \cdot \text{deg} & \theta_{p3} := 237.4 \cdot \text{deg}
 \end{array}$$

Solution: See Figure P5-3 and Mathcad file P0519.

1. Determine the angle changes between precision points from the body angles given.

$$\begin{array}{ll}
 \alpha_2 := \theta_{p2} - \theta_{p1} & \alpha_2 = 79.300 \cdot \text{deg} \\
 \alpha_3 := \theta_{p3} - \theta_{p1} & \alpha_3 = 125.600 \cdot \text{deg}
 \end{array}$$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$\begin{array}{llll}
 R_{1x} := -O_{2x} & R_{1x} = 1.788 & R_{1y} := -O_{2y} & R_{1y} = 1.994 \\
 R_{2x} := R_{1x} + P_{21x} & R_{2x} = 0.881 & & \\
 R_{2y} := R_{1y} + P_{21y} & R_{2y} = 1.994 & & \\
 R_{3x} := R_{1x} + P_{31x} & R_{3x} = 0.341 & & \\
 R_{3y} := R_{1y} + P_{31y} & R_{3y} = 1.994 & & \\
 R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} & R_1 = 2.678 & & \\
 R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} & R_2 = 2.180 & & \\
 R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} & R_3 = 2.023 & &
 \end{array}$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\begin{array}{ll}
 \zeta_1 := \text{atan2}(R_{1x}, R_{1y}) & \zeta_1 = 48.118 \cdot \text{deg} \\
 \zeta_2 := \text{atan2}(R_{2x}, R_{2y}) & \zeta_2 = 66.163 \cdot \text{deg} \\
 \zeta_3 := \text{atan2}(R_{3x}, R_{3y}) & \zeta_3 = 80.296 \cdot \text{deg}
 \end{array}$$

4. Solve for β_2 and β_3 using equations 5.34

$$\begin{array}{ll}
 C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) & C_1 = 0.238 \\
 C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) & C_2 = 1.150 \\
 C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) & C_3 = -3.003 \\
 C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) & C_4 = 1.701 \\
 C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) & C_5 = -2.508
 \end{array}$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2)$$

$$C_6 = -0.133$$

$$A_1 := -C_3^2 - C_4^2$$

$$A_1 = -11.912$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$$

$$A_2 = 4.666$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$$

$$A_3 = -7.307$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$$

$$A_4 = -3.048$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$$

$$A_5 = -4.666$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$$

$$A_6 = -2.671$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$$

$$K_1 = 5.293$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$$

$$K_2 = 34.731$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$$

$$K_3 = 25.158$$

$$\beta_{31} := 2 \cdot \operatorname{atan}\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\beta_{31} = 125.600 \cdot \text{deg}$$

$$\beta_{32} := 2 \cdot \operatorname{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\beta_{32} = 37.070 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\beta_3 := \beta_{32}$$

$$\beta_{21} := \operatorname{acos}\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right)$$

$$\beta_{21} = 18.241 \cdot \text{deg}$$

$$\beta_{22} := \operatorname{asin}\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right)$$

$$\beta_{22} = 18.241 \cdot \text{deg}$$

Since both values are the same,

$$\beta_2 := \beta_{21}$$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$R_{1x} := -O_{4x}$$

$$R_{1x} = -0.212$$

$$R_{1y} := -O_{4y}$$

$$R_{1y} = 1.994$$

$$R_{2x} := R_{1x} + P_{21x}$$

$$R_{2x} = -1.119$$

$$R_{2y} := R_{1y} + P_{21y}$$

$$R_{2y} = 1.994$$

$$R_{3x} := R_{1x} + P_{31x}$$

$$R_{3x} = -1.659$$

$$R_{3y} := R_{1y} + P_{31y}$$

$$R_{3y} = 1.994$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$$

$$R_1 = 2.005$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 2.287$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 2.594$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 96.069 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 119.300 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = 129.760 \cdot \text{deg}$$

7. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = -1.297$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = 0.811$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = 0.161$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = 3.327$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = -0.880$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = 1.832$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -11.096$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = 3.222$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = -5.954$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = -4.186$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = -3.222$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -2.906$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = 3.816$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = 34.288$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = 25.658$$

$$\gamma_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{31} = 125.600 \cdot \text{deg}$$

$$\gamma_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{32} = 41.699 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\gamma_3 := \gamma_{32}$$

$$\gamma_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1}\right)$$

$$\gamma_{21} = 31.159 \cdot \text{deg}$$

$$\gamma_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1}\right)$$

$$\gamma_{22} = 31.159 \cdot \text{deg}$$

Since both values are the same ,

$$\gamma_2 := \gamma_{21}$$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2}$$

$$p_{21} = 0.907$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y})$$

$$\delta_2 = 180.000 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2}$$

$$p_{31} = 1.447$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y})$$

$$\delta_3 = 180.000 \cdot \text{deg}$$

9. Evaluate terms in the $\mathbf{WZ}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1$$

$$B := \sin(\beta_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the $\mathbf{W}$ and $\mathbf{Z}$ vectors are:

$$W1x = 1.943$$

$$W1y = 1.529$$

$$Z1x = -0.155$$

$$Z1y = 0.465$$

11. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2}$

$$w = 2.472$$

12. Evaluate terms in the $\mathbf{US}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$U1x = -0.395$$

$$U1y = 2.460$$

$$S1x = 0.183$$

$$S1y = -0.466$$

14. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2}$ $u = 2.491$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x$$

$$V1x = -0.338$$

$$V1y := Z1y - S1y$$

$$V1y = 0.931$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$ $v = 0.991$

$$G1x := W1x + V1x - U1x$$

$$G1x = 2.000$$

$$G1y := W1y + V1y - U1y$$

$$G1y = 0.000$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$ $g = 2.000$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors **W**₁, **Z**₁, **U**₁, and **S**₁.

$$O2x := -Z1x - W1x$$

$$O2x = -1.788$$

$$O2y := -Z1y - W1y$$

$$O2y = -1.994$$

$$O4x := -S1x - U1x$$

$$O4x = 0.212$$

$$O4y := -S1y - U1y$$

$$O4y = -1.994$$

These check with Figure P5-3.

17. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $z := \sqrt{Z1x^2 + Z1y^2}$ $z = 0.491$ $r_P := z$

Angle BAP (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 0.501$

$$\psi := \text{atan2}(S1x, S1y) \quad \psi = -68.558 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z1x, Z1y) \quad \phi = 108.446 \cdot \text{deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = 109.959 \cdot \text{deg}$$

$$\delta_p := \phi - \theta_3$$

$$\delta_p = -1.513 \cdot \text{deg}$$

18. DESIGN SUMMARY

$$\text{Link 1:} \quad g = 2.000$$

$$\text{Link 2:} \quad w = 2.472$$

$$\text{Link 3:} \quad v = 0.991$$

$$\text{Link 4:} \quad u = 2.491$$

$$\text{Coupler point:} \quad r_P = 0.491 \quad \delta_p = -1.513 \cdot \text{deg}$$

19. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-20

Statement: Write a program to generate and plot the circle-point and center-point circles for Problem 5-19 using an equation solver or any program language.

Given:

$$\begin{array}{llll}
 P_{21x} := -0.907 & P_{21y} := 0.0 & P_{31x} := -1.447 & P_{31y} := 0.0 \\
 O_{2x} := -1.788 & O_{2y} := -1.994 & O_{4x} := 0.212 & O_{4y} := -1.994 \\
 \text{Body angles:} & \theta_{p1} := 111.8 \cdot \text{deg} & \theta_{p2} := 191.1 \cdot \text{deg} & \theta_{p3} := 237.4 \cdot \text{deg}
 \end{array}$$

Assumptions: Let the position 1 to position 2 rotation angles be: $\beta_2 := 18.241 \cdot \text{deg}$ and $\gamma_2 := 31.159 \cdot \text{deg}$

Let the position 1 to position 2 coupler rotation angle be: $\alpha_2 := 79.3 \cdot \text{deg}$

Solution: See Figure P5-3 and Mathcad file P0520.

1. Use the method of Section 5.6 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$\begin{array}{ll}
 p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} & p_{21} = 0.907 \\
 \delta_2 := \text{atan2}(P_{21x}, P_{21y}) & \delta_2 = 180.000 \cdot \text{deg} \\
 p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} & p_{31} = 1.447 \\
 \delta_3 := \text{atan2}(P_{31x}, P_{31y}) & \delta_3 = 180.000 \cdot \text{deg}
 \end{array}$$

2. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\begin{array}{lll}
 \alpha_3 := 125.6 \cdot \text{deg} & \beta_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg} & \\
 A := \cos(\beta_2) - 1 & B := \sin(\beta_2) & C := \cos(\alpha_2) - 1 \\
 D := \sin(\alpha_2) & E := p_{21} \cdot \cos(\delta_2) & F(\beta_3) := \cos(\beta_3) - 1 \\
 G(\beta_3) := \sin(\beta_3) & H(\alpha_3) := \cos(\alpha_3) - 1 & K(\alpha_3) := \sin(\alpha_3) \\
 L := p_{31} \cdot \cos(\delta_3) & M := p_{21} \cdot \sin(\delta_2) & N := p_{31} \cdot \sin(\delta_3)
 \end{array}$$

$$AA(\alpha_3, \beta_3) := \begin{pmatrix} A & -B & C & -D \\ F(\beta_3) & -G(\beta_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\beta_3) & F(\beta_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \beta_3) := AA(\alpha_3, \beta_3)^{-1} \cdot CC$$

$$W1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_1$$

$$W1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_2$$

$$Z1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_3$$

$$Z1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_4$$

3. Check this against the solutions in Problem 5-19:

$$W1x(\alpha_3, 37.070 \cdot \text{deg}) = 1.943$$

$$W1y(\alpha_3, 37.070 \cdot \text{deg}) = 1.529$$

$$Z1x(\alpha_3, 37.070 \cdot \text{deg}) = -0.155$$

$$Z1y(\alpha_3, 37.070 \cdot \text{deg}) = 0.465$$

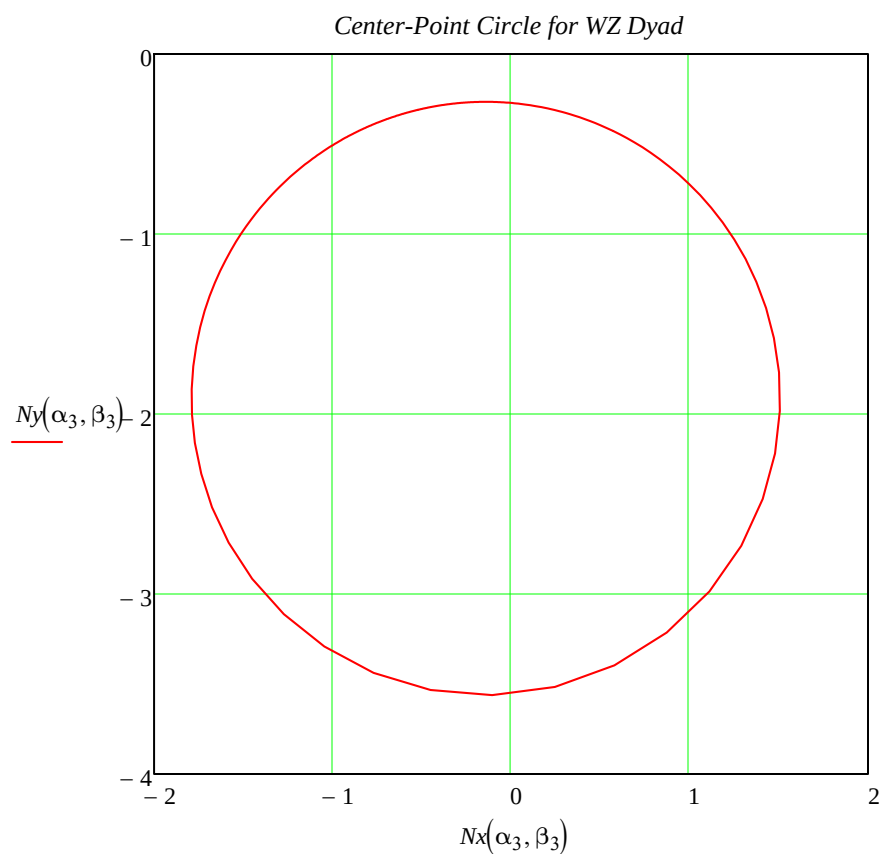
These are the same as the values calculated in Problem 5-19.

4. Form the vector **N**, whose tip describes the center-point circle for the **WZ** dyad.

$$Nx(\alpha_3, \beta_3) := -W1x(\alpha_3, \beta_3) - Z1x(\alpha_3, \beta_3)$$

$$Ny(\alpha_3, \beta_3) := -W1y(\alpha_3, \beta_3) - Z1y(\alpha_3, \beta_3)$$

5. Plot the center-point circle for the **WZ** dyad.



4. Form the vector **Z**, whose tip describes the center-point circle for the **WZ** dyad.

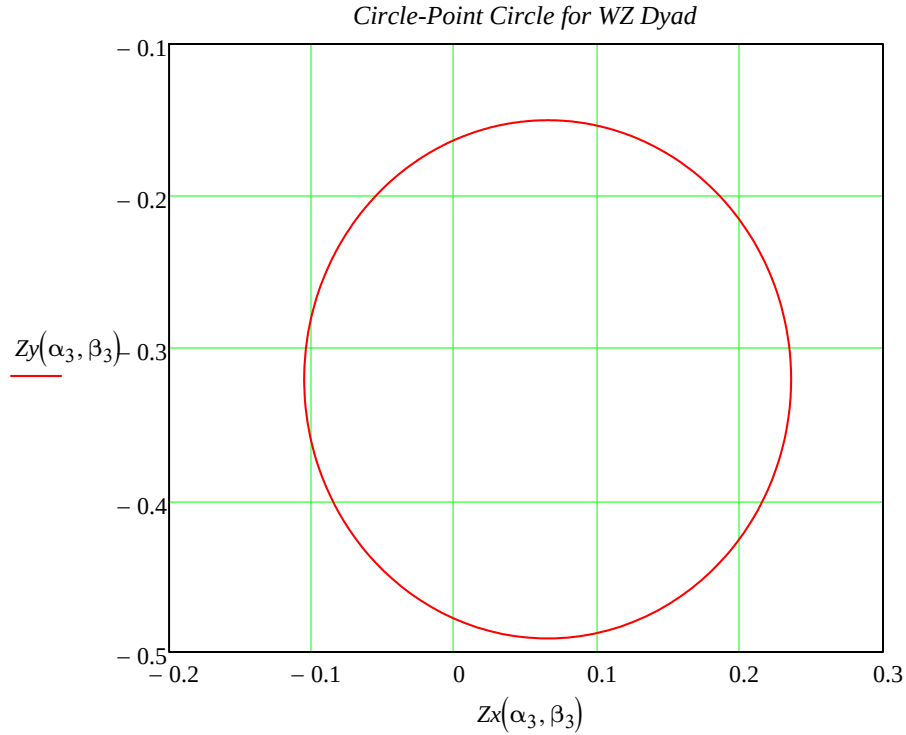
$$\beta_3 := 37.070 \cdot \text{deg}$$

$$\alpha_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

$$Zx(\alpha_3, \beta_3) := -Z1x(\alpha_3, \beta_3)$$

$$Zy(\alpha_3, \beta_3) := -Z1y(\alpha_3, \beta_3)$$

5. Plot the circle-point circle for the **WZ** dyad (see next page).



6. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.31) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\alpha_3 := 125.6 \cdot \text{deg}$$

$$\gamma_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

$$A := \cos(\gamma_2) - 1$$

$$B := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F(\gamma_3) := \cos(\gamma_3) - 1$$

$$G(\gamma_3) := \sin(\gamma_3)$$

$$H(\alpha_3) := \cos(\alpha_3) - 1$$

$$K(\alpha_3) := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA(\alpha_3, \gamma_3) := \begin{pmatrix} A & -B & C & -D \\ F(\gamma_3) & -G(\gamma_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\gamma_3) & F(\gamma_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \gamma_3) := AA(\alpha_3, \gamma_3)^{-1} \cdot CC$$

$$U1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_1$$

$$U1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_2$$

$$S1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_3$$

$$S1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_4$$

7. Check this against the solutions in Problem 5-19:

$$U1x(\alpha_3, 41.699 \cdot \text{deg}) = -0.395$$

$$U1y(\alpha_3, 41.699 \cdot \text{deg}) = 2.460$$

$$S1x(\alpha_3, 41.699\text{-deg}) = 0.183$$

$$S1y(\alpha_3, 41.699\text{-deg}) = -0.466$$

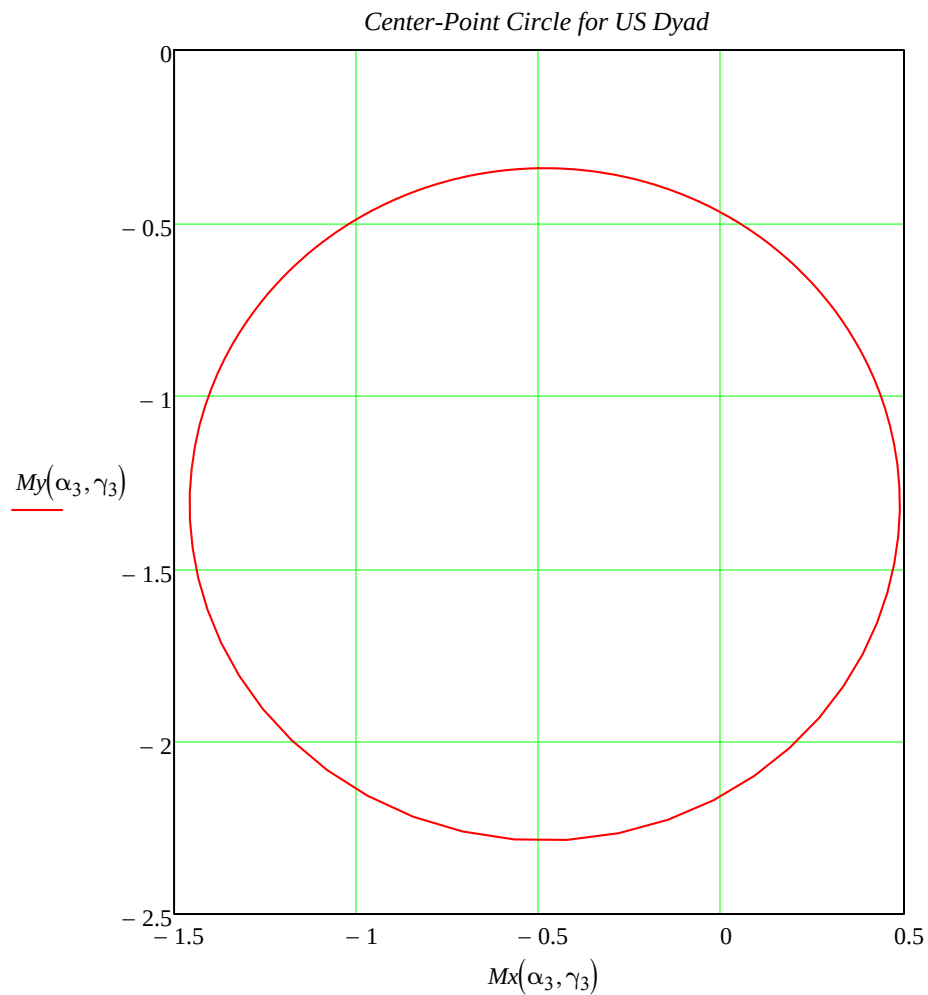
These are the same as the values calculated in Problem 5-19.

8. Form the vector **M**, whose tip describes the center-point circle for the **US** dyad.

$$Mx(\alpha_3, \gamma_3) := -U1x(\alpha_3, \gamma_3) - S1x(\alpha_3, \gamma_3)$$

$$My(\alpha_3, \gamma_3) := -U1y(\alpha_3, \gamma_3) - S1y(\alpha_3, \gamma_3)$$

9. Plot the center-point circle for the **US** dyad.



10. Form the vector **S**, whose tip describes the center-point circle for the **US** dyad.

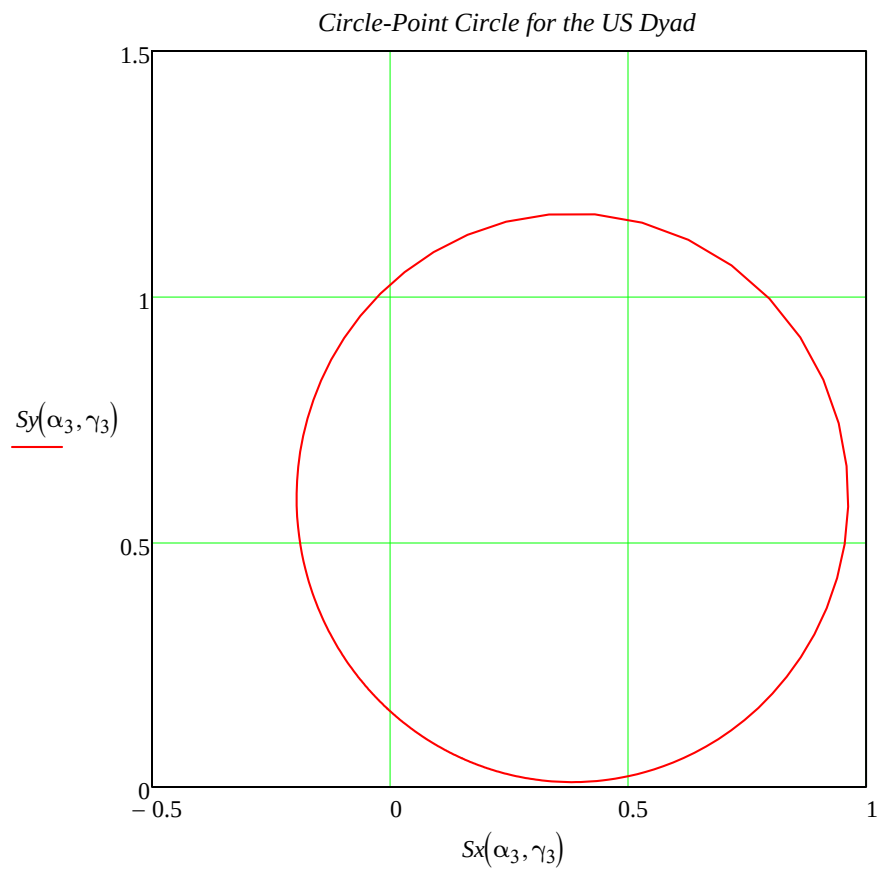
$$\gamma_3 := 41.699\text{-deg}$$

$$\alpha_3 := 0\text{-deg}, 1\text{-deg} \dots 360\text{-deg}$$

$$Sx(\alpha_3, \gamma_3) := -S1x(\alpha_3, \gamma_3)$$

$$Sy(\alpha_3, \gamma_3) := -S1y(\alpha_3, \gamma_3)$$

11. Plot the circle-point circle for the **WZ** dyad (see next page).



PROBLEM 5-21

Statement: Design a fourbar linkage to carry the box in Figure P5-4 from position 1 to 2 without regard for the fixed pivots shown. Use points *A* and *B* for your attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1 and P_2 with respect to P_1 :

$$P_{1x} := 0.0 \quad P_{1y} := 0.0 \quad P_{2x} := 184.0 \quad P_{2y} := -17.0$$

Angles made by the body in positions 1 and 2:

$$\theta_{p1} := 90.0 \cdot \text{deg} \quad \theta_{p2} := 45.0 \cdot \text{deg}$$

Coordinates of the points A_1 and B_1 with respect to P_1 :

$$A_{1x} := 17.0 \quad A_{1y} := -43.0 \quad B_{1x} := 69.0 \quad B_{1y} := -43.0$$

Free choice for the **WZ** dyad : $\beta_2 := -44.0 \cdot \text{deg}$

Free choice for the **US** dyad : $\gamma_2 := -55.0 \cdot \text{deg}$

Solution: See Figure P5-4 and Mathcad file P0521.

- Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$\mathbf{R}_1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \quad \mathbf{R}_2 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := \mathbf{R}_2 - \mathbf{R}_1 \quad P_{21x} = 184.000$$

$$P_{21y} = -17.000$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 184.784$$

- From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\alpha_2 := \theta_{p2} - \theta_{p1} \quad \alpha_2 = -45.000 \cdot \text{deg}$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = -5.279 \cdot \text{deg}$$

- Using Figure P5-4, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$z := \sqrt{(P_{1x} - A_{1x})^2 + (P_{1y} - A_{1y})^2} \quad z = 46.239$$

$$s := \sqrt{(P_{1x} - B_{1x})^2 + (P_{1y} - B_{1y})^2} \quad s = 81.302$$

$$v := \sqrt{(A_{1x} - B_{1x})^2 + (A_{1y} - B_{1y})^2} \quad v = 52.000$$

$$\phi := \text{acos}\left(\frac{v^2 + z^2 - s^2}{2 \cdot v \cdot z}\right) \quad \phi = 111.571 \cdot \text{deg}$$

$$\psi := \pi - \text{acos}\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) \quad \psi = 148.069 \cdot \text{deg}$$

- Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = -17.000 \quad Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = 43.000$$

$$A := \cos(\beta_2) - 1 \quad A = -0.281 \quad D := \sin(\alpha_2) \quad D = -0.707$$

$$B := \sin(\beta_2) \quad B = -0.695 \quad E := p_{21} \cdot \cos(\delta_2) \quad E = 184.000$$

$$C := \cos(\alpha_2) - 1 \quad C = -0.293 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = -17.000$$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \quad W_{1x} = -53.979$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \quad W_{1y} = 192.131$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \quad w = 199.570$$

$$\theta := \text{atan2}(W_{1x}, W_{1y}) \quad \theta = 105.693 \cdot \text{deg}$$

6. Solve for the **US** dyad using equations 5.12.

$$S_{1x} := s \cdot \cos(\psi) \quad S_{1x} = -69.000 \quad S_{1y} := s \cdot \sin(\psi) \quad S_{1y} = 43.000$$

$$A := \cos(\gamma_2) - 1 \quad A = -0.426 \quad D := \sin(\alpha_2) \quad D = -0.707$$

$$B := \sin(\gamma_2) \quad B = -0.819 \quad E := p_{21} \cdot \cos(\delta_2) \quad E = 184.000$$

$$C := \cos(\alpha_2) - 1 \quad C = -0.293 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = -17.000$$

$$U_{1x} := \frac{A \cdot (-C \cdot S_{1x} + D \cdot S_{1y} + E) + B \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F)}{-2 \cdot A} \quad U_{1x} = -15.598$$

$$U_{1y} := \frac{A \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F) + B \cdot (C \cdot S_{1x} - D \cdot S_{1y} - E)}{-2 \cdot A} \quad U_{1y} = 154.713$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2} \quad u = 155.497$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y}) \quad \sigma = 95.757 \cdot \text{deg}$$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := z \cdot \cos(\phi) - s \cdot \cos(\psi) \quad V_{1x} = 52.000$

$$V_{1y} := z \cdot \sin(\phi) - s \cdot \sin(\psi) \quad V_{1y} = 0.000$$

$$\theta_3 := \text{atan2}(V_{1x}, V_{1y}) \quad \theta_3 = 0.000 \cdot \text{deg}$$

$$v := \sqrt{V_{1x}^2 + V_{1y}^2} \quad v = 52.000$$

Link 1: $G_{1x} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma) \quad G_{1x} = 13.619$

$$G_{1y} := w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma) \quad G_{1y} = 37.418$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y}) \quad \theta_1 = 70.000 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2} \quad g = 39.819$$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \quad \theta_{2i} = 35.692 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_2 \quad \theta_{2f} = -8.308 \cdot \text{deg}$$

9. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z \quad \delta_p := \phi - \theta_3$$

$$r_p = 46.239 \quad \delta_p = 111.571 \cdot \text{deg}$$

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := 0 \cdot \text{deg} \quad \rho_1 = 0.000 \cdot \text{deg}$$

$$R_1 := \sqrt{P_{1x}^2 + P_{1y}^2} \quad R_1 = 0.000$$

$$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta) \quad O_{2x} = 70.979$$

$$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta) \quad O_{2y} = -235.131$$

$$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma) \quad O_{4x} = 84.598$$

$$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma) \quad O_{4y} = -197.713$$

These fixed pivot points fall on the base and are, therefore, acceptable.

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = 70.000 \cdot \text{deg}$$

12. Determine the Grashof condition.

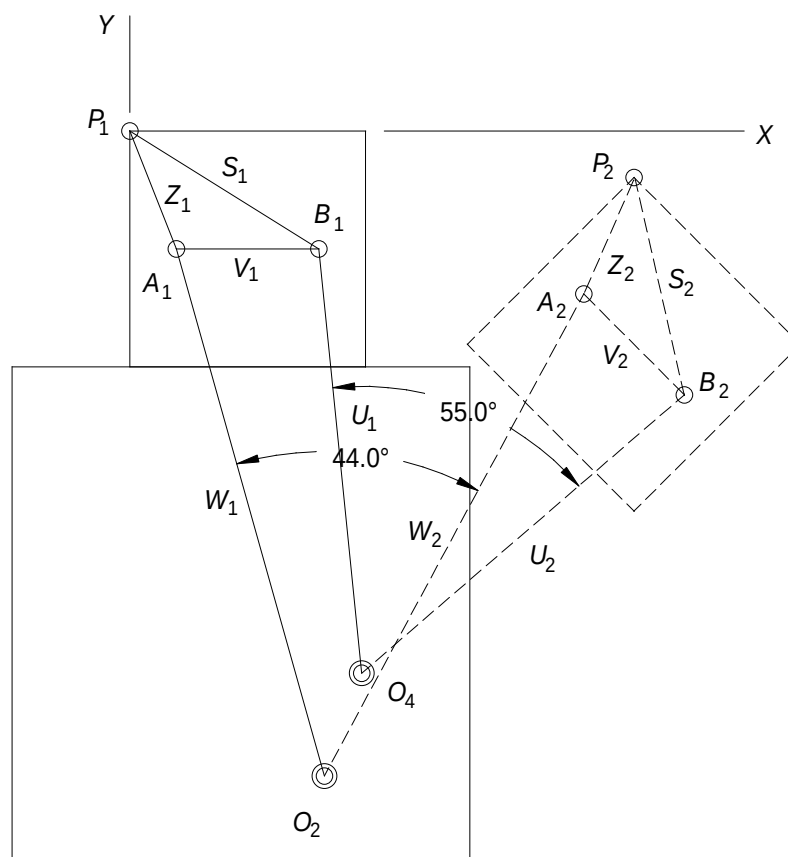
$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

13. DESIGN SUMMARY

Link 2:	$w = 199.570$	$\theta = 105.693 \cdot \text{deg}$
Link 3:	$v = 52.000$	$\theta_3 = 0.000 \cdot \text{deg}$
Link 4:	$u = 155.497$	$\sigma = 95.757 \cdot \text{deg}$
Link 1:	$g = 39.819$	$\theta_1 = 70.000 \cdot \text{deg}$
Coupler:	$r_p = 46.239$	$\delta_p = 111.571 \cdot \text{deg}$
Crank angles:	$\theta_{2i} = 35.692 \cdot \text{deg}$	$\theta_{2f} = -8.308 \cdot \text{deg}$

14. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-22

Statement: Design a fourbar linkage to carry the box in Figure P5-4 from position 1 to 3 without regard for the fixed pivots shown. Use points A and B for your attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1 and P_3 with respect to P_1 :

$$P_{1x} := 0.0 \quad P_{1y} := 0.0 \quad P_{3x} := 211.0 \quad P_{3y} := -180.0$$

Angles made by the body in positions 1 and 3:

$$\theta_{p1} := 90.0 \cdot \text{deg} \quad \theta_{p3} := 0.0 \cdot \text{deg}$$

Coordinates of the points A_1 and B_1 with respect to P_1 :

$$A_{1x} := 17.0 \quad A_{1y} := -43.0 \quad B_{1x} := 69.0 \quad B_{1y} := -43.0$$

Free choice for the **WZ** dyad : $\beta_2 := -70.0 \cdot \text{deg}$

Free choice for the **US** dyad : $\gamma_2 := -95.0 \cdot \text{deg}$

Solution: See Figure P5-4 and Mathcad file P0522.

- Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$\mathbf{R}_1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \quad \mathbf{R}_2 := \begin{pmatrix} P_{3x} \\ P_{3y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := \mathbf{R}_2 - \mathbf{R}_1 \quad \begin{array}{l} P_{21x} = 211.000 \\ P_{21y} = -180.000 \end{array}$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 277.346$$

- From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\alpha_2 := \theta_{p3} - \theta_{p1} \quad \alpha_2 = -90.000 \cdot \text{deg}$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = -40.467 \cdot \text{deg}$$

- Using Figure P5-4, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$z := \sqrt{(P_{1x} - A_{1x})^2 + (P_{1y} - A_{1y})^2} \quad z = 46.239$$

$$s := \sqrt{(P_{1x} - B_{1x})^2 + (P_{1y} - B_{1y})^2} \quad s = 81.302$$

$$v := \sqrt{(A_{1x} - B_{1x})^2 + (A_{1y} - B_{1y})^2} \quad v = 52.000$$

$$\phi := \text{acos}\left(\frac{v^2 + z^2 - s^2}{2 \cdot v \cdot z}\right) \quad \phi = 111.571 \cdot \text{deg}$$

$$\psi := \pi - \text{acos}\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) \quad \psi = 148.069 \cdot \text{deg}$$

- Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = -17.000 \quad Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = 43.000$$

$$\begin{aligned}
 A &:= \cos(\beta_2) - 1 & A &= -0.658 & D &:= \sin(\alpha_2) & D &= -1.000 \\
 B &:= \sin(\beta_2) & B &= -0.940 & E &:= p_{21} \cdot \cos(\delta_2) & E &= 211.000 \\
 C &:= \cos(\alpha_2) - 1 & C &= -1.000 & F &:= p_{21} \cdot \sin(\delta_2) & F &= -180.000 \\
 W_{1x} &:= \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} & W_{1x} &= 34.467 \\
 W_{1y} &:= \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} & W_{1y} &= 184.825 \\
 w &:= \sqrt{W_{1x}^2 + W_{1y}^2} & w &= 188.012 \\
 \theta &:= \text{atan2}(W_{1x}, W_{1y}) & \theta &= 79.436 \cdot \text{deg}
 \end{aligned}$$

6. Solve for the **US** dyad using equations 5.12.

$$\begin{aligned}
 S_{1x} &:= s \cdot \cos(\psi) & S_{1x} &= -69.000 & S_{1y} &:= s \cdot \sin(\psi) & S_{1y} &= 43.000 \\
 A &:= \cos(\gamma_2) - 1 & A &= -1.087 & D &:= \sin(\alpha_2) & D &= -1.000 \\
 B &:= \sin(\gamma_2) & B &= -0.996 & E &:= p_{21} \cdot \cos(\delta_2) & E &= 211.000 \\
 C &:= \cos(\alpha_2) - 1 & C &= -1.000 & F &:= p_{21} \cdot \sin(\delta_2) & F &= -180.000 \\
 U_{1x} &:= \frac{A \cdot (-C \cdot S_{1x} + D \cdot S_{1y} + E) + B \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F)}{-2 \cdot A} & U_{1x} &= 44.882 \\
 U_{1y} &:= \frac{A \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F) + B \cdot (C \cdot S_{1x} - D \cdot S_{1y} - E)}{-2 \cdot A} & U_{1y} &= 148.358 \\
 u &:= \sqrt{U_{1x}^2 + U_{1y}^2} & u &= 154.999 \\
 \sigma &:= \text{atan2}(U_{1x}, U_{1y}) & \sigma &= 73.168 \cdot \text{deg}
 \end{aligned}$$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\begin{aligned}
 \text{Link 3: } V_{1x} &:= z \cdot \cos(\phi) - s \cdot \cos(\psi) & V_{1x} &= 52.000 \\
 V_{1y} &:= z \cdot \sin(\phi) - s \cdot \sin(\psi) & V_{1y} &= 0.000 \\
 \theta_3 &:= \text{atan2}(V_{1x}, V_{1y}) & \theta_3 &= 0.000 \cdot \text{deg} \\
 v &:= \sqrt{V_{1x}^2 + V_{1y}^2} & v &= 52.000 \\
 \text{Link 1: } G_{1x} &:= w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma) & G_{1x} &= 41.585 \\
 G_{1y} &:= w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma) & G_{1y} &= 36.467 \\
 \theta_1 &:= \text{atan2}(G_{1x}, G_{1y}) & \theta_1 &= 41.248 \cdot \text{deg}
 \end{aligned}$$

$$g := \sqrt{G_{Ix}^2 + G_{Iy}^2} \quad g = 55.310$$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \quad \theta_{2i} = 38.188 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_2 \quad \theta_{2f} = -31.812 \cdot \text{deg}$$

9. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z \quad \delta_p := \phi - \theta_3$$

$$r_p = 46.239 \quad \delta_p = 111.571 \cdot \text{deg}$$

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := 0 \cdot \text{deg} \quad \rho_1 = 0.000 \cdot \text{deg}$$

$$R_1 := \sqrt{P_{1x}^2 + P_{1y}^2} \quad R_1 = 0.000$$

$$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta) \quad O_{2x} = -17.467$$

$$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta) \quad O_{2y} = -227.825$$

$$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma) \quad O_{4x} = 24.118$$

$$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma) \quad O_{4y} = -191.358$$

These fixed pivot points fall on the base and are, therefore, acceptable.

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = 41.248 \cdot \text{deg}$$

12. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

13. DESIGN SUMMARY

$$\text{Link 2:} \quad w = 188.012 \quad \theta = 79.436 \cdot \text{deg}$$

$$\text{Link 3:} \quad v = 52.000 \quad \theta_3 = 0.000 \cdot \text{deg}$$

$$\text{Link 4:} \quad u = 154.999 \quad \sigma = 73.168 \cdot \text{deg}$$

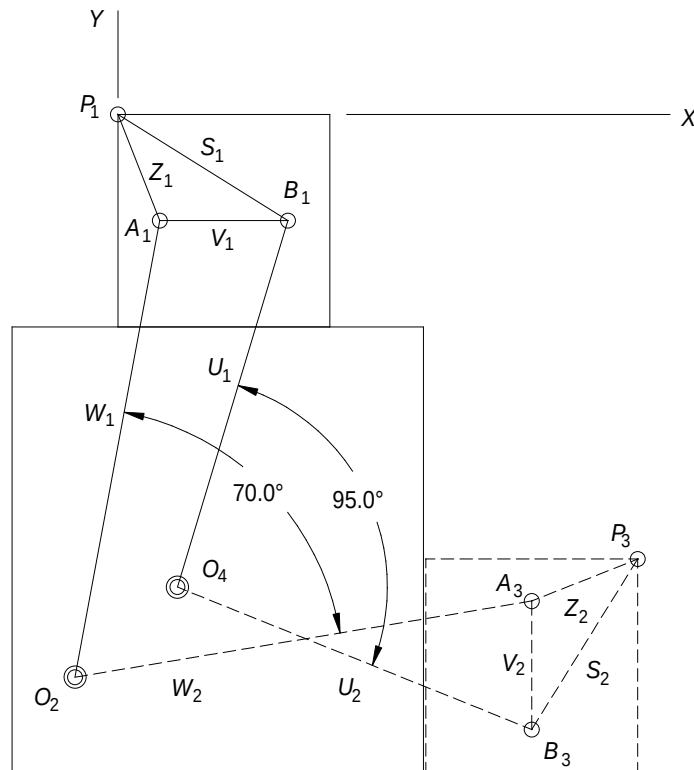
Link 1: $g = 55.310$ $\theta_1 = 41.248 \cdot \text{deg}$
 Coupler: $r_p = 46.239$ $\delta_p = 111.571 \cdot \text{deg}$

Crank angles:

$$\theta_{2i} = 38.188 \cdot \text{deg}$$

$$\theta_{2f} = -31.812 \cdot \text{deg}$$

14. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-23

Statement: Design a fourbar linkage to carry the box in Figure P5-4 from position 2 to 3 without regard for the fixed pivots shown. Use points *A* and *B* for your attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_2 and P_3 with respect to P_1 :

$$P_{2x} := 184.0 \quad P_{2y} := -17.0 \quad P_{3x} := 211.0 \quad P_{3y} := -180.0$$

Angles made by the body in positions 1 and 3:

$$\theta_{p2} := 45.0 \cdot \text{deg} \quad \theta_{p3} := 0.0 \cdot \text{deg}$$

Coordinates of the points A_1 and B_1 with respect to P_1 :

$$A_{1x} := 17.0 \quad A_{1y} := -43.0 \quad B_{1x} := 69.0 \quad B_{1y} := -43.0$$

Free choice for the **WZ** dyad : $\beta_2 := -60.0 \cdot \text{deg}$

Free choice for the **US** dyad : $\gamma_2 := -45.0 \cdot \text{deg}$

Solution: See Figure P5-4 and Mathcad file P0523.

- Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$\mathbf{R}_1 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \quad \mathbf{R}_2 := \begin{pmatrix} P_{3x} \\ P_{3y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := \mathbf{R}_2 - \mathbf{R}_1 \quad \begin{aligned} P_{21x} &= 27.000 \\ P_{21y} &= -163.000 \\ p_{21} &:= \sqrt{P_{21x}^2 + P_{21y}^2} = 165.221 \end{aligned}$$

- From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\begin{aligned} \alpha_2 &:= \theta_{p3} - \theta_{p2} & \alpha_2 &= -45.000 \cdot \text{deg} \\ \delta_2 &:= \text{atan2}(P_{21x}, P_{21y}) & \delta_2 &= -80.595 \cdot \text{deg} \end{aligned}$$

- Using Figure P5-4, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$\begin{aligned} z &:= \sqrt{(0.0 - A_{1x})^2 + (0.0 - A_{1y})^2} & z &= 46.239 \\ s &:= \sqrt{(0.0 - B_{1x})^2 + (0.0 - B_{1y})^2} & s &= 81.302 \\ v &:= \sqrt{(A_{1x} - B_{1x})^2 + (A_{1y} - B_{1y})^2} & v &= 52.000 \\ \phi &:= \text{acos}\left(\frac{v^2 + z^2 - s^2}{2 \cdot v \cdot z}\right) - 45 \cdot \text{deg} & \phi &= 66.571 \cdot \text{deg} \\ \psi &:= \pi - \text{acos}\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) - 45 \cdot \text{deg} & \psi &= 103.069 \cdot \text{deg} \end{aligned}$$

- Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = 18.385 \quad Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = 42.426$$

$$\begin{aligned}
 A &:= \cos(\beta_2) - 1 & A &= -0.500 & D &:= \sin(\alpha_2) & D &= -0.707 \\
 B &:= \sin(\beta_2) & B &= -0.866 & E &:= p_{21} \cdot \cos(\delta_2) & E &= 27.000 \\
 C &:= \cos(\alpha_2) - 1 & C &= -0.293 & F &:= p_{21} \cdot \sin(\delta_2) & F &= -163.000 \\
 W_{Ix} &:= \frac{A \cdot (-C \cdot Z_{Ix} + D \cdot Z_{Iy} + E) + B \cdot (-C \cdot Z_{Iy} - D \cdot Z_{Ix} + F)}{-2 \cdot A} & W_{Ix} &= 117.950 \\
 W_{Iy} &:= \frac{A \cdot (-C \cdot Z_{Iy} - D \cdot Z_{Ix} + F) + B \cdot (C \cdot Z_{Ix} - D \cdot Z_{Iy} - E)}{-2 \cdot A} & W_{Iy} &= 70.852 \\
 w &:= \sqrt{W_{Ix}^2 + W_{Iy}^2} & w &= 137.594 \\
 \theta &:= \text{atan2}(W_{Ix}, W_{Iy}) & \theta &= 30.993 \cdot \text{deg}
 \end{aligned}$$

6. Solve for the **US** dyad using equations 5.12.

$$\begin{aligned}
 S_{Ix} &:= s \cdot \cos(\psi) & S_{Ix} &= -18.385 & S_{Iy} &:= s \cdot \sin(\psi) & S_{Iy} &= 79.196 \\
 A &:= \cos(\gamma_2) - 1 & A &= -0.293 & D &:= \sin(\alpha_2) & D &= -0.707 \\
 B &:= \sin(\gamma_2) & B &= -0.707 & E &:= p_{21} \cdot \cos(\delta_2) & E &= 27.000 \\
 C &:= \cos(\alpha_2) - 1 & C &= -0.293 & F &:= p_{21} \cdot \sin(\delta_2) & F &= -163.000 \\
 U_{Ix} &:= \frac{A \cdot (-C \cdot S_{Ix} + D \cdot S_{Iy} + E) + B \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F)}{-2 \cdot A} & U_{Ix} &= 201.643 \\
 U_{Iy} &:= \frac{A \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F) + B \cdot (C \cdot S_{Ix} - D \cdot S_{Iy} - E)}{-2 \cdot A} & U_{Iy} &= 34.896 \\
 u &:= \sqrt{U_{Ix}^2 + U_{Iy}^2} & u &= 204.640 \\
 \sigma &:= \text{atan2}(U_{Ix}, U_{Iy}) & \sigma &= 9.818 \cdot \text{deg}
 \end{aligned}$$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\begin{aligned}
 \text{Link 3: } V_{Ix} &:= z \cdot \cos(\phi) - s \cdot \cos(\psi) & V_{Ix} &= 36.770 \\
 V_{Iy} &:= z \cdot \sin(\phi) - s \cdot \sin(\psi) & V_{Iy} &= -36.770 \\
 \theta_3 &:= \text{atan2}(V_{Ix}, V_{Iy}) & \theta_3 &= -45.000 \cdot \text{deg} \\
 v &:= \sqrt{V_{Ix}^2 + V_{Iy}^2} & v &= 52.000 \\
 \text{Link 1: } G_{Ix} &:= w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma) & G_{Ix} &= -46.924 \\
 G_{Iy} &:= w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma) & G_{Iy} &= -0.813
 \end{aligned}$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y})$$

$$\theta_1 = -179.007 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2}$$

$$g = 46.931$$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1$$

$$\theta_{2i} = 210.000 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_2$$

$$\theta_{2f} = 150.000 \cdot \text{deg}$$

9. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 46.239$$

$$\delta_p = 111.571 \cdot \text{deg}$$

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := \text{atan2}(P_{2x}, P_{2y})$$

$$\rho_1 = -5.279 \cdot \text{deg}$$

$$R_1 := \sqrt{P_{2x}^2 + P_{2y}^2}$$

$$R_1 = 184.784$$

$$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = 47.665$$

$$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = -130.278$$

$$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = 0.742$$

$$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = -131.092$$

These fixed pivot points fall on the base and are, therefore, acceptable.

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$$

$$\theta_{rot} = -179.007 \cdot \text{deg}$$

12. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

13. DESIGN SUMMARY

$$\text{Link 2: } w = 137.594$$

$$\theta = 30.993 \cdot \text{deg}$$

$$\text{Link 3: } v = 52.000$$

$$\theta_3 = -45.000 \cdot \text{deg}$$

$$\text{Link 4: } u = 204.640$$

$$\sigma = 9.818 \cdot \text{deg}$$

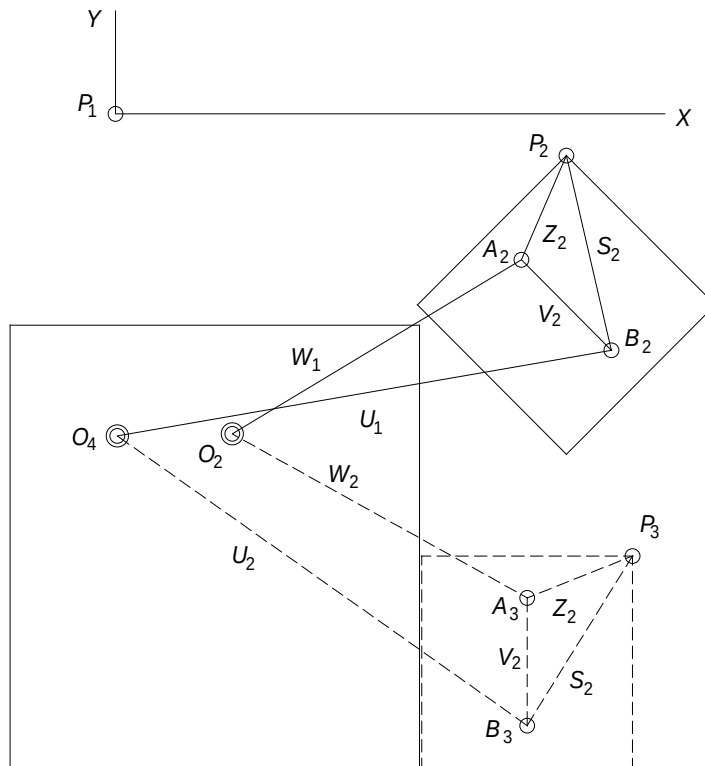
Link 1: $g = 46.931$ $\theta_1 = -179.007 \cdot \text{deg}$

Coupler: $r_p = 46.239$ $\delta_p = 111.571 \cdot \text{deg}$

Crank angles: $\theta_{2i} = 210.000 \cdot \text{deg}$

$\theta_{2f} = 150.000 \cdot \text{deg}$

14. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-24

Statement: Design a fourbar linkage to carry the box in Figure P5-4 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use any points on the object as attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to P_1 :

$$P_{1x} := 0.0 \quad P_{1y} := 0.0 \quad P_{2x} := 184.0 \quad P_{2y} := -17.0$$

$$P_{3x} := 211.0 \quad P_{3y} := -180.0$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 90.0 \cdot \text{deg} \quad \theta_{p2} := 45.0 \cdot \text{deg} \quad \theta_{p3} := 0.0 \cdot \text{deg}$$

Free choices for the **WZ** dyad :

$$\beta_2 := -80.0 \cdot \text{deg} \quad \beta_3 := -160.0 \cdot \text{deg}$$

Free choices for the **US** dyad :

$$\gamma_2 := -80.0 \cdot \text{deg} \quad \gamma_3 := -170.0 \cdot \text{deg}$$

Solution: See Figure P5-4 and Mathcad file P0524.

- Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 184.784 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= -5.279 \cdot \text{deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 277.346 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= -40.467 \cdot \text{deg} \end{aligned}$$

- Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -45.000 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -90.000 \cdot \text{deg} \end{aligned}$$

- Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$\begin{aligned} A &:= \cos(\beta_2) - 1 & B &:= \sin(\beta_2) & C &:= \cos(\alpha_2) - 1 \\ D &:= \sin(\alpha_2) & E &:= p_{21} \cdot \cos(\delta_2) & F &:= \cos(\beta_3) - 1 \\ G &:= \sin(\beta_3) & H &:= \cos(\alpha_3) - 1 & K &:= \sin(\alpha_3) \\ L &:= p_{31} \cdot \cos(\delta_3) & M &:= p_{21} \cdot \sin(\delta_2) & N &:= p_{31} \cdot \sin(\delta_3) \end{aligned}$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$$\begin{aligned} W_{1x} &= -51.854 & W_{1y} &= 109.176 & Z_{1x} &= -43.555 & Z_{1y} &= 29.523 \\ \theta &:= \text{atan2}(W_{1x}, W_{1y}) & \theta &= 115.406 \cdot \text{deg} & \phi &:= \text{atan2}(Z_{1x}, Z_{1y}) & \phi &= 145.869 \cdot \text{deg} \end{aligned}$$

The length of link 2 is: $w := \sqrt{W_{1x}^2 + W_{1y}^2}$, $w = 120.864$

The length of vector **Z** is: $z := \sqrt{(Z_{1x}^2 + Z_{1y}^2)}$, $z = 52.618$

4. Evaluate terms in the **US** coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$$\begin{array}{lll} A' := \cos(\gamma_2) - 1 & B' := \sin(\gamma_2) & C := \cos(\alpha_2) - 1 \\ D := \sin(\alpha_2) & E := p_{21} \cdot \cos(\delta_2) & F' := \cos(\gamma_3) - 1 \\ G' := \sin(\gamma_3) & H := \cos(\alpha_3) - 1 & K := \sin(\alpha_3) \\ L := p_{31} \cdot \cos(\delta_3) & M := p_{21} \cdot \sin(\delta_2) & N := p_{31} \cdot \sin(\delta_3) \end{array}$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$$\begin{array}{llll} U_{1x} = -35.056 & U_{1y} = 94.023 & S_{1x} = -62.812 & S_{1y} = 62.282 \\ \sigma := \text{atan2}(U_{1x}, U_{1y}) & \sigma = 110.448 \cdot \text{deg} & \psi := \text{atan2}(S_{1x}, S_{1y}) & \psi = 135.242 \cdot \text{deg} \end{array}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 100.345$

The length of vector **S** is: $s := \sqrt{(S_{1x}^2 + S_{1y}^2)}$, $s = 88.456$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\begin{array}{ll} \text{Link 3: } V_{1x} := Z_{1x} - S_{1x} & V_{1x} = 19.256 \\ V_{1y} := Z_{1y} - S_{1y} & V_{1y} = -32.759 \\ \theta_3 := \text{atan2}(V_{1x}, V_{1y}) & \theta_3 = -59.552 \cdot \text{deg} \\ v := \sqrt{V_{1x}^2 + V_{1y}^2} & v = 38.000 \\ \text{Link 1: } G_{1x} := W_{1x} + V_{1x} - U_{1x} & G_{1x} = 2.458 \\ G_{1y} := W_{1y} + V_{1y} - U_{1y} & G_{1y} = -17.606 \\ \theta_1 := \text{atan2}(G_{1x}, G_{1y}) & \theta_1 = -82.052 \cdot \text{deg} \\ g := \sqrt{G_{1x}^2 + G_{1y}^2} & g = 17.777 \end{array}$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\begin{array}{ll} \theta_{2i} := \theta - \theta_1 & \theta_{2i} = 197.458 \cdot \text{deg} \\ \theta_{2f} := \theta_{2i} + \beta_3 & \theta_{2f} = 37.458 \cdot \text{deg} \end{array}$$

8. Define the coupler point with respect to point A and the vector $\mathbf{V}$.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 52.618$$

$$\delta_p = 205.422 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = 95.410$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = -138.699$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = 97.868$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = -156.305$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$$

$$\theta_{rot} = -82.052 \cdot \text{deg}$$

11. Determine the Grashof condition.

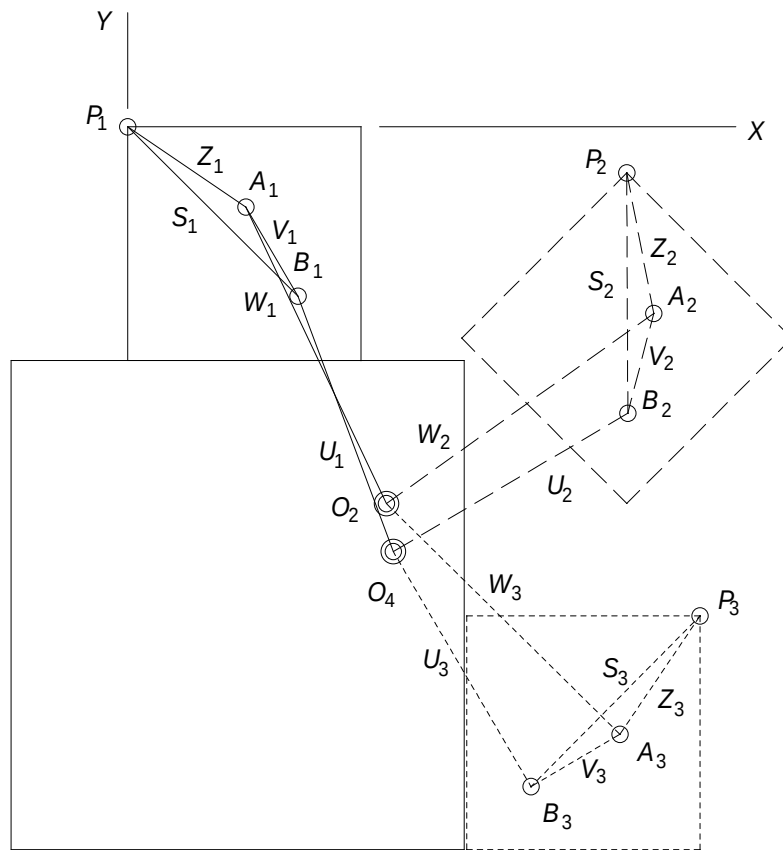
$$\begin{aligned} \text{Condition}(a, b, c, d) := & \left\{ \begin{array}{l} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} \quad \text{if } SL < PQ \\ \text{return "Special Grashof"} \quad \text{if } SL = PQ \\ \text{return "non-Grashof"} \quad \text{otherwise} \end{array} \right. \end{aligned}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

12. DESIGN SUMMARY

Link 2:	$w = 120.864$	$\theta = 115.406 \cdot \text{deg}$
Link 3:	$v = 38.000$	$\theta_3 = -59.552 \cdot \text{deg}$
Link 4:	$u = 100.345$	$\sigma = 110.448 \cdot \text{deg}$
Link 1:	$g = 17.777$	$\theta_1 = -82.052 \cdot \text{deg}$
Coupler:	$r_p = 52.618$	$\delta_p = 205.422 \cdot \text{deg}$
Crank angles:		
	$\theta_{2i} = 197.458 \cdot \text{deg}$	$\theta_{2f} = 37.458 \cdot \text{deg}$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-25

Statement: Design a fourbar linkage to carry the box in Figure P5-4 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use points A and B for your attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to P_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= 184.0 & P_{2y} &:= -17.0 \\ P_{3x} &:= 211.0 & P_{3y} &:= -180.0 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 90.0 \cdot \text{deg} \quad \theta_{p2} := 45.0 \cdot \text{deg} \quad \theta_{p3} := 0.0 \cdot \text{deg}$$

Coordinates of the points A_1 and B_1 with respect to P_1 :

$$A_{1x} := 17.0 \quad A_{1y} := -43.0 \quad B_{1x} := 69.0 \quad B_{1y} := -43.0$$

Solution: See Figure P5-4 and Mathcad file P0525.

1. Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 184.784 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= -5.279 \cdot \text{deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 277.346 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= -40.467 \cdot \text{deg} \end{aligned}$$

2. Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -45.000 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -90.000 \cdot \text{deg} \end{aligned}$$

3. Using Figure P5-4, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$\begin{aligned} z &:= \sqrt{(P_{1x} - A_{1x})^2 + (P_{1y} - A_{1y})^2} & z &= 46.239 \\ s &:= \sqrt{(P_{1x} - B_{1x})^2 + (P_{1y} - B_{1y})^2} & s &= 81.302 \\ v &:= \sqrt{(A_{1x} - B_{1x})^2 + (A_{1y} - B_{1y})^2} & v &= 52.000 \\ \phi &:= \text{acos}\left(\frac{v^2 + z^2 - s^2}{2 \cdot v \cdot z}\right) & \phi &= 111.571 \cdot \text{deg} \\ \psi &:= \pi - \text{acos}\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) & \psi &= 148.069 \cdot \text{deg} \\ Z_{1x} &:= z \cdot \cos(\phi) & Z_{1x} &= -17.000 \\ Z_{1y} &:= z \cdot \sin(\phi) & Z_{1y} &= 43.000 \\ S_{1x} &:= s \cdot \cos(\psi) & S_{1x} &= -69.000 \\ S_{1y} &:= s \cdot \sin(\psi) & S_{1y} &= 43.000 \end{aligned}$$

4. Use equations 5.24 to solve for w , θ , β_2 , and β_3 . Since the points A and B are to be used as pivots, z and ϕ are known from the calculations above.

$$\text{Guess:} \quad W_{1x} := -50 \quad W_{1y} := 200 \quad \beta_2 := -80 \cdot \text{deg} \quad \beta_3 := -160 \cdot \text{deg}$$

$$\text{Given} \quad W_{1x} \cdot (\cos(\beta_2) - 1) - W_{1y} \cdot \sin(\beta_2) \dots = p_{21} \cdot \cos(\delta_2) \\ + Z_{1x} \cdot (\cos(\alpha_2) - 1) - Z_{1y} \cdot \sin(\alpha_2)$$

$$W_{1x} \cdot (\cos(\beta_3) - 1) - W_{1y} \cdot \sin(\beta_3) \dots = p_{31} \cdot \cos(\delta_3) \\ + Z_{1x} \cdot (\cos(\alpha_3) - 1) - Z_{1y} \cdot \sin(\alpha_3)$$

$$W_{1y} \cdot (\cos(\beta_2) - 1) + W_{1x} \cdot \sin(\beta_2) \dots = p_{21} \cdot \sin(\delta_2) \\ + Z_{1y} \cdot (\cos(\alpha_2) - 1) + Z_{1x} \cdot \sin(\alpha_2)$$

$$W_{1y} \cdot (\cos(\beta_3) - 1) + W_{1x} \cdot \sin(\beta_3) \dots = p_{31} \cdot \sin(\delta_3) \\ + Z_{1y} \cdot (\cos(\alpha_3) - 1) + Z_{1x} \cdot \sin(\alpha_3)$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := \text{Find}(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$$\beta_2 = -86.887 \cdot \text{deg}$$

$$\beta_3 = -165.399 \cdot \text{deg}$$

The components of the **W** vector are:

$$W_{1x} = -65.636$$

$$W_{1y} = 86.672$$

$$\theta := \text{atan2}(W_{1x}, W_{1y})$$

$$\theta = 127.136 \cdot \text{deg}$$

$$\text{The length of link 2 is: } w := \sqrt{(W_{1x}^2 + W_{1y}^2)}, \quad w = 108.720$$

5. Use equations 5.28 to solve for u , σ , γ_2 , and γ_3 . Since the points *A* and *B* are to be used as pivots, s and ψ are known from the calculations above.

$$\text{Guess:} \quad U_{1x} := -30 \quad U_{1y} := 100 \quad \gamma_2 := -80 \cdot \text{deg} \quad \gamma_3 := -160 \cdot \text{deg}$$

$$\text{Given} \quad U_{1x} \cdot (\cos(\gamma_2) - 1) - U_{1y} \cdot \sin(\gamma_2) \dots = p_{21} \cdot \cos(\delta_2) \\ + S_{1x} \cdot (\cos(\alpha_2) - 1) - S_{1y} \cdot \sin(\alpha_2)$$

$$U_{1x} \cdot (\cos(\gamma_3) - 1) - U_{1y} \cdot \sin(\gamma_3) \dots = p_{31} \cdot \cos(\delta_3) \\ + S_{1x} \cdot (\cos(\alpha_3) - 1) - S_{1y} \cdot \sin(\alpha_3)$$

$$U_{1y} \cdot (\cos(\gamma_2) - 1) + U_{1x} \cdot \sin(\gamma_2) \dots = p_{21} \cdot \sin(\delta_2) \\ + S_{1y} \cdot (\cos(\alpha_2) - 1) + S_{1x} \cdot \sin(\alpha_2)$$

$$U_{1y} \cdot (\cos(\gamma_3) - 1) + U_{1x} \cdot \sin(\gamma_3) \dots = p_{31} \cdot \sin(\delta_3) \\ + S_{1y} \cdot (\cos(\alpha_3) - 1) + S_{1x} \cdot \sin(\alpha_3)$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := \text{Find}(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$$\gamma_2 = -76.700 \cdot \text{deg}$$

$$\gamma_3 = -161.878 \cdot \text{deg}$$

The components of the **U** vector are:

$$U_{1x} = -33.074$$

$$U_{1y} = 110.894$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y})$$

$$\sigma = 106.607 \cdot \text{deg}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 115.721$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$

$$V_{1x} = 52.000$$

$$V_{1y} := Z_{1y} - S_{1y}$$

$$V_{1y} = 0.000$$

$$\theta_3 := \text{atan2}(V_{1x}, V_{1y})$$

$$\theta_3 = 0.000 \cdot \text{deg}$$

$$v := \sqrt{V_{1x}^2 + V_{1y}^2}$$

$$v = 52.000$$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$

$$G_{1x} = 19.438$$

$$G_{1y} := W_{1y} + V_{1y} - U_{1y}$$

$$G_{1y} = -24.222$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y})$$

$$\theta_1 = -51.253 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2}$$

$$g = 31.057$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1$$

$$\theta_{2i} = 178.389 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_3$$

$$\theta_{2f} = 12.990 \cdot \text{deg}$$

8. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 46.239$$

$$\delta_p = 111.571 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = 82.636$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = -129.672$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = 102.074$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = -153.894$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis

to the line O_2O_4 .

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$$

$$\theta_{rot} = -51.253 \cdot \text{deg}$$

11. Determine the Grashof condition.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof"   if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Condition(g,u,v,w) = "Grashof"

12. DESIGN SUMMARY

Link 2:	$w = 108.720$	$\theta = 127.136 \cdot \text{deg}$
Link 3:	$v = 52.000$	$\theta_3 = 0.000 \cdot \text{deg}$
Link 4:	$u = 115.721$	$\sigma = 106.607 \cdot \text{deg}$
Link 1:	$g = 31.057$	$\theta_1 = -51.253 \cdot \text{deg}$
Coupler:	$r_p = 46.239$	$\delta_p = 111.571 \cdot \text{deg}$

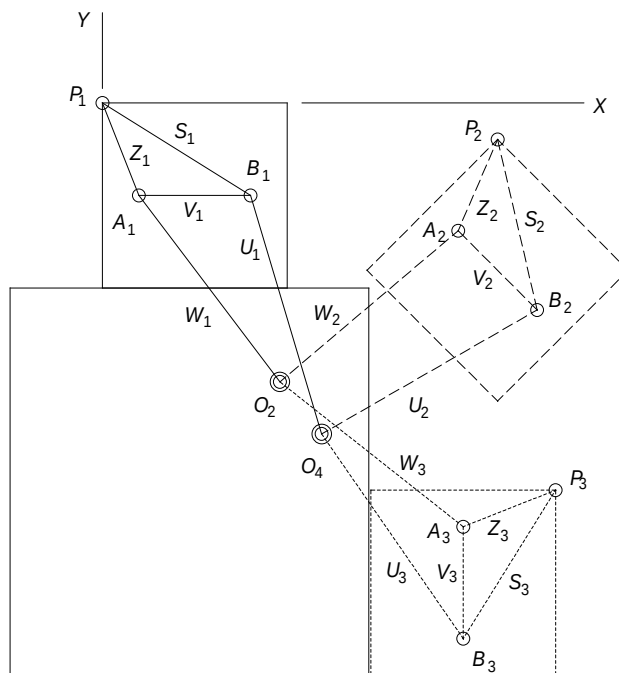
Crank angles:

$$\theta_{2i} = 178.389 \cdot \text{deg}$$

$$\theta_{2f} = 12.990 \cdot \text{deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. A driver dyad with a crank should be added to link 2 to control the motion of the fourbar so that it cannot move beyond positions 1 and 3.



PROBLEM 5-26

Statement: Design a fourbar linkage to carry the box in Figure P5-4 through the three positions shown in their numbered order using the fixed pivots shown. Determine the range of the transmission angle.

Given:

$P_{21x} := 184.0$	$P_{21y} := -17.0$	$P_{31x} := 211.0$	$P_{31y} := -180.0$
$O_{2x} := 86.0$	$O_{2y} := -132.0$	$O_{4x} := 104.0$	$O_{4y} := -155.0$
Body angles:	$\theta_{p1} := 90 \cdot \text{deg}$	$\theta_{p2} := 45 \cdot \text{deg}$	$\theta_{p3} := 0 \cdot \text{deg}$

Solution: See Figure P5-4 and Mathcad file P0526.

1. Determine the angle changes between precision points from the body angles given.

$$\begin{aligned}\alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -45.000 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -90.000 \cdot \text{deg}\end{aligned}$$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$\begin{aligned}R_{1x} &:= -O_{2x} & R_{1x} &= -86.000 & R_{1y} &:= -O_{2y} & R_{1y} &= 132.000 \\ R_{2x} &:= R_{1x} + P_{21x} & R_{2x} &= 98.000 \\ R_{2y} &:= R_{1y} + P_{21y} & R_{2y} &= 115.000 \\ R_{3x} &:= R_{1x} + P_{31x} & R_{3x} &= 125.000 \\ R_{3y} &:= R_{1y} + P_{31y} & R_{3y} &= -48.000 \\ R_1 &:= \sqrt{R_{1x}^2 + R_{1y}^2} & R_1 &= 157.544 \\ R_2 &:= \sqrt{R_{2x}^2 + R_{2y}^2} & R_2 &= 151.093 \\ R_3 &:= \sqrt{R_{3x}^2 + R_{3y}^2} & R_3 &= 133.899\end{aligned}$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\begin{aligned}\zeta_1 &:= \text{atan2}(R_{1x}, R_{1y}) & \zeta_1 &= 123.085 \cdot \text{deg} \\ \zeta_2 &:= \text{atan2}(R_{2x}, R_{2y}) & \zeta_2 &= 49.563 \cdot \text{deg} \\ \zeta_3 &:= \text{atan2}(R_{3x}, R_{3y}) & \zeta_3 &= -21.007 \cdot \text{deg}\end{aligned}$$

4. Solve for β_2 and β_3 using equations 5.34

$$\begin{aligned}C_1 &:= R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) & C_1 &= -60.553 \\ C_2 &:= R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) & C_2 &= -24.329 \\ C_3 &:= R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) & C_3 &= 7.000 \\ C_4 &:= -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) & C_4 &= -134.000 \\ C_5 &:= R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) & C_5 &= -65.473 \\ C_6 &:= -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) & C_6 &= -39.149\end{aligned}$$

$$A_1 := -C_3^2 - C_4^2$$

$$A_1 = -1.800 \times 10^4$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$$

$$A_2 = -9.047 \times 10^3$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$$

$$A_3 = -4.788 \times 10^3$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$$

$$A_4 = 7.944 \times 10^3$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$$

$$A_5 = 9.047 \times 10^3$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$$

$$A_6 = -3.684 \times 10^3$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$$

$$K_1 = -5.423 \times 10^7$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$$

$$K_2 = -7.136 \times 10^7$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$$

$$K_3 = 7.136 \times 10^7$$

$$\beta_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right)$$

$$\beta_{31} = -90.000 \cdot \text{deg}$$

$$\beta_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right)$$

$$\beta_{32} = -164.466 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\beta_3 := \beta_{32}$$

$$\beta_{21} := \text{acos} \left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1} \right)$$

$$\beta_{21} = 85.240 \cdot \text{deg}$$

$$\beta_{22} := \text{asin} \left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1} \right)$$

$$\beta_{22} = -85.240 \cdot \text{deg}$$

Since β_2 is not in the first quadrant,

$$\beta_2 := \beta_{22}$$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$R_{1x} := -O_{4x}$$

$$R_{1x} = -104.000$$

$$R_{1y} := -O_{4y}$$

$$R_{1y} = 155.000$$

$$R_{2x} := R_{1x} + P_{21x}$$

$$R_{2x} = 80.000$$

$$R_{2y} := R_{1y} + P_{21y}$$

$$R_{2y} = 138.000$$

$$R_{3x} := R_{1x} + P_{31x}$$

$$R_{3x} = 107.000$$

$$R_{3y} := R_{1y} + P_{31y}$$

$$R_{3y} = -25.000$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$$

$$R_1 = 186.657$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$$

$$R_2 = 159.512$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 109.882$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 123.860 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 59.899 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = -13.151 \cdot \text{deg}$$

7. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = -80.017$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = -13.338$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = 48.000$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = -129.000$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = -43.938$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = -45.141$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -1.894 \times 10^4$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = -7.835 \times 10^3$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = -3.714 \times 10^3$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = 9.682 \times 10^3$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = 7.835 \times 10^3$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -5.561 \times 10^3$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = -5.520 \times 10^7$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = -7.953 \times 10^7$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = 7.953 \times 10^7$$

$$\gamma_{31} := 2 \cdot \text{atan}\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \quad \gamma_{31} = -90.000 \cdot \text{deg}$$

$$\gamma_{32} := 2 \cdot \text{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \quad \gamma_{32} = -159.525 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\gamma_3 := \gamma_{32}$$

$$\gamma_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1}\right) \quad \gamma_{21} = 75.253 \cdot \text{deg}$$

$$\gamma_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1}\right) \quad \gamma_{22} = -75.253 \cdot \text{deg}$$

Since γ_2 is not in the first quadrant ,

$$\gamma_2 := \gamma_{22}$$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 184.784$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = -5.279 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \quad p_{31} = 277.346$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y}) \quad \delta_3 = -40.467 \cdot \text{deg}$$

9. Evaluate terms in the $\mathbf{WZ}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1$$

$$B := \sin(\beta_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the $\mathbf{W}$ and $\mathbf{Z}$ vectors are:

$$W1x = -62.394$$

$$W1y = 91.663$$

$$Z1x = -23.606$$

$$Z1y = 40.337$$

11. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2}$

$$w = 110.884$$

12. Evaluate terms in the $\mathbf{US}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **U** and **S** vectors are:

$$U1x = -29.920$$

$$U1y = 116.933$$

$$S1x = -74.080$$

$$S1y = 38.067$$

14. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2}$ $u = 120.700$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x$$

$$V1x = 50.474$$

$$V1y := Z1y - S1y$$

$$V1y = 2.270$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$ $v = 50.525$

$$G1x := W1x + V1x - U1x$$

$$G1x = 18.000$$

$$G1y := W1y + V1y - U1y$$

$$G1y = -23.000$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$ $g = 29.206$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors **W**₁, **Z**₁, **U**₁, and **S**₁.

$$O2x := -Z1x - W1x$$

$$O2x = 86.000$$

$$O2y := -Z1y - W1y$$

$$O2y = -132.000$$

$$O4x := -S1x - U1x$$

$$O4x = 104.000$$

$$O4y := -S1y - U1y$$

$$O4y = -155.000$$

These check with Figure P5-4.

17. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $z := \sqrt{Z1x^2 + Z1y^2}$ $z = 46.736$ $r_P := z$

Angle BAP (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 83.288$

$$\psi := \text{atan2}(S1x, S1y) \quad \psi = 152.803 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z1x, Z1y) \quad \phi = 120.337 \cdot \text{deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = 2.575 \cdot \text{deg}$$

$$\delta_p := \phi - \theta_3$$

$$\delta_p = 117.762 \cdot \text{deg}$$

18. DESIGN SUMMARY

Link 1: $g = 29.206$

Link 2: $w = 110.884$

Link 3: $v = 50.525$

Link 4: $u = 120.700$

Coupler point: $r_P = 46.736$ $\delta_p = 117.762 \cdot \text{deg}$

19. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-27

Statement: Design a fourbar linkage to carry the box in Figure P5-5 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use any points on the object as attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to P_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= 421.0 & P_{2y} &:= 963.0 \\ P_{3x} &:= 184.0 & P_{3y} &:= 1400.0 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 0.0 \cdot \text{deg} \quad \theta_{p2} := 27.0 \cdot \text{deg} \quad \theta_{p3} := 88.0 \cdot \text{deg}$$

Free choices for the **WZ** dyad :

$$\beta_2 := -50.0 \cdot \text{deg} \quad \beta_3 := -100.0 \cdot \text{deg}$$

Free choices for the **US** dyad :

$$\gamma_2 := -50.0 \cdot \text{deg} \quad \gamma_3 := -80.0 \cdot \text{deg}$$

Solution: See Figure P5-5 and Mathcad file P0527.

- Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 1.051 \times 10^3 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= 66.386 \cdot \text{deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 1.412 \times 10^3 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= 82.513 \cdot \text{deg} \end{aligned}$$

- Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= 27.000 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= 88.000 \cdot \text{deg} \end{aligned}$$

- Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$\begin{aligned} A &:= \cos(\beta_2) - 1 & B &:= \sin(\beta_2) & C &:= \cos(\alpha_2) - 1 \\ D &:= \sin(\alpha_2) & E &:= p_{21} \cdot \cos(\delta_2) & F &:= \cos(\beta_3) - 1 \\ G &:= \sin(\beta_3) & H &:= \cos(\alpha_3) - 1 & K &:= \sin(\alpha_3) \\ L &:= p_{31} \cdot \cos(\delta_3) & M &:= p_{21} \cdot \sin(\delta_2) & N &:= p_{31} \cdot \sin(\delta_3) \end{aligned}$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$$\begin{aligned} W_{1x} &= -784.602 & W_{1y} &= 362.803 & Z_{1x} &= 1.092 \times 10^3 & Z_{1y} &= 39.947 \\ \theta &:= \text{atan2}(W_{1x}, W_{1y}) & \theta &= 155.184 \cdot \text{deg} & \phi &:= \text{atan2}(Z_{1x}, Z_{1y}) & \phi &= 2.094 \cdot \text{deg} \end{aligned}$$

The length of link 2 is: $w := \sqrt{(W_{1x}^2 + W_{1y}^2)}$, $w = 864.423$

The length of vector **Z** is: $z := \sqrt{(Z_{1x}^2 + Z_{1y}^2)}$, $z = 1093.069$

4. Evaluate terms in the **US** coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$$\begin{array}{lll} A' := \cos(\gamma_2) - 1 & B' := \sin(\gamma_2) & C := \cos(\alpha_2) - 1 \\ D := \sin(\alpha_2) & E := p_{21} \cdot \cos(\delta_2) & F' := \cos(\gamma_3) - 1 \\ G' := \sin(\gamma_3) & H := \cos(\alpha_3) - 1 & K := \sin(\alpha_3) \\ L := p_{31} \cdot \cos(\delta_3) & M := p_{21} \cdot \sin(\delta_2) & N := p_{31} \cdot \sin(\delta_3) \end{array}$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$$\begin{array}{llll} U_{1x} = -924.539 & U_{1y} = 281.738 & S_{1x} = 802.719 & S_{1y} = 82.798 \\ \sigma := \text{atan2}(U_{1x}, U_{1y}) & \sigma = 163.052 \cdot \text{deg} & \psi := \text{atan2}(S_{1x}, S_{1y}) & \psi = 5.889 \cdot \text{deg} \end{array}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 966.514$

The length of vector **S** is: $s := \sqrt{(S_{1x}^2 + S_{1y}^2)}$, $s = 806.978$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\begin{array}{ll} \text{Link 3: } V_{1x} := Z_{1x} - S_{1x} & V_{1x} = 289.619 \\ V_{1y} := Z_{1y} - S_{1y} & V_{1y} = -42.852 \\ \theta_3 := \text{atan2}(V_{1x}, V_{1y}) & \theta_3 = -8.416 \cdot \text{deg} \\ v := \sqrt{V_{1x}^2 + V_{1y}^2} & v = 292.772 \end{array}$$

$$\begin{array}{ll} \text{Link 1: } G_{1x} := W_{1x} + V_{1x} - U_{1x} & G_{1x} = 429.556 \\ G_{1y} := W_{1y} + V_{1y} - U_{1y} & G_{1y} = 38.214 \\ \theta_1 := \text{atan2}(G_{1x}, G_{1y}) & \theta_1 = 5.084 \cdot \text{deg} \\ g := \sqrt{G_{1x}^2 + G_{1y}^2} & g = 431.252 \end{array}$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1$$

$$\theta_{2i} = 150.100 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_3$$

$$\theta_{2f} = 50.100 \cdot \text{deg}$$

8. Define the coupler point with respect to point A and the vector $\mathbf{V}$.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 1093.069$$

$$\delta_p = 10.511 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = -307.736$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = -402.750$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = 121.820$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = -364.536$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$$

$$\theta_{rot} = 5.084 \cdot \text{deg}$$

11. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"Grashof"}$$

12. DESIGN SUMMARY

$$\text{Link 2:} \quad w = 864.423 \quad \theta = 155.184 \cdot \text{deg}$$

$$\text{Link 3:} \quad v = 292.772 \quad \theta_3 = -8.416 \cdot \text{deg}$$

$$\text{Link 4:} \quad u = 966.514 \quad \sigma = 163.052 \cdot \text{deg}$$

$$\text{Link 1:} \quad g = 431.252 \quad \theta_1 = 5.084 \cdot \text{deg}$$

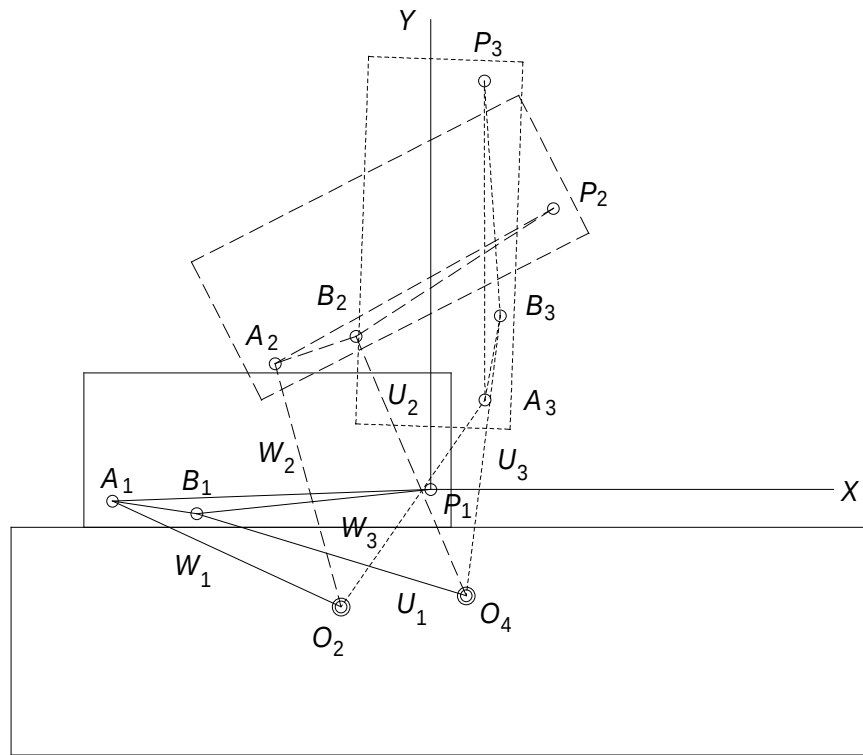
$$\text{Coupler:} \quad r_p = 1093.069 \quad \delta_p = 10.511 \cdot \text{deg}$$

Crank angles:

$$\theta_{2i} = 150.100 \cdot \text{deg}$$

$$\theta_{2f} = 50.100 \cdot \text{deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-28

Statement: Design a fourbar linkage to carry the box in Figure P5-5 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use points A and B for your attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to P_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= 421.0 & P_{2y} &:= 963.0 \\ P_{3x} &:= 184.0 & P_{3y} &:= 1400.0 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 0.0 \cdot \text{deg} \quad \theta_{p2} := 27.0 \cdot \text{deg} \quad \theta_{p3} := 88.0 \cdot \text{deg}$$

Coordinates of the points A_1 and B_1 with respect to P_1 :

$$\begin{aligned} A_{1x} &:= -1080 & A_{1y} &:= 0.0 & B_{1x} &:= -740 & B_{1y} &:= -60 \end{aligned}$$

Solution: See Figure P5-5 and Mathcad file P0528.

1. Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 1.051 \times 10^3 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= 66.386 \cdot \text{deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 1.412 \times 10^3 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= 82.513 \cdot \text{deg} \end{aligned}$$

2. Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= 27.000 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= 88.000 \cdot \text{deg} \end{aligned}$$

3. Using Figure P5-4, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$\begin{aligned} z &:= \sqrt{(P_{1x} - A_{1x})^2 + (P_{1y} - A_{1y})^2} & z &= 1080 \\ s &:= \sqrt{(P_{1x} - B_{1x})^2 + (P_{1y} - B_{1y})^2} & s &= 742.428 \\ v &:= \sqrt{(A_{1x} - B_{1x})^2 + (A_{1y} - B_{1y})^2} & v &= 345.254 \\ \phi &:= 0 & \phi &= 0.000 \cdot \text{deg} \\ \psi &:= \text{acos}\left(\frac{z^2 + s^2 - v^2}{2 \cdot z \cdot s}\right) & \psi &= 4.635 \cdot \text{deg} \\ Z_{1x} &:= z \cdot \cos(\phi) & Z_{1x} &= 1080 \\ Z_{1y} &:= z \cdot \sin(\phi) & Z_{1y} &= 0.000 \\ S_{1x} &:= s \cdot \cos(\psi) & S_{1x} &= 740.000 \\ S_{1y} &:= s \cdot \sin(\psi) & S_{1y} &= 60.000 \end{aligned}$$

4. Use equations 5.24 to solve for w , θ , β_2 , and β_3 . Since the points A and B are to be used as pivots, z and ϕ are known from the calculations above.

Guess: $W_{1x} := -50$ $W_{1y} := 200$ $\beta_2 := -80 \cdot \text{deg}$ $\beta_3 := -160 \cdot \text{deg}$

Given $W_{1x}(\cos(\beta_2) - 1) - W_{1y}\sin(\beta_2) \dots = p_{21} \cdot \cos(\delta_2)$
 $+ Z_{1x}(\cos(\alpha_2) - 1) - Z_{1y}\sin(\alpha_2)$

$W_{1x}(\cos(\beta_3) - 1) - W_{1y}\sin(\beta_3) \dots = p_{31} \cdot \cos(\delta_3)$
 $+ Z_{1x}(\cos(\alpha_3) - 1) - Z_{1y}\sin(\alpha_3)$

$W_{1y}(\cos(\beta_2) - 1) + W_{1x}\sin(\beta_2) \dots = p_{21} \cdot \sin(\delta_2)$
 $+ Z_{1y}(\cos(\alpha_2) - 1) + Z_{1x}\sin(\alpha_2)$

$W_{1y}(\cos(\beta_3) - 1) + W_{1x}\sin(\beta_3) \dots = p_{31} \cdot \sin(\delta_3)$
 $+ Z_{1y}(\cos(\alpha_3) - 1) + Z_{1x}\sin(\alpha_3)$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := \text{Find}(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$$\beta_2 = -54.243 \cdot \text{deg}$$

$$\beta_3 = -107.466 \cdot \text{deg}$$

The components of the **W** vector are:

$$W_{1x} = -730.785$$

$$W_{1y} = 289.533$$

$$\theta := \text{atan2}(W_{1x}, W_{1y})$$

$$\theta = 158.387 \cdot \text{deg}$$

The length of link 2 is: $w := \sqrt{W_{1x}^2 + W_{1y}^2}$, $w = 786.051$

5. Use equations 5.28 to solve for u , σ , γ_2 , and γ_3 . Since the points *A* and *B* are to be used as pivots, s and ψ are known from the calculations above.

Guess: $U_{1x} := -30$ $U_{1y} := 100$ $\gamma_2 := -80 \cdot \text{deg}$ $\gamma_3 := -160 \cdot \text{deg}$

Given $U_{1x}(\cos(\gamma_2) - 1) - U_{1y}\sin(\gamma_2) \dots = p_{21} \cdot \cos(\delta_2)$
 $+ S_{1x}(\cos(\alpha_2) - 1) - S_{1y}\sin(\alpha_2)$

$U_{1x}(\cos(\gamma_3) - 1) - U_{1y}\sin(\gamma_3) \dots = p_{31} \cdot \cos(\delta_3)$
 $+ S_{1x}(\cos(\alpha_3) - 1) - S_{1y}\sin(\alpha_3)$

$U_{1y}(\cos(\gamma_2) - 1) + U_{1x}\sin(\gamma_2) \dots = p_{21} \cdot \sin(\delta_2)$
 $+ S_{1y}(\cos(\alpha_2) - 1) + S_{1x}\sin(\alpha_2)$

$U_{1y}(\cos(\gamma_3) - 1) + U_{1x}\sin(\gamma_3) \dots = p_{31} \cdot \sin(\delta_3)$
 $+ S_{1y}(\cos(\alpha_3) - 1) + S_{1x}\sin(\alpha_3)$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := \text{Find}(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$$\gamma_2 = -411.378 \cdot \text{deg}$$

$$\gamma_3 = -437.949 \cdot \text{deg}$$

The components of the **U** vector are:

$$U_{1x} = -923.018 \quad U_{1y} = 232.957 \quad \sigma := \text{atan2}(U_{1x}, U_{1y}) \quad \sigma = 165.835 \cdot \text{deg}$$

The length of link 4 is: $u := \sqrt{U_{1x}^2 + U_{1y}^2}$, $u = 951.962$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\begin{aligned} \text{Link 3: } V_{1x} &:= Z_{1x} - S_{1x} & V_{1x} &= 340.000 \\ V_{1y} &:= Z_{1y} - S_{1y} & V_{1y} &= -60.000 \\ \theta_3 &:= \text{atan2}(V_{1x}, V_{1y}) & \theta_3 &= -10.008 \cdot \text{deg} \\ v &:= \sqrt{V_{1x}^2 + V_{1y}^2} & v &= 345.254 \end{aligned}$$

$$\begin{aligned} \text{Link 1: } G_{1x} &:= W_{1x} + V_{1x} - U_{1x} & G_{1x} &= 532.233 \\ G_{1y} &:= W_{1y} + V_{1y} - U_{1y} & G_{1y} &= -3.424 \\ \theta_1 &:= \text{atan2}(G_{1x}, G_{1y}) & \theta_1 &= -0.369 \cdot \text{deg} \\ g &:= \sqrt{G_{1x}^2 + G_{1y}^2} & g &= 532.244 \end{aligned}$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\begin{aligned} \theta_{2i} &:= \theta - \theta_1 & \theta_{2i} &= 158.755 \cdot \text{deg} \\ \theta_{2f} &:= \theta_{2i} + \beta_3 & \theta_{2f} &= 51.289 \cdot \text{deg} \end{aligned}$$

8. Define the coupler point with respect to point A and the vector **V**.

$$\begin{aligned} r_p &:= z & \delta_p &:= \phi - \theta_3 \\ r_p &= 1080.000 & \delta_p &= 10.008 \cdot \text{deg} \end{aligned}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\begin{aligned} O_{2x} &:= -z \cdot \cos(\phi) - w \cdot \cos(\theta) & O_{2x} &= -349.215 \\ O_{2y} &:= -z \cdot \sin(\phi) - w \cdot \sin(\theta) & O_{2y} &= -289.533 \\ O_{4x} &:= -s \cdot \cos(\psi) - u \cdot \cos(\sigma) & O_{4x} &= 183.018 \\ O_{4y} &:= -s \cdot \sin(\psi) - u \cdot \sin(\sigma) & O_{4y} &= -292.957 \end{aligned}$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{\text{rot}} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{\text{rot}} = -0.369 \cdot \text{deg}$$

11. Determine the Grashof condition.


```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof"   if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Condition(g,u,v,w) = "Grashof"

12. DESIGN SUMMARY

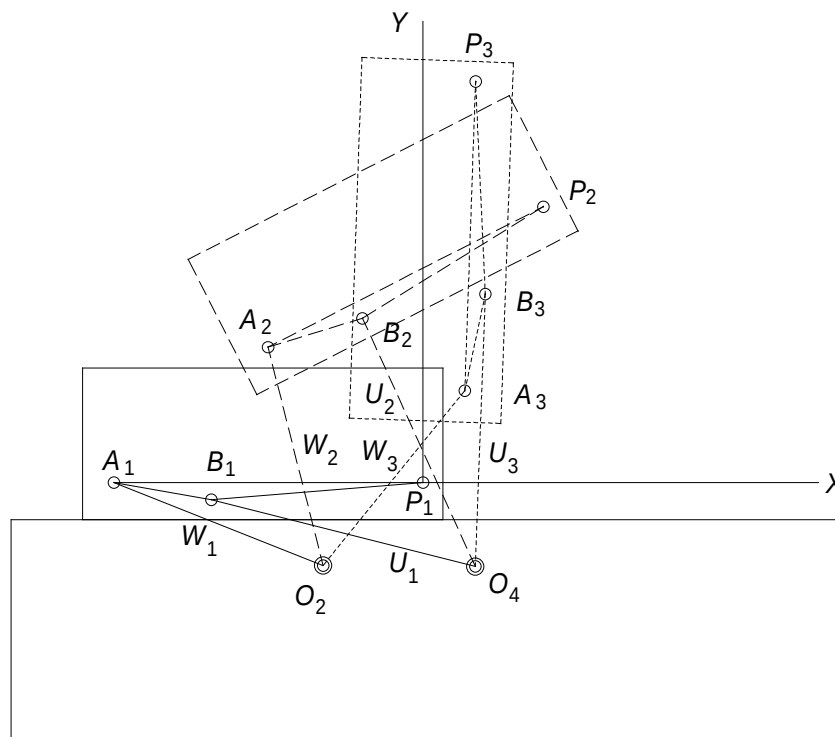
Link 2:	w = 786.051	$\theta = 158.387 \cdot \text{deg}$
Link 3:	v = 345.254	$\theta_3 = -10.008 \cdot \text{deg}$
Link 4:	u = 951.962	$\sigma = 165.835 \cdot \text{deg}$
Link 1:	g = 532.244	$\theta_1 = -0.369 \cdot \text{deg}$
Coupler:	$r_p = 1080.000$	$\delta_p = 10.008 \cdot \text{deg}$

Crank angles:

$$\theta_{2i} = 158.755 \cdot \text{deg}$$

$$\theta_{2f} = 51.289 \cdot \text{deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-29

Statement: Design a linkage to carry the object in Figure P5-5 through the three positions shown in their numbered order using the fixed pivots shown. Determine the range of the transmission angle.

Given:

$P_{21x} := 421.0$	$P_{21y} := 963.0$	$P_{31x} := 184.0$	$P_{31y} := 1400.0$
$O_{2x} := -362.0$	$O_{2y} := -291.0$	$O_{4x} := 182.0$	$O_{4y} := -291.0$
Body angles:	$\theta_{p1} := 0 \cdot \text{deg}$	$\theta_{p2} := 27 \cdot \text{deg}$	$\theta_{p3} := 88 \cdot \text{deg}$

Solution: See Figure P5-5 and Mathcad file P0529.

1. Determine the angle changes between precision points from the body angles given.

$$\begin{aligned}\alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= 27.000 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= 88.000 \cdot \text{deg}\end{aligned}$$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$\begin{aligned}R_{1x} &:= -O_{2x} & R_{1x} &= 362.000 & R_{1y} &:= -O_{2y} & R_{1y} &= 291.000 \\ R_{2x} &:= R_{1x} + P_{21x} & R_{2x} &= 783.000 \\ R_{2y} &:= R_{1y} + P_{21y} & R_{2y} &= 1.254 \times 10^3 \\ R_{3x} &:= R_{1x} + P_{31x} & R_{3x} &= 546.000 \\ R_{3y} &:= R_{1y} + P_{31y} & R_{3y} &= 1.691 \times 10^3 \\ R_1 &:= \sqrt{R_{1x}^2 + R_{1y}^2} & R_1 &= 464.462 \\ R_2 &:= \sqrt{R_{2x}^2 + R_{2y}^2} & R_2 &= 1.478 \times 10^3 \\ R_3 &:= \sqrt{R_{3x}^2 + R_{3y}^2} & R_3 &= 1.777 \times 10^3\end{aligned}$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\begin{aligned}\zeta_1 &:= \text{atan2}(R_{1x}, R_{1y}) & \zeta_1 &= 38.795 \cdot \text{deg} \\ \zeta_2 &:= \text{atan2}(R_{2x}, R_{2y}) & \zeta_2 &= 58.019 \cdot \text{deg} \\ \zeta_3 &:= \text{atan2}(R_{3x}, R_{3y}) & \zeta_3 &= 72.105 \cdot \text{deg}\end{aligned}$$

4. Solve for β_2 and β_3 using equations 5.34

$$\begin{aligned}C_1 &:= R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) & C_1 &= 944.701 \\ C_2 &:= R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) & C_2 &= 928.284 \\ C_3 &:= R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) & C_3 &= -824.189 \\ C_4 &:= -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) & C_4 &= 1.319 \times 10^3 \\ C_5 &:= R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) & C_5 &= -592.567 \\ C_6 &:= -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) & C_6 &= 830.373\end{aligned}$$

$$A_1 := -C_3^2 - C_4^2$$

$$A_1 = -2.419 \times 10^6$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$$

$$A_2 = 9.725 \times 10^4$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$$

$$A_3 = -1.584 \times 10^6$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$$

$$A_4 = 4.810 \times 10^5$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$$

$$A_5 = -9.725 \times 10^4$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$$

$$A_6 = -2.003 \times 10^6$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$$

$$K_1 = 3.219 \times 10^{12}$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$$

$$K_2 = -5.670 \times 10^{11}$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$$

$$K_3 = -4.543 \times 10^{11}$$

$$\beta_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right)$$

$$\beta_{31} = 88.000 \cdot \text{deg}$$

$$\beta_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right)$$

$$\beta_{32} = -107.980 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\beta_3 := \beta_{32}$$

$$\beta_{21} := \text{acos} \left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1} \right)$$

$$\beta_{21} = 54.008 \cdot \text{deg}$$

$$\beta_{22} := \text{asin} \left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1} \right)$$

$$\beta_{22} = -54.008 \cdot \text{deg}$$

Since β_2 is not in the first quadrant,

$$\beta_2 := \beta_{22}$$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$R_{1x} := -O_{4x}$$

$$R_{1x} = -182.000$$

$$R_{1y} := -O_{4y}$$

$$R_{1y} = 291.000$$

$$R_{2x} := R_{1x} + P_{21x}$$

$$R_{2x} = 239.000$$

$$R_{2y} := R_{1y} + P_{21y}$$

$$R_{2y} = 1.254 \times 10^3$$

$$R_{3x} := R_{1x} + P_{31x}$$

$$R_{3x} = 2.000$$

$$R_{3y} := R_{1y} + P_{31y}$$

$$R_{3y} = 1.691 \times 10^3$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$$

$$R_1 = 343.227$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$$

$$R_2 = 1.277 \times 10^3$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 1.691 \times 10^3$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 122.023 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 79.209 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = 89.932 \cdot \text{deg}$$

7. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = 478.979$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = 1.225 \times 10^3$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = -299.174$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = 1.863 \times 10^3$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = -533.274$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = 1.077 \times 10^3$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -3.559 \times 10^6$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = 6.710 \times 10^5$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = -2.166 \times 10^6$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = 5.257 \times 10^5$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = -6.710 \times 10^5$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -2.425 \times 10^6$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = 5.606 \times 10^{12}$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = 4.884 \times 10^{11}$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = 6.838 \times 10^{11}$$

$$\gamma_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{31} = 88.000 \cdot \text{deg}$$

$$\gamma_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{32} = -78.042 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\gamma_3 := \gamma_{32}$$

$$\gamma_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1}\right) \quad \gamma_{21} = 51.463 \cdot \text{deg}$$

$$\gamma_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1}\right) \quad \gamma_{22} = -51.463 \cdot \text{deg}$$

Since γ_2 is not in the first quadrant ,

$$\gamma_2 := \gamma_{22}$$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 1051.004$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = 66.386 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \quad p_{31} = 1412.040$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y}) \quad \delta_3 = 82.513 \cdot \text{deg}$$

9. Evaluate terms in the $\mathbf{WZ}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1$$

$$B := \sin(\beta_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the $\mathbf{W}$ and $\mathbf{Z}$ vectors are:

$$W1x = -728.089$$

$$W1y = 294.291$$

$$Z1x = 1.090 \times 10^3$$

$$Z1y = -3.291$$

11. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2}$

$$w = 785.316$$

12. Evaluate terms in the $\mathbf{US}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$U1x = -921.699$$

$$U1y = 231.572$$

$$S1x = 739.699$$

$$S1y = 59.428$$

14. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2}$ $u = 950.344$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x$$

$$V1x = 350.390$$

$$V1y := Z1y - S1y$$

$$V1y = -62.718$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$ $v = 355.959$

$$G1x := W1x + V1x - U1x$$

$$G1x = 544.000$$

$$G1y := W1y + V1y - U1y$$

$$G1y = -5.116 \times 10^{-13}$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$ $g = 544.000$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors **W**₁, **Z**₁, **U**₁, and **S**₁.

$$O2x := -Z1x - W1x$$

$$O2x = -362.000$$

$$O2y := -Z1y - W1y$$

$$O2y = -291.000$$

$$O4x := -S1x - U1x$$

$$O4x = 182.000$$

$$O4y := -S1y - U1y$$

$$O4y = -291.000$$

These check with Figure P5-5.

17. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $z := \sqrt{Z1x^2 + Z1y^2}$ $z = 1.090 \times 10^3$ $r_P := z$

Angle BAP (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 742.082$

$$\psi := \text{atan2}(S1x, S1y) \quad \psi = 4.593 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z1x, Z1y) \quad \phi = -0.173 \cdot \text{deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = -10.148 \cdot \text{deg}$$

$$\delta_p := \phi - \theta_3 \quad \delta_p = 9.975 \cdot \text{deg}$$

18. DESIGN SUMMARY

Link 1:	$g = 544.000$	
Link 2:	$w = 785.316$	
Link 3:	$v = 355.959$	
Link 4:	$u = 950.344$	
Coupler point:	$r_P = 1090.094$	$\delta_p = 9.975 \cdot \text{deg}$

19. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-30

Statement: To the linkage solution from Problem 5-29, add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions one and three.

Given: Solution to Problem 5-29:

Length of link 2	$w := 785.316$	
Angle of link 2 in first position	$\theta := 157.992 \cdot \text{deg}$	
Rotation angles for link 2	$\beta_2 := -54.008 \cdot \text{deg}$	$\beta_3 := -107.980 \cdot \text{deg}$
Coordinates of O_2	$O_{2x} := -362.0$	$O_{2y} := -291.0$

Solution: See Figure P5-5 and Mathcad file P0530.

- Link 2 of the solution to Problem 5-29 will become the driven link for the driver dyad. The driver dyad will be links 5 and 6 and the fixed pivot for the dyad will be at O_6 . Select a point on link 2 of Problem 5-29 and label it C . Let the distance O_2C be $R_2 := 200$. The solution that follows uses the algorithm presented in Section 5.2 with changes in nomenclature to account for the fact that the driven link is link 2 and the points A and B are already defined on the fourbar of Problem 5-29..
- Determine the coordinates of the points C_1 and C_3 using equations 5.0a. Determine the vector $\mathbf{M}$ using 5.0b.

$$\begin{aligned}
 C_{1x} &:= O_{2x} + R_2 \cdot \cos(\theta) & C_{1x} &= -547.426 \\
 C_{1y} &:= O_{2y} + R_2 \cdot \sin(\theta) & C_{1y} &= -216.053 \\
 C_{3x} &:= O_{2x} + R_2 \cdot \cos(\theta + \beta_3) & C_{3x} &= -233.475 \\
 C_{3y} &:= O_{2y} + R_2 \cdot \sin(\theta + \beta_3) & C_{3y} &= -137.764 \\
 \mathbf{R}_{C1} &:= \begin{pmatrix} C_{1x} \\ C_{1y} \end{pmatrix} & \mathbf{R}_{C3} &:= \begin{pmatrix} C_{3x} \\ C_{3y} \end{pmatrix} & \mathbf{M} &:= \mathbf{R}_{C3} - \mathbf{R}_{C1} & \mathbf{M} &= \begin{pmatrix} 313.952 \\ 78.289 \end{pmatrix}
 \end{aligned}$$

- Select a suitable value for the multiplier, K , in equation 5.0d say $K := 3.0$.
- Determine the coordinates of the crank pivot, O_6 using equation 5.0d. Place the pivot to the left of O_2 (by subtracting $K\mathbf{M}$ from $\mathbf{R}_{C3}$) so that it will be on the base below and to the left of O_2 .

$$\begin{aligned}
 \mathbf{R}_{O6} &:= \mathbf{R}_{C3} - K \cdot \mathbf{M} & O_{6x} &:= \mathbf{R}_{O6}_1 & O_{6y} &:= \mathbf{R}_{O6}_2 \\
 O_{6x} &= -1175.330 & O_{6y} &= -372.630
 \end{aligned}$$

- Determine the length of the driving crank using equation 5.0e.

$$R_6 := \left| R_2 \cdot \sin(0.5 \cdot \beta_3) \right| \quad R_6 = 161.783$$

- Determine the length of the driver dyad coupler, link 5, and the ground link from equation 5.0f.

$$\begin{aligned}
 R_5 &:= \left| \mathbf{R}_{C3} - \mathbf{R}_{O6} \right| - R_6 & R_5 &= 808.914 \\
 \mathbf{R}_{O2} &:= \begin{pmatrix} O_{2x} \\ O_{2y} \end{pmatrix} \\
 R_1 &:= \left| \mathbf{R}_{O2} - \mathbf{R}_{O6} \right| & R_1 &= 817.416
 \end{aligned}$$

7. Determine the Grashof condition.

$$R_1 = 817.416$$

$$R_2 = 200.000$$

$$R_5 = 808.914$$

$$R_6 = 161.783$$

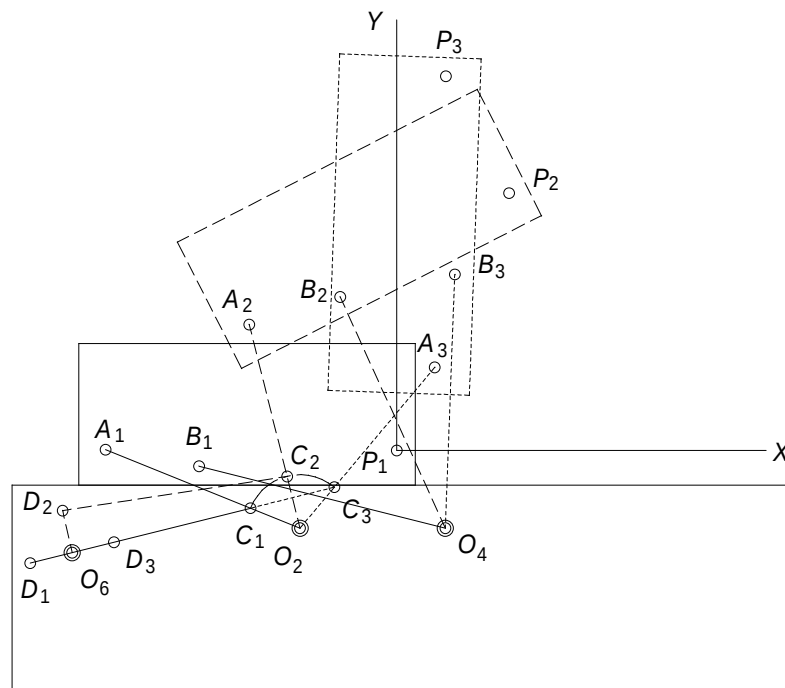
```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof"   if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$$\text{Condition}(R_1, R_2, R_5, R_6) = \text{"Grashof"}$$

8. Draw the linkage using the link lengths and fixed pivot coordinates calculated above to verify that the driver dyad will perform as required.



PROBLEM 5-31

Statement: Design a fourbar linkage to carry the box in Figure P5-6 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use points A and B for your attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to P_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= 130.0 & P_{2y} &:= -29.0 \\ P_{3x} &:= 148.0 & P_{3y} &:= -187.0 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 90.0 \cdot \text{deg} \quad \theta_{p2} := 65.0 \cdot \text{deg} \quad \theta_{p3} := -11.0 \cdot \text{deg}$$

Coordinates of the points A_1 and B_1 with respect to P_1 :

$$A_{1x} := -69.0 \quad A_{1y} := -43.0 \quad B_{1x} := -17.0 \quad B_{1y} := -43.0$$

Solution: See Figure P5-6 and Mathcad file P0531.

1. Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 133.195 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= -12.575 \cdot \text{deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 238.481 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= -51.640 \cdot \text{deg} \end{aligned}$$

2. Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -25.000 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -101.000 \cdot \text{deg} \end{aligned}$$

3. Using Figure P5-4, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$\begin{aligned} z &:= \sqrt{(P_{1x} - A_{1x})^2 + (P_{1y} - A_{1y})^2} & z &= 81.302 \\ s &:= \sqrt{(P_{1x} - B_{1x})^2 + (P_{1y} - B_{1y})^2} & s &= 46.239 \\ v &:= \sqrt{(A_{1x} - B_{1x})^2 + (A_{1y} - B_{1y})^2} & v &= 52.000 \\ \phi &:= \text{acos}\left(\frac{v^2 + z^2 - s^2}{2 \cdot v \cdot z}\right) & \phi &= 31.931 \cdot \text{deg} \\ \psi &:= \pi - \text{acos}\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) & \psi &= 68.429 \cdot \text{deg} \\ Z_{1x} &:= z \cdot \cos(\phi) & Z_{1x} &= 69.000 \\ Z_{1y} &:= z \cdot \sin(\phi) & Z_{1y} &= 43.000 \\ S_{1x} &:= s \cdot \cos(\psi) & S_{1x} &= 17.000 \\ S_{1y} &:= s \cdot \sin(\psi) & S_{1y} &= 43.000 \end{aligned}$$

4. Use equations 5.24 to solve for w , θ , β_2 , and β_3 . Since the points A and B are to be used as pivots, z and ϕ are known from the calculations above.

$$\text{Guess:} \quad W_{Ix} := -50 \quad W_{Iy} := 200 \quad \beta_2 := -80 \cdot \text{deg} \quad \beta_3 := -160 \cdot \text{deg}$$

$$\text{Given} \quad W_{Ix} \cdot (\cos(\beta_2) - 1) - W_{Iy} \cdot \sin(\beta_2) \dots = p_{21} \cdot \cos(\delta_2) \\ + Z_{Ix} \cdot (\cos(\alpha_2) - 1) - Z_{Iy} \cdot \sin(\alpha_2)$$

$$W_{Ix} \cdot (\cos(\beta_3) - 1) - W_{Iy} \cdot \sin(\beta_3) \dots = p_{31} \cdot \cos(\delta_3) \\ + Z_{Ix} \cdot (\cos(\alpha_3) - 1) - Z_{Iy} \cdot \sin(\alpha_3)$$

$$W_{Iy} \cdot (\cos(\beta_2) - 1) + W_{Ix} \cdot \sin(\beta_2) \dots = p_{21} \cdot \sin(\delta_2) \\ + Z_{Iy} \cdot (\cos(\alpha_2) - 1) + Z_{Ix} \cdot \sin(\alpha_2)$$

$$W_{Iy} \cdot (\cos(\beta_3) - 1) + W_{Ix} \cdot \sin(\beta_3) \dots = p_{31} \cdot \sin(\delta_3) \\ + Z_{Iy} \cdot (\cos(\alpha_3) - 1) + Z_{Ix} \cdot \sin(\alpha_3)$$

$$\begin{pmatrix} W_{Ix} \\ W_{Iy} \\ \beta_2 \\ \beta_3 \end{pmatrix} := \text{Find}(W_{Ix}, W_{Iy}, \beta_2, \beta_3)$$

$$\beta_2 = -52.277 \cdot \text{deg}$$

$$\beta_3 = -96.147 \cdot \text{deg}$$

The components of the **W** vector are:

$$W_{Ix} = -63.415$$

$$W_{Iy} = 118.432$$

$$\theta := \text{atan2}(W_{Ix}, W_{Iy})$$

$$\theta = 118.167 \cdot \text{deg}$$

$$\text{The length of link 2 is: } w := \sqrt{(W_{Ix}^2 + W_{Iy}^2)}, \quad w = 134.341$$

5. Use equations 5.28 to solve for u , σ , γ_2 , and γ_3 . Since the points A and B are to be used as pivots, s and ψ are known from the calculations above.

$$\text{Guess:} \quad U_{Ix} := -30 \quad U_{Iy} := 100 \quad \gamma_2 := -80 \cdot \text{deg} \quad \gamma_3 := -160 \cdot \text{deg}$$

$$\text{Given} \quad U_{Ix} \cdot (\cos(\gamma_2) - 1) - U_{Iy} \cdot \sin(\gamma_2) \dots = p_{21} \cdot \cos(\delta_2) \\ + S_{Ix} \cdot (\cos(\alpha_2) - 1) - S_{Iy} \cdot \sin(\alpha_2)$$

$$U_{Ix} \cdot (\cos(\gamma_3) - 1) - U_{Iy} \cdot \sin(\gamma_3) \dots = p_{31} \cdot \cos(\delta_3) \\ + S_{Ix} \cdot (\cos(\alpha_3) - 1) - S_{Iy} \cdot \sin(\alpha_3)$$

$$U_{Iy} \cdot (\cos(\gamma_2) - 1) + U_{Ix} \cdot \sin(\gamma_2) \dots = p_{21} \cdot \sin(\delta_2) \\ + S_{Iy} \cdot (\cos(\alpha_2) - 1) + S_{Ix} \cdot \sin(\alpha_2)$$

$$U_{Iy} \cdot (\cos(\gamma_3) - 1) + U_{Ix} \cdot \sin(\gamma_3) \dots = p_{31} \cdot \sin(\delta_3) \\ + S_{Iy} \cdot (\cos(\alpha_3) - 1) + S_{Ix} \cdot \sin(\alpha_3)$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := \text{Find}(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$$\gamma_2 = -79.044 \cdot \text{deg}$$

$$\gamma_3 = -147.982 \cdot \text{deg}$$

The components of the **U** vector are:

$$U_{1x} = -45.930$$

$$U_{1y} = 77.634$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y})$$

$$\sigma = 120.609 \cdot \text{deg}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 90.203$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$

$$V_{1x} = 52.000$$

$$V_{1y} := Z_{1y} - S_{1y}$$

$$V_{1y} = 0.000$$

$$\theta_3 := \text{atan2}(V_{1x}, V_{1y})$$

$$\theta_3 = 0.000 \cdot \text{deg}$$

$$v := \sqrt{V_{1x}^2 + V_{1y}^2}$$

$$v = 52.000$$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$

$$G_{1x} = 34.515$$

$$G_{1y} := W_{1y} + V_{1y} - U_{1y}$$

$$G_{1y} = 40.798$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y})$$

$$\theta_1 = 49.768 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2}$$

$$g = 53.439$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1$$

$$\theta_{2i} = 68.399 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_3$$

$$\theta_{2f} = -27.748 \cdot \text{deg}$$

8. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 81.302$$

$$\delta_p = 31.931 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = -5.585$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = -161.432$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = 28.930$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = -120.634$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}\left[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})\right]$$

$$\theta_{rot} = 49.768 \cdot \text{deg}$$

11. Determine the Grashof condition.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Condition(g,u,v,w) = "non-Grashof"

12. DESIGN SUMMARY

Link 2:	w = 134.341	$\theta = 118.167 \cdot \text{deg}$
Link 3:	v = 52.000	$\theta_3 = 0.000 \cdot \text{deg}$
Link 4:	u = 90.203	$\sigma = 120.609 \cdot \text{deg}$
Link 1:	g = 53.439	$\theta_1 = 49.768 \cdot \text{deg}$
Coupler:	r _p = 81.302	$\delta_p = 31.931 \cdot \text{deg}$

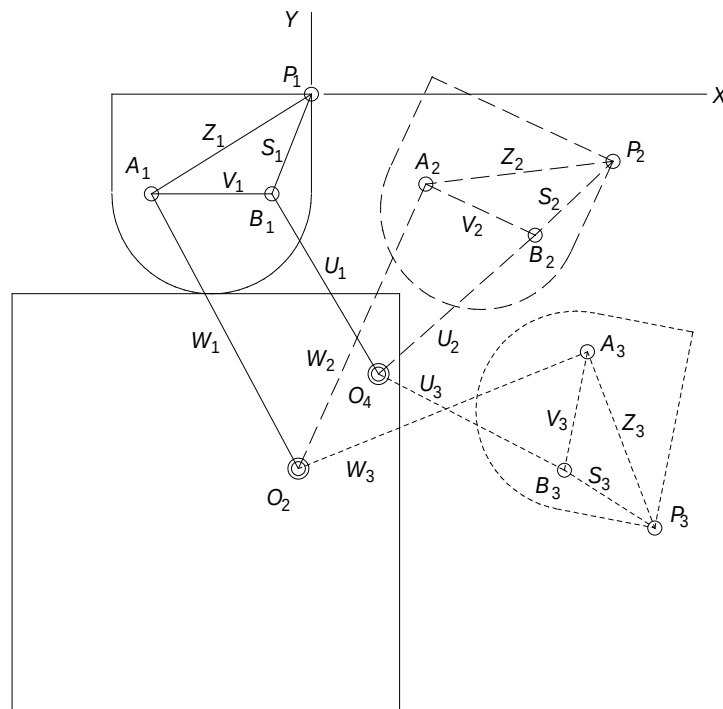
Crank angles:

$$\theta_{2i} = 68.399 \cdot \text{deg}$$

$$\theta_{2f} = -27.748 \cdot \text{deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. A driver dyad with a crank should be added to link 2 to control the motion of the fourbar so that it cannot move beyond positions 1 and 3.



PROBLEM 5-32

Statement: Design a fourbar linkage to carry the box in Figure P5-6 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use any points on the object as attachment points. Determine the range of the transmission angle. The fixed pivots should be on the base.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to P_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= 130.0 & P_{2y} &:= -29.0 \\ P_{3x} &:= 148.0 & P_{3y} &:= -187.0 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 90.0 \cdot \text{deg} \quad \theta_{p2} := 65.0 \cdot \text{deg} \quad \theta_{p3} := -11.0 \cdot \text{deg}$$

Free choices for the **WZ** dyad :

$$\beta_2 := -52.0 \cdot \text{deg} \quad \beta_3 := -95.0 \cdot \text{deg}$$

Free choices for the **US** dyad :

$$\gamma_2 := -76.0 \cdot \text{deg} \quad \gamma_3 := -145.0 \cdot \text{deg}$$

Solution: See Figure P5-6 and Mathcad file P0532.

- Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 133.195 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= -12.575 \cdot \text{deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 238.481 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= -51.640 \cdot \text{deg} \end{aligned}$$

- Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -25.000 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -101.000 \cdot \text{deg} \end{aligned}$$

- Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$\begin{aligned} A &:= \cos(\beta_2) - 1 & B &:= \sin(\beta_2) & C &:= \cos(\alpha_2) - 1 \\ D &:= \sin(\alpha_2) & E &:= p_{21} \cdot \cos(\delta_2) & F &:= \cos(\beta_3) - 1 \\ G &:= \sin(\beta_3) & H &:= \cos(\alpha_3) - 1 & K &:= \sin(\alpha_3) \\ L &:= p_{31} \cdot \cos(\delta_3) & M &:= p_{21} \cdot \sin(\delta_2) & N &:= p_{31} \cdot \sin(\delta_3) \end{aligned}$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$$\begin{aligned} W_{1x} &= -63.498 & W_{1y} &= 118.196 & Z_{1x} &= 69.575 & Z_{1y} &= 44.896 \\ \theta &:= \text{atan2}(W_{1x}, W_{1y}) & \theta &= 118.246 \cdot \text{deg} & \phi &:= \text{atan2}(Z_{1x}, Z_{1y}) & \phi &= 32.834 \cdot \text{deg} \end{aligned}$$

The length of link 2 is: $w := \sqrt{(W_{1x}^2 + W_{1y}^2)}$, $w = 134.173$

The length of vector **Z** is: $z := \sqrt{(Z_{1x}^2 + Z_{1y}^2)}$, $z = 82.803$

4. Evaluate terms in the **US** coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$$\begin{array}{lll} A' := \cos(\gamma_2) - 1 & B' := \sin(\gamma_2) & C := \cos(\alpha_2) - 1 \\ D := \sin(\alpha_2) & E := p_{21} \cdot \cos(\delta_2) & F' := \cos(\gamma_3) - 1 \\ G' := \sin(\gamma_3) & H := \cos(\alpha_3) - 1 & K := \sin(\alpha_3) \\ L := p_{31} \cdot \cos(\delta_3) & M := p_{21} \cdot \sin(\delta_2) & N := p_{31} \cdot \sin(\delta_3) \end{array}$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$$\begin{array}{llll} U_{1x} = -46.325 & U_{1y} = 82.956 & S_{1x} = 17.754 & S_{1y} = 37.985 \\ \sigma := \text{atan2}(U_{1x}, U_{1y}) & \sigma = 119.180 \cdot \text{deg} & \psi := \text{atan2}(S_{1x}, S_{1y}) & \psi = 64.949 \cdot \text{deg} \end{array}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 95.014$

The length of vector **S** is: $s := \sqrt{(S_{1x}^2 + S_{1y}^2)}$, $s = 41.930$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 51.820$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = 6.910$

$\theta_3 := \text{atan2}(V_{1x}, V_{1y})$ $\theta_3 = 7.596 \cdot \text{deg}$

$v := \sqrt{V_{1x}^2 + V_{1y}^2}$ $v = 52.279$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 34.647$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = 42.151$

$\theta_1 := \text{atan2}(G_{1x}, G_{1y})$ $\theta_1 = 50.580 \cdot \text{deg}$

$g := \sqrt{G_{1x}^2 + G_{1y}^2}$ $g = 54.563$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = 67.666 \cdot \text{deg}$

$$\theta_{2f} := \theta_{2i} + \beta_3$$

$$\theta_{2f} = -27.334 \cdot \text{deg}$$

8. Define the coupler point with respect to point A and the vector $\mathbf{V}$.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 82.803$$

$$\delta_p = 25.238 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = -6.076$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = -163.092$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = 28.571$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = -120.941$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$$

$$\theta_{rot} = 50.580 \cdot \text{deg}$$

11. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

12. DESIGN SUMMARY

$$\text{Link 2:} \quad w = 134.173 \quad \theta = 118.246 \cdot \text{deg}$$

$$\text{Link 3:} \quad v = 52.279 \quad \theta_3 = 7.596 \cdot \text{deg}$$

$$\text{Link 4:} \quad u = 95.014 \quad \sigma = 119.180 \cdot \text{deg}$$

$$\text{Link 1:} \quad g = 54.563 \quad \theta_1 = 50.580 \cdot \text{deg}$$

$$\text{Coupler:} \quad r_p = 82.803 \quad \delta_p = 25.238 \cdot \text{deg}$$

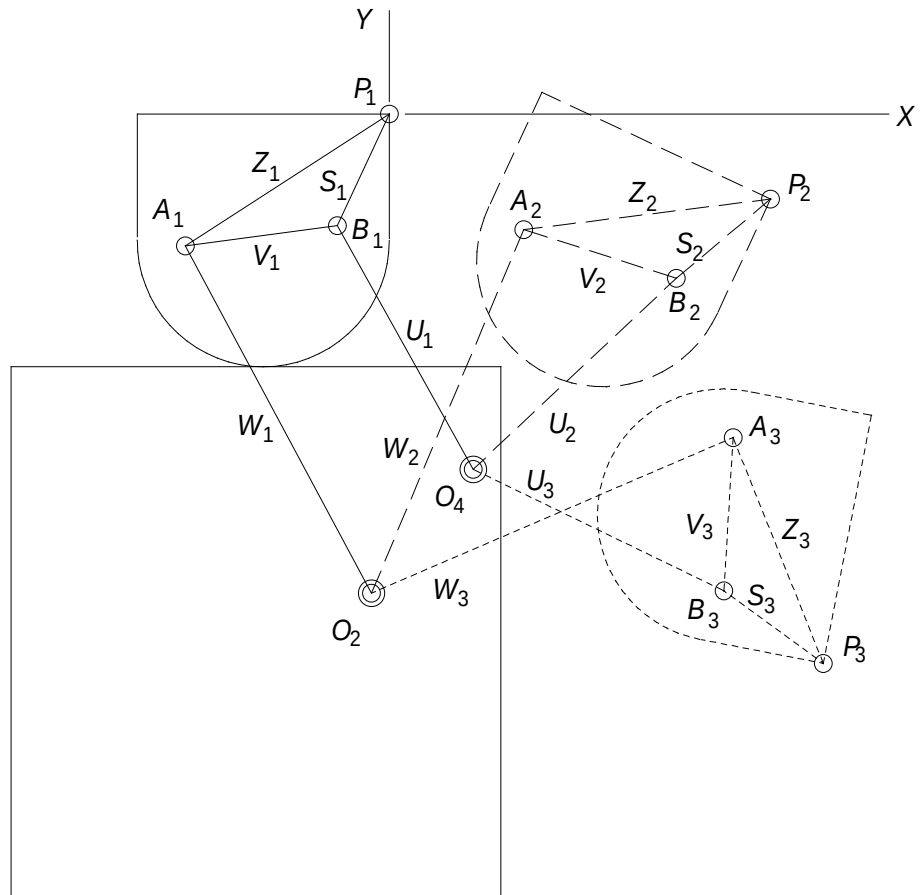
Crank angles:

$$\theta_{2i} = 67.666 \cdot \text{deg}$$

$$\theta_{2f} = -27.334 \cdot \text{deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. A driver dyad with a crank should be added to link 2 to control the motion of the fourbar so that it cannot move beyond positions 1 and 3.



PROBLEM 5-33

Statement: Design a linkage to carry the object in Figure P5-6 through the three positions shown in their numbered order using the fixed pivots shown. Determine the range of the transmission angle.

Given:

$P_{21x} := 130.0$	$P_{21y} := -29.0$	$P_{31x} := 148.0$	$P_{31y} := -187.0$
$O_{2x} := -6.2$	$O_{2y} := -164.0$	$O_{4x} := 28.0$	$O_{4y} := -121.0$
Body angles:	$\theta_{p1} := 90 \cdot \text{deg}$	$\theta_{p2} := 65 \cdot \text{deg}$	$\theta_{p3} := -11 \cdot \text{deg}$

Solution: See Figure P5-6 and Mathcad file P0533.

1. Determine the angle changes between precision points from the body angles given.

$$\begin{aligned}\alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -25.000 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -101.000 \cdot \text{deg}\end{aligned}$$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$\begin{aligned}R_{1x} &:= -O_{2x} & R_{1x} &= 6.200 & R_{1y} &:= -O_{2y} & R_{1y} &= 164.000 \\ R_{2x} &:= R_{1x} + P_{21x} & R_{2x} &= 136.200 \\ R_{2y} &:= R_{1y} + P_{21y} & R_{2y} &= 135.000 \\ R_{3x} &:= R_{1x} + P_{31x} & R_{3x} &= 154.200 \\ R_{3y} &:= R_{1y} + P_{31y} & R_{3y} &= -23.000 \\ R_1 &:= \sqrt{R_{1x}^2 + R_{1y}^2} & R_1 &= 164.117 \\ R_2 &:= \sqrt{R_{2x}^2 + R_{2y}^2} & R_2 &= 191.769 \\ R_3 &:= \sqrt{R_{3x}^2 + R_{3y}^2} & R_3 &= 155.906\end{aligned}$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\begin{aligned}\zeta_1 &:= \text{atan2}(R_{1x}, R_{1y}) & \zeta_1 &= 87.835 \cdot \text{deg} \\ \zeta_2 &:= \text{atan2}(R_{2x}, R_{2y}) & \zeta_2 &= 44.746 \cdot \text{deg} \\ \zeta_3 &:= \text{atan2}(R_{3x}, R_{3y}) & \zeta_3 &= -8.484 \cdot \text{deg}\end{aligned}$$

4. Solve for β_2 and β_3 using equations 5.34

$$\begin{aligned}C_1 &:= R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) & C_1 &= 23.501 \\ C_2 &:= R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) & C_2 &= 73.444 \\ C_3 &:= R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) & C_3 &= 5.604 \\ C_4 &:= -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) & C_4 &= 14.379 \\ C_5 &:= R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) & C_5 &= -61.271 \\ C_6 &:= -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) & C_6 &= -11.014\end{aligned}$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -238.152$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = 819.286$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = 501.727$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = 749.483$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = -819.286$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -924.339$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = 1.503 \times 10^5$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = 1.133 \times 10^6$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = -1.141 \times 10^6$$

$$\beta_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \beta_{31} = -101.000 \cdot \text{deg}$$

$$\beta_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \beta_{32} = -94.106 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\beta_3 := \beta_{32}$$

$$\beta_{21} := \text{acos} \left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1} \right) \quad \beta_{21} = 53.072 \cdot \text{deg}$$

$$\beta_{22} := \text{asin} \left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1} \right) \quad \beta_{22} = -53.072 \cdot \text{deg}$$

Since β_2 is not in the first quadrant,

$$\beta_2 := \beta_{22}$$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$R_{1x} := -O_{4x} \quad R_{1x} = -28.000 \quad R_{1y} := -O_{4y} \quad R_{1y} = 121.000$$

$$R_{2x} := R_{1x} + P_{21x} \quad R_{2x} = 102.000$$

$$R_{2y} := R_{1y} + P_{21y} \quad R_{2y} = 92.000$$

$$R_{3x} := R_{1x} + P_{31x} \quad R_{3x} = 120.000$$

$$R_{3y} := R_{1y} + P_{31y} \quad R_{3y} = -66.000$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \quad R_1 = 124.197$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 137.361$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 136.953$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 103.029 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 42.049 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = -28.811 \cdot \text{deg}$$

7. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = 10.017$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = 7.150$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = 4.120$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = -70.398$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = -76.240$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = -29.497$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -4.973 \times 10^3$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = -5.489 \times 10^3$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = -1.762 \times 10^3$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = -675.715$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = 5.489 \times 10^3$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = 544.601$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = 2.749 \times 10^6$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = 4.180 \times 10^6$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = -4.628 \times 10^6$$

$$\gamma_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{31} = -145.661 \cdot \text{deg}$$

$$\gamma_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{32} = -101.000 \cdot \text{deg}$$

The second value is the same as α_3 , so use the first value

$$\gamma_3 := \gamma_{31}$$

$$\gamma_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1}\right) \quad \gamma_{21} = 77.265 \cdot \text{deg}$$

$$\gamma_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1}\right) \quad \gamma_{22} = -77.265 \cdot \text{deg}$$

Since γ_2 is not in the first quadrant ,

$$\gamma_2 := \gamma_{22}$$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_2$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 133.195$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = -12.575 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \quad p_{31} = 238.481$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y}) \quad \delta_3 = -51.640 \cdot \text{deg}$$

9. Evaluate terms in the $\mathbf{WZ}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1$$

$$B := \sin(\beta_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the $\mathbf{W}$ and $\mathbf{Z}$ vectors are:

$$W1x = -61.917$$

$$W1y = 112.415$$

$$Z1x = 68.117$$

$$Z1y = 51.585$$

11. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2} \quad w = 128.339$

12. Evaluate terms in the $\mathbf{US}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$U1x = -46.374 \quad U1y = 80.382 \quad S1x = 18.374 \quad S1y = 40.618$$

14. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2}$ $u = 92.800$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x \quad V1x = 49.743$$

$$V1y := Z1y - S1y \quad V1y = 10.967$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$ $v = 50.938$

$$G1x := W1x + V1x - U1x \quad G1x = 34.200$$

$$G1y := W1y + V1y - U1y \quad G1y = 43.000$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$ $g = 54.942$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors **W**₁, **Z**₁, **U**₁, and **S**₁.

$$O2x := -Z1x - W1x \quad O2x = -6.200$$

$$O2y := -Z1y - W1y \quad O2y = -164.000$$

$$O4x := -S1x - U1x \quad O4x = 28.000$$

$$O4y := -S1y - U1y \quad O4y = -121.000$$

These check with Figure P5-6.

17. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $z := \sqrt{Z1x^2 + Z1y^2}$ $z = 85.446$ $r_P := z$

Angle BAP (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 44.581$

$$\psi := \text{atan2}(S1x, S1y) \quad \psi = 65.660 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z1x, Z1y) \quad \phi = 37.136 \cdot \text{deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = 12.433 \cdot \text{deg}$$

$$\delta_p := \phi - \theta_3 \quad \delta_p = 24.704 \cdot \text{deg}$$

18. DESIGN SUMMARYLink 1: $g = 54.942$ Link 2: $w = 128.339$ Link 3: $v = 50.938$ Link 4: $u = 92.800$ Coupler point: $r_p = 85.446$ $\delta_p = 24.704 \cdot \text{deg}$

19. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

20. A driver dyad with a crank should be added to link 2 to control the motion of the fourbar so that it cannot move beyond positions 1 and 3.

PROBLEM 5-34

Statement: Design a fourbar linkage to carry the bolt in Figure P5-7 from positions 1 to 2 to 3 without regard for the fixed pivots shown. The bolt is fed into the gripper in the z direction (into the paper). The gripper grabs the bolt, and your linkage moves it to position 3 to be inserted into the hole. A second degree of freedom within the gripper assembly (not shown) pushes the bolt into the hole. The moving pivots should be on, or close to, the gripper assembly, and the fixed pivots should be on the base. Use the free choices given below.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to P_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= 99.0 & P_{2y} &:= 13.0 \\ P_{3x} &:= 111.3 & P_{3y} &:= -151.8 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 272.3 \cdot \text{deg} \quad \theta_{p2} := 301.7 \cdot \text{deg} \quad \theta_{p3} := 270.0 \cdot \text{deg}$$

Free choices for the **WZ** dyad :

$$\beta_2 := 70 \cdot \text{deg} \quad \beta_3 := 140 \cdot \text{deg}$$

Free choices for the **US** dyad :

$$\gamma_2 := -5 \cdot \text{deg} \quad \gamma_3 := -49 \cdot \text{deg}$$

Solution: See Figure P5-7 and Mathcad file P0534.

- Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 99.850 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= 7.481 \cdot \text{deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 188.231 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= -53.751 \cdot \text{deg} \end{aligned}$$

- Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= 29.400 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -2.300 \cdot \text{deg} \end{aligned}$$

- Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$$\begin{aligned} A &:= \cos(\beta_2) - 1 & B &:= \sin(\beta_2) & C &:= \cos(\alpha_2) - 1 \\ D &:= \sin(\alpha_2) & E &:= p_{21} \cdot \cos(\delta_2) & F &:= \cos(\beta_3) - 1 \\ G &:= \sin(\beta_3) & H &:= \cos(\alpha_3) - 1 & K &:= \sin(\alpha_3) \\ L &:= p_{31} \cdot \cos(\delta_3) & M &:= p_{21} \cdot \sin(\delta_2) & N &:= p_{31} \cdot \sin(\delta_3) \end{aligned}$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$$W_{1x} = -86.624$$

$$W_{1y} = 50.030$$

$$Z_{1x} = 198.147$$

$$Z_{1y} = -233.314$$

$$\theta := \text{atan2}(W_{1x}, W_{1y})$$

$$\theta = 149.991 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z_{1x}, Z_{1y})$$

$$\phi = -49.660 \cdot \text{deg}$$

The length of link 2 is: $w := \sqrt{(W_{1x}^2 + W_{1y}^2)}$, $w = 100.033$

The length of vector **Z** is: $z := \sqrt{(Z_{1x}^2 + Z_{1y}^2)}$, $z = 306.100$

4. Evaluate terms in the **US** coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$$U_{1x} = 107.545$$

$$U_{1y} = 205.365$$

$$S_{1x} = 3.375$$

$$S_{1y} = -166.927$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y})$$

$$\sigma = 62.360 \cdot \text{deg}$$

$$\psi := \text{atan2}(S_{1x}, S_{1y})$$

$$\psi = -88.842 \cdot \text{deg}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 231.821$

The length of vector **S** is: $s := \sqrt{(S_{1x}^2 + S_{1y}^2)}$, $s = 166.961$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$

$$V_{1x} = 194.772$$

$$V_{1y} := Z_{1y} - S_{1y}$$

$$V_{1y} = -66.387$$

$$\theta_3 := \text{atan2}(V_{1x}, V_{1y})$$

$$\theta_3 = -18.821 \cdot \text{deg}$$

$$v := \sqrt{V_{1x}^2 + V_{1y}^2}$$

$$v = 205.775$$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$

$$G_{1x} = 0.602$$

$$G_{1y} := W_{1y} + V_{1y} - U_{1y}$$

$$G_{1y} = -221.722$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y})$$

$$\theta_1 = -89.844 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2}$$

$$g = 221.723$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \quad \theta_{2i} = 239.836 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_3 - 2 \cdot \pi \quad \theta_{2f} = 19.836 \cdot \text{deg}$$

8. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z \quad \delta_p := \phi - \theta_3$$

$$r_p = 306.100 \quad \delta_p = -30.838 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta) \quad O_{2x} = -111.523$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta) \quad O_{2y} = 183.284$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma) \quad O_{4x} = -110.920$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma) \quad O_{4y} = -38.438$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = -89.844 \cdot \text{deg}$$

11. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"Grashof"}$$

12. DESIGN SUMMARY

$$\text{Link 2:} \quad w = 100.033 \quad \theta = 149.991 \cdot \text{deg}$$

$$\text{Link 3:} \quad v = 205.775 \quad \theta_3 = -18.821 \cdot \text{deg}$$

$$\text{Link 4:} \quad u = 231.821 \quad \sigma = 62.360 \cdot \text{deg}$$

$$\text{Link 1:} \quad g = 221.723 \quad \theta_1 = -89.844 \cdot \text{deg}$$

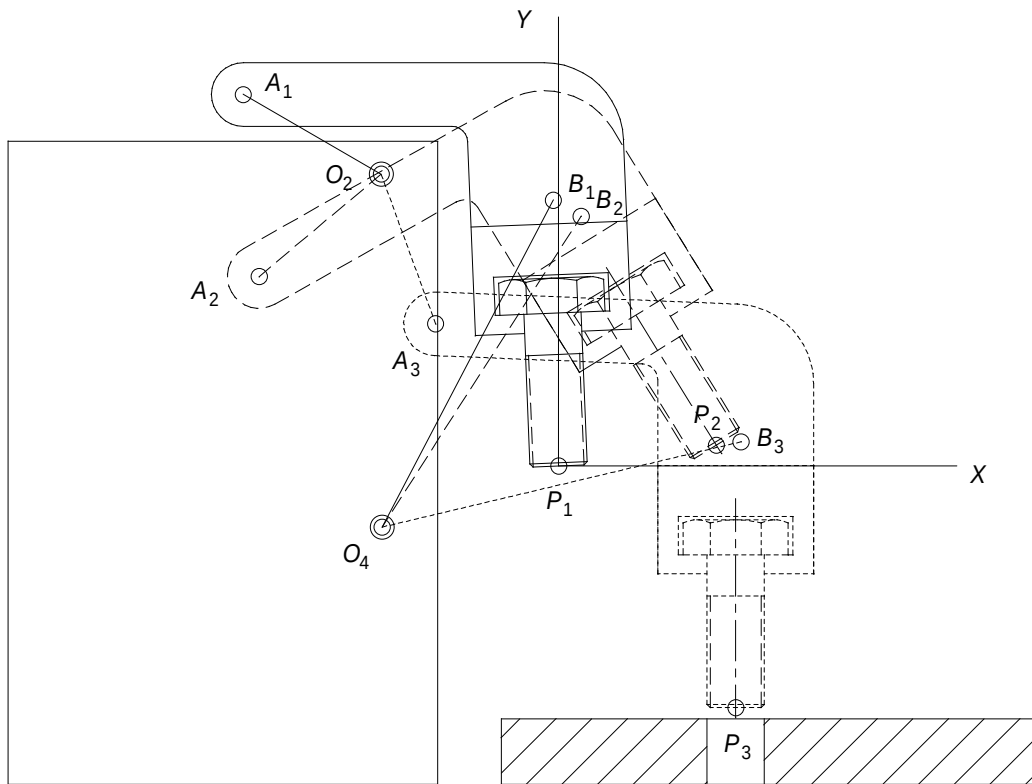
$$\text{Coupler:} \quad r_p = 306.100 \quad \delta_p = -30.838 \cdot \text{deg}$$

Crank angles:

$$\theta_{2i} = 239.836 \cdot \text{deg}$$

$$\theta_{2f} = 19.836 \cdot \text{deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-35

Statement: Design a linkage to carry the bolt in Figure P5-7 from positions 1 to 2 to 3 using the fixed pivots shown. See Problem 5-34 for more details.

Given:

$P_{21x} := 99.0$	$P_{21y} := 13.0$	$P_{31x} := 111.3$	$P_{31y} := -151.8$
$O_{2x} := -111.5$	$O_{2y} := 183.2$	$O_{4x} := -111.5$	$O_{4y} := -38.8$
Body angles:	$\theta_{p1} := 272.3 \cdot \text{deg}$	$\theta_{p2} := 301.7 \cdot \text{deg}$	$\theta_{p3} := 270 \cdot \text{deg}$

Solution: See Figure P5-7 and Mathcad file P0535.

1. Determine the angle changes between precision points from the body angles given.

$$\alpha_2 := \theta_{p2} - \theta_{p1} \quad \alpha_2 = 29.400 \cdot \text{deg}$$

$$\alpha_3 := \theta_{p3} - \theta_{p1} \quad \alpha_3 = -2.300 \cdot \text{deg}$$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$R_{1x} := -O_{2x} \quad R_{1x} = 111.500 \quad R_{1y} := -O_{2y} \quad R_{1y} = -183.200$$

$$R_{2x} := R_{1x} + P_{21x} \quad R_{2x} = 210.500$$

$$R_{2y} := R_{1y} + P_{21y} \quad R_{2y} = -170.200$$

$$R_{3x} := R_{1x} + P_{31x} \quad R_{3x} = 222.800$$

$$R_{3y} := R_{1y} + P_{31y} \quad R_{3y} = -335.000$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \quad R_1 = 214.463$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 270.700$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 402.324$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = -58.674 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = -38.957 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = -56.373 \cdot \text{deg}$$

4. Solve for β_2 and β_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = 155.059$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = -3.973$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = -118.742$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = -147.473$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = -23.426$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = -65.329$$

$$\begin{aligned}
 A_1 &:= -C_3^2 - C_4^2 & A_1 &= -3.585 \times 10^4 \\
 A_2 &:= C_3 \cdot C_6 - C_4 \cdot C_5 & A_2 &= 4.303 \times 10^3 \\
 A_3 &:= -C_4 \cdot C_6 - C_3 \cdot C_5 & A_3 &= -1.242 \times 10^4 \\
 A_4 &:= C_2 \cdot C_3 + C_1 \cdot C_4 & A_4 &= -2.240 \times 10^4 \\
 A_5 &:= C_4 \cdot C_5 - C_3 \cdot C_6 & A_5 &= -4.303 \times 10^3 \\
 A_6 &:= C_1 \cdot C_3 - C_2 \cdot C_4 & A_6 &= -1.900 \times 10^4 \\
 K_1 &:= A_2 \cdot A_4 + A_3 \cdot A_6 & K_1 &= 1.395 \times 10^8 \\
 K_2 &:= A_3 \cdot A_4 + A_5 \cdot A_6 & K_2 &= 3.598 \times 10^8 \\
 K_3 &:= \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} & K_3 &= 1.250 \times 10^8 \\
 \beta_{31} &:= 2 \cdot \operatorname{atan}\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) & \beta_{31} &= 139.911 \cdot \text{deg} \\
 \beta_{32} &:= 2 \cdot \operatorname{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) & \beta_{32} &= -2.300 \cdot \text{deg}
 \end{aligned}$$

The second value is the same as α_3 , so use the first value

$$\begin{aligned}
 \beta_{21} &:= \operatorname{acos}\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right) & \beta_{21} &= 69.984 \cdot \text{deg} \\
 \beta_{22} &:= \operatorname{asin}\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right) & \beta_{22} &= 69.984 \cdot \text{deg}
 \end{aligned}$$

Since both angles are the same,

$$\beta_2 := \beta_{21}$$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$\begin{aligned}
 R_{1x} &:= -O_{4x} & R_{1x} &= 111.500 & R_{1y} &:= -O_{4y} & R_{1y} &= 38.800 \\
 R_{2x} &:= R_{1x} + P_{21x} & R_{2x} &= 210.500 \\
 R_{2y} &:= R_{1y} + P_{21y} & R_{2y} &= 51.800 \\
 R_{3x} &:= R_{1x} + P_{31x} & R_{3x} &= 222.800 \\
 R_{3y} &:= R_{1y} + P_{31y} & R_{3y} &= -113.000 \\
 R_1 &:= \sqrt{R_{1x}^2 + R_{1y}^2} & R_1 &= 118.058 \\
 R_2 &:= \sqrt{R_{2x}^2 + R_{2y}^2} & R_2 &= 216.780
 \end{aligned}$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 249.818$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 19.187 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 13.825 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = -26.893 \cdot \text{deg}$$

7. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = 37.169$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = -32.384$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = -109.833$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = -147.294$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = -132.407$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = -36.739$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -3.376 \times 10^4$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = -1.547 \times 10^4$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = -1.995 \times 10^4$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = -1.918 \times 10^3$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = 1.547 \times 10^4$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -8.852 \times 10^3$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = 2.063 \times 10^8$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = -9.865 \times 10^7$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = 2.101 \times 10^8$$

$$\gamma_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{31} = -2.300 \cdot \text{deg}$$

$$\gamma_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{32} = -48.814 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\gamma_3 := \gamma_{32}$$

$$\gamma_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1}\right) \quad \gamma_{21} = 4.951 \cdot \text{deg}$$

$$\gamma_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1}\right) \quad \gamma_{22} = -4.951 \cdot \text{deg}$$

Since γ_2 is not in the first quadrant ,

$$\gamma_2 := \gamma_{22}$$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 99.850$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = 7.481 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \quad p_{31} = 188.231$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y}) \quad \delta_3 = -53.751 \cdot \text{deg}$$

9. Evaluate terms in the $\mathbf{WZ}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1$$

$$B := \sin(\beta_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the $\mathbf{W}$ and $\mathbf{Z}$ vectors are:

$$W1x = -86.684$$

$$W1y = 49.977$$

$$Z1x = 198.184$$

$$Z1y = -233.177$$

11. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2} \quad w = 100.059$

12. Evaluate terms in the $\mathbf{US}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$U1x = 108.268 \quad U1y = 205.938 \quad S1x = 3.232 \quad S1y = -167.138$$

14. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2}$ $u = 232.664$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x \quad V1x = 194.953$$

$$V1y := Z1y - S1y \quad V1y = -66.039$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$ $v = 205.834$

$$G1x := W1x + V1x - U1x \quad G1x = -1.990 \times 10^{-13}$$

$$G1y := W1y + V1y - U1y \quad G1y = -222.000$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$ $g = 222.000$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors **W**₁, **Z**₁, **U**₁, and **S**₁.

$$O2x := -Z1x - W1x \quad O2x = -111.500$$

$$O2y := -Z1y - W1y \quad O2y = 183.200$$

$$O4x := -S1x - U1x \quad O4x = -111.500$$

$$O4y := -S1y - U1y \quad O4y = -38.800$$

These check with Figure P5-7.

17. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $z := \sqrt{Z1x^2 + Z1y^2}$ $z = 306.020$ $r_p := z$

Angle BAP (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 167.169$

$$\psi := \text{atan2}(S1x, S1y) \quad \psi = -88.892 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z1x, Z1y) \quad \phi = -49.638 \cdot \text{deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = -18.714 \cdot \text{deg}$$

$$\delta_p := \phi - \theta_3 \quad \delta_p = -30.924 \cdot \text{deg}$$

18. DESIGN SUMMARYLink 1: $g = 222.000$ Link 2: $w = 100.059$ Link 3: $v = 205.834$ Link 4: $u = 232.664$ Coupler point: $r_p = 306.020$ $\delta_p = -30.924 \cdot \text{deg}$

- 19. VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

PROBLEM 5-36

Statement: To the linkage solution from Problem 5-35, add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions one and three.

Given: Solution to Problem 5-35:

$$\begin{array}{lll}
 \text{Length of link 4} & u := & 232.664 \\
 \text{Angle of link 4 in first position} & \theta := & 62.268 \cdot \text{deg} \\
 \text{Rotation angles for link 2} & \beta_2 := & -4.951 \cdot \text{deg} \quad \beta_3 := -48.814 \cdot \text{deg} \\
 \text{Coordinates of } O_4 & O_{4x} := & -111.5 \quad O_{4y} := -38.8
 \end{array}$$

Solution: See Figure P5-7 and Mathcad file P0536.

- Link 4 of the solution to Problem 5-35 will become the driven link for the driver dyad. The driver dyad will be links 5 and 6 and the fixed pivot for the dyad will be at O_6 . Select a point on link 4 of Problem 5-35 and label it C . Let the distance O_4C be $R_4 := 60$. The solution that follows uses the algorithm presented in Section 5.2 with changes in nomenclature to account for the fact that the driven link is link 2 and the points A and B are already defined on the fourbar of Problem 5-35.
- Determine the coordinates of the points C_1 and C_3 using equations 5.0a. Determine the vector $\mathbf{M}$ using 5.0b.

$$\begin{array}{ll}
 C_{1x} := O_{4x} + R_4 \cdot \cos(\theta) & C_{1x} = -83.580 \\
 C_{1y} := O_{4y} + R_4 \cdot \sin(\theta) & C_{1y} = 14.308 \\
 C_{3x} := O_{4x} + R_4 \cdot \cos(\theta + \beta_3) & C_{3x} = -53.147 \\
 C_{3y} := O_{4y} + R_4 \cdot \sin(\theta + \beta_3) & C_{3y} = -24.840 \\
 \mathbf{R}_{C1} := \begin{pmatrix} C_{1x} \\ C_{1y} \end{pmatrix} & \mathbf{R}_{C3} := \begin{pmatrix} C_{3x} \\ C_{3y} \end{pmatrix} & \mathbf{M} := \mathbf{R}_{C3} - \mathbf{R}_{C1} & \mathbf{M} = \begin{pmatrix} 30.433 \\ -39.148 \end{pmatrix}
 \end{array}$$

- Select a suitable value for the multiplier, K , in equation 5.0d say $K := 3.0$.
- Determine the coordinates of the crank pivot, O_6 using equation 5.0d. Place the pivot to the left of O_4 (by subtracting $K\mathbf{M}$ from $\mathbf{R}_{C3}$) so that it will be on the base above and to the left of O_2 .

$$\begin{array}{lll}
 \mathbf{R}_{O6} := \mathbf{R}_{C3} - K \cdot \mathbf{M} & O_{6x} := \mathbf{R}_{O6}_1 & O_{6y} := \mathbf{R}_{O6}_2 \\
 & O_{6x} = -144.446 & O_{6y} = 92.604
 \end{array}$$

- Determine the length of the driving crank using equation 5.0e.

$$R_6 := \left| R_4 \cdot \sin(0.5 \cdot \beta_3) \right| \quad R_6 = 24.793$$

- Determine the length of the driver dyad coupler, link 5, and the ground link from equation 5.0f.

$$\begin{array}{ll}
 R_5 := \left| \mathbf{R}_{C3} - \mathbf{R}_{O6} \right| - R_6 & R_5 = 123.965 \\
 \mathbf{R}_{O2} := \begin{pmatrix} O_{4x} \\ O_{4y} \end{pmatrix} & \\
 R_1 := \left| \mathbf{R}_{O2} - \mathbf{R}_{O6} \right| & R_1 = 135.472
 \end{array}$$

7. Determine the Grashof condition.

$$R_1 = 135.472$$

$$R_4 = 60.000$$

$$R_5 = 123.965$$

$$R_6 = 24.793$$

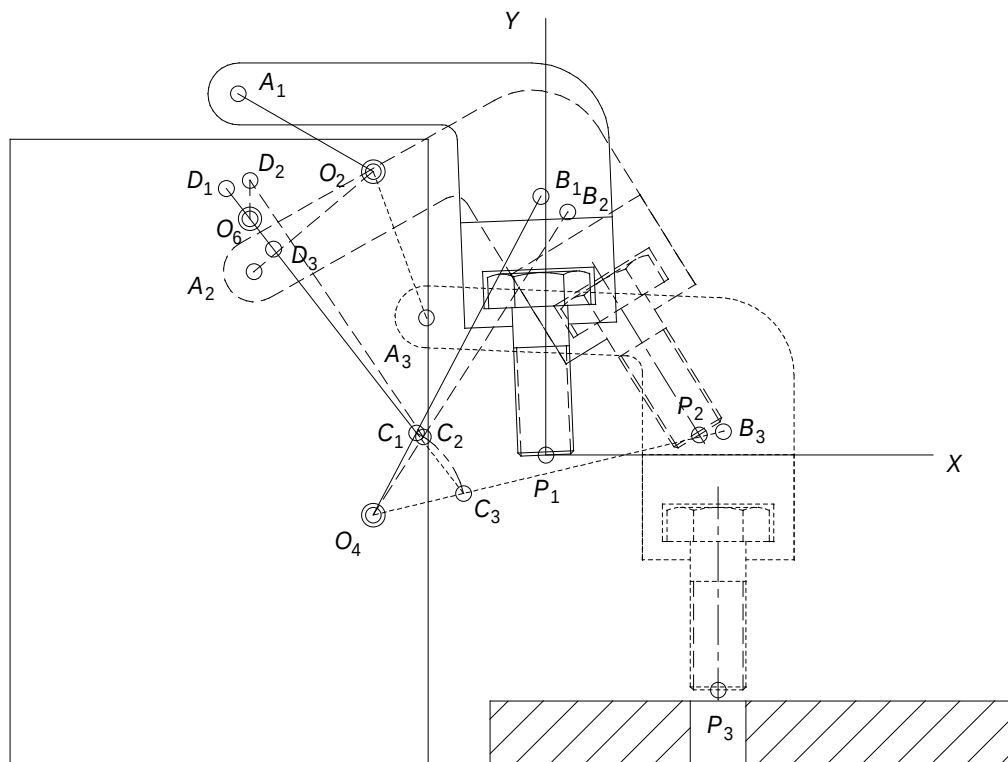
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Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof"   if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$$\text{Condition}(R_1, R_4, R_5, R_6) = \text{"Grashof"}$$

8. Draw the linkage using the link lengths and fixed pivot coordinates calculated above to verify that the driver dyad will perform as required.



PROBLEM 5-37

Statement: Figure P5-8 shows an off-loading mechanism for paper rolls. The V-link is rotated through 90 deg by an air-driven fourbar slider-crank linkage. Design a pin-jointed fourbar linkage to replace the existing off-loading station and perform essentially the same function. Choose three positions of the roll including its two end positions and synthesize a substitute mechanism. Use a link similar to the existing V-link as one of your links.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to P_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= 450 & P_{2y} &:= -140 \\ P_{3x} &:= 1000.0 & P_{3y} &:= -1000.0 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 90.0\text{-deg} \quad \theta_{p2} := 65.0\text{-deg} \quad \theta_{p3} := 0.0\text{-deg}$$

Coordinates of the points A_1 and B_1 with respect to P_1 :

$$A_{1x} := -400.0 \quad A_{1y} := -1035.0 \quad B_{1x} := 35.0 \quad B_{1y} := -600.0$$

Solution: See Figure P5-8 and Mathcad file P0537.

- Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 471.275 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= -17.281\text{-deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 1.414 \times 10^3 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= -45.000\text{-deg} \end{aligned}$$

- Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -25.000\text{-deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -90.000\text{-deg} \end{aligned}$$

- Using Figure P5-4, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$\begin{aligned} z &:= \sqrt{(P_{1x} - A_{1x})^2 + (P_{1y} - A_{1y})^2} & z &= 1109.606 \\ s &:= \sqrt{(P_{1x} - B_{1x})^2 + (P_{1y} - B_{1y})^2} & s &= 601.020 \\ v &:= \sqrt{(A_{1x} - B_{1x})^2 + (A_{1y} - B_{1y})^2} & v &= 615.183 \\ \phi &:= \text{atan2}(A_{1x}, A_{1y}) - \pi & \phi &= -291.130\text{-deg} \\ \psi &:= \pi + \text{atan2}(B_{1x}, B_{1y}) & \psi &= 93.338\text{-deg} \\ Z_{1x} &:= z \cdot \cos(\phi) & Z_{1x} &= 400.000 \\ Z_{1y} &:= z \cdot \sin(\phi) & Z_{1y} &= 1035.000 \\ S_{1x} &:= s \cdot \cos(\psi) & S_{1x} &= -35.000 \\ S_{1y} &:= s \cdot \sin(\psi) & S_{1y} &= 600.000 \end{aligned}$$

- Use equations 5.24 to solve for w , θ , β_2 , and β_3 . Since the points A and B are to be used as pivots, z and ϕ are known from the calculations above.

Guess: $W_{Ix} := -50$ $W_{Iy} := 200$ $\beta_2 := -80 \cdot \text{deg}$ $\beta_3 := -160 \cdot \text{deg}$

Given $W_{Ix}(\cos(\beta_2) - 1) - W_{Iy}\sin(\beta_2) \dots = p_{21} \cdot \cos(\delta_2)$
 $+ Z_{Ix}(\cos(\alpha_2) - 1) - Z_{Iy}\sin(\alpha_2)$

$W_{Ix}(\cos(\beta_3) - 1) - W_{Iy}\sin(\beta_3) \dots = p_{31} \cdot \cos(\delta_3)$
 $+ Z_{Ix}(\cos(\alpha_3) - 1) - Z_{Iy}\sin(\alpha_3)$

$W_{Iy}(\cos(\beta_2) - 1) + W_{Ix}\sin(\beta_2) \dots = p_{21} \cdot \sin(\delta_2)$
 $+ Z_{Iy}(\cos(\alpha_2) - 1) + Z_{Ix}\sin(\alpha_2)$

$W_{Iy}(\cos(\beta_3) - 1) + W_{Ix}\sin(\beta_3) \dots = p_{31} \cdot \sin(\delta_3)$
 $+ Z_{Iy}(\cos(\alpha_3) - 1) + Z_{Ix}\sin(\alpha_3)$

$$\begin{pmatrix} W_{Ix} \\ W_{Iy} \\ \beta_2 \\ \beta_3 \end{pmatrix} := \text{Find}(W_{Ix}, W_{Iy}, \beta_2, \beta_3)$$

$\beta_2 = -11.094 \cdot \text{deg}$ $\beta_3 = -47.757 \cdot \text{deg}$

The components of the **W** vector are:

$W_{Ix} = -673.809$ $W_{Iy} = 194.748$ $\theta := \text{atan2}(W_{Ix}, W_{Iy})$ $\theta = 163.879 \cdot \text{deg}$

The length of link 2 is: $w := \sqrt{(W_{Ix}^2 + W_{Iy}^2)}$, $w = 701.388$

5. Use equations 5.28 to solve for u , σ , γ_2 , and γ_3 . Since the points *A* and *B* are to be used as pivots, s and ψ are known from the calculations above.

Guess: $U_{Ix} := -30$ $U_{Iy} := 100$ $\gamma_2 := -80 \cdot \text{deg}$ $\gamma_3 := -160 \cdot \text{deg}$

Given $U_{Ix}(\cos(\gamma_2) - 1) - U_{Iy}\sin(\gamma_2) \dots = p_{21} \cdot \cos(\delta_2)$
 $+ S_{Ix}(\cos(\alpha_2) - 1) - S_{Iy}\sin(\alpha_2)$

$U_{Ix}(\cos(\gamma_3) - 1) - U_{Iy}\sin(\gamma_3) \dots = p_{31} \cdot \cos(\delta_3)$
 $+ S_{Ix}(\cos(\alpha_3) - 1) - S_{Iy}\sin(\alpha_3)$

$U_{Iy}(\cos(\gamma_2) - 1) + U_{Ix}\sin(\gamma_2) \dots = p_{21} \cdot \sin(\delta_2)$
 $+ S_{Iy}(\cos(\alpha_2) - 1) + S_{Ix}\sin(\alpha_2)$

$U_{Iy}(\cos(\gamma_3) - 1) + U_{Ix}\sin(\gamma_3) \dots = p_{31} \cdot \sin(\delta_3)$
 $+ S_{Iy}(\cos(\alpha_3) - 1) + S_{Ix}\sin(\alpha_3)$

$$\begin{pmatrix} U_{Ix} \\ U_{Iy} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := \text{Find}(U_{Ix}, U_{Iy}, \gamma_2, \gamma_3)$$

$\gamma_2 = -25.881 \cdot \text{deg}$ $\gamma_3 = -71.807 \cdot \text{deg}$

The components of the **U** vector are:

$$U_{1x} = 117.927 \quad U_{1y} = 469.583 \quad \sigma := \text{atan2}(U_{1x}, U_{1y}) \quad \sigma = 75.903 \cdot \text{deg}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}, \quad u = 484.164$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x} \quad V_{1x} = 435.000$

$$V_{1y} := Z_{1y} - S_{1y} \quad V_{1y} = 435.000$$

$$\theta_3 := \text{atan2}(V_{1x}, V_{1y}) \quad \theta_3 = 45.000 \cdot \text{deg}$$

$$v := \sqrt{V_{1x}^2 + V_{1y}^2} \quad v = 615.183$$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x} \quad G_{1x} = -356.736$

$$G_{1y} := W_{1y} + V_{1y} - U_{1y} \quad G_{1y} = 160.165$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y}) \quad \theta_1 = 155.821 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2} \quad g = 391.042$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \quad \theta_{2i} = 8.058 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_3 \quad \theta_{2f} = -39.699 \cdot \text{deg}$$

8. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z \quad \delta_p := \phi - \theta_3$$

$$r_p = 1109.606 \quad \delta_p = -336.130 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta) \quad O_{2x} = 273.809$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta) \quad O_{2y} = -1229.748$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma) \quad O_{4x} = -82.927$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma) \quad O_{4y} = -1069.583$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = 155.821 \cdot \text{deg}$$

11. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, w, u, v) = \text{"Grashof"}$$

12. DESIGN SUMMARY

Link 2:	$w = 701.388$	$\theta = 163.879 \cdot \text{deg}$
Link 3:	$v = 615.183$	$\theta_3 = 45.000 \cdot \text{deg}$
Link 4:	$u = 484.164$	$\sigma = 75.903 \cdot \text{deg}$
Link 1:	$g = 391.042$	$\theta_1 = 155.821 \cdot \text{deg}$
Coupler:	$r_p = 1109.606$	$\delta_p = -336.130 \cdot \text{deg}$

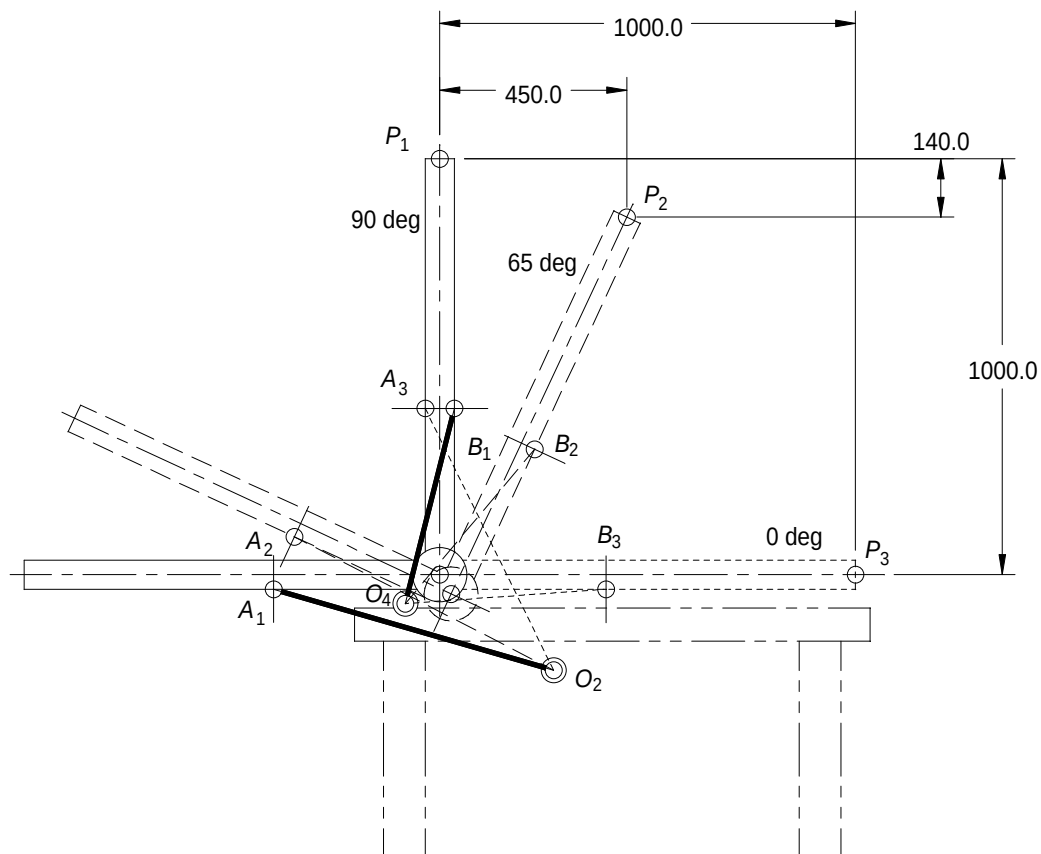
Crank angles:

$$\theta_{2i} = 8.058 \cdot \text{deg}$$

$$\theta_{2f} = -39.699 \cdot \text{deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. A driver dyad with a crank should be added to link 2 to control the motion of the fourbar so that it cannot move beyond positions 1 and 3.



PROBLEM 5-38

Statement: Design a fourbar linkage to carry the object in Figure P5-9 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use points *C* and *D* for your attachment points. Determine the range of the transmission angle.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to C_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= 4.500 & P_{2y} &:= 1.900 \\ P_{3x} &:= 7.600 & P_{3y} &:= 1.000 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 33.70 \cdot \text{deg} \quad \theta_{p2} := 14.60 \cdot \text{deg} \quad \theta_{p3} := 0.0 \cdot \text{deg}$$

Coordinates of the points C_1 and D_1 with respect to P_1 :

$$C_{1x} := 0.0 \quad C_{1y} := 0.0 \quad D_{1x} := 3.744 \quad D_{1y} := 2.497$$

Solution: See Figure P5-9 and Mathcad file P0538.

1. Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 4.885 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= 22.891 \cdot \text{deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 7.666 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= 7.496 \cdot \text{deg} \end{aligned}$$

2. Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -19.100 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -33.700 \cdot \text{deg} \end{aligned}$$

3. Using Figure P5-9, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$\begin{aligned} z &:= \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} & z &= 0.000 \\ s &:= \sqrt{(P_{1x} - D_{1x})^2 + (P_{1y} - D_{1y})^2} & s &= 4.500 \\ v &:= \sqrt{(C_{1x} - D_{1x})^2 + (C_{1y} - D_{1y})^2} & v &= 4.500 \\ \phi &:= \theta_{p1} & \phi &= 33.700 \cdot \text{deg} \\ \psi &:= \theta_{p1} + \pi & \psi &= 213.700 \cdot \text{deg} \\ Z_{1x} &:= z \cdot \cos(\phi) & Z_{1x} &= 0.000 \\ Z_{1y} &:= z \cdot \sin(\phi) & Z_{1y} &= 0.000 \\ S_{1x} &:= -s \cdot \cos(\theta_{p1}) & S_{1x} &= -3.744 \\ S_{1y} &:= -s \cdot \sin(\theta_{p1}) & S_{1y} &= -2.497 \end{aligned}$$

4. Use equations 5.24 to solve for w , θ , β_2 , and β_3 . Since the points *C* and *D* are to be used as pivots, z and ϕ are known from the calculations above. Use guess values from a graphical solution such as that for Problem 3-57.

$$\text{Guess:} \quad W_{1x} := -4 \quad W_{1y} := 4 \quad \beta_2 := -50 \cdot \text{deg} \quad \beta_3 := -80 \cdot \text{deg}$$

$$\begin{aligned} \text{Given} \quad & W_{1x}(\cos(\beta_2) - 1) - W_{1y}\sin(\beta_2) \dots = p_{21} \cdot \cos(\delta_2) \\ & + Z_{1x}(\cos(\alpha_2) - 1) - Z_{1y}\sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} & W_{1x}(\cos(\beta_3) - 1) - W_{1y}\sin(\beta_3) \dots = p_{31} \cdot \cos(\delta_3) \\ & + Z_{1x}(\cos(\alpha_3) - 1) - Z_{1y}\sin(\alpha_3) \end{aligned}$$

$$\begin{aligned} & W_{1y}(\cos(\beta_2) - 1) + W_{1x}\sin(\beta_2) \dots = p_{21} \cdot \sin(\delta_2) \\ & + Z_{1y}(\cos(\alpha_2) - 1) + Z_{1x}\sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} & W_{1y}(\cos(\beta_3) - 1) + W_{1x}\sin(\beta_3) \dots = p_{31} \cdot \sin(\delta_3) \\ & + Z_{1y}(\cos(\alpha_3) - 1) + Z_{1x}\sin(\alpha_3) \end{aligned}$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := \text{Find}(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$$\beta_2 = -47.370 \cdot \text{deg} \quad \beta_3 = -78.160 \cdot \text{deg}$$

The components of the **W** vector are:

$$W_{1x} = -4.416 \quad W_{1y} = 4.179 \quad \theta := \text{atan2}(W_{1x}, W_{1y}) \quad \theta = 136.576 \cdot \text{deg}$$

$$\text{The length of link 2 is: } w := \sqrt{(W_{1x}^2 + W_{1y}^2)}, \quad w = 6.080$$

5. Use equations 5.28 to solve for u , σ , γ_2 , and γ_3 . Since the points C and D are to be used as pivots, s and ψ are known from the calculations above.

$$\text{Guess:} \quad U_{1x} := -4 \quad U_{1y} := 4 \quad \gamma_2 := -44 \cdot \text{deg} \quad \gamma_3 := -76 \cdot \text{deg}$$

$$\begin{aligned} \text{Given} \quad & U_{1x}(\cos(\gamma_2) - 1) - U_{1y}\sin(\gamma_2) \dots = p_{21} \cdot \cos(\delta_2) \\ & + S_{1x}(\cos(\alpha_2) - 1) - S_{1y}\sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} & U_{1x}(\cos(\gamma_3) - 1) - U_{1y}\sin(\gamma_3) \dots = p_{31} \cdot \cos(\delta_3) \\ & + S_{1x}(\cos(\alpha_3) - 1) - S_{1y}\sin(\alpha_3) \end{aligned}$$

$$\begin{aligned} & U_{1y}(\cos(\gamma_2) - 1) + U_{1x}\sin(\gamma_2) \dots = p_{21} \cdot \sin(\delta_2) \\ & + S_{1y}(\cos(\alpha_2) - 1) + S_{1x}\sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} & U_{1y}(\cos(\gamma_3) - 1) + U_{1x}\sin(\gamma_3) \dots = p_{31} \cdot \sin(\delta_3) \\ & + S_{1y}(\cos(\alpha_3) - 1) + S_{1x}\sin(\alpha_3) \end{aligned}$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := \text{Find}(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$$\gamma_2 = -43.852 \cdot \text{deg} \quad \gamma_3 = -76.170 \cdot \text{deg}$$

The components of the **U** vector are:

$$U_{1x} = -3.223 \quad U_{1y} = 6.080 \quad \sigma := \text{atan2}(U_{1x}, U_{1y}) \quad \sigma = 117.929 \cdot \text{deg}$$

The length of link 4 is: $u := \sqrt{U_{1x}^2 + U_{1y}^2}$, $u = 6.881$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\begin{aligned} \text{Link 3: } V_{1x} &:= Z_{1x} - S_{1x} & V_{1x} &= 3.744 \\ V_{1y} &:= Z_{1y} - S_{1y} & V_{1y} &= 2.497 \\ \theta_3 &:= \text{atan2}(V_{1x}, V_{1y}) & \theta_3 &= 33.700 \cdot \text{deg} \\ v &:= \sqrt{V_{1x}^2 + V_{1y}^2} & v &= 4.500 \end{aligned}$$

$$\begin{aligned} \text{Link 1: } G_{1x} &:= W_{1x} + V_{1x} - U_{1x} & G_{1x} &= 2.551 \\ G_{1y} &:= W_{1y} + V_{1y} - U_{1y} & G_{1y} &= 0.596 \\ \theta_1 &:= \text{atan2}(G_{1x}, G_{1y}) & \theta_1 &= 13.155 \cdot \text{deg} \\ g &:= \sqrt{G_{1x}^2 + G_{1y}^2} & g &= 2.620 \end{aligned}$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\begin{aligned} \theta_{2i} &:= \theta - \theta_1 & \theta_{2i} &= 123.420 \cdot \text{deg} \\ \theta_{2f} &:= \theta_{2i} + \beta_3 & \theta_{2f} &= 45.261 \cdot \text{deg} \end{aligned}$$

8. Define the coupler point with respect to point **C** and the vector **V**.

$$\begin{aligned} r_p &:= z & \delta_p &:= \phi - \theta_3 \\ r_p &= 0.000 & \delta_p &= 0.000 \cdot \text{deg} \end{aligned}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\begin{aligned} O_{2x} &:= -z \cdot \cos(\phi) - w \cdot \cos(\theta) & O_{2x} &= 4.416 \\ O_{2y} &:= -z \cdot \sin(\phi) - w \cdot \sin(\theta) & O_{2y} &= -4.179 \\ O_{4x} &:= -s \cdot \cos(\psi) - u \cdot \cos(\sigma) & O_{4x} &= 6.967 \\ O_{4y} &:= -s \cdot \sin(\psi) - u \cdot \sin(\sigma) & O_{4y} &= -3.583 \end{aligned}$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{\text{rot}} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{\text{rot}} = 13.155 \cdot \text{deg}$$

11. Determine the Grashof condition.

$$\begin{aligned} \text{Condition}(a, b, c, d) &:= \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(g, u, v, w) = \text{"Grashof"}$$

12. DESIGN SUMMARY

Link 2:	$w = 6.080$	$\theta = 136.576 \cdot \text{deg}$
Link 3:	$v = 4.500$	$\theta_3 = 33.700 \cdot \text{deg}$
Link 4:	$u = 6.881$	$\sigma = 117.929 \cdot \text{deg}$
Link 1:	$g = 2.620$	$\theta_1 = 13.155 \cdot \text{deg}$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000 \cdot \text{deg}$

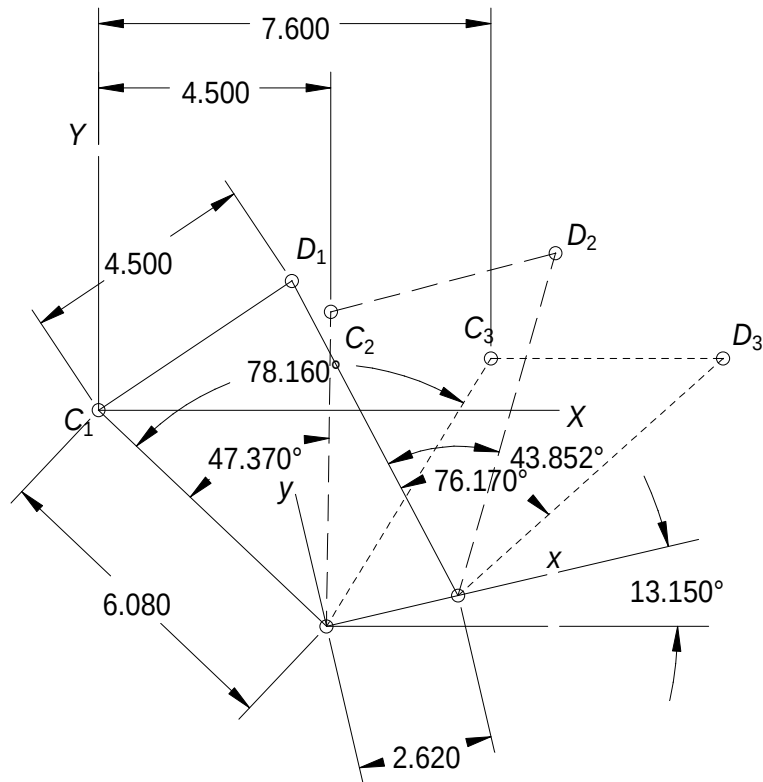
Crank angles:

$$\theta_{2i} = 123.420 \cdot \text{deg}$$

$$\theta_{2f} = 45.261 \cdot \text{deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. A driver dyad with a crank should be added to link 2 to control the motion of the fourbar so that it cannot move beyond positions 1 and 3.



PROBLEM 5-39

Statement: Design a fourbar linkage to carry the object in Figure P5-9 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use any points on the object as attachment points. Determine the range of the transmission angle.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to C_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= 4.500 & P_{2y} &:= 1.900 \\ P_{3x} &:= 7.600 & P_{3y} &:= 1.000 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 33.70 \cdot \text{deg} \quad \theta_{p2} := 14.60 \cdot \text{deg} \quad \theta_{p3} := 0.0 \cdot \text{deg}$$

Coordinates of the points C_1 and E_1 (used for attachment) with respect to P_1 :

$$C_{1x} := 0.0 \quad C_{1y} := 0.0 \quad E_{1x} := 3.744 \quad E_{1y} := 0.000$$

Solution: See Figure P5-9 and Mathcad file P0539.

1. Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 4.885 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= 22.891 \cdot \text{deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 7.666 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= 7.496 \cdot \text{deg} \end{aligned}$$

2. Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -19.100 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -33.700 \cdot \text{deg} \end{aligned}$$

3. Using Figure P5-9, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$\begin{aligned} z &:= \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} & z &= 0.000 \\ s &:= \sqrt{(P_{1x} - E_{1x})^2 + (P_{1y} - E_{1y})^2} & s &= 3.744 \\ v &:= \sqrt{(C_{1x} - E_{1x})^2 + (C_{1y} - E_{1y})^2} & v &= 3.744 \\ \phi &:= \text{atan2}(E_{1x} - C_{1x}, E_{1y} - C_{1y}) & \phi &= 0.000 \cdot \text{deg} \\ \psi &:= \phi + \pi & \psi &= 180.000 \cdot \text{deg} \\ Z_{1x} &:= z \cdot \cos(\phi) & Z_{1x} &= 0.000 \\ Z_{1y} &:= z \cdot \sin(\phi) & Z_{1y} &= 0.000 \\ S_{1x} &:= s \cdot \cos(\psi) & S_{1x} &= -3.744 \\ S_{1y} &:= s \cdot \sin(\psi) & S_{1y} &= 0.000 \end{aligned}$$

4. Use equations 5.24 to solve for w , θ , β_2 , and β_3 . Since the points C and D are to be used as pivots, z and ϕ are known from the calculations above. Use guess values from a graphical solution such as that for Problem 3-57.

$$\text{Guess:} \quad W_{1x} := -4 \quad W_{1y} := 4 \quad \beta_2 := -50 \cdot \text{deg} \quad \beta_3 := -80 \cdot \text{deg}$$

$$\begin{aligned} \text{Given} \quad & W_{1x}(\cos(\beta_2) - 1) - W_{1y}\sin(\beta_2) \dots = p_{21} \cdot \cos(\delta_2) \\ & + Z_{1x}(\cos(\alpha_2) - 1) - Z_{1y}\sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} & W_{1x}(\cos(\beta_3) - 1) - W_{1y}\sin(\beta_3) \dots = p_{31} \cdot \cos(\delta_3) \\ & + Z_{1x}(\cos(\alpha_3) - 1) - Z_{1y}\sin(\alpha_3) \end{aligned}$$

$$\begin{aligned} & W_{1y}(\cos(\beta_2) - 1) + W_{1x}\sin(\beta_2) \dots = p_{21} \cdot \sin(\delta_2) \\ & + Z_{1y}(\cos(\alpha_2) - 1) + Z_{1x}\sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} & W_{1y}(\cos(\beta_3) - 1) + W_{1x}\sin(\beta_3) \dots = p_{31} \cdot \sin(\delta_3) \\ & + Z_{1y}(\cos(\alpha_3) - 1) + Z_{1x}\sin(\alpha_3) \end{aligned}$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := \text{Find}(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$$\beta_2 = -47.370 \cdot \text{deg} \quad \beta_3 = -78.160 \cdot \text{deg}$$

The components of the **W** vector are:

$$W_{1x} = -4.416 \quad W_{1y} = 4.179 \quad \theta := \text{atan2}(W_{1x}, W_{1y}) \quad \theta = 136.576 \cdot \text{deg}$$

$$\text{The length of link 2 is: } w := \sqrt{(W_{1x}^2 + W_{1y}^2)}, \quad w = 6.080$$

5. Use equations 5.28 to solve for u , σ , γ_2 , and γ_3 . Since the points C and D are to be used as pivots, s and ψ are known from the calculations above.

$$\text{Guess:} \quad U_{1x} := -4 \quad U_{1y} := 4 \quad \gamma_2 := -44 \cdot \text{deg} \quad \gamma_3 := -76 \cdot \text{deg}$$

$$\begin{aligned} \text{Given} \quad & U_{1x}(\cos(\gamma_2) - 1) - U_{1y}\sin(\gamma_2) \dots = p_{21} \cdot \cos(\delta_2) \\ & + S_{1x}(\cos(\alpha_2) - 1) - S_{1y}\sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} & U_{1x}(\cos(\gamma_3) - 1) - U_{1y}\sin(\gamma_3) \dots = p_{31} \cdot \cos(\delta_3) \\ & + S_{1x}(\cos(\alpha_3) - 1) - S_{1y}\sin(\alpha_3) \end{aligned}$$

$$\begin{aligned} & U_{1y}(\cos(\gamma_2) - 1) + U_{1x}\sin(\gamma_2) \dots = p_{21} \cdot \sin(\delta_2) \\ & + S_{1y}(\cos(\alpha_2) - 1) + S_{1x}\sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} & U_{1y}(\cos(\gamma_3) - 1) + U_{1x}\sin(\gamma_3) \dots = p_{31} \cdot \sin(\delta_3) \\ & + S_{1y}(\cos(\alpha_3) - 1) + S_{1x}\sin(\alpha_3) \end{aligned}$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := \text{Find}(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$$\gamma_2 = -48.844 \cdot \text{deg} \quad \gamma_3 = -84.280 \cdot \text{deg}$$

The components of the **U** vector are:

$$U_{1x} = -2.890 \quad U_{1y} = 4.391 \quad \sigma := \text{atan2}(U_{1x}, U_{1y}) \quad \sigma = 123.354 \cdot \text{deg}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 5.256$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\begin{aligned} \text{Link 3: } V_{1x} &:= Z_{1x} - S_{1x} & V_{1x} &= 3.744 \\ V_{1y} &:= Z_{1y} - S_{1y} & V_{1y} &= 0.000 \\ \theta_3 &:= \text{atan2}(V_{1x}, V_{1y}) & \theta_3 &= 0.000 \cdot \text{deg} \\ v &:= \sqrt{V_{1x}^2 + V_{1y}^2} & v &= 3.744 \end{aligned}$$

$$\begin{aligned} \text{Link 1: } G_{1x} &:= W_{1x} + V_{1x} - U_{1x} & G_{1x} &= 2.218 \\ G_{1y} &:= W_{1y} + V_{1y} - U_{1y} & G_{1y} &= -0.211 \\ \theta_1 &:= \text{atan2}(G_{1x}, G_{1y}) & \theta_1 &= -5.444 \cdot \text{deg} \\ g &:= \sqrt{G_{1x}^2 + G_{1y}^2} & g &= 2.228 \end{aligned}$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\begin{aligned} \theta_{2i} &:= \theta - \theta_1 & \theta_{2i} &= 142.019 \cdot \text{deg} \\ \theta_{2f} &:= \theta_{2i} + \beta_3 & \theta_{2f} &= 63.860 \cdot \text{deg} \end{aligned}$$

8. Define the coupler point with respect to point **C** and the vector **V**.

$$\begin{aligned} r_p &:= z & \delta_p &:= \phi - \theta_3 \\ r_p &= 0.000 & \delta_p &= 0.000 \cdot \text{deg} \end{aligned}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\begin{aligned} O_{2x} &:= -z \cdot \cos(\phi) - w \cdot \cos(\theta) & O_{2x} &= 4.416 \\ O_{2y} &:= -z \cdot \sin(\phi) - w \cdot \sin(\theta) & O_{2y} &= -4.179 \\ O_{4x} &:= -s \cdot \cos(\psi) - u \cdot \cos(\sigma) & O_{4x} &= 6.634 \\ O_{4y} &:= -s \cdot \sin(\psi) - u \cdot \sin(\sigma) & O_{4y} &= -4.391 \end{aligned}$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis the line O_2O_4).

$$\theta_{\text{rot}} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{\text{rot}} = -5.444 \cdot \text{deg}$$

11. Determine the Grashof condition.

$$\begin{aligned} \text{Condition}(a, b, c, d) &:= \begin{array}{l} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array} \end{aligned}$$

$$\text{Condition}(g, u, v, w) = \text{"Grashof"}$$

12. DESIGN SUMMARY

Link 2:	$w = 6.080$	$\theta = 136.576 \cdot \text{deg}$
Link 3:	$v = 3.744$	$\theta_3 = 0.000 \cdot \text{deg}$
Link 4:	$u = 5.256$	$\sigma = 123.354 \cdot \text{deg}$
Link 1:	$g = 2.228$	$\theta_1 = -5.444 \cdot \text{deg}$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000 \cdot \text{deg}$

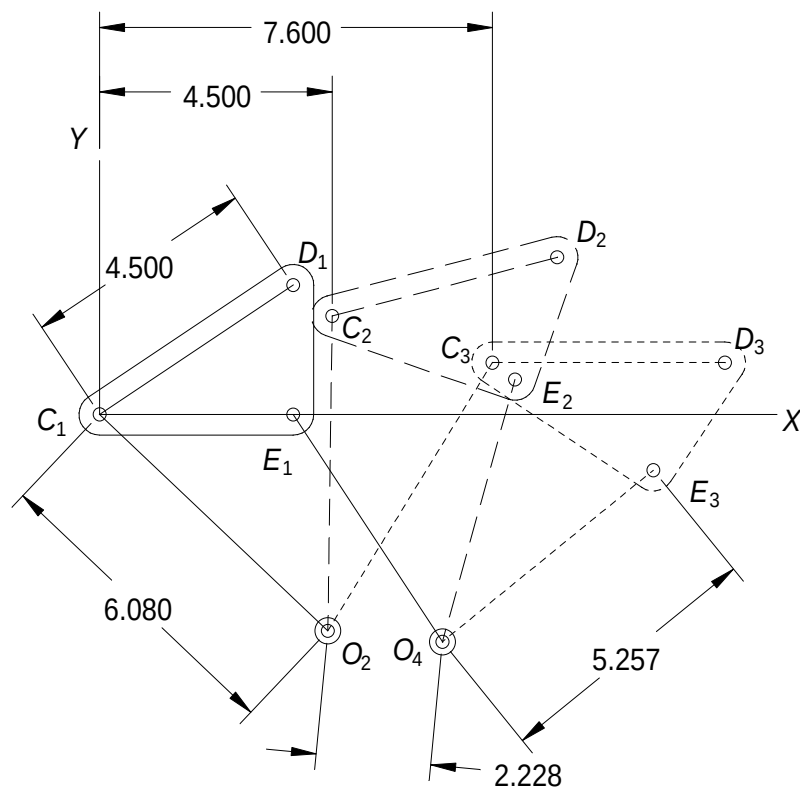
Crank angles:

$$\theta_{2i} = 142.019 \cdot \text{deg}$$

$$\theta_{2f} = 63.860 \cdot \text{deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. A driver dyad with a crank should be added to link 2 to control the motion of the fourbar so that it cannot move beyond positions 1 and 3.



PROBLEM 5-40

Statement: Design a fourbar linkage to carry the object in Figure P5-9 through the three positions shown in their numbered order using the fixed pivots shown. Determine the range of the transmission angle.

Given:

$P_{21x} := 4.500$	$P_{21y} := 1.900$	$P_{31x} := 7.600$	$P_{31y} := 1.000$
$O_{2x} := 2.900$	$O_{2y} := -5.100$	$O_{4x} := 5.900$	$O_{4y} := -5.100$
Body angles:	$\theta_{p1} := 33.70 \cdot \text{deg}$	$\theta_{p2} := 14.60 \cdot \text{deg}$	$\theta_{p3} := 0.0 \cdot \text{deg}$

Solution: See Figure P5-9 and Mathcad file P0540.

1. Determine the angle changes between precision points from the body angles given.

$$\begin{aligned}\alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -19.100 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= -33.700 \cdot \text{deg}\end{aligned}$$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$\begin{aligned}R_{1x} &:= -O_{2x} & R_{1x} &= -2.900 & R_{1y} &:= -O_{2y} & R_{1y} &= 5.100 \\ R_{2x} &:= R_{1x} + P_{21x} & R_{2x} &= 1.600 \\ R_{2y} &:= R_{1y} + P_{21y} & R_{2y} &= 7.000 \\ R_{3x} &:= R_{1x} + P_{31x} & R_{3x} &= 4.700 \\ R_{3y} &:= R_{1y} + P_{31y} & R_{3y} &= 6.100 \\ R_1 &:= \sqrt{R_{1x}^2 + R_{1y}^2} & R_1 &= 5.867 \\ R_2 &:= \sqrt{R_{2x}^2 + R_{2y}^2} & R_2 &= 7.181 \\ R_3 &:= \sqrt{R_{3x}^2 + R_{3y}^2} & R_3 &= 7.701\end{aligned}$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\begin{aligned}\zeta_1 &:= \text{atan2}(R_{1x}, R_{1y}) & \zeta_1 &= 119.624 \cdot \text{deg} \\ \zeta_2 &:= \text{atan2}(R_{2x}, R_{2y}) & \zeta_2 &= 77.125 \cdot \text{deg} \\ \zeta_3 &:= \text{atan2}(R_{3x}, R_{3y}) & \zeta_3 &= 52.386 \cdot \text{deg}\end{aligned}$$

4. Solve for β_2 and β_3 using equations 5.34

$$\begin{aligned}C_1 &:= R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) & C_1 &= 1.222 \\ C_2 &:= R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) & C_2 &= -0.710 \\ C_3 &:= R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) & C_3 &= -4.283 \\ C_4 &:= -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) & C_4 &= 0.248 \\ C_5 &:= R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) & C_5 &= -2.672\end{aligned}$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2)$$

$$C_6 = 1.232$$

$$A_1 := -C_3^2 - C_4^2$$

$$A_1 = -18.405$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$$

$$A_2 = -4.613$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$$

$$A_3 = -11.748$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$$

$$A_4 = 3.343$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$$

$$A_5 = 4.613$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$$

$$A_6 = -5.059$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$$

$$K_1 = 44.009$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$$

$$K_2 = -62.605$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$$

$$K_3 = 71.350$$

$$\beta_{31} := 2 \cdot \operatorname{atan}\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\beta_{31} = -33.700 \cdot \text{deg}$$

$$\beta_{32} := 2 \cdot \operatorname{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\beta_{32} = -76.089 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\beta_3 := \beta_{32}$$

$$\beta_{21} := \operatorname{acos}\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right)$$

$$\beta_{21} = 47.808 \cdot \text{deg}$$

$$\beta_{22} := \operatorname{asin}\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right)$$

$$\beta_{22} = -47.808 \cdot \text{deg}$$

Use the negative value,

$$\beta_2 := \beta_{22}$$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$R_{1x} := -O_{4x}$$

$$R_{1x} = -5.900$$

$$R_{1y} := -O_{4y}$$

$$R_{1y} = 5.100$$

$$R_{2x} := R_{1x} + P_{21x}$$

$$R_{2x} = -1.400$$

$$R_{2y} := R_{1y} + P_{21y}$$

$$R_{2y} = 7.000$$

$$R_{3x} := R_{1x} + P_{31x}$$

$$R_{3x} = 1.700$$

$$R_{3y} := R_{1y} + P_{31y}$$

$$R_{3y} = 6.100$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$$

$$R_1 = 7.799$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 7.139$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 6.332$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 139.160 \cdot \text{deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 101.310 \cdot \text{deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = 74.427 \cdot \text{deg}$$

7. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = 0.883$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = -1.393$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = -3.779$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = -1.417$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = -2.506$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = 0.250$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -16.286$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = -4.496$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = -9.117$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = 4.011$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = 4.496$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -5.310$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = 30.381$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = -60.441$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = 58.811$$

$$\gamma_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{31} = -33.700 \cdot \text{deg}$$

$$\gamma_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{32} = -92.928 \cdot \text{deg}$$

The first value is the same as α_3 , so use the second value

$$\gamma_3 := \gamma_{32}$$

$$\gamma_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1}\right)$$

$$\gamma_{21} = 55.029 \cdot \text{deg}$$

$$\gamma_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1}\right)$$

$$\gamma_{22} = -55.029 \cdot \text{deg}$$

Use the negative value ,

$$\gamma_2 := \gamma_{22}$$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2}$$

$$p_{21} = 4.885$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y})$$

$$\delta_2 = 22.891 \cdot \text{deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2}$$

$$p_{31} = 7.666$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y})$$

$$\delta_3 = 7.496 \cdot \text{deg}$$

9. Evaluate terms in the $\mathbf{WZ}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1$$

$$B := \sin(\beta_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the $\mathbf{W}$ and $\mathbf{Z}$ vectors are:

$$W1x = -5.043$$

$$W1y = 3.126$$

$$Z1x = 2.143$$

$$Z1y = 1.974$$

11. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2}$

$$w = 5.933$$

12. Evaluate terms in the $\mathbf{US}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$U1x = -2.773$$

$$U1y = 2.998$$

$$S1x = -3.127$$

$$S1y = 2.102$$

14. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2}$

$$u = 4.083$$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x$$

$$V1x = 5.271$$

$$V1y := Z1y - S1y$$

$$V1y = -0.128$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$

$$v = 5.272$$

$$G1x := W1x + V1x - U1x$$

$$G1x = 3.000$$

$$G1y := W1y + V1y - U1y$$

$$G1y = -2.176 \times 10^{-14}$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$

$$g = 3.000$$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors **W**₁, **Z**₁, **U**₁, and **S**₁.

$$O2x := -Z1x - W1x$$

$$O2x = 2.900$$

$$O2y := -Z1y - W1y$$

$$O2y = -5.100$$

$$O4x := -S1x - U1x$$

$$O4x = 5.900$$

$$O4y := -S1y - U1y$$

$$O4y = -5.100$$

These check with Figure P5-7.

17. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $z := \sqrt{Z1x^2 + Z1y^2}$

$$z = 2.914$$

$$r_P := z$$

Angle BAP (δ_p) $s := \sqrt{S1x^2 + S1y^2}$

$$s = 3.768$$

$$\psi := \text{atan2}(S1x, S1y)$$

$$\psi = 146.092 \cdot \text{deg}$$

$$\phi := \text{atan2}(Z1x, Z1y)$$

$$\phi = 42.651 \cdot \text{deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = -1.392 \cdot \text{deg}$$

$$\delta_p := \phi - \theta_3$$

$$\delta_p = 44.042 \cdot \text{deg}$$

18. DESIGN SUMMARY

Link 1: $g = 3.000$

Link 2: $w = 5.933$

Link 3: $v = 5.272$

Link 4: $u = 4.083$

Coupler point: $r_p = 2.914$ $\delta_p = 44.042 \cdot \text{deg}$

19. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

20. A driver dyad with a crank should be added to link 2 to control the motion of the fourbar so that it cannot move beyond positions 1 and 3.

PROBLEM 5-41

Statement: Design a fourbar linkage to carry the object in Figure P5-10 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use points *C* and *D* for your attachment points. Determine the range of the transmission angle.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to C_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= -0.743 & P_{2y} &:= 1.514 \\ P_{3x} &:= -1.750 & P_{3y} &:= 2.228 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 62.59 \cdot \text{deg} \quad \theta_{p2} := 68.25 \cdot \text{deg} \quad \theta_{p3} := 90.0 \cdot \text{deg}$$

Coordinates of the points C_1 and D_1 with respect to P_1 :

$$C_{1x} := 0.0 \quad C_{1y} := 0.0 \quad D_{1x} := 1.036 \quad D_{1y} := 1.998$$

Solution: See Figure P5-10 and Mathcad file P0541.

1. Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 1.686 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= 116.140 \text{ deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 2.833 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= 128.148 \text{ deg} \end{aligned}$$

2. Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= 5.660 \text{ deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= 27.410 \text{ deg} \end{aligned}$$

3. Using Figure P5-9, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$z := \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} \quad z = 0.000$$

$$s := \sqrt{(P_{1x} - D_{1x})^2 + (P_{1y} - D_{1y})^2} \quad s = 2.251$$

$$v := \sqrt{(C_{1x} - D_{1x})^2 + (C_{1y} - D_{1y})^2} \quad v = 2.251$$

$$\phi := \theta_{p1} \quad \phi = 62.590 \text{ deg}$$

$$\psi := \theta_{p1} + \pi \quad \psi = 242.590 \text{ deg}$$

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = 0.000$$

$$Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = 0.000$$

$$S_{1x} := -s \cdot \cos(\theta_{p1}) \quad S_{1x} = -1.036$$

$$S_{1y} := -s \cdot \sin(\theta_{p1}) \quad S_{1y} = -1.998$$

4. Use equations 5.24 to solve for w , θ , β_2 , and β_3 . Since the points *C* and *D* are to be used as pivots, z and ϕ are known from the calculations above. Use guess values from a graphical solution such as that for Problem 3-61.

Guess: $W_{1x} := 3$ $W_{1y} := 0.5$ $\beta_2 := 33.3 \cdot \text{deg}$ $\beta_3 := 57.0 \cdot \text{deg}$

Given $W_{1x}(\cos(\beta_2) - 1) - W_{1y}\sin(\beta_2) \dots = p_{21} \cdot \cos(\delta_2)$
 $+ Z_{1x}(\cos(\alpha_2) - 1) - Z_{1y}\sin(\alpha_2)$

$W_{1x}(\cos(\beta_3) - 1) - W_{1y}\sin(\beta_3) \dots = p_{31} \cdot \cos(\delta_3)$
 $+ Z_{1x}(\cos(\alpha_3) - 1) - Z_{1y}\sin(\alpha_3)$

$W_{1y}(\cos(\beta_2) - 1) + W_{1x}\sin(\beta_2) \dots = p_{21} \cdot \sin(\delta_2)$
 $+ Z_{1y}(\cos(\alpha_2) - 1) + Z_{1x}\sin(\alpha_2)$

$W_{1y}(\cos(\beta_3) - 1) + W_{1x}\sin(\beta_3) \dots = p_{31} \cdot \sin(\delta_3)$
 $+ Z_{1y}(\cos(\alpha_3) - 1) + Z_{1x}\sin(\alpha_3)$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := \text{Find}(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$\beta_2 = 33.028 \text{ deg}$ $\beta_3 = 57.045 \text{ deg}$

The components of the **W** vector are:

$W_{1x} = 2.925$ $W_{1y} = 0.496$ $\theta := \text{atan2}(W_{1x}, W_{1y})$ $\theta = 9.626 \text{ deg}$

The length of link 2 is: $w := \sqrt{(W_{1x}^2 + W_{1y}^2)}$, $w = 2.967$

5. Use equations 5.28 to solve for u , σ , γ_2 , and γ_3 . Since the points *C* and *D* are to be used as pivots, s and ψ are known from the calculations above.

Guess: $U_{1x} := 3.2$ $U_{1y} := 0.8$ $\gamma_2 := 32.3 \cdot \text{deg}$ $\gamma_3 := 68.4 \cdot \text{deg}$

Given $U_{1x}(\cos(\gamma_2) - 1) - U_{1y}\sin(\gamma_2) \dots = p_{21} \cdot \cos(\delta_2)$
 $+ S_{1x}(\cos(\alpha_2) - 1) - S_{1y}\sin(\alpha_2)$

$U_{1x}(\cos(\gamma_3) - 1) - U_{1y}\sin(\gamma_3) \dots = p_{31} \cdot \cos(\delta_3)$
 $+ S_{1x}(\cos(\alpha_3) - 1) - S_{1y}\sin(\alpha_3)$

$U_{1y}(\cos(\gamma_2) - 1) + U_{1x}\sin(\gamma_2) \dots = p_{21} \cdot \sin(\delta_2)$
 $+ S_{1y}(\cos(\alpha_2) - 1) + S_{1x}\sin(\alpha_2)$

$U_{1y}(\cos(\gamma_3) - 1) + U_{1x}\sin(\gamma_3) \dots = p_{31} \cdot \sin(\delta_3)$
 $+ S_{1y}(\cos(\alpha_3) - 1) + S_{1x}\sin(\alpha_3)$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := \text{Find}(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$\gamma_2 = 32.559 \text{ deg}$ $\gamma_3 = 68.259 \text{ deg}$

The components of the **U** vector are:

$$U_{Ix} = 3.223 \quad U_{Iy} = 0.815 \quad \sigma := \text{atan2}(U_{Ix}, U_{Iy}) \quad \sigma = 14.189 \text{ deg}$$

The length of link 4 is: $u := \sqrt{U_{Ix}^2 + U_{Iy}^2}$, $u = 3.324$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\text{Link 3: } V_{Ix} := Z_{Ix} - S_{Ix} \quad V_{Ix} = 1.036$$

$$V_{Iy} := Z_{Iy} - S_{Iy} \quad V_{Iy} = 1.998$$

$$\theta_3 := \text{atan2}(V_{Ix}, V_{Iy}) \quad \theta_3 = 62.590 \text{ deg}$$

$$v := \sqrt{V_{Ix}^2 + V_{Iy}^2} \quad v = 2.251$$

$$\text{Link 1: } G_{Ix} := W_{Ix} + V_{Ix} - U_{Ix} \quad G_{Ix} = 0.738$$

$$G_{Iy} := W_{Iy} + V_{Iy} - U_{Iy} \quad G_{Iy} = 1.679$$

$$\theta_1 := \text{atan2}(G_{Ix}, G_{Iy}) \quad \theta_1 = 66.275 \text{ deg}$$

$$g := \sqrt{G_{Ix}^2 + G_{Iy}^2} \quad g = 1.834$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \quad \theta_{2i} = -56.650 \text{ deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_3 \quad \theta_{2f} = 0.395 \text{ deg}$$

8. Define the coupler point with respect to point C and the vector **V**.

$$r_p := z \quad \delta_p := \phi - \theta_3$$

$$r_p = 0.000 \quad \delta_p = 0.000 \text{ deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta) \quad O_{2x} = -2.925$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta) \quad O_{2y} = -0.496$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma) \quad O_{4x} = -2.187$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma) \quad O_{4y} = 1.183$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = 66.275 \text{ deg}$$

11. Determine the Grashof condition.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof"   if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Condition(g,u,v,w) = "Grashof"

12. DESIGN SUMMARY

Link 2:	$w = 2.967$	$\theta = 9.626 \text{ deg}$
Link 3:	$v = 2.251$	$\theta_3 = 62.590 \text{ deg}$
Link 4:	$u = 3.324$	$\sigma = 14.189 \text{ deg}$
Link 1:	$g = 1.834$	$\theta_1 = 66.275 \text{ deg}$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000 \text{ deg}$

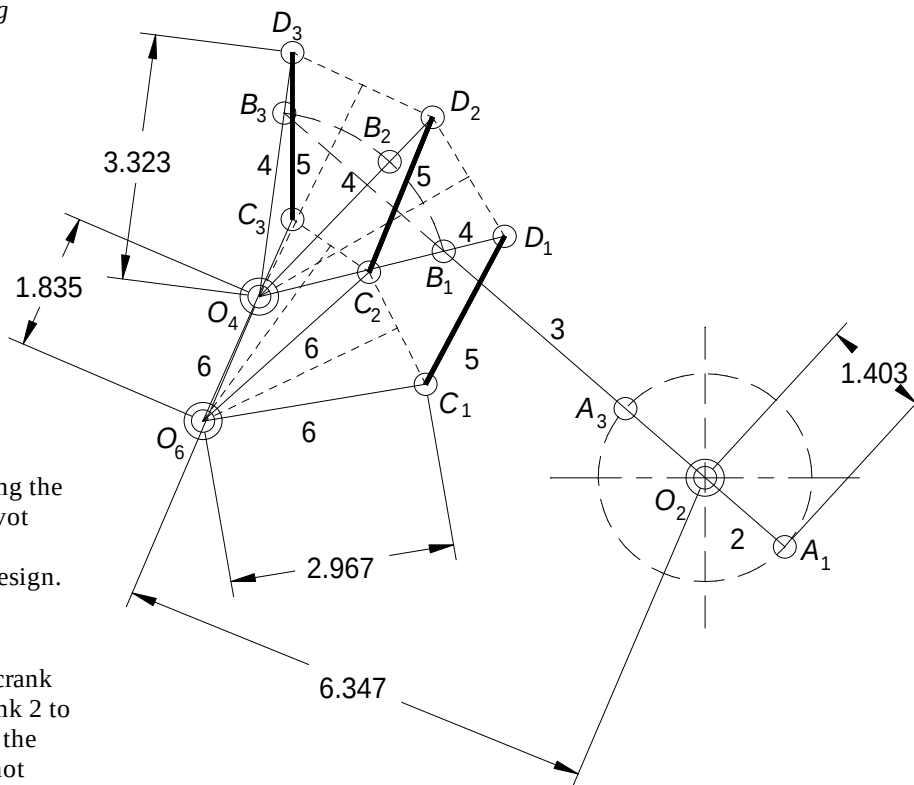
Crank angles:

$$\theta_{2i} = -56.650 \text{ deg}$$

$$\theta_{2f} = 0.395 \text{ deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. A driver dyad with a crank should be added to link 2 to control the motion of the fourbar so that it cannot move beyond positions 1 and 3.



PROBLEM 5-42

Statement: Design a fourbar linkage to carry the object in Figure P5-10 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use any points on the object as attachment points. Determine the range of the transmission angle.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to C_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= -0.743 & P_{2y} &:= 1.514 \\ P_{3x} &:= -1.750 & P_{3y} &:= 2.228 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 62.59 \cdot \text{deg} \quad \theta_{p2} := 68.25 \cdot \text{deg} \quad \theta_{p3} := 90.0 \cdot \text{deg}$$

Coordinates of the points C_1 and E_1 (used for attachment) with respect to P_1 :

$$C_{1x} := 0.0 \quad C_{1y} := 0.0 \quad E_{1x} := 1.036 \quad E_{1y} := 0.000$$

Solution: See Figure P5-10 and Mathcad file P0542.

1. Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 1.686 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= 116.140 \text{ deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 2.833 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= 128.148 \text{ deg} \end{aligned}$$

2. Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= 5.660 \text{ deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= 27.410 \text{ deg} \end{aligned}$$

3. Using Figure P5-9, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$\begin{aligned} z &:= \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} & z &= 0.000 \\ s &:= \sqrt{(P_{1x} - E_{1x})^2 + (P_{1y} - E_{1y})^2} & s &= 1.036 \\ v &:= \sqrt{(C_{1x} - E_{1x})^2 + (C_{1y} - E_{1y})^2} & v &= 1.036 \\ \phi &:= \text{atan2}(E_{1x} - C_{1x}, E_{1y} - C_{1y}) & \phi &= 0.000 \text{ deg} \\ \psi &:= \phi + \pi & \psi &= 180.000 \text{ deg} \\ Z_{1x} &:= z \cdot \cos(\phi) & Z_{1x} &= 0.000 \\ Z_{1y} &:= z \cdot \sin(\phi) & Z_{1y} &= 0.000 \\ S_{1x} &:= s \cdot \cos(\psi) & S_{1x} &= -1.036 \\ S_{1y} &:= s \cdot \sin(\psi) & S_{1y} &= 0.000 \end{aligned}$$

4. Use equations 5.24 to solve for w , θ , β_2 , and β_3 . Since the points C and D are to be used as pivots, z and ϕ are known from the calculations above. Use guess values from a graphical solution such as that for Problem 3-61.

$$\text{Guess:} \quad W_{1x} := 3 \quad W_{1y} := 0.5 \quad \beta_2 := 33.3 \cdot \text{deg} \quad \beta_3 := 57.0 \cdot \text{deg}$$

$$\text{Given} \quad \begin{aligned} W_{1x}(\cos(\beta_2) - 1) - W_{1y}\sin(\beta_2) \dots &= p_{21}\cos(\delta_2) \\ + Z_{1x}(\cos(\alpha_2) - 1) - Z_{1y}\sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} W_{1x}(\cos(\beta_3) - 1) - W_{1y}\sin(\beta_3) \dots &= p_{31}\cos(\delta_3) \\ + Z_{1x}(\cos(\alpha_3) - 1) - Z_{1y}\sin(\alpha_3) \end{aligned}$$

$$\begin{aligned} W_{1y}(\cos(\beta_2) - 1) + W_{1x}\sin(\beta_2) \dots &= p_{21}\sin(\delta_2) \\ + Z_{1y}(\cos(\alpha_2) - 1) + Z_{1x}\sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} W_{1y}(\cos(\beta_3) - 1) + W_{1x}\sin(\beta_3) \dots &= p_{31}\sin(\delta_3) \\ + Z_{1y}(\cos(\alpha_3) - 1) + Z_{1x}\sin(\alpha_3) \end{aligned}$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := \text{Find}(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$$\beta_2 = 33.028 \text{ deg} \quad \beta_3 = 57.045 \text{ deg}$$

The components of the **W** vector are:

$$W_{1x} = 2.925 \quad W_{1y} = 0.496 \quad \theta := \text{atan2}(W_{1x}, W_{1y}) \quad \theta = 9.626 \text{ deg}$$

$$\text{The length of link 2 is: } w := \sqrt{(W_{1x}^2 + W_{1y}^2)}, \quad w = 2.967$$

5. Use equations 5.28 to solve for u , σ , γ_2 , and γ_3 . Since the points *C* and *D* are to be used as pivots, s and ψ are known from the calculations above.

$$\text{Guess:} \quad U_{1x} := 3.2 \quad U_{1y} := 0.8 \quad \gamma_2 := 32.3 \text{ deg} \quad \gamma_3 := 68.4 \text{ deg}$$

$$\text{Given} \quad \begin{aligned} U_{1x}(\cos(\gamma_2) - 1) - U_{1y}\sin(\gamma_2) \dots &= p_{21}\cos(\delta_2) \\ + S_{1x}(\cos(\alpha_2) - 1) - S_{1y}\sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} U_{1x}(\cos(\gamma_3) - 1) - U_{1y}\sin(\gamma_3) \dots &= p_{31}\cos(\delta_3) \\ + S_{1x}(\cos(\alpha_3) - 1) - S_{1y}\sin(\alpha_3) \end{aligned}$$

$$\begin{aligned} U_{1y}(\cos(\gamma_2) - 1) + U_{1x}\sin(\gamma_2) \dots &= p_{21}\sin(\delta_2) \\ + S_{1y}(\cos(\alpha_2) - 1) + S_{1x}\sin(\alpha_2) \end{aligned}$$

$$\begin{aligned} U_{1y}(\cos(\gamma_3) - 1) + U_{1x}\sin(\gamma_3) \dots &= p_{31}\sin(\delta_3) \\ + S_{1y}(\cos(\alpha_3) - 1) + S_{1x}\sin(\alpha_3) \end{aligned}$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := \text{Find}(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$$\gamma_2 = 22.323 \text{ deg} \quad \gamma_3 = 41.857 \text{ deg}$$

The components of the **U** vector are:

$$U_{1x} = 4.470 \quad U_{1y} = 1.088 \quad \sigma := \text{atan2}(U_{1x}, U_{1y}) \quad \sigma = 13.676 \text{ deg}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 4.600$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $V_{1x} = 1.036$

$V_{1y} := Z_{1y} - S_{1y}$ $V_{1y} = 0.000$

$\theta_3 := \text{atan2}(V_{1x}, V_{1y})$ $\theta_3 = 0.000 \text{ deg}$

$v := \sqrt{V_{1x}^2 + V_{1y}^2}$ $v = 1.036$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = -0.509$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = -0.591$

$\theta_1 := \text{atan2}(G_{1x}, G_{1y})$ $\theta_1 = -130.699 \text{ deg}$

$g := \sqrt{G_{1x}^2 + G_{1y}^2}$ $g = 0.780$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = 140.325 \text{ deg}$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = 197.370 \text{ deg}$

8. Define the coupler point with respect to point C and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 0.000$ $\delta_p = 0.000 \text{ deg}$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta)$ $O_{2x} = -2.925$

$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta)$ $O_{2y} = -0.496$

$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma)$ $O_{4x} = -3.434$

$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma)$ $O_{4y} = -1.088$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})]$ $\theta_{rot} = -130.699 \text{ deg}$

11. Determine the Grashof condition.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Condition(g,u,v,w) = "non-Grashof"

12. DESIGN SUMMARY

Link 2:	w = 2.967	$\theta = 9.626 \text{ deg}$
Link 3:	v = 1.036	$\theta_3 = 0.000 \text{ deg}$
Link 4:	u = 4.600	$\sigma = 13.676 \text{ deg}$
Link 1:	g = 0.780	$\theta_1 = -130.699 \text{ deg}$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000 \text{ deg}$

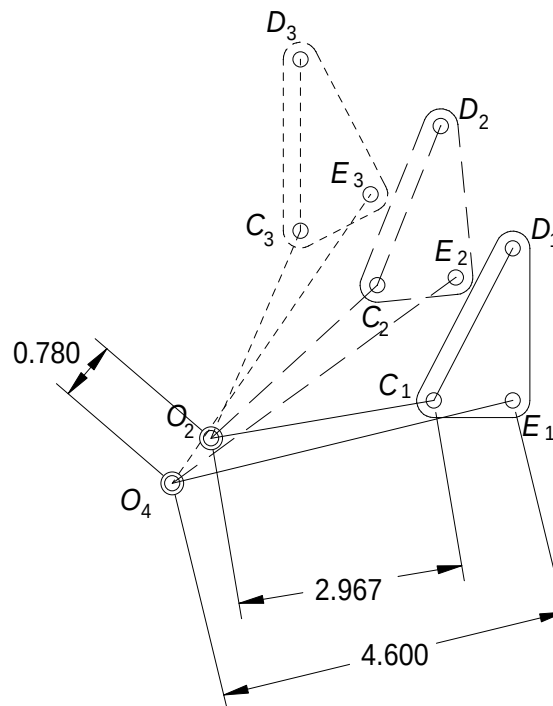
Crank angles:

$$\theta_{2i} = 140.325 \text{ deg}$$

$$\theta_{2f} = 197.370 \text{ deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. A driver dyad with a crank should be added to link 2 to control the motion of the fourbar so that it cannot move beyond positions 1 and 3.



PROBLEM 5-43

Statement: Design a fourbar linkage to carry the object in Figure P5-10 through the three positions shown in their numbered order using the fixed pivots shown. Determine the range of the transmission angle. Add a driver dyad with a crank to control the motion of your fourbar so that it cannot move beyond positions 1 and 3.

Given:

$P_{21x} := -0.743$	$P_{21y} := 1.514$	$P_{31x} := -1.750$	$P_{31y} := 2.228$
$O_{2x} := -3.100$	$O_{2y} := -1.200$	$O_{4x} := -0.100$	$O_{4y} := -1.200$
Body angles:	$\theta_{p1} := 62.59 \cdot \text{deg}$	$\theta_{p2} := 68.25 \cdot \text{deg}$	$\theta_{p3} := 90.0 \cdot \text{deg}$

Solution: See Figure P5-10 and Mathcad file P0543.

1. Determine the angle changes between precision points from the body angles given.

$$\begin{aligned}\alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= 5.660 \text{ deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= 27.410 \text{ deg}\end{aligned}$$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$\begin{aligned}R_{1x} &:= -O_{2x} & R_{1x} &= 3.100 & R_{1y} &:= -O_{2y} & R_{1y} &= 1.200 \\ R_{2x} &:= R_{1x} + P_{21x} & R_{2x} &= 2.357 \\ R_{2y} &:= R_{1y} + P_{21y} & R_{2y} &= 2.714 \\ R_{3x} &:= R_{1x} + P_{31x} & R_{3x} &= 1.350 \\ R_{3y} &:= R_{1y} + P_{31y} & R_{3y} &= 3.428 \\ R_1 &:= \sqrt{R_{1x}^2 + R_{1y}^2} & R_1 &= 3.324 \\ R_2 &:= \sqrt{R_{2x}^2 + R_{2y}^2} & R_2 &= 3.595 \\ R_3 &:= \sqrt{R_{3x}^2 + R_{3y}^2} & R_3 &= 3.684\end{aligned}$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\begin{aligned}\zeta_1 &:= \text{atan2}(R_{1x}, R_{1y}) & \zeta_1 &= 21.161 \text{ deg} \\ \zeta_2 &:= \text{atan2}(R_{2x}, R_{2y}) & \zeta_2 &= 49.027 \text{ deg} \\ \zeta_3 &:= \text{atan2}(R_{3x}, R_{3y}) & \zeta_3 &= 68.505 \text{ deg}\end{aligned}$$

4. Solve for β_2 and β_3 using equations 5.34

$$\begin{aligned}C_1 &:= R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) & C_1 &= 0.162 \\ C_2 &:= R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) & C_2 &= 0.050 \\ C_3 &:= R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) & C_3 &= 0.850 \\ C_4 &:= -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) & C_4 &= 0.936 \\ C_5 &:= R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) & C_5 &= 0.610\end{aligned}$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2)$$

$$C_6 = 1.214$$

$$A_1 := -C_3^2 - C_4^2$$

$$A_1 = -1.597$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$$

$$A_2 = 0.461$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$$

$$A_3 = -1.654$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$$

$$A_4 = 0.194$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$$

$$A_5 = -0.461$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$$

$$A_6 = 0.091$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$$

$$K_1 = -0.061$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$$

$$K_2 = -0.364$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$$

$$K_3 = -0.221$$

$$\beta_{31} := 2 \cdot \operatorname{atan}\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\beta_{31} = 27.410 \text{ deg}$$

$$\beta_{32} := 2 \cdot \operatorname{atan}\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\beta_{32} = 133.549 \text{ deg}$$

The first value is the same as α_3 so use the second value

$$\beta_3 := \beta_{32}$$

$$\beta_{21} := \operatorname{acos}\left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1}\right)$$

$$\beta_{21} = 124.137 \text{ deg}$$

$$\beta_{22} := \operatorname{asin}\left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1}\right)$$

$$\beta_{22} = 55.863 \text{ deg}$$

Use the first value,

$$\beta_2 := \beta_{21}$$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$R_{1x} := -O_{4x}$$

$$R_{1x} = 0.100$$

$$R_{1y} := -O_{4y}$$

$$R_{1y} = 1.200$$

$$R_{2x} := R_{1x} + P_{21x}$$

$$R_{2x} = -0.643$$

$$R_{2y} := R_{1y} + P_{21y}$$

$$R_{2y} = 2.714$$

$$R_{3x} := R_{1x} + P_{31x}$$

$$R_{3x} = -1.650$$

$$R_{3y} := R_{1y} + P_{31y}$$

$$R_{3y} = 3.428$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$$

$$R_1 = 1.204$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 2.789$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 3.804$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 85.236 \text{ deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 103.329 \text{ deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = 115.703 \text{ deg}$$

7. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = -0.160$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = 1.135$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = 1.186$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = 2.317$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = 0.624$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = 1.510$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -6.774$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = 0.345$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = -4.239$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = 0.977$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = -0.345$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -2.820$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = 12.289$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = -3.165$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = 9.452$$

$$\gamma_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{31} = 27.410 \text{ deg}$$

$$\gamma_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{32} = -56.299 \text{ deg}$$

The first value is the same as α_3 , so use the second value

$$\gamma_3 := \gamma_{32}$$

$$\gamma_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1}\right)$$

$$\gamma_{21} = 43.866 \text{ deg}$$

$$\gamma_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1}\right)$$

$$\gamma_{22} = -43.866 \text{ deg}$$

Use the negative value ,

$$\gamma_2 := \gamma_{22}$$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2}$$

$$p_{21} = 1.686$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y})$$

$$\delta_2 = 116.140 \text{ deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2}$$

$$p_{31} = 2.833$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y})$$

$$\delta_3 = 128.148 \text{ deg}$$

9. Evaluate terms in the $\mathbf{WZ}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1$$

$$B := \sin(\beta_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the $\mathbf{W}$ and $\mathbf{Z}$ vectors are:

$$W1x = 0.621$$

$$W1y = -0.489$$

$$Z1x = 2.479$$

$$Z1y = 1.689$$

11. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2}$

$$w = 0.790$$

12. Evaluate terms in the $\mathbf{US}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$U1x = -1.459$$

$$U1y = -1.294$$

$$S1x = 1.559$$

$$S1y = 2.494$$

14. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2}$ $u = 1.950$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x$$

$$V1x = 0.920$$

$$V1y := Z1y - S1y$$

$$V1y = -0.805$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$ $v = 1.222$

$$G1x := W1x + V1x - U1x$$

$$G1x = 3.000$$

$$G1y := W1y + V1y - U1y$$

$$G1y = 0.000$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$ $g = 3.000$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors **W**₁, **Z**₁, **U**₁, and **S**₁.

$$O2x := -Z1x - W1x$$

$$O2x = -3.100$$

$$O2y := -Z1y - W1y$$

$$O2y = -1.200$$

$$O4x := -S1x - U1x$$

$$O4x = -0.100$$

$$O4y := -S1y - U1y$$

$$O4y = -1.200$$

These check with Figure P5-7.

17. Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $z := \sqrt{Z1x^2 + Z1y^2}$ $z = 3.000$ $r_P := z$

Angle BAP (δ_p) $s := \sqrt{S1x^2 + S1y^2}$ $s = 2.941$

$$\psi := \text{atan2}(S1x, S1y) \quad \psi = 57.983 \text{ deg}$$

$$\phi := \text{atan2}(Z1x, Z1y) \quad \phi = 34.270 \text{ deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = -41.182 \text{ deg}$$

$$\delta_p := \phi - \theta_3$$

$$\delta_p = 75.452 \text{ deg}$$

18. DESIGN SUMMARY

$$\text{Link 1:} \quad g = 3.000$$

$$\text{Link 2:} \quad w = 0.790$$

$$\text{Link 3:} \quad v = 1.222$$

$$\text{Link 4:} \quad u = 1.950$$

$$\text{Coupler point:} \quad r_p = 3.000 \quad \delta_p = 75.452 \text{ deg}$$

19. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct. However, this mechanism as designed has a branch defect. The three positions can only be reached by changing the links from a crossed circuit to an open circuit.

PROBLEM 5-44

Statement: Design a fourbar linkage to carry the object in Figure P5-11 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use points *C* and *D* for your attachment points. Determine the range of the transmission angle.

Given: Coordinates of the points P_1, P_2 and P_3 with respect to C_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= -2.332 & P_{2y} &:= -0.311 \\ P_{3x} &:= -2.751 & P_{3y} &:= -2.015 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 45.0 \cdot \text{deg} \quad \theta_{p2} := 24.14 \cdot \text{deg} \quad \theta_{p3} := 86.84 \cdot \text{deg}$$

Coordinates of the points C_1 and D_1 with respect to P_1 :

$$C_{1x} := 0.0 \quad C_{1y} := 0.0 \quad D_{1x} := 1.591 \quad D_{1y} := 1.591$$

Solution: See Figure P5-11 and Mathcad file P0544.

- Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 2.353 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= -172.404 \text{ deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 3.410 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= -143.779 \text{ deg} \end{aligned}$$

- Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -20.860 \text{ deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= 41.840 \text{ deg} \end{aligned}$$

- Using Figure P5-9, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$\begin{aligned} z &:= \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} & z &= 0.000 \\ s &:= \sqrt{(P_{1x} - D_{1x})^2 + (P_{1y} - D_{1y})^2} & s &= 2.250 \\ v &:= \sqrt{(C_{1x} - D_{1x})^2 + (C_{1y} - D_{1y})^2} & v &= 2.250 \\ \phi &:= \theta_{p1} & \phi &= 45.000 \text{ deg} \\ \psi &:= \theta_{p1} + \pi & \psi &= 225.000 \text{ deg} \\ Z_{1x} &:= z \cdot \cos(\phi) & Z_{1x} &= 0.000 \\ Z_{1y} &:= z \cdot \sin(\phi) & Z_{1y} &= 0.000 \\ S_{1x} &:= -s \cdot \cos(\theta_{p1}) & S_{1x} &= -1.591 \\ S_{1y} &:= -s \cdot \sin(\theta_{p1}) & S_{1y} &= -1.591 \end{aligned}$$

- Use equations 5.24 to solve for w , θ , β_2 , and β_3 . Since the points *C* and *D* are to be used as pivots, z and ϕ are known from the calculations above. Use guess values from a graphical solution such as that for Problem 3-65.

Guess: $W_{Ix} := -1.75$ $W_{Iy} := -0.47$ $\beta_2 := 81 \cdot \text{deg}$ $\beta_3 := 138 \cdot \text{deg}$

Given $W_{Ix} \cdot (\cos(\beta_2) - 1) - W_{Iy} \cdot \sin(\beta_2) \dots = p_{21} \cdot \cos(\delta_2)$
 $+ Z_{Ix} \cdot (\cos(\alpha_2) - 1) - Z_{Iy} \cdot \sin(\alpha_2)$

$W_{Ix} \cdot (\cos(\beta_3) - 1) - W_{Iy} \cdot \sin(\beta_3) \dots = p_{31} \cdot \cos(\delta_3)$
 $+ Z_{Ix} \cdot (\cos(\alpha_3) - 1) - Z_{Iy} \cdot \sin(\alpha_3)$

$W_{Iy} \cdot (\cos(\beta_2) - 1) + W_{Ix} \cdot \sin(\beta_2) \dots = p_{21} \cdot \sin(\delta_2)$
 $+ Z_{Iy} \cdot (\cos(\alpha_2) - 1) + Z_{Ix} \cdot \sin(\alpha_2)$

$W_{Iy} \cdot (\cos(\beta_3) - 1) + W_{Ix} \cdot \sin(\beta_3) \dots = p_{31} \cdot \sin(\delta_3)$
 $+ Z_{Iy} \cdot (\cos(\alpha_3) - 1) + Z_{Ix} \cdot \sin(\alpha_3)$

$$\begin{pmatrix} W_{Ix} \\ W_{Iy} \\ \beta_2 \\ \beta_3 \end{pmatrix} := \text{Find}(W_{Ix}, W_{Iy}, \beta_2, \beta_3)$$

$$\beta_2 = 79.928 \text{ deg}$$

$$\beta_3 = 137.178 \text{ deg}$$

The components of the **W** vector are:

$$W_{Ix} = 0.980$$

$$W_{Iy} = 1.547$$

$$\theta := \text{atan2}(W_{Ix}, W_{Iy})$$

$$\theta = 57.632 \text{ deg}$$

The length of link 2 is: $w := \sqrt{W_{Ix}^2 + W_{Iy}^2}$, $w = 1.831$

5. Use equations 5.28 to solve for u , σ , γ_2 , and γ_3 . Since the points *C* and *D* are to be used as pivots, s and ψ are known from the calculations above.

Guess: $U_{Ix} := -0.1$ $U_{Iy} := -7.0$ $\gamma_2 := -17.5 \cdot \text{deg}$ $\gamma_3 := -37 \cdot \text{deg}$

Given $U_{Ix} \cdot (\cos(\gamma_2) - 1) - U_{Iy} \cdot \sin(\gamma_2) \dots = p_{21} \cdot \cos(\delta_2)$
 $+ S_{Ix} \cdot (\cos(\alpha_2) - 1) - S_{Iy} \cdot \sin(\alpha_2)$

$U_{Ix} \cdot (\cos(\gamma_3) - 1) - U_{Iy} \cdot \sin(\gamma_3) \dots = p_{31} \cdot \cos(\delta_3)$
 $+ S_{Ix} \cdot (\cos(\alpha_3) - 1) - S_{Iy} \cdot \sin(\alpha_3)$

$U_{Iy} \cdot (\cos(\gamma_2) - 1) + U_{Ix} \cdot \sin(\gamma_2) \dots = p_{21} \cdot \sin(\delta_2)$
 $+ S_{Iy} \cdot (\cos(\alpha_2) - 1) + S_{Ix} \cdot \sin(\alpha_2)$

$U_{Iy} \cdot (\cos(\gamma_3) - 1) + U_{Ix} \cdot \sin(\gamma_3) \dots = p_{31} \cdot \sin(\delta_3)$
 $+ S_{Iy} \cdot (\cos(\alpha_3) - 1) + S_{Ix} \cdot \sin(\alpha_3)$

$$\begin{pmatrix} U_{Ix} \\ U_{Iy} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := \text{Find}(U_{Ix}, U_{Iy}, \gamma_2, \gamma_3)$$

$$\gamma_2 = -17.457 \text{ deg}$$

$$\gamma_3 = -37.139 \text{ deg}$$

The components of the **U** vector are:

$$U_{1x} = 4.132 \quad U_{1y} = -5.598 \quad \sigma := \text{atan2}(U_{1x}, U_{1y}) \quad \sigma = -53.567 \text{ deg}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 6.958$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\text{Link 3: } V_{1x} := Z_{1x} - S_{1x} \quad V_{1x} = 1.591$$

$$V_{1y} := Z_{1y} - S_{1y} \quad V_{1y} = 1.591$$

$$\theta_3 := \text{atan2}(V_{1x}, V_{1y}) \quad \theta_3 = 45.000 \text{ deg}$$

$$v := \sqrt{V_{1x}^2 + V_{1y}^2} \quad v = 2.250$$

$$\text{Link 1: } G_{1x} := W_{1x} + V_{1x} - U_{1x} \quad G_{1x} = -1.561$$

$$G_{1y} := W_{1y} + V_{1y} - U_{1y} \quad G_{1y} = 8.736$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y}) \quad \theta_1 = 100.130 \text{ deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2} \quad g = 8.874$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \quad \theta_{2i} = -42.498 \text{ deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_3 \quad \theta_{2f} = 94.681 \text{ deg}$$

8. Define the coupler point with respect to point **C** and the vector **V**.

$$r_p := z \quad \delta_p := \phi - \theta_3$$

$$r_p = 0.000 \quad \delta_p = 0.000 \text{ deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta) \quad O_{2x} = -0.980$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta) \quad O_{2y} = -1.547$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma) \quad O_{4x} = -2.541$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma) \quad O_{4y} = 7.189$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = 100.130 \text{ deg}$$

11. Determine the Grashof condition.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

Condition(g,u,v,w) = "non-Grashof"

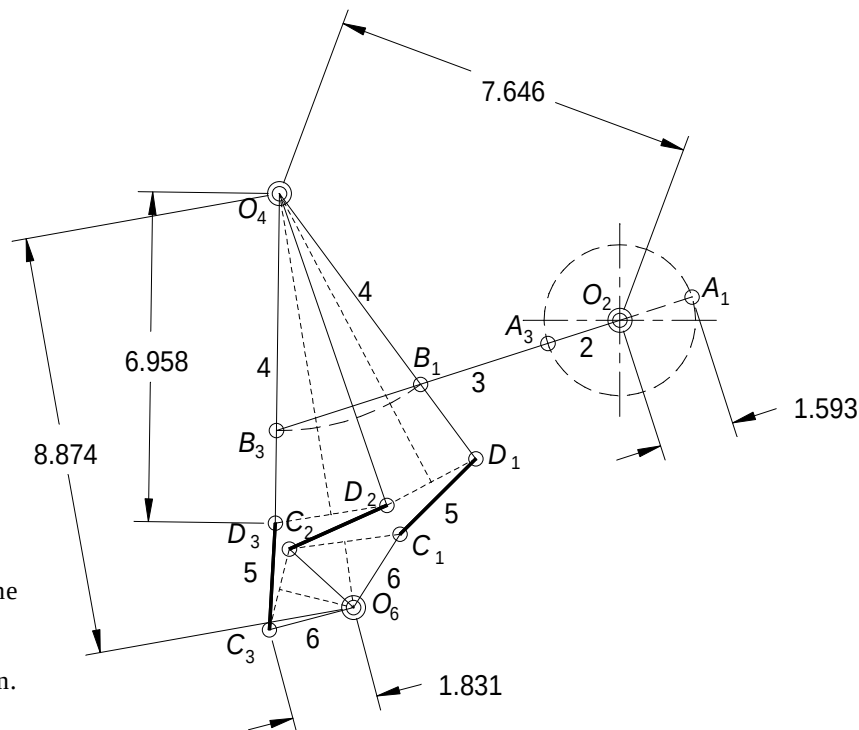
12. DESIGN SUMMARY

Link 2:	w = 1.831	$\theta = 57.632 \text{ deg}$
Link 3:	v = 2.250	$\theta_3 = 45.000 \text{ deg}$
Link 4:	u = 6.958	$\sigma = -53.567 \text{ deg}$
Link 1:	g = 8.874	$\theta_1 = 100.130 \text{ deg}$
Coupler:	$r_p = 0.000$	$\delta_p = 0.000 \text{ deg}$

Crank angles:

$$\theta_{2i} = -42.498 \text{ deg}$$

$$\theta_{2f} = 94.681 \text{ deg}$$



13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.

14. A driver dyad with a crank should be added to link 2 to control the motion of the fourbar so that it cannot move beyond positions 1 and 3.

PROBLEM 5-45

Statement: Design a fourbar linkage to carry the object in Figure P5-11 through the three positions shown in their numbered order without regard for the fixed pivots shown. Use any points on the object as attachment points. Determine the range of the transmission angle.

Given: Coordinates of the points P_1 , P_2 and P_3 with respect to C_1 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= -2.332 & P_{2y} &:= -0.311 \\ P_{3x} &:= -2.751 & P_{3y} &:= -2.015 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 45.0 \cdot \text{deg} \quad \theta_{p2} := 24.14 \cdot \text{deg} \quad \theta_{p3} := 86.84 \cdot \text{deg}$$

Coordinates of the points C_1 and E_1 (used for attachment) with respect to P_1 :

$$C_{1x} := 0.0 \quad C_{1y} := 0.0 \quad E_{1x} := 1.591 \quad E_{1y} := 0.000$$

Solution: See Figure P5-11 and Mathcad file P0545.

1. Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 2.353 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= -172.404 \text{ deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 3.410 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= -143.779 \text{ deg} \end{aligned}$$

2. Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -20.860 \text{ deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= 41.840 \text{ deg} \end{aligned}$$

3. Using Figure P5-9, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$\begin{aligned} z &:= \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} & z &= 0.000 \\ s &:= \sqrt{(P_{1x} - E_{1x})^2 + (P_{1y} - E_{1y})^2} & s &= 1.591 \\ v &:= \sqrt{(C_{1x} - E_{1x})^2 + (C_{1y} - E_{1y})^2} & v &= 1.591 \\ \phi &:= \text{atan2}(E_{1x} - C_{1x}, E_{1y} - C_{1y}) & \phi &= 0.000 \text{ deg} \\ \psi &:= \phi + \pi & \psi &= 180.000 \text{ deg} \\ Z_{1x} &:= z \cdot \cos(\phi) & Z_{1x} &= 0.000 \\ Z_{1y} &:= z \cdot \sin(\phi) & Z_{1y} &= 0.000 \\ S_{1x} &:= s \cdot \cos(\psi) & S_{1x} &= -1.591 \\ S_{1y} &:= s \cdot \sin(\psi) & S_{1y} &= 0.000 \end{aligned}$$

4. Use equations 5.24 to solve for w , θ , β_2 , and β_3 . Since the points C and D are to be used as pivots, z and ϕ are known from the calculations above. Use guess values from a graphical solution such as that for Problem 3-65.

$$\text{Guess:} \quad W_{1x} := -1.75 \quad W_{1y} := -0.47 \quad \beta_2 := 81 \cdot \text{deg} \quad \beta_3 := 138 \cdot \text{deg}$$

Given

$$\begin{aligned}
 W_{1x}(\cos(\beta_2) - 1) - W_{1y}\sin(\beta_2) \dots &= p_{21}\cos(\delta_2) \\
 + Z_{1x}(\cos(\alpha_2) - 1) - Z_{1y}\sin(\alpha_2) \\
 \\
 W_{1x}(\cos(\beta_3) - 1) - W_{1y}\sin(\beta_3) \dots &= p_{31}\cos(\delta_3) \\
 + Z_{1x}(\cos(\alpha_3) - 1) - Z_{1y}\sin(\alpha_3) \\
 \\
 W_{1y}(\cos(\beta_2) - 1) + W_{1x}\sin(\beta_2) \dots &= p_{21}\sin(\delta_2) \\
 + Z_{1y}(\cos(\alpha_2) - 1) + Z_{1x}\sin(\alpha_2) \\
 \\
 W_{1y}(\cos(\beta_3) - 1) + W_{1x}\sin(\beta_3) \dots &= p_{31}\sin(\delta_3) \\
 + Z_{1y}(\cos(\alpha_3) - 1) + Z_{1x}\sin(\alpha_3)
 \end{aligned}$$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := \text{Find}(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$$\beta_2 = 79.928 \text{ deg} \qquad \beta_3 = 137.178 \text{ deg}$$

The components of the **W** vector are:

$$W_{1x} = 0.980 \qquad W_{1y} = 1.547 \qquad \theta := \text{atan2}(W_{1x}, W_{1y}) \qquad \theta = 57.632 \text{ deg}$$

The length of link 2 is: $w := \sqrt{(W_{1x}^2 + W_{1y}^2)}$, $w = 1.831$

5. Use equations 5.28 to solve for u , σ , γ_2 , and γ_3 . Since the points *C* and *D* are to be used as pivots, s and ψ are known from the calculations above.

Guess: $U_{1x} := 1 \qquad U_{1y} := -5.0 \qquad \gamma_2 := -17.5 \text{ deg} \qquad \gamma_3 := -37 \text{ deg}$

Given

$$\begin{aligned}
 U_{1x}(\cos(\gamma_2) - 1) - U_{1y}\sin(\gamma_2) \dots &= p_{21}\cos(\delta_2) \\
 + S_{1x}(\cos(\alpha_2) - 1) - S_{1y}\sin(\alpha_2) \\
 \\
 U_{1x}(\cos(\gamma_3) - 1) - U_{1y}\sin(\gamma_3) \dots &= p_{31}\cos(\delta_3) \\
 + S_{1x}(\cos(\alpha_3) - 1) - S_{1y}\sin(\alpha_3) \\
 \\
 U_{1y}(\cos(\gamma_2) - 1) + U_{1x}\sin(\gamma_2) \dots &= p_{21}\sin(\delta_2) \\
 + S_{1y}(\cos(\alpha_2) - 1) + S_{1x}\sin(\alpha_2) \\
 \\
 U_{1y}(\cos(\gamma_3) - 1) + U_{1x}\sin(\gamma_3) \dots &= p_{31}\sin(\delta_3) \\
 + S_{1y}(\cos(\alpha_3) - 1) + S_{1x}\sin(\alpha_3)
 \end{aligned}$$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := \text{Find}(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$$\gamma_2 = -21.548 \text{ deg} \qquad \gamma_3 = -27.543 \text{ deg}$$

The components of the **U** vector are:

$$U_{1x} = 3.524 \qquad U_{1y} = -5.963 \qquad \sigma := \text{atan2}(U_{1x}, U_{1y}) \qquad \sigma = -59.417 \text{ deg}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 6.926$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\begin{aligned} \text{Link 3: } V_{1x} &:= Z_{1x} - S_{1x} & V_{1x} &= 1.591 \\ V_{1y} &:= Z_{1y} - S_{1y} & V_{1y} &= 0.000 \\ \theta_3 &:= \text{atan2}(V_{1x}, V_{1y}) & \theta_3 &= 0.000 \text{ deg} \\ v &:= \sqrt{V_{1x}^2 + V_{1y}^2} & v &= 1.591 \end{aligned}$$

$$\begin{aligned} \text{Link 1: } G_{1x} &:= W_{1x} + V_{1x} - U_{1x} & G_{1x} &= -0.952 \\ G_{1y} &:= W_{1y} + V_{1y} - U_{1y} & G_{1y} &= 7.510 \\ \theta_1 &:= \text{atan2}(G_{1x}, G_{1y}) & \theta_1 &= 97.228 \text{ deg} \\ g &:= \sqrt{G_{1x}^2 + G_{1y}^2} & g &= 7.570 \end{aligned}$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\begin{aligned} \theta_{2i} &:= \theta - \theta_1 & \theta_{2i} &= -39.596 \text{ deg} \\ \theta_{2f} &:= \theta_{2i} + \beta_3 & \theta_{2f} &= 97.582 \text{ deg} \end{aligned}$$

8. Define the coupler point with respect to point **C** and the vector **V**.

$$\begin{aligned} r_p &:= z & \delta_p &:= \phi - \theta_3 \\ r_p &= 0.000 & \delta_p &= 0.000 \text{ deg} \end{aligned}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\begin{aligned} O_{2x} &:= -z \cdot \cos(\phi) - w \cdot \cos(\theta) & O_{2x} &= -0.980 \\ O_{2y} &:= -z \cdot \sin(\phi) - w \cdot \sin(\theta) & O_{2y} &= -1.547 \\ O_{4x} &:= -s \cdot \cos(\psi) - u \cdot \cos(\sigma) & O_{4x} &= -1.933 \\ O_{4y} &:= -s \cdot \sin(\psi) - u \cdot \sin(\sigma) & O_{4y} &= 5.963 \end{aligned}$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = 97.228 \text{ deg}$$

11. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

PROBLEM 5-46

Statement: Design a fourbar linkage to carry the object in Figure P5-11 through the three positions shown in their numbered order using the fixed pivots shown. Determine the range of the transmission angle.

Given:

$P_{21x} := -2.332$	$P_{21y} := -0.311$	$P_{31x} := -2.751$	$P_{31y} := -2.015$
$O_{2x} := -3.679$	$O_{2y} := -3.282$	$O_{4x} := 0.321$	$O_{4y} := -3.282$
Body angles:	$\theta_{p1} := 45.0 \cdot \text{deg}$	$\theta_{p2} := 24.14 \cdot \text{deg}$	$\theta_{p3} := 86.84 \cdot \text{deg}$

Solution: See Figure P5-11 and Mathcad file P0546.

1. Determine the angle changes between precision points from the body angles given.

$$\begin{aligned}\alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= -20.860 \text{ deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= 41.840 \text{ deg}\end{aligned}$$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$$\begin{aligned}R_{1x} &:= -O_{2x} & R_{1x} &= 3.679 & R_{1y} &:= -O_{2y} & R_{1y} &= 3.282 \\ R_{2x} &:= R_{1x} + P_{21x} & R_{2x} &= 1.347 \\ R_{2y} &:= R_{1y} + P_{21y} & R_{2y} &= 2.971 \\ R_{3x} &:= R_{1x} + P_{31x} & R_{3x} &= 0.928 \\ R_{3y} &:= R_{1y} + P_{31y} & R_{3y} &= 1.267 \\ R_1 &:= \sqrt{R_{1x}^2 + R_{1y}^2} & R_1 &= 4.930 \\ R_2 &:= \sqrt{R_{2x}^2 + R_{2y}^2} & R_2 &= 3.262 \\ R_3 &:= \sqrt{R_{3x}^2 + R_{3y}^2} & R_3 &= 1.571\end{aligned}$$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\begin{aligned}\zeta_1 &:= \text{atan2}(R_{1x}, R_{1y}) & \zeta_1 &= 41.736 \text{ deg} \\ \zeta_2 &:= \text{atan2}(R_{2x}, R_{2y}) & \zeta_2 &= 65.611 \text{ deg} \\ \zeta_3 &:= \text{atan2}(R_{3x}, R_{3y}) & \zeta_3 &= 53.780 \text{ deg}\end{aligned}$$

4. Solve for β_2 and β_3 using equations 5.34

$$\begin{aligned}C_1 &:= R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) & C_1 &= 2.297 \\ C_2 &:= R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) & C_2 &= -2.258 \\ C_3 &:= R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) & C_3 &= -0.376 \\ C_4 &:= -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) & C_4 &= -3.632 \\ C_5 &:= R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) & C_5 &= 3.260 \\ C_6 &:= -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) & C_6 &= 1.214\end{aligned}$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -13.335$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = 11.382$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = 5.637$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = -7.492$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = -11.382$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = -9.068$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = -136.387$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = 60.979$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_5^2 - A_6^2}{2} \quad K_3 = -60.933$$

$$\beta_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \beta_{31} = -90.019 \text{ deg}$$

$$\beta_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \beta_{32} = 41.840 \text{ deg}$$

The second value is the same as α_3 so use the first value

$$\beta_3 := \beta_{31}$$

$$\beta_{21} := \text{acos} \left(\frac{A_5 \cdot \sin(\beta_3) + A_3 \cdot \cos(\beta_3) + A_6}{A_1} \right) \quad \beta_{21} = 99.989 \text{ deg}$$

$$\beta_{22} := \text{asin} \left(\frac{A_3 \cdot \sin(\beta_3) + A_2 \cdot \cos(\beta_3) + A_4}{A_1} \right) \quad \beta_{22} = 80.011 \text{ deg}$$

Use the first value,

$$\beta_2 := \beta_{21}$$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2 .

$$R_{1x} := -O_{4x} \quad R_{1x} = -0.321 \quad R_{1y} := -O_{4y} \quad R_{1y} = 3.282$$

$$R_{2x} := R_{1x} + P_{21x} \quad R_{2x} = -2.653$$

$$R_{2y} := R_{1y} + P_{21y} \quad R_{2y} = 2.971$$

$$R_{3x} := R_{1x} + P_{31x} \quad R_{3x} = -3.072$$

$$R_{3y} := R_{1y} + P_{31y} \quad R_{3y} = 1.267$$

$$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2} \quad R_1 = 3.298$$

$$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2} \quad R_2 = 3.983$$

$$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2} \quad R_3 = 3.323$$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := \text{atan2}(R_{1x}, R_{1y}) \quad \zeta_1 = 95.586 \text{ deg}$$

$$\zeta_2 := \text{atan2}(R_{2x}, R_{2y}) \quad \zeta_2 = 131.764 \text{ deg}$$

$$\zeta_3 := \text{atan2}(R_{3x}, R_{3y}) \quad \zeta_3 = 157.587 \text{ deg}$$

7. Solve for γ_2 and γ_3 using equations 5.34

$$C_1 := R_3 \cdot \cos(\alpha_2 + \zeta_3) - R_2 \cdot \cos(\alpha_3 + \zeta_2) \quad C_1 = 1.539$$

$$C_2 := R_3 \cdot \sin(\alpha_2 + \zeta_3) - R_2 \cdot \sin(\alpha_3 + \zeta_2) \quad C_2 = 1.834$$

$$C_3 := R_1 \cdot \cos(\alpha_3 + \zeta_1) - R_3 \cdot \cos(\zeta_3) \quad C_3 = 0.644$$

$$C_4 := -R_1 \cdot \sin(\alpha_3 + \zeta_1) + R_3 \cdot \sin(\zeta_3) \quad C_4 = -0.964$$

$$C_5 := R_1 \cdot \cos(\alpha_2 + \zeta_1) - R_2 \cdot \cos(\zeta_2) \quad C_5 = 3.522$$

$$C_6 := -R_1 \cdot \sin(\alpha_2 + \zeta_1) + R_2 \cdot \sin(\zeta_2) \quad C_6 = -0.210$$

$$A_1 := -C_3^2 - C_4^2 \quad A_1 = -1.343$$

$$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5 \quad A_2 = 3.260$$

$$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5 \quad A_3 = -2.469$$

$$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4 \quad A_4 = -0.303$$

$$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6 \quad A_5 = -3.260$$

$$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4 \quad A_6 = 2.758$$

$$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6 \quad K_1 = -7.799$$

$$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6 \quad K_2 = -8.243$$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2} \quad K_3 = -11.309$$

$$\gamma_{31} := 2 \cdot \text{atan} \left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{31} = 41.840 \text{ deg}$$

$$\gamma_{32} := 2 \cdot \text{atan} \left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3} \right) \quad \gamma_{32} = 51.335 \text{ deg}$$

The first value is the same as α_3 , so use the second value

$$\gamma_3 := \gamma_{32}$$

$$\gamma_{21} := \text{acos}\left(\frac{A_5 \cdot \sin(\gamma_3) + A_3 \cdot \cos(\gamma_3) + A_6}{A_1}\right) \quad \gamma_{21} = 8.321 \text{ deg}$$

$$\gamma_{22} := \text{asin}\left(\frac{A_3 \cdot \sin(\gamma_3) + A_2 \cdot \cos(\gamma_3) + A_4}{A_1}\right) \quad \gamma_{22} = 8.321 \text{ deg}$$

Both are the same so use the first value ,

$$\gamma_2 := \gamma_{21}$$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 2.353$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = -172.404 \text{ deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \quad p_{31} = 3.410$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y}) \quad \delta_3 = -143.779 \text{ deg}$$

9. Evaluate terms in the $\mathbf{WZ}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := \cos(\beta_2) - 1$$

$$B := \sin(\beta_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F := \cos(\beta_3) - 1$$

$$G := \sin(\beta_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

10. The components of the $\mathbf{W}$ and $\mathbf{Z}$ vectors are:

$$W1x = 1.732$$

$$W1y = 1.000$$

$$Z1x = 1.947$$

$$Z1y = 2.282$$

11. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2} \quad w = 2.000$

12. Evaluate terms in the $\mathbf{US}$ coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A' := \cos(\gamma_2) - 1$$

$$B' := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F' := \cos(\gamma_3) - 1$$

$$G' := \sin(\gamma_3)$$

$$H := \cos(\alpha_3) - 1$$

$$K := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$\begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$$U1x = -1.178$$

$$U1y = 6.903$$

$$S1x = 0.857$$

$$S1y = -3.621$$

14. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2}$

$$u = 7.002$$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$$V1x := Z1x - S1x$$

$$V1x = 1.090$$

$$V1y := Z1y - S1y$$

$$V1y = 5.903$$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$

$$v = 6.002$$

$$G1x := W1x + V1x - U1x$$

$$G1x = 4.000$$

$$G1y := W1y + V1y - U1y$$

$$G1y = -1.776 \times 10^{-15}$$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$

$$g = 4.000$$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors **W**₁, **Z**₁, **U**₁, and **S**₁.

$$O2x := -Z1x - W1x$$

$$O2x = -3.679$$

$$O2y := -Z1y - W1y$$

$$O2y = -3.282$$

$$O4x := -S1x - U1x$$

$$O4x = 0.321$$

$$O4y := -S1y - U1y$$

$$O4y = -3.282$$

These check with Figure P5-7.

17. Determine the location of the coupler point with respect to point A and line AB.

$$\text{Distance from A to P} \quad z := \sqrt{Z1x^2 + Z1y^2}$$

$$z = 3.000$$

$$r_P := z$$

$$\text{Angle BAP } (\delta_p) \quad s := \sqrt{S1x^2 + S1y^2}$$

$$s = 3.721$$

$$\psi := \text{atan2}(S1x, S1y)$$

$$\psi = -76.683 \text{ deg}$$

$$\phi := \text{atan2}(Z1x, Z1y)$$

$$\phi = 49.525 \text{ deg}$$

$$\theta_3 := \text{atan2}(z \cdot \cos(\phi) - s \cdot \cos(\psi), z \cdot \sin(\phi) - s \cdot \sin(\psi))$$

$$\theta_3 = 79.535 \text{ deg}$$

$$\delta_p := \phi - \theta_3$$

$$\delta_p = -30.010 \text{ deg}$$

18. DESIGN SUMMARY

Link 1:	$g = 4.000$	
Link 2:	$w = 2.000$	
Link 3:	$v = 6.002$	
Link 4:	$u = 7.002$	
Coupler point:	$r_p = 3.000$	$\delta_p = -30.010 \text{ deg}$

19. **VERIFICATION:** The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.

20. A driver dyad with a crank should be added to link 2 to control the motion of the fourbar so that it cannot move beyond positions 1 and 3.

PROBLEM 5-47

Statement: Write a program to generate and plot the circle-point and center-point circles for Problem 5-40 using an equation solver or any program language.

Given:

$P_{21x} := 4.500$	$P_{21y} := 1.900$	$P_{31x} := 7.600$	$P_{31y} := 1.000$
$O_{2x} := 2.900$	$O_{2y} := -5.100$	$O_{4x} := 5.900$	$O_{4y} := -5.100$
Body angles:	$\theta_{p1} := 33.70 \cdot \text{deg}$	$\theta_{p2} := 14.60 \cdot \text{deg}$	$\theta_{p3} := 0.0 \cdot \text{deg}$

Assumptions: Let the position 1 to position 2 rotation angles be: $\beta_2 := -47.808 \cdot \text{deg}$ and $\gamma_2 := -55.029 \cdot \text{deg}$

Let the position 1 to position 2 coupler rotation angle be: $\alpha_2 := -19.100 \cdot \text{deg}$

Solution: See Figure P5-9 and Mathcad file P0547.

1. Use the method of Section 5.6 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 4.885$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = 22.891 \text{ deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \quad p_{31} = 7.666$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y}) \quad \delta_3 = 7.496 \text{ deg}$$

2. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\alpha_3 := -33.700 \cdot \text{deg} \quad \beta_3 := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

$$A := \cos(\beta_2) - 1 \quad B := \sin(\beta_2) \quad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \quad E := p_{21} \cdot \cos(\delta_2) \quad F(\beta_3) := \cos(\beta_3) - 1$$

$$G(\beta_3) := \sin(\beta_3) \quad H(\alpha_3) := \cos(\alpha_3) - 1 \quad K(\alpha_3) := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \quad M := p_{21} \cdot \sin(\delta_2) \quad N := p_{31} \cdot \sin(\delta_3)$$

$$AA(\alpha_3, \beta_3) := \begin{pmatrix} A & -B & C & -D \\ F(\beta_3) & -G(\beta_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\beta_3) & F(\beta_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \beta_3) := AA(\alpha_3, \beta_3)^{-1} \cdot CC$$

$$W1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_1 \quad W1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_2$$

$$Z1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_3 \quad Z1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_4$$

3. Check this against the solutions in Problem 5-40:

$$W1x(\alpha_3, -76.089 \cdot \text{deg}) = -5.043 \quad W1y(\alpha_3, -76.089 \cdot \text{deg}) = 3.126$$

$$Z1x(\alpha_3, -76.089 \cdot \text{deg}) = 2.143$$

$$Z1y(\alpha_3, -76.089 \cdot \text{deg}) = 1.974$$

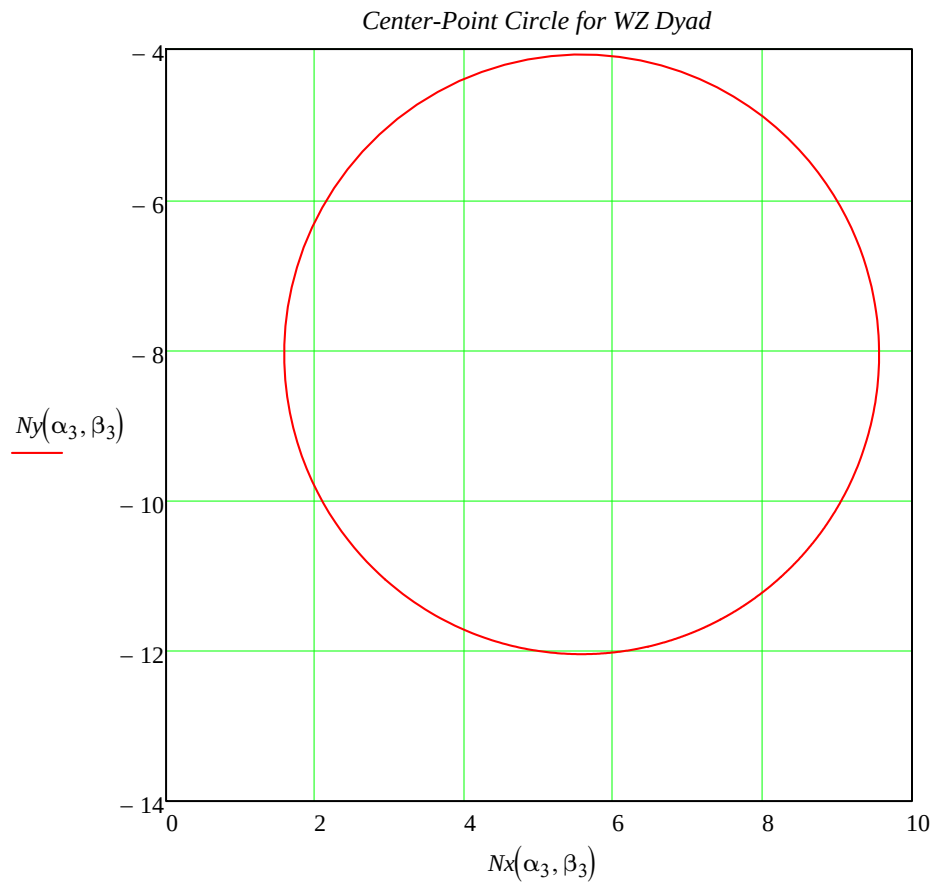
These are the same as the values calculated in Problem 5-40.

4. Form the vector **N**, whose tip describes the center-point circle for the **WZ** dyad.

$$Nx(\alpha_3, \beta_3) := -W1x(\alpha_3, \beta_3) - Z1x(\alpha_3, \beta_3)$$

$$Ny(\alpha_3, \beta_3) := -W1y(\alpha_3, \beta_3) - Z1y(\alpha_3, \beta_3)$$

5. Plot the center-point circle for the **WZ** dyad.



4. Form the vector **Z**, whose tip describes the circle-point circle for the **WZ** dyad.

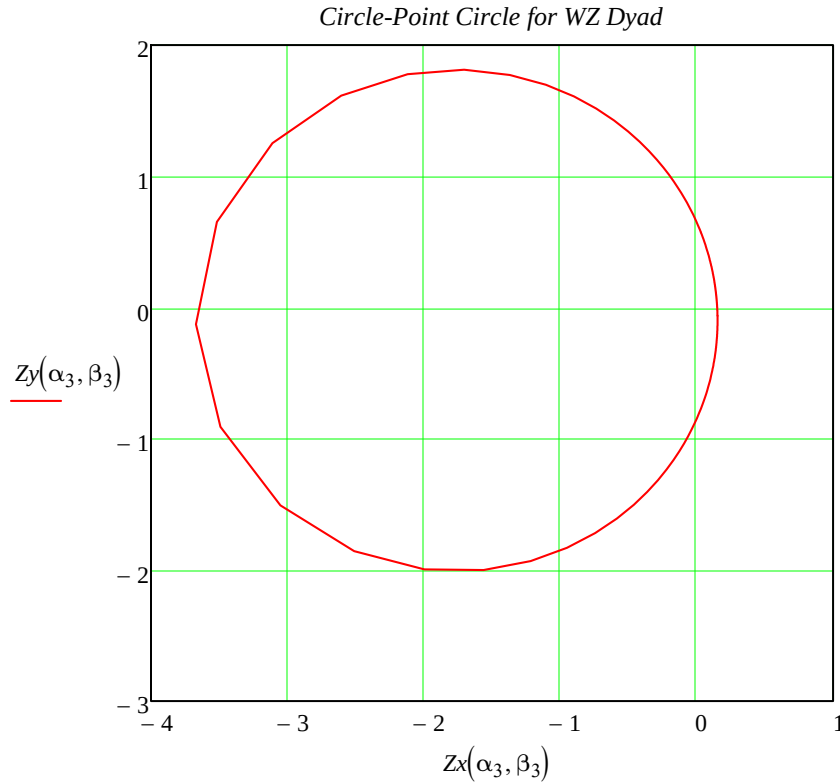
$$\beta_3 := -76.089 \cdot \text{deg}$$

$$\alpha_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

$$Zx(\alpha_3, \beta_3) := -Z1x(\alpha_3, \beta_3)$$

$$Zy(\alpha_3, \beta_3) := -Z1y(\alpha_3, \beta_3)$$

5. Plot the circle-point circle for the **WZ** dyad (see next page).



6. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.31) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\alpha_3 := -33.700 \cdot \text{deg}$$

$$\gamma_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

$$A := \cos(\gamma_2) - 1$$

$$B := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F(\gamma_3) := \cos(\gamma_3) - 1$$

$$G(\gamma_3) := \sin(\gamma_3)$$

$$H(\alpha_3) := \cos(\alpha_3) - 1$$

$$K(\alpha_3) := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA(\alpha_3, \gamma_3) := \begin{pmatrix} A & -B & C & -D \\ F(\gamma_3) & -G(\gamma_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\gamma_3) & F(\gamma_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \gamma_3) := AA(\alpha_3, \gamma_3)^{-1} \cdot CC$$

$$U1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_1$$

$$U1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_2$$

$$S1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_3$$

$$S1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_4$$

7. Check this against the solutions in Problem 5-40:

$$U1x(\alpha_3, -92.928 \cdot \text{deg}) = -2.773$$

$$U1y(\alpha_3, -92.928 \cdot \text{deg}) = 2.998$$

$$S1x(\alpha_3, -92.928 \cdot \text{deg}) = -3.128$$

$$S1y(\alpha_3, -92.928 \cdot \text{deg}) = 2.102$$

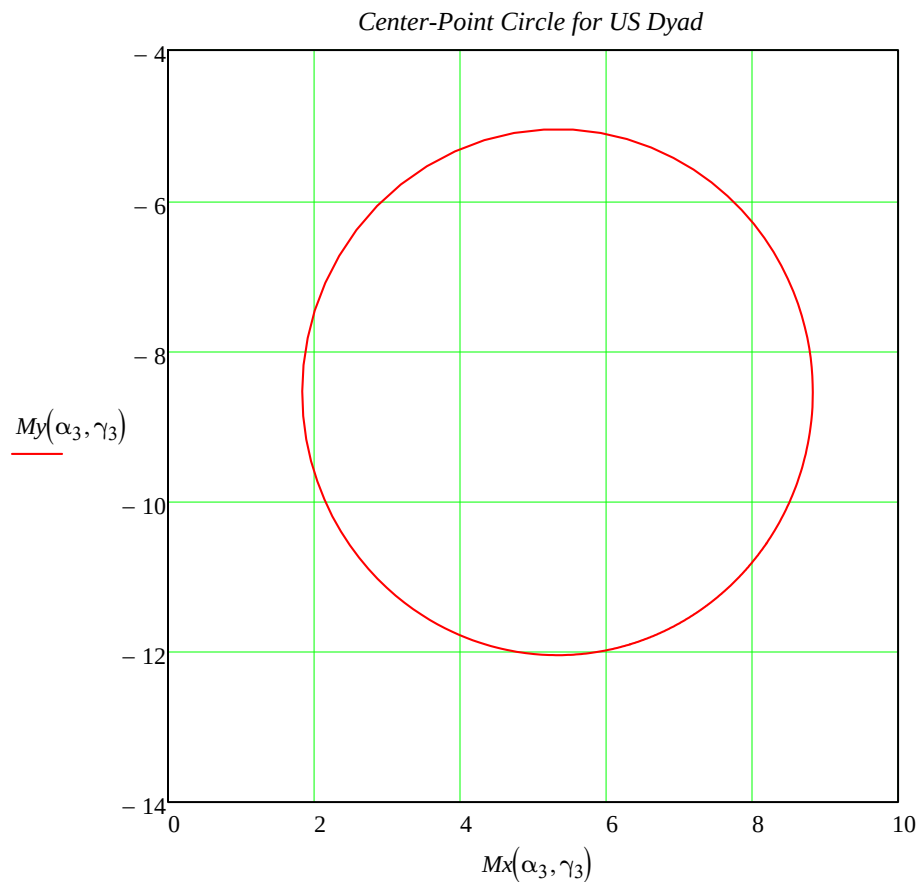
These are the same as the values calculated in Problem 5-40.

8. Form the vector **M**, whose tip describes the center-point circle for the **US** dyad.

$$Mx(\alpha_3, \gamma_3) := -U1x(\alpha_3, \gamma_3) - S1x(\alpha_3, \gamma_3)$$

$$My(\alpha_3, \gamma_3) := -U1y(\alpha_3, \gamma_3) - S1y(\alpha_3, \gamma_3)$$

9. Plot the center-point circle for the **US** dyad.



10. Form the vector **S**, whose tip describes the circle-point circle for the **US** dyad.

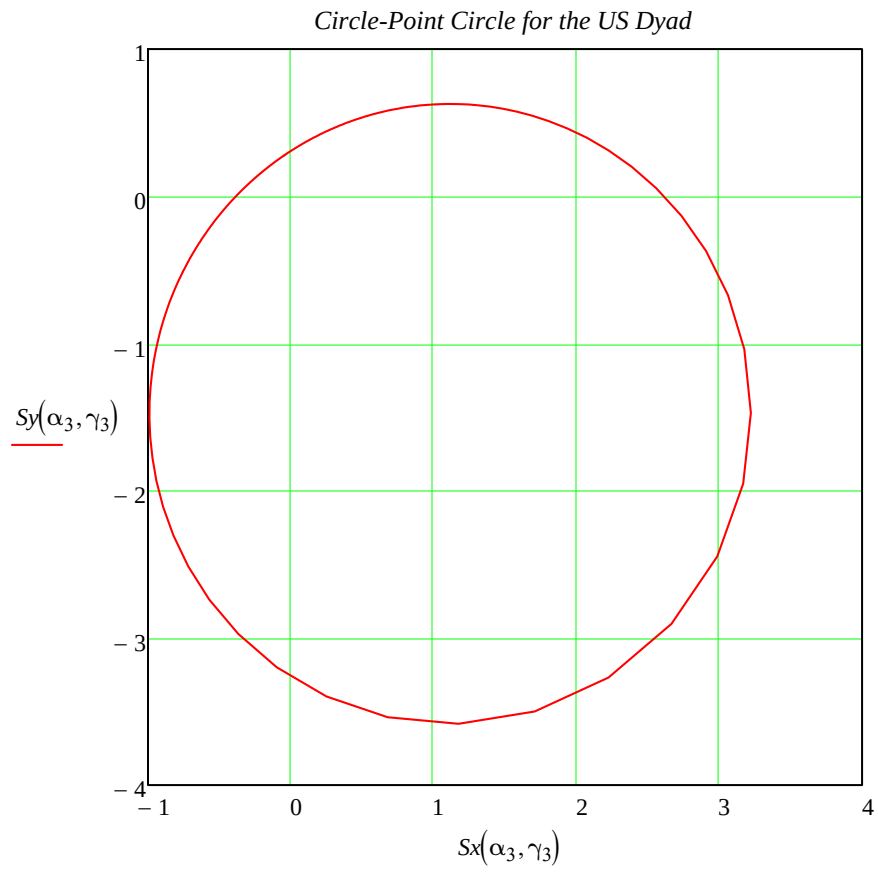
$$\gamma_3 := -92.928 \cdot \text{deg}$$

$$\alpha_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

$$Sx(\alpha_3, \gamma_3) := -S1x(\alpha_3, \gamma_3)$$

$$Sy(\alpha_3, \gamma_3) := -S1y(\alpha_3, \gamma_3)$$

11. Plot the circle-point circle for the **WZ** dyad (see next page).



PROBLEM 5-48

Statement: Write a program to generate and plot the circle-point and center-point circles for Problem 5-43 using an equation solver or any program language.

Given:

$P_{21x} := -0.743$	$P_{21y} := 1.514$	$P_{31x} := -1.750$	$P_{31y} := 2.228$
$O_{2x} := -3.100$	$O_{2y} := -1.200$	$O_{4x} := -0.100$	$O_{4y} := -1.200$
Body angles:	$\theta_{p1} := 62.59 \cdot \text{deg}$	$\theta_{p2} := 68.25 \cdot \text{deg}$	$\theta_{p3} := 90.0 \cdot \text{deg}$

Assumptions: Let the position 1 to position 2 rotation angles be: $\beta_2 := 124.137 \cdot \text{deg}$ and $\gamma_2 := -43.866 \cdot \text{deg}$
 Let the position 1 to position 2 coupler rotation angle be: $\alpha_2 := 5.660 \cdot \text{deg}$

Solution: See Figure P5-10 and Mathcad file P0548.

1. Use the method of Section 5.6 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 1.686$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = 116.140 \text{ deg}$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \quad p_{31} = 2.833$$

$$\delta_3 := \text{atan2}(P_{31x}, P_{31y}) \quad \delta_3 = 128.148 \text{ deg}$$

2. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\alpha_3 := 27.410 \cdot \text{deg} \quad \beta_3 := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. 360 \cdot \text{deg}$$

$$A := \cos(\beta_2) - 1 \quad B := \sin(\beta_2) \quad C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2) \quad E := p_{21} \cdot \cos(\delta_2) \quad F(\beta_3) := \cos(\beta_3) - 1$$

$$G(\beta_3) := \sin(\beta_3) \quad H(\alpha_3) := \cos(\alpha_3) - 1 \quad K(\alpha_3) := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3) \quad M := p_{21} \cdot \sin(\delta_2) \quad N := p_{31} \cdot \sin(\delta_3)$$

$$AA(\alpha_3, \beta_3) := \begin{pmatrix} A & -B & C & -D \\ F(\beta_3) & -G(\beta_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\beta_3) & F(\beta_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \beta_3) := AA(\alpha_3, \beta_3)^{-1} \cdot CC$$

$$W1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_1$$

$$W1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_2$$

$$Z1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_3$$

$$Z1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_4$$

3. Check this against the solutions in Problem 5-43:

$$W1x(\alpha_3, 133.549 \cdot \text{deg}) = 0.621$$

$$W1y(\alpha_3, 133.549 \cdot \text{deg}) = -0.489$$

$$Z1x(\alpha_3, 133.549 \cdot \text{deg}) = 2.479$$

$$Z1y(\alpha_3, 133.549 \cdot \text{deg}) = 1.689$$

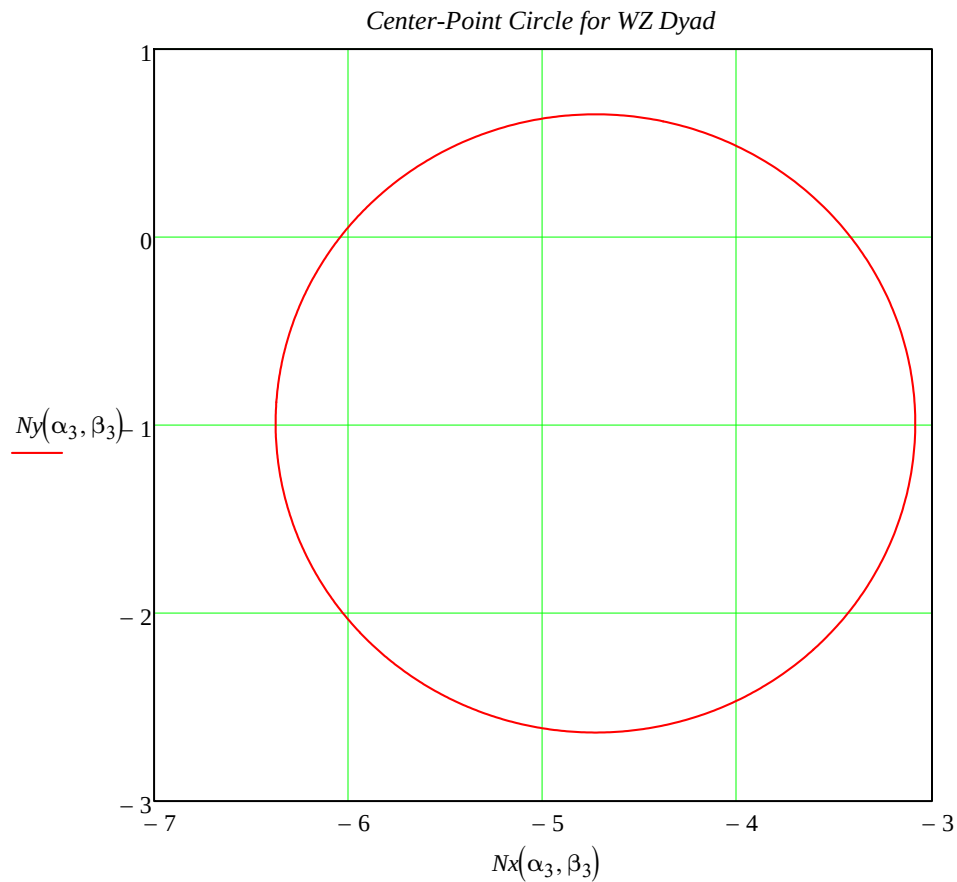
These are the same as the values calculated in Problem 5-43.

4. Form the vector **N**, whose tip describes the center-point circle for the **WZ** dyad.

$$Nx(\alpha_3, \beta_3) := -W1x(\alpha_3, \beta_3) - Z1x(\alpha_3, \beta_3)$$

$$Ny(\alpha_3, \beta_3) := -W1y(\alpha_3, \beta_3) - Z1y(\alpha_3, \beta_3)$$

5. Plot the center-point circle for the **WZ** dyad.



4. Form the vector **Z**, whose tip describes the circle-point circle for the **WZ** dyad.

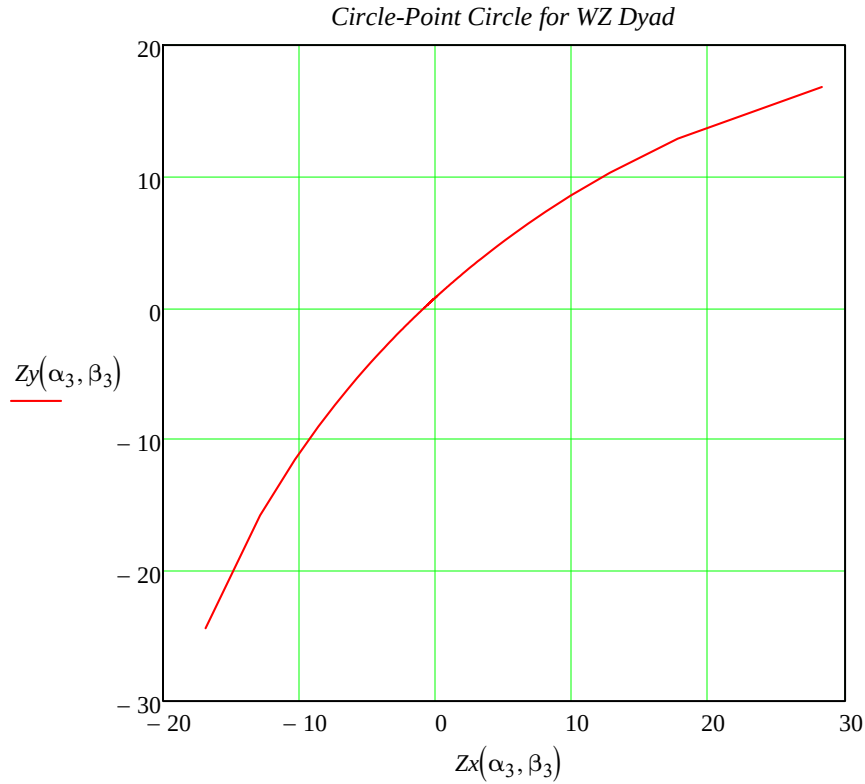
$$\beta_3 := 133.549 \cdot \text{deg}$$

$$\alpha_3 := 8 \cdot \text{deg}, 9 \cdot \text{deg} \dots 364 \cdot \text{deg}$$

$$Zx(\alpha_3, \beta_3) := -Z1x(\alpha_3, \beta_3)$$

$$Zy(\alpha_3, \beta_3) := -Z1y(\alpha_3, \beta_3)$$

5. Plot the circle-point arc for the **WZ** dyad (see next page).



6. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.31) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\alpha_3 := 27.410 \cdot \text{deg}$$

$$\gamma_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

$$A := \cos(\gamma_2) - 1$$

$$B := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F(\gamma_3) := \cos(\gamma_3) - 1$$

$$G(\gamma_3) := \sin(\gamma_3)$$

$$H(\alpha_3) := \cos(\alpha_3) - 1$$

$$K(\alpha_3) := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA(\alpha_3, \gamma_3) := \begin{pmatrix} A & -B & C & -D \\ F(\gamma_3) & -G(\gamma_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\gamma_3) & F(\gamma_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix}$$

$$CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \gamma_3) := AA(\alpha_3, \gamma_3)^{-1} \cdot CC$$

$$U1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_1$$

$$U1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_2$$

$$S1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_3$$

$$S1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_4$$

7. Check this against the solutions in Problem 5-43:

$$U1x(\alpha_3, -56.299 \cdot \text{deg}) = -1.459$$

$$U1y(\alpha_3, -56.299 \cdot \text{deg}) = -1.294$$

$$S1x(\alpha_3, -56.299 \cdot \text{deg}) = 1.559$$

$$S1y(\alpha_3, -56.299 \cdot \text{deg}) = 2.494$$

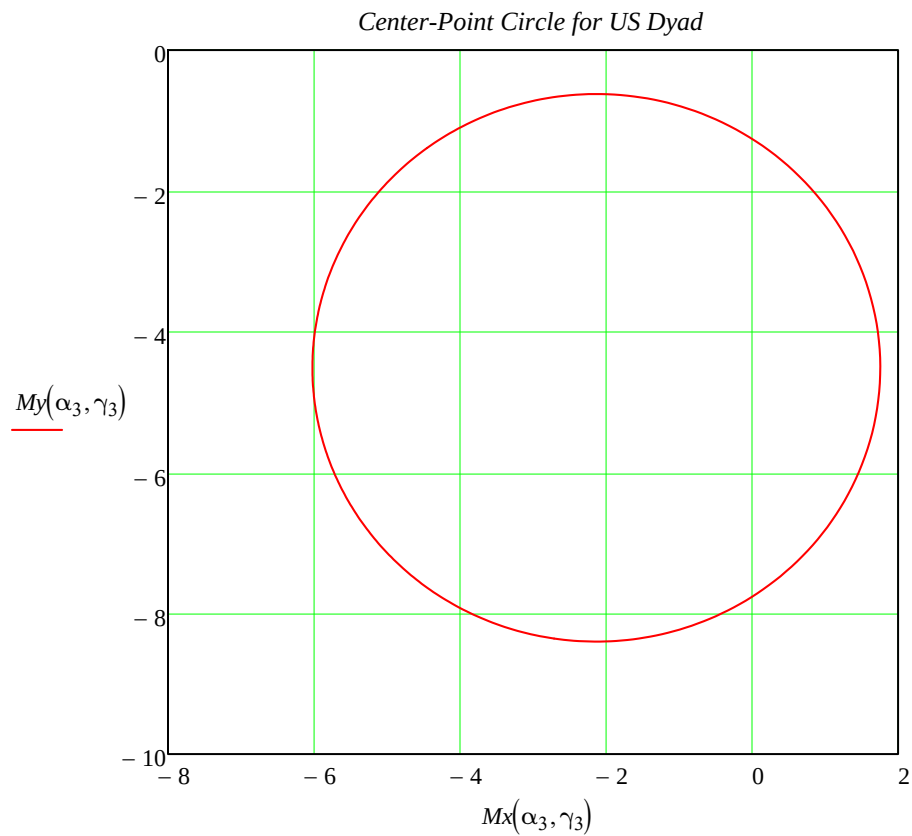
These are the same as the values calculated in Problem 5-43.

8. Form the vector **M**, whose tip describes the center-point circle for the **US** dyad.

$$Mx(\alpha_3, \gamma_3) := -U1x(\alpha_3, \gamma_3) - S1x(\alpha_3, \gamma_3)$$

$$My(\alpha_3, \gamma_3) := -U1y(\alpha_3, \gamma_3) - S1y(\alpha_3, \gamma_3)$$

9. Plot the center-point circle for the **US** dyad.



10. Form the vector **S**, whose tip describes the circle-point circle for the **US** dyad.

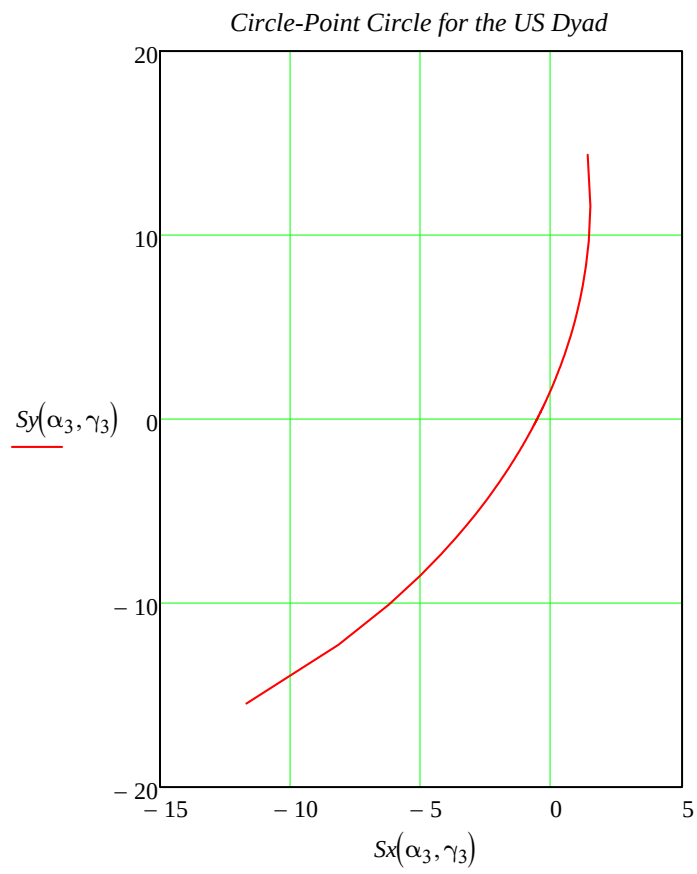
$$\gamma_3 := -56.299 \cdot \text{deg}$$

$$\alpha_3 := 10 \cdot \text{deg}, 11 \cdot \text{deg} .. 363 \cdot \text{deg}$$

$$Sx(\alpha_3, \gamma_3) := -S1x(\alpha_3, \gamma_3)$$

$$Sy(\alpha_3, \gamma_3) := -S1y(\alpha_3, \gamma_3)$$

11. Plot the circle-point arc for the **WZ** dyad (see next page).



PROBLEM 5-49

Statement: Write a program to generate and plot the circle-point and center-point circles for Problem 5-46 using an equation solver or any program language.

Given:

$$\begin{array}{llll} P_{21x} := -2.332 & P_{21y} := -0.311 & P_{31x} := -2.751 & P_{31y} := -2.015 \\ O_{2x} := -3.679 & O_{2y} := -3.282 & O_{4x} := 0.321 & O_{4y} := -3.282 \\ \text{Body angles:} & \theta_{p1} := 45.0 \cdot \text{deg} & \theta_{p2} := 24.14 \cdot \text{deg} & \theta_{p3} := 86.84 \cdot \text{deg} \end{array}$$

Assumptions: Let the position 1 to position 2 rotation angles be: $\beta_2 := 99.989 \cdot \text{deg}$ and $\gamma_2 := 8.321 \cdot \text{deg}$
Let the position 1 to position 2 coupler rotation angle be: $\alpha_2 := -20.860 \cdot \text{deg}$

Solution: See Figure P5-11 and Mathcad file P0549.

1. Use the method of Section 5.6 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$\begin{array}{ll} p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} & p_{21} = 2.353 \\ \delta_2 := \text{atan2}(P_{21x}, P_{21y}) & \delta_2 = -172.404 \text{ deg} \\ p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} & p_{31} = 3.410 \\ \delta_3 := \text{atan2}(P_{31x}, P_{31y}) & \delta_3 = -143.779 \text{ deg} \end{array}$$

2. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\begin{array}{lll} \alpha_3 := 41.840 \cdot \text{deg} & \beta_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg} & \\ A := \cos(\beta_2) - 1 & B := \sin(\beta_2) & C := \cos(\alpha_2) - 1 \\ D := \sin(\alpha_2) & E := p_{21} \cdot \cos(\delta_2) & F(\beta_3) := \cos(\beta_3) - 1 \\ G(\beta_3) := \sin(\beta_3) & H(\alpha_3) := \cos(\alpha_3) - 1 & K(\alpha_3) := \sin(\alpha_3) \\ L := p_{31} \cdot \cos(\delta_3) & M := p_{21} \cdot \sin(\delta_2) & N := p_{31} \cdot \sin(\delta_3) \end{array}$$

$$AA(\alpha_3, \beta_3) := \begin{pmatrix} A & -B & C & -D \\ F(\beta_3) & -G(\beta_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\beta_3) & F(\beta_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \beta_3) := AA(\alpha_3, \beta_3)^{-1} \cdot CC$$

$$W1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_1$$

$$W1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_2$$

$$Z1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_3$$

$$Z1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_4$$

3. Check this against the solutions in Problem 5-46:

$$W1x(\alpha_3, -90.019 \cdot \text{deg}) = 1.732$$

$$W1y(\alpha_3, -90.019 \cdot \text{deg}) = 1.000$$

$$Z1x(\alpha_3, -90.019 \cdot \text{deg}) = 1.947$$

$$Z1y(\alpha_3, -90.019 \cdot \text{deg}) = 2.282$$

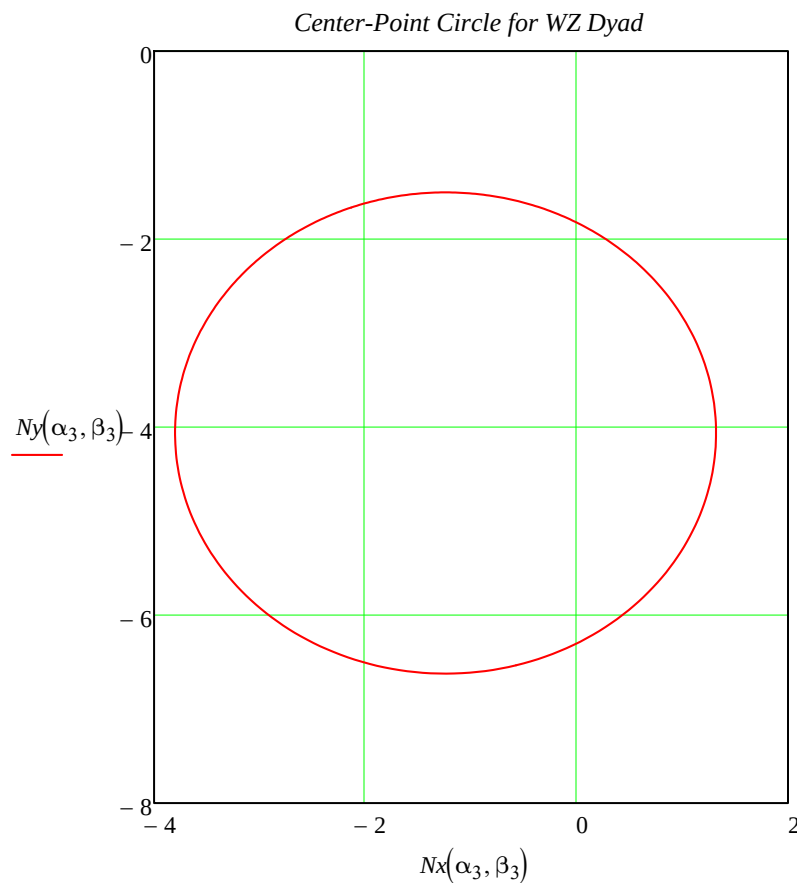
These are the same as the values calculated in Problem 5-46.

4. Form the vector **N**, whose tip describes the center-point circle for the **WZ** dyad.

$$Nx(\alpha_3, \beta_3) := -W1x(\alpha_3, \beta_3) - Z1x(\alpha_3, \beta_3)$$

$$Ny(\alpha_3, \beta_3) := -W1y(\alpha_3, \beta_3) - Z1y(\alpha_3, \beta_3)$$

5. Plot the center-point circle for the **WZ** dyad.



4. Form the vector **Z**, whose tip describes the circle-point circle for the **WZ** dyad.

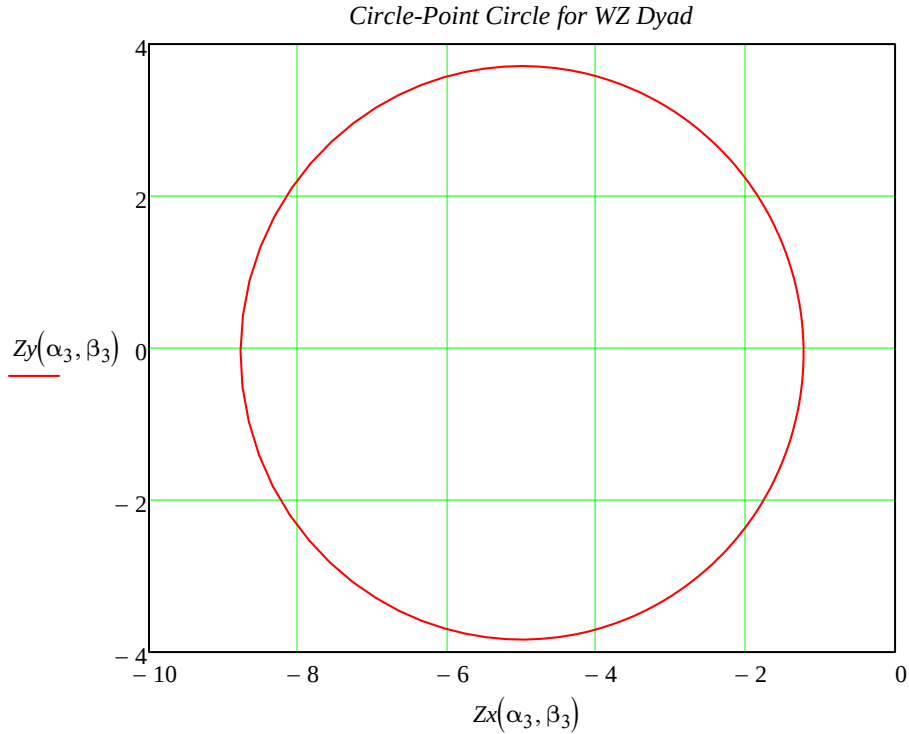
$$\beta_3 := -90.019 \cdot \text{deg}$$

$$\alpha_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

$$Zx(\alpha_3, \beta_3) := -Z1x(\alpha_3, \beta_3)$$

$$Zy(\alpha_3, \beta_3) := -Z1y(\alpha_3, \beta_3)$$

5. Plot the circle-point circle for the **WZ** dyad (see next page).



6. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.31) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\alpha_3 := 41.840 \cdot \text{deg}$$

$$\gamma_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

$$A := \cos(\gamma_2) - 1$$

$$B := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F(\gamma_3) := \cos(\gamma_3) - 1$$

$$G(\gamma_3) := \sin(\gamma_3)$$

$$H(\alpha_3) := \cos(\alpha_3) - 1$$

$$K(\alpha_3) := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA(\alpha_3, \gamma_3) := \begin{pmatrix} A & -B & C & -D \\ F(\gamma_3) & -G(\gamma_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\gamma_3) & F(\gamma_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \gamma_3) := AA(\alpha_3, \gamma_3)^{-1} \cdot CC$$

$$U1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_1$$

$$U1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_2$$

$$S1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_3$$

$$S1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_4$$

7. Check this against the solutions in Problem 5-46:

$$U1x(\alpha_3, 51.335 \cdot \text{deg}) = -1.178$$

$$U1y(\alpha_3, 51.335 \cdot \text{deg}) = 6.903$$

$$S1x(\alpha_3, 51.335 \cdot \text{deg}) = 0.857$$

$$S1y(\alpha_3, 51.335 \cdot \text{deg}) = -3.621$$

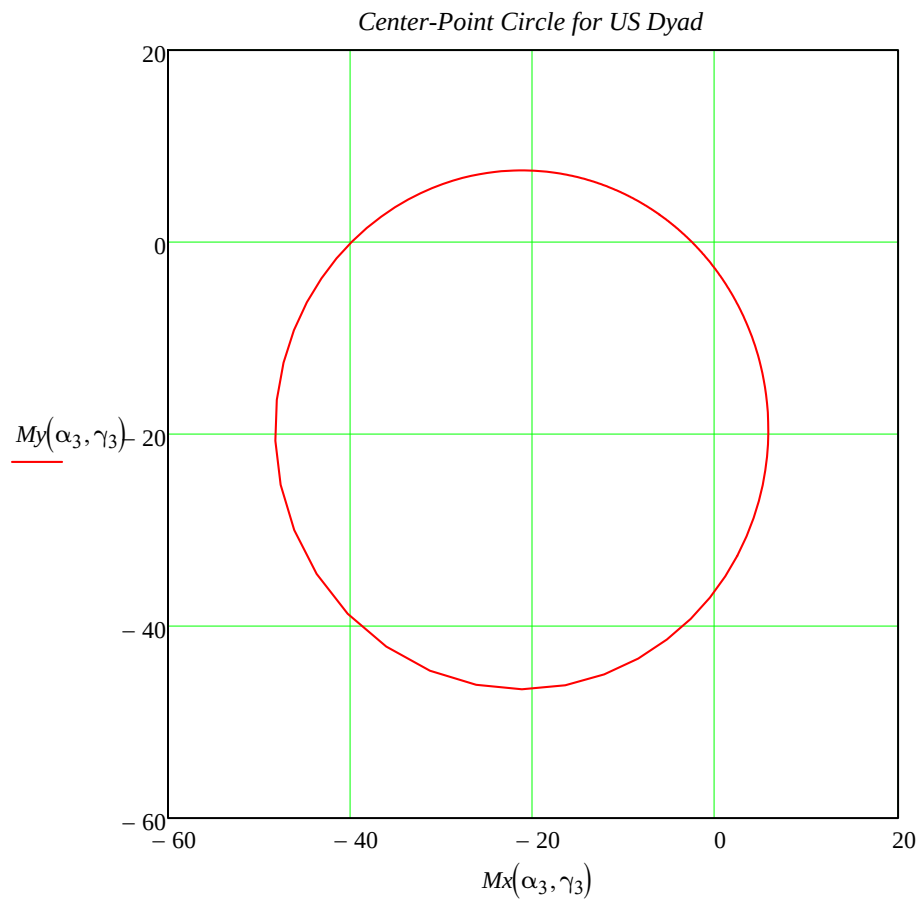
These are the same as the values calculated in Problem 5-46.

8. Form the vector **M**, whose tip describes the center-point circle for the **US** dyad.

$$Mx(\alpha_3, \gamma_3) := -U1x(\alpha_3, \gamma_3) - S1x(\alpha_3, \gamma_3)$$

$$My(\alpha_3, \gamma_3) := -U1y(\alpha_3, \gamma_3) - S1y(\alpha_3, \gamma_3)$$

9. Plot the center-point circle for the **US** dyad.



10. Form the vector **S**, whose tip describes the circle-point circle for the **US** dyad.

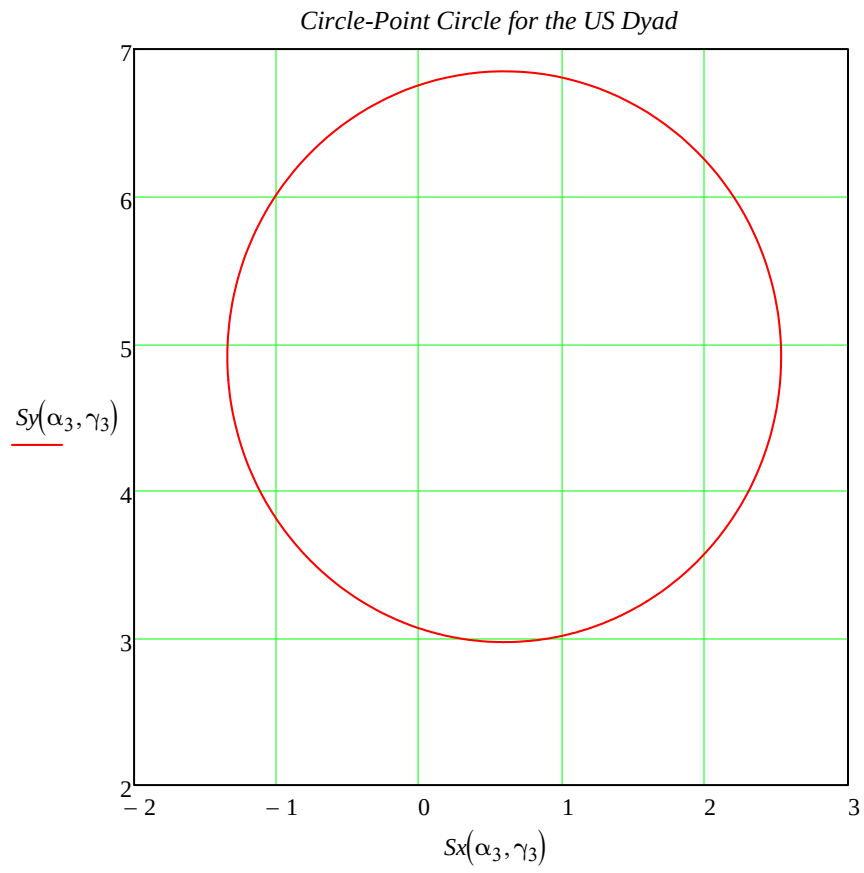
$$\gamma_3 := 51.335 \cdot \text{deg}$$

$$\alpha_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

$$Sx(\alpha_3, \gamma_3) := -S1x(\alpha_3, \gamma_3)$$

$$Sy(\alpha_3, \gamma_3) := -S1y(\alpha_3, \gamma_3)$$

11. Plot the circle-point circle for the **WZ** dyad (see next page).



PROBLEM 5-50

Statement: In Example 5-2 the precision points and rotation angles are specified while the input and output rotation angles β and γ are free choices. Using the choices given for β_2 and γ_2 , determine the radii and center coordinates of the center-point circles for O_2 and O_4 . Plot those circles (or portions of them) and show that the choices of β_3 and γ_3 give a solution that falls on the center-point circles.

Given:

$$\begin{array}{llll} P_{21x} := 2.394 & P_{21y} := -1.449 & P_{31x} := 3.761 & P_{31y} := -1.103 \\ O_{2x} := -1.234 & O_{2y} := -7.772 & O_{4x} := 2.737 & O_{4y} := 0.338 \\ \text{Body angles:} & \theta_{p1} := 38.565 \cdot \text{deg} & \theta_{p2} := -6.435 \cdot \text{deg} & \theta_{p3} := 47.865 \cdot \text{deg} \end{array}$$

Assumptions: Let the position 1 to position 2 rotation angles be: $\beta_2 := 342.3 \cdot \text{deg}$ and $\gamma_2 := 30.9 \cdot \text{deg}$
Let the position 1 to position 2 coupler rotation angle be: $\alpha_2 := -45.0 \cdot \text{deg}$

Solution: See Figure 5-5 and Mathcad file P0550.

1. Use the method of Section 5.6 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$\begin{array}{ll} p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} & p_{21} = 2.798 \\ \delta_2 := \text{atan2}(P_{21x}, P_{21y}) & \delta_2 = -31.185 \text{ deg} \\ p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} & p_{31} = 3.919 \\ \delta_3 := \text{atan2}(P_{31x}, P_{31y}) & \delta_3 = -16.345 \text{ deg} \end{array}$$

2. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector to get the center-point and circle-point circles for the left dyad. Since this is a non-Grashof linkage, the range of β_3 is limited to approximately $-56 \text{ deg} < \beta_3 < -3 \text{ deg}$.

$$\begin{array}{lll} \alpha_2 := 9.3 \cdot \text{deg} & \beta_3 := -56 \cdot \text{deg}, -55 \cdot \text{deg} .. -3 \cdot \text{deg} & \\ A := \cos(\beta_2) - 1 & B := \sin(\beta_2) & C := \cos(\alpha_2) - 1 \\ D := \sin(\alpha_2) & E := p_{21} \cdot \cos(\delta_2) & F(\beta_3) := \cos(\beta_3) - 1 \\ G(\beta_3) := \sin(\beta_3) & H(\alpha_3) := \cos(\alpha_3) - 1 & K(\alpha_3) := \sin(\alpha_3) \\ L := p_{31} \cdot \cos(\delta_3) & M := p_{21} \cdot \sin(\delta_2) & N := p_{31} \cdot \sin(\delta_3) \\ AA(\alpha_3, \beta_3) := \begin{pmatrix} A & -B & C & -D \\ F(\beta_3) & -G(\beta_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\beta_3) & F(\beta_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} & CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} & \end{array}$$

$$DD(\alpha_3, \beta_3) := AA(\alpha_3, \beta_3)^{-1} \cdot CC$$

$$W1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_1$$

$$W1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_2$$

$$Z1x(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_3$$

$$Z1y(\alpha_3, \beta_3) := DD(\alpha_3, \beta_3)_4$$

3. Check this against the solutions in Example 5-2:

$$W1x(\alpha_3, 324.8 \cdot \text{deg}) = 0.055$$

$$W1y(\alpha_3, 324.8 \cdot \text{deg}) = 6.832$$

$$Z1x(\alpha_3, 324.8 \cdot \text{deg}) = 1.179$$

$$Z1y(\alpha_3, 324.8 \cdot \text{deg}) = 0.940$$

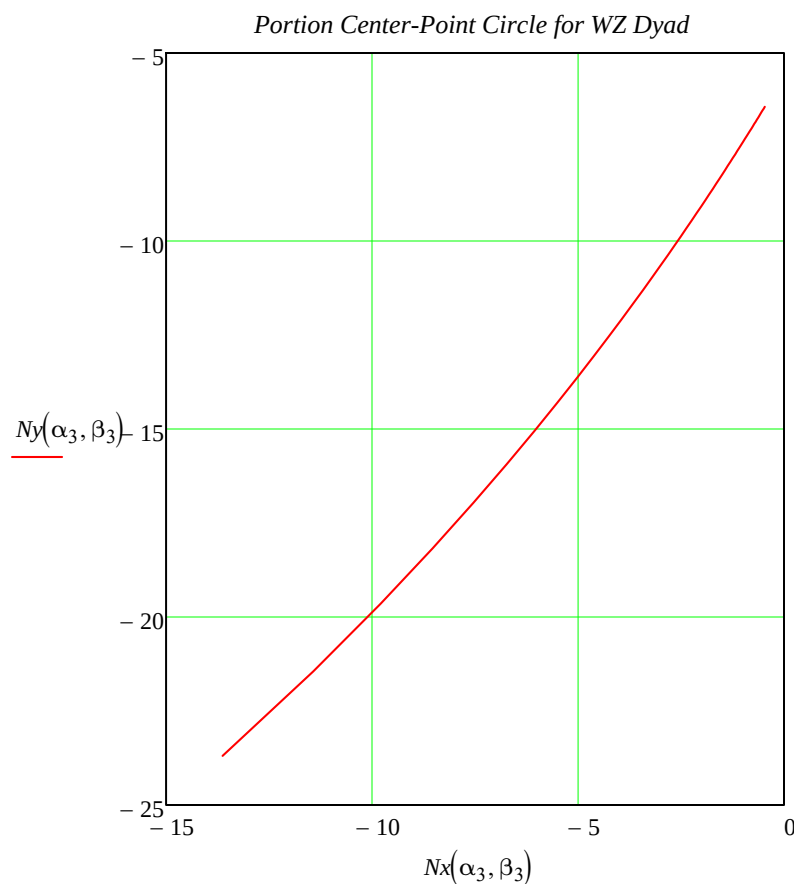
These are the same as the values calculated in Example 5-2.

4. Form the vector **N**, whose tip describes the center-point circle for the **WZ** dyad.

$$Nx(\alpha_3, \beta_3) := -W1x(\alpha_3, \beta_3) - Z1x(\alpha_3, \beta_3)$$

$$Ny(\alpha_3, \beta_3) := -W1y(\alpha_3, \beta_3) - Z1y(\alpha_3, \beta_3)$$

5. Plot the center-point circle for the **WZ** dyad.



4. Find the center and radius of this arc by taking three points on it and, from them, determine the radius and center point of the circle.

Three points:

$$x_1 := Nx(\alpha_3, -56 \cdot \text{deg})$$

$$x_1 = -0.470$$

$$y_1 := Ny(\alpha_3, -56 \cdot \text{deg})$$

$$y_1 = -6.430$$

$$x_2 := Nx(\alpha_3, -10 \cdot \text{deg})$$

$$x_2 = -5.506$$

$$y_2 := Ny(\alpha_3, -10 \cdot \text{deg})$$

$$y_2 = -14.302$$

$$x_3 := Nx(\alpha_3, -3 \cdot \text{deg})$$

$$x_3 = -13.631$$

$$y_3 := Ny(\alpha_3, -3 \cdot \text{deg})$$

$$y_3 = -23.695$$

$$\text{Form four ratios: } a := \frac{y_2^2 - y_1^2 + x_2^2 - x_1^2}{2 \cdot (y_2 - y_1)} \quad a = -12.278$$

$$b := \frac{x_2 - x_1}{y_2 - y_1} \quad b = 0.640$$

$$c := \frac{y_3^2 - y_2^2 + x_3^2 - x_2^2}{2 \cdot (y_2 - y_1)} \quad c = -32.547$$

$$d := \frac{x_3 - x_2}{y_3 - y_2} \quad d = 0.865$$

$$\text{Circle center x coordinate: } h := \frac{a - c}{b - d} \quad h = -89.999$$

$$\text{Circle center y coordinate: } k := a - b \cdot h \quad k = 45.303$$

$$\text{Center-point circle radius: } R := \sqrt{(x_1 - h)^2 + (y_1 - k)^2} \quad R = 103.40$$

4. Show that the point determined by $\beta_3 = 324.8 \text{ deg}$ falls on this circle.

$$\text{Coordinates of this point: } x_{b3} := Nx(\alpha_3, 324.8 \cdot \text{deg}) \quad x_{b3} = -1.234$$

$$y_{b3} := Ny(\alpha_3, 324.8 \cdot \text{deg}) \quad y_{b3} = -7.772$$

Substitute into radius equation:

$$R := \sqrt{(x_{b3} - h)^2 + (y_{b3} - k)^2} \quad R = 103.42$$

Since the radius is the same, this point does fall on the center=point circle for the **WZ** dyad.

6. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.31) and form the matrix and vector to get the center-point and circle-point circles for the left dyad.

$$\alpha_3 := 9.3 \cdot \text{deg}$$

$$\gamma_3 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

$$A := \cos(\gamma_2) - 1$$

$$B := \sin(\gamma_2)$$

$$C := \cos(\alpha_2) - 1$$

$$D := \sin(\alpha_2)$$

$$E := p_{21} \cdot \cos(\delta_2)$$

$$F(\gamma_3) := \cos(\gamma_3) - 1$$

$$G(\gamma_3) := \sin(\gamma_3)$$

$$H(\alpha_3) := \cos(\alpha_3) - 1$$

$$K(\alpha_3) := \sin(\alpha_3)$$

$$L := p_{31} \cdot \cos(\delta_3)$$

$$M := p_{21} \cdot \sin(\delta_2)$$

$$N := p_{31} \cdot \sin(\delta_3)$$

$$AA(\alpha_3, \gamma_3) := \begin{pmatrix} A & -B & C & -D \\ F(\gamma_3) & -G(\gamma_3) & H(\alpha_3) & -K(\alpha_3) \\ B & A & D & C \\ G(\gamma_3) & F(\gamma_3) & K(\alpha_3) & H(\alpha_3) \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix}$$

$$DD(\alpha_3, \gamma_3) := AA(\alpha_3, \gamma_3)^{-1} \cdot CC$$

$$U1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_1$$

$$U1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_2$$

$$S1x(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_3$$

$$S1y(\alpha_3, \gamma_3) := DD(\alpha_3, \gamma_3)_4$$

7. Check this against the solutions in Example 5-2:

$$U1x(\alpha_3, 80.6 \cdot \text{deg}) = -2.628$$

$$U1y(\alpha_3, 80.6 \cdot \text{deg}) = -1.825$$

$$S1x(\alpha_3, 80.6 \cdot \text{deg}) = -0.109$$

$$S1y(\alpha_3, 80.6 \cdot \text{deg}) = 1.487$$

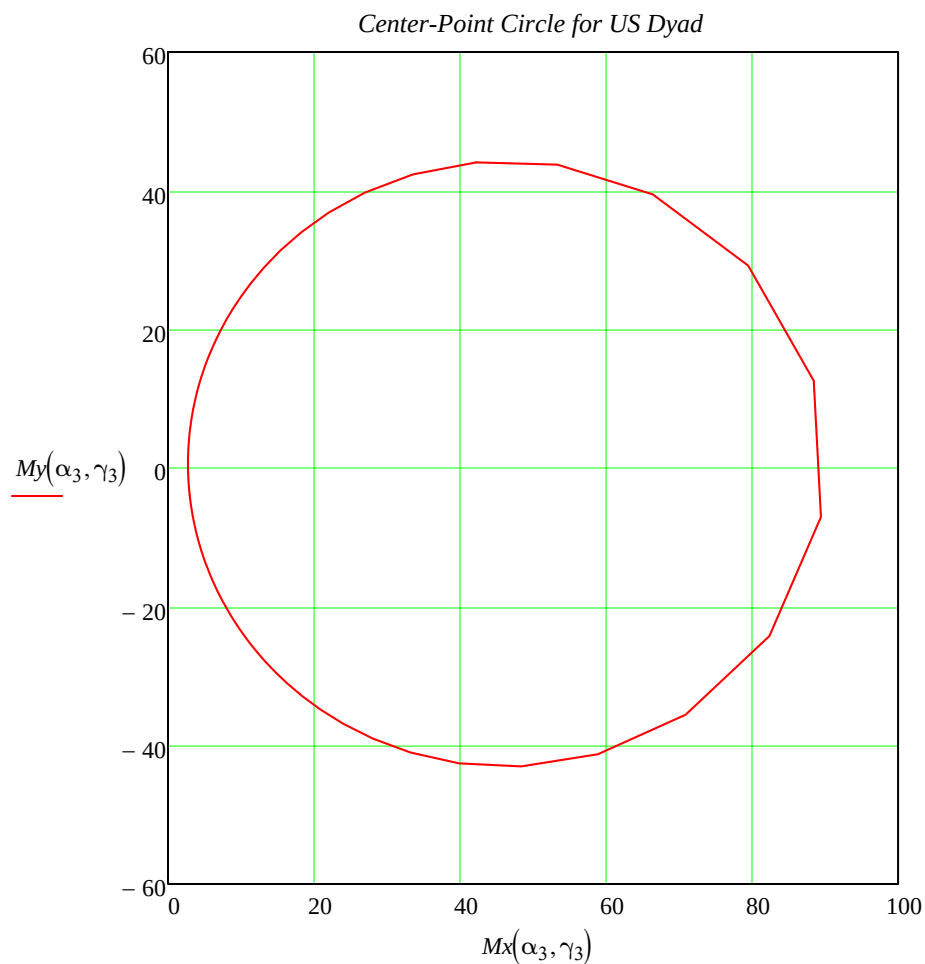
These are the same as the values calculated in Example 5-2.

8. Form the vector **M**, whose tip describes the center-point circle for the US dyad.

$$Mx(\alpha_3, \gamma_3) := -U1x(\alpha_3, \gamma_3) - S1x(\alpha_3, \gamma_3)$$

$$My(\alpha_3, \gamma_3) := -U1y(\alpha_3, \gamma_3) - S1y(\alpha_3, \gamma_3)$$

9. Plot the center-point circle for the US dyad.



10. Find the center and radius of this circle by taking three points on it and, from them, determine the radius and center point of the circle.

Three points:	$x_1 := Mx(\alpha_3, 0 \cdot \text{deg})$	$x_1 = 42.228$
	$y_1 := My(\alpha_3, 0 \cdot \text{deg})$	$y_1 = 44.088$

$$\begin{aligned}
 x_2 &:= Mx(\alpha_3, 90 \cdot \text{deg}) & x_2 &= 2.744 \\
 y_2 &:= My(\alpha_3, 90 \cdot \text{deg}) & y_2 &= -0.214 \\
 x_3 &:= Mx(\alpha_3, 350 \cdot \text{deg}) & x_3 &= 39.862 \\
 y_3 &:= My(\alpha_3, 350 \cdot \text{deg}) & y_3 &= -42.592
 \end{aligned}$$

Form four ratios: $a := \frac{y_2^2 - y_1^2 + x_2^2 - x_1^2}{2 \cdot (y_2 - y_1)} \quad a = 41.977$

$$b := \frac{x_2 - x_1}{y_2 - y_1} \quad b = 0.891$$

$$c := \frac{y_3^2 - y_2^2 + x_3^2 - x_2^2}{2 \cdot (y_3 - y_2)} \quad c = -38.321$$

$$d := \frac{x_3 - x_2}{y_3 - y_2} \quad d = -0.876$$

Circle center x coordinate: $h := \frac{a - c}{b - d} \quad h = 45.441$

Circle center y coordinate: $k := a - b \cdot h \quad k = 1.479$

Center-point circle radius: $R := \sqrt{(x_1 - h)^2 + (y_1 - k)^2} \quad R = 42.73$

11. Show that the point determined by $\gamma_3 = 80.6 \text{ deg}$ falls on this circle.

Coordinates of this point: $x_{g3} := Mx(\alpha_3, 80.6 \cdot \text{deg}) \quad x_{g3} = 2.738$
 $y_{g3} := My(\alpha_3, 80.6 \cdot \text{deg}) \quad y_{g3} = 0.338$

Substitute into radius equation:

$$R := \sqrt{(x_{g3} - h)^2 + (y_{g3} - k)^2} \quad R = 42.72$$

Since the radius is the same, this point does fall on the centerpoint circle for the **US** dyad.

PROBLEM 5-51

Statement: Design a driver dyad to move link 2 of Example 5-1 from position 1 to position 2 and return.

Given: Solution to Example 5-1:

$$\text{Length of link 2} \quad w := 2.467$$

$$\text{Angle of link 2 in first position} \quad \theta := 71.6 \cdot \text{deg}$$

$$\text{Rotation angle for link 2} \quad \beta_2 := 38.4 \cdot \text{deg}$$

Design Choice:

$$\text{Coordinates of } O_2 \quad O_{2x} := 0.00 \quad O_{2y} := 0.00$$

Solution: See Example 5-1, Figure 5-3 and Mathcad file P0551.

- Link 2 of the solution to Example 5-1 will become the driven link for the driver dyad. The driver dyad will be links 5 and 6 and the fixed pivot for the dyad will be at O_6 . Select a point on link 2 of Example 5-1 and label it C . Let the distance O_2C be $R_2 := 1.200$. The solution that follows uses the algorithm presented in Section 5.2 with changes in nomenclature to account for the fact that the driven link is link 2 and the points A and B are already defined on the fourbar of Example 5-1.
- Determine the coordinates of the points C_1 and C_2 using equations 5.0a. Determine the vector $\mathbf{M}$ using 5.0b.

$$C_{1x} := O_{2x} + R_2 \cdot \cos(\theta) \quad C_{1x} = 0.379$$

$$C_{1y} := O_{2y} + R_2 \cdot \sin(\theta) \quad C_{1y} = 1.139$$

$$C_{2x} := O_{2x} + R_2 \cdot \cos(\theta + \beta_2) \quad C_{2x} = -0.410$$

$$C_{2y} := O_{2y} + R_2 \cdot \sin(\theta + \beta_2) \quad C_{2y} = 1.128$$

$$\mathbf{R}_{C1} := \begin{pmatrix} C_{1x} \\ C_{1y} \end{pmatrix} \quad \mathbf{R}_{C2} := \begin{pmatrix} C_{2x} \\ C_{2y} \end{pmatrix} \quad \mathbf{M} := \mathbf{R}_{C2} - \mathbf{R}_{C1} \quad \mathbf{M} = \begin{pmatrix} -0.789 \\ -0.011 \end{pmatrix}$$

- Select a suitable value for the multiplier, K , in equation 5.0d say $K := 3.0$.
- Determine the coordinates of the crank pivot, O_6 using equation 5.0d.

$$\mathbf{R}_{O6} := \mathbf{R}_{C1} + K \cdot \mathbf{M} \quad O_{6x} := \mathbf{R}_{O6}_1 \quad O_{6y} := \mathbf{R}_{O6}_2$$

$$O_{6x} = -1.989 \quad O_{6y} = 1.106$$

- Determine the length of the driving crank using equation 5.0e.

$$R_6 := \left| R_2 \cdot \sin(0.5 \cdot \beta_2) \right| \quad R_6 = 0.395$$

- Determine the length of the driver dyad coupler, link 5, and the ground link from equation 5.0f.

$$R_5 := \left| \mathbf{R}_{C1} - \mathbf{R}_{O6} \right| - R_6 \quad R_5 = 1.973$$

$$\mathbf{R}_{O2} := \begin{pmatrix} O_{2x} \\ O_{2y} \end{pmatrix}$$

$$R_1 := \left| \mathbf{R}_{O2} - \mathbf{R}_{O6} \right| \quad R_1 = 2.275$$

- Determine the Grashof condition.

$$R_1 = 2.275$$

$$R_2 = 1.200$$

$$R_5 = 1.973$$

$$R_6 = 0.395$$

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$$\text{Condition}(R_1, R_2, R_5, R_6) = \text{"Grashof"}$$

8. Draw the linkage using the link lengths and fixed pivot coordinates calculated above to verify that the driver dyad will perform as required.

Solve for the coordinates of O_4 using equations 5.2:

Given: $z := 1.298$ $\phi := 26.5 \cdot \text{deg}$ $s := 1.035$ $\psi := 104.1 \cdot \text{deg}$

$u := 1.486$ $\sigma := 15.4 \cdot \text{deg}$

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = 1.162 \quad Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = 0.579$$

$$S_{1x} := s \cdot \cos(\psi) \quad S_{1x} = -0.252 \quad S_{1y} := s \cdot \sin(\psi) \quad S_{1y} = 1.004$$

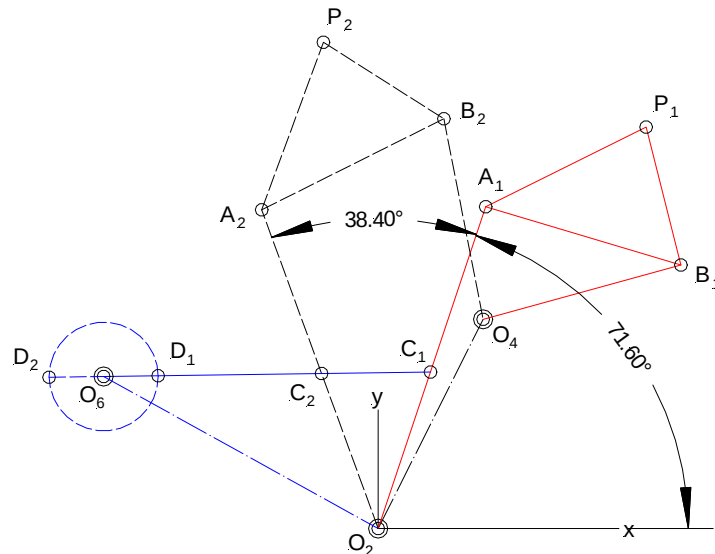
$$W_{1x} := w \cdot \cos(\theta) \quad W_{1x} = 0.779 \quad W_{1y} := w \cdot \sin(\theta) \quad W_{1y} = 2.341$$

$$U_{1x} := u \cdot \cos(\sigma) \quad U_{1x} = 1.433 \quad U_{1y} := u \cdot \sin(\sigma) \quad U_{1y} = 0.395$$

$$\mathbf{Z}_1 := \begin{pmatrix} Z_{1x} \\ Z_{1y} \end{pmatrix} \quad \mathbf{S}_1 := \begin{pmatrix} S_{1x} \\ S_{1y} \end{pmatrix} \quad \mathbf{W}_1 := \begin{pmatrix} W_{1x} \\ W_{1y} \end{pmatrix} \quad \mathbf{U}_1 := \begin{pmatrix} U_{1x} \\ U_{1y} \end{pmatrix}$$

$$\mathbf{V}_1 := \mathbf{Z}_1 - \mathbf{S}_1 \quad \mathbf{G}_1 := \mathbf{W}_1 + \mathbf{V}_1 - \mathbf{U}_1 \quad \mathbf{G}_1 = \begin{pmatrix} 0.760 \\ 1.522 \end{pmatrix}$$

$$g := |\mathbf{G}_1| \quad g = 1.701 \quad v := |\mathbf{V}_1| \quad v = 1.476$$



PROBLEM 5-52

Statement: Design a driver dyad to move link 2 of Example 5-2 from position 1 to position 3 and return.

Given: Solution to Example 5-2:

$$\begin{aligned} \text{Length of link 2} & w := 6.832 \\ \text{Angle of link 2 in first position} & \theta := \text{atan}\left(\frac{6.832}{0.055}\right) & \theta = 89.539 \text{ deg} \\ \text{Rotation angle for link 2} & \beta_3 := -35.20 \cdot \text{deg} \\ \text{Coordinates of } O_2 & O_{2x} := -1.234 & O_{2y} := -7.772 \end{aligned}$$

Solution: See Example 5-2, Figure 5-5, Table 5-1, and Mathcad file P0552.

- Link 2 of the solution to Example 5-2 will become the driven link for the driver dyad. The driver dyad will be links 5 and 6 and the fixed pivot for the dyad will be at O_6 . Select a point on link 2 of Example 5-2 and label it C . Let the distance O_2C be $R_2 := 4.000$. The solution that follows uses the algorithm presented in Section 5.2 with changes in nomenclature to account for the fact that the driven link is link 2 and the points A and B are already defined on the fourbar of Example 5-2.
- Determine the coordinates of the points C_1 and C_3 using equations 5.0a. Determine the vector $\mathbf{M}$ using 5.0b.

$$\begin{aligned} C_{1x} &:= O_{2x} + R_2 \cos(\theta) & C_{1x} &= -1.202 \\ C_{1y} &:= O_{2y} + R_2 \sin(\theta) & C_{1y} &= -3.772 \\ C_{3x} &:= O_{2x} + R_2 \cos(\theta + \beta_3) & C_{3x} &= 1.098 \\ C_{3y} &:= O_{2y} + R_2 \sin(\theta + \beta_3) & C_{3y} &= -4.522 \\ \mathbf{R}_{C1} &:= \begin{pmatrix} C_{1x} \\ C_{1y} \end{pmatrix} & \mathbf{R}_{C3} &:= \begin{pmatrix} C_{3x} \\ C_{3y} \end{pmatrix} & \mathbf{M} &:= \mathbf{R}_{C3} - \mathbf{R}_{C1} & \mathbf{M} &= \begin{pmatrix} 2.300 \\ -0.750 \end{pmatrix} \end{aligned}$$

- Select a suitable value for the multiplier, K , in equation 5.0d say $K := 3.0$.
- Determine the coordinates of the crank pivot, O_6 using equation 5.0d.

$$\begin{aligned} \mathbf{R}_{O6} &:= \mathbf{R}_{C1} + K \cdot \mathbf{M} & O_{6x} &:= \mathbf{R}_{O6}_1 & O_{6y} &:= \mathbf{R}_{O6}_2 \\ & & O_{6x} &= 5.698 & O_{6y} &= -6.022 \end{aligned}$$

- Determine the length of the driving crank using equation 5.0e.

$$R_6 := \left| R_2 \sin(0.5 \cdot \beta_3) \right| \quad R_6 = 1.209$$

- Determine the length of the driver dyad coupler, link 5, and the ground link from equation 5.0f.

$$\begin{aligned} R_5 &:= \left| \mathbf{R}_{C1} - \mathbf{R}_{O6} \right| - R_6 & R_5 &= 6.047 \\ \mathbf{R}_{O2} &:= \begin{pmatrix} O_{2x} \\ O_{2y} \end{pmatrix} \\ R_1 &:= \left| \mathbf{R}_{O2} - \mathbf{R}_{O6} \right| & R_1 &= 7.149 \end{aligned}$$

- Determine the Grashof condition.

$$R_1 = 7.149$$

$$R_2 = 4.000$$

$$R_5 = 6.047$$

$$R_6 = 1.209$$

```

Condition(a,b,c,d) :=
    S ← min(a,b,c,d)
    L ← max(a,b,c,d)
    SL ← S + L
    PQ ← a + b + c + d - SL
    return "Grashof"   if SL < PQ
    return "Special Grashof" if SL = PQ
    return "non-Grashof" otherwise

```

$$\text{Condition}(R_1, R_2, R_5, R_6) = \text{"Grashof"}$$

8. Draw the linkage using the link lengths and fixed pivot coordinates calculated above to verify that the driver dyad will perform as required.

Solve for the vectors **W**, **Z**, **U**, and **S**.

Given:

$$W_{1x} := 0.055$$

$$W_{1y} := 6.832$$

$$Z_{1x} := 1.179$$

$$Z_{1y} := 0.940$$

$$U_{1x} := -2.628$$

$$U_{1y} := -1.825$$

$$S_{1x} := -0.109$$

$$S_{1y} := 1.487$$

$$\mathbf{W}_1 := \begin{pmatrix} W_{1x} \\ W_{1y} \end{pmatrix}$$

$$\mathbf{Z}_1 := \begin{pmatrix} Z_{1x} \\ Z_{1y} \end{pmatrix}$$

$$\mathbf{U}_1 := \begin{pmatrix} U_{1x} \\ U_{1y} \end{pmatrix}$$

$$\mathbf{S}_1 := \begin{pmatrix} S_{1x} \\ S_{1y} \end{pmatrix}$$

$$w := |\mathbf{W}_1|$$

$$\theta := \text{atan2}(W_{1x}, W_{1y})$$

$$z := |\mathbf{Z}_1|$$

$$\phi := \text{atan2}(Z_{1x}, Z_{1y})$$

$$w = 6.832$$

$$\theta = 89.539 \text{ deg}$$

$$z = 1.508$$

$$\phi = 38.565 \text{ deg}$$

$$u := |\mathbf{U}_1|$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y})$$

$$s := |\mathbf{S}_1|$$

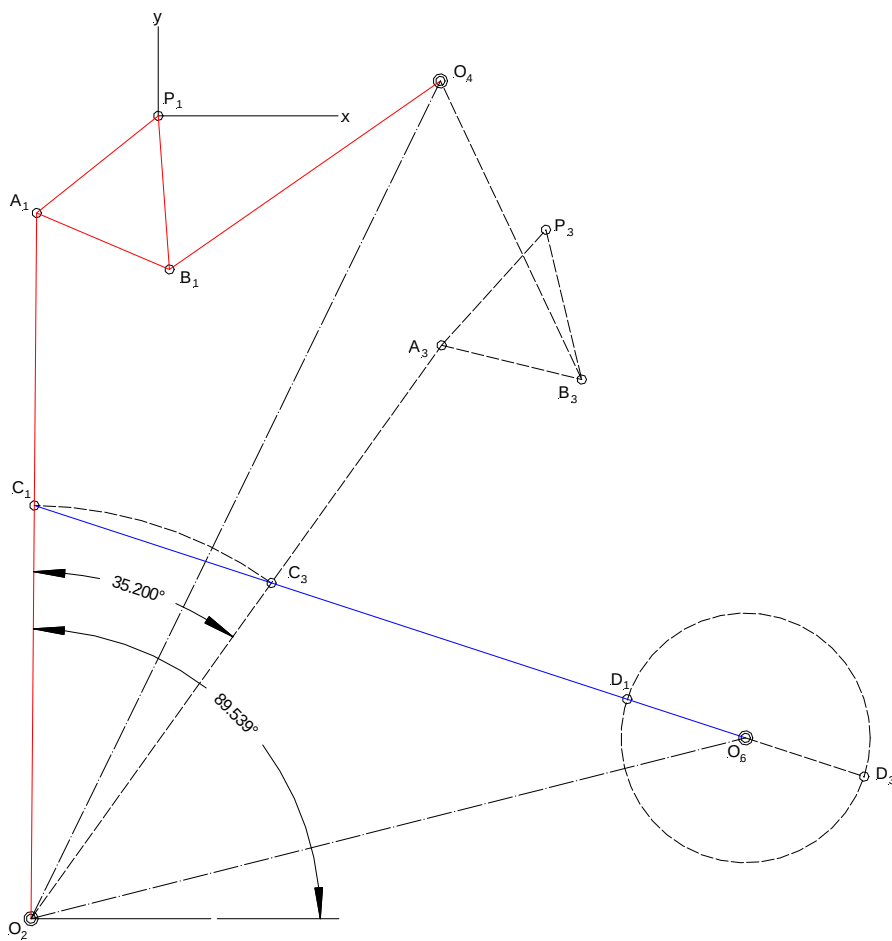
$$\psi := \text{atan2}(S_{1x}, S_{1y})$$

$$u = 3.200$$

$$\sigma = -145.222 \text{ deg}$$

$$s = 1.491$$

$$\psi = 94.192 \text{ deg}$$



PROBLEM 5-53

Statement: Design a driver dyad to move link 2 of Example 5-3 from position 1 to position 3 and return.

Given: Solution to Example 5-3:

$$\begin{aligned} \text{Length of link 2} & w := 1.000 \\ \text{Angle of link 2 in first position} & \theta := \text{atan}\left(\frac{0.500}{0.866}\right) & \theta = 30.001 \text{ deg} \\ \text{Rotation angle for link 2} & \beta_3 := 23.96 \cdot \text{deg} \\ \text{Coordinates of } O_2 & O_{2x} := -1.712 & O_{2y} := 0.033 \end{aligned}$$

Solution: See Example 5-3, Figure 5-7, Table 5-2, and Mathcad file P0553.

- Link 2 of the solution to Example 5-3 will become the driven link for the driver dyad. The driver dyad will be links 5 and 6 and the fixed pivot for the dyad will be at O_6 . Select a point on link 2 of Example 5-3 and label it C . Let the distance O_2C be $R_2 := 0.500$. The solution that follows uses the algorithm presented in Section 5.2 with changes in nomenclature to account for the fact that the driven link is link 2 and the points A and B are already defined on the fourbar of Example 5-3.
- Determine the coordinates of the points C_1 and C_3 using equations 5.0a. Determine the vector $\mathbf{M}$ using 5.0b.

$$\begin{aligned} C_{1x} &:= O_{2x} + R_2 \cdot \cos(\theta) & C_{1x} &= -1.279 \\ C_{1y} &:= O_{2y} + R_2 \cdot \sin(\theta) & C_{1y} &= 0.283 \\ C_{3x} &:= O_{2x} + R_2 \cdot \cos(\theta + \beta_3) & C_{3x} &= -1.418 \\ C_{3y} &:= O_{2y} + R_2 \cdot \sin(\theta + \beta_3) & C_{3y} &= 0.437 \\ \mathbf{R}_{C1} &:= \begin{pmatrix} C_{1x} \\ C_{1y} \end{pmatrix} & \mathbf{R}_{C3} &:= \begin{pmatrix} C_{3x} \\ C_{3y} \end{pmatrix} & \mathbf{M} &:= \mathbf{R}_{C3} - \mathbf{R}_{C1} & \mathbf{M} &= \begin{pmatrix} -0.139 \\ 0.154 \end{pmatrix} \end{aligned}$$

- Select a suitable value for the multiplier, K , in equation 5.0d say $K := 3.0$.
- Determine the coordinates of the crank pivot, O_6 using equation 5.0d.

$$\begin{aligned} \mathbf{R}_{O6} &:= \mathbf{R}_{C1} + K \cdot \mathbf{M} & O_{6x} &:= \mathbf{R}_{O6}_1 & O_{6y} &:= \mathbf{R}_{O6}_2 \\ & & O_{6x} &= -1.696 & O_{6y} &= 0.746 \end{aligned}$$

- Determine the length of the driving crank using equation 5.0e.

$$R_6 := \left| R_2 \cdot \sin(0.5 \cdot \beta_3) \right| \quad R_6 = 0.104$$

- Determine the length of the driver dyad coupler, link 5, and the ground link from equation 5.0f.

$$\begin{aligned} R_5 &:= \left| \mathbf{R}_{C1} - \mathbf{R}_{O6} \right| - R_6 & R_5 &= 0.519 \\ \mathbf{R}_{O2} &:= \begin{pmatrix} O_{2x} \\ O_{2y} \end{pmatrix} \\ R_1 &:= \left| \mathbf{R}_{O2} - \mathbf{R}_{O6} \right| & R_1 &= 0.713 \end{aligned}$$

- Determine the Grashof condition.

$$R_1 = 0.713$$

$$R_2 = 0.500$$

$$R_5 = 0.519$$

$$R_6 = 0.104$$

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof" if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$$\text{Condition}(R_1, R_2, R_5, R_6) = \text{"Grashof"}$$

8. Draw the linkage using the link lengths and fixed pivot coordinates calculated above to verify that the driver dyad will perform as required.

Solve for the vectors **W**, **Z**, **U**, and **S**.

Given:

$$W_{1x} := 0.866$$

$$Z_{1x} := 0.846$$

$$U_{1x} := -0.253$$

$$S_{1x} := -0.035$$

$$W_{1y} := 0.500$$

$$Z_{1y} := -0.533$$

$$U_{1y} := 0.973$$

$$S_{1y} := -1.006$$

$$\mathbf{W}_1 := \begin{pmatrix} W_{1x} \\ W_{1y} \end{pmatrix}$$

$$\mathbf{Z}_1 := \begin{pmatrix} Z_{1x} \\ Z_{1y} \end{pmatrix}$$

$$\mathbf{U}_1 := \begin{pmatrix} U_{1x} \\ U_{1y} \end{pmatrix}$$

$$\mathbf{S}_1 := \begin{pmatrix} S_{1x} \\ S_{1y} \end{pmatrix}$$

$$w := |\mathbf{W}_1|$$

$$w = 1.000$$

$$u := |\mathbf{U}_1|$$

$$u = 1.005$$

$$\theta := \text{atan2}(W_{1x}, W_{1y})$$

$$\theta = 30.001 \text{ deg}$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y})$$

$$\sigma = 104.575 \text{ deg}$$

$$z := |\mathbf{Z}_1|$$

$$z = 1.000$$

$$s := |\mathbf{S}_1|$$

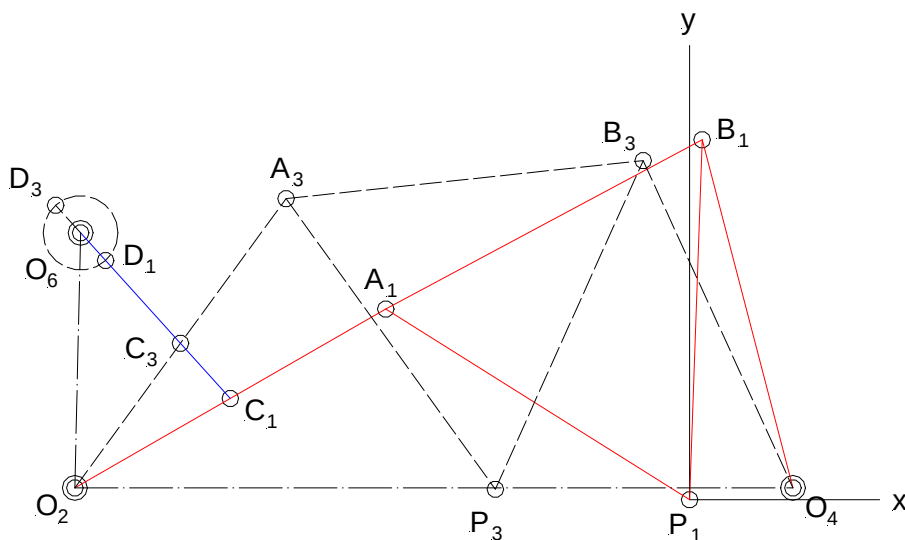
$$s = 1.007$$

$$\phi := \text{atan2}(Z_{1x}, Z_{1y})$$

$$\phi = -32.212 \text{ deg}$$

$$\psi := \text{atan2}(S_{1x}, S_{1y})$$

$$\psi = -91.993 \text{ deg}$$



PROBLEM 5-54

Statement: Design a fourbar linkage to carry the object in Figure P5-12 from position 1 to 2 using points C and D for your attachment points. The fixed pivots should be within the indicated area.

Given: Coordinates of the points P_1 (C_1) and P_2 (C_2):

$$P_{1x} := 0.0 \quad P_{1y} := 0.0 \quad P_{2x} := 13.871 \quad P_{2y} := -3.299$$

Length of the coupler (link 3): $v := 12.387$

Angles made by the body in positions 1 and 2:

$$\theta_{p1} := 0.0 \cdot \text{deg} \quad \theta_{p2} := 24.0 \cdot \text{deg}$$

Coordinates of the points C_1 and D_1 with respect to P_1 :

$$C_{1x} := 0.0 \quad C_{1y} := 0.0 \quad D_{1x} := C_{1x} + v \cdot \cos(\theta_{p1}) \quad D_{1y} := C_{1y} + v \cdot \sin(\theta_{p1})$$

Free choice for the **WZ** dyad: $\beta_2 := 45 \cdot \text{deg}$

$$D_{1x} = 12.387$$

Free choice for the **US** dyad: $\gamma_2 := 70 \cdot \text{deg}$

$$D_{1y} = 0.000$$

Solution: See Figure P5-12 and Mathcad file P0554.

- Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \quad R2 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \quad P_{21x} = 13.871$$

$$P_{21y} = -3.299$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 14.258$$

- From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\alpha_2 := \theta_{p2} - \theta_{p1} \quad \alpha_2 = 24.000 \cdot \text{deg}$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = -13.378 \cdot \text{deg}$$

- Using Figure P5-4, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$z := \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} \quad z = 0.000$$

$$s := \sqrt{(P_{1x} - D_{1x})^2 + (P_{1y} - D_{1y})^2} \quad s = 12.387$$

$$v := \sqrt{(C_{1x} - D_{1x})^2 + (C_{1y} - D_{1y})^2} \quad v = 12.387$$

$$\phi := \text{acos}\left(\frac{v^2 + z^2 - s^2}{2 \cdot v \cdot z}\right) + \theta_{p1} \quad \phi = 90.000 \cdot \text{deg}$$

$$\psi := \pi - \text{acos}\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) + \theta_{p1} \quad \psi = 180.000 \cdot \text{deg}$$

- Solve for the **WZ** dyad using equations 5.8.

$$\begin{array}{llll}
Z_{1x} := z \cdot \cos(\phi) & Z_{1x} = 0.000 & Z_{1y} := z \cdot \sin(\phi) & Z_{1y} = 0.000 \\
A := \cos(\beta_2) - 1 & A = -0.293 & D := \sin(\alpha_2) & D = 0.407 \\
B := \sin(\beta_2) & B = 0.707 & E := p_{21} \cdot \cos(\delta_2) & E = 13.871 \\
C := \cos(\alpha_2) - 1 & C = -0.086 & F := p_{21} \cdot \sin(\delta_2) & F = -3.299
\end{array}$$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \quad W_{1x} = -10.918$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \quad W_{1y} = -15.094$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \quad w = 18.629$$

$$\theta := \text{atan2}(W_{1x}, W_{1y}) \quad \theta = -125.878 \cdot \text{deg}$$

6. Solve for the **US** dyad using equations 5.12.

$$\begin{array}{llll}
S_{1x} := s \cdot \cos(\psi) & S_{1x} = -12.387 & S_{1y} := s \cdot \sin(\psi) & S_{1y} = 0.000 \\
A := \cos(\gamma_2) - 1 & A = -0.658 & D := \sin(\alpha_2) & D = 0.407 \\
B := \sin(\gamma_2) & B = 0.940 & E := p_{21} \cdot \cos(\delta_2) & E = 13.871 \\
C := \cos(\alpha_2) - 1 & C = -0.086 & F := p_{21} \cdot \sin(\delta_2) & F = -3.299
\end{array}$$

$$U_{1x} := \frac{A \cdot (-C \cdot S_{1x} + D \cdot S_{1y} + E) + B \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F)}{-2 \cdot A} \quad U_{1x} = -5.158$$

$$U_{1y} := \frac{A \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F) + B \cdot (C \cdot S_{1x} - D \cdot S_{1y} - E)}{-2 \cdot A} \quad U_{1y} = -10.010$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2} \quad u = 11.261$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y}) \quad \sigma = -117.262 \cdot \text{deg}$$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\begin{array}{ll}
\text{Link 3: } V_{1x} := z \cdot \cos(\phi) - s \cdot \cos(\psi) & V_{1x} = 12.387 \\
V_{1y} := z \cdot \sin(\phi) - s \cdot \sin(\psi) & V_{1y} = 0.000 \\
\theta_3 := \text{atan2}(V_{1x}, V_{1y}) & \theta_3 = 0.000 \cdot \text{deg} \\
v := \sqrt{V_{1x}^2 + V_{1y}^2} & v = 12.387 \\
\text{Link 1: } G_{1x} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma) & G_{1x} = 6.627
\end{array}$$

$$\begin{aligned}
 G_{1y} &:= w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma) & G_{1y} &= -5.084 \\
 \theta_1 &:= \text{atan2}(G_{1x}, G_{1y}) & \theta_1 &= -37.495 \cdot \text{deg} \\
 g &:= \sqrt{G_{1x}^2 + G_{1y}^2} & g &= 8.353
 \end{aligned}$$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\begin{aligned}
 \theta_{2i} &:= \theta - \theta_1 & \theta_{2i} &= -88.383 \cdot \text{deg} \\
 \theta_{2f} &:= \theta_{2i} + \beta_2 & \theta_{2f} &= -43.383 \cdot \text{deg}
 \end{aligned}$$

9. Define the coupler point with respect to point A and the vector **V**.

$$\begin{aligned}
 r_p &:= z & \delta_p &:= \phi - \theta_3 \\
 r_p &= 0.000 & \delta_p &= 90.000 \cdot \text{deg}
 \end{aligned}$$

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\begin{aligned}
 \rho_1 &:= 0.0 \cdot \text{deg} & \rho_1 &= 0.000 \cdot \text{deg} \\
 R_1 &:= \sqrt{P_{1x}^2 + P_{1y}^2} & R_1 &= 0.000 \\
 O_{2x} &:= R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta) & O_{2x} &= 10.918 \\
 O_{2y} &:= R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta) & O_{2y} &= 15.094 \\
 O_{4x} &:= R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma) & O_{4x} &= 17.545 \\
 O_{4y} &:= R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma) & O_{4y} &= 10.010
 \end{aligned}$$

These fixed pivot points fall on the base and are, therefore, acceptable.

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{\text{rot}} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{\text{rot}} = -37.495 \cdot \text{deg}$$

12. Determine the Grashof condition.

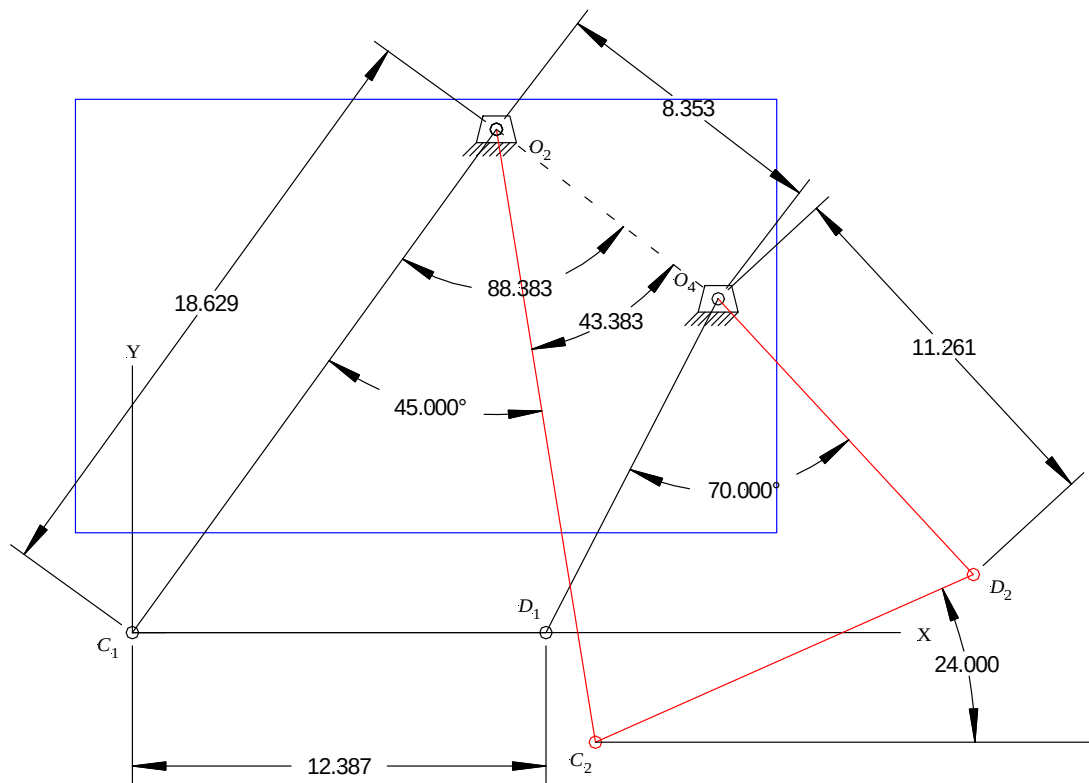
$$\begin{aligned}
 \text{Condition}(a, b, c, d) &:= \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}
 \end{aligned}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

13. DESIGN SUMMARY

Link 2:	$w = 18.629$	$\theta = -125.878 \cdot \text{deg}$
Link 3:	$v = 12.387$	$\theta_3 = 0.000 \cdot \text{deg}$
Link 4:	$u = 11.261$	$\sigma = -117.262 \cdot \text{deg}$
Link 1:	$g = 8.353$	$\theta_1 = -37.495 \cdot \text{deg}$
Coupler:	$r_p = 0.000$	$\delta_p = 90.000 \cdot \text{deg}$
Crank angles:	$\theta_{2i} = -88.383 \cdot \text{deg}$	$\theta_{2f} = -43.383 \cdot \text{deg}$

14. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-55

Statement: Design a fourbar linkage to carry the object in Figure P5-12 from position 1 to 3 using points C and D for your attachment points. The fixed pivots should be within the indicated area.

Given: Coordinates of the points P_1 (C_1) and P_2 (C_2):

$$P_{1x} := 0.0 \quad P_{1y} := 0.0 \quad P_{2x} := 19.544 \quad P_{2y} := -0.373$$

Length of the coupler (link 3) $v := 12.387$

Angles made by the body in positions 1 and 3:

$$\theta_{p1} := 00.0\text{-deg} \quad \theta_{p2} := 90.0\text{-deg}$$

Coordinates of the points C_1 and D_1 :

$$C_{1x} := 0.0 \quad C_{1y} := 0.0 \quad D_{1x} := C_{1x} + v \cdot \cos(\theta_{p1}) \quad D_{1y} := C_{1y} + v \cdot \sin(\theta_{p1})$$

Free choice for the **WZ** dyad: $\beta_2 := 80\text{-deg}$

$$D_{1x} = 12.387$$

Free choice for the **US** dyad: $\gamma_2 := 180\text{-deg}$

$$D_{1y} = 0.000$$

Solution: See Figure P5-12 and Mathcad file P0555.

- Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$\mathbf{R}_1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \quad \mathbf{R}_2 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := \mathbf{R}_2 - \mathbf{R}_1 \quad P_{21x} = 19.544$$

$$P_{21y} = -0.373$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 19.548$$

- From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\alpha_2 := \theta_{p2} - \theta_{p1} \quad \alpha_2 = 90.000\text{-deg}$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = -1.093\text{-deg}$$

- Using Figure P5-4, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$z := \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} \quad z = 0.000$$

$$s := \sqrt{(P_{1x} - D_{1x})^2 + (P_{1y} - D_{1y})^2} \quad s = 12.387$$

$$v := \sqrt{(C_{1x} - D_{1x})^2 + (C_{1y} - D_{1y})^2} \quad v = 12.387$$

$$\phi := \text{acos}\left(\frac{v^2 + z^2 - s^2}{2 \cdot v \cdot z}\right) + \theta_{p1} \quad \phi = 90.000\text{-deg}$$

$$\psi := \pi - \text{acos}\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) + \theta_{p1} \quad \psi = 180.000\text{-deg}$$

- Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = 0.000 \quad Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = 0.000$$

$$\begin{aligned}
 A &:= \cos(\beta_2) - 1 & A &= -0.826 & D &:= \sin(\alpha_2) & D &= 1.000 \\
 B &:= \sin(\beta_2) & B &= 0.985 & E &:= p_{21} \cdot \cos(\delta_2) & E &= 19.544 \\
 C &:= \cos(\alpha_2) - 1 & C &= -1.000 & F &:= p_{21} \cdot \sin(\delta_2) & F &= -0.373 \\
 W_{Ix} &:= \frac{A \cdot (-C \cdot Z_{Ix} + D \cdot Z_{Iy} + E) + B \cdot (-C \cdot Z_{Iy} - D \cdot Z_{Ix} + F)}{-2 \cdot A} & W_{Ix} &= -9.994 \\
 W_{Iy} &:= \frac{A \cdot (-C \cdot Z_{Iy} - D \cdot Z_{Ix} + F) + B \cdot (C \cdot Z_{Ix} - D \cdot Z_{Iy} - E)}{-2 \cdot A} & W_{Iy} &= -11.459 \\
 w &:= \sqrt{W_{Ix}^2 + W_{Iy}^2} & w &= 15.205 \\
 \theta &:= \text{atan2}(W_{Ix}, W_{Iy}) & \theta &= -131.093 \cdot \text{deg}
 \end{aligned}$$

6. Solve for the **US** dyad using equations 5.12.

$$\begin{aligned}
 S_{Ix} &:= s \cdot \cos(\psi) & S_{Ix} &= -12.387 & S_{Iy} &:= s \cdot \sin(\psi) & S_{Iy} &= 0.000 \\
 A &:= \cos(\gamma_2) - 1 & A &= -2.000 & D &:= \sin(\alpha_2) & D &= 1.000 \\
 B &:= \sin(\gamma_2) & B &= 0.000 & E &:= p_{21} \cdot \cos(\delta_2) & E &= 19.544 \\
 C &:= \cos(\alpha_2) - 1 & C &= -1.000 & F &:= p_{21} \cdot \sin(\delta_2) & F &= -0.373 \\
 U_{Ix} &:= \frac{A \cdot (-C \cdot S_{Ix} + D \cdot S_{Iy} + E) + B \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F)}{-2 \cdot A} & U_{Ix} &= -3.579 \\
 U_{Iy} &:= \frac{A \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F) + B \cdot (C \cdot S_{Ix} - D \cdot S_{Iy} - E)}{-2 \cdot A} & U_{Iy} &= -6.007 \\
 u &:= \sqrt{U_{Ix}^2 + U_{Iy}^2} & u &= 6.992 \\
 \sigma &:= \text{atan2}(U_{Ix}, U_{Iy}) & \sigma &= -120.783 \cdot \text{deg}
 \end{aligned}$$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\begin{aligned}
 \text{Link 3: } V_{Ix} &:= z \cdot \cos(\phi) - s \cdot \cos(\psi) & V_{Ix} &= 12.387 \\
 V_{Iy} &:= z \cdot \sin(\phi) - s \cdot \sin(\psi) & V_{Iy} &= 0.000 \\
 \theta_3 &:= \text{atan2}(V_{Ix}, V_{Iy}) & \theta_3 &= 0.000 \cdot \text{deg} \\
 v &:= \sqrt{V_{Ix}^2 + V_{Iy}^2} & v &= 12.387 \\
 \text{Link 1: } G_{Ix} &:= w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma) & G_{Ix} &= 5.971 \\
 G_{Iy} &:= w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma) & G_{Iy} &= -5.452 \\
 \theta_1 &:= \text{atan2}(G_{Ix}, G_{Iy}) & \theta_1 &= -42.399 \cdot \text{deg}
 \end{aligned}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2} \quad g = 8.086$$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\begin{aligned} \theta_{2i} &:= \theta - \theta_1 & \theta_{2i} &= -88.694 \cdot \text{deg} \\ \theta_{2f} &:= \theta_{2i} + \beta_2 & \theta_{2f} &= -8.694 \cdot \text{deg} \end{aligned}$$

9. Define the coupler point with respect to point A and the vector **V**.

$$\begin{aligned} r_p &:= z & \delta_p &:= \phi - \theta_3 \\ r_p &= 0.000 & \delta_p &= 90.000 \cdot \text{deg} \end{aligned}$$

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := 0.0 \cdot \text{deg} \quad \rho_1 = 0.000 \cdot \text{deg}$$

$$R_1 := \sqrt{P_{1x}^2 + P_{1y}^2} \quad R_1 = 0.000$$

$$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta) \quad O_{2x} = 9.994$$

$$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta) \quad O_{2y} = 11.459$$

$$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma) \quad O_{4x} = 15.966$$

$$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma) \quad O_{4y} = 6.007$$

These fixed pivot points fall on the base and are, therefore, acceptable.

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = -42.399 \cdot \text{deg}$$

12. Determine the Grashof condition.

$$\begin{aligned} \text{Condition}(a, b, c, d) &:= \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Condition}(g, u, v, w) = \text{"non-Grashof"}$$

13. DESIGN SUMMARY

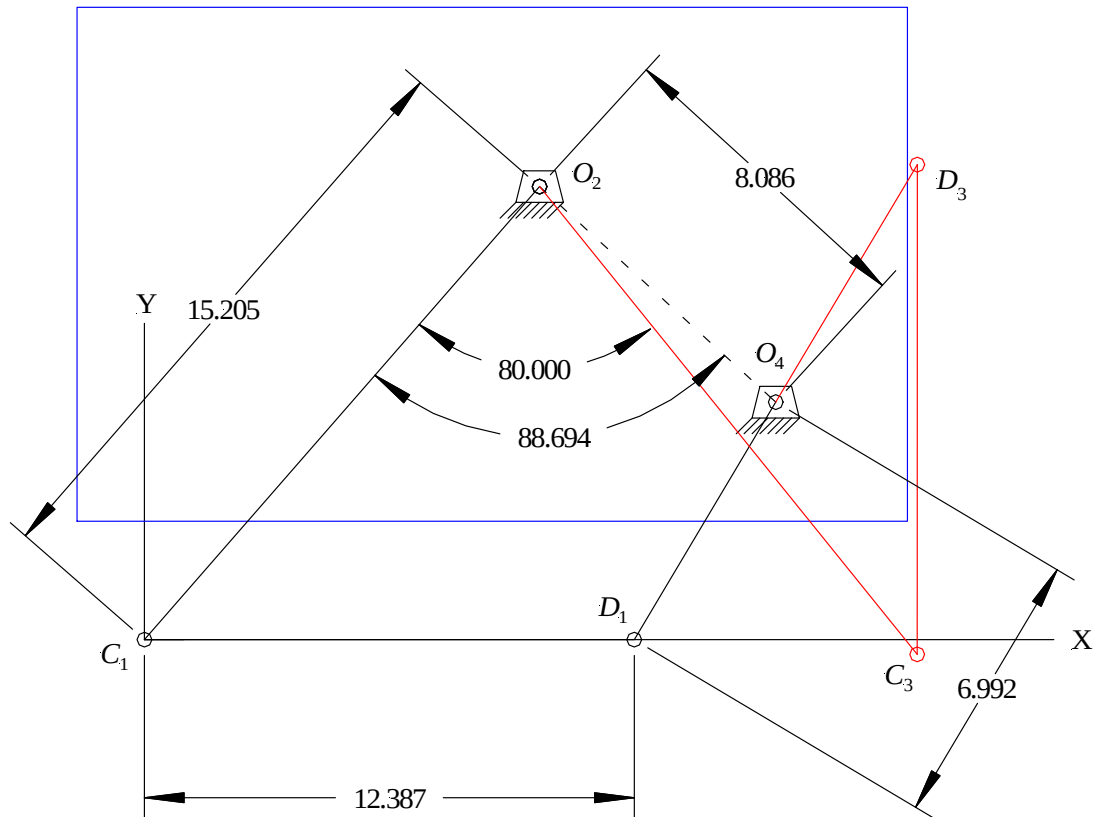
Link 2:	$w = 15.205$	$\theta = -131.093 \cdot \text{deg}$
Link 3:	$v = 12.387$	$\theta_3 = 0.000 \cdot \text{deg}$
Link 4:	$u = 6.992$	$\sigma = -120.783 \cdot \text{deg}$
Link 1:	$g = 8.086$	$\theta_1 = -42.399 \cdot \text{deg}$
Coupler:	$r_p = 0.000$	$\delta_p = 90.000 \cdot \text{deg}$

Crank angles:

$$\theta_{2i} = -88.694 \cdot \text{deg}$$

$$\theta_{2f} = -8.694 \cdot \text{deg}$$

14. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-56

Statement: Design a fourbar linkage to carry the object in Figure P5-12 from position 2 to 3 using points C and D for your attachment points. The fixed pivots should be within the indicated area.

Given: Coordinates of the points P_1 (C_2) and P_2 (C_3):

$$P_{1x} := 13.871 \quad P_{1y} := -3.299 \quad P_{2x} := 19.544 \quad P_{2y} := -0.373$$

Length of the coupler (link 3): $v := 12.387$

Angles made by the body in positions 2 and 3:

$$\theta_{p1} := 24.0 \cdot \text{deg} \quad \theta_{p2} := 90.0 \cdot \text{deg}$$

Coordinates of the points C_2 and D_2 :

$$C_{2x} := P_{1x} \quad C_{2y} := P_{1y} \quad D_{2x} := C_{2x} + v \cdot \cos(\theta_{p1}) \quad D_{2y} := C_{2y} + v \cdot \sin(\theta_{p1})$$

Free choice for the **WZ** dyad: $\beta_2 := 35 \cdot \text{deg}$

$$D_{2x} = 25.187$$

Free choice for the **US** dyad: $\gamma_2 := 90 \cdot \text{deg}$

$$D_{2y} = 1.739$$

Solution: See Figure P5-12 and Mathcad file P0556.

- Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. The second method of Section 5.3 will be used here.
- Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$\mathbf{R}_1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \quad \mathbf{R}_2 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \quad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := \mathbf{R}_2 - \mathbf{R}_1 \quad P_{21x} = 5.673$$

$$P_{21y} = 2.926$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \quad p_{21} = 6.383$$

- From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2 .

$$\alpha_2 := \theta_{p2} - \theta_{p1} \quad \alpha_2 = 66.000 \cdot \text{deg}$$

$$\delta_2 := \text{atan2}(P_{21x}, P_{21y}) \quad \delta_2 = 27.284 \cdot \text{deg}$$

- Using Figure P5-4, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$z := \sqrt{(P_{1x} - C_{2x})^2 + (P_{1y} - C_{2y})^2} \quad z = 0.000$$

$$s := \sqrt{(P_{1x} - D_{2x})^2 + (P_{1y} - D_{2y})^2} \quad s = 12.387$$

$$v := \sqrt{(C_{2x} - D_{2x})^2 + (C_{2y} - D_{2y})^2} \quad v = 12.387$$

$$\phi := \text{acos}\left(\frac{v^2 + z^2 - s^2}{2 \cdot v \cdot z}\right) + \theta_{p1} \quad \phi = 114.000 \cdot \text{deg}$$

$$\psi := \pi - \text{acos}\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) + \theta_{p1} \quad \psi = 204.000 \cdot \text{deg}$$

- Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot \cos(\phi) \quad Z_{1x} = 0.000 \quad Z_{1y} := z \cdot \sin(\phi) \quad Z_{1y} = 0.000$$

$$\begin{aligned}
 A &:= \cos(\beta_2) - 1 & A &= -0.181 & D &:= \sin(\alpha_2) & D &= 0.914 \\
 B &:= \sin(\beta_2) & B &= 0.574 & E &:= p_{21} \cdot \cos(\delta_2) & E &= 5.673 \\
 C &:= \cos(\alpha_2) - 1 & C &= -0.593 & F &:= p_{21} \cdot \sin(\delta_2) & F &= 2.926
 \end{aligned}$$

$$W_{Ix} := \frac{A \cdot (-C \cdot Z_{Ix} + D \cdot Z_{Iy} + E) + B \cdot (-C \cdot Z_{Iy} - D \cdot Z_{Ix} + F)}{-2 \cdot A} \quad W_{Ix} = 1.804$$

$$W_{Iy} := \frac{A \cdot (-C \cdot Z_{Iy} - D \cdot Z_{Ix} + F) + B \cdot (C \cdot Z_{Ix} - D \cdot Z_{Iy} - E)}{-2 \cdot A} \quad W_{Iy} = -10.459$$

$$w := \sqrt{W_{Ix}^2 + W_{Iy}^2} \quad w = 10.614$$

$$\theta := \text{atan2}(W_{Ix}, W_{Iy}) \quad \theta = -80.216 \cdot \text{deg}$$

6. Solve for the **US** dyad using equations 5.12.

$$S_{Ix} := s \cdot \cos(\psi) \quad S_{Ix} = -11.316 \quad S_{Iy} := s \cdot \sin(\psi) \quad S_{Iy} = -5.038$$

$$A := \cos(\gamma_2) - 1 \quad A = -1.000 \quad D := \sin(\alpha_2) \quad D = 0.914$$

$$B := \sin(\gamma_2) \quad B = 1.000 \quad E := p_{21} \cdot \cos(\delta_2) \quad E = 5.673$$

$$C := \cos(\alpha_2) - 1 \quad C = -0.593 \quad F := p_{21} \cdot \sin(\delta_2) \quad F = 2.926$$

$$U_{Ix} := \frac{A \cdot (-C \cdot S_{Ix} + D \cdot S_{Iy} + E) + B \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F)}{-2 \cdot A} \quad U_{Ix} = 7.959$$

$$U_{Iy} := \frac{A \cdot (-C \cdot S_{Iy} - D \cdot S_{Ix} + F) + B \cdot (C \cdot S_{Ix} - D \cdot S_{Iy} - E)}{-2 \cdot A} \quad U_{Iy} = -2.316$$

$$u := \sqrt{U_{Ix}^2 + U_{Iy}^2} \quad u = 8.289$$

$$\sigma := \text{atan2}(U_{Ix}, U_{Iy}) \quad \sigma = -16.224 \cdot \text{deg}$$

7. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

$$\text{Link 3: } V_{Ix} := z \cdot \cos(\phi) - s \cdot \cos(\psi) \quad V_{Ix} = 11.316$$

$$V_{Iy} := z \cdot \sin(\phi) - s \cdot \sin(\psi) \quad V_{Iy} = 5.038$$

$$\theta_3 := \text{atan2}(V_{Ix}, V_{Iy}) \quad \theta_3 = 24.000 \cdot \text{deg}$$

$$v := \sqrt{V_{Ix}^2 + V_{Iy}^2} \quad v = 12.387$$

$$\text{Link 1: } G_{Ix} := w \cdot \cos(\theta) + v \cdot \cos(\theta_3) - u \cdot \cos(\sigma) \quad G_{Ix} = 5.161$$

$$G_{Iy} := w \cdot \sin(\theta) + v \cdot \sin(\theta_3) - u \cdot \sin(\sigma) \quad G_{Iy} = -3.105$$

$$\theta_1 := \text{atan2}(G_{Ix}, G_{Iy}) \quad \theta_1 = -31.035 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2} \quad g = 6.023$$

8. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1 \quad \theta_{2i} = -49.181 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_2 \quad \theta_{2f} = -14.181 \cdot \text{deg}$$

9. Define the coupler point with respect to point **C** and the vector **V**.

$$r_p := z \quad \delta_p := \phi - \theta_3$$

$$r_p = 0.000 \quad \delta_p = 90.000 \cdot \text{deg}$$

10. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$\rho_1 := \text{atan2}(P_{1x}, P_{1y}) \quad \rho_1 = -13.378 \cdot \text{deg}$$

$$R_1 := \sqrt{P_{1x}^2 + P_{1y}^2} \quad R_1 = 14.258$$

$$O_{2x} := R_1 \cdot \cos(\rho_1) - z \cdot \cos(\phi) - w \cdot \cos(\theta) \quad O_{2x} = 12.067$$

$$O_{2y} := R_1 \cdot \sin(\rho_1) - z \cdot \sin(\phi) - w \cdot \sin(\theta) \quad O_{2y} = 7.160$$

$$O_{4x} := R_1 \cdot \cos(\rho_1) - s \cdot \cos(\psi) - u \cdot \cos(\sigma) \quad O_{4x} = 17.228$$

$$O_{4y} := R_1 \cdot \sin(\rho_1) - s \cdot \sin(\psi) - u \cdot \sin(\sigma) \quad O_{4y} = 4.055$$

These fixed pivot points fall on the base and are, therefore, acceptable.

11. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})] \quad \theta_{rot} = -31.035 \cdot \text{deg}$$

12. Determine the Grashof condition.

$$\text{Condition}(a, b, c, d) := \begin{cases} S \leftarrow \min(a, b, c, d) \\ L \leftarrow \max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ \text{return "Grashof"} & \text{if } SL < PQ \\ \text{return "Special Grashof"} & \text{if } SL = PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(g, u, v, w) = \text{"Grashof"}$$

13. DESIGN SUMMARY

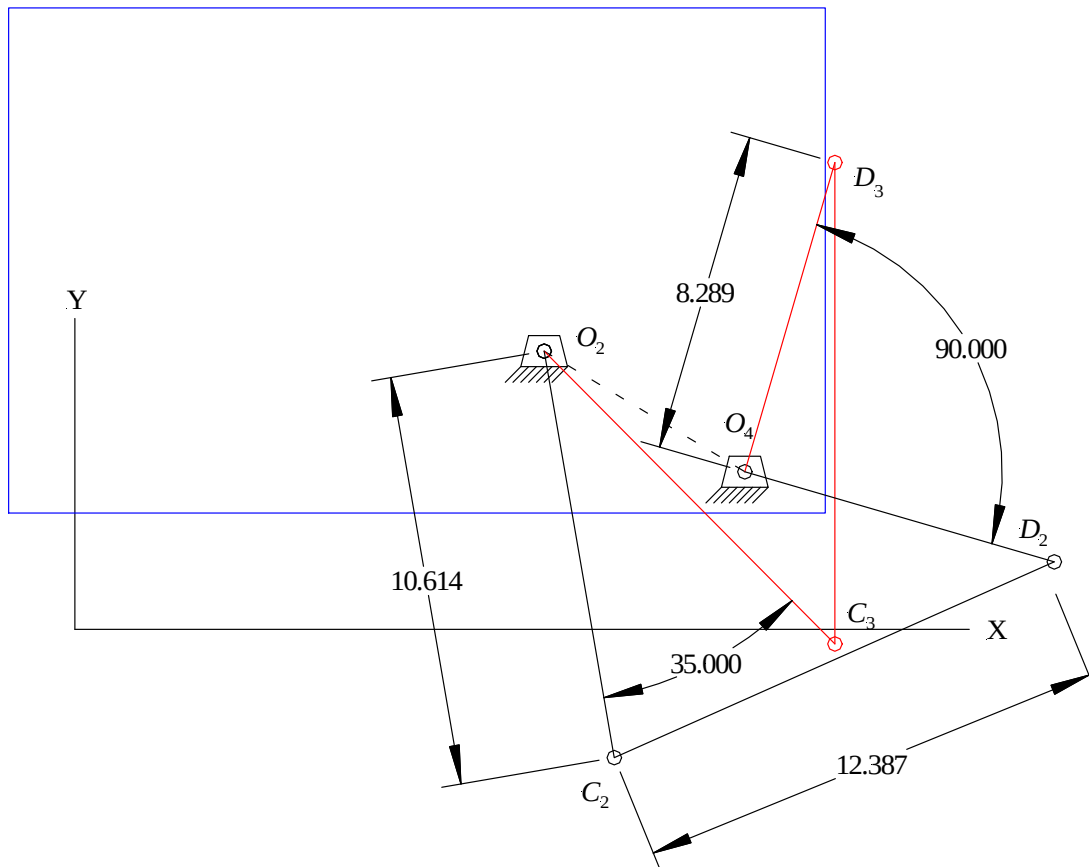
Link 2:	$w = 10.614$	$\theta = -80.216 \cdot \text{deg}$
Link 3:	$v = 12.387$	$\theta_3 = 24.000 \cdot \text{deg}$
Link 4:	$u = 8.289$	$\sigma = -16.224 \cdot \text{deg}$
Link 1:	$g = 6.023$	$\theta_1 = -31.035 \cdot \text{deg}$
Coupler:	$r_p = 0.000$	$\delta_p = 90.000 \cdot \text{deg}$

Crank angles:

$$\theta_{2i} = -49.181 \cdot \text{deg}$$

$$\theta_{2f} = -14.181 \cdot \text{deg}$$

14. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-57

Statement: Design a fourbar linkage to carry the object in Figure P5-12 through the three positions shown in their numbered order using points *C* and *D* for your attachment points. The fixed pivots should be within the indicated area.

Given: Coordinates of the points P_1 , P_2 and P_3 :

$$\begin{aligned} P_{1x} &:= 0.0 & P_{1y} &:= 0.0 & P_{2x} &:= 13.871 & P_{2y} &:= -3.299 \\ P_{3x} &:= 19.544 & P_{3y} &:= -0.373 \end{aligned}$$

Angles made by the body in positions 1, 2 and 3:

$$\theta_{p1} := 0.0 \cdot \text{deg} \quad \theta_{p2} := 24.0 \cdot \text{deg} \quad \theta_{p3} := 90.0 \cdot \text{deg}$$

Coordinates of the points C_1 and D_1 with respect to P_1 :

$$C_{1x} := 0.0 \quad C_{1y} := 0.0 \quad D_{1x} := 12.387 \quad D_{1y} := 0.0$$

Solution: See Figure P5-12 and Mathcad file P0557.

- Determine the magnitudes and orientation of the position difference vectors.

$$\begin{aligned} p_{21} &:= \sqrt{P_{2x}^2 + P_{2y}^2} & p_{21} &= 14.258 & \delta_2 &:= \text{atan2}(P_{2x}, P_{2y}) & \delta_2 &= -13.378 \cdot \text{deg} \\ p_{31} &:= \sqrt{P_{3x}^2 + P_{3y}^2} & p_{31} &= 19.548 & \delta_3 &:= \text{atan2}(P_{3x}, P_{3y}) & \delta_3 &= -1.093 \cdot \text{deg} \end{aligned}$$

- Determine the angle changes of the coupler between precision points.

$$\begin{aligned} \alpha_2 &:= \theta_{p2} - \theta_{p1} & \alpha_2 &= 24.000 \cdot \text{deg} \\ \alpha_3 &:= \theta_{p3} - \theta_{p1} & \alpha_3 &= 90.000 \cdot \text{deg} \end{aligned}$$

- Using Figure P5-4, the given data, and the law of cosines, determine z , s , ϕ , and ψ .

$$\begin{aligned} z &:= \sqrt{(P_{1x} - C_{1x})^2 + (P_{1y} - C_{1y})^2} & z &= 0.000 \\ s &:= \sqrt{(P_{1x} - D_{1x})^2 + (P_{1y} - D_{1y})^2} & s &= 12.387 \\ v &:= \sqrt{(C_{1x} - D_{1x})^2 + (C_{1y} - D_{1y})^2} & v &= 12.387 \\ \phi &:= \text{acos}\left(\frac{v^2 + z^2 - s^2}{2 \cdot v \cdot z}\right) + \theta_{p1} & \phi &= 90.000 \cdot \text{deg} \\ \psi &:= \pi - \text{acos}\left(\frac{v^2 + s^2 - z^2}{2 \cdot v \cdot s}\right) + \theta_{p1} & \psi &= 180.000 \cdot \text{deg} \\ Z_{1x} &:= z \cdot \cos(\phi) & Z_{1x} &= 0.000 \\ Z_{1y} &:= z \cdot \sin(\phi) & Z_{1y} &= 0.000 \\ S_{1x} &:= s \cdot \cos(\psi) & S_{1x} &= -12.387 \\ S_{1y} &:= s \cdot \sin(\psi) & S_{1y} &= 0.000 \end{aligned}$$

- Use equations 5.24 to solve for w , θ , β_2 , and β_3 . Since the points *C* and *D* are to be used as pivots, z and ϕ are known from the calculations above.

Guess: $W_{1x} := -2$ $W_{1y} := 15$ $\beta_2 := 45 \cdot \text{deg}$ $\beta_3 := 80 \cdot \text{deg}$

Given $W_{1x} \cdot (\cos(\beta_2) - 1) - W_{1y} \cdot \sin(\beta_2) \dots = p_{21} \cdot \cos(\delta_2)$
 $+ Z_{1x} \cdot (\cos(\alpha_2) - 1) - Z_{1y} \cdot \sin(\alpha_2)$

$W_{1x} \cdot (\cos(\beta_3) - 1) - W_{1y} \cdot \sin(\beta_3) \dots = p_{31} \cdot \cos(\delta_3)$
 $+ Z_{1x} \cdot (\cos(\alpha_3) - 1) - Z_{1y} \cdot \sin(\alpha_3)$

$W_{1y} \cdot (\cos(\beta_2) - 1) + W_{1x} \cdot \sin(\beta_2) \dots = p_{21} \cdot \sin(\delta_2)$
 $+ Z_{1y} \cdot (\cos(\alpha_2) - 1) + Z_{1x} \cdot \sin(\alpha_2)$

$W_{1y} \cdot (\cos(\beta_3) - 1) + W_{1x} \cdot \sin(\beta_3) \dots = p_{31} \cdot \sin(\delta_3)$
 $+ Z_{1y} \cdot (\cos(\alpha_3) - 1) + Z_{1x} \cdot \sin(\alpha_3)$

$$\begin{pmatrix} W_{1x} \\ W_{1y} \\ \beta_2 \\ \beta_3 \end{pmatrix} := \text{Find}(W_{1x}, W_{1y}, \beta_2, \beta_3)$$

$$\beta_2 = 56.754 \cdot \text{deg}$$

$$\beta_3 = 81.324 \cdot \text{deg}$$

The components of the **W** vector are:

$$W_{1x} = -9.989$$

$$W_{1y} = -11.190$$

$$\theta := \text{atan2}(W_{1x}, W_{1y})$$

$$\theta = -131.755 \cdot \text{deg}$$

The length of link 2 is: $w := \sqrt{(W_{1x}^2 + W_{1y}^2)}$, $w = 15.000$

5. Use equations 5.28 to solve for u , σ , γ_2 , and γ_3 . Since the points *C* and *D* are to be used as pivots, s and ψ are known from the calculations above.

Guess: $U_{1x} := 3$ $U_{1y} := 4$ $\gamma_2 := 70 \cdot \text{deg}$ $\gamma_3 := 90 \cdot \text{deg}$

Given $U_{1x} \cdot (\cos(\gamma_2) - 1) - U_{1y} \cdot \sin(\gamma_2) \dots = p_{21} \cdot \cos(\delta_2)$
 $+ S_{1x} \cdot (\cos(\alpha_2) - 1) - S_{1y} \cdot \sin(\alpha_2)$

$U_{1x} \cdot (\cos(\gamma_3) - 1) - U_{1y} \cdot \sin(\gamma_3) \dots = p_{31} \cdot \cos(\delta_3)$
 $+ S_{1x} \cdot (\cos(\alpha_3) - 1) - S_{1y} \cdot \sin(\alpha_3)$

$U_{1y} \cdot (\cos(\gamma_2) - 1) + U_{1x} \cdot \sin(\gamma_2) \dots = p_{21} \cdot \sin(\delta_2)$
 $+ S_{1y} \cdot (\cos(\alpha_2) - 1) + S_{1x} \cdot \sin(\alpha_2)$

$U_{1y} \cdot (\cos(\gamma_3) - 1) + U_{1x} \cdot \sin(\gamma_3) \dots = p_{31} \cdot \sin(\delta_3)$
 $+ S_{1y} \cdot (\cos(\alpha_3) - 1) + S_{1x} \cdot \sin(\alpha_3)$

$$\begin{pmatrix} U_{1x} \\ U_{1y} \\ \gamma_2 \\ \gamma_3 \end{pmatrix} := \text{Find}(U_{1x}, U_{1y}, \gamma_2, \gamma_3)$$

$$\gamma_2 = 119.119 \cdot \text{deg}$$

$$\gamma_3 = 222.077 \cdot \text{deg}$$

The components of the **U** vector are:

$$U_{1x} = -5.889$$

$$U_{1y} = -4.631$$

$$\sigma := \text{atan2}(U_{1x}, U_{1y})$$

$$\sigma = -141.822 \cdot \text{deg}$$

The length of link 4 is: $u := \sqrt{(U_{1x}^2 + U_{1y}^2)}$, $u = 7.492$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$

$$V_{1x} = 12.387$$

$$V_{1y} := Z_{1y} - S_{1y}$$

$$V_{1y} = 0.000$$

$$\theta_3 := \text{atan2}(V_{1x}, V_{1y})$$

$$\theta_3 = 0.000 \cdot \text{deg}$$

$$v := \sqrt{V_{1x}^2 + V_{1y}^2}$$

$$v = 12.387$$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$

$$G_{1x} = 8.287$$

$$G_{1y} := W_{1y} + V_{1y} - U_{1y}$$

$$G_{1y} = -6.559$$

$$\theta_1 := \text{atan2}(G_{1x}, G_{1y})$$

$$\theta_1 = -38.362 \cdot \text{deg}$$

$$g := \sqrt{G_{1x}^2 + G_{1y}^2}$$

$$g = 10.569$$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$$\theta_{2i} := \theta - \theta_1$$

$$\theta_{2i} = -93.393 \cdot \text{deg}$$

$$\theta_{2f} := \theta_{2i} + \beta_3$$

$$\theta_{2f} = -12.069 \cdot \text{deg}$$

8. Define the coupler point with respect to point A and the vector **V**.

$$r_p := z$$

$$\delta_p := \phi - \theta_3$$

$$r_p = 0.000$$

$$\delta_p = 90.000 \cdot \text{deg}$$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$$O_{2x} := -z \cdot \cos(\phi) - w \cdot \cos(\theta)$$

$$O_{2x} = 9.989$$

$$O_{2y} := -z \cdot \sin(\phi) - w \cdot \sin(\theta)$$

$$O_{2y} = 11.190$$

$$O_{4x} := -s \cdot \cos(\psi) - u \cdot \cos(\sigma)$$

$$O_{4x} = 18.276$$

$$O_{4y} := -s \cdot \sin(\psi) - u \cdot \sin(\sigma)$$

$$O_{4y} = 4.631$$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4).

$$\theta_{rot} := \text{atan2}\left[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})\right]$$

$$\theta_{rot} = -38.362 \cdot \text{deg}$$

11. Determine the Grashof condition.

```

Condition(a,b,c,d) :=
  S ← min(a,b,c,d)
  L ← max(a,b,c,d)
  SL ← S + L
  PQ ← a + b + c + d - SL
  return "Grashof"   if SL < PQ
  return "Special Grashof" if SL = PQ
  return "non-Grashof" otherwise

```

$$\text{Condition}(g,u,v,w) = \text{"Grashof"}$$

12. DESIGN SUMMARY

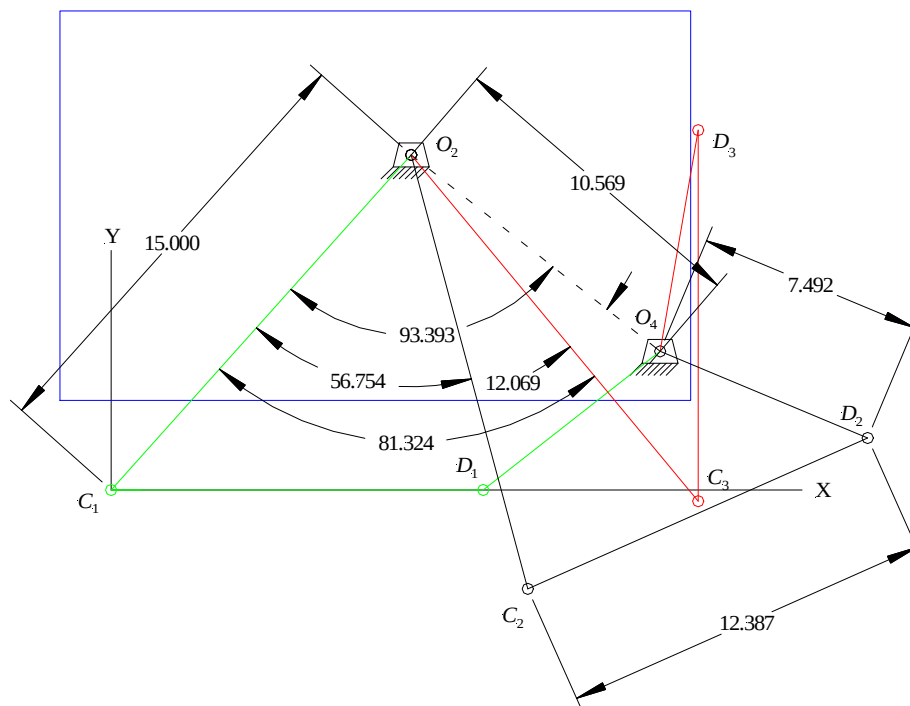
Link 2:	$w = 15.000$	$\theta = -131.755 \cdot \text{deg}$
Link 3:	$v = 12.387$	$\theta_3 = 0.000 \cdot \text{deg}$
Link 4:	$u = 7.492$	$\sigma = -141.822 \cdot \text{deg}$
Link 1:	$g = 10.569$	$\theta_1 = -38.362 \cdot \text{deg}$
Coupler:	$r_p = 0.000$	$\delta_p = 90.000 \cdot \text{deg}$

Crank angles:

$$\theta_{2i} = -93.393 \cdot \text{deg}$$

$$\theta_{2f} = -12.069 \cdot \text{deg}$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 6-1a

Statement: A ship is steaming due north at 20 knots (nautical miles per hour). A submarine is laying in wait 1/2 mile due west of the ship. The sub fires a torpedo on a course of 85 degrees. The torpedo travels at a constant speed of 30 knots. Will it strike the ship? If not, by how many nautical miles will it miss? Hint: Use the relative velocity equation and solve graphically or analytically.

Units: $naut\_mile := 1$ $knots := \frac{naut\_mile}{hr}$

Given: Speed of ship $V_s := 20 \cdot knots$ $\theta_s := 90 \cdot deg$

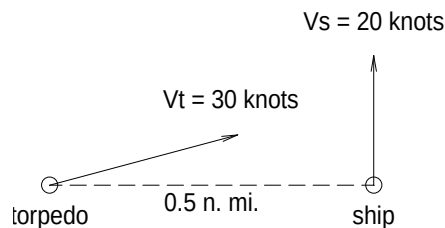
Speed of torpedo $V_t := 30 \cdot knots$ $\theta_t := 15 \cdot deg$

Initial distance between ship and torpedo $d_i := 0.5 \cdot naut\_mile$

Note that, for compass headings, due north is 0 degrees, due east 90 degrees, and the angle increases clockwise. However, in a right-handed Cartesian system, due north is 90 degrees (up) and due east is 0 degrees (to the right). The Cartesian system has been used above to define the ship and torpedo headings.

Solution: See Mathcad file P0601a.

- The key to this solution is to recognize that the only information of interest is the relative velocity of one vessel to the other. The ship captain wants to know the relative velocity of the torpedo versus the ship, $V_{ts} = V_t - V_s$. In effect, we want to resolve the situation with respect to a moving coordinate system attached to the ship.
- The figure below shows the initial positions of the torpedo and ship and their velocities.



- The figure below shows the vector diagram that solves the relative velocity equation $V_{ts} = V_t - V_s$. For the torpedo to hit the moving ship, the relative velocity vector has to be perpendicular to the ship's velocity vector (if you were on the ship observing the torpedo, it would appear to be headed directly for you). As the velocity diagram shows, the relative velocity vector is not perpendicular to the ship's velocity vector so it will miss and pass behind the ship.
- Determine the distance by which the torpedo will miss the ship.

Time required for torpedo to travel 0.5 nautical miles due east

$$t_{t\_east} := \frac{d_i}{V_t \cos(\theta_t)} \quad t_{t\_east} = 62.117 \text{ s}$$

Distance traveled by the torpedo due north in that time

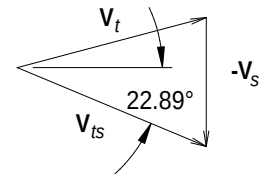
$$d_{t\_north} := V_t \sin(\theta_t) \cdot t_{t\_east} \quad d_{t\_north} = 0.134 \text{ naut\_mile}$$

Distance traveled by the ship due north in that time

$$d_{s\_north} := V_s \cdot t_{t\_east} \quad d_{s\_north} = 0.345 \text{ naut\_mile}$$

Distance by which the torpedo will miss the ship

$$d_{miss} := d_{s\_north} - d_{t\_north} \quad d_{miss} = 0.211 \text{ naut\_mile}$$



PROBLEM 6-1b

Statement: A plane is flying due south at 500 mph at 35,000 feet altitude, straight and level. A second plane is initially 40 miles due east of the first plane, also at 35,000 feet altitude, flying straight and level at 550 mph. Determine the compass angle at which the second plane would be on a collision course with the first. How long will it take for the second plane to catch the first? Hint: Use the relative velocity equation and solve graphically or analytically.

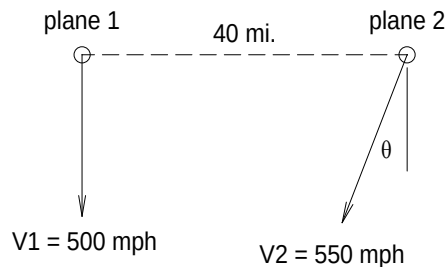
Given:

Speed of first plane $V_1 := 500 \cdot \text{mph}$ $\theta_1 := 270 \cdot \text{deg}$
 Speed of second plane $V_2 := 550 \cdot \text{mph}$
 Initial distance between planes $d_i := 40 \cdot \text{mi}$

Note that, for compass headings, due north is 0 degrees, due east 90 degrees, and the angle increases clockwise. However, in a right-handed Cartesian system, due north is 90 degrees (up) and due east is 0 degrees (to the right). The Cartesian system has been used above to define the plane headings.

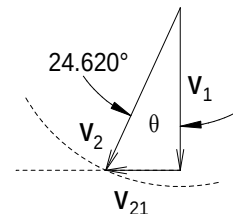
Solution: See Mathcad file P0601b.

1. The key to this solution is to recognize that the only information of interest is the relative velocity of one plane to the other. For a collision to occur, the relative velocity of the second plane with respect to the first must be perpendicular to the velocity vector of the first.
2. The figure below shows the initial positions of the two planes and their velocities.



3. The figure below shows the vector diagram that solves the relative velocity equation $\mathbf{V}_2 = \mathbf{V}_1 + \mathbf{V}_{21}$. To construct this diagram, chose a convenient velocity scale and draw $\mathbf{V}_1$ to its correct length with the arrow head pointing straight down (indicating due south). From the tip of the vector, layoff a horizontal construction line to the left (due west) an undetermined length. From the tail of the $\mathbf{V}_1$ vector, construct a circle whose radius is equal to the scaled length of vector $\mathbf{V}_2$. The intersection of the circle and the horizontal construction line determines the length of $\mathbf{V}_{21}$. Draw the arrowheads for $\mathbf{V}_2$ and $\mathbf{V}_{21}$ pointing toward the intersection of the circle and construction line. Label the horizontal vector $\mathbf{V}_{21}$ and the vector that joins the tail of $\mathbf{V}_1$ with the head of $\mathbf{V}_{21}$ as $\mathbf{V}_2$. The angle θ between $\mathbf{V}_1$ and $\mathbf{V}_2$ is the required direction for $\mathbf{V}_2$ in order that plane 2 collides with plane 1.
4. The angle θ can also be determined analytically from the velocity triangle as follows.

$$\theta := \arccos\left(\frac{V_1}{V_2}\right) \quad \theta = 24.620 \text{ deg}$$



5. The time it will take for the second plane to catch the first is the time that it will take plane 2 to travel the 40 miles to the west.

$$t := \frac{d_i}{V_2 \sin(\theta)} \quad t = 628.468 \text{ s} \quad t = 10.474 \text{ min}$$

PROBLEM 6-2

- Statement:** A point is at a 6.5-in radius on a body in pure rotation with $\omega = 100 \text{ rad/sec}$. The rotation center is at the origin of a coordinate system. When the point is at position A, its position vector makes a 45 deg angle with the X axis. At position B, its position vector makes a 75 deg angle with the X axis. Draw this system to some convenient scale and:
- Write an expression for the particle's velocity vector in position A using complex number notation, in both polar and Cartesian forms.
 - Write an expression for the particle's velocity vector in position B using complex number notation, in both polar and Cartesian forms.
 - Write a vector equation for the velocity difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the velocity difference numerically.
 - Check the result of part c with a graphical method.

Given:

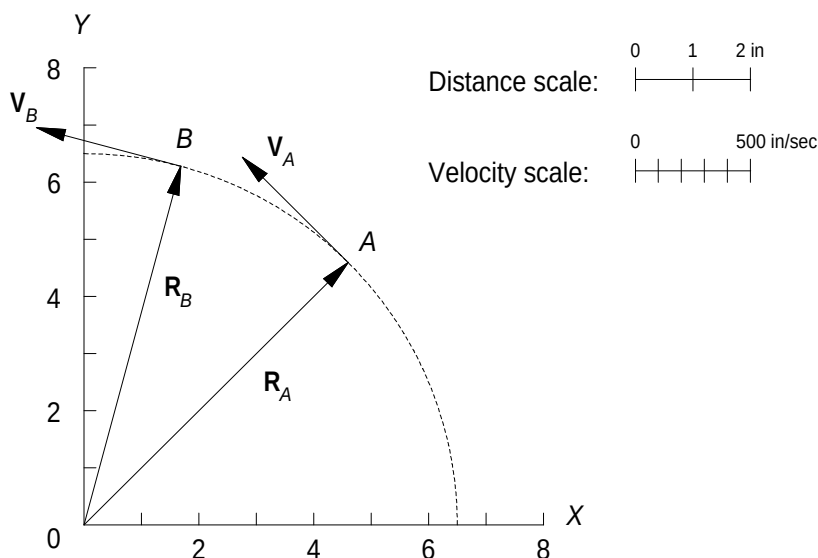
Rotation speed	$\omega := 100 \cdot \frac{\text{rad}}{\text{sec}}$	
Vector angles	$\theta_A := 45 \cdot \text{deg}$	$\theta_B := 75 \cdot \text{deg}$
Vector magnitude	$R := 6.5 \cdot \text{in}$	

Solution: See Mathcad file P0602.

- Calculate the magnitude of the velocity at points A and B using equation 6.3.

$$V := R \cdot \omega \qquad V = 650.000 \frac{\text{in}}{\text{sec}}$$

- Establish an X-Y coordinate frame and draw a circle with center at the origin and radius R.
- Draw lines from the origin that make angles of 45 and 75 deg with respect to the X axis. Label the intersections of the lines with the circles as A and B, respectively. Make the line segment OA a vector by putting an arrowhead at A, pointing away from the origin. Label the vector $\mathbf{R}_A$. Repeat for the line segment OB, labeling it $\mathbf{R}_B$.
- Choose a convenient velocity scale and draw the two velocity vectors $\mathbf{V}_A$ and $\mathbf{V}_B$ at the tips of $\mathbf{R}_A$ and $\mathbf{R}_B$, respectively. The velocity vectors will be perpendicular to their respective position vectors.



- Write an expression for the particle's velocity vector in position A using complex number notation, in both polar and Cartesian forms.

$$\begin{aligned}
 \text{Polar form:} \quad \mathbf{R}_A &:= R \cdot \mathbf{e}^{j \cdot \theta_A} & \mathbf{R}_A &:= 6.5 \cdot \mathbf{e}^{j \cdot \frac{\pi}{4}} \\
 \mathbf{V}_A &:= R \cdot j \cdot \omega \cdot \mathbf{e}^{j \cdot \theta_A} & \mathbf{V}_A &:= 650 \cdot j \cdot \mathbf{e}^{j \cdot \frac{\pi}{4}} \\
 \text{Cartesian form:} \quad \mathbf{V}_A &:= R \cdot j \cdot \omega \cdot (\cos(\theta_A) + j \cdot \sin(\theta_A)) \\
 \mathbf{V}_A &= (-459.619 + 459.619j) \frac{\text{in}}{\text{sec}}
 \end{aligned}$$

- b. Write an expression for the particle's velocity vector in position B using complex number notation, in both polar and Cartesian forms.

$$\begin{aligned}
 \text{Polar form:} \quad \mathbf{R}_B &:= R \cdot \mathbf{e}^{j \cdot \theta_B} & \mathbf{R}_B &:= 6.5 \cdot \mathbf{e}^{j \cdot \frac{\pi}{4}} \\
 \mathbf{V}_B &:= R \cdot j \cdot \omega \cdot \mathbf{e}^{j \cdot \theta_B} & \mathbf{V}_B &:= 650 \cdot j \cdot \mathbf{e}^{j \cdot \frac{75 \cdot \pi}{180}} \\
 \text{Cartesian form:} \quad \mathbf{V}_B &:= R \cdot j \cdot \omega \cdot (\cos(\theta_B) + j \cdot \sin(\theta_B)) \\
 \mathbf{V}_B &= (-627.852 + 168.232j) \frac{\text{in}}{\text{sec}}
 \end{aligned}$$

- c. Write a vector equation for the velocity difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.

$$\mathbf{V}_{BA} := \mathbf{V}_B - \mathbf{V}_A \quad \mathbf{V}_{BA} = (-168.232 - 291.387j) \frac{\text{in}}{\text{sec}}$$

- d. Check the result of part c with a graphical method. Solve the equation $\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$ using a velocity scale of 250 in/sec per drawing unit.

Velocity scale factor

$$k_v := 250 \cdot \frac{\text{in}}{\text{sec}}$$

Horizontal component

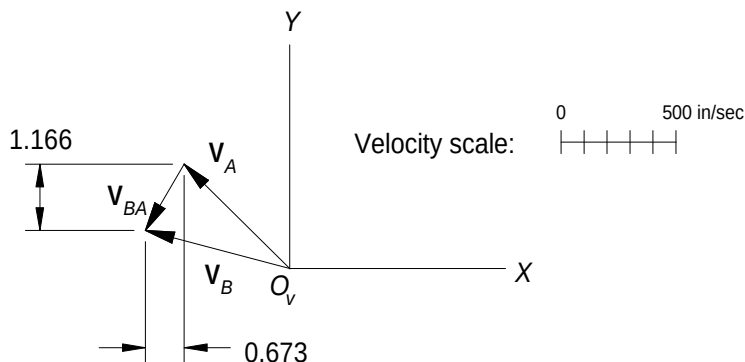
$$V_{BAx} := -0.673 \cdot k_v$$

$$V_{BAx} = -168.3 \frac{\text{in}}{\text{sec}}$$

Vertical component

$$V_{BAy} := -1.166 \cdot k_v$$

$$V_{BAy} = -291.5 \frac{\text{in}}{\text{sec}}$$



On the layout above the X and Y components of $\mathbf{V}_{BA}$ are equal to the real and imaginary components calculated, confirming that the calculation is correct.

PROBLEM 6-3

Statement: A point A is at a 6.5-in radius on a body in pure rotation with $\omega = -50$ rad/sec. The rotation center is at the origin of a coordinate system. At the instant considered its position vector makes a 45 deg angle with the X axis. A point B is at a 6.5-in radius on another body in pure rotation with $\omega = +75$ rad/sec. Its position vector makes a 75 deg angle with the X axis. Draw this system to some convenient scale and:

- Write an expression for the particle's velocity vector in position A using complex number notation, in both polar and Cartesian forms.
- Write an expression for the particle's velocity vector in position B using complex number notation, in both polar and Cartesian forms.
- Write a vector equation for the velocity difference between points B and A . Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.
- Check the result of part c with a graphical method.

Given:

$$\text{Rotation speeds} \quad \omega_A := -50 \cdot \frac{\text{rad}}{\text{sec}} \quad \omega_B := 75 \cdot \frac{\text{rad}}{\text{sec}}$$

$$\text{Vector angles} \quad \theta_A := 45 \cdot \text{deg} \quad \theta_B := 75 \cdot \text{deg}$$

$$\text{Vector magnitude} \quad R := 6.5 \cdot \text{in}$$

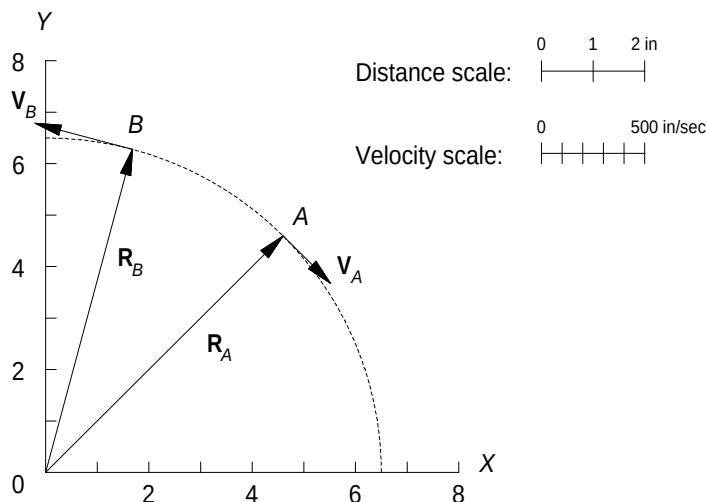
Solution: See Mathcad file P0603.

- Calculate the magnitude of the velocity at points A and B using equation 6.3.

$$V_A := R \cdot \omega_A \quad V_A = -325.000 \frac{\text{in}}{\text{sec}}$$

$$V_B := R \cdot \omega_B \quad V_B = 487.500 \frac{\text{in}}{\text{sec}}$$

- Establish an X - Y coordinate frame and draw a circle with center at the origin and radius R .
- Draw lines from the origin that make angles of 45 and 75 deg with respect to the X axis. Label the intersections of the lines with the circles as A and B , respectively. Make the line segment OA a vector by putting an arrowhead at A , pointing away from the origin. Label the vector $\mathbf{R}_A$. Repeat for the line segment OB , labeling it $\mathbf{R}_B$.
- Choose a convenient velocity scale and draw the two velocity vectors $\mathbf{V}_A$ and $\mathbf{V}_B$ at the tips of $\mathbf{R}_A$ and $\mathbf{R}_B$, respectively. The velocity vectors will be perpendicular to their respective position vectors.



- Write an expression for the particle's velocity vector on body A using complex number notation, in both polar and Cartesian forms.

$$\begin{aligned}
 \text{Polar form:} \quad \mathbf{R}_A &:= R \cdot e^{j \cdot \theta_A} & \mathbf{R}_A &:= 6.5 \cdot e^{j \cdot \frac{\pi}{4}} \\
 \mathbf{V}_A &:= R \cdot j \cdot \omega_A \cdot e^{j \cdot \theta_A} & \mathbf{V}_A &:= -325 \cdot j \cdot e^{j \cdot \frac{\pi}{4}} \\
 \text{Cartesian form:} \quad \mathbf{V}_A &:= R \cdot j \cdot \omega_A \cdot (\cos(\theta_A) + j \cdot \sin(\theta_A)) \\
 \mathbf{V}_A &= (229.810 - 229.810j) \frac{\text{in}}{\text{sec}}
 \end{aligned}$$

- b. Write an expression for the particle's velocity vector on body B using complex number notation, in both polar and Cartesian forms.

$$\begin{aligned}
 \text{Polar form:} \quad \mathbf{R}_B &:= R \cdot e^{j \cdot \theta_B} & \mathbf{R}_B &:= 6.5 \cdot e^{j \cdot \frac{\pi}{4}} \\
 \mathbf{V}_B &:= R \cdot j \cdot \omega_B \cdot e^{j \cdot \theta_B} & \mathbf{V}_B &:= 487.5 \cdot j \cdot e^{j \cdot \frac{75 \cdot \pi}{180}} \\
 \text{Cartesian form:} \quad \mathbf{V}_B &:= R \cdot j \cdot \omega_B \cdot (\cos(\theta_B) + j \cdot \sin(\theta_B)) \\
 \mathbf{V}_B &= (-470.889 + 126.174j) \frac{\text{in}}{\text{sec}}
 \end{aligned}$$

- c. Write a vector equation for the velocity difference between the points on bodies B and A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.

$$\mathbf{V}_{BA} := \mathbf{V}_B - \mathbf{V}_A \quad \mathbf{V}_{BA} = (-700.699 + 355.984j) \frac{\text{in}}{\text{sec}}$$

- d. Check the result of part c with a graphical method. Solve the equation $\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$ using a velocity scale of 250 in/sec per drawing unit.

Velocity scale factor

$$k_v := 250 \cdot \frac{\text{in}}{\text{sec}}$$

Horizontal component

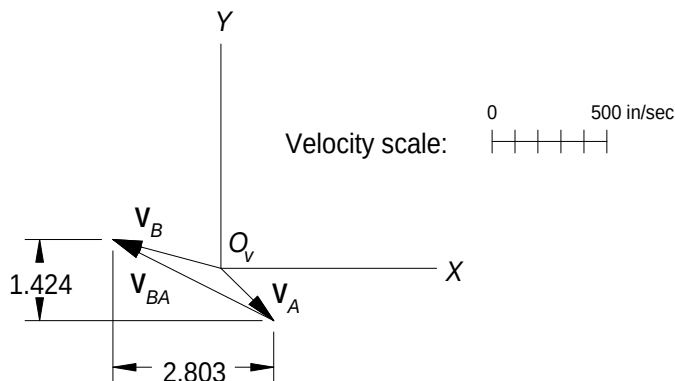
$$V_{BAx} := -2.803 \cdot k_v$$

$$V_{BAx} = -700.7 \frac{\text{in}}{\text{sec}}$$

Vertical component

$$V_{BAy} := 1.424 \cdot k_v$$

$$V_{BAy} = 356.0 \frac{\text{in}}{\text{sec}}$$



On the layout above the X and Y components of $\mathbf{V}_{BA}$ are equal to the real and imaginary components calculated, confirming that the calculation is correct.

PROBLEM 6-4a

Statement: For the fourbar defined in Table P6-1, line a , find the velocities of the pin joints A and B , and of the instant centers $I_{1,3}$ and $I_{2,4}$. Then calculate ω_3 and ω_4 and find the velocity of point P . Use a graphical method.

Given: Link lengths:

$$\text{Link 1} \quad d := 6 \cdot \text{in} \quad \text{Link 2} \quad a := 2 \cdot \text{in}$$

$$\text{Link 3} \quad b := 7 \cdot \text{in} \quad \text{Link 4} \quad c := 9 \cdot \text{in}$$

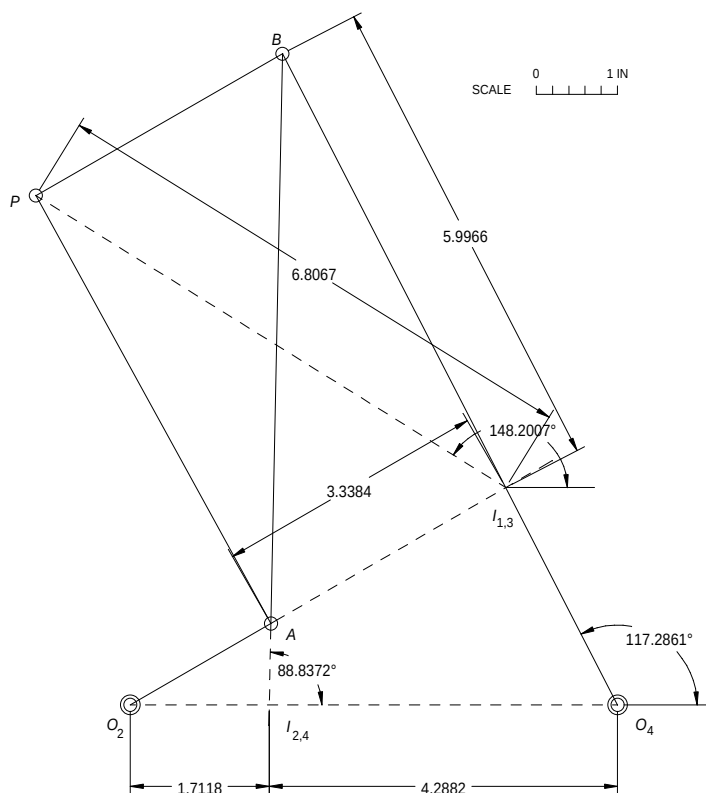
$$\text{Crank angle:} \quad \theta_2 := 30 \cdot \text{deg} \quad \text{Crank velocity:} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$$

Coupler point data:

$$R_{pa} := 6 \cdot \text{in} \quad \delta_3 := 30 \cdot \text{deg}$$

Solution: See Figure P6-1 and Mathcad file P0604a.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.



From the layout above:

$$O_2I_{2,4} := 1.7118 \cdot \text{in}$$

$$O_4I_{2,4} := 4.2882 \cdot \text{in}$$

$$AI_{1,3} := 3.3384 \cdot \text{in}$$

$$BI_{1,3} := 5.9966 \cdot \text{in}$$

$$PI_{1,3} := 6.8067 \cdot \text{in}$$

$$\theta_4 := 117.2861 \cdot \text{deg}$$

$$\theta_3 := 88.8372 \cdot \text{deg}$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A .

$$V_A := a \cdot \omega_2 \qquad V_A = 20.0 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 + 90 \cdot \text{deg} \qquad \theta_{VA} = 120.0 \text{ deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 5.991 \frac{\text{rad}}{\text{sec}} \qquad \text{CW}$$

4. Determine the magnitude of the velocity at point *B* using equation 6.9b. Determine its direction by inspection.

$$V_B := BI13 \cdot \omega_3 \qquad V_B = 35.925 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VB} := \theta_4 - 90 \cdot \text{deg} \qquad \theta_{VB} = 27.286 \text{ deg}$$

5. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 3.99 \frac{\text{rad}}{\text{sec}} \qquad \text{CW}$$

6. Use equation 6.9d and inspection of the layout to determine the magnitude and direction of the velocity at point *P*.

$$V_P := PI13 \cdot \omega_3 \qquad V_P = 40.778 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VP} := 148.2007 \cdot \text{deg} - 90 \cdot \text{deg} \qquad \theta_{VP} = 58.201 \text{ deg}$$

PROBLEM 6-5a

Statement: A general fourbar linkage configuration and its notation are shown in Figure P6-1. The link lengths, coupler point location, and the values of θ_2 and ω_2 for the same fourbar linkages as used for position analysis in Chapter 4 are redefined in Table P6-1, which is the same as Table P4-1. For row *a*, find the velocities of the pin joints A and B, and coupler point P. Calculate ω_3 and ω_4 . Draw the linkage to scale and label it before setting up the equations.

Given: Link lengths:

Link 1	$d := 6$	Link 2	$a := 2$	Link 3	$b := 7$	Link 4	$c := 9$
Coupler point:	$R_{pa} := 6$			$\delta_3 := 30\text{-deg}$			
Link 2 position and velocity:	$\theta_2 := 30\text{-deg}$			$\omega_2 := 10$			

Solution: See Mathcad file P0605a.

1. Draw the linkage to scale and label it.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 3.0000 \quad K_2 = 0.6667$$

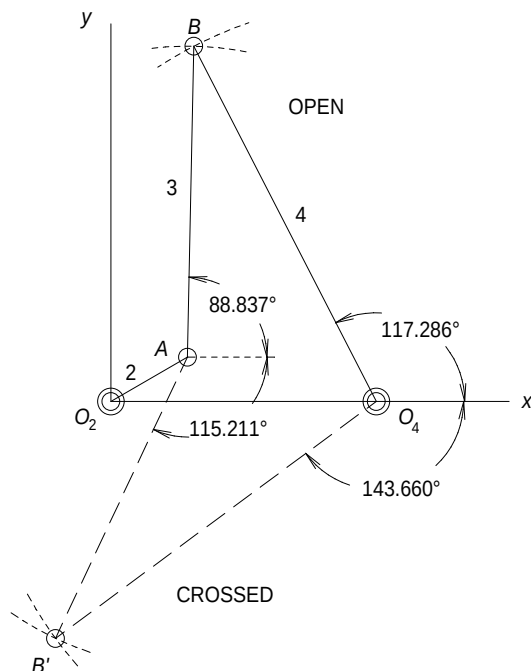
$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 2.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.7113 \quad B = -1.0000 \quad C = 3.5566$$



3. Use equation 4.10b to find values of θ_4 for the open and crossed circuits.

Open: $\theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right)$ $\theta_{41} = -242.714 \text{ deg}$

$\theta_{41} := \theta_{41} - 360 \cdot \text{deg}$ $\theta_{41} = -602.714 \text{ deg}$

Crossed: $\theta_{42} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right)$ $\theta_{42} = 216.340 \text{ deg}$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.8571 \quad K_5 = -0.2857$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -1.6774$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 2.5906$$

5. Use equation 4.13 to find values of θ_3 for the open and crossed circuits.

$$\text{Open:} \quad \theta_{31} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{31} = -271.163 \text{ deg}$$

$$\theta_{31} := \theta_{31} - 360 \cdot \text{deg} \quad \theta_{31} = -631.163 \text{ deg}$$

$$\text{Crossed:} \quad \theta_{32} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{32} = 244.789 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41} - \theta_2)}{\sin(\theta_{31} - \theta_{41})} \quad \omega_{31} = -5.991$$

$$\omega_{41} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31})}{\sin(\theta_{41} - \theta_{31})} \quad \omega_{41} = -3.992$$

7. Determine the velocity of points A and B for the open circuit using equations 6.19.

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = -10.000 + 17.321j \quad |\mathbf{V}_A| = 20 \quad \arg(\mathbf{V}_A) = 120 \text{ deg}$$

$$\mathbf{V}_B := c \cdot \omega_{41} \cdot (-\sin(\theta_{41}) + j \cdot \cos(\theta_{41}))$$

$$\mathbf{V}_B = 31.928 + 16.470j \quad |\mathbf{V}_B| = 35.926 \quad \arg(\mathbf{V}_B) = 27.286 \text{ deg}$$

8. Determine the velocity of the coupler point P for the open circuit using equations 6.36.

$$\mathbf{V}_{PA} := R_{pa} \cdot \omega_{31} \cdot (-\sin(\theta_{31} + \delta_3) + j \cdot \cos(\theta_{31} + \delta_3))$$

$$\mathbf{V}_{PA} = 31.488 + 17.337j$$

$$\mathbf{V}_P := \mathbf{V}_A + \mathbf{V}_{PA}$$

$$\mathbf{V}_P = 21.488 + 34.658j \quad |\mathbf{V}_P| = 40.779 \quad \arg(\mathbf{V}_P) = 58.201 \text{ deg}$$

9. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{42} - \theta_2)}{\sin(\theta_{32} - \theta_{42})} \quad \omega_{32} = -0.662$$

$$\omega_{42} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{32})}{\sin(\theta_{42} - \theta_{32})} \quad \omega_{42} = -2.662$$

10. Determine the velocity of point B for the crossed circuit using equations 6.19.

$$\mathbf{V}_B := c \cdot \omega_{42} \cdot (-\sin(\theta_{42}) + j \cdot \cos(\theta_{42}))$$

$$\mathbf{V}_B = -14.195 + 19.295j \quad |\mathbf{V}_B| = 23.954 \quad \arg(\mathbf{V}_B) = 126.340 \text{ deg}$$

11. Determine the velocity of the coupler point P for the crossed circuit using equations 6.36.

$$\mathbf{V}_{PA} := R_{pa} \cdot \omega_{32} \cdot (-\sin(\theta_{32} + \delta_3) + j \cdot \cos(\theta_{32} + \delta_3))$$

$$\mathbf{V}_{PA} = -3.960 - 0.332j$$

$$\mathbf{V}_P := \mathbf{V}_A + \mathbf{V}_{PA}$$

$$\mathbf{V}_P = -13.960 + 16.989j \quad \left| \mathbf{V}_P \right| = 21.989 \quad \arg(\mathbf{V}_P) = 129.411 \text{ deg}$$

PROBLEM 6-6a

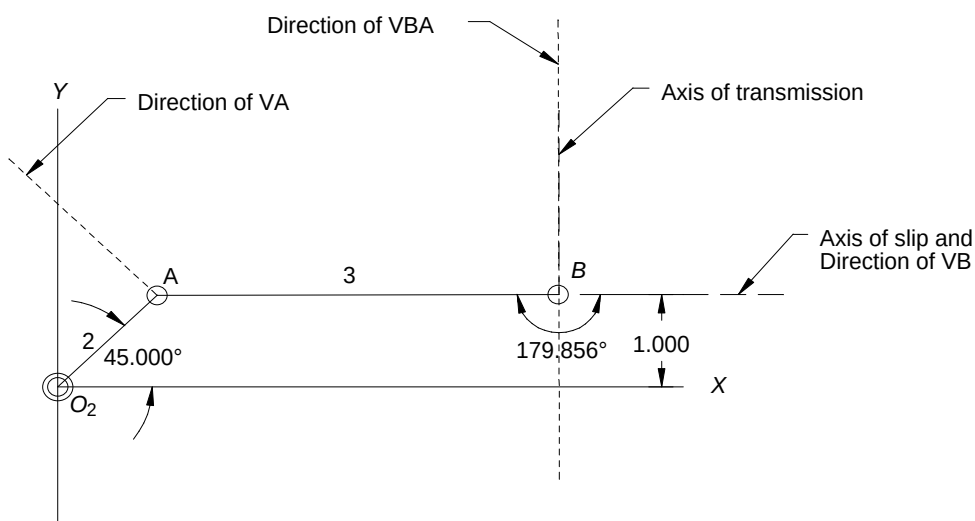
Statement: The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Fig P6-2. The link lengths and the values of θ_2 and ω_2 are defined in Table P6-2. For row *a*, find the velocities of the pin joints A and B and the velocity of slip at the sliding joint using a graphical method.

Given: Link lengths:

$$\begin{array}{ll} \text{Link 2} & a := 1.4 \cdot \text{in} \\ \text{Offset} & c := 1 \cdot \text{in} \end{array} \quad \begin{array}{ll} \text{Link 3} & b := 4 \cdot \text{in} \\ & \theta_2 := 45 \cdot \text{deg} \end{array} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$$

Solution: See Figure P6-2 and Mathcad file P0606a.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

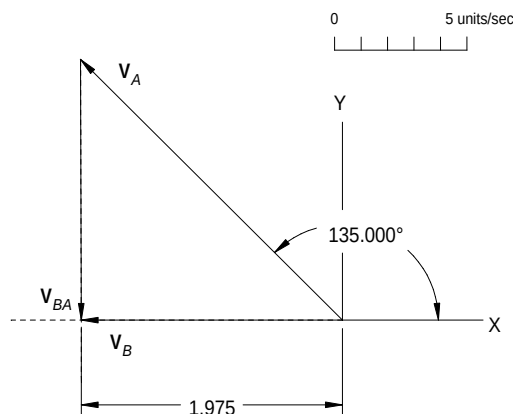
$$V_A := a \cdot \omega_2 \quad V_A = 14.000 \frac{\text{in}}{\text{sec}} \quad \theta_{VA} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{VA} = 135 \text{ deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.

4. From the velocity triangle we have:



Velocity scale factor: $k_v := \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_B := 1.975 \cdot \text{in} \cdot k_v \quad V_B = 9.875 \frac{\text{in}}{\text{sec}} \quad \theta_{VB} := 180 \cdot \text{deg}$$

5. Since the slip axis and the direction of the velocity of point B are parallel, $V_{\text{slip}} = V_B$.

PROBLEM 6-7a

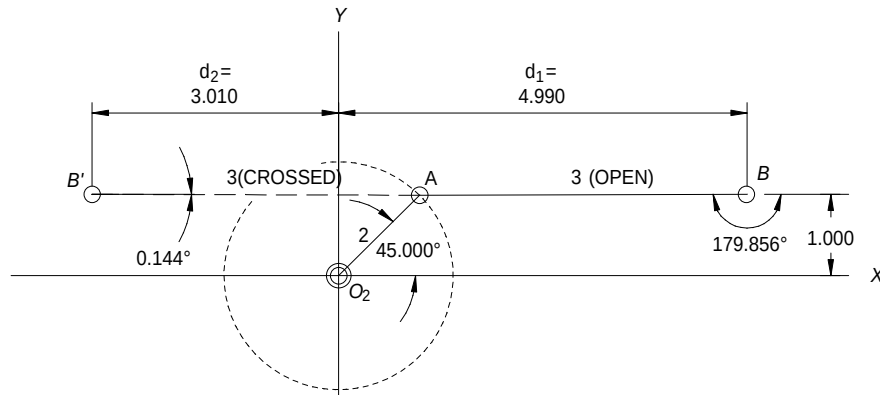
Statement: The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Fig P6-2. The link lengths and the values of θ_2 and ω_2 are defined in Table P6-2. For row *a*, find the velocities of the pin joints A and B and the velocity of slip at the sliding joint using the analytic method. Draw the linkage to scale and label it before setting up the equations.

Given: Link lengths:

Link 2 (O_2A)	$a := 1.4$	Link 3 (AB)	$b := 4$	Offset (y_B)	$c := 1$
Crank angle	$\theta_2 := 45\text{-deg}$	Crank angular velocity	$\omega_2 := 10$		

Solution: See Figure P6-2 and Mathcad file P0607a.

1. Draw the linkage to scale and label it.



2. Determine θ_3 and d using equations 4.16 and 4.17.

Crossed: $\theta_{32} := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right)$ $\theta_{32} = -0.144 \text{ deg}$

$d_2 := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_{32})$ $d_2 = -3.010$

Open: $\theta_{31} := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi$ $\theta_{31} = 180.144 \text{ deg}$

$d_1 := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_{31})$ $d_1 = 4.990$

3. Determine the angular velocity of link 3 using equation 6.22a:

Open $\omega_{31} := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_{31})} \cdot \omega_2$ $\omega_{31} = -2.475$

Crossed $\omega_{32} := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_{32})} \cdot \omega_2$ $\omega_{32} = 2.475$

4. Determine the velocity of pin A using equation 6.23a:

$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$

$\mathbf{V}_A = -9.899 + 9.899i$ $|\mathbf{V}_A| = 14.000$ $\arg(\mathbf{V}_A) = 135.000 \text{ deg}$

5. Determine the velocity of pin B using equation 6.22b:

$$\text{Open} \quad V_{B1} := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{31} \cdot \sin(\theta_{31}) \quad V_{B1} = -9.875$$

$$\text{Crossed} \quad V_{B2} := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{32} \cdot \sin(\theta_{32}) \quad V_{B2} = -9.924$$

The angle of $\mathbf{V}_B$ is 0 deg if V_B is positive and 180 deg if V_B negative.

6. The velocity of slip is the same as the velocity of pin B .

PROBLEM 6-8a

Statement: The general linkage configuration and terminology for an inverted fourbar slider-crank linkage are shown in Fig P6-3. The link lengths and the values of θ_2 and ω_2 and γ are defined in Table P6-3. For row *a*, using a graphical method, find the velocities of the pin joints A and B and the velocity of slip at the sliding joint. Draw the linkage to scale.

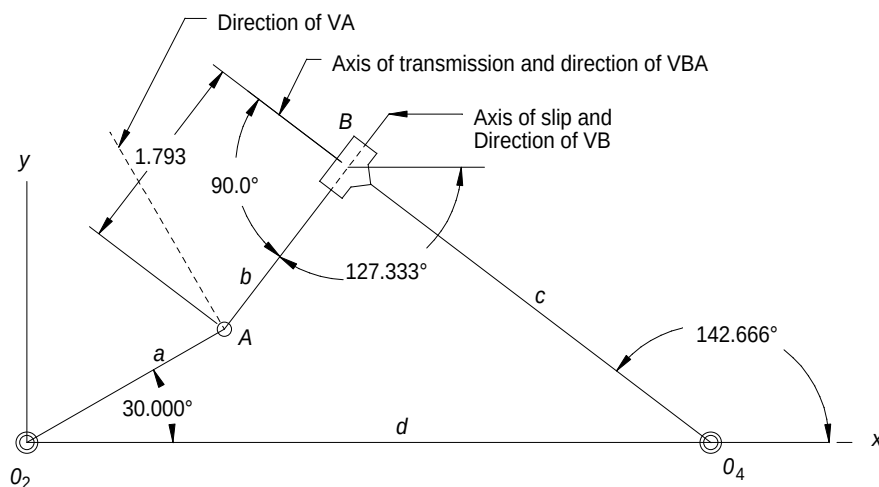
Given: Link lengths:

$$\text{Link 1} \quad d := 6 \cdot \text{in} \quad \text{Link 2} \quad a := 2 \cdot \text{in}$$

$$\text{Link 4} \quad c := 4 \cdot \text{in} \quad \gamma := 90 \cdot \text{deg} \quad \theta_2 := 30 \cdot \text{deg} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$$

Solution: See Figure P6-3 and Mathcad file P0608a.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



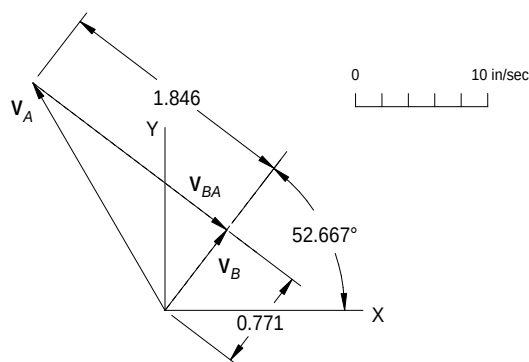
2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 20.000 \frac{\text{in}}{\text{sec}} \quad \theta_{VA} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{VA} = 120 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B on link 3. The equation to be solved graphically is

$$\mathbf{V}_{B3} = \mathbf{V}_{A3} + \mathbf{V}_{BA3}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

$$\text{Velocity scale factor:} \quad k_v := \frac{10 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_{B3} := 0.771 \cdot \text{in} \cdot k_v \quad V_{B3} = 7.710 \frac{\text{in}}{\text{sec}} \quad \theta_{VB3} := 52.667 \cdot \text{deg}$$

$$V_{BA3} := 1.846 \cdot \text{in} \cdot k_v \quad V_{BA3} = 18.460 \frac{\text{in}}{\text{sec}}$$

5. Determine the angular velocity of link 3 using equation 6.7.

From the linkage layout above $b := 1.793 \cdot \text{in}$ and $\theta_4 := 142.666 \cdot \text{deg}$

$$\omega_3 := \frac{V_{BA3}}{b} \quad \omega_3 = 10.296 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

The way in which link 3 slides in link 4 requires that $\omega_4 := \omega_3$

6. Determine the magnitude and sense of the vector $\mathbf{V}_{B4}$ using equation 6.7.

$$V_{B4} := |c \cdot \omega_4| \quad V_{B4} = 41.182 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VB4} := \theta_4 - 90 \cdot \text{deg} \quad \theta_{VB4} = 52.666 \text{ deg}$$

7. Note that V_{B3} and V_{B4} are in the same direction in this case. The velocity of slip is

$$V_{\text{slip}} := |V_{B3} - V_{B4}| \quad V_{\text{slip}} = 33.472 \frac{\text{in}}{\text{sec}}$$

$$\theta_{\text{slip}} := \theta_{VB4} + 180 \cdot \text{deg} \quad \theta_{\text{slip}} = 232.666 \text{ deg}$$

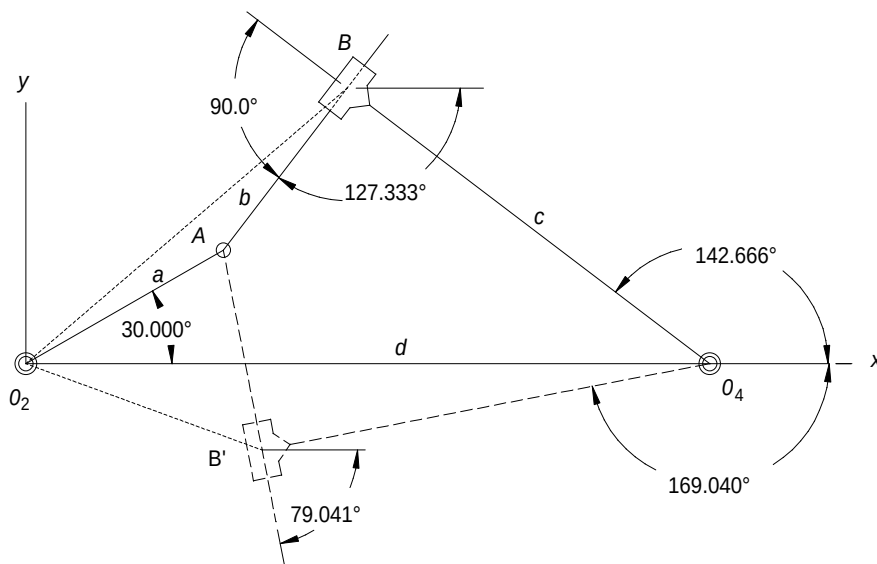
PROBLEM 6-9a

Statement: The general linkage configuration and terminology for an inverted fourbar slider-crank linkage are shown in Fig P6-3. The link lengths and the values of θ_2 and ω_2 and γ are defined in Table P6-3. For row *a*, using an analytic method, find the velocities of the pin joints *A* and *B* and the velocity of slip at the sliding joint. Draw the linkage to scale and label it before setting up the equations.

Given: Link lengths:
 Link 1 $d := 6$ Link 2 $a := 2$
 Link 4 $c := 4$ $\gamma := 90 \cdot \text{deg}$ $\theta_2 := 30 \cdot \text{deg}$ $\omega_2 := 10$

Solution: See Mathcad file P0609a.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.25 and 4.26.

$$P := a \cdot \sin(\theta_2) \cdot \sin(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \cos(\gamma) \quad P = 1.000$$

$$Q := -a \cdot \sin(\theta_2) \cdot \cos(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \sin(\gamma) \quad Q = -4.268$$

$$R := -c \cdot \sin(\gamma) \quad R = -4.000 \quad T := 2 \cdot P \quad T = 2.000$$

$$S := R - Q \quad S = 0.268 \quad U := Q + R \quad U = -8.268$$

3. Use equation 4.26 to find values of θ_4 for the open and crossed circuits.

$$\text{OPEN} \quad \theta_{41} := 2 \cdot \text{atan2}\left(2 \cdot S, -T + \sqrt{T^2 - 4 \cdot S \cdot U}\right) \quad \theta_{41} = 142.667 \text{ deg}$$

$$\text{CROSSED} \quad \theta_{42} := 2 \cdot \text{atan2}\left(2 \cdot S, -T - \sqrt{T^2 - 4 \cdot S \cdot U}\right) \quad \theta_{42} = -169.041 \text{ deg}$$

4. Use equation 4.22 to find values of θ_3 for the open and crossed circuits.

$$\text{OPEN} \quad \theta_{31} := \theta_{41} + \gamma \quad \theta_{31} = 232.667 \text{ deg}$$

$$\text{CROSSED} \quad \theta_{32} := \theta_{42} + \gamma \quad \theta_{32} = -79.041 \text{ deg}$$

5. Determine the magnitude of the instantaneous "length" of link 3 from equation 4.20a.

$$\text{OPEN} \quad b_1 := \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_{41})}{\sin(\theta_{41} + \gamma)} \quad b_1 = 1.793$$

$$\text{CROSSED} \quad b_2 := \left| \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_{42})}{\sin(\theta_{42} + \gamma)} \right| \quad b_2 = 1.793$$

6. Determine the angular velocity of link 4 using equation 6.30c:

$$\text{OPEN} \quad \omega_{41} := \frac{a \cdot \omega_2 \cdot \cos(\theta_2 - \theta_{31})}{b_1 + c \cdot \cos(\gamma)} \quad \omega_{41} = -10.292$$

$$\text{CROSSED} \quad \omega_{42} := \frac{a \cdot \omega_2 \cdot \cos(\theta_2 - \theta_{32})}{b_2 + c \cdot \cos(\gamma)} \quad \omega_{42} = -3.639$$

7. Determine the velocity of pin A using equation 6.23a:

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = -10.000 + 17.321i \quad |\mathbf{V}_A| = 20.000 \quad \arg(\mathbf{V}_A) = 120.000 \text{ deg}$$

8. Determine the velocity of point B on link 4 using equation 6.31:

$$\text{OPEN} \quad V_{B4x1} := -c \cdot \omega_{41} \cdot \sin(\theta_{41}) \quad V_{B4x1} = 24.966$$

$$V_{B4y1} := c \cdot \omega_{41} \cdot \cos(\theta_{41}) \quad V_{B4y1} = 32.734$$

$$V_{B41} := \sqrt{V_{B4x1}^2 + V_{B4y1}^2} \quad V_{B41} = 41.168$$

$$\theta_{VB1} := \text{atan2}(V_{B4x1}, V_{B4y1}) \quad \theta_{VB1} = 52.667 \text{ deg}$$

$$\text{CROSSED} \quad V_{B4x2} := -c \cdot \omega_{42} \cdot \sin(\theta_{42}) \quad V_{B4x2} = -2.767$$

$$V_{B4y2} := c \cdot \omega_{42} \cdot \cos(\theta_{42}) \quad V_{B4y2} = 14.289$$

$$V_{B42} := \sqrt{V_{B4x2}^2 + V_{B4y2}^2} \quad V_{B42} = 14.555$$

$$\theta_{VB2} := \text{atan2}(V_{B4x2}, V_{B4y2}) \quad \theta_{VB2} = 100.959 \text{ deg}$$

9. Determine the slip velocity using equation 6.30a:

$$\text{OPEN} \quad V_{slip1} := \frac{-a \cdot \omega_2 \cdot \sin(\theta_2) + \omega_{41} \cdot (b_1 \cdot \sin(\theta_{31}) + c \cdot \sin(\theta_{41}))}{\cos(\theta_{31})} \quad V_{slip1} = 33.461$$

$$\text{CROSSED} \quad V_{slip2} := \frac{-a \cdot \omega_2 \cdot \sin(\theta_2) + \omega_{42} \cdot (b_2 \cdot \sin(\theta_{32}) + c \cdot \sin(\theta_{42}))}{\cos(\theta_{32})} \quad V_{slip2} = -4.351$$

PROBLEM 6-10a

Statement: The link lengths, gear ratio (λ), phase angle (ϕ), and the values of θ_2 and ω_2 for a geared fivebar from row *a* of Table P6-4 are given below. Draw the linkage to scale and graphically find ω_3 and ω_4 , using a graphical method.

Given: Link lengths:

$$\begin{array}{lll} \text{Link 1} & f := 6 \cdot \text{in} & \text{Link 3} \quad b := 7 \cdot \text{in} \quad \text{Link 5} \quad d := 4 \cdot \text{in} \\ \text{Link 2} & a := 1 \cdot \text{in} & \text{Link 4} \quad c := 9 \cdot \text{in} \end{array}$$

Gear ratio, phase angle, and crank angle:

$$\lambda := 2 \quad \phi := 30 \cdot \text{deg} \quad \theta_2 := 60 \cdot \text{deg} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$$

Solution: See Figure P6-4 and Mathcad file P0610a.

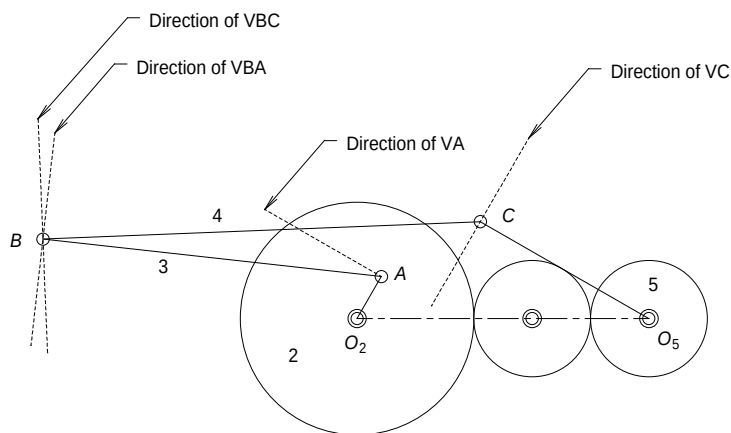
- Determine the angle of link 5 using the equation in Figure P6-4.

$$\theta_5 := \lambda \cdot \theta_2 + \phi \quad \theta_5 = 150 \cdot \text{deg}$$

- Choose the pitch radii of the gears. Since the gear ratio is positive, an idler must be used between gear 2 and gear 5. Let the idler be the same diameter as gear 5, and let all three gears be in line.

$$\begin{aligned} f &:= r_2 + 3 \cdot r_5 & \lambda &:= \frac{r_2}{r_5} \\ r_5 &:= \frac{f}{\lambda + 3} & r_5 &= 1.200 \text{ in} \\ r_2 &:= \lambda \cdot r_5 & r_2 &= 2.400 \text{ in} \end{aligned}$$

- Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



- Use equation 6.7 to calculate the magnitude of the velocity at points A and C.

$$\begin{aligned} V_A &:= a \cdot \omega_2 & V_A &= 10.00 \frac{\text{in}}{\text{sec}} & \theta_A &:= 60 \cdot \text{deg} + 90 \cdot \text{deg} \\ \omega_5 &:= \lambda \cdot \omega_2 & \omega_5 &= 20.000 \frac{\text{rad}}{\text{sec}} \\ V_C &:= d \cdot \omega_5 & V_C &= 80.00 \frac{\text{in}}{\text{sec}} & \theta_C &:= 150 \cdot \text{deg} + 90 \cdot \text{deg} \end{aligned}$$

5. Use equation 6.5 to (graphically) determine the magnitudes of the relative velocity vectors $\mathbf{V}_{BA}$ and $\mathbf{V}_{BC}$. The equation to be solved graphically is the last of the following three.

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \quad \mathbf{V}_B = \mathbf{V}_C + \mathbf{V}_{BC} \quad \mathbf{V}_A + \mathbf{V}_{BA} = \mathbf{V}_C + \mathbf{V}_{BC}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
 - From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
 - From the tail of $\mathbf{V}_A$, layout the known vector $\mathbf{V}_C$.
 - From the tip of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_{BC}$, magnitude unknown.
 - Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BC}$ construction line and drawing $\mathbf{V}_{BC}$ from the tip of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_{BA}$ construction line.
6. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{50 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_{BA} := 4.562 \cdot \text{in} \cdot k_v \quad V_{BA} = 228.100 \frac{\text{in}}{\text{sec}}$$

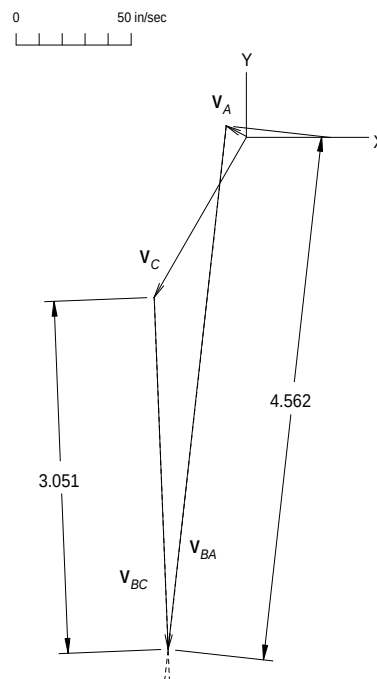
$$V_{BC} := 3.051 \cdot \text{in} \cdot k_v \quad V_{BC} = 152.550 \frac{\text{in}}{\text{sec}}$$

7. Determine the angular velocity of links 3 and 4 using equation 6.7.

$$\omega_3 := \frac{V_{BA}}{b} \quad \omega_3 = 32.586 \frac{\text{rad}}{\text{sec}}$$

$$\omega_3 := \frac{V_{BC}}{c} \quad \omega_3 = 16.950 \frac{\text{rad}}{\text{sec}}$$

Both links are rotating CCW.



PROBLEM 6-11a

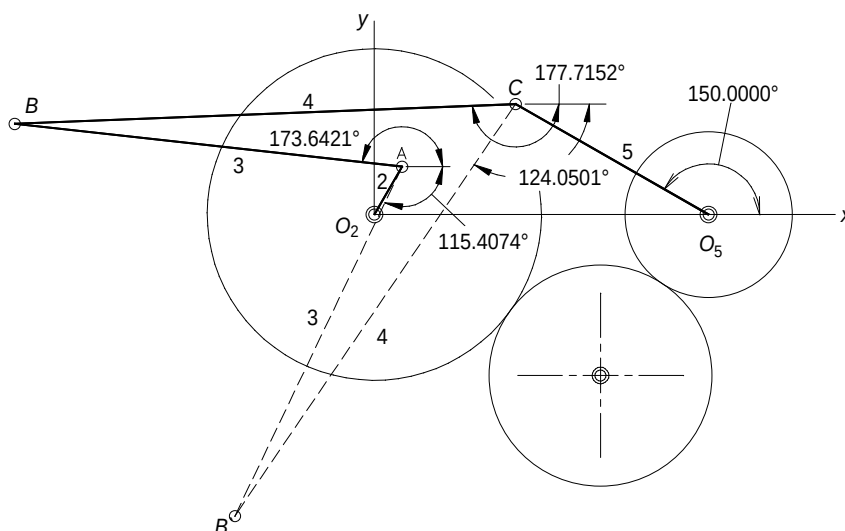
Statement: The general linkage configuration and terminology for a geared fivebar linkage are shown in Figure P6-4. The link lengths, gear ratio (λ), phase angle (ϕ), and the values of θ_2 and ω_2 are defined in Table P6-4. For row *a*, find ω_3 and ω_4 , using an analytic method. Draw the linkage to scale and label it before setting up the equations.

Given: Link lengths:

Link 1	$d := 4$	Link 2	$a := 1$	Link 3	$b := 7$
Link 4	$c := 9$	Link 5	$f := 6$	Input angle	$\theta_2 := 60\text{-deg}$
Gear ratio	$\lambda := 2.0$	Phase angle	$\phi := 30\text{-deg}$		$\omega_2 := 10$

Solution: See Figure P6-4 and Mathcad file P0611a.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_3 and θ_4 from equations 4.28h and 4.28i.

$$A := 2 \cdot c \cdot (d \cdot \cos(\lambda \cdot \theta_2 + \phi) - a \cdot \cos(\theta_2) + f) \quad A = 36.646$$

$$B := 2 \cdot c \cdot (d \cdot \sin(\lambda \cdot \theta_2 + \phi) - a \cdot \sin(\theta_2)) \quad B = 20.412$$

$$C := (a^2 - b^2 + c^2 + d^2 + f^2) - 2 \cdot a \cdot f \cdot \cos(\theta_2) \dots \\ + [-2 \cdot d \cdot (a \cdot \cos(\theta_2) - f) \cdot \cos(\lambda \cdot \theta_2 + \phi)] \dots \\ + -2 \cdot a \cdot d \cdot \sin(\theta_2) \cdot \sin(\lambda \cdot \theta_2 + \phi) \quad C = 37.431$$

$$D := C - A \quad D = 0.785$$

$$E := 2 \cdot B \quad E = 40.823$$

$$F := A + C \quad F = 74.077$$

$$G := 2 \cdot b \cdot [-d \cdot \cos(\lambda \cdot \theta_2 + \phi) + a \cdot \cos(\theta_2) - f] \quad G = -28.503$$

$$H := 2 \cdot b \cdot [-d \cdot \sin(\lambda \cdot \theta_2 + \phi) + a \cdot \sin(\theta_2)] \quad H = -15.876$$

$$K := (a^2 + b^2 - c^2 + d^2 + f^2) - 2 \cdot a \cdot f \cdot \cos(\theta_2) \dots \\ + [-2 \cdot d \cdot (a \cdot \cos(\theta_2) - f) \cdot \cos(\lambda \cdot \theta_2 + \phi)] \dots \\ + -2 \cdot a \cdot d \cdot \sin(\theta_2) \cdot \sin(\lambda \cdot \theta_2 + \phi) \quad K = -26.569$$

$$L := K - G \quad L = 1.933$$

$$M := 2 \cdot H$$

$$M = -31.751$$

$$N := G + K$$

$$N = -55.072$$

3. Use equations 4.28h and 4.28i to find values of θ_3 and θ_4 for the open and crossed circuits.

$$\text{OPEN} \quad \theta_{31} := 2 \cdot \left(\text{atan2} \left(2 \cdot L, -M + \sqrt{M^2 - 4 \cdot L \cdot N} \right) \right) \quad \theta_{31} = 173.642 \text{ deg}$$

$$\theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{41} = -177.715 \text{ deg}$$

$$\text{CROSSED} \quad \theta_{32} := 2 \cdot \left(\text{atan2} \left(2 \cdot L, -M - \sqrt{M^2 - 4 \cdot L \cdot N} \right) \right) \quad \theta_{32} = -115.407 \text{ deg}$$

$$\theta_{42} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{42} = -124.050 \text{ deg}$$

4. Determine the position and angular velocity of gear 5 from equations 4.27c and 6.32c

$$\theta_5 := \lambda \cdot \theta_2 + \phi$$

$$\theta_5 = 150.000 \text{ deg}$$

$$\omega_5 := \lambda \cdot \omega_2$$

$$\omega_5 = 20.000 \quad \text{CCW}$$

Angular velocity of links 3 and 4 from equations 6.33

$$\text{OPEN} \quad \omega_{31} := -\frac{2 \cdot \sin(\theta_{41}) \cdot (a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_{41}) + d \cdot \omega_5 \cdot \sin(\theta_{41} - \theta_5))}{b \cdot (\cos(\theta_{31} - 2 \cdot \theta_{41}) - \cos(\theta_{31}))}$$

$$\omega_{31} = 32.585 \quad \text{CCW}$$

$$\omega_{41} := \frac{a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{31} \cdot \sin(\theta_{31}) - d \cdot \omega_5 \cdot \sin(\theta_5)}{c \cdot \sin(\theta_{41})}$$

$$\omega_{41} = 16.948 \quad \text{CCW}$$

$$\text{CROSSED} \quad \omega_{32} := -\frac{2 \cdot \sin(\theta_{42}) \cdot (a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_{42}) + d \cdot \omega_5 \cdot \sin(\theta_{42} - \theta_5))}{b \cdot (\cos(\theta_{32} - 2 \cdot \theta_{42}) - \cos(\theta_{32}))}$$

$$\omega_{32} = -75.191 \quad \text{CW}$$

$$\omega_{42} := \frac{a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{32} \cdot \sin(\theta_{32}) - d \cdot \omega_5 \cdot \sin(\theta_5)}{c \cdot \sin(\theta_{42})}$$

$$\omega_{42} = -59.554 \quad \text{CW}$$

PROBLEM 6-12

Statement: Find all of the instant centers of the linkages shown in Figure P6-5.

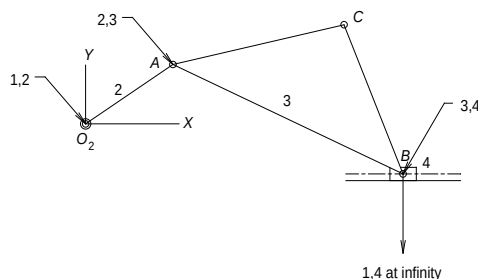
Solution: See Figure P6-5 and Mathcad file P0612.

a. This is a fourbar slider-crank with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

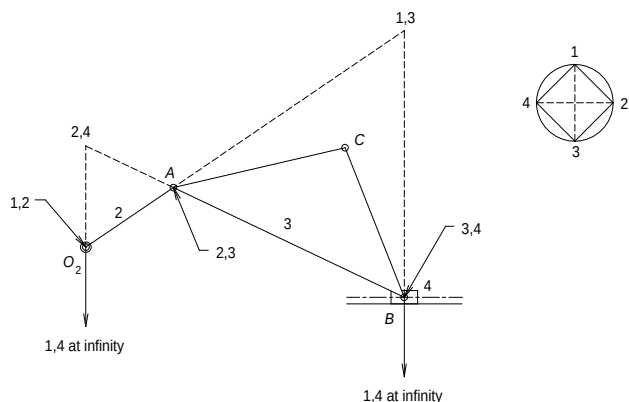
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



b. This is a fourbar with planetary motion (roller 3), $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

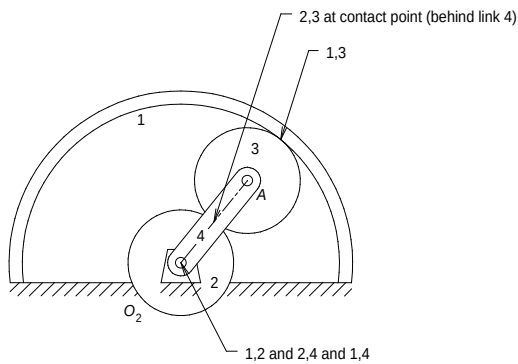
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection, which in this case, is all of them.

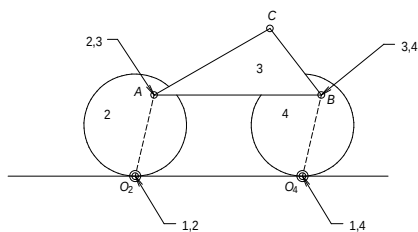
c. This is a fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$



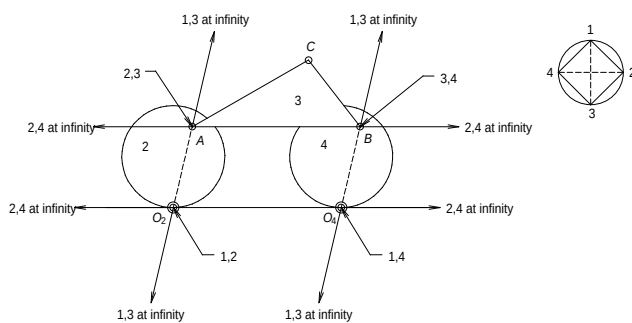
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



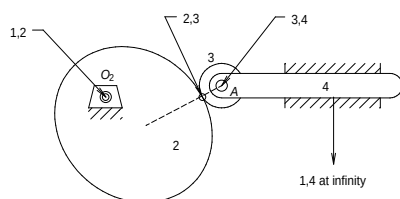
- d. This is a fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2}$$

$$C = 6$$

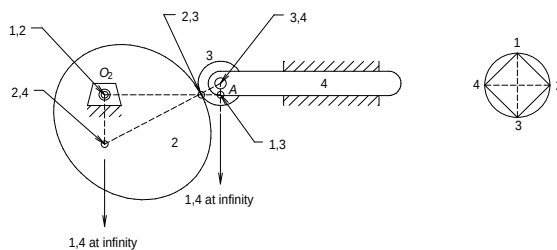
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$

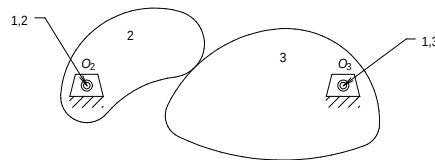


- e. This is a threebar with $n := 3$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

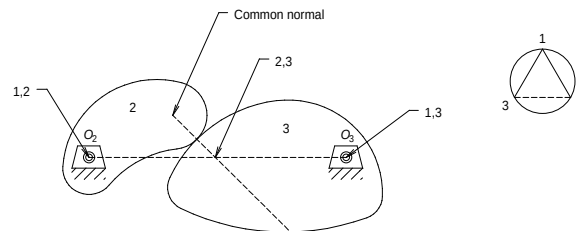
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 3$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining IC, $I_{2,3}$

$$I_{2,3}: I_{1,2}-I_{1,3}$$



- f. This is a fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

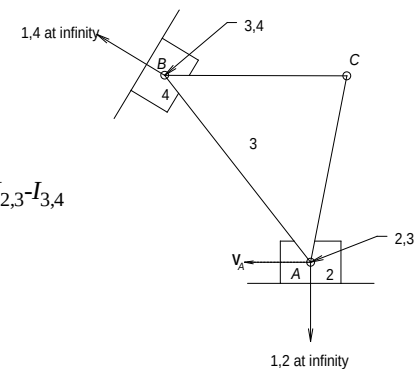
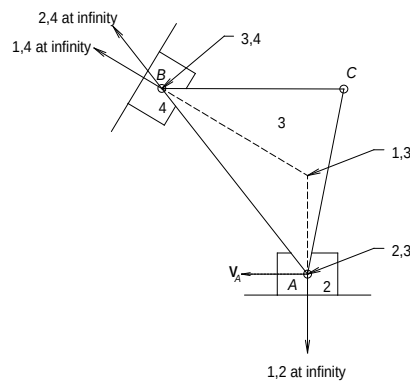
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



PROBLEM 6-13

Statement: Find all of the instant centers of the linkages shown in Figure P6-6.

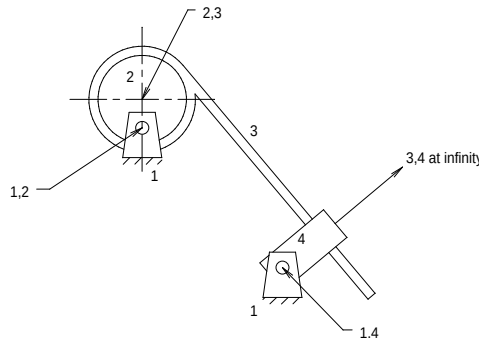
Solution: See Figure P6-6 and Mathcad file P0613.

a. This is a fourbar inverted slider-crank with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

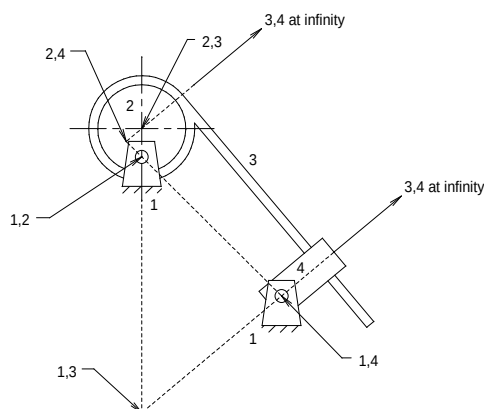
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



b. This is a sixbar with slider, $n := 6$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. Use Kennedy's Rule and a linear graph to find the remaining 7 ICs.

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{1,5}: I_{1,6}-I_{5,6} \text{ and } I_{1,4}-I_{4,5}$$

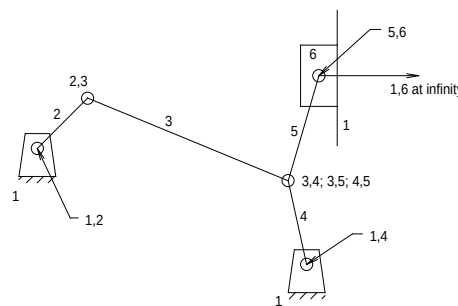
$$I_{2,5}: I_{1,2}-I_{1,5} \text{ and } I_{2,4}-I_{4,5}$$

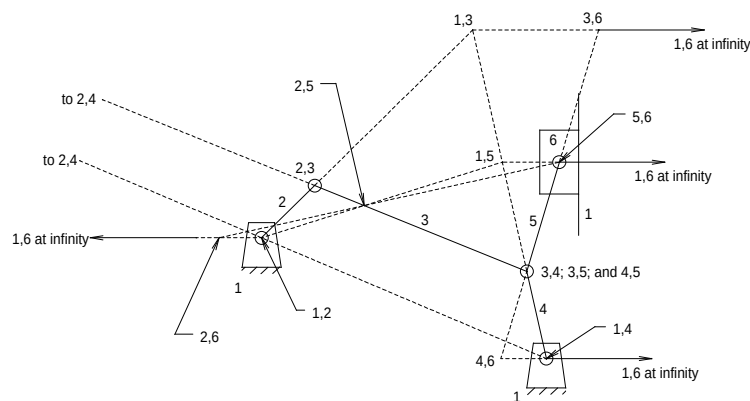
$$I_{3,6}: I_{1,6}-I_{1,3} \text{ and } I_{3,4}-I_{4,6}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$

$$I_{4,6}: I_{1,6}-I_{1,4} \text{ and } I_{4,5}-I_{5,6}$$

$$I_{2,6}: I_{1,2}-I_{1,6} \text{ and } I_{2,5}-I_{5,6}$$





c. This is a sixbar with slider and roller with $n := 6$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs.

$$I_{2,6}: I_{1,2}-I_{1,6} \text{ and } I_{2,5}-I_{5,6}$$

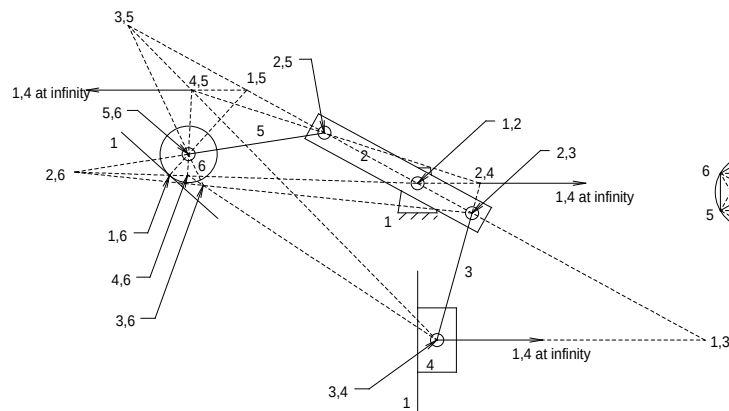
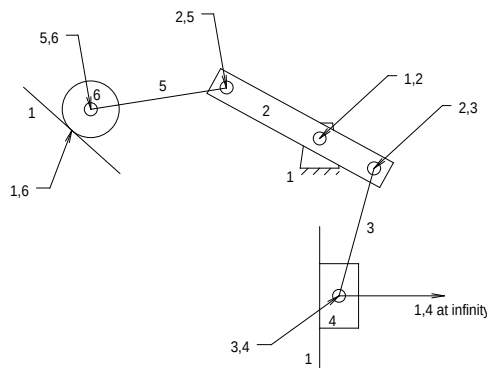
$$I_{1,5}: I_{1,6}-I_{5,6} \text{ and } I_{1,2}-I_{2,5}$$

$$I_{4,5}: I_{1,4}-I_{1,5} \text{ and } I_{2,4}-I_{2,5}$$

$$I_{3,6}: I_{3,6}-I_{5,6} \text{ and } I_{2,3}-I_{2,6}$$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4} \quad I_{3,5}: I_{3,4}-I_{4,5} \text{ and } I_{2,5}-I_{2,3}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,5}-I_{2,4} \quad I_{4,6}: I_{4,5}-I_{5,6} \text{ and } I_{3,4}-I_{3,6}$$

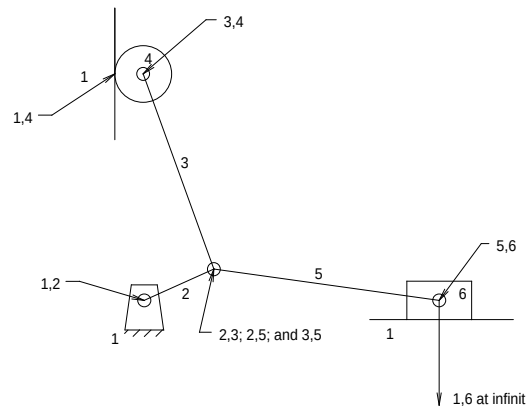


d. This is a sixbar with slider and roller with $n := 6$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 7 ICs.

$I_{1,3}$: $I_{1,2}-I_{2,3}$ and $I_{1,4}-I_{3,4}$

$I_{3,6}$: $I_{1,6}-I_{1,3}$ and $I_{3,5}-I_{5,6}$

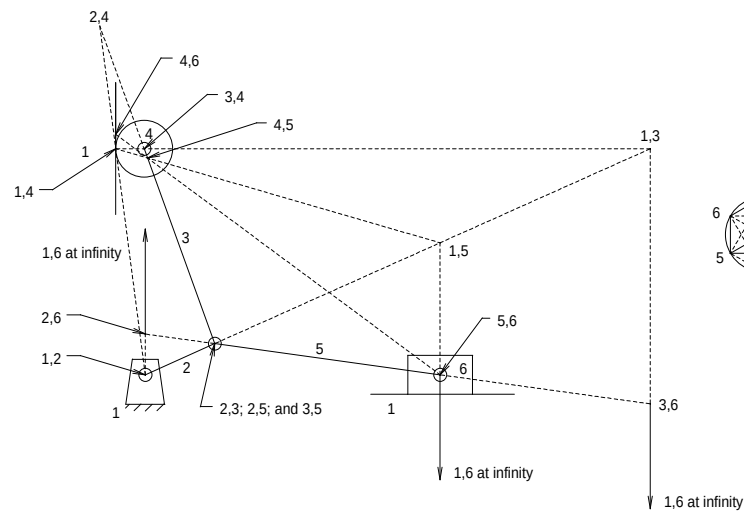
$I_{2,6}$: $I_{1,2}-I_{1,6}$ and $I_{2,5}-I_{5,6}$

$I_{1,5}$: $I_{1,6}-I_{5,6}$ and $I_{1,2}-I_{2,5}$

$I_{4,5}$: $I_{1,4}-I_{1,5}$ and $I_{3,5}-I_{3,4}$

$I_{2,4}$: $I_{1,2}-I_{1,4}$ and $I_{2,3}-I_{3,4}$

$I_{4,6}$: $I_{1,6}-I_{1,4}$ and $I_{4,5}-I_{5,6}$



PROBLEM 6-14

Statement: Find all of the instant centers of the linkages shown in Figure P6-7.

Solution: See Figure P6-7 and Mathcad file P0614.

a. This is a pin-jointed fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

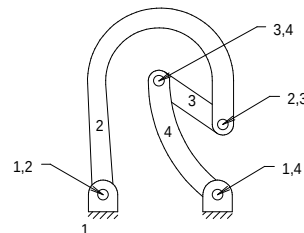
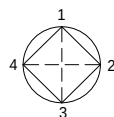
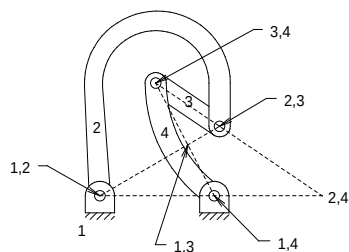
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$.

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



b. This is a fourbar inverted slider-crank, $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

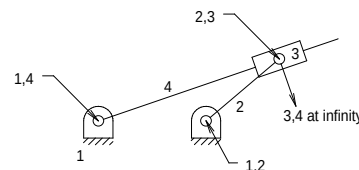
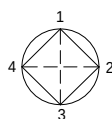
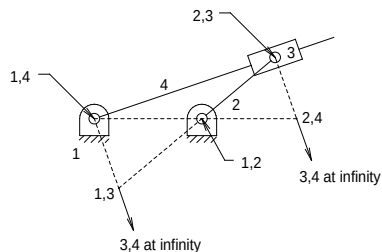
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection, which in this case, is all of them.

3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$.

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$

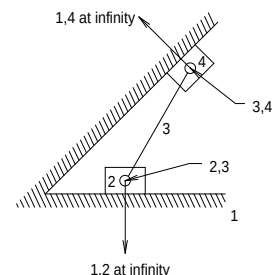


c. This is a fourbar double slider with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

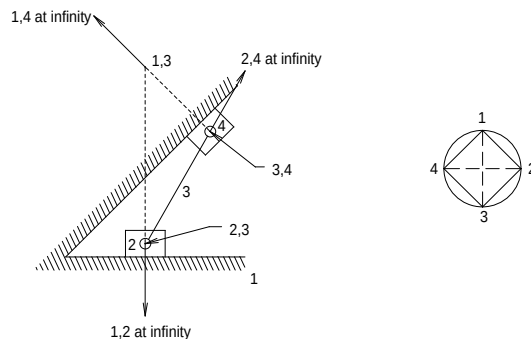
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



- d. This is a pin-jointed fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

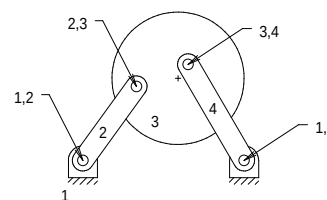
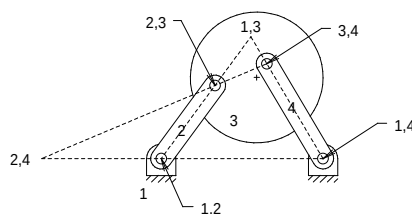
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



- e. This is a fourbar effective slider-crank with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

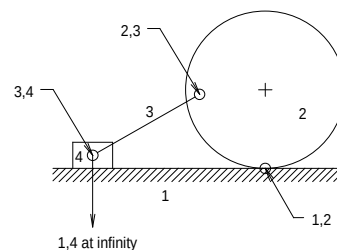
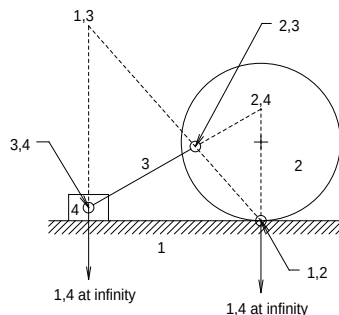
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



f. This is a fourbar cam-follower with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

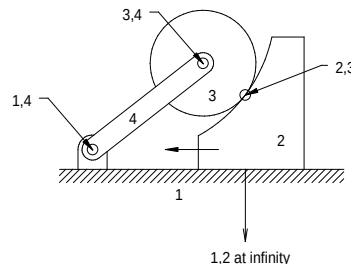
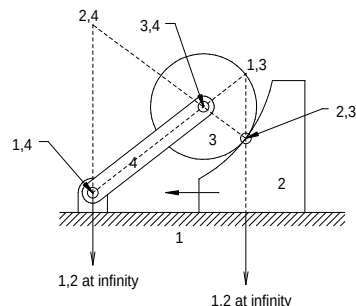
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



g. This is a fourbar slider-crank with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

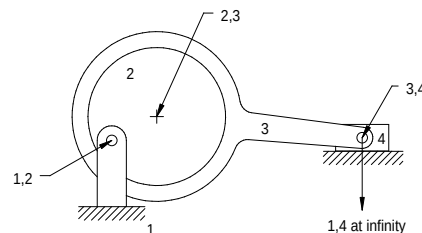
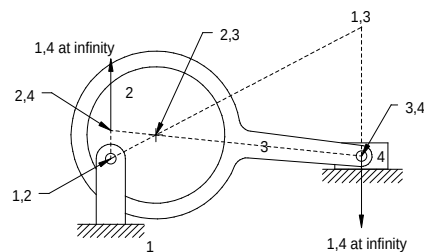
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$

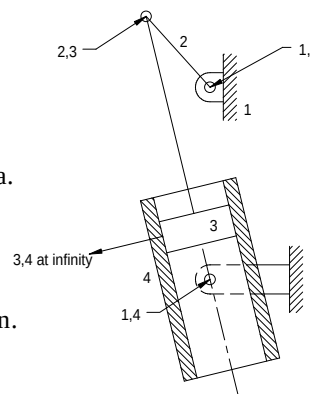


h. This is a fourbar inverted slider-crank with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

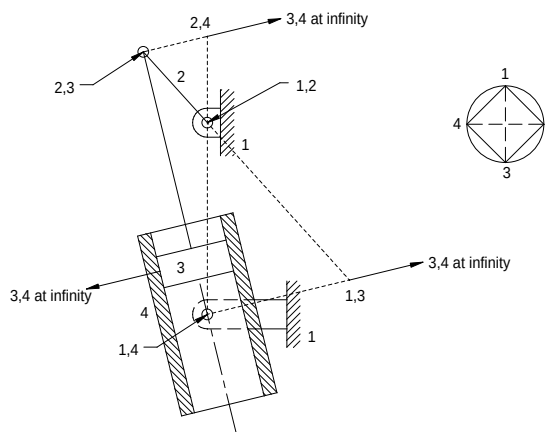
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$

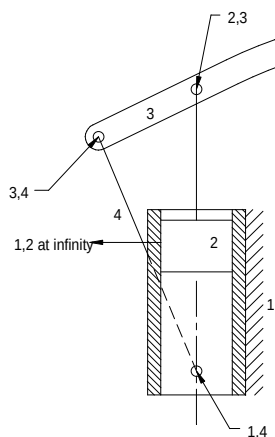


- i. This is a fourbar slider-crank (hand pump) with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

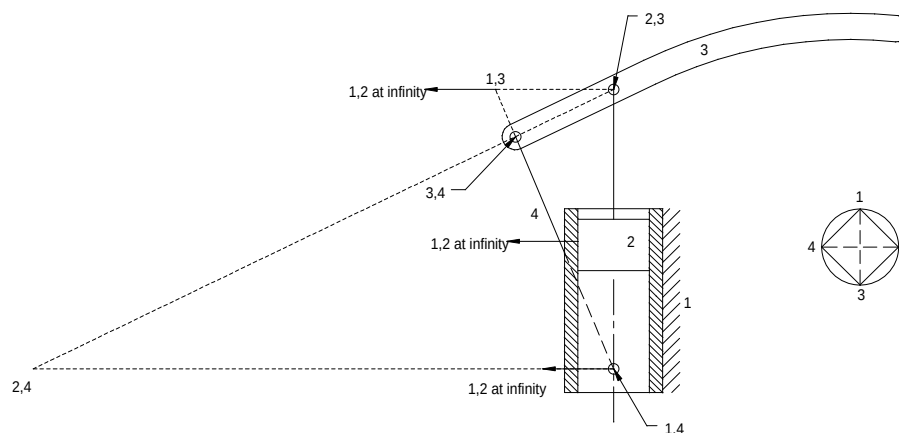
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



PROBLEM 6-15

Statement: Find all of the instant centers of the linkages shown in Figure P6-8.

Solution: See Figure P6-8 and Mathcad file P0615.

a. This is a pin-jointed fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

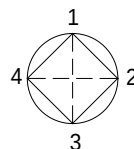
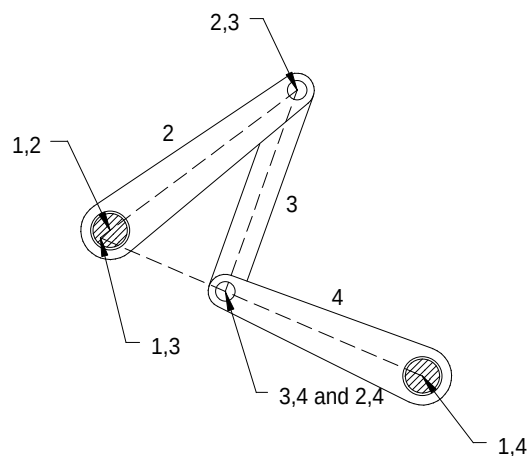
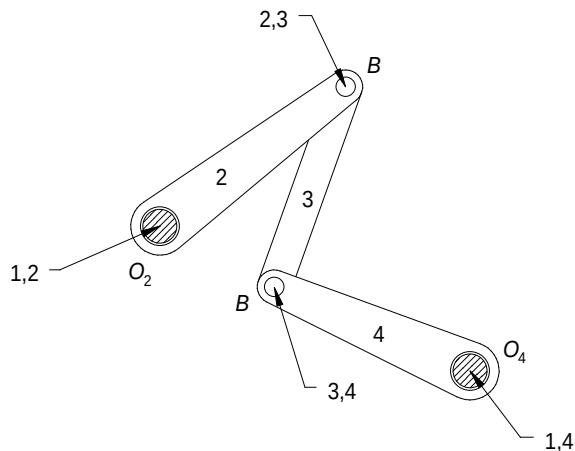
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



b. This is a pin-jointed fourbar with $n := 4$.

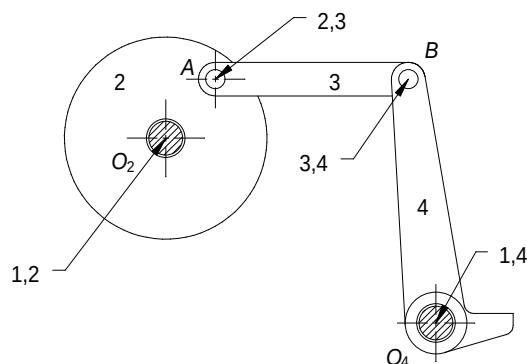
1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

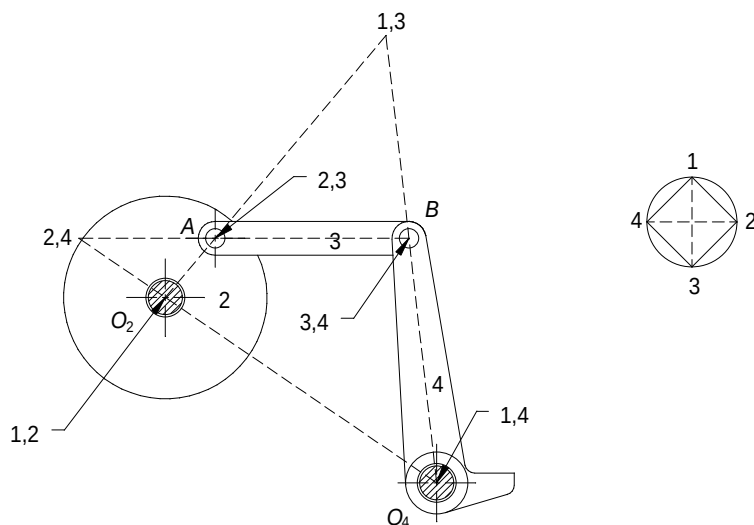
2. Draw the linkage to scale and identify those ICs that can be found by inspection, which in this case, is all of them.

3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$



$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



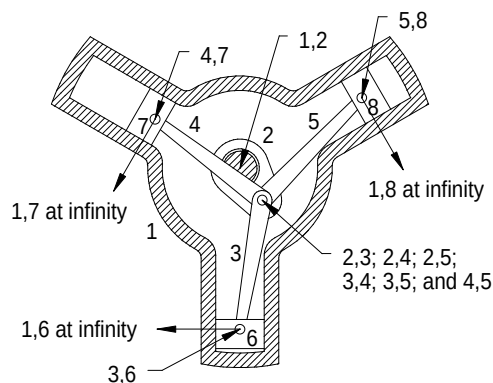
c. This is an eightbar (three slider-cranks with a common crank) with $n := 8$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 28$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. Use Kennedy's Rule and a linear graph to find the remaining 15 ICs.



$$I_{1,5}: I_{1,2}-I_{2,5} \text{ and } I_{1,8}-I_{5,8}$$

$$I_{2,6}: I_{1,2}-I_{1,6} \text{ and } I_{2,3}-I_{3,6}$$

$$I_{2,7}: I_{1,2}-I_{1,7} \text{ and } I_{2,4}-I_{4,7}$$

$$I_{3,7}: I_{1,3}-I_{1,7} \text{ and } I_{3,4}-I_{4,7}$$

$$I_{4,6}: I_{1,6}-I_{1,4} \text{ and } I_{3,4}-I_{3,6}$$

$$I_{5,6}: I_{2,5}-I_{2,6} \text{ and } I_{3,5}-I_{3,6}$$

$$I_{6,7}: I_{2,6}-I_{2,7} \text{ and } I_{3,6}-I_{3,7}$$

$$I_{6,8}: I_{2,7}-I_{2,8} \text{ and } I_{3,7}-I_{3,8}$$

$$I_{2,8}: I_{1,2}-I_{1,8} \text{ and } I_{2,5}-I_{5,8}$$

$$I_{1,4}: I_{1,7}-I_{4,7} \text{ and } I_{1,2}-I_{2,4}$$

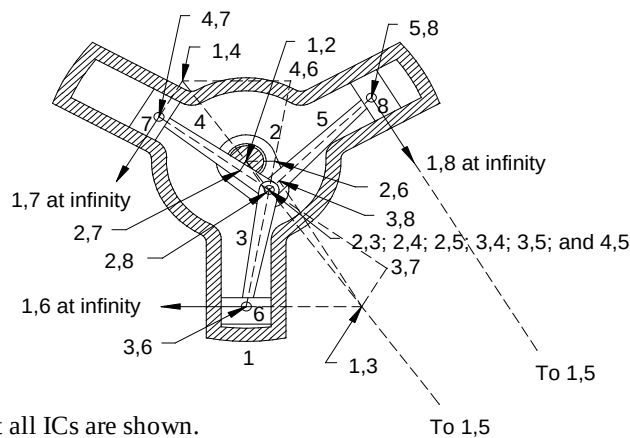
$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,6}-I_{3,6}$$

$$I_{3,8}: I_{1,3}-I_{1,8} \text{ and } I_{3,5}-I_{5,8}$$

$$I_{4,8}: I_{3,8}-I_{3,4} \text{ and } I_{4,5}-I_{5,8}$$

$$I_{5,7}: I_{2,5}-I_{2,7} \text{ and } I_{3,5}-I_{3,7}$$

$$I_{6,8}: I_{4,6}-I_{4,8} \text{ and } I_{3,6}-I_{3,8}$$



Note that, for clarity, not all ICs are shown.

d. This is a pin-jointed fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

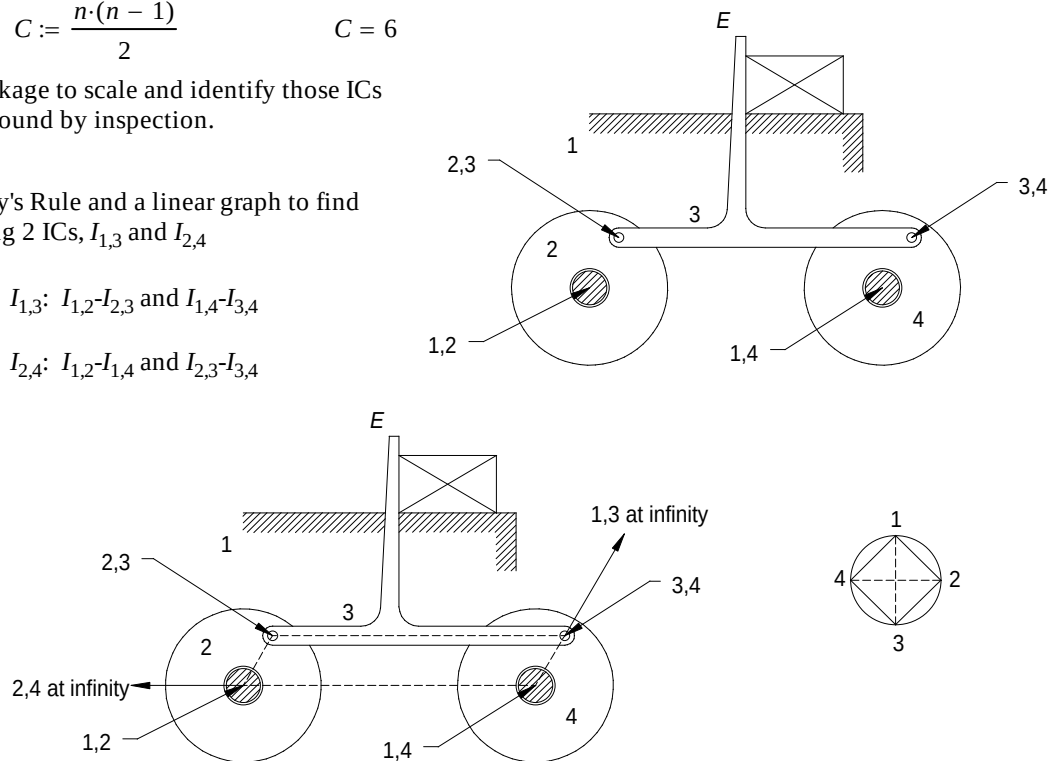
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



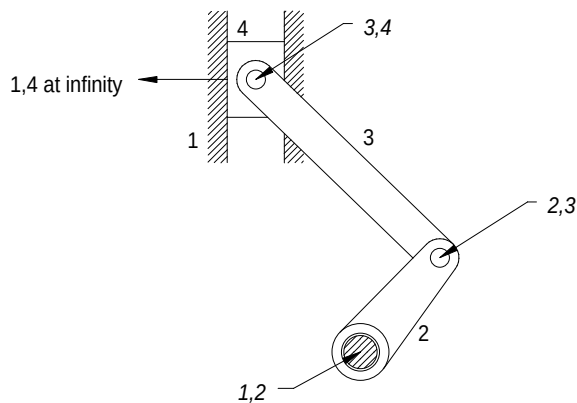
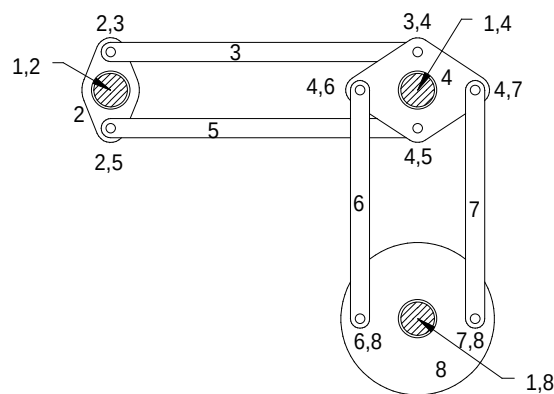
e. This is an eightbar with $n := 8$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 28$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. The remaining 17 ICs are at infinity.



f. This is an offset crank-slider with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

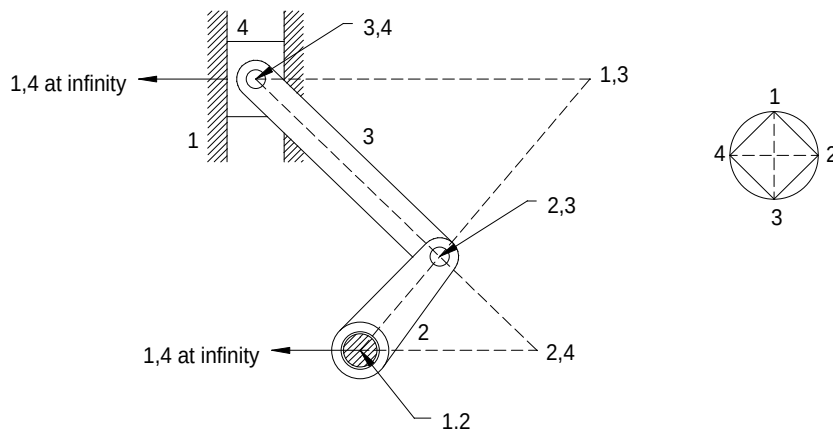
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.

3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$

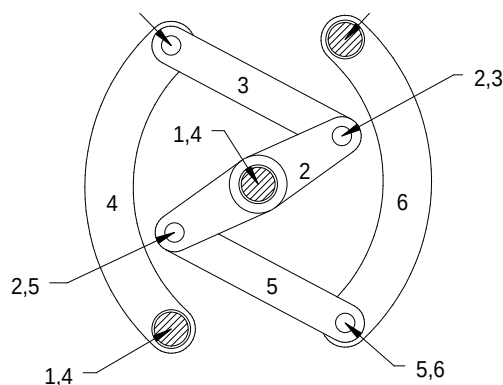


g. This is a sixbar with $n := 6$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.
3. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs.



$$I_{1,3}: I_{1,4}-I_{4,3} \text{ and } I_{1,2}-I_{3,2}$$

$$I_{1,5}: I_{1,6}-I_{5,6} \text{ and } I_{1,2}-I_{2,5}$$

$$I_{3,5}: I_{1,3}-I_{1,5} \text{ and } I_{3,2}-I_{2,5}$$

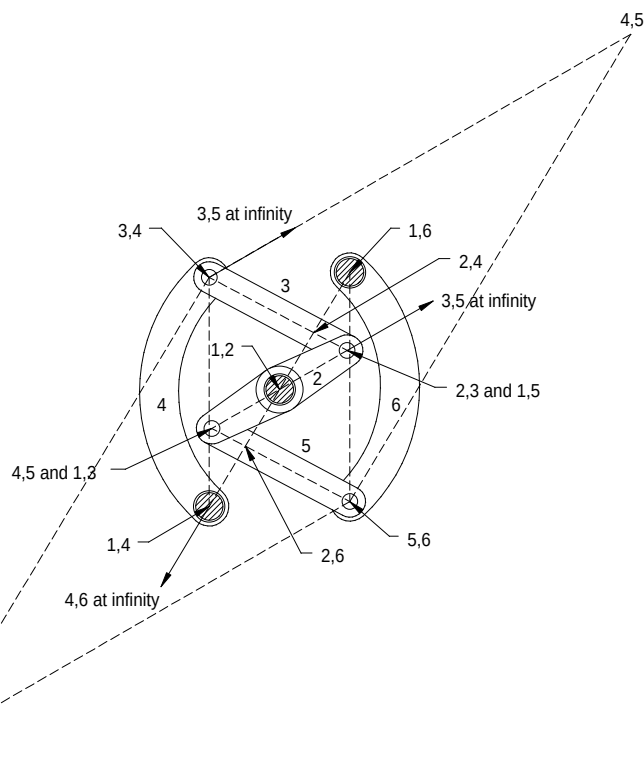
$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{3,4}-I_{2,3}$$

$$I_{2,6}: I_{1,6}-I_{1,2} \text{ and } I_{2,5}-I_{5,6}$$

$$I_{4,6}: I_{1,4}-I_{1,6} \text{ and } I_{2,4}-I_{2,6}$$

$$I_{4,5}: I_{4,6}-I_{5,6} \text{ and } I_{3,4}-I_{3,5}$$

$$I_{3,6}: I_{3,4}-I_{4,6} \text{ and } I_{3,5}-I_{5,6}$$



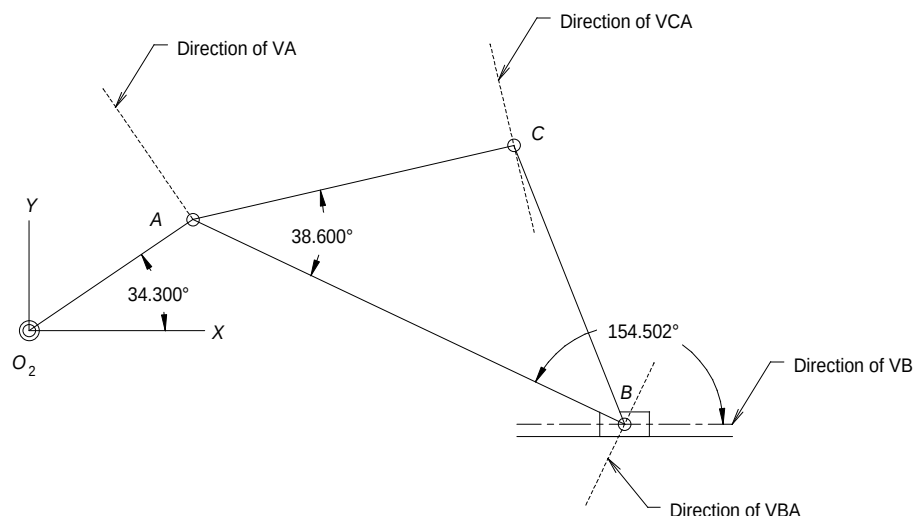
PROBLEM 6-16a

Statement: The linkage in Figure P6-5a has the dimensions and crank angle given below. Find ω_3 , $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ in the direction shown. Use the velocity difference graphical method.

Given: Link lengths: Link 2 (O_2 to A) $a := 0.80 \cdot \text{in}$ Coupler point data: Distance from A to C $p := 1.33 \cdot \text{in}$
 Link 3 (A to B) $b := 1.93 \cdot \text{in}$ Angle BAC $\delta := 38.6 \cdot \text{deg}$
 Offset $c := -0.38 \cdot \text{in}$ Crank angle: $\theta_2 := 34.3 \cdot \text{deg}$
 Input crank angular velocity $\omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW

Solution: See Figure P6-5a and Mathcad file P0616a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



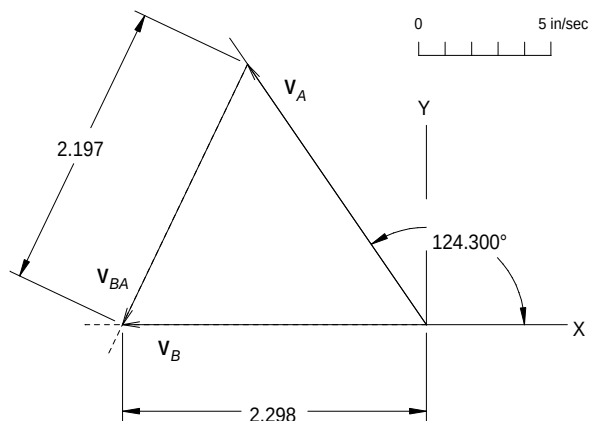
2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 12.00 \frac{\text{in}}{\text{sec}} \quad \theta_A := 34.3 \cdot \text{deg} + 90 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_B := 2.298 \cdot \text{in} \cdot k_v \quad V_B = 11.490 \frac{\text{in}}{\text{sec}} \quad \theta_B := 180 \cdot \text{deg}$$

$$V_{BA} := 2.197 \cdot \text{in} \cdot k_v \quad V_{BA} = 10.985 \frac{\text{in}}{\text{sec}}$$

5. Determine the angular velocity of link 3 using equation 6.7.

$$\omega_3 := \frac{-V_{BA}}{b} \quad \omega_3 = -5.692 \frac{\text{rad}}{\text{sec}}$$

6. Determine the magnitude and sense of the vector $\mathbf{V}_{CA}$ using equation 6.7.

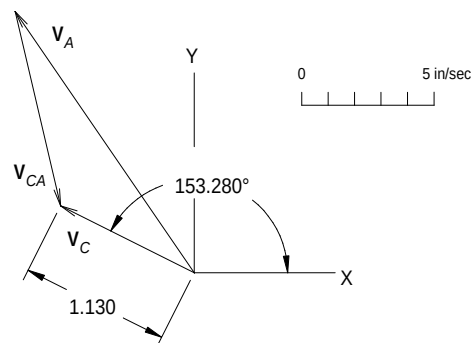
$$V_{CA} := |p \cdot \omega_3| \quad V_{CA} = 7.570 \frac{\text{in}}{\text{sec}}$$

$$\theta_{CA} := (154.502 - 180 + 38.6 - 90) \cdot \text{deg} \quad \theta_{CA} = -76.898 \text{ deg}$$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- From the tip of $\mathbf{V}_A$, layout the (now) known vector $\mathbf{V}_{CA}$.
- Complete the vector triangle by drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_A$ to the tip of the $\mathbf{V}_{CA}$ vector.



8. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_C := 1.130 \cdot \text{in} \cdot k_v \quad V_C = 5.650 \frac{\text{in}}{\text{sec}} \quad \theta_C := 153.28 \cdot \text{deg}$$

PROBLEM 6-16b

Statement: The linkage in Figure P6-5a has the dimensions and crank angle given below. Find ω_3 , $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ in the direction shown. Use the instant center graphical method.

Given: Link lengths: Coupler point data:

Link 2 (O_2 to A)	$a := 0.80 \cdot \text{in}$	Distance from A to C	$p := 1.33 \cdot \text{in}$
Link 3 (A to B)	$b := 1.93 \cdot \text{in}$	Angle BAC	$\delta := 38.6 \cdot \text{deg}$
Offset	$c := -0.38 \cdot \text{in}$	Crank angle:	$\theta_2 := 34.3 \cdot \text{deg}$

Input crank angular velocity $\omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW

Solution: See Figure P6-5a and Mathcad file P0616b.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.

From the layout:

$$AI13 := 2.109 \cdot \text{in}$$

$$BI13 := 2.019 \cdot \text{in}$$

$$CI13 := 0.993 \cdot \text{in}$$

$$\theta_C := (360 - 116.732) \cdot \text{deg}$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2$$

$$V_A = 12.000 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{VA} = 124.3 \text{ deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \quad \omega_3 = 5.690 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point B using equation 6.9b. Determine its direction by inspection.

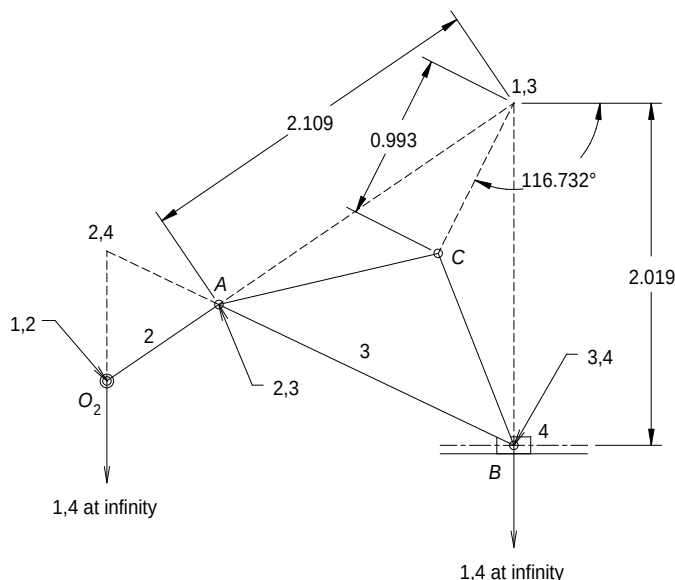
$$V_B := BI13 \cdot \omega_3 \quad V_B = 11.488 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VB} := 180 \cdot \text{deg}$$

5. Determine the magnitude of the velocity at point C using equation 6.9b. Determine its direction by inspection.

$$V_C := CI13 \cdot \omega_3 \quad V_C = 5.650 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VC} := \theta_C - 90 \cdot \text{deg} \quad \theta_{VC} = 153.268 \text{ deg}$$



PROBLEM 6-16c

Statement: The linkage in Figure P6-5a has the dimensions and crank angle given below. Find ω_3 , $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ in the direction shown. Use an analytical method.

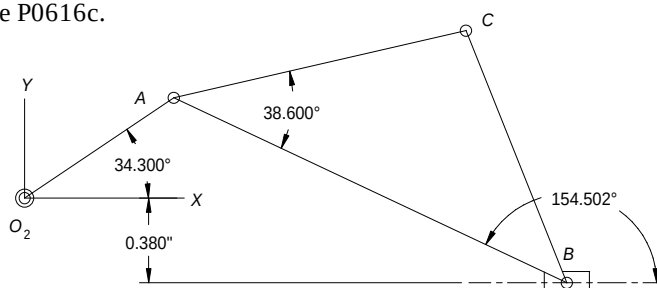
Given: Link lengths: Coupler point data:

Link 2 (O_2 to A)	$a := 0.80 \cdot \text{in}$	Distance from A to C	$R_{ca} := 1.33 \cdot \text{in}$
Link 3 (A to B)	$b := 1.93 \cdot \text{in}$	Angle BAC	$\delta_3 := 38.6 \cdot \text{deg}$
Offset	$c := -0.38 \cdot \text{in}$	Crank angle:	$\theta_2 := 34.3 \cdot \text{deg}$

Input crank angular velocity $\omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW

Solution: See Figure P6-5a and Mathcad file P0616c.

1. Draw the linkage to scale and label it.
2. Determine θ_3 and d using equation 4.17.



$$\theta_{31} := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi$$

$$\theta_{31} = 154.502 \text{ deg}$$

$$d_1 := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_{31}) \quad d_1 = 2.403 \text{ in}$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_{31} := \frac{a \cdot \cos(\theta_2)}{b \cdot \cos(\theta_{31})} \cdot \omega_2 \quad \omega_{31} = -5.691 \frac{\text{rad}}{\text{sec}}$$

4. Determine the velocity of pin A using equation 6.23a:

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = (-6.762 + 9.913j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_A| = 12.000 \frac{\text{in}}{\text{sec}} \quad \arg(\mathbf{V}_A) = 124.300 \text{ deg}$$

5. Determine the velocity of pin B using equation 6.22b:

$$\mathbf{V}_B := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{31} \cdot \sin(\theta_{31})$$

$$\mathbf{V}_B = -11.490 \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_B| = 11.490 \frac{\text{in}}{\text{sec}} \quad \arg(\mathbf{V}_B) = 180.000 \text{ deg}$$

6. Determine the velocity of the coupler point C for the open circuit using equations 6.36.

$$\mathbf{V}_{CA} := R_{ca} \cdot \omega_{31} \cdot (-\sin(\pi + \theta_{31} + \delta_3) + j \cdot \cos(\pi + \theta_{31} + \delta_3))$$

$$\mathbf{V}_{CA} = (1.716 - 7.371j) \frac{\text{in}}{\text{sec}}$$

$$\mathbf{V}_C := \mathbf{V}_A + \mathbf{V}_{CA}$$

$$\mathbf{V}_C = (-5.047 + 2.542j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_C| = 5.651 \frac{\text{in}}{\text{sec}} \quad \arg(\mathbf{V}_C) = 153.268 \text{ deg}$$

Note that θ_3 is defined at point B for the slider-crank and at point A for the pin-jointed fourbar. Thus, to use equation 6.36a for the slider-crank, 180 deg must be added to the calculated value of θ_3 .

PROBLEM 6-17a

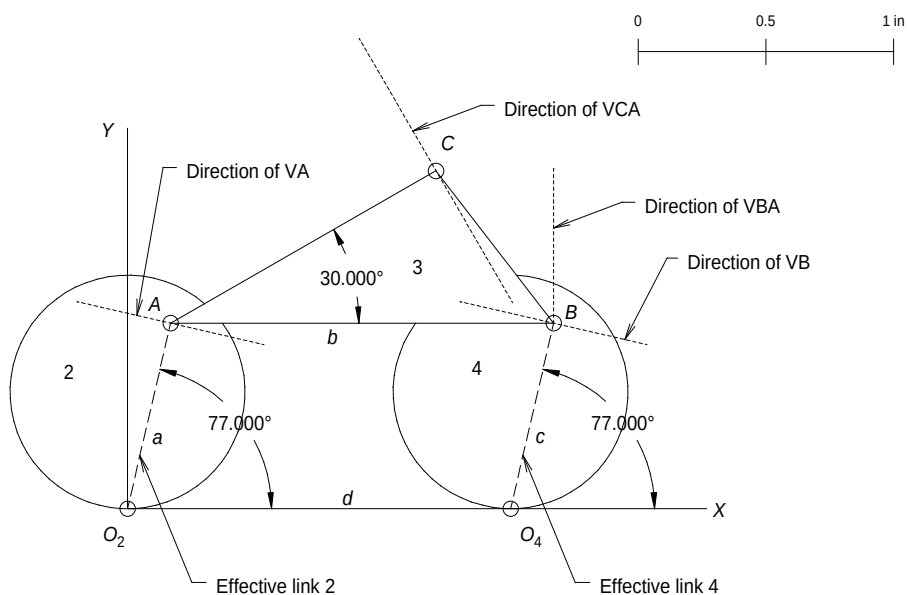
Statement: The linkage in Figure P6-5c has the dimensions and effective crank angle given below. Find ω_3 , ω_4 , $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ in the direction shown. Use the velocity difference graphical method.

Given: Link lengths: Coupler point:

Link 2 (point of contact to A)	$a := 0.75 \cdot \text{in}$	Distance A to C	$p := 1.2 \cdot \text{in}$
Link 3 (A to B)	$b := 1.5 \cdot \text{in}$	Angle BAC	$\delta := 30 \cdot \text{deg}$
Link 4 (point of contact to B)	$c := 0.75 \cdot \text{in}$		
Link 1 (between contact points)	$d := 1.5 \cdot \text{in}$	Crank angle:	$\theta_2 := 77 \cdot \text{deg}$
Input crank angular velocity	$\omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1}$		

Solution: See Figure P6-5c and Mathcad file P0617a.

- Although the mechanism shown in Figure P6-5c is not entirely pin-jointed, it can be analyzed for the position shown by its effective pin-jointed fourbar, which is shown below. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



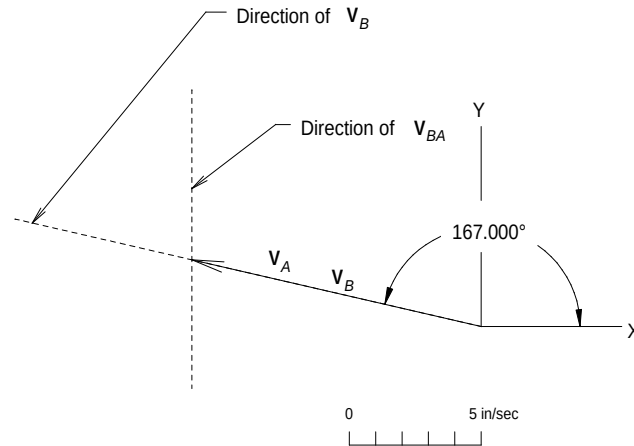
- Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 11.250 \frac{\text{in}}{\text{sec}} \quad \theta_A := 77 \cdot \text{deg} + 90 \cdot \text{deg}$$

- Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

$$V_B := V_A \quad V_B = 11.250 \frac{\text{in}}{\text{sec}} \quad \theta_B := 167 \cdot \text{deg}$$

$$V_{BA} := 0 \cdot \frac{\text{in}}{\text{sec}}$$

5. Determine the angular velocity of links 3 and 4 using equation 6.7.

$$\omega_3 := \frac{V_{BA}}{b} \quad \omega_3 = 0.000 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{V_B}{c} \quad \omega_4 = 15.000 \frac{\text{rad}}{\text{sec}}$$

6. Determine the magnitude of the vector $\mathbf{V}_{CA}$ using equation 6.7.

$$V_{CA} := p \cdot \omega_3 \quad V_{CA} = 0.000 \frac{\text{in}}{\text{sec}}$$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$$

Normally, we would draw the velocity triangle represented by this equation. However, since $\mathbf{V}_{CA}$ is zero,

$$V_C := V_A \quad V_C = 11.250 \frac{\text{in}}{\text{sec}}$$

$$\theta_C := \theta_A \quad \theta_C = 167.000 \text{ deg}$$

PROBLEM 6-17b

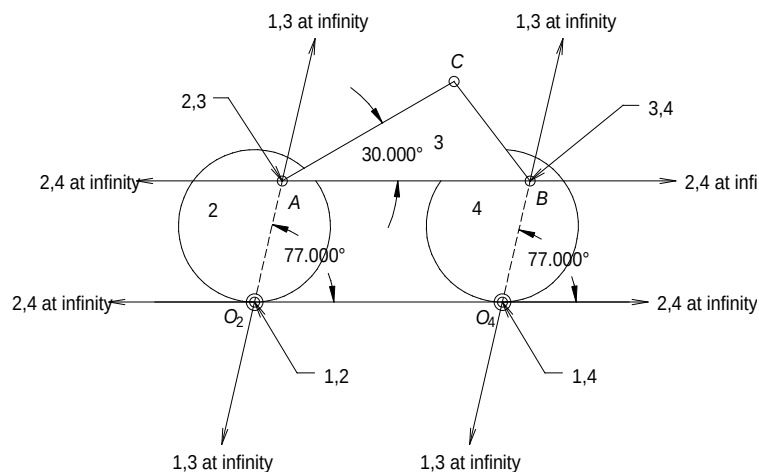
Statement: The linkage in Figure P6-5c has the dimensions and effective crank angle given below. Find ω_3 , ω_4 , $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ in the direction shown. Use the instant center graphical method.

Given: Link lengths: Coupler point:

Link 2 (point of contact to A)	$a := 0.75 \cdot \text{in}$	Distance A to C	$p := 1.2 \cdot \text{in}$
Link 3 (A to B)	$b := 1.5 \cdot \text{in}$	Angle BAC	$\delta := 30 \cdot \text{deg}$
Link 4 (point of contact to B)	$c := 0.75 \cdot \text{in}$		
Link 1 (between contact points)	$d := 1.5 \cdot \text{in}$	Crank angle:	$\theta_2 := 77 \cdot \text{deg}$
Input crank angular velocity	$\omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1}$	CCW	

Solution: See Figure P6-5c and Mathcad file P0617b.

- Although the mechanism shown in Figure P6-5c is not entirely pin-jointed, it can be analyzed for the position shown by its effective pin-jointed fourbar, which is shown below. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.



Since $I_{1,3}$ is at infinity, link 3 is not rotating ($\omega_3 := 0 \cdot \text{rad} \cdot \text{sec}^{-1}$). Thus, the velocity of every point on link 3 is the same.

From the layout above:

$$\theta_4 := 77.000 \cdot \text{deg}$$

$$\theta_3 := 0.000 \cdot \text{deg}$$

- Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 11.250 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{VA} = 167.0 \text{ deg}$$

- Determine the magnitude of the velocity at point B knowing that it is the same as that of point A.

$$V_B := V_A \quad V_B = 11.250 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VB} := \theta_4 + 90 \cdot \text{deg} \quad \theta_{VB} = 167.000 \text{ deg}$$

4. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 15 \frac{rad}{sec} \qquad CCW$$

5. Determine the magnitude of the velocity at point *C* knowing that it is the same as that of point *A*.

$$V_C := V_A \qquad V_C = 11.250 \frac{in}{sec}$$
$$\theta_{VC} := \theta_{VA} \qquad \theta_{VC} = 167.000 \deg$$

PROBLEM 6-17c

Statement: The linkage in Figure P6-5c has the dimensions and effective crank angle given below. Find ω_3 , ω_4 , $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ in the direction shown. Use an analytical method.

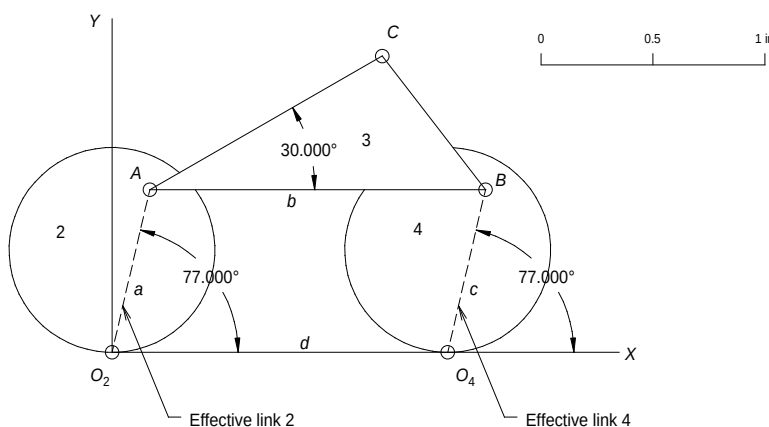
Given: Link lengths: Coupler point:

Link 2 (point of contact to A)	$a := 0.75 \cdot \text{in}$	Distance A to C	$R_{ca} := 1.2 \cdot \text{in}$
Link 3 (A to B)	$b := 1.5 \cdot \text{in}$	Angle BAC	$\delta_3 := 30 \cdot \text{deg}$
Link 4 (point of contact to B)	$c := 0.75 \cdot \text{in}$		
Link 1 (between contact points)	$d := 1.5 \cdot \text{in}$	Crank angle:	$\theta_2 := 77 \cdot \text{deg}$

Input crank angular velocity $\omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1}$

Solution: See Figure P6-5c and Mathcad file P0617c.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.0000$$

$$K_2 = 2.0000$$

$$K_3 = 1.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -1.2250 \quad B = -1.9487 \quad C = 2.3251$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_{41} = -283.000 \text{ deg}$$

$$\theta_{41} := \theta_{41} - 360 \cdot \text{deg} \quad \theta_{41} = -643.000 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0000$$

$$K_5 = -2.0000$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$D = -3.5501$$

$$E := -2 \cdot \sin(\theta_2)$$

$$E = -1.9487$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

$$F = 0.0000$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right)$$

$$\theta_{31} = 360.000 \text{ deg}$$

$$\theta_{31} := \theta_{31} - 360 \cdot \text{deg}$$

$$\theta_{31} = 0.000 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41} - \theta_2)}{\sin(\theta_{31} - \theta_{41})}$$

$$\omega_{31} = 0.000 \frac{\text{rad}}{\text{sec}}$$

$$\omega_{41} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31})}{\sin(\theta_{41} - \theta_{31})}$$

$$\omega_{41} = 15.000 \frac{\text{rad}}{\text{sec}}$$

7. Determine the velocity of points A and B for the open circuit using equations 6.19.

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = (-10.962 + 2.531j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_A| = 11.250 \frac{\text{in}}{\text{sec}}$$

$$\arg(\mathbf{V}_A) = 167.000 \text{ deg}$$

$$\mathbf{V}_B := c \cdot \omega_{41} \cdot (-\sin(\theta_{41}) + j \cdot \cos(\theta_{41}))$$

$$\mathbf{V}_B = (-10.962 + 2.531j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_B| = 11.250 \frac{\text{in}}{\text{sec}}$$

$$\arg(\mathbf{V}_B) = 167.000 \text{ deg}$$

8. Determine the velocity of the coupler point C for the open circuit using equations 6.36.

$$\mathbf{V}_{CA} := R_{ca} \cdot \omega_{31} \cdot (-\sin(\theta_{31} + \delta_3) + j \cdot \cos(\theta_{31} + \delta_3))$$

$$\mathbf{V}_{CA} = 0.000 \frac{\text{in}}{\text{sec}}$$

$$\mathbf{V}_C := \mathbf{V}_A + \mathbf{V}_{CA}$$

$$\mathbf{V}_C = (-10.962 + 2.531j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_C| = 11.250 \frac{\text{in}}{\text{sec}}$$

$$\arg(\mathbf{V}_C) = 167.000 \text{ deg}$$

PROBLEM 6-18a

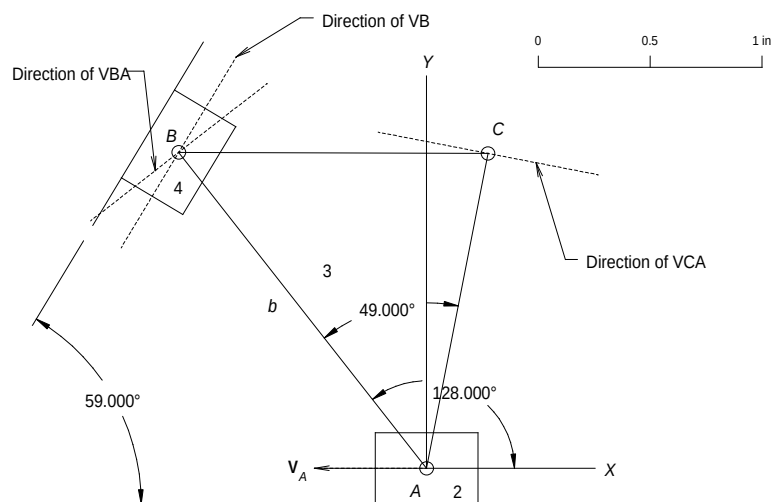
Statement: The linkage in Figure P6-5f has the dimensions and coupler angle given below. Find ω_3 , $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\mathbf{V}_A = 10$ in/sec in the direction shown. Use the velocity difference graphical method.

Given: Link lengths and angles: Coupler point:

Link 3 (A to B)	$b := 1.8 \cdot \text{in}$	Distance A to C	$p := 1.44 \cdot \text{in}$
Coupler angle	$\theta_3 := 128 \cdot \text{deg}$	Angle BAC	$\delta := 49 \cdot \text{deg}$
Slider 4 angle	$\theta_4 := 59 \cdot \text{deg}$	Input slider velocity	$V_A := 10 \cdot \text{in} \cdot \text{sec}^{-1}$

Solution: See Figure P6-5f and Mathcad file P0618a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



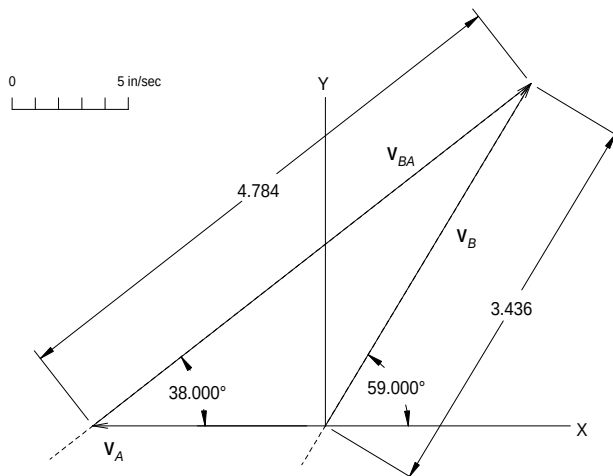
2. The magnitude and sense of the velocity at point A.

$$V_A = 10.000 \frac{\text{in}}{\text{sec}} \quad \theta_A := 180 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

$$\begin{aligned} \text{Velocity scale factor: } k_v &:= \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}} \\ V_B &:= 3.438 \cdot \text{in} \cdot k_v & V_B &= 17.190 \frac{\text{in}}{\text{sec}} & \theta_B &:= 59 \cdot \text{deg} \\ V_{BA} &:= 4.784 \cdot \text{in} \cdot k_v & V_{BA} &= 23.920 \frac{\text{in}}{\text{sec}} & \theta_{BA} &:= 38 \cdot \text{deg} \end{aligned}$$

5. Determine the angular velocity of link 3 using equation 6.7.

$$\omega_3 := \frac{-V_{BA}}{b} \quad \omega_3 = -13.289 \frac{\text{rad}}{\text{sec}}$$

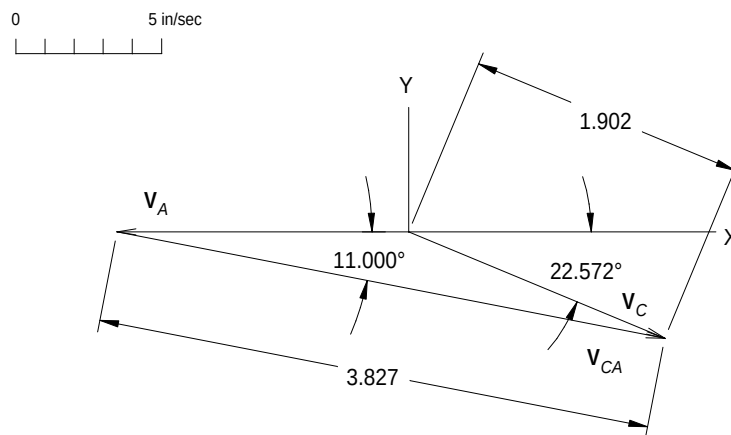
6. Determine the magnitude and sense of the vector $\mathbf{V}_{CA}$ using equation 6.7.

$$\begin{aligned} V_{CA} &:= |p \cdot \omega_3| & V_{CA} &= 19.136 \frac{\text{in}}{\text{sec}} \\ \theta_{CA} &:= (128 - 49 - 90) \cdot \text{deg} & \theta_{CA} &= -11.000 \text{ deg} \end{aligned}$$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- From the tip of $\mathbf{V}_A$, layout the (now) known vector $\mathbf{V}_{CA}$.
- Complete the vector triangle by drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_A$ to the tip of the $\mathbf{V}_{CA}$ vector.



8. From the velocity triangle we have:

$$\begin{aligned} \text{Velocity scale factor: } k_v &:= \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}} \\ V_C &:= 1.902 \cdot \text{in} \cdot k_v & V_C &= 9.510 \frac{\text{in}}{\text{sec}} & \theta_C &:= -22.572 \cdot \text{deg} \end{aligned}$$

PROBLEM 6-18b

Statement: The linkage in Figure P6-5f has the dimensions and coupler angle given below. Find ω_3 , $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\mathbf{V}_A = 10 \text{ in/sec}$ in the direction shown. Use the instant center graphical method.

Given: Link lengths and angles: Coupler point:

Link 3 (A to B)	$b := 1.8 \cdot \text{in}$	Distance A to C	$p := 1.44 \cdot \text{in}$
Coupler angle	$\theta_3 := 128 \cdot \text{deg}$	Angle BAC	$\delta := 49 \cdot \text{deg}$
Slider 4 angle	$\theta_4 := 59 \cdot \text{deg}$	Input slider velocity	$V_A := 10 \cdot \text{in} \cdot \text{sec}^{-1}$

Solution: See Figure P6-5f and Mathcad file P0618b.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.

From the layout:

$$AI13 := 0.753 \cdot \text{in} \quad BI13 := 1.293 \cdot \text{in}$$

$$CI13 := 0.716 \cdot \text{in} \quad \theta_C := 67.428 \cdot \text{deg}$$

2. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \quad \omega_3 = 13.280 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

3. Determine the magnitude of the velocity at point B using equation 6.9b. Determine its direction by inspection.

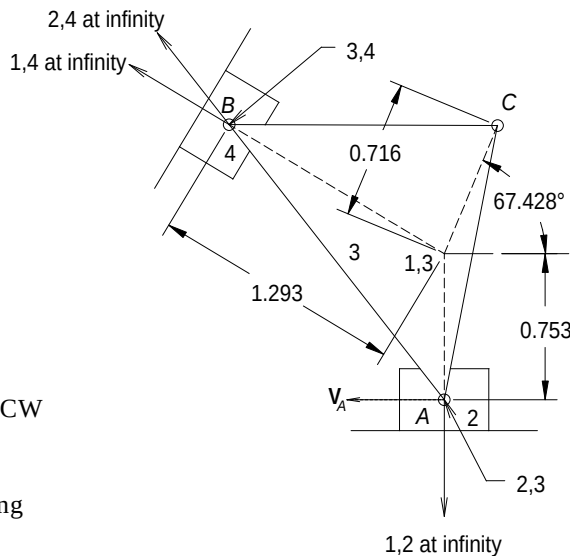
$$V_B := BI13 \cdot \omega_3 \quad V_B = 17.17 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VB} := \theta_4 \quad \theta_{VB} = 59.000 \text{ deg}$$

4. Determine the magnitude of the velocity at point C using equation 6.9b. Determine its direction by inspection.

$$V_C := CI13 \cdot \omega_3 \quad V_C = 9.509 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VC} := \theta_C - 90 \cdot \text{deg} \quad \theta_{VC} = -22.572 \text{ deg}$$



PROBLEM 6-18c

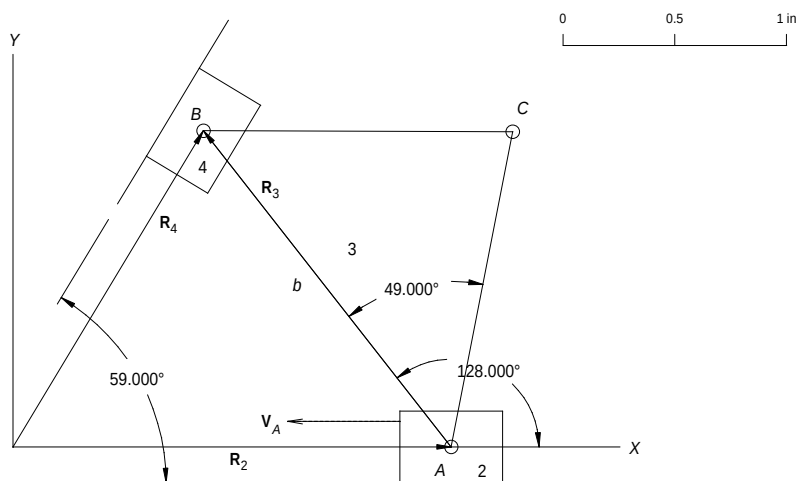
Statement: The linkage in Figure P6-5f has the dimensions and coupler angle given below. Find ω_3 , $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\mathbf{V}_A = 10 \text{ in/sec}$ in the direction shown. Use an analytical method.

Given: Link lengths and angles: Coupler point:

Link 3 (A to B)	$b := 1.8 \cdot \text{in}$	Distance A to C	$R_{ca} := 1.44 \cdot \text{in}$
Coupler angle	$\theta_3 := 128 \cdot \text{deg}$	Angle BAC	$\delta_3 := -49 \cdot \text{deg}$
Slider 4 angle	$\theta_4 := 59 \cdot \text{deg}$	Input slider velocity	$V_A := -10 \cdot \text{in} \cdot \text{sec}^{-1}$

Solution: See Figure P6-5f and Mathcad file P0618c.

1. Draw the mechanism to scale and define a vector loop using the fourbar slider-crank derivation in Section 6.7 as a model.



2. Write the vector loop equation, differentiate it, expand the result and separate into real and imaginary parts to solve for ω_3 and $\mathbf{V}_B$.

$$\mathbf{R}_2 + \mathbf{R}_3 := \mathbf{R}_4$$

$$a \cdot e^{j \cdot \theta_2} + b \cdot e^{j \cdot \theta_3} := c \cdot e^{j \cdot \theta_4}$$

where a is the distance from the origin to point A, a variable; b is the distance from A to B, a constant; and c is the distance from the origin to point B, a variable. Angle θ_2 is zero, θ_3 is the angle that AB makes with the x axis, and θ_4 is the constant angle that slider 4 makes with the x axis. Differentiating,

$$\frac{d}{dt} a + j \cdot b \cdot \omega_3 \cdot e^{j \cdot \theta_3} := \left(\frac{d}{dt} c \right) \cdot e^{j \cdot \theta_4}$$

Substituting the Euler equivalents,

$$\frac{d}{dt} a + b \cdot \omega_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3)) := \left(\frac{d}{dt} c \right) \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

Separating into real and imaginary components and solving for ω_3 and $\mathbf{V}_B$. Note that $dc/dt = V_B$ and $da/dt = V_A$

$$\omega_3 := \frac{V_A \cdot \tan(\theta_4)}{b \cdot (\sin(\theta_3) \cdot \tan(\theta_4) + \cos(\theta_3))} \quad \omega_3 = -13.288 \frac{\text{rad}}{\text{sec}}$$

$$V_B := \frac{V_A - b \cdot \omega_3 \cdot \sin(\theta_3)}{\cos(\theta_4)}$$

$$V_B = 17.180 \frac{\text{in}}{\text{sec}}$$

$$\mathbf{V}_B := V_B \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$\arg(\mathbf{V}_B) = 59.000 \text{ deg}$$

3. Determine the velocity of the coupler point C using equations 6.36.

$$\mathbf{V}_{CA} := R_{ca} \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3))$$

$$\mathbf{V}_{CA} = (18.783 - 3.651j) \frac{\text{in}}{\text{sec}}$$

$$\mathbf{V}_A := V_A$$

$$\mathbf{V}_C := \mathbf{V}_A + \mathbf{V}_{CA}$$

$$\mathbf{V}_C = (8.783 - 3.651j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_C| = 9.512 \frac{\text{in}}{\text{sec}} \quad \arg(\mathbf{V}_C) = -22.572 \text{ deg}$$

PROBLEM 6-19

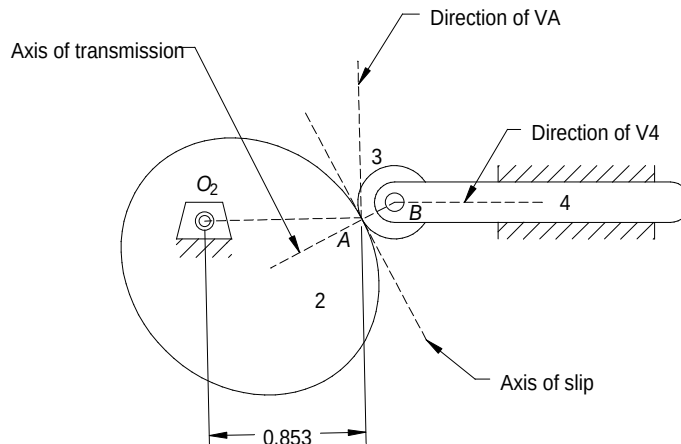
Statement: The cam-follower in Figure P6-5d has $O_2A = 0.853$ in. Find $\mathbf{V}_A$, $\mathbf{V}_{trans}$, and $\mathbf{V}_{slip}$ for the position shown with $\omega_2 = 20$ rad/sec in the direction (CCW) shown.

Given: $\omega_2 := 20 \cdot \text{rad} \cdot \text{sec}^{-1}$ $O_2A := 0.853 \cdot \text{in}$

Assumptions: Rolling contact (no sliding)

Solution: See Figure P6-5d and Mathcad file P0619.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



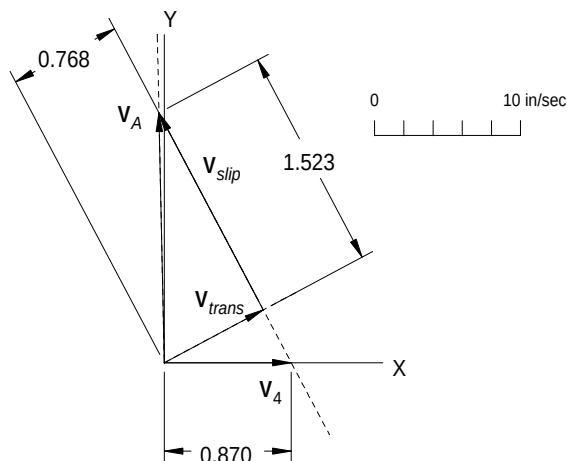
2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 17.060 \frac{\text{in}}{\text{sec}}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A. The equation to be solved graphically is

$$\mathbf{V}_A = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_A$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

$$\begin{aligned} \text{Velocity scale factor: } k_v &:= \frac{10 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}} & V_{trans} &:= 0.768 \cdot \text{in} \cdot k_v & V_{trans} &= 7.680 \frac{\text{in}}{\text{sec}} \\ V_{slip} &:= 1.523 \cdot \text{in} \cdot k_v & V_{slip} &= 15.230 \frac{\text{in}}{\text{sec}} & V_4 &:= 0.870 \cdot \text{in} \cdot k_v & V_4 &= 8.700 \frac{\text{in}}{\text{sec}} \end{aligned}$$

PROBLEM 6-20

Statement: The cam-follower in Figure P6-5e has $O_2A = 0.980$ in and $O_3A = 1.344$ in. Find ω_3 , V_{trans} , and V_{slip} for the position shown with $\omega_2 = 10$ rad/sec in the direction (CW) shown.

Given: $\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$ Distance from O_2 to A: $a := 0.980 \cdot \text{in}$
Distance from O_3 to A: $b := 1.344 \cdot \text{in}$

Assumptions: Roll-slide contact

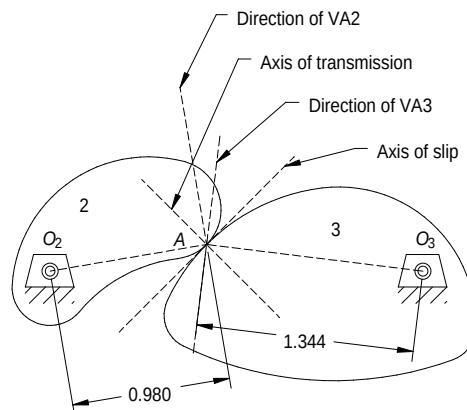
Solution: See Figure P6-5e and Mathcad file P0620.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.
2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 9.800 \frac{\text{in}}{\text{sec}}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A. The equation to be solved graphically is

$$\mathbf{V}_{A2} = \mathbf{V}_{trans} + \mathbf{V}_{A2slip}$$



- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_A$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{slip}$ construction line.

4. From the velocity triangle we have:

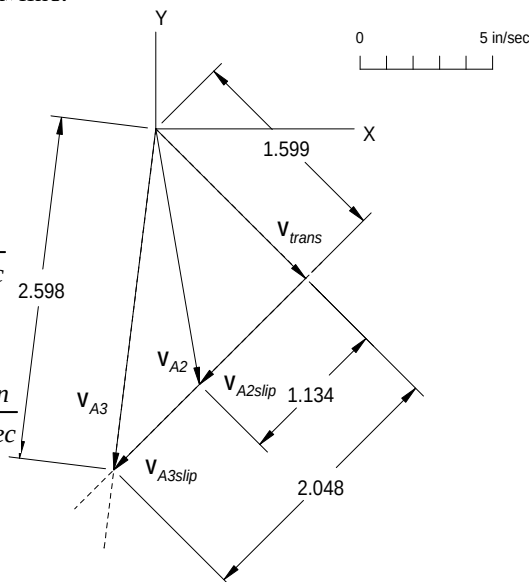
Velocity scale factor: $k_v := \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_{A2slip} := 1.134 \cdot \text{in} \cdot k_v \quad V_{A2slip} = 5.670 \frac{\text{in}}{\text{sec}}$$

$$V_{trans} := 1.599 \cdot \text{in} \cdot k_v \quad V_{trans} = 7.995 \frac{\text{in}}{\text{sec}}$$

$$V_{A3slip} := 2.048 \cdot \text{in} \cdot k_v \quad V_{A3slip} = 10.240 \frac{\text{in}}{\text{sec}}$$

$$V_{A3} := 2.598 \cdot \text{in} \cdot k_v \quad V_{A3} = 12.990 \frac{\text{in}}{\text{sec}}$$



5. Determine the angular velocity of link 3 using equation 6.7.

$$\omega_3 := \frac{V_{A3}}{b} \quad \omega_3 = 9.665 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

6. The relative slip velocity is $V_{slip} := V_{A3slip} - V_{A2slip} \quad V_{slip} = 4.570 \frac{\text{in}}{\text{sec}}$

PROBLEM 6-21a

Statement: The linkage in Figure P6-6b has $L_1 = 61.9$, $L_2 = 15$, $L_3 = 45.8$, $L_4 = 18.1$, $L_5 = 23.1$ mm. θ_2 is 68.3 deg in the xy coordinate system, which is at -23.3 deg in the XY coordinate system. The X component of O_2C is 59.2 mm. Find, for the position shown, the velocity ratio $V_{5,6}/V_{2,3}$ and the mechanical advantage from link 2 to link 6. Use the velocity difference graphical method.

Given: Link lengths:

Link 1 $d := 61.9 \cdot \text{mm}$

Link 2 $a := 15.0 \cdot \text{mm}$

Link 3 $b := 45.8 \cdot \text{mm}$

Link 4 $c := 18.1 \cdot \text{mm}$

Link 5 $e := 23.1 \cdot \text{mm}$

Offset $f := 59.2 \cdot \text{mm}$ from O_2

Crank angle: $\theta_2 := 45 \cdot \text{deg}$ Global XY system

Coordinate rotation angle $\alpha := -23.3 \cdot \text{deg}$ Global XY system to local xy system

Solution: See Figure P6-6b and Mathcad file P0621a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.

2. Choose an arbitrary value for the magnitude of the velocity at $I_{2,3}$ (point A). Let link 2 rotate CCW.

$$V_A := 10 \cdot \frac{\text{mm}}{\text{sec}}$$

$$\theta_{VC} := 45 \cdot \text{deg} + 90 \cdot \text{deg}$$

$$\theta_{VC} = 135.000 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity V_{BA} , and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.

4. From the velocity triangle we have:

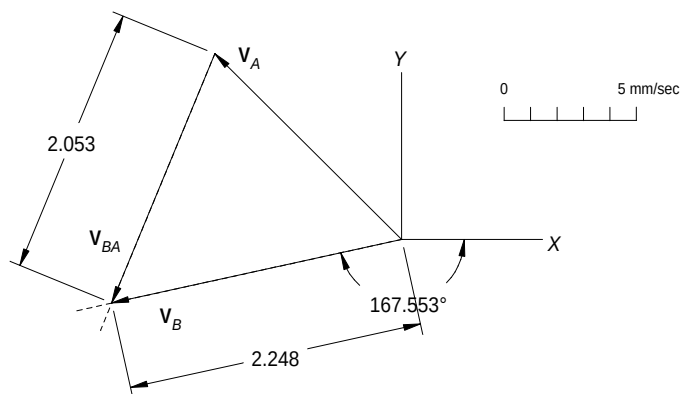
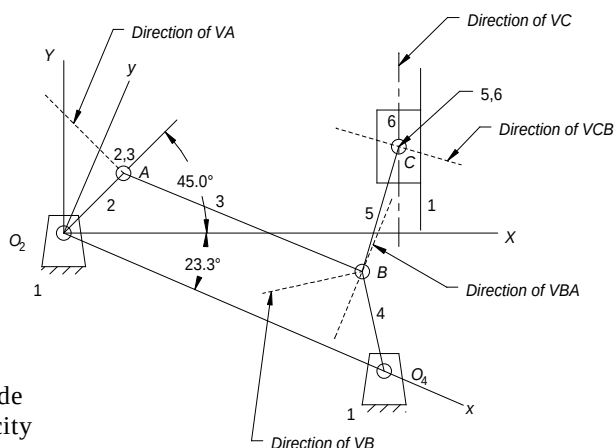
Velocity scale factor:

$$k_v := \frac{5 \cdot \text{mm} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_B := 2.248 \cdot \text{in} \cdot k_v$$

$$V_B = 11.2 \frac{\text{mm}}{\text{sec}}$$

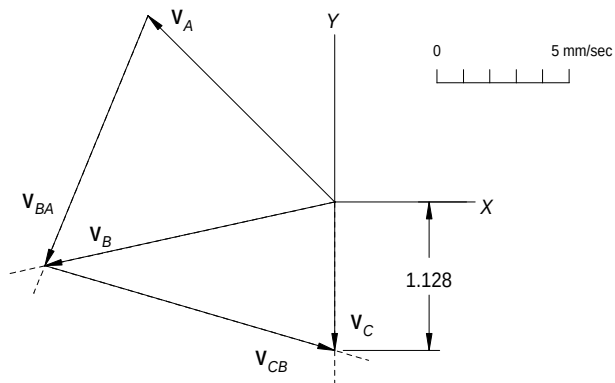
$$\theta_{VB} := (360 - 167.558) \cdot \text{deg}$$



5. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

- Choose a convenient velocity scale and layout the (now) known vector $\mathbf{V}_B$.
- From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.
- From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.



6. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{5 \cdot \text{mm} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_C := 1.128 \cdot \text{in} \cdot k_v \quad V_C = 5.6 \frac{\text{mm}}{\text{sec}} \quad \theta_{VC} := 270 \cdot \text{deg}$$

7. The ratio $V_{15,6} / V_{12,3}$ is $\frac{V_C}{V_A} = 0.564$

8. Use equations 6.12 and 6.13 to derive an expression for the mechanical advantage for this linkage where the input is a rotating crank and the output is a slider.

$$F_{in} = \frac{T_{in}}{r_{in}} = \frac{P_{in}}{r_{in} \cdot \omega_{in}} = \frac{P_{in}}{V_A}$$

$$F_{out} = \frac{P_{out}}{V_{out}} = \frac{P_{out}}{V_C}$$

$$m_A = \frac{F_{out}}{F_{in}} = \frac{P_{out}}{V_C} \cdot \frac{V_A}{P_{in}} \quad m_A := \frac{V_A}{V_C} \quad m_A = 1.773$$

PROBLEM 6-21b

Statement: The linkage in Figure P6-6b has $L_1 = 61.9$, $L_2 = 15$, $L_3 = 45.8$, $L_4 = 18.1$, $L_5 = 23.1$ mm. θ_2 is 68.3 deg in the xy coordinate system, which is at -23.3 deg in the XY coordinate system. The X component of O_2C is 59.2 mm. Find, for the position shown, the velocity ratio $V_{I5,6}/V_{I2,3}$ and the mechanical advantage from link 2 to link 6. Use the instant center graphical method.

Given: Link lengths:

Link 1 $d := 61.9 \cdot \text{mm}$

Link 2 $a := 15.0 \cdot \text{mm}$

Link 3 $b := 45.8 \cdot \text{mm}$

Link 4 $c := 18.1 \cdot \text{mm}$

Link 5 $e := 23.1 \cdot \text{mm}$

Offset $f := 59.2 \cdot \text{mm}$ from O_2

Crank angle: $\theta_2 := 45 \cdot \text{deg}$ Global XY system

Coordinate rotation angle $\alpha := -23.3 \cdot \text{deg}$ Global XY system to local xy system

Solution: See Figure P6-6b and Mathcad file P0621b.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.

From the layout:

$$AI13 := 44.594 \cdot \text{mm} \quad BI13 := 50.121 \cdot \text{mm}$$

$$BI15 := 22.683 \cdot \text{mm} \quad CI15 := 11.377 \cdot \text{mm}$$

2. Choose an arbitrary value for the magnitude of the velocity at $I_{2,3}$ (point A). Let link 2 rotate CCW.

$$V_A := 10 \cdot \frac{\text{mm}}{\text{sec}}$$

$$\theta_{VC} := \theta_2 + 90 \cdot \text{deg}$$

$$\theta_{VC} = 135.000 \cdot \text{deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \quad \omega_3 = 0.224 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point B using equation 6.9b. Determine its direction by inspection.

$$V_B := BI13 \cdot \omega_3 \quad V_B = 11.239 \frac{\text{mm}}{\text{sec}}$$

5. Determine the angular velocity of link 5 using equation 6.9a.

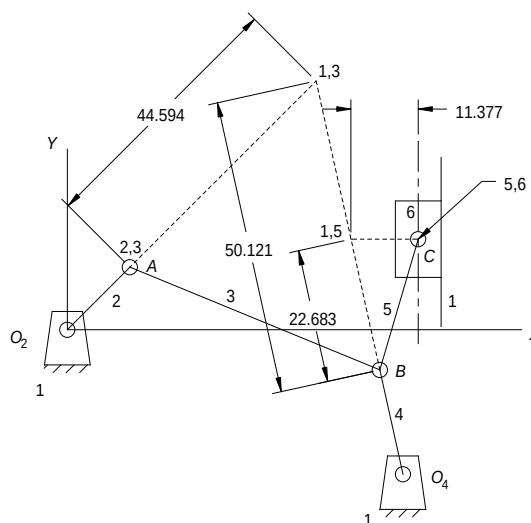
$$\omega_5 := \frac{V_B}{BI15} \quad \omega_5 = 0.495 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

6. Determine the magnitude of the velocity at point C using equation 6.9b. Determine its direction by inspection.

$$V_C := CI15 \cdot \omega_5 \quad V_C = 5.637 \frac{\text{mm}}{\text{sec}} \quad \text{downward}$$

7. The ratio $V_{I5,6}/V_{I2,3}$ is $\frac{V_C}{V_A} = 0.56$

8. Use equations 6.12 and 6.13 to derive an expression for the mechanical advantage for this linkage where the input is a rotating crank and the output is a slider.



$$F_{in} = \frac{T_{in}}{r_{in}} = \frac{P_{in}}{r_{in} \cdot \omega_{in}} = \frac{P_{in}}{V_A}$$

$$F_{out} = \frac{P_{out}}{V_{out}} = \frac{P_{out}}{V_C}$$

$$m_A = \frac{F_{out}}{F_{in}} = \frac{P_{out}}{V_C} \cdot \frac{V_A}{P_{in}} \quad m_A := \frac{V_A}{V_C} \quad m_A = 1.77$$

PROBLEM 6-22a

Statement: The linkage in Figure P6-6d has $L_2 = 15$, $L_3 = 40.9$, $L_5 = 44.7$ mm. θ_2 is 24.2 deg in the XY coordinate system. Find, for the position shown, the velocity ratio $V_{5,6}/V_{2,3}$ and the mechanical advantage from link 2 to link 6. Use the velocity difference graphical method.

Given: Link lengths:

Link 2 $a := 15.0 \cdot \text{mm}$

Link 3 $b := 40.9 \cdot \text{mm}$

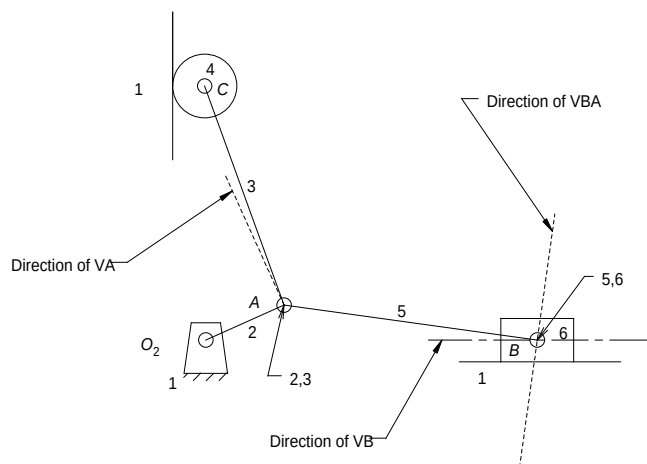
Link 5 $c := 44.7 \cdot \text{mm}$

Offset $f := 0 \cdot \text{mm}$ from O_2

Crank angle: $\theta_2 := 24.2 \cdot \text{deg}$

Solution: See Figure P6-6d and Mathcad file P0622a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Choose an arbitrary value for the magnitude of the velocity at $I_{2,3}$ (point A). Let link 2 rotate CCW.

$$V_A := 10 \cdot \frac{\text{mm}}{\text{sec}}$$

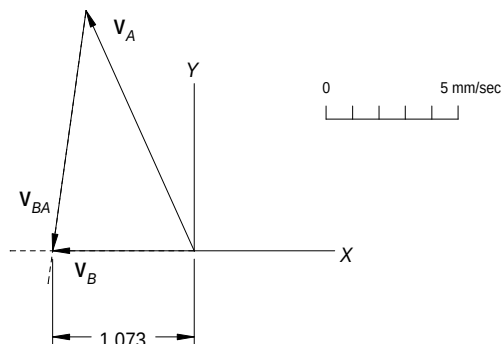
$$\theta_{VC} := \theta_2 + 90 \cdot \text{deg}$$

$$\theta_{VC} = 114.200 \text{ deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{5 \cdot \text{mm} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_B := 1.073 \cdot \text{in} \cdot k_v \quad V_B = 5.4 \frac{\text{mm}}{\text{sec}} \quad \theta_{VB} := 180 \cdot \text{deg}$$

5. The ratio $V_{I5,6} / V_{I2,3}$ is $\frac{V_B}{V_A} = 0.54$

6. Use equations 6.12 and 6.13 to derive an expression for the mechanical advantage for this linkage where the input is a rotating crank and the output is a slider.

$$F_{in} = \frac{T_{in}}{r_{in}} = \frac{P_{in}}{r_{in} \cdot \omega_{in}} = \frac{P_{in}}{V_A}$$

$$F_{out} = \frac{P_{out}}{V_{out}} = \frac{P_{out}}{V_B}$$

$$m_A = \frac{F_{out}}{F_{in}} = \frac{P_{out}}{V_B} \cdot \frac{V_A}{P_{in}} \quad m_A := \frac{V_A}{V_B} \quad m_A = 1.86$$

PROBLEM 6-22b

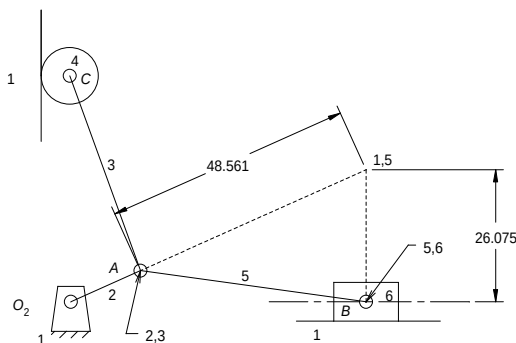
Statement: The linkage in Figure P6-6d has $L_2 = 15$, $L_3 = 40.9$, $L_5 = 44.7$ mm. θ_2 is 24.2 deg in the XY coordinate system. Find, for the position shown, the velocity ratio $V_{I5,6}/V_{I2,3}$ and the mechanical advantage from link 2 to link 6. Use the instant center graphical method.

Given: Link lengths:

Link 2	$a := 15.0 \cdot \text{mm}$	Link 3	$b := 40.9 \cdot \text{mm}$
Link 5	$c := 44.7 \cdot \text{mm}$	Offset	$f := 0 \cdot \text{mm}$ from O_2
Crank angle:	$\theta_2 := 24.2 \cdot \text{deg}$		

Solution: See Figure P6-6d and Mathcad file P0622b.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.



From the layout:

$$AI15 := 48.561 \cdot \text{mm} \quad BI15 := 26.075 \cdot \text{mm}$$

2. Choose an arbitrary value for the magnitude of the velocity at $I_{2,3}$ (point A). Let link 2 rotate CCW.

$$V_A := 10 \cdot \frac{\text{mm}}{\text{sec}} \quad \theta_{VC} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{VC} = 114.200 \cdot \text{deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_5 := \frac{V_A}{AI15} \quad \omega_5 = 0.206 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point B using equation 6.9b. Determine its direction by inspection.

$$V_B := BI15 \cdot \omega_5 \quad V_B = 5.370 \frac{\text{mm}}{\text{sec}} \quad \text{to the left}$$

5. The ratio $V_{I5,6}/V_{I2,3}$ is $\frac{V_B}{V_A} = 0.54$

6. Use equations 6.12 and 6.13 to derive an expression for the mechanical advantage for this linkage where the input is a rotating crank and the output is a slider.

$$F_{in} = \frac{T_{in}}{r_{in}} = \frac{P_{in}}{r_{in} \cdot \omega_{in}} = \frac{P_{in}}{V_A} \quad F_{out} = \frac{P_{out}}{V_{out}} = \frac{P_{out}}{V_C}$$

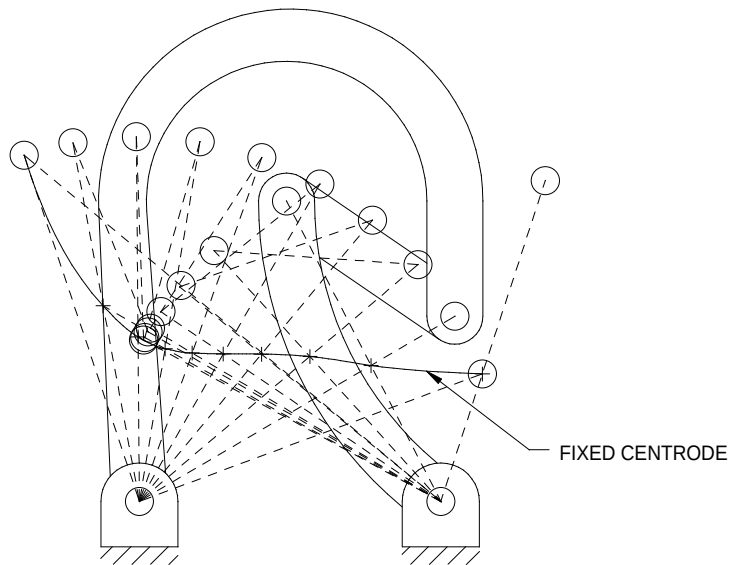
$$m_A = \frac{F_{out}}{F_{in}} = \frac{P_{out}}{V_C} \cdot \frac{V_A}{P_{in}} \quad m_A := \frac{V_A}{V_B} \quad m_A = 1.86$$

PROBLEM 6-23

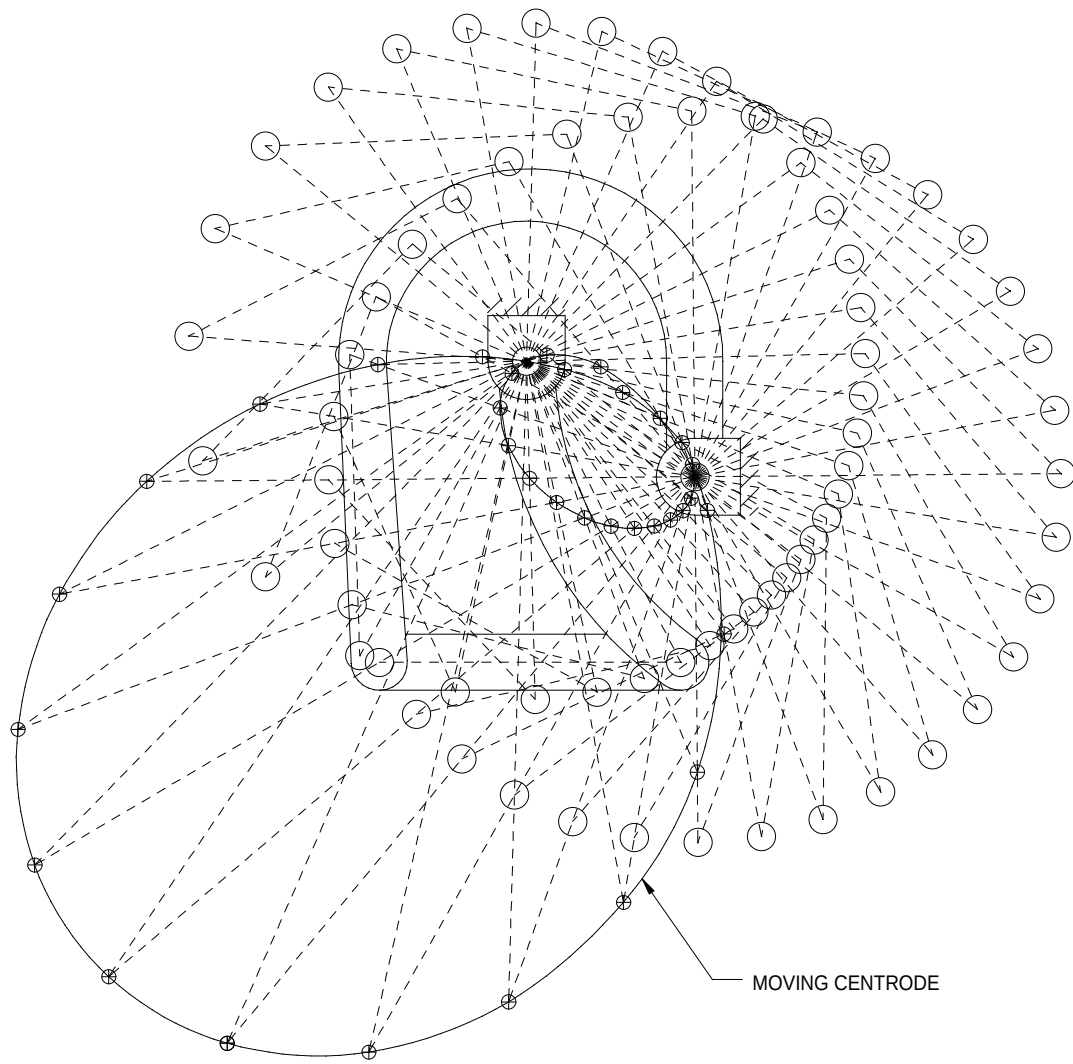
Statement: Generate and draw the fixed and moving centrodes of links 1 and 3 for the linkage in Figure P6-7a

Solution: See Figure P6-7a and Mathcad file P0623.

1. Draw the linkage to scale, find the instant center $I_{1,3}$, and repeat for several positions of the linkage. The locus of points $I_{1,3}$ is the fixed centrode.



2. Invert the linkage, grounding link 3. Draw the linkage to scale, find the instant center $I_{1,3}$, and repeat for several positions of the linkage. The locus of points $I_{1,3}$ is the moving centrode. (See next page.)



PROBLEM 6-24

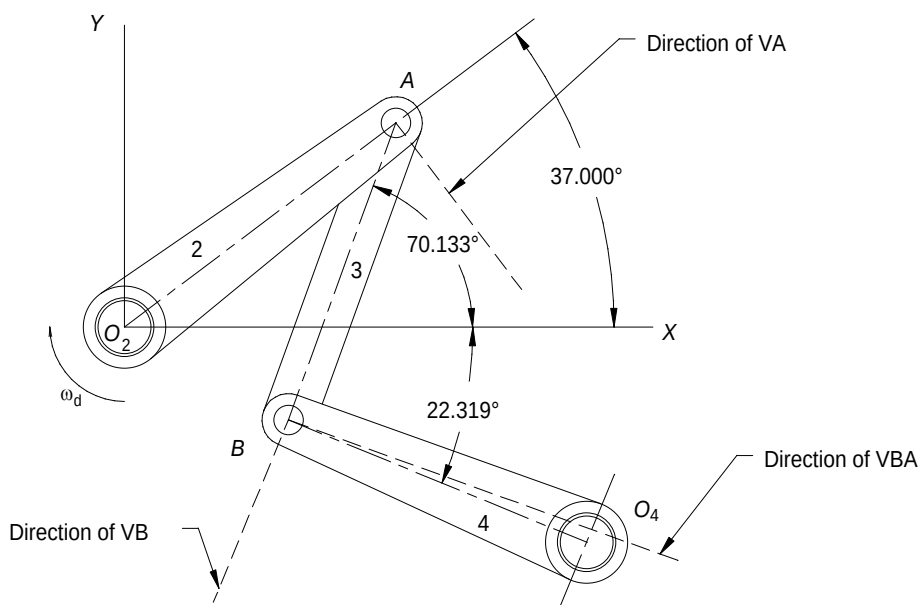
Statement: The linkage in Figure P6-8a has the dimensions and crank angle given below. Find ω_3 , $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW). Use the velocity difference graphical method.

Given: Link lengths: Crank angle:

Link 2 (O_2 to A)	$a := 116 \cdot \text{mm}$	$\theta_2 := 37 \cdot \text{deg}$ Global XY system
Link 3 (A to B)	$b := 108 \cdot \text{mm}$	Input crank angular velocity
Link 4 (B to O_4)	$c := 110 \cdot \text{mm}$	$\omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1}$
Link 1 (O_2 to O_4)	$d := 174 \cdot \text{mm}$	
Coordinate rotation angle	$\alpha := -25 \cdot \text{deg}$	Global XY system to local xy system

Solution: See Figure P6-8a and Mathcad file P0624.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



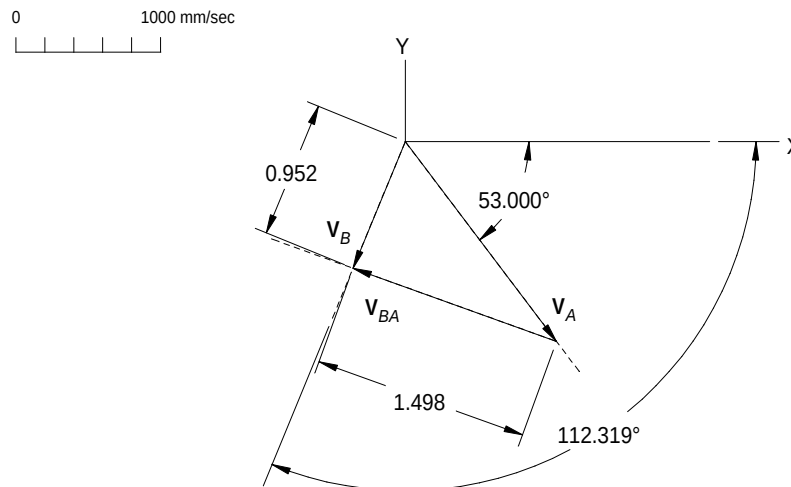
2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 1740.0 \frac{\text{mm}}{\text{sec}} \quad \theta_A := 37 \cdot \text{deg} - 90 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{1000 \cdot \text{mm} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_B := 0.952 \cdot \text{in} \cdot k_v \quad V_B = 952.0 \frac{\text{mm}}{\text{sec}} \quad \theta_B := -112.319 \cdot \text{deg}$$

$$V_{BA} := 1.498 \cdot \text{in} \cdot k_v \quad V_{BA} = 1498.0 \frac{\text{mm}}{\text{sec}}$$

5. Determine the angular velocity of link 3 using equation 6.7.

$$\omega_3 := \frac{-V_{BA}}{b} \quad \omega_3 = -13.87 \frac{\text{rad}}{\text{sec}}$$

6. Determine the angular velocity of link 4 using equation 6.7.

$$\omega_4 := \frac{V_B}{c} \quad \omega_4 = 8.655 \frac{\text{rad}}{\text{sec}}$$

PROBLEM 6-25

Statement: The linkage in Figure P6-8a has the dimensions and crank angle given below. Find ω_P , V_A , and V_B for the position shown for $\omega_2 = 15$ rad/sec clockwise (CW). Use the instant center graphical method.

Given: Link lengths: Crank angle:
 Link 2 (O_2 to A) $a := 116 \cdot mm$ $\theta_2 := 37 \cdot deg$ Global XY system
 Link 3 (A to B) $b := 108 \cdot mm$ Input crank angular velocity
 Link 4 (B to O_4) $c := 110 \cdot mm$ $\omega_2 := 15 \cdot rad \cdot sec^{-1}$
 Link 1 (O_2 to O_4) $d := 174 \cdot mm$
 Coordinate rotation angle $\alpha := -25 \cdot deg$ Global XY system to local xy system

Solution: See Figure P6-8a and Mathcad file P0625.

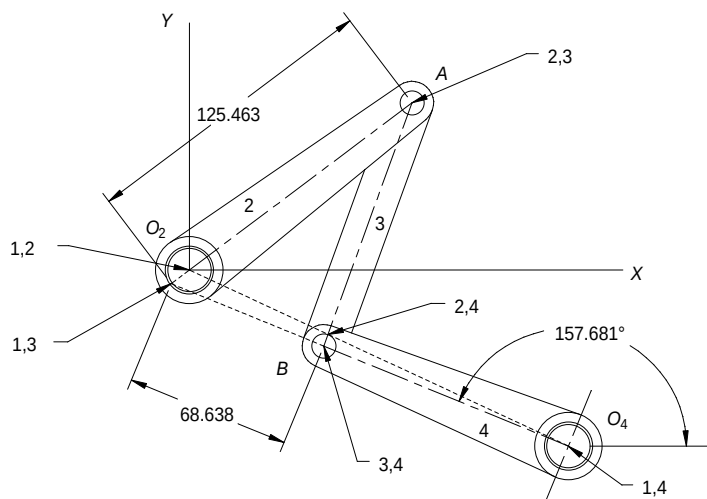
1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.

From the layout:

$$AI13 := 125.463 \cdot mm$$

$$BI13 := 68.638 \cdot mm$$

$$\theta_4 := 157.681 \cdot deg$$



2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 1740.0 \frac{mm}{sec}$$

$$\theta_{VA} := \theta_2 - 90 \cdot deg \quad \theta_{VA} = -53.0 \cdot deg$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \quad \omega_3 = 13.869 \frac{rad}{sec} \quad CW$$

4. Determine the magnitude of the velocity at point B using equation 6.9b. Determine its direction by inspection.

$$V_B := BI13 \cdot \omega_3 \quad V_B = 951.915 \frac{mm}{sec}$$

$$\theta_{VB} := \theta_4 + 90 \cdot deg \quad \theta_{VB} = 247.681 \cdot deg$$

5. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \quad \omega_4 = 8.654 \frac{rad}{sec} \quad CCW$$

PROBLEM 6-26

Statement: The linkage in Figure P6-8a has the dimensions and crank angle given below. Find ω_4 , $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW). Use an analytical method.

Given: Link lengths:

Crank angle:

Link 2 (O_2 to A)	$a := 116 \cdot \text{mm}$	$\theta_2 := 62 \cdot \text{deg}$	Global XY system
Link 3 (A to B)	$b := 108 \cdot \text{mm}$	Input crank angular velocity	
Link 4 (B to O_4)	$c := 110 \cdot \text{mm}$	$\omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1}$	CW
Link 1 (O_2 to O_4)	$d := 174 \cdot \text{mm}$		

Solution: See Figure P6-8a and Mathcad file P0626.

1. Draw the linkage to scale and label it.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_1 = 1.5000$$

$$K_2 := \frac{d}{c} \quad K_2 = 1.5818$$

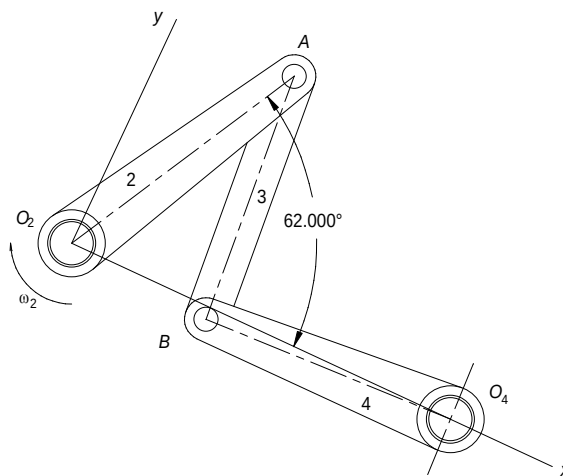
$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.7307$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.0424 \quad B = -1.7659 \quad C = 2.0186$$



3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{42} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_{42} = 182.681 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.6111 \quad K_5 = -1.7280$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -2.0021$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.7659$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.0589$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_{32} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{32} = 275.133 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{42} - \theta_2)}{\sin(\theta_{32} - \theta_{42})} \quad \omega_{32} = -13.869 \frac{\text{rad}}{\text{sec}}$$

$$\omega_{42} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{32})}{\sin(\theta_{42} - \theta_{32})} \qquad \omega_{42} = 8.654 \frac{rad}{sec}$$

7. Determine the velocity of points *A* and *B* for the crossed circuit using equations 6.19.

$$V_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_A = (1536.329 - 816.881j) \frac{mm}{sec} \quad |V_A| = 1740.000 \frac{mm}{sec} \quad arg(V_A) = -28.000 \deg$$

$$V_B := c \cdot \omega_{42} \cdot (-\sin(\theta_{42}) + j \cdot \cos(\theta_{42}))$$

$$V_B = (44.524 - 950.875j) \frac{mm}{sec} \quad |V_B| = 951.917 \frac{mm}{sec} \quad arg(V_B) = -87.319 \deg$$

PROBLEM 6-27

Statement: The linkage in Figure P6-8a has the dimensions given below. Find and plot ω_4 , $\mathbf{V}_A$, and $\mathbf{V}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 15$ rad/sec clockwise (CW).

Given: Link lengths:

Link 2 (O_2 to A)	$a := 116 \cdot \text{mm}$	Link 3 (A to B)	$b := 108 \cdot \text{mm}$
Link 4 (B to O_4)	$c := 110 \cdot \text{mm}$	Link 1 (O_2 to O_4)	$d := 174 \cdot \text{mm}$
Input crank angular velocity $\omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW			

Solution: See Figure P6-8a and Mathcad file P0627.

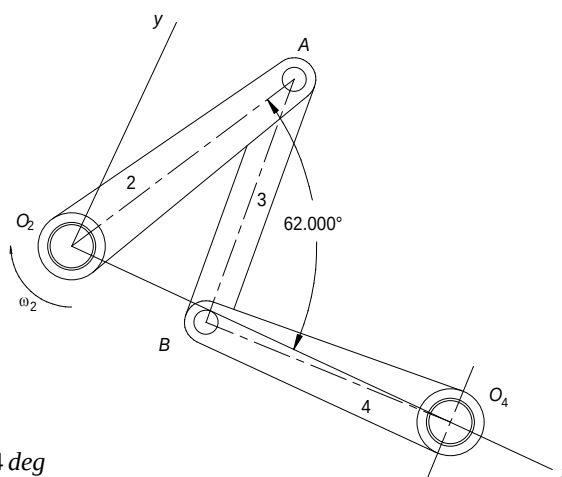
1. Draw the linkage to scale and label it.
2. Determine the range of motion for this non-Grashof triple rocker using equations 4.33.

$$\arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d}$$

$$\arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d}$$

$$\arg_1 = 1.083 \quad \arg_2 = -0.094$$

$$\theta_{2\text{toggle}} := \arccos(\arg_2) \quad \theta_{2\text{toggle}} = 95.4 \text{ deg}$$



The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2\text{toggle}} + 1 \cdot \text{deg}, -\theta_{2\text{toggle}} + 2 \cdot \text{deg} .. \theta_{2\text{toggle}} - 1 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 1.5000$$

$$K_2 = 1.5818$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.7307$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{42}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b}$$

$$K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 1.6111$$

$$K_5 = -1.7280$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_{32}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{42}(\theta_2) - \theta_2)}{\sin(\theta_{32}(\theta_2) - \theta_{42}(\theta_2))}$$

$$\omega_{42}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{32}(\theta_2))}{\sin(\theta_{42}(\theta_2) - \theta_{32}(\theta_2))}$$

8. Determine the velocity of points A and B for the crossed circuit using equations 6.19.

$$V_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_{Ax}(\theta_2) := \text{Re}(V_A(\theta_2))$$

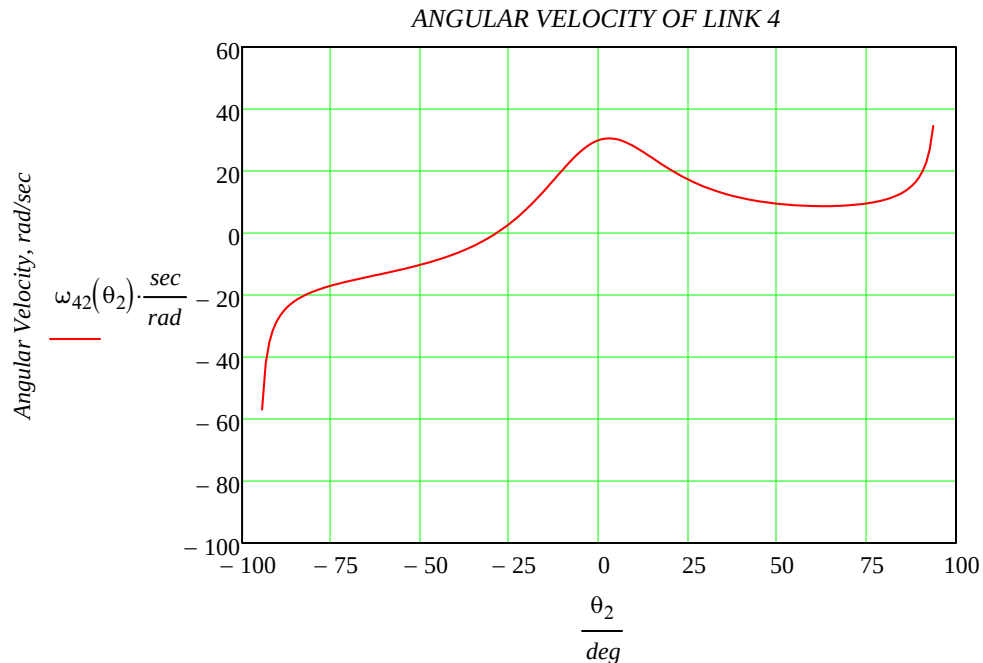
$$V_{Ay}(\theta_2) := \text{Im}(V_A(\theta_2))$$

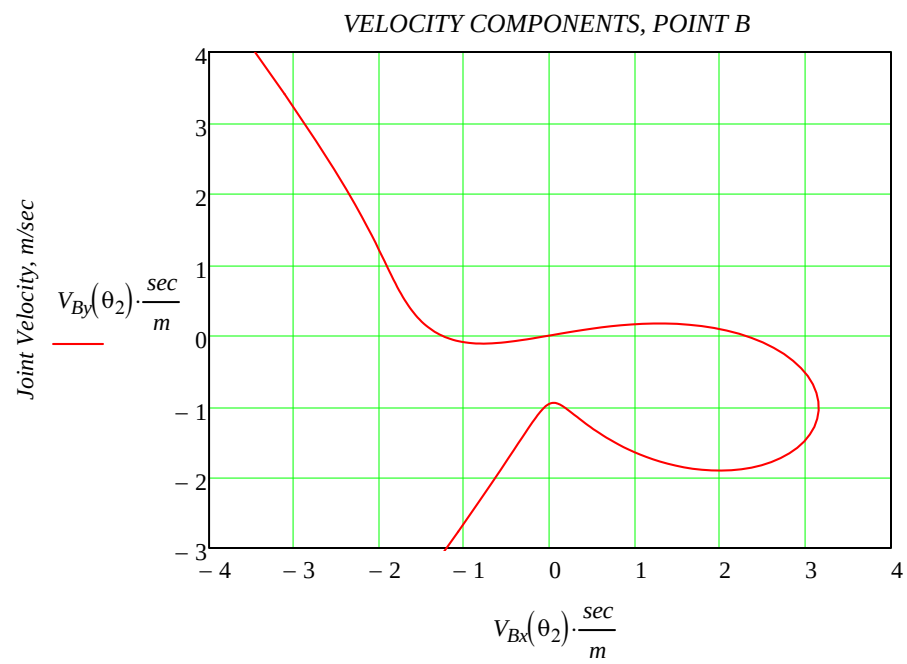
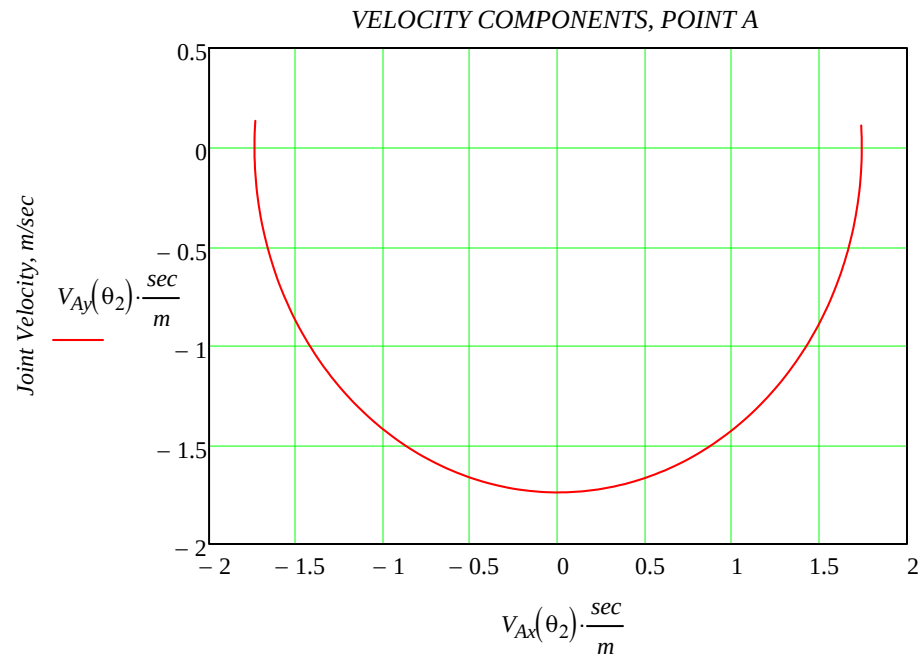
$$V_B(\theta_2) := c \cdot \omega_{42}(\theta_2) \cdot (-\sin(\theta_{42}(\theta_2)) + j \cdot \cos(\theta_{42}(\theta_2)))$$

$$V_{Bx}(\theta_2) := \text{Re}(V_B(\theta_2))$$

$$V_{By}(\theta_2) := \text{Im}(V_B(\theta_2))$$

9. Plot the angular velocity of the output link, ω_4 , and the x and y components of the velocities at points A and B.





PROBLEM 6-28

Statement: The linkage in Figure P6-8b has the dimensions and crank angle given below. Find ω_4 , $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for $\omega_2 = 20$ rad/sec counterclockwise (CCW). Use the velocity difference graphical method.

Given: Link lengths: Crank angle:

Link 2 (O_2 to A)	$a := 40\text{-mm}$	$\theta_2 := 57\text{-deg}$	Global XY system
Link 3 (A to B)	$b := 96\text{-mm}$		Input crank angular velocity
Link 4 (B to O_4)	$c := 122\text{-mm}$	$\omega_2 := 20\text{-rad}\cdot\text{sec}^{-1}$	
Link 1 (O_2 to O_4)	$d := 162\text{-mm}$		
Coordinate rotation angle	$\alpha := -36\text{-deg}$		Global XY system to local xy system

Solution: See Figure P6-8b and Mathcad file P0628.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.

2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2$$

$$V_A = 800.000 \frac{\text{mm}}{\text{sec}}$$

$$\theta_A := \theta_2 + 90\text{-deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.

4. From the velocity triangle we have:

Velocity scale factor:

$$k_v := \frac{400\text{-mm}\cdot\text{sec}^{-1}}{\text{in}}$$

$$V_B := 1.790\text{-in}\cdot k_v \quad V_B = 716.0 \frac{\text{mm}}{\text{sec}}$$

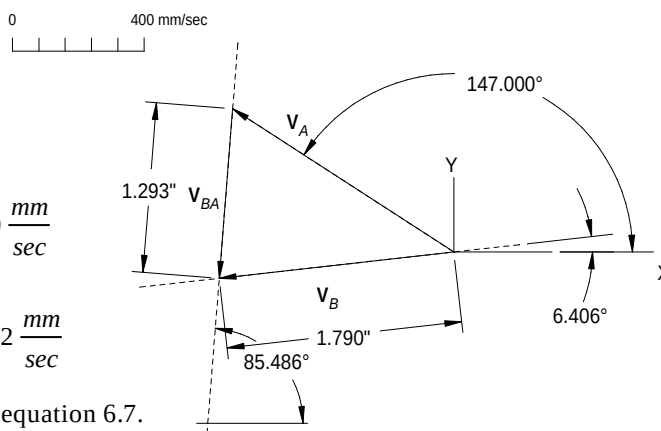
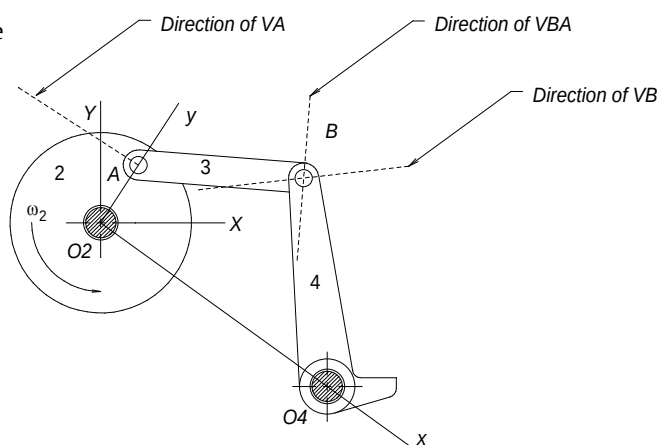
$$\theta_B := 186.406\text{-deg}$$

$$V_{BA} := 1.293\text{-in}\cdot k_v \quad V_{BA} = 517.2 \frac{\text{mm}}{\text{sec}}$$

5. Determine the angular velocity of link 3 using equation 6.7.

$$\omega_3 := \frac{V_{BA}}{b} \quad \omega_3 = 5.388 \frac{\text{rad}}{\text{sec}}$$

6. Determine the angular velocity of link 4 using equation 6.7. $\omega_4 := \frac{V_B}{c} \quad \omega_4 = 5.869 \frac{\text{rad}}{\text{sec}}$



PROBLEM 6-29

Statement: The linkage in Figure P6-8b has the dimensions and crank angle given below. Find ω_4 , V_A , and V_B for the position shown for $\omega_2 = 20$ rad/sec counterclockwise (CCW). Use the instant center graphical method.

Given: Link lengths: $a := 40\text{-mm}$ Crank angle:
 $\theta_2 := 57\text{-deg}$ Global XY system
 $b := 96\text{-mm}$ Input crank angular velocity
 $c := 122\text{-mm}$ $\omega_2 := 20\text{-rad}\cdot\text{sec}^{-1}$
 $d := 162\text{-mm}$
Coordinate rotation angle $\alpha := -36\text{-deg}$ Global XY system to local xy system

Solution: See Figure P6-8b and Mathcad file P0629.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.

From the layout:

$$AI13 := 148.700\text{-mm} \quad BI13 := 133.192\text{-mm}$$

$$\theta_4 := 96.406\text{-deg}$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 800.0 \frac{\text{mm}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 + 90\text{-deg} \quad \theta_{VA} = 147.0\text{ deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \quad \omega_3 = 5.380 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

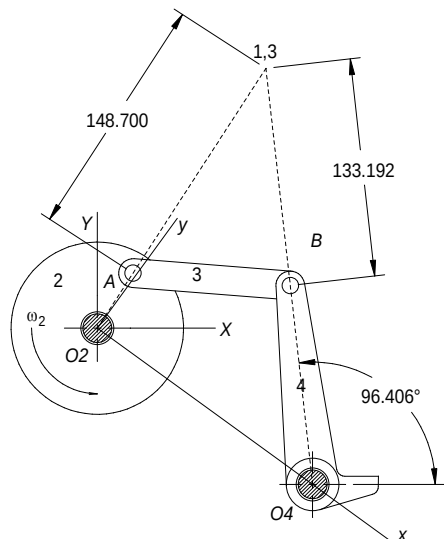
4. Determine the magnitude of the velocity at point B using equation 6.9b. Determine its direction by inspection.

$$V_B := BI13 \cdot \omega_3 \quad V_B = 716.6 \frac{\text{mm}}{\text{sec}}$$

$$\theta_{VB} := \theta_4 + 90\text{-deg} \quad \theta_{VB} = 186.406\text{ deg}$$

5. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \quad \omega_4 = 5.874 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$



PROBLEM 6-30

Statement: The linkage in Figure P6-8b has the dimensions and crank angle given below. Find ω_4 , $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for $\omega_2 = 20$ rad/sec counterclockwise (CCW). Use an analytical method.

Given: Link lengths: Crank angle:
 Link 2 (O_2 to A) $a := 40 \cdot \text{mm}$ $\theta_{21} := 57 \cdot \text{deg}$ Global XY system
 Link 3 (A to B) $b := 96 \cdot \text{mm}$ Input crank angular velocity
 Link 4 (B to O_4) $c := 122 \cdot \text{mm}$ $\omega_2 := 20 \cdot \text{rad} \cdot \text{sec}^{-1}$
 Link 1 (O_2 to O_4) $d := 162 \cdot \text{mm}$
 Coordinate rotation angle $\alpha := -36 \cdot \text{deg}$ Global XY system to local xy system

Solution: See Figure P6-8b and Mathcad file P0630.

1. Draw the linkage to scale and label it.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

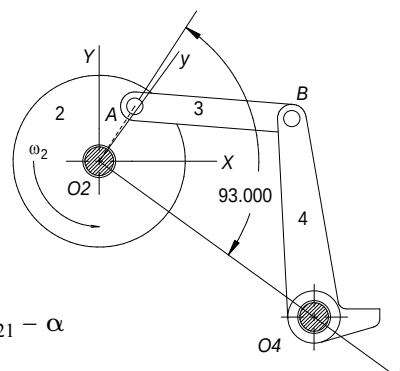
$$K_1 = 4.0500 \quad K_2 = 1.3279$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 3.4336 \quad \theta_2 := \theta_{21} - \alpha$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.5992 \quad B = -1.9973 \quad C = 7.6054$$



3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) - 2 \cdot \pi \quad \theta_4 = -587.614 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.6875 \quad K_5 = -2.8875$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -7.0782$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.9973$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 1.1265$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) - 2 \cdot \pi \quad \theta_3 = -688.496 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad \omega_3 = -5.385 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad \omega_4 = 5.868 \frac{\text{rad}}{\text{sec}}$$

7. Determine the velocity of points *A* and *B* for the open circuit using equations 6.19.

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = (-798.904 - 41.869j) \frac{mm}{sec} \quad |\mathbf{V}_A| = 800.000 \frac{mm}{sec} \quad \arg(\mathbf{V}_A) = -177.000 \text{ deg}$$

$$\mathbf{V}_B := c \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$$

$$\mathbf{V}_B = (-528.774 - 482.608j) \frac{mm}{sec} \quad |\mathbf{V}_B| = 715.900 \frac{mm}{sec} \quad \arg(\mathbf{V}_B) = -137.614 \text{ deg}$$

PROBLEM 6-31

Statement: The linkage in Figure P6-8b has the dimensions and crank angle given below. Find and plot ω_4 , $\mathbf{V}_A$, and $\mathbf{V}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 20 \text{ rad/sec}$ counterclockwise (CCW).

Given: Link lengths:

Link 2 (O_2 to A)	$a := 40 \cdot \text{mm}$	Link 3 (A to B)	$b := 96 \cdot \text{mm}$
Link 4 (B to O_4)	$c := 122 \cdot \text{mm}$	Link 1 (O_2 to O_4)	$d := 162 \cdot \text{mm}$

Input crank angular velocity $\omega_2 := 20 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW

Solution: See Figure P6-8b and Mathcad file P0631.

1. Draw the linkage to scale and label it.
2. Determine Grashof condition.

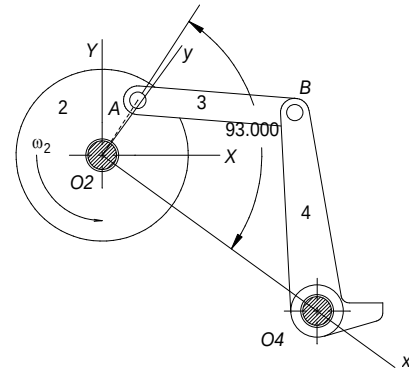
```

Condition(S,L,P,Q) :=
    SL ← S + L
    PQ ← P + Q
    return "Grashof"   if SL < PQ
    return "Special Grashof" if SL = PQ
    return "non-Grashof" otherwise

```

$\text{Condition}(a, d, b, c) = \text{"Grashof"}$ Crank-rocker

$\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$



3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 4.0500 \quad K_2 = 1.3279 \quad K_3 = 3.4336$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.6875 \quad K_5 = -2.8875$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Determine the velocity of points *B* and *C* for the open circuit using equations 6.19.

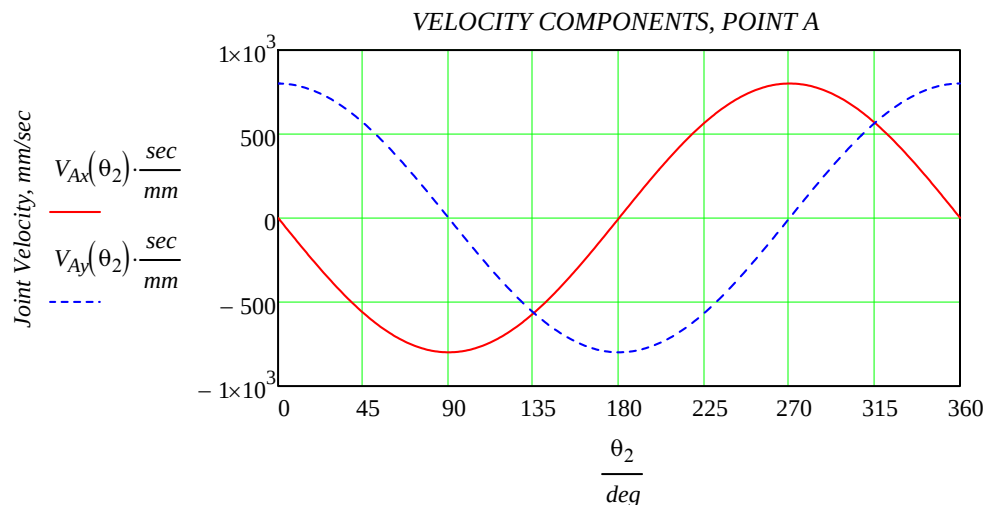
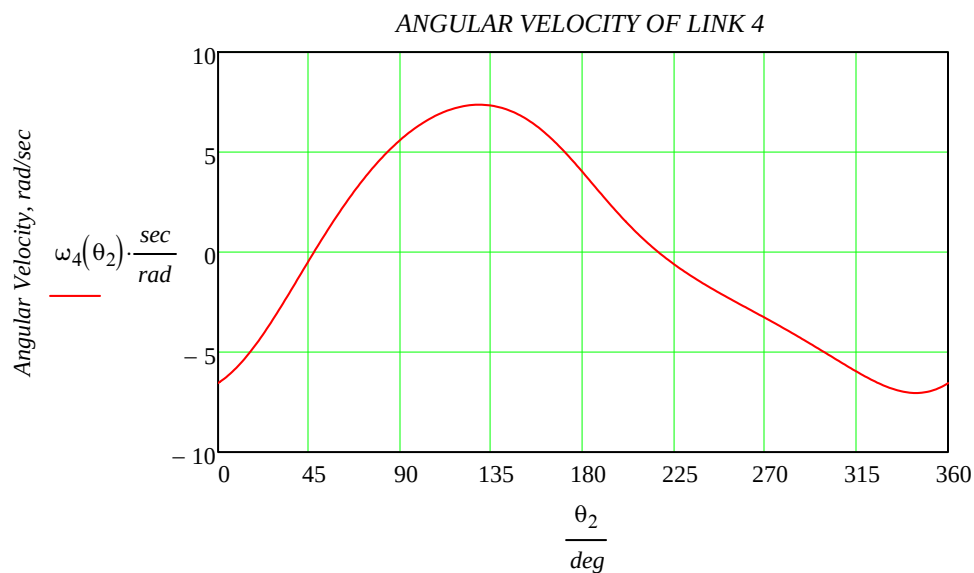
$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

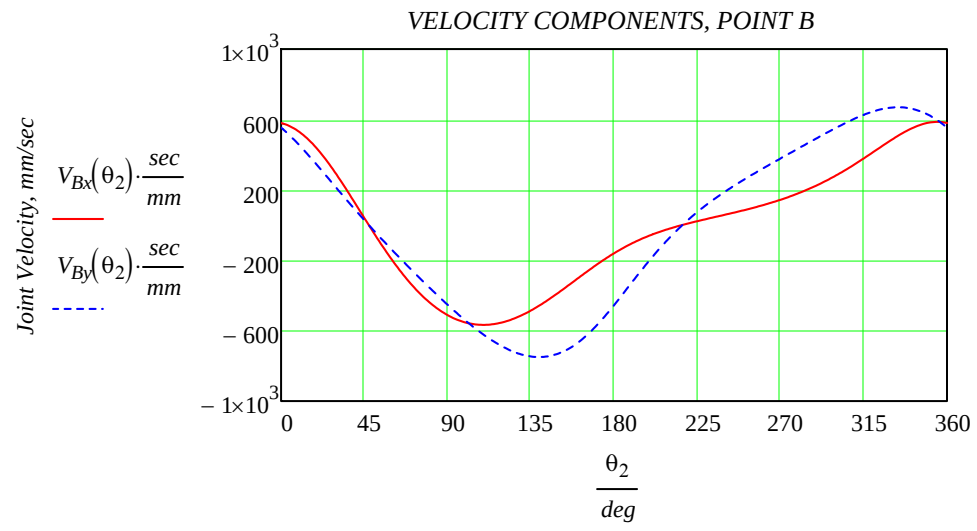
$$V_{Ax}(\theta_2) := \text{Re}(\mathbf{V}_A(\theta_2)) \quad V_{Ay}(\theta_2) := \text{Im}(\mathbf{V}_A(\theta_2))$$

$$\mathbf{V}_B(\theta_2) := c \cdot \omega_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2)))$$

$$V_{Bx}(\theta_2) := \text{Re}(\mathbf{V}_B(\theta_2)) \quad V_{By}(\theta_2) := \text{Im}(\mathbf{V}_B(\theta_2))$$

9. Plot the angular velocity of the output link, ω_4 , and the magnitudes of the velocities at points *B* and *C*.





PROBLEM 6-32

Statement: The offset slider-crank linkage in Figure P6-8f has the dimensions and crank angle given below. Find $\mathbf{V}_A$ and $\mathbf{V}_B$ for the position shown if $\omega_2 = 25 \text{ rad/sec CW}$. Use the velocity difference graphical method.

Given: Link lengths:

Link 2 $a := 63 \cdot \text{mm}$

Link 3 $b := 130 \cdot \text{mm}$

Offset $c := -52 \cdot \text{mm}$

Crank angle: $\theta_2 := 51 \cdot \text{deg}$

Input crank angular velocity

$\omega_2 := 25 \cdot \text{rad} \cdot \text{sec}^{-1} \text{ CW}$

Solution: See Figure P6-8f and Mathcad file P0632.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.

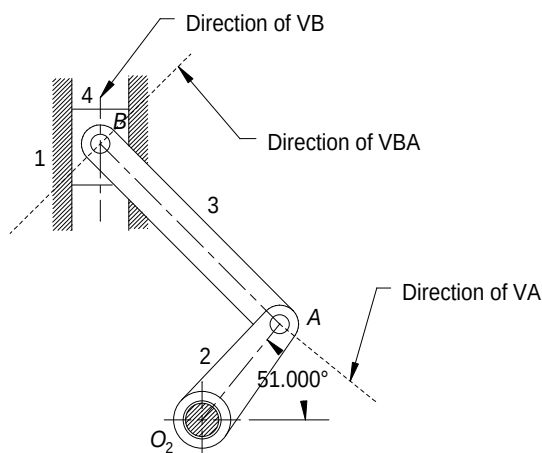
2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 1575.0 \frac{\text{mm}}{\text{sec}}$$

$$\theta_A := 51 \cdot \text{deg} - 90 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$



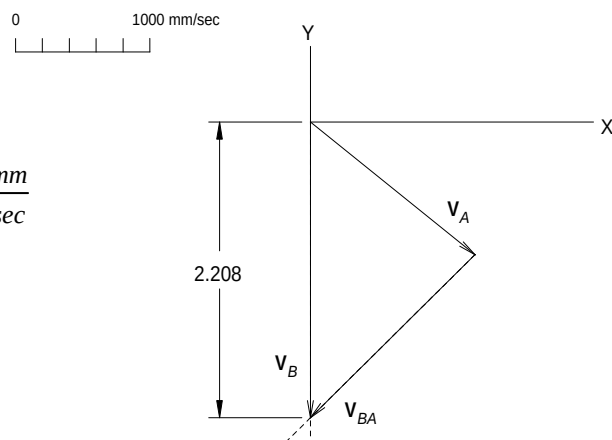
- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.

4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{1000 \cdot \text{mm} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_B := 2.208 \cdot \text{in} \cdot k_v \quad V_B = 2208 \frac{\text{mm}}{\text{sec}}$$

$$\theta_{VB} := 270 \cdot \text{deg}$$



PROBLEM 6-33

Statement: The offset crank-slider linkage in Figure P6-8f has the dimensions and crank angle given below. Find $\mathbf{V}_A$ and $\mathbf{V}_B$ for the position shown for $\omega_2 = 25 \text{ rad/sec CW}$. Use the instant center graphical method.

Given: Link lengths:

Link 2 $a := 63 \cdot \text{mm}$

Crank angle: $\theta_2 := 51 \cdot \text{deg}$

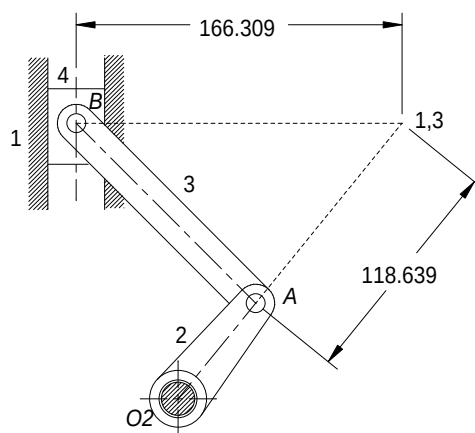
Link 3 $b := 130 \cdot \text{mm}$

Offset $c := -52 \cdot \text{mm}$

Input crank angular velocity $\omega_2 := 25 \cdot \text{rad} \cdot \text{sec}^{-1} \text{ CW}$

Solution: See Figure P6-8f and Mathcad file P0633.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.



From the layout above:

$$AI13 := 118.639 \cdot \text{mm}$$

$$BI13 := 166.309 \cdot \text{mm}$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 1575.0 \frac{\text{mm}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{VA} = -39.0 \text{ deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \quad \omega_3 = 13.276 \frac{\text{rad}}{\text{sec}} \text{ CCW}$$

4. Determine the magnitude of the velocity at point B using equation 6.9b. Determine its direction by inspection.

$$V_B := BI13 \cdot \omega_3 \quad V_B = 2207.8 \frac{\text{mm}}{\text{sec}}$$

$$\theta_{VB} := 270 \cdot \text{deg}$$

PROBLEM 6-34

Statement: The offset slider-crank linkage in Figure P6-8f has the dimensions and crank angle given below. Find $\mathbf{V}_A$ and $\mathbf{V}_B$ for the position shown for $\omega_2 = 25 \text{ rad/sec CW}$. Use an analytical method.

Given: Link lengths:

Link 2 $a := 63 \cdot \text{mm}$

Link 3 $b := 130 \cdot \text{mm}$

Offset $c := -52 \cdot \text{mm}$

Crank angle: $\theta_2 := 141 \cdot \text{deg}$

Local xy coordinate system

Input crank angular velocity

$$\omega_2 := -25 \cdot \text{rad} \cdot \text{sec}^{-1}$$

Solution: See Figure P6-8f and Mathcad file P0634.

1. Draw the linkage to a convenient scale.

2. Determine θ_3 and d using equations 4.16 for the crossed circuit.

$$\theta_3 := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right) \quad \theta_3 = 44.828 \text{ deg}$$

$$d := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_3) \quad d = -141.160 \text{ mm}$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3)} \cdot \omega_2 \quad \omega_3 = 13.276 \frac{\text{rad}}{\text{sec}}$$

4. Determine the velocity of pin A using equation 6.23a:

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = (991.180 + 1224.005j) \frac{\text{mm}}{\text{sec}} \quad |\mathbf{V}_A| = 1575.000 \frac{\text{mm}}{\text{sec}} \quad \arg(\mathbf{V}_A) = 51.000 \text{ deg}$$

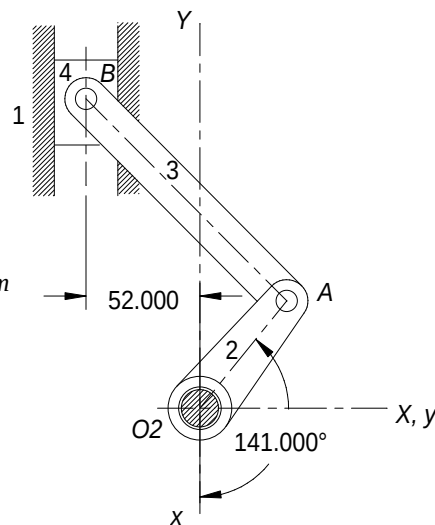
In the global coordinate system, $\theta_{VA} := \arg(\mathbf{V}_A) - 90 \cdot \text{deg} \quad \theta_{VA} = -39.000 \text{ deg}$

5. Determine the velocity of pin B using equation 6.22b:

$$V_B := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_3 \cdot \sin(\theta_3) \quad V_B = 2207.849 \frac{\text{mm}}{\text{sec}}$$

$$\mathbf{V}_B := V_B$$

In the global coordinate system, $\theta_{VB} := \arg(\mathbf{V}_B) - 90 \cdot \text{deg} \quad \theta_{VB} = -90.000 \text{ deg}$



PROBLEM 6-35

Statement: The offset crank-slider linkage in Figure P6-8f has the dimensions and crank angle given below. Find and plot $\mathbf{V}_A$ and $\mathbf{V}_B$ in the global coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 25 \text{ rad/sec CW}$.

Given: Link lengths:

Link 2 $a := 63 \cdot \text{mm}$ Offset $c := -52 \cdot \text{mm}$

Link 3 $b := 130 \cdot \text{mm}$

Input crank angular velocity $\omega_2 := -25 \cdot \text{rad} \cdot \text{sec}^{-1}$

Solution: See Figure P6-8f and Mathcad file P0635.

1. Draw the linkage to a convenient scale. The coordinate rotation angle is $\alpha := -90 \cdot \text{deg}$

2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg}$$

3. Determine θ_3 using equations 4.16 for the crossed circuit.

$$\theta_3(\theta_2) := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right)$$

4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3(\theta_2))} \cdot \omega_2$$

5. Determine the x and y components of the velocity of pin A using equation 6.23a:

$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_{Ax}(\theta_2) := \text{Re}(\mathbf{V}_A(\theta_2))$$

$$V_{Ay}(\theta_2) := \text{Im}(\mathbf{V}_A(\theta_2))$$

In the global coordinate system,

$$V_{AX}(\theta_2) := V_{Ay}(\theta_2)$$

$$V_{AY}(\theta_2) := -V_{Ax}(\theta_2)$$

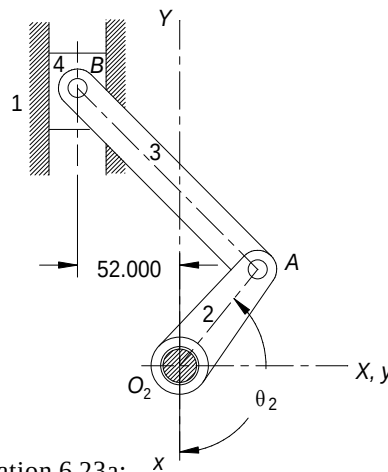
6. Determine the velocity of pin B using equation 6.22b:

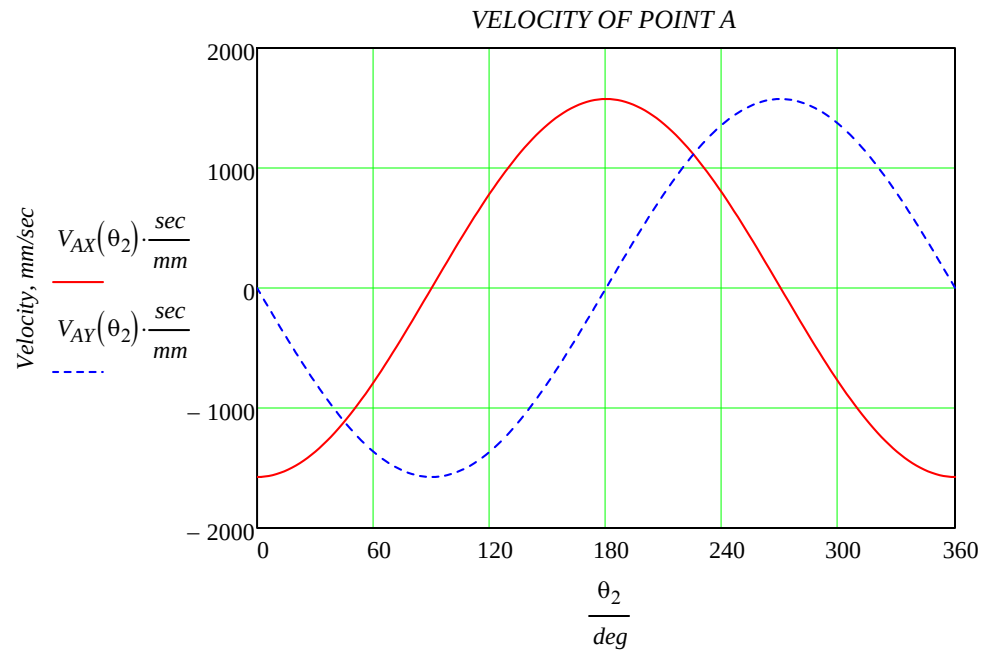
$$V_{Bx}(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_3(\theta_2) \cdot \sin(\theta_3(\theta_2))$$

In the global coordinate system,

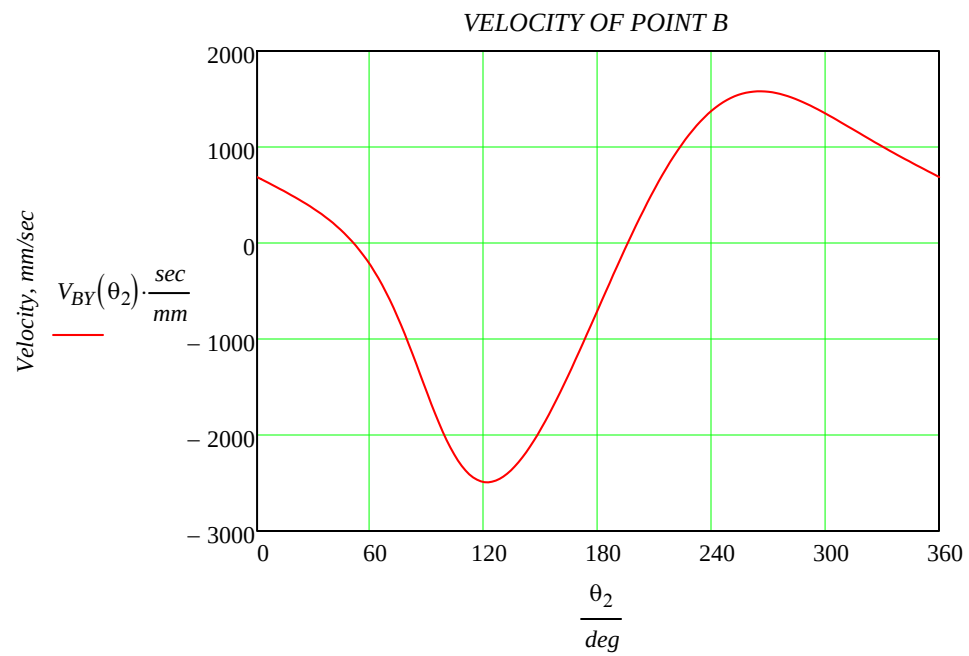
$$V_{BY}(\theta_2) := -V_{Bx}(\theta_2)$$

7. Plot the x and y components of the velocity of A. (See next page.)





7. Plot the velocity of point B.



PROBLEM 6-36

Statement: The linkage in Figure P6-8d has the dimensions and crank angle given below. Find $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_{box}$ for the position shown for $\omega_2 = 30 \text{ rad/sec}$ clockwise (CW). Use the velocity difference graphical method.

Given: Link lengths:

Link 2 $a := 30 \cdot \text{mm}$

Link 3 $b := 150 \cdot \text{mm}$

Link 4 $c := 30 \cdot \text{mm}$

Link 1 $d := 150 \cdot \text{mm}$

Crank angle: $\theta_2 := 58 \cdot \text{deg}$ Global XY system

Input crank angular velocity $\omega_2 := 30 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

Solution: See Figure P6-8d and Mathcad file P0636.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.
2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

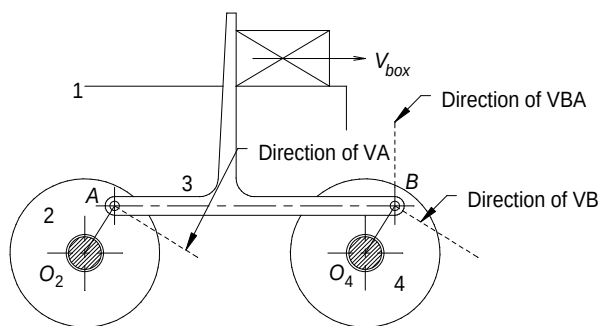
$$V_A := a \cdot \omega_2 \quad V_A = 900.000 \frac{\text{mm}}{\text{sec}}$$

$$\theta_A := 58 \cdot \text{deg} - 90 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.

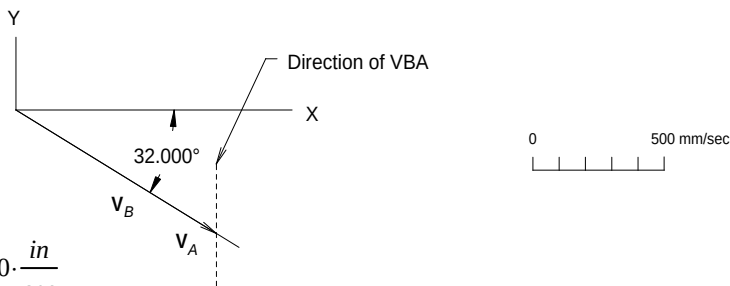


4. From the velocity triangle we have:

$$V_B := V_A$$

$$V_B = 900.000 \frac{\text{mm}}{\text{sec}}$$

$$\theta_B := -32 \cdot \text{deg} \quad V_{BA} := 0 \cdot \frac{\text{in}}{\text{sec}}$$



5. Determine the angular velocity of links 3 and 4 using equation 6.7.

$$\omega_3 := \frac{V_{BA}}{b}$$

$$\omega_3 = 0.000 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{-V_B}{c}$$

$$\omega_4 = -30.000 \frac{\text{rad}}{\text{sec}}$$

6. Determine the magnitude of the vector $\mathbf{V}_{box}$. This is a special case Grashof mechanism in the parallelogram configuration. Link 3 does not rotate, therefore all points on link 3 have the same velocity. The velocity $\mathbf{V}_{box}$ is the horizontal component of $\mathbf{V}_A$.

$$V_{box} := V_A \cdot \cos(\theta_A) \quad V_{box} = 763.243 \frac{\text{mm}}{\text{sec}}$$

PROBLEM 6-37

Statement: The linkage in Figure P6-8d has the dimensions and effective crank angle given below. Find $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_{box}$ in the global coordinate system for the position shown for $\omega_2 = 30 \text{ rad/sec CW}$. Use an analytical method.

Given: Link lengths:

Link 2	$a := 30 \cdot \text{mm}$	Link 3	$b := 150 \cdot \text{mm}$
Link 4	$c := 30 \cdot \text{mm}$	Link 1	$d := 150 \cdot \text{mm}$

Crank angle: $\theta_2 := 58 \cdot \text{deg}$

Input crank angular velocity $\omega_2 := -30 \cdot \text{rad} \cdot \text{sec}^{-1}$

Solution: See Figure P6-8d and Mathcad file P0637.

1. Draw the linkage to scale and label it.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 5.0000 \quad K_2 = 5.0000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -6.1197 \quad B = -1.6961 \quad C = 2.8205$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_{41} = -302.000 \text{ deg}$$

$$\theta_{41} := \theta_{41} - 360 \cdot \text{deg} \quad \theta_{41} = -662.000 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0000 \quad K_5 = -5.0000$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -8.9402$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.6961$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.0000$$

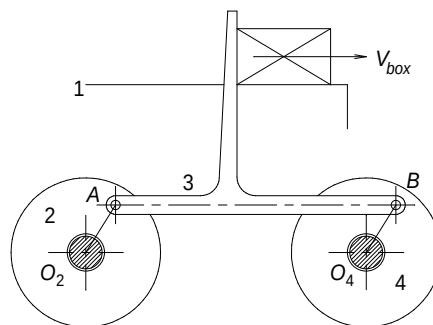
5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{31} = 360.000 \text{ deg}$$

$$\theta_{31} := \theta_{31} - 360 \cdot \text{deg} \quad \theta_{31} = 0.000 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41} - \theta_2)}{\sin(\theta_{31} - \theta_{41})} \quad \omega_{31} = 0.000 \frac{\text{rad}}{\text{sec}}$$



$$\omega_{41} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31})}{\sin(\theta_{41} - \theta_{31})} \qquad \omega_{41} = -30.000 \frac{rad}{sec}$$

7. Determine the velocity of points A and B for the open circuit using equations 6.19.

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = (763.243 - 476.927j) \frac{mm}{sec} \quad |\mathbf{V}_A| = 900.000 \frac{mm}{sec} \quad arg(\mathbf{V}_A) = -32.000 \deg$$

$$\mathbf{V}_B := c \cdot \omega_{41} \cdot (-\sin(\theta_{41}) + j \cdot \cos(\theta_{41}))$$

$$\mathbf{V}_B = (763.243 - 476.927j) \frac{mm}{sec} \quad |\mathbf{V}_B| = 900.000 \frac{mm}{sec} \quad arg(\mathbf{V}_B) = -32.000 \deg$$

8. Determine the velocity V_{box} . Since link 3 does not rotate (this is a special case Grashof linkage in the parallelogram mode), all points on it have the same velocity. Therefore,

$$V_{box} := |\mathbf{V}_A| \qquad V_{box} = 900.000 \frac{mm}{sec}$$

PROBLEM 6-38

Statement: The linkage in Figure P6-8d has the dimensions and effective crank angle given below. Find and plot $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_{box}$ in the global coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 30 \text{ rad/sec CW}$.

Given: Link lengths:

Link 2	$a := 30 \cdot \text{mm}$	Link 3	$b := 150 \cdot \text{mm}$
Link 4	$c := 30 \cdot \text{mm}$	Link 1	$d := 150 \cdot \text{mm}$

Input crank angular velocity $\omega_2 := -30 \cdot \text{rad} \cdot \text{sec}^{-1}$

Solution: See Figure P6-8d and Mathcad file P0638.

1. Draw the linkage to scale and label it.
2. Determine the range of motion for this special-case Grashof double crank.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 5.0000 \quad K_2 = 5.0000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.0000$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0000 \quad K_5 = -5.0000$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

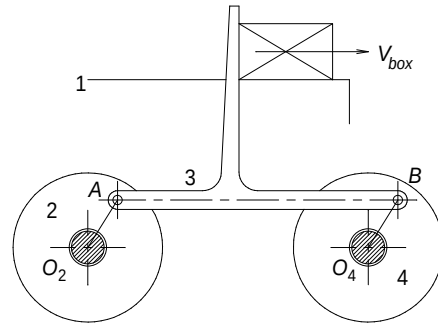
6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$



8. Determine the velocity of points A and B for the open circuit using equations 6.19.

$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_A(\theta_2) := |\mathbf{V}_A(\theta_2)|$$

$$\mathbf{V}_B(\theta_2) := c \cdot \omega_{41}(\theta_2) \cdot (-\sin(\theta_{41}(\theta_2)) + j \cdot \cos(\theta_{41}(\theta_2)))$$

$$V_B(\theta_2) := |\mathbf{V}_B(\theta_2)|$$

9. Plot the angular velocity of the output link, ω_4 , and the magnitudes of the velocities at points A and B .

Since this is a special-case Grashof linkage in the parallelogram configuration, $\omega_3 = 0$ and $\omega_4 = \omega_2$ for all values of θ_2 . Similarly, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_{box}$ all have the same constant magnitude through all values of θ_2 .

$$\omega_{41}(5 \cdot \text{deg}) = -30.000 \frac{\text{rad}}{\text{sec}}$$

$$\omega_{41}(135 \cdot \text{deg}) = -30.000 \frac{\text{rad}}{\text{sec}}$$

$$V_A(5 \cdot \text{deg}) = 900.000 \frac{\text{mm}}{\text{sec}}$$

$$V_A(135 \cdot \text{deg}) = 900.000 \frac{\text{mm}}{\text{sec}}$$

$$V_B(5 \cdot \text{deg}) = 900.000 \frac{\text{mm}}{\text{sec}}$$

$$V_B(135 \cdot \text{deg}) = 900.000 \frac{\text{mm}}{\text{sec}}$$

PROBLEM 6-39

Statement: The linkage in Figure P6-8g has the dimensions and crank angle given below. Find ω_4 , $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW). Use the velocity difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49\text{-mm}$	Link 2 (O_2 to C)	$a' := 49\text{-mm}$
Link 3 (A to B)	$b := 100\text{-mm}$	Link 5 (C to D)	$b' := 100\text{-mm}$
Link 4 (B to O_4)	$c := 153\text{-mm}$	Link 6 (D to O_6)	$c' := 153\text{-mm}$
Link 1 (O_2 to O_4)	$d := 87\text{-mm}$	Link 1 (O_2 to O_6)	$d' := 87\text{-mm}$

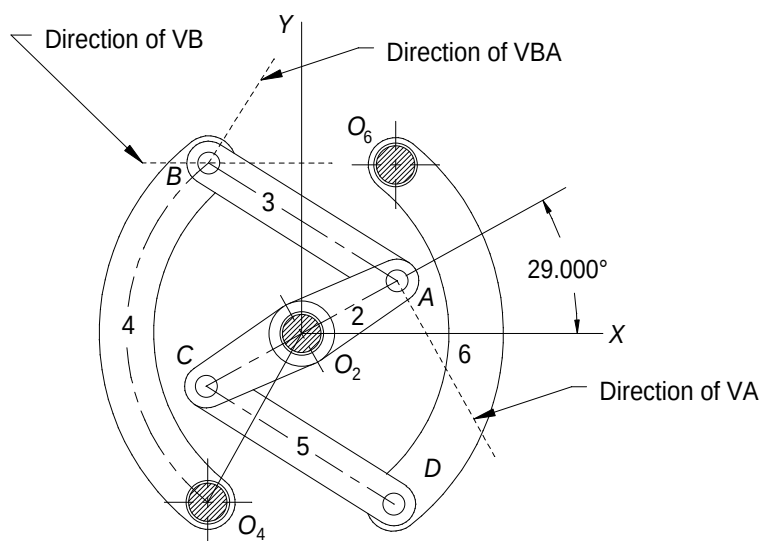
Crank angle: $\theta_2 := 29\text{-deg}$ Global XY system

Input crank angular velocity $\omega_2 := 15\text{-rad}\cdot\text{sec}^{-1}$ CW

Coordinate rotation angle $\alpha := 119\text{-deg}$ Global XY system to local xy system

Solution: See Figure P6-8g and Mathcad file P0639.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



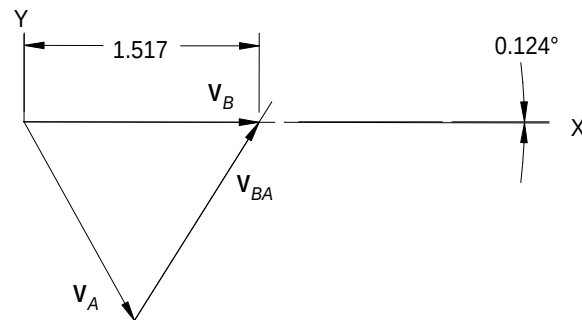
2. Use equation 6.7 to calculate the magnitude of the velocity at point B.

$$V_B := a \cdot \omega_2 \qquad V_B = 735.0 \frac{\text{mm}}{\text{sec}} \qquad \theta_B := 29\text{-deg} - 90\text{-deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point A, the magnitude of the relative velocity $\mathbf{V}_{AB}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_A = \mathbf{V}_B + \mathbf{V}_{AB}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
- b. From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{AB}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_A$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{AB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_A$ construction line and drawing $\mathbf{V}_A$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{AB}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{500 \cdot \text{mm} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_A := 1.517 \cdot \text{in} \cdot k_v \quad V_A = 758.5 \frac{\text{mm}}{\text{sec}} \quad \theta_A := -0.124 \cdot \text{deg}$$

5. Determine the angular velocity of link 4 using equation 6.7.

$$\omega_4 := \frac{-V_A}{c} \quad \omega_4 = -4.958 \frac{\text{rad}}{\text{sec}}$$

PROBLEM 6-40

Statement: The linkage in Figure P6-8g has the dimensions and crank angle given below. Find ω_4 , V_A , and V_B for the position shown for $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW). Use the instant center graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49\text{-mm}$	Link 2 (O_2 to C)	$a' := 49\text{-mm}$
Link 3 (A to B)	$b := 100\text{-mm}$	Link 5 (C to D)	$b' := 100\text{-mm}$
Link 4 (B to O_4)	$c := 153\text{-mm}$	Link 6 (D to O_6)	$c' := 153\text{-mm}$
Link 1 (O_2 to O_4)	$d := 87\text{-mm}$	Link 1 (O_2 to O_6)	$d' := 87\text{-mm}$

Crank angle: $\theta_2 := 29\text{-deg}$ Global XY system

Input crank angular velocity $\omega_2 := 15\text{-rad}\cdot\text{sec}^{-1}$ CW

Coordinate rotation angle $\alpha := 119\text{-deg}$ Global XY system to local xy system

Solution: See Figure P6-8g and Mathcad file P0640.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.

From the layout:

$$AI13 := 97.094\text{-mm} \quad BI13 := 100.224\text{-mm}$$

$$\theta_4 := 89.876\text{-deg}$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 735.0 \frac{\text{mm}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 - 90\text{-deg} \quad \theta_{VA} = -61.0\text{-deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \quad \omega_3 = 7.570 \frac{\text{rad}}{\text{sec}} \text{ CW}$$

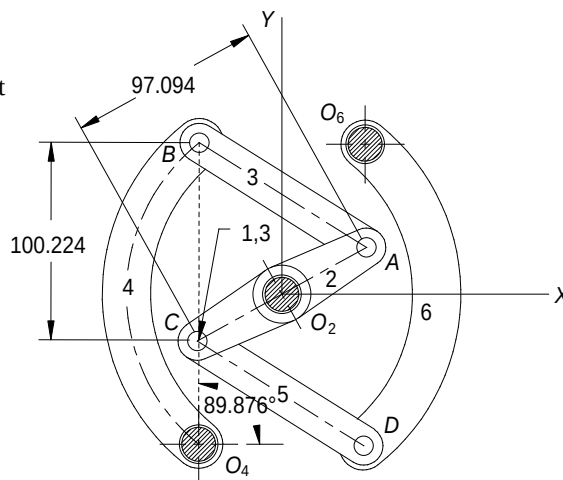
4. Determine the magnitude of the velocity at point B using equation 6.9b. Determine its direction by inspection.

$$V_B := BI13 \cdot \omega_3 \quad V_B = 758.694 \frac{\text{mm}}{\text{sec}}$$

$$\theta_{VB} := \theta_4 - 90\text{-deg} \quad \theta_{VB} = -0.124\text{-deg}$$

5. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \quad \omega_4 = 4.959 \frac{\text{rad}}{\text{sec}} \text{ CW}$$



PROBLEM 6-41

Statement: The linkage in Figure P6-8g has the dimensions and crank angle given below. Find ω_4 , $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW). Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49 \cdot \text{mm}$	Link 2 (O_2 to C)	$a' := 49 \cdot \text{mm}$
Link 3 (A to B)	$b := 100 \cdot \text{mm}$	Link 5 (C to D)	$b' := 100 \cdot \text{mm}$
Link 4 (B to O_4)	$c := 153 \cdot \text{mm}$	Link 6 (D to O_6)	$c' := 153 \cdot \text{mm}$
Link 1 (O_2 to O_4)	$d := 87 \cdot \text{mm}$	Link 1 (O_2 to O_6)	$d' := 87 \cdot \text{mm}$

Crank angle: $\theta_2 := 148 \cdot \text{deg}$ Local xy system

Input crank angular velocity $\omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1}$

Solution: See Figure P6-8g and Mathcad file P0641.

1. Draw the linkage to scale and label it.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 1.7755 \quad K_2 = 0.5686$$

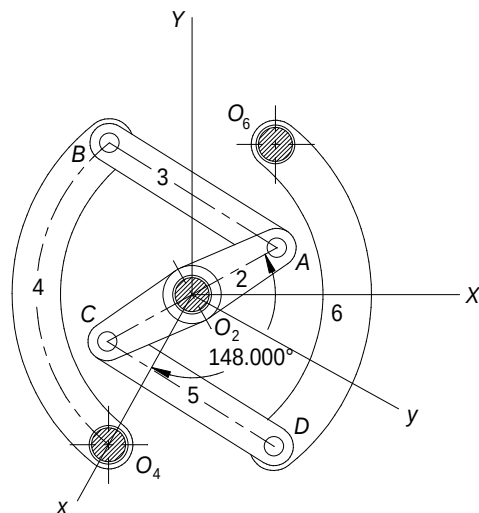
$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.5592$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.5821 \quad B = -1.0598 \quad C = 4.6650$$



3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{42} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_{42} = 208.876 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.8700 \quad K_5 = 0.3509$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -3.0104$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.0598$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 2.2367$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_{32} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{32} = 266.892 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{42} - \theta_2)}{\sin(\theta_{32} - \theta_{42})} \quad \omega_{32} = -7.570 \frac{\text{rad}}{\text{sec}}$$

$$\omega_{42} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{32})}{\sin(\theta_{42} - \theta_{32})} \qquad \omega_{42} = -4.959 \frac{rad}{sec}$$

7. Determine the velocity of points *B* and *A* for the crossed circuit using equations 6.19.

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = (389.491 + 623.315j) \frac{mm}{sec} \quad |\mathbf{V}_A| = 735.000 \frac{mm}{sec} \quad arg(\mathbf{V}_A) = 58.000 \deg$$

$$\mathbf{V}_B := c \cdot \omega_{42} \cdot (-\sin(\theta_{42}) + j \cdot \cos(\theta_{42}))$$

$$\mathbf{V}_B = (-366.389 + 664.362j) \frac{mm}{sec} \quad |\mathbf{V}_B| = 758.694 \frac{mm}{sec} \quad arg(\mathbf{V}_B) = 118.876 \deg$$

PROBLEM 6-42

Statement: The linkage in Figure P6-8g has the dimensions and crank angle given below. Find and plot ω_4 , $\mathbf{V}_A$, and $\mathbf{V}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW).

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49 \cdot \text{mm}$	Link 2 (O_2 to C)	$a' := 49 \cdot \text{mm}$
Link 3 (A to B)	$b := 100 \cdot \text{mm}$	Link 5 (C to D)	$b' := 100 \cdot \text{mm}$
Link 4 (B to O_4)	$c := 153 \cdot \text{mm}$	Link 6 (D to O_6)	$c' := 153 \cdot \text{mm}$
Link 1 (O_2 to O_4)	$d := 87 \cdot \text{mm}$	Link 1 (O_2 to O_6)	$d' := 87 \cdot \text{mm}$

Input crank angular velocity $\omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1}$

Solution: See Figure P6-8g and Mathcad file P0642.

1. Draw the linkage to scale and label it.
2. Determine the range of motion for this non-Grashof triple rocker using equations 4.37.

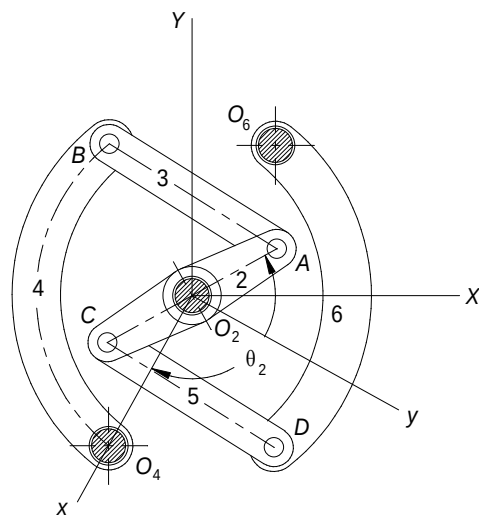
$$\arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad \arg_1 = 0.840$$

$$\arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad \arg_2 = -6.338$$

$$\theta_{2\text{toggle}} := \arccos(\arg_1) \quad \theta_{2\text{toggle}} = 32.9 \text{ deg}$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := \theta_{2\text{toggle}} + 1 \cdot \text{deg}, \theta_{2\text{toggle}} + 2 \cdot \text{deg} \dots 359 \cdot \text{deg} - \theta_{2\text{toggle}}$$



3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 1.7755 \quad K_2 = 0.5686 \quad K_3 = 1.5592$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{42}(\theta_2) := 2 \cdot \left(\operatorname{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.8700 \quad K_5 = 0.3509$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_{32}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{42}(\theta_2) - \theta_2)}{\sin(\theta_{32}(\theta_2) - \theta_{42}(\theta_2))}$$

$$\omega_{42}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{32}(\theta_2))}{\sin(\theta_{42}(\theta_2) - \theta_{32}(\theta_2))}$$

8. Determine the velocity of points A and B for the crossed circuit using equations 6.19.

$$\mathbf{V}_B(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_{Bx}(\theta_2) := \text{Re}(\mathbf{V}_B(\theta_2))$$

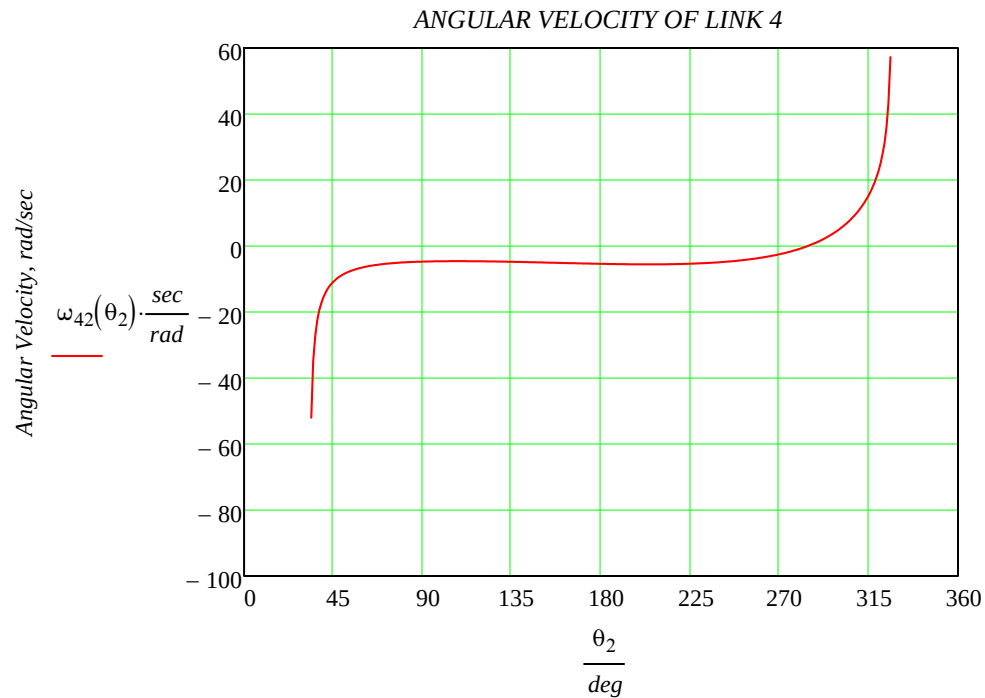
$$V_{By}(\theta_2) := \text{Im}(\mathbf{V}_B(\theta_2))$$

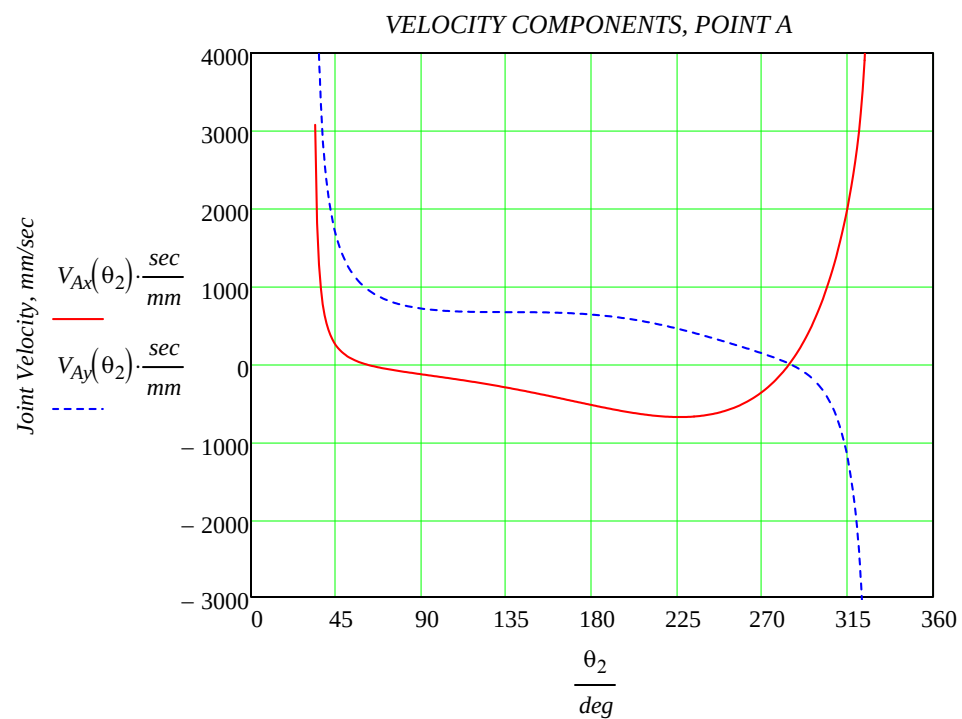
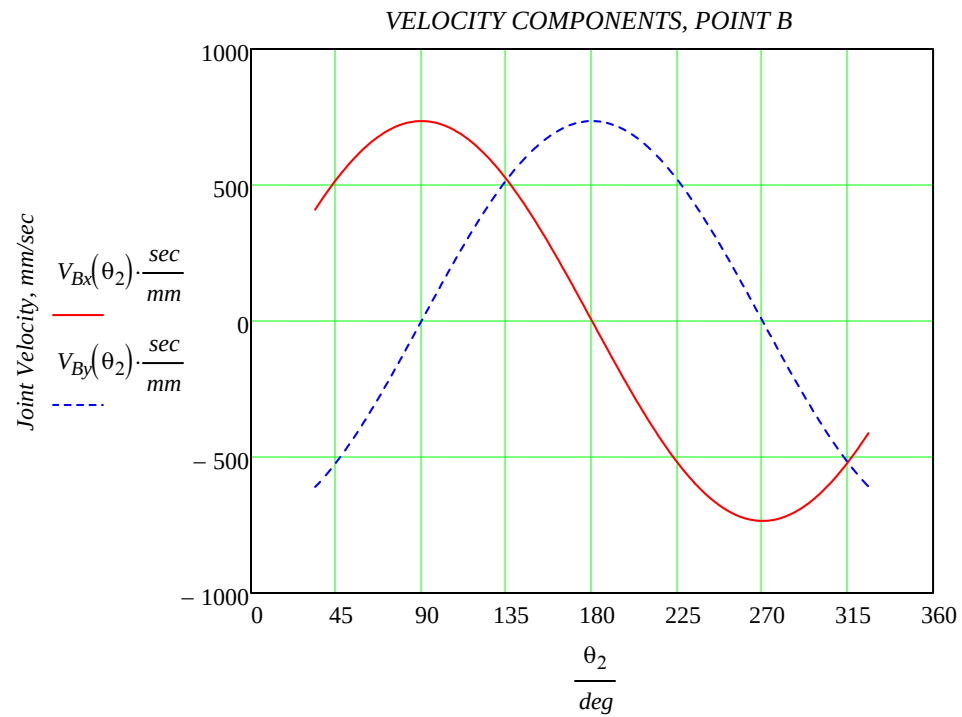
$$\mathbf{V}_A(\theta_2) := c \cdot \omega_{42}(\theta_2) \cdot (-\sin(\theta_{42}(\theta_2)) + j \cdot \cos(\theta_{42}(\theta_2)))$$

$$V_{Ax}(\theta_2) := \text{Re}(\mathbf{V}_A(\theta_2))$$

$$V_{Ay}(\theta_2) := \text{Im}(\mathbf{V}_A(\theta_2))$$

9. Plot the angular velocity of the output link, ω_4 , and the x and y components of the velocities at points B and A.





PROBLEM 6-43

Statement: The 3-cylinder radial compressor in Figure P6-8c has the dimensions and crank angle given below. Find piston velocities V_6 , V_7 , and V_8 for the position shown for $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW). Use the velocity difference graphical method.

Given: Link lengths:

Link 2 $a := 19\text{-mm}$ Offset $c := 0\text{-mm}$

Links 3, 4, and 5 $b := 70\text{-mm}$

Crank angle: $\theta_2 := -53\text{-deg}$ Global XY system

Input crank angular velocity $\omega_2 := 15\text{-rad}\cdot\text{sec}^{-1}$ CW

Cylinder angular spacing $\alpha := 120\text{-deg}$

Solution: See Figure P6-8c and Mathcad file P0643.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.
2. Use equation 6.7 to calculate the magnitude of the velocity at rod pin on link 2.

$$V_2 := a \cdot \omega_2 \quad V_2 = 285.0 \frac{\text{mm}}{\text{sec}}$$

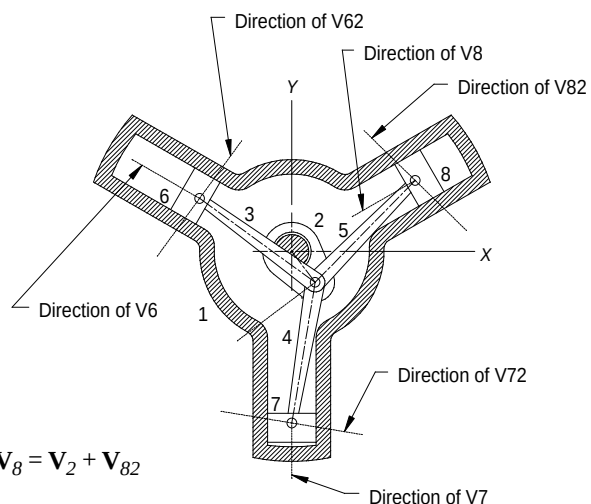
$$\theta_2 := -53\text{-deg} - 90\text{-deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at pistons 6, 7, and 8. The equations to be solved graphically are

$$\mathbf{V}_7 = \mathbf{V}_2 + \mathbf{V}_{72}$$

$$\mathbf{V}_6 = \mathbf{V}_2 + \mathbf{V}_{62}$$

$$\mathbf{V}_8 = \mathbf{V}_2 + \mathbf{V}_{82}$$



- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_2$.
- b. From the tip of $\mathbf{V}_2$, draw a construction line with the direction of $\mathbf{V}_{72}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_2$, draw a construction line with the direction of $\mathbf{V}_7$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{72}$ from the tip of $\mathbf{V}_2$ to the intersection of the $\mathbf{V}_7$ construction line and drawing $\mathbf{V}_7$ from the tail of $\mathbf{V}_2$ to the intersection of the $\mathbf{V}_{72}$ construction line.
- e. Repeat for $\mathbf{V}_6$ and $\mathbf{V}_8$.

4. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{200\text{-mm}\cdot\text{sec}^{-1}}{\text{in}}$$

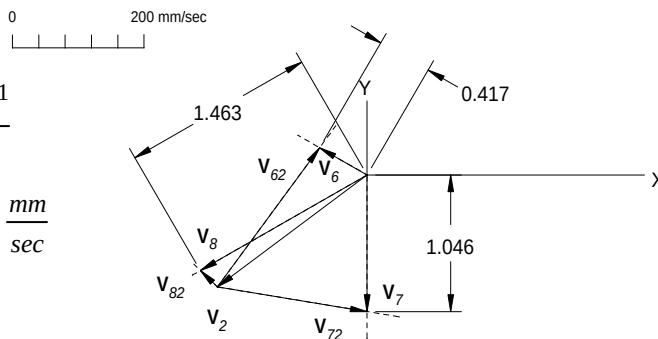
$$V_7 := 1.046\text{-in}\cdot k_v \quad V_7 = 209.2 \frac{\text{mm}}{\text{sec}}$$

$$\theta_{V7} := 270\text{-deg}$$

$$V_6 := 0.417\text{-in}\cdot k_v \quad V_6 = 83.4 \frac{\text{mm}}{\text{sec}}$$

$$\theta_{V6} := 150\text{-deg}$$

$$V_8 := 1.463\text{-in}\cdot k_v \quad V_8 = 292.6 \frac{\text{mm}}{\text{sec}} \quad \theta_{V8} := 210\text{-deg}$$



PROBLEM 6-44

Statement: The 3-cylinder radial compressor in Figure P6-8c has the dimensions and crank angle given below. Find $\mathbf{V}_6$, $\mathbf{V}_7$, and $\mathbf{V}_8$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW). Use an analytical method.

Given:

Link lengths:

Link 2 $a := 19 \text{ mm}$

Offset

$c := 0 \text{ mm}$

Links 3, 4, and 5 $b := 70 \text{ mm}$

Crank angle:

$\theta_2 := 37 \text{ deg}$

Input crank angular velocity $\omega_2 := -15 \text{ rad} \cdot \text{sec}^{-1}$ CW

Local $x'y'$ system

Cylinder angular spacing $\alpha := 120 \text{ deg}$

Solution: See Figure P6-8c and Mathcad file P0644.

1. Draw the linkage to scale and label it.

2. Determine θ_4 and d' using equation 4.17.

$$\theta_4 := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi$$

$$\theta_4 = 170.599 \text{ deg} \quad (x'y' \text{ system})$$

$$d' := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_4) \quad d' = 3.316 \text{ in}$$

3. Determine the angular velocity of link 4 using equation 6.22a:

$$\omega_4 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_4)} \cdot \omega_2 \quad \omega_4 = 3.296 \frac{\text{rad}}{\text{sec}}$$

4. Determine the velocity of the rod pin on link 2 using equation 6.23a:

$$\mathbf{V}_2 := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_2 = (171.517 - 227.611j) \frac{\text{mm}}{\text{sec}} \quad |\mathbf{V}_2| = 285.000 \frac{\text{mm}}{\text{sec}} \quad \arg(\mathbf{V}_2) = -53.000 \text{ deg}$$

$$\text{In the global coordinate system,} \quad \theta_{V2} := \arg(\mathbf{V}_2) - 90 \text{ deg} \quad \theta_{V2} = -143.000 \text{ deg}$$

5. Determine the velocity of piston 7 using equation 6.22b:

$$\mathbf{V}_7 := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_4 \cdot \sin(\theta_4)$$

$$\mathbf{V}_7 = 209.204 \frac{\text{mm}}{\text{sec}} \quad |\mathbf{V}_7| = 209.204 \frac{\text{mm}}{\text{sec}} \quad \arg(\mathbf{V}_7) = 0.000 \text{ deg}$$

$$\text{In the global coordinate system,} \quad \theta_{V7} := \arg(\mathbf{V}_7) - 90 \text{ deg} \quad \theta_{V7} = -90.000 \text{ deg}$$

6. Determine θ_3 and d'' using equation 4.17.

$$\theta_2 := \theta_2 + 120 \text{ deg}$$

$$\theta_2 = 157.000 \text{ deg} \quad (x''y'' \text{ system})$$

$$\theta_3 := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi$$

$$\theta_3 = 173.912 \text{ deg}$$

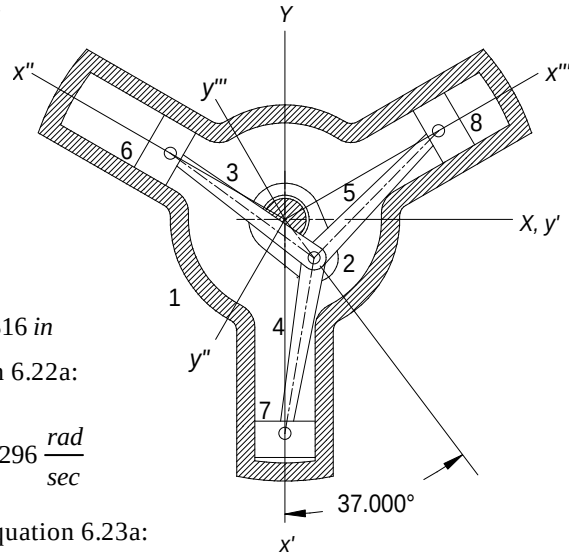
$$d'' := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_3)$$

$$d'' = 2.052 \text{ in}$$

7. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_3 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3)} \cdot \omega_2$$

$$\omega_3 = -3.769 \frac{\text{rad}}{\text{sec}}$$



8. Determine the velocity of the rod pin on link 2 using equation 6.23a:

$$\mathbf{V}_2 := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_2 = (111.358 + 262.344i) \frac{\text{mm}}{\text{sec}} \quad |\mathbf{V}_2| = 285.000 \frac{\text{mm}}{\text{sec}} \quad \arg(\mathbf{V}_2) = 67.000 \text{ deg}$$

$$\text{In the global coordinate system,} \quad \theta_{V2} := \arg(\mathbf{V}_2) + 150 \cdot \text{deg} \quad \theta_{V2} = 217.000 \text{ deg}$$

9. Determine the velocity of piston 6 using equation 6.22b:

$$\mathbf{V}_6 := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_3 \cdot \sin(\theta_3)$$

$$\mathbf{V}_6 = 83.378 \frac{\text{mm}}{\text{sec}} \quad |\mathbf{V}_6| = 83.378 \frac{\text{mm}}{\text{sec}} \quad \arg(\mathbf{V}_6) = 0.000 \text{ deg}$$

$$\text{In the global coordinate system,} \quad \theta_{V6} := \arg(\mathbf{V}_6) + 150 \cdot \text{deg} \quad \theta_{V6} = 150.000 \text{ deg}$$

10. Determine θ_5 and d''' using equation 4.17.

$$\theta_2 := \theta_2 + 120 \cdot \text{deg} \quad \theta_2 = 277.000 \text{ deg} \quad (x'''y''' \text{ system})$$

$$\theta_5 := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi \quad \theta_5 = 195.629 \text{ deg}$$

$$d''' := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_5) \quad d''' = 2.745 \text{ in}$$

11. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_5)} \cdot \omega_2 \quad \omega_5 = 0.515 \frac{\text{rad}}{\text{sec}}$$

12. Determine the velocity of the rod pin on link 2 using equation 6.23a:

$$\mathbf{V}_2 := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_2 = (-282.876 - 34.733i) \frac{\text{mm}}{\text{sec}} \quad |\mathbf{V}_2| = 285.000 \frac{\text{mm}}{\text{sec}} \quad \arg(\mathbf{V}_2) = -173.000 \text{ deg}$$

$$\text{In the global coordinate system,} \quad \theta_{V2} := \arg(\mathbf{V}_2) + 30 \cdot \text{deg} \quad \theta_{V2} = -143.000 \text{ deg}$$

13. Determine the velocity of piston 8 using equation 6.22b:

$$\mathbf{V}_8 := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_5 \cdot \sin(\theta_5)$$

$$\mathbf{V}_8 = -292.592 \frac{\text{mm}}{\text{sec}} \quad |\mathbf{V}_8| = 292.592 \frac{\text{mm}}{\text{sec}} \quad \arg(\mathbf{V}_8) = 180.000 \text{ deg}$$

$$\text{In the global coordinate system,} \quad \theta_{V8} := \arg(\mathbf{V}_8) + 30 \cdot \text{deg} \quad \theta_{V8} = 210.000 \text{ deg}$$

PROBLEM 6-45

Statement: The 3-cylinder radial compressor in Figure P6-8c has the dimensions and crank angle given below. Find and plot $\mathbf{V}_6$, $\mathbf{V}_7$, and $\mathbf{V}_8$ for one revolution of the crank if $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW).

Given: Link lengths:

Link 2 $a := 19 \cdot \text{mm}$ Offset $c := 0 \cdot \text{mm}$

Links 3, 4, and 5 $b := 70 \cdot \text{mm}$

Input crank angular velocity $\omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

Cylinder angular spacing $\alpha := 120 \cdot \text{deg}$

Solution: See Figure P6-8c and Mathcad file P0645.

1. Draw the linkage to scale and label it. Note that there are three local coordinate systems.
2. Determine the range of motion for this slider-crank linkage. This will be the same in each coordinate frame.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

3. Determine θ_4 using equation 4.17.

$$\theta_4(\theta_2) := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi \quad (x'y' \text{ system})$$

4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_4(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_4(\theta_2))} \cdot \omega_2$$

5. Determine the velocity of piston 7 using equation 6.22b:

$$V_7(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_4(\theta_2) \cdot \sin(\theta_4(\theta_2))$$

6. Determine θ_3 using equation 4.17.

$$\theta_3(\theta_2) := \text{asin}\left(-\frac{a \cdot \sin(\theta_2 + \alpha) - c}{b}\right) + \pi \quad (x''y'' \text{ system})$$

7. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2 + \alpha)}{\cos(\theta_3(\theta_2))} \cdot \omega_2$$

8. Determine the velocity of piston 6 using equation 6.22b:

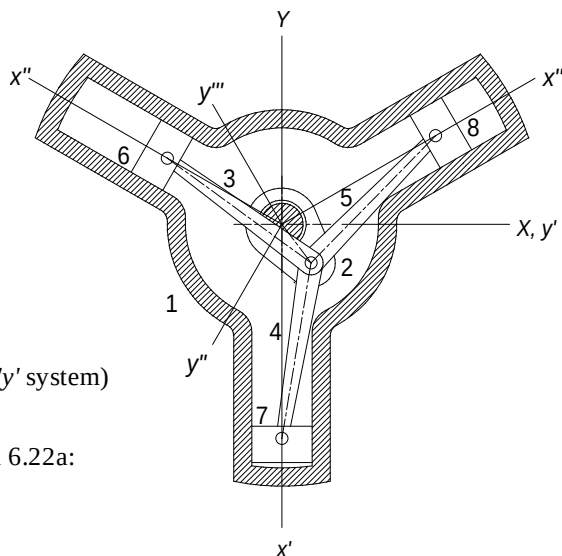
$$V_6(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2 + \alpha) + b \cdot \omega_3(\theta_2) \cdot \sin(\theta_3(\theta_2))$$

9. Determine θ_5 and d''' using equation 4.17.

$$\theta_5(\theta_2) := \text{asin}\left(-\frac{a \cdot \sin(\theta_2 + 2 \cdot \alpha) - c}{b}\right) + \pi \quad (x'''y''' \text{ system})$$

10. Determine the angular velocity of link 5 using equation 6.22a:

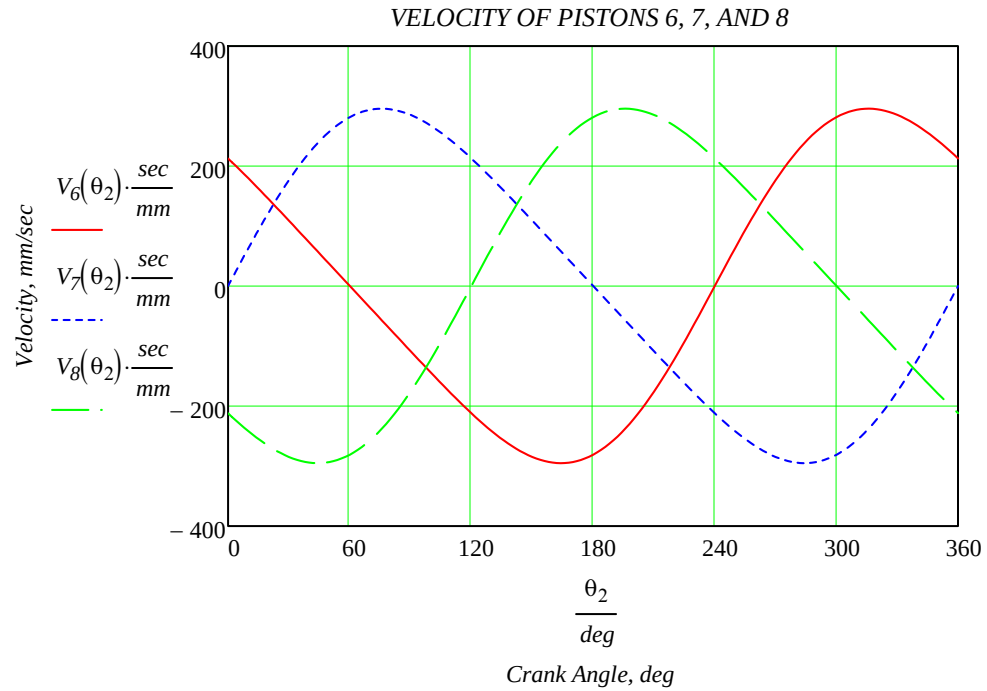
$$\omega_5(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2 + 2 \cdot \alpha)}{\cos(\theta_5(\theta_2))} \cdot \omega_2$$



11. Determine the velocity of piston 8 using equation 6.22b:

$$V_8(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2 + 2 \cdot \alpha) + b \cdot \omega_5(\theta_2) \cdot \sin(\theta_5(\theta_2))$$

12. Plot the velocities of pistons 6, 7, and 8.



PROBLEM 6-46

Statement: Figure P6-9 shows a linkage in one position. Find the instantaneous velocities of points A, B, and P if link O_2A is rotating CW at 40 rad/sec.

Given: Link lengths:

Link 2 $a := 5.00 \cdot \text{in}$

Link 3 $b := 4.40 \cdot \text{in}$

Link 4 $c := 5.00 \cdot \text{in}$

Link 1 $d := 9.50 \cdot \text{in}$

Coupler point: $R_{pa} := 8.90 \cdot \text{in}$

$\delta_3 := 56 \cdot \text{deg}$

Crank angle and speed: $\theta_2 := 50 \cdot \text{deg}$

$\omega_2 := -40 \cdot \text{rad} \cdot \text{sec}^{-1}$

Solution: See Figure P6-9 and Mathcad file P0646.

1. Draw the linkage to scale and label it. All calculated angles are in the local xy system.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 1.9000 \quad K_2 = 1.9000$$

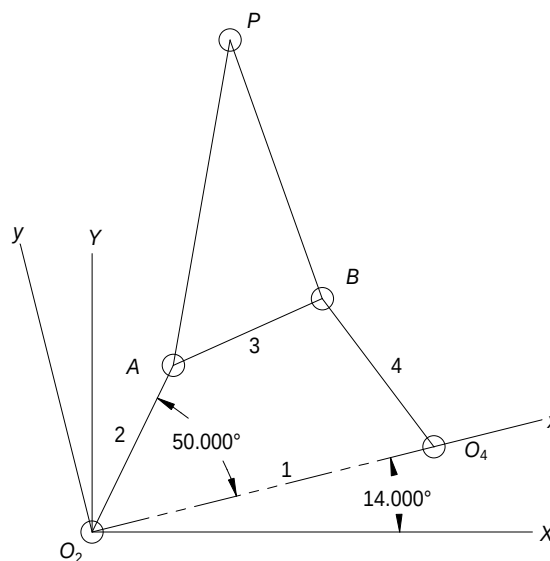
$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 2.4178$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.0607 \quad B = -1.5321 \quad C = 2.4537$$



3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_{41} = -246.992 \text{ deg}$$

$$\theta_{41} := \theta_{41} - 360 \cdot \text{deg} \quad \theta_{41} = -606.992 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 2.1591 \quad K_5 = -2.4911$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -2.3605$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.5321$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.1539$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{31} = -349.895 \text{ deg}$$

$$\theta_{31} := \theta_{31} - 360 \cdot \text{deg} \quad \theta_{31} = -709.895 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31} := \frac{a \cdot \omega_2 \cdot \sin(\theta_{41} - \theta_2)}{b \cdot \sin(\theta_{31} - \theta_{41})} \quad \omega_{31} = 41.552 \frac{\text{rad}}{\text{sec}}$$

$$\omega_{41} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31})}{\sin(\theta_{41} - \theta_{31})} \quad \omega_{41} = -26.320 \frac{\text{rad}}{\text{sec}}$$

7. Determine the velocity of points *A* and *B* for the open circuit using equations 6.19.

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = (153.209 - 128.558j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_A| = 200.000 \frac{\text{in}}{\text{sec}} \quad \arg(\mathbf{V}_A) = -40.000 \text{ deg}$$

$$\mathbf{V}_B := c \cdot \omega_{41} \cdot (-\sin(\theta_{41}) + j \cdot \cos(\theta_{41}))$$

$$\mathbf{V}_B = (121.130 + 51.437j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_B| = 131.598 \frac{\text{in}}{\text{sec}} \quad \arg(\mathbf{V}_B) = 23.008 \text{ deg}$$

8. Determine the velocity of the coupler point *P* for the open circuit using equations 6.36.

$$\mathbf{V}_{PA} := R_{pa} \cdot \omega_{31} \cdot (-\sin(\theta_{31} + \delta_3) + j \cdot \cos(\theta_{31} + \delta_3))$$

$$\mathbf{V}_{PA} = (-338.121 + 149.797j) \frac{\text{in}}{\text{sec}}$$

$$\mathbf{V}_P := \mathbf{V}_A + \mathbf{V}_{PA}$$

$$\mathbf{V}_P = (-184.912 + 21.239j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_P| = 186.128 \frac{\text{in}}{\text{sec}} \quad \arg(\mathbf{V}_P) = 173.448 \text{ deg}$$

PROBLEM 6-47

Statement: Figure P6-10 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of the coupler point P at 2-deg increments of crank angle for $\omega_2 = 100$ rpm. Check your results with program FOURBAR.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 1.00 \cdot \text{in}$$

$$\text{Link 3 (A to B)} \quad b := 2.06 \cdot \text{in}$$

$$\text{Link 4 (B to O}_4) \quad c := 2.33 \cdot \text{in}$$

$$\text{Link 1 (O}_2 \text{ to O}_4) \quad d := 2.22 \cdot \text{in}$$

$$\text{Coupler point:} \quad R_{pa} := 3.06 \cdot \text{in}$$

$$\delta_3 := -31 \cdot \text{deg}$$

$$\text{Crank speed:} \quad \omega_2 := 100 \cdot \text{rpm}$$

Solution: See Figure P6-10 and Mathcad file P0647.

1. Draw the linkage to scale and label it.
2. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 2.2200$$

$$K_2 = 0.9528$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.5265$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b}$$

$$K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 1.0777 \quad K_5 = -1.1512$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

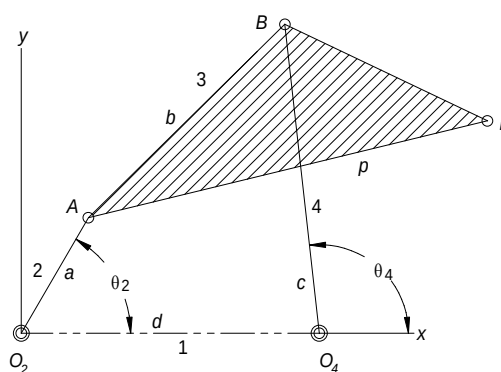
$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$



$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point A using equations 6.19.

$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) \quad V_A(\theta_2) := |\mathbf{V}_A(\theta_2)|$$

9. Determine the velocity of the coupler point P using equations 6.36.

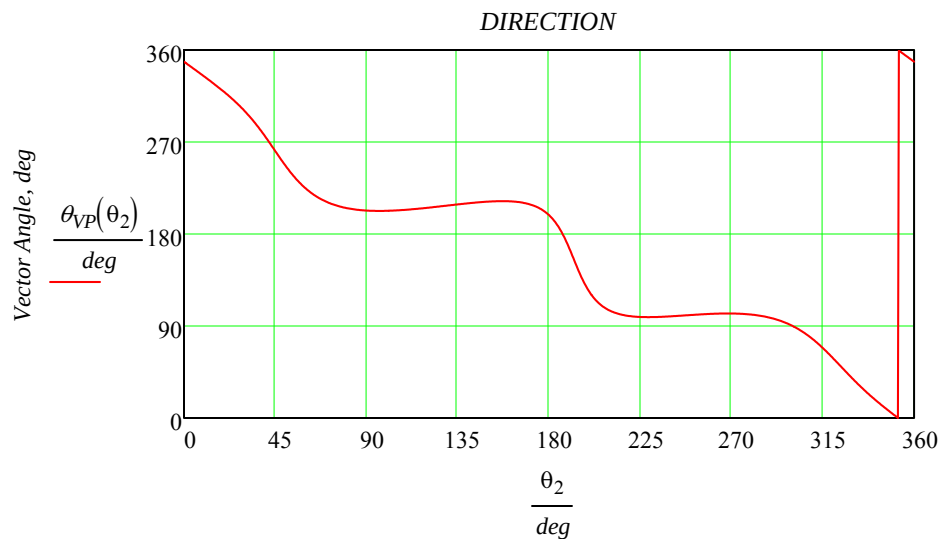
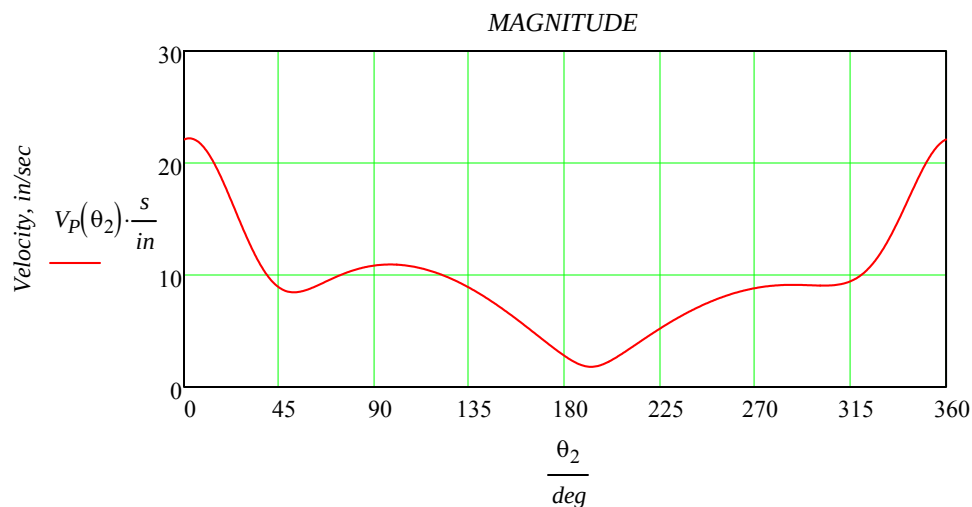
$$\mathbf{V}_{PA}(\theta_2) := R_{pa} \cdot \omega_{31}(\theta_2) \cdot (-\sin(\theta_{31}(\theta_2) + \delta_3) + j \cdot \cos(\theta_{31}(\theta_2) + \delta_3))$$

$$\mathbf{V}_P(\theta_2) := \mathbf{V}_A(\theta_2) + \mathbf{V}_{PA}(\theta_2)$$

10. Plot the magnitude and direction of the velocity at coupler point P.

$$\text{Magnitude: } V_P(\theta_2) := |\mathbf{V}_P(\theta_2)| \quad \text{Direction: } \theta_{VP}(\theta_2) := \arg(\mathbf{V}_P(\theta_2))$$

$$\theta_{VP}(\theta_2) := \text{if}(\theta_{VP1}(\theta_2) \geq 0, \theta_{VP1}(\theta_2), \theta_{VP1}(\theta_2) + 2 \cdot \pi)$$



PROBLEM 6-48

Statement: Figure P6-11 shows a linkage that operates at 500 crank rpm. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of point B at 2-deg increments of crank angle. Check your result with program FOURBAR.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 2.000 \cdot in$$

$$\text{Link 3 (A to B)} \quad b := 8.375 \cdot in$$

$$\text{Link 4 (B to O}_4) \quad c := 7.187 \cdot in$$

$$\text{Link 1 (O}_2 \text{ to O}_4) \quad d := 9.625 \cdot in$$

$$\text{Input crank angular velocity} \quad \omega_2 := 500 \cdot rpm \quad \omega_2 = 52.360 \cdot rad \cdot sec^{-1}$$

Solution: See Figure P6-11 and Mathcad file P0648.

1. Draw the linkage to scale and label it.
2. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot deg, 0.5 \cdot deg .. 360 \cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 4.8125$$

$$K_2 = 1.3392$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 2.7186$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.1493 \quad K_5 = -3.4367$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

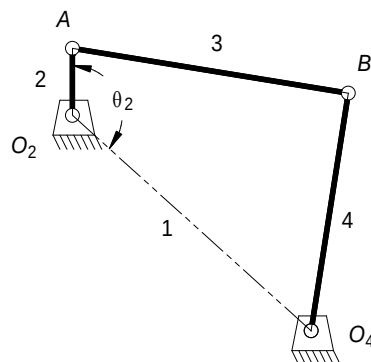
$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$



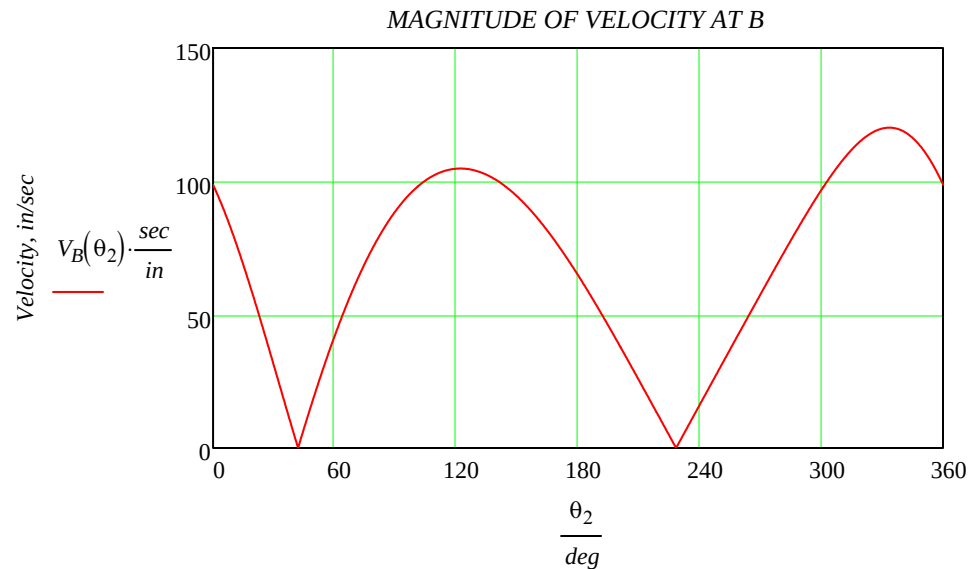
$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point B using equations 6.19.

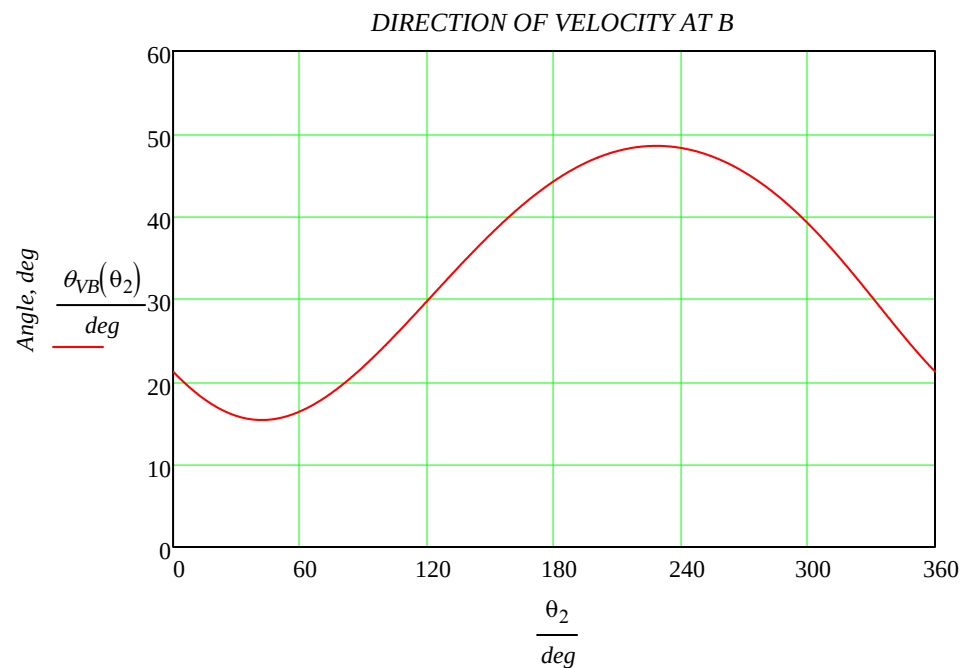
$$\mathbf{V}_B(\theta_2) := c \cdot \omega_{41}(\theta_2) \cdot (-\sin(\theta_{41}(\theta_2)) + j \cdot \cos(\theta_{41}(\theta_2)))$$

$$V_B(\theta_2) := |\mathbf{V}_B(\theta_2)| \quad \theta_{VB1}(\theta_2) := \arg(\mathbf{V}_B(\theta_2))$$

9. Plot the magnitude and angle of the velocity at point B.



$$\theta_{VB}(\theta_2) := \text{if}(\theta_{VB1}(\theta_2) < 0, \theta_{VB1}(\theta_2) + \pi, \theta_{VB1}(\theta_2))$$



PROBLEM 6-49

Statement: Figure P6-12 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of the coupler point P at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 0.785 \cdot \text{in}$$

$$\text{Link 3 (A to B)} \quad b := 0.356 \cdot \text{in}$$

$$\text{Link 4 (B to O}_4) \quad c := 0.950 \cdot \text{in}$$

$$\text{Link 1 (O}_2 \text{ to O}_4) \quad d := 0.544 \cdot \text{in}$$

$$\text{Coupler point:} \quad R_{pa} := 1.09 \cdot \text{in}$$

$$\delta_3 := 0 \cdot \text{deg}$$

$$\text{Crank speed:} \quad \omega_2 := 20 \cdot \text{rpm}$$

Solution: See Figure P6-12 and Mathcad file P0649.

1. Draw the linkage to scale and label it.
2. Using the geometry defined in Figure 3-1a in the text, determine the input crank angles (relative to the line O_2O_4) at which links 2 and 3, and 3 and 4 are in toggle.

$$\theta_{20} := \arccos \left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot a \cdot d} \right]$$

$$\theta_{20} = 158.286 \text{ deg}$$

$$\theta_2 := -\theta_{20} + 1 \cdot \text{deg}, -\theta_{20} + 2 \cdot \text{deg} \dots \theta_{20} - 1 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 0.6930$$

$$K_2 = 0.5726$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.1317$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b}$$

$$K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 1.5281$$

$$K_5 = -0.2440$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

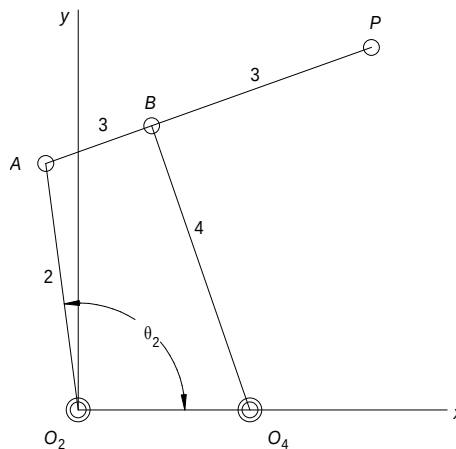
$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.



$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point A using equations 6.19.

$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

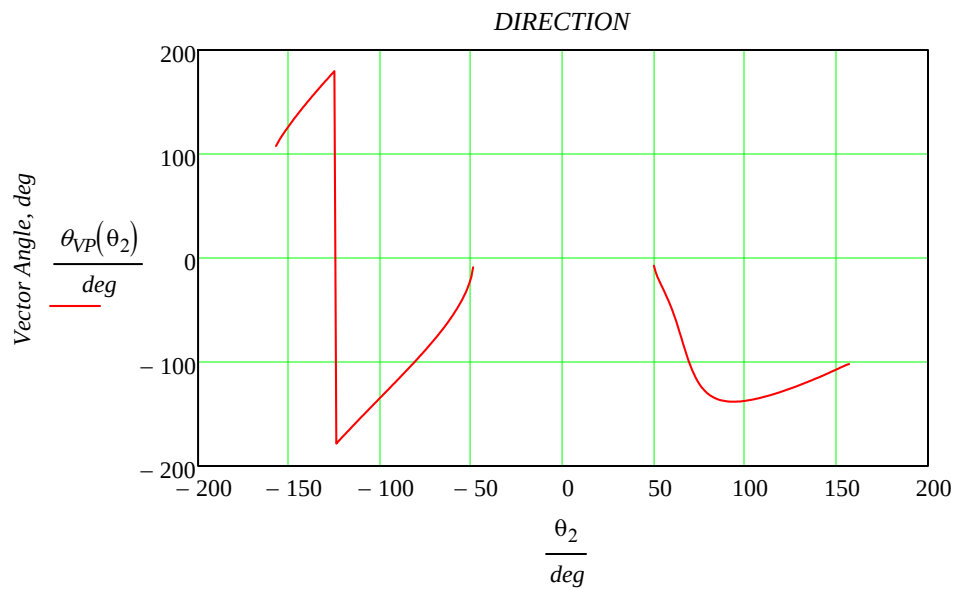
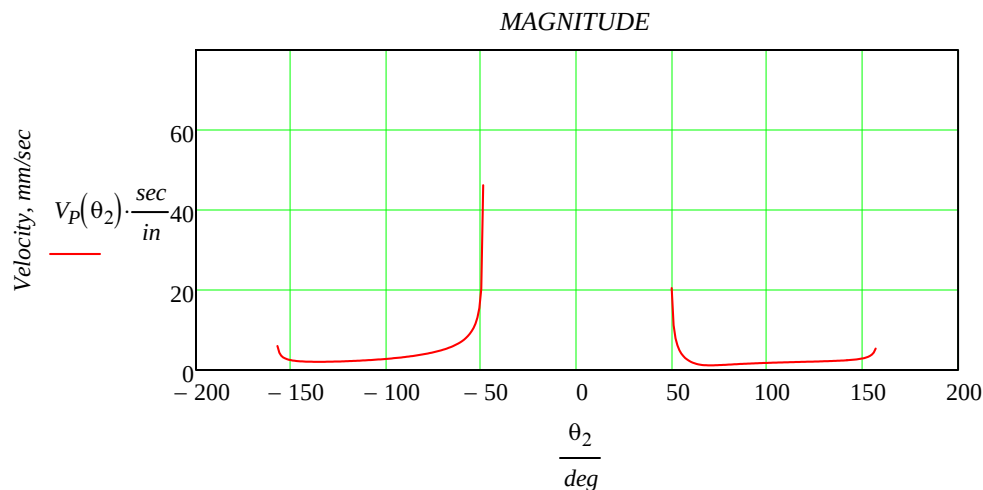
9. Determine the velocity of the coupler point P using equations 6.36.

$$\mathbf{V}_{PA}(\theta_2) := R_{pa} \cdot \omega_{31}(\theta_2) \cdot (-\sin(\theta_{31}(\theta_2) + \delta_3) + j \cdot \cos(\theta_{31}(\theta_2) + \delta_3))$$

$$\mathbf{V}_P(\theta_2) := \mathbf{V}_A(\theta_2) + \mathbf{V}_{PA}(\theta_2)$$

10. Plot the magnitude and direction of the coupler point P.

Magnitude: $V_P(\theta_2) := |\mathbf{V}_P(\theta_2)|$ Direction: $\theta_{VP}(\theta_2) := \arg(\mathbf{V}_P(\theta_2))$



PROBLEM 6-50

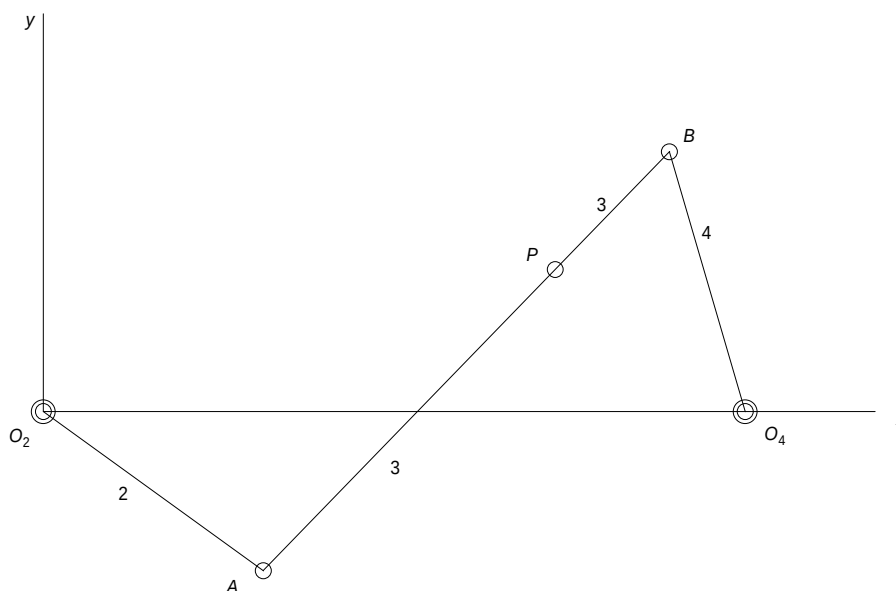
Statement: Figure P6-13 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of the coupler point P at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 0.86 \cdot \text{in}$	Link 3 (A to B)	$b := 1.85 \cdot \text{in}$
Link 4 (B to O_4)	$c := 0.86 \cdot \text{in}$	Link 1 (O_2 to O_4)	$d := 2.22 \cdot \text{in}$
Coupler point:	$R_{pa} := 1.33 \cdot \text{in}$	$\delta_3 := 0 \cdot \text{deg}$	
Crank speed:	$\omega_2 := 80 \cdot \text{rpm}$		

Solution: See Figure P6-13 and Mathcad file P0650.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.37.

$$arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad arg_1 = 1.228$$

$$arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad arg_2 = -0.439$$

$$\theta_{2toggle} := \arccos(arg_2) \quad \theta_{2toggle} = 116.0 \text{ deg}$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2toggle} + 1 \cdot \text{deg}, -\theta_{2toggle} + 2 \cdot \text{deg} \dots \theta_{2toggle} - 1 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.5814$$

$$K_2 = 2.5814$$

$$K_3 = 2.0181$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.2000 \quad K_5 = -2.6244$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))} \quad \omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point A using equations 6.19.

$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

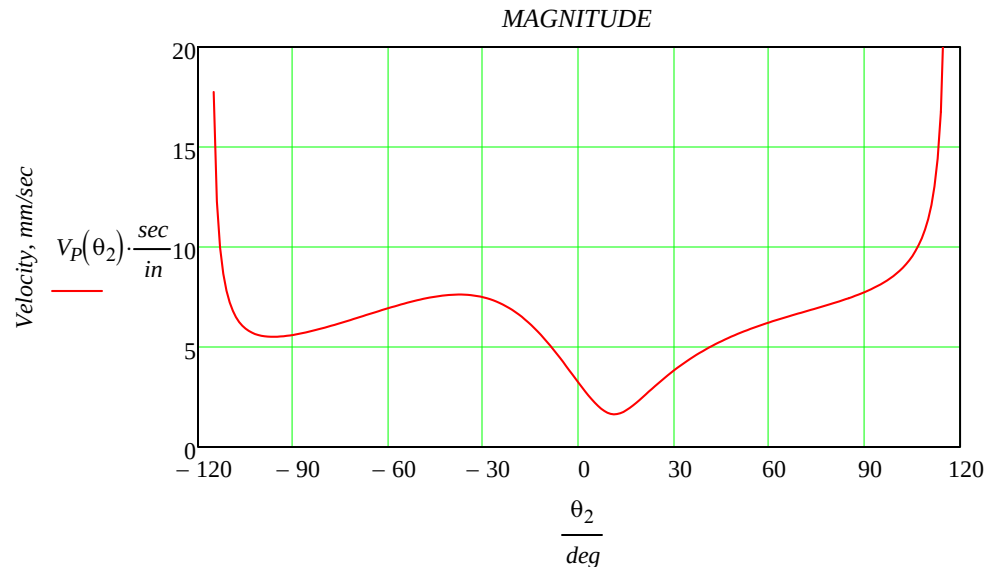
9. Determine the velocity of the coupler point P using equations 6.36.

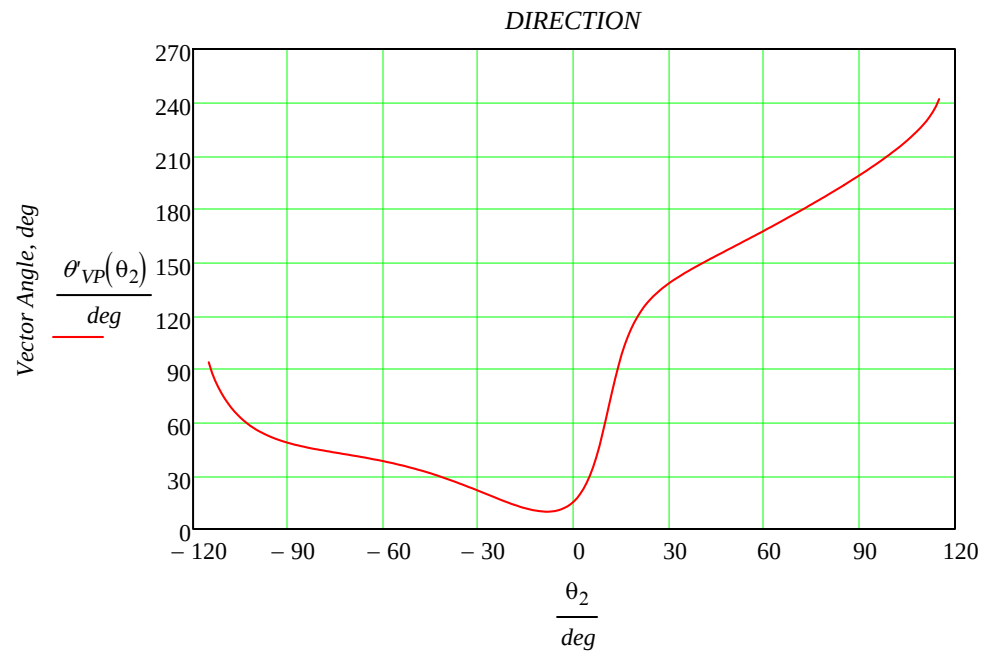
$$\mathbf{V}_{PA}(\theta_2) := R_{pa} \cdot \omega_{31}(\theta_2) \cdot (-\sin(\theta_{31}(\theta_2) + \delta_3) + j \cdot \cos(\theta_{31}(\theta_2) + \delta_3))$$

$$\mathbf{V}_P(\theta_2) := \mathbf{V}_A(\theta_2) + \mathbf{V}_{PA}(\theta_2)$$

10. Plot the magnitude and direction of the coupler point P.

$$\text{Magnitude: } V_P(\theta_2) := |\mathbf{V}_P(\theta_2)| \quad \text{Direction: } \theta_{VP}(\theta_2) := \arg(\mathbf{V}_P(\theta_2))$$





PROBLEM 6-51

Statement: Figure P6-14 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of the coupler point P at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 0.72 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 0.68 \cdot \text{in}$$

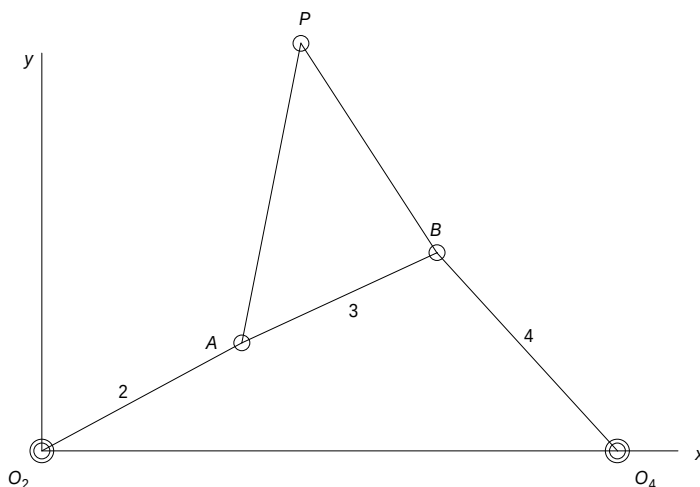
$$\text{Link 4 (B to O}_4) \quad c := 0.85 \cdot \text{in} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 1.82 \cdot \text{in}$$

$$\text{Coupler point:} \quad R_{pa} := 0.97 \cdot \text{in} \quad \delta_3 := 54 \cdot \text{deg}$$

$$\text{Crank speed:} \quad \omega_2 := 80 \cdot \text{rpm}$$

Solution: See Figure P6-14 and Mathcad file P0651.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.37.

$$arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad arg_1 = 1.451$$

$$arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad arg_2 = 0.568$$

$$\theta_{2toggle} := \text{acos}(arg_2) \quad \theta_{2toggle} = 55.4 \text{ deg}$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2toggle} + 1 \cdot \text{deg}, -\theta_{2toggle} + 2 \cdot \text{deg} .. \theta_{2toggle} - 1 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 2.5278 \quad K_2 = 2.1412$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 3.3422$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 2.6765$$

$$K_5 = -3.6465$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point A using equations 6.19.

$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

9. Determine the velocity of the coupler point P using equations 6.36.

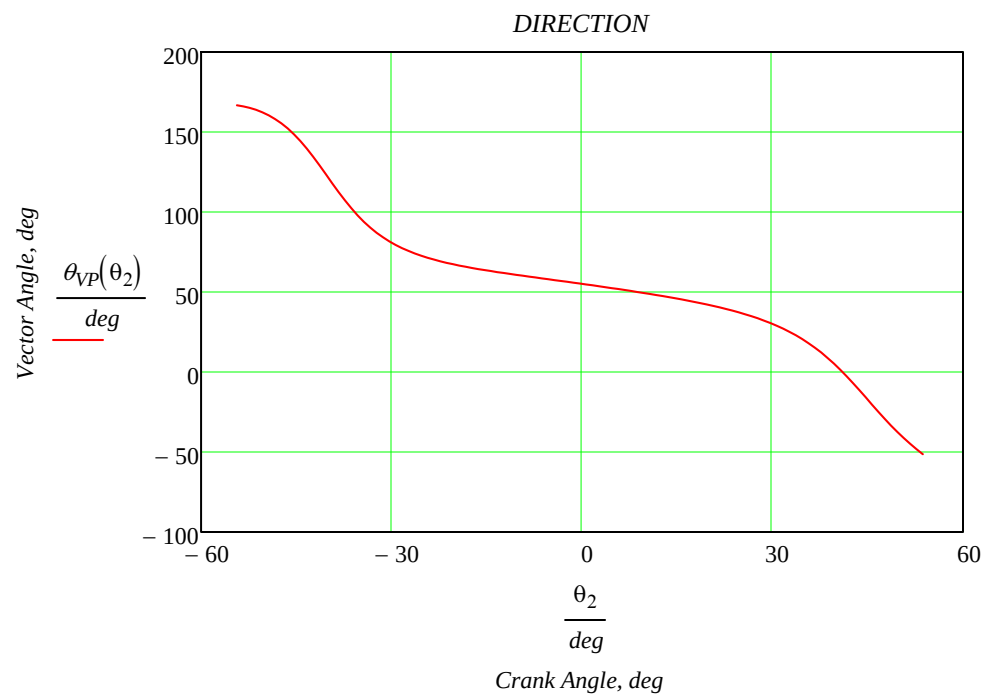
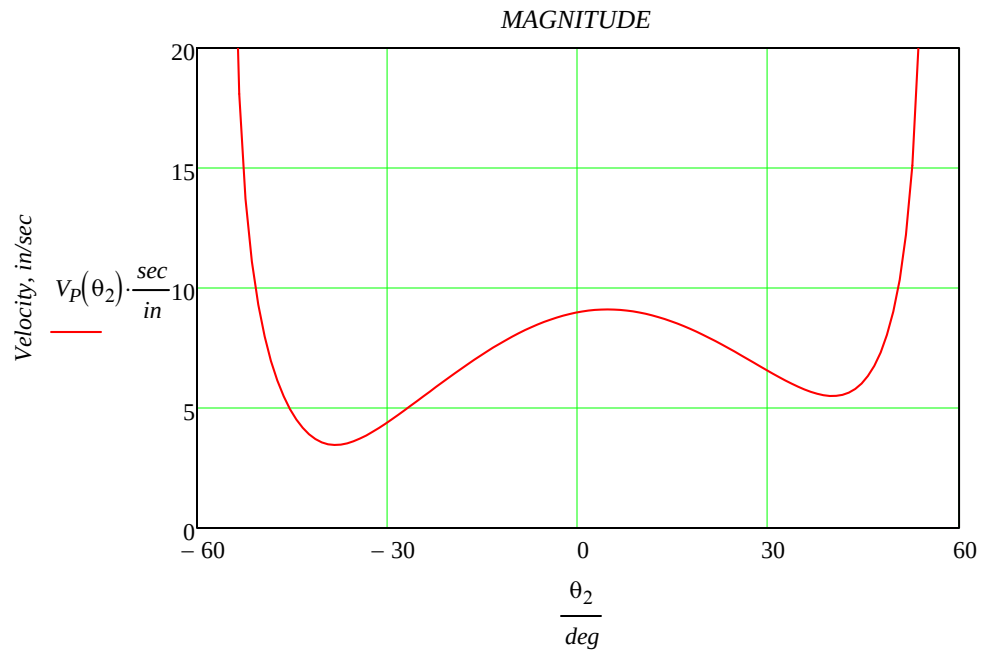
$$\mathbf{V}_{PA}(\theta_2) := R_{pa} \cdot \omega_{31}(\theta_2) \cdot (-\sin(\theta_{31}(\theta_2) + \delta_3) + j \cdot \cos(\theta_{31}(\theta_2) + \delta_3))$$

$$\mathbf{V}_P(\theta_2) := \mathbf{V}_A(\theta_2) + \mathbf{V}_{PA}(\theta_2)$$

10. Plot the magnitude and direction of the coupler point P.

$$\text{Magnitude:} \quad V_P(\theta_2) := |\mathbf{V}_P(\theta_2)|$$

$$\text{Direction:} \quad \theta_{VP}(\theta_2) := \arg(\mathbf{V}_P(\theta_2))$$



PROBLEM 6-52

Statement: Figure P6-15 shows a power hacksaw that is an offset crank-slider mechanism that has the dimensions given below. Draw an equivalent linkage diagram; then calculate and plot the velocity of the saw blade with respect to the piece being cut over one revolution of the crank, which rotates at 50 rpm.

Given: Link lengths:

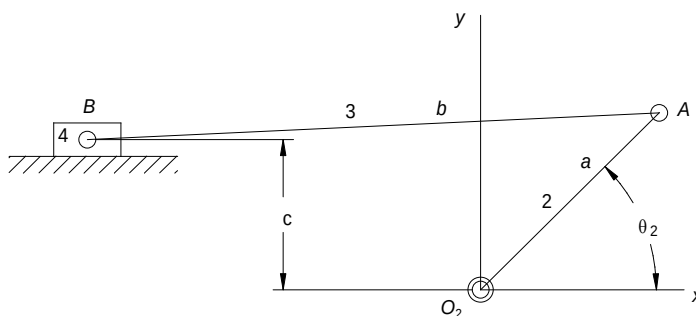
$$\text{Link 2} \quad a := 75 \cdot \text{mm} \quad \text{Offset} \quad c := 45 \cdot \text{mm}$$

$$\text{Link 3} \quad b := 170 \cdot \text{mm}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 50 \cdot \text{rpm}$$

Solution: See Figure P6-15 and Mathcad file P0652.

1. Draw the equivalent linkage to a convenient scale and label it.



2. Determine the range of motion for this crank-slider linkage.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg}$$

3. Determine θ_3 using equations 4.16 for the crossed circuit.

$$\theta_3(\theta_2) := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right)$$

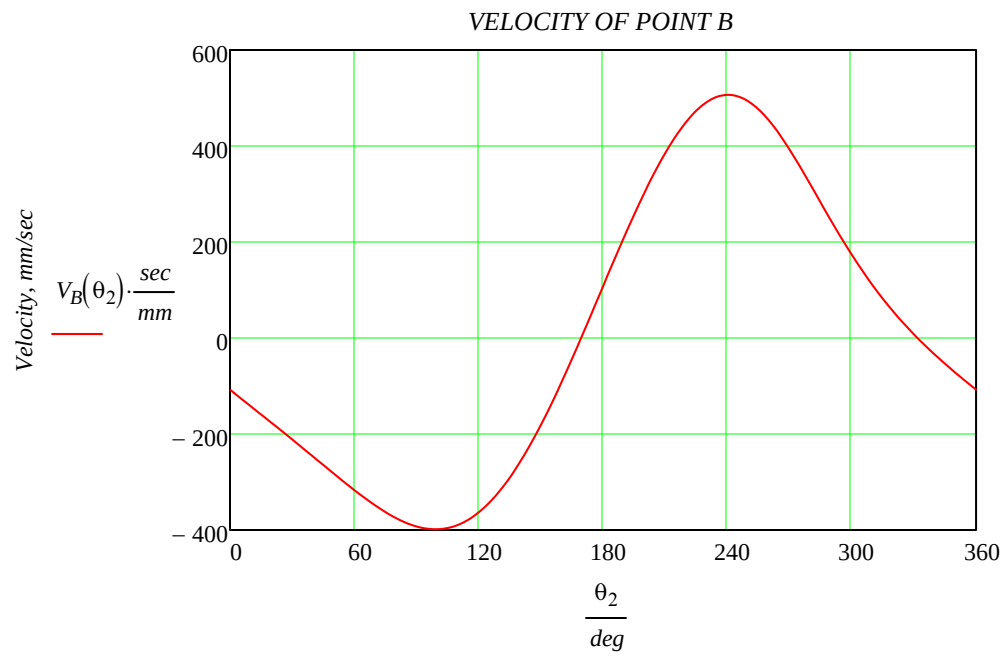
4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3(\theta_2))} \cdot \omega_2$$

5. Determine the velocity of pin B using equation 6.22b:

$$V_B(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_3(\theta_2) \cdot \sin(\theta_3(\theta_2))$$

7. Plot the magnitude of the velocity of B. (See next page.)



PROBLEM 6-53

Statement: Figure P6-16 shows a walking-beam indexing and pick-and-place mechanism that can be analyzed as two fourbar linkages driven by a common crank. Calculate and plot the absolute velocities of points *E* and *P* and the relative velocity between points *E* and *P* for one revolution of gear 2.

Given: Link lengths (walking-beam linkage):

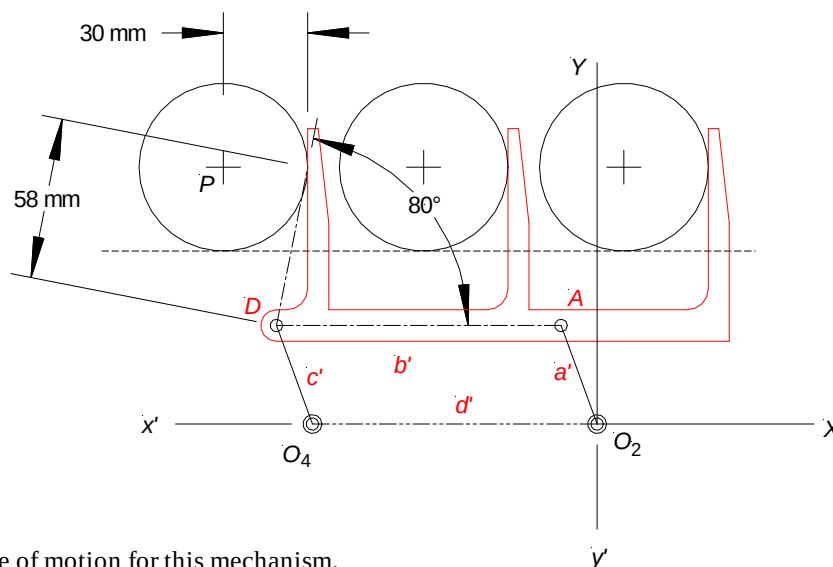
Link 2 (O_2 to A)	$a' := 40 \cdot \text{mm}$	Link 3 (A to D)	$b' := 108 \cdot \text{mm}$
Link 4 (O_4 to D)	$c' := 40 \cdot \text{mm}$	Link 1 (O_2 to O_4)	$d' := 108 \cdot \text{mm}$

Link lengths (pick and place linkage):

Link 5 (O_5 to B)	$a := 13 \cdot \text{mm}$	Link 7 (B to C)	$b := 193 \cdot \text{mm}$
Link 6 (C to O_6)	$c := 92 \cdot \text{mm}$	Link 1 (O_5 to O_6)	$d := 128 \cdot \text{mm}$
Rocker point <i>E</i> :	$u := 164 \cdot \text{mm}$	Crank speed:	$\omega_2 := 10 \cdot \text{rpm}$
Gears 4 & 5 phase angle	$\phi := 143 \cdot \text{deg}$		

Solution: See Figure P6-16 and Mathcad file P0653.

1. Draw the walking-beam linkage to scale and label it.



2. Determine the range of motion for this mechanism.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg} \quad (\text{local } x'y' \text{ coordinate system})$$

3. This part of the mechanism is a special-case Grashof in the parallelogram configuration. As such, the coupler does not rotate, but has curvilinear motion with every point on it having the same velocity. Therefore, it is only necessary to calculate the X-component of the velocity at point *A* in order to determine the velocity of the cylinder center, *P*.

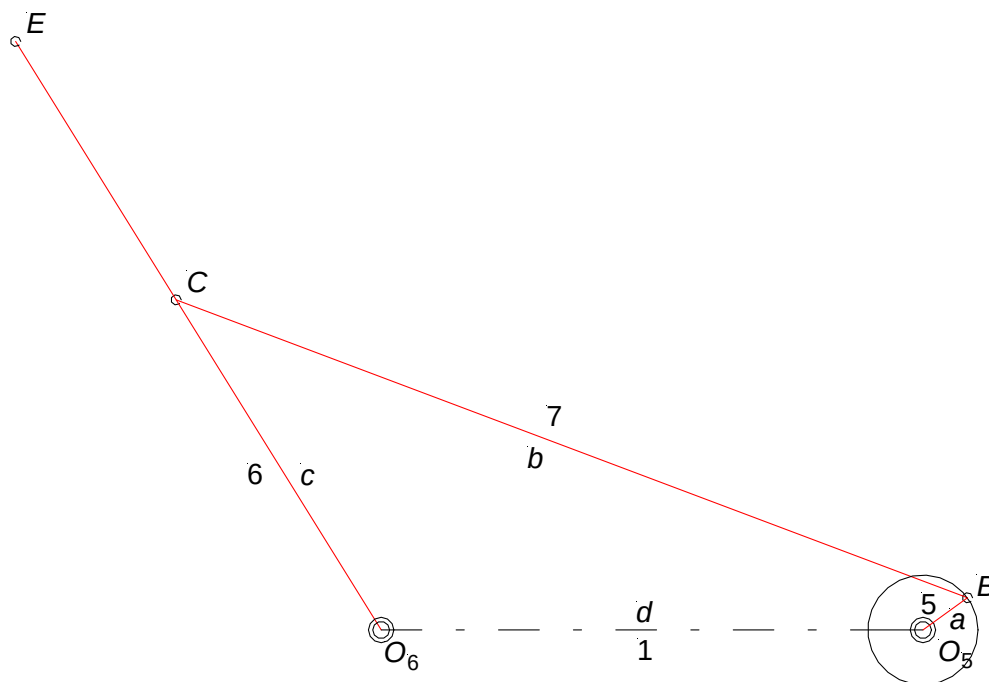
$$\mathbf{V}_A(\theta_2) := a' \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_{Ax}(\theta_2) := \text{Re}(\mathbf{V}_A(\theta_2))$$

In the global *X-Y* coordinate frame,

$$\mathbf{V}_P(\theta_2) := -V_{Ax}(\theta_2)$$

4. Draw the pick and place linkage to scale and label it.



5. Establish the relationship between θ_5 and θ_2 . Note that gear 5 is driven by gear 4 and that their ratio is -1 (i.e., they rotate in opposite directions with the same speed). Also, because the walking beam fourbar is special Grashof, $\theta_4 = \theta_2$. Thus,

$$\theta_5(\theta_2) := -\theta_2 + \phi \quad \text{and} \quad \omega_5 := -\omega_2$$

6. Determine the values of the constants needed for finding θ_6 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_1 = 9.8462 \quad K_2 := \frac{d}{c} \quad K_2 = 1.3913$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = -5.1137$$

$$A(\theta_2) := \cos(\theta_5(\theta_2)) - K_1 - K_2 \cdot \cos(\theta_5(\theta_2)) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_5(\theta_2))$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_5(\theta_2)) + K_3$$

7. Use equation 4.10b to find values of θ_6 for the crossed circuit.

$$\theta_6(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

8. Determine the values of the constants needed for finding θ_7 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.6632 \quad K_5 = -9.0351$$

$$D(\theta_2) := \cos(\theta_5(\theta_2)) - K_1 + K_4 \cdot \cos(\theta_5(\theta_2)) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_5(\theta_2))$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_5(\theta_2)) + K_5$$

9. Use equation 4.13 to find values of θ_7 for the crossed circuit.

$$\theta_7(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

10. Determine the angular velocity of links 6 and 7 using equations 6.18.

$$\omega_7(\theta_2) := \frac{a \cdot \omega_5}{b} \cdot \frac{\sin[\theta_6(\theta_2) - (\theta_5(\theta_2))]}{\sin(\theta_7(\theta_2) - \theta_6(\theta_2))}$$

$$\omega_6(\theta_2) := \frac{a \cdot \omega_5}{c} \cdot \frac{\sin(\theta_5(\theta_2) - \theta_7(\theta_2))}{\sin(\theta_6(\theta_2) - \theta_7(\theta_2))}$$

10. Determine the velocity of the rocker point E using equations 6.34.

$$\mathbf{V}_E(\theta_2) := u \cdot \omega_6(\theta_2) \cdot (-\sin(\theta_6(\theta_2)) + j \cdot \cos(\theta_6(\theta_2)))$$

11. Transform this into the global XY system.

$$V_{Ex}(\theta_2) := \text{Re}(\mathbf{V}_E(\theta_2))$$

$$V_{Ey}(\theta_2) := \text{Im}(\mathbf{V}_E(\theta_2))$$

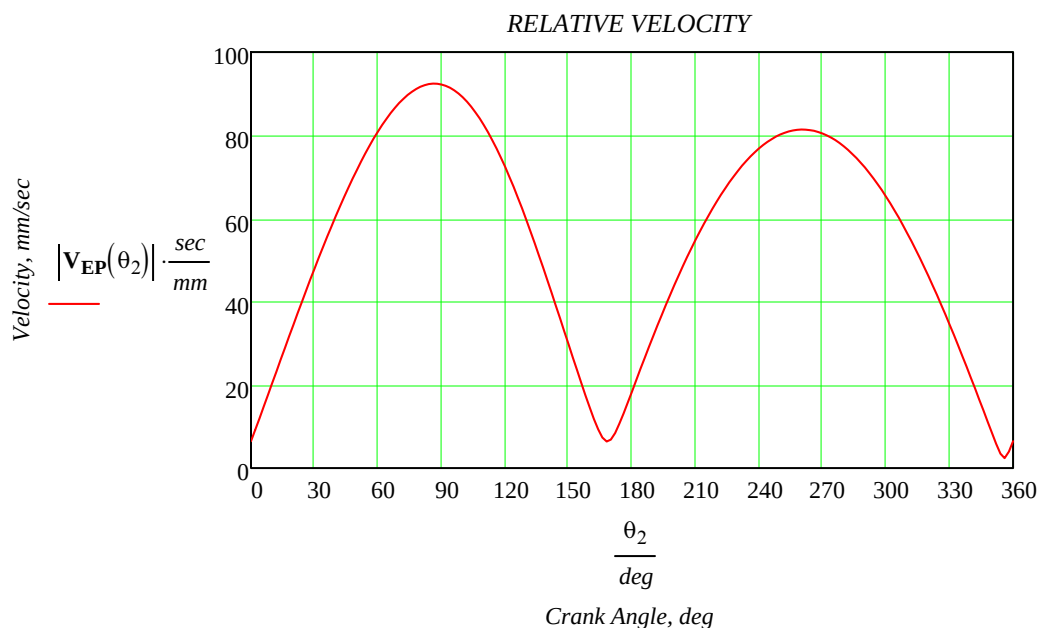
$$V_{EX}(\theta_2) := -V_{Ex}(\theta_2)$$

$$V_{EY}(\theta_2) := -V_{Ey}(\theta_2)$$

$$\mathbf{V}_{EXY}(\theta_2) := V_{EX}(\theta_2) + j \cdot V_{EY}(\theta_2)$$

12. Calculate and plot the velocity of E relative to P .

$$\mathbf{V}_{EP}(\theta_2) := \mathbf{V}_{EXY}(\theta_2) - \mathbf{V}_P(\theta_2)$$



PROBLEM 6-54

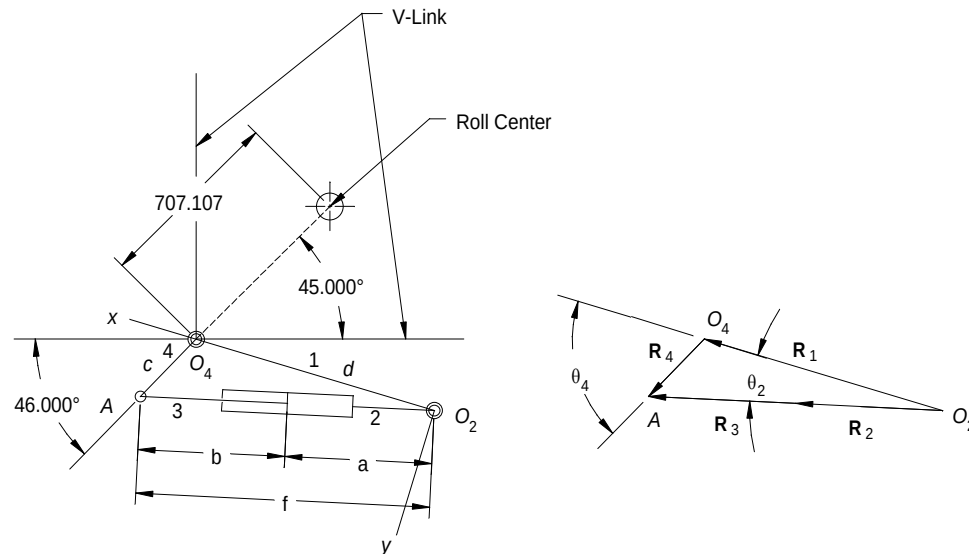
Statement: Figure P6-17 shows a paper roll off-loading mechanism driven by an air cylinder. In the position shown, it has the dimensions given below. The V-links are rigidly attached to O_4A . The air cylinder is retracted at a constant velocity of 0.2 m/sec. Draw a kinematic diagram of the mechanism, write the necessary equations, and calculate and plot the angular velocity of the paper roll and the linear velocity of its center as it rotates through 90 deg CCW from the position shown.

Given: Link lengths and angles: Paper roll location from O_4 :

Link 4 (O_4 to A)	$c := 300 \cdot \text{mm}$	$u := 707.1 \cdot \text{mm}$
Link 1 (O_2 to O_4)	$d := 930 \cdot \text{mm}$	$\delta_4 := -181 \cdot \text{deg}$
Link 4 initial angle	$\theta_{40} := 62.8 \cdot \text{deg}$	with respect to local x axis
Input cylinder velocity	$\dot{a} := -200 \cdot \text{mm} \cdot \text{sec}^{-1}$	

Solution: See Figure P6-17 and Mathcad file P0654.

1. Draw the mechanism to scale and define a vector loop using the fourbar derivation in Section 6.7 as a model.



2. Write the vector loop equation, differentiate it, expand the result and separate into real and imaginary parts to solve for $\dot{f}$, θ_2 , and ω_4 .

$$\mathbf{R}_1 + \mathbf{R}_4 := \mathbf{R}_2 + \mathbf{R}_3 \quad (a)$$

$$d \cdot e^{j \cdot \theta_1} + c \cdot e^{j \cdot \theta_4} := a \cdot e^{j \cdot \theta_2} + b \cdot e^{j \cdot \theta_2} \quad (b)$$

where a is the distance from the origin to the cylinder piston, a variable; b is the distance from the cylinder piston to A , a constant; and c is the distance from O_4 to point A , a constant. Angle θ_1 is zero, $\theta_3 = \theta_2$, and θ_4 is the variable angle that the rocker arm makes with the x axis. Solving the position equations:

Let $f := a + b$ then, making this substitution and substituting the Euler equivalents,

$$d + c \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4)) := f \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \quad (c)$$

Separating into real and imaginary components and solving for θ_2 and f ,

$$\theta_2(\theta_4) := \text{atan2}(d + c \cdot \cos(\theta_4), c \cdot \sin(\theta_4)) \quad (d)$$

$$f(\theta_4) := \frac{c \cdot \sin(\theta_4)}{\sin(\theta_2(\theta_4))} \quad (e)$$

Differentiate equation *b*.

$$j \cdot c \cdot \omega_4 \cdot e^{j \cdot \theta_4} := \left(\frac{d}{dt} f \right) \cdot e^{j \cdot \theta_2} + j \cdot f \cdot \omega_2 \cdot e^{j \cdot \theta_2} \quad (f)$$

Substituting the Euler equivalents,

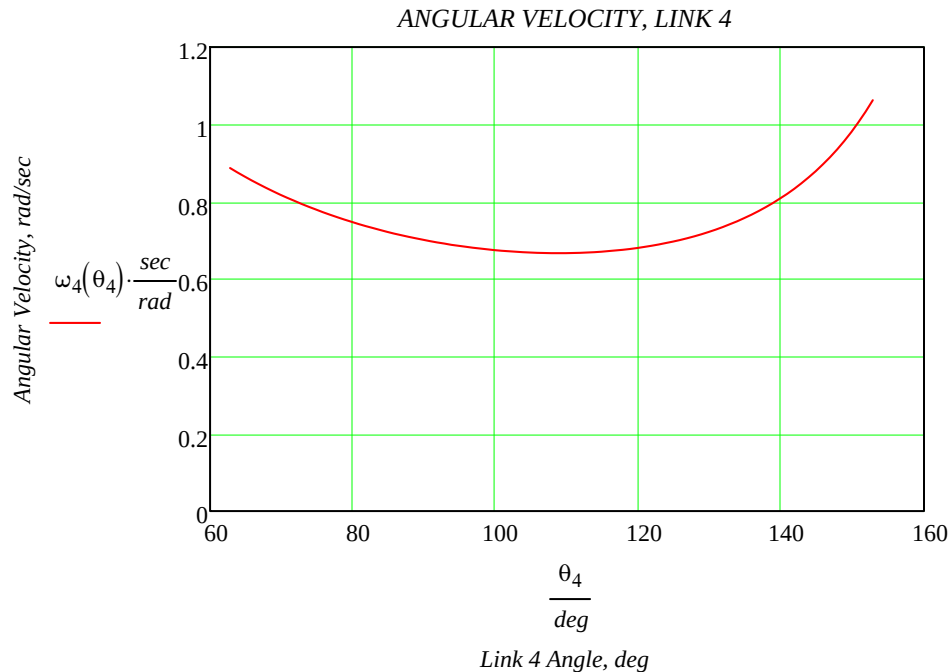
$$c \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) := \left(\frac{d}{dt} f \right) \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) + f \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) \quad (g)$$

Separating into real and imaginary components and solving for ω_4 . Note that $df/dt = \dot{a}$.

$$\omega_4(\theta_4) := \frac{\dot{a}}{c \cdot \sin(\theta_2(\theta_4) - \theta_4)} \quad (h)$$

3. Plot ω_4 over a range of θ_4 of

$$\theta_4 := \theta_{40}, \theta_{40} + 1 \cdot \text{deg} .. \theta_{40} + 90 \cdot \text{deg}$$

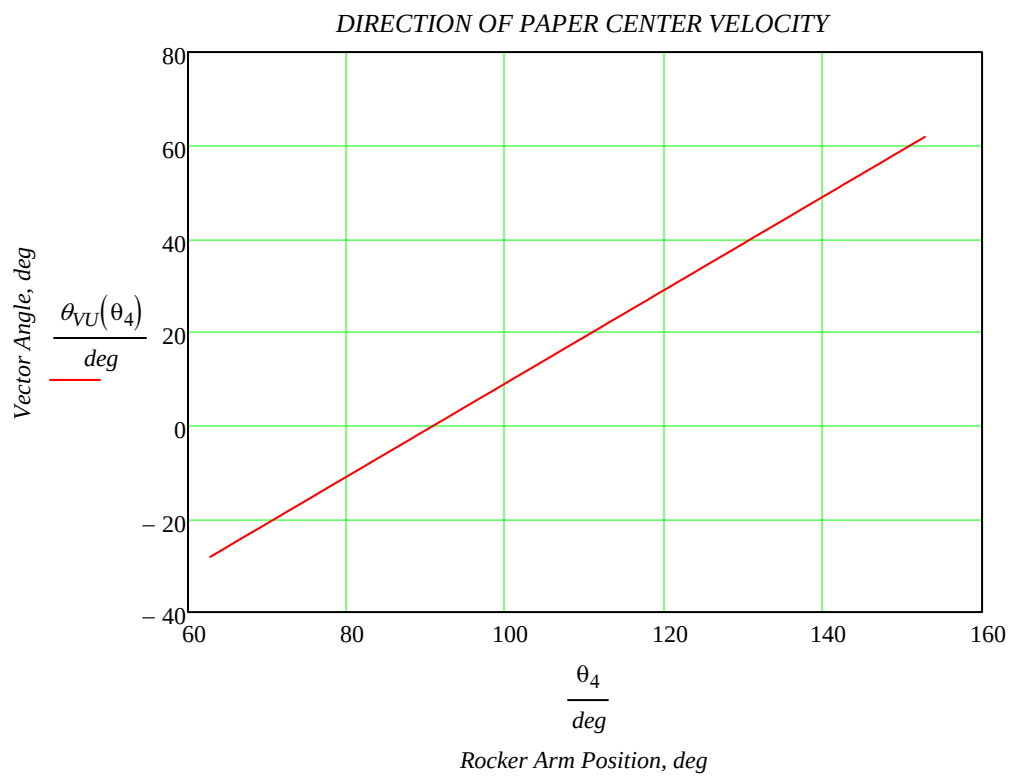
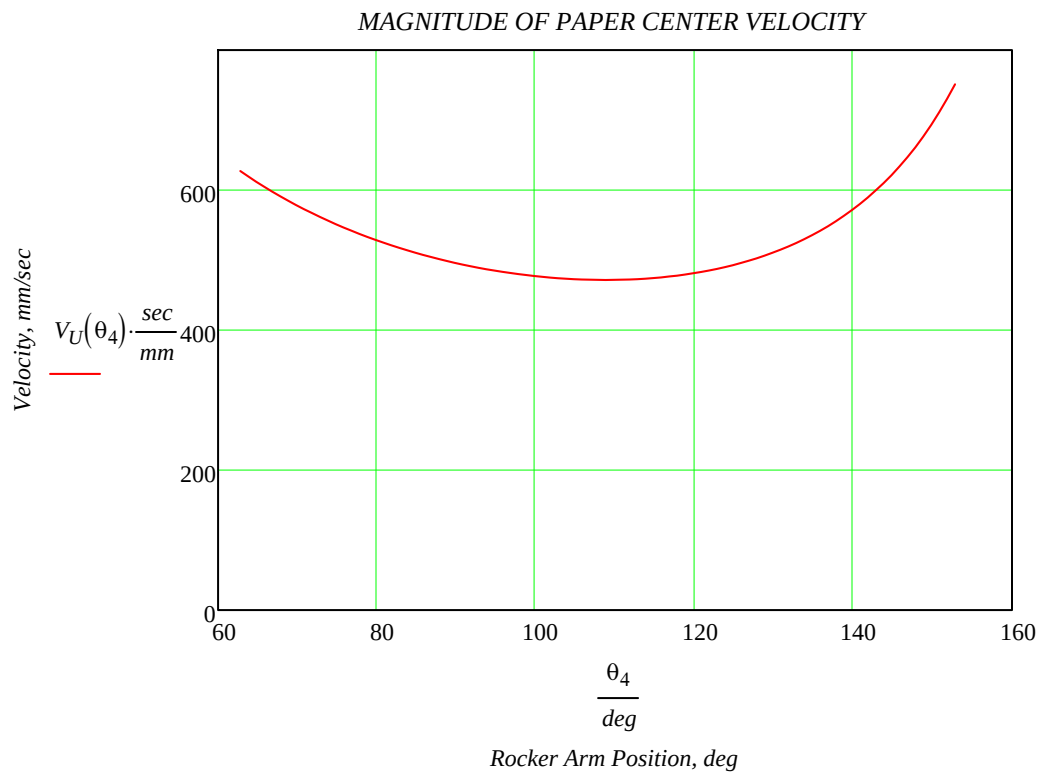


4. Determine the velocity of the center of the paper roll using equation 6.35. The direction is in the local *xy* coordinate system.

$$\mathbf{V}_U(\theta_4) := u \cdot \omega_4(\theta_4) \cdot (-\sin(\theta_4 + \delta_4) + j \cdot \cos(\theta_4 + \delta_4))$$

$$V_U(\theta_4) := |\mathbf{V}_U(\theta_4)| \quad \theta_{VU}(\theta_4) := \arg(\mathbf{V}_U(\theta_4))$$

5. Plot the magnitude and direction of the velocity of the paper roll center. (See next page.)



PROBLEM 6-55a

Statement: Figure P6-18 shows a powder compaction mechanism. Calculate its mechanical advantage for the position shown.

Given: Link lengths:

$$\text{Link 2 (A to B)} \quad a := 105 \cdot \text{mm}$$

$$\text{Offset} \quad c := 27 \cdot \text{mm}$$

$$\text{Link 3 (B to D)} \quad b := 172 \cdot \text{mm}$$

Distance to force application:

$$\text{Link 2 (AC)} \quad r_{in} := 301 \cdot \text{mm}$$

$$\text{Position of link 2:} \quad \theta_2 := 44 \cdot \text{deg}$$

$$\text{Let } \omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$$

Solution: See Figure P6-18 and Mathcad file P0655a.

1. Draw the linkage to scale and label it.

2. Determine θ_3 using equation 4.17.

$$\theta_3 := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi$$

$$\theta_3 = 164.509 \text{ deg}$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3)} \cdot \omega_2$$

4. Determine the velocity of pin D using equation 6.22b:

$$V_D := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_3 \cdot \sin(\theta_3) \quad \text{Positive upward}$$

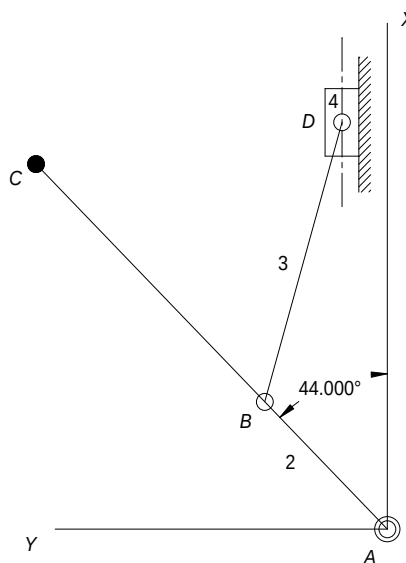
5. Calculate the velocity of point C using equation 6.23a:

$$\mathbf{V}_C := r_{in} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_C := |\mathbf{V}_C|$$

6. Calculate the mechanical advantage using equation 6.13.

$$m_A := \frac{V_C}{|V_D|} \quad m_A = 3.206$$



PROBLEM 6-55b

Statement: Figure P6-18 shows a powder compaction mechanism. Calculate and plot its mechanical advantage as a function of the angle of link AC as it rotates from 15 to 60 deg.

Given: Link lengths:

$$\text{Link 2 (A to B)} \quad a := 105 \cdot \text{mm} \quad \text{Offset} \quad c := 27 \cdot \text{mm}$$

$$\text{Link 3 (B to D)} \quad b := 172 \cdot \text{mm}$$

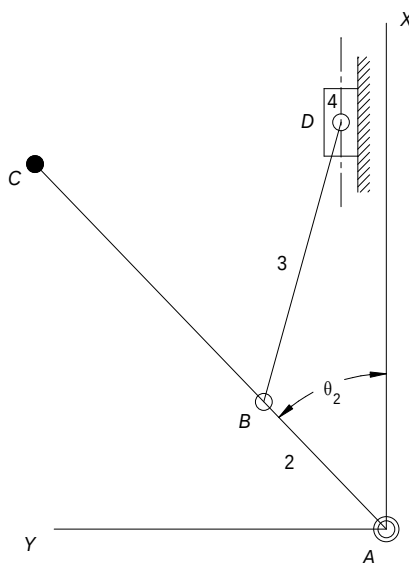
Distance to force application:

$$\text{Link 2 (AC)} \quad r_{in} := 301 \cdot \text{mm}$$

$$\text{Initial and final positions of link 2: } \theta_{20} := 15 \cdot \text{deg} \quad \theta_{21} := 60 \cdot \text{deg} \quad \text{Let } \omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$$

Solution: See Figure P6-18 and Mathcad file P0655b.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := \theta_{20}, \theta_{20} + 1 \cdot \text{deg} .. \theta_{21}$$

3. Determine θ_3 using equation 4.17.

$$\theta_3(\theta_2) := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi$$

4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3(\theta_2))} \cdot \omega_2$$

5. Determine the velocity of pin D using equation 6.22b:

$$V_D(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_3(\theta_2) \cdot \sin(\theta_3(\theta_2)) \quad \text{Positive upward}$$

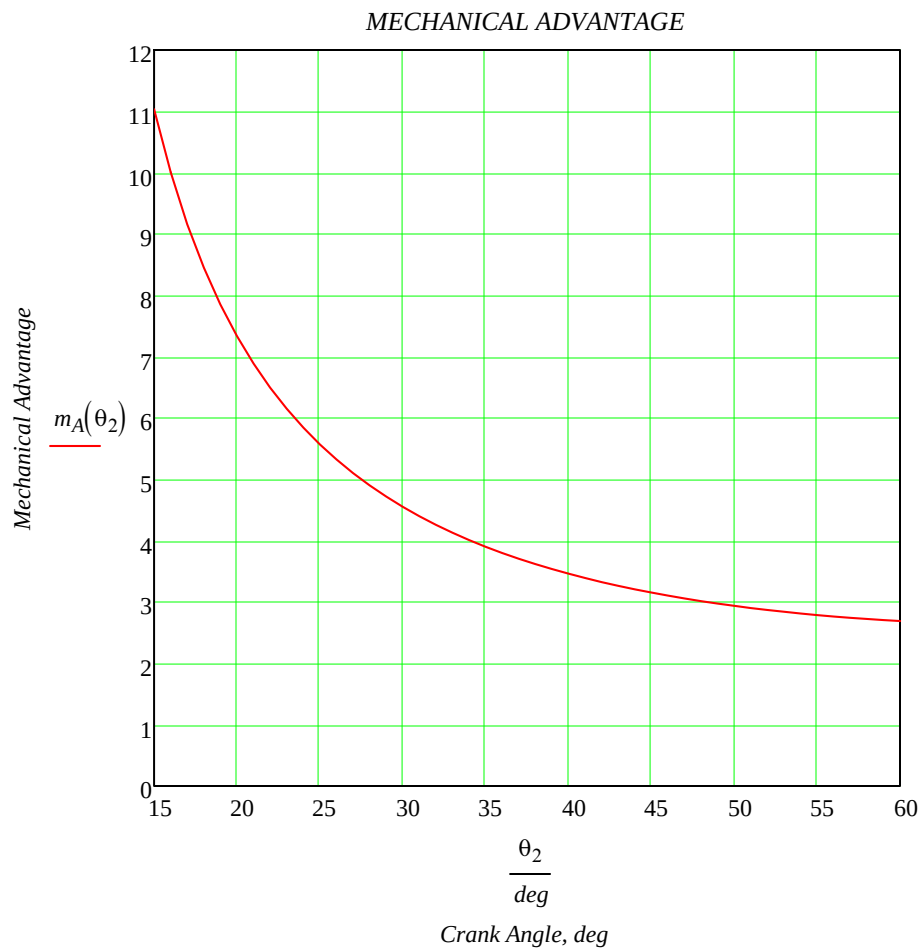
6. Calculate the velocity of point C using equation 6.23a:

$$\mathbf{V}_C(\theta_2) := r_{in} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_C(\theta_2) := |\mathbf{V}_C(\theta_2)|$$

7. Calculate the mechanical advantage using equation 6.13.

$$m_A(\theta_2) := \frac{V_C(\theta_2)}{|V_D(\theta_2)|}$$



PROBLEM 6-56

Statement: Figure P6-19 shows a walking beam mechanism. Calculate and plot the velocity V_{out} for one revolution of the input crank 2 rotating at 100 rpm.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 1.00 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 2.06 \cdot \text{in}$$

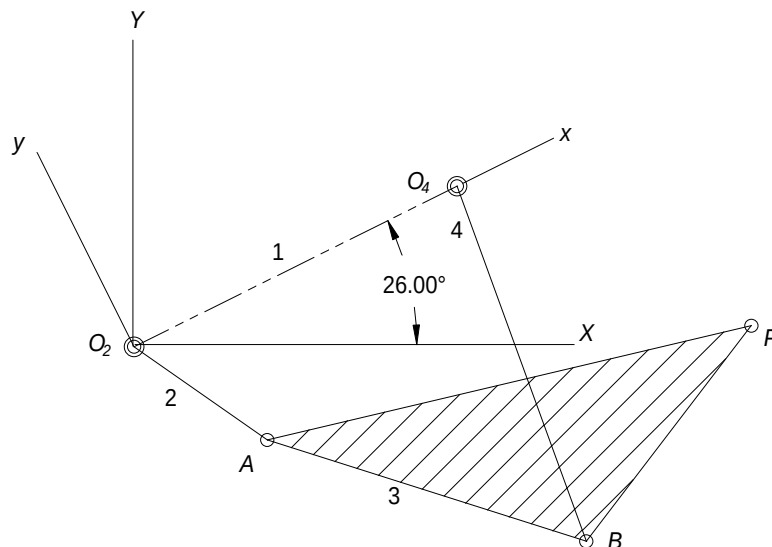
$$\text{Link 4 (B to O}_4) \quad c := 2.33 \cdot \text{in} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 2.22 \cdot \text{in}$$

$$\text{Coupler point:} \quad R_{pa} := 3.06 \cdot \text{in} \quad \delta_3 := 31 \cdot \text{deg}$$

$$\text{Crank speed:} \quad \omega_2 := 100 \cdot \text{rpm}$$

Solution: See Figure P6-19 and Mathcad file P0656.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 2.2200$$

$$K_2 = 0.9528$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.5265$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0777 \quad K_5 = -1.1512$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point A using equations 6.19.

$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

9. Determine the velocity of the coupler point P using equations 6.36.

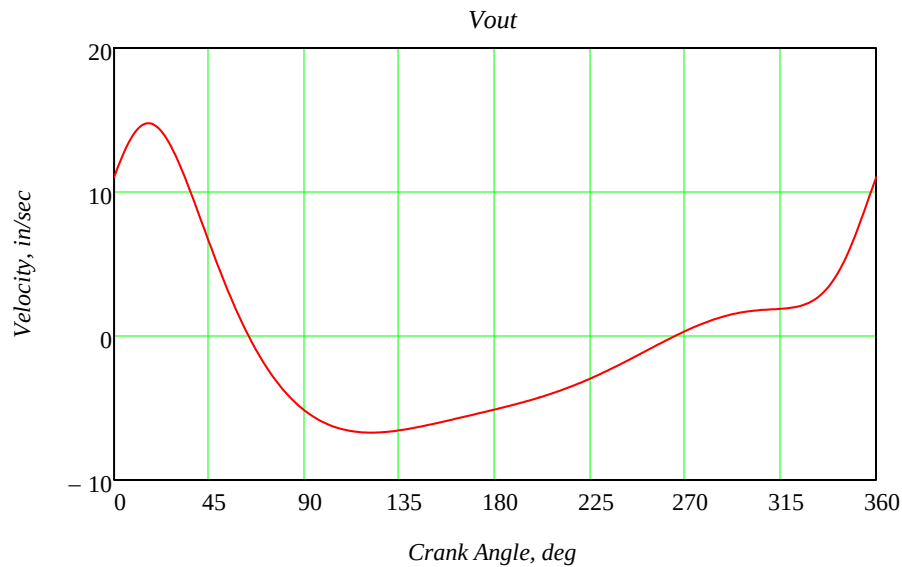
$$\mathbf{V}_{PA}(\theta_2) := R_{pa} \cdot \omega_{31}(\theta_2) \cdot (-\sin(\theta_{31}(\theta_2) + \delta_3) + j \cdot \cos(\theta_{31}(\theta_2) + \delta_3))$$

$$\mathbf{V}_P(\theta_2) := \mathbf{V}_A(\theta_2) + \mathbf{V}_{PA}(\theta_2)$$

10. Plot the X-component (global coordinate system) of the velocity of the coupler point P.

Coordinate rotation angle: $\alpha := 26 \cdot \text{deg}$

$$V_{out}(\theta_2) := \text{Re}(\mathbf{V}_P(\theta_2)) \cdot \cos(\alpha) - \text{Im}(\mathbf{V}_P(\theta_2)) \cdot \sin(\alpha)$$



PROBLEM 6-57a

Statement: Figure P6-20 shows a crimping tool. For the dimensions given below, calculate its mechanical advantage for the position shown.

Given: Link lengths:

$$\text{Link 2 (AB)} \quad a := 0.80 \cdot \text{in}$$

$$\text{Link 3 (BC)} \quad b := 1.23 \cdot \text{in}$$

$$\text{Link 4 (CD)} \quad c := 1.55 \cdot \text{in}$$

$$\text{Link 1 (AD)} \quad d := 2.40 \cdot \text{in}$$

Distance to force application:

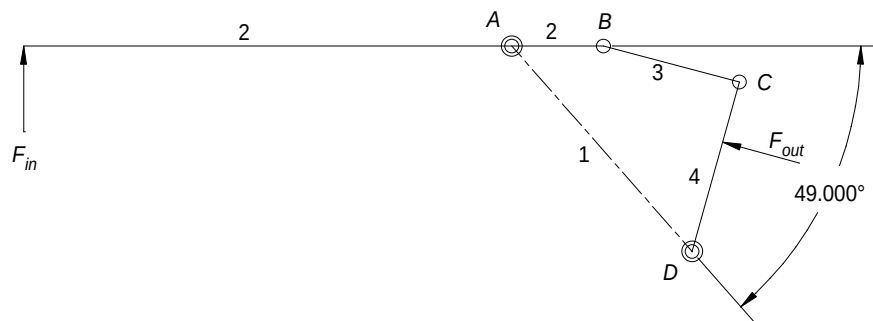
$$\text{Link 2 (AB)} \quad r_{in} := 4.26 \cdot \text{in}$$

$$\text{Link 4 (CD)} \quad r_{out} := 1.00 \cdot \text{in}$$

$$\text{Initial position of link 2:} \quad \theta_2 := 49 \cdot \text{deg}$$

Solution: See Figure P6-20 and Mathcad file P0657a.

1. Draw the mechanism to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 3.0000$$

$$K_2 = 1.5484$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 2.9394$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.4204 \quad B = -1.5094 \quad C = 4.2675$$

3. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_4 = -236.482 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b}$$

$$K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 1.9512 \quad K_5 = -2.8000$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -3.8638$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.5094$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.8241$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_3 = -325.961 \text{ deg}$$

6. Referring to Figure 6-10, calculate the values of the angles ν and μ .

$$\nu := \theta_2 - \theta_3 \quad \nu = 374.961 \text{ deg}$$

If $\nu > 360$ deg, subtract 360 deg from it.

$$\nu := \text{if}(\nu > 360 \cdot \text{deg}, \nu - 360 \cdot \text{deg}, \nu) \quad \nu = 14.961 \text{ deg}$$

$$\mu := \theta_4 - \theta_3 \quad \mu = 89.479 \text{ deg}$$

If $\mu > 90$ deg, subtract it from 180 deg.

$$\mu := \text{if}(\mu > 90 \cdot \text{deg}, 180 \cdot \text{deg} - \mu, \mu) \quad \mu = 89.479 \text{ deg}$$

7. Using equation 6.13e, calculate the mechanical advantage of the linkage in the position shown.

$$m_A := \frac{c \cdot \sin(\mu)}{a \cdot \sin(\nu)} \cdot \frac{r_{in}}{r_{out}} \quad m_A = 31.969$$

PROBLEM 6-57b

Statement: Figure P6-20 shows a crimping tool. For the dimensions given below, calculate and plot its mechanical advantage as a function of the angle of link AB as it rotates from 60 to 45 deg.

Given: Link lengths:

$$\text{Link 2 (AB)} \quad a := 0.80 \cdot \text{in}$$

$$\text{Link 3 (BC)} \quad b := 1.23 \cdot \text{in}$$

$$\text{Link 4 (CD)} \quad c := 1.55 \cdot \text{in}$$

$$\text{Link 1 (AD)} \quad d := 2.40 \cdot \text{in}$$

Distance to force application:

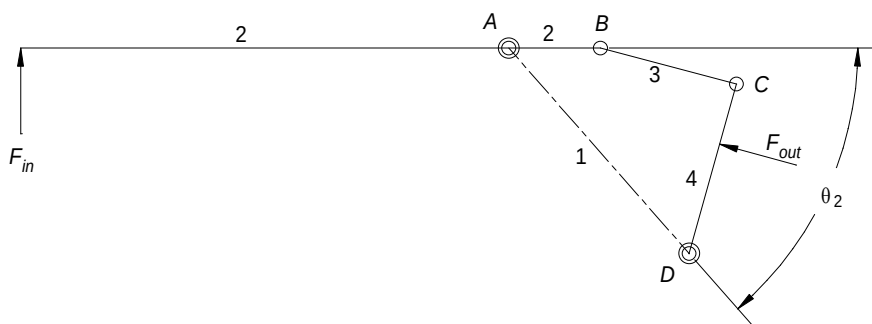
$$\text{Link 2 (AB)} \quad r_{in} := 4.26 \cdot \text{in}$$

$$\text{Link 4 (CD)} \quad r_{out} := 1.00 \cdot \text{in}$$

$$\text{Range of positions of link 2:} \quad \theta_{20} := 60 \cdot \text{deg} \quad \theta_{21} := 45 \cdot \text{deg}$$

Solution: See Figure P6-20 and Mathcad file P0657b.

1. Draw the mechanism to scale and label it.



2. Calculate the range of θ_2 in the local coordinate system (required to calculate θ_3 and θ_4).

$$\theta_2 := \theta_{20}, \theta_{20} - 1 \cdot \text{deg} .. \theta_{21}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 3.0000$$

$$K_2 = 1.5484$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_3 = 2.9394$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.9512 \quad K_5 = -2.8000$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

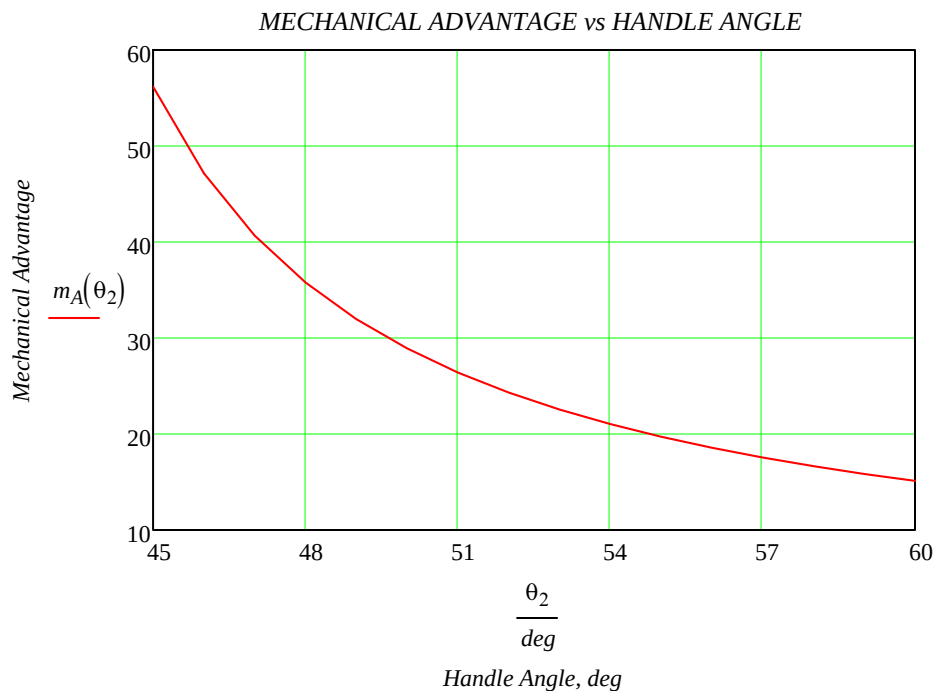
7. Referring to Figure 6-10, calculate the values of the angles ν and μ .

$$\nu(\theta_2) := \theta_2 - \theta_3(\theta_2)$$

$$\mu(\theta_2) := \theta_4(\theta_2) - \theta_3(\theta_2)$$

8. Using equation 6.13e, calculate and plot the mechanical advantage of the linkage over the given range.

$$m_A(\theta_2) := \frac{c \cdot \sin(\mu(\theta_2))}{a \cdot \sin(\nu(\theta_2))} \cdot \frac{r_{in}}{r_{out}}$$

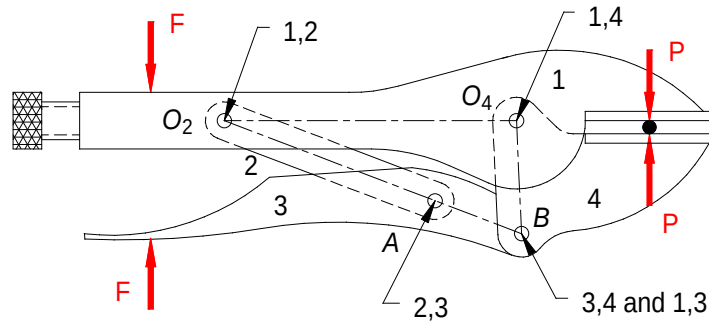


PROBLEM 6-58

Statement: Figure P6-21 shows a locking pliers. Calculate its mechanical advantage for the position shown. Scale any dimensions needed from the diagram.

Solution: See Figure P6-21 and Mathcad file P0658.

1. Draw the linkage to scale in the position given and find the instant centers.



2. Note that the linkage is in a toggle position (links 2 and 3 are in line) and the angle between links 2 and 3 is 0 deg. From the discussion below equation 6.13e in the text, we see that the mechanical advantage for this linkage in this position is theoretically infinite.

PROBLEM 6-59a

Statement: Figure P6-22 shows a fourbar toggle clamp used to hold a workpiece in place by clamping it at D. The linkage will toggle when link 2 reaches 90 deg. For the dimensions given below, calculate its mechanical advantage for the position shown.

Given:

Link lengths:

Link 2 (O_2A) $a := 70\text{ mm}$

Link 3 (AB) $b := 35\text{ mm}$

Link 4 (O_4B) $c := 34\text{ mm}$

Link 1 (O_2O_4) $d := 48\text{ mm}$

Distance to force application:

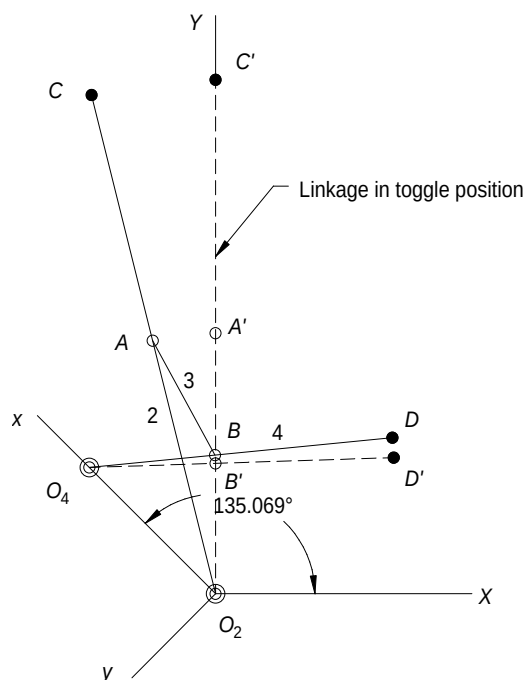
Link 2 (O_2C) $r_{in} := 138\text{ mm}$

Link 4 (O_4D) $r_{out} := 82\text{ mm}$

Initial position of link 2: $\theta_{20} := 104\text{ deg}$ Global XY system

Solution: See Figure P6-22 and Mathcad file P0659a.

1. Draw the mechanism to scale and label it. To establish the position of O_4 with respect to O_2 (in the global coordinate frame), draw the linkage in the toggle position with $\theta_2 = 90\text{ deg}$. The fixed pivot O_4 is then 48 mm from O_2 and 34 mm from B' (see layout).



2. Calculate the value of θ_2 in the local coordinate system (required to calculate θ_3 and θ_4).

Rotation angle of local xy system to global XY system: $\alpha := 135.069\text{ deg}$

$$\theta_2 := \theta_{20} - \alpha$$

$$\theta_2 = -31.069\text{ deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_1 = 0.6857 \quad K_2 := \frac{d}{c} \quad K_2 = 1.4118$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.4989$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = 0.4605 \quad B = 1.0321 \quad C = 0.1189$$

4. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_4 = -129.480 \text{ deg}$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.3714$$

$$K_5 = -1.4843$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -0.1388$$

$$E := -2 \cdot \sin(\theta_2) \quad E = 1.0321$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = -0.4804$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_3 = -196.400 \text{ deg}$$

7. Referring to Figure 6-10, calculate the values of the angles ν and μ .

$$\nu := \theta_2 - \theta_3 \quad \nu = 165.331 \text{ deg}$$

If $\nu > 90$ deg, subtract it from 180 deg.

$$\nu := \text{if}(\nu > 90 \cdot \text{deg}, 180 \cdot \text{deg} - \nu, \nu) \quad \nu = 14.669 \text{ deg}$$

$$\mu := \theta_4 - \theta_3 \quad \mu = 66.920 \text{ deg}$$

If $\mu > 90$ deg, subtract it from 180 deg.

$$\mu := \text{if}(\mu > 90 \cdot \text{deg}, 180 \cdot \text{deg} - \mu, \mu) \quad \mu = 66.920 \text{ deg}$$

8. Using equation 6.13e, calculate the mechanical advantage of the linkage in the position shown.

$$m_A := \frac{c \cdot \sin(\mu)}{a \cdot \sin(\nu)} \cdot \frac{r_{in}}{r_{out}} \quad m_A = 2.970$$

PROBLEM 6-59b

Statement: Figure P6-22 shows a fourbar toggle clamp used to hold a workpiece in place by clamping it at D. The linkage will toggle when link 2 reaches 90 deg. For the dimensions given below, calculate and plot its mechanical advantage as a function of the angle of link AB as link 2 rotates from 120 to 90 deg (in the global coordinate system).

Given: Link lengths:

Link 2 (O_2A)	$a := 70\text{-mm}$	Link 3 (AB)	$b := 35\text{-mm}$
Link 4 (O_4B)	$c := 34\text{-mm}$	Link 1 (O_2O_4)	$d := 48\text{-mm}$

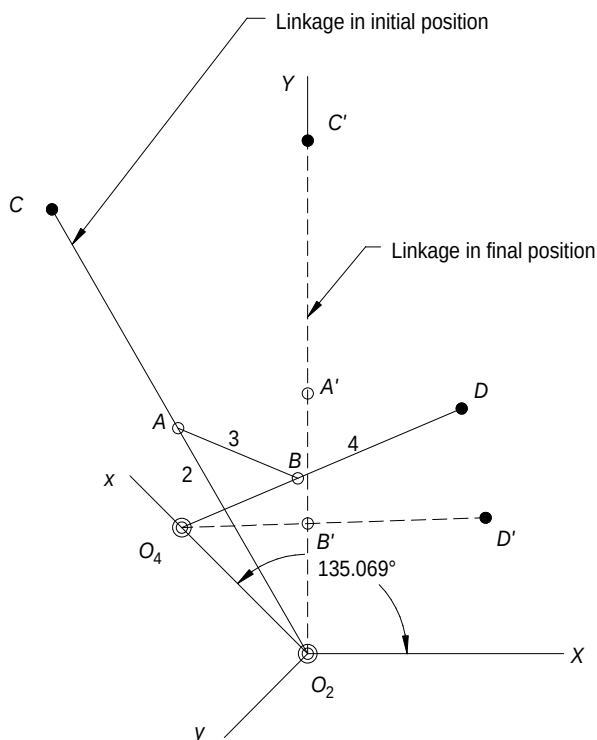
Distance to force application:

Link 2 (O_2C)	$r_{in} := 138\text{-mm}$	Link 4 (O_4D)	$r_{out} := 82\text{-mm}$

Range of positions of link 2: $\theta_{20} := 120\text{-deg}$ $\theta_{21} := 90.1\text{-deg}$ Global XY system

Solution: See Figure P6-22 and Mathcad file P0659b.

1. Draw the mechanism to scale and label it. To establish the position of O_4 with respect to O_2 (in the global coordinate frame), draw the linkage in the toggle position with $\theta_2 = 90$ deg. The fixed pivot O_4 is then 48 mm from O_2 and 34 mm from B' (see layout).



2. Calculate the range of θ_2 in the local coordinate system (required to calculate θ_3 and θ_4).

Rotation angle of local xy system to global XY system: $\alpha := 135.069\text{-deg}$

$$\theta_2 := \theta_{20} - \alpha, \theta_{20} - \alpha - 1\text{-deg} .. \theta_{21} - \alpha$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_1 = 0.6857 \quad K_2 := \frac{d}{c} \quad K_2 = 1.4118$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.4989$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.3714 \quad K_5 = -1.4843$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

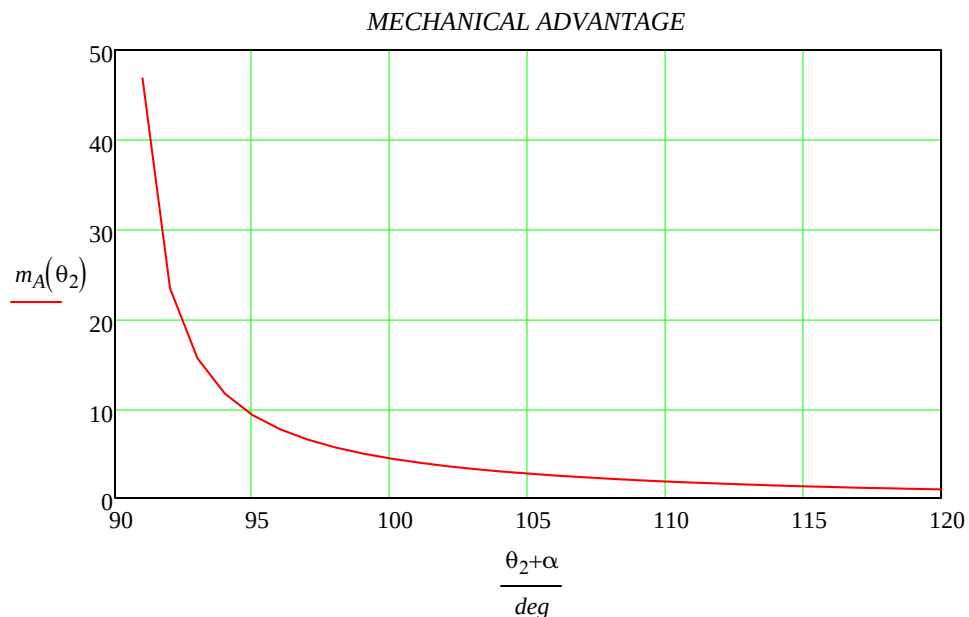
7. Referring to Figure 6-10, calculate the values of the angles ν and μ .

$$\nu(\theta_2) := \theta_2 - \theta_3(\theta_2)$$

$$\mu(\theta_2) := \theta_4(\theta_2) - \theta_3(\theta_2)$$

8. Using equation 6.13e, calculate and plot the mechanical advantage of the linkage over the given range.

$$m_A(\theta_2) := \frac{c \cdot \sin(\mu(\theta_2))}{a \cdot \sin(\nu(\theta_2))} \cdot \frac{r_{in}}{r_{out}}$$



PROBLEM 6-60

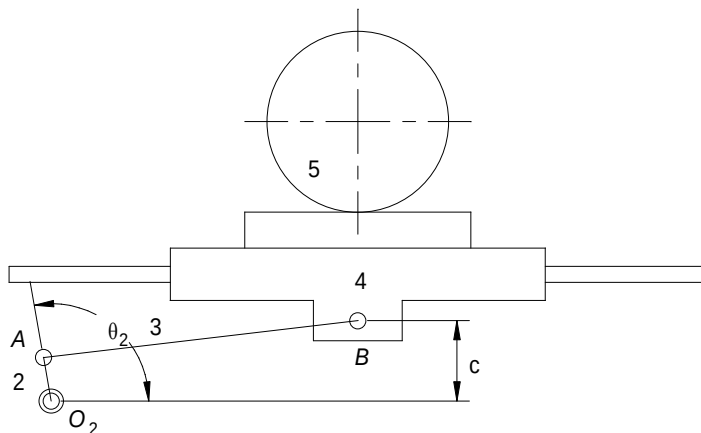
Statement: Figure P6-23 shows a surface grinder. The workpiece is oscillated under the spinning grinding wheel by the slider-crank linkage that has the dimensions given below. Calculate and plot the velocity of the grinding wheel contact point relative to the workpiece over one revolution of the crank.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 22 \cdot \text{mm}$	Offset	$c := 40 \cdot \text{mm}$
Link 3 (A to B)	$b := 157 \cdot \text{mm}$		
Grinding wheel diameter	$d := 90 \cdot \text{mm}$		
Input crank angular velocity	$\omega_2 := 120 \cdot \text{rpm}$	CCW	
Grinding wheel angular velocity	$\omega_5 := 3450 \cdot \text{rpm}$	CCW	

Solution: See Figure P6-23 and Mathcad file P0660.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

3. Determine θ_3 using equation 4.17.

$$\theta_3(\theta_2) := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi$$

4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3(\theta_2))} \cdot \omega_2$$

5. Determine the velocity of pin B using equation 6.22b:

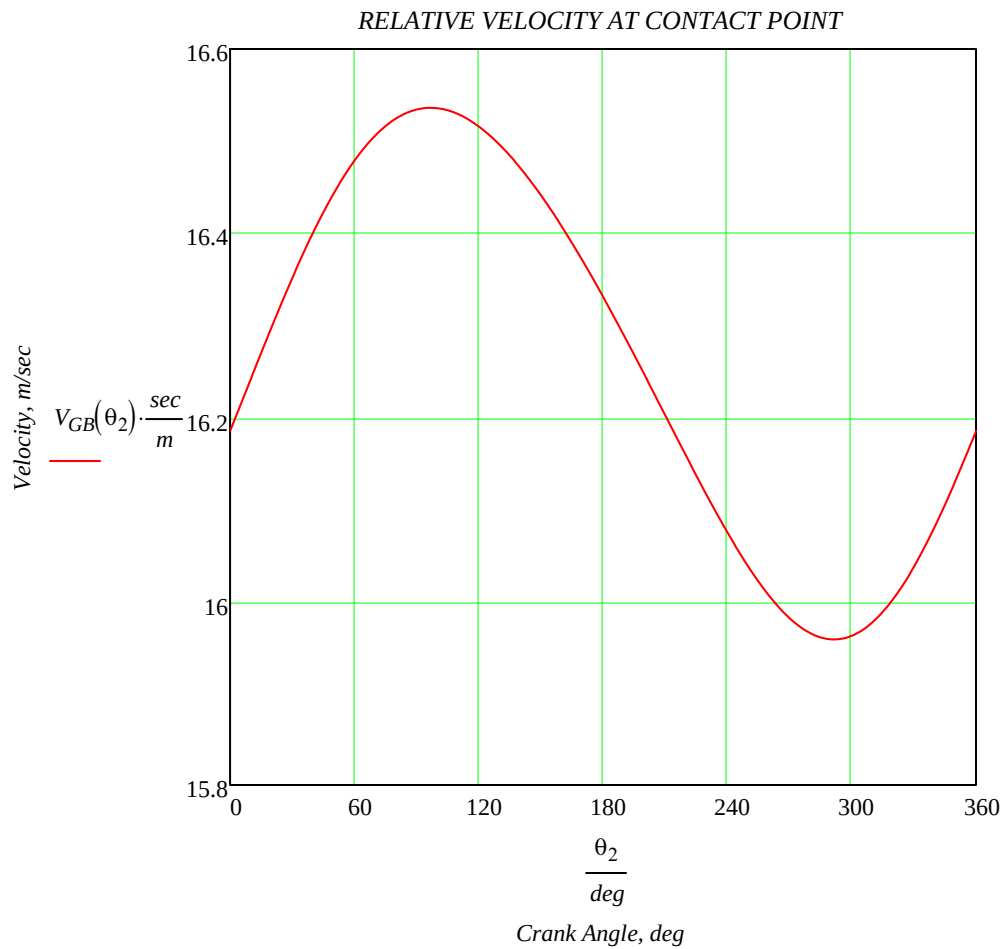
$$V_B(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_3(\theta_2) \cdot \sin(\theta_3(\theta_2)) \quad \text{Positive to the right}$$

6. Calculate the velocity of the grinding wheel contact point using equation 6.7:

$$V_G := \frac{d}{2} \cdot \omega_5 \quad V_G = 16.258 \frac{\text{m}}{\text{sec}} \quad \text{Directed to the right}$$

7. The velocity of the grinding wheel contact point relative to the workpiece, which has velocity V_B , is

$$V_{GB}(\theta_2) := V_G - V_B(\theta_2)$$



PROBLEM 6-61

Statement: Figure P6-24 shows an inverted slider-crank mechanism. Given the dimensions below, find ω_2 , ω_3 , ω_4 , V_{A4} , V_{trans} , and V_{slip} for the position shown with $V_{A2} = 20$ in/sec in the direction shown.

Given: Link lengths:

$$\text{Link 2 (O}_2\text{A)} \quad a := 2.5 \cdot \text{in} \quad \text{Link 4 (O}_4\text{A)} \quad c := 4.1 \cdot \text{in} \quad \text{Link 1 (O}_2\text{O}_4) \quad d := 3.9 \cdot \text{in}$$

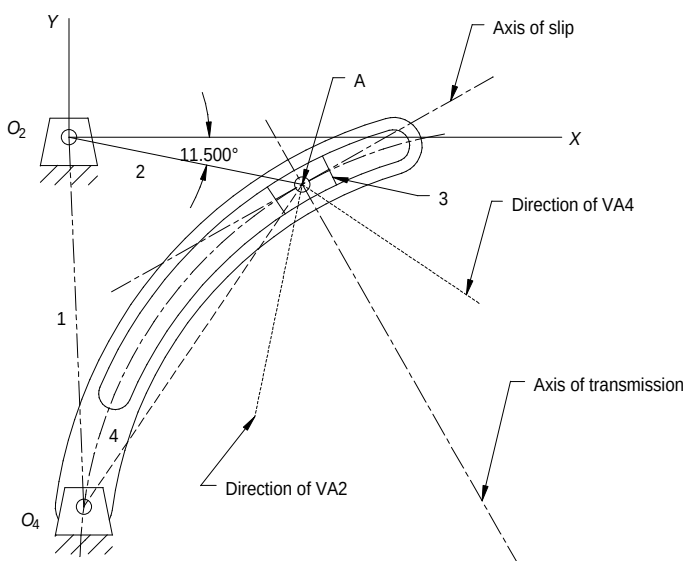
Measured angles:

$$\theta_2 := 75.5 \cdot \text{deg} \quad \theta_{trans} := 26.5 \cdot \text{deg} \quad \theta_{slip} := 116.5 \cdot \text{deg}$$

$$\text{Velocity of point A on links 2 and 3:} \quad V_{A2} := 20 \cdot \text{in} \cdot \text{sec}^{-1} \quad V_{A3} := V_{A2}$$

Solution: See Figure P6-24 and Mathcad file P0661.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



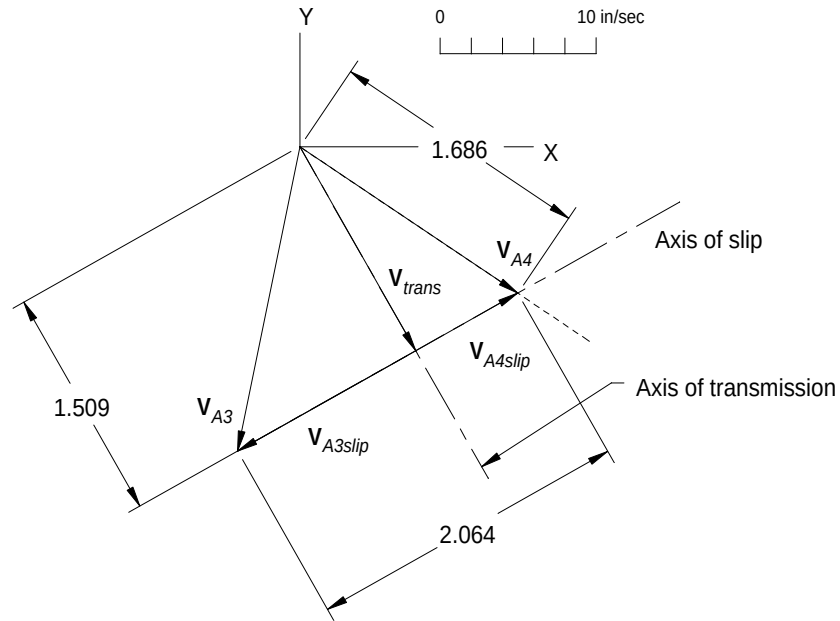
2. Use equation 6.7 to calculate the angular velocity of link 2.

$$\omega_2 := \frac{V_{A2}}{a} \quad \omega_2 = 8.000 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A on links 2 and 3. The equation to be solved graphically is

$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{A3slip}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.
- b. From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{A3slip}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{A3slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{A3slip}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{10 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$V_{A4} := 1.686 \cdot \text{in} \cdot k_v$ $V_{A4} = 16.9 \frac{\text{in}}{\text{sec}}$

$V_{slip} := 2.064 \cdot \text{in} \cdot k_v$ $V_{slip} = 20.6 \frac{\text{in}}{\text{sec}}$

$V_{trans} := 1.509 \cdot \text{in} \cdot k_v$ $V_{trans} = 15.1 \frac{\text{in}}{\text{sec}}$

5. Determine the angular velocity of link 4 using equation 6.7.

$\omega_4 := \frac{V_{A4}}{c}$ $\omega_4 = 4.1 \frac{\text{rad}}{\text{sec}}$ CW

Because link 3 slides within link 4, $\omega_3 = \omega_4$.

PROBLEM 6-62

Statement: Figure P6-25 shows a drag-link mechanism with dimensions. Write the necessary equations and solve them to calculate and plot the angular velocity of link 4 for an input of $\omega_2 = 1 \text{ rad/sec}$. Comment on the uses for this mechanism.

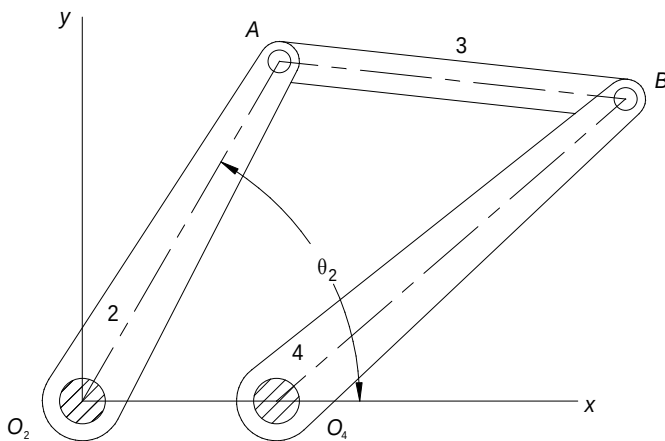
Given: Link lengths:

Link 2 (L_2)	$a := 1.38 \cdot \text{in}$	Link 3 (L_3)	$b := 1.22 \cdot \text{in}$
Link 4 (L_4)	$c := 1.62 \cdot \text{in}$	Link 1 (L_1)	$d := 0.68 \cdot \text{in}$

Input crank angular velocity $\omega_2 := -1 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$

Solution: See Figure P6-25 and Mathcad file P0662.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this Grashof double crank.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.4928 \qquad K_2 = 0.4198$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \qquad K_3 = 0.7834$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.5574 \quad K_5 = -0.3655$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

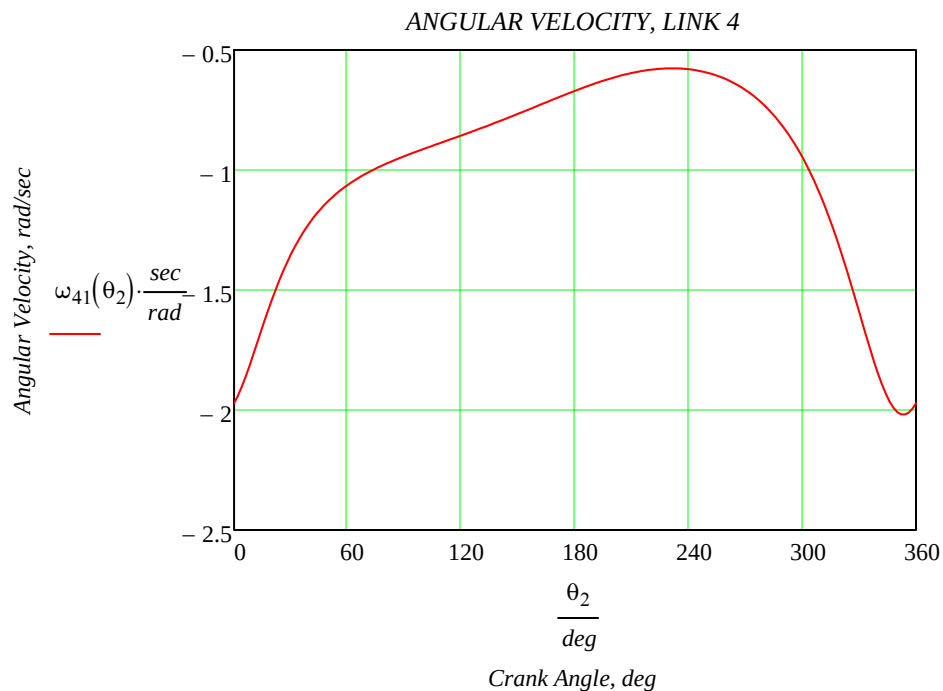
$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

9. Plot the angular velocity of link 4.



PROBLEM 6-63

Statement: Figure P6-25 shows a drag-link mechanism with dimensions. Write the necessary equations and solve them to calculate and plot the centroids of instant center $I_{2,4}$.

Given: Measured link lengths:

$$\text{Link 2 } (L_2) \quad L_2 := 1.38 \cdot \text{in} \quad \text{Link 3 } (L_3) \quad L_3 := 1.22 \cdot \text{in}$$

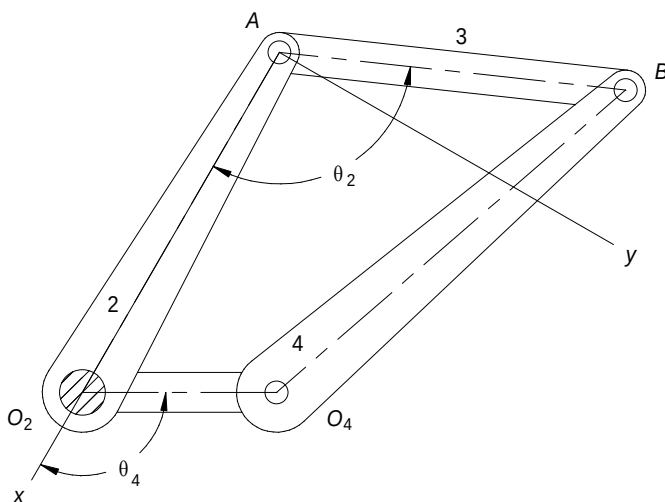
$$\text{Link 4 } (L_4) \quad L_4 := 1.62 \cdot \text{in} \quad \text{Link 1 } (L_1) \quad L_1 := 0.68 \cdot \text{in}$$

$$\text{Input crank angular velocity} \quad \omega_2 := -1 \cdot \text{rad} \cdot \text{sec}^{-1}$$

Solution: See Figure P6-25 and Mathcad file P0663.

1. Draw the linkage to scale and label it. Instant center $I_{2,4}$ is at the intersection of line AB with line O_2O_4 . To get the first centroid, ground link 2 and let link 3 be the input. Then we have

$$a := L_3 \quad b := L_4 \quad c := L_1 \quad d := L_2$$



2. Determine the range of motion for this Grashof double rocker. From Figure 3-1a on page 80, one toggle angle is

$$\theta_{20} := \arccos \left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot a \cdot d} \right] \quad \theta_{20} = 124.294 \text{ deg}$$

The other toggle angle is the negative of this. The range of motion is

$$\theta_2 := -\theta_{20} + 1 \cdot \text{deg}, -\theta_{20} + 2 \cdot \text{deg} .. \theta_{20} - 1 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 1.1311$$

$$K_2 = 2.0294$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 0.7418$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\operatorname{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Calculate the coordinates of the intersection of line BC with line AD . This will be the instant center $I_{2,4}$.

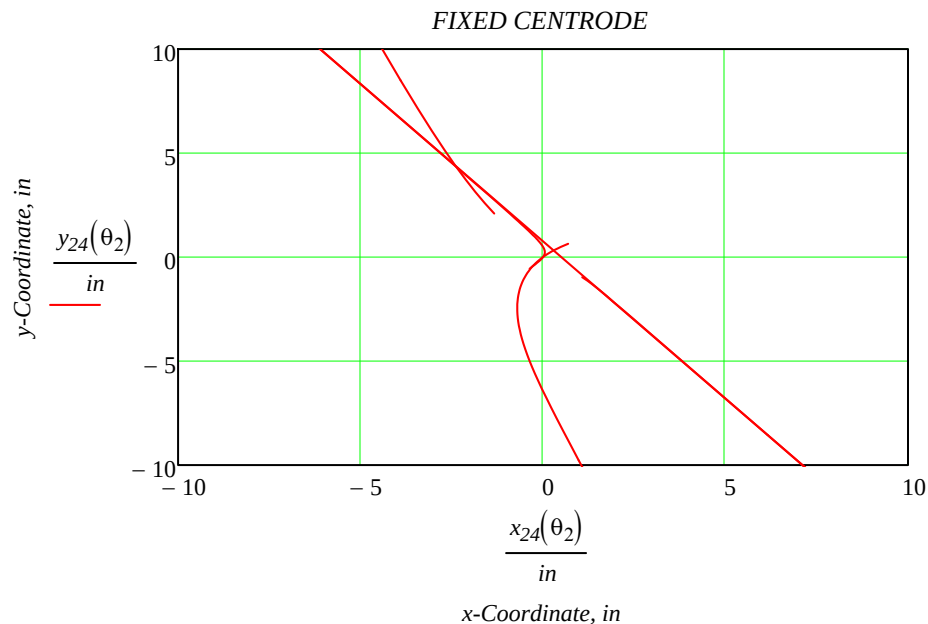
$$\text{Line } BC: \quad y := x \cdot \tan(\theta_2)$$

$$\text{Line } AD: \quad y - 0 := (x - d) \cdot \tan(\theta_4)$$

Eliminating y and solving for the x - and y -coordinates of the intersection,

$$x_{24}(\theta_2) := \frac{d \cdot \tan(\theta_{41}(\theta_2))}{\tan(\theta_{41}(\theta_2)) - \tan(\theta_2)} \quad y_{24}(\theta_2) := x_{24}(\theta_2) \cdot \tan(\theta_2)$$

6. Plot the fixed centrode.

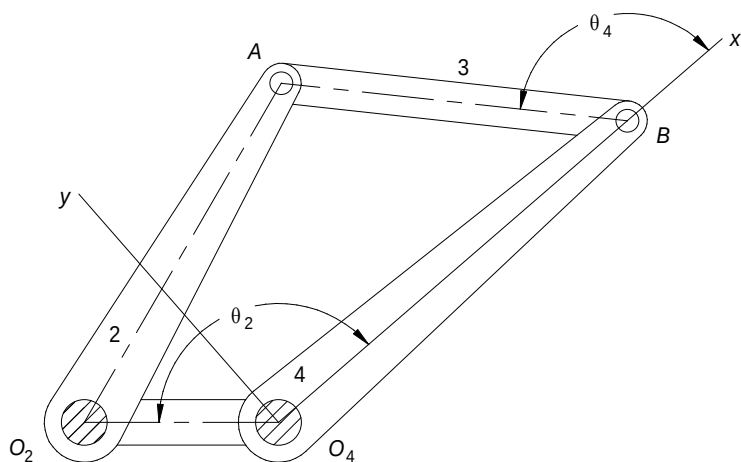


7. Invert the linkage, making C and D the fixed pivots. Then,

$$a := L_1 \quad b := L_2 \quad c := L_3 \quad d := L_4$$

8. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. 360 \cdot \text{deg}$$



9. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 2.3824 \quad K_2 = 1.3279$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.6097$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

10. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{42}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

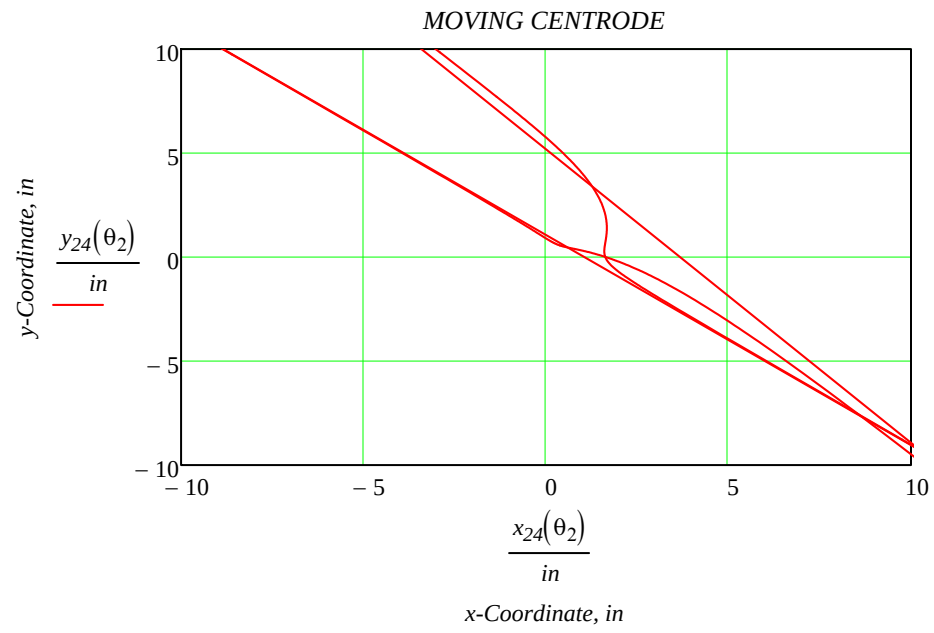
11. Calculate the coordinates of the intersection of line BC with line AD . This will be the instant center $I_{2,4}$.

$$\begin{aligned} \text{Line } AD: \quad y &:= x \cdot \tan(\theta_2) \\ \text{Line } BC: \quad y - 0 &:= (x - d) \cdot \tan(\theta_4) \end{aligned}$$

Eliminating y and solving for the x - and y -coordinates of the intersection,

$$x_{24}(\theta_2) := \frac{d \cdot \tan(\theta_{42}(\theta_2))}{\tan(\theta_{42}(\theta_2)) - \tan(\theta_2)} \quad y_{24}(\theta_2) := x_{24}(\theta_2) \cdot \tan(\theta_2)$$

6. Plot the moving centrod. (See next page.)



PROBLEM 6-64

Statement: Figure P6-26 shows a mechanism with dimensions. Use a graphical method to calculate the velocities of points A, B, and C and the velocity of slip for the position shown.

Given: Link lengths and angles:

Link 1 (O_2O_4) $d := 1.22 \cdot \text{in}$ Angle O_2O_4 makes with X axis $\theta_1 := 56.5 \cdot \text{deg}$

Link 2 (O_2A) $a := 1.35 \cdot \text{in}$ Angle θ_2 makes with X axis $\theta_2 := 14 \cdot \text{deg}$

Link 4 (O_4B) $e := 1.36 \cdot \text{in}$

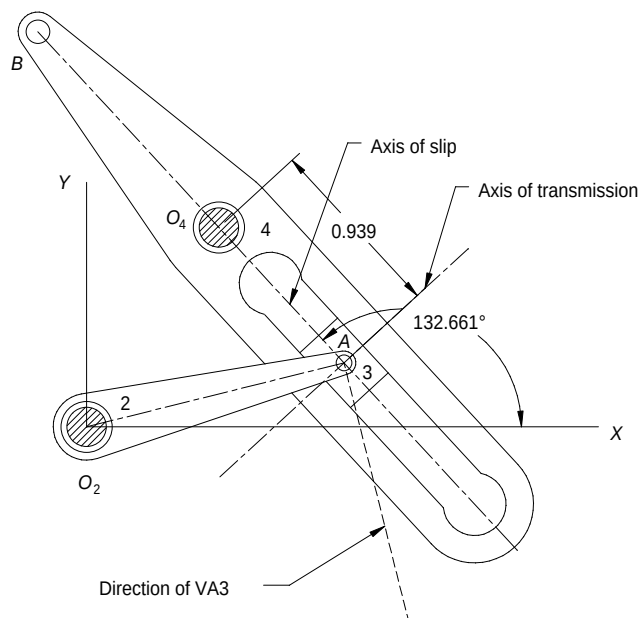
Link 5 (BC) $f := 2.69 \cdot \text{in}$

Link 6 (O_6C) $g := 1.80 \cdot \text{in}$ Angle O_6C makes with X axis $\theta_6 := 88 \cdot \text{deg}$

Angular velocity of link 2 $\omega_2 := 20 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

Solution: See Figure P6-26 and Mathcad file P0664.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



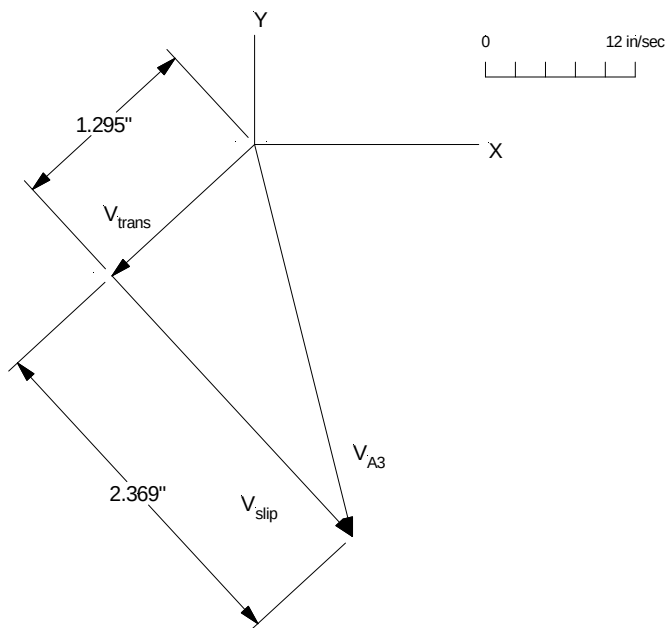
2. Use equation 6.7 to calculate the magnitude of the velocity at point A on links 2 and 3.

$$V_{A3} := a \cdot \omega_2 \quad V_{A3} = 27.000 \cdot \frac{\text{in}}{\text{sec}} \quad \theta_{VA3} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{VA3} = -76.0 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A on link 3. The equation to be solved graphically is

$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.
- b. From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

$$\begin{aligned} \text{Velocity scale factor: } k_v &:= \frac{12 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}} \\ V_{slip} &:= 2.369 \cdot \text{in} \cdot k_v & V_{slip} &= 28.428 \cdot \frac{\text{in}}{\text{sec}} \\ V_{trans} &:= 1.295 \cdot \text{in} \cdot k_v & V_{trans} &= 15.540 \cdot \frac{\text{in}}{\text{sec}} \end{aligned}$$

5. The true velocity of point A on link 4 is V_{trans} ,

$$V_{A4} := V_{trans} \quad V_{A4} = 15.54 \cdot \frac{\text{in}}{\text{sec}}$$

6. Determine the angular velocity of link 4 using equation 6.7.

$$\text{From the linkage layout above: } c := 0.939 \cdot \text{in} \quad \text{and} \quad \theta_4 := 132.661 \cdot \text{deg}$$

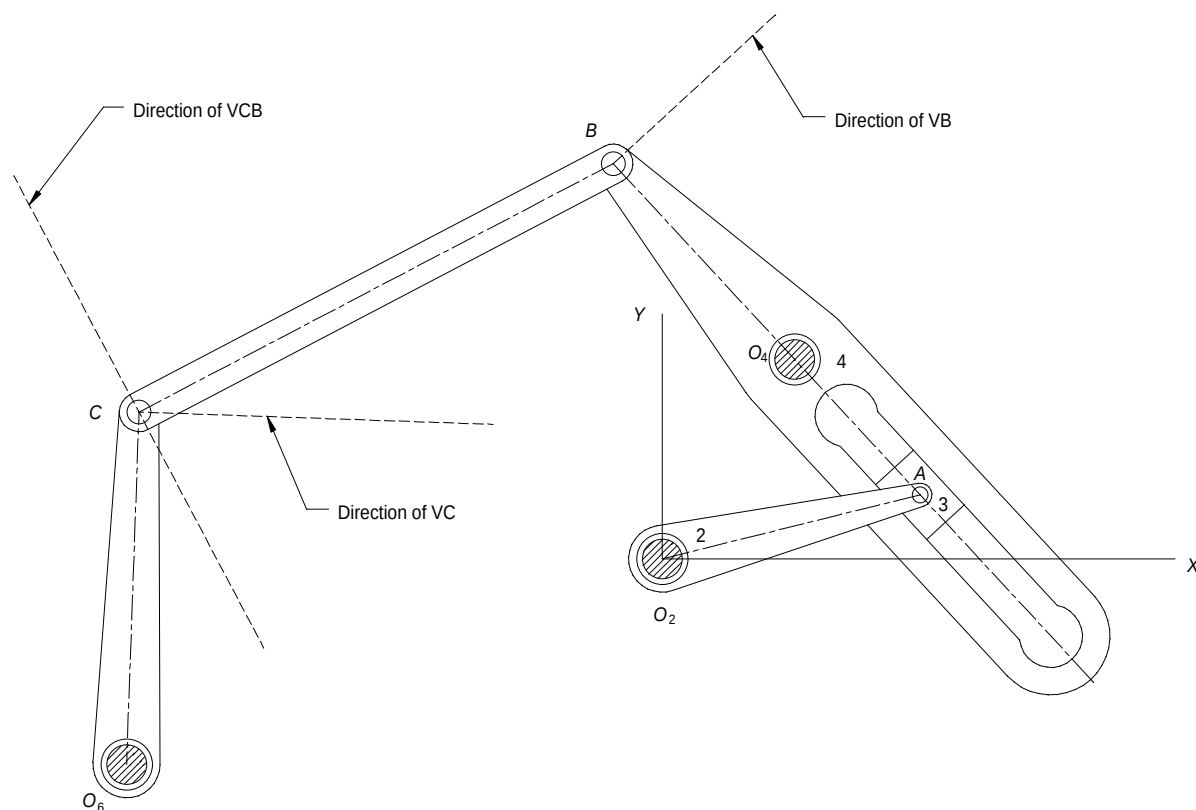
$$\omega_4 := \frac{V_{A4}}{c} \quad \omega_4 = 16.550 \cdot \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

7. Determine the magnitude and sense of the vector $\mathbf{V}_B$ using equation 6.7.

$$V_B := e \cdot \omega_4 \quad V_B = 22.507 \cdot \frac{\text{in}}{\text{sec}}$$

$$\theta_{V_{A4}} := \theta_4 - 90 \cdot \text{deg} \quad \theta_{V_{A4}} = 42.661 \cdot \text{deg}$$

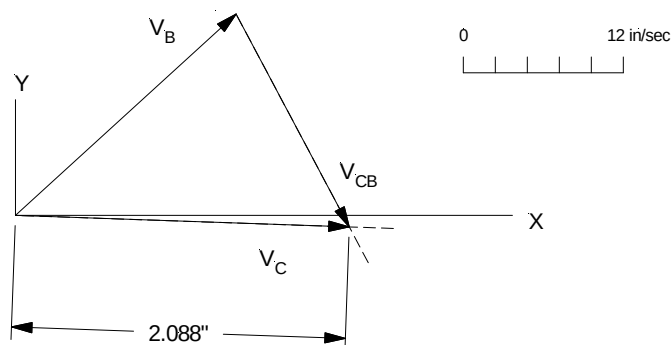
8. Draw links 1, 4, 5, and 6 to a convenient scale. Indicate the directions of the velocity vectors of interest. (See next page.)



9. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
- From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.
- From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.



10. From the velocity triangle we have:

$$\begin{aligned} \text{Velocity scale factor: } k_v &:= \frac{12 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}} & \theta_{VC} &:= \theta_6 - 90 \cdot \text{deg} \\ V_C &:= 2.088 \cdot \text{in} \cdot k_v & V_C &= 25.1 \cdot \frac{\text{in}}{\text{sec}} & \theta_{VC} &= -2.0 \cdot \text{deg} \end{aligned}$$

PROBLEM 6-65

Statement: Figure P6-27 shows a cam and follower. Distances are given below. Find the velocities of points A and B, the velocity of transmission, velocity of slip, and ω_3 if $\omega_2 = 50$ rad/sec (CW). Use a graphical method.

Given: $\omega_2 := 50 \cdot \text{rad} \cdot \text{sec}^{-1}$

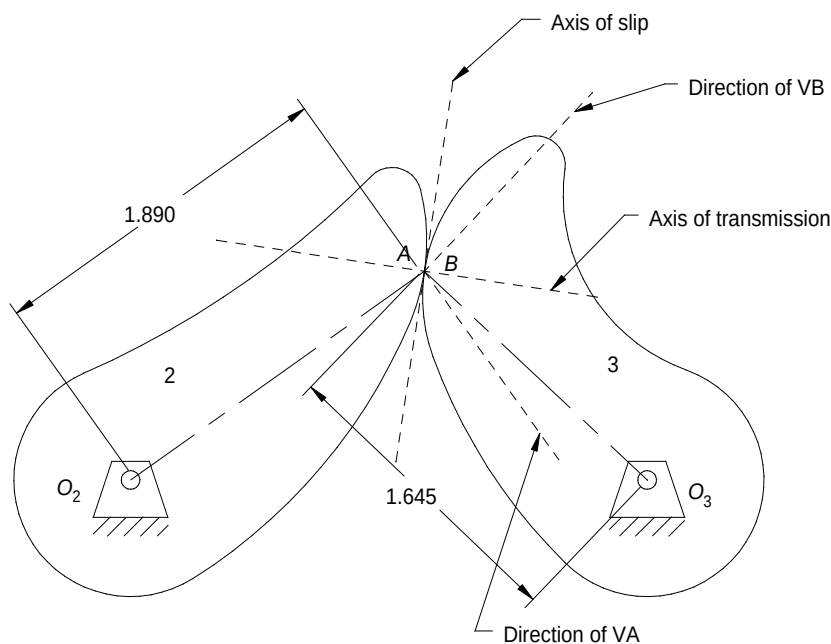
Distance from O_2 to A: $a := 1.890 \cdot \text{in}$

Distance from O_3 to B: $b := 1.645 \cdot \text{in}$

Assumptions: Roll-slide contact

Solution: See Figure P6-27 and Mathcad file P0665.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



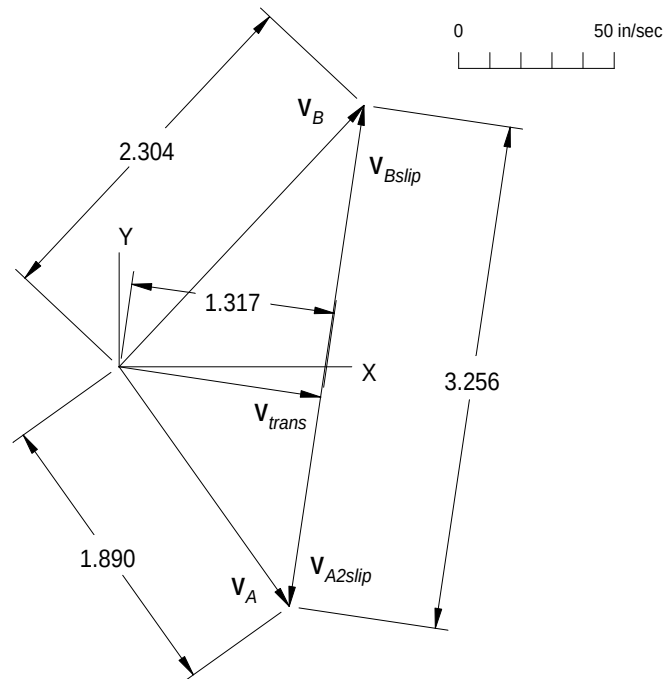
2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 94.500 \frac{\text{in}}{\text{sec}}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A. The equation to be solved graphically is

$$\mathbf{V}_A = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_A$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{50 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_A := 1.890 \cdot \text{in} \cdot k_v \quad V_A = 94.5 \frac{\text{in}}{\text{sec}}$$

$$V_B := 2.304 \cdot \text{in} \cdot k_v \quad V_B = 115.2 \frac{\text{in}}{\text{sec}}$$

$$V_{slip} := 3.256 \cdot \text{in} \cdot k_v \quad V_{slip} = 162.8 \frac{\text{in}}{\text{sec}}$$

$$V_{trans} := 1.317 \cdot \text{in} \cdot k_v \quad V_{trans} = 65.8 \frac{\text{in}}{\text{sec}}$$

5. Determine the angular velocity of link 3 using equation 6.7.

$$\omega_3 := \frac{V_B}{b} \quad \omega_3 = 70.0 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

PROBLEM 6-66

Statement: Figure P6-28 shows a quick-return mechanism with dimensions. Use a graphical method to calculate the velocities of points A, B, and C and the velocity of slip for the position shown.

Given: Link lengths and angles:

Link 1 (O_2O_4) $d := 1.69 \cdot \text{in}$ Angle O_2O_4 makes with X axis $\theta_1 := 15.5 \cdot \text{deg}$

Link 2 (L_2) $a := 1.00 \cdot \text{in}$ Angle link 2 makes with X axis $\theta_2 := 99 \cdot \text{deg}$

Link 4 (L_4) $e := 4.76 \cdot \text{in}$

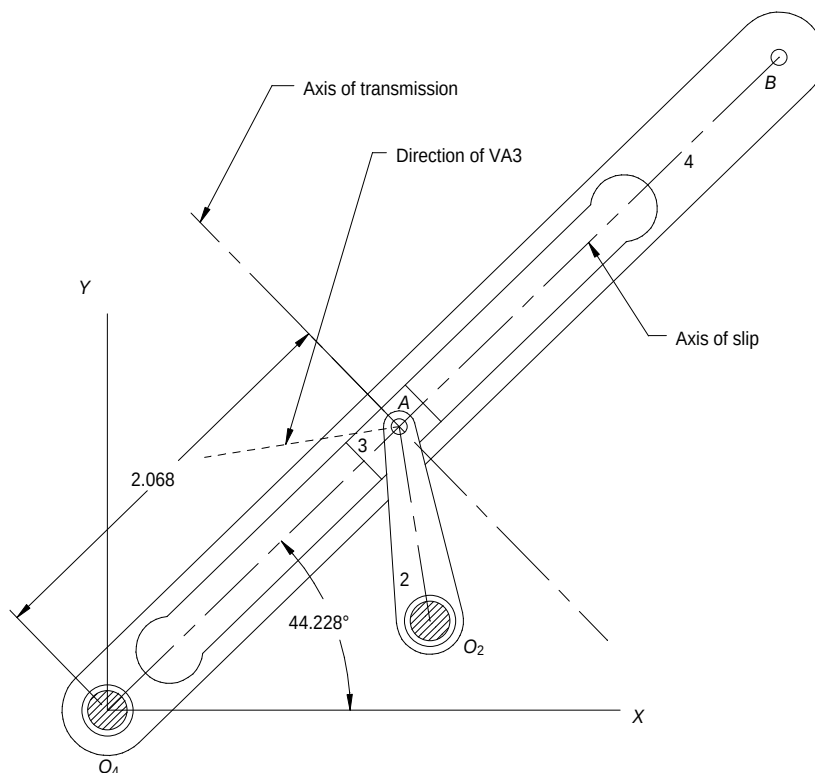
Link 5 (L_5) $f := 4.55 \cdot \text{in}$

Offset (O_2C) $g := 2.86 \cdot \text{in}$

Angular velocity of link 2 $\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW

Solution: See Figure P6-28 and Mathcad file P0666.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



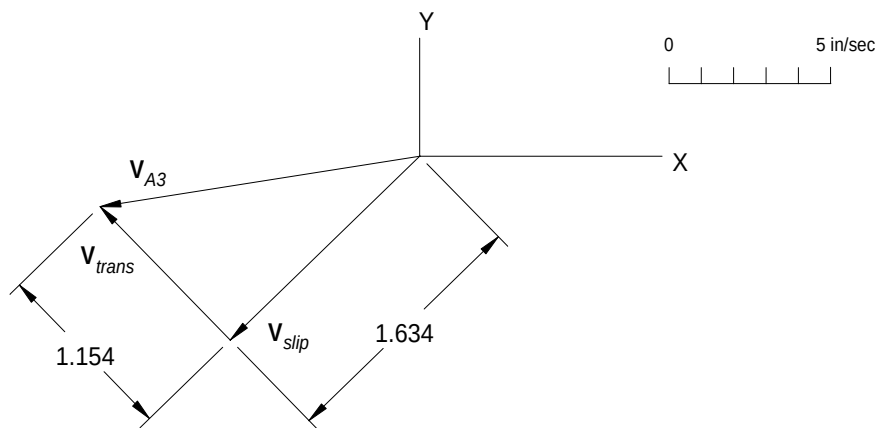
2. Use equation 6.7 to calculate the magnitude of the velocity at point A on links 2 and 3.

$$V_{A2} := a \cdot \omega_2 \quad V_{A2} = 10.000 \frac{\text{in}}{\text{sec}} \quad \theta_{VA2} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{VA2} = 189.0 \text{ deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A on link 3. The equation to be solved graphically is

$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.
- From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.
- From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

$$\begin{aligned} \text{Velocity scale factor: } k_v &:= \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}} \\ V_{slip} &:= 1.634 \cdot \text{in} \cdot k_v & V_{slip} &= 8.170 \frac{\text{in}}{\text{sec}} \\ V_{trans} &:= 1.154 \cdot \text{in} \cdot k_v & V_{trans} &= 5.770 \frac{\text{in}}{\text{sec}} \end{aligned}$$

5. The true velocity of point A on link 4 is V_{trans} ,

$$V_{A4} := V_{trans} \quad V_{A4} = 5.77 \frac{\text{in}}{\text{sec}}$$

6. Determine the angular velocity of link 4 using equation 6.7.

$$\text{From the linkage layout above: } c := 2.068 \cdot \text{in} \quad \text{and} \quad \theta_4 := 44.228 \cdot \text{deg}$$

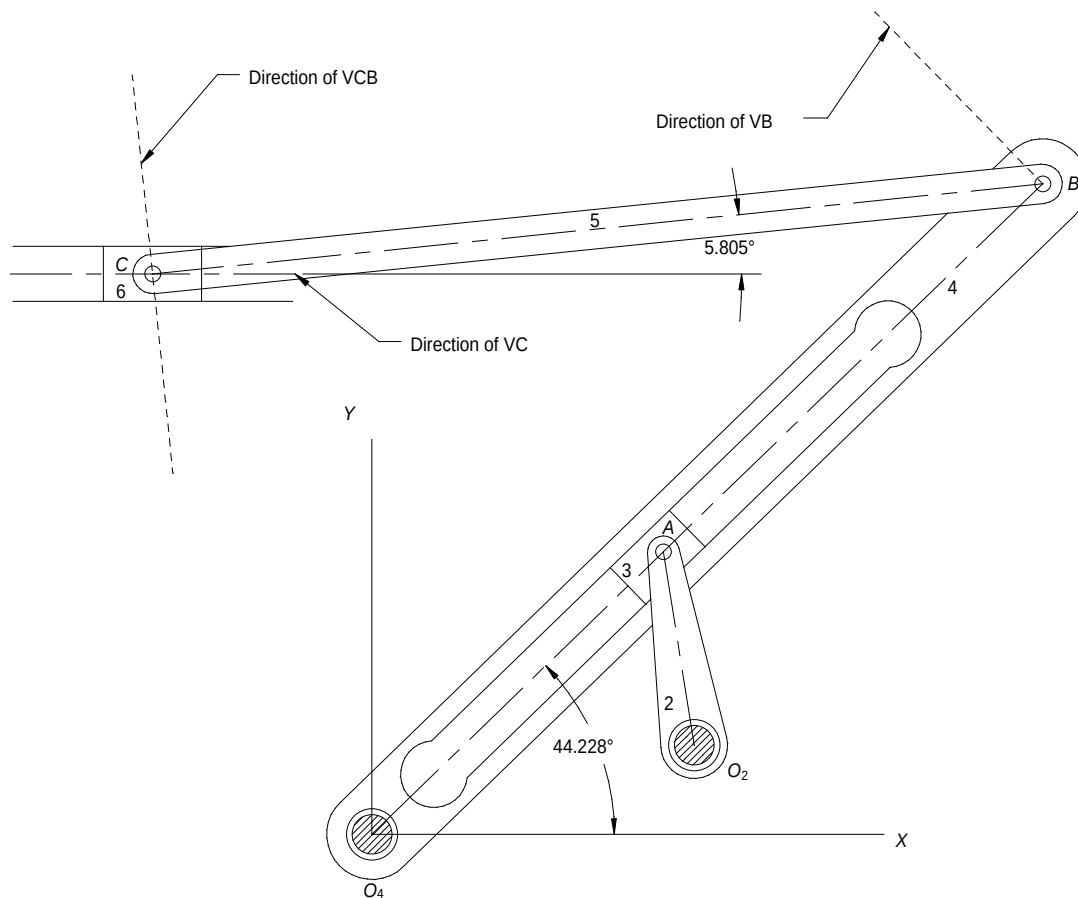
$$\omega_4 := \frac{V_{A4}}{c} \quad \omega_4 = 2.790 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

7. Determine the magnitude and sense of the vector $\mathbf{V}_B$ using equation 6.7.

$$V_B := e \cdot \omega_4 \quad V_B = 13.281 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VB} := \theta_4 + 90 \cdot \text{deg} \quad \theta_{VB} = 134.228 \text{ deg}$$

8. Draw links 1, 4, 5, and 6 to a convenient scale. Indicate the directions of the velocity vectors of interest. (See next page.)



9. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

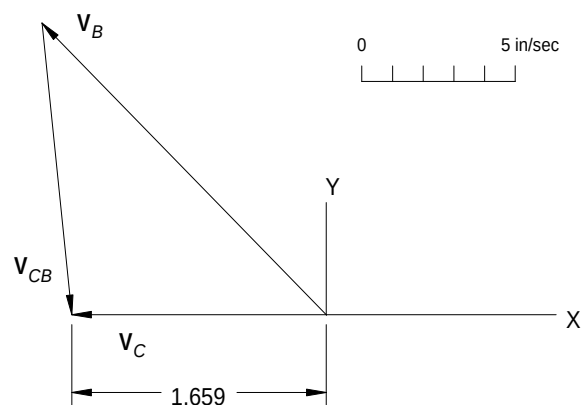
- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
- From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.
- From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.

10. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_C := 1.659 \cdot \text{in} \cdot k_v \quad V_C = 8.30 \frac{\text{in}}{\text{sec}}$$

$$\theta_{V_C} := 180 \cdot \text{deg}$$



PROBLEM 6-67

Statement: Figure P6-29 shows a drum pedal mechanism. For the dimensions given below, find and plot the mechanical advantage and the velocity ratio of the linkage over its range of motion. If the input velocity V_{in} is a constant and F_{in} is constant, find the output velocity, output force, and power in over the range of motion.

Given: Link lengths:

Link 2 (O_2A)	$a := 100 \cdot mm$	Link 3 (AB)	$b := 28 \cdot mm$
Link 4 (O_4B)	$c := 64 \cdot mm$	Link 1 (O_2O_4)	$d := 56 \cdot mm$

Distance to force application:

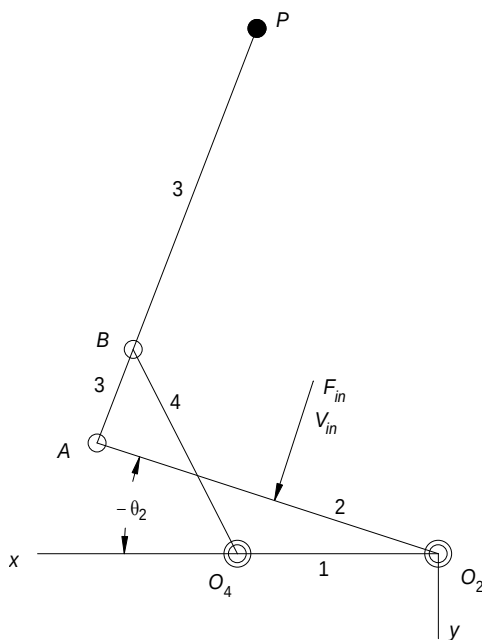
Link 2	$r_{in} := 48 \cdot mm$	Link 3 (AP)	$r_{out} := 124 \cdot mm$

Input force and velocity:	$F_{in} := 50 \cdot N$	$V_{in} := 3 \cdot m \cdot sec^{-1}$

Range of positions of link 2:	$\theta_{20} := 162 \cdot deg$	$\theta_{21} := 171 \cdot deg$

Solution: See Figure P6-29 and Mathcad file P0667.

1. Draw the mechanism to scale and label it.



2. Calculate the range of θ_2 in the local coordinate system (required to calculate θ_3 and θ_4).

Rotation angle of local xy system to global XY system: $\alpha := 180 \cdot deg$

$$\theta_2 := \theta_{20} - \alpha, (\theta_{20} - \alpha) + 1 \cdot deg .. \theta_{21} - \alpha$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 0.5600 \quad K_2 = 0.8750$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.2850$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 2.0000 \quad K_5 = -1.7543$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

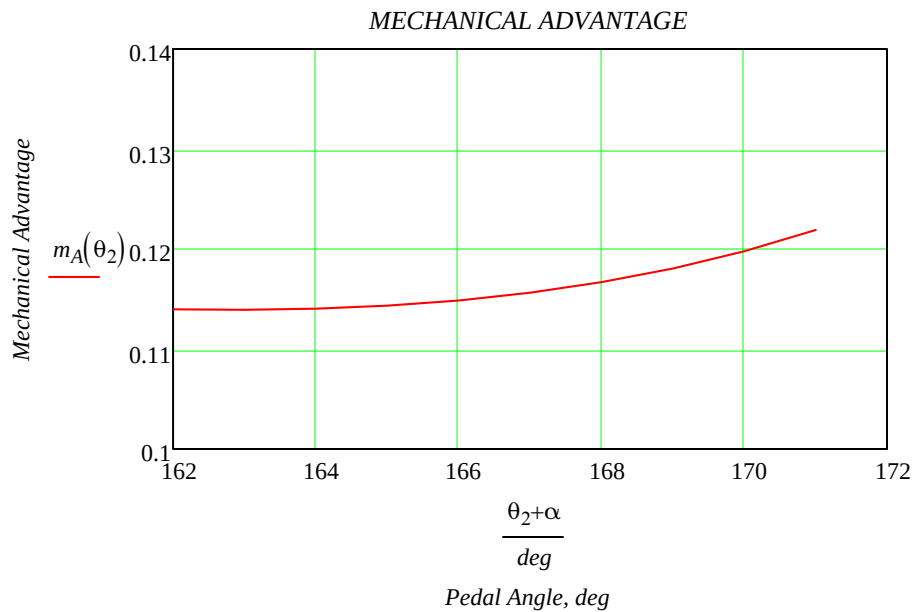
$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

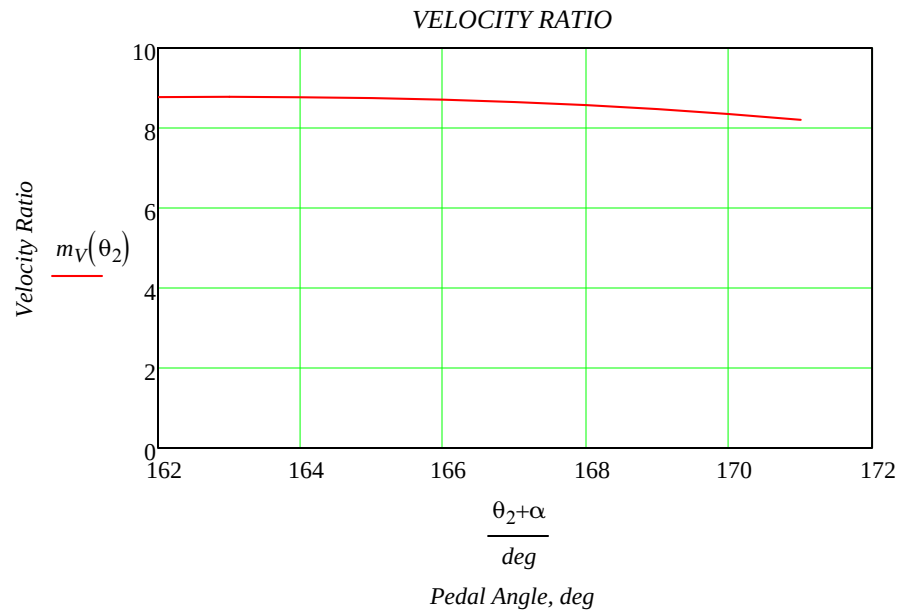
7. Using equations 6.13d and 6.18a, where ω_{out} is ω_3 , calculate and plot the mechanical advantage of the linkage over the given range.

$$m_A(\theta_2) := \frac{b \cdot \sin(\theta_3(\theta_2) - \theta_4(\theta_2))}{a \cdot \sin(\theta_4(\theta_2) - \theta_2)} \cdot \frac{r_{in}}{r_{out}}$$



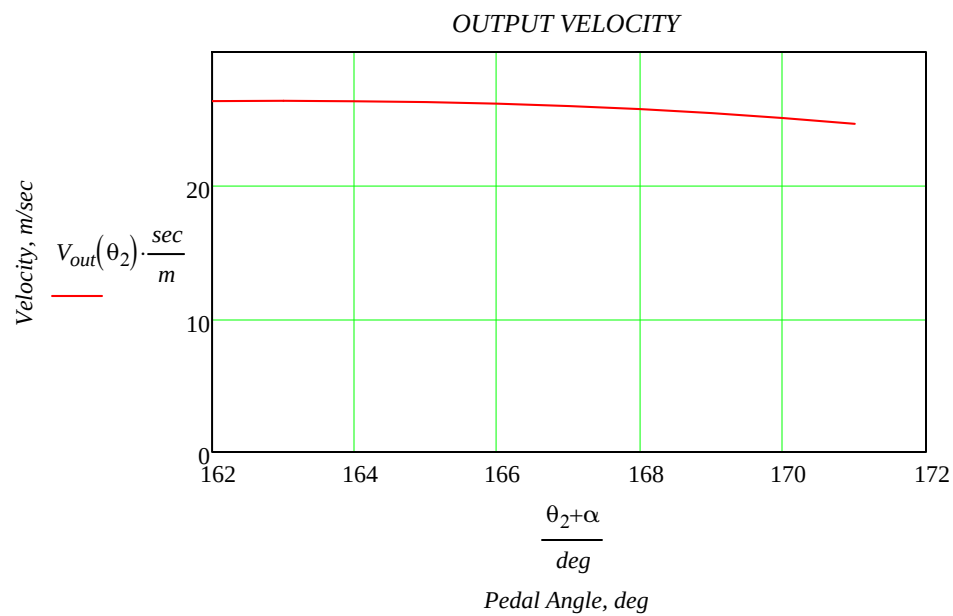
8. Calculate and plot the velocity ratio using equation 6.13d,

$$m_V(\theta_2) := \frac{1}{m_A(\theta_2)}$$



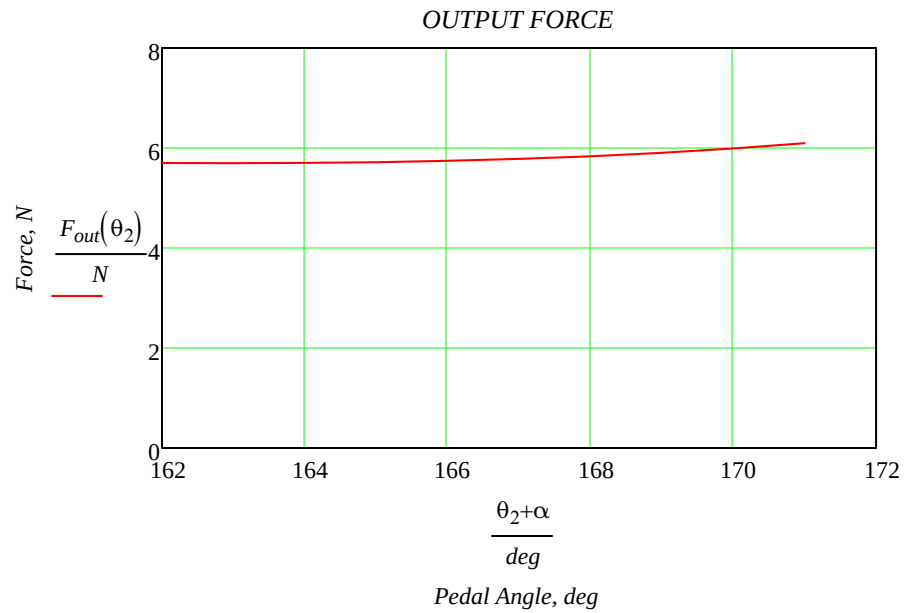
9. Calculate and plot the output velocity using equation 6.13a.

$$V_{out}(\theta_2) := V_{in} \cdot m_V(\theta_2)$$



10. Calculate and plot the output force using equation 6.13a.

$$F_{out}(\theta_2) := F_{in} \cdot m_A(\theta_2)$$



PROBLEM 6-68

Statement: Figure 3-33 shows a sixbar slider crank linkage. Find all of its instant centers in the position shown:

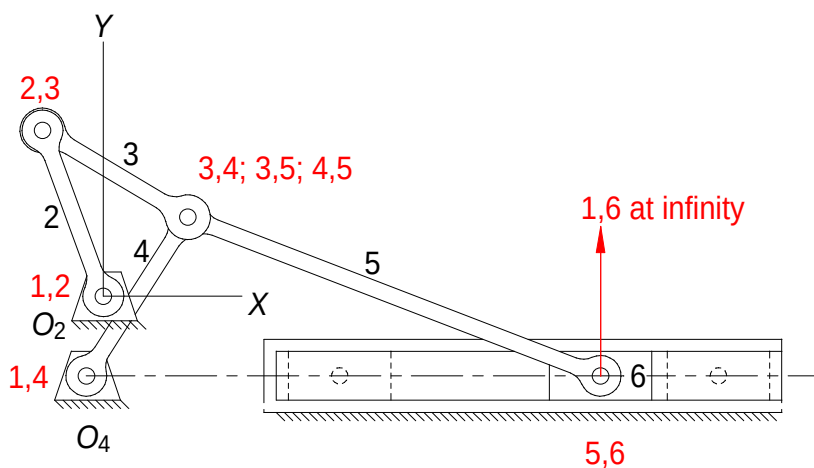
Given: Number of links $n := 6$

Solution: See Figure 3-33 and Mathcad file P0668.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection (8).



2. Use Kennedy's Rule and a linear graph to find the remaining 7 ICs:

$I_{1,3}; I_{1,5}; I_{2,4}; I_{2,5}; I_{2,6}; I_{3,6};$ and $I_{4,6}$

$I_{1,5}: I_{1,6}-I_{5,6}$ and $I_{1,4}-I_{4,5}$

$I_{2,5}: I_{1,2}-I_{1,5}$ and $I_{2,3}-I_{3,5}$

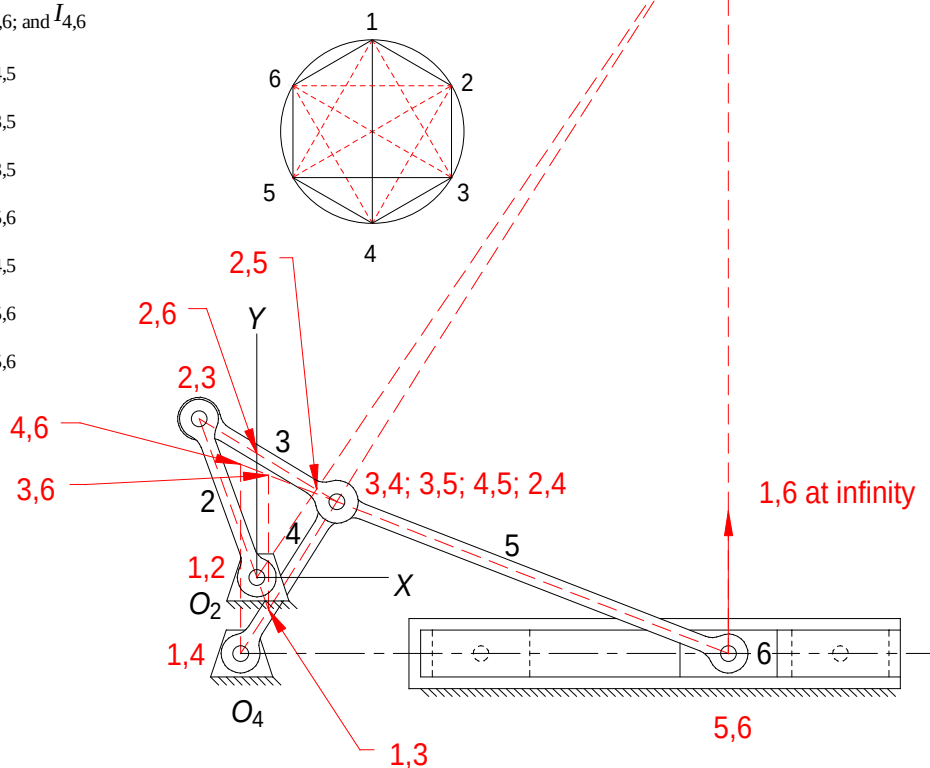
$I_{1,3}: I_{1,2}-I_{2,3}$ and $I_{1,5}-I_{3,5}$

$I_{3,6}: I_{1,6}-I_{1,3}$ and $I_{3,5}-I_{5,6}$

$I_{2,4}: I_{2,3}-I_{3,4}$ and $I_{2,5}-I_{4,5}$

$I_{2,6}: I_{1,2}-I_{1,6}$ and $I_{2,5}-I_{5,6}$

$I_{4,6}: I_{1,4}-I_{1,6}$ and $I_{4,5}-I_{5,6}$



PROBLEM 6-69

Statement: Calculate and plot the centroids of instant center I_{24} of the linkage in Figure 3-33 so that a pair of noncircular gears can be made to replace the driver dyad 23.

Given: Link lengths:

$$\text{Input crank } (L_2) \quad L_2 := 2.170 \quad \text{Fourbar coupler } (L_3) \quad L_3 := 2.067$$

$$\text{Output crank } (L_4) \quad L_4 := 2.310 \quad \text{Fourbar ground link } (L_1) \quad L_1 := 1.000$$

Solution: See Figure 3-33 and Mathcad file P0669.

1. Invert the linkage, grounding link 2 such that the input link is 3, the coupler is 4, and the output link is 1.

$$a := L_3 \quad b := L_4 \quad c := L_1 \quad d := L_2$$

2. Define the input crank motion for this inversion: $\theta_2 := 47 \cdot \text{deg}, 47.5 \cdot \text{deg} \dots 102.5 \cdot \text{deg}$

3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 1.0498$$

$$K_2 = 2.1700$$

$$K_3 = 1.1237$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

$$\theta_4(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

5. Calculate the coordinates of the intersection of links 1 and 3 in the xy coordinate system.

$$x_{242}(\theta_2) := -\frac{d \cdot \tan(\theta_4(\theta_2))}{\tan(\theta_2) - \tan(\theta_4(\theta_2))} \quad y_{242}(\theta_2) := x_{242}(\theta_2) \cdot \tan(\theta_2)$$

6. Invert the linkage, grounding link 4 such that the input link is 1, the coupler is 2, and the output link is 3.

$$a := L_1 \quad b := L_2 \quad c := L_3 \quad d := L_4$$

8. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.3100$$

$$K_2 = 1.1176$$

$$K_3 = 1.4271$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

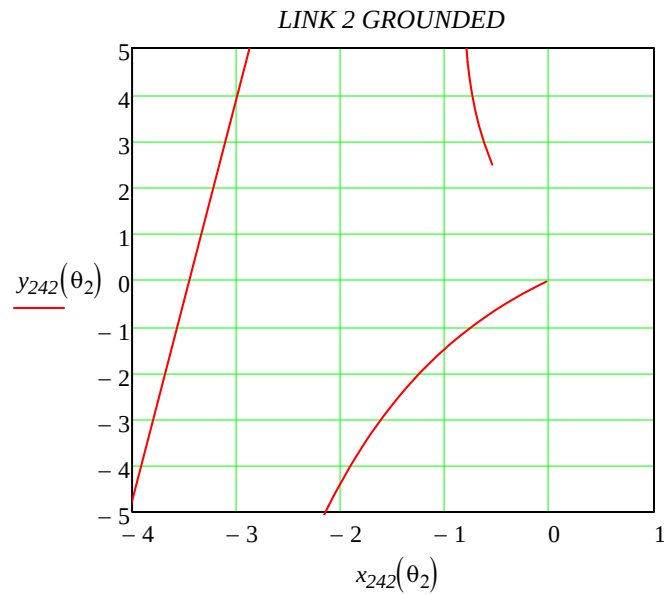
$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

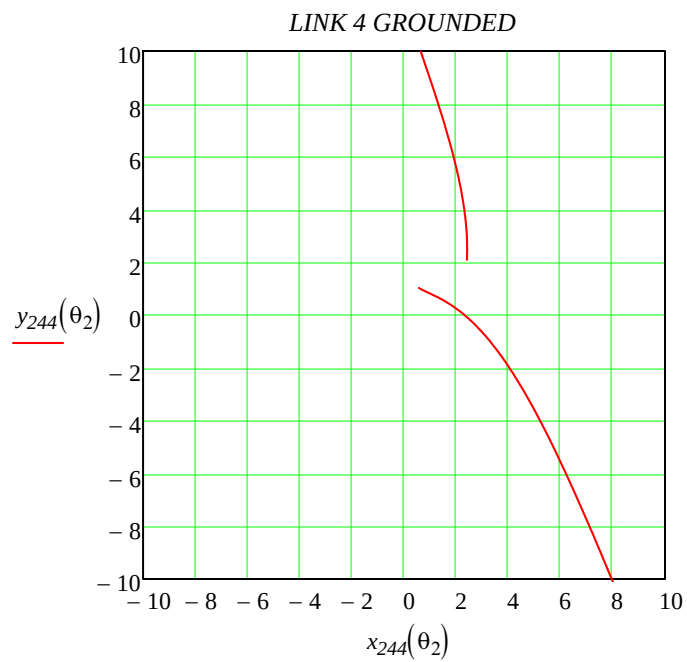
$$\theta_4(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

5. Calculate the coordinates of the intersection of links 1 and 3 in the xy coordinate system.

$$x_{244}(\theta_2) := -\frac{d \cdot \tan(\theta_4(\theta_2))}{\tan(\theta_2) - \tan(\theta_4(\theta_2))} \quad y_{244}(\theta_2) := x_{244}(\theta_2) \cdot \tan(\theta_2)$$



7. Define the input crank motion for this inversion: $\theta_2 := 41 \cdot \text{deg}, 42 \cdot \text{deg} \dots 241 \cdot \text{deg}$



PROBLEM 6-70a

Statement: Find the velocity of the slider in Figure 3-33 for $\theta_2 = 110^\circ$ with respect to the global X axis assuming $\omega_2 = 1 \text{ rad/sec CW}$. Use a graphical method.

Given: Link lengths:

Link 2 (O_2 to A) $a := 2.170 \cdot \text{in}$ Link 3 (A to B) $b := 2.067 \cdot \text{in}$

Link 4 (O_4 to B) $c := 2.310 \cdot \text{in}$ Link 1 (O_2 to O_4) $d := 1.000 \cdot \text{in}$

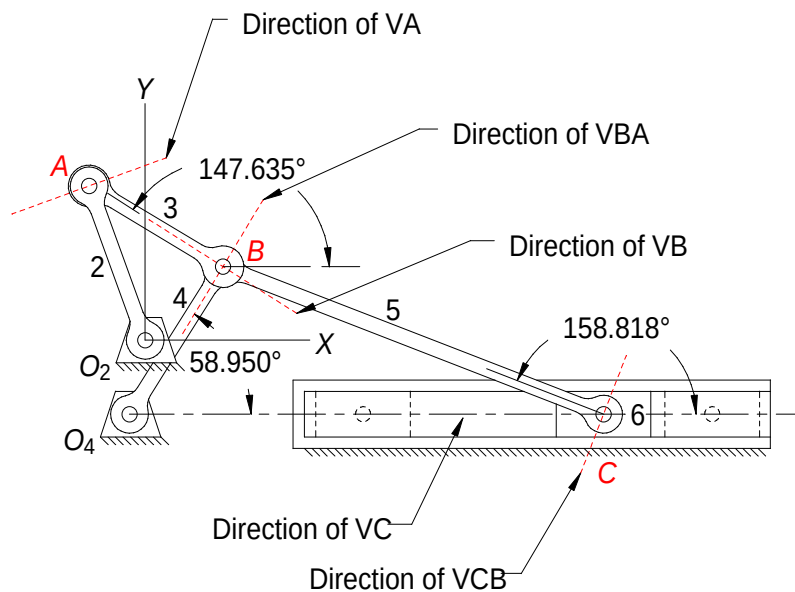
Link 5 (B to C) $e := 5.400$

Coordinate angle $\delta := -102 \cdot \text{deg}$ Crank angle: $\theta_2 := 110 \cdot \text{deg}$

Input crank angular velocity $\omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

Solution: See Figure P6-33 and Mathcad file P0670a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

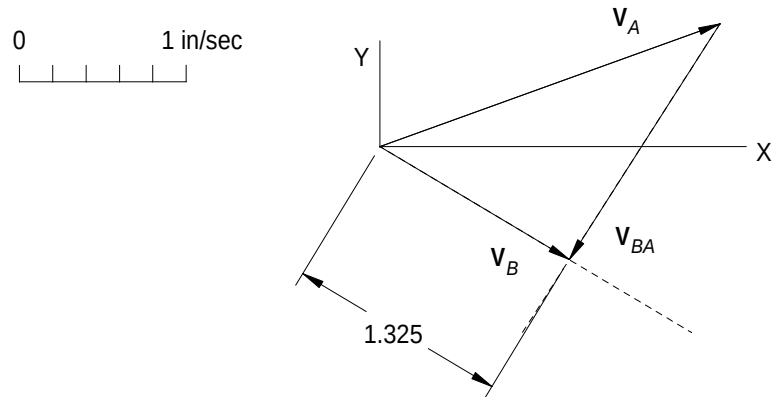
$$V_A := a \cdot \omega_2 \quad V_A = 2.170 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{VA} = 20.000 \text{ deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity V_{BA} , and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

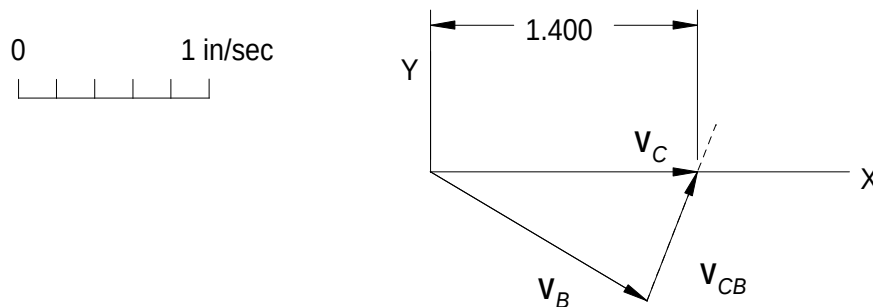
$$\text{Velocity scale factor: } k_v := \frac{1 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_B := 1.325 \cdot \text{in} \cdot k_v \quad V_B = 1.325 \frac{\text{in}}{\text{sec}} \quad \theta_{VB} := -31.050 \cdot \text{deg}$$

5. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C, the magnitude of the relative velocity V_{CB} , and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
- From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.
- From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.



4. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{1 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_C := 1.400 \cdot \text{in} \cdot k_v \quad V_C = 1.400 \frac{\text{in}}{\text{sec}} \quad \theta_{VC} := 0.0 \cdot \text{deg}$$

PROBLEM 6-70b

Statement: Find the velocity of the slider in Figure 3-33 for $\theta_2 = 110^\circ$ with respect to the global X axis assuming $\omega_2 = 1 \text{ rad/sec CW}$. Use the method of instant centers.

Given: Link lengths:

Link 2 (O_2 to A) $a := 2.170 \cdot \text{in}$ Link 3 (A to B) $b := 2.067 \cdot \text{in}$

Link 4 (O_4 to B) $c := 2.310 \cdot \text{in}$ Link 1 (O_2 to O_4) $d := 1.000 \cdot \text{in}$

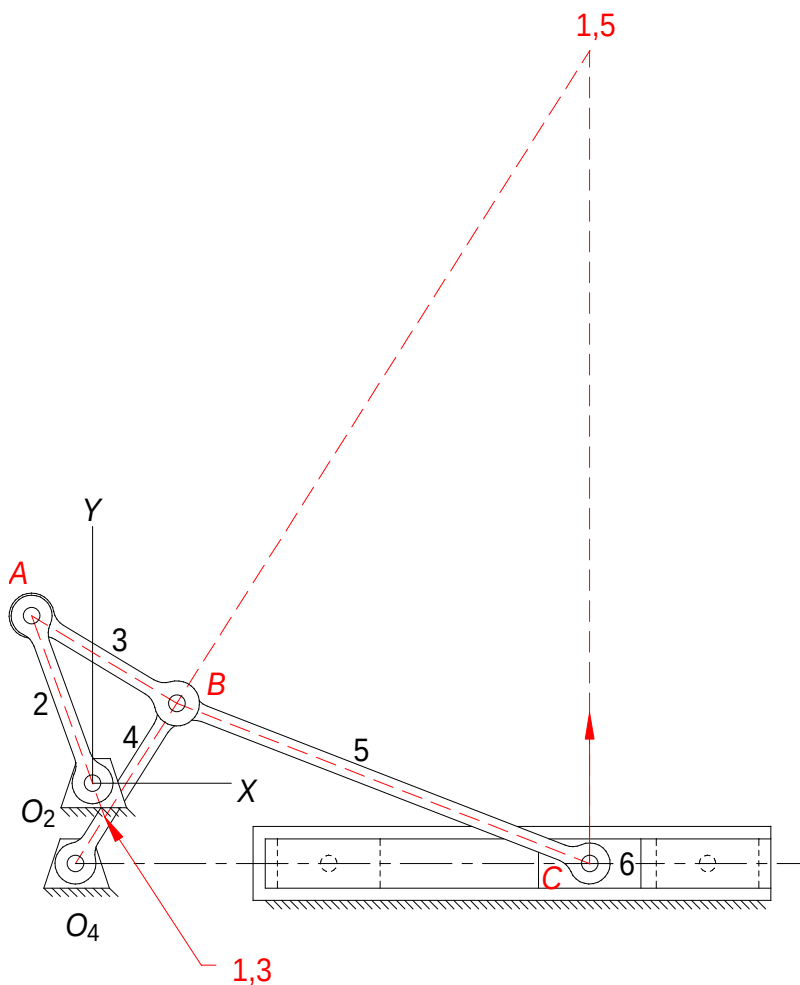
Link 5 (B to C) $e := 5.400$

Coordinate angle $\delta := -102^\circ$ Crank angle: $\theta_2 := 110^\circ$

Input crank angular velocity $\omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1} \text{ CW}$

Solution: See Figure 3-33 and Mathcad file P0670b.

1. Draw the linkage to scale in the position given, find instant centers $I_{1,3}$ and $I_{1,5}$, and the distances from the pin joints to the instant centers. See Problem 6-68 for the determination of IC locations.



From the layout above:

$$AI13 := 2.609 \cdot \text{in}$$

$$BI13 := 1.641 \cdot \text{in}$$

$$BI15 := 9.406 \cdot \text{in}$$

$$CI15 := 9.896 \cdot \text{in}$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A .

$$V_A := a \cdot \omega_2 \qquad V_A = 2.170 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 - 90 \cdot \text{deg} \qquad \theta_{VA} = 20.0 \text{ deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 0.832 \frac{\text{rad}}{\text{sec}} \qquad \text{CW}$$

4. Determine the magnitude of the velocity at point *B* using equation 6.9b. The direction of $\mathbf{V}_B$ is down and to the right

$$V_B := BI13 \cdot \omega_3 \qquad V_B = 1.365 \frac{\text{in}}{\text{sec}}$$

5. Use equation 6.9c to determine the angular velocity of link 5.

$$\omega_5 := \frac{V_B}{BI15} \qquad \omega_5 = 0.145 \frac{\text{rad}}{\text{sec}} \qquad \text{CCW}$$

6. Determine the magnitude of the velocity at point *C* using equation 6.9b.

$$V_C := CI15 \cdot \omega_5 \qquad V_C = 1.436 \frac{\text{in}}{\text{sec}} \qquad \text{to the right}$$

PROBLEM 6-70c

Statement: Find the velocity of the slider in Figure 3-33 for $\theta_2 = 110^\circ$ with respect to the global X axis assuming $\omega_2 = 1 \text{ rad/sec CW}$. Use an analytical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 2.170 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 2.067 \cdot \text{in}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 2.310 \cdot \text{in} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 1.000 \cdot \text{in}$$

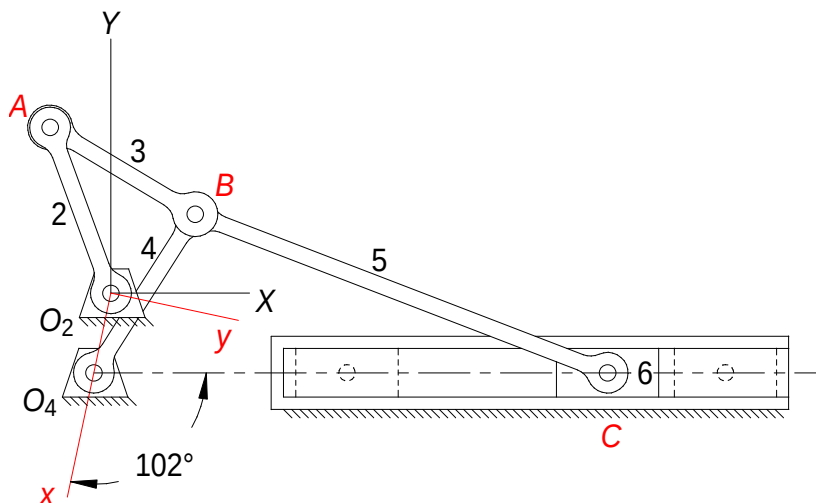
$$\text{Link 5 (B to C)} \quad e := 5.400 \cdot \text{in}$$

$$\text{Coordinate angle} \quad \delta := -102 \cdot \text{deg} \quad \text{Crank angle:} \quad \theta_{2XY} := 110 \cdot \text{deg}$$

$$\text{Input crank angular velocity} \quad \omega_2 := -1 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$$

Solution: See Figure P6-33 and Mathcad file P0670c.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

Transform crank angle to the local xy coordinate system:

$$\theta_2 := \theta_{2XY} - \delta \quad \theta_2 = 212.000 \text{ deg}$$

$$K_1 := \frac{d}{a} \quad K_1 = 0.4608 \quad K_2 := \frac{d}{c} \quad K_2 = 0.4329$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 0.6755$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.2662 \quad B = 1.0598 \quad C = 2.3515$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{4xy} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) - 2 \cdot \pi \quad \theta_{4xy} = -560.365 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.4838 \quad K_5 = -0.5178$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -2.2370$$

$$E := -2 \cdot \sin(\theta_2) \quad E = 1.0598$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.3808$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{3xy} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) - 2 \cdot \pi \quad \theta_{3xy} = -649.050 \text{ deg}$$

6. Determine the angular velocity of link 4 for the open circuit using equations 6.18.

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{3xy})}{\sin(\theta_{4xy} - \theta_{3xy})} \quad \omega_4 = -0.591 \frac{\text{rad}}{\text{sec}}$$

7. Transform θ_4 back to the global XY system.

$$\theta_4 := \theta_{4xy} + \delta \quad \theta_4 = -662.365 \text{ deg}$$

5. Determine θ_5 and d , with respect to O_4 , using equation 4.17.

$$\text{Offset: } cc := 0 \cdot \text{in}$$

$$\theta_5 := \text{asin} \left(-\frac{c \cdot \sin(\theta_4) - cc}{e} \right) + \pi \quad \theta_5 = 158.818 \text{ deg}$$

$$dd := c \cdot \cos(\theta_4) - e \cdot \cos(\theta_5) \quad dd = 6.272 \text{ in}$$

6. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5 := \frac{c}{e} \cdot \frac{\cos(\theta_4)}{\cos(\theta_5)} \cdot \omega_4 \quad \omega_5 = 0.145 \frac{\text{rad}}{\text{sec}}$$

7. Determine the velocity of pin C using equation 6.22b:

$$\mathbf{v}_C := -c \cdot \omega_4 \cdot \sin(\theta_4) + e \cdot \omega_5 \cdot \sin(\theta_5)$$

$$\mathbf{v}_C = 1.436 \frac{\text{in}}{\text{sec}} \quad |\mathbf{v}_C| = 1.436 \frac{\text{in}}{\text{sec}} \quad \arg(\mathbf{v}_C) = 0.000 \text{ deg}$$

PROBLEM 6-71

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular velocity of link 4 and the linear velocity of slider 6 in the sixbar slider-crank linkage of Figure 3-33 as a function of the angle of input link 2 for a constant $\omega_2 = 1$ rad/sec CW. Plot V_c both as a function of θ_2 and separately as a function of slider position as shown in the figure. What is the percent deviation from constant velocity over $240^\circ < \theta_2 < 270^\circ$ and over $190^\circ < \theta_2 < 315^\circ$?

Given: Link lengths:

Input crank (L_2)	$a := 2.170$	Fourbar coupler (L_3)	$b := 2.067$
Output crank (L_4)	$c := 2.310$	Slider coupler (L_5)	$e := 5.40$
Fourbar ground link (L_1)	$d := 1.000$		
Crank velocity:	$\omega_2 := -1 \cdot \frac{\text{rad}}{\text{sec}}$		

Solution: See Figure 3-33 and Mathcad file P0671.

1. This sixbar drag-link mechanism can be analyzed as a fourbar Grashof double crank in series with a crank-slider mechanism using the output of the fourbar, link 4, as the input to the crank-slider.
2. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit) in the global XY coordinate system.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 0.4608 \quad K_2 = 0.4329 \quad K_3 = 0.6755$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right) - 102 \cdot \text{deg}$$

4. If the calculated value of θ_4 is greater than 2π , subtract 2π from it and if it is negative, make it positive.

$$\theta_{42}(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := \text{if}(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$$

5. Determine the slider-crank motion using equations 4.16 and 4.17 with θ_4 as the input angle.

$$\theta_5(\theta_2) := \text{asin} \left(\frac{-c \cdot \sin(\theta_4(\theta_2))}{e} \right) + \pi$$

$$f(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) - e \cdot \cos(\theta_5(\theta_2))$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.4838 \quad K_5 = -0.5178$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

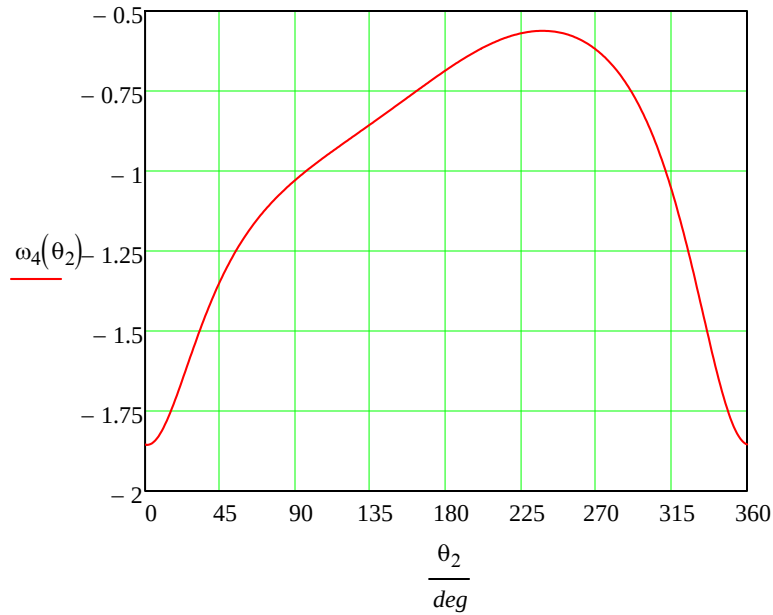
$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of link 4 for the open circuit using equations 6.18.

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) + 102 \cdot \text{deg} - \theta_3(\theta_2))}$$

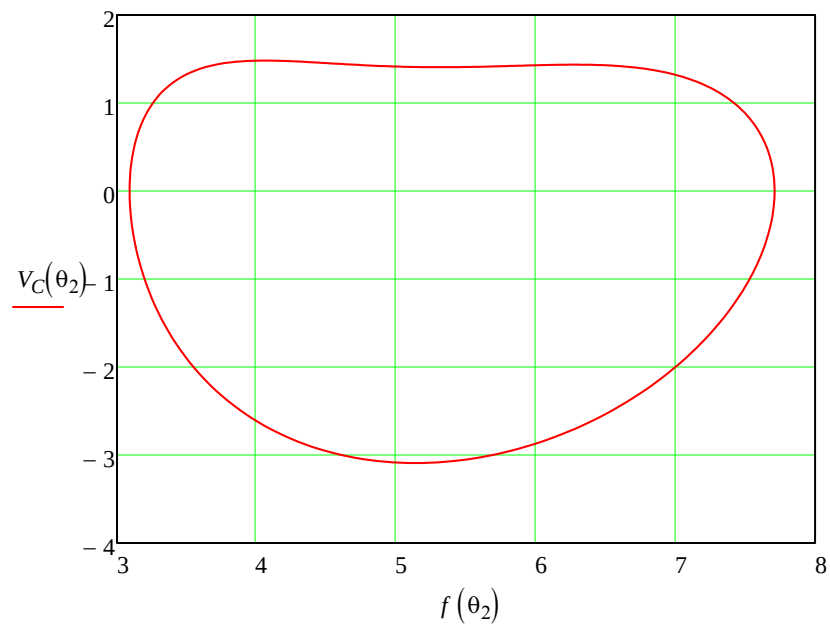
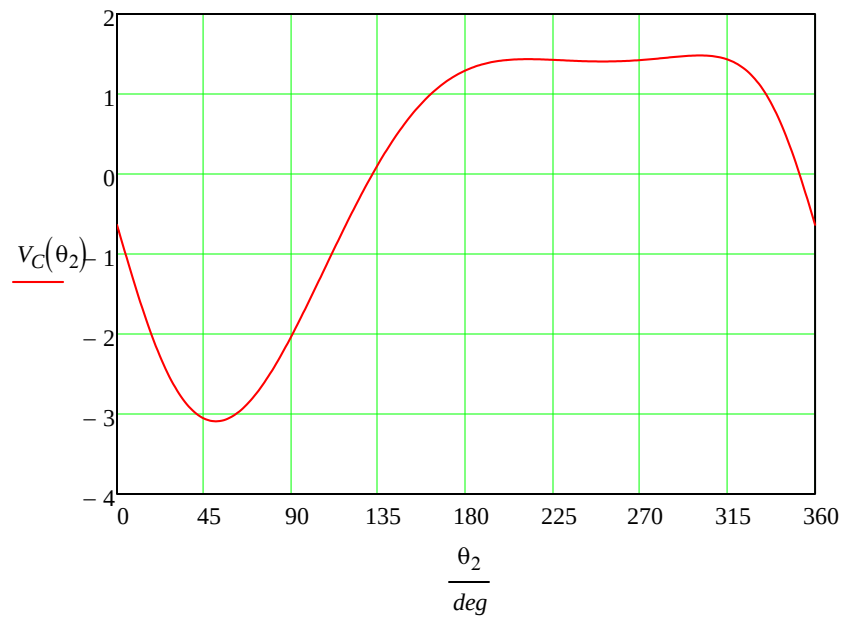


8. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5(\theta_2) := \frac{c}{e} \cdot \frac{\cos(\theta_4(\theta_2))}{\cos(\theta_5(\theta_2))} \cdot \omega_4(\theta_2)$$

9. Determine the velocity of pin C using equation 6.22b:

$$V_C(\theta_2) := -c \cdot \omega_4(\theta_2) \cdot \sin(\theta_4(\theta_2)) + e \cdot \omega_5(\theta_2) \cdot \sin(\theta_5(\theta_2))$$



PROBLEM 6-72

Statement: Figure 3-34 shows a Stephenson's sixbar mechanism. Find all of its instant centers in the position shown:

Given: Number of links $n := 6$

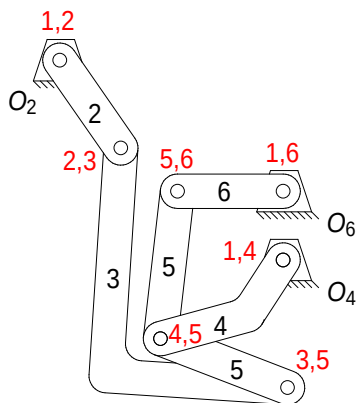
Solution: See Figure 3-34 and Mathcad file P0672.

- Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 15$$

a. In part (a) of the figure.

- Draw the linkage to scale and identify those ICs that can be found by inspection (7).



- Use Kennedy's Rule and a linear graph to find the remaining 8 ICs:

$I_{1,3}$; $I_{1,5}$; $I_{2,4}$; $I_{2,5}$; $I_{2,6}$; $I_{3,4}$; $I_{3,6}$; and $I_{4,6}$

$I_{1,5}$: $I_{1,6}$ - $I_{5,6}$ and $I_{1,4}$ - $I_{4,5}$

$I_{2,5}$: $I_{1,2}$ - $I_{1,5}$ and $I_{2,3}$ - $I_{3,5}$

$I_{1,3}$: $I_{1,2}$ - $I_{2,3}$ and $I_{1,5}$ - $I_{3,5}$

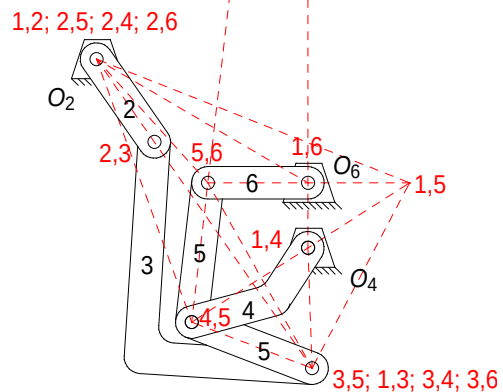
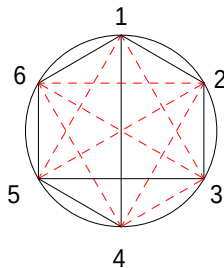
$I_{3,4}$: $I_{1,4}$ - $I_{1,3}$ and $I_{4,5}$ - $I_{3,5}$

$I_{2,8}$: $I_{2,3}$ - $I_{3,4}$ and $I_{2,5}$ - $I_{4,5}$

$I_{2,6}$: $I_{1,2}$ - $I_{1,6}$ and $I_{2,5}$ - $I_{5,6}$

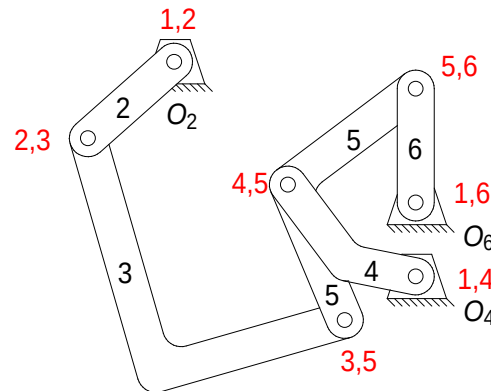
$I_{3,6}$: $I_{1,3}$ - $I_{1,6}$ and $I_{3,5}$ - $I_{5,6}$

$I_{4,6}$: $I_{1,4}$ - $I_{1,6}$ and $I_{4,5}$ - $I_{5,6}$

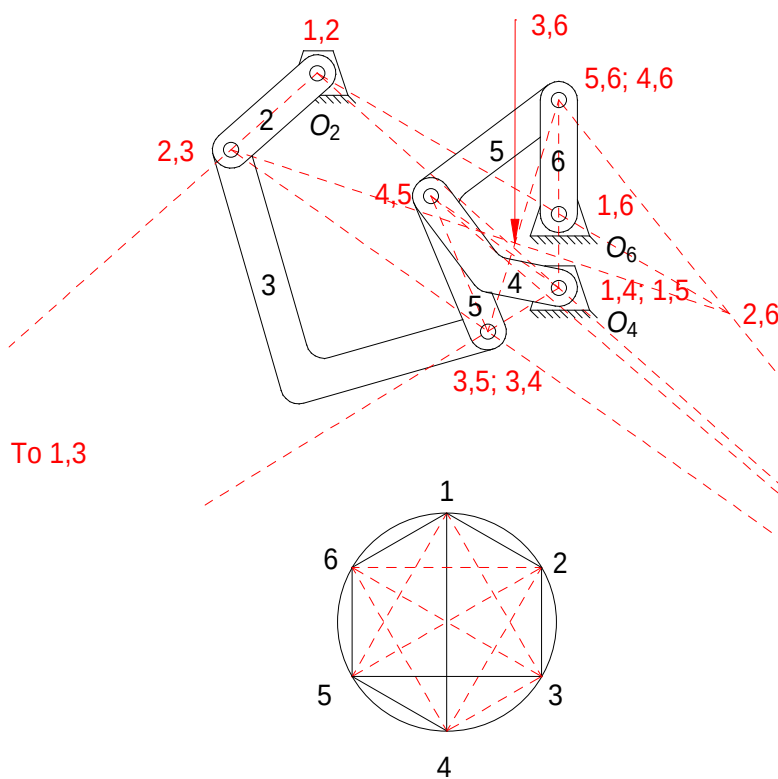


b. In part (b) of the figure.

1. Draw the linkage to scale and identify those ICs that can be found by inspection (7).



2. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs: $I_{1,3}$; $I_{1,5}$; $I_{2,4}$; $I_{2,5}$; $I_{2,6}$; $I_{3,4}$; $I_{3,6}$; and $I_{4,6}$



$I_{1,5}$: $I_{1,6}$ - $I_{5,6}$ and $I_{1,4}$ - $I_{4,5}$

$I_{2,5}$: $I_{1,2}$ - $I_{1,5}$ and $I_{2,3}$ - $I_{3,5}$

$I_{1,3}$: $I_{1,2}$ - $I_{2,3}$ and $I_{1,5}$ - $I_{3,5}$

$I_{3,4}$: $I_{1,4}$ - $I_{1,3}$ and $I_{4,5}$ - $I_{3,5}$

$I_{2,4}$: $I_{2,3}$ - $I_{3,4}$ and $I_{2,5}$ - $I_{4,5}$

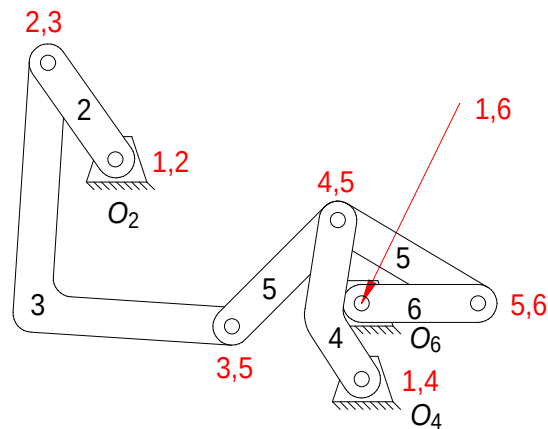
$I_{2,6}$: $I_{1,2}$ - $I_{1,6}$ and $I_{2,5}$ - $I_{5,6}$

$I_{3,6}$: $I_{1,3}$ - $I_{1,6}$ and $I_{3,5}$ - $I_{5,6}$

$I_{4,6}$: $I_{1,4}$ - $I_{1,6}$ and $I_{4,5}$ - $I_{5,6}$

c. In part (c) of the figure.

1. Draw the linkage to scale and identify those ICs that can be found by inspection (7).



2. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs: $I_{1,3}$; $I_{1,5}$; $I_{2,4}$; $I_{2,5}$; $I_{2,6}$; $I_{3,4}$; $I_{3,6}$; and $I_{4,6}$

$$I_{1,5}: I_{1,6}-I_{5,6} \text{ and } I_{1,4}-I_{4,5}$$

$$I_{2,5}: I_{1,2}-I_{1,5} \text{ and } I_{2,3}-I_{3,5}$$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,5}-I_{3,5}$$

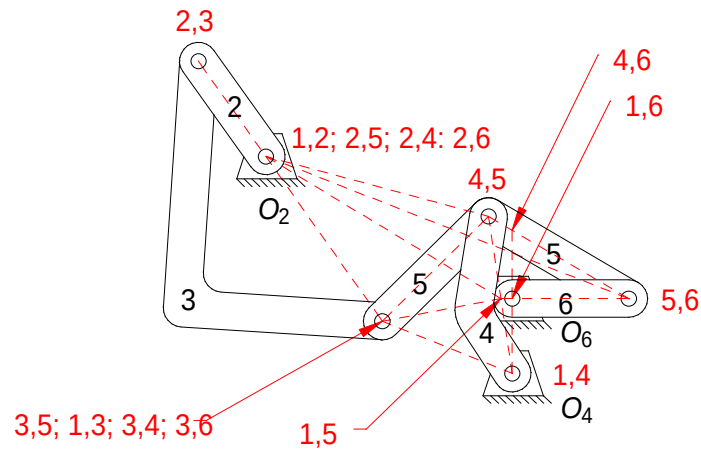
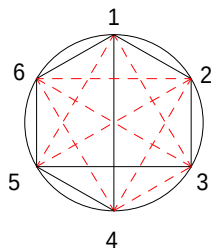
$$I_{3,4}: I_{1,4}-I_{1,3} \text{ and } I_{4,5}-I_{3,5}$$

$$I_{2,4}: I_{2,3}-I_{3,4} \text{ and } I_{2,5}-I_{4,5}$$

$$I_{2,6}: I_{1,2}-I_{1,6} \text{ and } I_{2,5}-I_{5,6}$$

$$I_{3,6}: I_{1,3}-I_{1,6} \text{ and } I_{3,5}-I_{5,6}$$

$$I_{4,6}: I_{1,4}-I_{1,6} \text{ and } I_{4,5}-I_{5,6}$$



PROBLEM 6-73a

Statement: Find the angular velocity of link 6 in Figure 3-34b for $\theta_6 = 90^\circ$ with respect to the global X axis assuming $\omega_2 = 10 \text{ rad/sec CW}$. Use a graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot \text{in}$	Link 3 (A to B)	$f := 4.248 \cdot \text{in}$
Link 4 (O_4 to C)	$c := 2.125 \cdot \text{in}$	Link 5 (C to D)	$b := 2.158 \cdot \text{in}$
Link 6 (O_6 to D)	$a := 1.542 \cdot \text{in}$	Link 5 (B to D)	$p := 3.274 \cdot \text{in}$
Link 1 X-offset	$d_X := 3.259 \cdot \text{in}$	Link 1 Y-offset	$d_Y := -2.905 \cdot \text{in}$

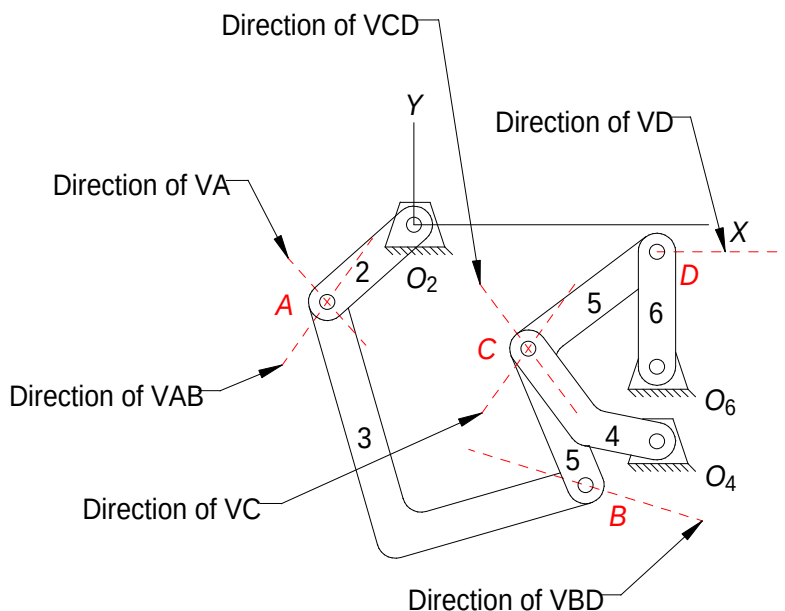
Angle CDB $\delta_5 := 36.0^\circ$

Output rocker angle: $\theta_6 := 90^\circ$ Global XY system

Input crank angular velocity $\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

Solution: See Figure 3-34b and Mathcad file P0673a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Since this linkage is a Stephenson's II sixbar, we will have to start at link 6 and work back to link 2. We will assume a value for ω_6 and eventually find a value for ω_2 . We will then multiply the magnitudes of all velocities by the ratio of the actual ω_2 to the found ω_2 . Use equation 6.7 to calculate the magnitude of the velocity at point D.

Assume: $\omega_6 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

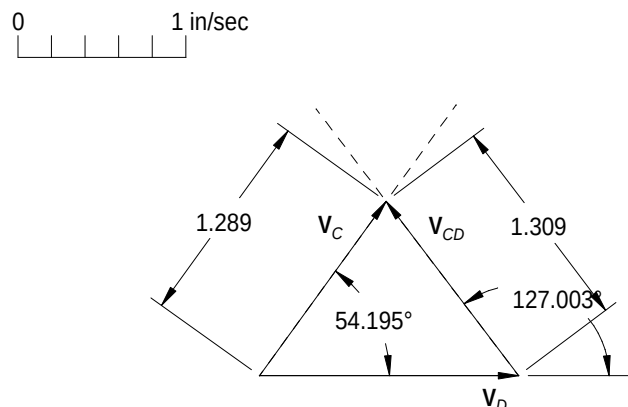
$$V_D := a \cdot \omega_6 \quad V_D = 1.542 \frac{\text{in}}{\text{sec}} \quad \theta_{VD} := 0^\circ$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C, the magnitude of the relative velocity $\mathbf{V}_{CD}$, and the angular velocity of link 5. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_D + \mathbf{V}_{CD}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_D$.
- b. From the tip of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_{CD}$, magnitude unknown.

- c. From the tail of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{CD}$ from the tip of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_{CD}$ construction line.



4. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{1 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_C := 1.289 \cdot \text{in} \cdot k_v \quad V_C = 1.289 \frac{\text{in}}{\text{sec}} \quad \theta_{VC} := 54.195 \cdot \text{deg}$$

$$V_{CD} := 1.309 \cdot \text{in} \cdot k_v \quad V_{CD} = 1.309 \frac{\text{in}}{\text{sec}} \quad \theta_{VCD} := 127.003 \cdot \text{deg}$$

5. Determine the angular velocity of links 5 and 4 using equation 6.7.

$$\omega_5 := \frac{-V_{CD}}{b} \quad \omega_5 = -0.607 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{-V_C}{c} \quad \omega_4 = -0.607 \frac{\text{rad}}{\text{sec}}$$

6. Determine the magnitude and sense of the vector $\mathbf{V}_{BD}$ using equation 6.7.

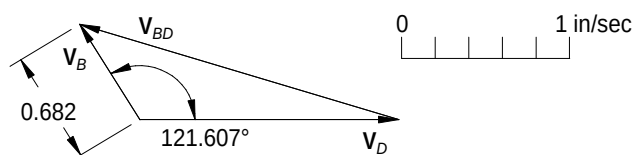
$$V_{BD} := |p \cdot \omega_5| \quad V_{BD} = 1.986 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VBD} := \theta_{VCD} + \delta_5 \quad \theta_{VBD} = 163.003 \text{ deg}$$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_D + \mathbf{V}_{BD}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_D$.
- b. From the tip of $\mathbf{V}_D$, layout the (now) known vector $\mathbf{V}_{BD}$.
- c. Complete the vector triangle by drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_D$ to the tip of the $\mathbf{V}_{BD}$ vector.



8. From the velocity triangle we have:

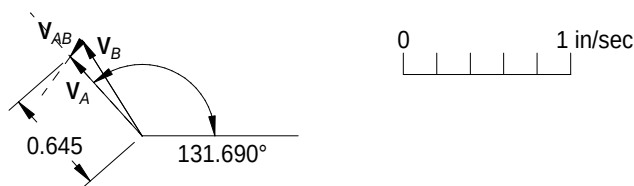
$$\text{Velocity scale factor: } k_v := \frac{1 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_B := 0.682 \cdot \text{in} \cdot k_v \quad V_B = 0.682 \frac{\text{in}}{\text{sec}} \quad \theta_{VB} := 121.607 \cdot \text{deg}$$

9. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point A, the magnitude of the relative velocity $\mathbf{V}_{AB}$, and the angular velocity of link 2. The equation to be solved graphically is

$$\mathbf{V}_A = \mathbf{V}_B + \mathbf{V}_{AB}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
- From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{AB}$, magnitude unknown.
- From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_A$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{AB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_A$ construction line and drawing $\mathbf{V}_A$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{AB}$ construction line.



10. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{1 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_A := 0.645 \cdot \text{in} \cdot k_v \quad V_A = 0.645 \frac{\text{in}}{\text{sec}} \quad \theta_{VA} := 131.690 \cdot \text{deg}$$

11. Determine the angular velocity of link 2 with respect to the assumed value of ω_6 using equation 6.7.

$$\omega_{26} := \frac{V_A}{g} \quad \omega_{26} = 0.415 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

12. Calculate the actual value of the angular velocity of link 6.

$$\omega_6 := \frac{\omega_2}{\omega_{26}} \cdot \frac{\text{rad}}{\text{sec}} \quad \omega_6 = 24.124 \frac{\text{rad}}{\text{sec}}$$

PROBLEM 6-73b

Statement: Find the angular velocity of link 6 in Figure 3-34b for $\theta_6 = 90^\circ$ with respect to the global X axis assuming $\omega_2 = 10 \text{ rad/sec CW}$. Use the method of instant centers.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot \text{in}$	Link 3 (A to B)	$f := 4.248 \cdot \text{in}$
Link 4 (O_4 to C)	$c := 2.125 \cdot \text{in}$	Link 5 (C to D)	$b := 2.158 \cdot \text{in}$
Link 6 (O_6 to D)	$a := 1.542 \cdot \text{in}$	Link 5 (B to D)	$p := 3.274 \cdot \text{in}$
Link 1 X -offset	$d_X := 3.259 \cdot \text{in}$	Link 1 Y -offset	$d_Y := -2.905 \cdot \text{in}$

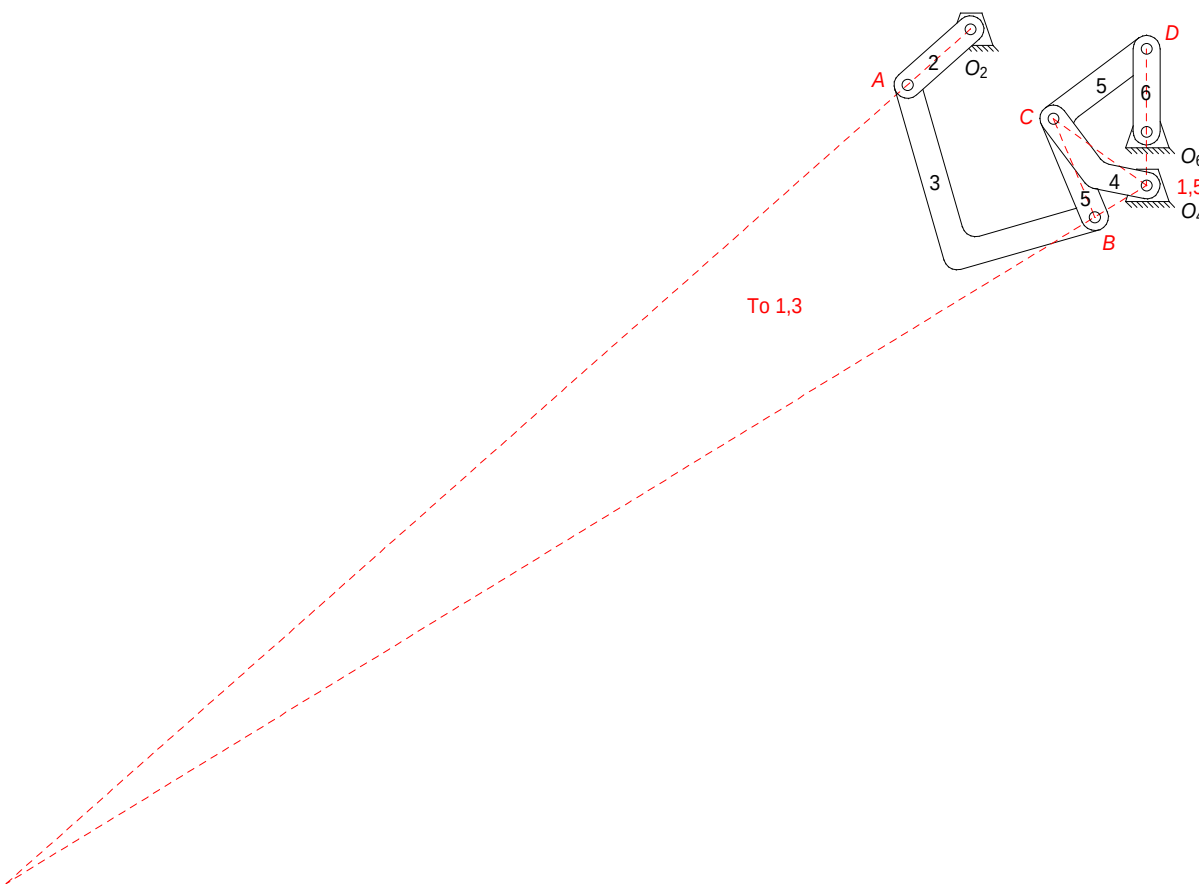
Angle CDB $\delta_5 := 36.0 \cdot \text{deg}$

Output rocker angle: $\theta_6 := 90 \cdot \text{deg}$ Global XY system

Input crank angular velocity $\omega_{21} := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

Solution: See Figure 3-34b and Mathcad file P0673b.

1. Draw the linkage to scale in the position given, find instant centers $I_{1,3}$ and $I_{1,5}$, and the distances from the pin joints to the instant centers. See Problem 6-72b for determination of IC locations.



From the layout above:

$$AI13 := 22.334 \cdot \text{in}$$

$$BI13 := 23.650 \cdot \text{in}$$

$$BI15 := 1.124 \cdot \text{in}$$

$$DI15 := 2.542 \cdot \text{in}$$

2. Start from point D with an assumed value for ω_6 and work to find ω_2 . Then, use the ratio of the actual value of ω_2 to the found value to calculate the actual value of ω_6 . Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point D .

$$\omega_6 := 1 \cdot \frac{\text{rad}}{\text{sec}}$$

$$V_D := a \cdot \omega_6$$

$$V_D = 1.542 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VD} := \theta_6 - 90 \cdot \text{deg}$$

$$\theta_{VD} = 0.0 \text{ deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_5 := \frac{V_D}{DI15}$$

$$\omega_5 = 0.607 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point B using equation 6.9b.

$$V_B := BI15 \cdot \omega_5$$

$$V_B = 0.682 \frac{\text{in}}{\text{sec}}$$

5. Use equation 6.9c to determine the angular velocity of link 3.

$$\omega_3 := \frac{V_B}{BI13}$$

$$\omega_3 = 0.029 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

6. Determine the magnitude of the velocity at point A using equation 6.9b.

$$V_A := AI13 \cdot \omega_3$$

$$V_A = 0.644 \frac{\text{in}}{\text{sec}} \quad \text{to the left}$$

7. Use equation 6.9c to determine the angular velocity of link 2 based on the assumed value of ω_6 .

$$\omega_{22} := \frac{V_A}{g}$$

$$\omega_{22} = 0.414 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

8. Multiply the assumed value of ω_6 by the ratio of ω_{21} over ω_{22} to get the value of ω_6 for $\omega_2 = 10 \text{ rad/sec}$.

$$\omega_6 := \frac{\omega_{21}}{\omega_{22}} \cdot \frac{\text{rad}}{\text{sec}}$$

$$\omega_6 = 24.166 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

PROBLEM 6-73c

Statement: Find the angular velocity of link 6 in Figure 3-34b for $\theta_6 = 90^\circ$ with respect to the global X axis assuming $\omega_2 = 10 \text{ rad/sec CW}$. Use an analytic method.

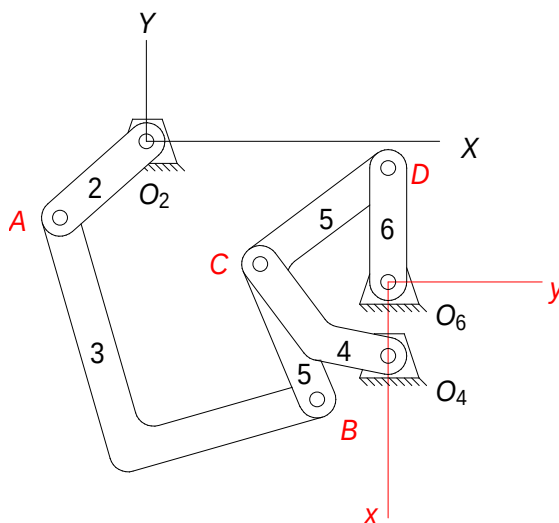
Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot \text{in}$	Link 3 (A to B)	$f := 4.248 \cdot \text{in}$
Link 4 (O_4 to C)	$c := 2.125 \cdot \text{in}$	Link 5 (C to D)	$b := 2.158 \cdot \text{in}$
Link 6 (O_6 to D)	$a := 1.542 \cdot \text{in}$	Link 5 (B to D)	$p := 3.274 \cdot \text{in}$
Link 1 X-offset	$d_X := 3.259 \cdot \text{in}$	Link 1 Y-offset	$d_Y := -2.905 \cdot \text{in}$
Angle CDB	$\delta_5 := 36.0 \cdot \text{deg}$	Link 1 (O_4 to O_6)	$d := 1.000 \cdot \text{in}$
Output rocker angle:	$\theta_{6XY} := 90 \cdot \text{deg}$	Global XY system	
Input crank angular velocity	$\omega_2 := -10 \cdot \text{rad} \cdot \text{sec}^{-1}$	CW	
Coordinate rotation angle	$\delta := -90 \cdot \text{deg}$		

Solution: See Figure 3-34b and Mathcad file P0673c.

1. Transform the crank angle to the local coordinate system. Draw the linkage to scale and label it.

$$\theta_6 := \theta_{6XY} - \delta \quad \theta_6 = 180.000 \text{ deg}$$



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$\begin{aligned}
 K_1 &:= \frac{d}{a} & K_2 &:= \frac{d}{c} & K_3 &:= \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \\
 K_1 &= 0.6485 & K_2 &= 0.4706 & K_3 &= 0.4938 \\
 A &:= \cos(\theta_6) - K_1 - K_2 \cdot \cos(\theta_6) + K_3 & A &= -0.6841 \\
 B &:= -2 \cdot \sin(\theta_6) & B &= 0.0000 \\
 C &:= K_1 - (K_2 + 1) \cdot \cos(\theta_6) + K_3 & C &= 2.6129
 \end{aligned}$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_4 = 234.195 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.4634 \quad K_5 = -0.5288$$

$$D := \cos(\theta_6) - K_1 + K_4 \cdot \cos(\theta_6) + K_5 \quad D = -2.6407$$

$$E := -2 \cdot \sin(\theta_6) \quad E = 0.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_6) + K_5 \quad F = 0.6563$$

5. Use equation 4.13 to find values of θ_5 for the crossed circuit.

$$\theta_{51} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{51} = 307.003 \text{ deg}$$

6. Determine the angular velocity of links 4 and 5 for the open circuit using equations 6.18. Initially assume $\omega_6 = 1 \text{ rad/sec}$. then by trial and error, change it to make $\omega_2 = 10 \text{ rad/sec CW}$.

$$\omega_6 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$$

$$\omega_5 := \frac{a \cdot \omega_6}{b} \cdot \frac{\sin(\theta_4 - \theta_6)}{\sin(\theta_{51} - \theta_4)} \quad \omega_5 = 0.607 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_6}{c} \cdot \frac{\sin(\theta_6 - \theta_{51})}{\sin(\theta_4 - \theta_{51})} \quad \omega_4 = 0.607 \frac{\text{rad}}{\text{sec}}$$

PROBLEM 6-74

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular velocity of link 6 in the sixbar linkage of Figure 3-34 as a function of θ_2 for a constant $\omega_2 = 1 \text{ rad/sec CW}$.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot \text{in}$	Link 3 (A to B)	$f := 4.248 \cdot \text{in}$
Link 4 (O_4 to C)	$c := 2.125 \cdot \text{in}$	Link 5 (C to D)	$b := 2.158 \cdot \text{in}$
Link 6 (O_6 to D)	$a := 1.542 \cdot \text{in}$	Link 5 (B to D)	$p := 3.274 \cdot \text{in}$
Link 1 X -offset	$d_X := 3.259 \cdot \text{in}$	Link 1 Y -offset	$d_Y := -2.905 \cdot \text{in}$
Angle CDB	$\delta_5 := 36.0 \cdot \text{deg}$	Link 1 (O_4 to O_6)	$d := 1.000 \cdot \text{in}$
Output rocker angle:	$\theta_{6XY} := 90 \cdot \text{deg}$	Global XY system	
Input crank angular velocity	$\omega_2 := -10 \cdot \text{rad} \cdot \text{sec}^{-1} \text{ CW}$		
Coordinate rotation angle	$\delta := -90 \cdot \text{deg}$		

Solution: See Figure P6-34 and Mathcad file P0674.

1. This problem is long and may be more appropriate for a project assignment. The solution involves defining vector loops and solving the resulting equations using a method such as Newton-Raphson.

PROBLEM 6-75

Statement: Figure 3-35 shows a Stephenson's sixbar mechanism. Find all of its instant centers in the position shown:

Given: Number of links $n := 6$

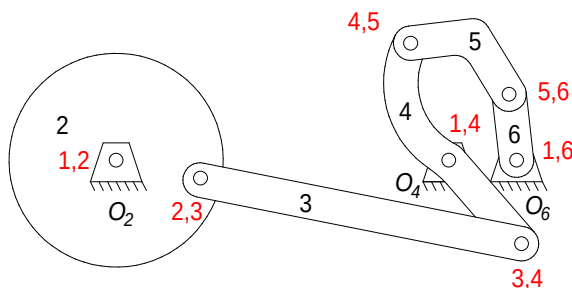
Solution: See Figure 3-35 and Mathcad file P0675.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

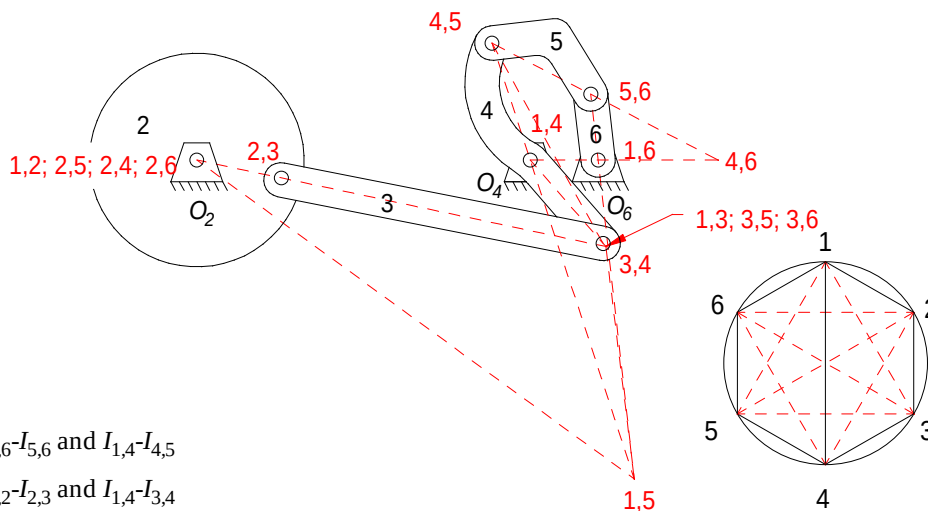
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 15$$

a. In part (a) of the figure.

1. Draw the linkage to scale and identify those ICs that can be found by inspection (7).



2. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs: $I_{1,3}$; $I_{1,5}$; $I_{2,4}$; $I_{2,5}$; $I_{2,6}$; $I_{3,5}$; $I_{3,6}$; and $I_{4,6}$



$I_{1,5}$: $I_{1,6}$ - $I_{5,6}$ and $I_{1,4}$ - $I_{4,5}$

$I_{1,3}$: $I_{1,2}$ - $I_{2,3}$ and $I_{1,4}$ - $I_{3,4}$

$I_{3,5}$: $I_{1,5}$ - $I_{1,3}$ and $I_{3,4}$ - $I_{4,5}$

$I_{2,5}$: $I_{1,2}$ - $I_{1,5}$ and $I_{2,3}$ - $I_{3,5}$

$I_{2,4}$: $I_{2,3}$ - $I_{3,4}$ and $I_{2,5}$ - $I_{4,5}$

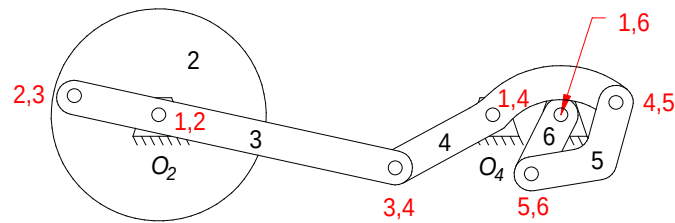
$I_{2,6}$: $I_{1,2}$ - $I_{1,6}$ and $I_{2,5}$ - $I_{5,6}$

$I_{3,6}$: $I_{1,3}$ - $I_{1,6}$ and $I_{3,5}$ - $I_{5,6}$

$I_{4,6}$: $I_{1,4}$ - $I_{1,6}$ and $I_{4,5}$ - $I_{5,6}$

b. In part (b) of the figure.

1. Draw the linkage to scale and identify those ICs that can be found by inspection (7).



2. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs: $I_{1,3}$; $I_{1,5}$; $I_{2,4}$; $I_{2,5}$; $I_{2,6}$; $I_{3,5}$; $I_{3,6}$; and $I_{4,6}$

$$I_{1,5}: I_{1,6}-I_{5,6} \text{ and } I_{1,4}-I_{4,5}$$

$$I_{2,4}: I_{2,3}-I_{3,4} \text{ and } I_{2,5}-I_{4,5}$$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

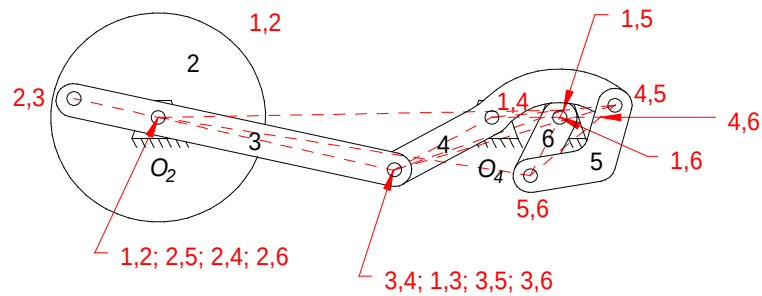
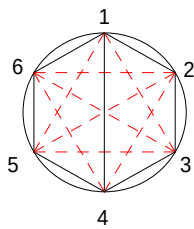
$$I_{2,6}: I_{1,2}-I_{1,6} \text{ and } I_{2,5}-I_{5,6}$$

$$I_{3,5}: I_{1,5}-I_{1,3} \text{ and } I_{3,4}-I_{4,5}$$

$$I_{3,6}: I_{1,3}-I_{1,6} \text{ and } I_{3,5}-I_{5,6}$$

$$I_{2,5}: I_{1,2}-I_{1,5} \text{ and } I_{2,3}-I_{3,5}$$

$$I_{4,6}: I_{1,4}-I_{1,6} \text{ and } I_{4,5}-I_{5,6}$$



PROBLEM 6-76a

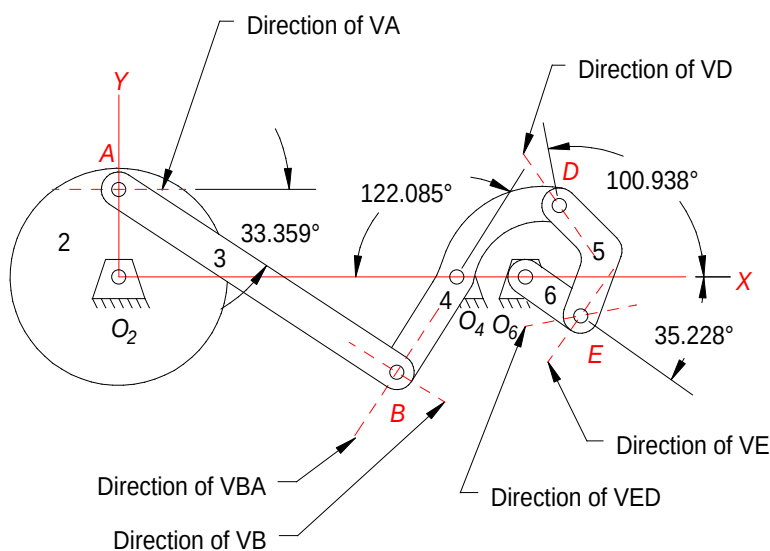
Statement: Use a compass and straightedge to draw the linkage in Figure 3-35 with link 2 at 90 deg and find the angular velocity of link 6 assuming $\omega_2 = 10 \text{ rad/sec CCW}$. Use a graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 1.000 \cdot \text{in}$	Link 3 (A to B)	$b := 3.800 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 1.286 \cdot \text{in}$	Link 1 (O_2 to O_4)	$d := 3.857 \cdot \text{in}$
Link 4 (O_4 to D)	$e := 1.429 \cdot \text{in}$	Link 5 (D to E)	$f := 1.286$
Link 6 (O_6 to E)	$g := 0.771 \cdot \text{in}$	Crank angle:	$\theta_2 := 90 \cdot \text{deg}$
Input crank angular velocity $\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW			

Solution: See Figure 3-35 and Mathcad file P0676a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

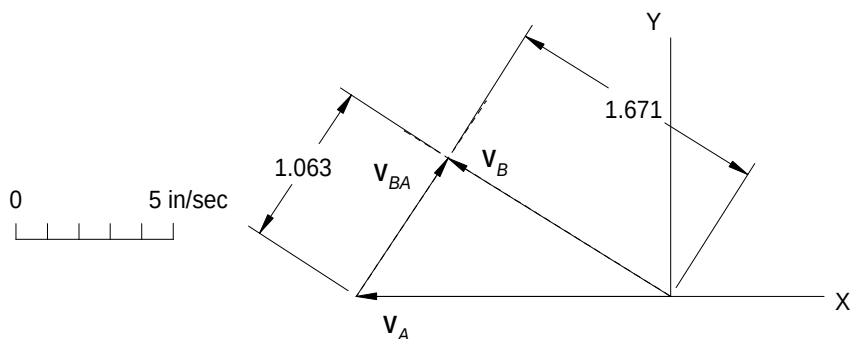
$$V_A := a \cdot \omega_2 \quad V_A = 10.000 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{VA} = 180.000 \text{ deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_B := 1.671 \cdot \text{in} \cdot k_v \quad V_B = 8.355 \frac{\text{in}}{\text{sec}} \quad \theta_{VB} := 147.915 \cdot \text{deg}$$

$$V_{BA} := 1.063 \cdot \text{in} \cdot k_v \quad V_{BA} = 5.315 \frac{\text{in}}{\text{sec}} \quad \theta_{VBA} := 56.641 \cdot \text{deg}$$

$$\omega_4 := \frac{V_B}{c} \quad \omega_4 = 6.497 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

5. Calculate the magnitude and direction of V_D .

$$V_D := e \cdot \omega_4 \quad V_D = 9.284 \frac{\text{in}}{\text{sec}} \quad \theta_{VD} := -55.085 \cdot \text{deg}$$

6. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point E, the magnitude of the relative velocity V_{ED} , and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_E = \mathbf{V}_D + \mathbf{V}_{ED}$$

- Choose a convenient velocity scale and layout the known vector V_D .
- From the tip of V_D , draw a construction line with the direction of V_{ED} , magnitude unknown.
- From the tail of V_D , draw a construction line with the direction of V_E , magnitude unknown.
- Complete the vector triangle by drawing V_{ED} from the tip of V_D to the intersection of the V_E construction line and drawing V_E from the tail of V_D to the intersection of the V_{ED} construction line.

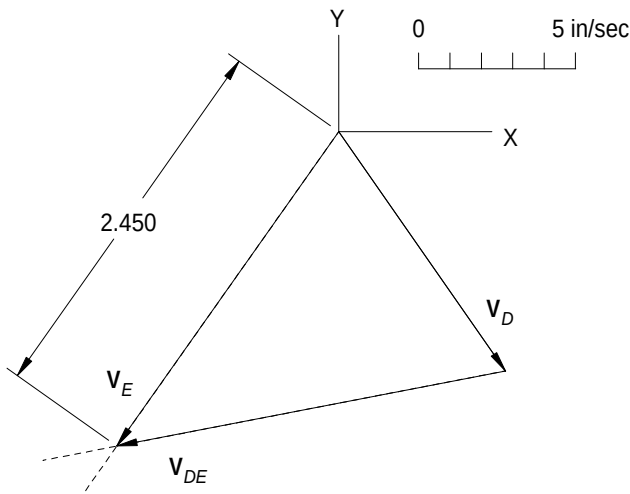
4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_E := 2.450 \cdot \text{in} \cdot k_v \quad V_E = 12.250 \frac{\text{in}}{\text{sec}}$$

$$\omega_6 := \frac{V_E}{g}$$

$$\omega_6 = 15.888 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$



PROBLEM 6-76b

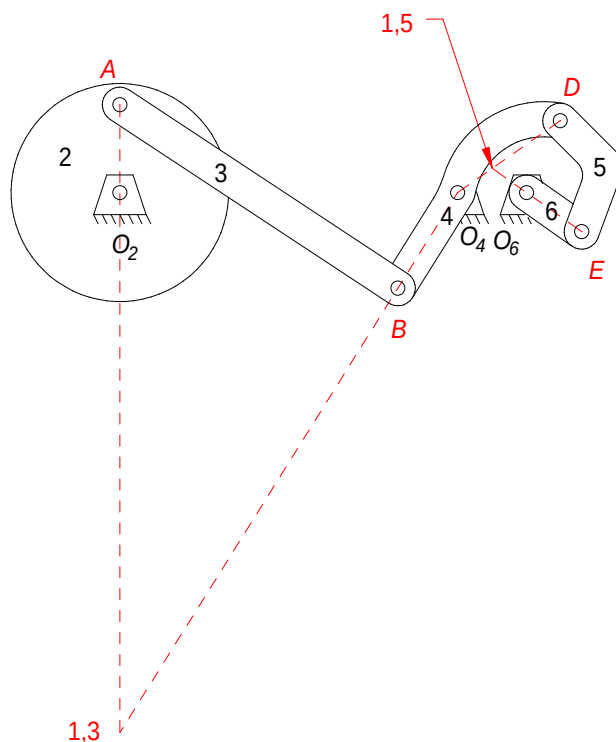
Statement: Use a compass and straightedge to draw the linkage in Figure 3-35 with link 2 at 90 deg and find the angular velocity of link 6 assuming $\omega_2 = 10 \text{ rad/sec CCW}$. Use the method of instant centers.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 1.000 \cdot \text{in}$	Link 3 (A to B)	$b := 3.800 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 1.286 \cdot \text{in}$	Link 1 (O_2 to O_4)	$d := 3.857 \cdot \text{in}$
Link 4 (O_4 to D)	$e := 1.429 \cdot \text{in}$	Link 5 (D to E)	$f := 1.286$
Link 6 (O_6 to E)	$g := 0.771 \cdot \text{in}$	Crank angle:	$\theta_2 := 90 \cdot \text{deg}$
Input crank angular velocity $\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW			

Solution: See Figure 3-35 and Mathcad file P0676a.

1. Draw the linkage to scale in the position given, find instant centers $I_{1,3}$ and $I_{1,5}$, and the distances from the pin joints to the instant centers. See Problem 6-68 for the determination of IC locations.



From the layout above:

$$AI13 := 7.152 \cdot \text{in} \quad BI13 := 5.975 \cdot \text{in} \quad BI15 := 1.740 \cdot \text{in} \quad DI15 := 0.947 \cdot \text{in} \quad EI15 := 1.249 \cdot \text{in}$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 10.000 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{VA} = 180.0 \text{ deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \quad \omega_3 = 1.398 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

4. Determine the magnitude of the velocity at point *B* using equation 6.9b.

$$V_B := BI13 \cdot \omega_3 \qquad V_B = 8.354 \frac{\text{in}}{\text{sec}}$$

5. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 6.496 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

6. Determine the magnitude of the velocity at point *D* using equation 6.9b.

$$V_D := e \cdot \omega_4 \qquad V_D = 9.283 \frac{\text{in}}{\text{sec}}$$

7. Use equation 6.9c to determine the angular velocity of link 5.

$$\omega_5 := \frac{V_D}{DI15} \qquad \omega_5 = 9.803 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

8. Determine the magnitude of the velocity at point *E* using equation 6.9b.

$$V_E := EI15 \cdot \omega_5 \qquad V_E = 12.244 \frac{\text{in}}{\text{sec}}$$

9. Use equation 6.9c to determine the angular velocity of link 6.

$$\omega_6 := \frac{V_E}{g} \qquad \omega_6 = 15.88 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

PROBLEM 6-76c

Statement: Use a compass and straightedge to draw the linkage in Figure 3-35 with link 2 at 90 deg and find the angular velocity of link 6 assuming $\omega_2 = 10 \text{ rad/sec CCW}$. Use an analytical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 1.000 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 3.800 \cdot \text{in}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 1.286 \cdot \text{in} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 3.857 \cdot \text{in}$$

$$\text{Link 4 (O}_4 \text{ to D)} \quad e := 1.429 \cdot \text{in} \quad \text{Link 5 (D to E)} \quad f := 1.286 \cdot \text{in}$$

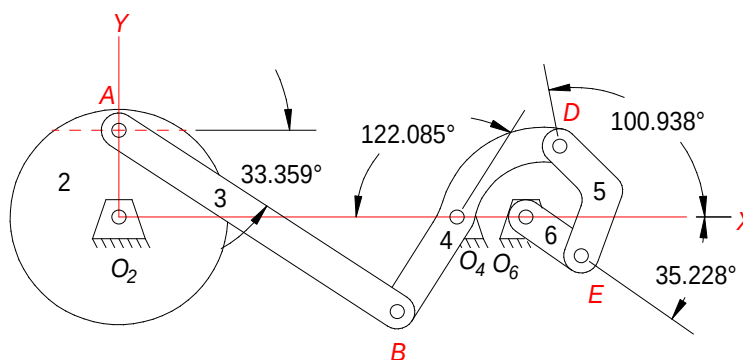
$$\text{Link 6 (O}_6 \text{ to E)} \quad g := 0.771 \cdot \text{in} \quad \text{Link 1 (O}_4 \text{ to O}_6) \quad h := 0.786 \cdot \text{in}$$

$$\text{Crank angle:} \quad \theta_2 := 90 \cdot \text{deg}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CCW}$$

Solution: See Figure 3-35 and Mathcad file P0676c.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_1 = 3.8570 \quad K_2 = 2.9992$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.2015$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -2.6555 \quad B = -2.0000 \quad C = 5.0585$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_{41} = 237.915 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0150 \quad K_5 = -3.7714$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -7.6284$$

$$E := -2 \cdot \sin(\theta_2)$$

$$E = -2.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

$$F = 0.0856$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right)$$

$$\theta_3 = 326.641 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41} - \theta_2)}{\sin(\theta_3 - \theta_{41})}$$

$$\omega_3 = 1.398 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_{41} - \theta_3)}$$

$$\omega_4 = -6.496 \frac{\text{rad}}{\text{sec}}$$

7. Link 4 will be the input to the second fourbar (links 4, 5, 6, and 1). Let the angle that link 4 makes in the second fourbar be

$$\theta_{42} := \theta_{41} + 157 \cdot \text{deg} - 360 \cdot \text{deg}$$

$$\theta_{42} = 34.915 \text{ deg}$$

8. Determine the values of the constants needed for finding θ_6 from equations 4.8a and 4.10a.

$$K_1 := \frac{h}{e}$$

$$K_2 := \frac{h}{g}$$

$$K_1 = 0.5500$$

$$K_2 = 1.0195$$

$$K_3 := \frac{e^2 - f^2 + g^2 + h^2}{2 \cdot e \cdot g} \quad K_3 = 0.7263$$

$$A := \cos(\theta_{42}) - K_1 - K_2 \cdot \cos(\theta_{42}) + K_3$$

$$B := -2 \cdot \sin(\theta_{42})$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_{42}) + K_3$$

$$A = 0.1603 \quad B = -1.1447 \quad C = -0.3796$$

9. Use equation 4.10b to find values of θ_6 for the open circuit.

$$\theta_6 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right)$$

$$\theta_6 = -35.228 \text{ deg}$$

10. Determine the values of the constants needed for finding θ_5 from equations 4.11b and 4.12.

$$K_4 := \frac{h}{f} \quad K_5 := \frac{g^2 - h^2 - e^2 - f^2}{2 \cdot e \cdot f}$$

$$K_4 = 0.6112$$

$$K_5 = -1.0119$$

$$D := \cos(\theta_{42}) - K_1 + K_4 \cdot \cos(\theta_{42}) + K_5$$

$$D = -0.2408$$

$$E := -2 \cdot \sin(\theta_{42})$$

$$E = -1.1447$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_{42}) + K_5$$

$$F = -0.7807$$

11. Use equation 4.13 to find values of θ_5 for the open circuit.

$$\theta_5 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right)$$

$$\theta_5 = 280.938 \text{ deg}$$

12. Determine the angular velocity of link 6 for the open circuit using equations 6.18.

$$\omega_6 := \left(\frac{e \cdot \omega_4}{g} \right) \cdot \frac{\sin(\theta_{42} - \theta_5)}{\sin(\theta_6 - \theta_5)}$$

$$\omega_6 = -15.885 \frac{\text{rad}}{\text{sec}}$$

PROBLEM 6-77

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular velocity of link 6 in the sixbar linkage of Figure 3-35 as a function of θ_2 for a constant $\omega_2 = 1 \text{ rad/sec CCW}$.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 1.000 \cdot \text{in}$	Link 3 (A to B)	$b := 3.800 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 1.286 \cdot \text{in}$	Link 1 (O_2 to O_4)	$d := 3.857 \cdot \text{in}$
Link 4 (O_4 to D)	$e := 1.429 \cdot \text{in}$	Link 5 (D to E)	$f := 1.286 \cdot \text{in}$
Link 6 (O_6 to E)	$g := 0.771 \cdot \text{in}$	Link 1 (O_4 to O_6)	$h := 0.786 \cdot \text{in}$
Input crank angular velocity $\omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CCW}$			

Solution: See Figure 3-35 and Mathcad file P0677.

1. Define the range of the input angle: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_1 = 3.8570 \quad K_2 = 2.9992$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.2015$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0150 \quad K_5 = -3.7714$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

7. Link 4 will be the input to the second fourbar (links 4, 5, 6, and 1). Let the angle that link 4 makes in the second fourbar be

$$\theta_{42}(\theta_2) := \theta_4(\theta_2) + 157 \cdot \text{deg} - 360 \cdot \text{deg}$$

8. Determine the values of the constants needed for finding θ_6 from equations 4.8a and 4.10a.

$$K_1 := \frac{h}{e} \quad K_2 := \frac{h}{g} \quad K_1 = 0.5500 \quad K_2 = 1.0195$$

$$K_3 := \frac{e^2 - f^2 + g^2 + h^2}{2 \cdot e \cdot g} \quad K_3 = 0.7263$$

$$A'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K_1 - K_2 \cdot \cos(\theta_{42}(\theta_2)) + K_3 \quad B'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

$$C'(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_{42}(\theta_2)) + K_3$$

9. Use equation 4.10b to find values of θ_6 for the open circuit.

$$\theta_6(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A'(\theta_2), B'(\theta_2) - \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)} \right) \right)$$

10. Determine the values of the constants needed for finding θ_5 from equations 4.11b and 4.12.

$$K_4 := \frac{h}{f} \quad K_5 := \frac{g^2 - h^2 - e^2 - f^2}{2 \cdot e \cdot f} \quad K_4 = 0.6112 \quad K_5 = -1.0119$$

$$D'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K_1 + K_4 \cdot \cos(\theta_{42}(\theta_2)) + K_5 \quad E'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

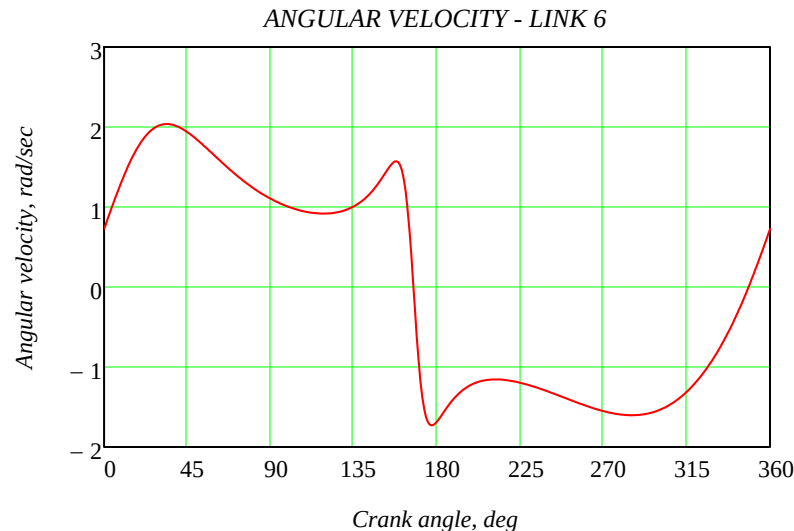
$$F'(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_{42}(\theta_2)) + K_5$$

11. Use equation 4.13 to find values of θ_5 for the open circuit.

$$\theta_5(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D'(\theta_2), -E'(\theta_2) - \sqrt{E'(\theta_2)^2 - 4 \cdot D'(\theta_2) \cdot F'(\theta_2)} \right) \right)$$

12. Determine the angular velocity of link6 for the open circuit using equations 6.18.

$$\omega_6(\theta_2) := \left(\frac{e \cdot \omega_4(\theta_2)}{g} \right) \cdot \frac{\sin(\theta_{42}(\theta_2) - \theta_5(\theta_2))}{\sin(\theta_6(\theta_2) - \theta_5(\theta_2))}$$



PROBLEM 6-78

Statement: Figure 3-36 shows an eightbar mechanism. Find all of its instant centers in the position shown in part (a) of the figure:

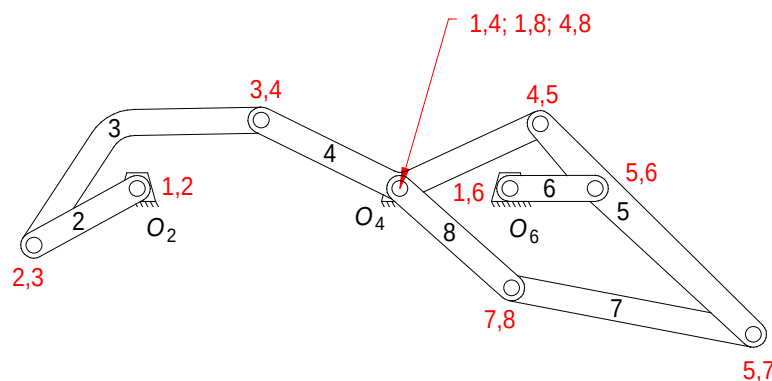
Given: Number of links $n := 8$

Solution: See Figure 3-36a and Mathcad file P0678.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 28$$

1. Draw the linkage to scale and identify those ICs that can be found by inspection (11).



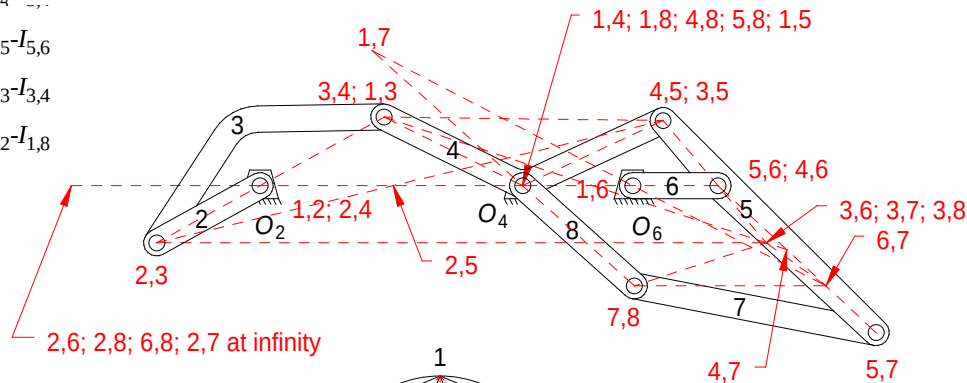
2. Use Kennedy's Rule and a linear graph to find the remaining 17 ICs.

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{2,4}$$

$$I_{4,6}: I_{1,4}-I_{1,6} \text{ and } I_{4,5}-I_{5,6}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$

$$I_{2,8}: I_{2,4}-I_{4,8} \text{ and } I_{1,2}-I_{1,8}$$



$$I_{2,6}: I_{1,2}-I_{1,6} \text{ and } I_{2,4}-I_{4,6}$$

$$I_{6,8}: I_{2,8}-I_{2,6} \text{ and } I_{1,8}-I_{1,6}$$

$$I_{6,7}: I_{5,6}-I_{5,7} \text{ and } I_{6,8}-I_{7,8}$$

$$I_{2,5}: I_{2,6}-I_{5,6} \text{ and } I_{2,3}-I_{4,5}$$

$$I_{2,7}: I_{2,8}-I_{7,8} \text{ and } I_{2,6}-I_{6,7}$$

$$I_{3,5}: I_{3,4}-I_{4,5} \text{ and } I_{2,3}-I_{2,5}$$

$$I_{3,6}: I_{2,3}-I_{2,6} \text{ and } I_{3,5}-I_{5,6}$$

$$I_{3,7}: I_{3,6}-I_{6,7} \text{ and } I_{2,3}-I_{2,7}$$

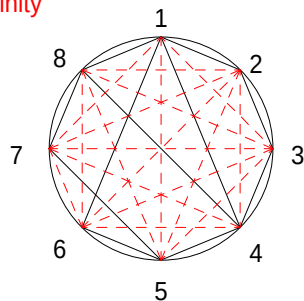
$$I_{3,8}: I_{3,7}-I_{7,8} \text{ and } I_{2,3}-I_{2,8}$$

$$I_{4,7}: I_{4,6}-I_{6,7} \text{ and } I_{3,4}-I_{3,7}$$

$$I_{5,8}: I_{5,6}-I_{6,8} \text{ and } I_{4,5}-I_{4,8}$$

$$I_{1,5}: I_{1,6}-I_{5,6} \text{ and } I_{1,4}-I_{4,5}$$

$$I_{1,7}: I_{1,8}-I_{7,8} \text{ and } I_{1,6}-I_{6,7}$$



PROBLEM 6-79

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular velocity of link 8 in the linkage of Figure 3-36 as a function of θ_2 for a constant $\omega_2 = 1 \text{ rad/sec CCW}$.

Given: Link lengths:

Input crank (L_2)	$a := 0.450$	First coupler (L_3)	$b := 0.990$
Common rocker (O_4B)	$c := 0.590$	First ground link (O_2O_4)	$d := 1.000$
Common rocker (O_4C)	$a' := 0.590$	Second coupler (CD)	$b' := 0.325$
Output rocker (L_6)	$c' := 0.325$	Second ground link (O_4O_6)	$d' := 0.419$
Link 7 (L_7)	$e := 0.938$	Link 8 (L_8)	$f := 0.572$
Link 5 extension (DE)	$p := 0.823$	Angle DCE	$\delta := 7.0 \cdot \text{deg}$
Angle BO_4C	$\alpha := 128.6 \cdot \text{deg}$		
Input crank angular velocity	$\omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$	CCW	

Solution: See Figure 3-36 and Mathcad file P0679.

1. See problem 4-43 for the position solution. The velocity solution will use the same vector loop equations for links 5, 6, 7, and 8, differentiated with respect to time. This problem is suitable for a project assignment and is probably too long for an overnight assignment.

PROBLEM 6-80

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the magnitude and direction of the velocity of point P in Figure 3-37a as a function of θ_2 for a constant $\omega_2 = 1$ rad/sec CCW. Also calculate and plot the velocity of point P versus point A .

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 0.136 \quad \text{Link 3 (A to B)} \quad b := 1.000$$

$$\text{Link 4 (B to O}_4) \quad c := 1.000 \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 1.414$$

$$\text{Coupler point:} \quad R_{pa} := 2.000 \quad \delta_3 := 0 \cdot \text{deg}$$

$$\text{Crank speed:} \quad \omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$$

Solution: See Figure 3-37a and Mathcad file P0680.

- Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

- Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 10.3971 \quad K_2 = 1.4140$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 7.4187$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

- Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

- Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.4140 \quad K_5 = -7.4187$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

- Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

- Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

7. Determine the velocity of point A using equations 6.19.

$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) \quad V_A(\theta_2) := |\mathbf{V}_A(\theta_2)|$$

8. Determine the velocity of the coupler point P using equations 6.36.

$$\mathbf{V}_{PA}(\theta_2) := R_{pa} \cdot \omega_{31}(\theta_2) \cdot (-\sin(\theta_{31}(\theta_2) + \delta_3) + j \cdot \cos(\theta_{31}(\theta_2) + \delta_3))$$

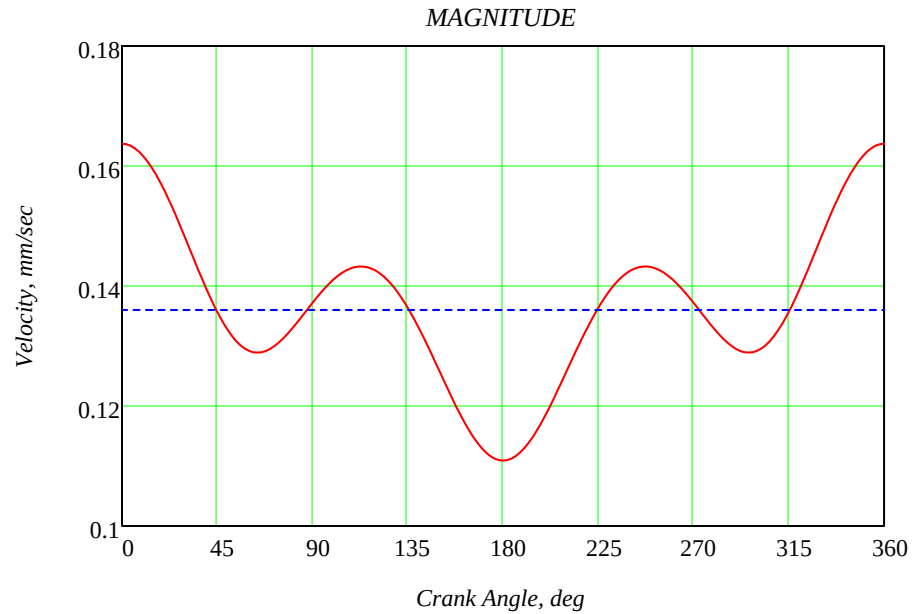
$$\mathbf{V}_P(\theta_2) := \mathbf{V}_A(\theta_2) + \mathbf{V}_{PA}(\theta_2)$$

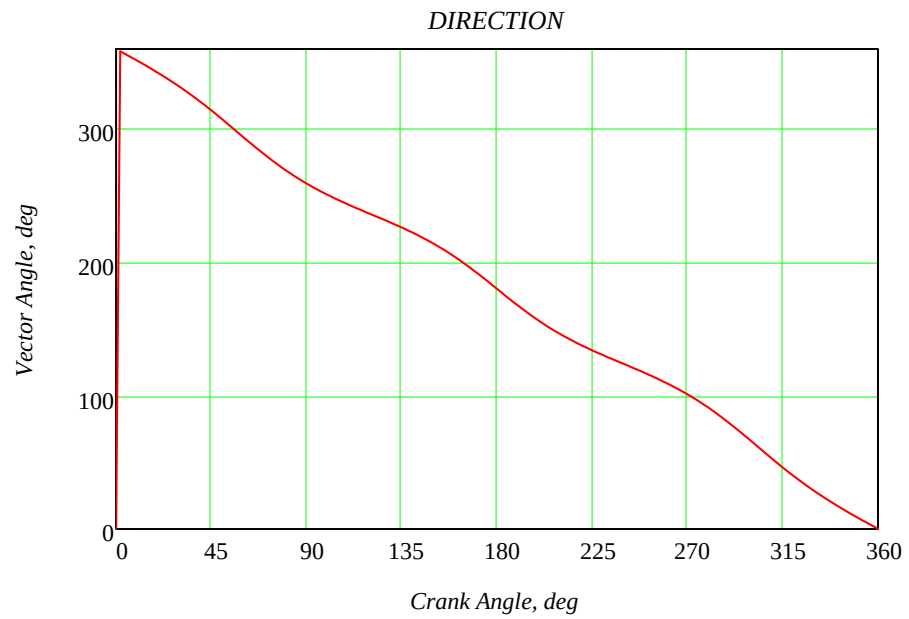
9. Plot the magnitude and direction of the velocity at coupler point P.

Magnitude: $V_P(\theta_2) := |\mathbf{V}_P(\theta_2)|$

Direction: $\theta_{VP1}(\theta_2) := \arg(\mathbf{V}_P(\theta_2))$

$$\theta_{VP}(\theta_2) := \text{if}(\theta_{VP1}(\theta_2) < 0, \theta_{VP1}(\theta_2) + 2 \cdot \pi, \theta_{VP1}(\theta_2))$$





PROBLEM 6-81

Statement: Calculate the percent error of the deviation from constant velocity magnitude of point P in Figure 3-37a.

Given: Link lengths:

$$\begin{array}{llll} \text{Link 2 (O}_2 \text{ to A)} & a := 0.136 & \text{Link 3 (A to B)} & b := 1.000 \\ \text{Link 4 (B to O}_4) & c := 1.000 & \text{Link 1 (O}_2 \text{ to O}_4) & d := 1.414 \\ \text{Coupler point:} & R_{pa} := 2.000 & \delta_3 := 0 \cdot \text{deg} & \\ \text{Crank speed:} & \omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1} & & \end{array}$$

Solution: See Figure 3-37a and Mathcad file P0681.

1. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg}$$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$\begin{array}{ll} K_1 := \frac{d}{a} & K_2 := \frac{d}{c} \\ K_1 = 10.3971 & K_2 = 1.4140 \\ K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} & K_3 = 7.4187 \end{array}$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$\begin{array}{llll} K_4 := \frac{d}{b} & K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} & K_4 = 1.4140 & K_5 = -7.4187 \\ D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 & & & \\ E(\theta_2) := -2 \cdot \sin(\theta_2) & & & \\ F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 & & & \end{array}$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

7. Determine the velocity of point A using equations 6.19.

$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) \quad V_A(\theta_2) := |\mathbf{V}_A(\theta_2)|$$

8. Determine the velocity of the coupler point P using equations 6.36.

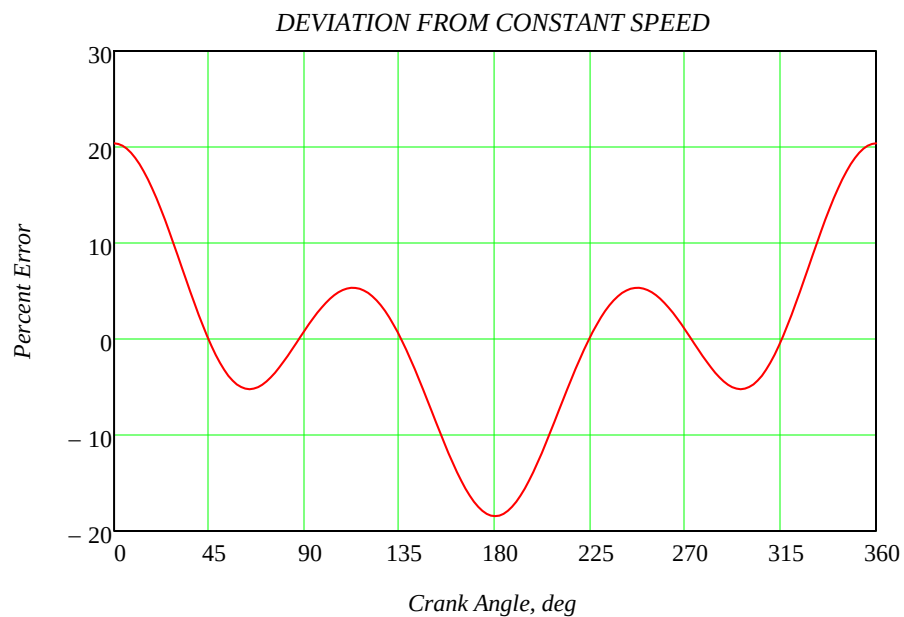
$$\mathbf{V}_{PA}(\theta_2) := R_{pa} \cdot \omega_{31}(\theta_2) \cdot (-\sin(\theta_{31}(\theta_2) + \delta_3) + j \cdot \cos(\theta_{31}(\theta_2) + \delta_3))$$

$$\mathbf{V}_P(\theta_2) := \mathbf{V}_A(\theta_2) + \mathbf{V}_{PA}(\theta_2)$$

9. Calculate the magnitude of the velocity at coupler point P.

Magnitude: $V_P(\theta_2) := |\mathbf{V}_P(\theta_2)|$

Deviation from constant speed: $err(\theta_2) := \frac{V_P(\theta_2) - V_A(\theta_2)}{V_A(\theta_2)}$



PROBLEM 6-82

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the magnitude and the direction of the velocity of point P in Figure 3-37b as a function of θ_2 . Also calculate and plot the velocity of point P versus point A .

Given: Link lengths:

Input crank (L_2)	$a := 0.50$	First coupler (AB)	$b := 1.00$
Rocker 4 (O_4B)	$c := 1.00$	Rocker 5 (L_5)	$c' := 1.00$
Ground link (O_2O_4)	$d := 0.75$	Second coupler 6 (CD)	$b' := 1.00$
Coupler point (DP)	$p := 1.00$	Distance to O_P (O_2O_P)	$d' := 1.50$
Crank speed:	$\omega_2 := 1 \cdot \frac{\text{rad}}{\text{sec}}$		

Solution: See Figure 3-37b and Mathcad file P0682.

- Links 4, 5, BC , and CD form a parallelogram whose opposite sides remain parallel throughout the motion of the fourbar 1, 2, AB , 4. Define a position vector whose tail is at point D and whose tip is at point P and another whose tail is at O_4 and whose tip is at point D . Then, since $\mathbf{R}_5 = \mathbf{R}_{AB}$ and $\mathbf{R}_{DP} = -\mathbf{R}_4$, the position vector from O_2 to P is $\mathbf{P} = \mathbf{R}_1 + \mathbf{R}_{AB} - \mathbf{R}_4$. Separating this vector equation into real and imaginary parts gives the equations for the X and Y coordinates of the coupler point P .

$$X_P = d + b \cdot \cos(\theta_3) - c \cdot \cos(\theta_4) \quad Y_P = b \cdot \sin(\theta_3) - c \cdot \sin(\theta_4)$$

- Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. 360 \cdot \text{deg}$
- Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 1.5000 \quad K_2 = 0.7500 \quad K_3 = 0.8125$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right) - 2 \cdot \pi$$

- Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.7500 \quad K_5 = -0.8125$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

- Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right) - 2 \cdot \pi$$

- Define a local xy coordinate system with origin at O_P and with the positive x axis to the right. The coordinates of P are transformed to $x_P = X_P - d'$, $y_P = Y_P$.

$$x_P(\theta_2) := d + b \cdot \cos(\theta_3(\theta_2)) - c \cdot \cos(\theta_4(\theta_2)) - d'$$

$$y_P(\theta_2) := b \cdot \sin(\theta_3(\theta_2)) - c \cdot \sin(\theta_4(\theta_2))$$

7. Use equations 6.18 to calculate ω_3 and ω_4 .

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

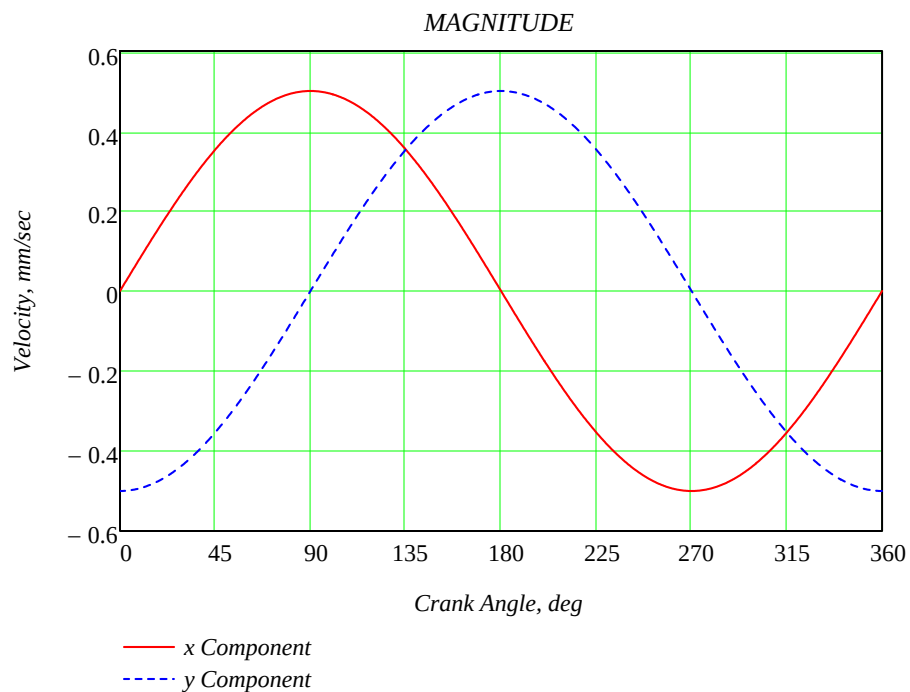
$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Differentiate the position equations with respect to time to get the velocity components.

$$V_{Px}(\theta_2) := -b \cdot \omega_3(\theta_2) \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2) \cdot \sin(\theta_4(\theta_2))$$

$$V_{Py}(\theta_2) := b \cdot \omega_3(\theta_2) \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2) \cdot \cos(\theta_4(\theta_2))$$

$$V_P(\theta_2) := \sqrt{V_{Px}(\theta_2)^2 + V_{Py}(\theta_2)^2}$$



PROBLEM 6-83

Statement: Find all instant centers of the linkage in Figure P6-30 in the position shown.

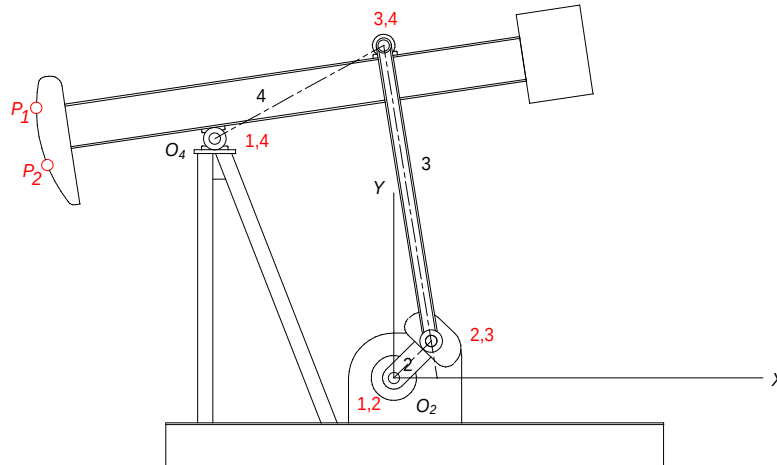
Given: Number of links $n := 4$

Solution: See Figure P6-30 and Mathcad file P0683.

- Determine the number of instant centers for this mechanism using equation 6.8a.

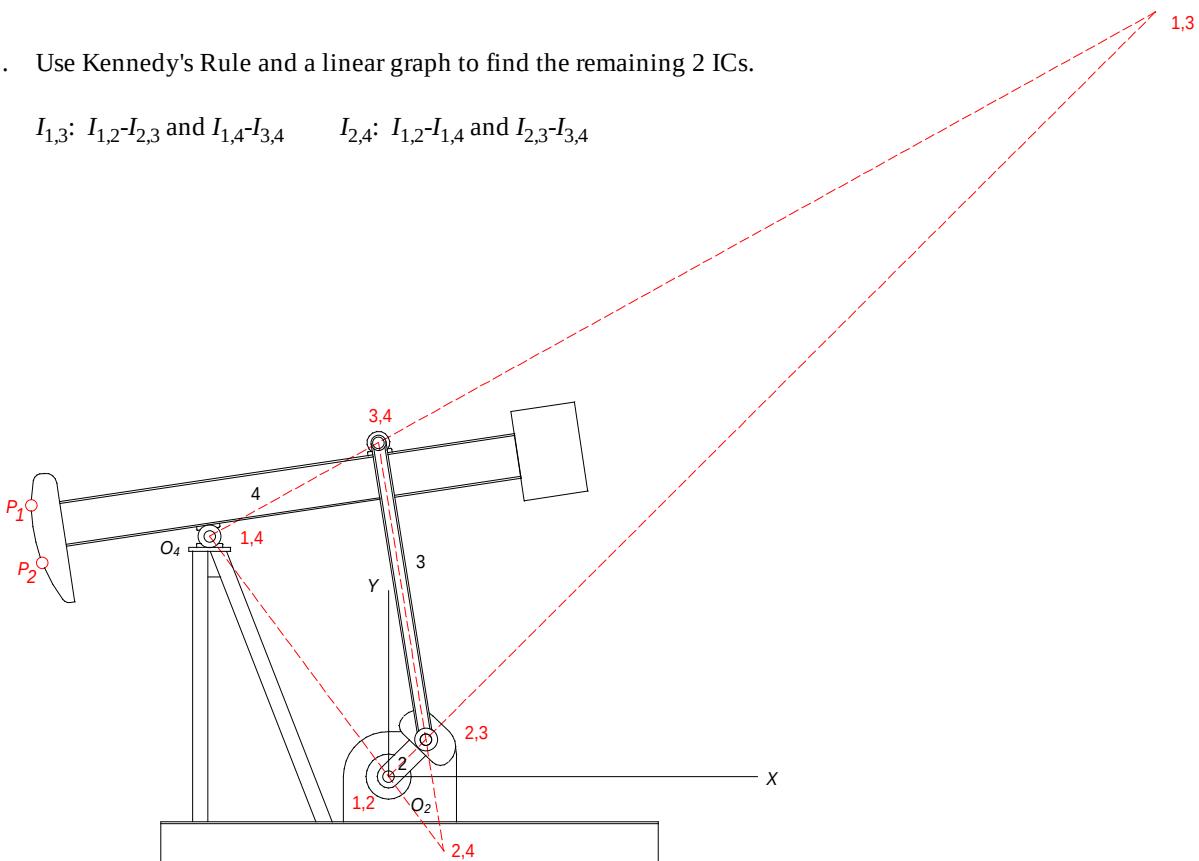
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

- Draw the linkage to scale and identify those ICs that can be found by inspection (4).



- Use Kennedy's Rule and a linear graph to find the remaining 2 ICs.

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4} \quad I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



PROBLEM 6-84a

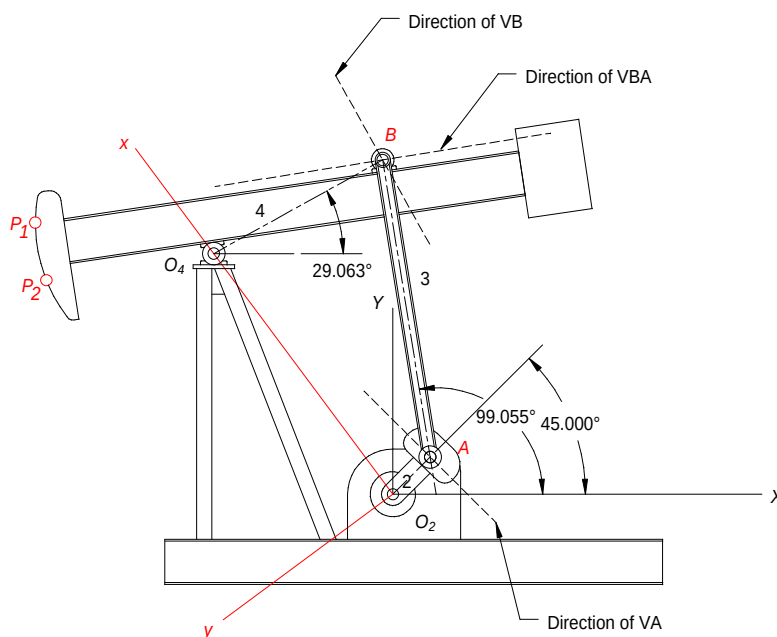
Statement: Find the angular velocities of links 3 and 4 and the linear velocity of points A, B and P_1 in the XY coordinate system for the linkage in Figure P6-30 in the position shown. Assume that $\theta_2 = 45^\circ$ in the XY coordinate system and $\omega_2 = 10 \text{ rad/sec}$. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the xy coordinate system. Use a graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 14.00 \cdot \text{in}$	Link 3 (A to B)	$b := 80.00 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 51.26 \cdot \text{in}$		
Link 1 X-offset	$d_X := 47.5 \cdot \text{in}$	Link 1 Y-offset	$d_Y := 76.00 \cdot \text{in} - 12.00 \cdot \text{in}$
Coupler point x-offset	$p_x := 114.68 \cdot \text{in}$	Coupler point y-offset	$p_y := 33.19 \cdot \text{in}$
Crank angle:	$\theta_2 := 45 \cdot \text{deg}$		
Input crank angular velocity	$\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$	CCW	

Solution: See Figure P6-30 and Mathcad file P0684a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



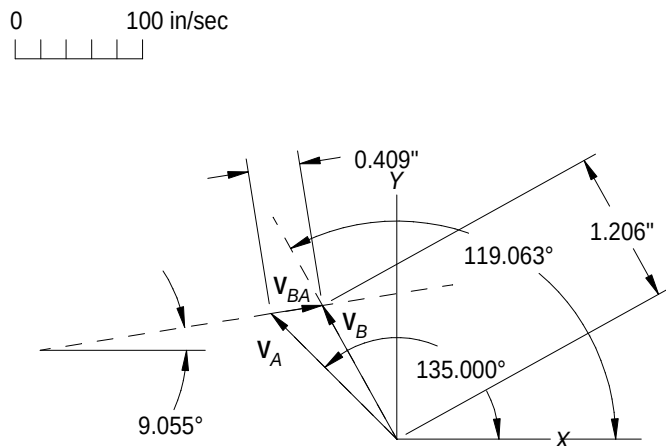
2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 140.000 \frac{\text{in}}{\text{sec}} \quad \theta_{VA} := 45 \cdot \text{deg} + 90 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity V_{BA} , and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{100 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_B := 1.206 \cdot \text{in} \cdot k_v \quad V_B = 120.600 \frac{\text{in}}{\text{sec}} \quad \theta_{VB} := 119.063 \cdot \text{deg}$$

$$V_{BA} := 0.409 \cdot \text{in} \cdot k_v \quad V_{BA} = 40.900 \frac{\text{in}}{\text{sec}}$$

5. Determine the angular velocity of links 3 and 4 using equation 6.7.

$$\omega_3 := \frac{-V_{BA}}{b} \quad \omega_3 = -0.511 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{V_B}{c} \quad \omega_4 = 2.353 \frac{\text{rad}}{\text{sec}}$$

6. Transform the xy coordinates of point P1 into the XY system using equations 4.0b.

$$\text{Coordinate transformation angle } \delta := \text{atan2}(-d_X, d_Y) \quad \delta = 126.582 \text{ deg}$$

$$P_X := p_x \cdot \cos(\delta) - p_y \cdot \sin(\delta) \quad P_X = -94.998 \text{ in}$$

$$P_Y := p_x \cdot \sin(\delta) + p_y \cdot \cos(\delta) \quad P_Y = 72.308 \text{ in}$$

7. Calculate the distance from O_4 to P_1 and use equation 6.7 to calculate the velocity at P_1 .

$$\text{Distance from } O_4 \text{ to } P_1: \quad e := \sqrt{(P_X + d_X)^2 + (P_Y - d_Y)^2} \quad e = 48.219 \text{ in}$$

$$V_{P1} := e \cdot \omega_4 \quad V_{P1} = 113.446 \frac{\text{in}}{\text{sec}}$$

PROBLEM 6-84b

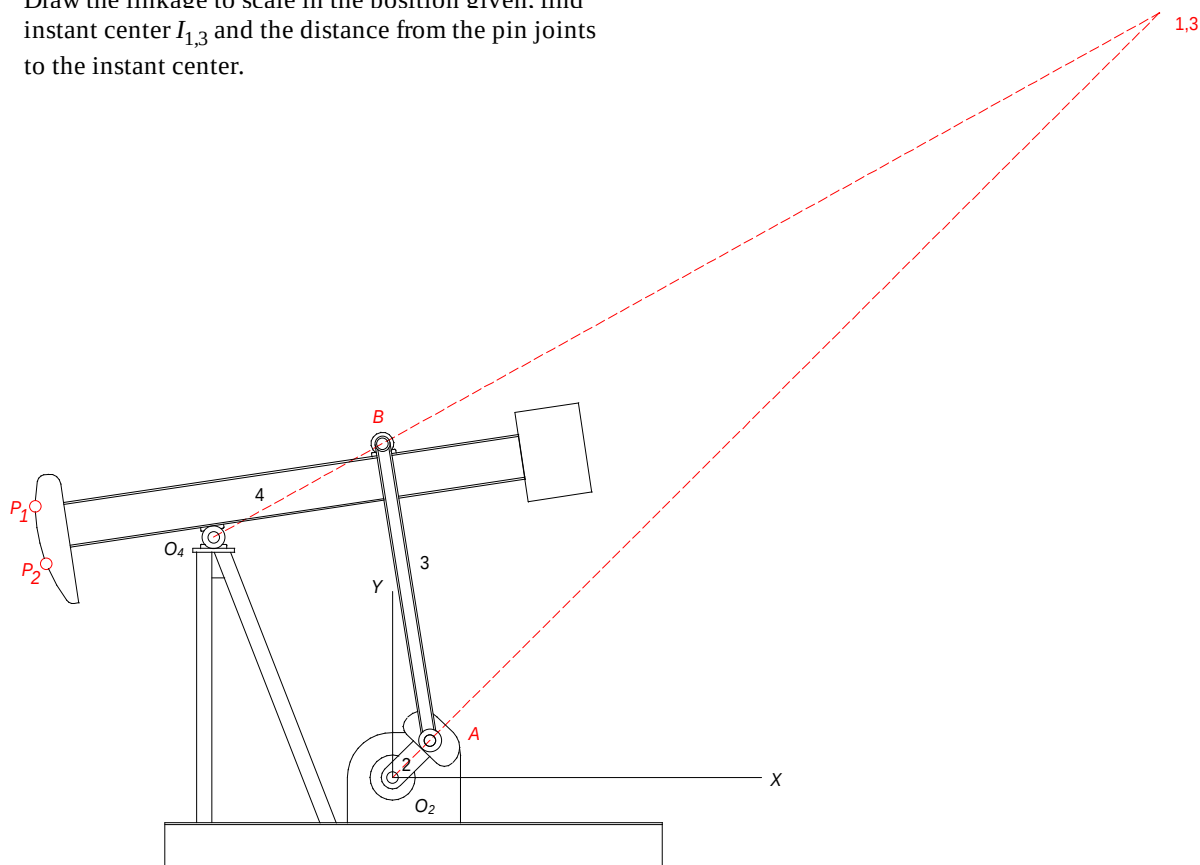
Statement: Find the angular velocities of links 3 and 4 and the linear velocity of points A, B and P_1 in the XY coordinate system for the linkage in Figure P6-30 in the position shown. Assume that $\theta_2 = 45^\circ$ in the XY coordinate system and $\omega_2 = 10 \text{ rad/sec}$. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the xy coordinate system. Use the method of instant centers.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 14.00 \cdot \text{in}$	Link 3 (A to B)	$b := 80.00 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 51.26 \cdot \text{in}$		
Link 1 X-offset	$d_x := 47.5 \cdot \text{in}$	Link 1 Y-offset	$d_y := 76.00 \cdot \text{in} - 12.00 \cdot \text{in}$
Coupler point x-offset	$p_x := 114.68 \cdot \text{in}$	Coupler point y-offset	$p_y := 33.19 \cdot \text{in}$
Crank angle:	$\theta_2 := 45^\circ$		
Input crank angular velocity	$\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$	CCW	

Solution: See Figure P6-30 and Mathcad file P0684b.

1. Draw the linkage to scale in the position given. find instant center $I_{1,3}$ and the distance from the pin joints to the instant center.



From the layout above:

$$AI_{13} := 273.768 \cdot \text{in} \quad BI_{13} := 235.874 \cdot \text{in}$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 140.000 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 + 90 \cdot \text{deg} \qquad \theta_{VA} = 135.0 \text{ deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 0.511 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point B using equation 6.9b.

$$V_B := BI13 \cdot \omega_3 \qquad V_B = 120.622 \frac{\text{in}}{\text{sec}}$$

5. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 2.353 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

6. Transform the xy coordinates of point P1 into the XY system using equations 4.0b.

$$\text{Coordinate transformation angle } \delta := \text{atan2}(-d_X, d_Y) \qquad \delta = 126.582 \text{ deg}$$

$$P_X := p_x \cdot \cos(\delta) - p_y \cdot \sin(\delta) \qquad P_X = -94.998 \text{ in}$$

$$P_Y := p_x \cdot \sin(\delta) + p_y \cdot \cos(\delta) \qquad P_Y = 72.308 \text{ in}$$

7. Calculate the distance from O_4 to P_1 and use equation 6.7 to calculate the velocity at P_1 .

$$\text{Distance from } O_4 \text{ to } P_1: \quad e := \sqrt{(P_X + d_X)^2 + (P_Y - d_Y)^2} \qquad e = 48.219 \text{ in}$$

$$V_{P1} := e \cdot \omega_4 \qquad V_{P1} = 113.467 \frac{\text{in}}{\text{sec}}$$

PROBLEM 6-84c

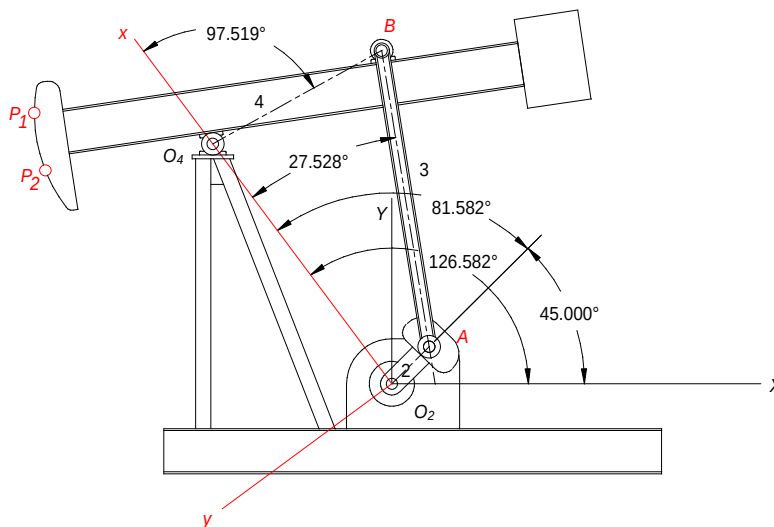
Statement: Find the angular velocities of links 3 and 4 and the linear velocity of points A, B and P_1 in the XY coordinate system for the linkage in Figure P6-30 in the position shown. Assume that $\theta_2 = 45^\circ$ in the XY coordinate system and $\omega_2 = 10 \text{ rad/sec}$. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the xy coordinate system. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 14.00 \cdot \text{in}$	Link 3 (A to B)	$b := 80.00 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 51.26 \cdot \text{in}$		
Link 1 X-offset	$d_X := 47.5 \cdot \text{in}$	Link 1 Y-offset	$d_Y := 76.00 \cdot \text{in} - 12.00 \cdot \text{in}$
Coupler point x-offset	$p_x := 114.68 \cdot \text{in}$	Coupler point y-offset	$p_y := 33.19 \cdot \text{in}$
Crank angle:	$\theta_{2XY} := 45 \cdot \text{deg}$	XY coord system	
Coordinate transformation angle:	$\delta := 126.582 \cdot \text{deg}$		
Input crank angular velocity	$\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$	CCW	

Solution: See Figure P6-30 and Mathcad file P0684c.

1. Draw the linkage to scale and label it.



Transform the crank angle from global to local coordinate system:

$$\theta_2 := \theta_{2XY} - \delta \quad \theta_2 = -81.582 \text{ deg}$$

$$\text{Distance } O_2O_4: \quad d := \sqrt{d_X^2 + d_Y^2} \quad d = 79.701 \text{ in}$$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_1 = 5.6929 \quad K_2 = 1.5548$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.9340$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -3.8401 \quad B = 1.9785 \quad C = 7.2529$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_4 = 262.482 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.9963 \quad K_5 = -4.6074$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -10.0081$$

$$E := -2 \cdot \sin(\theta_2) \quad E = 1.9785$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 1.0849$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_3 = 332.475 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_4 - \theta_2)}{b \cdot \sin(\theta_3 - \theta_4)} \quad \omega_3 = -0.511 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_4 - \theta_3)} \quad \omega_4 = 2.353 \frac{\text{rad}}{\text{sec}}$$

7. Determine the velocity of points A and B for the crossed circuit using equations 6.19.

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = (138.492 + 20.495j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_A| = 140.000 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VAXY} := \arg(\mathbf{V}_A) + \delta \quad \theta_{VAXY} = 135.000 \text{ deg}$$

$$\mathbf{V}_B := c \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$$

$$\mathbf{V}_B = (119.588 - 15.781j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_B| = 120.624 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VBXY} := \arg(\mathbf{V}_B) + \delta \quad \theta_{VBXY} = 119.064 \text{ deg}$$

8. Calculate the distance from O_4 to P_1 and the angle BO_4P_1 .

$$\text{Distance from } O_4 \text{ to } P_1: \quad e := \sqrt{(p_x - d)^2 + p_y^2} \quad e = 48.219 \text{ in}$$

$$\delta_4 := 180 \cdot \text{deg} - \text{atan} \left(\frac{p_y}{p_x - d} \right) \quad \delta_4 = 136.503 \text{ deg}$$

9. Determine the velocity of point P_1 using equations 6.35.

$$\mathbf{V}_{P1} := e \cdot \omega_4 \cdot (-\sin(\theta_4 + \delta) + j \cdot \cos(\theta_4 + \delta))$$

$$\mathbf{V}_{P1} = (-55.122 + 99.181j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_{P1}| = 113.469 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VPXY} := \arg(\mathbf{V}_{P1}) + \delta \quad \theta_{VPXY} = 245.646 \text{ deg}$$

PROBLEM 6-85

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the absolute velocity of point P_1 in Figure P6-30 as a function of θ_2 for $\omega_2 = 10 \text{ rad/sec}$. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the xy coordinate system.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 14.00 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 80.00 \cdot \text{in}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 51.26 \cdot \text{in}$$

$$\text{Link 1 X-offset} \quad d_X := 47.5 \cdot \text{in} \quad \text{Link 1 Y-offset} \quad d_Y := 76.00 \cdot \text{in} - 12.00 \cdot \text{in}$$

$$\text{Coupler point x-offset} \quad p_x := 114.68 \cdot \text{in} \quad \text{Coupler point y-offset} \quad p_y := 33.19 \cdot \text{in}$$

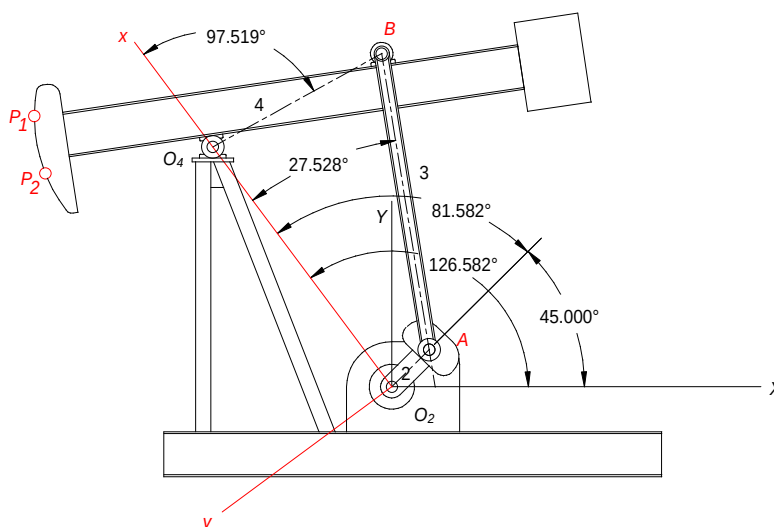
$$\text{Crank angle:} \quad \theta_{2XY} := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg} \quad \text{XY coord system}$$

$$\text{Coordinate transformation angle:} \quad \delta := 126.582 \cdot \text{deg}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CCW}$$

Solution: See Figure P6-30 and Mathcad file P0685.

1. Draw the linkage to scale and label it.



Transform the crank angle from global to local coordinate system:

$$\theta_2(\theta_{2XY}) := \theta_{2XY} - \delta$$

$$\text{Distance } O_2O_4: \quad d := \sqrt{d_X^2 + d_Y^2} \quad d = 79.701 \text{ in}$$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_1 = 5.6929 \quad K_2 = 1.5548$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.9340$$

$$A(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 - K_2 \cdot \cos(\theta_2(\theta_{2XY})) + K_3$$

$$B(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$C(\theta_{2XY}) := K_1 - (K_2 + 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_{2XY}) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_{2XY}), -B(\theta_{2XY}) + \sqrt{B(\theta_{2XY})^2 - 4 \cdot A(\theta_{2XY}) \cdot C(\theta_{2XY})} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.9963 \quad K_5 = -4.6074$$

$$D(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 + K_4 \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

$$E(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$F(\theta_{2XY}) := K_1 + (K_4 - 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_{2XY}) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_{2XY}), -E(\theta_{2XY}) + \sqrt{E(\theta_{2XY})^2 - 4 \cdot D(\theta_{2XY}) \cdot F(\theta_{2XY})} \right) \right)$$

6. Determine the angular velocity of link 4 for the crossed circuit using equations 6.18.

$$\omega_4(\theta_{2XY}) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2(\theta_{2XY}) - \theta_3(\theta_{2XY}))}{\sin(\theta_4(\theta_{2XY}) - \theta_3(\theta_{2XY}))}$$

7. Calculate the distance from O_4 to P_1 and the angle BO_4P_1 .

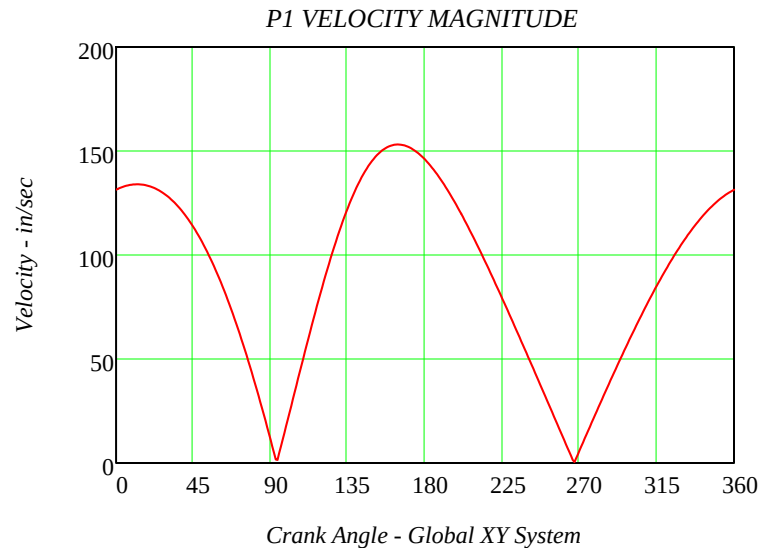
$$\text{Distance from } O_4 \text{ to } P_1: \quad e := \sqrt{(p_x - d)^2 + p_y^2} \quad e = 48.219 \text{ in}$$

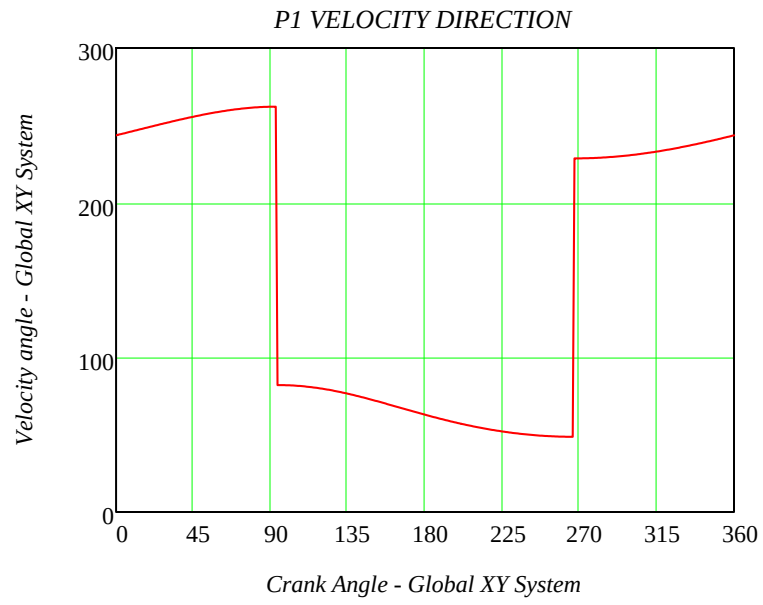
$$\delta_4 := 180 \cdot \text{deg} - \text{atan} \left(\frac{p_y}{p_x - d} \right) \quad \delta_4 = 136.503 \text{ deg}$$

8. Determine and plot the velocity of point P_1 using equations 6.35.

$$\mathbf{V}_{P1}(\theta_{2XY}) := e \cdot \omega_4(\theta_{2XY}) \cdot (-\sin(\theta_4(\theta_{2XY}) + \delta_4) + j \cdot \cos(\theta_4(\theta_{2XY}) + \delta_4))$$

$$V_{P1}(\theta_{2XY}) := |\mathbf{V}_{P1}(\theta_{2XY})| \quad \theta_{VPXY}(\theta_{2XY}) := \arg(\mathbf{V}_{P1}(\theta_{2XY})) + \delta$$





PROBLEM 6-86

Statement: Find all instant centers of the linkage in Figure P6-31 in the position shown.

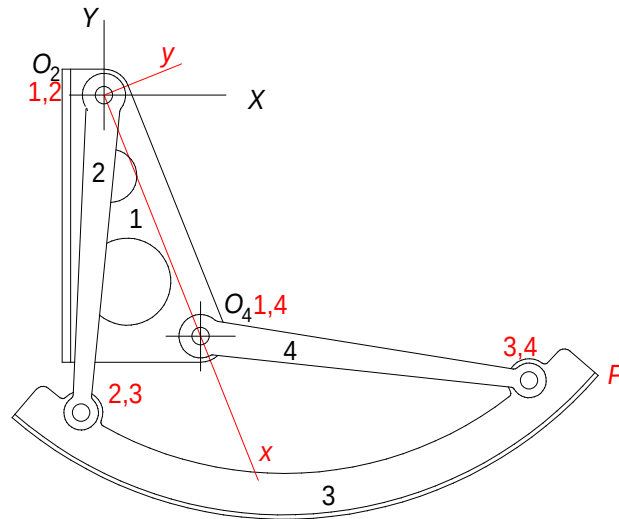
Given: Number of links $n := 4$

Solution: See Figure P6-31 and Mathcad file P0686.

- Determine the number of instant centers for this mechanism using equation 6.8a.

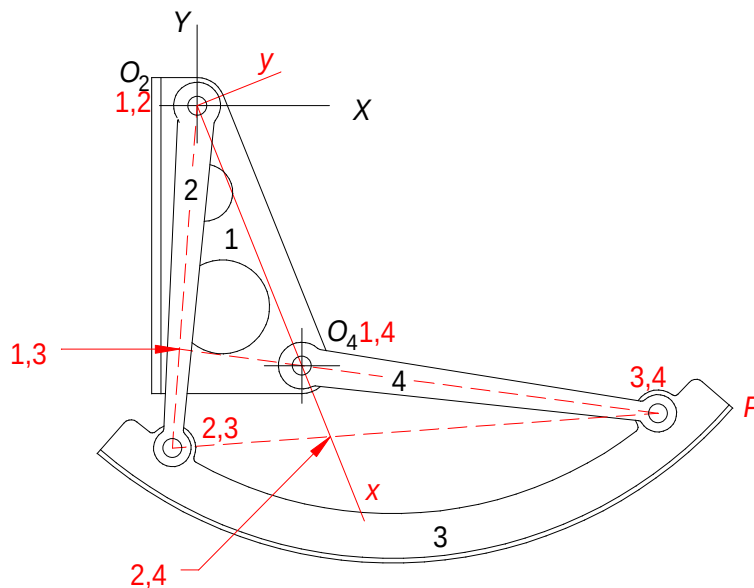
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

- Draw the linkage to scale and identify those ICs that can be found by inspection (4).



- Use Kennedy's Rule and a linear graph to find the remaining 2 ICs.

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4} \quad I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$



PROBLEM 6-87a

Statement: Find the angular velocities of links 3 and 4 and the linear velocity of point P in the XY coordinate system for the linkage in Figure P6-31 in the position shown. Assume that $\theta_2 = -94.121$ deg in the XY coordinate system and $\omega_2 = 1$ rad/sec. The position of the coupler point P on link 3 with respect to point A is: $p = 15.00$, $\delta_3 = 0$ deg. Use a graphical method.

Given: Link lengths:

Link 2 (O_2 to A) $a := 9.17 \cdot \text{in}$ Link 3 (A to B) $b := 12.97 \cdot \text{in}$

Link 4 (O_4 to B) $c := 9.57 \cdot \text{in}$

Link 1 X -offset $d_X := 2.79 \cdot \text{in}$ Link 1 Y -offset $d_Y := -6.95 \cdot \text{in}$

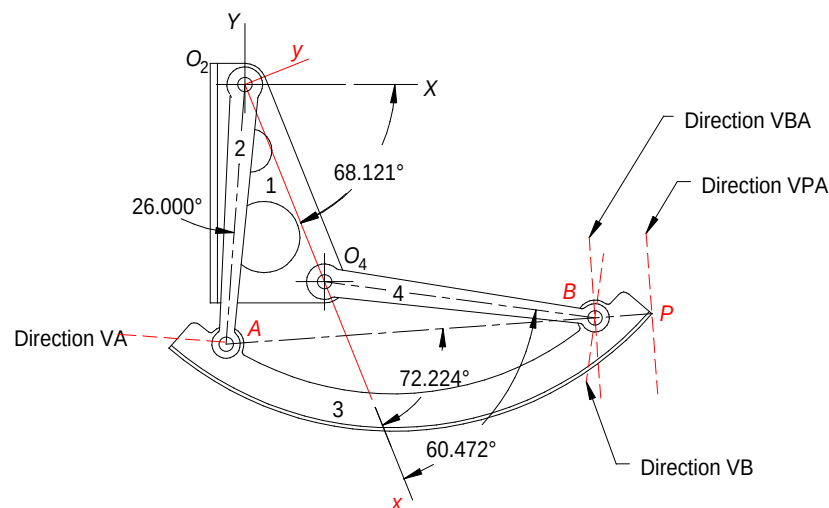
Coupler point data: $p := 15.00 \cdot \text{in}$ $\delta_3 := 0 \cdot \text{deg}$

Crank angle: $\theta_2 := -94.121 \cdot \text{deg}$

Input crank angular velocity $\omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

Solution: See Figure P6-31 and Mathcad file P0687a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A .

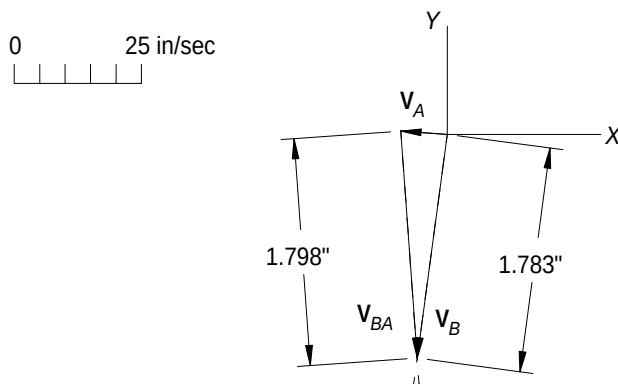
$$V_A := a \cdot \omega_2 \quad V_A = 9.170 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{VA} = -184.121 \text{ deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B , the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{25 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_B := 1.783 \cdot \text{in} \cdot k_v \quad V_B = 44.575 \frac{\text{in}}{\text{sec}} \quad \theta_{VB} := 262.352 \cdot \text{deg}$$

$$V_{BA} := 1.798 \cdot \text{in} \cdot k_v \quad V_{BA} = 44.950 \frac{\text{in}}{\text{sec}} \quad \theta_{VBA} := 274.103 \cdot \text{deg}$$

5. Determine the angular velocity of links 3 and 4 using equation 6.7.

$$\omega_3 := \frac{-V_{BA}}{b} \quad \omega_3 = -3.466 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{-V_B}{c} \quad \omega_4 = -4.658 \frac{\text{rad}}{\text{sec}}$$

6. Determine the magnitude and sense of the vector $\mathbf{V}_{PA}$ using equation 6.7.

$$V_{PA} := |p \cdot \omega_3| \quad V_{PA} = 51.985 \frac{\text{in}}{\text{sec}}$$

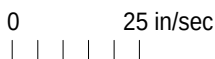
$$\theta_{VPA} := \theta_{VBA} \quad \theta_{VPA} = 274.103 \text{ deg}$$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point P . The equation to be solved graphically is

$$\mathbf{V}_P = \mathbf{V}_A + \mathbf{V}_{PA}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- From the tip of $\mathbf{V}_A$, layout the (now) known vector $\mathbf{V}_{PA}$.
- Complete the vector triangle by drawing $\mathbf{V}_P$ from the tail of $\mathbf{V}_A$ to the tip of the $\mathbf{V}_{PA}$ vector.

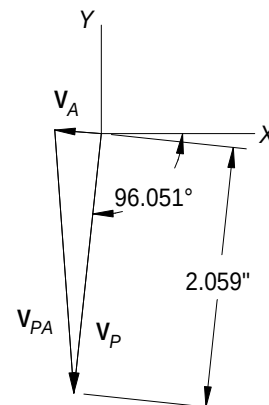
8. From the velocity triangle we have:



Velocity scale factor: $k_v := \frac{25 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_P := 2.059 \cdot \text{in} \cdot k_v \quad V_P = 51.475 \frac{\text{in}}{\text{sec}}$$

$$\theta_P := -96.051 \cdot \text{deg}$$



PROBLEM 6-87b

Statement: Find the angular velocities of links 3 and 4 and the linear velocity of point P in the XY coordinate system for the linkage in Figure P6-31 in the position shown. Assume that $\theta_2 = -94.121$ deg in the XY coordinate system and $\omega_2 = 1$ rad/sec. The position of the coupler point P on link 3 with respect to point A is: $p = 15.00$, $\delta_3 = 0$ deg. Use the method of instant centers.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 9.174 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 12.971 \cdot \text{in}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 9.573 \cdot \text{in}$$

$$\text{Link 1 X-offset} \quad d_X := 2.790 \cdot \text{in} \quad \text{Link 1 Y-offset} \quad d_Y := -6.948 \cdot \text{in}$$

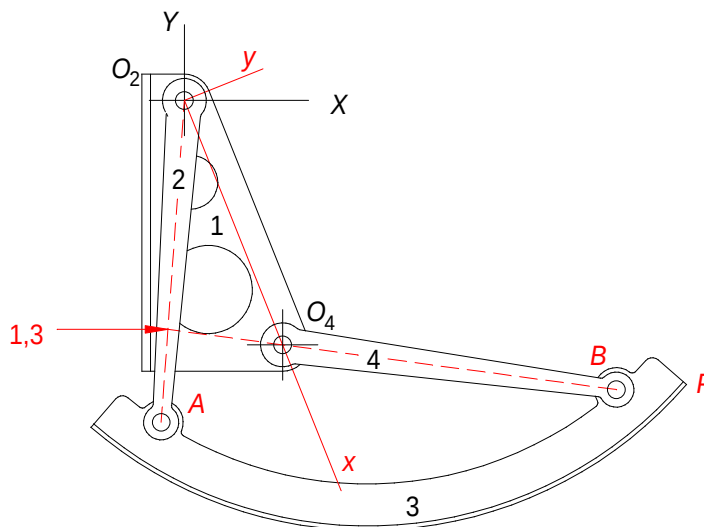
$$\text{Coupler point data:} \quad p := 15.00 \cdot \text{in} \quad \delta_3 := 0 \cdot \text{deg}$$

$$\text{Crank angle:} \quad \theta_2 := -94.121 \cdot \text{deg}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$$

Solution: See Figure P6-31 and Mathcad file P0687b.

1. Draw the linkage to scale in the position given, find instant center $I_{1,3}$ and the distance from the pin joints to the instant center.



From the layout above:

$$AI13 := 2.647 \cdot \text{in} \quad BI13 := 12.862 \cdot \text{in} \quad PI13 := 14.825 \cdot \text{in}$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A .

$$V_A := a \cdot \omega_2 \quad V_A = 9.174 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{VA} = -184.121 \text{ deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \quad \omega_3 = 3.466 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point B using equation 6.9b.

$$V_B := BI13 \cdot \omega_3 \quad V_B = 44.577 \frac{\text{in}}{\text{sec}}$$

5. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 4.657 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

6. Determine the magnitude of the velocity at point P using equation 6.9b.

$$V_P := PI13 \cdot \omega_3 \qquad V_P = 51.381 \frac{\text{in}}{\text{sec}}$$

PROBLEM 6-87c

Statement: Find the angular velocities of links 3 and 4 and the linear velocity of point P in the XY coordinate system for the linkage in Figure P6-31 in the position shown. Assume that $\theta_2 = -94.121$ deg in the XY coordinate system and $\omega_2 = 1$ rad/sec. The position of the coupler point P on link 3 with respect to point A is: $p = 15.00$, $\delta_3 = 0$ deg. Use an analytical method.

Given:

Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 9.174 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 12.971 \cdot \text{in}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 9.573 \cdot \text{in}$$

$$\text{Link 1 X-offset} \quad d_X := 2.790 \cdot \text{in} \quad \text{Link 1 Y-offset} \quad d_Y := -6.948 \cdot \text{in}$$

$$\text{Coupler point data:} \quad p := 15.00 \cdot \text{in} \quad \delta_3 := 0 \cdot \text{deg}$$

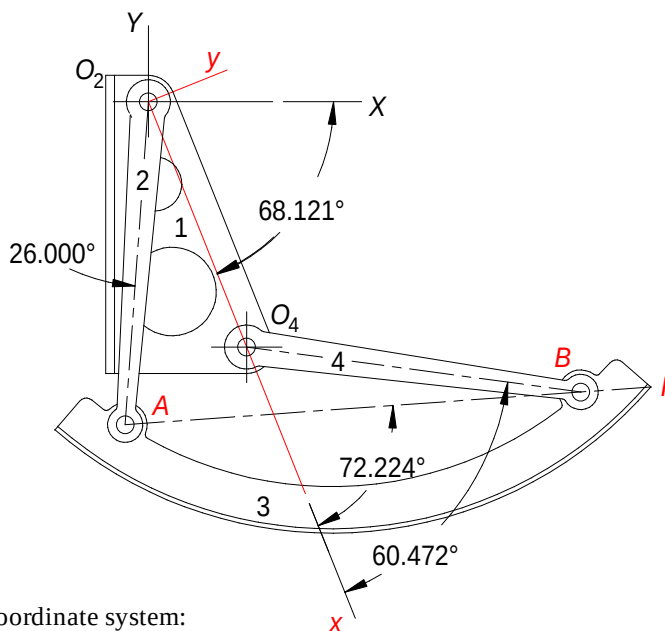
$$\text{Crank angle:} \quad \theta_{2XY} := -94.121 \cdot \text{deg} \quad \text{Global } XY \text{ coord system}$$

$$\text{Coordinate transformation angle:} \quad \delta := -68.121 \cdot \text{deg}$$

$$\text{Input crank angular velocity} \quad \omega_2 := -1 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$$

Solution: See Figure P6-31 and Mathcad file P0687c.

1. Draw the linkage to scale and label it.



Transform crank angle into local xy coordinate system:

$$\theta_2 := \theta_{2XY} - \delta \quad \theta_2 = -26.000 \text{ deg}$$

$$\text{Calculate distance } O_2O_4: \quad d := \sqrt{d_X^2 + d_Y^2} \quad d = 7.487 \text{ in}$$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_1 = 0.8161 \quad K_2 = 0.7821$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 0.3622$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.2581 \quad B = 0.8767 \quad C = -0.4234$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) + 2 \cdot \pi \quad \theta_4 = 60.488 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$\begin{aligned} K_4 &:= \frac{d}{b} & K_5 &:= \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} & K_4 &= 0.5772 & K_5 &= -0.9111 \\ D &:= \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 & D &= -0.3096 \\ E &:= -2 \cdot \sin(\theta_2) & E &= 0.8767 \\ F &:= K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 & F &= -0.4749 \end{aligned}$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) + 2 \cdot \pi \quad \theta_3 = 72.236 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\begin{aligned} \omega_3 &:= \frac{a \cdot \omega_2 \cdot \sin(\theta_4 - \theta_2)}{b \cdot \sin(\theta_3 - \theta_4)} & \omega_3 &= -3.467 \frac{\text{rad}}{\text{sec}} \\ \omega_4 &:= \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_4 - \theta_3)} & \omega_4 &= -4.658 \frac{\text{rad}}{\text{sec}} \end{aligned}$$

7. Determine the velocity of points A and B using equations 6.19.

$$\begin{aligned} \mathbf{V}_A &:= a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) \\ \mathbf{V}_A &= (-4.022 - 8.246j) \frac{\text{in}}{\text{sec}} & |\mathbf{V}_A| &= 9.174 \frac{\text{in}}{\text{sec}} \\ \theta_{VAXY} &:= \arg(\mathbf{V}_A) + \delta & \theta_{VAXY} &= -184.121 \text{ deg} \\ \mathbf{V}_B &:= c \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) \\ \mathbf{V}_B &= (38.807 - 21.967j) \frac{\text{in}}{\text{sec}} & |\mathbf{V}_B| &= 44.593 \frac{\text{in}}{\text{sec}} \\ \theta_{VBXY} &:= \arg(\mathbf{V}_B) + \delta & \theta_{VBXY} &= -97.633 \text{ deg} \end{aligned}$$

8. Determine the velocity of the coupler point P for the crossed circuit using equations 6.36.

$$\begin{aligned} \mathbf{V}_{PA} &:= p \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3)) \\ \mathbf{V}_{PA} &= (49.529 - 15.867j) \frac{\text{in}}{\text{s}} \\ \mathbf{V}_P &:= \mathbf{V}_A + \mathbf{V}_{PA} & \mathbf{V}_P &= (45.507 - 24.113j) \frac{\text{in}}{\text{sec}} & |\mathbf{V}_P| &= 51.501 \frac{\text{in}}{\text{sec}} \\ \theta_{VPXY} &:= \arg(\mathbf{V}_P) + \delta & \theta_{VPXY} &= -96.039 \text{ deg} \end{aligned}$$

PROBLEM 6-88

Statement: Figure P6-32 shows a fourbar double slider known as an elliptical trammel. Find all its instant centers in the position shown.

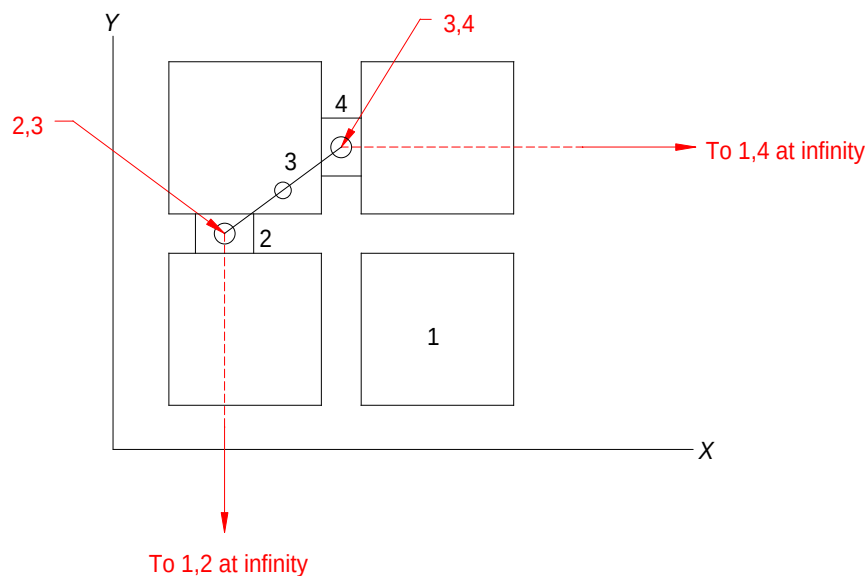
Given: Number of links $n := 4$

Solution: See Figure P6-32 and Mathcad file P0688.

- Determine the number of instant centers for this mechanism using equation 6.8a.

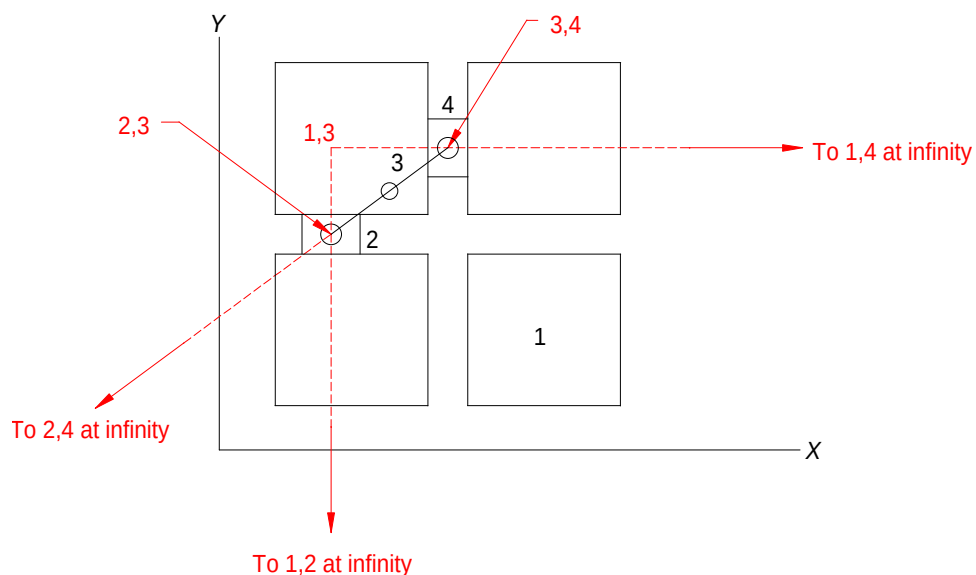
$$C := \frac{n \cdot (n - 1)}{2} \quad C = 6$$

- Draw the linkage to scale and identify those ICs that can be found by inspection (4).



- Use Kennedy's Rule and a linear graph to find the remaining 2 ICs.

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4} \quad I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$

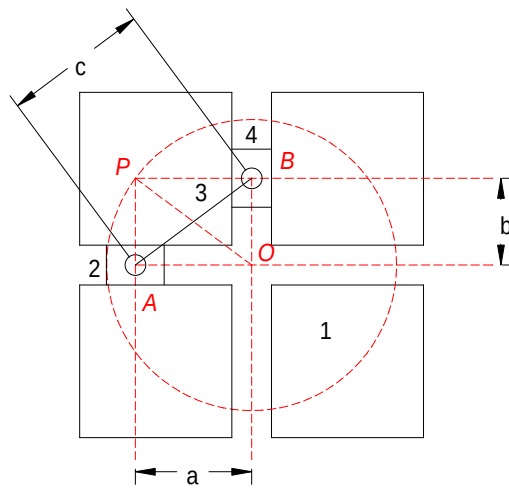


PROBLEM 6-89

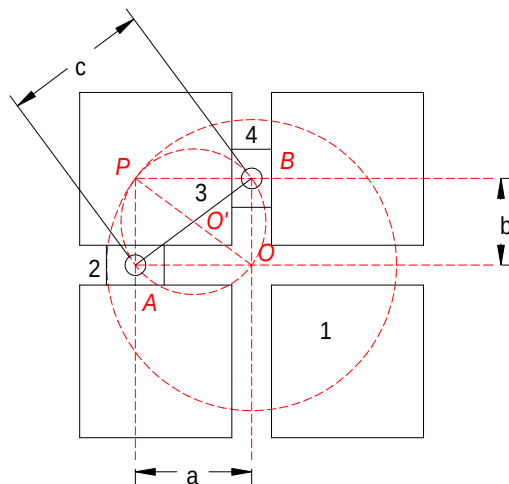
Statement: The elliptical trammel in Figure P6-32 must be driven by rotating link 3 in a full circle. Points on line AB describe ellipses. Find and draw (manually or with a computer) the fixed and moving centrodes of instant center I_{13} . (Hint: These are called the Cardan circles.)

Solution: See Figure P6-32 and Mathcad file P0689.

1. The instant center I_{13} is shown as the point P in the diagram below. The length of link 3, c , is constant and it is a diagonal of the rectangle $OBPA$. Therefore, the other diagonal, OP , has a constant length, c , regardless of the current lengths a and b . Thus, the point P (I_{13}) travels on a circle of radius c . This circle is the fixed centrode.



2. Now, invert the mechanism by holding link 3 fixed and allowing the slots to move, which can only be in a circle about O . The locus of points P will then be a circle of radius $0.5c$ with center at O' . This is the moving centrode.

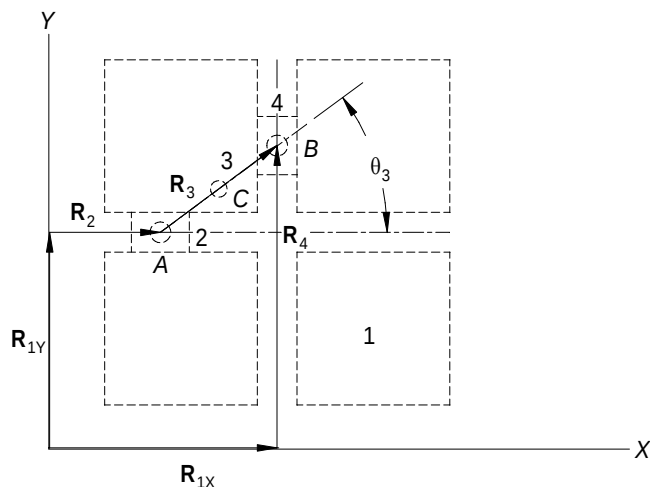


PROBLEM 6-90

Statement: Derive analytical expressions for the velocities of points A and B in Figure P6-32 as a function of θ_3 , ω_3 and the length AB of link 3. Use a vector loop equation.

Solution: See Figure P6-32 and Mathcad file P0690.

1. Establish the global XY system such that the coordinates of the intersection of the slot centerlines is at (d_X, d_Y) . Then, define position vectors $\mathbf{R}_{1X}$, $\mathbf{R}_{1Y}$, $\mathbf{R}_2$, $\mathbf{R}_3$, and $\mathbf{R}_4$ as shown below.



2. Write the vector loop equation: $\mathbf{R}_{1Y} + \mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_{1X} - \mathbf{R}_4 = 0$ then substitute the complex number notation for each position vector. The equation then becomes:

$$d_Y \cdot e^{j\left(\frac{\pi}{2}\right)} + a \cdot e^{j(0)} + c \cdot e^{j(\theta_3)} - d_X \cdot e^{j(0)} - b \cdot e^{j\left(\frac{\pi}{2}\right)} = 0$$

3. Differentiate this equation with respect to time.

$$\frac{d}{dt} a + j \cdot c \cdot \omega_3 \cdot e^{j(\theta_3)} - \frac{d}{dt} \left(b \cdot e^{j\left(\frac{\pi}{2}\right)} \right) = 0$$

4. Substituting the Euler identity into this equation gives:

$$V_a + j c \cdot \omega_3 \cdot (\cos(\theta_3) + j \cdot \sin(\theta_3)) - V_b \cdot j = 0$$

5. Separate this equation into its real (x component) and imaginary (y component) parts, setting each equal to zero.

$$V_a - c \cdot \omega_3 \cdot \sin(\theta_3) = 0 \qquad c \cdot \omega_3 \cdot \cos(\theta_3) - V_b = 0$$

6. Solve for the two unknowns V_a and V_b in terms of the independent variables θ_3 and ω_3

$$V_a = c \cdot \omega_3 \cdot \sin(\theta_3) \qquad V_b = c \cdot \omega_3 \cdot \cos(\theta_3)$$

PROBLEM 6-91

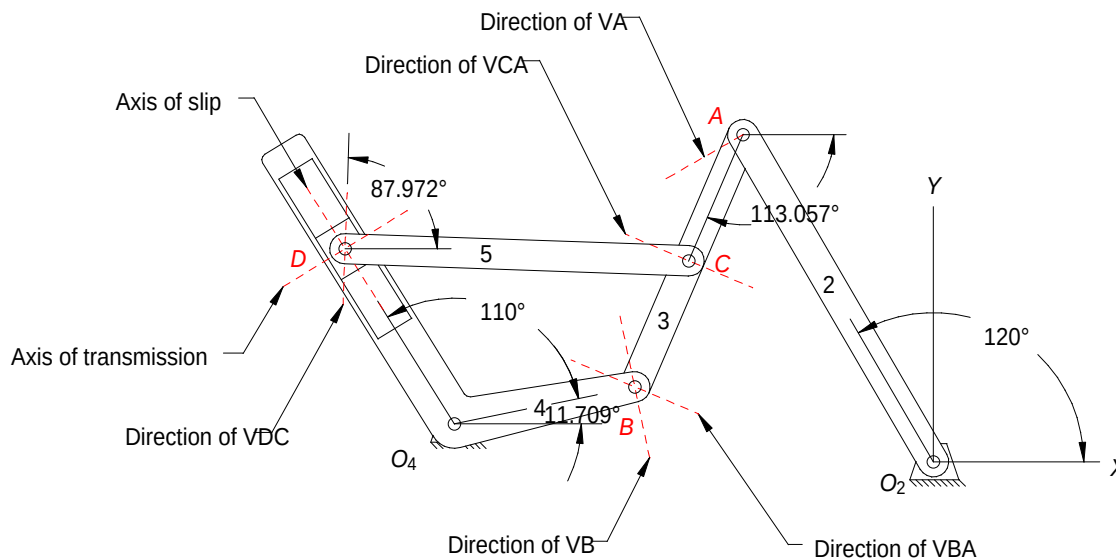
Statement: The linkage in Figure P6-33a has link 2 at 120 deg in the global XY coordinate system. Find ω_6 and $\mathbf{V}_D$ in the global XY coordinate system for the position shown if $\omega_2 = 10 \text{ rad/sec CCW}$. Use the velocity difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 6.20 \cdot \text{in}$	Link 3 (A to B)	$b := 4.50 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 3.00 \cdot \text{in}$	Link 5 (C to D)	$e := 5.60 \cdot \text{in}$
Link 3 (A to C)	$p := 2.25 \cdot \text{in}$	Link 4 (O_4 to D)	$f := 3.382 \cdot \text{in}$
Link 1 X-offset	$d_X := -7.80 \cdot \text{in}$	Link 1 Y-offset	$d_Y := 0.62 \cdot \text{in}$
Angle ACB	$\delta_3 := 0.0 \cdot \text{deg}$	Angle BO_4D	$\delta_4 := 110.0 \cdot \text{deg}$
Input rocker angle:	$\theta_2 := 120 \cdot \text{deg}$	Global XY system	
Input crank angular velocity	$\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$	CW	

Solution: See Figure P6-33a and Mathcad file P0691.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



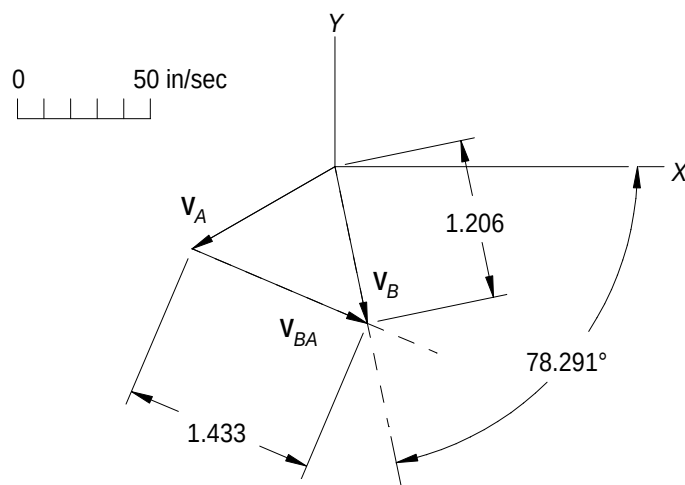
2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 62.000 \frac{\text{in}}{\text{sec}} \quad \theta_{VA} := 120 \cdot \text{deg} + 90 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 5. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{50 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_B := 1.206 \cdot \text{in} \cdot k_v \quad V_B = 60.300 \frac{\text{in}}{\text{sec}} \quad \theta_{VB} := -78.291 \cdot \text{deg}$$

$$V_{BA} := 1.433 \cdot \text{in} \cdot k_v \quad V_{BA} = 71.650 \frac{\text{in}}{\text{sec}} \quad \theta_{VBA} := -23.057 \cdot \text{deg}$$

5. Determine the angular velocity of links 3 and 4 using equation 6.7.

$$\omega_3 := \frac{V_{BA}}{b} \quad \omega_3 = 15.922 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

$$\omega_4 := \frac{-V_B}{c} \quad \omega_4 = -20.100 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

6. Determine the magnitude and sense of the vector $\mathbf{V}_{CA}$ using equation 6.7.

$$V_{CA} := |p \cdot \omega_3| \quad V_{CA} = 35.825 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VCA} := \theta_{VBA} + \delta_3 \quad \theta_{VCA} = -23.057 \text{ deg}$$

7. Determine the magnitude and sense of the vector $\mathbf{V}_{trans}$ using equation 6.7.

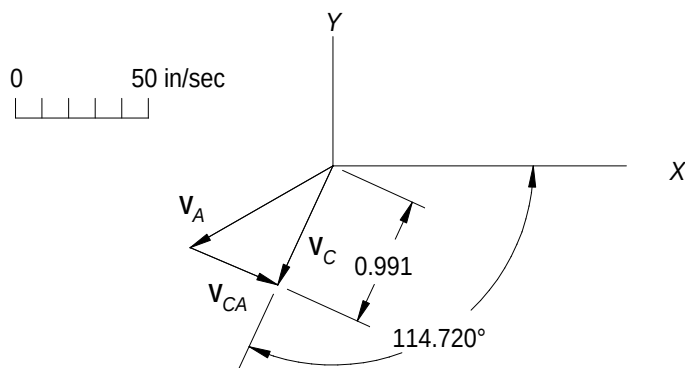
$$V_{trans} := |f \cdot \omega_4| \quad V_{trans} = 67.978 \frac{\text{in}}{\text{sec}}$$

$$\theta_{trans} := \theta_{VB} + \delta_4 \quad \theta_{trans} = 31.709 \text{ deg}$$

8. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- From the tip of $\mathbf{V}_A$, layout the (now) known vector $\mathbf{V}_{CA}$.
- Complete the vector triangle by drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_A$ to the tip of the $\mathbf{V}_{CA}$ vector.



9. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{50 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_C := 0.991 \cdot \text{in} \cdot k_v \quad V_C = 49.550 \frac{\text{in}}{\text{sec}} \quad \theta_{VC} := 114.720 \cdot \text{deg}$$

9. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point D , the magnitude of the relative velocity $\mathbf{V}_{DC}$. The equations to be solved (simultaneously) graphically are

$$\mathbf{V}_D = \mathbf{V}_C + \mathbf{V}_{DC} \quad \text{and} \quad \mathbf{V}_D = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_C$.
- From the tip of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_{DC}$, magnitude unknown.
- Repeat steps a and b with $\mathbf{V}_{trans}$ and $\mathbf{V}_{slip}$.
- From the tail of $\mathbf{V}_C$, draw a construction line to the intersection of the two construction lines.
- Complete the vector polygon by drawing $\mathbf{V}_{DC}$ from the tip of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_D$ construction line and drawing $\mathbf{V}_D$ from the tail of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_{DC}$ construction line.

10. From the velocity polygon we have:

Velocity scale factor:

$$k_v := \frac{50 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

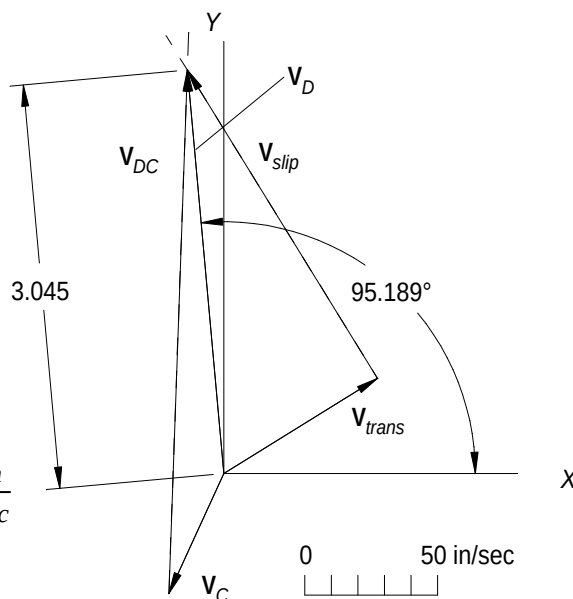
$$V_D := 3.045 \cdot \text{in} \cdot k_v$$

$$\theta_{VD} := 95.189 \cdot \text{deg}$$

$$\omega_6 := \omega_4$$

$$V_D = 152.250 \frac{\text{in}}{\text{sec}}$$

$$\omega_6 = -20.100 \frac{\text{rad}}{\text{sec}}$$



PROBLEM 6-92

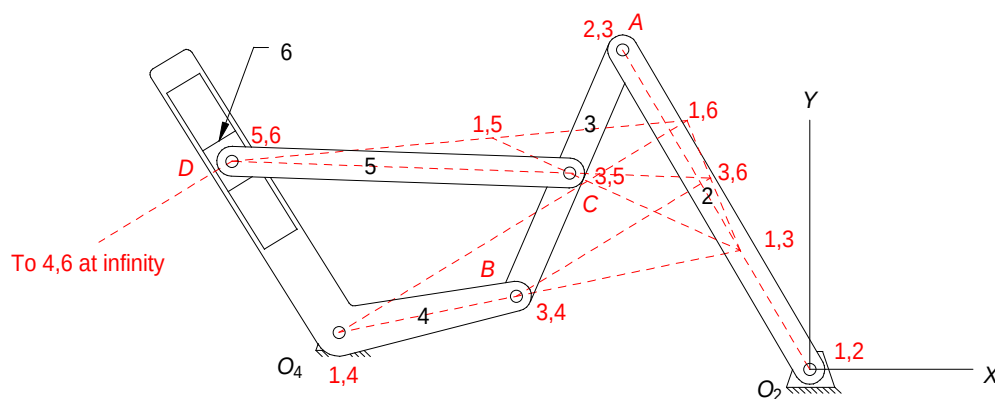
Statement: The linkage in Figure P6-33a has link 2 at 120 deg in the global XY coordinate system. Find ω_6 and $\mathbf{V}_D$ in the global XY coordinate system for the position shown if $\omega_2 = 10$ rad/sec CCW. Use the instant center graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 6.20 \cdot \text{in}$	Link 3 (A to B)	$b := 4.50 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 3.00 \cdot \text{in}$	Link 5 (C to D)	$e := 5.60 \cdot \text{in}$
Link 3 (A to C)	$p := 2.25 \cdot \text{in}$	Link 4 (O_4 to D)	$f := 3.382 \cdot \text{in}$
Link 1 X-offset	$d_X := -7.80 \cdot \text{in}$	Link 1 Y-offset	$d_Y := 0.62 \cdot \text{in}$
Angle ACB	$\delta_3 := 0.0 \cdot \text{deg}$	Angle BO_4D	$\delta_4 := 110.0 \cdot \text{deg}$
Input rocker angle:	$\theta_2 := 120 \cdot \text{deg}$	Global XY system	
Input crank angular velocity	$\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$	CW	

Solution: See Figure P6-33a and Mathcad file P0692.

1. Draw the linkage to scale in the position given, find instant centers $I_{1,3}$ and $I_{1,5}$, and the distances from the pin joints to the instant centers.



From the layout above:

$$AI13 := 3.893 \cdot \text{in} \quad BI13 := 3.788 \cdot \text{in} \quad CI13 := 3.113 \cdot \text{in} \quad CI15 := 1.411 \cdot \text{in} \quad DI15 := 4.333 \cdot \text{in} \quad DI16 := 7.573 \cdot \text{in}$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 62.000 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{VA} = 210.0 \text{ deg}$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \quad \omega_3 = 15.926 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

4. Determine the magnitude of the velocity at point B using equation 6.9b.

$$V_B := BI13 \cdot \omega_3 \quad V_B = 60.328 \frac{\text{in}}{\text{sec}}$$

5. Use equation 6.9c to determine the angular velocity of links 4 and 6.

$$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 20.109 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

$$w_6 := \omega_4 \qquad w_6 = 20.109 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

6. Determine the magnitude of the velocity at point *C* using equation 6.9b.

$$V_C := CI13 \cdot \omega_3 \qquad V_C = 49.578 \frac{\text{in}}{\text{sec}}$$

7. Determine the angular velocity of link 5 using equation 6.9a.

$$\omega_5 := \frac{V_C}{CI15} \qquad \omega_5 = 35.137 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

8. Determine the magnitude of the velocity at point *D* using equation 6.9b.

$$V_D := DI15 \cdot \omega_5 \qquad V_D = 152.247 \frac{\text{in}}{\text{sec}} \quad @ 95.185 \text{ deg}$$

PROBLEM 6-93

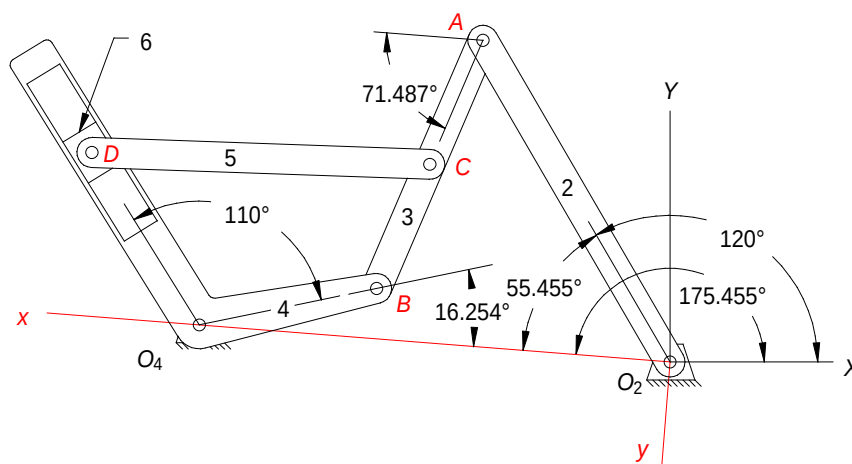
Statement: The linkage in Figure P6-33a has link 2 at 120 deg in the global XY coordinate system. Find ω_6 and $\mathbf{V}_D$ in the global XY coordinate system for the position shown if $\omega_2 = 10 \text{ rad/sec CCW}$. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 6.20 \cdot \text{in}$	Link 3 (A to B)	$b := 4.50 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 3.00 \cdot \text{in}$	Link 5 (C to D)	$e := 5.60 \cdot \text{in}$
Link 3 (A to C)	$p := 2.25 \cdot \text{in}$	Link 4 (O_4 to D)	$f := 3.382 \cdot \text{in}$
Link 1 X-offset	$d_X := -7.80 \cdot \text{in}$	Link 1 Y-offset	$d_Y := 0.62 \cdot \text{in}$
Angle ACB	$\delta_3 := 0.0 \cdot \text{deg}$	Angle BO_4D	$\delta_4 := 110.0 \cdot \text{deg}$
Input rocker angle:	$\theta_{2XY} := 120 \cdot \text{deg}$	Global XY system	
Coordinate transformation angle:	$\delta := 175.455 \cdot \text{deg}$		
Input crank angular velocity	$\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$	CCW	

Solution: See Figure P6-33a and Mathcad file P0691.

1. Draw the linkage to scale and label it.



Calculate the distance O_2O_4 : $d := \sqrt{d_X^2 + d_Y^2}$ $d = 7.825 \text{ in}$

Transform θ_{2XY} into the local coordinate system: $\theta_2 := \theta_{2XY} - \delta$ $\theta_2 = -55.455 \text{ deg}$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_1 = 1.2620 \quad K_2 = 2.6082$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 2.3767$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = 0.2028 \quad B = 1.6474 \quad C = 1.5927$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_{41} = -163.746 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.7388 \quad K_5 = -1.9877$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -1.6967$$

$$E := -2 \cdot \sin(\theta_2) \quad E = 1.6474$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = -0.3067$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) - 2 \cdot \pi \quad \theta_3 = -648.512 \text{ deg}$$

6. At this point write vector loop equations for links 3, 4, 5, and 6 to solve for the position and velocity of link 6.

PROBLEM 6-94

Statement: The linkage in Figure P6-33b has link 3 perpendicular to the X-axis and links 2 and 4 are parallel to each other. Find ω_3 , $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_P$ if $\omega_2 = 15 \text{ rad/sec CW}$. Use the velocity difference graphical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 2.75 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 3.26 \cdot \text{in}$$

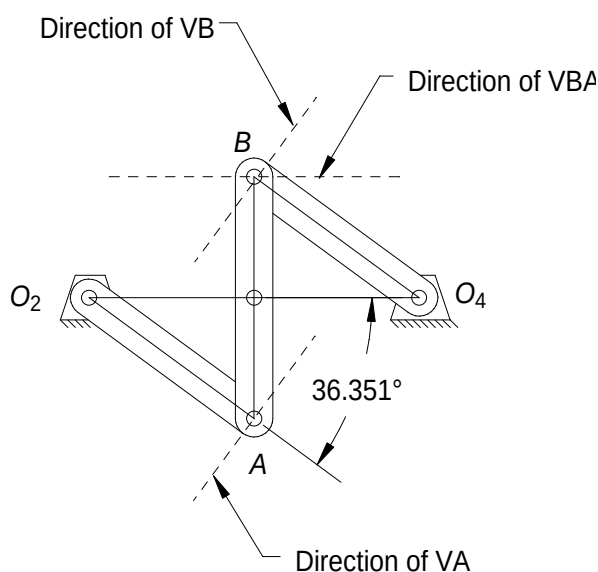
$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 2.75 \cdot \text{in} \quad \text{Link 1 (O}_4 \text{ to B)} \quad d := 4.43 \cdot \text{in}$$

$$\text{Coupler point data:} \quad p := 1.63 \cdot \text{in} \quad \delta_3 := 0 \cdot \text{deg}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$$

Solution: See Figure P6-33b and Mathcad file P0694.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



$$\theta_2 := -36.352 \cdot \text{deg}$$

2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \quad V_A = 41.250 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{VA} = -126.352 \text{ deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
 - b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
 - c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
 - d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.
4. Without actually drawing the vector polygon, we see that $\mathbf{V}_A$ and $\mathbf{V}_B$ have the same angle with respect to the X axis. Therefore,

$$V_B := V_A \quad \text{and} \quad V_{BA} := 0 \cdot \frac{\text{in}}{\text{sec}} \quad \theta_{VB} := \theta_{VA} \quad \theta_{VBA} := 0 \cdot \text{deg}$$

5. Determine the angular velocity of links 3 and 4 using equation 6.7.

$$\omega_3 := \frac{-V_{BA}}{b} \quad \omega_3 = 0.000 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{-V_B}{c} \quad \omega_4 = -15.000 \frac{\text{rad}}{\text{sec}}$$

6. Determine the magnitude and sense of the vector $\mathbf{V}_{PA}$ using equation 6.7.

$$V_{PA} := |p \cdot \omega_3| \quad V_{PA} = 0.000 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VPA} := \theta_{VBA} \quad \theta_{VPA} = 0.000 \text{ deg}$$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point P . The equation to be solved graphically is

$$\mathbf{V}_P = \mathbf{V}_A + \mathbf{V}_{PA}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
 - From the tip of $\mathbf{V}_A$, layout the (now) known vector $\mathbf{V}_{PA}$.
 - Complete the vector triangle by drawing $\mathbf{V}_P$ from the tail of $\mathbf{V}_A$ to the tip of the $\mathbf{V}_{PA}$ vector.
8. Again, without drawing the vector polygon we see that,

$$V_P := V_A$$

At this instant, link 3 is in pure translation with every point on it having the same velocity.

PROBLEM 6-95

Statement: The linkage in Figure P6-33b has link 3 perpendicular to the X-axis and links 2 and 4 are parallel to each other. Find ω_3 , $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_P$ if $\omega_2 = 15 \text{ rad/sec CW}$. Use the instant center graphical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 2.75 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 3.26 \cdot \text{in}$$

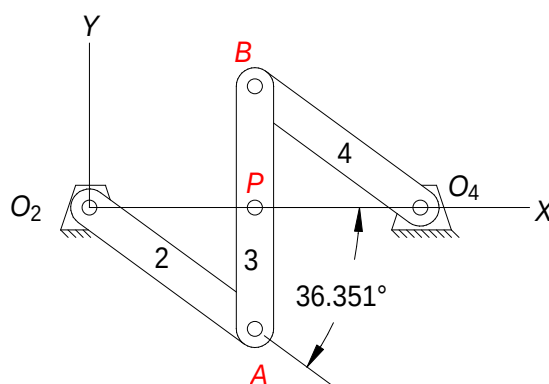
$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 2.75 \cdot \text{in} \quad \text{Link 1 (O}_4 \text{ to B)} \quad d := 4.43 \cdot \text{in}$$

$$\text{Coupler point data:} \quad p := 1.63 \cdot \text{in} \quad \delta_3 := 0 \cdot \text{deg}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$$

Solution: See Figure P6-33b and Mathcad file P0695.

1. Draw the linkage to scale in the position given, find instant center $I_{1,3}$ and the distance from the pin joints to the instant center.



Since O_2A and O_4B are parallel, $I_{1,3}$ is at infinity and link 3 does not rotate for this instantaneous position of the linkage. Thus, link 3 is in pure translation and all points on it will have the same velocity.

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at points A, B, and P, which will be the same.

$$V_A := a \cdot \omega_2 \quad V_A = 41.250 \frac{\text{in}}{\text{sec}}$$

$$\theta_2 := -36.351 \cdot \text{deg}$$

$$\theta_{VA} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{VA} = -126.351 \text{ deg}$$

The velocity vectors for points B and P are identical to $\mathbf{V}_A$.

PROBLEM 6-96

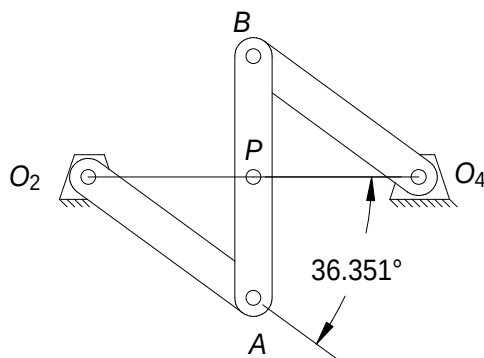
Statement: The linkage in Figure P6-33b has link 3 perpendicular to the X-axis and links 2 and 4 are parallel to each other. Find ω_3 , $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_P$ if $\omega_2 = 15 \text{ rad/sec CW}$. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 2.75 \cdot \text{in}$	Link 3 (A to B)	$b := 3.26 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 2.75 \cdot \text{in}$	Link 1 (O_4 to B)	$d := 4.43 \cdot \text{in}$
Coupler point data:	$p := 1.63 \cdot \text{in}$	$\delta_3 := 0 \cdot \text{deg}$	
Input crank angular velocity	$w_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1}$	CW	

Solution: See Figure P6-33b and Mathcad file P0696.

1. Draw the linkage to a convenient scale and label it.



$$\theta_2 := -36.351 \cdot \text{deg}$$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_1 = 1.6109 \quad K_2 = 1.6109$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.5949$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad A = -0.5081$$

$$B := -2 \cdot \sin(\theta_2) \quad B = 1.1855$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3 \quad C = 1.1029$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) + 2 \cdot \pi \quad \theta_4 = 143.649 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.3589 \quad K_5 = -1.6873$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -1.3983$$

$$E := -2 \cdot \sin(\theta_2) \quad E = 1.1855$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.2127$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) + 2 \cdot \pi \quad \theta_3 = 89.995 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad \omega_3 = -0.000 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad \omega_4 = 15.000 \frac{\text{rad}}{\text{sec}}$$

7. Determine the velocity of points A and B using equations 6.19.

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = (-24.450 - 33.223i) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_A| = 41.250 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VAXY} := \arg(\mathbf{V}_A) \quad \theta_{VAXY} = -126.351 \text{ deg}$$

$$\mathbf{V}_B := c \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$$

$$\mathbf{V}_B = (-24.450 - 33.223i) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_B| = 41.250 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VBXY} := \arg(\mathbf{V}_B) \quad \theta_{VBXY} = -126.351 \text{ deg}$$

8. Determine the velocity of the coupler point P using equations 6.36.

$$\mathbf{V}_{PA} := p \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3))$$

$$\mathbf{V}_{PA} = (1.936 \times 10^{-4} - 1.676i \times 10^{-8}) \frac{\text{in}}{\text{sec}}$$

$$\mathbf{V}_P := \mathbf{V}_A + \mathbf{V}_{PA} \quad |\mathbf{V}_P| = 41.250 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VPXY} := \arg(\mathbf{V}_P) \quad \theta_{VPXY} = -126.351 \text{ deg}$$

PROBLEM 6-97

Statement: The crosshead linkage shown in Figure P6-33c has 2 *DOF* with inputs at crossheads 2 and 5. Find instant centers $I_{1,3}$ and $I_{1,4}$.

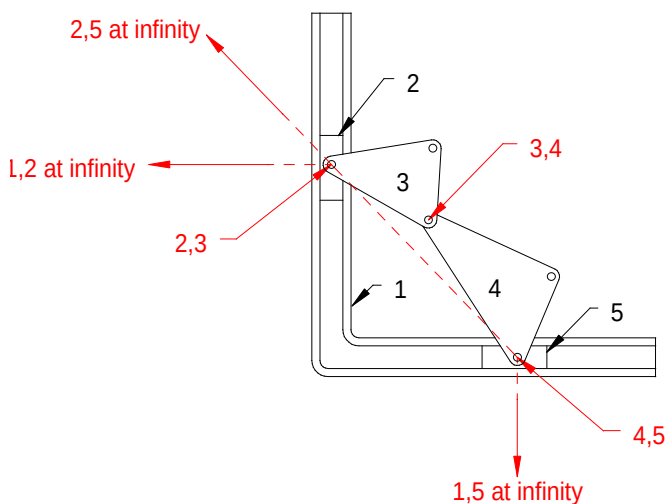
Given: Number of links $n := 5$

Solution: See Figure P6-33c and Mathcad file P06976.

- Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 10$$

- Draw the linkage to scale and identify those ICs that can be found by inspection (4).



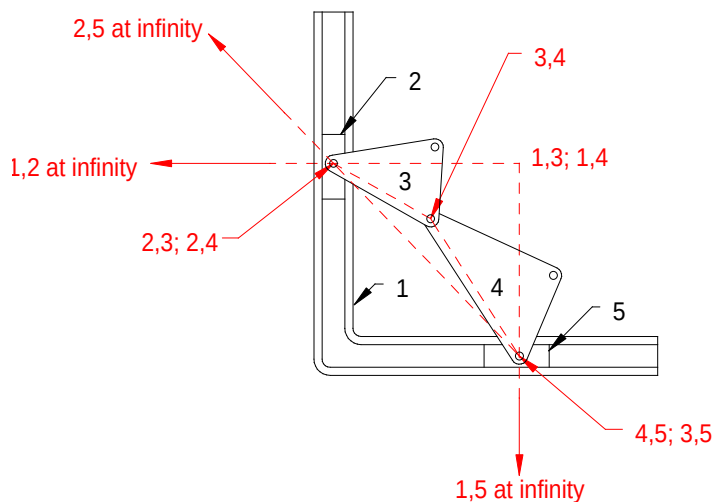
- Use Kennedy's Rule and a linear graph to find the remaining ICs required to find $I_{1,3}$ and $I_{1,4}$.

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$

$$I_{3,5}: I_{2,3}-I_{2,5} \text{ and } I_{4,5}-I_{3,4}$$

$$I_{1,4}: I_{1,2}-I_{2,4} \text{ and } I_{1,5}-I_{4,5}$$

$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$



PROBLEM 6-98

Statement: The crosshead linkage shown in Figure P6-33c has 2 *DOF* with inputs at cross heads 2 and 5. Find $\mathbf{V}_B$, $\mathbf{V}_{P_3}$, and $\mathbf{V}_{P_4}$ if the crossheads are each moving toward the origin of the *XY* coordinate system with a speed of 20 in/sec. Use a graphical method.

Given: Link lengths:

$$\text{Link 3 (A to B)} \quad b := 34.32 \cdot \text{in} \quad \text{Link 4 (B to C)} \quad c := 50.4 \cdot \text{in}$$

$$\text{Link 2 Y-offset} \quad a := 59.5 \cdot \text{in} \quad \text{Link 5 X-offset} \quad d := 57 \cdot \text{in}$$

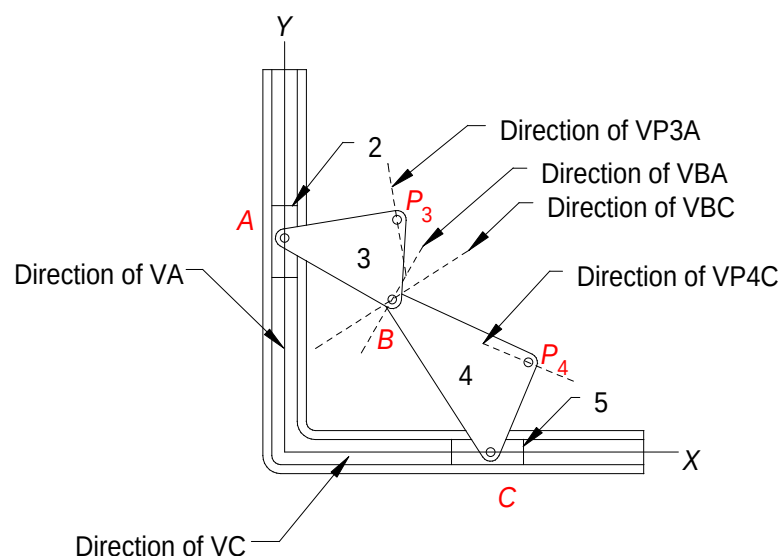
$$\text{Coupler point data:} \quad AP_3 := 31.5 \cdot \text{in} \quad BP_3 := 22.2 \cdot \text{in}$$

$$BP_4 := 41.52 \cdot \text{in} \quad CP_4 := 27.0 \cdot \text{in}$$

$$\text{Input velocities:} \quad V_{2Y} := -20 \cdot \text{in} \cdot \text{sec}^{-1} \quad V_{5X} := -20 \cdot \text{in} \cdot \text{sec}^{-1}$$

Solution: See Figure P6-33c and Mathcad file P0698.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.

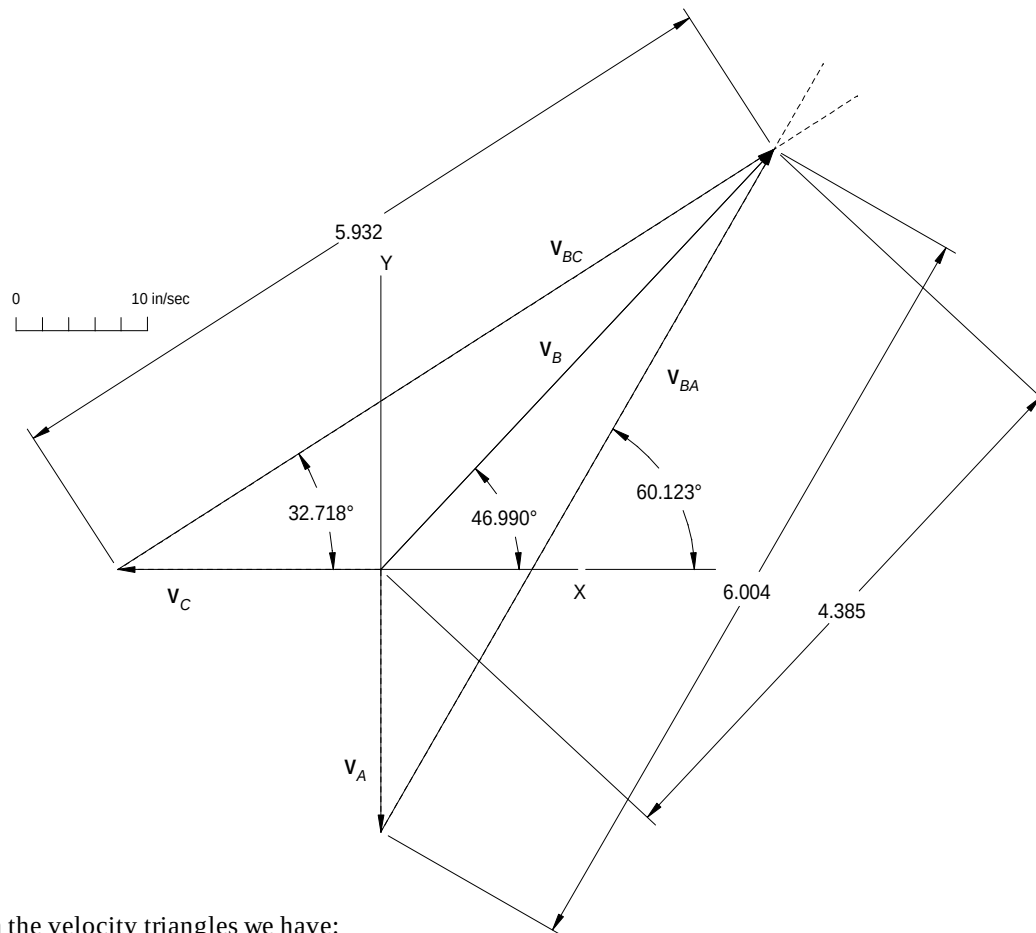


2. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B*, the magnitude of the relative velocities $\mathbf{V}_{BA}$, $\mathbf{V}_{BC}$ and the angular velocities of links 3 and 4. The equations to be solved (simultaneously) graphically are

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \quad \mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

- a. Choose a convenient velocity scale and layout the known vectors $\mathbf{V}_A$ and $\mathbf{V}_C$
- b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tip of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.
- d. From the origin, draw a construction line to the intersection of the two construction lines in steps b and c.
- e. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line. And then do the same with the $\mathbf{V}_{CB}$ vector

(See next page)



3. From the velocity triangles we have:

Velocity scale factor: $k_v := \frac{10 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_B := 4.385 \cdot \text{in} \cdot k_v \quad V_B = 43.850 \frac{\text{in}}{\text{sec}} \quad \theta_{VB} := 46.990 \cdot \text{deg}$$

$$V_{BA} := 6.004 \cdot \text{in} \cdot k_v \quad V_{BA} = 60.040 \frac{\text{in}}{\text{sec}} \quad \theta_{VBA} := 60.123 \cdot \text{deg}$$

$$V_{BC} := 5.932 \cdot \text{in} \cdot k_v \quad V_{BC} = 59.320 \frac{\text{in}}{\text{sec}} \quad \theta_{VBC} := 32.718 \cdot \text{deg}$$

4. Determine the angular velocity of links 3 and 4 using equation 6.7.

$$\omega_3 := \frac{V_{BA}}{b} \quad \omega_3 = 1.749 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

$$\omega_4 := \frac{-V_{BC}}{c} \quad \omega_4 = -1.177 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

6. Determine the magnitudes of the vectors $\mathbf{V}_{P3A}$ and $\mathbf{V}_{P4C}$ using equation 6.7.

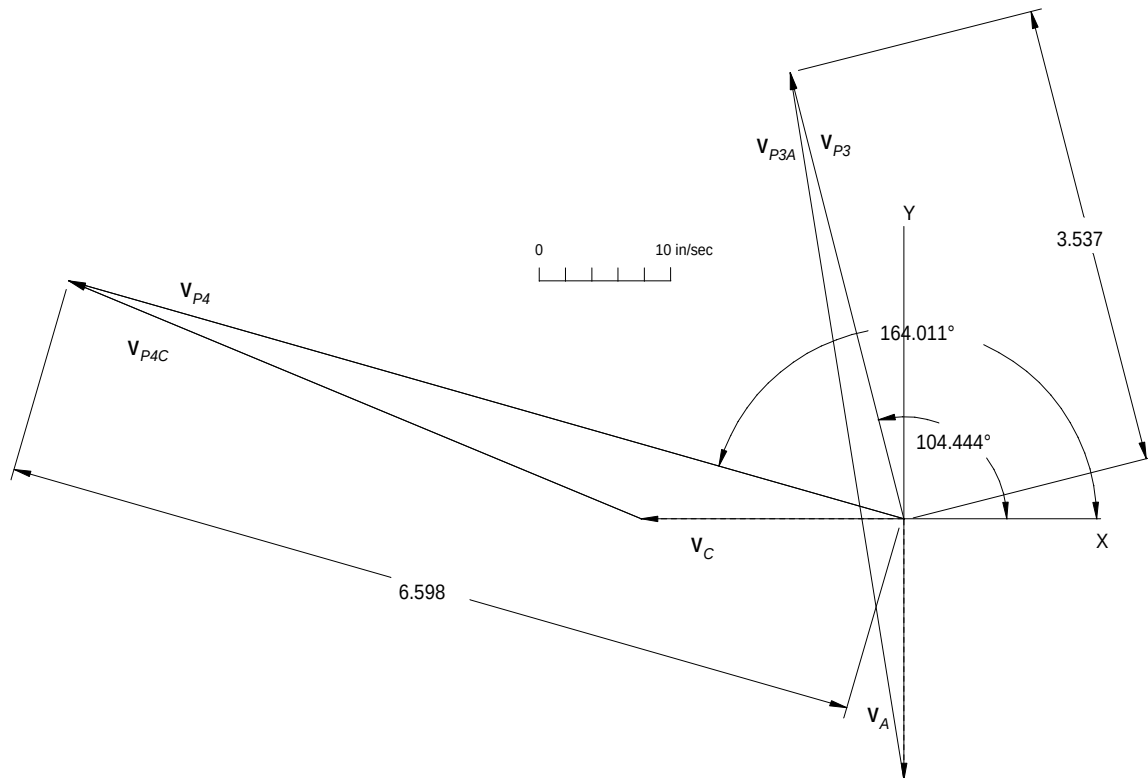
$$V_{P3A} := |AP_3 \cdot \omega_3| \quad V_{P3A} = 55.107 \frac{\text{in}}{\text{sec}}$$

$$V_{P4C} := |CP_4 \cdot \omega_3| \quad V_{P4C} = 47.234 \frac{\text{in}}{\text{sec}}$$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocities at points P_3 and P_4 . The equations be solved graphically are

$$\mathbf{V}_{P3} = \mathbf{V}_A + \mathbf{V}_{P3A} \quad \mathbf{V}_{P4} = \mathbf{V}_C + \mathbf{V}_{P4C}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- From the tip of $\mathbf{V}_A$, layout the (now) known vector $\mathbf{V}_{P3A}$.
- Complete the vector triangle by drawing $\mathbf{V}_{P3}$ from the tail of $\mathbf{V}_A$ to the tip of the $\mathbf{V}_{P3A}$ vector.
- Repeat for $\mathbf{V}_{P4}$.



8. From the velocity triangles we have:

$$\text{Velocity scale factor: } k_v := \frac{10 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_{P3} := 3.537 \cdot \text{in} \cdot k_v \quad V_{P3} = 35.370 \frac{\text{in}}{\text{sec}} \quad \theta_{P3} := 104.444 \cdot \text{deg}$$

$$V_{P4} := 6.598 \cdot \text{in} \cdot k_v \quad V_{P4} = 65.980 \frac{\text{in}}{\text{sec}} \quad \theta_{P4} := 164.011 \cdot \text{deg}$$

PROBLEM 6-99

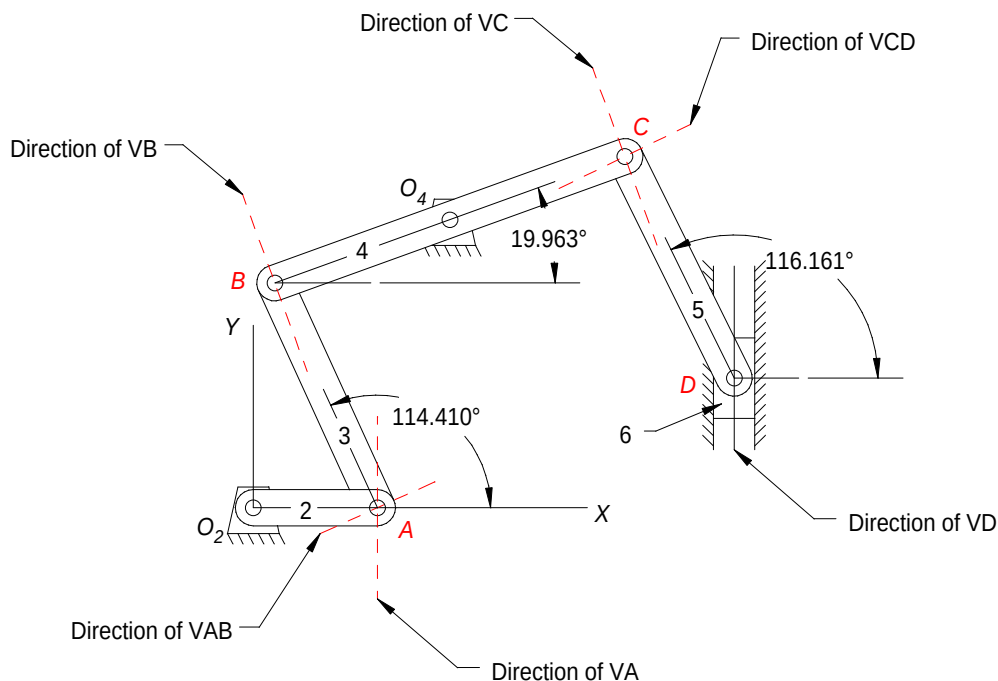
Statement: The linkage in Figure P6-33d has the path of slider 6 perpendicular to the global X-axis and link 2 aligned with the global X-axis. Find $\mathbf{V}_A$ in the position shown if the velocity of the slider is 20 in/sec downward. Use the velocity difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 12 \cdot \text{in}$	Link 3 (A to B)	$b := 24 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 18 \cdot \text{in}$	Link 5 (C to D)	$f := 24 \cdot \text{in}$
Link 4 (O_4 to C)	$e := 18 \cdot \text{in}$		
Link 1 X-offset	$d_X := 19 \cdot \text{in}$	Link 1 Y-offset	$d_Y := 28 \cdot \text{in}$
Angle BO_4C	$\delta_4 := 0.0 \cdot \text{deg}$		
Output crank angle:	$\theta_2 := 0 \cdot \text{deg}$	Global XY system	
Input slider velocity	$V_D := 20 \cdot \text{in} \cdot \text{sec}^{-1}$	$\theta_{VD} := -90.0 \cdot \text{deg}$	

Solution: See Figure P6-33d and Mathcad file P0699.

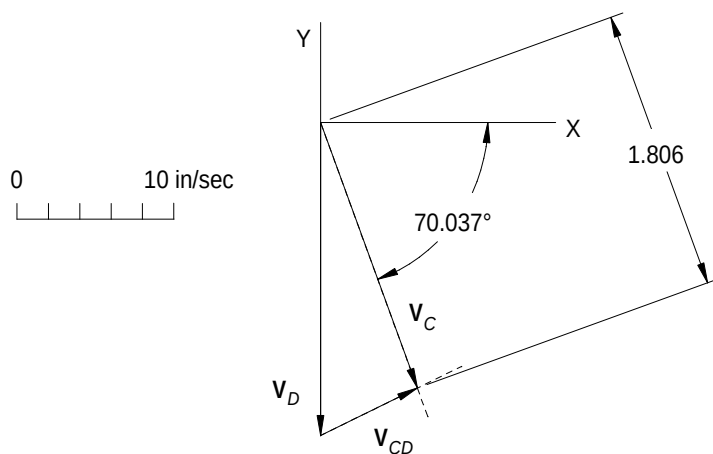
1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_D + \mathbf{V}_{CD}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_D$.
- b. From the tip of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_{CD}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{CD}$ from the tip of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_{CD}$ construction line.



3. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{10 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_C := 1.806 \cdot \text{in} \cdot k_v \quad V_C = 18.060 \frac{\text{in}}{\text{sec}} \quad \theta_{VC} := -70.037 \cdot \text{deg}$$

4. Determine the magnitude and sense of the vector $\mathbf{V}_B$.

$$V_B := V_C \quad V_B = 18.060 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VB} := \theta_{VC} + 180 \cdot \text{deg} \quad \theta_{VB} = 109.963 \text{ deg}$$

5. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point A. The equation to be solved graphically is

$$\mathbf{V}_A = \mathbf{V}_B + \mathbf{V}_{AB}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
- From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{AB}$, magnitude unknown.
- From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_A$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{AB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_A$ construction line and drawing $\mathbf{V}_A$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{AB}$ construction line.

6. From the velocity triangle we have:

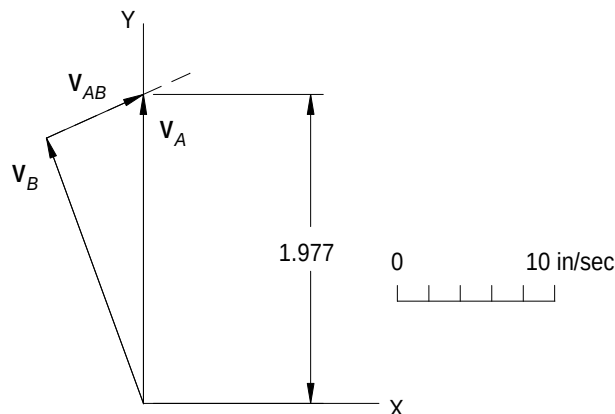
Velocity scale factor:

$$k_v := \frac{10 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_A := 1.977 \cdot \text{in} \cdot k_v$$

$$V_A = 19.770 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := 90.0 \cdot \text{deg}$$



PROBLEM 6-100

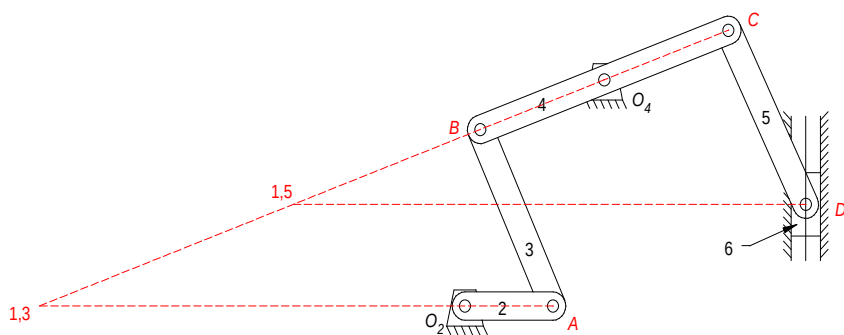
Statement: The linkage in Figure P6-33a has the path of slider 6 perpendicular to the global X-axis and link 2 aligned with the global X-axis. Find V_A in the position shown if the velocity of the slider is 20 in/sec downward. Use the instant center graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 12 \cdot \text{in}$	Link 3 (A to B)	$b := 24 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 18 \cdot \text{in}$	Link 5 (C to D)	$f := 24 \cdot \text{in}$
Link 3 (A to C)	$p := 2.25 \cdot \text{in}$	Link 4 (O_4 to C)	$e := 18 \cdot \text{in}$
Link 1 X-offset	$d_X := 19 \cdot \text{in}$	Link 1 Y-offset	$d_Y := 28 \cdot \text{in}$
Angle BO_4C	$\delta_4 := 0.0 \cdot \text{deg}$		
Output crank angle:	$\theta_2 := 0 \cdot \text{deg}$	Global XY system	
Input slider velocity	$V_D := 20 \cdot \text{in} \cdot \text{sec}^{-1}$	$\theta_{VD} := -90.0 \cdot \text{deg}$	

Solution: See Figure P6-33d and Mathcad file P06100.

1. Draw the linkage to scale in the position given, find instant centers $I_{1,3}$ and $I_{1,5}$, and the distances from the pin joints to the instant centers.



From the layout above:

$$AI13 := 70.085 \cdot \text{in} \quad BI13 := 64.013 \cdot \text{in} \quad CI15 := 63.095 \cdot \text{in} \quad DI15 := 69.886 \cdot \text{in}$$

2. Use equation 6.9c to determine the angular velocity of link 5.

$$\omega_5 := \frac{V_D}{DI15} \quad \omega_5 = 0.286 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

3. Determine the magnitude of the velocity at point C using equation 6.9b.

$$V_C := CI15 \cdot \omega_5 \quad V_C = 18.057 \frac{\text{in}}{\text{sec}}$$

4. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_C}{e} \quad \omega_4 = 1.003 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

5. Determine the magnitude of the velocity at point B using equation 6.9b.

$$V_B := c \cdot \omega_4 \quad V_B = 18.057 \frac{\text{in}}{\text{sec}}$$

6. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_B}{BI13} \quad \omega_3 = 0.282 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

7. Determine the magnitude of the velocity at point A using equation 6.9b.

$$V_A := AI13 \cdot \omega_3 \quad V_A = 19.769 \frac{\text{in}}{\text{sec}} \quad \text{Upward}$$

PROBLEM 6-101

Statement: For the linkage of Figure P6-33e, write a computer program or use an equation solver to find and plot $\mathbf{V}_D$ in the global coordinate system for one revolution of link 2 if $\omega_2 = 10$ rad/sec CW.

Given: Link lengths:

Input crank (L_2)	$a := 5.00$	Fourbar coupler (L_3)	$b := 5.00$
Output crank (O_4B)	$c := 6.00$	Slider coupler (L_5)	$e := 15.00$
Fourbar ground link (L_1)	$d := 2.500$	Angle BO_4C	$\delta := 83.621 \cdot \text{deg}$
Crank velocity:	$\omega_2 := -10 \cdot \frac{\text{rad}}{\text{sec}}$		

Solution: See Figure P6-33e and Mathcad file P06101.

1. This sixbar drag-link mechanism can be analyzed as a fourbar Grashof double crank in series with a crank-slider mechanism using the output of the fourbar, link 4, as the input to the crank-slider.
2. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit) in the global XY coordinate system.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 0.5000 \quad K_2 = 0.4167 \quad K_3 = 0.7042$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. If the calculated value of θ_4 is greater than 2π , subtract 2π from it and if it is negative, make it positive.

$$\theta_{42}(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := \text{if}(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$$

5. Determine the slider-crank motion using equations 4.16 and 4.17 with θ_4 as the input angle.

$$\theta_5(\theta_2) := \text{asin} \left(\frac{-c \cdot \sin(\theta_4(\theta_2) - \delta)}{e} \right) + \pi$$

$$f(\theta_2) := c \cdot \cos(\theta_4(\theta_2) - \delta) - e \cdot \cos(\theta_5(\theta_2))$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 0.5000 \quad K_5 = -0.4050$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\operatorname{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of link 4 for the open circuit using equations 6.18.

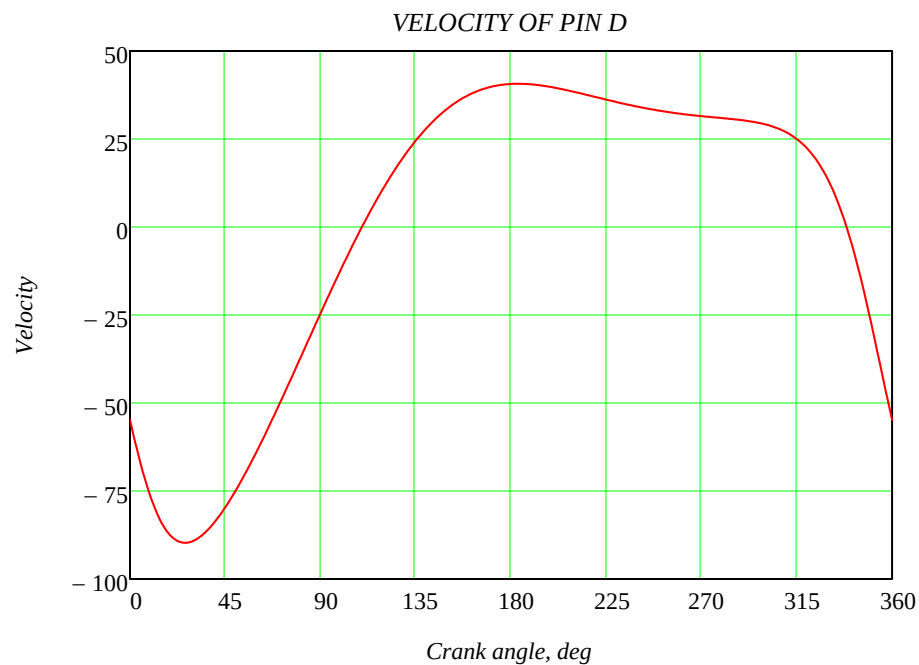
$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5(\theta_2) := \frac{c}{e} \cdot \frac{\cos(\theta_4(\theta_2) - \delta)}{\cos(\theta_5(\theta_2))} \cdot \omega_4(\theta_2)$$

9. Determine the velocity of pin D using equation 6.22b:

$$V_D(\theta_2) := -c \cdot \omega_4(\theta_2) \cdot \sin(\theta_4(\theta_2) - \delta) + e \cdot \omega_5(\theta_2) \cdot \sin(\theta_5(\theta_2))$$



PROBLEM 6-102

Statement: For the linkage of Figure P6-33f, locate and identify all instant centers.

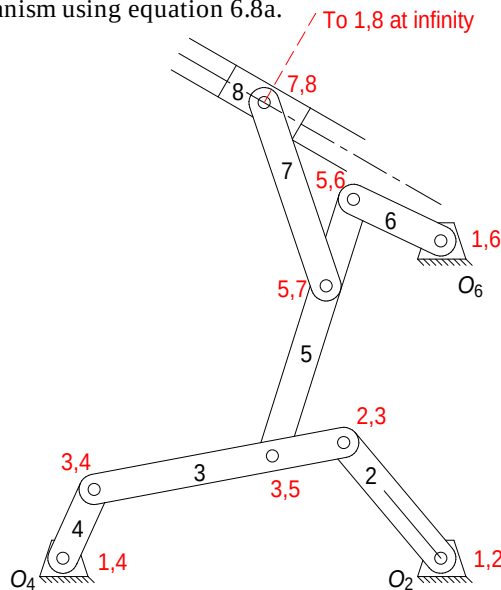
Given: Number of links $n := 8$

Solution: See Figure P6-33f and Mathcad file P06102.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 28$$

1. Draw the linkage to scale and identify those ICs that can be found by inspection (10).
2. Use Kennedy's Rule and a linear graph to find the remaining 18 ICs.



$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{1.5}: I_{1.6}-I_{5.6} \text{ and } I_{1.3}-I_{3.5}$$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$

$$I_{4,5}: I_{1,4}-I_{1,5} \text{ and } I_{3,5}-I_{3,4}$$

$$I_{2,5}: I_{1,2}-I_{1,5} \text{ and } I_{2,4}-I_{4,5}$$

$$I_{2,6}: I_{1,2}-I_{1,6} \text{ and } I_{2,5}-I_{5,6}$$

$$I_{3.6}: I_{2.3}-I_{2.6} \text{ and } I_{1.2}-I_{1.6}$$

$I_{5.8}$: $I_{5.7}$ - $I_{7.8}$ and $I_{1.5}$ - $I_{1.8}$

$$I_{4.8}: I_{4.5}-I_{5.8} \text{ and } I_{1.8}-I_{1.4}$$

$$I_{2,8}: I_{2,4}-I_{4,8} \text{ and } I_{1,2}-I_{1,8}$$

$$I_{3.8}: I_{3.4}-I_{4.8} \text{ and } I_{1.3}-I_{1.8}$$

$$I_{6,8}: I_{5,8}-I_{5,6} \text{ and } I_{2,8}-I_{2,6}$$

$I_{2.7}: I_{2.8}-I_{7.8} \text{ and } I_{2.5}-I_{5.7}$

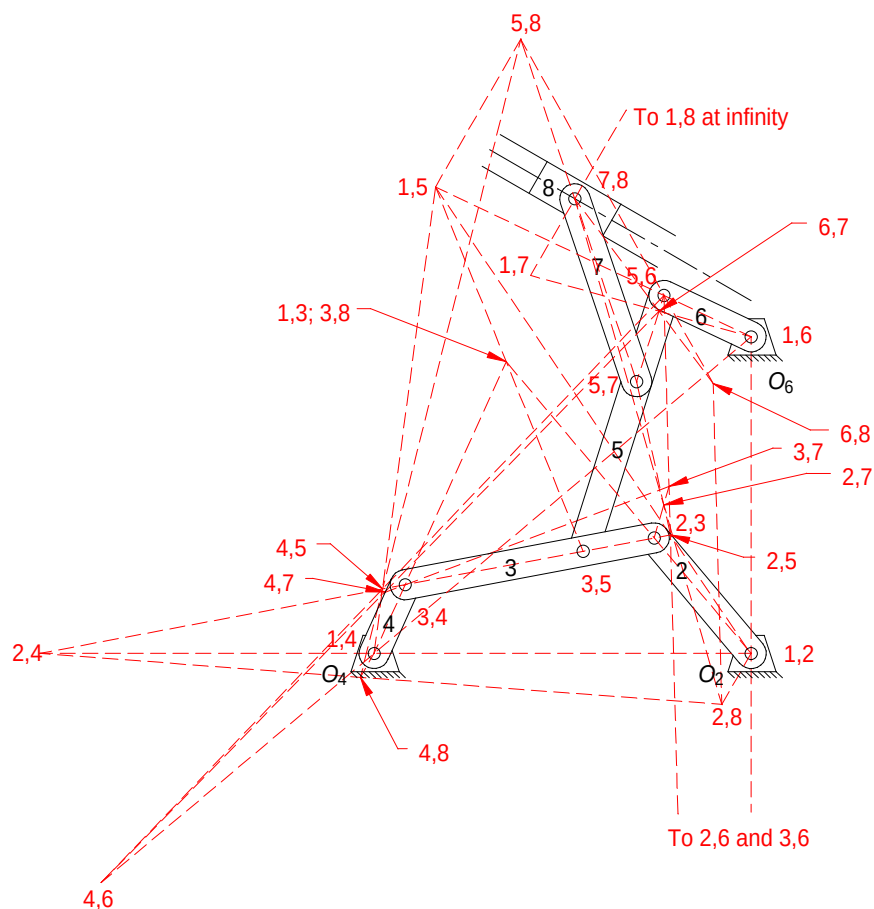
$I_{6.7}: I_{5.6}-I_{5.7}$ and $I_{6.8}-I_{7.8}$

$$I_{3.7}: I_{3.6}-I_{6.7} \text{ and } I_{2.3}-I_{2.7}$$

$$I_{4.6}: I_{1.4}-I_{1.6} \text{ and } I_{4.5}-I_{5.6}$$

$I_{4.7}$: $I_{4.6}$ - $I_{6.7}$ and $I_{3.4}$ - $I_{3.7}$

$I_{1.7}$: $I_{1.8}$ - $I_{7.8}$ and $I_{1.6}$ - $I_{6.7}$



PROBLEM 6-103

Statement: The linkage of Figure P6-33f has link 2 at 130 deg in the global XY coordinate system. Find V_D in the global coordinate system for the position shown if $\omega_2 = 15 \text{ rad/sec CW}$. Use any graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 5.0 \cdot \text{in}$	Link 3 (A to B)	$b := 8.4 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 2.5 \cdot \text{in}$	Link 1 (O_2 to O_4)	$d_1 := 12.5 \cdot \text{in}$
Link 5 (C to E)	$e := 8.9 \cdot \text{in}$	Link 5 (C to D)	$h := 5.9 \cdot \text{in}$
Link 6 (O_6 to E)	$f := 3.2 \cdot \text{in}$	Link 7 (D to F)	$k := 6.4 \cdot \text{in}$
Link 1 (O_2 to O_6)	$d_2 := 10.5 \cdot \text{in}$	Link 3 (A to C)	$g := 2.4 \cdot \text{in}$

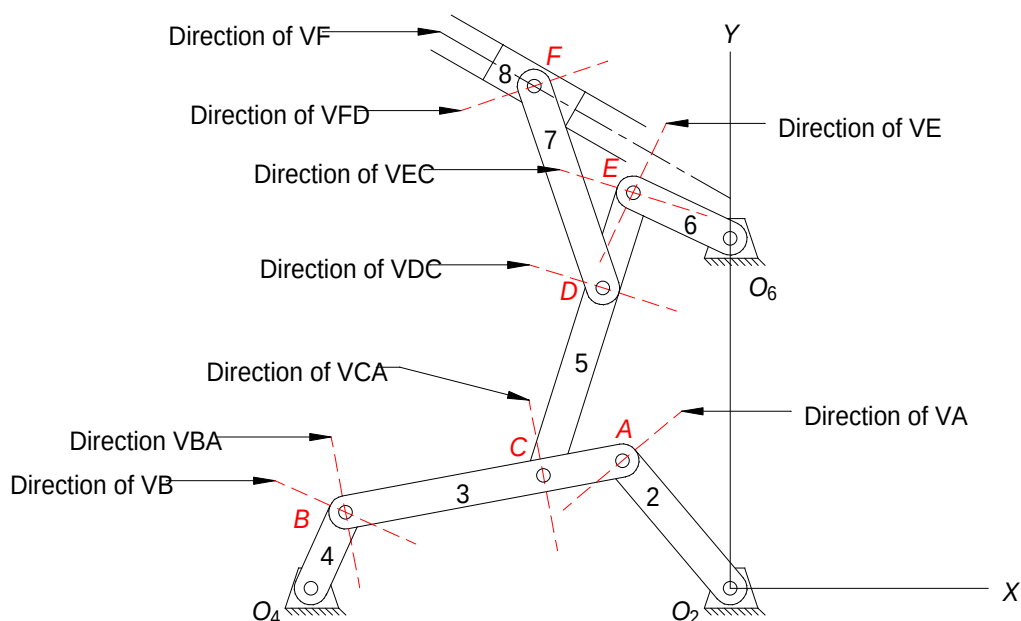
Crank angle: $\theta_2 := 130 \cdot \text{deg}$

Slider axis offset and angle: $s := 11.7 \cdot \text{in}$ $\delta_7 := 150 \cdot \text{deg}$

Input crank angular velocity $\omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

Solution: See Figure P6-33f and Mathcad file P06103.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



Angles measured from layout:

$$\begin{aligned} \theta_{VB} &:= -24.351 \cdot \text{deg} & \theta_{VBA} &:= -79.348 \cdot \text{deg} & \theta_{VCA} &:= \theta_{VBA} \\ \theta_{VE} &:= 64.594 \cdot \text{deg} & \theta_{VEC} &:= 162.461 \cdot \text{deg} & \theta_{VDC} &:= \theta_{VEC} \\ \theta_{VF} &:= 150.00 \cdot \text{deg} & \theta_{VFD} &:= 198.690 \cdot \text{deg} \end{aligned}$$

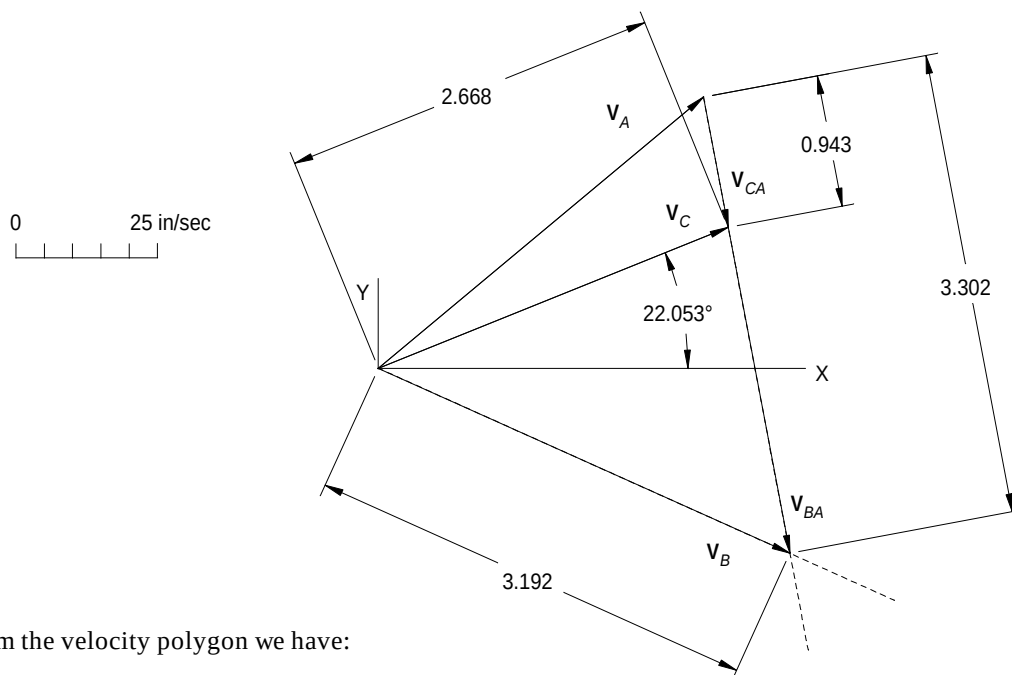
2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$\begin{aligned} V_A &:= a \cdot \omega_2 & V_A &= 75.000 \frac{\text{in}}{\text{sec}} \\ \theta_{VA} &:= \theta_2 - 90 \cdot \text{deg} & \theta_{VA} &= 40.000 \text{ deg} \end{aligned}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equations to be solved graphically are

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \quad \text{and} \quad \mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity polygon we have:

Velocity scale factor: $k_v := \frac{25 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_B := 3.192 \cdot \text{in} \cdot k_v \quad V_B = 79.800 \frac{\text{in}}{\text{sec}} \quad \theta_{VB} = -24.351 \text{ deg}$$

$$V_{BA} := 3.302 \cdot \text{in} \cdot k_v \quad V_{BA} = 82.550 \frac{\text{in}}{\text{sec}} \quad \theta_{VBA} = -79.348 \text{ deg}$$

$$\omega_3 := \frac{V_{BA}}{b} \quad \omega_3 = 9.827 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

5. Calculate the magnitude and direction of $\mathbf{V}_{CA}$ and determine the magnitude and velocity of $\mathbf{V}_C$ from the velocity polygon above.

$$V_{CA} := g \cdot \omega_3 \quad V_{CA} = 23.586 \frac{\text{in}}{\text{sec}} \quad \theta_{VCA} = -79.348 \text{ deg}$$

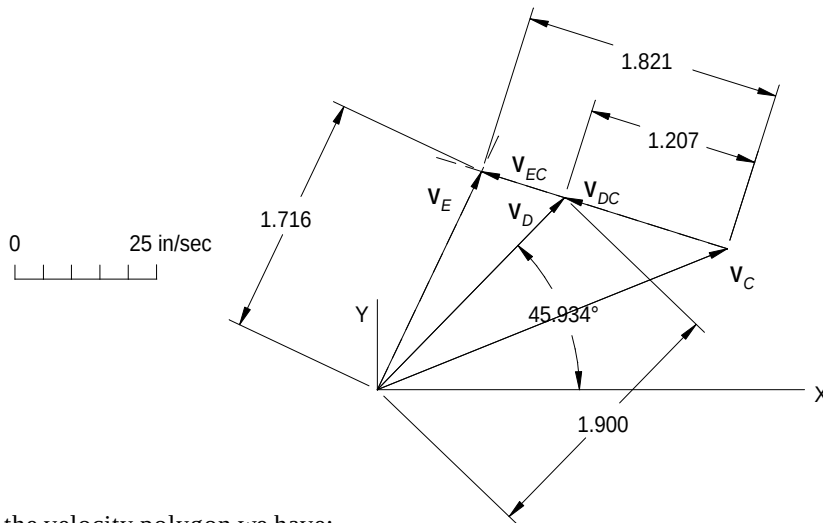
$$\text{Length of } \mathbf{V}_{CA} \text{ on velocity polygon } v_{CA} := \frac{V_{CA}}{k_v} \quad v_{CA} = 0.943 \text{ in}$$

$$V_C := 2.668 \cdot \text{in} \cdot k_v \quad V_C = 66.700 \frac{\text{in}}{\text{sec}} \quad \theta_{VC} = 22.053 \text{ deg}$$

6. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point E , the magnitude of the relative velocity V_{ED} , and the angular velocity of link 5. The equations to be solved graphically are

$$\mathbf{V}_E = \mathbf{V}_C + \mathbf{V}_{EC} \quad \text{and} \quad \mathbf{V}_D = \mathbf{V}_C + \mathbf{V}_{DC}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_C$.
- From the tip of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_{EC}$, magnitude unknown.
- From the tail of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_E$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{EC}$ from the tip of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_E$ construction line and drawing $\mathbf{V}_E$ from the tail of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_{EC}$ construction line.



7. From the velocity polygon we have:

$$\begin{aligned} \text{Velocity scale factor: } k_v &:= \frac{25 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}} \\ V_E &:= 1.716 \cdot \text{in} \cdot k_v & V_E &= 42.900 \frac{\text{in}}{\text{sec}} & \theta_{VE} &= 64.594 \text{ deg} \\ V_{EC} &:= 1.821 \cdot \text{in} \cdot k_v & V_{EC} &= 45.525 \frac{\text{in}}{\text{sec}} & \theta_{VEC} &= 162.461 \text{ deg} \\ \omega_5 &:= \frac{V_{EC}}{e} & \omega_5 &= 5.115 \frac{\text{rad}}{\text{sec}} & \text{CCW} \end{aligned}$$

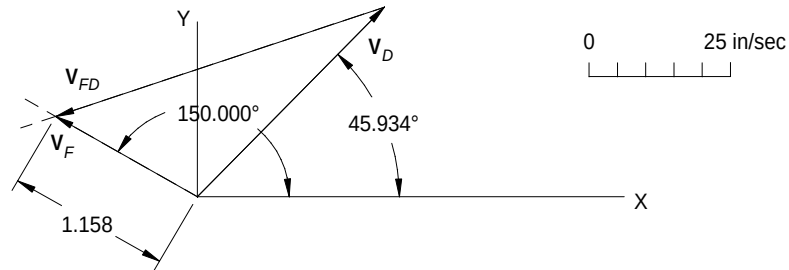
8. Calculate the magnitude and direction of $\mathbf{V}_{DE}$ and determine the magnitude and velocity of $\mathbf{V}_D$ from the velocity polygon above.

$$\begin{aligned} V_{DC} &:= h \cdot \omega_5 & V_{DC} &= 30.179 \frac{\text{in}}{\text{sec}} & \theta_{VDC} &= 162.461 \text{ deg} \\ \text{Length of } \mathbf{V}_{DC} \text{ on velocity polygon: } v_{DC} &:= \frac{V_{DC}}{k_v} & v_{DC} &= 1.207 \text{ in} \\ V_D &:= 1.900 \cdot \text{in} \cdot k_v & V_D &= 47.500 \frac{\text{in}}{\text{sec}} & \theta_{VD} &:= 45.934 \text{ deg} \end{aligned}$$

9. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point F , the magnitude of the relative velocity V_{FD} , and the angular velocity of link 5. The equation to be solved graphically is

$$\mathbf{V}_F = \mathbf{V}_D + \mathbf{V}_{FD}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_D$.
- From the tip of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_{FD}$, magnitude unknown.
- From the tail of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_F$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{FD}$ from the tip of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_F$ construction line and drawing $\mathbf{V}_F$ from the tail of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_{FD}$ construction line.



10. From the velocity polygon we have:

Velocity scale factor: $k_v := \frac{25 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_F := 1.158 \cdot \text{in} \cdot k_v$$

$$V_F = 28.950 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VF} = 150.000 \text{ deg}$$

PROBLEM 6-104

Statement: For the linkage of Figure P6-34, locate and identify all instant centers.

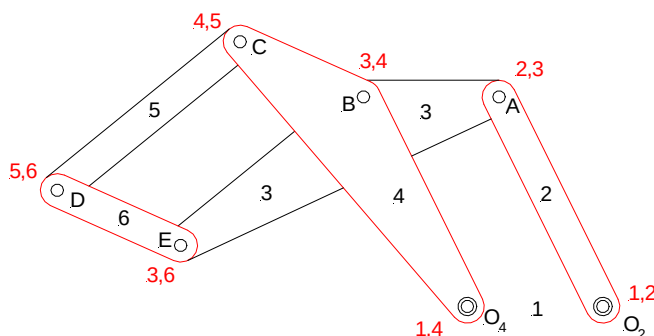
Given: Number of links $n := 6$

Solution: See Figure P6-34 and Mathcad file P06104.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n - 1)}{2} \quad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection (7).



2. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs: $I_{1,3}$; $I_{1,5}$; $I_{1,6}$; $I_{2,4}$; $I_{2,5}$; $I_{2,6}$; $I_{3,5}$; and $I_{4,6}$

$$I_{2,4}: I_{1,2}-I_{1,4} \text{ and } I_{2,3}-I_{3,4}$$

$$I_{3,5}: I_{2,3}-I_{2,5} \text{ and } I_{3,4}-I_{4,5}$$

$$I_{4,6}: I_{3,4}-I_{3,6} \text{ and } I_{4,5}-I_{5,6}$$

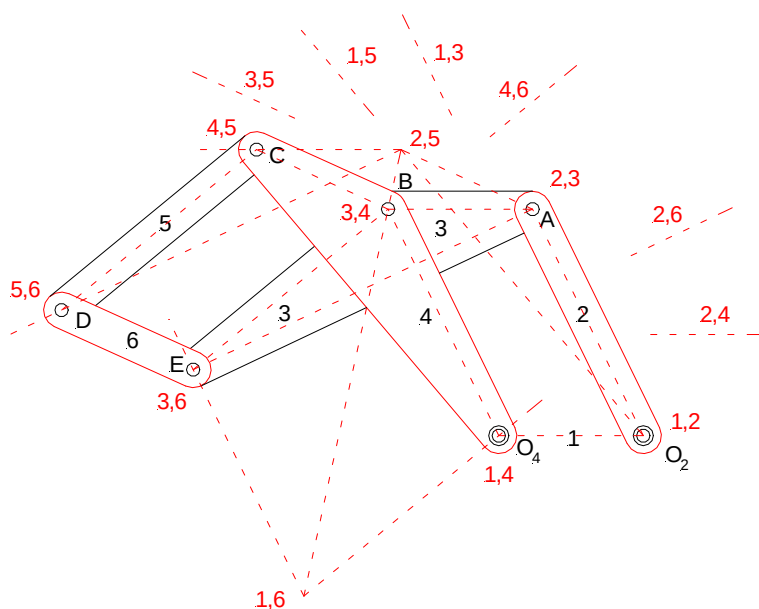
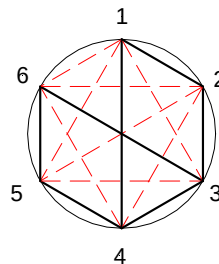
$$I_{1,3}: I_{1,2}-I_{2,3} \text{ and } I_{1,4}-I_{3,4}$$

$$I_{2,6}: I_{1,4}-I_{4,6} \text{ and } I_{2,3}-I_{3,6}$$

$$I_{1,5}: I_{1,2}-I_{2,5} \text{ and } I_{1,3}-I_{3,5}$$

$$I_{2,5}: I_{2,6}-I_{5,6} \text{ and } I_{2,4}-I_{4,5}$$

$$I_{1,6}: I_{1,3}-I_{3,6} \text{ and } I_{1,4}-I_{4,6}$$



PROBLEM 6-105

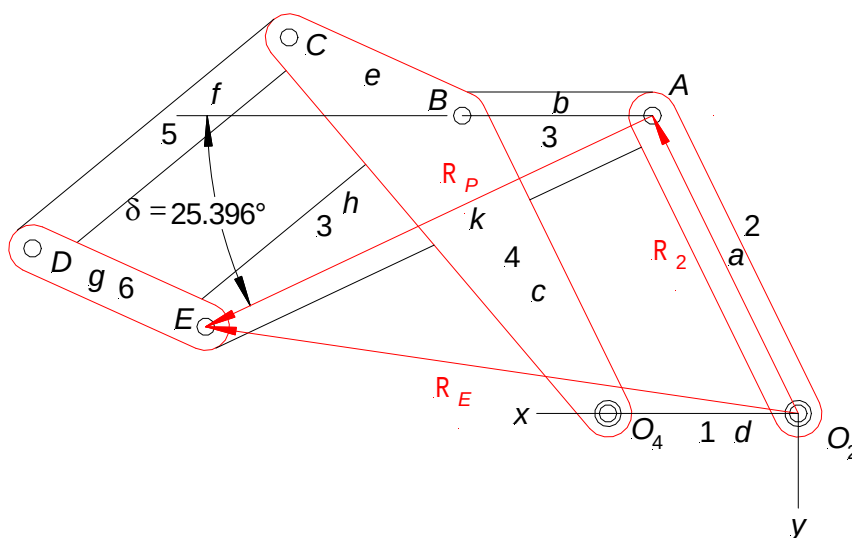
Statement: For the linkage of Figure P6-34, show that $I_{1,6}$ is stationary for all positions of link 2.

Given: Link lengths:

Input crank (L_2)	$a := 1.75$	First coupler (AB)	$b := 1.00$
First rocker (O_4B)	$c := 1.75$	Ground link (O_2O_4)	$d := 1.00$
Second input (BC)	$e := 1.00$	Second coupler (L_5)	$f := 1.75$
Output rocker (L_6)	$g := 1.00$	Third coupler (BE)	$h := 1.75$
Ternary link (AE)	$k := 2.60$		

Solution: See Figure P6-34 and Mathcad file P06105.

1. Because the linkage is symmetrical and composed of two parallelograms the analysis can be done with simple trigonometry. We will show that the path of point E on link 6 is a circle and that the center of the circle is the instant center $I_{1,6}$.



2. Calculate the fixed angle that line AB (b) makes with line AE (k) using the law of cosines. This is the angle that the coupler point, E , makes with link 3 in the fourbar made up of links 1, 2, 3, and 4. Because links 1, 2, 3, and 4 are a parallelogram, link 4 will have the same angle as link 2 and AB will always be parallel to O_2O_4 .

$$\delta := \arccos\left(\frac{b^2 + k^2 - h^2}{2 \cdot b \cdot k}\right) \quad \delta = 25.396 \text{ deg}$$

3. Define the vectors $\mathbf{R}_2$, $\mathbf{R}_p$, and $\mathbf{R}_E$ as shown above. Then, $\mathbf{R}_E = \mathbf{R}_2 + \mathbf{R}_p$ or,

$$\mathbf{R}_E(\theta_2) = a \cdot (\cos(\theta_2) + j \sin(\theta_2)) + k \cdot (\cos(\delta) + j \sin(\delta))$$

Separating into real and imaginary parts, the coordinates of point E for all positions of link 2 are:

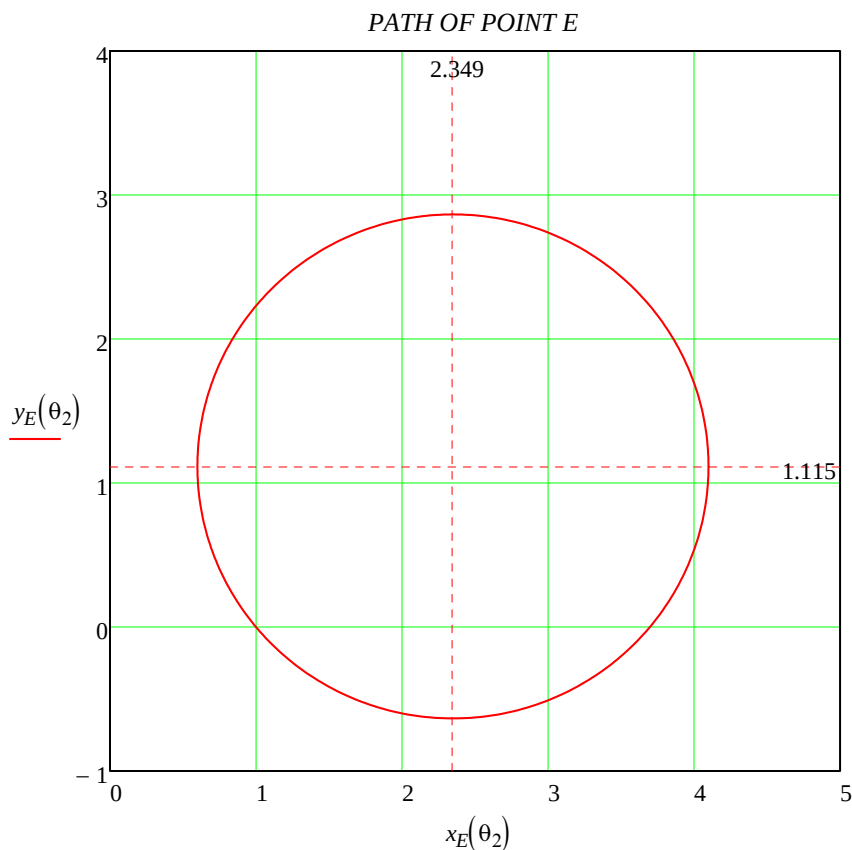
$$x_E(\theta_2) := a \cdot \cos(\theta_2) + k \cdot \cos(\delta)$$

$$y_E(\theta_2) := a \cdot \sin(\theta_2) + k \cdot \sin(\delta)$$

This pair of equations represent the parametric equation of a circle with radius a and center at

$$\begin{aligned} x_{EC} &:= k \cdot \cos(\delta) & x_{EC} &= 2.349 \\ y_{EC} &:= k \cdot \sin(\delta) & y_{EC} &= 1.115 \end{aligned}$$

4. Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$
5. Plot the position of point E as a function of the angle of input link 2.



6. From Problem 6-104, the instant center $I_{1,6}$ is located at the intersection of a line through O_4 that is parallel to BE (angle α) with a line through E that is parallel to O_2A (angle θ_2). (See figure on next page). The coordinates of the intersection of these two lines for the position shown ($\theta_2 := -64.036 \cdot \text{deg}$) are:

$$\begin{aligned} \text{Angle } BAE: & & \alpha &:= \pi - \arccos\left(\frac{b^2 + h^2 - k^2}{2 \cdot b \cdot h}\right) & \alpha &= 39.582 \text{ deg} \end{aligned}$$

$$\begin{aligned} \text{Coordinates of point } E: & & x_E &:= a \cdot \cos(\theta_2) + k \cdot \cos(\delta) & x_E &= 3.115 \\ & & y_E &:= a \cdot \sin(\theta_2) + k \cdot \sin(\delta) & y_E &= -0.458 \end{aligned}$$

$$\begin{aligned} \text{Line through } E \text{ with angle } \theta_2: & & y(x) &:= (x - x_E) \cdot \tan(\theta_2) + y_E \end{aligned}$$

$$\begin{aligned} \text{Coordinates of point } O_4: & & x_{O4} &:= d & y_{O4} &:= 0 \end{aligned}$$

PROBLEM 6-106

Statement: Figure P6-26 shows a mechanism with dimensions. Use a graphical method to determine the velocities of points A and B and the velocity of slip for the position shown. Ignore links 5 and 6. $\omega_2 = 24 \text{ rad/s}$ clock-wise.

Given: Link lengths and angles:

Link 1 (O_2O_4) $d := 1.22 \cdot \text{in}$ Angle O_2O_4 makes with X axis $\theta_1 := 56.5 \cdot \text{deg}$

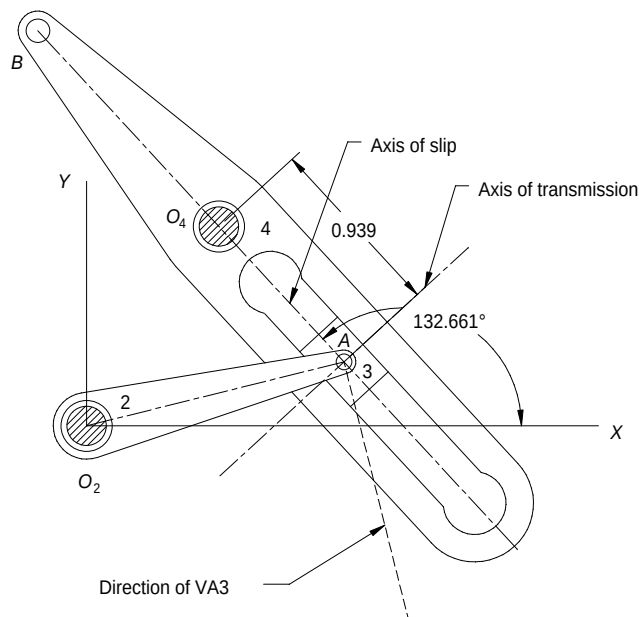
Link 2 (O_2A) $a := 1.35 \cdot \text{in}$ Angle θ_2 makes with X axis $\theta_2 := 14 \cdot \text{deg}$

Link 4 (O_4B) $e := 1.36 \cdot \text{in}$

Angular velocity of link 2 $\omega_2 := 24 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

Solution: See Figure P6-26 and Mathcad file P06106.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



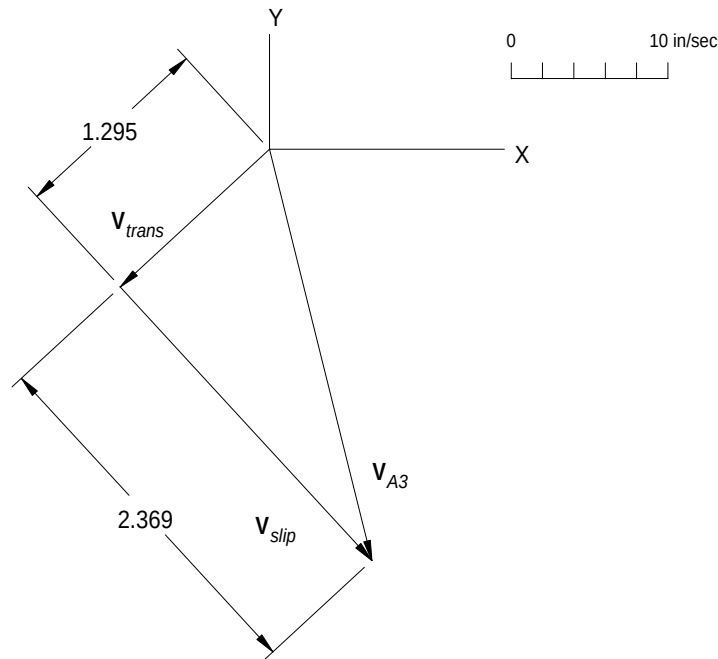
2. Use equation 6.7 to calculate the magnitude of the velocity at point A on links 2 and 3.

$$V_{A3} := a \cdot \omega_2 \quad V_{A3} = 32.400 \cdot \frac{\text{in}}{\text{sec}} \quad \theta_{VA3} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{VA3} = -76.0 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A on link 3. The equation to be solved graphically is

$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.
- b. From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{12 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_{slip} := 2.369 \cdot \text{in} \cdot k_v \quad V_{slip} = 28.43 \cdot \frac{\text{in}}{\text{sec}}$$

$$V_{trans} := 1.295 \cdot \text{in} \cdot k_v \quad V_{trans} = 15.54 \cdot \frac{\text{in}}{\text{sec}}$$

5. The true velocity of point A on link 4 is V_{trans} ,

$$V_{A4} := V_{trans} \quad V_{A4} = 15.54 \cdot \frac{\text{in}}{\text{sec}}$$

6. Determine the angular velocity of link 4 using equation 6.7.

$$\text{From the linkage layout above: } c := 0.939 \cdot \text{in} \quad \text{and} \quad \theta_4 := 132.661 \cdot \text{deg}$$

$$\omega_4 := \frac{V_{A4}}{c} \quad \omega_4 = 16.55 \cdot \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

7. Determine the magnitude and sense of the vector V_B using equation 6.7.

$$V_B := e \cdot \omega_4 \quad V_B = 22.51 \cdot \frac{\text{in}}{\text{sec}}$$

$$\theta_{V_{A4}} := \theta_4 - 90 \cdot \text{deg}$$

$$\theta_{V_{A4}} = 42.66 \cdot \text{deg}$$

PROBLEM 6-107

Statement: Figure P6-26 shows a mechanism with dimensions. Use an analytic method to calculate the velocities of points A and B and the velocity of slip for the position shown. Ignore links 5 and 6. $\omega_2 = 24 \text{ rad/s}$ clock-wise.

Given: Link lengths and angles:

$$\text{Link 1 (} O_2O_4 \text{)} \quad d := 1.22 \cdot \text{in} \quad \text{Angle } O_2O_4 \text{ makes with } X \text{ axis} \quad \theta_{10} := 56.5 \cdot \text{deg}$$

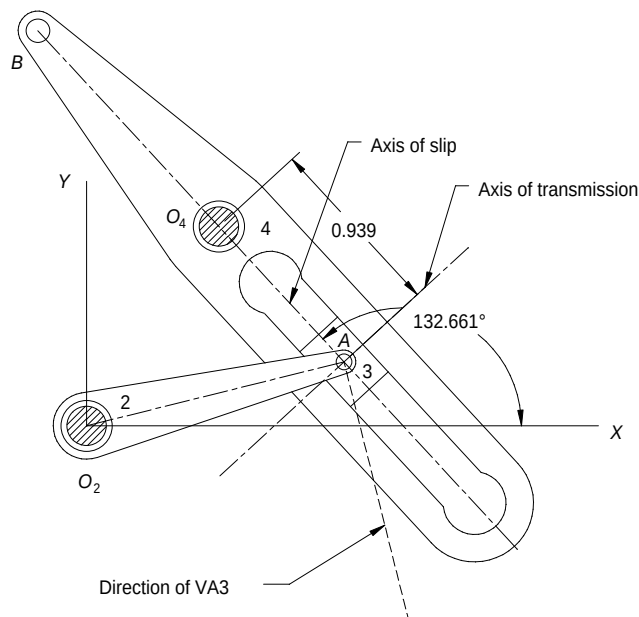
$$\text{Link 2 (} O_2A \text{)} \quad a := 1.35 \cdot \text{in} \quad \text{Angle } \theta_2 \text{ makes with } X \text{ axis} \quad \theta_{20} := 14 \cdot \text{deg}$$

$$\text{Link 4 (} O_4B \text{)} \quad e := 1.36 \cdot \text{in}$$

$$\text{Angular velocity of link 2} \quad \omega_2 := -24 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$$

Solution: See Figure P6-26 and Mathcad file P06107.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Establish an x-y frame with origin at O_2 and positive x-axis through O_4 . Using the given data, calculate b , θ_2 , θ_3 , and θ_4 with respect to the x-y frame. From the vector loop equation for this geometry,

$$\theta_2 := \theta_{20} - \theta_{10} \quad \theta_2 = -42.500 \cdot \text{deg}$$

$$\theta_4 := 180 \cdot \text{deg} - \text{atan}\left(\frac{a \cdot \sin(\theta_2)}{d - a \cdot \cos(\theta_2)}\right) \quad \theta_4 = 256.161 \cdot \text{deg} \quad \theta_3 := \theta_4$$

$$b := a \cdot \frac{\sin(\theta_2)}{\sin(\theta_4)} \quad b = 0.939 \text{ in}$$

3. Using Example 6-5, determine V_{A2} , V_{trans} , and V_{slip} .

Velocity on link 2 at A:

$$V_{A2} := |a \cdot \omega_2| \quad \text{at an angle of} \quad \theta_{VA2} := \theta_2 + \text{sign}(\omega_2) \cdot 90 \cdot \text{deg}$$

$$V_{A2} = 32.400 \frac{\text{in}}{\text{s}} \qquad \theta_{VA2} = -132.500 \cdot \text{deg}$$

Angle between VA2 vector and link 4 axis:

$$\alpha := \theta_{VA2} - \theta_4 + 2 \cdot \pi \qquad \alpha = -28.661 \cdot \text{deg}$$

$$V_{\text{slip}} := V_{A2} \cdot \cos(\alpha) \qquad V_{\text{slip}} = 28.430 \frac{\text{in}}{\text{s}}$$

$$V_{\text{trans}} := V_{A2} \cdot \sin(\alpha) \qquad V_{\text{trans}} = -15.540 \frac{\text{in}}{\text{s}}$$

4. Calculate the angular velocity of link 4 and the velocity at point B.

$$\text{Angular velocity of link 4:} \qquad \omega_4 := \frac{V_{\text{trans}}}{b} \qquad \omega_4 = -16.544 \cdot \frac{\text{rad}}{\text{s}}$$

$$\text{Velocity at point B:} \qquad V_B := e \cdot \omega_4 \qquad V_B = -22.500 \frac{\text{in}}{\text{s}}$$

PROBLEM 6-108

Statement: Figure P6-28 shows a quick-return mechanism with dimensions. Use a graphical method to determine the velocities of points A and B and the velocity of slip for the position shown. Ignore links 5 and 6. $\omega_2 = 16 \text{ rad/s}$.

Given: Link lengths and angles:

Link 1 (O_2O_4) $d := 1.69 \cdot \text{in}$ Angle O_2O_4 makes with X axis $\theta_1 := 15.5 \cdot \text{deg}$

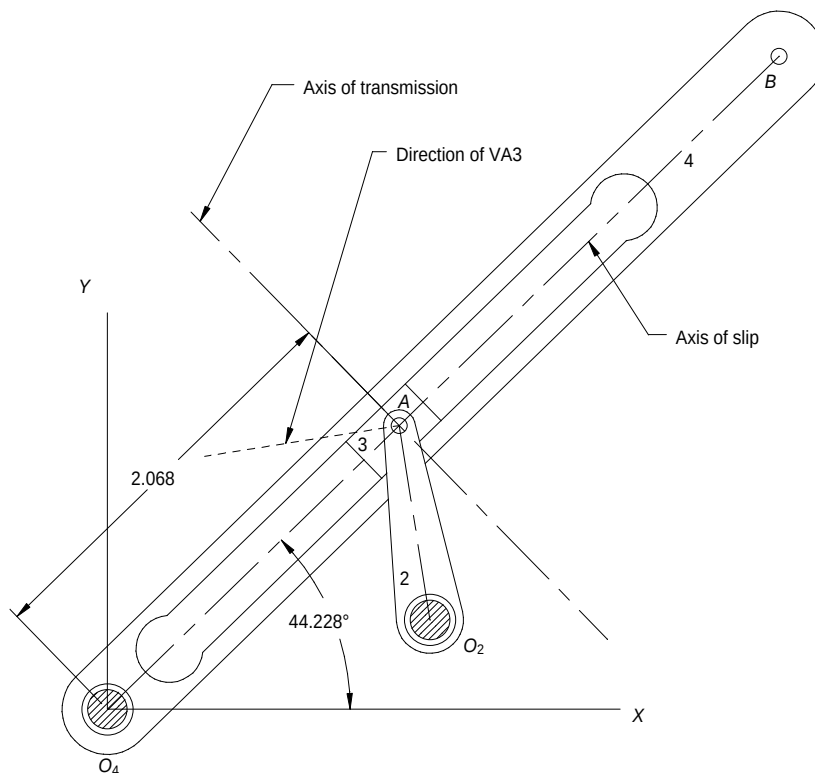
Link 2 (L_2) $a := 1.00 \cdot \text{in}$ Angle link 2 makes with X axis $\theta_2 := 99 \cdot \text{deg}$

Link 4 (L_4) $e := 4.76 \cdot \text{in}$

Angular velocity of link 2 $\omega_2 := 16 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW

Solution: See Figure P6-28 and Mathcad file P06108.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A on links 2 and 3.

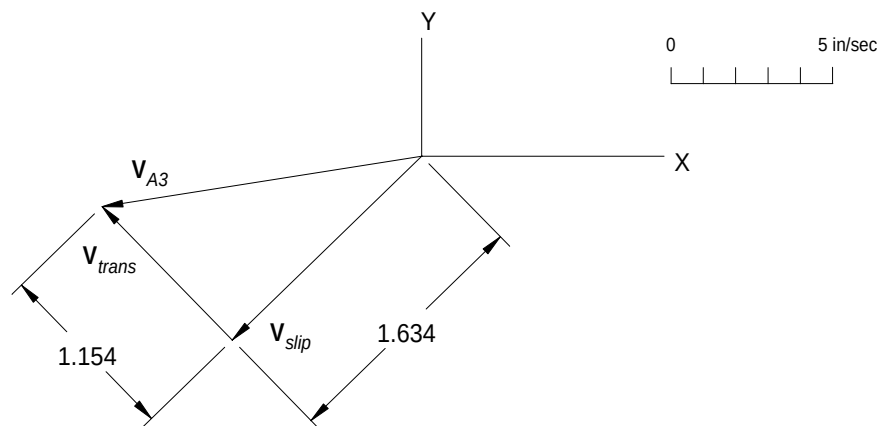
$$V_{A2} := a \cdot \omega_2 \quad V_{A2} = 16.000 \cdot \frac{\text{in}}{\text{sec}} \quad \theta_{VA2} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{VA2} = 189.0 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A on link 3. The equation to be solved graphically is

$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.
- b. From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.

- c. From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_{slip} := 1.634 \cdot \text{in} \cdot k_v \quad V_{slip} = 8.17 \cdot \frac{\text{in}}{\text{sec}}$$

$$V_{trans} := 1.154 \cdot \text{in} \cdot k_v \quad V_{trans} = 5.77 \cdot \frac{\text{in}}{\text{sec}}$$

5. The true velocity of point A on link 4 is V_{trans} ,

$$V_{A4} := V_{trans} \quad V_{A4} = 5.77 \cdot \frac{\text{in}}{\text{sec}}$$

6. Determine the angular velocity of link 4 using equation 6.7.

$$\text{From the linkage layout above: } c := 2.068 \cdot \text{in} \quad \text{and} \quad \theta_4 := 44.228 \cdot \text{deg}$$

$$\omega_4 := \frac{V_{A4}}{c} \quad \omega_4 = 2.790 \cdot \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

7. Determine the magnitude and sense of the vector $\mathbf{V}_B$ using equation 6.7.

$$V_B := e \cdot \omega_4 \quad V_B = 13.28 \cdot \frac{\text{in}}{\text{sec}}$$

$$\theta_{VB} := \theta_4 + 90 \cdot \text{deg} \quad \theta_{VB} = 134.228 \cdot \text{deg}$$

PROBLEM 6-109

Statement: Figure P6-28 shows a quick-return mechanism with dimensions. Use an analytic method to calculate the velocities of points A and B and the velocity of slip for the position shown. Ignore links 5 and 6. $\omega_2 = 16 \text{ rad/s}$.

Given: Link lengths and angles:

Link 1 (O_2O_4) $d := 1.69 \cdot \text{in}$ Angle O_2O_4 makes with X axis $\theta_{10} := 15.5 \cdot \text{deg}$

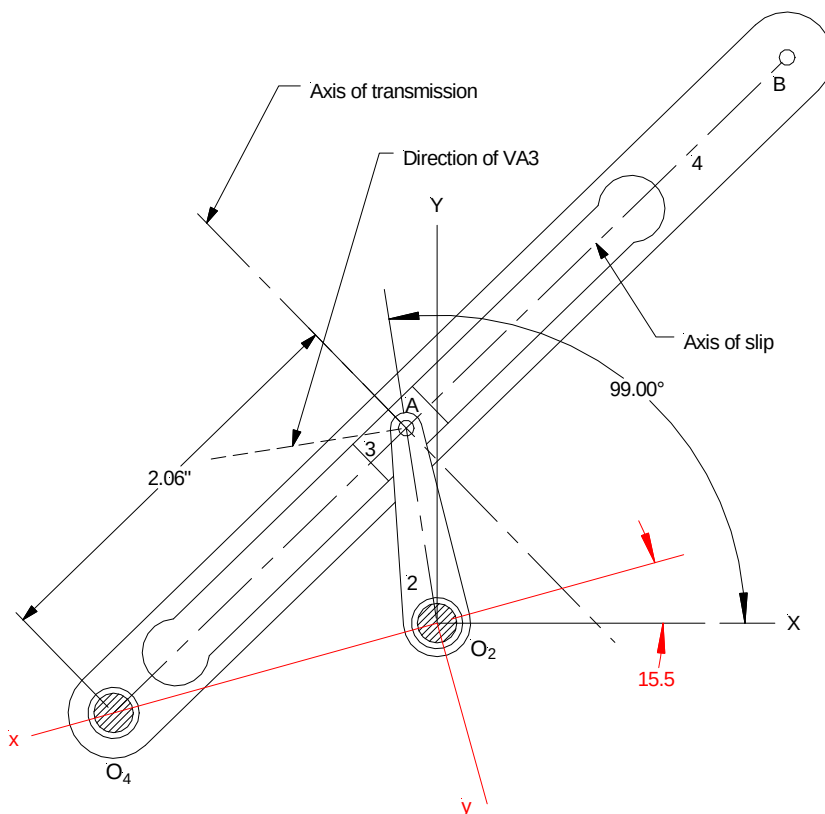
Link 2 (L_2) $a := 1.00 \cdot \text{in}$ Angle link 2 makes with X axis $\theta_{20} := 99 \cdot \text{deg}$

Link 4 (L_4) $e := 4.76 \cdot \text{in}$

Angular velocity of link 2 $\omega_2 := 16 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW

Solution: See Figure P6-28 and Mathcad file P06108.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Establish an x-y frame with origin at O_2 and positive x-axis through O_4 . Using the given data, calculate b , θ_2 , θ_3 , and θ_4 with respect to the x-y frame. From the vector loop equation for this geometry,

$$\theta_2 := (\theta_{20} - \theta_{10}) - 180 \cdot \text{deg} \quad \theta_2 = -96.500 \cdot \text{deg}$$

$$\theta_4 := 180 \cdot \text{deg} - \text{atan}\left(\frac{a \cdot \sin(\theta_2)}{d - a \cdot \cos(\theta_2)}\right) \quad \theta_4 = 208.855 \cdot \text{deg} \quad \theta_3 := \theta_4$$

$$b := a \cdot \frac{\sin(\theta_2)}{\sin(\theta_4)} \quad b = 2.059 \text{ in}$$

3. Using Example 6-5, determine V_{A2} , V_{trans} , and V_{slip} .

Velocity on link 2 at A:

$$V_{A2} := |a \cdot \omega_2| \quad \text{at an angle of} \quad \theta_{VA2} := \theta_2 + \text{sign}(\omega_2) \cdot 90 \cdot \text{deg}$$

$$V_{A2} = 16.000 \frac{\text{in}}{\text{s}} \quad \theta_{VA2} = -6.500 \cdot \text{deg}$$

Angle between VA2 vector and link 4 axis:

$$\alpha := \theta_{VA2} - \theta_4 + 2 \cdot \pi \quad \alpha = 144.645 \cdot \text{deg}$$

$$V_{slip} := V_{A2} \cdot \cos(\alpha) \quad V_{slip} = -13.049 \frac{\text{in}}{\text{s}}$$

$$V_{trans} := V_{A2} \cdot \sin(\alpha) \quad V_{trans} = 9.258 \frac{\text{in}}{\text{s}}$$

4. Calculate the angular velocity of link 4 and the velocity at point B.

$$\text{Angular velocity of link 4:} \quad \omega_4 := \frac{V_{trans}}{b} \quad \omega_4 = 4.497 \cdot \frac{\text{rad}}{\text{s}}$$

$$\text{Velocity at point B:} \quad V_B := e \cdot \omega_4 \quad V_B = 21.405 \frac{\text{in}}{\text{s}}$$

PROBLEM 6-110a

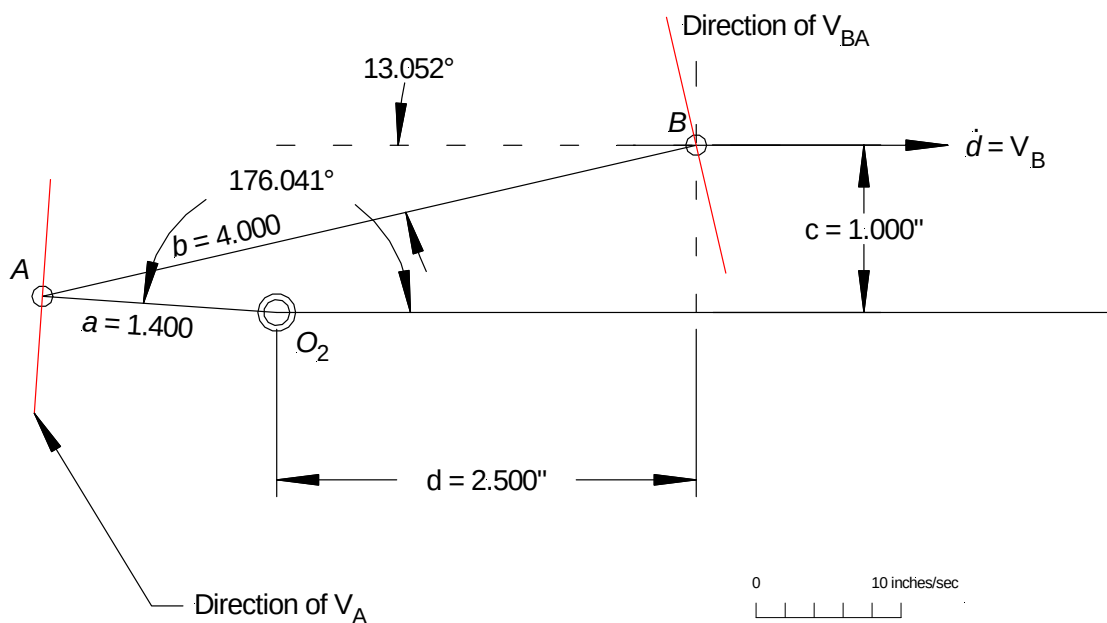
Statement: The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Fig P6-2. The link lengths and the values of d and $d\text{-dot}$ are defined in Table P6-5. For row a , find the velocity of the pin joint A and the angular velocity of the crank using a graphical method.

Given: Link lengths:

Link 2	$a := 1.4 \cdot \text{in}$	Link 3	$b := 4 \cdot \text{in}$
Offset	$c := 1 \cdot \text{in}$	$\theta_2 := 176.041 \cdot \text{deg}$	$d\text{-dot} := 10 \cdot \text{in} \cdot \text{sec}^{-1}$

Solution: See Figure P6-2, Table P6-5, and Mathcad file P06110a.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.

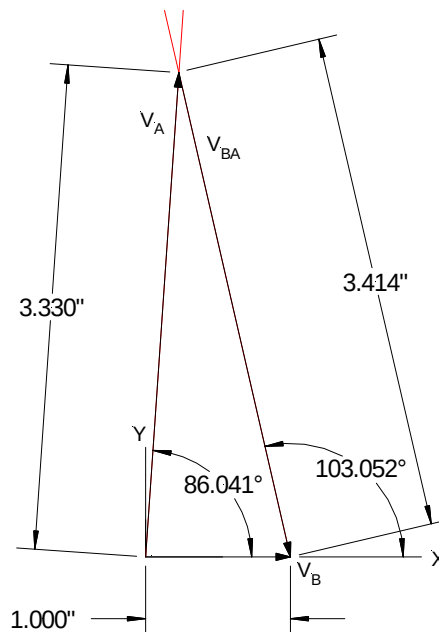


2. Use equation 6.6.24b to (graphically) determine the magnitude of the velocity at point A. The equation to be solved graphically is

$$\mathbf{V}_A = \mathbf{V}_B + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
- b. From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_A$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_A$ construction line and drawing $\mathbf{V}_A$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{BA}$ construction line.

3. From the velocity triangle we have:



Velocity scale factor: $k_v := \frac{10 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$

$$V_A := 3.330 \cdot \text{in} \cdot k_v \quad V_A = 33.300 \cdot \frac{\text{in}}{\text{sec}} \quad \theta_{VB} := 86.041 \cdot \text{deg}$$

4. Use equation 6.7 to find the angular velocity of the crank (link 2).

$$\omega_2 := \frac{V_A}{a} \quad \omega_2 = 23.786 \cdot \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

PROBLEM 6-111a

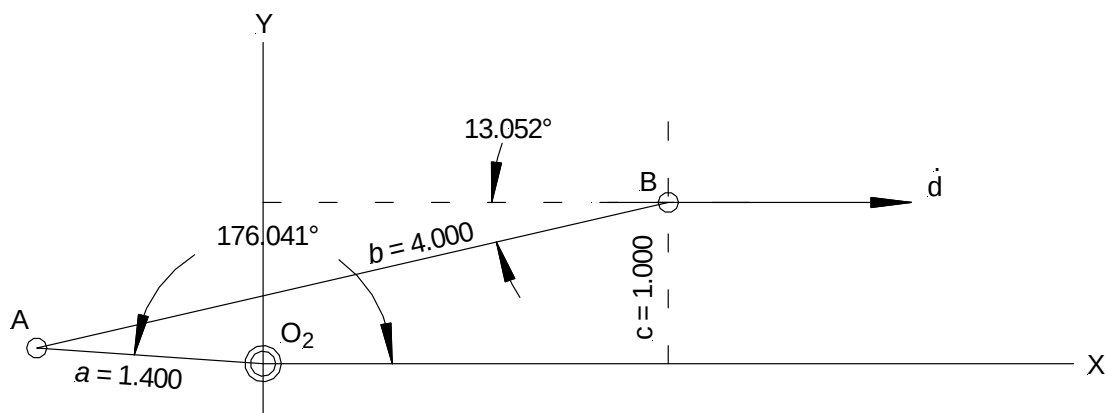
Statement: The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Fig P6-2. The link lengths and the values of d and $\dot{d}$ are defined in Table P6-5. For row a , find the velocity of pin joint A and the angular velocity of the crank using the analytic method. Draw the linkage to scale and label it before setting up the equations.

Given: Link lengths:

Link 2 (O_2A)	$a := 1.4 \text{ in}$	Link 3 (AB)	$b := 4 \text{ in}$	Offset (y_B)	$c := 1 \text{ in}$
Slider position	$d := 2.5 \text{ in}$	Slider velocity	$\dot{d} := 10 \frac{\text{in}}{\text{s}}$		

Solution: See Figure P6-2, Table P6-5, and Mathcad file P06111a.

1. Draw the linkage to scale and label it.



2. Determine the open value of θ_2 using equations 4.20 and 4.21.

$$K_1 := a^2 - b^2 + c^2 + d^2 \quad K_1 = -6.79 \text{ in}^2$$

$$K_2 := -2 \cdot a \cdot c \quad K_2 = -2.8 \text{ in}^2$$

$$K_3 := -2 \cdot a \cdot d \quad K_3 = -7 \text{ in}^2$$

$$A := K_1 - K_3 \quad A = 0.21 \text{ in}^2$$

$$B := 2 \cdot K_2 \quad B = -5.6 \text{ in}^2$$

$$C := K_1 + K_3 \quad C = -13.79 \text{ in}^2$$

$$\theta_2 := 2 \cdot \text{atan2}(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}) \quad \theta_2 = 176.041 \cdot \text{deg}$$

3. Determine the value of θ_3 using equation 4.16a or 4.17.

$$\beta(a, b, c, d, \alpha) := \begin{cases} \theta \leftarrow \text{asin}\left(\frac{a \cdot \sin(\alpha) - c}{b}\right) \\ d_1 \leftarrow a \cdot \cos(\alpha) - b \cdot \cos(\theta) \\ \text{return } \theta \text{ if } d_1 = d \\ \text{asin}\left(-\frac{a \cdot \sin(\alpha) - c}{b}\right) + \pi \text{ otherwise} \end{cases}$$

$$\theta_3 := \beta(a, b, c, d, \theta_2) \quad \theta_3 = 193.052 \cdot \text{deg}$$

4. Determine the value of ω_2 using equation 6.24b.

$$\omega_2 := \frac{\ddot{\theta}_3 \cdot \cos(\theta_3)}{a \cdot (\cos(\theta_2) \cdot \sin(\theta_3) - \sin(\theta_2) \cdot \cos(\theta_3))} \quad \omega_2 = -23.785 \cdot \frac{\text{rad}}{\text{s}}$$

5. Determine the value of V_A using equation 6.7.

$$V_A := |a \cdot \omega_2| \quad V_A = 33.299 \frac{\text{in}}{\text{s}}$$

$$\theta_{VA} := \theta_2 + \text{sign}(\omega_2) \cdot 90 \cdot \text{deg} \quad \theta_{VA} = 86.041 \cdot \text{deg}$$

PROBLEM 7-1

- Statement:** A point at a 6.5-in radius is on a body that is in pure rotation with $\omega = 100$ rad/sec and a constant $\alpha = -500$ rad/sec² at point A. The rotation center is at the origin of a coordinate system. When the point is at position A, its position vector makes a 45 deg angle with the X axis. It takes 0.01 sec to reach point B. Draw this system to some convenient scale, calculate the θ and ω of position B, and:
- Write an expression for the particle's acceleration vector in position A using complex number notation, in both polar and Cartesian forms.
 - Write an expression for the particle's acceleration vector in position B using complex number notation, in both polar and Cartesian forms.
 - Write a vector equation for the acceleration difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the acceleration difference numerically.
 - Check the result of part c with a graphical method.

Given:

Initial rotation speed $\omega_A := 100 \cdot \frac{\text{rad}}{\text{sec}}$

Rotation acceleration $\alpha := -500 \cdot \frac{\text{rad}}{\text{sec}^2}$

Time to reach point B $t := 0.01 \cdot \text{sec}$

Vector angle $\theta_A := 45 \cdot \text{deg}$

Vector magnitude $R := 6.5 \cdot \text{in}$

Solution: See Mathcad file P0701.

1. Calculate the position and angular velocity at point B using equations 6.1 and 7.1.

$$\omega_B := \alpha \cdot t + \omega_A \quad \omega_B = 95.000 \frac{\text{rad}}{\text{sec}}$$

$$\theta_B := \frac{\alpha \cdot t^2}{2} + \omega_A \cdot t + \theta_A \quad \theta_B = 100.863 \text{ deg}$$

2. Calculate the magnitudes and directions of the normal and tangential components of acceleration at points A and B using equations 7.2.

$$a_{An} := R \cdot \omega_A^2 \quad a_{An} = 65000 \frac{\text{in}}{\text{sec}^2} \quad \theta_{An} := \theta_A + 180 \cdot \text{deg} \quad \theta_{An} = 225.000 \text{ deg}$$

$$a_{At} := |R \cdot \alpha| \quad a_{At} = 3250 \frac{\text{in}}{\text{sec}^2} \quad \theta_{At} := \theta_A + \text{sign}(\alpha) \cdot 90 \cdot \text{deg}$$

$$\theta_{At} = -45.000 \text{ deg}$$

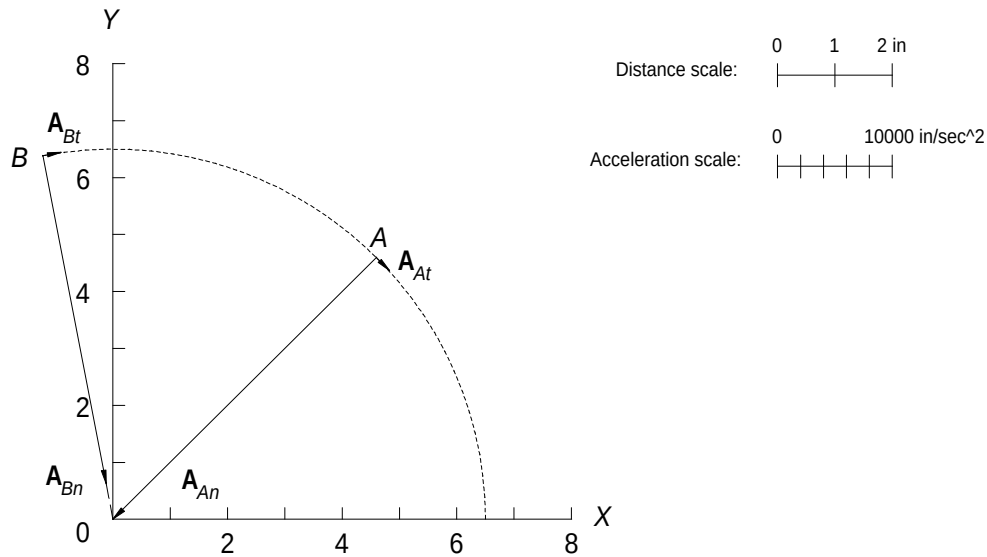
$$a_{Bn} := R \cdot \omega_B^2 \quad a_{Bn} = 58663 \frac{\text{in}}{\text{sec}^2} \quad \theta_{Bn} := \theta_B + 180 \cdot \text{deg} \quad \theta_{Bn} = 280.863 \text{ deg}$$

$$a_{Bt} := |R \cdot \alpha| \quad a_{Bt} = 3250 \frac{\text{in}}{\text{sec}^2} \quad \theta_{Bt} := \theta_B + \text{sign}(\alpha) \cdot 90 \cdot \text{deg}$$

$$\theta_{Bt} = 10.863 \text{ deg}$$

3. Establish an X-Y coordinate frame and draw a circle with center at the origin and radius R.

4. Draw lines from the origin that make angles of 45 and 100.863 deg with respect to the X axis. Label the intersections of the lines with the circles as A and B , respectively.
5. Choose a convenient acceleration scale and draw the two acceleration component vectors at points A and B .



- a. Write an expression for the particle's acceleration vector in position A using complex number notation, in both polar and Cartesian forms.

Polar form:

$$\mathbf{R}_A := R \cdot \mathbf{e}^{j \cdot \theta_A} \qquad \mathbf{R}_A := 6.5 \cdot \mathbf{e}^{j \cdot \frac{\pi}{4}}$$

$$\mathbf{V}_A := R \cdot j \cdot \omega_A \cdot \mathbf{e}^{j \cdot \theta_A} \qquad \mathbf{V}_A := 650 \cdot j \cdot \mathbf{e}^{j \cdot \frac{\pi}{4}}$$

$$\mathbf{A}_A := R \cdot j \cdot \alpha \cdot \mathbf{e}^{j \cdot \theta_A} - R \cdot j \cdot \omega_A^2 \cdot \mathbf{e}^{j \cdot \theta_A}$$

$$\mathbf{A}_A := 6.50 \cdot j \cdot \alpha \cdot \mathbf{e}^{j \cdot \frac{\pi}{4}} - 6.50 \cdot \omega_A^2 \cdot \mathbf{e}^{j \cdot \frac{\pi}{4}}$$

Cartesian form:

$$\mathbf{A}_A := R \cdot \alpha \cdot (-\sin(\theta_A) + j \cdot \cos(\theta_A)) - R \cdot \omega_A^2 \cdot (\cos(\theta_A) + j \cdot \sin(\theta_A))$$

$$\mathbf{A}_A = (-43664 - 48260j) \frac{\text{in}}{\text{sec}^2}$$

- b. Write an expression for the particle's velocity vector in position B using complex number notation, in both polar and Cartesian forms.

Polar form:

$$\mathbf{R}_B := R \cdot \mathbf{e}^{j \cdot \theta_B} \qquad \mathbf{R}_B := 6.5 \cdot \mathbf{e}^{j \cdot \frac{\pi}{180} \cdot 100.863}$$

$$\mathbf{V}_B := R \cdot j \cdot \omega_B \cdot \mathbf{e}^{j \cdot \theta_B} \qquad \mathbf{V}_B := 650 \cdot j \cdot \mathbf{e}^{j \cdot \frac{\pi}{180} \cdot 100.863}$$

$$\mathbf{A}_B := R \cdot j \cdot \alpha \cdot e^{j \cdot \theta_B} - R \cdot j \cdot \omega_B^2 \cdot e^{j \cdot \theta_B}$$

$$\mathbf{A}_B := 6.50 \cdot j \cdot \alpha \cdot e^{j \cdot \frac{\pi}{180} \cdot 100.863} - 6.50 \cdot \omega_B^2 \cdot e^{j \cdot \frac{\pi}{180} \cdot 100.863}$$

Cartesian form: $\mathbf{A}_B := R \cdot \alpha \cdot (-\sin(\theta_B) + j \cdot \cos(\theta_B)) - R \cdot \omega_B^2 \cdot (\cos(\theta_B) + j \cdot \sin(\theta_B))$

$$\mathbf{A}_B = (14248 - 56999j) \frac{\text{in}}{\text{sec}^2}$$

- c. Write a vector equation for the velocity difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.

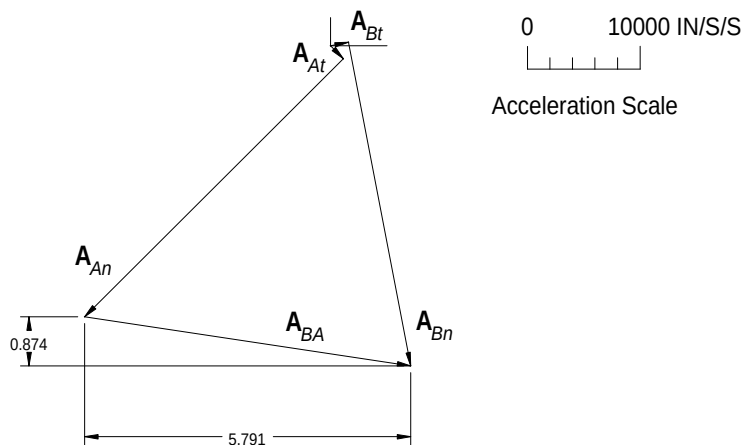
$$\mathbf{A}_{BA} := \mathbf{A}_B - \mathbf{A}_A \quad \mathbf{A}_{BA} = (57912 - 8739j) \frac{\text{in}}{\text{sec}^2}$$

- d. Check the result of part c with a graphical method. Solve the equation $\mathbf{A}_{Bt} + \mathbf{A}_{Bn} = \mathbf{A}_{At} + \mathbf{A}_{An} + \mathbf{A}_{BA}$ using an acceleration scale of 10000 in/sec² per drawing unit.

Acceleration scale factor $k_a := 10000 \cdot \frac{\text{in}}{\text{sec}^2}$

Horizontal component $A_{BAx} := 5.791 \cdot k_a \quad A_{BAx} = 57910 \frac{\text{in}}{\text{sec}^2}$

Vertical component $A_{BAy} := -0.874 \cdot k_a \quad A_{BAy} = -8740 \frac{\text{in}}{\text{sec}^2}$



On the layout above the X and Y components of $\mathbf{A}_{BA}$ are equal (to three significant figures) to the real and imaginary components calculated, confirming that the calculation is correct.

PROBLEM 7-2

Statement: In Problem 7-1 let A and B represent points on separate, rotating bodies both having the given ω and α at $t = 0$, $\theta_A = 45 \text{ deg}$, and $\theta_B = 120 \text{ deg}$. Find their relative acceleration.

Given:

Initial rotation speed	$\omega := 100 \cdot \frac{\text{rad}}{\text{sec}}$	
Rotation acceleration	$\alpha := -500 \cdot \frac{\text{rad}}{\text{sec}^2}$	
Vector angles	$\theta_A := 45 \cdot \text{deg}$	$\theta_B := 120 \cdot \text{deg}$
Vector magnitude	$R := 6.5 \cdot \text{in}$	

Solution: See Mathcad file P0702.

1. Calculate the acceleration of points A and B using equation 7.3.

$$\mathbf{A}_A := R \cdot \alpha \cdot (-\sin(\theta_A) + j \cdot \cos(\theta_A)) - R \cdot \omega^2 \cdot (\cos(\theta_A) + j \cdot \sin(\theta_A))$$

$$\mathbf{A}_A = (-43664 - 48260j) \frac{\text{in}}{\text{sec}^2}$$

$$\mathbf{A}_B := R \cdot \alpha \cdot (-\sin(\theta_B) + j \cdot \cos(\theta_B)) - R \cdot \omega^2 \cdot (\cos(\theta_B) + j \cdot \sin(\theta_B))$$

$$\mathbf{A}_B = (35315 - 54667j) \frac{\text{in}}{\text{sec}^2}$$

2. Calculate the relative acceleration of point B with respect to A using equation 7.4.

$$\mathbf{A}_{BA} := \mathbf{A}_B - \mathbf{A}_A \quad \mathbf{A}_{BA} = (78978 - 6407j) \frac{\text{in}}{\text{sec}^2}$$

$$\text{Magnitude: } A_{BA} := |\mathbf{A}_{BA}| \quad A_{BA} = 79238 \frac{\text{in}}{\text{sec}^2}$$

$$\text{Direction: } \theta_{ABA} := \arg(\mathbf{A}_{BA}) \quad \theta_{ABA} = -4.638 \text{ deg}$$

PROBLEM 7-3a

Statement: The link lengths, coupler point location, and the values of θ_2 , ω_2 , and α_2 for the same fourbar linkages as used for position and velocity analysis in Chapters 4 and 6 are redefined in Table P7-1, which is the same as Table P6-1. The general linkage configuration and terminology are shown in Figure P7-1. For row *a*, draw the linkage to scale and graphically find the accelerations of points A and B. Then, calculate α_3 and α_4 and the acceleration of point P.

Given: Link lengths: Link 1 $d := 6 \cdot \text{in}$ Link 2 $a := 2 \cdot \text{in}$
 Link 3 $b := 7 \cdot \text{in}$ Link 4 $c := 9 \cdot \text{in}$
 Coupler point: $R_{PA} := 6 \cdot \text{in}$ $\delta_3 := 30 \cdot \text{deg}$
 Link 2 position, velocity, and acceleration: $\theta_2 := 30 \cdot \text{deg}$ $\omega_2 := 10 \cdot \frac{\text{rad}}{\text{sec}}$ $\alpha_2 := 0 \cdot \frac{\text{rad}}{\text{sec}^2}$

Solution: See Figure P7-1 and Mathcad file P0703a.

1. In order to solve for the accelerations at points A and B, we will need θ_3 , ω_3 , and θ_4 . From the graphical position solution below,

$$\theta_3 := 88.837 \cdot \text{deg}$$

$$\theta_4 := 117.286 \cdot \text{deg}$$

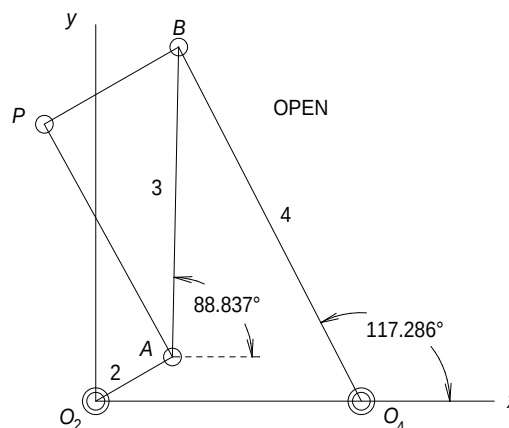
Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_4 - \theta_2)}{b \cdot \sin(\theta_3 - \theta_4)}$$

$$\omega_3 = -5.991 \text{ rad} \cdot \text{sec}^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_4 - \theta_3)}$$

$$\omega_4 = -3.992 \text{ rad} \cdot \text{sec}^{-1}$$



2. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

3. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_{Bn} := c \cdot \omega_4^2 \quad A_{Bn} = 143.403 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 + 180 \cdot \text{deg} \quad \theta_{ABn} = 297.286 \text{ deg}$$

$$A_{An} := a \cdot \omega_2^2 \quad A_{An} = 200.000 \text{ in} \cdot \text{sec}^{-2}$$

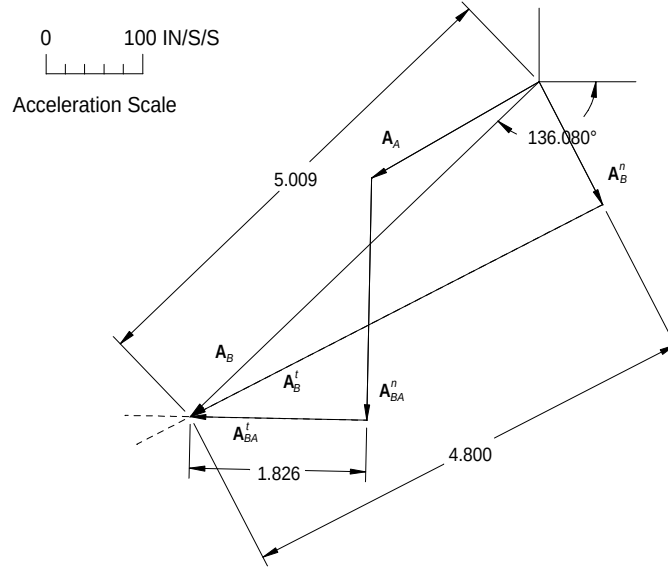
$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AAn} = 210.000 \text{ deg}$$

$$A_{At} := a \cdot \alpha_2 \quad A_{At} = 0.000 \text{ in} \cdot \text{sec}^{-2}$$

$$A_{BA n} := b \cdot \omega_3^2 \quad A_{BA n} = 251.239 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABAn} := \theta_3 + 180 \cdot \text{deg} \quad \theta_{ABAn} = 268.837 \text{ deg}$$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of $\theta_4 + 180$ deg and $\mathbf{A}_A^n$ at an angle of $\theta_2 + 180$ deg. From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_{BA}^n$ at an angle of $\theta_3 + 180$ deg. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B^n$ and $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B^t$, $\mathbf{A}_{BA}^t$, and $\mathbf{A}_B$.



5. From the graphical solution above,

$$\begin{aligned} \text{Acceleration scale factor} \quad k_a &:= 100 \cdot \frac{\text{in}}{\text{sec}^2} \\ A_{BA}^t &:= 1.826 \cdot k_a & A_{BA}^t &= 182.6 \text{ in} \cdot \text{sec}^{-2} \\ A_{Bt} &:= 4.800 \cdot k_a & A_{Bt} &= 480.0 \text{ in} \cdot \text{sec}^{-2} \\ A_B &:= 5.009 \cdot k_a & A_B &= 500.9 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -136.08 \text{ deg} \end{aligned}$$

6. Calculate α_3 and α_4 using equation 7.6.

$$\begin{aligned} \alpha_3 &:= \frac{A_{BA}^t}{b} & \alpha_3 &= 26.086 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CCW} \\ \alpha_4 &:= \frac{A_{Bt}}{c} & \alpha_4 &= 53.333 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CCW} \end{aligned}$$

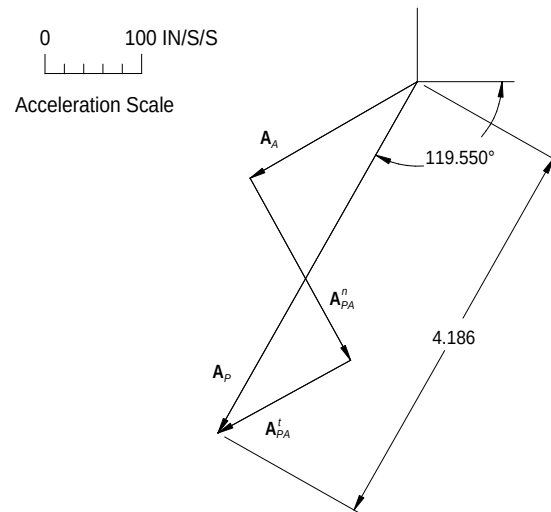
From the graphical solution we see that both of the tangential acceleration vectors for links 3 and 4 point to the left, so both of the angular accelerations are positive.

7. For point P , equation 7.4 becomes: $\mathbf{A}_P = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$, where

$$\begin{aligned} A_{PA}^n &:= R_{PA} \cdot \omega_3^2 & A_{PA}^n &= 215.348 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{PA}^n &:= \theta_3 + \delta_3 + 180 \text{ deg} & \theta_{PA}^n &= 298.837 \text{ deg} \\ A_{PA}^t &:= R_{PA} \cdot \alpha_3 & A_{PA}^t &= 156.514 \text{ in} \cdot \text{sec}^{-2} \end{aligned}$$

$$\theta_{APAt} := \theta_3 + \delta_3 + 90 \cdot deg \quad \theta_{APAt} = 208.837 \deg$$

8. Repeat procedure of step 4 for the equation in step 7.



9. From the graphical solution above,

Acceleration scale factor $k_a := 100 \cdot \frac{in}{sec^2}$

$$A_P := 4.186 \cdot k_a \quad A_P = 418.6 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -119.6 \text{ deg}$$

PROBLEM 7-4a

Statement: The link lengths, coupler point location, and the values of θ_2 , ω_2 , and α_2 for the same fourbar linkages as used for position and velocity analysis in Chapters 4 and 6 are redefined in Table P7-1, which is the same as Table P6-1. The general linkage configuration and terminology are shown in Figure P7-1. For row *a*, find the accelerations of points *A* and *B* using the analytic method. Then, calculate α_3 and α_4 and the acceleration of point *P*.

Given: Link lengths:

$$\text{Link 1} \quad d := 6$$

$$\text{Link 2} \quad a := 2$$

$$\text{Link 3} \quad b := 7$$

$$\text{Link 4} \quad c := 9$$

$$\text{Coupler point:} \quad R_{PA} := 6$$

$$\delta_3 := 30 \cdot \text{deg}$$

$$\text{Link 2 position, velocity, and acceleration:}$$

$$\theta_2 := 30 \cdot \text{deg}$$

$$\omega_2 := 10$$

$$\alpha_2 := 0$$

Solution: See Figure P7-1 and Mathcad file P0704a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 3.0000$$

$$K_2 = 0.6667$$

$$K_3 = 2.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$A = -0.7113$$

$$B := -2 \cdot \sin(\theta_2)$$

$$B = -1.0000$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$C = 3.5566$$

2. Use equation 4.10b to find values of θ_4 for the open and crossed circuits.

$$\text{Open:} \quad \theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \qquad \theta_{41} = -242.714 \text{ deg}$$

$$\theta_{41} := \theta_{41} - 360 \cdot \text{deg} \qquad \theta_{41} = -602.714 \text{ deg}$$

$$\text{Crossed:} \quad \theta_{42} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \qquad \theta_{42} = 216.340 \text{ deg}$$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \qquad K_4 = 0.8571 \qquad K_5 = -0.2857$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -1.6774$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = -1.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = 2.5906$$

4. Use equation 4.13 to find values of θ_3 for the open and crossed circuits.

$$\text{Open:} \quad \theta_{31} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \qquad \theta_{31} = -271.163 \text{ deg}$$

$$\theta_{31} := \theta_{31} - 360 \cdot \text{deg} \qquad \theta_{31} = -631.163 \text{ deg}$$

$$\text{Crossed: } \theta_{32} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{32} = 244.789 \text{ deg}$$

5. Determine the angular velocity of links 3 and 4 for the open and crossed circuits using equations 6.18.

$$\text{OPEN} \quad \omega_{31} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41} - \theta_2)}{\sin(\theta_{31} - \theta_{41})} \quad \omega_{31} = -5.991$$

$$\omega_{41} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31})}{\sin(\theta_{41} - \theta_{31})} \quad \omega_{41} = -3.992$$

$$\text{CROSSED} \quad \omega_{32} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{42} - \theta_2)}{\sin(\theta_{32} - \theta_{42})} \quad \omega_{32} = -0.662$$

$$\omega_{42} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{32})}{\sin(\theta_{42} - \theta_{32})} \quad \omega_{42} = -2.662$$

6. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Separate into real and imaginary parts to give the x and y components

$$\mathbf{A}_A = a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{Ax} := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) \quad A_{Ax} = -173.205$$

$$A_{Ay} := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) \quad A_{Ay} = -100.000$$

$$A_A := \sqrt{A_{Ax}^2 + A_{Ay}^2} \quad \theta_{AA} := \text{atan2}(A_{Ax}, A_{Ay})$$

$$\text{The acceleration of pin A is } A_A = 200.000 \quad \text{at} \quad \theta_{AA} = -150 \text{ deg}$$

7. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the open and crossed circuits.

$$\text{OPEN} \quad A := c \cdot \sin(\theta_{41}) \quad B := b \cdot \sin(\theta_{31}) \quad D := c \cdot \cos(\theta_{41}) \quad E := b \cdot \cos(\theta_{31})$$

$$A = 7.999 \quad B = 6.999 \quad D = -4.126 \quad E = 0.142$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_{31}^2 \cdot \cos(\theta_{31}) - c \cdot \omega_{41}^2 \cdot \cos(\theta_{41})$$

$$C = 244.045$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_{31}^2 \cdot \sin(\theta_{31}) + c \cdot \omega_{41}^2 \cdot \sin(\theta_{41})$$

$$F = -223.741$$

$$\alpha_{31} := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_{31} = 26.080 \quad \alpha_{41} := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_{41} = 53.331$$

$$\text{CROSS} \quad A := c \cdot \sin(\theta_{42}) \quad B := b \cdot \sin(\theta_{32}) \quad D := c \cdot \cos(\theta_{42}) \quad E := b \cdot \cos(\theta_{32})$$

$$A = -5.333 \quad B = -6.333 \quad D = -7.250 \quad E = -2.982$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_{32}^2 \cdot \cos(\theta_{32}) - c \cdot \omega_{42}^2 \cdot \cos(\theta_{42})$$

$$C = 223.253$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_{32}^2 \cdot \sin(\theta_{32}) + c \cdot \omega_{42}^2 \cdot \sin(\theta_{42})$$

$$F = -135.002$$

$$\alpha_{32} := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_{32} = 77.920 \quad \alpha_{42} := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_{42} = 50.669$$

8. Use equation 7.13c to determine the acceleration of point B for the open and crossed circuits.

$$\text{OPEN } A_{Bx1} := c \cdot \left(-\alpha_{41} \cdot \sin(\theta_{41}) - \omega_{41}^2 \cdot \cos(\theta_{41}) \right) \quad A_{Bx1} = -360.826$$

$$A_{By1} := c \cdot \left(\alpha_{41} \cdot \cos(\theta_{41}) - \omega_{41}^2 \cdot \sin(\theta_{41}) \right) \quad A_{By1} = -347.485$$

$$A_{B1} := \sqrt{A_{Bx1}^2 + A_{By1}^2} \quad A_{B1} = 500.941$$

$$\theta_{AB1} := \text{atan2}(A_{Bx1}, A_{By1}) \quad \theta_{AB1} = -136.1 \text{ deg}$$

$$\text{CROSS } A_{Bx2} := c \cdot \left(-\alpha_{42} \cdot \sin(\theta_{42}) - \omega_{42}^2 \cdot \cos(\theta_{42}) \right) \quad A_{Bx2} = 321.587$$

$$A_{By2} := c \cdot \left(\alpha_{42} \cdot \cos(\theta_{42}) - \omega_{42}^2 \cdot \sin(\theta_{42}) \right) \quad A_{By2} = -329.551$$

$$A_{B2} := \sqrt{A_{Bx2}^2 + A_{By2}^2} \quad A_{B2} = 460.459$$

$$\theta_{AB2} := \text{atan2}(A_{Bx2}, A_{By2}) \quad \theta_{AB2} = -45.7 \text{ deg}$$

9. Use equations 7.32 to find the acceleration of the point P for the open and crossed circuits.

$$\text{OPEN } \mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_{PA1} := R_{PA} \cdot \alpha_{31} \cdot (-\sin(\theta_{31} + \delta_3) + j \cdot \cos(\theta_{31} + \delta_3)) \dots \\ + R_{PA} \cdot \omega_{31}^2 \cdot (\cos(\theta_{31} + \delta_3) + j \cdot \sin(\theta_{31} + \delta_3))$$

$$\mathbf{A}_{P1} := \mathbf{A}_A + \mathbf{A}_{PA1}$$

$$A_{P1} := |\mathbf{A}_{P1}| \quad A_{P1} = 418.556 \quad \arg(\mathbf{A}_{P1}) = -119.548 \text{ deg}$$

$$\text{CROSS } \mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_{PA2} := R_{PA} \cdot \alpha_{32} \cdot (-\sin(\theta_{32} + \delta_3) + j \cdot \cos(\theta_{32} + \delta_3)) \dots \\ + R_{PA} \cdot \omega_{32}^2 \cdot (\cos(\theta_{32} + \delta_3) + j \cdot \sin(\theta_{32} + \delta_3))$$

$$\mathbf{A}_{P2} := \mathbf{A}_A + \mathbf{A}_{PA2}$$

$$A_{P2} := |\mathbf{A}_{P2}| \quad A_{P2} = 298.225 \quad \arg(\mathbf{A}_{P2}) = -11.282 \text{ deg}$$

PROBLEM 7-5a

Statement: The link lengths and the values of θ_2 , ω_2 , and α_2 for the some non inverted offset fourbar slider-crank linkages are defined in Table P7-2. The general linkage configuration and termin-olc are shown in Figure P7-2. For row *a*, draw the linkage to scale and graphically find the accelerations of the pin joints A and B the acceleration of slip at the sliding joint.

Given:

Link lengths: Link 2 $a := 1.4 \cdot \text{in}$ Link 3 $b := 4 \cdot \text{in}$

Offset: $c := 1 \cdot \text{in}$

Link 2 position, velocity, and acceleration: $\theta_2 := 45 \cdot \text{deg}$ $\omega_2 := 10 \cdot \frac{\text{rad}}{\text{sec}}$ $\alpha_2 := 0 \cdot \frac{\text{rad}}{\text{sec}^2}$

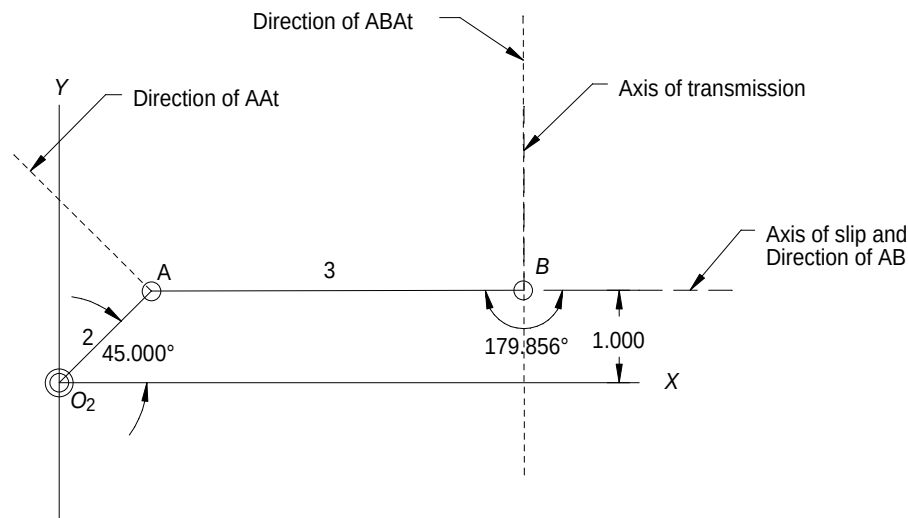
Solution: See Figure P7-2 and Mathcad file P0705a.

1. In order to solve for the accelerations at point B, we will need θ_3 , and ω_3 . From the graphical position solution below,

$$\theta_3 := 360 \cdot \text{deg} - 179.856 \cdot \text{deg}$$

$$\theta_3 = 180.144 \cdot \text{deg}$$

Using equation 6.22a,
$$\omega_3 := \frac{a \cdot \omega_2 \cdot \cos(\theta_2)}{b \cdot \cos(\theta_3)} \quad \omega_3 = -2.475 \cdot \text{rad} \cdot \text{sec}^{-1}$$



2. The graphical solution for accelerations uses equation 7.4:

$$(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$$

3. For point B, this becomes:

$$\mathbf{A}_B = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n), \text{ where}$$

$$A_{An} := a \cdot \omega_2^2 \quad A_{An} = 140.000 \cdot \text{in} \cdot \text{sec}^{-2}$$

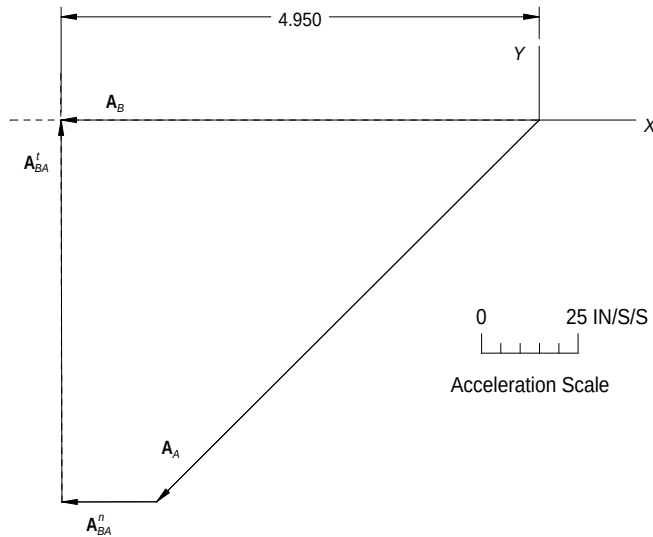
$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AAn} = 225.000 \cdot \text{deg}$$

$$A_{At} := a \cdot \alpha_2 \quad A_{At} = 0.000 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$A_{BAAn} := b \cdot \omega_3^2 \quad A_{BAAn} = 24.500 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{ABAn} := \theta_3 \quad \theta_{ABAn} = 180.144 \cdot \text{deg}$$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_A^n$ at an angle of θ_{AA^n} . From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_{BA}^n$ at an angle of θ_{ABA^n} . Now that the vectors with known magnitudes are drawn, from the origin and the tip of $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B$ (horizontal) and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_{BA}^t$ and $\mathbf{A}_B$.



5. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 25 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_B := 4.950 \cdot k_a \quad A_B = 123.8 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 180 \text{ deg}$$

6. The acceleration of slip is equal to $\mathbf{A}_B$ since link 1 is stationary.

PROBLEM 7-6a

Statement: The link lengths and the values of θ_2 , ω_2 , and α_2 for the some non inverted offset fourbar slider-crank linkages are defined in Table P7-2. The general linkage configuration and terminology are shown in Figure P7-2. For row *a*, draw the linkage to scale and find the accelerations of the pin joints A and B the acceleration of slip at the sliding joint using an analytical method.

Given: Link lengths:

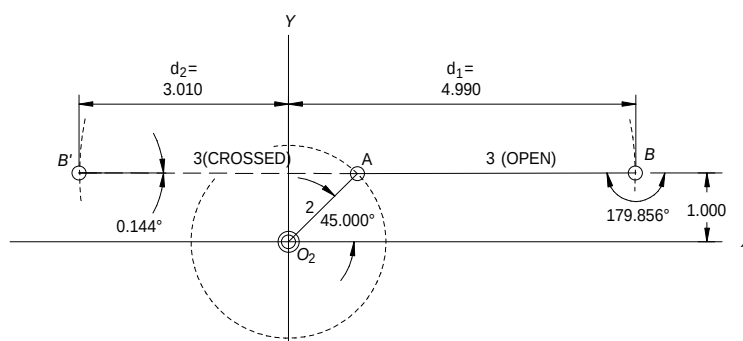
$$\text{Link 2 } a := 1.4 \cdot \text{in} \quad \text{Link 3 } b := 4 \cdot \text{in}$$

$$\text{Offset: } c := 1 \cdot \text{in}$$

$$\text{Link 2 position, velocity, and acceleration: } \theta_2 := 45 \cdot \text{deg} \quad \omega_2 := 10 \cdot \frac{\text{rad}}{\text{sec}} \quad \alpha_2 := 0 \cdot \frac{\text{rad}}{\text{sec}^2}$$

Solution: See Figure P7-2 and Mathcad file P0706a.

1. Draw the linkage to scale and label it.



2. Determine θ_3 and d using equations 4.16 and 4.17.

Open:

$$\theta_{32} := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi \quad \theta_{32} = 180.144 \text{ deg}$$

$$d_2 := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_{32}) \quad d_2 = 4.990 \text{ in}$$

Crossed:

$$\theta_{31} := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right) \quad \theta_{31} = -0.144 \text{ deg}$$

$$d_1 := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_{31}) \quad d_1 = -3.010 \text{ in}$$

3. Determine the angular velocity of link 3 using equation 6.22a.

$$\text{Open} \quad \omega_{32} := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_{32})} \cdot \omega_2 \quad \omega_{32} = -2.475 \frac{\text{rad}}{\text{sec}}$$

$$\text{Crossed} \quad \omega_{31} := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_{31})} \cdot \omega_2 \quad \omega_{31} = 2.475 \frac{\text{rad}}{\text{sec}}$$

4. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its magnitude, and direction.

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (-98.995 - 98.995j) \frac{\text{in}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A)$$

The acceleration of pin A is $A_A = 140.0 \frac{\text{in}}{\text{sec}^2}$ at $\theta_{AA} = -135.0 \text{ deg}$

5. Determine the angular acceleration of link 3 using equation 7.16d.

$$\text{Open} \quad \alpha_{32} := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_{32}^2 \cdot \sin(\theta_{32})}{b \cdot \cos(\theta_{32})}$$

$$\alpha_{32} = 24.764 \frac{\text{rad}}{\text{sec}^2}$$

$$\text{Crossed} \quad \alpha_{31} := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_{31}^2 \cdot \sin(\theta_{31})}{b \cdot \cos(\theta_{31})}$$

$$\alpha_{31} = -24.764 \frac{\text{rad}}{\text{sec}^2}$$

6. Use equation 7.16e for the acceleration of pin B.

Open:

$$A_{B2} := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \alpha_{32} \cdot \sin(\theta_{32}) + b \cdot \omega_{32}^2 \cdot \cos(\theta_{32})$$

$$A_{B2} = -123.7 \frac{\text{in}}{\text{sec}^2} \quad \text{A negative sign means that } \mathbf{A}_B \text{ is to the left}$$

Crossed:

$$A_{B1} := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \alpha_{31} \cdot \sin(\theta_{31}) + b \cdot \omega_{31}^2 \cdot \cos(\theta_{31})$$

$$A_{B1} = -74.2 \frac{\text{in}}{\text{sec}^2} \quad \text{A negative sign means that } \mathbf{A}_B \text{ is to the left}$$

PROBLEM 7-7a

Statement: The link lengths and the values of θ_2 , ω_2 , and γ for an inverted fourbar slider-crank linkage are defined in Table P7-3, row *a*, and are given below. Find the acceleration of the pin joints A and B and the acceleration of slip at the sliding joint. Solve by the analytic vector loop method of Section 7.3 for the open configuration of the linkage.

Given: Link lengths:

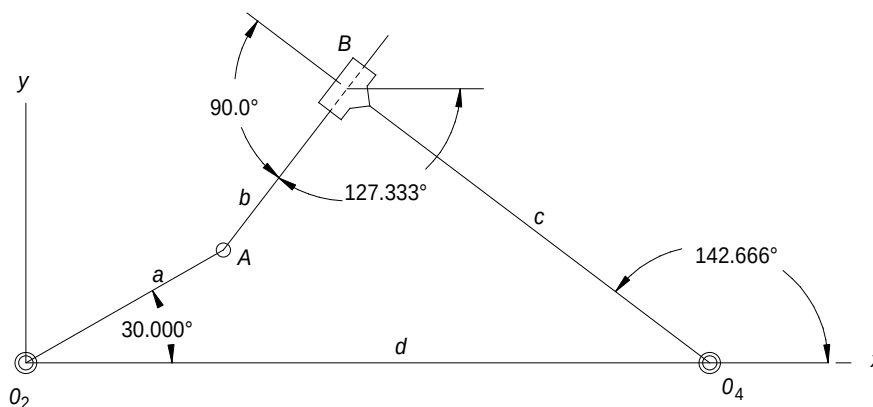
$$\text{Link 2} \quad a := 2 \cdot \text{in} \quad \text{Link 4} \quad c := 4 \cdot \text{in} \quad \text{Link 1} \quad d := 6 \cdot \text{in}$$

$$\text{Angle between links 3 and 4} \quad \gamma := 90 \cdot \text{deg}$$

$$\text{Link 2 position, velocity, and accel.} \quad \theta_2 := 30 \cdot \text{deg} \quad \omega_2 := 10 \cdot \frac{\text{rad}}{\text{sec}} \quad \alpha_2 := -25 \cdot \frac{\text{rad}}{\text{sec}^2}$$

Solution: See Figure P7-3 and Mathcad file P0707a.

1. Draw the linkage to scale and label it.



2. Use the equations in Section 4.7 to solve for the positions of links 3 and 4 and for the length *b*.

$$P := a \cdot \sin(\theta_2) \cdot \sin(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \cos(\gamma) \quad P = 1.000 \cdot \text{in}$$

$$Q := -a \cdot \sin(\theta_2) \cdot \cos(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \sin(\gamma) \quad Q = -4.268 \cdot \text{in}$$

$$R := -c \cdot \sin(\gamma) \quad S := R - Q \quad T := 2 \cdot P \quad U := Q + R$$

$$R = -4.000 \cdot \text{in} \quad S = 0.268 \cdot \text{in} \quad T = 2.000 \cdot \text{in} \quad U = -8.268 \cdot \text{in}$$

$$\theta_4 := 2 \cdot \text{atan2} \left[(2 \cdot S), (-T + \sqrt{T^2 - 4 \cdot S \cdot U}) \right] \quad \theta_4 = 142.667 \cdot \text{deg}$$

$$\theta_3 := \theta_4 + \gamma \quad \theta_3 = 232.667 \cdot \text{deg}$$

$$b := \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_4)}{\sin(\theta_3)} \quad b = 1.793 \cdot \text{in}$$

3. Calculate the angular velocity of links 3 and 4 and the slip velocity using equations 6.30.

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \cos(\theta_2 - \theta_3)}{b + c \cdot \cos(\gamma)}$$

$$\omega_4 = -10.292 \cdot \frac{\text{rad}}{\text{sec}}$$

$$b\dot{\omega} := \frac{-a \cdot \omega_2 \cdot \sin(\theta_2) + \omega_4 \cdot (b \cdot \sin(\theta_3) + c \cdot \sin(\theta_4))}{\cos(\theta_3)}$$

$$b\dot{\omega} = 33.461 \cdot \frac{\text{in}}{\text{sec}}$$

$$\omega_3 := \omega_4$$

4. Solve for the accelerations using equations (7.26) and (7.27).

$$P := a \cdot \alpha_2 \cdot \cos(\theta_3 - \theta_2)$$

$$P = 46.138 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$Q := a \cdot \omega_2^2 \cdot \sin(\theta_3 - \theta_2)$$

$$Q = -77.075 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$R := c \cdot \omega_4^2 \cdot \sin(\theta_4 - \theta_3)$$

$$R = -423.705 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$S := 2 \cdot b\dot{\omega} \cdot \omega_3$$

$$S = -688.757 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$T := b + c \cdot \cos(\theta_3 - \theta_4)$$

$$T = 1.793 \cdot \text{in}$$

$$\alpha_4 := \frac{P + Q + R - S}{T}$$

$$\alpha_4 = 130.56 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$K := a \cdot \omega_2^2 \cdot (b \cdot \cos(\theta_3 - \theta_2) + c \cdot \cos(\theta_4 - \theta_2))$$

$$K = -639.230 \cdot \text{in}^2 \cdot \text{sec}^{-2}$$

$$L := a \cdot \alpha_2 \cdot (b \cdot \sin(\theta_2 - \theta_3) - c \cdot \sin(\theta_4 - \theta_2))$$

$$L = 150.000 \cdot \text{in}^2 \cdot \text{sec}^{-2}$$

$$M := -\omega_4^2 \cdot (b^2 + c^2 + 2 \cdot b \cdot c \cdot \cos(\theta_4 - \theta_3))$$

$$M = -2.035 \times 10^3 \cdot \text{in}^2 \cdot \text{sec}^{-2}$$

$$N := 2 \cdot b\dot{\omega} \cdot c \cdot \omega_4 \cdot \sin(\theta_4 - \theta_3)$$

$$N = 2.755 \times 10^3 \cdot \text{in}^2 \cdot \text{sec}^{-2}$$

$$b\ddot{\omega} := -\frac{K + L + M + N}{T}$$

$$b\ddot{\omega} = -128.48 \cdot \frac{\text{in}}{\text{sec}^2}$$

The acceleration of slip is $b\ddot{\omega}$. It is directed along link 3, positive inward from B towards A, so that its angle is θ_3 . For the acceleration of the pin joints A and B,

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_A := |\mathbf{A}_A| \quad A_A = 206.155 \cdot \frac{\text{in}}{\text{sec}^2} \quad \theta_{AA} := \arg(\mathbf{A}_A) \quad \theta_{AA} = -135.964 \cdot \text{deg}$$

$$\mathbf{A}_B := -c \cdot \alpha_4 \cdot (\sin(\theta_4) - j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$A_B := |\mathbf{A}_B| \quad A_B = 672.505 \cdot \frac{\text{in}}{\text{sec}^2} \quad \theta_{AB} := \arg(\mathbf{A}_B) \quad \theta_{AB} = -88.280 \cdot \text{deg}$$

PROBLEM 7-8a

Statement: The link lengths and the values of θ_2 , ω_2 , and γ for an inverted fourbar slider-crank linkage are defined in Table P7-3, row *a*, and are given below. Find the acceleration of the pin joints A and B and the acceleration of slip at the sliding joint. Solve by the analytic vector loop method of Section 7.3 for the crossed configuration of the linkage.

Given: Link lengths:

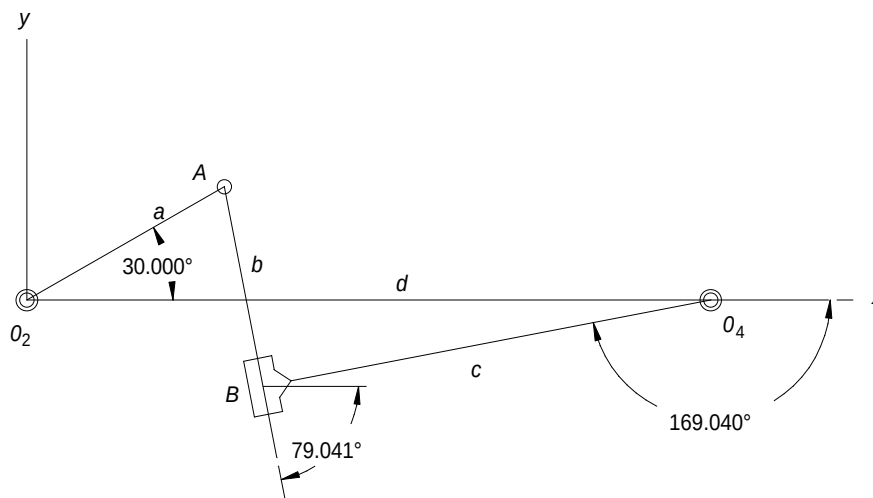
$$\text{Link 2} \quad a := 2 \cdot \text{in} \quad \text{Link 4} \quad c := 4 \cdot \text{in} \quad \text{Link 1} \quad d := 6 \cdot \text{in}$$

$$\text{Angle between links 3 and 4} \quad \gamma := 90 \cdot \text{deg}$$

$$\text{Link 2 position, velocity, and accel.} \quad \theta_2 := 30 \cdot \text{deg} \quad \omega_2 := 10 \cdot \frac{\text{rad}}{\text{sec}} \quad \alpha_2 := -25 \cdot \frac{\text{rad}}{\text{sec}^2}$$

Solution: See Figure P7-3 and Mathcad file P0708a.

1. Draw the linkage to scale and label it.



2. Use the equations in Section 4.7 to solve for the positions of links 3 and 4 and for the length *b*.

$$P := a \cdot \sin(\theta_2) \cdot \sin(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \cos(\gamma) \quad P = 1.000 \text{ in}$$

$$Q := -a \cdot \sin(\theta_2) \cdot \cos(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \sin(\gamma) \quad Q = -4.268 \text{ in}$$

$$R := -c \cdot \sin(\gamma) \quad S := R - Q \quad T := 2 \cdot P \quad U := Q + R$$

$$R = -4.000 \text{ in} \quad S = 0.268 \text{ in} \quad T = 2.000 \text{ in} \quad U = -8.268 \text{ in}$$

$$\theta_4 := 2 \cdot \text{atan2}[(2 \cdot S), -T - \sqrt{T^2 - 4 \cdot S \cdot U}] \quad \theta_4 = -169.041 \text{ deg}$$

$$\theta_3 := \theta_4 + \gamma \quad \theta_3 = -79.041 \text{ deg}$$

$$b := \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_4)}{\sin(\theta_3)} \quad b = -1.793 \text{ in}$$

3. Calculate the angular velocity of links 3 and 4 and the slip velocity using equations 6.30.

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \cos(\theta_2 - \theta_3)}{b + c \cdot \cos(\gamma)}$$

$$\omega_4 = 3.639 \frac{\text{rad}}{\text{sec}}$$

$$b\dot{\omega} := \frac{-a \cdot \omega_2 \cdot \sin(\theta_2) + \omega_4 \cdot (b \cdot \sin(\theta_3) + c \cdot \sin(\theta_4))}{\cos(\theta_3)}$$

$$b\dot{\omega} = -33.461 \frac{\text{in}}{\text{sec}}$$

$$\omega_3 := \omega_4$$

4. Solve for the accelerations using equations (7.26) and (7.27).

$$P := a \cdot \alpha_2 \cdot \cos(\theta_3 - \theta_2)$$

$$P = 16.312 \text{ in} \cdot \text{sec}^{-2}$$

$$Q := a \cdot \omega_2^2 \cdot \sin(\theta_3 - \theta_2)$$

$$Q = -189.057 \text{ in} \cdot \text{sec}^{-2}$$

$$R := c \cdot \omega_4^2 \cdot \sin(\theta_4 - \theta_3)$$

$$R = -52.961 \text{ in} \cdot \text{sec}^{-2}$$

$$S := 2 \cdot b\dot{\omega} \cdot \omega_3$$

$$S = -243.508 \text{ in} \cdot \text{sec}^{-2}$$

$$T := b + c \cdot \cos(\theta_3 - \theta_4)$$

$$T = -1.793 \text{ in}$$

$$\alpha_4 := \frac{P + Q + R - S}{T}$$

$$\alpha_4 = -9.93 \text{ rad} \cdot \text{sec}^{-2}$$

$$K := a \cdot \omega_2^2 \cdot (b \cdot \cos(\theta_3 - \theta_2) + c \cdot \cos(\theta_4 - \theta_2))$$

$$K = -639.230 \text{ in}^2 \cdot \text{sec}^{-2}$$

$$L := a \cdot \alpha_2 \cdot (b \cdot \sin(\theta_2 - \theta_3) - c \cdot \sin(\theta_4 + \theta_2))$$

$$L = -46.352 \text{ in}^2 \cdot \text{sec}^{-2}$$

$$M := -\omega_4^2 \cdot (b^2 + c^2 + 2 \cdot b \cdot c \cdot \cos(\theta_4 - \theta_3))$$

$$M = -254.418 \text{ in}^2 \cdot \text{sec}^{-2}$$

$$N := 2 \cdot b\dot{\omega} \cdot c \cdot \omega_4 \cdot \sin(\theta_4 - \theta_3)$$

$$N = 974.033 \text{ in}^2 \cdot \text{sec}^{-2}$$

$$b\ddot{\omega} := -\frac{K + L + M + N}{T}$$

$$b\ddot{\omega} = 18.98 \frac{\text{in}}{\text{sec}^2}$$

The acceleration of slip is $b\ddot{\omega}$. It is directed along link 3, positive inward from B towards A, so that its angle is θ_3 . For the acceleration of the pin joints A and B,

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_A := |\mathbf{A}_A| \quad A_A = 206.155 \frac{\text{in}}{\text{sec}^2} \quad \theta_{AA} := \arg(\mathbf{A}_A) \quad \theta_{AA} = -135.964 \text{ deg}$$

$$\mathbf{A}_B := -c \cdot \alpha_4 \cdot (\sin(\theta_4) - j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$A_B := |\mathbf{A}_B| \quad A_B = 66.195 \frac{\text{in}}{\text{sec}^2} \quad \theta_{AB} := \arg(\mathbf{A}_B) \quad \theta_{AB} = 47.822 \text{ deg}$$

PROBLEM 7-9a

Statement: The link lengths, gear ratio (λ), phase angle (ϕ), and the values of θ_2 , ω_2 , and α_2 for a geared five-bar from row *a* of Table P7-4 are given below. Find α_3 and α_4 and the linear acceleration of point *P*.

Given:

Link lengths:

$$\text{Link 1} \quad f := 6 \cdot \text{in}$$

$$\text{Link 3} \quad b := 7 \cdot \text{in}$$

$$\text{Link 5} \quad d := 4 \cdot \text{in}$$

$$\text{Link 2} \quad a := 1 \cdot \text{in}$$

$$\text{Link 4} \quad c := 9 \cdot \text{in}$$

Gear ratio, phase angle, and crank angle:

$$\lambda := 2 \quad \phi := 30 \cdot \text{deg} \quad \theta_2 := 60 \cdot \text{deg} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\text{Coupler data:} \quad R_{pa} := 6 \cdot \text{in} \quad \delta_3 := 30 \cdot \text{deg}$$

Solution: See Figure P7-4 and Mathcad file P0709a.

1. Determine the angle of link 5 using the equation in Figure P6-4.

$$\theta_5 := \lambda \cdot \theta_2 + \phi \quad \theta_5 = 150 \cdot \text{deg}$$

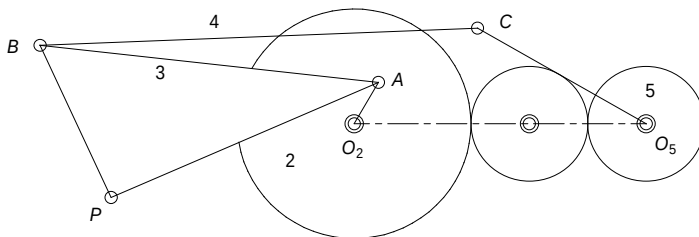
2. Choose the pitch radii of the gears. Since the gear ratio is positive, an idler must be used between gear 2 and gear 5. Let the idler be the same diameter as gear 5, and let all three gears be in line.

$$f := r_2 + 3 \cdot r_5 \quad \lambda := \frac{r_2}{r_5}$$

$$r_5 := \frac{f}{\lambda + 3} \quad r_5 = 1.200 \text{ in}$$

$$r_2 := \lambda \cdot r_5 \quad r_2 = 2.400 \text{ in}$$

3. Draw the linkage to scale and label it.



4. Determine the values of the constants needed for finding θ_3 and θ_4 from equations 4.24h and 4.24i.

$$A := 2 \cdot c \cdot (d \cdot \cos(\lambda \cdot \theta_2 + \phi) - a \cdot \cos(\theta_2) + f) \quad A = 36.6462 \text{ in}^2$$

$$B := 2 \cdot c \cdot (d \cdot \sin(\lambda \cdot \theta_2 + \phi) - a \cdot \sin(\theta_2)) \quad B = 20.412 \text{ in}^2$$

$$C := (a^2 - b^2 + c^2 + d^2 + f^2) - 2 \cdot a \cdot f \cdot \cos(\theta_2) \dots$$

$$+ [-2 \cdot d \cdot (a \cdot \cos(\theta_2) - f) \cdot \cos(\lambda \cdot \theta_2 + \phi)] \dots$$

$$+ -2 \cdot a \cdot d \cdot \sin(\theta_2) \cdot \sin(\lambda \cdot \theta_2 + \phi) \quad C = 37.4308 \text{ in}^2$$

$$D := C - A \quad D = 0.78461 \text{ in}^2$$

$$E := 2 \cdot B \quad E = 40.823 \text{ in}^2$$

$$\begin{aligned}
 F &:= A + C & F &= 74.077 \text{ in}^2 \\
 G &:= 2 \cdot b \cdot \left[-\left(d \cdot \cos(\lambda \cdot \theta_2 + \phi) \right) + a \cdot \cos(\theta_2) - f \right] & G &= -28.503 \text{ in}^2 \\
 H &:= 2 \cdot b \cdot \left[-\left(d \cdot \sin(\lambda \cdot \theta_2 + \phi) \right) + a \cdot \sin(\theta_2) \right] & H &= -15.876 \text{ in}^2 \\
 K &:= \left(a^2 + b^2 - c^2 + d^2 + f^2 \right) - 2 \cdot a \cdot f \cdot \cos(\theta_2) \dots \\
 &\quad + \left[2 \cdot d \cdot \left(a \cdot \cos(\theta_2) - f \right) \cdot \cos(\lambda \cdot \theta_2 + \phi) \right] \dots \\
 &\quad + -2 \cdot a \cdot d \cdot \sin(\theta_2) \cdot \sin(\lambda \cdot \theta_2 + \phi) & K &= -26.569 \text{ in}^2 \\
 L &:= K - G & L &= 1.933 \text{ in}^2 \\
 M &:= 2 \cdot H & M &= -31.751 \text{ in}^2 \\
 N &:= G + K & N &= -55.072 \text{ in}^2
 \end{aligned}$$

5. Use equations 4.24h and 4.24i to find values of θ_3 and θ_4 for the open and crossed circuits.

$$\begin{aligned}
 \text{OPEN} \quad \theta_{31} &:= 2 \cdot \left(\text{atan2} \left(2 \cdot L, -M + \sqrt{M^2 - 4 \cdot L \cdot N} \right) \right) & \theta_{31} &= 173.642 \text{ deg} \\
 \theta_{41} &:= 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) & \theta_{41} &= -177.715 \text{ deg} \\
 \text{CROSSED} \quad \theta_{32} &:= 2 \cdot \left(\text{atan2} \left(2 \cdot L, -M - \sqrt{M^2 - 4 \cdot L \cdot N} \right) \right) & \theta_{32} &= -115.407 \text{ deg} \\
 \theta_{42} &:= 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) & \theta_{42} &= -124.050 \text{ deg}
 \end{aligned}$$

6. Use equation 6.32c to find ω_5 . $\omega_5 := \lambda \cdot \omega_2$

7. Calculate ω_3 and ω_4 using equations 6.33.

$$\begin{aligned}
 \text{OPEN} \quad \omega_{31} &:= \frac{-\left[2 \cdot \sin(\theta_{41}) \cdot \left(a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_{41}) + d \cdot \omega_5 \cdot \sin(\theta_{41} - \theta_5) \right) \right]}{b \cdot \left(\cos(\theta_{31} - 2 \cdot \theta_{41}) - \cos(\theta_{31}) \right)} \\
 \omega_{31} &= 32.585 \frac{\text{rad}}{\text{sec}} \\
 \omega_{41} &:= \frac{a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{31} \cdot \sin(\theta_{31}) - d \cdot \omega_5 \cdot \sin(\theta_5)}{c \cdot \sin(\theta_{41})} \\
 \omega_{41} &= 16.948 \frac{\text{rad}}{\text{sec}} \\
 \text{CROSSED} \quad \omega_{32} &:= \frac{-\left[2 \cdot \sin(\theta_{42}) \cdot \left(a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_{42}) + d \cdot \omega_5 \cdot \sin(\theta_{42} - \theta_5) \right) \right]}{b \cdot \left(\cos(\theta_{32} - 2 \cdot \theta_{42}) - \cos(\theta_{32}) \right)} \\
 \omega_{32} &= -75.191 \frac{\text{rad}}{\text{sec}}
 \end{aligned}$$

$$\omega_{42} := \frac{a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{32} \cdot \sin(\theta_{32}) - d \cdot \omega_5 \cdot \sin(\theta_5)}{c \cdot \sin(\theta_{42})}$$

$$\omega_{42} = -59.554 \frac{\text{rad}}{\text{sec}}$$

8. Use equation 7.28c to find α_5 . $\alpha_5 := \lambda \cdot \alpha_2$

9. Calculate α_3 and α_4 using equations 7.29.

$$\begin{aligned} & -a \cdot \alpha_2 \cdot \sin(\theta_2 - \theta_{41}) - a \cdot \omega_2^2 \cdot \cos(\theta_2 - \theta_{41}) \dots \\ & + -b \cdot \omega_{31}^2 \cdot \cos(\theta_{31} - \theta_{41}) + d \cdot \omega_5^2 \cdot \cos(\theta_5 - \theta_{41}) \dots \\ & + d \cdot \alpha_5 \cdot \sin(\theta_5 - \theta_{41}) + c \cdot \omega_{41}^2 \end{aligned}$$

OPEN $\alpha_{31} := \frac{\dots}{b \cdot \sin(\theta_{31} - \theta_{41})}$

$$\alpha_{31} = 3191 \frac{\text{rad}}{\text{sec}^2}$$

$$\begin{aligned} & a \cdot \alpha_2 \cdot \sin(\theta_2 - \theta_{31}) + a \cdot \omega_2^2 \cdot \cos(\theta_2 - \theta_{31}) \dots \\ & + -c \cdot \omega_{41}^2 \cdot \cos(\theta_{31} - \theta_{41}) - d \cdot \omega_5^2 \cdot \cos(\theta_{31} - \theta_5) \dots \\ & + d \cdot \alpha_5 \cdot \sin(\theta_{31} - \theta_5) + b \cdot \omega_{31}^2 \end{aligned}$$

$$\alpha_{41} := \frac{\dots}{c \cdot \sin(\theta_{41} - \theta_{31})}$$

$$\alpha_{41} = 2492 \frac{\text{rad}}{\text{sec}^2}$$

$$\begin{aligned} & -a \cdot \alpha_2 \cdot \sin(\theta_2 - \theta_{42}) - a \cdot \omega_2^2 \cdot \cos(\theta_2 - \theta_{42}) \dots \\ & + -b \cdot \omega_{32}^2 \cdot \cos(\theta_{32} - \theta_{42}) + d \cdot \omega_5^2 \cdot \cos(\theta_5 - \theta_{42}) \dots \\ & + d \cdot \alpha_5 \cdot \sin(\theta_5 - \theta_{42}) + c \cdot \omega_{42}^2 \end{aligned}$$

CROSSED $\alpha_{32} := \frac{\dots}{b \cdot \sin(\theta_{32} - \theta_{42})}$

$$\alpha_{32} = -6648 \frac{\text{rad}}{\text{sec}^2}$$

$$\begin{aligned} & a \cdot \alpha_2 \cdot \sin(\theta_2 - \theta_{32}) + a \cdot \omega_2^2 \cdot \cos(\theta_2 - \theta_{32}) \dots \\ & + -c \cdot \omega_{42}^2 \cdot \cos(\theta_{32} - \theta_{42}) - d \cdot \omega_5^2 \cdot \cos(\theta_{32} - \theta_5) \dots \\ & + d \cdot \alpha_5 \cdot \sin(\theta_{32} - \theta_5) + b \cdot \omega_{32}^2 \end{aligned}$$

$$\alpha_{42} := \frac{\dots}{c \cdot \sin(\theta_{42} - \theta_{32})}$$

$$\alpha_{42} = -5950 \frac{\text{rad}}{\text{sec}^2}$$

PROBLEM 7-10

Statement: An automobile driver took a curve too fast. The car spun out of control about its CG and slid off the road in a northeasterly direction. The friction of the skidding tires provided a $0.25g$ linear deceleration. The car rotated at 100 rpm. When the car hit the tree head-on at 30 mph, it took 0.1 sec to come to rest.

Given: $V := 30 \cdot \text{mph}$ $A_c := -0.25 \cdot g$ $\omega := 100 \cdot \text{rpm}$

Solution: See Mathcad file P0710.

- a. What was the acceleration experienced by the child seated on the middle of the rear seat, 2 ft behind the car's CG, just prior to impact?

$$r := 2 \cdot \text{ft} \quad \omega = 10.472 \text{ rad} \cdot \text{sec}^{-1} \quad A_c = -8.044 \text{ ft} \cdot \text{sec}^{-2}$$

We want to solve the vector equation $\mathbf{A}_p = \mathbf{A}_c + \mathbf{A}_{pc}^n + \mathbf{A}_{pc}^t$ where P is the position of the child and C is the CG. Since α is zero, $\mathbf{A}_{pc}^t$ is zero. $\mathbf{A}_c$ and $\mathbf{A}_{pc}^n$ both have the same direction, but are opposite in sense (see diagram).

$$A_{pcn} := r \cdot \omega^2$$

$$A_{pcn} = 6.817 g$$

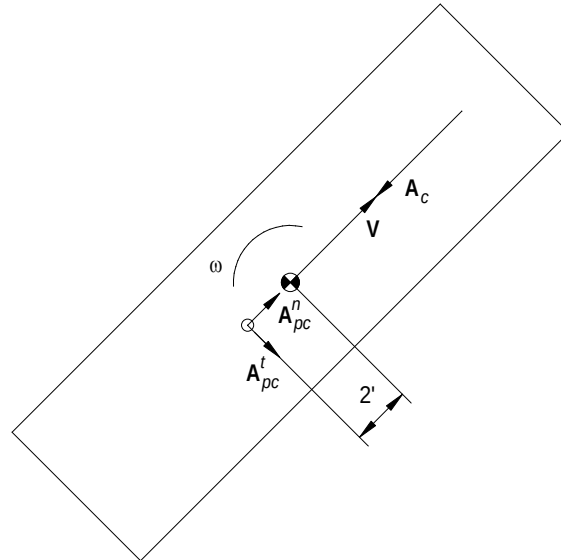
$$A_p := A_c + A_{pcn}$$

$$A_p = 6.567 g$$

This acceleration is toward the tree.

$$A_p = 211.3 \text{ ft} \cdot \text{sec}^{-2}$$

$$A_p = 2535 \text{ in} \cdot \text{sec}^{-2}$$



- b. What force did the 100 lb child exert on her seat belt harness as a result of the acceleration just prior to impact?

$$W := 100 \cdot \text{lbf}$$

The seat back must provide a force of $F := \frac{W}{g} \cdot A_p$ $F = 657 \text{ lbf}$

- c. Assuming a constant deceleration during the impact, what was the magnitude of the average deceleration felt by the passengers in that interval? $t := 0.1 \cdot \text{sec}$

Assume that the rotation has ceased. Then, the average velocity during impact is

$$V_{avg} := \frac{V}{2} \quad V_{avg} = 22.0 \frac{\text{ft}}{\text{sec}}$$

and the average deceleration is $A_{avg} := \frac{V_{avg}}{t}$ $A_{avg} = 6.838 g$ $A_{avg} = 220.0 \text{ ft} \cdot \text{sec}^{-2}$

Adding the deceleration due to skidding, $A_{dec} := A_c - A_{avg}$

$$A_{dec} = -7.088 g \quad A_{dec} = -228 \text{ ft} \cdot \text{sec}^{-2} \quad A_{dec} = -2737 \text{ in} \cdot \text{sec}^{-2}$$

PROBLEM 7-11a

Statement: For row *a* in Table P7-1, find the angular jerk of links 3 and 4 and the linear jerk of the pin between links 3 and 4 (point *B*). Assume an angular jerk of zero on link 2. The linkage configuration and terminology are shown in Figure P7-1.

Given: Link lengths:

$$\text{Link 1 } d := 6$$

$$\text{Link 2 } a := 2$$

$$\text{Link 3 } b := 7$$

$$\text{Link 4 } c := 9$$

$$\text{Coupler point: } R_{PA} := 6$$

$$\delta_3 := 30 \cdot \text{deg}$$

$$\text{Link 2 position, velocity, and acceleration } \theta_2 := 30 \cdot \text{deg} \quad \omega_2 := 10 \quad \alpha_2 := 0 \quad \phi_2 := 0$$

Solution: See Figure P7-1 and Mathcad file P0711a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 3.0000$$

$$K_2 = 0.6667$$

$$K_3 = 2.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$A = -0.7113$$

$$B := -2 \cdot \sin(\theta_2)$$

$$B = -1.0000$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$C = 3.5566$$

2. Use equation 4.10b to find values of θ_4 for the open and crossed circuits.

$$\text{Open: } \theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_{41} = -242.714 \text{ deg}$$

$$\theta_{41} := \theta_{41} - 360 \cdot \text{deg} \quad \theta_{41} = -602.714 \text{ deg}$$

$$\text{Crossed: } \theta_{42} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_{42} = 216.340 \text{ deg}$$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.8571 \quad K_5 = -0.2857$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -1.6774$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 2.5906$$

4. Use equation 4.13 to find values of θ_3 for the open and crossed circuits.

$$\text{Open: } \theta_{31} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{31} = -271.163 \text{ deg}$$

$$\theta_{31} := \theta_{31} - 360 \cdot \text{deg} \quad \theta_{31} = -631.163 \text{ deg}$$

$$\text{Crossed: } \theta_{32} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{32} = 244.789 \text{ deg}$$

5. Determine the angular velocity of links 3 and 4 for the open and crossed circuits using equations 6.18.

$$\text{OPEN } \omega_{31} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41} - \theta_2)}{\sin(\theta_{31} - \theta_{41})} \quad \omega_{31} = -5.991$$

$$\omega_{41} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31})}{\sin(\theta_{41} - \theta_{31})} \quad \omega_{41} = -3.992$$

$$\text{CROSS } \omega_{32} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{42} - \theta_2)}{\sin(\theta_{32} - \theta_{42})} \quad \omega_{32} = -0.662$$

$$\omega_{42} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{32})}{\sin(\theta_{42} - \theta_{32})} \quad \omega_{42} = -2.662$$

6. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the open and crossed circuits.

$$\text{OPEN } A := c \cdot \sin(\theta_{41}) \quad B := b \cdot \sin(\theta_{31}) \quad D := c \cdot \cos(\theta_{41}) \quad E := b \cdot \cos(\theta_{31})$$

$$A = 7.999 \quad B = 6.999 \quad D = -4.126 \quad E = 0.142$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_{31}^2 \cdot \cos(\theta_{31}) - c \cdot \omega_{41}^2 \cdot \cos(\theta_{41})$$

$$C = 244.045$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_{31}^2 \cdot \sin(\theta_{31}) + c \cdot \omega_{41}^2 \cdot \sin(\theta_{41})$$

$$F = -223.741$$

$$\alpha_{31} := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_{31} = 26.080 \quad \alpha_{41} := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_{41} = 53.331$$

$$\text{CROSS } A := c \cdot \sin(\theta_{42}) \quad B := b \cdot \sin(\theta_{32}) \quad D := c \cdot \cos(\theta_{42}) \quad E := b \cdot \cos(\theta_{32})$$

$$A = -5.333 \quad B = -6.333 \quad D = -7.250 \quad E = -2.982$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_{32}^2 \cdot \cos(\theta_{32}) - c \cdot \omega_{42}^2 \cdot \cos(\theta_{42})$$

$$C = 223.253$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_{32}^2 \cdot \sin(\theta_{32}) + c \cdot \omega_{42}^2 \cdot \sin(\theta_{42})$$

$$F = -135.002$$

$$\alpha_{32} := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_{32} = 77.920 \quad \alpha_{42} := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_{42} = 50.669$$

7. Use equations 7.36 and 7.37 to determine the angular jerk of links 3 and 4 for the open and crossed circuits.

$$\text{OPEN } A := a \cdot \omega_2^3 \cdot \sin(\theta_2) \quad D := b \cdot \omega_{31}^3 \cdot \sin(\theta_{31}) \quad G := 3 \cdot c \cdot \omega_{41} \cdot \alpha_{41} \cdot \cos(\theta_{41})$$

$$B := 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot \cos(\theta_2) \quad E := 3 \cdot b \cdot \omega_{31} \cdot \alpha_{31} \cdot \cos(\theta_{31}) \quad H := c \cdot \sin(\theta_{41})$$

$$C := a \cdot \phi_2 \cdot \sin(\theta_2) \quad F := c \cdot \omega_{41}^3 \cdot \sin(\theta_{41}) \quad K := b \cdot \sin(\theta_{31})$$

$$\begin{aligned}
 L &:= a \cdot \omega_2^3 \cdot \cos(\theta_2) & P &:= b \cdot \omega_{31}^3 \cdot \cos(\theta_{31}) & S &:= c \cdot \omega_{41}^3 \cdot \cos(\theta_{41}) \\
 M &:= 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot \sin(\theta_2) & Q &:= 3 \cdot b \cdot \omega_{31} \cdot \alpha_{31} \cdot \sin(\theta_{31}) & T &:= 3 \cdot c \cdot \omega_{41} \cdot \alpha_{41} \cdot \sin(\theta_{41}) \\
 N &:= a \cdot \phi_2 \cdot \cos(\theta_2) & R &:= b \cdot \cos(\theta_{31}) & U &:= c \cdot \cos(\theta_{41})
 \end{aligned}$$

$$\phi_{41} := \frac{K \cdot (N - L - M - P - Q + S + T) \dots + R \cdot (A - B - C + D - E - F + G)}{K \cdot U - H \cdot R} \quad \phi_{41} = 749.012$$

$$\phi_{31} := \frac{A - B - C + D - E - F + G + H \cdot \phi_{41}}{K} \quad \phi_{31} = 1242.6$$

$$\mathbf{J}_A := -a \cdot \omega_2^3 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \dots + a \cdot \phi_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{J}_{BA1} := -b \cdot \omega_{31}^3 \cdot (-\sin(\theta_{31}) + j \cdot \cos(\theta_{31})) - 3 \cdot b \cdot \omega_{31} \cdot \alpha_{31} \cdot (\cos(\theta_{31}) + j \cdot \sin(\theta_{31})) \dots + b \cdot \phi_{31} \cdot (-\sin(\theta_{31}) + j \cdot \cos(\theta_{31}))$$

$$\mathbf{J}_{B1} := \mathbf{J}_A + \mathbf{J}_{BA1}$$

$$J_{B1} := |\mathbf{J}_{B1}| \quad J_{B1x} := \text{Re}(\mathbf{J}_{B1}) \quad J_{B1y} := \text{Im}(\mathbf{J}_{B1})$$

$$J_{B1} = 9301.9 \quad J_{B1x} = -9134.7 \quad J_{B1y} = 1755.5$$

$$\begin{aligned}
 \text{CROSSED } A &:= a \cdot \omega_2^3 \cdot \sin(\theta_2) & D &:= b \cdot \omega_{32}^3 \cdot \sin(\theta_{32}) & G &:= 3 \cdot c \cdot \omega_{42} \cdot \alpha_{42} \cdot \cos(\theta_{42}) \\
 B &:= 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot \cos(\theta_2) & E &:= 3 \cdot b \cdot \omega_{32} \cdot \alpha_{32} \cdot \cos(\theta_{32}) & H &:= c \cdot \sin(\theta_{42}) \\
 C &:= a \cdot \phi_2 \cdot \sin(\theta_2) & F &:= c \cdot \omega_{42}^3 \cdot \sin(\theta_{42}) & K &:= b \cdot \sin(\theta_{32})
 \end{aligned}$$

$$\begin{aligned}
 L &:= a \cdot \omega_2^3 \cdot \cos(\theta_2) & P &:= b \cdot \omega_{32}^3 \cdot \cos(\theta_{32}) & S &:= c \cdot \omega_{42}^3 \cdot \cos(\theta_{42}) \\
 M &:= 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot \sin(\theta_2) & Q &:= 3 \cdot b \cdot \omega_{32} \cdot \alpha_{32} \cdot \sin(\theta_{32}) & T &:= 3 \cdot c \cdot \omega_{42} \cdot \alpha_{42} \cdot \sin(\theta_{42}) \\
 N &:= a \cdot \phi_2 \cdot \cos(\theta_2) & R &:= b \cdot \cos(\theta_{32}) & U &:= c \cdot \cos(\theta_{42})
 \end{aligned}$$

$$\phi_{42} := \frac{K \cdot (N - L - M - P - Q + S + T) \dots + R \cdot (A - B - C + D - E - F + G)}{K \cdot U - H \cdot R} \quad \phi_{42} = -246.639$$

$$\phi_{32} := \frac{A - B - C + D - E - F + G + H \cdot \phi_{42}}{K} \quad \phi_{32} = -740.2$$

$$\mathbf{J}_{BA2} := -b \cdot \omega_{32}^3 \cdot (-\sin(\theta_{32}) + j \cdot \cos(\theta_{32})) - 3 \cdot b \cdot \omega_{32} \cdot \alpha_{32} \cdot (\cos(\theta_{32}) + j \cdot \sin(\theta_{32})) \dots + b \cdot \phi_{32} \cdot (-\sin(\theta_{32}) + j \cdot \cos(\theta_{32}))$$

$$\mathbf{J}_{B2} := \mathbf{J}_A + \mathbf{J}_{BA2}$$

$$J_{B2} := |\mathbf{J}_{B2}| \quad J_{B2x} := \text{Re}(\mathbf{J}_{B2}) \quad J_{B2y} := \text{Im}(\mathbf{J}_{B2})$$

$$J_{B2} = 4178.7 \quad J_{B2x} = -4147.9 \quad J_{B2y} = -506.4$$

PROBLEM 7-12

Statement: You are riding on a carousel that is rotating at a constant 12 rpm. It has an inside radius of 4 ft and an outside radius of 12 ft. You begin to run from the inside to the outside along a radius. Your peak velocity with respect to the carousel is 4 mph and occurs at a radius of 8 ft. What is your maximum Coriolis acceleration magnitude and its direction with respect to the carousel?

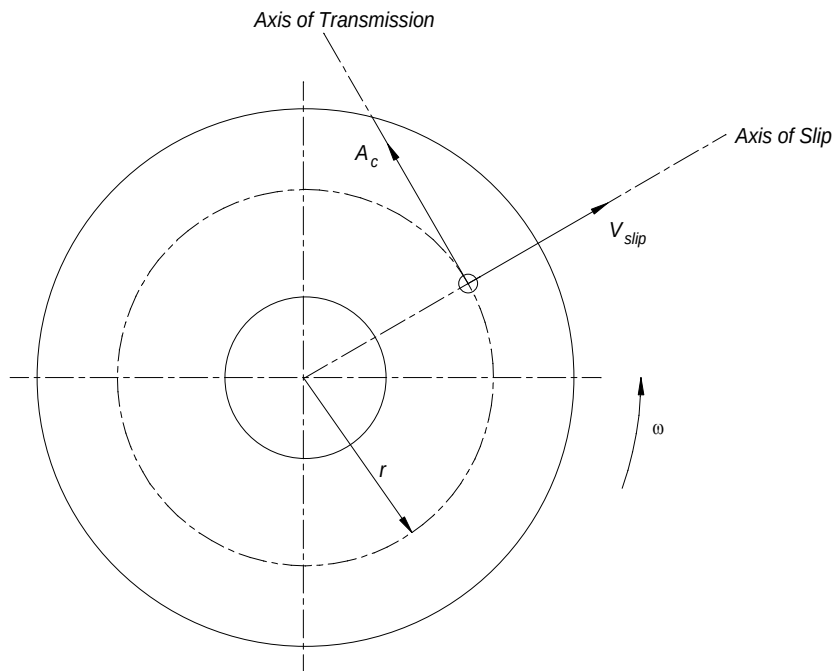
Given: Carousel angular velocity $\omega := 12 \cdot rpm$ $\omega = 1.257 \cdot \frac{rad}{sec}$

Peak velocity $V_{slip} := 4 \cdot mph$ $V_{slip} = 5.867 \cdot \frac{ft}{sec}$

Radius at peak velocity $r := 8 \cdot ft$

Solution: See Mathcad file P0712.

1. Draw a plan view of the carousel floor showing your position at peak velocity and the velocity and Coriolis acceleration vectors.



2. The direction of your path defines the axis of slip. The transmission axis is perpendicular to the axis of slip and positive in the direction of the tangential velocity of the carousel. Thus, the direction of the Coriolis acceleration vector is along the positive transmission axis.
3. Use equation 7.19 to calculate the magnitude of the Coriolis component of your acceleration.

$$A_c := 2 \cdot V_{slip} \cdot \omega \qquad A_c = 14.745 \cdot \frac{ft}{sec^2} \qquad A_c = 176.93 \cdot \frac{in}{sec^2}$$

PROBLEM 7-13a

Statement: The linkage in Figure P7-5a has the dimensions and crank angle given below. Find α_3 , $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ and $\alpha_2 = 10 \text{ rad/sec}^2$ in the direction shown. Use the acceleration difference graphical method.

Given:

Link lengths: Link 2 $a := 0.8 \cdot \text{in}$ Link 3 $b := 1.93 \cdot \text{in}$

Offset: $c := -0.38 \cdot \text{in}$

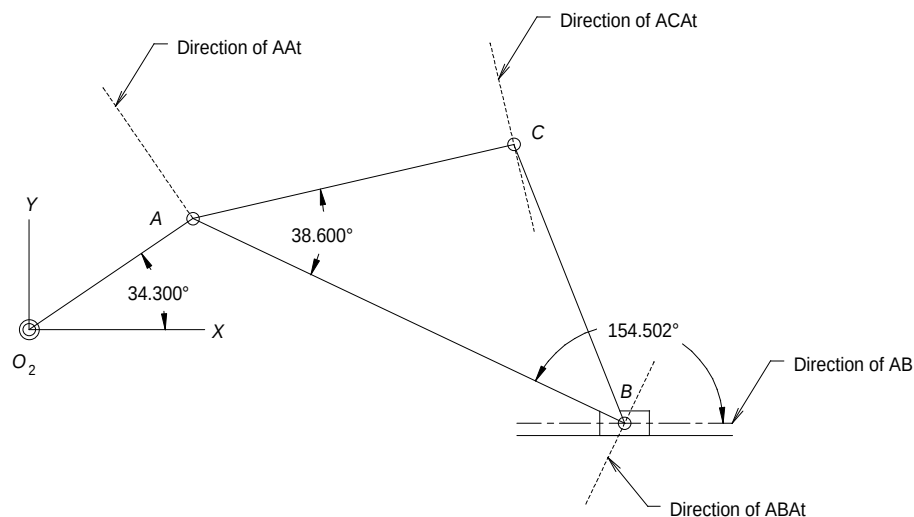
Coupler point data: $p := 1.33 \cdot \text{in}$ $\delta_3 := 38.6 \cdot \text{deg}$

Link 2 position, velocity, and acceleration $\theta_2 := 34.3 \cdot \text{deg}$ $\omega_2 := 15 \cdot \frac{\text{rad}}{\text{sec}}$ $\alpha_2 := 10 \cdot \frac{\text{rad}}{\text{sec}^2}$

Solution: See Figure P7-5a and Mathcad file P0713a.

1. In order to solve for the accelerations at points A, B and C, we will need θ_3 , and ω_3 . From the graphical position solution below, $\theta_3 := 154.502 \cdot \text{deg}$

Using equation 6.22a,
$$\omega_3 := \frac{a \cdot \omega_2 \cdot \cos(\theta_2)}{b \cdot \cos(\theta_3)} \quad \omega_3 = -5.691 \text{ rad} \cdot \text{sec}^{-1}$$



2. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$
3. For point B, this becomes: $\mathbf{A}_B = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_{An} := a \cdot \omega_2^2 \quad A_{An} = 180.000 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AAn} = 214.300 \text{ deg}$$

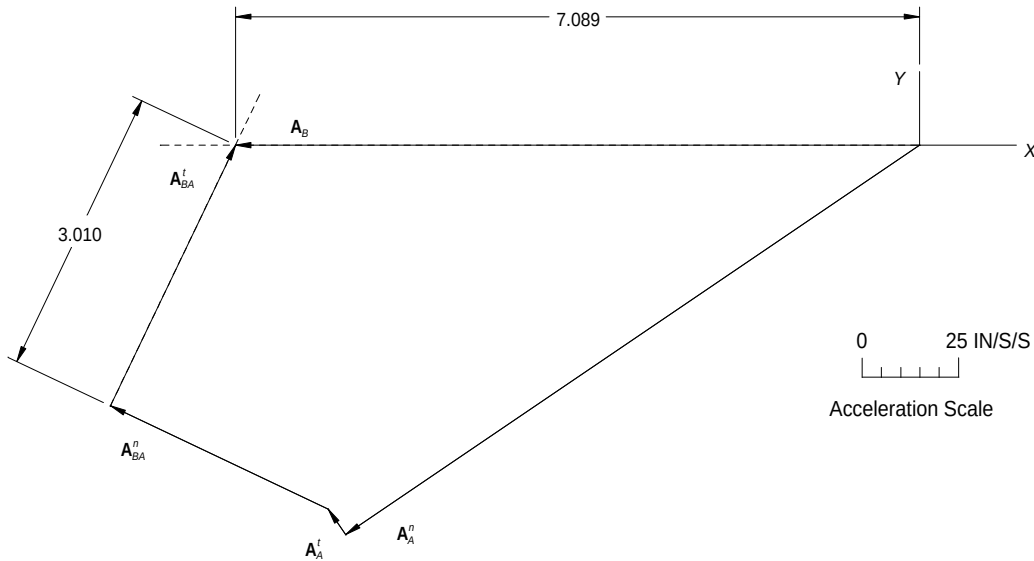
$$A_{At} := a \cdot \alpha_2 \quad A_{At} = 8.000 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AA_t} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{AA_t} = 124.300 \text{ deg}$$

$$A_{BA_n} := b \cdot \omega_3^2 \quad A_{BA_n} = 62.500 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABA_n} := \theta_3 \quad \theta_{ABA_n} = 154.502 \text{ deg}$$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_A^n$ at an angle of θ_{AA^n} . From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_A^t$ at an angle of θ_{AA^t} . From the tip of $\mathbf{A}_A^t$, draw $\mathbf{A}_{BA}^n$ at an angle of θ_{ABAn} . Now that the vectors with known magnitudes are drawn, from the origin and the tip of $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B$ (horizontal) and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_{BA}^t$, and $\mathbf{A}_B$.



5. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 25 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_B := 7.089 \cdot k_a \quad A_B = 177.2 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 180 \text{ deg}$$

$$A_{BA^t} := 3.010 \cdot k_a \quad A_{BA^t} = 75.3 \text{ in} \cdot \text{sec}^{-2}$$

6. Calculate α_3 using equation 7.6.

$$\alpha_3 := \frac{A_{BA^t}}{b} \quad \alpha_3 = 38.990 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CCW}$$

7. For point C, equation 7.4 becomes: $\mathbf{AC} = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{CA}^t + \mathbf{A}_{CA}^n)$, where

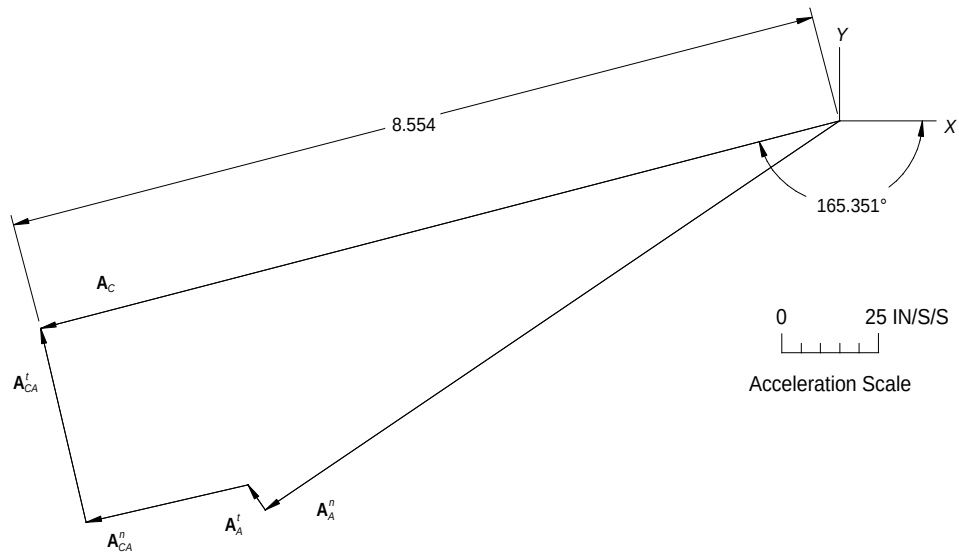
$$A_{CA^n} := p \cdot \omega_3^2 \quad A_{CA^n} = 43.070 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{APAn} := \theta_3 + \delta_3 \quad \theta_{APAn} = 193.102 \text{ deg}$$

$$A_{CA^t} := p \cdot \alpha_3 \quad A_{CA^t} = 51.856 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{APAt} := (\theta_3 + \delta_3) - 90 \cdot \text{deg} \quad \theta_{APAt} = 103.102 \text{ deg}$$

8. Repeat procedure of step 4 for the equation in step 7.



9. From the graphical solution above,

Acceleration scale factor

$$k_a := 25 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_C := 8.554 \cdot k_a$$

$$A_C = 213.9 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -165.35 \text{ deg}$$

PROBLEM 7-13b

Statement: The linkage in Figure P7-5a has the dimensions and crank angle given below. Find α_3 , $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ and $\alpha_2 = 10 \text{ rad/sec}^2$ in the direction shown. Use an analytical method.

Given: Link lengths:

$$\text{Link 2} \quad a := 0.8 \cdot \text{in} \quad \text{Link 3} \quad b := 1.93 \cdot \text{in}$$

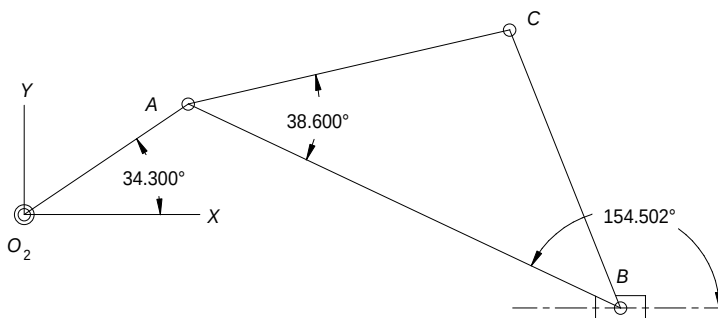
$$\text{Offset:} \quad c := -0.38 \cdot \text{in}$$

$$\text{Coupler point data:} \quad p := 1.33 \cdot \text{in} \quad \delta_3 := 38.6 \cdot \text{deg}$$

$$\text{Link 2 position, velocity, and acceleration } \theta_2 := 34.3 \cdot \text{deg} \quad \omega_2 := 15 \cdot \frac{\text{rad}}{\text{sec}} \quad \alpha_2 := 10 \cdot \frac{\text{rad}}{\text{sec}^2}$$

Solution: See Figure P7-5a and Mathcad file P0713b.

1. Draw the linkage to scale and label it.



2. Determine θ_3 and d using equations 4.16 and 4.17.

$$\theta_3 := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi \quad \theta_3 = 154.502 \text{ deg}$$

$$d := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_3) \quad d = 2.403 \text{ in}$$

3. Determine the angular velocity of link 3 using equation 6.22a.

$$\omega_3 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3)} \cdot \omega_2 \quad \omega_3 = -5.691 \frac{\text{rad}}{\text{sec}}$$

4. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its magnitude, and direction.

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (-153.206 - 94.826j) \frac{\text{in}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A)$$

$$\text{The acceleration of pin A is } A_A = 180 \frac{\text{in}}{\text{sec}^2} \quad \text{at } \theta_{AA} = -148.2 \text{ deg}$$

5. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3 := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_3^2 \cdot \sin(\theta_3)}{b \cdot \cos(\theta_3)} \quad \alpha_3 = 38.990 \frac{\text{rad}}{\text{sec}^2}$$

6. Use equation 7.16e for the acceleration of pin B.

$$A_B := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \alpha_3 \cdot \sin(\theta_3) + b \cdot \omega_3^2 \cdot \cos(\theta_3)$$

$$A_B = -177.2 \frac{\text{in}}{\text{sec}^2} \quad \text{A negative sign means that } \mathbf{A}_B \text{ is to the left}$$

7. Determine the acceleration of the coupler point C using equations 7.32. Note that θ_3 is defined from point B in Figure 7-6 and from point A in Figure 7-9. To use equation 7.32 for a slider-crank we must redefine θ_3 .

$$\theta_3 := \theta_3 - 180 \cdot \text{deg} \quad \theta_3 = -25.498 \text{ deg}$$

$$\mathbf{A}_{CA} := p \cdot \alpha_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3)) \dots \\ + -p \cdot \omega_3^2 \cdot (\cos(\theta_3 + \delta_3) + j \cdot \sin(\theta_3 + \delta_3))$$

$$\mathbf{A}_C := \mathbf{A}_A + \mathbf{A}_{CA}$$

$$\mathbf{A}_C = (-206.910 - 54.083i) \frac{\text{in}}{\text{sec}^2} \quad A_C := |\mathbf{A}_C| \quad \theta_{AC} := \arg(\mathbf{A}_C)$$

$$\text{The acceleration of point C is } A_C = 213.861 \frac{\text{in}}{\text{sec}^2} \quad \text{at } \theta_{AC} = -165.352 \text{ deg}$$

PROBLEM 7-14a

Statement: The linkage in Figure P7-5b has the dimensions and effective crank angle given below. Find α_3 , $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ and $\alpha_2 = 10 \text{ rad/sec}^2$ in the directions shown. Use the acceleration difference graphical method.

Given: Link lengths:

Link 2 (point of contact to A) $a := 0.75 \cdot \text{in}$

Link 3 (A to B) $b := 1.5 \cdot \text{in}$

Link 4 (point of contact to B) $c := 0.75 \cdot \text{in}$

Link 1 (between contact points) $d := 1.5 \cdot \text{in}$

Coupler point:

Distance A to C $p := 1.2 \cdot \text{in}$

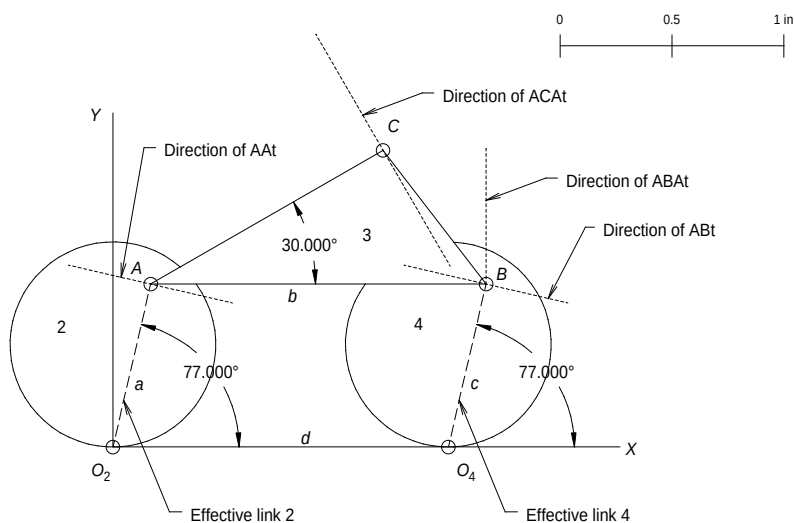
Angle BAC $\delta := 30 \cdot \text{deg}$

Crank angle: $\theta_2 := 77 \cdot \text{deg}$

Input crank angular velocity $\omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1}$ $\alpha_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure P7-5b and Mathcad file P0714a.

- Although the mechanism shown in Figure P6-5b is not entirely pin-jointed, it can be analyzed for the position shown by its effective pin-jointed fourbar, which is shown below. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



- In order to solve for the accelerations at points A and B, we will need θ_3 , ω_3 , and θ_4 . From the graphical position solution above,

$$\theta_3 := 0.0 \cdot \text{deg}$$

$$\theta_4 := 77.0 \cdot \text{deg}$$

This is a special-case Grashof in the parallelogram configuration. Therefore,

$$\omega_3 := 0.0 \cdot \text{rad} \cdot \text{sec}^{-1}$$

$$\omega_4 := \omega_2$$

- The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$
- For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$\begin{aligned}
 A_{Bn} &:= c \cdot \omega_4^2 & A_{Bn} &= 168.750 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{ABn} &:= \theta_4 + 180 \cdot \text{deg} & \theta_{ABn} &= 257.000 \text{ deg} \\
 A_{An} &:= a \cdot \omega_2^2 & A_{An} &= 168.750 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{AAn} &:= \theta_2 + 180 \cdot \text{deg} & \theta_{AAn} &= 257.000 \text{ deg} \\
 A_{At} &:= a \cdot \alpha_2 & A_{At} &= 7.500 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{At} &:= \theta_2 + 90 \cdot \text{deg} & \theta_{At} &= 167.000 \text{ deg} \\
 A_{BAAn} &:= b \cdot \omega_3^2 & A_{BAAn} &= 0.000 \text{ in} \cdot \text{sec}^{-2}
 \end{aligned}$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of $\theta_4 + 180$ deg and $\mathbf{A}_A^n$ at an angle of $\theta_2 + 180$ deg. From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_A^t$ at an angle of $\theta_2 + 90$ deg. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B^n$ and $\mathbf{A}_A^t$, draw construction lines in the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B^t$, $\mathbf{A}_{BA}^t$, and $\mathbf{A}_B$.

0 25 IN/S/S
Acceleration Scale

From the layout $\mathbf{A}_B^t = \mathbf{A}_A^t$ and $\mathbf{A}_{BA}^t = 0$. Also, $\mathbf{A}_B = \mathbf{A}_A$.

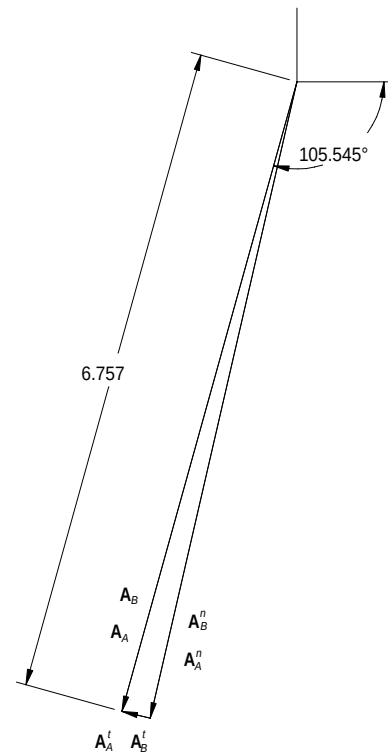
Acceleration scale factor

$$k_a := 25 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_B := 6.757 \cdot k_a$$

$$A_B = 168.9 \text{ in} \cdot \text{sec}^{-2}$$

at an angle of -105.55 deg



6. Since $\mathbf{A}_{BA}^t = 0$, $\alpha_3 = 0$ and, since $\mathbf{A}_B = \mathbf{A}_A$, $\alpha_4 = \alpha_2$.
7. For point P , equation 7.4 becomes: $\mathbf{A}_P = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$, where $\mathbf{A}_{PA}^t$ and $\mathbf{A}_{PA}^n$ are both zero since α and ω_3 are zero. Therefore, $\mathbf{A}_P = \mathbf{A}_A$.

PROBLEM 7-14b

Statement: The linkage in Figure P7-5b has the dimensions and effective crank angle given below. Find α_3 , $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\omega_2 = 15 \text{ rad/sec}$ and $\alpha_2 = 10 \text{ rad/sec}^2$ in the directions shown. Use an analytical method.

Given: Link lengths:

Link 2 (point of contact to A) $a := 0.75 \cdot \text{in}$ Link 3 (A to B) $b := 1.5 \cdot \text{in}$

Link 4 (point of contact to B) $c := 0.75 \cdot \text{in}$ Link 1 (between contact points) $d := 1.5 \cdot \text{in}$

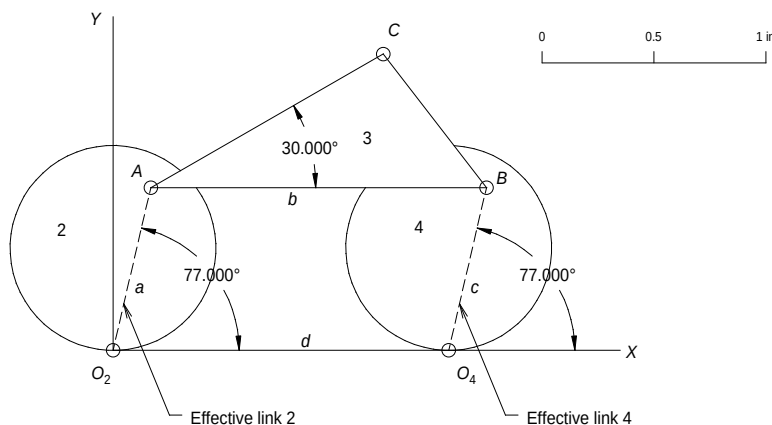
Coupler point:

Distance A to C $R_{ca} := 1.2 \cdot \text{in}$ Crank angle: $\theta_2 := 77 \cdot \text{deg}$ $\alpha_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-2}$

Angle BAC $\delta_3 := 30 \cdot \text{deg}$ Input crank angular velocity $\omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1}$

Solution: See Figure P7-5b and Mathcad file P0714b.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_1 = 2.0000$$

$$K_2 := \frac{d}{c}$$

$$K_2 = 2.0000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_3 = 1.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -1.2250 \quad B = -1.9487 \quad C = 2.3251$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right)$$

$$\theta_4 = -283.000 \text{ deg}$$

$$\theta_4 := \theta_4 + 360 \cdot \text{deg}$$

$$\theta_4 = 77.000 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0000 \quad K_5 = -2.0000$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -3.5501$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.9487$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.0000$$

5. Use equation 4.13 to find values of θ_3 .

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_3 = 360.000 \text{ deg}$$

$$\theta_3 := \theta_3 - 360 \cdot \text{deg} \quad \theta_3 = 0.000 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_4 - \theta_2)}{b \cdot \sin(\theta_3 - \theta_4)} \quad \omega_3 = 0.000 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_4 - \theta_3)} \quad \omega_4 = 15.000 \frac{\text{rad}}{\text{sec}}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (-45.268 - 162.738j) \frac{\text{in}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A)$$

$$\text{The acceleration of pin A is } A_A = 168.917 \frac{\text{in}}{\text{sec}^2} \text{ at } \theta_{AA} = -105.5 \text{ deg}$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A := c \cdot \sin(\theta_4) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_4) \quad E := b \cdot \cos(\theta_3)$$

$$A = 0.731 \text{ in} \quad B = 0.000 \text{ in} \quad D = 0.169 \text{ in} \quad E = 1.500 \text{ in}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$C = 7.308 \text{ in} \cdot \text{sec}^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = 1.687 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = 0.000 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = 10.000 \frac{\text{rad}}{\text{sec}^2}$$

9. Use equation 7.13c to determine the acceleration of point B.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$\mathbf{A_B} = (-45.268 - 162.738j) \frac{\text{in}}{\text{sec}^2} \quad A_B := |\mathbf{A_B}| \quad \theta_{AB} := \arg(\mathbf{A_B})$$

$$\text{The acceleration of pin B is } A_B = 168.917 \frac{\text{in}}{\text{sec}^2} \text{ at } \theta_{AB} = -105.5 \text{ deg}$$

10. Use equations 7.32 to find the acceleration of the point C.

$$\begin{aligned} \mathbf{A_{CA}} := & R_{ca} \cdot \alpha_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3)) \dots \\ & + -R_{ca} \cdot \omega_3^2 \cdot (\cos(\theta_3 + \delta_3) + j \cdot \sin(\theta_3 + \delta_3)) \end{aligned}$$

$$\mathbf{A_C} := \mathbf{A_A} + \mathbf{A_{CA}}$$

$$A_C := |\mathbf{A_C}| \quad A_C = 168.917 \frac{\text{in}}{\text{sec}^2} \quad \arg(\mathbf{A_C}) = -105.545 \text{ deg}$$

PROBLEM 7-15a

Statement: The linkage in Figure P7-5c has the dimensions and coupler angle given below. Find α_3 , $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\mathbf{V}_A = 10$ in/sec and $\mathbf{A}_A = 15$ in/sec² in the directions shown. Use the acceleration difference graphical method.

Given: Link lengths and angles:

$$\text{Link 3 (A to B)} \quad b := 1.8 \cdot \text{in}$$

$$\text{Coupler angle} \quad \theta_3 := 128 \cdot \text{deg} \quad \text{Slider 4 angle} \quad \theta_4 := 59 \cdot \text{deg}$$

Coupler point:

$$\text{Distance A to C} \quad p := 1.44 \cdot \text{in} \quad \text{Angle BAC} \quad \delta_3 := -49 \cdot \text{deg}$$

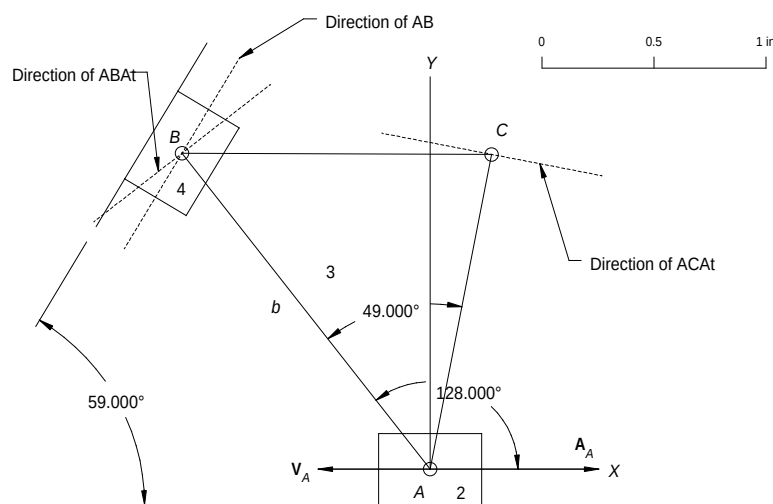
$$\text{Input slider motion} \quad V_A := -10 \cdot \text{in} \cdot \text{sec}^{-1} \quad A_A := 15 \cdot \text{in} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-5c and Mathcad file P0715a.

- In order to solve for the accelerations at points B and C, we will need ω_3 . From the layout below and Problem 6-18,

$$\theta_3 := 128 \cdot \text{deg}$$

$$\omega_3 := -13.288 \cdot \frac{\text{rad}}{\text{sec}}$$



- The graphical solution for accelerations uses equation 7.4:

$$(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$$

- For point B, this becomes: $\mathbf{A}_B = \mathbf{A}_A + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_A = 15.000 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AA} := 0 \cdot \text{deg}$$

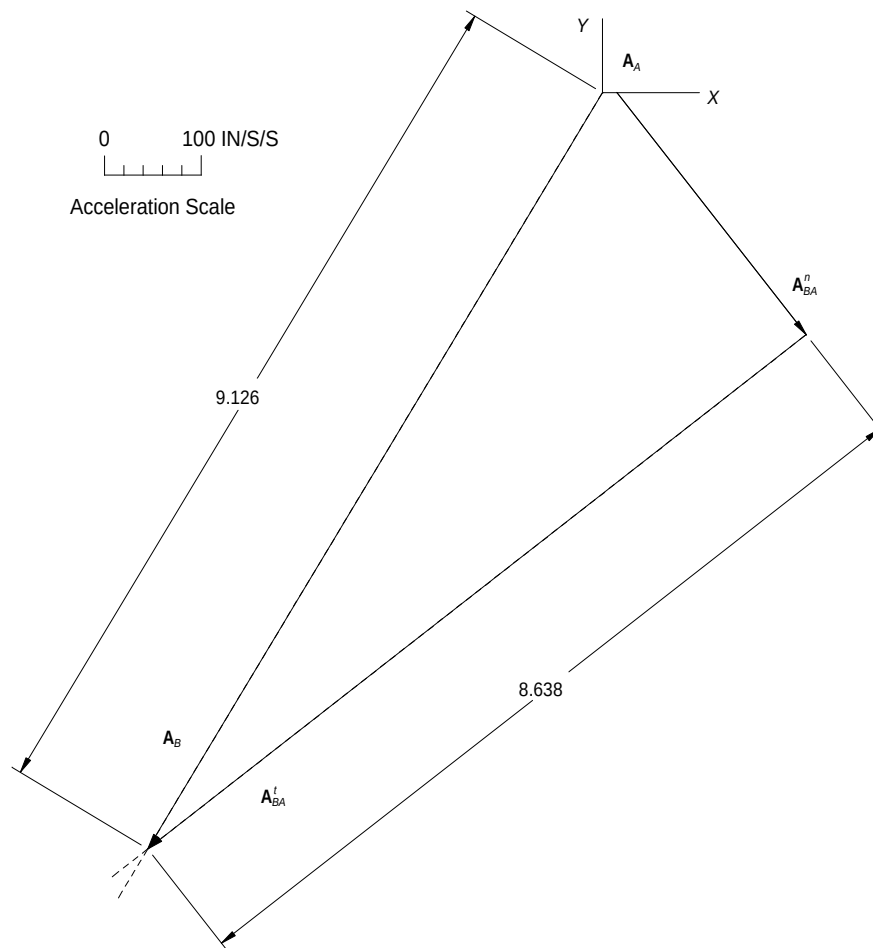
$$A_{BA^n} := b \cdot \omega_3^2$$

$$A_{BA^n} = 317.828 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABA^n} := \theta_3 + 180 \cdot \text{deg}$$

$$\theta_{ABA^n} = 308.000 \text{ deg}$$

- Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_A$ at an angle of θ_{AA} . From the tip of $\mathbf{A}_A$, draw $\mathbf{A}_{BA}^n$ at an angle of θ_{ABA^n} . Now that the vectors with known magnitudes are drawn, from the origin and the tip of $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_{BA}^t$, and $\mathbf{A}_B$.



5. From the graphical solution above,

$$\begin{aligned} \text{Acceleration scale factor} \quad k_a &:= 100 \cdot \frac{\text{in}}{\text{sec}^2} \\ A_B &:= 9.126 \cdot k_a & A_B &= 912.6 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 239 \text{ deg} \\ A_{BA^t} &:= 8.638 \cdot k_a & A_{BA^t} &= 863.8 \text{ in} \cdot \text{sec}^{-2} \end{aligned}$$

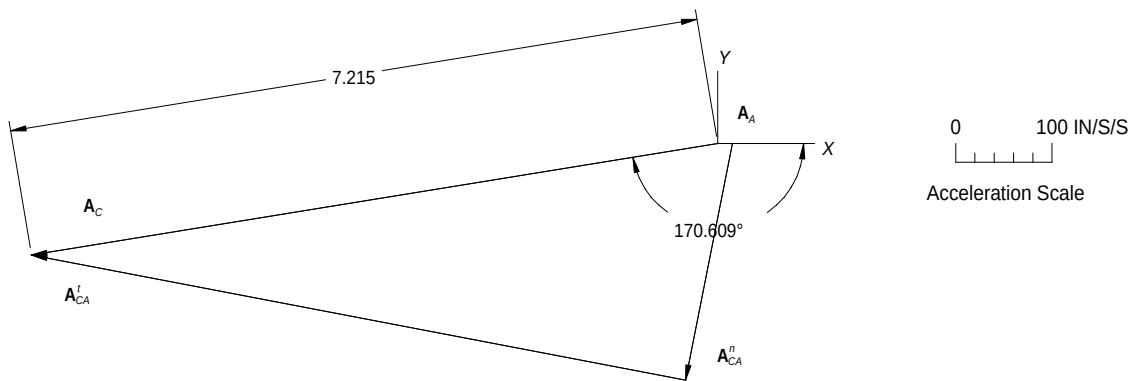
6. Calculate α_3 using equation 7.6.

$$\alpha_3 := \frac{A_{BA^t}}{b} \quad \alpha_3 = 479.9 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CW}$$

7. For point C, equation 7.4 becomes: $\mathbf{A}_C = \mathbf{A}_A + (\mathbf{A}_{CA}^t + \mathbf{A}_{CA}^n)$, where

$$\begin{aligned} A_{CA^t} &:= p \cdot \omega_3^2 & A_{CA^t} &= 254.262 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{APAn} &:= \theta_3 + \delta_3 + 180 \cdot \text{deg} & \theta_{APAn} &= 259.000 \text{ deg} \\ A_{CA^t} &:= p \cdot \alpha_3 & A_{CA^t} &= 691.040 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{APAt} &:= (\theta_3 + \delta_3) + 90 \cdot \text{deg} & \theta_{APAt} &= 169.000 \text{ deg} \end{aligned}$$

8. Repeat procedure of step 4 for the equation in step 7.



9. From the graphical solution above,

Acceleration scale factor $k_a := 100 \cdot \frac{\text{in}}{\text{sec}^2}$

$A_C := 7.215 \cdot k_a$ $A_C = 721.5 \text{ in} \cdot \text{sec}^{-2}$ at an angle of -170.61 deg

PROBLEM 7-15b

Statement: The linkage in Figure P7-5c has the dimensions and coupler angle given below. Find α_3 , $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\mathbf{V}_A = 10 \text{ in/sec}$ and $\mathbf{A}_A = 15 \text{ in/sec}^2$ in the directions shown. Use an analytical method.

Given: Link lengths and angles:

Link 3 (A to B) $b := 1.8 \cdot \text{in}$

Coupler angle $\theta_3 := 128 \cdot \text{deg}$

Slider 4 angle $\theta_4 := 59 \cdot \text{deg}$

Coupler point:

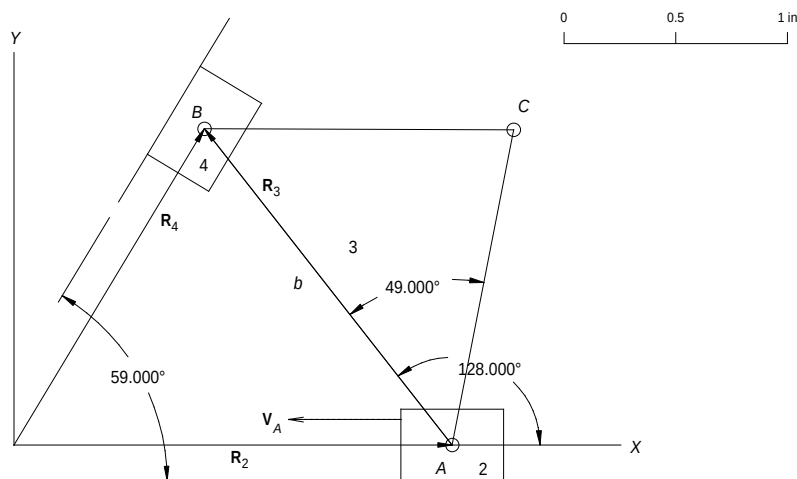
Distance A to C $R_{ca} := 1.44 \cdot \text{in}$

Angle BAC $\delta_3 := -49 \cdot \text{deg}$

Input slider motion $V_A := -10 \cdot \text{in} \cdot \text{sec}^{-1}$ $A_A := 15 \cdot \text{in} \cdot \text{sec}^{-2}$

Solution: See Figure P7-5c and Mathcad file P0715b.

1. Draw the mechanism to scale and define a vector loop using the fourbar slider-crank derivation in Section 7.3 as a model.



2. Write the vector loop equation, differentiate it, expand the result and separate into real and imaginary parts to solve for ω_3 and $\mathbf{V}_B$.

$$\mathbf{R}_2 + \mathbf{R}_3 := \mathbf{R}_4 \quad a \cdot e^{j \cdot \theta_2} + b \cdot e^{j \cdot \theta_3} := c \cdot e^{j \cdot \theta_4}$$

where a is the distance from the origin to point A, a variable; b is the distance from A to B, a constant; and c is the distance from the origin to point B, a variable. Angle θ_2 is zero, θ_3 is the angle that AB makes with the x axis, and θ_4 is the constant angle that slider 4 makes with the x axis. Differentiating,

$$\frac{d}{dt}a + j \cdot b \cdot \omega_3 \cdot e^{j \cdot \theta_3} := \left(\frac{d}{dt}c \right) \cdot e^{j \cdot \theta_4}$$

Substituting the Euler equivalents,

$$\frac{d}{dt}a + b \cdot \omega_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3)) := \left(\frac{d}{dt}c \right) \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

Separating into real and imaginary components and solving for ω_3 . Note that $dc/dt = V_B$ and $da/dt = V_A$

$$\omega_3 := \frac{V_A \cdot \tan(\theta_4)}{b \cdot (\sin(\theta_3) \cdot \tan(\theta_4) + \cos(\theta_3))} \quad \omega_3 = -13.288 \frac{\text{rad}}{\text{sec}}$$

3. Differentiate the velocity equation, expand it and solve for α_3 and A_B .

$$\frac{d^2}{dt^2} \left(a + b \cdot \alpha_3 \cdot j \cdot e^{j \cdot \theta_3} + b \cdot \omega_3^2 \cdot j \cdot e^{j \cdot \theta_3} \right) := \frac{d^2}{dt^2} c \cdot e^{j \cdot \theta_4}$$

Substituting the Euler equivalents,

$$\begin{aligned} \frac{d^2}{dt^2} a + b \cdot \alpha_3 (-\sin(\theta_3) + j \cdot \cos(\theta_3)) \dots \\ + b \cdot \omega_3^2 (\cos(\theta_3) + j \cdot \sin(\theta_3)) - \frac{d^2}{dt^2} c (\cos(\theta_4) + j \cdot \sin(\theta_4)) \end{aligned} := 0$$

Separating into real and imaginary components and solving for α_3 and A_B . Note that $d^2c/dt^2 = A_B$ and $d^2a/dt^2 = A_A$

$$\alpha_3 := \frac{A_A \cdot \sin(\theta_4) - b \cdot \omega_3^2 \cdot \sin(\theta_4 - \theta_3)}{b \cdot \cos(\theta_4 - \theta_3)} \quad \alpha_3 = 479.924 \frac{\text{rad}}{\text{sec}^2}$$

$$A_B := \frac{b \cdot \alpha_3 \cdot \cos(\theta_3) - b \cdot \omega_3^2 \cdot \sin(\theta_3)}{\sin(\theta_4)} \quad A_B = -912.662 \frac{\text{in}}{\text{sec}^2}$$

4. Determine the acceleration of the coupler point C using equations 7.32.

$$\mathbf{A}_{CA} := R_{ca} \cdot \alpha_3 (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3)) - R_{ca} \cdot \omega_3^2 (\cos(\theta_3 + \delta_3) + j \cdot \sin(\theta_3 + \delta_3))$$

$$\mathbf{A}_{CA} = (-726.910 - 117.729j) \frac{\text{in}}{\text{sec}^2}$$

$$\mathbf{A}_A := A_A$$

$$\mathbf{A}_C := \mathbf{A}_A + \mathbf{A}_{CA}$$

$$\mathbf{A}_C = (-711.910 - 117.729j) \frac{\text{in}}{\text{sec}^2} \quad |\mathbf{A}_C| = 721.579 \frac{\text{in}}{\text{sec}^2}$$

$$\arg(\mathbf{A}_C) = -170.610 \text{ deg}$$

PROBLEM 7-16

Statement: For the linkage shown in Figure P7-6a, write the vector loop equations; differentiate them, and do a complete position, velocity, and acceleration analysis of the linkage. Measure the linkage geometry from the figure. Assume $\omega_2 = 10 \text{ rad/sec}$ and $\alpha_2 = 20 \text{ rad/sec}^2$.

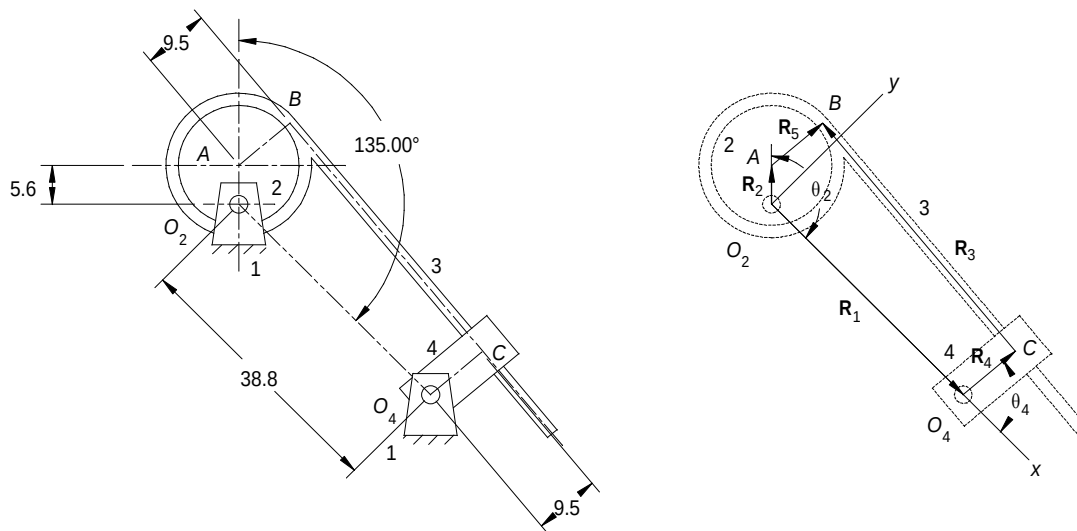
Given: Link lengths and angles:

Link 2 (O_2 to A)	$a := 5.6 \cdot \text{mm}$	Link 4 (O_4 to C)	$c := 9.5 \cdot \text{mm}$
Link 3 offset (A to B)	$f := 9.5 \cdot \text{mm}$	Link 1 (O_2 to O_4)	$d := 38.8 \cdot \text{mm}$

Link 2 position, velocity and acceleration $\theta_2 := 135 \cdot \text{deg}$ $\omega_2 := 10 \cdot \frac{\text{rad}}{\text{sec}}$ $\alpha_2 := 20 \cdot \frac{\text{rad}}{\text{sec}^2}$

Solution: See Figure P7-6a and Mathcad file P0716.

1. Draw the mechanism to scale and define a vector loop using the fourbar inverted slider-crank derivation in Section 7.3 as a model.



2. Write the vector loop equation, expand the result and separate into real and imaginary parts to solve for θ_3 and b .

$$\mathbf{R}_2 + \mathbf{R}_5 := \mathbf{R}_1 + \mathbf{R}_4 + \mathbf{R}_3$$

$$a \cdot e^{j \cdot \theta_2} + f \cdot e^{j \cdot \theta_5} := d \cdot e^{j \cdot \theta_1} + c \cdot e^{j \cdot \theta_4} + b \cdot e^{j \cdot \theta_3}$$

where a is the distance from pivot O_2 to point A, a constant; f is the distance from A to B, a constant; b is the distance from B to C, a variable; and c is the distance from pivot O_4 to point C, a constant. Angle θ_2 is the input crank angle, θ_3 is the angle that BC makes with the x axis (measured from C), and θ_4 is the variable angle that link 4 makes with the x axis. The angular relationships are

$$\theta_5 := \theta_4 \quad \text{and} \quad \theta_3 := \theta_4 + 90 \cdot \text{deg}$$

Substituting the Euler equivalents,

$$a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) + f \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4)) := d + c \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4)) \dots + b \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

If we stipulate that $f = c$, this reduces to

$$a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) := d + b \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

Separating into real and imaginary components and solving for θ_3 and b .

$$\theta_3 := \text{atan2}(a \cdot \cos(\theta_2) - d, a \cdot \sin(\theta_2)) \quad \theta_3 = 174.7 \text{ deg}$$

$$\theta_4 := \theta_3 - 90 \text{ deg} \quad \theta_4 = 84.7 \text{ deg}$$

$$b := \frac{a \cdot \sin(\theta_2)}{\sin(\theta_3)} \quad b = 42.9 \text{ mm}$$

3. Differentiate the position equation to get the velocity equation.

$$\text{Position (f and c terms cancel)} \quad a \cdot e^{j \cdot \theta_2} := d + b \cdot e^{j \cdot \theta_3}$$

$$\text{Velocity} \quad a \cdot j \cdot \omega_2 \cdot e^{j \cdot \theta_2} := b \dot{\theta}_3 \cdot e^{j \cdot \theta_3} + b \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3}$$

Separating into real and imaginary components and solving for ω_3 and $b \dot{\theta}_3$.

$$\omega_3 := \frac{a \cdot \cos(\theta_2 - \theta_3)}{b} \cdot \omega_2 \quad \omega_3 = 1.00 \frac{\text{rad}}{\text{sec}}$$

$$b \dot{\theta}_3 := \frac{b \cdot \omega_3 \cdot \sin(\theta_3) - a \cdot \omega_2 \cdot \sin(\theta_2)}{\cos(\theta_3)} \quad b \dot{\theta}_3 = 35.8 \frac{\text{mm}}{\text{sec}}$$

4. Differentiate the velocity equation to get the acceleration equation.

$$\text{Velocity} \quad a \cdot j \cdot \omega_2 \cdot e^{j \cdot \theta_2} := b \dot{\theta}_3 \cdot e^{j \cdot \theta_3} + b \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3}$$

$$\begin{aligned} \text{Acceleration} \quad a \cdot j \cdot \alpha_2 \cdot e^{j \cdot \theta_2} + a \cdot j^2 \cdot \omega_2^2 \cdot e^{j \cdot \theta_2} &:= b \ddot{\theta}_3 \cdot e^{j \cdot \theta_3} + b \dot{\theta}_3 \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} \dots \\ &+ b \cdot j \cdot \alpha_3 \cdot e^{j \cdot \theta_3} + b \cdot j^2 \cdot \omega_3^2 \cdot e^{j \cdot \theta_3} \end{aligned}$$

Separating into real and imaginary components and solving for α_3 and $b \ddot{\theta}_3$.

$$\alpha_3 := \frac{1}{b} \cdot (a \cdot \alpha_2 \cdot \cos(\theta_2 - \theta_3) - a \cdot \omega_2^2 \cdot \sin(\theta_2 - \theta_3) - b \dot{\theta}_3 \cdot \omega_3)$$

$$\alpha_3 = 9.50 \frac{\text{rad}}{\text{sec}^2}$$

$$b \ddot{\theta}_3 := \frac{1}{\sin(\theta_3)} \cdot \left[a \cdot (\alpha_2 \cdot \cos(\theta_2) - \omega_2^2 \cdot \sin(\theta_2)) \dots \right. \\ \left. + -b \dot{\theta}_3 \cdot \omega_3 \cdot \cos(\theta_3) - b \cdot (\alpha_3 \cdot \cos(\theta_3) - \omega_3^2 \cdot \sin(\theta_3)) \right]$$

$$b \ddot{\theta}_3 = -316.0 \frac{\text{mm}}{\text{sec}^2}$$

PROBLEM 7-17

Statement: For the linkage shown in Figure P7-6b, write the vector loop equations; differentiate them, and do a complete position, velocity, and acceleration analysis of the linkage. Use the linkage geometry given below. Assume $\omega_2 = 10 \text{ rad/sec}$ and $\alpha_2 = 20 \text{ rad/sec}^2$.

Given: Measured lengths and angles:

Link 1	$d := 61.9 \cdot \text{mm}$	Link 2	$a := 15.0 \cdot \text{mm}$
Link 3	$b := 45.8 \cdot \text{mm}$	Link 4	$c := 18.1 \cdot \text{mm}$
Link 5	$e := 23.1 \cdot \text{mm}$	Offset	$f := -2.6 \cdot \text{mm}$ from x' axis

Crank angle: $\theta_{20} := 45 \cdot \text{deg}$ Global XY system

Coordinate rotation angles $\alpha := -23.3 \cdot \text{deg}$ Global XY system to local xy system

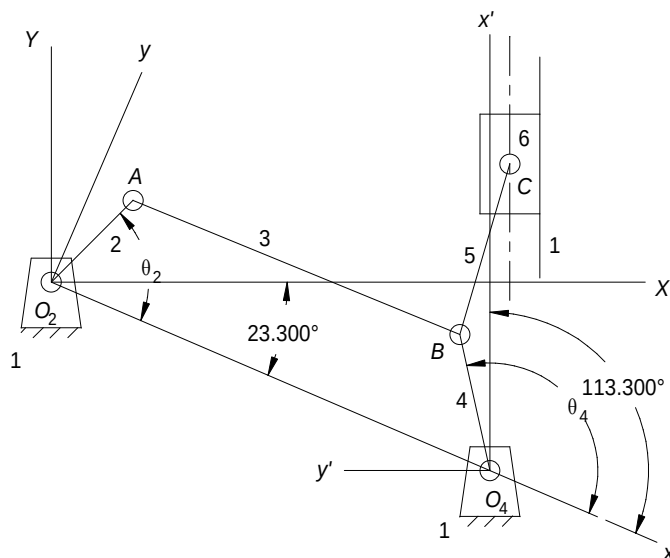
$\beta := 113.3 \cdot \text{deg}$ Local xy system to local $x'y'$ system

$\gamma := 90 \cdot \text{deg}$ Global XY system to local $x'y'$ system

Link 2 position, velocity and acceleration $\theta_2 := 68.3 \cdot \text{deg}$ $\omega_2 := 10 \cdot \frac{\text{rad}}{\text{sec}}$ $\alpha_2 := 20 \cdot \frac{\text{rad}}{\text{sec}^2}$

Solution: See Figure P7-6b and Mathcad file P0717.

1. Draw the mechanism to scale and label it.



2. This mechanism can be analyzed as a pin-jointed fourbar (links 1, 2, 3, and 4) and a fourbar slider-crank (links 1, 4, 5, and 6). Link 4 is the common, non-stationary link in the two branches and is the input to the slider-crank. Since the vector loops are the same as those defined for the pin-jointed and slider-crank linkages, we can use the equations derived in the text with slight modification of variable names for the slider-crank.

There are three coordinate systems: the global XY frame, the local xy frame for the pin-jointed fourbar, and the local $x'y'$ frame for the slider crank. Inputs to the fourbars must be in their respective local coordinates.

All output will be given in local coordinate system.

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_1 = 4.1267 \quad K_2 := \frac{d}{c} \quad K_2 = 3.4199$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 4.2110$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad A = -0.8104$$

$$B := -2 \cdot \sin(\theta_2) \quad B = -1.8583$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3 \quad C = 6.7034$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \text{atan2}(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}) + 360 \cdot \text{deg} \quad \theta_4 = 125.692 \text{ deg}$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.3515 \quad K_5 = -4.2406$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -7.4978$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.8583$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.0160$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2}(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}) \right) + 360 \cdot \text{deg} \quad \theta_3 = 0.955 \text{ deg}$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad \omega_3 = -3.357 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad \omega_4 = 9.307 \frac{\text{rad}}{\text{sec}}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_B$. Determine the magnitude, and direction (in the local coordinate system).

$$\mathbf{A}_B := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_B = (-833 - 1283j) \frac{\text{mm}}{\text{sec}^2} \quad A_B := |\mathbf{A}_B| \quad \theta_{AB} := \arg(\mathbf{A}_B)$$

$$\text{The acceleration of pin } B \text{ is } A_B = 1530 \frac{\text{mm}}{\text{sec}^2} \quad \text{at } \theta_{AB} = -123.01 \text{ deg}$$

9. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c \cdot \sin(\theta_4) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_4) \quad E := b \cdot \cos(\theta_3)$$

$$A = 14.700 \text{ mm} \quad B = 0.763 \text{ mm} \quad D = -10.560 \text{ mm} \quad E = 45.794 \text{ mm}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$C = 2.264 \times 10^3 \text{ mm} \cdot \text{sec}^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = -18.160 \text{ mm} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = -34.706 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = 152.221 \frac{\text{rad}}{\text{sec}^2}$$

9. Use equation 7.13c to determine the acceleration of point C.

$$\mathbf{A}_C := c \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$\mathbf{A}_C = (-1323 - 2881j) \frac{\text{mm}}{\text{sec}^2} \quad A_C := |\mathbf{A}_C| \quad \theta_{AC} := \arg(\mathbf{A}_C)$$

$$\text{The acceleration of pin C is } A_C = 3170 \frac{\text{mm}}{\text{sec}^2} \quad \text{at } \theta_{AC} = -114.7 \text{ deg}$$

10. Determine θ_5 and the vertical distance from O_4 to C (d') for the slider-crank using equations 4.16 and 4.17.

$$\theta_5 := \text{asin}\left(-\frac{c \cdot \sin(\theta_4 - \beta) - f}{e}\right) + \pi \quad \theta_5 = 163.7 \text{ deg}$$

$$d' := c \cdot \cos(\theta_4 - \beta) - e \cdot \cos(\theta_5) \quad d' = 39.8 \text{ mm}$$

11. Determine the angular velocity of link 5 using equation 6.22a.

$$\omega_5 := \frac{c}{e} \cdot \frac{\cos(\theta_4 - \beta)}{\cos(\theta_5)} \cdot \omega_4 \quad \omega_5 = -7.421 \frac{\text{rad}}{\text{sec}}$$

12. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5 := \frac{c \cdot \alpha_4 \cdot \cos(\theta_4 - \beta) - c \cdot \omega_4^2 \cdot \sin(\theta_4 - \beta) + e \cdot \omega_5^2 \cdot \sin(\theta_5)}{e \cdot \cos(\theta_5)} \quad \alpha_5 = -122.305 \frac{\text{rad}}{\text{sec}^2}$$

13. Use equation 7.16e for the acceleration of pin C.

$$A_C := -c \cdot \alpha_4 \cdot \sin(\theta_4 - \beta) - c \cdot \omega_4^2 \cdot \cos(\theta_4 - \beta) + e \cdot \alpha_5 \cdot \sin(\theta_5) + e \cdot \omega_5^2 \cdot \cos(\theta_5)$$

$$A_C = -162.9 \frac{\text{in}}{\text{sec}^2} \quad \text{A negative sign means that } \mathbf{A}_C \text{ is downward}$$

PROBLEM 7-18

Statement: For the linkage shown in Figure P7-6c, write the vector loop equations; differentiate them, and do a complete position, velocity, and acceleration analysis of the linkage. Use the linkage geometry given below. Assume $\omega_2 = 10 \text{ rad/sec}$ and $\alpha_2 = 20 \text{ rad/sec}^2$.

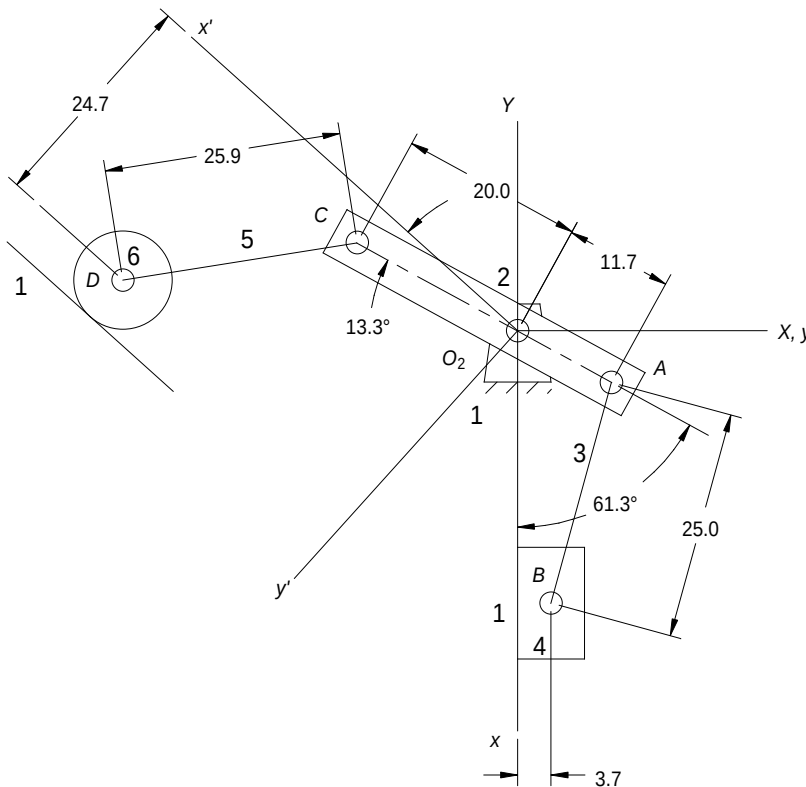
Given: Measured lengths and angles:

Link 21	$a1 := 11.7\text{-mm}$	Link 22	$a2 := 20.0\text{-mm}$
Link 3	$b1 := 25.0\text{-mm}$	Link 5	$b2 := 25.9\text{-mm}$
Offset	$c1 := 3.7\text{-mm}$	Offset'	$c2 := 24.7\text{-mm}$ from x' axis
Crank angle:	$\theta_{21} := 61.3\text{-deg}$		$\theta_{22} := 13.3\text{-deg}$

Link 2 velocity and acceleration $\omega_2 := 10 \cdot \frac{\text{rad}}{\text{sec}}$ $\alpha_2 := 20 \cdot \frac{\text{rad}}{\text{sec}^2}$

Solution: See Figure P7-6c and Mathcad file P0718.

1. Draw the mechanism to scale and label it.



2. This mechanism can be analyzed as a fourbar slider-crank (links 1, 2, 3, and 4) and a fourbar slider-crank (links 1, 2, 5, and 6). Link 2 is the common, non-stationary link in the two branches and is the input to both slider-cranks. Since the vector loops are the same as those defined for the slider-crank linkage, we can use the equations derived in the text with slight modification of variable names.

There are three coordinate systems: the global XY frame, the local xy frame for the first slider-crank, and the local $x'y'$ frame for the second slider crank. Inputs to the fourbars must be in their respective local coordinates. **All output will be given in local coordinate system.**

3. Determine θ_3 and the vertical distance from O_2 to B ($d1$) for the first slider-crank using equations 4.16 and 4.17.

$$\theta_3 := \operatorname{asin}\left(-\frac{a1 \cdot \sin(\theta_{21}) - c1}{b1}\right) + \pi \quad \theta_3 = 164.8 \text{ deg}$$

$$d1 := a1 \cdot \cos(\theta_{21}) - b1 \cdot \cos(\theta_3) \quad d1 = 29.7 \text{ mm}$$

4. Determine the angular velocity of link 3 using equation 6.22a.

$$\omega_3 := \frac{a1}{b1} \cdot \frac{\cos(\theta_{21})}{\cos(\theta_3)} \cdot \omega_2 \quad \omega_3 = -2.329 \frac{\text{rad}}{\text{sec}}$$

5. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its magnitude, and direction.

$$\mathbf{A}_A := a1 \cdot \alpha_2 \cdot (-\sin(\theta_{21}) + j \cdot \cos(\theta_{21})) - a1 \cdot \omega_2^2 \cdot (\cos(\theta_{21}) + j \cdot \sin(\theta_{21}))$$

$$\mathbf{A}_A = (-767.114 - 913.889i) \frac{\text{mm}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A)$$

The acceleration of pin A is $A_A = 1193 \frac{\text{mm}}{\text{sec}^2}$ at $\theta_{AA} = -130.0 \text{ deg}$

6. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3 := \frac{a1 \cdot \alpha_2 \cdot \cos(\theta_{21}) - a1 \cdot \omega_2^2 \cdot \sin(\theta_{21}) + b1 \cdot \omega_3^2 \cdot \sin(\theta_3)}{b1 \cdot \cos(\theta_3)} \quad \alpha_3 = 36.4 \frac{\text{rad}}{\text{sec}^2}$$

7. Use equation 7.16e for the acceleration of pin B.

$$A_B := -a1 \cdot \alpha_2 \cdot \sin(\theta_{21}) - a1 \cdot \omega_2^2 \cdot \cos(\theta_{21}) + b1 \cdot \alpha_3 \cdot \sin(\theta_3) + b1 \cdot \omega_3^2 \cdot \cos(\theta_3)$$

$$A_B = -659 \frac{\text{mm}}{\text{sec}^2} \quad \text{A negative sign means that } \mathbf{A}_B \text{ is upward}$$

8. Determine θ_5 and the distance perpendicular to the x' axis from O_2 to BD ($d2$) for the second slider-crank using equations 4.16 and 4.17.

$$\theta_5 := \operatorname{asin}\left(-\frac{a2 \cdot \sin(\theta_{22}) - c2}{b2}\right) + \pi \quad \theta_5 = 230.9 \text{ deg}$$

$$d2 := a2 \cdot \cos(\theta_{22}) - b2 \cdot \cos(\theta_5) \quad d2 = 35.8 \text{ mm}$$

9. Determine the angular velocity of link 5 using equation 6.22a.

$$\omega_5 := \frac{a2}{b2} \cdot \frac{\cos(\theta_{22})}{\cos(\theta_5)} \cdot \omega_2 \quad \omega_5 = -11.915 \frac{\text{rad}}{\text{sec}}$$

10. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_C$, determine its magnitude, and direction.

$$\mathbf{A}_C := a2 \cdot \alpha_2 \cdot (-\sin(\theta_{22}) + j \cdot \cos(\theta_{22})) - a2 \cdot \omega_2^2 \cdot (\cos(\theta_{22}) + j \cdot \sin(\theta_{22}))$$

$$\mathbf{A}_C = (-2038.378 - 70.828i) \frac{\text{mm}}{\text{sec}^2} \quad A_C := |\mathbf{A}_C| \quad \theta_{AC} := \arg(\mathbf{A}_C)$$

The acceleration of pin C is $A_C = 2040 \frac{mm}{sec^2}$ $\theta_{AC} = -179.0 \text{ deg}$

11. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5 := \frac{a_2 \cdot \alpha_2 \cdot \cos(\theta_{22}) - a_2 \cdot \omega_2^2 \cdot \sin(\theta_{22}) + b_2 \cdot \omega_5^2 \cdot \sin(\theta_5)}{b_2 \cdot \cos(\theta_5)} \quad \alpha_5 = 179.0 \frac{rad}{sec^2}$$

12. Use equation 7.16e for the acceleration of pin D .

$$A_D := -a_2 \cdot \alpha_2 \cdot \sin(\theta_{22}) - a_2 \cdot \omega_2^2 \cdot \cos(\theta_{22}) + b_2 \cdot \alpha_5 \cdot \sin(\theta_5) + b_2 \cdot \omega_5^2 \cdot \cos(\theta_5)$$

$$A_D = -7956 \frac{mm}{sec^2} \quad \text{A negative sign means that } \mathbf{A}_D \text{ is downward and to the right}$$

PROBLEM 7-19

Statement: For the linkage shown in Figure P7-6d, write the vector loop equations; differentiate them, and do a complete position, velocity, and acceleration analysis of the linkage. Use the linkage geometry given below. Assume $\omega_2 = 10 \text{ rad/sec}$ and $\alpha_2 = 20 \text{ rad/sec}^2$.

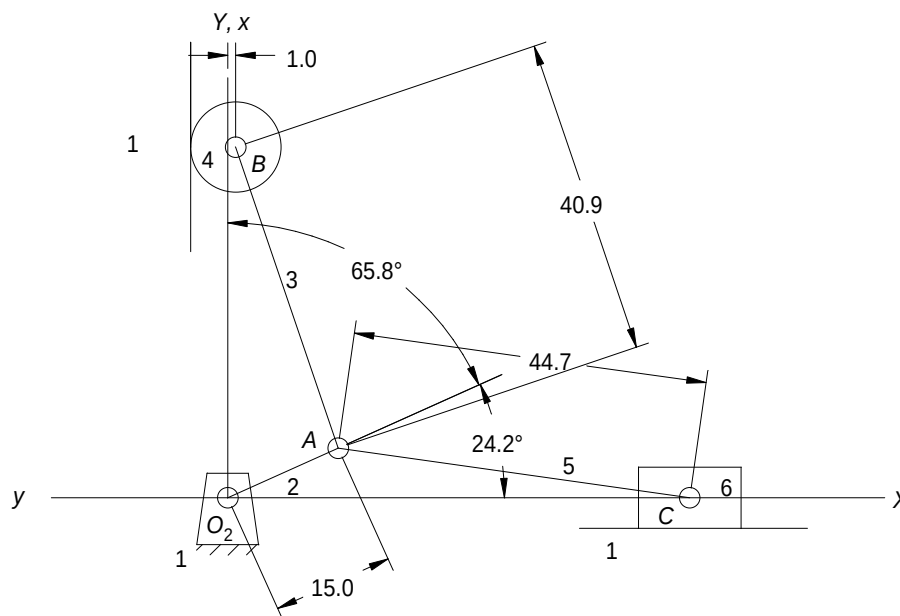
Given: Measured lengths and angles:

Link 21	$a1 := 15.0\text{-mm}$	Link 22	$a2 := 15\text{-mm}$
Link 3	$b1 := 40.9\text{-mm}$	Link 5	$b2 := 44.7\text{-mm}$
Offset	$c1 := -1.0\text{-mm}$	Offset'	$c2 := 0.0\text{-mm}$ from x' axis
Crank angle:	$\theta_{21} := -65.8\text{-deg}$		$\theta_{22} := 24.2\text{-deg}$

Link 2 velocity and acceleration $\omega_2 := 10 \cdot \frac{\text{rad}}{\text{sec}}$ $\alpha_2 := 20 \cdot \frac{\text{rad}}{\text{sec}^2}$

Solution: See Figure P7-6d and Mathcad file P0719.

1. Draw the mechanism to scale and label it.



2. This mechanism can be analyzed as a fourbar slider-crank (links 1, 2, 3, and 4) and a fourbar slider-crank (links 1, 2, 5, and 6). Link 2 is the common, non-stationary link in the two branches and is the input to both slider-cranks. Since the vector loops are the same as those defined for the slider-crank linkage, we can use the equations derived in the text with slight modification of variable names.

There are two coordinate systems: the global XY frame and the local xy frame for the first slider-crank. Inputs to the fourbars must be in their respective local coordinates. **All output will be given in local coordinate system.**

3. Determine θ_3 and the vertical distance from O_2 to B ($d1$) for the first slider-crank using equations 4.16 and 4.17.

$$\theta_3 := \text{asin}\left(-\frac{a1 \cdot \sin(\theta_{21}) - c1}{b1}\right) + \pi \quad \theta_3 = 198.1 \text{ deg}$$

$$d1 := a1 \cdot \cos(\theta_{21}) - b1 \cdot \cos(\theta_3) \quad d1 = 45.0 \text{ mm}$$

4. Determine the angular velocity of link 3 using equation 6.22a.

$$\omega_3 := \frac{a1}{b1} \cdot \frac{\cos(\theta_{21})}{\cos(\theta_3)} \cdot \omega_2 \quad \omega_3 = -1.581 \frac{\text{rad}}{\text{sec}}$$

5. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its magnitude, and direction.

$$\mathbf{A}_A := a1 \cdot \alpha_2 \cdot (-\sin(\theta_{21}) + j \cdot \cos(\theta_{21})) - a1 \cdot \omega_2^2 \cdot (\cos(\theta_{21}) + j \cdot \sin(\theta_{21}))$$

$$\mathbf{A}_A = (-341.249 + 1491.157i) \frac{\text{mm}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A)$$

The acceleration of pin A is $A_A = 1530 \frac{\text{mm}}{\text{sec}^2}$ at $\theta_{AA} = 102.9 \text{ deg}$

6. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3 := \frac{a1 \cdot \alpha_2 \cdot \cos(\theta_{21}) - a1 \cdot \omega_2^2 \cdot \sin(\theta_{21}) + b1 \cdot \omega_3^2 \cdot \sin(\theta_3)}{b1 \cdot \cos(\theta_3)} \quad \alpha_3 = -37.5 \frac{\text{rad}}{\text{sec}^2}$$

7. Use equation 7.16e for the acceleration of pin B.

$$A_B := -a1 \cdot \alpha_2 \cdot \sin(\theta_{21}) - a1 \cdot \omega_2^2 \cdot \cos(\theta_{21}) + b1 \cdot \alpha_3 \cdot \sin(\theta_3) + b1 \cdot \omega_3^2 \cdot \cos(\theta_3)$$

$$A_B = 37.5 \frac{\text{mm}}{\text{sec}^2} \quad \text{A positive sign means that } \mathbf{A}_B \text{ is upward}$$

8. Determine θ_5 and the distance perpendicular to the x' axis from O_2 to BD ($d2$) for the second slider-crank using equations 4.16 and 4.17.

$$\theta_5 := \text{asin}\left(-\frac{a2 \cdot \sin(\theta_{22}) - c2}{b2}\right) + \pi \quad \theta_5 = 172.1 \text{ deg}$$

$$d2 := a2 \cdot \cos(\theta_{22}) - b2 \cdot \cos(\theta_5) \quad d2 = 58.0 \text{ mm}$$

9. Determine the angular velocity of link 5 using equation 6.22a.

$$\omega_5 := \frac{a2}{b2} \cdot \frac{\cos(\theta_{22})}{\cos(\theta_5)} \cdot \omega_2 \quad \omega_5 = -3.090 \frac{\text{rad}}{\text{sec}}$$

10. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5 := \frac{a2 \cdot \alpha_2 \cdot \cos(\theta_{22}) - a2 \cdot \omega_2^2 \cdot \sin(\theta_{22}) + b2 \cdot \omega_5^2 \cdot \sin(\theta_5)}{b2 \cdot \cos(\theta_5)} \quad \alpha_5 = 6.4 \frac{\text{rad}}{\text{sec}^2}$$

11. Use equation 7.16e for the acceleration of pin C.

$$A_C := -a2 \cdot \alpha_2 \cdot \sin(\theta_{22}) - a2 \cdot \omega_2^2 \cdot \cos(\theta_{22}) + b2 \cdot \alpha_5 \cdot \sin(\theta_5) + b2 \cdot \omega_5^2 \cdot \cos(\theta_5)$$

$$A_C = -1875 \frac{\text{mm}}{\text{sec}^2} \quad \text{A negative sign means that } \mathbf{A}_C \text{ is to the left}$$

PROBLEM 7-20

Statement: Figure P7-7 shows a sixbar linkage with the dimensions and crank angle given below. Find the angular acceleration of link 6 if ω_2 is a constant 1 rad/sec.

Given: Link lengths:

$$\text{Link 2 } a := 1.00 \cdot \text{in} \quad \text{Link 3 } b := 5.00 \cdot \text{in} \quad \text{Offset: } c := 0.0 \cdot \text{in}$$

$$\text{Link 6 } d := 3.00 \cdot \text{in} \quad \text{Distance DB } L_{DB} := 1.50 \cdot \text{in}$$

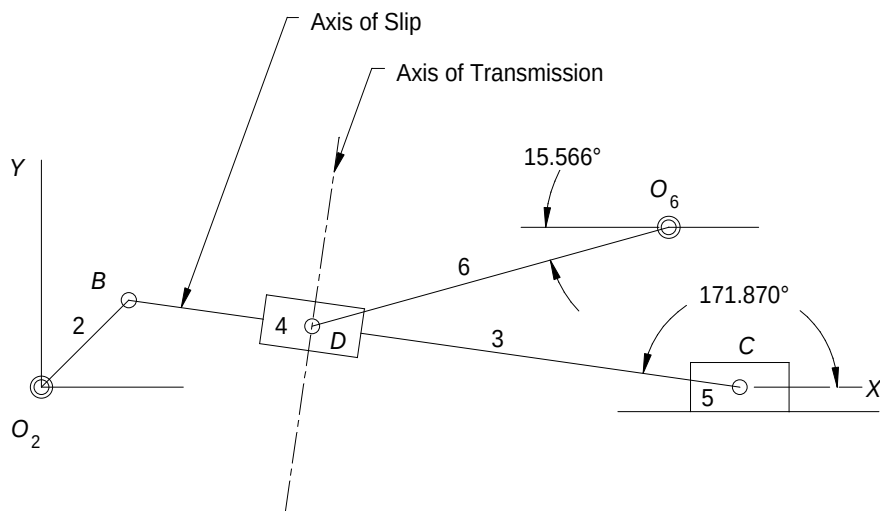
$$\text{Link 2 position, velocity, and acceleration } \theta_2 := 45 \cdot \text{deg} \quad \omega_2 := 1 \cdot \frac{\text{rad}}{\text{sec}} \quad \alpha_2 := 0 \cdot \frac{\text{rad}}{\text{sec}^2}$$

Solution: See Figure P7-7 and Mathcad file P0720.

- Links 1, 2, 3, and 5 constitute a fourbar slider-crank. In order to solve for the accelerations at points B, C and D, we will need θ_3 , and ω_3 . From the graphical position solution below,

$$\theta_3 := 171.870 \cdot \text{deg}$$

Using equation 6.22a,
$$\omega_3 := \frac{a \cdot \omega_2 \cdot \cos(\theta_2)}{b \cdot \cos(\theta_3)} \quad \omega_3 = -0.143 \text{ rad} \cdot \text{sec}^{-1} \quad \text{CW}$$



- The graphical solution for accelerations of pin-jointed linkages uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$
- For point C, this becomes: $\mathbf{A}_C = (\mathbf{A}_B^t + \mathbf{A}_B^n) + (\mathbf{A}_{CB}^t + \mathbf{A}_{CB}^n)$, where

$$A_{Bn} := a \cdot \omega_2^2 \quad A_{Bn} = 1.000 \text{ in} \cdot \text{sec}^{-2}$$

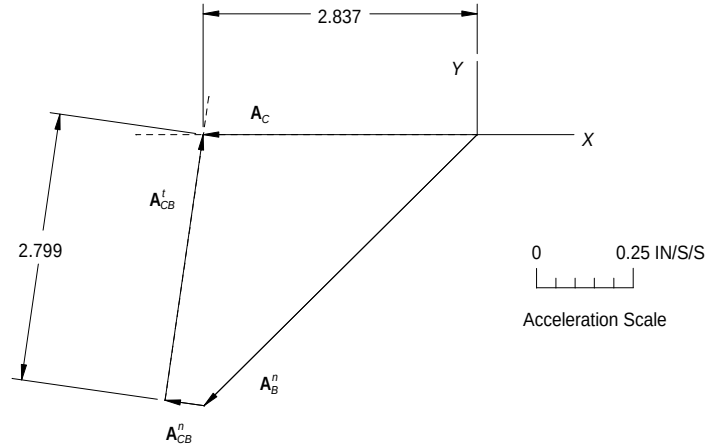
$$\theta_{ABn} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{ABn} = 225.000 \text{ deg}$$

$$A_{Bt} := a \cdot \alpha_2 \quad A_{Bt} = 0.000 \text{ in} \cdot \text{sec}^{-2}$$

$$A_{CBn} := b \cdot \omega_3^2 \quad A_{CBn} = 0.102 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ACBn} := \theta_3 \quad \theta_{ACBn} = 171.870 \text{ deg}$$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of θ_{ABn} . From the tip of $\mathbf{A}_B^n$, draw $\mathbf{A}_{CB}^n$ at an angle of θ_{ACBn} . Now that the vectors with known magnitudes are drawn, from the origin and the tip of $\mathbf{A}_{CB}^n$, draw construction lines in the directions of $\mathbf{A}_C$ (horizontal) and $\mathbf{A}_{CB}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_{CB}^t$, and $\mathbf{A}_C$.



5. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 0.25 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_C := 2.837 \cdot k_a \quad A_C = 0.709 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 180 \text{ deg}$$

$$A_{CBt} := 2.799 \cdot k_a \quad A_{CBt} = 0.700 \text{ in} \cdot \text{sec}^{-2}$$

6. Calculate α_3 using equation 7.6.

$$\alpha_3 := \frac{A_{CBt}}{b} \quad \alpha_3 = 0.140 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CCW}$$

7. In order to solve for the acceleration at points D , we will need θ_6 , and ω_6 . From the graphical position solution above,

$$\theta_6 := 15.566 \cdot \text{deg} + 180 \cdot \text{deg} \quad \theta_6 = 195.566 \text{ deg}$$

From the vector diagram in Figure P7-7,

$$V_D := 0.40 \cdot \frac{\text{in}}{\text{sec}} \quad \text{and} \quad \omega_6 := \frac{V_D}{d} \quad \omega_6 = 0.133 \frac{\text{rad}}{\text{sec}}$$

$$V_{slip} := 0.65 \cdot \frac{\text{in}}{\text{sec}}$$

8. For point D , use equation 7.19, referenced to point B instead of O_2 :

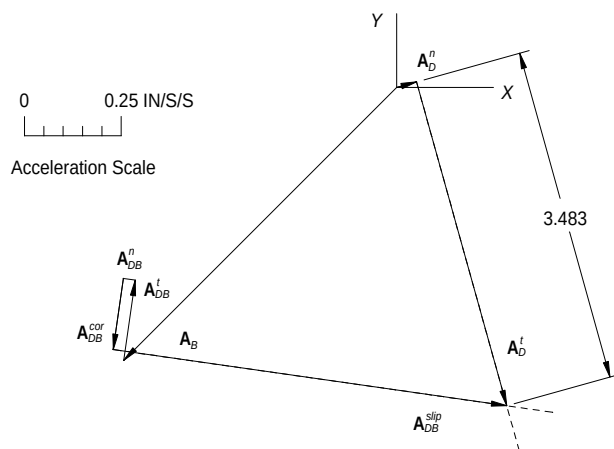
$$(\mathbf{A}_D^t + \mathbf{A}_D^n) = \mathbf{A}_B + (\mathbf{A}_{DB}^t + \mathbf{A}_{DB}^n + \mathbf{A}_{DB}^{cor} + \mathbf{A}_{DB}^{slip}), \text{ where}$$

$$A_{Dn} := d \cdot \omega_6^2 \quad A_{Dn} = 0.053 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ADn} := \theta_6 - 180 \cdot \text{deg} \quad \theta_{ADn} = 15.566 \text{ deg}$$

$$\begin{aligned}
 A_B &:= a \cdot \omega_2^2 & A_B &= 1.000 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{AB} &:= \theta_2 + 180 \cdot \text{deg} & \theta_{AB} &= 225.000 \text{ deg} \\
 A_{DBt} &:= L_{DB} \cdot \alpha_3 & A_{DBt} &= 0.210 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{ADBt} &:= \theta_3 - 90 \cdot \text{deg} & \theta_{ADBt} &= 81.870 \text{ deg} \\
 A_{DBn} &:= L_{DB} \cdot \omega_3^2 & A_{DBn} &= 0.031 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{ADBn} &:= \theta_3 & \theta_{ADBn} &= 171.870 \text{ deg} \\
 A_{DBcor} &:= |2 \cdot V_{slip} \cdot \omega_3| & A_{DBcor} &= 0.186 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{ADBcor} &:= \theta_3 + 90 \cdot \text{deg} & \theta_{ADBcor} &= 261.870 \text{ deg}
 \end{aligned}$$

9. Repeat procedure of step 4 for the equation in step 8.



10. From the graphical solution above,

$$\begin{aligned}
 \text{Acceleration scale factor} \quad k_a &:= 0.25 \cdot \frac{\text{in}}{\text{sec}^2} \\
 A_{Dt} &:= 3.483 \cdot k_a & A_{Dt} &= 0.871 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -74.434 \text{ deg}
 \end{aligned}$$

The angular acceleration of link 6 is

$$\alpha_6 := \frac{A_{Dt}}{d} \quad \alpha_6 = 0.290 \frac{\text{rad}}{\text{sec}^2} \quad \text{CCW}$$

PROBLEM 7-21

Statement: The linkage in Figure P7-8a has the dimensions and crank angle given below. Find α_4 , $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown for $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW) and $\alpha_2 = 25 \text{ rad/sec}^2$ CCW. Use the acceleration difference graphical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 116 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad b := 108 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4) \quad c := 110 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 174 \cdot \text{mm}$$

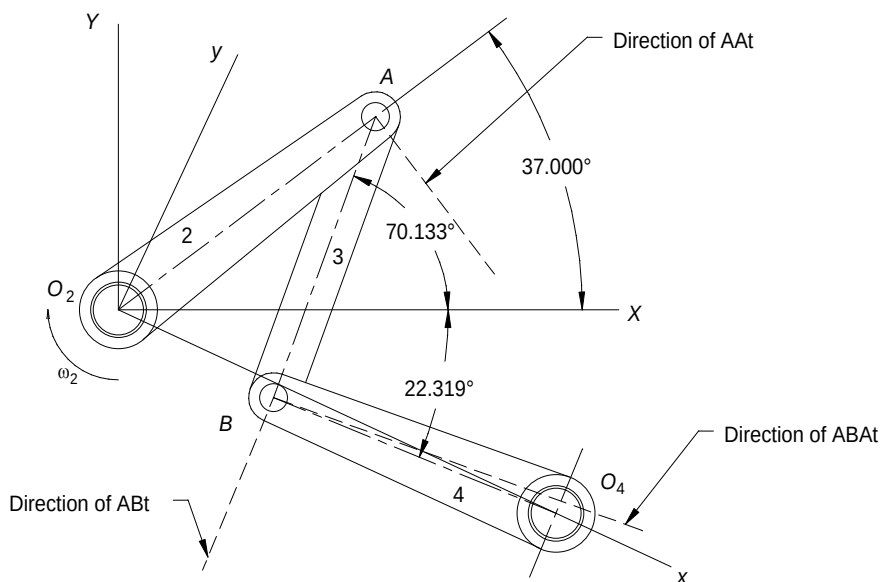
$$\text{Crank angle:} \quad \theta_2 := 62 \cdot \text{deg} \quad \text{Local } xy \text{ system}$$

$$\text{Input crank angular velocity} \quad \omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 25 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\text{Coordinate rotation angle} \quad \alpha := -25 \cdot \text{deg} \quad \text{Global } XY \text{ system to local } xy \text{ system}$$

Solution: See Figure P7-8a and Mathcad file P0721.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A and B, we will need θ_3 , ω_3 , and θ_4 . From the graphical position solution above (in the local xy coordinate system),

$$\theta_3 := 180 \cdot \text{deg} + 70.133 \cdot \text{deg} - \alpha \quad \theta_3 = 275.133 \cdot \text{deg}$$

$$\theta_4 := 180 \cdot \text{deg} - 22.319 \cdot \text{deg} - \alpha \quad \theta_4 = 182.681 \cdot \text{deg}$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_4 - \theta_2)}{b \cdot \sin(\theta_3 - \theta_4)} \quad \omega_3 = -13.869 \cdot \text{rad} \cdot \text{sec}^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_4 - \theta_3)} \quad \omega_4 = 8.654 \cdot \text{rad} \cdot \text{sec}^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

4. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_{Bn} := c \cdot \omega_4^2 \quad A_{Bn} = 8237.9 \text{ mm} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 - 180 \cdot \text{deg} \quad \theta_{ABn} = 2.681 \text{ deg}$$

$$A_{An} := a \cdot \omega_2^2 \quad A_{An} = 26100.0 \text{ mm} \cdot \text{sec}^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AAn} = 242.000 \text{ deg}$$

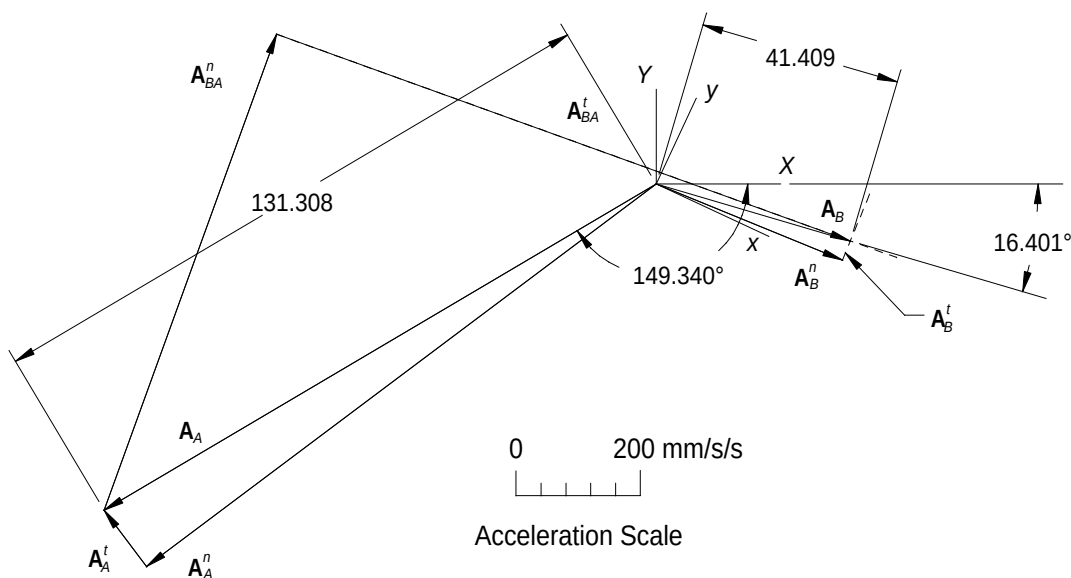
$$A_{At} := a \cdot \alpha_2 \quad A_{At} = 2900.0 \text{ mm} \cdot \text{sec}^{-2}$$

$$\theta_{AA_t} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{AA_t} = 152.000 \text{ deg}$$

$$A_{BA_n} := b \cdot \omega_3^2 \quad A_{BA_n} = 20773 \text{ mm} \cdot \text{sec}^{-2}$$

$$\theta_{ABA_n} := \theta_3 - 180 \cdot \text{deg} \quad \theta_{ABA_n} = 95.133 \text{ deg}$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of $\theta_{ABn} + \alpha$ and $\mathbf{A}_A^n$ at an angle of $\theta_{AAn} + \alpha$. From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_A^t$ at an angle of $\theta_{AA_t} + \alpha$. From the tip of $\mathbf{A}_A^t$, draw $\mathbf{A}_{BA}^n$ at an angle of $\theta_{BA_n} + \alpha$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B^n$ and $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B^t$, $\mathbf{A}_B^t$, and $\mathbf{A}_{BA}^t$.



6. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 200 \cdot \frac{\text{mm}}{\text{sec}^2}$$

$$A_A := 131.308 \cdot k_a \quad A_A = 26262 \text{ mm} \cdot \text{sec}^{-2} \quad \text{at an angle of } -149.34 \text{ deg}$$

$$A_B := 41.409 \cdot k_a \quad A_B = 8282 \text{ mm} \cdot \text{sec}^{-2} \quad \text{at an angle of } -16.40 \text{ deg}$$

PROBLEM 7-22

Statement: The linkage in Figure P7-8a has the dimensions and crank angle given below. Find α_4 , A_A , and A_B in the global coordinate system for the position shown for $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW) and $\alpha_2 = 25 \text{ rad/sec}^2$ CCW. Use an analytical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 116 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad b := 108 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4) \quad c := 110 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 174 \cdot \text{mm}$$

$$\text{Crank angle:} \quad \theta_2 := 62 \cdot \text{deg} \quad \text{Local } xy \text{ system}$$

$$\text{Input crank angular velocity} \quad \omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 25 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\text{Coordinate rotation angle} \quad \beta := -25 \cdot \text{deg} \quad \text{Global } XY \text{ system to local } xy \text{ system}$$

Solution: See Figure P7-8a and Mathcad file P0722.

1. Draw the linkage to scale and label it.

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 1.5000 \quad K_2 = 1.5818$$

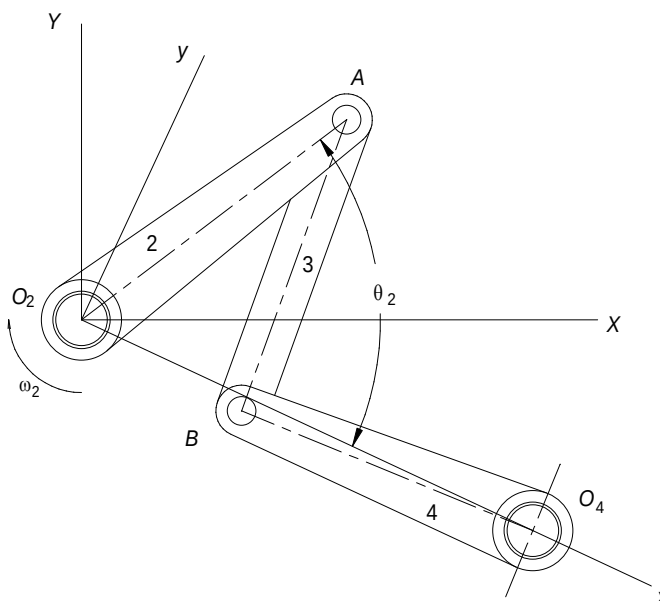
$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.7307$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.0424 \quad B = -1.7659 \quad C = 2.0186$$



3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_4 = 182.681 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.6111 \quad K_5 = -1.7280$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -2.0021$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.7659$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.0589$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_3 = 275.133 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad \omega_3 = -13.869 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad \omega_4 = 8.654 \frac{\text{rad}}{\text{sec}}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction (in the global coordinate system).

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (-14814 - 21683i) \frac{\text{mm}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A) + \beta$$

$$\text{The acceleration of pin A is } A_A = 26261 \frac{\text{mm}}{\text{sec}^2} \quad \text{at } \theta_{AA} = -149.3 \text{ deg}$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c \cdot \sin(\theta_4) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_4) \quad E := b \cdot \cos(\theta_3)$$

$$A = -5.145 \text{ mm} \quad B = -107.567 \text{ mm} \quad D = -109.880 \text{ mm} \quad E = 9.662 \text{ mm}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$C = 2.490 \times 10^4 \text{ mm} \cdot \text{sec}^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = -1.380 \times 10^3 \text{ mm} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = 231.119 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = -7.768 \frac{\text{rad}}{\text{sec}^2}$$

9. Use equation 7.13c to determine the acceleration of point B for the crossed circuit.

$$\mathbf{A}_B := c \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$\mathbf{A}_B = (8189 + 1239i) \frac{\text{mm}}{\text{sec}^2} \quad A_B := |\mathbf{A}_B| \quad \theta_{AB} := \arg(\mathbf{A}_B) + \beta$$

$$\text{The acceleration of pin B is } A_B = 8282 \frac{\text{mm}}{\text{sec}^2} \quad \text{at } \theta_{AB} = -16.4 \text{ deg} \quad (\text{Global})$$

PROBLEM 7-23

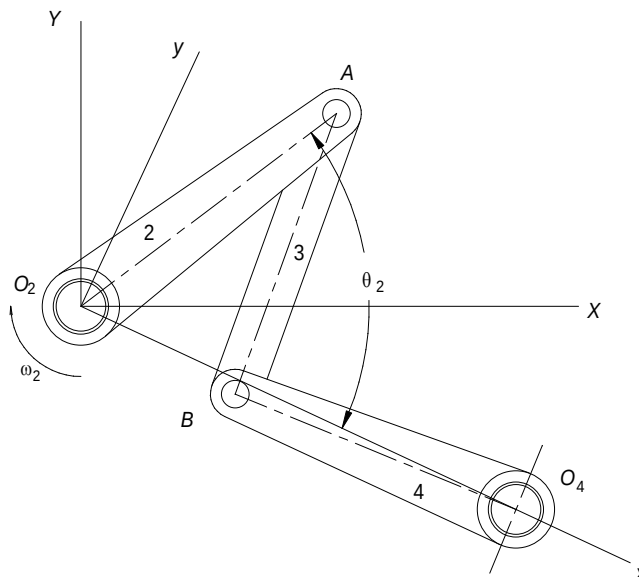
Statement: The linkage in Figure P7-8a has the dimensions and crank angle given below. Find and plot α_4 , $\mathbf{A}_A$, and $\mathbf{A}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 15 \text{ rad/sec CW}$ and $\alpha_2 = 25 \text{ rad/sec}^2 \text{ CCW}$.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 116 \cdot \text{mm}$	Link 3 (A to B)	$b := 108 \cdot \text{mm}$
Link 4 (B to O_4)	$c := 110 \cdot \text{mm}$	Link 1 (O_2 to O_4)	$d := 174 \cdot \text{mm}$
Input crank angular velocity	$\omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1}$	$\alpha_2 := 25 \cdot \text{rad} \cdot \text{sec}^{-2}$	

Solution: See Figure P7-8a and Mathcad file P0723.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.37.

$$\arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad \arg_1 = 1.083$$

$$\arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad \arg_2 = -0.094$$

$$\theta_{2\text{toggle}} := \text{acos}(\arg_2) \quad \theta_{2\text{toggle}} = 95.4 \text{ deg}$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2\text{toggle}} + 0.5 \cdot \text{deg}, -\theta_{2\text{toggle}} + 1 \cdot \text{deg} .. \theta_{2\text{toggle}} - 0.5 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 1.5000 \quad K_2 = 1.5818$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.7307$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.6111 \quad K_5 = -1.7280$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the x and y components in the local coordinate system.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{Ax}(\theta_2) := \text{Re}(\mathbf{A}_A(\theta_2)) \quad A_{Ay}(\theta_2) := \text{Im}(\mathbf{A}_A(\theta_2))$$

9. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

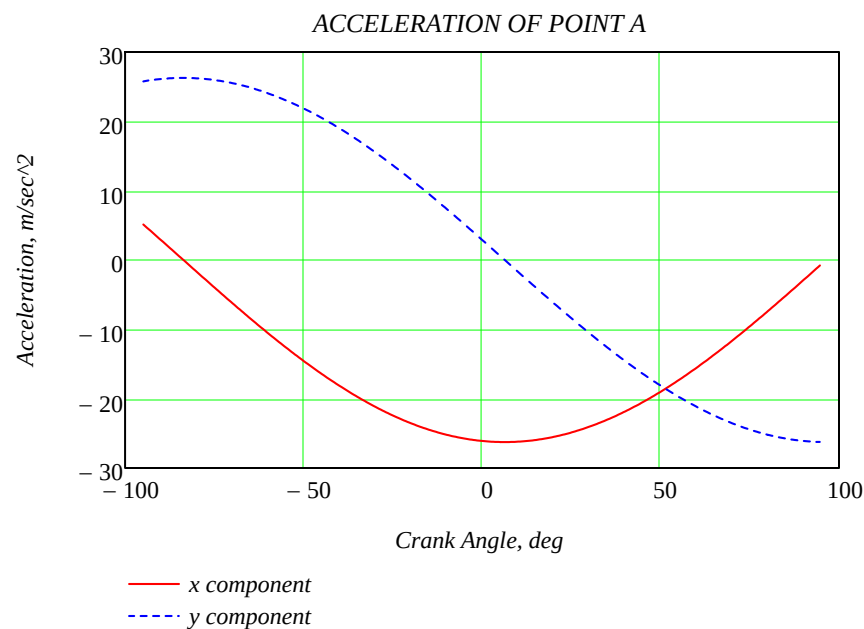
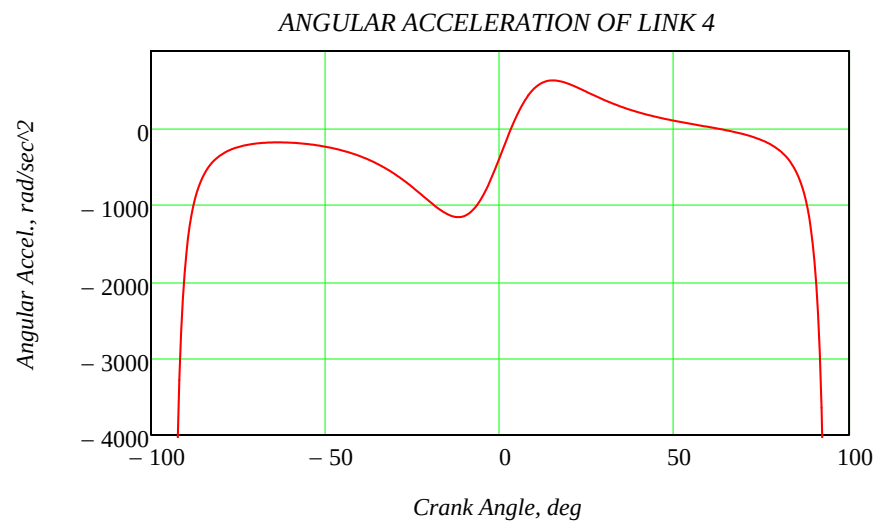
$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

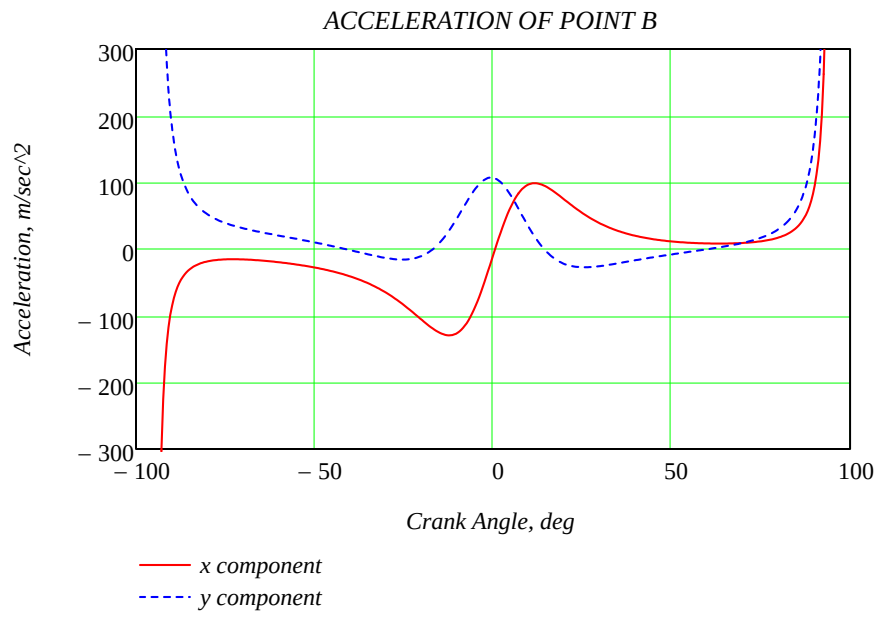
10. Use equation 7.13c to determine the acceleration of point B. Determine the x and y components in the local coordinate system.

$$\mathbf{A_B}(\theta_2) := c \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) - c \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$A_{Bx}(\theta_2) := \text{Re}(\mathbf{A_B}(\theta_2)) \quad A_{By}(\theta_2) := \text{Im}(\mathbf{A_B}(\theta_2))$$

11. Plot the angular acceleration for link 4 and the acceleration components for pins B and C.





PROBLEM 7-24

Statement: The linkage in Figure P7-8b has the dimensions and crank angle given below. Find α_4 , $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown for $\omega_2 = 20 \text{ rad/sec CCW}$. Use the acceleration difference graphical method.

Given: Link lengths:

Link 2 (A to B)	$a := 40 \cdot \text{mm}$	Link 3 (B to C)	$b := 96 \cdot \text{mm}$
Link 4 (C to D)	$c := 122 \cdot \text{mm}$	Link 1 (A to D)	$d := 162 \cdot \text{mm}$

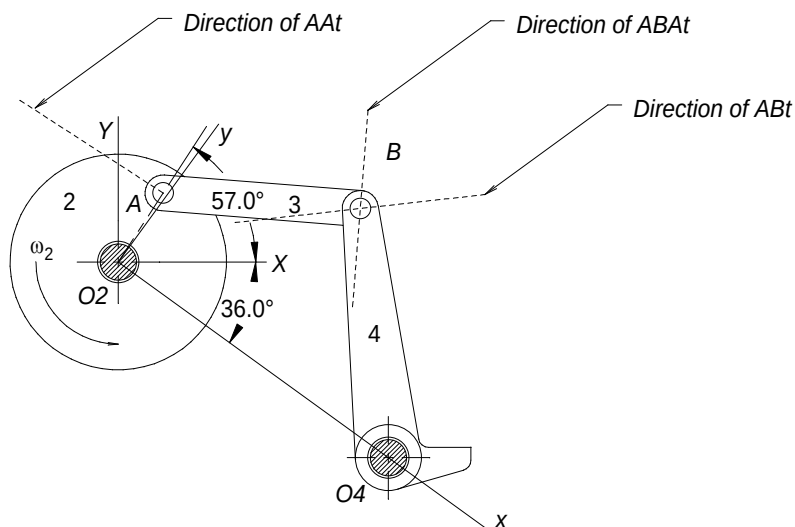
Crank angle: $\theta_2 := 93 \cdot \text{deg}$ Local xy system

Input crank angular velocity $\omega_2 := 20 \cdot \text{rad} \cdot \text{sec}^{-1}$ $\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Coordinate rotation angle $\alpha := -36 \cdot \text{deg}$ Global XY system to local xy system

Solution: See Figure P7-8b and Mathcad file P0724.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A and B, we will need θ_3 , ω_3 , and θ_4 . From the graphical position solution above (in the local xy coordinate system),

$$\theta_3 := 31.486 \cdot \text{deg}$$

$$\theta_4 := 132.406 \cdot \text{deg}$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_4 - \theta_2)}{b \cdot \sin(\theta_3 - \theta_4)} \quad \omega_3 = -5.388 \text{ rad} \cdot \text{sec}^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_4 - \theta_3)} \quad \omega_4 = 5.870 \text{ rad} \cdot \text{sec}^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

4. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_{Bn} := c \cdot \omega_4^2 \quad A_{Bn} = 4203 \text{ mm} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 - 180 \cdot \text{deg}$$

$$\theta_{ABn} = -47.594 \text{ deg}$$

$$A_{An} := a \cdot \omega_2^2$$

$$A_{An} = 16000 \text{ mm} \cdot \text{sec}^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg}$$

$$\theta_{AAn} = 273.000 \text{ deg}$$

$$A_{At} := a \cdot \alpha_2$$

$$A_{At} = 0.000 \text{ mm} \cdot \text{sec}^{-2}$$

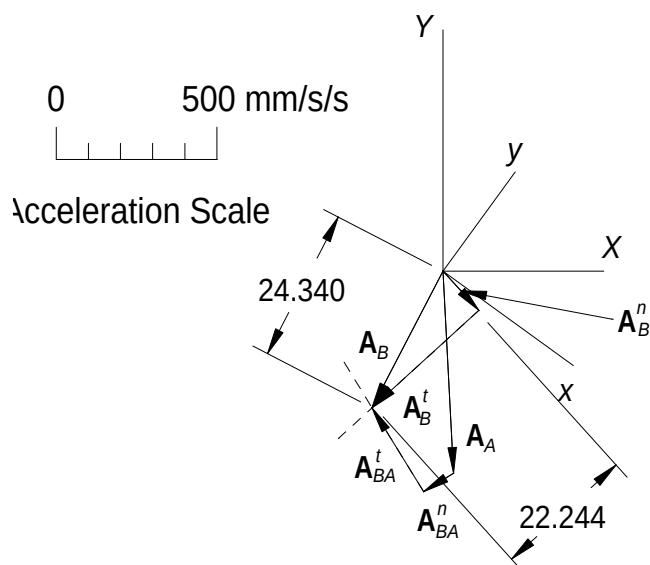
$$A_{BAAn} := b \cdot \omega_3^2$$

$$A_{BAAn} = 2787 \text{ mm} \cdot \text{sec}^{-2}$$

$$\theta_{ABAn} := \theta_3 - 180 \cdot \text{deg}$$

$$\theta_{ABAn} = -148.514 \text{ deg}$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of θ_{ABn} and $\mathbf{A}_A^n$ at an angle of θ_{AAn} . From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_A^t$ at an angle of θ_{AA^t} . From the tip of $\mathbf{A}_A^t$, draw $\mathbf{A}_{BA}^n$ at an angle of θ_{BAAn} . Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B^n$ and $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B^t$, $\mathbf{A}_B^t$, and $\mathbf{A}_{BA}^t$.



6. From the graphical solution above,

Acceleration scale factor

$$k_a := 500 \cdot \frac{\text{mm}}{\text{sec}^2}$$

$$A_A := A_{An}$$

$$A_A = 16000 \text{ mm} \cdot \text{sec}^{-2} \quad \text{at an angle of } 273.0 \text{ deg (Global)}$$

$$A_B := 24.340 \cdot k_a$$

$$A_B = 12170 \text{ mm} \cdot \text{sec}^{-2} \quad \text{at an angle of } 242.610 \text{ deg}$$

$$A_{Bt} := 22.244 \cdot k_a$$

$$A_{Bt} = 11122 \text{ mm} \cdot \text{sec}^{-2}$$

7. Calculate α_4 using equation 7.6.

$$\alpha_4 := \frac{A_{Bt}}{c}$$

$$\alpha_4 = 91.2 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CCW}$$

PROBLEM 7-25

Statement: The linkage in Figure P7-8b has the dimensions and crank angle given below. Find α_4 , $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown for $\omega_2 = 20 \text{ rad/sec}$ CCW constant. Use an analytical method.

Given: Link lengths:

Link 2 (A to B)	$a := 40 \cdot \text{mm}$	Link 3 (B to C)	$b := 96 \cdot \text{mm}$
Link 4 (C to D)	$c := 122 \cdot \text{mm}$	Link 1 (A to D)	$d := 162 \cdot \text{mm}$

Crank angle: $\theta_2 := 93 \cdot \text{deg}$ Local xy system

Input crank angular velocity $\omega_2 := 20 \cdot \text{rad} \cdot \text{sec}^{-1}$ $\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Coordinate rotation angle $\beta := -36 \cdot \text{deg}$ Global XY system to local xy system

Solution: See Figure P7-8b and Mathcad file P0725.

1. Draw the linkage to scale and label it.

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 4.0500 \quad K_2 = 1.3279$$

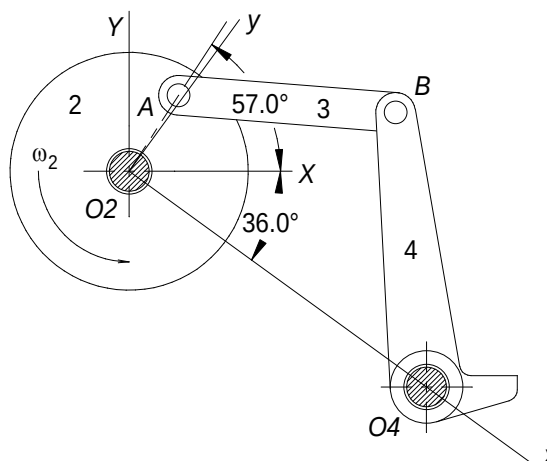
$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 3.4336$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.5992 \quad B = -1.9973 \quad C = 7.6054$$



3. Use equation 4.10b to find values of θ_4 for the open circuit in the local xy coordinate system.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) + 2 \cdot \pi \quad \theta_4 = 132.386 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.6875 \quad K_5 = -2.8875$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -7.0782$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.9973$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 1.1265$$

5. Use equation 4.13 to find values of θ_3 for the open circuit in the local xy coordinate system.

$$\theta_3 := 2 \cdot \left(\text{atan2}(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}) \right) + 2 \cdot \pi \quad \theta_3 = 31.504 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad \omega_3 = -5.385 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad \omega_4 = 5.868 \frac{\text{rad}}{\text{sec}}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction (in the global coordinate system).

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (837 - 15978i) \frac{\text{mm}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A) + \beta$$

$$\text{The acceleration of pin A is } A_A = 16000 \frac{\text{mm}}{\text{sec}^2} \quad \text{at } \theta_{AA} = -123.0 \text{ deg}$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c \cdot \sin(\theta_4) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_4) \quad E := b \cdot \cos(\theta_3)$$

$$A = 90.111 \text{ mm} \quad B = 50.166 \text{ mm} \quad D = -82.244 \text{ mm} \quad E = 81.850 \text{ mm}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$C = 4.368 \times 10^3 \text{ mm} \cdot \text{sec}^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = -1.433 \times 10^4 \text{ mm} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = 81.037 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = 93.586 \frac{\text{rad}}{\text{sec}^2}$$

9. Use equation 7.13c to determine the acceleration of point B for the open circuit.

$$\mathbf{A}_B := c \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$\mathbf{A}_B = (-5601 - 10800i) \frac{\text{mm}}{\text{sec}^2} \quad A_B := |\mathbf{A}_B| \quad \theta_{AB} := \arg(\mathbf{A}_B) + \beta$$

$$\text{The acceleration of pin B is } A_B = 12166 \frac{\text{mm}}{\text{sec}^2} \quad \text{at } \theta_{AB} = -153.4 \text{ deg} \quad (\text{Global})$$

PROBLEM 7-26

Statement: The linkage in Figure P7-8b has the dimensions and crank angle given below. Find and plot α_4 , $\mathbf{A}_A$, and $\mathbf{A}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 20 \text{ rad/sec}$ CCW constant.

Given: Link lengths:

Link 2 (A to B)	$a := 40 \cdot \text{mm}$	Link 3 (B to C)	$b := 96 \cdot \text{mm}$
Link 4 (C to D)	$c := 122 \cdot \text{mm}$	Link 1 (A to D)	$d := 162 \cdot \text{mm}$

Input crank angular velocity $\omega_2 := 20 \cdot \text{rad} \cdot \text{sec}^{-1}$ $\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure P7-8b and Mathcad file P0726.

1. Draw the linkage to scale and label it.

2. The range of θ_2 for this Grashof crank-rocker is:

$$\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

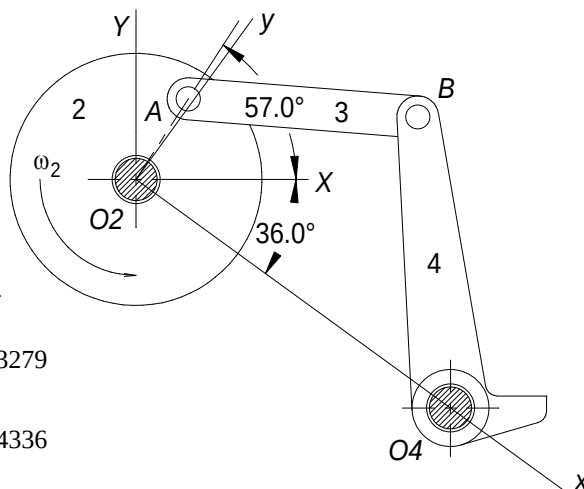
$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 4.0500 \qquad K_2 = 1.3279$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \qquad K_3 = 3.4336$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$



4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \qquad K_4 = 1.6875 \qquad K_5 = -2.8875$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the x and y components in the local coordinate system.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{Ax}(\theta_2) := \text{Re}(\mathbf{A}_A(\theta_2)) \quad A_{Ay}(\theta_2) := \text{Im}(\mathbf{A}_A(\theta_2))$$

9. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

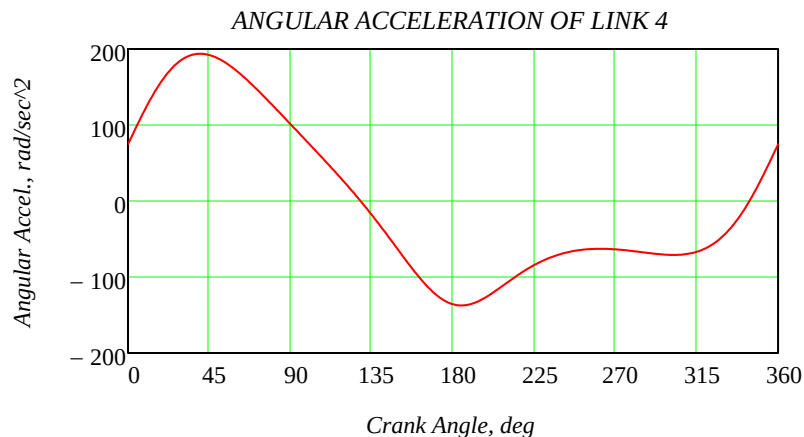
$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

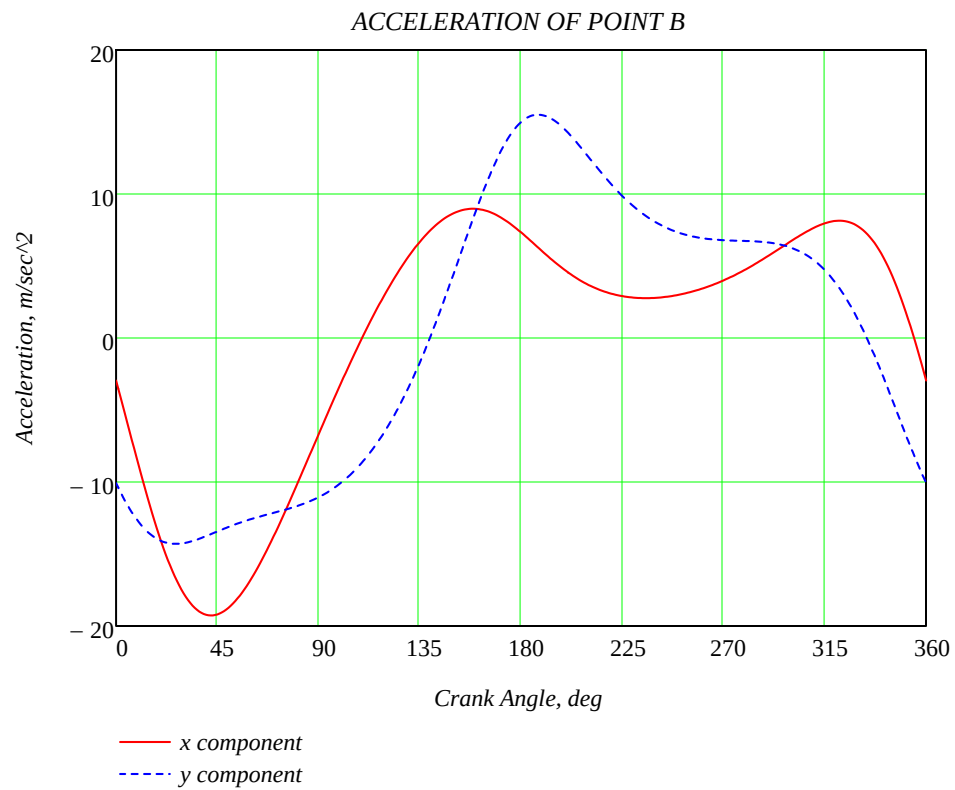
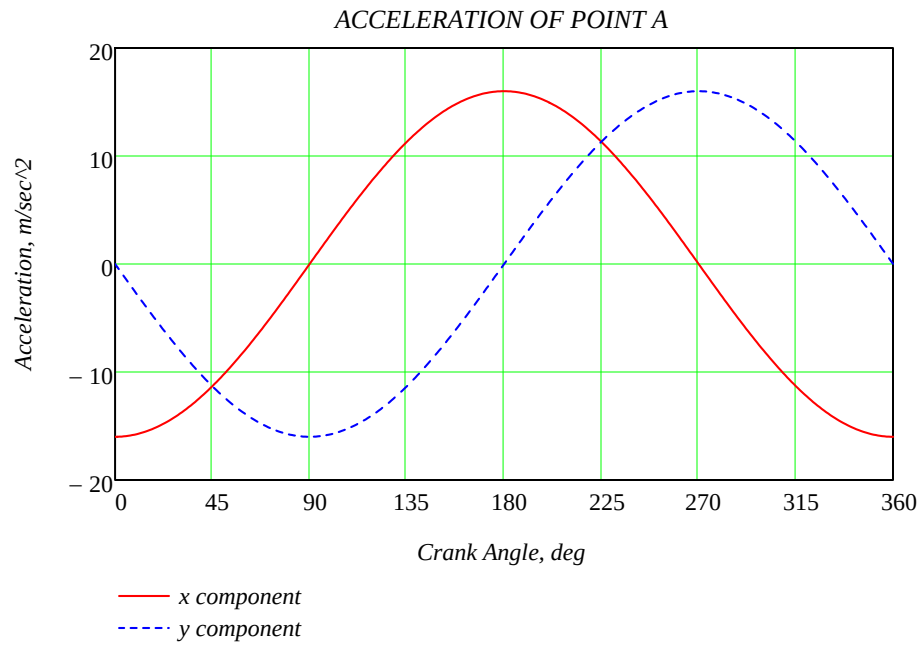
10. Use equation 7.13c to determine the acceleration of point B. Determine the x and y components in the local coordinate system.

$$\mathbf{A}_B(\theta_2) := c \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) - c \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$A_{Bx}(\theta_2) := \text{Re}(\mathbf{A}_B(\theta_2)) \quad A_{By}(\theta_2) := \text{Im}(\mathbf{A}_B(\theta_2))$$

11. Plot the angular acceleration for link 4 and the acceleration components for pins A and B.





PROBLEM 7-27

Statement: The offset crank-slider linkage in Figure P7-8f has the dimensions and crank angle given below. Find $\mathbf{A}_A$ and $\mathbf{A}_B$ in the global coordinate system for the position shown for $\omega_2 = 25 \text{ rad/sec}$ CW, constant. Use the acceleration difference graphical method.

Given: Link lengths:

$$\text{Link 2 (D to E)} \quad a := 63 \cdot \text{mm} \quad \text{Link 3 (E to F)} \quad b := 130 \cdot \text{mm}$$

$$\text{Offset} \quad c := -52 \cdot \text{mm}$$

$$\text{Link 2 position, velocity, and acceleration: } \theta_2 := 141 \cdot \text{deg} \quad \omega_2 := -25 \cdot \frac{\text{rad}}{\text{sec}} \quad \alpha_2 := 0 \cdot \frac{\text{rad}}{\text{sec}^2}$$

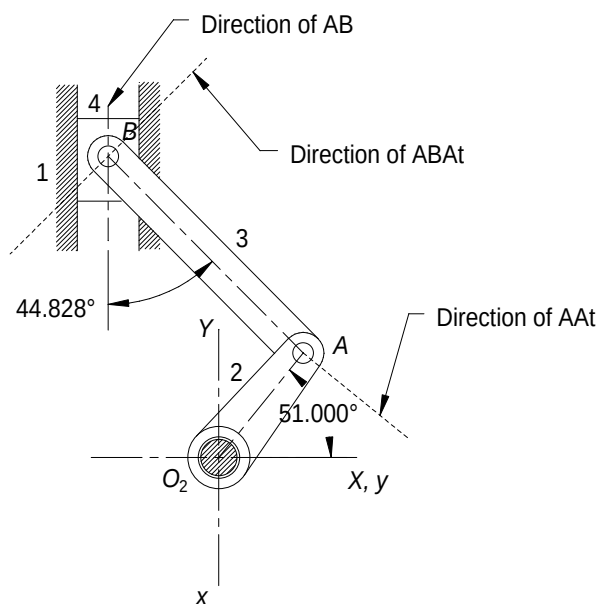
$$\text{Coordinate rotation angle: } \alpha := -90 \cdot \text{deg}$$

Solution: See Figure P7-8f and Mathcad file P0727.

- In order to solve for the accelerations at point F , we will need θ_3 , and ω_3 . From the graphical position solution below (in the local coordinate system),

$$\theta_3 := 44.828 \cdot \text{deg}$$

$$\text{Using equation 6.22a,} \quad \omega_3 := \frac{a \cdot \omega_2 \cdot \cos(\theta_2)}{b \cdot \cos(\theta_3)} \quad \omega_3 = 13.276 \cdot \text{rad} \cdot \text{sec}^{-1}$$



- The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_p^t + \mathbf{A}_p^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$
- For point F , this becomes: $\mathbf{A}_F = (\mathbf{A}_E^t + \mathbf{A}_E^n) + (\mathbf{A}_{FE}^t + \mathbf{A}_{FE}^n)$, where (angles in the global coordinate system)

$$A_{En} := a \cdot \omega_2^2 \quad A_{En} = 39375 \cdot \text{mm} \cdot \text{sec}^{-2}$$

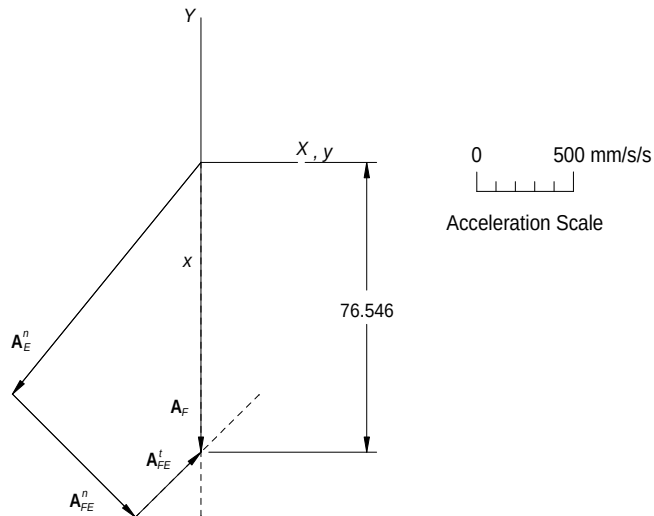
$$\theta_{AEn} := \theta_2 + \alpha + 180 \cdot \text{deg} \quad \theta_{AEn} = 231.000 \cdot \text{deg}$$

$$A_{Et} := a \cdot \alpha_2 \quad A_{Et} = 0.000 \cdot \text{mm} \cdot \text{sec}^{-2}$$

$$A_{FEn} := b \cdot \omega_3^2 \quad A_{FEn} = 22911 \cdot \text{mm} \cdot \text{sec}^{-2}$$

$$\theta_{AFEn} := \theta_3 + \alpha \quad \theta_{AFEn} = -45.172 \cdot \text{deg}$$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_E^n$ at an angle of θ_{AE^n} . From the tip of $\mathbf{A}_E^n$, draw $\mathbf{A}_{FE}^n$ at an angle of θ_{AFE^n} . Now that the vectors with known magnitudes are drawn, from the origin and the tip of $\mathbf{A}_{FE}^n$, draw construction lines in the directions of $\mathbf{A}_F$ (vertical) and $\mathbf{A}_{FE}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_{FE}^t$ and $\mathbf{A}_F$.



5. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 500 \cdot \frac{\text{mm}}{\text{sec}^2}$$

$$A_F := 76.546 \cdot k_a \quad A_F = 38273 \cdot \text{mm} \cdot \text{sec}^{-2} \quad \text{at an angle of } -90 \text{ deg (global)}$$

PROBLEM 7-28

Statement: The offset crank-slider linkage in Figure P7-8f has the dimensions and crank angle given below. Find $\mathbf{A}_A$ and $\mathbf{A}_B$ in the global coordinate system for the position shown for $\omega_2 = 25 \text{ rad/sec}$ CW, constant. Use an analytical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 63 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad b := 130 \cdot \text{mm}$$

$$\text{Offset} \quad c := -52 \cdot \text{mm}$$

$$\text{Link 2 position, velocity, and acceleration: } \theta_2 := 141 \cdot \text{deg} \quad \omega_2 := -25 \cdot \frac{\text{rad}}{\text{sec}} \quad \alpha_2 := 0 \cdot \frac{\text{rad}}{\text{sec}^2}$$

$$\text{Coordinate rotation angle: } \alpha := -90 \cdot \text{deg}$$

Solution: See Figure P7-8f and Mathcad file P0728.

1. Draw the linkage to a convenient scale.
2. Determine θ_3 (in the local coordinate system) and d using equations 4.16 for the crossed circuit.

$$\theta_3 := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right) \quad \theta_3 = 44.828 \cdot \text{deg}$$

$$d := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_3) \quad d = -141.160 \cdot \text{mm}$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3)} \cdot \omega_2 \quad \omega_3 = 13.276 \cdot \frac{\text{rad}}{\text{sec}}$$

4. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_B$, determine its magnitude, and direction (global).

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (30600.122 - 24779.490j) \cdot \frac{\text{mm}}{\text{sec}^2} \quad \mathbf{A}_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A) + \alpha$$

$$\text{The acceleration of pin A is } A_A = 39375 \cdot \frac{\text{mm}}{\text{sec}^2} \quad \text{at } \theta_{AA} = -129.0 \cdot \text{deg}$$

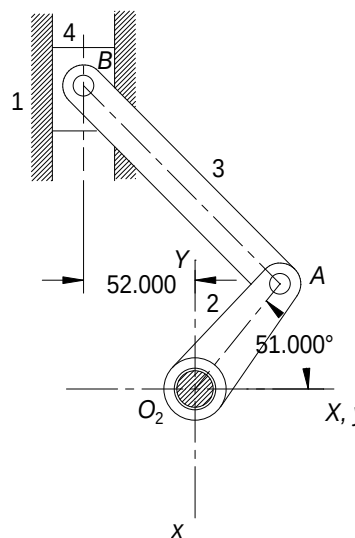
5. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3 := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_3^2 \cdot \sin(\theta_3)}{b \cdot \cos(\theta_3)} \quad \alpha_3 = -93.574 \cdot \frac{\text{rad}}{\text{sec}^2}$$

6. Use equation 7.16e for the acceleration of pin B.

$$A_B := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \alpha_3 \cdot \sin(\theta_3) + b \cdot \omega_3^2 \cdot \cos(\theta_3)$$

$$A_B = 38274 \cdot \frac{\text{mm}}{\text{sec}^2} \quad \text{A positive sign means that } \mathbf{A}_B \text{ is downward}$$



PROBLEM 7-29

Statement: The offset crank-slider linkage in Figure P7-8f has the dimensions and crank angle given below. Find and plot $\mathbf{A}_A$ and $\mathbf{A}_B$ in the global coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 25 \text{ rad/sec CW}$, constant..

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 63 \cdot \text{mm}$$

$$\text{Link 3 (A to B)} \quad b := 130 \cdot \text{mm}$$

$$\text{Offset} \quad c := -52 \cdot \text{mm}$$

$$\text{Link 2 velocity, and acceleration: } \omega_2 := -25 \cdot \frac{\text{rad}}{\text{sec}} \quad \alpha_2 := 0 \cdot \frac{\text{rad}}{\text{sec}^2}$$

$$\text{Coordinate rotation angle: } \alpha := -90 \cdot \text{deg}$$

Solution: See Figure P7-8f and Mathcad file P0729.

1. Draw the linkage to a convenient scale.
2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

3. Determine θ_3 using equations 4.16 for the crossed circuit.

$$\theta_3(\theta_2) := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right)$$

4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3(\theta_2))} \cdot \omega_2$$

5. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its X and Y components in the global coordinate system.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{AX}(\theta_2) := \text{Re}(\mathbf{A}_A(\theta_2)) \cdot \cos(\alpha) - \text{Im}(\mathbf{A}_A(\theta_2)) \cdot \sin(\alpha)$$

$$A_{AY}(\theta_2) := \text{Re}(\mathbf{A}_A(\theta_2)) \cdot \sin(\alpha) + \text{Im}(\mathbf{A}_A(\theta_2)) \cdot \cos(\alpha)$$

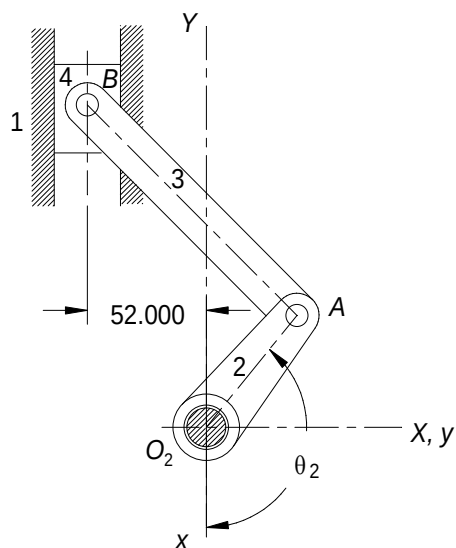
6. Determine the angular acceleration of link 3 using equation 7.16d.

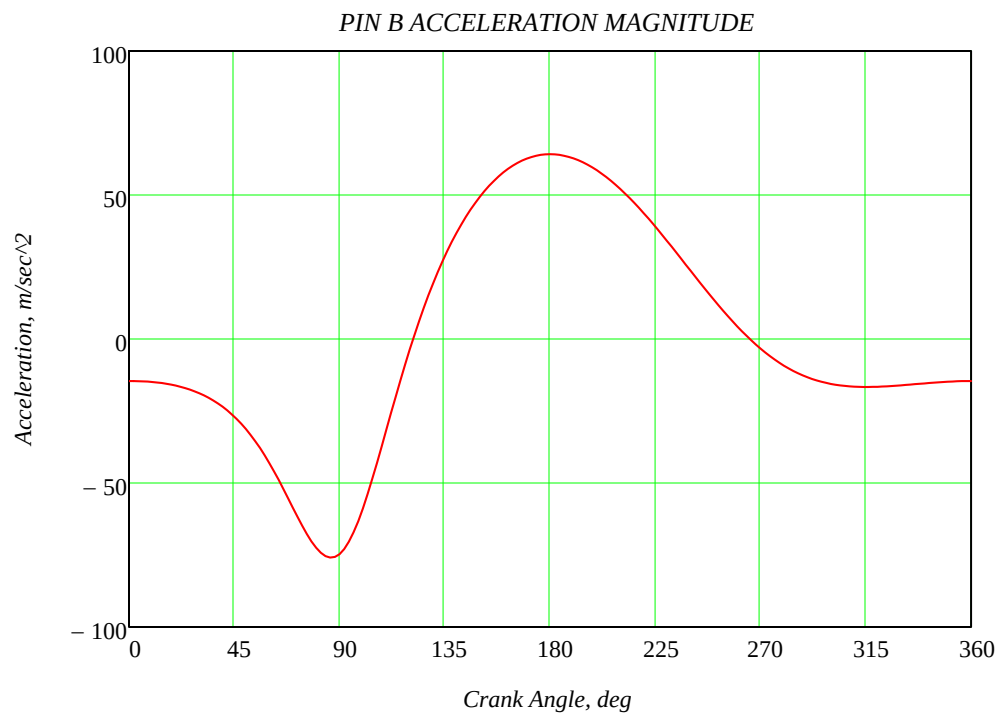
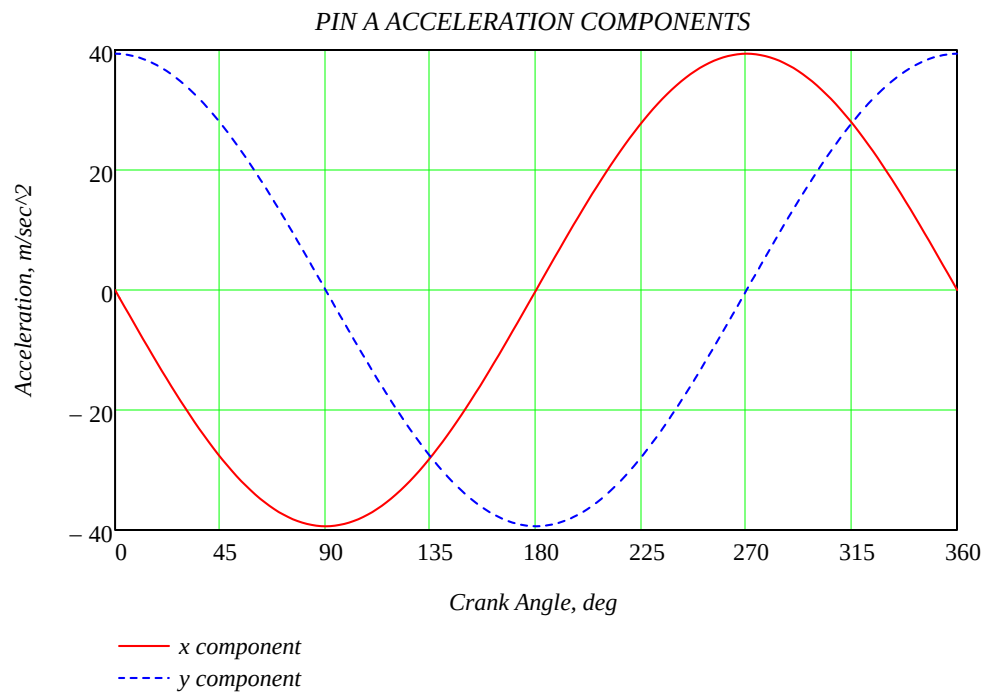
$$\alpha_3(\theta_2) := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2))}{b \cdot \cos(\theta_3(\theta_2))}$$

7. Use equation 7.16e for the acceleration of pin B.

$$\begin{aligned} A_B(\theta_2) := & -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) \dots \\ & + b \cdot \alpha_3(\theta_2) \cdot \sin(\theta_3(\theta_2)) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) \end{aligned}$$

8. Plot the components of $\mathbf{A}_A$ and the magnitude of $\mathbf{A}_B$. A positive value for $\mathbf{A}_B$ is downward, a negative value, upward.





PROBLEM 7-30

Statement: The linkage in Figure P7-8d has the dimensions and crank angle given below. Find $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_{box}$ in the global coordinate system for the position shown for $\omega_2 = 30 \text{ rad/sec CW}$, constant. Use the acceleration difference graphical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 30 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad b := 150 \cdot \text{mm}$$

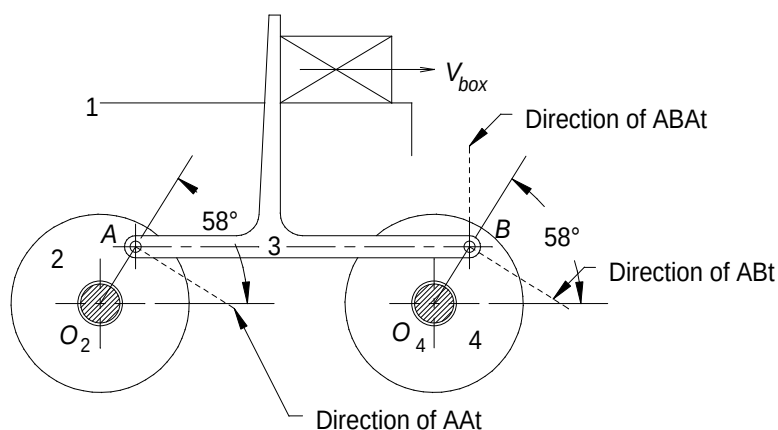
$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 30 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 150 \cdot \text{mm}$$

$$\text{Crank angle:} \quad \theta_2 := 58 \cdot \text{deg} \quad \text{Global XY system}$$

$$\text{Input crank angular velocity} \quad \omega_2 := -30 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-8d and Mathcad file P0730.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A and B, we will need θ_3 , ω_3 , and θ_4 . From the graphical position solution above (in the global XY coordinate system),

$$\theta_3 := 0 \cdot \text{deg} \quad \theta_3 = 0.000 \text{ deg}$$

$$\theta_4 := \theta_2 \quad \theta_4 = 58.000 \text{ deg}$$

This is a special-case Grashof in the parallelogram configuration. Therefore,

$$\omega_3 := 0.0 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \omega_4 := \omega_2$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_p^t + \mathbf{A}_p^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{pA}^t + \mathbf{A}_{pA}^n)$

4. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_{Bn} := c \cdot \omega_4^2 \quad A_{Bn} = 27000 \text{ mm} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 + 180 \cdot \text{deg} \quad \theta_{ABn} = 238.000 \text{ deg}$$

$$A_{An} := a \cdot \omega_2^2 \quad A_{An} = 27000 \text{ mm} \cdot \text{sec}^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AAn} = 238.000 \text{ deg}$$

$$A_{At} := a \cdot \alpha_2 \quad A_{At} = 0.0 \text{ mm} \cdot \text{sec}^{-2}$$

$$A_{BA n} := b \cdot \omega_3^2 \quad A_{BA n} = 0 \text{ mm} \cdot \text{sec}^{-2}$$

5. In this case, there is no need to draw an acceleration diagram since $\mathbf{A}_A^t$ and $\mathbf{A}_{BA}^n$ are zero. This means that $\mathbf{A}_{BA}^t$ and $\mathbf{A}_B^t$ will also be zero and $\mathbf{A}_A = \mathbf{A}_A^n$, $\mathbf{A}_B = \mathbf{A}_B^n$.
6. Since $\mathbf{A}_{BA}^t = 0$, $\alpha_3 = 0$ and, since $\mathbf{A}_B = \mathbf{A}_A$, $\alpha_4 = \alpha_2 = 0$.
7. For the box, equation 7.4 becomes: $\mathbf{A}_{box} = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{boxA}^t + \mathbf{A}_{boxA}^n)$, where $\mathbf{A}_{boxA}^t$ and $\mathbf{A}_{boxA}^n$ are both zero since α_3 and ω_3 are zero. Therefore, $\mathbf{A}_{box} = \mathbf{A}_A$.

PROBLEM 7-31

Statement: The linkage in Figure P7-8d has the dimensions and crank angle given below. Find $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_{box}$ in the global coordinate system for the position shown for $\omega_2 = 30 \text{ rad/sec CW}$, constant. Use an analytical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 30 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad b := 150 \cdot \text{mm}$$

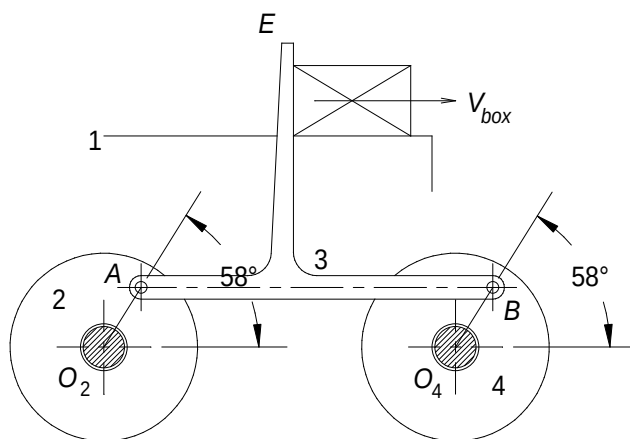
$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 30 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 150 \cdot \text{mm}$$

$$\text{Crank angle:} \quad \theta_2 := 58 \cdot \text{deg}$$

$$\text{Input crank angular velocity} \quad \omega_2 := -30 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-8d and Mathcad file P0731.

1. Draw the linkage to a convenient scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 5.0000 \quad K_2 = 5.0000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -6.1197 \quad B = -1.6961 \quad C = 2.8205$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_4 = -302.000 \text{ deg}$$

$$\theta_4 := \theta_4 + 360 \cdot \text{deg} \quad \theta_4 = 58.000 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0000 \quad K_5 = -5.0000$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -8.9402$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.6961$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.0000$$

5. Use equation 4.13 to find values of θ_3 .

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_3 = 360.000 \text{ deg}$$

$$\theta_3 := \theta_3 - 360 \cdot \text{deg} \quad \theta_3 = 0.000 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_4 - \theta_2)}{b \cdot \sin(\theta_3 - \theta_4)} \quad \omega_3 = 0.000 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_4 - \theta_3)} \quad \omega_4 = -30.000 \frac{\text{rad}}{\text{sec}}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (-14308 - 22897j) \frac{\text{mm}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A)$$

$$\text{The acceleration of pin A is } A_A = 27000 \frac{\text{mm}}{\text{sec}^2} \quad \text{at } \theta_{AA} = -122.0 \text{ deg}$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A := c \cdot \sin(\theta_4) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_4) \quad E := b \cdot \cos(\theta_3)$$

$$A = 1.002 \text{ in} \quad B = 0.000 \text{ in} \quad D = 0.626 \text{ in} \quad E = 5.906 \text{ in}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$C = 0.000 \text{ mm} \cdot \text{sec}^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = 0.000 \text{ mm} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = 0.000 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = 0.000 \frac{\text{rad}}{\text{sec}^2}$$

9. Use equation 7.13c to determine the acceleration of point B.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$\mathbf{A_B} = (-14308 - 22897j) \frac{mm}{sec^2} \quad A_B := |\mathbf{A_B}| \quad \theta_{AB} := \arg(\mathbf{A_B})$$

$$\text{The acceleration of pin B is } A_B = 27000 \frac{mm}{sec^2} \quad \text{at } \theta_{AB} = -122.0 \text{ deg}$$

10. This is a special case Grashof in the parallelogram configuration. All points on link 3 have the same velocity and acceleration. The acceleration of the box will be equal to the X-component of the acceleration of any point on link 3.

$$A_{box} := \text{Re}(\mathbf{A_A})$$

$$\text{The acceleration of the box is } A_{box} = -14308 \frac{mm}{sec^2} \quad \text{a negative sign means } \mathbf{A_{box}} \text{ is to the left}$$

PROBLEM 7-32

Statement: The linkage in Figure P7-8d has the dimensions and crank angle given below. Write a computer program or use an equation solver to find and plot $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_{box}$ in the global coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 30 \text{ rad/sec CW}$, constant.

Given: Link lengths:

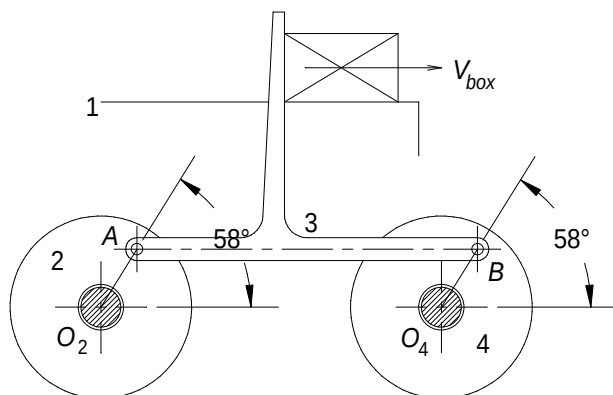
$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 30 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad b := 150 \cdot \text{mm}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 30 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 150 \cdot \text{mm}$$

$$\text{Input crank angular velocity} \quad \omega_2 := -30 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-8d and Mathcad file P0732.

1. Draw the linkage to a convenient scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 5.0000 \quad K_2 = 5.0000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.0000$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0000 \quad K_5 = -5.0000$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 .

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the XY components.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{AX}(\theta_2) := \text{Re}(\mathbf{A}_A(\theta_2)) \quad A_{AY}(\theta_2) := \text{Im}(\mathbf{A}_A(\theta_2))$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)} \quad \alpha_4(\theta_2) := \frac{C'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

9. Use equation 7.13c to determine the acceleration of point B.

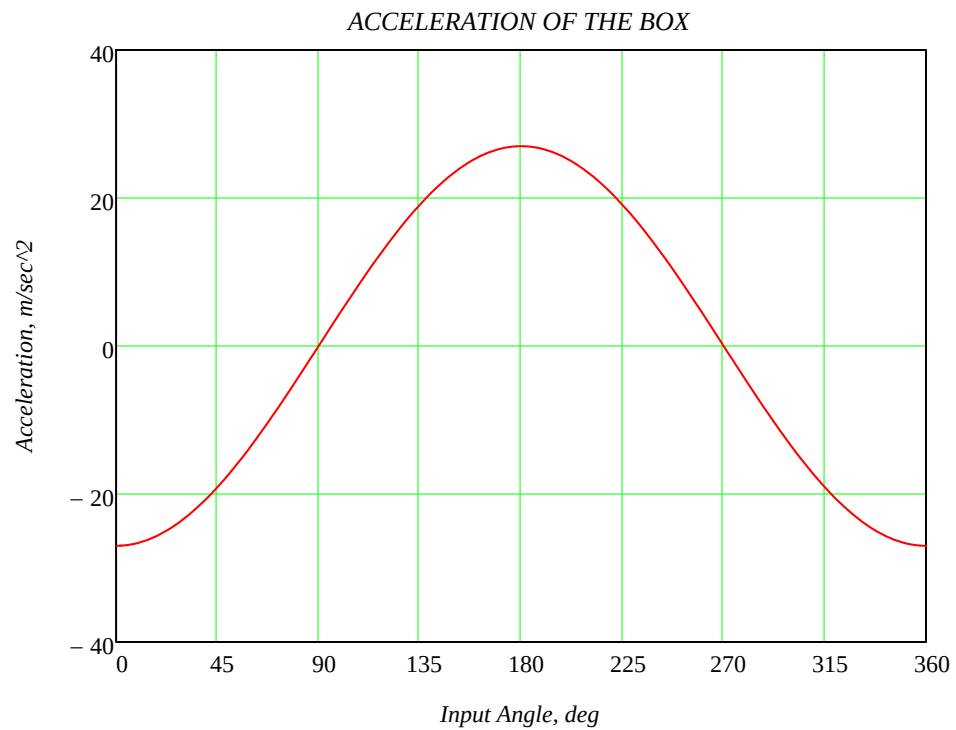
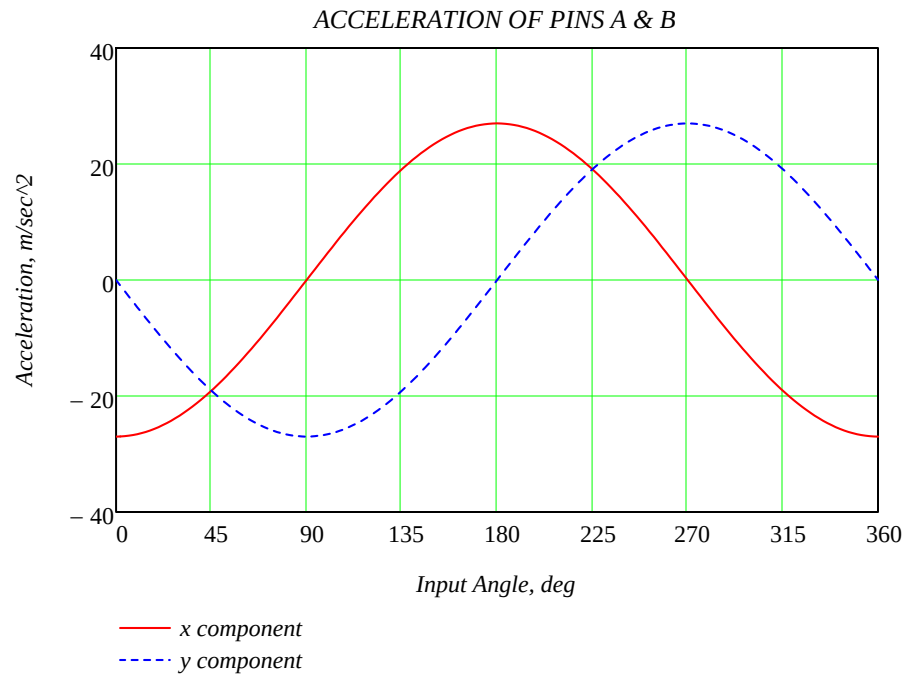
$$\mathbf{A}_B(\theta_2) := c \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) \dots \\ + -c \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$A_{BX}(\theta_2) := \text{Re}(\mathbf{A}_B(\theta_2)) \quad A_{BY}(\theta_2) := \text{Im}(\mathbf{A}_B(\theta_2))$$

10. This is a special case Grashof in the parallelogram configuration. All points on link 3 have the same velocity and acceleration. The acceleration of the box will be equal to the X-component of the acceleration of any point on link 3.

$$A_{box}(\theta_2) := \text{Re}(\mathbf{A}_A(\theta_2))$$

11. Plot the accelerations over the range: $\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$



PROBLEM 7-33

Statement: The linkage in Figure P7-8g has the dimensions and crank angle given below. Find α_4 , $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown if $\omega_2 = 15 \text{ rad/sec CW}$, constant. Use the acceleration difference graphical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 49 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad b := 100 \cdot \text{mm}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 153 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 87 \cdot \text{mm}$$

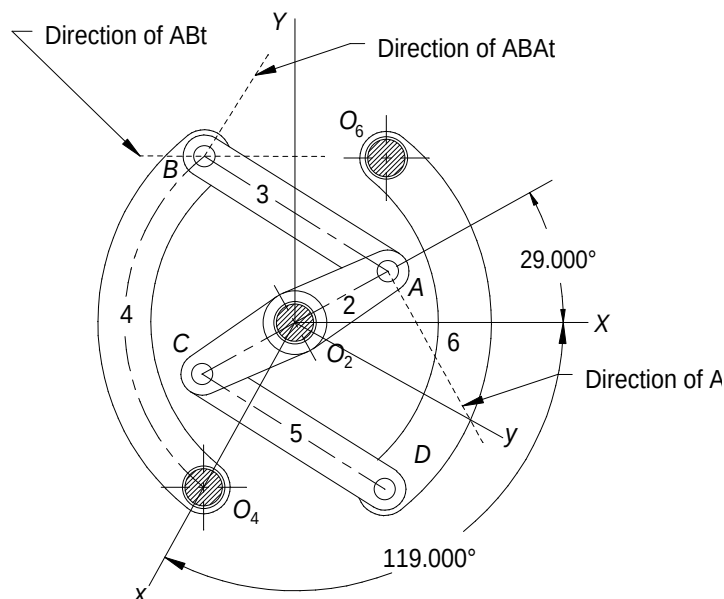
$$\text{Crank angle:} \quad \theta_2 := 148 \cdot \text{deg} \quad \text{Local } xy \text{ system}$$

$$\text{Input crank angular velocity} \quad \omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\text{Coordinate rotation angle} \quad \beta := -119 \cdot \text{deg} \quad \text{Global } XY \text{ system to local } xy \text{ system}$$

Solution: See Figure P7-8e and Mathcad file P0733.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A and B, we will need θ_3 , ω_3 , and θ_4 . From the graphical position solution above (in the local xy coordinate system),

$$\theta_3 := 266.812 \cdot \text{deg}$$

$$\theta_4 := 208.876 \cdot \text{deg}$$

Using equation (6.18),

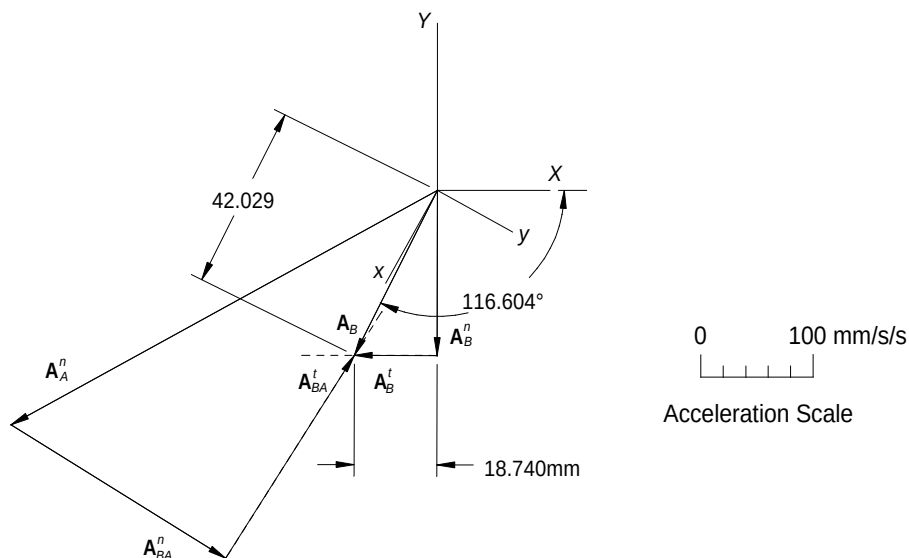
$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_4 - \theta_2)}{b \cdot \sin(\theta_3 - \theta_4)} \quad \omega_3 = -7.576 \text{ rad} \cdot \text{sec}^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_4 - \theta_3)} \quad \omega_4 = -4.967 \text{ rad} \cdot \text{sec}^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$
4. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where (angles in the global coordinate system)

$$\begin{aligned}
 A_{Bn} &:= c \cdot \omega_4^2 & A_{Bn} &= 3775 \text{ mm} \cdot \text{sec}^{-2} \\
 \theta_{ABn} &:= \theta_4 - 180 \cdot \text{deg} + \beta & \theta_{ABn} &= -90.124 \text{ deg} \\
 A_{An} &:= a \cdot \omega_2^2 & A_{An} &= 11025 \text{ mm} \cdot \text{sec}^{-2} \\
 \theta_{AAn} &:= \theta_2 + 180 \cdot \text{deg} + \beta & \theta_{AAn} &= 209.000 \text{ deg} \\
 A_{At} &:= a \cdot \alpha_2 & A_{At} &= 0.000 \text{ mm} \cdot \text{sec}^{-2} \\
 A_{BA n} &:= b \cdot \omega_3^2 & A_{BA n} &= 5740 \text{ mm} \cdot \text{sec}^{-2} \\
 \theta_{ABn} &:= \theta_3 - 180 \cdot \text{deg} + \beta & \theta_{ABn} &= -32.188 \text{ deg}
 \end{aligned}$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of θ_{ABn} and $\mathbf{A}_A^n$ at an angle of θ_{AAn} . From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_A^t$ at an angle of θ_{AA^t} . From the tip of $\mathbf{A}_A^t$, draw $\mathbf{A}_{BA}^n$ at an angle of $\theta_{ABA n}$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B^n$ and $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B^t$, $\mathbf{A}_B^t$, and $\mathbf{A}_{BA}^t$.



6. From the graphical solution above,

$$\text{Acceleration scale factor } k_a := 100 \cdot \frac{\text{mm}}{\text{sec}^2}$$

$$\begin{aligned}
 A_A &:= A_{An} & A_A &= 11025 \text{ mm} \cdot \text{sec}^{-2} & \text{at an angle of } 209 \text{ deg} \\
 A_B &:= 42.029 \cdot k_a & A_B &= 4203 \text{ mm} \cdot \text{sec}^{-2} & \text{at an angle of } -116.6 \text{ deg} \\
 A_{Bt} &:= 18.740 \cdot k_a & A_{Bt} &= 1874 \text{ mm} \cdot \text{sec}^{-2}
 \end{aligned}$$

7. Calculate α_4 using equation 7.6.

$$\alpha_4 := \frac{A_{Bt}}{c} \quad \alpha_4 = 12.2 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CCW}$$

PROBLEM 7-34

Statement: The linkage in Figure P7-8g has the dimensions and crank angle given below. Find α_4 , $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown if $\omega_2 = 15 \text{ rad/sec CW}$ and $\alpha_2 = 10 \text{ rad/sec CCW}$, constant. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49 \cdot \text{mm}$	Link 3 (A to B)	$b := 100 \cdot \text{mm}$
Link 4 (O_4 to B)	$c := 153 \cdot \text{mm}$	Link 1 (O_2 to O_4)	$d := 87 \cdot \text{mm}$

Crank angle: $\theta_2 := 148 \cdot \text{deg}$ Local xy system (see layout below)

Input crank angular velocity $\omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1}$ $\alpha_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-2}$
 Coordinate rotation angle $\beta := -119 \cdot \text{deg}$ Global XY system to local xy system

Solution: See Figure P7-8g and Mathcad file P0734.

1. Draw the linkage to scale and label it.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 1.7755 \quad K_2 = 0.5686$$

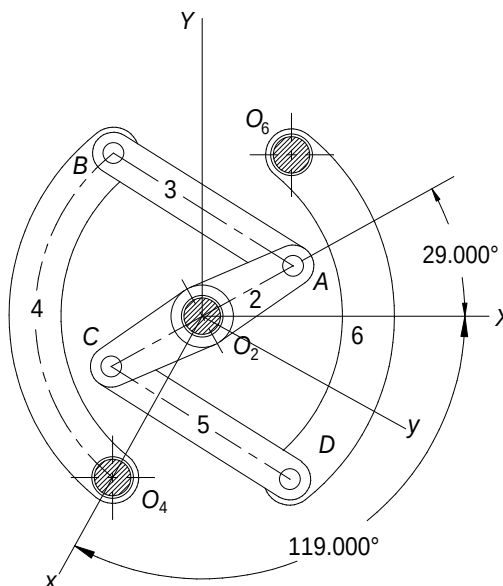
$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.5592$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.5821 \quad B = -1.0598 \quad C = 4.6650$$



3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_4 = 208.876 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.8700 \quad K_5 = 0.3509$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -3.0104$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.0598$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 2.2367$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_3 = 266.892 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad \omega_3 = -7.570 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad \omega_4 = -4.959 \frac{\text{rad}}{\text{sec}}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction (in the local coordinate system).

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (9090 - 6258j) \frac{\text{mm}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A) - \beta$$

$$\text{The acceleration of pin A is } A_A = 11036 \frac{\text{mm}}{\text{sec}^2} \quad \text{at } \theta_{AA} = 84.46 \text{ deg}$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c \cdot \sin(\theta_4) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_4) \quad E := b \cdot \cos(\theta_3)$$

$$A = -73.887 \text{ mm} \quad B = -99.853 \text{ mm} \quad D = -133.977 \text{ mm} \quad E = -5.422 \text{ mm}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$C = -6.106 \times 10^3 \text{ mm} \cdot \text{sec}^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = -2.353 \times 10^3 \text{ mm} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = -49.646 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = 15.552 \frac{\text{rad}}{\text{sec}^2}$$

9. Use equation 7.13c to determine the acceleration of point B for the crossed circuit.

$$\mathbf{A}_B := c \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$\mathbf{A}_B = (4444 - 267j) \frac{\text{mm}}{\text{sec}^2} \quad A_B := |\mathbf{A}_B| \quad \theta_{AB} := \arg(\mathbf{A}_B) - \beta$$

$$\text{The acceleration of pin B is } A_B = 4452 \frac{\text{mm}}{\text{sec}^2} \quad \text{at } \theta_{AB} = 115.6 \text{ deg}$$

PROBLEM 7-35

Statement: At $t = 0$, the non-Grashof linkage in Figure P7-8g has the local axis at -119° and O_2A at 29° in the global XY coordinate system and $\omega_2 = 0$. Write a computer program or use an equation solver to find and plot ω_4 , $\mathbf{V}_A$, $\mathbf{A}_A$, $\mathbf{V}_B$, and $\mathbf{A}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if $\alpha_2 = 15 \text{ rad/sec CCW, constant}$.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49 \cdot \text{mm}$	Link 3 (A to B)	$b := 100 \cdot \text{mm}$
Link 4 (O_4 to B)	$c := 153 \cdot \text{mm}$	Link 1 (O_2 to O_4)	$d := 87 \cdot \text{mm}$
Input crank angular velocity	$\omega_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-1}$	$\alpha_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-2}$	
Initial crank angle:	$\theta_{20} := 148 \cdot \text{deg}$	Local xy system (see layout below)	

Solution: See Figure P7-8g and Mathcad file P0735.

1. Draw the linkage to scale and label it.
2. Determine the range of motion for this non-Grashof triple rocker using equations 4.37.

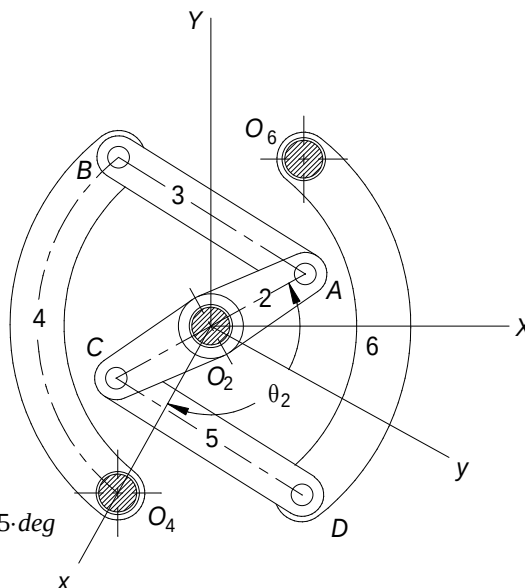
$$\arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad \arg_1 = 0.840$$

$$\arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad \arg_2 = -6.338$$

$$\theta_{2\text{toggle}} := \arccos(\arg_1) \quad \theta_{2\text{toggle}} = 32.9^\circ$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := \theta_{20} + 0.5^\circ, \theta_{20} + 1^\circ \dots 360^\circ - \theta_{2\text{toggle}} - 0.5^\circ$$



3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 1.7755 \quad K_2 = 0.5686$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.5592$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\operatorname{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.8700 \quad K_5 = 0.3509$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 2, 3, and 4 using equations 6.18.

$$\omega_2(\theta_2) := \sqrt{2 \cdot (\theta_2 - \theta_{20})} \cdot \alpha_2$$

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2(\theta_2)}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2(\theta_2)}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$, and determine its magnitude.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2(\theta_2)^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_A(\theta_2) := |\mathbf{A}_A(\theta_2)|$$

9. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2(\theta_2)^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2(\theta_2)^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

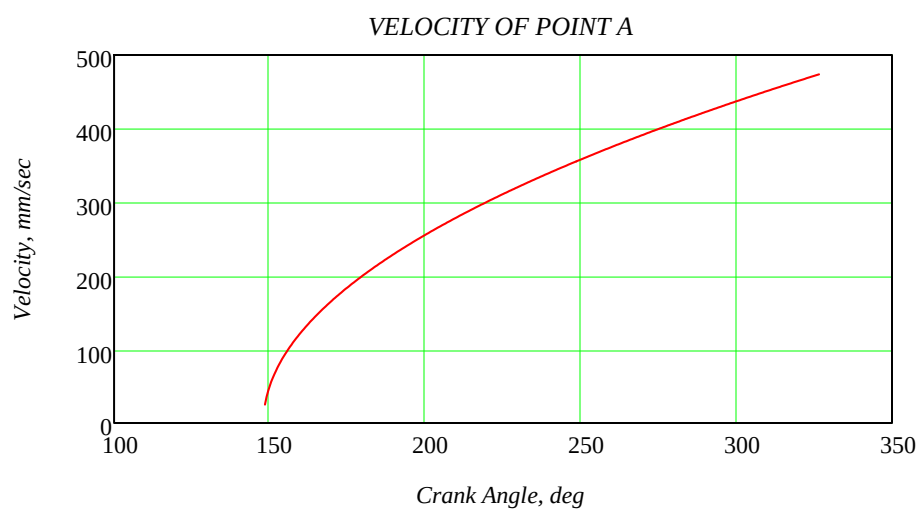
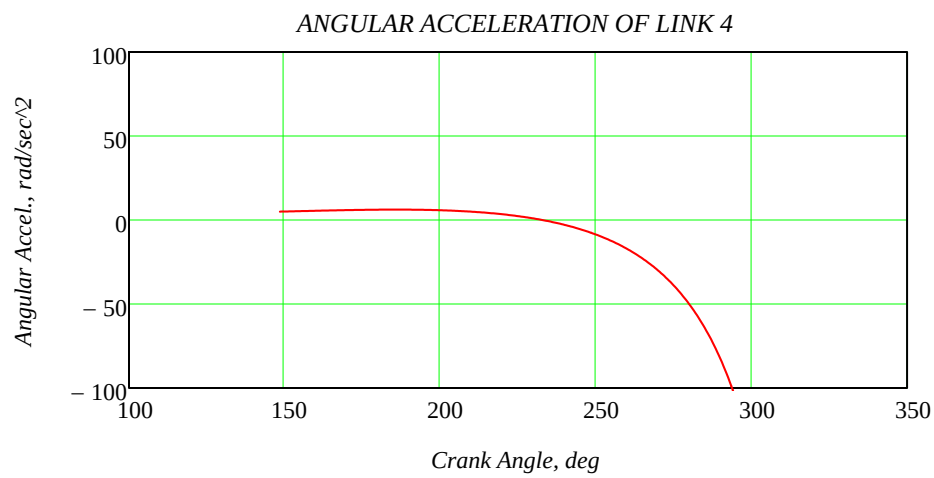
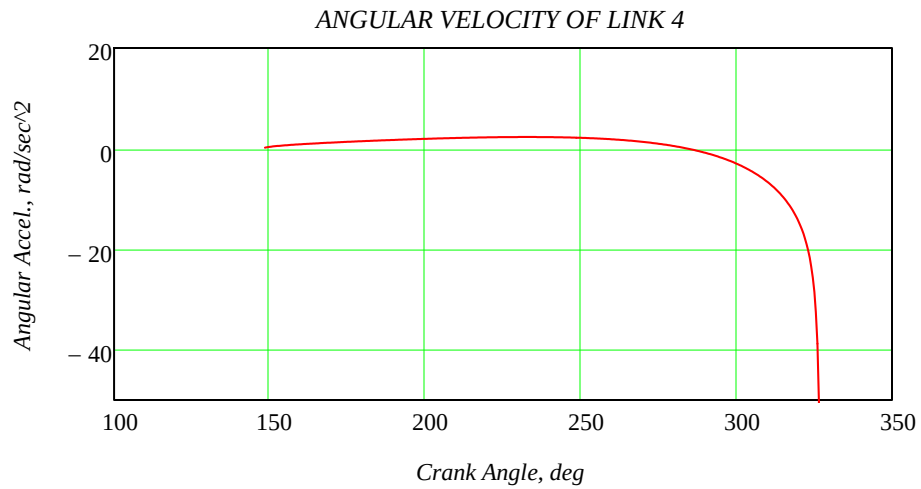
$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

10. Use equation 7.13c to determine the acceleration of point B and determine its magnitude.

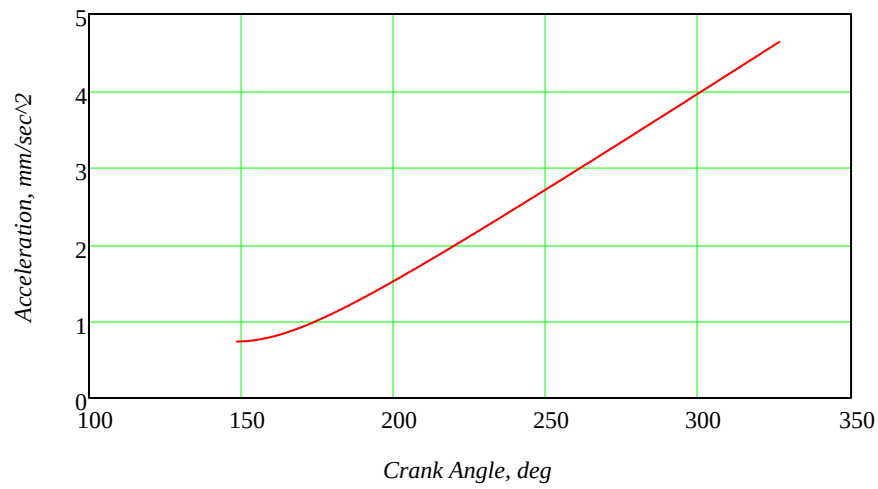
$$\mathbf{A}_B(\theta_2) := c \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) - c \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$A_B(\theta_2) := |\mathbf{A}_B(\theta_2)|$$

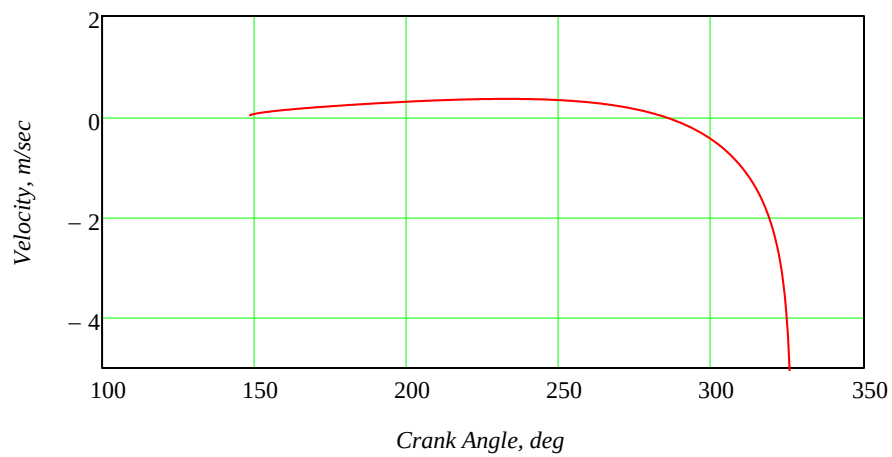
11. Plot the angular velocity and acceleration for link 4 and the velocity and acceleration components for pins A and B.



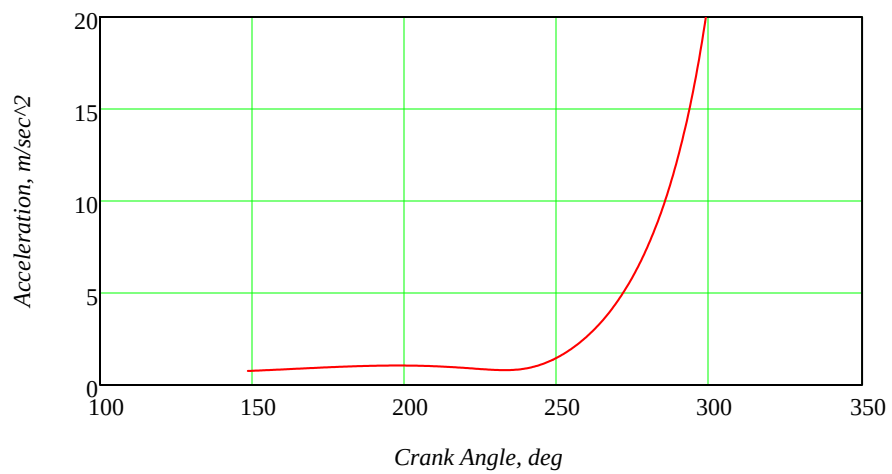
ACCELERATION OF POINT A



VELOCITY OF POINT B



ACCELERATION OF POINT B



PROBLEM 7-36

Statement: The 3-cylinder radial compressor in Figure P7-8c has the dimensions and crank angle given below. Find the piston accelerations $\mathbf{A}_6$, $\mathbf{A}_7$, and $\mathbf{A}_8$ for the position shown for $\omega = 15$ rad/sec CW, constant. Use the acceleration difference graphical method.

Given: Link lengths:

Link 2 $a := 19\text{-mm}$

Link 3 $b := 70\text{-mm}$

Offset $c := 0\text{-mm}$

Link 2 position, velocity, and acceleration: $\theta_2 := -53\text{-deg}$ $\omega_2 := -15 \frac{\text{rad}}{\text{sec}}$ $\alpha_2 := 0 \frac{\text{rad}}{\text{sec}^2}$

Cylinder angular spacing: $\beta := 120\text{-deg}$

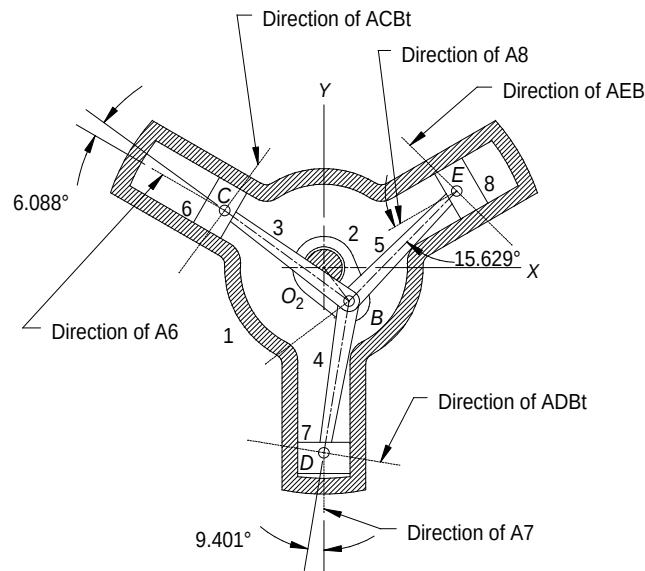
Solution: See Figure P7-8c and Mathcad file P0736.

1. In order to solve for the accelerations at piston 7, we will need θ_4 , and ω_4 . From the graphical position solution below (in the local coordinate system),

Coordinate system rotation angle: $\alpha := -90\text{-deg}$

$\theta_{21} := \theta_2 - \alpha$ $\theta_{21} = 37.000\text{ deg}$ $\theta_4 := 180\text{-deg} - 9.401\text{-deg}$ $\theta_4 = 170.599\text{ deg}$

Using equation 6.22a, $\omega_4 := \frac{a \cdot \omega_2 \cdot \cos(\theta_{21})}{b \cdot \cos(\theta_4)}$ $\omega_4 = 3.296 \text{ rad} \cdot \text{sec}^{-1}$



2. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_p^t + \mathbf{A}_p^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$
3. For point D, this becomes: $\mathbf{A}_D = (\mathbf{A}_B^t + \mathbf{A}_B^n) + (\mathbf{A}_{DB}^t + \mathbf{A}_{DB}^n)$, where (angles in the global coordinate system)

$A_{Bn} := a \cdot \omega_2^2$ $A_{Bn} = 4275 \text{ mm} \cdot \text{sec}^{-2}$

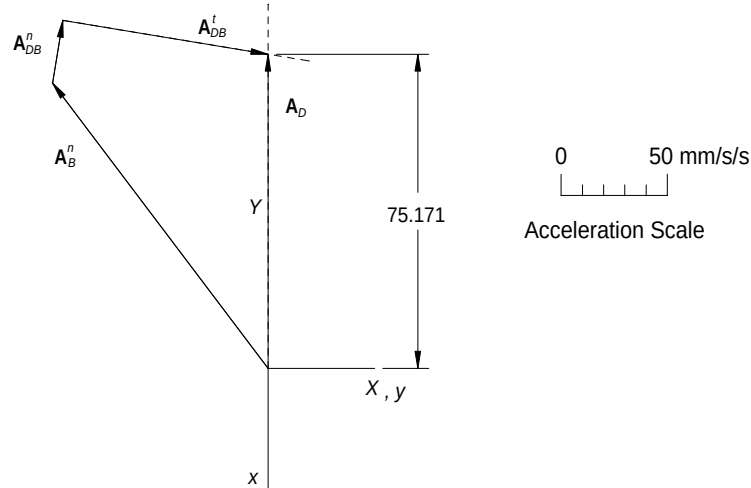
$\theta_{ABn} := \theta_{21} + \alpha + 180\text{-deg}$ $\theta_{ABn} = 127.000\text{ deg}$

$A_{Bt} := a \cdot \alpha_2$ $A_{Bt} = 0.000 \text{ mm} \cdot \text{sec}^{-2}$

$A_{DBn} := b \cdot \omega_4^2$ $A_{DBn} = 760 \text{ mm} \cdot \text{sec}^{-2}$

$\theta_{ADBn} := \theta_4 + \alpha$ $\theta_{ADBn} = 80.599\text{ deg}$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of θ_{ABn} . From the tip of $\mathbf{A}_B^n$, draw $\mathbf{A}_{DB}^n$ at an angle of θ_{ADBn} . Now that the vectors with known magnitudes are drawn, from the origin and the tip of $\mathbf{A}_{DB}^n$, draw construction lines in the directions of $\mathbf{A}_D$ (vertical) and $\mathbf{A}_{DB}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_{DB}^t$ and $\mathbf{A}_D$.



5. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 50 \cdot \frac{\text{mm}}{\text{sec}^2}$$

$$A_D := 75.171 \cdot k_a \quad A_D = 3759 \text{ mm} \cdot \text{sec}^{-2} \quad \text{at an angle of } 90 \text{ deg (global)}$$

6. In order to solve for the accelerations at piston 6, we will need θ_3 , and ω_3 . From the graphical position solution above (in the local coordinate system),

$$\text{Coordinate system rotation angle:} \quad \alpha := -90 \cdot \text{deg} - \beta$$

$$\theta_{22} := \theta_2 - \alpha \quad \theta_{22} = 157.000 \text{ deg} \quad \theta_3 := 180 \cdot \text{deg} - 6.088 \cdot \text{deg} \quad \theta_3 = 173.912 \text{ deg}$$

$$\text{Using equation 6.22a,} \quad \omega_3 := \frac{a \cdot \omega_2 \cdot \cos(\theta_{22})}{b \cdot \cos(\theta_3)} \quad \omega_3 = -3.769 \text{ rad} \cdot \text{sec}^{-1}$$

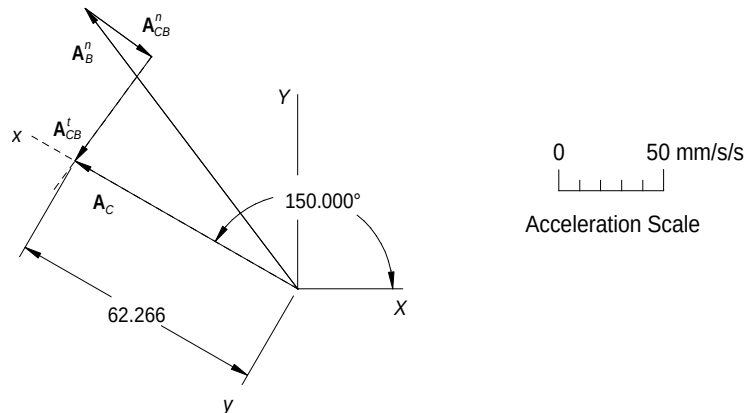
7. For point C, equation 7.4 becomes: $\mathbf{A}_C = (\mathbf{A}_B^t + \mathbf{A}_B^n) + (\mathbf{A}_{CB}^t + \mathbf{A}_{CB}^n)$, where (angles in the global coordinate system)

$$A_{CBn} := b \cdot \omega_3^2$$

$$A_{CBn} = 994.4 \text{ mm} \cdot \text{sec}^{-2}$$

$$\theta_{ACBn} := \theta_3 + \alpha$$

$$\theta_{ACBn} = -36.088 \text{ deg}$$



8. Repeat the procedure of step 4 for the equation in step 7 using $\mathbf{A}_B^n$ and $\mathbf{A}_B^t$ from step 3.

9. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 50 \cdot \frac{\text{mm}}{\text{sec}^2}$$

$$A_C := 62.266 \cdot k_a \quad A_C = 3113 \text{ mm} \cdot \text{sec}^{-2} \quad \text{at an angle of } 150 \text{ deg (global)}$$

10. In order to solve for the accelerations at piston 8, we will need θ_5 , and ω_5 . From the graphical position solution above (in the local coordinate system),

$$\text{Coordinate system rotation angle:} \quad \alpha := 360 \cdot \text{deg} + (-90 \cdot \text{deg} - 2 \cdot \beta)$$

$$\theta_{23} := \theta_2 - \alpha \quad \theta_{23} = -83.000 \text{ deg} \quad \theta_5 := 180 \cdot \text{deg} + 15.629 \cdot \text{deg} \quad \theta_5 = 195.629 \text{ deg}$$

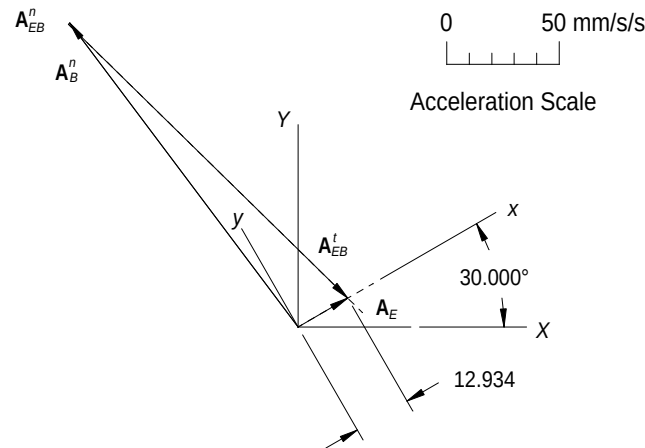
$$\text{Using equation 6.22a,} \quad \omega_5 := \frac{a \cdot \omega_2 \cdot \cos(\theta_{23})}{b \cdot \cos(\theta_5)} \quad \omega_5 = 0.515 \text{ rad} \cdot \text{sec}^{-1}$$

11. For point E, equation 7.4 becomes: $\mathbf{A}_E = (\mathbf{A}_B^t + \mathbf{A}_B^n) + (\mathbf{A}_{EB}^t + \mathbf{A}_{EB}^n)$, where (angles in the global coordinate system)

$$A_{EBn} := b \cdot \omega_5^2 \quad A_{EBn} = 18.583 \text{ mm} \cdot \text{sec}^{-2}$$

$$\theta_{AEBn} := \theta_5 + \alpha \quad \theta_{AEBn} = 225.629 \text{ deg}$$

12. Repeat the procedure of step 4 for the equation in step 11 using $\mathbf{A}_B^n$ and $\mathbf{A}_B^t$ from step 3.



Note that, at the acceleration scale chosen, $\mathbf{A}_{EB}^n$ is so small that it can hardly be seen in the layout above.

13. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 50 \cdot \frac{\text{mm}}{\text{sec}^2}$$

$$A_E := 12.934 \cdot k_a \quad A_E = 647 \text{ mm} \cdot \text{sec}^{-2} \quad \text{at an angle of } 30 \text{ deg (global)}$$

PROBLEM 7-37

Statement: The 3-cylinder radial compressor in Figure P7-8c has the dimensions and crank angle given below. Find the piston accelerations $\mathbf{A}_6$, $\mathbf{A}_7$, and $\mathbf{A}_8$ for the position shown for $\omega = 15$ rad/sec CW, constant. Use an analytical method.

Given: Link lengths:

Link 2 $a := 19\text{-mm}$

Link 3 $b := 70\text{-mm}$

Offset $c := 0\text{-mm}$

Link 2 position, velocity, and acceleration: $\theta_2 := 37\text{-deg}$ $\omega_2 := -15 \frac{\text{rad}}{\text{sec}}$ $\alpha_2 := 0 \frac{\text{rad}}{\text{sec}^2}$

Cylinder angular spacing: $\beta := 120\text{-deg}$

Solution: See Figure P7-8c and Mathcad file P0737.

1. Draw the linkage to scale and label it.

2. Determine θ_4 and d' using equation 4.17.

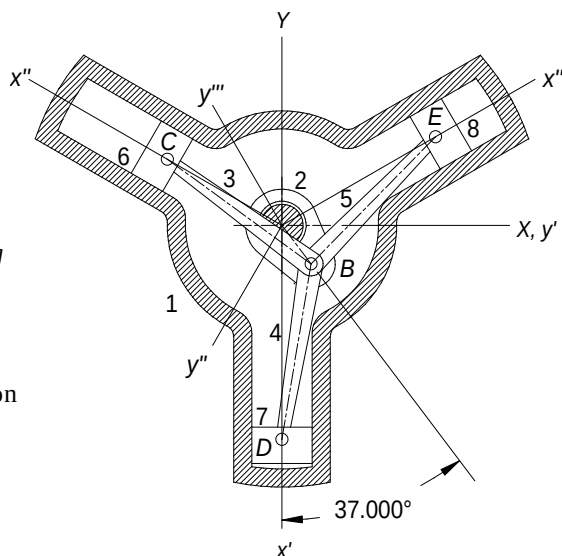
Coordinate rotation angle $\alpha := -90\text{-deg}$
($x'y'$ system)

$$\theta_4 := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi \quad \theta_4 = 170.599 \text{ deg}$$

$$d' := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_4) \quad d' = 3.316 \text{ in}$$

3. Determine the angular velocity of link 4 using equation 6.22a:

$$\omega_4 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_4)} \cdot \omega_2 \quad \omega_4 = 3.296 \frac{\text{rad}}{\text{sec}}$$



4. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_B$, determine its magnitude, and direction (global).

$$\mathbf{A}_B := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_B = (-3414.167 - 2572.759j) \frac{\text{mm}}{\text{sec}^2} \quad A_B := |\mathbf{A}_B| \quad \theta_{AB} := \arg(\mathbf{A}_B) + \alpha + 360 \cdot \text{deg}$$

The acceleration of pin B is $A_B = 4275 \frac{\text{mm}}{\text{sec}^2}$ at $\theta_{AB} = 127 \text{ deg}$

5. Determine the angular acceleration of link 4 using equation 7.16d.

$$\alpha_4 := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_4^2 \cdot \sin(\theta_4)}{b \cdot \cos(\theta_4)} \quad \alpha_4 = 35.456 \frac{\text{rad}}{\text{sec}^2}$$

6. Use equation 7.16e for the acceleration of pin D.

$$A_D := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \alpha_4 \cdot \sin(\theta_4) + b \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$A_D = -3759 \frac{mm}{sec^2} \quad \text{A negative sign means that } \mathbf{A}_D \text{ is inward}$$

7. Determine θ_3 and d'' using equation 4.17.

$$\theta_2 := \theta_2 + 120 \cdot deg \quad \theta_2 = 157.000 \ deg \quad (x''y'' \text{ system})$$

$$\theta_3 := asin\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi \quad \theta_3 = 173.912 \ deg$$

$$d'' := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_3) \quad d'' = 2.052 \ in$$

8. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3)} \cdot \omega_2 \quad \omega_3 = -3.769 \frac{rad}{sec}$$

9. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3 := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_3^2 \cdot \sin(\theta_3)}{b \cdot \cos(\theta_3)} \quad \alpha_3 = 22.483 \frac{rad}{sec^2}$$

10. Use equation 7.16e for the acceleration of pin C.

$$A_C := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \alpha_3 \cdot \sin(\theta_3) + b \cdot \omega_3^2 \cdot \cos(\theta_3)$$

$$A_C = 3113 \frac{mm}{sec^2} \quad \text{A positive sign means that } \mathbf{A}_C \text{ is outward}$$

11. Determine θ_5 and d''' using equation 4.17.

$$\theta_2 := \theta_2 + 120 \cdot deg \quad \theta_2 = 277.000 \ deg \quad (x'''y''' \text{ system})$$

$$\theta_5 := asin\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi \quad \theta_5 = 195.629 \ deg$$

$$d''' := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_5) \quad d''' = 2.745 \ in$$

12. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_5)} \cdot \omega_2 \quad \omega_5 = 0.515 \frac{rad}{sec}$$

13. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5 := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_5^2 \cdot \sin(\theta_5)}{b \cdot \cos(\theta_5)} \quad \alpha_5 = -62.869 \frac{rad}{sec^2}$$

14. Use equation 7.16e for the acceleration of pin E.

$$A_E := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \alpha_5 \cdot \sin(\theta_5) + b \cdot \omega_5^2 \cdot \cos(\theta_5)$$

$$A_E = 646.72 \frac{mm}{sec^2} \quad \text{A positive sign means that } \mathbf{A}_E \text{ is outward}$$

PROBLEM 7-38

Statement: The 3-cylinder radial compressor in Figure P7-8c has the dimensions and crank angle given below. Find and plot the piston accelerations A_6 , A_7 , and A_8 for one revolution of the crank if $\omega = 15 \text{ rad/sec}$ CW, constant.

Given: Link lengths:

Link 2 $a := 19 \cdot \text{mm}$

Link 3 $b := 70 \cdot \text{mm}$

Offset $c := 0 \cdot \text{mm}$

Link 2 velocity, and acceleration: $\omega_2 := -15 \cdot \frac{\text{rad}}{\text{sec}}$ $\alpha_2 := 0 \cdot \frac{\text{rad}}{\text{sec}^2}$

Cylinder angular spacing: $\alpha := 120 \cdot \text{deg}$

Solution: See Figure P7-8c and Mathcad file P0738.

1. Draw the linkage to scale and label it. Note that there are three local coordinate systems.

2. Determine the range of motion for this slider-crank linkage. This will be the same in each coordinate frame.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

3. Determine θ_4 using equation 4.17.

$$\theta_4(\theta_2) := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi \quad (x'y' \text{ system})$$

4. Determine the angular velocity of link 4 using equation 6.22a:

$$\omega_4(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_4(\theta_2))} \cdot \omega_2$$

5. Determine the angular acceleration of link 4 using equation 7.16d.

$$\alpha_4(\theta_2) := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))}{b \cdot \cos(\theta_4(\theta_2))}$$

6. Use equation 7.16e for the acceleration of pin D.

$$A_D(\theta_2) := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) \dots \\ + b \cdot \alpha_4(\theta_2) \cdot \sin(\theta_4(\theta_2)) + b \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

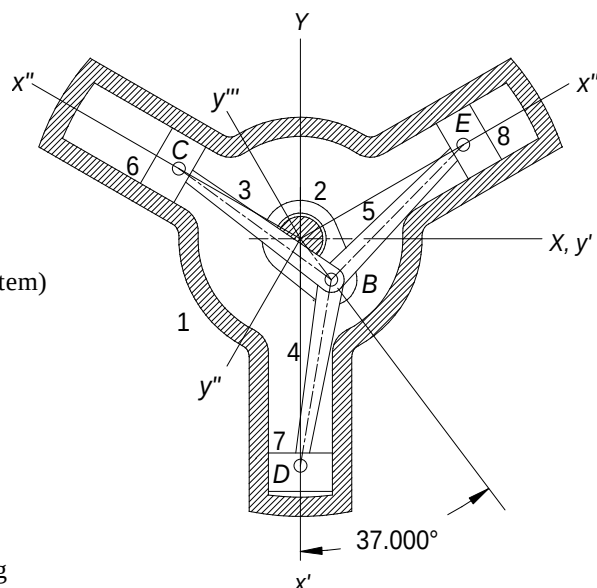
7. Determine θ_3 using equation 4.17.

$$\theta_3(\theta_2) := \text{asin}\left(-\frac{a \cdot \sin(\theta_2 + \alpha) - c}{b}\right) + \pi \quad (x''y'' \text{ system})$$

8. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2 + \alpha)}{\cos(\theta_3(\theta_2))} \cdot \omega_2$$

9. Determine the angular acceleration of link 3 using equation 7.16d.



$$\alpha_3(\theta_2) := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2 + \alpha) - a \cdot \omega_2^2 \cdot \sin(\theta_2 + \alpha) + b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2))}{b \cdot \cos(\theta_3(\theta_2))}$$

10. Use equation 7.16e for the acceleration of pin C.

$$A_C(\theta_2) := -a \cdot \alpha_2 \cdot \sin(\theta_2 + \alpha) - a \cdot \omega_2^2 \cdot \cos(\theta_2 + \alpha) \dots \\ + b \cdot \alpha_3(\theta_2) \cdot \sin(\theta_3(\theta_2)) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2))$$

11. Determine θ_5 and d''' using equation 4.17.

$$\theta_5(\theta_2) := \operatorname{asin}\left(-\frac{a \cdot \sin(\theta_2 + 2 \cdot \alpha) - c}{b}\right) + \pi \quad (x'''y''' \text{ system})$$

12. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2 + 2 \cdot \alpha)}{\cos(\theta_5(\theta_2))} \cdot \omega_2$$

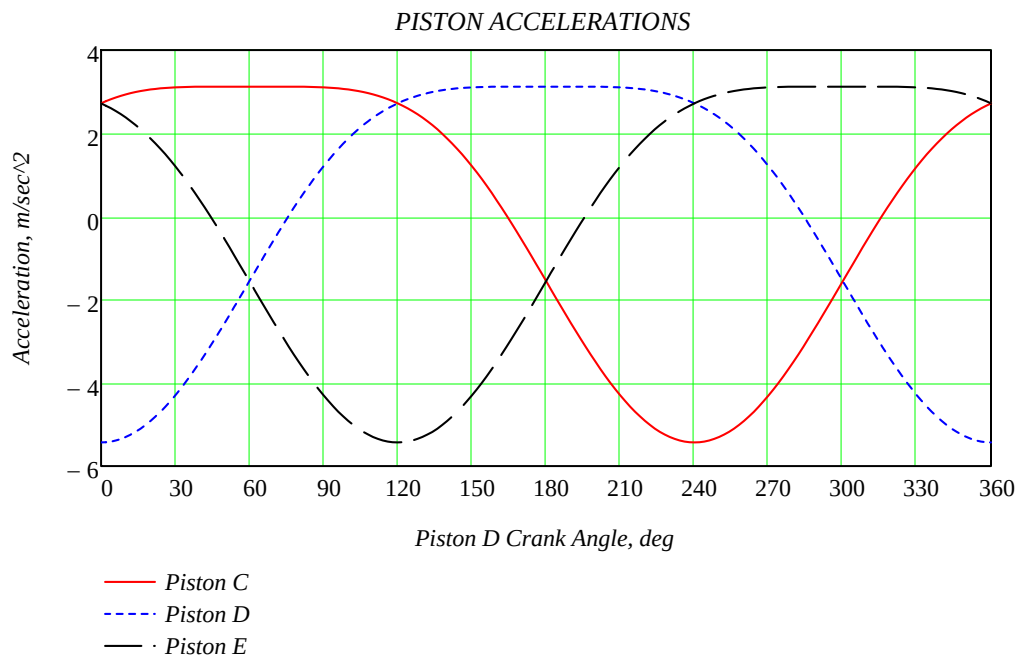
13. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5(\theta_2) := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2 + 2 \cdot \alpha) - a \cdot \omega_2^2 \cdot \sin(\theta_2 + 2 \cdot \alpha) + b \cdot \omega_5(\theta_2)^2 \cdot \sin(\theta_5(\theta_2))}{b \cdot \cos(\theta_5(\theta_2))}$$

14. Use equation 7.16e for the acceleration of pin E.

$$A_E(\theta_2) := -a \cdot \alpha_2 \cdot \sin(\theta_2 + 2 \cdot \alpha) - a \cdot \omega_2^2 \cdot \cos(\theta_2 + 2 \cdot \alpha) \dots \\ + b \cdot \alpha_5(\theta_2) \cdot \sin(\theta_5(\theta_2)) + b \cdot \omega_5(\theta_2)^2 \cdot \cos(\theta_5(\theta_2))$$

15. Plot the piston accelerations.



PROBLEM 7-39

Statement: Figure P7-9 shows a linkage in one position. Find the instantaneous accelerations of points A, B, and P if link O_2A is rotating CW at 40 rad/sec.

Given: Link lengths:

$$\text{Link 2 (A to B)} \quad a := 5.00 \cdot \text{in} \quad \text{Link 3 (B to C)} \quad b := 4.40 \cdot \text{in}$$

$$\text{Link 4 (C to D)} \quad c := 5.00 \cdot \text{in} \quad \text{Link 1 (A to D)} \quad d := 9.50 \cdot \text{in}$$

$$\text{Coupler point:} \quad R_{pa} := 8.90 \cdot \text{in} \quad \delta_3 := 56 \cdot \text{deg}$$

$$\text{Crank angle and motion:} \quad \theta_2 := 50 \cdot \text{deg} \quad \omega_2 := -40 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-9 and Mathcad file P0739.

1. Draw the linkage to scale and label it.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 1.9000 \quad K_2 = 1.9000$$

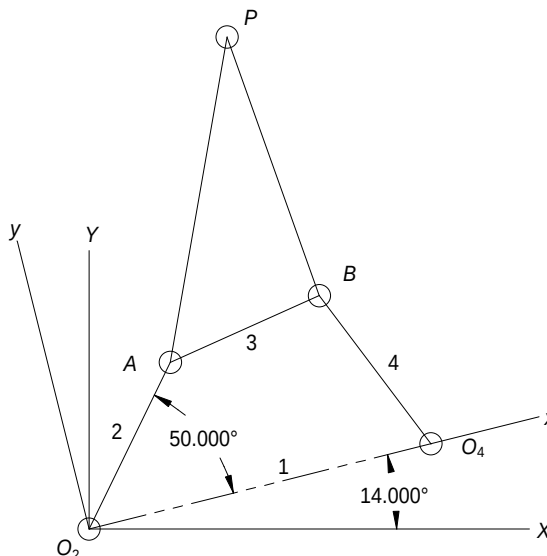
$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 2.4178$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.0607 \quad B = -1.5321 \quad C = 2.4537$$



3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_4 = -246.992 \text{ deg}$$

$$\theta_4 := \theta_4 + 360 \cdot \text{deg} \quad \theta_4 = 113.008 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 2.1591 \quad K_5 = -2.4911$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -2.3605$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.5321$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.1539$$

5. Use equation 4.13 to find values of θ_3 .

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_3 = -349.895 \text{ deg}$$

$$\theta_3 := \theta_3 + 360 \cdot \text{deg} \quad \theta_3 = 10.105 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad \omega_3 = 41.552 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad \omega_4 = -26.320 \frac{\text{rad}}{\text{sec}}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (-5142.3 - 6128.4i) \frac{\text{in}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A)$$

$$\text{The acceleration of pin A is } A_A = 8000 \frac{\text{in}}{\text{sec}^2} \quad \text{at } \theta_{AA} = -130.0 \text{ deg}$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A := c \cdot \sin(\theta_4) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_4) \quad E := b \cdot \cos(\theta_3)$$

$$A = 4.602 \text{ in} \quad B = 0.772 \text{ in} \quad D = -1.954 \text{ in} \quad E = 4.332 \text{ in}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$C = 1.398 \times 10^4 \text{ in} \cdot \text{sec}^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = -4.273 \times 10^3 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = -356.538 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = 2.977 \times 10^3 \frac{\text{rad}}{\text{sec}^2}$$

9. Use equation 7.13c to determine the acceleration of point B.

$$\mathbf{A}_B := c \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$\mathbf{A}_B = (-12346.3 - 9005.8i) \frac{\text{in}}{\text{sec}^2} \quad A_B := |\mathbf{A}_B| \quad \theta_{AB} := \arg(\mathbf{A}_B)$$

$$\text{The acceleration of pin B is } A_B = 15282 \frac{\text{in}}{\text{sec}^2} \quad \text{at } \theta_{AB} = -143.9 \text{ deg}$$

10. Use equations 7.32 to find the acceleration of the point C.

$$\mathbf{A}_{CA} := R_{pa} \cdot \alpha_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3)) \dots \\ + -R_{pa} \cdot \omega_3^2 \cdot (\cos(\theta_3 + \delta_3) + j \cdot \sin(\theta_3 + \delta_3))$$

$$\mathbf{A}_C := \mathbf{A}_A + \mathbf{A}_{CA}$$

$$A_C := |\mathbf{A}_C| \quad A_C = 23073 \frac{\text{in}}{\text{sec}^2} \quad \arg(\mathbf{A}_C) = -111.525 \text{ deg}$$

PROBLEM 7-40

Statement: Figure P7-10 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the acceleration of the coupler point P at 2-deg increments of crank angle. Check your results with program FOURBAR.

Units: $\text{rpm} := 2 \cdot \pi \cdot \text{rad} \cdot \text{min}^{-1}$

Given: Link lengths:

Link 2 (O_2 to A)	$a := 10 \cdot \text{mm}$	Link 3 (A to B)	$b := 20.6 \cdot \text{mm}$
Link 4 (B to O_4)	$c := 23.3 \cdot \text{mm}$	Link 1 (O_2 to O_4)	$d := 22.2 \cdot \text{mm}$
Coupler point:	$R_{pa} := 30.6 \cdot \text{mm}$	$\delta_3 := -31 \cdot \text{deg}$	
Crank motion:	$\omega_2 := 100 \cdot \text{rpm}$	$\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$	

Solution: See Figure P7-10 and Mathcad file P0740.

1. Draw the linkage to scale and label it.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

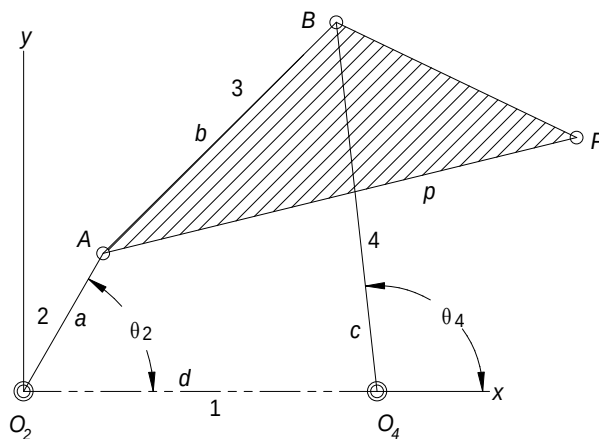
$$K_1 = 2.2200 \quad K_2 = 0.9528$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.5265$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$



3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0777 \quad K_5 = -1.1512$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 .

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

8. Use equations 7.12 to determine the angular accelerations of link 3.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

9. Use equations 7.32 to find the acceleration of the point P .

$$\mathbf{A}_{PA}(\theta_2) := R_{pa} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots$$

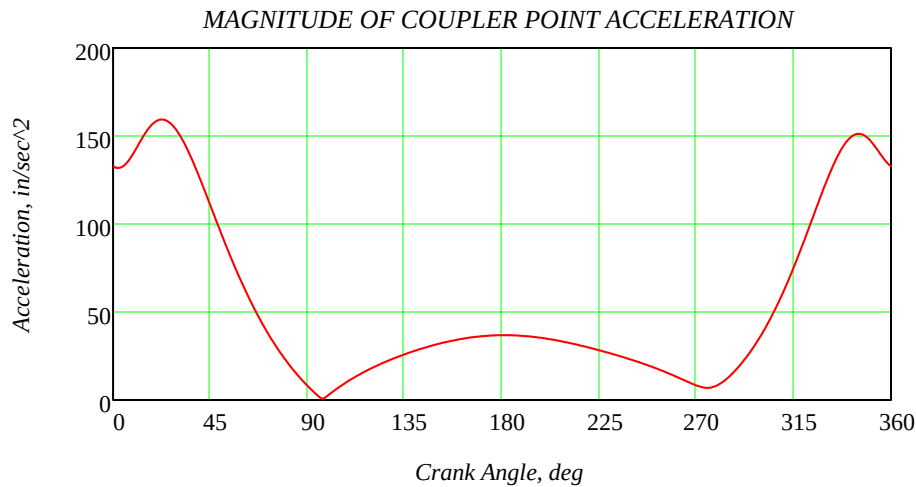
$$+ R_{pa} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3))$$

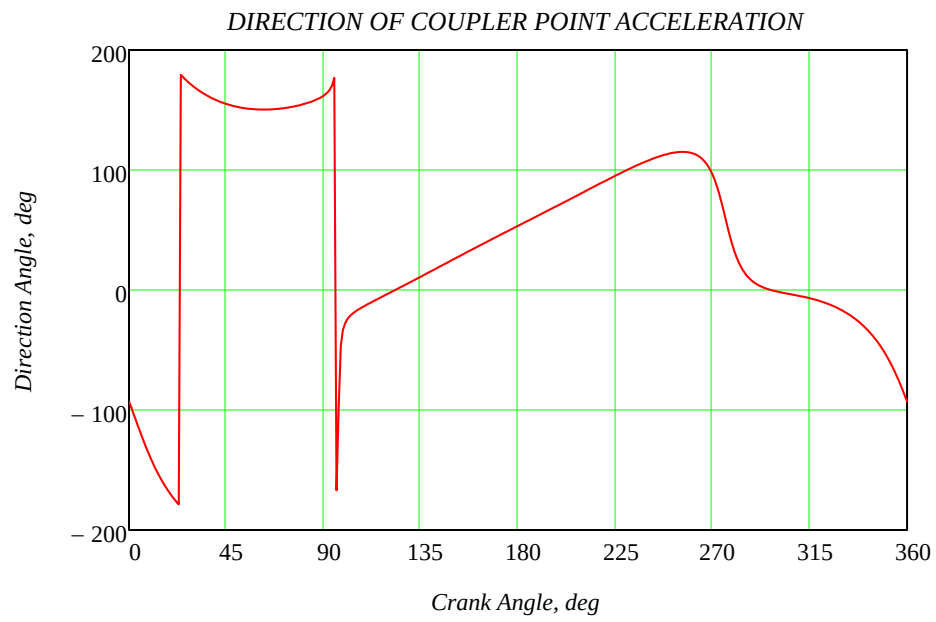
$$\mathbf{A}_P(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2)$$

$$A_P(\theta_2) := |\mathbf{A}_P(\theta_2)| \quad \theta_{AP}(\theta_2) := \arg(\mathbf{A}_P(\theta_2))$$

10. Plot the magnitude and direction of the coupler point P . (See next page.)

$$\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$$





PROBLEM 7-41

Statement: Figure P7-11 shows a linkage that operates at 500 crank rpm. Find and plot the magnitude and direction of the acceleration of point B at 2-deg increments of crank angle. Check your result with program FOURBAR.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 2.000 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 8.375 \cdot \text{in}$$

$$\text{Link 4 (B to O}_4) \quad c := 7.187 \cdot \text{in} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 9.625 \cdot \text{in}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 500 \cdot \text{rpm} \quad \omega_2 = 52.360 \text{ rad} \cdot \text{sec}^{-1}$$

$$\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-11 and Mathcad file P0741.

1. Draw the linkage to scale and label it.
2. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

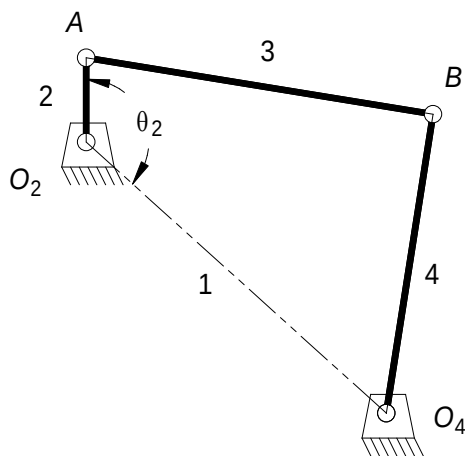
$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 4.8125 \quad K_2 = 1.3392$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 2.7186$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$



4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.1493 \quad K_5 = -3.4367$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

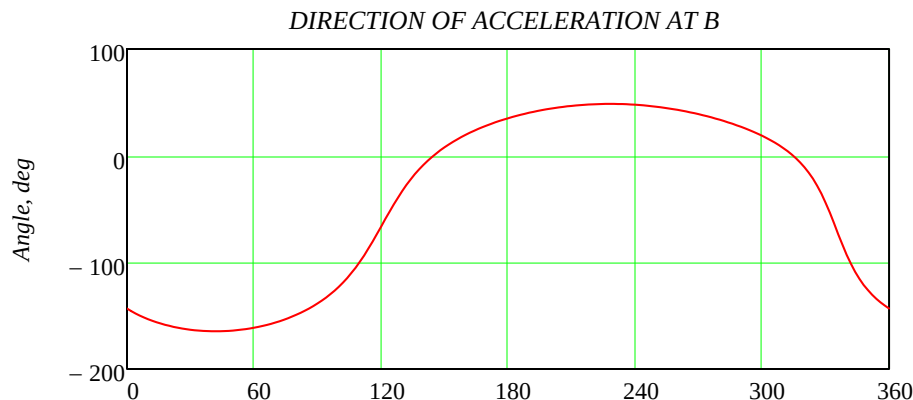
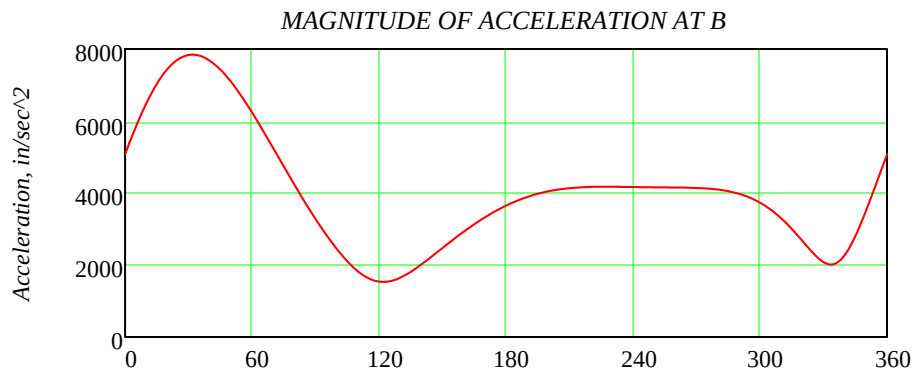
$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

9. Determine the acceleration of point *B* using equations 7.13c.

$$\mathbf{A_B}(\theta_2) := c \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) - c \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$A_B(\theta_2) := |\mathbf{A_B}(\theta_2)| \quad \theta_{AB}(\theta_2) := \arg(\mathbf{A_B}(\theta_2))$$

10. Plot the magnitude and angle of the acceleration at point *B*.



PROBLEM 7-42

Statement: Figure P7-12 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the acceleration of the coupler point P at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 0.785 \cdot \text{in}$	Link 3 (A to B)	$b := 0.356 \cdot \text{in}$
Link 4 (B to O_4)	$c := 0.950 \cdot \text{in}$	Link 1 (O_2 to O_4)	$d := 0.544 \cdot \text{in}$
Coupler point:	$R_{pa} := 1.09 \cdot \text{in}$	$\delta_3 := 0 \cdot \text{deg}$	
Crank speed:	$\omega_2 := 20 \cdot \text{rpm}$	$\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$	

Solution: See Figure P7-12 and Mathcad file P0742.

1. Draw the linkage to scale and label it.
2. Using the geometry defined in Figure 3-1a in the text, determine the input crank angles (relative to the line O_2O_4) at which links 2 and 3, and 3 and 4 are in toggle.

$$\theta_{20} := \arccos \left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot a \cdot d} \right] \quad \theta_{20} = 158.286 \text{ deg}$$

$$\theta_2 := -\theta_{20} + 0.5 \cdot \text{deg}, -\theta_{20} + 1 \cdot \text{deg} .. \theta_{20} - 0.5 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 0.6930$$

$$K_2 = 0.5726$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_3 = 1.1317$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b}$$

$$K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 1.5281$$

$$K_5 = -0.2440$$

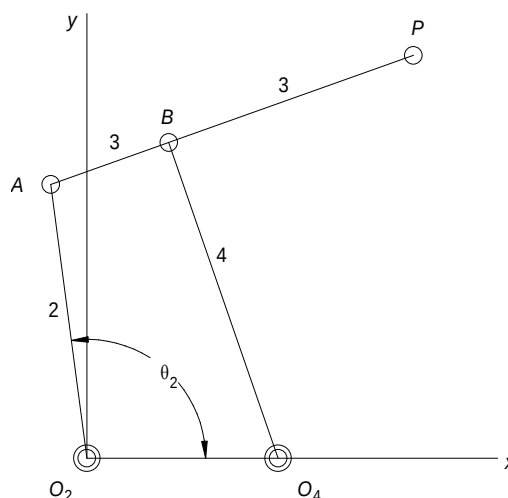
$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 .

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$



7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

9. Use equations 7.12 to determine the angular accelerations of link 3.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

10. Use equations 7.32 to find the acceleration of the point P .

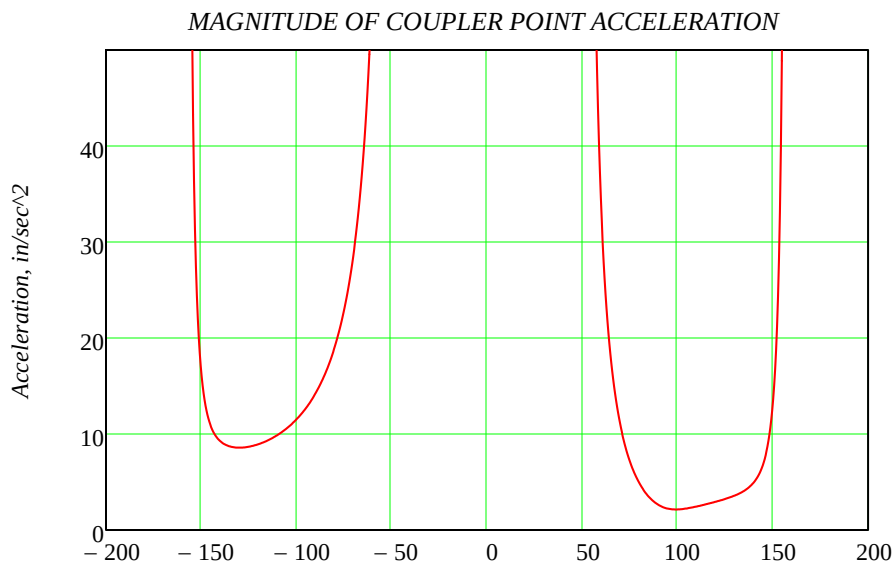
$$\mathbf{A}_{PA}(\theta_2) := R_{pa} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots$$

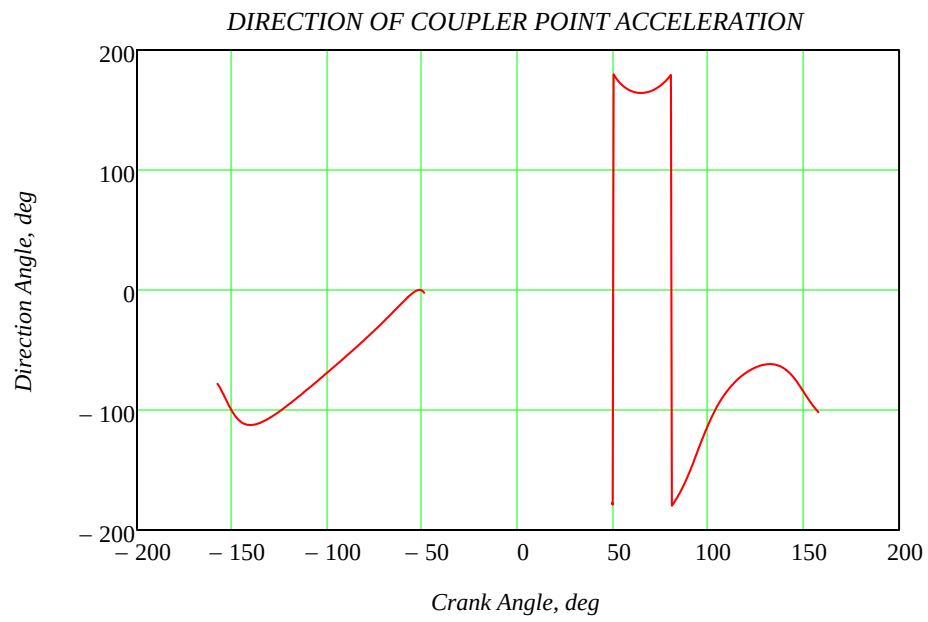
$$+ R_{pa} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{A}_P(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2)$$

$$A_P(\theta_2) := |\mathbf{A}_P(\theta_2)| \quad \theta_{AP}(\theta_2) := \arg(\mathbf{A}_P(\theta_2))$$

11. Plot the magnitude and direction of the coupler point P .





PROBLEM 7-43

Statement: Figure P7-13 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the acceleration of the coupler point P at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 0.86 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 1.85 \cdot \text{in}$$

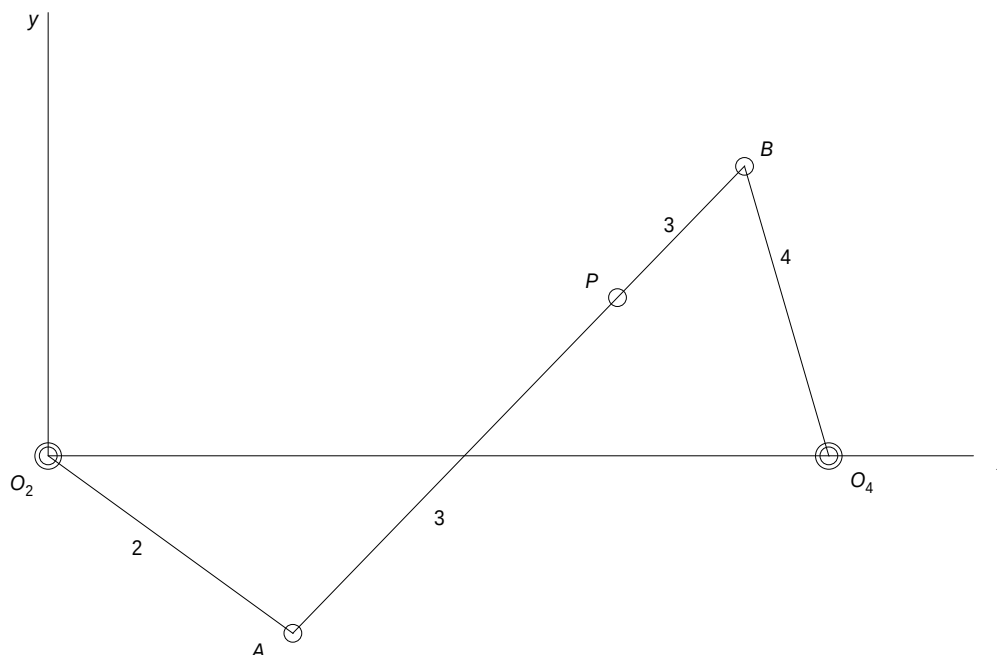
$$\text{Link 4 (B to O}_4) \quad c := 0.86 \cdot \text{in} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 2.22 \cdot \text{in}$$

$$\text{Coupler point:} \quad R_{pa} := 1.33 \cdot \text{in} \quad \delta_3 := 0 \cdot \text{deg}$$

$$\text{Crank motion:} \quad \omega_2 := 80 \cdot \text{rpm} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-13 and Mathcad file P0743.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.37.

$$\arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad \arg_1 = 1.228$$

$$\arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad \arg_2 = -0.439$$

$$\theta_{2\text{toggle}} := \text{acos}(\arg_2) \quad \theta_{2\text{toggle}} = 116.0 \text{ deg}$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2\text{toggle}} + 0.5 \cdot \text{deg}, -\theta_{2\text{toggle}} + 1 \cdot \text{deg} .. \theta_{2\text{toggle}} - 0.5 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 2.5814 \quad K_2 = 2.5814$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 2.0181$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.2000 \quad K_5 = -2.6244$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 .

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

9. Use equations 7.12 to determine the angular accelerations of link 3.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

10. Use equations 7.32 to find the acceleration of the point P .

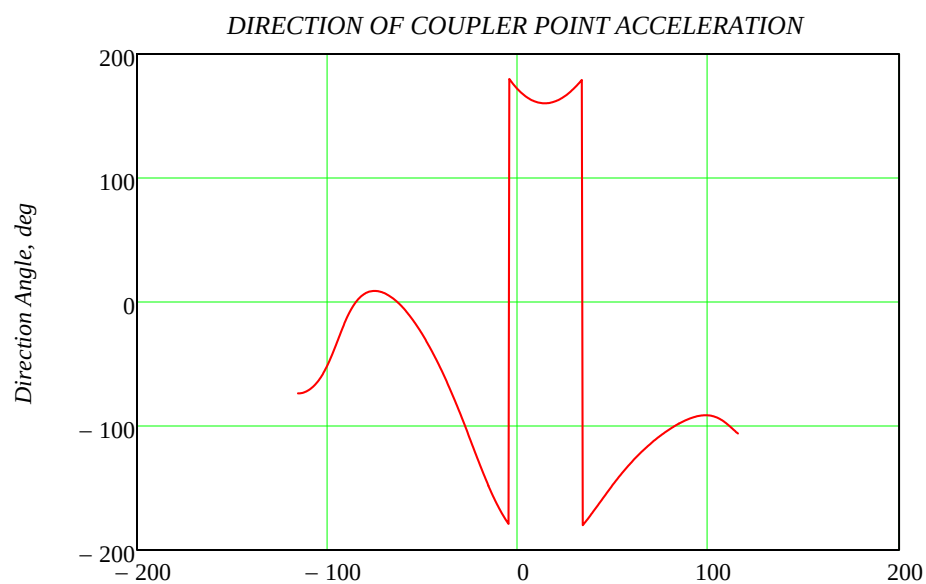
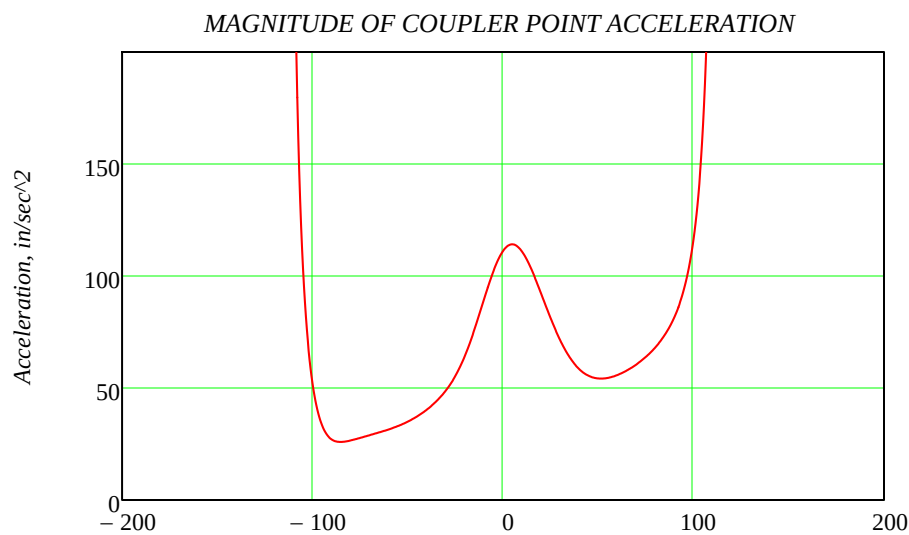
$$\mathbf{A}_{PA}(\theta_2) := R_{pa} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots$$

$$+ -R_{pa} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{A}_P(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2)$$

$$A_P(\theta_2) := |\mathbf{A}_P(\theta_2)| \quad \theta_{AP}(\theta_2) := \arg(\mathbf{A}_P(\theta_2))$$

11. Plot the magnitude and direction of the coupler point P .



PROBLEM 7-44

Statement: Figure P7-14 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the acceleration of the coupler point P at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 0.72 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 0.68 \cdot \text{in}$$

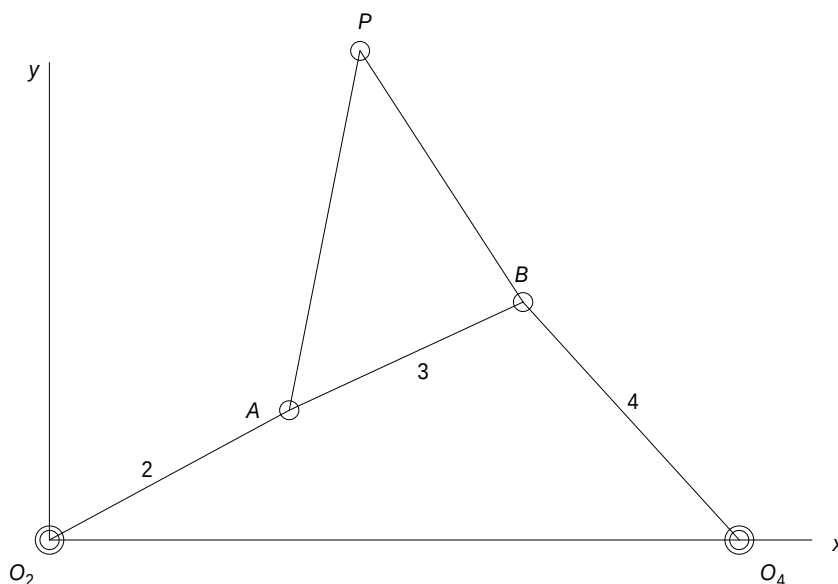
$$\text{Link 4 (B to O}_4) \quad c := 0.85 \cdot \text{in} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 1.82 \cdot \text{in}$$

$$\text{Coupler point:} \quad R_{pa} := 0.97 \cdot \text{in} \quad \delta_3 := 54 \cdot \text{deg}$$

$$\text{Crank motion:} \quad \omega_2 := 80 \cdot \text{rpm} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-14 and Mathcad file P0744.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.37.

$$\arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \quad \arg_1 = 1.451$$

$$\arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \quad \arg_2 = 0.568$$

$$\theta_{2\text{toggle}} := \text{acos}(\arg_2) \quad \theta_{2\text{toggle}} = 55.4 \text{ deg}$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2\text{toggle}} + 0.5 \cdot \text{deg}, -\theta_{2\text{toggle}} + 1 \cdot \text{deg} .. \theta_{2\text{toggle}} - 0.5 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 2.5278 \quad K_2 = 2.1412$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 3.3422$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 2.6765 \quad K_5 = -3.6465$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 .

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

9. Use equations 7.12 to determine the angular accelerations of link 3.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

10. Use equations 7.32 to find the acceleration of the point P.

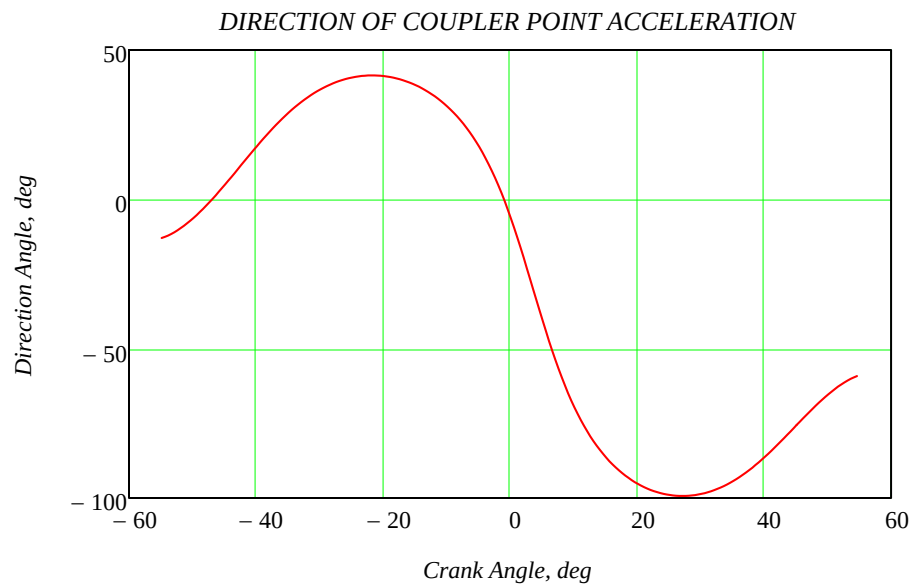
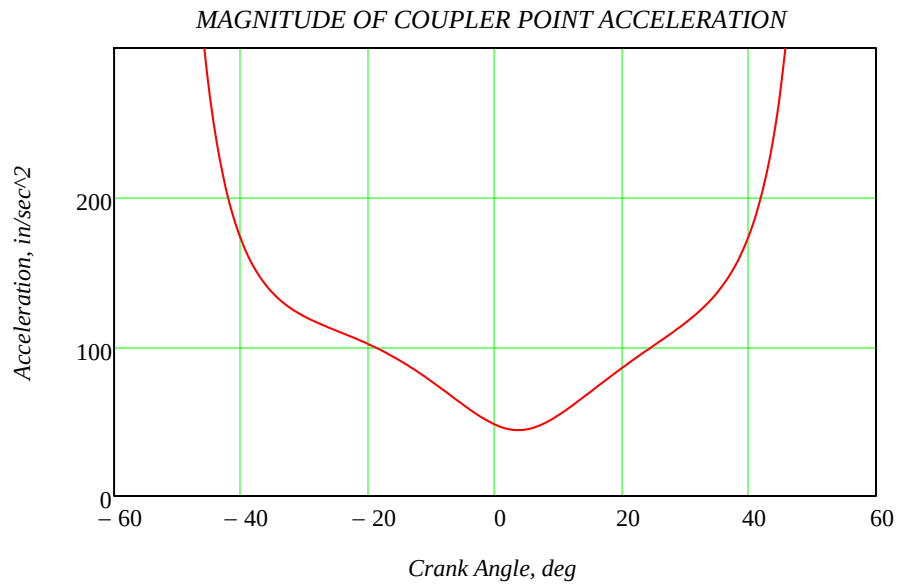
$$\begin{aligned} \mathbf{A}_{PA}(\theta_2) &:= R_{pa} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots \\ &\quad + -R_{pa} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3)) \end{aligned}$$

$$\mathbf{A}_P(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2)$$

$$A_P(\theta_2) := |\mathbf{A_P}(\theta_2)|$$

$$\theta_{AP}(\theta_2) := \arg(\mathbf{A_P}(\theta_2))$$

11. Plot the magnitude and direction of the coupler point P .



PROBLEM 7-45

Statement: Figure P7-15 shows a power hacksaw that is an offset crank-slider mechanism that has the dimensions given below. Draw an equivalent linkage diagram and then calculate and plot the acceleration of the saw blade with respect to the piece being cut over one revolution of the crank, which rotates at 50 rpm.

Given: Link lengths:

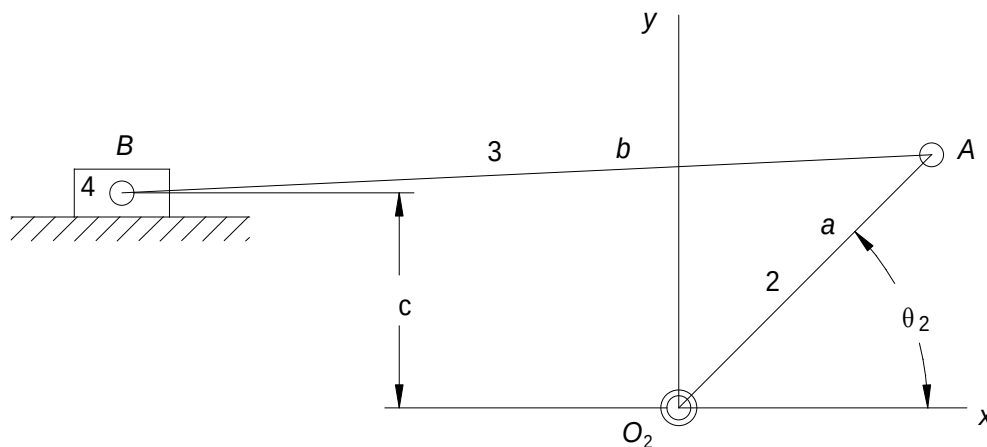
$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 75 \cdot \text{mm} \quad \text{Offset} \quad c := 45 \cdot \text{mm}$$

$$\text{Link 3 (A to B)} \quad b := 170 \cdot \text{mm}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 50 \cdot \text{rpm} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-15 and Mathcad file P0745.

1. Draw the equivalent linkage to a convenient scale and label it.



2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

3. Determine θ_3 using equations 4.16 for the crossed circuit.

$$\theta_3(\theta_2) := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right)$$

4. Determine the angular velocity of link 3 using equation 6.22a:

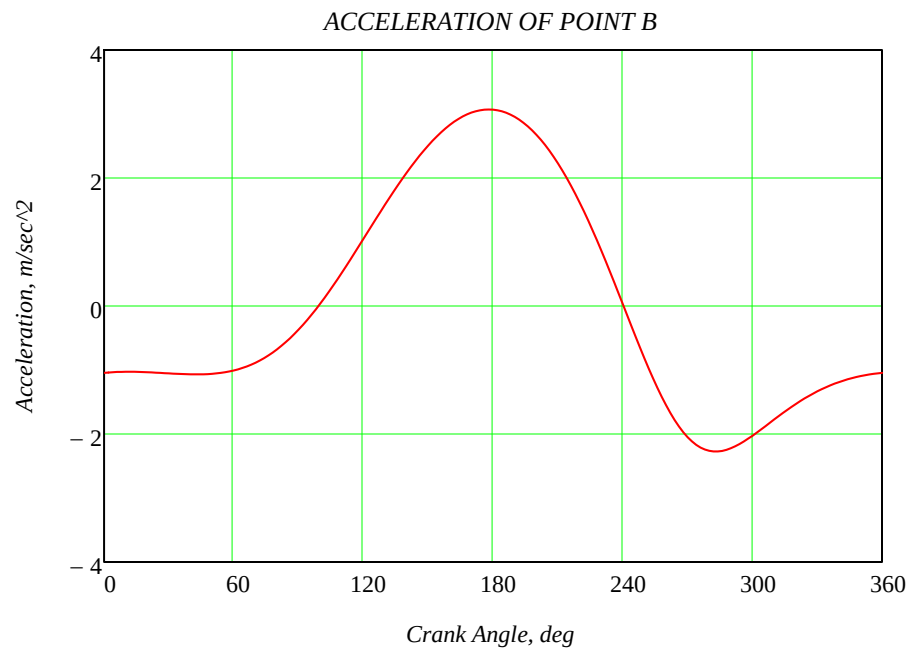
$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3(\theta_2))} \cdot \omega_2$$

5. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3(\theta_2) := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2))}{b \cdot \cos(\theta_3(\theta_2))}$$

6. Use equation 7.16e for the acceleration of pin B.

$$A_B(\theta_2) := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) \dots \\ + b \cdot \alpha_3(\theta_2) \cdot \sin(\theta_3(\theta_2)) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2))$$



PROBLEM 7-46

Statement: Figure P7-16 shows a walking beam indexing and pick-and-place mechanism that can be analyzed as two fourbar linkages driven by a common crank. Calculate and plot the relative acceleration between points E and P for one revolution of gear 2.

Given: Link lengths (walking-beam linkage):

Link 2 (O_2 to A)	$a' := 40 \cdot \text{mm}$	Link 3 (A to D)	$b' := 108 \cdot \text{mm}$
Link 4 (O_4 to D)	$c' := 40 \cdot \text{mm}$	Link 1 (O_2 to O_4)	$d' := 108 \cdot \text{mm}$

Link lengths (pick and place linkage):

Link 2 (O_5 to B)	$a := 13 \cdot \text{mm}$	Link 7 (B to C)	$b := 193 \cdot \text{mm}$
Link 6 (C to O_6)	$c := 92 \cdot \text{mm}$	Link 1 (O_5 to O_6)	$d := 128 \cdot \text{mm}$

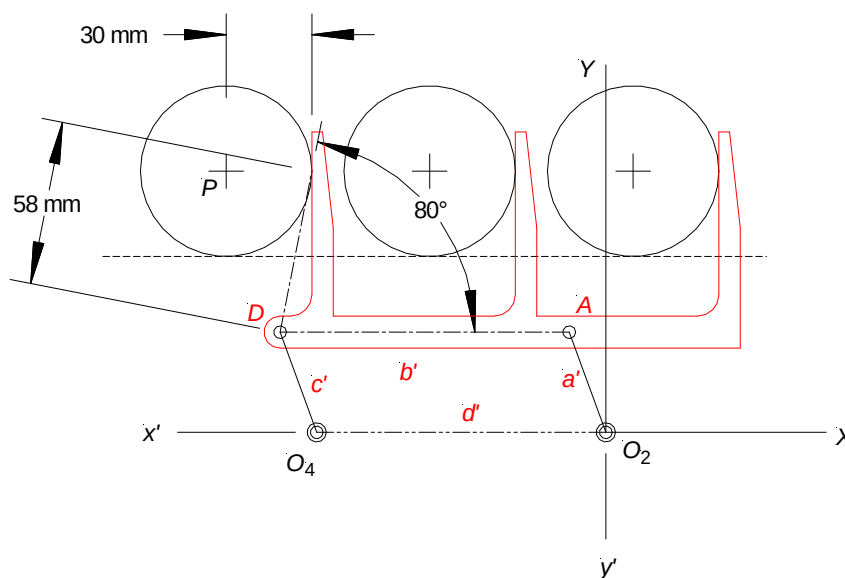
Rocker point E : $u := 164 \cdot \text{mm}$

Crank speed: $\omega_2 := 10 \cdot \text{rpm}$ $\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Gears 4 & 5 phase angle $\phi := 143 \cdot \text{deg}$

Solution: See Figure P7-16 and Mathcad file P0746.

1. Draw the walking-beam linkage to scale and label it.



2. Determine the range of motion for this mechanism.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg} \quad (\text{local } x'y' \text{ coordinate system})$$

3. This part of the mechanism is a special-case Grashof in the parallelogram configuration. As such, the coupler does not rotate, but has curvilinear motion with every point on it having the same velocity and acceleration. Therefore, it is only necessary to calculate the X -component of the acceleration at point A in order to determine the acceleration of the cylinder center, P .

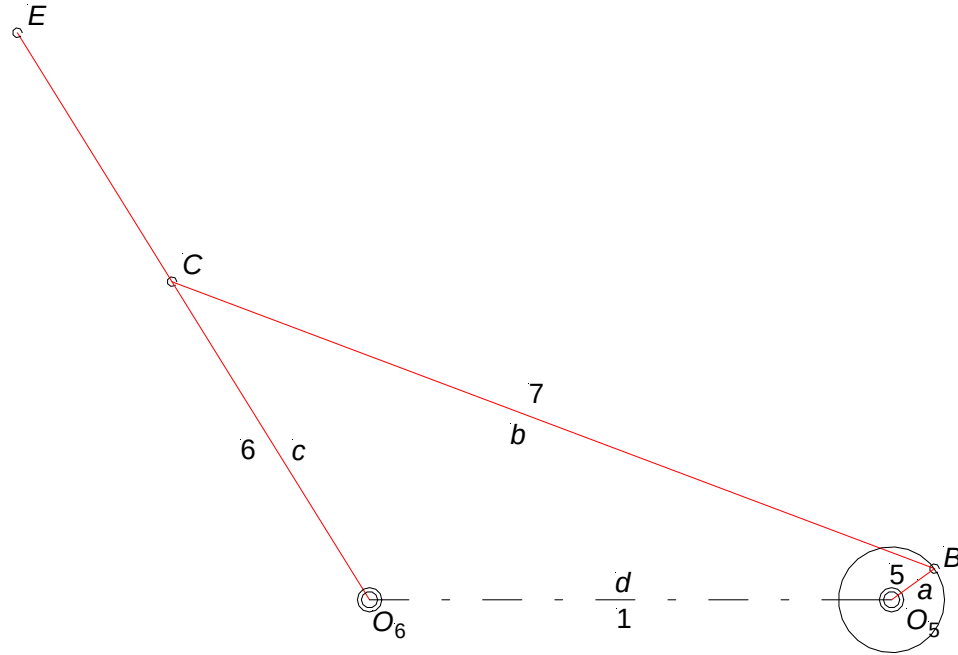
$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{Ax}(\theta_2) := \text{Re}(\mathbf{A}_A(\theta_2))$$

In the global coordinate frame,

$$\mathbf{A}_P(\theta_2) := -A_{Ax}(\theta_2)$$

4. Draw the pick-and-place linkage to scale and label it.



5. Establish the relationship between θ_5 and θ_2 . Note that gear 5 is driven by gear 4 and that their ratio is -1 (i.e., they rotate in opposite directions with the same speed). Also, because the walking beam fourbar is a special Grashof, $\theta_4 = \theta_2$. Thus,

$$\theta_5(\theta_2) := -\theta_2 + \phi \quad \text{and} \quad \omega_5 := -\omega_2 \quad \alpha_5 := -\alpha_2$$

6. Determine the values of the constants needed for finding θ_6 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 9.8462 \quad K_2 = 1.3913$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = -5.1137$$

$$A(\theta_2) := \cos(\theta_5(\theta_2)) - K_1 - K_2 \cdot \cos(\theta_5(\theta_2)) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_5(\theta_2))$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_5(\theta_2)) + K_3$$

7. Use equation 4.10b to find values of θ_6 for the crossed circuit.

$$\theta_6(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

8. Determine the values of the constants needed for finding θ_7 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.6632 \quad K_5 = -9.0351$$

$$D(\theta_2) := \cos(\theta_5(\theta_2)) - K_1 + K_4 \cdot \cos(\theta_5(\theta_2)) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_5(\theta_2))$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_5(\theta_2)) + K_5$$

9. Use equation 4.13 to find values of θ_7 for the crossed circuit.

$$\theta_7(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

10. Determine the angular velocity of links 7 and 6 using equations 6.18.

$$\omega_7(\theta_2) := \frac{a \cdot \omega_5}{b} \cdot \frac{\sin[\theta_6(\theta_2) - \theta_5(\theta_2)]}{\sin(\theta_7(\theta_2) - \theta_6(\theta_2))}$$

$$\omega_6(\theta_2) := \frac{a \cdot \omega_5}{c} \cdot \frac{\sin(\theta_5(\theta_2) - \theta_7(\theta_2))}{\sin(\theta_6(\theta_2) - \theta_7(\theta_2))}$$

11. Use equations 7.12 to determine the angular acceleration of link 6.

$$A(\theta_2) := c \cdot \sin(\theta_6(\theta_2)) \quad B(\theta_2) := b \cdot \sin(\theta_7(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_6(\theta_2)) \quad E(\theta_2) := b \cdot \cos(\theta_7(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_5 \cdot \sin(\theta_5(\theta_2)) + a \cdot \omega_5^2 \cdot \cos(\theta_5(\theta_2)) \dots \\ + b \cdot \omega_7(\theta_7(\theta_2))^2 \cdot \cos(\theta_7(\theta_2)) - c \cdot \omega_6(\theta_2)^2 \cdot \cos(\theta_6(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_5 \cdot \cos(\theta_5(\theta_2)) - a \cdot \omega_5^2 \cdot \sin(\theta_5(\theta_2)) \dots \\ + b \cdot \omega_7(\theta_7(\theta_2))^2 \cdot \sin(\theta_7(\theta_2)) + c \cdot \omega_6(\theta_2)^2 \cdot \sin(\theta_6(\theta_2))$$

$$\alpha_6(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

12. Determine the acceleration of the rocker point E using equations 7.13c (substituting the distance to point E from O_6 for the distance c).

$$\mathbf{A_E}(\theta_2) := u \cdot \alpha_6(\theta_2) \cdot (-\sin(\theta_6(\theta_2)) + j \cdot \cos(\theta_6(\theta_2))) - u \cdot \omega_6(\theta_2)^2 \cdot (\cos(\theta_6(\theta_2)) + j \cdot \sin(\theta_6(\theta_2)))$$

13. Transform this into the global XY system.

$$A_{Ex}(\theta_2) := \text{Re}(\mathbf{A_E}(\theta_2)) \quad A_{Ey}(\theta_2) := \text{Im}(\mathbf{A_E}(\theta_2))$$

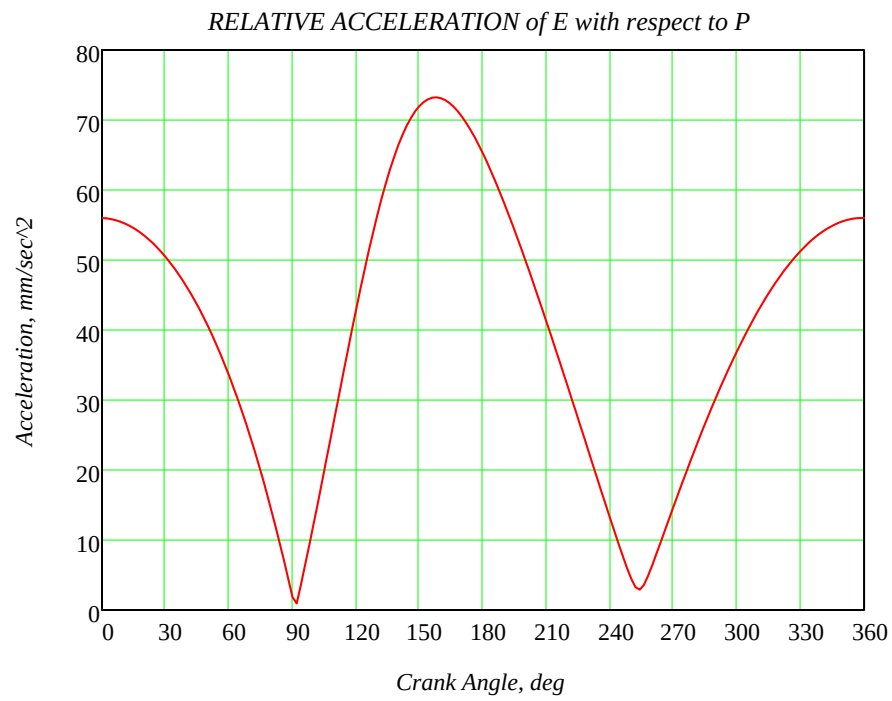
$$A_{EX}(\theta_2) := -A_{Ex}(\theta_2) \quad A_{EY}(\theta_2) := -A_{Ey}(\theta_2)$$

$$\mathbf{A_{EXY}}(\theta_2) := A_{EX}(\theta_2) + j \cdot A_{EY}(\theta_2)$$

14. Calculate and plot the acceleration of E relative to P . (See next page).

$$\mathbf{A_{EP}}(\theta_2) := \mathbf{A_{EXY}}(\theta_2) - \mathbf{A_P}(\theta_2)$$

$$A_{EPX}(\theta_2) := \text{Re}(\mathbf{A_{EP}}(\theta_2)) \quad A_{EPY}(\theta_2) := \text{Im}(\mathbf{A_{EP}}(\theta_2))$$



PROBLEM 7-47

Statement: Figure P6-17 shows a paper roll off-loading mechanism driven by an air cylinder. In the position shown, it has the dimensions given below. The V-links are rigidly attached to O_4A . The air cylinder is retracted at a constant acceleration of 0.1 m/sec^2 . Draw a kinematic diagram of the mechanism, write the necessary equations, and calculate and plot the angular acceleration of the paper roll and the linear acceleration of its center as it rotates through 90° CCW from the position shown.

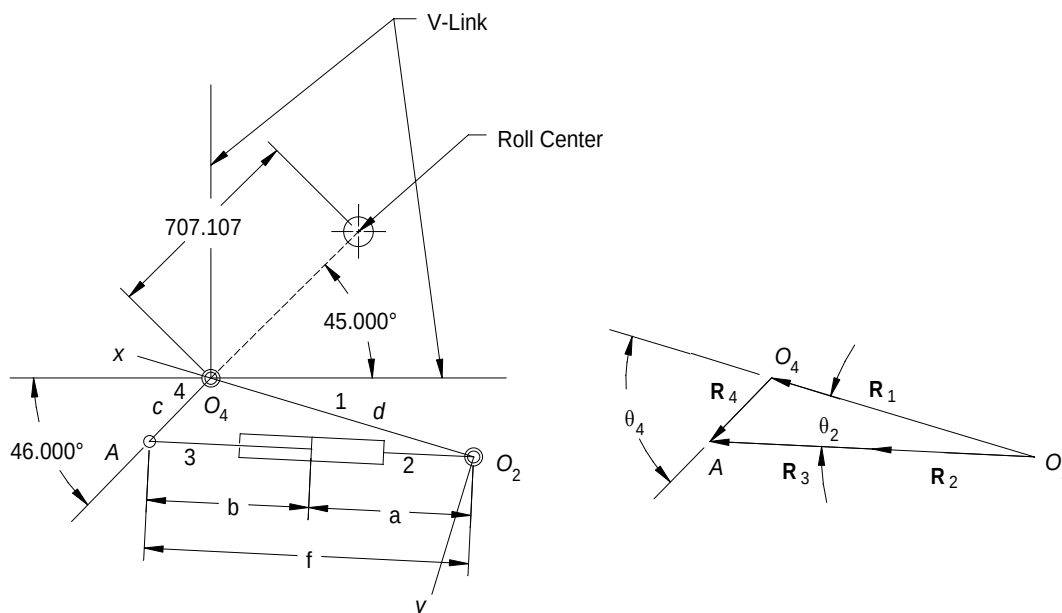
Given: Link lengths and angles: Paper roll location from O_4 :

Link 4 (O_4 to A)	$c := 300 \cdot \text{mm}$	$u := 707.1 \cdot \text{mm}$
Link 1 (O_2 to O_4)	$d := 930 \cdot \text{mm}$	$\delta_4 := -181 \cdot \text{deg}$
Link 4 initial angle	$\theta_{40} := 62.8 \cdot \text{deg}$	with respect to local x axis

Input cylinder acceleration $\text{addot} := -100 \cdot \text{mm} \cdot \text{sec}^{-2}$

Solution: See Figure P7-17 and Mathcad file P0747.

1. Draw the mechanism to scale and define a vector loop using the fourbar derivation in Section 6.7 as a model.



2. Write the vector loop equation, differentiate it, expand the result and separate into real and imaginary parts to solve for f , θ_2 , and ω_4 .

$$\mathbf{R}_1 + \mathbf{R}_4 := \mathbf{R}_2 + \mathbf{R}_3 \quad (a)$$

$$d \cdot e^{j \cdot \theta_1} + c \cdot e^{j \cdot \theta_4} := a \cdot e^{j \cdot \theta_2} + b \cdot e^{j \cdot \theta_2} \quad (b)$$

where a is the distance from the origin to the cylinder piston, a variable; b is the distance from the cylinder piston to A , a constant; and c is the distance from θ_4 to point A , a constant. Angle θ_1 is zero, $\theta_3 = \theta_2$, and θ_4 is the variable angle that the rocker arm makes with the x axis. Solving the position equations:

Let $f := a + b$ then, making this substitution and substituting the Euler equivalents,

$$d + c \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4)) := f \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \quad (c)$$

Separating into real and imaginary components and solving for θ_2 and f ,

$$\theta_2(\theta_4) := \text{atan2}(d + c \cdot \cos(\theta_4), c \cdot \sin(\theta_4)) \quad (d)$$

$$f(\theta_4) := \frac{c \cdot \sin(\theta_4)}{\sin(\theta_2(\theta_4))} \quad (e)$$

For constant deceleration of the hydraulic cylinder rod, the velocity of the rod is

$$\text{adot}(\theta_4) := \text{adot} \cdot \sqrt{\frac{2 \cdot (f(\theta_4) - f(\theta_{40}))}{\text{adot}}}$$

Differentiate equation b.

$$j \cdot c \cdot \omega_4 \cdot e^{j \cdot \theta_4} := \left(\frac{d}{dt} f \right) \cdot e^{j \cdot \theta_2} + j \cdot f \cdot \omega_2 \cdot e^{j \cdot \theta_2} \quad (f)$$

Substituting the Euler equivalents,

$$c \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) := \left(\frac{d}{dt} f \right) \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) + f \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) \quad (g)$$

Separating into real and imaginary components and solving for ω_4 and ω_2 . Note that $df/dt = \text{adot}$.

$$\omega_4(\theta_4) := \frac{\text{adot}(\theta_4)}{c \cdot \sin(\theta_2(\theta_4) - \theta_4)} \quad (h)$$

$$\omega_2(\theta_4) := \frac{c \cdot \omega_4(\theta_4) + \text{adot}(\theta_4) \cdot \cos(\theta_2(\theta_4))}{f(\theta_4) \cdot \sin(\theta_2(\theta_4))} \quad (i)$$

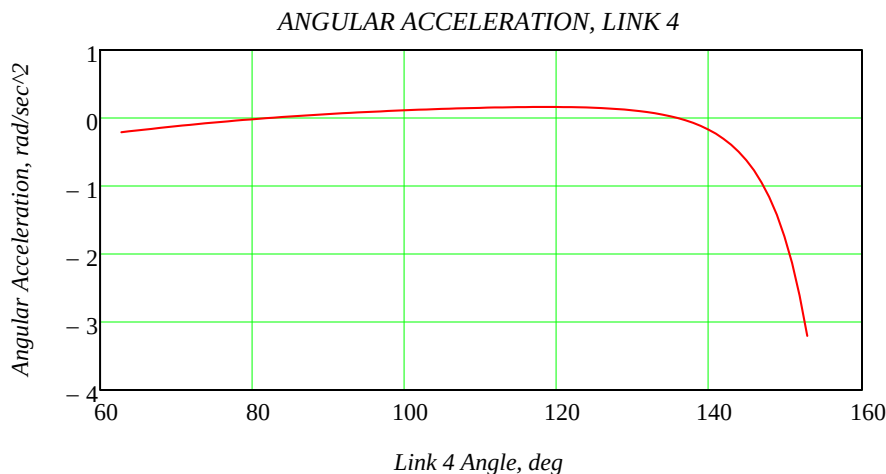
3. Differentiate equation f, expand the result and separate into real and imaginary parts to solve for α_4 .

$$j \cdot c \cdot \alpha_4 \cdot e^{j \cdot \theta_4} + j^2 \cdot c \cdot \omega_4^2 \cdot e^{j \cdot \theta_4} := \text{adot} \cdot e^{j \cdot \theta_2} + j \cdot \text{adot} \cdot \omega_2 \cdot e^{j \cdot \theta_2} + j \cdot \text{adot} \cdot \omega_2 \cdot e^{j \cdot \theta_2} + j \cdot f \cdot \alpha_2 \cdot e^{j \cdot \theta_2} + j^2 \cdot f \cdot \omega_2^2 \cdot e^{j \cdot \theta_2} \quad (f)$$

Separating into real and imaginary components and solving for α_4 .

$$\alpha_4(\theta_4) := \frac{\left(\text{adot} + 2 \cdot \text{adot}(\theta_4) \cdot \omega_2(\theta_4) - f(\theta_4) \cdot \omega_2(\theta_4)^2 \right) \cdot \sin(2 \cdot \theta_2(\theta_4)) + c \cdot \omega_4(\theta_4)^2 \cdot (\sin(\theta_2(\theta_4) + \theta_4))}{c \cdot \sin(\theta_4 - \theta_2(\theta_4))}$$

4. Plot α_4 over a range of θ_4 of $\theta_4 := \theta_{40}, \theta_{40} + 1 \cdot \text{deg} \dots \theta_{40} + 90 \cdot \text{deg}$

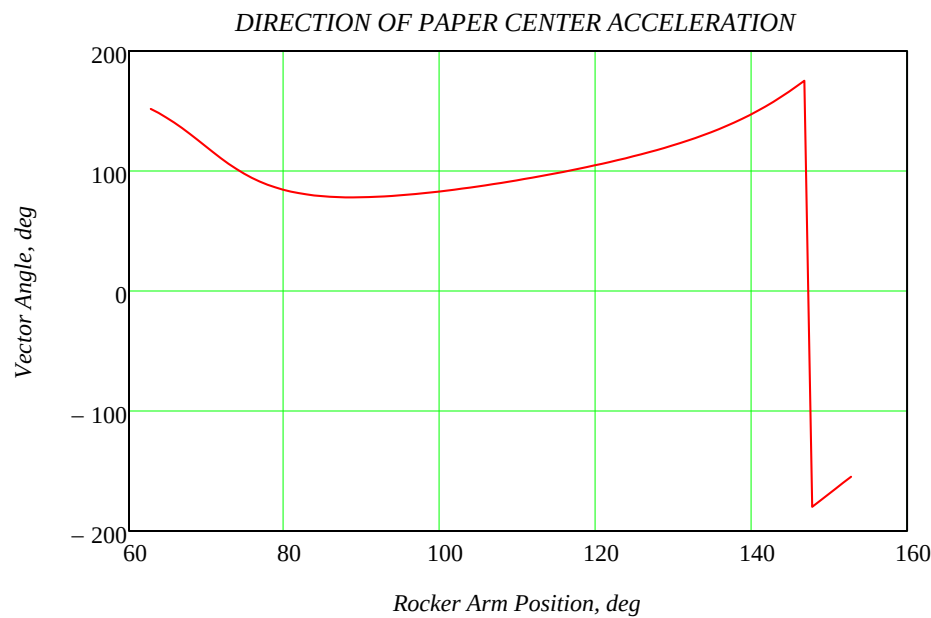
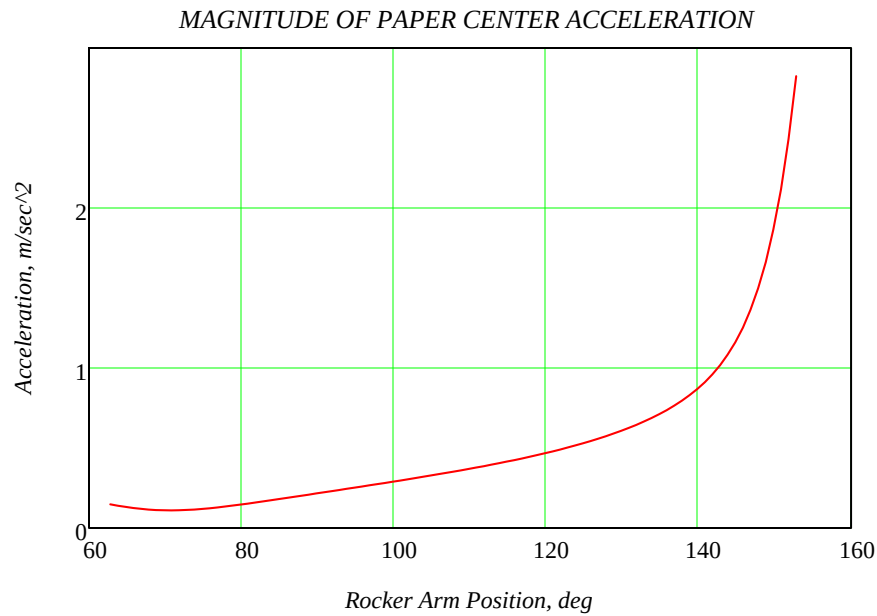


5. Determine the acceleration of the center of the paper roll using equation 7.31. The direction is in the local xy coordinate system.

$$\mathbf{A_U}(\theta_4) := u \cdot \alpha_4(\theta_4) \cdot (-\sin(\theta_4 + \delta_4) + j \cdot \cos(\theta_4 + \delta_4)) - u \cdot \omega_4(\theta_4)^2 \cdot (\cos(\theta_4 + \delta_4) + j \cdot \sin(\theta_4 + \delta_4))$$

$$A_U(\theta_4) := |\mathbf{A_U}(\theta_4)| \quad \theta_{AU}(\theta_4) := \arg(\mathbf{A_U}(\theta_4))$$

6. Plot the magnitude and direction of the acceleration of the paper roll center.



PROBLEM 7-48

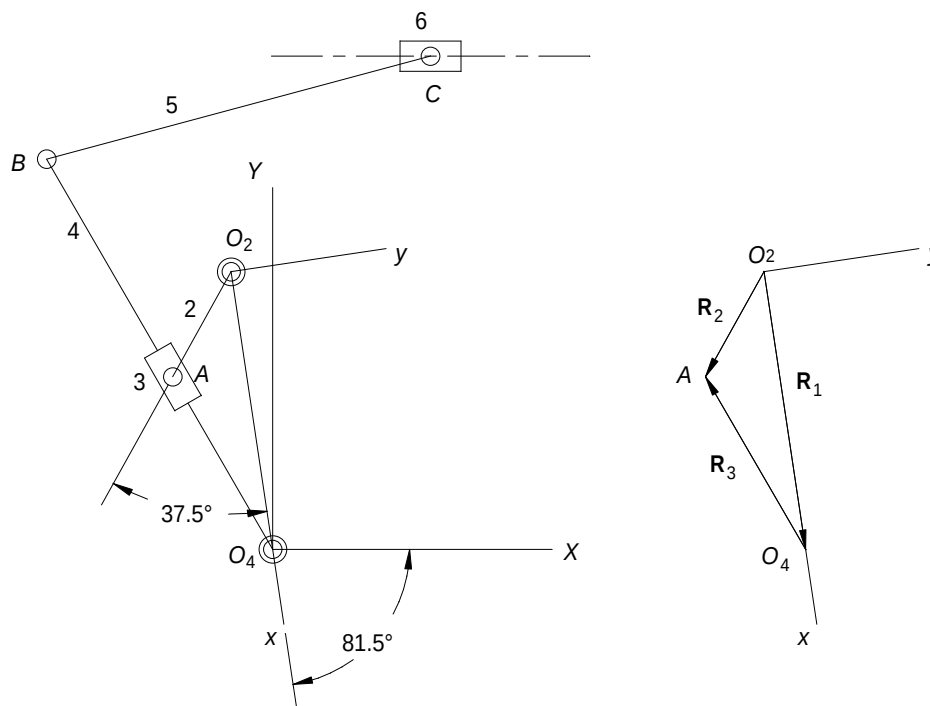
Statement: Figure P7-18 shows a mechanism and its dimensions. Find the accelerations of points A, B, and C for the position shown.

Given: Lengths and angles:

Link 2 (O_2 to A)	$a := 0.80 \cdot \text{in}$	Link 1 (O_2 to O_4)	$d := 1.85 \cdot \text{in}$
Link 4 (O_4 to B)	$a' := 2.97 \cdot \text{in}$	Link 5 (B to C)	$b' := 2.61 \cdot \text{in}$
Crank angle	$\theta_2 := -37.5 \cdot \text{deg}$	Local xy system	
Coordinate rotation angle	$\beta := -81.5 \cdot \text{deg}$	Slider-crank offset	$c' := 3.25 \cdot \text{in}$
Input crank motion	$\omega_2 := 40 \cdot \frac{\text{rad}}{\text{min}}$	$\alpha_2 := -1500 \cdot \frac{\text{rad}}{\text{min}^2}$	

Solution: See Figure P7-18 and Mathcad file P0748.

1. Draw the mechanism to scale and define a vector loop for the input portion (links 1, 2, 3, and 4) using the four-bar crank-slider derivation in Section 7.3 as a model.



2. Write the vector loop equation, expand the result and separate into real and imaginary parts to solve for θ_3 and b (distance from O_4 to A).

$$\mathbf{R}_2 := \mathbf{R}_1 + \mathbf{R}_3$$

$$a \cdot e^{j \cdot \theta_2} := d + b \cdot e^{j \cdot \theta_3}$$

Substituting the Euler equivalents,

$$a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) := d + b (\cos(\theta_3) + j \cdot \sin(\theta_3))$$

Separating into real and imaginary components and solving for θ_3 and b .

$$\theta_3 := \text{atan2}(a \cdot \cos(\theta_2) - d, a \cdot \sin(\theta_2)) \quad \theta_3 = -158.2 \text{ deg}$$

$$b := \frac{a \cdot \sin(\theta_2)}{\sin(\theta_3)} \quad b = 1.31 \text{ in}$$

3. Differentiate the position equation, expand it and solve for ω_3 and $b\dot{\theta}$.

$$a \cdot j \cdot \omega_2 \cdot e^{j \cdot \theta_2} := b \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + b\dot{\theta} \cdot e^{j \cdot \theta_3}$$

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \cos(\theta_2 - \theta_3) \quad \omega_3 = -0.208 \frac{\text{rad}}{\text{sec}}$$

$$b\dot{\theta} := \frac{a \cdot \omega_2 \cdot \cos(\theta_2) - b \cdot \omega_3 \cdot \cos(\theta_3)}{\sin(\theta_3)} \quad b\dot{\theta} = -0.459 \frac{\text{in}}{\text{sec}}$$

4. Differentiate the velocity equation, expand it and solve for α_3 .

$$a \cdot j \cdot \alpha_2 \cdot e^{j \cdot \theta_2} + a \cdot j^2 \cdot \omega_2^2 \cdot e^{j \cdot \theta_2} := b\dot{\theta} \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + b \cdot j \cdot \alpha_3 \cdot e^{j \cdot \theta_3} \dots$$

$$+ b \cdot j^2 \cdot \omega_3^2 \cdot e^{j \cdot \theta_3} + b\ddot{\theta} \cdot e^{j \cdot \theta_3} + b\dot{\theta} \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3}$$

$$\alpha_3 := \frac{1}{b} \cdot \left(a \cdot \alpha_2 \cdot \cos(\theta_2 - \theta_3) + a \cdot \omega_2^2 \cdot \sin(\theta_3 - \theta_2) - 2 \cdot b\dot{\theta} \cdot \omega_3 \right)$$

$$\alpha_3 = -0.249 \frac{\text{rad}}{\text{sec}^2}$$

5. Determine the acceleration of point A on link 2 (in the local xy coordinate system) using equations 7.13a.

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_A := |\mathbf{A}_A| \quad A_A = 0.487 \frac{\text{in}}{\text{sec}^2} \quad \theta_{AA} := \arg(\mathbf{A}_A) \quad \theta_{AA} = -174.348 \text{ deg}$$

6. Determine the acceleration of points A and B on link 3 (in the Global XY coordinate system) using equations 7.13a.

$$\text{Transform } \theta_3 \text{ to the global coordinate frame: } \theta_3 := \theta_3 + \beta + 360 \cdot \text{deg} \quad \theta_3 = 120.337 \text{ deg}$$

$$\mathbf{A}_{A3} := b \cdot \alpha_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3)) - b \cdot \omega_3^2 \cdot (\cos(\theta_3) + j \cdot \sin(\theta_3))$$

$$A_{A3} := |\mathbf{A}_{A3}| \quad A_{A3} = 0.331 \frac{\text{in}}{\text{sec}^2} \quad \theta_{AA3} := \arg(\mathbf{A}_{A3}) \quad \theta_{AA3} = 20.518 \text{ deg}$$

$$\mathbf{A}_B := a' \cdot \alpha_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3)) - a' \cdot \omega_3^2 \cdot (\cos(\theta_3) + j \cdot \sin(\theta_3))$$

$$A_B := |\mathbf{A}_B| \quad A_B = 0.752 \frac{\text{in}}{\text{sec}^2} \quad \theta_{AB} := \arg(\mathbf{A}_B) \quad \theta_{AB} = 20.518 \text{ deg}$$

7. Determine θ_5 using equation 4.16.

$$\theta_5 := \text{asin}\left(-\frac{a' \cdot \sin(\theta_3) - c'}{b'}\right) + \pi \quad \theta_5 = 195.254 \text{ deg}$$

8. Determine the angular velocity of link 5 using equation 6.22a.

$$\omega_5 := \frac{a'}{b'} \cdot \frac{\cos(\theta_3)}{\cos(\theta_5)} \cdot \omega_3 \qquad \omega_5 = -0.124 \frac{\text{rad}}{\text{sec}}$$

9. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5 := \frac{a' \cdot \alpha_3 \cdot \cos(\theta_3) - a' \cdot \omega_3^2 \cdot \sin(\theta_3) + b' \cdot \omega_5^2 \cdot \sin(\theta_5)}{b' \cdot \cos(\theta_5)} \qquad \alpha_5 = -0.100 \frac{\text{rad}}{\text{sec}^2}$$

10. Use equation 7.16e for the acceleration of pin C.

$$A_C := -a' \cdot \alpha_3 \cdot \sin(\theta_3) - a' \cdot \omega_3^2 \cdot \cos(\theta_3) + b' \cdot \alpha_5 \cdot \sin(\theta_5) + b' \cdot \omega_5^2 \cdot \cos(\theta_5)$$

$$A_C = 0.73 \frac{\text{in}}{\text{sec}^2} \qquad \text{A positive sign means that } A_C \text{ is to the right}$$

PROBLEM 7-49

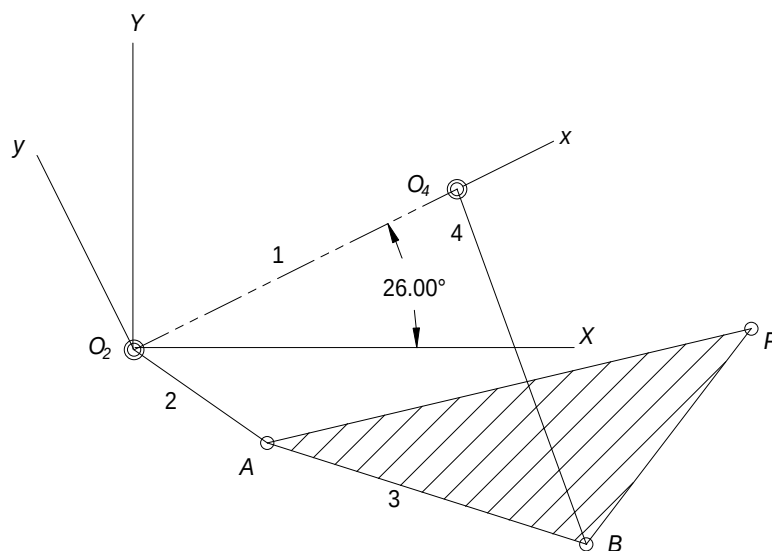
Statement: Figure P7-19 shows a walking beam mechanism. Calculate and plot the acceleration A_{out} for one revolution of the input crank 2 rotating at 100 rpm.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 1.00 \cdot in$	Link 3 (A to B)	$b := 2.06 \cdot in$
Link 4 (B to O_4)	$c := 2.33 \cdot in$	Link 1 (O_2 to O_4)	$d := 2.22 \cdot in$
Coupler point:	$p := 3.06 \cdot in$	$\delta_3 := 31 \cdot deg$	
Crank speed:	$\omega_2 := 100 \cdot rpm$	$\alpha_2 := 0 \cdot rad \cdot sec^{-2}$	

Solution: See Figure P7-19 and Mathcad file P0749.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 2.2200$$

$$K_2 = 0.9528$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_3 = 1.5265$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0777 \quad K_5 = -1.1512$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Determine the acceleration of point A using equation 7.13a.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

9. Use equations 7.12 to determine the angular acceleration of link 3.

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C(\theta_2) \cdot D(\theta_2) - A(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

10. Determine the acceleration of the coupler point P using equations 7.32.

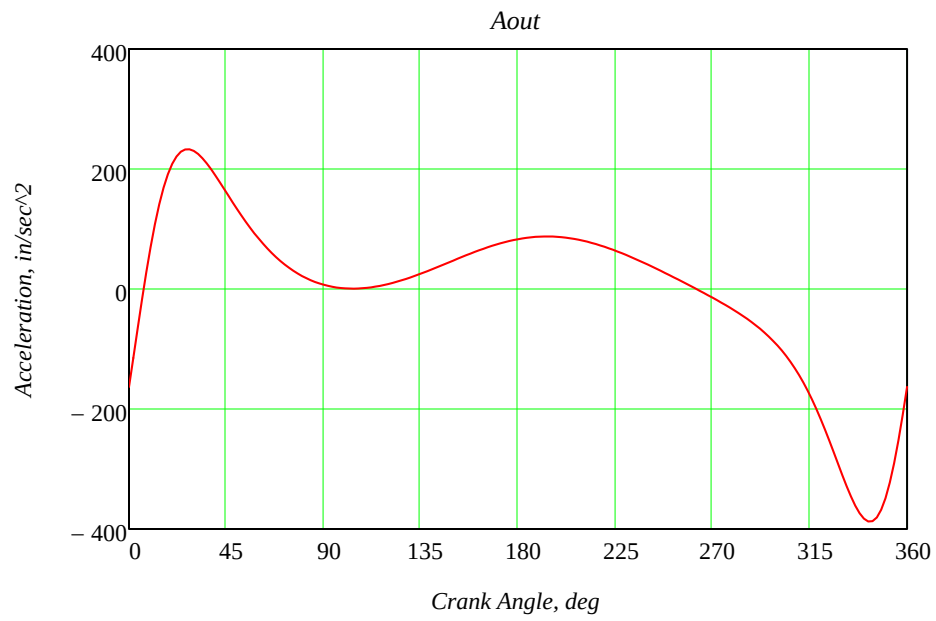
$$\begin{aligned} \mathbf{A}_{PA}(\theta_2) := & p \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots \\ & + p \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3)) \end{aligned}$$

$$\mathbf{A}_P(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2)$$

11. Plot the X-component (global coordinate system) of the acceleration of the coupler point P. (See next page).

$$\text{Coordinate rotation angle:} \quad \alpha := 26 \cdot \text{deg}$$

$$A_{out}(\theta_2) := \text{Re}(\mathbf{A}_P(\theta_2)) \cdot \cos(\alpha) - \text{Im}(\mathbf{A}_P(\theta_2)) \cdot \sin(\alpha)$$



PROBLEM 7-50

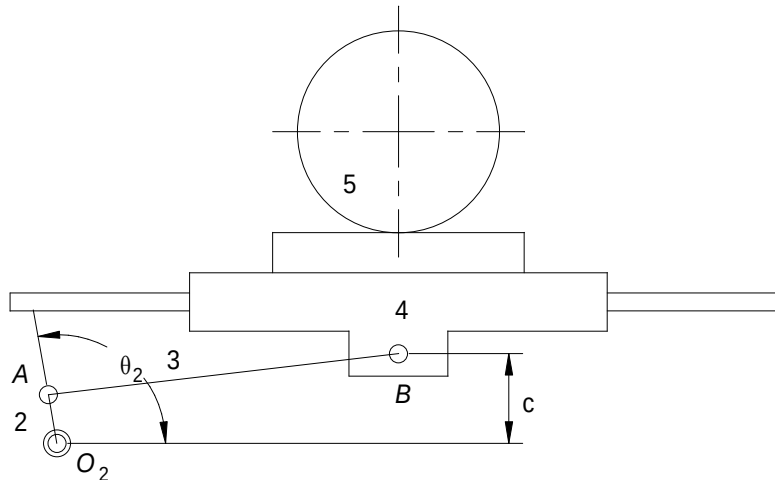
Statement: Figure P7-20 shows a surface grinder. The workpiece is oscillated under the spinning grinding wheel by the slider-crank linkage that has the dimensions given below. Calculate and plot the acceleration of the grinding wheel contact point relative to the workpiece over one revolution of the crank.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 22 \cdot \text{mm}$	Offset	$c := 40 \cdot \text{mm}$
Link 3 (A to B)	$b := 157 \cdot \text{mm}$		
Grinding wheel diameter	$d := 90 \cdot \text{mm}$		
Input crank angular velocity	$\omega_2 := 30 \cdot \text{rpm}$	CCW	$\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$
Grinding wheel angular velocity	$\omega_5 := 3450 \cdot \text{rpm}$	CCW	

Solution: See Figure P7-20 and Mathcad file P0750.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg}$$

3. Determine θ_3 using equation 4.17.

$$\theta_3(\theta_2) := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi$$

4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3(\theta_2))} \cdot \omega_2$$

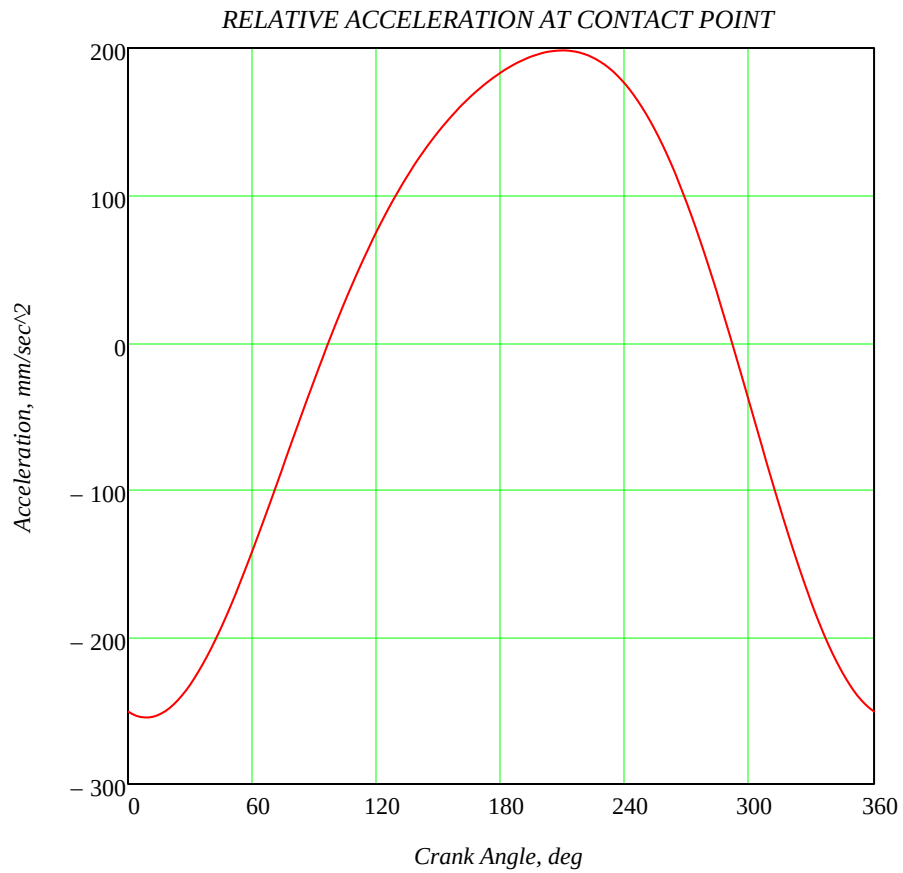
5. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3(\theta_2) := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2))}{b \cdot \cos(\theta_3(\theta_2))}$$

6. Use equation 7.16e for the acceleration of pin B.

$$\begin{aligned} A_B(\theta_2) := & -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) \dots \\ & + b \cdot \alpha_3(\theta_2) \cdot \sin(\theta_3(\theta_2)) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) \end{aligned}$$

7. The acceleration of the grinding wheel contact point relative to the workpiece, which has zero tangential acceleration due to constant angular velocity, is A_B (positive to the right).



PROBLEM 7-51

Statement: Figure P7-21 shows a drag-link mechanism with dimensions. Write the necessary equations and solve them to calculate and plot the angular acceleration of link 4 for an input of $\omega_2 = 1 \text{ rad/sec}$. Comment on the uses for this mechanism.

Given:

Link lengths:

Link 2 (L_2)

$a := 1.38 \cdot \text{in}$

Link 3 (L_3)

$b := 1.22 \cdot \text{in}$

Link 4 (L_4)

$c := 1.62 \cdot \text{in}$

Link 1 (L_1)

$d := 0.68 \cdot \text{in}$

Input crank angular velocity

$\omega_2 := -1 \cdot \text{rad} \cdot \text{sec}^{-1} \text{ CW}$

$\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure P7-21 and Mathcad file P0751.

1. Draw the linkage to scale and label it.

2. Determine the range of motion for this Grashof double crank.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 0.4928$$

$$K_2 = 0.4198$$

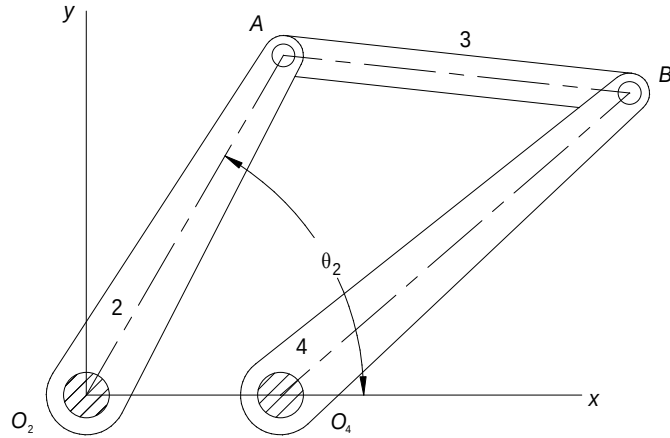
$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_3 = 0.7834$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$



4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b}$$

$$K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 0.5574$$

$$K_5 = -0.3655$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

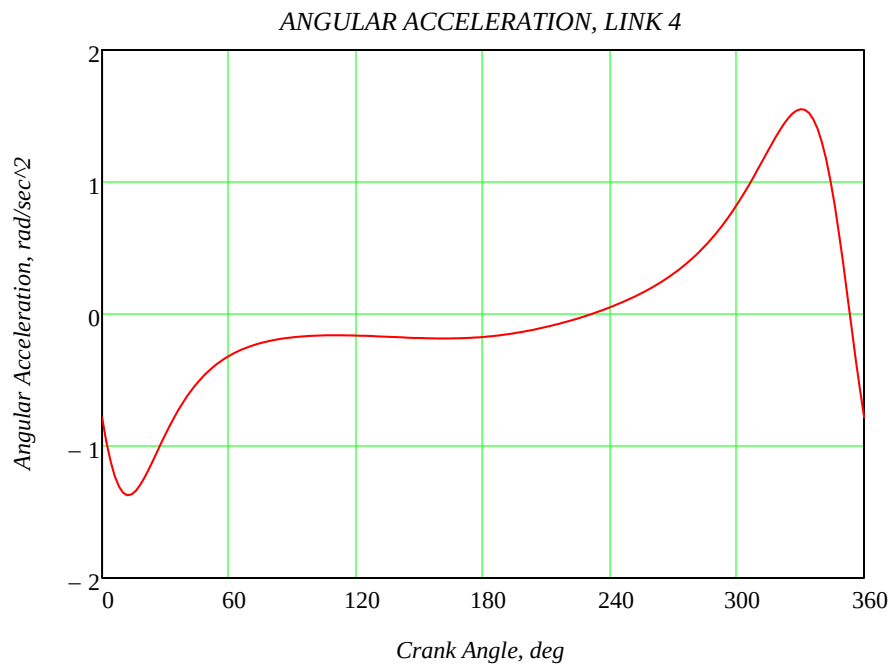
$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

9. Plot the angular acceleration of link 4.



PROBLEM 7-52

Statement: Figure P7-22 shows a mechanism with dimensions. Use a graphical method to calculate the accelerations of points A, B, and C for the position shown.

Given: Link lengths and angles:

Link 1 (O_2O_4) $d := 1.22 \cdot \text{in}$ Angle O_2O_4 makes with X axis $\theta_1 := 56.5 \cdot \text{deg}$

Link 2 (O_2A) $a := 1.35 \cdot \text{in}$ Angle O_2A makes with X axis $\theta_2 := 14 \cdot \text{deg}$

Link 4 (O_4B) $e := 1.36 \cdot \text{in}$

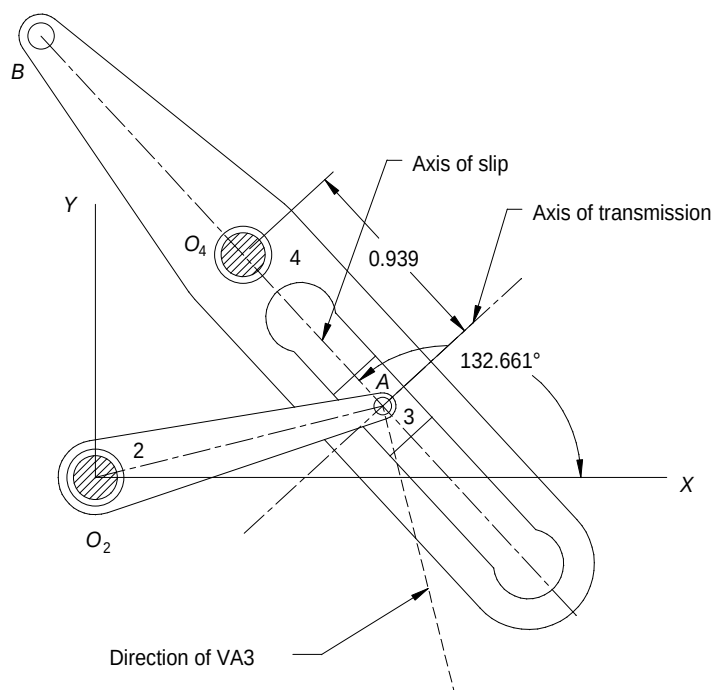
Link 5 (BC) $f := 2.69 \cdot \text{in}$

Link 6 (CO_6) $g := 1.80 \cdot \text{in}$ Angle CO_6 makes with X axis $\theta_6 := 88 \cdot \text{deg}$

Motion of link 2 $\omega_2 := 20 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW $\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure P7-22 and Mathcad file P0752.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



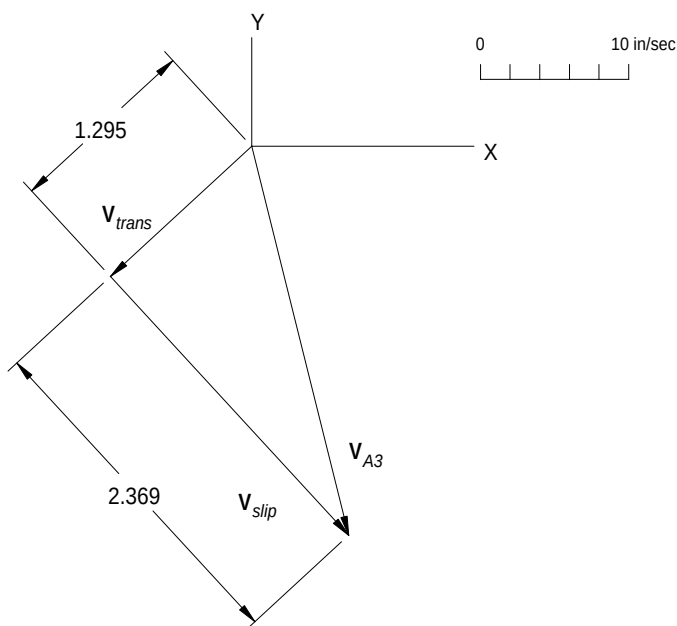
2. Use equation 6.7 to calculate the magnitude of the velocity at point A on links 2 and 3.

$$V_{A3} := a \cdot \omega_2 \quad V_{A3} = 27.000 \frac{\text{in}}{\text{sec}} \quad \theta_{VA3} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{VA3} = -76.0 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A on link 3. The equation to be solved graphically is

$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.
- b. From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{10 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_{slip} := 2.369 \cdot \text{in} \cdot k_v \quad V_{slip} = 23.690 \frac{\text{in}}{\text{sec}}$$

$$V_{trans} := 1.295 \cdot \text{in} \cdot k_v \quad V_{trans} = 12.950 \frac{\text{in}}{\text{sec}}$$

5. The true velocity of point A on link 4 is V_{trans} ,

$$V_{A4} := V_{trans} \quad V_{A4} = 12.95 \frac{\text{in}}{\text{sec}}$$

6. Determine the angular velocity of link 4 using equation 6.7.

$$\text{From the linkage layout above: } c := 0.939 \cdot \text{in} \quad \text{and} \quad \theta_4 := 132.661 \cdot \text{deg}$$

$$\omega_4 := \frac{V_{A4}}{c} \quad \omega_4 = 13.791 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

7. Determine the magnitude and sense of the vector $\mathbf{V}_B$ using equation 6.7.

$$V_B := e \cdot \omega_4 \quad V_B = 18.756 \frac{\text{in}}{\text{sec}}$$

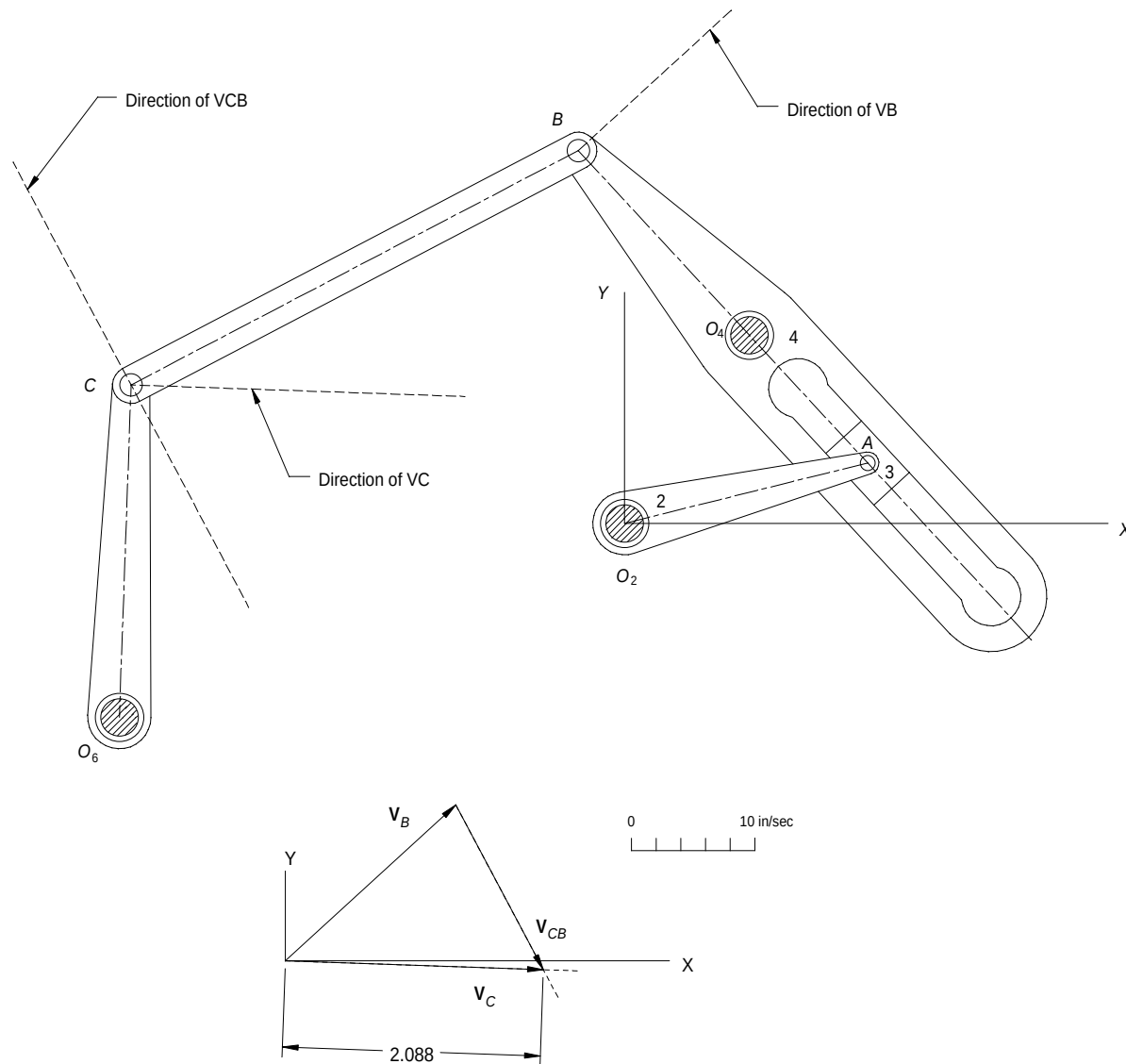
$$\theta_{VA4} := \theta_4 - 90 \cdot \text{deg} \quad \theta_{VA4} = 42.661 \cdot \text{deg}$$

8. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
- From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.
- From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.

Draw links 1, 4, 5, and 6 to a convenient scale. Indicate the directions of the velocity vectors of interest.



9. From the velocity triangle we have:

$$\begin{aligned} \text{Velocity scale factor: } k_v &:= \frac{10 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}} & \theta_{VC} &:= \theta_6 - 90 \cdot \text{deg} \\ V_C &:= 2.088 \cdot \text{in} \cdot k_v & V_C &= 20.9 \frac{\text{in}}{\text{sec}} & \theta_{VC} &= -2.0 \text{ deg} \\ V_{CB} &:= 1.519 \cdot \text{in} \cdot k_v & V_{CB} &= 15.2 \frac{\text{in}}{\text{sec}} \end{aligned}$$

10. Determine the angular velocity of links 5 and 6 using equation 6.7.

$$\begin{aligned} \omega_5 &:= \frac{V_{CB}}{f} & \omega_5 &= 5.647 \frac{\text{rad}}{\text{sec}} & \text{CCW} \\ \omega_6 &:= \frac{V_C}{g} & \omega_6 &= 11.600 \frac{\text{rad}}{\text{sec}} & \text{CCW} \end{aligned}$$

11. For the acceleration of point A, use equation 7.19: $(\mathbf{A}_{A2}^t + \mathbf{A}_{A2}^n) = \mathbf{A}_{A4}^t + \mathbf{A}_{A4}^n + \mathbf{A}_A^{cor} + \mathbf{A}_A^{slip}$, where

$$A_{A2n} := a \cdot \omega_2^2 \quad A_{A2n} = 540.000 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AA} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AA} = 194.000 \text{ deg}$$

$$A_{A2t} := a \cdot \alpha_2 \quad A_{A2t} = 0.000 \text{ in} \cdot \text{sec}^{-2}$$

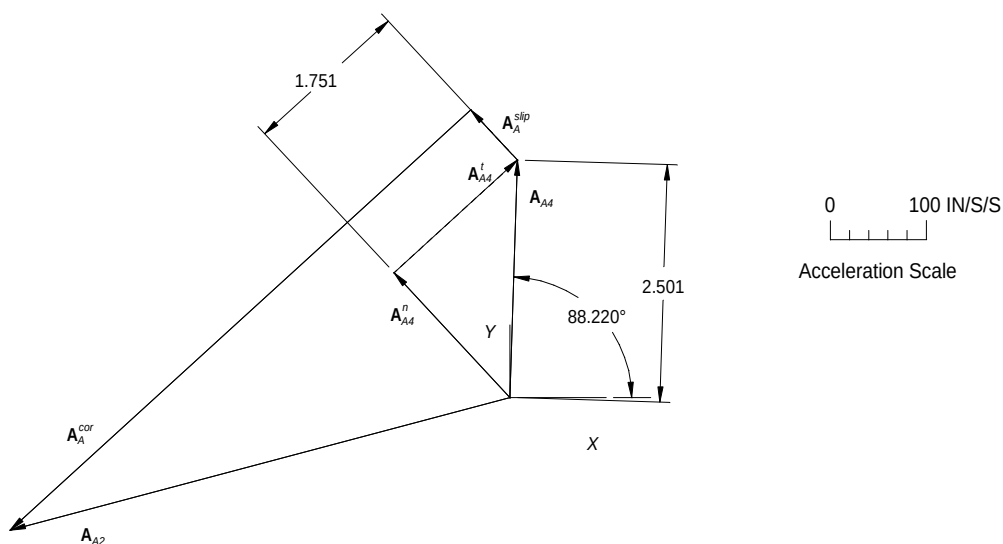
$$A_{A4n} := c \cdot \omega_4^2 \quad A_{A4n} = 178.6 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AA4n} := \theta_4 \quad \theta_{AA4n} = 132.661 \text{ deg}$$

$$A_{Acor} := |2 \cdot V_{slip} \cdot \omega_4| \quad A_{Acor} = 653.430 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AAcor} := \theta_4 + 90 \cdot \text{deg} \quad \theta_{AAcor} = 222.661 \text{ deg}$$

12. Repeat procedure of steps 3 and 7 for the equation in step 11.



13. From the acceleration polygon above,

$$\text{Acceleration scale factor} \quad k_a := 100 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_{A4} := 2.501 \cdot k_a \quad A_{A4} = 250.1 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 88.22 \text{ deg}$$

$$A_{A4t} := 1.751 \cdot k_a \quad A_{A4t} = 175.1 \text{ in} \cdot \text{sec}^{-2}$$

The angular acceleration of link 4 is

$$\alpha_4 := \frac{A_{A4t}}{c} \quad \alpha_4 = 186.5 \frac{\text{rad}}{\text{sec}^2} \quad \text{CCW}$$

14. The graphical solution for accelerations in pin-jointed fourbars uses equation 7.4:

$$(\mathbf{A}_p^t + \mathbf{A}_p^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{pA}^t + \mathbf{A}_{pA}^n)$$

15. For point C, this becomes: $(\mathbf{A}_C^t + \mathbf{A}_C^n) = (\mathbf{A}_B^t + \mathbf{A}_B^n) + (\mathbf{A}_{CB}^t + \mathbf{A}_{CB}^n)$, where

$$A_{Cn} := g \cdot \omega_6^2 \quad A_{Cn} = 242.2 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ACn} := \theta_6 + 180 \cdot \text{deg} \quad \theta_{ACn} = 268.000 \text{ deg}$$

$$A_{Bn} := e \cdot \omega_4^2 \quad A_{Bn} = 258.7 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 + 180 \cdot \text{deg} \quad \theta_{ABn} = 312.661 \text{ deg}$$

$$A_{Bt} := e \cdot \alpha_4 \quad A_{Bt} = 253.6 \text{ in} \cdot \text{sec}^{-2}$$

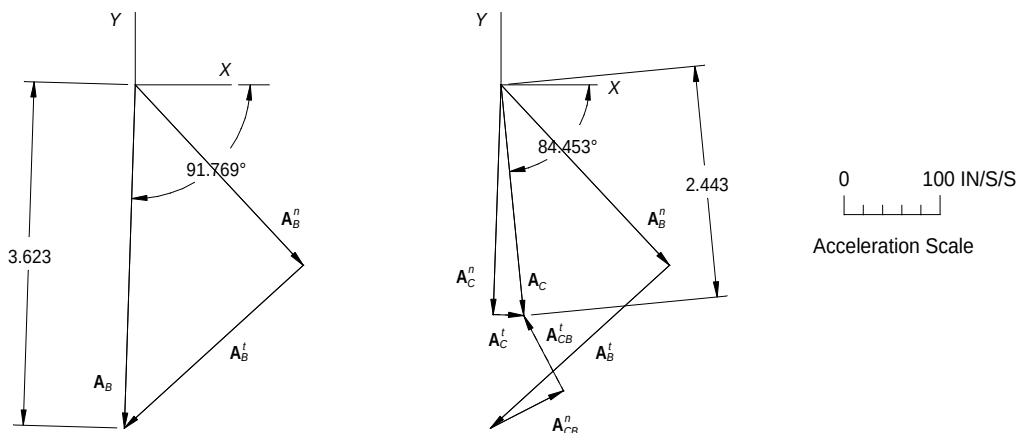
$$\theta_{ABt} := \theta_4 + 90 \cdot \text{deg} \quad \theta_{ABt} = 222.661 \text{ deg}$$

$$A_{CBn} := f \cdot \omega_5^2 \quad A_{CBn} = 85.78 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_5 := 27.5 \cdot \text{deg}$$

$$\theta_{ACBn} := \theta_5 \quad \theta_{ACBn} = 27.500 \text{ deg}$$

16. Repeat procedure of step 12 for the equation in step 15.



17. From the acceleration polygon above,

$$\text{Acceleration scale factor} \quad k_a := 100 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_B := 3.623 \cdot k_a \quad A_B = 362.3 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -91.8 \text{ deg}$$

$$A_C := 2.443 \cdot k_a \quad A_C = 244.3 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -84.5 \text{ deg}$$

PROBLEM 7-53

Statement: Figure P7-23 shows a quick-return mechanism with dimensions. Use a graphical method to calculate the accelerations of points A, B, and C for the position shown.

Given: Link lengths and angles:

Link 1 (O_2O_4) $d := 1.69 \cdot \text{in}$ Angle O_2O_4 makes with X axis $\theta_1 := 195.5 \cdot \text{deg}$

Link 2 (L_2) $a := 1.00 \cdot \text{in}$ Angle link 2 makes with X axis $\theta_2 := 99 \cdot \text{deg}$

Link 4 (L_4) $e := 4.76 \cdot \text{in}$

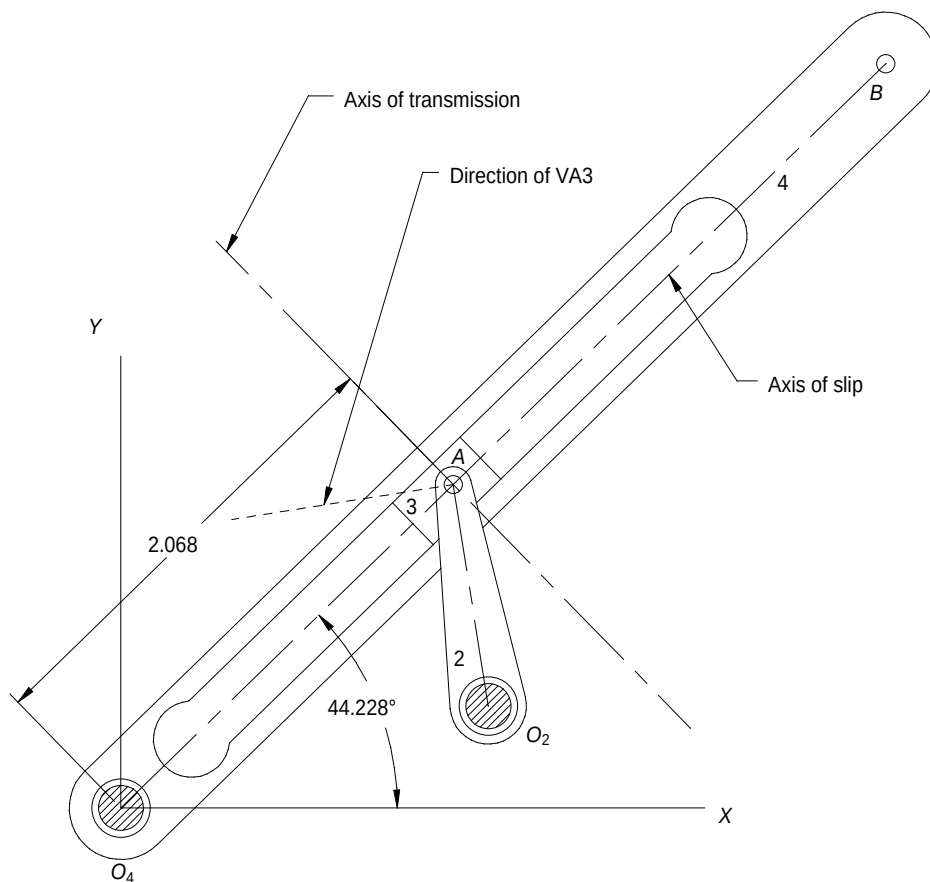
Link 5 (L_5) $f := 4.55 \cdot \text{in}$

Offset (O_2C) $g := 2.86 \cdot \text{in}$

Angular velocity of link 2 $\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW $\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure P7-23 and Mathcad file P0753.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



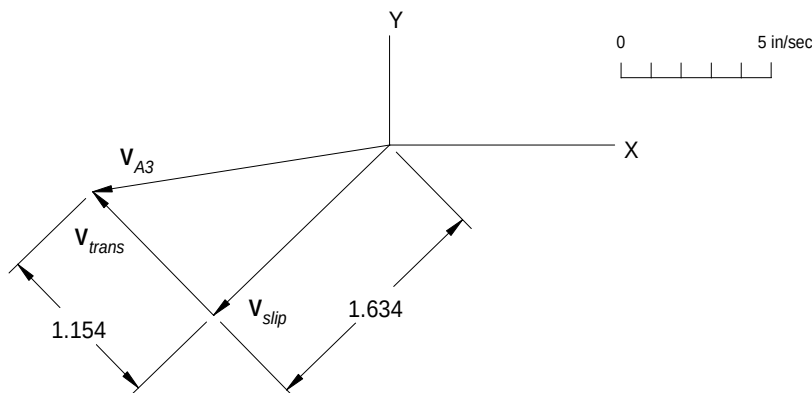
2. Use equation 6.7 to calculate the magnitude of the velocity at point A on links 2 and 3.

$$V_{A3} := a \cdot \omega_2 \quad V_{A3} = 10.000 \frac{\text{in}}{\text{sec}} \quad \theta_{VA3} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{VA3} = 189.0 \text{ deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A on link 3. The equation to be solved graphically is

$$\mathbf{V}_{A3} = \mathbf{V}_{\text{trans}} + \mathbf{V}_{\text{slip}}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.
- From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.
- From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

$$\begin{aligned} \text{Velocity scale factor: } k_v &:= \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}} \\ V_{slip} &:= 1.634 \cdot \text{in} \cdot k_v & V_{slip} &= 8.170 \frac{\text{in}}{\text{sec}} \\ V_{trans} &:= 1.154 \cdot \text{in} \cdot k_v & V_{trans} &= 5.770 \frac{\text{in}}{\text{sec}} \end{aligned}$$

5. The true velocity of point A on link 4 is V_{trans} ,

$$V_{A4} := V_{trans} \quad V_{A4} = 5.77 \frac{\text{in}}{\text{sec}}$$

6. Determine the angular velocity of link 4 using equation 6.7.

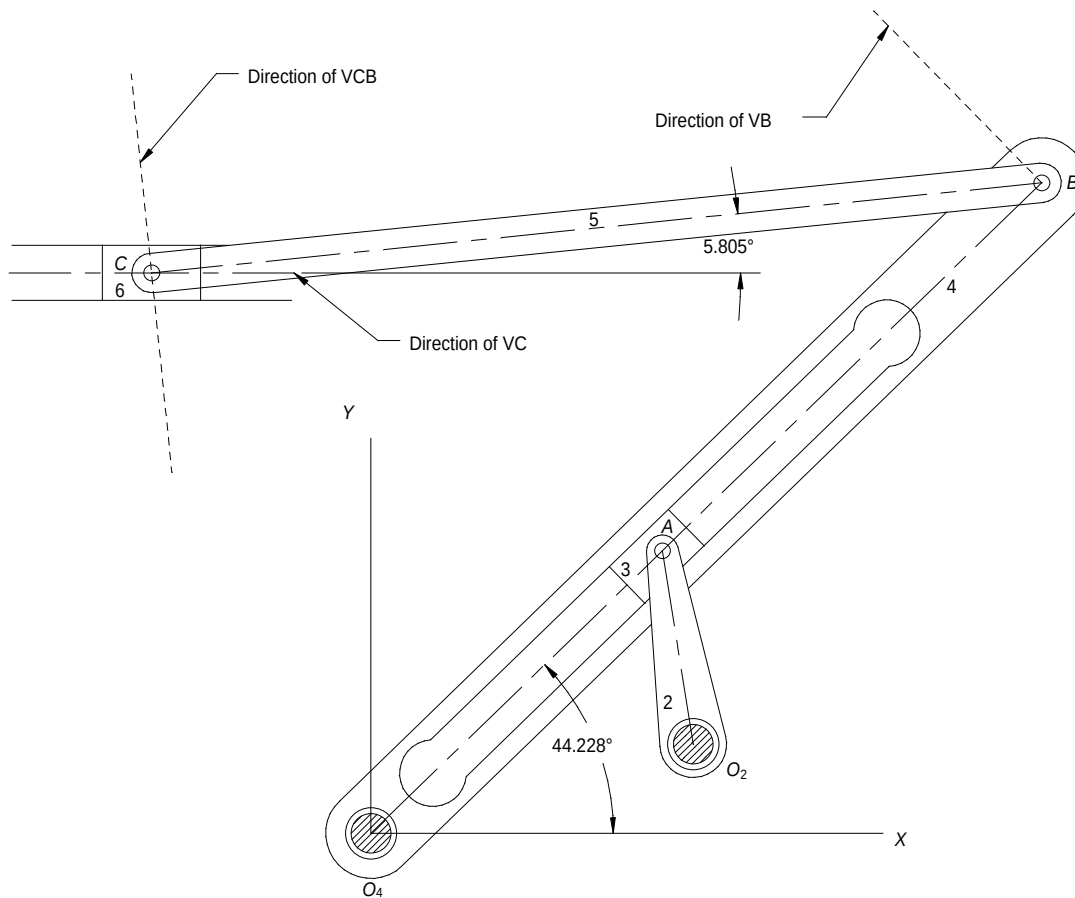
$$\text{From the linkage layout above: } c := 2.068 \cdot \text{in} \quad \text{and} \quad \theta_4 := 44.228 \cdot \text{deg}$$

$$\omega_4 := \frac{V_{A4}}{c} \quad \omega_4 = 2.790 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

- Draw links 1, 4, 5, and 6 to a convenient scale. Indicate the directions of the velocity vectors of interest. (See next page.)
- Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
- From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.
- From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.



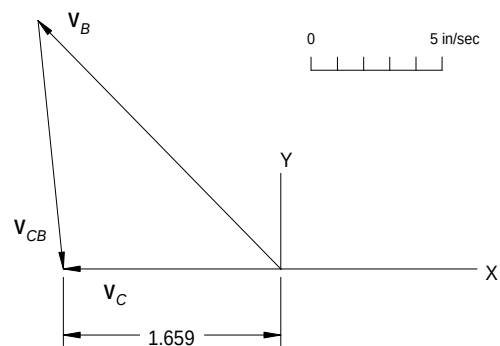
9. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_C := 1.659 \cdot \text{in} \cdot k_v \quad V_C = 8.30 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VC} := 180 \cdot \text{deg}$$

$$V_{CB} := 1.913 \cdot \text{in} \cdot k_v \quad V_{CB} = 9.57 \frac{\text{in}}{\text{sec}}$$



10. Determine the angular velocity of link 5 using equation 6.7.

$$\text{From the linkage layout above: } \theta_5 := 5.805 \cdot \text{deg}$$

$$\omega_5 := \frac{V_{CB}}{f} \quad \omega_5 = 2.102 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

11. For the acceleration of point A, use equation 7.19: $(\mathbf{A}_{A2}^t + \mathbf{A}_{A2}^n) = \mathbf{A}_{A4}^t + \mathbf{A}_{A4}^n + \mathbf{A}_A^{\text{cor}} + \mathbf{A}_A^{\text{slip}}$, where

$$A_{A2n} := a \cdot \omega_2^2 \quad A_{A2n} = 100.000 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AA} := \theta_2 + 180 \cdot \text{deg}$$

$$\theta_{AA} = 279.000 \text{ deg}$$

$$A_{A2t} := a \cdot \alpha_2$$

$$A_{A2t} = 0.000 \text{ in} \cdot \text{sec}^{-2}$$

$$A_{A4n} := c \cdot \omega_4^2$$

$$A_{A4n} = 16.099 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AA4n} := \theta_4 + 180 \cdot \text{deg}$$

$$\theta_{AA4n} = 224.228 \text{ deg}$$

$$A_{Acor} := |2 \cdot V_{slip} \cdot \omega_4|$$

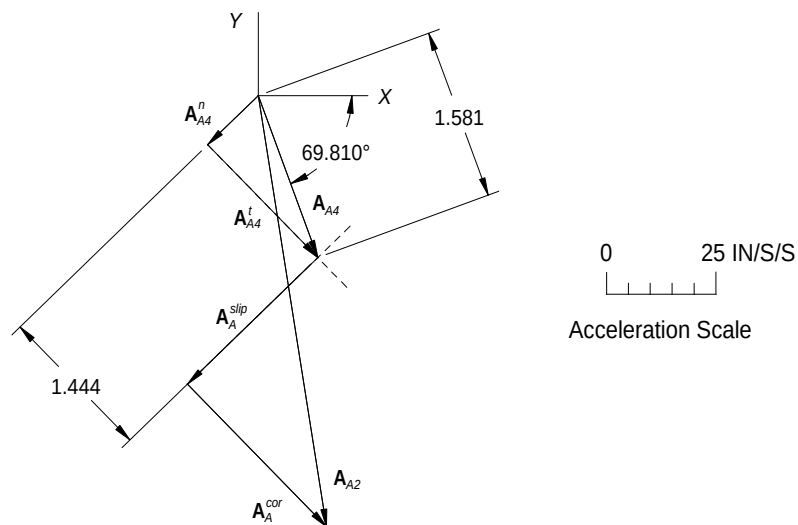
$$A_{Acor} = 45.591 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AAcor} := \theta_4 - 90 \cdot \text{deg}$$

$$\theta_{AAcor} = -45.772 \text{ deg}$$

(V_{slip} is negative)

12. Repeat procedure of steps 3 and 7 for the equation in step 11.



13. From the acceleration polygon above,

$$\text{Acceleration scale factor} \quad k_a := 25 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_{A4} := 1.581 \cdot k_a \quad A_{A4} = 39.5 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -69.81 \text{ deg}$$

$$A_{A4t} := 1.444 \cdot k_a \quad A_{A4t} = 36.1 \text{ in} \cdot \text{sec}^{-2}$$

The angular acceleration of link 4 is

$$\alpha_4 := \frac{A_{A4t}}{c} \quad \alpha_4 = 17.5 \frac{\text{rad}}{\text{sec}^2} \quad \text{CW}$$

14. The graphical solution for accelerations in pin-jointed fourbars uses equation 7.4:

$$(\mathbf{A}_p^t + \mathbf{A}_p^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$$

15. For point C, this becomes: $\mathbf{A}_C = (\mathbf{A}_B^t + \mathbf{A}_B^n) + (\mathbf{A}_{CB}^t + \mathbf{A}_{CB}^n)$, where

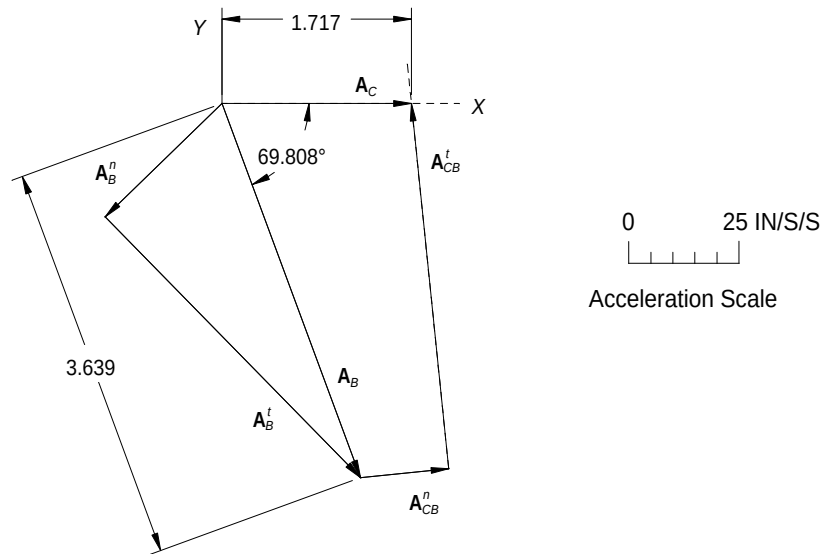
$$A_{Bn} := e \cdot \omega_4^2 \quad A_{Bn} = 37.056 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 + 180 \cdot \text{deg}$$

$$\theta_{ABn} = 224.228 \text{ deg}$$

$$\begin{aligned}
 A_{Bt} &:= e \cdot \alpha_4 & A_{Bt} &= 83.09 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{ABt} &:= \theta_4 - 90 \cdot \text{deg} & \theta_{ABt} &= -45.772 \text{ deg} \\
 A_{CBn} &:= f \cdot \omega_5^2 & A_{CBn} &= 20.108 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{ACBn} &:= \theta_5 & \theta_{ACBn} &= 5.805 \text{ deg}
 \end{aligned}$$

16. Repeat procedure of step 12 for the equation in step 15.



17. From the acceleration polygon above,

$$\begin{aligned}
 \text{Acceleration scale factor} \quad k_a &:= 25 \cdot \frac{\text{in}}{\text{sec}^2} \\
 A_B &:= 3.639 \cdot k_a & A_B &= 91.0 \text{ in} \cdot \text{sec}^{-2} & \text{at an angle of } -69.81 \text{ deg} \\
 A_C &:= 1.717 \cdot k_a & A_C &= 42.9 \text{ in} \cdot \text{sec}^{-2} & \text{at an angle of } 0 \text{ deg}
 \end{aligned}$$

PROBLEM 7-54

Statement: Figure P7-23 shows a quick-return mechanism with dimensions. Use an analytical method to calculate the accelerations of points A, B, and C for one revolution of the input link.

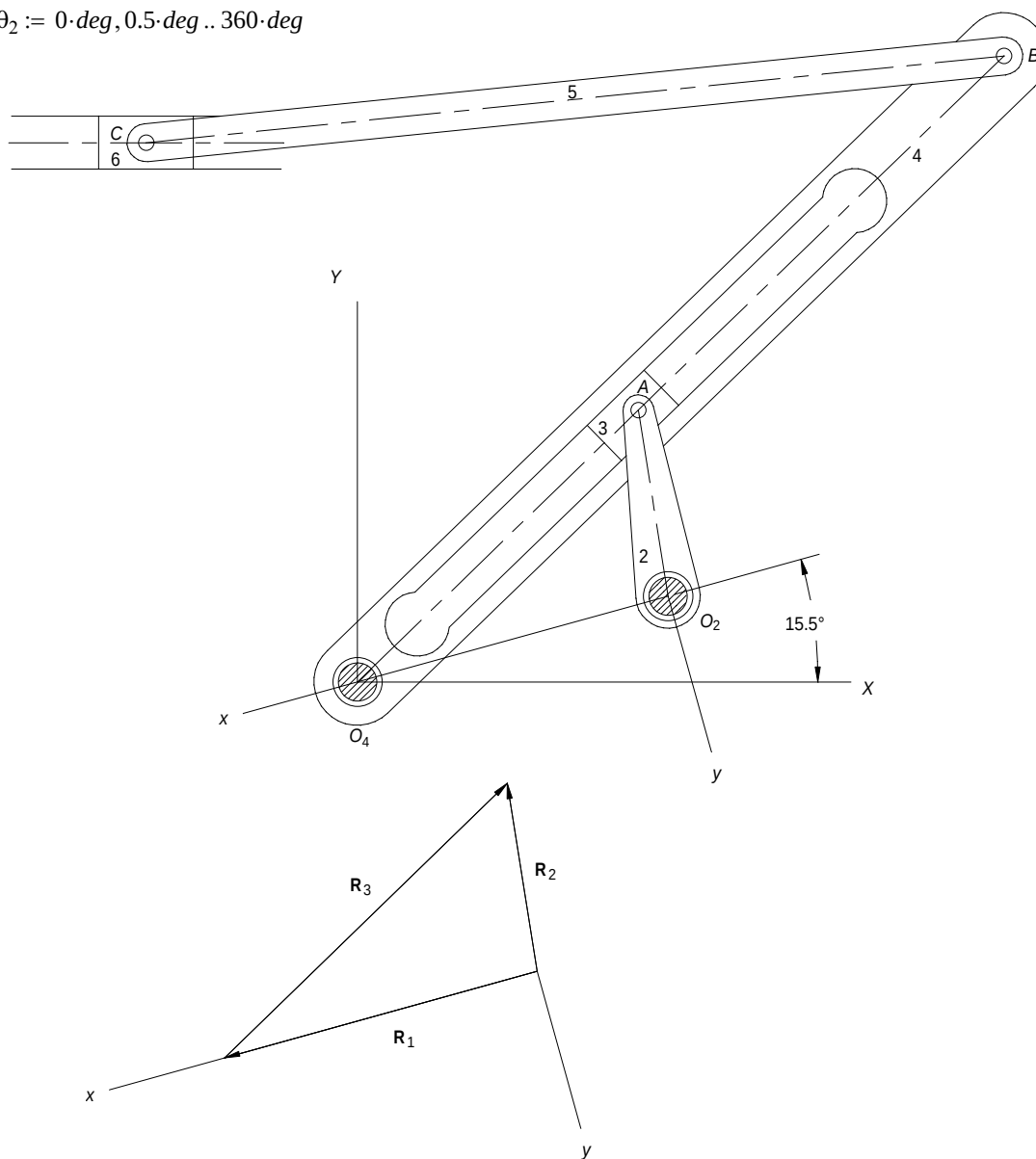
Given: Link lengths and angles:

Link 1 (O_2O_4)	$d := 1.69 \cdot \text{in}$	Link 4 (L_4)	$a' := 4.76 \cdot \text{in}$
Link 2 (L_2)	$a := 1.00 \cdot \text{in}$	Link 5 (L_5)	$b' := 4.55 \cdot \text{in}$
Input crank motion	$\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW		$\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$
Coordinate rotation angle	$\beta := -164.5 \cdot \text{deg}$	Offset (AE)	$c' := 2.86 \cdot \text{in}$

Solution: See Figure P7-23 and Mathcad file P0754.

1. Draw the mechanism to scale and define a vector loop for the input portion (links 1, 2, 3, and 4) using the four-bar crank-slider derivation in Section 7.3 as a model.

$$\theta_2 := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



2. Write the vector loop equation, expand the result and separate into real and imaginary parts to solve for θ_3 and b (distance from O_4 to A).

$$\mathbf{R}_2 := \mathbf{R}_1 + \mathbf{R}_3$$

$$a \cdot e^{j \cdot \theta_2} := d + b \cdot e^{j \cdot \theta_3}$$

Substituting the Euler equivalents,

$$a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) := d + b (\cos(\theta_3) + j \cdot \sin(\theta_3))$$

Separating into real and imaginary components and solving for θ_3 and b .

$$\theta_3(\theta_2) := \text{atan2}(a \cdot \cos(\theta_2) - d, a \cdot \sin(\theta_2))$$

$$b(\theta_2) := \frac{a \cdot \sin(\theta_2)}{\sin(\theta_3(\theta_2))}$$

3. Differentiate the position equation, expand it and solve for ω_3 and $b\dot{\theta}$.

$$a \cdot j \cdot \omega_2 \cdot e^{j \cdot \theta_2} := b \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + b\dot{\theta} \cdot e^{j \cdot \theta_3}$$

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b(\theta_2)} \cdot \cos(\theta_2 - \theta_3(\theta_2))$$

$$b\dot{\theta}(\theta_2) := \frac{a \cdot \omega_2 \cdot \cos(\theta_2) - b(\theta_2) \cdot \omega_3(\theta_2) \cdot \cos(\theta_3(\theta_2))}{\sin(\theta_3(\theta_2))}$$

4. Differentiate the velocity equation, expand it and solve for α_3 .

$$a \cdot j \cdot \alpha_2 \cdot e^{j \cdot \theta_2} + a \cdot j^2 \cdot \omega_2^2 \cdot e^{j \cdot \theta_2} := b\dot{\theta} \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + b \cdot j \cdot \alpha_3 \cdot e^{j \cdot \theta_3} \dots$$

$$+ b \cdot j^2 \cdot \omega_3^2 \cdot e^{j \cdot \theta_3} + b\ddot{\theta} \cdot e^{j \cdot \theta_3} + b\dot{\theta} \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3}$$

$$\alpha_3(\theta_2) := \frac{1}{b(\theta_2)} \cdot (a \cdot \alpha_2 \cdot \cos(\theta_2 - \theta_3(\theta_2)) + a \cdot \omega_2^2 \cdot \sin(\theta_3(\theta_2) - \theta_2) - 2 \cdot b\dot{\theta}(\theta_2) \cdot \omega_3(\theta_2))$$

5. Determine the acceleration of point A on link 2 (in the global XY coordinate system) using equations 7.13a.

$$\mathbf{A}_{A2}(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{A2}(\theta_2) := |\mathbf{A}_{A2}(\theta_2)| \quad \theta_{AA2}(\theta_2) := \arg(\mathbf{A}_{A2}(\theta_2)) + \beta$$

6. Determine the acceleration of points A and B on link 4 (in the Global XY coordinate system) using equations 7.13a.

Transform and rename θ_3 to the global coordinate frame:	$\theta_4(\theta_2) := \theta_3(\theta_2) + \beta$
Rename α_3 to α_4 :	$\alpha_4(\theta_2) := \alpha_3(\theta_2)$
	$\omega_4(\theta_2) := \omega_3(\theta_2)$

$$\mathbf{A}_{A4}(\theta_2) := b(\theta_2) \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) \dots \\ + -b(\theta_2) \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$A_{A4}(\theta_2) := |\mathbf{A}_{A4}(\theta_2)| \quad \theta_{A44}(\theta_2) := \arg(\mathbf{A}_{A4}(\theta_2))$$

$$\mathbf{A}_B(\theta_2) := a' \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) \dots \\ + -a' \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$A_B(\theta_2) := |\mathbf{A}_B(\theta_2)| \quad \theta_{AB}(\theta_2) := \arg(\mathbf{A}_B(\theta_2))$$

7. Determine θ_5 using equation 4.16.

$$\theta_5(\theta_2) := \text{asin}\left(\frac{a' \cdot \sin(\theta_4(\theta_2)) - c'}{b'}\right)$$

8. Determine the angular velocity of link 5 using equation 6.22a:

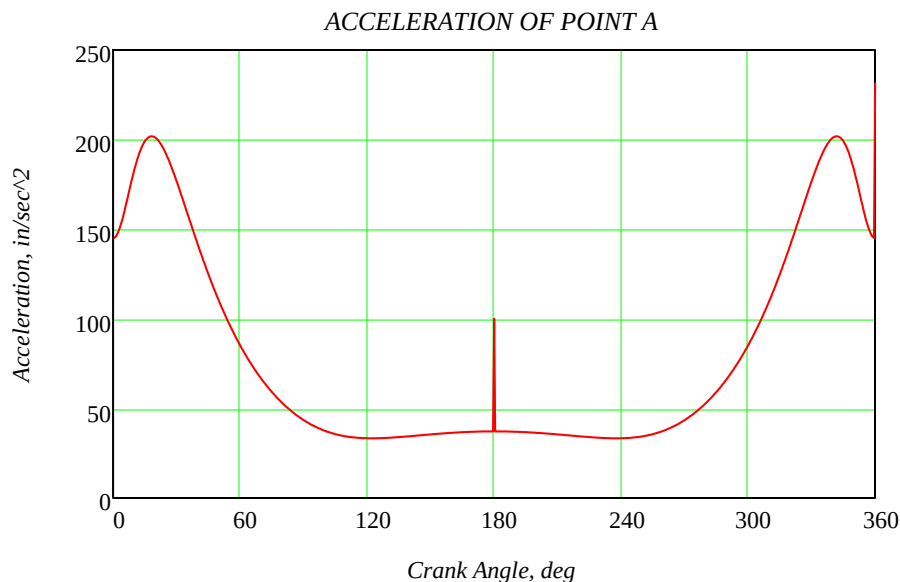
$$\omega_5(\theta_2) := \frac{a' \cdot \cos(\theta_4(\theta_2))}{b' \cdot \cos(\theta_5(\theta_2))} \cdot \omega_4(\theta_2)$$

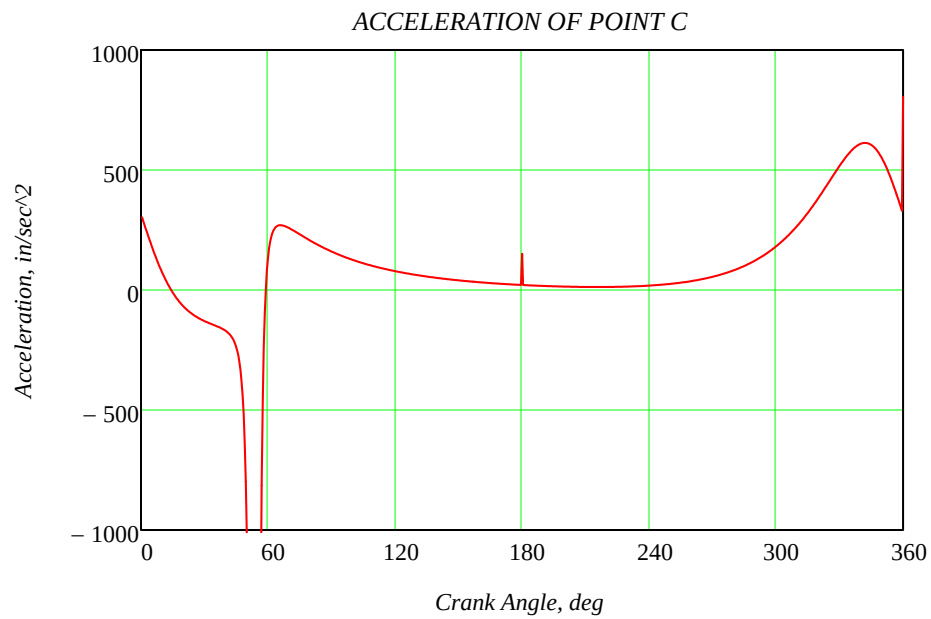
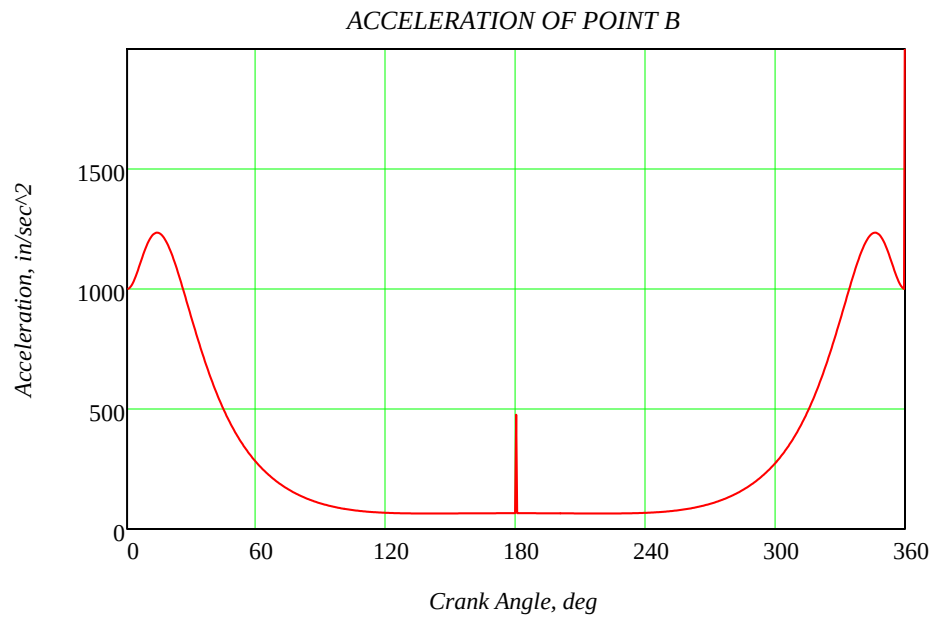
9. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5(\theta_2) := \frac{a' \cdot \alpha_4(\theta_2) \cdot \cos(\theta_4(\theta_2)) - a' \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2)) + b' \cdot \omega_5(\theta_2)^2 \cdot \sin(\theta_5(\theta_2))}{b' \cdot \cos(\theta_5(\theta_2))}$$

10. Use equation 7.16e for the acceleration of pin C.

$$A_C(\theta_2) := -a' \cdot \alpha_4(\theta_2) \cdot \sin(\theta_4(\theta_2)) - a' \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2)) \dots \\ + b' \cdot \alpha_5(\theta_2) \cdot \sin(\theta_5(\theta_2)) + b' \cdot \omega_5(\theta_2)^2 \cdot \cos(\theta_5(\theta_2))$$





PROBLEM 7-55

Statement: Figure P7-24 shows a drum pedal mechanism. For the dimensions given below, find and plot the output acceleration of the linkage over its range of motion if the input velocity V_{in} is a constant.

Given: Link lengths:

$$\text{Link 2 (O}_2\text{A)} \quad a := 100 \cdot \text{mm}$$

$$\text{Link 3 (AB)} \quad b := 28 \cdot \text{mm}$$

$$\text{Link 4 (O}_4\text{B)} \quad c := 64 \cdot \text{mm}$$

$$\text{Link 1 (O}_2\text{O}_4) \quad d := 56 \cdot \text{mm}$$

Distance to force application:

$$\text{Link 2} \quad r_{in} := 48 \cdot \text{mm}$$

$$\text{Link 3 (AP)} \quad r_{out} := 124 \cdot \text{mm}$$

$$\text{Input velocity:} \quad V_{in} := 3 \cdot \text{m} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2} \quad \delta_3 := 0 \cdot \text{deg}$$

$$\text{Range of positions of link 2:} \quad \theta_{20} := 162 \cdot \text{deg} \quad \theta_{21} := 171 \cdot \text{deg}$$

Solution: See Figure P7-24 and Mathcad file P0755.

1. Draw the mechanism to scale and label it.

2. Calculate the range of θ_2 in the local coordinate system (required to calculate θ_3 and θ_4).

Rotation angle of local xy system to global XY system:

$$\alpha := 180 \cdot \text{deg}$$

$$\theta_2 := \theta_{20} - \alpha, (\theta_{20} - \alpha) + 1 \cdot \text{deg} .. \theta_{21} - \alpha$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 0.5600$$

$$K_2 = 0.8750$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_3 = 1.2850$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b}$$

$$K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 2.0000$$

$$K_5 = -1.7543$$

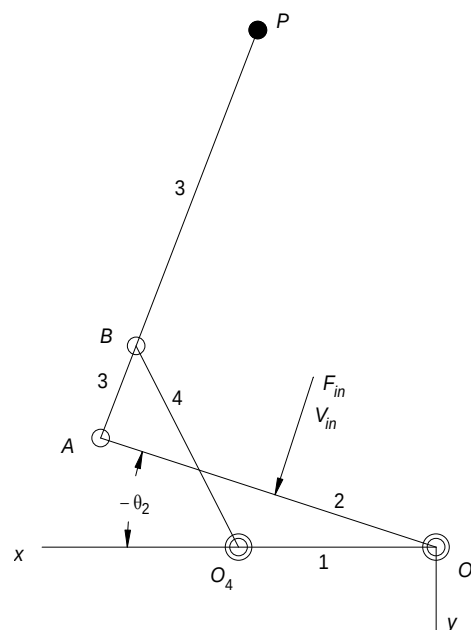
$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$



7. Determine the angular velocity of link 2 using equation 6.7 and links 3 and 4 using equations 6.18.

$$\omega_2 := \frac{V_{in}}{r_{in}} \quad \omega_2 = 62.500 \frac{rad}{sec}$$

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Determine the acceleration of point A using equation 7.13a.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

9. Use equations 7.12 to determine the angular acceleration of link 3.

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C(\theta_2) \cdot D(\theta_2) - A(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

10. Determine the acceleration of the coupler point P using equations 7.32.

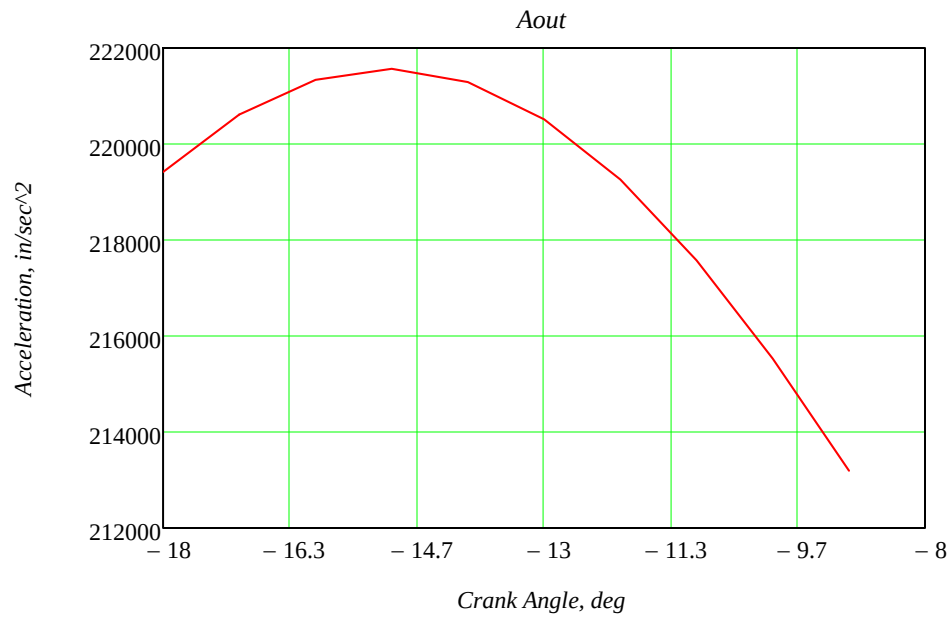
$$\mathbf{A}_{PA}(\theta_2) := r_{out} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots$$

$$+ r_{out} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{A}_P(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2)$$

11. Plot the magnitude of the acceleration of the coupler point P. (See next page).

$$A_{out}(\theta_2) := |\mathbf{A}_P(\theta_2)|$$



Note the acceleration scale values. If the scale went to zero, A_{out} would be a horizontal (constant) line over the range of θ_2 plotted.

PROBLEM 7-56

Statement: A tractor-trailer tipped over while negotiating an on-ramp to the New York Thruway. The road has a 50-ft radius at that point and tilts 3 deg toward the outside of the curve. The 45-ft-long by 8-ft-wide by 8.5-ft-high trailer box (13 ft from ground to top) was loaded with 44 415 lb of paper rolls in two rows by two high as shown in Figure P7-25. The rolls are 40-in-dia by 38-in-long and weigh about 900 lb each. They are wedged against rolling backward but not against sliding sideways. The empty trailer weighed 14 000 lb. The driver claims that he was traveling at less than 15 mph and that the load of paper shifted inside the trailer, struck the trailer sidewall, and tipped the truck. The paper company that loaded the truck claims the load was properly stowed and would not shift at that speed. Independent test of the coefficient of friction between similar paper rolls and a similar trailer floor give a value of 0.43 ± 0.08 . The composite center of gravity of the loaded trailer is estimated to be 7.5 ft above the road. Determine the truck speed that would cause the truck to just begin to tip and the speed at which the rolls will just begin to slide sideways. What do you think caused the accident?

Given:	Weight of paper	$W_p := 44415 \cdot \text{lb}$
	Weight of trailer	$W_t := 14000 \cdot \text{lb}$
	Radius of curve	$r := 50 \cdot \text{ft}$
	Nominal coefficient of friction	$\mu_{nom} := 0.43$
	Coefficient of friction uncertainty	$u_\mu := 0.08$
	Trailer width	$w := 8 \cdot \text{ft}$
	Height of CG from pavement	$h := 7.5 \cdot \text{ft}$

- Assumptions:**
1. The paper rolls act as a monolith since they are tightly strapped together with steel bands.
 2. The tractor has about 15 deg of potential roll freedom versus the trailer due to their relative angle in plan during the turn combined with the substantial pitch freedom in the fifth wheel. So the trailer can tip independently of the tractor.
 3. The outside track width of the trailer tires is equal to the width of the trailer.

Solution: See Figure P7-25 and Mathcad file P0756.

1. First, calculate the location of the trailer's CG with respect to the outside wheel when it is on the reverse-banked curve. From the figure at right,

$$\text{Tilt angle} \quad \theta := 3 \cdot \text{deg}$$

$$a := h \cdot \tan(\theta) \quad a = 0.393 \text{ ft}$$

$$b := \frac{w}{2} - a \quad b = 3.607 \text{ ft}$$

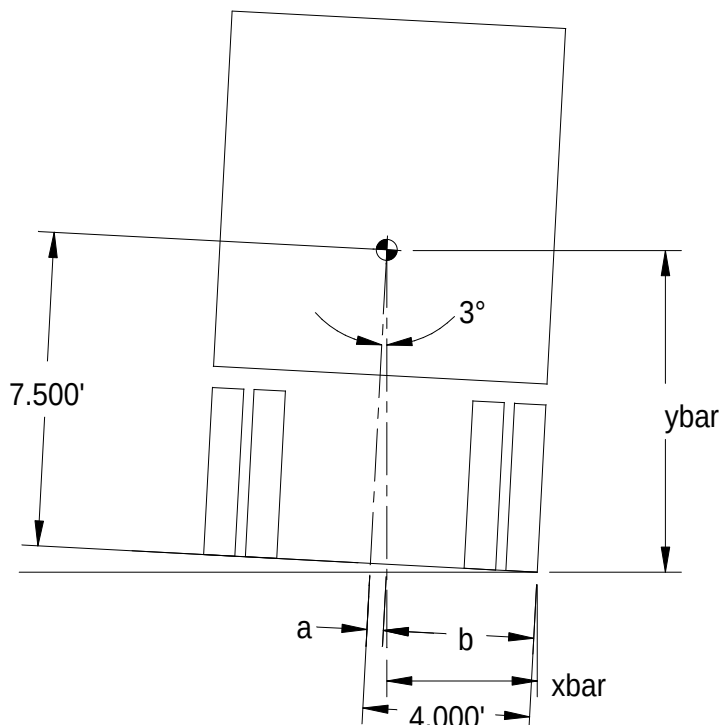
$$\bar{x} := b \cdot \cos(\theta) \quad \bar{x} = 3.602 \text{ ft}$$

$$\bar{y} := b \cdot \sin(\theta) + \frac{h}{\cos(\theta)}$$

$$\bar{y} = 7.699 \text{ ft}$$

The coordinates of the CG of the loaded trailer with respect to the lower outside corner of the tires are:

$$\bar{x} = 3.602 \text{ ft} \quad \bar{y} = 7.699 \text{ ft}$$



2. The trailer is on the verge of tipping over when the couple due to centrifugal force is equal to the couple formed by the weight of the loaded trailer acting through its CG and the vertical reaction at the outside edge of the tires. At this instant, it is assumed that the entire weight of the trailer is reacted at the outside tires with the inside tires carrying none of the weight. Any increase in tangential velocity of the tractor and trailer will result in tipping. Summing moments about the tire edge (see figure below),

$$\sum M \quad F_w \cdot x_{bar} - F_c \cdot y_{bar} = 0 \quad (1)$$

where F_c is the centrifugal force due to the normal acceleration as the tractor and trailer go through the curve. The normal acceleration is

$$a_{tip} = \frac{v_{tip}^2}{r} \quad (2)$$

and the force necessary to keep the tractor trailer following a circular path is

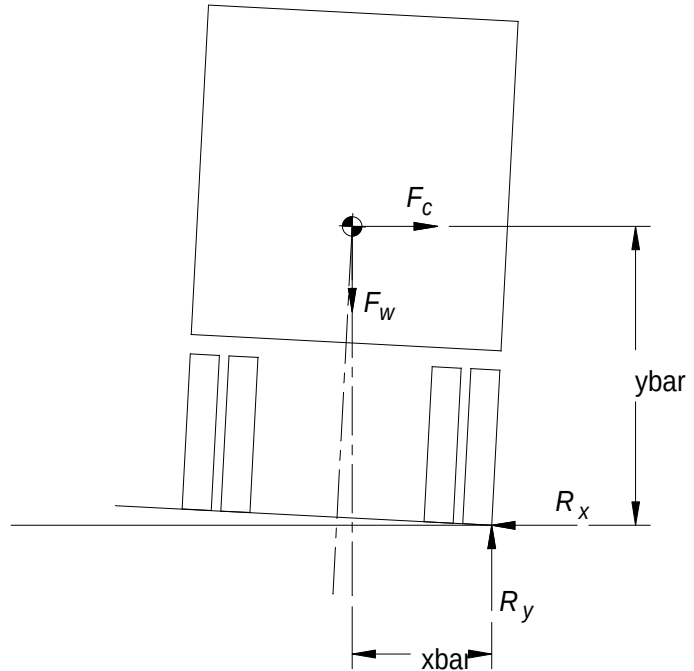
$$F_c = m_{tot} \cdot a_{tip} \quad (3)$$

where m_{tot} is the total mass of the trailer and its payload. Combining equations (2) and (3) and solving for v_{tip} , we have

$$v_{tip} = \sqrt{\frac{F_c \cdot r}{m_{tot}}} \quad (4)$$

or,

$$v_{tip} = \sqrt{\frac{F_c \cdot r \cdot g}{F_w}} \quad (5)$$



3. Calculate the minimum tipping velocity of the tractor/trailer. From equations (1) and (5),

Total weight	$F_w := W_t + W_p$	$F_w = 58415 \text{ lbf}$

Centrifugal force required to tip the trailer	$F_c := \frac{x_{bar}}{y_{bar}} \cdot F_w$	$F_c = 27329 \text{ lbf}$

Minimum tipping speed	$v_{tip} := \sqrt{\frac{F_c \cdot r \cdot g}{F_w}}$	$v_{tip} = 18.7 \text{ mph}$

Thus, with the assumptions that we have made, the trailer would not begin to tip over until it reached a speed of

$$v_{tip} = 18.7 \text{ mph}$$

4. The load will slip when the friction force between the paper rolls and the trailer floor is no longer sufficient to react the centrifugal force on the paper rolls. Looking at the FBD of the paper rolls below, we see that

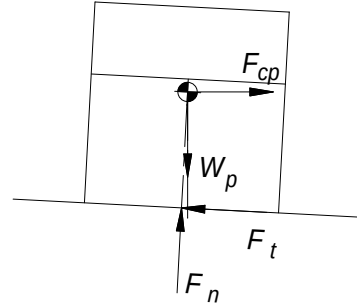
Normal force between paper and floor	$F_n = W_p \cdot \cos(\theta) - F_{cp} \cdot \sin(\theta)$	(6)

Tangential force tending to slide the paper

$$F_t = W_p \cdot \sin(\theta) + F_{cp} \cdot \cos(\theta) \quad (7)$$

Centrifugal force on the paper

$$F_{cp} = \frac{W_p}{g} \cdot a_s = \frac{W_p}{g} \cdot \frac{v_s^2}{r} \quad (8)$$



But, the maximum friction force is $F_f = \mu \cdot F_n = F_t$ (9)

Substituting equation (9) into (7), then combining (6) and (7) to eliminate F_n , and solving for F_{cp} yields

$$F_{cp} = \frac{W_p \cdot (\mu \cdot \cos(\theta) - \sin(\theta))}{\mu \cdot \sin(\theta) + \cos(\theta)} \quad (10)$$

Substituting equation (10) into (8), to eliminate F_{cp} , and solving for v_s yields

$$v_s = \sqrt{\left(\frac{\mu \cdot \cos(\theta) - \sin(\theta)}{\mu \cdot \sin(\theta) + \cos(\theta)} \right) \cdot r \cdot g} \quad (11)$$

5. Use the upper and lower limit on the coefficient of friction to determine an upper and lower limit on the speed necessary to cause sliding.

Maximum coefficient $\mu_{max} := \mu_{nom} + u_\mu$ $\mu_{max} = 0.510$

Minimum coefficient $\mu_{min} := \mu_{nom} - u_\mu$ $\mu_{min} = 0.350$

Maximum velocity to cause sliding

$$v_{smax} := \sqrt{\left(\frac{\mu_{max} \cdot \cos(\theta) - \sin(\theta)}{\mu_{max} \cdot \sin(\theta) + \cos(\theta)} \right) \cdot r \cdot g} \quad v_{smax} = 18.3 \text{ mph}$$

Minimum velocity to cause sliding

$$v_{smin} := \sqrt{\left(\frac{\mu_{min} \cdot \cos(\theta) - \sin(\theta)}{\mu_{min} \cdot \sin(\theta) + \cos(\theta)} \right) \cdot r \cdot g} \quad v_{smin} = 14.8 \text{ mph}$$

6. This very rough analysis shows that, if the coefficient of friction was at or near the low end of its measured value, the paper load could slide at a tractor/trailer speed of 15 mph, which would lead to the trailer tipping over. In any case, it appears that the paper load would slide before the truck would tip with the load in place.

PROBLEM 7-57

Statement: Figure P7-26 shows a V-belt drive. The sheaves (pulleys) have pitch diameters of 150 and 300 mm, respectively. The smaller sheave is driven at a constant 1750 rpm. For a cross-sectional, differential element of the belt, write the equations of its acceleration for one complete trip around both pulleys including its travel between the pulleys. Compute and plot the acceleration of the differential element versus time for one circuit around the belt path. What does your analysis tell you about the dynamic behavior of the belt?

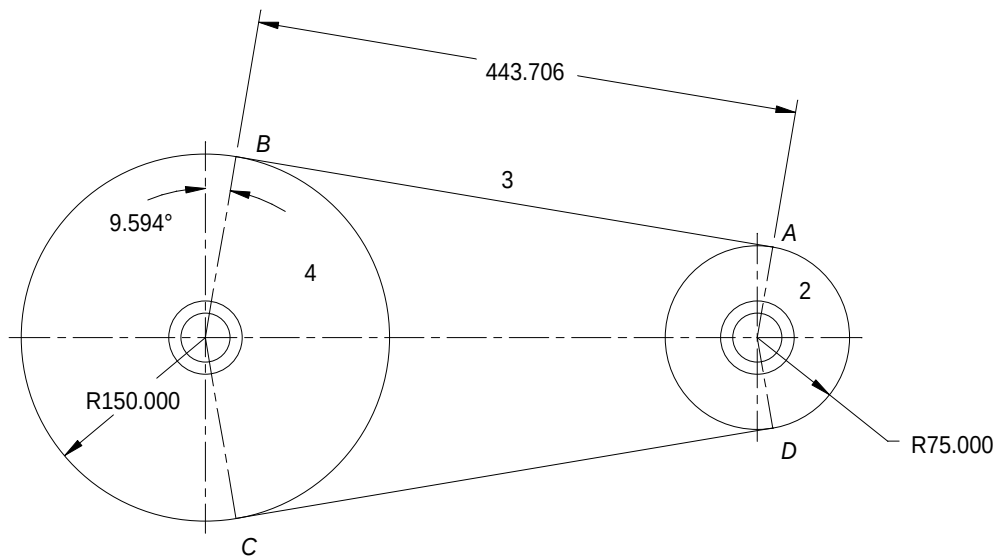
Given: Sheave radii and speed:

$$r_2 := 75 \cdot \text{mm} \quad r_4 := 150 \cdot \text{mm} \quad \omega_2 := 1750 \cdot \text{rpm}$$

Assumptions: Center distance: $C := 450 \cdot \text{mm}$

Solution: See Figure P7-26 and Mathcad file P0757.

1. Draw a schematic representation of the V-belt drive to scale and label it.



From the layout,

Angle to point of tangency	$\beta := 9.594 \cdot \text{deg}$	
Distance from A to B	$L_{AB} := 443.706 \cdot \text{mm}$	
Distance from B to C	$L_{BC} := (\pi + 2 \cdot \beta) \cdot r_4$	$L_{BC} = 521.473 \text{ mm}$
Distance from C to D	$L_{CD} := 443.706 \cdot \text{mm}$	
Distance from D to A	$L_{DA} := (\pi - 2 \cdot \beta) \cdot r_2$	$L_{DA} = 210.502 \text{ mm}$
Total path length	$L := L_{AB} + L_{BC} + L_{CD} + L_{DA}$	$L = 1619.387 \text{ mm}$

2. Calculate the angular velocity of each sheave.

Sheave 2	$\omega_2 = 183.260 \frac{\text{rad}}{\text{sec}}$	
Sheave 4	$\omega_4 := \frac{r_2}{r_4} \cdot \omega_2$	$\omega_4 = 91.630 \frac{\text{rad}}{\text{sec}}$

3. Calculate the acceleration of an element on the belt for each portion of the path, starting at A.

From *A* to *B* the acceleration is zero since the belt velocity is constant $A_{AB} := 0 \cdot \frac{mm}{sec^2}$

From *B* to *C* the tangential acceleration is zero since the belt velocity is constant The normal component is

$$A_{BC} := r_4 \cdot \omega_4^2 \quad A_{BC} = 1.259 \times 10^6 \frac{mm}{sec^2}$$

From *C* to *D* the acceleration is zero since the belt velocity is constant $A_{CD} := 0 \cdot \frac{mm}{sec^2}$

From *D* to *A* the tangential acceleration is zero since the belt velocity is constant The normal component is

$$A_{DA} := r_2 \cdot \omega_2^2 \quad A_{DA} = 2.519 \times 10^6 \frac{mm}{sec^2}$$

4. Plot the acceleration over the entire path using a path variable of *s*.

$$s := 0 \cdot mm, 5 \cdot mm .. L$$

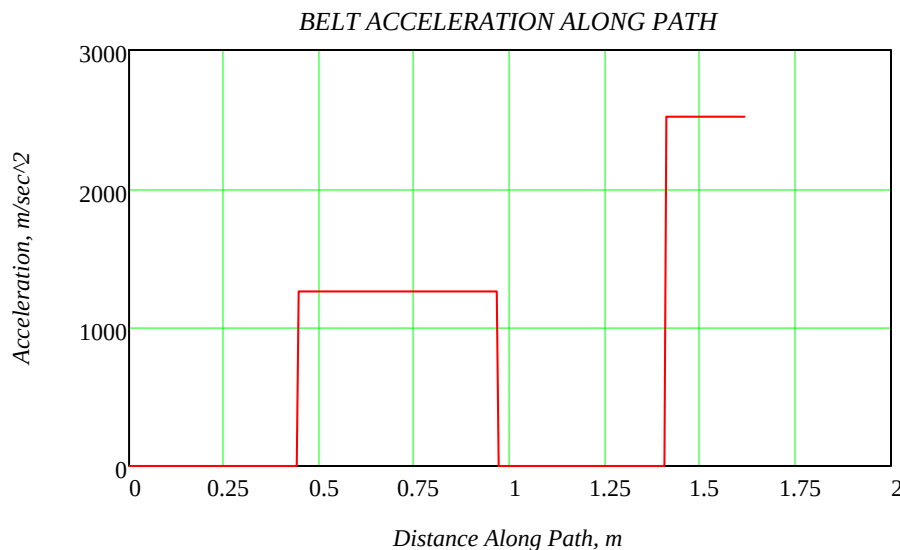
Define a range function that will plot a variable only between two limits using the Heaviside step function Φ .

$$R(s, a, b) := \Phi(s - a) - \Phi(s - b)$$

The acceleration function for the entire path is.

$$\text{Let} \quad s_1 := L_{AB} \quad s_2 := L_{AB} + L_{BC} \quad s_3 := L_{AB} + L_{BC} + L_{CD}$$

$$A(s) := A_{AB} \cdot R(s, 0 \cdot mm, s_1) + A_{BC} \cdot R(s, s_1, s_2) + A_{CD} \cdot R(s, s_2, s_3) + A_{DA} \cdot R(s, s_3, L)$$



5. The graph shows the sudden change in acceleration as the belt enters and leaves a sheave. This results in infinite jerk at these points. This results in belt "hop" at these points, a condition that will cause eventual fatigue failure and wear in the belt. Because of the belt hop caused by infinite jerk at the initial belt/sheave tangency points the span of the belt is continuously vibrating.

PROBLEM 7-58

Statement: Write a program using an equation solver or any computer language to solve for the displacements, velocities, and accelerations in an offset crank-slider linkage as shown in Figure P7-2. Plot the variation in all link's angular and pin's linear positions, velocities, and accelerations with a constant angular velocity input to the crank over one revolution for both open and crossed configurations of the linkage. To test the program, use data from row *a* of Table P7-2. Check your results with program SLIDER.

Enter: Link lengths:

$$\text{Link 2 } a := 1.4 \cdot \text{in} \quad \text{Link 3 } b := 4 \cdot \text{in}$$

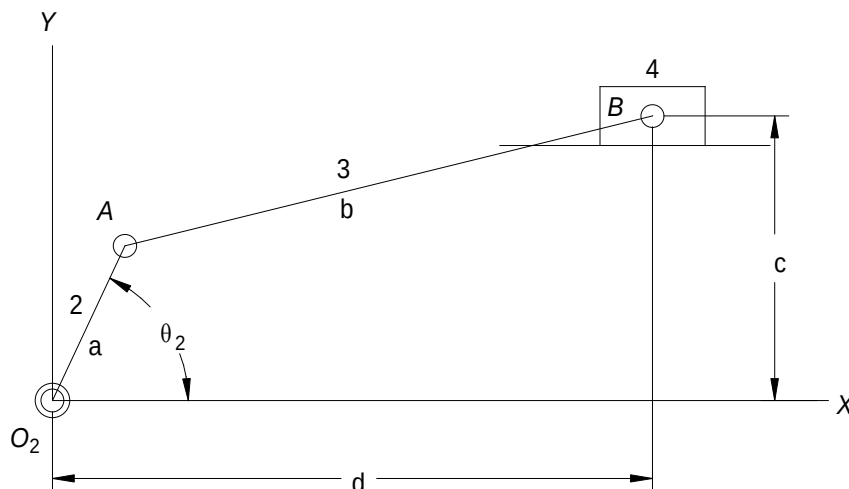
$$\text{Offset: } c := 1 \cdot \text{in}$$

$$\text{Link 2 velocity and acceleration: } \omega_2 := 10 \cdot \frac{\text{rad}}{\text{sec}} \quad \alpha_2 := 0 \cdot \frac{\text{rad}}{\text{sec}^2}$$

$$\text{Crank position range: } \theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

Solution: See Figure P7-2 and Mathcad file P0758.

1. Draw the linkage and label it.



2. Determine θ_3 and d using equations 4.16 and 4.17.

$$\text{Open: } \theta_{32}(\theta_2) := \text{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi$$

$$d_2(\theta_2) := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_{32}(\theta_2))$$

$$\text{Crossed: } \theta_{31}(\theta_2) := \text{asin}\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right)$$

$$d_1(\theta_2) := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_{31}(\theta_2))$$

3. Determine the angular velocity of link 3 using equation 6.22a.

$$\text{Open } \omega_{32}(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_{32}(\theta_2))} \cdot \omega_2$$

$$\text{Crossed} \quad \omega_{31}(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_{31}(\theta_2))} \cdot \omega_2$$

4. Determine the velocity of pin A using equation 6.23a:

$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_{AX}(\theta_2) := \text{Re}(\mathbf{V}_A(\theta_2)) \quad V_{AY}(\theta_2) := \text{Im}(\mathbf{V}_A(\theta_2))$$

5. Determine the velocity of pin B using equation 6.22b:

$$\text{Open} \quad V_{B1}(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{31}(\theta_2) \cdot \sin(\theta_{31}(\theta_2))$$

$$\text{Crossed} \quad V_{B2}(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{32}(\theta_2) \cdot \sin(\theta_{32}(\theta_2))$$

6. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its x and y components.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{AX}(\theta_2) := \text{Re}(\mathbf{A}_A(\theta_2)) \quad A_{AY}(\theta_2) := \text{Im}(\mathbf{A}_A(\theta_2))$$

7. Determine the angular acceleration of link 3 using equation 7.16d.

$$\text{Open} \quad \alpha_{32}(\theta_2) := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_{32}(\theta_2)^2 \cdot \sin(\theta_{32}(\theta_2))}{b \cdot \cos(\theta_{32}(\theta_2))}$$

$$\text{Crossed} \quad \alpha_{31}(\theta_2) := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_{31}(\theta_2)^2 \cdot \sin(\theta_{31}(\theta_2))}{b \cdot \cos(\theta_{31}(\theta_2))}$$

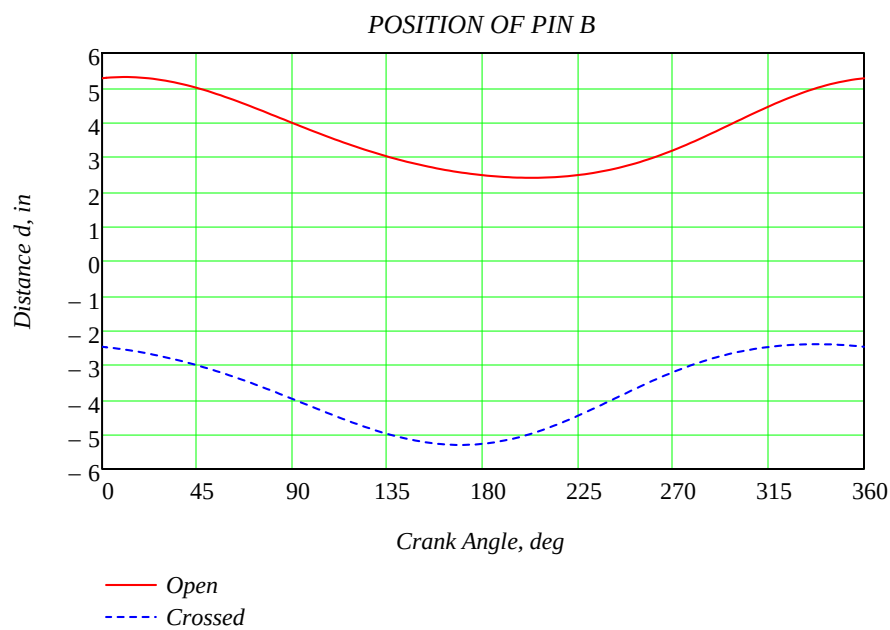
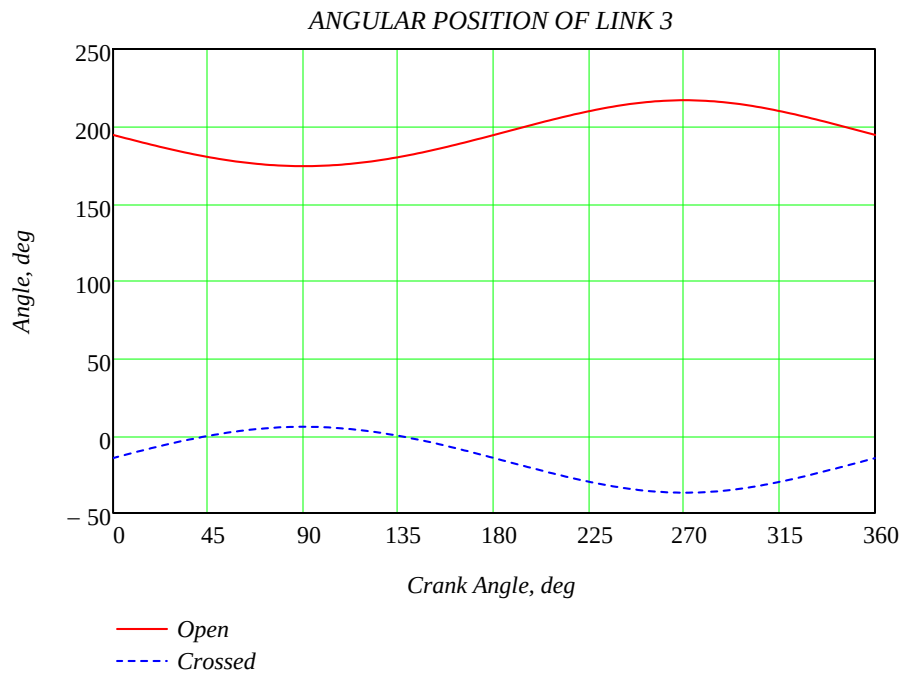
8. Use equation 7.16e for the acceleration of pin B.

$$\begin{aligned} \text{Open:} \quad A_{B2}(\theta_2) &:= -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) \dots \\ &\quad + b \cdot \alpha_{32}(\theta_2) \cdot \sin(\theta_{32}(\theta_2)) + b \cdot \omega_{32}(\theta_2)^2 \cdot \cos(\theta_{32}(\theta_2)) \end{aligned}$$

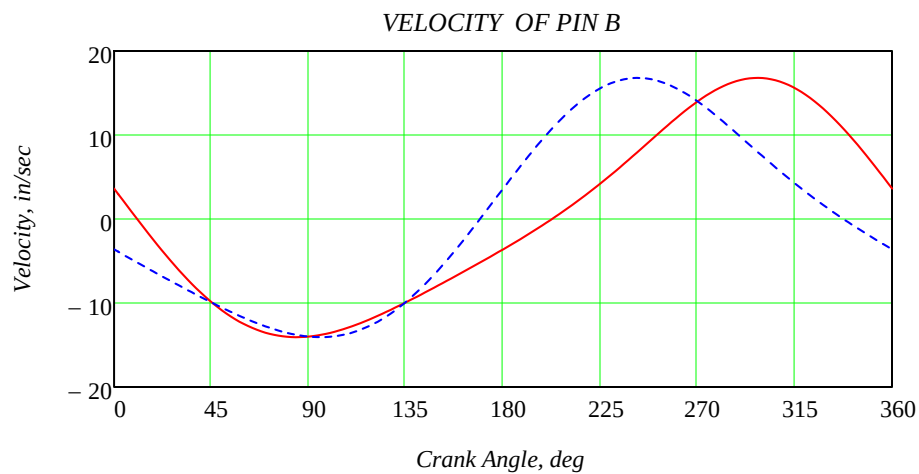
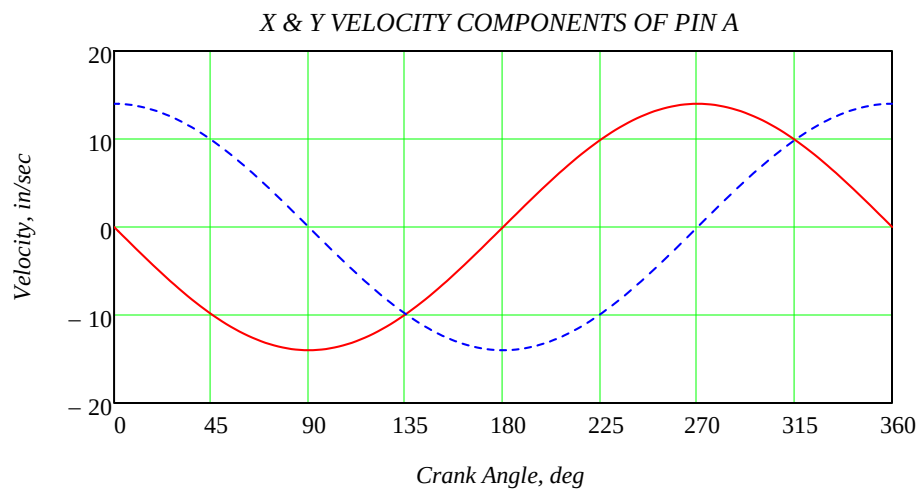
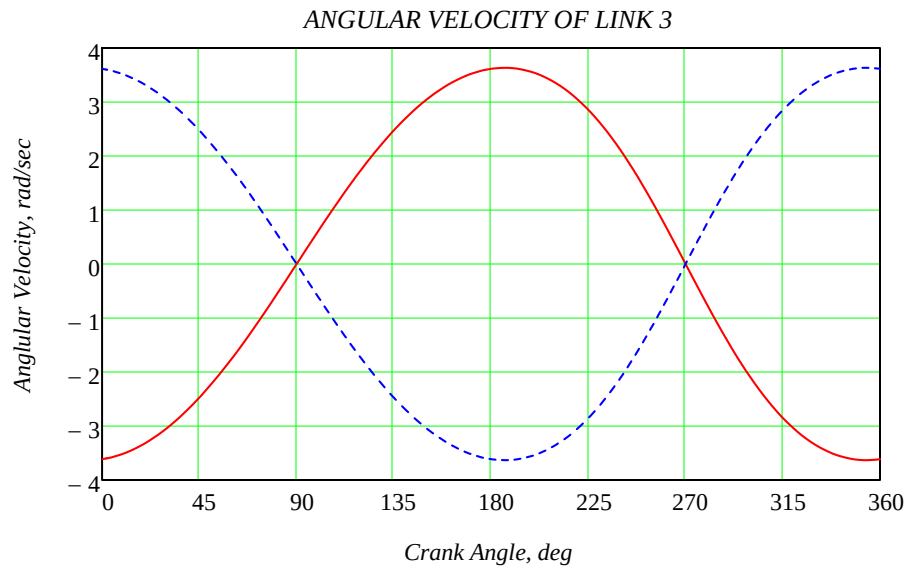
$$\begin{aligned} \text{Crossed:} \quad A_{B1}(\theta_2) &:= -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) \dots \\ &\quad + b \cdot \alpha_{31}(\theta_2) \cdot \sin(\theta_{31}(\theta_2)) + b \cdot \omega_{31}(\theta_2)^2 \cdot \cos(\theta_{31}(\theta_2)) \end{aligned}$$

(See plots on the following pages.)

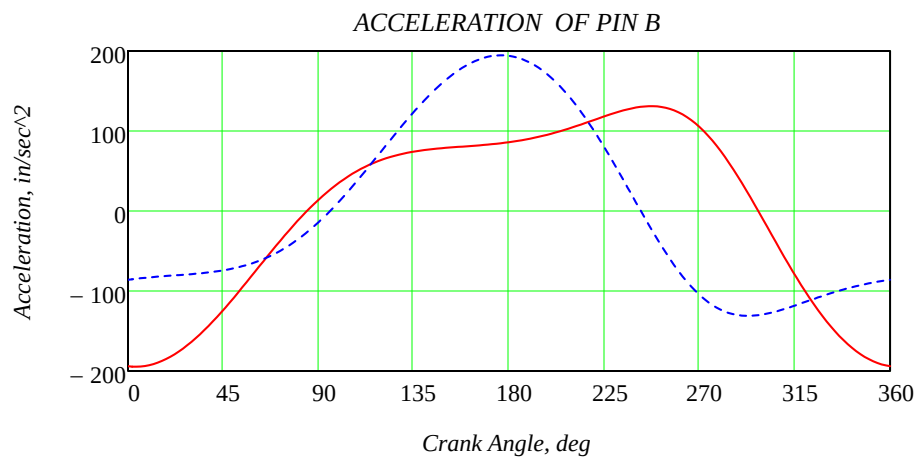
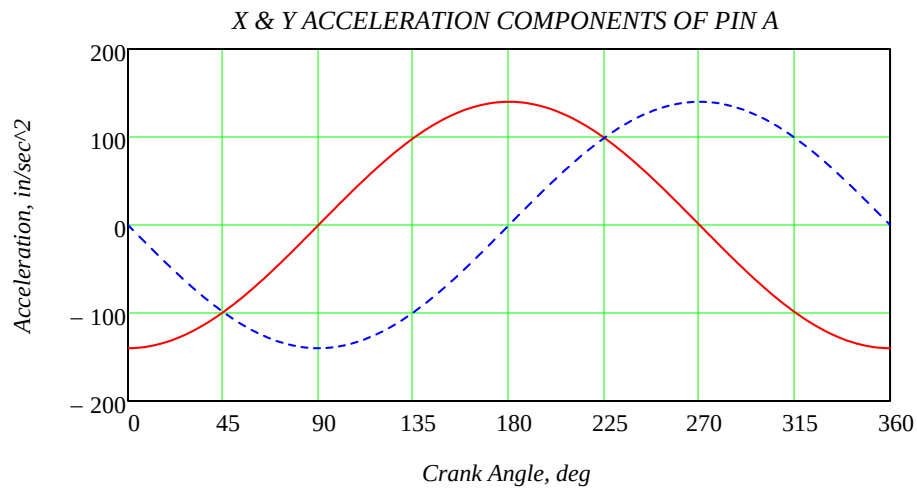
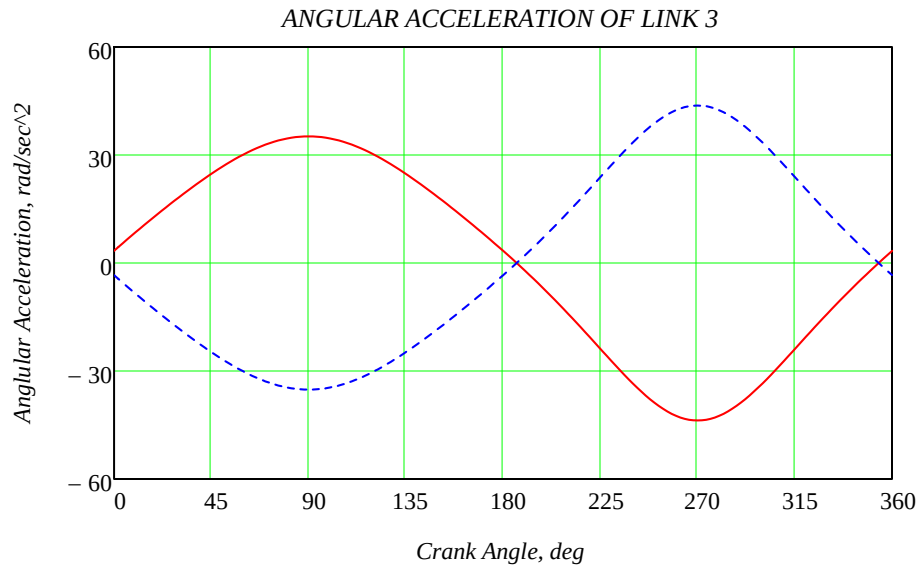
9. Plot the angular position of link 3 and the distance of pin *B* from the *y* axis for open (solid) and crossed (dashed) configurations.



10. Plot the angular velocity of link 3 and the velocities of pins A and B for open (solid) and crossed (dashed) configurations.



11. Plot the angular acceleration of link 3 and the accelerations of pins A and B for open (solid) and crossed (dashed) configurations.



PROBLEM 7-59

Statement: Write a program using an equation solver or any computer language to solve for the displacements, velocities, and accelerations in an inverted slider-crank linkage as shown in Figure P7-3. Plot the variation in all link's angular and pin's linear positions, velocities, and accelerations with a constant angular velocity input to the crank over one revolution for both open and crossed configurations of the linkage. To test the program, use data from row *e* of Table P7-3 except for the value of α_2 , which will be set to zero for this exercise.

Enter: Link lengths:

$$\text{Link 2} \quad a := 4 \cdot \text{in} \quad \text{Link 4} \quad c := 2 \cdot \text{in} \quad \text{Link 1} \quad d := 8 \cdot \text{in}$$

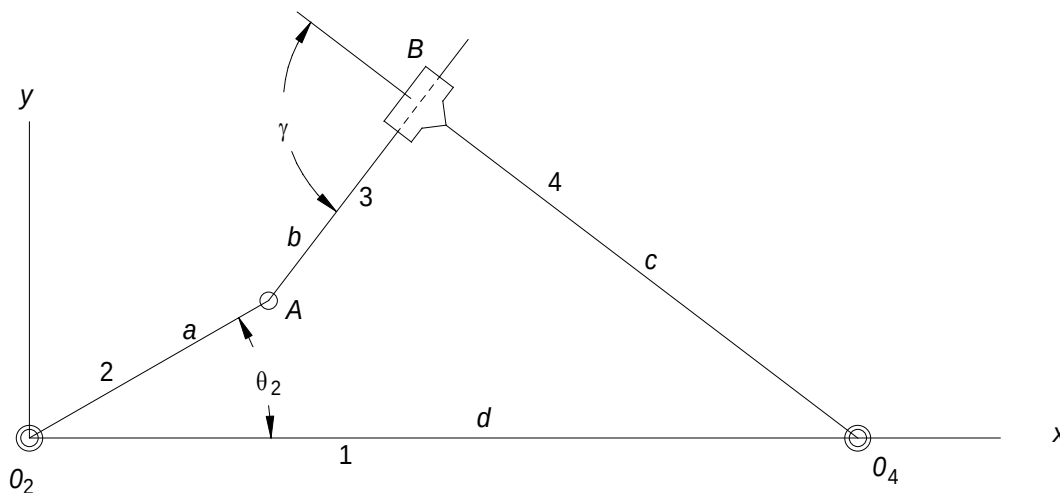
$$\text{Angle between links 3 and 4} \quad \gamma := 30 \cdot \text{deg}$$

$$\text{Link 2 velocity and accel.} \quad \omega_2 := -45 \cdot \frac{\text{rad}}{\text{sec}} \quad \alpha_2 := 0 \cdot \frac{\text{rad}}{\text{sec}^2}$$

$$\text{Crank position range:} \quad \theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

Solution: See Figure P7-3 and Mathcad file P0759.

1. Draw the linkage and label it.



2. Use the equations in Section 4.7 to solve for the positions of links 3 and 4 and for the length *b*.

$$P(\theta_2) := a \cdot \sin(\theta_2) \cdot \sin(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \cos(\gamma)$$

$$Q(\theta_2) := -a \cdot \sin(\theta_2) \cdot \cos(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \sin(\gamma)$$

$$R := -c \cdot \sin(\gamma) \quad S(\theta_2) := R - Q(\theta_2) \quad T(\theta_2) := 2 \cdot P(\theta_2) \quad U(\theta_2) := Q(\theta_2) + R$$

$$\text{Open:} \quad \theta_{41}(\theta_2) := 2 \cdot \text{atan2} \left[(2 \cdot S(\theta_2)), \left(-T(\theta_2) + \sqrt{T(\theta_2)^2 - 4 \cdot S(\theta_2) \cdot U(\theta_2)} \right) \right]$$

$$\theta_{31}(\theta_2) := \theta_{41}(\theta_2) + \gamma$$

$$b_1(\theta_2) := \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_{41}(\theta_2))}{\sin(\theta_{31}(\theta_2))}$$

$$\text{Crossed: } \theta_{42}(\theta_2) := 2 \cdot \text{atan2} \left[(2 \cdot S(\theta_2)), -T(\theta_2) - \sqrt{T(\theta_2)^2 - 4 \cdot S(\theta_2) \cdot U(\theta_2)} \right]$$

$$\theta_{32}(\theta_2) := \theta_{42}(\theta_2) + \gamma$$

$$b_2(\theta_2) := \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_{42}(\theta_2))}{\sin(\theta_{32}(\theta_2))}$$

3. Calculate the angular velocity of links 3 and 4 and the slip velocity using equations 6.30.

$$\text{Open: } \omega_{41}(\theta_2) := \frac{a \cdot \omega_2 \cdot \cos(\theta_2 - \theta_{31}(\theta_2))}{b_1(\theta_2) + c \cdot \cos(\gamma)}$$

$$b\dot{\theta}_1(\theta_2) := \frac{-a \cdot \omega_2 \cdot \sin(\theta_2) + \omega_{41}(\theta_2) \cdot (b_1(\theta_2) \cdot \sin(\theta_{31}(\theta_2)) + c \cdot \sin(\theta_{41}(\theta_2)))}{\cos(\theta_{31}(\theta_2))}$$

$$\omega_{31}(\theta_2) := \omega_{41}(\theta_2)$$

$$\text{Crossed: } \omega_{42}(\theta_2) := \frac{a \cdot \omega_2 \cdot \cos(\theta_2 - \theta_{32}(\theta_2))}{b_2(\theta_2) + c \cdot \cos(\gamma)}$$

$$b\dot{\theta}_2(\theta_2) := \frac{-a \cdot \omega_2 \cdot \sin(\theta_2) + \omega_{42}(\theta_2) \cdot (b_2(\theta_2) \cdot \sin(\theta_{32}(\theta_2)) + c \cdot \sin(\theta_{42}(\theta_2)))}{\cos(\theta_{32}(\theta_2))}$$

$$\omega_{32}(\theta_2) := \omega_{42}(\theta_2)$$

4. Solve for the accelerations using equations (7.26) and (7.27).

Open:

$$P_1(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_{31}(\theta_2) - \theta_2) \quad Q_1(\theta_2) := a \cdot \omega_2^2 \cdot \sin(\theta_{31}(\theta_2) - \theta_2)$$

$$R_1(\theta_2) := c \cdot \omega_{41}(\theta_2)^2 \cdot \sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2)) \quad S_1(\theta_2) := 2 \cdot b\dot{\theta}_1(\theta_2) \cdot \omega_{31}(\theta_2)$$

$$T_1(\theta_2) := b_1(\theta_2) + c \cdot \cos(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))$$

$$\alpha_{41}(\theta_2) := \frac{P_1(\theta_2) + Q_1(\theta_2) + R_1(\theta_2) - S_1(\theta_2)}{T_1(\theta_2)}$$

$$K_1(\theta_2) := a \cdot \omega_2^2 \cdot (b_1(\theta_2) \cdot \cos(\theta_{31}(\theta_2) - \theta_2) + c \cdot \cos(\theta_{41}(\theta_2) - \theta_2))$$

$$L_1(\theta_2) := a \cdot \alpha_2 \cdot (b_1(\theta_2) \cdot \sin(\theta_2 - \theta_{31}(\theta_2)) - c \cdot \sin(\theta_{41}(\theta_2) + \theta_2))$$

$$M_1(\theta_2) := -\omega_{41}(\theta_2)^2 \cdot (b_1(\theta_2)^2 + c^2 + 2 \cdot b_1(\theta_2) \cdot c \cdot \cos(\theta_{41}(\theta_2) - \theta_{31}(\theta_2)))$$

$$N_1(\theta_2) := 2 \cdot b\dot{\theta}_1(\theta_2) \cdot c \cdot \omega_{41}(\theta_2) \cdot \sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))$$

$$b\dot{d}d\dot{o}t_1(\theta_2) := -\frac{K_1(\theta_2) + L_1(\theta_2) + M_1(\theta_2) + N_1(\theta_2)}{T_1(\theta_2)}$$

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{AX}(\theta_2) := \text{Re}(\mathbf{A}_A(\theta_2)) \quad A_{AY}(\theta_2) := \text{Im}(\mathbf{A}_A(\theta_2))$$

$$\begin{aligned} \mathbf{A}_{B1}(\theta_2) := & -c \cdot \alpha_{41}(\theta_2) \cdot (\sin(\theta_{41}(\theta_2)) - j \cdot \cos(\theta_{41}(\theta_2))) \dots \\ & + -c \cdot \omega_{41}(\theta_2)^2 \cdot (\cos(\theta_{41}(\theta_2)) + j \cdot \sin(\theta_{41}(\theta_2))) \end{aligned}$$

$$A_{B1}(\theta_2) := |\mathbf{A}_{B1}(\theta_2)| \quad \theta_{AB1}(\theta_2) := \arg(\mathbf{A}_{B1}(\theta_2))$$

Crossed:

$$P_2(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_{32}(\theta_2) - \theta_2) \quad Q_2(\theta_2) := a \cdot \omega_2^2 \cdot \sin(\theta_{32}(\theta_2) - \theta_2)$$

$$R_2(\theta_2) := c \cdot \omega_{42}(\theta_2)^2 \cdot \sin(\theta_{42}(\theta_2) - \theta_{32}(\theta_2)) \quad S_2(\theta_2) := 2 \cdot b\dot{d}d\dot{o}t_2(\theta_2) \cdot \omega_{32}(\theta_2)$$

$$T_2(\theta_2) := b_2(\theta_2) + c \cdot \cos(\theta_{32}(\theta_2) - \theta_{42}(\theta_2))$$

$$\alpha_{42}(\theta_2) := \frac{P_2(\theta_2) + Q_2(\theta_2) + R_2(\theta_2) - S_2(\theta_2)}{T_2(\theta_2)}$$

$$K_2(\theta_2) := a \cdot \omega_2^2 \cdot (b_2(\theta_2) \cdot \cos(\theta_{32}(\theta_2) - \theta_2) + c \cdot \cos(\theta_{42}(\theta_2) - \theta_2))$$

$$L_2(\theta_2) := a \cdot \alpha_2 \cdot (b_2(\theta_2) \cdot \sin(\theta_2 - \theta_{32}(\theta_2)) - c \cdot \sin(\theta_{42}(\theta_2) + \theta_2))$$

$$M_2(\theta_2) := -\omega_{42}(\theta_2)^2 \cdot (b_2(\theta_2)^2 + c^2 + 2 \cdot b_2(\theta_2) \cdot c \cdot \cos(\theta_{42}(\theta_2) - \theta_{32}(\theta_2)))$$

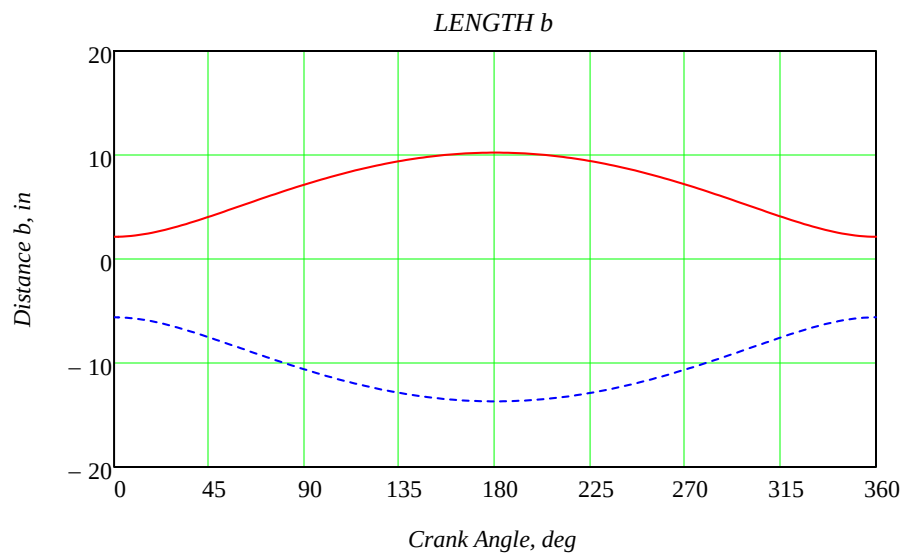
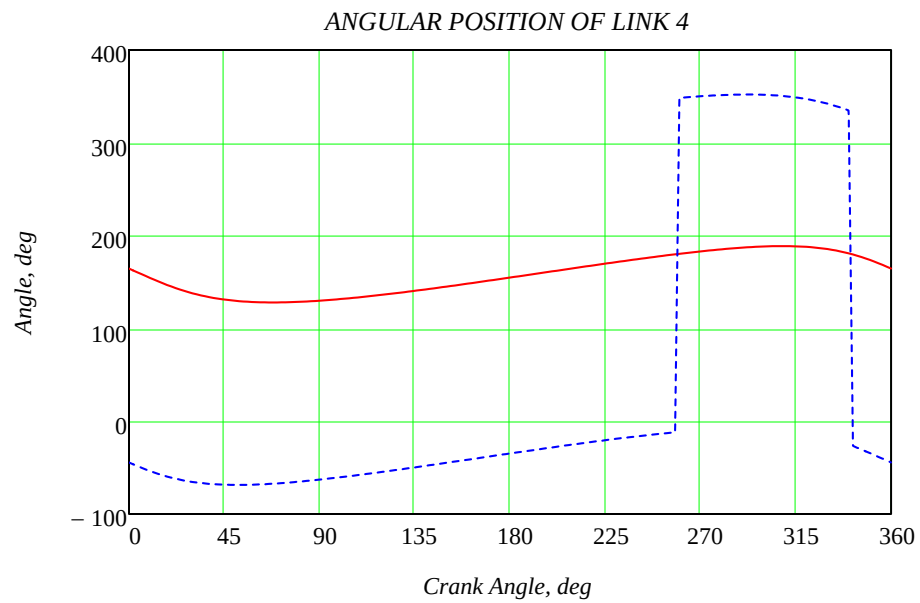
$$N_2(\theta_2) := 2 \cdot b\dot{d}d\dot{o}t_2(\theta_2) \cdot c \cdot \omega_{42}(\theta_2) \cdot \sin(\theta_{42}(\theta_2) - \theta_{32}(\theta_2))$$

$$b\dot{d}d\dot{o}t_2(\theta_2) := -\frac{K_2(\theta_2) + L_2(\theta_2) + M_2(\theta_2) + N_2(\theta_2)}{T_2(\theta_2)}$$

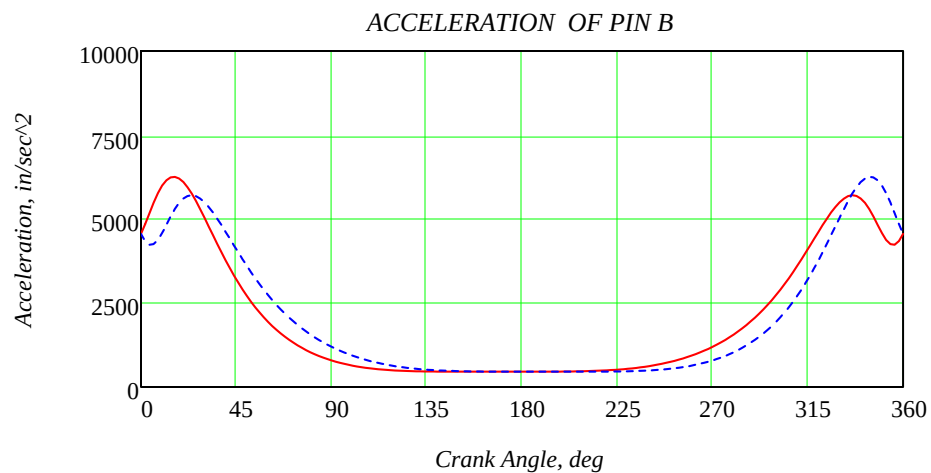
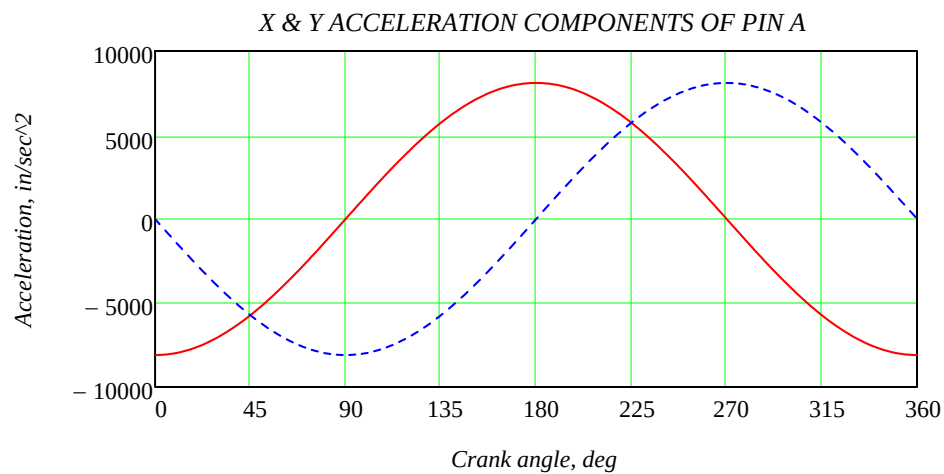
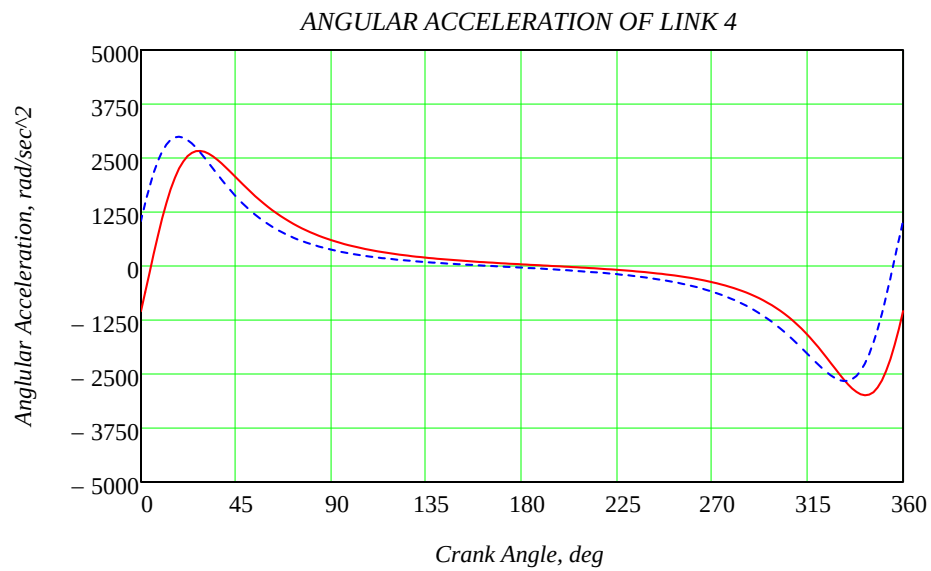
$$\begin{aligned} \mathbf{A}_{B2}(\theta_2) := & -c \cdot \alpha_{42}(\theta_2) \cdot (\sin(\theta_{42}(\theta_2)) - j \cdot \cos(\theta_{42}(\theta_2))) \dots \\ & + -c \cdot \omega_{42}(\theta_2)^2 \cdot (\cos(\theta_{42}(\theta_2)) + j \cdot \sin(\theta_{42}(\theta_2))) \end{aligned}$$

$$A_{B2}(\theta_2) := |\mathbf{A}_{B2}(\theta_2)| \quad \theta_{AB2}(\theta_2) := \arg(\mathbf{A}_{B2}(\theta_2))$$

5. Plot the angular position of link 4 and the distance b for open (solid) and crossed (dashed) configurations.



6. Plot the angular acceleration of link 4 and the accelerations of points A and B for open (solid) and crossed (dashed) configurations.



PROBLEM 7-60

Statement: Write a program using an equation solver or any computer language to solve for the displacements, velocities, and accelerations in a geared fivebar linkage as shown in Figure P7-4. Plot the variation in all link's angular and pin's linear positions, velocities, and accelerations with a constant angular velocity input to the crank over one revolution for both open and crossed configurations of the linkage. To test the program, use data from row *a* of Table P7-4. Check your results with program FIVEBAR.

Enter: Link lengths:

$$\text{Link 1 } f := 6 \cdot \text{in} \quad \text{Link 3 } b := 7 \cdot \text{in} \quad \text{Link 5 } d := 4 \cdot \text{in}$$

$$\text{Link 2 } a := 1 \cdot \text{in} \quad \text{Link 4 } c := 9 \cdot \text{in}$$

Gear ratio, phase angle, and crank velocity and acceleration:

$$\lambda := 2 \quad \phi := 30 \cdot \text{deg} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\text{Coupler data: } R_{pa} := 6 \cdot \text{in} \quad \delta_3 := 30 \cdot \text{deg}$$

Solution: See Figure P7-4 and Mathcad file P0760.

- Determine the angle of link 5 using the equation in Figure P6-4.

$$\theta_5(\theta_2) := \lambda \cdot \theta_2 + \phi$$

- Determine the values of the constants needed for finding θ_3 and θ_4 from equations 4.28h and 4.28i.

$$A(\theta_2) := 2 \cdot c \cdot (d \cdot \cos(\lambda \cdot \theta_2 + \phi) - a \cdot \cos(\theta_2) + f)$$

$$B(\theta_2) := 2 \cdot c \cdot (d \cdot \sin(\lambda \cdot \theta_2 + \phi) - a \cdot \sin(\theta_2))$$

$$C(\theta_2) := (a^2 - b^2 + c^2 + d^2 + f^2) - 2 \cdot a \cdot f \cdot \cos(\theta_2) \dots \\ + [-2 \cdot d \cdot (a \cdot \cos(\theta_2) - f) \cdot \cos(\lambda \cdot \theta_2 + \phi)] \dots \\ + [-2 \cdot a \cdot d \cdot \sin(\theta_2) \cdot \sin(\lambda \cdot \theta_2 + \phi)]$$

$$D(\theta_2) := C(\theta_2) - A(\theta_2) \quad E(\theta_2) := 2 \cdot B(\theta_2) \quad F(\theta_2) := A(\theta_2) + C(\theta_2)$$

$$G(\theta_2) := 2 \cdot b \cdot [- (d \cdot \cos(\lambda \cdot \theta_2 + \phi)) + a \cdot \cos(\theta_2) - f]$$

$$H(\theta_2) := 2 \cdot b \cdot [- (d \cdot \sin(\lambda \cdot \theta_2 + \phi)) + a \cdot \sin(\theta_2)]$$

$$K(\theta_2) := (a^2 + b^2 - c^2 + d^2 + f^2) - 2 \cdot a \cdot f \cdot \cos(\theta_2) \dots \\ + [-2 \cdot d \cdot (a \cdot \cos(\theta_2) - f) \cdot \cos(\lambda \cdot \theta_2 + \phi)] \dots \\ + [-2 \cdot a \cdot d \cdot \sin(\theta_2) \cdot \sin(\lambda \cdot \theta_2 + \phi)]$$

$$L(\theta_2) := K(\theta_2) - G(\theta_2) \quad M(\theta_2) := 2 \cdot H(\theta_2) \quad N(\theta_2) := G(\theta_2) + K(\theta_2)$$

- Use equations 4.28h and 4.28i to find values of θ_3 and θ_4 for the open and crossed circuits.

$$\text{OPEN} \quad \theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot L(\theta_2), -M(\theta_2) + \sqrt{M(\theta_2)^2 - 4 \cdot L(\theta_2) \cdot N(\theta_2)} \right) \right)$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

$$\text{CROSSED} \quad \theta_{32}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot L(\theta_2), -M(\theta_2) - \sqrt{M(\theta_2)^2 - 4 \cdot L(\theta_2) \cdot N(\theta_2)} \right) \right)$$

$$\theta_{42}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

4. Use equation 6.32c to find ω_5 . $\omega_5 := \lambda \cdot \omega_2$
5. Calculate ω_3 and ω_4 and the pin velocities using equations 6.33.

OPEN

$$\omega_{31}(\theta_2) := \frac{-[2 \cdot \sin(\theta_{41}(\theta_2)) \cdot (a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_{41}(\theta_2)) + d \cdot \omega_5 \cdot \sin(\theta_{41}(\theta_2) - \theta_5(\theta_2)))]}{b \cdot (\cos(\theta_{31}(\theta_2) - 2 \cdot \theta_{41}(\theta_2)) - \cos(\theta_{31}(\theta_2)))}$$

$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{31}(\theta_2) \cdot \sin(\theta_{31}(\theta_2)) - d \cdot \omega_5 \cdot \sin(\theta_5(\theta_2))}{c \cdot \sin(\theta_{41}(\theta_2))}$$

$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_{Ax}(\theta_2) := \text{Re}(\mathbf{V}_A(\theta_2))$$

$$V_{Ay}(\theta_2) := \text{Im}(\mathbf{V}_A(\theta_2))$$

$$\mathbf{V}_{BA1}(\theta_2) := b \cdot \omega_{31}(\theta_2) \cdot (-\sin(\theta_{31}(\theta_2)) + j \cdot \cos(\theta_{31}(\theta_2)))$$

$$\mathbf{V}_C(\theta_2) := d \cdot \omega_5 \cdot (-\sin(\theta_5(\theta_2)) + j \cdot \cos(\theta_5(\theta_2)))$$

$$\mathbf{V}_{B1}(\theta_2) := \mathbf{V}_A(\theta_2) + \mathbf{V}_{BA1}(\theta_2)$$

$$V_{B1x}(\theta_2) := \text{Re}(\mathbf{V}_{B1}(\theta_2))$$

$$V_{B1y}(\theta_2) := \text{Im}(\mathbf{V}_{B1}(\theta_2))$$

CROSSED

$$\omega_{32}(\theta_2) := \frac{-[2 \cdot \sin(\theta_{42}(\theta_2)) \cdot (a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_{42}(\theta_2)) + d \cdot \omega_5 \cdot \sin(\theta_{42}(\theta_2) - \theta_5(\theta_2)))]}{b \cdot (\cos(\theta_{32}(\theta_2) - 2 \cdot \theta_{42}(\theta_2)) - \cos(\theta_{32}(\theta_2)))}$$

$$\omega_{42}(\theta_2) := \frac{a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{32}(\theta_2) \cdot \sin(\theta_{32}(\theta_2)) - d \cdot \omega_5 \cdot \sin(\theta_5(\theta_2))}{c \cdot \sin(\theta_{42}(\theta_2))}$$

$$\mathbf{V}_{BA2}(\theta_2) := b \cdot \omega_{32}(\theta_2) \cdot (-\sin(\theta_{32}(\theta_2)) + j \cdot \cos(\theta_{32}(\theta_2)))$$

$$\mathbf{V}_{B2}(\theta_2) := \mathbf{V}_A(\theta_2) + \mathbf{V}_{BA2}(\theta_2)$$

$$V_{B2x}(\theta_2) := \text{Re}(\mathbf{V}_{B2}(\theta_2))$$

$$V_{B2y}(\theta_2) := \text{Im}(\mathbf{V}_{B2}(\theta_2))$$

6. Use equation 7.28c to find α_5 . $\alpha_5 := \lambda \cdot \alpha_2$
7. Calculate α_3 and α_4 and the pin accelerations using equations 7.29.

OPEN

$$\alpha_{31}(\theta_2) := \frac{-a \cdot \alpha_2 \cdot \sin(\theta_2 - \theta_{41}(\theta_2)) - a \cdot \omega_2^2 \cdot \cos(\theta_2 - \theta_{41}(\theta_2)) \dots \\ + b \cdot \omega_{31}(\theta_2)^2 \cdot \cos(\theta_{31}(\theta_2) - \theta_{41}(\theta_2)) + d \cdot \omega_5^2 \cdot \cos(\theta_5(\theta_2) - \theta_{41}(\theta_2)) \dots \\ + d \cdot \alpha_5 \cdot \sin(\theta_5(\theta_2) - \theta_{41}(\theta_2)) + c \cdot \omega_{41}(\theta_2)^2}{b \cdot \sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\alpha_{41}(\theta_2) := \frac{a \cdot \alpha_2 \cdot \sin(\theta_2 - \theta_{31}(\theta_2)) + a \cdot \omega_2^2 \cdot \cos(\theta_2 - \theta_{31}(\theta_2)) \dots \\ + c \cdot \omega_{41}(\theta_2)^2 \cdot \cos(\theta_{31}(\theta_2) - \theta_{41}(\theta_2)) - d \cdot \omega_5^2 \cdot \cos(\theta_{31}(\theta_2) - \theta_5(\theta_2)) \dots \\ + d \cdot \alpha_5 \cdot \sin(\theta_{31}(\theta_2) - \theta_5(\theta_2)) + b \cdot \omega_{31}(\theta_2)^2}{c \cdot \sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{Ax}(\theta_2) := \text{Re}(\mathbf{A}_A(\theta_2)) \quad A_{Ay}(\theta_2) := \text{Im}(\mathbf{A}_A(\theta_2))$$

$$\begin{aligned} \mathbf{A}_{BA1}(\theta_2) := & b \cdot \alpha_{31}(\theta_2) \cdot (-\sin(\theta_{31}(\theta_2)) + j \cdot \cos(\theta_{31}(\theta_2))) \dots \\ & + b \cdot \omega_{31}(\theta_2)^2 \cdot (\cos(\theta_{31}(\theta_2)) + j \cdot \sin(\theta_{31}(\theta_2))) \end{aligned}$$

$$\mathbf{A}_{B1}(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{BA1}(\theta_2)$$

$$A_{B1x}(\theta_2) := \text{Re}(\mathbf{A}_{B1}(\theta_2)) \quad A_{B1y}(\theta_2) := \text{Im}(\mathbf{A}_{B1}(\theta_2))$$

$$\begin{aligned} \mathbf{A}_C(\theta_2) := & c \cdot \alpha_5 \cdot (-\sin(\theta_5(\theta_2)) + j \cdot \cos(\theta_5(\theta_2))) \dots \\ & + c \cdot \omega_5^2 \cdot (\cos(\theta_5(\theta_2)) + j \cdot \sin(\theta_5(\theta_2))) \end{aligned}$$

$$A_{Cx}(\theta_2) := \text{Re}(\mathbf{A}_C(\theta_2)) \quad A_{Cy}(\theta_2) := \text{Im}(\mathbf{A}_C(\theta_2))$$

CROSSED

$$\begin{aligned} & -a \cdot \alpha_2 \cdot \sin(\theta_2 - \theta_{42}(\theta_2)) - a \cdot \omega_2^2 \cdot \cos(\theta_2 - \theta_{42}(\theta_2)) \dots \\ & + b \cdot \omega_{32}(\theta_2)^2 \cdot \cos(\theta_{32}(\theta_2) - \theta_{42}(\theta_2)) + d \cdot \omega_5^2 \cdot \cos(\theta_5(\theta_2) - \theta_{42}(\theta_2)) \dots \\ & + d \cdot \alpha_5 \cdot \sin(\theta_5(\theta_2) - \theta_{42}(\theta_2)) + c \cdot \omega_{42}(\theta_2)^2 \\ \alpha_{32}(\theta_2) := & \frac{b \cdot \sin(\theta_{32}(\theta_2) - \theta_{42}(\theta_2))}{\dots} \end{aligned}$$

$$\begin{aligned} & a \cdot \alpha_2 \cdot \sin(\theta_2 - \theta_{32}(\theta_2)) + a \cdot \omega_2^2 \cdot \cos(\theta_2 - \theta_{32}(\theta_2)) \dots \\ & + c \cdot \omega_{42}(\theta_2)^2 \cdot \cos(\theta_{32}(\theta_2) - \theta_{42}(\theta_2)) - d \cdot \omega_5^2 \cdot \cos(\theta_{32}(\theta_2) - \theta_5(\theta_2)) \dots \\ & + d \cdot \alpha_5 \cdot \sin(\theta_{32}(\theta_2) - \theta_5(\theta_2)) + b \cdot \omega_{32}(\theta_2)^2 \\ \alpha_{42}(\theta_2) := & \frac{c \cdot \sin(\theta_{42}(\theta_2) - \theta_{32}(\theta_2))}{\dots} \end{aligned}$$

$$\begin{aligned} \mathbf{A}_{BA2}(\theta_2) := & b \cdot \alpha_{32}(\theta_2) \cdot (-\sin(\theta_{32}(\theta_2)) + j \cdot \cos(\theta_{32}(\theta_2))) \dots \\ & + b \cdot \omega_{32}(\theta_2)^2 \cdot (\cos(\theta_{32}(\theta_2)) + j \cdot \sin(\theta_{32}(\theta_2))) \end{aligned}$$

$$\mathbf{A}_{B2}(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{BA2}(\theta_2)$$

$$A_{B2x}(\theta_2) := \text{Re}(\mathbf{A}_{B2}(\theta_2)) \quad A_{B2y}(\theta_2) := \text{Im}(\mathbf{A}_{B2}(\theta_2))$$

PROBLEM 7-61a

Statement: Find the acceleration of the slider in Figure 3-33 for $\theta_2 = 110^\circ$ deg with respect to the global X axis assuming a constant $\omega_2 = 1$ rad/sec CW. Use a graphical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 2.170 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 2.067 \cdot \text{in}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 2.310 \cdot \text{in} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 1.000 \cdot \text{in}$$

$$\text{Link 5 (B to C)} \quad e := 5.400 \cdot \text{in}$$

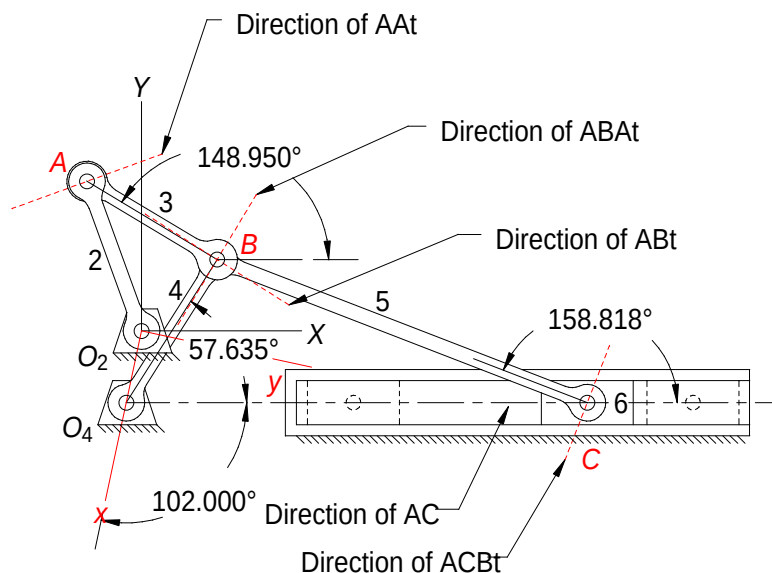
$$\text{Coordinate angle} \quad \delta := -102 \cdot \text{deg} \quad \text{Crank angle:} \quad \theta_{2XY} := 110 \cdot \text{deg}$$

$$\text{Input crank angular velocity} \quad \omega_2 := -1 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$$

$$\text{Input crank angular acceleration} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P6-33 and Mathcad file P0761a.

1. Draw the linkage to scale and label it.



$$\theta_2 := \theta_{2XY} - \delta \quad \theta_2 = 212.000 \text{ deg}$$

2. In order to solve for the acceleration at point B, we will need θ_3 , ω_3 , and θ_4 . From the graphical position solution above (in the local xy coordinate system),

$$\theta_3 := 148.950 \cdot \text{deg} - 180 \cdot \text{deg} - \delta \quad \theta_3 = 70.950 \text{ deg}$$

$$\theta_4 := 57.635 \cdot \text{deg} - \delta \quad \theta_4 = 159.635 \text{ deg}$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_4 - \theta_2)}{b \cdot \sin(\theta_3 - \theta_4)} \quad \omega_3 = -0.832 \text{ rad} \cdot \text{sec}^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_4 - \theta_3)} \quad \omega_4 = -0.591 \text{ rad} \cdot \text{sec}^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

4. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_{Bn} := c \cdot \omega_4^2 \quad A_{Bn} = 0.806 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 - 180 \cdot \text{deg} + \delta \quad \theta_{ABn} = -122.365 \text{ deg}$$

$$A_{An} := a \cdot \omega_2^2 \quad A_{An} = 2.170 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg} + \delta \quad \theta_{AAn} = 290.000 \text{ deg}$$

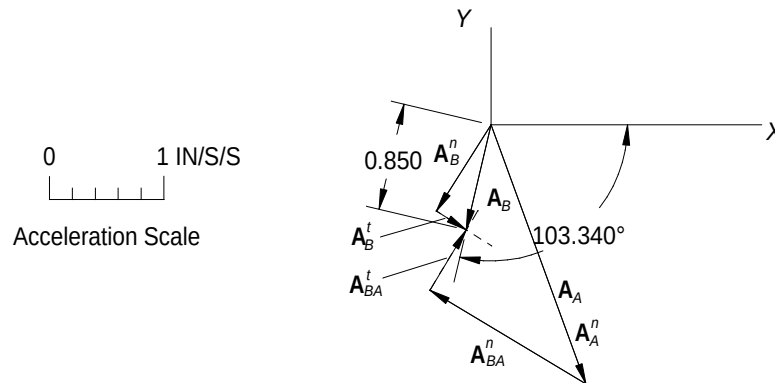
$$A_{At} := a \cdot \alpha_2 \quad A_{At} = 0.0 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AA_t} := \theta_2 + 90 \cdot \text{deg} + \delta \quad \theta_{AA_t} = 200.000 \text{ deg}$$

$$A_{BA_n} := b \cdot \omega_3^2 \quad A_{BA_n} = 1.429 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABA_n} := \theta_3 + 180 \cdot \text{deg} + \delta \quad \theta_{ABA_n} = 148.950 \text{ deg}$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of $\theta_{ABn} + \alpha$ and $\mathbf{A}_A^n$ at an angle of $\theta_{AAn} + \alpha$. From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_A^t$ at an angle of $\theta_{AA_t} + \alpha$. From the tip of $\mathbf{A}_A^t$, draw $\mathbf{A}_{BA}^n$ at an angle of $\theta_{BA_n} + \alpha$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B^n$ and $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B^t$, $\mathbf{A}_B^t$, and $\mathbf{A}_{BA}^t$.



6. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 1.000 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_A := A_{An} \quad A_A = 2.170 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 290.0 \text{ deg}$$

$$A_B := 0.850 \cdot k_a \quad A_B = 0.850 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -103.340 \text{ deg}$$

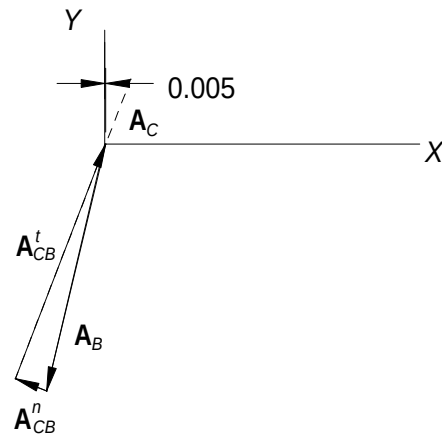
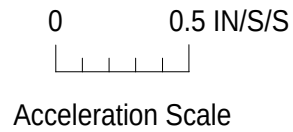
7. From Problem 6-70a, the angular velocity of link 5 is $\omega_5 := 0.145 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW. From the graphical layout, $\theta_5 := 158.818 \cdot \text{deg}$ in the global XY coordinate system.

8. For point C, equation 7.4 becomes: $\mathbf{A}_C = \mathbf{A}_B + (\mathbf{A}_{CB}^t + \mathbf{A}_{CB}^n)$, where $\mathbf{A}_B$ is known and

$$A_{CBn} := e \cdot \omega_5^2 \quad A_{CBn} = 0.114 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ACBn} := \theta_5$$

$$\theta_{ACBn} = 158.818 \text{ deg}$$



9. From the graphical solution above,

Acceleration scale factor

$$k_a := 0.50 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_C := 0.005 \cdot k_a$$

$$A_C = 0.0025 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 0.000 \text{ deg}$$

PROBLEM 7-61b

Statement: Find the acceleration of the slider in Figure 3-33 for $\theta_2 = 110^\circ$ deg with respect to the global X axis assuming a constant $\omega_2 = 1$ rad/sec CW. Use an analytical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 2.170 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 2.067 \cdot \text{in}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 2.310 \cdot \text{in} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 1.000 \cdot \text{in}$$

$$\text{Link 5 (B to C)} \quad e := 5.400 \cdot \text{in}$$

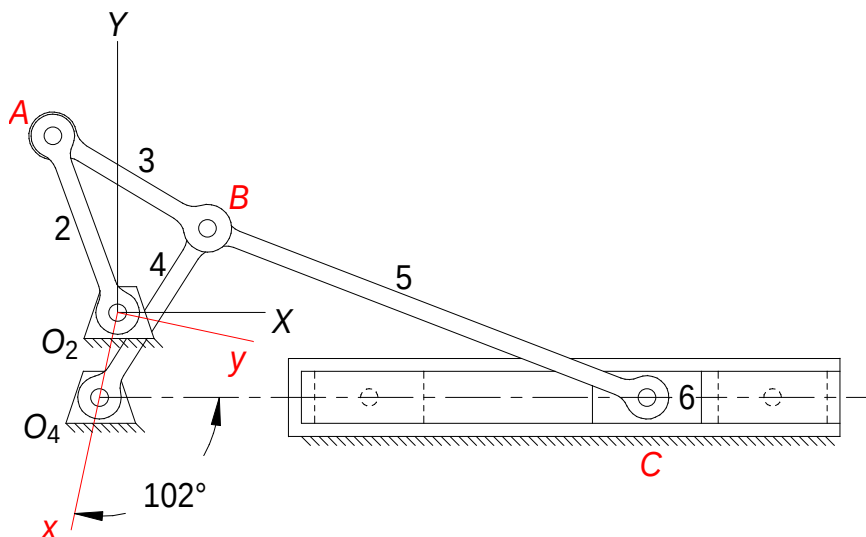
$$\text{Coordinate angle} \quad \delta := -102 \cdot \text{deg} \quad \text{Crank angle:} \quad \theta_{2XY} := 110 \cdot \text{deg}$$

$$\text{Input crank angular velocity} \quad \omega_2 := -1 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$$

$$\text{Input crank angular acceleration} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P6-33 and Mathcad file P0761b.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

Transform crank angle to the local xy coordinate system:

$$\theta_2 := \theta_{2XY} - \delta$$

$$\theta_2 = 212.000 \text{ deg}$$

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 0.4608$$

$$K_2 = 0.4329$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_3 = 0.6755$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.2662 \quad B = 1.0598 \quad C = 2.3515$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{4xy} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_{4xy} = -200.365 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.4838$$

$$K_5 = -0.5178$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -2.2370$$

$$E := -2 \cdot \sin(\theta_2) \quad E = 1.0598$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.3808$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{3xy} := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{3xy} = -289.050 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{4xy} - \theta_2)}{\sin(\theta_{3xy} - \theta_{4xy})} \quad \omega_3 = -0.832 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{3xy})}{\sin(\theta_{4xy} - \theta_{3xy})} \quad \omega_4 = -0.591 \frac{\text{rad}}{\text{sec}}$$

7. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A := c \cdot \sin(\theta_{4xy}) \quad B := b \cdot \sin(\theta_{3xy}) \quad D := c \cdot \cos(\theta_{4xy}) \quad E := b \cdot \cos(\theta_{3xy})$$

$$A = 0.804 \text{ in} \quad B = 1.954 \text{ in} \quad D = -2.166 \text{ in} \quad E = 0.675 \text{ in}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_{3xy}) - c \cdot \omega_4^2 \cdot \cos(\theta_{4xy})$$

$$C = -0.618 \frac{\text{in}}{\text{sec}^2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_{3xy}) + c \cdot \omega_4^2 \cdot \sin(\theta_{4xy})$$

$$F = 0.079 \frac{\text{in}}{\text{sec}^2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = 0.267 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = -0.120 \frac{\text{rad}}{\text{sec}^2}$$

8. Transform θ_4 back to the global XY system.

$$\theta_4 := \theta_{4xy} + \delta \quad \theta_4 = -302.365 \text{ deg}$$

9. Determine θ_5 using equation 4.17.

$$\text{Offset:} \quad cc := 0 \cdot \text{in}$$

$$\theta_5 := \operatorname{asin}\left(-\frac{c \cdot \sin(\theta_4) - cc}{e}\right) + \pi \quad \theta_5 = 158.818 \text{ deg}$$

10. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5 := \frac{c}{e} \cdot \frac{\cos(\theta_4)}{\cos(\theta_5)} \cdot \omega_4 \quad \omega_5 = 0.145 \frac{\text{rad}}{\text{sec}}$$

11. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5 := \frac{c \cdot \alpha_4 \cdot \cos(\theta_4) - c \cdot \omega_4^2 \cdot \sin(\theta_4) + e \cdot \omega_5^2 \cdot \sin(\theta_5)}{e \cdot \cos(\theta_5)} \quad \alpha_5 = 0.156 \frac{\text{rad}}{\text{sec}^2}$$

12. Use equation 7.16e for the acceleration of pin C.

$$A_C := -c \cdot \alpha_4 \cdot \sin(\theta_4) - c \cdot \omega_4^2 \cdot \cos(\theta_4) + e \cdot \alpha_5 \cdot \sin(\theta_5) + e \cdot \omega_5^2 \cdot \cos(\theta_5)$$

$$A_C = 0.00161 \frac{\text{in}}{\text{sec}^2} \quad \text{A positive sign means that } \mathbf{A}_C \text{ is to the right.}$$

PROBLEM 7-62

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular acceleration of link 4 and the linear acceleration of slider 6 in the sixbar crank-slider linkage of Figure 3-33 as a function of the angle of input link 2 for a constant $\omega_2 = 1 \text{ rad/sec CW}$. Plot A_c both as a function of θ_2 and separately as a function of slider position as shown in the figure.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 2.170 \cdot \text{in}$	Link 3 (A to B)	$b := 2.067 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 2.310 \cdot \text{in}$	Link 1 (O_2 to O_4)	$d := 1.000 \cdot \text{in}$
Link 5 (B to C)	$e := 5.400 \cdot \text{in}$		
Coordinate angle	$\delta := -102 \cdot \text{deg}$	Crank angle:	$\theta_{2XY} := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$
Input crank angular velocity	$\omega_2 := -1 \cdot \text{rad} \cdot \text{sec}^{-1}$		CW
Input crank angular acceleration	$\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$		

Solution: See Figure 3-33 and Mathcad file P0762.

1. Transform crank angle to the local xy coordinate system: $\theta_2(\theta_{2XY}) := \theta_{2XY} - \delta$
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_1 = 0.4608 \quad K_2 := \frac{d}{c} \quad K_2 = 0.4329$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 0.6755$$

$$A(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 - K_2 \cdot \cos(\theta_2(\theta_{2XY})) + K_3$$

$$B(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$C(\theta_{2XY}) := K_1 - (K_2 + 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{4xy}(\theta_{2XY}) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_{2XY}), -B(\theta_{2XY}) - \sqrt{B(\theta_{2XY})^2 - 4 \cdot A(\theta_{2XY}) \cdot C(\theta_{2XY})} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.4838 \quad K_5 = -0.5178$$

$$D(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 + K_4 \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

$$E(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$F(\theta_{2XY}) := K_1 + (K_4 - 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{3xy}(\theta_{2XY}) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_{2XY}), -E(\theta_{2XY}) - \sqrt{E(\theta_{2XY})^2 - 4 \cdot D(\theta_{2XY}) \cdot F(\theta_{2XY})} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3(\theta_{2XY}) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{4xy}(\theta_{2XY}) - \theta_2(\theta_{2XY}))}{\sin(\theta_{3xy}(\theta_{2XY}) - \theta_{4xy}(\theta_{2XY}))}$$

$$\omega_4(\theta_{2XY}) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2(\theta_{2XY}) - \theta_{3xy}(\theta_{2XY}))}{\sin(\theta_{4xy}(\theta_{2XY}) - \theta_{3xy}(\theta_{2XY}))}$$

7. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_{2XY}) := c \cdot \sin(\theta_{4xy}(\theta_{2XY})) \quad B(\theta_{2XY}) := b \cdot \sin(\theta_{3xy}(\theta_{2XY}))$$

$$D(\theta_{2XY}) := c \cdot \cos(\theta_{4xy}(\theta_{2XY})) \quad E(\theta_{2XY}) := b \cdot \cos(\theta_{3xy}(\theta_{2XY}))$$

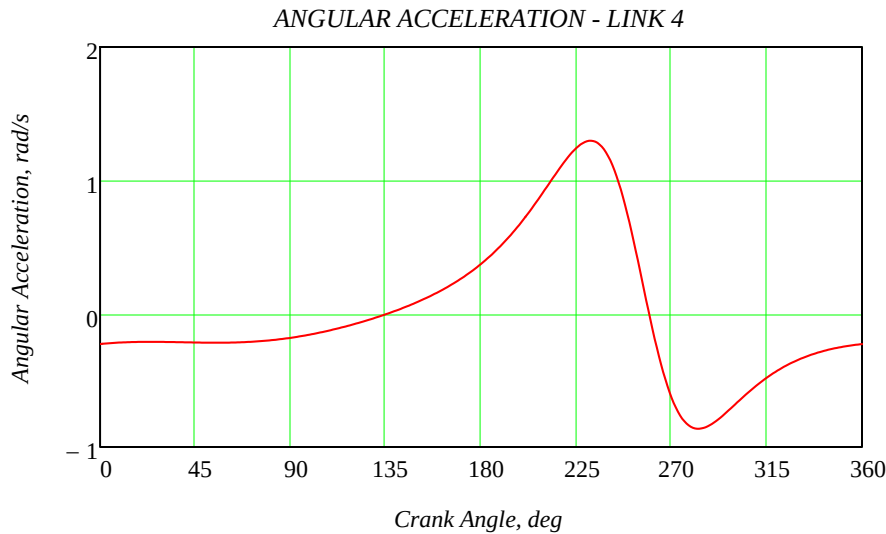
$$C(\theta_{2XY}) := a \cdot \alpha_2 \cdot \sin(\theta_2(\theta_{2XY})) + a \cdot \omega_2^2 \cdot \cos(\theta_2(\theta_{2XY})) \dots$$

$$+ b \cdot \omega_3(\theta_{2XY})^2 \cdot \cos(\theta_{3xy}(\theta_{2XY})) - c \cdot \omega_4(\theta_{2XY})^2 \cdot \cos(\theta_{4xy}(\theta_{2XY}))$$

$$F(\theta_{2XY}) := a \cdot \alpha_2 \cdot \cos(\theta_2(\theta_{2XY})) - a \cdot \omega_2^2 \cdot \sin(\theta_2(\theta_{2XY})) \dots$$

$$+ -b \cdot \omega_3(\theta_{2XY})^2 \cdot \sin(\theta_{3xy}(\theta_{2XY})) + c \cdot \omega_4(\theta_{2XY})^2 \cdot \sin(\theta_{4xy}(\theta_{2XY}))$$

$$\alpha_4(\theta_{2XY}) := \frac{C(\theta_{2XY}) \cdot E(\theta_{2XY}) - B(\theta_{2XY}) \cdot F(\theta_{2XY})}{A(\theta_{2XY}) \cdot E(\theta_{2XY}) - B(\theta_{2XY}) \cdot D(\theta_{2XY})}$$



8. Transform θ_4 back to the global XY system.

$$\theta_4(\theta_{2XY}) := \theta_{4xy}(\theta_{2XY}) + \delta$$

9. Determine θ_5 using equation 4.17. Offset: $cc := 0 \cdot \text{in}$

$$\theta_5(\theta_{2XY}) := \text{asin}\left(-\frac{c \cdot \sin(\theta_4(\theta_{2XY})) - cc}{e}\right) + \pi$$

10. Determine the angular velocity of link 5 using equation 6.22a:

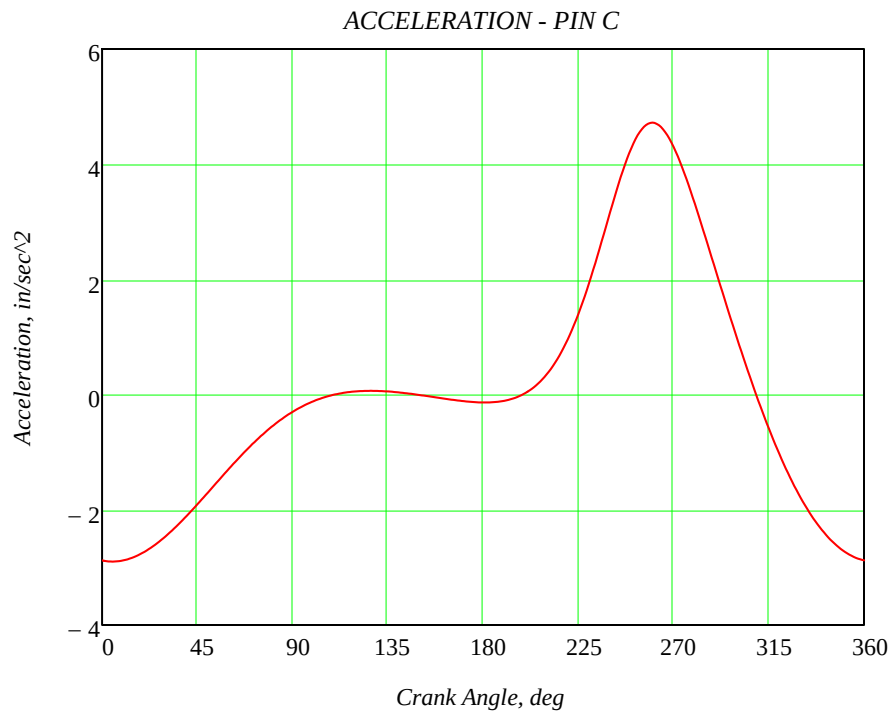
$$\omega_5(\theta_{2XY}) := \frac{c}{e} \cdot \frac{\cos(\theta_4(\theta_{2XY}))}{\cos(\theta_5(\theta_{2XY}))} \cdot \omega_4(\theta_{2XY})$$

11. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5(\theta_{2XY}) := \frac{c \cdot \alpha_4(\theta_{2XY}) \cdot \cos(\theta_4(\theta_{2XY})) - c \cdot \omega_4(\theta_{2XY})^2 \cdot \sin(\theta_4(\theta_{2XY})) \dots}{e \cdot \cos(\theta_5(\theta_{2XY}))} + \frac{e \cdot \omega_5(\theta_{2XY})^2 \cdot \sin(\theta_5(\theta_{2XY}))}{e \cdot \cos(\theta_5(\theta_{2XY}))}$$

12. Use equation 7.16e for the acceleration of pin C.

$$A_C(\theta_{2XY}) := -c \cdot \alpha_4(\theta_{2XY}) \cdot \sin(\theta_4(\theta_{2XY})) - c \cdot \omega_4(\theta_{2XY})^2 \cdot \cos(\theta_4(\theta_{2XY})) \dots + e \cdot \alpha_5(\theta_{2XY}) \cdot \sin(\theta_5(\theta_{2XY})) + e \cdot \omega_5(\theta_{2XY})^2 \cdot \cos(\theta_5(\theta_{2XY}))$$



PROBLEM 7-63a

Statement: Find the angular acceleration of link 6 in Figure 3-34b for $\theta_6 = 90^\circ$ with respect to the global X axis assuming constant $\omega_2 = 10 \text{ rad/sec CW}$. Use a graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot \text{in}$	Link 3 (A to B)	$f := 4.248 \cdot \text{in}$
Link 4 (O_4 to C)	$c := 2.125 \cdot \text{in}$	Link 5 (C to D)	$b := 2.158 \cdot \text{in}$
Link 6 (O_6 to D)	$a := 1.542 \cdot \text{in}$	Link 5 (B to D)	$p := 3.274 \cdot \text{in}$
Link 1 X-offset	$d_X := 3.259 \cdot \text{in}$	Link 1 Y-offset	$d_Y := -2.905 \cdot \text{in}$

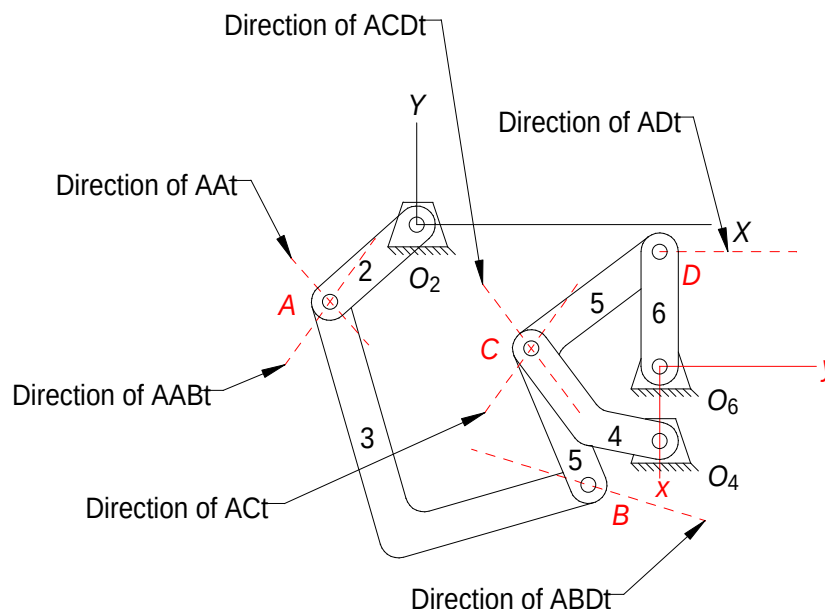
Angle CDB $\delta_5 := 36.0 \cdot \text{deg}$

Output rocker angle: $\theta_6 := 90 \cdot \text{deg}$ Global XY system

Input crank angular velocity $\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

Solution: See Figure 3-34b and Mathcad file P0763a.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. Since this linkage is a Stephenson's II sixbar, we will have to start at link 6 and work back to link 2. We will assume a value for α_6 and eventually find a value for α_2 . We will then multiply the magnitudes of all velocities by the ratio of the actual α_2 to the found α_2 . Use Problem 6-73 for the link angular velocities.

Assume: $\alpha_6 := 100 \cdot \text{rad} \cdot \text{sec}^{-2}$ CCW

From Problem 6-73 and the graphical layout (angles in the global XY coordinate system):

$\omega_6 := 24.124 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW	$\theta_6 := 90.000 \cdot \text{deg}$	
$\omega_5 := 14.64 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW	$\theta_{52} := 217.003 \cdot \text{deg}$	$\theta_{51} := 253.003 \cdot \text{deg}$
$\omega_4 := 14.64 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW	$\theta_4 := 144.195 \cdot \text{deg}$	
$\omega_3 := 3.016 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW	$\theta_3 := 144.628 \cdot \text{deg}$	

$$\omega_2 := 10.000 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$$

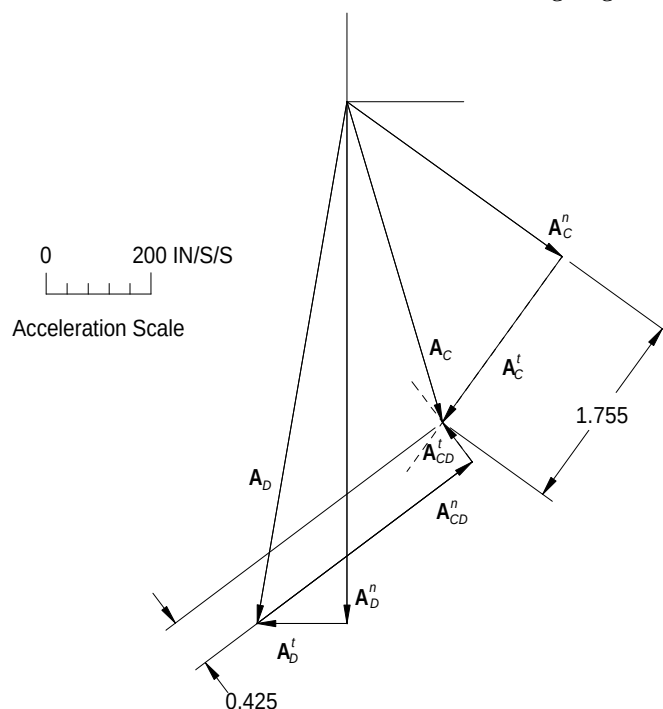
$$\theta_2 := 221.690 \cdot \text{deg}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

4. For point C, this becomes: $(\mathbf{A}_C^t + \mathbf{A}_C^n) = (\mathbf{A}_D^t + \mathbf{A}_D^n) + (\mathbf{A}_{CD}^t + \mathbf{A}_{CD}^n)$, where

$$\begin{aligned} A_{Cn} &:= c \cdot \omega_4^2 & A_{Cn} &= 455.450 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{ACn} &:= \theta_4 + 180 \cdot \text{deg} & \theta_{ACn} &= 324.195 \text{ deg} \\ A_{Dn} &:= a \cdot \omega_6^2 & A_{Dn} &= 897.394 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{ADn} &:= \theta_6 + 180 \cdot \text{deg} & \theta_{ADn} &= 270.000 \text{ deg} \\ A_{Dt} &:= a \cdot \alpha_6 & A_{Dt} &= 154.200 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{ADt} &:= \theta_6 + 90 \cdot \text{deg} & \theta_{ADt} &= 180.000 \text{ deg} \\ A_{CDn} &:= b \cdot \omega_5^2 & A_{CDn} &= 462.523 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{ACDn} &:= \theta_{52} - 180 \cdot \text{deg} & \theta_{ACDn} &= 37.003 \text{ deg} \end{aligned}$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_C^n$ at an angle of θ_{ACn} and $\mathbf{A}_D^n$ at an angle of θ_{ADn} . From the tip of $\mathbf{A}_D^n$, draw $\mathbf{A}_D^t$ at an angle of $\theta_{ADt} + \alpha$. From the tip of $\mathbf{A}_D^t$, draw $\mathbf{A}_{CD}^n$ at an angle of θ_{CDn} . Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_C^n$ and $\mathbf{A}_{CD}^n$, draw construction lines in the directions of $\mathbf{A}_C^t$ and $\mathbf{A}_{CD}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_C^t$, $\mathbf{A}_C^t$, and $\mathbf{A}_{CD}^t$.



6. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 200 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_{Ct} := 1.755 \cdot k_a \quad A_{Ct} = 351.00 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_4 := \frac{A_{Ct}}{c} \quad \alpha_4 = 165.176 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CW}$$

$$A_{CDt} := 0.425 \cdot k_a \quad A_{CDt} = 85.00 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_5 := \frac{A_{CDt}}{b} \quad \alpha_5 = 39.388 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CW}$$

7. For point B, equation 7.4 becomes: $\mathbf{A}_B = \mathbf{A}_D + (\mathbf{A}_{BD}^t + \mathbf{A}_{BD}^n)$, where

$$A_{BDn} := p \cdot \omega_5^2 \quad A_{BDn} = 701.715 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABDn} := \theta_{51} - 180 \cdot \text{deg} \quad \theta_{ABDn} = 73.003 \text{ deg}$$

$$A_{BDt} := p \cdot \alpha_5 \quad A_{BDt} = 128.957 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABDt} := \theta_{ABDn} + 90 \cdot \text{deg} \quad \theta_{ABDt} = 163.003 \text{ deg}$$

8. For point A, equation 7.4 becomes:

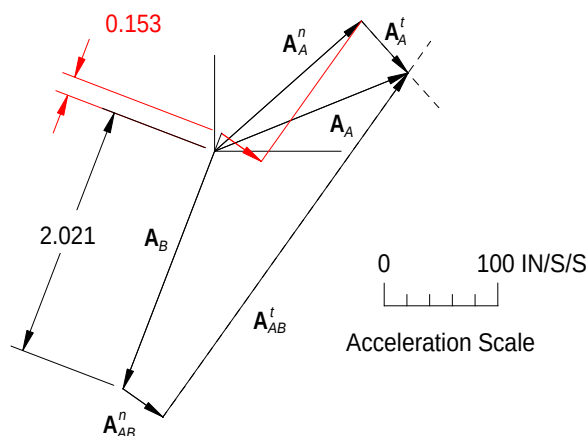
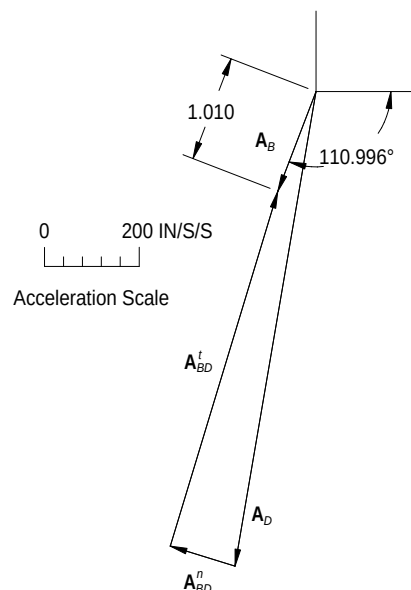
$$(\mathbf{A}_A^t + \mathbf{A}_A^n) = \mathbf{A}_B + (\mathbf{A}_{AB}^t + \mathbf{A}_{AB}^n), \text{ where}$$

$$A_{An} := g \cdot \omega_2^2 \quad A_{An} = 155.600 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AAn} := \theta_2 - 180 \cdot \text{deg} \quad \theta_{AAn} = 41.690 \text{ deg}$$

$$A_{ABn} := f \cdot \omega_3^2 \quad A_{ABn} = 38.641 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AABn} := \theta_3 - 180 \cdot \text{deg} \quad \theta_{AABn} = -35.372 \text{ deg}$$



9. The requirement for this problem is that $\alpha_2 = 0$ (constant ω_2). For this to be so $\mathbf{A}_A^t$ must be zero. To do this the magnitude of $\mathbf{A}_B$ must be smaller as shown above.

$$\text{Correction ratio: } r := \frac{-0.153}{2.021} \quad r = -0.076$$

$$\text{Corrected value of } \alpha_6: \quad \alpha_6 := r \cdot \alpha_6 \quad \alpha_6 = -7.571 \frac{\text{rad}}{\text{sec}^2}$$

PROBLEM 7-63b

Statement: Find the angular acceleration of link 6 in Figure 3-34b for $\theta_6 = 90$ deg with respect to the global X axis assuming constant $\omega_2 = 10$ rad/sec CW. Use an analytic method.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot \text{in}$	Link 3 (A to B)	$f := 4.248 \cdot \text{in}$
Link 4 (O_4 to C)	$c := 2.125 \cdot \text{in}$	Link 5 (C to D)	$b := 2.158 \cdot \text{in}$
Link 6 (O_6 to D)	$a := 1.542 \cdot \text{in}$	Link 5 (B to D)	$p := 3.274 \cdot \text{in}$
Link 1 X -offset	$d_X := 3.259 \cdot \text{in}$	Link 1 Y -offset	$d_Y := -2.905 \cdot \text{in}$
Angle CDB	$\delta_5 := 36.0 \cdot \text{deg}$		
Output rocker angle:	$\theta_6 := 90 \cdot \text{deg}$	Global XY system	
Input crank angular velocity	$\omega_2 := -10 \cdot \text{rad} \cdot \text{sec}^{-1}$	CW	
Input crank angular acceleration	$\alpha_2 := 0.0 \cdot \text{rad} \cdot \text{sec}^{-2}$		

Solution: See Figure P6-34b and Mathcad file P0763b.

1. Since this linkage is a Stephenson's II sixbar an iterative solution must be used. This problem is suited to a longer-term project rather than a daily homework problem.

PROBLEM 7-64

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular acceleration of link 6 in Figure 3-34b as a function of θ_2 for a constant $\omega_2 = 10$ rad/sec CW.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot in$	Link 3 (A to B)	$f := 4.248 \cdot in$
Link 4 (O_4 to C)	$c := 2.125 \cdot in$	Link 5 (C to D)	$b := 2.158 \cdot in$
Link 6 (O_6 to D)	$a := 1.542 \cdot in$	Link 5 (B to D)	$p := 3.274 \cdot in$
Link 1 X-offset	$d_X := 3.259 \cdot in$	Link 1 Y-offset	$d_Y := -2.905 \cdot in$
Angle CDB	$\delta_5 := 36.0 \cdot deg$		
Output rocker angle:	$\theta_6 := 90 \cdot deg$	Global XY system	
Input crank angular velocity	$\omega_2 := -10 \cdot rad \cdot sec^{-1}$		CW
Input crank angular acceleration	$\alpha_2 := 0.0 \cdot rad \cdot sec^{-2}$		

Solution: See Figure P6-34b and Mathcad file P0764.

1. Since this linkage is a Stephenson's II sixbar an iterative solution must be used. This problem is suited to a longer-term project rather than a daily homework problem.

PROBLEM 7-65a

Statement: Use a compass and straightedge to draw the linkage in Figure 3-35 with link 2 at 90 deg and find the angular acceleration of link 6 assuming a constant $\omega_2 = 10 \text{ rad/sec}$ CCW. Use a graphical method.

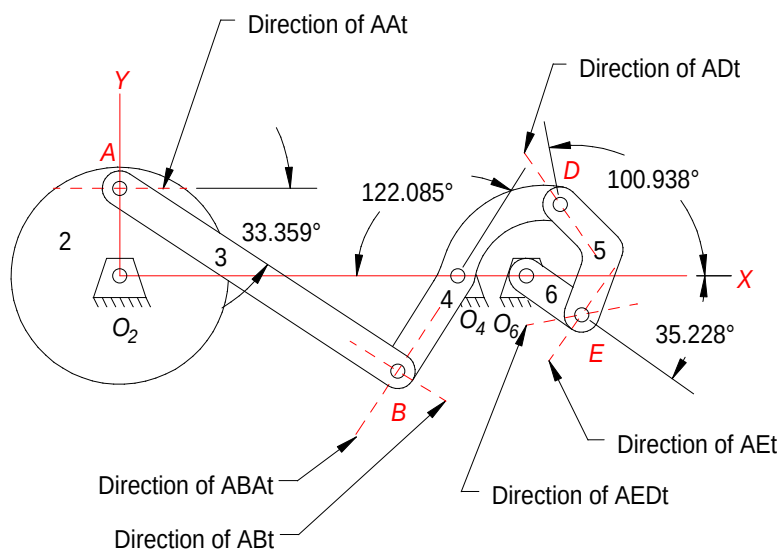
Given:

Link lengths:

Link 2 (O_2 to A)	$a := 1.000 \cdot \text{in}$	Link 3 (A to B)	$b := 3.800 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 1.286 \cdot \text{in}$	Link 1 (O_2 to O_4)	$d := 3.857 \cdot \text{in}$
Link 4 (O_4 to D)	$e := 1.429 \cdot \text{in}$	Link 5 (D to E)	$f := 1.286 \cdot \text{in}$
Link 6 (O_6 to E)	$g := 0.771 \cdot \text{in}$	Link 1 (O_4 to O_6)	$h := 0.786 \cdot \text{in}$
Crank angle:	$\theta_2 := 90 \cdot \text{deg}$	Angle BO_4D	$\delta_4 := 157 \cdot \text{deg}$
Input crank angular velocity	$\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$		CCW
Input crank angular acceleration	$\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$		

Solution: See Figure 3-35 and Mathcad file P0765a.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A and B, we will need θ_3 , ω_3 , and θ_4 . From the graphical position solution above,

$$\theta_3 := -33.359 \cdot \text{deg}$$

$$\theta_3 = -33.359 \text{ deg}$$

$$\theta_{41} := -122.085 \cdot \text{deg}$$

$$\theta_{41} = -122.085 \text{ deg}$$

$$\theta_{42} := -122.085 \cdot \text{deg} + \delta_4$$

$$\theta_{42} = 34.915 \text{ deg}$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_{41} - \theta_2)}{b \cdot \sin(\theta_3 - \theta_{41})}$$

$$\omega_3 = 1.398 \text{ rad} \cdot \text{sec}^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_{41} - \theta_3)}$$

$$\omega_4 = -6.496 \text{ rad} \cdot \text{sec}^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$
4. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_{Bn} := c \cdot \omega_4^2 \quad A_{Bn} = 54.275 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_{41} + 180 \cdot \text{deg} \quad \theta_{ABn} = 57.915 \text{ deg}$$

$$A_{An} := a \cdot \omega_2^2 \quad A_{An} = 100.000 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AAn} = 270.000 \text{ deg}$$

$$A_{At} := |a \cdot \alpha_2| \quad A_{At} = 0.000 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AA_t} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{AA_t} = 0.000 \text{ deg}$$

$$A_{BA_n} := b \cdot \omega_3^2 \quad A_{BA_n} = 7.429 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABA_n} := \theta_3 + 180 \cdot \text{deg} \quad \theta_{ABA_n} = 146.641 \text{ deg}$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of θ_{ABn} and $\mathbf{A}_A^n$ at an angle of θ_{AAn} . From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_A^t$ at an angle of $\theta_{AA_t} + \alpha$. From the tip of $\mathbf{A}_A^t$, draw $\mathbf{A}_{BA}^n$ at an angle of θ_{ABA_n} . Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B^n$ and $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B$, $\mathbf{A}_B^t$, and $\mathbf{A}_{BA}^t$.
6. From the graphical solution at right,

$$\text{Acceleration scale factor} \quad k_a := 25 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_B := 2.919 \cdot k_a \quad A_B = 73.0 \text{ in} \cdot \text{sec}^{-2}$$

at an angle of 15.969 deg

$$A_{Bt} := 1.951 \cdot k_a \quad A_{Bt} = 48.8 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_4 := \frac{A_{Bt}}{c} \quad \alpha_4 = 37.928 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CCW}$$

6. From the graphical layout,

$$\theta_5 := 100.938 \cdot \text{deg} - 180 \cdot \text{deg} \quad \theta_5 = -79.062 \text{ deg}$$

$$\theta_6 := -35.228 \cdot \text{deg} \quad \theta_6 = -35.228 \text{ deg}$$

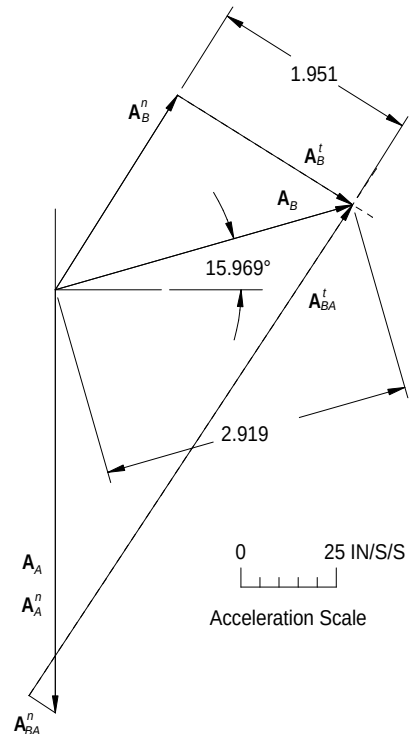
Using equation (6.18),

$$\omega_5 := \frac{e \cdot \omega_4}{f} \cdot \frac{\sin(\theta_6 - \theta_{42})}{\sin(\theta_5 - \theta_6)}$$

$$\omega_6 := \frac{e \cdot \omega_4}{g} \cdot \frac{\sin(\theta_{42} - \theta_5)}{\sin(\theta_6 - \theta_5)}$$

$$\omega_5 = -9.804 \text{ rad} \cdot \text{sec}^{-1}$$

$$\omega_6 = -15.885 \text{ rad} \cdot \text{sec}^{-1}$$



7. For point E, this becomes: $(\mathbf{A}_E^t + \mathbf{A}_E^n) = (\mathbf{A}_D^t + \mathbf{A}_D^n) + (\mathbf{A}_{ED}^t + \mathbf{A}_{ED}^n)$, where

$$A_{En} := g \cdot \omega_6^2 \quad A_{En} = 194.560 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AEn} := \theta_6 + 180 \cdot \text{deg} \quad \theta_{AEn} = 144.772 \text{ deg}$$

$$A_{Dn} := e \cdot \omega_4^2 \quad A_{Dn} = 60.310 \text{ in} \cdot \text{sec}^{-2}$$

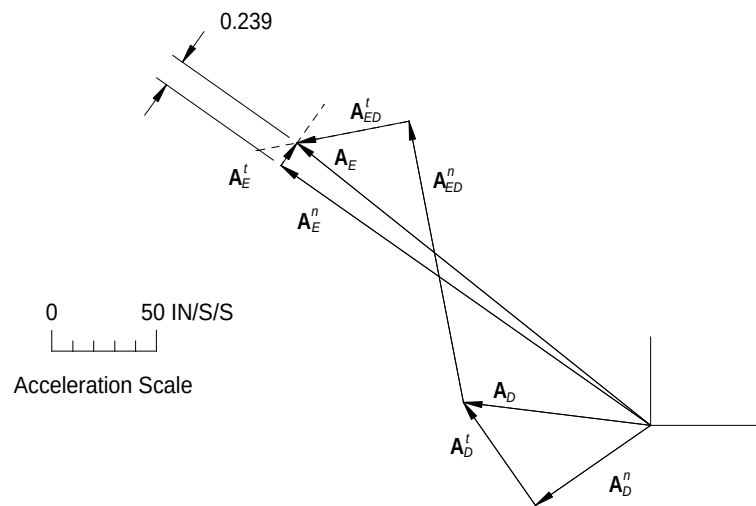
$$\theta_{ADn} := \theta_{42} + 180 \cdot \text{deg} \quad \theta_{ADn} = 214.915 \text{ deg}$$

$$A_{Dt} := e \cdot \alpha_4 \quad A_{Dt} = 54.199 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ADt} := \theta_{42} + 90 \cdot \text{deg} \quad \theta_{ADt} = 124.915 \text{ deg}$$

$$A_{EDn} := f \cdot \omega_5^2 \quad A_{EDn} = 123.597 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AEDn} := \theta_5 + 180 \cdot \text{deg} \quad \theta_{AEDn} = 100.938 \text{ deg}$$



8. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 50 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_{Et} := 0.239 \cdot k_a \quad A_{Et} = 11.950 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_6 := \frac{A_{Et}}{g} \quad \alpha_6 = 15.499 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CCW}$$

PROBLEM 7-65b

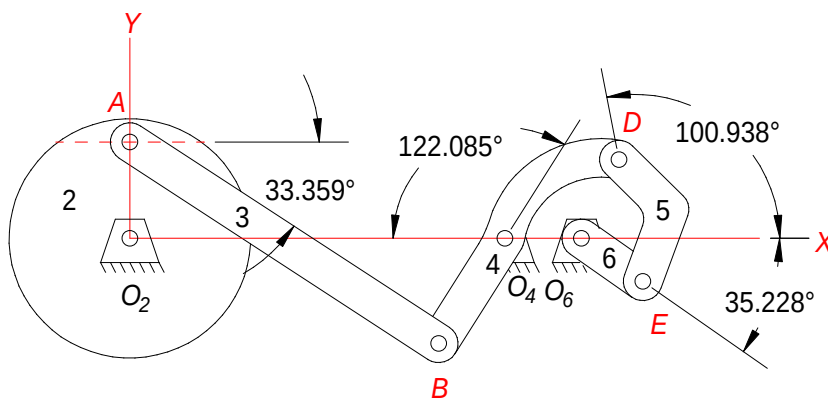
Statement: Use a compass and straightedge to draw the linkage in Figure 3-35 with link 2 at 90 deg and find the angular acceleration of link 6 assuming a constant $\omega_2 = 10 \text{ rad/sec}$ CCW. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 1.000 \cdot \text{in}$	Link 3 (A to B)	$b := 3.800 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 1.286 \cdot \text{in}$	Link 1 (O_2 to O_4)	$d := 3.857 \cdot \text{in}$
Link 4 (O_4 to D)	$e := 1.429 \cdot \text{in}$	Link 5 (D to E)	$f := 1.286 \cdot \text{in}$
Link 6 (O_6 to E)	$g := 0.771 \cdot \text{in}$	Link 1 (O_4 to O_6)	$h := 0.786 \cdot \text{in}$
Crank angle:	$\theta_2 := 90 \cdot \text{deg}$	Angle BO_4D	$\delta_4 := 157 \cdot \text{deg}$
Input crank angular velocity	$\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$	CCW	
Input crank angular acceleration	$\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$		

Solution: See Figure P6-35 and Mathcad file P0765b.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 3.8570 \quad K_2 = 2.9992$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.2015$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -2.6555 \quad B = -2.0000 \quad C = 5.0585$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{41} := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_{41} = 237.915 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0150 \quad K_5 = -3.7714$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -7.6284$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -2.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.0856$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_3 = 326.641 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_{41} - \theta_2)}{b \cdot \sin(\theta_3 - \theta_{41})} \quad \omega_3 = 1.398 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_{41} - \theta_3)} \quad \omega_4 = -6.496 \frac{\text{rad}}{\text{sec}}$$

7. Link 4 will be the input to the second fourbar (links 4, 5, 6, and 1). Let the angle that link 4 makes in the second fourbar be

$$\theta_{42} := \theta_{41} + 157 \cdot \text{deg} - 360 \cdot \text{deg} \quad \theta_{42} = 34.915 \text{ deg}$$

8. Determine the values of the constants needed for finding θ_6 from equations 4.8a and 4.10a.

$$K_1 := \frac{h}{e} \quad K_2 := \frac{h}{g}$$

$$K_1 = 0.5500 \quad K_2 = 1.0195$$

$$K_3 := \frac{e^2 - f^2 + g^2 + h^2}{2 \cdot e \cdot g} \quad K_3 = 0.7263$$

$$A := \cos(\theta_{42}) - K_1 - K_2 \cdot \cos(\theta_{42}) + K_3$$

$$B := -2 \cdot \sin(\theta_{42})$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_{42}) + K_3$$

$$A = 0.1603 \quad B = -1.1447 \quad C = -0.3796$$

9. Use equation 4.10b to find values of θ_6 for the open circuit.

$$\theta_6 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_6 = -35.228 \text{ deg}$$

10. Determine the values of the constants needed for finding θ_5 from equations 4.11b and 4.12.

$$K_4 := \frac{h}{f} \quad K_5 := \frac{g^2 - h^2 - e^2 - f^2}{2 \cdot e \cdot f} \quad K_4 = 0.6112$$

$$K_5 = -1.0119$$

$$D := \cos(\theta_{42}) - K_1 + K_4 \cdot \cos(\theta_{42}) + K_5 \quad D = -0.2408$$

$$E := -2 \cdot \sin(\theta_{42}) \quad E = -1.1447$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_{42}) + K_5 \quad F = -0.7807$$

11. Use equation 4.13 to find values of θ_5 for the open circuit.

$$\theta_5 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_5 = 280.938 \text{ deg}$$

12. Determine the angular velocity of links 5 and 6 for the open circuit using equations 6.18.

$$\omega_5 := \left(\frac{e \cdot \omega_4}{f} \right) \cdot \frac{\sin(\theta_6 - \theta_{42})}{\sin(\theta_5 - \theta_6)} \quad \omega_5 = -9.804 \frac{\text{rad}}{\text{sec}}$$

$$\omega_6 := \left(\frac{e \cdot \omega_4}{g} \right) \cdot \frac{\sin(\theta_{42} - \theta_5)}{\sin(\theta_6 - \theta_5)} \quad \omega_6 = -15.885 \frac{\text{rad}}{\text{sec}}$$

13. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c \cdot \sin(\theta_{41}) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_{41}) \quad E := b \cdot \cos(\theta_3)$$

$$A = -1.090 \text{ in} \quad B = -2.090 \text{ in} \quad D = -0.683 \text{ in} \quad E = 3.174 \text{ in}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_{41})$$

$$C = 35.034 \frac{\text{in}}{\text{sec}^2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_{41})$$

$$F = -141.900 \frac{\text{in}}{\text{sec}^2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = 36.545 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = 37.931 \frac{\text{rad}}{\text{sec}^2}$$

14. Use equations 7.12 to determine the angular accelerations of links 5 and 6 for the open circuit. Use link 4 as the input to the second fourbar stage.

$$A := g \cdot \sin(\theta_6) \quad B := f \cdot \sin(\theta_5) \quad D := g \cdot \cos(\theta_6) \quad E := f \cdot \cos(\theta_5)$$

$$A = -0.037 \text{ ft} \quad B = -0.105 \text{ ft} \quad D = 0.052 \text{ ft} \quad E = 0.020 \text{ ft}$$

$$C := e \cdot \alpha_4 \cdot \sin(\theta_{42}) + e \cdot \omega_4^2 \cdot \cos(\theta_{42}) + f \cdot \omega_5^2 \cdot \cos(\theta_5) - g \cdot \omega_6^2 \cdot \cos(\theta_6)$$

$$C = -4.583 \frac{\text{ft}}{\text{s}^2}$$

$$F := e \cdot \alpha_4 \cdot \cos(\theta_{42}) - e \cdot \omega_4^2 \cdot \sin(\theta_{42}) - f \cdot \omega_5^2 \cdot \sin(\theta_5) + g \cdot \omega_6^2 \cdot \sin(\theta_6)$$

$$F = 1.588 \frac{\text{ft}}{\text{s}^2}$$

$$\alpha_5 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_5 = -38.104 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_6 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_6 = 15.486 \frac{\text{rad}}{\text{sec}^2}$$

PROBLEM 7-66

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular acceleration of link 6 in the sixbar linkage of Figure 3-35 as a function of θ_2 for a constant $\omega_2 = 1 \text{ rad/sec CCW}$.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 1.000 \cdot \text{in}$	Link 3 (A to B)	$b := 3.800 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 1.286 \cdot \text{in}$	Link 1 (O_2 to O_4)	$d := 3.857 \cdot \text{in}$
Link 4 (O_4 to D)	$e := 1.429 \cdot \text{in}$	Link 5 (D to E)	$f := 1.286 \cdot \text{in}$
Link 6 (O_6 to E)	$g := 0.771 \cdot \text{in}$	Link 1 (O_4 to O_6)	$h := 0.786 \cdot \text{in}$
Input crank angular velocity	$\omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$	CCW	$\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure 3-35 and Mathcad file P0766.

- Define the range of the input angle: $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$
- Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 3.8570 \quad K_2 = 2.9992$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.2015$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

- Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

- Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.0150 \quad K_5 = -3.7714$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

- Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

- Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

7. Link 4 will be the input to the second fourbar (links 4, 5, 6, and 1). Let the angle that link 4 makes in the second fourbar be

$$\theta_{42}(\theta_2) := \theta_4(\theta_2) + 157 \cdot \text{deg} - 360 \cdot \text{deg}$$

8. Determine the values of the constants needed for finding θ_6 from equations 4.8a and 4.10a.

$$\begin{aligned} K_1 &:= \frac{h}{e} & K_2 &:= \frac{h}{g} \\ K_1 &= 0.5500 & K_2 &= 1.0195 \\ K_3 &:= \frac{e^2 - f^2 + g^2 + h^2}{2 \cdot e \cdot g} & K_3 &= 0.7263 \end{aligned}$$

$$A'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K_1 - K_2 \cdot \cos(\theta_{42}(\theta_2)) + K_3$$

$$B'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

$$C'(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_{42}(\theta_2)) + K_3$$

9. Use equation 4.10b to find values of θ_6 for the open circuit.

$$\theta_6(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A'(\theta_2), -B'(\theta_2) - \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)} \right) \right)$$

10. Determine the values of the constants needed for finding θ_5 from equations 4.11b and 4.12.

$$\begin{aligned} K_4 &:= \frac{h}{f} & K_5 &:= \frac{g^2 - h^2 - e^2 - f^2}{2 \cdot e \cdot f} & K_4 &= 0.6112 & K_5 &= -1.0119 \end{aligned}$$

$$D'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K_1 + K_4 \cdot \cos(\theta_{42}(\theta_2)) + K_5$$

$$E'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

$$F'(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_{42}(\theta_2)) + K_5$$

11. Use equation 4.13 to find values of θ_5 for the open circuit.

$$\theta_5(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D'(\theta_2), -E'(\theta_2) - \sqrt{E'(\theta_2)^2 - 4 \cdot D'(\theta_2) \cdot F'(\theta_2)} \right) \right)$$

12. Determine the angular velocity of link 6 for the open circuit using equations 6.18.

$$\omega_5(\theta_2) := \left(\frac{e \cdot \omega_4(\theta_2)}{f} \right) \cdot \frac{\sin(\theta_6(\theta_2) - \theta_{42}(\theta_2))}{\sin(\theta_5(\theta_2) - \theta_6(\theta_2))}$$

$$\omega_6(\theta_2) := \left(\frac{e \cdot \omega_4(\theta_2)}{g} \right) \cdot \frac{\sin(\theta_{42}(\theta_2) - \theta_5(\theta_2))}{\sin(\theta_6(\theta_2) - \theta_5(\theta_2))}$$

13. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$\begin{aligned} C(\theta_2) &:= a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) \dots \\ &\quad + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2)) \end{aligned}$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) \dots$$

$$+ -b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

14. Use equations 7.12 to determine the angular accelerations of links 5 and 6 for the open circuit. Use link 4 as the input to the second fourbar stage.

$$A'(\theta_2) := g \cdot \sin(\theta_6(\theta_2)) \quad B'(\theta_2) := f \cdot \sin(\theta_5(\theta_2))$$

$$D'(\theta_2) := g \cdot \cos(\theta_6(\theta_2)) \quad E'(\theta_2) := f \cdot \cos(\theta_5(\theta_2))$$

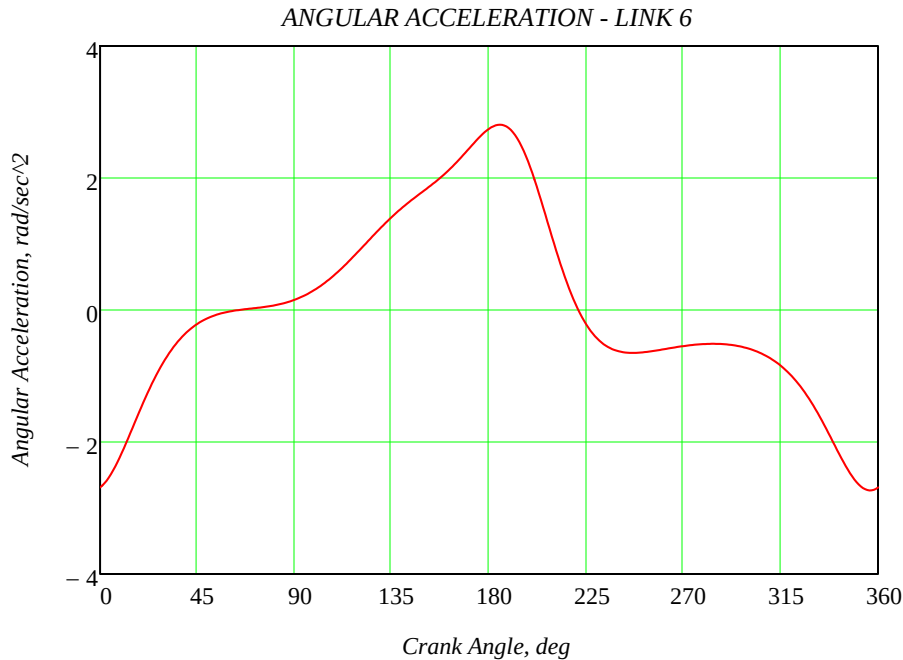
$$C'(\theta_2) := e \cdot \alpha_4(\theta_2) \cdot \sin(\theta_{42}(\theta_2)) + e \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_{42}(\theta_2)) \dots$$

$$+ f \cdot \omega_5(\theta_2)^2 \cdot \cos(\theta_5(\theta_2)) - g \cdot \omega_6(\theta_2)^2 \cdot \cos(\theta_6(\theta_2))$$

$$F'(\theta_2) := e \cdot \alpha_4(\theta_2) \cdot \cos(\theta_{42}(\theta_2)) - e \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_{42}(\theta_2)) \dots$$

$$+ -f \cdot \omega_5(\theta_2)^2 \cdot \sin(\theta_5(\theta_2)) + g \cdot \omega_6(\theta_2)^2 \cdot \sin(\theta_6(\theta_2))$$

$$\alpha_6(\theta_2) := \frac{C'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$



PROBLEM 7-67

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular acceleration of link 8 in the linkage of Figure 3-36 as a function of θ_2 for a constant $\omega_2 = 1 \text{ rad/sec CCW}$.

Given: Link lengths:

Input crank (L_2)	$a := 0.450$	First coupler (L_3)	$b := 0.990$
Common rocker (O_4B)	$c := 0.590$	First ground link (O_2O_4)	$d := 1.000$
Common rocker (O_4C)	$a' := 0.590$	Second coupler (CD)	$b' := 0.325$
Output rocker (L_6)	$c' := 0.325$	Second ground link (O_4O_6)	$d' := 0.419$
Link 7 (L_7)	$e := 0.938$	Link 8 (L_8)	$f := 0.572$
Link 5 extension (DE)	$p := 0.823$	Angle DCE	$\delta := 7.0 \cdot \text{deg}$
Angle BO_4C	$\alpha := 128.6 \cdot \text{deg}$		

Input crank angular velocity $\omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW

Solution: See Figure 3-36 and Mathcad file P0767.

1. See problem 4-43 for the position solution. The velocity and acceleration solutions will use the same vector loop equations for links 5, 6, 7, and 8, differentiated with respect to time. This problem is suitable for a project assignment and is probably too long for an overnight assignment.

PROBLEM 7-68

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the magnitude and direction of the acceleration of point P in Figure 3-37a as a function of θ_2 for a constant $\omega_2 = 1$ rad/sec CCW. Also calculate and plot the acceleration of point P versus point A .

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 0.136 \quad \text{Link 3 (A to B)} \quad b := 1.000$$

$$\text{Link 4 (B to O}_4) \quad c := 1.000 \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 1.414$$

$$\text{Coupler point:} \quad R_{pa} := 2.000 \quad \delta_3 := 0 \cdot \text{deg}$$

$$\text{Crank speed:} \quad \omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure 3-37a and Mathcad file P0768.

1. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 10.3971 \quad K_2 = 1.4140$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 7.4187$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.4140 \quad K_5 = -7.4187$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2))$$

$$B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2))$$

$$E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)} \quad \alpha_4(\theta_2) := \frac{C'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

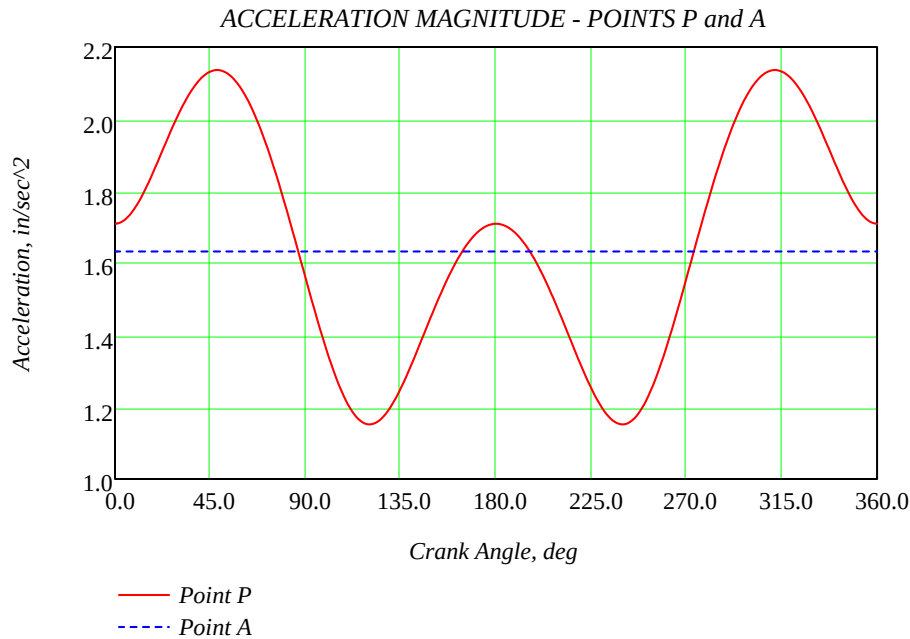
9. Use equations 7.32 to find the acceleration of the point P.

$$\begin{aligned} \mathbf{A}_{PA}(\theta_2) := & R_{pa} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots \\ & + R_{pa} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3)) \end{aligned}$$

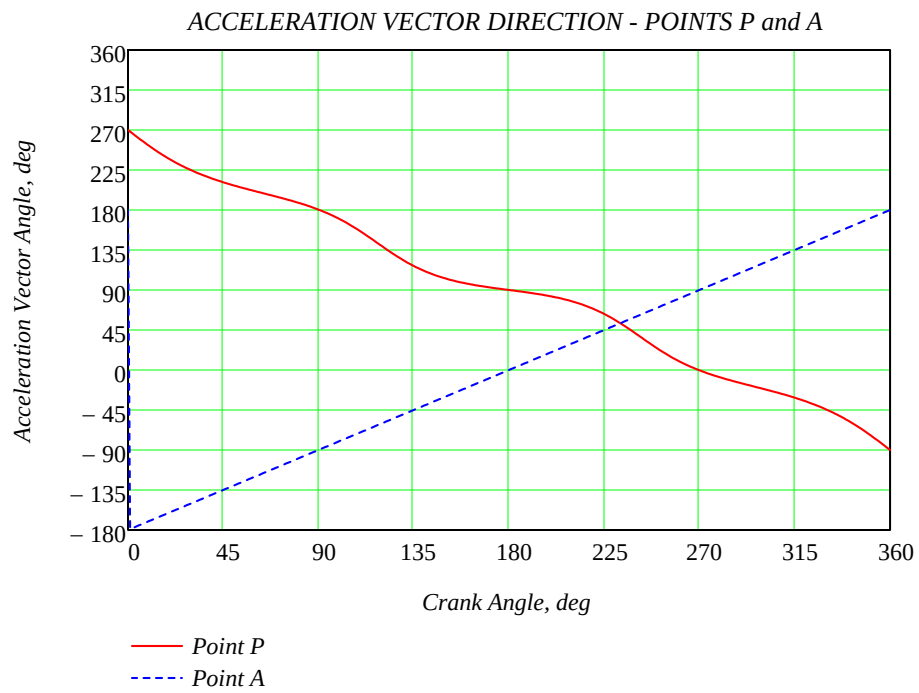
$$\mathbf{A}_P(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2)$$

$$A_P(\theta_2) := |\mathbf{A}_P(\theta_2)| \quad \theta_{Ap}(\theta_2) := \arg(\mathbf{A}_P(\theta_2)) \quad A_A(\theta_2) := |\mathbf{A}_A(\theta_2)|$$

$$\theta_{Ap}(\theta_2) := \text{if}(\theta_2 < 91 \cdot \text{deg}, \theta_{Ap}(\theta_2) + 2 \cdot \pi, \theta_{Ap}(\theta_2))$$



$$\theta_{AA}(\theta_2) := \arg(\mathbf{A}_A(\theta_2))$$



PROBLEM 7-69

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the magnitude and the direction of the acceleration of point P in Figure 3-37b as a function of θ_2 . Also calculate and plot the velocity of point P versus point A .

Given: Link lengths:

Input crank (L_2)	$a := 0.50$	First coupler (AB)	$b := 1.00$
Rocker 4 (O_4B)	$c := 1.00$	Rocker 5 (L_5)	$c' := 1.00$
Ground link (O_2O_4)	$d := 0.75$	Second coupler 6 (CD)	$b' := 1.00$
Coupler point (DP)	$p := 1.00$	Distance to O_P (O_2O_P)	$d' := 1.50$
Crank speed:	$\omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$	$\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$	

Solution: See Figure 3-37b and Mathcad file P07-69.

- Links 4, 5, BC , and CD form a parallelogram whose opposite sides remain parallel throughout the motion of the fourbar 1, 2, AB , 4. Define a position vector whose tail is at point D and whose tip is at point P and another whose tail is at O_4 and whose tip is at point D . Then, since $\mathbf{R}_5 = \mathbf{R}_{AB}$ and $\mathbf{R}_{DP} = -\mathbf{R}_4$, the position vector from O_2 to P is $\mathbf{P} = \mathbf{R}_1 + \mathbf{R}_{AB} - \mathbf{R}_4$. Separating this vector equation into real and imaginary parts gives the equations for the X and Y coordinates of the coupler point P .

$$X_P = d + b \cdot \cos(\theta_3) - c \cdot \cos(\theta_4) \quad Y_P = b \cdot \sin(\theta_3) - c \cdot \sin(\theta_4)$$

- Define one revolution of the input crank: $\theta_2 := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. 360 \cdot \text{deg}$
- Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 1.5000 \quad K_2 = 0.7500 \quad K_3 = 0.8125$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right) - 2 \cdot \pi$$

- Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.7500 \quad K_5 = -0.8125$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

- Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right) - 2 \cdot \pi$$

- Define a local xy coordinate system with origin at O_P and with the positive x axis to the right. The coordinates of P are transformed to $x_P = X_P - d'$, $y_P = Y_P$.

$$x_P(\theta_2) := d + b \cdot \cos(\theta_3(\theta_2)) - c \cdot \cos(\theta_4(\theta_2)) - d'$$

$$y_P(\theta_2) := b \cdot \sin(\theta_3(\theta_2)) - c \cdot \sin(\theta_4(\theta_2))$$

7. Use equations 6.18 to calculate ω_3 and ω_4 .

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)} \quad \alpha_4(\theta_2) := \frac{C'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

9. Differentiate the position equations twice with respect to time to get the acceleration components.

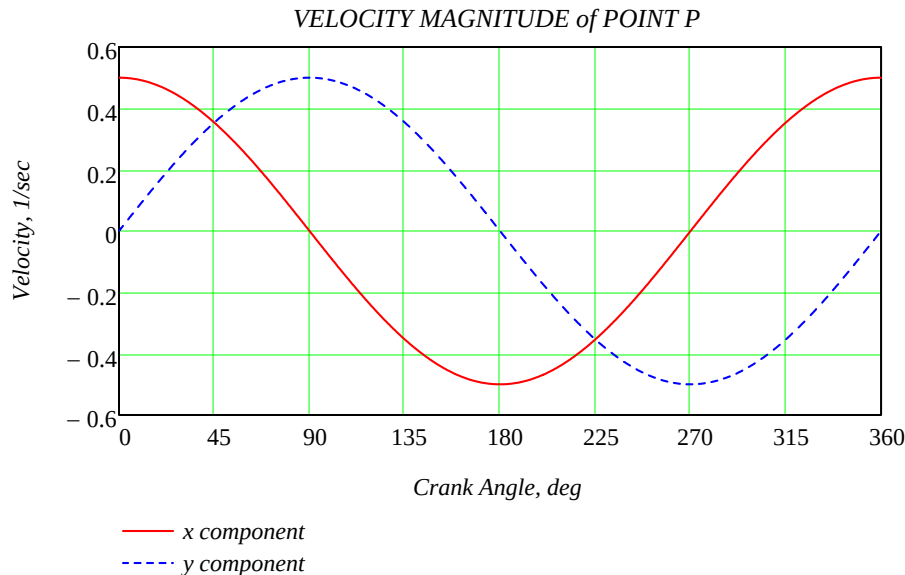
$$A_{Px}(\theta_2) := -b \cdot \left(\alpha_3(\theta_2) \cdot \sin(\theta_3(\theta_2)) + \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) \right) \dots$$

$$+ c \cdot \left(\alpha_4(\theta_2) \cdot \sin(\theta_4(\theta_2)) + \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2)) \right)$$

$$A_{Py}(\theta_2) := b \cdot \left(\alpha_3(\theta_2) \cdot \cos(\theta_3(\theta_2)) - \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) \right) \dots$$

$$+ -c \cdot \left(\alpha_4(\theta_2) \cdot \cos(\theta_4(\theta_2)) - \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2)) \right)$$

$$A_P(\theta_2) := \sqrt{A_{Px}(\theta_2)^2 + A_{Py}(\theta_2)^2}$$



PROBLEM 7-70a

Statement: Find the angular accelerations of links 3 and 4 and the linear accelerations of points A, B and P_1 in the XY coordinate system for the linkage in Figure P7-27 in the position shown. Assume that $\theta_2 = 45^\circ$ in the XY coordinate system and $\omega_2 = 10 \text{ rad/sec}$, constant. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the xy coordinate system. Use a graphical method.

Given:

Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 14.00 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 80.00 \cdot \text{in}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 51.26 \cdot \text{in}$$

$$\text{Link 1 X-offset} \quad d_X := 47.5 \cdot \text{in} \quad \text{Link 1 Y-offset} \quad d_Y := 76.00 \cdot \text{in} - 12.00 \cdot \text{in}$$

$$\text{Coupler point x-offset} \quad p_x := 114.68 \cdot \text{in} \quad \text{Coupler point y-offset} \quad p_y := 33.19 \cdot \text{in}$$

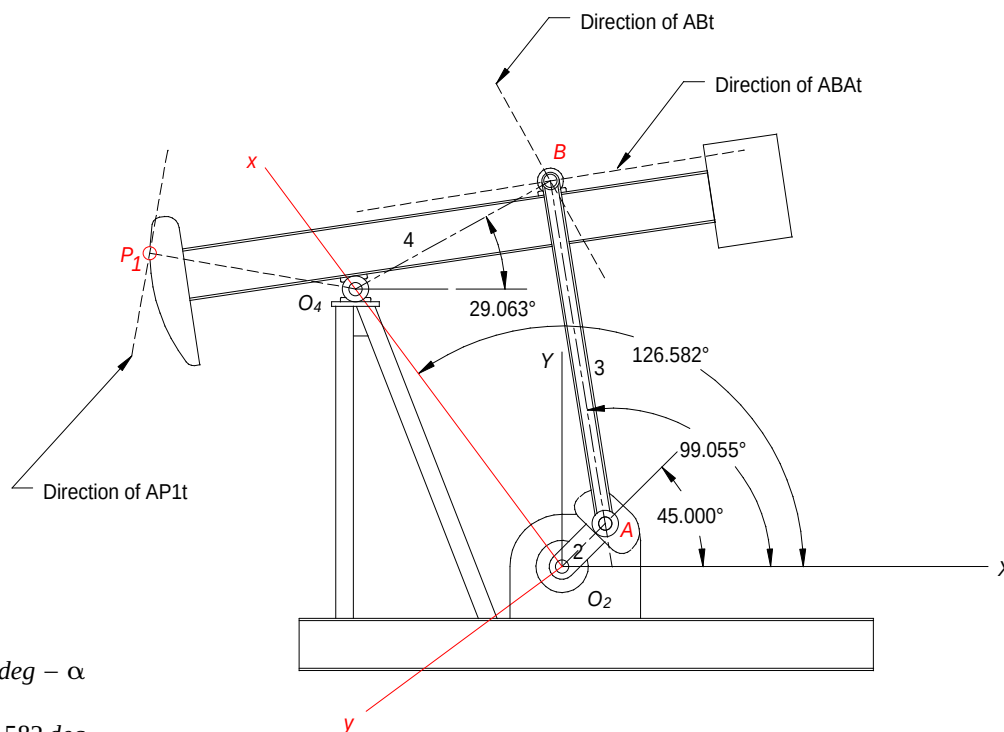
$$\text{Crank angle:} \quad \theta_2 := 45 \cdot \text{deg}$$

$$\text{Coordinate rotation angle} \quad \alpha := 126.582 \cdot \text{deg} \quad \text{Global XY system to local xy system}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CCW} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-27 and Mathcad file P0770a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity acceleration of interest.



$$\theta_2 := 45 \cdot \text{deg} - \alpha$$

$$\theta_2 = -81.582 \text{ deg}$$

2. In order to solve for the accelerations at points A and B, we will need θ_3 , ω_3 , and θ_4 . From the graphical position solution above (in the local xy coordinate system),

$$\theta_3 := 99.055 \cdot \text{deg} - \alpha \quad \theta_3 = -27.527 \text{ deg}$$

$$\theta_4 := 29.063 \cdot \text{deg} - \alpha \quad \theta_4 = -97.519 \text{ deg}$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad \omega_3 = -0.511 \text{ rad} \cdot \text{sec}^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad \omega_4 = 2.353 \text{ rad} \cdot \text{sec}^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

4. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_{Bn} := c \cdot \omega_4^2 \quad A_{Bn} = 283.838 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 - 180 \cdot \text{deg} \quad \theta_{ABn} = -277.519 \text{ deg}$$

$$A_{An} := a \cdot \omega_2^2 \quad A_{An} = 1400.0 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AAn} = 98.418 \text{ deg}$$

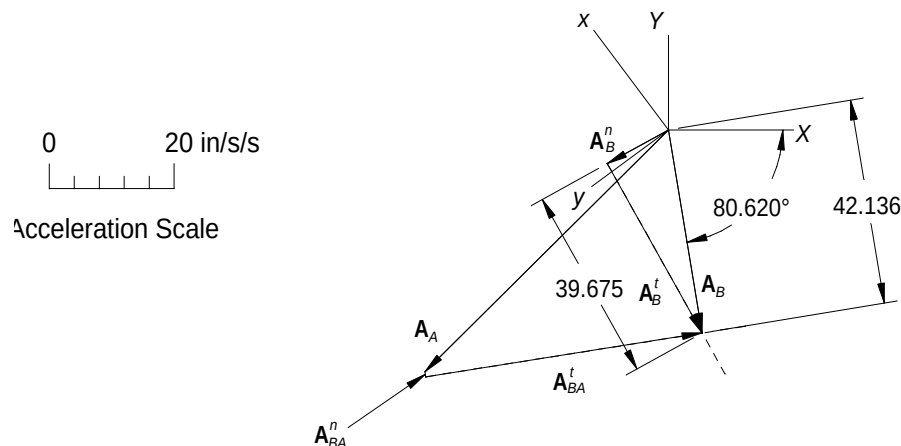
$$A_{At} := a \cdot \alpha_2 \quad A_{At} = 0.0 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AAt} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{AAt} = 8.418 \text{ deg}$$

$$A_{BA n} := b \cdot \omega_3^2 \quad A_{BA n} = 20.921 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABAn} := \theta_3 - 180 \cdot \text{deg} \quad \theta_{ABAn} = -207.527 \text{ deg}$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of $\theta_{ABn} + \alpha$ and $\mathbf{A}_A^n$ at an angle of $\theta_{AAn} + \alpha$. From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_A^t$ at an angle of $\theta_{AAt} + \alpha$. From the tip of $\mathbf{A}_A^t$, draw $\mathbf{A}_{BA}^n$ at an angle of $\theta_{BA n} + \alpha$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B^n$ and $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B^t$, $\mathbf{A}_B^t$, and $\mathbf{A}_{BA}^t$.



6. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 20.0 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_A := A_{An} \quad A_A = 1400 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -135.00 \text{ deg}$$

$$\begin{aligned}
 A_B &:= 42.136 \cdot k_a & A_B &= 842.7 \text{ in} \cdot \text{sec}^{-2} & \text{at an angle of } -80.62 \text{ deg} \\
 A_{Bt} &:= 39.675 \cdot k_a & A_{Bt} &= 793.500 \text{ in} \cdot \text{sec}^{-2} & \text{at an angle of } -60.937 \text{ deg} \\
 \alpha_4 &:= \frac{A_{Bt}}{c} & \alpha_4 &= 15.480 \text{ rad} \cdot \text{sec}^{-2}
 \end{aligned}$$

7. Determine the distance from O_4 to P_1 and the angle O_4P_1 makes with the x axis.

$$\begin{aligned}
 \text{Distance } O_2O_4: & \quad d := \sqrt{d_x^2 + d_y^2} & d &= 79.701 \text{ in} \\
 \text{Distance } P_1O_4: & \quad u := \sqrt{(p_x - d)^2 + p_y^2} & u &= 48.219 \text{ in} \\
 \text{Angle } O_4P_1: & \quad \delta_4 := \text{atan}\left(\frac{p_y}{p_x - d}\right) & \delta_4 &= 43.497 \text{ deg}
 \end{aligned}$$

4. Determine the acceleration at point P_1 .

$$\begin{aligned}
 A_{P1t} &:= u \cdot \alpha_4 & A_{P1t} &= 746.431 \text{ in} \cdot \text{sec}^{-2} & \text{at an angle of } 80.079 \text{ deg} \\
 A_{P1n} &:= u \cdot \omega_4^2 & A_{P1n} &= 267.001 \text{ in} \cdot \text{sec}^{-2} & \text{at an angle of } -9.921 \text{ deg} \\
 A_{P1} &:= \sqrt{A_{P1t}^2 + A_{P1n}^2} & A_{P1} &= 792.748 \text{ in} \cdot \text{sec}^{-2} & \text{at an angle of } 60.397 \text{ deg} \\
 \text{atan}\left(\frac{A_{P1t}}{A_{P1n}}\right) - 9.921 \cdot \text{deg} &= 60.397 \text{ deg}
 \end{aligned}$$

PROBLEM 7-70b

Statement: Find the angular accelerations of links 3 and 4 and the linear accelerations of points A, B and P_1 in the XY coordinate system for the linkage in Figure P7-27 in the position shown. Assume that $\theta_2 = 45$ deg in the XY coordinate system and $\omega_2 = 10$ rad/sec, constant. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the xy coordinate system. Use an analytical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 14.00 \cdot \text{in}$$

$$\text{Link 3 (A to B)} \quad b := 80.00 \cdot \text{in}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 51.26 \cdot \text{in}$$

$$\text{Link 1 X-offset} \quad d_X := 47.5 \cdot \text{in}$$

$$\text{Link 1 Y-offset} \quad d_Y := 76.00 \cdot \text{in} - 12.00 \cdot \text{in}$$

$$\text{Coupler point x-offset} \quad p_x := 114.68 \cdot \text{in}$$

$$\text{Coupler point y-offset} \quad p_y := 33.19 \cdot \text{in}$$

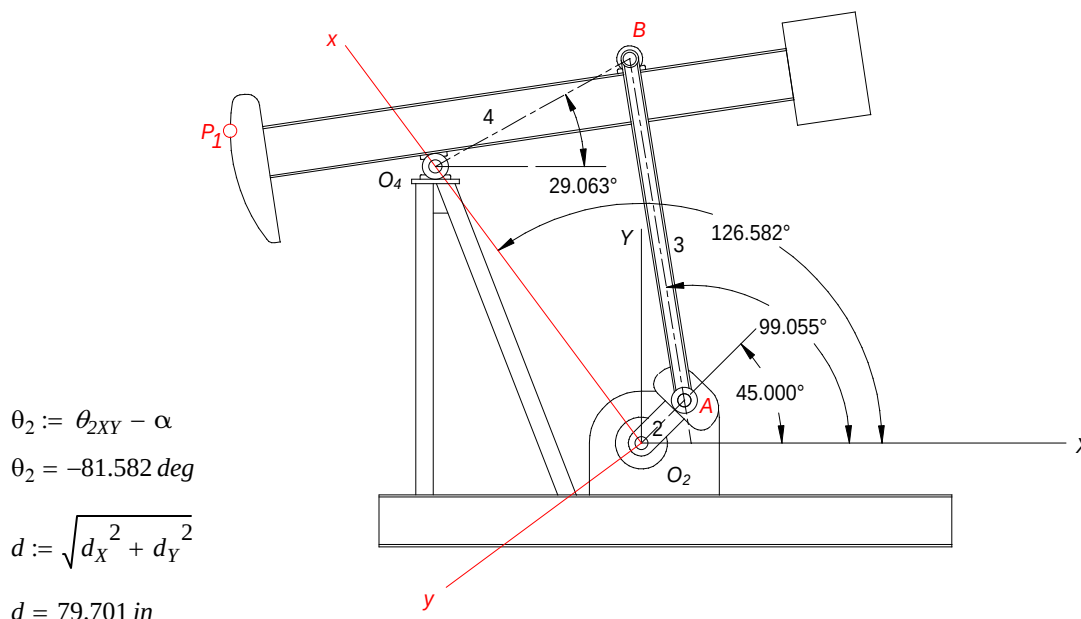
$$\text{Crank angle:} \quad \theta_{2XY} := 45 \cdot \text{deg}$$

$$\text{Coordinate rotation angle} \quad \alpha := 126.582 \cdot \text{deg} \quad \text{Global XY system to local xy system}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CCW} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-27 and Mathcad file P0770b.

1. Draw the linkage to scale and label it.



$$\theta_2 := \theta_{2XY} - \alpha$$

$$\theta_2 = -81.582 \text{ deg}$$

$$d := \sqrt{d_X^2 + d_Y^2}$$

$$d = 79.701 \text{ in}$$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 5.6929$$

$$K_2 = 1.5548$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_3 = 1.9340$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -3.8401 \quad B = 1.9785 \quad C = 7.2529$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_4 = 262.482 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.9963 \quad K_5 = -4.6074$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -10.0081$$

$$E := -2 \cdot \sin(\theta_2) \quad E = 1.9785$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 1.0849$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_3 = 332.475 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad \omega_3 = -0.511 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad \omega_4 = 2.353 \frac{\text{rad}}{\text{sec}}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction (in the global coordinate system).

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (-205 + 1385j) \frac{\text{in}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A) + \alpha$$

$$\text{The acceleration of pin A is } A_A = 1400 \frac{\text{in}}{\text{sec}^2} \quad \text{at } \theta_{AA} = 225 \text{ deg}$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c \cdot \sin(\theta_4) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_4) \quad E := b \cdot \cos(\theta_3)$$

$$A = -1.291 \times 10^3 \text{ mm} \quad B = -939.044 \text{ mm} \quad D = -170.343 \text{ mm} \quad E = 1.802 \times 10^3 \text{ mm}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$C = 260.638 \text{ in} \cdot \text{sec}^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = 1.113 \times 10^3 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = -14.227 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = -15.479 \frac{\text{rad}}{\text{sec}^2}$$

9. Use equation 7.13c to determine the acceleration of point B for the crossed circuit.

$$\mathbf{A}_B := c \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$\mathbf{A}_B = (-749 + 385j) \frac{\text{in}}{\text{sec}^2} \quad A_B := |\mathbf{A}_B| \quad \theta_{AB} := \arg(\mathbf{A}_B) + \alpha$$

$$\text{The acceleration of pin } B \text{ is } A_B = 842.7 \frac{\text{in}}{\text{sec}^2} \quad \text{at } \theta_{AB} = 279.4 \text{ deg} \quad (\text{Global})$$

10. Use equation 7.31 to determine the acceleration of the point P_1 on link 4.

$$u := \sqrt{(p_x - d)^2 + p_y^2} \quad u = 48.219 \text{ in}$$

$$\delta_4 := 360 \cdot \text{deg} - \theta_4 + \text{atan}\left(\frac{p_y}{p_x - d}\right) \quad \delta_4 = 141.014 \text{ deg}$$

$$\mathbf{A}_{P1} := u \cdot \alpha_4 \cdot (-\sin(\theta_4 + \delta_4) + j \cdot \cos(\theta_4 + \delta_4)) - u \cdot \omega_4^2 \cdot (\cos(\theta_4 + \delta_4) + j \cdot \sin(\theta_4 + \delta_4))$$

$$\mathbf{A}_{P1} = (320 - 725j) \frac{\text{in}}{\text{sec}^2} \quad A_{P1} := |\mathbf{A}_{P1}| \quad \theta_{AP1} := \arg(\mathbf{A}_{P1}) + \alpha$$

$$\text{The acceleration of point } P_1 \text{ is } A_{P1} = 792.7 \frac{\text{in}}{\text{sec}^2} \quad \text{at } \theta_{AP1} = 60.39 \text{ deg} \quad (\text{Global})$$

PROBLEM 7-71

Statement: Find the angular accelerations of links 3 and 4 and the linear accelerations of points A, B and P_1 in the XY coordinate system for the linkage in Figure P7-27 in the position shown. Assume that $\theta_2 = 45^\circ$ in the XY coordinate system and $\omega_2 = 10 \text{ rad/sec}$, constant. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the xy coordinate system. Use an analytical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 14.00 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 80.00 \cdot \text{in}$$

$$\text{Link 4 (O}_2 \text{ to B)} \quad c := 51.26 \cdot \text{in}$$

$$\text{Link 1 X-offset} \quad d_X := 47.5 \cdot \text{in} \quad \text{Link 1 Y-offset} \quad d_Y := 76.00 \cdot \text{in} - 12.00 \cdot \text{in}$$

$$\text{Link 1 (O}_2 \text{ to O}_4) \quad d := \sqrt{d_X^2 + d_Y^2} \quad d = 79.701 \text{ in}$$

$$\text{Coupler point x-offset} \quad p_x := 114.68 \cdot \text{in} \quad \text{Coupler point y-offset} \quad p_y := 33.19 \cdot \text{in}$$

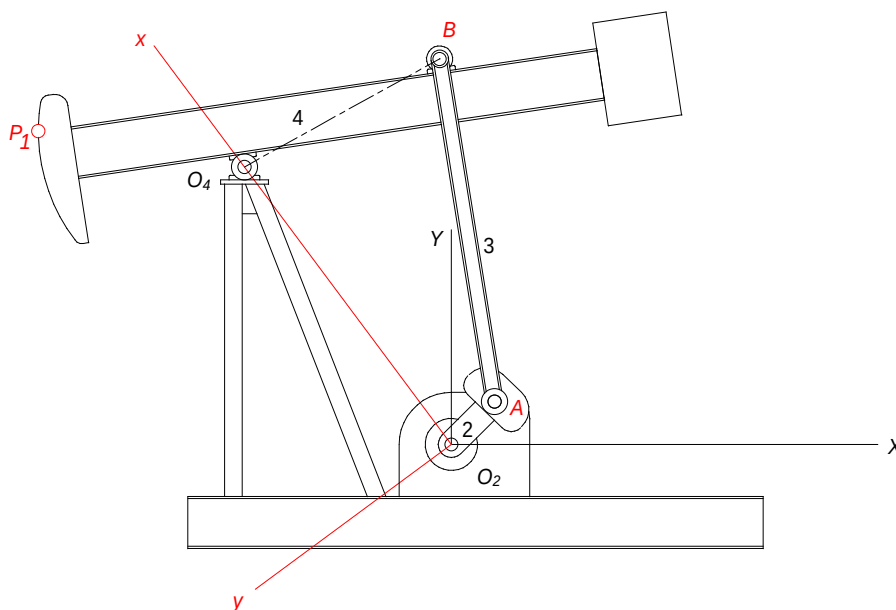
$$\text{Crank angle:} \quad \theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

$$\text{Coordinate rotation angle} \quad \alpha := 126.582 \cdot \text{deg} \quad \text{Global XY system to local xy system}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CCW} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-27 and Mathcad file P0771.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 5.6929$$

$$K_2 = 1.5548$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_3 = 1.9340$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.9963 \quad K_5 = -4.6074$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

7. Use equations 7.12 to determine the angular acceleration of link 4.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2))$$

$$B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2))$$

$$E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

8. Use equation 7.31 to determine and plot the acceleration of the point P_1 on link 4.

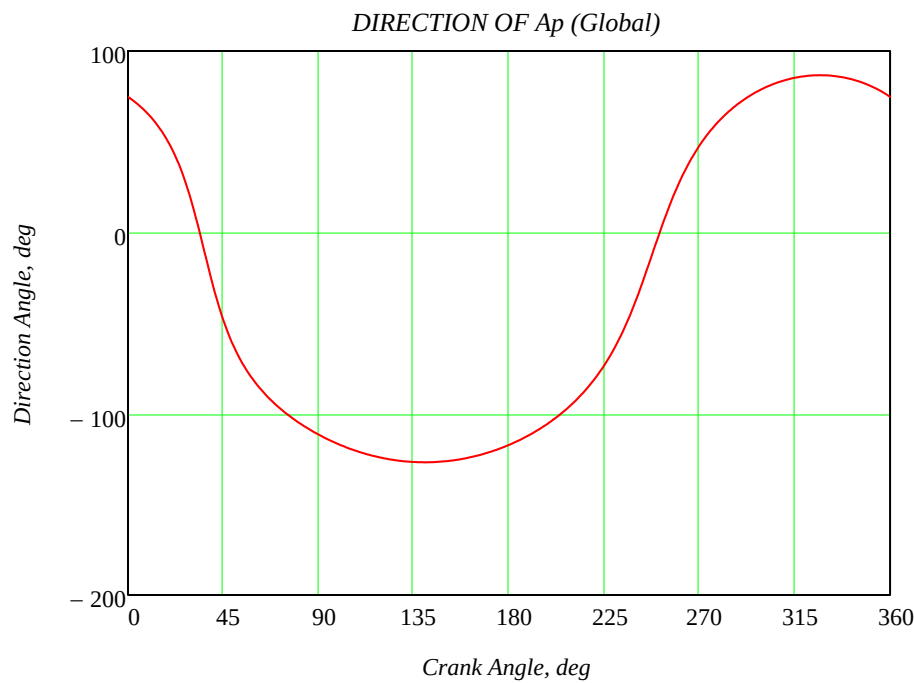
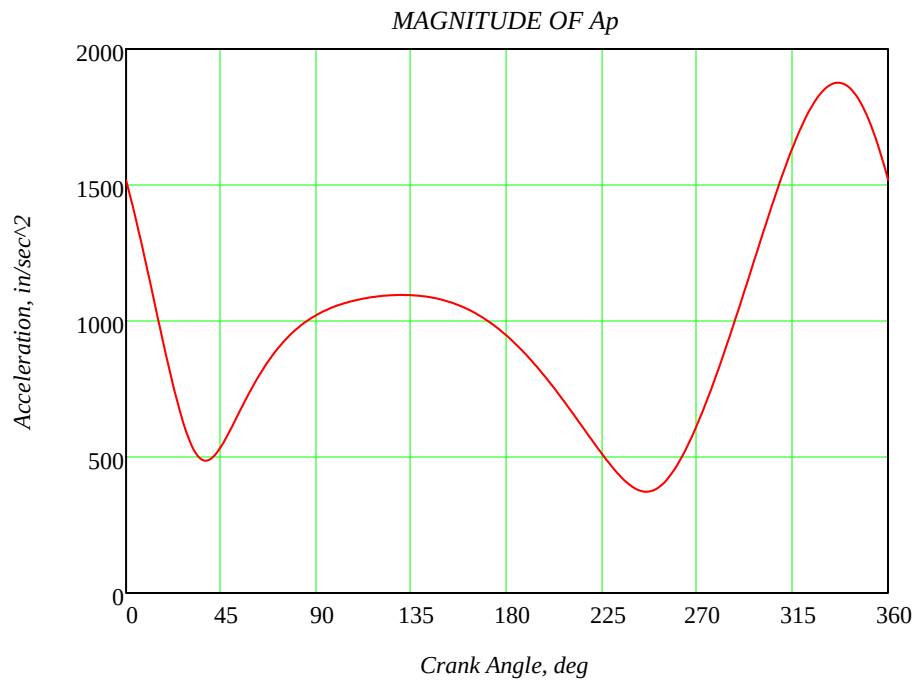
$$u := \sqrt{(p_x - d)^2 + p_y^2} \quad u = 48.219 \text{ in}$$

$$\delta_4 := 141.067 \cdot \text{deg}$$

$$\begin{aligned} \mathbf{A_{P1}}(\theta_2) &:= u \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2) + \delta_4) + j \cdot \cos(\theta_4(\theta_2) + \delta_4)) \dots \\ &\quad + -u \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2) + \delta_4) + j \cdot \sin(\theta_4(\theta_2) + \delta_4)) \end{aligned}$$

$$A_{PI}(\theta_2) := |\mathbf{A}_{P1}(\theta_2)| \quad \theta_{AP1}(\theta_2) := \arg(\mathbf{A}_{P1}(\theta_2)) + \alpha$$

$$\theta_{AP1}(\theta_2) := \text{if}(\theta_{AP1}(\theta_2) > 100 \cdot \text{deg}, \theta_{AP1}(\theta_2) - 2 \cdot \pi, \theta_{AP1}(\theta_2))$$



$$A_{PI}(-81.582 \cdot \text{deg}) = 792.706 \frac{\text{in}}{\text{sec}^2} \quad \theta_{AP1}(-81.582 \cdot \text{deg}) = 60.447 \text{ deg}$$

PROBLEM 7-72a

Statement: Find the angular accelerations of links 3 and 4 and the linear acceleration of point P in the XY coordinate system for the linkage in Figure P7-28 in the position shown. Assume that $\theta_2 = -94.121^\circ$ in the XY coordinate system, $\omega_2 = 1 \text{ rad/sec}$, and $\alpha_2 = 10 \text{ rad/sec}^2$. The position of the coupler point P on link 3 with respect to point A is: $p = 15.00$, $\delta_3 = 0^\circ$. Use a graphical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 9.17 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 12.97 \cdot \text{in}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 9.57 \cdot \text{in}$$

$$\text{Link 1 X-offset} \quad d_X := 2.79 \cdot \text{in} \quad \text{Link 1 Y-offset} \quad d_Y := -6.95 \cdot \text{in}$$

$$\text{Coupler point data:} \quad p := 15.00 \cdot \text{in} \quad \delta_3 := 0^\circ$$

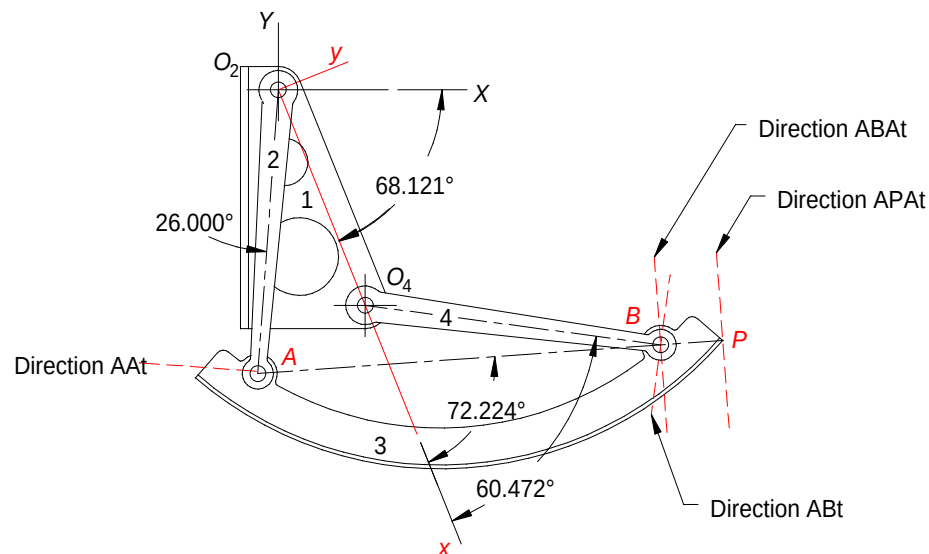
$$\text{Crank angle:} \quad \theta_2 := -26.000^\circ$$

$$\text{Coordinate rotation angle} \quad \alpha := -68.121^\circ \quad \text{Global } XY \text{ system to local } xy \text{ system}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW} \quad \alpha_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-28 and Mathcad file P0772a.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A and B , we will need θ_3 , ω_3 , and θ_4 . From the graphical position solution above (in the local xy coordinate system),

$$\theta_3 := 72.224^\circ$$

$$\theta_3 = 72.224^\circ$$

$$\theta_4 := 60.472^\circ$$

$$\theta_4 = 60.472^\circ$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_4 - \theta_2)}{b \cdot \sin(\theta_3 - \theta_4)} \quad \omega_3 = 3.465 \text{ rad} \cdot \text{sec}^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_4 - \theta_3)} \quad \omega_4 = 4.656 \text{ rad} \cdot \text{sec}^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

4. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_{Bn} := c \cdot \omega_4^2 \quad A_{Bn} = 207.476 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 + 180 \cdot \text{deg} + \alpha \quad \theta_{ABn} = 172.351 \text{ deg}$$

$$A_{An} := a \cdot \omega_2^2 \quad A_{An} = 9.170 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg} + \alpha \quad \theta_{AAn} = 85.879 \text{ deg}$$

$$A_{At} := a \cdot \alpha_2 \quad A_{At} = 91.700 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AA_t} := \theta_2 + 90 \cdot \text{deg} + \alpha \quad \theta_{AA_t} = -4.121 \text{ deg}$$

$$A_{BA_n} := b \cdot \omega_3^2 \quad A_{BA_n} = 155.694 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABA_n} := \theta_3 + 180 \cdot \text{deg} + \alpha \quad \theta_{ABA_n} = 184.103 \text{ deg}$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of $\theta_{ABn} + \alpha$ and $\mathbf{A}_A^n$ at an angle of $\theta_{AAn} + \alpha$. From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_A^t$ at an angle of $\theta_{AA_t} + \alpha$. From the tip of $\mathbf{A}_A^t$, draw $\mathbf{A}_{BA}^n$ at an angle of $\theta_{BA_n} + \alpha$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B^n$ and $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B^t$, $\mathbf{A}_B^t$, and $\mathbf{A}_{BA}^t$.

6. From the graphical solution at right,

$$\text{Acceleration scale factor} \quad k_a := 10.0 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_A := 0.922 \cdot k_a \quad A_A = 9.220 \text{ in} \cdot \text{sec}^{-2} \text{ at an angle of } 1.590 \text{ deg}$$

$$A_B := 7.156 \cdot k_a \quad A_B = 71.560 \text{ in} \cdot \text{sec}^{-2} \text{ at an angle of } 99.204 \text{ deg}$$

$$A_{BA_t} := 7.169 \cdot k_a \quad A_{BA_t} = 71.690 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{A_{BA_t}}{b} \quad \alpha_3 = 5.527 \text{ rad} \cdot \text{sec}^{-2}$$

7. For point P, equation 7.4 becomes:

$$\mathbf{A}_P = \mathbf{A}_A + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n), \text{ where}$$

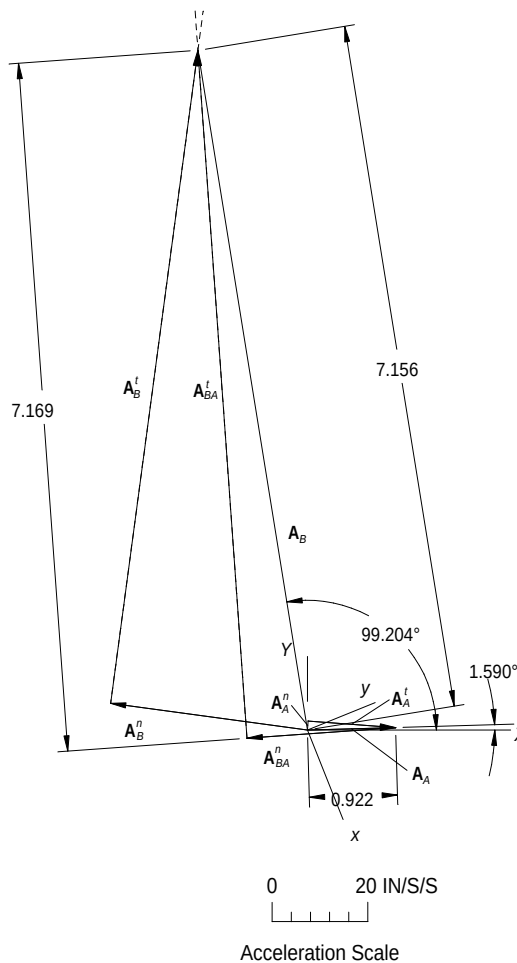
$$A_{PA_n} := p \cdot \omega_3^2 \quad A_{PA_n} = 180.062 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{APA_n} := \theta_3 + \delta_3 + 180 \cdot \text{deg} + \alpha$$

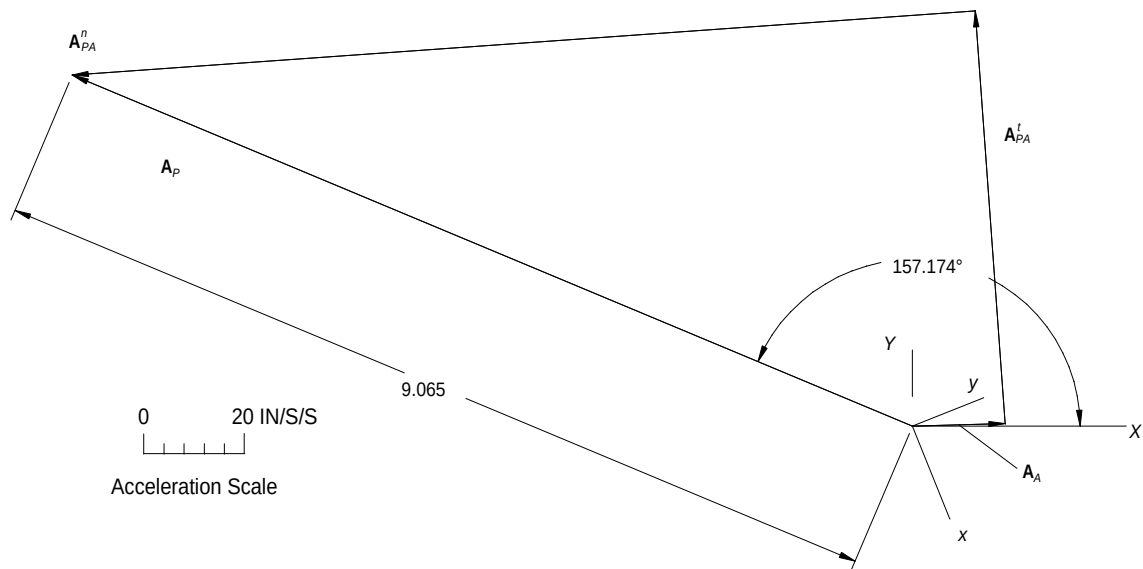
$$\theta_{APA_n} = 184.103 \text{ deg}$$

$$A_{PA_t} := p \cdot \alpha_3 \quad A_{PA_t} = 82.911 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{APA_t} := \theta_{APA_n} - 90 \cdot \text{deg} \quad \theta_{APA_t} = 94.103 \text{ deg}$$



8. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction.



9. From the graphical solution above,

Acceleration scale factor $k_a := 10.0 \cdot \frac{in}{sec^2}$

$$A_P := 9.065 \cdot k_a \quad A_P = 90.650 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 157.174 \text{ deg}$$

PROBLEM 7-72b

Statement: Find the angular accelerations of links 3 and 4 and the linear acceleration of point P in the XY coordinate system for the linkage in Figure P7-28 in the position shown. Assume that $\theta_2 = -94.121^\circ$ in the XY coordinate system, $\omega_2 = 1 \text{ rad/sec}$, and $\alpha_2 = 10 \text{ rad/sec}^2$. The position of the coupler point P on link 3 with respect to point A is: $p = 15.00$, $\delta_3 = 0^\circ$. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 9.174 \cdot \text{in}$	Link 3 (A to B)	$b := 12.971 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 9.573 \cdot \text{in}$		
Link 1 X-offset	$d_X := 2.790 \cdot \text{in}$	Link 1 Y-offset	$d_Y := -6.948 \cdot \text{in}$
Link 1 (O_2 to O_4)	$d := \sqrt{d_X^2 + d_Y^2}$		$d = 7.487 \text{ in}$

Coupler point data: $R_{pa} := 15.00 \cdot \text{in}$ $\delta_3 := 0^\circ$

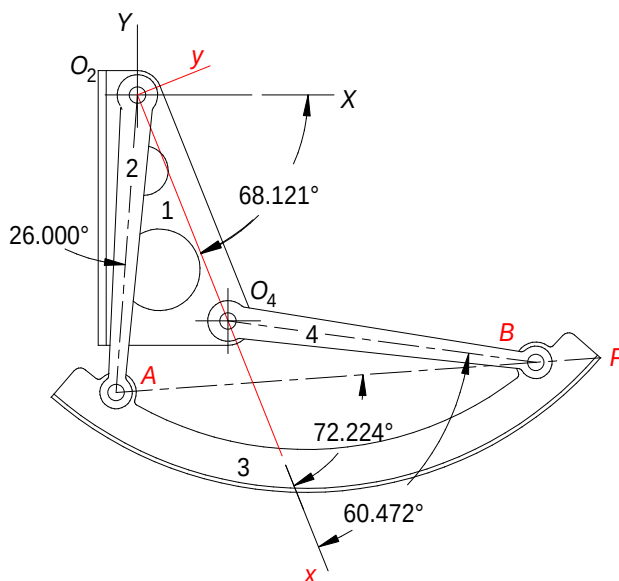
Crank angle: $\theta_2 := -26.000^\circ$

Coordinate rotation angle $\alpha := -68.121^\circ$ Global XY system to local xy system

Input crank angular velocity $\omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW $\alpha_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure P7-28 and Mathcad file P0772b.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_1 = 0.8161$$

$$K_2 := \frac{d}{c}$$

$$K_2 = 0.7821$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_3 = 0.3622$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.2581 \quad B = 0.8767 \quad C = -0.4234$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_4 = -299.512 \text{ deg}$$

$$\theta_4 := \theta_4 + 360 \cdot \text{deg} \quad \theta_4 = 60.488 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.5772 \quad K_5 = -0.9111$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -0.3096$$

$$E := -2 \cdot \sin(\theta_2) \quad E = 0.8767$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = -0.4749$$

5. Use equation 4.13 to find values of θ_3 .

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_3 = -287.764 \text{ deg}$$

$$\theta_3 := \theta_3 + 360 \cdot \text{deg} \quad \theta_3 = 72.236 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad \omega_3 = 3.467 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad \omega_4 = 4.658 \frac{\text{rad}}{\text{sec}}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (32.0 + 86.5j) \frac{\text{in}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A) + \alpha$$

$$\text{The acceleration of pin A is } A_A = 92.198 \frac{\text{in}}{\text{sec}^2} \text{ at } \theta_{AA} = 1.590 \text{ deg} \quad (\text{Global})$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A := c \cdot \sin(\theta_4) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_4) \quad E := b \cdot \cos(\theta_3)$$

$$A = 8.331 \text{ in} \quad B = 12.353 \text{ in} \quad D = 4.716 \text{ in} \quad E = 3.957 \text{ in}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$C = -86.722 \text{ in} \cdot \text{sec}^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = 118.753 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = 55.307 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = 71.596 \frac{\text{rad}}{\text{sec}^2}$$

9. Use equation 7.13c to determine the acceleration of point B.

$$\mathbf{A}_B := c \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$\mathbf{A}_B = (-698.8 + 156.9i) \frac{\text{in}}{\text{sec}^2} \quad A_B := |\mathbf{A}_B| \quad \theta_{AB} := \arg(\mathbf{A}_B) + \alpha$$

$$\text{The acceleration of pin B is } A_B = 716.18 \frac{\text{in}}{\text{sec}^2} \quad \text{at } \theta_{AB} = 99.228 \text{ deg}$$

10. Use equations 7.32 to find the acceleration of the point P.

$$\mathbf{A}_{PA} := R_{pa} \cdot \alpha_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3)) \dots \\ + -R_{pa} \cdot \omega_3^2 \cdot (\cos(\theta_3 + \delta_3) + j \cdot \sin(\theta_3 + \delta_3))$$

$$\mathbf{A}_P := \mathbf{A}_A + \mathbf{A}_{PA}$$

$$A_P := |\mathbf{A}_P| \quad A_P = 830 \frac{\text{in}}{\text{sec}^2} \quad \arg(\mathbf{A}_P) + \alpha = 100.214 \text{ deg}$$

PROBLEM 7-73

Statement: For the linkage in Figure P7-28, write a computer program or use an equation solver to calculate and plot the angular velocity and acceleration of links 3 and 4, and the magnitude and direction of the velocity and acceleration of point P as a function of θ_2 through its possible range of motion starting at the position shown. The position of the coupler point P on link 3 with respect to point A is: $p = 15.00$, $\delta_3 = 0$ deg. Assume that, at $t = 0$, $\theta_2 = -94.121$ deg in the XY coordinate system, $\omega_2 = 0$, and $\alpha_2 = 10$ rad/sec², constant.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 9.174 \cdot \text{in}$	Link 3 (A to B)	$b := 12.971 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 9.573 \cdot \text{in}$		
Link 1 X -offset	$d_x := 2.790 \cdot \text{in}$	Link 1 Y -offset	$d_y := -6.948 \cdot \text{in}$
Link 1 (O_2 to O_4)	$d := \sqrt{d_x^2 + d_y^2}$	$d = 7.487 \text{ in}$	

Coupler point data: $R_{pa} := 15.00 \cdot \text{in}$ $\delta_3 := 0 \cdot \text{deg}$

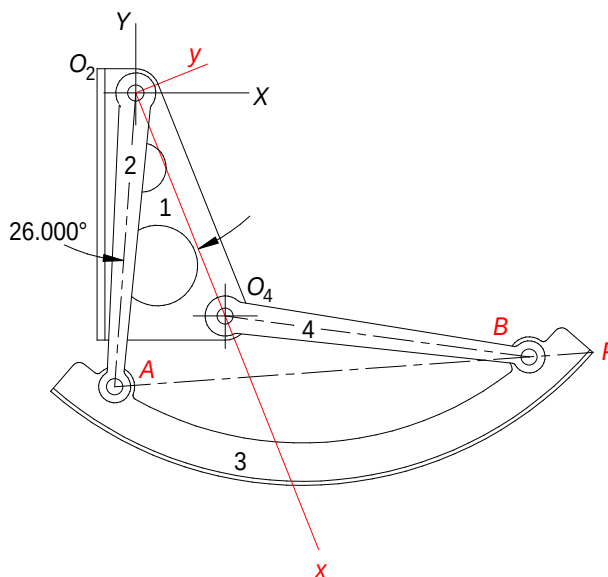
Crank angle: $\theta_2 := -26.000 \cdot \text{deg}, -25.9 \cdot \text{deg} \dots -15 \cdot \text{deg}$ $\theta_{20} := -26 \cdot \text{deg}$

Coordinate rotation angle $\alpha := -68.121 \cdot \text{deg}$ Global XY system to local xy system

Input crank angular velocity $\omega_{20} := 0 \cdot \text{rad} \cdot \text{sec}^{-1}$ $\alpha_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure P7-28 and Mathcad file P0773.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 0.8161 \quad K_2 = 0.7821$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 0.3622$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \quad B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.5772 \quad K_5 = -0.9111$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 .

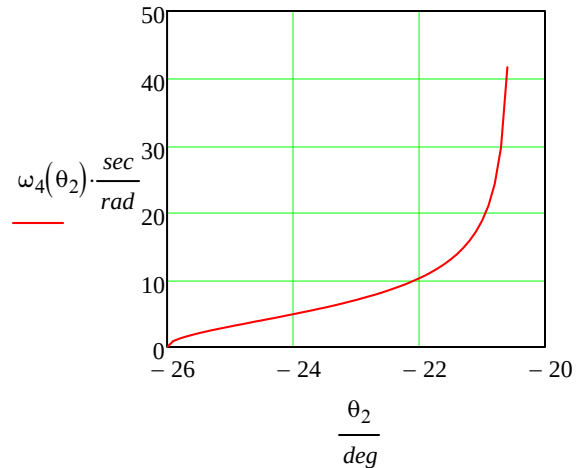
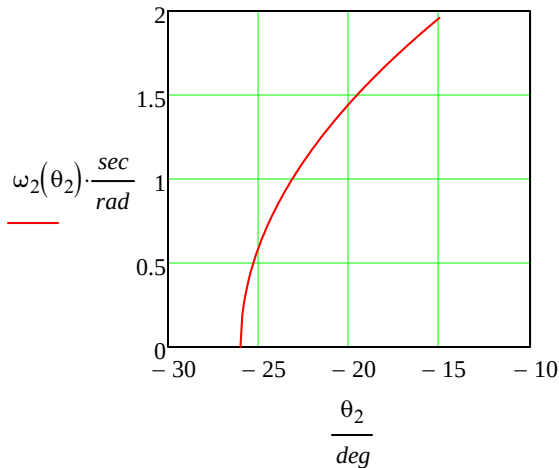
$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_2(\theta_2) := \sqrt{2 \cdot (\theta_2 - \theta_{20}) \cdot \alpha_2}$$

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2(\theta_2)}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2(\theta_2)}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$



7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2(\theta_2)^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2))$$

$$B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

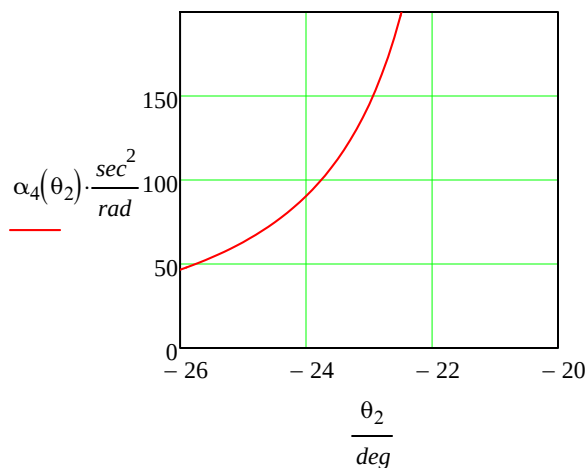
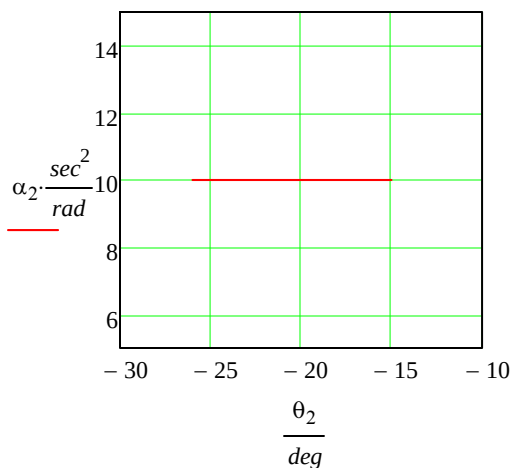
$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2))$$

$$E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2(\theta_2)^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2(\theta_2)^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)} \quad \alpha_4(\theta_2) := \frac{C'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$



9. Use equations 7.32 to find the acceleration of the point P .

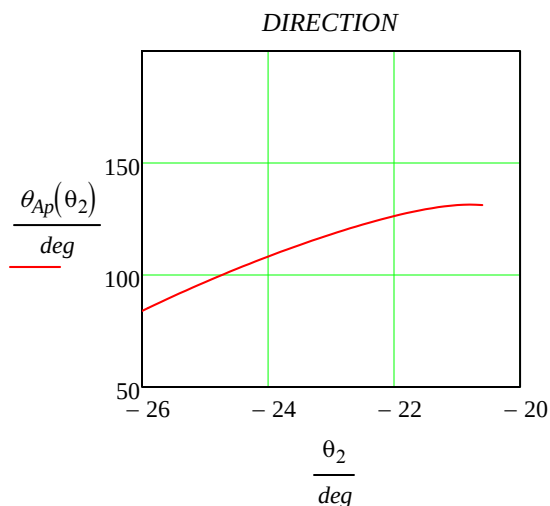
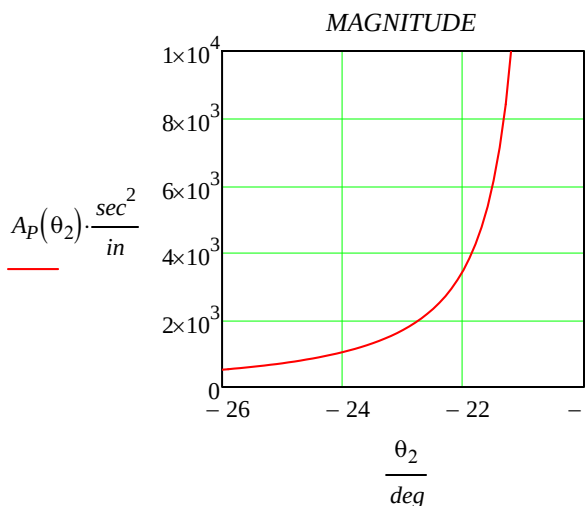
$$\mathbf{A}_{PA}(\theta_2) := R_{pa} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots$$

$$+ R_{pa} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{A}_P(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2)$$

$$A_P(\theta_2) := |\mathbf{A}_P(\theta_2)| \quad \theta_{Ap}(\theta_2) := \arg(\mathbf{A}_P(\theta_2)) + \alpha$$

$$\theta_{Ap}(\theta_2) := \text{if}(\theta_{Ap}(\theta_2) < 0, \theta_{Ap}(\theta_2) + 2 \cdot \pi, \theta_{Ap}(\theta_2))$$

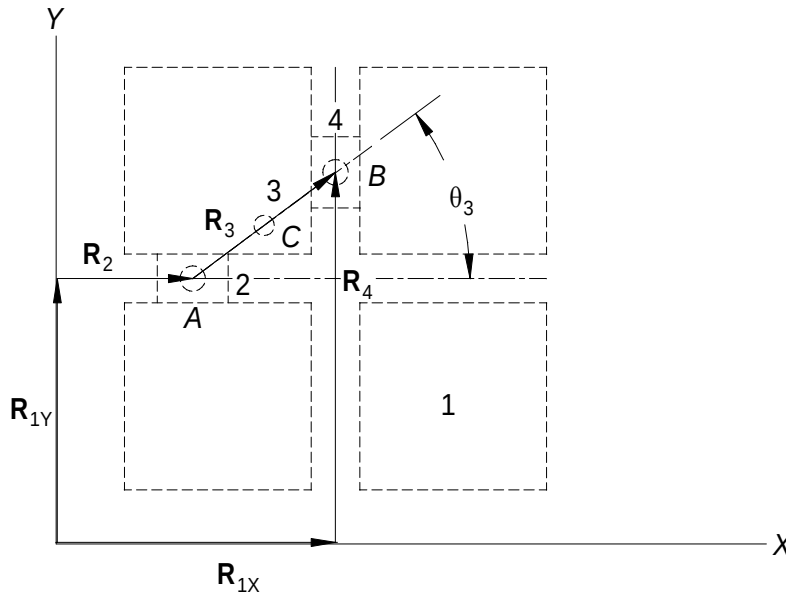


PROBLEM 7-74

Statement: Derive analytical expressions for the accelerations of points A and B in Figure P7-29 as a function of θ_3 , ω_3 , α_3 and the length AB of link 3. Use a vector loop equation.

Solution: See Figure P7-29 and Mathcad file P0774.

1. Establish the global XY system such that the coordinates of the intersection of the slot centerlines is at (d_X, d_Y) . Then, define position vectors $\mathbf{R}_{1X}$, $\mathbf{R}_{1Y}$, $\mathbf{R}_2$, $\mathbf{R}_3$, and $\mathbf{R}_4$ as shown below.



2. Write the vector loop equation: $\mathbf{R}_{1Y} + \mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_{1X} - \mathbf{R}_4 = 0$ then substitute the complex number notation for each position vector. The equation then becomes:

$$d_Y \cdot e^{j\left(\frac{\pi}{2}\right)} + a \cdot e^{j(0)} + c \cdot e^{j(\theta_3)} - d_X \cdot e^{j(0)} - b \cdot e^{j\left(\frac{\pi}{2}\right)} = 0$$

3. Differentiate this equation with respect to time.

$$\frac{d}{dt} a + j \cdot c \cdot \omega_3 \cdot e^{j(\theta_3)} - \frac{d}{dt} \left(b \cdot e^{j\left(\frac{\pi}{2}\right)} \right) = 0$$

3. Substituting the Euler identity into this equation gives:

$$V_a + j c \cdot \omega_3 \cdot (\cos(\theta_3) + j \cdot \sin(\theta_3)) - V_b \cdot j = 0$$

4. Separate this equation into its real (x component) and imaginary (y component) parts, setting each equal to zero.

$$V_a - c \cdot \omega_3 \cdot \sin(\theta_3) = 0 \quad c \cdot \omega_3 \cdot \cos(\theta_3) - V_b = 0$$

5. Solve for the two unknowns V_a and V_b in terms of the independent variables θ_3 and ω_3

$$V_a = c \cdot \omega_3 \cdot \sin(\theta_3) \quad V_b = c \cdot \omega_3 \cdot \cos(\theta_3)$$

6. Differentiate again with respect to time to get the accelerations.

$$A_a = c \cdot \omega_3^2 \cdot \cos(\theta_3) \quad A_b = -c \cdot \omega_3^2 \cdot \sin(\theta_3)$$

PROBLEM 7-75

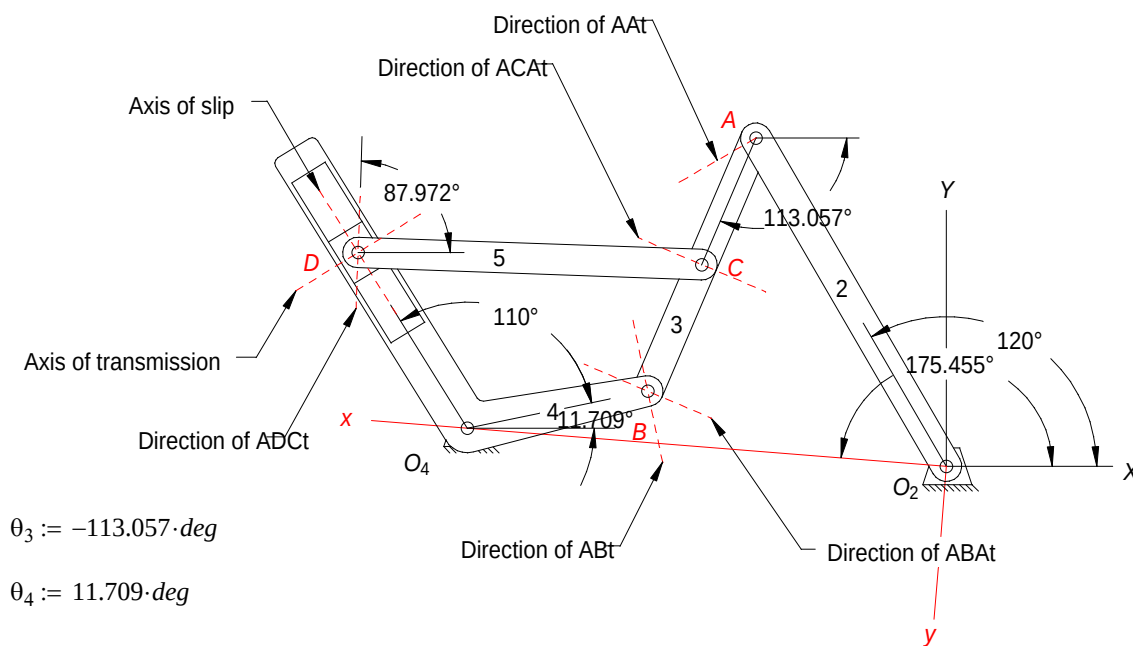
Statement: The linkage in Figure P7-30a has link 2 at 120 deg in the global XY coordinate system. Find α_6 and $\mathbf{A}_D$ in the global XY coordinate system for the position shown if $\omega_2 = 10 \text{ rad/sec}$ CCW and $\alpha_2 = 50 \text{ rad/sec}^2$ CW. Use the acceleration difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 6.20 \cdot \text{in}$	Link 3 (A to B)	$b := 4.50 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 3.00 \cdot \text{in}$	Link 5 (C to D)	$e := 5.60 \cdot \text{in}$
Link 3 (A to C)	$p := 2.25 \cdot \text{in}$	Link 4 (O_4 to D)	$f := 3.382 \cdot \text{in}$
Link 1 X-offset	$d_X := -7.80 \cdot \text{in}$	Link 1 Y-offset	$d_Y := 0.62 \cdot \text{in}$
Angle ACB	$\delta_3 := 0.0 \cdot \text{deg}$	Angle BO_4D	$\delta_4 := 110.0 \cdot \text{deg}$
Input rocker angle:	$\theta_2 := 120 \cdot \text{deg}$	Global XY system	
Input crank angular velocity	$\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$	CCW	$\alpha_2 := -50 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure P7-30a and Mathcad file P0775.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



$$\theta_3 := -113.057 \cdot \text{deg}$$

$$\theta_4 := 11.709 \cdot \text{deg}$$

Coordinate rotation angle: $\delta := 175.455 \cdot \text{deg}$

2. In order to solve for the accelerations at points A and B, we will need θ_3 , ω_3 , and θ_4 . From the graphical position solution above (in the local xy coordinate system),

$$\theta_{2xy} := \theta_2 + \delta$$

$$\theta_{2xy} = 295.455 \text{ deg}$$

$$\theta_{3xy} := \theta_3 + \delta$$

$$\theta_{3xy} = 62.398 \text{ deg}$$

$$\theta_{4xy} := \theta_4 + \delta$$

$$\theta_{4xy} = 187.164 \text{ deg}$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_{4xy} - \theta_{2xy})}{b \cdot \sin(\theta_{3xy} - \theta_{4xy})}$$

$$\omega_3 = 15.924 \text{ rad} \cdot \text{sec}^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_{2xy} - \theta_{3xy})}{c \cdot \sin(\theta_{4xy} - \theta_{3xy})}$$

$$\omega_4 = -20.107 \text{ rad} \cdot \text{sec}^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

4. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_{Bn} := c \cdot \omega_4^2 \quad A_{Bn} = 1212.852 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 + 180 \cdot \text{deg} \quad \theta_{ABn} = 191.709 \text{ deg}$$

$$A_{An} := a \cdot \omega_2^2 \quad A_{An} = 620.000 \text{ in} \cdot \text{sec}^{-2}$$

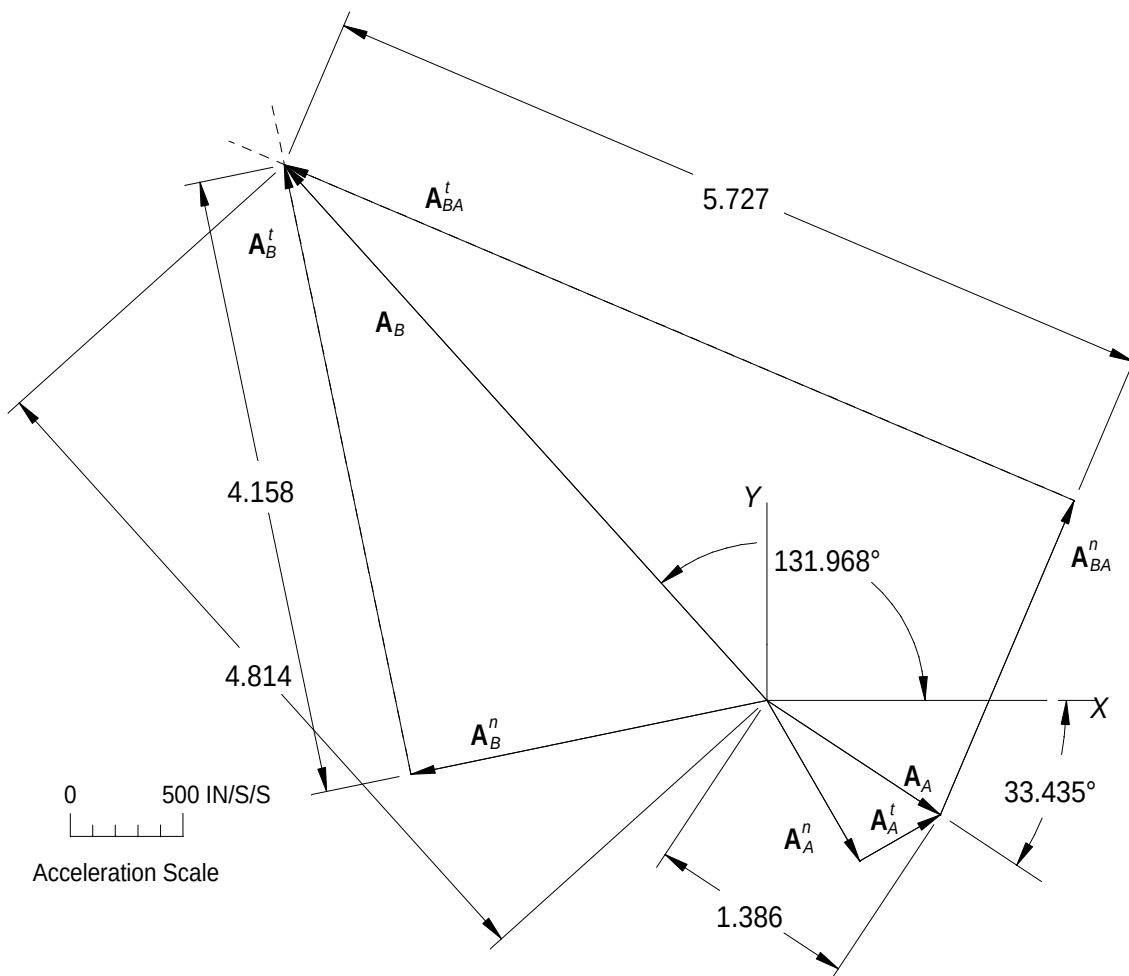
$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AAn} = 300.000 \text{ deg}$$

$$A_{At} := |a \cdot \alpha_2| \quad A_{At} = 310.000 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AA_t} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{AA_t} = 30.000 \text{ deg}$$

$$A_{BA_n} := b \cdot \omega_3^2 \quad A_{BA_n} = 1141.131 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABA_n} := \theta_3 + 180 \cdot \text{deg} \quad \theta_{ABA_n} = 66.943 \text{ deg}$$

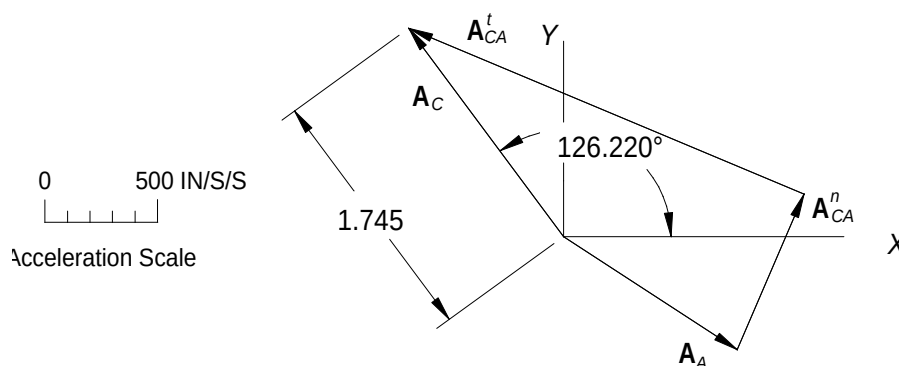


6. From the graphical solution above,

Acceleration scale factor	$k_a := 500 \cdot \frac{\text{in}}{\text{sec}^2}$	
$A_A := 1.386 \cdot k_a$	$A_A = 693.0 \text{ in} \cdot \text{sec}^{-2}$	at an angle of -33.435 deg
$A_B := 4.814 \cdot k_a$	$A_B = 2407.0 \text{ in} \cdot \text{sec}^{-2}$	at an angle of 131.968 deg
$A_{BA_t} := 5.727 \cdot k_a$	$A_{BA_t} = 2863.5 \text{ in} \cdot \text{sec}^{-2}$	
$\alpha_3 := \frac{A_{BA_t}}{b}$	$\alpha_3 = 636.333 \text{ rad} \cdot \text{sec}^{-2}$	CW
$A_{B_t} := 4.158 \cdot k_a$	$A_{B_t} = 2079.0 \text{ in} \cdot \text{sec}^{-2}$	
$\alpha_4 := \frac{A_{B_t}}{c}$	$\alpha_4 = 693.000 \text{ rad} \cdot \text{sec}^{-2}$	CCW

7. For point C, equation 7.4 becomes: $\mathbf{A}_C = \mathbf{A}_A + (\mathbf{A}_{CA}^t + \mathbf{A}_{CA}^n)$, where

$A_{CA_n} := p \cdot \omega_3^2$	$A_{CA_n} = 570.566 \text{ in} \cdot \text{sec}^{-2}$
$\theta_{ACA_n} := \theta_3 + 180 \cdot \text{deg}$	$\theta_{ACA_n} = 66.943 \text{ deg}$
$A_{CA_t} := p \cdot \alpha_3$	$A_{CA_t} = 1431.8 \text{ in} \cdot \text{sec}^{-2}$
$\theta_{ACA_t} := \theta_{ACA_n} + 90 \cdot \text{deg}$	$\theta_{ACA_t} = 156.943 \text{ deg}$



8. From the graphical solution above,

Acceleration scale factor	$k_a := 500 \cdot \frac{\text{in}}{\text{sec}^2}$	
$A_C := 1.745 \cdot k_a$	$A_C = 872.50 \text{ in} \cdot \text{sec}^{-2}$	at an angle of 126.220 deg

9. Determining the acceleration of point D requires the (graphical) solution of two equations simultaneously. They are: $\mathbf{A}_D = \mathbf{A}_C + (\mathbf{A}_{DC}^t + \mathbf{A}_{DC}^n)$ and $\mathbf{A}_D = \mathbf{A}_D^t + \mathbf{A}_D^n + \mathbf{A}_D^{cor} + \mathbf{A}_D^{slip}$, where $\mathbf{A}_C$ is known from the above analysis and

From the velocity analysis (see Problem 6-91) $\omega_5 := -35.13 \cdot \text{rad} \cdot \text{sec}^{-1}$ $V_{slip} := 136.20 \cdot \text{in} \cdot \text{sec}^{-1}$

$A_{DC_n} := e \cdot \omega_5^2$	$A_{DC_n} = 6911 \text{ in} \cdot \text{sec}^{-2}$

$$\theta_{ADCn} := 177.972 \cdot \text{deg} + 180 \cdot \text{deg}$$

$$\theta_{ADCn} = 357.972 \text{ deg}$$

$$A_{Dt} := f \cdot \alpha_4$$

$$A_{Dt} = 2343.7 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ADt} := \theta_4 + \delta_4 + 90 \cdot \text{deg}$$

$$\theta_{ADt} = 211.709 \text{ deg}$$

$$A_{Dn} := f \cdot \omega_4^2$$

$$A_{Dn} = 1367.3 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ADn} := \theta_{ADt} + 90 \cdot \text{deg}$$

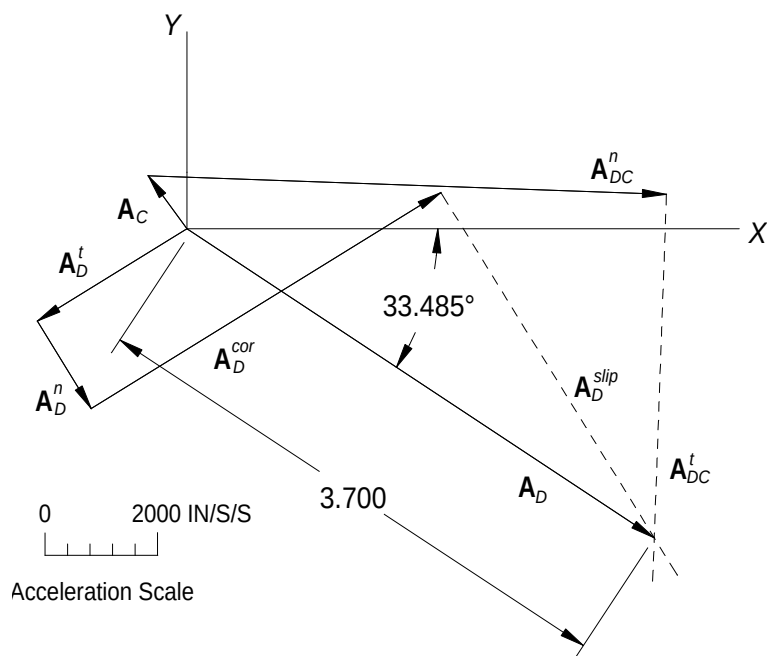
$$\theta_{ADn} = 301.709 \text{ deg}$$

$$A_{Dcor} := |2 \cdot V_{slip} \cdot \omega_4|$$

$$A_{Dcor} = 5477.1 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ADcor} := \theta_{ADt} - 180 \cdot \text{deg}$$

$$\theta_{ADcor} = 31.709 \text{ deg}$$



8. From the graphical solution above,

Acceleration scale factor

$$k_a := 2000 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_D := 3.700 \cdot k_a$$

$$A_D = 7400 \text{ in} \cdot \text{sec}^{-2}$$

at an angle of -33.485 deg

$$\alpha_6 := \alpha_4$$

$$\alpha_6 = 693.0 \text{ rad} \cdot \text{sec}^{-2}$$

PROBLEM 7-76

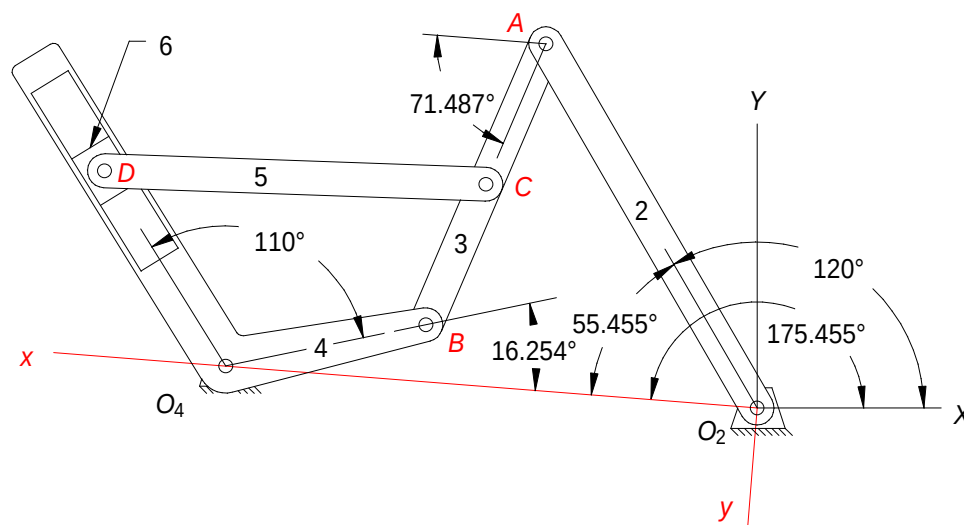
Statement: The linkage in Figure P7-30a has link 2 at 120 deg in the global XY coordinate system. Find α_6 and $\mathbf{A}_D$ in the global XY coordinate system for the position shown if $\omega_2 = 10 \text{ rad/sec}$ CCW and $\alpha_2 = 50 \text{ rad/sec}^2$ CW. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 6.20 \cdot in$	Link 3 (A to B)	$b := 4.50 \cdot in$
Link 4 (O_4 to B)	$c := 3.00 \cdot in$	Link 5 (C to D)	$h := 5.60 \cdot in$
Link 3 (A to C)	$p := 2.25 \cdot in$	Link 4 (O_4 to D)	$f := 3.382 \cdot in$
Link 1 X-offset	$d_X := -7.80 \cdot in$	Link 1 Y-offset	$d_Y := 0.62 \cdot in$
Angle ACB	$\delta_3 := 0.0 \cdot deg$	Angle BO_4D	$\delta_4 := 110.0 \cdot deg$
Input rocker angle:	$\theta_{2XY} := 120 \cdot deg$	Global XY system	
Coordinate transformation angle:	$\delta := 175.455 \cdot deg$		
Input crank angular velocity	$\omega_2 := 10 \cdot rad \cdot sec^{-1}$ (CCW)		
Input crank angular acceleration	$\alpha_2 := -50 \cdot rad \cdot sec^{-2}$ (CW)		

Solution: See Figure P7-30a and Mathcad file P0776.

1. Draw the linkage to scale and label it.



Calculate the distance O_2O_4 : $d := \sqrt{d_X^2 + d_Y^2}$ $d = 7.825 \text{ in}$

Transform θ_{2XY} into the local coordinate system: $\theta_2 := \theta_{2XY} - \delta$ $\theta_2 = -55.455 \text{ deg}$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 1.2620 \quad K_2 = 2.6082$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 2.3767$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = 0.2028 \quad B = 1.6474 \quad C = 1.5927$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2} \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_4 = -163.746 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.7388 \quad K_5 = -1.9877$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \quad D = -1.6967$$

$$E := -2 \cdot \sin(\theta_2) \quad E = 1.6474$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = -0.3067$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2} \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) - 2 \cdot \pi \quad \theta_3 = -648.512 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad \omega_3 = 15.924 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad \omega_4 = -20.107 \frac{\text{rad}}{\text{sec}}$$

7. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the open circuit.

$$A := c \cdot \sin(\theta_4) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_4) \quad E := b \cdot \cos(\theta_3)$$

$$A = -21.328 \text{ mm} \quad B = 108.386 \text{ mm} \quad D = -73.154 \text{ mm} \quad E = 36.291 \text{ mm}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

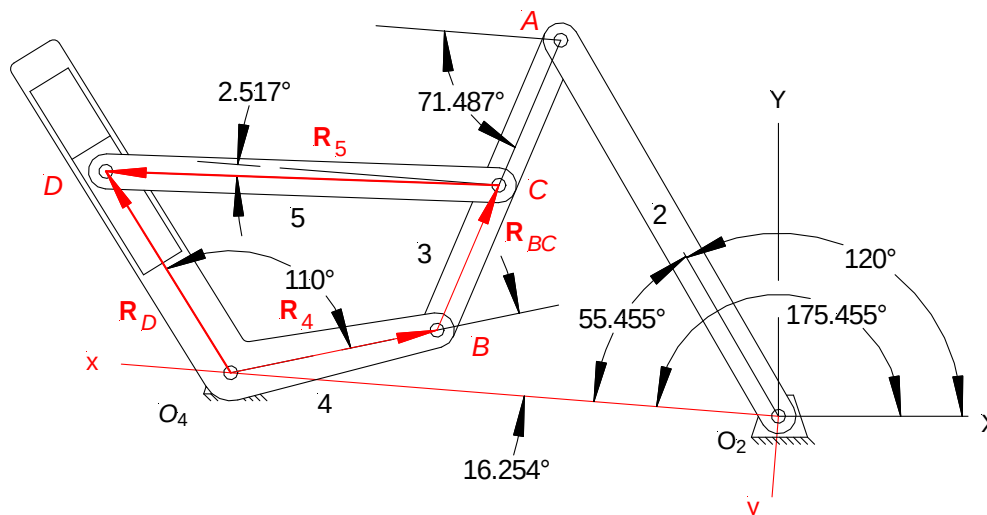
$$C = 2.134 \times 10^3 \text{ in} \cdot \text{sec}^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = -1.087 \times 10^3 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = -636.368 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = 692.987 \frac{\text{rad}}{\text{sec}^2}$$

8. Establish a vector loop that defines the position of point D with respect to O_4 and write the loop equation.



$$\mathbf{R}_D = \mathbf{R}_4 + \mathbf{R}_{BC} + \mathbf{R}_5 \quad f \cdot e^{j(\theta_4 + \delta_4)} = c \cdot e^{j\theta_4} + p \cdot e^{j(\theta_3 + \pi)} + h \cdot e^{j\theta_5}$$

where f and the angles are functions of time and c, p, h , and δ_4 are constants.

9. Solve the vector loop equation for θ_5 by substituting the Euler identity for the unit vectors and then separating into real and imaginary parts.

$$f \cdot (\cos(\theta_4 + \delta_4) + j \cdot \sin(\theta_4 + \delta_4)) = c \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4)) + p \cdot (\cos(\theta_3 + \pi) + j \cdot \sin(\theta_3 + \pi)) \dots \\ + h \cdot (\cos(\theta_5) + j \cdot \sin(\theta_5))$$

$$f \cdot \cos(\theta_4 + \delta_4) = c \cdot \cos(\theta_4) + p \cdot \cos(\theta_3 + \pi) + h \cdot \cos(\theta_5)$$

$$f \cdot \sin(\theta_4 + \delta_4) = c \cdot \sin(\theta_4) + p \cdot \sin(\theta_3 + \pi) + h \cdot \sin(\theta_5)$$

$$\tan(\theta_5) = \frac{f \cdot \sin(\theta_4 + \delta_4) - c \cdot \sin(\theta_4) - p \cdot \sin(\theta_3 + \pi)}{f \cdot \cos(\theta_4 + \delta_4) - c \cdot \cos(\theta_4) - p \cdot \cos(\theta_3 + \pi)}$$

$$\theta_5 := \text{atan2}(f \cdot \cos(\theta_4 + \delta_4) - c \cdot \cos(\theta_4) - p \cdot \cos(\theta_3 + \pi), f \cdot \sin(\theta_4 + \delta_4) - c \cdot \sin(\theta_4) - p \cdot \sin(\theta_3 + \pi))$$

$$f = 3.382 \text{ in}$$

$$\theta_5 = 2.518 \text{ deg}$$

10. Differentiate the position equation and solve for ω_5 and $\dot{f}$ by substituting the Euler identity for the unit vectors and then separating into real and imaginary parts.

$$\begin{aligned} f \cdot \omega_4 \cdot (-\sin(\theta_4 + \delta_4) + j \cdot \cos(\theta_4 + \delta_4)) + f_{dot} \cdot (\cos(\theta_4 + \delta_4) + j \cdot \sin(\theta_4 + \delta_4)) &= \\ c \cdot \omega_4 [-\sin(\theta_4) + j \cdot \cos(\theta_4)] + p \cdot \omega_3 [-\sin(\theta_3 + \pi) + j \cdot \cos(\theta_3 + \pi)] + h \cdot \omega_5 \cdot (-\sin(\theta_5) + j \cdot \cos(\theta_5)) \\ -f \cdot \omega_4 \cdot \sin(\theta_4 + \delta_4) + f_{dot} \cdot \cos(\theta_4 + \delta_4) &= -c \cdot \omega_4 \cdot \sin(\theta_4) - p \cdot \omega_3 \cdot \sin(\theta_3 + \pi) - h \cdot \omega_5 \cdot \sin(\theta_5) \\ f \cdot \omega_4 \cdot \cos(\theta_4 + \delta_4) + f_{dot} \cdot \sin(\theta_4 + \delta_4) &= c \cdot \omega_4 \cdot \cos(\theta_4) + p \cdot \omega_3 \cdot \cos(\theta_3 + \pi) + h \omega_5 \cdot \cos(\theta_5) \end{aligned}$$

Let $A := f \cdot \omega_4 \cdot \sin(\theta_4 + \delta_4) - c \cdot \omega_4 \cdot \sin(\theta_4) - p \cdot \omega_3 \cdot \sin(\theta_3 + \pi)$ $A = 71.929 \frac{in}{s}$

$$B := -f \cdot \omega_4 \cdot \cos(\theta_4 + \delta_4) + c \cdot \omega_4 \cdot \cos(\theta_4) + p \cdot \omega_3 \cdot \cos(\theta_3 + \pi) \quad B = 86.747 \frac{\text{in}}{\text{s}}$$

then

$$f\dot{d}ot \cdot \cos(\theta_4 + \delta_4) = A - h \cdot \omega_5 \cdot \sin(\theta_5) \quad f\dot{d}ot \cdot \sin(\theta_4 + \delta_4) = B + h \cdot \omega_5 \cdot \cos(\theta_5)$$

Eliminating $f\dot{d}ot$ and solving for ω_5 ,

$$\omega_5 := \frac{A \cdot \tan(\theta_4 + \delta_4) - B}{h \cdot (\cos(\theta_5) + \sin(\theta_5) \cdot \tan(\theta_4 + \delta_4))} \quad \omega_5 = -35.145 \frac{\text{rad}}{\text{s}}$$

and

$$f\dot{d}ot := \frac{A - h \cdot \omega_5 \cdot \sin(\theta_5)}{\cos(\theta_4 + \delta_4)} \quad f\dot{d}ot = 136.252 \frac{\text{in}}{\text{s}}$$

11. Differentiate the velocity equation and solve for α_5 and $f\ddot{d}ot$ by substituting the Euler identity for the unit vectors and then separating into real and imaginary parts.

$$\begin{aligned} f \cdot \alpha_4 \cdot j \cdot e^{j \cdot (\theta_4 + \delta_4)} - f \cdot \omega_4^2 \cdot e^{j \cdot (\theta_4 + \delta_4)} + 2 \cdot f\dot{d}ot \cdot \omega_4 \cdot j \cdot e^{j \cdot (\theta_4 + \delta_4)} + f\ddot{d}ot \cdot e^{j \cdot (\theta_4 + \delta_4)} = \\ c \cdot (j \cdot \alpha_4 \cdot e^{j \cdot \theta_4} - \omega_4^2 \cdot e^{j \cdot \theta_4}) + p \cdot [j \cdot e^{j \cdot (\theta_3 + \pi)} - \omega_3^2 \cdot e^{j \cdot (\theta_3 + \pi)}] + h \cdot (j \cdot \alpha_5 \cdot e^{j \cdot \theta_5} - \omega_5^2 \cdot e^{j \cdot \theta_5}) \end{aligned}$$

$$\begin{aligned} f \cdot \alpha_4 \cdot (-\sin(\theta_4 + \delta_4) + j \cdot \cos(\theta_4 + \delta_4)) - f \cdot \omega_4^2 \cdot (\cos(\theta_4 + \delta_4) + j \cdot \sin(\theta_4 + \delta_4)) \dots \\ + 2 \cdot f\dot{d}ot \cdot \omega_4 \cdot (-\sin(\theta_4 + \delta_4) + j \cdot \cos(\theta_4 + \delta_4)) + f\ddot{d}ot \cdot (\cos(\theta_4 + \delta_4) + j \cdot \sin(\theta_4 + \delta_4)) = \end{aligned}$$

$$\begin{aligned} c \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4)) \dots \\ + p \cdot \alpha_3 \cdot (-\sin(\theta_3 + \pi) + j \cdot \cos(\theta_3 + \pi)) - p \cdot \omega_3^2 \cdot (\cos(\theta_3 + \pi) + j \cdot \sin(\theta_3 + \pi)) \dots \\ + h \cdot \alpha_5 \cdot (-\sin(\theta_5) + j \cdot \cos(\theta_5)) - h \cdot \omega_5^2 \cdot (\cos(\theta_5) + j \cdot \sin(\theta_5)) \end{aligned}$$

$$\begin{aligned} \text{Let } C := f \cdot \alpha_4 \cdot \sin(\theta_4 + \delta_4) + f \cdot \omega_4^2 \cdot \cos(\theta_4 + \delta_4) + 2 \cdot f\dot{d}ot \cdot \omega_4 \cdot \sin(\theta_4 + \delta_4) \dots \\ + -c \cdot (\alpha_4 \cdot \sin(\theta_4) + \omega_4^2 \cdot \cos(\theta_4)) - p \cdot (\alpha_3 \cdot \sin(\theta_3 + \pi) + \omega_3^2 \cdot \cos(\theta_3 + \pi)) \dots \\ + -h \cdot \omega_5^2 \cdot \cos(\theta_5) \end{aligned}$$

$$C = -3003.51 \frac{\text{in}}{\text{s}^2}$$

$$\begin{aligned} D := -f \cdot \alpha_4 \cdot \cos(\theta_4 + \delta_4) + f \cdot \omega_4^2 \cdot \sin(\theta_4 + \delta_4) - 2 \cdot f\dot{d}ot \cdot \omega_4 \cdot \cos(\theta_4 + \delta_4) \dots \\ + c \cdot (\alpha_4 \cdot \cos(\theta_4) - \omega_4^2 \cdot \sin(\theta_4)) + p \cdot (\alpha_3 \cdot \cos(\theta_3 + \pi) + \omega_3^2 \cdot \sin(\theta_3 + \pi)) \dots \\ + -h \cdot \omega_5^2 \cdot \sin(\theta_5) \end{aligned}$$

$$D = -1295.03 \frac{\text{in}}{\text{s}^2}$$

then, similar to the velocity solution,

$$\alpha_5 := \frac{C \cdot \tan(\theta_4 + \delta_4) - D}{h \cdot (\cos(\theta_5) + \sin(\theta_5) \cdot \tan(\theta_4 + \delta_4))} \quad \alpha_5 = 1025.02 \frac{\text{rad}}{\text{s}^2}$$

$$f\ddot{d}ot := \frac{C - h \cdot \alpha_5 \cdot \sin(\theta_5)}{\cos(\theta_4 + \delta_4)} \quad f\ddot{d}ot = -5505.35 \frac{\text{in}}{\text{s}^2}$$

12. Using the terms in the acceleration equation in step 10, define the components of $\mathbf{A}_D$ and then combine them to determine the total acceleration of point D .

$$\mathbf{A}_{Dtangential} := f \cdot \alpha_4 \cdot (-\sin(\theta_4 + \delta_4) + j \cdot \cos(\theta_4 + \delta_4))$$

$$\mathbf{A}_{Dnormal} := -f \cdot \omega_4^2 \cdot (\cos(\theta_4 + \delta_4) + j \cdot \sin(\theta_4 + \delta_4))$$

$$\mathbf{A}_{Dcoriolis} := 2 \cdot f \cdot \dot{\omega}_4 \cdot (-\sin(\theta_4 + \delta_4) + j \cdot \cos(\theta_4 + \delta_4))$$

$$\mathbf{A}_{Dslip} := f \cdot \ddot{\delta}_4 \cdot (\cos(\theta_4 + \delta_4) + j \cdot \sin(\theta_4 + \delta_4))$$

$$\mathbf{A}_D := \mathbf{A}_{Dtangential} + \mathbf{A}_{Dnormal} + \mathbf{A}_{Dcoriolis} + \mathbf{A}_{Dslip}$$

$$\mathbf{A}_D = (-6592.71 + 3687.93j) \frac{\text{in}}{\text{s}^{2.00}} \quad \text{in the local coordinate frame}$$

$$\text{Magnitude:} \quad A_D := |\mathbf{A}_D| \quad A_D = 7554.11 \frac{\text{in}}{\text{s}^{2.00}}$$

$$\text{Angle in local coordinate frame:} \quad \theta_{AD} := \arg(\mathbf{A}_D) \quad \theta_{AD} = 150.778 \text{ deg}$$

Angle in global coordinate frame:

$$\theta_{ADXY} := \theta_{AD} + \delta \quad \theta_{ADXY} = 326.233 \text{ deg}$$

The angular acceleration of block 6 is the same as link 4, that is,

$$\alpha_6 := \alpha_4 \quad \alpha_6 = 692.987 \frac{\text{rad}}{\text{s}^2}$$

PROBLEM 7-77

Statement: The linkage in Figure P7-30b has link 3 perpendicular to the X-axis and links 2 and 4 are parallel to each other. Find α_4 , $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_P$ if $\omega_2 = 15 \text{ rad/sec CW}$ and $\alpha_2 = 100 \text{ rad/sec}^2 \text{ CW}$. Use the acceleration difference graphical method.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 2.75 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 3.26 \cdot \text{in}$$

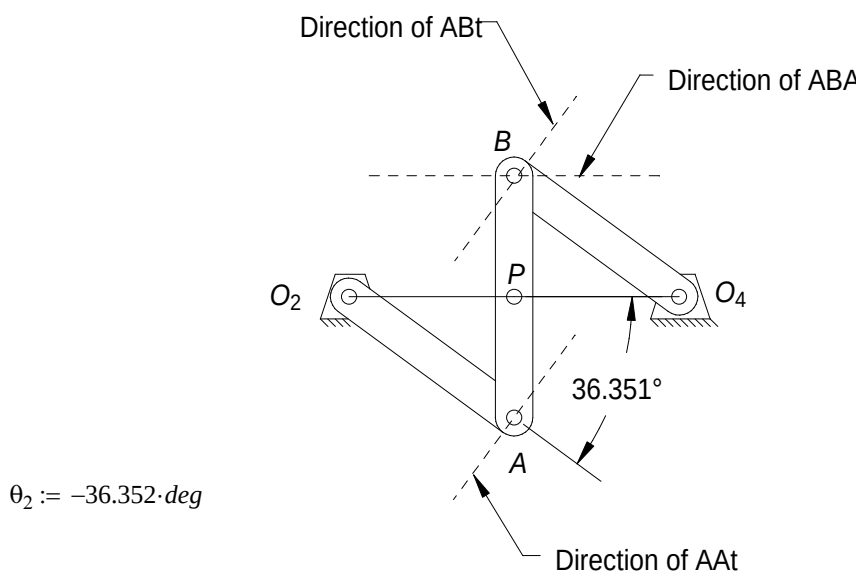
$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 2.75 \cdot \text{in} \quad \text{Link 1 (O}_4 \text{ to O}_2) \quad d := 4.43 \cdot \text{in}$$

$$\text{Coupler point data:} \quad p := 1.63 \cdot \text{in} \quad \delta_3 := 0 \cdot \text{deg}$$

$$\text{Input crank angular velocity} \quad \omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1} \text{ CW} \quad \alpha_2 := -100 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-30b and Mathcad file P0777.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A and B, we will need θ_3 , ω_3 , and θ_4 . From the graphical position solution above (in the local xy coordinate system),

$$\theta_3 := 90.000 \cdot \text{deg}$$

$$\theta_3 = 90.000 \text{ deg}$$

$$\theta_4 := 180 \cdot \text{deg} + \theta_2$$

$$\theta_4 = 143.648 \text{ deg}$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_4 - \theta_2)}{b \cdot \sin(\theta_3 - \theta_4)}$$

$$\omega_3 = 1.924 \times 10^{-15} \text{ rad} \cdot \text{sec}^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_4 - \theta_3)}$$

$$\omega_4 = 15.000 \text{ rad} \cdot \text{sec}^{-1}$$

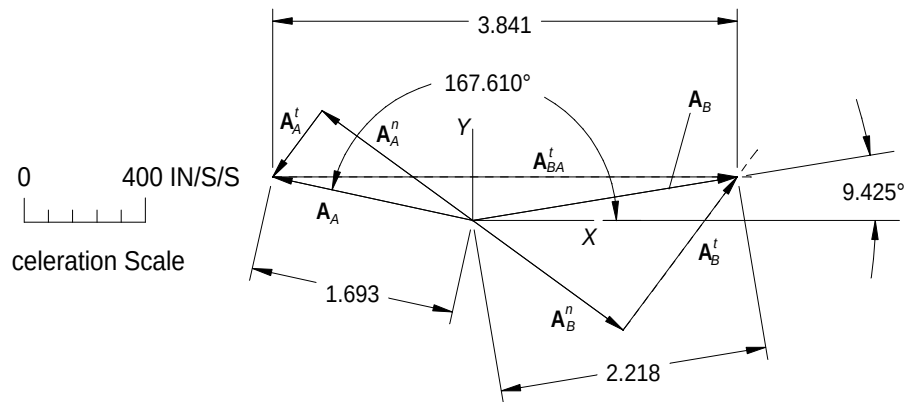
3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$
4. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_{Bn} := c \cdot \omega_4^2$$

$$A_{Bn} = 618.750 \text{ in} \cdot \text{sec}^{-2}$$

$$\begin{aligned}
 \theta_{ABn} &:= \theta_4 + 180 \cdot \text{deg} & \theta_{ABn} &= 323.648 \text{ deg} \\
 A_{An} &:= a \cdot \omega_2^2 & A_{An} &= 618.750 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{AAn} &:= \theta_2 + 180 \cdot \text{deg} & \theta_{AAn} &= 143.648 \text{ deg} \\
 A_{At} &:= |a \cdot \alpha_2| & A_{At} &= 275.000 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{AA_t} &:= \theta_2 - 90 \cdot \text{deg} & \theta_{AA_t} &= -126.352 \text{ deg} \\
 A_{BA_n} &:= b \cdot \omega_3^2 & A_{BA_n} &= 0.000 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{ABA_n} &:= \theta_3 + 180 \cdot \text{deg} & \theta_{ABA_n} &= 270.000 \text{ deg}
 \end{aligned}$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of $\theta_{ABn} + \alpha$ and $\mathbf{A}_A^n$ at an angle of $\theta_{AAn} + \alpha$. From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_A^t$ at an angle of $\theta_{AA_t} + \alpha$. From the tip of $\mathbf{A}_A^t$, draw $\mathbf{A}_{BA}^n$ at an angle of $\theta_{BA_n} + \alpha$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B^n$ and $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B^t$, $\mathbf{A}_B^t$, and $\mathbf{A}_{BA}^t$.



6. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 400 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_A := 1.693 \cdot k_a \quad A_A = 677.2 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 167.610 \text{ deg}$$

$$A_B := 2.218 \cdot k_a \quad A_B = 887.2 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 9.425 \text{ deg}$$

$$A_{BA_t} := 3.841 \cdot k_a \quad A_{BA_t} = 1536.4 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{A_{BA_t}}{b} \quad \alpha_3 = 471.288 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CW}$$

7. For point P, equation 7.4 becomes: $\mathbf{A}_P = \mathbf{A}_A + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$, where

$$A_{PA_n} := p \cdot \omega_3^2 \quad A_{PA_n} = 0.000 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{APA_n} := \theta_3 + \delta_3 + 180 \cdot \text{deg} \quad \theta_{APA_n} = 270.000 \text{ deg}$$

$$A_{PA_t} := p \cdot \alpha_3 \quad A_{PA_t} = 768.2 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{APA_t} := \theta_{APA_n} + 90 \cdot \text{deg} \quad \theta_{APA_t} = 360.000 \text{ deg}$$

8. Since $\omega_3 = 0$ the acceleration at P is one-half that at B and is in the same direction as $\mathbf{A}_B$.

PROBLEM 7-78

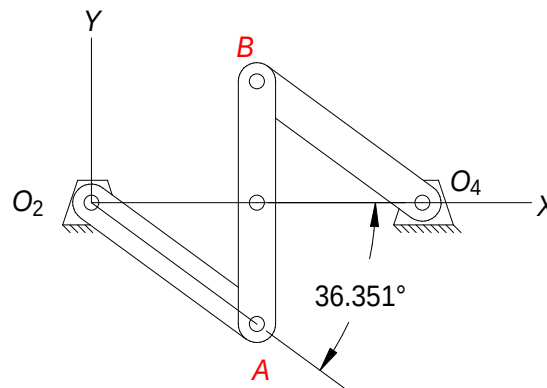
Statement: The linkage in Figure P6-33b has link 3 perpendicular to the X-axis and links 2 and 4 are parallel to each other. Find α_4 , $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_P$ if $\omega_2 = 15 \text{ rad/sec CW}$ and $\alpha_2 = 100 \text{ rad/sec}^2 \text{ CW}$. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 2.75 \cdot \text{in}$	Link 3 (A to B)	$b := 3.26 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 2.75 \cdot \text{in}$	Link 1 (O_4 to B)	$d := 4.43 \cdot \text{in}$
Coupler point data:	$R_{PA} := 1.63 \cdot \text{in}$	$\delta_3 := 0 \cdot \text{deg}$	
Input crank angular velocity	$\omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1}$	CW	
Input crank angular acceleration	$\alpha_2 := -100 \cdot \text{rad} \cdot \text{sec}^{-2}$	CW	

Solution: See Figure P6-33b and Mathcad file P0778.

1. Draw the linkage to scale and label it.



From the figure, $\theta_2 := 36.351 \cdot \text{deg}$

2. From the problem statement we have:

$$\theta_3 := 90 \cdot \text{deg} \quad \theta_4 := \theta_2 + 180 \cdot \text{deg} \quad \theta_4 = 216.351 \cdot \text{deg}$$

3. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \sin(\theta_4 - \theta_2)}{b \cdot \sin(\theta_3 - \theta_4)} \quad \omega_3 = 1.924 \times 10^{-15} \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \sin(\theta_2 - \theta_3)}{c \cdot \sin(\theta_4 - \theta_3)} \quad \omega_4 = 15.000 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

4. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (-335 - 588j) \frac{\text{in}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A)$$

$$\text{The acceleration of pin A is } A_A = 677.1 \frac{\text{in}}{\text{sec}^2} \quad \text{at } \theta_{AA} = -119.7 \cdot \text{deg}$$

5. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A := c \cdot \sin(\theta_4) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_4) \quad E := b \cdot \cos(\theta_3)$$

$$A = -1.630 \text{ in} \quad B = 3.260 \text{ in} \quad D = -2.215 \text{ in} \quad E = 0.0 \text{ in}$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$C = 833.683 \text{ in} \cdot \text{sec}^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = -954.989 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \quad \alpha_3 = -471.320 \frac{\text{rad}}{\text{sec}^2} \quad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_4 = 431.175 \frac{\text{rad}}{\text{sec}^2}$$

6. Use equation 7.13c to determine the acceleration of point B.

$$\mathbf{A}_B := c \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$\mathbf{A}_B = (30509 - 14941j) \frac{\text{mm}}{\text{sec}^2} \quad A_B := |\mathbf{A}_B| \quad \theta_{AB} := \arg(\mathbf{A}_B)$$

The acceleration of pin B is $A_B = 1337.5 \frac{\text{in}}{\text{sec}^2}$ at $\theta_{AB} = -26.092 \text{ deg}$

7. Use equations 7.32 to find the acceleration of the point P.

$$\mathbf{A}_{PA} := R_{PA} \cdot \alpha_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3)) \dots$$

$$+ -R_{PA} \cdot \omega_3^2 \cdot (\cos(\theta_3 + \delta_3) + j \cdot \sin(\theta_3 + \delta_3))$$

$$\mathbf{A}_P := \mathbf{A}_A + \mathbf{A}_{PA}$$

$$A_P := |\mathbf{A}_P| \quad A_P = 730.37 \frac{\text{in}}{\text{sec}^2} \quad \arg(\mathbf{A}_P) = -53.649 \text{ deg}$$

PROBLEM 7-79

Statement: The crosshead linkage shown in Figure P7-30c has 2 *DOF* with inputs at cross heads 2 and 5. Find $\mathbf{A}_B$, $\mathbf{A}_{P_3}$, and $\mathbf{A}_{P_4}$ if the crossheads are each moving toward the origin of the *XY* coordinate system with a speed of 20 in/sec and are decelerating at 75 in/sec². Use a graphical method.

Given: Link lengths:

$$\text{Link 3 (A to B)} \quad b := 34.32 \cdot \text{in} \quad \text{Link 4 (B to C)} \quad c := 50.4 \cdot \text{in}$$

$$\text{Link 2 Y-offset} \quad a := 59.5 \cdot \text{in} \quad \text{Link 5 X-offset} \quad d := 57 \cdot \text{in}$$

$$\text{Coupler point data:} \quad AP_3 := 31.5 \cdot \text{in} \quad BP_3 := 22.2 \cdot \text{in}$$

$$BP_4 := 41.52 \cdot \text{in} \quad CP_4 := 27.0 \cdot \text{in}$$

$$\text{Input velocities:} \quad V_{2Y} := -20 \cdot \text{in} \cdot \text{sec}^{-1} \quad V_{5X} := -20 \cdot \text{in} \cdot \text{sec}^{-1}$$

$$\text{Input accelerations} \quad A_A := -75 \cdot \text{in} \cdot \text{sec}^{-2} \quad A_C := -75 \cdot \text{in} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-30c and Mathcad file P0779.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.
2. In order to solve for the accelerations at point B, we will need θ_3 , ω_3 , θ_4 , and ω_4 .
From the velocity solution (Problem 6-98),

$$\theta_3 := 320.123 \cdot \text{deg}$$

$$\omega_3 := 1.749 \cdot \text{rad} \cdot \text{sec}^{-1} \text{ CCW}$$

$$\theta_4 := 122.718 \cdot \text{deg}$$

$$\omega_4 := -1.177 \cdot \text{rad} \cdot \text{sec}^{-1} \text{ CW}$$

3. Use equation 7.4 to (graphically) determine the magnitude of the acceleration at point B, the magnitude of the relative accelerations $\mathbf{A}_{BA}$, $\mathbf{A}_{BC}$ and the angular accelerations of links 3 and 4. The equations to be solved (simultaneously) graphically are

$$\mathbf{A}_B = \mathbf{A}_A + \mathbf{A}_{BA}$$

$$\mathbf{A}_B = \mathbf{A}_C + \mathbf{A}_{BC}$$

Eliminating $\mathbf{A}_B$ by combining equations and expanding,

$$\mathbf{A}_A + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n) = \mathbf{A}_C + (\mathbf{A}_{BC}^t + \mathbf{A}_{BC}^n)$$

$$|\mathbf{A}_A| = 75.000 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{AA} := 270 \cdot \text{deg}$$

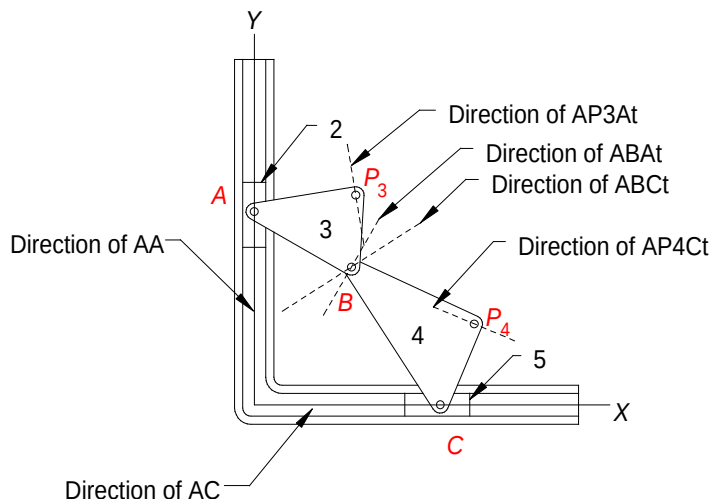
$$|\mathbf{A}_C| = 75.000 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{AC} := 180 \cdot \text{deg}$$

$$A_{BA_n} := b \cdot \omega_3^2 \quad A_{BA_n} = 104.985 \text{ in} \cdot \text{sec}^{-2}$$

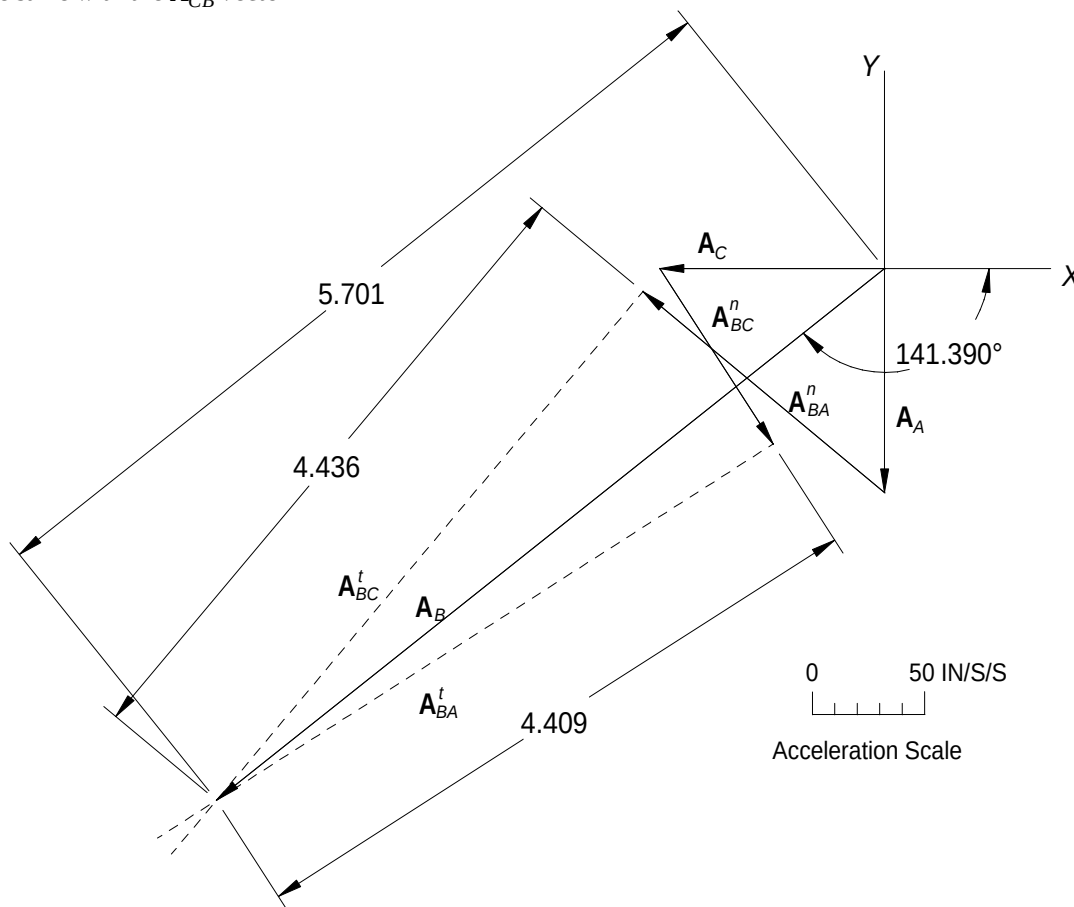
$$\theta_{BA_n} := \theta_3 - 180 \cdot \text{deg} \quad \theta_{BA_n} = 140.123 \text{ deg}$$

$$A_{BC_n} := c \cdot \omega_4^2 \quad A_{BC_n} = 69.821 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{BC_n} := \theta_4 - 180 \cdot \text{deg} \quad \theta_{BC_n} = -57.282 \text{ deg}$$



- Choose a convenient velocity scale and layout the known vectors $\mathbf{A}_A$ and $\mathbf{A}_C$
- From the tip of $\mathbf{A}_A$, draw known vector $\mathbf{A}_{BA}^n$.
- From the tip of $\mathbf{A}_C$, draw known vector $\mathbf{A}_{BC}^n$.
- From the tips of $\mathbf{A}_{BA}^n$ and $\mathbf{A}_{BC}^n$ draw construction lines in the known directions of these vectors.
- From the origin, draw a construction line to the intersection of the two construction lines in step d.
- Complete the vector triangle by drawing $\mathbf{A}_{BA}$ from the tip of $\mathbf{A}_A$ to the intersection of the $\mathbf{A}_B$ construction line and drawing $\mathbf{A}_B$ from the tail of $\mathbf{A}_A$ to the intersection of the $\mathbf{A}_{BA}$ construction line. And then do the same with the $\mathbf{A}_{CB}$ vector



6. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 50 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_B := 5.701 \cdot k_a \quad A_B = 285.1 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -141.390 \text{ deg}$$

$$A_{BA_t} := 4.409 \cdot k_a \quad A_{BA_t} = 220.5 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_3 := \frac{A_{BA_t}}{b} \quad \alpha_3 = 6.423 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CW}$$

$$A_{BC_t} := 4.436 \cdot k_a \quad A_{BC_t} = 221.8 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_4 := \frac{A_{BC_t}}{c} \quad \alpha_4 = 4.401 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CCW}$$

7. From the graphical solution, the angles from A to P_3 and C to P_4 are

$$\theta_{P3} := 9.250 \cdot \text{deg}$$

$$\theta_{P4} := 67.313 \cdot \text{deg}$$

8. The accelerations of points P_3 and P_4 with respect to A are:

$$A_{P3An} := AP_3 \cdot \omega_3^2 \quad A_{P3An} = 96.359 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{AP3An} := \theta_{P3} - 180 \cdot \text{deg} \quad \theta_{AP3An} = -170.750 \text{ deg}$$

$$A_{P3At} := AP_3 \cdot \alpha_3 \quad A_{P3At} = 202.336 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{AP3At} := \theta_{P3} - 90 \cdot \text{deg} \quad \theta_{AP3At} = -80.750 \text{ deg}$$

$$A_{P4Cn} := CP_4 \cdot \omega_4^2 \quad A_{P4Cn} = 37.404 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{AP4Cn} := \theta_{P4} - 180 \cdot \text{deg} \quad \theta_{AP4Cn} = -112.687 \text{ deg}$$

$$A_{P4Ct} := CP_4 \cdot \alpha_4 \quad A_{P4Ct} = 118.821 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{AP4Ct} := \theta_{P4} + 90 \cdot \text{deg} \quad \theta_{AP4Ct} = 157.313 \text{ deg}$$

3. Use equation 7.4 to (graphically) determine the magnitude of the acceleration at points P_3 and P_4 .

$$\mathbf{A}_{P3} = \mathbf{A}_A + (\mathbf{A}_{P3A}^t + \mathbf{A}_{P3A}^n)$$

$$\mathbf{A}_{P4} = \mathbf{A}_C + (\mathbf{A}_{P4C}^t + \mathbf{A}_{P4C}^n)$$

6. From the graphical solution at right,

Acceleration scale factor

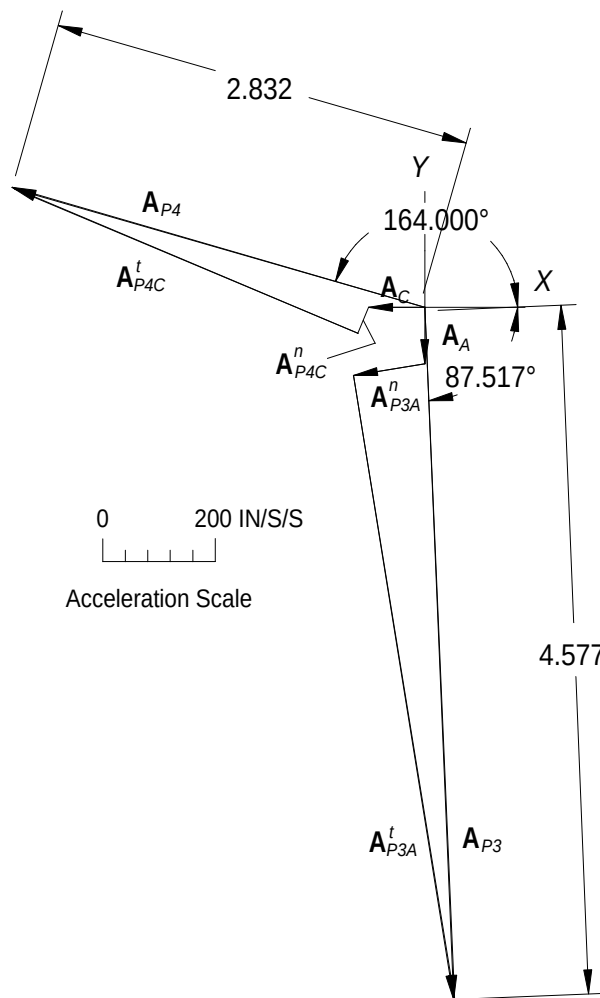
$$k_a := 200 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_{P3} := 4.577 \cdot k_a \quad A_{P3} = 915.4 \text{ in} \cdot \text{sec}^{-2}$$

at an angle of -87.517 deg

$$A_{P4} := 2.832 \cdot k_a \quad A_{P4} = 566.4 \text{ in} \cdot \text{sec}^{-2}$$

at an angle of 164.000 deg



PROBLEM 7-80

Statement: The crosshead linkage shown in Figure P7-30c has 2 *DOF* with inputs at cross heads 2 and 5. Find $\mathbf{A}_B$, $\mathbf{A}_{P_3}$, and $\mathbf{A}_{P_4}$ if the crossheads are each moving toward the origin of the *XY* coordinate system with a speed of 20 in/sec and are decelerating at 75 in/sec². Use an analytical method.

Given: Link lengths:

$$\text{Link 3 (A to B)} \quad b := 34.32 \cdot \text{in} \quad \text{Link 4 (B to C)} \quad c := 50.4 \cdot \text{in}$$

$$\text{Link 2 Y-offset} \quad a := 59.5 \cdot \text{in} \quad \text{Link 5 X-offset} \quad d := 57 \cdot \text{in}$$

$$\text{Coupler point data:} \quad AP_3 := 31.5 \cdot \text{in} \quad BP_3 := 22.2 \cdot \text{in}$$

$$BP_4 := 41.52 \cdot \text{in} \quad CP_4 := 27.0 \cdot \text{in}$$

$$\text{Input velocities:} \quad V_{2Y} := -20 \cdot \text{in} \cdot \text{sec}^{-1} \quad V_{5X} := -20 \cdot \text{in} \cdot \text{sec}^{-1}$$

$$\text{Input accelerations} \quad A_A := -75 \cdot \text{in} \cdot \text{sec}^{-2} \quad A_C := -75 \cdot \text{in} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-30c and Mathcad file P0780.

1. Draw the linkage to a convenient scale and label it.
2. In order to solve for the accelerations at point B, we will need θ_3 , ω_3 , θ_4 , and ω_4 . From the velocity solution (Problem 6-98),

$$\theta_3 := 320.123 \cdot \text{deg} \quad \theta_4 := 122.718 \cdot \text{deg}$$

$$\omega_3 := 1.749 \cdot \text{rad} \cdot \text{sec}^{-1} \text{ CCW} \quad \omega_4 := -1.177 \cdot \text{rad} \cdot \text{sec}^{-1} \text{ CW}$$

3. Use equation 7.4 to (analytically) determine the magnitude of the acceleration at point B, the magnitude of the relative accelerations $\mathbf{A}_{BA}$, $\mathbf{A}_{BC}$ and the angular accelerations of links 3 and 4. The equations to be solved simultaneously are

$$\mathbf{A}_B = \mathbf{A}_A + \mathbf{A}_{BA} \quad \mathbf{A}_B = \mathbf{A}_C + \mathbf{A}_{BC}$$

Eliminating $\mathbf{A}_B$ by combining equations and expanding,

$$\mathbf{A}_A + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n) = \mathbf{A}_C + (\mathbf{A}_{BC}^t + \mathbf{A}_{BC}^n)$$

$$|A_A| = 75.000 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{AA} := 270 \cdot \text{deg}$$

$$|A_C| = 75.000 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{AC} := 180 \cdot \text{deg}$$

$$A_{BA_n} := b \cdot \omega_3^2 \quad A_{BA_n} = 104.985 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{BA_n} := \theta_3 - 180 \cdot \text{deg} \quad \theta_{BA_n} = 140.123 \text{ deg}$$

$$A_{BC_n} := c \cdot \omega_4^2 \quad A_{BC_n} = 69.821 \text{ in} \cdot \text{sec}^{-2}$$

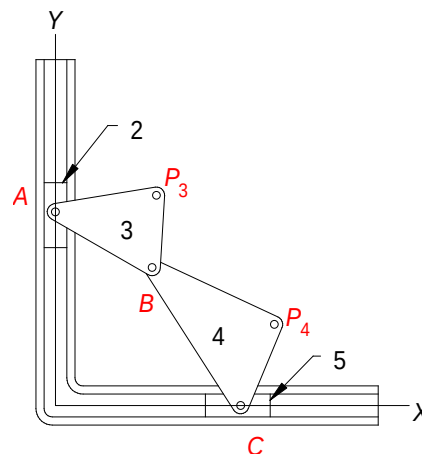
$$\theta_{BC_n} := \theta_4 - 180 \cdot \text{deg} \quad \theta_{BC_n} = -57.282 \text{ deg}$$

Let

$$A_{Ay} := -|A_A| \quad A_{Ay} = -75.000 \text{ in} \cdot \text{sec}^{-2}$$

$$A_{BA_{nx}} := A_{BA_n} \cdot \cos(\theta_{BA_n}) \quad A_{BA_{nx}} = -80.568 \text{ in} \cdot \text{sec}^{-2}$$

$$A_{BA_{ny}} := A_{BA_n} \cdot \sin(\theta_{BA_n}) \quad A_{BA_{ny}} = 67.310 \text{ in} \cdot \text{sec}^{-2}$$



$$\begin{aligned}
 A_{Cx} &:= -|A_C| & A_{Cx} &= -75.000 \text{ in} \cdot \text{sec}^{-2} \\
 A_{BCnx} &:= A_{BCn} \cdot \cos(\theta_{ABCn}) & A_{BCnx} &= 37.738 \text{ in} \cdot \text{sec}^{-2} \\
 A_{BCny} &:= A_{BCn} \cdot \sin(\theta_{ABCn}) & A_{BCny} &= -58.743 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{ABAt} &:= \theta_{ABAn} - 90 \cdot \text{deg} & \theta_{ABAt} &= 50.123 \text{ deg} \\
 \theta_{ABct} &:= \theta_{ABCn} + 90 \cdot \text{deg} & \theta_{ABct} &= 32.718 \text{ deg} \\
 m_{BCt} &:= \tan(\theta_{ABct}) & m_{BCt} &= 0.642 \\
 m_{BAAt} &:= \tan(\theta_{ABAt}) & m_{BAAt} &= 1.197
 \end{aligned}$$

Write the equations for the two lines that represent the vectors $\mathbf{A}_{BAAt}$ and $\mathbf{A}_{BCt}$ and solve them simultaneously to find the coordinates of $\mathbf{A}_B$:

$$\begin{aligned}
 A_{Bx} &:= \frac{m_{BCt}(A_{Cx} + A_{BCnx}) - m_{BAAt}A_{BAAnx} - A_{BCny} + (A_{Ay} + A_{BAAny})}{m_{BCt} - m_{BAAt}} \\
 A_{By} &:= m_{BCt}[A_{Bx} - (A_{Cx} + A_{BCnx})] + A_{BCny} \\
 A_{Bx} &= -222.804 \text{ in} \cdot \text{sec}^{-2} & A_{By} &= -177.941 \text{ in} \cdot \text{sec}^{-2} \\
 A_B &:= \sqrt{A_{Bx}^2 + A_{By}^2} & A_B &= 285.140 \text{ in} \cdot \text{sec}^{-2} \\
 \theta_{AB} &:= \text{atan2}(A_{Bx}, A_{By}) & \theta_{AB} &= -141.388 \text{ deg}
 \end{aligned}$$

A similar approach can be taken to find $\mathbf{A}_{P3}$ and $\mathbf{A}_{P4}$.

PROBLEM 7-81

Statement: The crosshead linkage shown in Figure P7-30c has 2 *DOF* with inputs at cross heads 2 and 5. At $t = 0$, crosshead 2 is at rest at the origin of the global XY coordinate system and crosshead 5 is at rest at (70,0). Write a computer program to find and plot $\mathbf{A}_{P_3}$ and $\mathbf{A}_{P_4}$ for the first 5 sec of motion if $\mathbf{A}_2 = 0.5 \text{ in/sec}^2$ upward and $\mathbf{A}_5 = 0.5 \text{ in/sec}^2$ to the left.

Given:

Link lengths:

Link 3 (A to B) $b := 34.32 \cdot \text{in}$

Link 4 (B to C) $c := 50.4 \cdot \text{in}$

Link 2 Y-offset $a := 59.5 \cdot \text{in}$

Link 5 X-offset $d := 57 \cdot \text{in}$

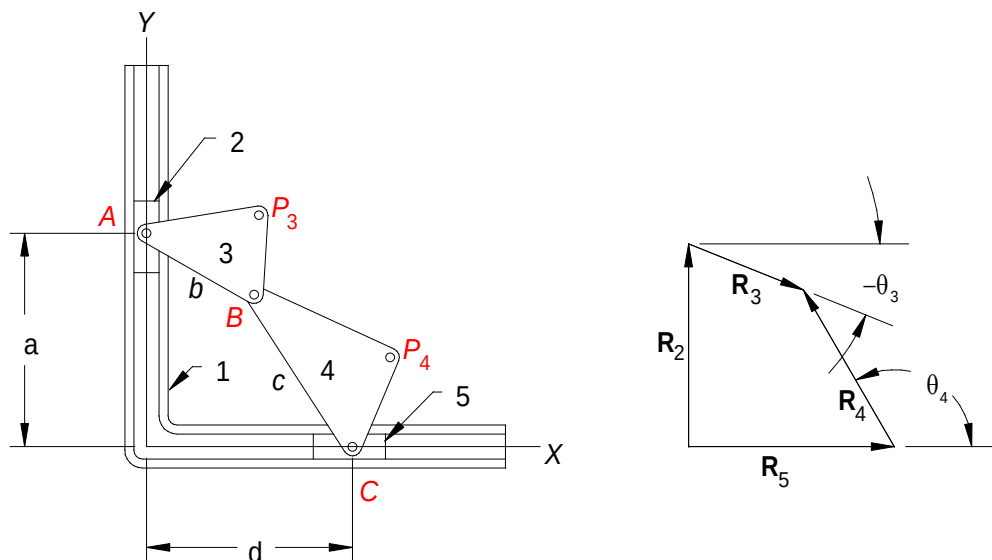
Coupler point data: $AP_3 := 31.5 \cdot \text{in}$ $BP_3 := 22.2 \cdot \text{in}$ $\delta_3 := 39.128 \cdot \text{deg}$

$BP_4 := 41.52 \cdot \text{in}$ $CP_4 := 27.0 \cdot \text{in}$ $\delta_4 := -55.405 \cdot \text{deg}$

Input accelerations $A_2 := 0.5 \cdot \text{in} \cdot \text{sec}^{-2}$ $A_5 := -0.5 \cdot \text{in} \cdot \text{sec}^{-2}$

Solution: See Figure P7-30c and Mathcad file P0781.

1. Draw the linkage to a convenient scale and label it.



$$t := 0 \cdot \text{sec}, 0.1 \cdot \text{sec} \dots 5 \cdot \text{sec} \quad a_0 := 0 \cdot \text{in} \quad d_0 := 70 \cdot \text{in}$$

$$a(t) := a_0 + 0.5 \cdot A_2 \cdot t^2 \quad d(t) := d_0 + 0.5 A_5 \cdot t^2$$

$$V_2(t) := A_2 \cdot t \quad V_5(t) := A_5 \cdot t$$

2. Define a vector loop as shown above. Summing the vectors around the loop,

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_5 = 0$$

$$b \cdot \cos(\theta_3) - c \cdot \cos(\theta_4) - d = 0$$

$$a + b \cdot \sin(\theta_3) - c \cdot \sin(\theta_4) = 0$$

3. Substitute the polar notation and Euler equivalents and solve for θ_3 and θ_4 using the method of Section 4.5 and the identities in equations 4.9.

$$K_1(t) := \frac{a(t)^2 - b^2 + c^2 + d(t)^2}{2 \cdot c}$$

$$A(t) := K_1(t) - d(t) \quad B(t) := -2 \cdot a(t) \quad C(t) := K_1(t) + d(t)$$

$$\theta_4(t) := 2 \cdot \text{atan2}\left(2 \cdot A(t), -B(t) - \sqrt{B(t)^2 - 4 \cdot A(t) \cdot C(t)}\right)$$

$$K_2(t) := -\frac{a(t)^2 + b^2 - c^2 + d(t)^2}{2 \cdot b}$$

$$D(t) := K_2(t) - d(t) \quad E(t) := -2 \cdot a(t) \quad F(t) := K_2(t) + d(t)$$

$$\theta_3(t) := 2 \cdot \text{atan2}\left(2 \cdot D(t), -E(t) - \sqrt{E(t)^2 - 4 \cdot D(t) \cdot F(t)}\right)$$

4. Differentiate the position equations with respect to time and solve for ω_3 and ω_4 .

$$-b \cdot \omega_3 \cdot \sin(\theta_3) - c \cdot \omega_4 \cdot \sin(\theta_4) - V_5 = 0$$

$$V_2 + b \cdot \omega_3 \cdot \cos(\theta_3) - c \cdot \omega_4 \cdot \cos(\theta_4) = 0$$

$$\omega_3(t) := -\frac{V_2(t) \cdot \sin(\theta_4(t)) + V_5(t) \cdot \cos(\theta_4(t))}{b \cdot \sin(\theta_3(t) + \theta_4(t))}$$

$$\omega_4(t) := \frac{V_2(t) \cdot \sin(\theta_3(t)) - V_5(t) \cdot \cos(\theta_3(t))}{c \cdot \sin(\theta_3(t) + \theta_4(t))}$$

5. Differentiate the velocity equations with respect to time and solve for α_3 and α_4 .

$$-b \cdot \left(\alpha_3 \cdot \sin(\theta_3) + \omega_3^2 \cdot \cos(\theta_3) \right) - c \cdot \left(\alpha_4 \cdot \sin(\theta_4) + \omega_4^2 \cdot \cos(\theta_4) \right) - A_5 = 0$$

$$A_2 + b \cdot \left(\alpha_3 \cdot \cos(\theta_3) - \omega_3^2 \cdot \sin(\theta_3) \right) - c \cdot \left(\alpha_4 \cdot \cos(\theta_4) - \omega_4^2 \cdot \sin(\theta_4) \right) = 0$$

$$A'(t) := -b \cdot \sin(\theta_3(t)) \quad B'(t) := -c \cdot \sin(\theta_4(t))$$

$$C'(t) := -b \cdot \omega_3(t)^2 \cdot \cos(\theta_3(t)) - c \cdot \omega_3(t)^2 \cdot \cos(\theta_4(t)) - A_5$$

$$D'(t) := b \cdot \cos(\theta_3(t)) \quad E'(t) := -c \cdot \cos(\theta_4(t))$$

$$F'(t) := -b \cdot \omega_3(t)^2 \cdot \sin(\theta_3(t)) + c \cdot \omega_3(t)^2 \cdot \sin(\theta_4(t)) + A_2$$

$$\alpha_3(t) := \frac{B'(t) \cdot F'(t) - C'(t) \cdot E'(t)}{A'(t) \cdot E'(t) - B'(t) \cdot D'(t)} \quad \alpha_4(t) := \frac{C'(t) \cdot D'(t) - A'(t) \cdot F'(t)}{A'(t) \cdot E'(t) - B'(t) \cdot D'(t)}$$

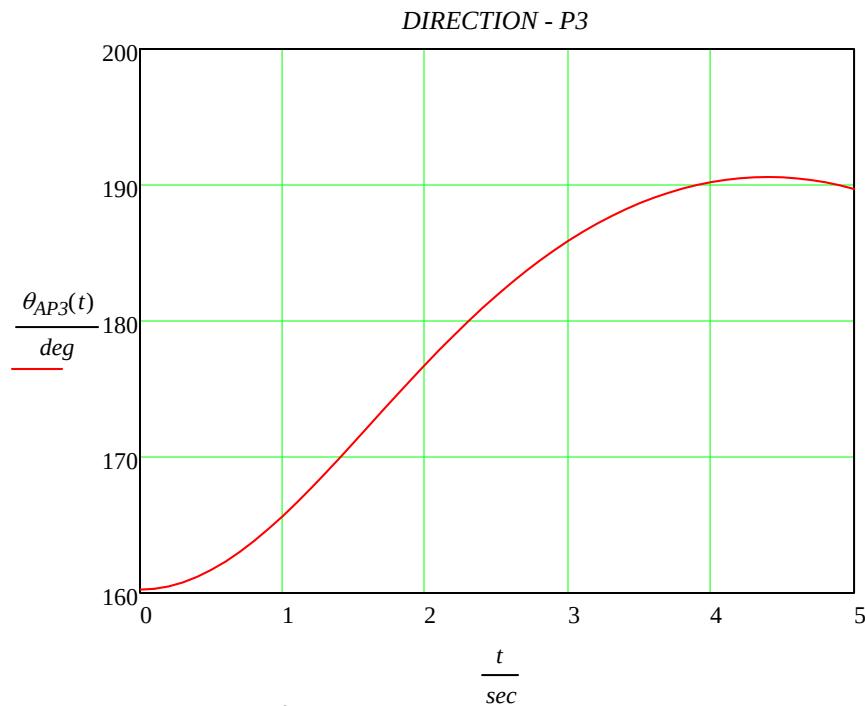
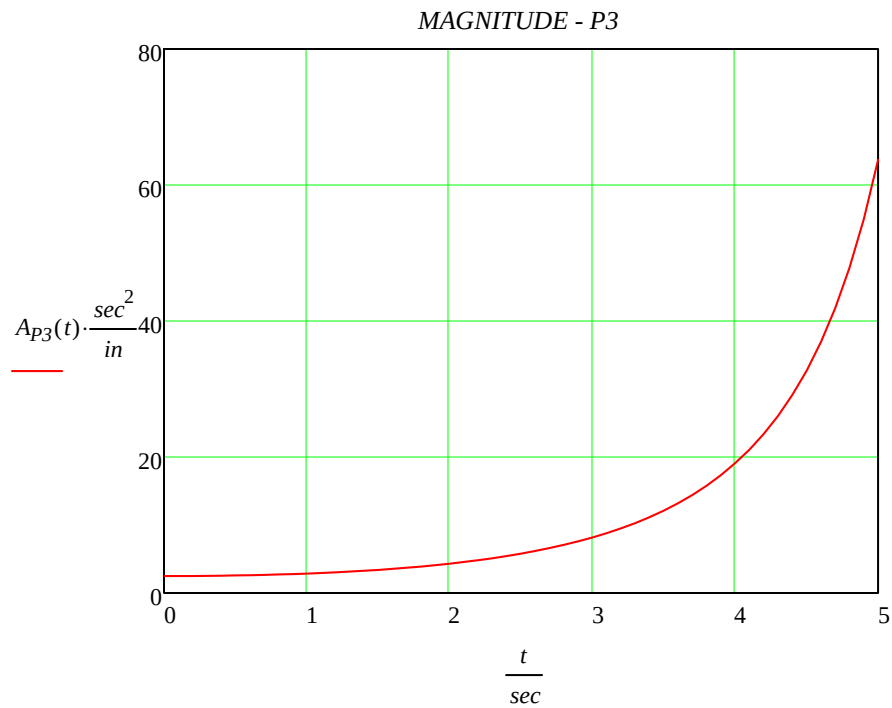
6. Using equations 6.32, solve for the accelerations at P_3 .

$$\mathbf{A}_{P3A}(t) := AP_3 \cdot \left[\alpha_3(t) \cdot \left(-\sin(\theta_3(t) + \delta_3) + j \cdot \cos(\theta_3(t) + \delta_3) \right) \dots \right. \\ \left. + -\omega_3(t)^2 \cdot \left(\cos(\theta_3(t) + \delta_3) + j \cdot \sin(\theta_3(t) + \delta_3) \right) \right]$$

$$\mathbf{A}_{P3}(t) := j \cdot A_2 + \mathbf{A}_{P3A}(t) \quad A_{P3}(t) := |\mathbf{A}_{P3}(t)| \quad \theta_{AP3}(t) := \arg(\mathbf{A}_{P3}(t))$$

$$\theta_{AP3}(t) := \text{if}(\theta_{AP3}(t) \leq 0, \theta_{AP3}(t) + 2 \cdot \pi, \theta_{AP3}(t))$$

7. Plot the magnitude and direction of the acceleration at P_3 .



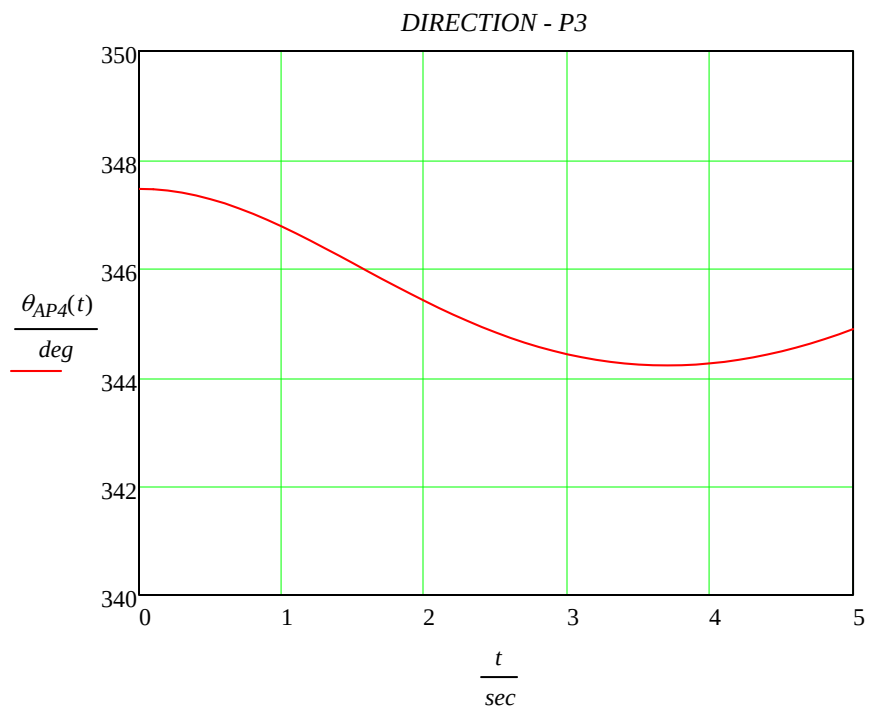
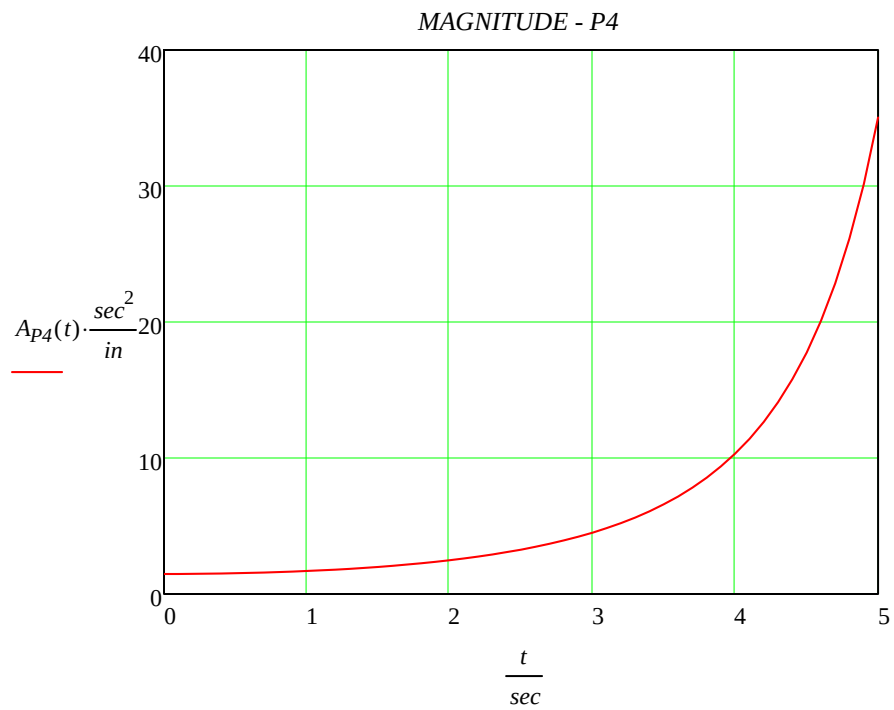
8. Using equations 6.32, solve for the accelerations at P_4 .

$$\mathbf{A}_{P4A}(t) := CP_4 \cdot \begin{bmatrix} \alpha_4(t) \cdot (-\sin(\theta_4(t) + \delta_4) + j \cdot \cos(\theta_4(t) + \delta_4)) \dots \\ + \omega_4(t)^2 \cdot (\cos(\theta_4(t) + \delta_4) + j \cdot \sin(\theta_4(t) + \delta_4)) \end{bmatrix}$$

$$\mathbf{A}_{P4}(t) := j \cdot A_5 + \mathbf{A}_{P4A}(t) \quad A_{P4}(t) := |\mathbf{A}_{P4}(t)| \quad \theta_{AP4}(t) := \arg(\mathbf{A}_{P4}(t))$$

$$\theta_{AP4}(t) := \text{if}(\theta_{AP4}(t) \leq 0, \theta_{AP4}(t) + 2 \cdot \pi, \theta_{AP4}(t))$$

9. Plot the magnitude and direction of the acceleration at P_4 .



PROBLEM 7-82

Statement: The linkage in Figure P7-30d has the path of slider 6 perpendicular to the global X-axis and link 2 aligned with the global X-axis. Find α_2 and $\mathbf{A}_A$ in the position shown if the velocity of the slider is constant at 20 in/sec downward. Use the acceleration difference graphical method.

Given:

Link lengths:

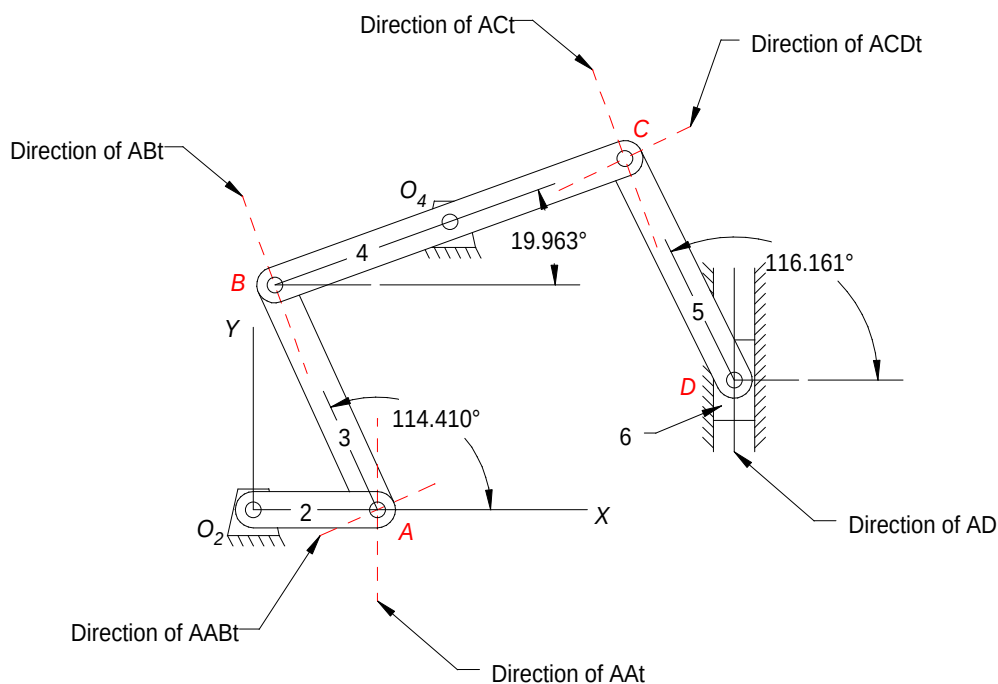
Link 2 (O_2 to A)	$a := 12 \cdot \text{in}$	Link 3 (A to B)	$b := 24 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 18 \cdot \text{in}$	Link 5 (C to D)	$f := 24 \cdot \text{in}$
Link 4 (O_4 to C)	$e := 18 \cdot \text{in}$		
Link 1 X-offset	$d_X := 19 \cdot \text{in}$	Link 1 Y-offset	$d_Y := 28 \cdot \text{in}$

Angle BO_4C $\delta_4 := 0.0 \cdot \text{deg}$

Input slider velocity $V_D := 20 \cdot \text{in} \cdot \text{sec}^{-1}$ $\theta_{VD} := -90.0 \cdot \text{deg}$

Solution: See Figure P7-30d and Mathcad file P0782.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A , B and C , we will need θ_2 , ω_2 , θ_3 , ω_3 , and θ_4 , ω_4 . From the graphical position solution above and the velocity solution (see Problem 6-99),

$$\theta_2 := 0.000 \cdot \text{deg} \quad \omega_2 := 1.648 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CCW}$$

$$\theta_3 := 114.410 \cdot \text{deg} \quad \omega_3 := 0.282 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CCW}$$

$$\theta_{41} := 199.963 \cdot \text{deg} \quad \omega_4 := 1.003 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$$

$$\theta_{42} := 19.963 \cdot \text{deg}$$

$$\theta_5 := 116.161 \cdot \text{deg} \quad \omega_5 := 0.286 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

4. For point C , this becomes: $(\mathbf{A}_C^t + \mathbf{A}_C^n) = \mathbf{A}_D + (\mathbf{A}_{CD}^t + \mathbf{A}_{CD}^n)$, where

$$A_{Cn} := e \cdot \omega_4^2 \quad A_{Cn} = 18.108 \text{ in} \cdot \text{sec}^{-2}$$

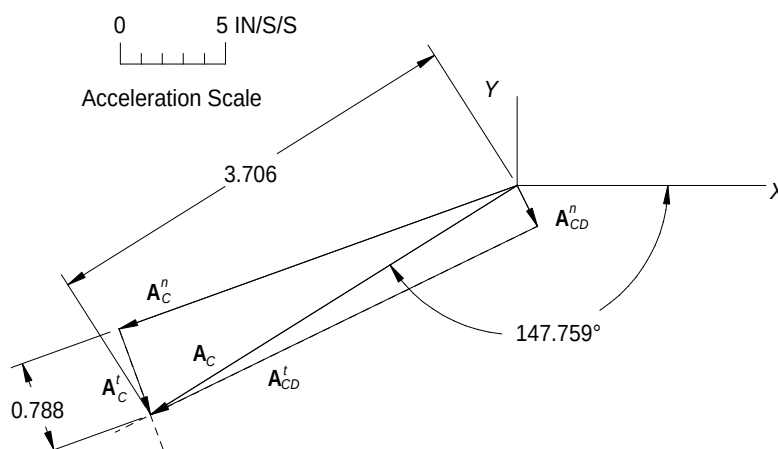
$$\theta_{ACn} := \theta_{42} + 180 \cdot \text{deg} \quad \theta_{ACn} = 199.963 \text{ deg}$$

$$A_D := 0 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$A_{CDn} := f \cdot \omega_5^2 \quad A_{CDn} = 1.963 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ACDn} := \theta_5 + 180 \cdot \text{deg} \quad \theta_{ACDn} = 296.161 \text{ deg}$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_C^n$ at an angle of $\theta_{ACn} + \alpha$ and $\mathbf{A}_D^n$ at an angle of $\theta_{ADn} + \alpha$. From the tip of $\mathbf{A}_D^n$, draw $\mathbf{A}_D^t$ at an angle of $\theta_{ADt} + \alpha$. From the tip of $\mathbf{A}_D^t$, draw $\mathbf{A}_{CD}^n$ at an angle of $\theta_{CDn} + \alpha$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_C^n$ and $\mathbf{A}_{CD}^n$, draw construction lines in the directions of $\mathbf{A}_C^t$ and $\mathbf{A}_{CD}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_C^t$, $\mathbf{A}_C$, and $\mathbf{A}_{CD}^t$.



6. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 5.0 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_C := 3.706 \cdot k_a \quad A_C = 18.530 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -147.759 \text{ deg}$$

$$A_{Ct} := 0.788 \cdot k_a \quad A_{Ct} = 3.940 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_4 := \frac{A_{Ct}}{e} \quad \alpha_4 = 0.219 \text{ rad} \cdot \text{sec}^{-2} \quad \text{CW}$$

Since link 4 is straight and the distance from O_4 to B is the same as to C ,

$$A_B := A_C \quad A_B = 18.530 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 32.241 \text{ deg}$$

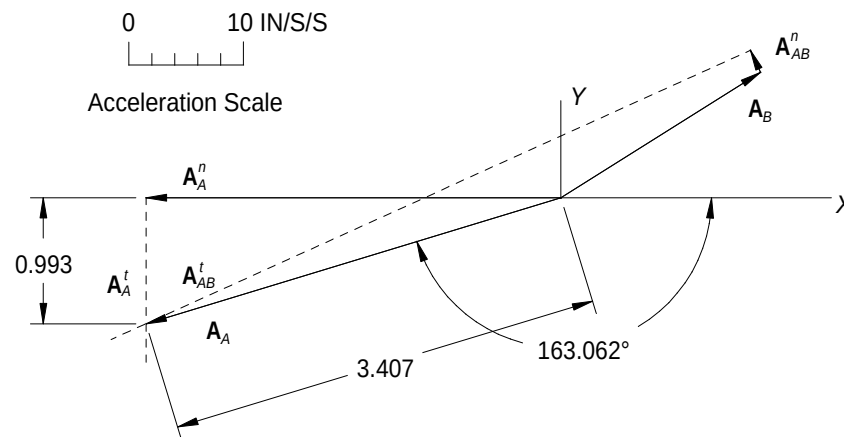
7. For point A , equation 7.4 becomes: $(\mathbf{A}_A^t + \mathbf{A}_A^n) = \mathbf{A}_B + (\mathbf{A}_{AB}^t + \mathbf{A}_{AB}^n)$, where

$$A_{An} := a \cdot \omega_2^2 \quad A_{An} = 32.591 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AAn} = 180.000 \text{ deg}$$

$$A_{ABn} := b \cdot \omega_3^2 \quad A_{ABn} = 1.909 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AABn} := \theta_3 \quad \theta_{AABn} = 114.410 \text{ deg}$$



8. From the graphical solution above,

Acceleration scale factor $k_a := 10 \cdot \frac{\text{in}}{\text{sec}^2}$

$A_A := 3.407 \cdot k_a$ $A_A = 34.070 \text{ in} \cdot \text{sec}^{-2}$ at an angle of -163.062 deg

$A_{At} := 0.993 \cdot k_a$ $A_{At} = 9.930 \text{ in} \cdot \text{sec}^{-2}$

$\alpha_2 := \frac{A_{At}}{a}$ $\alpha_2 = 0.828 \text{ rad} \cdot \text{sec}^{-2}$ CW

PROBLEM 7-83

Statement: The linkage in Figure P7-30d has the path of slider 6 perpendicular to the global X-axis and link 2 aligned with the global X-axis. Find α_2 and $\mathbf{A}_A$ in the position shown if the velocity of the slider is constant at 20 in/sec downward. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$c := 12 \cdot \text{in}$	Link 3 (A to B)	$b := 24 \cdot \text{in}$
Link 4 (O_4 to B)	$a := 18 \cdot \text{in}$	Link 5 (C to D)	$f := 24 \cdot \text{in}$
Link 4 (O_4 to C)	$e := 18 \cdot \text{in}$		
Link 1 X-offset	$d_X := 19 \cdot \text{in}$	Link 1 Y-offset	$d_Y := 28 \cdot \text{in}$
Link 1 (O_2 to O_4)	$d := \sqrt{d_X^2 + d_Y^2}$	$d = 33.838 \cdot \text{in}$	

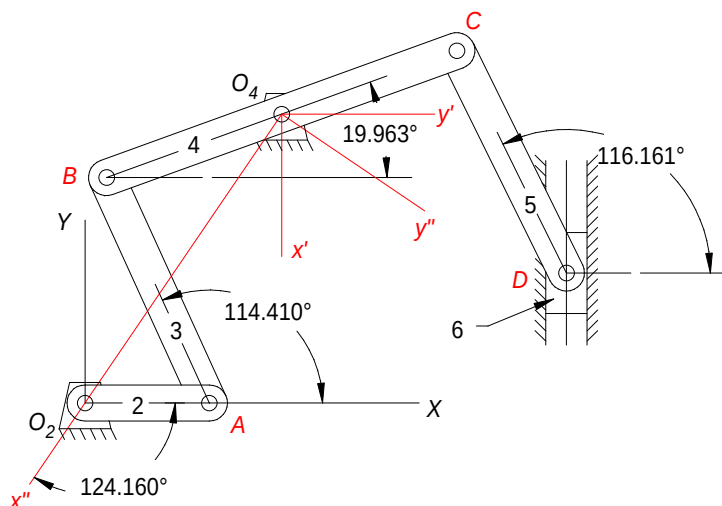
Axis rotation angle: $\delta := -124.160 \cdot \text{deg}$

Input slider velocity $V_D := 20 \cdot \text{in} \cdot \text{sec}^{-1}$ Downward (positive x' direction)

Input slider acceleration $A_D := 0 \cdot \text{in} \cdot \text{sec}^{-2}$

Solution: See Figure P7-30d and Mathcad file P0783.

1. Draw the linkage to a convenient scale and label it.



2. From the graphical position solution above and the velocity solution (see Problem 6-99),

Local $x''y''$ coordinate system:

Output $\theta_2 := 124.160 \cdot \text{deg}$ $\omega_2 := 1.648 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW

$\theta_3 := 58.570 \cdot \text{deg}$ $\omega_3 := 0.282 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW

Input $\theta_{41} := 324.123 \cdot \text{deg}$ $\omega_4 := -1.003 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

Local $x'y'$ coordinate system:

$\theta_{42} := 109.963 \cdot \text{deg}$ $\omega_4 := -1.003 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

$\theta_5 := 206.161 \cdot \text{deg}$ $\omega_5 := -0.286 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW

3. Solve equations 7.16b and c for α_2 (which is α_4 in this case):

$$\alpha_4 := \frac{A_D \cdot \cos(\theta_5) + e \cdot \omega_4^2 \cdot \cos(\theta_5 - \theta_{42}) - f \cdot \omega_5^2}{e \cdot \sin(\theta_5 - \theta_{42})} \quad \alpha_4 = -0.219 \cdot \text{rad} \cdot \text{sec}^{-2}$$

4. Use equations 7.12 to determine the angular accelerations of link 2.

$$A := c \cdot \sin(\theta_2) \quad B := b \cdot \sin(\theta_3) \quad D := c \cdot \cos(\theta_2) \quad E := b \cdot \cos(\theta_3)$$

$$A = 9.930 \cdot \text{in} \quad B = 20.479 \cdot \text{in} \quad D = -6.738 \cdot \text{in} \quad E = 12.515 \cdot \text{in}$$

$$C := a \cdot \alpha_4 \cdot \sin(\theta_{41}) + a \cdot \omega_4^2 \cdot \cos(\theta_{41}) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_2^2 \cdot \cos(\theta_2)$$

$$C = 36.278 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$F := a \cdot \alpha_4 \cdot \cos(\theta_{41}) - a \cdot \omega_4^2 \cdot \sin(\theta_{41}) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_2^2 \cdot \sin(\theta_2)$$

$$F = 32.758 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\alpha_2 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \quad \alpha_2 = -0.827 \cdot \frac{\text{rad}}{\text{sec}^2}$$

9. Use equation 7.13c to determine the acceleration of point A.

$$\mathbf{A}_A := c \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - c \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_A = (26.510 - 21.397j) \cdot \frac{\text{in}}{\text{sec}^2} \quad A_A := |\mathbf{A}_A| \quad \theta_{AA} := \arg(\mathbf{A}_A) + \delta$$

$$\text{The acceleration of pin A is } A_A = 34.068 \cdot \frac{\text{in}}{\text{sec}^2} \text{ at } \theta_{AA} = -163.068 \cdot \text{deg} \quad (\text{Global})$$

PROBLEM 7-84

Statement: The linkage in Figure P7-30a has the path of slider 6 perpendicular to the global X-axis and link 2 aligned with the global X-axis at $t = 0$. Write a computer program or use an equation solver to find and plot $\mathbf{A}_D$ as a function of θ_2 , over the possible range of motion of link 2, in the global XY coordinate system.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 12 \cdot \text{in}$	Link 3 (A to B)	$b := 24 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 18 \cdot \text{in}$	Link 5 (C to D)	$f := 24 \cdot \text{in}$
Link 4 (O_4 to C)	$e := 18 \cdot \text{in}$		
Link 1 X-offset	$d_X := 19 \cdot \text{in}$	Link 1 Y-offset	$d_Y := 28 \cdot \text{in}$

$$\text{Link 1 } (O_2 \text{ to } O_4) \quad d := \sqrt{d_X^2 + d_Y^2} \quad d = 33.838 \text{ in}$$

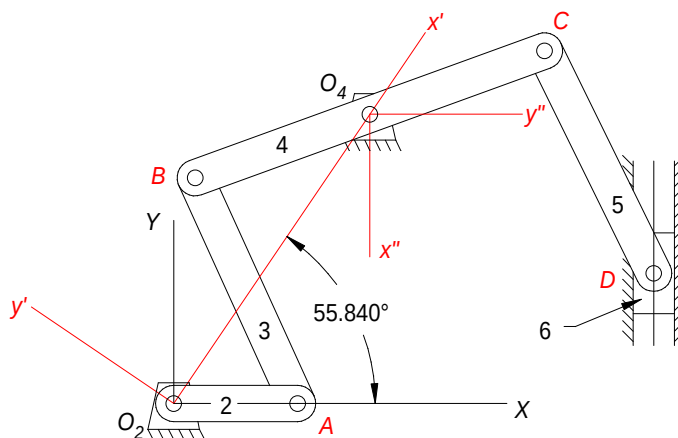
$$\text{Axis rotation angle:} \quad \delta_1 := 55.840 \cdot \text{deg} \quad \delta_2 := -90.000 \cdot \text{deg}$$

$$\text{Input velocity \& accel:} \quad \omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\text{Crank angle:} \quad \theta_{2XY} := -70 \cdot \text{deg}, -69 \cdot \text{deg} \dots 180 \cdot \text{deg} \quad \theta_2(\theta_{2XY}) := \theta_{2XY} - \delta_1$$

Solution: See Figure P7-30d and Mathcad file P0784.

1. Draw the linkage to a convenient scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_1 = 2.8198 \quad K_2 := \frac{d}{c} \quad K_2 = 1.8799$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 2.4005$$

$$A(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 - K_2 \cdot \cos(\theta_2(\theta_{2XY})) + K_3$$

$$B(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$C(\theta_{2XY}) := K_1 - (K_2 + 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{4xy}(\theta_{2XY}) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_{2XY}), -B(\theta_{2XY}) - \sqrt{B(\theta_{2XY})^2 - 4 \cdot A(\theta_{2XY}) \cdot C(\theta_{2XY})} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.4099 \quad K_5 = -2.6753$$

$$D(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 + K_4 \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

$$E(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$F(\theta_{2XY}) := K_1 + (K_4 - 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{3xy}(\theta_{2XY}) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_{2XY}), -E(\theta_{2XY}) - \sqrt{E(\theta_{2XY})^2 - 4 \cdot D(\theta_{2XY}) \cdot F(\theta_{2XY})} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3(\theta_{2XY}) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{4xy}(\theta_{2XY}) - \theta_2(\theta_{2XY}))}{\sin(\theta_{3xy}(\theta_{2XY}) - \theta_{4xy}(\theta_{2XY}))}$$

$$\omega_4(\theta_{2XY}) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2(\theta_{2XY}) - \theta_{3xy}(\theta_{2XY}))}{\sin(\theta_{4xy}(\theta_{2XY}) - \theta_{3xy}(\theta_{2XY}))}$$

7. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_{2XY}) := c \cdot \sin(\theta_{4xy}(\theta_{2XY})) \quad B(\theta_{2XY}) := b \cdot \sin(\theta_{3xy}(\theta_{2XY}))$$

$$D(\theta_{2XY}) := c \cdot \cos(\theta_{4xy}(\theta_{2XY})) \quad E(\theta_{2XY}) := b \cdot \cos(\theta_{3xy}(\theta_{2XY}))$$

$$C(\theta_{2XY}) := a \cdot \alpha_2 \cdot \sin(\theta_2(\theta_{2XY})) + a \cdot \omega_2^2 \cdot \cos(\theta_2(\theta_{2XY})) \dots \\ + b \cdot \omega_3(\theta_{2XY})^2 \cdot \cos(\theta_{3xy}(\theta_{2XY})) - c \cdot \omega_4(\theta_{2XY})^2 \cdot \cos(\theta_{4xy}(\theta_{2XY}))$$

$$F(\theta_{2XY}) := a \cdot \alpha_2 \cdot \cos(\theta_2(\theta_{2XY})) - a \cdot \omega_2^2 \cdot \sin(\theta_2(\theta_{2XY})) \dots \\ + b \cdot \omega_3(\theta_{2XY})^2 \cdot \sin(\theta_{3xy}(\theta_{2XY})) + c \cdot \omega_4(\theta_{2XY})^2 \cdot \sin(\theta_{4xy}(\theta_{2XY}))$$

$$\alpha_4(\theta_{2XY}) := \frac{C(\theta_{2XY}) \cdot E(\theta_{2XY}) - B(\theta_{2XY}) \cdot F(\theta_{2XY})}{A(\theta_{2XY}) \cdot E(\theta_{2XY}) - B(\theta_{2XY}) \cdot D(\theta_{2XY})}$$

8. Transform θ_4 to the local x'y' coordinate system and add 180 deg to get the link from O_4 to C, which is the input angle to the crank-slider portion of the linkage.

$$\theta_4(\theta_{2XY}) := \theta_{4xy}(\theta_{2XY}) + 180 \cdot \text{deg} + \delta_1 - \delta_2$$

9. Determine θ_5 in the x'y' coordinate system using equation 4.17. Offset: $cc := 27.5 \cdot \text{in}$

$$\theta_5(\theta_{2XY}) := \text{asin} \left(-\frac{e \cdot \sin(\theta_4(\theta_{2XY})) - cc}{f} \right) + \pi$$

10. Determine the angular velocity of link 5 using equation 6.22a:

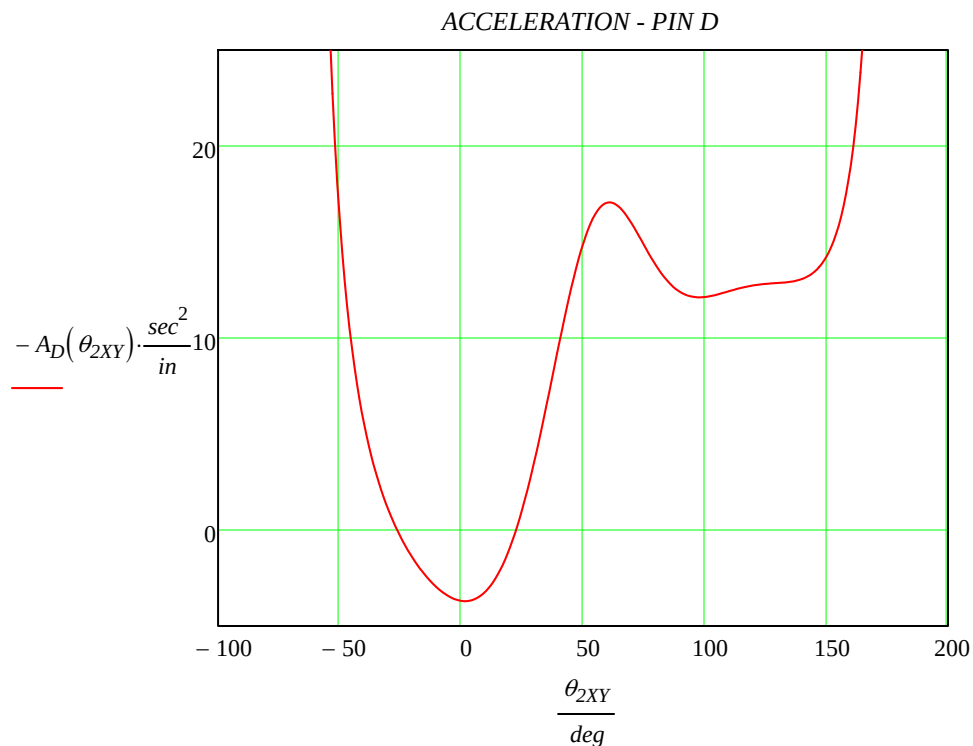
$$\omega_5(\theta_{2XY}) := \frac{e}{f} \cdot \frac{\cos(\theta_4(\theta_{2XY}))}{\cos(\theta_5(\theta_{2XY}))} \cdot \omega_4(\theta_{2XY})$$

11. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5(\theta_{2XY}) := \frac{e \cdot \alpha_4(\theta_{2XY}) \cdot \cos(\theta_4(\theta_{2XY})) - e \cdot \omega_4(\theta_{2XY})^2 \cdot \sin(\theta_4(\theta_{2XY})) \dots + f \cdot \omega_5(\theta_{2XY})^2 \cdot \sin(\theta_5(\theta_{2XY}))}{f \cdot \cos(\theta_5(\theta_{2XY}))}$$

12. Use equation 7.16e for the acceleration of pin D.

$$A_D(\theta_{2XY}) := -e \cdot \alpha_4(\theta_{2XY}) \cdot \sin(\theta_4(\theta_{2XY})) - e \cdot \omega_4(\theta_{2XY})^2 \cdot \cos(\theta_4(\theta_{2XY})) \dots + f \cdot \alpha_5(\theta_{2XY}) \cdot \sin(\theta_5(\theta_{2XY})) + f \cdot \omega_5(\theta_{2XY})^2 \cdot \cos(\theta_5(\theta_{2XY}))$$



Positive is up (positive Y direction in the global coordinate system).

PROBLEM 7-85

Statement: For the linkage of Figure P7-30e, write a computer program or use an equation solver to find and plot $\mathbf{A}_D$ in the global coordinate system for one revolution of link 2 if ω_2 is constant at 10 rad/sec CW.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 5.0 \cdot \text{in}$$

$$\text{Link 3 (A to B)} \quad b := 5.0 \cdot \text{in}$$

$$\text{Link 4 (O}_4 \text{ to B)} \quad c := 6.0 \cdot \text{in}$$

$$\text{Link 1 (O}_2 \text{ to O}_4) \quad d := 2.5 \cdot \text{in}$$

$$\text{Link 5 (B to C)} \quad e := 15 \cdot \text{in}$$

$$\text{Angle } BO_4C \quad \delta := 83.621 \cdot \text{deg}$$

$$\text{Crank angle: } \theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

$$\text{Input crank angular velocity} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Solution: See Figure P7-30e and Mathcad file P0785.

1. This sixbar drag-link mechanism can be analyzed as a fourbar Grashof double crank in series with a crank-slider mechanism using the output of the fourbar, link 4, as the input to the slider-crank.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_1 = 0.5000 \quad K_2 := \frac{d}{c} \quad K_2 = 0.4167$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 0.7042$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.5000 \quad K_5 = -0.4050$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_3(\theta_2))}$$

7. Use equations 7.12 to determine the angular acceleration of link 4.

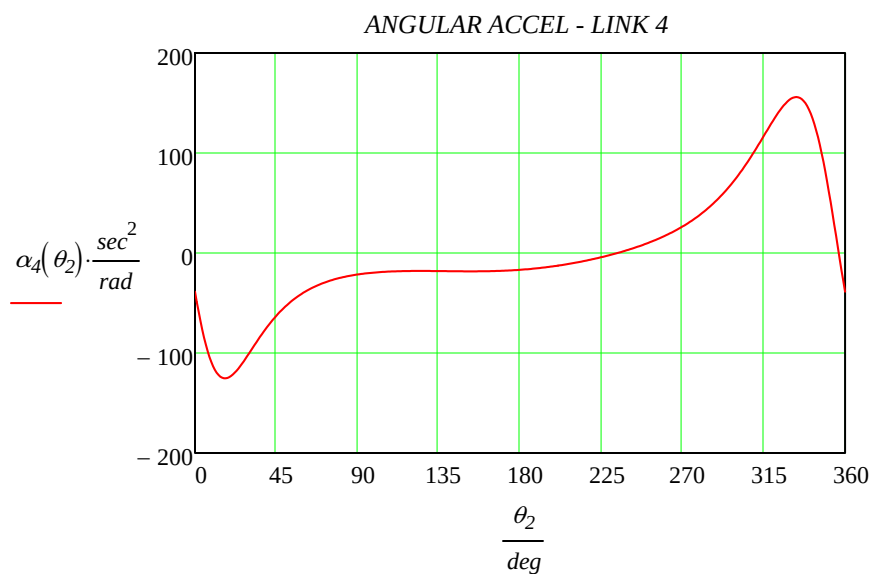
$$A(\theta_2) := c \cdot \sin(\theta_{41}(\theta_2)) \quad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_{41}(\theta_2)) \quad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) \dots \\ + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_{41}(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) \dots \\ + b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_{41}(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$



8. Transform θ_4 to the slider-crank coordinate system (origin at O_4) system.

$$\theta_{42}(\theta_2) := \theta_{41}(\theta_2) - \delta$$

9. Determine θ_5 using equation 4.17. Offset: $cc := 0 \cdot \text{in}$

$$\theta_5(\theta_2) := \text{asin}\left(-\frac{c \cdot \sin(\theta_{42}(\theta_2)) - cc}{e}\right) + \pi$$

10. Determine the angular velocity of link 5 using equation 6.22a:

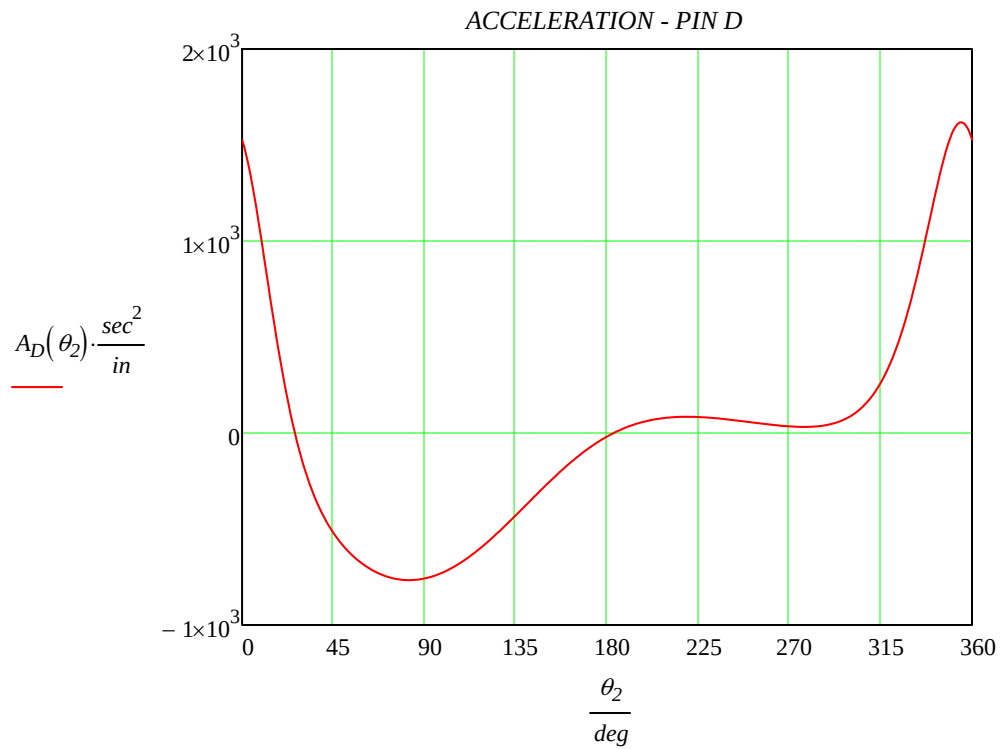
$$\omega_5(\theta_2) := \frac{c}{e} \cdot \frac{\cos(\theta_{42}(\theta_2))}{\cos(\theta_5(\theta_2))} \cdot \omega_4(\theta_2)$$

11. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5(\theta_2) := \frac{c \cdot \alpha_4(\theta_2) \cdot \cos(\theta_{42}(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_{42}(\theta_2)) \dots + e \cdot \omega_5(\theta_2)^2 \cdot \sin(\theta_5(\theta_2))}{e \cdot \cos(\theta_5(\theta_2))}$$

12. Use equation 7.16e for the acceleration of pin D.

$$A_D(\theta_2) := -c \cdot \alpha_4(\theta_2) \cdot \sin(\theta_{42}(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_{42}(\theta_2)) \dots + e \cdot \alpha_5(\theta_2) \cdot \sin(\theta_5(\theta_2)) + e \cdot \omega_5(\theta_2)^2 \cdot \cos(\theta_5(\theta_2))$$



PROBLEM 7-86

Statement: The linkage of Figure P6-33f has link 2 at 130 deg in the global XY coordinate system. Find A_D in the global coordinate system for the position shown if $\omega_2 = 15 \text{ rad/sec CW}$ and $\alpha_2 = 50 \text{ rad/sec}^2 \text{ CW}$. Use the acceleration difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 5.0 \cdot \text{in}$	Link 3 (A to B)	$b := 8.4 \cdot \text{in}$
Link 4 (O_4 to B)	$c := 2.5 \cdot \text{in}$	Link 1 (O_2 to O_4)	$d_1 := 12.5 \cdot \text{in}$
Link 5 (C to E)	$e := 8.9 \cdot \text{in}$	Link 5 (C to D)	$h := 5.9 \cdot \text{in}$
Link 6 (O_6 to E)	$f := 3.2 \cdot \text{in}$	Link 7 (D to F)	$k := 6.4 \cdot \text{in}$
Link 1 (O_2 to O_6)	$d_2 := 10.5 \cdot \text{in}$	Link 3 (A to C)	$g := 2.4 \cdot \text{in}$

Crank angle: $\theta_2 := 130 \cdot \text{deg}$

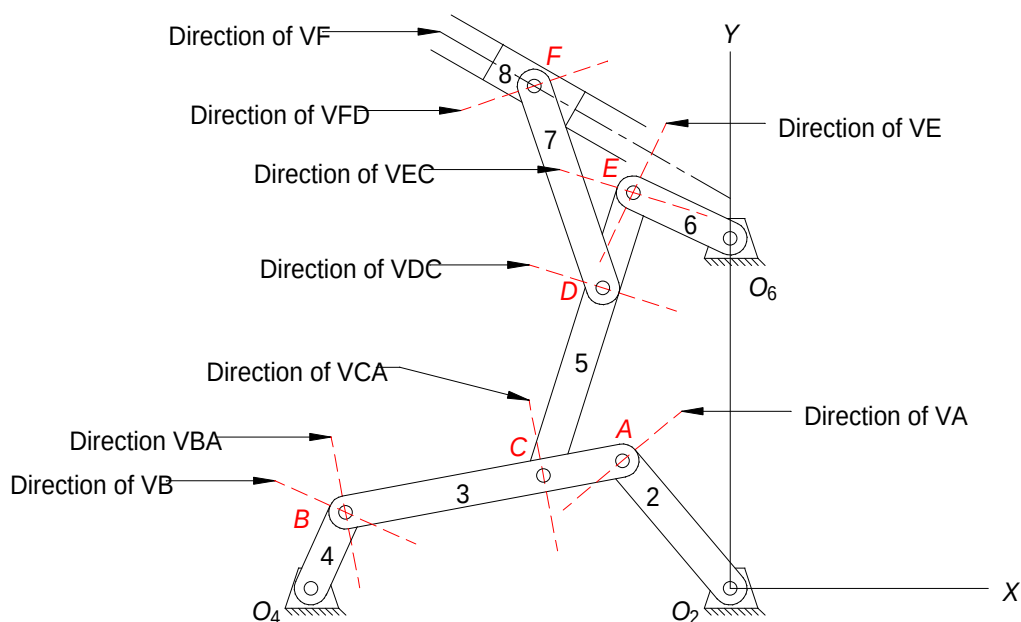
Slider axis offset and angle: $s := 11.7 \cdot \text{in}$ $\delta_7 := 150 \cdot \text{deg}$

Input crank angular velocity $\omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \text{ CW}$

Input crank angular acceleration $\alpha_2 := 50 \cdot \text{rad} \cdot \text{sec}^{-2} \text{ CW}$

Solution: See Figure P6-33f and Mathcad file P0786.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



Angles measured from layout:

$\theta_{VB} := -24.351 \cdot \text{deg}$	$\theta_{VBA} := -79.348 \cdot \text{deg}$	$\theta_{VCA} := \theta_{VBA}$
$\theta_{VE} := 64.594 \cdot \text{deg}$	$\theta_{VEC} := 162.461 \cdot \text{deg}$	$\theta_{VDC} := \theta_{VEC}$
$\theta_{VF} := 150.00 \cdot \text{deg}$	$\theta_{VFD} := 198.690 \cdot \text{deg}$	$\theta_3 := -169.348 \cdot \text{deg}$
$\theta_4 := 65.649 \cdot \text{deg}$	$\theta_5 := 72.461 \cdot \text{deg}$	$\theta_6 := 154.594 \cdot \text{deg}$ $\theta_7 := 108.690 \cdot \text{deg}$

2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

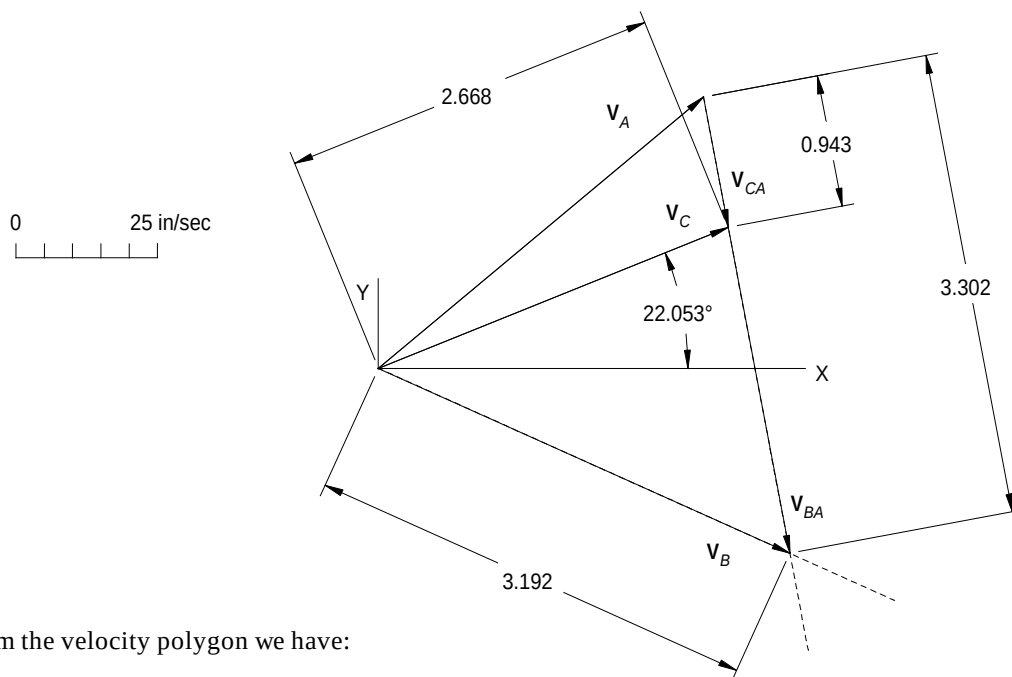
$$V_A := a \cdot \omega_2 \quad V_A = 75.000 \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{VA} = 40.000 \text{ deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equations to be solved graphically are

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \quad \text{and} \quad \mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
- From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity polygon we have:

$$\text{Velocity scale factor: } k_v := \frac{25 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_B := 3.192 \cdot \text{in} \cdot k_v \quad V_B = 79.800 \frac{\text{in}}{\text{sec}} \quad \theta_{VB} = -24.351 \text{ deg}$$

$$\omega_4 := \frac{V_B}{c} \quad \omega_4 = 31.920 \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

$$V_{BA} := 3.302 \cdot \text{in} \cdot k_v \quad V_{BA} = 82.550 \frac{\text{in}}{\text{sec}} \quad \theta_{VBA} = -79.348 \text{ deg}$$

$$\omega_3 := \frac{V_{BA}}{b} \quad \omega_3 = 9.827 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

5. Calculate the magnitude and direction of $\mathbf{V}_{CA}$ and determine the magnitude and velocity of $\mathbf{V}_C$ from the velocity polygon above.

$$V_{CA} := g \cdot \omega_3 \quad V_{CA} = 23.586 \frac{\text{in}}{\text{sec}} \quad \theta_{VCA} = -79.348 \text{ deg}$$

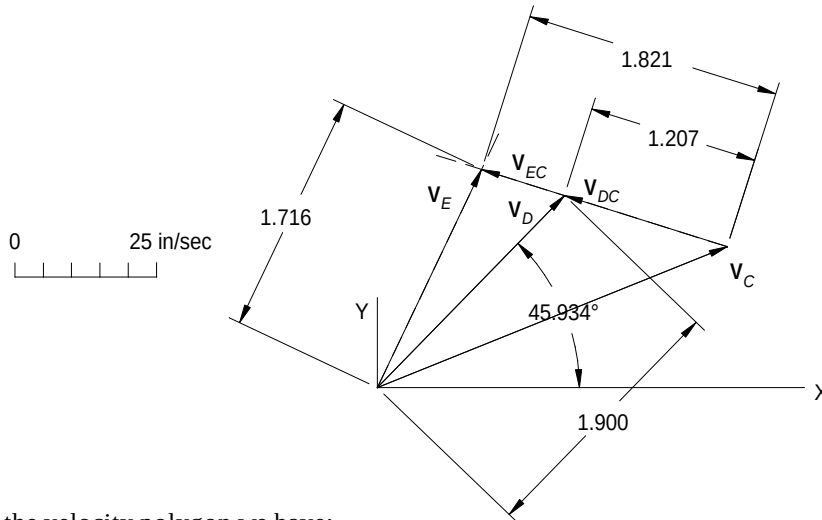
$$\text{Length of } \mathbf{V}_{CA} \text{ on velocity polygon: } v_{CA} := \frac{V_{CA}}{k_v} \quad v_{CA} = 0.943 \text{ in}$$

$$V_C := 2.668 \cdot \text{in} \cdot k_v \quad V_C = 66.700 \frac{\text{in}}{\text{sec}} \quad \theta_{VC} := 22.053 \text{ deg}$$

6. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point E , the magnitude of the relative velocity $\mathbf{V}_{ED}$, and the angular velocity of link 5. The equations to be solved graphically are

$$\mathbf{V}_E = \mathbf{V}_C + \mathbf{V}_{EC} \quad \text{and} \quad \mathbf{V}_D = \mathbf{V}_C + \mathbf{V}_{DC}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_C$.
- From the tip of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_{EC}$, magnitude unknown.
- From the tail of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_E$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{EC}$ from the tip of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_E$ construction line and drawing $\mathbf{V}_E$ from the tail of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_{EC}$ construction line.



7. From the velocity polygon we have:

$$\begin{aligned} \text{Velocity scale factor: } k_v &:= \frac{25 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}} \\ V_E &:= 1.716 \cdot \text{in} \cdot k_v & V_E &= 42.900 \frac{\text{in}}{\text{sec}} & \theta_{VE} &= 64.594 \text{ deg} \\ \omega_6 &:= \frac{V_E}{f} & \omega_6 &= 13.406 \frac{\text{rad}}{\text{sec}} & \text{CW} \\ V_{EC} &:= 1.821 \cdot \text{in} \cdot k_v & V_{EC} &= 45.525 \frac{\text{in}}{\text{sec}} & \theta_{VEC} &= 162.461 \text{ deg} \\ \omega_5 &:= \frac{V_{EC}}{e} & \omega_5 &= 5.115 \frac{\text{rad}}{\text{sec}} & \text{CCW} \end{aligned}$$

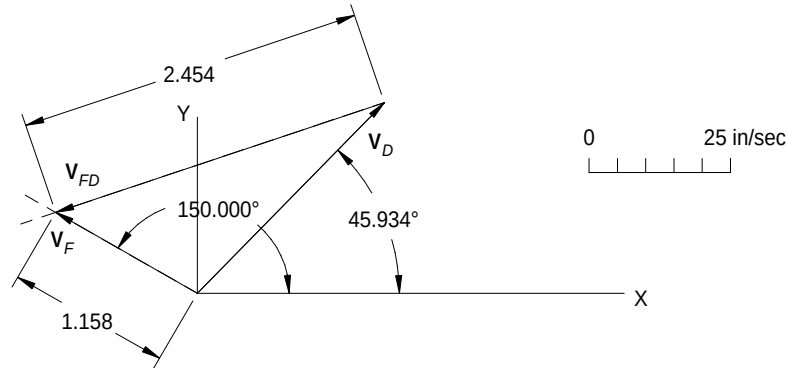
8. Calculate the magnitude and direction of $\mathbf{V}_{DE}$ and determine the magnitude and velocity of $\mathbf{V}_D$ from the velocity polygon above.

$$\begin{aligned} V_{DC} &:= h \cdot \omega_5 & V_{DC} &= 30.179 \frac{\text{in}}{\text{sec}} & \theta_{VDC} &= 162.461 \text{ deg} \\ \text{Length of } \mathbf{V}_{DC} \text{ on velocity polygon: } v_{DC} &:= \frac{V_{DC}}{k_v} & v_{DC} &= 1.207 \text{ in} \\ V_D &:= 1.900 \cdot \text{in} \cdot k_v & V_D &= 47.500 \frac{\text{in}}{\text{sec}} & \theta_{VD} &= 45.934 \text{ deg} \end{aligned}$$

9. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point F , the magnitude of the relative velocity $\mathbf{V}_{FD}$, and the angular velocity of link 5. The equation to be solved graphically is

$$\mathbf{V}_F = \mathbf{V}_D + \mathbf{V}_{FD}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_D$.
- From the tip of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_{FD}$, magnitude unknown.
- From the tail of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_F$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{FD}$ from the tip of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_F$ construction line and drawing $\mathbf{V}_F$ from the tail of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_{FD}$ construction line.



10. From the velocity polygon we have:

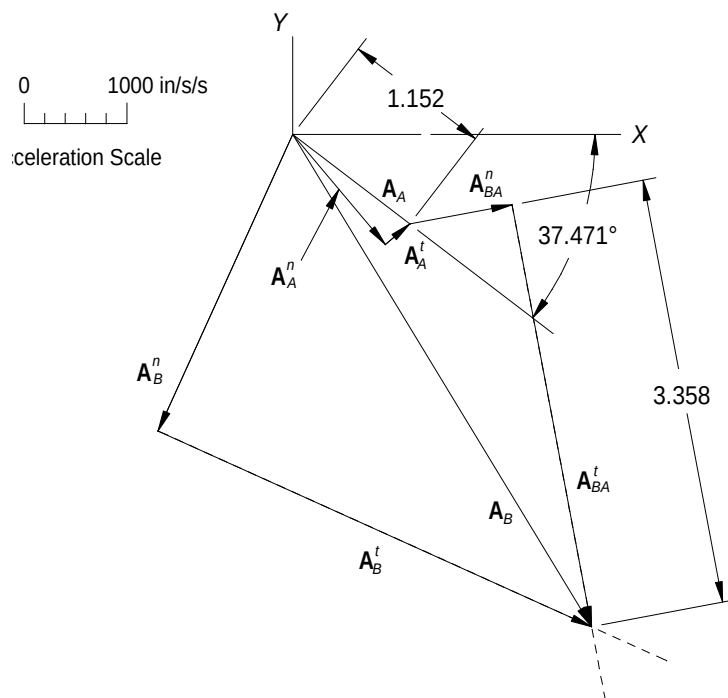
$$\begin{aligned} \text{Velocity scale factor: } k_v &:= \frac{25 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}} \\ V_F &:= 1.158 \cdot \text{in} \cdot k_v & V_F &= 28.950 \frac{\text{in}}{\text{sec}} & \theta_{VF} &= 150.000 \text{ deg} \\ V_{FD} &:= 2.454 \cdot \text{in} \cdot k_v & V_{FD} &= 61.350 \frac{\text{in}}{\text{sec}} \\ \omega_7 &:= \frac{V_{FD}}{k} & \omega_7 &= 9.586 \frac{\text{rad}}{\text{sec}} & \text{CCW} \end{aligned}$$

11. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

12. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$\begin{aligned} A_{Bn} &:= c \cdot \omega_4^2 & A_{Bn} &= 2547.2 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{ABn} &:= \theta_4 - 180 \cdot \text{deg} & \theta_{ABn} &= -114.351 \text{ deg} \\ A_{An} &:= a \cdot \omega_2^2 & A_{An} &= 1125.0 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{AAn} &:= \theta_2 - 180 \cdot \text{deg} & \theta_{AAn} &= -50.000 \text{ deg} \\ A_{At} &:= a \cdot \alpha_2 & A_{At} &= 250.0 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{AA_t} &:= \theta_2 - 90 \cdot \text{deg} & \theta_{AA_t} &= 40.000 \text{ deg} \\ A_{BA_n} &:= b \cdot \omega_3^2 & A_{BA_n} &= 811.3 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{ABA_n} &:= \theta_3 - 180 \cdot \text{deg} & \theta_{ABA_n} &= -349.348 \text{ deg} \end{aligned}$$

13. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_{Bn}$ at an angle of θ_{ABn} and $\mathbf{A}_A^n$ at an angle of θ_{AAn} . From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_A^t$ at an angle of θ_{AA_t} . From the tip of $\mathbf{A}_A^t$, draw $\mathbf{A}_{BA}^n$ at an angle of θ_{ABA_n} . Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B^n$ and $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B$, $\mathbf{A}_B^t$, and $\mathbf{A}_{BA}^t$.

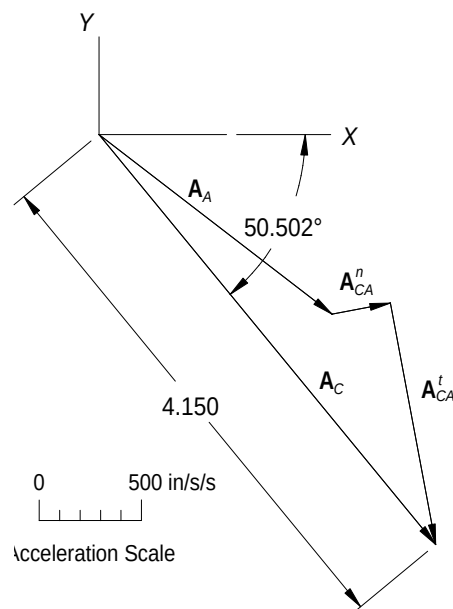


14. From the graphical solution above,

$$\begin{aligned} \text{Acceleration scale factor} \quad k_a &:= 1000 \cdot \frac{\text{in}}{\text{sec}^2} \\ A_A &:= 1.152 \cdot k_a & A_A &= 1152 \text{ in} \cdot \text{sec}^{-2} & \theta_{AA} &:= -37.471 \cdot \text{deg} \\ A_{BA_t} &:= 3.358 \cdot k_a & A_{BA_t} &= 3358 \text{ in} \cdot \text{sec}^{-2} \\ \alpha_3 &:= \frac{A_{BA_t}}{b} & \alpha_3 &= 399.762 \frac{\text{rad}}{\text{sec}^2} & \text{CW} \end{aligned}$$

15. For point C, : $\mathbf{A}_C = \mathbf{A}_A + (\mathbf{A}_{CA}^t + \mathbf{A}_{CA}^n)$, where

$$\begin{aligned} A_A &= 1.152 \times 10^3 \text{ in} \cdot \text{sec}^{-2} & \theta_{AA} &= -37.471 \text{ deg} \\ \theta_{AA_n} &:= \theta_2 - 180 \cdot \text{deg} & \theta_{AA_n} &= -50.000 \text{ deg} \\ A_{CA_t} &:= g \cdot \alpha_3 & A_{CA_t} &= 959.4 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{CA_t} &:= \theta_3 + 90 \cdot \text{deg} & \theta_{CA_t} &= -79.348 \text{ deg} \\ A_{CA_n} &:= g \cdot \omega_3^2 & A_{CA_n} &= 231.8 \text{ in} \cdot \text{sec}^{-2} \\ \theta_{CA_n} &:= \theta_3 - 180 \cdot \text{deg} & \theta_{CA_n} &= -349.348 \text{ deg} \end{aligned}$$



16. From the graphical solution above,

$$\text{Acceleration scale factor} \quad k_a := 500 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_C := 4.150 \cdot k_a \quad A_C = 2075 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{AC} := -50.502 \cdot \text{deg}$$

17. For point E: $(\mathbf{A}_E^t + \mathbf{A}_E^n) = \mathbf{A}_C + (\mathbf{A}_{EC}^t + \mathbf{A}_{EC}^n)$, where

$$A_{En} := f \cdot \omega_6^2 \quad A_{En} = 575.1 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AEn} := \theta_6 - 180 \cdot \text{deg} \quad \theta_{AEn} = -25.406 \text{ deg}$$

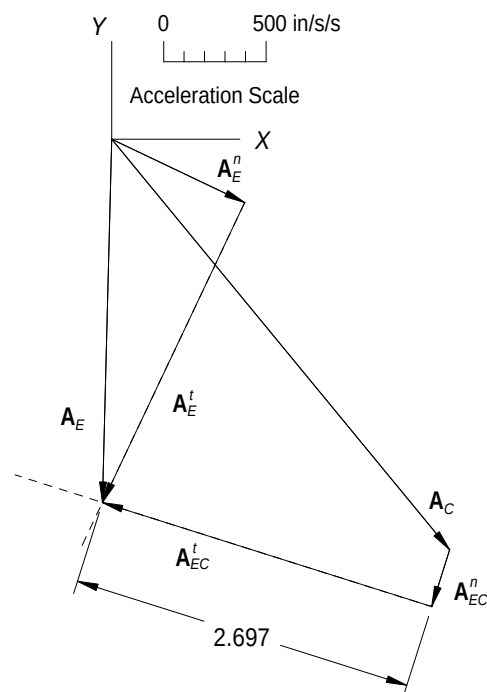
$$A_{ECn} := e \cdot \omega_5^2 \quad A_{ECn} = 232.9 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AECn} := \theta_5 - 180 \cdot \text{deg} \quad \theta_{AECn} = -107.539 \text{ deg}$$

$$\text{Acceleration scale factor} \quad k_a := 500 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_{ECt} := 2.697 \cdot k_a \quad A_{ECt} = 1349 \text{ in} \cdot \text{sec}^{-2}$$

$$\alpha_5 := \frac{A_{ECt}}{e} \quad \alpha_5 = 151.517 \frac{\text{rad}}{\text{sec}^2} \quad \text{CCW}$$



18. For point D: $\mathbf{A}_D = \mathbf{A}_C + (\mathbf{A}_{DC}^t + \mathbf{A}_{DC}^n)$, where

$$A_C = 2075.0 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{AC} = -50.502 \text{ deg}$$

$$A_{DCt} := h \cdot \alpha_5 \quad A_{DCt} = 893.9 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ADCt} := \theta_5 + 90 \cdot \text{deg} \quad \theta_{ADCt} = 162.461 \text{ deg}$$

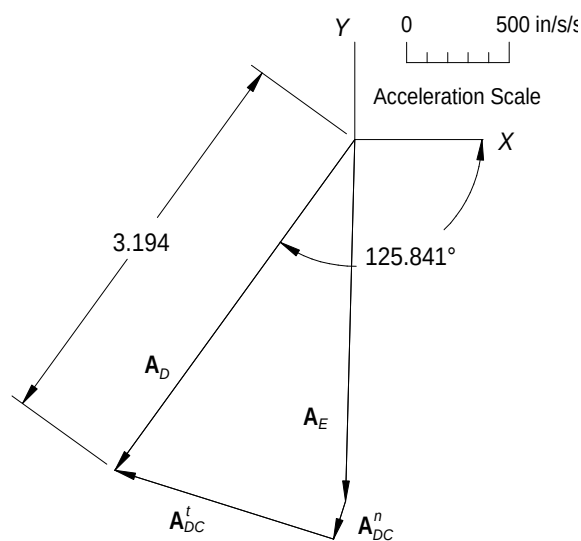
$$A_{DCn} := h \cdot \omega_5^2 \quad A_{DCn} = 154.4 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ADCn} := \theta_5 - 180 \cdot \text{deg} \quad \theta_{ADCn} = -107.539 \text{ deg}$$

$$\text{Acceleration scale factor} \quad k_a := 500 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_D := 3.194 \cdot k_a \quad A_C = 2075 \text{ in} \cdot \text{sec}^{-2}$$

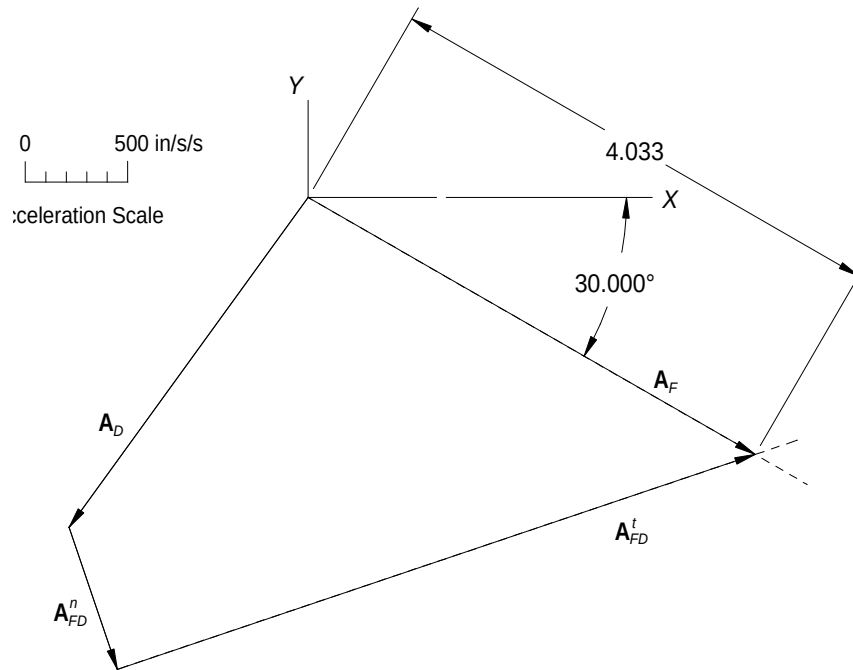
$$\theta_{AD} := -125.841 \cdot \text{deg}$$



19. For point F: $\mathbf{A}_F = \mathbf{A}_D + (\mathbf{A}_{FD}^t + \mathbf{A}_{FD}^n)$, where

$$A_{FDn} := k \cdot \omega_7^2 \quad A_{FDn} = 588.10 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AFDn} := \theta_7 - 180 \cdot \text{deg} \quad \theta_{AFDn} = -71.310 \text{ deg}$$



Acceleration scale factor

$$k_a := 500 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_F := 4.033 \cdot k_a$$

$$A_F = 2017 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AD} := -30.00 \cdot \text{deg}$$

PROBLEM 7-87

Statement: Figure 3-14 shows a crank-shaper quick-return mechanism with the following dimensions: $L_2 = 4.80$ in, $L_4 = 24.00$ in, $L_5 = 19.50$ in, the distance from link 4's fixed pivot (O_4) to link 2's fixed pivot (O_2) is 16.50 in, and the vertical distance from O_2 to the pivot point on link 6 is 6.465 in. Use a graphical method to calculate the acceleration of link 6 when $\theta_2 = 45$ deg. Assume that link 2 has a constant angular velocity of 2 rad/sec CW.

Given: Link lengths and angles:

Link 2 $L_2 := 4.80 \cdot \text{in}$ Angle link 2 makes with X axis $\theta_2 := 45 \cdot \text{deg}$

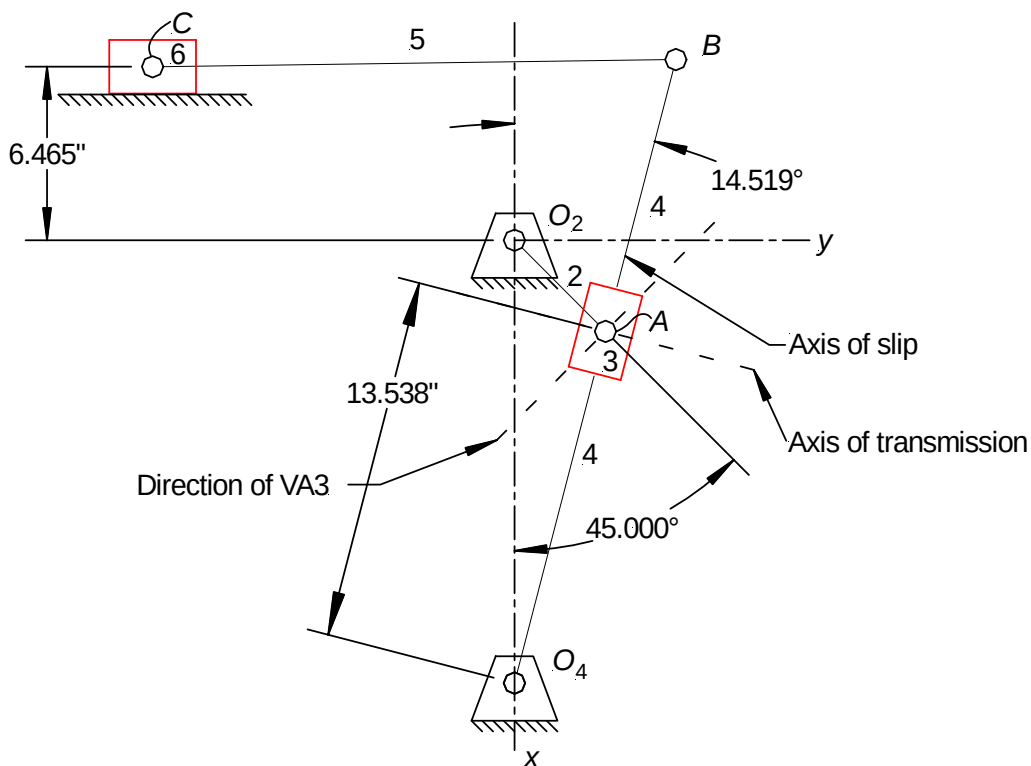
Link 4 $L_4 := 24.00 \cdot \text{in}$

Link 5 $L_5 := 19.50 \cdot \text{in}$

Angular velocity of link 2 $\omega_2 := 2 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW $\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure P3-14 and Mathcad file P0787.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A on links 2 and 3.

$$V_{A3} := L_2 \cdot \omega_2 \quad V_{A3} = 9.600 \frac{\text{in}}{\text{sec}} \quad \theta_{VA3} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{VA3} = -45.0 \text{ deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A on link 3. The equation to be solved graphically is

$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

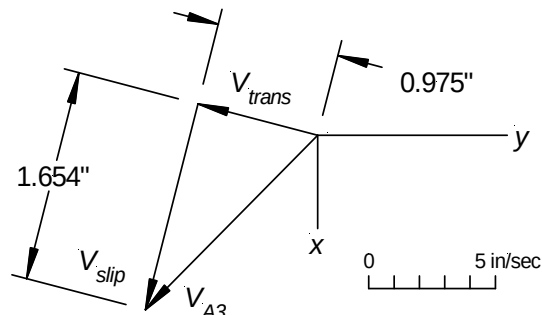
- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.
- From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.
- From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.

4. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_{slip} := 1.654 \cdot \text{in} \cdot k_v \quad V_{slip} = 8.270 \frac{\text{in}}{\text{sec}}$$

$$V_{trans} := 0.975 \cdot \text{in} \cdot k_v \quad V_{trans} = 4.875 \frac{\text{in}}{\text{sec}}$$



5. The true velocity of point A on link 4 is $\mathbf{V}_{trans}$,

$$V_{A4} := V_{trans} \quad V_{A4} = 4.875 \frac{\text{in}}{\text{sec}}$$

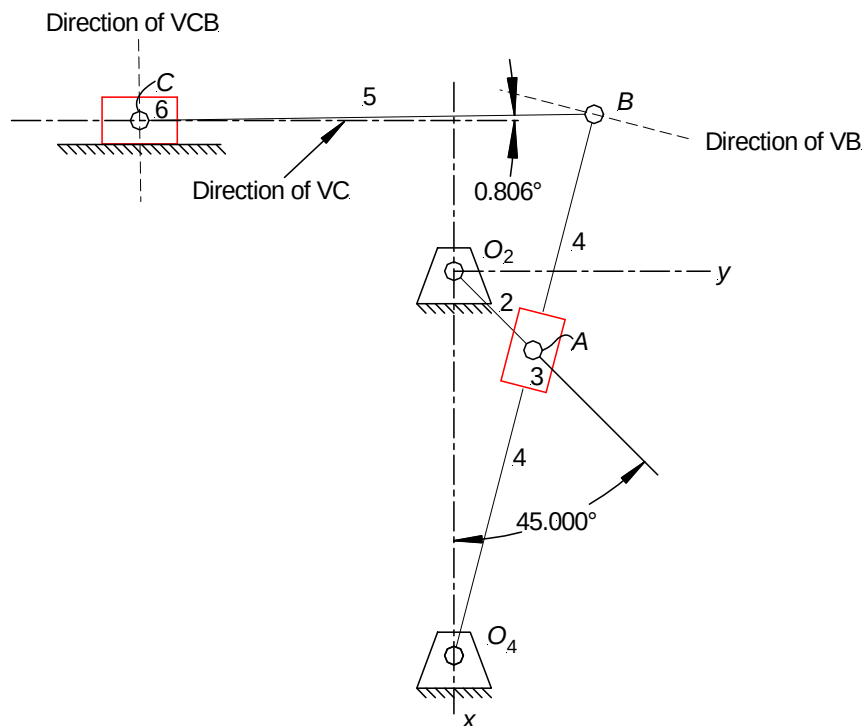
6. Determine the angular velocity of link 4 using equation 6.7.

$$\text{From the linkage layout above: } c := 13.538 \cdot \text{in} \quad \text{and} \quad \theta_4 := 180 \cdot \text{deg} - 14.519 \cdot \text{deg}$$

$$\omega_4 := \frac{V_{A4}}{c} \quad \omega_4 = 0.360 \frac{\text{rad}}{\text{sec}} \quad \text{CCW} \quad \theta_4 = 165.481 \text{ deg}$$

$$V_B := L_4 \cdot \omega_4 \quad V_B = 8.642 \frac{\text{in}}{\text{sec}}$$

7. Draw links 1, 4, 5, and 6 to a convenient scale. Indicate the directions of the velocity vectors of interest.



8. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

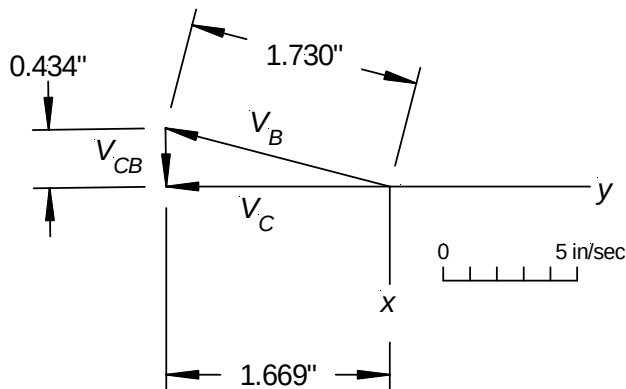
- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
 - From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.
 - From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.
 - Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.
9. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{5 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_C := 1.669 \cdot \text{in} \cdot k_v \quad V_C = 8.345 \frac{\text{in}}{\text{sec}}$$

$$\theta_{V_C} := 180 \cdot \text{deg}$$

$$V_{CB} := 0.434 \cdot \text{in} \cdot k_v \quad V_{CB} = 2.170 \frac{\text{in}}{\text{sec}}$$



10. Determine the angular velocity of link 5 using equation 6.7.

$$\text{From the linkage layout above: } \theta_5 := 270.806 \cdot \text{deg}$$

$$\omega_5 := \frac{V_{CB}}{L_5} \quad \omega_5 = 0.111 \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

11. For the acceleration of point A, use equation 7.19: $(\mathbf{A}_{A2}^t + \mathbf{A}_{A2}^n) = \mathbf{A}_{A4}^t + \mathbf{A}_{A4}^n + \mathbf{A}_A^{\text{cor}} + \mathbf{A}_A^{\text{slip}}$, where

$$A_{A2n} := L_2 \cdot \omega_2^2 \quad A_{A2n} = 19.200 \text{ in} \cdot \text{sec}^{-2} \quad A_{A2t} := L_2 \cdot \alpha_2 \quad A_{A2t} = 0.000 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AA} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AA} = 225.000 \text{ deg}$$

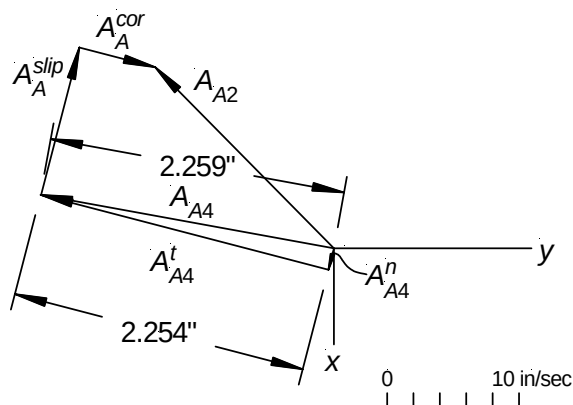
$$A_{A4n} := c \cdot \omega_4^2 \quad A_{A4n} = 1.755 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AA4n} := \theta_4 + 180 \cdot \text{deg} \quad \theta_{AA4n} = 345.481 \text{ deg}$$

$$A_{Acor} := |2 \cdot V_{\text{slip}} \cdot \omega_4| \quad A_{Acor} = 5.956 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{AAcor} := \theta_4 - 90 \cdot \text{deg} \quad \theta_{AAcor} = 75.481 \text{ deg}$$

(V_{slip} is negative)



12. Repeat procedure of steps 3 and 7 for the equation in step 11. (See next page).
13. From the acceleration polygon above,

$$\text{Acceleration scale factor} \quad k_a := 10 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_{A4} := 2.259 \cdot k_a \quad A_{A4} = 22.6 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -100.285 \text{ deg}$$

$$A_{A4t} := 2.254 \cdot k_a \quad A_{A4t} = 22.5 \text{ in} \cdot \text{sec}^{-2}$$

The angular acceleration of link 4 is

$$\alpha_4 := \frac{A_{A4t}}{c} \quad \alpha_4 = 1.665 \frac{\text{rad}}{\text{sec}^2} \quad \text{CCW}$$

14. The graphical solution for accelerations in pin-jointed fourbars uses equation 7.4:

$$(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$$

15. For point C, this becomes: $\mathbf{A}_C = (\mathbf{A}_B^t + \mathbf{A}_B^n) + (\mathbf{A}_{CB}^t + \mathbf{A}_{CB}^n)$, where

$$A_{Bn} := L_4 \cdot \omega_4^2 \quad A_{Bn} = 3.112 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 + 180 \cdot \text{deg} \quad \theta_{ABn} = 345.481 \text{ deg}$$

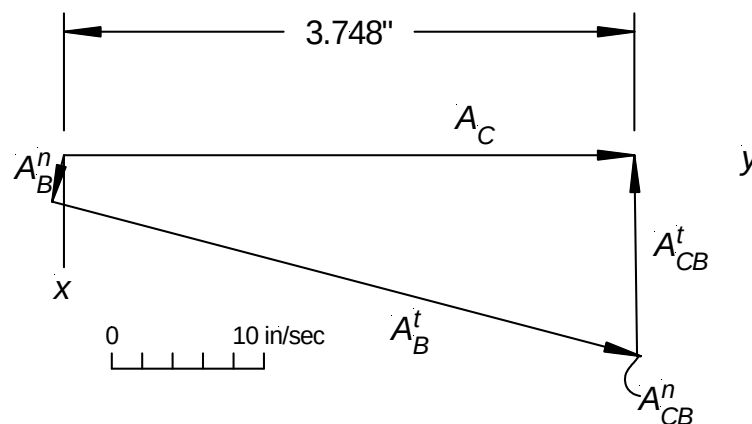
$$A_{Bt} := L_4 \cdot \alpha_4 \quad A_{Bt} = 39.96 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ABt} := \theta_4 - 90 \cdot \text{deg} \quad \theta_{ABt} = 75.481 \text{ deg}$$

$$A_{CBn} := L_5 \cdot \omega_5^2 \quad A_{CBn} = 0.241 \text{ in} \cdot \text{sec}^{-2}$$

$$\theta_{ACBn} := \theta_5 \quad \theta_{ACBn} = 270.806 \text{ deg}$$

16. Repeat procedure of step 12 for the equation in step 15.



17. From the acceleration polygon above,

$$\text{Acceleration scale factor} \quad k_a := 10 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_C := 3.748 \cdot k_a \quad A_C = 37.5 \text{ in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 90 \text{ deg}$$

PROBLEM 7-88

Statement: Repeat Problem 7-87 using an analytical method to calculate the acceleration of point C for one revolution of the input crank (link 2).

Given: Link lengths and angles:

Link 1 (O_2O_4) $d := 16.50 \cdot \text{in}$ Vertical offset from O_4 to C $d' := 22.965 \cdot \text{in}$

Link 2 (L_2) $a := 4.80 \cdot \text{in}$

Link 4 (L_4) $a' := 24.00 \cdot \text{in}$

Link 5 (L_5) $b' := 19.50 \cdot \text{in}$

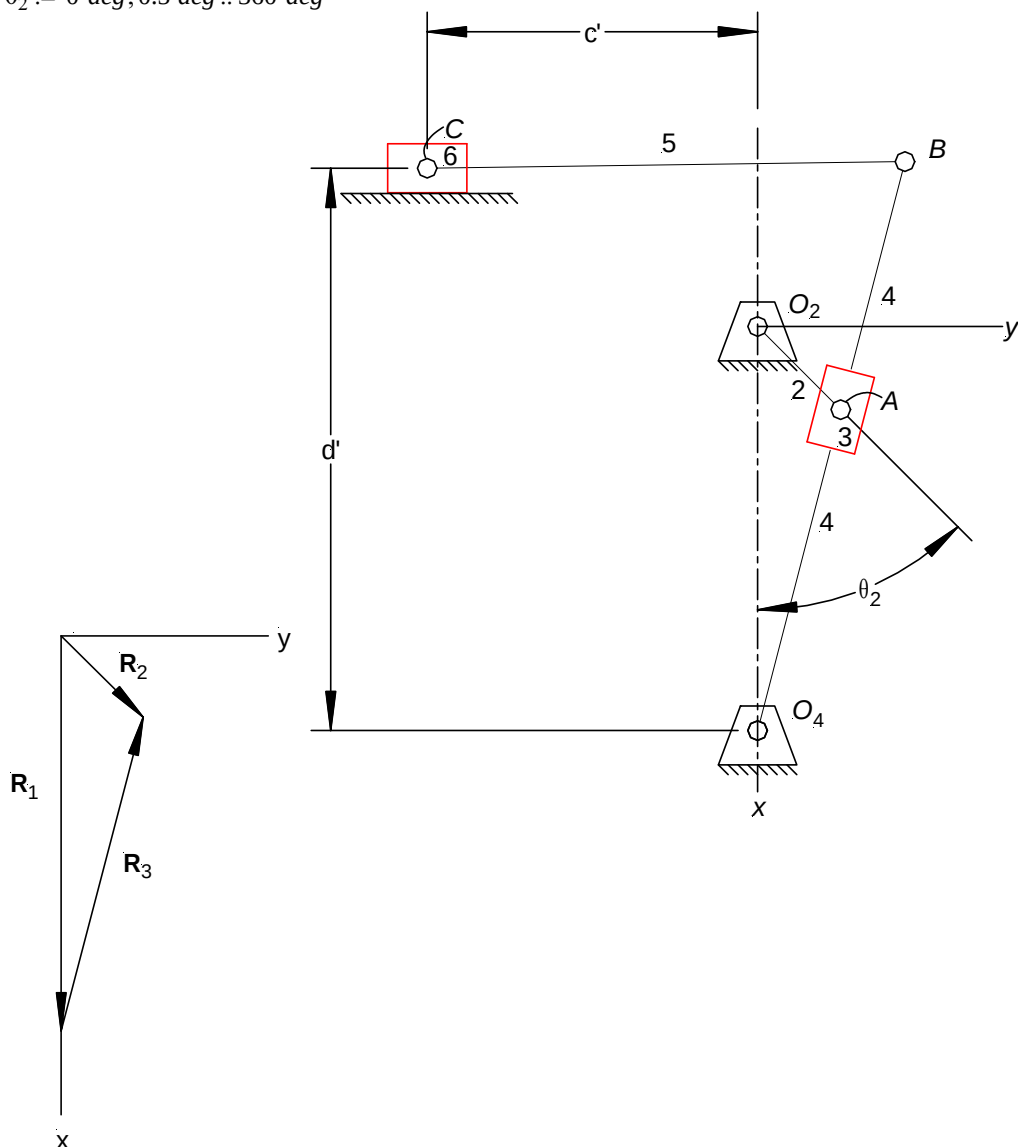
Angular velocity of link 2 $\omega_2 := -2 \cdot \text{rad} \cdot \text{sec}^{-1}$

$\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure 3-14 and Mathcad file P0788.

1. Draw the mechanism to scale and define a vector loop for the input portion (links 1, 2, 3, and 4) using the four-bar crank-slider derivation in Section 7.3 as a model.

$$\theta_2 := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



2. Write the vector loop equation, expand the result and separate into real and imaginary parts to solve for θ_3 and b (distance from O_4 to A).

$$\mathbf{R}_2 := \mathbf{R}_1 + \mathbf{R}_3$$

$$a \cdot e^{j \cdot \theta_2} := d + b \cdot e^{j \cdot \theta_3}$$

Substituting the Euler equivalents,

$$a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) := d + b (\cos(\theta_3) + j \cdot \sin(\theta_3))$$

Separating into real and imaginary components and solving for θ_3 and b .

$$\theta_3(\theta_2) := \text{atan2}(a \cdot \cos(\theta_2) - d, a \cdot \sin(\theta_2))$$

$$b(\theta_2) := \frac{a \cdot \sin(\theta_2)}{\sin(\theta_3(\theta_2))}$$

3. Differentiate the position equation, expand it and solve for ω_3 and $b\dot{\theta}$.

$$a \cdot j \cdot \omega_2 \cdot e^{j \cdot \theta_2} := b \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + b\dot{\theta} \cdot e^{j \cdot \theta_3}$$

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b(\theta_2)} \cdot \cos(\theta_2 - \theta_3(\theta_2))$$

$$b\dot{\theta}(\theta_2) := \frac{a \cdot \omega_2 \cdot \cos(\theta_2) - b(\theta_2) \cdot \omega_3(\theta_2) \cdot \cos(\theta_3(\theta_2))}{\sin(\theta_3(\theta_2))}$$

4. Differentiate the velocity equation, expand it and solve for α_3 .

$$a \cdot j \cdot \alpha_2 \cdot e^{j \cdot \theta_2} + a \cdot j^2 \cdot \omega_2^2 \cdot e^{j \cdot \theta_2} := b\dot{\theta} \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + b \cdot j \cdot \alpha_3 \cdot e^{j \cdot \theta_3} \dots$$

$$+ b \cdot j^2 \cdot \omega_3^2 \cdot e^{j \cdot \theta_3} + b\ddot{\theta} \cdot e^{j \cdot \theta_3} + b\dot{\theta} \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3}$$

$$\alpha_3(\theta_2) := \frac{1}{b(\theta_2)} \cdot (a \cdot \alpha_2 \cdot \cos(\theta_2 - \theta_3(\theta_2)) + a \cdot \omega_2^2 \cdot \sin(\theta_3(\theta_2) - \theta_2) - 2 \cdot b\dot{\theta}(\theta_2) \cdot \omega_3(\theta_2))$$

5. The position, angular velocity, and angular acceleration are the same for links 3 and 4.

$$\theta_4(\theta_2) := \theta_3(\theta_2) \quad \omega_4(\theta_2) := \omega_3(\theta_2) \quad \alpha_4(\theta_2) := \alpha_3(\theta_2)$$

6. Write a vector loop equation for links 1, 4, 5, and 6, expand the result and separate into real and imaginary parts to solve for θ_5 and c' (horizontal distance from O_4 to C).

$$\mathbf{R}_6 + \mathbf{R}_7 = \mathbf{R}_4 + \mathbf{R}_5 \quad c' \cdot e^{j \left(\frac{3 \cdot \pi}{2} \right)} + d' \cdot e^{j \cdot \pi} = a' \cdot e^{j \cdot \theta_4} + b' \cdot e^{j \cdot \theta_5}$$

Substituting the Euler equivalents,

$$-j \cdot c' - d' = a' (\cos(\theta_4) + j \cdot \sin(\theta_4)) + b' (\cos(\theta_5) + j \cdot \sin(\theta_5))$$

Separating into real and imaginary components and solving for θ_5 and c' .

$$\theta_5(\theta_2) := -\text{acos}\left(\frac{a' \cdot \cos(\theta_4(\theta_2)) + d'}{b'}\right)$$

$$c'(\theta_2) := -(a' \cdot \sin(\theta_4(\theta_2)) + b' \cdot \sin(\theta_5(\theta_2)))$$

7. Differentiate the position equation, expand it and solve for ω_5 and V_C .

$$V_C \cdot e^{j \cdot \left(\frac{3 \cdot \pi}{2} \right)} = a' \cdot \omega_4 \cdot j \cdot e^{j \cdot \theta_4} + b' \cdot \omega_5 \cdot j \cdot e^{j \cdot \theta_5}$$

Substituting the Euler equivalents,

$$-V_C = a' \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) + b' \cdot \omega_5 \cdot (-\sin(\theta_5) + j \cdot \cos(\theta_5))$$

Separating into real and imaginary components and solving for ω_5 and V_C .

$$\omega_5(\theta_2) := -\frac{a' \cdot \sin(\theta_4(\theta_2))}{b' \cdot \sin(\theta_5(\theta_2))} \cdot \omega_4(\theta_2)$$

$$V_C(\theta_2) := a' \cdot \omega_4(\theta_2) \cdot \cos(\theta_4(\theta_2)) + b' \cdot \omega_5(\theta_2) \cdot \cos(\theta_5(\theta_2))$$

8. Differentiate the position equation, expand it and solve for α_5 and A_C .

$$A_C \cdot e^{j \cdot \left(\frac{3 \cdot \pi}{2} \right)} = a' \cdot j \cdot \left(j \cdot \omega_4^2 \cdot e^{j \cdot \theta_4} + \alpha_4 \cdot e^{j \cdot \theta_4} \right) + b' \cdot j \cdot \left(j \cdot \omega_5^2 \cdot e^{j \cdot \theta_5} + \alpha_5 \cdot e^{j \cdot \theta_5} \right)$$

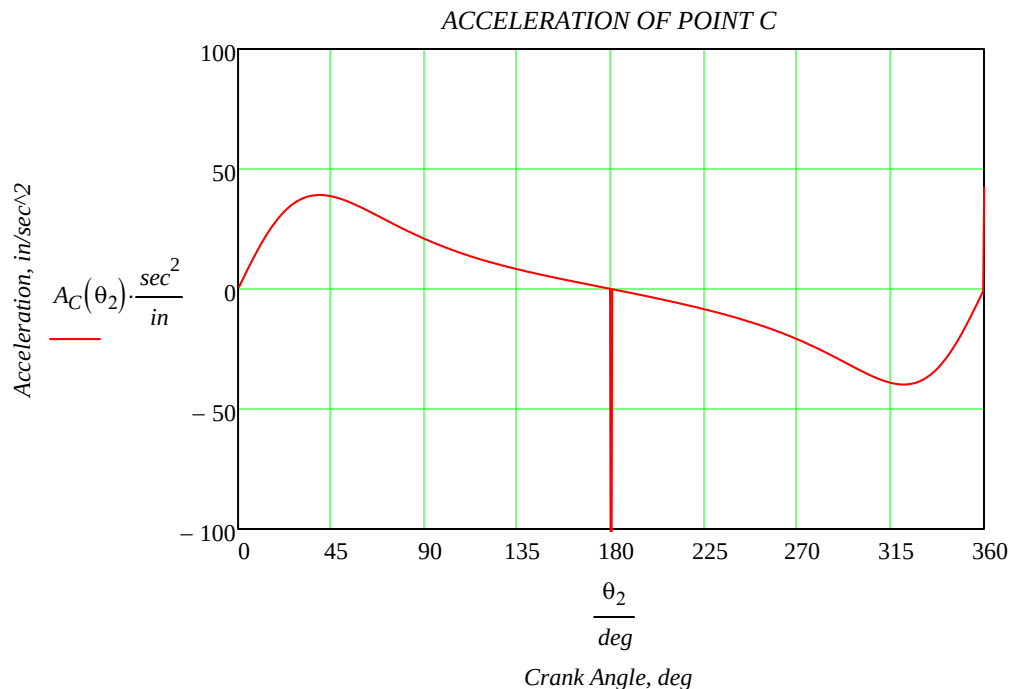
Substituting the Euler equivalents,

$$\begin{aligned} -A_C = & -a' \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4)) + a' \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) \dots \\ & + b' \cdot \omega_5^2 \cdot (\cos(\theta_5) + j \cdot \sin(\theta_5)) + b' \cdot \alpha_5 \cdot (-\sin(\theta_5) + j \cdot \cos(\theta_5)) \end{aligned}$$

Separating into real and imaginary components and solving for α_5 and A_C .

$$\alpha_5(\theta_2) := \frac{-a' \cdot \alpha_4(\theta_2) \cdot \cos(\theta_4(\theta_2)) - a' \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2)) - b' \cdot \omega_5(\theta_2)^2 \cdot \cos(\theta_5(\theta_2))}{b' \cdot \sin(\theta_5(\theta_2))}$$

$$\begin{aligned} A_C(\theta_2) = & -a' \cdot \alpha_4(\theta_2) \cdot \cos(\theta_4(\theta_2)) + a' \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2)) \dots \\ & + b' \cdot \alpha_5(\theta_2) \cdot \cos(\theta_5(\theta_2)) + b' \cdot \omega_5(\theta_2)^2 \cdot \sin(\theta_5(\theta_2)) \end{aligned}$$



PROBLEM 7-89

Statement: Figure P7-22 shows a mechanism with dimensions. Use a graphical method to determine the accelerations of points A and B for the position shown. Ignore links 5 and 6. $\omega_2 = 24 \text{ rad/s}$ clock-wise.

Given: Link lengths and angles:

Link 1 (O_2O_4) $d := 1.22 \cdot \text{in}$ Angle O_2O_4 makes with X axis $\theta_1 := 56.5 \cdot \text{deg}$

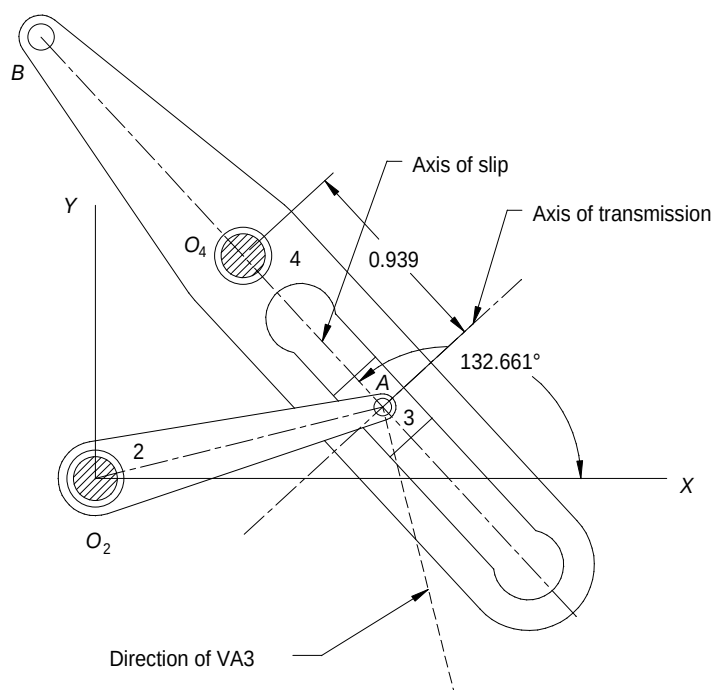
Link 2 (O_2A) $a := 1.35 \cdot \text{in}$ Angle O_2A makes with X axis $\theta_2 := 14 \cdot \text{deg}$

Link 4 (O_4B) $e := 1.36 \cdot \text{in}$

Motion of link 2 $\omega_2 := 24 \cdot \text{rad} \cdot \text{sec}^{-1}$ CW $\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure P7-22 and Mathcad file P0789.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



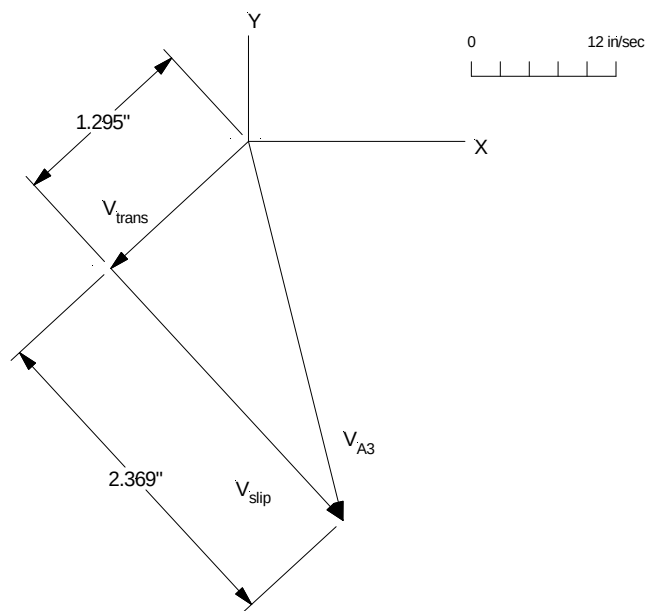
2. Use equation 6.7 to calculate the magnitude of the velocity at point A on links 2 and 3.

$$V_{A3} := a \cdot \omega_2 \quad V_{A3} = 32.400 \cdot \frac{\text{in}}{\text{sec}} \quad \theta_{VA3} := \theta_2 - 90 \cdot \text{deg} \quad \theta_{VA3} = -76.0 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A on link 3. The equation to be solved graphically is

$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.
- b. From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

$$\text{Velocity scale factor: } k_v := \frac{12 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_{slip} := 2.369 \cdot \text{in} \cdot k_v \quad V_{slip} = 28.428 \cdot \frac{\text{in}}{\text{sec}}$$

$$V_{trans} := 1.295 \cdot \text{in} \cdot k_v \quad V_{trans} = 15.540 \cdot \frac{\text{in}}{\text{sec}}$$

5. The true velocity of point A on link 4 is V_{trans} ,

$$V_{A4} := V_{trans} \quad V_{A4} = 15.54 \cdot \frac{\text{in}}{\text{sec}}$$

6. Determine the angular velocity of link 4 using equation 6.7.

$$\text{From the linkage layout above: } c := 0.939 \cdot \text{in} \quad \text{and} \quad \theta_4 := 132.661 \cdot \text{deg}$$

$$\omega_4 := \frac{V_{A4}}{c} \quad \omega_4 = 16.550 \cdot \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

7. Determine the magnitude and sense of the vector $\mathbf{V}_B$ using equation 6.7.

$$V_B := e \cdot \omega_4 \quad V_B = 22.507 \cdot \frac{\text{in}}{\text{sec}}$$

$$\theta_{VA4} := \theta_4 - 90 \cdot \text{deg} \quad \theta_{VA4} = 42.661 \cdot \text{deg}$$

8. For the acceleration of point A, use equation 7.19: $(\mathbf{A}_{A2}^t + \mathbf{A}_{A2}^n) = \mathbf{A}_{A4}^t + \mathbf{A}_{A4}^n + \mathbf{A}_A^{cor} + \mathbf{A}_A^{slip}$, where

$$A_{A2n} := a \cdot \omega_2^2 \quad A_{A2n} = 777.600 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{AA} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AA} = 194.000 \cdot \text{deg}$$

$$A_{A2t} := a \cdot \alpha_2 \quad A_{A2t} = 0.000 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$A_{A4n} := c \cdot \omega_4^2 \quad A_{A4n} = 257.2 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{AA4n} := \theta_4$$

$$\theta_{AA4n} = 132.661 \cdot \text{deg}$$

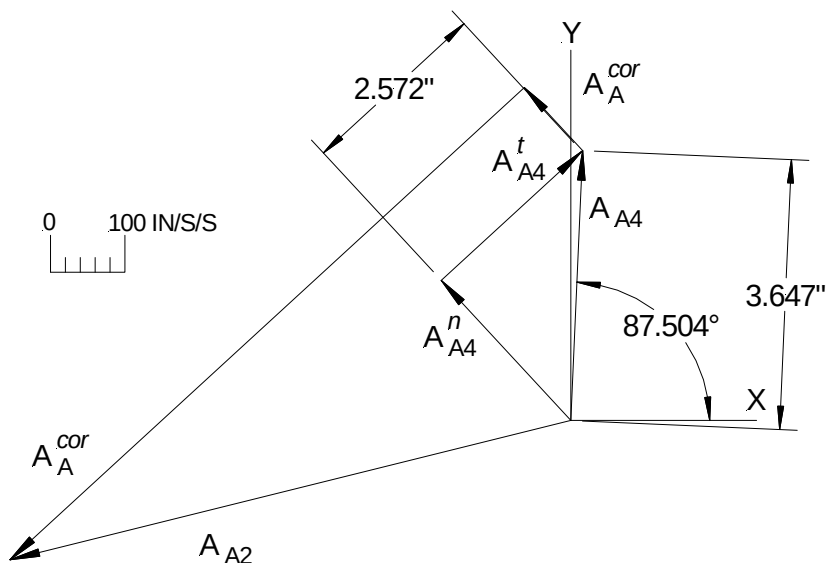
$$A_{Acor} := |2 \cdot V_{slip} \cdot \omega_4|$$

$$A_{Acor} = 940.940 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{AAcor} := \theta_4 + 90 \cdot \text{deg}$$

$$\theta_{AAcor} = 222.661 \cdot \text{deg}$$

9. Repeat procedure of steps 3 and 7 for the equation in step 8.



10. From the acceleration polygon above,

$$\text{Acceleration scale factor} \quad k_a := 100 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_{A4} := 3.647 \cdot k_a \quad A_{A4} = 364.7 \cdot \text{in} \cdot \text{sec}^{-2} \quad \text{at an angle of } 87.50 \text{ deg}$$

$$A_{A4t} := 2.572 \cdot k_a \quad A_{A4t} = 257.2 \cdot \text{in} \cdot \text{sec}^{-2}$$

The angular acceleration of link 4 is

$$\alpha_4 := \frac{A_{A4t}}{c} \quad \alpha_4 = 273.9 \cdot \frac{\text{rad}}{\text{sec}^2} \quad \text{CCW}$$

11. For point B:

$$A_{Bn} := e \cdot \omega_4^2 \quad A_{Bn} = 372.5 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 + 180 \cdot \text{deg} \quad \theta_{ABn} = 312.661 \cdot \text{deg}$$

$$A_{Bt} := e \cdot \alpha_4 \quad A_{Bt} = 372.5 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{ABt} := \theta_4 + 90 \cdot \text{deg} \quad \theta_{ABt} = 222.661 \cdot \text{deg}$$

PROBLEM 7-90

Statement: Figure P7-22 shows a mechanism with dimensions. Use an analytical method to calculate the accelerations of points A and B for the position shown. Ignore links 5 and 6. $\omega_2 = 24 \text{ rad/s}$ clock-wise.

Given: Link lengths and angles:

Link 1 (O_2O_4) $d := 1.22\text{-in}$ Angle link 2 makes with X axis $\theta_2 := -42.5\text{-deg}$

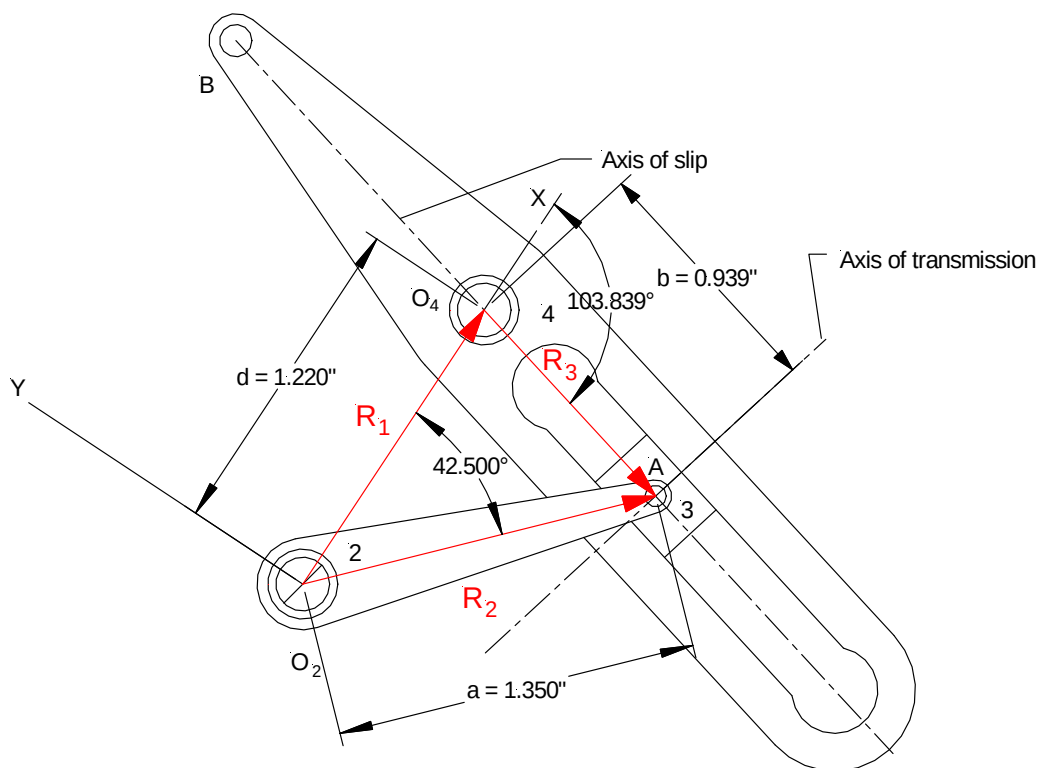
Link 2 (O_2A) $a := 1.35\text{-in}$ Angle link 3 makes with X axis $\theta_3 := -103.839\text{-deg}$

Link 4 (O_4B) $c := 1.36\text{-in}$

Motion of link 2 $\omega_2 := -24\text{-rad}\cdot\text{sec}^{-1}$ $\alpha_2 := 0\text{-rad}\cdot\text{sec}^{-2}$

Solution: See Figure P7-22 and Mathcad file P0790.

1. Draw the mechanism to scale and define a vector loop for links 1, 2, 3, and 4 using the four-bar crank-slider derivation in Section 7.3 as a model.



2. Write the vector loop equation, expand the result and separate into real and imaginary parts to solve for θ_3 and b (distance from O_4 to A).

$$\mathbf{R}_2 = \mathbf{R}_1 + \mathbf{R}_3$$

$$a \cdot e^{j \cdot \theta_2} = d + b \cdot e^{j \cdot \theta_3}$$

Substituting the Euler equivalents,

$$a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) = d + b (\cos(\theta_3) + j \cdot \sin(\theta_3))$$

Separating into real and imaginary components and solving for b .

$$b := \frac{a \cdot \sin(\theta_2)}{\sin(\theta_3)} \quad b = 0.939 \text{ in}$$

3. Differentiate the position equation, expand it and solve for ω_3 and $b\dot{\theta}$.

$$a \cdot j \cdot \omega_2 \cdot e^{j \cdot \theta_2} = b \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + b\dot{\theta} \cdot e^{j \cdot \theta_3}$$

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \cos(\theta_2 - \theta_3) \quad \omega_3 = -16.544 \cdot \frac{\text{rad}}{\text{s}}$$

$$b\dot{\theta} := \frac{a \cdot \omega_2 \cdot \cos(\theta_2) - b \cdot \omega_3 \cdot \cos(\theta_3)}{\sin(\theta_3)} \quad b\dot{\theta} = 28.430 \frac{\text{in}}{\text{s}}$$

4. Differentiate the velocity equation, expand it and solve for α_3 .

$$a \cdot j \cdot \alpha_2 \cdot e^{j \cdot \theta_2} + a \cdot j^2 \cdot \omega_2^2 \cdot e^{j \cdot \theta_2} = b\dot{\theta} \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + b \cdot j \cdot \alpha_3 \cdot e^{j \cdot \theta_3} \dots$$

$$+ b \cdot j^2 \cdot \omega_3^2 \cdot e^{j \cdot \theta_3} + b\ddot{\theta} \cdot e^{j \cdot \theta_3} + b\dot{\theta} \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3}$$

$$\alpha_3 := \frac{1}{b} \cdot \left(a \cdot \alpha_2 \cdot \cos(\theta_2 - \theta_3) + a \cdot \omega_2^2 \cdot \sin(\theta_3 - \theta_2) - 2 \cdot b\dot{\theta} \cdot \omega_3 \right) \quad \alpha_3 = 275.059 \cdot \frac{\text{rad}}{\text{s}^2}$$

5. Determine the acceleration of point A on link 2 (in the local XY coordinate system) using equations 7.13a.

$$\mathbf{A}_{A2} := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{A2} := |\mathbf{A}_{A2}| \quad A_{A2} = 777.600 \cdot \frac{\text{in}}{\text{s}^2} \quad \theta_{AA2} := \arg(\mathbf{A}_{A2}) \quad \theta_{AA2} = 137.500 \cdot \text{deg}$$

6. Determine the acceleration of points A and B on link 4 (in the local XY coordinate system) using equations 7.13a.

$$\text{Rename } \theta_3 \text{ to } \theta_4: \quad \theta_4 := \theta_3$$

$$\text{Rename } \alpha_3 \text{ to } \alpha_4: \quad \alpha_4 := \alpha_3$$

$$\omega_4 := \omega_3$$

$$\mathbf{A}_{A4} := b \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - b \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$A_{A4} := |\mathbf{A}_{A4}| \quad A_{A4} = 364.484 \cdot \frac{\text{in}}{\text{s}^2} \quad \theta_{AA4} := \arg(\mathbf{A}_{A4}) \quad \theta_{AA4} = 31.019 \cdot \text{deg}$$

$$\mathbf{A}_B := a \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - a \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$A_B := |\mathbf{A}_B| \quad A_B = 523.844 \cdot \frac{\text{in}}{\text{s}^2} \quad \theta_{AB} := \arg(\mathbf{A}_B) \quad \theta_{AB} = 31.019 \cdot \text{deg}$$

PROBLEM 7-91

Statement: Figure P7-23 shows a quick-return mechanism with dimensions. Use a graphical method to calculate the accelerations of points A, B, and C for the position shown. $\omega_2 = 16 \text{ rad/s}$. Ignore links 5 and 6.

Given: Link lengths and angles:

Link 1 (O_2O_4) $d := 1.69 \cdot \text{in}$ Angle O_2O_4 makes with X axis $\theta_1 := 195.5 \cdot \text{deg}$

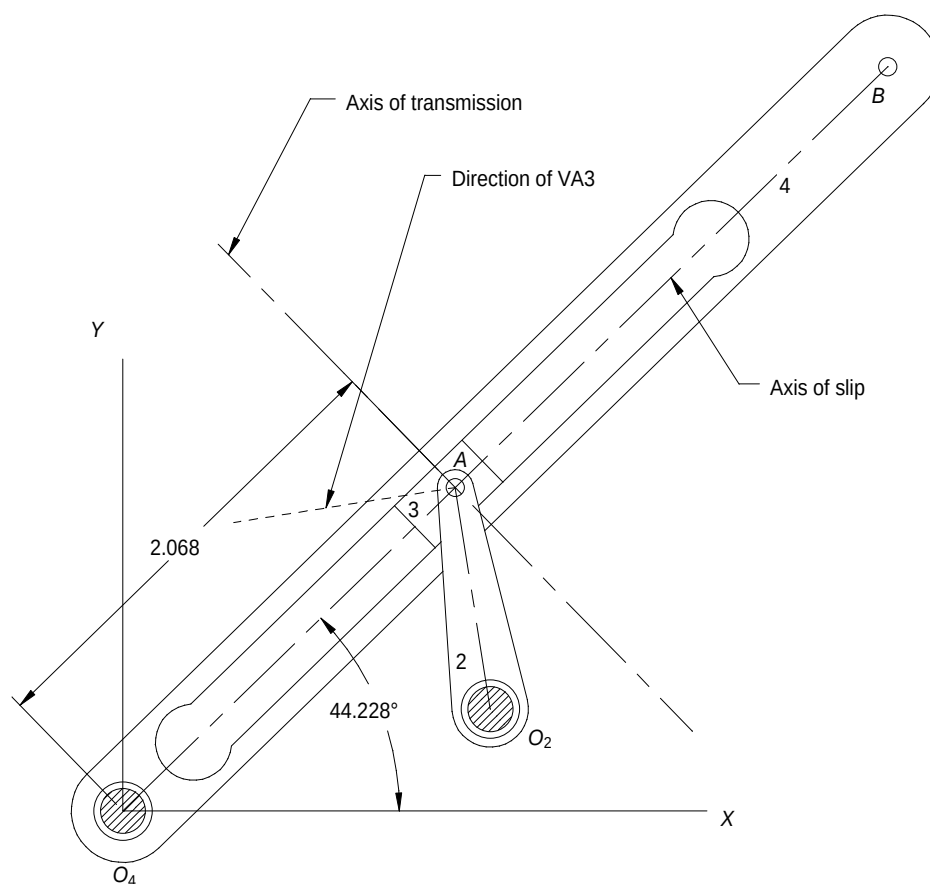
Link 2 (L_2) $a := 1.00 \cdot \text{in}$ Angle link 2 makes with X axis $\theta_2 := 99 \cdot \text{deg}$

Link 4 (L_4) $e := 4.76 \cdot \text{in}$

Angular velocity of link 2 $\omega_2 := 16 \cdot \text{rad} \cdot \text{sec}^{-1}$ CCW $\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure P7-23 and Mathcad file P0791.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



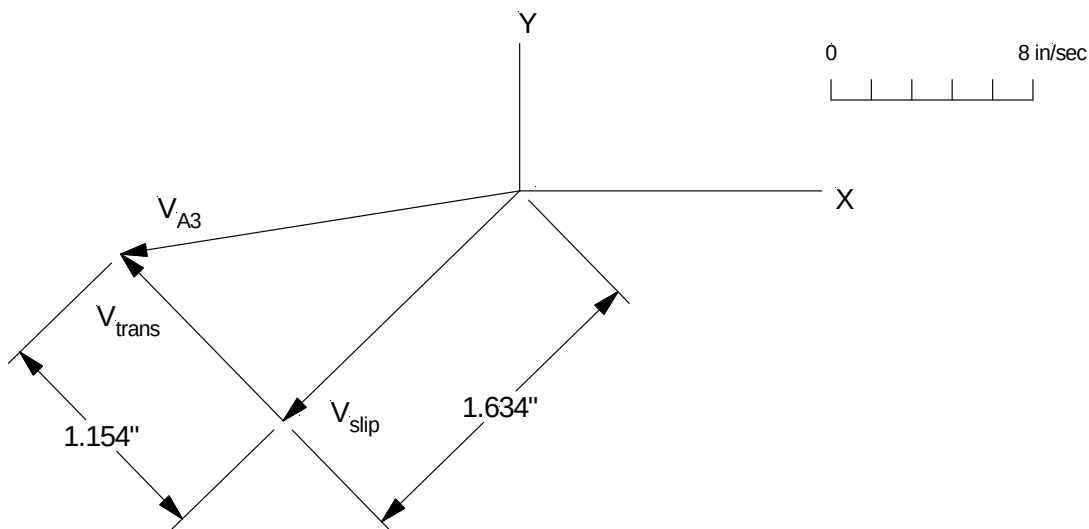
2. Use equation 6.7 to calculate the magnitude of the velocity at point A on links 2 and 3.

$$V_{A3} := a \cdot \omega_2 \quad V_{A3} = 16.000 \cdot \frac{\text{in}}{\text{sec}} \quad \theta_{VA3} := \theta_2 + 90 \cdot \text{deg} \quad \theta_{VA3} = 189.0 \cdot \text{deg}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A on link 3. The equation to be solved graphically is

$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

- Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.
- From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.
- From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.
- Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

$$\begin{aligned} \text{Velocity scale factor: } k_v &:= \frac{8 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}} \\ V_{slip} &:= 1.634 \cdot \text{in} \cdot k_v & V_{slip} &= 13.072 \cdot \frac{\text{in}}{\text{sec}} \\ V_{trans} &:= 1.154 \cdot \text{in} \cdot k_v & V_{trans} &= 9.232 \cdot \frac{\text{in}}{\text{sec}} \end{aligned}$$

5. The true velocity of point A on link 4 is V_{trans} ,

$$V_{A4} := V_{trans} \quad V_{A4} = 9.23 \cdot \frac{\text{in}}{\text{sec}}$$

6. Determine the angular velocity of link 4 using equation 6.7.

$$\text{From the linkage layout above: } c := 2.068 \cdot \text{in} \quad \text{and} \quad \theta_4 := 44.228 \cdot \text{deg}$$

$$\omega_4 := \frac{V_{A4}}{c} \quad \omega_4 = 4.464 \cdot \frac{\text{rad}}{\text{sec}} \quad \text{CCW}$$

7. Determine the magnitude and sense of the vector $\mathbf{V}_B$ using equation 6.7.

$$\begin{aligned} V_B &:= e \cdot \omega_4 & V_B &= 21.250 \cdot \frac{\text{in}}{\text{sec}} \\ \theta_{VA4} &:= \theta_4 - 90 \cdot \text{deg} & \theta_{VA4} &= -45.772 \cdot \text{deg} \end{aligned}$$

8. For the acceleration of point A, use equation 7.19: $(\mathbf{A}_{A2}^t + \mathbf{A}_{A2}^n) = \mathbf{A}_{A4}^t + \mathbf{A}_{A4}^n + \mathbf{A}_A^{cor} + \mathbf{A}_A^{slip}$, where

$$A_{A2n} := a \cdot \omega_2^2 \quad A_{A2n} = 256.000 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{AA} := \theta_2 + 180 \cdot \text{deg}$$

$$\theta_{AA} = 279.000 \cdot \text{deg}$$

$$A_{A2t} := a \cdot \alpha_2$$

$$A_{A2t} = 0.000 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$A_{A4n} := c \cdot \omega_4^2$$

$$A_{A4n} = 41.214 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{AA4n} := \theta_4 + 180 \cdot \text{deg}$$

$$\theta_{AA4n} = 224.228 \cdot \text{deg}$$

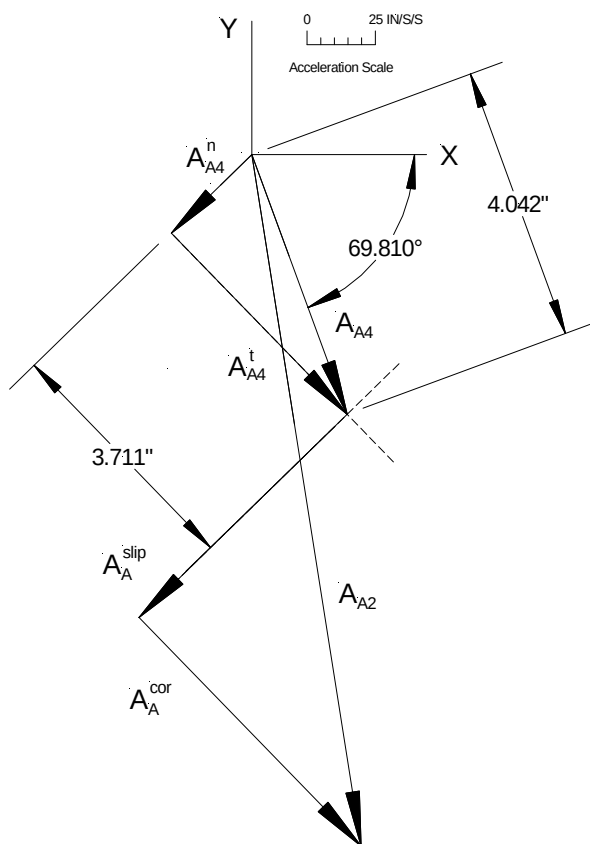
$$A_{Acor} := |2 \cdot V_{slip} \cdot \omega_4|$$

$$A_{Acor} = 116.712 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{AAcor} := \theta_4 + 90 \cdot \text{deg}$$

$$\theta_{AAcor} = 134.228 \cdot \text{deg}$$

9. Repeat procedure of steps 3 and 7 for the equation in step 8.



10. From the acceleration polygon above,

$$\text{Acceleration scale factor} \quad k_a := 25 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_{A4} := 4.042 \cdot k_a \quad A_{A4} = 101.0 \cdot \text{in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -69.81 \text{ deg}$$

$$A_{A4t} := 3.711 \cdot k_a \quad A_{A4t} = 92.8 \cdot \text{in} \cdot \text{sec}^{-2}$$

The angular acceleration of link 4 is

$$\alpha_4 := \frac{A_{A4t}}{c} \quad \alpha_4 = 44.9 \cdot \frac{\text{rad}}{\text{sec}^2} \quad \text{CW}$$

11. For point B:

$$A_{Bn} := e \cdot \omega_4^2$$

$$A_{Bn} = 94.9 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{ABn} := \theta_4 + 180 \cdot \text{deg}$$

$$\theta_{ABn} = 224.228 \cdot \text{deg}$$

$$A_{Bt} := e \cdot \alpha_4$$

$$A_{Bt} = 213.5 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{ABt} := \theta_4 + 90 \cdot \text{deg}$$

$$\theta_{ABt} = 134.228 \cdot \text{deg}$$

PROBLEM 7-92

Statement: Figure P7-23 shows a quick-return mechanism with dimensions. Use an analytical method to calculate the accelerations of points A and B for the position shown. Ignore links 5 and 6. $\omega_2 = 16 \text{ rad/s}$.

Given: Link lengths and angles:

Link 1 (O_2O_4) $d := 1.69 \cdot \text{in}$ Angle link 2 makes with X axis $\theta_2 := -96.517 \cdot \text{deg}$

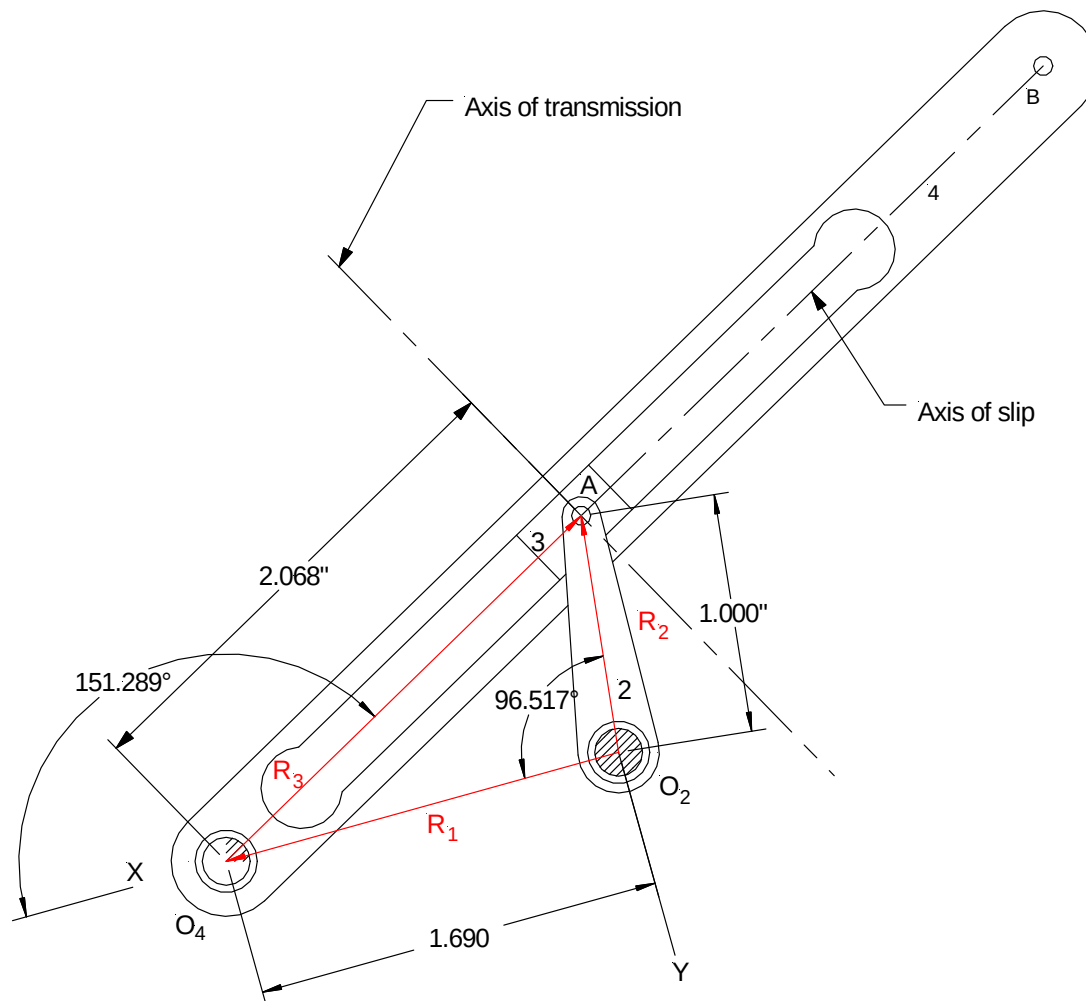
Link 2 (O_2A) $a := 1.00 \cdot \text{in}$ Angle link 3 makes with X axis $\theta_3 := -151.289 \cdot \text{deg}$

Link 4 (O_4B) $e := 4.76 \cdot \text{in}$

Motion of link 2 $\omega_2 := 16 \cdot \text{rad} \cdot \text{sec}^{-1}$ $\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Solution: See Figure P7-23 and Mathcad file P0792.

1. Draw the mechanism to scale and define a vector loop for links 1, 2, 3, and 4 using the four-bar crank-slider derivation in Section 7.3 as a model.



2. Write the vector loop equation, expand the result and separate into real and imaginary parts to solve for θ_3 and b (distance from O_4 to A).

$$\mathbf{R}_2 = \mathbf{R}_1 + \mathbf{R}_3$$

$$a \cdot e^{j \cdot \theta_2} = d + b \cdot e^{j \cdot \theta_3}$$

Substituting the Euler equivalents,

$$a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) = d + b(\cos(\theta_3) + j \cdot \sin(\theta_3))$$

Separating into real and imaginary components and solving for b .

$$b := \frac{a \cdot \sin(\theta_2)}{\sin(\theta_3)} \quad b = 2.068 \text{ in}$$

3. Differentiate the position equation, expand it and solve for ω_3 and $b\dot{\theta}$.

$$a \cdot j \cdot \omega_2 \cdot e^{j \cdot \theta_2} = b \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + b\dot{\theta} \cdot e^{j \cdot \theta_3}$$

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \cos(\theta_2 - \theta_3) \quad \omega_3 = 4.463 \cdot \frac{\text{rad}}{\text{s}}$$

$$b\dot{\theta} := \frac{a \cdot \omega_2 \cdot \cos(\theta_2) - b \cdot \omega_3 \cdot \cos(\theta_3)}{\sin(\theta_3)} \quad b\dot{\theta} = -13.070 \frac{\text{in}}{\text{s}}$$

4. Differentiate the velocity equation, expand it and solve for α_3 .

$$a \cdot j \cdot \alpha_2 \cdot e^{j \cdot \theta_2} + a \cdot j^2 \cdot \omega_2^2 \cdot e^{j \cdot \theta_2} = b\dot{\theta} \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + b \cdot j \cdot \alpha_3 \cdot e^{j \cdot \theta_3} \dots \\ + b \cdot j^2 \cdot \omega_3^2 \cdot e^{j \cdot \theta_3} + b\ddot{\theta} \cdot e^{j \cdot \theta_3} + b\dot{\theta} \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3}$$

$$\alpha_3 := \frac{1}{b} \cdot (a \cdot \alpha_2 \cdot \cos(\theta_2 - \theta_3) + a \cdot \omega_2^2 \cdot \sin(\theta_3 - \theta_2) - 2 \cdot b\dot{\theta} \cdot \omega_3) \quad \alpha_3 = -44.710 \cdot \frac{\text{rad}}{\text{s}^2}$$

5. Determine the acceleration of point A on link 2 (in the local XY coordinate system) using equations 7.13a.

$$\mathbf{A}_{A2} := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{A2} := |\mathbf{A}_{A2}| \quad A_{A2} = 256.000 \cdot \frac{\text{in}}{\text{s}^2} \quad \theta_{AA2} := \arg(\mathbf{A}_{A2}) \quad \theta_{AA2} = 83.483 \cdot \text{deg}$$

6. Determine the acceleration of points A and B on link 4 (in the local XY coordinate system) using equations 7.13a.

Rename θ_3 to θ_4 :

$$\theta_4 := \theta_3$$

Rename α_3 to α_4 :

$$\alpha_4 := \alpha_3$$

$$\omega_4 := \omega_3$$

$$\mathbf{A}_{A4} := b \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - b \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$A_{A4} := |\mathbf{A}_{A4}| \quad A_{A4} = 101.226 \cdot \frac{\text{in}}{\text{s}^2} \quad \theta_{AA4} := \arg(\mathbf{A}_{A4}) \quad \theta_{AA4} = 94.703 \cdot \text{deg}$$

$$\mathbf{A}_B := a \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - a \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$A_B := |\mathbf{A}_B| \quad A_B = 48.944 \cdot \frac{\text{in}}{\text{s}^2} \quad \theta_{AB} := \arg(\mathbf{A}_B) \quad \theta_{AB} = 94.703 \cdot \text{deg}$$

PROBLEM 7-93a

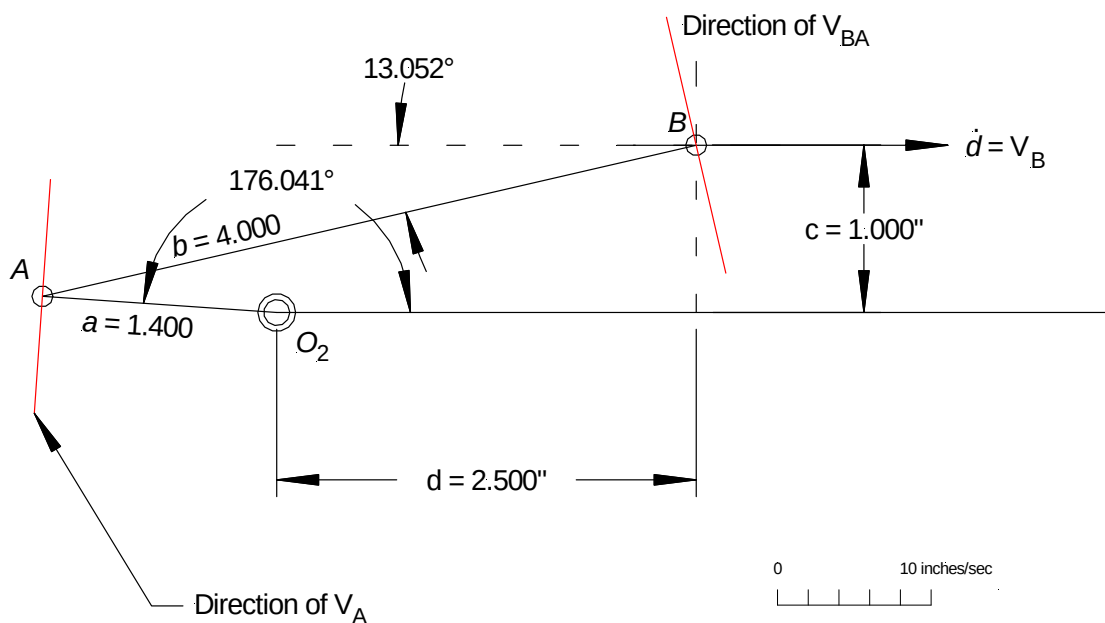
Statement: The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Fig P7-2. The link lengths and the values of d , $d\dot{\cdot}$, and $d\ddot{\cdot}$ are defined in Table P7-5. For row a , find the acceleration of the pin joint A and the angular acceleration of the crank using a graphical method.

Given: Link lengths:

$$\begin{aligned} \text{Link 2} \quad a &:= 1.4 \cdot \text{in} & \text{Link 3} \quad b &:= 4 \cdot \text{in} \\ \text{Offset} \quad c &:= 1 \cdot \text{in} & \theta_2 &:= 176.041 \cdot \text{deg} & d\dot{\cdot} &:= 10 \cdot \text{in} \cdot \text{sec}^{-1} & d\ddot{\cdot} &:= 0 \cdot \text{in} \cdot \text{sec}^{-2} \end{aligned}$$

Solution: See Figure P7-2, Table P7-5, and Mathcad file P0793a.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.

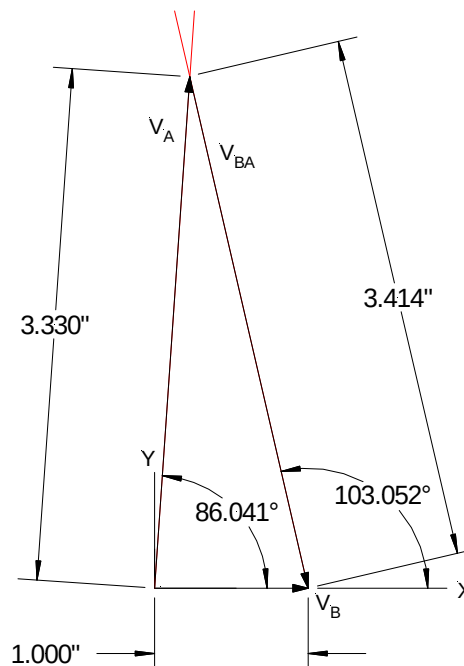


2. Use equation 6.6.24b to (graphically) determine the magnitude of the velocity at point A . The equation to be solved graphically is

$$\mathbf{V}_A = \mathbf{V}_B + \mathbf{V}_{BA}$$

- a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
- b. From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
- c. From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_A$, magnitude unknown.
- d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_A$ construction line and drawing $\mathbf{V}_A$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{BA}$ construction line.

3. From the velocity triangle we have:



$$\text{Velocity scale factor: } k_v := \frac{10 \cdot \text{in} \cdot \text{sec}^{-1}}{\text{in}}$$

$$V_A := 3.330 \cdot \text{in} \cdot k_v \quad V_A = 33.300 \cdot \frac{\text{in}}{\text{sec}} \quad \theta_{VB} := 86.041 \cdot \text{deg}$$

$$V_{BA} := 3.414 \cdot \text{in} \cdot k_v \quad V_{BA} = 34.140 \cdot \frac{\text{in}}{\text{sec}} \quad \theta_{VBA} := 103.052 \cdot \text{deg} - 180 \text{deg}$$

$$\theta_{VBA} = -76.948 \cdot \text{deg}$$

4. Use equation 6.7 to find the angular velocity of the crank (link 2).

$$\omega_2 := \frac{V_A}{a} \quad \omega_2 = 23.786 \cdot \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

$$\omega_3 := \frac{V_{BA}}{b} \quad \omega_3 = 8.535 \cdot \frac{\text{rad}}{\text{sec}} \quad \text{CW}$$

5. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

6. For point A, this becomes: $\mathbf{A}_A^t + \mathbf{A}_A^n = \mathbf{A}_B^t - (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$$A_{An} := a \cdot \omega_2^2 \quad A_{An} = 792.064 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot \text{deg} \quad \theta_{AAn} = 356.041 \cdot \text{deg}$$

$$A_{Bt} := \text{dddot} \quad A_{Bt} = 0.000 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{ABt} := 0 \cdot \text{deg} \quad \theta_{ABt} = 0.000 \cdot \text{deg}$$

$$A_{BAAn} := b \cdot \omega_3^2 \quad A_{BAAn} = 291.385 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\theta_{ABAn} := 13.052 \cdot \text{deg} + 180 \cdot \text{deg} \quad \theta_{ABAn} = 193.052 \cdot \text{deg}$$

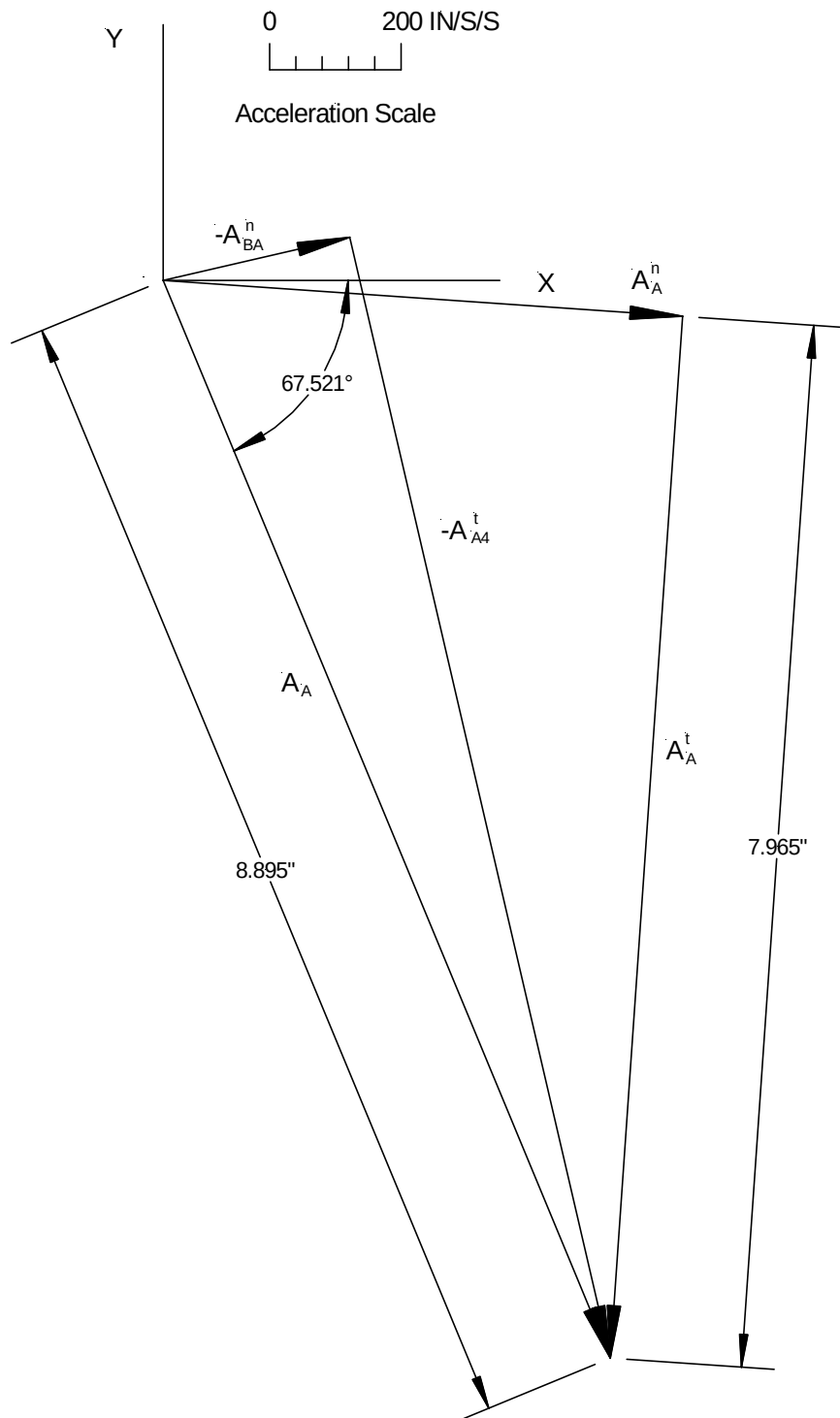
4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_A^n$ at an angle of θ_{AAn} . From the origin, draw $-\mathbf{A}_{BA}^t$ at an angle of θ_{ABt} . Now that the vectors with known magnitudes are drawn, from the tip $\mathbf{A}_A^n$ and the tip of $-\mathbf{A}_{BA}^t$, draw construction lines in the directions of $\mathbf{A}_A^t$ and $\mathbf{A}_{BA}^n$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_A^t$ and $\mathbf{A}_{BA}^n$.
5. From the graphical solution below,

$$\text{Acceleration scale factor } k_a := 200 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$A_A := 8.895 \cdot k_a \quad A_A = 1779 \cdot \text{in} \cdot \text{sec}^{-2} \quad \text{at an angle of } -67.52 \text{ deg}$$

$$A_{At} := 7.965 \cdot k_a \quad A_{At} = 1593 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\alpha_2 := \frac{A_{At}}{a} \quad \alpha_2 = 1138 \cdot \frac{\text{rad}}{\text{sec}^2}$$



PROBLEM 7-94a

Statement: The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Fig P7-2. The link lengths and the values of d , $\dot{d}$ and $\ddot{d}$ are defined in Table P7-5. For row a , find the acceleration of pin joint A and the angular acceleration of the crank using the analytic method. Draw the linkage to scale and label it before setting up the equations.

Given: Link lengths:

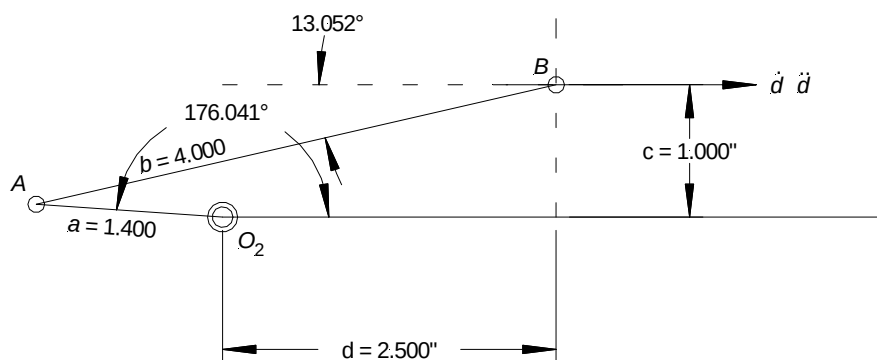
$$\text{Link 2 (O}_2\text{A)} \quad a := 1.4 \cdot \text{in} \quad \text{Link 3 (AB)} \quad b := 4 \cdot \text{in} \quad \text{Offset (y}_B\text{)} \quad c := 1 \cdot \text{in}$$

$$\text{Slider position} \quad d := 2.5 \cdot \text{in} \quad \text{Slider velocity} \quad \dot{d} := 10 \cdot \frac{\text{in}}{\text{s}}$$

$$\text{Slider acceleration:} \quad \ddot{d} := 0 \cdot \frac{\text{in}}{\text{s}^2}$$

Solution: See Figure P7-2, Table P7-5, and Mathcad file P0794a.

1. Draw the linkage to scale and label it.



2. Determine the open value of θ_2 using equations 4.20 and 4.21.

$$K_1 := a^2 - b^2 + c^2 + d^2 \quad K_1 = -6.79 \text{ in}^2$$

$$K_2 := -2 \cdot a \cdot c \quad K_2 = -2.8 \text{ in}^2$$

$$K_3 := -2 \cdot a \cdot d \quad K_3 = -7 \text{ in}^2$$

$$A := K_1 - K_3 \quad A = 0.21 \text{ in}^2$$

$$B := 2 \cdot K_2 \quad B = -5.6 \text{ in}^2$$

$$C := K_1 + K_3 \quad C = -13.79 \text{ in}^2$$

$$\theta_2 := 2 \cdot \text{atan2}(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}) \quad \theta_2 = 176.041 \cdot \text{deg}$$

3. Determine the value of θ_3 using equation 4.16a or 4.17.

$$\beta(a, b, c, d, \alpha) := \begin{cases} \theta \leftarrow \text{asin}\left(\frac{a \cdot \sin(\alpha) - c}{b}\right) \\ d_1 \leftarrow a \cdot \cos(\alpha) - b \cdot \cos(\theta) \\ \text{return } \theta \text{ if } d_1 = d \\ \text{asin}\left(-\frac{a \cdot \sin(\alpha) - c}{b}\right) + \pi \text{ otherwise} \end{cases}$$

$$\theta_3 := \beta(a, b, c, d, \theta_2) \quad \theta_3 = 193.052 \cdot \text{deg}$$

4. Determine the value of ω_2 and ω_3 using equations 6.24.

$$\omega_2 := \frac{ddot{\theta}_3 \cdot \cos(\theta_3)}{a \cdot (\cos(\theta_2) \cdot \sin(\theta_3) - \sin(\theta_2) \cdot \cos(\theta_3))} \quad \omega_2 = -23.785 \cdot \frac{\text{rad}}{\text{s}}$$

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \cos(\theta_2)}{b \cdot \cos(\theta_3)} \quad \omega_3 = -8.525 \cdot \frac{\text{rad}}{\text{s}}$$

5. Determine the value of V_A using equation 6.7.

$$V_A := |a \cdot \omega_2| \quad V_A = 33.299 \frac{\text{in}}{\text{s}}$$

$$\theta_{VA} := \theta_2 + \text{sign}(\omega_2) \cdot 90 \cdot \text{deg} \quad \theta_{VA} = 86.041 \cdot \text{deg}$$

6. Determine the value of α_2 using equation 7.17b.

$$\alpha_2 := \frac{a \cdot \omega_2^2 \cdot (\cos(\theta_2) \cdot \cos(\theta_3) + \sin(\theta_2) \cdot \sin(\theta_3)) - b \cdot \omega_3^2 + dddot{\theta}_3 \cdot \cos(\theta_3)}{a \cdot (\cos(\theta_2) \cdot \sin(\theta_3) - \sin(\theta_2) \cdot \cos(\theta_3))}$$

$$\alpha_2 = 1139.37 \cdot \frac{\text{rad}}{\text{s}^2}$$

7. Determine the value of $\mathbf{A}_A$ using equation 7.3.

$$\mathbf{A}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_A := |\mathbf{A}_A| \quad A_A = 1780.93 \frac{\text{in}}{\text{sec}^2}$$

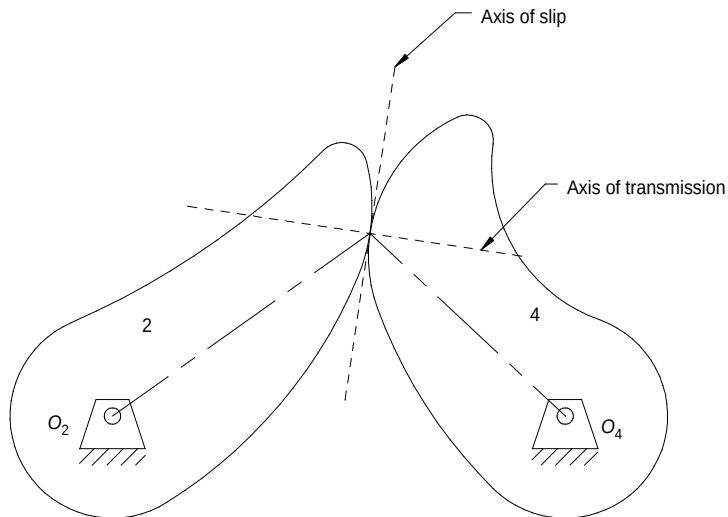
$$\theta_{AA} := \arg(\mathbf{A}_A) \quad \theta_{AA} = -67.553 \text{ deg}$$

PROBLEM 8-1

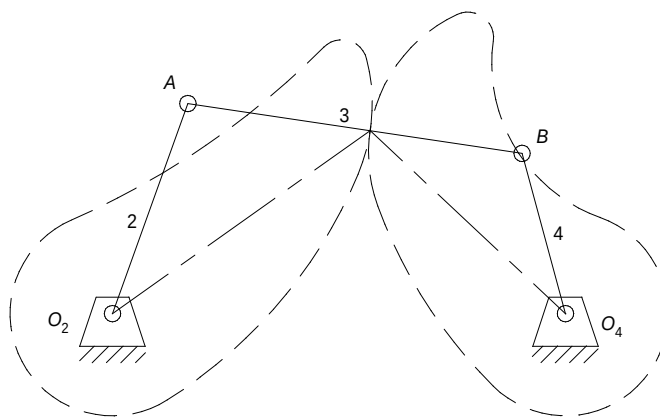
Statement: Figure P8-1 shows the cam and follower from Problem 6-65. Using graphical methods, find and sketch the equivalent fourbar linkage for this position of the cam and follower.

Solution: See Figure P8-1 and Mathcad file P0801.

1. Draw the cam and follower to scale.



2. As described in Section 8.1 and Figure 8-1, find the centers of curvature of the cams, which are located on the common normal (axis of transmission). They will be the locations of the moving pivots of the effective pin-jointed fourbar linkage.

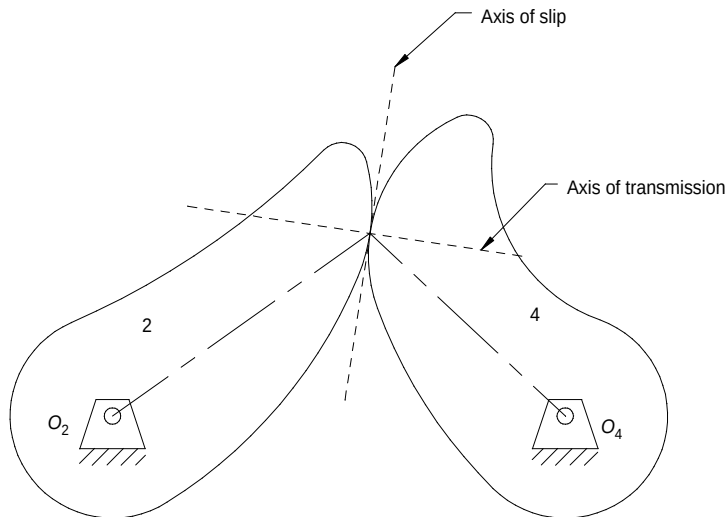


PROBLEM 8-2

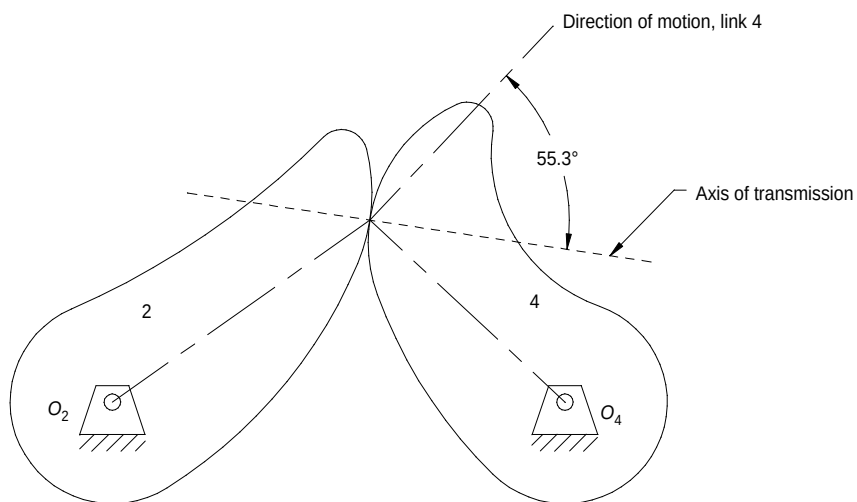
Statement: Figure P8-1 shows the cam and follower from Problem 6-65. Using graphical methods, find the pressure angle at the position shown.

Solution: See Figure P8-1 and Mathcad file P0802.

1. Draw the cam and follower to scale.



2. As defined in Section 8.6 and Figure 8-41, the pressure angle is the angle between the direction of motion of the follower and the direction of the axis of transmission. Establish those two directions and measure the angle between them.

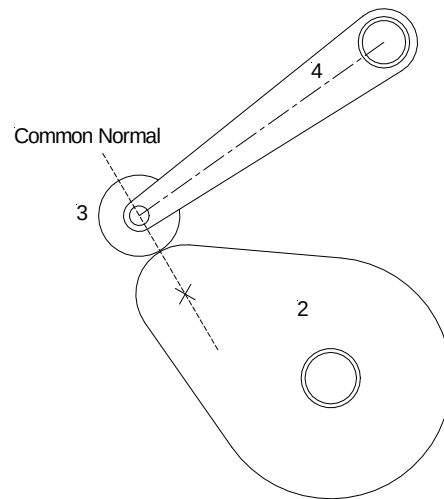


PROBLEM 8-3

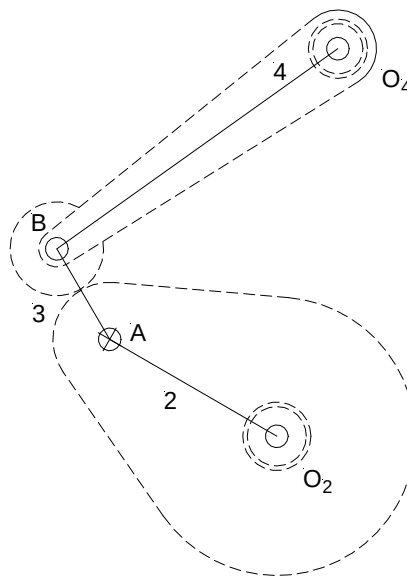
Statement: Figure P8-2 shows the cam and follower. Using graphical methods, find and sketch the equivalent fourbar linkage for this position of the cam and follower.

Solution: See Figure P8-2 and Mathcad file P0803.

1. Draw the cam and follower to scale.



2. As described in Section 8.1 and Figure 8-1, find the centers of curvature of the cams, which are located on the common normal (axis of transmission). They will be the locations of the moving pivots of the effective pin-jointed fourbar linkage.

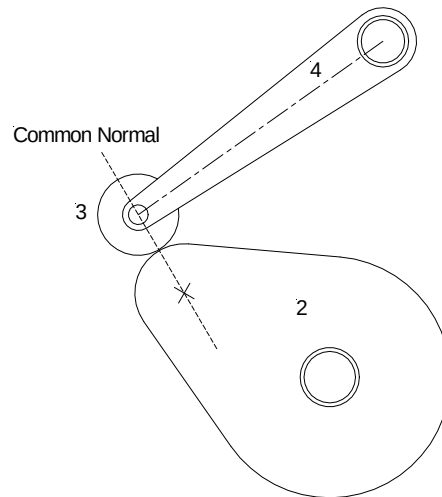


PROBLEM 8-4

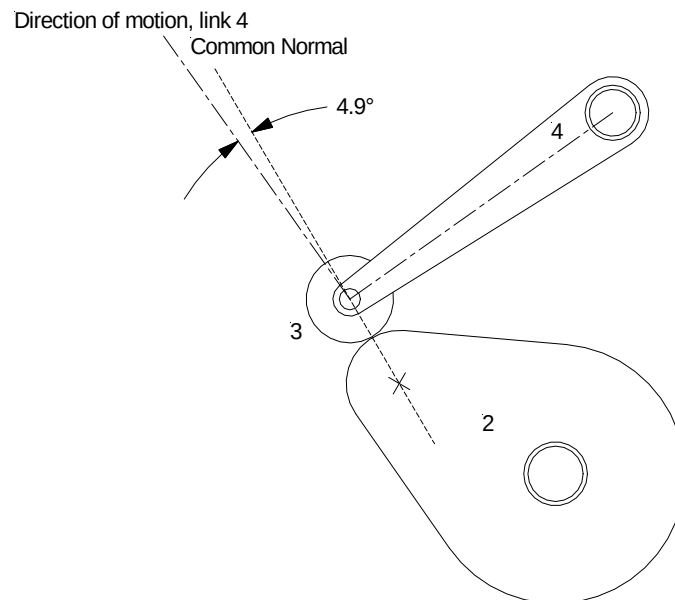
Statement: Figure P8-2 shows the cam and follower. Using graphical methods, find the pressure angle at the position shown.

Solution: See Figure P8-2 and Mathcad file P0804.

1. Draw the cam and follower to scale.



2. As defined in Section 8.6 and Figure 8-41, the pressure angle is the angle between the direction of motion of the follower and the direction of the axis of transmission. Establish those two directions and measure the angle between them.

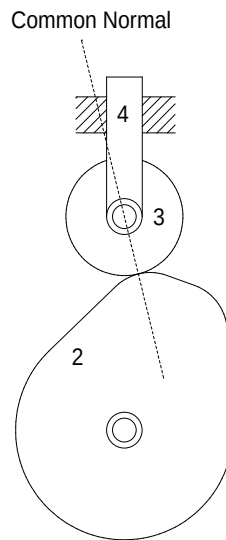


PROBLEM 8-5

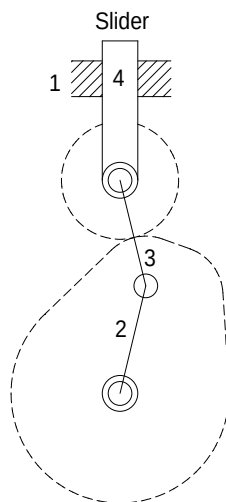
Statement: Figure P8-3 shows the cam and follower. Using graphical methods, find and sketch the equivalent fourbar linkage for this position of the cam and follower.

Solution: See Figure P8-3 and Mathcad file P0805.

1. Draw the cam and follower to scale.



2. As described in Section 8.1 and Figure 8-1, find the centers of curvature of the cams, which are located on the common normal (axis of transmission). They will be the locations of the moving pivots of the effective pin-jointed fourbar linkage.

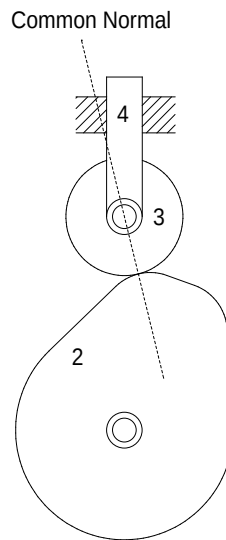


PROBLEM 8-6

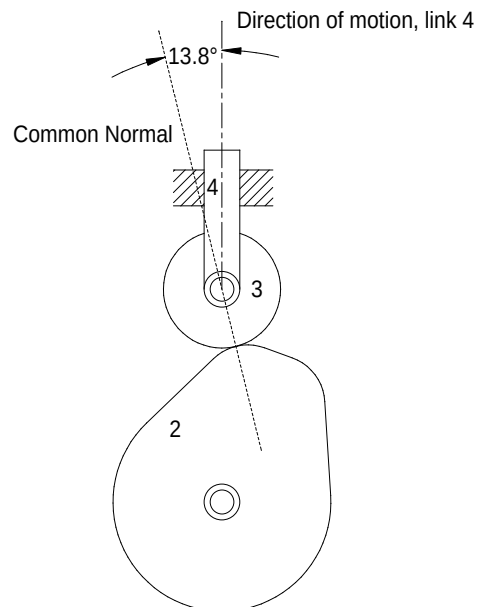
Statement: Figure P8-3 shows the cam and follower. Using graphical methods, find the pressure angle at the position shown.

Solution: See Figure P8-3 and Mathcad file P0806.

1. Draw the cam and follower to scale.



2. As defined in Section 8.6 and Figure 8-41, the pressure angle is the angle between the direction of motion of the follower and the direction of the axis of transmission. Establish those two directions and measure the angle between them.



PROBLEM 8-7

Statement: Design a double-dwell cam to move a follower from 0 to 2.5 in in 60 deg, dwell for 120 deg, fall 2.5 in in 30 deg and dwell for the remainder. The total cycle must take 4 sec. Choose suitable programs for rise and fall to minimize accelerations. Plot the $s v a j$ diagrams.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 60 \cdot \text{deg}$	$\beta_2 := 120 \cdot \text{deg}$	$\beta_3 := 30 \cdot \text{deg}$	$\beta_4 := 150 \cdot \text{deg}$
$h_1 := 2.5 \cdot \text{in}$	$h_2 := 0 \cdot \text{in}$	$h_3 := 2.5 \cdot \text{in}$	$h_4 := 0 \cdot \text{in}$

Cycle time: $\tau := 4 \cdot \text{sec}$

Solution: See Mathcad file P0807.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.571 \frac{\text{rad}}{\text{sec}}$$

2. From Table 8-3, the motion program with lowest acceleration that does not have infinite jerk is the modified trapezoidal. The modified trapezoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$\begin{aligned} b &:= 0.25 & c &:= 0.50 & d &:= 0.25 \\ C_v &:= 2.0000 & C_a &:= 4.8881 & C_j &:= 61.426 \end{aligned}$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 \leq x \leq b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$\begin{aligned} y_1(x) &:= C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi} \right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \right] & y'_1(x) &:= C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x\right) \right) \\ y''_1(x) &:= C_a \cdot \sin\left(\frac{\pi}{b} \cdot x\right) & y'''_1(x) &:= C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x\right) \end{aligned}$$

for $b/2 \leq x \leq (1-d)/2$

$$\begin{aligned} y_2(x) &:= C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \right] & y'_2(x) &:= C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \right] \\ y''_2(x) &:= C_a & y'''_2(x) &:= 0 \end{aligned}$$

for $(1-d)/2 \leq x \leq (1+d)/2$

$$\begin{aligned} y_3(x) &:= C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2} \right) \cdot x + \left(\frac{d}{\pi} \right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi} \right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y'_3(x) &:= C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y''_3(x) &:= C_a \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] & y'''_3(x) &:= -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \end{aligned}$$

for $(1 + d)/2 \leq x \leq 1 - b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + (2 \cdot d^2 - b^2) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) \quad y''_4(x) := -C_a \quad y'''_4(x) := 0$$

for $1 - b/2 \leq x \leq 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot (d^2 - b^2)}{\pi^2} + \frac{(1 - b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos \left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin \left[\frac{\pi}{b} \cdot (x - 1) \right] \quad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos \left[\frac{\pi}{b} \cdot (x - 1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the *SVAJ* curves, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range $0 \leq \theta \leq 360$ deg.
6. Write the local *svaj* equations for the first interval, $0 \leq \theta \leq \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y'_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y'_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y'_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y'_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y''_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y''_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y''_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y''_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y'''_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y'''_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y'''_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y'''_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y'''_5(x) \right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 \leq \theta \leq \beta_1 + \beta_2$. For this interval, the value of *S* is the value of *S* at the end of the previous interval and the values of *V*, *A*, and *J* are zero because of the dwell.

$$\text{For } \beta_1 \leq \theta \leq \beta_1 + \beta_2$$

$$s_2(x) := h_1 \quad v_2(x) := 0 \quad a_2(x) := 0 \quad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$.

$$\text{For } \beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$$

$$s_3(x) := h_3 \left[1 - \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right) \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right]$$

$$v_3(x) := -\frac{h_3}{\beta_3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_3(x) := -\frac{h_3}{\beta_3^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_3(x) := -\frac{h_3}{\beta_3^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

$$\text{For } \beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$$

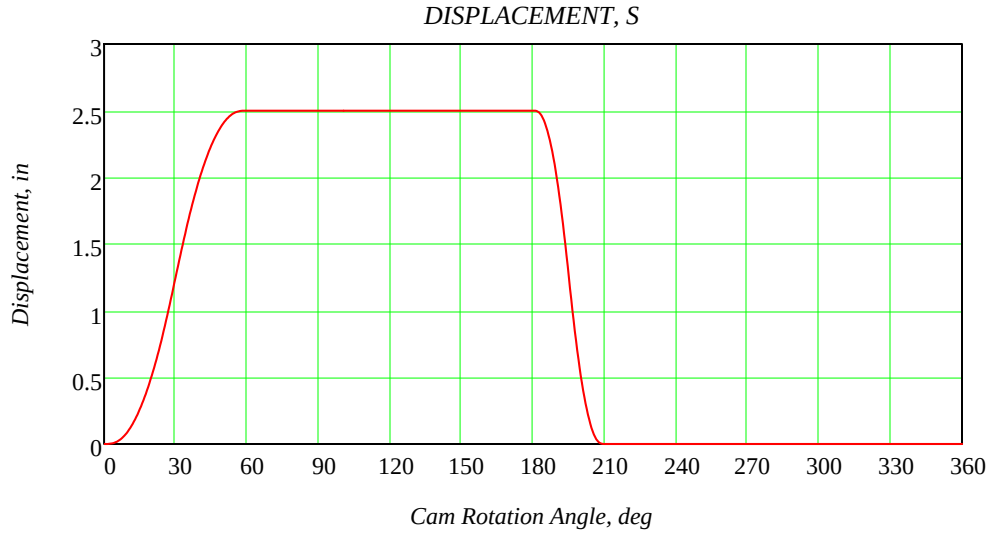
$$s_4(x) := 0 \quad v_4(x) := 0 \quad a_4(x) := 0 \quad j_4(x) := 0$$

10. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$\text{Let } \theta_1 := \beta_1 \quad \theta_2 := \theta_1 + \beta_2 \quad \theta_3 := \theta_2 + \beta_3 \quad \theta_4 := \theta_3 + \beta_4$$

$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2) \cdot s_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \dots \\ + R(\theta, \theta_2, \theta_3) \cdot s_3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4) \cdot s_4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$

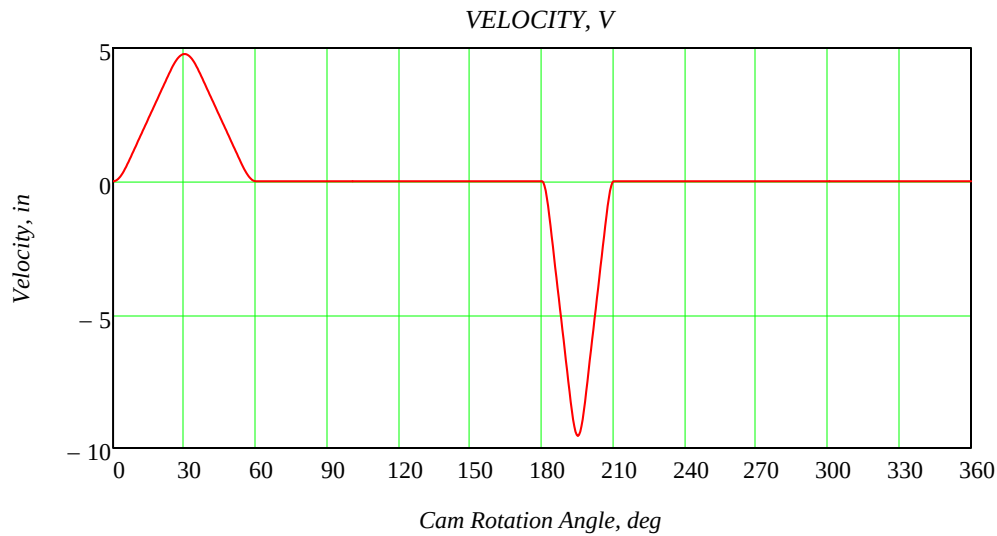
$$\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



11. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$V(\theta) := v_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot v_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

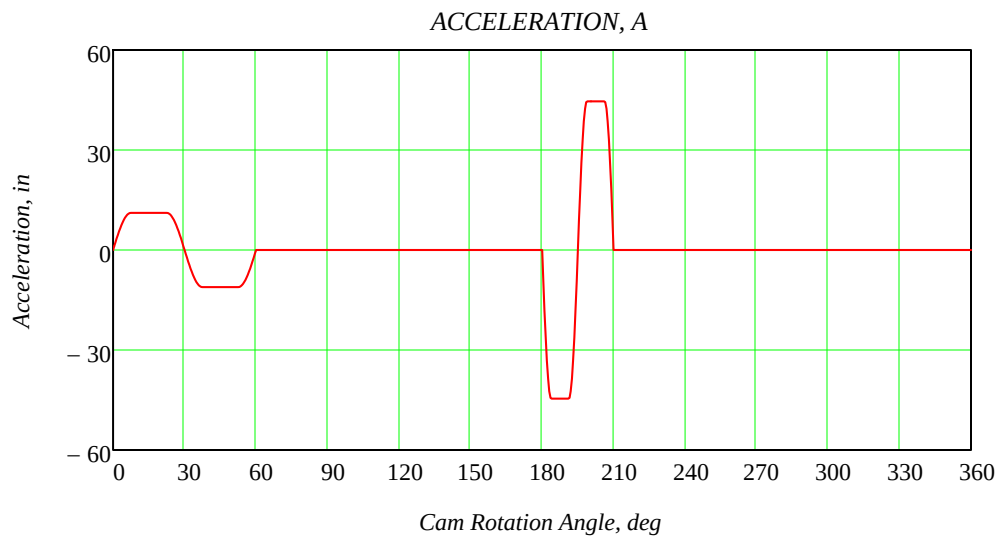
$$+ R(\theta, \theta_2, \theta_3) \cdot v_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot v_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



12. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := a_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot a_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

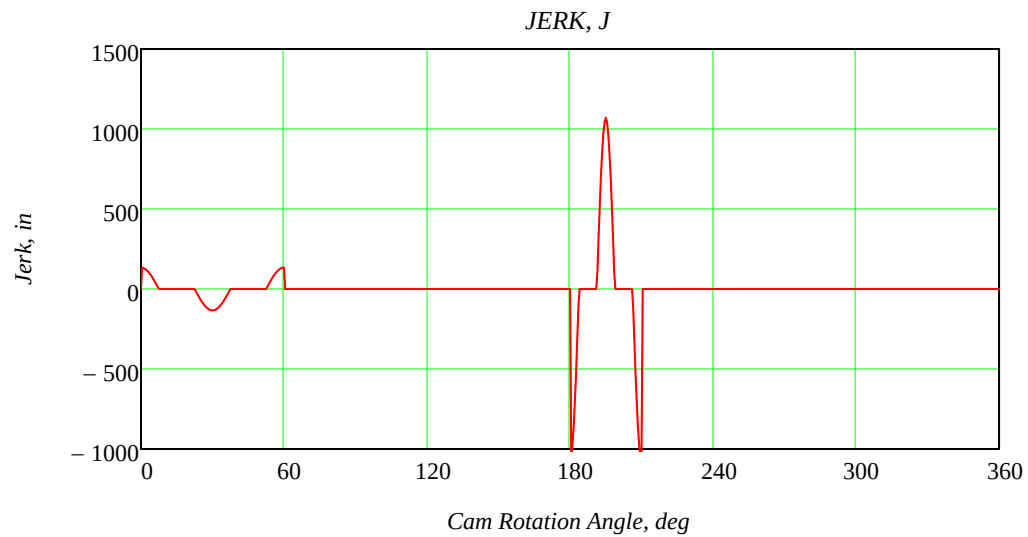
$$+ R(\theta, \theta_2, \theta_3) \cdot a_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot a_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := j_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot j_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

$$+ R(\theta, \theta_2, \theta_3) \cdot j_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot j_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



PROBLEM 8-8

Statement: Design a double-dwell cam to move a follower from 0 to 1.5 in in 45 deg, dwell for 150 deg, fall 1.5 in in 90 deg and dwell for the remainder. The total cycle must take 6 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the SVAJ diagrams.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 45 \cdot \text{deg}$	$\beta_2 := 150 \cdot \text{deg}$	$\beta_3 := 90 \cdot \text{deg}$	$\beta_4 := 75 \cdot \text{deg}$
$h_1 := 1.5 \cdot \text{in}$	$h_2 := 0 \cdot \text{in}$	$h_3 := 1.5 \cdot \text{in}$	$h_4 := 0 \cdot \text{in}$
Cycle time: $\tau := 6 \cdot \text{sec}$			

Solution: See Mathcad file P0808.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.047 \frac{\text{rad}}{\text{sec}}$$

2. The modified sinusoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$b := 0.25 \quad c := 0.00 \quad d := 0.75$$

$$C_a := 5.5280$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 \leq x \leq b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi} \right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x \right) \right] \quad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x \right) \right)$$

$$y''_1(x) := C_a \cdot \sin\left(\frac{\pi}{b} \cdot x \right) \quad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x \right)$$

for $b/2 \leq x \leq (1-d)/2$

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \right] \quad y'_2(x) := C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \right]$$

$$y''_2(x) := C_a \quad y'''_2(x) := 0$$

for $(1-d)/2 \leq x \leq (1+d)/2$

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2} \right) \cdot x + \left(\frac{d}{\pi} \right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi} \right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right]$$

$$y'_3(x) := C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right]$$

$$y''_3(x) := C_a \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \quad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right]$$

for $(1 + d)/2 \leq x \leq 1 - b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + (2 \cdot d^2 - b^2) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) \quad y''_4(x) := -C_a \quad y'''_4(x) := 0$$

for $1 - b/2 \leq x \leq 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot (d^2 - b^2)}{\pi^2} + \frac{(1 - b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos \left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin \left[\frac{\pi}{b} \cdot (x - 1) \right] \quad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos \left[\frac{\pi}{b} \cdot (x - 1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the *SVAJ* curves, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range $0 \leq \theta \leq 360$ deg.
6. Write the local *svaj* equations for the first interval, $0 \leq \theta \leq \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y'_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y'_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y'_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y'_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y''_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y''_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y''_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y''_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y'''_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y'''_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y'''_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y'''_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y'''_5(x) \right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 \leq \theta \leq \beta_1 + \beta_2$. For this interval, the value of S is the value of S at the end of the previous interval and the values of V , A , and J are zero because of the dwell.

For $\beta_1 \leq \theta \leq \beta_1 + \beta_2$

$$s_2(x) := h_1 \quad v_2(x) := 0 \quad a_2(x) := 0 \quad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$

$$s_3(x) := h_3 \left[1 - \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right) \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right]$$

$$v_3(x) := -\frac{h_3}{\beta_3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right) \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x)$$

$$a_3(x) := -\frac{h_3}{\beta_3^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right) \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x)$$

$$j_3(x) := -\frac{h_3}{\beta_3^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right) \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x)$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$. For this interval, the values of S , V , A , and J are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$

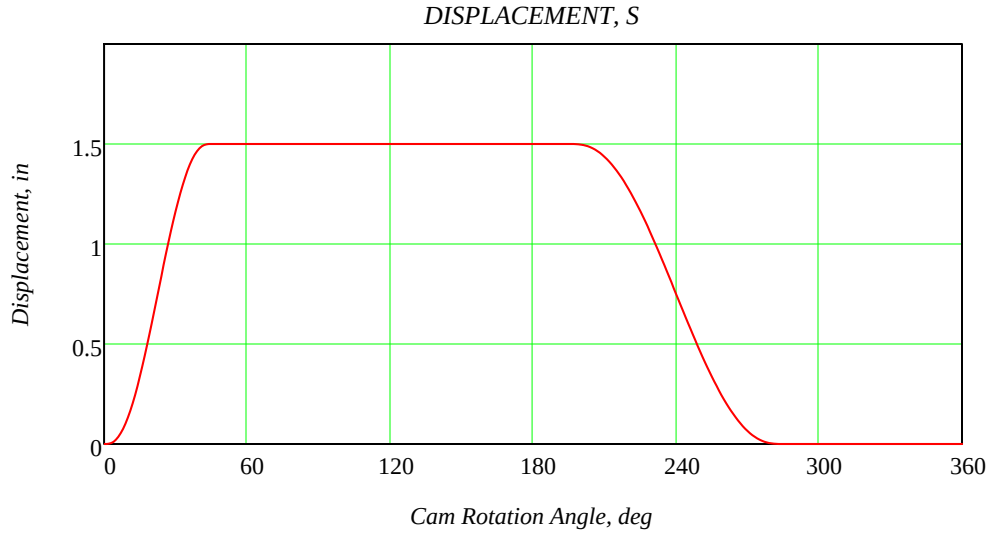
$$s_4(x) := 0 \quad v_4(x) := 0 \quad a_4(x) := 0 \quad j_4(x) := 0$$

10. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$\text{Let} \quad \theta_1 := \beta_1 \quad \theta_2 := \theta_1 + \beta_2 \quad \theta_3 := \theta_2 + \beta_3 \quad \theta_4 := \theta_3 + \beta_4$$

$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2) \cdot s_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \dots \\ + R(\theta, \theta_2, \theta_3) \cdot s_3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4) \cdot s_4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$

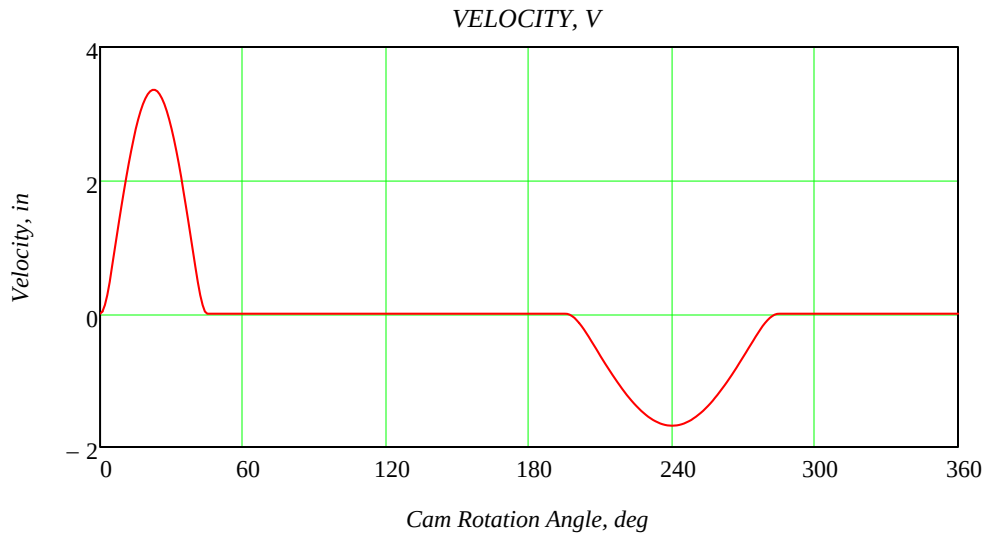
$$\theta := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



11. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$V(\theta) := v_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot v_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

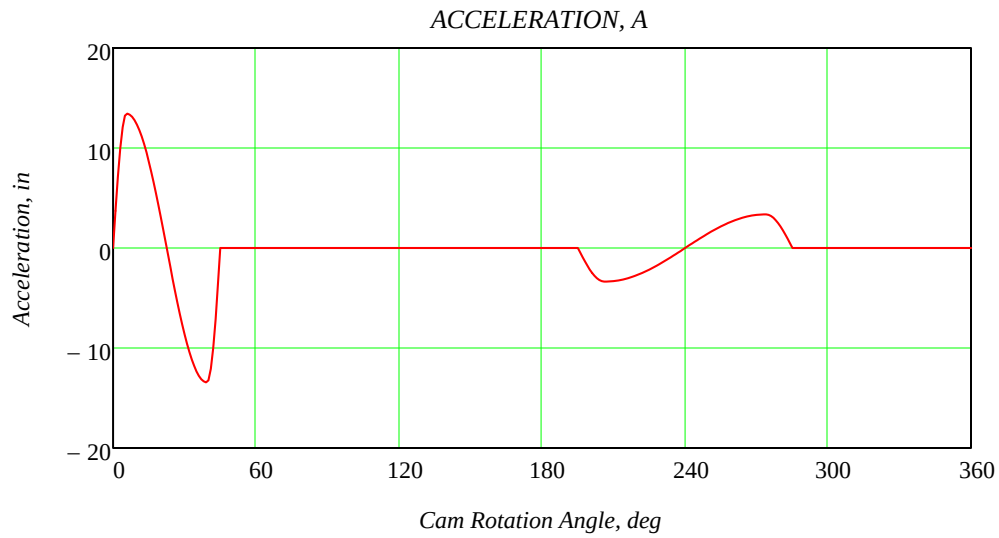
$$+ R(\theta, \theta_2, \theta_3) \cdot v_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot v_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



12. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := a_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot a_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

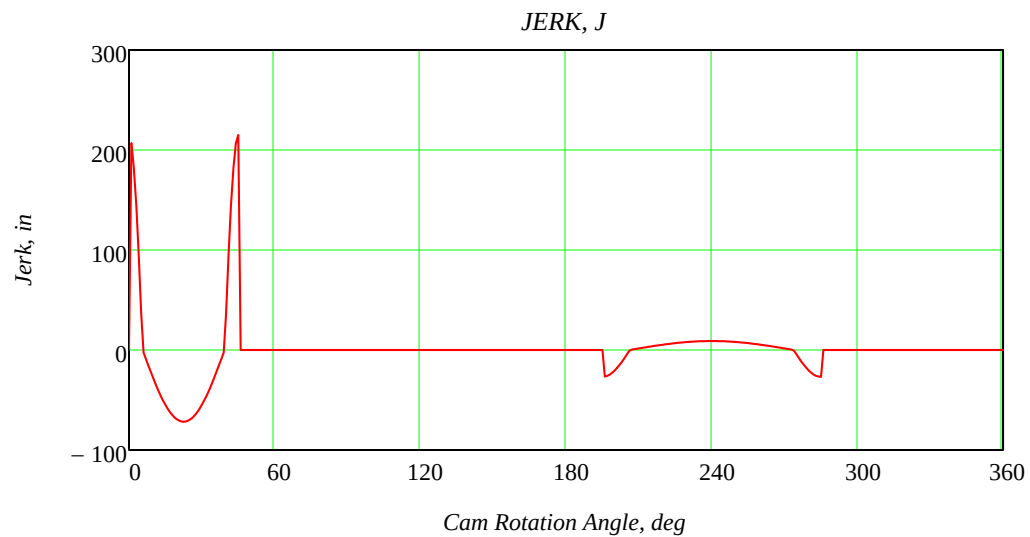
$$+ R(\theta, \theta_2, \theta_3) \cdot a_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot a_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := j_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot j_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

$$+ R(\theta, \theta_2, \theta_3) \cdot j_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot j_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



PROBLEM 8-9

Statement: Design a single-dwell cam to move a follower from 0 to 2.0 in in 60 deg, fall 2.0 in in 90 deg and dwell for the remainder. The total cycle must take 2 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the SVAJ diagrams.

Given: RISE/FALL DWELL

$\beta := 150 \cdot \text{deg}$ $\beta_3 := 210 \cdot \text{deg}$

$h := 2.0 \cdot \text{in}$ $h_3 := 0.0 \cdot \text{in}$

Cycle time: $\tau := 2 \cdot \text{sec}$

Solution: See Mathcad file P0809.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \qquad \omega = 3.142 \frac{\text{rad}}{\text{sec}}$$

2. Use a two-segment polynomial. Let the rise and fall, together, be one segment and the dwell be the second segment. Then, the boundary conditions are:

$$\begin{array}{llll} \text{at} & \theta = 0: & s = 0, & v = 0, \quad a = 0 \\ & \theta = \beta_1: & s = h, & v = 0 \\ & \theta = \beta: & s = 0, & v = 0, \quad a = 0 \end{array}$$

This is a minimum set of 8 BCs. The $v = 0$ condition at $\theta = \beta_1$ is required to keep the displacement from overshooting the lift, h . Define the total lift, the rise interval, the fall interval, and the ratio of rise to the total interval.

Total lift: $h = 2.000 \text{ in}$

$$\begin{array}{llll} \text{Rise interval:} & \beta_1 := 60 \cdot \text{deg} & A := \frac{\beta_1}{\beta} & A = 0.400 \end{array}$$

Fall interval: $\beta_2 := 90 \cdot \text{deg}$

3. Use the 8 BCs and equation 8.23 to write 8 equations in s , v , and a similar to those in example 8-9 but with 8 terms in the equation for s (the highest term will be seventh degree).

For $\theta = 0$: $s = v = a = 0$

$$0 := c_0 \quad 0 := c_1 \quad 0 := c_2$$

For $\theta = \beta_1$: $s = h$, $v = 0$

$$\begin{aligned} h &:= c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5 + c_6 \cdot A^6 + c_7 \cdot A^7 \\ 0 &:= 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4 + 6 \cdot c_6 \cdot A^5 + 7 \cdot c_7 \cdot A^6 \end{aligned}$$

For $\theta = \beta$: $s = v = a = 0$

$$\begin{aligned} 0 &:= c_3 + c_4 + c_5 + c_6 + c_7 \\ 0 &:= 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5 + 6 \cdot c_6 + 7 \cdot c_7 \\ 0 &:= 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5 + 30 \cdot c_6 + 42 \cdot c_7 \end{aligned}$$

4. Solve for the unknown polynomial coefficients. Note that C_0 through C_2 are zero

$$C := \begin{pmatrix} A^3 & A^4 & A^5 & A^6 & A^7 \\ 3 \cdot A^2 & 4 \cdot A^3 & 5 \cdot A^4 & 6 \cdot A^5 & 7 \cdot A^6 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 & 7 \\ 6 & 12 & 20 & 30 & 42 \end{pmatrix} \quad H := \begin{pmatrix} h \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{pmatrix} := C^{-1} \cdot H$$

$$c_3 = 289.352 \text{ in}$$

$$c_4 = -1229.745 \text{ in}$$

$$c_5 = 1953.125 \text{ in}$$

$$c_6 = -1374.421 \text{ in}$$

$$c_7 = 361.690 \text{ in}$$

5. Write the SVAJ equations for the rise/fall segment.

$$s(\theta) := c_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + c_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + c_5 \cdot \left(\frac{\theta}{\beta}\right)^5 + c_6 \cdot \left(\frac{\theta}{\beta}\right)^6 + c_7 \cdot \left(\frac{\theta}{\beta}\right)^7$$

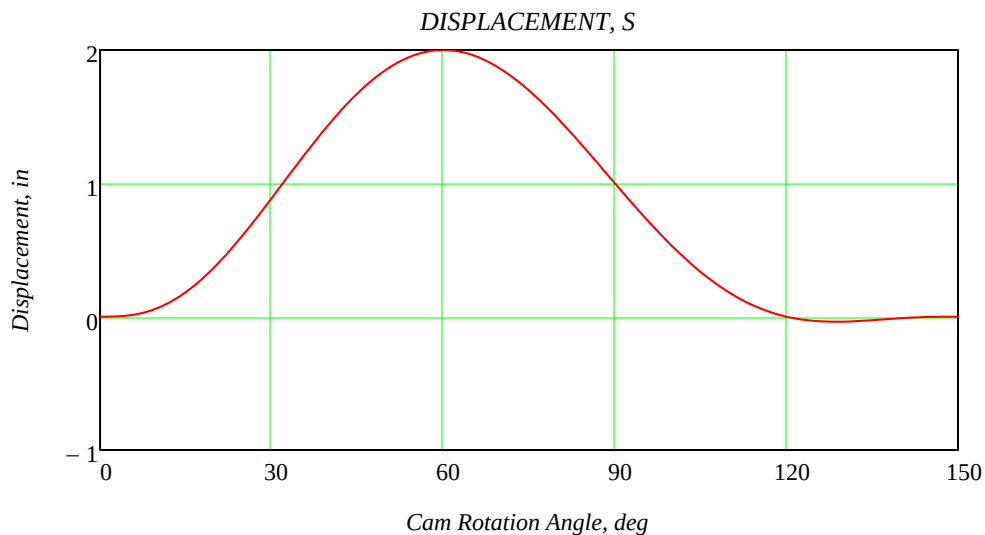
$$v(\theta) := \frac{1}{\beta} \cdot \left[3 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^3 + 5 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^4 + 6 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^5 + 7 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^6 \right]$$

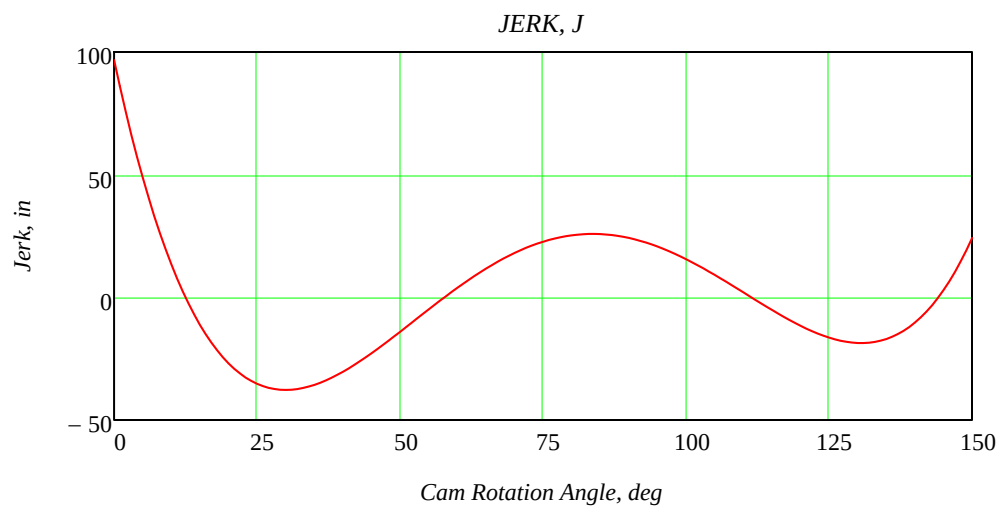
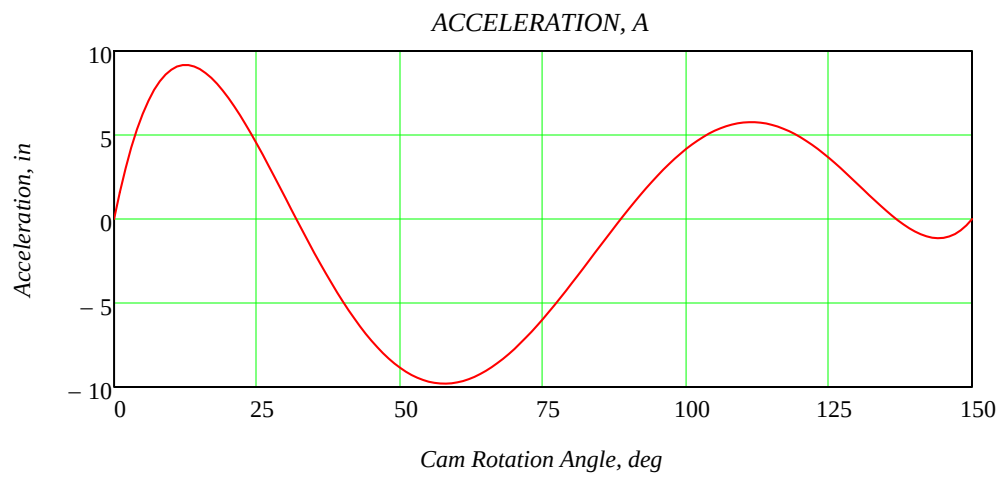
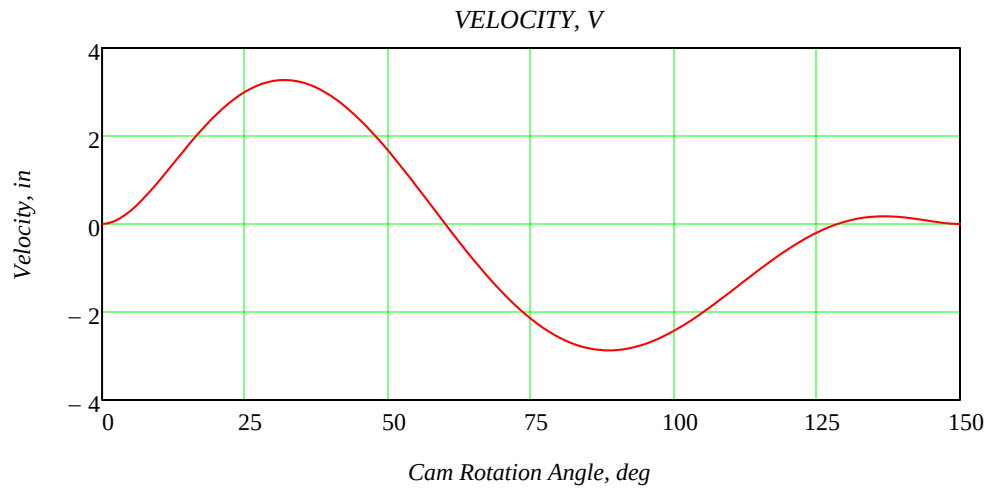
$$A(\theta) := \frac{1}{\beta^2} \cdot \left[6 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right) + 12 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^2 + 20 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^3 + 30 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^4 + 42 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^5 \right]$$

$$J(\theta) := \frac{1}{\beta^3} \cdot \left[6 \cdot c_3 + 24 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right) + 60 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^2 + 120 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^3 + 210 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^4 \right]$$

6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 \leq \theta \leq \beta$.

$$\theta := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots \beta$$





PROBLEM 8-10

Statement: Design a three-dwell cam to move a follower from 0 to 2.5 in in 40 deg, dwell for 100 deg, fall 1.5 in in 90 deg, dwell for 20 deg, fall 1.0 in in 30 deg, and dwell for the remainder. The total cycle must take 10 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the SVAJ diagrams.

Given:

RISE/FALL	DWELL	FALL	DWELL
$\beta_1 := 40 \cdot \text{deg}$	$\beta_2 := 100 \cdot \text{deg}$	$\beta_3 := 90 \cdot \text{deg}$	$\beta_4 := 20 \cdot \text{deg}$
$h_1 := 2.5 \cdot \text{in}$	$h_2 := 0.0 \cdot \text{in}$	$h_3 := 1.5 \cdot \text{in}$	$h_4 := 0.0 \cdot \text{in}$
$\beta_5 := 30 \cdot \text{deg}$	$\beta_6 := 80 \cdot \text{deg}$		
$h_5 := 1.0 \cdot \text{in}$	$h_6 := 0.0 \cdot \text{in}$		
Cycle time: $\tau := 15 \cdot \text{sec}$			

Solution: See Mathcad file P0810.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 0.419 \frac{\text{rad}}{\text{sec}}$$

2. From Table 8-3, the motion program with lowest velocity that does not have infinite jerk is the modified sinusoidal. The modified sinusoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$\begin{aligned} b &:= 0.25 & c &:= 0.00 & d &:= 0.75 \\ C_v &:= 1.7596 & C_a &:= 5.5280 & C_j &:= 69.466 \end{aligned}$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 \leq x \leq b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$\begin{aligned} y_1(x) &:= C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi} \right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x \right) \right] & y'_1(x) &:= C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x \right) \right) \\ y''_1(x) &:= C_a \cdot \sin\left(\frac{\pi}{b} \cdot x \right) & y'''_1(x) &:= C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x \right) \end{aligned}$$

for $b/2 \leq x \leq (1-d)/2$

$$\begin{aligned} y_2(x) &:= C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \right] & y'_2(x) &:= C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \right] \\ y''_2(x) &:= C_a & y'''_2(x) &:= 0 \end{aligned}$$

for $(1-d)/2 \leq x \leq (1+d)/2$

$$\begin{aligned} y_3(x) &:= C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2} \right) \cdot x + \left(\frac{d}{\pi} \right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi} \right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y'_3(x) &:= C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \end{aligned}$$

$$y''_3(x) := C_a \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right]$$

$$y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right]$$

for $(1+d)/2 \leq x \leq 1-b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2}\right) \cdot x + (2 \cdot d^2 - b^2) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8}\right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) \quad y''_4(x) := -C_a \quad y'''_4(x) := 0$$

for $1-b/2 \leq x \leq 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot (d^2 - b^2)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left[\frac{\pi}{b} \cdot (x-1)\right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos\left[\frac{\pi}{b} \cdot (x-1)\right] \right]$$

$$y''_5(x) := C_a \cdot \sin\left[\frac{\pi}{b} \cdot (x-1)\right] \quad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left[\frac{\pi}{b} \cdot (x-1)\right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the *SVAJ* curves, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

5. The global *SVAJ* equations are composed of six intervals (rise, dwell, fall, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range $0 \leq \theta \leq 360$ deg.
6. Write the local *svaj* equations for the first interval, $0 \leq \theta \leq \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 \leq \theta \leq \beta_1 + \beta_2$. For this interval, the value of S is the value of S at the end of the previous interval and the values of V , A , and J are zero because of the dwell.

For $\beta_1 \leq \theta \leq \beta_1 + \beta_2$ (dwell)

$$s_2(x) := h_1 \quad v_2(x) := 0 \quad a_2(x) := 0 \quad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$ (fall)

$$s_3(x) := h_3 \left[1 - \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right) \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right] + h_1 - h_3$$

$$v_3(x) := -\frac{h_3}{\beta_3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_3(x) := -\frac{h_3}{\beta_3^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_3(x) := -\frac{h_3}{\beta_3^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$. For this interval, the values of S , V , A , and J are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$ (dwell)

$$s_4(x) := h_1 - h_3 \quad v_4(x) := 0 \quad a_4(x) := 0 \quad j_4(x) := 0$$

10. Write the local *svaj* equations for the fifth interval,

For $\beta_1 + \beta_2 + \beta_3 + \beta_4 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5$ (fall)

$$s_5(x) := h_5 \left[1 - \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right) \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right]$$

$$v_5(x) := -\frac{h_5}{\beta_5} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_5(x) := -\frac{h_5}{\beta_5^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1-\frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1-\frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_5(x) := -\frac{h_5}{\beta_5^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1-\frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1-\frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

11. Write the local *svaj* equations for the sixth interval. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6$ (dwell)

$$s_6(x) := 0 \quad v_6(x) := 0 \quad a_6(x) := 0 \quad j_6(x) := 0$$

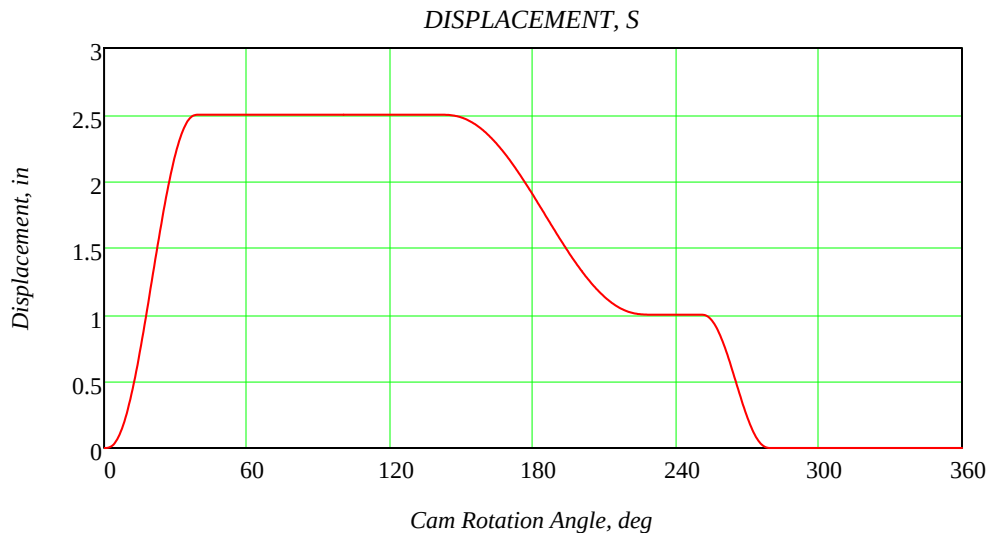
12. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

$$\text{Let} \quad \theta_1 := \beta_1 \quad \theta_2 := \theta_1 + \beta_2 \quad \theta_3 := \theta_2 + \beta_3 \quad \theta_4 := \theta_3 + \beta_4$$

$$\theta_5 := \theta_4 + \beta_5 \quad \theta_6 := \theta_5 + \beta_6$$

$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2) \cdot s_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \dots \\ + R(\theta, \theta_2, \theta_3) \cdot s_3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4) \cdot s_4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right) \dots \\ + R(\theta, \theta_4, \theta_5) \cdot s_5\left(\frac{\theta - \theta_4}{\theta_5 - \theta_4}\right) + R(\theta, \theta_5, \theta_6) \cdot s_6\left(\frac{\theta - \theta_5}{\theta_6 - \theta_5}\right)$$

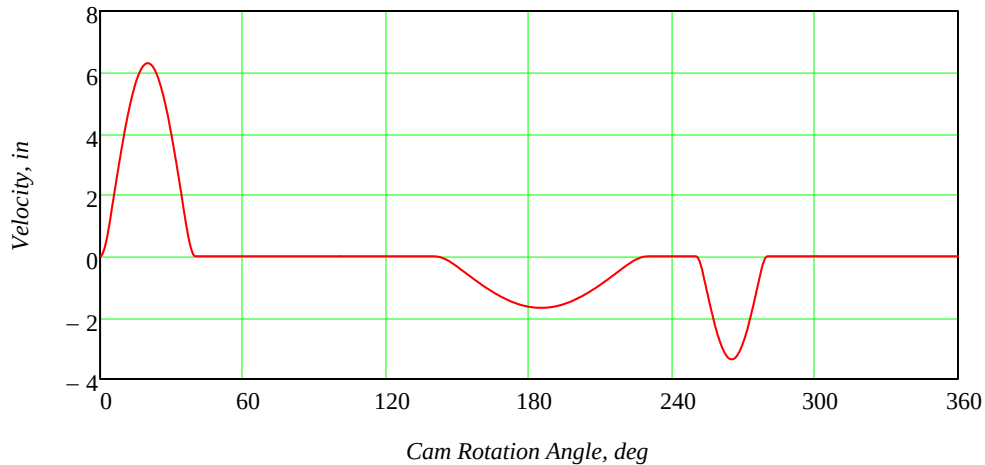
$$\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



15. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

$$\begin{aligned}
 V(\theta) := & v_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot v_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots \\
 & + R(\theta, \theta_2, \theta_3) \cdot v_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot v_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right) \dots \\
 & + R(\theta, \theta_4, \theta_5) \cdot v_5 \left(\frac{\theta - \theta_4}{\theta_5 - \theta_4} \right) + R(\theta, \theta_5, \theta_6) \cdot v_6 \left(\frac{\theta - \theta_5}{\theta_6 - \theta_5} \right)
 \end{aligned}$$

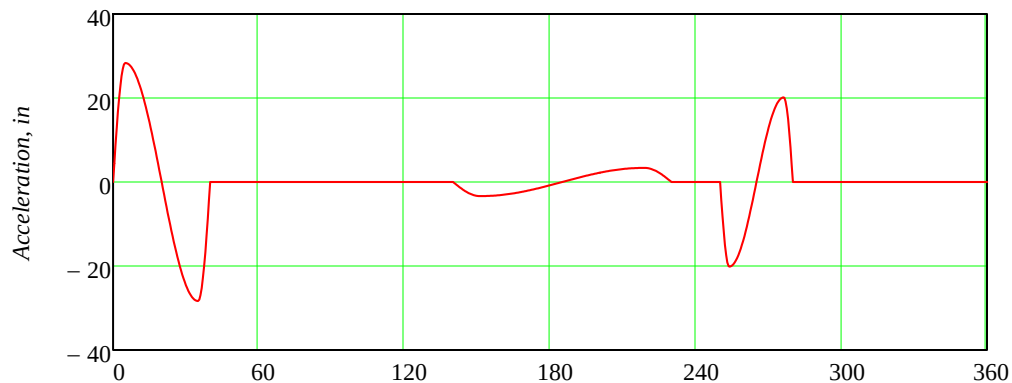
VELOCITY, V



16. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

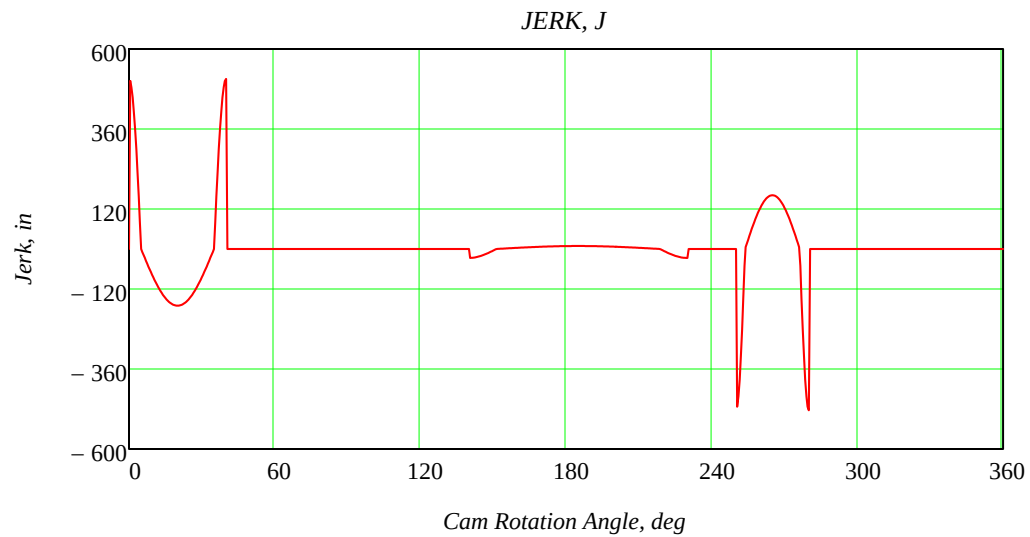
$$\begin{aligned}
 A(\theta) := & a_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot a_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots \\
 & + R(\theta, \theta_2, \theta_3) \cdot a_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot a_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right) \dots \\
 & + R(\theta, \theta_4, \theta_5) \cdot a_5 \left(\frac{\theta - \theta_4}{\theta_5 - \theta_4} \right) + R(\theta, \theta_5, \theta_6) \cdot a_6 \left(\frac{\theta - \theta_5}{\theta_6 - \theta_5} \right)
 \end{aligned}$$

ACCELERATION, A



17. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

$$\begin{aligned}
 J(\theta) := & j_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot j_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots \\
 & + R(\theta, \theta_2, \theta_3) \cdot j_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot j_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right) \dots \\
 & + R(\theta, \theta_4, \theta_5) \cdot j_5 \left(\frac{\theta - \theta_4}{\theta_5 - \theta_4} \right) + R(\theta, \theta_5, \theta_6) \cdot j_6 \left(\frac{\theta - \theta_5}{\theta_6 - \theta_5} \right)
 \end{aligned}$$



PROBLEM 8-11

Statement: Design a four-dwell cam to move a follower from 0 to 2.5 in in 40 deg, dwell for 100 deg, fall 1.5 in in 90 deg, dwell for 20 deg, fall 0.5 in in 30 deg, dwell for 40 deg, fall 0.5 in in 30 deg, and dwell for the remainder. The total cycle must take 15 sec. Choose suitable programs for rise and fall to minimize accelerations. Plot the SVAJ diagrams.

RISE/FALL	DWELL	FALL	DWELL
$\beta_1 := 40\text{-deg}$	$\beta_2 := 100\text{-deg}$	$\beta_3 := 90\text{-deg}$	$\beta_4 := 20\text{-deg}$
$h_1 := 2.5\text{-in}$	$h_2 := 0.0\text{-in}$	$h_3 := 1.5\text{-in}$	$h_4 := 0.0\text{-in}$
$\beta_5 := 30\text{-deg}$	$\beta_6 := 40\text{-deg}$	$\beta_7 := 30\text{-deg}$	$\beta_8 := 10\text{-deg}$
$h_5 := 0.5\text{-in}$	$h_6 := 0.0\text{-in}$	$h_7 := 0.5\text{-in}$	$h_8 := 0.0\text{-in}$
Cycle time: $\tau := 15\text{-sec}$			

Solution: See Mathcad file P0811.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 0.419 \frac{\text{rad}}{\text{sec}}$$

2. From Table 8-3, the motion program with lowest acceleration that does not have infinite jerk is the modified trapezoidal. The modified trapezoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$\begin{aligned} b &:= 0.25 & c &:= 0.50 & d &:= 0.25 \\ C_v &:= 2.0000 & C_a &:= 4.8881 & C_j &:= 61.426 \end{aligned}$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 \leq x \leq b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$\begin{aligned} y_1(x) &:= C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi} \right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x \right) \right] & y'_1(x) &:= C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x \right) \right) \\ y''_1(x) &:= C_a \cdot \sin\left(\frac{\pi}{b} \cdot x \right) & y'''_1(x) &:= C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x \right) \end{aligned}$$

for $b/2 \leq x \leq (1-d)/2$

$$\begin{aligned} y_2(x) &:= C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \right] & y'_2(x) &:= C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \right] \\ y''_2(x) &:= C_a & y'''_2(x) &:= 0 \end{aligned}$$

for $(1-d)/2 \leq x \leq (1+d)/2$

$$\begin{aligned} y_3(x) &:= C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2} \right) \cdot x + \left(\frac{d}{\pi} \right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi} \right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y'_3(x) &:= C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \end{aligned}$$

$$y''_3(x) := C_a \cdot \cos \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right]$$

$$y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right]$$

for $(1+d)/2 \leq x \leq 1-b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + (2 \cdot d^2 - b^2) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right)$$

$$y''_4(x) := -C_a$$

$$y'''_4(x) := 0$$

for $1-b/2 \leq x \leq 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot (d^2 - b^2)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left[\frac{\pi}{b} \cdot (x-1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos \left[\frac{\pi}{b} \cdot (x-1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin \left[\frac{\pi}{b} \cdot (x-1) \right]$$

$$y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos \left[\frac{\pi}{b} \cdot (x-1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the *SVAJ* curves, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

5. The global *SVAJ* equations are composed of eight intervals (rise, dwell, fall, dwell, fall, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range $0 \leq \theta \leq 360$ deg.
6. Write the local *svaj* equations for the first interval, $0 \leq \theta \leq \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y'_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y'_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y'_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y'_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y''_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y''_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y''_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y''_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y'''_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y'''_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y'''_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y'''_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y'''_5(x) \right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 \leq \theta \leq \beta_1 + \beta_2$. For this interval, the value of *S* is the value of *S* at the end of the previous interval and the values of *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 \leq \theta \leq \beta_1 + \beta_2$ (dwell)

$$s_2(x) := h_1 \quad v_2(x) := 0 \quad a_2(x) := 0 \quad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$ (fall)

$$s_3(x) := h_3 \left[1 - \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right) \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right] + h_1 - h_3$$

$$v_3(x) := -\frac{h_3}{\beta_3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_3(x) := -\frac{h_3}{\beta_3^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_3(x) := -\frac{h_3}{\beta_3^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$ (dwell)

$$s_4(x) := h_1 - h_3 \quad v_4(x) := 0 \quad a_4(x) := 0 \quad j_4(x) := 0$$

10. Write the local *svaj* equations for the fifth interval,

For $\beta_1 + \beta_2 + \beta_3 + \beta_4 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5$ (fall)

$$s_5(x) := h_5 \left[1 - \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right) \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right] + h_7$$

$$v_5(x) := -\frac{h_5}{\beta_5} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_5(x) := -\frac{h_5}{\beta_5^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_5(x) := -\frac{h_5}{\beta_5^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

11. Write the local *svaj* equations for the sixth interval. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6$ (dwell)

$$s_6(x) := h_5 \quad v_6(x) := 0 \quad a_6(x) := 0 \quad j_6(x) := 0$$

12. Write the local *svaj* equations for the seventh interval,

For $\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7$ (fall)

$$s_7(x) := h_7 \left[1 - \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right) \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right]$$

$$v_7(x) := -\frac{h_7}{\beta_7} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_7(x) := -\frac{h_7}{\beta_7^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_7(x) := -\frac{h_7}{\beta_7^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

13. Write the local *svaj* equations for the eighth interval. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8$ (dwell)

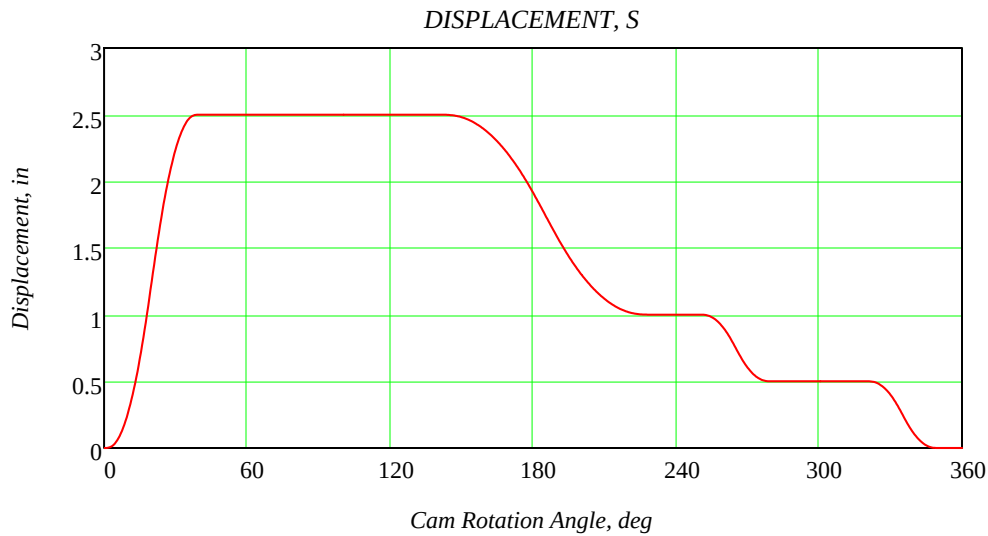
$$s_8(x) := 0 \quad v_8(x) := 0 \quad a_8(x) := 0 \quad j_8(x) := 0$$

14. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

$$\begin{aligned} \text{Let } \theta_1 &:= \beta_1 & \theta_2 &:= \theta_1 + \beta_2 & \theta_3 &:= \theta_2 + \beta_3 & \theta_4 &:= \theta_3 + \beta_4 \\ \theta_5 &:= \theta_4 + \beta_5 & \theta_6 &:= \theta_5 + \beta_6 & \theta_7 &:= \theta_6 + \beta_7 & \theta_8 &:= \theta_7 + \beta_8 \end{aligned}$$

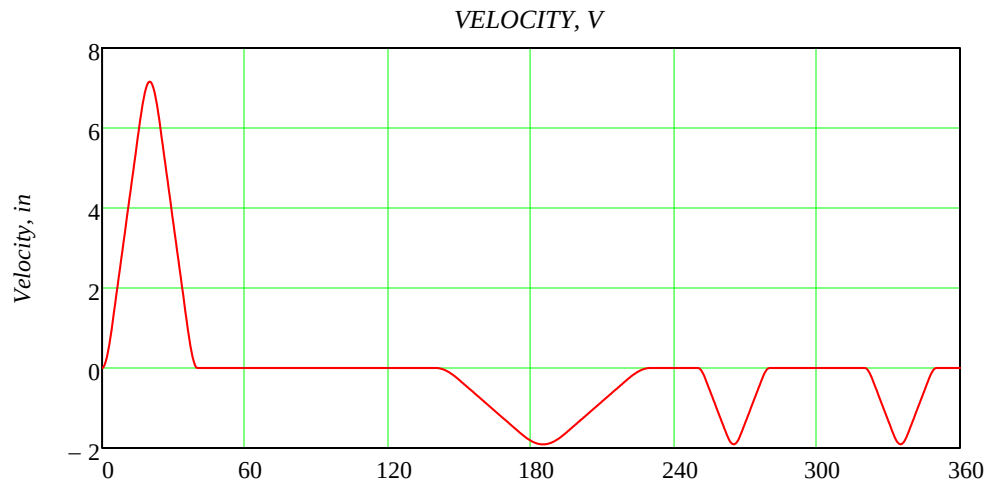
$$\begin{aligned} S(\theta) &:= s_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot s_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots \\ &+ R(\theta, \theta_2, \theta_3) \cdot s_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot s_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right) \dots \\ &+ R(\theta, \theta_4, \theta_5) \cdot s_5 \left(\frac{\theta - \theta_4}{\theta_5 - \theta_4} \right) + R(\theta, \theta_5, \theta_6) \cdot s_6 \left(\frac{\theta - \theta_5}{\theta_6 - \theta_5} \right) \dots \\ &+ R(\theta, \theta_6, \theta_7) \cdot s_7 \left(\frac{\theta - \theta_6}{\theta_7 - \theta_6} \right) + R(\theta, \theta_7, \theta_8) \cdot s_8 \left(\frac{\theta - \theta_7}{\theta_8 - \theta_7} \right) \end{aligned}$$

$$\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



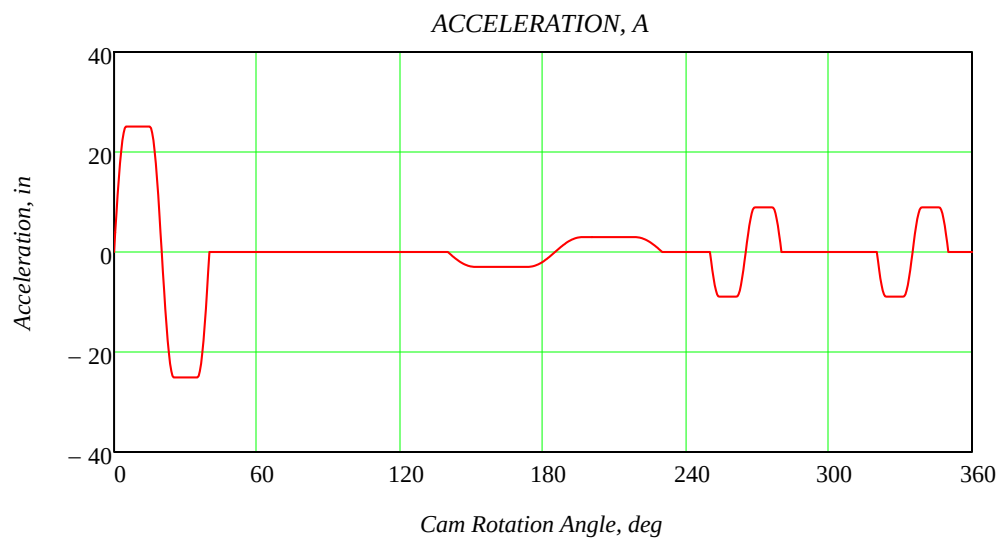
15. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

$$\begin{aligned} V(\theta) &:= v_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot v_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots \\ &+ R(\theta, \theta_2, \theta_3) \cdot v_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot v_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right) \dots \\ &+ R(\theta, \theta_4, \theta_5) \cdot v_5 \left(\frac{\theta - \theta_4}{\theta_5 - \theta_4} \right) + R(\theta, \theta_5, \theta_6) \cdot v_6 \left(\frac{\theta - \theta_5}{\theta_6 - \theta_5} \right) \dots \\ &+ R(\theta, \theta_6, \theta_7) \cdot v_7 \left(\frac{\theta - \theta_6}{\theta_7 - \theta_6} \right) + R(\theta, \theta_7, \theta_8) \cdot v_8 \left(\frac{\theta - \theta_7}{\theta_8 - \theta_7} \right) \end{aligned}$$



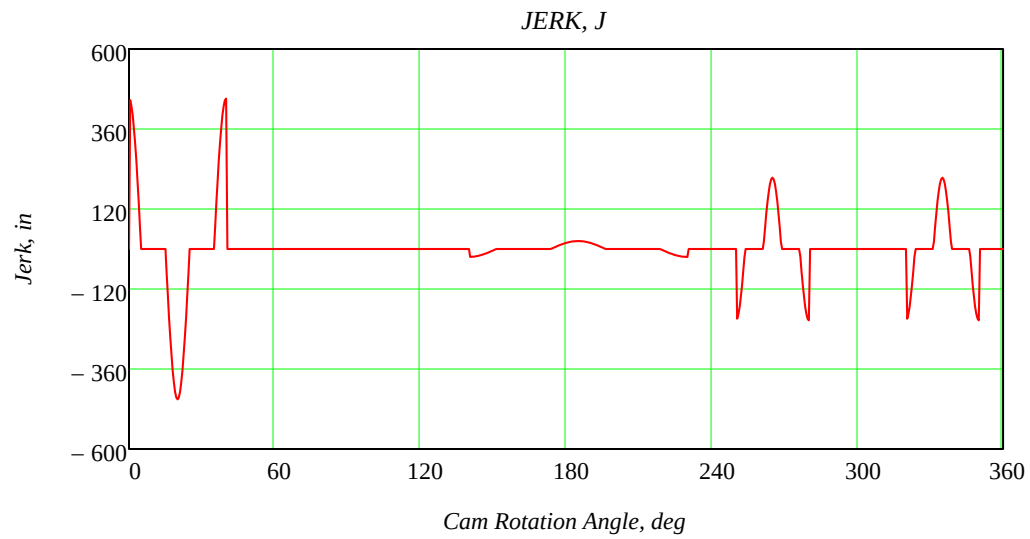
16. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

$$\begin{aligned}
 A(\theta) := & a_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot a_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots \\
 & + R(\theta, \theta_2, \theta_3) \cdot a_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot a_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right) \dots \\
 & + R(\theta, \theta_4, \theta_5) \cdot a_5 \left(\frac{\theta - \theta_4}{\theta_5 - \theta_4} \right) + R(\theta, \theta_5, \theta_6) \cdot a_6 \left(\frac{\theta - \theta_5}{\theta_6 - \theta_5} \right) \dots \\
 & + R(\theta, \theta_6, \theta_7) \cdot a_7 \left(\frac{\theta - \theta_6}{\theta_7 - \theta_6} \right) + R(\theta, \theta_7, \theta_8) \cdot a_8 \left(\frac{\theta - \theta_7}{\theta_8 - \theta_7} \right)
 \end{aligned}$$



17. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the eight intervals defined above.

$$\begin{aligned}
 J(\theta) := & j_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot j_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots \\
 & + R(\theta, \theta_2, \theta_3) \cdot j_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot j_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right) \dots \\
 & + R(\theta, \theta_4, \theta_5) \cdot j_5 \left(\frac{\theta - \theta_4}{\theta_5 - \theta_4} \right) + R(\theta, \theta_5, \theta_6) \cdot j_6 \left(\frac{\theta - \theta_5}{\theta_6 - \theta_5} \right) \dots \\
 & + R(\theta, \theta_6, \theta_7) \cdot j_7 \left(\frac{\theta - \theta_6}{\theta_7 - \theta_6} \right) + R(\theta, \theta_7, \theta_8) \cdot j_8 \left(\frac{\theta - \theta_7}{\theta_8 - \theta_7} \right)
 \end{aligned}$$



PROBLEM 8-12

Statement: Size the cam from Problem 8-7 for a 1-in-radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Draw the cam profile. Repeat for a flat faced follower. Which would you use?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 60 \cdot deg$	$\beta_2 := 120 \cdot deg$	$\beta_3 := 30 \cdot deg$	$\beta_4 := 150 \cdot deg$
$h_1 := 2.5 \cdot in$	$h_2 := 0 \cdot in$	$h_3 := 2.5 \cdot in$	$h_4 := 0 \cdot in$
Cycle time: $\tau := 4 \cdot sec$			

Solution: See Mathcad file P0812.

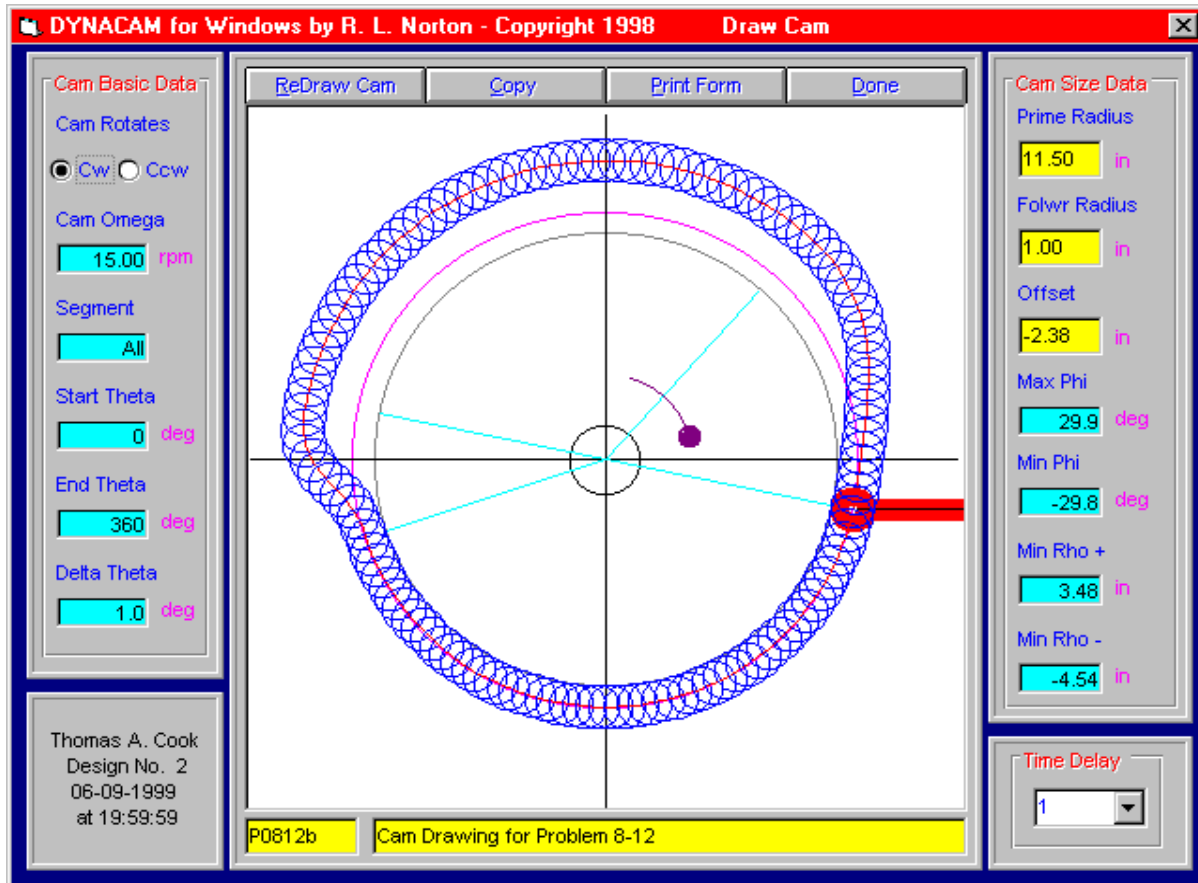
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \quad \omega = 1.571 \frac{rad}{sec} \quad \omega = 15.000 rpm$$

2. Problem 8-7 used a four-segment cam with modified trapezoidal acceleration for the rise and fall. Enter the above data into program DYNACAM. The input screen is shown below.

Angles		Cam Contour		Position (in)			
Seg	Beta	Start	End	Motion	Program	Start	End
1	60	0	60	Rise	MT - Mod Trap	0	2.5
2	120	60	180	Dwell	DW - Dwell	2.5	2.5
3	30	180	210	Fall	MT - Mod Trap	2.5	0
4	150	210	360	Dwell	DW - Dwell	0	0
5							
6							
7							
8							
9							
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12							
13							
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16							

3. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 30 deg or less by increasing the prime circle radius. The resulting cam is shown below.



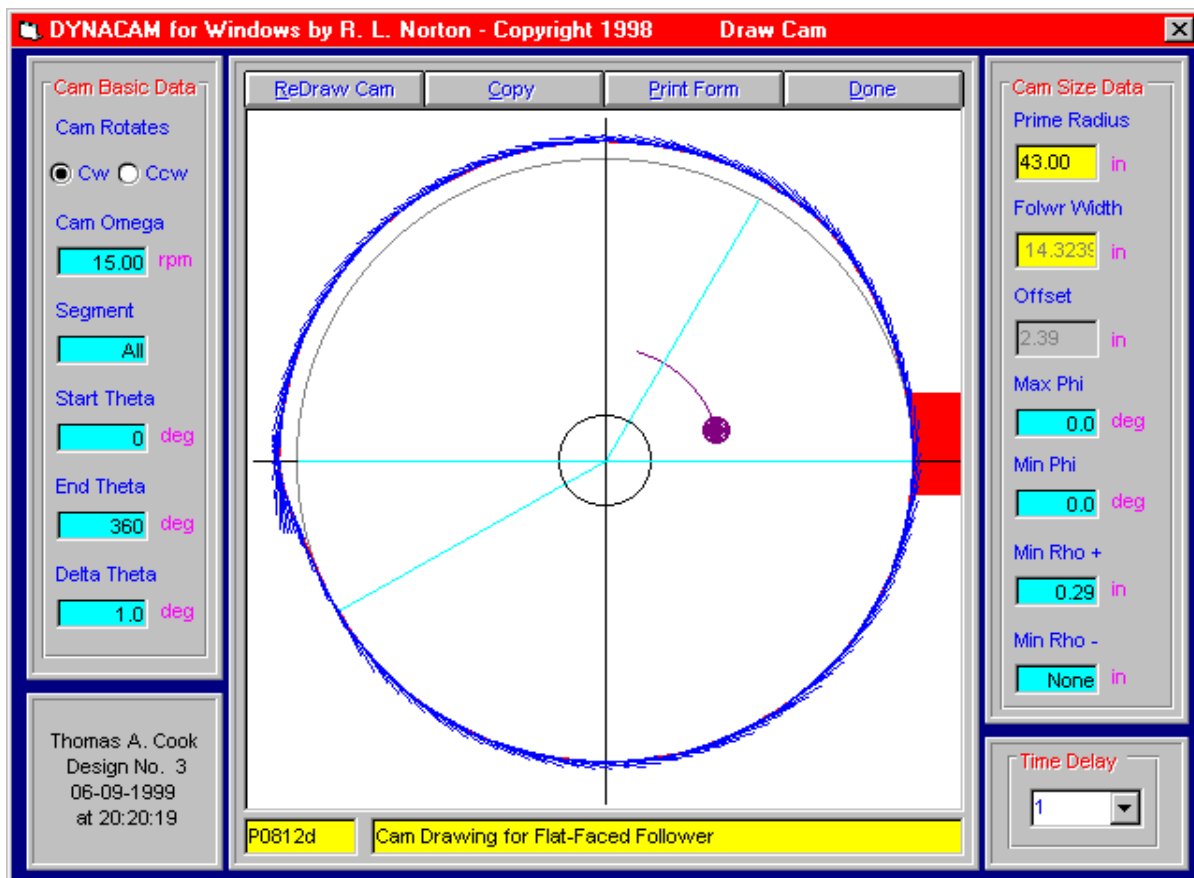
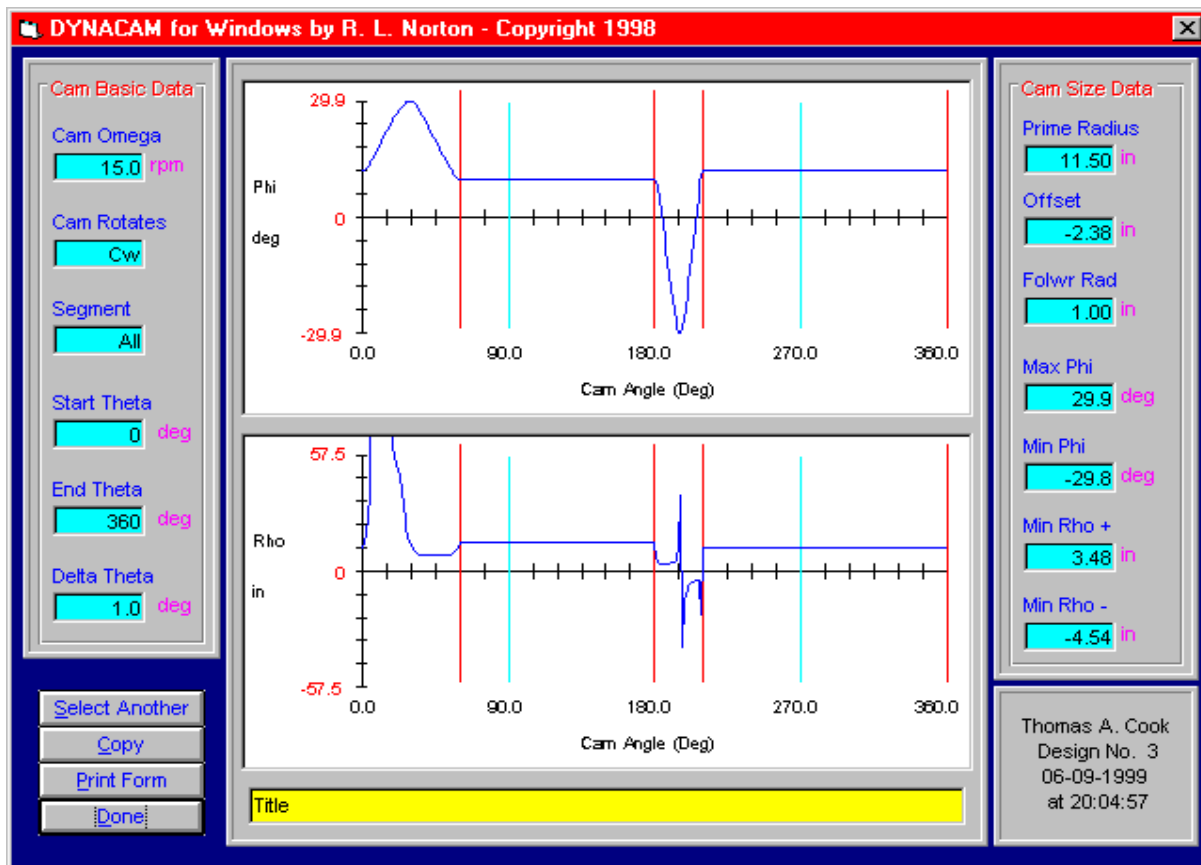
4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

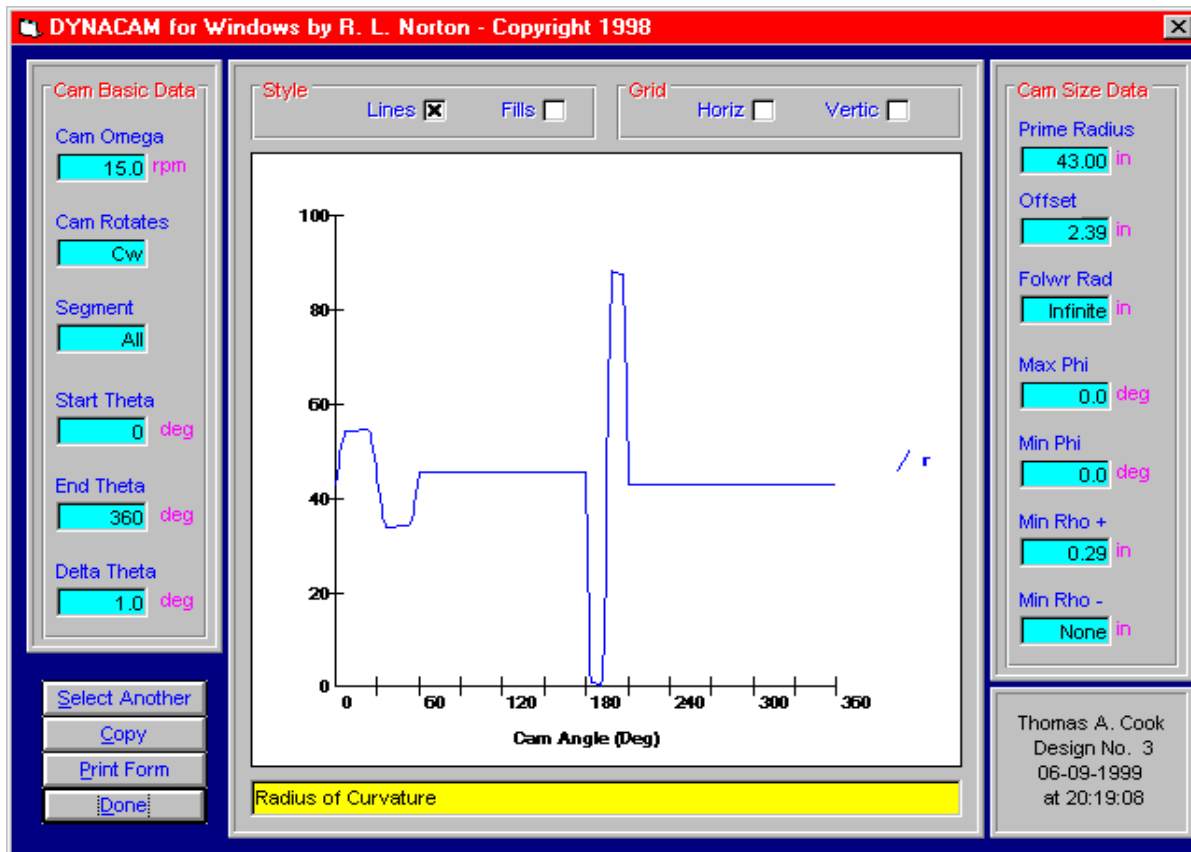
$$\text{Prime circle radius} \quad R_p := 11.5 \cdot \text{in}$$

$$\text{Roller follower radius} \quad R_f := 1.00 \cdot \text{in}$$

$$\text{Follower eccentricity} \quad \varepsilon := -2.375 \cdot \text{in}$$

5. Graphs of ϕ and ρ for the roller follower are shown on the following page.
6. The cam was sized for a flat-faced follower and the cam drawing is shown on the next page. In order to avoid undercutting, a base circle radius of 43 in is required. Obviously, the roller follower is to be preferred over the flat face follower in this case.





PROBLEM 8-13

Statement: Size the cam from Problem 8-8 for a 1.5-in-radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Draw the cam profile. Repeat for a flat faced follower. Which would you use?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 45 \cdot deg$	$\beta_2 := 150 \cdot deg$	$\beta_3 := 90 \cdot deg$	$\beta_4 := 75 \cdot deg$
$h_1 := 1.5 \cdot in$	$h_2 := 0 \cdot in$	$h_3 := 1.5 \cdot in$	$h_4 := 0 \cdot in$
Cycle time: $\tau := 6 \cdot sec$			

Solution: See Mathcad file P0813.

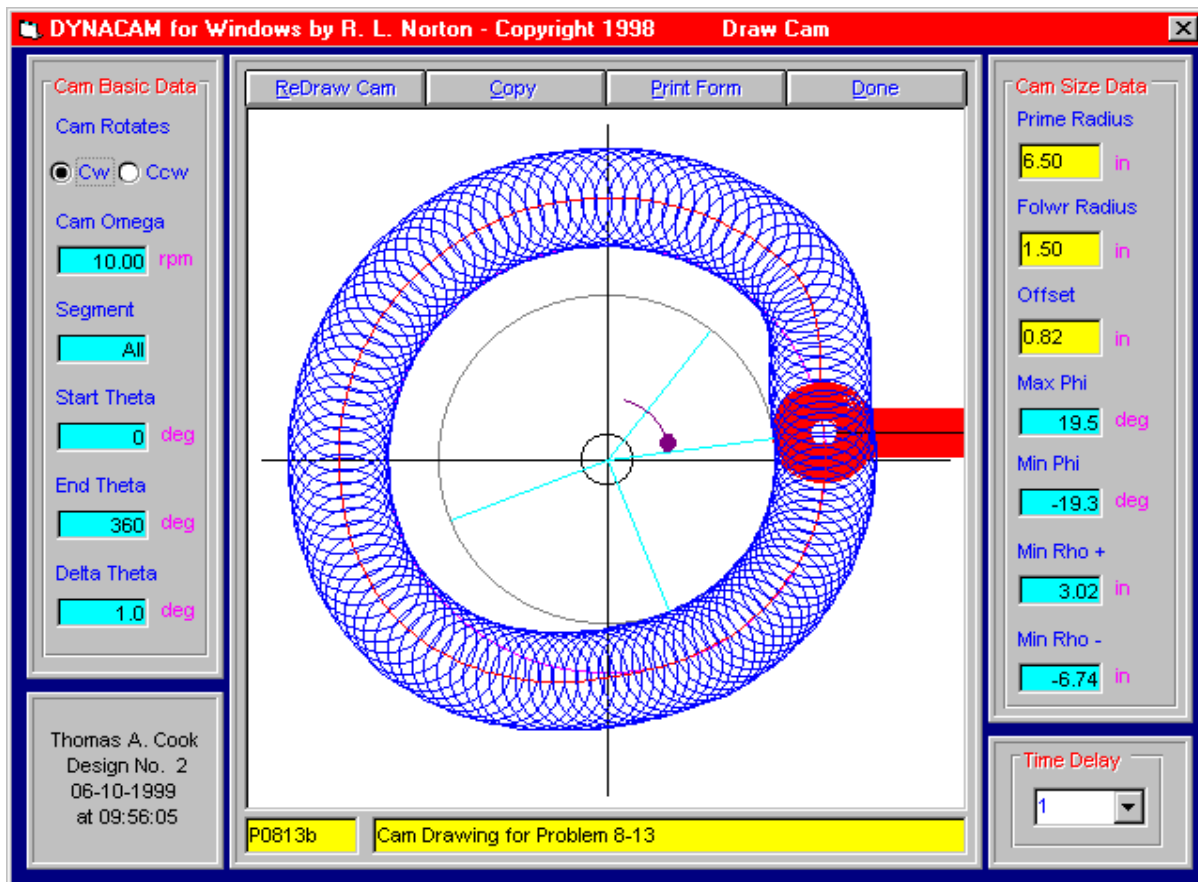
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \quad \omega = 1.047 \frac{rad}{sec} \quad \omega = 10.000 rpm$$

2. Problem 8-8 used a four-segment cam with modified sinusoidal acceleration for the rise and fall. Enter the above data into program DYNACAM. The input screen is shown below.

Angles		Cam Contour		Position (in)			
Seg	Beta	Start	End	Motion	Program	Start	End
1	45	0	45	Rise	MS - Mod Sine	0	1.5
2	150	45	195	Dwell	DW - Dwell	1.5	1.5
3	90	195	285	Fall	MS - Mod Sine	1.5	0
4	75	285	360	Dwell	DW - Dwell	0	0
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

3. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 30 deg or less by increasing the prime circle radius. The resulting cam is shown below.



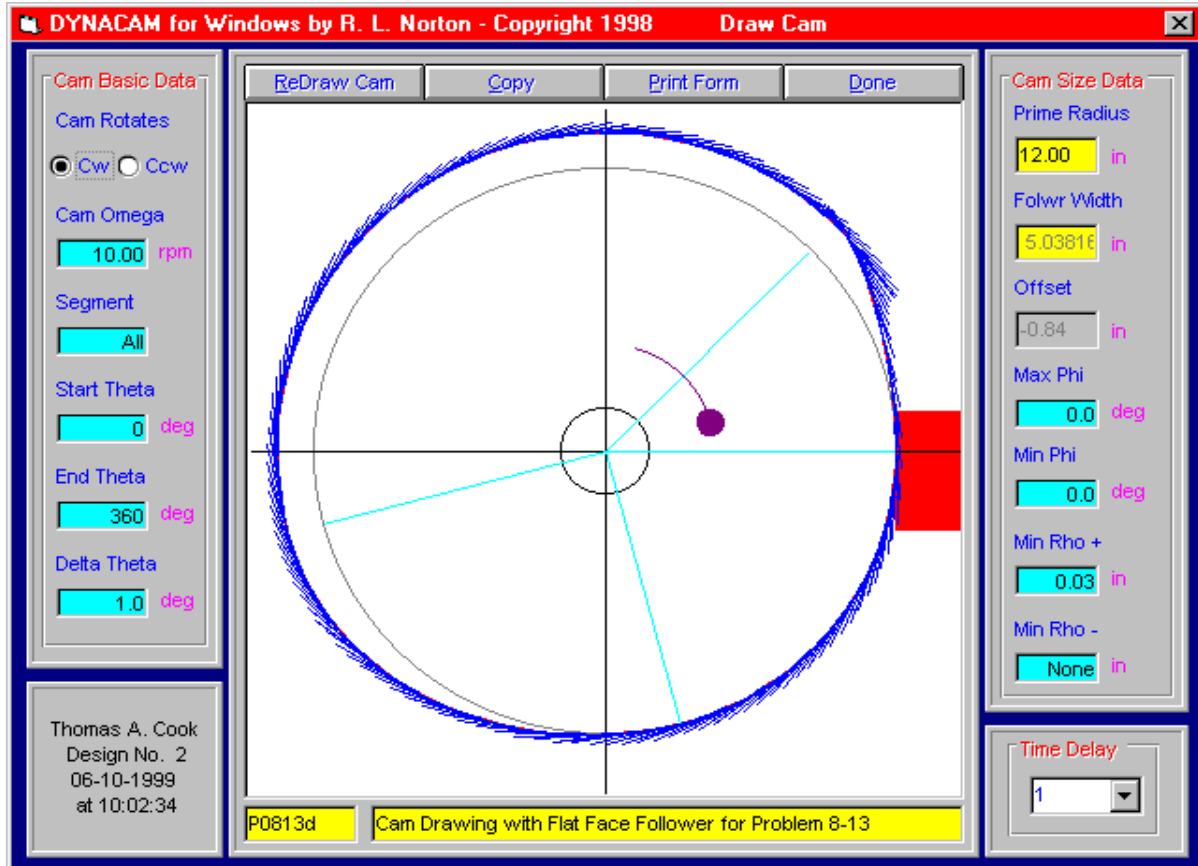
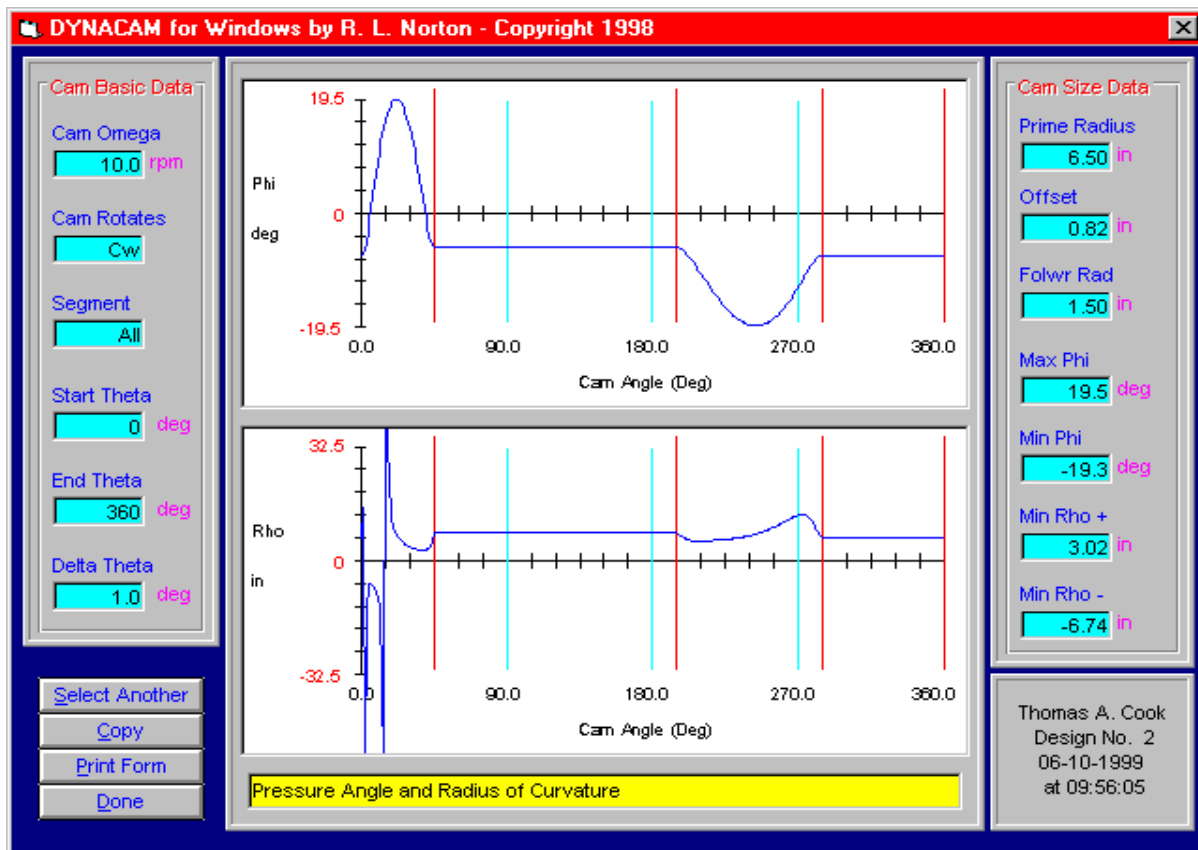
4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The prime radius had to be increased above that which was necessary for the pressure angle constraint in order to provide a minimum radius of curvature that was twice the roller follower radius. The design has the following dimensions:

Prime circle radius $R_p := 6.5 \cdot \text{in}$

Roller follower radius $R_f := 1.50 \cdot \text{in}$

Follower eccentricity $\varepsilon := 0.82 \cdot \text{in}$

5. Graphs of ϕ and ρ for the roller follower are shown on the following page.
6. The cam was sized for a flat-faced follower and the cam drawing is shown on the next page. In order to avoid undercutting, a base circle radius of 12 in is required. Obviously, the roller follower is to be preferred over the flat face follower in this case.



PROBLEM 8-14

Statement: Size the cam from Problem 8-9 for a 0.5-in-radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Draw the cam profile. Repeat for a flat faced follower. Which would you use?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: RISE/FALL DWELL
 $\beta := 150 \cdot deg$ $\beta_2 := 150 \cdot deg$
 $h_1 := 2.0 \cdot in$ $h_2 := 0.0 \cdot in$
 Cycle time: $\tau := 2 \cdot sec$

Solution: See Mathcad file P0814.

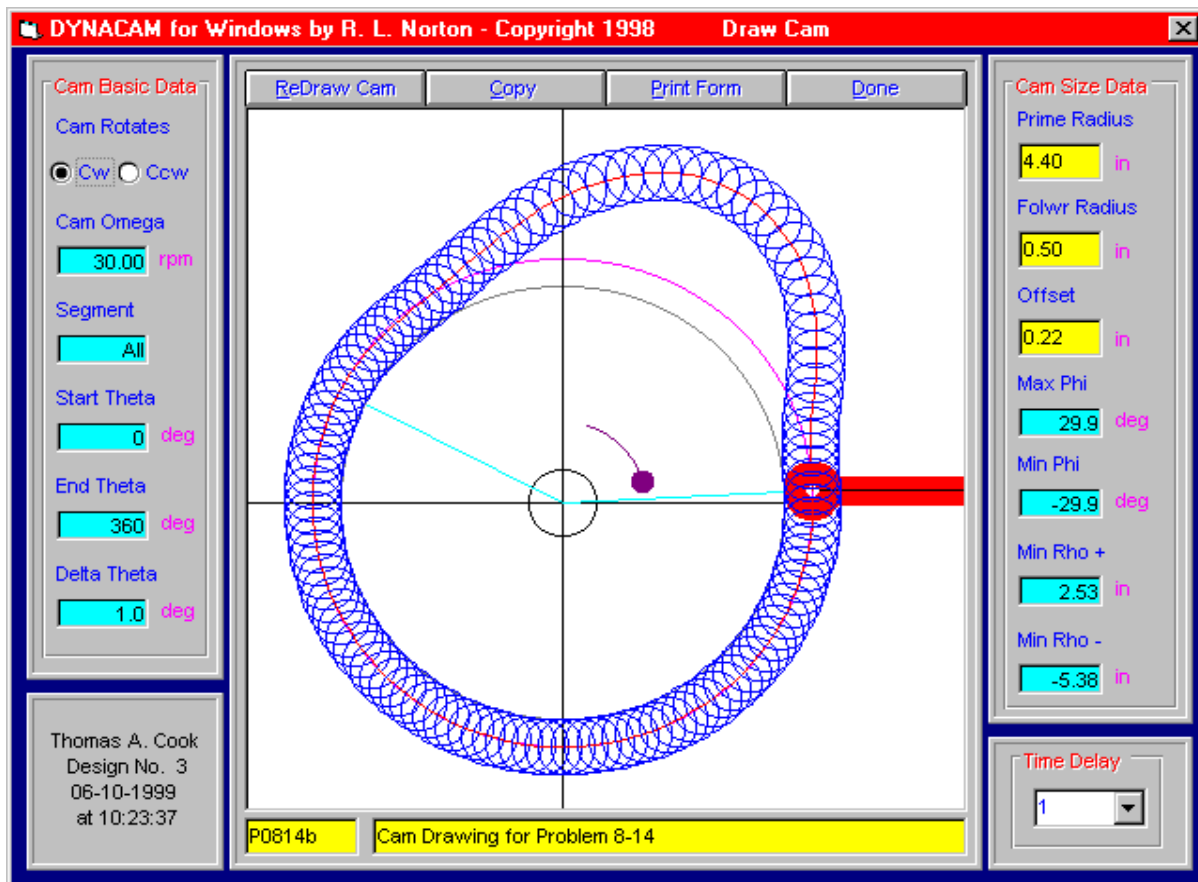
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \quad \omega = 3.142 \frac{rad}{sec} \quad \omega = 30.000 rpm$$

2. Problem 8-9 used a two-segment cam with one polynomial segment for the rise and fall. Enter the above data in program DYNACAM. The input screen is shown below.

Angles		Cam Contour		Position (in)			
Seg	Beta	Start	End	Motion	Program	Start	End
1	150	0	150	Poly	PO - Polynomial	0	0
2	210	150	360	Dwell	DW - Dwell	0	0
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

3. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 30 deg or less by increasing the prime circle radius. The resulting cam is shown below.



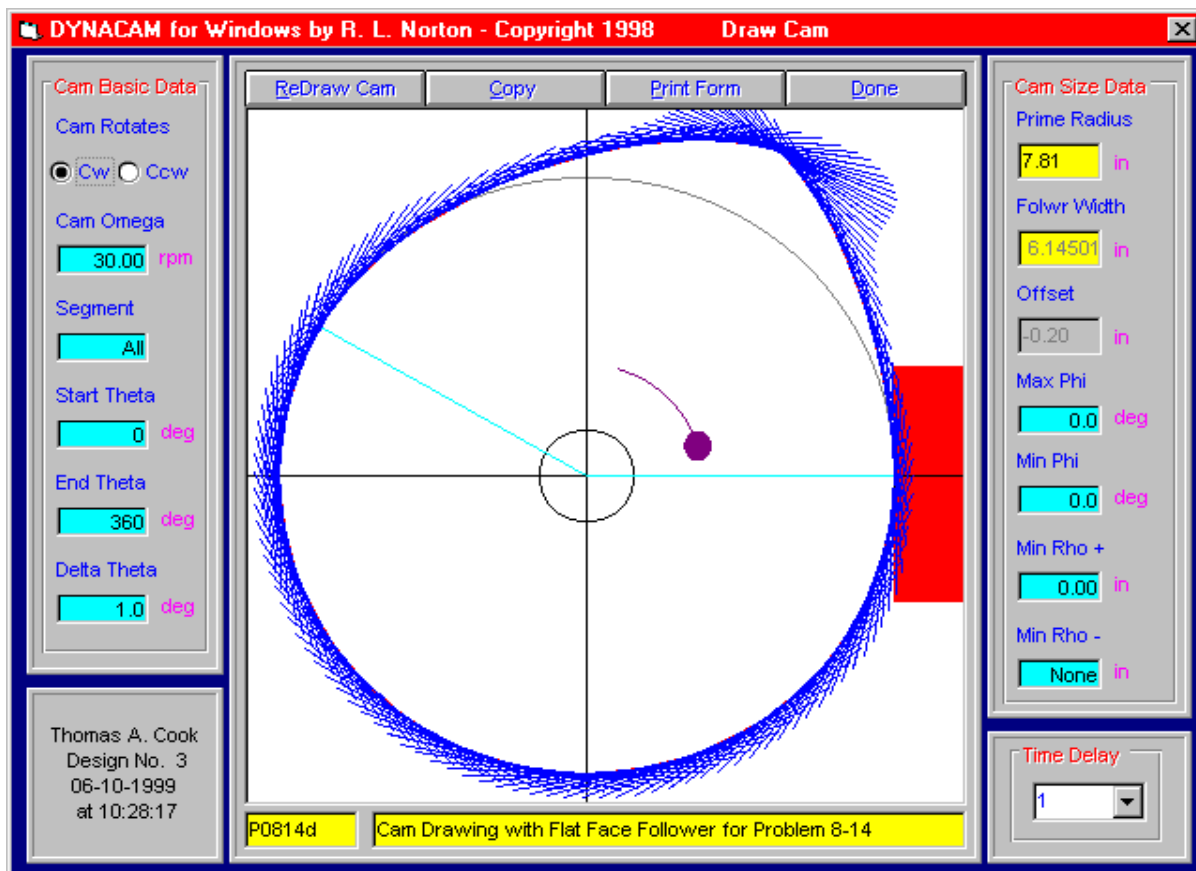
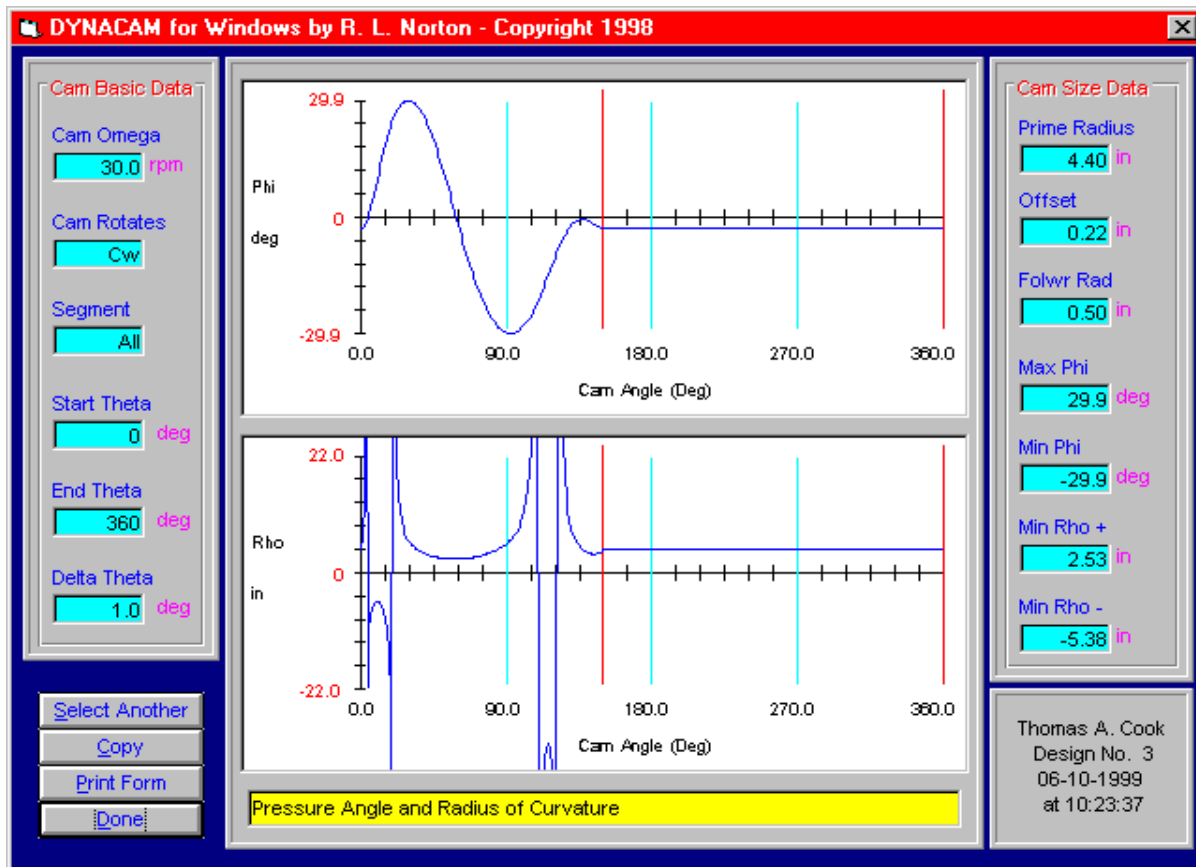
4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

Prime circle radius $R_p := 4.40 \cdot \text{in}$

Roller follower radius $R_f := 0.50 \cdot \text{in}$

Follower eccentricity $\varepsilon := 0.22 \cdot \text{in}$

5. Graphs of ϕ and ρ for the roller follower are shown on the following page.
6. The cam was sized for a flat-faced follower and the cam drawing is shown on the next page. In order to avoid undercutting, a base circle radius of 7.81 in is required. Obviously, the roller follower is to be preferred over the flat face follower in this case.



PROBLEM 8-15

Statement: Size the cam from Problem 8-10 for a 2.0-in-radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Draw the cam profile. Repeat for a flat faced follower. Which would you use?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

RISE/FALL	DWELL	FALL	DWELL
$\beta_1 := 40 \cdot deg$	$\beta_2 := 100 \cdot deg$	$\beta_3 := 90 \cdot deg$	$\beta_4 := 20 \cdot deg$
$h_1 := 2.5 \cdot in$	$h_2 := 0.0 \cdot in$	$h_3 := 1.5 \cdot in$	$h_4 := 0.0 \cdot in$
$\beta_5 := 30 \cdot deg$	$\beta_6 := 80 \cdot deg$		
$h_5 := 1.0 \cdot in$	$h_6 := 0.0 \cdot in$		
Cycle time: $\tau := 10 \cdot sec$			

Solution: See Mathcad file P0815.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \quad \omega = 0.628 \frac{rad}{sec} \quad \omega = 6.000 rpm$$

2. Problem 8-10 used a six-segment cam with modified sinusoidal acceleration for the rise and fall. Enter the above data into program DYNACAM. The input screen is shown below.

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Input Screen**

Cam Data		Segment Data				Position (in)		
Cam Omega (rpm)	No. Segmts	Beta	Start	End	Motion	Program	Start	End
6	6	40	0	40	Rise	MS - Mod Sine	0	2.5
		100	40	140	Dwell	DW - Dwell	2.5	2.5
		90	140	230	Fall	MS - Mod Sine	2.5	1
		20	230	250	Dwell	DW - Dwell	1	1
		30	250	280	Fall	MS - Mod Sine	1	0
		80	280	360	Dwell	DW - Dwell	0	0
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								

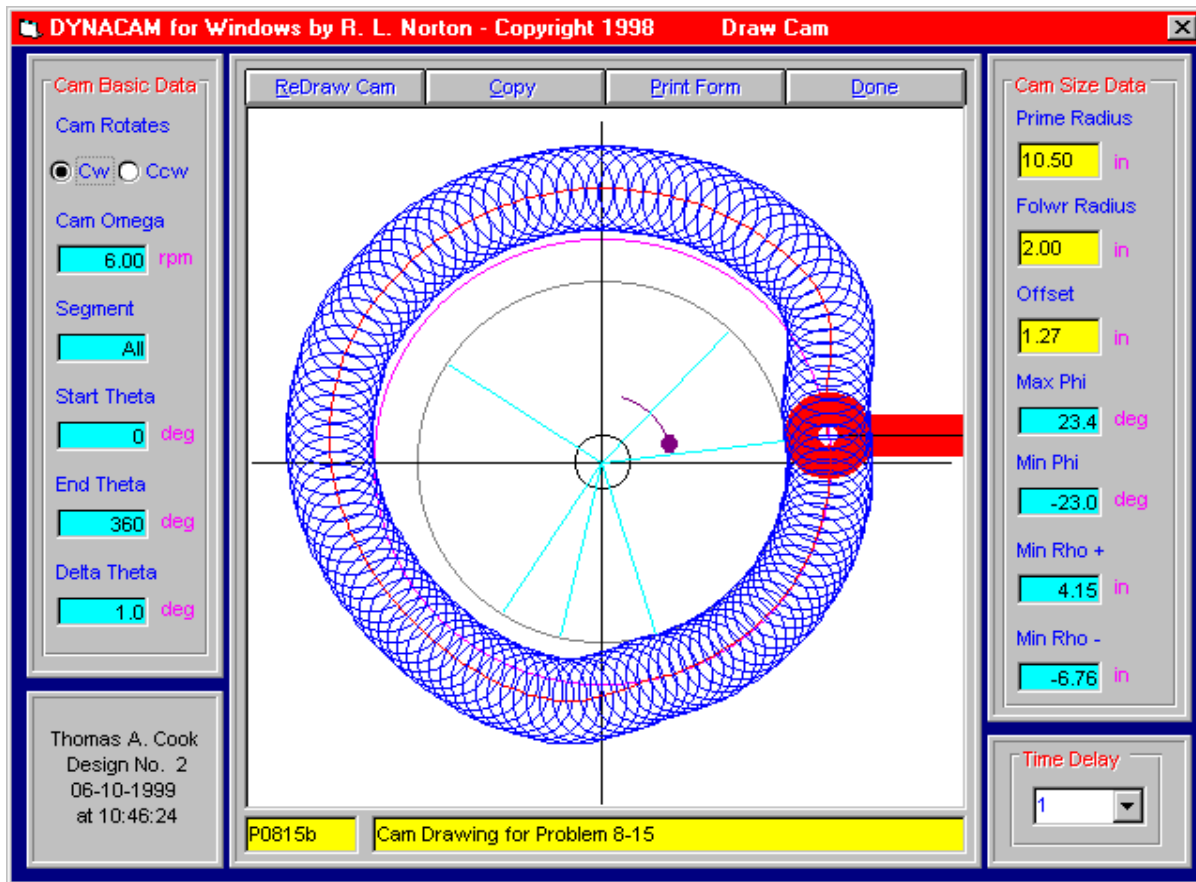
Delta Theta (deg): 1.0

Follower: ☒ Trans ☐ Oscill

Buttons: Calc All, Clear Segs, Clear All, Copy, Print Form, Done, Cancel

Examples: [Dropdown] Calc Plot Print

3. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 30 deg or less by increasing the prime circle radius. The resulting cam is shown below.



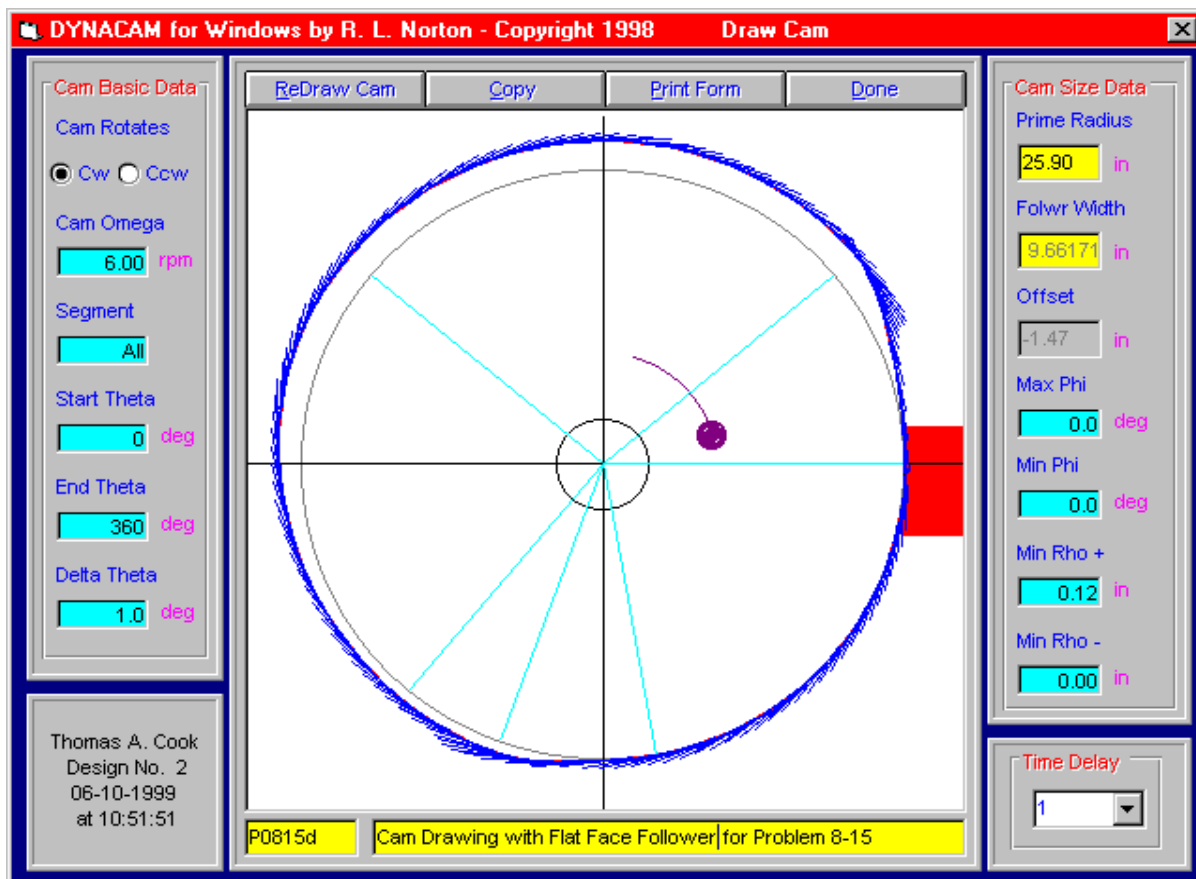
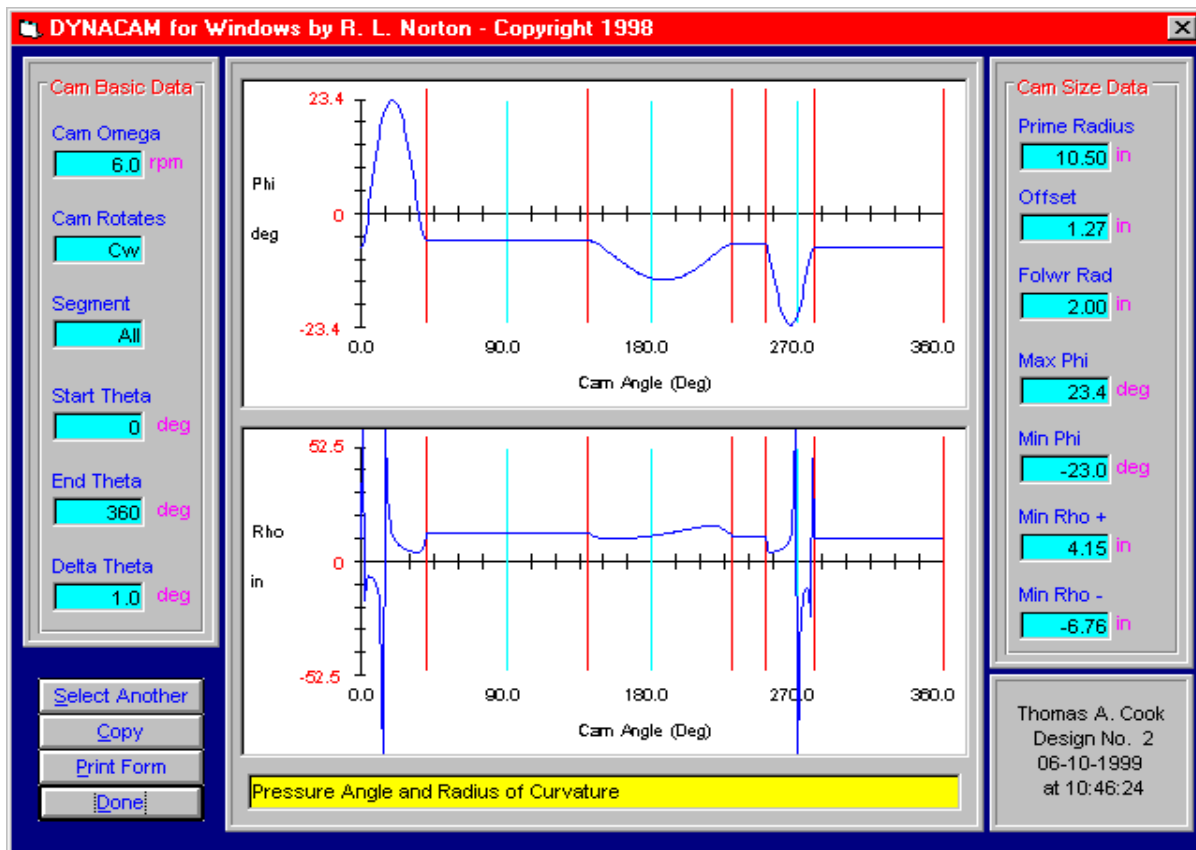
4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The prime radius had to be increased above that which was necessary for the pressure angle constraint in order to provide a minimum radius of curvature that was twice the roller follower radius. The design has the following dimensions:

$$\text{Prime circle radius} \quad R_p := 10.50 \cdot \text{in}$$

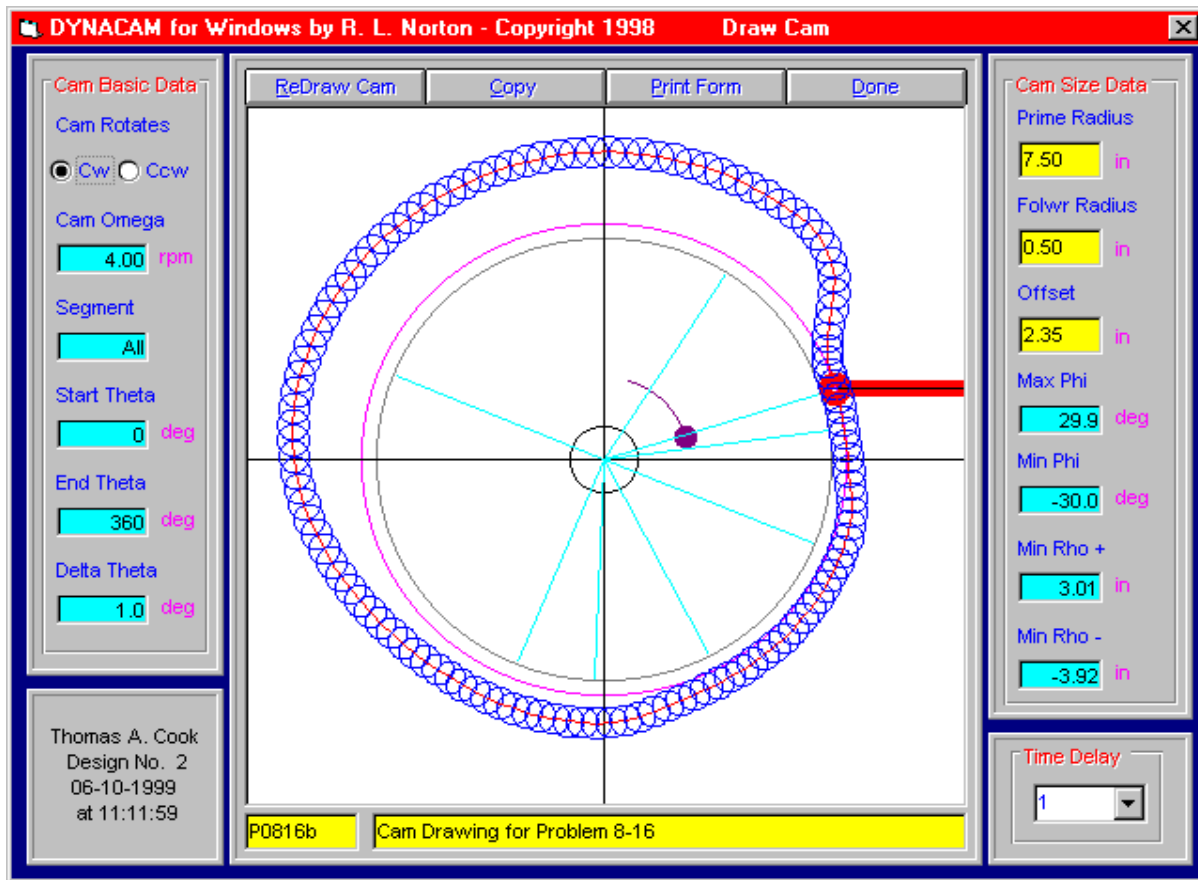
$$\text{Roller follower radius} \quad R_f := 2.00 \cdot \text{in}$$

$$\text{Follower eccentricity} \quad \varepsilon := 1.27 \cdot \text{in}$$

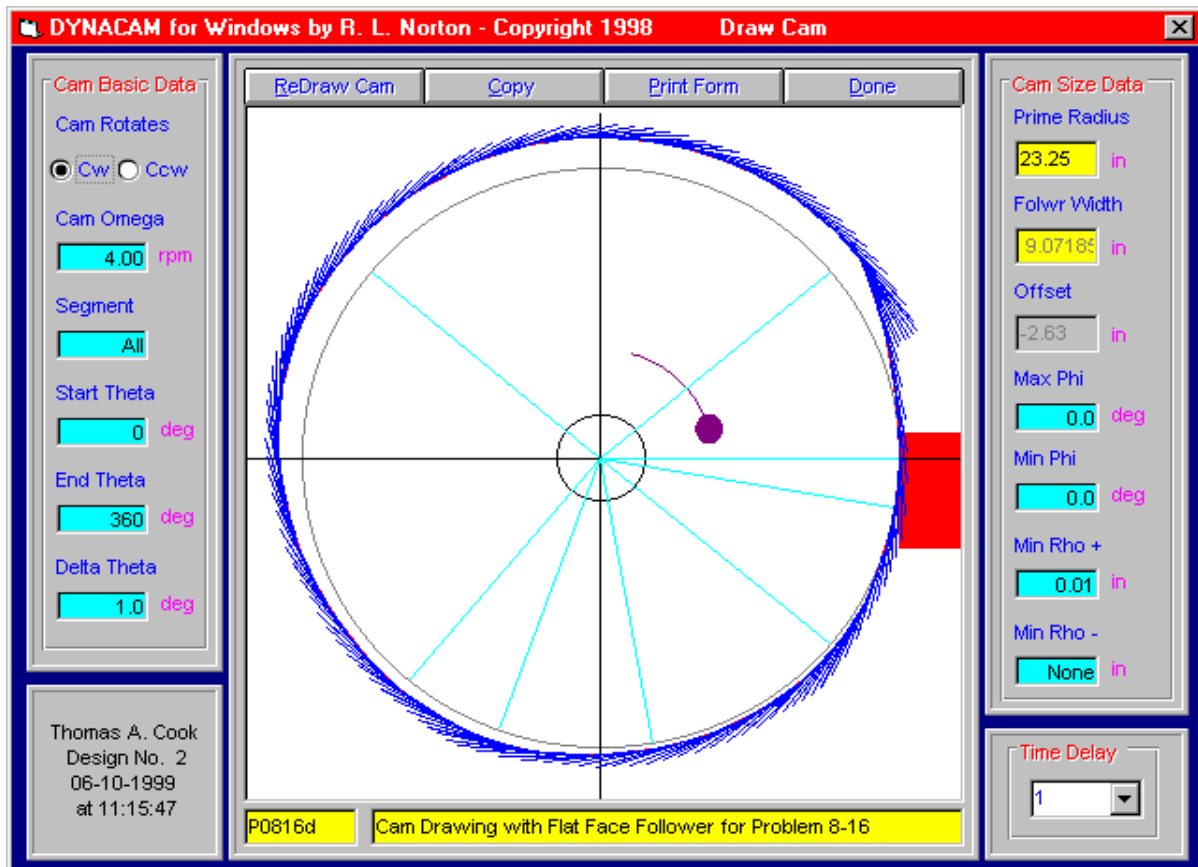
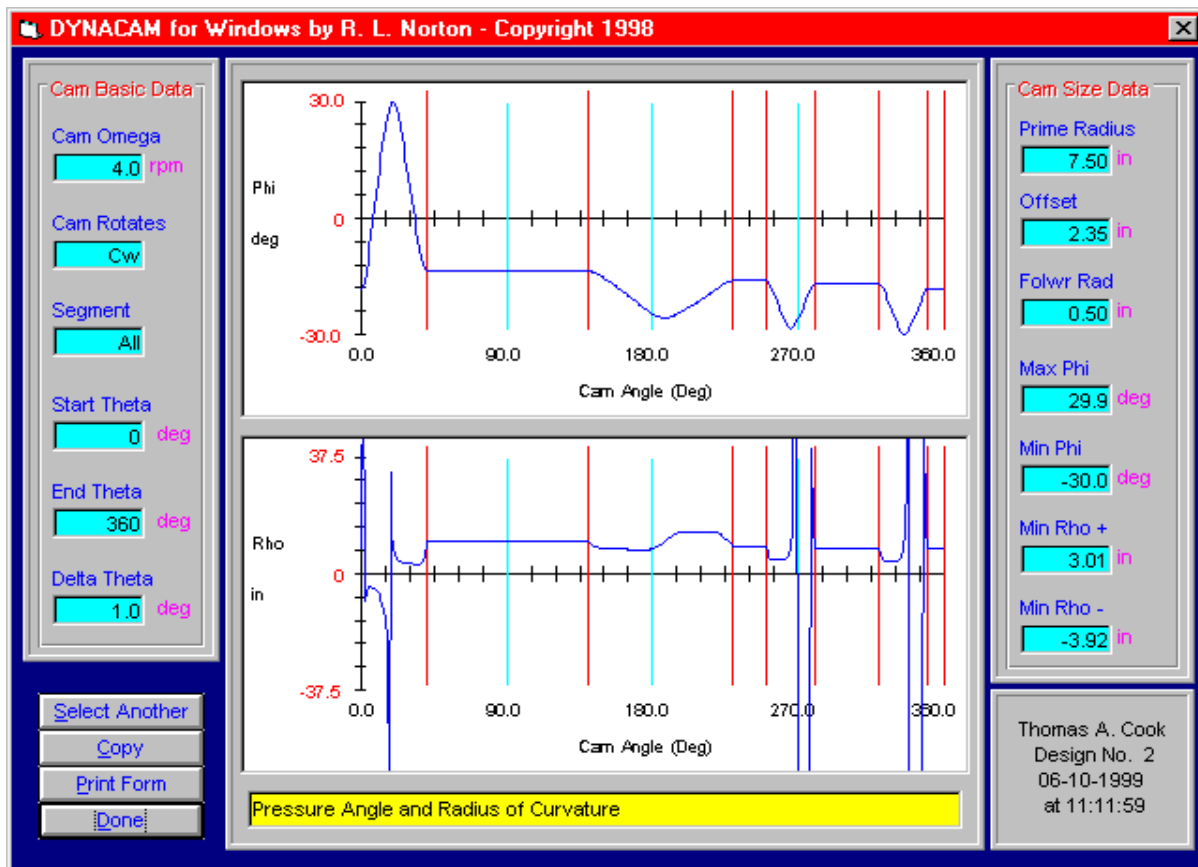
5. Graphs of ϕ and ρ for the roller follower are shown on the following page.
6. The cam was sized for a flat-faced follower and the cam drawing is shown on the next page. In order to avoid undercutting, a base circle radius of 25.9 in is required. Obviously, the roller follower is to be preferred over the flat face follower in this case.



3. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 30 deg or less by increasing the prime circle radius. The resulting cam is shown below.



4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:
- Prime circle radius $R_p := 7.50 \cdot \text{in}$
- Roller follower radius $R_f := 0.50 \cdot \text{in}$
- Follower eccentricity $\varepsilon := 2.35 \cdot \text{in}$
5. Graphs of ϕ and ρ for the roller follower are shown on the following page.
6. The cam was sized for a flat-faced follower and the cam drawing is shown on the next page. In order to avoid undercutting, a base circle radius of 23.25 in is required. Obviously, the roller follower is to be preferred over the flat face follower in this case.



PROBLEM 8-17

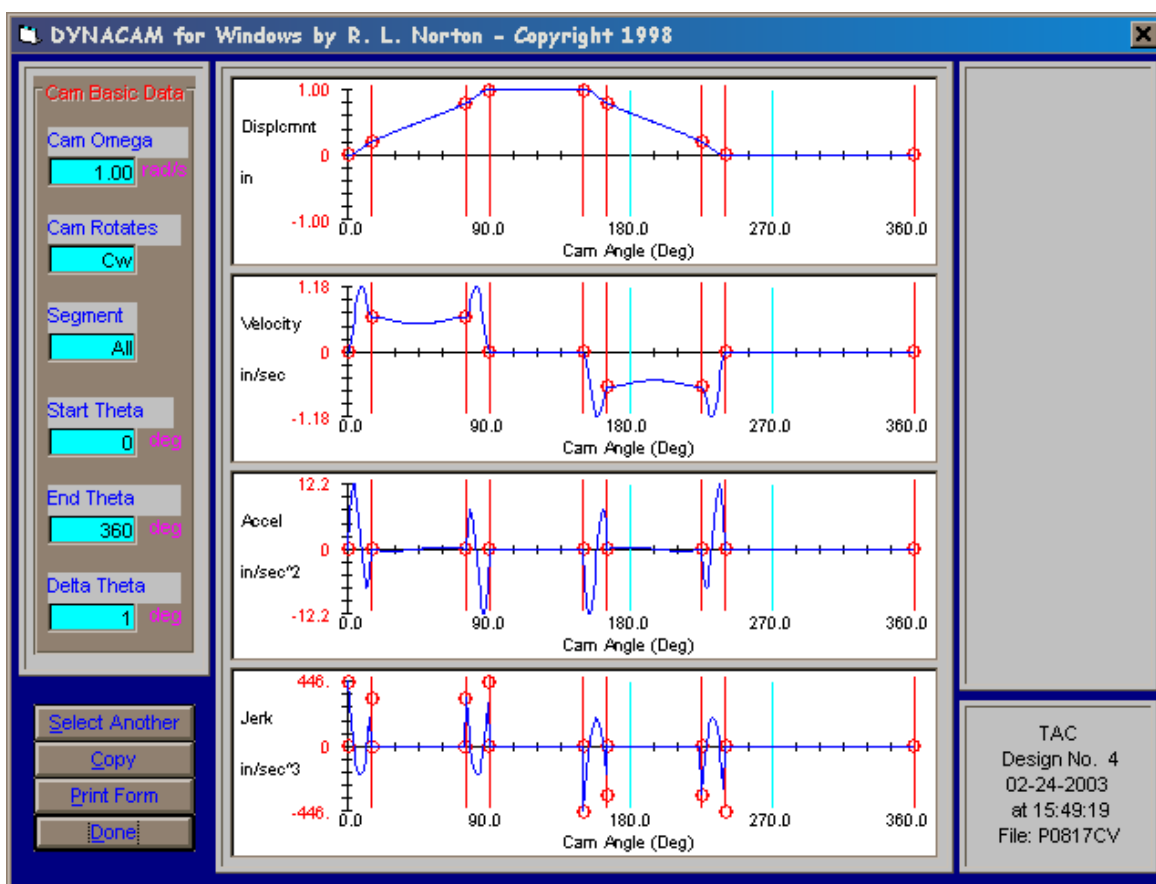
Statement: A high friction, high inertia load is to be driven. We wish to keep peak velocity low. Combine segments of polynomial displacements with a constant velocity segment on both rise and fall to reduce the maximum velocity below that obtainable with a full period modified sine acceleration alone (i.e., with no constant velocity portion). Compare the two designs and comment. Use an ω of one for comparison.

Given:

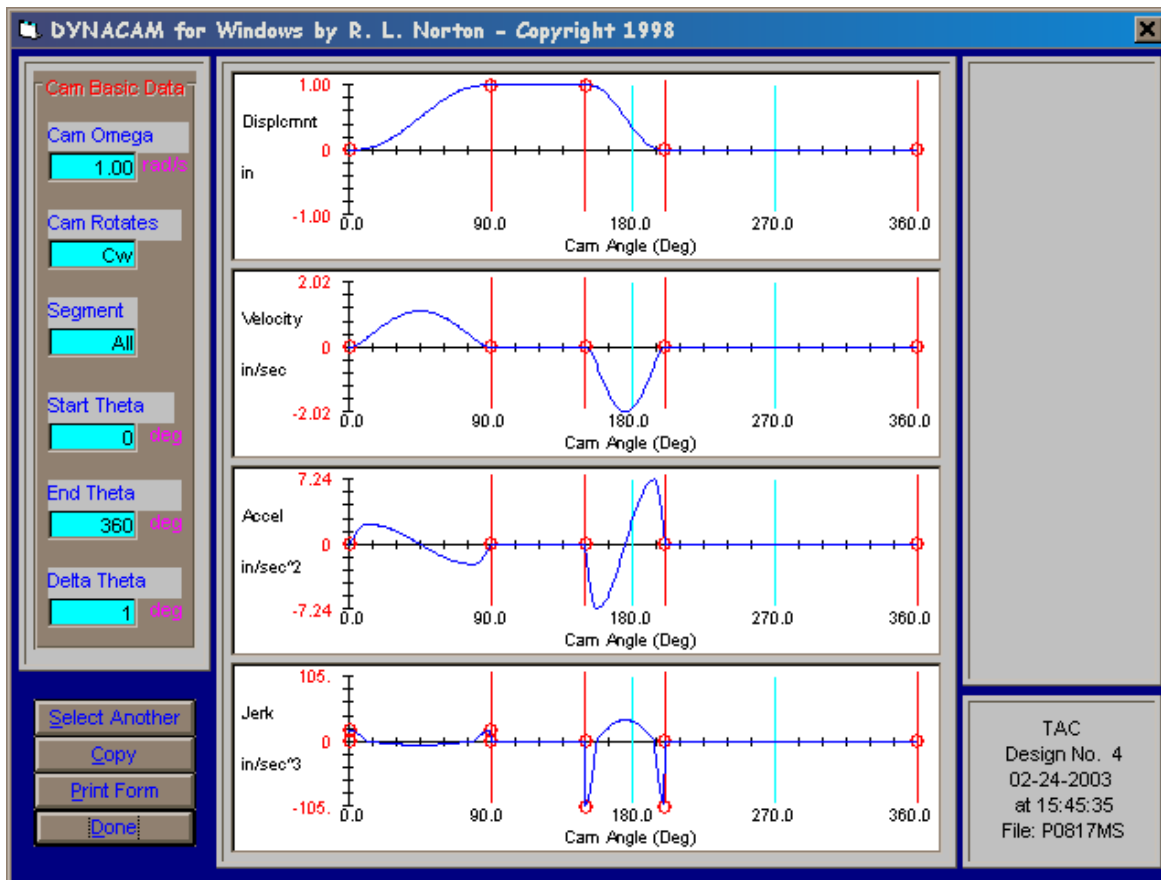
RISE	DWELL	FALL	DWELL
$\beta_1 := 90\text{-deg}$	$\beta_2 := 60\text{-deg}$	$\beta_3 := 50\text{-deg}$	$\beta_4 := 160\text{-deg}$
$h_1 := 1.0\text{-in}$	$h_2 := 0\text{-in}$	$h_3 := 1.0\text{-in}$	$h_4 := 0\text{-in}$

Solution: See Mathcad file P0817.

1. Use DYNACAM to get the data for comparison of these two cam profiles. Start with the CV-poly profile, which is shown below. In this design eight segments were used: three each for the rise and fall and one each for the two dwells. The (nearly) constant velocity segments were each 54 deg. The maximum acceleration and jerk are 12.2 and 446, respectively.



1. The DYNACAM output screen for the modified sine profile is shown below. The maximum acceleration and jerk are 74.4 and 105, respectively. Obviously, for the CV-poly design choices that were made, the modified sine profile results in lower inertia forces and a smoother follower motion.



PROBLEM 8-18

Statement: A constant velocity of 0.4 in/sec must be matched for 1.5 sec. Then the follower must return to your choice of starting point and dwell for 2 sec. The total cycle time is 6 sec. Design a cam for a follower radius of 0.75 in and a maximum pressure angle of 30 deg, absolute value.

Given: Required constant velocity $V_{cv} := 0.4 \cdot \text{in} \cdot \text{sec}^{-1}$ $t_{cv} := 1.5 \cdot \text{sec}$
 Dwell time $t_{dwell} := 2 \cdot \text{sec}$
 Cycle time $t_{cycle} := 6 \cdot \text{sec}$
 Follower radius $R_f := 0.75 \cdot \text{in}$

Solution: See Mathcad file P0818.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{t_{cycle}} \quad \omega = 1.047 \frac{\text{rad}}{\text{sec}}$$

2. Use a four-segment polynomial cam as described below.

Segment	Function	Motion
1	Constant Velocity	Rise
2	Polynomial (5 deg)	Fall
3	None	Dwell
4	Polynomial (5 deg)	Rise

3. Calculate the two known cam rotation interval widths.

$$\text{Constant velocity segment} \quad \beta_1 := \frac{t_{cv}}{t_{cycle}} \cdot 2 \cdot \pi \quad \beta_1 = 90.000 \text{ deg}$$

$$\text{Dwell segment} \quad \beta_3 := \frac{t_{dwell}}{t_{cycle}} \cdot 2 \cdot \pi \quad \beta_3 = 120.000 \text{ deg}$$

4. This leaves 150 deg to allocate to segments 2 and 4. Tentatively, let

$$\beta_2 := 100 \cdot \text{deg} \quad \beta_4 := 50 \cdot \text{deg}$$

5. Calculate the lift for the constant velocity segment.

$$L_1 := V_{cv} \cdot t_{cv} \quad L_1 = 0.600 \text{ in}$$

6. Make other initial design choices.

$$\text{Lift of segment 4 (and starting position for segment 1)} \quad L_4 := 0.1 \cdot \text{in}$$

$$\text{Prime circle radius} \quad R_p := 2.50 \cdot \text{in}$$

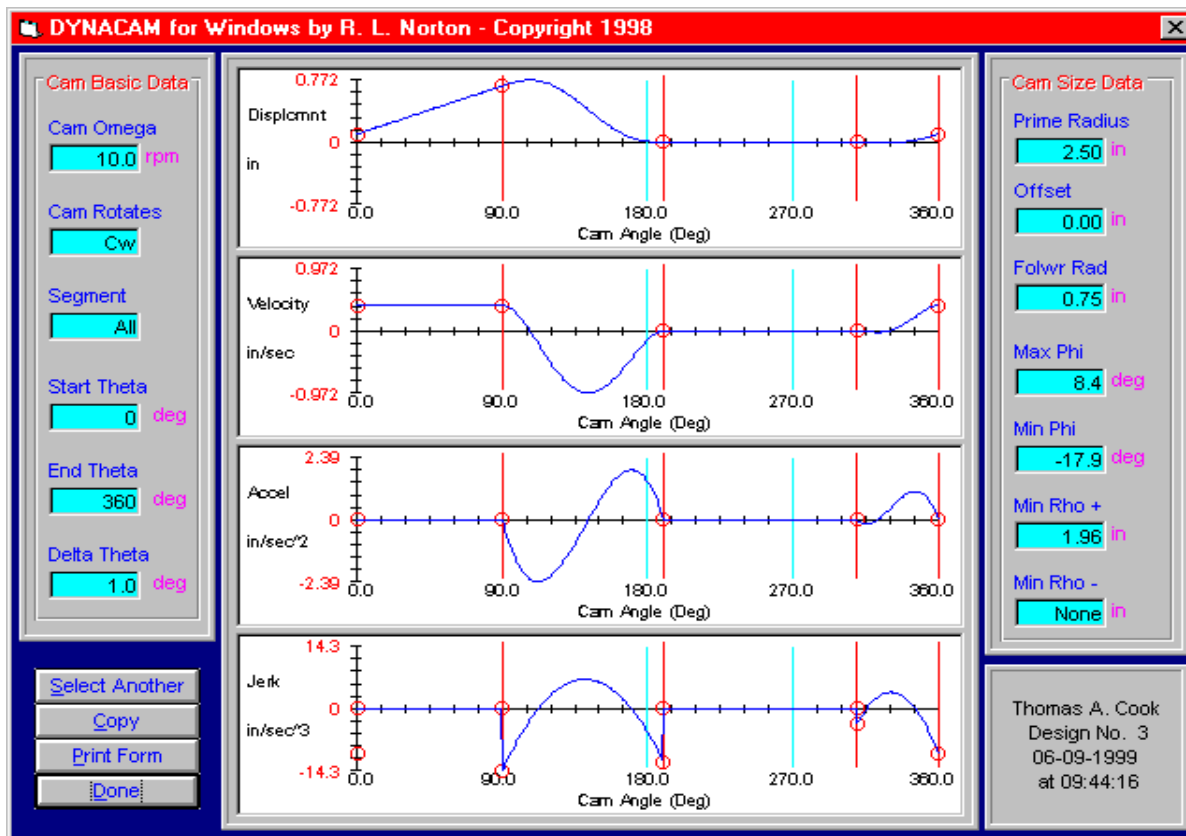
7. Enter the above data into program DYNACAM. The input screen and resulting SVAJ diagrams are shown on the next page.

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Input Screen**

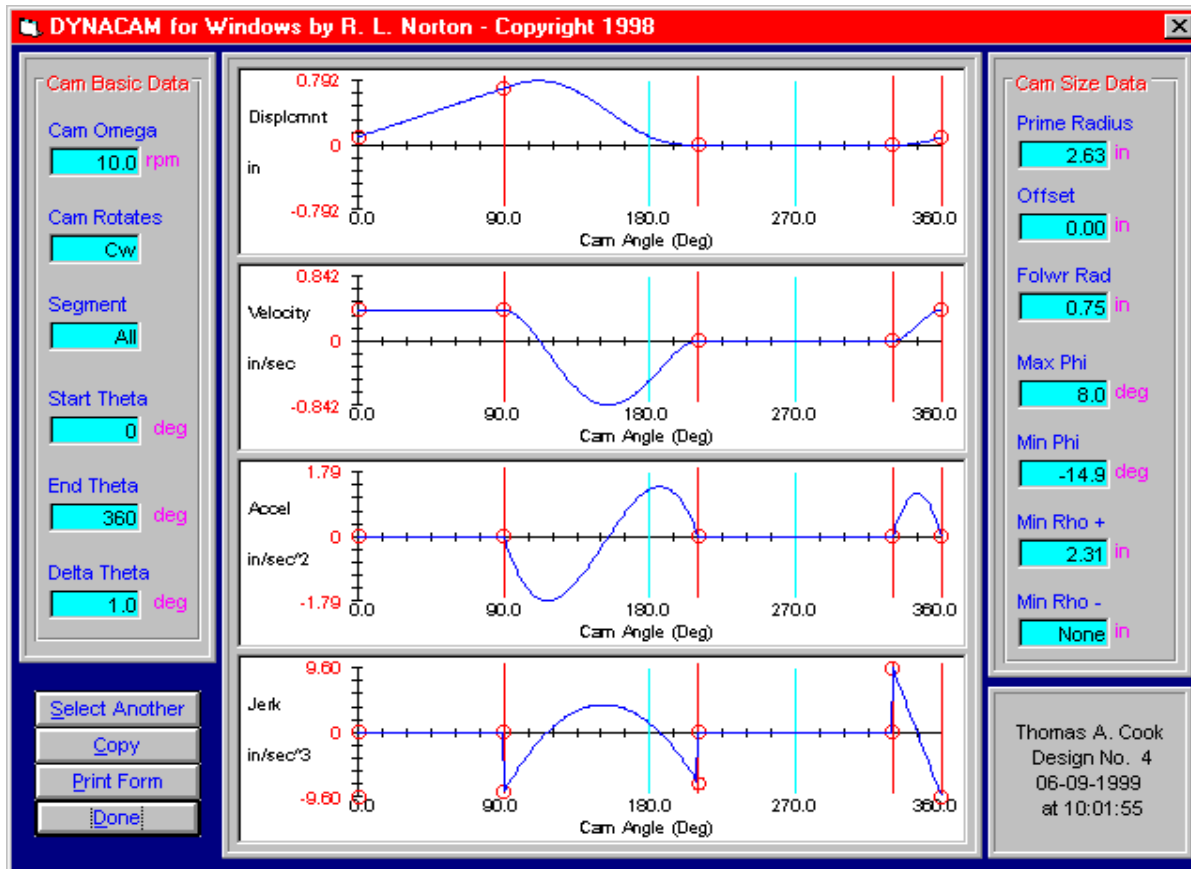
Cam Data				Segment Data				Cam Contour		Position (in)	
Cam Omega (rpm)	No. Segmts	Delta Theta (deg)	Follower	Seg	Beta	Start	End	Motion	Program	Start	End
10	4	1.0	☒ Trans ☐ Oscit	1	90	0	90	Poly	PO - Polynomial	0.1	0.7
				2	100	90	190	Poly	PO - Polynomial	0.7	0
				3	120	190	310	Dwell	DW - Dwell	0	0
				4	50	310	360	Poly	PO - Polynomial	0	0.1
				5							
				6							
				7							
				8							
				9							
				10							
				11							
				12							
				13							
				14							
				15							
				16							

Buttons: Calc All, Clear Segs, Clear All, Copy, Print Form, Done, Cancel

Examples:



8. From SVAJ diagrams we see that all design goals are met but the peak acceleration in segment 2 is higher than the peak in segment 4. These can be more evenly balanced by increasing the segment 2 interval and decreasing the segment 4 interval. Iterate on these intervals until the peak accelerations are more nearly the same.
9. The second interval was changed to 120 deg and the fourth to 30 deg, and the prime circle radius was increased to 2.625 in (to maintain $\rho_{min}/R_f > 3$). The resulting peak accelerations are more in balance and are shown in the SVAJ diagram below. The maximum pressure angle is well within the required limit.



PROBLEM 8-19

Statement: A constant velocity of 0.25 in/sec must be matched for 3 sec. Then the follower must return to your choice of starting point and dwell for 3 sec. The total cycle time is 12 sec. Design a cam for a follower radius of 1.25 in and a maximum pressure angle of 35 deg, absolute value.

Given: Required constant velocity $V_{cv} := 0.25 \cdot \text{in} \cdot \text{sec}^{-1}$ $t_{cv} := 3 \cdot \text{sec}$
 Dwell time $t_{dwell} := 3 \cdot \text{sec}$
 Cycle time $t_{cycle} := 12 \cdot \text{sec}$
 Follower radius $R_f := 1.25 \cdot \text{in}$

Solution: See Mathcad file P0819.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{t_{cycle}} \quad \omega = 0.524 \frac{\text{rad}}{\text{sec}}$$

2. Use a four-segment polynomial cam as described below.

Segment	Function	Motion
1	Constant Velocity	Rise
2	Polynomial (5 deg)	Fall
3	None	Dwell
4	Polynomial (5 deg)	Rise

3. Calculate the two known cam rotation interval widths.

$$\text{Constant velocity segment} \quad \beta_1 := \frac{t_{cv}}{t_{cycle}} \cdot 2 \cdot \pi \quad \beta_1 = 90.000 \text{ deg}$$

$$\text{Dwell segment} \quad \beta_3 := \frac{t_{dwell}}{t_{cycle}} \cdot 2 \cdot \pi \quad \beta_3 = 90.000 \text{ deg}$$

4. This leaves 180 deg to allocate to segments 2 and 4. Tentatively, let

$$\beta_2 := 120 \cdot \text{deg} \quad \beta_4 := 60 \cdot \text{deg}$$

5. Calculate the lift for the constant velocity segment.

$$L_1 := V_{cv} \cdot t_{cv} \quad L_1 = 0.750 \text{ in}$$

6. Make other initial design choices.

$$\text{Lift of segment 4 (and starting position for segment 1)} \quad L_4 := 0.15 \cdot \text{in}$$

$$\text{Prime circle radius} \quad R_p := 4.00 \cdot \text{in}$$

7. Enter the above data into program DYNACAM. The input screen and resulting SVAJ diagrams are shown on the next page.

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Cam Data

Cam Omega (rpm)

No. Segmts

Delta Theta (deg)

Follower

☒ Trans
☐ Oscit

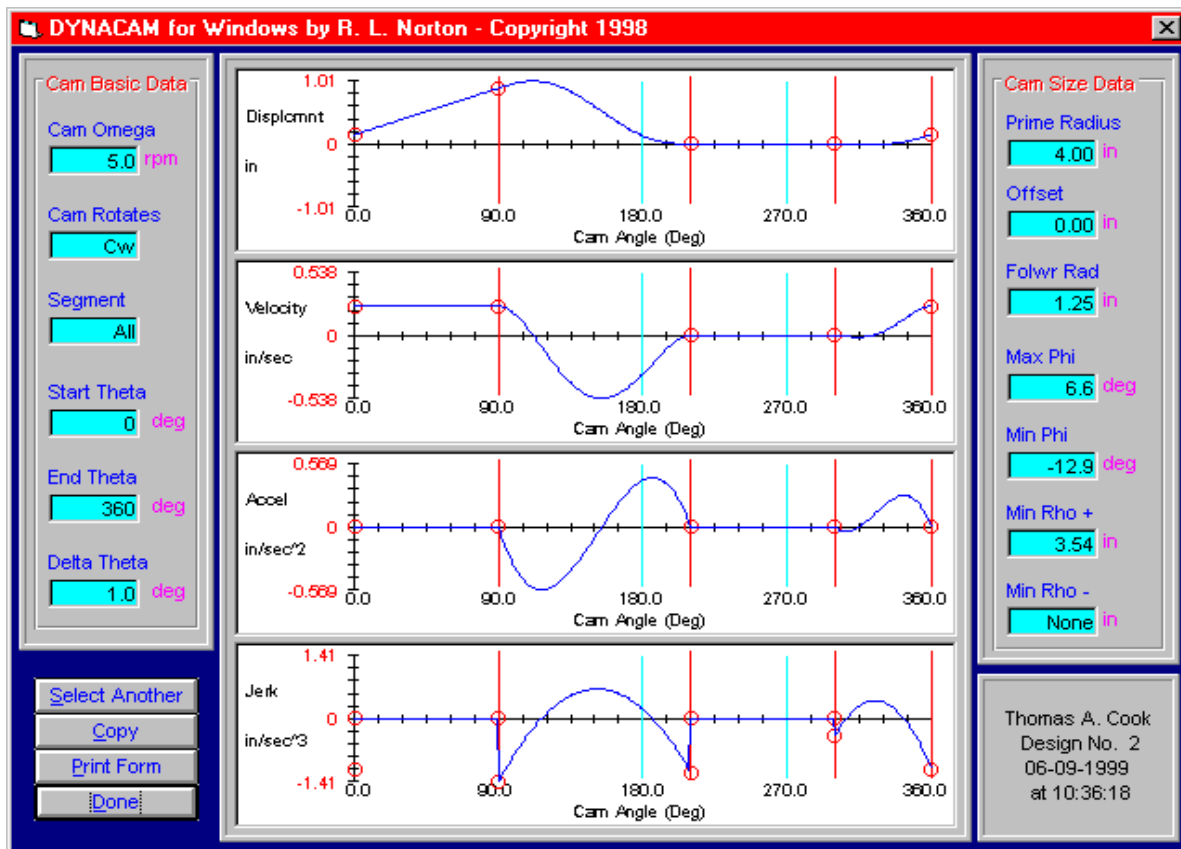
Calc All
Clear Segs
Clear All
Copy
Print Form
Done
Cancel

Segment Data

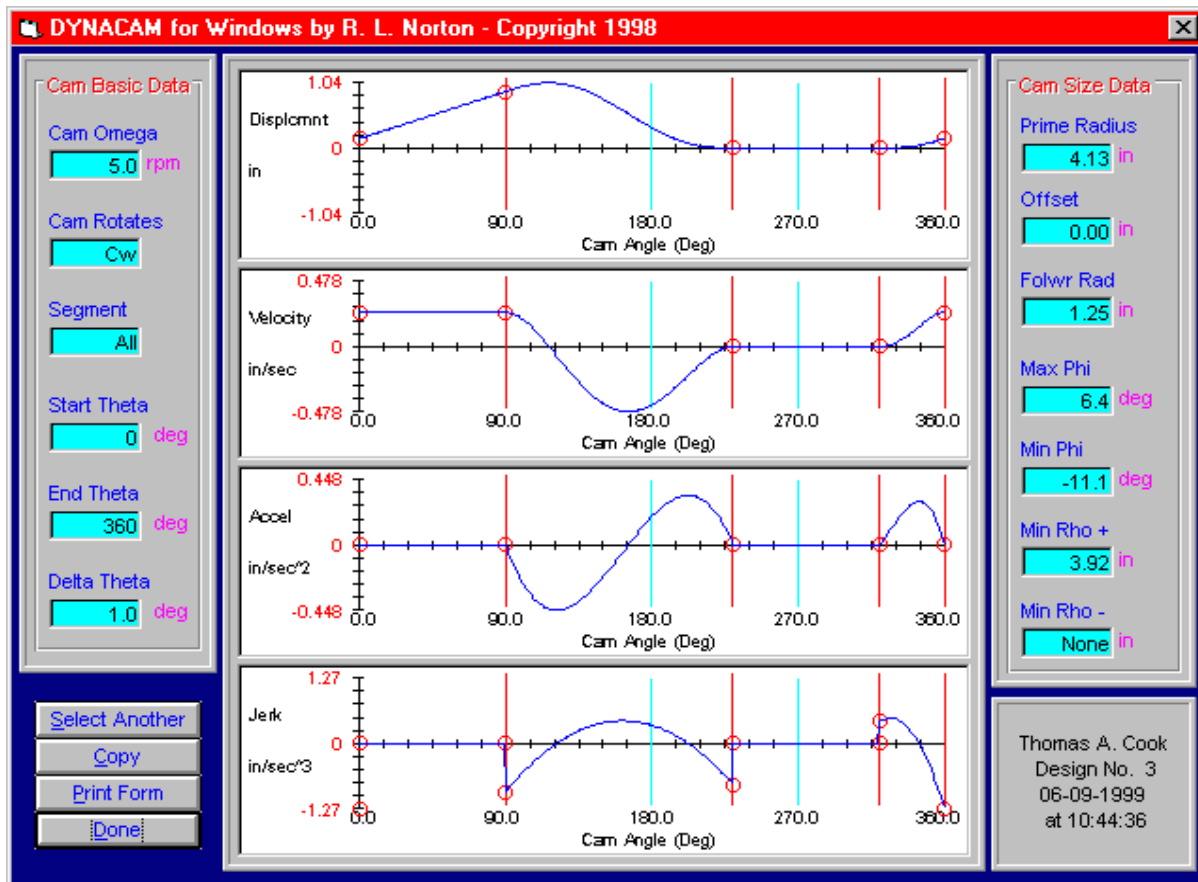
Seg	Angles			Cam Contour		Position (in)	
	Beta	Start	End	Motion	Program	Start	End
1	90	0	90	Poly	PO - Polynomial	0.15	0.90
2	120	90	210	Poly	PO - Polynomial	0.90	0
3	90	210	300	Dwell	DW - Dwell	0	0
4	60	300	360	Poly	PO - Polynomial	0	0.15
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

Examples

Calc Plot Print
Calc Plot Print
Calc Plot Print
Calc Plot Print



8. From SVAJ diagrams we see that all design goals are met but the peak acceleration in segment 2 is higher than the peak in segment 4. These can be more evenly balanced by increasing the segment 2 interval and decreasing the segment 4 interval. Iterate on these intervals until the peak accelerations are more nearly the same. Note also that ρ_{min} is not greater than 3 times R_f .
9. The second interval was changed to 140 deg and the fourth to 40 deg, and the prime circle radius was increased to 4.125 in (to maintain $\rho_{min}/R_f > 3$). The resulting peak accelerations are more in balance and are shown in the SVAJ diagram below. The maximum pressure angle is well within the required limit.



PROBLEM 8-20

Statement: A constant velocity of 2 in/sec must be matched for 1 sec. Then the follower must return to your choice of starting point. The total cycle time is 2.75 sec. Design a cam for a follower radius of 0.5 in and a maximum pressure angle of 25 deg, absolute value.

Given: Required constant velocity $V_{cv} := 2.0 \cdot \text{in} \cdot \text{sec}^{-1}$ $t_{cv} := 1 \cdot \text{sec}$
 Cycle time $t_{cycle} := 2.75 \cdot \text{sec}$
 Follower radius $R_f := 0.5 \cdot \text{in}$

Solution: See Mathcad file P0820.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{t_{cycle}} \quad \omega = 2.285 \frac{\text{rad}}{\text{sec}}$$

2. Use a two-segment polynomial cam similar to that used in Example 8-10 and described below.

Segment	Function	Motion
1	Constant Velocity	Rise
2	Polynomial (5 deg)	Fall

3. Calculate the known cam rotation interval width.

$$\text{Constant velocity segment} \quad \beta_1 := \frac{t_{cv}}{t_{cycle}} \cdot 2 \cdot \pi \quad \beta_1 = 130.909 \text{ deg}$$

4. This leaves 229.091 deg for segments 2.

$$\beta_2 := 360 \cdot \text{deg} - \beta_1 \quad \beta_2 = 229.091 \text{ deg}$$

5. Calculate the lift for the constant velocity segment.

$$L_1 := V_{cv} \cdot t_{cv} \quad L_1 = 2.000 \text{ in}$$

6. Make other initial design choices.

$$\text{Prime circle radius} \quad R_p := 4.00 \cdot \text{in}$$

7. Enter the above data into program DYNACAM. The input screen and resulting SVAJ diagrams are shown on the next page.

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Input Screen**

Cam Data

Cam Omega (rpm)
21.818

No. Segmts
2

Delta Theta (deg)
1.0

Follower

☒ Trans
☐ Osc

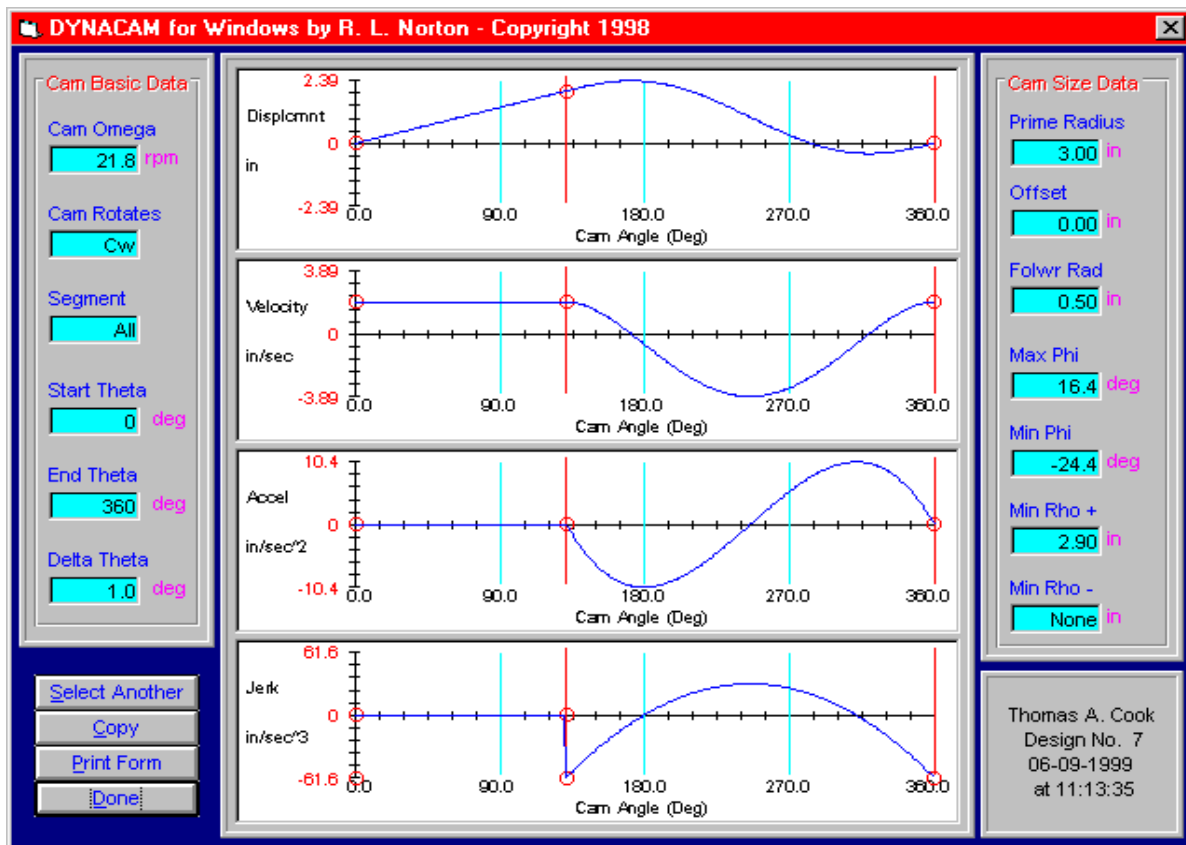
Calc All
Clear Segs
Clear All
Copy
Print Form
Done
Cancel

Segment Data

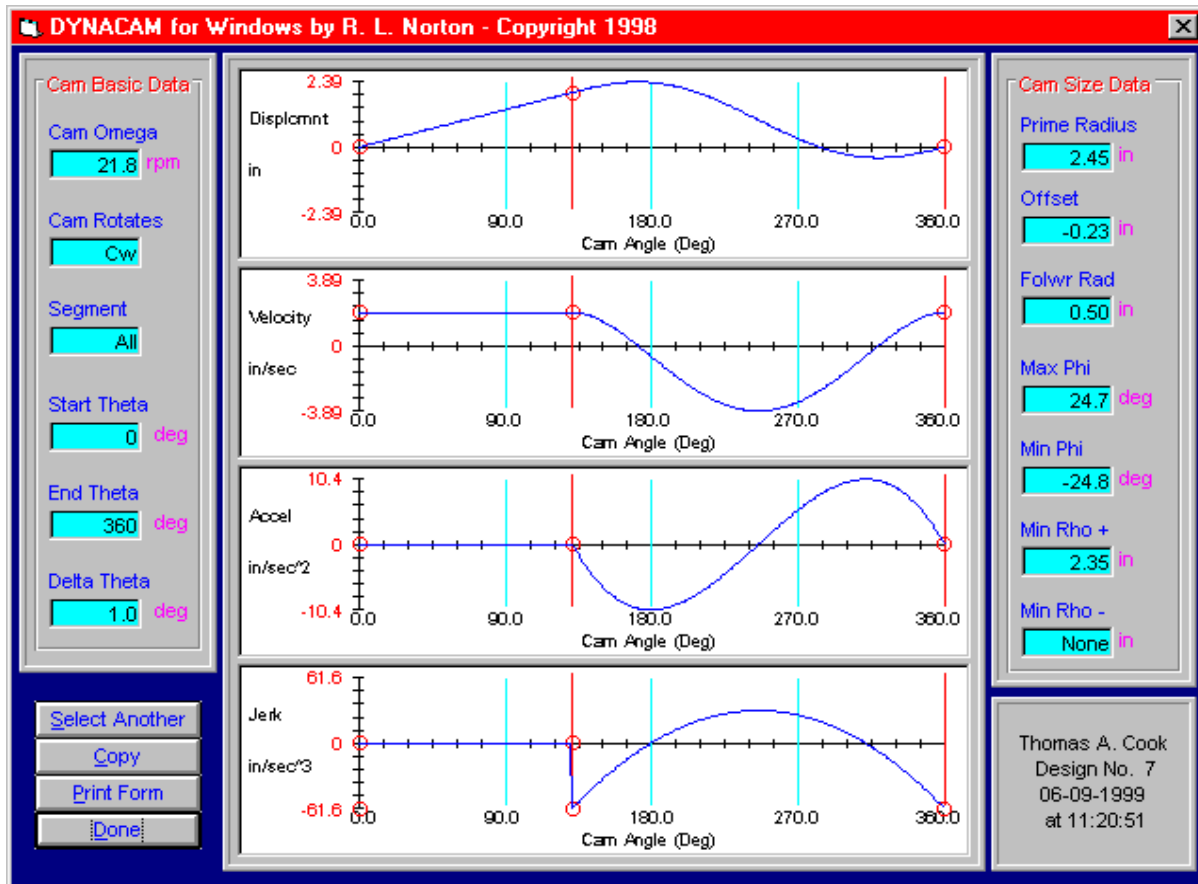
Seg	Angles			Cam Contour		Position (in)	
	Beta	Start	End	Motion	Program	Start	End
1	131	0	131	Poly	PO - Polynomial	0	
2	229	131	360	Poly	PO - Polynomial	2.001	0
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

Examples

Calc Plot Print
Calc Plot Print



8. From *SVAJ* diagrams we see that all design goals are met but the maximum pressure angle is considerably lower (in absolute value) than the minimum pressure angle. These can be balanced by making the follower eccentric with respect to the cam centerline. This will also allow the prime radius to be reduced. Iterate on these parameters until the maximum and minimum pressure angles are nearly the same (numerically) and reduce the prime circle radius until the pressure angle is at, or below, the required limit.
9. The eccentricity was iterated to -0.23 in and the prime circle radius was reduced to 2.45 in. This resulted in a balanced maximum and minimum pressure angle that is just below the required limit. The minimum radius of curvature is almost 5 times the follower radius, which is ample. The *SVAJ* diagram for the final iteration is shown below.



PROBLEM 8-21

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the *s v a j* diagrams for a modified trapezoidal acceleration cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec.

Enter: Lift: $h_1 := 20 \cdot \text{mm}$

Duration: $\beta_1 := 60 \cdot \text{deg}$

Solution: See Mathcad file P0821.

1. Enter values for lift and duration above.
2. The numerical constants in these SCCA for the modified trapezoidal equations are given in Table 8-2.

$$\begin{aligned} b &:= 0.25 & c &:= 0.50 & d &:= 0.25 \\ C_v &:= 2.0000 & C_a &:= 4.8881 & C_j &:= 61.426 \end{aligned}$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 \leq x \leq b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$\begin{aligned} y_1(x) &:= C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi} \right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \right] & y'_1(x) &:= C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x\right) \right) \\ y''_1(x) &:= C_a \cdot \sin\left(\frac{\pi}{b} \cdot x\right) & y'''_1(x) &:= C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x\right) \end{aligned}$$

for $b/2 \leq x \leq (1-d)/2$

$$\begin{aligned} y_2(x) &:= C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \right] & y'_2(x) &:= C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \right] \\ y''_2(x) &:= C_a & y'''_2(x) &:= 0 \end{aligned}$$

for $(1-d)/2 \leq x \leq (1+d)/2$

$$\begin{aligned} y_3(x) &:= C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2} \right) \cdot x + \left(\frac{d}{\pi} \right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi} \right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y'_3(x) &:= C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y''_3(x) &:= C_a \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] & y'''_3(x) &:= -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \end{aligned}$$

for $(1+d)/2 \leq x \leq 1-b/2$

$$\begin{aligned} y_4(x) &:= C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + (2 \cdot d^2 - b^2) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right] \\ y'_4(x) &:= C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) & y''_4(x) &:= -C_a & y'''_4(x) &:= 0 \end{aligned}$$

for $1-b/2 \leq x \leq 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot (d^2 - b^2)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left[\frac{\pi}{b} \cdot (x-1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos \left[\frac{\pi}{b} \cdot (x-1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin \left[\frac{\pi}{b} \cdot (x-1) \right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos \left[\frac{\pi}{b} \cdot (x-1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the *SVAJ* curves, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for S , V , A , and J that applies over the range $0 \leq \theta \leq 360$ deg.
6. Write the local *svaj* equations for the first interval, $0 \leq \theta \leq \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$ (Rise)

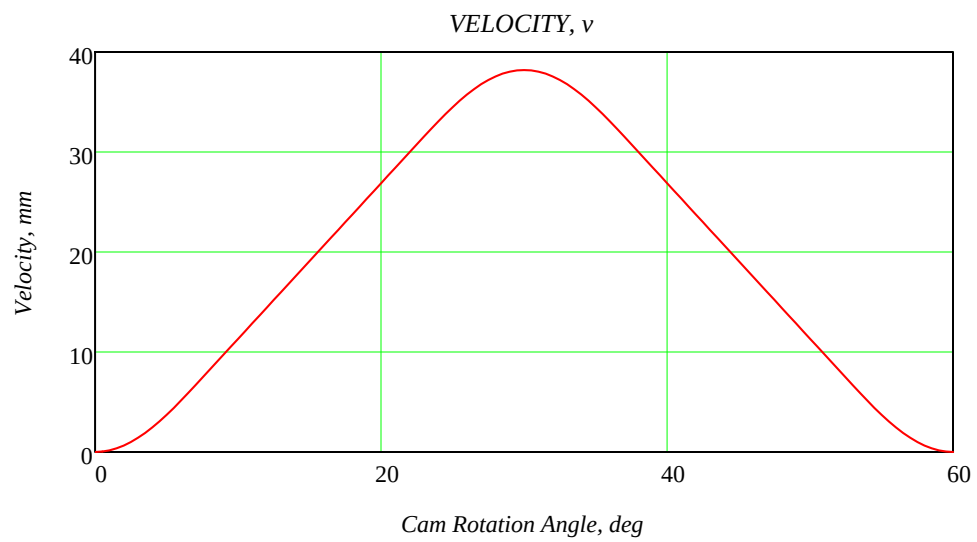
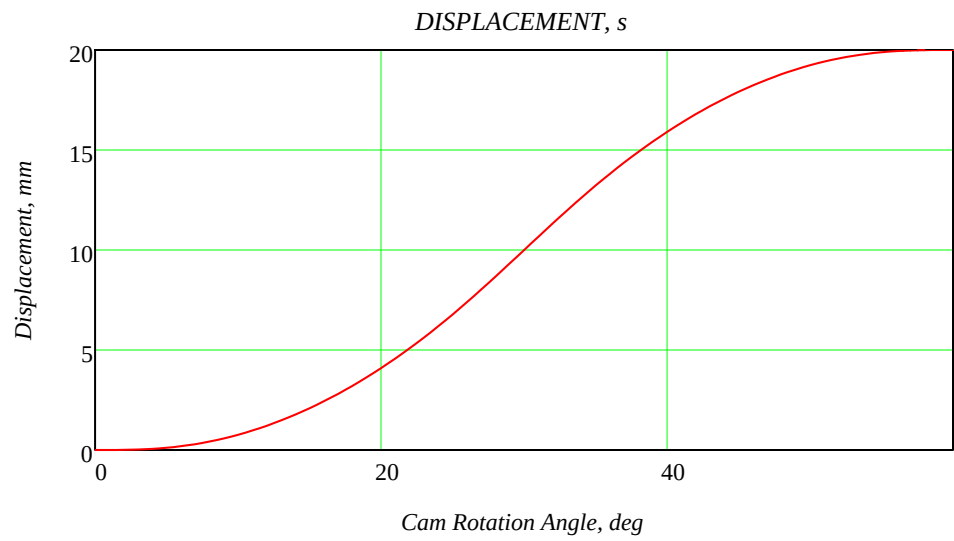
$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

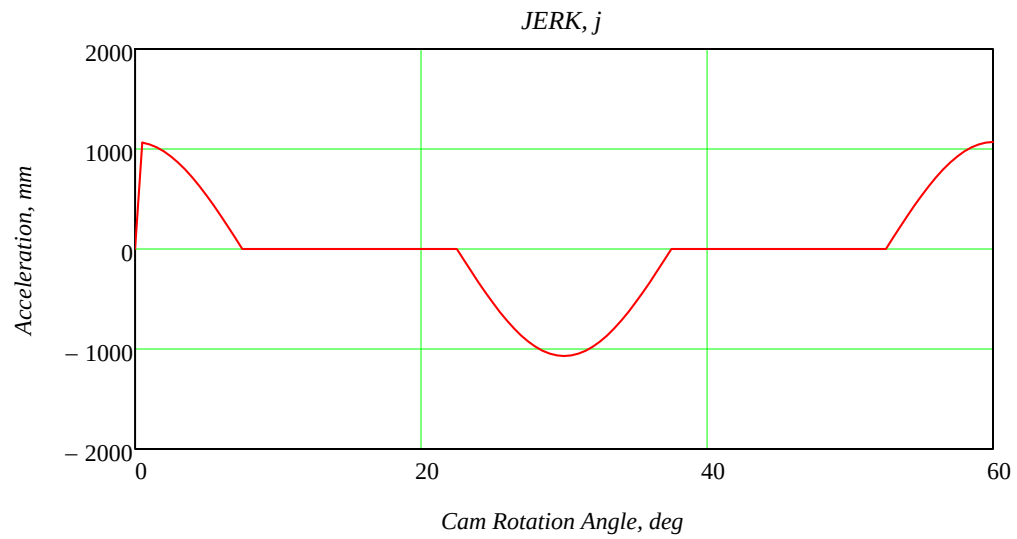
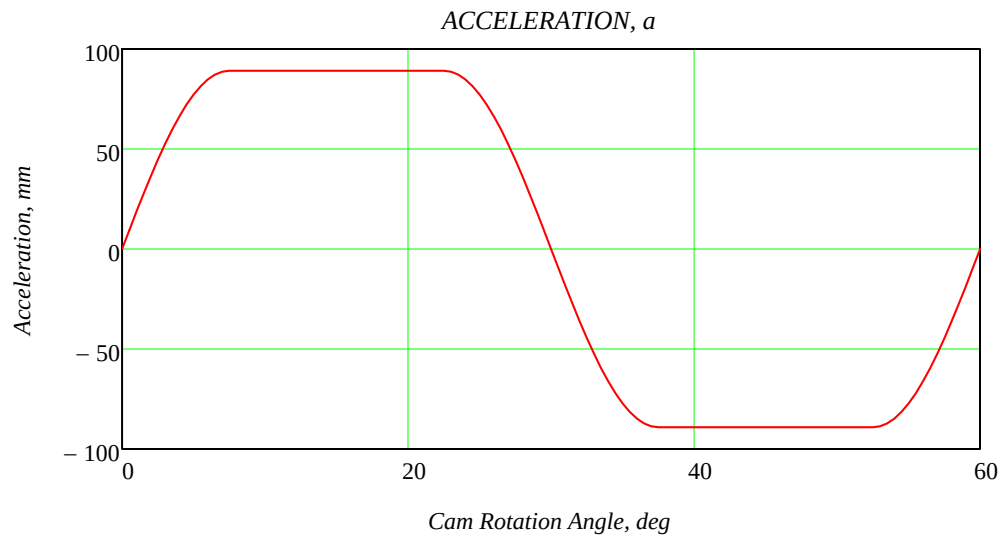
$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

5. Plot the displacement, velocity, acceleration, and jerk functions over the lift interval: $\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots \beta_1$





PROBLEM 8-22

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the *s v a j* diagrams for a modified sinusoidal acceleration cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec.

Enter: Lift: $h_1 := 20 \cdot \text{mm}$
Duration: $\beta_1 := 60 \cdot \text{deg}$

Solution: See Mathcad file P0822.

1. Enter values for lift and duration above.
2. The numerical constants in these SCCA for the modified trapezoidal equations are given in Table 8-2.

$$\begin{aligned} b &:= 0.25 & c &:= 0.00 & d &:= 0.75 \\ C_v &:= 1.7596 & C_a &:= 5.5280 & C_j &:= 69.466 \end{aligned}$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:
for $0 \leq x \leq b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$\begin{aligned} y_1(x) &:= C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left(\frac{\pi}{b} \cdot x \right) \right] & y'_1(x) &:= C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos \left(\frac{\pi}{b} \cdot x \right) \right) \\ y''_1(x) &:= C_a \cdot \sin \left(\frac{\pi}{b} \cdot x \right) & y'''_1(x) &:= C_a \cdot \frac{\pi}{b} \cdot \cos \left(\frac{\pi}{b} \cdot x \right) \end{aligned}$$

for $b/2 \leq x \leq (1-d)/2$

$$\begin{aligned} y_2(x) &:= C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \right] & y'_2(x) &:= C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \right] \\ y''_2(x) &:= C_a & y'''_2(x) &:= 0 \end{aligned}$$

for $(1-d)/2 \leq x \leq (1+d)/2$

$$\begin{aligned} y_3(x) &:= C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2} \right) \cdot x + \left(\frac{d}{\pi} \right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi} \right)^2 \cdot \cos \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y'_3(x) &:= C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y''_3(x) &:= C_a \cdot \cos \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] & y'''_3(x) &:= -C_a \cdot \frac{\pi}{d} \cdot \sin \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \end{aligned}$$

for $(1+d)/2 \leq x \leq 1-b/2$

$$\begin{aligned} y_4(x) &:= C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + (2 \cdot d^2 - b^2) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right] \\ y'_4(x) &:= C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) & y''_4(x) &:= -C_a & y'''_4(x) &:= 0 \end{aligned}$$

for $1-b/2 \leq x \leq 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot (d^2 - b^2)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left[\frac{\pi}{b} \cdot (x-1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos \left[\frac{\pi}{b} \cdot (x-1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin \left[\frac{\pi}{b} \cdot (x-1) \right]$$

$$y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos \left[\frac{\pi}{b} \cdot (x-1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the *SVAJ* curves, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for S , V , A , and J that applies over the range $0 \leq \theta \leq 360$ deg.
6. Write the local *svaj* equations for the first interval, $0 \leq \theta \leq \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

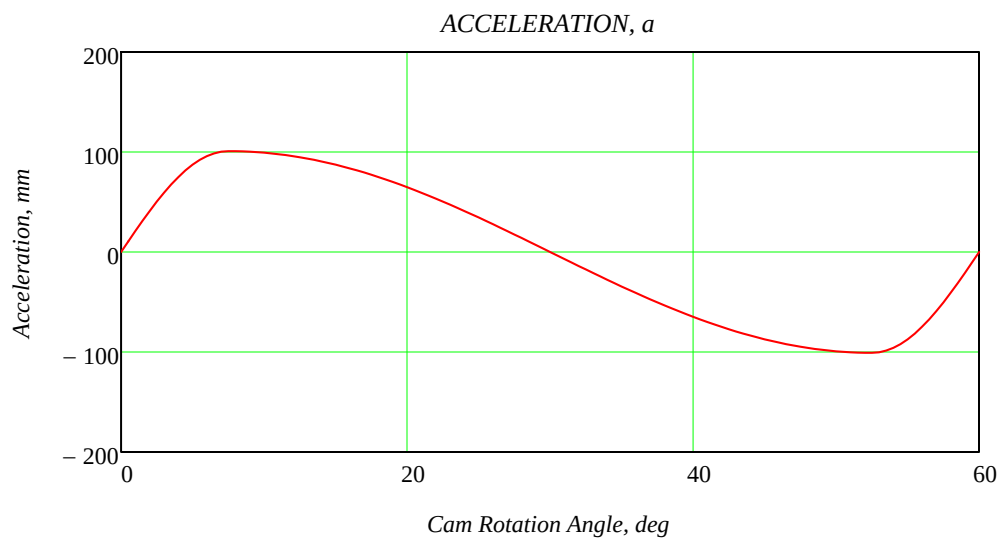
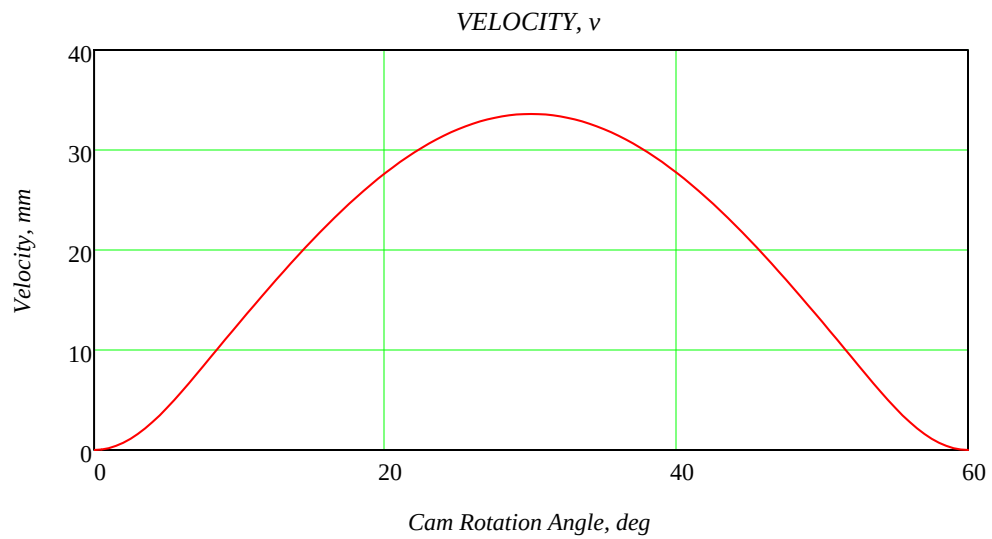
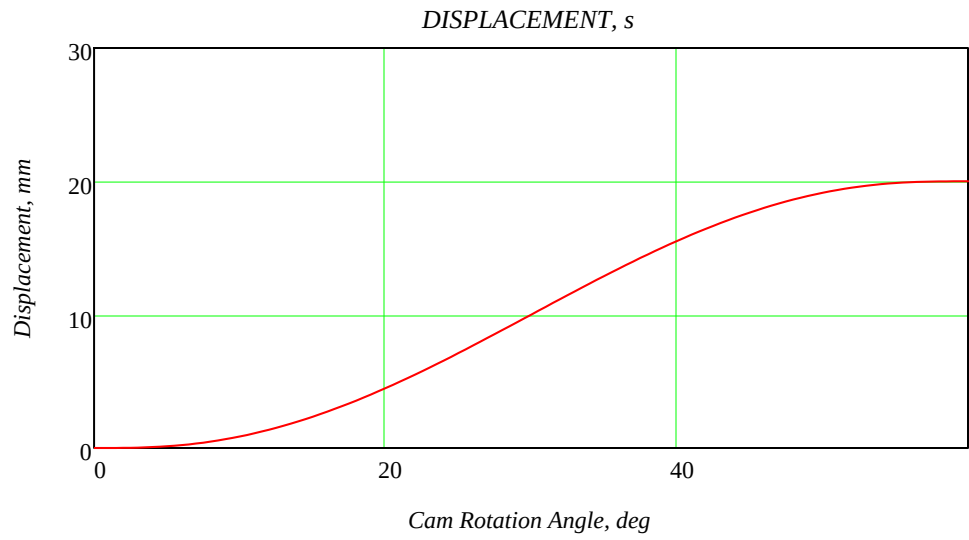
$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

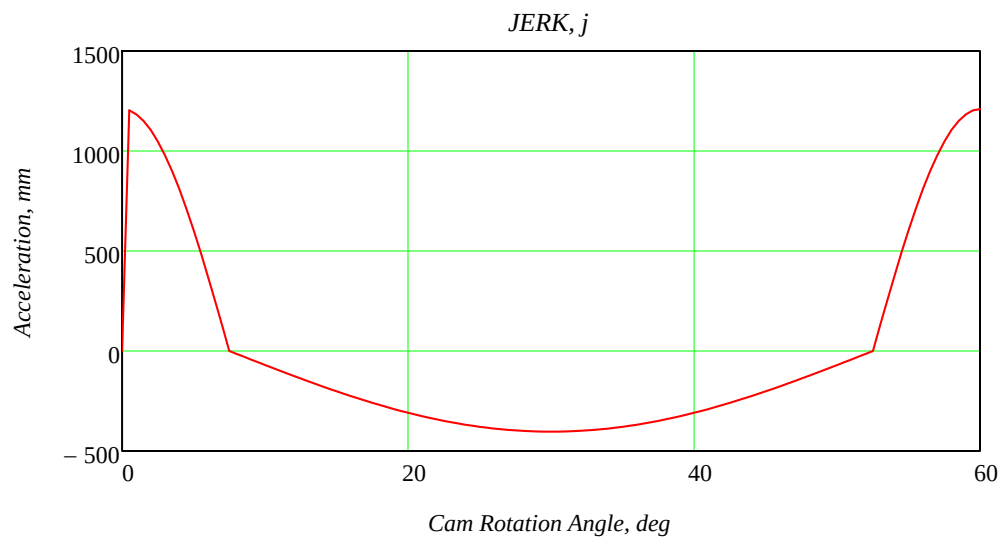
$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

5. Plot the displacement, velocity, acceleration, and jerk functions over the lift interval:

$$\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots \beta_1$$





PROBLEM 8-23

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the $s v a j$ diagrams for a cycloidal displacement cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec.

Enter:

Lift: $h := 20 \cdot \text{mm}$

Duration: $\beta := 60 \cdot \text{deg}$

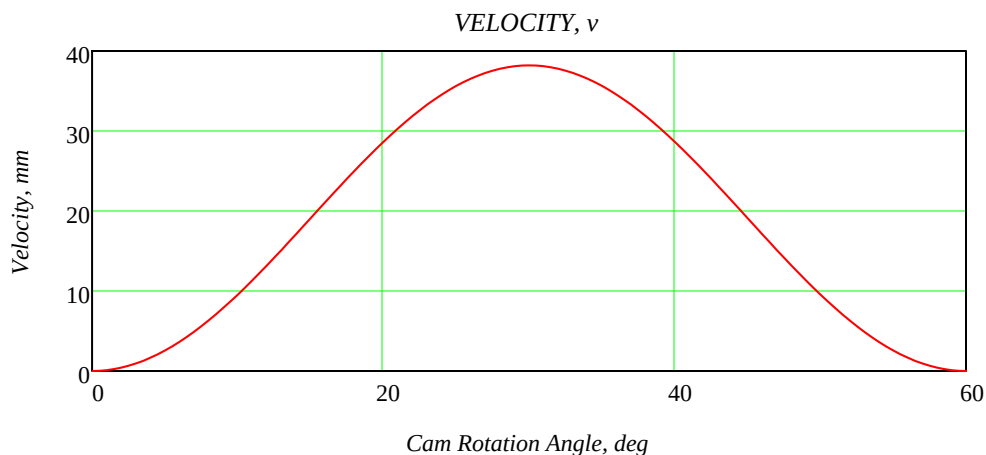
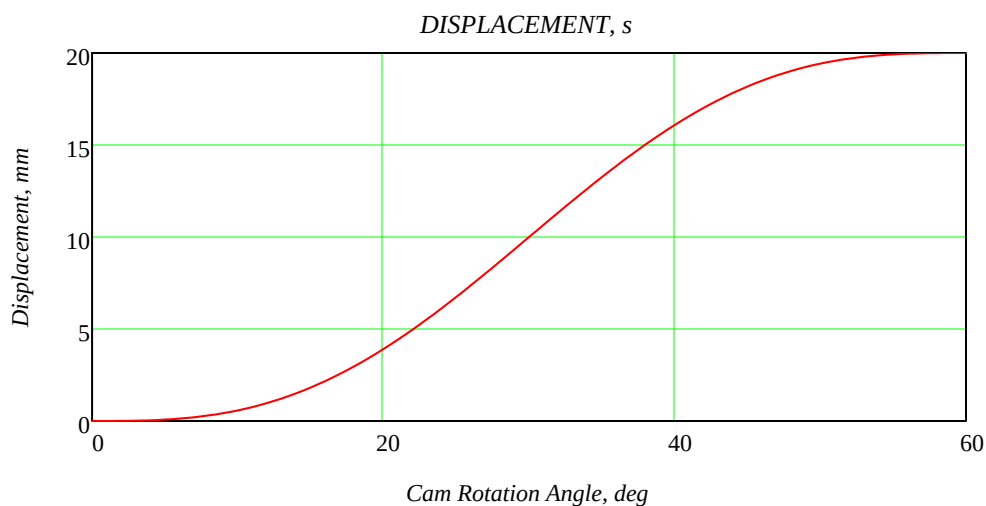
Solution: See Mathcad file P0823.

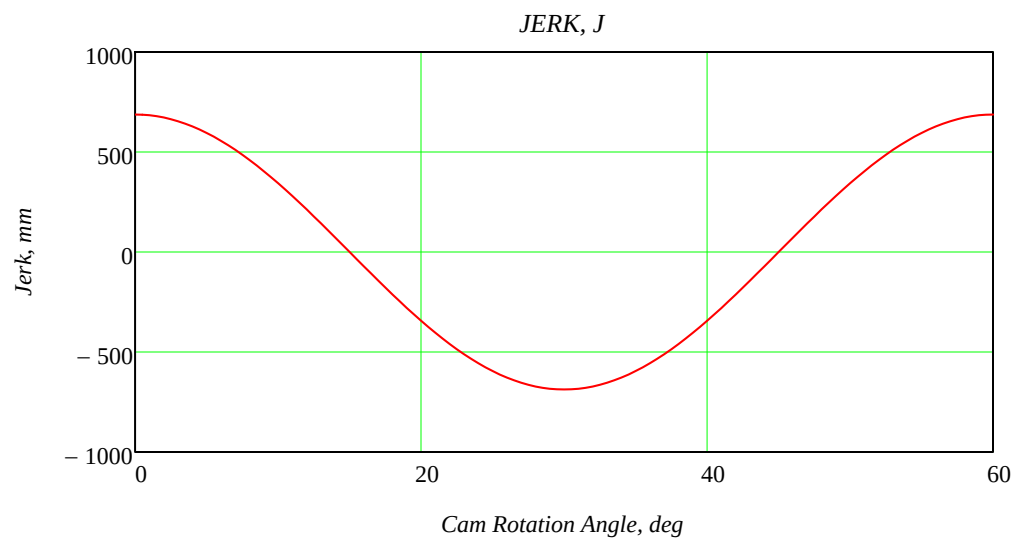
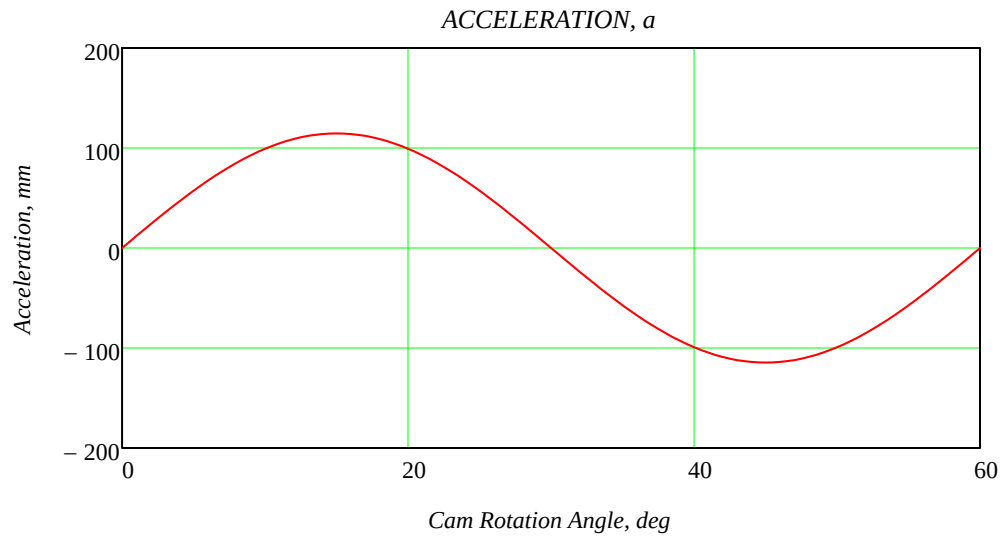
1. Cycloidal motion is defined in local coordinates by equations 8.12. They are:

$$s(\theta) := h \cdot \left(\frac{\theta}{\beta} - \frac{1}{2 \cdot \pi} \cdot \sin \left(2 \cdot \pi \cdot \frac{\theta}{\beta} \right) \right) \quad v(\theta) := \frac{h}{\beta} \cdot \left(1 - \cos \left(2 \cdot \pi \cdot \frac{\theta}{\beta} \right) \right)$$

$$a(\theta) := 2 \cdot \pi \cdot \frac{h}{\beta^2} \cdot \sin \left(2 \cdot \pi \cdot \frac{\theta}{\beta} \right) \quad j(\theta) := 4 \cdot \pi^2 \cdot \frac{h}{\beta^3} \cdot \cos \left(2 \cdot \pi \cdot \frac{\theta}{\beta} \right)$$

2. Plot the displacement, velocity, acceleration, and jerk functions over the lift interval: $\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots \beta$





PROBLEM 8-24

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the $s v a j$ diagrams for a 3-4-5 polynomial displacement cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec.

Enter: Lift: $h := 20 \cdot \text{mm}$
Duration: $\beta := 60 \cdot \text{deg}$

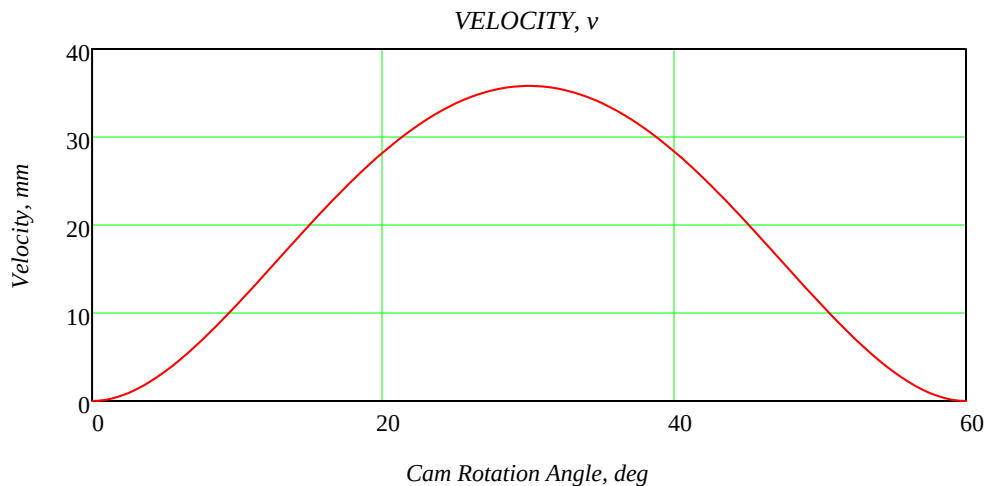
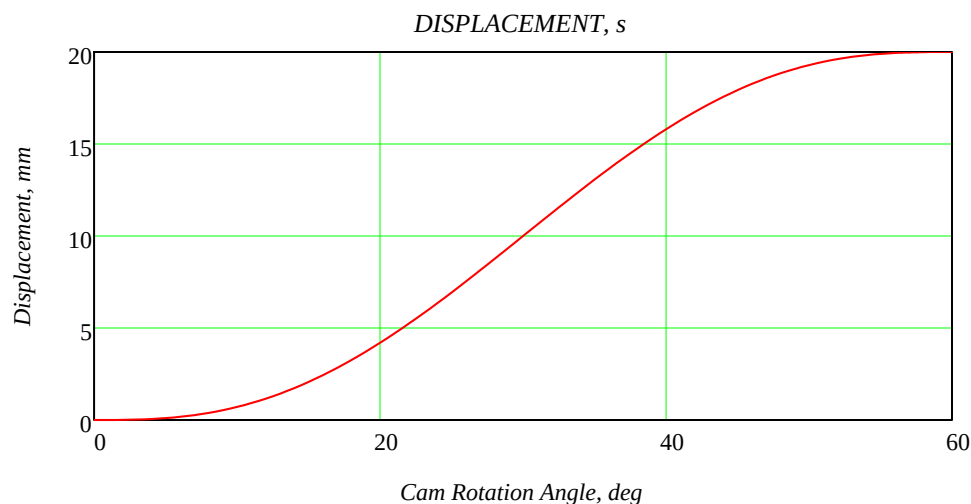
Solution: See Mathcad file P0824.

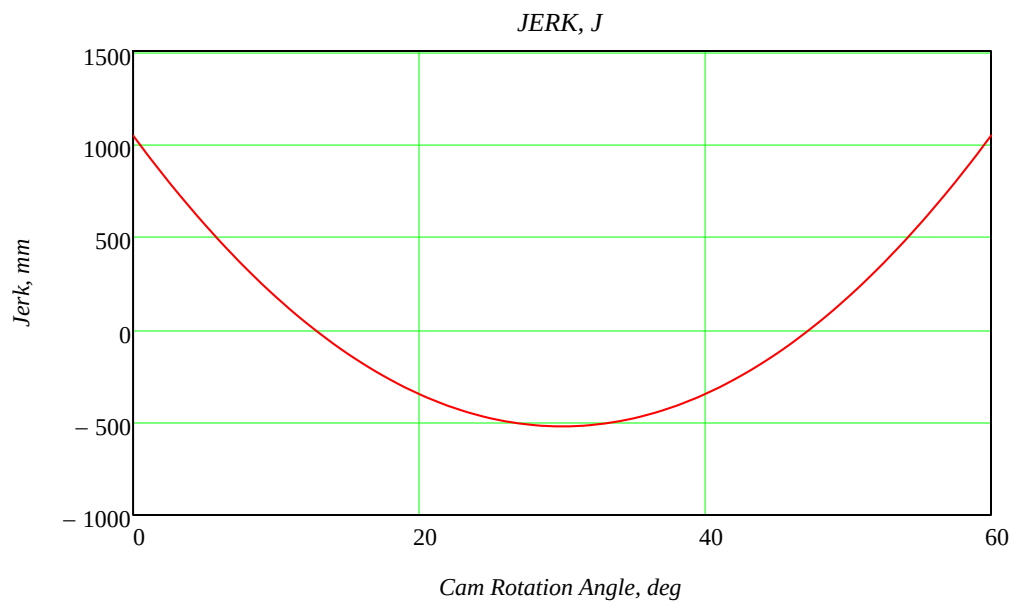
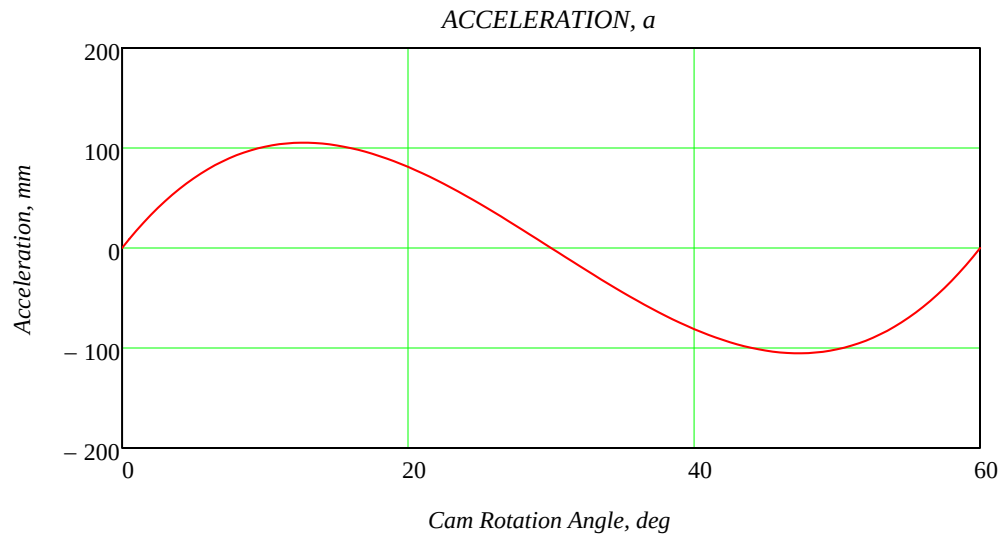
1. The 3-4-5 polynomial is defined in local coordinates by equations 8.24. They are:

$$s(\theta) := h \cdot \left[10 \cdot \left(\frac{\theta}{\beta} \right)^3 - 15 \cdot \left(\frac{\theta}{\beta} \right)^4 + 6 \cdot \left(\frac{\theta}{\beta} \right)^5 \right] \quad v(\theta) := \frac{h}{\beta} \cdot \left[30 \cdot \left(\frac{\theta}{\beta} \right)^2 - 60 \cdot \left(\frac{\theta}{\beta} \right)^3 + 30 \cdot \left(\frac{\theta}{\beta} \right)^4 \right]$$

$$a(\theta) := \frac{h}{\beta^2} \cdot \left[60 \cdot \left(\frac{\theta}{\beta} \right) - 180 \cdot \left(\frac{\theta}{\beta} \right)^2 + 120 \cdot \left(\frac{\theta}{\beta} \right)^3 \right] \quad j(\theta) := \frac{h}{\beta^3} \cdot \left[60 - 360 \cdot \left(\frac{\theta}{\beta} \right) + 360 \cdot \left(\frac{\theta}{\beta} \right)^2 \right]$$

2. Plot the displacement, velocity, acceleration, and jerk functions over the lift interval: $\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots \beta$





PROBLEM 8-25

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the $s v a j$ diagrams for a 4-5-6-7 polynomial displacement cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec.

Enter: Lift: $h := 20 \cdot \text{mm}$

Duration: $\beta := 60 \cdot \text{deg}$

Solution: See Mathcad file P0825.

1. The 4-5-6-7 polynomial is defined in local coordinates by equation 8.25. Differentiate it to get v , a , and j .

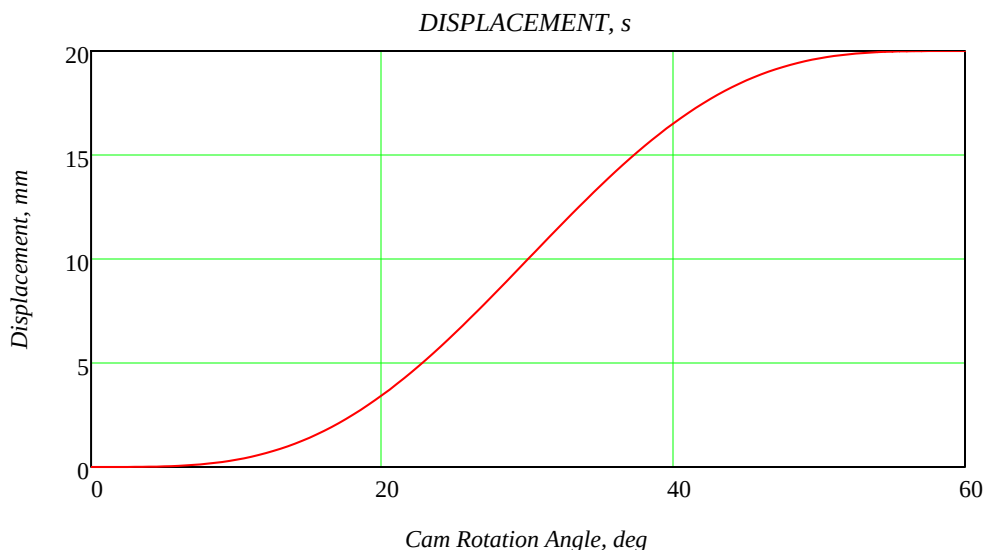
$$s(\theta) := h \cdot \left[35 \cdot \left(\frac{\theta}{\beta} \right)^4 - 84 \cdot \left(\frac{\theta}{\beta} \right)^5 + 70 \cdot \left(\frac{\theta}{\beta} \right)^6 - 20 \cdot \left(\frac{\theta}{\beta} \right)^7 \right]$$

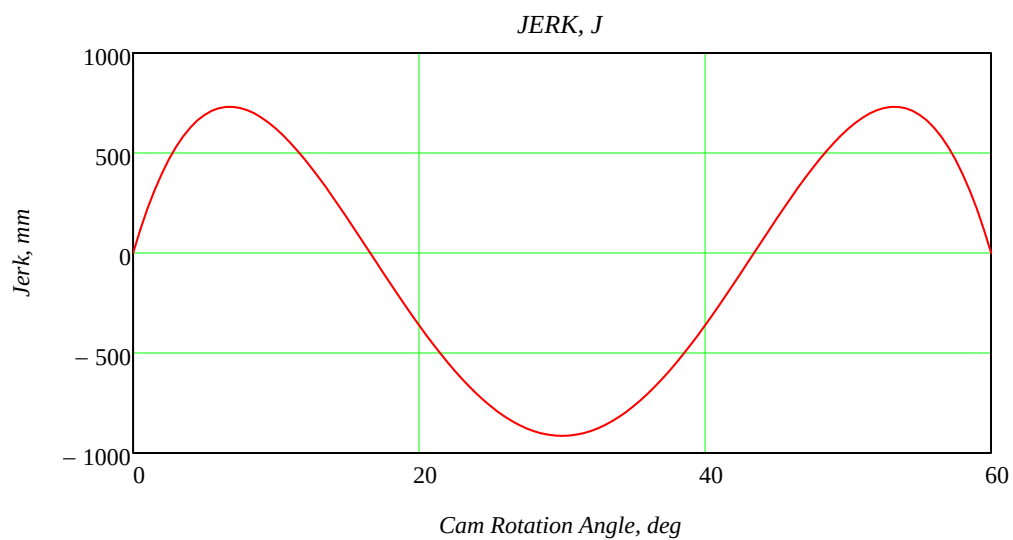
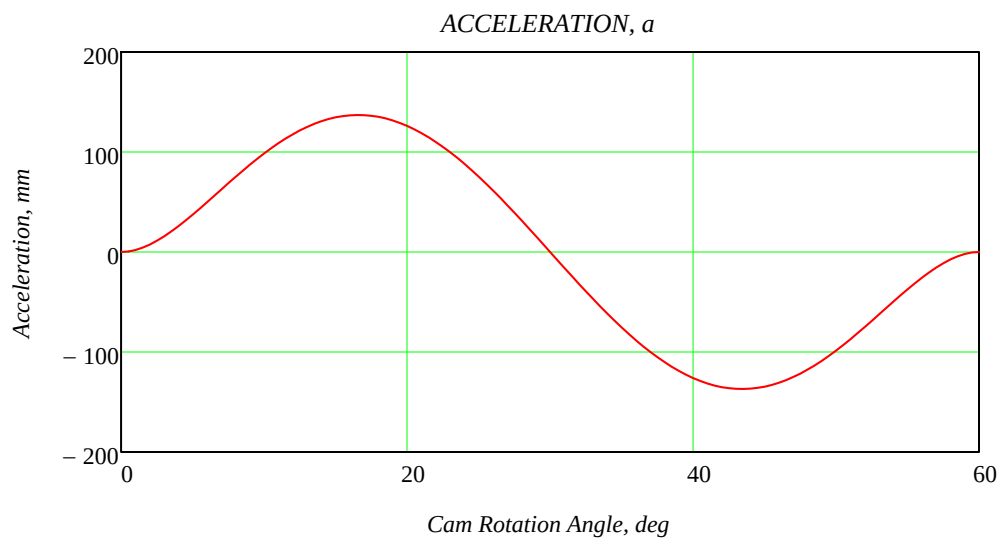
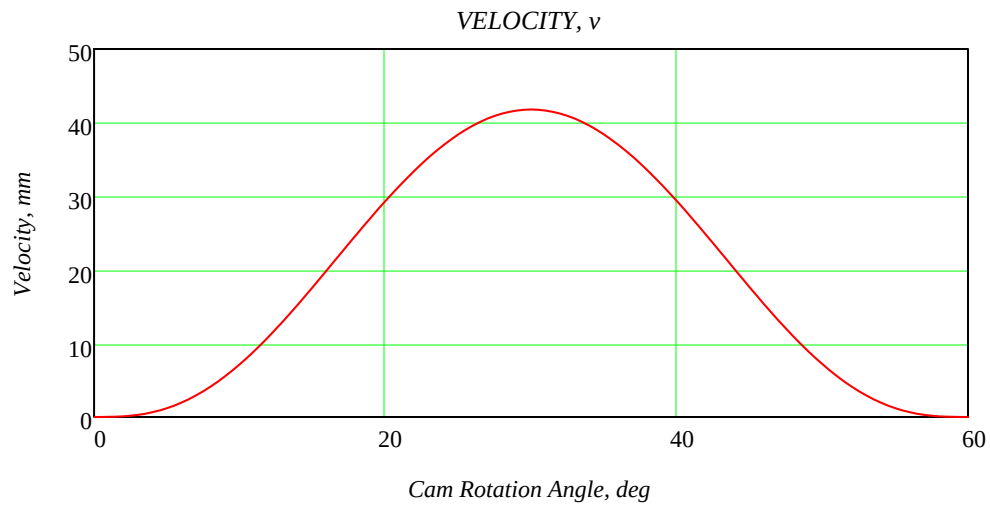
$$v(\theta) := \frac{h}{\beta} \cdot \left[140 \cdot \left(\frac{\theta}{\beta} \right)^3 - 420 \cdot \left(\frac{\theta}{\beta} \right)^4 + 420 \cdot \left(\frac{\theta}{\beta} \right)^5 - 140 \cdot \left(\frac{\theta}{\beta} \right)^6 \right]$$

$$a(\theta) := \frac{h}{\beta^2} \cdot \left[420 \cdot \left(\frac{\theta}{\beta} \right)^2 - 1680 \cdot \left(\frac{\theta}{\beta} \right)^3 + 2100 \cdot \left(\frac{\theta}{\beta} \right)^4 - 840 \cdot \left(\frac{\theta}{\beta} \right)^5 \right]$$

$$j(\theta) := \frac{h}{\beta^3} \cdot \left[840 \cdot \left(\frac{\theta}{\beta} \right) - 5040 \cdot \left(\frac{\theta}{\beta} \right)^2 + 8400 \cdot \left(\frac{\theta}{\beta} \right)^3 - 4200 \cdot \left(\frac{\theta}{\beta} \right)^4 \right]$$

2. Plot the displacement, velocity, acceleration, and jerk functions over the lift interval: $\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. \beta$





PROBLEM 8-26

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the $s v a j$ diagrams for a simple harmonic displacement cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec.

Enter: Lift: $h := 20 \cdot \text{mm}$
Duration: $\beta := 60 \cdot \text{deg}$

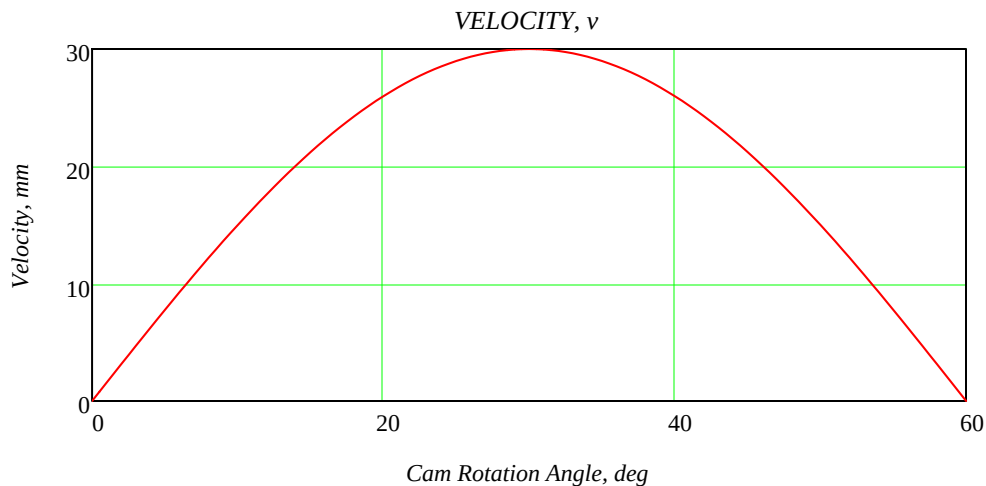
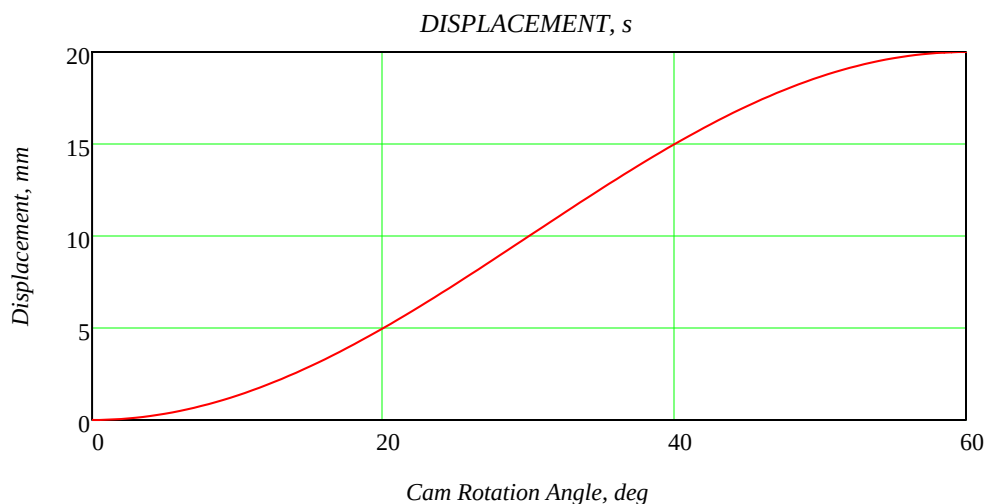
Solution: See Mathcad file P0826.

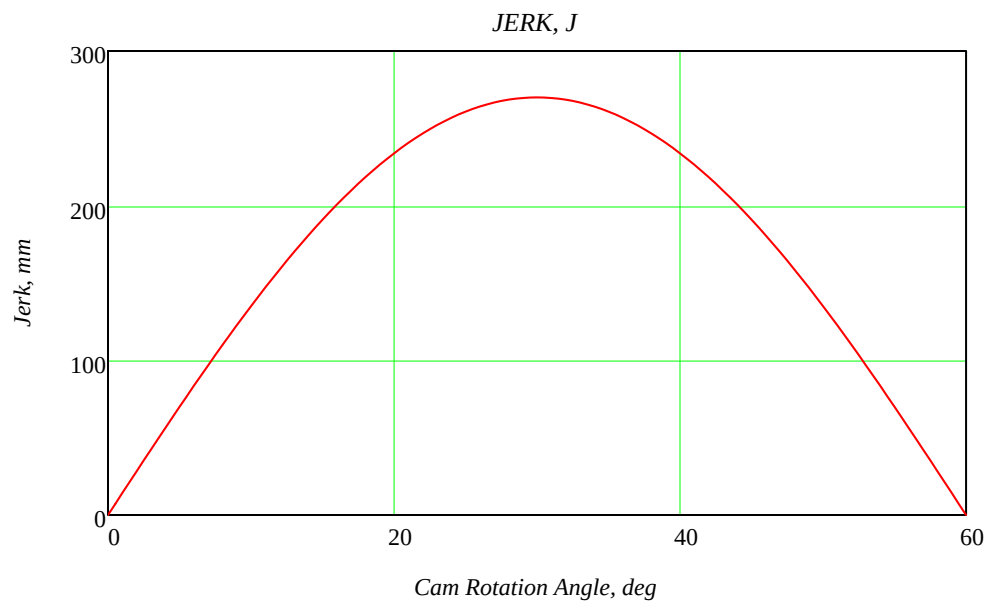
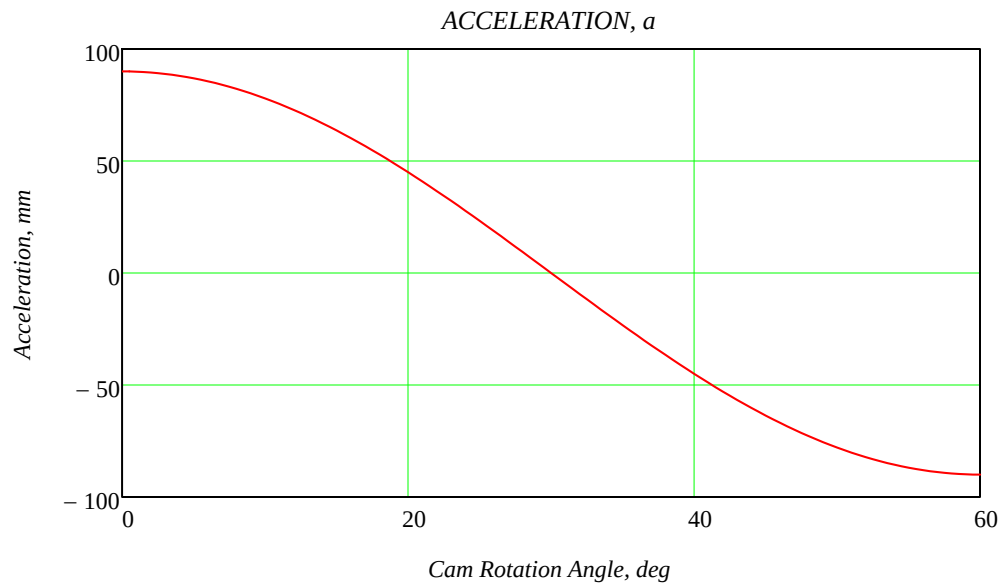
1. The simple harmonic motion (SHM) is defined in local coordinates by equations 8.6. They are:

$$s(\theta) := \frac{h}{2} \cdot \left(1 - \cos\left(\pi \cdot \frac{\theta}{\beta}\right) \right) \qquad v(\theta) := \frac{\pi}{\beta} \cdot \frac{h}{2} \cdot \sin\left(\pi \cdot \frac{\theta}{\beta}\right)$$

$$a(\theta) := \frac{\pi^2}{\beta^2} \cdot \frac{h}{2} \cdot \cos\left(\pi \cdot \frac{\theta}{\beta}\right) \qquad j(\theta) := \frac{\pi^3}{\beta^3} \cdot \frac{h}{2} \cdot \sin\left(\pi \cdot \frac{\theta}{\beta}\right)$$

2. Plot the displacement, velocity, acceleration, and jerk functions over the lift interval: $\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots \beta$





PROBLEM 8-27

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the pressure angle and radius of curvature for a modified trapezoidal acceleration cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20 deg. What is the minimum diameter roller follower needed to avoid undercutting with these data?

Enter: Lift: $h_1 := 20 \cdot \text{mm}$ Eccentricity $\epsilon := 4 \cdot \text{mm}$
 Duration: $\beta_1 := 60 \cdot \text{deg}$ Prime circle radius $R_p := 50 \cdot \text{mm}$

Solution: See Mathcad file P0827.

1. Enter values for lift and duration above.
2. The numerical constants in these SCCA for the modified trapezoidal equations are given in Table 8-2.

$$\begin{array}{lll} b := 0.25 & c := 0.50 & d := 0.25 \\ C_v := 2.0000 & C_a := 4.8881 & C_j := 61.426 \end{array}$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 \leq x \leq b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$\begin{aligned} y_1(x) &:= C_a \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left(\frac{\pi}{b} \cdot x \right) \right] & y'_1(x) &:= C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos \left(\frac{\pi}{b} \cdot x \right) \right) \\ y''_1(x) &:= C_a \cdot \sin \left(\frac{\pi}{b} \cdot x \right) & y'''_1(x) &:= C_a \cdot \frac{\pi}{b} \cdot \cos \left(\frac{\pi}{b} \cdot x \right) \end{aligned}$$

for $b/2 \leq x \leq (1-d)/2$

$$\begin{aligned} y_2(x) &:= C_a \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \right] & y'_2(x) &:= C_a \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \right] \\ y''_2(x) &:= C_a & y'''_2(x) &:= 0 \end{aligned}$$

for $(1-d)/2 \leq x \leq (1+d)/2$

$$\begin{aligned} y_3(x) &:= C_a \left[\left(\frac{b}{\pi} + \frac{c}{2} \right) \cdot x + \left(\frac{d}{\pi} \right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi} \right)^2 \cdot \cos \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y'_3(x) &:= C_a \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y''_3(x) &:= C_a \cdot \cos \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] & y'''_3(x) &:= -C_a \cdot \frac{\pi}{d} \cdot \sin \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \end{aligned}$$

for $(1+d)/2 \leq x \leq 1-b/2$

$$\begin{aligned} y_4(x) &:= C_a \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + (2 \cdot d^2 - b^2) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right] \\ y'_4(x) &:= C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) & y''_4(x) &:= -C_a & y'''_4(x) &:= 0 \end{aligned}$$

for $1-b/2 \leq x \leq 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot (d^2 - b^2)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left[\frac{\pi}{b} \cdot (x-1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos \left[\frac{\pi}{b} \cdot (x-1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin \left[\frac{\pi}{b} \cdot (x-1) \right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos \left[\frac{\pi}{b} \cdot (x-1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the *SVAJ* curves, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range $0 \leq \theta \leq 360$ deg.
6. Write the local *svaj* equations for the first interval, $0 \leq \theta \leq \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

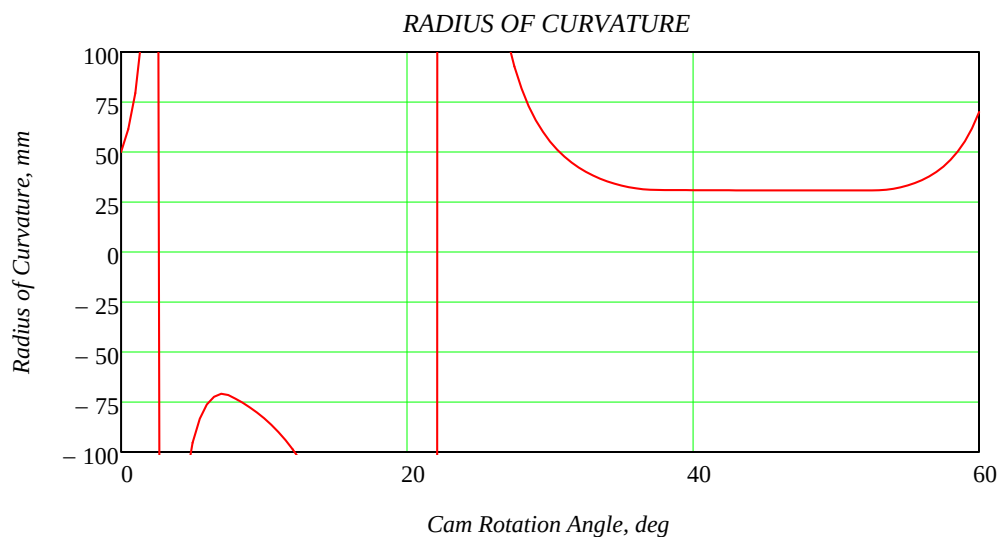
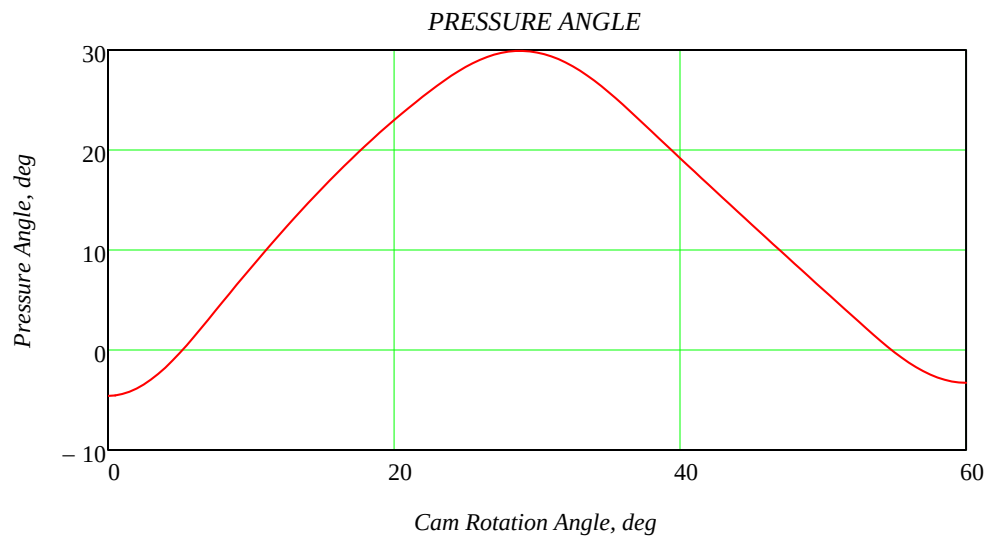
$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

5. Using equations 8.31d and 8.33, write the pressure angle and radius of curvature functions.

$$\phi(\theta, R_p, \varepsilon) := \text{atan} \left(\frac{v_1\left(\frac{\theta}{\beta_1}\right) - \varepsilon}{s_1\left(\frac{\theta}{\beta_1}\right) + \sqrt{R_p^2 - \varepsilon^2}} \right)$$

$$\rho(\theta, R_p) := \frac{\left[\left(R_p + s_1 \left(\frac{\theta}{\beta_1} \right) \right)^2 + v_1 \left(\frac{\theta}{\beta_1} \right)^2 \right]^{\frac{3}{2}}}{\left(R_p + s_1 \left(\frac{\theta}{\beta_1} \right) \right)^2 + 2 \cdot v_1 \left(\frac{\theta}{\beta_1} \right)^2 - a_1 \left(\frac{\theta}{\beta_1} \right) \cdot \left(R_p + s_1 \left(\frac{\theta}{\beta_1} \right) \right)}$$

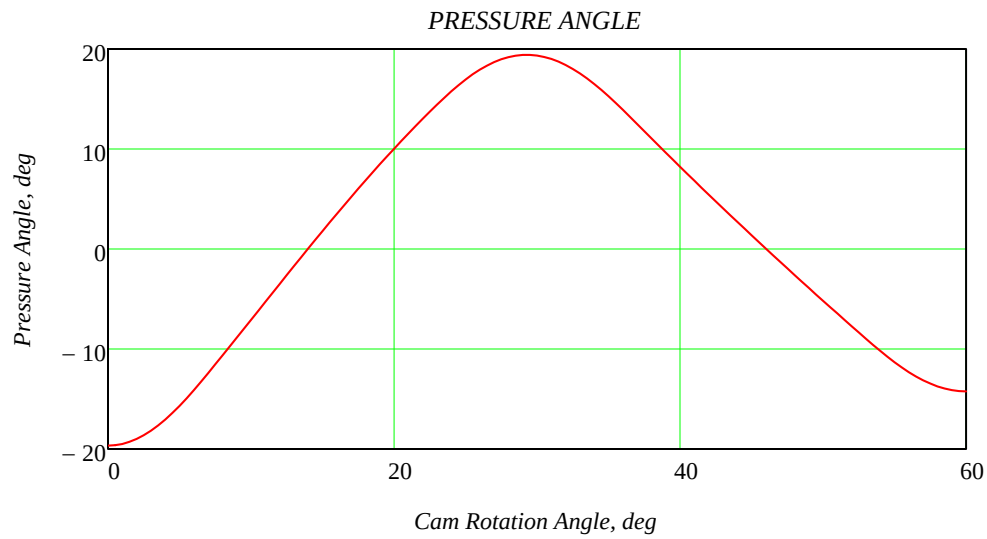
6. Plot the pressure angle and radius of curvature functions over the lift interval: $\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. \beta_1$



7. The graphs above show the pressure angle and radius of curvature for the values of R_p and ε entered on the first page. These values will be iterated below to obtain a balanced pressure angle whose absolute value is not greater than 20 deg.

$$R_p := 52 \cdot \text{mm}$$

$$\varepsilon := 17.5 \cdot \text{mm}$$



8. From the graph of radius of curvature above, we see that the minimum value occurs at a cam angle of about 45 deg.

$$\rho_{min} := \rho(45 \cdot \text{deg}, R_p)$$

$$\rho_{min} = 32.123 \text{ mm}$$

Using a multiple of 3, the maximum roller follower radius is

$$R_f := \frac{\rho_{min}}{3}$$

$$R_f = 10.7 \text{ mm}$$

PROBLEM 8-28

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the pressure angle and radius of curvature for a modified sinusoidal acceleration cam function for any specified values of lift and duration. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20 deg. What is the minimum diameter roller follower needed to avoid undercutting with these data?

Enter: Lift: $h_1 := 20 \cdot \text{mm}$ Eccentricity $\varepsilon := 4 \cdot \text{mm}$
 Duration: $\beta_1 := 60 \cdot \text{deg}$ Prime circle radius $R_p := 50 \cdot \text{mm}$

Solution: See Mathcad file P0828.

1. Enter values for lift and duration above.
2. The numerical constants in these SCCA for the modified sinusoidal equations are given in Table 8-2.

$$\begin{aligned} b &:= 0.25 & c &:= 0.00 & d &:= 0.75 \\ C_v &:= 1.7596 & C_a &:= 5.5280 & C_j &:= 69.466 \end{aligned}$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:
 for $0 \leq x \leq b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$\begin{aligned} y_1(x) &:= C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left(\frac{\pi}{b} \cdot x \right) \right] & y'_1(x) &:= C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos \left(\frac{\pi}{b} \cdot x \right) \right) \\ y''_1(x) &:= C_a \cdot \sin \left(\frac{\pi}{b} \cdot x \right) & y'''_1(x) &:= C_a \cdot \frac{\pi}{b} \cdot \cos \left(\frac{\pi}{b} \cdot x \right) \end{aligned}$$

for $b/2 \leq x \leq (1-d)/2$

$$\begin{aligned} y_2(x) &:= C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \right] & y'_2(x) &:= C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \right] \\ y''_2(x) &:= C_a & y'''_2(x) &:= 0 \end{aligned}$$

for $(1-d)/2 \leq x \leq (1+d)/2$

$$\begin{aligned} y_3(x) &:= C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2} \right) \cdot x + \left(\frac{d}{\pi} \right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi} \right)^2 \cdot \cos \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y'_3(x) &:= C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y''_3(x) &:= C_a \cdot \cos \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] & y'''_3(x) &:= -C_a \cdot \frac{\pi}{d} \cdot \sin \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \end{aligned}$$

for $(1+d)/2 \leq x \leq 1-b/2$

$$\begin{aligned} y_4(x) &:= C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + (2 \cdot d^2 - b^2) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right] \\ y'_4(x) &:= C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) & y''_4(x) &:= -C_a & y'''_4(x) &:= 0 \end{aligned}$$

for $1-b/2 \leq x \leq 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot (d^2 - b^2)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left[\frac{\pi}{b} \cdot (x-1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos \left[\frac{\pi}{b} \cdot (x-1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin \left[\frac{\pi}{b} \cdot (x-1) \right]$$

$$y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos \left[\frac{\pi}{b} \cdot (x-1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the *SVAJ* curves, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range $0 \leq \theta \leq 360$ deg.
6. Write the local *svaj* equations for the first interval, $0 \leq \theta \leq \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

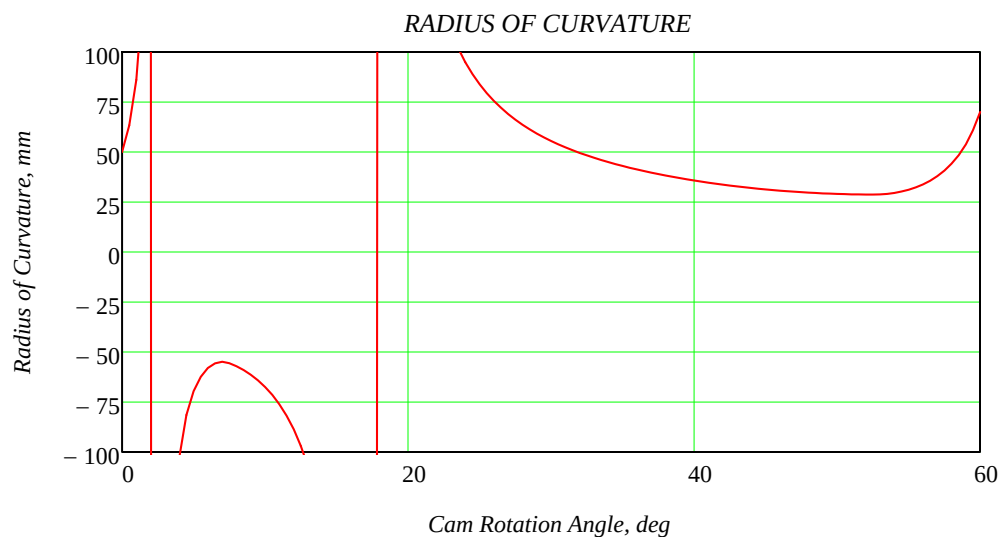
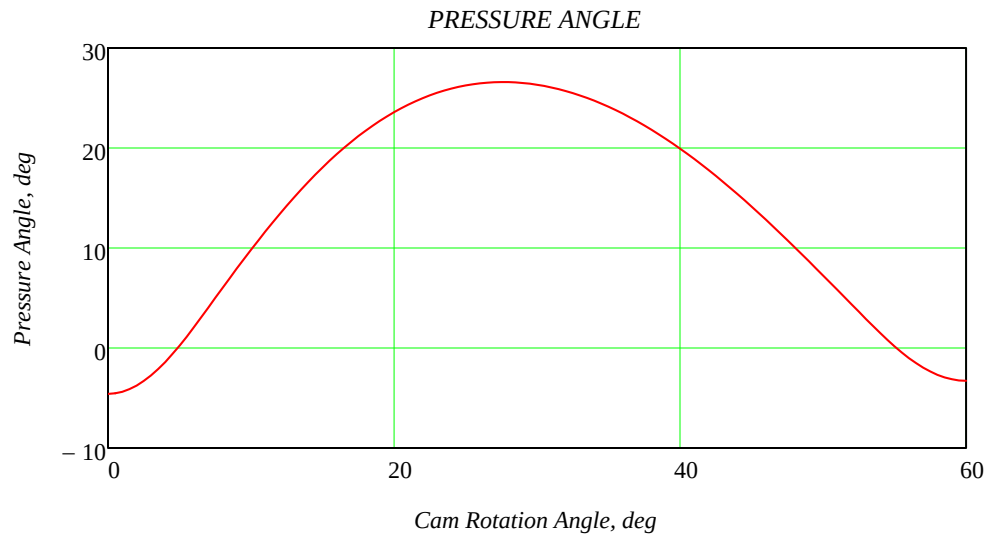
$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

5. Using equations 8.31d and 8.33, write the pressure angle and radius of curvature functions.

$$\phi(\theta, R_p, \epsilon) := \text{atan} \left(\frac{v_1\left(\frac{\theta}{\beta_1}\right) - \epsilon}{s_1\left(\frac{\theta}{\beta_1}\right) + \sqrt{R_p^2 - \epsilon^2}} \right)$$

$$\rho(\theta, R_p) := \frac{\left[\left(R_p + s_1 \left(\frac{\theta}{\beta_1} \right) \right)^2 + v_1 \left(\frac{\theta}{\beta_1} \right)^2 \right]^{\frac{3}{2}}}{\left(R_p + s_1 \left(\frac{\theta}{\beta_1} \right) \right)^2 + 2 \cdot v_1 \left(\frac{\theta}{\beta_1} \right)^2 - a_1 \left(\frac{\theta}{\beta_1} \right) \cdot \left(R_p + s_1 \left(\frac{\theta}{\beta_1} \right) \right)}$$

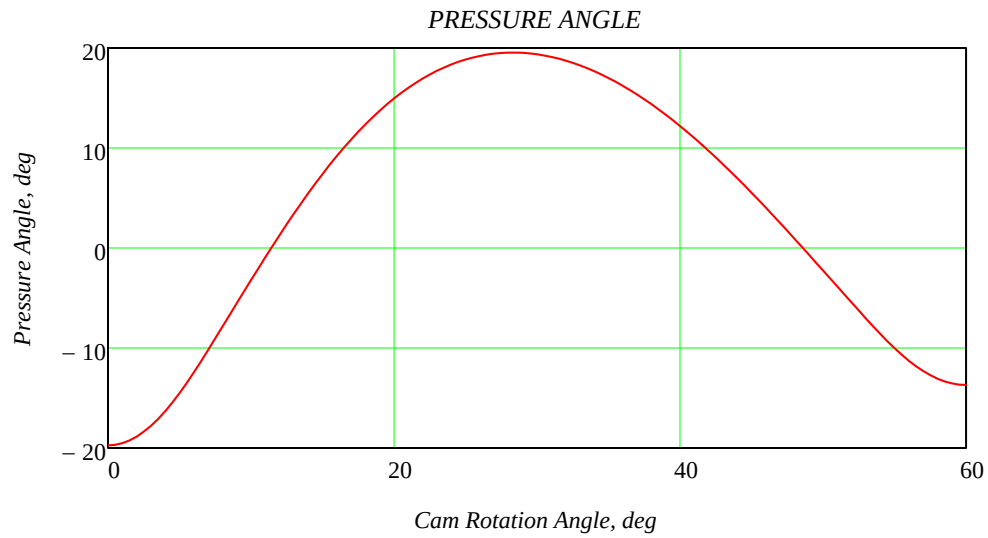
6. Plot the pressure angle and radius of curvature functions over the lift interval: $\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots \beta_1$



7. The graphs above show the pressure angle and radius of curvature for the values of R_p and ε entered on the first page. These values will be iterated below to obtain a balanced pressure angle whose absolute value is not greater than 20 deg.

$$R_p := 45 \cdot \text{mm}$$

$$\varepsilon := 15.2 \cdot \text{mm}$$



8. From the graph of radius of curvature above, we see that the minimum value occurs at a cam angle of about 52 deg.

$$\rho_{\min} := \rho(52 \cdot \text{deg}, R_p)$$

$$\rho_{\min} = 25.562 \text{ mm}$$

Using a multiple of 3, the maximum roller follower radius is $R_f := \frac{\rho_{\min}}{3}$ $R_f = 8.5 \text{ mm}$

PROBLEM 8-29

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the pressure angle and radius of curvature for a cycloidal displacement cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20 deg. What is the minimum diameter roller follower needed to avoid undercutting with these data?

Enter:

Lift: $h := 20 \cdot \text{mm}$ Eccentricity $\varepsilon := 4 \cdot \text{mm}$
 Duration: $\beta := 60 \cdot \text{deg}$ Prime circle radius $R_p := 50 \cdot \text{mm}$

Solution: See Mathcad file P0829.

1. Cycloidal motion is defined in local coordinates by equations 8.12. They are:

$$s(\theta) := h \cdot \left(\frac{\theta}{\beta} - \frac{1}{2\pi} \cdot \sin \left(2\pi \cdot \frac{\theta}{\beta} \right) \right) \quad v(\theta) := \frac{h}{\beta} \cdot \left(1 - \cos \left(2\pi \cdot \frac{\theta}{\beta} \right) \right)$$

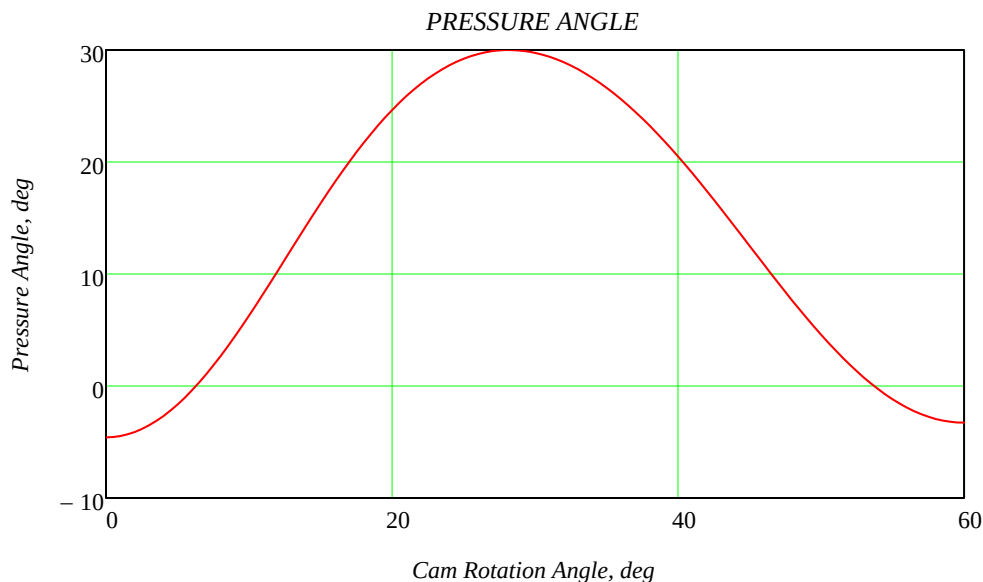
$$a(\theta) := 2\pi \cdot \frac{h}{\beta^2} \cdot \sin \left(2\pi \cdot \frac{\theta}{\beta} \right) \quad j(\theta) := 4\pi^2 \cdot \frac{h}{\beta^3} \cdot \cos \left(2\pi \cdot \frac{\theta}{\beta} \right)$$

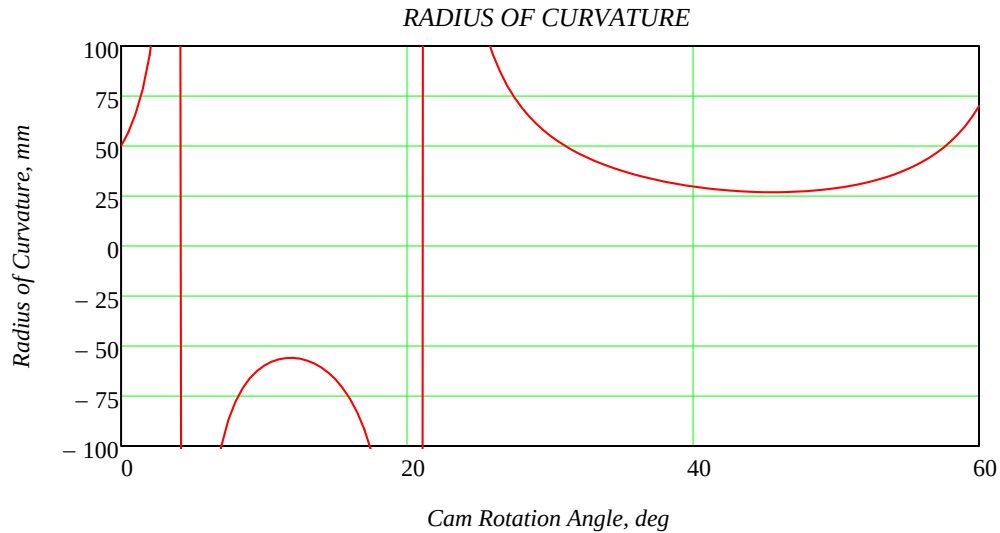
2. Using equations 8.31d and 8.33, write the pressure angle and radius of curvature functions.

$$\phi(\theta, R_p, \varepsilon) := \text{atan} \left(\frac{v(\theta) - \varepsilon}{s(\theta) + \sqrt{R_p^2 - \varepsilon^2}} \right)$$

$$\rho(\theta, R_p) := \frac{\left[(R_p + s(\theta))^2 + v(\theta)^2 \right]^{\frac{3}{2}}}{(R_p + s(\theta))^2 + 2 \cdot v(\theta)^2 - a(\theta) \cdot (R_p + s(\theta))}$$

3. Plot the pressure angle and radius of curvature functions over the lift interval: $\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. \beta$

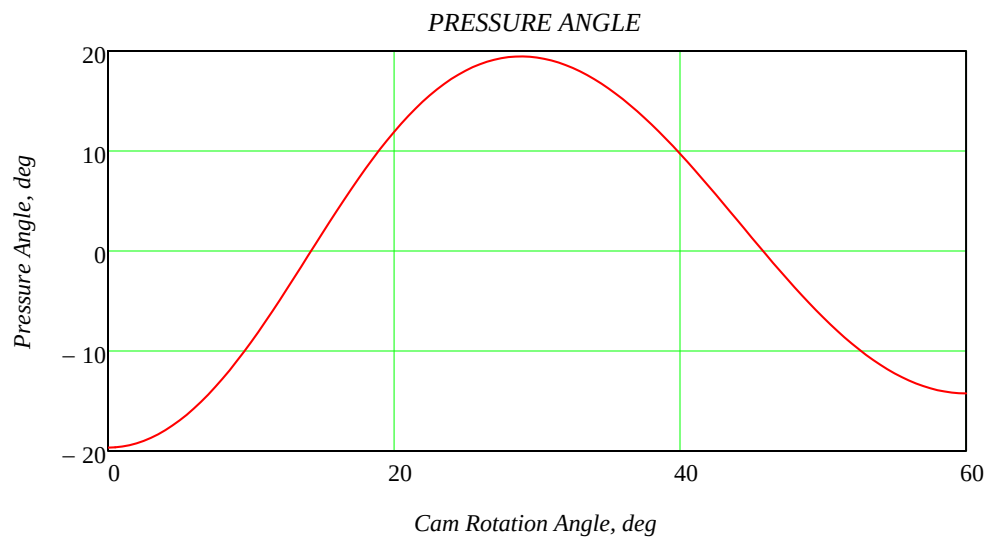




4. The graphs above show the pressure angle and radius of curvature for the values of R_p and ε entered on the first page. These values will be iterated below to obtain a balanced pressure angle whose absolute value is not greater than 20 deg.

$$R_p := 52 \cdot \text{mm}$$

$$\varepsilon := 17.5 \cdot \text{mm}$$



5. From the graph of radius of curvature above, we see that the minimum value occurs at a cam angle of about 45 deg.

$$\rho_{min} := \rho(45 \cdot \text{deg}, R_p)$$

$$\rho_{min} = 28.093 \text{ mm}$$

Using a multiple of 3, the maximum roller follower radius is $R_f := \frac{\rho_{min}}{3}$ $R_f = 9.4 \text{ mm}$

PROBLEM 8-30

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the pressure angle and radius of curvature for a 3-4-5 polynomial displacement cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20 deg. What is the minimum diameter roller follower needed to avoid undercutting with these data?

Enter:

Lift: $h := 20 \cdot \text{mm}$ Eccentricity $\varepsilon := 4 \cdot \text{mm}$
 Duration: $\beta := 60 \cdot \text{deg}$ Prime circle radius $R_p := 50 \cdot \text{mm}$

Solution: See Mathcad file P0830.

1. The 3-4-5 polynomial is defined in local coordinates by equations 8.24. They are:

$$s(\theta) := h \cdot \left[10 \cdot \left(\frac{\theta}{\beta} \right)^3 - 15 \cdot \left(\frac{\theta}{\beta} \right)^4 + 6 \cdot \left(\frac{\theta}{\beta} \right)^5 \right] \quad v(\theta) := \frac{h}{\beta} \cdot \left[30 \cdot \left(\frac{\theta}{\beta} \right)^2 - 60 \cdot \left(\frac{\theta}{\beta} \right)^3 + 30 \cdot \left(\frac{\theta}{\beta} \right)^4 \right]$$

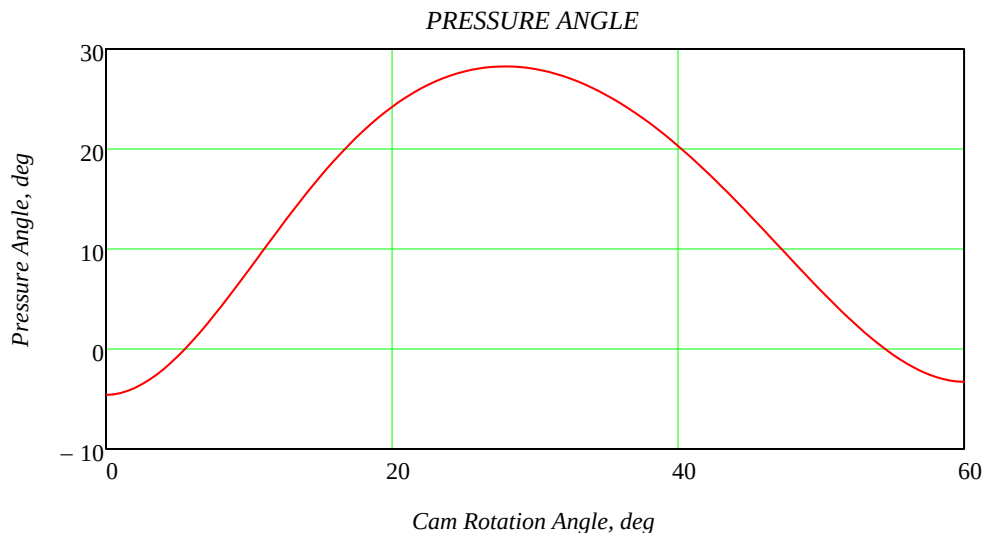
$$a(\theta) := \frac{h}{\beta^2} \cdot \left[60 \cdot \left(\frac{\theta}{\beta} \right) - 180 \cdot \left(\frac{\theta}{\beta} \right)^2 + 120 \cdot \left(\frac{\theta}{\beta} \right)^3 \right] \quad j(\theta) := \frac{h}{\beta^3} \cdot \left[60 - 360 \cdot \left(\frac{\theta}{\beta} \right) + 360 \cdot \left(\frac{\theta}{\beta} \right)^2 \right]$$

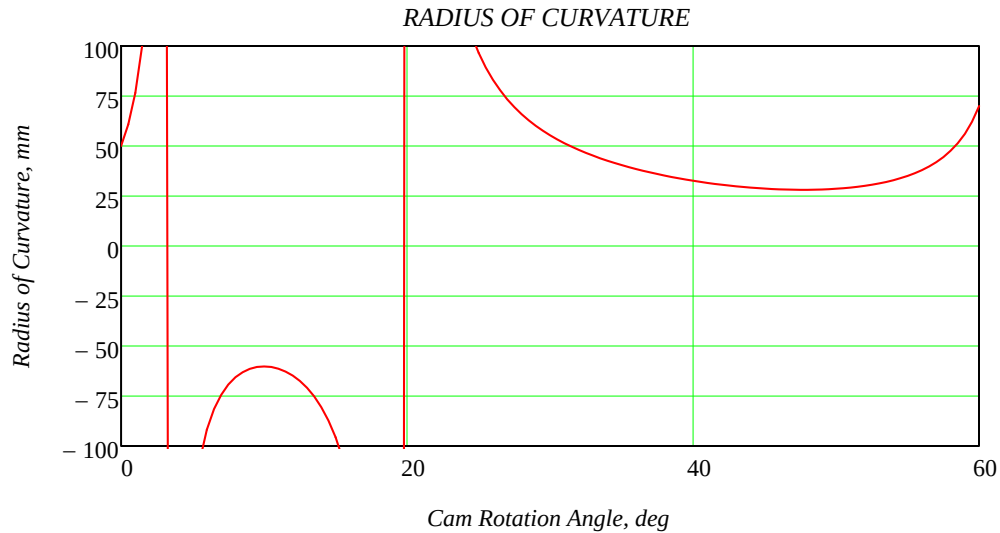
2. Using equations 8.31d and 8.33, write the pressure angle and radius of curvature functions.

$$\phi(\theta, R_p, \varepsilon) := \text{atan} \left(\frac{v(\theta) - \varepsilon}{s(\theta) + \sqrt{R_p^2 - \varepsilon^2}} \right)$$

$$\rho(\theta, R_p) := \frac{\left[(R_p + s(\theta))^2 + v(\theta)^2 \right]^{\frac{3}{2}}}{(R_p + s(\theta))^2 + 2 \cdot v(\theta)^2 - a(\theta) \cdot (R_p + s(\theta))}$$

3. Plot the pressure angle and radius of curvature functions over the lift interval: $\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. \beta$

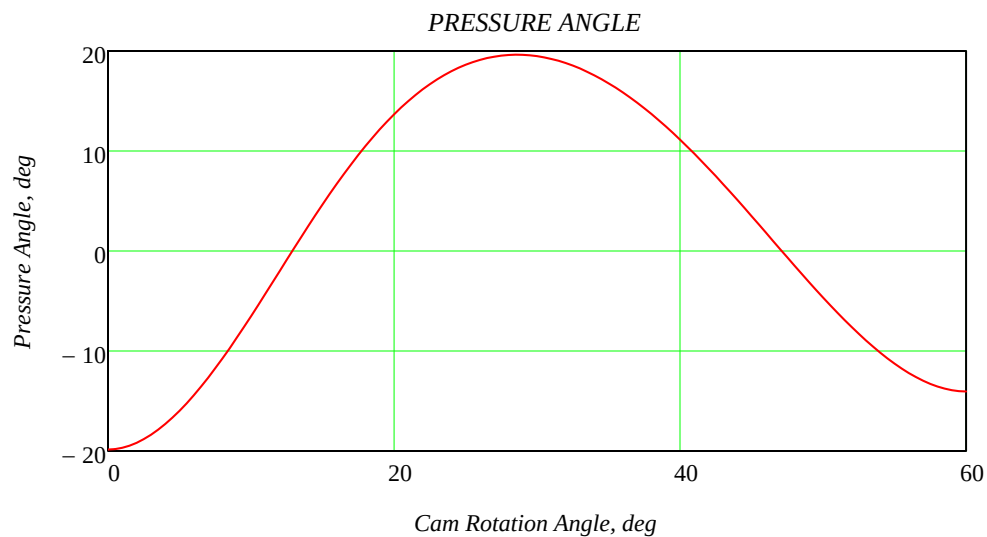




4. The graphs above show the pressure angle and radius of curvature for the values of R_p and ε entered on the first page. These values will be iterated below to obtain a balanced pressure angle whose absolute value is not greater than 20 deg.

$$R_p := 48 \cdot \text{mm}$$

$$\varepsilon := 16.3 \cdot \text{mm}$$



5. From the graph of radius of curvature above, we see that the minimum value occurs at a cam angle of about 48 deg.

$$\rho_{\min} := \rho(48 \cdot \text{deg}, R_p)$$

$$\rho_{\min} = 26.885 \text{ mm}$$

Using a multiple of 3, the maximum roller follower radius is $R_f := \frac{\rho_{\min}}{3}$ $R_f = 9.0 \text{ mm}$

PROBLEM 8-31

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the pressure angle and radius of curvature for a 4-5-6-7 polynomial displacement cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20 deg. What is the minimum diameter roller follower needed to avoid undercutting with these data?

Enter:

Lift: $h := 20 \cdot \text{mm}$ Eccentricity $\varepsilon := 4 \cdot \text{mm}$
 Duration: $\beta := 60 \cdot \text{deg}$ Prime circle radius $R_p := 50 \cdot \text{mm}$

Solution: See Mathcad file P0831.

1. The 4-5-6-7 polynomial is defined in local coordinates by equation 8.25. Differentiate it to get v , a , and j .

$$s(\theta) := h \cdot \left[35 \cdot \left(\frac{\theta}{\beta} \right)^4 - 84 \cdot \left(\frac{\theta}{\beta} \right)^5 + 70 \cdot \left(\frac{\theta}{\beta} \right)^6 - 20 \cdot \left(\frac{\theta}{\beta} \right)^7 \right]$$

$$v(\theta) := \frac{h}{\beta} \cdot \left[140 \cdot \left(\frac{\theta}{\beta} \right)^3 - 420 \cdot \left(\frac{\theta}{\beta} \right)^4 + 420 \cdot \left(\frac{\theta}{\beta} \right)^5 - 140 \cdot \left(\frac{\theta}{\beta} \right)^6 \right]$$

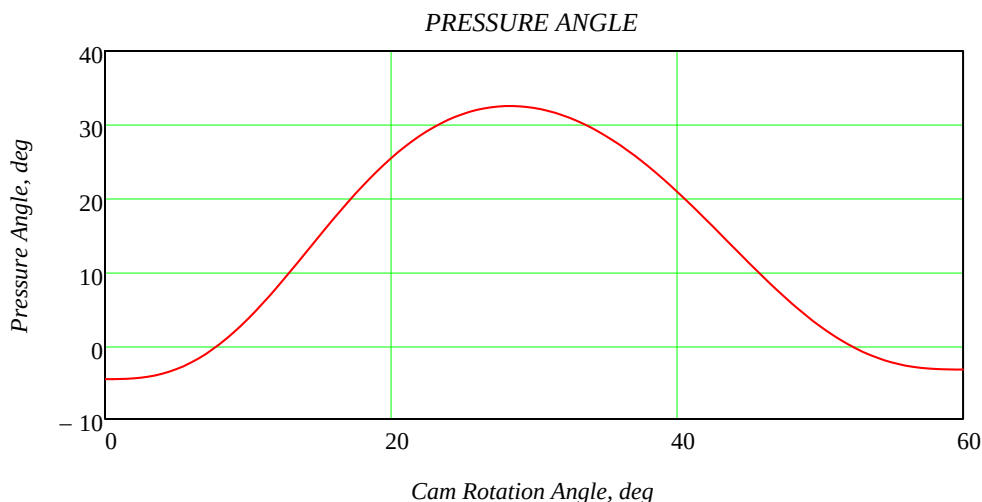
$$a(\theta) := \frac{h}{\beta^2} \cdot \left[420 \cdot \left(\frac{\theta}{\beta} \right)^2 - 1680 \cdot \left(\frac{\theta}{\beta} \right)^3 + 2100 \cdot \left(\frac{\theta}{\beta} \right)^4 - 840 \cdot \left(\frac{\theta}{\beta} \right)^5 \right]$$

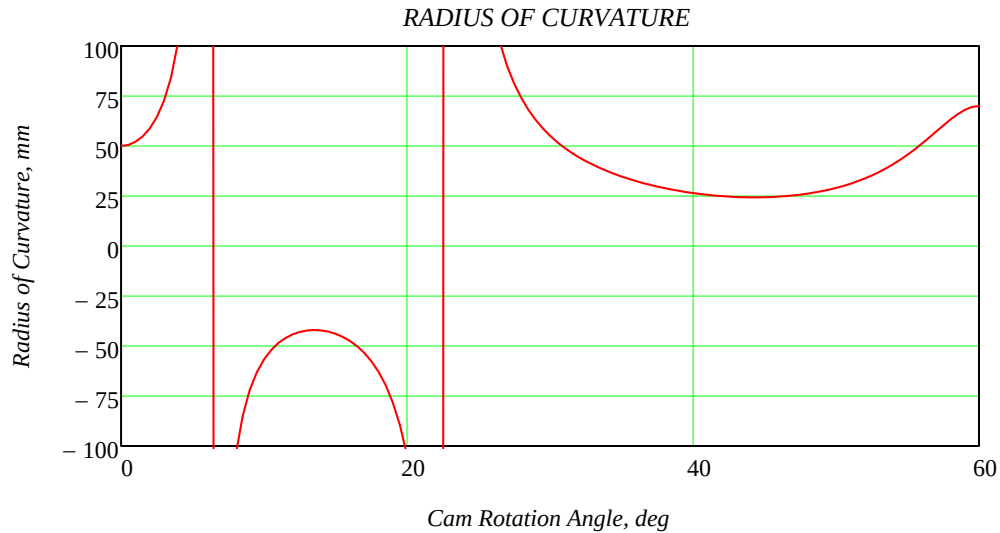
2. Using equations 8.31d and 8.33, write the pressure angle and radius of curvature functions.

$$\phi(\theta, R_p, \varepsilon) := \text{atan} \left(\frac{v(\theta) - \varepsilon}{s(\theta) + \sqrt{R_p^2 - \varepsilon^2}} \right)$$

$$\rho(\theta, R_p) := \frac{\left[(R_p + s(\theta))^2 + v(\theta)^2 \right]^{\frac{3}{2}}}{(R_p + s(\theta))^2 + 2 \cdot v(\theta)^2 - a(\theta) \cdot (R_p + s(\theta))}$$

3. Plot the pressure angle and radius of curvature functions over the lift interval: $\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. \beta$

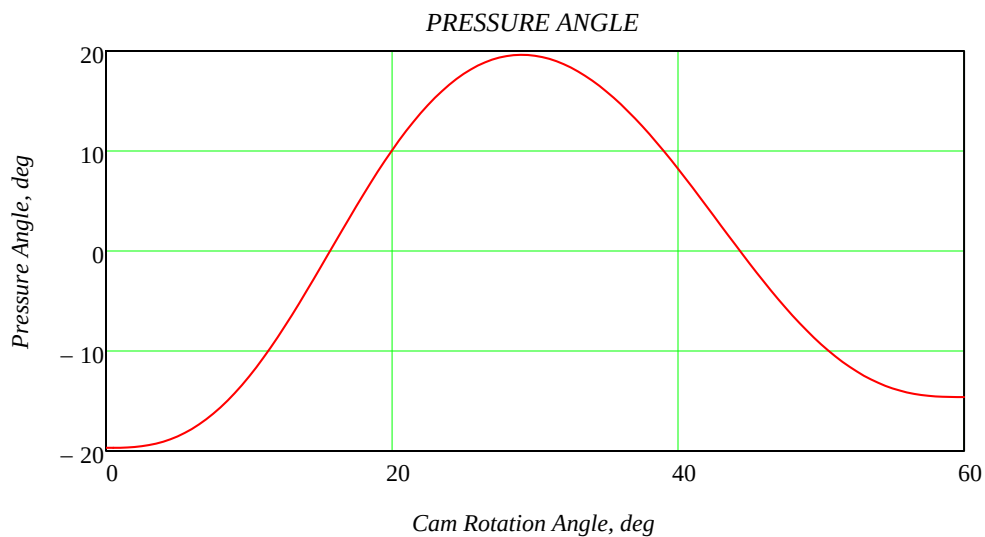




4. The graphs above show the pressure angle and radius of curvature for the values of R_p and ε entered on the first page. These values will be iterated below to obtain a balanced pressure angle whose absolute value is not greater than 20 deg.

$$R_p := 57 \cdot \text{mm}$$

$$\varepsilon := 19.2 \cdot \text{mm}$$



5. From the graph of radius of curvature above, we see that the minimum value occurs at a cam angle of about 45 deg.

$$\rho_{\min} := \rho(45 \cdot \text{deg}, R_p)$$

$$\rho_{\min} = 28.317 \text{ mm}$$

Using a multiple of 3, the maximum roller follower radius is $R_f := \frac{\rho_{\min}}{3}$ $R_f = 9.4 \text{ mm}$

PROBLEM 8-32

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the pressure angle and radius of curvature for a simple harmonic displacement cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20 deg. What is the minimum diameter roller follower needed to avoid undercutting with these data?

Enter:

Lift: $h := 20 \cdot \text{mm}$ Eccentricity $\varepsilon := 4 \cdot \text{mm}$
 Duration: $\beta := 60 \cdot \text{deg}$ Prime circle radius $R_p := 50 \cdot \text{mm}$

Solution: See Mathcad file P0832.

1. The simple harmonic motion (SHM) is defined in local coordinates by equations 8.6. They are:

$$s(\theta) := \frac{h}{2} \cdot \left(1 - \cos\left(\pi \cdot \frac{\theta}{\beta}\right) \right) \quad v(\theta) := \frac{\pi}{\beta} \cdot \frac{h}{2} \cdot \sin\left(\pi \cdot \frac{\theta}{\beta}\right)$$

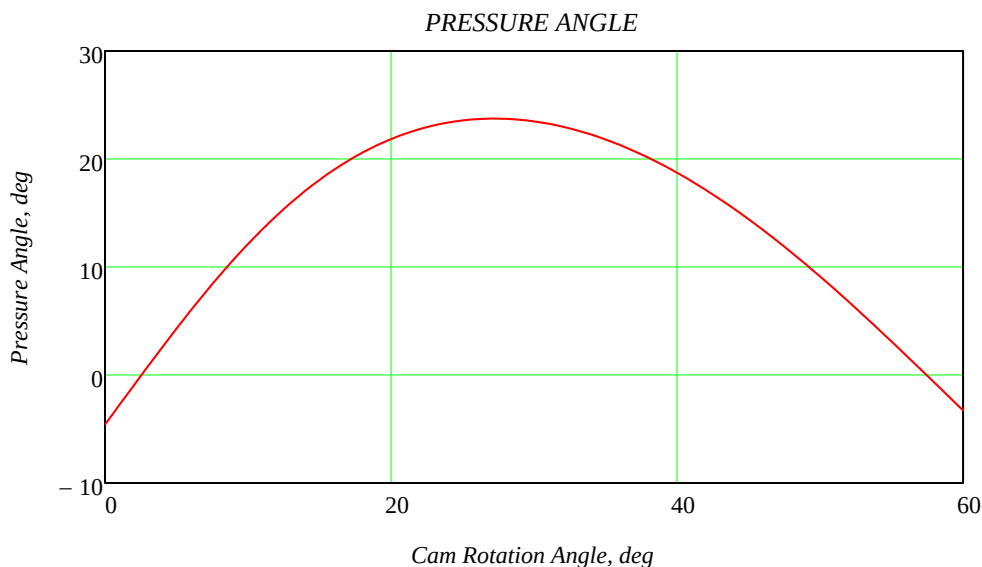
$$a(\theta) := \frac{\pi^2}{\beta^2} \cdot \frac{h}{2} \cdot \cos\left(\pi \cdot \frac{\theta}{\beta}\right) \quad j(\theta) := \frac{\pi^3}{\beta^3} \cdot \frac{h}{2} \cdot \sin\left(\pi \cdot \frac{\theta}{\beta}\right)$$

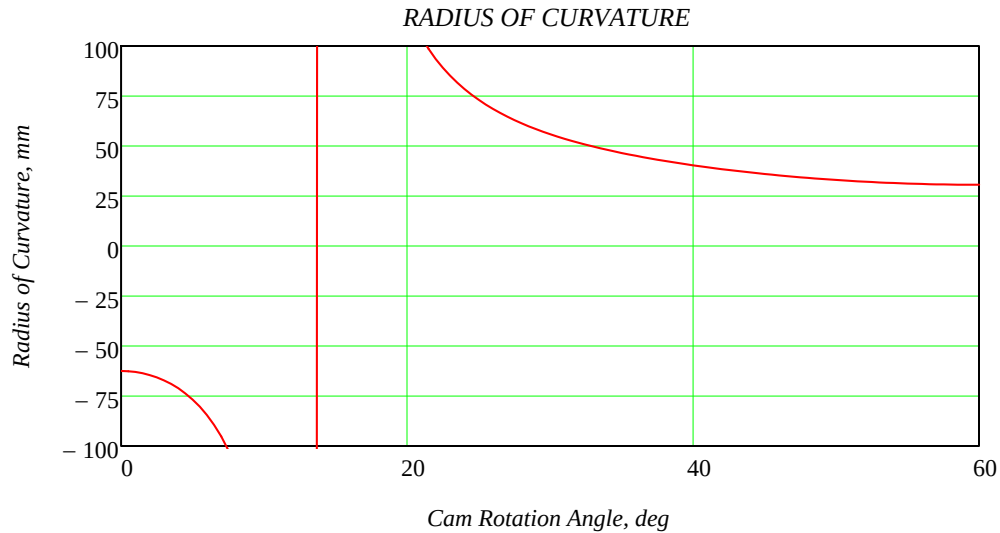
2. Using equations 8.31d and 8.33, write the pressure angle and radius of curvature functions.

$$\phi(\theta, R_p, \varepsilon) := \text{atan}\left(\frac{v(\theta) - \varepsilon}{s(\theta) + \sqrt{R_p^2 - \varepsilon^2}} \right)$$

$$\rho(\theta, R_p) := \frac{\left[(R_p + s(\theta))^2 + v(\theta)^2 \right]^{\frac{3}{2}}}{(R_p + s(\theta))^2 + 2 \cdot v(\theta)^2 - a(\theta) \cdot (R_p + s(\theta))}$$

3. Plot the pressure angle and radius of curvature functions over the lift interval: $\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. \beta$

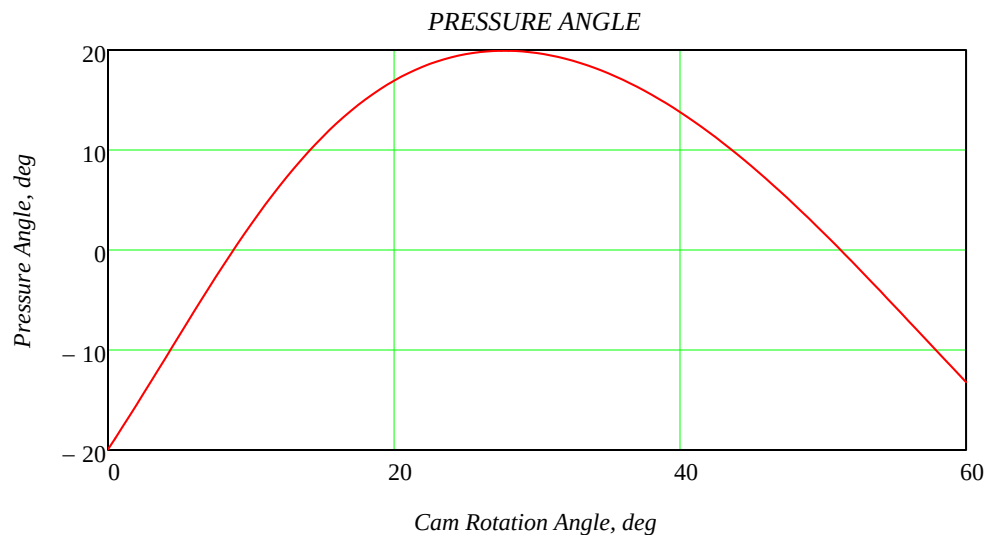




4. The graphs above show the pressure angle and radius of curvature for the values of R_p and ε entered on the first page. These values will be iterated below to obtain a balanced pressure angle whose absolute value is not greater than 20 deg.

$$R_p := 39 \cdot \text{mm}$$

$$\varepsilon := 13.3 \cdot \text{mm}$$



5. From the graph of radius of curvature above, we see that the minimum value occurs at a cam angle of about 60 deg.

$$\rho_{\min} := \rho(60 \cdot \text{deg}, R_p)$$

$$\rho_{\min} = 23.362 \text{ mm}$$

Using a multiple of 3, the maximum roller follower radius is $R_f := \frac{\rho_{\min}}{3}$ $R_f = 7.8 \text{ mm}$

PROBLEM 8-33

Statement: Derive equation 8.25 for the 4-5-6-7 polynomial.

Solution: See Mathcad file P0833.

1. There are eight boundary conditions. They are:

$$\begin{array}{lllll} \text{At } \theta = 0: & s = 0, & v = 0, & a = 0, & j = 0 \\ \text{At } \theta = \beta: & s = h, & v = 0, & a = 0, & j = 0 \end{array}$$

2. Write equation 8.23 in the form of equation c in Example 8-5, but with eight terms. Differentiate it repeatedly to get equations for v , a , and j that are similar to equations d and e . Write eight equations in the unknown coefficients C_0 through C_7 , applying the boundary conditions above. These eight equations are.

For $\theta = 0$: $s = 0, v = 0, a = 0, j = 0$

$$0 := C_0 \quad C_0 := 0$$

$$0 := C_1 \quad C_1 := 0$$

$$0 := 2 \cdot C_2 \quad C_2 := 0$$

$$0 := 6 \cdot C_3 \quad C_3 := 0$$

For $\theta = \beta$: $s = h, v = 0, a = 0, j = 0$

$$h := C_4 + C_5 + C_6 + C_7$$

$$0 := 4 \cdot C_4 + 5 \cdot C_5 + 6 \cdot C_6 + 7 \cdot C_7$$

$$0 := 12 \cdot C_4 + 20 \cdot C_5 + 30 \cdot C_6 + 42 \cdot C_7$$

$$0 := 24 \cdot C_4 + 60 \cdot C_5 + 120 \cdot C_6 + 210 \cdot C_7$$

3. Solve the last four equations for the four unknown coefficients C_4, C_5, C_6 , and C_7 . Set $h = 1$ then multiply all coefficients by h .

$$C := \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 5 & 6 & 7 \\ 12 & 20 & 30 & 42 \\ 24 & 60 & 120 & 210 \end{pmatrix} \quad H := \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} C_4 \\ C_5 \\ C_6 \\ C_7 \end{pmatrix} := C^{-1} \cdot H$$

$$C_4 = 35$$

$$C_5 = -84$$

$$C_6 = 70$$

$$C_7 = -20$$

PROBLEM 8-34

Statement: Derive an expression for the pressure angle of a barrel cam with zero eccentricity.

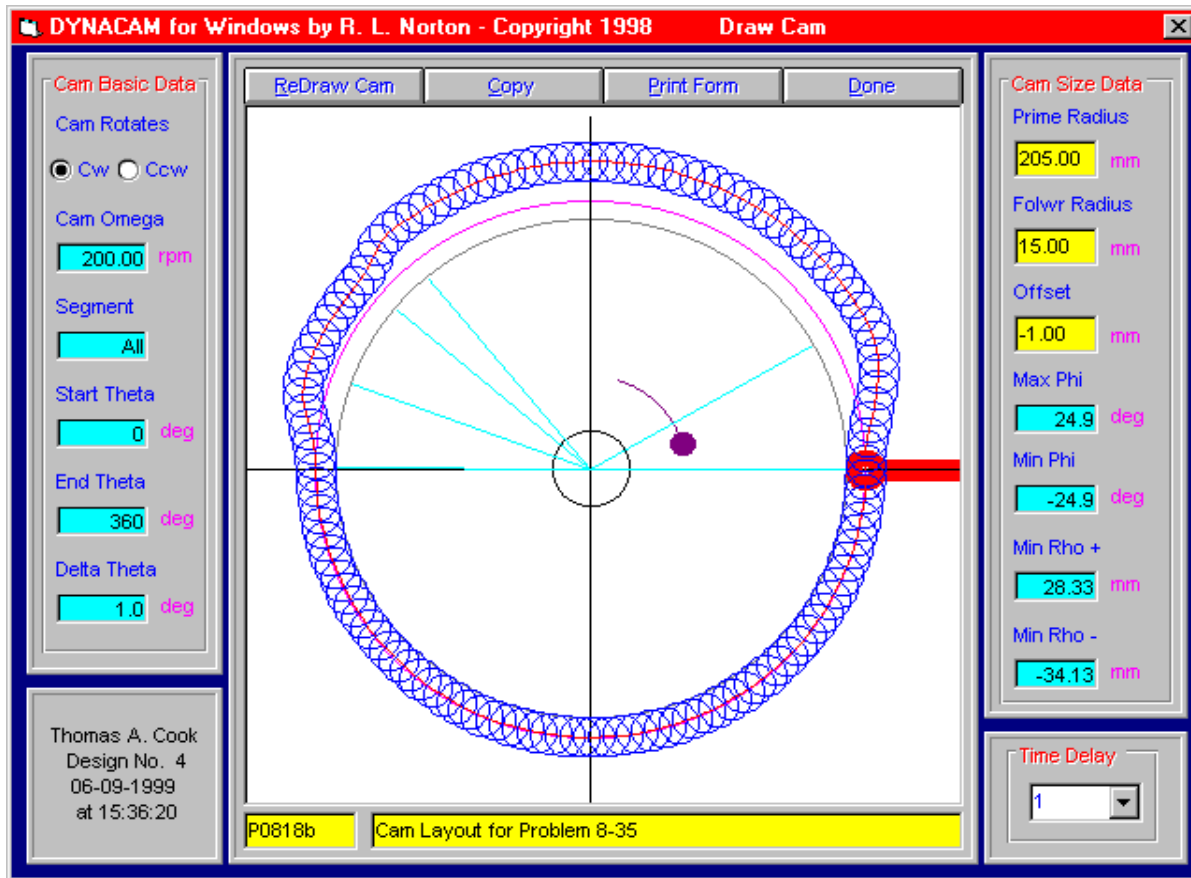
Solution: See Mathcad file P0834.

1. A barrel cam is shown in Figure 8-4. The motion of the follower is parallel to the axis of rotation of the cam. The transmission axis is the common normal between the follower and the groove in which it travels. The common normal will go through and be perpendicular to the center of rotation of the follower and will be normal to the wall of the groove at the point of contact. Let
 s = axial displacement of the follower from some reference point, length units
 v = velocity of the follower, length/rad units
 r = mean radius of the groove, length units

then, for an infinitesimal rotation of the cam, the follower will advance a distance ds while the distance along the groove will have advanced an amount $rd\theta$. These two quantities form the legs of a right triangle whose hypotenuse is perpendicular to the common normal. Because of this relationship, we have

$$\tan(\phi) = \frac{1}{r} \cdot \left(\frac{ds}{d\theta} \right) = \frac{v}{r} \quad \phi := \operatorname{atan}\left(\frac{v}{r}\right) \blacksquare$$

2. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 25 deg or less by increasing the prime circle radius. The resulting cam is shown below.



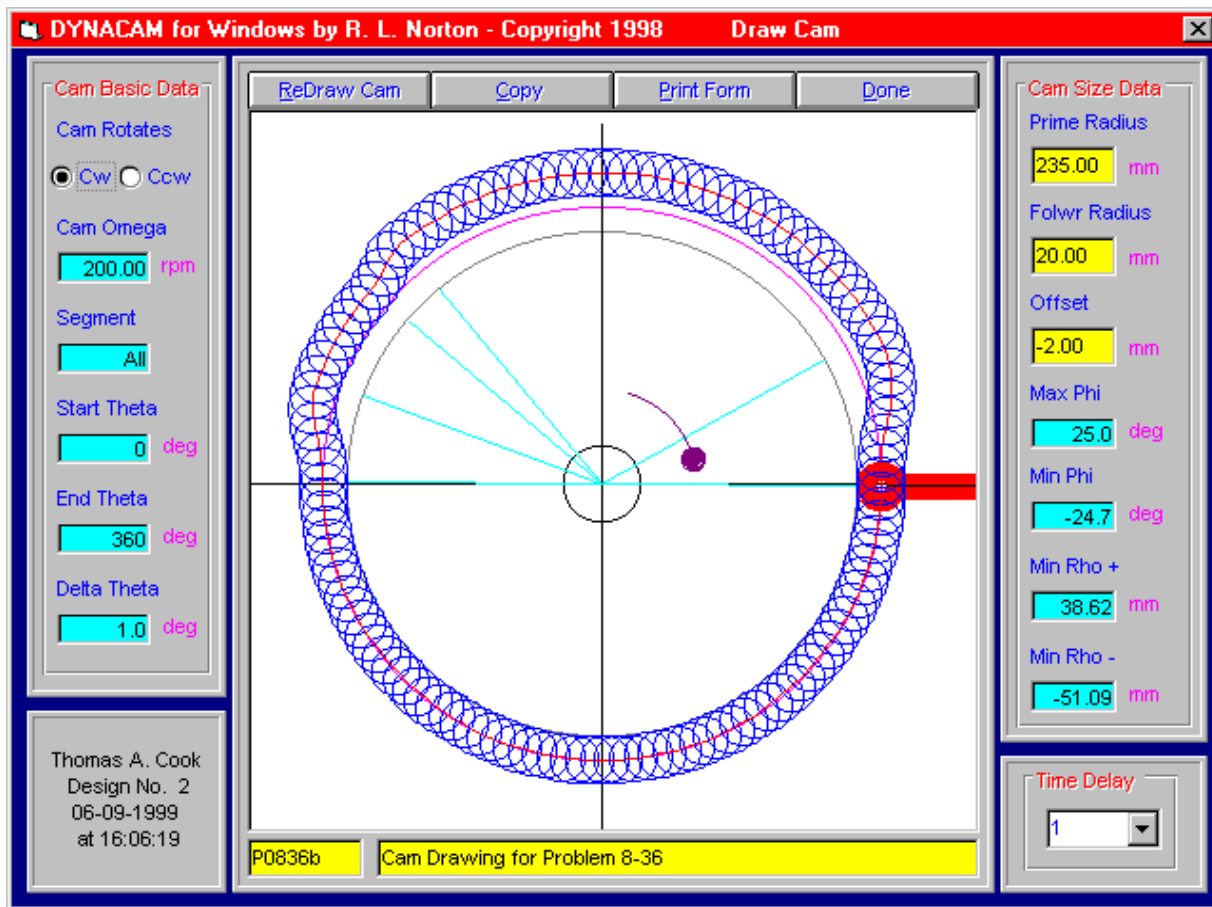
3. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

Prime circle radius $R_p := 205 \cdot \text{mm}$

Roller follower radius $R_f := 15 \cdot \text{mm}$

Follower eccentricity $\varepsilon := -1.00 \cdot \text{mm}$

2. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 25 deg or less by increasing the prime circle radius. The resulting cam is shown below.



3. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

Prime circle radius $R_p := 235 \cdot \text{mm}$

Roller follower radius $R_f := 20 \cdot \text{mm}$

Follower eccentricity $\varepsilon := -2.00 \cdot \text{mm}$

PROBLEM 8-37

Statement: Design a radial plate cam to move a translating roller follower through 30 mm in 30 deg, dwell for 100 deg, fall 10 mm in 10 deg, dwell for 20 deg, fall 20 mm in 20 deg, and dwell for the remainder. Camshaft $\omega = 200$ rpm. Minimize the follower's peak jerk and determine the minimum prime circle radius that will give a maximum 25-deg pressure angle. Determine the minimum radii of curvature on the pitch curve.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:	RISE/FALL	DWELL	FALL	DWELL
	$\beta_1 := 30 \cdot deg$	$\beta_2 := 100 \cdot deg$	$\beta_3 := 10 \cdot deg$	$\beta_4 := 20 \cdot deg$
	$h_1 := 30 \cdot mm$	$h_2 := 0.0 \cdot in$	$h_3 := 10 \cdot mm$	$h_4 := 0.0 \cdot in$
	$\beta_5 := 20 \cdot deg$	$\beta_6 := 180 \cdot deg$		
	$h_5 := 20 \cdot mm$	$h_6 := 0.0 \cdot in$		
	Shaft speed	$\omega := 200 \cdot rpm$		

Solution: See Mathcad file P0837.

1. From Table 8-3, the motion program with lowest jerk that does not have infinite jerk is the cycloidal displacement. Use the cycloidal for segments 1, 3, and 5. Segments 2, 4, and 6 are dwells. Enter the above data into program DYNACAM. The input screen is shown below.

DYNACAM for Windows by R. L. Norton - Copyright 1998 Input Screen

Cam Data

Cam Omega (rpm)

No. Segmts

Delta Theta (deg)

Follower

☒ Trans
☐ Oscit

Segment Data

Seg	Angles			Cam Contour		Position (mm)	
	Beta	Start	End	Motion	Program	Start	End
1	30	0	30	Rise	CF - Cycloid Full	0	30
2	100	30	130	Dwell	DW - Dwell	30	30
3	10	130	140	Fall	CF - Cycloid Full	30	20
4	20	140	160	Dwell	DW - Dwell	20	20
5	20	160	180	Fall	CF - Cycloid Full	20	0
6	180	180	360	Dwell	DW - Dwell	0	0
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

Calc All

Clear Segs

Clear All

Copy

Print Form

Done

Cancel

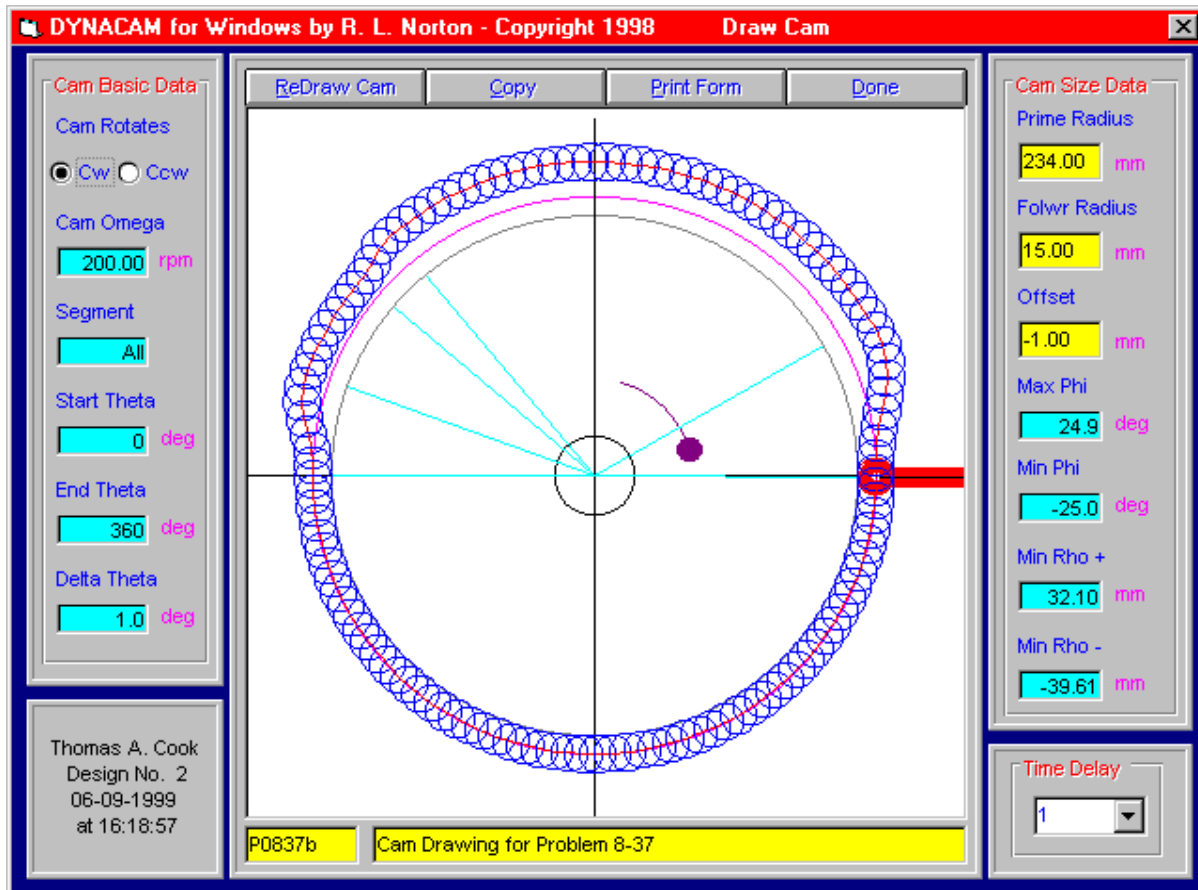
Examples

Calc

Plot

Print

2. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 25 deg or less by increasing the prime circle radius. The resulting cam is shown below.



3. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

Prime circle radius $R_p := 234 \cdot \text{mm}$

Roller follower radius $R_f := 15 \cdot \text{mm}$

Follower eccentricity $\varepsilon := -1.00 \cdot \text{mm}$

PROBLEM 8-38

Statement: Design a radial plate cam to lift a translating roller follower through 10 mm in 65 deg, return to 0 in 65 deg, and dwell for the remainder. Camshaft $\omega = 3500$ rpm. Minimize the cam size while not exceeding a 25-deg pressure angle. What size roller follower is needed?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

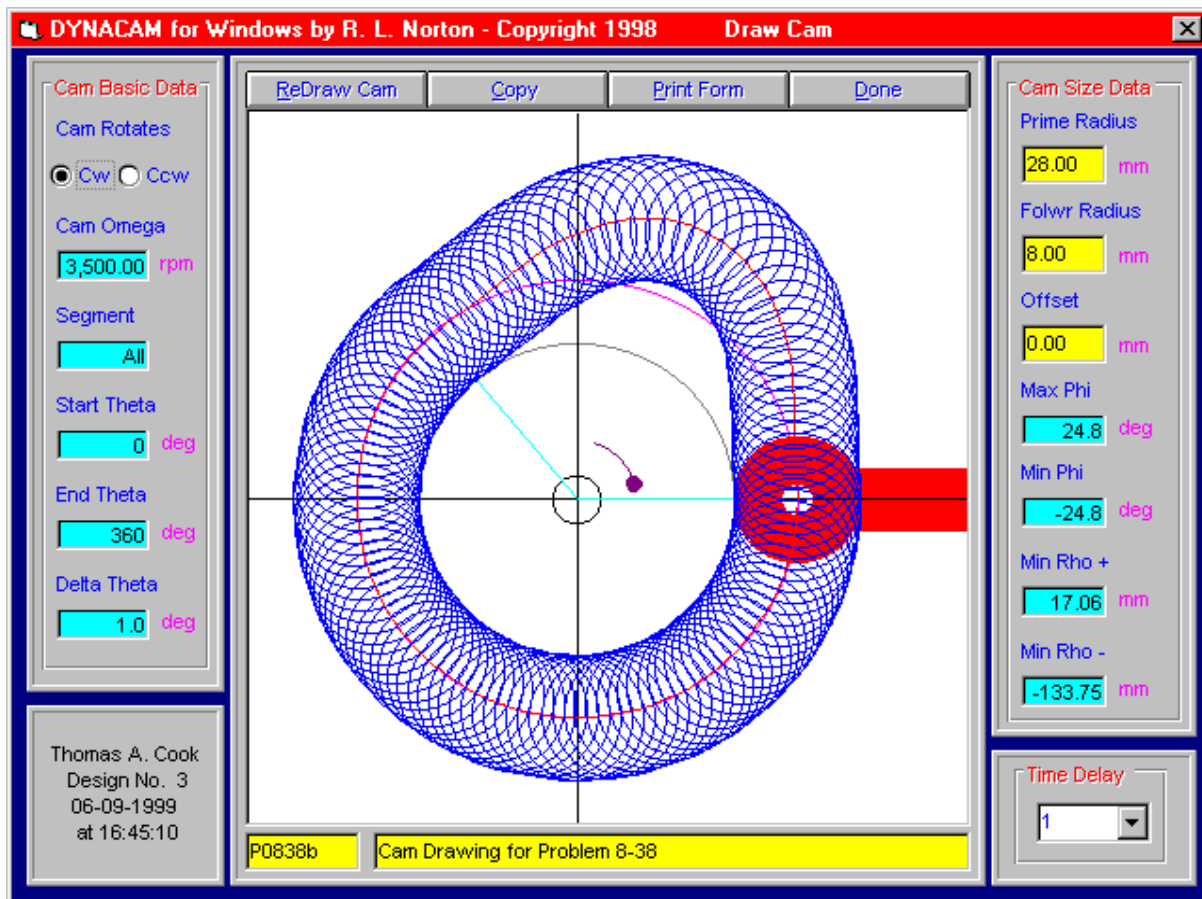
RISE/FALL	DWELL
$\beta_1 := 130 \cdot deg$	$\beta_2 := 230 \cdot deg$
$h_1 := 10 \cdot mm$	$h_2 := 0 \cdot mm$
Shaft speed	$\omega := 3500 \cdot rpm$

Solution: See Mathcad file P0838.

1. Use a two-segment polynomial as described in Example 8-8. This will result in a 6-deg polynomial requiring 7 boundary conditions. Enter the above data into program DYNACAM. The input boundary conditions screen for the polynomial segment is shown below.

Boundary Cond.	0	65	130	Intermediate 1	Intermediate 2
S	0	10	0		
V	0				0
A	0				0
J					

2. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 25 deg or less by increasing the prime circle radius. The resulting cam is shown below.



3. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

Prime circle radius $R_p := 28 \cdot \text{mm}$

Roller follower radius $R_f := 8.0 \cdot \text{mm}$

Follower eccentricity $\varepsilon := 0.0 \cdot \text{mm}$

PROBLEM 8-39

Statement: Design a cam-driven quick-return mechanism for a 3:1 time ratio. The translating roller follower should move forward and back 50 mm and dwell in the back position for 80 deg. It should take one-third the time to return as to move forward. Camshaft $\omega = 100$ rpm. Minimize the package size while maintaining a 25-deg maximum pressure angle. Draw a sketch of your design and provide s , v , a , ϕ , and ρ diagrams.

Units: $\text{rpm} := 2 \cdot \pi \cdot \text{rad} \cdot \text{min}^{-1}$

Given:

Time ratio	$t_r := 3$	Dwell interval	$\beta_2 := 80 \cdot \text{deg}$
Lift	$L := 50 \cdot \text{mm}$	Shaft speed	$\omega := 100 \cdot \text{rpm}$

Solution: See Mathcad file P0839.

1. Calculate the rise/fall interval widths.

First segment interval $\beta_1 := 360 \cdot \text{deg} - \beta_2$ $\beta_1 = 280.000 \text{ deg}$

Rise subinterval $\beta_r := \beta_1 \cdot \frac{t_r}{t_r + 1}$ $\beta_r = 210.000 \text{ deg}$

2. Use a three-segment polynomial. Enter the above data into program DYNACAM. The input screen is shown below.

DYNACAM for Windows by R. L. Norton - Copyright 1998 Input Screen

Cam Data

Cam Omega (rpm): 100

No. Segmts: 3

Delta Theta (deg): 1.0

Follower: ☒ Trans ☐ Oscit

Segment Data

Seg	Angles			Cam Contour		Position (mm)	
	Beta	Start	End	Motion	Program	Start	End
1	210	0	210	Poly	PO - Polynomial	0	50
2	70	210	280	Poly	PO - Polynomial	50	0
3	80	280	360	Dwell	Dwell - Dwell	0	0
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

Examples

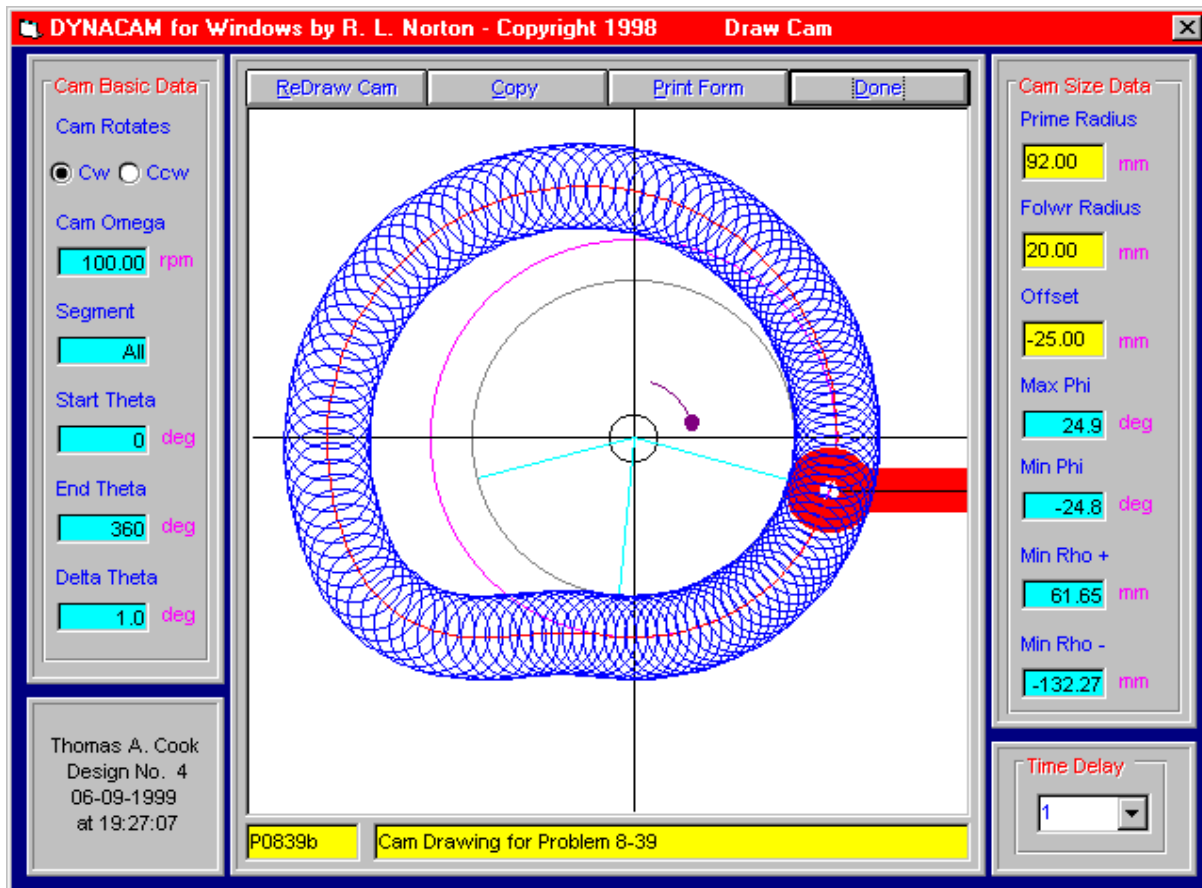
Calc Plot Print

Calc Plot Print

Calc Plot Print

Calc All Clear Segs Clear All Copy Print Form Done Cancel

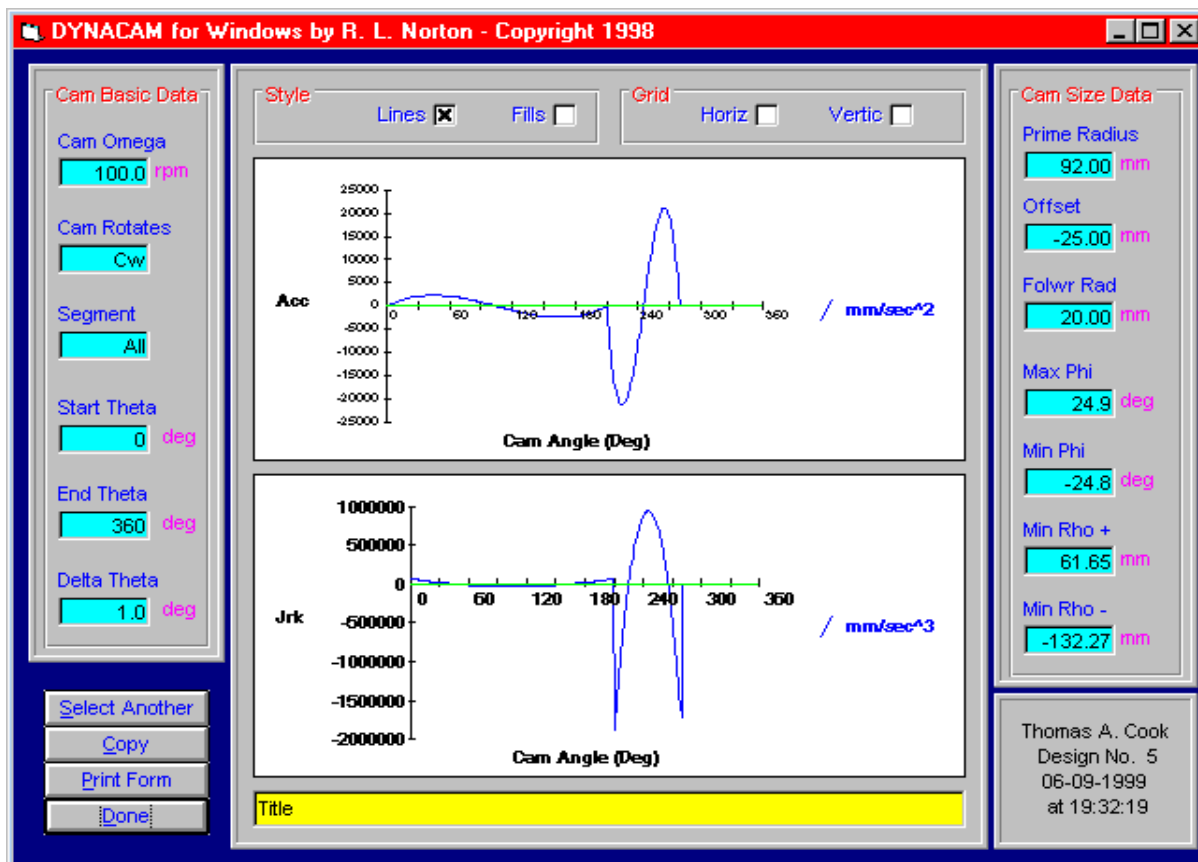
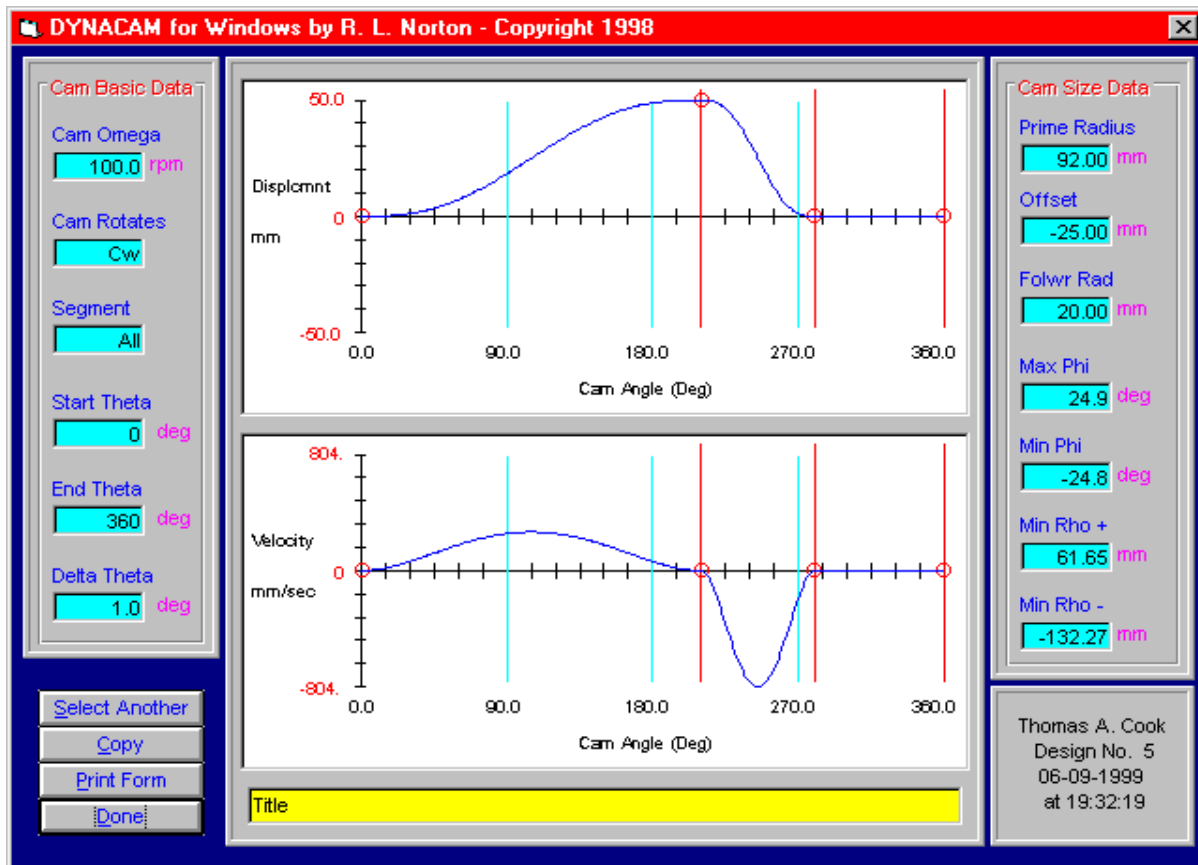
3. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 25 deg or less by increasing the prime circle radius. The resulting cam is shown below.

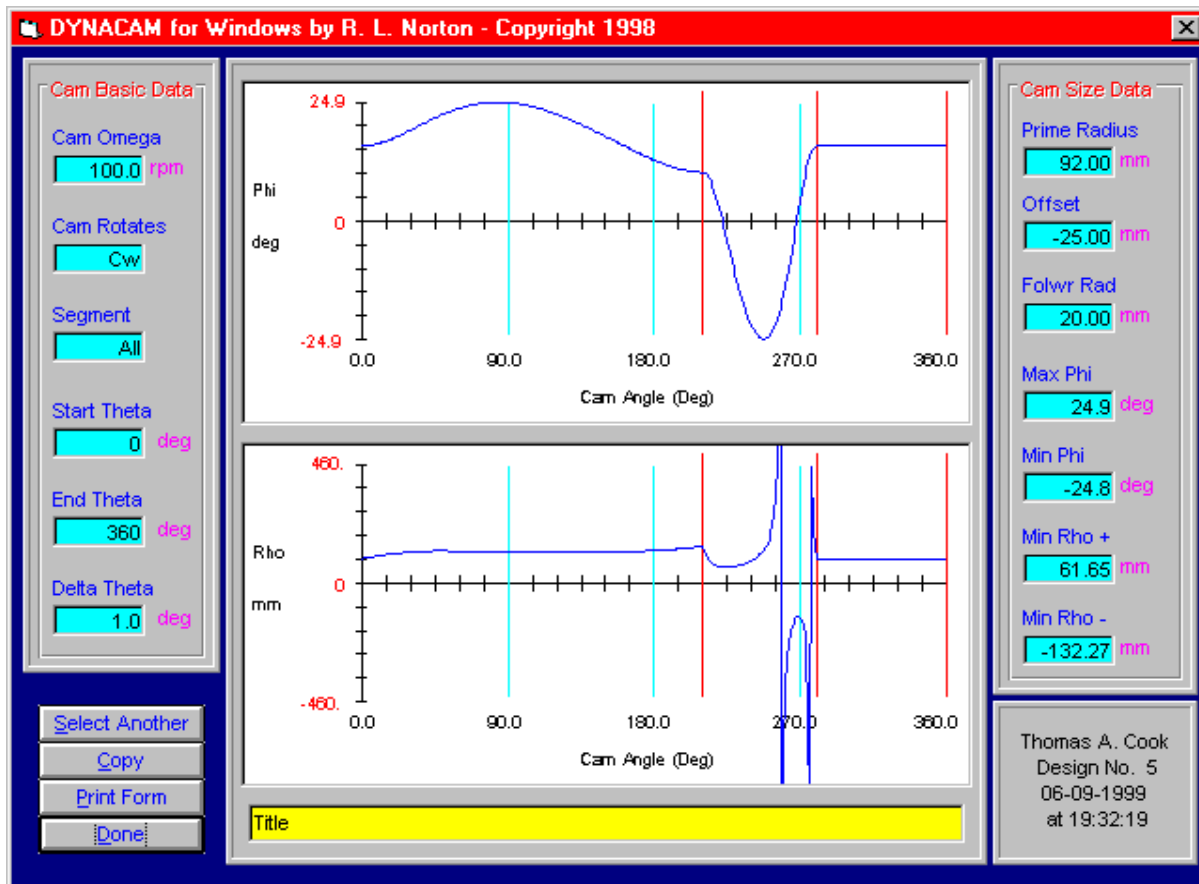


4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

$$\begin{aligned} \text{Prime circle radius} \quad R_p &:= 92 \cdot \text{mm} \\ \text{Roller follower radius} \quad R_f &:= 20.0 \cdot \text{mm} \\ \text{Follower eccentricity} \quad \varepsilon &:= -25.0 \cdot \text{mm} \end{aligned}$$

5. Graphs of s , v , a , j , ϕ , and ρ are shown on the following pages.





PROBLEM 8-40

Statement: Design a cam-follower system to drive a linear translating piston at a constant velocity for 200 deg through a stroke of 100 mm at 60 rpm. Minimize the package size while maintaining a 25-deg maximum pressure angle. Draw a sketch of your design and provide $svaj$, ϕ , and ρ diagrams.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Constant velocity interval width and stroke $\beta_1 := 200 \cdot deg$ $h_1 := 100 \cdot mm$
Cam rotation speed $\omega := 60 \cdot rpm$

Assumptions: A roller follower can be attached to the translating piston to interface with the cam.

Solution: See Mathcad file P0840.

1. Calculate the cycle time and time for the cam to rotate through the CV segment.

$$t_{cycle} := \frac{2 \cdot \pi \cdot rad}{\omega} \quad t_{cycle} = 1.000 \text{ s}$$

$$t_1 := \frac{\beta_1}{360 \cdot deg} \cdot t_{cycle} \quad t_1 = 0.556 \text{ s}$$

2. Calculate the required constant velocity.

$$V_{cv} := \frac{h_1}{t_1} \quad V_{cv} = 180.000 \frac{mm}{sec}$$

3. Use a two-segment cam with both segments polynomials. Let the first segment be a 1-deg polynomial and the second be a 7-deg (8 boundary conditions). There are two design choices to be made concerning the cam motion. They are:

Lift at end of second segment (and beginning of first) $L_2 := 10 \cdot mm$

Subinterval of second segment at which $s = 0$ $\beta_f := 120 \cdot deg$

4. Enter the above data into program DYNACAM. The segment segment boundary condition input screen is shown on the next page followed by a cam drawing for the design choices made.
5. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 25 deg or less by increasing the prime circle radius. The resulting cam is shown below.
6. The minimum and maximum pressure angles and radius of curvature are shown in the cam drawing. The design has the following dimensions:

Prime circle radius $R_p := 122 \cdot mm$

Roller follower radius $R_f := 20 \cdot mm$

Follower eccentricity $\epsilon := -30.6 \cdot mm$

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Input Boundary Conditions**

Cam Data

Segment Number:

Number of Conditions Requested:

Number of Conditions Selected:

Clear Copy Print Form Done Cancel

	Global Theta (deg)	Local Theta (deg)	Boundary Cond.
S	200	0	110
V	320	120	0
A	360	160	180
J			0

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Draw Cam**

ReDraw Cam Copy Print Form Done

Cam Basic Data

Cam Rotates: ☒ Cw ☐ Ccw

Cam Omega: rpm

Segment:

Start Theta: deg

End Theta: deg

Delta Theta: deg

Thomas A. Cook
Design No. 4
06-10-1999
at 13:01:24
File: P0840

Cam Size Data

Prime Radius: mm

Folwr Radius: mm

Offset: mm

Max Phi: deg

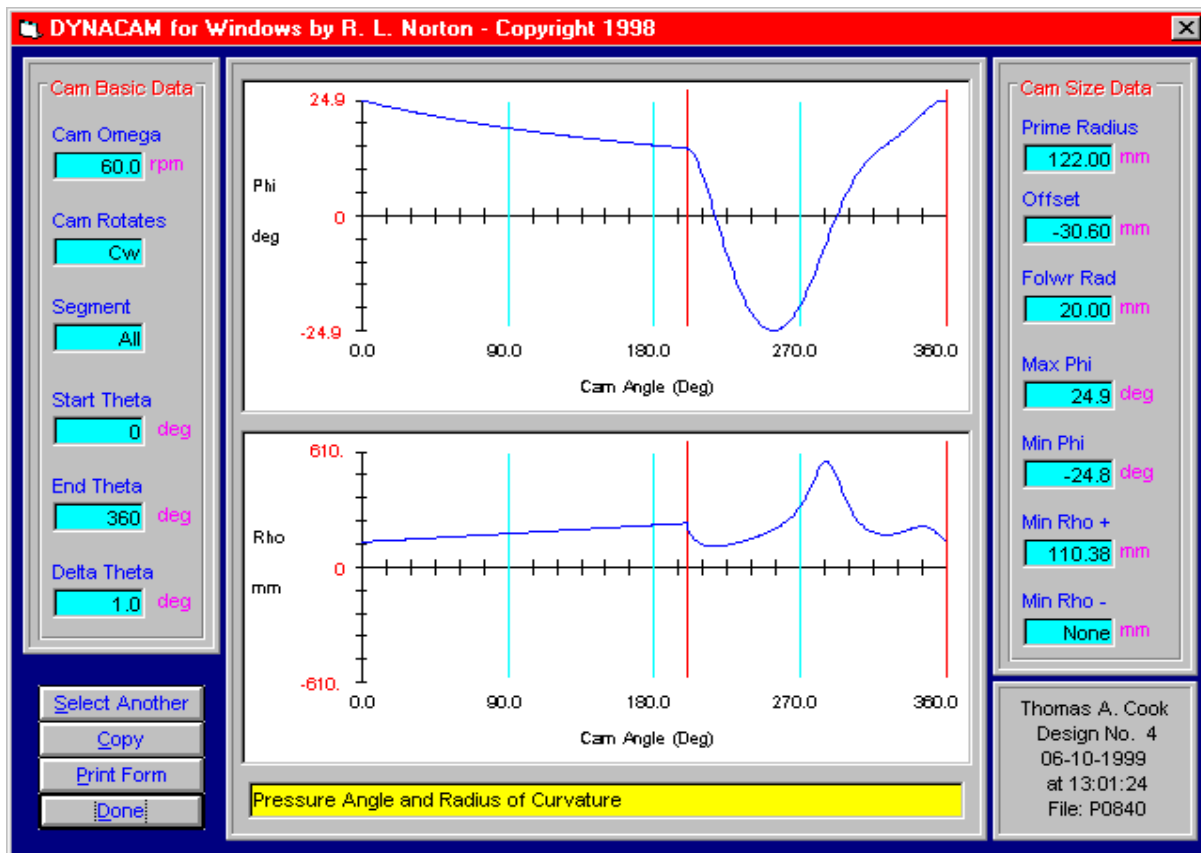
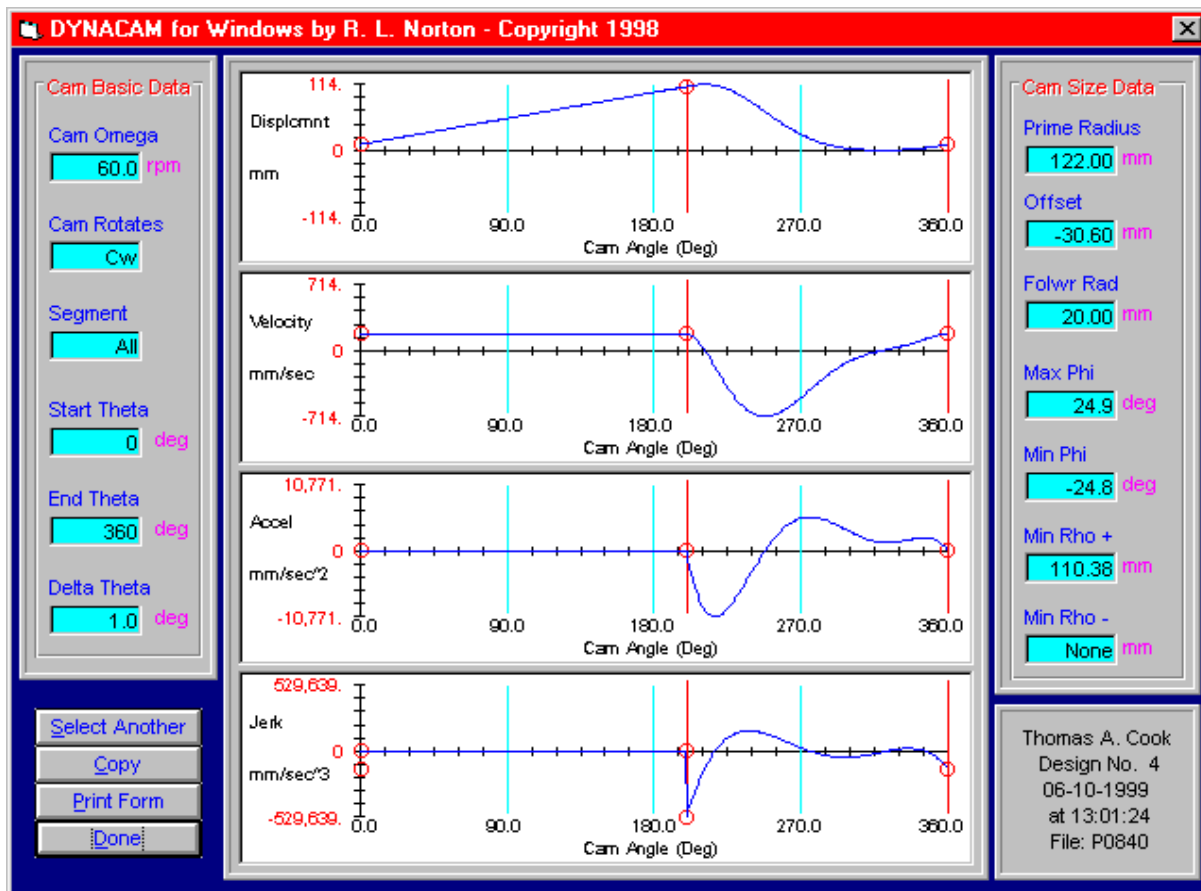
Min Phi: deg

Min Rho +: mm

Min Rho -: mm

Time Delay

P0840b Cam Drawing for Problem 8-40



PROBLEM 8-41

Statement: Design a cam-follower system to rise 20 mm in 80 deg, fall 10 mm in 100 deg, dwell at 10 mm for 100 deg, fall 10 mm in 50 deg, and dwell at 0 for 30 deg. The total cycle must take 4 sec. Avoid unnecessary returns to zero acceleration. Minimize the package size and maximize the roller follower diameter while maintaining a 25 deg maximum pressure angle. Draw a sketch of your design and provide s , v , a , j , ϕ , and ρ diagrams.

Given:

RISE	FALL	DWELL	FALL
$\beta_1 := 80 \cdot \text{deg}$	$\beta_2 := 100 \cdot \text{deg}$	$\beta_3 := 100 \cdot \text{deg}$	$\beta_4 := 50 \cdot \text{deg}$
$h_1 := 20 \cdot \text{mm}$	$h_2 := 10 \cdot \text{mm}$	$h_3 := 10 \cdot \text{mm}$	$h_4 := 0 \cdot \text{mm}$
Cycle time: $\tau := 4 \cdot \text{sec}$			

Solution: See Mathcad file P0841.

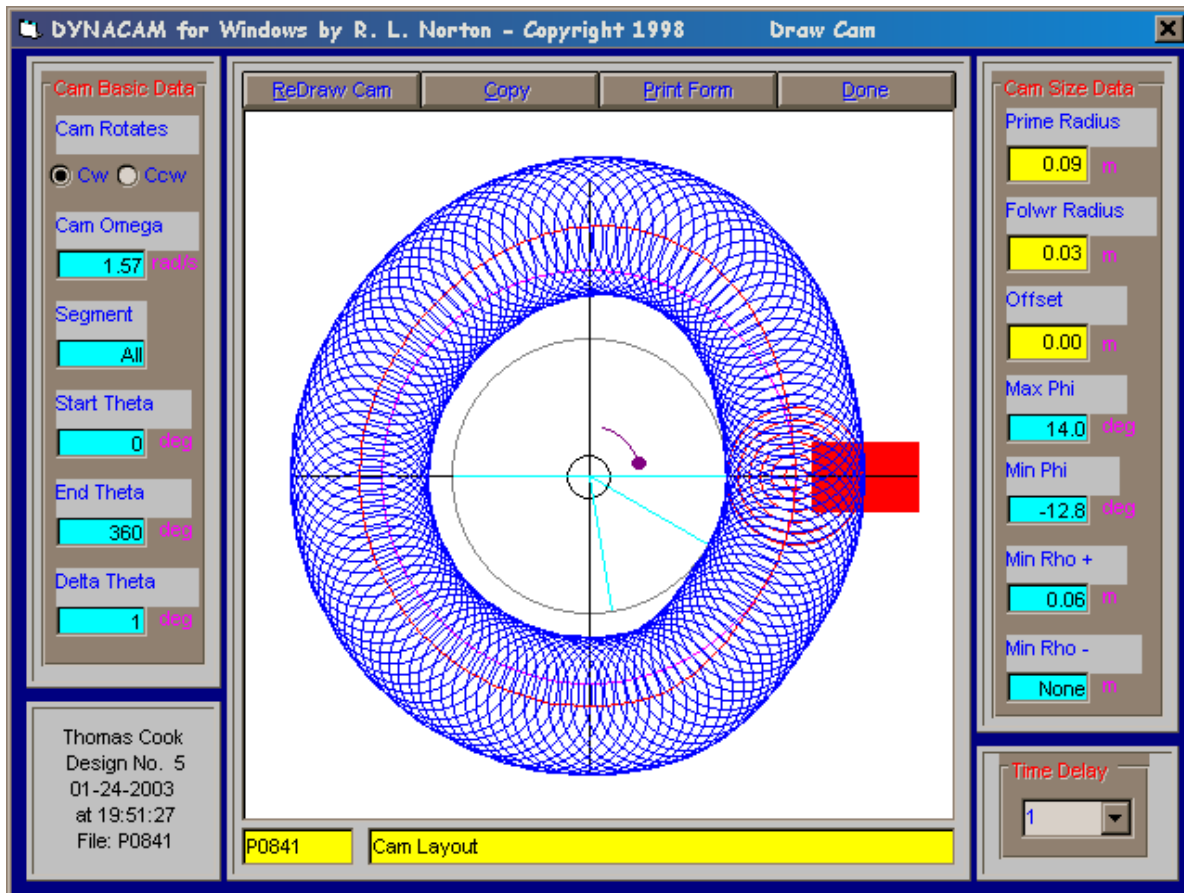
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.571 \frac{\text{rad}}{\text{sec}}$$

2. Use a four-segment for the rise, fall, dwell, fall, and dwell. Enter the above data into program DYNACAM. The input screen is shown below.

Cam Data		Segment Data								Examples		
		Angles			Cam Contour		Position (mm)					
Seg	Beta	Start	End	Motion	Program	Start	End					
1	180	0	180	Poly	PO - Polynomial	0	0.010			Calc	Plot	Print
2	100	180	280	Poly	PO - Polynomial	0.010	0.010			Calc	Plot	Print
3	50	280	330	Poly	PO - Polynomial	0.010	0			Calc	Plot	Print
4	30	330	360	Poly	PO - Polynomial	0	0			Calc	Plot	Print
5												
6												
7												
8												
9												
10												
11												
12												
13												
14												
15												
16												

3. The cam was sized iteratively to reduce package size while keeping the follower radius large, the pressure angle low, and the follower radius at least two times the radius of curvature. The resulting cam is shown below.



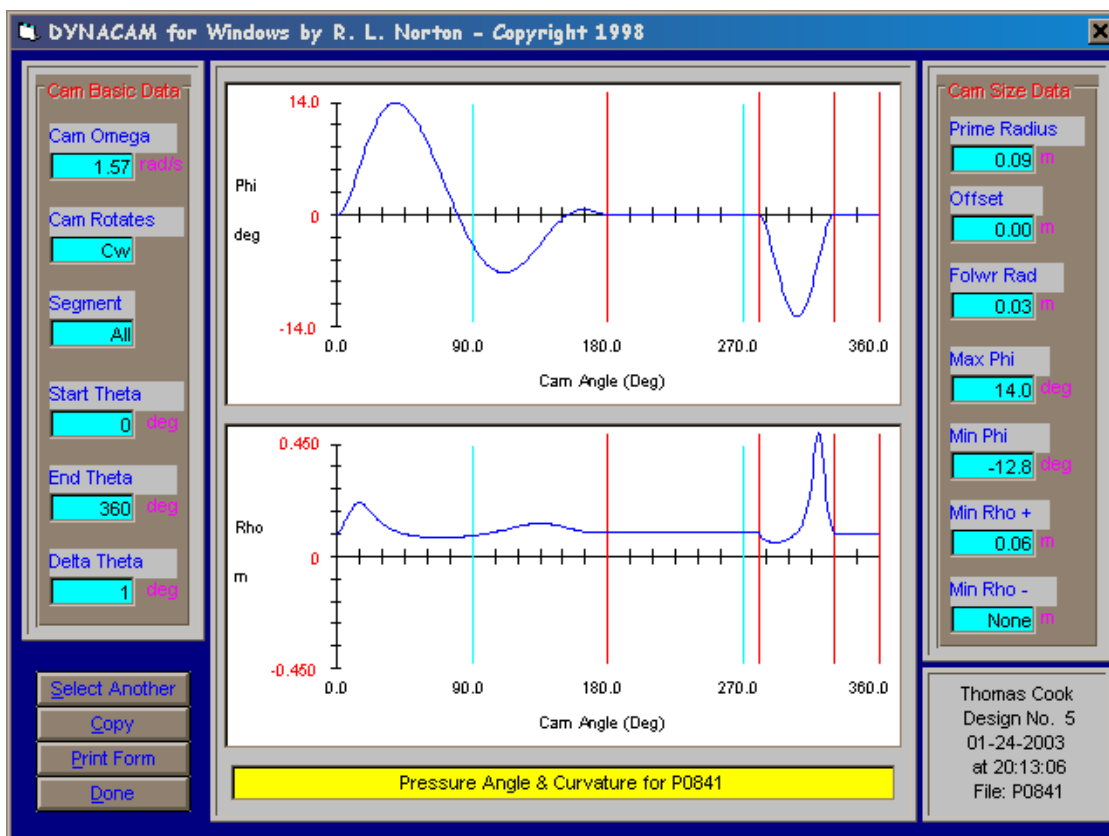
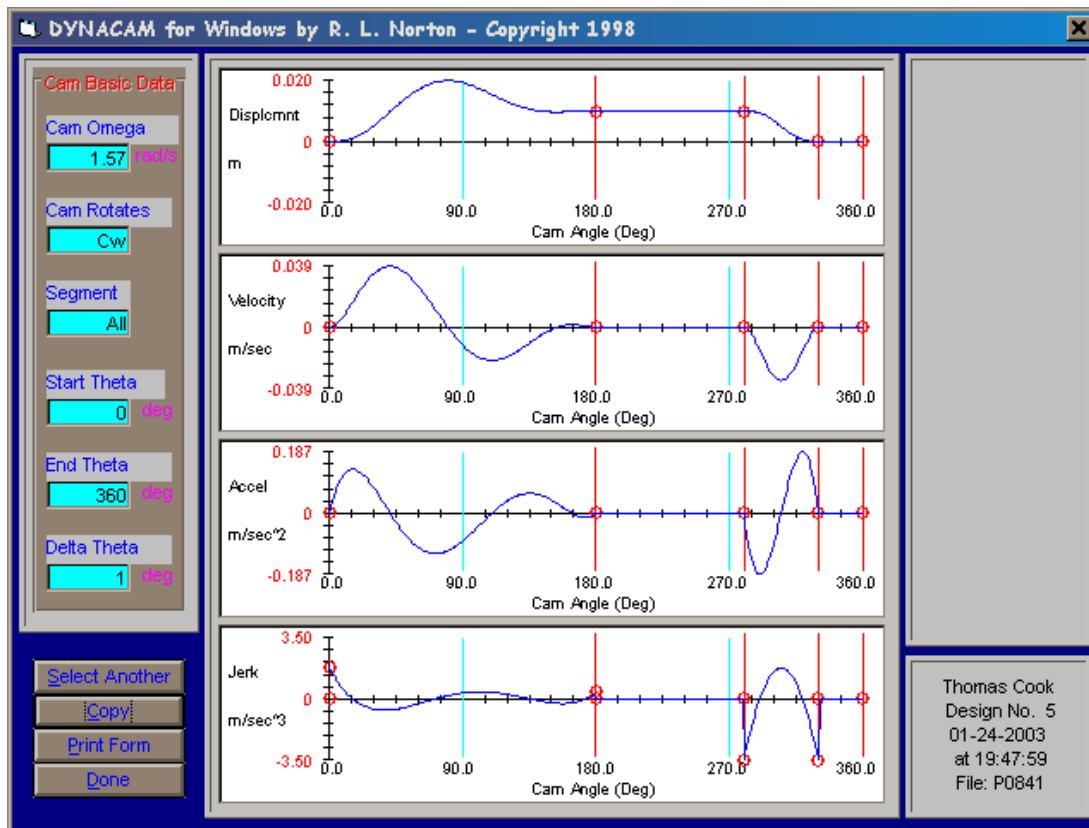
4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

Prime circle radius $R_p := 90 \cdot mm$

Roller follower radius $R_f := 30 \cdot mm$

Follower eccentricity $\epsilon := 0 \cdot mm$

5. Graphs of $svaj$, ϕ and ρ for the roller follower are shown on the following page.



PROBLEM 8-42

Statement: Design a single-dwell cam to move a follower from 0 to 35 mm in 75 deg, fall 35 mm in 120 deg and dwell for the remainder. The total cycle must take 3 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the SVAJ diagrams.

Given:

RISE/FALL	DWELL	
$\beta := 195 \cdot \text{deg}$	$\beta_3 := 165 \cdot \text{deg}$	
$h := 35 \cdot \text{mm}$	$h_3 := 0.0 \cdot \text{mm}$	
Cycle time: $\tau := 3 \cdot \text{sec}$		

Solution: See Mathcad file P0842.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 2.094 \frac{\text{rad}}{\text{sec}}$$

2. Use a two-segment polynomial. Let the rise and fall, together, be one segment and the dwell be the second segment. Then, the boundary conditions are:

$$\begin{aligned} \text{at } \theta = 0: \quad & s = 0, \quad v = 0, \quad a = 0 \\ \theta = \beta_1: \quad & s = h, \quad v = 0 \\ \theta = \beta: \quad & s = 0, \quad v = 0, \quad a = 0 \end{aligned}$$

This is a minimum set of 8 BCs. The $v = 0$ condition at $\theta = \beta_1$ is required to keep the displacement from overshooting the lift, h . Define the total lift, the rise interval, the fall interval, and the ratio of rise to the total interval.

$$\text{Total lift:} \quad h = 35.000 \text{ mm}$$

$$\text{Rise interval:} \quad \beta_1 := 75 \cdot \text{deg} \quad A := \frac{\beta_1}{\beta} \quad A = 0.385$$

$$\text{Fall interval:} \quad \beta_2 := 120 \cdot \text{deg}$$

3. Use the 8 BCs and equation 8.23 to write 8 equations in s , v , and a similar to those in example 8-9 but with 8 terms in the equation for s (the highest term will be seventh degree).

$$\text{For } \theta = 0: \quad s = v = a = 0$$

$$0 := c_0^{\blacksquare} \quad 0 := c_1^{\blacksquare} \quad 0 := c_2^{\blacksquare}$$

$$\text{For } \theta = \beta_1: \quad s = h, \quad v = 0$$

$$h := c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5 + c_6 \cdot A^6 + c_7 \cdot A^7^{\blacksquare}$$

$$0 := 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4 + 6 \cdot c_6 \cdot A^5 + 7 \cdot c_7 \cdot A^6^{\blacksquare}$$

$$\text{For } \theta = \beta: \quad s = v = a = 0$$

$$0 := c_3 + c_4 + c_5 + c_6 + c_7^{\blacksquare}$$

$$0 := 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5 + 6 \cdot c_6 + 7 \cdot c_7^{\blacksquare}$$

$$0 := 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5 + 30 \cdot c_6 + 42 \cdot c_7^{\blacksquare}$$

4. Solve for the unknown polynomial coefficients. Note that C_0 through C_2 are zero

$$C := \begin{pmatrix} A^3 & A^4 & A^5 & A^6 & A^7 \\ 3 \cdot A^2 & 4 \cdot A^3 & 5 \cdot A^4 & 6 \cdot A^5 & 7 \cdot A^6 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 & 7 \\ 6 & 12 & 20 & 30 & 42 \end{pmatrix} \quad H := \begin{pmatrix} h \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{pmatrix} := C^{-1} \cdot H$$

$$c_3 = 5609.280 \text{ mm}$$

$$c_4 = -24548.849 \text{ mm}$$

$$c_5 = 39990.867 \text{ mm}$$

$$c_6 = -28772.307 \text{ mm}$$

$$c_7 = 7721.009 \text{ mm}$$

5. Write the SVAJ equations for the rise/fall segment.

$$s(\theta) := c_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + c_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + c_5 \cdot \left(\frac{\theta}{\beta}\right)^5 + c_6 \cdot \left(\frac{\theta}{\beta}\right)^6 + c_7 \cdot \left(\frac{\theta}{\beta}\right)^7$$

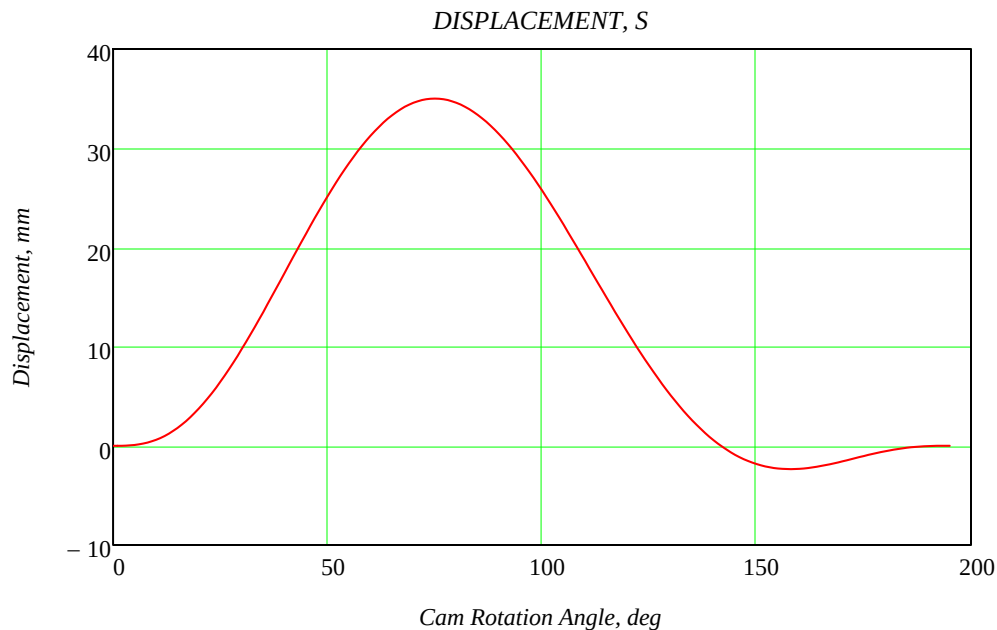
$$v(\theta) := \frac{1}{\beta} \cdot \left[3 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^3 + 5 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^4 + 6 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^5 + 7 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^6 \right]$$

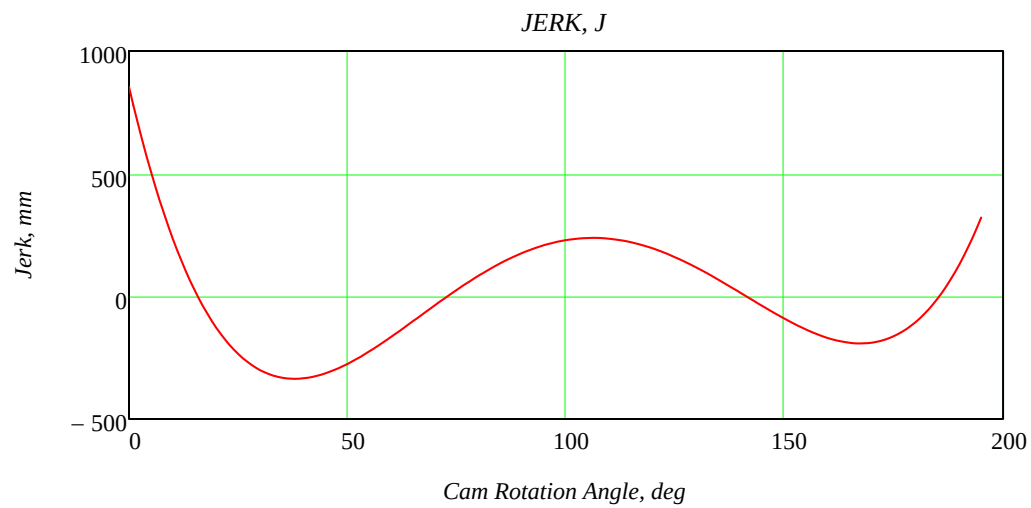
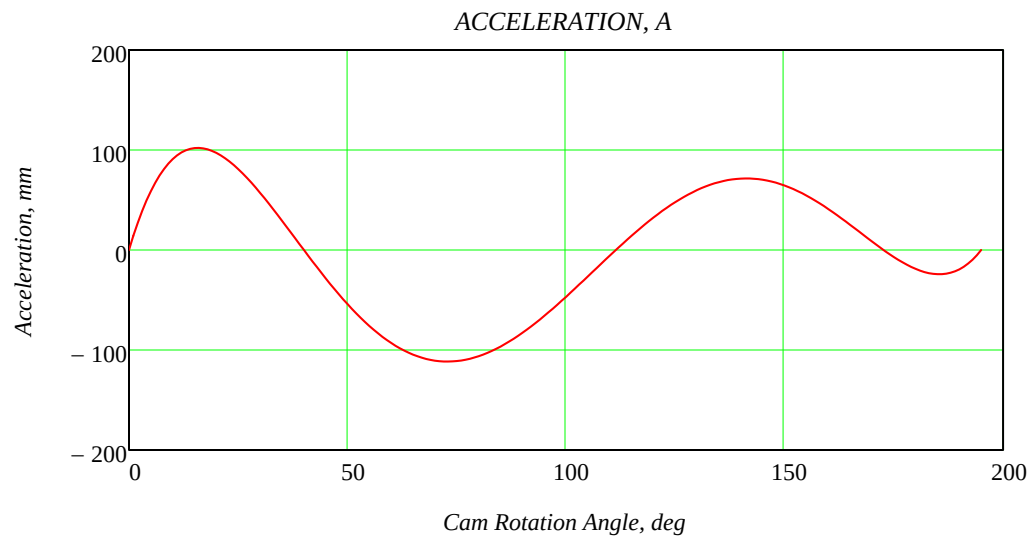
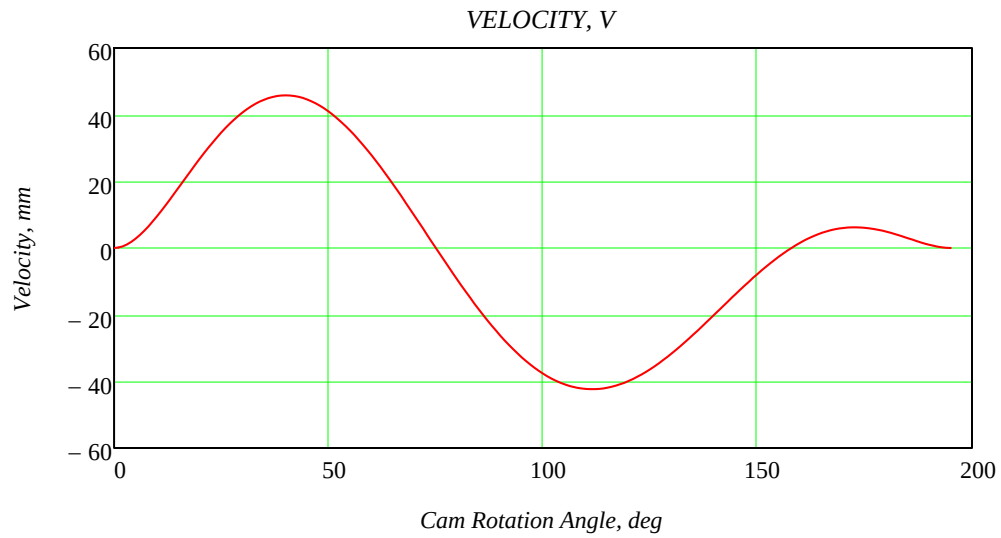
$$A(\theta) := \frac{1}{\beta^2} \cdot \left[6 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right) + 12 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^2 + 20 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^3 + 30 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^4 + 42 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^5 \right]$$

$$J(\theta) := \frac{1}{\beta^3} \cdot \left[6 \cdot c_3 + 24 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right) + 60 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^2 + 120 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^3 + 210 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^4 \right]$$

6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 \leq \theta \leq \beta$.

$$\theta := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. \beta$$





PROBLEM 8-43

Statement: Design a cam to move a follower at a constant velocity of 100 mm/sec for 2 sec then return to its starting position with a total cycle time of 3 sec.

Given: Constant velocity: $v_c := 100 \cdot \text{mm} \cdot \text{sec}^{-1}$
 Time duration of cv segment: $t_{cv} := 2 \cdot \text{sec}$ Cycle time:
 $\tau := 3 \cdot \text{sec}$

Solution: See Mathcad file P0843.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 2.094 \frac{\text{rad}}{\text{sec}}$$

2. Use a two-segment polynomial as demonstrated in Example 8-12. The lift during the first segment and the *svaj* equations for the first segment are:

$$\begin{aligned} \text{Normalized velocity: } v_{cv} &:= \frac{v_c}{\omega} & v_{cv} &= 47.746 \text{ mm} \\ h_{cv} &:= v_c \cdot t_{cv} & h_{cv} &= 200.000 \text{ mm} & \beta_1 &:= \frac{t_{cv}}{\tau} \cdot 360 \cdot \text{deg} & \beta_1 &= 240 \text{ deg} \\ s_1(\theta) &:= h_{cv} \cdot \frac{\theta}{\beta_1} & v_1(\theta) &:= v_{cv} & a_1(\theta) &:= 0 \cdot \text{mm} & j_1(\theta) &:= 0 \cdot \text{mm} \end{aligned}$$

2. The boundary conditions for the second segment are:

$$\begin{aligned} \text{at } \theta = \beta_1: & \quad s = h_{cv}, \quad v = v_{cv}, \quad a = 0 \\ \theta = 360 \text{ deg:} & \quad s = 0, \quad v = v_{cv}, \quad a = 0 \end{aligned}$$

This is a minimum set of 6 BCs. Define the total interval and the constant velocity interval, and the ratio of constant velocity interval to the total interval.

$$\begin{aligned} \text{Total interval: } \beta &:= 360 \cdot \text{deg} \\ \text{CV interval: } \beta_1 &= 240 \text{ deg} & A &:= \frac{\beta_1}{\beta} & A &= 0.667 \end{aligned}$$

3. Use the 6 BCs and equation 8.23 to write 6 equations in s , v , and a similar to those in example 8-9 but with 6 terms in the equation for s (the highest term will be fifth degree).

$$\text{For } \theta = \beta_1: \quad s = h_{cv}, \quad v = v_{cv}, \quad a = 0$$

$$h_{cv} = c_0 + c_1 \cdot A + c_2 \cdot A^2 + c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5$$

$$v_{cv} = \frac{1}{\beta} \cdot (c_1 + 2 \cdot c_2 \cdot A + 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4)$$

$$0 = 2 \cdot c_2 + 6 \cdot c_3 \cdot A + 12 \cdot c_4 \cdot A^2 + 20 \cdot c_5 \cdot A^3$$

$$\text{For } \theta = \beta: \quad s = 0, \quad v = v_{cv}, \quad a = 0$$

$$0 = c_0 + c_1 + c_2 + c_3 + c_4 + c_5$$

$$0 = \frac{1}{\beta} \cdot (c_1 + 2 \cdot c_2 + 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5)$$

$$0 = 2 \cdot c_2 + 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5$$

4. Solve for the unknown polynomial coefficients.

$$C := \begin{pmatrix} 1 & A & A^2 & A^3 & A^4 & A^5 \\ 0 & 1 & 2 \cdot A & 3 \cdot A^2 & 4 \cdot A^3 & 5 \cdot A^4 \\ 0 & 0 & 2 & 6 \cdot A & 12 \cdot A^2 & 20 \cdot A^3 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 2 & 6 & 12 & 20 \end{pmatrix} \quad H := \begin{pmatrix} h_{cv} \\ \beta \cdot v_{cv} \\ 0 \\ 0 \\ \beta \cdot v_{cv} \\ 0 \end{pmatrix} \quad \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix} := C^{-1} \cdot H$$

$$\begin{aligned} c_0 &= 1.536 \times 10^5 \text{ mm} & c_1 &= -9.717 \times 10^5 \text{ mm} & c_2 &= 2.430 \times 10^6 \text{ mm} \\ c_3 &= -2.997 \times 10^6 \text{ mm} & c_4 &= 1.822 \times 10^6 \text{ mm} & c_5 &= -4.374 \times 10^5 \text{ mm} \end{aligned}$$

5. Write the *svaj* equations for the second segment.

$$\begin{aligned} s_2(\theta) &:= c_0 + c_1 \cdot \left(\frac{\theta}{\beta}\right) + c_2 \cdot \left(\frac{\theta}{\beta}\right)^2 + c_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + c_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + c_5 \cdot \left(\frac{\theta}{\beta}\right)^5 \\ v_2(\theta) &:= \frac{1}{\beta} \cdot \left[c_1 + 2 \cdot c_2 \cdot \left(\frac{\theta}{\beta}\right) + 3 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^3 + 5 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^4 \right] \\ a_2(\theta) &:= \frac{1}{\beta^2} \cdot \left[2 \cdot c_2 + 6 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right) + 12 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^2 + 20 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^3 \right] \\ j_2(\theta) &:= \frac{1}{\beta^3} \cdot \left[6 \cdot c_3 + 24 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right) + 60 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^2 \right] \end{aligned}$$

4. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(\theta, a, b) := \text{if}[(\theta > a) \wedge (\theta \leq b), 1, 0]$$

$$S(\theta) := R(\theta, 0, \beta_1) \cdot s_1(\theta) + R(\theta, \beta_1, \beta) \cdot s_2(\theta)$$

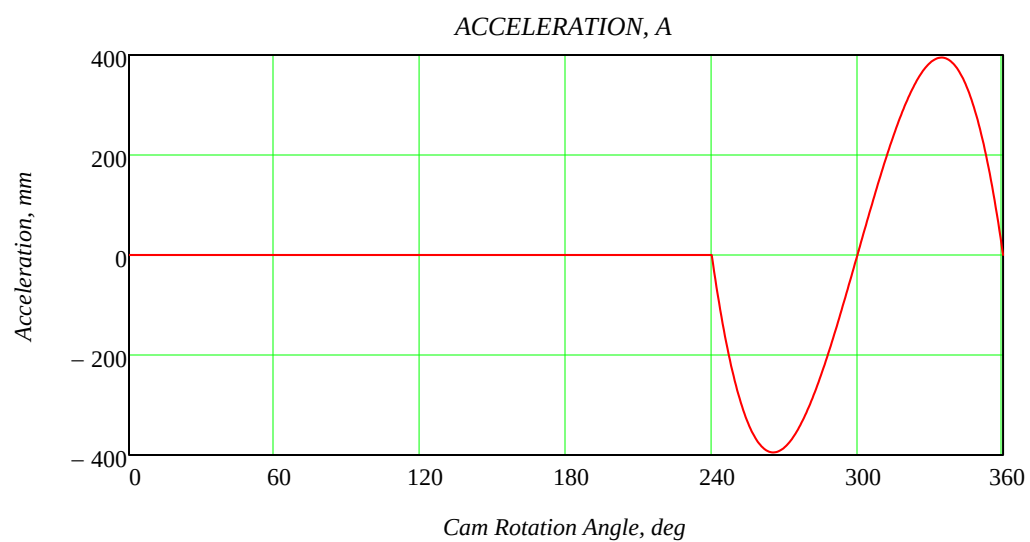
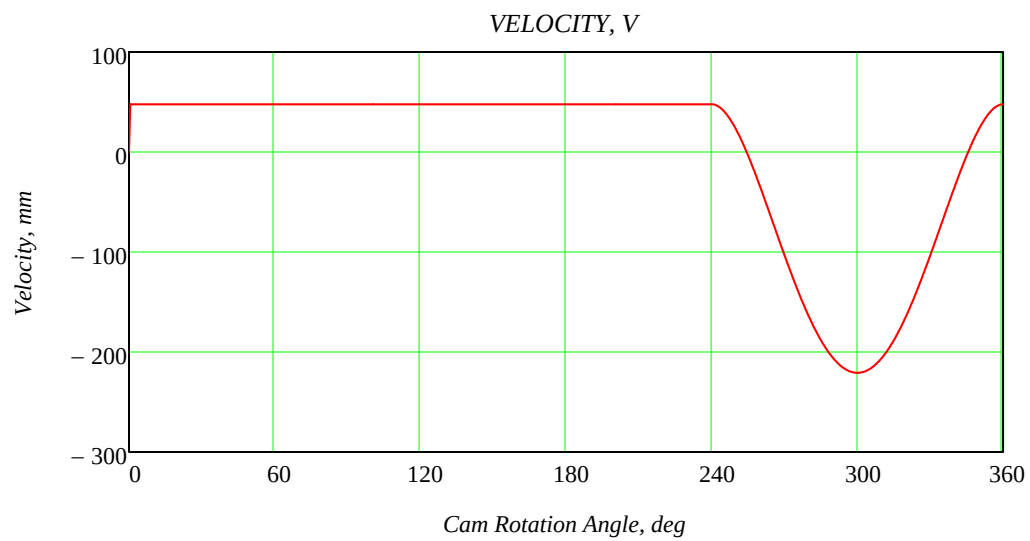
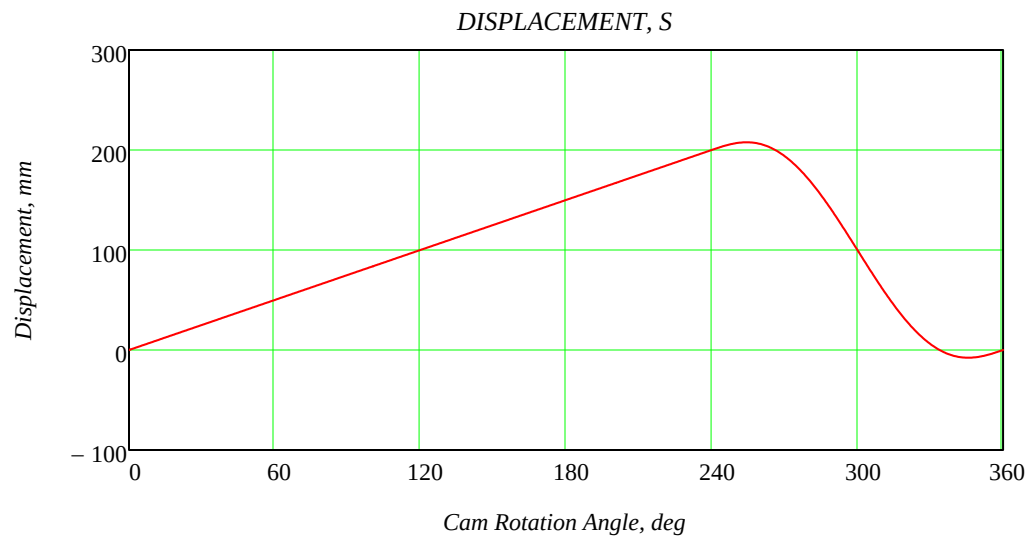
$$V(\theta) := R(\theta, 0, \beta_1) \cdot v_1(\theta) + R(\theta, \beta_1, \beta) \cdot v_2(\theta)$$

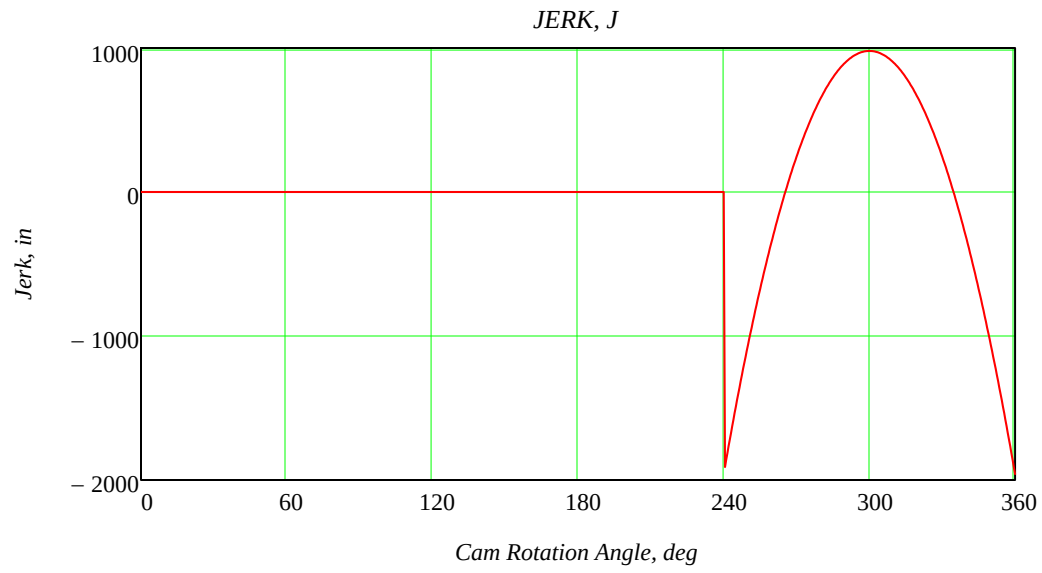
$$A(\theta) := R(\theta, 0, \beta_1) \cdot a_1(\theta) + R(\theta, \beta_1, \beta) \cdot a_2(\theta)$$

$$J(\theta) := R(\theta, 0, \beta_1) \cdot j_1(\theta) + R(\theta, \beta_1, \beta) \cdot j_2(\theta)$$

6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 \leq \theta \leq \beta$.

$$\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. \beta$$





PROBLEM 8-44

Statement: Design a double-dwell cam to move a follower from 0 to 50 mm in 75 deg, dwell for 75 deg, fall 50 mm in 75 deg and dwell for the remainder. The total cycle must take 5 sec. Use a modified trapezoidal function for rise and fall and plot the $s v a j$ diagrams.

Given:	RISE	DWELL	FALL	DWELL
	$\beta_1 := 75 \cdot \text{deg}$	$\beta_2 := 75 \cdot \text{deg}$	$\beta_3 := 75 \cdot \text{deg}$	$\beta_4 := 135 \cdot \text{deg}$
	$h_1 := 50 \cdot \text{mm}$	$h_2 := 0 \cdot \text{mm}$	$h_3 := 50 \cdot \text{mm}$	$h_4 := 0 \cdot \text{mm}$
	Cycle time: $\tau := 5 \cdot \text{sec}$			

Solution: See Mathcad file P0844.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.257 \frac{\text{rad}}{\text{sec}}$$

2. The modified trapezoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$\begin{aligned} b &:= 0.25 & c &:= 0.50 & d &:= 0.25 \\ C_v &:= 2.0000 & C_a &:= 4.8881 & C_j &:= 61.426 \end{aligned}$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 \leq x \leq b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$\begin{aligned} y_1(x) &:= C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi} \right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \right] & y'_1(x) &:= C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x\right) \right) \\ y''_1(x) &:= C_a \cdot \sin\left(\frac{\pi}{b} \cdot x\right) & y'''_1(x) &:= C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x\right) \end{aligned}$$

for $b/2 \leq x \leq (1-d)/2$

$$\begin{aligned} y_2(x) &:= C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \right] & y'_2(x) &:= C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \right] \\ y''_2(x) &:= C_a & y'''_2(x) &:= 0 \end{aligned}$$

for $(1-d)/2 \leq x \leq (1+d)/2$

$$\begin{aligned} y_3(x) &:= C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2} \right) \cdot x + \left(\frac{d}{\pi} \right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi} \right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y'_3(x) &:= C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right] \\ y''_3(x) &:= C_a \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] & y'''_3(x) &:= -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \end{aligned}$$

for $(1+d)/2 \leq x \leq 1-b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + (2 \cdot d^2 - b^2) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) \quad y''_4(x) := -C_a \quad y'''_4(x) := 0$$

for $1 - b/2 \leq x \leq 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot (d^2 - b^2)}{\pi^2} + \frac{(1 - b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos \left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin \left[\frac{\pi}{b} \cdot (x - 1) \right] \quad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos \left[\frac{\pi}{b} \cdot (x - 1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the *SVAJ* curves, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for S , V , A , and J that applies over the range $0 \leq \theta \leq 360$ deg.
6. Write the local *svaj* equations for the first interval, $0 \leq \theta \leq \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y'_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y'_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y'_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y'_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y''_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y''_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y''_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y''_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y'''_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y'''_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y'''_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y'''_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y'''_5(x) \right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 \leq \theta \leq \beta_1 + \beta_2$. For this interval, the value of S is the value of S at the end of the previous interval and the values of V , A , and J are zero because of the dwell.

For $\beta_1 \leq \theta \leq \beta_1 + \beta_2$

$$s_2(x) := h_1 \quad v_2(x) := 0 \quad a_2(x) := 0 \quad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$

$$s_3(x) := h_3 \left[1 - \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right) \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right]$$

$$v_3(x) := -\frac{h_3}{\beta_3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right) \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x)$$

$$a_3(x) := -\frac{h_3}{\beta_3^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right) \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x)$$

$$j_3(x) := -\frac{h_3}{\beta_3^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right) \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x)$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$

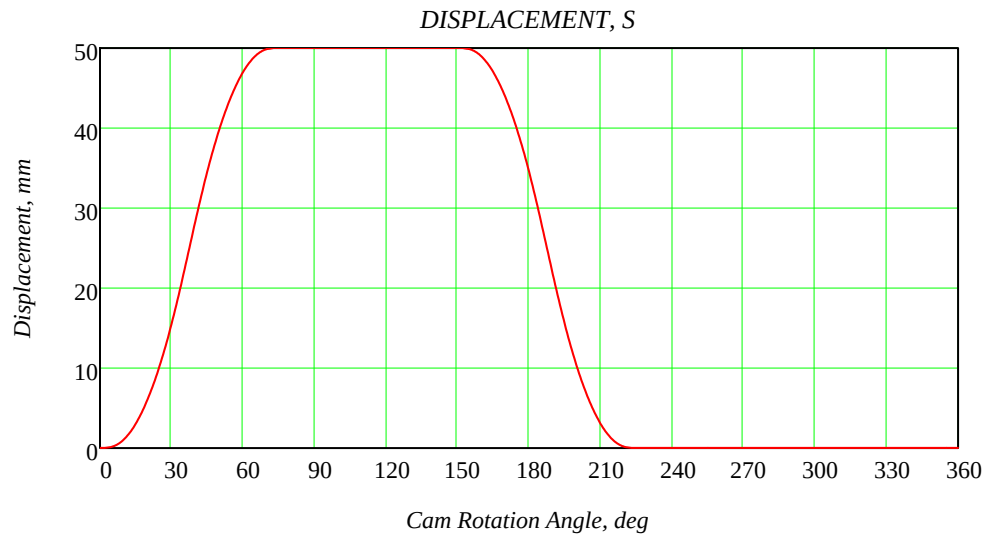
$$s_4(x) := 0 \quad v_4(x) := 0 \quad a_4(x) := 0 \quad j_4(x) := 0$$

10. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$\text{Let} \quad \theta_1 := \beta_1 \quad \theta_2 := \theta_1 + \beta_2 \quad \theta_3 := \theta_2 + \beta_3 \quad \theta_4 := \theta_3 + \beta_4$$

$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2) \cdot s_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \dots \\ + R(\theta, \theta_2, \theta_3) \cdot s_3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4) \cdot s_4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$

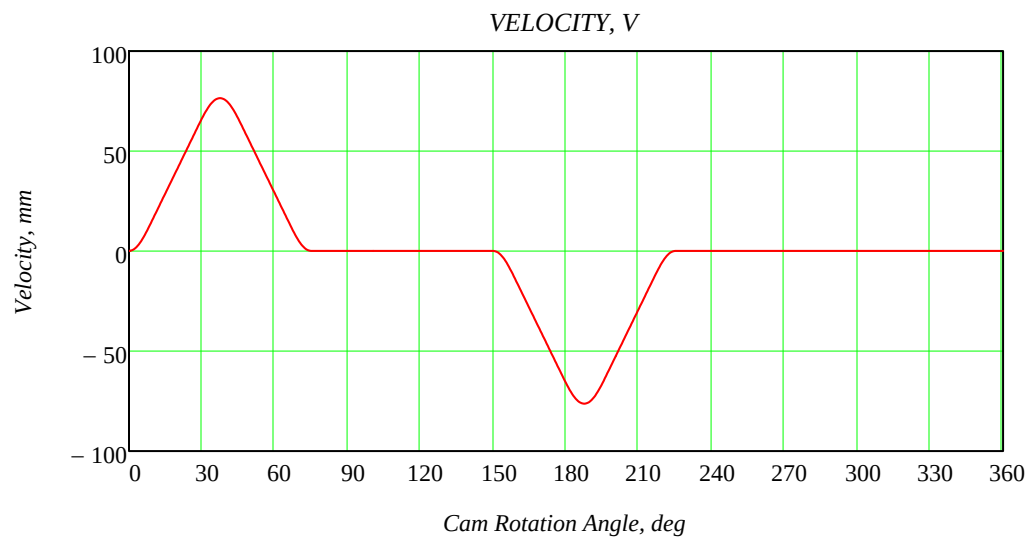
$$\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



11. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$V(\theta) := v_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot v_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

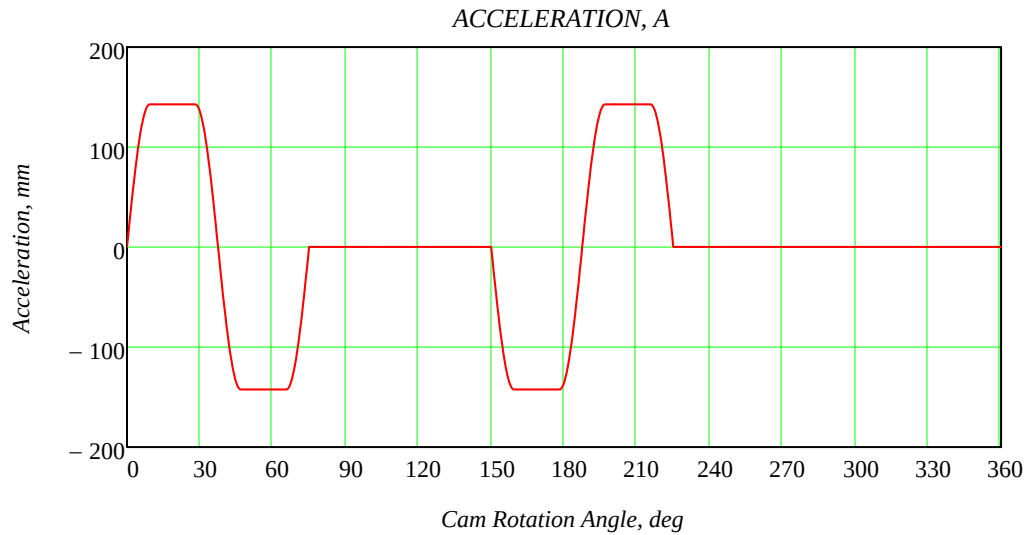
$$+ R(\theta, \theta_2, \theta_3) \cdot v_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot v_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



12. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := a_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot a_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

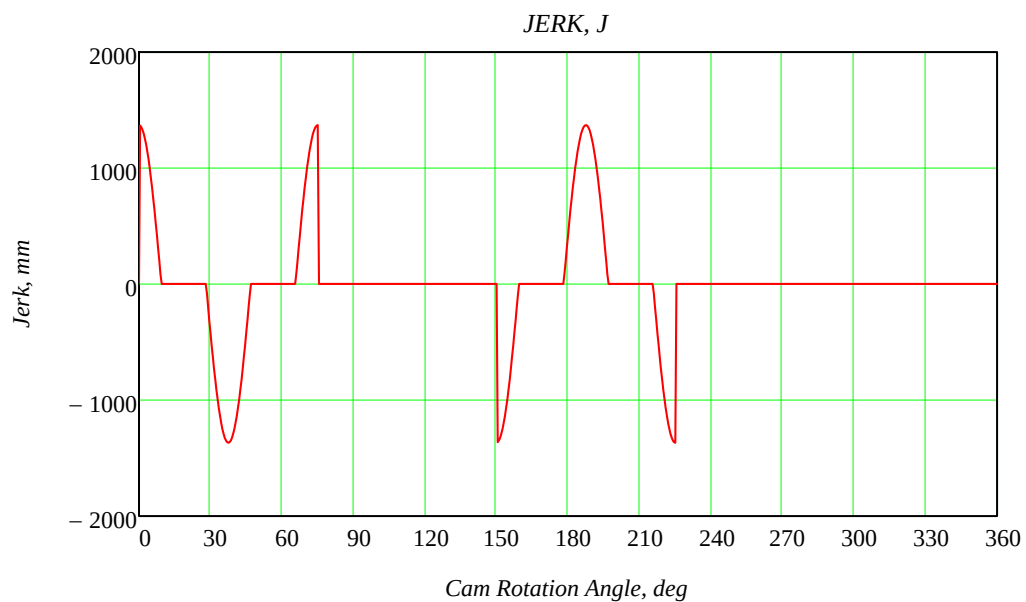
$$+ R(\theta, \theta_2, \theta_3) \cdot a_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot a_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := j_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot j_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

$$+ R(\theta, \theta_2, \theta_3) \cdot j_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot j_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



PROBLEM 8-45

Statement: Design a double-dwell cam to move a follower from 0 to 50 mm in 75 deg, dwell for 75 deg, fall 50 mm in 75 deg and dwell for the remainder. The total cycle must take 5 sec. Use a modified sinusoidal function for rise and fall and plot the $s v a j$ diagrams.

Given:	RISE	DWELL	FALL	DWELL
	$\beta_1 := 75 \cdot \text{deg}$	$\beta_2 := 75 \cdot \text{deg}$	$\beta_3 := 75 \cdot \text{deg}$	$\beta_4 := 135 \cdot \text{deg}$
	$h_1 := 50 \cdot \text{mm}$	$h_2 := 0 \cdot \text{mm}$	$h_3 := 50 \cdot \text{mm}$	$h_4 := 0 \cdot \text{mm}$
	Cycle time: $\tau := 5 \cdot \text{sec}$			

Solution: See Mathcad file P0845.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.257 \frac{\text{rad}}{\text{sec}}$$

2. The modified sinusoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$b := 0.25 \quad c := 0.00 \quad d := 0.75$$

$$C_a := 5.5280$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 \leq x \leq b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left(\frac{\pi}{b} \cdot x \right) \right] \quad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos \left(\frac{\pi}{b} \cdot x \right) \right)$$

$$y''_1(x) := C_a \cdot \sin \left(\frac{\pi}{b} \cdot x \right) \quad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot \cos \left(\frac{\pi}{b} \cdot x \right)$$

for $b/2 \leq x \leq (1-d)/2$

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \right] \quad y'_2(x) := C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \right]$$

$$y''_2(x) := C_a \quad y'''_2(x) := 0$$

for $(1-d)/2 \leq x \leq (1+d)/2$

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2} \right) \cdot x + \left(\frac{d}{\pi} \right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi} \right)^2 \cdot \cos \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right]$$

$$y'_3(x) := C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right]$$

$$y''_3(x) := C_a \cdot \cos \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \quad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right]$$

for $(1+d)/2 \leq x \leq 1-b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + (2 \cdot d^2 - b^2) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y_4'(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) \quad y_4''(x) := -C_a \quad y_4'''(x) := 0$$

for $1 - b/2 \leq x \leq 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot (d^2 - b^2)}{\pi^2} + \frac{(1 - b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y_5'(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos \left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y_5''(x) := C_a \cdot \sin \left[\frac{\pi}{b} \cdot (x - 1) \right] \quad y_5'''(x) := C_a \cdot \frac{\pi}{b} \cdot \cos \left[\frac{\pi}{b} \cdot (x - 1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the *SVAJ* curves, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range $0 \leq \theta \leq 360$ deg.
6. Write the local *svaj* equations for the first interval, $0 \leq \theta \leq \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y_1(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y_2(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y_3(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y_4(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y_1'(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y_2'(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y_3'(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y_4'(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y_5'(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y_1''(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y_2''(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y_3''(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y_4''(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y_5''(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R \left(x, 0, \frac{b}{2} \right) \cdot y_1'''(x) + R \left(x, \frac{b}{2}, \frac{1-d}{2} \right) \cdot y_2'''(x) + R \left(x, \frac{1-d}{2}, \frac{1+d}{2} \right) \cdot y_3'''(x) \dots \right. \\ \left. + R \left(x, \frac{1+d}{2}, 1 - \frac{b}{2} \right) \cdot y_4'''(x) + R \left(x, 1 - \frac{b}{2}, 1 \right) \cdot y_5'''(x) \right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 \leq \theta \leq \beta_1 + \beta_2$. For this interval, the value of *S* is the value of *S* at the end of the previous interval and the values of *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 \leq \theta \leq \beta_1 + \beta_2$

$$s_2(x) := h_1 \quad v_2(x) := 0 \quad a_2(x) := 0 \quad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$

$$s_3(x) := h_3 \left[1 - \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right) \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right]$$

$$v_3(x) := -\frac{h_3}{\beta_3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_3(x) := -\frac{h_3}{\beta_3^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_3(x) := -\frac{h_3}{\beta_3^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$

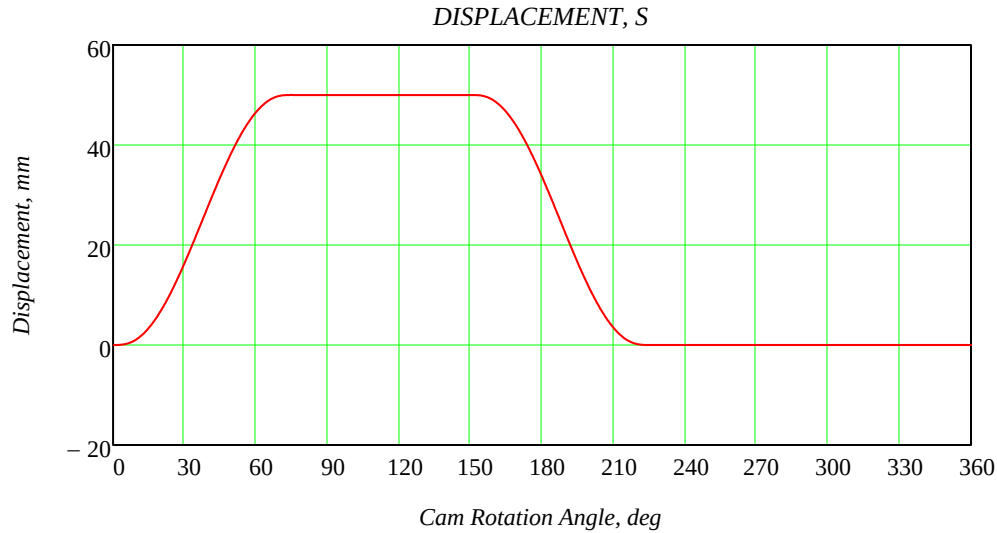
$$s_4(x) := 0 \quad v_4(x) := 0 \quad a_4(x) := 0 \quad j_4(x) := 0$$

10. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$\text{Let} \quad \theta_1 := \beta_1 \quad \theta_2 := \theta_1 + \beta_2 \quad \theta_3 := \theta_2 + \beta_3 \quad \theta_4 := \theta_3 + \beta_4$$

$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2) \cdot s_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \dots \\ + R(\theta, \theta_2, \theta_3) \cdot s_3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4) \cdot s_4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$

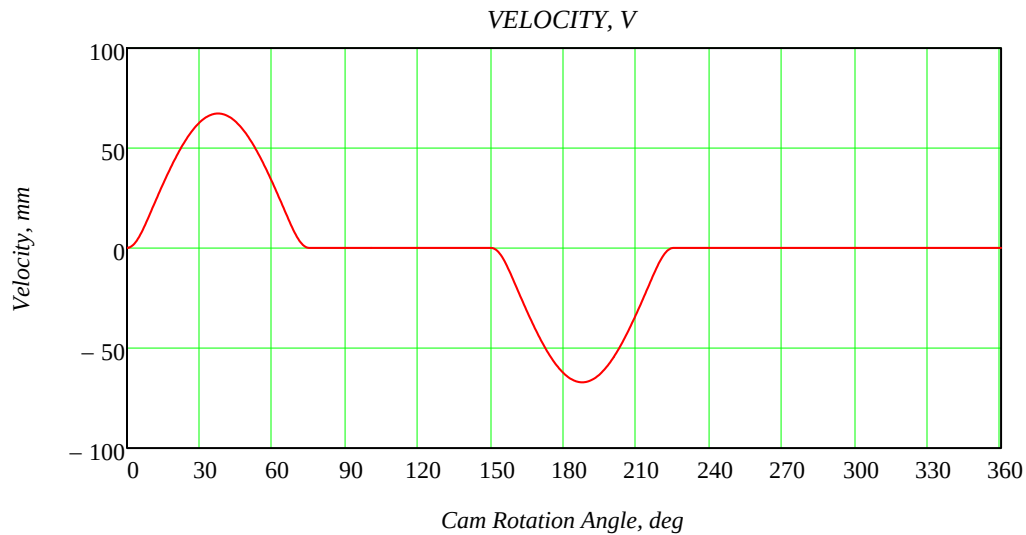
$$\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



11. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$V(\theta) := v_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot v_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

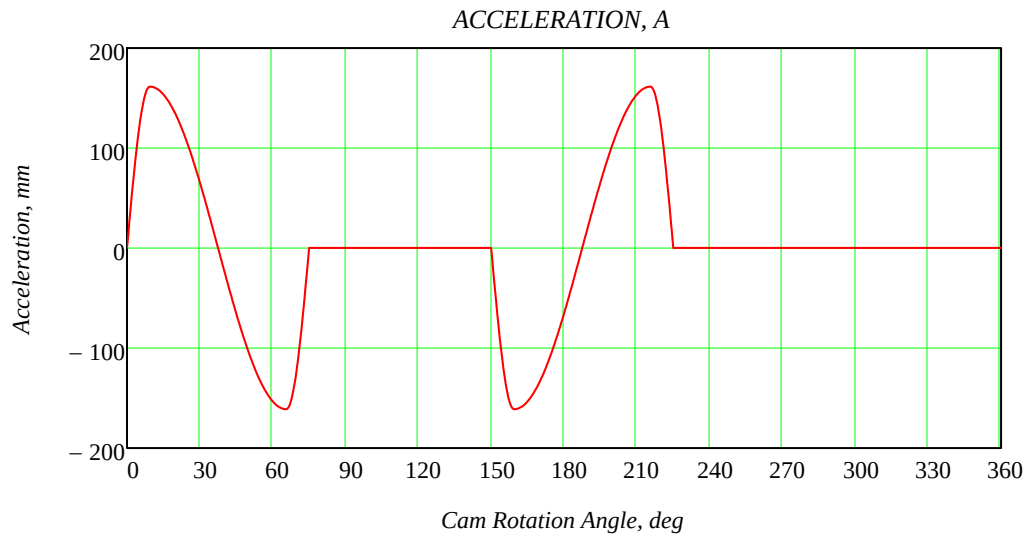
$$+ R(\theta, \theta_2, \theta_3) \cdot v_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot v_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



12. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := a_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot a_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

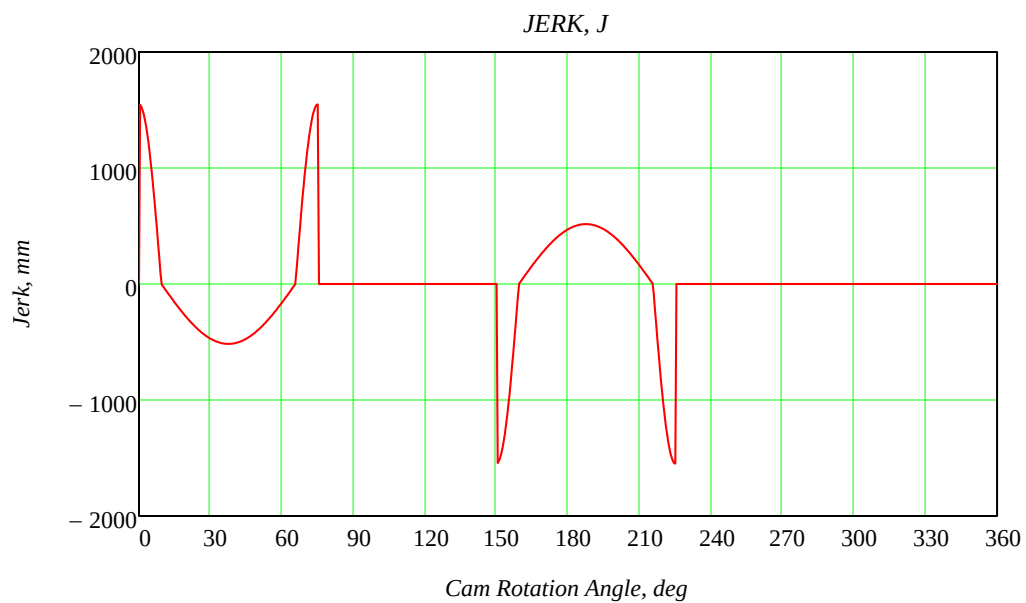
$$+ R(\theta, \theta_2, \theta_3) \cdot a_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot a_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := j_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot j_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

$$+ R(\theta, \theta_2, \theta_3) \cdot j_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot j_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



PROBLEM 8-46

Statement: Design a double-dwell cam to move a follower from 0 to 50 mm in 75 deg, dwell for 75 deg, fall 50 mm in 75 deg and dwell for the remainder. The total cycle must take 5 sec. Use a 4-5-6-7 polynomial function for rise and fall and plot the $s v a j$ diagrams.

Given:	RISE	DWELL	FALL	DWELL
	$\beta_1 := 75\text{-deg}$	$\beta_2 := 75\text{-deg}$	$\beta_3 := 75\text{-deg}$	$\beta_4 := 135\text{-deg}$
	$h_1 := 50\text{-mm}$	$h_2 := 0\text{-mm}$	$h_3 := 50\text{-mm}$	$h_4 := 0\text{-mm}$
	Cycle time: $\tau := 5\text{-sec}$			

Solution: See Mathcad file P0846.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.257 \frac{\text{rad}}{\text{sec}}$$

2. The 4-5-6-7 polynomial is defined in local coordinates by equation 8.25. Differentiate it to get v , a , and j .

$$s(\theta, \beta, h) := h \cdot \left[35 \cdot \left(\frac{\theta}{\beta} \right)^4 - 84 \cdot \left(\frac{\theta}{\beta} \right)^5 + 70 \cdot \left(\frac{\theta}{\beta} \right)^6 - 20 \cdot \left(\frac{\theta}{\beta} \right)^7 \right]$$

$$v(\theta, \beta, h) := \frac{h}{\beta} \cdot \left[140 \cdot \left(\frac{\theta}{\beta} \right)^3 - 420 \cdot \left(\frac{\theta}{\beta} \right)^4 + 420 \cdot \left(\frac{\theta}{\beta} \right)^5 - 140 \cdot \left(\frac{\theta}{\beta} \right)^6 \right]$$

$$a(\theta, \beta, h) := \frac{h}{\beta^2} \cdot \left[420 \cdot \left(\frac{\theta}{\beta} \right)^2 - 1680 \cdot \left(\frac{\theta}{\beta} \right)^3 + 2100 \cdot \left(\frac{\theta}{\beta} \right)^4 - 840 \cdot \left(\frac{\theta}{\beta} \right)^5 \right]$$

$$j(\theta, \beta, h) := \frac{h}{\beta^3} \cdot \left[840 \cdot \left(\frac{\theta}{\beta} \right) - 5040 \cdot \left(\frac{\theta}{\beta} \right)^2 + 8400 \cdot \left(\frac{\theta}{\beta} \right)^3 - 4200 \cdot \left(\frac{\theta}{\beta} \right)^4 \right]$$

3. The above equations can be used for a rise or fall by inserting the proper values of θ , β , and h . To plot the $SVAJ$ curves, first define a range function that has a value of one between the values of x and y and zero elsewhere.

$$R(\theta, x, y) := \text{if}[(\theta > x) \cdot (\theta \leq y), 1, 0]$$

4. Write the global $SVAJ$ equations for the first interval, $0 \leq \theta \leq \beta_1$. For this interval, the local and global frames are coincident so the local equations can be used as written, substituting only for h_1 for h and β_1 for β . Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$

$$S_1(\theta) := R(\theta, 0, \beta_1) \cdot s(\theta, \beta_1, h_1) \quad V_1(\theta) := R(\theta, 0, \beta_1) \cdot v(\theta, \beta_1, h_1)$$

$$A_1(\theta) := R(\theta, 0, \beta_1) \cdot a(\theta, \beta_1, h_1) \quad J_1(\theta) := R(\theta, 0, \beta_1) \cdot j(\theta, \beta_1, h_1)$$

5. Write the global $SVAJ$ equations for the second interval, $\beta_1 \leq \theta \leq \beta_1 + \beta_2$. For this interval, the value of S is the value of S at the end of the previous interval and the values of V , A , and J are zero because of the dwell.

For $\beta_1 \leq \theta \leq \beta_1 + \beta_2$

$$S_2(\theta) := R(\theta, \beta_1, \beta_1 + \beta_2) \cdot h_1 \quad V_2(\theta) := R(\theta, \beta_1, \beta_1 + \beta_2) \cdot 0\text{-mm}$$

$$A_2(\theta) := R(\theta, \beta_1, \beta_1 + \beta_2) \cdot 0 \cdot mm$$

$$J_2(\theta) := R(\theta, \beta_1, \beta_1 + \beta_2) \cdot 0 \cdot mm$$

6. Write the global SVAJ equations for the third interval, $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$ Let $\alpha := \beta_1 + \beta_2$

$$S_3(\theta) := R(\theta, \alpha, \alpha + \beta_3) \cdot (h_1 - s(\theta - \alpha, \beta_3, h_3)) \quad V_3(\theta) := -R(\theta, \alpha, \alpha + \beta_3) \cdot v(\theta - \alpha, \beta_3, h_3)$$

$$A_3(\theta) := -R(\theta, \alpha, \alpha + \beta_3) \cdot a(\theta - \alpha, \beta_3, h_3) \quad J_3(\theta) := -R(\theta, \alpha, \alpha + \beta_3) \cdot j(\theta - \alpha, \beta_3, h_3)$$

7. Write the global SVAJ equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$. For this interval, the values of S , V , A , and J are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$ Let $\alpha := \beta_1 + \beta_2 + \beta_3$

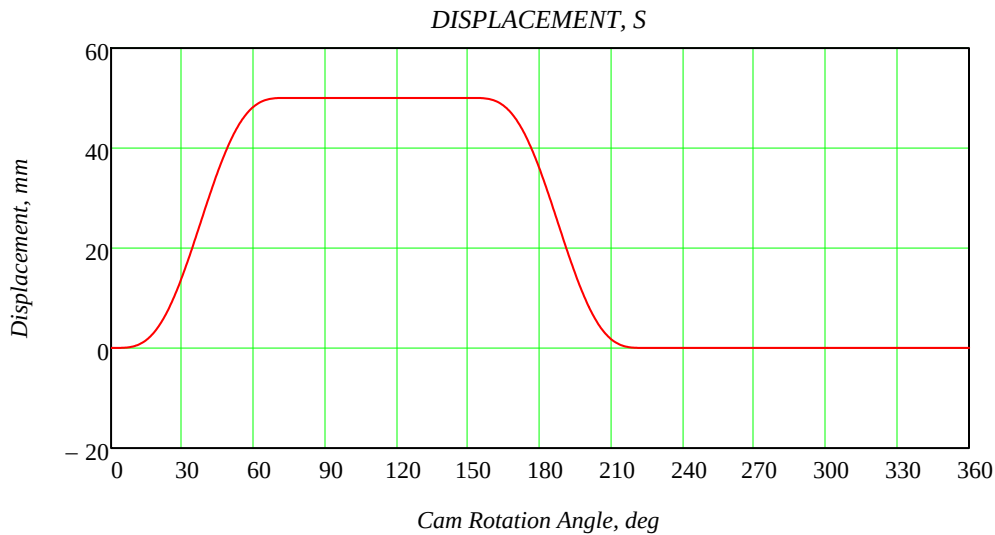
$$S_4(\theta) := R(\theta, \alpha, \alpha + \beta_4) \cdot 0 \cdot mm \quad V_4(\theta) := R(\theta, \alpha, \alpha + \beta_4) \cdot 0 \cdot mm$$

$$A_4(\theta) := R(\theta, \alpha, \alpha + \beta_4) \cdot 0 \cdot mm \quad J_4(\theta) := R(\theta, \alpha, \alpha + \beta_4) \cdot 0 \cdot mm$$

8. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

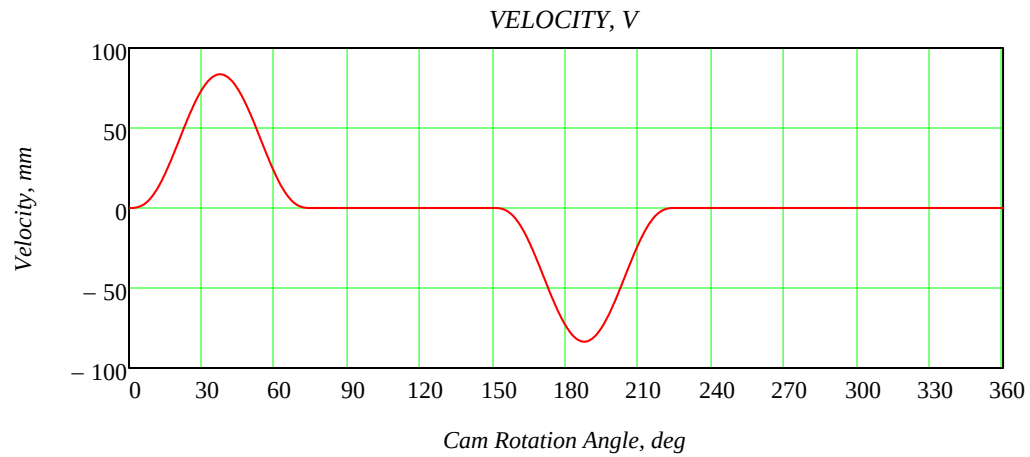
$$S(\theta) := S_1(\theta) + S_2(\theta) + S_3(\theta) + S_4(\theta)$$

$$\theta := 0 \cdot deg, 0.5 \cdot deg .. 360 \cdot deg$$



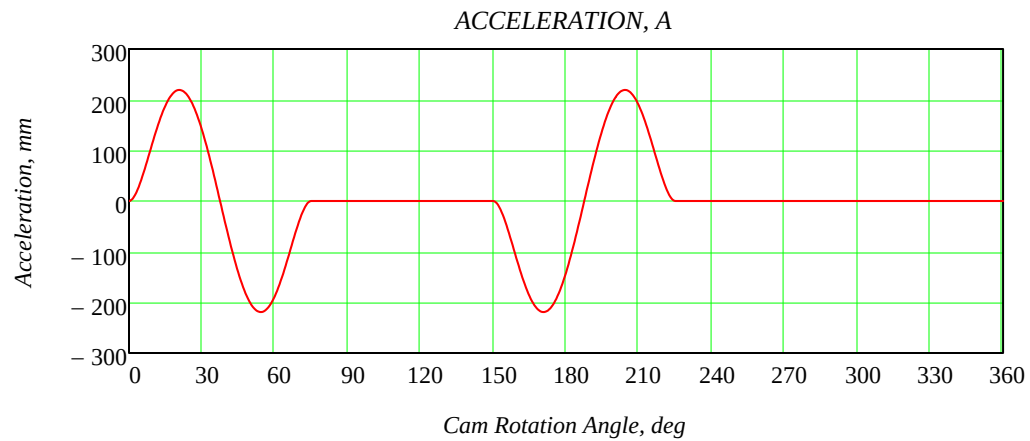
9. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$v(\theta) := v_1(\theta) + v_2(\theta) + v_3(\theta) + v_4(\theta)$$



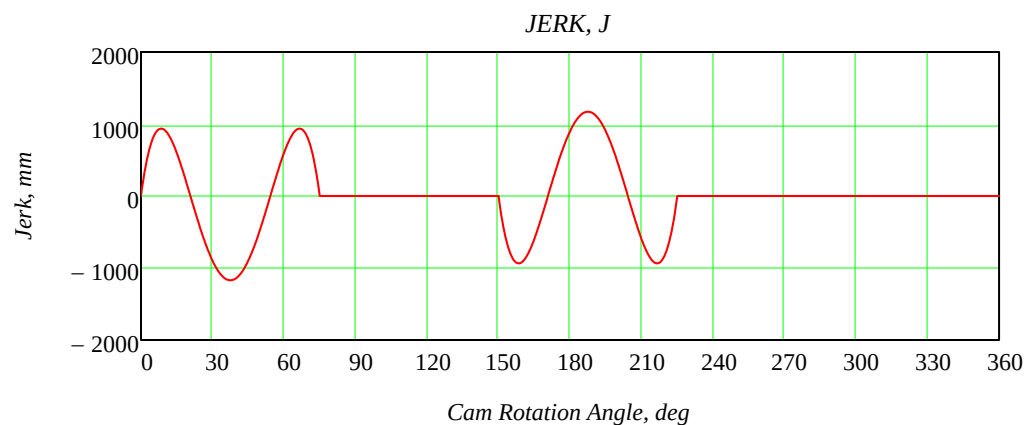
10. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := A_1(\theta) + A_2(\theta) + A_3(\theta) + A_4(\theta)$$



13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := J_1(\theta) + J_2(\theta) + J_3(\theta) + J_4(\theta)$$



PROBLEM 8-47

Statement: Design a single-dwell cam to move a follower from 0 to 65 mm in 90 deg, fall 65 mm in 180 deg and dwell for the remainder. The total cycle must take 2 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the SVAJ diagrams.

Given: RISE/FALL DWELL

$\beta := 270 \cdot \text{deg}$ $\beta_3 := 90 \cdot \text{deg}$

$h := 65 \cdot \text{mm}$ $h_3 := 0.0 \cdot \text{mm}$

Cycle time: $\tau := 2 \cdot \text{sec}$

Solution: See Mathcad file P0847.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \qquad \omega = 3.142 \frac{\text{rad}}{\text{sec}}$$

2. Use a two-segment polynomial. Let the rise and fall, together, be one segment and the dwell be the second segment. Then, the boundary conditions are:

$$\begin{aligned} \text{at } \theta = 0: \quad & s = 0, \quad v = 0, \quad a = 0 \\ \theta = \beta_1: \quad & s = h, \quad v = 0 \\ \theta = \beta: \quad & s = 0, \quad v = 0, \quad a = 0 \end{aligned}$$

This is a minimum set of 8 BCs. The $v = 0$ condition at $\theta = \beta_1$ is required to keep the displacement from overshooting the lift, h . Define the total lift, the rise interval, the fall interval, and the ratio of rise to the total interval.

Total lift: $h = 65.000 \text{ mm}$

Rise interval: $\beta_1 := 90 \cdot \text{deg}$ $A := \frac{\beta_1}{\beta}$ $A = 0.333$

Fall interval: $\beta_2 := 180 \cdot \text{deg}$

3. Use the 8 BCs and equation 8.23 to write 8 equations in s , v , and a similar to those in example 8-9 but with 8 terms in the equation for s (the highest term will be seventh degree).

For $\theta = 0$: $s = v = a = 0$

$$0 := c_0 \quad 0 := c_1 \quad 0 := c_2$$

For $\theta = \beta_1$: $s = h$, $v = 0$

$$h := c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5 + c_6 \cdot A^6 + c_7 \cdot A^7$$

$$0 := 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4 + 6 \cdot c_6 \cdot A^5 + 7 \cdot c_7 \cdot A^6$$

For $\theta = \beta$: $s = v = a = 0$

$$0 := c_3 + c_4 + c_5 + c_6 + c_7$$

$$0 := 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5 + 6 \cdot c_6 + 7 \cdot c_7$$

$$0 := 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5 + 30 \cdot c_6 + 42 \cdot c_7$$

4. Solve for the unknown polynomial coefficients. Note that C_0 through C_2 are zero

$$C := \begin{pmatrix} A^3 & A^4 & A^5 & A^6 & A^7 \\ 3 \cdot A^2 & 4 \cdot A^3 & 5 \cdot A^4 & 6 \cdot A^5 & 7 \cdot A^6 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 & 7 \\ 6 & 12 & 20 & 30 & 42 \end{pmatrix} \quad H := \begin{pmatrix} h \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{pmatrix} := C^{-1} \cdot H$$

$$c_3 = 582.985 \text{ in}$$

$$c_4 = -2798.327 \text{ in}$$

$$c_5 = 4897.072 \text{ in}$$

$$c_6 = -3731.102 \text{ in}$$

$$c_7 = 1.049 \times 10^3 \text{ in}$$

5. Write the SVAJ equations for the rise/fall segment.

$$s(\theta) := c_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + c_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + c_5 \cdot \left(\frac{\theta}{\beta}\right)^5 + c_6 \cdot \left(\frac{\theta}{\beta}\right)^6 + c_7 \cdot \left(\frac{\theta}{\beta}\right)^7$$

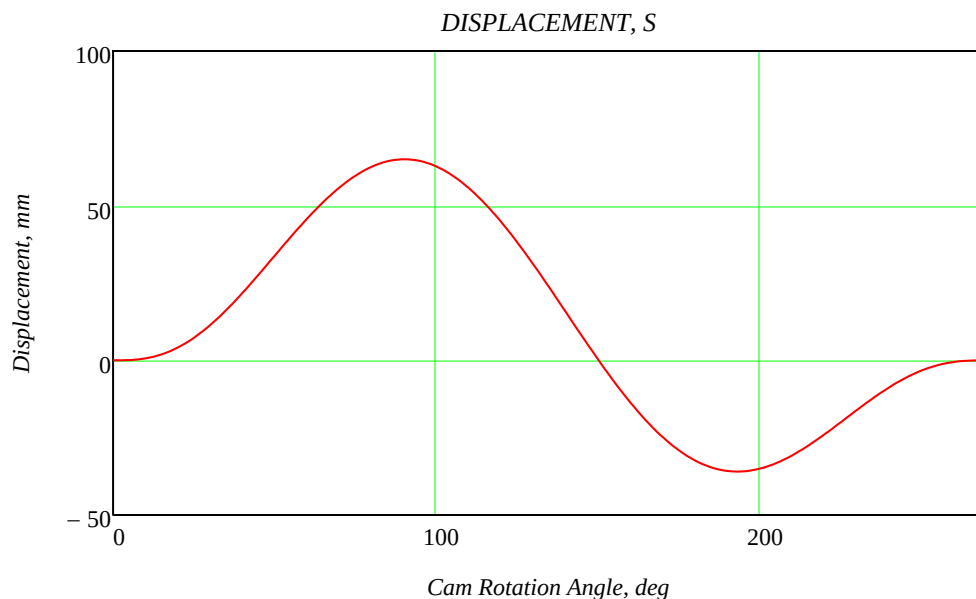
$$v(\theta) := \frac{1}{\beta} \cdot \left[3 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^3 + 5 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^4 + 6 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^5 + 7 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^6 \right]$$

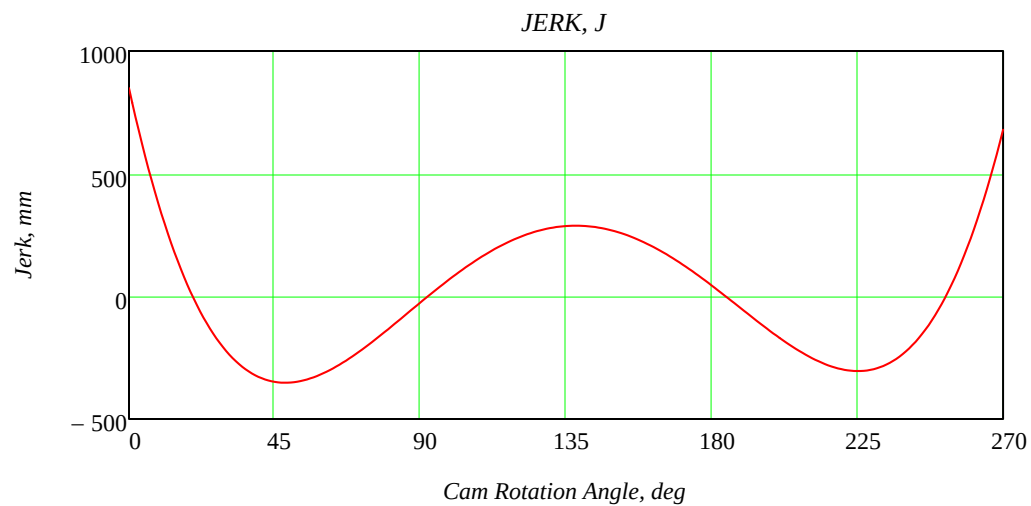
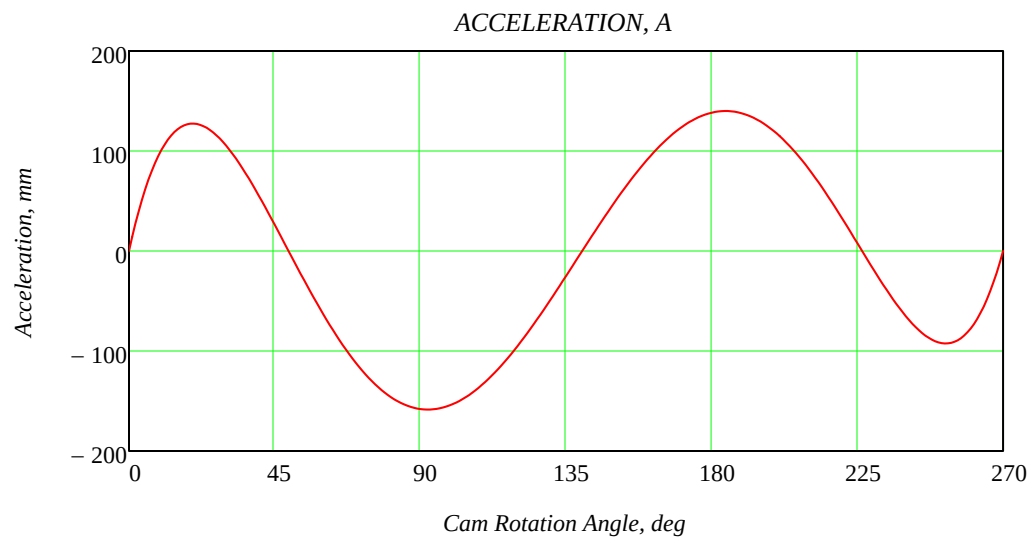
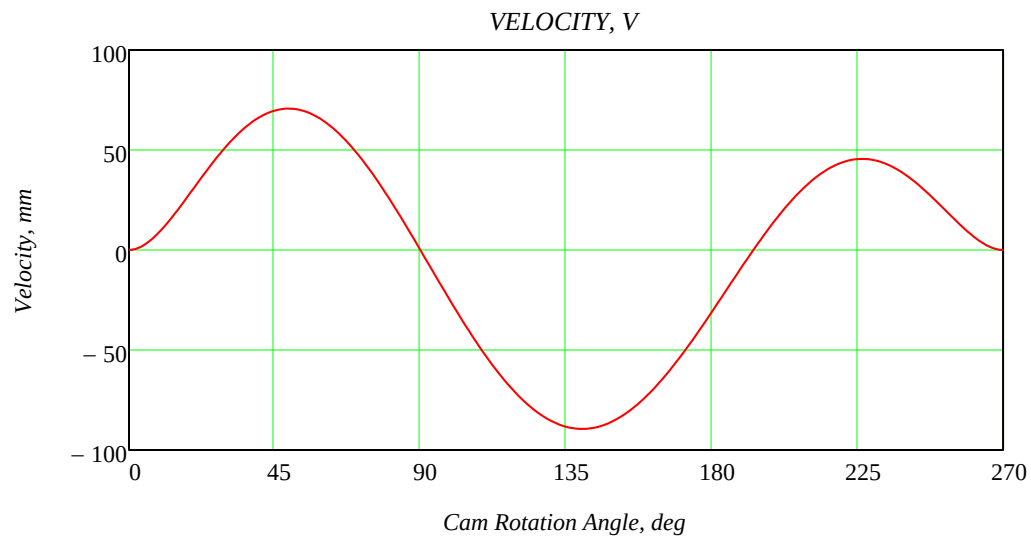
$$A(\theta) := \frac{1}{\beta^2} \cdot \left[6 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right) + 12 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^2 + 20 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^3 + 30 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^4 + 42 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^5 \right]$$

$$J(\theta) := \frac{1}{\beta^3} \cdot \left[6 \cdot c_3 + 24 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right) + 60 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^2 + 120 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^3 + 210 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^4 \right]$$

6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 \leq \theta \leq \beta$.

$$\theta := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. \beta$$





PROBLEM 8-48

Statement: Design a cam to move a follower at a constant velocity of 200 mm/sec for 3 sec then return to its starting position with a total cycle time of 6 sec.

Given: Constant velocity: $v_c := 200 \cdot \text{mm} \cdot \text{sec}^{-1}$
 Time duration of cv segment: $t_{cv} := 3 \cdot \text{sec}$ Cycle time:
 $\tau := 6 \cdot \text{sec}$

Solution: See Mathcad file P0848.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.047 \frac{\text{rad}}{\text{sec}}$$

2. Use a two-segment polynomial as demonstrated in Example 8-12. The lift during the first segment and the *svaj* equations for the first segment are:

$$\begin{aligned} \text{Normalized velocity: } v_{cv} &:= \frac{v_c}{\omega} & v_{cv} &= 190.986 \text{ mm} \\ h_{cv} &:= v_c \cdot t_{cv} & h_{cv} &= 600.000 \text{ mm} & \beta_1 &:= \frac{t_{cv}}{\tau} \cdot 360 \cdot \text{deg} & \beta_1 &= 180 \text{ deg} \\ s_1(\theta) &:= h_{cv} \cdot \frac{\theta}{\beta_1} & v_1(\theta) &:= v_{cv} & a_1(\theta) &:= 0 \cdot \text{mm} & j_1(\theta) &:= 0 \cdot \text{mm} \end{aligned}$$

3. The boundary conditions for the second segment are:

$$\begin{aligned} \text{at } \theta = \beta_1: & \quad s = h_{cv}, \quad v = v_{cv}, \quad a = 0 \\ \theta = 360 \text{ deg:} & \quad s = 0, \quad v = v_{cv}, \quad a = 0 \end{aligned}$$

This is a minimum set of 6 BCs. Define the total interval and the constant velocity interval, and the ratio of constant velocity interval to the total interval.

$$\begin{aligned} \text{Total interval: } \beta &:= 360 \cdot \text{deg} \\ \text{CV interval: } \beta_1 &= 180 \text{ deg} & A &:= \frac{\beta_1}{\beta} & A &= 0.500 \end{aligned}$$

4. Use the 6 BCs and equation 8.23 to write 6 equations in s , v , and a similar to those in example 8-9 but with 6 terms in the equation for s (the highest term will be fifth degree).

$$\text{For } \theta = \beta_1: \quad s = h_{cv}, \quad v = v_{cv}, \quad a = 0$$

$$h_{cv} = c_0 + c_1 \cdot A + c_2 \cdot A^2 + c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5$$

$$v_{cv} = \frac{1}{\beta} \cdot (c_1 + 2 \cdot c_2 \cdot A + 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4)$$

$$0 = 2 \cdot c_2 + 6 \cdot c_3 \cdot A + 12 \cdot c_4 \cdot A^2 + 20 \cdot c_5 \cdot A^3$$

$$\text{For } \theta = \beta: \quad s = 0, \quad v = v_{cv}, \quad a = 0$$

$$0 = c_0 + c_1 + c_2 + c_3 + c_4 + c_5$$

$$0 = \frac{1}{\beta} \cdot (c_1 + 2 \cdot c_2 + 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5)$$

$$0 = 2 \cdot c_2 + 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5$$

5. Solve for the unknown polynomial coefficients.

$$C := \begin{pmatrix} 1 & A & A^2 & A^3 & A^4 & A^5 \\ 0 & 1 & 2 \cdot A & 3 \cdot A^2 & 4 \cdot A^3 & 5 \cdot A^4 \\ 0 & 0 & 2 & 6 \cdot A & 12 \cdot A^2 & 20 \cdot A^3 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 2 & 6 & 12 & 20 \end{pmatrix} \quad H := \begin{pmatrix} h_{cv} \\ \beta \cdot v_{cv} \\ 0 \\ 0 \\ \beta \cdot v_{cv} \\ 0 \end{pmatrix} \quad \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix} := C^{-1} \cdot H$$

$$\begin{aligned} c_0 &= 3.720 \times 10^4 \text{ mm} & c_1 &= -2.868 \times 10^5 \text{ mm} & c_2 &= 8.640 \times 10^5 \text{ mm} \\ c_3 &= -1.248 \times 10^6 \text{ mm} & c_4 &= 8.64 \times 10^5 \text{ mm} & c_5 &= -2.304 \times 10^5 \text{ mm} \end{aligned}$$

6. Write the *svaj* equations for the second segment.

$$\begin{aligned} s_2(\theta) &:= c_0 + c_1 \cdot \left(\frac{\theta}{\beta}\right) + c_2 \cdot \left(\frac{\theta}{\beta}\right)^2 + c_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + c_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + c_5 \cdot \left(\frac{\theta}{\beta}\right)^5 \\ v_2(\theta) &:= \frac{1}{\beta} \cdot \left[c_1 + 2 \cdot c_2 \cdot \left(\frac{\theta}{\beta}\right) + 3 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^3 + 5 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^4 \right] \\ a_2(\theta) &:= \frac{1}{\beta^2} \cdot \left[2 \cdot c_2 + 6 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right) + 12 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^2 + 20 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^3 \right] \\ j_2(\theta) &:= \frac{1}{\beta^3} \cdot \left[6 \cdot c_3 + 24 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right) + 60 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^2 \right] \end{aligned}$$

7. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(\theta, a, b) := \text{if}[(\theta > a) \wedge (\theta \leq b), 1, 0]$$

$$S(\theta) := R(\theta, 0, \beta_1) \cdot s_1(\theta) + R(\theta, \beta_1, \beta) \cdot s_2(\theta)$$

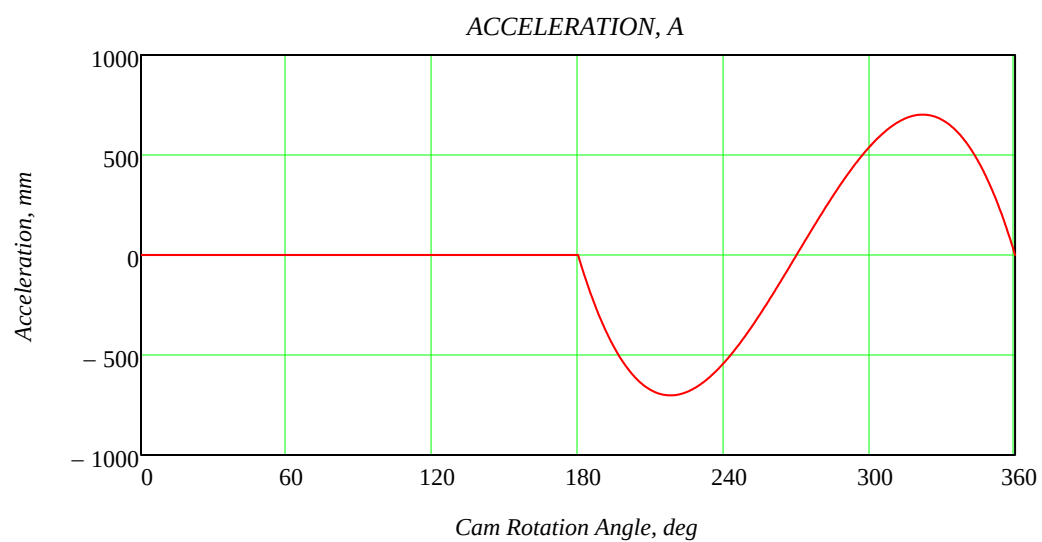
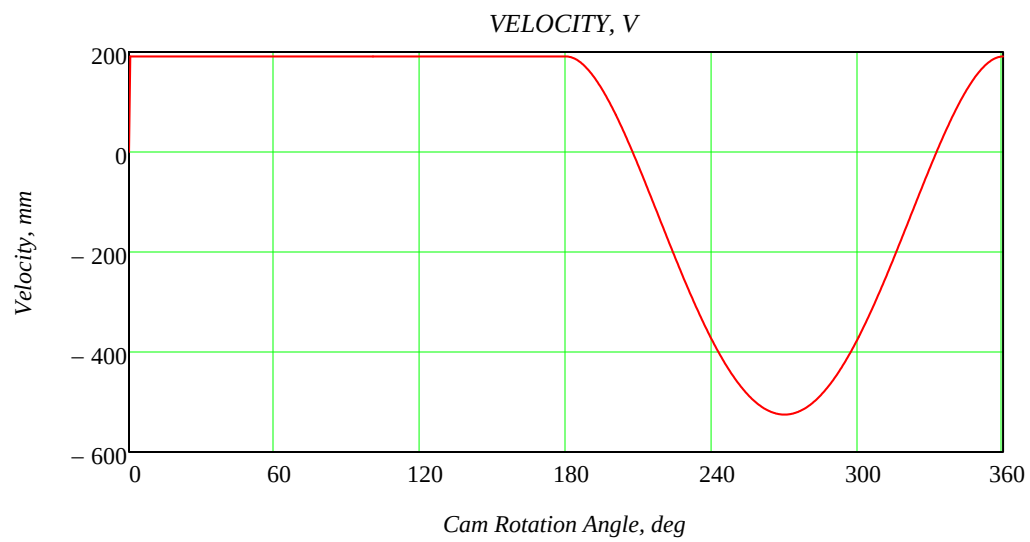
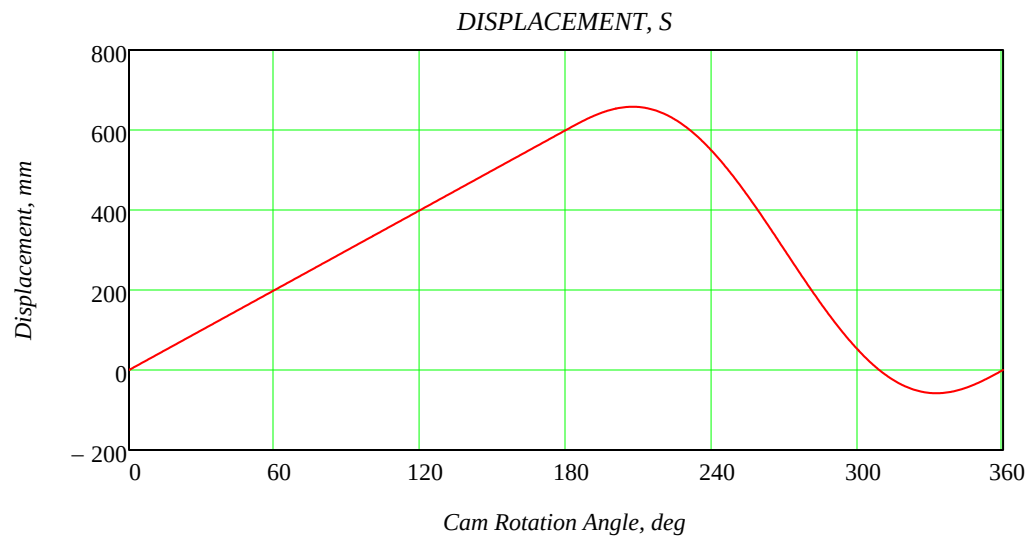
$$V(\theta) := R(\theta, 0, \beta_1) \cdot v_1(\theta) + R(\theta, \beta_1, \beta) \cdot v_2(\theta)$$

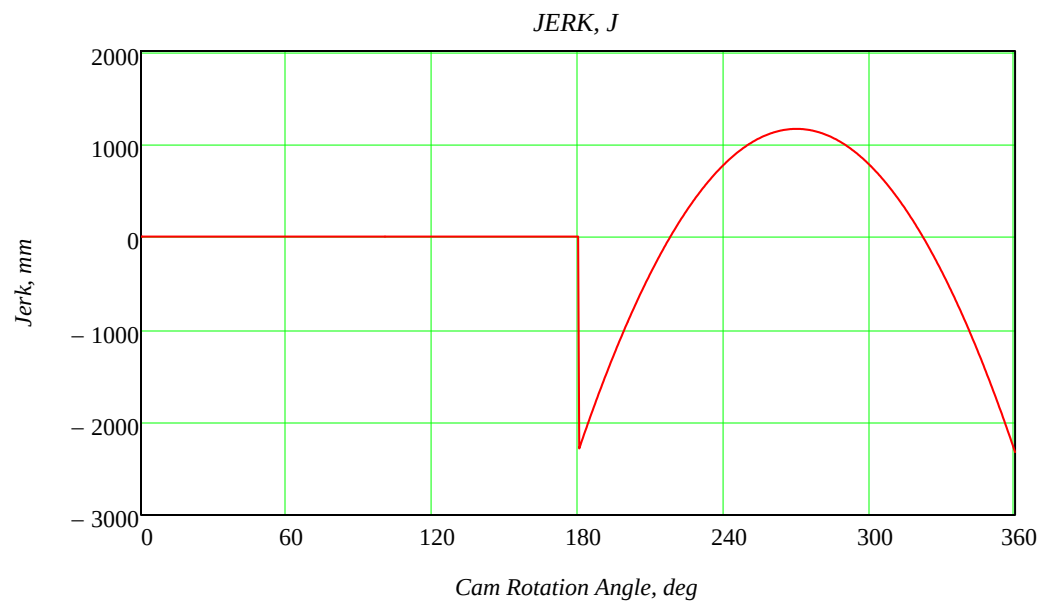
$$A(\theta) := R(\theta, 0, \beta_1) \cdot a_1(\theta) + R(\theta, \beta_1, \beta) \cdot a_2(\theta)$$

$$J(\theta) := R(\theta, 0, \beta_1) \cdot j_1(\theta) + R(\theta, \beta_1, \beta) \cdot j_2(\theta)$$

8. Plot the displacement, velocity, acceleration, and jerk over the interval $0 \leq \theta \leq \beta$.

$$\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. \beta$$





PROBLEM 8-49

Statement: Size the cam from Problem 8-42 for a flat-faced follower considering follower face width and radius of curvature. Plot the radius of curvature and draw the cam profile.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: RISE/FALL DWELL

$\beta := 195 \cdot deg$ $\beta_3 := 165 \cdot deg$

$h := 35 \cdot mm$ $h_3 := 0.0 \cdot mm$

Cycle time: $\tau := 3 \cdot sec$

Solution: See Mathcad file P0849.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \quad \omega = 2.094 \frac{rad}{sec} \quad \omega = 20.000 \text{ rpm}$$

2. Problem 8-42 used a two-segment polynomial with the rise and fall together in one segment and the dwell in the second segment. Enter the above data into program DYNACAM. The input screen is shown below.

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Input Screen**

Cam Data

Cam Omega (rad/sec): 2.094

No. Segmts: 2

Delta Theta (deg): 1

Follower: ☒ Trans ☐ Oscit

Buttons: Calc All, Clear Segs, Clear All, Copy, Print Form, Done, Cancel

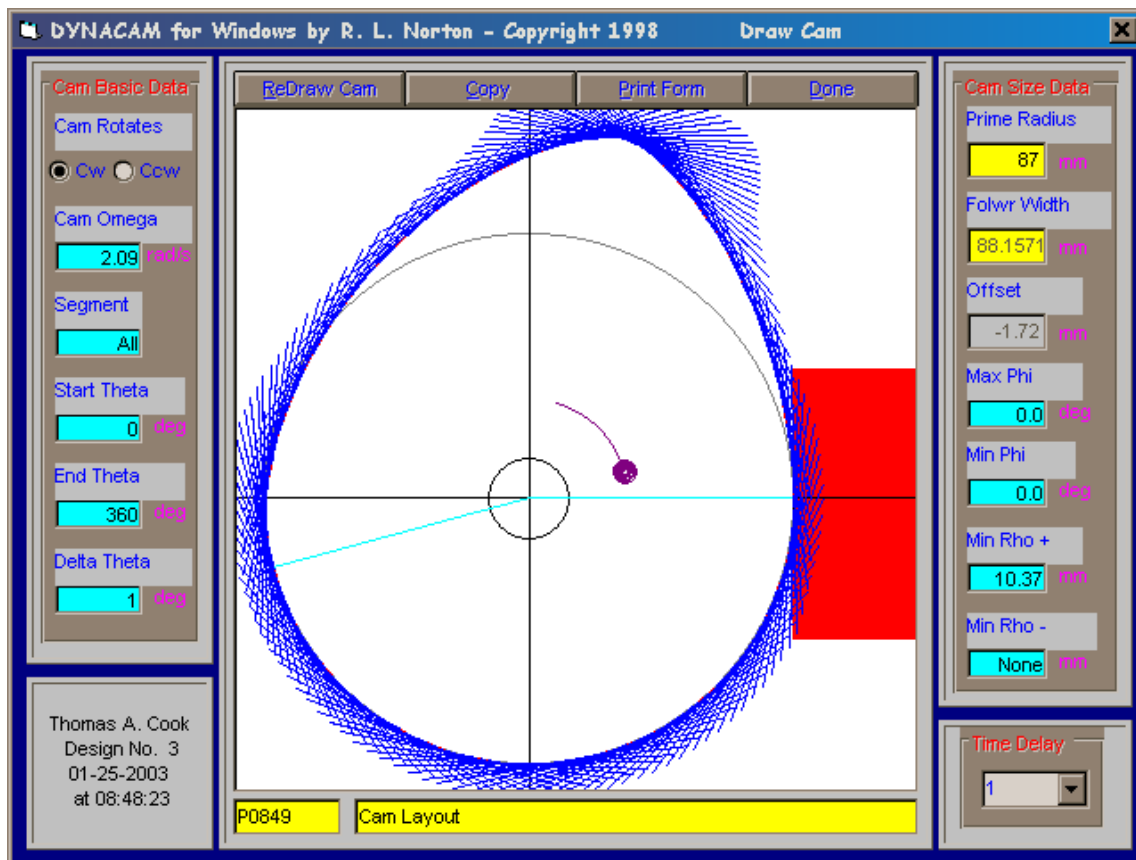
Segment Data

Seg	Angles			Motion	Program	Position (mm)	
	Beta	Start	End			Start	End
1	195	0	195	Poly	PO - Polynomial	0	0
2	165	195	360	Dwell	DW - Dwell	0	0
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

Examples

Buttons: Calc, Plot, Print

3. The cam was sized iteratively to have the smallest base circle diameter while having no negative curvature and at least a 10 mm positive curvature. The resulting cam is shown below.

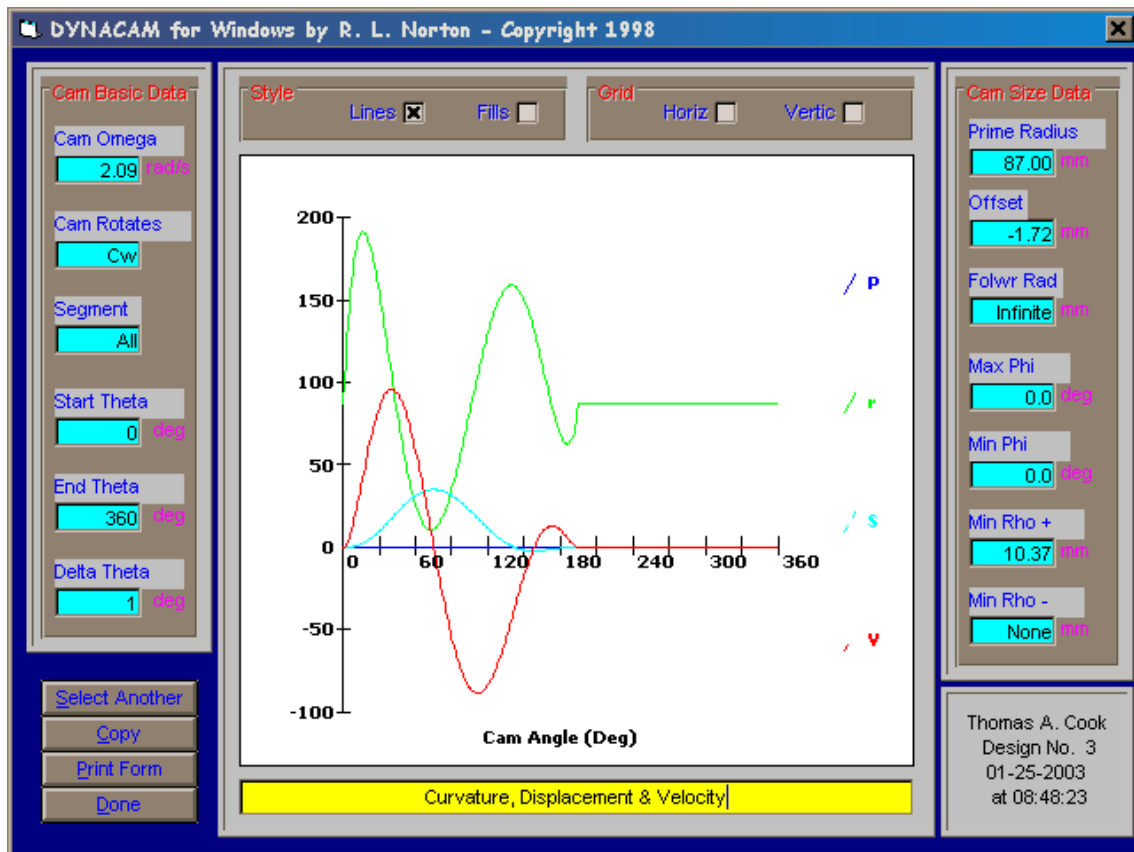


4. The minimum and maximum radius of curvature are shown in the figure above. The design has the following dimensions:

Base circle radius $R_b := 87 \cdot \text{mm}$

Follower width $\text{facewidth} := 89 \cdot \text{mm}$

5. Graphs of ρ , s , and v for the flat-faced follower are shown on the following page.



PROBLEM 8-50

Statement: Size the cam from Problem 8-44 for a translating flat-faced follower considering follower face width and radius of curvature. Plot the radius of curvature and draw the cam profile.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 75 \cdot \text{deg}$	$\beta_2 := 75 \cdot \text{deg}$	$\beta_3 := 75 \cdot \text{deg}$	$\beta_4 := 135 \cdot \text{deg}$
$h_1 := 50 \cdot \text{mm}$	$h_2 := 0 \cdot \text{mm}$	$h_3 := 50 \cdot \text{mm}$	$h_4 := 0 \cdot \text{mm}$
Cycle time: $\tau := 5 \cdot \text{sec}$			

Solution: See Mathcad file P0850.

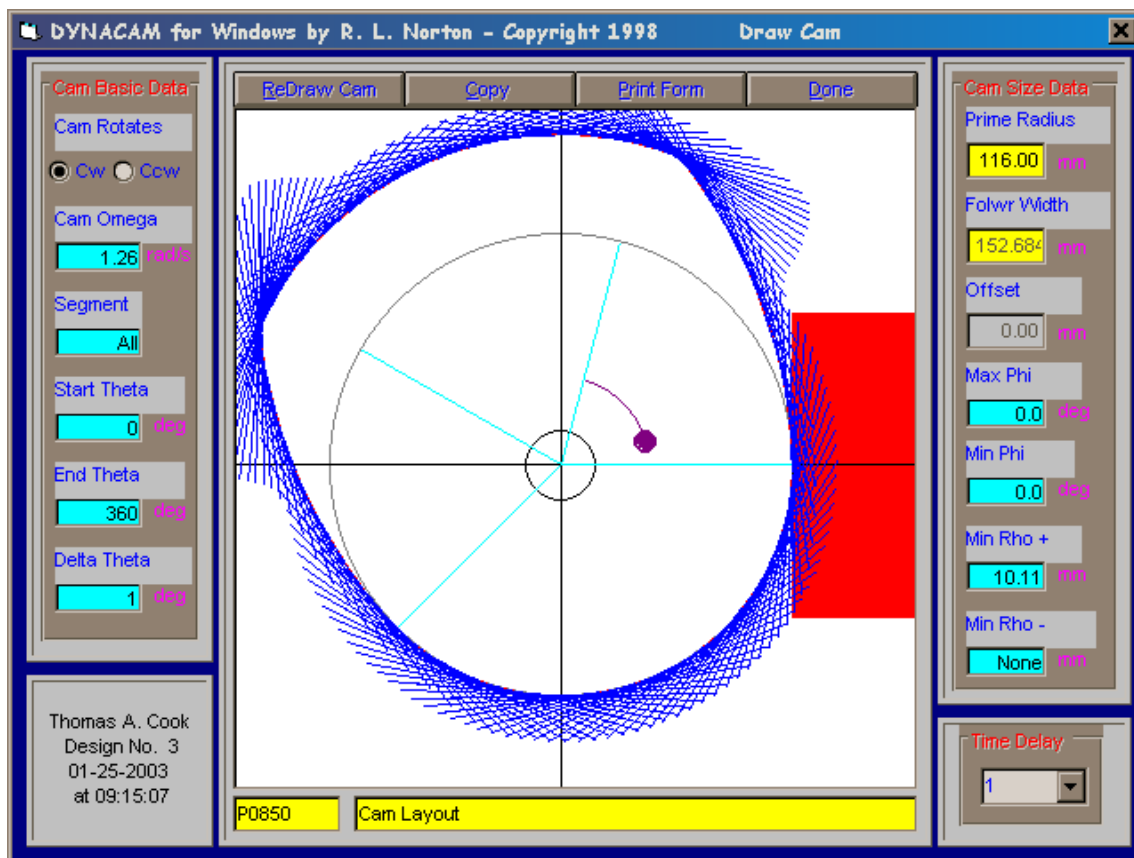
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.257 \frac{\text{rad}}{\text{sec}}$$

2. Problem 8-44 used modified trapezoidal rise and fall segments with two dwell segments. Enter the above data into program DYNACAM. The input screen is shown below.

Segment Data				Cam Contour		Position (mm)	
Seg	Beta	Start	End	Motion	Program	Start	End
1	75	0	75	Rise	MT - Mod Trap	0	50
2	75	75	150	Dwell	DW - Dwell	50	
3	75	150	225	Fall	MT - Mod Trap	50	0
4	135	225	360	Dwell	DW - Dwell	0	
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

3. The cam was sized iteratively to have the smallest base circle diameter while having no negative curvature and at least a 10 mm positive curvature. The resulting cam is shown below.

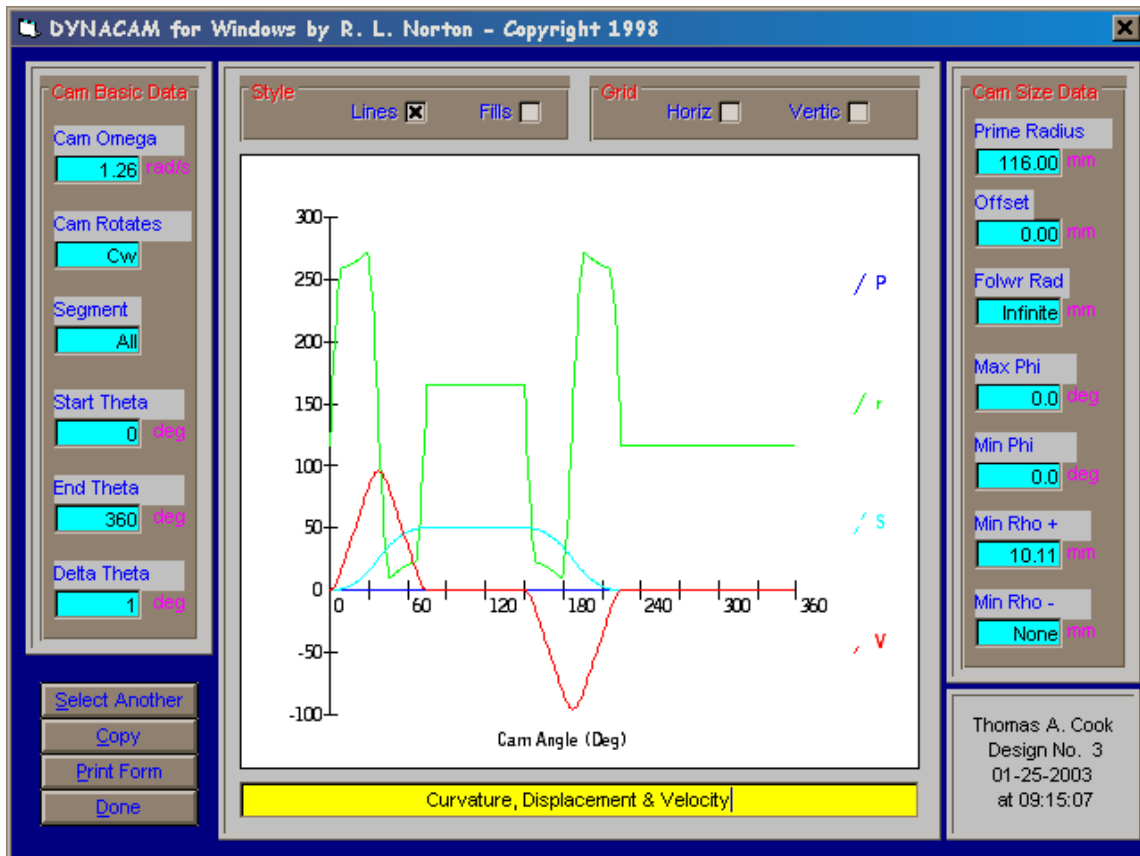


4. The minimum and maximum radius of curvature are shown in the figure above. The design has the following dimensions:

Base circle radius $R_b := 116 \cdot \text{mm}$

Follower width $\text{facewidth} := 153 \cdot \text{mm}$

5. Graphs of ρ , s , and v for the flat-faced follower are shown on the following page.



PROBLEM 8-51

Statement: Size the cam from Problem 8-45 for a translating flat-faced follower considering follower face width and radius of curvature. Plot the radius of curvature and draw the cam profile.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 75 \cdot \text{deg}$	$\beta_2 := 75 \cdot \text{deg}$	$\beta_3 := 75 \cdot \text{deg}$	$\beta_4 := 135 \cdot \text{deg}$
$h_1 := 50 \cdot \text{mm}$	$h_2 := 0 \cdot \text{mm}$	$h_3 := 50 \cdot \text{mm}$	$h_4 := 0 \cdot \text{mm}$
Cycle time: $\tau := 5 \cdot \text{sec}$			

Solution: See Mathcad file P0851.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.257 \frac{\text{rad}}{\text{sec}}$$

2. Problem 8-45 used modified sinusoidal rise and fall segments with two dwell segments. Enter the above data into program DYNACAM. The input screen is shown below.

Cam Data

Cam Omega (rad/sec): 1.257

No. Segmts: 4

Delta Theta (deg): 1

Follower:

☒ Trans

☐ Oscit

Segment Data

Seg	Angles			Motion	Cam Contour Program	Position (mm)	
	Beta	Start	End			Start	End
1	75	0	75	Rise	MS - Mod Sine	0	50
2	75	75	150	Dwell	DW - Dwell	50	50
3	75	150	225	Fall	MS - Mod Sine	50	0
4	135	225	360	Dwell	DW - Dwell	0	0
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

Examples

Calc Plot Print

Calc Plot Print

Calc Plot Print

Calc Plot Print

Calc All

Clear Segs

Clear All

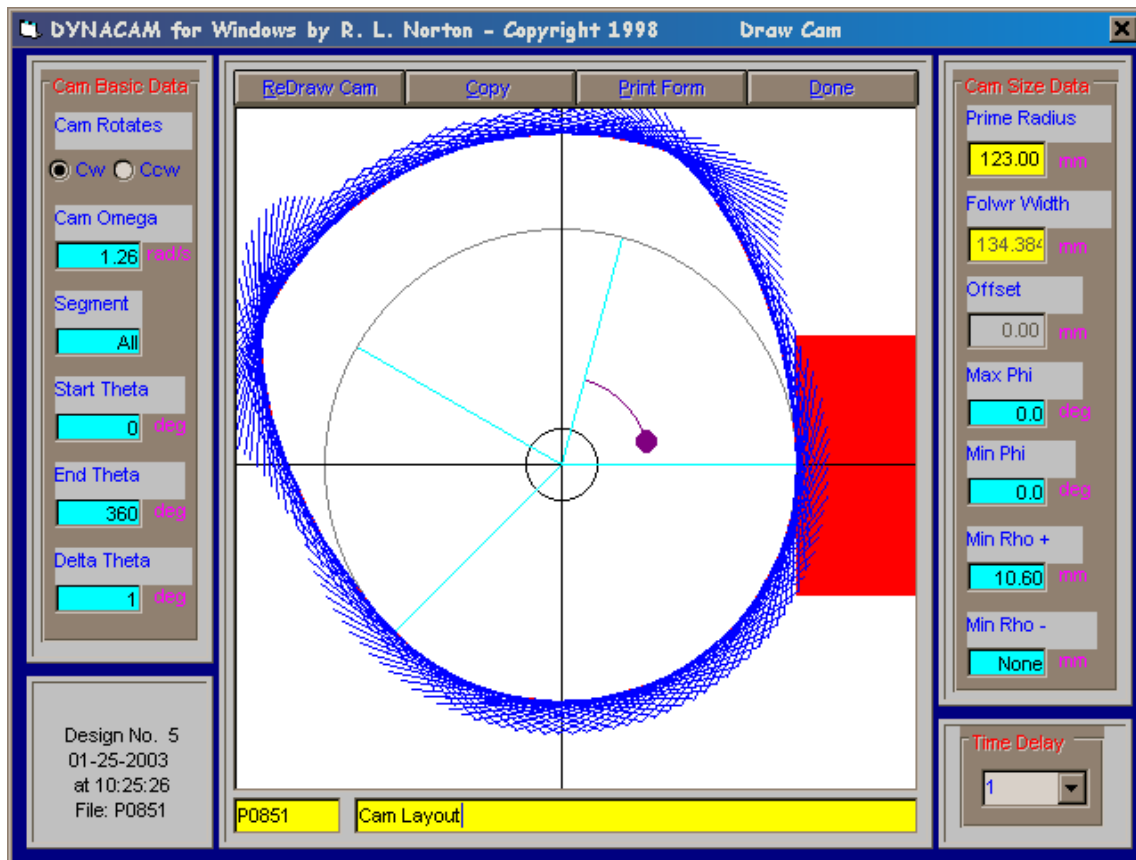
Copy

Print Form

Done

Cancel

3. The cam was sized iteratively to have the smallest base circle diameter while having no negative curvature and at least a 10 mm positive curvature. The resulting cam is shown below.

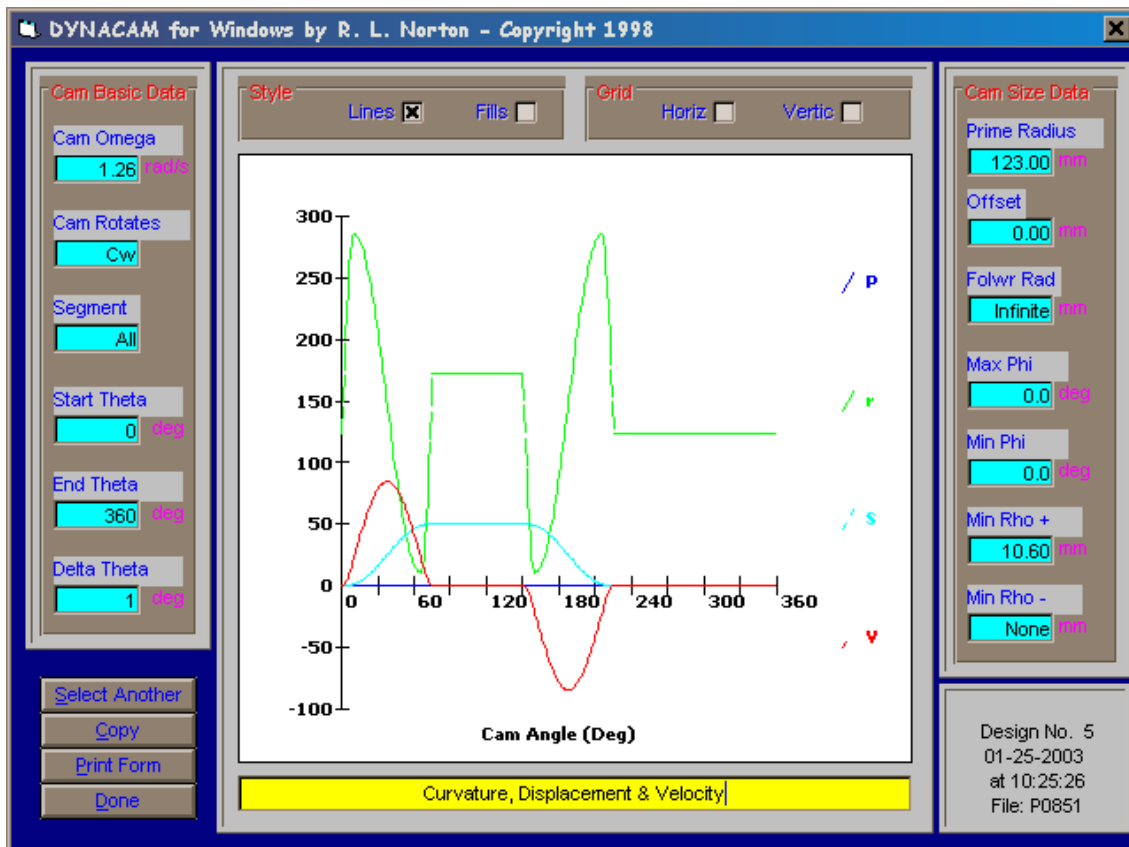


4. The minimum and maximum radius of curvature are shown in the figure above. The design has the following dimensions:

Base circle radius $R_b := 123 \cdot \text{mm}$

Follower width $\text{facewidth} := 135 \cdot \text{mm}$

5. Graphs of ρ , s , and v for the flat-faced follower are shown on the following page.



PROBLEM 8-52

Statement: Size the cam from Problem 8-46 for a translating flat-faced follower considering follower face width and radius of curvature. Plot the radius of curvature and draw the cam profile.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 75\text{-deg}$	$\beta_2 := 75\text{-deg}$	$\beta_3 := 75\text{-deg}$	$\beta_4 := 135\text{-deg}$
$h_1 := 50\text{-mm}$	$h_2 := 0\text{-mm}$	$h_3 := 50\text{-mm}$	$h_4 := 0\text{-mm}$
Cycle time: $\tau := 5\text{-sec}$			

Solution: See Mathcad file P0852.

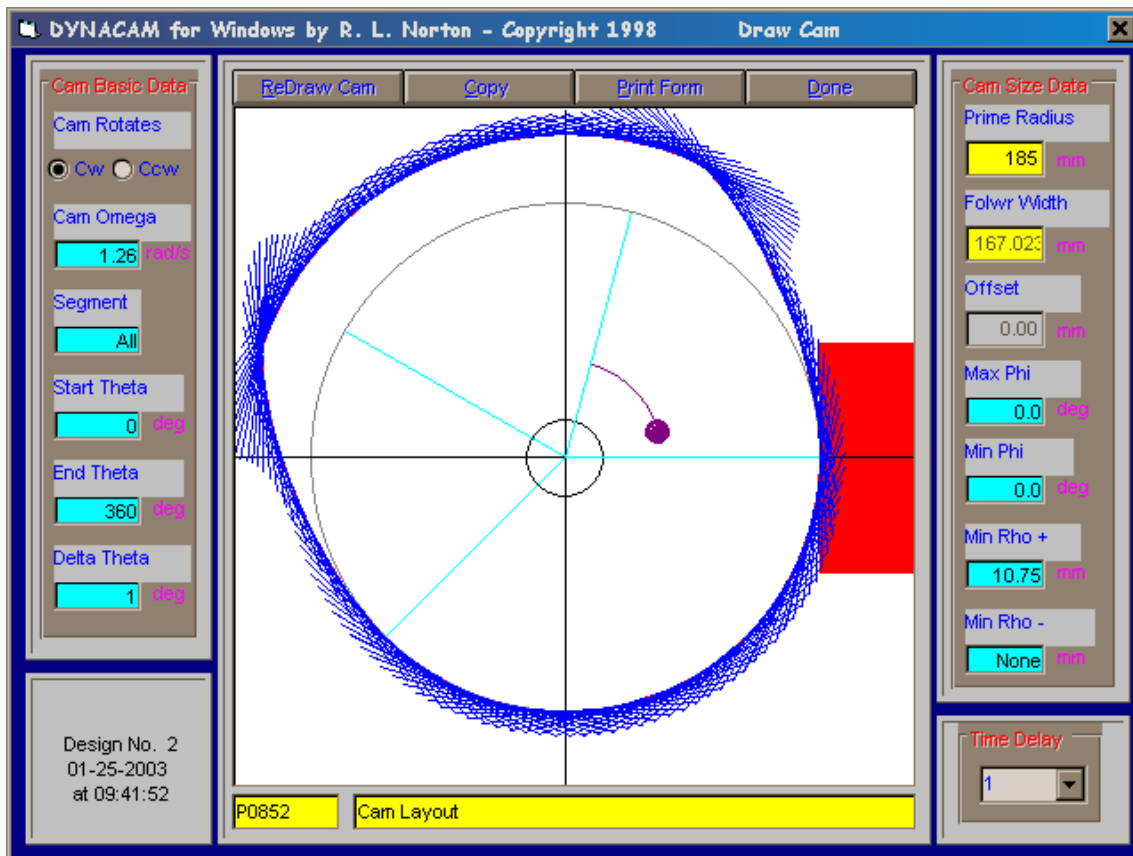
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.257 \frac{\text{rad}}{\text{sec}}$$

2. Problem 8-46 used 4-5-6-7 polynomials for the rise and fall segments with two dwell segments. Enter the above data into program DYNACAM. The input screen is shown below.

Segment Data		Angles		Cam Contour		Position (mm)	
Seg	Beta	Start	End	Motion	Program	Start	End
1	75	0	75	Poly	PO - Polynomial	0	50
2	75	75	150	Poly	PO - Polynomial	50	50
3	75	150	225	Poly	PO - Polynomial	50	0
4	135	225	360	Poly	PO - Polynomial	0	0
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

3. The cam was sized iteratively to have the smallest base circle diameter while having no negative curvature and at least a 10 mm positive curvature. The resulting cam is shown below.

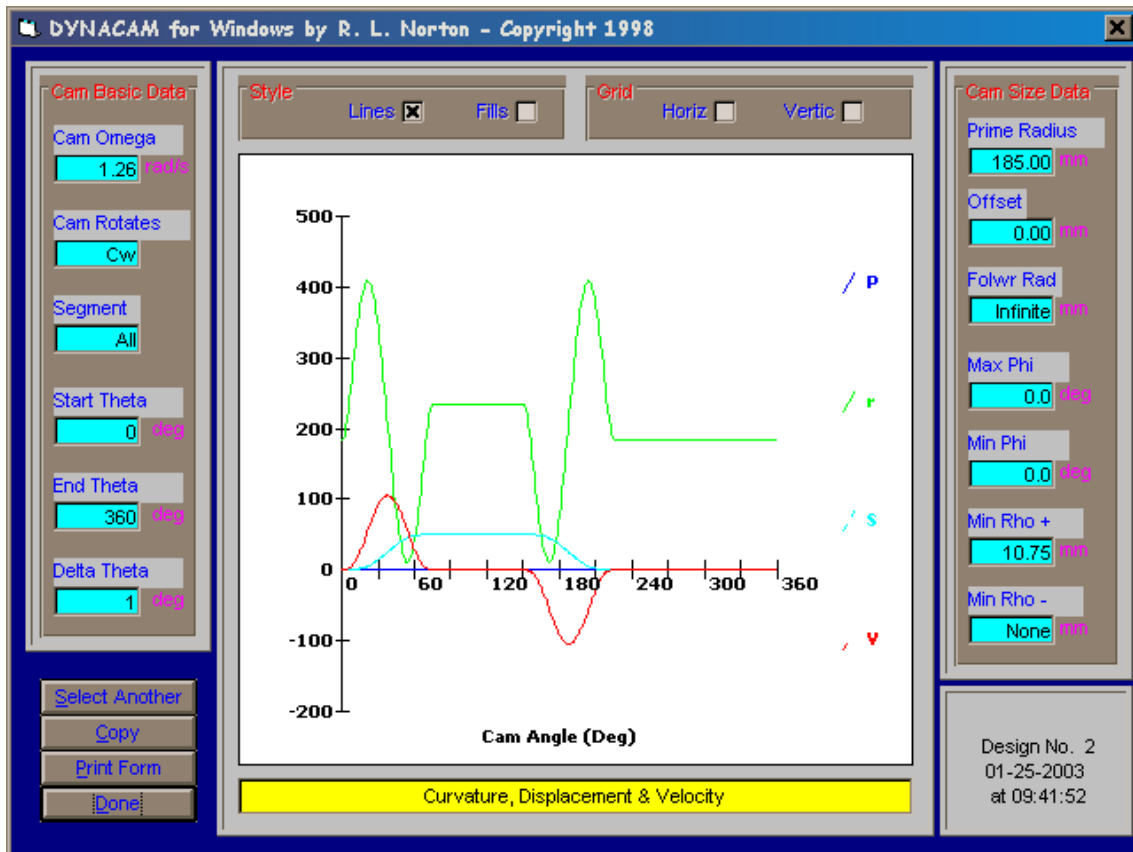


4. The minimum and maximum radius of curvature are shown in the figure above. The design has the following dimensions:

Base circle radius $R_b := 185 \cdot \text{mm}$

Follower width $\text{facewidth} := 168 \cdot \text{mm}$

5. Graphs of ρ , s , and v for the flat-faced follower are shown on the following page.



PROBLEM 8-53

Statement: Design a single-dwell cam to move a follower from 0 to 50 mm in 100 deg, fall 50 mm in 120 deg and dwell for the remainder. The total cycle must take 1 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the SVAJ diagrams.

Given:

RISE/FALL	DWELL
$\beta := 220 \cdot \text{deg}$	$\beta_3 := 140 \cdot \text{deg}$
$h := 50 \cdot \text{mm}$	$h_3 := 0.0 \cdot \text{mm}$
Cycle time: $\tau := 1 \cdot \text{sec}$	

Solution: See Mathcad file P0853.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 6.283 \frac{\text{rad}}{\text{sec}}$$

2. Use a two-segment polynomial. Let the rise and fall, together, be one segment and the dwell be the second segment. Then, the boundary conditions are:

$$\begin{aligned} \text{at } \theta = 0: \quad & s = 0, \quad v = 0, \quad a = 0 \\ \theta = \beta_1: \quad & s = h, \quad v = 0 \\ \theta = \beta: \quad & s = 0, \quad v = 0, \quad a = 0 \end{aligned}$$

This is a minimum set of 8 BCs. The $v = 0$ condition at $\theta = \beta_1$ is required to keep the displacement from overshooting the lift, h . Define the total lift, the rise interval, the fall interval, and the ratio of rise to the total interval.

$$\text{Total lift:} \quad h = 50.000 \text{ mm}$$

$$\text{Rise interval:} \quad \beta_1 := 100 \cdot \text{deg} \quad A := \frac{\beta_1}{\beta} \quad A = 0.455$$

$$\text{Fall interval:} \quad \beta_2 := 120 \cdot \text{deg}$$

3. Use the 8 BCs and equation 8.23 to write 8 equations in s , v , and a similar to those in example 8-9 but with 8 terms in the equation for s (the highest term will be seventh degree).

$$\text{For } \theta = 0: \quad s = v = a = 0$$

$$0 := c_0^{\blacksquare} \quad 0 := c_1^{\blacksquare} \quad 0 := c_2^{\blacksquare}$$

$$\text{For } \theta = \beta_1: \quad s = h, \quad v = 0$$

$$h := c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5 + c_6 \cdot A^6 + c_7 \cdot A^7^{\blacksquare}$$

$$0 := 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4 + 6 \cdot c_6 \cdot A^5 + 7 \cdot c_7 \cdot A^6^{\blacksquare}$$

$$\text{For } \theta = \beta: \quad s = v = a = 0$$

$$0 := c_3 + c_4 + c_5 + c_6 + c_7^{\blacksquare}$$

$$0 := 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5 + 6 \cdot c_6 + 7 \cdot c_7^{\blacksquare}$$

$$0 := 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5 + 30 \cdot c_6 + 42 \cdot c_7^{\blacksquare}$$

4. Solve for the unknown polynomial coefficients. Note that C_0 through C_2 are zero

$$C := \begin{pmatrix} A^3 & A^4 & A^5 & A^6 & A^7 \\ 3 \cdot A^2 & 4 \cdot A^3 & 5 \cdot A^4 & 6 \cdot A^5 & 7 \cdot A^6 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 & 7 \\ 6 & 12 & 20 & 30 & 42 \end{pmatrix} \quad H := \begin{pmatrix} h \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{pmatrix} := C^{-1} \cdot H$$

$$c_3 = 193.740 \text{ in}$$

$$c_4 = -723.297 \text{ in}$$

$$c_5 = 1007.449 \text{ in}$$

$$c_6 = -619.969 \text{ in}$$

$$c_7 = 142.076 \text{ in}$$

5. Write the SVAJ equations for the rise/fall segment.

$$s(\theta) := c_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + c_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + c_5 \cdot \left(\frac{\theta}{\beta}\right)^5 + c_6 \cdot \left(\frac{\theta}{\beta}\right)^6 + c_7 \cdot \left(\frac{\theta}{\beta}\right)^7$$

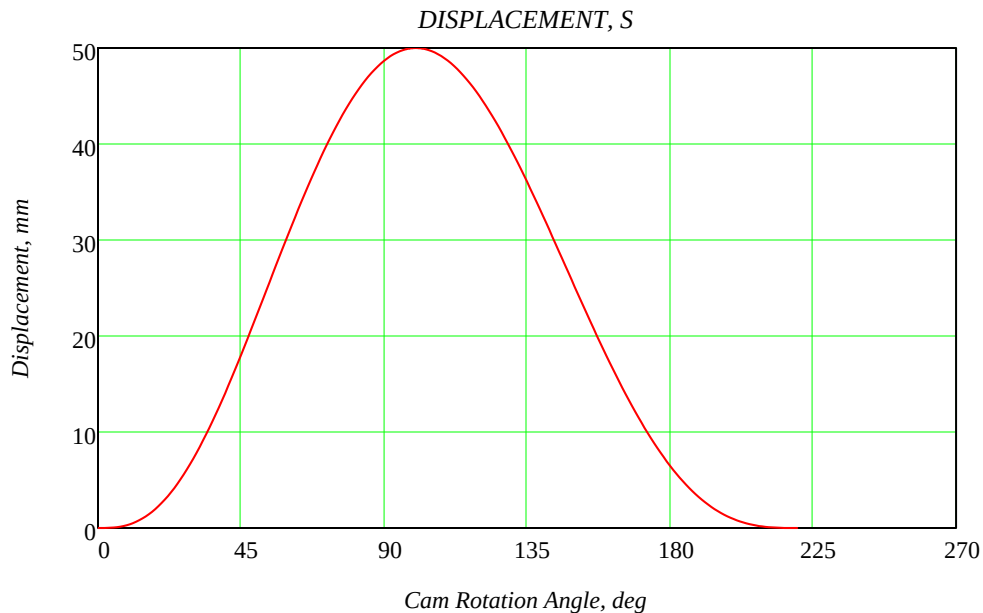
$$v(\theta) := \frac{1}{\beta} \cdot \left[3 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^3 + 5 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^4 + 6 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^5 + 7 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^6 \right]$$

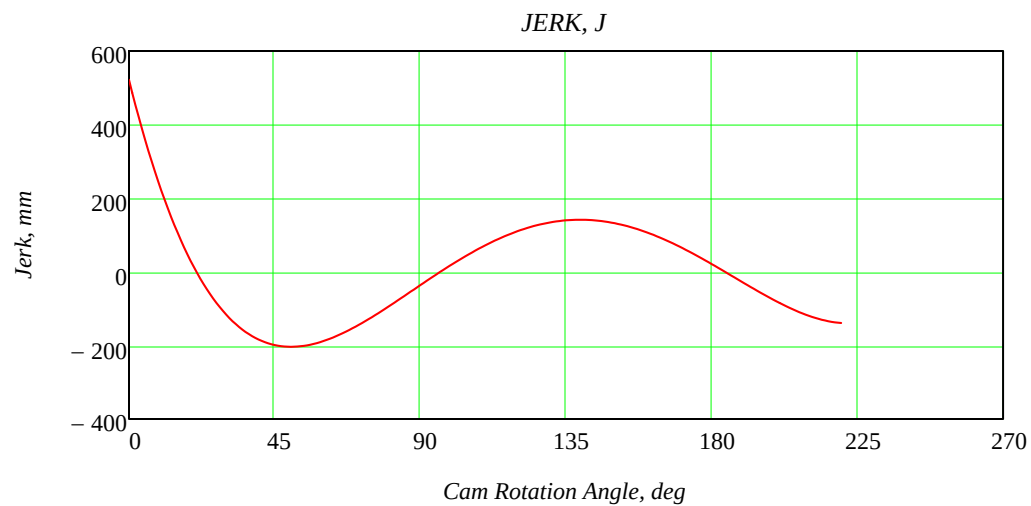
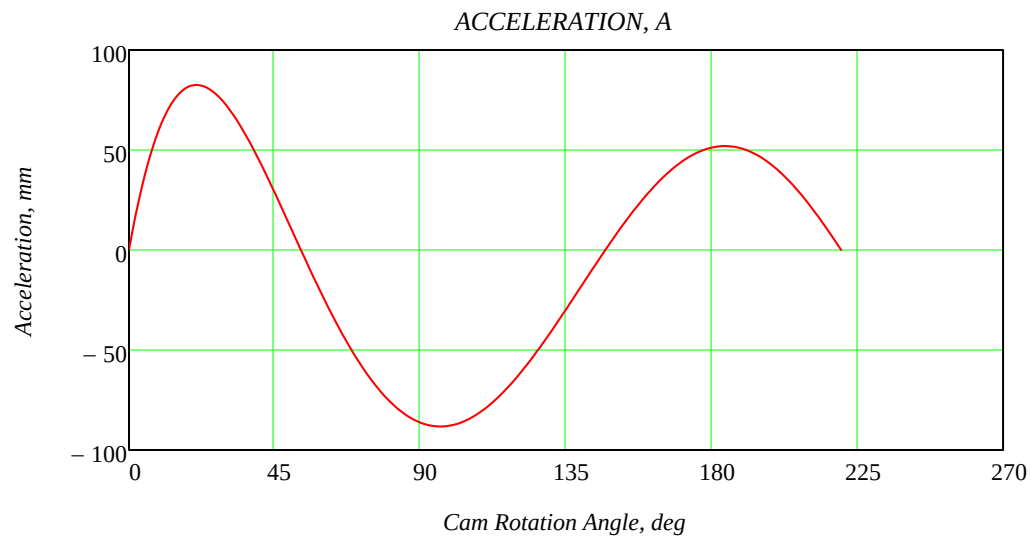
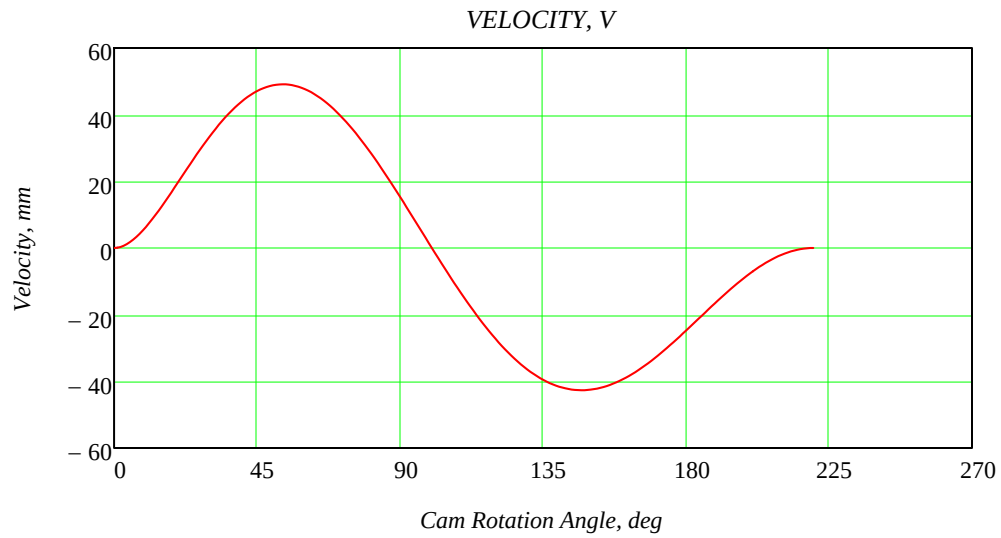
$$A(\theta) := \frac{1}{\beta^2} \cdot \left[6 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right) + 12 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^2 + 20 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^3 + 30 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^4 + 42 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^5 \right]$$

$$J(\theta) := \frac{1}{\beta^3} \cdot \left[6 \cdot c_3 + 24 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right) + 60 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^2 + 120 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^3 + 210 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^4 \right]$$

6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 \leq \theta \leq \beta$.

$$\theta := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. \beta$$





PROBLEM 8-54

Statement: Design a cam to move a follower at a constant velocity of 300 mm/sec for 2 sec then return to its starting position with a total cycle time of 4 sec.

Given: Constant velocity: $v_c := 300 \cdot \text{mm} \cdot \text{sec}^{-1}$
 Time duration of cv segment: $t_{cv} := 2 \cdot \text{sec}$ Cycle time:
 $\tau := 4 \cdot \text{sec}$

Solution: See Mathcad file P0854.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.571 \frac{\text{rad}}{\text{sec}}$$

2. Use a two-segment polynomial as demonstrated in Example 8-12. The lift during the first segment and the *svaj* equations for the first segment are:

$$\begin{aligned} \text{Normalized velocity: } v_{cv} &:= \frac{v_c}{\omega} & v_{cv} &= 190.986 \text{ mm} \\ h_{cv} &:= v_c \cdot t_{cv} & h_{cv} &= 600.000 \text{ mm} & \beta_1 &:= \frac{t_{cv}}{\tau} \cdot 360 \cdot \text{deg} & \beta_1 &= 180 \text{ deg} \\ s_1(\theta) &:= h_{cv} \cdot \frac{\theta}{\beta_1} & v_1(\theta) &:= v_{cv} & a_1(\theta) &:= 0 \cdot \text{mm} & j_1(\theta) &:= 0 \cdot \text{mm} \end{aligned}$$

2. The boundary conditions for the second segment are:

$$\begin{aligned} \text{at } \theta = \beta_1: & \quad s = h_{cv}, \quad v = v_{cv}, \quad a = 0 \\ \theta = 360 \text{ deg}: & \quad s = 0, \quad v = v_{cv}, \quad a = 0 \end{aligned}$$

This is a minimum set of 6 BCs. Define the total interval and the constant velocity interval, and the ratio of constant velocity interval to the total interval.

$$\begin{aligned} \text{Total interval: } \beta &:= 360 \cdot \text{deg} \\ \text{CV interval: } \beta_1 &= 180 \text{ deg} & A &:= \frac{\beta_1}{\beta} & A &= 0.500 \end{aligned}$$

3. Use the 6 BCs and equation 8.23 to write 6 equations in s , v , and a similar to those in example 8-9 but with 6 terms in the equation for s (the highest term will be fifth degree).

$$\text{For } \theta = \beta_1: \quad s = h_{cv}, \quad v = v_{cv}, \quad a = 0$$

$$h_{cv} = c_0 + c_1 \cdot A + c_2 \cdot A^2 + c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5$$

$$v_{cv} = \frac{1}{\beta} \cdot (c_1 + 2 \cdot c_2 \cdot A + 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4)$$

$$0 = 2 \cdot c_2 + 6 \cdot c_3 \cdot A + 12 \cdot c_4 \cdot A^2 + 20 \cdot c_5 \cdot A^3$$

$$\text{For } \theta = \beta: \quad s = 0, \quad v = v_{cv}, \quad a = 0$$

$$0 = c_0 + c_1 + c_2 + c_3 + c_4 + c_5$$

$$0 = \frac{1}{\beta} \cdot (c_1 + 2 \cdot c_2 + 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5)$$

$$0 = 2 \cdot c_2 + 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5$$

4. Solve for the unknown polynomial coefficients.

$$C := \begin{pmatrix} 1 & A & A^2 & A^3 & A^4 & A^5 \\ 0 & 1 & 2 \cdot A & 3 \cdot A^2 & 4 \cdot A^3 & 5 \cdot A^4 \\ 0 & 0 & 2 & 6 \cdot A & 12 \cdot A^2 & 20 \cdot A^3 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 2 & 6 & 12 & 20 \end{pmatrix} \quad H := \begin{pmatrix} h_{cv} \\ \beta \cdot v_{cv} \\ 0 \\ 0 \\ \beta \cdot v_{cv} \\ 0 \end{pmatrix} \quad \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix} := C^{-1} \cdot H$$

$$\begin{aligned} c_0 &= 3.720 \times 10^4 \text{ mm} & c_1 &= -2.868 \times 10^5 \text{ mm} & c_2 &= 8.640 \times 10^5 \text{ mm} \\ c_3 &= -1.248 \times 10^6 \text{ mm} & c_4 &= 8.64 \times 10^5 \text{ mm} & c_5 &= -2.304 \times 10^5 \text{ mm} \end{aligned}$$

5. Write the *svaj* equations for the second segment.

$$\begin{aligned} s_2(\theta) &:= c_0 + c_1 \cdot \left(\frac{\theta}{\beta}\right) + c_2 \cdot \left(\frac{\theta}{\beta}\right)^2 + c_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + c_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + c_5 \cdot \left(\frac{\theta}{\beta}\right)^5 \\ v_2(\theta) &:= \frac{1}{\beta} \cdot \left[c_1 + 2 \cdot c_2 \cdot \left(\frac{\theta}{\beta}\right) + 3 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^3 + 5 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^4 \right] \\ a_2(\theta) &:= \frac{1}{\beta^2} \cdot \left[2 \cdot c_2 + 6 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right) + 12 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^2 + 20 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^3 \right] \\ j_2(\theta) &:= \frac{1}{\beta^3} \cdot \left[6 \cdot c_3 + 24 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right) + 60 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^2 \right] \end{aligned}$$

4. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(\theta, a, b) := \text{if}[(\theta > a) \wedge (\theta \leq b), 1, 0]$$

$$S(\theta) := R(\theta, 0, \beta_1) \cdot s_1(\theta) + R(\theta, \beta_1, \beta) \cdot s_2(\theta)$$

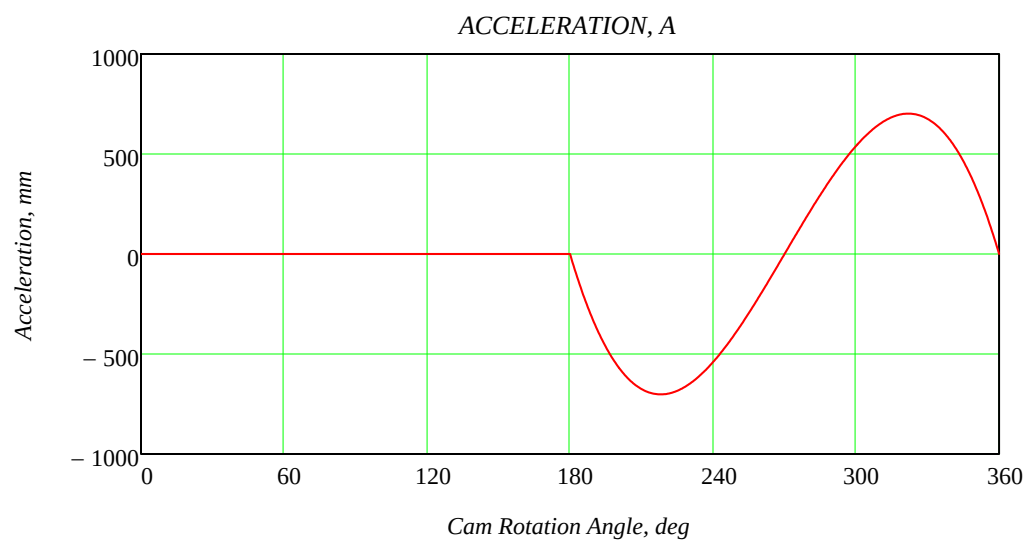
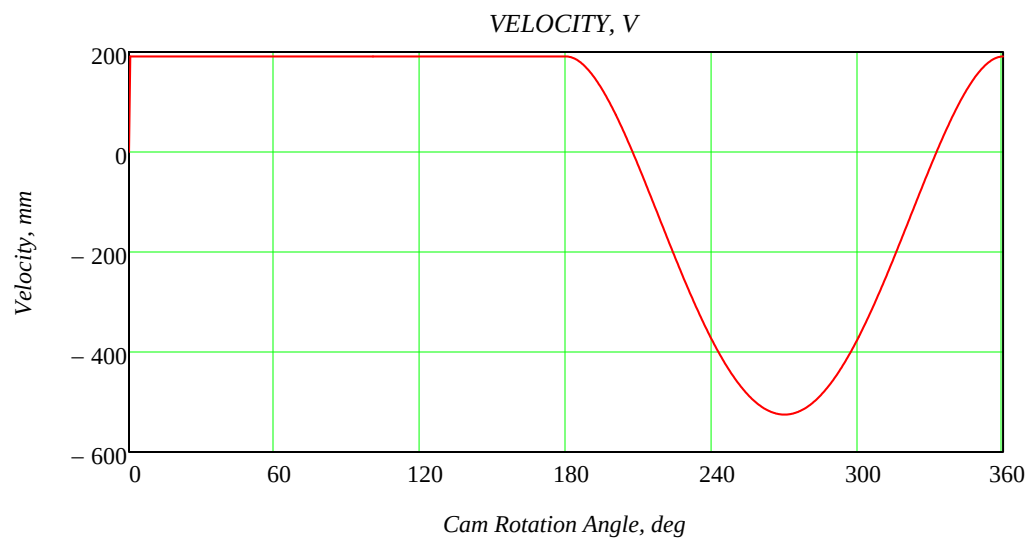
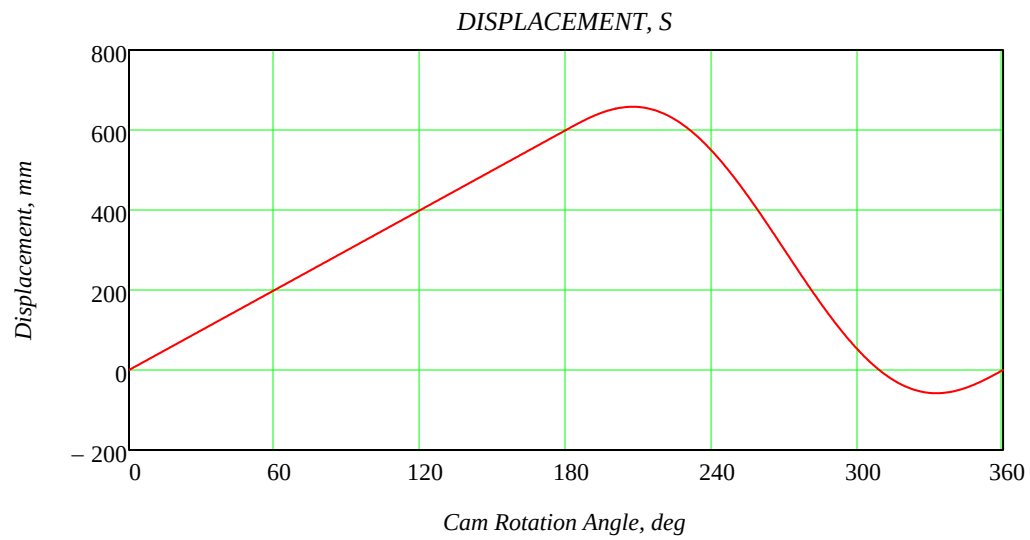
$$V(\theta) := R(\theta, 0, \beta_1) \cdot v_1(\theta) + R(\theta, \beta_1, \beta) \cdot v_2(\theta)$$

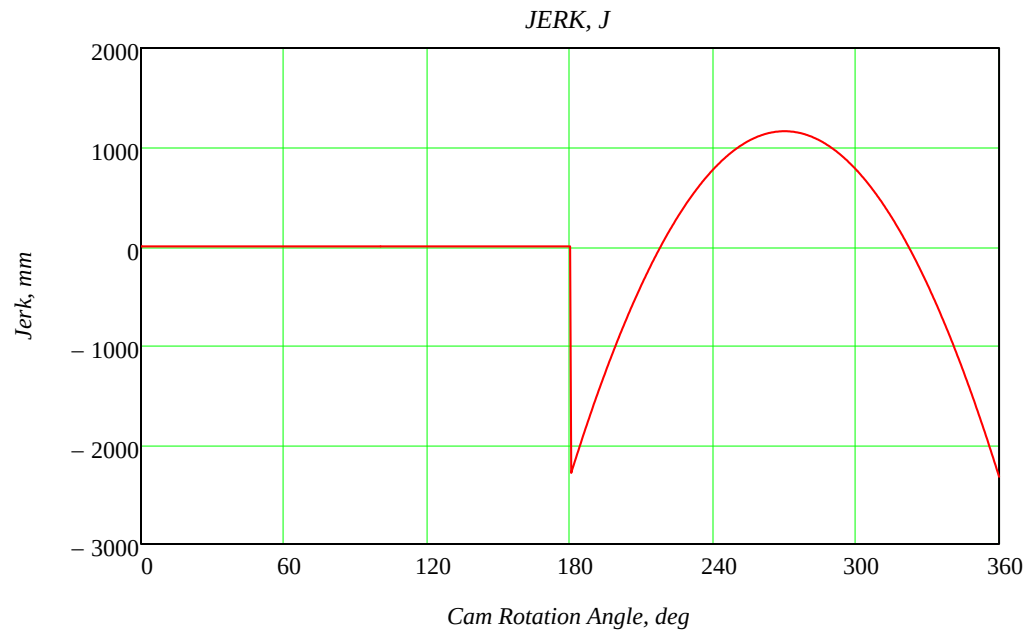
$$A(\theta) := R(\theta, 0, \beta_1) \cdot a_1(\theta) + R(\theta, \beta_1, \beta) \cdot a_2(\theta)$$

$$J(\theta) := R(\theta, 0, \beta_1) \cdot j_1(\theta) + R(\theta, \beta_1, \beta) \cdot j_2(\theta)$$

6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 \leq \theta \leq \beta$.

$$\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. \beta$$





PROBLEM 8-55

Statement: Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to calculate and plot *svaj* diagrams for the family of SCCA cam functions for any specified values of lift and duration. It should allow the user to change the values of the SCCA parameters b , c , d , and C_a to generate and plot any member of the family. Test it using cycloidal motion with a lift of 100 mm in 100 deg, a dwell of 80 deg, return to zero in 120 deg, and dwell for the remainder of the cycle at 1 rad/sec.

Input:

RISE	DWELL	FALL	DWELL
$\beta_1 := 100 \cdot \text{deg}$	$\beta_2 := 80 \cdot \text{deg}$	$\beta_3 := 120 \cdot \text{deg}$	$\beta_4 := 60 \cdot \text{deg}$

Total rise: $h := 100 \cdot \text{mm}$ Cam rotational velocity: $\omega := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$

The numerical constants for the SCCA equations are given in Table 8-2 and listed below. Enter values of b , c , d , and C_a for the motion type that you desire.

Function	b	c	d	C_a
Modified trapezoidal	0.25	0.50	0.25	4.8881
Modified sine	0.25	0.00	0.75	5.5280
Cycloidal	0.50	0.00	0.50	6.2832

Enter values: $b := 0.50$ $c := 0.00$ $d := 0.50$ $C_a := 6.2832$

Program: See Mathcad file P0855.

1. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 \leq x \leq b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left(\frac{\pi}{b} \cdot x \right) \right] \quad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos \left(\frac{\pi}{b} \cdot x \right) \right)$$

$$y''_1(x) := C_a \cdot \sin \left(\frac{\pi}{b} \cdot x \right) \quad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot \cos \left(\frac{\pi}{b} \cdot x \right)$$

for $b/2 \leq x \leq (1-d)/2$

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \right] \quad y'_2(x) := C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \right]$$

$$y''_2(x) := C_a \quad y'''_2(x) := 0$$

for $(1-d)/2 \leq x \leq (1+d)/2$

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2} \right) \cdot x + \left(\frac{d}{\pi} \right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi} \right)^2 \cdot \cos \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right]$$

$$y'_3(x) := C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right]$$

$$y''_3(x) := C_a \cdot \cos \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \quad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right]$$

for $(1+d)/2 \leq x \leq 1-b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + (2 \cdot d^2 - b^2) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) \quad y''_4(x) := -C_a \quad y'''_4(x) := 0$$

for $1 - b/2 \leq x \leq 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot (d^2 - b^2)}{\pi^2} + \frac{(1 - b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos \left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin \left[\frac{\pi}{b} \cdot (x - 1) \right] \quad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos \left[\frac{\pi}{b} \cdot (x - 1) \right]$$

2. The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the *SVAJ* curves, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

3. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range $0 \leq \theta \leq 360$ deg.
4. Write the local *svaj* equations for the first interval, $0 \leq \theta \leq \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$ (Rise)

$$s_1(x) := h \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

5. Write the local *svaj* equations for the second interval, $\beta_1 \leq \theta \leq \beta_1 + \beta_2$. For this interval, the value of *S* is the value of *S* at the end of the previous interval and the values of *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 \leq \theta \leq \beta_1 + \beta_2$

$$s_2(x) := h \quad v_2(x) := 0 \quad a_2(x) := 0 \quad j_2(x) := 0$$

6. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$

$$s_3(x) := h \left[1 - \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right) \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right]$$

$$v_3(x) := -\frac{h}{\beta_3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \dots \right) \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x)$$

$$a_3(x) := -\frac{h}{\beta_3^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \dots \right) \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x)$$

$$j_3(x) := -\frac{h}{\beta_3^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \dots \right) \\ + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x)$$

7. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 + \beta_4$

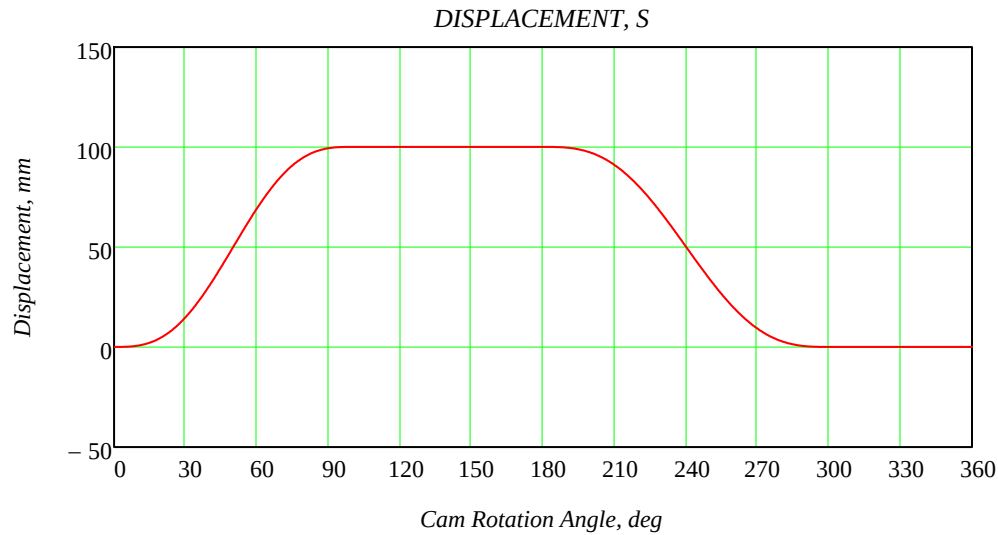
$$s_4(x) := 0 \quad v_4(x) := 0 \quad a_4(x) := 0 \quad j_4(x) := 0$$

8. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$\text{Let} \quad \theta_1 := \beta_1 \quad \theta_2 := \theta_1 + \beta_2 \quad \theta_3 := \theta_2 + \beta_3 \quad \theta_4 := \theta_3 + \beta_4$$

$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2) \cdot s_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \dots \\ + R(\theta, \theta_2, \theta_3) \cdot s_3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4) \cdot s_4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$

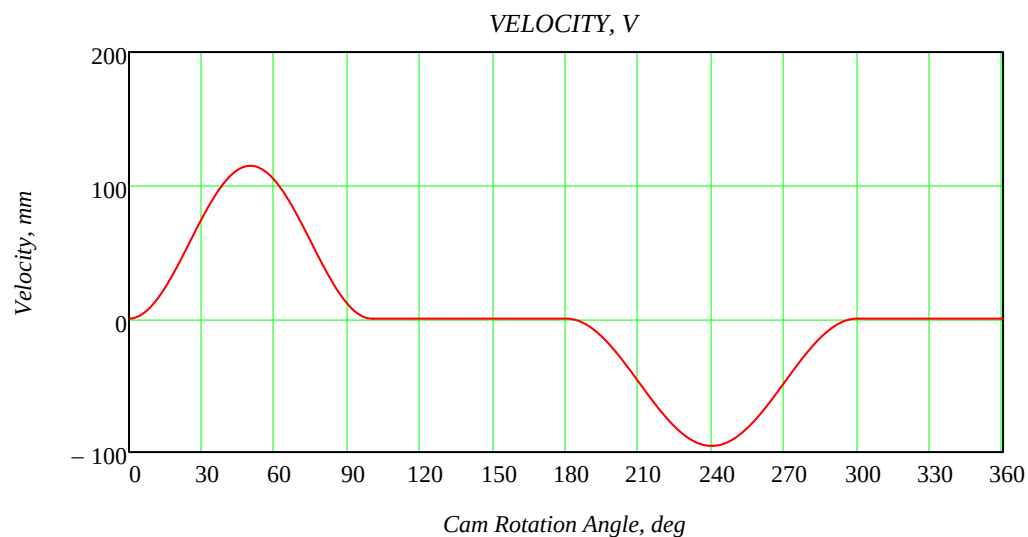
$$\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



11. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$V(\theta) := v_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot v_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

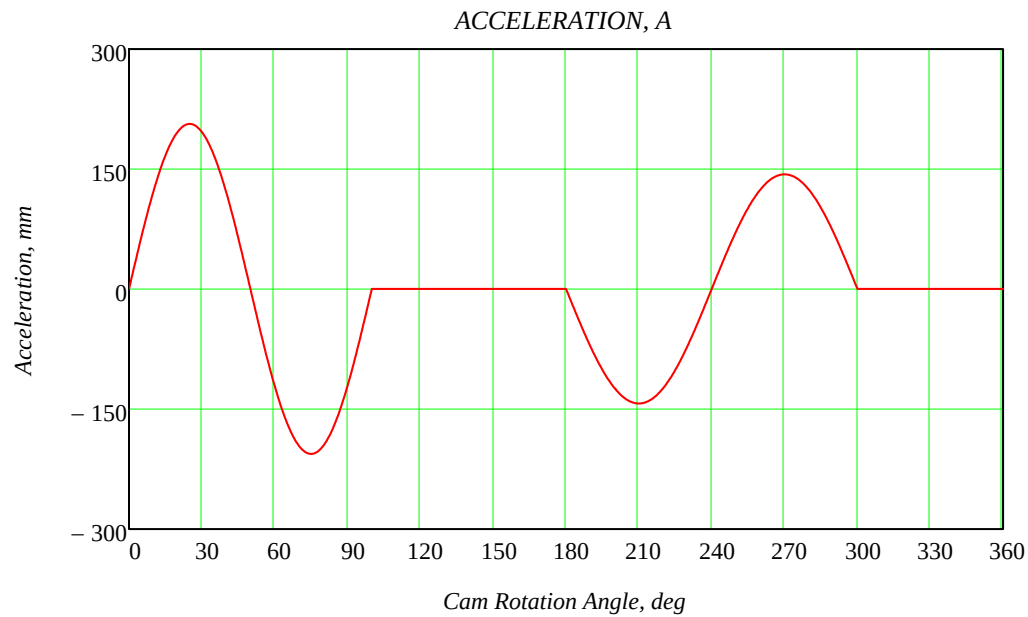
$$+ R(\theta, \theta_2, \theta_3) \cdot v_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot v_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



12. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := a_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot a_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

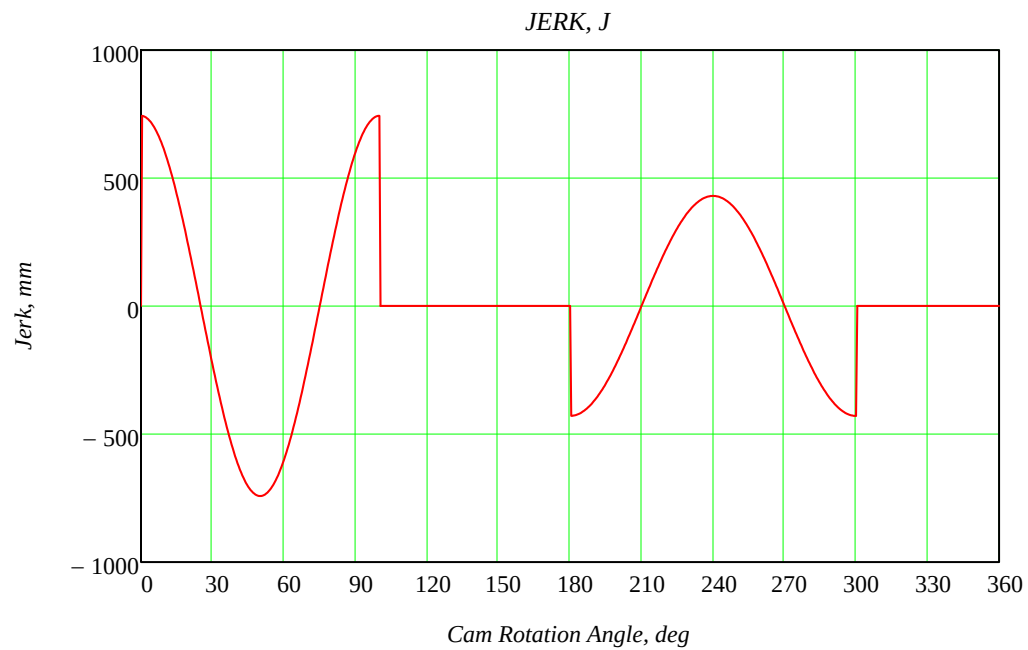
$$+ R(\theta, \theta_2, \theta_3) \cdot a_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot a_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := j_1 \left(\frac{\theta}{\theta_1} \right) + R(\theta, \theta_1, \theta_2) \cdot j_2 \left(\frac{\theta - \theta_1}{\theta_2 - \theta_1} \right) \dots$$

$$+ R(\theta, \theta_2, \theta_3) \cdot j_3 \left(\frac{\theta - \theta_2}{\theta_3 - \theta_2} \right) + R(\theta, \theta_3, \theta_4) \cdot j_4 \left(\frac{\theta - \theta_3}{\theta_4 - \theta_3} \right)$$



PROBLEM 8-56

Statement: Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to calculate and plot the pressure angle for the cam of Problem 8-42 for any given prime circle radius and follower eccentricity. Test it using $R_p = 45 \text{ mm}$ and $\varepsilon = 10 \text{ mm}$.

Given: $R_p := 45 \cdot \text{mm}$ $\varepsilon := 10 \cdot \text{mm}$

Solution: See Mathcad file P0856.

1. The cam of Problem 8-42 used a two-segment polynomial with the following constants:

$$c_3 := 5609.280 \cdot \text{mm} \qquad c_4 := -24548.849 \cdot \text{mm} \qquad c_5 := 39990.867 \cdot \text{mm}$$

$$c_6 := -28772.307 \cdot \text{mm} \qquad c_7 := 7721.009 \cdot \text{mm} \qquad \beta := 195 \cdot \text{deg}$$

2. From Problem 8-42 the s and v equations for the rise/fall segment are:

$$s(\theta) := c_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + c_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + c_5 \cdot \left(\frac{\theta}{\beta}\right)^5 + c_6 \cdot \left(\frac{\theta}{\beta}\right)^6 + c_7 \cdot \left(\frac{\theta}{\beta}\right)^7$$

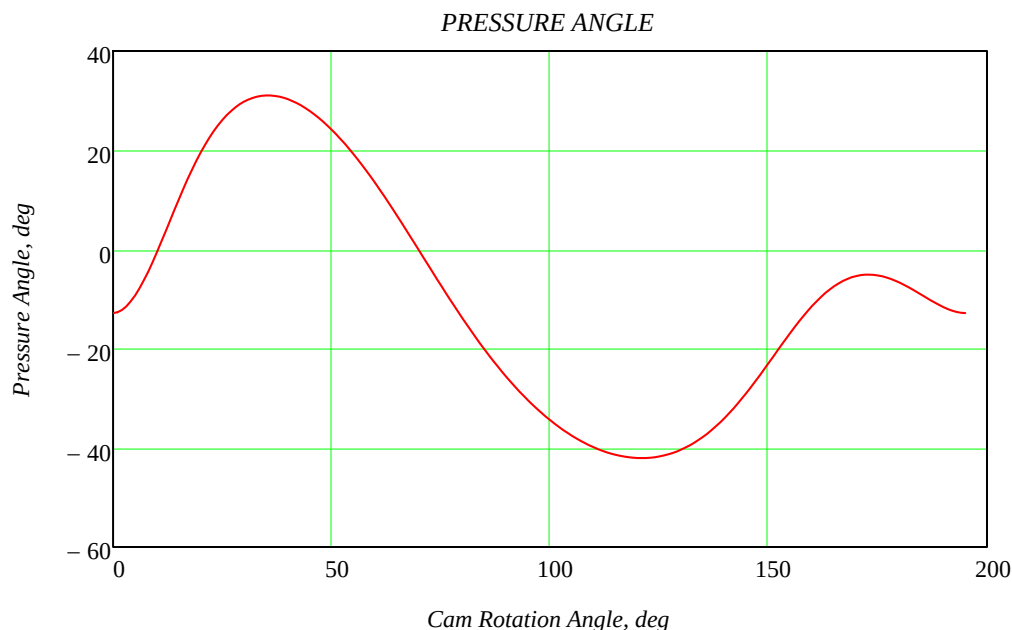
$$v(\theta) := \frac{1}{\beta} \cdot \left[3 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^3 + 5 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^4 + 6 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^5 + 7 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^6 \right]$$

3. Use equation 8-31d to calculate the pressure angle over the rise and fall segment.

$$\phi(\theta) := \text{atan} \left(\frac{v(\theta) - \varepsilon}{s(\theta) + \sqrt{R_p^2 - \varepsilon^2}} \right)$$

4. Plot the pressure angle over the interval $0 \leq \theta \leq \beta$.

$$\theta := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. \beta$$



PROBLEM 8-57

Statement: Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to calculate and plot the pressure angle for the cam of Problem 8-43 for any given prime circle radius and follower eccentricity. Test it using $R_p = 100 \text{ mm}$ and $\varepsilon = -15 \text{ mm}$.

Given: $R_p := 120 \cdot \text{mm}$ $\varepsilon := -15 \cdot \text{mm}$

Solution: See Mathcad file P0857.

1. The cam of Problem 8-43 used a two-segment polynomial with the following constants:

$$c_0 := 153600 \cdot \text{mm} \qquad c_1 := -971700 \cdot \text{mm} \qquad c_2 := 2430000 \cdot \text{mm}$$

$$c_3 := -2997000 \cdot \text{mm} \qquad c_4 := 1822500 \cdot \text{mm} \qquad c_5 := -437400 \cdot \text{mm}$$

$$h_{cv} := 95.492 \cdot \text{mm} \qquad v_{cv} := 47.746 \cdot \text{mm} \qquad \beta_1 := 240 \cdot \text{deg}$$

$$\beta := 360 \cdot \text{deg}$$

2. From Problem 8-43 the s and v equations for the constant velocity rise segment are:

$$s_1(\theta) := h_{cv} \cdot \frac{\theta}{\beta_1} \qquad v_1(\theta) := v_{cv}$$

3. From Problem 8-42 the s and v equations for the second segment are:

$$s_2(\theta) := c_0 + c_1 \cdot \left(\frac{\theta}{\beta}\right) + c_2 \cdot \left(\frac{\theta}{\beta}\right)^2 + c_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + c_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + c_5 \cdot \left(\frac{\theta}{\beta}\right)^5$$

$$v_2(\theta) := \frac{1}{\beta} \cdot \left[c_1 + 2 \cdot c_2 \cdot \left(\frac{\theta}{\beta}\right) + 3 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^3 + 5 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^4 \right]$$

4. To combine the s and v equations for both segments, first define a range function that has a value of one between the values of a and b and zero elsewhere.

$$R(\theta, a, b) := \text{if}[(\theta > a) \wedge (\theta \leq b), 1, 0]$$

$$s(\theta) := R(\theta, 0, \beta_1) \cdot s_1(\theta) + R(\theta, \beta_1, \beta) \cdot s_2(\theta)$$

$$v(\theta) := R(\theta, 0, \beta_1) \cdot v_1(\theta) + R(\theta, \beta_1, \beta) \cdot v_2(\theta)$$

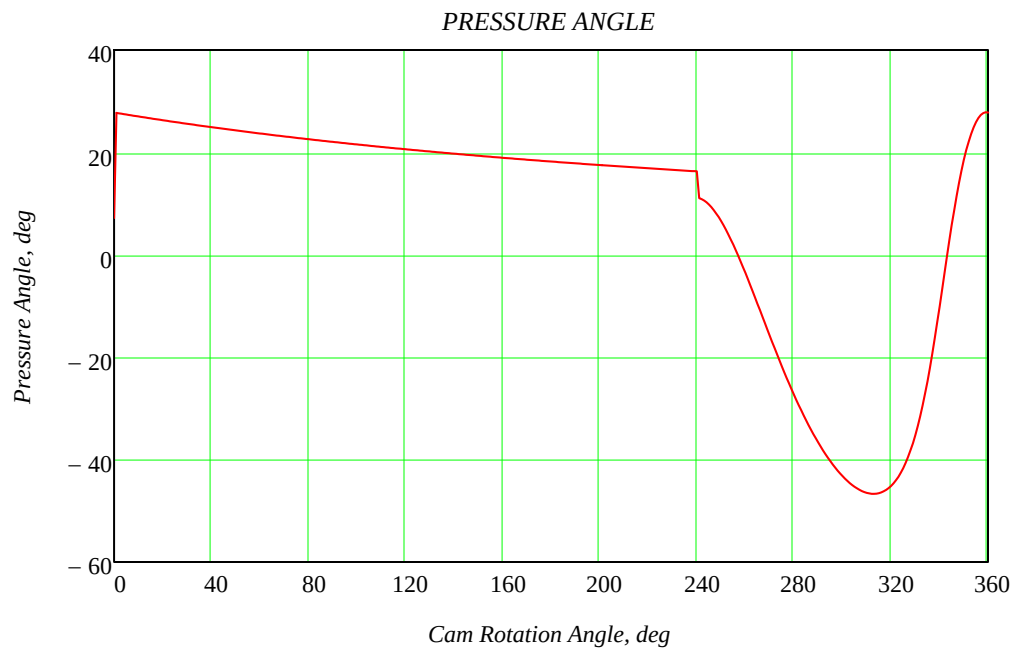
5. Use equation 8-31d to calculate the pressure angle over the rise and fall segment.

$$\phi(\theta) := \text{atan}\left(\frac{v(\theta) - \varepsilon}{s(\theta) + \sqrt{R_p^2 - \varepsilon^2}}\right)$$

4. Plot the pressure angle over the interval $0 \leq \theta \leq \beta$.

$$\theta := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. \beta$$

See next page



PROBLEM 8-58

Statement: Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to calculate and plot the pressure angle for the rise segment of the cam of Problem 8-46 for any given prime circle radius and follower eccentricity. Test it using $R_p = 75$ mm and $\varepsilon = 20$ mm.

Given: $R_p := 75 \cdot \text{mm}$ $\varepsilon := 20 \cdot \text{mm}$

Solution: See Mathcad file P0858.

1. The cam of Problem 8-46 used a 4-5-6-7 polynomial for the rise with the following constants:

$$h := 50 \cdot \text{mm} \quad \beta := 75 \cdot \text{deg}$$

2. From Problem 8-46 the s and v equations for the rise segment are:

$$s(\theta) := h \cdot \left[35 \cdot \left(\frac{\theta}{\beta} \right)^4 - 84 \cdot \left(\frac{\theta}{\beta} \right)^5 + 70 \cdot \left(\frac{\theta}{\beta} \right)^6 - 20 \cdot \left(\frac{\theta}{\beta} \right)^7 \right]$$

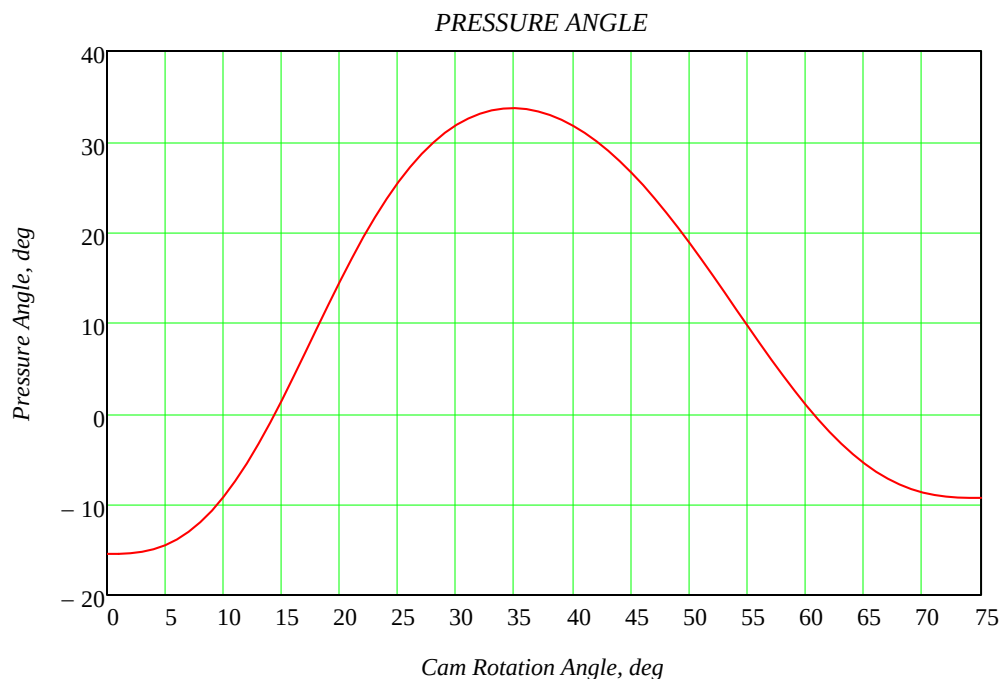
$$v(\theta) := \frac{h}{\beta} \cdot \left[140 \cdot \left(\frac{\theta}{\beta} \right)^3 - 420 \cdot \left(\frac{\theta}{\beta} \right)^4 + 420 \cdot \left(\frac{\theta}{\beta} \right)^5 - 140 \cdot \left(\frac{\theta}{\beta} \right)^6 \right]$$

3. Use equation 8-31d to calculate the pressure angle for the rise segment.

$$\phi(\theta) := \text{atan} \left(\frac{v(\theta) - \varepsilon}{s(\theta) + \sqrt{R_p^2 - \varepsilon^2}} \right)$$

4. Plot the pressure angle over the interval $0 \leq \theta \leq \beta$.

$$\theta := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. \beta$$



PROBLEM 8-59

Statement: Design a cam to move a follower from 20.5 to 15 mm in 60 deg, fall an additional 15 mm in 90 deg, rise 20.5 mm in 110 deg, and dwell for the remainder. Use polynomial functions for the rise and falls. Some of the boundary conditions are given in Table P8-1, however, in order to make the polynomials piecewise continuous, other boundary conditions will have to be determined. The shaft speed is 250 rpm. Plot the $s v a j$ diagrams.

Given:

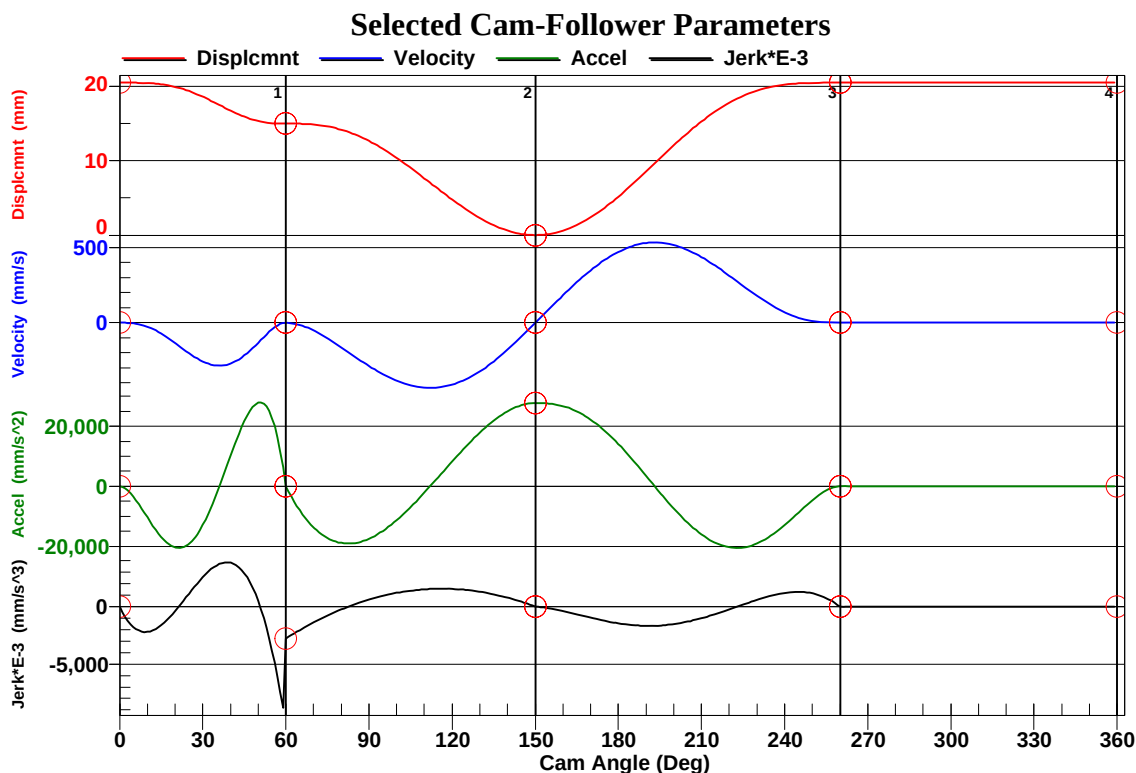
FALL	FALL	RISE	DWELL
$\beta_1 := 60\text{-deg}$	$\beta_2 := 90\text{-deg}$	$\beta_3 := 110\text{-deg}$	$\beta_4 := 100\text{-deg}$
$h_1 := 5.5\text{-mm}$	$h_2 := 15.0\text{-mm}$	$h_3 := 20.5\text{-mm}$	$h_4 := 20.5\text{-mm}$
Shaft speed $\omega := 250\text{-rpm}$			

Solution: See Mathcad file P0859.

1. Calculate the shaft speed in rad/sec for input into DYNACAM.

$$\omega = 26.180 \frac{\text{rad}}{\text{sec}}$$

2. Input the number of segments, which in this case is 4.
3. Input 60 deg for the first segment and choose "poly" for the motion. Input 20.5 and 15 for the positions. Click on "Calculate". There are seven ICs to enter, all zeroes.
4. Input 90 deg for the second segment and choose "poly" for the motion. Input 15 and 0 for the positions. Click on "Calculate". There 6 ICs to enter, again all zeroes. Click on "Plot". From the plot screen get the maximum value of the acceleration, which is 27,777.90.
5. Input 110 deg for the third segment and choose "poly" for the motion. Input 0 and 20.5 for the positions. Click on "Calculate". There 8 ICs to enter, all zeroes except for the starting acceleration, which is 27,777.90.
6. Input 100 deg for the last segment and choose "dwell" for the motion. Input 20.5 and 20.5 for the positions. Click on "Calculate". Click on "Next" in the command line. Plotting the results yields the diagram below.



PROBLEM 8-60

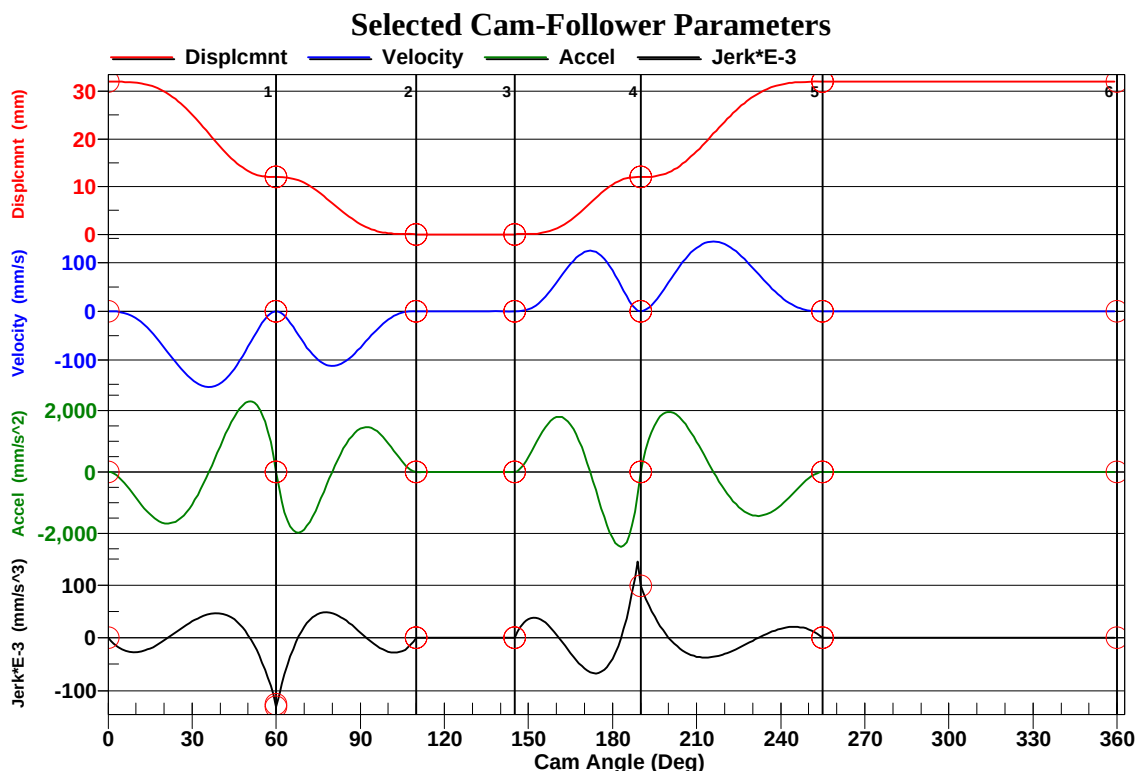
Statement: Design a cam to move a follower from 32 to 12 mm in 60 deg, fall an additional 12 mm in 50 deg, dwell 35 deg, rise 12 mm in 45 deg, rise an additional 20mm in 65 deg, and dwell for the remainder. Use polynomial functions for the rises and falls. Velocity and acceleration are zero at the beginning and end of each event and jerk is zero at $\theta = 0, 110, 145$, and 255 deg. The shaft speed is 37.5 rpm. Plot the $s v a j$ diagrams.

Given:

FALL	FALL	DWELL	RISE	RISE	DWELL
$\beta_1 := 60\text{-deg}$	$\beta_2 := 50\text{-deg}$	$\beta_3 := 35\text{-deg}$	$\beta_4 := 45\text{-deg}$	$\beta_5 := 65\text{-deg}$	$\beta_6 := 105\text{-deg}$
$h_1 := 20\text{-mm}$	$h_2 := 12\text{-mm}$	$h_3 := 0\text{-mm}$	$h_4 := 12\text{-mm}$	$h_5 := 20\text{-mm}$	$h_6 := 32\text{-mm}$
Shaft speed $\omega := 37.5\text{-rpm}$					

Solution: See Mathcad file P0860.

1. Calculate the shaft speed in rad/sec for input into DYNACAM. $\omega = 3.927 \frac{\text{rad}}{\text{sec}}$
2. Input the number of segments, which in this case is 6.
3. Input 60 deg for the first segment and choose "poly" for the motion. Input 32 and 12 for the positions. Click on "Calculate". There are 7 ICs to enter, all zeroes.
4. Input 50 deg for the second segment and choose "poly" for the motion. Input 12 and 0 for the positions. Click on "Calculate". There 7 ICs to enter, again all zeroes.
5. Input 35 deg for the third segment and choose "dwell" for the motion. Input 0 and 0 for the positions. Click on "Calculate".
6. Input 45 deg for the fourth segment and choose "poly" for the motion. Input 0 and 12 for the positions. Click on "Calculate". There are 7 ICs to enter, all zeroes.
7. Input 65 deg for the fifth segment and choose "poly" for the motion. Input 12 and 32 for the positions. Click on "Calculate". There 7 ICs to enter, again all zeroes.
6. Input 105 deg for the last segment and choose "dwell" for the motion. Input 32 and 32 for the positions. Click on "Calculate". Click on "Next" in the command line. Plotting the results yields the diagram below.



PROBLEM 9-1

Statement: A 24-tooth gear has AGMA standard full-depth involute teeth with diametral pitch of 5. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance.

Given: Tooth number $N := 24$ Diametral pitch $p_d := 5 \cdot \text{in}^{-1}$

Solution: See Table 9-1 and Mathcad file P0901.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter	$d := \frac{N}{p_d}$	$d = 4.8000 \cdot \text{in}$

Circular pitch	$p_c := \frac{\pi}{p_d}$	$p_c = 0.6283 \cdot \text{in}$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum	$a := \frac{1.0000}{p_d}$	$a = 0.2000 \cdot \text{in}$

Dedendum	$b := \frac{1.2500}{p_d}$	$b = 0.2500 \cdot \text{in}$

Tooth thickness	$t := 0.5 \cdot p_c$	$t = 0.3142 \cdot \text{in}$

Clearance	$c := \frac{0.2500}{p_d}$	$c = 0.0500 \cdot \text{in}$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions should be displayed to four decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in ten-thousandths of an inch..

PROBLEM 9-2

Statement: A 40-tooth gear has AGMA standard full-depth involute teeth with diametral pitch of 10. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance.

Given: Tooth number $N := 40$ Diametral pitch $p_d := 10 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0902.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter	$d := \frac{N}{p_d}$	$d = 4.0000 \text{ in}$

Circular pitch	$p_c := \frac{\pi}{p_d}$	$p_c = 0.3142 \text{ in}$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum	$a := \frac{1.0000}{p_d}$	$a = 0.1000 \text{ in}$

Dedendum	$b := \frac{1.2500}{p_d}$	$b = 0.1250 \text{ in}$

Tooth thickness	$t := 0.5 \cdot p_c$	$t = 0.1571 \text{ in}$

Clearance	$c := \frac{0.2500}{p_d}$	$c = 0.0250 \text{ in}$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions should be displayed to four decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in ten-thousandths of an inch..

PROBLEM 9-3

Statement: A 30-tooth gear has AGMA standard full-depth involute teeth with diametral pitch of 12. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance.

Given: Tooth number $N := 30$ Diametral pitch $p_d := 12 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0903.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter	$d := \frac{N}{p_d}$	$d = 2.5000 \text{ in}$

Circular pitch	$p_c := \frac{\pi}{p_d}$	$p_c = 0.2618 \text{ in}$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum	$a := \frac{1.0000}{p_d}$	$a = 0.0833 \text{ in}$

Dedendum	$b := \frac{1.2500}{p_d}$	$b = 0.1042 \text{ in}$

Tooth thickness	$t := 0.5 \cdot p_c$	$t = 0.1309 \text{ in}$

Clearance	$c := \frac{0.2500}{p_d}$	$c = 0.0208 \text{ in}$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions should be displayed to four decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in ten-thousandths of an inch..

PROBLEM 9-4

Statement: Using any available string, some tape, a pencil, and a drinking glass or tin can, generate and draw an involute curve on a piece of paper. With your protractor, show that all normals to the curve are tangent to the base circle.

Solution: This is a "hands-on" student demonstration project. The result should look like Figure 9-5.

PROBLEM 9-5

Statement: A spur gearset must have pitch diameters of 2.5 and 8 in. What is the largest standard tooth size, in terms of diametral pitch, that can be used without having any interference or undercutting and what are the number of teeth on each gear that result from using this diametral pitch? Assume that both gears are cut with a hob.

- a. For a 20-deg pressure angle
- b. For a 25-deg pressure angle

Given: Pitch diameters: $d_1 := 2.5 \cdot \text{in}$ $d_2 := 8 \cdot \text{in}$

Solution: See Table 9-4 and Mathcad file P0905.

1. To avoid undercutting, use the minimum tooth numbers given in Table 9-4b.

a. Pressure angle of 20 deg.

$$N_{min} := 21 \quad p_{dmin} := \frac{N_{min}}{d_1} \quad p_{dmin} = 8.400 \cdot \text{in}^{-1}$$

From Table 9-2, the smallest standard diametral pitch (largest tooth size) that can be used is 9. But, since the pinion pitch diameter is not an integer, using 9 would result in a noninteger number of teeth.

Therefore, we must go to the next larger (even) pitch (smaller tooth size) of $p_d := 10 \cdot \text{in}^{-1}$. The resulting tooth numbers are:

$$N_1 := p_d \cdot d_1 \quad N_1 = 25 \quad N_2 := p_d \cdot d_2 \quad N_2 = 80$$

b. Pressure angle of 25 deg

$$N_{min} := 14 \quad p_{dmin} := \frac{N_{min}}{d_1} \quad p_{dmin} = 5.600 \cdot \text{in}^{-1}$$

From Table 9-2, the smallest standard diametral pitch (largest tooth size) that can be used is 6.

Let $p_d := 6 \cdot \text{in}^{-1}$. The resulting tooth numbers are:

$$N_1 := p_d \cdot d_1 \quad N_1 = 15 \quad N_2 := p_d \cdot d_2 \quad N_2 = 48$$

PROBLEM 9-6

Statement: Design a simple, spur gear train for a ratio of -7:1 and a diametral pitch of 10. Specify pitch diameters and numbers of teeth. Calculate the contact ratio.

Given: Gear ratio $m_G := 7$ Diametral pitch $p_d := 10 \cdot \text{in}^{-1}$

Assumptions: The pinion is not cut by a hob and can, therefore, have fewer than 21 teeth (see Table 9-4b) for a 20-deg pressure angle.

Design Choice: Pressure angle $\phi := 20 \cdot \text{deg}$

Solution: See Mathcad file P0906.

- From inspection of Table 9-5a, we see that 16 teeth is the least number that the pinion can have for a gear ratio of 7. therefore, let the number of teeth on the pinion be

$$N_p := 16 \quad \text{and} \quad N_g := m_G \cdot N_p \quad N_g = 112$$

- Using equation 9.4c, calculate the pitch diameters of the pinion and gear.

$$d_p := \frac{N_p}{p_d} \quad d_p = 1.6000 \cdot \text{in} \quad d_g := \frac{N_g}{p_d} \quad d_g = 11.2000 \cdot \text{in}$$

- Calculate the contact ratio using equations 9.2 and 9.6b and those from Table 9-1.

$$r_p := 0.5 \cdot d_p \quad r_p = 0.8000 \cdot \text{in} \quad r_g := 0.5 \cdot d_g \quad r_g = 5.6000 \cdot \text{in}$$

$$a_p := \frac{1}{p_d} \quad a_p = 0.1000 \cdot \text{in} \quad a_g := \frac{1}{p_d} \quad a_g = 0.1000 \cdot \text{in}$$

$$\text{Center distance} \quad C := r_p + r_g \quad C = 6.4000 \cdot \text{in}$$

$$Z := \sqrt{(r_p + a_p)^2 - (r_p \cdot \cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot \cos(\phi))^2} - C \cdot \sin(\phi) \quad Z = 0.4964 \cdot \text{in}$$

$$\text{Contact ratio} \quad m_p := \frac{p_d \cdot Z}{\pi \cdot \cos(\phi)} \quad m_p = 1.682$$

PROBLEM 9-7

Statement: Design a simple, spur gear train for a ratio of +6:1 and a diametral pitch of 5. Specify pitch diameters and numbers of teeth. Calculate the contact ratio.

Given: Gear ratio $m_G := 6$ Diametral pitch $p_d := 5 \cdot \text{in}^{-1}$

Assumptions: The pinion is not cut by a hob and can, therefore, have fewer than 21 teeth for a 20-deg pressure angle (see Table 9-4b).

Design Choice: Pressure angle $\phi := 20 \cdot \text{deg}$

Solution: See Mathcad file P0907.

- From inspection of Table 9-5a, we see that 16 teeth is the least number that the pinion can have for a gear ratio of 6. therefore, let the number of teeth on the pinion be

$$N_p := 16 \quad \text{and} \quad N_g := m_G \cdot N_p \quad N_g = 96$$

- Using equation 9.4c, calculate the pitch diameters of the pinion and gear.

$$d_p := \frac{N_p}{p_d} \quad d_p = 3.2000 \cdot \text{in} \quad d_g := \frac{N_g}{p_d} \quad d_g = 19.2000 \cdot \text{in}$$

- Calculate the contact ratio using equations 9.2 and 9.6b and those from Table 9-1.

$$r_p := 0.5 \cdot d_p \quad r_p = 1.6000 \cdot \text{in} \quad r_g := 0.5 \cdot d_g \quad r_g = 9.6000 \cdot \text{in}$$

$$a_p := \frac{1}{p_d} \quad a_p = 0.2000 \cdot \text{in} \quad a_g := \frac{1}{p_d} \quad a_g = 0.2000 \cdot \text{in}$$

$$\text{Center distance} \quad C := r_p + r_g \quad C = 11.2000 \cdot \text{in}$$

$$Z := \sqrt{(r_p + a_p)^2 - (r_p \cdot \cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot \cos(\phi))^2} - C \cdot \sin(\phi) \quad Z = 0.9880 \text{ in}$$

$$\text{Contact ratio} \quad m_p := \frac{p_d \cdot Z}{\pi \cdot \cos(\phi)} \quad m_p = 1.673$$

- An idler gear of any diameter is needed to get the positive ratio. If the idler does not have the same number of teeth as the gear, the calculation of contact ratio (step 3) will not be correct.

PROBLEM 9-8

Statement: Design a simple, spur gear train for a ratio of -7:1 and a diametral pitch of 8. Specify pitch diameters and numbers of teeth. Calculate the contact ratio.

Given: Gear ratio $m_G := 7$ Diametral pitch $p_d := 8 \cdot \text{in}^{-1}$

Assumptions: The pinion is not cut by a hob and can, therefore, have fewer than 21 teeth for a 20-deg pressure angle (see Table 9-4b).

Design Choice: Pressure angle $\phi := 20 \cdot \text{deg}$

Solution: See Mathcad file P0908.

- From inspection of Table 9-5a, we see that 17 teeth is the least number that the pinion can have for a gear ratio of 7. therefore, let the number of teeth on the pinion be

$$N_p := 17 \quad \text{and} \quad N_g := m_G \cdot N_p \quad N_g = 119$$

- Using equation 9.4c, calculate the pitch diameters of the pinion and gear.

$$d_p := \frac{N_p}{p_d} \quad d_p = 2.1250 \text{ in} \quad d_g := \frac{N_g}{p_d} \quad d_g = 14.8750 \text{ in}$$

- Calculate the contact ratio using equations 9.2 and 9.6b and those from Table 9-1.

$$r_p := 0.5 \cdot d_p \quad r_p = 1.0625 \text{ in} \quad r_g := 0.5 \cdot d_g \quad r_g = 7.4375 \text{ in}$$

$$a_p := \frac{1}{p_d} \quad a_p = 0.1250 \text{ in} \quad a_g := \frac{1}{p_d} \quad a_g = 0.1250 \text{ in}$$

$$\text{Center distance} \quad C := r_p + r_g \quad C = 8.5000 \text{ in}$$

$$Z := \sqrt{(r_p + a_p)^2 - (r_p \cdot \cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot \cos(\phi))^2} - C \cdot \sin(\phi) \quad Z = 0.6246 \text{ in}$$

$$\text{Contact ratio} \quad m_p := \frac{p_d \cdot Z}{\pi \cdot \cos(\phi)} \quad m_p = 1.693$$

PROBLEM 9-9

Statement: Design a simple, spur gear train for a ratio of +6.5:1 and a diametral pitch of 5. Specify pitch diameters and numbers of teeth. Calculate the contact ratio.

Given: Gear ratio $m_G := 6.5$ Diametral pitch $p_d := 5 \cdot \text{in}^{-1}$

Assumptions: The pinion is not cut by a hob and can, therefore, have fewer than 21 teeth for a 20-deg pressure angle (see Table 9-4b).

Design Choice: Pressure angle $\phi := 20 \cdot \text{deg}$

Solution: See Mathcad file P0909.

- From inspection of Table 9-5a, we see that 17 teeth is the least number that the pinion can have for a gear ratio of 6.5. therefore, let the number of teeth on the pinion be (an even number so the gear tooth number will be an integer).

$$N_p := 18 \quad \text{and} \quad N_g := m_G \cdot N_p \quad N_g = 117$$

- Using equation 9.4c, calculate the pitch diameters of the pinion and gear.

$$d_p := \frac{N_p}{p_d} \quad d_p = 3.6000 \text{ in} \quad d_g := \frac{N_g}{p_d} \quad d_g = 23.4000 \text{ in}$$

- Calculate the contact ratio using equations 9.2 and 9.6b and those from Table 9-1.

$$r_p := 0.5 \cdot d_p \quad r_p = 1.8000 \text{ in} \quad r_g := 0.5 \cdot d_g \quad r_g = 11.7000 \text{ in}$$

$$a_p := \frac{1}{p_d} \quad a_p = 0.2000 \text{ in} \quad a_g := \frac{1}{p_d} \quad a_g = 0.2000 \text{ in}$$

$$\text{Center distance} \quad C := r_p + r_g \quad C = 13.5000 \text{ in}$$

$$Z := \sqrt{(r_p + a_p)^2 - (r_p \cdot \cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot \cos(\phi))^2} - C \cdot \sin(\phi) \quad Z = 1.0033 \text{ in}$$

$$\text{Contact ratio} \quad m_p := \frac{p_d \cdot Z}{\pi \cdot \cos(\phi)} \quad m_p = 1.699$$

- An idler gear of any diameter is needed to get the positive ratio. If the idler does not have the same number of teeth as the gear, the calculation of contact ratio (step 3) will not be correct.

PROBLEM 9-10

Statement: Design a compound, spur gear train for a ratio of -80:1 and diametral pitch of 12. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio: $m_G := 80$ Diametral pitch: $p_d := 12 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0910.

1. Since the ratio is negative, we want to have an odd number of stages or an even number with an idler. Let the number of stages be

Possible number of stages $j := 2, 3 \dots 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

then	$j =$	$r(j) =$
	2	8.944
	3	4.309
	4	2.991

2. Two stages would result in a stage ratio less than 10 but will require an idler, so we will use three stages. The average ratio for three stages is about 21:5. Using a pressure angle of 20 deg, let the stage ratios be

$$\text{Stage 1 ratio } r_1 := \frac{20}{5} \quad \text{Stage 2 ratio } r_2 := \frac{20}{5} \quad \text{Stage 3 ratio } r_3 := \frac{30}{6}$$

and let the driver gears have tooth numbers of

Tooth number index $i := 2, 3 \dots 7$

$$N_2 := 20$$

$$N_4 := 20$$

$$N_6 := 18$$

then the driven gears will have tooth numbers of

$$N_3 := r_1 \cdot N_2$$

$$N_5 := r_2 \cdot N_4$$

$$N_7 := r_3 \cdot N_6$$

$$N_3 = 80$$

$$N_5 = 80$$

$$N_7 = 90$$

The pitch diameters are: $d_i := \frac{N_i}{p_d}$

$i =$	$\frac{d_i}{\text{in}} =$
2	1.6667
3	6.6667
4	1.6667
5	6.6667
6	1.5000
7	7.5000

	Tooth numbers:
$i =$	$N_i =$
2	20
3	80
4	20
5	80
6	18
7	90

Checking the overall gear ratio: $\frac{N_3 \cdot N_5 \cdot N_7}{N_2 \cdot N_4 \cdot N_6} = 80.000$

PROBLEM 9-11

Statement: Design a compound, spur gear train for a ratio of 50:1 and diametral pitch of 8. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 50$ Diametral pitch $p_d := 8 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0911.

1. Since the ratio is positive, we want to have an even number of stages or an odd number with an idler. Let the number of stages be

Possible number of stages $j := 2, 3 \dots 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

then $j =$ $r(j) =$

2	7.071
3	3.684
4	2.659

2. Two stages would result in a stage ratio less than 10 and about 7, so we will use two stages. The average ratio for two stages is about 50:7. Using a pressure angle of 20 deg, let the stage ratios be

Stage 1 ratio $r_1 := \frac{50}{7}$ Stage 2 ratio $r_2 := 7$

and let the driver gears have tooth numbers of (note that N_2 must be a multiple of 7)

Tooth number index $i := 2, 3 \dots 5$

$N_2 := 21$ $N_4 := 18$

then the driven gears will have tooth numbers of

$N_3 := r_1 \cdot N_2$ $N_5 := r_2 \cdot N_4$

$N_3 = 150$ $N_5 = 126$

The pitch diameters are: $d_i := \frac{N_i}{p_d}$

$i =$	$\frac{d_i}{\text{in}} =$	Tooth numbers:	$N_i =$
2	2.6250	2	21
3	18.7500	3	150
4	2.2500	4	18
5	15.7500	5	126

Checking the overall gear ratio: $\frac{N_3 \cdot N_5}{N_2 \cdot N_4} = 50.000$

PROBLEM 9-12

Statement: Design a compound, spur gear train for a ratio of 120:1 and diametral pitch of 5. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 120$ Diametral pitch $p_d := 5 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0912.

1. Since the ratio is positive, we want to have an even number of stages or an odd number with an idler. Let the number of stages be

Possible number of stages $j := 2, 3 \dots 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

then	$j =$	$r(j) =$
	2	10.954
	3	4.932
	4	3.310

2. Two stages would result in a stage ratio greater than 10, so we will use three stages with an idler to get the required output direction. The average ratio for three stages is about 15:3. Using a pressure angle of 20 deg, let the stage ratios be

Stage 1 ratio $r_1 := 5$ Stage 2 ratio $r_2 := 6$ Stage 3 ratio $r_3 := 4$

and let the driver gears have tooth numbers of

Tooth number index $i := 2, 3 \dots 8$

$N_2 := 18$ $N_4 := 18$ $N_6 := 18$

then the driven gears will have tooth numbers of

$N_3 := r_1 \cdot N_2$ $N_5 := r_2 \cdot N_4$ $N_8 := r_3 \cdot N_6$

$N_3 = 90$ $N_5 = 108$ $N_8 = 72$

The idler gear will have $N_7 := 18$

The pitch diameters are: $d_i := \frac{N_i}{p_d}$

$i =$	$\frac{d_i}{\text{in}} =$
2	3.6000
3	18.0000
4	3.6000
5	21.6000
6	3.6000
7	3.6000
8	14.4000

Tooth numbers:

$i =$	$N_i =$
2	18
3	90
4	18
5	108
6	18
7	18
8	72

Checking the overall gear ratio: $\frac{N_3 \cdot N_5 \cdot N_8}{N_2 \cdot N_4 \cdot N_6} = 120.000$

PROBLEM 9-13

Statement: Design a compound, spur gear train for a ratio of -250:1 and diametral pitch of 9. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 250$ Diametral pitch $p_d := 9 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0913.

1. Since the ratio is negative, we want to have an odd number of stages or an even number with an idler. Let the number of stages be

Possible number of stages $j := 2, 3 \dots 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

then	$j =$	$r(j) =$						
	<table><tr><td>2</td></tr><tr><td>3</td></tr><tr><td>4</td></tr></table>	2	3	4	<table><tr><td>15.811</td></tr><tr><td>6.300</td></tr><tr><td>3.976</td></tr></table>	15.811	6.300	3.976
2								
3								
4								
15.811								
6.300								
3.976								

2. Two stages would result in a stage ratio greater than 10 and will require an idler, so we will use three stages. The average ratio for three stages is about 20:3. Using a pressure angle of 20 deg, let the stage ratios be

Stage 1 ratio $r_1 := \frac{20}{3}$ Stage 2 ratio $r_2 := \frac{18}{3}$ Stage 3 ratio $r_3 := \frac{25}{4}$

and let the driver gears have tooth numbers of

Tooth number index $i := 2, 3 \dots 7$

$N_2 := 18$ $N_4 := 18$ $N_6 := 20$

then the driven gears will have tooth numbers of

$N_3 := r_1 \cdot N_2$ $N_5 := r_2 \cdot N_4$ $N_7 := r_3 \cdot N_6$
 $N_3 = 120$ $N_5 = 108$ $N_7 = 125$

The pitch diameters are: $d_i := \frac{N_i}{p_d}$

$i =$	$\frac{d_i}{in} =$
2	2.0000
3	13.3333
4	2.0000
5	12.0000
6	2.2222
7	13.8889

Tooth numbers:

$i =$	$N_i =$
2	18
3	120
4	18
5	108
6	20
7	125

Checking the overall gear ratio: $\frac{N_3 \cdot N_5 \cdot N_7}{N_2 \cdot N_4 \cdot N_6} = 250.000$

PROBLEM 9-14

Statement: Design a compound, reverted, spur gear train for a ratio of 28:1 and diametral pitch of 8. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 28$ Diametral pitch $p_d := 8 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0914.

1. Since the ratio is positive, we want to have an even number of stages. Let the number of stages be

Possible number of stages $j := 2, 3 \dots 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

then $j =$

2	5.292
3	3.037
4	2.300

2. Two stages would result in a stage ratio less than 10, so we will use two stages. The average ratio for two stages is about 21:4. Using a pressure angle of 25 deg, let the stage ratios be

Stage 1 ratio $r_1 := 4$ Stage 2 ratio $r_2 := 7$

3. Following the procedure of Example 9-3,

Tooth number index $i := 2, 3 \dots 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \frac{N_3}{N_2}$ $r_2 := \frac{N_5}{N_4}$

Solving independently for N_2 and N_4 , $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 40.000$

By iteration, find a multiple of K_{min} that will result in a minimum number of teeth on N_2 and N_4 .

$K := 3 \cdot K_{min}$ $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

$K = 120.000$ $N_2 = 24$ $N_4 = 15$

These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears will have tooth numbers of

$N_3 := r_1 \cdot N_2$ $N_3 = 96$ $N_5 := r_2 \cdot N_4$ $N_5 = 105$

The pitch diameters are: $d_i := \frac{N_i}{p_d}$

$i =$	$\frac{d_i}{\text{in}} =$	$N_i =$
2	3.0000	24
3	12.0000	96
4	1.8750	15
5	13.1250	105

Checking the overall gear ratio: $\left(-\frac{N_3}{N_2} \right) \cdot \left(-\frac{N_5}{N_4} \right) = 28.000$

PROBLEM 9-15

Statement: Design a compound, reverted, spur gear train for a ratio of 40:1 and diametral pitch of 8. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 40$ Diametral pitch $p_d := 8 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0915.

1. Since the ratio is positive, we want to have an even number of stages. Let the number of stages be

Possible number of stages $j := 2, 3 \dots 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

then $j =$

2	6.325
3	3.420
4	2.515

2. Two stages would result in a stage ratio less than 10, so we will use two stages. The average ratio for two stages is about 20:3. Using a pressure angle of 25 deg, let the stage ratios be

Stage 1 ratio $r_1 := 5$ Stage 2 ratio $r_2 := 8$

3. Following the procedure of Example 9-3,

Tooth number index $i := 2, 3 \dots 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \frac{N_3}{N_2}$ $r_2 := \frac{N_5}{N_4}$

Solving independently for N_2 and N_4 , $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 54.000$

By iteration, find a multiple of K_{min} that will result in a minimum number of teeth on N_2 and N_4 .

$K := 3 \cdot K_{min}$ $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

$K = 162.000$ $N_2 = 27$ $N_4 = 18$

These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears will have tooth numbers of

$N_3 := r_1 \cdot N_2$ $N_3 = 135$ $N_5 := r_2 \cdot N_4$ $N_5 = 144$

The pitch diameters are: $d_i := \frac{N_i}{p_d}$

$i =$

2	3.3750	27
3	16.8750	135
4	2.2500	18
5	18.0000	144

Checking the overall gear ratio: $\left(-\frac{N_3}{N_2} \right) \cdot \left(-\frac{N_5}{N_4} \right) = 40.000$

PROBLEM 9-16

Statement: Design a compound, reverted, spur gear train for a ratio of 65:1 and diametral pitch of 8. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 65$ Diametral pitch $p_d := 8 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0916.

1. Since the ratio is positive, we want to have an even number of stages. Let the number of stages be

Possible number of stages $j := 2, 3 \dots 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

then $j =$

2
3
4

 $r(j) =$

8.062
4.021
2.839

2. Two stages would result in a stage ratio less than 10, so we will use two stages. Using a pressure angle of 25 deg, let the stage ratios be

Stage 1 ratio $r_1 := 6.5$ Stage 2 ratio $r_2 := 10$

3. Following the procedure of Example 9-3,

Tooth number index $i := 2, 3 \dots 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \frac{N_3}{N_2}$ $r_2 := \frac{N_5}{N_4}$

Solving independently for N_2 and N_4 , $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 82.500$

By iteration, find a multiple of K_{min} that will result in a minimum, integer number of teeth on N_2 and N_4 .

$K := 2 \cdot K_{min}$ $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

$K = 165.000$ $N_2 = 22.0$ $N_4 = 15.0$

These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears will have tooth numbers of

$N_3 := r_1 \cdot N_2$ $N_3 = 143$ $N_5 := r_2 \cdot N_4$ $N_5 = 150$

The pitch diameters are: $d_i := \frac{N_i}{p_d}$

$i =$	$\frac{d_i}{\text{in}} =$	$N_i =$												
<table border="1"><tr><td>2</td></tr><tr><td>3</td></tr><tr><td>4</td></tr><tr><td>5</td></tr></table>	2	3	4	5	<table border="1"><tr><td>2.7500</td></tr><tr><td>17.8750</td></tr><tr><td>1.8750</td></tr><tr><td>18.7500</td></tr></table>	2.7500	17.8750	1.8750	18.7500	<table border="1"><tr><td>22</td></tr><tr><td>143</td></tr><tr><td>15</td></tr><tr><td>150</td></tr></table>	22	143	15	150
2														
3														
4														
5														
2.7500														
17.8750														
1.8750														
18.7500														
22														
143														
15														
150														

Checking the overall gear ratio: $\left(-\frac{N_3}{N_2} \right) \cdot \left(-\frac{N_5}{N_4} \right) = 65.000$

PROBLEM 9-17

Statement: Design a compound, reverted, spur gear train for a ratio of 7:1 and diametral pitch of 4. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 7$ Diametral pitch $p_d := 4 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0917.

1. Since the ratio is positive, we want to have an even number of stages. Let the number of stages be 2.

2. Using a pressure angle of 25 deg, let the stage ratios be

$$\text{Stage 1 ratio} \quad r_1 := 3.5 \quad \text{Stage 2 ratio} \quad r_2 := 2$$

3. Following the procedure of Example 9-3,

$$\text{Tooth number index} \quad i := 2, 3 \dots 5 \quad N_2 + N_3 = N_4 + N_5 = K \text{ and,} \quad r_1 := \frac{N_3}{N_2} \quad r_2 := \frac{N_5}{N_4}$$

$$\text{Solving independently for } N_2 \text{ and } N_4, \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$\text{where} \quad K_{\min} := (r_1 + 1) \cdot (r_2 + 1) \quad K_{\min} = 13.500$$

By iteration, find a multiple of $K_{\min}$ that will result in a minimum, integer number of teeth on N_2 and N_4 .

$$K := 6 \cdot K_{\min} \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$K = 81.000 \quad N_2 = 18 \quad N_4 = 27$$

These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears will have tooth numbers of

$$N_3 := r_1 \cdot N_2 \quad N_3 = 63 \quad N_5 := r_2 \cdot N_4 \quad N_5 = 54$$

The pitch diameters are: $d_i := \frac{N_i}{p_d}$

$i =$	$\frac{d_i}{\text{in}} =$	$N_i =$
2	4.5000	18
3	15.7500	63
4	6.7500	27
5	13.5000	54

$$\text{Checking the overall gear ratio:} \quad \left(-\frac{N_3}{N_2} \right) \cdot \left(-\frac{N_5}{N_4} \right) = 7.000$$

PROBLEM 9-18

Statement: Design a compound, reverted, spur gear train for a ratio of 12:1 and diametral pitch of 6. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 12$ Diametral pitch $p_d := 6 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0918.

1. Since the ratio is positive, we want to have an even number of stages. Let the number of stages be 2.

2. Using a pressure angle of 25 deg, let the stage ratios be

$$\text{Stage 1 ratio} \quad r_1 := 4 \quad \text{Stage 2 ratio} \quad r_2 := 3$$

3. Following the procedure of Example 9-3,

$$\text{Tooth number index} \quad i := 2, 3 \dots 5 \quad N_2 + N_3 = N_4 + N_5 = K \text{ and,} \quad r_1 := \frac{N_3}{N_2} \quad r_2 := \frac{N_5}{N_4}$$

$$\text{Solving independently for } N_2 \text{ and } N_4, \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$\text{where} \quad K_{\min} := (r_1 + 1) \cdot (r_2 + 1) \quad K_{\min} = 20.000$$

By iteration, find a multiple of $K_{\min}$ that will result in a minimum, integer number of teeth on N_2 and N_4 .

$$K := 4 \cdot K_{\min} \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$K = 80.000 \quad N_2 = 16 \quad N_4 = 20$$

These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears will have tooth numbers of

$$N_3 := r_1 \cdot N_2 \quad N_3 = 64 \quad N_5 := r_2 \cdot N_4 \quad N_5 = 60$$

The pitch diameters are:

$$d_i := \frac{N_i}{p_d}$$

$i =$
2
3
4
5

$$\frac{d_i}{\text{in}} =$$

2.6667
10.6667
3.3333
10.0000

$$N_i =$$

16
64
20
60

Checking the overall gear ratio: $\left(-\frac{N_3}{N_2} \right) \cdot \left(-\frac{N_5}{N_4} \right) = 12.000$

PROBLEM 9-19

Statement: Design a compound, reverted, spur gear transmission that will give two shiftable ratios of +3:1 forward and -4.5:1 reverse with diametral pitch of 6. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 3$ Diametral pitch $p_d := 6 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0919.

- Since the forward ratio is positive, we want to have an even number of stages. Let the number of stages be 2.
- Using a pressure angle of 25 deg, let the stage ratios be

$$\text{Stage 1 ratio} \quad r_1 := 2 \quad \text{Stage 2 ratio} \quad r_2 := 1.5$$

- Following the procedure of Example 9-3,

$$\text{Tooth number index} \quad i := 2, 3 \dots 5 \quad N_2 + N_3 = N_4 + N_5 = K \text{ and}, \quad r_1 := \frac{N_3}{N_2} \quad r_2 := \frac{N_5}{N_4}$$

$$\text{Solving independently for } N_2 \text{ and } N_4, \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$\text{where} \quad K_{\min} := (r_1 + 1) \cdot (r_2 + 1) \quad K_{\min} = 7.500$$

By iteration, find a multiple of $K_{\min}$ that will result in a minimum, integer number of teeth on N_2 and N_4 .

$$K := 6 \cdot K_{\min} \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$K = 45.000 \quad N_2 = 15 \quad N_4 = 18$$

These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

- The driven gears for the forward train will have tooth numbers of

$$N_3 := r_1 \cdot N_2 \quad N_3 = 30 \quad N_5 := r_2 \cdot N_4 \quad N_5 = 27$$

The pitch diameters are: $d_i := \frac{N_i}{p_d}$

$i =$	$\frac{d_i}{\text{in}} =$	$N_i =$
2	2.5000	15
3	5.0000	30
4	3.0000	18
5	4.5000	27

Checking the overall gear ratio:

$$\left(-\frac{N_3}{N_2} \right) \cdot \left(-\frac{N_5}{N_4} \right) = 3.000$$

- The reverse train will also have two stages and use the first forward stage and an idler gear to get the change in direction. Let the stage ratios be

$$\text{Stage 1 ratio} \quad r_1 := 2 \quad \text{Stage 2 ratio} \quad r_2 := 2.25$$

- Let the number of teeth on the reverse stage driver gear be $N_6 := 12$ then the number of teeth on the driven gear will be

$$\text{Driven reverse gear} \quad N_7 := r_2 \cdot N_6 \quad N_7 = 27$$

PROBLEM 9-20

Statement: Design a compound, reverted, spur gear transmission that will give two shiftable ratios of +5:1 forward and -3.5:1 reverse with diametral pitch of 6. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 5$ Diametral pitch $p_d := 6 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0920.

- Since the forward ratio is positive, we want to have an even number of stages. Let the number of stages be 2.
- Using a pressure angle of 25 deg, let the stage ratios be

$$\text{Stage 1 ratio} \quad r_1 := 2 \quad \text{Stage 2 ratio} \quad r_2 := 2.5$$

- Following the procedure of Example 9-3,

$$\text{Tooth number index} \quad i := 2, 3 \dots 5 \quad N_2 + N_3 = N_4 + N_5 = K \text{ and,} \quad r_1 := \frac{N_3}{N_2} \quad r_2 := \frac{N_5}{N_4}$$

$$\text{Solving independently for } N_2 \text{ and } N_4, \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$\text{where} \quad K_{\min} := (r_1 + 1) \cdot (r_2 + 1) \quad K_{\min} = 10.500$$

By iteration, find a multiple of $K_{\min}$ that will result in a minimum, integer number of teeth on N_2 and N_4 .

$$K := 4 \cdot K_{\min} \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$K = 42.000 \quad N_2 = 14 \quad N_4 = 12$$

These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

- The driven gears for the forward train will have tooth numbers of

$$N_3 := r_1 \cdot N_2 \quad N_3 = 28 \quad N_5 := r_2 \cdot N_4 \quad N_5 = 30$$

$$\text{The pitch diameters are:} \quad d_i := \frac{N_i}{p_d} \quad i = \quad \frac{d_i}{\text{in}} = \quad N_i =$$

Checking the overall gear ratio:

$$\left(-\frac{N_3}{N_2} \right) \cdot \left(-\frac{N_5}{N_4} \right) = 5.000$$

2	2.3333	14
3	4.6667	28
4	2.0000	12
5	5.0000	30

- The reverse train will also have two stages and use the first forward stage and an idler gear to get the change in direction. Let the stage ratios be

$$\text{Stage 1 ratio} \quad r_1 := 2 \quad \text{Stage 2 ratio} \quad r_2 := 1.75$$

- Let the number of teeth on the reverse stage driver gear be $N_6 := 12$ then the number of teeth on the driven gear will be

$$\text{Driven reverse gear} \quad N_7 := r_2 \cdot N_6 \quad N_7 = 21$$

PROBLEM 9-21

Statement: Design a compound, reverted, spur gear transmission that will give three shiftable ratios of +6:1, +3.5:1 forward and -4:1 reverse with diametral pitch of 8. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 6$ Diametral pitch $p_d := 8 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0921.

- Since the forward ratio is positive, we want to have an even number of stages. Let the number of stages be 2.
- Using a pressure angle of 25 deg, let the stage ratios be

$$\text{Stage 1 ratio} \quad r_1 := \frac{7}{3} \quad \text{Stage 2 ratio} \quad r_2 := \frac{18}{7}$$

- Following the procedure of Example 9-3,

$$\text{Tooth number index} \quad i := 2, 3 \dots 5 \quad N_2 + N_3 = N_4 + N_5 = K \text{ and,} \quad r_1 := \frac{N_3}{N_2} \quad r_2 := \frac{N_5}{N_4}$$

$$\text{Solving independently for } N_2 \text{ and } N_4, \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$\text{where} \quad K_{\min} := (r_1 + 1) \cdot (r_2 + 1) \quad K_{\min} = 11.905$$

By iteration, find a multiple of $K_{\min}$ that will result in a minimum, integer number of teeth on N_2 and N_4 .

$$K := 4.2 \cdot K_{\min} \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$K = 50.000 \quad N_2 = 15 \quad N_4 = 14$$

These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

- The driven gears for the forward train will have tooth numbers of

$$N_3 := r_1 \cdot N_2 \quad N_3 = 35 \quad N_5 := r_2 \cdot N_4 \quad N_5 = 36$$

The pitch diameters are: $d_i := \frac{N_i}{p_d}$

Checking the overall gear ratio:

$$\left(-\frac{N_3}{N_2} \right) \cdot \left(-\frac{N_5}{N_4} \right) = 6.000$$

$i =$	$\frac{d_i}{\text{in}} =$	$N_i =$
2	1.8750	15
3	4.3750	35
4	1.7500	14
5	4.5000	36

- The second forward train will also have two stages and use the first forward stage to get the overall ratio. Let the stage ratios be

$$\text{Stage 1 ratio} \quad r_1 := \frac{7}{3} \quad \text{Stage 2 ratio} \quad r_2 := 1.5$$

- Let the number of teeth on the second forward stage driver gear be $N_6 := 20$ then the number of teeth on the driven gear will be

$$\text{Second forward driven gear} \quad N_7 := r_2 \cdot N_6 \quad N_7 = 30$$

7. The reverse train will also have two stages and use the first forward stage and an idler gear to get the change in direction. Let the stage ratios be

$$\text{Stage 1 ratio} \quad r_1 := \frac{7}{3} \quad \text{Stage 2 ratio} \quad r_2 := \frac{12}{7}$$

8. Let the number of teeth on the reverse stage driver gear be $N_8 := 14$ then the number of teeth on the driven gear will be

$$\text{Driven reverse gear} \quad N_9 := r_2 \cdot N_8 \quad N_9 = 24$$

PROBLEM 9-22

Statement: Design a compound, reverted, spur gear transmission that will give three shiftable ratios of +4.5:1, +2.5:1 forward and -3.5:1 reverse with diametral pitch of 5. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 4.5$ Diametral pitch $p_d := 5 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0922.

- Since the forward ratio is positive, we want to have an even number of stages. Let the number of stages be 2.
- Using a pressure angle of 25 deg, let the stage ratios be

$$\text{Stage 1 ratio} \quad r_1 := \frac{9}{6} \quad \text{Stage 2 ratio} \quad r_2 := \frac{m_G}{r_1} \quad r_2 = 3.000$$

- Following the procedure of Example 9-3,

$$\text{Tooth number index} \quad i := 2, 3 \dots 5 \quad N_2 + N_3 = N_4 + N_5 = K \text{ and,} \quad r_1 := \frac{N_3}{N_2} \quad r_2 := \frac{N_5}{N_4}$$

$$\text{Solving independently for } N_2 \text{ and } N_4, \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$\text{where} \quad K_{\min} := (r_1 + 1) \cdot (r_2 + 1) \quad K_{\min} = 10.000$$

By iteration, find a multiple of $K_{\min}$ that will result in a minimum, integer number of teeth on N_2 and N_4 .

$$K := 12 \cdot K_{\min} \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$K = 120.000 \quad N_2 = 48 \quad N_4 = 30$$

These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

- The driven gears for the forward train will have tooth numbers of

$$N_3 := r_1 \cdot N_2 \quad N_3 = 72 \quad N_5 := r_2 \cdot N_4 \quad N_5 = 90$$

The pitch diameters are: $d_i := \frac{N_i}{p_d}$

Checking the overall gear ratio:

$$\left(-\frac{N_3}{N_2} \right) \cdot \left(-\frac{N_5}{N_4} \right) = 4.500$$

$i =$	$\frac{d_i}{\text{in}} =$	$N_i =$
2	9.6000	48
3	14.4000	72
4	6.0000	30
5	18.0000	90

- The second forward train will also have two stages and use the first forward stage to get the overall ratio. Let the stage ratios be

$$\text{Stage 1 ratio} \quad r_1 = 1.500 \quad \text{Stage 2 ratio} \quad r_2 := \frac{2.5}{r_1} \quad r_2 = 1.667$$

- Let the number of teeth on the second forward stage driver gear be $N_6 := 45$ then the number of teeth on the driven gear will be

$$\text{Second forward driven gear} \quad N_7 := r_2 \cdot N_6 \quad N_7 = 75$$

7. The reverse train will also have two stages and use the first forward stage and an idler gear to get the change in direction. Let the stage ratios be

$$\text{Stage 1 ratio} \quad r_1 = 1.500 \quad \text{Stage 2 ratio} \quad r_2 := \frac{3.5}{r_1} \quad r_2 = 2.333$$

8. Let the number of teeth on the reverse stage driver gear be $N_8 := 12$ then the number of teeth on the driven gear will be

$$\text{Driven reverse gear} \quad N_9 := r_2 \cdot N_8 \quad N_9 = 28$$

PROBLEM 9-23

Statement: Design the rolling cones for a -3:1 ratio and a 60-deg included angle between shafts. Sketch the train to scale.

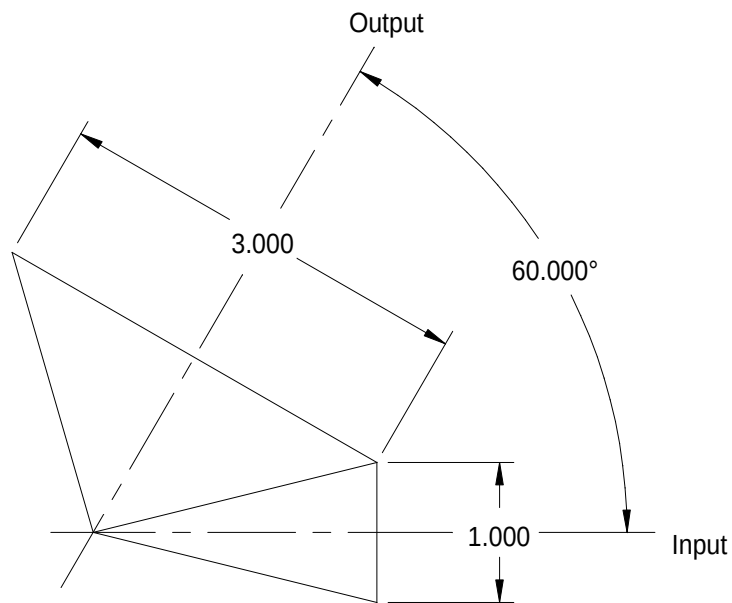
Given: Train ratio $m_T := -3$

Solution: See Figure 9-20 and Mathcad file P0923.

1. Choose a back-cone pitch diameter for the input cone of 1.000 in. Then, from equation 9.1,

$$d_{in} := 1.000 \cdot \text{in} \quad d_{out} := |m_T| \cdot d_{in} \quad d_{out} = 3.000 \text{ in}$$

2. Using these back-cone pitch diameters and the given shaft included angle, draw the cones to scale.



PROBLEM 9-24

Statement: Design the rolling cones for a -4.5:1 ratio and a 40-deg included angle between shafts. Sketch the train to scale.

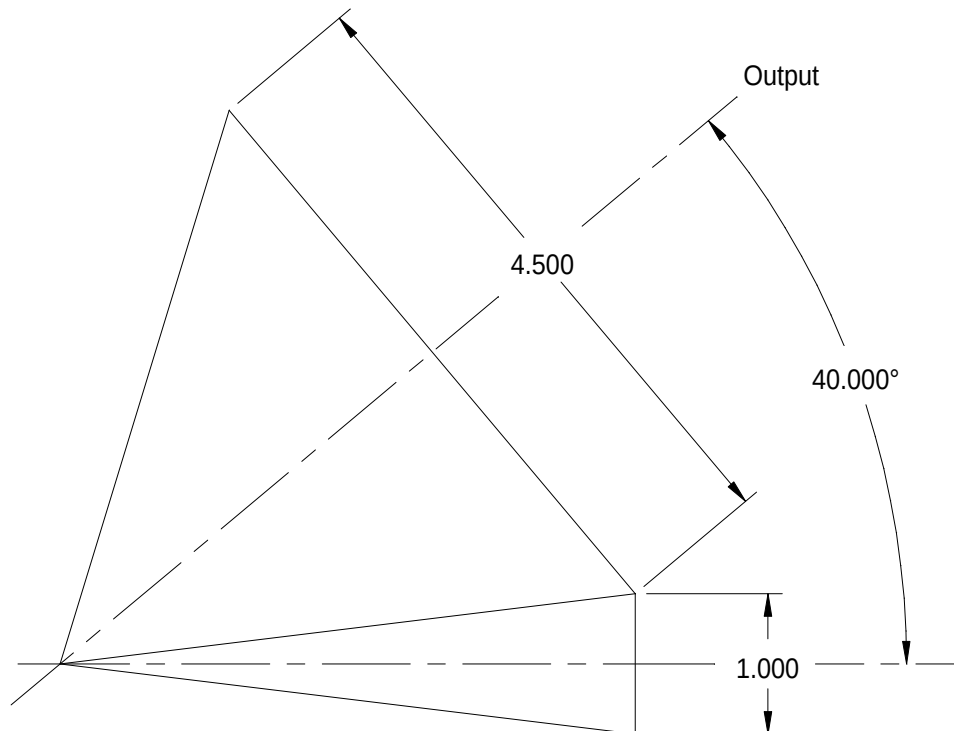
Given: Train ratio $m_T := -4.5$

Solution: See Figure 9-20 and Mathcad file P0924.

1. Choose a back-cone pitch diameter for the input cone of 1.000 in. Then, from equation 9.1,

$$d_{in} := 1.000 \cdot \text{in} \quad d_{out} := |m_T| \cdot d_{in} \quad d_{out} = 4.500 \text{ in}$$

2. Using these back-cone pitch diameters and the given shaft included angle, draw the cones to scale.



PROBLEM 9-25a

Statement: Figure P9-1 shows a compound planetary gear train (not to scale). Table P9-1 gives data for gear numbers of teeth and input velocities. For the data in row *a*, find ω_2 .

Given: Tooth numbers:

$$N_2 := 30 \quad N_3 := 25 \quad N_4 := 45 \quad N_5 := 50 \quad N_6 := 200$$

Input velocities: $\omega_6 := 20 \quad \omega_{arm} := -50$

Solution: See Figure P9-1 and Mathcad file P0925a.

1. The formula method will be used in this solution. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be 2 and last be 6. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_6 - \omega_{arm}}{\omega_2 - \omega_{arm}} = R$$

2. Calculate R using equation 9.14 and inspection of Figure P9-1.

$$R := \frac{N_2 \cdot N_3 \cdot N_5}{N_4 \cdot N_5 \cdot N_6} \quad R = 0.08333$$

Note that N_5 appears both in the numerator (a driver) and the denominator (driven). It could be left out completely because it is an idler. R is positive in this case because gears 2 and 6 rotate in the same direction.

3. Solve the right-hand equation in step 1 for ω_2 .

$$\omega_2 := \frac{\omega_6 - \omega_{arm}}{R} + \omega_{arm} \quad \omega_2 = 790.000$$

ω_2 will be in the same direction as ω_6 .

PROBLEM 9-26a

Statement: Figure P9-2 shows a compound planetary gear train (not to scale). Table P9-2 gives data for gear numbers of teeth and input velocities. For the data in row *a*, find ω_2 .

Given: Tooth numbers:

$$N_2 := 50 \quad N_3 := 25 \quad N_4 := 45 \quad N_5 := 30 \quad N_6 := 40$$

$$\text{Input velocities:} \quad \omega_6 := 20 \quad \omega_{arm} := -50$$

Solution: See Figure P9-2 and Mathcad file P0926a.

1. The formula method will be used in this solution. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be 3 and last be 6. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_6 - \omega_{arm}}{\omega_3 - \omega_{arm}} = R$$

2. Calculate R using equation 9.14 and inspection of Figure P9-2.

$$R := \frac{N_3 \cdot N_5}{N_4 \cdot N_6} \quad R = 0.4167$$

R is positive in this case because gears 3 and 6 rotate in the same direction.

3. Solve the right-hand equation in step 1 for ω_3 .

$$\omega_3 := \frac{\omega_6 - \omega_{arm}}{R} + \omega_{arm} \quad \omega_3 = 118.000$$

4. Solve for ω_2 using equation 9.7.

$$\omega_2 := -\frac{N_3}{N_2} \cdot \omega_3 \quad \omega_2 = -59.000$$

ω_2 will be in the opposite direction as ω_6 .

PROBLEM 9-27

Statement: Figure P9-3 shows a planetary gear train used in an automotive rear-end differential (not to scale). The car has wheels with a 16-in rolling radius and is moving forward in a straight line at 55 mph. The engine is turning 2500 rpm. The transmission is in direct drive (1:1) with the driveshaft.

- What is the rear wheel's rpm and the gear ratio between ring and pinion?
- As the car hits a patch of ice, the right wheel speeds up to 800 rpm. What is the speed of the left wheel? Hint: The average of both wheel's rpm is a constant.
- Calculate the fundamental train value of the epicyclic stage.

Given: $r := 16 \cdot \text{in}$ $V := 55 \cdot \text{mph}$ $\omega_{eng} := 2500 \cdot \text{rpm}$

Solution: See Figure P9-3 and Mathcad file P0927.

- Calculate the rear wheel rpm and the gear ratio between ring and pinion.

$$\omega_{wheel} := \frac{V}{r} \qquad \omega_{wheel} = 577.732 \cdot \text{rpm}$$

$$m_G := \frac{\omega_{eng}}{\omega_{wheel}} \qquad m_G = 4.327$$

- Using the hint given, calculate the speed of the left wheel when the right wheel hits the ice.

$$\omega_{avg} := \omega_{wheel} \qquad \omega_{right} := 800 \cdot \text{rpm}$$

$$\omega_{left} := 2 \cdot \omega_{avg} - \omega_{right} \qquad \omega_{left} = 355.465 \cdot \text{rpm}$$

PROBLEM 9-28

Statement: Design a speed-reducing planetary gearbox to be used to lift a 5-ton load 50 ft with a motor that develops 20 lb-ft of torque at its operating speed of 1750 rpm. The available winch drum has no more than a 16-in diameter when full of its steel cable. The speed reducer should be no larger in diameter than the winch drum. Gears of no more than 75 teeth are desired, and diametral pitch needs to be no larger than 6 to stand the stresses. Make multiview sketches of your design and show all calculations. How long will it take to raise the load with your design?

Units: $\text{rpm} := 2 \cdot \pi \cdot \text{rad} \cdot \text{min}^{-1}$

Given:

Load $W := 10000 \cdot \text{lb}$

Motor torque $T_m := 20 \cdot \text{ft} \cdot \text{lb}$ Motor speed $\omega_m := 1750 \cdot \text{rpm}$

Maximum drum diameter $d_{\max} := 16 \cdot \text{in}$

Assumptions: Minimum drum diameter (with no cable) is $d_{\min} := 8 \cdot \text{in}$

Solution: See Figure 9-33 and Mathcad file P0901.

1. Determine the minimum gear ratio required.

$$\text{Worst load torque: } T_L := W \cdot \frac{d_{\max}}{2} \quad T_L = 6667 \text{ ft} \cdot \text{lb}$$

Minimum gear ratio required to lift load:

$$m_G := \frac{T_L}{T_m} \quad m_G = 333.333$$

2. Use a combination of basic planetary gearsets, such as that shown in Figure 9-33, compounded in the following way. The input to each stage will be the sun gear. The output will be the arm, and the ring gear will be stationary. All stages will mesh with the same ring gear, which will be elongated to reach the planets of all stages. Except for the last stage, the arm of one stage will be coupled directly to the sun gear of the next stage. All stages will have the same number of teeth on the sun gears and the same (different from the sun gears) for the planets. Since they all mesh with the same ring gear, they must all have the same diametral pitch and pressure angle. Let the pressure angle be 25 deg.
3. Calculate the number of stages required to achieve the required gear ratio.

$$\text{Let the diametral pitch be } p_d := 4 \cdot \text{in}^{-1}$$

$$\text{Ring gear pitch diameter } d_r := 16 \cdot \text{in} \quad N_r := p_d \cdot d_r \quad N_r = 64$$

$$\text{Number of teeth on each sun gear } N_s := 12$$

$$\text{Number of teeth on each planet gear } N_p := \frac{N_r - N_s}{2} \quad N_p = 26$$

For a single stage, the gear ratio for the planetary configuration of Figure 9-33 is $-(N_r - N_s)/N_s$. For multiple stages,

$$m_G := \left(\frac{N_r - N_s}{N_s} \right)^n$$

where n is the number of stages. Solving for n ,

$$n := \text{ceil} \left(\frac{\log(m_G)}{\log \left(\frac{N_r - N_s}{N_s} \right)} \right) \quad n = 4 \quad \text{stages}$$

4. Calculate the actual gear ratio achieved.

$$m_G := \left(-\frac{N_r - N_s}{N_s} \right)^n \quad m_G = 352.605$$

5. Calculate the time required to raise the load $L := 50 \cdot \text{ft}$.

$$\text{Average drum diameter} \quad d_{avg} := \frac{1}{2} \cdot (d_{max} + d_{min}) \quad d_{avg} = 12.000 \text{ in}$$

$$\text{Average cable velocity} \quad V_{avg} := \frac{d_{avg}}{2} \cdot \frac{\omega_m}{m_G} \quad V_{avg} = 15.592 \frac{\text{ft}}{\text{min}}$$

$$\text{Average time to raise load} \quad t_{avg} := \frac{L}{V_{avg}} \quad t_{avg} = 3.2 \text{ min}$$

PROBLEM 9-29

Statement: Determine all possible two-stage compound gear combinations that will give an approximation to the Napierian base 2.71828. Limit tooth numbers to between 18 and 80. Determine the arrangement that gives the smallest error.

Given:

$$\text{ratio} := 2.71828 \quad N_{\min} := 18 \quad N_{\max} := 80 \quad \text{err} := 0.001\%$$

Solution: See Mathcad file P0929.

1. Input the given data into the TK-Solver file *compound.tkw*. The $n := 8$ tooth number combinations given below are the result. Let $i := 1, 2 \dots n$

$N2_i :=$	$N3_i :=$	$N4_i :=$	$N5_i :=$
25	67	70	71
29	57	47	65
30	32	31	79
30	64	62	79
31	48	45	79
31	64	60	79
31	79	75	80
35	67	50	71

$$\text{least}(r, n) := \begin{cases} \text{low} \leftarrow r_1 \\ \text{index} \leftarrow 1 \\ \text{for } i \in 2 \dots n \\ \quad \left| \begin{array}{l} \text{index} \leftarrow i \text{ if } r_i < \text{low} \\ \text{low} \leftarrow r_i \text{ if } r_i < \text{low} \end{array} \right. \\ \text{index} \end{cases}$$

2. Determine the stage ratios and the overall ratio for each set.

$$\text{Stage 1 ratio} \quad \text{ratio1}_i := \frac{N3_i}{N2_i} \quad \text{Stage 2 ratio} \quad \text{ratio2}_i := \frac{N5_i}{N4_i}$$

$$\text{Actual ratio for given set:} \quad \text{Ratio}_i := \text{ratio1}_i \cdot \text{ratio2}_i$$

$$\text{Error in ratio:} \quad \text{Error}_i := \left| \text{ratio} - \text{Ratio}_i \right|$$

3. Find the set with the least error.

$$R := \text{least}(\text{Error}, n)$$

$$R = 3$$

$$N2_R = 30$$

$$N3_R = 32$$

$$N4_R = 31$$

$$N5_R = 79$$

$$\text{ratio1}_R = 1.06667$$

$$\text{ratio2}_R = 2.54839$$

$$\text{Ratio}_R = 2.718280$$

$$\text{Error}_R = 4.30108 \times 10^{-7}$$

PROBLEM 9-30

Statement: Determine all possible two-stage compound gear combinations that will give an approximation to 2π . Limit tooth numbers to between 18 and 80. Determine the arrangement that gives the smallest error.

Given:

$$ratio := 6.283185 \quad N_{min} := 15 \quad N_{max} := 90 \quad err := 0.001\%$$

Solution: See Mathcad file P0930.

1. Input the given data into the *TK-Solver* file *compound.tkw*. The $n := 4$ tooth number combinations given below are the result. Let $i := 1, 2 \dots n$

$N2_i :=$	$N3_i :=$	$N4_i :=$	$N5_i :=$	$least(r, n) :=$	$low \leftarrow r_1$ $index \leftarrow 1$ for $i \in 2 \dots n$ $index \leftarrow i$ if $r_i < low$ $low \leftarrow r_i$ if $r_i < low$ $index$																
<table border="1"> <tr><td>16</td></tr> <tr><td>23</td></tr> <tr><td>25</td></tr> <tr><td>28</td></tr> </table>	16	23	25	28	<table border="1"> <tr><td>63</td></tr> <tr><td>75</td></tr> <tr><td>51</td></tr> <tr><td>85</td></tr> </table>	63	75	51	85	<table border="1"> <tr><td>47</td></tr> <tr><td>41</td></tr> <tr><td>25</td></tr> <tr><td>43</td></tr> </table>	47	41	25	43	<table border="1"> <tr><td>75</td></tr> <tr><td>79</td></tr> <tr><td>77</td></tr> <tr><td>89</td></tr> </table>	75	79	77	89		
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2. Determine the stage ratios and the overall ratio for each set.

$$\text{Stage 1 ratio} \quad ratio1_i := \frac{N3_i}{N2_i} \quad \text{Stage 2 ratio} \quad ratio2_i := \frac{N5_i}{N4_i}$$

$$\text{Actual ratio for given set:} \quad Ratio_i := ratio1_i \cdot ratio2_i$$

$$\text{Error in ratio:} \quad Error_i := |ratio - Ratio_i|$$

3. Find the set with the least error.

$$R := least(Error, n)$$

$$R = 3$$

$$N2_R = 25$$

$$N3_R = 51$$

$$N4_R = 25$$

$$N5_R = 77$$

$$ratio1_R = 2.04000$$

$$ratio2_R = 3.08000$$

$$Ratio_R = 6.283200$$

$$Error_R = 1.50000 \times 10^{-5}$$

PROBLEM 9-31

Statement: Determine all possible two-stage compound gear combinations that will give an approximation to $3\pi/2$. Limit tooth numbers to between 18 and 80. Determine the arrangement that gives the smallest error.

Given:

$$\text{ratio} := 1.570796 \quad N_{\min} := 20 \quad N_{\max} := 100 \quad \text{err} := 0.001\%$$

Solution: See Mathcad file P0931.

1. Input the given data into the *TK-Solver* file *compound.tkw*. The $n := 29$ tooth number combinations given below are the result. Let $i := 1, 2 \dots n$

$N2_i :=$	$N3_i :=$	$N4_i :=$	$N5_i :=$	$\text{least}(r, n) :=$
22	31	61	68	$\text{low} \leftarrow r_1$
22	34	61	62	$\text{index} \leftarrow 1$
22	49	95	67	for $i \in 2 \dots n$
25	51	100	77	$\text{index} \leftarrow i$ if $r_i < \text{low}$
29	44	85	88	$\text{low} \leftarrow r_i$ if $r_i < \text{low}$
32	63	94	75	index
33	34	61	93	
33	51	61	62	
36	55	71	73	
37	59	67	66	
38	49	55	67	
41	57	77	87	
41	75	92	79	
43	50	57	77	
43	55	57	70	
43	58	79	92	
44	62	61	68	
44	67	95	98	
45	64	67	74	
46	75	82	79	
47	63	64	75	
47	65	81	92	
50	51	50	77	
55	62	61	85	
55	67	76	98	
56	85	86	89	
57	77	86	100	
58	88	85	88	
61	68	66	93	

2. Determine the stage ratios and the overall ratio for each set.

$$\text{Stage 1 ratio} \quad \text{ratio1}_i := \frac{N3_i}{N2_i} \quad \text{Stage 2 ratio} \quad \text{ratio2}_i := \frac{N5_i}{N4_i}$$

$$\text{Actual ratio for given set:} \quad \text{Ratio}_i := \text{ratio1}_i \cdot \text{ratio2}_i$$

Error in ratio:

$$Error_i := \left| ratio - Ratio_i \right|$$

3. Find the set with the least error.

$$R := \text{least}(Error, n)$$

$$R = 12$$

$$N2_R = 41$$

$$N3_R = 57$$

$$N4_R = 77$$

$$N5_R = 87$$

$$ratio1_R = 1.39024$$

$$ratio2_R = 1.12987$$

$$Ratio_R = 1.570795$$

$$Error_R = 9.41400 \times 10^{-7}$$

PROBLEM 9-32

Statement: Determine all possible two-stage compound gear combinations that will give an approximation to $3\pi/2$. Limit tooth numbers to between 18 and 80. Determine the arrangement that gives the smallest error.

Given:

$$ratio := 4.71239 \quad N_{min} := 20 \quad N_{max} := 100 \quad err := 0.001\%$$

Solution: See Mathcad file P0932.

1. Input the given data into the *TK-Solver* file *compound.tkw*. The $n := 5$ tooth number combinations given below are the result. Let $i := 1, 2 \dots n$

$N2_i :=$	$N3_i :=$	$N4_i :=$	$N5_i :=$	$least(r, n) :=$
22	68	61	93	$low \leftarrow r_1$
27	65	47	92	$index \leftarrow 1$
37	68	39	100	for $i \in 2 \dots n$
37	80	39	85	$index \leftarrow i$ if $r_i < low$
38	77	43	100	$low \leftarrow r_i$ if $r_i < low$
				$index$

2. Determine the stage ratios and the overall ratio for each set.

$$\text{Stage 1 ratio} \quad ratio1_i := \frac{N3_i}{N2_i} \quad \text{Stage 2 ratio} \quad ratio2_i := \frac{N5_i}{N4_i}$$

$$\text{Actual ratio for given set:} \quad Ratio_i := ratio1_i \cdot ratio2_i$$

$$\text{Error in ratio:} \quad Error_i := |ratio - Ratio_i|$$

3. Find the set with the least error.

$$R := least(Error, n)$$

$$R = 4$$

$$N2_R = 37$$

$$N3_R = 80$$

$$N4_R = 39$$

$$N5_R = 85$$

$$ratio1_R = 2.16216$$

$$ratio2_R = 2.17949$$

$$Ratio_R = 4.712405$$

$$Error_R = 1.47124 \times 10^{-5}$$

PROBLEM 9-33

Statement: Figure P-9-4a shows a reverted clock train. Design it using 25-deg nominal pressure angle gears of 24 p_d having between 12 and 150 teeth. Determine the tooth numbers and nominal center distance. If the center distance has a manufacturing tolerance of plus or minus 0.006 in, what will the pressure angle and the backlash at the minute hand be at each extreme of tolerance?

Given: Diametral pitch $p_d := 24 \cdot \text{in}^{-1}$ Tolerance (one side) $t := 0.006 \cdot \text{in}$

Solution: See Mathcad file P0933.

1. Write the speeds of the minute and hour hands and their ratio.

$$\begin{array}{ll} \text{Minute hand} & \omega_1 := \frac{2 \cdot \pi \cdot \text{rad}}{\text{hr}} \\ \text{Hour hand} & \omega_2 := \frac{2 \cdot \pi \cdot \text{rad}}{12 \cdot \text{hr}} \\ & \text{ratio} := \frac{\omega_1}{\omega_2} \quad \text{ratio} = 12.000 \end{array}$$

Assume some input speed ω_{in} to the shaft with gears B and C on it. Then,

$$\begin{array}{lll} \omega_1 := \frac{N_B}{N_A} \cdot \omega_{in} & \omega_2 := \frac{N_C}{N_E} \cdot \omega_{in} & \frac{\omega_1}{\omega_2} = \frac{N_B}{N_A} \cdot \frac{N_E}{N_C} = \text{ratio} \\ \frac{N_B}{N_A} = \frac{N_3}{N_2} = r_1 & \frac{N_E}{N_C} = \frac{N_5}{N_4} = r_2 & \text{ratio} := r_1 \cdot r_2 \end{array}$$

2. Let r_1 and r_2 be factors of ratio. Say,

$$\begin{array}{ll} \text{Minute ratio} & r_1 := 3 \\ \text{Hour ratio} & r_2 := 4 \end{array}$$

3. Following the procedure of Example 9-3,

$$\text{Tooth number index } i := 2, 3 \dots 5 \quad N_2 + N_3 = N_4 + N_5 = K \text{ and, } r_1 := \frac{N_3}{N_2} \quad r_2 := \frac{N_5}{N_4}$$

$$\text{Solving independently for } N_2 \text{ and } N_4, \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$\text{where } K_{min} := (r_1 + 1) \cdot (r_2 + 1) \quad K_{min} = 20.000$$

By iteration, find a multiple of K_{min} that will result in a minimum number of teeth on N_2 and N_4 .

$$K := 3 \cdot K_{min} \quad N_2 := \frac{K}{r_1 + 1} \quad N_4 := \frac{K}{r_2 + 1}$$

$$K = 60.000 \quad N_2 = 15 \quad N_4 = 12$$

4. The other gears will have tooth numbers of

$$N_3 := r_1 \cdot N_2 \quad N_3 = 45 \quad N_5 := r_2 \cdot N_4 \quad N_5 = 48$$

The pitch diameters are: $d_i := \frac{N_i}{p_d}$

$i =$

2
3
4
5

$\frac{d_i}{in} =$

0.6250
1.8750
0.5000
2.0000

$N_i =$

15
45
12
48

A
B
C
E

5. The nominal center distance is $C := 0.5 \cdot (d_2 + d_3)$ $C = 1.250 \text{ in}$

6. Calculate the extreme values of center distance and their ratios with respect to the nominal center distance.

$$C_{max} := C + t \quad C_{max} = 1.2560 \text{ in} \quad r_{max} := \frac{C_{max}}{C} \quad r_{max} = 1.0048$$

$$C_{min} := C - t \quad C_{min} = 1.2440 \text{ in} \quad r_{min} := \frac{C_{min}}{C} \quad r_{min} = 0.9952$$

7. Using the equation in step 11 of Example 9-1, calculate the pressure angle that would result if the center distance were at its extreme values.

$$\phi_{max} := \arccos\left(\frac{\cos(25 \cdot \text{deg})}{r_{max}}\right) \quad \phi_{max} = 25.581 \text{ deg}$$

$$\phi_{min} := \arccos\left(\frac{\cos(25 \cdot \text{deg})}{r_{min}}\right) \quad \phi_{min} = 24.401 \text{ deg}$$

8. The backlash in the minute hand is found using equation 9.3,

$$\theta_{Bmax} := 43200 \cdot t \cdot \frac{\tan(\phi_{max})}{\pi \cdot d_2} \quad \theta_{Bmax} = 63 \quad \text{Minutes of arc}$$

Depending on the nominal clearance in the gearset, decreasing the center distance will eventually cause the teeth to jamb and the gears may not be capable of assembly.

PROBLEM 9-34

Statement: Figure P9-4b shows a three-speed shiftable transmission. Shaft F, with the cluster of gears E, G, and H, is capable of sliding left and right to engage and disengage the three gearsets in turn. Design the three reverted stages to give output speeds at shaft F of 150, 350, and 550 rpm for an input speed of 450 rpm on shaft D.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Output speeds:

$$\omega_{F1} := 150 \cdot rpm \quad \omega_{F2} := 350 \cdot rpm \quad \omega_{F3} := 550 \cdot rpm$$

Input speed: $\omega_D := 450 \cdot rpm$

Solution: See Figure P9-4b and Mathcad file P0934.

1. Determine the gearset ratios

$$r_1 := \frac{\omega_D}{\omega_{F1}} \quad r_2 := \frac{\omega_D}{\omega_{F2}} \quad r_3 := \frac{\omega_D}{\omega_{F3}}$$

$$r_1 = 3.0000 \quad r_2 = 1.2857 \quad r_3 = 0.8182$$

2. Following the procedure of Example 9-3,

$$N_A + N_E = N_B + N_G = N_C + N_H = K, \quad r_1 := \frac{N_E}{N_A} \quad r_2 := \frac{N_H}{N_C} \quad r_3 := \frac{N_G}{N_B}$$

Solving independently for N_A , N_C and N_B , $N_A := \frac{K}{r_1 + 1} \quad N_C := \frac{K}{r_2 + 1} \quad N_B := \frac{K}{r_3 + 1}$

where $K_{min} := (r_1 + 1) \cdot (r_2 + 1) \cdot (r_3 + 1) \quad K_{min} = 1.000 \frac{1280}{77}$

By iteration, find a multiple of K_{min} that will result in a minimum, integer number of teeth on the drivers:

$$K := \frac{77}{16} \cdot K_{min} \quad N_A := \frac{K}{r_1 + 1} \quad N_C := \frac{K}{r_2 + 1} \quad N_B := \frac{K}{r_3 + 1}$$

$$K = 80.000 \quad N_A = 20 \quad N_C = 35 \quad N_B = 44$$

These are acceptable tooth numbers for gears with a 25- or 20-deg pressure angle that are cut by a hob.

3. The driven gears on shaft F will have tooth numbers of

$$N_E := r_1 \cdot N_A \quad N_E = 60 \quad N_H := r_2 \cdot N_C \quad N_H = 45$$

$$N_G := r_3 \cdot N_B \quad N_G = 36$$

PROBLEM 9-35

Statement: Figure P9-5a shows an epicyclic train used to drive a winch drum. Gear A, on shaft 1, is driven at 18 rpm CCW and gear D, on shaft 2, is fixed to ground. The tooth numbers are indicated in the figure. Determine the speed and direction of the drum. What is the efficiency of this train if the basic gearsets have $E_0 = 0.97$?

Given: Tooth numbers:

$$N_A := 72 \quad N_B := 16 \quad N_C := 48 \quad N_D := 40$$

$$\text{Input speeds:} \quad \omega_A := 18 \cdot \text{rpm} \quad \omega_D := 0 \cdot \text{rpm}$$

$$\text{Basic gearset efficiency:} \quad E_0 := 0.97$$

Solution: See Figure P9-5a and Mathcad file P0935.

1. Determine the speed of the arm using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be D. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{F arm}} = \frac{\omega_D - \omega_{arm}}{\omega_A - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-5a.

$$R := \left(-\frac{N_A}{N_B} \right) \cdot \left(-\frac{N_C}{N_D} \right) \quad R = 5.40000$$

Solve the right-hand equation above for ω_{arm} with $\omega_D = 0$.

$$\omega_{arm} := \frac{R}{R - 1} \cdot \omega_A \quad \omega_{arm} = 22.09 \cdot \text{rpm}$$

The arm drives the drum, so

$$\omega_{drum} := \omega_{arm} \quad \omega_{drum} = 22.1 \cdot \text{rpm}$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := R \quad \rho = 5.400$$

3. The combination of $\rho > 1$, shaft 2 fixed, and input to shaft 1 corresponds to Case 1 in Table 9-12 giving an efficiency of

$$\eta := \frac{\rho \cdot E_0 - 1}{\rho - 1} \quad \eta = 0.963$$

PROBLEM 9-36

Statement: Figure P9-5b shows an epicyclic train. The arm is driven CCW at 20 rpm. Gear A is driven at 40 rpm CW. The tooth numbers are indicated in the figure. Find the speed of the ring gear D.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$$N_A := 17 \quad N_B := 28 \quad N_C := 15 \quad N_D := 60$$

$$\text{Input speeds:} \quad \omega_A := -40 \cdot rpm \quad \omega_{arm} := 20 \cdot rpm$$

Solution: See Figure P9-5b and Mathcad file P0936.

1. Determine the speed of the ring gear using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be D. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_D - \omega_{arm}}{\omega_A - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-5b.

$$R := \left(-\frac{N_A}{N_B} \right) \cdot \left(\frac{N_C}{N_D} \right) \quad R = -0.15179$$

Solve the right-hand equation above for ω_D .

$$\omega_D := R \cdot (\omega_A - \omega_{arm}) + \omega_{arm} \quad \omega_D = 29.11 \, rpm$$

PROBLEM 9-37

Statement: Figure P9-6a shows an epicyclic train. The arm is driven at 50 rpm CCW and gear A, on shaft 1, is fixed to ground. The tooth numbers are indicated in the figure. Find the speed of gear D, on shaft 2. What is the efficiency of this train if the basic gearsets have $E_0 = 0.96$?

Given: Tooth numbers:

$$N_A := 108 \quad N_B := 27 \quad N_C := 100 \quad N_D := 35$$

$$\text{Input speeds:} \quad \omega_{arm} := 50 \cdot rpm \quad \omega_A := 0 \cdot rpm$$

$$\text{Basic gearset efficiency:} \quad E_0 := 0.96$$

Solution: See Figure P9-6a and Mathcad file P0937.

1. Determine the speed of the sun gear using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be D. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{F arm}} = \frac{\omega_D - \omega_{arm}}{\omega_A - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-6a.

$$R := \left(-\frac{N_A}{N_B} \right) \cdot \left(-\frac{N_C}{N_D} \right) \quad R = 11.42857$$

Solve the right-hand equation above for ω_D with $\omega_A = 0$.

$$\omega_D := (1 - R) \cdot \omega_{arm} \quad \omega_D = -521.43 \cdot rpm$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := R \quad \rho = 11.429$$

3. The combination of $\rho > 1$, shaft 1 fixed, and input to the arm corresponds to Case 4 in Table 9-12 giving an efficiency of

$$\eta := \frac{\rho - 1}{\rho - E_0} \quad \eta = 0.996$$

PROBLEM 9-38

Statement: Figure P9-6b shows a differential. Gear A is driven at 10 rpm CCW and gear B is driven CW at 24 rpm. The tooth numbers are indicated in the figure. Find the speed of gear D.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$$N_{A'} := 18 \quad N_{B'} := 18 \quad N_C := 18 \quad N_D := 30$$

$$\text{Input speeds:} \quad \omega_A := 10 \cdot rpm \quad \omega_B := -24 \cdot rpm$$

Solution: See Figure P9-6b and Mathcad file P0938.

1. Determine the speed of the sun gear using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A' and last be B'. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{F arm}} = \frac{\omega_{B'} - \omega_{arm}}{\omega_{A'} - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-6b.

$$R := \left(-\frac{N_{A'}}{N_{B'}} \right) \quad R = -1.00000$$

Solve the right-hand equation above for ω_D with $\omega_A = 0$.

$$\omega_{A'} := \omega_A$$

$$\omega_{B'} := \omega_B$$

$$\omega_{arm} := \frac{R \cdot \omega_{A'} - \omega_{B'}}{R - 1}$$

$$\omega_{arm} = -7.000 \text{ rpm}$$

Gear D is attached to the arm shaft so,

$$\omega_D := \omega_{arm}$$

$$\omega_D = -7.00 \text{ rpm}$$

PROBLEM 9-39

Statement: Figure P9-7a shows a gear train containing both compound-reverted and epicyclic stages. The tooth numbers are indicated in the figure. The motor is driven CW at 1500 rpm. Find the speeds of shafts 1 and 2.

Given: Tooth numbers:

$$N_A := 15 \quad N_B := 40 \quad N_C := 20 \quad N_D := 35 \quad N_E := 48$$

$$N_F := 12 \quad N_G := 72 \quad N_H := 18 \quad N_J := 42 \quad N_K := 15$$

Motor speed: $\omega_m := -1500 \cdot \text{rpm}$ CW

Solution: See Figure P9-7a and Mathcad file P0939.

1. The motor furnishes two inputs to the epicyclic train, which is composed of gears E (sun), F (planets), and G (ring). One input is to the sun gear through the reverted train ABCD. The other is directly from the motor to the arm.
2. Determine the speed of the shaft containing gears D and E using equations 9.8.

$$\omega_D := \left(-\frac{N_A}{N_B} \right) \cdot \left(-\frac{N_C}{N_D} \right) \cdot \omega_m \quad \omega_D = -321.429 \cdot \text{rpm}$$

$$\omega_E := \omega_D \quad \omega_E = -321.429 \cdot \text{rpm}$$

3. Determine the speed of the ring gear G using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be E and last be G. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_G - \omega_{arm}}{\omega_E - \omega_{arm}} = R \quad \omega_{arm} := \omega_m$$

Calculate R using equation 9.14 and inspection of Figure P9-7a.

$$R := -\frac{N_E}{N_G} \quad R = -0.66667$$

Solve the right-hand equation above for ω_G .

$$\omega_G := R \cdot (\omega_E - \omega_{arm}) + \omega_{arm} \quad \omega_G = -2285.71 \cdot \text{rpm}$$

4. Gears G and H rotate at the same speed and gear H drives the two output shafts. Calculate their speed using equation 9.7.

$$\omega_1 := -\frac{N_H}{N_J} \cdot \omega_G \quad \omega_1 = 979.6 \cdot \text{rpm}$$

$$\omega_2 := -\frac{N_H}{N_K} \cdot \omega_G \quad \omega_2 = 2742.9 \cdot \text{rpm}$$

PROBLEM 9-40

Statement: Figure P9-35b shows (schematically) a compound epicyclic train. The tooth numbers are 50, 25, 35, and 90 for gears 2, 3, 4, and 5, respectively. The arm is driven at 180 rpm CW and gear 5 is fixed to ground. Determine the speed and direction of gear 2. What is the efficiency of this train if the basic gearsets have $E_0 = 0.98$?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$$N_2 := 50 \quad N_3 := 25 \quad N_4 := 35 \quad N_5 := 90$$

$$\text{Input speeds:} \quad \omega_{arm} := -180 \cdot rpm \quad \omega_5 := 0 \cdot rpm$$

$$\text{Basic gearset efficiency:} \quad E_0 := 0.98$$

Solution: See Figure P9-7b and Mathcad file P0940.

1. Determine the speed of the sun gear 2 using the formula method for analyzing an epicyclic train. To start, choose a first and last gears that mesh with gears that have planetary motion. Let the first gear be 2 and last be 5. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_5 - \omega_{arm}}{\omega_2 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-7b.

$$R := \left(-\frac{N_2}{N_3} \right) \cdot \left(\frac{N_4}{N_5} \right) \quad R = -0.77778$$

Solve the right-hand equation above for ω_2 with $\omega_5 = 0$.

$$\omega_2 := \frac{R - 1}{R} \cdot \omega_{arm} \quad \omega_2 = -411.43 \, rpm$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := \frac{1}{R} \quad \rho = -1.286$$

3. The combination of $\rho < -1$, shaft 1 fixed, and input to the arm corresponds to Case 8 in Table 9-12 giving an efficiency of

$$\eta := \frac{E_0(\rho - 1)}{\rho \cdot E_0 - 1} \quad \eta = 0.991$$

PROBLEM 9-41

Statement: Figure P9-8 shows a compound epicyclic train. Gear 2 is driven at 600 rpm CW and gear D is fixed to ground. The tooth numbers are indicated in the figure. Determine the speed and direction of gears 1 and 3.

Given: Tooth numbers:

$$N_A := 24 \quad N_B := 30 \quad N_C := 72 \quad N_D := 60 \quad N_E := 120 \quad N_F := 100$$

$$\text{Gear 2 speed: } \omega_2 := -600 \cdot \text{rpm} \quad \text{CW}$$

Solution: See Figure P9-8 and Mathcad file P0941.

1. Determine the speed of the arm using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be D. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_D - \omega_{arm}}{\omega_A - \omega_{arm}} = R \quad \omega_A := \omega_2$$

Calculate R using equation 9.14 and inspection of Figure P9-8.

$$R := -\frac{N_A}{N_B} \cdot \left(\frac{N_C}{N_D} \right) \quad R = -0.96000$$

Solve the right-hand equation above for ω_{arm} with $\omega_D = 0$.

$$\omega_{arm} := \frac{R}{R - 1} \cdot \omega_A \quad \omega_{arm} = -293.88 \cdot \text{rpm}$$

The arm drives gear 1, so

$$\omega_1 := \omega_{arm} \quad \omega_1 = -293.9 \cdot \text{rpm}$$

2. Determine the speed of gear F using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be F. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_F - \omega_{arm}}{\omega_A - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-1.

$$R := \left(\frac{N_A}{N_B} \right) \cdot \left(\frac{N_E}{N_F} \right) \quad R = 0.96000$$

Solve the right-hand equation above for ω_F .

$$\omega_F := R \cdot (\omega_A - \omega_{arm}) + \omega_{arm} \quad \omega_F = -587.755 \cdot \text{rpm}$$

Gear F drives gear 3, so

$$\omega_3 := \omega_F \quad \omega_3 = -587.8 \cdot \text{rpm}$$

PROBLEM 9-42

Statement: Figure P9-9a shows a compound epicyclic train. Shaft 1 is driven at 300 rpm CCW and gear A is fixed to ground. The tooth numbers are indicated in the figure. Determine the speed and direction of shaft 2.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$$N_A := 56 \quad N_B := 18 \quad N_C := 48 \quad N_D := 26 \quad N_E := 60 \quad N_F := 18 \quad N_G := 68$$

$$\text{Shaft 1 speed: } \omega_1 := 300 \cdot rpm \quad \text{CCW}$$

Solution: See Figure P9-9a and Mathcad file P0942.

1. Shaft 1 drives arm-1, the first stage arm, and arm-2, the second stage arm. The first stage is composed of gears A, B, C, and D, with gear A fixed. The second stage is composed of gears D, E, F, and G. Second stage inputs are from gear D and arm-2.

$$\omega_{arm} := \omega_1$$

2. Determine the speed of gear D using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be D. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_D - \omega_{arm}}{\omega_A - \omega_{arm}} = R \quad \omega_A := 0 \cdot rpm$$

Calculate R using equation 9.14 and inspection of Figure P9-9a.

$$R := \left(-\frac{N_A}{N_B} \right) \cdot \left(-\frac{N_C}{N_D} \right) \quad R = 5.74359$$

Solve the right-hand equation above for ω_D with $\omega_A = 0$.

$$\omega_D := (1 - R) \cdot \omega_{arm} \quad \omega_D = -1423.08 \text{ rpm}$$

3. Determine the speed of gear G using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be D and last be G. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_G - \omega_{arm}}{\omega_D - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-9a.

$$R := \left(-\frac{N_D}{N_E} \right) \cdot \left(-\frac{N_F}{N_G} \right) \quad R = 0.11471$$

Solve the right-hand equation above for ω_G .

$$\omega_G := R \cdot (\omega_D - \omega_{arm}) + \omega_{arm} \quad \omega_G = 102.4 \text{ rpm}$$

Gear G drives shaft 2, so

$$\omega_2 := \omega_G \quad \omega_2 = 102.4 \text{ rpm}$$

PROBLEM 9-43

Statement: Figure P9-10 shows a compound epicyclic train. Shaft 1 is driven at 60 rpm. The tooth numbers are indicated in the figure. Determine the speed and direction of gears G and M.

Given: Tooth numbers:

$$N_A := 36 \quad N_B := 60 \quad N_C := 40 \quad N_{Dout} := 64 \quad N_{Din} := 58 \quad N_E := 18 \quad N_F := 26$$

$$N_G := 66 \quad N_H := 24 \quad N_J := 54 \quad N_K := 40 \quad N_L := 38 \quad N_M := 96$$

$$\text{Shaft 1 speed: } \omega_1 := 60 \cdot \text{rpm} \quad \text{CCW}$$

Solution: See Figure P9-10 and Mathcad file P0943.

1. The motor furnishes inputs to both epicyclic trains. To the first stage, it drives the ring gear D through gear A, and it drives arm-1 through gears B and C. The second stage inputs are from shaft 1 through gears B and C to sun J, and from the output of the first stage through gear G to arm-2
2. Determine the speed of gear D using equations 9.7.

$$\omega_D := \left(-\frac{N_A}{N_{Dout}} \right) \cdot \omega_1 \quad \omega_D = -33.750 \cdot \text{rpm}$$

3. Determine the speed of arm-1 and gear J using equations 9.7.

$$\omega_{arm1} := \left(-\frac{N_B}{N_C} \right) \cdot \omega_1 \quad \omega_{arm1} = -90.000 \cdot \text{rpm} \quad \omega_J := \omega_{arm1}$$

4. Determine the speed of the ring gear G using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be D and last be G. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_G - \omega_{arm1}}{\omega_D - \omega_{arm1}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-9b.

$$R := \frac{N_{Din}}{N_E} \cdot \frac{N_F}{N_G} \quad R = 1.26936$$

Solve the right-hand equation above for ω_G .

$$\omega_G := R \cdot (\omega_D - \omega_{arm1}) + \omega_{arm1} \quad \omega_G = -18.60 \cdot \text{rpm}$$

5. Determine the speed of gear L using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be J and last be L. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_L - \omega_{arm2}}{\omega_J - \omega_{arm2}} = R \quad \omega_{arm2} := \omega_G$$

Calculate R using equation 9.14 and inspection of Figure P9-9b.

$$R := \left(-\frac{N_J}{N_H} \right) \cdot \left(-\frac{N_K}{N_L} \right) \quad R = 2.36842$$

Solve the right-hand equation above for ω_L .

$$\omega_L := R \cdot (\omega_J - \omega_{arm2}) + \omega_{arm2} \quad \omega_L = -187.71 \cdot \text{rpm}$$

$$\omega_M := \omega_L \quad \omega_M = -187.71 \cdot \text{rpm}$$

PROBLEM 9-44

Statement: Calculate the ratios in the Model T transmission shown in Figure 9-46 (p. 476).

Given: Tooth numbers:

$$N_3 := 27 \quad N_4 := 33 \quad N_5 := 24 \quad N_6 := 27 \quad N_7 := 21 \quad N_8 := 30$$

Solution: See Figure P9-46 and Mathcad file P0944.

1. For low gear, the gearset consists of gears 3, 4, 6, and 7. Determine the ratio of input (arm) to output (gear 6) using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be gear 7 and last be gear 6. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_6 - \omega_{arm}}{\omega_7 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure 9-46.

$$R := \left(-\frac{N_3}{N_6} \right) \cdot \left(-\frac{N_7}{N_4} \right) \quad R = 0.63636$$

Solve the right-hand equation above for the ratio $\omega_{arm}/\omega_6 = m_G$ for $\omega_7 = 0$.

$$m_{GL} := \frac{1}{1 - R} \quad m_{GL} = 2.750$$

2. For reverse gear, the gearset consists of gears 3, 5, 6, and 8. Determine the ratio of input (arm) to output (gear 6) using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be gear 8 and last be gear 6. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_6 - \omega_{arm}}{\omega_8 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure 9-46.

$$R := \left(-\frac{N_3}{N_6} \right) \cdot \left(-\frac{N_8}{N_5} \right) \quad R = 1.25000$$

Solve the right-hand equation above for the ratio $\omega_{arm}/\omega_6 = m_G$ for $\omega_8 = 0$.

$$m_{GR} := \frac{1}{1 - R} \quad m_{GR} = -4.000$$

PROBLEM 9-45

Statement: Figure P7-26 shows a V-belt drive. The sheaves (pulleys) have pitch diameters of 150 and 300 mm, respectively. The smaller sheave is driven at a constant 1750 rpm. For a cross-sectional, differential element of the belt, write the equations of its acceleration for one complete trip around both pulleys including its travel between the pulleys. Compute and plot the acceleration of the differential element versus time for one circuit around the belt path. What does your analysis tell you about the dynamic behavior of the belt?

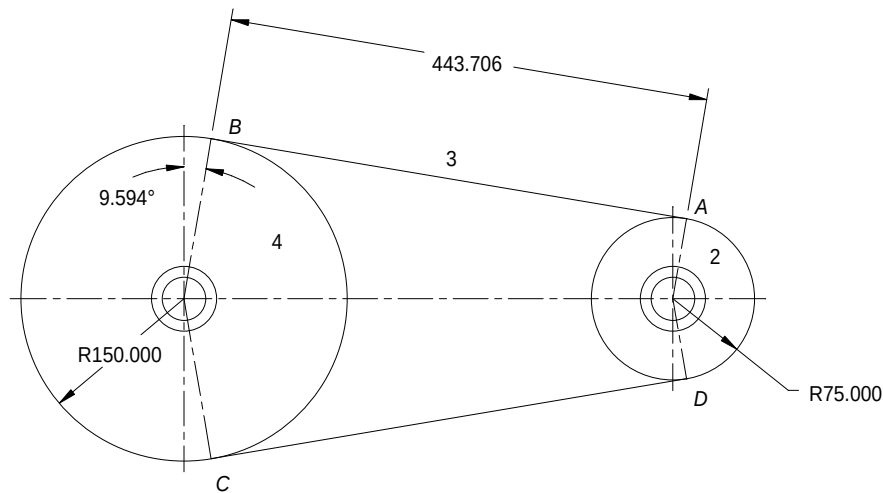
Given: Sheave radii and speed:

$$r_2 := 75 \cdot \text{mm} \quad r_4 := 150 \cdot \text{mm} \quad \omega_2 := 1750 \cdot \text{rpm}$$

Assumptions: Center distance: $C := 450 \cdot \text{mm}$

Solution: See Figure P7-26 and Mathcad file P0945.

1. Draw a schematic representation of the V-belt drive to scale and label it.



From the layout,

Angle to point of tangency	$\beta := 9.594 \cdot \text{deg}$	
Distance from A to B	$L_{AB} := 443.706 \cdot \text{mm}$	
Distance from B to C	$L_{BC} := (\pi + 2 \cdot \beta) \cdot r_4$	$L_{BC} = 521.473 \text{ mm}$
Distance from C to D	$L_{CD} := 443.706 \cdot \text{mm}$	
Distance from D to A	$L_{DA} := (\pi - 2 \cdot \beta) \cdot r_2$	$L_{DA} = 210.502 \text{ mm}$
Total path length	$L := L_{AB} + L_{BC} + L_{CD} + L_{DA}$	$L = 1619.387 \text{ mm}$

2. Calculate the angular velocity of each sheave.

Sheave 2	$\omega_2 = 183.260 \frac{\text{rad}}{\text{sec}}$	
Sheave 4	$\omega_4 := \frac{r_2}{r_4} \cdot \omega_2$	$\omega_4 = 91.630 \frac{\text{rad}}{\text{sec}}$

3. Calculate the acceleration of an element on the belt for each portion of the path, starting at A.

From A to B the acceleration is zero since the belt velocity is constant

$$A_{AB} := 0 \cdot \frac{\text{mm}}{\text{sec}^2}$$

From *B* to *C* the tangential acceleration is zero since the belt velocity is constant The normal component is

$$A_{BC} := r_4 \cdot \omega_4^2 \quad A_{BC} = 1.259 \times 10^6 \frac{mm}{sec^2}$$

From *C* to *D* the acceleration is zero since the belt velocity is constant $A_{CD} := 0 \cdot \frac{mm}{sec^2}$

From *D* to *A* the tangential acceleration is zero since the belt velocity is constant The normal component is

$$A_{DA} := r_2 \cdot \omega_2^2 \quad A_{DA} = 2.519 \times 10^6 \frac{mm}{sec^2}$$

4. Plot the acceleration over the entire path using a path variable of *s*.

$$s := 0 \cdot mm, 5 \cdot mm .. L$$

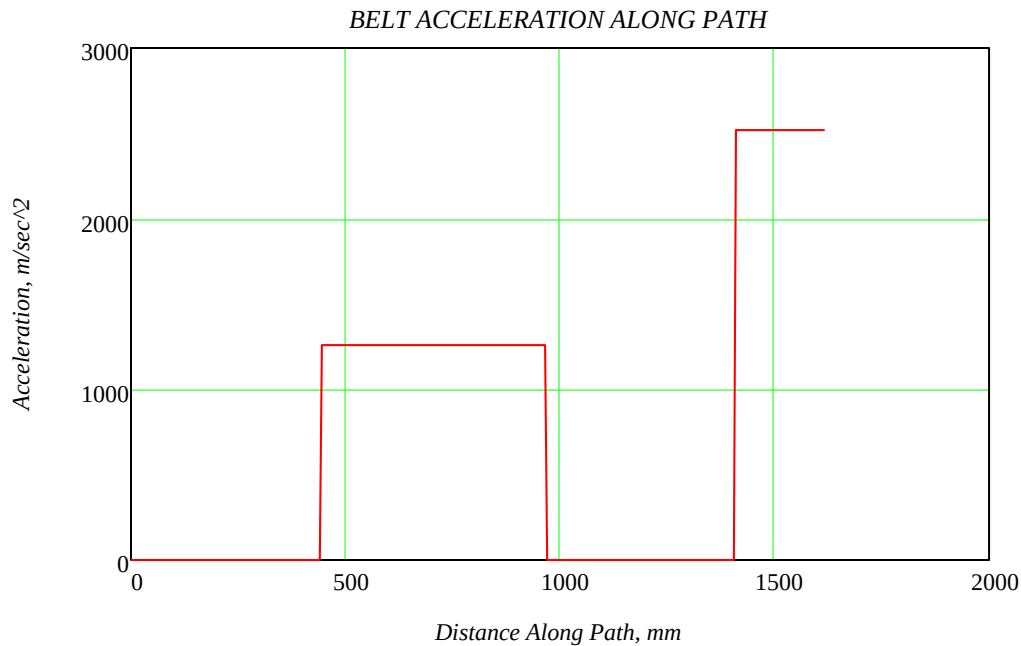
Define a range function that will plot a variable only between two limits using the Heaviside step function Φ .

$$R(s, a, b) := \Phi(s - a) - \Phi(s - b)$$

The acceleration function for the entire path is.

$$\text{Let} \quad s_1 := L_{AB} \quad s_2 := L_{AB} + L_{BC} \quad s_3 := L_{AB} + L_{BC} + L_{CD}$$

$$A(s) := A_{AB} \cdot R(s, 0 \cdot mm, s_1) + A_{BC} \cdot R(s, s_1, s_2) + A_{CD} \cdot R(s, s_2, s_3) + A_{DA} \cdot R(s, s_3, L)$$



5. The graph shows the sudden change in acceleration as the belt enters and leaves a sheave. This results in infinite jerk at these points, which results in belt "hop" at these points, a condition that will cause eventual fatigue failure and wear in the belt. Because of the belt hop caused by infinite jerk at the initial belt/sheave tangency points the span of the belt is continuously vibrating.

PROBLEM 9-46

Statement: Figure P9-11 shows an involute that has been generated from a base circle of radius r_b . Point A is simultaneously on the base circle and on the involute. Point B is any point on the involute curve and point C is on the base circle where a line drawn from point B is tangent to the base circle. The angle ϕ_B (angle BOC) is known as the *involute pressure angle* corresponding to point B (not to be confused with the *pressure angle of two gears in mesh*, which is defined in the text). The angle AOB is known as the involute of ϕ_B and is often designated as $inv \phi_B$. Using the definitions of the involute tooth form and Figure 9-5, derive an equation for $inv \phi_B$ as a function of ϕ_B alone.

Solution: See Figures P9-11 and 9-5 and Mathcad file P0946.

From Figure 9-5 and the definition of the involute curve, arc AC is equal to length BC . Therefore

$$\text{Angle } AOC = (\text{arc } AC) / OC = BC / OC \text{ and thus,}$$

$$\text{Angle } AOC = \tan \phi_B \text{ also,}$$

$$\text{Angle } AOB = \text{angle } AOC - \phi_B = \tan \phi_B - \phi_B \text{ thus,}$$

$$inv \phi_B = \tan \phi_B - \phi_B$$

PROBLEM 9-47

Statement: Using the data and definitions from Problem 9-46, show that when the point B is at the pitch circle the *involute pressure angle* is equal to the *pressure angle of two gears in mesh*.

Solution: See Figures P9-11, 9-6 and 9-7; and Mathcad file P0947.

When the point B in Figure P9-11 is at the pitch point, points B and P (of Figures 9-6 and 9-7) are coincident. The angle between a line through the tangency point of the line of action with the base circle and point O (in Figure 9-6) is the *pressure angle of two gears in mesh*, ϕ . Looking at Figure P9-11, we see that this is also the angle designated as ϕ_B , which is the *involute pressure angle* of point B .

PROBLEM 9-48

Statement: Using the data and definitions from Problem 9-46 and with the point B at the pitch circle where the involute pressure angle ϕ_B is equal to the pressure angle ϕ of two gears in mesh, derive equation 9.4b.

Solution: See Figures P9-11, 9-6 and 9-9; and Mathcad file P0948.

From equation 9.4a and Figure 9-9 $p_c = \frac{\pi \cdot d}{N} = \frac{2\pi \cdot r_p}{N}$. Similarly, $p_b = \frac{\pi \cdot d_b}{N} = \frac{2\pi \cdot r_b}{N}$

Taking the ratio of p_b to p_c , $\frac{p_b}{p_c} = \frac{r_b}{r_c}$. But, from Figure 9-6, we see that $r_b = r_p \cdot \cos(\phi)$. Thus,

$$\frac{p_b}{p_c} = \frac{r_p \cdot \cos(\phi)}{r_p} \quad \text{and} \quad p_b = r_p \cdot \cos(\phi)$$

PROBLEM 9-49

Statement: Using Figures 9-6 and 9-7, derive equation 9.2, which is used to calculate the length of action of a pair of meshing gears.

Solution: See Figures 9-6 and 9-7 and Mathcad file P0949.

1. Define points E_1 and E_2 as shown in the figure below.

Note that the distance

$$O_1E_1 = r_g \cos \phi \text{ and } O_2E_2 = r_p \cos \phi.$$

$$Z = AB = E_1B + E_2A - E_1E_2$$

$$E_1E_2 = r_g \sin \phi + r_p \sin \phi = (r_g + r_p) \sin \phi = C \sin \phi$$

$$O_1B = r_g + a_g \text{ and } O_2A = r_p + a_p$$

From triangles

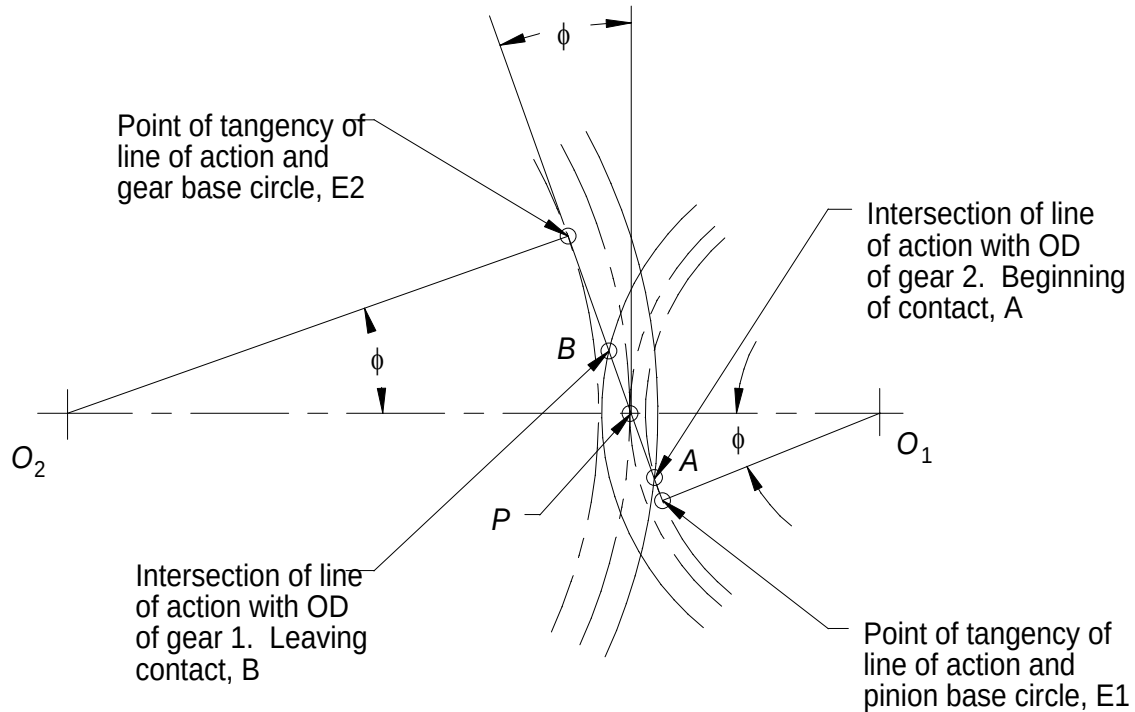
O_1E_1B and O_2E_2A we have

$$E_1B = \sqrt{(r_g + a_g)^2 - (r_g \cos(\phi))^2}$$

$$E_2A = \sqrt{(r_p + a_p)^2 - (r_p \cos(\phi))^2}$$

Thus,

$$Z = \sqrt{(r_p + a_p)^2 - (r_p \cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cos(\phi))^2} - C \sin(\phi)$$



PROBLEM 9-50

Statement: For lubrication purposes it is desired to have a backlash of 0.03 mm measured on the pitch circle of a 40-mm-diameter pinion in mesh with a 100-mm-diameter gear. if the gears are standard, full-depth, with 25 deg pressure angles, by how much should the center distance be increased?

Units: Arc minute: $\text{arcmin} := \frac{1}{60} \cdot \text{deg}$

Given: Pitch diameters: $d_p := 40 \cdot \text{mm}$ $d_g := 100 \cdot \text{mm}$ Pressure angle: $\phi := 25 \cdot \text{deg}$
Desired backlash: $b := 0.03 \cdot \text{mm}$

Solution: See Figure 9-8 and Mathcad file P0950.

1. Convert the desired backlash from a linear to an angular measurement on the 40-mm-diameter pinion.

$$\theta_B := \frac{2 \cdot b}{d_p} \quad \theta_B = 5.1566 \text{ arcmin}$$

2. Use equation 9.3 to calculate the change in center distance required.

$$\Delta C := \frac{\pi \cdot d_p \cdot \theta_B}{43200 \cdot \tan(\phi) \cdot \text{arcmin}} \quad \Delta C = 0.032 \text{ mm}$$

PROBLEM 9-51

Statement: For lubrication purposes it is desired to have a backlash of 0.0012 in measured on the pitch circle of a 2.000-in-diameter pinion in mesh with a 5.000-in-diameter gear. if the gears are standard, full-depth, with 25 deg pressure angles, by how much should the center distance be increased?

Units: Arc minute: $\text{arcmin} := \frac{1}{60} \cdot \text{deg}$

Given: Pitch diameters: $d_p := 2.000 \cdot \text{in}$ $d_g := 5.000 \cdot \text{in}$ Pressure angle: $\phi := 25 \cdot \text{deg}$
Desired backlash: $b := 0.0012 \cdot \text{in}$

Solution: See Figure 9-8 and Mathcad file P0951.

1. Convert the desired backlash from a linear to an angular measurement on the 40-mm-diameter pinion.

$$\theta_B := \frac{2 \cdot b}{d_p} \quad \theta_B = 4.1253 \text{ arcmin}$$

2. Use equation 9.3 to calculate the change in center distance required.

$$\Delta C := \frac{\pi \cdot d_p \cdot \theta_B}{43200 \cdot \tan(\phi) \cdot \text{arcmin}} \quad \Delta C = 1.287 \times 10^{-3} \text{ in}$$

PROBLEM 9-52

Statement: A 22-tooth gear has standard full-depth involute teeth with module of 6. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance using the AGMA specifications in Table 9-1, substituting m for $1/p_d$.

Given: Tooth number $N := 22$ Module $m := 6 \cdot mm$

Solution: See Table 9-1 and Mathcad file P0952.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter	$d := N \cdot m$	$d = 132.00 \text{ mm}$

Circular pitch	$p_c := \pi \cdot m$	$p_c = 18.85 \text{ mm}$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum	$a := 1.0000 \cdot m$	$a = 6.00 \text{ mm}$

Dedendum	$b := 1.2500 \cdot m$	$b = 7.50 \text{ mm}$

Tooth thickness	$t := 0.5 \cdot p_c$	$t = 9.42 \text{ mm}$

Clearance	$c := 0.2500 \cdot m$	$c = 1.50 \text{ mm}$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions in mm should be displayed to two decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in hundredths of a millimeter.

PROBLEM 9-53

Statement: A 40-tooth gear has standard full-depth involute teeth with module of 3. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance using the AGMA specifications in Table 9-1, substituting m for $1/p_d$.

Given: Tooth number $N := 40$ Module $m := 3 \cdot mm$

Solution: See Table 9-1 and Mathcad file P0953.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter	$d := N \cdot m$	$d = 120.00 \text{ mm}$

Circular pitch	$p_c := \pi \cdot m$	$p_c = 9.42 \text{ mm}$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum	$a := 1.0000 \cdot m$	$a = 3.00 \text{ mm}$

Dedendum	$b := 1.2500 \cdot m$	$b = 3.75 \text{ mm}$

Tooth thickness	$t := 0.5 \cdot p_c$	$t = 4.71 \text{ mm}$

Clearance	$c := 0.2500 \cdot m$	$c = 0.75 \text{ mm}$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions in mm should be displayed to two decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in hundredths of a millimeter.

PROBLEM 9-54

Statement: A 30-tooth gear has standard full-depth involute teeth with module of 2. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance using the AGMA specifications in Table 9-1, substituting m for $1/p_d$.

Given: Tooth number $N := 30$ Module $m := 2 \cdot mm$

Solution: See Table 9-1 and Mathcad file P0954.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter	$d := N \cdot m$	$d = 60.00 \text{ mm}$

Circular pitch	$p_c := \pi \cdot m$	$p_c = 6.28 \text{ mm}$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum	$a := 1.0000 \cdot m$	$a = 2.00 \text{ mm}$

Dedendum	$b := 1.2500 \cdot m$	$b = 2.50 \text{ mm}$

Tooth thickness	$t := 0.5 \cdot p_c$	$t = 3.14 \text{ mm}$

Clearance	$c := 0.2500 \cdot m$	$c = 0.50 \text{ mm}$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions in mm should be displayed to two decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in hundredths of a millimeter.

PROBLEM 9-55

Statement: Determine the minimum number of teeth on a pinion with a 20 deg pressure angle for the following gear-to-pinion ratios such that there will be no tooth-to-tooth interference: 1:1, 2:1, 3:1, 4:1, and 5:1.

Given: Pressure angle $\phi := 20\text{-deg}$

Solution: See Tables 9-4 and 9-5 and Mathcad file P0955.

1. From Table 9-5a, the minimum number of teeth on a 20-deg pressure angle pinion that can mesh with another gear is 13 if there are no more than 16 teeth on the other gear. So, for a 1:1 ratio, both gears can have 13 teeth. Looking further at Table 9-5a, we see that there must be more teeth on the pinion as the ratio increases. For instance, for ratios of 2:1 and 3:1, the pinion must have at least 15 teeth since a 15-tooth pinion can mesh with a gear with up to 45 teeth. If the ratio is 4:1 or 5:1, the minimum number of teeth on the pinion is 16 since these ratios would require more than 45 teeth on the gear.

SUMMARY

Ratio	Min teeth on pinion	Number teeth on gear
1:1	13	13
2:1	15	30
3:1	15	45
4:1	16	64
5:1	16	80

However, from Table 9-4b, we see that none of these pinions could be cut with a hob without undercutting. They would have to be produced by other methods to avoid weakening the tooth by undercutting.

PROBLEM 9-56

Statement: Determine the minimum number of teeth on a pinion with a 25 deg pressure angle for the following gear-to-pinion ratios such that there will be no tooth-to-tooth interference: 1:1, 2:1, 3:1, 4:1, and 5:1.

Given: Pressure angle $\phi := 25\text{-deg}$

Solution: See Tables 9-4 and 9-5 and Mathcad file P0956.

- From Table 9-5b, the minimum number of teeth on a 25-deg pressure angle pinion that can mesh with another gear is 9 if there are no more than 13 teeth on the other gear. So, for a 1:1 ratio, both gears can have 9 teeth. Looking further at Table 9-5b, we see that there must be more teeth on the pinion as the ratio increases. For instance, for ratios of 2:1 and 3:1, the pinion must have at least 10 teeth since a 10-tooth pinion can mesh with a gear with up to 32 teeth. If the ratio is 4:1 or 5:1, the minimum number of teeth on the pinion is 11 since these ratios would require more than 32 teeth on the gear.

SUMMARY

Ratio	Min teeth on pinion	Number teeth on gear
1:1	9	9
2:1	10	20
3:1	10	30
4:1	11	44
5:1	11	55

However, from Table 9-4b, we see that none of these pinions could be cut with a hob without undercutting. They would have to be produced by other methods to avoid weakening the tooth by undercutting.

PROBLEM 9-57

Statement: A pinion with a 3.000-in pitch diameter is to mesh with a rack. What is the largest standard tooth size, in terms of diametral pitch, that can be used without having any interference?

- For a 20-deg pressure angle
- For a 25-deg pressure angle

Given: Pitch diameter: $d := 3.000 \cdot \text{in}$

Solution: See Table 9-4 and Mathcad file P0957.

1. Assume that the pinion is generated by means other than being cut by a hob.

a. Pressure angle of 20 deg.

$$N_{\min} := 18 \quad p_{d\min} := \frac{N_{\min}}{d} \quad p_{d\min} = 6.000 \text{ in}^{-1}$$

From Table 9-2, the smallest standard diametral pitch (largest tooth size) that can be used is 6. $p_d := 6 \cdot \text{in}^{-1}$.
The resulting number of teeth on the pinion is:

$$N := p_d \cdot d \quad N = 18$$

b. Pressure angle of 25 deg

$$N_{\min} := 12 \quad p_{d\min} := \frac{N_{\min}}{d} \quad p_{d\min} = 4.000 \text{ in}^{-1}$$

From Table 9-2, the smallest standard diametral pitch (largest tooth size) that can be used is 4. $p_d := 4 \cdot \text{in}^{-1}$.
The resulting number of teeth on the pinion is:

$$N := p_d \cdot d \quad N = 12$$

PROBLEM 9-58

Statement: A pinion with a 75-mm pitch diameter is to mesh with a rack. What is the largest standard tooth size, in terms of diametral pitch, that can be used without having any interference?

- a. For a 20-deg pressure angle
- b. For a 25-deg pressure angle

Given: Pitch diameter: $d := 75 \cdot mm$

Solution: See Table 9-4 and Mathcad file P0958.

1. Assume that the pinion is generated by means other than being cut by a hob.

a. Pressure angle of 20 deg.

$$N_{min} := 18 \quad m := \frac{d}{N_{min}} \quad m = 4.167 \text{ mm}$$

From Table 9-3, the largest standard module (largest tooth size) that can be used is 4 but, in order to have a whole number of teeth, the module must be 3 or 5. Since a module of 5 would result in fewer than 18 teeth, let $m := 3 \cdot mm$. The resulting number of teeth on the pinion is:

$$N := \frac{d}{m} \quad N = 25$$

b. Pressure angle of 25 deg

$$N_{min} := 12 \quad m := \frac{d}{N_{min}} \quad m = 6.250 \text{ mm}$$

From Table 9-3, the largest standard module (largest tooth size) that can be used is 6 but, in order to have a whole number of teeth, the module must be 3 or 5. Since a module of 3 would result in more teeth than is necessary, let $m := 5 \cdot mm$. The resulting number of teeth on the pinion is:

$$N := \frac{d}{m} \quad N = 15$$

PROBLEM 9-59

Statement: In order to have a smooth-running gearset it is desired to have a contact ratio of at least 1.5. If the gears have a pressure angle of 25 deg and gear ratio of 4, what is the minimum number of teeth on the pinion that will yield the required minimum contact ratio?

Given: Contact ratio: $m_p := 1.5$ Gear ratio: $m_G := 4$ Pressure angle: $\phi := 25\text{-deg}$

Solution: See Mathcad file P0959.

1. Write equations 9.6b and 9.2 for the contact ratio and length of action, respectively.

$$m_p = \frac{p_d \cdot Z}{\pi \cdot \cos(\phi)} \quad Z = \sqrt{(r_p + a_p)^2 - (r_p \cdot \cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot \cos(\phi))^2} - C \cdot \sin(\phi)$$

Note that

$$p_d = \frac{N}{d} \quad a_p = a_g = \frac{1}{p_d} \quad r_g = m_G \cdot r_p \quad r_p = \frac{d}{2} \quad C = r_p + r_g = r_p \cdot (1 + m_G)$$

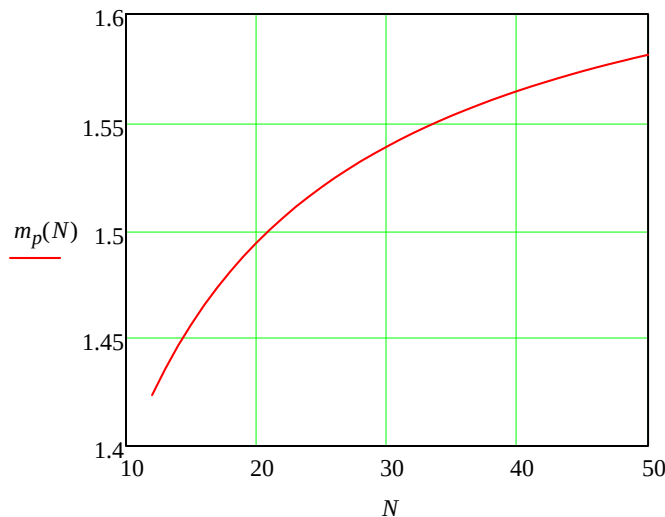
Substituting these identities into the equation for Z and collecting terms gives:

$$Z = \frac{1}{p_d} \cdot \left[\sqrt{\left(\frac{N}{2} + 1\right)^2 - \left(\frac{N}{2} \cdot \cos(\phi)\right)^2} + \sqrt{\left(\frac{m_G \cdot N}{2} + 1\right)^2 - \left(\frac{m_G \cdot N}{2} \cdot \cos(\phi)\right)^2} - \frac{N}{2} \cdot (1 + m_G) \cdot \sin(\phi) \right]$$

Substituting this into the equation for m_p and we have:

$$m_p(N) := \frac{\sqrt{\left(\frac{N}{2} + 1\right)^2 - \left(\frac{N}{2} \cdot \cos(\phi)\right)^2} + \sqrt{\left(\frac{m_G \cdot N}{2} + 1\right)^2 - \left(\frac{m_G \cdot N}{2} \cdot \cos(\phi)\right)^2} - \frac{N}{2} \cdot (1 + m_G) \cdot \sin(\phi)}{\pi \cdot \cos(\phi)}$$

2. Notice that m_p is independent of p_d and depends only on the pressure angle, the number of teeth on the pinion, and the gear ratio. To determine the minimum number of teeth on the pinion, plot the function on the right side of the equation above over the range $N := 12, 13 \dots 50$.



From the graph we see that we need about 21 teeth. Try various values of N in the function until the calculated value of m_p is greater than or equal to the required value.

$$m_p(21) = 1.500$$

Let $N := 21$

PROBLEM 9-60

Statement: In order to have a smooth-running gearset it is desired to have a contact ratio of at least 1.5. If the gears have a pressure angle of 25 deg and 20-tooth pinion, what is the minimum gear ratio that will yield the required minimum contact ratio?

Given: Contact ratio: $m_p := 1.5$ Number of teeth: $N := 20$ Pressure angle: $\phi := 25\text{-deg}$

Solution: See Mathcad file P0960.

1. Write equations 9.6b and 9.2 for the contact ratio and length of action, respectively.

$$m_p = \frac{p_d \cdot Z}{\pi \cdot \cos(\phi)} \quad Z = \sqrt{(r_p + a_p)^2 - (r_p \cdot \cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot \cos(\phi))^2} - C \cdot \sin(\phi)$$

Note that

$$p_d = \frac{N}{d} \quad a_p = a_g = \frac{1}{p_d} \quad r_g = m_G \cdot r_p \quad r_p = \frac{d}{2} \quad C = r_p + r_g = r_p \cdot (1 + m_G)$$

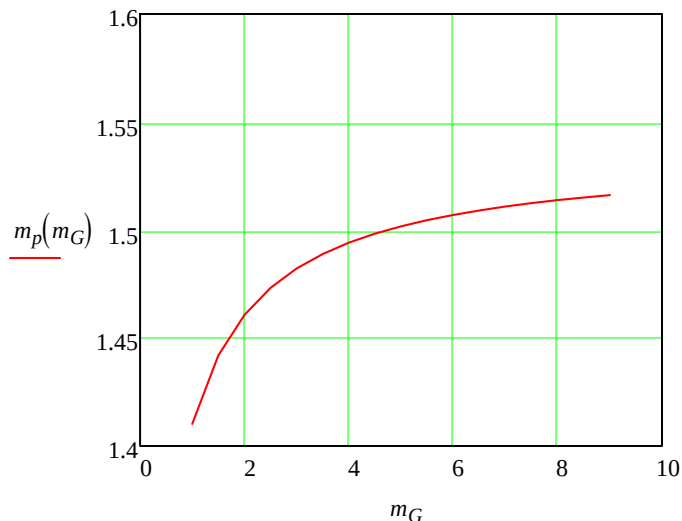
Substituting these identities into the equation for Z and collecting terms gives:

$$Z = \frac{1}{p_d} \cdot \left[\sqrt{\left(\frac{N}{2} + 1\right)^2 - \left(\frac{N}{2} \cdot \cos(\phi)\right)^2} + \sqrt{\left(\frac{m_G \cdot N}{2} + 1\right)^2 - \left(\frac{m_G \cdot N}{2} \cdot \cos(\phi)\right)^2} - \frac{N}{2} \cdot (1 + m_G) \cdot \sin(\phi) \right]$$

Substituting this into the equation for m_p and we have:

$$m_p(m_G) := \frac{\sqrt{\left(\frac{N}{2} + 1\right)^2 - \left(\frac{N}{2} \cdot \cos(\phi)\right)^2} + \sqrt{\left(\frac{m_G \cdot N}{2} + 1\right)^2 - \left(\frac{m_G \cdot N}{2} \cdot \cos(\phi)\right)^2} - \frac{N}{2} \cdot (1 + m_G) \cdot \sin(\phi)}{\pi \cdot \cos(\phi)}$$

2. Notice that m_p is independent of p_d and depends only on the pressure angle, the number of teeth on the pinion, and the gear ratio. To determine the minimum number of teeth on the pinion, plot the function on the right side of the equation above over the range $m_G := 1, 1.5 \dots 9$.



From the graph we see that we need a gear ratio of about 5. Try various values of m_G in the function until the calculated value of m_p is greater than or equal to the required value.

$$m_p(5) = 1.502$$

Let $m_G := 5$

PROBLEM 9-61

Statement: Calculate and plot the train ratio of a noncircular gearset, as a function of input angle, that is based on the centrodes of Figure 6-15b. The link length ratios are $L_1/L_2 = 1.60$, $L_3/L_2 = 1.60$, and $L_4/L_2 = 1.00$.

Given: Link lengths (assume $L_2 = 1.00$):

Link 2 (L_2)	$L_2 := 1.00$	Link 3 (L_3)	$L_3 := 1.60$
Link 4 (L_4)	$L_4 := 1.00$	Link 1 (L_1)	$L_1 := 1.60$

Solution: See Figure 6-15b and Mathcad file P0961.

1. In Figure 6-15b links 2 and 4 are the short links and 1 and 3 are the long links with link 1 fixed. To find the fixed and moving centrodes of I_{24} we will fix link 2 and find the intersection of links 1 and 3. The locus of these intersections is an ellipse that will be attached to link 2 when the linkage is reinverted such that link 1 is again grounded.
2. Invert the linkage, fixing link 2 to ground and determine the range of motion for this Grashof double crank.

$$\begin{aligned} a &:= L_3 & b &:= L_4 & c &:= L_1 & d &:= L_2 \\ a &= 1.600 & b &= 1.000 & c &= 1.600 & d &= 1.000 \\ \theta_3 &:= 0.5 \cdot \text{deg}, 1 \cdot \text{deg} \dots 359.5 \cdot \text{deg} \end{aligned}$$

3. Determine the values of the constants needed for finding θ_4 on the inverted linkage (actually θ_1 on the original linkage) from equations 4.8a, 4.10a, 4.11b and 4.12.

$$\begin{aligned} K_1 &:= \frac{d}{a} & K_2 &:= \frac{d}{c} \\ K_1 &= 0.6250 & K_2 &= 0.6250 \\ K_3 &:= \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} & K_3 &= 1.0000 \\ A(\theta_3) &:= \cos(\theta_3) - K_1 - K_2 \cdot \cos(\theta_3) + K_3 \\ B(\theta_3) &:= -2 \cdot \sin(\theta_3) \\ C(\theta_3) &:= K_1 - (K_2 + 1) \cdot \cos(\theta_3) + K_3 \end{aligned}$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\begin{aligned} \theta_{11}(\theta_3) &:= 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_3), -B(\theta_3) + \sqrt{B(\theta_3)^2 - 4 \cdot A(\theta_3) \cdot C(\theta_3)} \right) \right) \\ \theta_{12}(\theta_3) &:= 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_3), -B(\theta_3) - \sqrt{B(\theta_3)^2 - 4 \cdot A(\theta_3) \cdot C(\theta_3)} \right) \right) \end{aligned}$$

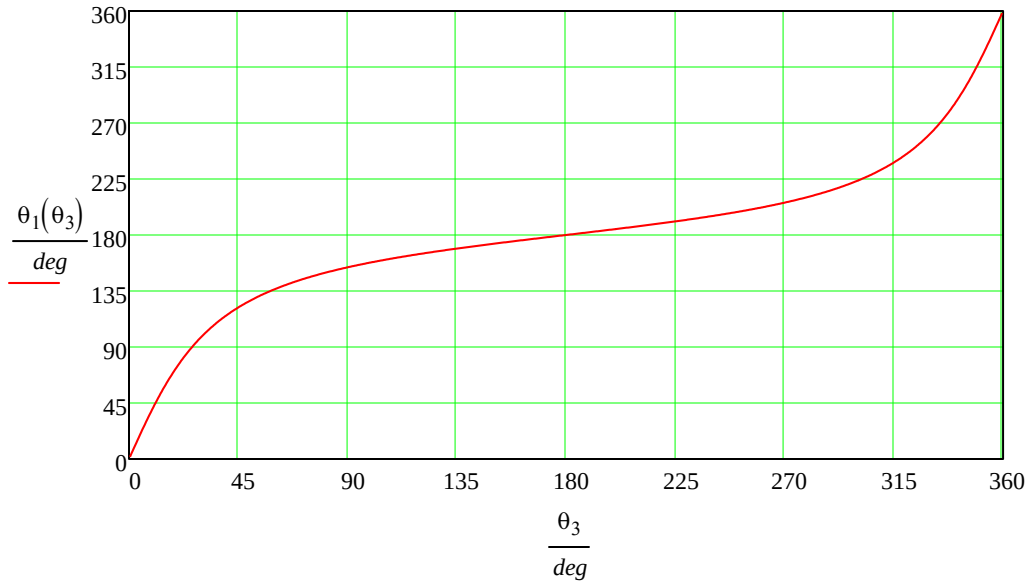
Use the crossed branch equation for the first 180 deg of crank motion and the open branch equation for the last 180 deg.

$$\theta_{10}(\theta_3) := \text{if}(\theta_3 \leq \pi, \theta_{11}(\theta_3), \theta_{12}(\theta_3))$$

If the calculated angle is negative, make it positive.

$$\theta_1(\theta_3) := \text{if}(\theta_3 \leq \pi, \theta_{10}(\theta_3), \theta_{10}(\theta_3) + 2 \cdot \pi)$$

Plot θ_1 as a function of θ_3 (link 3 is the driving crank and link 1 is the driven crank in the inverted linkage).



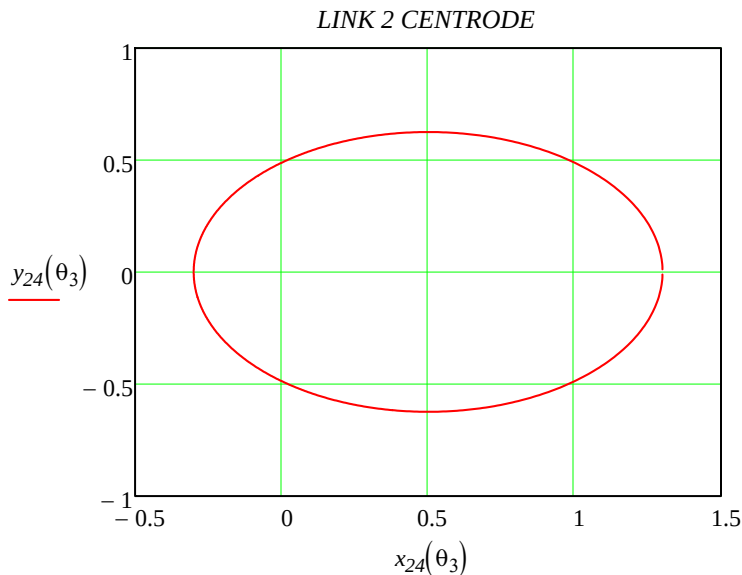
5. Define the coordinates of the intersection of links 1 and 3 for the inverted linkage.

Equation of link 3: $y = x \cdot m_3$, where $m_3(\theta_3) := \tan(\theta_3)$

Equation of link 1: $y = (x - d) \cdot m_1$, where $m_1(\theta_3) := \tan(\theta_1(\theta_3))$

The coordinates of the intersection (I_{24}) of these two lines are

$$x_{24}(\theta_3) := \frac{m_1(\theta_3)}{m_1(\theta_3) - m_3(\theta_3)} \cdot d \quad y_{24}(\theta_3) := x_{24}(\theta_3) \cdot m_3(\theta_3)$$



$$x_{24}(0.0001 \cdot \text{deg}) = 1.300$$

$$y_{24}(0.0001 \cdot \text{deg}) = 0.000$$

$$x_{24}(51.318 \cdot \text{deg}) = 0.500$$

$$y_{24}(51.318 \cdot \text{deg}) = 0.624$$

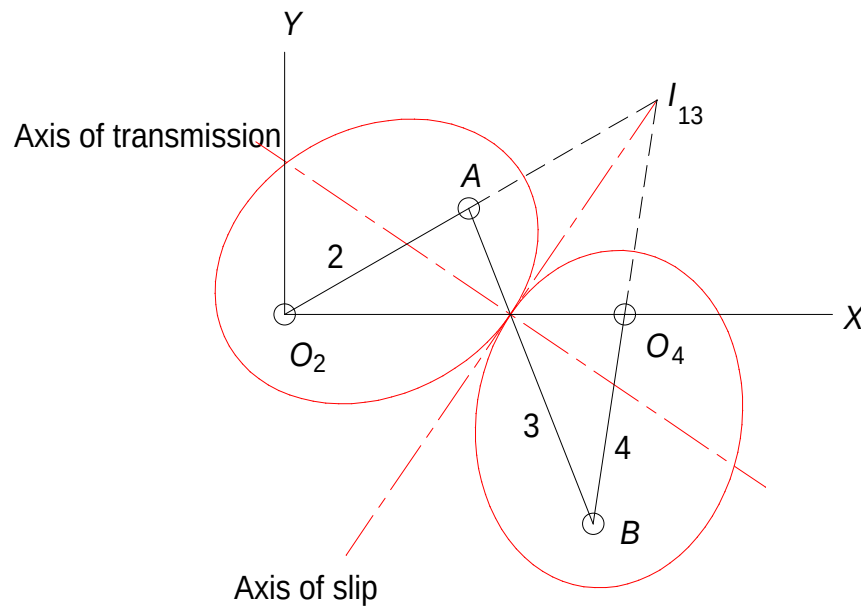
$$x_{24}(179.0001 \cdot \text{deg}) = -0.300$$

$$y_{24}(179.0001 \cdot \text{deg}) = 0.005$$

$$\theta_1(51.318 \cdot \text{deg}) = 128.682 \text{ deg}$$

The ellipse has a major axis of 1.600 and a minor axis of 1.248 and the origin on the graph above is the point O . The centroid for link 4 is a mirror image of this centroid.

6. The linkage is shown below, reinverted such that link 1 is fixed, with the centres attached to links 2 and 4. They are tangent to each other at the instant center I_{24} . The axis of slip is found by extending a line from the instant center I_{13} to I_{24} . The axis of transmission is perpendicular to this line as shown.



PROBLEM 9-62

Statement: Calculate and plot the train ratio of a noncircular gearset, as a function of input angle, that is based on the centrodes of Figure 6-15b. The link length ratios are $L_1/L_2 = 1.80$, $L_3/L_2 = 1.80$, and $L_4/L_2 = 1.00$.

Given: Link lengths (assume $L_2 = 1.00$):

Link 2 (L_2)	$L_2 := 1.00$	Link 3 (L_3)	$L_3 := 1.80$
Link 4 (L_4)	$L_4 := 1.00$	Link 1 (L_1)	$L_1 := 1.80$

Solution: See Figure 6-15b and Mathcad file P0962.

1. In Figure 6-15b links 2 and 4 are the short links and 1 and 3 are the long links with link 1 fixed. To find the fixed and moving centrodes of I_{24} we will fix link 2 and find the intersection of links 1 and 3. The locus of these intersections is an ellipse that will be attached to link 2 when the linkage is reinverted such that link 1 is again grounded.
2. Invert the linkage, fixing link 2 to ground and determine the range of motion for this Grashof double crank.

$$\begin{aligned} a &:= L_3 & b &:= L_4 & c &:= L_1 & d &:= L_2 \\ a &= 1.800 & b &= 1.000 & c &= 1.800 & d &= 1.000 \\ \theta_3 &:= 0.5 \cdot \text{deg}, 1 \cdot \text{deg} \dots 359.5 \cdot \text{deg} \end{aligned}$$

3. Determine the values of the constants needed for finding θ_4 on the inverted linkage (actually θ_1 on the original linkage) from equations 4.8a, 4.10a, 4.11b and 4.12.

$$\begin{aligned} K_1 &:= \frac{d}{a} & K_2 &:= \frac{d}{c} \\ K_1 &= 0.5556 & K_2 &= 0.5556 \\ K_3 &:= \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} & K_3 &= 1.0000 \\ A(\theta_3) &:= \cos(\theta_3) - K_1 - K_2 \cdot \cos(\theta_3) + K_3 \\ B(\theta_3) &:= -2 \cdot \sin(\theta_3) \\ C(\theta_3) &:= K_1 - (K_2 + 1) \cdot \cos(\theta_3) + K_3 \end{aligned}$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\begin{aligned} \theta_{11}(\theta_3) &:= 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_3), -B(\theta_3) + \sqrt{B(\theta_3)^2 - 4 \cdot A(\theta_3) \cdot C(\theta_3)} \right) \right) \\ \theta_{12}(\theta_3) &:= 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_3), -B(\theta_3) - \sqrt{B(\theta_3)^2 - 4 \cdot A(\theta_3) \cdot C(\theta_3)} \right) \right) \end{aligned}$$

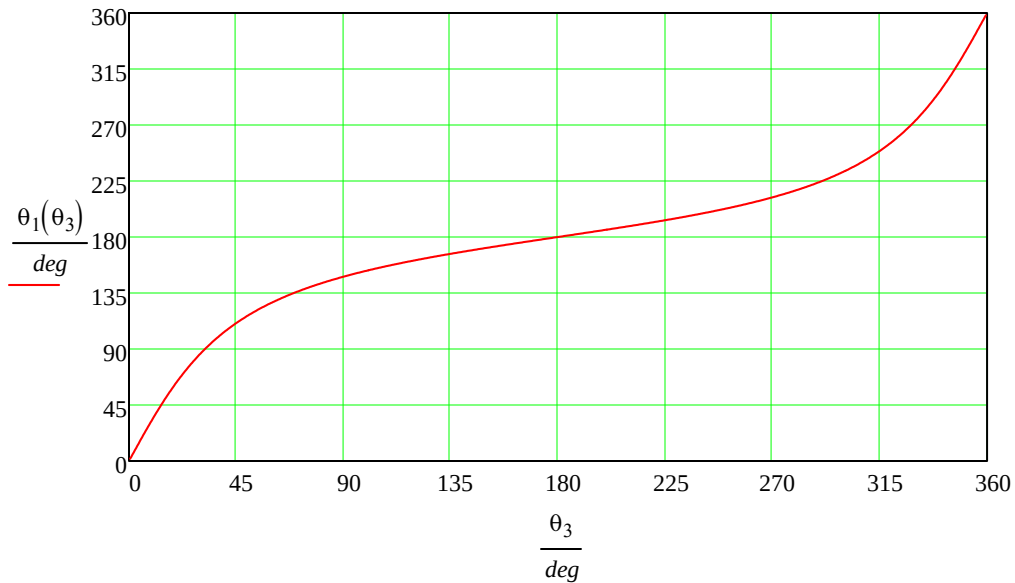
Use the crossed branch equation for the first 180 deg of crank motion and the open branch equation for the last 180 deg.

$$\theta_{10}(\theta_3) := \text{if}(\theta_3 \leq \pi, \theta_{11}(\theta_3), \theta_{12}(\theta_3))$$

If the calculated angle is negative, make it positive.

$$\theta_1(\theta_3) := \text{if}(\theta_3 \leq \pi, \theta_{10}(\theta_3), \theta_{10}(\theta_3) + 2 \cdot \pi)$$

Plot θ_1 as a function of θ_3 (link 3 is the driving crank and link 1 is the driven crank in the inverted linkage).



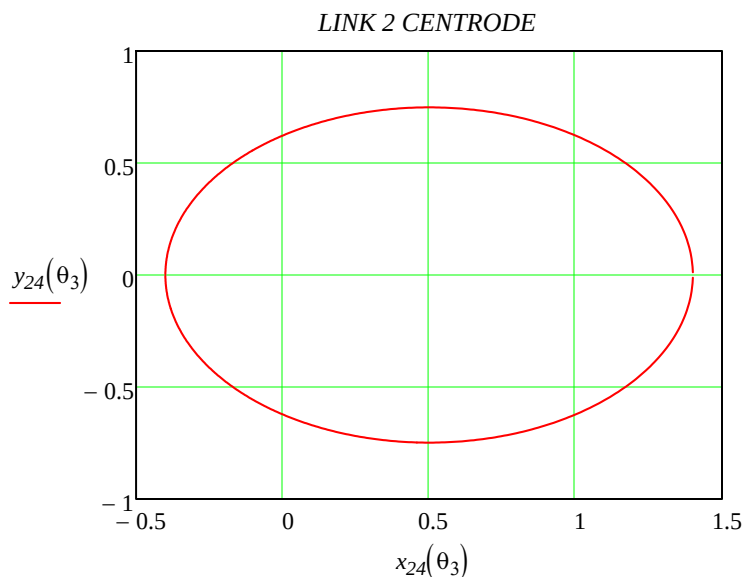
5. Define the coordinates of the intersection of links 1 and 3 for the inverted linkage.

Equation of link 3: $y = x \cdot m_3$, where $m_3(\theta_3) := \tan(\theta_3)$

Equation of link 1: $y = (x - d) \cdot m_1$, where $m_1(\theta_3) := \tan(\theta_1(\theta_3))$

The coordinates of the intersection (I_{24}) of these two lines are

$$x_{24}(\theta_3) := \frac{m_1(\theta_3)}{m_1(\theta_3) - m_3(\theta_3)} \cdot d \quad y_{24}(\theta_3) := x_{24}(\theta_3) \cdot m_3(\theta_3)$$



$$x_{24}(0.0001 \cdot \text{deg}) = 1.400$$

$$y_{24}(0.0001 \cdot \text{deg}) = 0.000$$

$$x_{24}(56.251 \cdot \text{deg}) = 0.500$$

$$y_{24}(56.251 \cdot \text{deg}) = 0.748$$

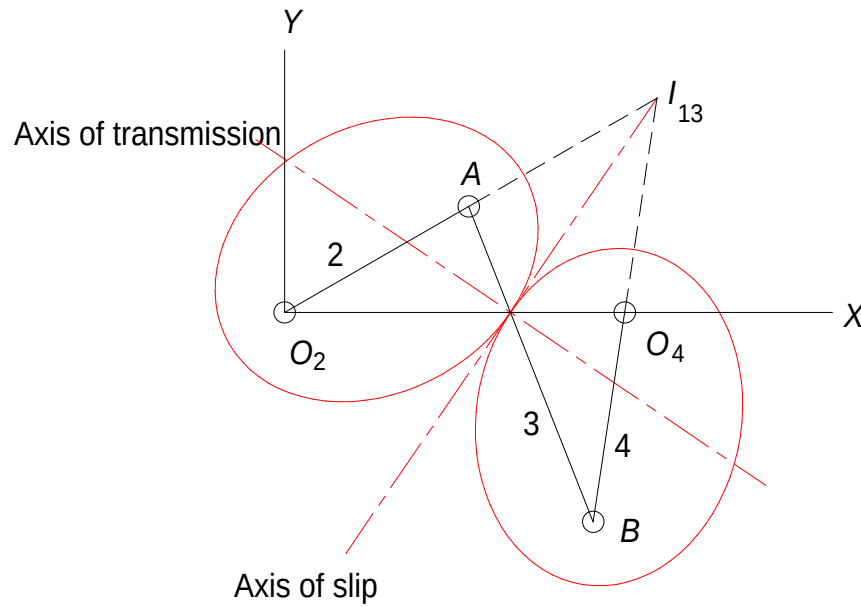
$$x_{24}(179.0001 \cdot \text{deg}) = -0.400$$

$$y_{24}(179.0001 \cdot \text{deg}) = 0.007$$

$$\theta_1(56.251 \cdot \text{deg}) = 123.749 \text{ deg}$$

The ellipse has a major axis of 1.800 and a minor axis of 1.496 and the origin on the graph above is the point O . The centroid for link 4 is a mirror image of this centroid.

6. The linkage is shown below, reinverted such that link 1 is fixed, with the centrodes attached to links 2 and 4. They are tangent to each other at the instant center I_{24} . The axis of slip is found by extending a line from the instant center I_{13} to I_{24} . The axis of transmission is perpendicular to this line as shown.



PROBLEM 9-63

Statement: Figure P9-35b shows (schematically) a compound epicyclic train. The tooth numbers are 50, 25, 35, and 90 for gears 2, 3, 4, and 5, respectively. The arm is driven at 180 rpm CW and gear 5 is fixed to ground. Determine the speed and direction of gear 2. What is the efficiency of this train if the basic gearsets have $E_0 = 0.98$?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$$N_2 := 50 \quad N_3 := 25 \quad N_4 := 35 \quad N_5 := 90$$

$$\text{Input speeds:} \quad \omega_{arm} := -180 \cdot rpm \quad \omega_5 := 0 \cdot rpm$$

$$\text{Basic gearset efficiency:} \quad E_0 := 0.98$$

Solution: See Figure P9-35b and Mathcad file P0963.

1. Determine the speed of the sun gear 2 using the formula method for analyzing an epicyclic train. To start, choose a first and last gears that mesh with gears that have planetary motion. Let the first gear be 2 and last be 5. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_5 - \omega_{arm}}{\omega_2 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-7b.

$$R := \left(-\frac{N_2}{N_3} \right) \cdot \left(\frac{N_4}{N_5} \right) \quad R = -0.77778$$

Solve the right-hand equation above for ω_2 with $\omega_5 = 0$.

$$\omega_2 := \frac{R - 1}{R} \cdot \omega_{arm} \quad \omega_2 = -411.43 \text{ rpm}$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := \frac{1}{R} \quad \rho = -1.286$$

3. The combination of $\rho < -1$, shaft 1 fixed, and input to the arm corresponds to Case 8 in Table 9-12 giving an efficiency of

$$\eta := \frac{E_0(\rho - 1)}{\rho \cdot E_0 - 1} \quad \eta = 0.991$$

PROBLEM 9-64

Statement: Figure P9-35h shows (schematically) a compound epicyclic train. The tooth numbers are 80, 20, 25, and 85 for gears 2, 3, 4, and 5, respectively. Gear 2 is driven at 200 rpm CCW and gear 5 is fixed to ground. Determine the speed and direction of the arm. What is the efficiency of this train if the basic gearsets have $E_0 = 0.98$?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$$N_2 := 80 \quad N_3 := 20 \quad N_4 := 25 \quad N_5 := 85$$

Input speeds: $\omega_2 := 200 \cdot rpm \quad \omega_5 := 0 \cdot rpm$

Basic gearset efficiency: $E_0 := 0.98$

Solution: See Figure P9-35h and Mathcad file P0964.

1. Determine the speed of the arm using the formula method for analyzing an epicyclic train. To start, choose first and last gears that mesh with gears that have planetary motion. Let the first gear be 2 and last be 5. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_5 - \omega_{arm}}{\omega_2 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-7b.

$$R := \left(\frac{N_2}{N_3} \right) \cdot \left(\frac{N_4}{N_5} \right) \quad R = 1.17647$$

Solve the right-hand equation above for ω_{arm} with $\omega_5 = 0$.

$$\omega_{arm} := \frac{R}{R - 1} \cdot \omega_2 \quad \omega_{arm} = 1333 \cdot rpm$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := R \quad \rho = 1.176$$

3. The combination of $\rho > 1$, shaft 1 fixed, and input to gear 2 corresponds to Case 3 in Table 9-12 giving an efficiency of

$$\eta := \frac{\rho \cdot E_0 - 1}{E_0 \cdot (\rho - 1)} \quad \eta = 0.884$$

PROBLEM 9-65

Statement: Figure P9-35i shows (schematically) a compound epicyclic train. The tooth numbers are 24, 18, 20, and 90 for gears 2, 3, 4, and 5, respectively. The arm is driven at 100 rpm CCW and gear 2 is fixed to ground. Determine the speed and direction of gear 5. What is the efficiency of this train if the basic gearsets have $E_0 = 0.98$?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$$N_2 := 24 \quad N_3 := 18 \quad N_4 := 20 \quad N_5 := 90$$

Input speeds: $\omega_{arm} := 100 \cdot rpm \quad \omega_2 := 0 \cdot rpm$

Basic gearset efficiency: $E_0 := 0.98$

Solution: See Figure P9-35i and Mathcad file P0965.

1. Determine the speed of the arm using the formula method for analyzing an epicyclic train. To start, choose first and last gears that mesh with gears that have planetary motion. Let the first gear be 2 and last be 5. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_5 - \omega_{arm}}{\omega_2 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-7b.

$$R := \left(-\frac{N_2}{N_3} \right) \cdot \left(-\frac{N_3}{N_4} \right) \cdot \left(\frac{N_4}{N_5} \right) \quad R = 0.26667$$

Solve the right-hand equation above for ω_5 with $\omega_2 = 0$.

$$\omega_5 := (R + 1) \cdot \omega_{arm} \quad \omega_5 = 126.67 \text{ rpm}$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := \frac{1}{R} \quad \rho = 3.750$$

3. The combination of $\rho > 1$, shaft 2 fixed, and input to the arm corresponds to Case 2 in Table 9-12 giving an efficiency of

$$\eta := \frac{E_0(\rho - 1)}{\rho - E_0} \quad \eta = 0.973$$

PROBLEM 9-66

Statement: Using Figure 9-8, derive an equation for the operating pressure angle of two gears in mesh as a function of their base circle radii, the standard center distance, and the change in center distance.

Solution: See Figure 9-8 and Mathcad file P0966.

1. Let:	Base circle radius of the gear be r_{bg}	Standard pressure angle be ϕ
	Base circle radius of the pinion be r_{bp}	Standard center distance be C
	Change in center distance be ΔC	Operating pressure angle be ϕ_O

2. Then from Figure 9-8, the standard center distance is $C = \frac{r_{bg} + r_{bp}}{\cos(\phi)}$

3. As the center distance increases the line of action remains tangent to the base circles as shown in Figure 9-8b. Thus, the operating pressure angle is $\phi_O = \arccos\left(\frac{r_{bg} + r_{bp}}{C + \Delta C}\right)$

PROBLEM 9-67

Statement: A pinion and gear in mesh have base circle radii of 1.8126 and 3.6252 in, respectively. If they were cut with a standard pressure angle of 25° , determine their operating pressure angle if the standard center distance is increased by 0.062 in.

Given: Base circle radii: $r_{bp} := 1.8126 \cdot \text{in}$ $r_{bg} := 3.6252 \cdot \text{in}$ Pressure angle: $\phi := 25 \cdot \text{deg}$
Change in center distance: $\Delta := 0.062 \cdot \text{in}$

Solution: See Figure 9-8 and Mathcad file P0967.

- From Figure 9-8, the standard center distance is

$$C := \frac{r_{bp} + r_{bg}}{\cos(\phi)} \quad C = 6.000 \cdot \text{in}$$

- As the center distance increases the line of action remains tangent to the base circles as shown in Figure 9-8b. Thus, the operating pressure angle is

$$\phi_O := \arccos\left(\frac{r_{bp} + r_{bg}}{C + \Delta}\right) \quad \phi_O = 26.229 \cdot \text{deg}$$

PROBLEM 9-68

Statement: A pinion and gear in mesh have base circle radii of 1.35946 and 2.26577 in, respectively. If they have a standard center distance of 4.000 in, determine the standard pressure angle and the operating pressure angle if the standard center distance is increased by 0.050 in.

Given: Base circle radii: $r_{bp} := 1.35946 \cdot \text{in}$ $r_{bg} := 2.26577 \cdot \text{in}$ Pressure angle: $C := 4.000 \cdot \text{in}$
 Change in center distance: $\Delta := 0.050 \cdot \text{in}$

Solution: See Figure 9-8 and Mathcad file P0968.

1. From Figure 9-8, the standard pressure angle is

$$\phi := \arccos\left(\frac{r_{bp} + r_{bg}}{C}\right) \quad \phi = 25.000 \text{ deg}$$

2. As the center distance increases the line of action remains tangent to the base circles as shown in Figure 9-8b. Thus, the operating pressure angle is

$$\phi_O := \arccos\left(\frac{r_{bp} + r_{bg}}{C + \Delta}\right) \quad \phi_O = 26.476 \text{ deg}$$

PROBLEM 9-69

Statement: A 25-tooth pinion meshes with a 60-tooth gear. They have a diametral pitch of 4, a pressure angle of 20°, and AGMA full-depth involute profiles. Find the gear ratio, circular pitch, base pitch, pitch diameters, standard center distance, addendum, dedendum, whole depth, clearance, outside diameters and contact ratio of the gearset.

Given: Number of teeth: $N_p := 25$ $N_g := 60$ Pressure angle: $\phi := 20\text{-deg}$
 Diametral pitch: $p_d := 4\text{-in}^{-1}$

Solution: See Mathcad file P0969.

1. From equation 9.5b, the gear ratio is:

$$m_G := \frac{N_g}{N_p} \qquad m_G = 2.400$$

2. From equations 9.4a and 9.4c, the circular pitch is:

$$p_c := \frac{\pi}{p_d} \qquad p_c = 0.785\text{ in}$$

3. From equation 9.4b, the base pitch is

$$p_b := p_c \cdot \cos(\phi) \qquad p_b = 0.738\text{ in}$$

4. From equation 9.4c, the pitch diameters and radii are:

$$d_p := \frac{N_p}{p_d} \qquad d_p = 6.250\text{ in}$$

$$d_g := \frac{N_g}{p_d} \qquad d_g = 15.000\text{ in}$$

5. The standard center distance is one half the sum of the pitch diameters. Using equation 9.4c, this can be written as:

$$C := \frac{N_p + N_g}{2 \cdot p_d} \qquad C = 10.625\text{ in}$$

6. The tooth dimensions are found from the specifications in Table 9-1 for $\phi = 20\text{ deg}$, coarse pitch:

Addendum	$a := \frac{1.000}{p_d}$	$a = 0.2500\text{ in}$

Dedendum	$b := \frac{1.250}{p_d}$	$b = 0.3125\text{ in}$

Whole depth	$h_t := a + b$	$h_t = 0.5625\text{ in}$

Clearance	$c := b - a$	$c = 0.0625\text{ in}$

Outside diameter	$D_{Op} := d_p + 2 \cdot a$	$D_{Op} = 6.7500\text{ in}$

	$D_{Og} := d_g + 2 \cdot a$	$D_{Og} = 15.5000\text{ in}$

7. From equations 9.2 and 9.6a, the contact ratio is:

Pitch radii	$r_p := 0.5 \cdot d_p$	$r_g := 0.5 \cdot d_g$

	$r_p = 3.125\text{ in}$	$r_g = 7.500\text{ in}$

Length of contact

$$Z := \sqrt{(r_p + a)^2 - (r_p \cdot \cos(\phi))^2} + \sqrt{(r_g + a)^2 - (r_g \cdot \cos(\phi))^2} - C \cdot \sin(\phi)$$

$$Z = 1.2533 \text{ in}$$

Contact ratio

$$m_p := \frac{Z}{p_b}$$

$$m_p = 1.698$$

PROBLEM 9-70

Statement: A 15-tooth pinion meshes with a 45-tooth gear. They have a diametral pitch of 5, a pressure angle of 25°, and AGMA full-depth involute profiles. Find the gear ratio, circular pitch, base pitch, pitch diameters, standard center distance, addendum, dedendum, whole depth, clearance, outside diameters and contact ratio of the gearset.

Given: Number of teeth: $N_p := 15$ $N_g := 45$ Pressure angle: $\phi := 25 \cdot \text{deg}$
 Diametral pitch: $p_d := 5 \cdot \text{in}^{-1}$

Solution: See Mathcad file P0970.

1. From equation 9.5b, the gear ratio is:

$$m_G := \frac{N_g}{N_p} \qquad m_G = 3.000$$

2. From equations 9.4a and 9.4c, the circular pitch is:

$$p_c := \frac{\pi}{p_d} \qquad p_c = 0.628 \text{ in}$$

3. From equation 9.4b, the base pitch is

$$p_b := p_c \cdot \cos(\phi) \qquad p_b = 0.569 \text{ in}$$

4. From equation 9.4c, the pitch diameters and radii are:

$$d_p := \frac{N_p}{p_d} \qquad d_p = 3.000 \text{ in}$$

$$d_g := \frac{N_g}{p_d} \qquad d_g = 9.000 \text{ in}$$

5. The standard center distance is one half the summ of the pitch diameters. Using equation 9.4c, this can be written as:

$$C := \frac{N_p + N_g}{2 \cdot p_d} \qquad C = 6.000 \text{ in}$$

6. The tooth dimensions are found from the specifications in Table 9-1 for $\phi = 25 \text{ deg}$, coarse pitch:

Addendum	$a := \frac{1.000}{p_d}$	$a = 0.2000 \text{ in}$

Dedendum	$b := \frac{1.250}{p_d}$	$b = 0.2500 \text{ in}$

Whole depth	$h_t := a + b$	$h_t = 0.4500 \text{ in}$

Clearance	$c := b - a$	$c = 0.0500 \text{ in}$

Outside diameter	$D_{Op} := d_p + 2 \cdot a$	$D_{Op} = 3.4000 \text{ in}$

	$D_{Og} := d_g + 2 \cdot a$	$D_{Og} = 9.4000 \text{ in}$

7. From equations 9.2 and 9.6a, the contact ratio is:

Pitch radii	$r_p := 0.5 \cdot d_p$	$r_g := 0.5 \cdot d_g$

	$r_p = 1.500 \text{ in}$	$r_g = 4.500 \text{ in}$

Length of contact

$$Z := \sqrt{(r_p + a)^2 - (r_p \cdot \cos(\phi))^2} + \sqrt{(r_g + a)^2 - (r_g \cdot \cos(\phi))^2} - C \cdot \sin(\phi)$$

$$Z = 0.8210 \text{ in}$$

Contact ratio

$$m_p := \frac{Z}{p_b}$$

$$m_p = 1.442$$

PROBLEM 9-71

Statement: Design a compound, spur gear train that will reduce the speed of a 900-rpm synchronous AC motor to exactly 72 revolutions per hour with the output rotating in the same direction as the motor. Use gears with a pressure angle of 25 deg and minimize the package size.

Given: Motor speed: $n_m := 900 \cdot rpm$ Output speed: $n_{out} := 72 \cdot \frac{rev}{hr}$

Solution: See Mathcad file P0971.

1. Calculate the required gear ratio. $m_G := \frac{n_m}{n_{out}} \quad m_G = 750$
2. Since the ratio must be positive, we want to have an even number of stages or an odd number with an idler. Let the number of stages be

Possible number of stages	$j := 2, 3 \dots 5$	Ideal, theoretical stage ratios	$r(j) := m_G^{\frac{1}{j}}$
then	$j =$	$r(j) =$	
	2	27.386	
	3	9.086	
	4	5.233	
	5	3.758	

3. Four stages would result in a stage ratio less than 10 and about 5.25, so we will use four stages. Using a pressure angle of 25 deg, let the stage ratios be (choosing integer ratios whose product is $m_G = 750$)

Stage 1 ratio	$r_1 := 6$	Stage 2 ratio	$r_2 := 5$
Stage 3 ratio	$r_3 := 5$	Stage 4 ratio	$r_4 := 5$

and let the driver gears have tooth numbers of

$$\text{Tooth number index } i := 2, 3 \dots 9 \quad N_2 := 14 \quad N_4 := 14 \quad N_6 := 14 \quad N_8 := 14$$

then the driven gears will have tooth numbers of

$$\begin{aligned} N_3 &:= r_1 \cdot N_2 & N_5 &:= r_2 \cdot N_4 & N_7 &:= r_3 \cdot N_6 & N_9 &:= r_4 \cdot N_8 \\ N_3 &= 84 & N_5 &= 70 & N_7 &= 70 & N_9 &= 70 \end{aligned}$$

$$\text{Checking the resulting gear ratio: } \frac{N_3 \cdot N_5 \cdot N_7 \cdot N_9}{N_2 \cdot N_4 \cdot N_6 \cdot N_8} = 750$$

PROBLEM 9-72

Statement: A gear-train designer wants to have a contact ratio of at least 1.5 for a gearset. If the gears have a pressure angle of 25 deg and the pinion has 21 teeth, what is the minimum gear-ratio that will yield the required minimum contact ratio?

Given: Contact ratio: $m_p := 1.5$ Number of teeth: $N := 21$ Pressure angle:

Solution: $\phi := 25 \cdot \text{deg}$
See Mathcad file P0972.

1. Write equations 9.6b and 9.2 for the contact ratio and length of action, respectively.

$$m_p = \frac{p_d \cdot Z}{\pi \cdot \cos(\phi)} \quad Z = \sqrt{(r_p + a_p)^2 - (r_p \cdot \cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot \cos(\phi))^2} - C \cdot \sin(\phi)$$

Note that

$$p_d = \frac{N}{d} \quad a_p = a_g = \frac{1}{p_d} \quad r_g = m_G \cdot r_p \quad r_p = \frac{d}{2} \quad C = r_p + r_g = r_p \cdot (1 + m_G)$$

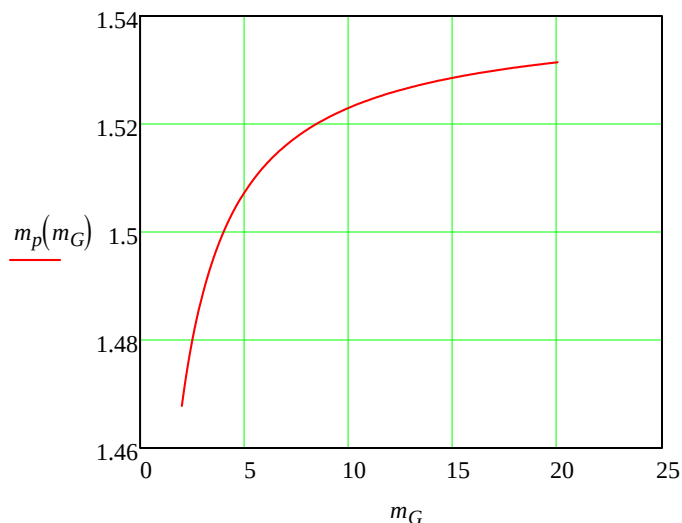
Substituting these identities into the equation for Z and collecting terms gives:

$$Z = \frac{1}{p_d} \cdot \left[\sqrt{\left(\frac{N}{2} + 1\right)^2 - \left(\frac{N}{2} \cdot \cos(\phi)\right)^2} + \sqrt{\left(\frac{m_G \cdot N}{2} + 1\right)^2 - \left(\frac{m_G \cdot N}{2} \cdot \cos(\phi)\right)^2} - \frac{N}{2} \cdot (1 + m_G) \cdot \sin(\phi) \right]$$

Substituting this into the equation for m_p and we have:

$$m_p(m_G) := \frac{\sqrt{\left(\frac{N}{2} + 1\right)^2 - \left(\frac{N}{2} \cdot \cos(\phi)\right)^2} + \sqrt{\left(\frac{m_G \cdot N}{2} + 1\right)^2 - \left(\frac{m_G \cdot N}{2} \cdot \cos(\phi)\right)^2} - \frac{N}{2} \cdot (1 + m_G) \cdot \sin(\phi)}{\pi \cdot \cos(\phi)}$$

2. Notice that m_p is independent of p_d and depends only on the pressure angle, the number of teeth on the pinion, and the gear ratio. To determine the minimum gear ratio, plot the function on the right side of the equation above over the range $m_G := 2, 2.1 \dots 20$.



From the graph we see that we need a gear-ratio of about 4. Try various values of m_G in the function until the calculated value of m_p is greater than or equal to the required value.

$$m_p(4) = 1.500$$

so, the minimum is 4.

PROBLEM 9-7

Statement: Provide a preliminary design (pitch diameters and numbers of teeth) for a gear set that will have a gear ratio of $m_G = 4$, a diametral pitch $p_d = 8$, and a contact ratio of at least 1.5.

Given: Gear ratio $m_G := 4$ Diametral pitch $p_d := 8 \cdot \text{in}^{-1}$

Design Choice: Pressure angle $\phi := 25 \cdot \text{deg}$

Assumptions: The pinion is cut by a hob and can, therefore, have no fewer than 14 teeth for a 25-deg pressure angle (see Table 9-4b).

Solution: See Mathcad file P0907.

- From inspection of Table 9-5b, we see that 11 teeth is the least number that the pinion can have for a gear ratio of $m_G = 4$. Therefore, let the number of teeth on the pinion be (an even number > 11). Iterate on the number of teeth until a satisfactory contact ratio is achieved.

$$N_p := 22 \quad \text{and} \quad N_g := m_G \cdot N_p \quad N_g = 88$$

- Using equation 9.4c, calculate the pitch diameters of the pinion and gear.

$$d_p := \frac{N_p}{p_d} \quad d_p = 2.7500 \cdot \text{in} \quad d_g := \frac{N_g}{p_d} \quad d_g = 11.0000 \cdot \text{in}$$

- Calculate the contact ratio using equations 9.2 and 9.6b and those from Table 9-1.

$$r_p := 0.5 \cdot d_p \quad r_p = 1.3750 \cdot \text{in} \quad r_g := 0.5 \cdot d_g \quad r_g = 5.5000 \cdot \text{in}$$

$$a_p := \frac{1}{p_d} \quad a_p = 0.1250 \cdot \text{in} \quad a_g := \frac{1}{p_d} \quad a_g = 0.1250 \cdot \text{in}$$

$$\text{Center distance} \quad C := r_p + r_g \quad C = 6.8750 \cdot \text{in}$$

$$Z := \sqrt{(r_p + a_p)^2 - (r_p \cdot \cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot \cos(\phi))^2} - C \cdot \sin(\phi) \quad Z = 0.5358 \text{ in}$$

$$\text{Contact ratio} \quad m_p := \frac{p_d \cdot Z}{\pi \cdot \cos(\phi)} \quad m_p = 1.506$$

PROBLEM 10-1

Statement: The mallet shown in Figure 10-2 has the specifications given below. Find the location of its composite CG, and its moment of inertia and radius of gyration about axis ZZ. Assume the wood has a density equal to 0.9 times that of water.

Given: Brass hammer head has specific weight: $\gamma_d := 0.31 \cdot \text{lb} \cdot \text{in}^{-3}$

Wood handle has specific weight: $\gamma_h := 0.9 \cdot 0.036 \cdot \text{lb} \cdot \text{in}^{-3}$

Head dimensions: $r_d := 0.625 \cdot \text{in}$ $h_d := 3.5 \cdot \text{in}$

Handle dimensions: $r_{h1} := 0.750 \cdot \text{in}$ $L_{h1} := 10 \cdot \text{in}$ $r_{h2} := 0.3875 \cdot \text{in}$ $L_{h2} := 2 \cdot r_d$

Solution: See Figure 10-2 and Mathcad file P1001.

1. Calculate the volume and weight of each component.

$$\text{Head: } V_d := \pi \cdot r_d^2 \cdot h_d - \pi \cdot r_{h2}^2 \cdot L_{h2} \quad V_d = 3.705 \cdot \text{in}^3$$

$$W_d := \gamma_d \cdot V_d \quad W_d = 1.149 \cdot \text{lb} \cdot \text{f}$$

$$\text{Handle: } V_{h1} := \pi \cdot r_{h1}^2 \cdot L_{h1} \quad V_{h1} = 17.671 \cdot \text{in}^3$$

$$V_{h2} := \pi \cdot r_{h2}^2 \cdot L_{h2} \quad V_{h2} = 0.590 \cdot \text{in}^3$$

$$V_h := V_{h1} + V_{h2} \quad V_h = 18.261 \cdot \text{in}^3$$

$$W_{h1} := \gamma_h \cdot V_{h1} \quad W_{h1} = 0.573 \cdot \text{lb} \cdot \text{f}$$

$$W_{h2} := \gamma_h \cdot V_{h2} \quad W_{h2} = 0.019 \cdot \text{lb} \cdot \text{f}$$

$$W_h := W_{h1} + W_{h2} \quad W_h = 0.592 \cdot \text{lb} \cdot \text{f}$$

2. The CG of each component will lie along the XX axis. Find the distance from the origin to the CGs (distance to axes HH and DD).

$$\text{Head: } X_{cg_d} := L_{h1} + r_d \quad X_{cg_d} = 10.625 \cdot \text{in}$$

$$\text{Handle: } X_{cg_h} := \frac{0.5 \cdot L_{h1} \cdot V_{h1} + (L_{h1} + 0.5 \cdot L_{h2}) \cdot V_{h2}}{V_h}$$

$$X_{cg_h} = 5.182 \cdot \text{in}$$

3. Find the location of the composite CG using equation 10.3d.

$$X_{Cg} := \frac{X_{cg_d} \cdot W_d + X_{cg_h} \cdot W_h}{W_d + W_h} \quad X_{Cg} = 8.774 \cdot \text{in}$$

4. Calculate the moment of inertia of the head with respect to the ZZ axis.

$$I_{DDd} := \frac{W_d}{12 \cdot g} \cdot (3 \cdot r_d^2 + h_d^2) \quad I_{DDd} = 3.328 \times 10^{-3} \cdot \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}$$

$$I_{ZZd} := I_{DDd} + \frac{W_d}{g} \cdot X_{cg_d}^2 \quad I_{ZZd} = 0.339 \cdot \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}$$

5. Calculate the moment of inertia of the handle with respect to the ZZ axis.

$$I_{h1} := \frac{W_{h1}}{12 \cdot g} \cdot (3 \cdot r_{h1}^2 + L_{h1}^2) \quad I_{h1} = 0.013 \cdot \text{lb} \cdot \text{sec}^2 \cdot \text{in}$$

$$I_{ZZh1} := I_{h1} + \frac{W_{h1}}{g} \cdot \left(\frac{L_{h1}}{2} \right)^2 \quad I_{ZZh1} = 0.050 \cdot \text{lb} \cdot \text{sec}^2 \cdot \text{in}$$

$$I_{h2} := \frac{W_{h2}}{12 \cdot g} \cdot (3 \cdot r_{h2}^2 + L_{h2}^2) \quad I_{h2} = 8.301 \times 10^{-6} \cdot \text{lb} \cdot \text{sec}^2 \cdot \text{in}$$

$$I_{ZZh2} := I_{h2} + \frac{W_{h2}}{g} \cdot \left(L_{h1} + \frac{L_{h2}}{2} \right)^2 \quad I_{ZZh2} = 5.595 \times 10^{-3} \cdot \text{lb} \cdot \text{sec}^2 \cdot \text{in}$$

$$I_{ZZh} := I_{ZZh1} + I_{ZZh2} \quad I_{ZZh} = 0.055 \cdot \text{lb} \cdot \text{sec}^2 \cdot \text{in}$$

6. Add the moments of the two components about the ZZ axis to get the moment of inertia of the mallet about the ZZ axis.

$$I_{ZZ} := I_{ZZd} + I_{ZZh} \quad I_{ZZ} = 0.394 \cdot \text{lb} \cdot \text{sec}^2 \cdot \text{in}$$

7. Use equation 10.10b to calculate the radius of gyration about the ZZ axis.

$$k := \sqrt{\frac{I_{ZZ} \cdot g}{W_d + W_h}} \quad k = 9.354 \cdot \text{in}$$

PROBLEM 10-2

Statement: The mallet shown in Figure 10-2 has the specifications given below. Find the location of its composite CG, and its moment of inertia and radius of gyration about axis ZZ. Assume the wood has a density equal to 0.9 times that of water.

Given: Wood has specific weight: $\gamma := 0.85 \cdot 0.036 \cdot \text{lb} \cdot \text{in}^{-3}$

Head dimensions: $r_d := 1.25 \cdot \text{in}$ $h_d := 3.5 \cdot \text{in}$

Handle dimensions: $r_h := 0.750 \cdot \text{in}$ $L_h := 10 \cdot \text{in}$

Solution: See Figure 10-2 and Mathcad file P1002.

1. Calculate the volume and weight of each component.

$$\text{Head: } V_d := \pi \cdot r_d^2 \cdot h_d \quad V_d = 17.181 \cdot \text{in}^3$$

$$W_d := \gamma \cdot V_d \quad W_d = 0.526 \cdot \text{lb} \cdot \text{f}$$

$$\text{Handle: } V_h := \pi \cdot r_h^2 \cdot L_h \quad V_h = 17.671 \cdot \text{in}^3$$

$$W_h := \gamma \cdot V_h \quad W_h = 0.541 \cdot \text{lb} \cdot \text{f}$$

2. The CG of each component will lie along the XX axis. Find the distance from the origin to the CGs (distance to axes HH and DD).

$$\text{Head: } X_{cg_d} := L_h + r_d \quad X_{cg_d} = 11.250 \cdot \text{in}$$

$$\text{Handle: } X_{cg_h} := \frac{L_h}{2} \quad X_{cg_h} = 5.000 \cdot \text{in}$$

3. Find the location of the composite CG using equation 10.3d.

$$X_{Cg} := \frac{X_{cg_d} \cdot W_d + X_{cg_h} \cdot W_h}{W_d + W_h} \quad X_{Cg} = 8.081 \cdot \text{in}$$

4. Calculate the moment of inertia of the head with respect to the ZZ axis.

$$I_{DDd} := \frac{W_d}{12 \cdot g} \cdot (3 \cdot r_d^2 + h_d^2) \quad I_{DDd} = 1.922 \times 10^{-3} \cdot \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}$$

$$I_{ZZd} := I_{DDd} + \frac{W_d}{g} \cdot X_{cg_d}^2 \quad I_{ZZd} = 0.174 \cdot \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}$$

5. Calculate the moment of inertia of the handle with respect to the ZZ axis.

$$I_{HHh} := \frac{W_h}{12 \cdot g} \cdot (3 \cdot r_h^2 + L_h^2) \quad I_{HHh} = 0.012 \cdot \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}$$

$$I_{ZZh} := I_{HHh} + \frac{W_h}{g} \cdot X_{cg_h}^2 \quad I_{ZZh} = 0.047 \cdot \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}$$

6. Add the moments of the two components about the ZZ axis to get the moment of inertia of the mallet about the ZZ axis.

$$I_{ZZ} := I_{ZZd} + I_{ZZh} \quad I_{ZZ} = 0.221 \cdot \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}$$

7. Use equation 10.10b to calculate the radius of gyration about the ZZ axis.

$$k := \sqrt{\frac{I_{ZZ} \cdot g}{W_d + W_h}} \quad k = 8.948 \cdot \text{in}$$

PROBLEM 10-3

Statement: Calculate the location of the composite center of gravity, the mass moment of inertia and the radius of gyration with respect to the specified axis, for whichever of the following commonly available items that are assigned. (Note these are not short problems).

- a. A good-quality writing pen, about the pivot point at which you grip it to write. (How does placing the cap on the upper end of the pen affect these parameters when you write?)
- b. Two table knives, one metal and one plastic, about the pivot axis when held for cutting. Compare the calculated results and comment on what they tell you about the dynamic usability of the two knives (ignore sharpness considerations).
- c. A ball-peen hammer (available for inspection in any university machine shop), about the center of rotation (after you calculate it for the proper center of percussion).
- d. A baseball bat (see the coach) about the center of rotation (after you calculate its location for the proper center of percussion).
- e. A cylindrical coffee mug, about the handle hole.

Solution: No solution is provided here due to the wide variety of possibilities in this open-ended problem.

PROBLEM 10-4

Statement: Set up these equations in matrix form. Use program MATRIX, Mathcad, or a calculator that has matrix math capability to solve them.

$$\begin{array}{rclclcl}
 \text{a.} & -5x & -2y & +12z & -w & = & -9 \\
 & x & +3y & -2z & +4w & = & 10 \\
 & -x & -y & +z & & = & -7 \\
 & 3x & -3y & +7z & +9w & = & -6 \\
 \text{b.} & 3x & -5y & +17z & -5w & = & -5 \\
 & -2x & +9y & -14z & +6w & = & 22 \\
 & -x & -y & & -2w & = & 13 \\
 & 4x & -7y & +8z & +4w & = & -9
 \end{array}$$

Solution: See Mathcad file P1004.

1. Place the coefficients of the unknowns in a 4 x 4 matrix.

$$Ca := \begin{pmatrix} -5 & -2 & 12 & -1 \\ 1 & 3 & -2 & 4 \\ -1 & -1 & 1 & 0 \\ 3 & -3 & 7 & 9 \end{pmatrix} \qquad Cb := \begin{pmatrix} 3 & -5 & 17 & -5 \\ -2 & 9 & -14 & 6 \\ -1 & -1 & 0 & -2 \\ 4 & -7 & 8 & 4 \end{pmatrix}$$

2. Place the constants on the right-hand side of the equal sign in a 4 x 1 array.

$$Ba := \begin{pmatrix} -9 \\ 10 \\ -7 \\ -6 \end{pmatrix} \qquad Bb := \begin{pmatrix} -5 \\ 22 \\ 13 \\ -9 \end{pmatrix}$$

3. Get the solution array by premultiplying the constant array by the inverse of the coefficient matrix.

$$\begin{pmatrix} xa \\ ya \\ za \\ wa \end{pmatrix} := Ca^{-1} \cdot Ba \qquad xa = 3.547 \qquad ya = 4.884 \qquad za = 1.431 \qquad wa = -1.334$$

$$\begin{pmatrix} xb \\ yb \\ zb \\ wb \end{pmatrix} := Cb^{-1} \cdot Bb \qquad xb = -62.029 \qquad yb = 0.235 \qquad zb = 17.897 \qquad wb = 24.397$$

PROBLEM 10-5

- Statement:** Figure P10-1 shows a bracket made of steel.
- Find the location of its centroid referred to point B.
 - Find its mass moment of inertia I_{xx} about the x axis through point B.
 - Find its mass moment of inertia I_{yy} about the y axis through point B.

Given: Dimensions in the figures below. Density: $\rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3}$

Solution: See Figure P10-1 and Mathcad file P1005.

- Divide the bracket into five volumes, find the location of the CG and the mass moments for each of them and then add the results to get the CG and mass moments for the entire bracket.

$$i := 1, 2 \dots 5$$

- Volume 1 is the rectangular prism with two negative cylinders shown below.

Dimensions:

$$a := 64 \cdot \text{mm}$$

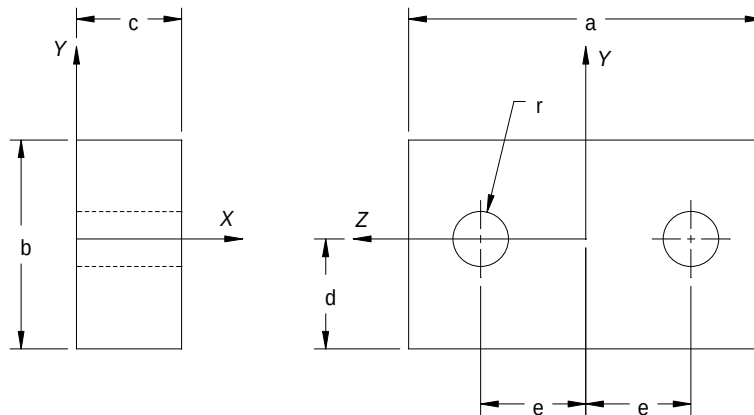
$$b := 38 \cdot \text{mm}$$

$$c := 19 \cdot \text{mm}$$

$$d := 20 \cdot \text{mm}$$

$$e := 19 \cdot \text{mm}$$

$$r := 5 \cdot \text{mm}$$



Determine the location of the CG in global coordinates.

$$Xcg_1 := \frac{c}{2} \quad Xcg_1 = 9.500 \text{ mm} \quad Zcg_1 := 0 \cdot \text{mm}$$

$$Ycg_1 := \frac{a \cdot b \cdot \frac{b}{2} - 2 \cdot \pi \cdot r^2 \cdot d}{a \cdot b - 2 \cdot \pi \cdot r^2} - d \quad Ycg_1 = -1.069 \text{ mm}$$

Determine the volume and mass of this segment.

$$V_1 := a \cdot b \cdot c - 2 \cdot \pi \cdot r^2 \cdot c \quad V_1 = 4.322 \times 10^4 \text{ mm}^3 \quad M_1 := \rho \cdot V_1 \quad M_1 = 0.337 \text{ kg}$$

Determine the mass moments about the local axes through the CGs.

$$\text{Rectangular prism: } m_a := a \cdot b \cdot c \cdot \rho \quad m_a = 0.360 \text{ kg}$$

$$I_{ax} := \frac{m_a}{12} \cdot (a^2 + b^2) \quad I_{ax} = 1.664 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_{ay} := \frac{m_a}{12} \cdot (a^2 + c^2) \quad I_{ay} = 1.339 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_{az} := \frac{m_a}{12} \cdot (b^2 + c^2) \quad I_{az} = 5.421 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

Cylinder (one): $m_b := \pi \cdot r^2 \cdot c \cdot \rho$ $m_b = -0.012 \text{ kg}$

$$I_{bx} := \frac{m_b \cdot r^2}{2} \quad I_{bx} = -1.455 \times 10^{-7} \text{ kg} \cdot \text{m}^2$$

$$I_{by} := \frac{m_b}{12} \cdot (3 \cdot r^2 + c^2) \quad I_{by} = -4.229 \times 10^{-7} \text{ kg} \cdot \text{m}^2$$

$$I_{bz} := \frac{m_b}{12} \cdot (3 \cdot r^2 + c^2) \quad I_{bz} = -4.229 \times 10^{-7} \text{ kg} \cdot \text{m}^2$$

Determine the mass moments about the global axes.

$$I_{xx_1} := I_{ax} + m_a \cdot \left[(Ycg_1)^2 + (Zcg_1)^2 \right] + 2 \cdot (I_{bx} + m_b \cdot e^2) \quad I_{xx_1} = 1.581 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_{yy_1} := I_{ay} + m_a \cdot \left[(Xcg_1)^2 + (Zcg_1)^2 \right] + 2 \cdot \left[I_{by} + m_b \cdot \left(\frac{c}{2} \right)^2 \right] \quad I_{yy_1} = 1.634 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_{zz_1} := I_{az} + m_a \cdot \left[(Xcg_1)^2 + (Ycg_1)^2 \right] + 2 \cdot (I_{bz}) \quad I_{zz_1} = 8.631 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

3. Volume 2 is the bend just above segment 1. It is a quarter hollow cylinder with dimensions shown below.

Dimensions:

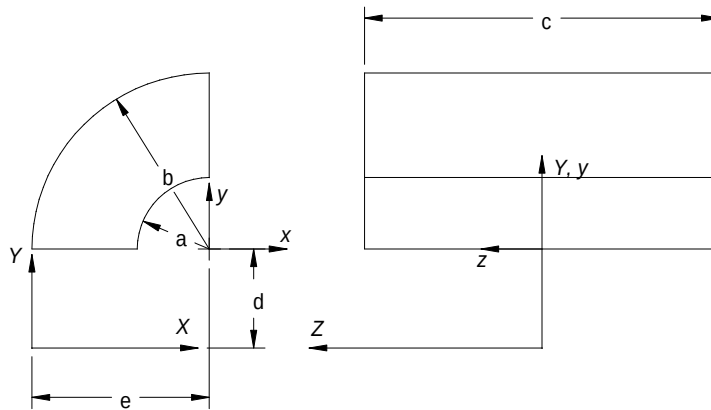
$$a := 13 \cdot \text{mm}$$

$$b := 32 \cdot \text{mm}$$

$$c := 64 \cdot \text{mm}$$

$$d := 18 \cdot \text{mm}$$

$$e := 32 \cdot \text{mm}$$



Determine the location of the CG in global coordinates.

$$Xcg_2 := e - \frac{4}{3 \cdot \pi} \cdot \frac{b^3 - a^3}{b^2 - a^2} \quad Xcg_2 = 16.825 \text{ mm} \quad Zcg_2 := 0 \cdot \text{mm}$$

$$Ycg_2 := d + \frac{4}{3 \cdot \pi} \cdot \frac{b^3 - a^3}{b^2 - a^2} \quad Ycg_2 = 33.175 \text{ mm}$$

Determine the volume and mass of this segment.

$$V_2 := \frac{\pi \cdot b^2 \cdot c - \pi \cdot a^2 \cdot c}{4} \quad V_2 = 4.298 \times 10^4 \text{ mm}^3 \quad M_2 := \rho \cdot V_2 \quad M_2 = 0.335 \text{ kg}$$

Determine the mass moments about the local axes noted on the drawing of the segment.

$$I_x := \frac{M_2}{12} \cdot (3 \cdot a^2 + 3 \cdot b^2 + c^2) \quad I_x = 2.144 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_y := \frac{M_2}{12} \cdot (3 \cdot a^2 + 3 \cdot b^2 + c^2) \quad I_y = 2.144 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_z := \frac{M_2}{2} \cdot (a^2 + b^2) \quad I_z = 2.000 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

Determine the mass moments about the global axes.

$$I_{xx_2} := I_x + M_2 \cdot d^2 \quad I_{xx_2} = 3.230 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_{yy_1} := I_{ay} + M_2 \cdot e^2 \quad I_{yy_1} = 4.771 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_{zz_1} := I_{az} + M_2 \cdot (d^2 + e^2) \quad I_{zz_1} = 5.061 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

4. Volume 3 is a rectangular prism with dimensions shown below.

Dimensions:

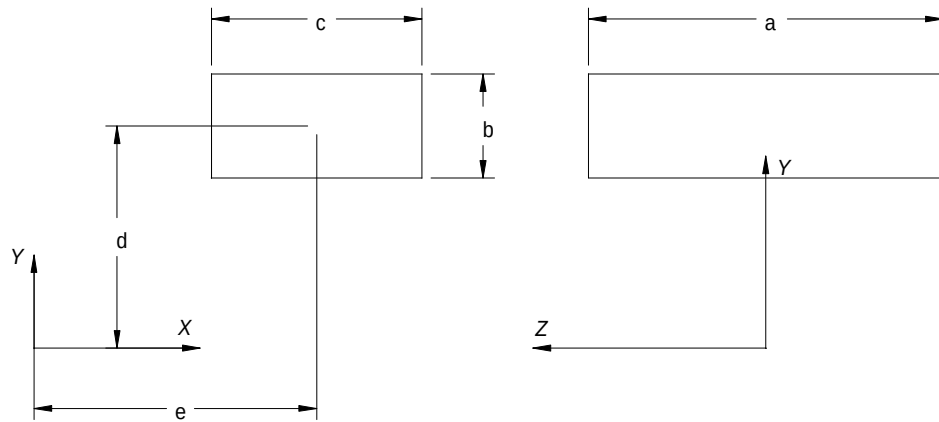
$$a := 64 \cdot \text{mm}$$

$$b := 19 \cdot \text{mm}$$

$$c := 38 \cdot \text{mm}$$

$$d := 40.5 \cdot \text{mm}$$

$$e := 51 \cdot \text{mm}$$



Determine the location of the CG in global coordinates.

$$Xcg_3 := e \quad Xcg_3 = 51.000 \text{ mm} \quad Ycg_3 := d \quad Ycg_3 = 40.500 \text{ mm}$$

$$Zcg_3 := 0 \cdot \text{mm}$$

Determine the volume and mass of this segment.

$$V_3 := a \cdot b \cdot c \quad V_3 = 4.621 \times 10^4 \text{ mm}^3 \quad M_3 := \rho \cdot V_3 \quad M_3 = 0.360 \text{ kg}$$

Determine the mass moments about the local axes through the CGs.

$$I_x := \frac{M_3}{12} \cdot (a^2 + b^2) \quad I_x = 1.339 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_y := \frac{M_3}{12} \cdot (a^2 + c^2) \quad I_y = 1.664 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_z := \frac{M_3}{12} \cdot (b^2 + c^2) \quad I_z = 5.421 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

Determine the mass moments about the global axes.

$$I_{xx_3} := I_x + M_3 \cdot \left[(Ycg_3)^2 + (Zcg_3)^2 \right] \quad I_{xx_3} = 7.250 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_{yy_3} := I_y + M_3 \cdot \left[(Xcg_3)^2 + (Zcg_3)^2 \right] \quad I_{yy_3} = 1.104 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_{zz_3} := I_z + M_3 \cdot \left[(Xcg_3)^2 + (Ycg_3)^2 \right] \quad I_{zz_1} = 5.061 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

5. Volume 4 is a half cylinder with dimensions shown below.

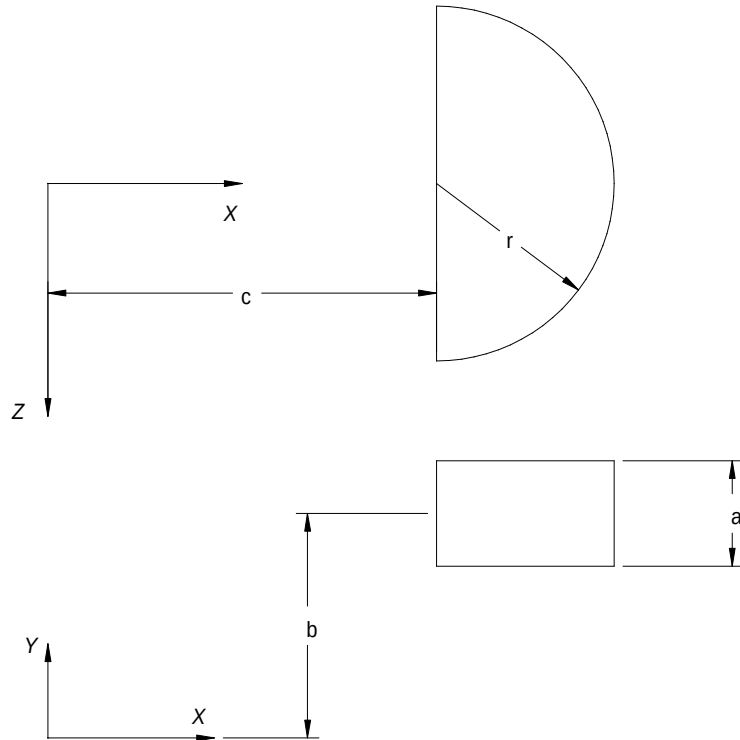
Dimensions:

$$a := 19 \cdot \text{mm}$$

$$b := 40.5 \cdot \text{mm}$$

$$c := 70 \cdot \text{mm}$$

$$r := 32 \cdot \text{mm}$$



Determine the location of the CG in global coordinates.

$$Xcg_4 := c + \frac{4 \cdot r}{3 \cdot \pi} \quad Xcg_4 = 83.581 \text{ mm} \quad Ycg_4 := b \quad Ycg_3 = 40.500 \text{ mm}$$

$$Zcg_4 := 0 \cdot \text{mm}$$

Determine the volume and mass of this segment.

$$V_4 := a \cdot \frac{\pi \cdot r^2}{2} \quad V_4 = 3.056 \times 10^4 \text{ mm}^3 \quad M_4 := \rho \cdot V_4 \quad M_4 = 0.238 \text{ kg}$$

Determine the mass moments about the local axes of the segment.

$$I_x := \frac{M_4}{12} \cdot (3 \cdot r^2 + a^2) \quad I_x = 6.820 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

$$I_y := \frac{M_4}{2} \cdot r^2 \quad I_y = 1.221 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_z := \frac{M_4}{12} \cdot (3 \cdot r^2 + a^2) \quad I_z = 6.820 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

Determine the mass moments about the global axes.

$$I_{xx_4} := I_x + M_4 \cdot \left[(Y_{cg_4})^2 + (Z_{cg_4})^2 \right] \quad I_{xx_4} = 4.592 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_{yy_4} := I_y + M_4 \cdot \left[(X_{cg_4})^2 + (Z_{cg_4})^2 \right] \quad I_{yy_4} = 1.787 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_{zz_4} := I_z + M_4 \cdot \left[(X_{cg_4})^2 + (Y_{cg_4})^2 \right] \quad I_{zz_4} = 2.124 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

6. Volume 5 is a negative cylinder with dimensions shown below.

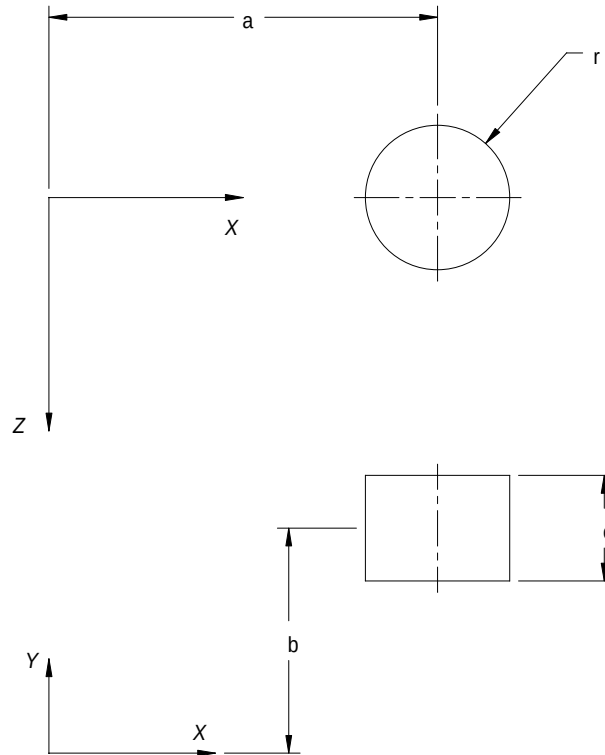
Dimensions:

$$a := 70 \cdot \text{mm}$$

$$b := 40.5 \cdot \text{mm}$$

$$c := 19 \cdot \text{mm}$$

$$r := 13 \cdot \text{mm}$$



Determine the location of the CG in global coordinates.

$$X_{cg_5} := a$$

$$X_{cg_5} = 70.000 \text{ mm}$$

$$Y_{cg_5} := b$$

$$Y_{cg_5} = 40.500 \text{ mm}$$

$$Z_{cg_5} := 0 \cdot \text{mm}$$

Determine the volume and mass of this segment.

$$V_5 := c \cdot \pi \cdot r^2$$

$$V_5 = 1.009 \times 10^4 \text{ mm}^3$$

$$M_5 := -\rho \cdot V_5$$

$$M_5 = -0.079 \text{ kg}$$

Determine the mass moments about the local axes of the segment.

$$I_x := \frac{M_5}{12} \cdot (3 \cdot r^2 + c^2)$$

$$I_x = -5.691 \times 10^{-6} \text{ kg} \cdot \text{m}^2$$

$$I_y := \frac{M_5}{2} \cdot r^2$$

$$I_y = -6.649 \times 10^{-6} \text{ kg} \cdot \text{m}^2$$

$$I_z := \frac{M_5}{12} \cdot (3 \cdot r^2 + c^2)$$

$$I_z = -5.691 \times 10^{-6} \text{ kg} \cdot \text{m}^2$$

Determine the mass moments about the global axes.

$$I_{xx_5} := I_x + M_5 \cdot \left[(Y_{cg_5})^2 + (Z_{cg_5})^2 \right] \quad I_{xx_5} = -1.348 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_{yy_5} := I_y + M_5 \cdot \left[(X_{cg_5})^2 + (Z_{cg_5})^2 \right] \quad I_{yy_5} = -3.922 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$I_{zz_5} := I_z + M_5 \cdot \left[(X_{cg_5})^2 + (Y_{cg_5})^2 \right] \quad I_{zz_5} = -5.203 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

a. Find the location of its centroid referred to point B.

$$X_{Cg} := \frac{\sum_i (X_{cg_i} \cdot M_i)}{\sum_i M_i} \quad X_{Cg} = 34.919 \text{ mm}$$

$$Y_{Cg} := \frac{\sum_i (Y_{cg_i} \cdot M_i)}{\sum_i M_i} \quad Y_{Cg} = 26.688 \text{ mm}$$

$$Z_{Cg} := \frac{\sum_i (Z_{cg_i} \cdot M_i)}{\sum_i M_i} \quad Z_{Cg} = 0.000 \text{ mm}$$

b. Find its mass moment of inertia I_{xx} about the X axis through point B.

$$I_{xx} := \sum_i I_{xx_i} \quad I_{xx} = 1.531 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

c. Find its mass moment of inertia I_{yy} about the Y axis through point B.

$$I_{yy} := \sum_i I_{yy_i} \quad I_{yy} = 2.976 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

PROBLEM 10-6

Statement: Two springs are connected in series. One has a k of 34 and the other a k of 3.4. Calculate their effective spring constant. Which spring dominates? Repeat with the two springs in parallel. Which spring dominates? (Use any unit system.)

Given: Spring constants:

$$k_1 := 34 \cdot \frac{N}{mm} \quad k_2 := 3.4 \cdot \frac{N}{mm}$$

Solution: See Mathcad file P1006.

1. For springs in series, use equation 10.19c to find the effective spring rate.

$$k_{eff} := \left(\frac{1}{k_1} + \frac{1}{k_2} \right)^{-1} \quad k_{eff} = 3.091 \frac{N}{mm}$$

The spring with the smaller rate dominates.

2. For springs in parallel, use equation 10.20b to find the effective spring rate.

$$k_{eff} := k_1 + k_2 \quad k_{eff} = 37.400 \frac{N}{mm}$$

The spring with the larger rate dominates.

PROBLEM 10-7

Statement: Two springs are connected in series. One has a k of 125 and the other a k of 25. Calculate their effective spring constant. Which spring dominates? Repeat with the two springs in parallel. Which spring dominates? (Use any unit system.)

Given: Spring constants:

$$k_1 := 125 \cdot \frac{N}{mm} \quad k_2 := 25 \cdot \frac{N}{mm}$$

Solution: See Mathcad file P1007.

1. For springs in series, use equation 10.19c to find the effective spring rate.

$$k_{eff} := \left(\frac{1}{k_1} + \frac{1}{k_2} \right)^{-1} \quad k_{eff} = 20.833 \frac{N}{mm}$$

The spring with the smaller rate dominates.

2. For springs in parallel, use equation 10.20b to find the effective spring rate.

$$k_{eff} := k_1 + k_2 \quad k_{eff} = 150.000 \frac{N}{mm}$$

The spring with the larger rate dominates.

PROBLEM 10-8

Statement: Two springs are connected in series. One has a k of 125 and the other a k of 115. Calculate their effective spring constant. Which spring dominates? Repeat with the two springs in parallel. Which spring dominates? (Use any unit system.)

Given: Spring constants:

$$k_1 := 125 \cdot \frac{N}{mm} \quad k_2 := 115 \cdot \frac{N}{mm}$$

Solution: See Mathcad file P1008.

1. For springs in series, use equation 10.19c to find the effective spring rate.

$$k_{eff} := \left(\frac{1}{k_1} + \frac{1}{k_2} \right)^{-1} \quad k_{eff} = 59.896 \frac{N}{mm}$$

Neither spring dominates since they are both about equally stiff.

2. For springs in parallel, use equation 10.20b to find the effective spring rate.

$$k_{eff} := k_1 + k_2 \quad k_{eff} = 240.000 \frac{N}{mm}$$

Neither spring dominates since they are both about equally stiff.

PROBLEM 10-9

Statement: Two dampers are connected in series. One has a damping factor of $c_1 = 12.5$ and the other, $c_2 = 1.2$. Calculate their effective damping constant. Which damper dominates? Repeat with the two dampers in parallel. Which damper dominates? (Use any unit system.)

Given: Damping factors:

$$c_1 := 12.5 \cdot \frac{N \cdot \text{sec}}{\text{mm}} \quad c_2 := 1.2 \cdot \frac{N \cdot \text{sec}}{\text{mm}}$$

Solution: See Mathcad file P1009.

1. For dampers in series, use equation 10.18a to find the effective damping factor.

$$c_{\text{eff}} := \left(\frac{1}{c_1} + \frac{1}{c_2} \right)^{-1} \quad c_{\text{eff}} = 1.095 \frac{N \cdot \text{sec}}{\text{mm}}$$

The softer damper dominates.

2. For dampers in parallel, use equation 10.18b to find the effective damping factor.

$$c_{\text{eff}} := c_1 + c_2 \quad c_{\text{eff}} = 13.700 \frac{N \cdot \text{sec}}{\text{mm}}$$

The stiffer damper dominates.

PROBLEM 10-10

Statement: Two dampers are connected in series. One has a damping factor of $c_1 = 12.5$ and the other, $c_2 = 2.5$. Calculate their effective damping constant. Which damper dominates? Repeat with the two dampers in parallel. Which damper dominates? (Use any unit system.)

Given: Damping factors:

$$c_1 := 12.5 \cdot \frac{N \cdot \text{sec}}{\text{mm}} \quad c_2 := 2.5 \cdot \frac{N \cdot \text{sec}}{\text{mm}}$$

Solution: See Mathcad file P1010.

1. For dampers in series, use equation 10.18a to find the effective damping factor.

$$c_{\text{eff}} := \left(\frac{1}{c_1} + \frac{1}{c_2} \right)^{-1} \quad c_{\text{eff}} = 2.083 \frac{N \cdot \text{sec}}{\text{mm}}$$

The softer damper dominates.

2. For dampers in parallel, use equation 10.18b to find the effective damping factor.

$$c_{\text{eff}} := c_1 + c_2 \quad c_{\text{eff}} = 15.000 \frac{N \cdot \text{sec}}{\text{mm}}$$

The stiffer damper dominates.

PROBLEM 10-11

Statement: Two dampers are connected in series. One has a damping factor of $c_1 = 12.5$ and the other, $c_2 = 10$. Calculate their effective damping constant. Which damper dominates? Repeat with the two dampers in parallel. Which damper dominates? (Use any unit system.)

Given: Damping factors:

$$c_1 := 12.5 \cdot \frac{N \cdot sec}{mm} \quad c_2 := 10 \cdot \frac{N \cdot sec}{mm}$$

Solution: See Mathcad file P1011.

1. For dampers in series, use equation 10.18a to find the effective damping factor.

$$c_{eff} := \left(\frac{1}{c_1} + \frac{1}{c_2} \right)^{-1} \quad c_{eff} = 5.556 \frac{N \cdot sec}{mm}$$

Neither damper dominates since they are both about equally stiff.

2. For dampers in parallel, use equation 10.18b to find the effective damping factor.

$$c_{eff} := c_1 + c_2 \quad c_{eff} = 22.500 \frac{N \cdot sec}{mm}$$

Neither damper dominates since they are both about equally stiff.

PROBLEM 10-12

Statement: A mass of $m = 2.75$ and a spring with $k = 48$ are attached to one end of a lever at a radius of 5. Calculate the effective mass and effective spring constant at a radius of 10 on the same lever. (Use any unit system.)

Given: Mass: Spring: Radius:

$$M := 2.75 \cdot kg \quad k := 48 \cdot \frac{N}{mm} \quad r_1 := 5 \cdot mm \quad r_2 := 10 \cdot mm$$

Solution: See Figure 10-8 and Mathcad file P1012.

1. The effective mass and spring must have the same energy as the original. Taking the mass first and using equation 10.22b,

$$m_{eff} := \left(\frac{r_1}{r_2} \right)^2 \cdot M \quad m_{eff} = 0.688 \cdot kg$$

2. To find the effective spring rate, use equation 10.23b.

$$k_{eff} := \left(\frac{r_1}{r_2} \right)^2 \cdot k \quad k_{eff} = 12.000 \cdot \frac{N}{mm}$$

PROBLEM 10-13

Statement: A mass of $m = 1.5$ and a spring with $k = 24$ are attached to one end of a lever at a radius of 3. Calculate the effective mass and effective spring constant at a radius of 10 on the same lever. (Use any unit system.)

Given: Mass: Spring: Radius:

$$M := 1.5 \cdot kg \quad k := 24 \cdot \frac{N}{mm} \quad r_1 := 3 \cdot mm \quad r_2 := 10 \cdot mm$$

Solution: See Figure 10-8 and Mathcad file P1013.

1. The effective mass and spring must have the same energy as the original. Taking the mass first and using equation 10.22b,

$$m_{eff} := \left(\frac{r_1}{r_2} \right)^2 \cdot M \quad m_{eff} = 0.135 \, kg$$

2. To find the effective spring rate, use equation 10.23b.

$$k_{eff} := \left(\frac{r_1}{r_2} \right)^2 \cdot k \quad k_{eff} = 2.160 \frac{N}{mm}$$

PROBLEM 10-14

Statement: A mass of $m = 6.5$ and a spring with $k = 25$ are attached to one end of a lever at a radius of 15. Calculate the effective mass and effective spring constant at a radius of 5 on the same lever. (Use any unit system.)

Given: Mass: Spring: Radius:

$$M := 6.5 \cdot kg \qquad k := 25 \cdot \frac{N}{mm} \qquad r_1 := 15 \cdot mm \qquad r_2 := 5 \cdot mm$$

Solution: See Figure 10-8 and Mathcad file P1014.

1. The effective mass and spring must have the same energy as the original. Taking the mass first and using equation 10.22b,

$$m_{eff} := \left(\frac{r_1}{r_2} \right)^2 \cdot M \qquad m_{eff} = 58.500 \, kg$$

2. To find the effective spring rate, use equation 10.23b.

$$k_{eff} := \left(\frac{r_1}{r_2} \right)^2 \cdot k \qquad k_{eff} = 225.000 \cdot \frac{N}{mm}$$

PROBLEM 10-15

Statement: Refer to Figure 10-9 and Example 10-1. The data for the valve train are given below. Calculate the effective spring constant and effective mass of a single-DOF equivalent system placed on the cam side of the rocker arm. (Use ips unit system.)

Given: Tappet (solid cylinder): $d_{tp} := 0.75 \cdot \text{in}$ $L_{tp} := 1.25 \cdot \text{in}$
 Pushrod (hollow cylinder): $d_{pod} := 0.375 \cdot \text{in}$ $d_{pid} := 0.250 \cdot \text{in}$ $L_{pr} := 12 \cdot \text{in}$
 Rocker arm: $w := 1.00 \cdot \text{in}$ $h := 1.50 \cdot \text{in}$ $a := 2.00 \cdot \text{in}$ $b := 3.00 \cdot \text{in}$
 Cam shaft (cam in center): $d_{cs} := 1.00 \cdot \text{in}$ $L_{cs} := 3.00 \cdot \text{in}$
 Valve spring: $k_{vs} := 200 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-1}$
 All parts are steel: Modulus of elasticity $E := 30 \cdot 10^6 \cdot \text{psi}$ Spec. weight $\gamma := 0.3 \cdot \frac{\text{lb} \cdot \text{f}}{\text{in}^3}$

Solution: See Figure 10-9 and Mathcad file P1015.

1. Break the system into individual elements as shown in Figure 10-9b.
2. Define the individual spring constants of each of the six elements.

Cam shaft (simply-supported beam with central load):

$$\text{Moment of inertia} \quad I_{cs} := \frac{\pi \cdot d_{cs}^4}{64}$$

$$\text{Spring constant} \quad k_{cs} := \frac{48 \cdot E \cdot I_{cs}}{L_{cs}^3} \quad k_{cs} = 2.618 \times 10^6 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

Tappet (solid cylinder):

$$\text{Area} \quad A_{tp} := \frac{\pi \cdot d_{tp}^2}{4}$$

$$\text{Spring constant} \quad k_{tp} := \frac{A_{tp} \cdot E}{L_{tp}} \quad k_{tp} = 1.060 \times 10^7 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

Pushrod (hollow cylinder):

$$\text{Area} \quad A_{pr} := \frac{\pi \cdot (d_{pod}^2 - d_{pid}^2)}{4}$$

$$\text{Spring constant} \quad k_{pr} := \frac{A_{pr} \cdot E}{L_{pr}} \quad k_{pr} = 1.534 \times 10^5 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

Rocker arm (side A):

$$\text{Moment of inertia} \quad I_r := \frac{w \cdot h^3}{12}$$

$$\text{Spring constant} \quad k_{ra} := \frac{3 \cdot E \cdot I_r}{a^3} \quad k_{ra} = 3.164 \times 10^6 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

Rocker arm (side B):

$$\text{Spring constant} \quad k_{rb} := \frac{3 \cdot E \cdot I_r}{b^3} \quad k_{rb} = 9.375 \times 10^5 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

3. Damping will be neglected.
4. Determine the mass of each of the elements.

Tappet (solid cylinder):

$$\text{Volume} \quad V_{tp} := A_{tp} \cdot L_{tp}$$

$$\text{Mass} \quad m_{tp} := \frac{\gamma \cdot V_{tp}}{g} \quad m_{tp} = 4.291 \times 10^{-4} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Pushrod (hollow cylinder):

$$\text{Volume} \quad V_{pr} := A_{pr} \cdot L_{pr}$$

$$\text{Mass} \quad m_{pr} := \frac{\gamma \cdot V_{pr}}{g} \quad m_{pr} = 5.721 \times 10^{-4} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Rocker arm (side A):

$$\text{Volume} \quad V_{ra} := w \cdot h \cdot a$$

$$\text{Mass} \quad m_{ra} := \frac{\gamma \cdot V_{ra}}{g} \quad m_{ra} = 2.331 \times 10^{-3} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Rocker arm (side B):

$$\text{Volume} \quad V_{rb} := w \cdot h \cdot b$$

$$\text{Mass} \quad m_{rb} := \frac{\gamma \cdot V_{rb}}{g} \quad m_{rb} = 3.497 \times 10^{-3} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Omit the valve and valve spring because no data are available.

5. Determine the effective mass and spring constant on either side of the rocker arm..

$$\text{Left side:} \quad m_L := m_{tp} + m_{pr} + m_{ra} \quad m_L = 3.332 \times 10^{-3} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_L := \left(\frac{1}{k_{cs}} + \frac{1}{k_{tp}} + \frac{1}{k_{pr}} + \frac{1}{k_{ra}} \right)^{-1} \quad k_L = 1.368 \times 10^5 \frac{\text{lbf}}{\text{in}}$$

$$\text{Right side:} \quad m_R := m_{rb} \quad m_R = 3.497 \times 10^{-3} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_R := \left(\frac{1}{k_{rb}} + \frac{1}{k_{vs}} \right)^{-1} \quad k_R = 199.957 \frac{\text{lbf}}{\text{in}}$$

6. Reduce the system to a single DOF.

$$m_{eff} := m_L + \left(\frac{b}{a} \right)^2 \cdot m_R \quad m_{eff} = 0.011 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_{eff} := k_L + \left(\frac{b}{a} \right)^2 \cdot k_R \quad k_{eff} = 1.372 \times 10^5 \frac{\text{lbf}}{\text{in}}$$

PROBLEM 10-16

Statement: Figure P10-2 shows a cam-follower system. The dimensions and other data are given below. Find the arm's mass, center of gravity location and mass moment of inertia about both its CG and arm pivot. Create a linear, one-DOF lumped mass model of the dynamic system referenced to the cam follower. Ignore damping.

Given:

Arm dimensions:

Cutout dimensions:

Width (Z-direction): $a := 2 \cdot \text{in}$

Width (Z-direction): $a' := 1.5 \cdot \text{in}$

Height (Y-direction): $b := 2.5 \cdot \text{in}$

Height (Y-direction): $b' := 2.5 \cdot \text{in}$

Length (X-direction): $c := 28 \cdot \text{in}$

Length (X-direction): $c' := 3 \cdot \text{in}$

Distance from left end of arm to roller center: $L_r := 22 \cdot \text{in}$

Distance from left end of arm to arm pivot: $L_p := 10 \cdot \text{in}$

Specific weight of aluminum: $\gamma := 0.1 \cdot \text{lb} \cdot \text{in}^{-3}$ $E := 10.4 \cdot 10^6 \cdot \text{psi}$

Solution: See Figure P10-2 and Mathcad file P1016.

1. Determine the volume and weight of the arm.

$$V_{\text{solid}} := a \cdot b \cdot c \quad V_{\text{solid}} = 140.000 \text{ in}^3$$

$$V_{\text{slot}} := a' \cdot b' \cdot c' \quad V_{\text{slot}} = 11.250 \text{ in}^3$$

$$V_a := V_{\text{solid}} - V_{\text{slot}} \quad V_a = 128.750 \text{ in}^3$$

$$W_{\text{solid}} := \gamma \cdot V_{\text{solid}} \quad W_{\text{solid}} = 14.000 \text{ lbf}$$

$$W_{\text{slot}} := -\gamma \cdot V_{\text{slot}} \quad W_{\text{slot}} = -1.125 \text{ lbf}$$

$$W_a := W_{\text{solid}} + W_{\text{slot}} \quad W_a = 12.875 \text{ lbf}$$

2. Calculate the location of the arm CG with respect to the left end.

$$X_{\text{Cg}} := \frac{W_{\text{solid}} \cdot \frac{c}{2} + W_{\text{slot}} \cdot L_r}{W_a} \quad X_{\text{Cg}} = 13.301 \text{ in}$$

3. Determine the moment of inertia of the arm about the its CG.

$$\text{Solid portion about its own CG} \quad I_{\text{Zsolid}} := \frac{W_{\text{solid}}}{12 \cdot g} \cdot (b^2 + c^2) \quad I_{\text{Zsolid}} = 2.388 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

$$\text{Slot portion about its own CG} \quad I_{\text{Zslot}} := \frac{W_{\text{slot}}}{12 \cdot g} \cdot (b^2 + c'^2) \quad I_{\text{Zslot}} = -3.703 \times 10^{-3} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

$$\text{Solid portion about arm CG} \quad I_{\text{Zsolid}} := I_{\text{Zsolid}} + \frac{W_{\text{solid}}}{g} \cdot \left(X_{\text{Cg}} - \frac{c}{2} \right)^2$$

$$I_{\text{Zsolid}} = 2.406 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

$$\text{Slot portion about arm CG} \quad I_{\text{Zslot}} := I_{\text{Zslot}} + \frac{W_{\text{slot}}}{g} \cdot (L_r - X_{\text{Cg}})^2$$

$$I_{Zslot} = -0.224 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

Arm about its CG

$$I_{ZZ} := I_{Zsolid} + I_{Zslot}$$

$$I_{ZZ} = 2.181 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

4. Determine the moment of inertia of the arm about the arm pivot.

$$I_{pivot} := I_{ZZ} + \frac{W_a}{g} \cdot (X_{Cg} - L_p)^2 \quad I_{pivot} = 2.545 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

5. Determine the mass and spring constant of the arm on the left side of the pivot.

$$\text{Moment of inertia} \quad I_{arm} := \frac{a \cdot b^3}{12}$$

$$\text{Spring constant} \quad k_L := \frac{3 \cdot E \cdot I_{arm}}{L_p^3} \quad k_L = 8.125 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

$$\text{Volume} \quad V_L := a \cdot b \cdot L_p$$

$$\text{Mass} \quad m_L := \frac{\gamma \cdot V_L}{g} \quad m_L = 0.013 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

6. Determine the mass and spring constant of the arm on the right side of the pivot.

$$\text{Spring constant} \quad k_R := \frac{3 \cdot E \cdot I_{arm}}{(c - L_p)^3} \quad k_R = 1.393 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

$$\text{Volume} \quad V_R := a \cdot b \cdot (c - L_p)$$

$$\text{Mass} \quad m_R := \frac{\gamma \cdot V_R}{g} \quad m_R = 0.023 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

6. Find the effective mass and spring constant referenced to the cam-follower..

$$m_{eff} := m_L + \left(\frac{L_r - L_p}{L_p} \right)^2 \cdot m_R \quad m_{eff} = 0.04652 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_{eff} := k_L + \left(\frac{L_r - L_p}{L_p} \right)^2 \cdot k_R \quad k_{eff} = 1.0131 \times 10^5 \frac{\text{lbf}}{\text{in}}$$

PROBLEM 10-17

Statement: Use the method of virtual work to find the torque required to rotate the cam in Figure P10-2 through one revolution. The cam is a pure eccentric.

Given: Cam eccentricity and speed: $e := 0.5 \cdot \text{in}$ $\omega_2 := 500 \cdot \text{rpm}$
 Cam radius and thickness: $r_c := 3 \cdot \text{in}$ $w_c := 0.75 \cdot \text{in}$ (measured from Figure)
 Spring rate and preload: $k := 123 \cdot \text{lb} \cdot \text{in}^{-1}$ $F_{50} := 173 \cdot \text{lb}$
 Arm dimensions: Cutout dimensions:
 Width (Z-direction): $a := 2 \cdot \text{in}$ Width (Z-direction): $a' := 1.5 \cdot \text{in}$
 Height (Y-direction): $b := 2.5 \cdot \text{in}$ Height (Y-direction): $b' := 2.5 \cdot \text{in}$
 Length (X-direction): $c := 28 \cdot \text{in}$ Length (X-direction): $c' := 3 \cdot \text{in}$
 Roller follower dimensions: $r_f := 1 \cdot \text{in}$ $w_f := 1.5 \cdot \text{in}$
 Distance from left end of arm to roller center: $L_r := 22 \cdot \text{in}$
 Distance from left end of arm to spring: $L_s := 29 \cdot \text{in}$
 Distance from left end of arm to pivot: $L_p := 10 \cdot \text{in}$
 Specific weight of aluminum: $\gamma_a := 0.1 \cdot \text{lb} \cdot \text{in}^{-3}$
 Specific weight of steel: $\gamma_s := 0.3 \cdot \text{lb} \cdot \text{in}^{-3}$

Assumptions: Small angle theory applies.

Solution: See Figure P10-2, Problem 10-16, and Mathcad file P1017.

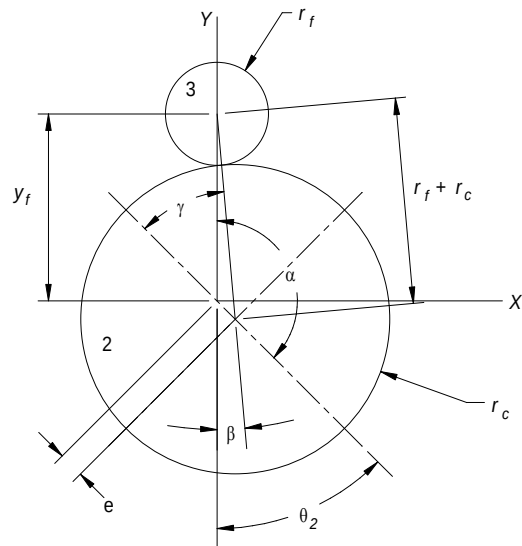
1. Start with a kinematic analysis of the mechanism. Let the cam be link 2, the roller follower link 3, the arm link 4, and the spring link 5. Let the point where the spring connects to the arm be point B, and the center of the roller follower be point A.
2. Calculate the motion of the roller follower center with respect to the X axis, which goes through the center of rotation of the cam.

The cam and follower are shown at right. The origin of the coordinate frame is at the center of rotation. The cam, link 2, is shown rotated an amount θ_2 from the position at which the follower, link 3, is at its lowest position. A triangle is formed with sides e , $r_f + r_c$, and y_f . This triangle will be used to determine the displacement of the follower, y_f , with respect to the cam rotation angle, θ_2 .

Using the law of sines,

$$\beta(\theta_2) := \text{asin}\left(\frac{e}{r_f + r_c} \cdot \sin(\theta_2)\right)$$

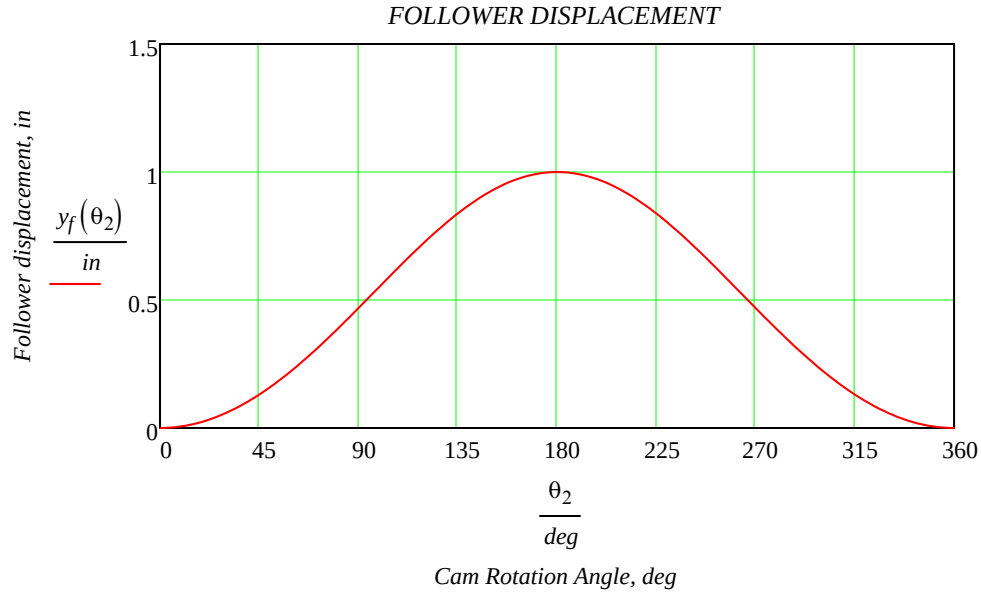
From the relationship among the angles of a triangle,



$$\gamma(\theta_2) := \theta_2 - \beta(\theta_2)$$

And, using the law of cosines,
$$y_f(\theta_2) := \left[e^2 + (r_f + r_c)^2 - 2 \cdot e \cdot (r_f + r_c) \cdot \cos(\gamma(\theta_2)) \right]^{\frac{1}{2}} \dots$$

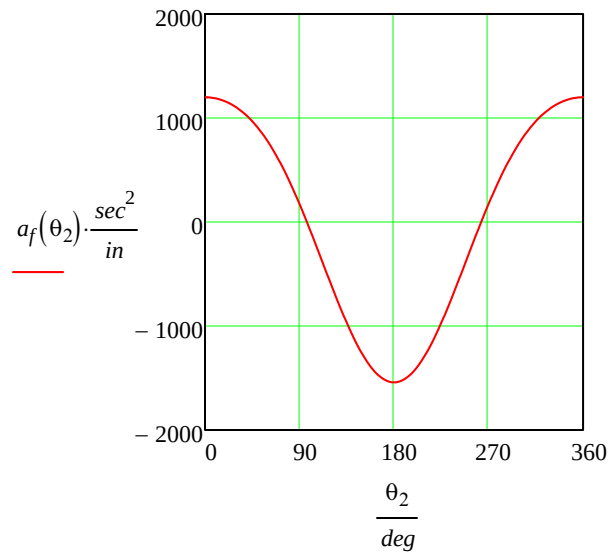
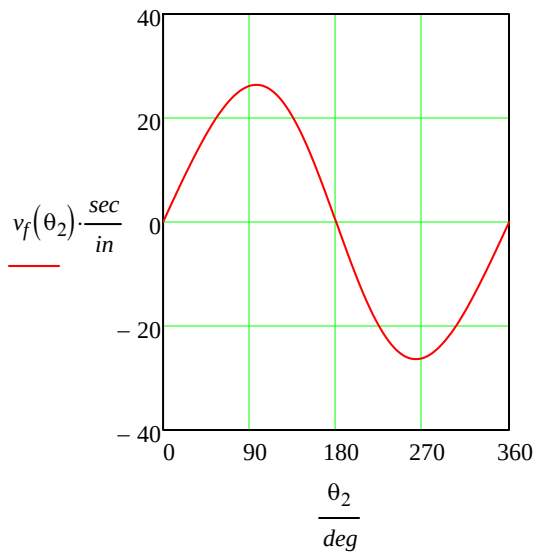
Plotting y_f as a function of θ_2 : $\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$



3. Differentiate the displacement function twice to get the velocity and acceleration functions.

$$v_f(\theta_2) := \left(\frac{d}{d\theta_2} y_f(\theta_2) \right) \cdot \omega_2$$

$$a_f(\theta_2) := \left(\frac{d}{d\theta_2} v_f(\theta_2) \right) \cdot \omega_2$$



4. Determine the angular displacement, velocity, and acceleration of the arm, link 4.

$$\theta_4(\theta_2) := \frac{1}{L_r - L_p} \cdot y_f(\theta_2) \quad \omega_4(\theta_2) := \frac{1}{L_r - L_p} \cdot v_f(\theta_2)$$

$$\alpha_4(\theta_2) := \frac{1}{L_r - L_p} \cdot a_f(\theta_2)$$

5. Determine the displacement and velocity of point A, where the spring attaches to the arm.

$$y_5(\theta_2) := \frac{L_s - L_p}{L_r - L_p} \cdot y_f(\theta_2) \quad v_5(\theta_2) := \frac{L_s - L_p}{L_r - L_p} \cdot v_f(\theta_2)$$

6. Calculate the spring force as a function of θ_2 .

$$F_5(\theta_2) := k \cdot y_5(\theta_2) + F_{50}$$

7. The spring force is the only external force on the linkage that is not applied at a joint and is, therefore, the only force to be considered in the first term of equation 10.28b. Determine the first term in the equation.

$$EF(\theta_2) := F_5(\theta_2) \cdot v_5(\theta_2)$$

8. Determine the components that make up the third term in equation 10.28b. Links 2 and 4 are rotating about fixed pivots so their input can be taken care of in the fourth term. The roller follower has both rotation and translation. The translation motion will be accounted for in this term.

$$\text{Mass of follower:} \quad m_f := \pi \cdot r_f^2 \cdot w_f \cdot \frac{\gamma_s}{g} \quad m_f = 1.414 \text{ lb}$$

$$IF(\theta_2) := m_f \cdot a_f(\theta_2) \cdot v_f(\theta_2)$$

9. Determine the components that make up the fourth term in equation 10.28b. There will be input from the rotation of the arm. The cam and follower have no angular acceleration so they have no contribution. From Problem 10-26, we have the following data:

$$\text{Mass of roller and arm combined:} \quad m_{ra} := \frac{14.289 \cdot \text{lb} \cdot \text{ft}}{g} \quad m_{ra} = 14.289 \text{ lb}$$

$$\text{Distance from left end of arm to composite CG:} \quad X_{Cg} := 14.162 \cdot \text{in}$$

$$\text{Composite moment of inertia with respect to CG:} \quad I_{CG} := 2.433 \cdot \text{lb} \cdot \text{ft} \cdot \text{sec}^2 \cdot \text{in}$$

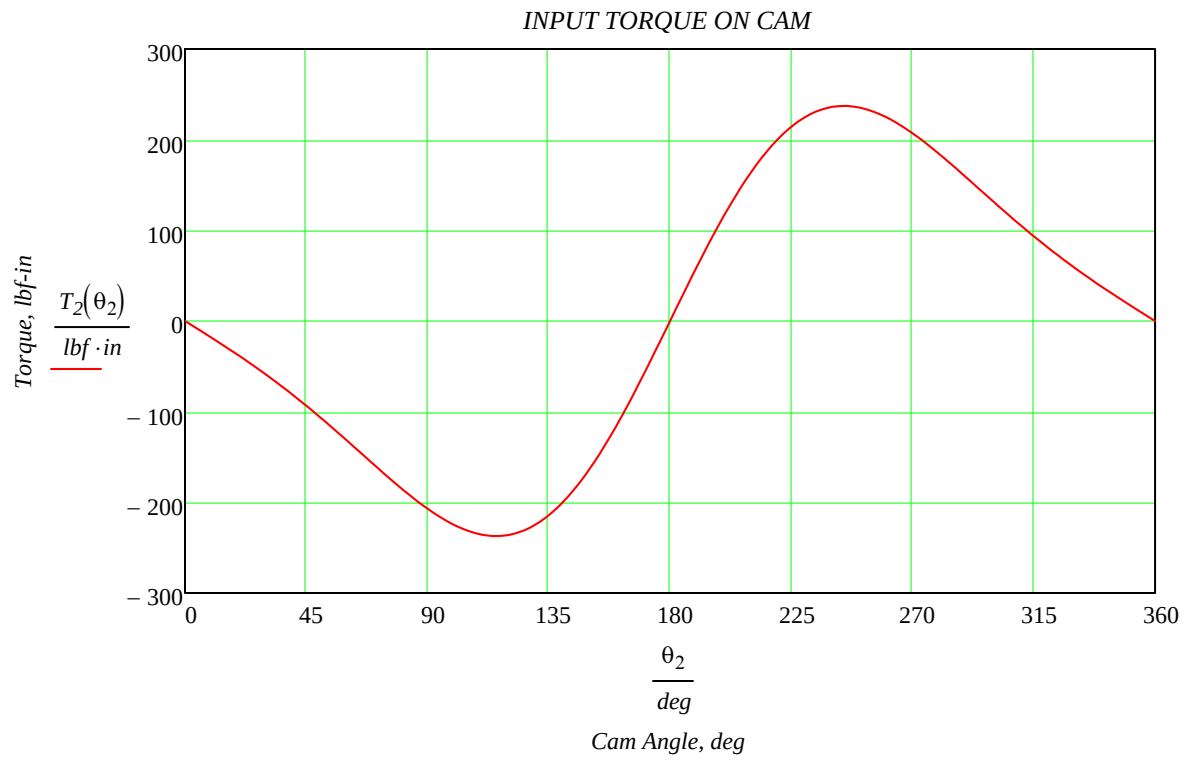
Moment with respect to the arm pivot:

$$I_4 := I_{CG} + m_{ra} \cdot (X_{Cg} - L_p)^2 \quad I_4 = 3.074 \text{ lb} \cdot \text{ft} \cdot \text{sec}^2 \cdot \text{in}$$

$$IT(\theta_2) := I_4 \cdot \alpha_4(\theta_2) \cdot \omega_4(\theta_2)$$

10. Using equation 10.28b, solve for and plot the input torque as a function of cam angle.

$$T_2(\theta_2) := \frac{1}{\omega_2} \cdot (IF(\theta_2) + IT(\theta_2) - EF(\theta_2))$$



PROBLEM 10-18

Statement: Use the method of virtual work to find the torque required to rotate the cam in Figure P10-3 through one revolution. The cam is a pure eccentric.

Given: Cam eccentricity and speed: $e := 20 \cdot \text{mm}$ $\omega_2 := 200 \cdot \text{rpm}$
 Cam radius and thickness: $r_c := 3 \cdot \text{in}$ $w_c := 0.75 \cdot \text{in}$ (measured from Figure)
 Spring rate and preload: $k := 10 \cdot \text{N} \cdot \text{m}^{-1}$ $F_{50} := 0.2 \cdot \text{N}$
 Arm dimensions: Cutout dimensions:
 Width (Z-direction): $a := 2 \cdot \text{in}$ Width (Z-direction): $a' := 1.5 \cdot \text{in}$
 Height (Y-direction): $b := 2.5 \cdot \text{in}$ Height (Y-direction): $b' := 2.5 \cdot \text{in}$
 Length (X-direction): $c := 28 \cdot \text{in}$ Length (X-direction): $c' := 3 \cdot \text{in}$
 Roller follower dimensions: $r_f := 1 \cdot \text{in}$ $w_f := 1.5 \cdot \text{in}$
 Distance from left end of arm to roller center: $L_r := 22 \cdot \text{in}$
 Distance from left end of arm to spring: $L_s := 29 \cdot \text{in}$
 Distance from left end of arm to pivot: $L_p := 10 \cdot \text{in}$
 Specific weight of aluminum: $\gamma_a := 0.1 \cdot \text{lb} \cdot \text{ft} \cdot \text{in}^{-3}$
 Specific weight of steel: $\gamma_s := 0.3 \cdot \text{lb} \cdot \text{ft} \cdot \text{in}^{-3}$

Assumptions: Small angle theory applies.

Solution: See Figure P10-3 and Mathcad file P1018.

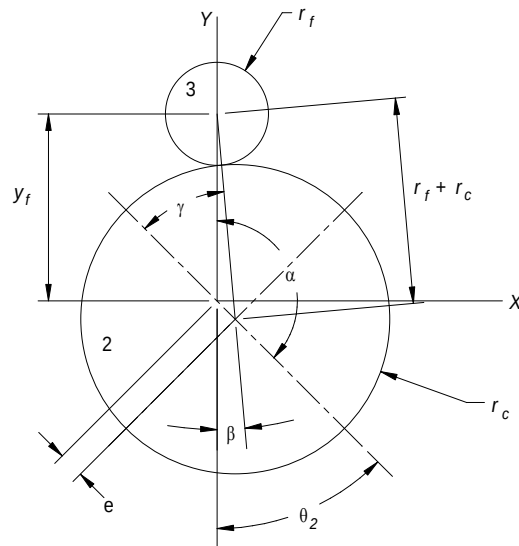
1. Start with a kinematic analysis of the mechanism. Let the cam be link 2, the roller follower link 3, the arm link 4, and the spring link 5. Let the point where the spring connects to the arm be point B, and the center of the roller follower be point A.
2. Calculate the motion of the roller follower center with respect to the X axis, which goes through the center of rotation of the cam.

The cam and follower are shown at right. The origin of the coordinate frame is at the center of rotation. The cam, link 2, is shown rotated an amount θ_2 from the position at which the follower, link 3, is at its lowest position. A triangle is formed with sides e , $r_f + r_c$, and y_f . This triangle will be used to determine the displacement of the follower, y_f , with respect to the cam rotation angle, θ_2 .

Using the law of sines,

$$\beta(\theta_2) := \text{asin}\left(\frac{e}{r_f + r_c} \cdot \sin(\theta_2)\right)$$

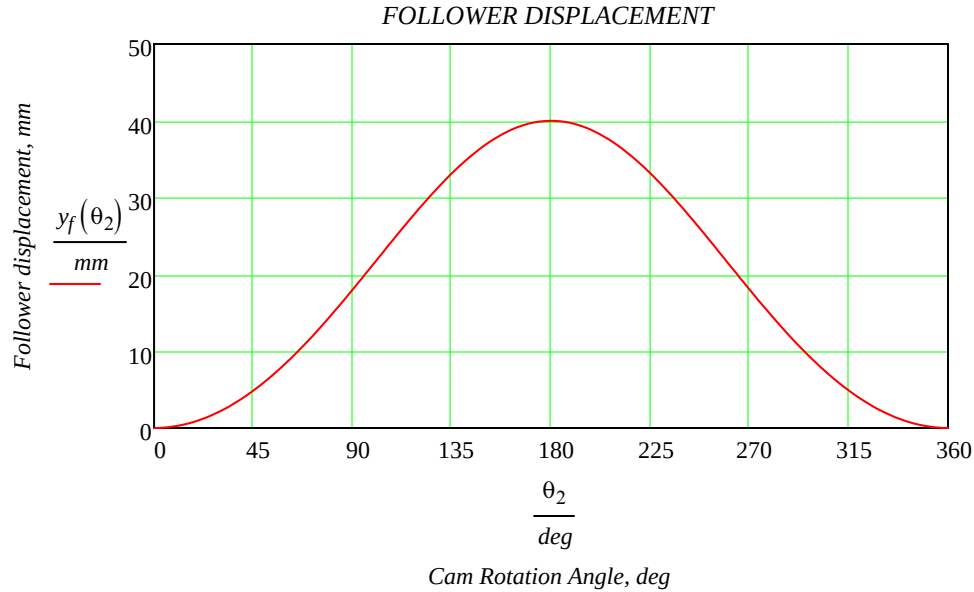
From the relationship among the angles of a triangle,



$$\gamma(\theta_2) := \theta_2 - \beta(\theta_2)$$

And, using the law of cosines,
$$y_f(\theta_2) := \left[e^2 + (r_f + r_c)^2 - 2 \cdot e \cdot (r_f + r_c) \cdot \cos(\gamma(\theta_2)) \right]^{\frac{1}{2}} \dots$$

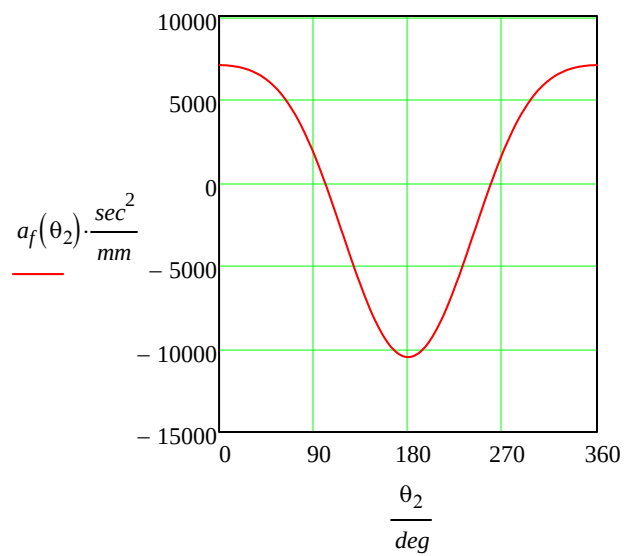
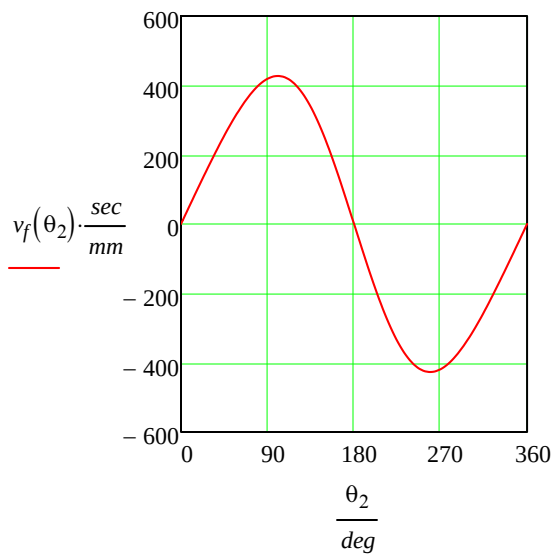
Plotting y_f as a function of θ_2 : $\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$



3. Differentiate the displacement function twice to get the velocity and acceleration functions.

$$v_f(\theta_2) := \left(\frac{d}{d\theta_2} y_f(\theta_2) \right) \cdot \omega_2$$

$$a_f(\theta_2) := \left(\frac{d}{d\theta_2} v_f(\theta_2) \right) \cdot \omega_2$$



4. Determine the angular displacement, velocity, and acceleration of the arm, link 4.

$$\theta_4(\theta_2) := \frac{1}{L_r - L_p} \cdot y_f(\theta_2) \quad \omega_4(\theta_2) := \frac{1}{L_r - L_p} \cdot v_f(\theta_2)$$

$$\alpha_4(\theta_2) := \frac{1}{L_r - L_p} \cdot a_f(\theta_2)$$

5. Determine the displacement and velocity of point A, where the spring attaches to the arm.

$$y_5(\theta_2) := \frac{L_s - L_p}{L_r - L_p} \cdot y_f(\theta_2) \quad v_5(\theta_2) := \frac{L_s - L_p}{L_r - L_p} \cdot v_f(\theta_2)$$

6. Calculate the spring force as a function of θ_2 .

$$F_5(\theta_2) := k \cdot y_5(\theta_2) + F_{50}$$

7. The spring force is the only external force on the linkage that is not applied at a joint and is, therefore, the only force to be considered in the first term of equation 10.28b. Determine the first term in the equation.

$$EF(\theta_2) := F_5(\theta_2) \cdot v_5(\theta_2)$$

8. Determine the components that make up the third term in equation 10.28b. Links 2 and 4 are rotating about fixed pivots so their input can be taken care of in the fourth term. The roller follower has both rotation and translation. The translation motion will be accounted for in this term.

$$\text{Mass of follower:} \quad m_f := 1 \cdot \text{kg}$$

$$IF(\theta_2) := m_f \cdot a_f(\theta_2) \cdot v_f(\theta_2)$$

9. Determine the components that make up the fourth term in equation 10.28b. There will be input from the rotation of the arm. The cam and follower have no angular acceleration so they have no contribution. From Problem 10-26, we have the following data:

$$\text{Mass of roller and arm combined:} \quad m_{ra} := \frac{14.289 \cdot \text{lbf}}{g} \quad m_{ra} = 6.481 \text{ kg}$$

$$\text{Distance from left end of arm to composite CG:} \quad X_{Cg} := 14.162 \cdot \text{in}$$

$$\text{Composite moment of inertia with respect to CG:} \quad I_{CG} := 2.433 \cdot \text{lbf} \cdot \text{sec}^2 \cdot \text{in}$$

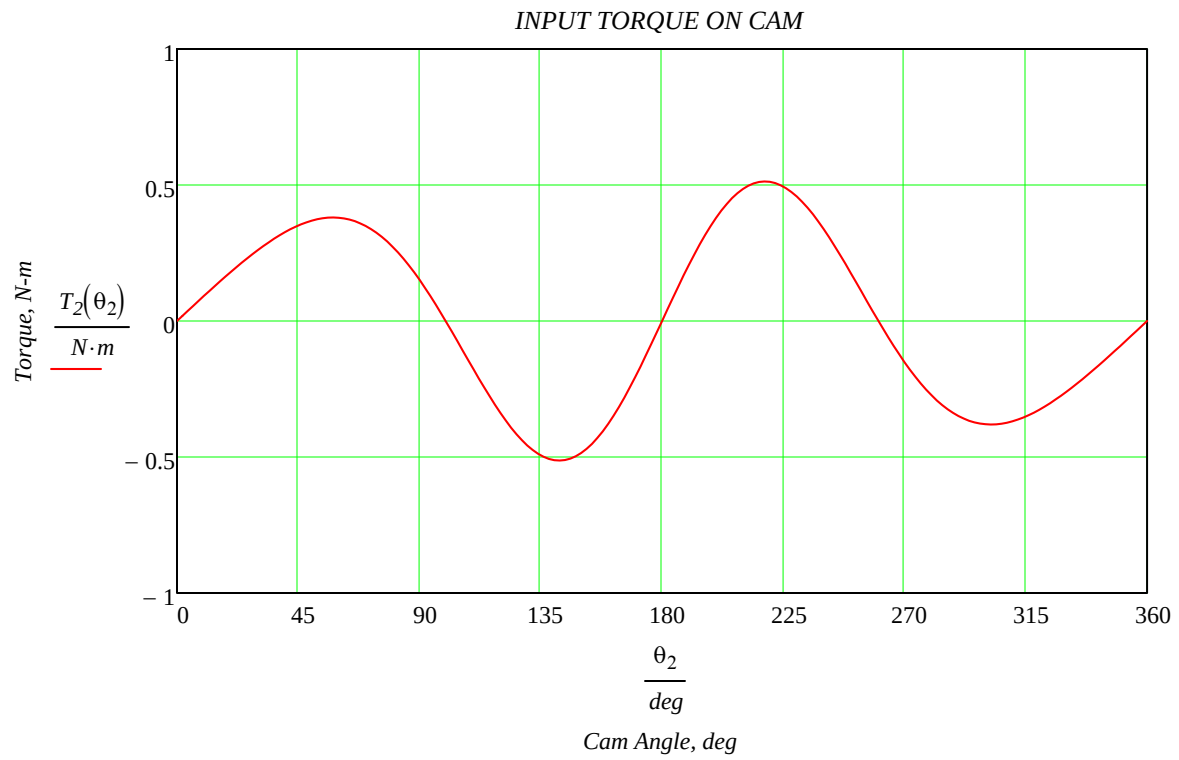
Moment with respect to the arm pivot:

$$I_4 := I_{CG} + m_{ra} \cdot (X_{Cg} - L_p)^2 \quad I_4 = 0.347 \text{ kg} \cdot \text{m}^2$$

$$IT(\theta_2) := I_4 \cdot \alpha_4(\theta_2) \cdot \omega_4(\theta_2)$$

10. Using equation 10.28b, solve for and plot the input torque as a function of cam angle.

$$T_2(\theta_2) := \frac{1}{\omega_2} \cdot (IF(\theta_2) + IT(\theta_2) - EF(\theta_2))$$



PROBLEM 10-19

Statement: Use the method of virtual work to find the torque required to rotate the cam in Figure P10-3 through one revolution. The cam motion is a double harmonic.

Given: Cam lift and speed: $h := 20 \cdot \text{mm}$ $\omega_2 := 200 \cdot \text{rpm}$
 Cam radius and thickness: $r_c := 3 \cdot \text{in}$ $w_c := 0.75 \cdot \text{in}$ (measured from Figure)
 Spring rate and preload: $k := 10 \cdot \text{N} \cdot \text{m}^{-1}$ $F_{50} := 0.2 \cdot \text{N}$
 Arm dimensions: Cutout dimensions:
 Width (Z-direction): $a := 2 \cdot \text{in}$ Width (Z-direction): $a' := 1.5 \cdot \text{in}$
 Height (Y-direction): $b := 2.5 \cdot \text{in}$ Height (Y-direction): $b' := 2.5 \cdot \text{in}$
 Length (X-direction): $c := 28 \cdot \text{in}$ Length (X-direction): $c' := 3 \cdot \text{in}$
 Roller follower dimensions: $r_f := 1 \cdot \text{in}$ $w_f := 1.5 \cdot \text{in}$
 Distance from left end of arm to roller center: $L_r := 22 \cdot \text{in}$
 Distance from left end of arm to spring: $L_s := 29 \cdot \text{in}$
 Distance from left end of arm to pivot: $L_p := 10 \cdot \text{in}$
 Specific weight of aluminum: $\gamma_a := 0.1 \cdot \text{lb} \cdot \text{ft}^{-3}$
 Specific weight of steel: $\gamma_s := 0.3 \cdot \text{lb} \cdot \text{ft}^{-3}$

Solution: See Figure P10-3 and Mathcad file P1019.

1. Start with a kinematic analysis of the mechanism. Let the cam be link 2, the roller follower link 3, the arm link 4, and the spring link 5. Let the point where the spring connects to the arm be point B, and the center of the roller follower be point A.
2. Calculate the motion of the roller follower using equations 8.25. Define β and a range function so the rise and fall can be combined in a single equation.

$$\beta := 180 \cdot \text{deg} \quad R(\theta, a, b) := \text{if}[(\theta > a) \cdot (\theta \leq b), 1, 0]$$

$$y_f(\theta_2) := R(\theta_2, 0, \beta) \cdot \frac{h}{2} \cdot \left[\left(1 - \cos\left(\pi \cdot \frac{\theta_2}{\beta}\right) \right) - \frac{1}{4} \cdot \left(1 - \cos\left(2 \cdot \pi \cdot \frac{\theta_2}{\beta}\right) \right) \right] \dots$$

$$+ R(\theta_2, \beta, 2 \cdot \pi) \cdot \frac{h}{2} \cdot \left[\left(1 + \cos\left(\pi \cdot \frac{\theta_2 - \beta}{\beta}\right) \right) - \frac{1}{4} \cdot \left(1 - \cos\left(2 \cdot \pi \cdot \frac{\theta_2 - \beta}{\beta}\right) \right) \right]$$

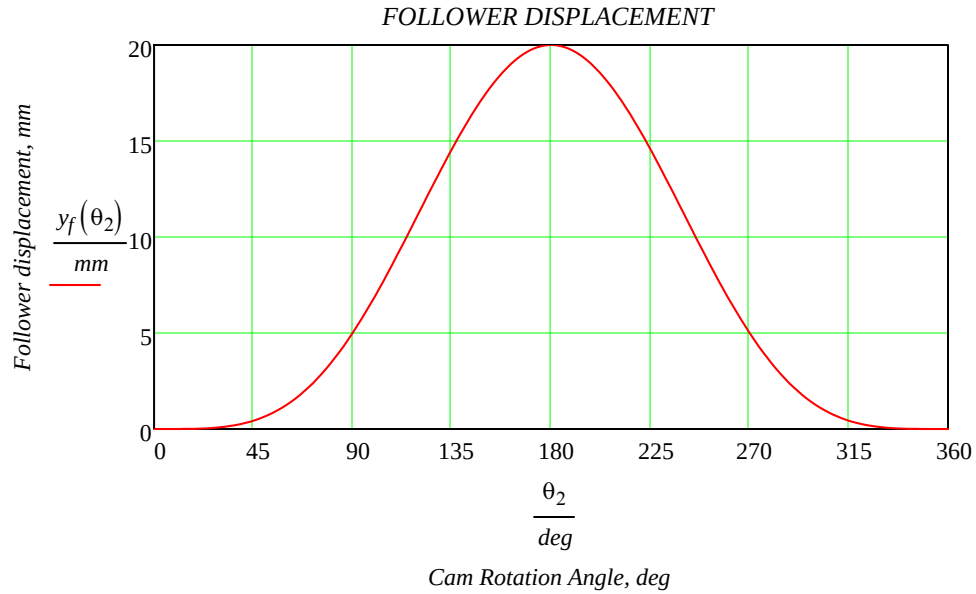
$$v_f(\theta_2) := \left[R(\theta_2, 0, \beta) \cdot \frac{\pi}{\beta} \cdot \frac{h}{2} \cdot \left(\sin\left(\pi \cdot \frac{\theta_2}{\beta}\right) - \frac{1}{2} \cdot \sin\left(2 \cdot \pi \cdot \frac{\theta_2}{\beta}\right) \right) \dots \right] \cdot \omega_2$$

$$+ R(\theta_2, \beta, 2 \cdot \pi) \cdot \frac{\pi}{\beta} \cdot \frac{h}{2} \cdot \left(\sin\left(\pi \cdot \frac{\theta_2 - \beta}{\beta}\right) + \frac{1}{2} \cdot \sin\left(2 \cdot \pi \cdot \frac{\theta_2 - \beta}{\beta}\right) \right)$$

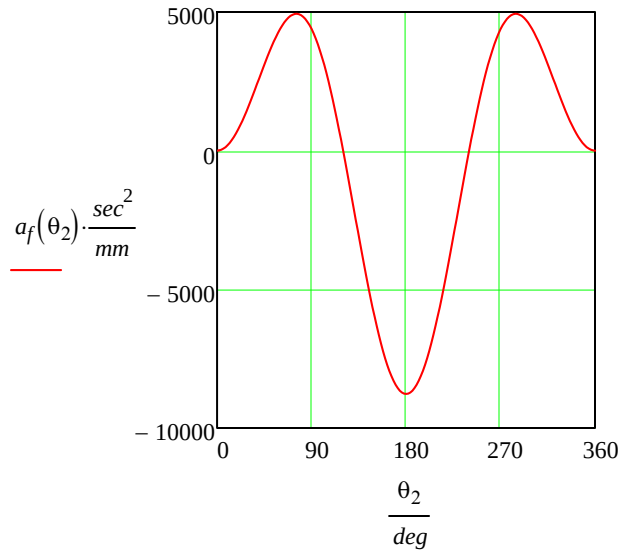
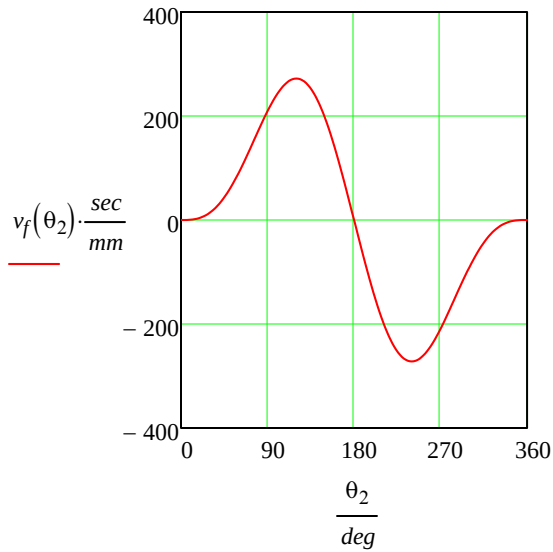
$$a_f(\theta_2) := \left[R(\theta_2, 0, \beta) \cdot \frac{\pi^2}{\beta^2} \cdot \frac{h}{2} \cdot \left(\cos\left(\pi \cdot \frac{\theta_2}{\beta}\right) - \cos\left(2 \cdot \pi \cdot \frac{\theta_2}{\beta}\right) \right) \dots \right] \cdot \omega_2^2$$

$$+ R(\theta_2, \beta, 2 \cdot \pi) \cdot \frac{\pi^2}{\beta^2} \cdot \frac{h}{2} \cdot \left(\cos\left(\pi \cdot \frac{\theta_2 - \beta}{\beta}\right) + \cos\left(2 \cdot \pi \cdot \frac{\theta_2 - \beta}{\beta}\right) \right)$$

Plotting y_f as a function of θ_2 : $\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$



3. Plot the velocity and acceleration functions.



4. Determine the angular displacement, velocity, and acceleration of the arm, link 4.

$$\theta_4(\theta_2) := \frac{1}{L_r - L_p} \cdot y_f(\theta_2) \qquad \omega_4(\theta_2) := \frac{1}{L_r - L_p} \cdot v_f(\theta_2)$$

$$\alpha_4(\theta_2) := \frac{1}{L_r - L_p} \cdot a_f(\theta_2)$$

5. Determine the displacement and velocity of point A, where the spring attaches to the arm.

$$y_5(\theta_2) := \frac{L_s - L_p}{L_r - L_p} \cdot y_f(\theta_2) \qquad v_5(\theta_2) := \frac{L_s - L_p}{L_r - L_p} \cdot v_f(\theta_2)$$

6. Calculate the spring force as a function of θ_2 .

$$F_5(\theta_2) := k \cdot y_5(\theta_2) + F_{50}$$

7. The spring force is the only external force on the linkage that is not applied at a joint and is, therefore, the only force to be considered in the first term of equation 10.28b. Determine the first term in the equation.

$$EF(\theta_2) := F_5(\theta_2) \cdot v_5(\theta_2)$$

8. Determine the components that make up the third term in equation 10.28b. Links 2 and 4 are rotating about fixed pivots so their input can be taken care of in the fourth term. The roller follower has both rotation and translation. The translation motion will be accounted for in this term.

$$\text{Mass of follower: } m_f := 1 \cdot \text{kg}$$

$$IF(\theta_2) := m_f \cdot a_f(\theta_2) \cdot v_f(\theta_2)$$

9. Determine the components that make up the fourth term in equation 10.28b. There will be input from the rotation of the arm. The cam and follower have no angular acceleration so they have no contribution. From Problem 10-26, we have the following data:

$$\text{Mass of roller and arm combined: } m_{ra} := \frac{14.289 \cdot \text{lb} \cdot \text{f}}{g} \quad m_{ra} = 6.481 \text{ kg}$$

$$\text{Distance from left end of arm to composite CG: } X_{Cg} := 14.162 \cdot \text{in}$$

$$\text{Composite moment of inertia with respect to CG: } I_{CG} := 2.433 \cdot \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}$$

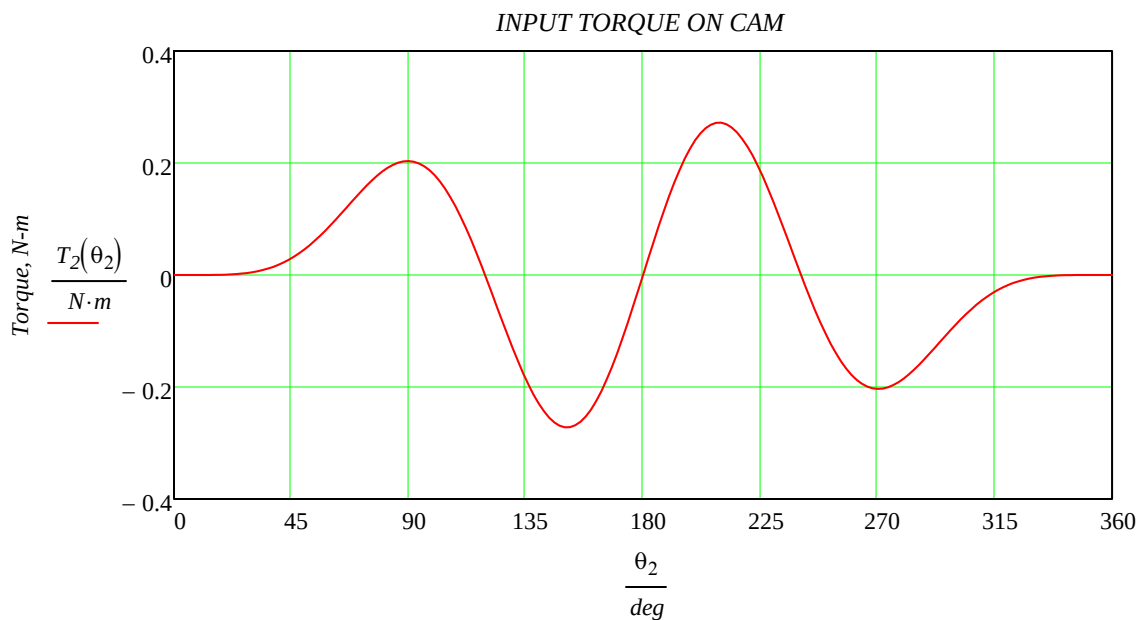
Moment with respect to the arm pivot:

$$I_4 := I_{CG} + m_{ra} \cdot (X_{Cg} - L_p)^2 \quad I_4 = 0.347 \text{ kg} \cdot \text{m}^2$$

$$IT(\theta_2) := I_4 \cdot \alpha_4(\theta_2) \cdot \omega_4(\theta_2)$$

10. Using equation 10.28b, solve for and plot the input torque as a function of cam angle.

$$T_2(\theta_2) := \frac{1}{\omega_2} \cdot (IF(\theta_2) + IT(\theta_2) - EF(\theta_2))$$



PROBLEM 10-20

Statement: A 3000-lb automobile has a final drive ratio of 1:3 and transmission gear ratios of 1:4, 1:3, 1:2, and 1:1 in first through fourth speeds, respectively. What is the effective mass of the vehicle as felt by the engine flywheel in each gear?

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Gear ratios:

Final $m_{Gf} := 3$ First $m_{G1} := 4$ Second $m_{G2} := 3$

Third $m_{G3} := 2$ Fourth $m_{G4} := 1$

Weight of automobile: $W := 3000 \cdot lbf$

Solution: See Mathcad file P1020.

1. Calculate the mass of the vehicle.

$$M := \frac{W}{g} \qquad M = 7.770 \text{ blob}$$

2. The effective mass of the vehicle at the flywheel while in first gear is,

$$m_{eff1} := (m_{Gf} \cdot m_{G1})^{-2} \cdot M \qquad m_{eff1} = 0.0540 \text{ blob}$$

3. The effective mass of the vehicle at the flywheel while in second gear is,

$$m_{eff2} := (m_{Gf} \cdot m_{G2})^{-2} \cdot M \qquad m_{eff2} = 0.0959 \text{ blob}$$

4. The effective mass of the vehicle at the flywheel while in third gear is,

$$m_{eff3} := (m_{Gf} \cdot m_{G3})^{-2} \cdot M \qquad m_{eff3} = 0.216 \text{ blob}$$

5. The effective mass of the vehicle at the flywheel while in fourth gear is,

$$m_{eff4} := (m_{Gf} \cdot m_{G4})^{-2} \cdot M \qquad m_{eff4} = 0.863 \text{ blob}$$

PROBLEM 10-21

Statement: Determine the effective spring constant and effective preload of the spring in Figure P10-2 as reflected back to the cam follower. See Problem 10-17 for additional data.

Given: Spring data: $k := 123 \cdot \text{lb} \cdot \text{in}^{-1}$ $F_o := 173 \cdot \text{lb} \cdot \text{f}$
Distance from arm pivot to spring: $a := 19 \cdot \text{in}$
Distance from arm pivot to follower: $b := 12 \cdot \text{in}$

Solution: See Figure P10-2, Problem 10-17, and Mathcad file P1021.

1. The effective spring constant at the follower is,

$$k_{eff} := \left(\frac{a}{b} \right)^2 \cdot k \qquad k_{eff} = 308.354 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

2. The effective preload at the follower is,

$$F_{oeff} := \frac{a}{b} \cdot F_o \qquad F_{oeff} = 273.917 \text{ lb} \cdot \text{f}$$

PROBLEM 10-22

Statement: What is the effective inertia of a load applied at the drum of Figure P9-5 as reflected back to gear A?

Given: From Problem 9-35:

$$\text{Ratio of drum speed to speed of gear A} \quad ratio := \frac{5.4}{5.4 - 1} \quad ratio = 1.227$$

Solution: See Figure P9-5, Problem 9-35, and Mathcad file P1022.

1. The energy of the effective inertia at gear A must be the same as that at the drum,

$$\frac{1}{2} \cdot I_{drum} \cdot \omega_{drum}^2 := \frac{1}{2} \cdot I_{eff} \cdot \omega_A^2$$

$$\frac{I_{eff}}{I_{drum}} = \left(\frac{\omega_{drum}}{\omega_A} \right)^2 = ratio^2$$

Thus, the ratio of the effective inertia to the drum inertia will be the square the ratio of drum speed to the speed of gear A.

$$Ratio_{inertia} := ratio^2 \quad Ratio_{inertia} = 1.506$$

PROBLEM 10-23

Statement: What is the effective inertia of a load applied at the drum of Figure P9-7 as reflected back to the arm?

Given: From Problem 9-40:

Ratio of drum speed to speed of the arm $ratio := 16.5$

Solution: See Figure P9-7, Problem 9-40, and Mathcad file P1023.

1. The energy of the effective inertia at the arm must be the same as that at the drum,

$$\frac{1}{2} \cdot I_{drum} \cdot \omega_{drum}^2 := \frac{1}{2} \cdot I_{eff} \cdot \omega_{arm}^2$$

$$\frac{I_{eff}}{I_{drum}} = \left(\frac{\omega_{drum}}{\omega_{arm}} \right)^2 = ratio^2$$

Thus, the ratio of the effective inertia to the drum inertia will be the square the ratio of drum speed to the speed of the arm.

$$Ratio_{inertia} := ratio^2 \qquad Ratio_{inertia} = 272.250$$

PROBLEM 10-24

Statement: Refer to Figure 10-8a (p. 507). For the data given below, find the equivalent mass at point A and the equivalent spring at point B.

Given: $a := 100 \cdot mm$ $b := 150 \cdot mm$ $k_A := 2000 \cdot N \cdot m^{-1}$ $m_B := 2 \cdot kg$

Solution: See Figure 10-8a and Mathcad file P1024.

1. Use equation 10.22b to calculate the equivalent mass at point A.

$$m_{eff} := \left(\frac{b}{a} \right)^2 \cdot m_B \qquad m_{eff} = 4.5 \, kg$$

2. Use equation 10.23b to calculate the equivalent spring constant at point B.

$$k_{eff} := \left(\frac{a}{b} \right)^2 \cdot k_A \qquad k_{eff} = 888.9 \, \frac{N}{m}$$

PROBLEM 10-25

Statement: Refer to Figure 10-8a (p. 507). For the data given below, find the equivalent mass at point A and the equivalent spring at point B.

Given: $a := 50 \cdot mm$ $b := 150 \cdot mm$ $k_A := 1000 \cdot N \cdot m^{-1}$ $m_B := 3 \cdot kg$

Solution: See Figure 10-8a and Mathcad file P1025.

1. Use equation 10.22b to calculate the equivalent mass at point A.

$$m_{eff} := \left(\frac{b}{a}\right)^2 \cdot m_B \qquad m_{eff} = 27 \text{ kg}$$

2. Use equation 10.23b to calculate the equivalent spring constant at point B.

$$k_{eff} := \left(\frac{a}{b}\right)^2 \cdot k_A \qquad k_{eff} = 111.1 \frac{N}{m}$$

PROBLEM 10-26

Statement: For the cam-follower arm in Figure P10-2, determine the location of its fixed pivot that will have zero reaction force when the cam applies its force to the follower.

Given: Arm dimensions:

Cutout dimensions:

Width (Z-direction): $a := 2 \cdot in$

Width (Z-direction): $a' := 1.5 \cdot in$

Height (Y-direction): $b := 2.5 \cdot in$

Height (Y-direction): $b' := 2.5 \cdot in$

Length (X-direction): $c := 28 \cdot in$

Length (X-direction): $c' := 3 \cdot in$

Roller dimensions: $r := 1 \cdot \text{in}$ $w := 1.5 \cdot \text{in}$

Distance from left end of arm to roller center: $L_r := 22 \cdot in$

Specific weight of aluminum: $\gamma_a := 0.1 \cdot \text{lb} \cdot \text{in}^{-3}$

Specific weight of steel: $\gamma_s := 0.3 \cdot \text{lb} \cdot \text{in}^{-3}$

Assumptions: The center of the steel roller is located on the horizontal centerline (X axis) of the arm.

Solution: See Figure P10-2 and Mathcad file P1026.

1. Determine the volume and weight of the roller and the arm.

Roller: $V_r := \pi \cdot r^2 \cdot w$ $V_r = 4.712 \text{ in}^3$

$$W_r := \gamma_s \cdot V_r \qquad W_r = 1.414 \text{ lbf}$$

Arm: $V_{solid} := a \cdot b \cdot c$ $V_{solid} = 140.000 \text{ in}^3$

$$V_{slot} := a' \cdot b' \cdot c' \qquad V_{slot} = 11.250 \text{ in}^3$$

$$V_g := V_{solid} - V_{slot} \quad V_g = 128.750 \text{ in}^3$$

$$W_{solid} := \gamma_a \cdot V_{solid} \qquad W_{solid} = 14.000 \text{ lbf}$$

$$W_{slot} := -\gamma_a \cdot V_{slot} \qquad W_{slot} = -1.125 \text{ lbf}$$

$$W_g := W_{solid} + W_{slot} \quad W_g = 12.875 \text{ lbf}$$

2. Calculate the location of the arm CG with respect to the left end.

$$Xcga := \frac{W_{solid} \cdot \frac{c}{2} + W_{slot} \cdot L_r}{W_g} \quad Xcga = 13.301 \text{ in}$$

3. Calculate the location of the composite CG with respect to the left end.

$$X_{Cg} := \frac{W_r \cdot L_r + W_a \cdot X_{cg_a}}{W_r + W_a} \quad X_{Cg} = 14.162 \text{ in}$$

4. Determine the moment of inertia of the arm about the composite CG

$$\text{Solid portion about its own CG} \quad I_{z_{solid}} := \frac{W_{solid}}{12 \cdot q} \cdot (b^2 + c^2) \quad I_{z_{solid}} = 2.388 \text{ } lb \cdot sec^2 \cdot in$$

$$\text{Slot portion about its own CG} \quad I_{zslot} := \frac{W_{slot}}{12 \cdot g} \cdot (b^2 + c^2) \quad I_{zslot} = -0.0037 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

$$\text{Solid portion about composite CG} \quad I_{Zsolid} := I_{solid} + \frac{W_{solid}}{g} \cdot \left(X_{Cg} - \frac{c}{2} \right)^2$$

$$I_{Zsolid} = 2.389 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

$$\text{Slot portion about composite CG} \quad I_{Zslot} := I_{zslot} + \frac{W_{slot}}{g} \cdot (L_r - X_{Cg})^2$$

$$I_{Zslot} = -0.183 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

$$\text{Arm about composite CG} \quad I_{ZZa} := I_{Zsolid} + I_{Zslot}$$

$$I_{ZZa} = 2.206 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

5. Determine the moment of inertia of the roller about the composite CG

$$\text{Roller about its own CG} \quad I_{Zr} := \frac{W_r}{2 \cdot g} \cdot r^2 \quad I_{Zr} = 1.831 \times 10^{-3} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

$$\text{Roller about composite CG} \quad I_{ZZr} := I_{Zr} + \frac{W_r}{g} \cdot (L_r - X_{Cg})^2$$

$$I_{ZZr} = 0.227 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

6. Determine the moment of inertia of the assembly about the composite CG

$$I_{ZZ} := I_{ZZa} + I_{ZZr} \quad I_{ZZ} = 2.433 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

7. Calculate the radius of gyration using equation 10.11b.

$$k := \sqrt{\frac{I_{ZZ} \cdot g}{W_a + W_r}} \quad k = 8.108 \text{ in}$$

8. Use equation 10.15b to calculate the location of the pivot point with respect to the left end.

$$X_{pivot} := X_{Cg} - \frac{k^2}{L_r - X_{Cg}} \quad X_{pivot} = 5.775 \text{ in}$$

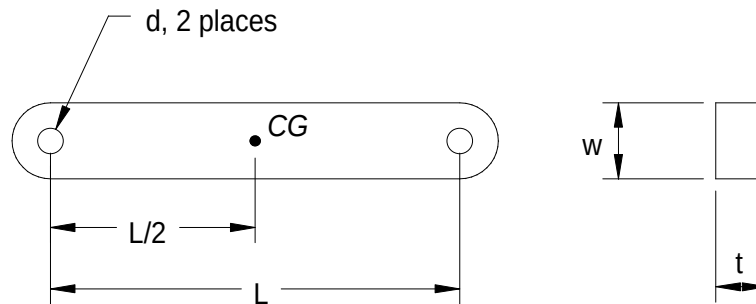
PROBLEM 10-27

Statement: Figure P10-4 shows a fourbar mechanism. The crank is 1.00in wide by 0.5 in thick. The coupler and rocker are both 0.75 in wide by 0.5 in thick. All links are made from steel. The ends of the links have a full radius equal to one half of the link width. The pivot pins all have a diameter of 0.25 in. Find the moment of inertia of the crank and rocker about their fixed pivots and the moment of inertia of the coupler about its CG.

Given: Dimensions in the figure below. Specific weight of steel: $\gamma := 0.28 \cdot \text{lb} \cdot \text{in}^{-3}$
 Pivot pin diameter: $d := 0.25 \cdot \text{in}$

Solution: See Figure P10-4 and Mathcad file P1027.

1. Divide each link into 3 volumes: 1) the main, rectangular body; 2) the half-cylindrical ends; and 3) the pivot pin holes.
2. All links have the same basic shape, which is shown below.



2. Volume 1 is the main rectangular body of length L, width w, and thickness t.

$$\text{Mass: } m_1(L, w, t) := \frac{L \cdot w \cdot t \cdot \gamma}{g}$$

$$\text{Moment of inertia about CG: } I_1(L, w, t) := \frac{m_1(L, w, t)}{12} \cdot (L^2 + w^2)$$

3. Volume 2 is the half-cylinder of diameter w, and length t. There are two of these.

$$\text{Mass: } m_2(w, t) := \frac{\pi \cdot w^2 \cdot t \cdot \gamma}{8 \cdot g}$$

$$\text{Moment of inertia about CG: } I_2(L, w, t) := 2 \cdot \left[\frac{m_2(w, t) \cdot w^2}{8} + m_2(w, t) \cdot \left(\frac{L}{2} \right)^2 \right]$$

4. Volume 3 is the cylinder of diameter d, and length t. There are two of these and they are negative.

$$\text{Mass: } m_3(t) := -\frac{\pi \cdot d^2 \cdot t \cdot \gamma}{8 \cdot g}$$

$$\text{Moment of inertia about CG: } I_3(L, t) := 2 \cdot \left[\frac{m_3(t) \cdot d^2}{8} + m_3(t) \cdot \left(\frac{L}{2} \right)^2 \right]$$

5. For the crank: $L := 2.00 \cdot \text{in}$, $w := 1.00 \cdot \text{in}$, $t := 0.50 \cdot \text{in}$

$$\text{Mass: } m_{\text{crank}} := m_1(L, w, t) + m_2(w, t) + m_3(t) \quad m_{\text{crank}} = 8.587 \times 10^{-4} \text{ lb} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Moment of inertia about CG:

$$I_{crankCG} := I_1(L, w, t) + I_2(L, w, t) + I_3(L, t) \quad I_{crankCG} = 6.046 \times 10^{-4} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

Moment of inertia about pivot axis:

$$I_{crank} := I_{crankCG} + m_{crank} \cdot \left(\frac{L}{2} \right)^2 \quad I_{crank} = 1.463 \times 10^{-3} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

6. For the rocker: $L := 7.187 \cdot \text{in}$, $w := 0.75 \cdot \text{in}$, $t := 0.50 \cdot \text{in}$

$$\text{Mass: } m_{rocker} := m_1(L, w, t) + m_2(w, t) + m_3(t) \quad m_{rocker} = 2.026 \times 10^{-3} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Moment of inertia about CG:

$$I_{rockerCG} := I_1(L, w, t) + I_2(L, w, t) + I_3(L, t) \quad I_{rockerCG} = 0.010 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

Moment of inertia about pivot axis:

$$I_{rocker} := I_{rockerCG} + m_{rocker} \cdot \left(\frac{L}{2} \right)^2 \quad I_{rocker} = 0.037 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

7. For the coupler $L := 8.375 \cdot \text{in}$, $w := 0.75 \cdot \text{in}$, $t := 0.50 \cdot \text{in}$

$$\text{Mass: } m_{coupler} := m_1(L, w, t) + m_2(w, t) + m_3(t) \quad m_{coupler} = 2.349 \times 10^{-3} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Moment of inertia about CG:

$$I_{couplerCG} := I_1(L, w, t) + I_2(L, w, t) + I_3(L, t) \quad I_{couplerCG} = 0.016 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

PROBLEM 10-28

Statement: The rocker in Figure 10-11a has the dimensions given below. When supported on knife-edges at A and B, the weights at the supports were found to be 4.3 N and 5.8 N, respectively. The rocker was supported at its pivot point with a low-friction ball bearing and the period of oscillation was found. What is the approximate moment of inertia of the rocker with respect to its pivot axis?

Given: Dimensions: $a := 50.8 \cdot \text{mm}$ $b := 76.2 \cdot \text{mm}$
 Total weight: $W := 10.1 \cdot \text{N}$ $W_A := 4.3 \cdot \text{N}$ $W_B := 5.8 \cdot \text{N}$
 Period of oscillation: $\tau := 0.75 \cdot \text{sec}$

Solution: See Figure 10-11a and Mathcad file P1028.

1. Determine the location of the CG by summing moments on the rocker about point A. The distance, r, from the pivot axis to the CG is:

$$W \cdot (a + r) - W_B \cdot (a + b) = 0$$

$$r := \frac{W_B \cdot (a + b) - W \cdot a}{W} \quad r = 22.131 \text{ mm}$$

2. Use equation 10.10g to calculate the approximate moment of inertia with respect to the pivot axis.

$$I_{\text{pivot}} := W \cdot r \cdot \left(\frac{\tau}{2 \cdot \pi} \right)^2 \quad I_{\text{pivot}} = 3185 \text{ kg} \cdot \text{mm}^2$$

PROBLEM 10-29

Statement: The arm in Problem 10-16 and Figure P10-2 has been redesigned such that the cross-section is no longer uniform and the material changed from aluminum to steel. However, the dimensions shown in the figure remain unchanged. The new arm has a total weight of 15.3 lb and, when supported on knife-edges at points 9.5 in to the left of the pivot and 17.5 in to the right of the pivot, the weights at the supports were found to be 7.1 lb and 8.2 lb, respectively. The arm was supported at its pivot point with a low-friction ball bearing and the period of oscillation was found. What is the approximate moment of inertia of the rocker with respect to its pivot axis?

Given: Let the weight measurement point on the left of the pivot be A and the distance from the pivot axis to point A be a . Similarly, let the point on the right be B and its distance b .

Dimensions: $a := 9.5 \cdot \text{in}$ $b := 17.5 \cdot \text{in}$

Total weight: $W := 15.3 \cdot \text{lbf}$ $W_A := 7.1 \cdot \text{lbf}$ $W_B := 8.2 \cdot \text{lbf}$

Period of oscillation: $\tau := 2.0 \cdot \text{sec}$

Solution: See Figure P10-2 and Mathcad file P1029.

1. Determine the location of the CG by summing moments on the rocker about point A . The distance, r , from the pivot axis to the CG is:

$$W \cdot (a + r) - W_B \cdot (a + b) = 0$$

$$r := \frac{W_B \cdot (a + b) - W \cdot a}{W} \quad r = 4.971 \text{ in}$$

2. Use equation 10.10g to calculate the approximate moment of inertia with respect to the pivot axis.

$$I_{\text{pivot}} := W \cdot r \cdot \left(\frac{\tau}{2 \cdot \pi} \right)^2 \quad I_{\text{pivot}} = 7.7 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}$$

PROBLEM 10-30

Statement: Figure P10-5 shows a cam-follower system that drives slider 6 through an external output arm 3. Arms 2 and 3 are both rigidly attached to the 0.75-in-dia shaft X-X, which rotates in bearings that are supported by the housing. The pin-to-pin dimensions of the links are shown. The cross-section of arms 2, 3, and 5 are solid, rectangular 1.5 x 0.75 in steel. The ends of these links have a full radius equal to one half the link width. Link 4 is 1-in-dia x 0.125 in-wall, round steel tubing. Link 6 is a 2-in-dia x 6-in long solid steel cylinder. Find the effective mass and effective spring constant of the follower train referenced to the cam-follower roller.

Given: Link 6 (solid cylinder): $d_6 := 2.00 \cdot \text{in}$ $L_6 := 6.00 \cdot \text{in}$

Rocker arm 5: $w := 0.75 \cdot \text{in}$ $h := 1.50 \cdot \text{in}$ $a := 10.0 \cdot \text{in}$ $b := 8.0 \cdot \text{in}$

Pushrod 4 (hollow cylinder): $d_{4od} := 1.00 \cdot \text{in}$ $d_{4id} := 0.75 \cdot \text{in}$ $L_4 := 22 \cdot \text{in}$

Output arm 3: $w := 0.75 \cdot \text{in}$ $h := 1.50 \cdot \text{in}$ $L_3 := 16 \cdot \text{in}$

Spring: $k_s := 150 \cdot \text{lbf} \cdot \text{in}^{-1}$

Roller arm 2: $w := 0.75 \cdot \text{in}$ $h := 1.50 \cdot \text{in}$ $L_2 := 8 \cdot \text{in}$

All parts are steel: Modulus of elasticity $E := 30 \cdot 10^6 \cdot \text{psi}$ Spec. weight $\gamma := 0.28 \cdot \frac{\text{lbf}}{\text{in}^3}$

Solution: See Figures P10-5 and 10-9, and Mathcad file P1030.

1. Break the system into individual elements as shown in Figure 10-9b.
2. Define the individual spring constants of each of the six elements.

Roller arm (cantilever beam with end load):

$$\text{Moment of inertia} \quad I_2 := \frac{w \cdot h^3}{12}$$

$$\text{Spring constant} \quad k_2 := \frac{3 \cdot E \cdot I_2}{L_2^3} \quad k_2 = 3.708 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

Output arm:

$$\text{Moment of inertia} \quad I_3 := \frac{w \cdot h^3}{12}$$

$$\text{Spring constant} \quad k_3 := \frac{3 \cdot E \cdot I_3}{L_3^3} \quad k_3 = 4.635 \times 10^3 \frac{\text{lbf}}{\text{in}}$$

Pushrod (hollow cylinder):

$$\text{Area} \quad A_4 := \frac{\pi \cdot (d_{4od}^2 - d_{4id}^2)}{4}$$

$$\text{Spring constant} \quad k_4 := \frac{A_4 \cdot E}{L_4} \quad k_4 = 4.686 \times 10^5 \frac{\text{lbf}}{\text{in}}$$

Rocker arm 5 (side B):

$$\text{Moment of inertia} \quad I_5 := \frac{w \cdot h^3}{12}$$

$$\text{Spring constant} \quad k_{5a} := \frac{3 \cdot E \cdot I_5}{a^3} \quad k_{5a} = 1.898 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

Rocker arm 5 (side C):

$$\text{Spring constant} \quad k_{5b} := \frac{3 \cdot E \cdot I_5}{b^3} \quad k_{5b} = 3.708 \times 10^4 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

3. Damping will be neglected.
4. Determine the mass of each of the elements.

Roller arm 2 (cantilever beam with end load):

$$\text{Volume} \quad V_2 := w \cdot h \cdot L_2$$

$$\text{Mass} \quad m_2 := \frac{\gamma \cdot V_2}{g} \quad m_2 = 6.527 \times 10^{-3} \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Outout arm 3 (cantilever beam with end load):

$$\text{Volume} \quad V_3 := w \cdot h \cdot L_3$$

$$\text{Mass} \quad m_3 := \frac{\gamma \cdot V_3}{g} \quad m_3 = 0.013 \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Pushrod 4 (hollow cylinder):

$$\text{Volume} \quad V_4 := A_4 \cdot L_4$$

$$\text{Mass} \quad m_4 := \frac{\gamma \cdot V_4}{g} \quad m_4 = 5.482 \times 10^{-3} \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Rocker arm 5 (side B):

$$\text{Volume} \quad V_{5a} := w \cdot h \cdot a$$

$$\text{Mass} \quad m_{5a} := \frac{\gamma \cdot V_{5a}}{g} \quad m_{5a} = 8.159 \times 10^{-3} \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Rocker arm 5 (side C):

$$\text{Volume} \quad V_{5b} := w \cdot h \cdot b$$

$$\text{Mass} \quad m_{5b} := \frac{\gamma \cdot V_{5b}}{g} \quad m_{5b} = 6.527 \times 10^{-3} \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Cylinder 6:

$$\text{Volume} \quad V_6 := \frac{\pi \cdot d_6^2}{4} \cdot L_6$$

$$\text{Mass} \quad m_6 := \frac{\gamma \cdot V_6}{g} \quad m_6 = 0.014 \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Omit the spring because no data are available.

5. Determine the effective mass and spring constant on either side of the rocker arm 5, reflected to point A.

$$\text{Upper side:} \quad m_U := m_4 + m_{5a} \quad m_U = 0.014 \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_U := \left(\frac{1}{k_s} + \frac{1}{k_4} + \frac{1}{k_{5a}} \right)^{-1}$$

$$k_U = 148.777 \frac{\text{lbf}}{\text{in}}$$

Lower side: $m_L := m_6 + m_{5b}$

$$m_L = 0.020 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_L := k_{5b}$$

$$k_L = 3.708 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

$$m_{Aeff} := m_U + \left(\frac{b}{a} \right)^2 \cdot m_L$$

$$m_{Aeff} = 0.027 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_{Aeff} := k_U + \left(\frac{b}{a} \right)^2 \cdot k_L$$

$$k_{Aeff} = 2.388 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

6. Determine the effective mass and spring constant on either side of the output and roller arms, reflected to the roller.

Output side: $m_O := m_{Aeff} + m_3$

$$m_O = 0.040 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_O := \left(\frac{1}{k_{Aeff}} + \frac{1}{k_3} \right)^{-1}$$

$$k_O = 3.881 \times 10^3 \frac{\text{lbf}}{\text{in}}$$

Roller side: $m_R := m_2$

$$m_R = 6.527 \times 10^{-3} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_R := k_2$$

$$k_R = 3.708 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

$$m_{eff} := m_R + \left(\frac{L_3}{L_2} \right)^2 \cdot m_O$$

$$m_{eff} = 0.165 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_{eff} := k_R + \left(\frac{L_3}{L_2} \right)^2 \cdot k_O$$

$$k_{eff} = 5.260 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

PROBLEM 10-31

Statement: The spring in Figure P10-5 has a rate of 150 lb/in with a preload of 60 lb. Determine the effective spring constant and preload of the spring as reflected back to the cam follower. See Problem 10-30 for a description of the system.

Given: Spring data: $k := 150 \cdot \text{lb} \cdot \text{in}^{-1}$ $F_o := 60 \cdot \text{lb}$
 Distance from arm pivot to spring (L_3): $a := 16 \cdot \text{in}$
 Distance from arm pivot to follower (L_2): $b := 8 \cdot \text{in}$

Solution: See Figure P10-5, Problem 10-30, and Mathcad file P1031.

1. The effective spring constant at the follower is,

$$k_{eff} := \left(\frac{a}{b} \right)^2 \cdot k \qquad k_{eff} = 600 \frac{\text{lb}}{\text{in}}$$

2. The effective preload at the follower is,

$$F_{oeff} := \frac{a}{b} \cdot F_o \qquad F_{oeff} = 120 \text{ lb}$$

PROBLEM 10-32

Statement: A company wants to manufacture chimes that are made from hollow tubes of various lengths. Regardless of length they will be hung from a hole that is 25 mm from one end of the tube. Develop an equation that will give the distance from this hole to the point where the chime should be struck such that there will be zero reaction force at the hole where the chime is hung. The distance should be a function of the length (L), outside diameter (OD), and inside diameter (ID) of the chime as well as the distance from the end to the hanging hole (25 mm) only. Solve your equation for the following dimensions: $L = 300$ mm, $OD = 35$ mm, $ID = 30$ mm.

Given: Distance to hanging hole from end of tube: $d := 25 \cdot \text{mm}$

Test dimensions: $L_t := 300 \cdot \text{mm}$ $OD_t := 35 \cdot \text{mm}$ $ID_t := 30 \cdot \text{mm}$

Solution: See Mathcad file P1032.

1. Referring to Figure 10-3, let the distance from the hanging hole to the point where the chime should be struck be L_s . Then,

$$L_s = x + l_p \quad \text{where} \quad x = \frac{I_G}{m \cdot l_p} \quad \text{from equation 10.15a}$$

2. The CG of the chime will be located at $L/2$ from either end. From Appendix C, for a hollow tube

$$\frac{I_G}{m} = \frac{0.75 \cdot ID^2 + 0.75 \cdot OD^2 + L^2}{12}$$

and, the distance from the CG to the hanging hole is $x = \frac{L}{2} - d$

3. Combine these equations to derive the equation for the strike distance, L_s , which is measured from the hanging hole.

$$L_s(L, ID, OD) := \frac{0.75 \cdot ID^2 + 0.75 \cdot OD^2 + L^2}{12 \cdot (0.5 \cdot L - d)} + \left(\frac{L}{2} - d \right) \quad \text{where} \quad d = 25 \text{ mm}$$

4. Test the equation with the given values.

$$L_{stest} := L_s(L_t, ID_t, OD_t) \quad L_{stest} = 186.1 \text{ mm}$$

PROBLEM 10-33

Statement: What is the amount by which the roller arm of Problem 10-30 must be extended on the opposite side of the pivot axis O_2 in order to make the pivot axis a *center of rotation* if the point where the cam-follower is mounted is a *center of percussion*?

Given: Length of roller arm without extension: $L_2 := 8 \cdot \text{in}$

Cross-section dimensions: $w := 1.50 \cdot \text{in}$ $t := 0.125 \cdot \text{in}$

Solution: See Figure P10-5 and Mathcad file P1033.

1. Referring to Figure 10-3, let x be the distance from O_2 to the *CG* of the link after extension and l_p be the distance from the *CG* (after extension) to the roller axis. The amount by which the link is extended is ΔL . The extended length of the link is $L_2 + \Delta L$.

$$L_2 = x + l_p \quad \text{where} \quad x = \frac{I_G}{m \cdot l_p} = \frac{L_2 + \Delta L}{2} \quad \text{from equation 10.15a}$$

2. Eliminating x and l_p from these three equations gives (ignoring the rounded ends on the link)

$$\Delta L = \sqrt{L_2^2 - 4 \cdot \frac{I_G}{m}} \quad \text{where} \quad \frac{I_G}{m} = \frac{(L_2 + \Delta L)^2 + w^2}{12} \quad (\text{See Appendix C})$$

3. Solve for ΔL iteratively.

$$f(\Delta L) := \sqrt{L_2^2 - 4 \cdot \left[\frac{(L_2 + \Delta L)^2 + w^2}{12} \right]}$$

$$\text{Try} \quad \Delta L := 3.953 \cdot \text{in} \quad f(\Delta L) = 3.953 \cdot \text{in}$$

$$\text{Let} \quad \Delta L := 4.0 \cdot \text{in}$$

$$\text{Then the new length of link 2 is} \quad L_{\text{new}} := L_2 + \Delta L \quad L_{\text{new}} = 12.0 \cdot \text{in}$$

PROBLEM 10-34

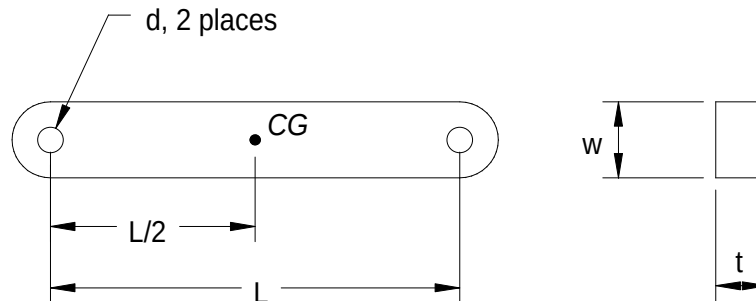
Statement: Figure P10-6 shows a sixbar mechanism with link lengths given in centimeters. The crank (2) is 30 mm wide by 10 mm thick. The couplers (3 and 5) are both 24 mm wide by 8 mm thick. The rocker (4) is 40 mm wide by 12 mm thick. All links are made from steel. The ends of the links have a full radius equal to one half of the link width. The pivot pins all have a diameter of 8 mm. Find the moment of inertia of the crank and rocker about their fixed pivots and the moment of inertia of the couplers about their CGs.

Given: Dimensions in the figure below. Mass density of steel: $\rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3}$

Pivot pin diameter: $d := 8 \cdot \text{mm}$

Solution: See Figure P10-6 and Mathcad file P1034.

1. Divide each link into 3 volumes: 1) the main, rectangular body of length L ; 2) the half-cylindrical ends; and 3) the pivot pin holes.
2. All links have the same basic shape, which is shown below.



2. Volume 1 is the main rectangular body of length L , width w , and thickness t .

Mass: $m_1(L, w, t) := L \cdot w \cdot t \cdot \rho$

Moment of inertia about CG: $I_1(L, w, t) := \frac{m_1(L, w, t)}{12} \cdot (L^2 + w^2)$

3. Volume 2 is the half-cylinder of diameter w , and length t . There are two of these.

Mass: $m_2(w, t) := \frac{\pi \cdot w^2 \cdot t \cdot \rho}{8}$

Moment of inertia about CG: $I_2(L, w, t) := 2 \cdot \left[\frac{m_2(w, t) \cdot w^2}{8} + m_2(w, t) \cdot \left(\frac{L}{2} \right)^2 \right]$

4. Volume 3 is the cylinder of diameter d , and length t . There are two of these and they are negative.

Mass: $m_3(t) := -\frac{\pi \cdot d^2 \cdot t \cdot \rho}{8}$

Moment of inertia about CG: $I_3(L, t) := 2 \cdot \left[\frac{m_3(t) \cdot d^2}{8} + m_3(t) \cdot \left(\frac{L}{2} \right)^2 \right]$

5. For the crank (2): $L := 120 \cdot \text{mm}$, $w := 30 \cdot \text{mm}$, $t := 10 \cdot \text{mm}$

Mass: $m_{crank} := m_1(L, w, t) + m_2(w, t) + m_3(t)$ $m_{crank} = 0.306 \text{ kg}$

Moment of inertia about CG:

$$I_{crankCG} := I_1(L, w, t) + I_2(L, w, t) + I_3(L, t) \quad I_{crankCG} = 548.563 \text{ kg}\cdot\text{mm}^2$$

Moment of inertia about pivot axis:

$$I_{crank} := I_{crankCG} + m_{crank} \cdot \left(\frac{L}{2}\right)^2 \quad I_{crank} = 1.652 \times 10^3 \text{ kg}\cdot\text{mm}^2$$

6. For the rocker (4): $L := 360\cdot\text{mm}$, $w := 40\cdot\text{mm}$, $t := 12\cdot\text{mm}$

Mass: $m_{rocker} := m_1(L, w, t) + m_2(w, t) + m_3(t)$ $m_{rocker} = 1.404 \text{ kg}$

Moment of inertia about CG (which is also the pivot axis):

$$I_{rocker} := I_1(L, w, t) + I_2(L, w, t) + I_3(L, t) \quad I_{rocker} = 1.842 \times 10^4 \text{ kg}\cdot\text{mm}^2$$

7. For the couplers (3 and 5) each $L := 240\cdot\text{mm}$, $w := 24\cdot\text{mm}$, $t := 8\cdot\text{mm}$

Mass: $m_{coupler} := m_1(L, w, t) + m_2(w, t) + m_3(t)$ $m_{coupler} = 0.372 \text{ kg}$

Moment of inertia about CG:

$$I_{couplerCG} := I_1(L, w, t) + I_2(L, w, t) + I_3(L, t) \quad I_{couplerCG} = 2.106 \times 10^3 \text{ kg}\cdot\text{mm}^2$$

PROBLEM 10-35

Statement: A certain bat has a mass of 1 kg and a mass moment of inertia about its CG of 0.08 kg-m². It's CG is located 630 mm from the end closest to the grip. If the center of a batter's grip is located 75 mm from the same end of the bat, at what point on the bat (measured from the end closest to the grip) should the batter hit the ball to produce no reaction at the grip?

Given: Mass of the bat: $m_{bat} := 1.0 \cdot kg$ Mass moment of inertia: $I := 0.08 \cdot kg \cdot m^2$
 Distance from end to: $x_{grip} := 75 \cdot mm$ $x_{CG} := 630 \cdot mm$

Solution: See Mathcad file P1035.

1. Using equation 10.11b, calculate the radius of gyration of the bat: $k := \sqrt{\frac{I}{m_{bat}}}$ $k = 282.843 \text{ mm}$

2. The distance from the CG to the center of the grip is $x := x_{grip} - x_{CG}$ $x = -555.000 \text{ mm}$

3. Calculate the distance from the CG to the center of percussion for the given grip point using equation 10.15b:

$$l_p := \frac{-k^2}{x} \quad l_p = 144.144 \text{ mm}$$

4. The distance from the grip-end of the bat to the center of percussion is

$$x_{ball} := x_{CG} + l_p \quad x_{ball} = 774 \text{ mm}$$

PROBLEM 10-36

Statement: The cam of Example 8-8 drives an aligned translating roller follower. The effective mass of the follower and the mechanism that it actuates is 0.45 kg. The follower spring has a rate of 8 N/m with a preload at zero displacement of 0.3 N. Use the method of virtual work to find and plot the torque required to rotate the cam through one rise-fall segment.

Given: Cam speed: $\omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1}$

Spring rate and preload: $k := 8 \cdot \text{N} \cdot \text{m}^{-1}$ $F_0 := 0.3 \cdot \text{N}$

Roller follower mass: $m_f := 0.45 \cdot \text{kg}$

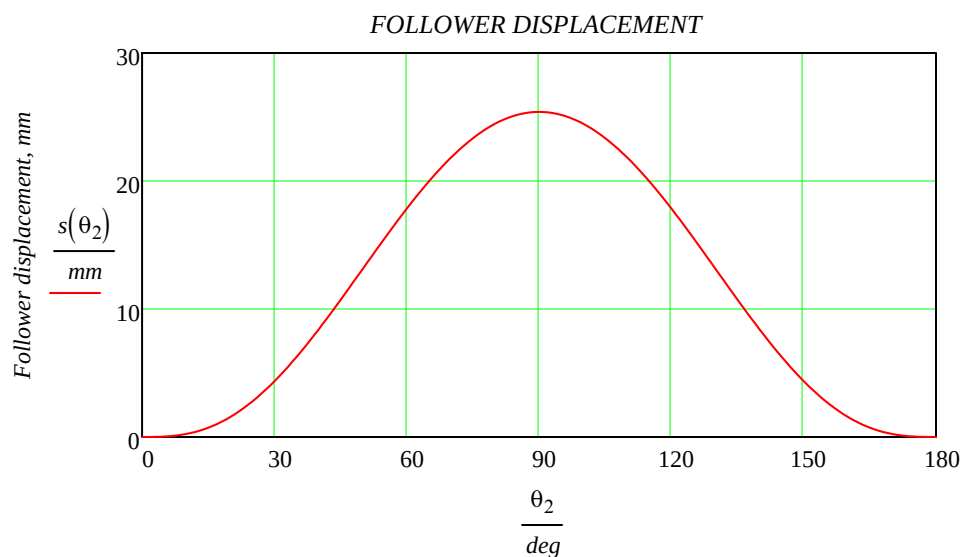
Rise-fall angle: $\beta := 180 \cdot \text{deg}$

Solution: See Example 8-8 and Mathcad file P1036.

1. From Example 8-8, the follower displacement is

$$s(\theta) := 1 \cdot \left[64 \cdot \left(\frac{\theta}{\beta} \right)^3 - 192 \cdot \left(\frac{\theta}{\beta} \right)^4 + 192 \cdot \left(\frac{\theta}{\beta} \right)^5 - 64 \cdot \left(\frac{\theta}{\beta} \right)^6 \right] \cdot \text{in}$$

Plotting s as a function of $\theta_2 := 0 \cdot \text{deg}, 1 \cdot \text{deg} \dots 180 \cdot \text{deg}$
 θ_2 :



2. Differentiate the displacement function twice to get the velocity and acceleration functions.

$$v_f(\theta_2) := \left(\frac{d}{d\theta_2} s(\theta_2) \right) \cdot \omega_2 \qquad a_f(\theta_2) := \left(\frac{d}{d\theta_2} v_f(\theta_2) \right) \cdot \omega_2$$

3. Calculate the spring force as a function of θ_2 .

$$F_f(\theta_2) := k \cdot s(\theta_2) + F_0$$

4. The spring force is the only external force on the linkage that is not applied at a joint and is, therefore, the only force to be considered in the first term of equation 10.28b. Determine the first term in the equation.

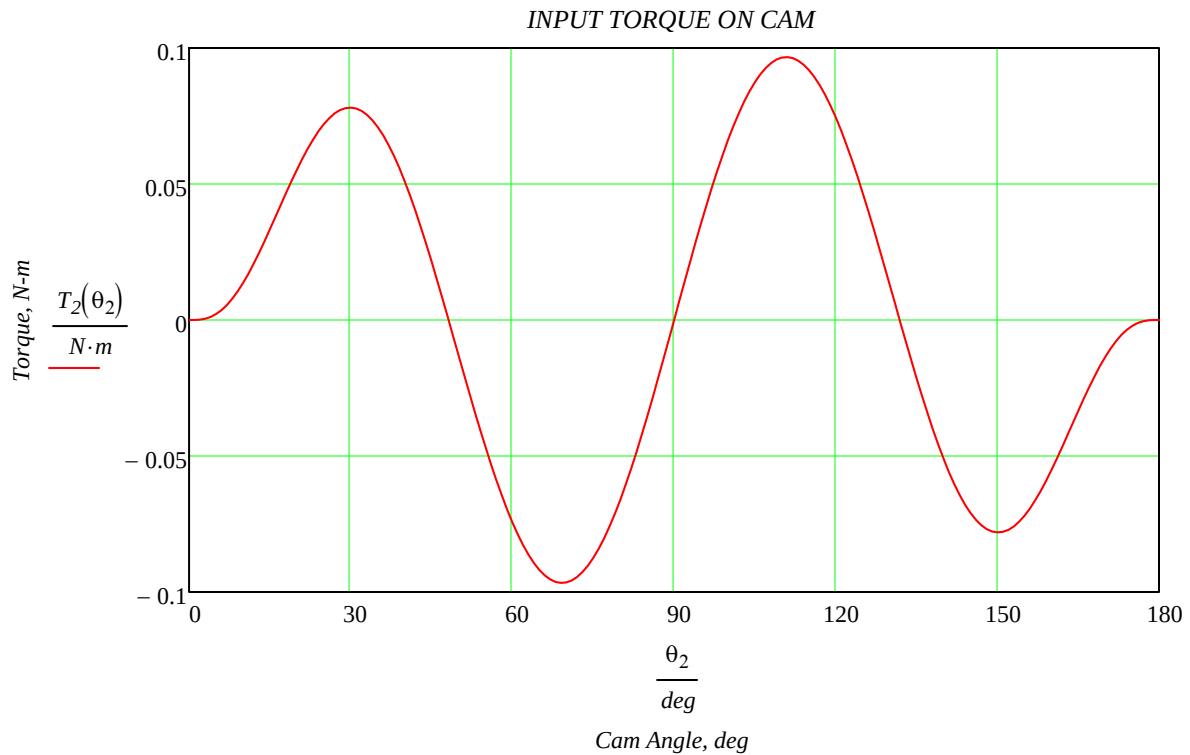
$$EF(\theta_2) := F_f(\theta_2) \cdot v_f(\theta_2)$$

5. Determine the components that make up the third term in equation 10.28b. The roller follower translates but does not rotate. The translation motion will be accounted for in this term.

$$IF(\theta_2) := m_f \cdot a_f(\theta_2) \cdot v_f(\theta_2)$$

6. The fourth term in equation 10.28b is zero. The only rotating component is the cam itself and it rotates at constant speed.
7. Using equation 10.28b, solve for and plot the input torque as a function of cam angle over the complete rise and fall. During the dwell the input torque is zero..

$$T_2(\theta_2) := \frac{1}{\omega_2} \cdot (IF(\theta_2) - EF(\theta_2))$$



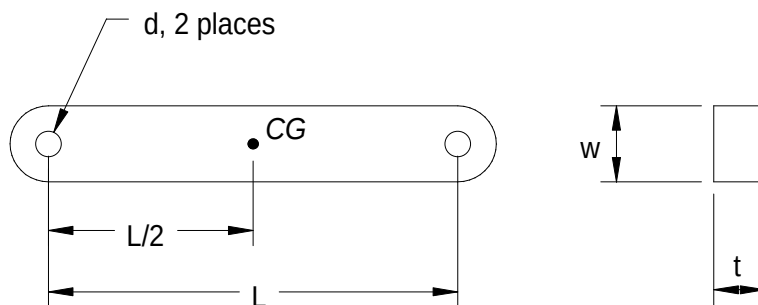
PROBLEM 10-37a

Statement: Figure P10-6a shows a typical binary link with full-radius ends. Figure P10-6b shows a full-radius end and gives moments of inertia about its CG and about an axis through the center of the radius, R . Table P10-1 gives data for the length, L , between holes of diameter d ; the end radius, R ; the thickness, t ; and the material of the link. For the row(s) assigned, find the moment of inertia of the link about one fixed pivot and its CG.

Given: Mass density of steel: $\rho := 7.8 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^{-3}$
 Length: $L := 225 \cdot \text{mm}$
 Width: $R := 13 \cdot \text{mm}$ $w := 2 \cdot R$ $w = 26 \text{ mm}$
 Thickness: $t := 12 \cdot \text{mm}$
 Pivot pin diameter: $d := 8 \cdot \text{mm}$

Solution: See Figure P10-6, Table P10-1, and Mathcad file P1037a.

1. Divide each link into 3 volumes: 1) the main, rectangular body; 2) the half-cylindrical ends; and 3) the pivot pin holes.
2. All links have the same basic shape, which is shown below.



2. Volume 1 is the main rectangular body of length L , width w , and thickness t .

$$\text{Mass: } m_1 := L \cdot w \cdot t \cdot \rho \quad m_1 = 0.548 \text{ kg}$$

$$\text{Moment of inertia about link CG: } I_1 := \frac{m_1}{12} \cdot (L^2 + w^2) \quad I_1 = 2341 \text{ kg} \cdot \text{mm}^2$$

3. Volume 2 is the half-cylinder of diameter w , and length t . There are two of these.

$$\text{Mass (each): } m_2 := \frac{\pi \cdot R^2 \cdot t \cdot \rho}{2} \quad m_2 = 0.025 \text{ kg}$$

$$\text{Moment of inertia about link CG: } I_2 := 2 \cdot \left[\frac{m_2 \cdot R^2}{2} + m_2 \cdot \left(\frac{L}{2} \right)^2 \right]$$

$$I_2 = 633.151 \text{ kg} \cdot \text{mm}^2$$

4. Volume 3 is the cylinder of diameter d , and length t . There are two of these and they are negative.

$$\text{Mass (each): } m_3 := -\frac{\pi \cdot d^2 \cdot t \cdot \rho}{8} \quad m_3 = -0.0024 \text{ kg}$$

$$\text{Moment of inertia about link CG: } I_3 := 2 \cdot \left[\frac{m_3 \cdot d^2}{8} + m_3 \cdot \left(\frac{L}{2} \right)^2 \right]$$

$$I_3 = -59.583 \text{ kg}\cdot\text{mm}^2$$

5. The total moment of inertia about the link *CG* is:

$$I_{CG} := I_1 + I_2 + I_3$$

$$I_{CG} = 2914 \text{ kg}\cdot\text{mm}^2$$

6. Use equation 10.8 to calculate the moment of inertia about one pivot hole:

Total mass of link:

$$m_{link} := m_1 + 2 \cdot (m_2 + m_3)$$

$$m_{link} = 0.593 \text{ kg}$$

Moment of inertia about
pivot hole:

$$I_{pivot} := I_{CG} + m_{link} \cdot \left(\frac{L}{2} \right)^2$$

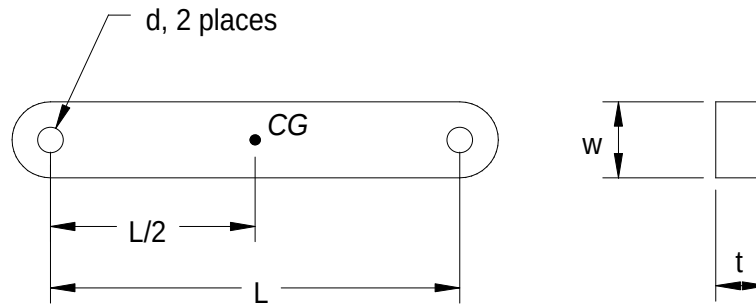
$$I_{pivot} = 10414 \text{ kg}\cdot\text{mm}^2$$

PROBLEM 10-38

Statement: Using the definition of the binary link in Problem 10-37, write a computer program or use an equation solver to calculate the moment of inertia of the link about its fixed pivots and about its CG. Use the data in row *a* of Table P10-1 to test your program.

Solution: See Figure P10-6, Table P10-1, and Mathcad file P1038.

1. Divide each link into 3 volumes: 1) the main, rectangular body; 2) the half-cylindrical ends; and 3) the pivot pin holes.
2. All links have the same basic shape, which is shown below.



2. Volume 1 is the main rectangular body of length L , width w , and thickness t .

$$\text{Mass: } m_1(L, w, t, \rho) := L \cdot w \cdot t \cdot \rho$$

$$\text{Moment of inertia about link CG: } I_1(L, w, t, \rho) := \frac{m_1(L, w, t, \rho)}{12} \cdot (L^2 + w^2)$$

3. Volume 2 is the half-cylinder of diameter w , and length t . There are two of these.

$$\text{Mass (each): } m_2(R, t, \rho) := \frac{\pi \cdot R^2 \cdot t \cdot \rho}{2}$$

$$\text{Moment of inertia about link CG: } I_2(L, R, t, \rho) := 2 \cdot \left[\frac{m_2(R, t, \rho) \cdot R^2}{2} + m_2(R, t, \rho) \cdot \left(\frac{L}{2} \right)^2 \right]$$

4. Volume 3 is the cylinder of diameter d , and length t . There are two of these and they are negative.

$$\text{Mass (each): } m_3(t, d, \rho) := -\frac{\pi \cdot d^2 \cdot t \cdot \rho}{8}$$

$$\text{Moment of inertia about link CG: } I_3(L, t, d, \rho) := 2 \cdot \left[\frac{m_3(t, d, \rho) \cdot d^2}{8} + m_3(t, d, \rho) \cdot \left(\frac{L}{2} \right)^2 \right]$$

5. The total moment of inertia about the link CG is:

$$I_{CG}(L, R, w, t, d, \rho) := I_1(L, w, t, \rho) + I_2(L, R, t, \rho) + I_3(L, t, d, \rho)$$

6. Use equation 10.8 to calculate the moment of inertia about one pivot hole:

Total mass of link:

$$m_{link}(L, R, w, t, d, \rho) := m_1(L, w, t, \rho) + 2 \cdot (m_2(R, t, \rho) + m_3(t, d, \rho))$$

Moment of inertia about pivot hole:

$$I_{pivot}(L, R, w, t, d, \rho) := I_{CG}(L, R, w, t, d, \rho) + m_{link}(L, R, w, t, d, \rho) \cdot \left(\frac{L}{2}\right)^2$$

7. Use the data of Table 10-1, row *a* to test the program.

Mass density of steel: $\rho := 7.8 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^{-3}$

Length: $L := 225 \cdot \text{mm}$

Width: $R := 13 \cdot \text{mm}$ $w := 2 \cdot R$ $w = 26 \cdot \text{mm}$

Thickness: $t := 12 \cdot \text{mm}$

Pivot pin diameter: $d := 8 \cdot \text{mm}$

$$m_{link}(L, R, w, t, d, \rho) = 0.593 \text{ kg}$$

$$I_{CG}(L, R, w, t, d, \rho) = 2914 \text{ kg} \cdot \text{mm}^2$$

$$I_{pivot}(L, R, w, t, d, \rho) = 10414 \text{ kg} \cdot \text{mm}^2$$

These results agree with the solution to Problem 10-37a.

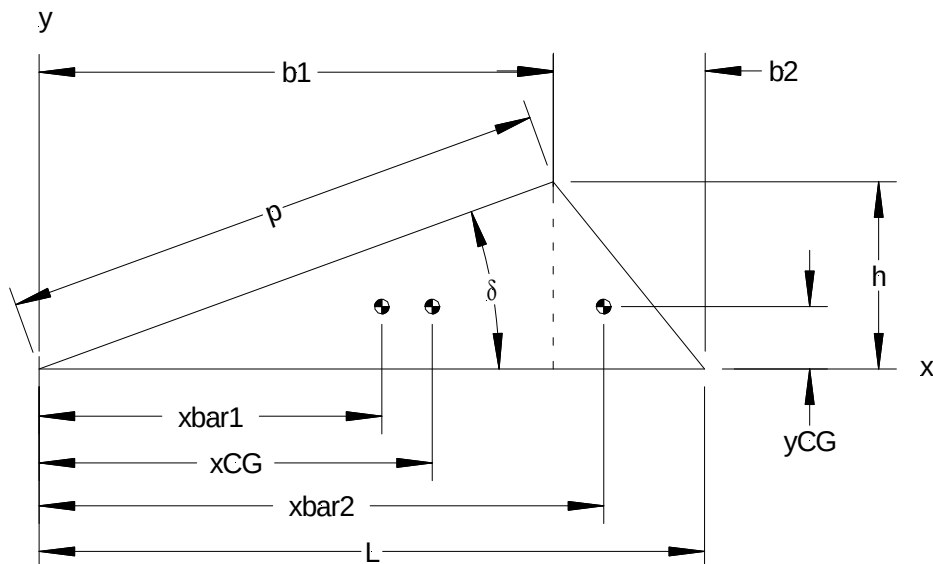
PROBLEM 10-39a

Statement: Figure P10-7a shows a simplified ternary link (the vertices would normally be rounded). Figure P10-7b shows a triangular plate and gives the moment of inertia about its CG. Table P10-1 gives data for the length L , of the base of the link; the angle, δ , between the base and one side; the distance p along that side; the diameter, d , of the three holes; the thickness, t ; and the material of the link. For the row a , find the moment of inertia of the link about its CG. As a first approximation, ignore the holes.

Given: Mass density of steel: $\rho := 7.8 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^{-3}$
 Length: $L := 225 \cdot \text{mm}$
 Distance to apex: $p := 185 \cdot \text{mm}$
 Angle: $\delta := 20 \cdot \text{deg}$
 Thickness: $t := 12 \cdot \text{mm}$
 Pivot pin diameter: $d := 8 \cdot \text{mm}$

Solution: See Figure P10-7, Table P10-1, and Mathcad file P1037.

1. Divide the link into 2 volumes: 1) the left right triangle, and 2) the remaining right triangle.
2. All links have the same basic shape, which is shown below.



2. Locate the CG of the constant thickness solid using the triangular areas.

Base 1: $b_1 := p \cdot \cos(\delta) \quad b_1 = 173.843 \text{ mm}$

Height: $h := b_1 \cdot \tan(\delta) \quad h = 63.274 \text{ mm}$

Distance to CG from origin of xy coordinate frame for triangle 1:

$$xbar_1 := \frac{2}{3} \cdot b_1 \quad xbar_1 = 115.895 \text{ mm}$$

$$ybar_1 := \frac{1}{3} \cdot h \quad ybar_1 = 21.091 \text{ mm}$$

Base 2: $b_2 := L - b_1 \quad b_2 = 51.157 \text{ mm}$

Distance to CG from origin of xy coordinate frame for triangle 2:

$$\bar{x}_2 := b_1 + \frac{1}{3} \cdot b_2 \quad \bar{x}_2 = 190.895 \text{ mm}$$

$$\bar{y}_2 := \bar{y}_1 \quad \bar{y}_2 = 21.091 \text{ mm}$$

Area of triangles:

$$A_1 := 0.5 \cdot b_1 \cdot h \quad A_1 = 5499.9 \text{ mm}^2$$

$$A_2 := 0.5 \cdot b_2 \cdot h \quad A_2 = 1618.4 \text{ mm}^2$$

$$A := A_1 + A_2 \quad A = 7118.3 \text{ mm}^2$$

Location of overall CG with respect to xy axes:

$$\bar{x}_{CG} := \frac{\bar{x}_1 \cdot A_1 + \bar{x}_2 \cdot A_2}{A} \quad \bar{x}_{CG} = 132.948 \text{ mm}$$

$$\bar{y}_{CG} := \bar{y}_1 \quad \bar{y}_{CG} = 21.091 \text{ mm}$$

3. Determine the mass and moment of inertia of volume 1 using given moment of inertia equation and equation 10.8:

$$\text{Mass:} \quad m_1 := 0.5 \cdot b_1 \cdot h \cdot t \cdot \rho \quad m_1 = 0.515 \text{ kg}$$

Distance from volume 1 CG to CG of entire volume

$$d_1 := \bar{x}_{CG} - \bar{x}_1 \quad d_1 = 17.052 \text{ mm}$$

Moment of inertia of volume 1 about overall CG:

$$I_1 := \frac{m_1}{18} \cdot (b_1^2 + h^2) + m_1 \cdot d_1^2 \quad I_1 = 1128.5 \text{ kg} \cdot \text{mm}^2$$

4. Determine the mass and moment of inertia of volume 2 using given moment of inertia equation and equation 10.8:

$$\text{Mass:} \quad m_2 := 0.5 \cdot b_2 \cdot h \cdot t \cdot \rho \quad m_2 = 0.151 \text{ kg}$$

Distance from volume 2 CG to CG of entire volume

$$d_2 := \bar{x}_{CG} - \bar{x}_2 \quad d_2 = -57.948 \text{ mm}$$

Moment of inertia of volume 2 about overall CG:

$$I_2 := \frac{m_2}{18} \cdot (b_2^2 + h^2) + m_2 \cdot d_2^2 \quad I_2 = 564.4 \text{ kg} \cdot \text{mm}^2$$

5. Add the moments of inertia to obtain the overall moment of inertia about the CG of the ternary link.

$$I := I_1 + I_2 \quad I = 1693 \text{ kg} \cdot \text{mm}^2$$

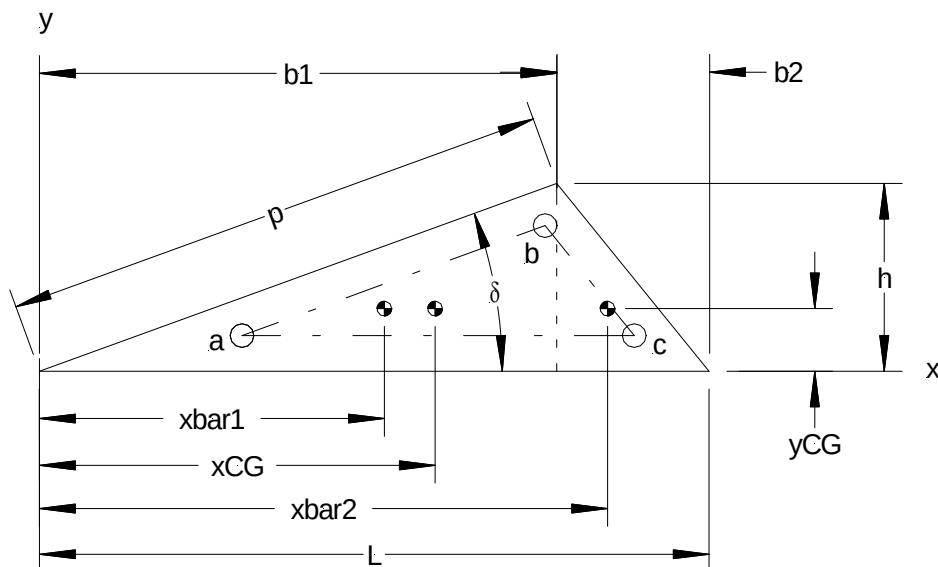
PROBLEM 10-40

Statement: Using the definition of the ternary link in Problem 10-39, write a computer program or use an equation solver to calculate the moment of inertia of the link about its CG. Use the data in row *a* of Table P10-1 to test your program. Include the holes in your calculation.

Input: Mass density of steel: $\rho := 7.8 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^{-3}$
 Length: $L := 225 \cdot \text{mm}$
 Distance to apex: $p := 185 \cdot \text{mm}$
 Angle: $\delta := 20 \cdot \text{deg}$
 Thickness: $t := 12 \cdot \text{mm}$
 Pivot pin diameter: $d := 8 \cdot \text{mm}$

Solution: See Figure P10-7, Table P10-1, and Mathcad file P1040.

1. Divide the link into 2 volumes: 1) the left right triangle, and 2) the remaining right triangle.



2. Locate the CG of the constant thickness solid using the triangular areas.

Base 1: $b_1 := p \cdot \cos(\delta) \quad b_1 = 173.843 \text{ mm}$

Height: $h := b_1 \cdot \tan(\delta) \quad h = 63.274 \text{ mm}$

Distance to CG from origin of xy coordinate frame for triangle 1:

$$xbar_1 := \frac{2}{3} \cdot b_1 \quad xbar_1 = 115.895 \text{ mm}$$

$$ybar_1 := \frac{1}{3} \cdot h \quad ybar_1 = 21.091 \text{ mm}$$

Base 2: $b_2 := L - b_1 \quad b_2 = 51.157 \text{ mm}$

Distance to CG from origin of xy coordinate frame for triangle 2:

$$xbar_2 := b_1 + \frac{1}{3} \cdot b_2 \quad xbar_2 = 190.895 \text{ mm}$$

$$ybar_2 := ybar_1 \quad ybar_2 = 21.091 \text{ mm}$$

Area of triangles:

$$A_1 := 0.5 \cdot b_1 \cdot h \quad A_1 = 5499.9 \text{ mm}^2$$

$$A_2 := 0.5 \cdot b_2 \cdot h \quad A_2 = 1618.4 \text{ mm}^2$$

$$A := A_1 + A_2 \quad A = 7118.3 \text{ mm}^2$$

Location of overall CG with respect to xy axes:

$$x_{CG} := \frac{xbar_1 \cdot A_1 + xbar_2 \cdot A_2}{A} \quad x_{CG} = 132.948 \text{ mm}$$

$$y_{CG} := ybar_1 \quad y_{CG} = 21.091 \text{ mm}$$

3. Determine the mass and moment of inertia of volume 1 using given moment of inertia equation and equation 10.8:

$$\text{Mass:} \quad m_1 := 0.5 \cdot b_1 \cdot h \cdot t \cdot \rho \quad m_1 = 0.515 \text{ kg}$$

Distance from volume 1 CG to CG of entire volume

$$d_1 := x_{CG} - xbar_1 \quad d_1 = 17.052 \text{ mm}$$

Moment of inertia of volume 1 about overall CG:

$$I_1 := \frac{m_1}{18} \cdot (b_1^2 + h^2) + m_1 \cdot d_1^2 \quad I_1 = 1128.5 \text{ kg} \cdot \text{mm}^2$$

4. Determine the mass and moment of inertia of volume 2 using given moment of inertia equation and equation 10.8:

$$\text{Mass:} \quad m_2 := 0.5 \cdot b_2 \cdot h \cdot t \cdot \rho \quad m_2 = 0.151 \text{ kg}$$

Distance from volume 2 CG to CG of entire volume

$$d_2 := x_{CG} - xbar_2 \quad d_2 = -57.948 \text{ mm}$$

Moment of inertia of volume 2 about overall CG:

$$I_2 := \frac{m_2}{18} \cdot (b_2^2 + h^2) + m_2 \cdot d_2^2 \quad I_2 = 564.4 \text{ kg} \cdot \text{mm}^2$$

5. Determine the locations and moments of inertia of the hole centers.

The equations of the lines parallel to and offset by 1.5d from the link boundaries are:

$$y = 1.5 \cdot d \quad y = x \cdot \tan(\delta) - e \quad y = x \cdot \tan(\beta) + f$$

$$\text{where} \quad e := \frac{1.5 \cdot d}{\cos(\delta)} \quad e = 12.770 \text{ mm}$$

$$\alpha := \text{atan}\left(\frac{h}{b_2}\right) \quad \alpha = 51.044 \text{ deg}$$

$$\beta := 180 \cdot \text{deg} - \alpha \quad \beta = 128.956 \text{ deg}$$

$$f := \frac{L \cdot h}{b_2} - \frac{1.5 \cdot d}{\cos(\alpha)} \quad f = 259.206 \text{ mm}$$

Solving for the intersections of these three lines:

$$x_a := \frac{1.5 \cdot d + e}{\tan(\delta)} \quad x_a = 68.055 \text{ mm}$$

$$y_a := 1.5 \cdot d \quad y_a = 12.000 \text{ mm}$$

$$x_b := \frac{e + f}{\tan(\delta) - \tan(\beta)} \quad x_b = 169.897 \text{ mm}$$

$$y_b := x_b \cdot \tan(\delta) - e \quad y_b = 49.067 \text{ mm}$$

$$x_c := \frac{1.5 \cdot d - f}{\tan(\beta)} \quad x_c = 199.867 \text{ mm}$$

$$y_c := 1.5 \cdot d \quad y_c = 12.000 \text{ mm}$$

The distances from the hole centers to the link CG are:

$$d_a := \sqrt{(x_{CG} - x_a)^2 + (y_{CG} - y_a)^2} \quad d_a = 65.526 \text{ mm}$$

$$d_b := \sqrt{(x_{CG} - x_b)^2 + (y_{CG} - y_b)^2} \quad d_b = 46.346 \text{ mm}$$

$$d_c := \sqrt{(x_{CG} - x_c)^2 + (y_{CG} - y_c)^2} \quad d_c = 67.534 \text{ mm}$$

The moments of inertia of the holes with respect to the link CG are:

$$I_a := -\left(\frac{d^2}{8} + d_a^2\right) \cdot \frac{\pi \cdot d^2}{4} \cdot t \cdot \rho \quad I_a = -20.239 \text{ kg} \cdot \text{mm}^2$$

$$I_b := -\left(\frac{d^2}{8} + d_b^2\right) \cdot \frac{\pi \cdot d^2}{4} \cdot t \cdot \rho \quad I_b = -10.143 \text{ kg} \cdot \text{mm}^2$$

$$I_c := -\left(\frac{d^2}{8} + d_c^2\right) \cdot \frac{\pi \cdot d^2}{4} \cdot t \cdot \rho \quad I_c = -21.495 \text{ kg} \cdot \text{mm}^2$$

6. Add the moments of inertia to obtain the overall moment of inertia about the CG of the ternary link.

$$I := I_1 + I_2 + I_a + I_b + I_c \quad I = 1641 \text{ kg} \cdot \text{mm}^2$$

PROBLEM 11-1

Statement: Draw free body diagrams of the links in the geared fivebar linkage shown in Figure 4-11 and write the dynamic equations to solve for all forces plus the driving torque. Assemble the symbolic equations in matrix form for solution.

Solution: No solution is provided to this algebraic exercise.

PROBLEM 11-2

Statement: Draw free body diagrams of the links in the sixbar linkage shown in Figure 4-12 and write the dynamic equations to solve for all forces plus the driving torque. Assemble the symbolic equations in matrix form for solution.

Solution: No solution is provided to this algebraic exercise.

PROBLEM 11-3a

Statement: Table P11-1 shows kinematic and geometric data for several slider-crank linkages of the type and orientation shown in Figure P11-1. The point locations are defined as described in the text. For row *a* in the table, solve for forces and torques at the position shown. Also, compute the shaking force and the shaking torque. Consider the coefficient of friction μ between slider and ground to be zero.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

Link 2 (O_2 to A)	$a := 4.00 \cdot in$	Link 3 (A to B)	$b := 12.00 \cdot in$
Offset	$c := 0.00 \cdot in$	Friction:	$\mu := 0$
Crank angle and motion:	$\theta_2 := 45 \cdot deg$	$\omega_2 := 10 \cdot rad \cdot sec^{-1}$	$\theta_3 := 166.40 \cdot deg$
Coupler point:	$R_{P3} := 0.0 \cdot in$	$\delta_{RP3} := 0.0 \cdot deg$	
Mass:	$m_2 := 0.002 \cdot blob$	$m_3 := 0.020 \cdot blob$	$m_4 := 0.060 \cdot blob$
Moment of inertia:	$I_{G2} := 0.10 \cdot blob \cdot in^2$	$I_{G3} := 0.20 \cdot blob \cdot in^2$	
Mass center:	$R_{CG2} := 2.00 \cdot in$	$\delta_2 := 0 \cdot deg$	$R_{CG3} := 5.00 \cdot in$ $\delta_3 := 0 \cdot deg$
Force and torque:	$F_{P3} := 0 \cdot lbf$	$\delta_{FP3} := 0 \cdot deg$	$T_3 := 20 \cdot lbf \cdot in$
Accelerations:	$\alpha_2 := 20 \cdot rad \cdot sec^{-2}$	$a_{G2} := 203.96 \cdot in \cdot sec^{-2}$	$\theta_{AG2} := 213.69 \cdot deg$
	$\alpha_3 := -2.40 \cdot rad \cdot sec^{-2}$	$a_{G3} := 371.08 \cdot in \cdot sec^{-2}$	$\theta_{AG3} := 200.84 \cdot deg$
		$a_{G4} := 357.17 \cdot in \cdot sec^{-2}$	$\theta_{AG4} := 180.0 \cdot deg$

Solution: See Figure P11-1, Table P11-1, and Mathcad file P1103a.

1. Calculate the x and y components of the position vectors.

$R_{12x} := R_{CG2} \cdot \cos(\theta_2 + 180 \cdot deg)$	$R_{12x} = -1.414 \cdot in$
$R_{12y} := R_{CG2} \cdot \sin(\theta_2 + 180 \cdot deg)$	$R_{12y} = -1.414 \cdot in$
$R_{32x} := R_{CG2} \cdot \cos(\theta_2)$	$R_{32x} = 1.414 \cdot in$
$R_{32y} := R_{CG2} \cdot \sin(\theta_2)$	$R_{32y} = 1.414 \cdot in$
$R_{23x} := R_{CG3} \cdot \cos(\theta_3)$	$R_{23x} = -4.860 \cdot in$
$R_{23y} := R_{CG3} \cdot \sin(\theta_3)$	$R_{23y} = 1.176 \cdot in$
$R_{43x} := (b - R_{CG3}) \cdot \cos(\theta_3 + 180 \cdot deg)$	$R_{43x} = 6.804 \cdot in$
$R_{43y} := (b - R_{CG3}) \cdot \sin(\theta_3 + 180 \cdot deg)$	$R_{43y} = -1.646 \cdot in$
$R_{P3x} := R_{P3} \cdot \cos(\theta_3 + 180 \cdot deg + \delta_{RP3})$	$R_{P3x} = 0.000 \cdot in$
$R_{P3y} := R_{P3} \cdot \sin(\theta_3 + 180 \cdot deg + \delta_{RP3})$	$R_{P3y} = 0.000 \cdot in$

2. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\begin{aligned}
 a_{G2x} &:= a_{G2} \cdot \cos(\theta_{AG2}) & a_{G2x} &= -169.705 \text{ in} \cdot \text{sec}^{-2} \\
 a_{G2y} &:= a_{G2} \cdot \sin(\theta_{AG2}) & a_{G2y} &= -113.136 \text{ in} \cdot \text{sec}^{-2} \\
 a_{G3x} &:= a_{G3} \cdot \cos(\theta_{AG3}) & a_{G3x} &= -346.803 \text{ in} \cdot \text{sec}^{-2} \\
 a_{G3y} &:= a_{G3} \cdot \sin(\theta_{AG3}) & a_{G3y} &= -132.015 \text{ in} \cdot \text{sec}^{-2} \\
 a_{G4x} &:= a_{G4} \cdot \cos(\theta_{AG4}) & a_{G4x} &= -357.170 \text{ in} \cdot \text{sec}^{-2}
 \end{aligned}$$

3. Calculate the x and y components of the external force at P on link 3 in the CGS.

$$\begin{aligned}
 F_{P3x} &:= F_{P3} \cdot \cos(\delta_{FP3}) & F_{P3x} &= 0.000 \text{ lbf} \\
 F_{P3y} &:= F_{P3} \cdot \sin(\delta_{FP3}) & F_{P3y} &= 0.000 \text{ lbf}
 \end{aligned}$$

4. Substitute these given and calculated values into the matrix equation 11.10g, modified for this problem. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.10g will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{\text{in}} & \frac{R_{12x}}{\text{in}} & \frac{-R_{32y}}{\text{in}} & \frac{R_{32x}}{\text{in}} & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{\text{in}} & \frac{-R_{23x}}{\text{in}} & \frac{-R_{43y}}{\text{in}} & \frac{R_{43x}}{\text{in}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot \text{lbf}^{-1} \\ m_2 \cdot a_{G2y} \cdot \text{lbf}^{-1} \\ I_{G2} \cdot \alpha_2 \cdot \text{lbf}^{-1} \cdot \text{in}^{-1} \\ (m_3 \cdot a_{G3x} - F_{P3x}) \cdot \text{lbf}^{-1} \\ (m_3 \cdot a_{G3y} - F_{P3y}) \cdot \text{lbf}^{-1} \\ (I_{G3} \cdot \alpha_3 - R_{P3x} \cdot F_{P3y} + R_{P3y} \cdot F_{P3x} - T_3) \cdot \text{lbf}^{-1} \cdot \text{in}^{-1} \\ m_4 \cdot a_{G4x} \cdot \text{lbf}^{-1} \\ 0 \end{bmatrix} \quad R := C^{-1} \cdot F$$

$$F_{12x} := R_1 \cdot \text{lbf} \quad F_{12x} = -28.7 \text{ lbf} \quad F_{12y} := R_2 \cdot \text{lbf} \quad F_{12y} = 5.87 \text{ lbf}$$

$$\begin{aligned}
 F_{32x} &:= R_3 \cdot lbf & F_{32x} &= 28.4 \text{ lbf} & F_{32y} &:= R_4 \cdot lbf & F_{32y} &= -6.10 \text{ lbf} \\
 F_{43x} &:= R_5 \cdot lbf & F_{43x} &= 21.4 \text{ lbf} & F_{43y} &:= R_6 \cdot lbf & F_{43y} &= -8.74 \text{ lbf} \\
 F_{14y} &:= R_7 \cdot lbf & F_{14y} &= -8.74 \text{ lbf} \\
 T_{12} &:= R_8 \cdot lbf \cdot in & T_{12} &= 99.6 \text{ lbf} \cdot in
 \end{aligned}$$

5. Calculate the shaking force and shaking torque using equations 11.15.

$$\begin{aligned}
 \mathbf{F}_{21} &:= -F_{12x} - j \cdot F_{12y} & \mathbf{F}_{41} &:= -j \cdot F_{14y} \\
 \mathbf{F}_s &:= \mathbf{F}_{21} + \mathbf{F}_{41} & \mathbf{F}_s &= (28.706 + 2.867j) \text{ lbf} \\
 \text{Magnitude:} & & F_s &:= |\mathbf{F}_s| & F_s &= 28.848 \text{ lbf} \\
 \text{Angle:} & & \theta_{F_s} &:= \arg(\mathbf{F}_s) & \theta_{F_s} &= 5.703 \text{ deg} \\
 \mathbf{T}_s &:= -T_{12} & \mathbf{T}_s &= -99.6 \text{ lbf} \cdot in
 \end{aligned}$$

PROBLEM 11-4a

Statement: Table P11-5 gives kinematic and geometric data for a slider-crank linkage of the type and orientation shown in Figure 11-4. For the row(s) assigned in the table, solve for the input torque on link 2 using the method of virtual work at the position shown.

Units: $\text{blob} := \text{lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Link lengths:

Link 2 (O_2 to A) $a := 4.00 \cdot \text{in}$ Link 3 (A to B) $b := 12.00 \cdot \text{in}$

Offset $c := 0.00 \cdot \text{in}$ Friction: $\mu := 0$

Crank angle and motion: $\theta_2 := 45 \cdot \text{deg}$ $\theta_3 := 166.40 \cdot \text{deg}$

Coupler point: $R_{P3} := 0.0 \cdot \text{in}$ $\delta_{RP3} := 0.0 \cdot \text{deg}$

Mass: $m_2 := 0.002 \cdot \text{blob}$ $m_3 := 0.020 \cdot \text{blob}$ $m_4 := 0.060 \cdot \text{blob}$

Moment of inertia: $I_{G2} := 0.10 \cdot \text{blob} \cdot \text{in}^2$ $I_{G3} := 0.20 \cdot \text{blob} \cdot \text{in}^2$

Mass center: $R_{CG2} := 2.00 \cdot \text{in}$ $\delta_2 := 0 \cdot \text{deg}$ $R_{CG3} := 5.00 \cdot \text{in}$ $\delta_3 := 0 \cdot \text{deg}$

Force and torque: $F_{P3} := 0 \cdot \text{lbf}$ $\delta_{FP3} := 0 \cdot \text{deg}$ $T_3 := 20 \cdot \text{lbf} \cdot \text{in}$

Accelerations: $\alpha_2 := 20 \cdot \text{rad} \cdot \text{sec}^{-2}$ $a_{G2} := 203.96 \cdot \text{in} \cdot \text{sec}^{-2}$ $\theta_{AG2} := 213.69 \cdot \text{deg}$

$\alpha_3 := -2.40 \cdot \text{rad} \cdot \text{sec}^{-2}$ $a_{G3} := 371.08 \cdot \text{in} \cdot \text{sec}^{-2}$ $\theta_{AG3} := 200.84 \cdot \text{deg}$

$a_{G4} := 357.17 \cdot \text{in} \cdot \text{sec}^{-2}$ $\theta_{AG4} := 180.0 \cdot \text{deg}$

Velocities: $\omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1}$ $v_{G2} := 20.0 \cdot \text{in} \cdot \text{sec}^{-1}$ $\theta_{VG2} := 135.0 \cdot \text{deg}$

$\omega_3 := -2.43 \cdot \text{rad} \cdot \text{sec}^{-1}$ $v_{G3} := 35.24 \cdot \text{in} \cdot \text{sec}^{-1}$ $\theta_{VG3} := 152.09 \cdot \text{deg}$

$v_{G4} := 35.14 \cdot \text{in} \cdot \text{sec}^{-1}$ $\theta_{VG4} := 180.0 \cdot \text{deg}$

$v_{P3} := 35.24 \cdot \text{in} \cdot \text{sec}^{-1}$ $\theta_{VP3} := 152.04 \cdot \text{deg}$

Solution: See Figure P11-1, Table P11-1, Table P11-2, and Mathcad file P1103a.

1. Calculate the x and y components of the velocity vectors.

$$v_{G2x} := v_{G2} \cdot \cos(\theta_{VG2}) \quad v_{G2x} = -14.142 \cdot \text{in} \cdot \text{sec}^{-1}$$

$$v_{G2y} := v_{G2} \cdot \sin(\theta_{VG2}) \quad v_{G2y} = 14.142 \cdot \text{in} \cdot \text{sec}^{-1}$$

$$v_{G3x} := v_{G3} \cdot \cos(\theta_{VG3}) \quad v_{G3x} = -31.141 \cdot \text{in} \cdot \text{sec}^{-1}$$

$$v_{G3y} := v_{G3} \cdot \sin(\theta_{VG3}) \quad v_{G3y} = 16.495 \cdot \text{in} \cdot \text{sec}^{-1}$$

$$v_{G4x} := v_{G4} \cdot \cos(\theta_{VG4}) \quad v_{G4x} = -35.140 \cdot \text{in} \cdot \text{sec}^{-1}$$

$$v_{P3x} := v_{P3} \cdot \cos(\theta_{VP3}) \quad v_{P3x} = -31.127 \cdot \text{in} \cdot \text{sec}^{-1}$$

$$v_{P3y} := v_{P3} \cdot \sin(\theta_{VP3})$$

$$v_{P3y} = 16.522 \cdot \text{in} \cdot \text{sec}^{-1}$$

2. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$a_{G2x} := a_{G2} \cdot \cos(\theta_{AG2})$$

$$a_{G2x} = -169.705 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$a_{G2y} := a_{G2} \cdot \sin(\theta_{AG2})$$

$$a_{G2y} = -113.136 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$a_{G3x} := a_{G3} \cdot \cos(\theta_{AG3})$$

$$a_{G3x} = -346.803 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$a_{G3y} := a_{G3} \cdot \sin(\theta_{AG3})$$

$$a_{G3y} = -132.015 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$a_{G4x} := a_{G4} \cdot \cos(\theta_{AG4})$$

$$a_{G4x} = -357.170 \cdot \text{in} \cdot \text{sec}^{-2}$$

3. Calculate the x and y components of the external force at P in the CGS.

$$F_{P3x} := F_{P3} \cdot \cos(\delta_{FP3})$$

$$F_{P3x} = 0.000 \cdot \text{lbf}$$

$$F_{P3y} := F_{P3} \cdot \sin(\delta_{FP3})$$

$$F_{P3y} = 0.000 \cdot \text{lbf}$$

4. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \left[m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \dots \right. \\ \left. + m_4 \cdot (a_{G4x} \cdot v_{G4x}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3) \dots \right. \\ \left. + -(F_{P3x} \cdot v_{P3x} + F_{P3y} \cdot v_{P3y}) - T_3 \cdot \omega_3 \right]$$

$$T_{12} = 99.687 \cdot \text{lbf} \cdot \text{in}$$

PROBLEM 11-5a

Statement: Table P11-3 shows kinematic and geometric data for several pin-jointed fourbar linkages of the type and orientation shown in Figure P11-2. All have $\theta_1 = 0$. The point locations are defined as described in the text. For row *a* in the table, solve for forces and torques at the position shown. Also, compute the shaking force and the shaking torque. Work in any units system you prefer.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 4.00 \cdot in \quad \text{Link 3 (A to B)} \quad b := 12.00 \cdot in$$

$$\text{Link 4 (B to O}_4) \quad c := 8.00 \cdot in \quad \text{Link 3 (O}_2 \text{ to O}_4) \quad d := 15.00 \cdot in$$

$$\text{Crank angle and motion: } \theta_2 := 45 \cdot deg \quad \omega_2 := 20 \cdot rad \cdot sec^{-1}$$

$$\text{Other link angles: } \theta_3 := 24.97 \cdot deg \quad \omega_3 := -5.62 \cdot rad \cdot sec^{-1} \quad \theta_4 := 99.30 \cdot deg \quad \omega_4 := 3.56 \cdot rad \cdot sec^{-1}$$

$$\text{Coupler point: } R_{P3} := 0.0 \cdot in \quad \delta_{RP3} := 0.0 \cdot deg$$

$$\text{Rocker point: } R_{P4} := 8.0 \cdot in \quad \delta_{RP4} := 0.0 \cdot deg$$

$$\text{Mass: } m_2 := 0.002 \cdot blob \quad m_3 := 0.020 \cdot blob \quad m_4 := 0.100 \cdot blob$$

$$\text{Moment of inertia: } I_{G2} := 0.10 \cdot blob \cdot in^2 \quad I_{G3} := 0.20 \cdot blob \cdot in^2 \quad I_{G4} := 0.50 \cdot blob \cdot in^2$$

$$\text{Mass center: } R_{CG2} := 2.00 \cdot in \quad \delta_2 := 0 \cdot deg \quad R_{CG3} := 5.00 \cdot in \quad \delta_3 := 0 \cdot deg$$

$$R_{CG4} := 4.00 \cdot in \quad \delta_4 := 30 \cdot deg$$

$$\text{Force and torque: } F_{P3} := 0 \cdot lbf \quad \delta_{FP3} := 0 \cdot deg \quad F_{P4} := 40 \cdot lbf \quad \delta_{FP4} := -30 \cdot deg$$

$$T_3 := -15 \cdot lbf \cdot in \quad T_4 := 25 \cdot lbf \cdot in$$

$$\text{Accelerations: } \alpha_2 := 20 \cdot rad \cdot sec^{-2} \quad a_{G2} := 801.00 \cdot in \cdot sec^{-2} \quad \theta_{AG2} := 222.14 \cdot deg$$

$$\alpha_3 := 75.29 \cdot rad \cdot sec^{-2} \quad a_{G3} := 1691.49 \cdot in \cdot sec^{-2} \quad \theta_{AG3} := 208.24 \cdot deg$$

$$\alpha_4 := 244.43 \cdot rad \cdot sec^{-2} \quad a_{G4} := 979.02 \cdot in \cdot sec^{-2} \quad \theta_{AG4} := 222.27 \cdot deg$$

Solution: See Figure P11-2, Table P11-3 and Mathcad file P1105a.

1. Calculate the x and y components of the position vectors.

$$R_{12x} := R_{CG2} \cdot \cos(\theta_2 + \delta_2 + 180 \cdot deg) \quad R_{12x} = -1.414 \cdot in$$

$$R_{12y} := R_{CG2} \cdot \sin(\theta_2 + \delta_2 + 180 \cdot deg) \quad R_{12y} = -1.414 \cdot in$$

$$R_{32} := \sqrt{(R_{CG2} \cdot \sin(\delta_2))^2 + (a - R_{CG2} \cdot \cos(\delta_2))^2} \quad R_{32} = 2.000 \cdot in$$

$$\delta_{32} := \text{atan2}(a - R_{CG2} \cdot \cos(\delta_2), R_{CG2} \cdot \sin(\delta_2)) \quad \delta_{32} = 0.000 \cdot deg$$

$$R_{32x} := R_{32} \cdot \cos(\theta_2 - \delta_{32}) \quad R_{32x} = 1.414 \cdot in$$

$$R_{32y} := R_{32} \cdot \sin(\theta_2 - \delta_{32}) \quad R_{32y} = 1.414 \cdot in$$

$$R_{23x} := R_{CG3} \cdot \cos(\theta_3 + \delta_3 + 180 \cdot deg) \quad R_{23x} = -4.533 \cdot in$$

$$R_{23y} := R_{CG3} \cdot \sin(\theta_3 + \delta_3 + 180 \cdot deg) \quad R_{23y} = -2.111 \cdot in$$

$$\begin{aligned}
 R_{43} &:= \sqrt{(R_{CG3} \cdot \sin(\delta_3))^2 + (b - R_{CG3} \cdot \cos(\delta_3))^2} & R_{43} &= 7.000 \text{ in} \\
 \delta_{43} &:= \text{atan2}(b - R_{CG3} \cdot \cos(\delta_3), R_{CG3} \cdot \sin(\delta_3)) & \delta_{43} &= 0.000 \text{ deg} \\
 R_{43x} &:= R_{43} \cdot \cos(\theta_3 - \delta_{43}) & R_{43x} &= 6.346 \text{ in} \\
 R_{43y} &:= R_{43} \cdot \sin(\theta_3 - \delta_{43}) & R_{43y} &= 2.955 \text{ in} \\
 R_{14x} &:= R_{CG4} \cdot \cos(\theta_4 + \delta_4 + 180 \cdot \text{deg}) & R_{14x} &= 2.534 \text{ in} \\
 R_{14y} &:= R_{CG4} \cdot \sin(\theta_4 + \delta_4 + 180 \cdot \text{deg}) & R_{14y} &= -3.095 \text{ in} \\
 R_{34} &:= \sqrt{(R_{CG4} \cdot \sin(\delta_4))^2 + (c - R_{CG4} \cdot \cos(\delta_4))^2} & R_{34} &= 4.957 \text{ in} \\
 \delta_{34} &:= \text{atan2}(c - R_{CG4} \cdot \cos(\delta_4), R_{CG4} \cdot \sin(\delta_4)) & \delta_{34} &= 23.794 \text{ deg} \\
 R_{34x} &:= R_{34} \cdot \cos(\theta_4 - \delta_{34}) & R_{34x} &= 1.241 \text{ in} \\
 R_{34y} &:= R_{34} \cdot \sin(\theta_4 - \delta_{34}) & R_{34y} &= 4.799 \text{ in} \\
 R_{P3x} &:= R_{P3} \cdot \cos(\theta_3 + \delta_{RP3}) & R_{P3x} &= 0.000 \text{ in} \\
 R_{P3y} &:= R_{P3} \cdot \sin(\theta_3 + \delta_{RP3}) & R_{P3y} &= 0.000 \text{ in} \\
 R_{P4x} &:= R_{P4} \cdot \cos(\theta_4 + \delta_{RP4}) & R_{P4x} &= -1.293 \text{ in} \\
 R_{P4y} &:= R_{P4} \cdot \sin(\theta_4 + \delta_{RP4}) & R_{P4y} &= 7.895 \text{ in}
 \end{aligned}$$

2. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\begin{aligned}
 a_{G2x} &:= a_{G2} \cdot \cos(\theta_{AG2}) & a_{G2x} &= -593.948 \text{ in} \cdot \text{sec}^{-2} \\
 a_{G2y} &:= a_{G2} \cdot \sin(\theta_{AG2}) & a_{G2y} &= -537.427 \text{ in} \cdot \text{sec}^{-2} \\
 a_{G3x} &:= a_{G3} \cdot \cos(\theta_{AG3}) & a_{G3x} &= -1.490 \times 10^3 \text{ in} \cdot \text{sec}^{-2} \\
 a_{G3y} &:= a_{G3} \cdot \sin(\theta_{AG3}) & a_{G3y} &= -800.355 \text{ in} \cdot \text{sec}^{-2} \\
 a_{G4x} &:= a_{G4} \cdot \cos(\theta_{AG4}) & a_{G4x} &= -724.459 \text{ in} \cdot \text{sec}^{-2} \\
 a_{G4y} &:= a_{G4} \cdot \sin(\theta_{AG4}) & a_{G4y} &= -658.513 \text{ in} \cdot \text{sec}^{-2}
 \end{aligned}$$

3. Calculate the x and y components of the external force on links 3 and 4 in the CGS.

$$\begin{aligned}
 F_{P3x} &:= F_{P3} \cdot \cos(\delta_{FP3}) & F_{P3x} &= 0.000 \text{ lbf} \\
 F_{P3y} &:= F_{P3} \cdot \sin(\delta_{FP3}) & F_{P3y} &= 0.000 \text{ lbf} \\
 F_{P4x} &:= F_{P4} \cdot \cos(\delta_{FP4}) & F_{P4x} &= 34.641 \text{ lbf} \\
 F_{P4y} &:= F_{P4} \cdot \sin(\delta_{FP4}) & F_{P4y} &= -20.000 \text{ lbf}
 \end{aligned}$$

4. Substitute these given and calculated values into the matrix equation 11.9, modified for additional terms. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.10g will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{in} & \frac{R_{12x}}{in} & \frac{-R_{32y}}{in} & \frac{R_{32x}}{in} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{in} & \frac{-R_{23x}}{in} & \frac{-R_{43y}}{in} & \frac{R_{43x}}{in} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{in} & \frac{-R_{34x}}{in} & \frac{-R_{14y}}{in} & \frac{R_{14x}}{in} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot lbf^{-1} \\ m_2 \cdot a_{G2y} \cdot lbf^{-1} \\ I_{G2} \cdot \alpha_2 \cdot lbf^{-1} \cdot in^{-1} \\ (m_3 \cdot a_{G3x} - F_{P3x}) \cdot lbf^{-1} \\ (m_3 \cdot a_{G3y} - F_{P3y}) \cdot lbf^{-1} \\ (I_{G3} \cdot \alpha_3 - R_{P3x} \cdot F_{P3y} + R_{P3y} \cdot F_{P3x} - T_3) \cdot lbf^{-1} \cdot in^{-1} \\ (m_4 \cdot a_{G4x} - F_{P4x}) \cdot lbf^{-1} \\ (m_4 \cdot a_{G4y} - F_{P4y}) \cdot lbf^{-1} \\ (I_{G4} \cdot \alpha_4 - R_{P4x} \cdot F_{P4y} + R_{P4y} \cdot F_{P4x} - T_4) \cdot lbf^{-1} \cdot in^{-1} \end{bmatrix}$$

$$R := C^{-1} \cdot F$$

$$\begin{aligned} F_{12x} &:= R_1 \cdot lbf & F_{12x} &= -124.0 \text{ lbf} & F_{12y} &:= R_2 \cdot lbf & F_{12y} &= -62.3 \text{ lbf} \\ F_{32x} &:= R_3 \cdot lbf & F_{32x} &= 122.8 \text{ lbf} & F_{32y} &:= R_4 \cdot lbf & F_{32y} &= 61.2 \text{ lbf} \\ F_{43x} &:= R_5 \cdot lbf & F_{43x} &= 93.0 \text{ lbf} & F_{43y} &:= R_6 \cdot lbf & F_{43y} &= 45.2 \text{ lbf} \\ F_{14x} &:= R_7 \cdot lbf & F_{14x} &= -14.10 \text{ lbf} & F_{14y} &:= R_8 \cdot lbf & F_{14y} &= -0.676 \text{ lbf} \\ T_{12} &:= R_9 \cdot lbf \cdot in & T_{12} &= 176.4 \text{ lbf} \cdot in \end{aligned}$$

5. Calculate the shaking force and shaking torque using equations 11.15.

$$\mathbf{F}_{21} := -F_{12x} - j \cdot F_{12y} \quad \mathbf{F}_{41} := F_{14x} + j \cdot F_{14y}$$

$$\mathbf{F}_s := \mathbf{F}_{21} + \mathbf{F}_{41} \quad \mathbf{F}_s = (109.868 + 61.581j) \text{ lbf}$$

$$\text{Magnitude: } F_s := |\mathbf{F}_s| \quad F_s = 125.949 \text{ lbf} \quad \text{Angle: } \theta_{F_s} := \arg(\mathbf{F}_s) \quad \theta_{F_s} = 29.271 \text{ deg}$$

$$\mathbf{T}_s := -T_{12} \quad \mathbf{T}_s = -176.4 \text{ lbf} \cdot in$$

PROBLEM 11-6a

Statement: Tables P11-3 and P11-4 show kinematic and geometric data for several pin-jointed fourbar linkages of the type and orientation shown in Figure P11-2. All have $\theta_1 = 0$. The point locations are defined as described in the text. For row *a* in the table, solve for input torque on link 2, using the method of virtual work, at the position shown. Work in any units system you prefer.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

Link 2 (O_2 to A)	$a := 4.00 \cdot in$	Link 3 (A to B)	$b := 12.00 \cdot in$
Link 4 (B to O_4)	$c := 8.00 \cdot in$	Link 3 (O_2 to O_4)	$d := 15.00 \cdot in$

Link angles: $\theta_2 := 45 \cdot deg$ $\theta_3 := 24.97 \cdot deg$ $\theta_4 := 99.30 \cdot deg$

Coupler point: $R_{P3} := 0.0 \cdot in$ $\delta_{RP3} := 0.0 \cdot deg$

Rocker point: $R_{P4} := 8.0 \cdot in$ $\delta_{RP4} := 0.0 \cdot deg$

Mass: $m_2 := 0.002 \cdot blob$ $m_3 := 0.020 \cdot blob$ $m_4 := 0.100 \cdot blob$

Moment of inertia: $I_{G2} := 0.10 \cdot blob \cdot in^2$ $I_{G3} := 0.20 \cdot blob \cdot in^2$ $I_{G4} := 0.50 \cdot blob \cdot in^2$

Mass center: $R_{CG2} := 2.00 \cdot in$ $\delta_2 := 0 \cdot deg$ $R_{CG3} := 5.00 \cdot in$ $\delta_3 := 0 \cdot deg$

$R_{CG4} := 4.00 \cdot in$ $\delta_4 := 30 \cdot deg$

Force and torque: $F_{P3} := 0 \cdot lbf$ $\delta_{FP3} := 0 \cdot deg$ $F_{P4} := 40 \cdot lbf$ $\delta_{FP4} := -30 \cdot deg$

$T_3 := -15 \cdot lbf \cdot in$ $T_4 := 25 \cdot lbf \cdot in$

Accelerations: $\alpha_2 := 20 \cdot rad \cdot sec^{-2}$ $a_{G2} := 801.00 \cdot in \cdot sec^{-2}$ $\theta_{AG2} := 222.14 \cdot deg$

$\alpha_3 := 75.29 \cdot rad \cdot sec^{-2}$ $a_{G3} := 1691.49 \cdot in \cdot sec^{-2}$ $\theta_{AG3} := 208.24 \cdot deg$

$\alpha_4 := 244.43 \cdot rad \cdot sec^{-2}$ $a_{G4} := 979.02 \cdot in \cdot sec^{-2}$ $\theta_{AG4} := 222.27 \cdot deg$

Velocities: $\omega_2 := 20 \cdot rad \cdot sec^{-1}$ $v_{G2} := 40.0 \cdot in \cdot sec^{-1}$ $\theta_{VG2} := 135.0 \cdot deg$

$\omega_3 := -5.62 \cdot rad \cdot sec^{-1}$ $v_{G3} := 54.44 \cdot in \cdot sec^{-1}$ $\theta_{VG3} := 145.19 \cdot deg$

$\omega_4 := 3.56 \cdot rad \cdot sec^{-1}$ $v_{G4} := 14.23 \cdot in \cdot sec^{-1}$ $\theta_{VG4} := 219.30 \cdot deg$

$v_{P3} := 80.00 \cdot in \cdot sec^{-1}$ $\theta_{VP3} := 135.00 \cdot deg$

$v_{P4} := 28.45 \cdot in \cdot sec^{-1}$ $\theta_{VP4} := 189.30 \cdot deg$

Solution: See Figure P11-2, Table P11-3, Table P11-4, and Mathcad file P1106a.

1. Calculate the x and y components of the velocity vectors.

$v_{G2x} := v_{G2} \cdot \cos(\theta_{VG2})$ $v_{G2x} = -28.284 \cdot in \cdot sec^{-1}$

$v_{G2y} := v_{G2} \cdot \sin(\theta_{VG2})$ $v_{G2y} = 28.284 \cdot in \cdot sec^{-1}$

$v_{G3x} := v_{G3} \cdot \cos(\theta_{VG3})$ $v_{G3x} = -44.698 \cdot in \cdot sec^{-1}$

$v_{G3y} := v_{G3} \cdot \sin(\theta_{VG3})$ $v_{G3y} = 31.077 \cdot in \cdot sec^{-1}$

$v_{G4x} := v_{G4} \cdot \cos(\theta_{VG4})$ $v_{G4x} = -11.012 \cdot in \cdot sec^{-1}$

$$v_{G4y} := v_{G4} \cdot \sin(\theta_{VG4})$$

$$v_{G4y} = -9.013 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{P3x} := v_{P3} \cdot \cos(\theta_{VP3})$$

$$v_{P3x} = -56.569 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{P3y} := v_{P3} \cdot \sin(\theta_{VP3})$$

$$v_{P3y} = 56.569 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{P4x} := v_{P4} \cdot \cos(\theta_{VP4})$$

$$v_{P4x} = -28.076 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{P4y} := v_{P4} \cdot \sin(\theta_{VP4})$$

$$v_{P4y} = -4.598 \text{ in} \cdot \text{sec}^{-1}$$

2. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$a_{G2x} := a_{G2} \cdot \cos(\theta_{AG2})$$

$$a_{G2x} = -593.948 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G2y} := a_{G2} \cdot \sin(\theta_{AG2})$$

$$a_{G2y} = -537.427 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G3x} := a_{G3} \cdot \cos(\theta_{AG3})$$

$$a_{G3x} = -1.490 \times 10^3 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G3y} := a_{G3} \cdot \sin(\theta_{AG3})$$

$$a_{G3y} = -800.355 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G4x} := a_{G4} \cdot \cos(\theta_{AG4})$$

$$a_{G4x} = -724.459 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G4y} := a_{G4} \cdot \sin(\theta_{AG4})$$

$$a_{G4y} = -658.513 \text{ in} \cdot \text{sec}^{-2}$$

3. Calculate the x and y components of the external force on links 3 and 4 in the CGS.

$$F_{P3x} := F_{P3} \cdot \cos(\delta_{FP3})$$

$$F_{P3x} = 0.000 \text{ lbf}$$

$$F_{P3y} := F_{P3} \cdot \sin(\delta_{FP3})$$

$$F_{P3y} = 0.000 \text{ lbf}$$

$$F_{P4x} := F_{P4} \cdot \cos(\delta_{FP4})$$

$$F_{P4x} = 34.641 \text{ lbf}$$

$$F_{P4y} := F_{P4} \cdot \sin(\delta_{FP4})$$

$$F_{P4y} = -20.000 \text{ lbf}$$

4. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \left[m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \dots \right. \\ \left. + m_4 \cdot (a_{G4x} \cdot v_{G4x} + a_{G4y} \cdot v_{G4y}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3 + I_{G4} \cdot \alpha_4 \cdot \omega_4) \dots \right. \\ \left. + -(F_{P3x} \cdot v_{P3x} + F_{P3y} \cdot v_{P3y}) - (F_{P4x} \cdot v_{P4x} + F_{P4y} \cdot v_{P4y}) \dots \right. \\ \left. + -T_3 \cdot \omega_3 - T_4 \cdot \omega_4 \right]$$

$$T_{12} = 166.3 \text{ lbf} \cdot \text{in}$$

PROBLEM 11-7a

Statement: For row *a* in Table P11-3, input the associated disk file to program FOURBAR, calculate the linkage parameters for crank angles from zero to 360 deg by 5 deg increments, and design a steel disk flywheel to smooth the input torque using a coefficient of fluctuation of 0.05. Minimize the flywheel weight.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\begin{array}{llll} \text{Link 2 (O}_2 \text{ to A)} & a := 4.00 \cdot in & \text{Link 3 (A to B)} & b := 12.00 \cdot in \\ \text{Link 4 (B to O}_4) & c := 8.00 \cdot in & \text{Link 3 (O}_2 \text{ to O}_4) & d := 15.00 \cdot in \end{array}$$

$$\text{Crank angle and motion: } \theta_2 := 45 \cdot deg \quad \omega_2 := 20 \cdot rad \cdot sec^{-1}$$

$$\text{Other link angles: } \theta_3 := 24.97 \cdot deg \quad \omega_3 := -5.62 \cdot rad \cdot sec^{-1} \quad \theta_4 := 99.30 \cdot deg \quad \omega_4 := 3.56 \cdot rad \cdot sec^{-1}$$

$$\text{Coupler point: } R_{P3} := 0.0 \cdot in \quad \delta_{RP3} := 0.0 \cdot deg$$

$$\text{Rocker point: } R_{P4} := 8.0 \cdot in \quad \delta_{RP4} := 0.0 \cdot deg$$

$$\text{Mass: } m_2 := 0.002 \cdot blob \quad m_3 := 0.020 \cdot blob \quad m_4 := 0.100 \cdot blob$$

$$\text{Moment of inertia: } I_{G2} := 0.10 \cdot blob \cdot in^2 \quad I_{G3} := 0.20 \cdot blob \cdot in^2 \quad I_{G4} := 0.50 \cdot blob \cdot in^2$$

$$\begin{array}{llll} \text{Mass center: } R_{CG2} := 2.00 \cdot in & \delta_2 := 0 \cdot deg & R_{CG3} := 5.00 \cdot in & \delta_3 := 0 \cdot deg \\ & R_{CG4} := 4.00 \cdot in & \delta_4 := 30 \cdot deg & \end{array}$$

$$\begin{array}{llll} \text{Force and torque: } F_{P3} := 0 \cdot lbf & \delta_{FP3} := 0 \cdot deg & F_{P4} := 40 \cdot lbf & \delta_{FP4} := -30 \cdot deg \\ & T_3 := -15 \cdot lbf \cdot in & T_4 := 25 \cdot lbf \cdot in & \end{array}$$

$$\begin{array}{llll} \text{Accelerations: } \alpha_2 := 20 \cdot rad \cdot sec^{-2} & a_{G2} := 801.00 \cdot in \cdot sec^{-2} & \theta_{AG2} := 222.14 \cdot deg \\ & \alpha_3 := 75.29 \cdot rad \cdot sec^{-2} & a_{G3} := 1691.49 \cdot in \cdot sec^{-2} & \theta_{AG3} := 208.24 \cdot deg \\ & \alpha_4 := 244.43 \cdot rad \cdot sec^{-2} & a_{G4} := 979.02 \cdot in \cdot sec^{-2} & \theta_{AG4} := 222.27 \cdot deg \end{array}$$

$$\text{Coefficient of fluctuation: } k := 0.05$$

$$\text{Desired average speed: } \omega_{avg} := \omega_2$$

Solution: See Figure P11-2, Table P11-3 and Mathcad file P1107a.

1. Enter the above data into program FOURBAR and determine the energy change from minimum to maximum speed by calculating the dynamic values and then plotting the input torque vs. crank angle. See screens below.
2. Integrate the torque function using the values in the "Areas of Crank Torque Pulses" shown in the Linkage Balancing screen below as related to the Torque vs. Crank Angle screen above (see Table 11-1 in text).

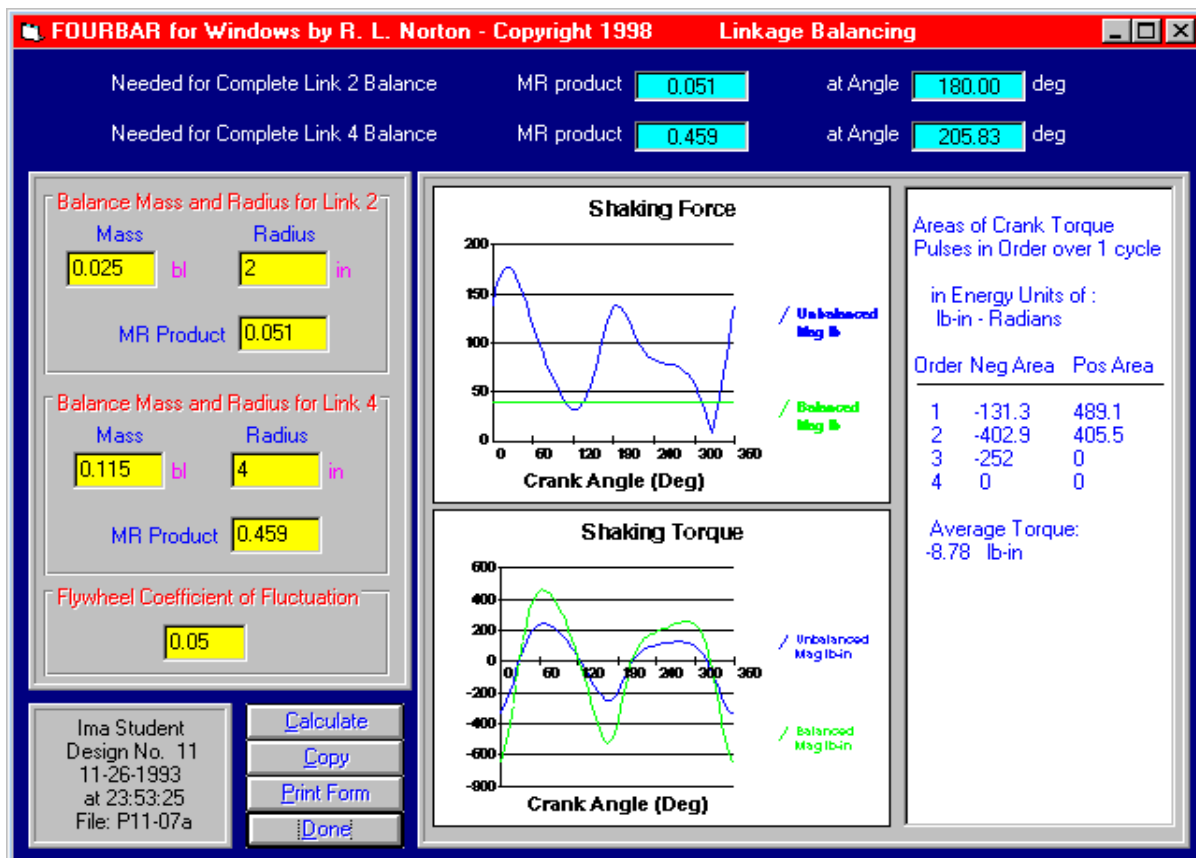
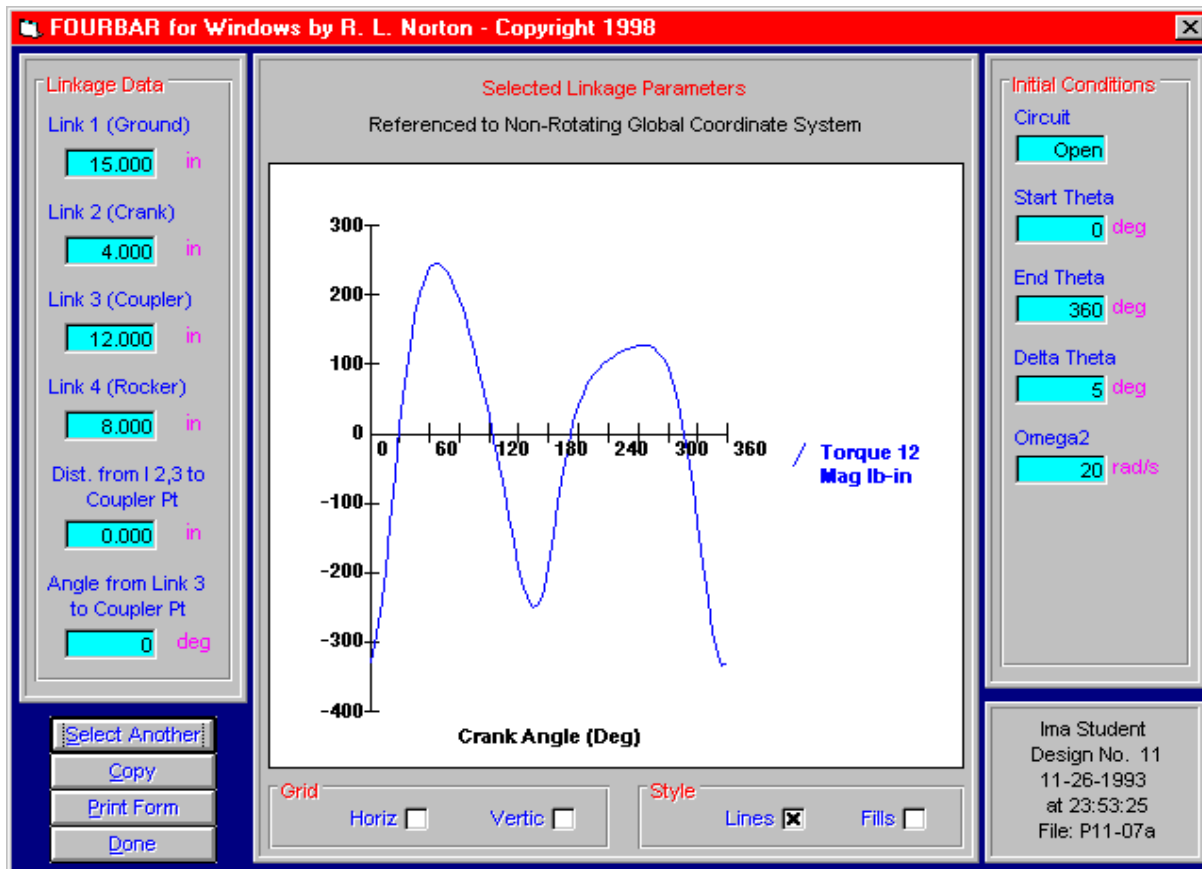
$$\begin{array}{llll} \text{A to B} & +489.1 & +489.1 & \\ \text{B to C} & -402.9 & +86.2 & \omega_{max} \text{ at C} \\ \text{C to D} & +405.5 & +491.7 & \omega_{min} \text{ at D} \\ \text{D to A} & -383.3 & +108.4 & \end{array}$$

$$E_{maxspeed} := 86.2 \cdot in \cdot lbf \quad E_{minspeed} := 491.7 \cdot in \cdot lbf$$

$$E := |E_{maxspeed} - E_{minspeed}| \quad E = 405.5 \cdot in \cdot lbf$$

$$I_s := \frac{E}{k \cdot \omega_{avg}^2} \quad I_s = 20.3 \cdot blob \cdot in^2$$

The moment of inertia of the input crank and the motor armature can be subtracted from this value to obtain the required flywheel moment. There are an infinity of possible size/shape solutions for this problem.



PROBLEM 11-8

Statement: Figure P11-3 shows a fourbar linkage and its dimensions. The steel crank and rocker have uniform cross sections 1 in wide by 0.5 in thick. The aluminum coupler is 0.75 in thick. In the instantaneous position shown, the crank O_2A has $\omega = 40 \text{ rad/sec}$ and $\alpha = -20 \text{ rad/sec}^2$. There is a horizontal force at P of $F = 50 \text{ lb}$. Find all pin forces and the torque needed to drive the crank at this instant.

Units: $\text{blob} := \text{lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 5.00 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 4.40 \cdot \text{in}$$

$$\text{Link 4 (B to O}_4) \quad c := 5.00 \cdot \text{in} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 9.50 \cdot \text{in}$$

$$\text{Coupler point:} \quad R_{pa} := 8.90 \cdot \text{in} \quad \delta_3 := 56 \cdot \text{deg} \quad F := 50 \cdot \text{lbf} \quad T_4 := 0 \cdot \text{lbf} \cdot \text{in}$$

$$\text{Crank angle and motion:} \quad \theta_2 := 50 \cdot \text{deg} \quad \omega_2 := 40 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := -20 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 1.00 \cdot \text{in} \quad t_2 := 0.50 \cdot \text{in} \quad t_3 := 0.75 \cdot \text{in} \quad w_4 := 1.00 \cdot \text{in} \quad t_4 := 0.50 \cdot \text{in}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lbf} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Solution: See Figure P11-3 and Mathcad file P1108.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$$\theta_3 := 10.105 \cdot \text{deg} \quad \omega_3 := -41.552 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_3 := -335.762 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\theta_4 := 113.008 \cdot \text{deg} \quad \omega_4 := 26.320 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_4 := 2963.667 \cdot \text{rad} \cdot \text{sec}^{-2}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 2.500 \text{ in} \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 2.500 \text{ in}$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 3.126 \text{ in}$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = 2.459 \text{ in}$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 3.977 \text{ in}$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = 38.199 \text{ deg}$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot R_{pa} \cdot \sin(\delta_3) \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 1.943 \times 10^{-3} \text{ blob} \quad m_3 = 3.153 \times 10^{-3} \text{ blob} \quad m_4 = 1.943 \times 10^{-3} \text{ blob}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 4.209 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

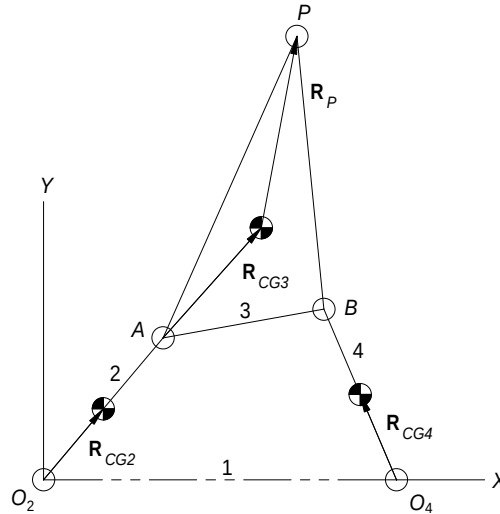
$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2]$$

$$I_{G3} = 0.039 \text{ blob} \cdot \text{in}^2$$

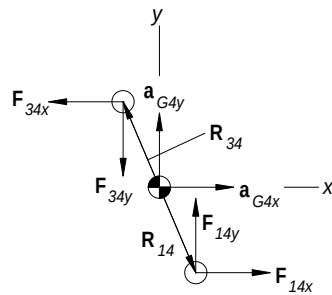
$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2)$$

$$I_{G4} = 4.209 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

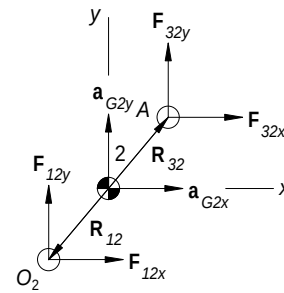
4. Set up an LNCS xy coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.



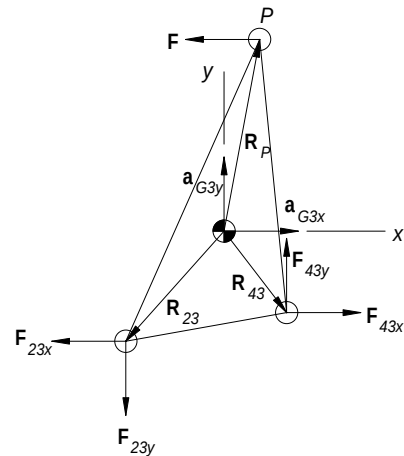
(a) The complete linkage with GCS



(d) FBD of Link 4



(b) FBD of Link 2



(c) FBD of Link 3

5. Calculate the x and y components of the position vectors.

$$R_{12x} := R_{CG2} \cdot \cos(\theta_2 + 180 \cdot \text{deg})$$

$$R_{12x} = -1.607 \text{ in}$$

$$R_{12y} := R_{CG2} \cdot \sin(\theta_2 + 180 \cdot \text{deg})$$

$$R_{12y} = -1.915 \text{ in}$$

$$R_{32x} := R_{CG2} \cdot \cos(\theta_2)$$

$$R_{32x} = 1.607 \text{ in}$$

$$R_{32y} := R_{CG2} \cdot \sin(\theta_2)$$

$$R_{32y} = 1.915 \text{ in}$$

$$R_{23x} := R_{CG3} \cdot \cos(\delta_{33} + \theta_3 + 180 \cdot \text{deg})$$

$$R_{23x} = -2.646 \text{ in}$$

$$R_{23y} := R_{CG3} \cdot \sin(\delta_{33} + \theta_3 + 180 \cdot \text{deg})$$

$$R_{23y} = -2.970 \text{ in}$$

$$R_{43x} := b \cdot \cos(\theta_3) - R_{CG3} \cdot \cos(\theta_3 + \delta_{33}) \quad R_{43x} = 1.686 \text{ in}$$

$$R_{43y} := -(R_{CG3} \cdot \sin(\theta_3 + \delta_{33}) - b \cdot \sin(\theta_3)) \quad R_{43y} = -2.198 \text{ in}$$

$$R_{34x} := R_{CG4} \cdot \cos(\theta_4) \quad R_{34x} = -0.977 \text{ in}$$

$$R_{34y} := R_{CG4} \cdot \sin(\theta_4) \quad R_{34y} = 2.301 \text{ in}$$

$$R_{14x} := R_{CG4} \cdot \cos(\theta_4 + 180 \cdot \text{deg}) \quad R_{14x} = 0.977 \text{ in}$$

$$R_{14y} := R_{CG4} \cdot \sin(\theta_4 + 180 \cdot \text{deg}) \quad R_{14y} = -2.301 \text{ in}$$

$$R_{Px} := R_{pa} \cdot \cos(\theta_3 + \delta_3) - |R_{23x}| \quad R_{Px} = 0.959 \text{ in}$$

$$R_{Py} := R_{pa} \cdot \sin(\theta_3 + \delta_3) - |R_{23y}| \quad R_{Py} = 5.167 \text{ in}$$

6. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2} := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$a_{G2x} := \text{Re}(\mathbf{a}_{G2}) \quad a_{G2x} = -5.104 \times 10^3 \frac{\text{in}}{\text{sec}^2}$$

$$a_{G2y} := \text{Im}(\mathbf{a}_{G2}) \quad a_{G2y} = -6.160 \times 10^3 \frac{\text{in}}{\text{sec}^2}$$

$$\mathbf{a}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{a}_{CG3A} := R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_3 + \delta_{33}) + j \cdot \cos(\theta_3 + \delta_{33})) \dots \\ + -R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_3 + \delta_{33}) + j \cdot \sin(\theta_3 + \delta_{33}))$$

$$\mathbf{a}_{G3} := \mathbf{a}_A + \mathbf{a}_{CG3A} \quad a_{G3x} := \text{Re}(\mathbf{a}_{G3}) \quad a_{G3x} = -8.636 \times 10^3 \frac{\text{in}}{\text{sec}^2}$$

$$a_{G3y} := \text{Im}(\mathbf{a}_{G3}) \quad a_{G3y} = -1.221 \times 10^4 \frac{\text{in}}{\text{sec}^2}$$

$$\mathbf{a}_{G4} := R_{CG4} \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$a_{G4x} := \text{Re}(\mathbf{a}_{G4}) \quad a_{G4x} = -5.466 \times 10^3 \frac{\text{in}}{\text{sec}^2}$$

$$a_{G4y} := \text{Im}(\mathbf{a}_{G4}) \quad a_{G4y} = -6.084 \times 10^3 \frac{\text{in}}{\text{sec}^2}$$

7. Calculate the x and y components of the external force at P in the CGS.

$$F_{Px} := -F \quad F_{Py} := 0 \cdot \text{lbf}$$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{in} & \frac{R_{12x}}{in} & \frac{-R_{32y}}{in} & \frac{R_{32x}}{in} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{in} & \frac{-R_{23x}}{in} & \frac{-R_{43y}}{in} & \frac{R_{43x}}{in} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{in} & \frac{-R_{34x}}{in} & \frac{-R_{14y}}{in} & \frac{R_{14x}}{in} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot lbf^{-1} \\ m_2 \cdot a_{G2y} \cdot lbf^{-1} \\ I_{G2} \cdot \alpha_2 \cdot lbf^{-1} \cdot in^{-1} \\ (m_3 \cdot a_{G3x} - F_{Px}) \cdot lbf^{-1} \\ (m_3 \cdot a_{G3y} - F_{Py}) \cdot lbf^{-1} \\ (I_{G3} \cdot \alpha_3 - R_{Px} \cdot F_{Py} + R_{Py} \cdot F_{Px}) \cdot lbf^{-1} \cdot in^{-1} \\ m_4 \cdot a_{G4x} \cdot lbf^{-1} \\ m_4 \cdot a_{G4y} \cdot lbf^{-1} \\ (I_{G4} \cdot \alpha_4 - T_4) \cdot lbf^{-1} \cdot in^{-1} \end{bmatrix}$$

$$R := C^{-1} \cdot F$$

$$F_{12x} := R_1 \cdot lbf \quad F_{12x} = -37.1 \, lbf$$

$$F_{12y} := R_2 \cdot lbf \quad F_{12y} = 42.4 \, lbf$$

$$F_{32x} := R_3 \cdot lbf \quad F_{32x} = 27.2 \, lbf$$

$$F_{32y} := R_4 \cdot lbf \quad F_{32y} = -54.4 \, lbf$$

$$F_{43x} := R_5 \cdot lbf \quad F_{43x} = 50.0 \, lbf$$

$$F_{43y} := R_6 \cdot lbf \quad F_{43y} = -92.9 \, lbf$$

$$F_{14x} := R_7 \cdot lbf \quad F_{14x} = 39.3 \, lbf$$

$$F_{14y} := R_8 \cdot lbf \quad F_{14y} = -104.7 \, lbf$$

$$T_{12} := R_9 \cdot lbf \cdot in \quad T_{12} = 279 \, lbf \cdot in$$

PROBLEM 11-9

Statement: Figure P11-4a shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 10$ rad/sec and $\alpha = 5$ rad/sec². There is a vertical force at P of $F = 100$ N. Find all pin forces and the torque needed to drive the crank at this instant.

Given:

Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 1.00 \cdot m \quad \text{Link 3 (A to B)} \quad b := 2.06 \cdot m$$

$$\text{Link 4 (B to O}_4) \quad c := 2.33 \cdot m \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 2.22 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 3.06 \cdot m \quad \delta_3 := -31 \cdot \text{deg} \quad F := 100 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank angle and motion:} \quad \theta_2 := 60 \cdot \text{deg} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 5 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3}$$

Solution: See Figure P11-4a and Mathcad file P1109.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$$\theta_3 := 44.732 \cdot \text{deg} \quad \omega_3 := -3.669 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_3 := 55.752 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\theta_4 := 96.322 \cdot \text{deg} \quad \omega_4 := 1.442 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_4 := 67.103 \cdot \text{rad} \cdot \text{sec}^{-2}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.500 \cdot m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 1.165 \cdot m$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 1.561 \cdot m$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = -0.525 \cdot m$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 1.647 \cdot m$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = -18.600 \cdot \text{deg}$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot |R_{pa} \cdot \sin(\delta_3)| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 10.380 \cdot \text{kg} \quad m_3 = 112.332 \cdot \text{kg} \quad m_4 = 24.185 \cdot \text{kg}$$

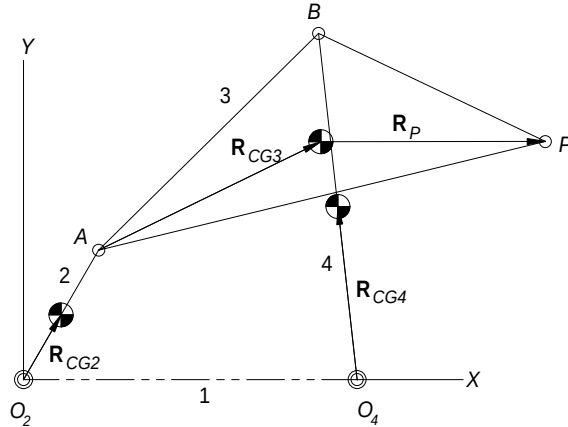
$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.867 \cdot \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2] \quad I_{G3} = 125.951 \cdot \text{kg} \cdot \text{m}^2$$

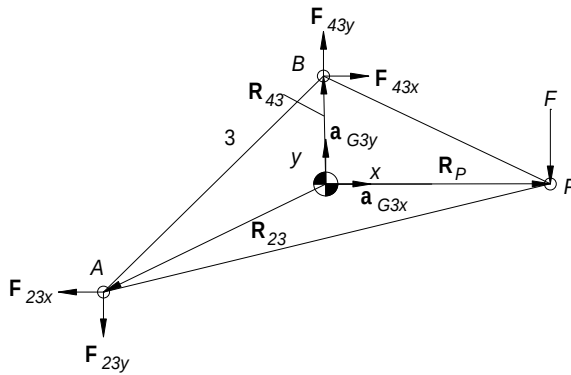
$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2)$$

$$I_{G4} = 10.947 \text{ kg} \cdot \text{m}^2$$

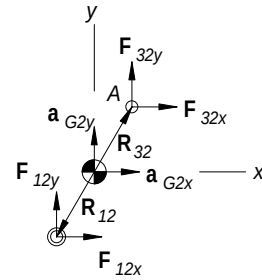
4. Set up an LNCS xy coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.



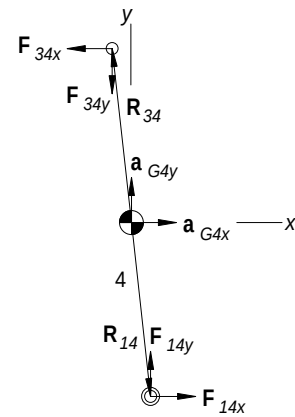
(a) The complete linkage with GCS



(c) FBD of Link 3



(b) FBD of Link 2



(d) FBD of Link 4

5. Calculate the x and y components of the position vectors.

$$R_{12x} := R_{CG2} \cdot \cos(\theta_2 + 180 \cdot \text{deg})$$

$$R_{12x} = -0.250 \text{ m}$$

$$R_{12y} := R_{CG2} \cdot \sin(\theta_2 + 180 \cdot \text{deg})$$

$$R_{12y} = -0.433 \text{ m}$$

$$R_{32x} := R_{CG2} \cdot \cos(\theta_2)$$

$$R_{32x} = 0.250 \text{ m}$$

$$R_{32y} := R_{CG2} \cdot \sin(\theta_2)$$

$$R_{32y} = 0.433 \text{ m}$$

$$R_{23x} := R_{CG3} \cdot \cos(\delta_{33} + \theta_3 + 180 \cdot \text{deg})$$

$$R_{23x} = -1.479 \text{ m}$$

$$R_{23y} := R_{CG3} \cdot \sin(\delta_{33} + \theta_3 + 180 \cdot \text{deg})$$

$$R_{23y} = -0.725 \text{ m}$$

$$R_{43x} := b \cdot \cos(\theta_3) - R_{CG3} \cdot \cos(\theta_3 + \delta_{33})$$

$$R_{43x} = -0.015 \text{ m}$$

$$R_{43y} := -(R_{CG3} \cdot \sin(\theta_3 + \delta_{33}) - b \cdot \sin(\theta_3))$$

$$R_{43y} = 0.724 \text{ m}$$

$$\begin{aligned}
 R_{34x} &:= R_{CG4} \cdot \cos(\theta_4) & R_{34x} &= -0.128 \text{ m} \\
 R_{34y} &:= R_{CG4} \cdot \sin(\theta_4) & R_{34y} &= 1.158 \text{ m} \\
 R_{14x} &:= R_{CG4} \cdot \cos(\theta_4 + 180 \cdot \text{deg}) & R_{14x} &= 0.128 \text{ m} \\
 R_{14y} &:= R_{CG4} \cdot \sin(\theta_4 + 180 \cdot \text{deg}) & R_{14y} &= -1.158 \text{ m} \\
 R_{Px} &:= R_{pa} \cdot \cos(\theta_3 + \delta_3) - |R_{23x}| & R_{Px} &= 1.494 \text{ m} \\
 R_{Py} &:= R_{pa} \cdot \sin(\theta_3 + \delta_3) - |R_{23y}| & R_{Py} &= 9.865 \times 10^{-4} \text{ m}
 \end{aligned}$$

6. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\begin{aligned}
 \mathbf{a}_{G2} &:= R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \\
 a_{G2x} &:= \text{Re}(\mathbf{a}_{G2}) & a_{G2x} &= -52.165 \frac{\text{m}}{\text{sec}^2} \\
 a_{G2y} &:= \text{Im}(\mathbf{a}_{G2}) & a_{G2y} &= -85.353 \frac{\text{m}}{\text{sec}^2} \\
 \mathbf{a}_A &:= a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \\
 \mathbf{a}_{CG3A} &:= R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_3 + \delta_{33}) + j \cdot \cos(\theta_3 + \delta_{33})) \dots \\
 &\quad + -R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_3 + \delta_{33}) + j \cdot \sin(\theta_3 + \delta_{33})) \\
 \mathbf{a}_{G3} &:= \mathbf{a}_A + \mathbf{a}_{CG3A} & a_{G3x} &:= \text{Re}(\mathbf{a}_{G3}) & a_{G3x} &= -114.678 \frac{\text{m}}{\text{sec}^2} \\
 & & a_{G3y} &:= \text{Im}(\mathbf{a}_{G3}) & a_{G3y} &= -11.429 \frac{\text{m}}{\text{sec}^2} \\
 \mathbf{a}_{G4} &:= R_{CG4} \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4)) \\
 a_{G4x} &:= \text{Re}(\mathbf{a}_{G4}) & a_{G4x} &= -77.166 \frac{\text{m}}{\text{sec}^2} \\
 a_{G4y} &:= \text{Im}(\mathbf{a}_{G4}) & a_{G4y} &= -13.424 \frac{\text{m}}{\text{sec}^2}
 \end{aligned}$$

7. Calculate the x and y components of the external force at P in the CGS.

$$F_{Px} := 0 \cdot N \quad F_{Py} := -F$$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{m} & \frac{R_{12x}}{m} & \frac{-R_{32y}}{m} & \frac{R_{32x}}{m} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{m} & \frac{-R_{23x}}{m} & \frac{-R_{43y}}{m} & \frac{R_{43x}}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{m} & \frac{-R_{34x}}{m} & \frac{-R_{14y}}{m} & \frac{R_{14x}}{m} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot N^{-1} \\ m_2 \cdot a_{G2y} \cdot N^{-1} \\ I_{G2} \cdot \alpha_2 \cdot N^{-1} \cdot m^{-1} \\ (m_3 \cdot a_{G3x} - F_{Px}) \cdot N^{-1} \\ (m_3 \cdot a_{G3y} - F_{Py}) \cdot N^{-1} \\ (I_{G3} \cdot \alpha_3 - R_{Px} \cdot F_{Py} + R_{Py} \cdot F_{Px}) \cdot N^{-1} \cdot m^{-1} \\ m_4 \cdot a_{G4x} \cdot N^{-1} \\ m_4 \cdot a_{G4y} \cdot N^{-1} \\ (I_{G4} \cdot \alpha_4 - T_4) \cdot N^{-1} \cdot m^{-1} \end{bmatrix}$$

$$R := C^{-1} \cdot F$$

$$F_{12x} := R_1 \cdot N \quad F_{12x} = -13559 \, N \quad F_{12y} := R_2 \cdot N \quad F_{12y} = -12294 \, N$$

$$F_{32x} := R_3 \cdot N \quad F_{32x} = 13018 \, N \quad F_{32y} := R_4 \cdot N \quad F_{32y} = 11408 \, N$$

$$F_{43x} := R_5 \cdot N \quad F_{43x} = 136 \, N \quad F_{43y} := R_6 \cdot N \quad F_{43y} = 10224 \, N$$

$$F_{14x} := R_7 \cdot N \quad F_{14x} = -1731 \, N \quad F_{14y} := R_8 \cdot N \quad F_{14y} = 9899 \, N$$

$$T_{12} := R_9 \cdot N \cdot m \quad T_{12} = 5587 \, N \cdot m$$

PROBLEM 11-10

Statement: Figure P11-4b shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 15 \text{ rad/sec}$ and $\alpha = -10 \text{ rad/sec}^2$. There is a horizontal force at P of $F = 200 \text{ N}$. Find all pin forces and the torque needed to drive the crank at this instant.

Given:

Link lengths:

$$\text{Link 2 (} O_2 \text{ to } A) \quad a := 0.72 \cdot m \quad \text{Link 3 (} A \text{ to } B) \quad b := 0.68 \cdot m$$

$$\text{Link 4 (} B \text{ to } O_4) \quad c := 0.85 \cdot m \quad \text{Link 1 (} O_2 \text{ to } O_4) \quad d := 1.82 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 0.97 \cdot m \quad \delta_3 := 54 \cdot \text{deg} \quad F := 200 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank angle and motion:} \quad \theta_2 := 30 \cdot \text{deg} \quad \omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := -10 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lbf} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Solution: See Figure P11-4b and Mathcad file P1110.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$$\theta_3 := 23.290 \cdot \text{deg} \quad \omega_3 := -16.412 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_3 := -138.628 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\theta_4 := 132.283 \cdot \text{deg} \quad \omega_4 := 1.570 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_4 := 427.881 \cdot \text{rad} \cdot \text{sec}^{-2}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.360 \text{ m} \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.425 \text{ m}$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 0.417 \text{ m}$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = 0.262 \text{ m}$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 0.492 \text{ m}$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = 32.117 \text{ deg}$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot \left| R_{pa} \cdot \sin(\delta_3) \right| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 7.474 \text{ kg}$$

$$m_3 = 18.463 \text{ kg}$$

$$m_4 = 8.823 \text{ kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2)$$

$$I_{G2} = 0.324 \text{ kg} \cdot \text{m}^2$$

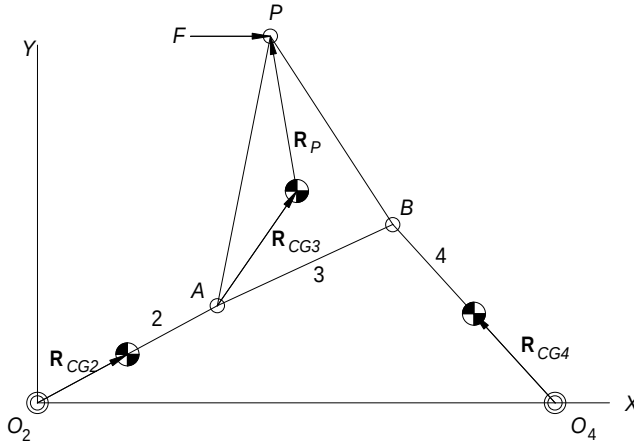
$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2]$$

$$I_{G3} = 3.318 \text{ kg} \cdot \text{m}^2$$

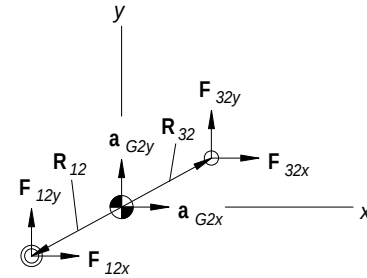
$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2)$$

$$I_{G4} = 0.533 \text{ kg} \cdot \text{m}^2$$

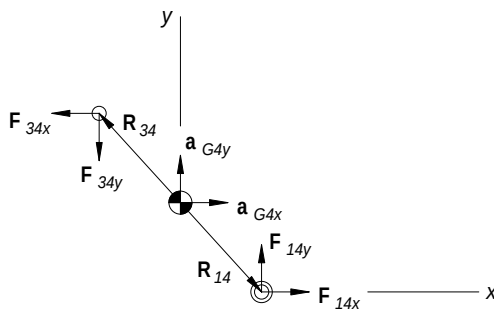
4. Set up an LNCS xy coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.



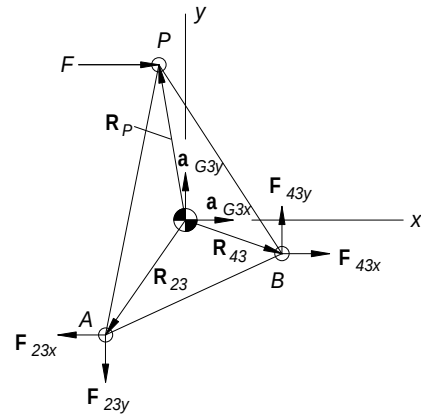
(a) The complete linkage with GCS



(b) FBD of Link 2



(d) FBD of Link 4



(c) FBD of Link 3

5. Calculate the x and y components of the position vectors.

$$R_{12x} := R_{CG2} \cdot \cos(\theta_2 + 180 \cdot \text{deg})$$

$$R_{12x} = -0.312 \text{ m}$$

$$R_{12y} := R_{CG2} \cdot \sin(\theta_2 + 180 \cdot \text{deg})$$

$$R_{12y} = -0.180 \text{ m}$$

$$R_{32x} := R_{CG2} \cdot \cos(\theta_2)$$

$$R_{32x} = 0.312 \text{ m}$$

$$R_{32y} := R_{CG2} \cdot \sin(\theta_2)$$

$$R_{32y} = 0.180 \text{ m}$$

$$R_{23x} := R_{CG3} \cdot \cos(\delta_{33} + \theta_3 + 180 \cdot \text{deg})$$

$$R_{23x} = -0.279 \text{ m}$$

$$\begin{aligned}
 R_{23y} &:= R_{CG3} \cdot \sin(\delta_{33} + \theta_3 + 180 \cdot \text{deg}) & R_{23y} &= -0.405 \text{ m} \\
 R_{43x} &:= b \cdot \cos(\theta_3) - R_{CG3} \cdot \cos(\theta_3 + \delta_{33}) & R_{43x} &= 0.345 \text{ m} \\
 R_{43y} &:= -(R_{CG3} \cdot \sin(\theta_3 + \delta_{33}) - b \cdot \sin(\theta_3)) & R_{43y} &= -0.136 \text{ m} \\
 R_{34x} &:= R_{CG4} \cdot \cos(\theta_4) & R_{34x} &= -0.286 \text{ m} \\
 R_{34y} &:= R_{CG4} \cdot \sin(\theta_4) & R_{34y} &= 0.314 \text{ m} \\
 R_{14x} &:= R_{CG4} \cdot \cos(\theta_4 + 180 \cdot \text{deg}) & R_{14x} &= 0.286 \text{ m} \\
 R_{14y} &:= R_{CG4} \cdot \sin(\theta_4 + 180 \cdot \text{deg}) & R_{14y} &= -0.314 \text{ m} \\
 R_{Px} &:= R_{pa} \cdot \cos(\theta_3 + \delta_3) - |R_{23x}| & R_{Px} &= -0.066 \text{ m} \\
 R_{Py} &:= R_{pa} \cdot \sin(\theta_3 + \delta_3) - |R_{23y}| & R_{Py} &= 0.541 \text{ m}
 \end{aligned}$$

6. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\begin{aligned}
 \mathbf{a}_{G2} &:= R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \\
 a_{G2x} &:= \text{Re}(\mathbf{a}_{G2}) & a_{G2x} &= -138.496 \frac{\text{m}}{\text{sec}^2} \\
 a_{G2y} &:= \text{Im}(\mathbf{a}_{G2}) & a_{G2y} &= -84.118 \frac{\text{m}}{\text{sec}^2} \\
 \mathbf{a}_A &:= a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \\
 \mathbf{a}_{CG3A} &:= R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_3 + \delta_{33}) + j \cdot \cos(\theta_3 + \delta_{33})) \dots \\
 &\quad + -R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_3 + \delta_{33}) + j \cdot \sin(\theta_3 + \delta_{33})) \\
 \mathbf{a}_{G3} &:= \mathbf{a}_A + \mathbf{a}_{CG3A} & a_{G3x} &:= \text{Re}(\mathbf{a}_{G3}) & a_{G3x} &= -155.788 \frac{\text{m}}{\text{sec}^2} \\
 & & a_{G3y} &:= \text{Im}(\mathbf{a}_{G3}) & a_{G3y} &= -235.056 \frac{\text{m}}{\text{sec}^2} \\
 \mathbf{a}_{G4} &:= R_{CG4} \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4)) \\
 a_{G4x} &:= \text{Re}(\mathbf{a}_{G4}) & a_{G4x} &= -133.128 \frac{\text{m}}{\text{sec}^2} \\
 a_{G4y} &:= \text{Im}(\mathbf{a}_{G4}) & a_{G4y} &= -123.897 \frac{\text{m}}{\text{sec}^2}
 \end{aligned}$$

7. Calculate the x and y components of the external force at P in the CGS.

$$F_{Px} := F \quad F_{Py} := 0 \cdot N$$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{m} & \frac{R_{12x}}{m} & \frac{-R_{32y}}{m} & \frac{R_{32x}}{m} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{m} & \frac{-R_{23x}}{m} & \frac{-R_{43y}}{m} & \frac{R_{43x}}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{m} & \frac{-R_{34x}}{m} & \frac{-R_{14y}}{m} & \frac{R_{14x}}{m} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot N^{-1} \\ m_2 \cdot a_{G2y} \cdot N^{-1} \\ I_{G2} \cdot \alpha_2 \cdot N^{-1} \cdot m^{-1} \\ (m_3 \cdot a_{G3x} - F_{Px}) \cdot N^{-1} \\ (m_3 \cdot a_{G3y} - F_{Py}) \cdot N^{-1} \\ (I_{G3} \cdot \alpha_3 - R_{Px} \cdot F_{Py} + R_{Py} \cdot F_{Px}) \cdot N^{-1} \cdot m^{-1} \\ m_4 \cdot a_{G4x} \cdot N^{-1} \\ m_4 \cdot a_{G4y} \cdot N^{-1} \\ (I_{G4} \cdot \alpha_4 - T_4) \cdot N^{-1} \cdot m^{-1} \end{bmatrix}$$

$$R := C^{-1} \cdot F$$

$$F_{12x} := R_1 \cdot N$$

$$F_{12x} = -5484 \text{ N}$$

$$F_{12y} := R_2 \cdot N$$

$$F_{12y} = -5050 \text{ N}$$

$$F_{32x} := R_3 \cdot N$$

$$F_{32x} = 4449 \text{ N}$$

$$F_{32y} := R_4 \cdot N$$

$$F_{32y} = 4422 \text{ N}$$

$$F_{43x} := R_5 \cdot N$$

$$F_{43x} = 1373 \text{ N}$$

$$F_{43y} := R_6 \cdot N$$

$$F_{43y} = 81.8 \text{ N}$$

$$F_{14x} := R_7 \cdot N$$

$$F_{14x} = 198 \text{ N}$$

$$F_{14y} := R_8 \cdot N$$

$$F_{14y} = -1011 \text{ N}$$

$$T_{12} := R_9 \cdot N \cdot m$$

$$T_{12} = -1168 \text{ N} \cdot m$$

PROBLEM 11-11

Statement: Figure P11-5a shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections 50 mm wide by 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 15 \text{ rad/sec}$ and $\alpha = -10 \text{ rad/sec}^2$. There is a vertical force at P of $F = 500 \text{ N}$. Find all pin forces and the torque needed to drive the crank at this instant.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 0.785 \cdot m \quad \text{Link 3 (A to B)} \quad b := 0.356 \cdot m$$

$$\text{Link 4 (B to O}_4) \quad c := 0.950 \cdot m \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 0.544 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 1.09 \cdot m \quad \delta_3 := 0 \cdot \text{deg} \quad F := 500 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank angle and motion:} \quad \theta_2 := 96 \cdot \text{deg} \quad \omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := -10 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad w_3 := 50 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \gamma_s := 0.3 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Solution: See Figure P11-5a and Mathcad file P1111.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$$\theta_3 := 20.261 \cdot \text{deg} \quad \omega_3 := -6.830 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_3 := 106.282 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\theta_4 := 107.906 \cdot \text{deg} \quad \omega_4 := 12.023 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_4 := 49.372 \cdot \text{rad} \cdot \text{sec}^{-2}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.393 \cdot m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.475 \cdot m$$

$$\text{Link 3:} \quad R_{CG3} := 0.5 \cdot R_{pa} \quad R_{CG3} = 0.545 \cdot m$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := w_3 \cdot t_3 \cdot R_{pa} \cdot \frac{\gamma_s}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

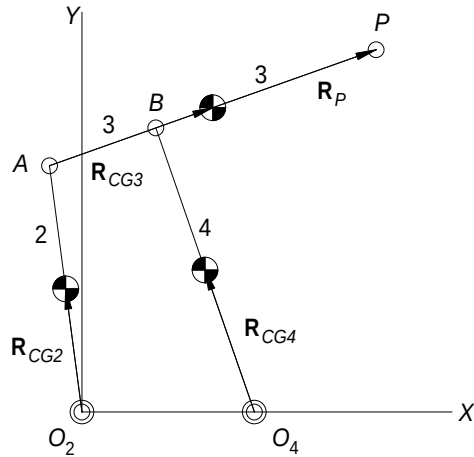
$$m_2 = 8.148 \cdot \text{kg} \quad m_3 = 11.314 \cdot \text{kg} \quad m_4 = 9.861 \cdot \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.420 \cdot \text{kg} \cdot \text{m}^2$$

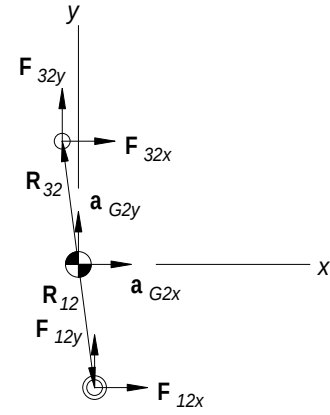
$$I_{G3} := \frac{m_3}{12} \cdot (w_3^2 + R_{pa}^2) \quad I_{G3} = 1.123 \cdot \text{kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 0.744 \cdot \text{kg} \cdot \text{m}^2$$

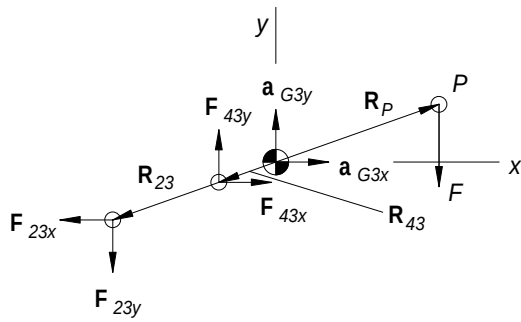
4. Set up an LNCS xy coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.



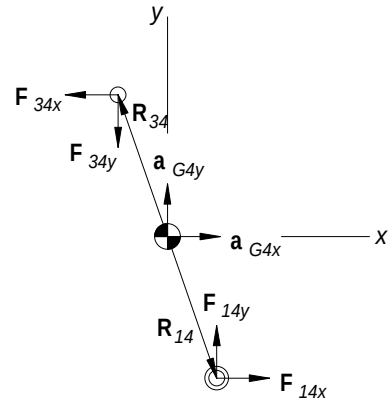
(a) The complete linkage with GCS



(b) FBD of Link 2



(c) FBD of Link 3



(d) FBD of Link 4

5. Calculate the x and y components of the position vectors.

$$R_{12x} := R_{CG2} \cdot \cos(\theta_2 + 180 \cdot \text{deg})$$

$$R_{12y} := R_{CG2} \cdot \sin(\theta_2 + 180 \cdot \text{deg})$$

$$R_{32x} := R_{CG2} \cdot \cos(\theta_2)$$

$$R_{32y} := R_{CG2} \cdot \sin(\theta_2)$$

$$R_{23x} := R_{CG3} \cdot \cos(\theta_3 + 180 \cdot \text{deg})$$

$$R_{23y} := R_{CG3} \cdot \sin(\theta_3 + 180 \cdot \text{deg})$$

$$R_{43x} := (R_{CG3} - b) \cdot \cos(\theta_3 + 180 \cdot \text{deg})$$

$$R_{43y} := (R_{CG3} - b) \cdot \sin(\theta_3 + 180 \cdot \text{deg})$$

$$R_{34x} := R_{CG4} \cdot \cos(\theta_4)$$

$$R_{34y} := R_{CG4} \cdot \sin(\theta_4)$$

$$R_{14x} := R_{CG4} \cdot \cos(\theta_4 + 180 \cdot \text{deg})$$

$$R_{12x} = 0.041 \text{ m}$$

$$R_{12y} = -0.390 \text{ m}$$

$$R_{32x} = -0.041 \text{ m}$$

$$R_{32y} = 0.390 \text{ m}$$

$$R_{23x} = -0.511 \text{ m}$$

$$R_{23y} = -0.189 \text{ m}$$

$$R_{43x} = -0.177 \text{ m}$$

$$R_{43y} = -0.065 \text{ m}$$

$$R_{34x} = -0.146 \text{ m}$$

$$R_{34y} = 0.452 \text{ m}$$

$$R_{14x} = 0.146 \text{ m}$$

$$R_{14y} := R_{CG4} \cdot \sin(\theta_4 + 180 \cdot \text{deg}) \quad R_{14y} = -0.452 \text{ m}$$

$$R_{Px} := (R_{pa} - R_{CG3}) \cdot \cos(\theta_3) \quad R_{Px} = 0.511 \text{ m}$$

$$R_{Py} := (R_{pa} - R_{CG3}) \cdot \sin(\theta_3) \quad R_{Py} = 0.189 \text{ m}$$

6. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2} := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$a_{G2x} := \text{Re}(\mathbf{a}_{G2}) \quad a_{G2x} = 22.366 \frac{\text{m}}{\text{sec}^2}$$

$$a_{G2y} := \text{Im}(\mathbf{a}_{G2}) \quad a_{G2y} = -175.247 \frac{\text{m}}{\text{sec}^2}$$

$$\mathbf{a}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{a}_{CG3A} := R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3)) \dots \\ + -R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_3) + j \cdot \sin(\theta_3))$$

$$\mathbf{a}_{G3} := \mathbf{a}_A + \mathbf{a}_{CG3A} \quad a_{G3x} := \text{Re}(\mathbf{a}_{G3}) \quad a_{G3x} = -17.640 \frac{\text{m}}{\text{sec}^2}$$

$$a_{G3y} := \text{Im}(\mathbf{a}_{G3}) \quad a_{G3y} = -129.301 \frac{\text{m}}{\text{sec}^2}$$

$$\mathbf{a}_{G4} := R_{CG4} \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$a_{G4x} := \text{Re}(\mathbf{a}_{G4}) \quad a_{G4x} = 19.906 \frac{\text{m}}{\text{sec}^2}$$

$$a_{G4y} := \text{Im}(\mathbf{a}_{G4}) \quad a_{G4y} = -137.884 \frac{\text{m}}{\text{sec}^2}$$

7. Calculate the x and y components of the external force at P in the CGS.

$$F_{Px} := 0 \cdot N \quad F_{Py} := -F$$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{m} & \frac{R_{12x}}{m} & \frac{-R_{32y}}{m} & \frac{R_{32x}}{m} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{m} & \frac{-R_{23x}}{m} & \frac{-R_{43y}}{m} & \frac{R_{43x}}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{m} & \frac{-R_{34x}}{m} & \frac{-R_{14y}}{m} & \frac{R_{14x}}{m} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot N^{-1} \\ m_2 \cdot a_{G2y} \cdot N^{-1} \\ I_{G2} \cdot \alpha_2 \cdot N^{-1} \cdot m^{-1} \\ (m_3 \cdot a_{G3x} - F_{Px}) \cdot N^{-1} \\ (m_3 \cdot a_{G3y} - F_{Py}) \cdot N^{-1} \\ (I_{G3} \cdot \alpha_3 - R_{Px} \cdot F_{Py} + R_{Py} \cdot F_{Px}) \cdot N^{-1} \cdot m^{-1} \\ m_4 \cdot a_{G4x} \cdot N^{-1} \\ m_4 \cdot a_{G4y} \cdot N^{-1} \\ (I_{G4} \cdot \alpha_4 - T_4) \cdot N^{-1} \cdot m^{-1} \end{bmatrix}$$

$$R := C^{-1} \cdot F$$

$$F_{12x} := R_1 \cdot N$$

$$F_{12x} = -231 \, N$$

$$F_{12y} := R_2 \cdot N$$

$$F_{12y} = -2231 \, N$$

$$F_{32x} := R_3 \cdot N$$

$$F_{32x} = 413 \, N$$

$$F_{32y} := R_4 \cdot N$$

$$F_{32y} = 803 \, N$$

$$F_{43x} := R_5 \cdot N$$

$$F_{43x} = 214 \, N$$

$$F_{43y} := R_6 \cdot N$$

$$F_{43y} = -160 \, N$$

$$F_{14x} := R_7 \cdot N$$

$$F_{14x} = 410 \, N$$

$$F_{14y} := R_8 \cdot N$$

$$F_{14y} = -1519 \, N$$

$$T_{12} := R_9 \cdot N \cdot m$$

$$T_{12} = 372 \, N \cdot m$$

PROBLEM 11-12

Statement: Figure P11-5b shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections of 60 mm diameter. In the instantaneous position shown, the crank O_2A has $\omega = -10 \text{ rad/sec}$ and $\alpha = 10 \text{ rad/sec}^2$. There is a horizontal force at P of $F = 500 \text{ N}$. Find all pin forces and the torque needed to drive the crank at this instant.

Given: Link lengths:

$$\text{Link 2 (} O_2 \text{ to } A) \quad a := 0.86 \cdot m \quad \text{Link 3 (} A \text{ to } B) \quad b := 1.85 \cdot m$$

$$\text{Link 4 (} B \text{ to } O_4) \quad c := 0.86 \cdot m \quad \text{Link 1 (} O_2 \text{ to } O_4) \quad d := 2.22 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 1.33 \cdot m \quad \delta_3 := 0 \cdot \text{deg} \quad F := 500 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank angle and motion:} \quad \theta_2 := -36 \cdot \text{deg} \quad \omega_2 := -10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$d_{\text{link}} := 60 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \gamma_s := 0.3 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Solution: See Figure P11-5b and Mathcad file P1112.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$$\theta_3 := 46.028 \cdot \text{deg} \quad \omega_3 := 3.285 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_3 := -109.287 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\theta_4 := 106.189 \cdot \text{deg} \quad \omega_4 := 11.417 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_4 := -43.426 \cdot \text{rad} \cdot \text{sec}^{-2}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.430 \cdot m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.430 \cdot m$$

$$\text{Link 3:} \quad R_{CG3} := 0.5 \cdot b \quad R_{CG3} = 0.925 \cdot m$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot b \cdot \frac{\gamma_s}{g} \quad m_4 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot c \cdot \frac{\gamma_s}{g}$$

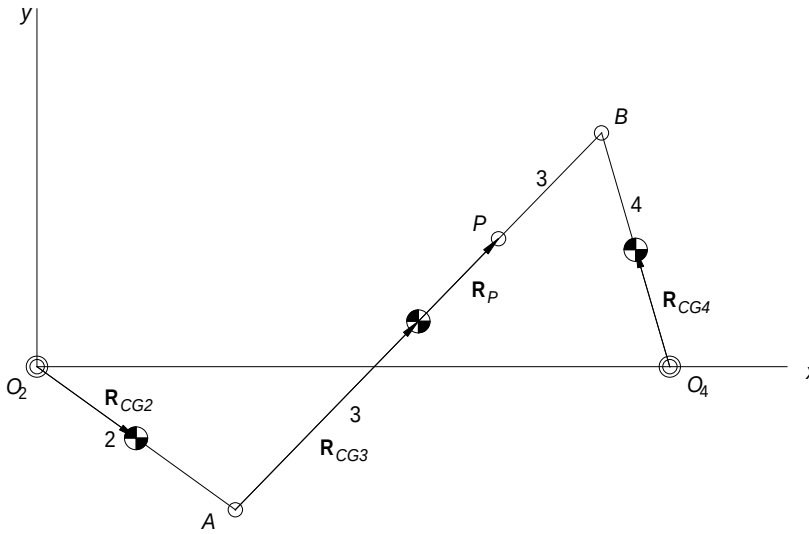
$$m_2 = 20.192 \cdot \text{kg} \quad m_3 = 43.436 \cdot \text{kg} \quad m_4 = 20.192 \cdot \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + a^2 \right) \quad I_{G2} = 1.249 \cdot \text{kg} \cdot \text{m}^2$$

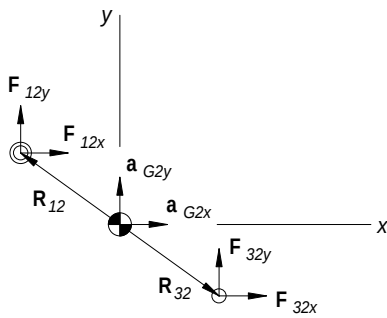
$$I_{G3} := \frac{m_3}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + b^2 \right) \quad I_{G3} = 12.398 \cdot \text{kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + c^2 \right) \quad I_{G4} = 1.249 \cdot \text{kg} \cdot \text{m}^2$$

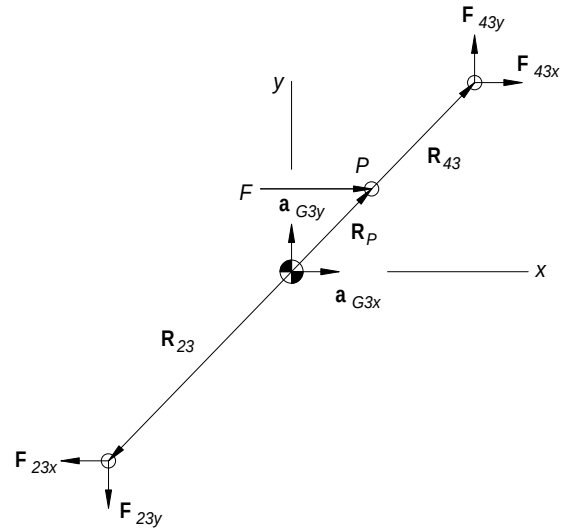
4. Set up an LNCS xy coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.



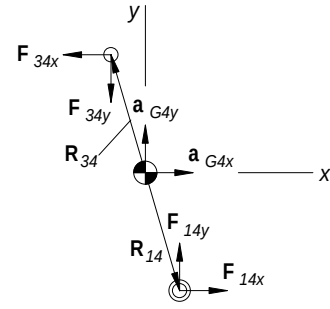
(a) The complete linkage with GCS



(b) FBD of Link 2



(c) FBD of Link 3



(d) FBD of Link 4

5. Calculate the x and y components of the position vectors.

$$R_{12x} := R_{CG2} \cdot \cos(\theta_2 + 180 \cdot \text{deg})$$

$$R_{12x} = -0.348 \cdot m$$

$$R_{12y} := R_{CG2} \cdot \sin(\theta_2 + 180 \cdot \text{deg})$$

$$R_{12y} = 0.253 \cdot m$$

$$R_{32x} := R_{CG2} \cdot \cos(\theta_2)$$

$$R_{32x} = 0.348 \cdot m$$

$$R_{32y} := R_{CG2} \cdot \sin(\theta_2)$$

$$R_{32y} = -0.253 \cdot m$$

$$R_{23x} := R_{CG3} \cdot \cos(\theta_3 + 180 \cdot \text{deg})$$

$$R_{23x} = -0.642 \cdot m$$

$$R_{23y} := R_{CG3} \cdot \sin(\theta_3 + 180 \cdot \text{deg})$$

$$R_{23y} = -0.666 \cdot m$$

$$R_{43x} := (R_{CG3} - b) \cdot \cos(\theta_3 + 180 \cdot \text{deg})$$

$$R_{43x} = 0.642 \cdot m$$

$$R_{43y} := (R_{CG3} - b) \cdot \sin(\theta_3 + 180 \cdot \text{deg})$$

$$R_{43y} = 0.666 \cdot m$$

$$\begin{aligned}
 R_{34x} &:= R_{CG4} \cdot \cos(\theta_4) & R_{34x} &= -0.120 \cdot m \\
 R_{34y} &:= R_{CG4} \cdot \sin(\theta_4) & R_{34y} &= 0.413 \cdot m \\
 R_{14x} &:= R_{CG4} \cdot \cos(\theta_4 + 180 \cdot \text{deg}) & R_{14x} &= 0.120 \cdot m \\
 R_{14y} &:= R_{CG4} \cdot \sin(\theta_4 + 180 \cdot \text{deg}) & R_{14y} &= -0.413 \cdot m \\
 R_{Px} &:= (R_{pa} - R_{CG3}) \cdot \cos(\theta_3) & R_{Px} &= 0.281 \cdot m \\
 R_{Py} &:= (R_{pa} - R_{CG3}) \cdot \sin(\theta_3) & R_{Py} &= 0.291 \cdot m
 \end{aligned}$$

6. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2} := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$a_{G2x} := \text{Re}(\mathbf{a}_{G2}) \quad a_{G2x} = -67.048 \cdot \frac{m}{\text{sec}^2}$$

$$a_{G2y} := \text{Im}(\mathbf{a}_{G2}) \quad a_{G2y} = 54.028 \cdot \frac{m}{\text{sec}^2}$$

$$\mathbf{a}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{a}_{CG3A} := R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3)) \dots \\ + -R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_3) + j \cdot \sin(\theta_3))$$

$$\mathbf{a}_{G3} := \mathbf{a}_A + \mathbf{a}_{CG3A} \quad a_{G3x} := \text{Re}(\mathbf{a}_{G3}) \quad a_{G3x} = 1.302 \cdot \frac{m}{\text{sec}^2}$$

$$a_{G3y} := \text{Im}(\mathbf{a}_{G3}) \quad a_{G3y} = -19.864 \cdot \frac{m}{\text{sec}^2}$$

$$\mathbf{a}_{G4} := R_{CG4} \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$a_{G4x} := \text{Re}(\mathbf{a}_{G4}) \quad a_{G4x} = 49.187 \cdot \frac{m}{\text{sec}^2}$$

$$a_{G4y} := \text{Im}(\mathbf{a}_{G4}) \quad a_{G4y} = -102.448 \cdot \frac{m}{\text{sec}^2}$$

7. Calculate the x and y components of the external force at P in the CGS.

$$F_{Px} := F \quad F_{Py} := 0 \cdot N$$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{m} & \frac{R_{12x}}{m} & \frac{-R_{32y}}{m} & \frac{R_{32x}}{m} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{m} & \frac{-R_{23x}}{m} & \frac{-R_{43y}}{m} & \frac{R_{43x}}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{m} & \frac{-R_{34x}}{m} & \frac{-R_{14y}}{m} & \frac{R_{14x}}{m} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot N^{-1} \\ m_2 \cdot a_{G2y} \cdot N^{-1} \\ I_{G2} \cdot \alpha_2 \cdot N^{-1} \cdot m^{-1} \\ (m_3 \cdot a_{G3x} - F_{Px}) \cdot N^{-1} \\ (m_3 \cdot a_{G3y} - F_{Py}) \cdot N^{-1} \\ (I_{G3} \cdot \alpha_3 - R_{Px} \cdot F_{Py} + R_{Py} \cdot F_{Px}) \cdot N^{-1} \cdot m^{-1} \\ m_4 \cdot a_{G4x} \cdot N^{-1} \\ m_4 \cdot a_{G4y} \cdot N^{-1} \\ (I_{G4} \cdot \alpha_4 - T_4) \cdot N^{-1} \cdot m^{-1} \end{bmatrix}$$

$$R := C^{-1} \cdot F$$

$$F_{12x} := R_1 \cdot N$$

$$F_{12x} = -1851 \cdot N$$

$$F_{12y} := R_2 \cdot N$$

$$F_{12y} = 1315 \cdot N$$

$$F_{32x} := R_3 \cdot N$$

$$F_{32x} = 497 \cdot N$$

$$F_{32y} := R_4 \cdot N$$

$$F_{32y} = -225 \cdot N$$

$$F_{43x} := R_5 \cdot N$$

$$F_{43x} = 53.7 \cdot N$$

$$F_{43y} := R_6 \cdot N$$

$$F_{43y} = -1087 \cdot N$$

$$F_{14x} := R_7 \cdot N$$

$$F_{14x} = 1047 \cdot N$$

$$F_{14y} := R_8 \cdot N$$

$$F_{14y} = -3156 \cdot N$$

$$T_{12} := R_9 \cdot N \cdot m$$

$$T_{12} = -45.28 \cdot N \cdot m$$

PROBLEM 11-13

Statement: Figure P11-6 shows a water jet loom laybar drive mechanism driven by a pair of Grashof crank rocker fourbar linkages. The crank rotates at 500 rpm. The laybar is carried between the coupler-rocker joints of the two linkages at their respective instant centers $I_{3,4}$. The combined weight of the reed and laybar is 29 lb. A 540-lb beat-up force from the cloth is applied to the reed as shown. The steel links have a 2 x 1 in uniform cross-section. Find the forces on the pins for one revolution of the crank. Find the torque-time function required to drive the system.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

Link 2 (A to B) $a := 2.00 \cdot in$ Link 3 (B to C) $b := 8.375 \cdot in$

Link 4 (C to D) $c := 7.187 \cdot in$ Link 1 (A to D) $d := 9.625 \cdot in$

Coupler point: $R_{pa} := 0.0 \cdot in$ $\delta_3 := 0 \cdot deg$

Crank angle and motion: $\omega_2 := 500 \cdot rpm$ $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Link cross-section dims: $w := 2.00 \cdot in$ $t := 1.00 \cdot in$

Material specific weight: steel $\gamma := 0.3 \cdot lbf \cdot in^{-3}$

Solution: See Figure P11-6 and Mathcad file P1113.

1. Determine the distance to the CG in the LRCS on each of the three moving links. All three are located on the x' axis in the LRCS and their angle is zero deg.

$$R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 1.000 \text{ in} \quad R_{CG3} := 0.5 \cdot b \quad R_{CG3} = 4.188 \text{ in}$$

$$R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 3.594 \text{ in}$$

2. Determine the mass and moment of inertia of each link.

$$m_2 := w \cdot t \cdot a \cdot \frac{\gamma}{g} \quad m_3 := w \cdot t \cdot b \cdot \frac{\gamma}{g} \quad m_4 := w \cdot t \cdot c \cdot \frac{\gamma}{g}$$

$$m_2 = 3.108 \times 10^{-3} \text{ blob} \quad m_3 = 0.013 \text{ blob} \quad m_4 = 0.011 \text{ blob}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w^2 + a^2) \quad I_{G2} = 0.00207 \text{ blob} \cdot in^2$$

$$I_{G3} := \frac{m_3}{12} \cdot (w^2 + b^2) \quad I_{G3} = 0.01920 \text{ blob} \cdot in^2$$

Include one half of the mass of the laybar as a lumped mass at the end of link 4.

$$I_{G4} := \frac{m_4}{12} \cdot (w^2 + c^2) + R_{CG4}^2 \cdot \frac{14.5 \cdot lbf}{g} \quad I_{G4} = 0.53677 \text{ blob} \cdot in^2$$

3. Define any external forces, their locations and directions.

Beat-up force $F := 590 \cdot lbf$ acting on link 4 at a distance $R := c + 3.75 \cdot in$

The angle in the CGS is 180 deg. $R = 10.937 \text{ in}$

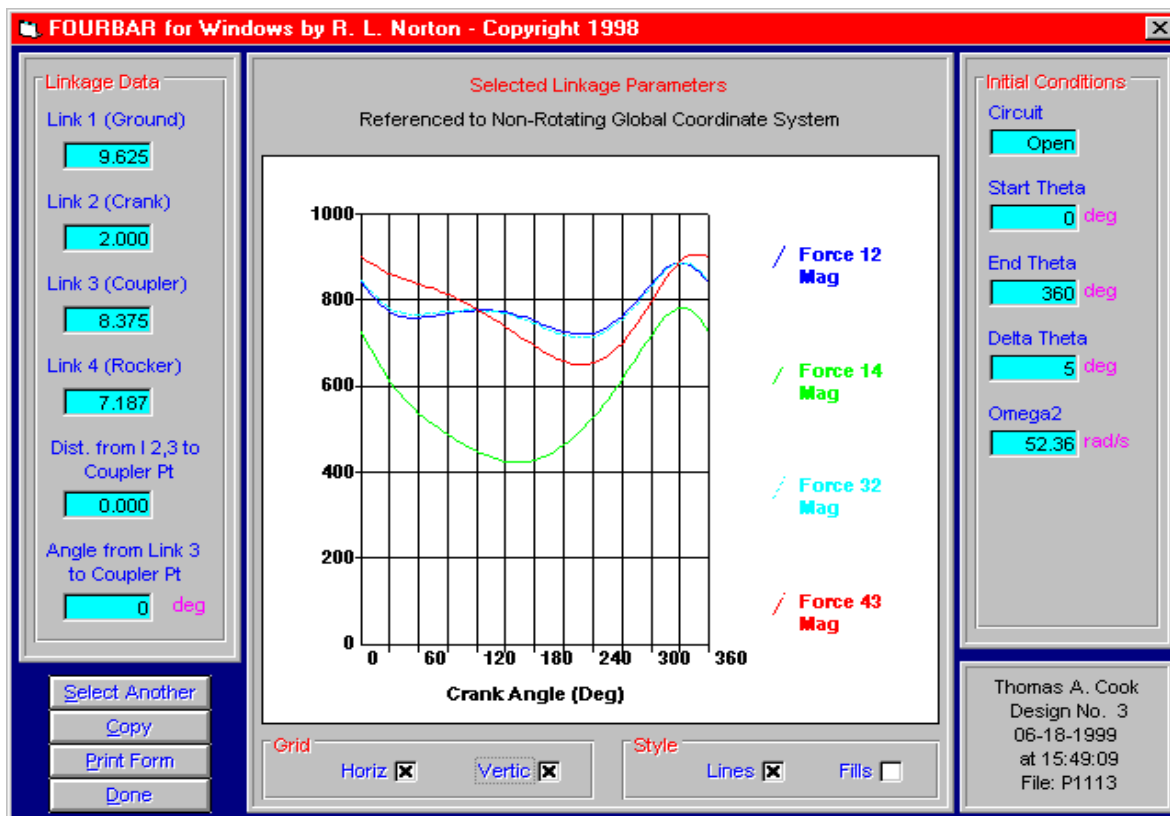
4. Enter the above data into program FOURBAR and solve for the pin forces and driving torque. The dynamic input screen is shown below followed by a plot of dynamic pin forces..

FOURBAR for Windows by R. L. Norton - Copyright 1998 **Dynamics Screen**

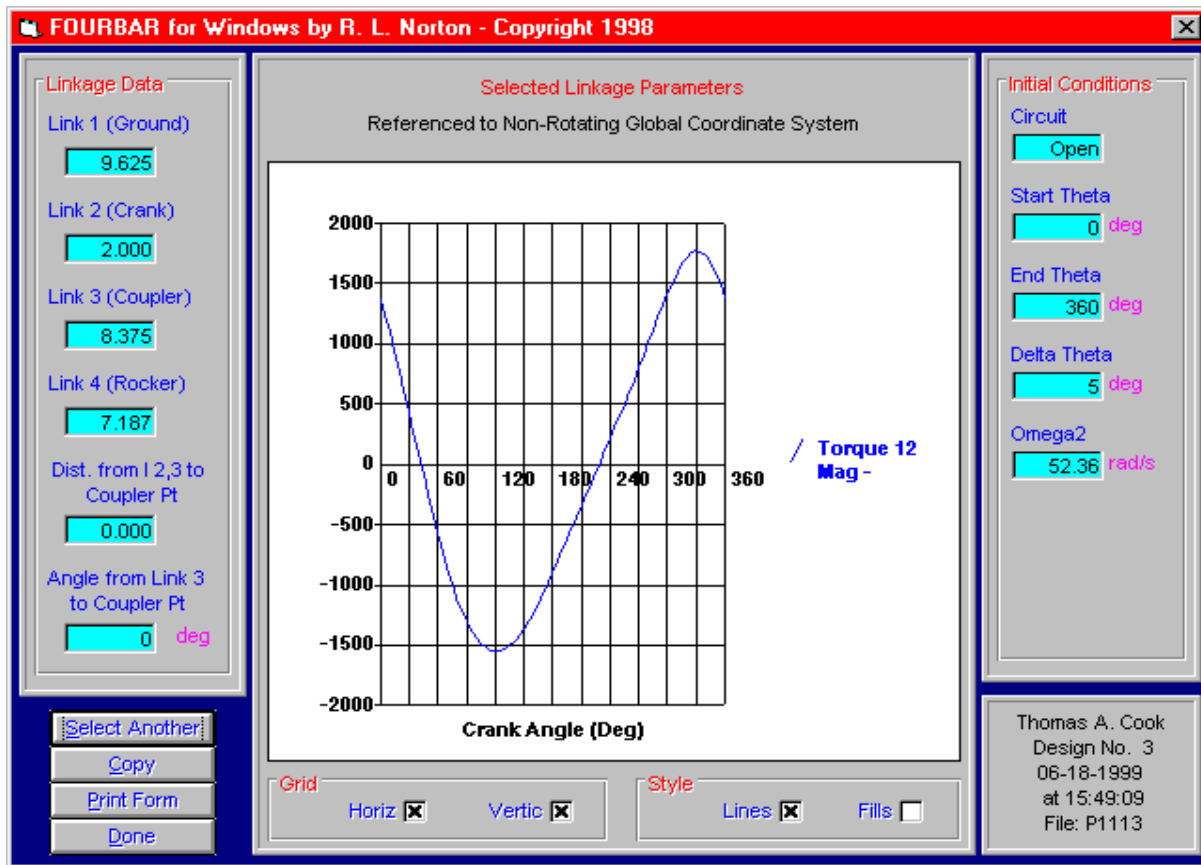
	Link	2	3	4	Units
Mass Properties					
Mass of Link		0.00311	0.013	0.011	
Mass Moment of Inertia about CG		0.00207	0.0192	0.537	
CG Locations					
Distance to CG from Pivot		1.000	4.188	3.594	
Angle to CG in LRCS*		0.00	0.0	0.00	degrees
External Forces					
Force Magnitude		0	590		
Angle of Force in GCS*		0	180		degrees
External Force Locations					
Distance to Force Point from CG		0	7.343		
Angle to Force Point in LRCS*		0	0		degrees
External Torques					
Torque on Link		0	0		
Friction					
Friction Force on Link		0			

* LRCS = Local Rotating Coordinate System. GCS = Global Coordinate System

Solve
Show Matrix
Copy
Print Form
Done
Cancel



5. The input torque is plotted below.



PROBLEM 11-14

Statement: Figure P11-7 shows a crimping tool. Find the force F_{hand} needed to generate a 2000 lb F_{crimp} . Find the pin forces. What is the linkage's joint force transmission index (JFI) in this position?

Given: Link lengths:

$$\text{Link 2 (A to B)} \quad a := 0.80 \cdot \text{in}$$

$$\text{Link 3 (B to C)} \quad b := 1.23 \cdot \text{in}$$

$$\text{Link 2 (C to D)} \quad c := 1.55 \cdot \text{in}$$

$$\text{Link 2 (A to D)} \quad d := 2.40 \cdot \text{in}$$

$$\text{Link 2 angle:} \quad \theta_2 := 49 \cdot \text{deg}$$

$$\text{Distance to crimp force from pivot D:} \quad R_{p4} := 1.00 \cdot \text{in} \quad \delta_4 := 0 \cdot \text{deg}$$

$$\text{Crimp force:} \quad F_{p4} := 2000 \cdot \text{lbf} \quad (\text{perpendicular to link 4})$$

$$\text{Distance to hand force from pivot A:} \quad R_{p2} := 4.26 \cdot \text{in} \quad \delta_2 := 0 \cdot \text{deg}$$

Solution: See Figure P11-7 and Mathcad file P1114.

1. Enter the above data into program FOURBAR to determine link 3 and 4 angles and calculate the angle that the crimping force makes with respect to the fourbar coordinate frame..

$$\theta_3 := 34.039 \cdot \text{deg}$$

$$\theta_4 := 123.518 \cdot \text{deg}$$

$$\theta_{FP4} := \theta_4 + 90 \cdot \text{deg}$$

$$\theta_{FP4} = 213.518 \cdot \text{deg}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.400 \cdot \text{in} \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.775 \cdot \text{in}$$

$$\text{Link 3:} \quad R_{CG3} := 0.5 \cdot b \quad R_{CG3} = 0.615 \cdot \text{in}$$

3. Calculate the x and y components of the position vectors.

$$R_{12x} := R_{CG2} \cdot \cos(\theta_2 + 180 \cdot \text{deg})$$

$$R_{12x} = -0.262 \cdot \text{in}$$

$$R_{12y} := R_{CG2} \cdot \sin(\theta_2 + 180 \cdot \text{deg})$$

$$R_{12y} = -0.302 \cdot \text{in}$$

$$R_{32x} := R_{CG2} \cdot \cos(\theta_2)$$

$$R_{32x} = 0.262 \cdot \text{in}$$

$$R_{32y} := R_{CG2} \cdot \sin(\theta_2)$$

$$R_{32y} = 0.302 \cdot \text{in}$$

$$R_{23x} := R_{CG3} \cdot \cos(\theta_3 + 180 \cdot \text{deg})$$

$$R_{23x} = -0.510 \cdot \text{in}$$

$$R_{23y} := R_{CG3} \cdot \sin(\theta_3 + 180 \cdot \text{deg})$$

$$R_{23y} = -0.344 \cdot \text{in}$$

$$R_{43x} := (b - R_{CG3}) \cdot \cos(\theta_3)$$

$$R_{43x} = 0.510 \cdot \text{in}$$

$$R_{43y} := (b - R_{CG3}) \cdot \sin(\theta_3)$$

$$R_{43y} = 0.344 \cdot \text{in}$$

$$R_{34x} := R_{CG4} \cdot \cos(\theta_4)$$

$$R_{34x} = -0.428 \cdot \text{in}$$

$$R_{34y} := R_{CG4} \cdot \sin(\theta_4)$$

$$R_{34y} = 0.646 \cdot \text{in}$$

$$R_{14x} := R_{CG4} \cdot \cos(\theta_4 + 180 \cdot \text{deg})$$

$$R_{14x} = 0.428 \cdot \text{in}$$

$$R_{14y} := R_{CG4} \cdot \sin(\theta_4 + 180 \cdot \text{deg})$$

$$R_{14y} = -0.646 \cdot \text{in}$$

$$R_{p2y} := (R_{p2} + R_{CG2}) \cdot \sin(\theta_2 + 180 \cdot \text{deg})$$

$$R_{p2y} = -3.517 \cdot \text{in}$$

$$R_{P2x} := (R_{P2} + R_{CG2}) \cdot \cos(\theta_2 + 180 \cdot \text{deg}) \quad R_{P2x} = -3.057 \cdot \text{in}$$

$$R_{P4x} := (R_{P4} - R_{CG4}) \cdot \cos(\theta_4) \quad R_{P4x} = -0.124 \cdot \text{in}$$

$$R_{P4y} := (R_{P4} - R_{CG4}) \cdot \sin(\theta_4) \quad R_{P4y} = 0.188 \cdot \text{in}$$

4. Calculate the x and y components of the external crimp force at *P* on link 4 in the CGS.

$$F_{P4x} := F_{P4} \cdot \cos(\theta_{FP4}) \quad F_{P4x} = -1667.4 \cdot \text{lbf}$$

$$F_{P4y} := F_{P4} \cdot \sin(\theta_{FP4}) \quad F_{P4y} = -1104.4 \cdot \text{lbf}$$

5. Substitute these given and calculated values into the matrix equation 11.9 modified to omit all mass and inertia terms. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-R_{12y}}{\text{in}} & \frac{R_{12x}}{\text{in}} & \frac{-R_{32y}}{\text{in}} & \frac{R_{32x}}{\text{in}} & 0 & 0 & 0 & 0 & \frac{-R_{P2x}}{\text{in}} & \frac{R_{P2y}}{\text{in}} \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{\text{in}} & \frac{-R_{23x}}{\text{in}} & \frac{-R_{43y}}{\text{in}} & \frac{R_{43x}}{\text{in}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{\text{in}} & \frac{-R_{34x}}{\text{in}} & \frac{-R_{14y}}{\text{in}} & \frac{R_{14x}}{\text{in}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \tan(\theta_2) \end{pmatrix}$$

$$F := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -F_{P4x} \cdot \text{lbf}^{-1} \\ -F_{P4y} \cdot \text{lbf}^{-1} \\ (-R_{P4x} \cdot F_{P4y} + R_{P4y} \cdot F_{P4x}) \cdot \text{lbf}^{-1} \cdot \text{in}^{-1} \\ 0 \end{bmatrix} \quad R := C^{-1} \cdot F$$

$$F_{12x} := R_1 \cdot \text{lbf} \quad F_{12x} = 1117 \cdot \text{lbf} \quad F_{12y} := R_2 \cdot \text{lbf} \quad F_{12y} = 681 \cdot \text{lbf}$$

$$F_{32x} := R_3 \cdot \text{lbf} \quad F_{32x} = -1069 \cdot \text{lbf} \quad F_{32y} := R_4 \cdot \text{lbf} \quad F_{32y} = -722 \cdot \text{lbf}$$

$$F_{43x} := R_5 \cdot lbf \quad F_{43x} = -1069 \cdot lbf$$

$$F_{14x} := R_7 \cdot lbf \quad F_{14x} = 598 \cdot lbf$$

$$F_{handx} := R_9 \cdot lbf \quad F_{handx} = -47.7 \cdot lbf$$

$$F_{43y} := R_6 \cdot lbf \quad F_{43y} = -722 \cdot lbf$$

$$F_{14y} := R_8 \cdot lbf \quad F_{14y} = 382 \cdot lbf$$

$$F_{handy} := R_{10} \cdot lbf \quad F_{handy} = 41.5 \cdot lbf$$

6. Calculate the pin forces.

$$\text{Pin at A:} \quad F_{12} := \sqrt{F_{12x}^2 + F_{12y}^2}$$

$$F_{12} = 1308 \cdot lbf$$

$$\text{Pin at B:} \quad F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2}$$

$$F_{32} = 1290 \cdot lbf$$

$$\text{Pin at C:} \quad F_{43} := \sqrt{F_{43x}^2 + F_{43y}^2}$$

$$F_{43} = 1290 \cdot lbf$$

$$\text{Pin at D:} \quad F_{14} := \sqrt{F_{14x}^2 + F_{14y}^2}$$

$$F_{14} = 710 \cdot lbf$$

7. Calculate the hand force.

$$F_{hand} := \sqrt{F_{handx}^2 + F_{handy}^2}$$

$$F_{hand} = 63.2 \cdot lbf$$

8. Use equation 11.23a to calculate the joint force index.

$$JFI := \frac{F_{32}}{F_{P4}} \quad JFI = 0.645$$

PROBLEM 11-15

Statement: Figure P11-8 shows a walking beam conveyor mechanism that operates a slow speed (25 rpm). The boxes being pushed each weigh 50 lb. Determine the pin forces in the linkage and the torque to drive the mechanism through one revolution. Neglect the masses of the links.

Solution: No solution is given for this problem, which is suited to solution using the *Working Model* program.

PROBLEM 11-16

Statement: Figure P11-9 shows a surface grinder table drive that operates at 120 rpm. The crank radius is 22 mm, the coupler is 157 mm, and its offset is 40 mm. The mass of the table and workpiece combined is 50 kg. Find the pin forces, slider side loads, and driving torque over one revolution.

Solution: No solution is given for this problem, which is suited to solution using the *Working Model* program.

PROBLEM 11-17

Statement: Figure P11-10 shows a power hacksaw that operates at 50 rpm. The crank is 75 mm, the coupler is 170 mm, and its offset is 45 mm. Find the pin forces, slider side loads, and driving torque over one revolution for a cutting force of 250 N in the forward direction and 50 N during the return stroke.

Solution: No solution is given for this problem, which is suited to solution using the *Working Model* program.

PROBLEM 11-18

Statement: Figure P11-11 shows a paper roll off-loading station. The paper rolls have a 0.9-m OD, 0.22-m ID, are 3.23 m long, and have a density of 984 kg/m^3 . The forks that support the roll are 1.2 m long. The motion is slow so inertial loading can be neglected. Find the force required of the air cylinder to rotate the roll through 90 deg.

Solution: No solution is given for this problem, which is suited to solution using the *Working Model* program.

PROBLEM 11-19

Statement: Derive an expression for the relationship between flywheel mass and the dimensionless parameter radius/thickness (r/t) for a solid disk flywheel of moment of inertia I . Plot this function for an arbitrary value of I and determine the optimum r/t ratio to minimize flywheel weight for that I .

Solution: No solution is provided to this algebraic exercise.

PROBLEM 11-20

Statement: Figure P11-5a shows an oil field pump mechanism. The head of the rocker arm is shaped such that the lower end of a flexible cable attached to it will always be directly over the well head regardless of the position of the rocker arm 4. The pump rod, which connects to the pump in the well casing, is connected to the lower end of the cable. The force in the pump rod on the up stroke is 2970 lb and the force on the down stroke is 2300 lb. Link 2 weighs 598.3 lb and has a moment of inertia of 11.8 blob-in²; both including the counterweight. Its CG is on the link centerline, 13.2 in from O₂. Link 3 weighs 108 lb and its CG is on the link centerline, 40 in from A. It has a mass moment of inertia of 150 blob-in². Link 4 weighs 2706 lb and has a moment of inertia of 10700 blob-in²; both include the counterweight. Its CG is on the link centerline where shown. The crank turns at a constant speed of 4 rpm CCW. At the instant shown in the figure the crank angle is at 45 deg with respect to the global coordinate system. Find all pin forces and the torque needed to drive the crank for the position shown. Include gravity forces.

Units: $\text{blob} := \text{lb} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A): } a := 14.0 \cdot \text{in} \quad \text{Link 3 (A to B): } b := 80.0 \cdot \text{in}$$

$$\text{Link 4 (B to O}_4\text{): } c := 51.3 \cdot \text{in} \quad \text{Link 1 (O}_2 \text{ to O}_4\text{): } d := 79.7 \cdot \text{in}$$

$$\text{Link 1 offsets: } d_X := -47.5 \cdot \text{in} \quad d_Y := 64 \cdot \text{in}$$

$$\text{External load data: } F := 2300 \cdot \text{lb} \quad T_4 := 0 \cdot \text{lb} \cdot \text{in}$$

$$\text{Crank angle and motion: } \theta_{2xy} := -81.582 \cdot \text{deg} \quad \omega_2 := 4 \cdot \text{rpm} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\text{Link CG positions: } R_{CG2} := 13.2 \cdot \text{in} \quad R_{CG3} := 40.0 \cdot \text{in} \quad R_{CG4} := 79.22 \cdot \text{in}$$

$$\text{Link weights: } W_2 := 598.3 \cdot \text{lb} \quad W_3 := 108 \cdot \text{lb} \quad W_4 := 2706 \cdot \text{lb}$$

$$\text{Moments of inertia: } I_{G2} := 11.8 \cdot \text{blob} \cdot \text{in}^2 \quad I_{G3} := 150 \cdot \text{blob} \cdot \text{in}^2 \quad I_{G4} := 10700 \cdot \text{blob} \cdot \text{in}^2$$

$$\text{Angle between O}_4\text{B and CG}_4\text{B: } \alpha := 143.11 \cdot \text{deg} \quad R_{34} := 32.00 \cdot \text{in}$$

$$\text{Angle between O}_4\text{B and CG}_4\text{O}_4: \beta := -14.03 \cdot \text{deg} \quad R_{14} := 79.22 \cdot \text{in}$$

$$\text{Angle between O}_4\text{B and CG}_4\text{P: } \delta := 156.62 \cdot \text{deg} \quad R_p := 124.44 \cdot \text{in}$$

Solution: See Figure P11-12 and Mathcad file P1120.

- Use Problems 6.84c and 7.70b with $\omega_2 = 4$ rpm to determine the position, velocity, and acceleration of links 3 and 4. The angles are calculated in the xy coordinate system and then rotated into the XY coordinate system after calculating accelerations.

$$\text{Coordinate rotation angle: } \gamma := \text{atan2}(d_X, d_Y) \quad \gamma = 126.582 \text{ deg}$$

$$\theta_{3xy} := 332.475 \cdot \text{deg} \quad \omega_3 := -0.0214 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_3 := -0.0250 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\theta_{4xy} := 262.482 \cdot \text{deg} \quad \omega_4 := 0.0986 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_4 := -0.0272 \cdot \text{rad} \cdot \text{sec}^{-2}$$

- Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2} := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_{2xy}) + j \cdot \cos(\theta_{2xy})) - R_{CG2} \cdot \omega_2^2 \cdot (\cos(\theta_{2xy}) + j \cdot \sin(\theta_{2xy}))$$

$$a_{G2} := |\mathbf{a}_{G2}| \quad a_{G2} = 2.316 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{aG2} := \arg(\mathbf{a}_{G2}) + \gamma \quad \theta_{aG2} = 225.000 \text{ deg}$$

$$a_{G2x} := a_{G2} \cdot \cos(\theta_{aG2}) \quad a_{G2y} := a_{G2} \cdot \sin(\theta_{aG2})$$

$$a_{G2x} = -1.638 \text{ in} \cdot \text{sec}^{-2} \quad a_{G2y} = -1.638 \text{ in} \cdot \text{sec}^{-2}$$

$$\mathbf{a}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_{2xy}) + j \cdot \cos(\theta_{2xy})) - a \cdot \omega_2^2 \cdot (\cos(\theta_{2xy}) + j \cdot \sin(\theta_{2xy}))$$

$$\mathbf{a}_{CG3A} := R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_{3xy}) + j \cdot \cos(\theta_{3xy})) - R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_{3xy}) + j \cdot \sin(\theta_{3xy}))$$

$$\mathbf{a}_{G3} := \mathbf{a}_A + \mathbf{a}_{CG3A}$$

$$a_{G3} := |\mathbf{a}_{G3}| \quad a_{G3} = 1.763 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{aG3} := \arg(\mathbf{a}_{G3}) + \gamma \quad \theta_{aG3} = 244.955 \text{ deg}$$

$$a_{G3x} := a_{G3} \cdot \cos(\theta_{aG3}) \quad a_{G3y} := a_{G3} \cdot \sin(\theta_{aG3})$$

$$a_{G3x} = -0.747 \text{ in} \cdot \text{sec}^{-2} \quad a_{G3y} = -1.598 \text{ in} \cdot \text{sec}^{-2}$$

$$\mathbf{a}_{G4} := R_{CG4} \cdot \alpha_4 \cdot (-\sin(\theta_{4xy} + \beta) + j \cdot \cos(\theta_{4xy} + \beta)) \dots$$

$$+ -R_{CG4} \cdot \omega_4^2 \cdot (\cos(\theta_{4xy} + \beta) + j \cdot \sin(\theta_{4xy} + \beta))$$

$$a_{G4} := |\mathbf{a}_{G4}| \quad a_{G4} = 2.288 \text{ in} \cdot \text{sec}^{-2} \quad \theta_{aG4} := \arg(\mathbf{a}_{G4}) + \gamma \quad \theta_{aG4} = 265.366 \text{ deg}$$

$$a_{G4x} := a_{G4} \cdot \cos(\theta_{aG4}) \quad a_{G4y} := a_{G4} \cdot \sin(\theta_{aG4})$$

$$a_{G4x} = -0.185 \text{ in} \cdot \text{sec}^{-2} \quad a_{G4y} = -2.281 \text{ in} \cdot \text{sec}^{-2}$$

Transform the link angles to the global XY system:

$$\theta_2 := \theta_{2xy} + \gamma \quad \theta_3 := \theta_{3xy} + \gamma - 360 \cdot \text{deg} \quad \theta_4 := \theta_{4xy} + \gamma - 360 \cdot \text{deg}$$

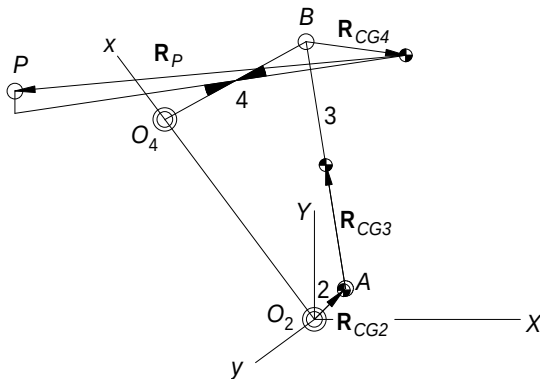
$$\theta_2 = 45.000 \text{ deg} \quad \theta_3 = 99.057 \text{ deg} \quad \theta_4 = 29.064 \text{ deg}$$

- Set up an LNCS xy coordinate system at the CG of each link, and draw all applicable vectors acting on the system as shown in Figure 11-3. Draw a free-body diagram of each moving link as shown in Figure 11-3.
- Calculate the x and y components of the position vectors.

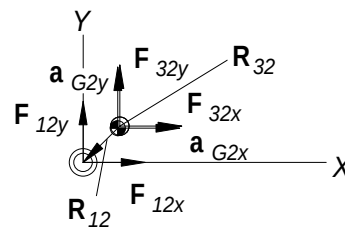
$$R_{12x} := R_{CG2} \cdot \cos(\theta_2 + 180 \cdot \text{deg}) \quad R_{12x} = -9.334 \text{ in}$$

$$R_{12y} := R_{CG2} \cdot \sin(\theta_2 + 180 \cdot \text{deg}) \quad R_{12y} = -9.334 \text{ in}$$

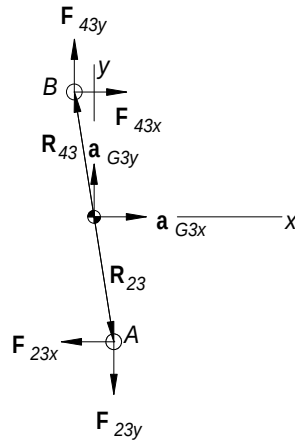
$$R_{32x} := (a - R_{CG2}) \cdot \cos(\theta_2) \quad R_{32x} = 0.566 \text{ in}$$



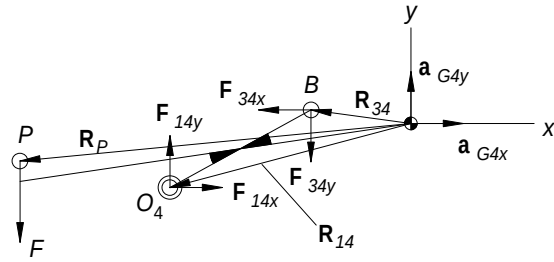
(a) The complete linkage with GCS



(b) FBD of Link 2



(c) FBD of Link 3



(d) FBD of Link 4

$$R_{32y} := (a - R_{CG2}) \cdot \sin(\theta_2)$$

$$R_{32y} = 0.566 \text{ in}$$

$$R_{23x} := R_{CG3} \cdot \cos(\theta_3 + 180 \cdot \text{deg})$$

$$R_{23x} = 6.297 \text{ in}$$

$$R_{23y} := R_{CG3} \cdot \sin(\theta_3 + 180 \cdot \text{deg})$$

$$R_{23y} = -39.501 \text{ in}$$

$$R_{43x} := (R_{CG3} - b) \cdot \cos(\theta_3 + 180 \cdot \text{deg})$$

$$R_{43x} = -6.297 \text{ in}$$

$$R_{43y} := (R_{CG3} - b) \cdot \sin(\theta_3 + 180 \cdot \text{deg})$$

$$R_{43y} = 39.501 \text{ in}$$

$$R_{34x} := R_{34} \cdot \cos(\theta_4 + \alpha)$$

$$R_{34x} = -31.702 \text{ in}$$

$$R_{34y} := R_{34} \cdot \sin(\theta_4 + \alpha)$$

$$R_{34y} = 4.357 \text{ in}$$

$$R_{14x} := R_{14} \cdot \cos(\theta_4 + \beta + 180 \cdot \text{deg})$$

$$R_{14x} = -76.508 \text{ in}$$

$$R_{14y} := R_{14} \cdot \sin(\theta_4 + \beta + 180 \cdot \text{deg})$$

$$R_{14y} = -20.550 \text{ in}$$

$$R_{Px} := (R_P) \cdot \cos(\theta_4 + \delta)$$

$$R_{Px} = -123.828 \text{ in}$$

$$R_{Py} := (R_P) \cdot \sin(\theta_4 + \delta)$$

$$R_{Py} = -12.326 \text{ in}$$

7. Calculate the x and y components of the external force at *P* and the weight forces at the CGs. in the CGS.

$$F_{Px} := 0 \cdot \text{lbf}$$

$$F_{Py} := -F$$

$$F_{G2y} := -W_2$$

$$F_{G3y} := -W_3$$

$$F_{G4y} := -W_4$$

8. Substitute these given and calculated values into the matrix equation 11.9. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.9 will be made dimensionless and the dimensions will be put back in after solving it.

$$m_2 := \frac{W_2}{g}$$

$$m_2 = 1.55 \text{ blob}$$

$$m_3 := \frac{W_3}{g}$$

$$m_3 = 0.28 \text{ blob}$$

$$m_4 := \frac{W_4}{g}$$

$$m_4 = 7.01 \text{ blob}$$

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{in} & \frac{R_{12x}}{in} & \frac{-R_{32y}}{in} & \frac{R_{32x}}{in} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{in} & \frac{-R_{23x}}{in} & \frac{-R_{43y}}{in} & \frac{R_{43x}}{in} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{in} & \frac{-R_{34x}}{in} & \frac{-R_{14y}}{in} & \frac{R_{14x}}{in} & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot lbf^{-1} \\ (m_2 \cdot a_{G2y} - F_{G2y}) \cdot lbf^{-1} \\ I_{G2} \cdot \alpha_2 \cdot lbf^{-1} \cdot in^{-1} \\ m_3 \cdot a_{G3x} \cdot lbf^{-1} \\ (m_3 \cdot a_{G3y} - F_{G3y}) \cdot lbf^{-1} \\ I_{G3} \cdot \alpha_3 \cdot lbf^{-1} \cdot in^{-1} \\ (m_4 \cdot a_{G4x} - F_{Px}) \cdot lbf^{-1} \\ (m_4 \cdot a_{G4y} - F_{Py} - F_{G4y}) \cdot lbf^{-1} \\ [I_{G4} \cdot \alpha_4 - (R_{Px} \cdot F_{Py} - R_{Py} \cdot F_{Px})] \cdot lbf^{-1} \cdot in^{-1} \end{bmatrix} \quad R := C^{-1} \cdot F$$

$$\begin{array}{llll} F_{12x} := R_1 \cdot lbf & F_{12x} = -327 \text{ lbf} & F_{12y} := R_2 \cdot lbf & F_{12y} = 2682 \text{ lbf} \\ F_{32x} := R_3 \cdot lbf & F_{32x} = 324 \text{ lbf} & F_{32y} := R_4 \cdot lbf & F_{32y} = -2086 \text{ lbf} \\ F_{43x} := R_5 \cdot lbf & F_{43x} = 324 \text{ lbf} & F_{43y} := R_6 \cdot lbf & F_{43y} = -1978 \text{ lbf} \\ F_{14x} := R_7 \cdot lbf & F_{14x} = 323 \text{ lbf} & F_{14y} := R_8 \cdot lbf & F_{14y} = 3012 \text{ lbf} \\ T_{12} := R_9 \cdot lbf \cdot in & T_{12} = 29442 \text{ lbf} \cdot in & & \end{array}$$

PROBLEM 11-21

Statement: Figure P11-5a shows an oil field pump mechanism. The head of the rocker arm is shaped such that the lower end of a flexible cable attached to it will always be directly over the well head regardless of the position of the rocker arm 4. The pump rod, which connects to the pump in the well casing, is connected to the lower end of the cable. The force in the pump rod on the up stroke is 2970 lb and the force on the down stroke is 2300 lb. Link 2 weighs 598.3 lb and has a moment of inertia of 11.8 blob·in²; both including the counterweight. Its CG is on the link centerline, 13.2 in from O₂. Link 3 weighs 108 lb and its CG is on the link centerline, 40 in from A. It has a mass moment of inertia of 150 blob·in². Link 4 weighs 2706 lb and has a moment of inertia of 10700 blob·in²; both include the counterweight. Its CG is on the link centerline where shown. The crank turns at a constant speed of 4 rpm CCW. Find and plot all pin forces and the torque needed to drive the crank for one revolution of the crank. Include gravity forces.

Units: $\text{blob} := \text{lb} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Link lengths:

Link 2 (O₂ to A): $a := 14.0 \cdot \text{in}$ Link 3 (A to B): $b := 80.0 \cdot \text{in}$

Link 4 (B to O₄): $c := 51.3 \cdot \text{in}$ Link 1 (O₂ to O₄): $d := 79.7 \cdot \text{in}$

Link 1 offsets: $d_X := -47.5 \cdot \text{in}$ $d_Y := 64 \cdot \text{in}$

External load data: $F := 2300 \cdot \text{lb}$ $T_4 := 0 \cdot \text{lb} \cdot \text{in}$

Crank angle and motion: $\theta_2 := 45 \cdot \text{deg}$ $\omega_2 := 4 \cdot \text{rpm}$ $\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Link CG positions: $R_{CG2} := 13.2 \cdot \text{in}$ $R_{CG3} := 40.0 \cdot \text{in}$ $R_{CG4} := 32.0 \cdot \text{in}$

Link weights: $W_2 := 598.3 \cdot \text{lb}$ $W_3 := 108 \cdot \text{lb}$ $W_4 := 2706 \cdot \text{lb}$

Moments of inertia: $I_{G2} := 11.8 \cdot \text{blob} \cdot \text{in}^2$ $I_{G3} := 150 \cdot \text{blob} \cdot \text{in}^2$ $I_{G4} := 10700 \cdot \text{blob} \cdot \text{in}^2$

Angle between O₄B and CG₄B: $\alpha := 143.11 \cdot \text{deg}$ $R_{34} := 32.00 \cdot \text{in}$

Angle between O₄B and CG₄O₄: $\beta := -14.03 \cdot \text{deg}$ $R_{14} := 79.22 \cdot \text{in}$

Angle between O₄B and CG₄P: $\delta := 156.62 \cdot \text{deg}$ $R_p := 124.44 \cdot \text{in}$

Solution: See Figure P11-12 and Mathcad file P1121.

1. No solution is given for this problem, which is suited to solution using the *Working Model* program.

PROBLEM 11-22

Statement: Figure P11-5a shows an oil field pump mechanism. The head of the rocker arm is shaped such that the lower end of a flexible cable attached to it will always be directly over the well head regardless of the position of the rocker arm 4. The pump rod, which connects to the pump in the well casing, is connected to the lower end of the cable. The force in the pump rod on the up stroke is 2970 lb and the force on the down stroke is 2300 lb. Link 2 weighs 598.3 lb and has a moment of inertia of 11.8 blob-in²; both including the counterweight. Its CG is on the link centerline, 13.2 in from O₂. Link 3 weighs 108 lb and its CG is on the link centerline, 40 in from A. It has a mass moment of inertia of 150 blob-in². Link 4 weighs 2706 lb and has a moment of inertia of 10700 blob-in²; both include the counterweight. Its CG is on the link centerline where shown. The crank turns at a constant speed of 4 rpm CCW. At the instant shown in the figure the crank angle is at 45 deg with respect to the global coordinate system. Find the torque needed to drive the crank for the position shown using the method of virtual work. Include gravity forces.

Units: $\text{blob} := \text{lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Link lengths:

Link 2 (O₂ to A): $a := 14.0 \cdot \text{in}$ Link 3 (A to B): $b := 80.0 \cdot \text{in}$

Link 4 (B to O₄): $c := 51.3 \cdot \text{in}$ Link 1 (O₂ to O₄): $d := 79.7 \cdot \text{in}$

Link 1 offsets: $d_X := -47.5 \cdot \text{in}$ $d_Y := 64 \cdot \text{in}$

External load data: $F_{p4y} := -2300 \cdot \text{lbf}$

Crank angle and motion: $\theta_2 := 45 \cdot \text{deg}$ $\omega_2 := 4 \cdot \text{rpm}$ $\alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$

Link CG positions: $R_{CG2} := 13.2 \cdot \text{in}$ $R_{CG3} := 40.0 \cdot \text{in}$ $R_{CG4} := 32.0 \cdot \text{in}$

Link weights: $W_2 := 598.3 \cdot \text{lbf}$ $W_3 := 108 \cdot \text{lbf}$ $W_4 := 2706 \cdot \text{lbf}$

Moments of inertia: $I_{G2} := 11.8 \cdot \text{blob} \cdot \text{in}^2$ $I_{G3} := 150 \cdot \text{blob} \cdot \text{in}^2$ $I_{G4} := 10700 \cdot \text{blob} \cdot \text{in}^2$

Angle between O₄B and CG₄B: $\alpha := 143.11 \cdot \text{deg}$ $R_{34} := 32.00 \cdot \text{in}$

Angle between O₄B and CG₄O₄: $\beta := -14.03 \cdot \text{deg}$ $R_{14} := 79.22 \cdot \text{in}$

Angle between O₄B and CG₄P: $\delta := 156.62 \cdot \text{deg}$ $R_p := 124.44 \cdot \text{in}$

Solution: See Figure P11-12 and Mathcad file P1122.

1. Use Problems 6.84c and 7.70b with $\omega_2 = 4$ rpm to determine the position, velocity, and acceleration of links 3 and 4. The angles are calculated in the xy coordinate system and then rotated into the XY coordinate system, which is used in the remainder of the problem.

Coordinate rotation angle: $\gamma := \text{atan2}(d_X, d_Y)$ $\gamma = 126.582 \cdot \text{deg}$

$\theta_3 := 99.057 \cdot \text{deg}$ $\omega_3 := -0.0214 \cdot \text{rad} \cdot \text{sec}^{-1}$ $\alpha_3 := -0.0250 \cdot \text{rad} \cdot \text{sec}^{-2}$

$\theta_4 := 29.064 \cdot \text{deg}$ $\omega_4 := 0.0986 \cdot \text{rad} \cdot \text{sec}^{-1}$ $\alpha_4 := -0.0272 \cdot \text{rad} \cdot \text{sec}^{-2}$

Accelerations: $a_{G2} := 2.316 \cdot \text{in} \cdot \text{sec}^{-2}$ $\theta_{AG2} := 225.00 \cdot \text{deg}$

$a_{G3} := 1.764 \cdot \text{in} \cdot \text{sec}^{-2}$ $\theta_{AG3} := 244.915 \cdot \text{deg}$

$a_{G4} := 2.288 \cdot \text{in} \cdot \text{sec}^{-2}$ $\theta_{AG4} := 265.366 \cdot \text{deg}$

$a_{p4} := 1.391 \cdot \text{in} \cdot \text{sec}^{-2}$ $\theta_{AP4} := 60.394 \cdot \text{deg}$

Velocities: $v_{G2} := 5.529 \cdot \text{in} \cdot \text{sec}^{-1}$ $\theta_{VG2} := 135.0 \cdot \text{deg}$

$$v_{G3} := 5.406 \cdot \text{in} \cdot \text{sec}^{-1} \quad \theta_{VG3} := 127.628 \cdot \text{deg}$$

$$v_{G4} := 7.809 \cdot \text{in} \cdot \text{sec}^{-1} \quad \theta_{VG4} := 105.034 \cdot \text{deg}$$

$$v_{P4} := 4.753 \cdot \text{in} \cdot \text{sec}^{-1} \quad \theta_{VP4} := 245.646 \cdot \text{deg}$$

2. Calculate the x and y components of the velocity vectors.

$$v_{G2x} := v_{G2} \cdot \cos(\theta_{VG2}) \quad v_{G2x} = -3.910 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{G2y} := v_{G2} \cdot \sin(\theta_{VG2}) \quad v_{G2y} = 3.910 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{G3x} := v_{G3} \cdot \cos(\theta_{VG3}) \quad v_{G3x} = -3.301 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{G3y} := v_{G3} \cdot \sin(\theta_{VG3}) \quad v_{G3y} = 4.282 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{G4x} := v_{G4} \cdot \cos(\theta_{VG4}) \quad v_{G4x} = -2.026 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{G4y} := v_{G4} \cdot \sin(\theta_{VG4}) \quad v_{G4y} = 7.542 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{P4x} := v_{P4} \cdot \cos(\theta_{VP4}) \quad v_{P4x} = -1.960 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{P4y} := v_{P4} \cdot \sin(\theta_{VP4}) \quad v_{P4y} = -4.330 \text{ in} \cdot \text{sec}^{-1}$$

3. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$a_{G2x} := a_{G2} \cdot \cos(\theta_{AG2}) \quad a_{G2x} = -1.638 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G2y} := a_{G2} \cdot \sin(\theta_{AG2}) \quad a_{G2y} = -1.638 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G3x} := a_{G3} \cdot \cos(\theta_{AG3}) \quad a_{G3x} = -0.748 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G3y} := a_{G3} \cdot \sin(\theta_{AG3}) \quad a_{G3y} = -1.598 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G4x} := a_{G4} \cdot \cos(\theta_{AG4}) \quad a_{G4x} = -0.185 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G4y} := a_{G4} \cdot \sin(\theta_{AG4}) \quad a_{G4y} = -2.281 \text{ in} \cdot \text{sec}^{-2}$$

4. Calculate the mass of each link.

$$m_2 := \frac{W_2}{g} \quad m_3 := \frac{W_3}{g} \quad m_4 := \frac{W_4}{g}$$

5. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \left[m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \dots \right. \\ \left. + m_4 \cdot (a_{G4x} \cdot v_{G4x} + a_{G4y} \cdot v_{G4y}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3 + I_{G4} \cdot \alpha_4 \cdot \omega_4) \dots \right. \\ \left. + (W_2 \cdot v_{G2y} + W_3 \cdot v_{G3y} + W_4 \cdot v_{G4y}) - (F_{P4y} \cdot v_{P4y}) \right]$$

$$T_{12} = 10219 \text{ lbf} \cdot \text{in}$$

PROBLEM 11-23

Statement: Use the information in problem 11-20 to find and plot the torque needed to drive the crank for one revolution of the crank using the method of virtual work.

Solution: See Figure P11-12 and Mathcad file P1123.

No solution for this problem is provided. This problem is more suitable for a longer-term project than for a short-term homework problem.

PROBLEM 11-24

Statement: In Figure P11-13, links 2 and 4 each weigh 2 lb and there are 2 of each (another set behind). Their CGs are at their midpoints. Link 3 weighs 10 lb. The moments of inertia of links 2, 3, and 4 are 0.071, 0.430, and 0.077 blob·in², respectively. Find the torque needed to begin a slow CCW rotation of link 2 from the position shown using the method of virtual work. Include gravity forces.

Units: $\text{blob} := \text{lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 9.174 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 12.971 \cdot \text{in}$$

$$\text{Link 4 (B to O}_4) \quad c := 9.573 \cdot \text{in} \quad \text{Link 3 (O}_2 \text{ to O}_4) \quad d := 7.487 \cdot \text{in}$$

$$\text{Link angles (LCS):} \quad \theta_2 := -26.0 \cdot \text{deg} \quad \theta_3 := 72.239 \cdot \text{deg} \quad \theta_4 := 60.491 \cdot \text{deg}$$

$$\text{Weight:} \quad W_2 := 4 \cdot \text{lbf} \quad W_3 := 10 \cdot \text{lbf} \quad W_4 := 4 \cdot \text{lbf}$$

$$\text{Mass:} \quad m_2 := \frac{W_2}{g} \quad m_3 := \frac{W_3}{g} \quad m_4 := \frac{W_4}{g}$$

$$m_2 = 0.010 \text{ blob} \quad m_3 = 0.026 \text{ blob} \quad m_4 = 0.010 \text{ blob}$$

$$\text{Moment of inertia:} \quad I_{G2} := 0.071 \cdot \text{blob} \cdot \text{in}^2 \quad I_{G3} := 0.430 \cdot \text{blob} \cdot \text{in}^2 \quad I_{G4} := 0.077 \cdot \text{blob} \cdot \text{in}^2$$

$$\text{Mass center:} \quad R_{CG2} := 4.587 \cdot \text{in} \quad \delta_2 := 0 \cdot \text{deg} \quad R_{CG3} := 7.086 \cdot \text{in} \quad \delta_3 := -23.758 \cdot \text{deg}$$

$$R_{CG4} := 4.786 \cdot \text{in} \quad \delta_4 := 0 \cdot \text{deg}$$

$$\text{Force and torque:} \quad F_{P3} := 0 \cdot \text{lbf} \quad \delta_{FP3} := 0 \cdot \text{deg} \quad F_{P4} := 0 \cdot \text{lbf} \quad \delta_{FP4} := 0 \cdot \text{deg}$$

$$T_3 := 0 \cdot \text{lbf} \cdot \text{in} \quad T_4 := 0 \cdot \text{lbf} \cdot \text{in}$$

$$\text{Accelerations:} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2} \quad a_{G2} := 0.0 \cdot \text{in} \cdot \text{sec}^{-2} \quad \theta_{AG2} := 85.879 \cdot \text{deg}$$

$$\alpha_3 := 0.00206 \cdot \text{rad} \cdot \text{sec}^{-2} \quad a_{G3} := 0.0174 \cdot \text{in} \cdot \text{sec}^{-2} \quad \theta_{AG3} := 100.159 \cdot \text{deg}$$

$$\alpha_4 := 0.0025 \cdot \text{rad} \cdot \text{sec}^{-2} \quad a_{G4} := 0.0159 \cdot \text{in} \cdot \text{sec}^{-2} \quad \theta_{AG4} := 123.308 \cdot \text{deg}$$

$$\text{Velocities:} \quad \omega_2 := 0.010 \cdot \text{rad} \cdot \text{sec}^{-1} \quad v_{G2} := 0.0459 \cdot \text{in} \cdot \text{sec}^{-1} \quad \theta_{VG2} := -41.21 \cdot \text{deg}$$

$$\omega_3 := 0.0347 \cdot \text{rad} \cdot \text{sec}^{-1} \quad v_{G3} := 0.284 \cdot \text{in} \cdot \text{sec}^{-1} \quad \theta_{VG3} := 52.251 \cdot \text{deg}$$

$$\omega_4 := 0.0466 \cdot \text{rad} \cdot \text{sec}^{-1} \quad v_{G4} := 0.0223 \cdot \text{in} \cdot \text{sec}^{-1} \quad \theta_{VG4} := 82.367 \cdot \text{deg}$$

Solution: See Figure P11-13 and Mathcad file P1124.

1. Calculate the x and y components of the velocity vectors (global coordinate system).

$$v_{G2x} := v_{G2} \cdot \cos(\theta_{VG2}) \quad v_{G2x} = 0.035 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{G2y} := v_{G2} \cdot \sin(\theta_{VG2}) \quad v_{G2y} = -0.030 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{G3x} := v_{G3} \cdot \cos(\theta_{VG3}) \quad v_{G3x} = 0.174 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{G3y} := v_{G3} \cdot \sin(\theta_{VG3}) \quad v_{G3y} = 0.225 \text{ in} \cdot \text{sec}^{-1}$$

$$v_{G4x} := v_{G4} \cdot \cos(\theta_{VG4}) \quad v_{G4x} = 2.962 \times 10^{-3} \text{ in} \cdot \text{sec}^{-1}$$

$$v_{G4y} := v_{G4} \cdot \sin(\theta_{VG4})$$

$$v_{G4y} = 0.022 \text{ in} \cdot \text{sec}^{-1}$$

2. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$a_{G2x} := a_{G2} \cdot \cos(\theta_{AG2})$$

$$a_{G2x} = 0.000 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G2y} := a_{G2} \cdot \sin(\theta_{AG2})$$

$$a_{G2y} = 0.000 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G3x} := a_{G3} \cdot \cos(\theta_{AG3})$$

$$a_{G3x} = -3.069 \times 10^{-3} \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G3y} := a_{G3} \cdot \sin(\theta_{AG3})$$

$$a_{G3y} = 0.017 \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G4x} := a_{G4} \cdot \cos(\theta_{AG4})$$

$$a_{G4x} = -8.731 \times 10^{-3} \text{ in} \cdot \text{sec}^{-2}$$

$$a_{G4y} := a_{G4} \cdot \sin(\theta_{AG4})$$

$$a_{G4y} = 0.013 \text{ in} \cdot \text{sec}^{-2}$$

3. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \left[m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \dots \right. \\ \left. + m_4 \cdot (a_{G4x} \cdot v_{G4x} + a_{G4y} \cdot v_{G4y}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3 + I_{G4} \cdot \alpha_4 \cdot \omega_4) \dots \right. \\ \left. + -T_3 \cdot \omega_3 - T_4 \cdot \omega_4 + (W_2 \cdot v_{G2y} + W_3 \cdot v_{G3y} + W_4 \cdot v_{G4y}) \right]$$

$$T_{12} = 221.32 \text{ lbf} \cdot \text{in}$$

PROBLEM 11-25

Statement: Figure P11-3 shows a fourbar linkage and its dimensions. The steel crank and rocker have uniform cross sections 1 in wide by 0.5 in thick. The aluminum coupler is 0.75 in thick. In the instantaneous position shown, the crank O_2A has $\omega = 40 \text{ rad/sec}$ and $\alpha = -20 \text{ rad/sec}^2$. There is a horizontal force at P of $F = 50 \text{ lb}$. Find the torque needed to drive the crank at this instant using the method of virtual work.

Units: $\text{blob} := \text{lb} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Link lengths:

$$\text{Link 2 (} O_2 \text{ to } A) \quad a := 5.0 \cdot \text{in} \quad \text{Link 3 (} A \text{ to } B) \quad b := 7.4 \cdot \text{in}$$

$$\text{Link 4 (} B \text{ to } O_4) \quad c := 8.0 \cdot \text{in} \quad \text{Link 1 (} O_2 \text{ to } O_4) \quad d := 9.5 \cdot \text{in}$$

$$\text{Coupler point:} \quad R_{pa} := 8.90 \cdot \text{in} \quad \delta_3 := 56 \cdot \text{deg} \quad F := 50 \cdot \text{lb} \quad T_4 := 0 \cdot \text{lb} \cdot \text{in}$$

$$\text{Crank angle and motion:} \quad \theta_2 := 50 \cdot \text{deg} \quad \omega_2 := 40 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := -20 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 1.0 \cdot \text{in} \quad t_2 := 0.50 \cdot \text{in} \quad t_3 := 0.75 \cdot \text{in} \quad w_4 := 1.0 \cdot \text{in} \quad t_4 := 0.50 \cdot \text{in}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lb} \cdot \text{in}^{-3}$$

Solution: See Figure P11-3 and Mathcad file P1125.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$$\theta_3 := 10.105 \cdot \text{deg} \quad \omega_3 := -41.552 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_3 := -335.762 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\theta_4 := 113.008 \cdot \text{deg} \quad \omega_4 := 26.320 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_4 := 2963.667 \cdot \text{rad} \cdot \text{sec}^{-2}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 2.500 \cdot \text{in} \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 4.000 \cdot \text{in}$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 4.126 \cdot \text{in}$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = 2.459 \cdot \text{in}$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 4.803 \cdot \text{in}$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = 30.801 \cdot \text{deg}$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot R_{pa} \cdot \sin(\delta_3) \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 1.943 \times 10^{-3} \cdot \text{blob} \quad m_3 = 5.303 \times 10^{-3} \cdot \text{blob} \quad m_4 = 3.108 \times 10^{-3} \cdot \text{blob}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 4.209 \times 10^{-3} \cdot \text{blob} \cdot \text{in}^2$$

$$I_{G3} := \frac{m_3}{6} \cdot \left[b^2 + (R_{pa} \cdot \sin(\delta_3))^2 \right]$$

$$I_{G3} = 0.097 \cdot \text{blob} \cdot \text{in}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2)$$

$$I_{G4} = 0.017 \cdot \text{blob} \cdot \text{in}^2$$

4. Calculate the x and y components of the velocity of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{v}_{G2} := R_{CG2} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$v_{G2x} := \text{Re}(\mathbf{v}_{G2})$$

$$v_{G2x} = -76.604 \frac{\text{in}}{\text{s}}$$

$$v_{G2y} := \text{Im}(\mathbf{v}_{G2})$$

$$v_{G2y} = 64.279 \frac{\text{in}}{\text{s}}$$

$$\mathbf{v}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{v}_{CG3A} := R_{CG3} \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_{33}) + j \cdot \cos(\theta_3 + \delta_{33}))$$

$$\mathbf{v}_{G3} := \mathbf{v}_A + \mathbf{v}_{CG3A}$$

$$v_{G3x} := \text{Re}(\mathbf{v}_{G3})$$

$$v_{G3x} = -22.521 \frac{\text{in}}{\text{s}}$$

$$v_{G3y} := \text{Im}(\mathbf{v}_{G3})$$

$$v_{G3y} = -22.280 \frac{\text{in}}{\text{s}}$$

$$\mathbf{v}_{PA} := R_{pa} \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3))$$

$$\mathbf{v}_P := \mathbf{v}_A + \mathbf{v}_{PA}$$

$$v_{Px} := \text{Re}(\mathbf{v}_P)$$

$$v_{Px} = 184.907 \frac{\text{in}}{\text{s}}$$

$$v_{Py} := \text{Im}(\mathbf{v}_P)$$

$$v_{Py} = -21.240 \frac{\text{in}}{\text{s}}$$

$$\mathbf{v}_{G4} := R_{CG4} \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$$

$$v_{G4x} := \text{Re}(\mathbf{v}_{G4})$$

$$v_{G4x} = -96.905 \frac{\text{in}}{\text{s}}$$

$$v_{G4y} := \text{Im}(\mathbf{v}_{G4})$$

$$v_{G4y} = -41.150 \frac{\text{in}}{\text{s}}$$

5. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2} := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$a_{G2x} := \text{Re}(\mathbf{a}_{G2})$$

$$a_{G2x} = -5.104 \times 10^3 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$a_{G2y} := \text{Im}(\mathbf{a}_{G2})$$

$$a_{G2y} = -6.160 \times 10^3 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$\mathbf{a}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{a}_{CG3A} := R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_3 + \delta_{33}) + j \cdot \cos(\theta_3 + \delta_{33})) \dots$$

$$+ -R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_3 + \delta_{33}) + j \cdot \sin(\theta_3 + \delta_{33}))$$

$$\mathbf{a}_{G3} := \mathbf{a}_A + \mathbf{a}_{CG3A}$$

$$a_{G3x} := \text{Re}(\mathbf{a}_{G3})$$

$$a_{G3x} = -10277.3 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$a_{G3y} := \text{Im}(\mathbf{a}_{G3})$$

$$a_{G3y} = -12841.8 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$\mathbf{a}_{G4} := R_{CG4} \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$a_{G4x} := \text{Re}(\mathbf{a}_{G4})$$

$$a_{G4x} = -8745.5 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$a_{G4y} := \text{Im}(\mathbf{a}_{G4})$$

$$a_{G4y} = -9734.6 \cdot \frac{\text{in}}{\text{sec}^2}$$

6. Calculate the x and y components of the external force at P in the CGS.

$$F_{Px} := -F$$

$$F_{Py} := 0 \cdot \text{lbf}$$

7. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \left[m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \dots \right. \\ \left. + m_4 \cdot (a_{G4x} \cdot v_{G4x} + a_{G4y} \cdot v_{G4y}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3 + I_{G4} \cdot \alpha_4 \cdot \omega_4) \dots \right. \\ \left. + -F_{Px} \cdot v_{Px} - F_{Py} \cdot v_{Py} \right]$$

$$T_{12} = 463 \cdot \text{lbf} \cdot \text{in}$$

PROBLEM 11-26

Statement: Figure P11-3 shows a fourbar linkage and its dimensions. The steel crank and rocker have uniform cross sections 1 in wide by 0.5 in thick. The aluminum coupler is 0.75 in thick. In the instantaneous position shown, the crank O_2A has $\omega = 40$ rad/sec. There is a horizontal force at P of $F = 50$ lb. Find and plot all pin forces and the torque needed to drive the crank at a constant speed of 40 rad/sec for one revolution of the crank using the program FOURBAR. Change the lengths of links 3 and 4 to 7.4 and 8.0 in, respectively.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 5.00 \cdot in \quad \text{Link 3 (A to B)} \quad b := 7.40 \cdot in$$

$$\text{Link 4 (B to O}_4) \quad c := 8.00 \cdot in \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 9.50 \cdot in$$

$$\text{Coupler point:} \quad R_{pa} := 8.90 \cdot in \quad \delta_3 := 56 \cdot deg \quad F_P := 50 \cdot lbf \quad T_4 := 0 \cdot lbf \cdot in$$

$$\text{Crank motion:} \quad \omega_2 := 40 \cdot rad \cdot sec^{-1} \quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$$

Link cross-section dims:

$$w_2 := 1.00 \cdot in \quad t_2 := 0.50 \cdot in \quad t_3 := 0.75 \cdot in \quad w_4 := 1.00 \cdot in \quad t_4 := 0.50 \cdot in$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot lbf \cdot in^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot lbf \cdot in^{-3}$$

Solution: See Figure P11-3 and Mathcad file P1127.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 2.500 \cdot in \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 4.000 \cdot in$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 4.126 \cdot in$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = 2.459 \cdot in$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 4.803 \cdot in$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = 30.801 \cdot deg$$

2. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot R_{pa} \cdot \sin(\delta_3) \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 1.943 \times 10^{-3} \cdot blob \quad m_3 = 5.303 \times 10^{-3} \cdot blob \quad m_4 = 3.108 \times 10^{-3} \cdot blob$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.0042 \cdot in \cdot lbf \cdot sec^2$$

$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2] \quad I_{G3} = 0.097 \cdot in \cdot lbf \cdot sec^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 0.017 \cdot in \cdot lbf \cdot sec^2$$

3. Calculate the x and y components of the external force at P in the CGS and the magnitude and angle of the position vector defining point P in the LRCS.

$$F_{Px} := -F_P \quad F_{Py} := 0 \cdot lbf$$

$$\begin{aligned}
 R_{pax'} &:= R_{pa} \cdot \cos(\delta_3) & R_{pay'} &:= R_{pa} \cdot \sin(\delta_3) & R_{px'} &:= R_{pax'} - R_{CG3x'} & R_{py'} &:= R_{pay'} - R_{CG3y'} \\
 R_{pax'} &= 4.977 \text{ in} & R_{pay'} &= 7.378 \text{ in} & R_{px'} &= 0.851 \text{ in} & R_{py'} &= 4.919 \text{ in} \\
 R_p &:= \sqrt{R_{px'}^2 + R_{py'}^2} & R_p &= 4.992 \text{ in} & \theta_p &:= \text{atan}\left(\frac{R_{py'}}{R_{px'}}\right) & \theta_p &= 80.182 \text{ deg}
 \end{aligned}$$

4. Enter the above data into program FOURBAR and solve for the pin forces and driving torque. The dynamic input screen is shown below followed by a plot of dynamic pin forces..

FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06 Dynamics Screen

Help

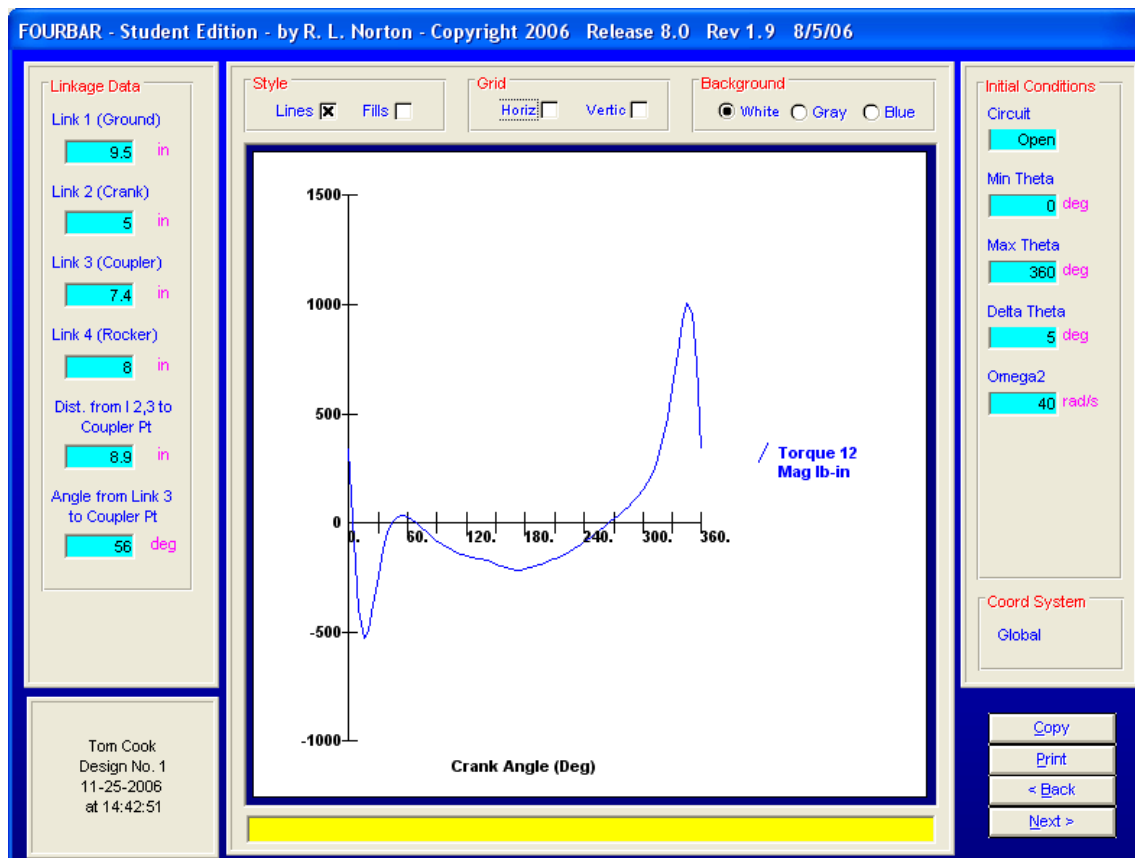
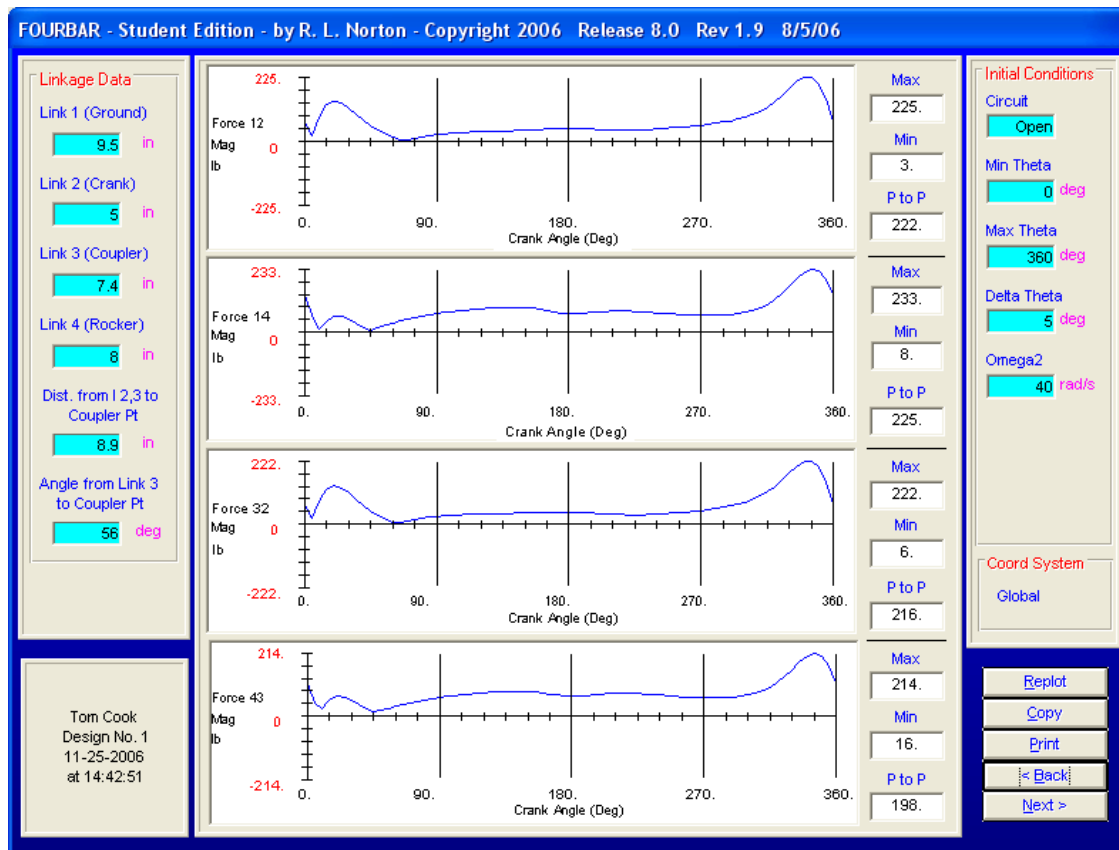
	Link	2	3	4	Units
Mass Properties					
Mass of Link		0.0019	0.0053	0.0031	lb
Mass Moment of Inertia about CG		0.0042	0.0970	0.0170	in-lb-s ²
CG Locations					
Distance to CG from Pivot		2.500	4.803	4.000	in
Angle to CG in LRCS*		0.00	30.80	0.00	degrees
External Forces					
Force Magnitude		50.00	0.00	0.00	lb
Angle of Force in GCS*		180	0	0	degrees
External Force Locations					
Distance to Force Point from CG		4.992	0.000	0.000	in
Angle to Force Point in LRCS*		80.182	0	0	degrees
External Torques					
Torque on Link		0.00	0.00	0.00	lb-in
Import a Torque Function File to Apply to Link 4		☐			
Friction					
Friction Force on Link		0.00	0.00	0.00	lb

* LRCS = Local Rotating Coordinate System. GCS = Global Coordinate System

Solve
Show Matrix
Copy
Print
< Back
Next >

Tom Cook
Design No. 1
11-25-2006
at 14:42:51

See next page for plots of pin forces and torque.



PROBLEM 11-27

Statement: Figure P11-3 shows a fourbar linkage and its dimensions. The steel crank and rocker have uniform cross sections 1 in wide by 0.5 in thick. The aluminum coupler is 0.75 in thick. In the instantaneous position shown, the crank O_2A has $\omega = 40$ rad/sec. There is a horizontal force at P of $F = 50$ lb. Find and plot the torque needed to drive the crank at a constant speed of 40 rad/sec for one revolution of the crank using the method of virtual work. Change the lengths of links 3 and 4 to 7.4 and 8.0 in, respectively.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (} O_2 \text{ to } A) \quad a := 5.00 \cdot in \quad \text{Link 3 (} A \text{ to } B) \quad b := 7.40 \cdot in$$

$$\text{Link 4 (} B \text{ to } O_4) \quad c := 8.00 \cdot in \quad \text{Link 1 (} O_2 \text{ to } O_4) \quad d := 9.50 \cdot in$$

$$\text{Coupler point:} \quad R_{pa} := 8.90 \cdot in \quad \delta_3 := 56 \cdot deg \quad F_P := 50 \cdot lbf \quad T_4 := 0 \cdot lbf \cdot in$$

$$\text{Crank motion:} \quad \omega_2 := 40 \cdot rad \cdot sec^{-1} \quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$$

Link cross-section dims:

$$w_2 := 1.00 \cdot in \quad t_2 := 0.50 \cdot in \quad t_3 := 0.75 \cdot in \quad w_4 := 1.00 \cdot in \quad t_4 := 0.50 \cdot in$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot lbf \cdot in^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot lbf \cdot in^{-3}$$

Solution: See Figure P11-3 and Mathcad file P1127.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 2.500 \cdot in \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 4.000 \cdot in$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 4.126 \cdot in$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = 2.459 \cdot in$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 4.803 \cdot in$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = 30.801 \cdot deg$$

2. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot R_{pa} \cdot \sin(\delta_3) \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 1.943 \times 10^{-3} \cdot blob \quad m_3 = 5.303 \times 10^{-3} \cdot blob \quad m_4 = 3.108 \times 10^{-3} \cdot blob$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 4.209 \times 10^{-3} \cdot blob \cdot in^2$$

$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2] \quad I_{G3} = 0.097 \cdot blob \cdot in^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 0.017 \cdot blob \cdot in^2$$

3. Calculate the position, angular velocity, and acceleration of links 3 and 4 as functions of θ_2 .

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 1.9000$$

$$K_2 = 1.1875$$

$$K_3 = 1.5561$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))} \quad \omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2))$$

$$B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2))$$

$$E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2))$$

$$B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2))$$

$$E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_{PA}(\theta_2) := R_{pa} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots \\ + -R_{pa} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{A}_P(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2)$$

$$A_P(\theta_2) := |\mathbf{A}_P(\theta_2)| \quad \theta_{AP}(\theta_2) := \arg(\mathbf{A}_P(\theta_2))$$

4. Calculate the x and y components of the velocity of the CGs of all moving links in the global coordinate system (GCS).

$$\begin{aligned}
 \mathbf{v}_{G2}(\theta_2) &:= R_{CG2} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) \\
 v_{G2x}(\theta_2) &:= \text{Re}(\mathbf{v}_{G2}(\theta_2)) & v_{G2y}(\theta_2) &:= \text{Im}(\mathbf{v}_{G2}(\theta_2)) \\
 \mathbf{v}_A(\theta_2) &:= a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) \\
 \mathbf{v}_{CG3A}(\theta_2) &:= R_{CG3} \cdot \omega_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_{33}) + j \cdot \cos(\theta_3(\theta_2) + \delta_{33})) \\
 \mathbf{v}_{G3}(\theta_2) &:= \mathbf{v}_A(\theta_2) + \mathbf{v}_{CG3A}(\theta_2) \\
 v_{G3x}(\theta_2) &:= \text{Re}(\mathbf{v}_{G3}(\theta_2)) & v_{G3y}(\theta_2) &:= \text{Im}(\mathbf{v}_{G3}(\theta_2)) \\
 \mathbf{v}_{PA}(\theta_2) &:= R_{pa} \cdot \omega_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \\
 \mathbf{v}_P(\theta_2) &:= \mathbf{v}_A(\theta_2) + \mathbf{v}_{PA}(\theta_2) \\
 v_{Px}(\theta_2) &:= \text{Re}(\mathbf{v}_P(\theta_2)) & v_{Py}(\theta_2) &:= \text{Im}(\mathbf{v}_P(\theta_2)) \\
 \mathbf{v}_{G4}(\theta_2) &:= R_{CG4} \cdot \omega_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) \\
 v_{G4x}(\theta_2) &:= \text{Re}(\mathbf{v}_{G4}(\theta_2)) & v_{G4y}(\theta_2) &:= \text{Im}(\mathbf{v}_{G4}(\theta_2))
 \end{aligned}$$

5. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\begin{aligned}
 \mathbf{a}_{G2}(\theta_2) &:= R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \\
 a_{G2x}(\theta_2) &:= \text{Re}(\mathbf{a}_{G2}(\theta_2)) & a_{G2y}(\theta_2) &:= \text{Im}(\mathbf{a}_{G2}(\theta_2)) \\
 \mathbf{a}_A(\theta_2) &:= a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \\
 \mathbf{a}_{CG3A}(\theta_2) &:= R_{CG3} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_{33}) + j \cdot \cos(\theta_3(\theta_2) + \delta_{33})) \dots \\
 &\quad + -R_{CG3} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_{33}) + j \cdot \sin(\theta_3(\theta_2) + \delta_{33})) \\
 \mathbf{a}_{G3}(\theta_2) &:= \mathbf{a}_A(\theta_2) + \mathbf{a}_{CG3A}(\theta_2) \\
 a_{G3x}(\theta_2) &:= \text{Re}(\mathbf{a}_{G3}(\theta_2)) & a_{G3y}(\theta_2) &:= \text{Im}(\mathbf{a}_{G3}(\theta_2)) \\
 \mathbf{a}_{G4}(\theta_2) &:= R_{CG4} \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) - c \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2))) \\
 a_{G4x}(\theta_2) &:= \text{Re}(\mathbf{a}_{G4}(\theta_2)) & a_{G4y}(\theta_2) &:= \text{Im}(\mathbf{a}_{G4}(\theta_2))
 \end{aligned}$$

6. Calculate the x and y components of the external force at P in the CGS.

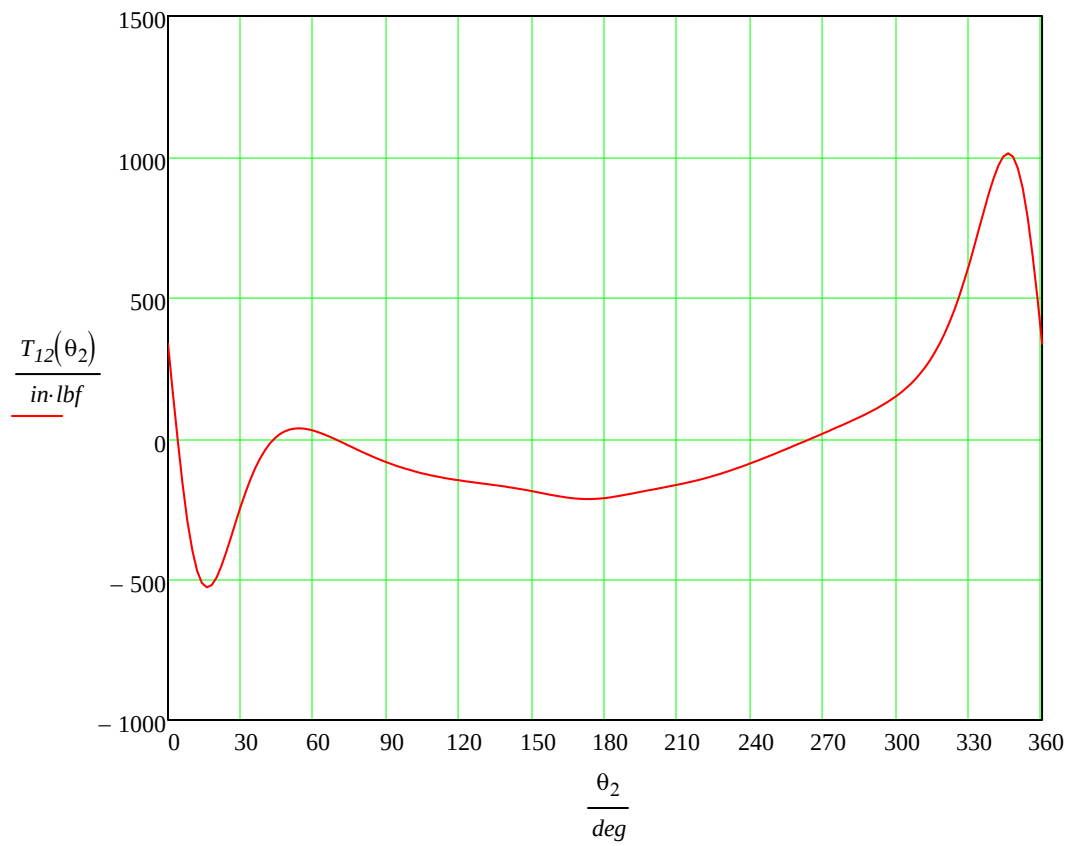
$$F_{Px} := -F_P \quad F_{Py} := 0 \cdot \text{lb}f$$

7. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12}(\theta_2) := \frac{1}{\omega_2} \cdot \left[\begin{aligned} &m_2 \cdot (a_{G2x}(\theta_2) \cdot v_{G2x}(\theta_2) + a_{G2y}(\theta_2) \cdot v_{G2y}(\theta_2)) \dots \\ &+ m_3 \cdot (a_{G3x}(\theta_2) \cdot v_{G3x}(\theta_2) + a_{G3y}(\theta_2) \cdot v_{G3y}(\theta_2)) \dots \\ &+ m_4 \cdot (a_{G4x}(\theta_2) \cdot v_{G4x}(\theta_2) + a_{G4y}(\theta_2) \cdot v_{G4y}(\theta_2)) \dots \\ &+ (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3(\theta_2) \cdot \omega_3(\theta_2) + I_{G4} \cdot \alpha_4(\theta_2) \cdot \omega_4(\theta_2)) \dots \\ &+ -F_{Px} \cdot v_{Px}(\theta_2) - F_{Py} \cdot v_{Py}(\theta_2) \end{aligned} \right]$$

8. Plot the input torque for one revolution of the crank.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg}$$



PROBLEM 11-28

Statement: Figure P11-4a shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 10 \text{ rad/sec}$ and $\alpha = 5 \text{ rad/sec}^2$. There is a vertical force at P of $F = 100 \text{ N}$. Find the torque needed to drive the crank at this instant using the method of virtual work.

Given:

Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 1.00 \cdot m \quad \text{Link 3 (A to B)} \quad b := 2.06 \cdot m$$

$$\text{Link 4 (B to O}_4) \quad c := 2.33 \cdot m \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 2.22 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 3.06 \cdot m \quad \delta_3 := -31 \cdot \text{deg} \quad F := 100 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank angle and motion:} \quad \theta_2 := 60 \cdot \text{deg} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 5 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3}$$

Solution: See Figure P11-4a and Mathcad file P1128.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$$\theta_3 := 44.732 \cdot \text{deg} \quad \omega_3 := -3.669 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_3 := 55.752 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\theta_4 := 96.322 \cdot \text{deg} \quad \omega_4 := 1.442 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_4 := 67.103 \cdot \text{rad} \cdot \text{sec}^{-2}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.500 \text{ m} \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 1.165 \text{ m}$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 1.561 \text{ m}$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = -0.525 \text{ m}$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 1.647 \text{ m}$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = -18.600 \text{ deg}$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot \left| R_{pa} \cdot \sin(\delta_3) \right| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 10.380 \text{ kg} \quad m_3 = 112.332 \text{ kg} \quad m_4 = 24.185 \text{ kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.867 \text{ kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{6} \cdot \left[b^2 + (R_{pa} \cdot \sin(\delta_3))^2 \right] \quad I_{G3} = 125.951 \text{ kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2)$$

$$I_{G4} = 10.947 \text{ kg} \cdot \text{m}^2$$

4. Calculate the x and y components of the velocity of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{v}_{G2} := R_{CG2} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$v_{G2x} := \text{Re}(\mathbf{v}_{G2})$$

$$v_{G2x} = -4.330 \frac{\text{m}}{\text{s}}$$

$$v_{G2y} := \text{Im}(\mathbf{v}_{G2})$$

$$v_{G2y} = 2.500 \frac{\text{m}}{\text{s}}$$

$$\mathbf{v}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{v}_{CG3A} := R_{CG3} \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_{33}) + j \cdot \cos(\theta_3 + \delta_{33}))$$

$$\mathbf{v}_{G3} := \mathbf{v}_A + \mathbf{v}_{CG3A}$$

$$v_{G3x} := \text{Re}(\mathbf{v}_{G3})$$

$$v_{G3x} = -5.999 \frac{\text{m}}{\text{s}}$$

$$v_{G3y} := \text{Im}(\mathbf{v}_{G3})$$

$$v_{G3y} = -0.425 \frac{\text{m}}{\text{s}}$$

$$\mathbf{v}_{PA} := R_{pa} \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3))$$

$$\mathbf{v}_P := \mathbf{v}_A + \mathbf{v}_{PA}$$

$$v_{Px} := \text{Re}(\mathbf{v}_P)$$

$$v_{Px} = -5.995 \frac{\text{m}}{\text{s}}$$

$$v_{Py} := \text{Im}(\mathbf{v}_P)$$

$$v_{Py} = -5.906 \frac{\text{m}}{\text{s}}$$

$$\mathbf{v}_{G4} := R_{CG4} \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$$

$$v_{G4x} := \text{Re}(\mathbf{v}_{G4})$$

$$v_{G4x} = -1.670 \frac{\text{m}}{\text{s}}$$

$$v_{G4y} := \text{Im}(\mathbf{v}_{G4})$$

$$v_{G4y} = -0.185 \frac{\text{m}}{\text{s}}$$

5. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2} := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$a_{G2x} := \text{Re}(\mathbf{a}_{G2})$$

$$a_{G2x} = -52.165 \frac{\text{m}}{\text{sec}^2}$$

$$a_{G2y} := \text{Im}(\mathbf{a}_{G2})$$

$$a_{G2y} = -85.353 \frac{\text{m}}{\text{sec}^2}$$

$$\mathbf{a}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{a}_{CG3A} := R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_3 + \delta_{33}) + j \cdot \cos(\theta_3 + \delta_{33})) \dots$$

$$+ R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_3 + \delta_{33}) + j \cdot \sin(\theta_3 + \delta_{33}))$$

$$\mathbf{a}_{G3} := \mathbf{a}_A + \mathbf{a}_{CG3A}$$

$$a_{G3x} := \text{Re}(\mathbf{a}_{G3})$$

$$a_{G3x} = -114.678 \frac{\text{m}}{\text{sec}^2}$$

$$a_{G3y} := \text{Im}(\mathbf{a}_{G3}) \quad a_{G3y} = -11.429 \frac{m}{\text{sec}^2}$$

$$\mathbf{a}_{G4} := R_{CG4} \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$a_{G4x} := \text{Re}(\mathbf{a}_{G4}) \quad a_{G4x} = -77.166 \frac{m}{\text{sec}^2}$$

$$a_{G4y} := \text{Im}(\mathbf{a}_{G4}) \quad a_{G4y} = -13.424 \frac{m}{\text{sec}^2}$$

6. Calculate the x and y components of the external force at P in the CGS.

$$F_{Px} := 0 \cdot N \quad F_{Py} := -F$$

7. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \left[m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \dots \right. \\ \left. + m_4 \cdot (a_{G4x} \cdot v_{G4x} + a_{G4y} \cdot v_{G4y}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3 + I_{G4} \cdot \alpha_4 \cdot \omega_4) \dots \right. \\ \left. + -F_{Px} \cdot v_{Px} - F_{Py} \cdot v_{Py} \right]$$

$$T_{12} = 5588 \text{ N} \cdot \text{m}$$

PROBLEM 11-29

Statement: Figure P11-4a shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 10$ rad/sec. There is a vertical force at P of $F = 100$ N. Find and plot all pin forces and the torque needed to drive the crank at a constant speed of 10 rad/sec for one revolution of the crank using the program FOURBAR.

Given:

Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 1.00 \cdot m \quad \text{Link 3 (A to B)} \quad b := 2.06 \cdot m$$

$$\text{Link 4 (B to O}_4) \quad c := 2.33 \cdot m \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 2.22 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 3.06 \cdot m \quad \delta_3 := -31 \cdot \text{deg} \quad F_P := 100 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank motion:} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lbf} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Solution: See Figure P11-4a and Mathcad file P1129.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.500 \, m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 1.165 \, m$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 1.561 \, m$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = -0.525 \, m$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 1.647 \, m$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = -18.600 \, \text{deg}$$

2. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot |R_{pa} \cdot \sin(\delta_3)| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 10.380 \, \text{kg} \quad m_3 = 112.332 \, \text{kg} \quad m_4 = 24.185 \, \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.867 \, \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2] \quad I_{G3} = 125.951 \, \text{kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 10.947 \, \text{kg} \cdot \text{m}^2$$

3. Calculate the x and y components of the external force at P in the CGS and the magnitude and angle of the position vector defining point P in the LRCS.

$$F_{Px} := 0 \cdot N \quad F_{Py} := -F_P$$

$$\begin{aligned}
 R_{pax'} &:= R_{pa} \cdot \cos(\delta_3) & R_{pay'} &:= R_{pa} \cdot \sin(\delta_3) & R_{px'} &:= R_{pax'} - R_{CG3x'} & R_{py'} &:= R_{pay'} - R_{CG3y'} \\
 R_{pax'} &= 2.623 \text{ m} & R_{pay'} &= -1.576 \text{ m} & R_{px'} &= 1.062 \text{ m} & R_{py'} &= -1.051 \text{ m} \\
 R_p &:= \sqrt{R_{px'}^2 + R_{py'}^2} & R_p &= 1.494 \text{ m} & \theta_p &:= \text{atan}\left(\frac{R_{py'}}{R_{px'}}\right) & \theta_p &= -44.694 \text{ deg}
 \end{aligned}$$

4. Enter the above data into program FOURBAR and solve for the pin forces and driving torque. The dynamic input screen is shown below followed by a plot of dynamic pin forces..

FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06 Dynamics Screen

Help

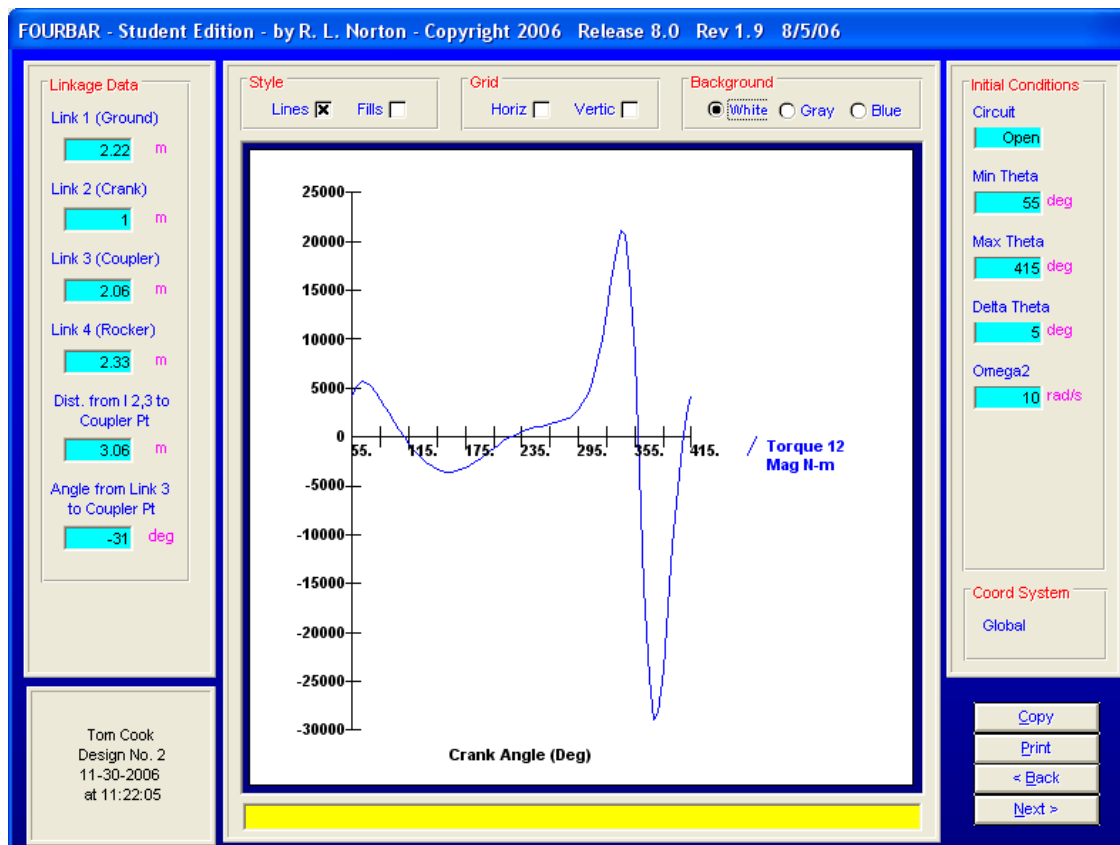
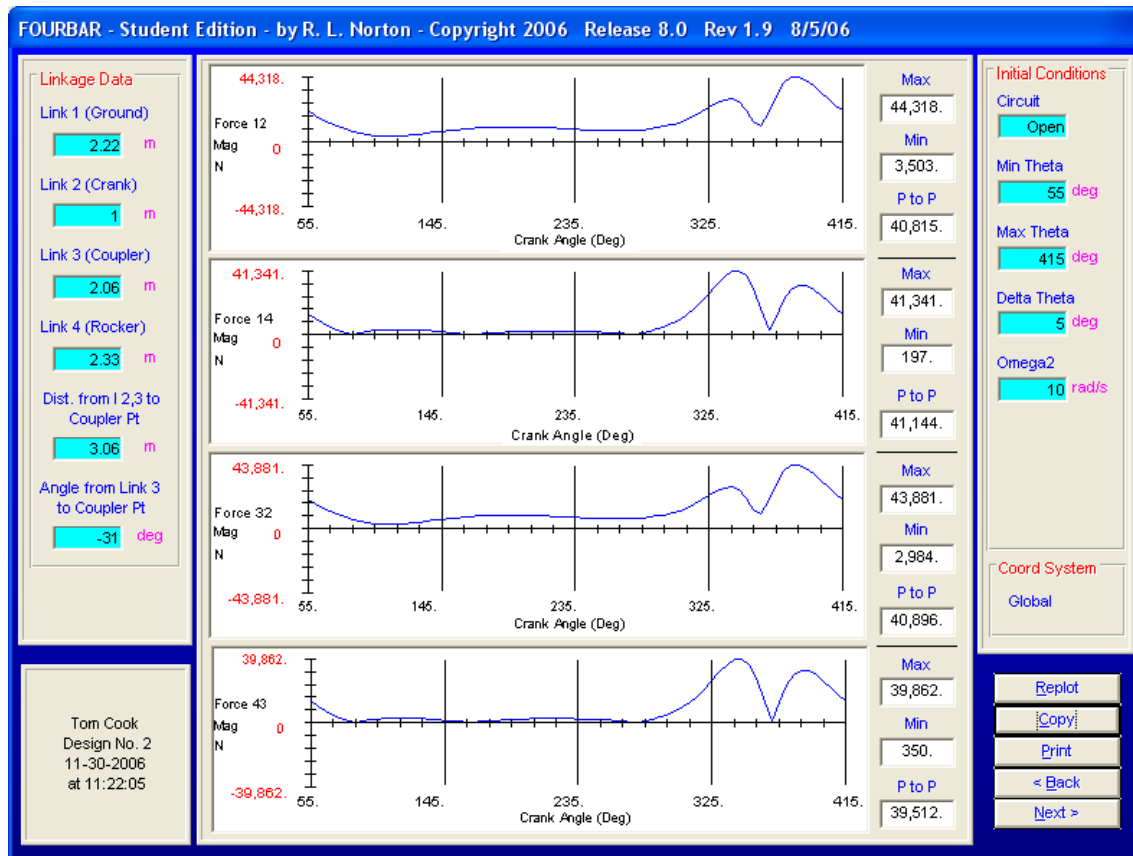
	Link 2	Link 3	Link 4	Units
Mass Properties				
Mass of Link	10.380	112.33	24.185	kg
Mass Moment of Inertia about CG	0.867	125.95	10.947	kg-m ²
CG Locations				
Distance to CG from Pivot	0.50	1.647	1.165	m
Angle to CG in LRCS*	0.00	-18.600	0.00	degrees
External Forces				
Force Magnitude	100	0.00		N
Angle of Force in GCS*	-90	0		degrees
External Force Locations				
Distance to Force Point from CG	1.494	0.0000		m
Angle to Force Point in LRCS*	-44.694	0		degrees
External Torques				
Torque on Link	0.000	0.000		N-m
Import a Torque Function File to Apply to Link 4 ☐				
Friction				
Friction Force on Link	0.00			N

* LRCS = Local Rotating Coordinate System. GCS = Global Coordinate System

Solve
Show Matrix
Copy
Print
< Back
Next >

Tom Cook
Design No. 2
11-30-2006
at 11:22:05

See next page for plots of pin forces and torque.



PROBLEM 11-30

Statement: Figure P11-4a shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 10$ rad/sec. There is a vertical force at P of $F = 100$ N. Find and plot the torque needed to drive the crank at a constant speed of 10 rad/sec for one revolution of the crank using the method of virtual work.

Given:

Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 1.00 \cdot m \quad \text{Link 3 (A to B)} \quad b := 2.06 \cdot m$$

$$\text{Link 4 (B to O}_4) \quad c := 2.33 \cdot m \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 2.22 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 3.06 \cdot m \quad \delta_3 := -31 \cdot \text{deg} \quad F_P := 100 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank motion:} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3}$$

Solution: See Figure P11-4a and Mathcad file P1130.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.500 \, m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 1.165 \, m$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 1.561 \, m$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = -0.525 \, m$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 1.647 \, m$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = -18.600 \, \text{deg}$$

2. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot |R_{pa} \cdot \sin(\delta_3)| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 10.380 \, \text{kg} \quad m_3 = 112.332 \, \text{kg} \quad m_4 = 24.185 \, \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.867 \, \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2] \quad I_{G3} = 125.951 \, \text{kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 10.947 \, \text{kg} \cdot \text{m}^2$$

3. Calculate the position, angular velocity, and acceleration of links 3 and 4 as functions of θ_2 .

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.2200$$

$$K_2 = 0.9528$$

$$K_3 = 1.5265$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))} \quad \omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2))$$

$$B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2))$$

$$E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2))$$

$$B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2))$$

$$E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_{PA}(\theta_2) := R_{pa} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots \\ + R_{pa} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{A}_P(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2)$$

$$A_P(\theta_2) := |\mathbf{A}_P(\theta_2)|$$

$$\theta_{AP}(\theta_2) := \arg(\mathbf{A}_P(\theta_2))$$

4. Calculate the x and y components of the velocity of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{v}_{G2}(\theta_2) := R_{CG2} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$v_{G2x}(\theta_2) := \text{Re}(\mathbf{v}_{G2}(\theta_2)) \quad v_{G2y}(\theta_2) := \text{Im}(\mathbf{v}_{G2}(\theta_2))$$

$$\mathbf{v}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{v}_{CG3A}(\theta_2) := R_{CG3} \cdot \omega_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_{33}) + j \cdot \cos(\theta_3(\theta_2) + \delta_{33}))$$

$$\mathbf{v}_{G3}(\theta_2) := \mathbf{v}_A(\theta_2) + \mathbf{v}_{CG3A}(\theta_2)$$

$$v_{G3x}(\theta_2) := \text{Re}(\mathbf{v}_{G3}(\theta_2)) \quad v_{G3y}(\theta_2) := \text{Im}(\mathbf{v}_{G3}(\theta_2))$$

$$\mathbf{v}_{PA}(\theta_2) := R_{pa} \cdot \omega_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{v}_P(\theta_2) := \mathbf{v}_A(\theta_2) + \mathbf{v}_{PA}(\theta_2)$$

$$v_{Px}(\theta_2) := \text{Re}(\mathbf{v}_P(\theta_2)) \quad v_{Py}(\theta_2) := \text{Im}(\mathbf{v}_P(\theta_2))$$

$$\mathbf{v}_{G4}(\theta_2) := R_{CG4} \cdot \omega_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2)))$$

$$v_{G4x}(\theta_2) := \text{Re}(\mathbf{v}_{G4}(\theta_2)) \quad v_{G4y}(\theta_2) := \text{Im}(\mathbf{v}_{G4}(\theta_2))$$

5. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2}(\theta_2) := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$a_{G2x}(\theta_2) := \text{Re}(\mathbf{a}_{G2}(\theta_2)) \quad a_{G2y}(\theta_2) := \text{Im}(\mathbf{a}_{G2}(\theta_2))$$

$$\mathbf{a}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{a}_{CG3A}(\theta_2) := R_{CG3} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_{33}) + j \cdot \cos(\theta_3(\theta_2) + \delta_{33})) \dots \\ + -R_{CG3} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_{33}) + j \cdot \sin(\theta_3(\theta_2) + \delta_{33}))$$

$$\mathbf{a}_{G3}(\theta_2) := \mathbf{a}_A(\theta_2) + \mathbf{a}_{CG3A}(\theta_2)$$

$$a_{G3x}(\theta_2) := \text{Re}(\mathbf{a}_{G3}(\theta_2)) \quad a_{G3y}(\theta_2) := \text{Im}(\mathbf{a}_{G3}(\theta_2))$$

$$\mathbf{a}_{G4}(\theta_2) := R_{CG4} \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) - c \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$a_{G4x}(\theta_2) := \text{Re}(\mathbf{a}_{G4}(\theta_2)) \quad a_{G4y}(\theta_2) := \text{Im}(\mathbf{a}_{G4}(\theta_2))$$

6. Calculate the x and y components of the external force at P in the CGS.

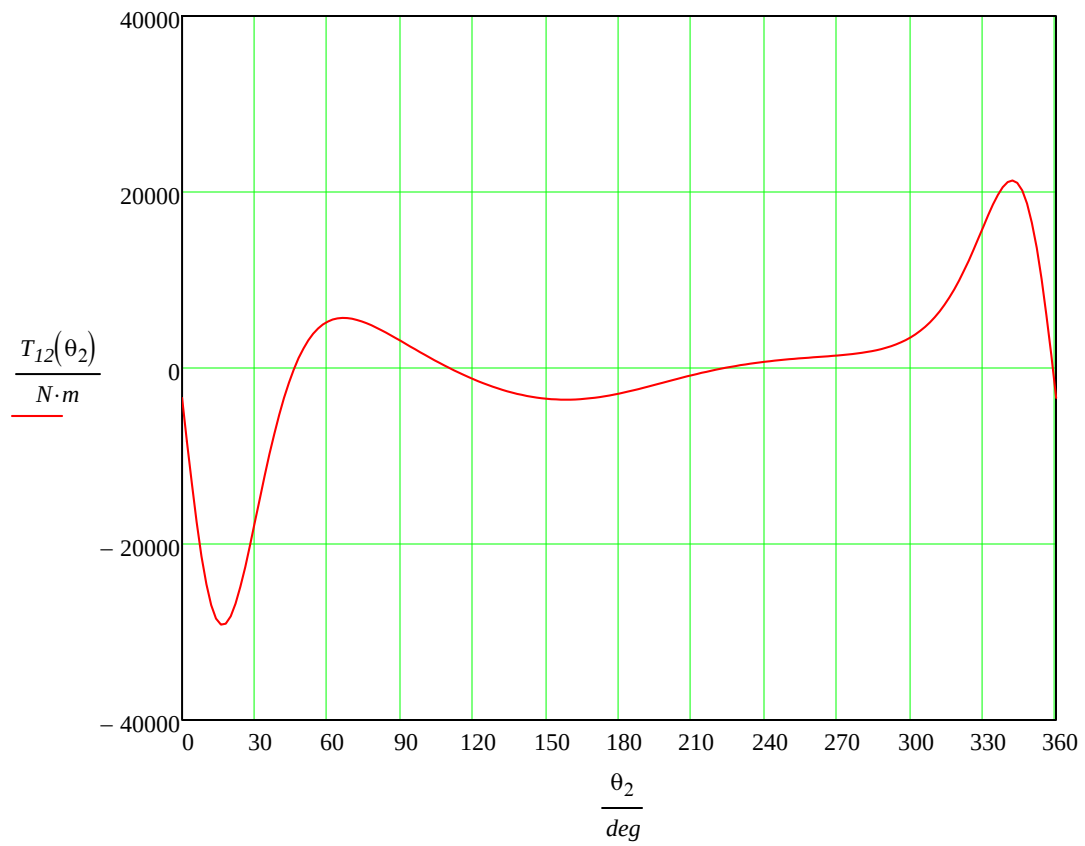
$$F_{Px} := 0 \cdot N \quad F_{Py} := -F_P$$

7. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12}(\theta_2) := \frac{1}{\omega_2} \cdot \left[m_2 \cdot (a_{G2x}(\theta_2) \cdot v_{G2x}(\theta_2) + a_{G2y}(\theta_2) \cdot v_{G2y}(\theta_2)) \dots \right. \\ \left. + m_3 \cdot (a_{G3x}(\theta_2) \cdot v_{G3x}(\theta_2) + a_{G3y}(\theta_2) \cdot v_{G3y}(\theta_2)) \dots \right. \\ \left. + m_4 \cdot (a_{G4x}(\theta_2) \cdot v_{G4x}(\theta_2) + a_{G4y}(\theta_2) \cdot v_{G4y}(\theta_2)) \dots \right. \\ \left. + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3(\theta_2) \cdot \omega_3(\theta_2) + I_{G4} \cdot \alpha_4(\theta_2) \cdot \omega_4(\theta_2)) \dots \right. \\ \left. + -F_{Px} \cdot v_{Px}(\theta_2) - F_{Py} \cdot v_{Py}(\theta_2) \right]$$

8. Plot the input torque for one revolution of the crank.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg}$$



PROBLEM 11-31

Statement: Figure P11-4b shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 15 \text{ rad/sec}$ and $\alpha = -10 \text{ rad/sec}^2$. There is a horizontal force at P of $F = 200 \text{ N}$. Find the torque needed to drive the crank at this instant using the method of virtual work.

Given:

Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 0.72 \cdot m \quad \text{Link 3 (A to B)} \quad b := 0.68 \cdot m$$

$$\text{Link 4 (B to O}_4) \quad c := 0.85 \cdot m \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 1.82 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 0.97 \cdot m \quad \delta_3 := 54 \cdot \text{deg} \quad F := 200 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank angle and motion:} \quad \theta_2 := 30 \cdot \text{deg} \quad \omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := -10 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3}$$

Solution: See Figure P11-4b and Mathcad file P1131.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$$\theta_3 := 23.290 \cdot \text{deg} \quad \omega_3 := -16.412 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_3 := -138.628 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\theta_4 := 132.283 \cdot \text{deg} \quad \omega_4 := 1.570 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_4 := 427.881 \cdot \text{rad} \cdot \text{sec}^{-2}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.360 \text{ m} \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.425 \text{ m}$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 0.417 \text{ m}$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = 0.262 \text{ m}$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 0.492 \text{ m}$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = 32.117 \text{ deg}$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot |R_{pa} \cdot \sin(\delta_3)| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 7.474 \text{ kg} \quad m_3 = 18.463 \text{ kg} \quad m_4 = 8.823 \text{ kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.324 \text{ kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2] \quad I_{G3} = 3.318 \text{ kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2)$$

$$I_{G4} = 0.533 \text{ kg} \cdot \text{m}^2$$

4. Calculate the x and y components of the velocity of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{v}_{G2} := R_{CG2} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$v_{G2x} := \text{Re}(\mathbf{v}_{G2})$$

$$v_{G2x} = -2.700 \frac{\text{m}}{\text{s}}$$

$$v_{G2y} := \text{Im}(\mathbf{v}_{G2})$$

$$v_{G2y} = 4.677 \frac{\text{m}}{\text{s}}$$

$$\mathbf{v}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{v}_{CG3A} := R_{CG3} \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_{33}) + j \cdot \cos(\theta_3 + \delta_{33}))$$

$$\mathbf{v}_{G3} := \mathbf{v}_A + \mathbf{v}_{CG3A}$$

$$v_{G3x} := \text{Re}(\mathbf{v}_{G3})$$

$$v_{G3x} = 1.247 \frac{\text{m}}{\text{s}}$$

$$v_{G3y} := \text{Im}(\mathbf{v}_{G3})$$

$$v_{G3y} = 4.769 \frac{\text{m}}{\text{s}}$$

$$\mathbf{v}_{PA} := R_{pa} \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3))$$

$$\mathbf{v}_P := \mathbf{v}_A + \mathbf{v}_{PA}$$

$$v_{Px} := \text{Re}(\mathbf{v}_P)$$

$$v_{Px} = 10.130 \frac{\text{m}}{\text{s}}$$

$$v_{Py} := \text{Im}(\mathbf{v}_P)$$

$$v_{Py} = 5.850 \frac{\text{m}}{\text{s}}$$

$$\mathbf{v}_{G4} := R_{CG4} \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$$

$$v_{G4x} := \text{Re}(\mathbf{v}_{G4})$$

$$v_{G4x} = -0.494 \frac{\text{m}}{\text{s}}$$

$$v_{G4y} := \text{Im}(\mathbf{v}_{G4})$$

$$v_{G4y} = -0.449 \frac{\text{m}}{\text{s}}$$

5. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2} := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$a_{G2x} := \text{Re}(\mathbf{a}_{G2})$$

$$a_{G2x} = -138.496 \frac{\text{m}}{\text{sec}^2}$$

$$a_{G2y} := \text{Im}(\mathbf{a}_{G2})$$

$$a_{G2y} = -84.118 \frac{\text{m}}{\text{sec}^2}$$

$$\mathbf{a}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{a}_{CG3A} := R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_3 + \delta_{33}) + j \cdot \cos(\theta_3 + \delta_{33})) \dots$$

$$+ -R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_3 + \delta_{33}) + j \cdot \sin(\theta_3 + \delta_{33}))$$

$$\mathbf{a}_{G3} := \mathbf{a}_A + \mathbf{a}_{CG3A}$$

$$a_{G3x} := \text{Re}(\mathbf{a}_{G3})$$

$$a_{G3x} = -155.788 \frac{\text{m}}{\text{sec}^2}$$

$$a_{G3y} := \text{Im}(\mathbf{a}_{G3}) \quad a_{G3y} = -235.056 \frac{m}{\text{sec}^2}$$

$$\mathbf{a}_{G4} := R_{CG4} \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$a_{G4x} := \text{Re}(\mathbf{a}_{G4}) \quad a_{G4x} = -133.128 \frac{m}{\text{sec}^2}$$

$$a_{G4y} := \text{Im}(\mathbf{a}_{G4}) \quad a_{G4y} = -123.897 \frac{m}{\text{sec}^2}$$

6. Calculate the x and y components of the external force at P in the CGS.

$$F_{Px} := F \quad F_{Py} := 0 \cdot N$$

7. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \left[m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \dots \right. \\ \left. + m_4 \cdot (a_{G4x} \cdot v_{G4x} + a_{G4y} \cdot v_{G4y}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3 + I_{G4} \cdot \alpha_4 \cdot \omega_4) \dots \right. \\ \left. + -F_{Px} \cdot v_{Px} - F_{Py} \cdot v_{Py} \right]$$

$$T_{12} = -1168 N \cdot m$$

PROBLEM 11-32

Statement: Figure P11-4b shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 15$ rad/sec. There is a horizontal force at P of $F = 200$ N. Find and plot all pin forces and the torque needed to drive the crank at a constant speed of 15 rad/sec for one revolution of the crank using the program FOURBAR. Change the lengths of links 3 and 4 to 1.43 and 1.60 m, respectively.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 0.72 \cdot \text{m} \quad \text{Link 3 (A to B)} \quad b := 1.43 \cdot \text{m}$$

$$\text{Link 4 (B to O}_4) \quad c := 1.60 \cdot \text{m} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 1.82 \cdot \text{m}$$

$$\text{Coupler point:} \quad R_{pa} := 0.97 \cdot \text{m} \quad \delta_3 := 54 \cdot \text{deg} \quad F_P := 200 \cdot \text{N} \quad T_4 := 0 \cdot \text{N} \cdot \text{m}$$

$$\text{Crank motion:} \quad \omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3}$$

Solution: See Figure P11-4b and Mathcad file P1132.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.360 \text{ m} \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.800 \text{ m}$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 0.667 \text{ m}$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = 0.262 \text{ m}$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 0.716 \text{ m}$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = 21.422 \text{ deg}$$

2. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot |R_{pa} \cdot \sin(\delta_3)| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 7.474 \text{ kg} \quad m_3 = 38.828 \text{ kg} \quad m_4 = 16.608 \text{ kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.324 \text{ kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2] \quad I_{G3} = 17.218 \text{ kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 3.546 \text{ kg} \cdot \text{m}^2$$

3. Calculate the x and y components of the external force at P in the CGS and the magnitude and angle of the position vector defining point P in the LRCS.

$$F_{Px} := F_P \quad F_{Py} := 0 \cdot \text{N}$$

$$\begin{aligned}
 R_{pax'} &:= R_{pa} \cdot \cos(\delta_3) & R_{pay'} &:= R_{pa} \cdot \sin(\delta_3) & R_{px'} &:= R_{pax'} - R_{CG3x'} & R_{py'} &:= R_{pay'} - R_{CG3y'} \\
 R_{pax'} &= 0.570 \text{ m} & R_{pay'} &= 0.785 \text{ m} & R_{px'} &= -0.097 \text{ m} & R_{py'} &= 0.523 \text{ m} \\
 R_p &:= \sqrt{R_{px'}^2 + R_{py'}^2} & R_p &= 0.532 \text{ m} & \theta_p &:= \text{atan2}(R_{px'}, R_{py'}) & \theta_p &= 100.458 \text{ deg}
 \end{aligned}$$

4. Enter the above data into program FOURBAR and solve for the pin forces and driving torque. The dynamic input screen is shown below followed by a plot of dynamic pin forces..

FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06 Dynamics Screen

Help

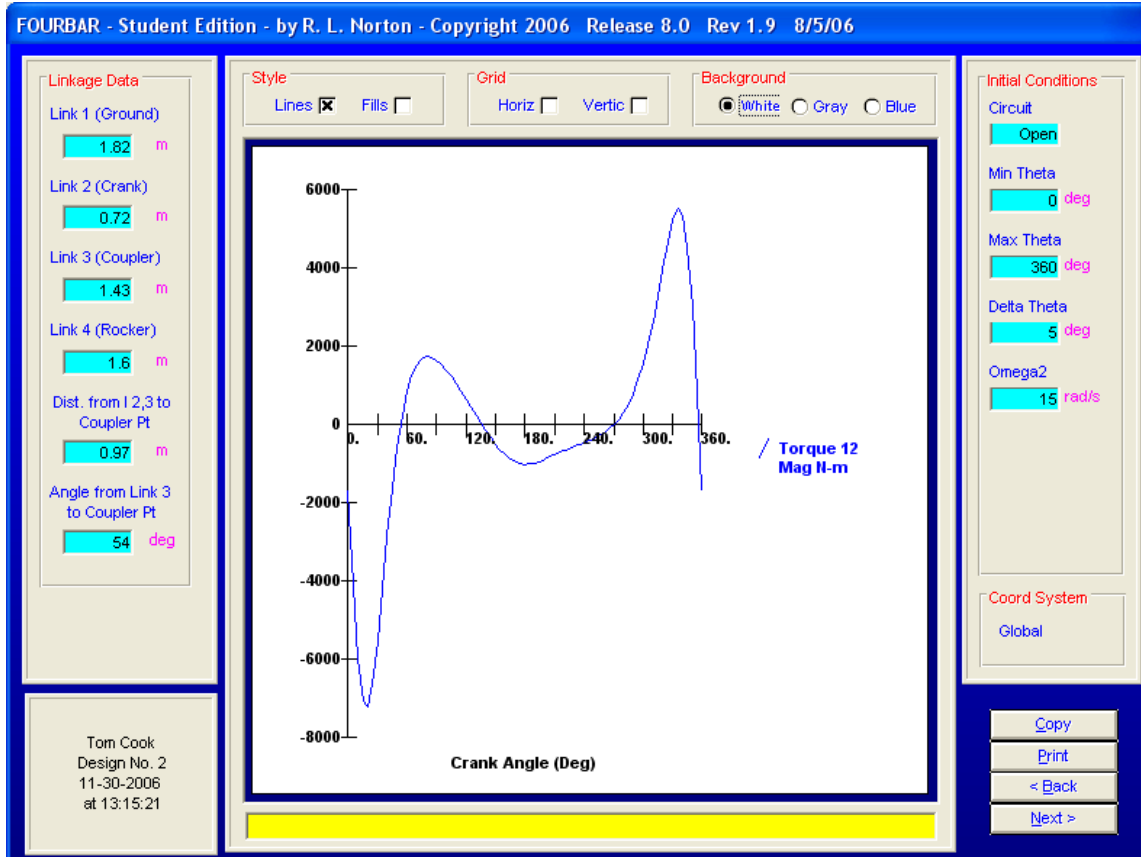
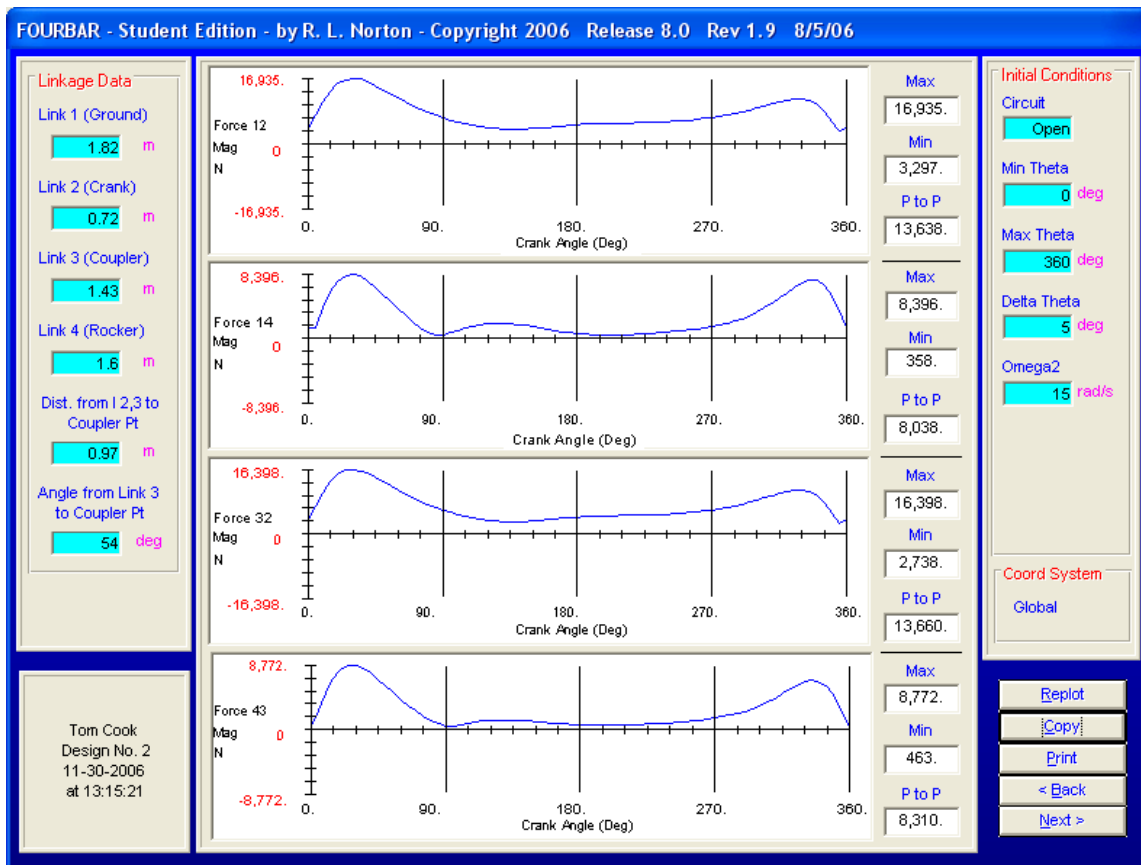
	Link 2	Link 3	Link 4	Units
Mass Properties				
Mass of Link	7.474	38.828	16.608	kg
Mass Moment of Inertia about CG	0.324	17.218	3.546	kg-m ²
CG Locations				
Distance to CG from Pivot	0.360	0.716	0.800	m
Angle to CG in LRCS*	0.00	21.422	0.00	degrees
External Forces				
Force Magnitude		200	0.00	N
Angle of Force in GCS*		0	0	degrees
External Force Locations				
Distance to Force Point from CG		0.532	0.0000	m
Angle to Force Point in LRCS*		100.458	0	degrees
External Torques				
Torque on Link		0.000	0.000	N-m
Import a Torque Function File to Apply to Link 4	☐			
Friction				
Friction Force on Link		0.00		N

* LRCS = Local Rotating Coordinate System. GCS = Global Coordinate System

Solve
Show Matrix
Copy
Print
< Back
Next >

Tom Cook
Design No. 2
11-30-2006
at 13:15:21

See next page for plots of pin forces and torque.



PROBLEM 11-33

Statement: Figure P11-4b shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 15$ rad/sec. There is a horizontal force at P of $F = 200$ N. Find and plot the torque needed to drive the crank at a constant speed of 15 rad/sec for one revolution of the crank using the method of virtual work. Change the lengths of links 3 and 4 to 1.43 and 1.60 m, respectively.

Given: Link lengths:

$$\text{Link 2 (} O_2 \text{ to } A) \quad a := 0.72 \cdot m \quad \text{Link 3 (} A \text{ to } B) \quad b := 1.43 \cdot m$$

$$\text{Link 4 (} B \text{ to } O_4) \quad c := 1.60 \cdot m \quad \text{Link 1 (} O_2 \text{ to } O_4) \quad d := 1.82 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 0.97 \cdot m \quad \delta_3 := 54 \cdot \text{deg} \quad F_P := 200 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank motion:} \quad \omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lbf} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Solution: See Figure P11-4b and Mathcad file P1133.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.360 \, m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.800 \, m$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 0.667 \, m$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = 0.262 \, m$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 0.716 \, m$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = 21.422 \, \text{deg}$$

2. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot |R_{pa} \cdot \sin(\delta_3)| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 7.474 \, \text{kg} \quad m_3 = 38.828 \, \text{kg} \quad m_4 = 16.608 \, \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.324 \, \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2] \quad I_{G3} = 17.218 \, \text{kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 3.546 \, \text{kg} \cdot \text{m}^2$$

3. Calculate the position, angular velocity, and acceleration of links 3 and 4 as functions of θ_2 .

$$\begin{aligned}
K_1 &:= \frac{d}{a} & K_2 &:= \frac{d}{c} & K_3 &:= \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \\
K_1 &= 2.5278 & K_2 &= 1.1375 & K_3 &= 1.8862 \\
A(\theta_2) &:= \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \\
B(\theta_2) &:= -2 \cdot \sin(\theta_2) & C(\theta_2) &:= K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3 \\
\theta_4(\theta_2) &:= 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right) \\
K_4 &:= \frac{d}{b} & K_5 &:= \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} & D(\theta_2) &:= \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \\
E(\theta_2) &:= -2 \cdot \sin(\theta_2) & F(\theta_2) &:= K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \\
\theta_3(\theta_2) &:= 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right) \\
\omega_3(\theta_2) &:= \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))} & \omega_4(\theta_2) &:= \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))} \\
A(\theta_2) &:= c \cdot \sin(\theta_4(\theta_2)) & B(\theta_2) &:= b \cdot \sin(\theta_3(\theta_2)) \\
D(\theta_2) &:= c \cdot \cos(\theta_4(\theta_2)) & E(\theta_2) &:= b \cdot \cos(\theta_3(\theta_2)) \\
C(\theta_2) &:= a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2)) \\
F(\theta_2) &:= a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2)) \\
\alpha_4(\theta_2) &:= \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)} \\
A'(\theta_2) &:= c \cdot \sin(\theta_4(\theta_2)) & B'(\theta_2) &:= b \cdot \sin(\theta_3(\theta_2)) \\
D'(\theta_2) &:= c \cdot \cos(\theta_4(\theta_2)) & E'(\theta_2) &:= b \cdot \cos(\theta_3(\theta_2)) \\
C'(\theta_2) &:= a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2)) \\
F'(\theta_2) &:= a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2)) \\
\alpha_3(\theta_2) &:= \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)} \\
\mathbf{A}_A(\theta_2) &:= a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \\
\mathbf{A}_{PA}(\theta_2) &:= R_{pa} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots \\
&\quad + -R_{pa} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3)) \\
\mathbf{A}_P(\theta_2) &:= \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2) \\
A_P(\theta_2) &:= |\mathbf{A}_P(\theta_2)| & \theta_{AP}(\theta_2) &:= \arg(\mathbf{A}_P(\theta_2))
\end{aligned}$$

4. Calculate the x and y components of the velocity of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{v}_{G2}(\theta_2) := R_{CG2} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$v_{G2x}(\theta_2) := \text{Re}(\mathbf{v}_{G2}(\theta_2)) \quad v_{G2y}(\theta_2) := \text{Im}(\mathbf{v}_{G2}(\theta_2))$$

$$\mathbf{v}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{v}_{CG3A}(\theta_2) := R_{CG3} \cdot \omega_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_{33}) + j \cdot \cos(\theta_3(\theta_2) + \delta_{33}))$$

$$\mathbf{v}_{G3}(\theta_2) := \mathbf{v}_A(\theta_2) + \mathbf{v}_{CG3A}(\theta_2)$$

$$v_{G3x}(\theta_2) := \text{Re}(\mathbf{v}_{G3}(\theta_2)) \quad v_{G3y}(\theta_2) := \text{Im}(\mathbf{v}_{G3}(\theta_2))$$

$$\mathbf{v}_{PA}(\theta_2) := R_{pa} \cdot \omega_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{v}_P(\theta_2) := \mathbf{v}_A(\theta_2) + \mathbf{v}_{PA}(\theta_2)$$

$$v_{Px}(\theta_2) := \text{Re}(\mathbf{v}_P(\theta_2)) \quad v_{Py}(\theta_2) := \text{Im}(\mathbf{v}_P(\theta_2))$$

$$\mathbf{v}_{G4}(\theta_2) := R_{CG4} \cdot \omega_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2)))$$

$$v_{G4x}(\theta_2) := \text{Re}(\mathbf{v}_{G4}(\theta_2)) \quad v_{G4y}(\theta_2) := \text{Im}(\mathbf{v}_{G4}(\theta_2))$$

5. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2}(\theta_2) := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$a_{G2x}(\theta_2) := \text{Re}(\mathbf{a}_{G2}(\theta_2)) \quad a_{G2y}(\theta_2) := \text{Im}(\mathbf{a}_{G2}(\theta_2))$$

$$\mathbf{a}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\begin{aligned} \mathbf{a}_{CG3A}(\theta_2) &:= R_{CG3} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_{33}) + j \cdot \cos(\theta_3(\theta_2) + \delta_{33})) \dots \\ &\quad + -R_{CG3} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_{33}) + j \cdot \sin(\theta_3(\theta_2) + \delta_{33})) \end{aligned}$$

$$\mathbf{a}_{G3}(\theta_2) := \mathbf{a}_A(\theta_2) + \mathbf{a}_{CG3A}(\theta_2)$$

$$a_{G3x}(\theta_2) := \text{Re}(\mathbf{a}_{G3}(\theta_2)) \quad a_{G3y}(\theta_2) := \text{Im}(\mathbf{a}_{G3}(\theta_2))$$

$$\mathbf{a}_{G4}(\theta_2) := R_{CG4} \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) - c \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$a_{G4x}(\theta_2) := \text{Re}(\mathbf{a}_{G4}(\theta_2)) \quad a_{G4y}(\theta_2) := \text{Im}(\mathbf{a}_{G4}(\theta_2))$$

6. Calculate the x and y components of the external force at P in the CGS.

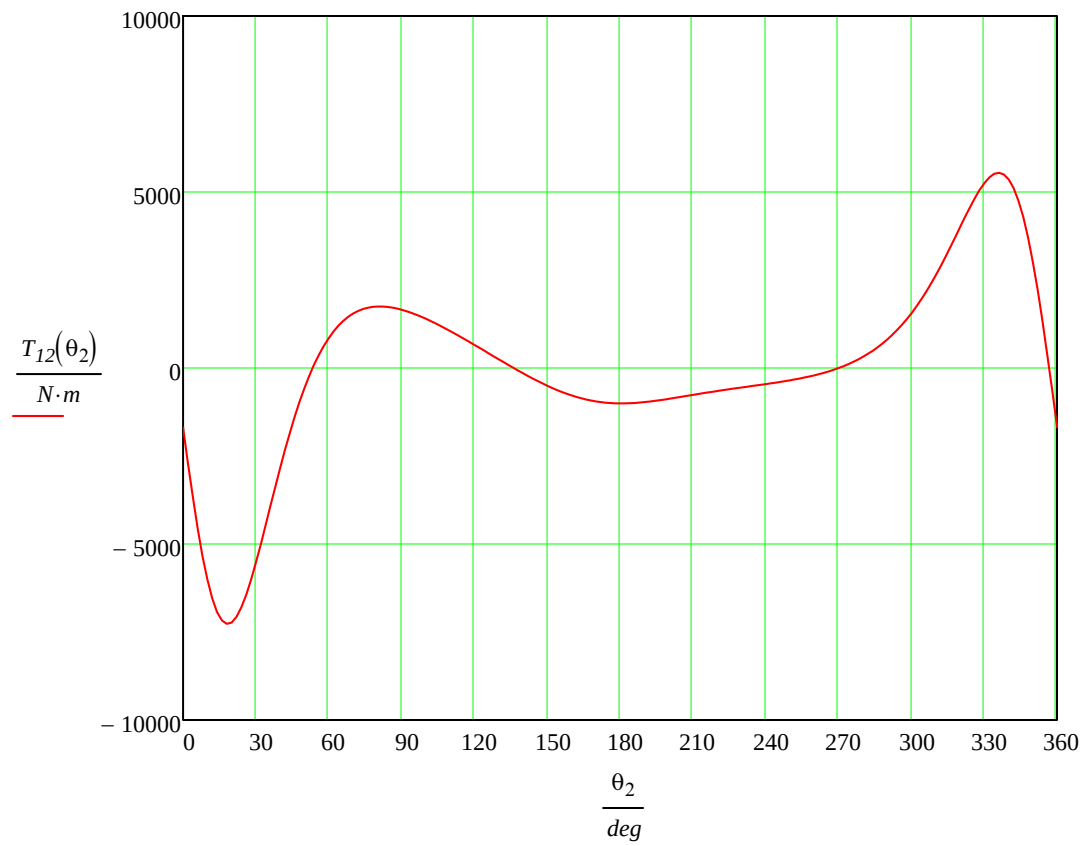
$$F_{Px} := F_P \quad F_{Py} := 0.N$$

7. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12}(\theta_2) := \frac{1}{\omega_2} \cdot \left[\begin{aligned} &m_2 \cdot (a_{G2x}(\theta_2) \cdot v_{G2x}(\theta_2) + a_{G2y}(\theta_2) \cdot v_{G2y}(\theta_2)) \dots \\ &+ m_3 \cdot (a_{G3x}(\theta_2) \cdot v_{G3x}(\theta_2) + a_{G3y}(\theta_2) \cdot v_{G3y}(\theta_2)) \dots \\ &+ m_4 \cdot (a_{G4x}(\theta_2) \cdot v_{G4x}(\theta_2) + a_{G4y}(\theta_2) \cdot v_{G4y}(\theta_2)) \dots \\ &+ (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3(\theta_2) \cdot \omega_3(\theta_2) + I_{G4} \cdot \alpha_4(\theta_2) \cdot \omega_4(\theta_2)) \dots \\ &+ -F_{Px} \cdot v_{Px}(\theta_2) - F_{Py} \cdot v_{Py}(\theta_2) \end{aligned} \right]$$

8. Plot the input torque for one revolution of the crank.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} .. 360 \cdot \text{deg}$$



PROBLEM 11-34

Statement: Figure P11-5a shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections 50 mm wide by 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 15 \text{ rad/sec}$ and $\alpha = -10 \text{ rad/sec}^2$. There is a vertical force at P of $F = 500 \text{ N}$. Find the torque needed to drive the crank at this instant using the method of virtual work.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 0.785 \cdot m \quad \text{Link 3 (A to B)} \quad b := 0.356 \cdot m$$

$$\text{Link 4 (B to O}_4) \quad c := 0.950 \cdot m \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 0.544 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 1.09 \cdot m \quad \delta_3 := 0 \cdot \text{deg} \quad F := 500 \cdot \text{N} \quad T_4 := 0 \cdot \text{N} \cdot m$$

$$\text{Crank angle and motion:} \quad \theta_2 := 96 \cdot \text{deg} \quad \omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := -10 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad w_3 := 50 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{in}^{-3}$$

Solution: See Figure P11-5a and Mathcad file P1134.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$$\theta_3 := 20.261 \cdot \text{deg} \quad \omega_3 := -6.830 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_3 := 106.282 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\theta_4 := 107.906 \cdot \text{deg} \quad \omega_4 := 12.023 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_4 := 49.372 \cdot \text{rad} \cdot \text{sec}^{-2}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.393 \cdot m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.475 \cdot m$$

$$\text{Link 3:} \quad R_{CG3} := 0.5 \cdot R_{pa} \quad R_{CG3} = 0.545 \cdot m$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := w_3 \cdot t_3 \cdot R_{pa} \cdot \frac{\gamma_s}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 8.148 \cdot \text{kg} \quad m_3 = 11.314 \cdot \text{kg} \quad m_4 = 9.861 \cdot \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.420 \cdot \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{12} \cdot (w_3^2 + R_{pa}^2) \quad I_{G3} = 1.123 \cdot \text{kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 0.744 \cdot \text{kg} \cdot \text{m}^2$$

4. Calculate the x and y components of the velocity of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{v}_{G2} := R_{CG2} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$v_{G2x} := \text{Re}(\mathbf{v}_{G2}) \quad v_{G2x} = -5.855 \frac{\text{m}}{\text{s}}$$

$$v_{G2y} := \text{Im}(\mathbf{v}_{G2}) \quad v_{G2y} = -0.615 \frac{\text{m}}{\text{s}}$$

$$\mathbf{v}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) \quad \mathbf{v}_{CG3A} := R_{CG3} \cdot \omega_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3))$$

$$\mathbf{v}_{G3} := \mathbf{v}_A + \mathbf{v}_{CG3A}$$

$$v_{G3x} := \text{Re}(\mathbf{v}_{G3}) \quad v_{G3x} = -10.421 \frac{m}{s}$$

$$v_{G3y} := \text{Im}(\mathbf{v}_{G3}) \quad v_{G3y} = -4.723 \frac{m}{s}$$

$$\mathbf{v}_{PA} := R_{pa} \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3))$$

$$\mathbf{v}_P := \mathbf{v}_A + \mathbf{v}_{PA}$$

$$v_{Px} := \text{Re}(\mathbf{v}_P) \quad v_{Px} = -9.132 \frac{m}{s}$$

$$v_{Py} := \text{Im}(\mathbf{v}_P) \quad v_{Py} = -8.215 \frac{m}{s}$$

$$\mathbf{v}_{G4} := R_{CG4} \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$$

$$v_{G4x} := \text{Re}(\mathbf{v}_{G4}) \quad v_{G4x} = -5.434 \frac{m}{s}$$

$$v_{G4y} := \text{Im}(\mathbf{v}_{G4}) \quad v_{G4y} = -1.756 \frac{m}{s}$$

5. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2} := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$a_{G2x} := \text{Re}(\mathbf{a}_{G2}) \quad a_{G2x} = 22.366 \frac{m}{sec^2}$$

$$a_{G2y} := \text{Im}(\mathbf{a}_{G2}) \quad a_{G2y} = -175.247 \frac{m}{sec^2}$$

$$\mathbf{a}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{a}_{CG3A} := R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3)) \dots \\ + -R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_3) + j \cdot \sin(\theta_3))$$

$$\mathbf{a}_{G3} := \mathbf{a}_A + \mathbf{a}_{CG3A} \quad a_{G3x} := \text{Re}(\mathbf{a}_{G3}) \quad a_{G3x} = -17.640 \frac{m}{sec^2}$$

$$a_{G3y} := \text{Im}(\mathbf{a}_{G3}) \quad a_{G3y} = -129.301 \frac{m}{sec^2}$$

$$\mathbf{a}_{G4} := R_{CG4} \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$a_{G4x} := \text{Re}(\mathbf{a}_{G4}) \quad a_{G4x} = 19.906 \frac{m}{sec^2}$$

$$a_{G4y} := \text{Im}(\mathbf{a}_{G4}) \quad a_{G4y} = -137.884 \frac{m}{sec^2}$$

6. Calculate the x and y components of the external force at P in the CGS.

$$F_{Px} := 0 \cdot N \quad F_{Py} := -F$$

7. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \left[m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \dots \right. \\ \left. + m_4 \cdot (a_{G4x} \cdot v_{G4x} + a_{G4y} \cdot v_{G4y}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3 + I_{G4} \cdot \alpha_4 \cdot \omega_4) \dots \right. \\ \left. + -F_{Px} \cdot v_{Px} - F_{Py} \cdot v_{Py} \right]$$

$$T_{12} = 372 \text{ N} \cdot \text{m}$$

PROBLEM 11-35

Statement: Figure P11-5a shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections 50 mm wide by 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 15$ rad/sec. There is a vertical force at P of $F = 500$ N. Find and plot all pin forces and the torque needed to drive the crank at a constant speed of 15 rad/sec for one revolution of the crank using the program FOURBAR. Change the lengths of links 1, 2, and 3 to 1.000, 0.356, and 0.785 m, respectively.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 0.356 \cdot m \quad \text{Link 3 (A to B)} \quad b := 0.785 \cdot m$$

$$\text{Link 4 (B to O}_4) \quad c := 0.950 \cdot m \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 1.000 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 1.09 \cdot m \quad \delta_3 := 0 \cdot \text{deg} \quad F_P := 500 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank motion:} \quad \omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad w_3 := 50 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \gamma_s := 0.3 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Solution: See Figure P11-5a and Mathcad file P1135.

- Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.178 \, m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.475 \, m$$

$$\text{Link 3:} \quad R_{CG3} := 0.5 \cdot R_{pa} \quad R_{CG3} = 0.545 \, m \quad \delta_{33} := 0 \cdot \text{deg}$$

- Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := w_3 \cdot t_3 \cdot R_{pa} \cdot \frac{\gamma_s}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 3.695 \, \text{kg} \quad m_3 = 11.314 \, \text{kg} \quad m_4 = 9.861 \, \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.040 \, \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{12} \cdot (w_3^2 + R_{pa}^2) \quad I_{G3} = 1.123 \, \text{kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 0.744 \, \text{kg} \cdot \text{m}^2$$

- Calculate the x and y components of the external force at P in the CGS and the magnitude and angle of the position vector defining point P in the LRCS.

$$F_{Px} := 0 \cdot N \quad F_{Py} := -F_P$$

$$R_{pax'} := R_{pa} \cdot \cos(\delta_3) \quad R_{px'} := R_{pax'} - R_{CG3} \quad R_{pax'} = 1.090 \, m \quad R_{px'} = 0.545 \, m$$

- Enter the above data into program FOURBAR and solve for the pin forces and driving torque. The dynamic input screen is shown below followed by a plot of dynamic pin forces..

FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06 Dynamics Screen

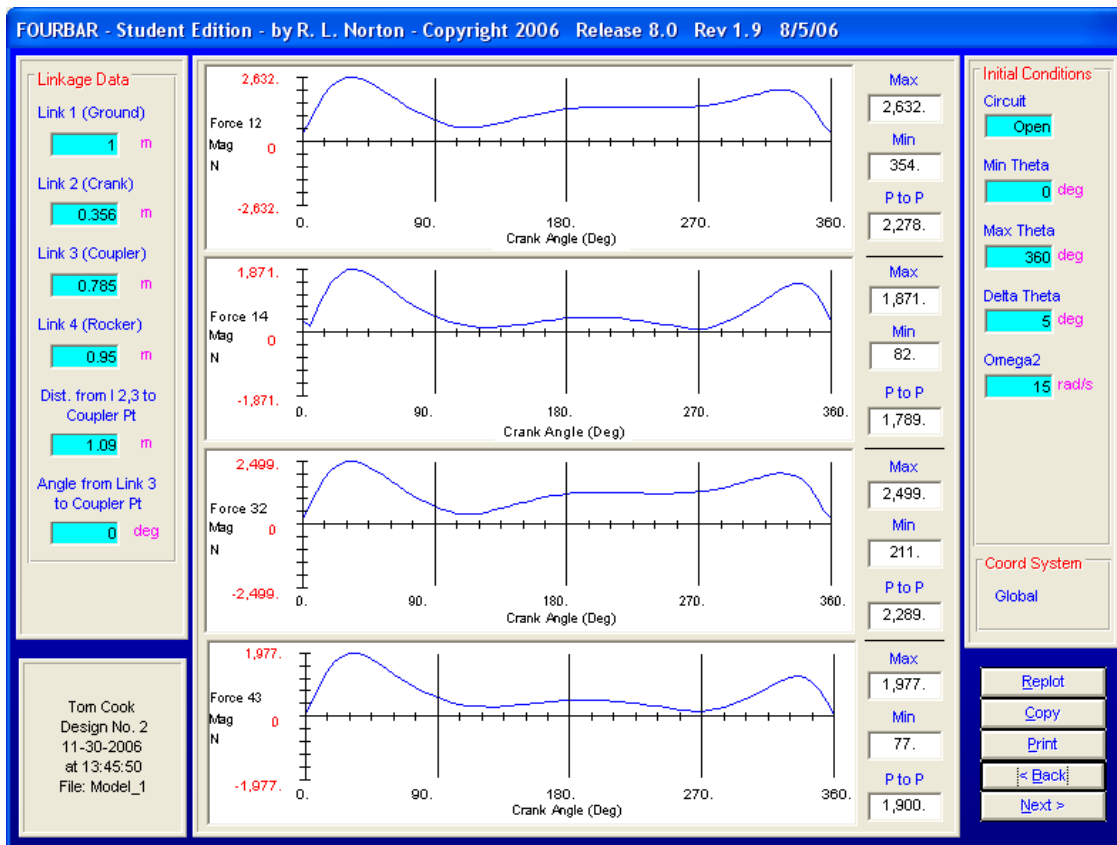
Help

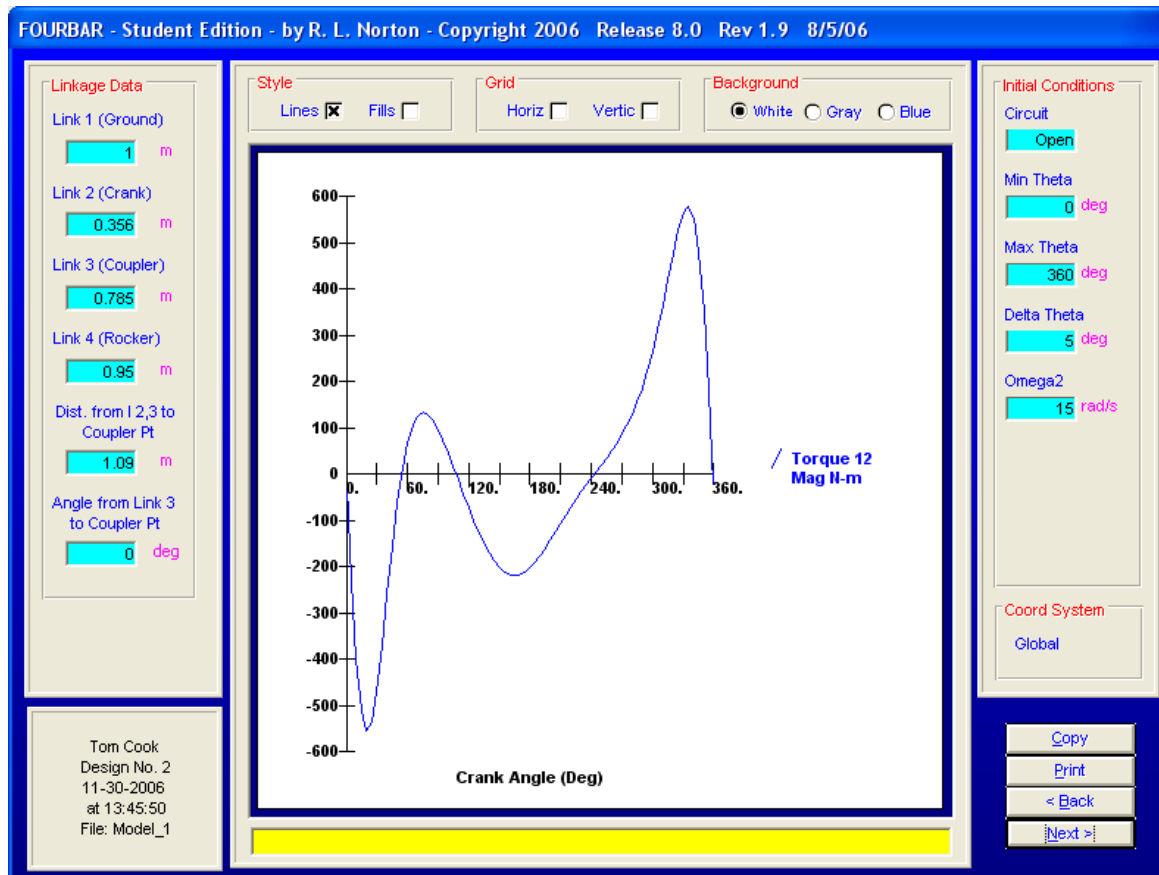
Link	2	3	4	Units
Mass Properties				
Mass of Link	3.695	11.314	9.861	kg
Mass Moment of Inertia about CG	0.040	1.123	0.744	kg-m ²
CG Locations				
Distance to CG from Pivot	0.178	0.545	0.475	m
Angle to CG in LRCS*	0.00	0.00	0.00	degrees
External Forces				
Force Magnitude		500	0.00	N
Angle of Force in GCS*		-90	0	degrees
External Force Locations				
Distance to Force Point from CG		0.545	0.0000	m
Angle to Force Point in LRCS*		0	0	degrees
External Torques				
Torque on Link		0.000	0.000	N-m
Import a Torque Function File to Apply to Link 4 ☐				
Friction				
Friction Force on Link		0.00		N

* LRCS = Local Rotating Coordinate System. GCS = Global Coordinate System

Tom Cook
Design No. 2
11-30-2006
at 13:45:50
File: Model_1

Solve
Show Matrix
Copy
Print
< Back
Next >





PROBLEM 11-36

Statement: Figure P11-5a shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections 50 mm wide by 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 15$ rad/sec. There is a vertical force at P of $F = 500$ N. Find and plot the torque needed to drive the crank at a constant speed of 15 rad/sec for one revolution of the crank using the method of virtual work. Change the lengths of links 1, 2, and 3 to 1.000, 0.356, and 0.785 m, respectively.

Given: Link lengths:

$$\text{Link 2 (} O_2 \text{ to } A \text{)} \quad a := 0.356 \cdot m \quad \text{Link 3 (} A \text{ to } B \text{)} \quad b := 0.785 \cdot m$$

$$\text{Link 4 (} B \text{ to } O_4 \text{)} \quad c := 0.950 \cdot m \quad \text{Link 1 (} O_2 \text{ to } O_4 \text{)} \quad d := 1.000 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 1.09 \cdot m \quad \delta_3 := 0 \cdot \text{deg} \quad F_P := 500 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank motion:} \quad \omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad w_3 := 50 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \gamma_s := 0.3 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Solution: See Figure P11-5a and Mathcad file P1136.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.178 \cdot m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.475 \cdot m$$

$$\text{Link 3:} \quad R_{CG3} := 0.5 \cdot R_{pa} \quad R_{CG3} = 0.545 \cdot m \quad \delta_{33} := 0 \cdot \text{deg}$$

2. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := w_3 \cdot t_3 \cdot R_{pa} \cdot \frac{\gamma_s}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 3.695 \cdot \text{kg} \quad m_3 = 11.314 \cdot \text{kg} \quad m_4 = 9.861 \cdot \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.040 \cdot \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{12} \cdot (w_3^2 + R_{pa}^2) \quad I_{G3} = 1.123 \cdot \text{kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 0.744 \cdot \text{kg} \cdot \text{m}^2$$

3. Calculate the position, angular velocity, and acceleration of links 3 and 4 as functions of θ_2 .

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.8090 \quad K_2 = 1.0526 \quad K_3 = 2.0890$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))} \quad \omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_{PA}(\theta_2) := R_{pa} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots \\ + R_{pa} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{A}_P(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2)$$

$$A_P(\theta_2) := |\mathbf{A}_P(\theta_2)| \quad \theta_{AP}(\theta_2) := \arg(\mathbf{A}_P(\theta_2))$$

4. Calculate the x and y components of the velocity of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{v}_{G2}(\theta_2) := R_{CG2} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$v_{G2x}(\theta_2) := \text{Re}(\mathbf{v}_{G2}(\theta_2)) \quad v_{G2y}(\theta_2) := \text{Im}(\mathbf{v}_{G2}(\theta_2))$$

$$\mathbf{v}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{v}_{CG3A}(\theta_2) := R_{CG3} \cdot \omega_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_{33}) + j \cdot \cos(\theta_3(\theta_2) + \delta_{33}))$$

$$\mathbf{v}_{G3}(\theta_2) := \mathbf{v}_A(\theta_2) + \mathbf{v}_{CG3A}(\theta_2)$$

$$v_{G3x}(\theta_2) := \text{Re}(\mathbf{v}_{G3}(\theta_2)) \quad v_{G3y}(\theta_2) := \text{Im}(\mathbf{v}_{G3}(\theta_2))$$

$$\mathbf{v}_{PA}(\theta_2) := R_{pa} \cdot \omega_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{v}_P(\theta_2) := \mathbf{v}_A(\theta_2) + \mathbf{v}_{PA}(\theta_2)$$

$$v_{Px}(\theta_2) := \text{Re}(\mathbf{v}_P(\theta_2)) \quad v_{Py}(\theta_2) := \text{Im}(\mathbf{v}_P(\theta_2))$$

$$\mathbf{v}_{G4}(\theta_2) := R_{CG4} \cdot \omega_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2)))$$

$$v_{G4x}(\theta_2) := \text{Re}(\mathbf{v}_{G4}(\theta_2)) \quad v_{G4y}(\theta_2) := \text{Im}(\mathbf{v}_{G4}(\theta_2))$$

5. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2}(\theta_2) := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$a_{G2x}(\theta_2) := \text{Re}(\mathbf{a}_{G2}(\theta_2)) \quad a_{G2y}(\theta_2) := \text{Im}(\mathbf{a}_{G2}(\theta_2))$$

$$\mathbf{a}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\begin{aligned} \mathbf{a}_{CG3A}(\theta_2) &:= R_{CG3} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_{33}) + j \cdot \cos(\theta_3(\theta_2) + \delta_{33})) \dots \\ &+ -R_{CG3} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_{33}) + j \cdot \sin(\theta_3(\theta_2) + \delta_{33})) \end{aligned}$$

$$\mathbf{a}_{G3}(\theta_2) := \mathbf{a}_A(\theta_2) + \mathbf{a}_{CG3A}(\theta_2)$$

$$a_{G3x}(\theta_2) := \text{Re}(\mathbf{a}_{G3}(\theta_2)) \quad a_{G3y}(\theta_2) := \text{Im}(\mathbf{a}_{G3}(\theta_2))$$

$$\mathbf{a}_{G4}(\theta_2) := R_{CG4} \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) - c \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$a_{G4x}(\theta_2) := \text{Re}(\mathbf{a}_{G4}(\theta_2)) \quad a_{G4y}(\theta_2) := \text{Im}(\mathbf{a}_{G4}(\theta_2))$$

6. Calculate the x and y components of the external force at P in the CGS.

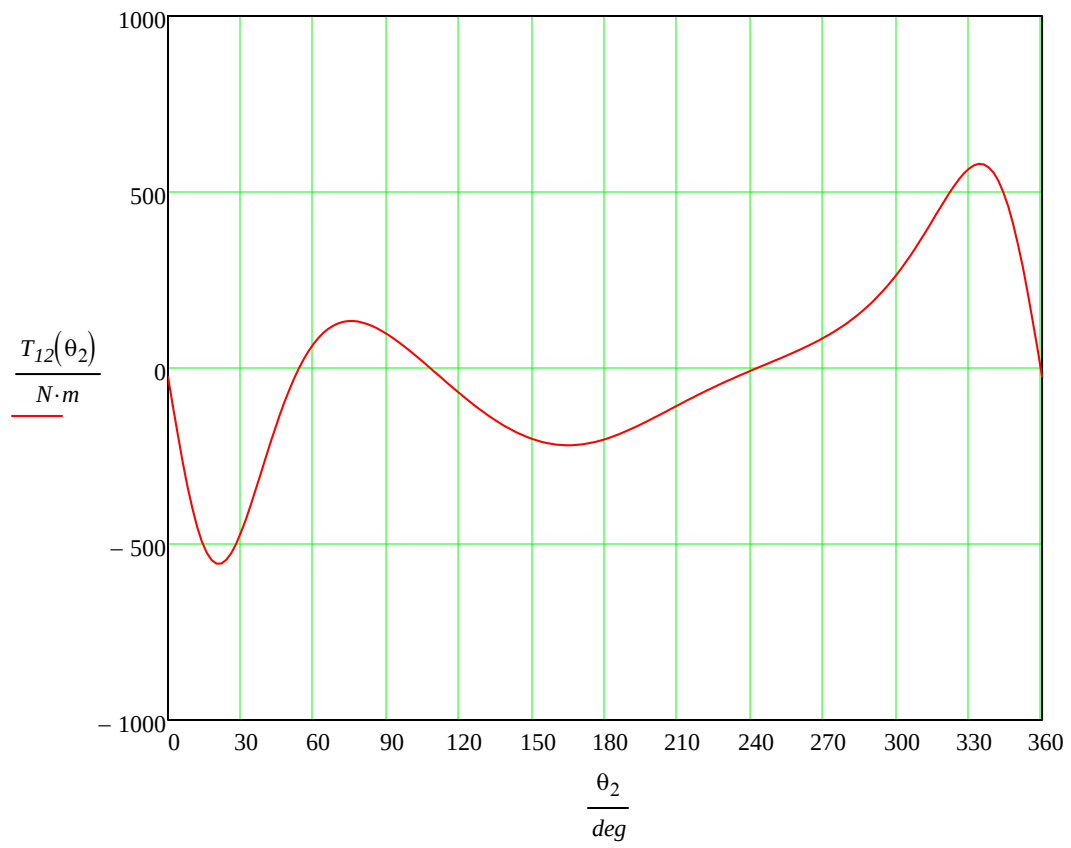
$$F_{Px} := 0 \cdot N \quad F_{Py} := -F_P$$

7. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12}(\theta_2) := \frac{1}{\omega_2} \cdot \left[\begin{aligned} &m_2 \cdot (a_{G2x}(\theta_2) \cdot v_{G2x}(\theta_2) + a_{G2y}(\theta_2) \cdot v_{G2y}(\theta_2)) \dots \\ &+ m_3 \cdot (a_{G3x}(\theta_2) \cdot v_{G3x}(\theta_2) + a_{G3y}(\theta_2) \cdot v_{G3y}(\theta_2)) \dots \\ &+ m_4 \cdot (a_{G4x}(\theta_2) \cdot v_{G4x}(\theta_2) + a_{G4y}(\theta_2) \cdot v_{G4y}(\theta_2)) \dots \\ &+ (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3(\theta_2) \cdot \omega_3(\theta_2) + I_{G4} \cdot \alpha_4(\theta_2) \cdot \omega_4(\theta_2)) \dots \\ &+ -F_{Px} \cdot v_{Px}(\theta_2) - F_{Py} \cdot v_{Py}(\theta_2) \end{aligned} \right]$$

8. Plot the input torque for one revolution of the crank.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



PROBLEM 11-37

Statement: Figure P11-5b shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections of 50 mm diameter. In the instantaneous position shown, the crank O_2A has $\omega = -10$ rad/sec and $\alpha = 10$ rad/sec². There is a horizontal force at P of $F = 300$ N. Find torque needed to drive the crank at this instant using the method of virtual work.

Given: Link lengths:

$$\text{Link 2 (} O_2 \text{ to } A) \quad a := 0.86 \cdot m \quad \text{Link 3 (} A \text{ to } B) \quad b := 1.85 \cdot m$$

$$\text{Link 4 (} B \text{ to } O_4) \quad c := 0.86 \cdot m \quad \text{Link 1 (} O_2 \text{ to } O_4) \quad d := 2.22 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 1.33 \cdot m \quad \delta_3 := 0 \cdot \text{deg} \quad F := 300 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank angle and motion:} \quad \theta_2 := -36 \cdot \text{deg} \quad \omega_2 := -10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$d_{\text{link}} := 50 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{in}^{-3}$$

Solution: See Figure P11-5b and Mathcad file P1137.

1. Use program FOURBAR to determine the position, velocity, and acceleration of links 3 and 4.

$$\theta_3 := 46.028 \cdot \text{deg} \quad \omega_3 := 3.285 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_3 := -109.287 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\theta_4 := 106.189 \cdot \text{deg} \quad \omega_4 := 11.417 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_4 := -43.426 \cdot \text{rad} \cdot \text{sec}^{-2}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.430 \cdot m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.430 \cdot m$$

$$\text{Link 3:} \quad R_{CG3} := 0.5 \cdot b \quad R_{CG3} = 0.925 \cdot m$$

3. Determine the mass and moment of inertia of each link.

$$m_2 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot b \cdot \frac{\gamma_s}{g} \quad m_4 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 14.022 \cdot \text{kg} \quad m_3 = 30.164 \cdot \text{kg} \quad m_4 = 14.022 \cdot \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + a^2 \right) \quad I_{G2} = 0.866 \cdot \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + b^2 \right) \quad I_{G3} = 8.608 \cdot \text{kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + c^2 \right) \quad I_{G4} = 0.866 \cdot \text{kg} \cdot \text{m}^2$$

4. Calculate the x and y components of the velocity of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{v}_{G2} := R_{CG2} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$v_{G2x} := \text{Re}(\mathbf{v}_{G2})$$

$$v_{G2x} = -2.527 \frac{m}{s}$$

$$v_{G2y} := \text{Im}(\mathbf{v}_{G2})$$

$$v_{G2y} = -3.479 \frac{m}{s}$$

$$\mathbf{v}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) \quad \mathbf{v}_{CG3A} := R_{CG3} \cdot \omega_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3))$$

$$\mathbf{v}_{G3} := \mathbf{v}_A + \mathbf{v}_{CG3A}$$

$$v_{G3x} := \text{Re}(\mathbf{v}_{G3}) \quad v_{G3x} = -7.242 \frac{m}{s}$$

$$v_{G3y} := \text{Im}(\mathbf{v}_{G3}) \quad v_{G3y} = -4.848 \frac{m}{s}$$

$$\mathbf{v}_{PA} := R_{pa} \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3))$$

$$\mathbf{v}_P := \mathbf{v}_A + \mathbf{v}_{PA}$$

$$v_{Px} := \text{Re}(\mathbf{v}_P) \quad v_{Px} = -8.199 \frac{m}{s}$$

$$v_{Py} := \text{Im}(\mathbf{v}_P) \quad v_{Py} = -3.924 \frac{m}{s}$$

$$\mathbf{v}_{G4} := R_{CG4} \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$$

$$v_{G4x} := \text{Re}(\mathbf{v}_{G4}) \quad v_{G4x} = -4.715 \frac{m}{s}$$

$$v_{G4y} := \text{Im}(\mathbf{v}_{G4}) \quad v_{G4y} = -1.369 \frac{m}{s}$$

5. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2} := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$a_{G2x} := \text{Re}(\mathbf{a}_{G2}) \quad a_{G2x} = -67.048 \frac{m}{sec^2}$$

$$a_{G2y} := \text{Im}(\mathbf{a}_{G2}) \quad a_{G2y} = 54.028 \frac{m}{sec^2}$$

$$\mathbf{a}_A := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{a}_{CG3A} := R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3)) \dots \\ + -R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_3) + j \cdot \sin(\theta_3))$$

$$\mathbf{a}_{G3} := \mathbf{a}_A + \mathbf{a}_{CG3A} \quad a_{G3x} := \text{Re}(\mathbf{a}_{G3}) \quad a_{G3x} = 1.302 \frac{m}{sec^2}$$

$$a_{G3y} := \text{Im}(\mathbf{a}_{G3}) \quad a_{G3y} = -19.864 \frac{m}{sec^2}$$

$$\mathbf{a}_{G4} := R_{CG4} \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$a_{G4x} := \text{Re}(\mathbf{a}_{G4}) \quad a_{G4x} = 49.187 \frac{m}{sec^2}$$

$$a_{G4y} := \text{Im}(\mathbf{a}_{G4}) \quad a_{G4y} = -102.448 \frac{m}{sec^2}$$

6. Calculate the x and y components of the external force at P in the CGS.

$$F_{Px} := F \qquad F_{Py} := 0 \cdot N$$

7. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \left[m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \dots \right. \\ \left. + m_4 \cdot (a_{G4x} \cdot v_{G4x} + a_{G4y} \cdot v_{G4y}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3 + I_{G4} \cdot \alpha_4 \cdot \omega_4) \dots \right. \\ \left. + -F_{Px} \cdot v_{Px} - F_{Py} \cdot v_{Py} \right]$$

$$T_{12} = 7.10 \, N \cdot m$$

PROBLEM 11-38

Statement: Figure P11-5b shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections of 50 mm diameter. In the instantaneous position shown, the crank O_2A has $\omega = 10$ rad/sec. There is a horizontal force at P of $F = 300$ N. Find and plot all pin forces and the torque needed to drive the crank at a constant speed of 15 rad/sec for one revolution of the crank using the program FOURBAR. Change the length of link 4 to 1.86 m.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 0.86 \cdot m \quad \text{Link 3 (A to B)} \quad b := 1.85 \cdot m$$

$$\text{Link 4 (B to O}_4) \quad c := 1.86 \cdot m \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 2.22 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 1.33 \cdot m \quad \delta_3 := 0 \cdot \text{deg} \quad F_P := 300 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank motion:} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\text{Link cross-section dims:} \quad d_{\text{link}} := 50 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{ft} \cdot \text{in}^{-3}$$

Solution: See Figure P11-5b and Mathcad file P1138.

- Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.430 \, m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.930 \, m$$

$$\text{Link 3:} \quad R_{CG3} := 0.5 \cdot b \quad R_{CG3} = 0.925 \, m \quad \delta_{33} := 0 \cdot \text{deg}$$

- Determine the mass and moment of inertia of each link.

$$m_2 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot b \cdot \frac{\gamma_s}{g} \quad m_4 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 14.022 \, \text{kg} \quad m_3 = 30.164 \, \text{kg} \quad m_4 = 30.327 \, \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + a^2 \right) \quad I_{G2} = 0.866 \, \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + b^2 \right) \quad I_{G3} = 8.608 \, \text{kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + c^2 \right) \quad I_{G4} = 8.748 \, \text{kg} \cdot \text{m}^2$$

- Calculate the x and y components of the external force at P in the CGS and the magnitude and angle of the position vector defining point P in the LRCS.

$$F_{Px} := F_P \quad F_{Py} := 0 \cdot N$$

$$R_{pax'} := R_{pa} \cdot \cos(\delta_3) \quad R_{px'} := R_{pax'} - R_{CG3} \quad R_{pax'} = 1.330 \, m \quad R_{px'} = 0.405 \, m$$

- Enter the above data into program FOURBAR and solve for the pin forces and driving torque. The dynamic input screen is shown below followed by a plot of dynamic pin forces..

FOURBAR - Student Edition - by R. L. Norton - Copyright 2006 Release 8.0 Rev 1.9 8/5/06 Dynamics Screen

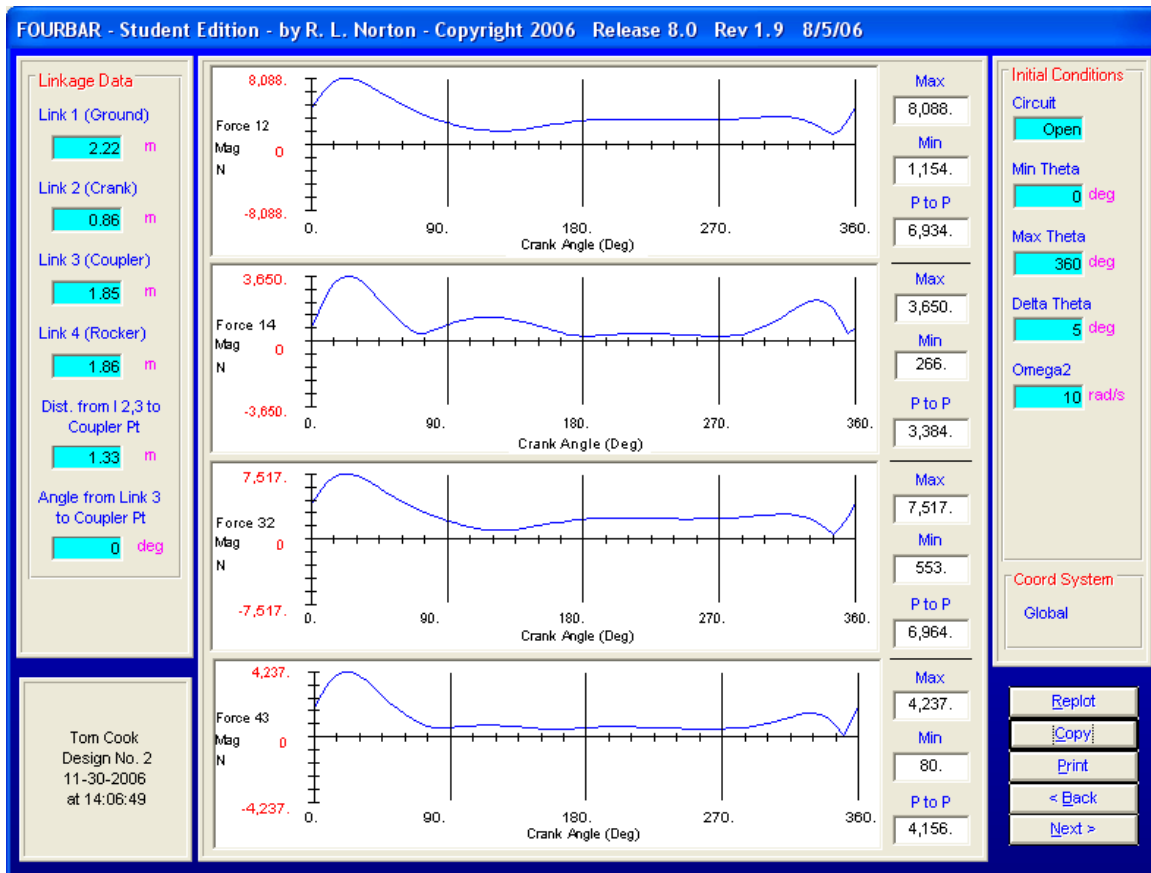
Help

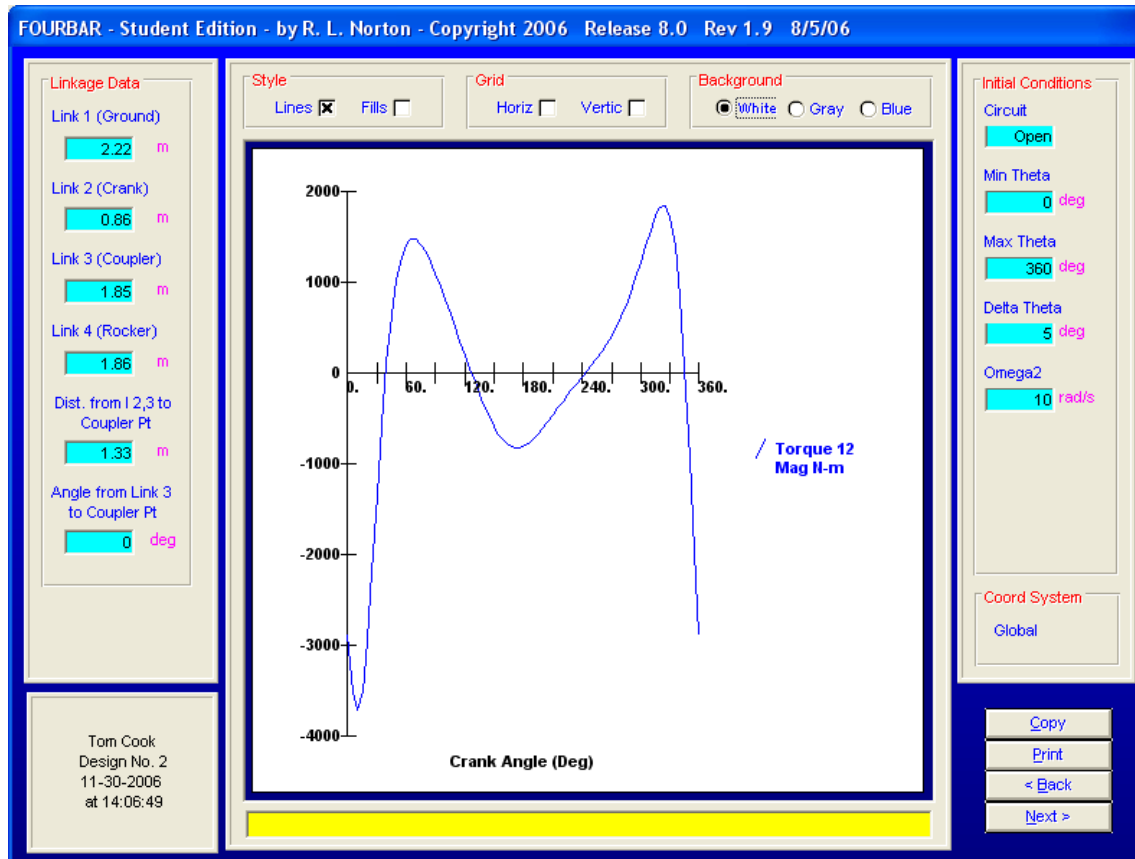
	Link 2	Link 3	Link 4	Units
Mass Properties				
Mass of Link	14.022	30.164	30.327	kg
Mass Moment of Inertia about CG	0.866	8.608	8.748	kg-m ²
CG Locations				
Distance to CG from Pivot	0.430	0.925	0.930	m
Angle to CG in LRCS*	0.00	0	0.00	degrees
External Forces				
Force Magnitude	300	0.00		N
Angle of Force in GCS*	0	0		degrees
External Force Locations				
Distance to Force Point from CG	0.405	0.0000		m
Angle to Force Point in LRCS*	0	0		degrees
External Torques				
Torque on Link	0.000	0.000		N-m
Import a Torque Function File to Apply to Link 4	☐			
Friction				
Friction Force on Link	0.00			N

* LRCS = Local Rotating Coordinate System. GCS = Global Coordinate System

Solve
Show Matrix
Copy
Print
< Back
Next >

Tom Cook
Design No. 2
11-30-2006
at 14:06:49





PROBLEM 11-39

Statement: Figure P11-5b shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections of 50 mm diameter. In the instantaneous position shown, the crank O_2A has $\omega = 10$ rad/sec. There is a horizontal force at P of $F = 300$ N. Find torque needed to drive the crank at this instant using the method of virtual work. Change the length of link 4 to 1.86 m.

Given: Link lengths:

$$\begin{aligned} \text{Link 2 (O}_2 \text{ to A)} \quad a &:= 0.86 \cdot m & \text{Link 3 (A to B)} \quad b &:= 1.85 \cdot m \\ \text{Link 4 (B to O}_4) \quad c &:= 1.86 \cdot m & \text{Link 1 (O}_2 \text{ to O}_4) \quad d &:= 2.22 \cdot m \\ \text{Coupler point:} \quad R_{pa} &:= 1.33 \cdot m & \delta_3 &:= 0 \cdot \text{deg} & F_P &:= 300 \cdot \text{N} & T_4 &:= 0 \cdot \text{N} \cdot m \\ \text{Crank motion:} \quad \omega_2 &:= 10 \cdot \text{rad} \cdot \text{sec}^{-1} & \alpha_2 &:= 0 \cdot \text{rad} \cdot \text{sec}^{-2} \\ \text{Link cross-section dims:} \quad d_{\text{link}} &:= 50 \cdot \text{mm} \\ \text{Material specific weight:} \quad \gamma_s &:= 0.3 \cdot \text{lb} \cdot \text{ft}^{-3} \end{aligned}$$

Solution: See Figure P11-5b and Mathcad file P1139.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.430 \, m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.930 \, m$$

$$\text{Link 3:} \quad R_{CG3} := 0.5 \cdot b \quad R_{CG3} = 0.925 \, m \quad \delta_{33} := 0 \cdot \text{deg}$$

2. Determine the mass and moment of inertia of each link.

$$m_2 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot b \cdot \frac{\gamma_s}{g} \quad m_4 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 14.022 \, \text{kg} \quad m_3 = 30.164 \, \text{kg} \quad m_4 = 30.327 \, \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + a^2 \right) \quad I_{G2} = 0.866 \, \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + b^2 \right) \quad I_{G3} = 8.608 \, \text{kg} \cdot \text{m}^2$$

$$I_{G4} := \frac{m_4}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + c^2 \right) \quad I_{G4} = 8.748 \, \text{kg} \cdot \text{m}^2$$

3. Calculate the position, angular velocity, and acceleration of links 3 and 4 as functions of θ_2 .

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 2.5814 \quad K_2 = 1.1935 \quad K_3 = 1.7833$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

$$\theta_3(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))} \quad \omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \quad B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \quad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

$$\mathbf{A}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A}_{PA}(\theta_2) := R_{pa} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \dots \\ + R_{pa} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{A}_P(\theta_2) := \mathbf{A}_A(\theta_2) + \mathbf{A}_{PA}(\theta_2)$$

$$A_P(\theta_2) := |\mathbf{A}_P(\theta_2)| \quad \theta_{AP}(\theta_2) := \arg(\mathbf{A}_P(\theta_2))$$

4. Calculate the x and y components of the velocity of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{v}_{G2}(\theta_2) := R_{CG2} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$v_{G2x}(\theta_2) := \text{Re}(\mathbf{v}_{G2}(\theta_2)) \quad v_{G2y}(\theta_2) := \text{Im}(\mathbf{v}_{G2}(\theta_2))$$

$$\mathbf{v}_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{v}_{CG3A}(\theta_2) := R_{CG3} \cdot \omega_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_{33}) + j \cdot \cos(\theta_3(\theta_2) + \delta_{33}))$$

$$\mathbf{v}_{G3}(\theta_2) := \mathbf{v}_A(\theta_2) + \mathbf{v}_{CG3A}(\theta_2)$$

$$v_{G3x}(\theta_2) := \text{Re}(\mathbf{v}_{G3}(\theta_2)) \quad v_{G3y}(\theta_2) := \text{Im}(\mathbf{v}_{G3}(\theta_2))$$

$$\mathbf{v}_{PA}(\theta_2) := R_{pa} \cdot \omega_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{v}_P(\theta_2) := \mathbf{v}_A(\theta_2) + \mathbf{v}_{PA}(\theta_2)$$

$$v_{Px}(\theta_2) := \text{Re}(\mathbf{v}_P(\theta_2)) \quad v_{Py}(\theta_2) := \text{Im}(\mathbf{v}_P(\theta_2))$$

$$\mathbf{v}_{G4}(\theta_2) := R_{CG4} \cdot \omega_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2)))$$

$$v_{G4x}(\theta_2) := \text{Re}(\mathbf{v}_{G4}(\theta_2)) \quad v_{G4y}(\theta_2) := \text{Im}(\mathbf{v}_{G4}(\theta_2))$$

5. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\mathbf{a}_{G2}(\theta_2) := R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$a_{G2x}(\theta_2) := \text{Re}(\mathbf{a}_{G2}(\theta_2)) \quad a_{G2y}(\theta_2) := \text{Im}(\mathbf{a}_{G2}(\theta_2))$$

$$\mathbf{a}_A(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{a}_{CG3A}(\theta_2) := R_{CG3} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_{33}) + j \cdot \cos(\theta_3(\theta_2) + \delta_{33})) \dots \\ + R_{CG3} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_{33}) + j \cdot \sin(\theta_3(\theta_2) + \delta_{33}))$$

$$\mathbf{a}_{G3}(\theta_2) := \mathbf{a}_A(\theta_2) + \mathbf{a}_{CG3A}(\theta_2)$$

$$a_{G3x}(\theta_2) := \text{Re}(\mathbf{a}_{G3}(\theta_2)) \quad a_{G3y}(\theta_2) := \text{Im}(\mathbf{a}_{G3}(\theta_2))$$

$$\mathbf{a}_{G4}(\theta_2) := R_{CG4} \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) - c \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$a_{G4x}(\theta_2) := \text{Re}(\mathbf{a}_{G4}(\theta_2)) \quad a_{G4y}(\theta_2) := \text{Im}(\mathbf{a}_{G4}(\theta_2))$$

6. Calculate the x and y components of the external force at P in the CGS.

$$F_{Px} := F_P \quad F_{Py} := 0 \cdot N$$

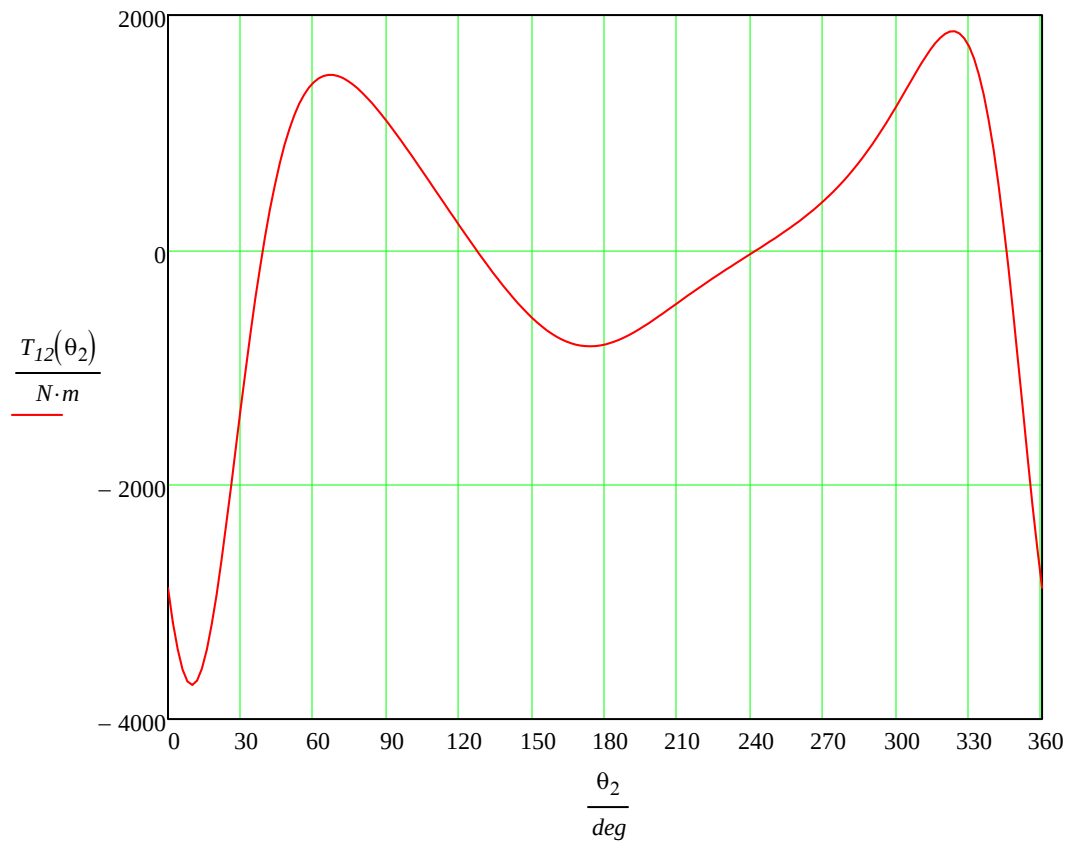
7. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12}(\theta_2) := \frac{1}{\omega_2} \cdot \left[m_2 \cdot (a_{G2x}(\theta_2) \cdot v_{G2x}(\theta_2) + a_{G2y}(\theta_2) \cdot v_{G2y}(\theta_2)) \dots \right. \\ \left. + m_3 \cdot (a_{G3x}(\theta_2) \cdot v_{G3x}(\theta_2) + a_{G3y}(\theta_2) \cdot v_{G3y}(\theta_2)) \dots \right. \\ \left. + m_4 \cdot (a_{G4x}(\theta_2) \cdot v_{G4x}(\theta_2) + a_{G4y}(\theta_2) \cdot v_{G4y}(\theta_2)) \dots \right. \\ \left. + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3(\theta_2) \cdot \omega_3(\theta_2) + I_{G4} \cdot \alpha_4(\theta_2) \cdot \omega_4(\theta_2)) \dots \right. \\ \left. + -F_{Px} \cdot v_{Px}(\theta_2) - F_{Py} \cdot v_{Py}(\theta_2) \right]$$

8. Plot the input torque for one revolution of the crank.

$$\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$$

See next page



PROBLEM 11-40

Statement: Figure P11-3 shows a fourbar linkage and its dimensions. The steel crank and rocker have uniform cross sections 1 in wide by 0.5 in thick. The aluminum coupler is 0.75 in thick. In the instantaneous position shown, the crank O_2A has $\omega = 40$ rad/sec. There is a horizontal force at P of $F = 50$ lb. Design a steel disc flywheel to smooth the input torque for the crank of Problem 11-26 using a coefficient of fluctuation of 0.05 while minimizing the flywheel weight.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 5.00 \cdot in \quad \text{Link 3 (A to B)} \quad b := 7.40 \cdot in$$

$$\text{Link 4 (B to O}_4) \quad c := 8.00 \cdot in \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 9.50 \cdot in$$

$$\text{Coupler point:} \quad R_{pa} := 8.90 \cdot in \quad \delta_3 := 56 \cdot deg \quad F_P := 50 \cdot lbf \quad T_4 := 0 \cdot lbf \cdot in$$

$$\text{Crank motion:} \quad \omega_2 := 40 \cdot rad \cdot sec^{-1} \quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$$

Link cross-section dims:

$$w_2 := 1.00 \cdot in \quad t_2 := 0.50 \cdot in \quad t_3 := 0.75 \cdot in \quad w_4 := 1.00 \cdot in \quad t_4 := 0.50 \cdot in$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot lbf \cdot in^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot lbf \cdot in^{-3}$$

$$\text{Coefficient of fluctuation:} \quad k := 0.05$$

$$\text{Desired average speed:} \quad \omega_{avg} := \omega_2$$

Solution: See Figure P11-3 and Mathcad file P1140.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 2.500 \cdot in \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 4.000 \cdot in$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 4.126 \cdot in$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = 2.459 \cdot in$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 4.803 \cdot in$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = 30.801 \cdot deg$$

2. Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot R_{pa} \cdot \sin(\delta_3) \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 1.943 \times 10^{-3} \cdot blob \quad m_3 = 5.303 \times 10^{-3} \cdot blob \quad m_4 = 3.108 \times 10^{-3} \cdot blob$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.0042 \cdot in \cdot lbf \cdot sec^2$$

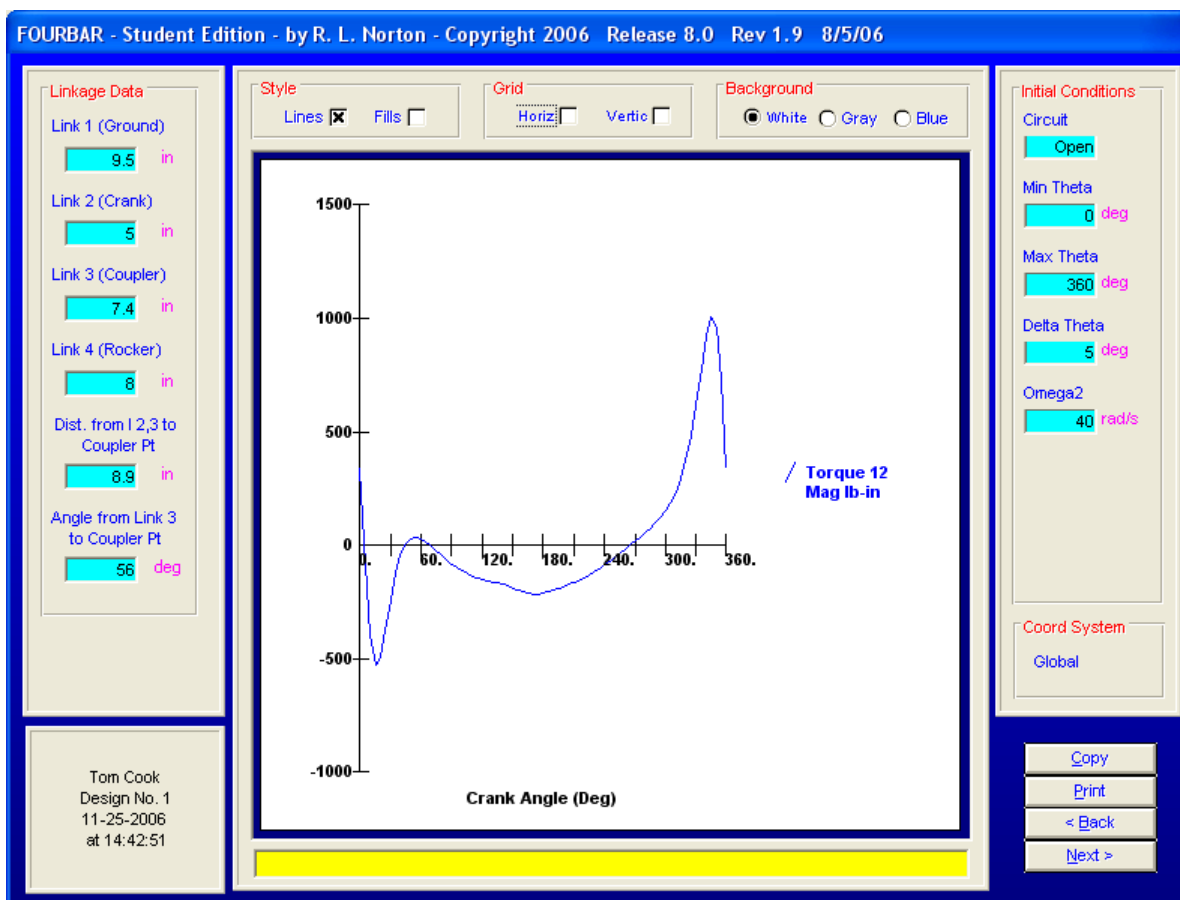
$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2] \quad I_{G3} = 0.097 \cdot in \cdot lbf \cdot sec^2$$

$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 0.017 \cdot in \cdot lbf \cdot sec^2$$

3. Calculate the x and y components of the external force at P in the CGS and the magnitude and angle of the position vector defining point P in the LRCS.

$$\begin{aligned}
 F_{Px} &:= -F_P & F_{Py} &:= 0 \cdot \text{lbf} \\
 R_{pax'} &:= R_{pa} \cdot \cos(\delta_3) & R_{pay'} &:= R_{pa} \cdot \sin(\delta_3) & R_{px'} &:= R_{pax'} - R_{CG3x'} & R_{py'} &:= R_{pay'} - R_{CG3y'} \\
 R_{pax'} &= 4.977 \text{ in} & R_{pay'} &= 7.378 \text{ in} & R_{px'} &= 0.851 \text{ in} & R_{py'} &= 4.919 \text{ in} \\
 R_p &:= \sqrt{R_{px'}^2 + R_{py'}^2} & R_p &= 4.992 \text{ in} & \theta_p &:= \text{atan}\left(\frac{R_{py'}}{R_{px'}}\right) & \theta_p &= 80.182 \cdot \text{deg}
 \end{aligned}$$

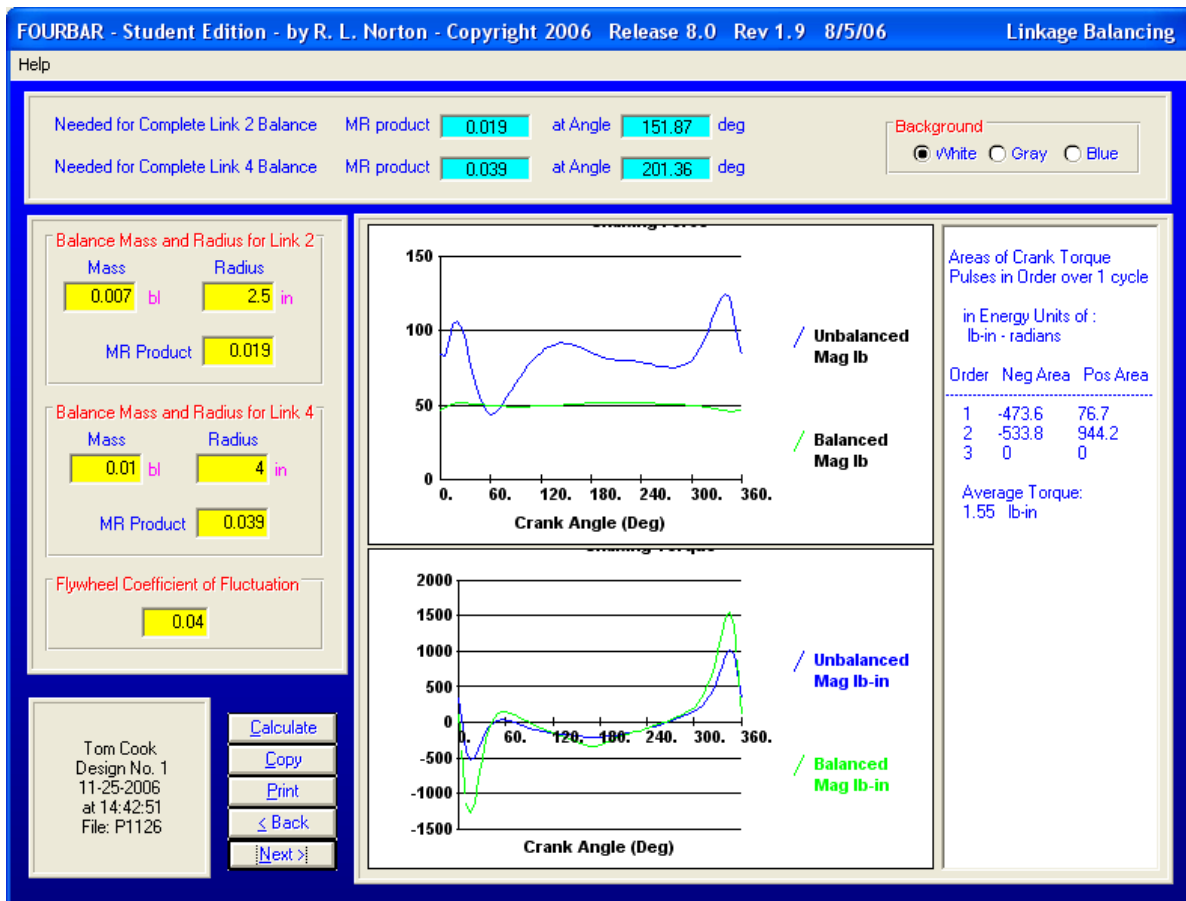
4. Enter the above data into program FOURBAR and determine the energy change from minimum to maximum speed by calculating the dynamic values and then plotting the input torque vs. crank angle. See screens below.



5. Integrate the torque function using the values in the "Areas of Crank Torque Pulses" shown in the Linkage Balancing screen below as related to the Torque vs. Crank Angle screen above (see Table 11-1 in text).

A to B	-473.6	-473.6	
B to C	+76.7	-396.9	
C to D	-533.8	-930.7	$\omega_{\max}$ at D
D to A	+944.2	+13.5	$\omega_{\min}$ at A

$$\begin{aligned}
 E_{\max\text{speed}} &:= -930.7 \cdot \text{in} \cdot \text{lbf} & E_{\min\text{speed}} &:= 13.5 \cdot \text{in} \cdot \text{lbf} \\
 E &:= |E_{\max\text{speed}} - E_{\min\text{speed}}| & E &= 944.2 \cdot \text{in} \cdot \text{lbf}
 \end{aligned}$$



6. Use equation 11.22 to determine the required system mass moment of inertia.

$$I_s := \frac{E}{k \cdot \omega_{avg}^2} \quad I_s = 11.8 \cdot \text{blob} \cdot \text{in}^2$$

The moment of inertia of the input crank and the motor armature can be subtracted from this value to obtain the required flywheel moment. However, in this case they are small compared with the required moment of inertia.

7. There is an infinity of solutions for the dimensions of the flywheel for the required moment of inertia. One possible solution follows:

$$\text{Let the flywheel radius be } r_f := 1.8 \cdot a \quad r_f = 9.000 \text{ in}$$

From Appendix C the moment of inertia of a cylinder about its central axis is $I = mr^2/2$. Solving for m ,

$$\text{The required mass is } m_f := \frac{2 \cdot I_s}{r_f^2} \quad m_f = 0.291 \cdot \text{blob}$$

The volume of a cylindrical disk is $V = \pi r^2 t$ and the mass is $m = V\gamma/g$. Eliminating V and solving for the thickness, t , for a steel disk

$$t_f := \frac{m_f \cdot g}{\pi r_f^2 \cdot \gamma_s} \quad t_f = 1.474 \text{ in}$$

PROBLEM 11-41

Statement: Figure P11-4a shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 10$ rad/sec. There is a vertical force at P of $F = 100$ N. Design a steel disc flywheel to smooth the input torque for the crank of Problem 11-29 using a coefficient of fluctuation of 0.05 while minimizing the flywheel weight.

Given:

Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 1.00 \cdot m \quad \text{Link 3 (A to B)} \quad b := 2.06 \cdot m$$

$$\text{Link 4 (B to O}_4) \quad c := 2.33 \cdot m \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad d := 2.22 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 3.06 \cdot m \quad \delta_3 := -31 \cdot \text{deg} \quad F_P := 100 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank motion:} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3}$$

$$\text{Coefficient of fluctuation:} \quad k := 0.05 \quad \text{Desired average speed:} \quad \omega_{avg} := \omega_2$$

Solution: See Figure P11-4a and Mathcad file P1141.

- Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.500 \, m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 1.165 \, m$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 1.561 \, m$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = -0.525 \, m$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 1.647 \, m$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = -18.600 \, \text{deg}$$

- Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot |R_{pa} \cdot \sin(\delta_3)| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 10.380 \, \text{kg} \quad m_3 = 112.332 \, \text{kg} \quad m_4 = 24.185 \, \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.867 \, \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2] \quad I_{G3} = 125.951 \, \text{kg} \cdot \text{m}^2$$

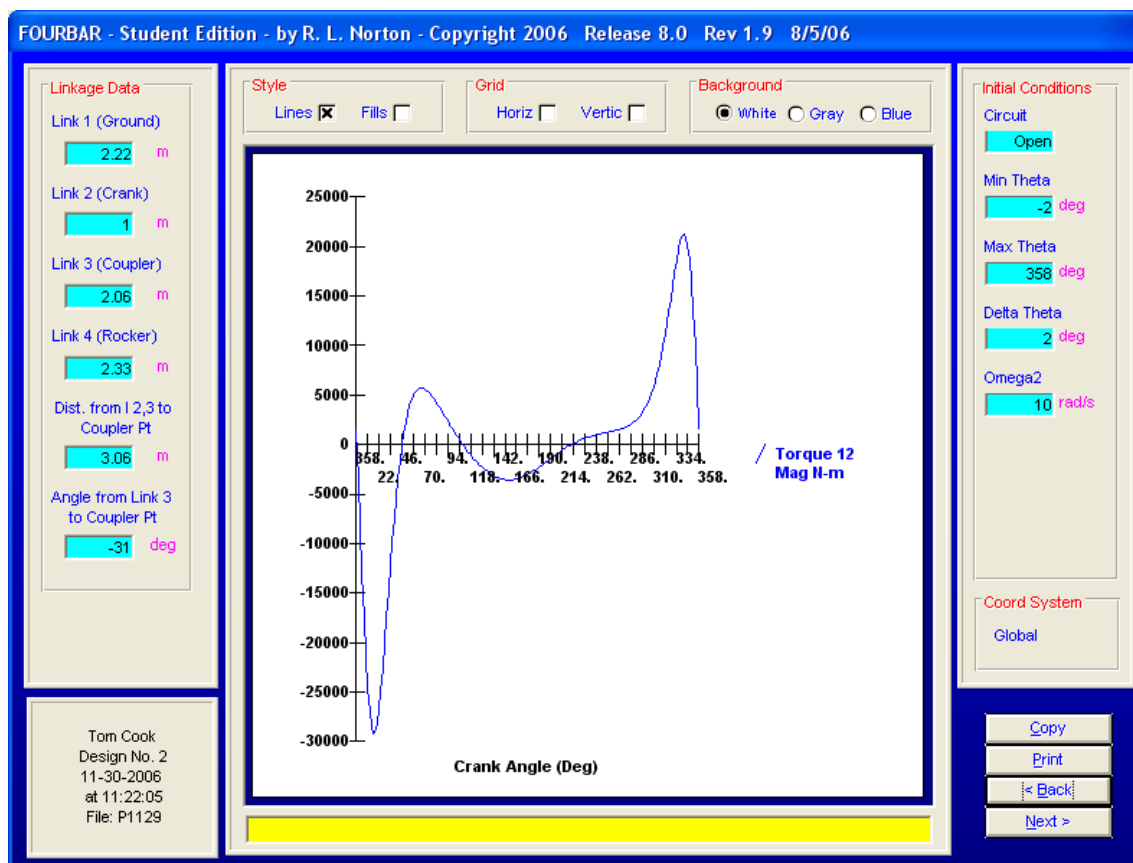
$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 10.947 \, \text{kg} \cdot \text{m}^2$$

- Calculate the x and y components of the external force at P in the CGS and the magnitude and angle of the position vector defining point P in the LRCS.

$$F_{Px} := 0 \cdot N \quad F_{Py} := -F_P$$

$$\begin{aligned}
 R_{pax'} &:= R_{pa} \cdot \cos(\delta_3) & R_{pay'} &:= R_{pa} \cdot \sin(\delta_3) & R_{px'} &:= R_{pax'} - R_{CG3x'} & R_{py'} &:= R_{pay'} - R_{CG3y'} \\
 R_{pax'} &= 2.623 \text{ m} & R_{pay'} &= -1.576 \text{ m} & R_{px'} &= 1.062 \text{ m} & R_{py'} &= -1.051 \text{ m} \\
 R_p &:= \sqrt{R_{px'}^2 + R_{py'}^2} & R_p &= 1.494 \text{ m} & \theta_p &:= \text{atan}\left(\frac{R_{py'}}{R_{px'}}\right) & \theta_p &= -44.694 \text{ deg}
 \end{aligned}$$

4. Enter the above data into program FOURBAR and determine the energy change from minimum to maximum speed by calculating the dynamic values and then plotting the input torque vs. crank angle. See screens below.

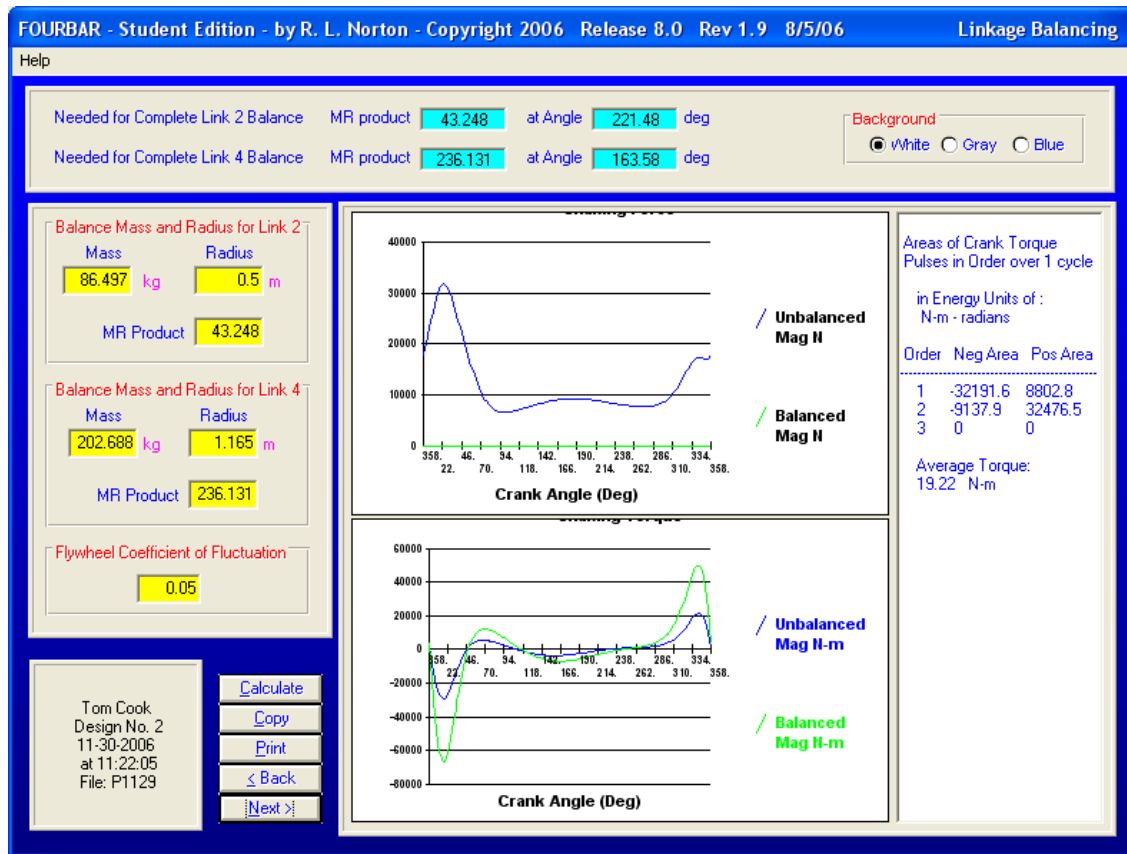


5. Integrate the torque function using the values in the "Areas of Crank Torque Pulses" shown in the Linkage Balancing screen below as related to the Torque vs. Crank Angle screen above (see Table 11-1 in text).

A to B	-32191.6	-32191.6	
B to C	+8802.8	-23388.8	
C to D	-9137.9	-32526.7	$\omega_{\max}$ at D
D to A	+32476.5	-50.2	$\omega_{\min}$ at A

$$E_{\max\text{speed}} := -32526.7 \cdot N \cdot m \quad E_{\min\text{speed}} := -50.2 \cdot N \cdot m$$

$$E := |E_{\max\text{speed}} - E_{\min\text{speed}}| \quad E = 32476.5 \text{ N} \cdot m$$



6. Use equation 11.22 to determine the required system mass moment of inertia.

$$I_s := \frac{E}{k \cdot \omega_{avg}^2} \quad I_s = 6495.3 \text{ kg} \cdot \text{m}^2$$

The moment of inertia of the input crank and the motor armature can be subtracted from this value to obtain the required flywheel moment. There are an infinity of possible size/shape solutions for this problem.

PROBLEM 11-42

Statement: Figure P11-4b shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 15 \text{ rad/sec}$. There is a horizontal force at P of $F = 200 \text{ N}$. Design a steel disc flywheel to smooth the input torque for the crank of Problem 11-32 using a coefficient of fluctuation of 0.07 while minimizing the flywheel weight. Change the lengths of links 3 and 4 to 1.43 and 1.60 m, respectively.

Given: Link lengths:

$$\text{Link 2 (} O_2 \text{ to } A) \quad a := 0.72 \cdot m \quad \text{Link 3 (} A \text{ to } B) \quad b := 1.43 \cdot m$$

$$\text{Link 4 (} B \text{ to } O_4) \quad c := 1.60 \cdot m \quad \text{Link 1 (} O_2 \text{ to } O_4) \quad d := 1.82 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 0.97 \cdot m \quad \delta_3 := 54 \cdot \text{deg} \quad F_P := 200 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank motion:} \quad \omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lb} \cdot \text{f} \cdot \text{in}^{-3}$$

$$\text{Coefficient of fluctuation:} \quad k := 0.07 \quad \text{Desired average speed:} \quad \omega_{\text{avg}} := \omega_2$$

Solution: See Figure P11-4b and Mathcad file P1142.

- Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.360 \text{ m} \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.800 \text{ m}$$

$$\text{Link 3:} \quad R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x'} = 0.667 \text{ m}$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y'} = 0.262 \text{ m}$$

$$R_{CG3} := \sqrt{R_{CG3x'}^2 + R_{CG3y'}^2} \quad R_{CG3} = 0.716 \text{ m}$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x'}, R_{CG3y'}) \quad \delta_{33} = 21.422 \text{ deg}$$

- Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot |R_{pa} \cdot \sin(\delta_3)| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 7.474 \text{ kg} \quad m_3 = 38.828 \text{ kg} \quad m_4 = 16.608 \text{ kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.324 \text{ kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{6} \cdot [b^2 + (R_{pa} \cdot \sin(\delta_3))^2] \quad I_{G3} = 17.218 \text{ kg} \cdot \text{m}^2$$

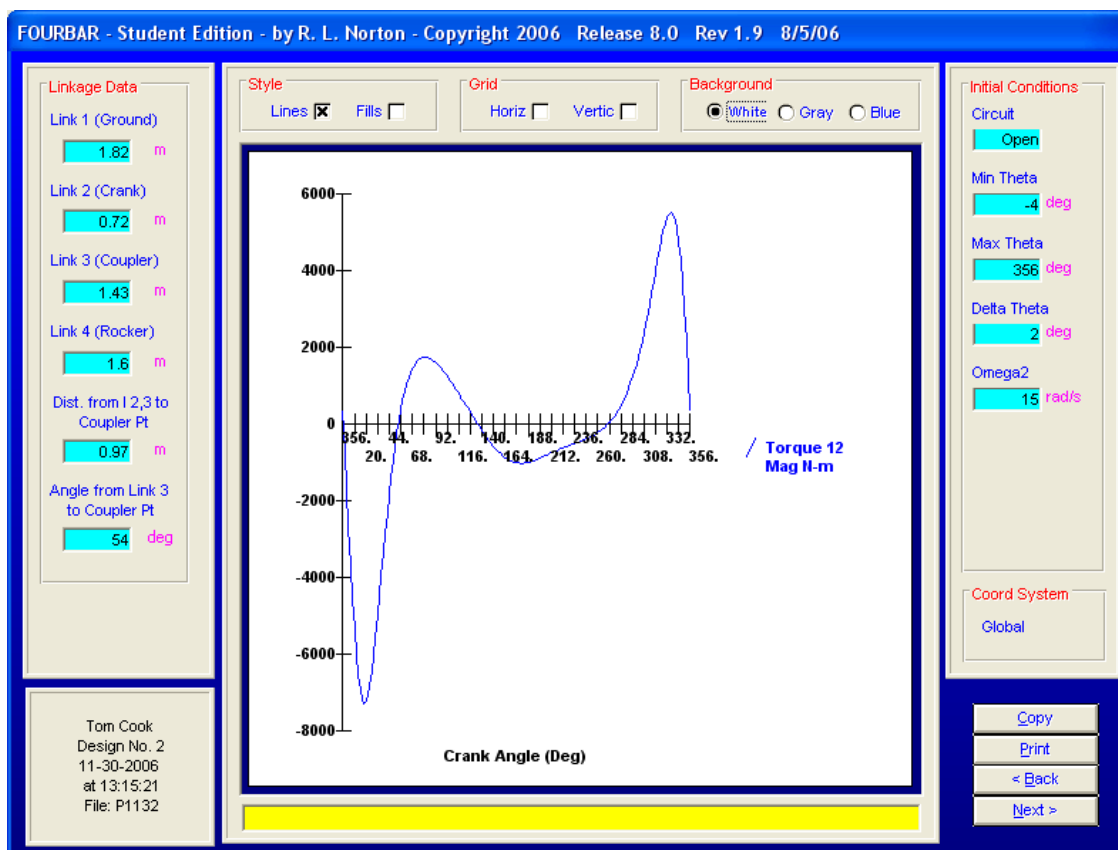
$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 3.546 \text{ kg} \cdot \text{m}^2$$

- Calculate the x and y components of the external force at P in the CGS and the magnitude and angle of the position vector defining point P in the LRCS.

$$F_{Px} := F_P \quad F_{Py} := 0 \cdot N$$

$$\begin{aligned}
 R_{pax'} &:= R_{pa} \cdot \cos(\delta_3) & R_{pay'} &:= R_{pa} \cdot \sin(\delta_3) & R_{px'} &:= R_{pax'} - R_{CG3x'} & R_{py'} &:= R_{pay'} - R_{CG3y'} \\
 R_{pax'} &= 0.570 \text{ m} & R_{pay'} &= 0.785 \text{ m} & R_{px'} &= -0.097 \text{ m} & R_{py'} &= 0.523 \text{ m} \\
 R_p &:= \sqrt{R_{px'}^2 + R_{py'}^2} & R_p &= 0.532 \text{ m} & \theta_p &:= \text{atan2}(R_{px'}, R_{py'}) & \theta_p &= 100.458 \text{ deg}
 \end{aligned}$$

4. Enter the above data into program FOURBAR and determine the energy change from minimum to maximum speed by calculating the dynamic values and then plotting the input torque vs. crank angle. See screens below.

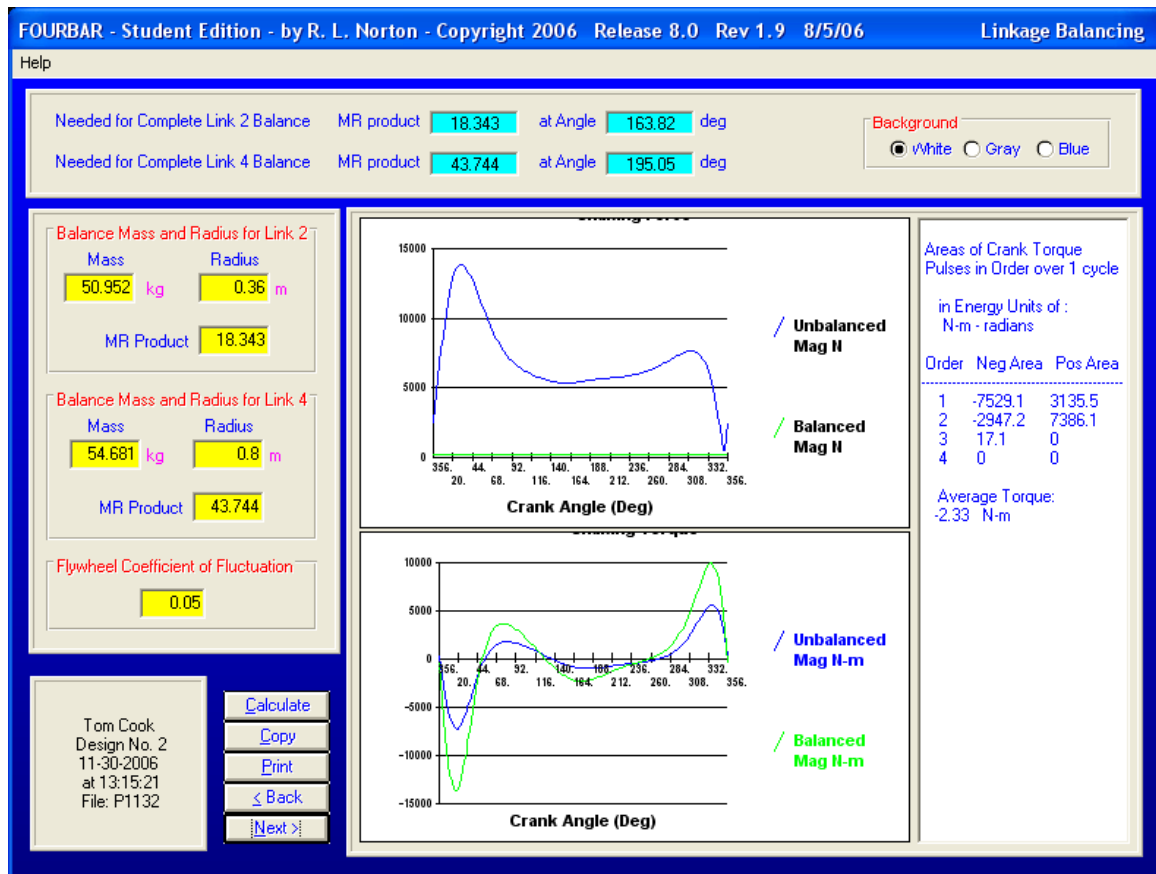


5. Integrate the torque function using the values in the "Areas of Crank Torque Pulses" shown in the Linkage Balancing screen below as related to the Torque vs. Crank Angle screen above (see Table 11-1 in text).

A to B	-7512.0	-7512.0
B to C	+3135.5	-4376.5
C to D	-2947.2	-7323.7 $\omega_{\max}$ at D
D to A	+7386.1	+62.4 $\omega_{\min}$ at A

$$E_{\max\text{speed}} := -7323.7 \cdot N \cdot m \quad E_{\min\text{speed}} := 62.4 \cdot N \cdot m$$

$$E := |E_{\max\text{speed}} - E_{\min\text{speed}}| \quad E = 7386.1 \text{ N} \cdot m$$



6. Use equation 11.22 to determine the required system mass moment of inertia.

$$I_s := \frac{E}{k \cdot \omega_{avg}^2} \quad I_s = 469.0 \text{ kg} \cdot \text{m}^2$$

The moment of inertia of the input crank and the motor armature can be subtracted from this value to obtain the required flywheel moment. There are an infinity of possible size/shape solutions for this problem.

PROBLEM 11-43

Statement: Figure P11-5a shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections 50 mm wide by 25 mm thick. In the instantaneous position shown, the crank O_2A has $\omega = 15$ rad/sec. There is a vertical force at P of $F = 500$ N. Design a steel disc flywheel to smooth the input torque for the crank of Problem 11-35 using a coefficient of fluctuation of 0.05 while minimizing the flywheel weight. Change the lengths of links 1, 2, and 3 to 1.000, 0.356, and 0.785 m, respectively.

Given: Link lengths:

$$\text{Link 2 (} O_2 \text{ to } A) \quad a := 0.356 \cdot m \quad \text{Link 3 (} A \text{ to } B) \quad b := 0.785 \cdot m$$

$$\text{Link 4 (} B \text{ to } O_4) \quad c := 0.950 \cdot m \quad \text{Link 1 (} O_2 \text{ to } O_4) \quad d := 1.000 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 1.09 \cdot m \quad \delta_3 := 0 \cdot \text{deg} \quad F_P := 500 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank motion:} \quad \omega_2 := 15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad w_3 := 50 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{ft}^{-3}$$

$$\text{Coefficient of fluctuation:} \quad k := 0.05 \quad \text{Desired average speed:} \quad \omega_{avg} := \omega_2$$

Solution: See Figure P11-5a and Mathcad file P1143.

- Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.178 \cdot m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.475 \cdot m$$

$$\text{Link 3:} \quad R_{CG3} := 0.5 \cdot R_{pa} \quad R_{CG3} = 0.545 \cdot m \quad \delta_{33} := 0 \cdot \text{deg}$$

- Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := w_3 \cdot t_3 \cdot R_{pa} \cdot \frac{\gamma_s}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 3.695 \cdot \text{kg} \quad m_3 = 11.314 \cdot \text{kg} \quad m_4 = 9.861 \cdot \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot (w_2^2 + a^2) \quad I_{G2} = 0.040 \cdot \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{12} \cdot (w_3^2 + R_{pa}^2) \quad I_{G3} = 1.123 \cdot \text{kg} \cdot \text{m}^2$$

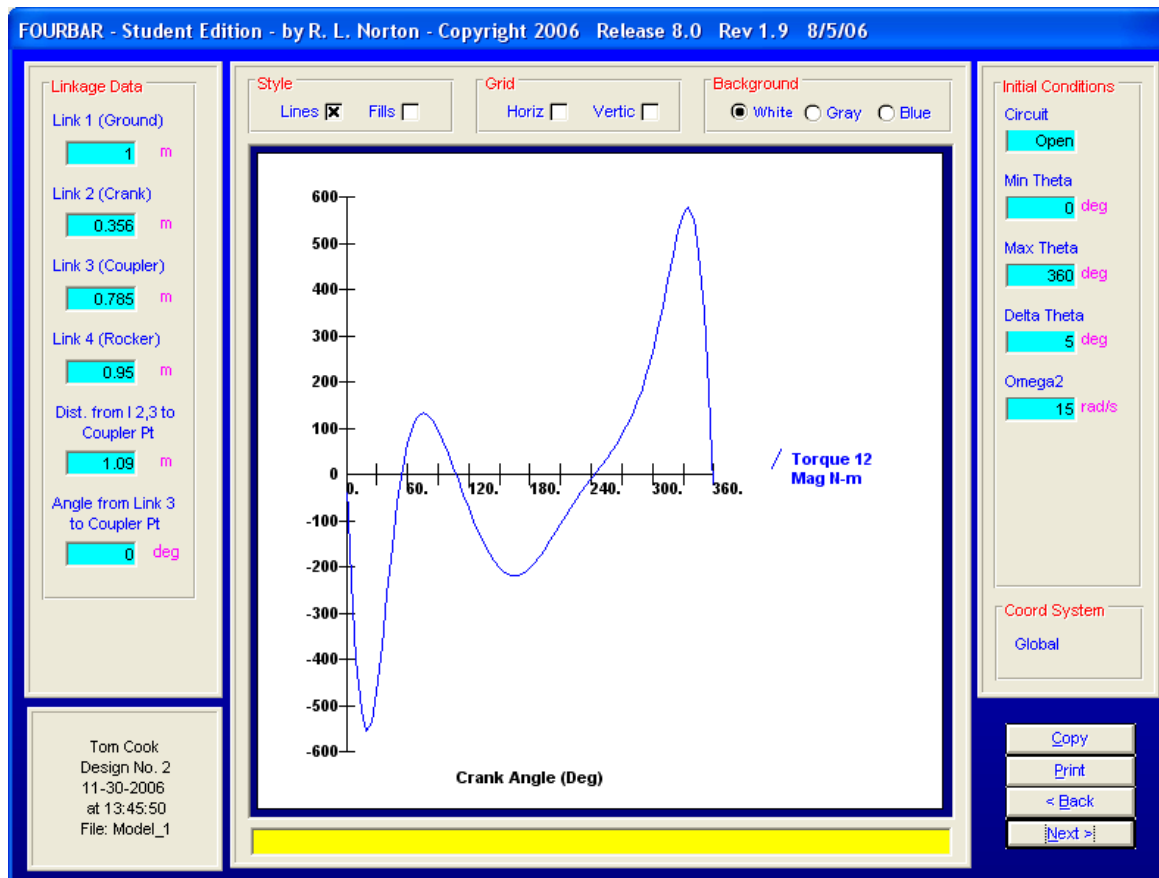
$$I_{G4} := \frac{m_4}{12} \cdot (w_4^2 + c^2) \quad I_{G4} = 0.744 \cdot \text{kg} \cdot \text{m}^2$$

- Calculate the x and y components of the external force at P in the CGS and the magnitude and angle of the position vector defining point P in the LRCS.

$$F_{Px} := 0 \cdot N \quad F_{Py} := -F_P$$

$$R_{pax'} := R_{pa} \cdot \cos(\delta_3) \quad R_{px'} := R_{pax'} - R_{CG3} \quad R_{pax'} = 1.090 \cdot m \quad R_{px'} = 0.545 \cdot m$$

- Enter the above data into program FOURBAR and determine the energy change from minimum to maximum speed by calculating the dynamic values and then plotting the input torque vs. crank angle. See screens below.

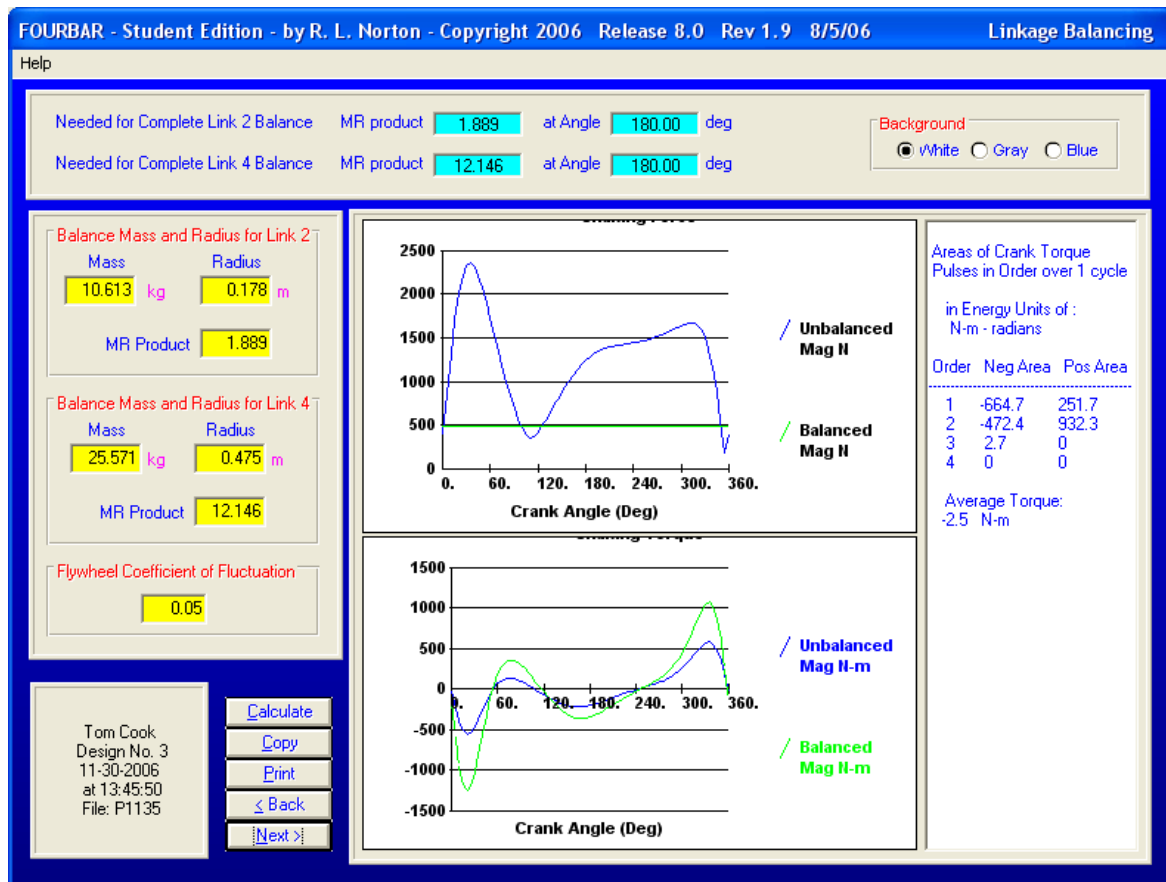


5. Integrate the torque function using the values in the "Areas of Crank Torque Pulses" shown in the Linkage Balancing screen below as related to the Torque vs. Crank Angle screen above (see Table 11-1 in text).

A to B	-662.0	-662.0	
B to C	+251.7	-410.3	
C to D	-472.4	-882.7	$\omega_{\max}$ at D
D to A	+932.3	+49.6	$\omega_{\min}$ at A

$$E_{\max\text{speed}} := -882.7 \cdot N \cdot m \qquad E_{\min\text{speed}} := 49.6 \cdot N \cdot m$$

$$E := |E_{\max\text{speed}} - E_{\min\text{speed}}| \qquad E = 932.3 \, N \cdot m$$



6. Use equation 11.22 to determine the required system mass moment of inertia.

$$I_s := \frac{E}{k \cdot \omega_{avg}^2} \quad I_s = 82.9 \text{ kg} \cdot \text{m}^2$$

The moment of inertia of the input crank and the motor armature can be subtracted from this value to obtain the required flywheel moment. There are an infinity of possible size/shape solutions for this problem.

PROBLEM 11-44

Statement: Figure P11-5b shows a fourbar linkage and its dimensions in meters. The steel crank, coupler, and rocker have uniform cross sections of 50 mm diameter. In the instantaneous position shown, the crank O_2A has $\omega = 10$ rad/sec. There is a horizontal force at P of $F = 300$ N.

Design a steel disc flywheel to smooth the input torque for the crank of Problem 11-38 using a coefficient of fluctuation of 0.06 while minimizing the flywheel weight. Change the length of link 4 to 1.86 m.

Given: Link lengths:

$$\text{Link 2 (} O_2 \text{ to } A) \quad a := 0.86 \cdot m \quad \text{Link 3 (} A \text{ to } B) \quad b := 1.85 \cdot m$$

$$\text{Link 4 (} B \text{ to } O_4) \quad c := 1.86 \cdot m \quad \text{Link 1 (} O_2 \text{ to } O_4) \quad d := 2.22 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 1.33 \cdot m \quad \delta_3 := 0 \cdot \text{deg} \quad F_P := 300 \cdot N \quad T_4 := 0 \cdot N \cdot m$$

$$\text{Crank motion:} \quad \omega_2 := 10 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

$$\text{Link cross-section dims:} \quad d_{\text{link}} := 50 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \gamma_s := 0.3 \cdot \text{lb} \cdot \text{in}^{-3}$$

$$\text{Coefficient of fluctuation:} \quad k := 0.06 \quad \text{Desired average speed:} \quad \omega_{\text{avg}} := \omega_2$$

Solution: See Figure P11-5b and Mathcad file P1144.

- Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.430 \, m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.930 \, m$$

$$\text{Link 3:} \quad R_{CG3} := 0.5 \cdot b \quad R_{CG3} = 0.925 \, m \quad \delta_{33} := 0 \cdot \text{deg}$$

- Determine the mass and moment of inertia of each link.

$$m_2 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot b \cdot \frac{\gamma_s}{g} \quad m_4 := \frac{\pi \cdot d_{\text{link}}^2}{4} \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 14.022 \, \text{kg} \quad m_3 = 30.164 \, \text{kg} \quad m_4 = 30.327 \, \text{kg}$$

$$I_{G2} := \frac{m_2}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + a^2 \right) \quad I_{G2} = 0.866 \, \text{kg} \cdot \text{m}^2$$

$$I_{G3} := \frac{m_3}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + b^2 \right) \quad I_{G3} = 8.608 \, \text{kg} \cdot \text{m}^2$$

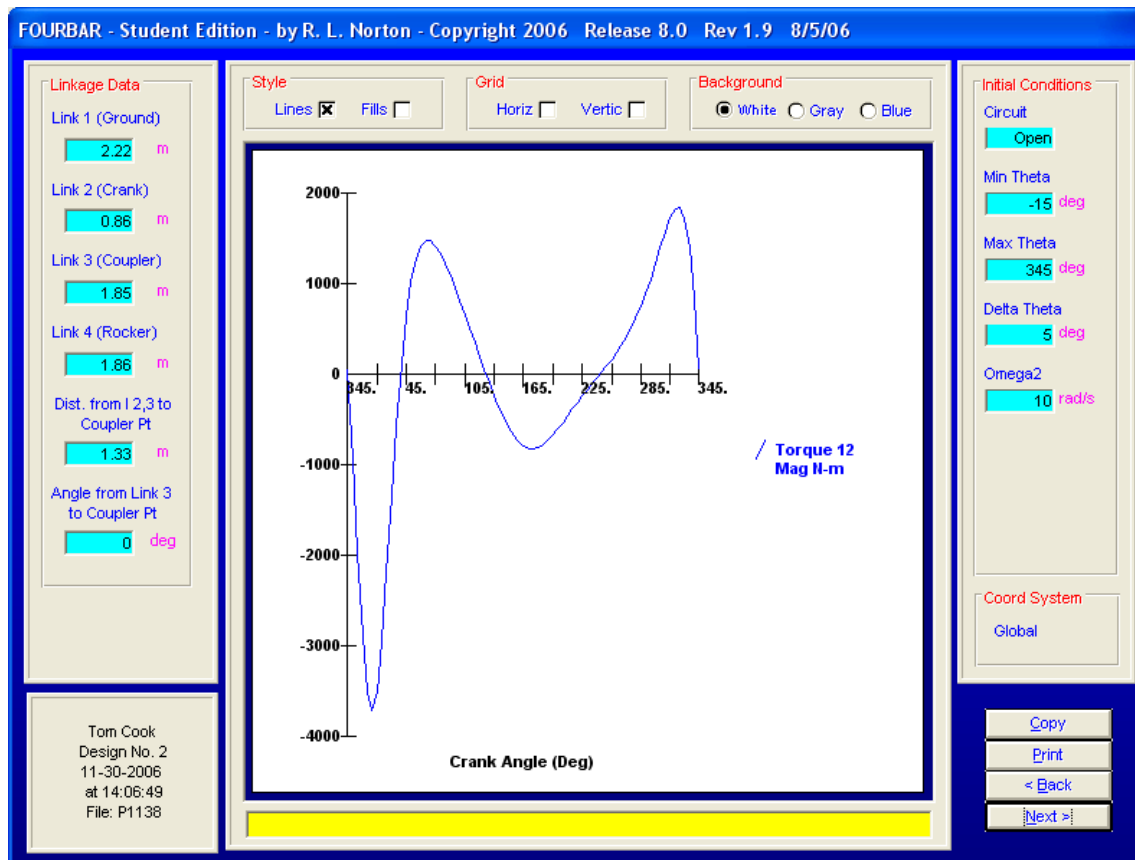
$$I_{G4} := \frac{m_4}{12} \cdot \left(\frac{3}{4} \cdot d_{\text{link}}^2 + c^2 \right) \quad I_{G4} = 8.748 \, \text{kg} \cdot \text{m}^2$$

- Calculate the x and y components of the external force at P in the CGS and the magnitude and angle of the position vector defining point P in the LRCS.

$$F_{Px} := F_P \quad F_{Py} := 0 \cdot N$$

$$R_{pax'} := R_{pa} \cdot \cos(\delta_3) \quad R_{px'} := R_{pax'} - R_{CG3} \quad R_{pax'} = 1.330 \, m \quad R_{px'} = 0.405 \, m$$

- Enter the above data into program FOURBAR and determine the energy change from minimum to maximum speed by calculating the dynamic values and then plotting the input torque vs. crank angle. See screens below.

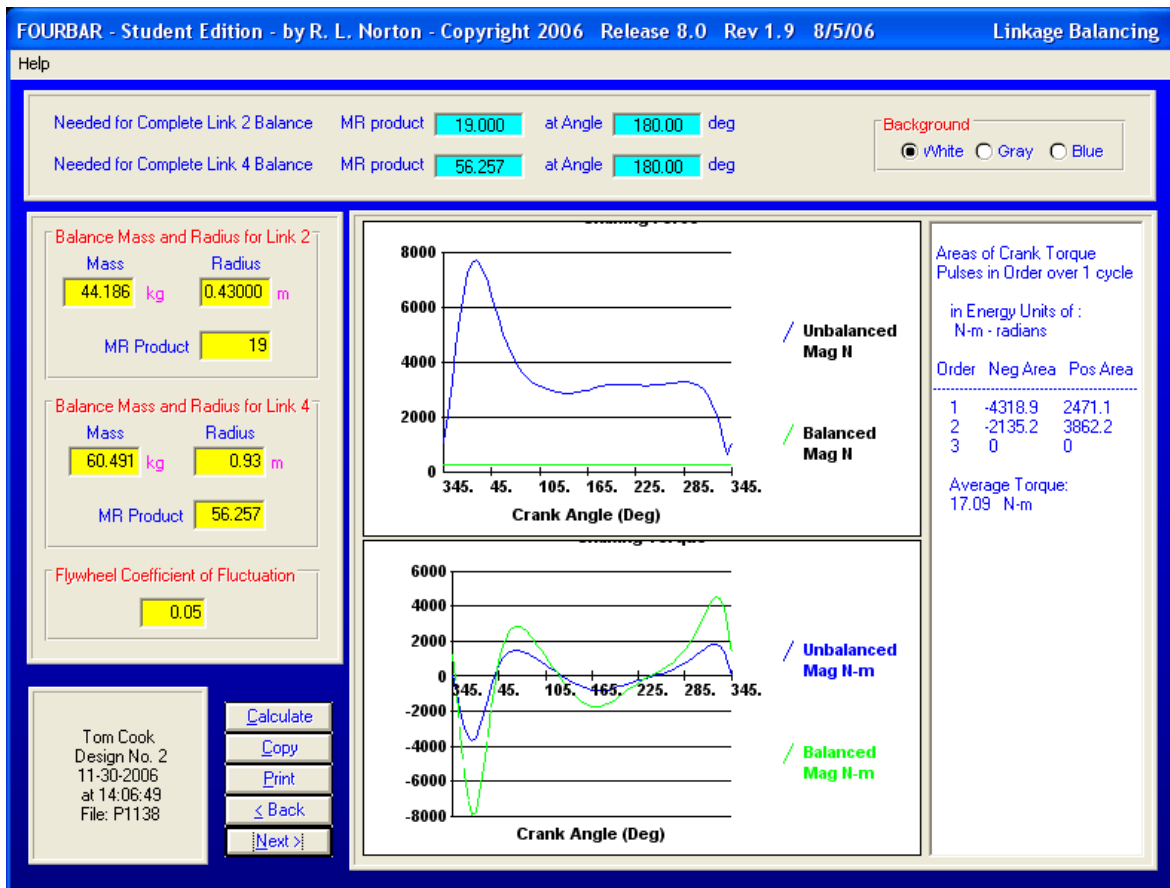


5. Integrate the torque function using the values in the "Areas of Crank Torque Pulses" shown in the Linkage Balancing screen below as related to the Torque vs. Crank Angle screen above (see Table 11-1 in text).

A to B	-4318.9	-4318.9 $\omega_{\max}$ at B
B to C	+2471.1	-1847.8
C to D	-2135.2	-3983.0
D to A	+3862.2	-120.8 $\omega_{\min}$ at A

$$E_{\max\text{speed}} := -4318.9 \cdot N \cdot m \quad E_{\min\text{speed}} := -120.8 \cdot N \cdot m$$

$$E := |E_{\max\text{speed}} - E_{\min\text{speed}}| \quad E = 4198.1 \, N \cdot m$$



6. Use equation 11.22 to determine the required system mass moment of inertia.

$$I_s := \frac{E}{k \cdot \omega_{avg}^2} \quad I_s = 699.7 \text{ kg} \cdot \text{m}^2$$

The moment of inertia of the input crank and the motor armature can be subtracted from this value to obtain the required flywheel moment. There are an infinity of possible size/shape solutions for this problem.

PROBLEM 11-45a

Statement: Table P11-5 gives kinematic and geometric data for several slider-crank linkages of the type and orientation shown in Figure 11-4 (p. 597). For the row(s) in the table assigned, solve for all pin forces and the resisting torque on the crank for the position shown.

Units: $\text{blob} := \text{lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 4.00 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 12.00 \cdot \text{in}$$

$$\text{Offset} \quad c := 0.00 \cdot \text{in} \quad \text{Friction:} \quad \mu := 0.15$$

$$\text{Slider position and motion:} \quad d := 14 \cdot \text{in} \quad \ddot{d} := 400 \cdot \text{in} \cdot \text{sec}^{-1} \quad \dddot{d} := -22760 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\text{Mass:} \quad m_2 := 0.002 \cdot \text{blob} \quad m_3 := 0.020 \cdot \text{blob} \quad m_4 := 0.060 \cdot \text{blob}$$

$$\text{Moment of inertia:} \quad I_{G2} := 0.10 \cdot \text{blob} \cdot \text{in}^2 \quad I_{G3} := 0.20 \cdot \text{blob} \cdot \text{in}^2$$

$$\text{Mass center:} \quad R_{CG2} := 1.30 \cdot \text{in} \quad \delta_2 := 180 \cdot \text{deg} \quad R_{CG3} := 3.00 \cdot \text{in} \quad \delta_3 := 0 \cdot \text{deg}$$

$$\text{Force:} \quad F_P := 60 \cdot \text{lbf} \quad \delta_{FP} := 180 \cdot \text{deg}$$

Solution: See Figure 11-4, Table P11-5, and Mathcad file P1145a.

1. Calculate the position of links 2 and 3 using equations 4.21, and 4.16 or 4.17.

$$K_1 := a^2 - b^2 + c^2 + d^2 \quad K_1 = 68 \text{ in}^2$$

$$K_2 := -2 \cdot a \cdot c \quad K_2 = 0$$

$$K_3 := -2 \cdot a \cdot d \quad K_3 = -112 \text{ in}^2$$

$$A := K_1 - K_3 \quad A = 180 \text{ in}^2$$

$$B := 2 \cdot K_2 \quad B = 0$$

$$C := K_1 + K_3 \quad C = -44 \text{ in}^2$$

$$\theta_2 := 2 \cdot \text{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right) \quad \theta_2 = 52.617 \cdot \text{deg}$$

$$\beta(a, b, c, d, \alpha) := \begin{cases} \theta \leftarrow \text{asin}\left(\frac{a \cdot \sin(\alpha) - c}{b}\right) \\ d_1 \leftarrow a \cdot \cos(\alpha) - b \cdot \cos(\theta) \\ \text{return } \theta \text{ if } d_1 = d \\ \text{asin}\left(-\frac{a \cdot \sin(\alpha) - c}{b}\right) + \pi \text{ otherwise} \end{cases}$$

$$\theta_3 := \beta(a, b, c, d, \theta_2) \quad \theta_3 = 164.641 \cdot \text{deg}$$

2. Calculate the x and y components of the position vectors.

$$R_{12x} := R_{CG2} \cdot \cos(\theta_2 + 180 \cdot \text{deg}) \quad R_{12x} = -0.789 \cdot \text{in}$$

$$R_{12y} := R_{CG2} \cdot \sin(\theta_2 + 180 \cdot \text{deg}) \quad R_{12y} = -1.033 \cdot \text{in}$$

$$R_{32x} := (a - R_{CG2}) \cdot \cos(\theta_2) \quad R_{32x} = 1.639 \cdot \text{in}$$

$$\begin{aligned}
 R_{32y} &:= (a - R_{CG2}) \cdot \sin(\theta_2) & R_{32y} &= 2.145 \cdot \text{in} \\
 R_{23x} &:= R_{CG3} \cdot \cos(\theta_3) & R_{23x} &= -2.893 \cdot \text{in} \\
 R_{23y} &:= R_{CG3} \cdot \sin(\theta_3) & R_{23y} &= 0.795 \cdot \text{in} \\
 R_{43x} &:= (b - R_{CG3}) \cdot \cos(\theta_3 + 180 \cdot \text{deg}) & R_{43x} &= 8.679 \cdot \text{in} \\
 R_{43y} &:= (b - R_{CG3}) \cdot \sin(\theta_3 + 180 \cdot \text{deg}) & R_{43y} &= -2.384 \cdot \text{in}
 \end{aligned}$$

3. Calculate the angular velocity of links 2 and 3 using equations 6.24.

$$\begin{aligned}
 \omega_2 &:= \frac{\ddot{\theta}_2 \cdot \cos(\theta_3)}{a \cdot (\cos(\theta_2) \cdot \sin(\theta_3) - \sin(\theta_2) \cdot \cos(\theta_3))} & \omega_2 &= -104.019 \cdot \frac{\text{rad}}{\text{s}} \\
 \omega_3 &:= \frac{a \cdot \omega_2 \cdot \cos(\theta_2)}{b \cdot \cos(\theta_3)} & \omega_3 &= 21.831 \cdot \frac{\text{rad}}{\text{s}}
 \end{aligned}$$

4. Calculate the angular acceleration of links 2 and 3 using equations 7.17.

$$\begin{aligned}
 \alpha_2 &:= \frac{a \cdot \omega_2^2 \cdot (\cos(\theta_2) \cdot \cos(\theta_3) + \sin(\theta_2) \cdot \sin(\theta_3)) - b \cdot \omega_3^2 + \ddot{\theta}_2 \cdot \cos(\theta_3)}{a \cdot (\cos(\theta_2) \cdot \sin(\theta_3) - \sin(\theta_2) \cdot \cos(\theta_3))} & \alpha_2 &= -0.58 \cdot \frac{\text{rad}}{\text{s}^2} \\
 \alpha_3 &:= \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_3^2 \cdot \sin(\theta_3)}{b \cdot \cos(\theta_3)} & \alpha_3 &= 2841 \cdot \frac{\text{rad}}{\text{s}^2}
 \end{aligned}$$

5. Calculate the value of $\mathbf{a}_{G2}$ and $\mathbf{a}_{G3}$ using equations 7.3 and 7.32.

$$\begin{aligned}
 \mathbf{a}_{G2} &:= R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - R_{CG2} \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \\
 a_{G2} &:= |\mathbf{a}_{G2}| & a_{G2} &= 14066.05 \cdot \frac{\text{in}}{\text{sec}^2} \\
 \theta_{AG2} &:= \arg(\mathbf{a}_{G2}) & \theta_{AG2} &= -127.380 \cdot \text{deg} \\
 \mathbf{a}_A &:= a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \\
 \mathbf{a}_{G3} &:= R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3)) - R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_3) + j \cdot \sin(\theta_3)) \\
 a_{G3} &:= |\mathbf{a}_{G3}| & a_{G3} &= 8642.67 \cdot \frac{\text{in}}{\text{sec}^2} \\
 \theta_{AG3} &:= \arg(\mathbf{a}_{G3}) & \theta_{AG3} &= -95.836 \cdot \text{deg}
 \end{aligned}$$

6. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$\begin{aligned}
 a_{G2x} &:= a_{G2} \cdot \cos(\theta_{AG2}) & a_{G2x} &= -8540 \cdot \text{in} \cdot \text{sec}^{-2} \\
 a_{G2y} &:= a_{G2} \cdot \sin(\theta_{AG2}) & a_{G2y} &= -11177 \cdot \text{in} \cdot \text{sec}^{-2} \\
 a_{G3x} &:= a_{G3} \cdot \cos(\theta_{AG3}) & a_{G3x} &= -879 \cdot \text{in} \cdot \text{sec}^{-2}
 \end{aligned}$$

$$a_{G3y} := a_{G3} \cdot \sin(\theta_{AG3}) \quad a_{G3y} = -8598 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$a_{G4x} := dddot{\theta} \quad a_{G4x} = -22760 \cdot \text{in} \cdot \text{sec}^{-2}$$

7. Calculate the x and y components of the external force on link 4 in the CGS.

$$F_{Px} := F_P \cdot \cos(\delta_{FP}) \quad F_{Px} = -60.000 \cdot \text{lbf}$$

$$F_{Py} := F_P \cdot \sin(\delta_{FP}) \quad F_{Py} = 0.000 \cdot \text{lbf}$$

8. Substitute these given and calculated values into the matrix equation 11.10g, modified for this problem. Note that Mathcad requires that all elements in a matrix have the same dimension. Thus, the matrix and array in equation 11.10g will be made dimensionless and the dimensions will be put back in after solving it.

$$C := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{\text{in}} & \frac{R_{12x}}{\text{in}} & \frac{-R_{32y}}{\text{in}} & \frac{R_{32x}}{\text{in}} & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{\text{in}} & \frac{-R_{23x}}{\text{in}} & \frac{-R_{43y}}{\text{in}} & \frac{R_{43x}}{\text{in}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{pmatrix}$$

$$F := \begin{bmatrix} m_2 \cdot a_{G2x} \cdot \text{lbf}^{-1} \\ m_2 \cdot a_{G2y} \cdot \text{lbf}^{-1} \\ I_{G2} \cdot \alpha_2 \cdot \text{lbf}^{-1} \cdot \text{in}^{-1} \\ (m_3 \cdot a_{G3x}) \cdot \text{lbf}^{-1} \\ (m_3 \cdot a_{G3y}) \cdot \text{lbf}^{-1} \\ (I_{G3} \cdot \alpha_3) \cdot \text{lbf}^{-1} \cdot \text{in}^{-1} \\ (m_4 \cdot a_{G4x} - F_{Px}) \cdot \text{lbf}^{-1} \\ m_4 \cdot g \cdot \text{lbf}^{-1} \end{bmatrix}$$

$$R := C^{-1} \cdot F$$

$$F_{12x} := R_1 \cdot \text{lbf} \quad F_{12x} = -1392 \cdot \text{lbf}$$

$$F_{12y} := R_2 \cdot \text{lbf} \quad F_{12y} = 173.6 \cdot \text{lbf}$$

$$F_{32x} := R_3 \cdot \text{lbf} \quad F_{32x} = 1375 \cdot \text{lbf}$$

$$F_{32y} := R_4 \cdot \text{lbf} \quad F_{32y} = -195.9 \cdot \text{lbf}$$

$$F_{43x} := R_5 \cdot \text{lbf} \quad F_{43x} = 1357 \cdot \text{lbf}$$

$$F_{43y} := R_6 \cdot \text{lbf} \quad F_{43y} = -367.9 \cdot \text{lbf}$$

$$F_{14y} := R_7 \cdot lbf \quad F_{14y} = -344.7 \cdot lbf$$

$$T_{12} := R_8 \cdot lbf \cdot in \quad T_{12} = 4846 \cdot lbf \cdot in$$

$$\mathbf{F}_{12} := F_{12x} + j \cdot F_{12y}$$

$$F_{12} := |\mathbf{F}_{12}|$$

$$F_{12} = 1403 \cdot lbf$$

$$\theta_{F12} := \arg(\mathbf{F}_{12})$$

$$\theta_{F12} = 172.891 \cdot deg$$

$$\mathbf{F}_{32} := F_{32x} + j \cdot F_{32y}$$

$$F_{32} := |\mathbf{F}_{32}|$$

$$F_{32} = 1389 \cdot lbf$$

$$\theta_{F32} := \arg(\mathbf{F}_{32})$$

$$\theta_{F32} = -8.111 \cdot deg$$

$$\mathbf{F}_{43} := F_{43x} + j \cdot F_{43y}$$

$$F_{43} := |\mathbf{F}_{43}|$$

$$F_{43} = 1406 \cdot lbf$$

$$\theta_{F43} := \arg(\mathbf{F}_{43})$$

$$\theta_{F43} = -15.166 \cdot deg$$

PROBLEM 11-46a

Statement: Table P11-5 gives kinematic and geometric data for several slider-crank linkages of the type and orientation shown in Figure 11-4 (p. 597). For the row(s) in the table assigned, solve for the torque available at the crank using the method of virtual work for the position shown assuming no friction loss.

Units: $\text{blob} := \text{lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a := 4.00 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad b := 12.00 \cdot \text{in}$$

$$\text{Offset} \quad c := 0.00 \cdot \text{in}$$

$$\text{Slider position and motion:} \quad d := 14 \cdot \text{in} \quad \ddot{d} := 400 \cdot \text{in} \cdot \text{sec}^{-1} \quad \dddot{d} := -22760 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$\text{Mass:} \quad m_2 := 0.002 \cdot \text{blob} \quad m_3 := 0.020 \cdot \text{blob} \quad m_4 := 0.060 \cdot \text{blob}$$

$$\text{Moment of inertia:} \quad I_{G2} := 0.10 \cdot \text{blob} \cdot \text{in}^2 \quad I_{G3} := 0.20 \cdot \text{blob} \cdot \text{in}^2$$

$$\text{Mass center:} \quad R_{CG2} := 1.30 \cdot \text{in} \quad \delta_2 := 0 \cdot \text{deg} \quad R_{CG3} := 3.00 \cdot \text{in} \quad \delta_3 := 0 \cdot \text{deg}$$

$$\text{Force:} \quad F_P := 60 \cdot \text{lbf} \quad \delta_{FP} := 180 \cdot \text{deg}$$

Solution: See Figure 11-4, Table P11-5, and Mathcad file P1146a.

1. Calculate the position of links 2 and 3 using equations 4.21, and 4.16 or 4.17.

$$K_1 := a^2 - b^2 + c^2 + d^2 \quad K_1 = 68 \text{ in}^2$$

$$K_2 := -2 \cdot a \cdot c \quad K_2 = 0$$

$$K_3 := -2 \cdot a \cdot d \quad K_3 = -112 \text{ in}^2$$

$$A := K_1 - K_3 \quad A = 180 \text{ in}^2$$

$$B := 2 \cdot K_2 \quad B = 0$$

$$C := K_1 + K_3 \quad C = -44 \text{ in}^2$$

$$\theta_2 := 2 \cdot \text{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right) \quad \theta_2 = 52.617 \cdot \text{deg}$$

$$\beta(a, b, c, d, \alpha) := \begin{cases} \theta \leftarrow \text{asin}\left(\frac{a \cdot \sin(\alpha) - c}{b}\right) \\ d_1 \leftarrow a \cdot \cos(\alpha) - b \cdot \cos(\theta) \\ \text{return } \theta \text{ if } d_1 = d \\ \text{asin}\left(-\frac{a \cdot \sin(\alpha) - c}{b}\right) + \pi \text{ otherwise} \end{cases}$$

$$\theta_3 := \beta(a, b, c, d, \theta_2) \quad \theta_3 = 164.641 \cdot \text{deg}$$

2. Calculate the angular velocity of links 2 and 3 using equations 6.24.

$$\omega_2 := \frac{\ddot{d} \cdot \cos(\theta_3)}{a \cdot (\cos(\theta_2) \cdot \sin(\theta_3) - \sin(\theta_2) \cdot \cos(\theta_3))} \quad \omega_2 = -104.019 \cdot \frac{\text{rad}}{\text{s}}$$

$$\omega_3 := \frac{a \cdot \omega_2 \cdot \cos(\theta_2)}{b \cdot \cos(\theta_3)} \quad \omega_3 = 21.831 \cdot \frac{\text{rad}}{\text{s}}$$

3. Calculate the value of $\mathbf{V}_{G2}$ and $\mathbf{V}_{G3}$ using equations 6.23 and 6.36.

$$\begin{aligned} \mathbf{V}_{G2} &:= R_{CG2} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) \\ v_{G2} &:= |\mathbf{V}_{G2}| \quad v_{G2} = 135.225 \frac{\text{in}}{\text{s}} \\ \theta_{VG2} &:= \arg(\mathbf{V}_{G2}) \quad \theta_{VG2} = -37.383 \cdot \text{deg} \\ \mathbf{V}_A &:= a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) \\ \mathbf{V}_{G3} &:= R_{CG3} \cdot \omega_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3)) - \mathbf{V}_A \\ v_{G3} &:= |\mathbf{V}_{G3}| \quad v_{G3} = 396.197 \frac{\text{in}}{\text{s}} \\ \theta_{VG3} &:= \arg(\mathbf{V}_{G3}) - 180 \cdot \text{deg} \quad \theta_{VG3} = -28.568 \cdot \text{deg} \end{aligned}$$

4. Calculate the x and y components of the velocity vectors.

$$\begin{aligned} v_{G2x} &:= v_{G2} \cdot \cos(\theta_{VG2}) \quad v_{G2x} = 107.449 \cdot \text{in} \cdot \text{sec}^{-1} \\ v_{G2y} &:= v_{G2} \cdot \sin(\theta_{VG2}) \quad v_{G2y} = -82.101 \cdot \text{in} \cdot \text{sec}^{-1} \\ v_{G3x} &:= v_{G3} \cdot \cos(\theta_{VG3}) \quad v_{G3x} = 347.959 \cdot \text{in} \cdot \text{sec}^{-1} \\ v_{G3y} &:= v_{G3} \cdot \sin(\theta_{VG3}) \quad v_{G3y} = -189.464 \cdot \text{in} \cdot \text{sec}^{-1} \\ v_{G4x} &:= \text{ddot} \quad v_{G4x} = 400.000 \cdot \text{in} \cdot \text{sec}^{-1} \\ v_{G4y} &:= 0.0 \cdot \text{in} \cdot \text{sec}^{-1} \end{aligned}$$

5. Calculate the angular acceleration of links 2 and 3 using equations 7.17.

$$\begin{aligned} \alpha_2 &:= \frac{a \cdot \omega_2^2 \cdot (\cos(\theta_2) \cdot \cos(\theta_3) + \sin(\theta_2) \cdot \sin(\theta_3)) - b \cdot \omega_3^2 + \text{dddot} \cdot \cos(\theta_3)}{a \cdot (\cos(\theta_2) \cdot \sin(\theta_3) - \sin(\theta_2) \cdot \cos(\theta_3))} \quad \alpha_2 = -0.58 \cdot \frac{\text{rad}}{\text{s}^2} \\ \alpha_3 &:= \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) + b \cdot \omega_3^2 \cdot \sin(\theta_3)}{b \cdot \cos(\theta_3)} \quad \alpha_3 = 2841 \cdot \frac{\text{rad}}{\text{s}^2} \end{aligned}$$

6. Calculate the value of $\mathbf{a}_{G2}$ and $\mathbf{a}_{G3}$ using equations 7.3 and 7.32.

$$\begin{aligned} \mathbf{a}_{G2} &:= R_{CG2} \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - R_{CG2} \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \\ a_{G2} &:= |\mathbf{a}_{G2}| \quad a_{G2} = 14066.05 \cdot \frac{\text{in}}{\text{sec}^2} \\ \theta_{AG2} &:= \arg(\mathbf{a}_{G2}) \quad \theta_{AG2} = -127.380 \cdot \text{deg} \\ \mathbf{a}_A &:= a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \\ \mathbf{a}_{G3} &:= R_{CG3} \cdot \alpha_3 \cdot (-\sin(\theta_3) + j \cdot \cos(\theta_3)) - R_{CG3} \cdot \omega_3^2 \cdot (\cos(\theta_3) + j \cdot \sin(\theta_3)) \end{aligned}$$

$$a_{G3} := |\mathbf{a}_{G3}| \quad a_{G3} = 8642.67 \cdot \frac{\text{in}}{\text{sec}^2}$$

$$\theta_{AG3} := \arg(\mathbf{a}_{G3}) \quad \theta_{AG3} = -95.836 \cdot \text{deg}$$

7. Calculate the x and y components of the acceleration of the CGs of all moving links in the global coordinate system (GCS).

$$a_{G2x} := a_{G2} \cdot \cos(\theta_{AG2}) \quad a_{G2x} = -8540 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$a_{G2y} := a_{G2} \cdot \sin(\theta_{AG2}) \quad a_{G2y} = -11177 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$a_{G3x} := a_{G3} \cdot \cos(\theta_{AG3}) \quad a_{G3x} = -879 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$a_{G3y} := a_{G3} \cdot \sin(\theta_{AG3}) \quad a_{G3y} = -8598 \cdot \text{in} \cdot \text{sec}^{-2}$$

$$a_{G4x} := \text{ddd} \quad a_{G4x} = -22760 \cdot \text{in} \cdot \text{sec}^{-2}$$

8. Calculate the x and y components of the external force on link 4 in the CGS.

$$F_{Px} := F_P \cdot \cos(\delta_{FP}) \quad F_{Px} = -60.000 \cdot \text{lbf}$$

$$F_{Py} := F_P \cdot \sin(\delta_{FP}) \quad F_{Py} = 0.000 \cdot \text{lbf}$$

9. Substitute these given and calculated values into equation 11.16c and solve for the input torque.

$$T_{12} := \frac{1}{\omega_2} \cdot \left[m_2 \cdot (a_{G2x} \cdot v_{G2x} + a_{G2y} \cdot v_{G2y}) + m_3 \cdot (a_{G3x} \cdot v_{G3x} + a_{G3y} \cdot v_{G3y}) \dots \right]$$

$$\left[+ m_4 \cdot (a_{G4x} \cdot v_{G4x}) + (I_{G2} \cdot \alpha_2 \cdot \omega_2 + I_{G3} \cdot \alpha_3 \cdot \omega_3) \dots \right]$$

$$\left[+ -(F_{Px} \cdot v_{G4x} + F_{Py} \cdot v_{G4y}) \right]$$

$$T_{12} = 4647 \cdot \text{lbf} \cdot \text{in}$$

PROBLEM 12-1a

Statement: A system of two coplanar arms on a common shaft, as shown in Figure 12-1, is to be designed. For row *a* in Table P12-1, find the shaking force of the linkage when run unbalanced and design a counterweight to statically balance the system. Work in a any consistent unit system you prefer.

Given: Masses and radii:

$$\begin{aligned} m_1 &:= 0.20 \cdot \text{kg} & r_1 &:= 1.25 \cdot \text{m} & \theta_1 &:= 30 \cdot \text{deg} \\ m_2 &:= 0.40 \cdot \text{kg} & r_2 &:= 2.25 \cdot \text{m} & \theta_2 &:= 120 \cdot \text{deg} \end{aligned}$$

Solution: See Figure 12-1, Table P12-1, and Mathcad file P1201a.

1. Resolve the position vectors into *xy* components in the arbitrary coordinate system associated with the freeze- frame position of the linkage chosen for analysis.

$$\begin{aligned} R_{1x} &:= r_1 \cdot \cos(\theta_1) & R_{1x} &= 1.083 \text{ m} & R_{1y} &:= r_1 \cdot \sin(\theta_1) & R_{1y} &= 0.625 \text{ m} \\ R_{2x} &:= r_2 \cdot \cos(\theta_2) & R_{2x} &= -1.125 \text{ m} & R_{2y} &:= r_2 \cdot \sin(\theta_2) & R_{2y} &= 1.949 \text{ m} \end{aligned}$$

2. Solve equation 12.2c for the mass-radius product components.

$$\begin{aligned} mR_{bx} &:= -m_1 \cdot R_{1x} - m_2 \cdot R_{2x} & mR_{bx} &= 0.233 \text{ kg} \cdot \text{m} \\ mR_{by} &:= -m_1 \cdot R_{1y} - m_2 \cdot R_{2y} & mR_{by} &= -0.904 \text{ kg} \cdot \text{m} \end{aligned}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required.

$$\begin{aligned} \theta_b &:= \text{atan2}(mR_{bx}, mR_{by}) & \theta_b &= -75.524 \text{ deg} \\ mR_b &:= \sqrt{mR_{bx}^2 + mR_{by}^2} & mR_b &= 0.934 \text{ kg} \cdot \text{m} \end{aligned}$$

PROBLEM 12-2

Statement: The minute hand on Big Ben weighs 40 lb and is 10 ft long. Its CG is 4 ft from the pivot. Calculate the mR product and angular location needed to statically balance this link and design a physical counterweight, positioned close to the center. Select material and design the detailed shape of the counterweight, which is of 2 in uniform thickness in the Z direction.

Given: Weight and radius data:

$$W_1 := 40 \cdot \text{lbf} \quad r_1 := 4 \cdot \text{ft}$$

$$\text{Counterweight thickness: } t := 2 \cdot \text{in}$$

Design choices:

$$\text{Counterweight material is steel: } \gamma := 0.28 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Counter weight shape is cylindrical

Solution: See Mathcad file P1202.

1. Since there is only one mass to balance, the balancing mass will be on the same centerline as the minute hand, on the side opposite the center of rotation of the hand.
2. Solve equation 12.2c for the required mass-radius for balancing.

$$mR_b := \frac{W_1}{g} \cdot r_1 \quad mR_b = 4.973 \text{ slug} \cdot \text{ft}$$

3. The mass of a solid cylinder is $m_b := \pi \cdot r_c^2 \cdot t \cdot \frac{\gamma}{g}$
4. Combine the equations in steps 2 and 3 and solve for the cylinder radius as a function of the balance radius. Plot the result over a suitable range of R_b to get an idea of the sizes involved.

$$r_c(R_b) := \sqrt{\frac{mR_b \cdot g}{\pi \cdot t \cdot R_b \cdot \gamma}} \quad R_{bal} := 5 \cdot \text{in}, 6 \cdot \text{in} \dots 20 \cdot \text{in}$$

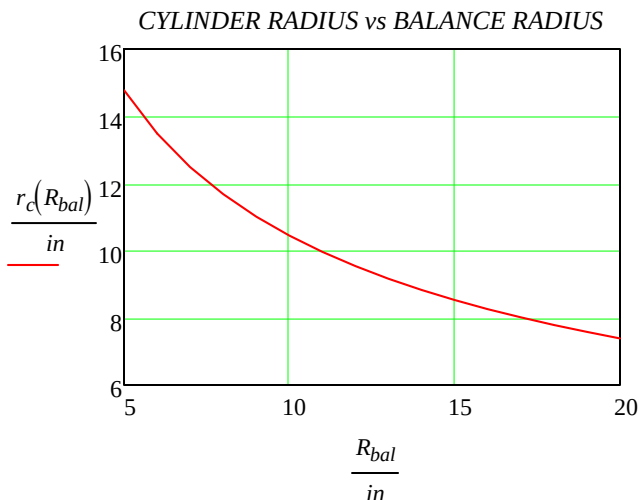
5. Choose a balance radius of $R_b := 15 \cdot \text{in}$. Then the required mass is

$$m_b := \frac{mR_b}{R_b} \quad m_b = 3.978 \text{ slug}$$

$$W_b := m_b \cdot g \quad W_b = 128.000 \text{ lbf}$$

6. Calculate the required cylinder diameter.

$$d_c := 2 \cdot \sqrt{\frac{mR_b \cdot g}{\pi \cdot t \cdot R_b \cdot \gamma}} \quad d_c = 17.059 \text{ in}$$



PROBLEM 12-3

Statement: A "V for victory" advertising sign is being designed to be oscillated about the apex of the V, on the billboard, as the rocker of a fourbar linkage. The angle between the legs of the V is 20 deg. Each leg is 8 ft long and 1.5 ft wide. Material is 0.25-in thick aluminum. Design the V-link for static balance.

Given: V dimensions:

$$\text{Length: } l_v := 8 \cdot \text{ft} \quad \text{Width: } w_v := 1.5 \cdot \text{ft} \quad \text{Thickness: } t_v := 0.25 \cdot \text{in}$$

$$\text{Specific weight of aluminum: } \gamma_a := 0.1 \cdot \text{lb} \cdot \text{ft} \cdot \text{in}^{-3}$$

Design choices:

$$\text{Counterweight material is steel: } \gamma_s := 0.28 \cdot \text{lb} \cdot \text{ft} \cdot \text{in}^{-3}$$

$$\text{Counter weight shape is cylindrical disk with thickness } t_c := 3 \cdot \text{in}$$

Solution: See Mathcad file P1203.

1. Since the V is symmetrical in geometry and mass, the counterweight will be located on the centerline of the V opposite the pivot point from the V.
2. Assume that the centerlines of each leg of the V intersect at the pivot point. Then, the mass, distance to the CG, and angle with respect to the V centerline of each leg is:

$$\text{Mass of one leg} \quad m_{leg} := l_v \cdot w_v \cdot t_v \cdot \frac{\gamma_a}{g} \quad m_{leg} = 43.200 \text{ lb}$$

$$m_1 := m_{leg} \quad r_1 := 4 \cdot \text{ft} \quad \theta_1 := 10 \cdot \text{deg}$$

$$m_2 := m_{leg} \quad r_2 := 4 \cdot \text{ft} \quad \theta_2 := -10 \cdot \text{deg}$$

3. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze- frame position of the linkage chosen for analysis.

$$R_{1x} := r_1 \cdot \cos(\theta_1) \quad R_{1x} = 3.939 \text{ ft} \quad R_{1y} := r_1 \cdot \sin(\theta_1) \quad R_{1y} = 0.695 \text{ ft}$$

$$R_{2x} := r_2 \cdot \cos(\theta_2) \quad R_{2x} = 3.939 \text{ ft} \quad R_{2y} := r_2 \cdot \sin(\theta_2) \quad R_{2y} = -0.695 \text{ ft}$$

4. Solve equation 12.2c for the mass-radius product.

$$mR_{bx} := -m_1 \cdot R_{1x} - m_2 \cdot R_{2x} \quad mR_{bx} = -4.084 \times 10^3 \text{ in} \cdot \text{lb}$$

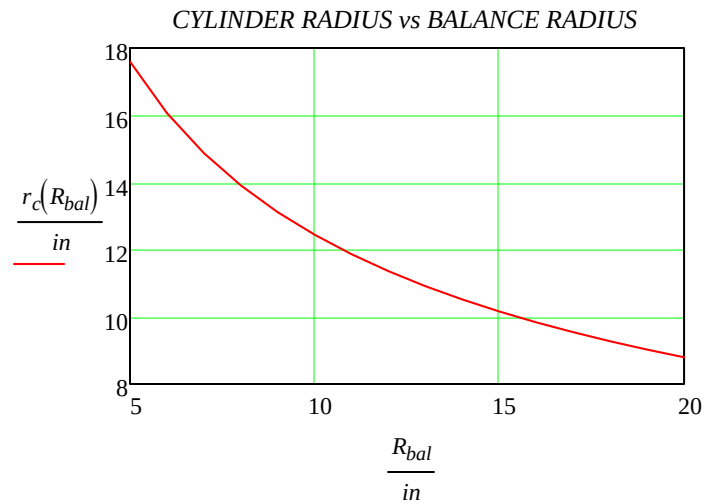
$$mR_{by} := -m_1 \cdot R_{1y} - m_2 \cdot R_{2y} \quad mR_{by} = 0.000 \text{ in} \cdot \text{lb}$$

$$mR_b := |mR_{bx}| \quad mR_b = 4084 \text{ in} \cdot \text{lb}$$

5. The mass of a solid cylinder is $m_b := \pi \cdot r_c^2 \cdot t \cdot \frac{\gamma}{g}$

6. Combine the equations in steps 4 and 5 and solve for the cylinder radius as a function of the balance radius. Plot the result over a suitable range of R_b to get an idea of the sizes involved.

$$r_c(R_b) := \sqrt{\frac{mR_b \cdot g}{\pi \cdot t_c \cdot R_b \cdot \gamma_s}} \quad R_{bal} := 5 \cdot \text{in}, 6 \cdot \text{in} \dots 20 \cdot \text{in}$$



7. Choose a balance radius of $R_b := 18 \cdot in$. Then the required mass is

$$m_b := \frac{mR_b}{R_b} \quad m_b = 226.900 \text{ lb}$$

8. Calculate the required cylinder diameter.

$$d_c := 2 \cdot \sqrt{\frac{mR_b \cdot g}{\pi \cdot t_c \cdot R_b \cdot \gamma_s}} \quad d_c = 18.5 \text{ in}$$

PROBLEM 12-4

Statement: A three-bladed ceiling fan has 1.5 ft by 0.25 ft equispaced rectangular blades that normally weigh 2 lb each. Manufacturing tolerances will cause the blade weight to vary up to plus or minus 5%. The mounting accuracy of the blades will vary the location of the CG versus the spin axis by plus or minus 10% of the blades' diameters. Calculate the weight of the largest steel counterweight needed at a 2-in radius to statically balance the worst-case blade assembly.

Given: Blade dimensions:

$$\text{Length: } l_b := 1.5 \cdot \text{ft} \quad \text{Width: } w_b := 0.25 \cdot \text{ft} \quad \text{Nominal weight: } W_{bnom} := 2 \cdot \text{lbf}$$

$$\text{Manufacturing tolerances: } \text{Weight } t_w := 0.05 \quad \text{CG offset } t_{CG} := 0.10$$

Assumptions: 1. The blades are held in place by a bracket such that their base is 6 in from the center of rotation making the tip 24 in from the center. Thus, the blade sweep diameter is 48 in.
2. There is one heavy (maximum weight) blade at 0 deg and two light (minimum weight) ones at 120 and 240 deg, respectively. Thus,

$$W_1 := (1 + t_w) \cdot W_{bnom} \quad W_1 = 2.100 \text{ lbf} \quad r_1 := 15 \cdot \text{in} \quad \theta_1 := 0 \cdot \text{deg}$$

$$W_2 := (1 - t_w) \cdot W_{bnom} \quad W_2 = 1.900 \text{ lbf} \quad r_2 := 15 \cdot \text{in} \quad \theta_2 := 120 \cdot \text{deg}$$

$$W_3 := (1 - t_w) \cdot W_{bnom} \quad W_3 = 1.900 \text{ lbf} \quad r_3 := 15 \cdot \text{in} \quad \theta_3 := 240 \cdot \text{deg}$$

Solution: See Mathcad file P1204.

- There are two factors to be taken into account, the variation in blade weight and the error or eccentricity in the location of the global CG. The variation in blade weight about its spin axis will be considered first.
- Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze- frame position of the linkage chosen for analysis.

$$R_{1x} := r_1 \cdot \cos(\theta_1) \quad R_{1x} = 15.000 \text{ in} \quad R_{1y} := r_1 \cdot \sin(\theta_1) \quad R_{1y} = 0.000 \text{ in}$$

$$R_{2x} := r_2 \cdot \cos(\theta_2) \quad R_{2x} = -7.500 \text{ in} \quad R_{2y} := r_2 \cdot \sin(\theta_2) \quad R_{2y} = 12.990 \text{ in}$$

$$R_{3x} := r_3 \cdot \cos(\theta_3) \quad R_{3x} = -7.500 \text{ in} \quad R_{3y} := r_3 \cdot \sin(\theta_3) \quad R_{3y} = -12.990 \text{ in}$$

- Solve equation 12.2c for the mass-radius product components.

$$mR_{bx} := \frac{-W_1 \cdot R_{1x} - W_2 \cdot R_{2x} - W_3 \cdot R_{3x}}{g} \quad mR_{bx} = -3.000 \text{ in} \cdot \text{lb}$$

$$mR_{by} := \frac{-W_1 \cdot R_{1y} - W_2 \cdot R_{2y} - W_3 \cdot R_{3y}}{g} \quad mR_{by} = 0.000 \text{ in} \cdot \text{lb}$$

- Solve equations 12.2d and 12.2e for the position angle and mass-radius product required.

$$\theta_b := \text{atan2}(mR_{bx}, mR_{by}) \quad \theta_b = -180.000 \text{ deg}$$

$$mR_{bb} := \sqrt{mR_{bx}^2 + mR_{by}^2} \quad mR_{bb} = 3.000 \text{ in} \cdot \text{lb}$$

- Now, account for the fact that the blades' spin axis can be eccentric from their CG.

$$\text{Maximum eccentricity: } r_e := 48 \cdot \text{in} \cdot t_{CG} \quad r_e = 4.800 \text{ in}$$

mR product due to eccentricity:

$$mR_{be} := \frac{W_1 + W_2 + W_3}{g} \cdot r_e \quad mR_{be} = 28.320 \text{ in} \cdot \text{lb}$$

6. Add the two mR products and divide by the 2-in radius specified for the counterweight to find the maximum weight required.

$$R_{cw} := 2.0 \cdot \text{in}$$

$$mR_b := mR_{bb} + mR_{be}$$

$$mR_b = 31.320 \text{ in} \cdot \text{lb}$$

$$m_{cw} := \frac{mR_b}{R_{cw}}$$

$$m_{cw} = 15.66 \text{ lb}$$

Note that 90% of the counterweight is required to balance the eccentricity. The manufacturer would be well advised to try to control this variation more tightly.

PROBLEM 12-5a

Statement: A system of three non coplanar weights is arranged on a shaft generally as shown in Figure 12-3. For row *a* in Table P12-2, find the shaking forces and shaking moment when run unbalanced at 100 rpm and specify the *mR* product and angle of the counterweights in planes A and B needed to dynamically balance the system. The correction planes are 20 units apart. Work in a any consistent unit system you prefer.

Given: Masses and radii:

$$\begin{array}{llll} m_1 := 0.20 \cdot \text{kg} & r_1 := 1.25 \cdot \text{m} & \theta_1 := 30 \cdot \text{deg} & l_1 := 2 \cdot \text{m} \\ m_2 := 0.40 \cdot \text{kg} & r_2 := 2.25 \cdot \text{m} & \theta_2 := 120 \cdot \text{deg} & l_2 := 8 \cdot \text{m} \\ m_3 := 1.24 \cdot \text{kg} & r_3 := 5.50 \cdot \text{m} & \theta_3 := -30 \cdot \text{deg} & l_3 := 17 \cdot \text{m} \end{array}$$

Distance between correction planes: $l_B := 20 \cdot \text{m}$

Solution: See Figure 12-3, Table P12-2, and Mathcad file P1205a.

1. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze- frame position of the linkage chosen for analysis.

$$\begin{array}{llll} R_{1x} := r_1 \cdot \cos(\theta_1) & R_{1x} = 1.083 \text{ m} & R_{1y} := r_1 \cdot \sin(\theta_1) & R_{1y} = 0.625 \text{ m} \\ R_{2x} := r_2 \cdot \cos(\theta_2) & R_{2x} = -1.125 \text{ m} & R_{2y} := r_2 \cdot \sin(\theta_2) & R_{2y} = 1.949 \text{ m} \\ R_{3x} := r_3 \cdot \cos(\theta_3) & R_{3x} = 4.763 \text{ m} & R_{3y} := r_3 \cdot \sin(\theta_3) & R_{3y} = -2.750 \text{ m} \end{array}$$

2. Solve equations 12.4e for summation of moments about O.

$$\begin{array}{ll} mR_{Bx} := \frac{-(m_1 \cdot R_{1x}) \cdot l_1 - (m_2 \cdot R_{2x}) \cdot l_2 - (m_3 \cdot R_{3x}) \cdot l_3}{l_B} & mR_{Bx} = -4.862 \text{ kg} \cdot \text{m} \\ mR_{By} := \frac{-(m_1 \cdot R_{1y}) \cdot l_1 - (m_2 \cdot R_{2y}) \cdot l_2 - (m_3 \cdot R_{3y}) \cdot l_3}{l_B} & mR_{By} = 2.574 \text{ kg} \cdot \text{m} \end{array}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B.

$$\begin{array}{ll} \theta_B := \text{atan2}(mR_{Bx}, mR_{By}) & \theta_B = 152.101 \text{ deg} \\ mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} & mR_B = 5.501 \text{ kg} \cdot \text{m} \end{array}$$

4. Solve equations 12.4c for forces in x and y directions in plane A.

$$\begin{array}{ll} mR_{Ax} := -m_1 \cdot R_{1x} - m_2 \cdot R_{2x} - m_3 \cdot R_{3x} - mR_{Bx} & mR_{Ax} = -0.811 \text{ kg} \cdot \text{m} \\ mR_{Ay} := -m_1 \cdot R_{1y} - m_2 \cdot R_{2y} - m_3 \cdot R_{3y} - mR_{By} & mR_{Ay} = -0.069 \text{ kg} \cdot \text{m} \end{array}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A.

$$\begin{array}{ll} \theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) & \theta_A = -175.160 \text{ deg} \\ mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} & mR_A = 0.814 \text{ kg} \cdot \text{m} \end{array}$$

PROBLEM 12-6

Statement: A wheel and tire assembly has been run at 100 rpm on a dynamic balancing machine as shown in Figure 12-12. The force measured at the left bearing had a peak of 5 lb at a phase angle of 45 deg with respect to the zero reference angle on the tire. The force measured at the right bearing had a peak of 2 lb at a phase angle of -120 deg with respect to the zero reference on the tire. The center distance between the two bearings on the machine is 10 in. The left edge of the wheel rim is 4 in from the centerline of the closest bearing. The wheel is 7-in wide at the rim. Calculate the size and location with respect to the tire's zero reference angle, of balance weights needed on each side of the rim to dynamically balance the tire assembly. The wheel rim diameter is 15 in.

Given: Bearing forces and plane locations with respect to correction plane A:

$$\text{Left: } F_1 := 5 \cdot \text{lb} \quad \theta_1 := 45 \cdot \text{deg} \quad l_1 := 14 \cdot \text{in}$$

$$\text{Right: } F_2 := 2 \cdot \text{lb} \quad \theta_2 := -120 \cdot \text{deg} \quad l_2 := 4 \cdot \text{in}$$

$$\text{Distance between correction planes: } l_B := 7 \cdot \text{in}$$

$$\text{Correction weight radii: } R_A := 7.5 \cdot \text{in} \quad R_B := 7.5 \cdot \text{in}$$

$$\text{Tire rotational speed: } \omega := 100 \cdot \text{rpm}$$

Solution: See Figure 12-12 and Mathcad file P1206.

1. Resolve the force vectors into xy components with respect to the zero reference angle of the tire.

$$F_{1x} := F_1 \cdot \cos(\theta_1) \quad F_{1x} = 3.536 \cdot \text{lb} \quad F_{1y} := F_1 \cdot \sin(\theta_1) \quad F_{1y} = 3.536 \cdot \text{lb}$$

$$F_{2x} := F_2 \cdot \cos(\theta_2) \quad F_{2x} = -1.000 \cdot \text{lb} \quad F_{2y} := F_2 \cdot \sin(\theta_2) \quad F_{2y} = -1.732 \cdot \text{lb}$$

2. Solve equations 12.4e for summation of moments about O, modified for the bearing forces.

$$mR_{Bx} := -\frac{F_{1x} \cdot l_1 + F_{2x} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{Bx} = -22.883 \cdot \text{in} \cdot \text{lb}$$

$$mR_{By} := -\frac{F_{1y} \cdot l_1 + F_{2y} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{By} = -21.411 \cdot \text{in} \cdot \text{lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B. Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = -136.904 \cdot \text{deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 31.338 \cdot \text{in} \cdot \text{lb}$$

$$W_B := \frac{mR_B}{R_B} \cdot g \quad W_B = 4.18 \cdot \text{lb}$$

4. Solve equations 12.4c for forces in x and y directions in plane A.

$$mR_{Ax} := -\frac{F_{1x} + F_{2x}}{\omega^2} - mR_{Bx} \quad mR_{Ax} = 13.956 \cdot \text{in} \cdot \text{lb}$$

$$mR_{Ay} := -\frac{F_{1y} + F_{2y}}{\omega^2} - mR_{By} \quad mR_{Ay} = 15.061 \cdot \text{in} \cdot \text{lb}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A.

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay})$$

$$\theta_A = 47.180 \cdot \text{deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2}$$

$$mR_A = 20.533 \cdot \text{in} \cdot \text{lb}$$

$$W_A := \frac{mR_A \cdot g}{R_A}$$

$$W_A = 2.74 \cdot \text{lbf}$$

PROBLEM 12-7

Statement: A wheel and tire assembly has been run at 100 rpm on a dynamic balancing machine as shown in Figure 12-12. The force measured at the left bearing had a peak of 6 lb at a phase angle of -60 deg with respect to the zero reference angle on the tire. The force measured at the right bearing had a peak of 4 lb at a phase angle of 150 deg with respect to the zero reference on the tire. The center distance between the two bearings on the machine is 10 in. The left edge of the wheel rim is 4 in from the centerline of the closest bearing. The wheel is 7-in wide at the rim. Calculate the size and location with respect to the tire's zero reference angle, of balance weights needed on each side of the rim to dynamically balance the tire assembly. The wheel rim diameter is 16 in.

Given: Bearing forces and plane locations with respect to correction plane A:

$$\text{Left: } F_1 := 6 \cdot \text{lb} \quad \theta_1 := -60 \cdot \text{deg} \quad l_1 := 14 \cdot \text{in}$$

$$\text{Right: } F_2 := 4 \cdot \text{lb} \quad \theta_2 := 150 \cdot \text{deg} \quad l_2 := 4 \cdot \text{in}$$

$$\text{Distance between correction planes: } l_B := 7 \cdot \text{in}$$

$$\text{Correction weight radii: } R_A := 8 \cdot \text{in} \quad R_B := 8 \cdot \text{in}$$

$$\text{Tire rotational speed: } \omega := 100 \cdot \text{rpm}$$

Solution: See Figure 12-12 and Mathcad file P1207.

1. Resolve the force vectors into xy components with respect to the zero reference angle of the tire.

$$F_{1x} := F_1 \cdot \cos(\theta_1) \quad F_{1x} = 3.000 \cdot \text{lb} \quad F_{1y} := F_1 \cdot \sin(\theta_1) \quad F_{1y} = -5.196 \cdot \text{lb}$$

$$F_{2x} := F_2 \cdot \cos(\theta_2) \quad F_{2x} = -3.464 \cdot \text{lb} \quad F_{2y} := F_2 \cdot \sin(\theta_2) \quad F_{2y} = 2.000 \cdot \text{lb}$$

2. Solve equations 12.4e for summation of moments about O, modified for the bearing forces.

$$mR_{Bx} := -\frac{F_{1x} \cdot l_1 + F_{2x} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{Bx} = -14.155 \cdot \text{in} \cdot \text{lb}$$

$$mR_{By} := -\frac{F_{1y} \cdot l_1 + F_{2y} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{By} = 32.565 \cdot \text{in} \cdot \text{lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B. Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 113.493 \cdot \text{deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 35.508 \cdot \text{in} \cdot \text{lb}$$

$$W_B := \frac{mR_B}{R_B} \cdot g \quad W_B = 4.44 \cdot \text{lb}$$

4. Solve equations 12.4c for forces in x and y directions in plane A.

$$mR_{Ax} := -\frac{F_{1x} + F_{2x}}{\omega^2} - mR_{Bx} \quad mR_{Ax} = 15.789 \cdot \text{in} \cdot \text{lb}$$

$$mR_{Ay} := -\frac{F_{1y} + F_{2y}}{\omega^2} - mR_{By} \quad mR_{Ay} = -21.312 \cdot \text{in} \cdot \text{lb}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A.

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay})$$

$$\theta_A = -53.467 \cdot \text{deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2}$$

$$mR_A = 26.523 \cdot \text{in} \cdot \text{lb}$$

$$W_A := \frac{mR_A \cdot g}{R_A}$$

$$W_A = 3.32 \cdot \text{lbf}$$

PROBLEM 12-8a

Statement: Table P11-3 shows the geometry and kinematic data for several fourbar linkages. For row *a* in Table P11-3:

- Calculate the size and angular locations of the counterbalance mass-radius products needed on links 2 and 4 to completely force balance the linkage by the method of Berkof and Lowen. Check your manual calculation with program FOURBAR.
- Calculate the input torque for the linkage both with and without the added balance weights and compare the results. Use program FOURBAR.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

Link 2 (O_2 to A)	$l_2 := 4.00 \cdot in$	Link 3 (A to B)	$l_3 := 12.00 \cdot in$
Link 4 (B to O_4)	$l_4 := 8.00 \cdot in$	Link 3 (O_2 to O_4)	$l_1 := 15.00 \cdot in$

Link angles: $\theta_2 := 45 \cdot deg$ $\theta_3 := 24.97 \cdot deg$ $\theta_4 := 99.30 \cdot deg$

Mass: $m_2 := 0.002 \cdot blob$ $m_3 := 0.020 \cdot blob$ $m_4 := 0.100 \cdot blob$

Moment of inertia: $I_{G2} := 0.10 \cdot blob \cdot in^2$ $I_{G3} := 0.20 \cdot blob \cdot in^2$ $I_{G4} := 0.50 \cdot blob \cdot in^2$

Mass center: $b_2 := 2.00 \cdot in$ $\phi_2 := 0 \cdot deg$ $b_3 := 5.00 \cdot in$ $\phi_3 := 0 \cdot deg$
 $b_4 := 4.00 \cdot in$ $\phi_4 := 30 \cdot deg$

Solution: See Table P11-3 and Mathcad file P1208a.

- Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \cos(\phi_3) - l_2 \right) \quad mb_{2x} = -0.0467 \cdot in \cdot blob$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.0000 \cdot in \cdot blob$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -0.0667 \cdot in \cdot blob$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.0000 \cdot in \cdot blob$$

- Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -0.0507 \cdot in \cdot blob$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.0000 \cdot in \cdot blob$$

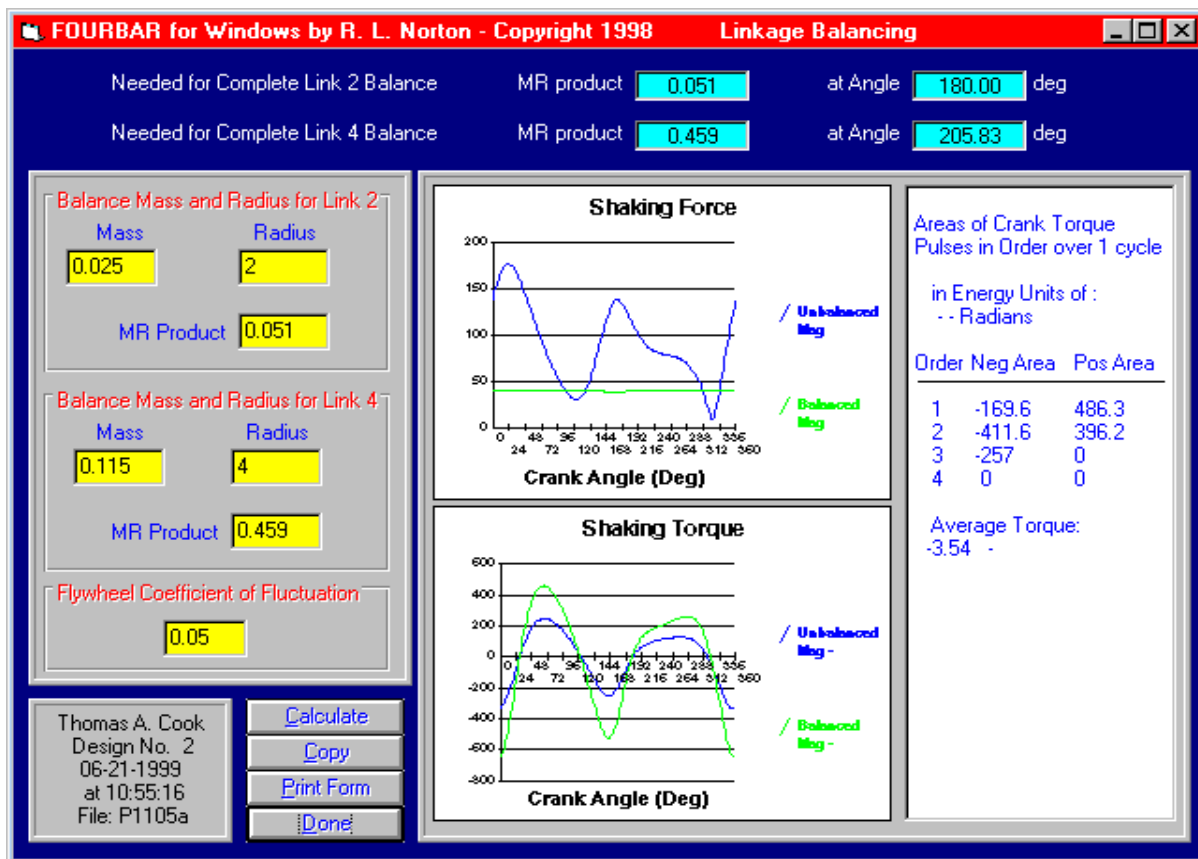
$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -0.4131 \cdot in \cdot blob$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = -0.2000 \cdot in \cdot blob$$

3. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$\begin{aligned}\theta_{b2} &:= \text{atan2}(mR_{2x}, mR_{2y}) & \theta_{b2} &= 180.000 \text{ deg} \\ mR_{b2} &:= \sqrt{mR_{2x}^2 + mR_{2y}^2} & mR_{b2} &= 0.0507 \text{ blob}\cdot\text{in} \\ \theta_{b4} &:= \text{atan2}(mR_{4x}, mR_{4y}) & \theta_{b4} &= -154.165 \text{ deg} \\ mR_{b4} &:= \sqrt{mR_{4x}^2 + mR_{4y}^2} & mR_{b4} &= 0.459 \text{ blob}\cdot\text{in}\end{aligned}$$

4. Check the result using program FOURBAR. The linkage balancing screen below confirms the calculations.



5. The maximum positive input torque without force balancing is 245.7 lb-in. After force balancing it is 460.7 lb-in.

PROBLEM 12-9

Statement: Link 2 in Figure P12-1 rotates at 500 rpm. The links are steel with cross-sections of 1 x 2 in. Half of the 29-lb weight of the laybar and reed are supported by the linkage at point B. Design counterweights to force balance the linkage and determine its change in peak torque versus unbalanced condition. See Problem 11-13 for more information on the overall mechanism.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (A to B)} \quad l_2 := 2.00 \cdot in \quad \text{Link 3 (B to C)} \quad l_3 := 8.375 \cdot in$$

$$\text{Link 4 (C to D)} \quad l_4 := 7.187 \cdot in \quad \text{Link 1 (A to D)} \quad l_1 := 9.625 \cdot in$$

$$\text{Coupler point:} \quad R_{pa} := 0.0 \cdot in \quad \delta_3 := 0 \cdot deg$$

$$\text{Crank angle and motion:} \quad \omega_2 := 500 \cdot rpm \quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$$

$$\text{Link cross-section dims:} \quad w := 2.00 \cdot in \quad t := 1.00 \cdot in$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma := 0.28 \cdot lbf \cdot in^{-3}$$

Solution: See Figure P12-1, Problem 11-13, and Mathcad file P1209.

1. Determine the distance to the CG in the LRCS on each of the three moving links. All three are located on the x' axis in the LRCS and their angle is zero deg.

$$b_2 := 0.5 \cdot l_2 \quad b_2 = 1.000 \cdot in \quad b_3 := 0.5 \cdot l_3 \quad b_3 = 4.188 \cdot in$$

$$\phi_2 := 0 \cdot deg \quad \phi_3 := 0 \cdot deg \quad \phi_4 := 0 \cdot deg$$

2. Determine the mass and moment of inertia of each link.

$$m_2 := w \cdot t \cdot l_2 \cdot \frac{\gamma}{g} \quad m_3 := w \cdot t \cdot l_3 \cdot \frac{\gamma}{g} \quad m_{41} := w \cdot t \cdot l_4 \cdot \frac{\gamma}{g} \quad m_{42} := \frac{29 \cdot lbf}{2 \cdot g}$$

$$m_2 = 0.0029 \cdot blob \quad m_3 = 0.012 \cdot blob \quad m_{41} = 0.010 \cdot blob$$

$$m_4 := m_{41} + m_{42} \quad m_4 = 0.048 \cdot blob \quad b_4 := \frac{l_4}{m_4} \cdot \left(\frac{m_{41}}{2} + m_{42} \right) \quad b_4 = 6.406 \cdot in$$

$$I_{G2} := \frac{m_2}{12} \cdot (w^2 + l_2^2) \quad I_{G2} = 0.00193 \cdot blob \cdot in^2$$

$$I_{G3} := \frac{m_3}{12} \cdot (w^2 + l_3^2) \quad I_{G3} = 0.01792 \cdot blob \cdot in^2$$

$$I_{G4} := (l_4 - b_4)^2 \cdot m_{42} + \frac{m_{41}}{12} \cdot (w^2 + l_4^2) + m_{41} \cdot \left(b_4 - \frac{l_4}{2} \right)^2 \quad I_{G4} = 0.154 \cdot blob \cdot in^2$$

3. Define any external forces, their locations and directions.

$$\text{Beat-up force} \quad F := 590 \cdot lbf \quad \text{acting on link 4 at a distance} \quad R := l_4 + 3.75 \cdot in$$

$$\text{The angle in the CGS is } 180 \cdot deg. \quad R = 10.937 \cdot in$$

4. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \cos(\phi_3) - l_2 \right) \quad mb_{2x} = -0.0121 \cdot in \cdot blob$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.0000 \cdot in \cdot blob$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -0.0437 \cdot in \cdot blob$$

$$mb_{4y} := -m_3 \left(b_3 \cdot \frac{l_4}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.0000 \text{ in} \cdot \text{blob}$$

5. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -0.0150 \text{ in} \cdot \text{blob}$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.0000 \text{ in} \cdot \text{blob}$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -0.3510 \text{ in} \cdot \text{blob}$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 0.0000 \text{ in} \cdot \text{blob}$$

6. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

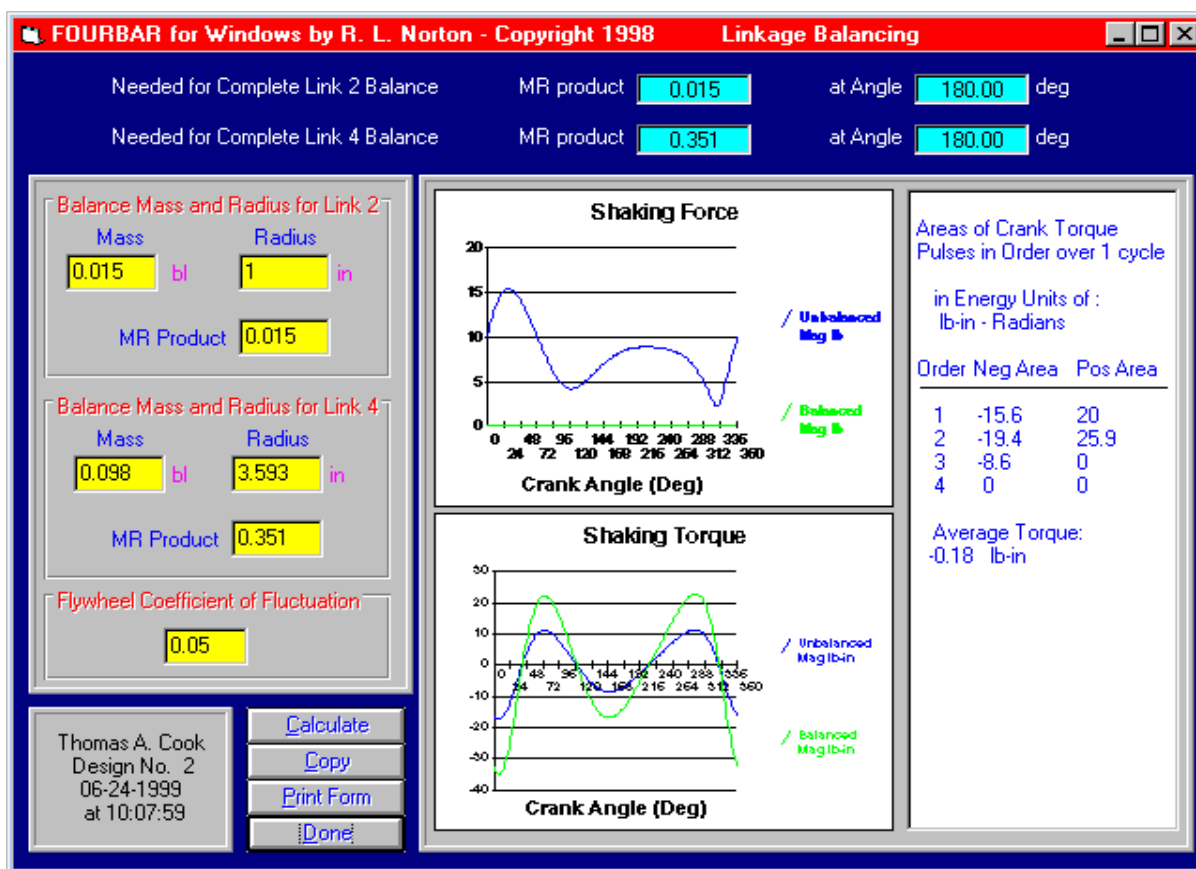
$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \text{ deg}$$

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 0.0150 \text{ blob} \cdot \text{in}$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 180.000 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 0.351 \text{ blob} \cdot \text{in}$$

7. Check the result using program FOURBAR. The linkage balancing screen below confirms the calculations.
 8. The maximum peak input torque without force balancing is 17.3 lb-in. After force balancing it is 34.9 lb-in.



PROBLEM 12-10

Statement: Figure P12-2a shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. The crank O_2A rotates at a constant speed of $\omega = 40$ rad/sec. Design counterweights to force balance the linkage and determine its change in peak torque versus the unbalanced condition.

Given: Link lengths:

$$\text{Link 2 (} O_2 \text{ to } A) \quad a := 1.00 \cdot m \quad \text{Link 3 (} A \text{ to } B) \quad b := 2.06 \cdot m$$

$$\text{Link 4 (} B \text{ to } O_4) \quad c := 2.33 \cdot m \quad \text{Link 1 (} O_2 \text{ to } O_4) \quad d := 2.22 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 3.06 \cdot m \quad \delta_3 := -31 \cdot \text{deg}$$

$$\text{Crank angle and motion:} \quad \theta_2 := 60 \cdot \text{deg} \quad \omega_2 := 40 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lbf} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Solution: See Figure P12-2a and Mathcad file P1210.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.500 \cdot m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 1.165 \cdot m$$

$$\text{Link 3:} \quad R_{CG3x} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x} = 1.561 \cdot m$$

$$R_{CG3y} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y} = -0.525 \cdot m$$

$$R_{CG3} := \sqrt{R_{CG3x}^2 + R_{CG3y}^2} \quad R_{CG3} = 1.647 \cdot m$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x}, R_{CG3y}) \quad \delta_{33} = -18.600 \cdot \text{deg}$$

2. Determine the mass of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot |R_{pa} \cdot \sin(\delta_3)| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 10.380 \cdot \text{kg} \quad m_3 = 112.332 \cdot \text{kg} \quad m_4 = 24.185 \cdot \text{kg}$$

3. Convert the above data for use in equations 12.8c and 12.8d.

$$\text{Link lengths:} \quad l_2 := a \quad l_3 := b \quad l_4 := c$$

$$\text{CG position vectors:} \quad b_2 := R_{CG2} \quad b_3 := R_{CG3} \quad b_4 := R_{CG4}$$

$$\phi_2 := 0 \cdot \text{deg} \quad \phi_3 := \delta_{33} \quad \phi_4 := 0 \cdot \text{deg}$$

4. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \cos(\phi_3) - l_2 \right) \quad mb_{2x} = -27.2117 \cdot \text{kg} \cdot \text{m}$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = -28.6467 \cdot \text{kg} \cdot \text{m}$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -198.3298 \text{ kg}\cdot\text{m}$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 66.7469 \text{ kg}\cdot\text{m}$$

5. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -32.4017 \text{ kg}\cdot\text{m}$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = -28.6467 \text{ kg}\cdot\text{m}$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -226.5057 \text{ kg}\cdot\text{m}$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 66.7469 \text{ kg}\cdot\text{m}$$

6. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = -138.520 \text{ deg}$$

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 43.25 \text{ kg}\cdot\text{m}$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 163.581 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 236.14 \text{ kg}\cdot\text{m}$$

7. The maximum peak input torque without force balancing is found from program FOURBAR to be 464.9 kN-m. After force balancing it is 719.4 kN-m.

PROBLEM 12-11

Statement: Figure P12-2b shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. The crank O_2A rotates at a constant speed of $\omega = 50$ rad/sec. Design counterweights to force balance the linkage and determine its change in peak torque versus the unbalanced condition.

Given: Link lengths:

$$\text{Link 2 (} O_2 \text{ to } A) \quad a := 0.72 \cdot m \quad \text{Link 3 (} A \text{ to } B) \quad b := 0.68 \cdot m$$

$$\text{Link 4 (} B \text{ to } O_4) \quad c := 0.85 \cdot m \quad \text{Link 1 (} O_2 \text{ to } O_4) \quad d := 1.82 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 0.97 \cdot m \quad \delta_3 := 54 \cdot \text{deg}$$

$$\text{Crank angle and motion:} \quad \theta_2 := 30 \cdot \text{deg} \quad \omega_2 := 50 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lbf} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Solution: See Figure P12-2b and Mathcad file P1211.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.360 \, m \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.425 \, m$$

$$\text{Link 3:} \quad R_{CG3x} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x} = 0.417 \, m$$

$$R_{CG3y} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y} = 0.262 \, m$$

$$R_{CG3} := \sqrt{R_{CG3x}^2 + R_{CG3y}^2} \quad R_{CG3} = 0.492 \, m$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x}, R_{CG3y}) \quad \delta_{33} = 32.117 \, \text{deg}$$

2. Determine the mass of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot |R_{pa} \cdot \sin(\delta_3)| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 7.474 \, \text{kg} \quad m_3 = 18.463 \, \text{kg} \quad m_4 = 8.823 \, \text{kg}$$

3. Convert the above data for use in equations 12.8c and 12.8d.

$$\text{Link lengths:} \quad l_2 := a \quad l_3 := b \quad l_4 := c$$

$$\text{CG position vectors:} \quad b_2 := R_{CG2} \quad b_3 := R_{CG3} \quad b_4 := R_{CG4}$$

$$\phi_2 := 0 \cdot \text{deg} \quad \phi_3 := \delta_{33} \quad \phi_4 := 0 \cdot \text{deg}$$

4. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \cos(\phi_3) - l_2 \right) \quad mb_{2x} = -5.1471 \, \text{kg} \cdot \text{m}$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 5.1138 \, \text{kg} \cdot \text{m}$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -9.6175 \text{ kg}\cdot\text{m}$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = -6.0371 \text{ kg}\cdot\text{m}$$

5. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -7.8375 \text{ kg}\cdot\text{m}$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 5.1138 \text{ kg}\cdot\text{m}$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -13.3673 \text{ kg}\cdot\text{m}$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = -6.0371 \text{ kg}\cdot\text{m}$$

6. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 146.877 \text{ deg}$$

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 9.36 \text{ kg}\cdot\text{m}$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = -155.694 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 14.67 \text{ kg}\cdot\text{m}$$

7. The maximum peak input torque without force balancing is found from program FOURBAR to be 5.34 MN-m. After force balancing it is 10.63 MN-m.

PROBLEM 12-12

Statement: Write a computer program or use an equation solver such as *Mathcad*, *Matlab*, or *TKSolver* to solve for the mass-radius products that will force balance any fourbar linkage for which the geometry and mass properties are known.

Units: Use any set of compatible units.

Enter data: Link lengths:

Link 2 (O ₂ to A)	$l_2 := 4.00$	Link 3 (A to B)	$l_3 := 12.00$
Link 4 (B to O ₄)	$l_4 := 8.00$	Link 1 (O ₂ to O ₄)	$l_1 := 15.00$
Mass:	$m_2 := 0.002$	$m_3 := 0.020$	$m_4 := 0.100$
Mass center:	$b_2 := 2.00$	$b_3 := 5.00$	$b_4 := 4.00$
	$\phi_2 := 0 \cdot \text{deg}$	$\phi_3 := 0 \cdot \text{deg}$	$\phi_4 := 30 \cdot \text{deg}$

Solution: See Mathcad file P1212.

1. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \cos(\phi_3) - l_2 \right) \quad mb_{2x} = -0.0467$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.000$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -0.0667$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.000$$

2. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -0.0507$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.000$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -0.413$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = -0.200$$

3. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 0.0507$$

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 0.459$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = -154.165 \text{ deg}$$

PROBLEM 12-13

Statement: Figure P12-3 shows a system with two weights on a rotating shaft. For the given data below, determine the magnitudes and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$$W_1 := 15 \cdot \text{lb} \quad r_1 := 6 \cdot \text{in} \quad \theta_1 := 0 \cdot \text{deg} \quad l_1 := -7 \cdot \text{in}$$

$$W_2 := 20 \cdot \text{lb} \quad r_2 := 5 \cdot \text{in} \quad \theta_2 := 270 \cdot \text{deg} \quad l_2 := 9 \cdot \text{in}$$

$$\text{Distance between correction planes:} \quad l_B := 9 \cdot \text{in}$$

$$\text{Correction weight radii:} \quad \text{Plane 4} \quad R_A := 8 \cdot \text{in} \quad \text{Plane 3} \quad R_B := 5 \cdot \text{in}$$

Solution: See Figure P12-3 and Mathcad file P1213.

1. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$R_{1x} := r_1 \cdot \cos(\theta_1) \quad R_{1x} = 6.000 \text{ in} \quad R_{1y} := r_1 \cdot \sin(\theta_1) \quad R_{1y} = 0.000 \text{ in}$$

$$R_{2x} := r_2 \cdot \cos(\theta_2) \quad R_{2x} = 0.000 \text{ in} \quad R_{2y} := r_2 \cdot \sin(\theta_2) \quad R_{2y} = -5.000 \text{ in}$$

2. Solve equations 12.4e for summation of moments about O, which is at plane 4.

$$mR_{Bx} := \frac{-(W_1 \cdot R_{1x}) \cdot l_1 - (W_2 \cdot R_{2x}) \cdot l_2}{l_B \cdot g} \quad mR_{Bx} = 70.000 \text{ in} \cdot \text{lb}$$

$$mR_{By} := \frac{-(W_1 \cdot R_{1y}) \cdot l_1 - (W_2 \cdot R_{2y}) \cdot l_2}{l_B \cdot g} \quad mR_{By} = 100.000 \text{ in} \cdot \text{lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (3). Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 55.008 \text{ deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 122.066 \text{ in} \cdot \text{lb}$$

$$W_3 := \frac{mR_B \cdot g}{R_B} \quad W_3 = 24.4 \text{ lbf}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (4).

$$mR_{Ax} := \frac{-W_1 \cdot R_{1x} - W_2 \cdot R_{2x}}{g} - mR_{Bx} \quad mR_{Ax} = -160.000 \text{ in} \cdot \text{lb}$$

$$mR_{Ay} := \frac{-W_1 \cdot R_{1y} - W_2 \cdot R_{2y}}{g} - mR_{By} \quad mR_{Ay} = 0.000 \text{ in} \cdot \text{lb}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (4).

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = 180.000 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 160.000 \text{ in} \cdot \text{lb}$$

$$W_4 := \frac{mR_A \cdot g}{R_A} \quad W_4 = 20.0 \text{ lbf}$$

PROBLEM 12-14

Statement: Figure P12-4 shows a system with two weights on a rotating shaft. For the given data below, determine the radii and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$$W_1 := 20 \cdot \text{lb}f \quad r_1 := 6 \cdot \text{in} \quad \theta_1 := 45 \cdot \text{deg} \quad l_1 := 0 \cdot \text{in}$$

$$W_2 := 15 \cdot \text{lb}f \quad r_2 := 4 \cdot \text{in} \quad \theta_2 := 300 \cdot \text{deg} \quad l_2 := 9 \cdot \text{in}$$

$$\text{Distance between correction planes:} \quad l_B := 12 \cdot \text{in}$$

$$\text{Correction weights:} \quad \text{Plane 3} \quad W_3 := 20 \cdot \text{lb}f \quad \text{Plane 4} \quad W_4 := 40 \cdot \text{lb}f$$

Solution: See Figure P12-4 and Mathcad file P1214.

1. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$\begin{aligned} R_{1x} &:= r_1 \cdot \cos(\theta_1) & R_{1x} &= 4.243 \cdot \text{in} & R_{1y} &:= r_1 \cdot \sin(\theta_1) & R_{1y} &= 4.243 \cdot \text{in} \\ R_{2x} &:= r_2 \cdot \cos(\theta_2) & R_{2x} &= 2.000 \cdot \text{in} & R_{2y} &:= r_2 \cdot \sin(\theta_2) & R_{2y} &= -3.464 \cdot \text{in} \end{aligned}$$

2. Solve equations 12.4e for summation of moments about O, which is at plane 3.

$$mR_{Bx} := \frac{-(W_1 \cdot R_{1x}) \cdot l_1 - (W_2 \cdot R_{2x}) \cdot l_2}{l_B \cdot g} \quad mR_{Bx} = -22.500 \cdot \text{in} \cdot \text{lb}$$

$$mR_{By} := \frac{-(W_1 \cdot R_{1y}) \cdot l_1 - (W_2 \cdot R_{2y}) \cdot l_2}{l_B \cdot g} \quad mR_{By} = 38.971 \cdot \text{in} \cdot \text{lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (4). Also, solve for the radius required for the given weight.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 120.000 \cdot \text{deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 45.000 \cdot \text{in} \cdot \text{lb}$$

$$R_4 := \frac{mR_B \cdot g}{W_4} \quad R_4 = 1.13 \cdot \text{in}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (3).

$$mR_{Ax} := \frac{-W_1 \cdot R_{1x} - W_2 \cdot R_{2x}}{g} - mR_{Bx} \quad mR_{Ax} = -92.353 \cdot \text{in} \cdot \text{lb}$$

$$mR_{Ay} := \frac{-W_1 \cdot R_{1y} - W_2 \cdot R_{2y}}{g} - mR_{By} \quad mR_{Ay} = -71.862 \cdot \text{in} \cdot \text{lb}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (3).

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = -142.112 \cdot \text{deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 117.018 \cdot \text{in} \cdot \text{lb}$$

$$R_3 := \frac{mR_A \cdot g}{W_3} \quad R_3 = 5.85 \cdot \text{in}$$

PROBLEM 12-15

Statement: Figure P12-5 shows a system with two weights on a rotating shaft. For the given data below, determine the magnitudes and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$$\begin{array}{llll} W_1 := 10 \cdot \text{lb}f & r_1 := 3 \cdot \text{in} & \theta_1 := 90 \cdot \text{deg} & l_1 := 3 \cdot \text{in} \\ W_2 := 15 \cdot \text{lb}f & r_2 := 3 \cdot \text{in} & \theta_2 := 240 \cdot \text{deg} & l_2 := 7 \cdot \text{in} \end{array}$$

$$\text{Distance between correction planes: } l_B := 12 \cdot \text{in}$$

$$\text{Correction weight radii: } \text{Plane 4 } R_A := 3 \cdot \text{in} \quad \text{Plane 3 } R_B := 3 \cdot \text{in}$$

Solution: See Figure P12-5 and Mathcad file P1215.

1. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$\begin{array}{llll} R_{1x} := r_1 \cdot \cos(\theta_1) & R_{1x} = 0.000 \text{ in} & R_{1y} := r_1 \cdot \sin(\theta_1) & R_{1y} = 3.000 \text{ in} \\ R_{2x} := r_2 \cdot \cos(\theta_2) & R_{2x} = -1.500 \text{ in} & R_{2y} := r_2 \cdot \sin(\theta_2) & R_{2y} = -2.598 \text{ in} \end{array}$$

2. Solve equations 12.4e for summation of moments about O, which is at plane 4.

$$\begin{array}{ll} mR_{Bx} := \frac{-(W_1 \cdot R_{1x}) \cdot l_1 - (W_2 \cdot R_{2x}) \cdot l_2}{l_B \cdot g} & mR_{Bx} = 13.125 \text{ in} \cdot \text{lb} \\ mR_{By} := \frac{-(W_1 \cdot R_{1y}) \cdot l_1 - (W_2 \cdot R_{2y}) \cdot l_2}{l_B \cdot g} & mR_{By} = 15.233 \text{ in} \cdot \text{lb} \end{array}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (3). Also, solve for the weight required at the given radius.

$$\begin{array}{ll} \theta_B := \text{atan2}(mR_{Bx}, mR_{By}) & \theta_B = 49.252 \text{ deg} \\ mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} & mR_B = 20.108 \text{ in} \cdot \text{lb} \\ W_3 := \frac{mR_B \cdot g}{R_B} & W_3 = 6.70 \text{ lb}f \end{array}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (4).

$$\begin{array}{ll} mR_{Ax} := \frac{-W_1 \cdot R_{1x} - W_2 \cdot R_{2x}}{g} - mR_{Bx} & mR_{Ax} = 9.375 \text{ in} \cdot \text{lb} \\ mR_{Ay} := \frac{-W_1 \cdot R_{1y} - W_2 \cdot R_{2y}}{g} - mR_{By} & mR_{Ay} = -6.262 \text{ in} \cdot \text{lb} \end{array}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (4).

$$\begin{array}{ll} \theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) & \theta_A = -33.741 \text{ deg} \\ mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} & mR_A = 11.274 \text{ in} \cdot \text{lb} \\ W_4 := \frac{mR_A \cdot g}{R_A} & W_4 = 3.76 \text{ lb}f \end{array}$$

PROBLEM 12-16

Statement: Figure P12-6 shows a system with three weights on a rotating shaft. For the given data below, determine the magnitudes and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$$\begin{array}{llll} W_1 := 6 \cdot \text{lb}f & r_1 := 5 \cdot \text{in} & \theta_1 := 120 \cdot \text{deg} & l_1 := -14 \cdot \text{in} \\ W_2 := 12 \cdot \text{lb}f & r_2 := 4 \cdot \text{in} & \theta_2 := 240 \cdot \text{deg} & l_2 := 4 \cdot \text{in} \\ W_3 := 9 \cdot \text{lb}f & r_3 := 8 \cdot \text{in} & \theta_3 := 300 \cdot \text{deg} & l_3 := -4 \cdot \text{in} \end{array}$$

$$\text{Distance between correction planes: } l_B := 14 \cdot \text{in}$$

$$\text{Correction weight radii: } \text{Plane 4 } R_A := 4 \cdot \text{in} \quad \text{Plane 5 } R_B := 4 \cdot \text{in}$$

Solution: See Figure P12-6 and Mathcad file P1216.

1. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$\begin{array}{llll} R_{1x} := r_1 \cdot \cos(\theta_1) & R_{1x} = -2.500 \cdot \text{in} & R_{1y} := r_1 \cdot \sin(\theta_1) & R_{1y} = 4.330 \cdot \text{in} \\ R_{2x} := r_2 \cdot \cos(\theta_2) & R_{2x} = -2.000 \cdot \text{in} & R_{2y} := r_2 \cdot \sin(\theta_2) & R_{2y} = -3.464 \cdot \text{in} \\ R_{3x} := r_3 \cdot \cos(\theta_3) & R_{3x} = 4.000 \cdot \text{in} & R_{3y} := r_3 \cdot \sin(\theta_3) & R_{3y} = -6.928 \cdot \text{in} \end{array}$$

2. Solve equations 12.4e for summation of moments about O, which is at plane 4.

$$\begin{array}{ll} mR_{Bx} := \frac{-(W_1 \cdot R_{1x}) \cdot l_1 - (W_2 \cdot R_{2x}) \cdot l_2 - (W_3 \cdot R_{3x}) \cdot l_3}{l_B \cdot g} & mR_{Bx} = 2.143 \cdot \text{in} \cdot \text{lb} \\ mR_{By} := \frac{-(W_1 \cdot R_{1y}) \cdot l_1 - (W_2 \cdot R_{2y}) \cdot l_2 - (W_3 \cdot R_{3y}) \cdot l_3}{l_B \cdot g} & mR_{By} = 20.042 \cdot \text{in} \cdot \text{lb} \end{array}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (5). Also, solve for the weight required at the given radius.

$$\begin{array}{ll} \theta_B := \text{atan2}(mR_{Bx}, mR_{By}) & \theta_B = 83.897 \cdot \text{deg} \\ mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} & mR_B = 20.157 \cdot \text{in} \cdot \text{lb} \\ W_5 := \frac{mR_B \cdot g}{R_B} & W_5 = 5.04 \cdot \text{lb}f \end{array}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (4).

$$\begin{array}{ll} mR_{Ax} := \frac{-W_1 \cdot R_{1x} - W_2 \cdot R_{2x} - W_3 \cdot R_{3x}}{g} - mR_{Bx} & mR_{Ax} = 0.857 \cdot \text{in} \cdot \text{lb} \\ mR_{Ay} := \frac{-W_1 \cdot R_{1y} - W_2 \cdot R_{2y} - W_3 \cdot R_{3y}}{g} - mR_{By} & mR_{Ay} = 57.900 \cdot \text{in} \cdot \text{lb} \end{array}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (4).

$$\begin{array}{ll} \theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) & \theta_A = 89.152 \cdot \text{deg} \\ mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} & mR_A = 57.906 \cdot \text{in} \cdot \text{lb} \\ W_4 := \frac{mR_A \cdot g}{R_A} & W_4 = 14.48 \cdot \text{lb}f \end{array}$$

PROBLEM 12-17

Statement: Figure P12-6 shows a system with three weights on a rotating shaft. For the given data below, determine the magnitudes and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$$\begin{array}{llll} W_2 := 10 \cdot \text{lb}f & r_2 := 3 \cdot \text{in} & \theta_2 := 90 \cdot \text{deg} & l_2 := 6 \cdot \text{in} \\ W_3 := 10 \cdot \text{lb}f & r_3 := 4 \cdot \text{in} & \theta_3 := 180 \cdot \text{deg} & l_3 := 12 \cdot \text{in} \\ W_4 := 8 \cdot \text{lb}f & r_4 := 4 \cdot \text{in} & \theta_4 := 315 \cdot \text{deg} & l_4 := 8 \cdot \text{in} \end{array}$$

Distance between correction planes: $l_B := 8 \cdot \text{in}$

Correction weight radii: Plane 1 $R_A := 4 \cdot \text{in}$ Plane 5 $R_B := 3 \cdot \text{in}$

Solution: See Figure P12-7 and Mathcad file P1217.

1. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$\begin{array}{llll} R_{2x} := r_2 \cdot \cos(\theta_2) & R_{2x} = 0.000 \text{ in} & R_{2y} := r_2 \cdot \sin(\theta_2) & R_{2y} = 3.000 \text{ in} \\ R_{3x} := r_3 \cdot \cos(\theta_3) & R_{3x} = -4.000 \text{ in} & R_{3y} := r_3 \cdot \sin(\theta_3) & R_{3y} = 0.000 \text{ in} \\ R_{4x} := r_4 \cdot \cos(\theta_4) & R_{4x} = 2.828 \text{ in} & R_{4y} := r_4 \cdot \sin(\theta_4) & R_{4y} = -2.828 \text{ in} \end{array}$$

2. Solve equations 12.4e for summation of moments about O, which is at plane 1.

$$\begin{array}{ll} mR_{Bx} := \frac{-(W_2 \cdot R_{2x}) \cdot l_2 - (W_3 \cdot R_{3x}) \cdot l_3 - (W_4 \cdot R_{4x}) \cdot l_4}{l_B \cdot g} & mR_{Bx} = 37.373 \text{ in} \cdot \text{lb} \\ mR_{By} := \frac{-(W_2 \cdot R_{2y}) \cdot l_2 - (W_3 \cdot R_{3y}) \cdot l_3 - (W_4 \cdot R_{4y}) \cdot l_4}{l_B \cdot g} & mR_{By} = 0.127 \text{ in} \cdot \text{lb} \end{array}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (5). Also, solve for the weight required at the given radius.

$$\begin{array}{ll} \theta_B := \text{atan2}(mR_{Bx}, mR_{By}) & \theta_B = 0.195 \text{ deg} \\ mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} & mR_B = 37.373 \text{ in} \cdot \text{lb} \\ W_5 := \frac{mR_B \cdot g}{R_B} & W_5 = 12.46 \text{ lb}f \end{array}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (1).

$$\begin{array}{ll} mR_{Ax} := \frac{-W_2 \cdot R_{2x} - W_3 \cdot R_{3x} - W_4 \cdot R_{4x}}{g} - mR_{Bx} & mR_{Ax} = -20.000 \text{ in} \cdot \text{lb} \\ mR_{Ay} := \frac{-W_2 \cdot R_{2y} - W_3 \cdot R_{3y} - W_4 \cdot R_{4y}}{g} - mR_{By} & mR_{Ay} = -7.500 \text{ in} \cdot \text{lb} \end{array}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (1).

$$\begin{array}{ll} \theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) & \theta_A = -159.444 \text{ deg} \\ mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} & mR_A = 21.360 \text{ in} \cdot \text{lb} \\ W_1 := \frac{mR_A \cdot g}{R_A} & W_1 = 5.34 \text{ lb}f \end{array}$$

PROBLEM 12-18

Statement: The 400-mm-dia steel roller in Figure P12-8 has been tested on a dynamic balancing machine at 100 rpm and shows the unbalance forces given below. Determine the angular locations and required diameters of 25-mm deep holes drilled radially inward from the surface in planes 2 and 3 to dynamically balance the system.

Given: Forces and locations:

$$F_1 := 0.291 \cdot N \quad \theta_1 := 45 \cdot \text{deg} \quad l_1 := -200 \cdot \text{mm}$$

$$F_4 := 0.514 \cdot N \quad \theta_4 := 210 \cdot \text{deg} \quad l_4 := 550 \cdot \text{mm}$$

$$\text{Distance between correction planes:} \quad l_B := 300 \cdot \text{mm}$$

$$\text{Correction weight radii:} \quad \text{Plane 2} \quad R_A := 187.5 \cdot \text{mm} \quad \text{Plane 3} \quad R_B := 187.5 \cdot \text{mm}$$

$$\text{Test speed:} \quad \omega := 100 \cdot \text{rpm}$$

Solution: See Figure P12-8 and Mathcad file P1218.

1. Resolve the force vectors into xy components in the arbitrary coordinate system associated with the test apparatus.

$$\begin{aligned} F_{1x} &:= F_1 \cdot \cos(\theta_1) & F_{1x} &= 0.206 \, N & F_{1y} &:= F_1 \cdot \sin(\theta_1) & F_{1y} &= 0.206 \, N \\ F_{4x} &:= F_4 \cdot \cos(\theta_4) & F_{4x} &= -0.445 \, N & F_{4y} &:= F_4 \cdot \sin(\theta_4) & F_{4y} &= -0.257 \, N \end{aligned}$$

2. Solve equations 12.3 for summation of moments about O, which is at plane 1. Negative mass is used.

$$mR_{Bx} := \frac{F_{1x} \cdot l_1 + F_{4x} \cdot l_4}{l_B \cdot \omega^2} \quad mR_{Bx} = -8.693 \times 10^{-3} \, \text{kg} \cdot \text{m}$$

$$mR_{By} := \frac{F_{1y} \cdot l_1 + F_{4y} \cdot l_4}{l_B \cdot \omega^2} \quad mR_{By} = -5.547 \times 10^{-3} \, \text{kg} \cdot \text{m}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (3). Also, solve for the weight and drill diameter required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = -147.455 \, \text{deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 0.0103 \, \text{kg} \cdot \text{m}$$

$$W_3 := \frac{mR_B \cdot g}{R_B} \quad W_3 = 0.539 \, N \quad (\text{removed})$$

$$\rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3} \quad \text{depth} := 25 \cdot \text{mm}$$

$$d_3 := 2 \cdot \sqrt{\frac{W_3}{\pi \cdot \rho \cdot \text{depth} \cdot g}} \quad d_3 = 18.9 \, \text{mm}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (2).

$$mR_{Ax} := \frac{-F_{1x} - F_{4x}}{\omega^2} - mR_{Bx} \quad mR_{Ax} = 0.0109 \, \text{kg} \cdot \text{m}$$

$$mR_{Ay} := \frac{-F_{1y} - F_{4y}}{\omega^2} - mR_{By} \quad mR_{Ay} = 6.015 \times 10^{-3} \, \text{kg} \cdot \text{m}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (2).

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay})$$

$$\theta_A = 28.944 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2}$$

$$mR_A = 0.0124 \text{ kg}\cdot\text{m}$$

$$W_2 := \frac{mR_A \cdot g}{R_A}$$

$$W_2 = 0.650 \text{ N} \quad (\text{removed})$$

$$d_2 := 2 \cdot \sqrt{\frac{W_2}{\pi \cdot \rho \cdot \text{depth} \cdot g}}$$

$$d_2 = 20.8 \text{ mm}$$

PROBLEM 12-19

Statement: The 500-mm-dia steel roller in Figure P12-8 has been tested on a dynamic balancing machine at 100 rpm and shows the unbalance forces given below. Determine the angular locations and required diameters of 25-mm deep holes drilled radially inward from the surface in planes 2 and 3 to dynamically balance the system.

Given: Forces and locations:

$$F_1 := 0.230 \cdot N \quad \theta_1 := 30 \cdot \text{deg} \quad l_1 := -200 \cdot \text{mm}$$

$$F_4 := 0.620 \cdot N \quad \theta_4 := 135 \cdot \text{deg} \quad l_4 := 550 \cdot \text{mm}$$

$$\text{Distance between correction planes:} \quad l_B := 300 \cdot \text{mm}$$

$$\text{Correction weight radii:} \quad \text{Plane 2} \quad R_A := 237.5 \cdot \text{mm} \quad \text{Plane 3} \quad R_B := 237.5 \cdot \text{mm}$$

$$\text{Test speed:} \quad \omega := 100 \cdot \text{rpm}$$

Solution: See Figure P12-8 and Mathcad file P1219.

1. Resolve the force vectors into xy components in the arbitrary coordinate system associated with the test apparatus.

$$F_{1x} := F_1 \cdot \cos(\theta_1) \quad F_{1x} = 0.199 \, N \quad F_{1y} := F_1 \cdot \sin(\theta_1) \quad F_{1y} = 0.115 \, N$$

$$F_{4x} := F_4 \cdot \cos(\theta_4) \quad F_{4x} = -0.438 \, N \quad F_{4y} := F_4 \cdot \sin(\theta_4) \quad F_{4y} = 0.438 \, N$$

2. Solve equations 12.3 for summation of moments about O, which is at plane 1. Negative mass is used.

$$mR_{Bx} := \frac{F_{1x} \cdot l_1 + F_{4x} \cdot l_4}{l_B \cdot \omega^2} \quad mR_{Bx} = -8.540 \times 10^{-3} \, \text{kg} \cdot \text{m}$$

$$mR_{By} := \frac{F_{1y} \cdot l_1 + F_{4y} \cdot l_4}{l_B \cdot \omega^2} \quad mR_{By} = 6.630 \times 10^{-3} \, \text{kg} \cdot \text{m}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (3). Also, solve for the weight and drill diameter required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 142.176 \, \text{deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 0.0108 \, \text{kg} \cdot \text{m}$$

$$W_3 := \frac{mR_B \cdot g}{R_B} \quad W_3 = 0.446 \, N \quad (\text{removed})$$

$$\rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3} \quad \text{depth} := 25 \cdot \text{mm}$$

$$d_3 := 2 \cdot \sqrt{\frac{W_3}{\pi \cdot \rho \cdot \text{depth} \cdot g}} \quad d_3 = 17.241 \, \text{mm}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (4).

$$mR_{Ax} := \frac{-F_{1x} - F_{4x}}{\omega^2} - mR_{Bx} \quad mR_{Ax} = 0.0107 \, \text{kg} \cdot \text{m}$$

$$mR_{Ay} := \frac{-F_{1y} - F_{4y}}{\omega^2} - mR_{By} \quad mR_{Ay} = -0.012 \, \text{kg} \cdot \text{m}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (4).

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay})$$

$$\theta_A = -47.441 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2}$$

$$mR_A = 0.0159 \text{ kg}\cdot\text{m}$$

$$W_4 := \frac{mR_A \cdot g}{R_A}$$

$$W_4 = 0.655 \text{ N} \quad (\text{removed})$$

$$d_4 := 2 \cdot \sqrt{\frac{W_4}{\pi \cdot \rho \cdot \text{depth} \cdot g}}$$

$$d_4 = 20.876 \text{ mm}$$

PROBLEM 12-20

Statement: The linkage in Figure P12-9a has rectangular steel links of 20 x 10 mm cross-section similar to that shown in Figure 12-10a. Design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment. State all assumptions.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a_2 := 78 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad a_3 := 109 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4) \quad a_4 := 121 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad a_1 := 54 \cdot \text{mm}$$

$$\text{Link cross-section dims: } h := 20 \cdot \text{mm} \quad t := 10 \cdot \text{mm}$$

$$\text{Material density: steel } \rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3}$$

Solution: See Figure P12-9a and Mathcad file P1220.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \cdot \sqrt{3 \cdot \left(\frac{a_3}{h}\right)^2 - 1} - \frac{a_3}{2} \quad e = 39.366 \text{ mm}$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 187.731 \text{ mm}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 39.000 \text{ mm}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 54.500 \text{ mm}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 60.500 \text{ mm}$$

3. Determine the mass of each link.

$$m_2 := h \cdot t \cdot a_2 \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := h \cdot t \cdot a_4 \cdot \rho$$

$$m_2 = 0.122 \text{ kg} \quad m_3 = 0.293 \text{ kg} \quad m_4 = 0.189 \text{ kg}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 54.500 \text{ mm}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3}\right) \quad mr_2 = 0.0114 \text{ kg} \cdot \text{m}$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3}\right) \quad mr_4 = 0.0177 \text{ kg} \cdot \text{m}$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 6.6760 \times 10^{-3} \text{ kg} \cdot \text{m}$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 6.2981 \times 10^{-3} \text{ kg} \cdot \text{m}$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 75 \cdot \text{mm} \quad R_4 := 70.8 \cdot \text{mm}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 0.0890 \text{ kg}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 0.0890 \text{ kg}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 0.211 \text{ kg} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 54.2 \text{ mm}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 0.278 \text{ kg} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 63.8 \text{ mm}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 1.161 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 74.244 \text{ mm}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 3.157 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 106.625 \text{ mm}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2^2 + r_2^2 + a_2 r_2) \quad I_{cw2} = 2.671 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_{cw4} := m'_4(k'_4^2 + r_4^2 + a_4 r_4) \quad I_{cw4} = 6.432 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

PROBLEM 12-21

Statement: The linkage in Figure P12-9a has rectangular steel links of 20 x 10 mm cross-section with "dog bone" ends of 50 mm dia, similar to that shown in Figure 12-10b. Design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment. State all assumptions.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a_2 := 78 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad a_3 := 109 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4) \quad a_4 := 121 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad a_1 := 54 \cdot \text{mm}$$

$$\text{Link cross-section dims: } h := 20 \cdot \text{mm} \quad t := 10 \cdot \text{mm} \quad c := 25 \cdot \text{mm}$$

$$\text{Material density: steel } \rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3}$$

Assumption: Only link 3 has the "dog bone" configuration.

Solution: See Figure P12-9a and Mathcad file P1221.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equations 12.13.

$$B := 12 \cdot \left(\frac{a_3}{c} \right) + 24 \quad B = 76.320 \quad C := 24 \cdot \left(\frac{a_3}{c} \right) + 26 \quad C = 130.640$$

$$D := -2 \cdot \left(\frac{a_3}{c} \right)^3 + 13 \cdot \left(\frac{a_3}{c} \right) + 12 \cdot \pi - 10 \quad D = -81.385$$

$$v := \begin{pmatrix} D \\ C \\ B \\ 8 \end{pmatrix} \quad r := \text{polyroots}(v) \quad r = \begin{pmatrix} -6.999 \\ -3.022 \\ 0.481 \end{pmatrix} \quad \text{Choosing the positive root,}$$

$$e := h \cdot r_3 \quad e = 9.620 \text{ mm}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 39.000 \text{ mm}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 54.500 \text{ mm}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 60.500 \text{ mm}$$

3. Determine the mass of each link.

$$m_2 := h \cdot t \cdot a_2 \cdot \rho \quad m_3 := t \cdot \left[h \cdot (a_3 - 2 \cdot c + 2 \cdot e) + 2 \cdot \pi \cdot c^2 \right] \cdot \rho \quad m_4 := h \cdot t \cdot a_4 \cdot \rho$$

$$m_2 = 0.122 \text{ kg} \quad m_3 = 0.428 \text{ kg} \quad m_4 = 0.189 \text{ kg}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 54.500 \text{ mm}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3} \right) \quad mr_2 = 0.0167 \text{ kg} \cdot \text{m}$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3} \right) \quad mr_4 = 0.0259 \text{ kg} \cdot \text{m}$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 0.0120 \text{ kg} \cdot \text{m}$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 0.0145 \text{ kg} \cdot \text{m}$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 75 \cdot mm$$

$$R_4 := 91 \cdot mm$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2}$$

$$m_{2b} = 0.159 \text{ kg}$$

$$m_{4b} := \frac{mR_4}{R_4}$$

$$m_{4b} = 0.159 \text{ kg}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 0.281 \text{ kg} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 59.4 \text{ mm}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 0.348 \text{ kg} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 74.5 \text{ mm}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 1.283 \times 10^{-3} \text{ kg} \cdot m^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 67.555 \text{ mm}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 3.718 \times 10^{-3} \text{ kg} \cdot m^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 103.358 \text{ mm}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2^2 + r_2^2 + a_2 \cdot r_2) \quad I_{cw2} = 3.579 \times 10^{-3} \text{ kg} \cdot m^2$$

$$I_{cw4} := m'_4(k'_4^2 + r_4^2 + a_4 \cdot r_4) \quad I_{cw4} = 8.784 \times 10^{-3} \text{ kg} \cdot m^2$$

PROBLEM 12-22

Statement: The linkage in Figure P12-9b has rectangular steel links of 20 x 10 mm cross-section similar to that shown in Figure 12-10a. Design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment. State all assumptions.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a_2 := 50 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad a_3 := 185 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4) \quad a_4 := 90 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad a_1 := 172 \cdot \text{mm}$$

$$\text{Link cross-section dims: } h := 20 \cdot \text{mm} \quad t := 10 \cdot \text{mm}$$

$$\text{Material density: steel } \rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3}$$

Solution: See Figure P12-9b and Mathcad file P1222.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \cdot \sqrt{3 \cdot \left(\frac{a_3}{h}\right)^2 - 1} - \frac{a_3}{2} \quad e = 67.402 \text{ mm}$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 319.805 \text{ mm}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 25.000 \text{ mm}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 92.500 \text{ mm}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 45.000 \text{ mm}$$

3. Determine the mass of each link.

$$m_2 := h \cdot t \cdot a_2 \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := h \cdot t \cdot a_4 \cdot \rho$$

$$m_2 = 0.078 \text{ kg} \quad m_3 = 0.499 \text{ kg} \quad m_4 = 0.140 \text{ kg}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 92.500 \text{ mm}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3}\right) \quad mr_2 = 0.0125 \text{ kg} \cdot \text{m}$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3}\right) \quad mr_4 = 0.0225 \text{ kg} \cdot \text{m}$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 0.0105 \text{ kg} \cdot \text{m}$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 0.0161 \text{ kg} \cdot \text{m}$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 75 \cdot \text{mm} \quad R_4 := 115 \cdot \text{mm}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 0.140 \text{ kg}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 0.140 \text{ kg}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 0.218 \text{ kg} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 57.1 \text{ mm}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 0.281 \text{ kg} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 80.0 \text{ mm}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 5.898 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 51.980 \text{ mm}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 2.465 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 93.707 \text{ mm}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2^2 + r_2^2 + a_2 \cdot r_2) \quad I_{cw2} = 1.926 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_{cw4} := m'_4(k'_4^2 + r_4^2 + a_4 \cdot r_4) \quad I_{cw4} = 6.281 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

PROBLEM 12-23

Statement: The linkage in Figure P12-9b has rectangular steel links of 20 x 10 mm cross-section with "dog bone" ends of 50 mm dia, similar to that shown in Figure 12-10b. Design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment. State all assumptions.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a_2 := 50 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad a_3 := 185 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4) \quad a_4 := 90 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad a_1 := 172 \cdot \text{mm}$$

$$\text{Link cross-section dims: } h := 20 \cdot \text{mm} \quad t := 10 \cdot \text{mm} \quad c := 25 \cdot \text{mm}$$

$$\text{Material density: steel } \rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3}$$

Assumption: Only link 3 has the "dog bone" configuration.

Solution: See Figure P12-9b and Mathcad file P1223.

- Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equations 12.13.

$$B := 12 \cdot \left(\frac{a_3}{c} \right) + 24 \quad B = 112.800 \quad C := 24 \cdot \left(\frac{a_3}{c} \right) + 26 \quad C = 203.600$$

$$D := -2 \cdot \left(\frac{a_3}{c} \right)^3 + 13 \cdot \left(\frac{a_3}{c} \right) + 12 \cdot \pi - 10 \quad D = -686.549$$

$$v := \begin{pmatrix} D \\ C \\ B \\ 8 \end{pmatrix} \quad r := \text{polyroots}(v) \quad r = \begin{pmatrix} -11.116 \\ -4.646 \\ 1.662 \end{pmatrix} \quad \text{Choosing the positive root,}$$

$$e := h \cdot r_3 \quad e = 33.236 \text{ mm}$$

- Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 25.000 \text{ mm}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 92.500 \text{ mm}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 45.000 \text{ mm}$$

- Determine the mass of each link.

$$m_2 := h \cdot t \cdot a_2 \cdot \rho \quad m_3 := t \cdot \left[h \cdot (a_3 - 2 \cdot c + 2 \cdot e) + 2 \cdot \pi \cdot c^2 \right] \cdot \rho \quad m_4 := h \cdot t \cdot a_4 \cdot \rho$$

$$m_2 = 0.078 \text{ kg} \quad m_3 = 0.621 \text{ kg} \quad m_4 = 0.140 \text{ kg}$$

- Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 92.500 \text{ mm}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3} \right) \quad mr_2 = 0.0155 \text{ kg} \cdot \text{m}$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3} \right) \quad mr_4 = 0.0279 \text{ kg} \cdot \text{m}$$

- Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 0.0136 \text{ kg} \cdot \text{m}$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 0.0216 \text{ kg} \cdot \text{m}$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 75 \cdot \text{mm} \quad R_4 := 119.5 \cdot \text{mm}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 0.181 \text{ kg}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 0.181 \text{ kg}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 0.259 \text{ kg} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 59.9 \text{ mm}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 0.321 \text{ kg} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 86.9 \text{ mm}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 6.226 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 49.041 \text{ mm}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 2.735 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 92.276 \text{ mm}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2^2 + r_2^2 + a_2 \cdot r_2) \quad I_{cw2} = 2.328 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_{cw4} := m'_4(k'_4^2 + r_4^2 + a_4 \cdot r_4) \quad I_{cw4} = 7.677 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

PROBLEM 12-24

Statement: The device in Figure P12-10 is used to balance fan blade/hub assemblies. The center distance between the two bearings on the machine is 250 mm. The left edge of the fan hub (plane A) is 100 mm from the centerline of the closest bearing (at F_2). The hub is 75 mm wide and has a diameter of 200 mm along the surfaces where balancing weights are fastened. The peak magnitude of force F_1 is 0.5 N at a phase angle of 30 deg with respect to the rotating x' axis. Force F_2 had a peak of 0.2 N at a phase angle of 130 deg. Calculate the magnitudes and locations, with respect to the x' axis, of balance weights placed in planes A and B of the hub to dynamically balance the fan assembly.

Given: Bearing forces and plane locations with respect to correction plane A:

$$\text{Left: } F_1 := 0.5 \cdot N \quad \theta_1 := 30 \cdot \text{deg} \quad l_1 := 350 \cdot \text{mm}$$

$$\text{Right: } F_2 := 0.2 \cdot N \quad \theta_2 := 130 \cdot \text{deg} \quad l_2 := 100 \cdot \text{mm}$$

$$\text{Distance between correction planes: } l_B := 75 \cdot \text{mm}$$

$$\text{Correction weight radii: } R_A := 100 \cdot \text{mm} \quad R_B := 100 \cdot \text{mm}$$

$$\text{Fan rotational speed: } \omega := 600 \cdot \text{rpm}$$

Solution: See Figure P12-10 and Mathcad file P1224.

1. Resolve the force vectors into xy components with respect to the zero reference angle of the tire.

$$F_{1x} := F_1 \cdot \cos(\theta_1) \quad F_{1x} = 0.433 \, N \quad F_{1y} := F_1 \cdot \sin(\theta_1) \quad F_{1y} = 0.250 \, N$$

$$F_{2x} := F_2 \cdot \cos(\theta_2) \quad F_{2x} = -0.129 \, N \quad F_{2y} := F_2 \cdot \sin(\theta_2) \quad F_{2y} = 0.153 \, N$$

2. Solve equations 12.4e for summation of moments about O, modified for the bearing forces.

$$mR_{Bx} := \frac{F_{1x} \cdot l_1 + F_{2x} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{Bx} = 0.468 \, \text{mm} \cdot \text{kg}$$

$$mR_{By} := \frac{F_{1y} \cdot l_1 + F_{2y} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{By} = 0.347 \, \text{mm} \cdot \text{kg}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B. Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 36.550 \, \text{deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 0.583 \, \text{mm} \cdot \text{kg}$$

$$W_B := \frac{mR_B}{R_B} \cdot g \quad W_B = 0.057 \, N$$

4. Solve equations 12.4c for forces in x and y directions in plane A.

$$mR_{Ax} := \frac{F_{1x} + F_{2x}}{\omega^2} - mR_{Bx} \quad mR_{Ax} = -0.391 \, \text{mm} \cdot \text{kg}$$

$$mR_{Ay} := \frac{F_{1y} + F_{2y}}{\omega^2} - mR_{By} \quad mR_{Ay} = -0.245 \, \text{mm} \cdot \text{kg}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A.

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = -147.936 \, \text{deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 0.462 \, \text{mm} \cdot \text{kg}$$

$$W_A := \frac{mR_A \cdot g}{R_A} \quad W_A = 0.045 \, N$$

PROBLEM 12-25

Statement: The device in Figure P12-10 is used to balance fan blade/hub assemblies. The center distance between the two bearings on the machine is 250 mm. The left edge of the fan hub (plane A) is 100 mm from the centerline of the closest bearing (at F_2). The hub is 55 mm wide and has a diameter of 150 mm along the surfaces where balancing weights are fastened. The peak magnitude of force F_1 is 1.5 N at a phase angle of 60 deg with respect to the rotating x' axis. Force F_2 had a peak of 2.0 N at a phase angle of -180 deg. Calculate the magnitudes and locations, with respect to the x' axis, of balance weights placed in planes A and B of the hub to dynamically balance the fan assembly.

Given: Bearing forces and plane locations with respect to correction plane A:

$$\text{Left: } F_1 := 1.5 \cdot N \quad \theta_1 := 60 \cdot \text{deg} \quad l_1 := 350 \cdot \text{mm}$$

$$\text{Right: } F_2 := 2.0 \cdot N \quad \theta_2 := -180 \cdot \text{deg} \quad l_2 := 100 \cdot \text{mm}$$

$$\text{Distance between correction planes: } l_B := 55 \cdot \text{mm}$$

$$\text{Correction weight radii: } R_A := 75 \cdot \text{mm} \quad R_B := 75 \cdot \text{mm}$$

$$\text{Fan rotational speed: } \omega := 600 \cdot \text{rpm}$$

Solution: See Figure P12-10 and Mathcad file P1225.

1. Resolve the force vectors into xy components with respect to the zero reference angle of the tire.

$$F_{1x} := F_1 \cdot \cos(\theta_1) \quad F_{1x} = 0.750 \, N \quad F_{1y} := F_1 \cdot \sin(\theta_1) \quad F_{1y} = 1.299 \, N$$

$$F_{2x} := F_2 \cdot \cos(\theta_2) \quad F_{2x} = -2.000 \, N \quad F_{2y} := F_2 \cdot \sin(\theta_2) \quad F_{2y} = 0.000 \, N$$

2. Solve equations 12.4e for summation of moments about O, modified for the bearing forces.

$$mR_{Bx} := \frac{F_{1x} \cdot l_1 + F_{2x} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{Bx} = 0.288 \, \text{mm} \cdot \text{kg}$$

$$mR_{By} := \frac{F_{1y} \cdot l_1 + F_{2y} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{By} = 2.094 \, \text{mm} \cdot \text{kg}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B. Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 82.173 \, \text{deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 2.114 \, \text{mm} \cdot \text{kg}$$

$$W_B := \frac{mR_B}{R_B} \cdot g \quad W_B = 0.28 \, N$$

4. Solve equations 12.4c for forces in x and y directions in plane A.

$$mR_{Ax} := \frac{F_{1x} + F_{2x}}{\omega^2} - mR_{Bx} \quad mR_{Ax} = -0.604 \, \text{mm} \cdot \text{kg}$$

$$mR_{Ay} := \frac{F_{1y} + F_{2y}}{\omega^2} - mR_{By} \quad mR_{Ay} = -1.765 \, \text{mm} \cdot \text{kg}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A.

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = -108.906 \, \text{deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2}$$

$$mR_A = 1.866 \text{ mm} \cdot \text{kg}$$

$$W_A := \frac{mR_A \cdot g}{R_A}$$

$$W_A = 0.24 \text{ N}$$

PROBLEM 12-26

Statement: The device in Figure P12-10 is used to balance fan blade/hub assemblies. The center distance between the two bearings on the machine is 250 mm. The left edge of the fan hub (plane A) is 100 mm from the centerline of the closest bearing (at F_2). The hub is 125 mm wide and has a diameter of 250 mm along the surfaces where balancing weights are fastened. The peak magnitude of force F_1 is 1.1 N at a phase angle of 120 deg with respect to the rotating x' axis. Force F_2 had a peak of 1.8 N at a phase angle of -93 deg. Calculate the magnitudes and locations, with respect to the x' axis, of balance weights placed in planes A and B of the hub to dynamically balance the fan assembly.

Given: Bearing forces and plane locations with respect to correction plane A:

$$\text{Left: } F_1 := 1.1 \cdot \text{N} \quad \theta_1 := 120 \cdot \text{deg} \quad l_1 := 350 \cdot \text{mm}$$

$$\text{Right: } F_2 := 1.8 \cdot \text{N} \quad \theta_2 := -93 \cdot \text{deg} \quad l_2 := 100 \cdot \text{mm}$$

$$\text{Distance between correction planes: } l_B := 125 \cdot \text{mm}$$

$$\text{Correction weight radii: } R_A := 125 \cdot \text{mm} \quad R_B := 125 \cdot \text{mm}$$

$$\text{Fan rotational speed: } \omega := 600 \cdot \text{rpm}$$

Solution: See Figure P12-10 and Mathcad file P1226.

1. Resolve the force vectors into xy components with respect to the zero reference angle of the tire.

$$F_{1x} := F_1 \cdot \cos(\theta_1) \quad F_{1x} = -0.550 \text{ N} \quad F_{1y} := F_1 \cdot \sin(\theta_1) \quad F_{1y} = 0.953 \text{ N}$$

$$F_{2x} := F_2 \cdot \cos(\theta_2) \quad F_{2x} = -0.094 \text{ N} \quad F_{2y} := F_2 \cdot \sin(\theta_2) \quad F_{2y} = -1.798 \text{ N}$$

2. Solve equations 12.4e for summation of moments about O, modified for the bearing forces.

$$mR_{Bx} := \frac{F_{1x} \cdot l_1 + F_{2x} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{Bx} = -0.409 \text{ mm} \cdot \text{kg}$$

$$mR_{By} := \frac{F_{1y} \cdot l_1 + F_{2y} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{By} = 0.311 \text{ mm} \cdot \text{kg}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B. Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 142.728 \text{ deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 0.514 \text{ mm} \cdot \text{kg}$$

$$W_B := \frac{mR_B}{R_B} \cdot g \quad W_B = 0.040 \text{ N}$$

4. Solve equations 12.4c for forces in x and y directions in plane A.

$$mR_{Ax} := \frac{F_{1x} + F_{2x}}{\omega^2} - mR_{Bx} \quad mR_{Ax} = 0.246 \text{ mm} \cdot \text{kg}$$

$$mR_{Ay} := \frac{F_{1y} + F_{2y}}{\omega^2} - mR_{By} \quad mR_{Ay} = -0.525 \text{ mm} \cdot \text{kg}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A.

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = -64.911 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2}$$

$$mR_A = 0.580 \text{ mm} \cdot \text{kg}$$

$$W_A := \frac{mR_A \cdot g}{R_A}$$

$$W_A = 0.046 \text{ N}$$

PROBLEM 12-27

Statement: Figure P12-11a shows a fourbar linkage and its dimensions in mm. All links are 4-mm-thick steel. Link 3 has a uniform cross section and is 20 mm wide with ends that extend 10 mm beyond the pivot hole. Links 2 and 4 have a 10-mm radius at each end. Design counterweights to force balance the linkage using the method of Berkof and Lowen.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad L_2 := 58 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad L_3 := 108 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4) \quad L_4 := 110 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad L_1 := 160 \cdot \text{mm}$$

$$\text{Link cross section: } h := 20 \cdot \text{mm} \quad t := 4 \cdot \text{mm} \quad r := 10 \cdot \text{mm}$$

$$\text{Mass center: } b_2 := 0.5 \cdot L_2 \quad b_3 := 0.5 \cdot L_3 \quad b_4 := 0.5 \cdot L_4$$

$$\phi_2 := 0 \cdot \text{deg} \quad \phi_3 := 0 \cdot \text{deg} \quad \phi_4 := 0 \cdot \text{deg}$$

$$\text{Mass density of steel: } \rho := 7.8 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^{-3}$$

Solution: See Figure P12-11 and Mathcad file P1227.

1. Calculate the mass of each link.

$$m_2 := (L_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_2 = 0.046 \text{ kg}$$

$$m_3 := (L_3 + 2 \cdot r) \cdot h \cdot t \cdot \rho \quad m_3 = 0.080 \text{ kg}$$

$$m_4 := (L_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_4 = 0.078 \text{ kg}$$

2. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \cos(\phi_3) - L_2 \right) \quad mb_{2x} = -2.316 \text{ kg} \cdot \text{mm}$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.000 \text{ kg} \cdot \text{mm}$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -4.393 \text{ kg} \cdot \text{mm}$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.000 \text{ kg} \cdot \text{mm}$$

3. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -3.650 \text{ kg} \cdot \text{mm}$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.000 \text{ kg} \cdot \text{mm}$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -8.707 \text{ kg} \cdot \text{mm}$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 0.000 \text{ kg} \cdot \text{mm}$$

4. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 3.6501 \text{ kg} \cdot \text{mm}$$

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 8.707 \text{ kg} \cdot \text{mm}$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 180.000 \text{ deg}$$

PROBLEM 12-28

Statement: Use the data of Problem 12-27 to design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment the linkage exerts on the ground link.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a_2 := 58 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad a_3 := 108 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4) \quad a_4 := 110 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad a_1 := 160 \cdot \text{mm}$$

$$\text{Link cross-section dims: } h := 20 \cdot \text{mm} \quad t := 4 \cdot \text{mm}$$

$$\text{Material density: steel } \rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3}$$

Solution: See Figure P12-11a and Mathcad file P1228.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \cdot \sqrt{3 \cdot \left(\frac{a_3}{h} \right)^2 - 1} - \frac{a_3}{2} \quad e = 38.995 \text{ mm}$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 185.989 \text{ mm}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 29.000 \text{ mm}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 54.000 \text{ mm}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 55.000 \text{ mm}$$

3. Determine the mass of each link.

$$m_2 := h \cdot t \cdot a_2 \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := h \cdot t \cdot a_4 \cdot \rho$$

$$m_2 = 0.036 \text{ kg} \quad m_3 = 0.116 \text{ kg} \quad m_4 = 0.069 \text{ kg}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 54.000 \text{ mm}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3} \right) \quad mr_2 = 3.366 \text{ kg} \cdot \text{mm}$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3} \right) \quad mr_4 = 6.383 \text{ kg} \cdot \text{mm}$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 2.316 \text{ kg} \cdot \text{mm}$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 2.608 \text{ kg} \cdot \text{mm}$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 75 \cdot \text{mm} \quad R_4 := 84.5 \cdot \text{mm}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 0.031 \text{ kg}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 0.031 \text{ kg}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 0.067 \text{ kg} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 50.2 \text{ mm}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 0.100 \text{ kg} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 64.2 \text{ mm}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 2.573 \times 10^{-4} \text{ kg}\cdot\text{m}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 61.933 \text{ mm}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 1.059 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 103.152 \text{ mm}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2^2 + r_2^2 + a_2 \cdot r_2) \quad I_{cw2} = 6.214 \times 10^{-4} \text{ kg}\cdot\text{m}^2$$

$$I_{cw4} := m'_4(k'_4^2 + r_4^2 + a_4 \cdot r_4) \quad I_{cw4} = 2.170 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

PROBLEM 12-29

Statement: Figure P12-11b shows a fourbar linkage and its dimensions in inches. All links have a uniform 0.5-in wide x 0.2-in thick cross-section and are made from steel. Link 3 has squared ends that extend 0.25 in from the pivot point centers. Links 2 and 4 have rounded ends that have a radius of 0.25 in. Design counterweights to force balance the linkage using the method of Berkof and Lowen.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad L_2 := 2.75 \cdot in \quad \text{Link 3 (A to B)} \quad L_3 := 3.26 \cdot in$$

$$\text{Link 4 (B to O}_4) \quad L_4 := 2.95 \cdot in \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad L_1 := 3.26 \cdot in$$

$$\text{Link cross section: } h := 0.50 \cdot in \quad t := 0.20 \cdot in \quad r := 0.25 \cdot in$$

$$\text{Mass center: } b_2 := 0.5 \cdot L_2 \quad b_3 := 0.5 \cdot L_3 \quad b_4 := 0.5 \cdot L_4$$

$$\phi_2 := 0 \cdot deg \quad \phi_3 := 0 \cdot deg \quad \phi_4 := 0 \cdot deg$$

$$\text{Mass density of steel: } \rho := 7.8 \cdot 10^3 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-11 and Mathcad file P1229.

1. Calculate the mass of each link.

$$m_2 := (L_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_3 := (L_3 + 2 \cdot r) \cdot h \cdot t \cdot \rho \quad m_4 := (L_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho$$

$$m_2 = 2.294 \times 10^{-4} \cdot blob \quad m_3 = 2.744 \times 10^{-4} \cdot blob \quad m_4 = 2.440 \times 10^{-4} \cdot blob$$

2. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \cos(\phi_3) - L_2 \right) \quad mb_{2x} = -3.773 \times 10^{-4} \cdot blob \cdot in$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.000 \cdot blob \cdot in$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -4.048 \times 10^{-4} \cdot blob \cdot in$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.000 \cdot blob \cdot in$$

3. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -6.927 \times 10^{-4} \cdot blob \cdot in$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.000 \cdot blob \cdot in$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -7.646 \times 10^{-4} \cdot blob \cdot in$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 0.000 \cdot blob \cdot in$$

4. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 6.927 \times 10^{-4} \cdot blob \cdot in$$

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \cdot deg$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 7.646 \times 10^{-4} \cdot blob \cdot in$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 180.000 \cdot deg$$

PROBLEM 12-30

Statement: Use the data of Problem 12-29 to design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment the linkage exerts on the ground link.

Units: $blob := lb \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a_2 := 2.75 \cdot in \quad \text{Link 3 (A to B)} \quad a_3 := 3.26 \cdot in$$

$$\text{Link 4 (B to O}_4) \quad a_4 := 2.95 \cdot in \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad a_1 := 3.26 \cdot in$$

$$\text{Link cross-section dims:} \quad h := 0.50 \cdot in \quad t := 0.20 \cdot in \quad r := 0.25 \cdot in$$

$$\text{Material density: steel} \quad \rho := 7800 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-11b and Mathcad file P1230.

- Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \cdot \sqrt{3 \cdot \left(\frac{a_3}{h}\right)^2 - 1} - \frac{a_3}{2} \quad e = 1.182 \cdot in$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 5.624 \cdot in$$

- Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 1.375 \cdot in$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 1.630 \cdot in$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 1.475 \cdot in$$

- Determine the mass of each link.

$$m_2 := \left(a_2 \cdot h + \pi \cdot r^2\right) \cdot t \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := \left(a_4 \cdot h + \pi \cdot r^2\right) \cdot t \cdot \rho$$

$$m_2 = 2.294 \times 10^{-4} \cdot blob \quad m_3 = 4.105 \times 10^{-4} \cdot blob \quad m_4 = 2.440 \times 10^{-4} \cdot blob$$

- Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 1.630 \cdot in$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3}\right) \quad mr_2 = 5.644 \times 10^{-4} \cdot blob \cdot in$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3}\right) \quad mr_4 = 6.055 \times 10^{-4} \cdot blob \cdot in$$

- Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 2.490 \times 10^{-4} \cdot blob \cdot in$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 2.456 \times 10^{-4} \cdot blob \cdot in$$

- Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 2 \cdot \text{in}$$

$$R_4 := 1.973 \cdot \text{in}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 1.245 \times 10^{-4} \text{ blob}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 1.245 \times 10^{-4} \text{ blob}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 3.539 \times 10^{-4} \text{ blob} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 1.595 \text{ in}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 3.685 \times 10^{-4} \text{ blob} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 1.6 \text{ in}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 2.193 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 2.489 \text{ in}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 2.568 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 2.640 \text{ in}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2^2 + r_2^2 + a_2 \cdot r_2) \quad I_{cw2} = 4.645 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

$$I_{cw4} := m'_4(k'_4^2 + r_4^2 + a_4 \cdot r_4) \quad I_{cw4} = 5.349 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

PROBLEM 12-31

Statement: Figure P12-12 shows a fourbar linkage and its dimensions in inches. All links have a uniform 0.5-in wide x 0.2-in thick cross-section and are made from aluminum. Link 3 has squared ends that extend 0.25 in from the pivot point centers. Links 2 and 4 have rounded ends that have a radius of 0.25 in. Design counterweights to force balance the linkage using the method of Berkof and Lowen.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad L_2 := 3.44 \cdot in \quad \text{Link 3 (A to B)} \quad L_3 := 7.40 \cdot in$$

$$\text{Link 4 (B to O}_4) \quad L_4 := 5.44 \cdot in \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad L_1 := 8.88 \cdot in$$

$$\text{Link cross section:} \quad h := 0.50 \cdot in \quad t := 0.20 \cdot in \quad r := 0.25 \cdot in$$

$$\text{Mass center:} \quad b_2 := 0.5 \cdot L_2 \quad b_3 := 0.5 \cdot L_3 \quad b_4 := 0.5 \cdot L_4$$

$$\phi_2 := 0 \cdot deg \quad \phi_3 := 0 \cdot deg \quad \phi_4 := 0 \cdot deg$$

$$\text{Mass density of aluminum: } \rho := 2.8 \cdot 10^3 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-12 and Mathcad file P1231.

1. Calculate the mass of each link.

$$m_2 := (L_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_3 := (L_3 + 2 \cdot r) \cdot h \cdot t \cdot \rho \quad m_4 := (L_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho$$

$$m_2 = 1.004 \times 10^{-4} \cdot blob \quad m_3 = 2.070 \times 10^{-4} \cdot blob \quad m_4 = 1.528 \times 10^{-4} \cdot blob$$

2. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \cos(\phi_3) - L_2 \right) \quad mb_{2x} = -3.560 \times 10^{-4} \cdot blob \cdot in$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.000 \cdot blob \cdot in$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -5.630 \times 10^{-4} \cdot blob \cdot in$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.000 \cdot blob \cdot in$$

3. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -5.287 \times 10^{-4} \cdot blob \cdot in$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.000 \cdot blob \cdot in$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -9.787 \times 10^{-4} \cdot blob \cdot in$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 0.000 \cdot blob \cdot in$$

4. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 5.287 \times 10^{-4} \cdot blob \cdot in$$

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \cdot deg$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 9.787 \times 10^{-4} \cdot blob \cdot in$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 180.000 \cdot deg$$

PROBLEM 12-32

Statement: Use the data of Problem 12-31 to design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment the linkage exerts on the ground link.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a_2 := 3.44 \cdot in \quad \text{Link 3 (A to B)} \quad a_3 := 7.40 \cdot in$$

$$\text{Link 4 (B to O}_4) \quad a_4 := 5.44 \cdot in \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad a_1 := 8.88 \cdot in$$

$$\text{Link cross-section dims:} \quad h := 0.50 \cdot in \quad t := 0.20 \cdot in \quad r := 0.25 \cdot in$$

$$\text{Material density: alum} \quad \rho := 2800 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-12 and Mathcad file P1232.

- Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \cdot \sqrt{3 \cdot \left(\frac{a_3}{h}\right)^2 - 1} - \frac{a_3}{2} \quad e = 2.704 \cdot in$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 12.807 \cdot in$$

- Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 1.720 \cdot in$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 3.700 \cdot in$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 2.720 \cdot in$$

- Determine the mass of each link.

$$m_2 := \left(a_2 \cdot h + \pi \cdot r^2\right) \cdot t \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := \left(a_4 \cdot h + \pi \cdot r^2\right) \cdot t \cdot \rho$$

$$m_2 = 1.004 \times 10^{-4} \cdot blob \quad m_3 = 3.356 \times 10^{-4} \cdot blob \quad m_4 = 1.528 \times 10^{-4} \cdot blob$$

- Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 3.700 \cdot in$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3}\right) \quad mr_2 = 5.772 \times 10^{-4} \cdot blob \cdot in$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3}\right) \quad mr_4 = 9.127 \times 10^{-4} \cdot blob \cdot in$$

- Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 4.044 \times 10^{-4} \cdot blob \cdot in$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 4.971 \times 10^{-4} \cdot blob \cdot in$$

- Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 2 \cdot in \quad R_4 := 2.458 \cdot in$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 2.022 \times 10^{-4} \text{ blob}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 2.022 \times 10^{-4} \text{ blob}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 3.026 \times 10^{-4} \text{ blob} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 1.907 \text{ in}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 3.550 \times 10^{-4} \text{ blob} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 2.571 \text{ in}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 1.424 \times 10^{-3} \text{ blob}\cdot\text{in}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 2.169 \text{ in}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 4.660 \times 10^{-3} \text{ blob}\cdot\text{in}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 3.623 \text{ in}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2^2 + r_2^2 + a_2 \cdot r_2) \quad I_{cw2} = 4.510 \times 10^{-3} \text{ blob}\cdot\text{in}^2$$

$$I_{cw4} := m'_4(k'_4^2 + r_4^2 + a_4 \cdot r_4) \quad I_{cw4} = 0.0120 \text{ blob}\cdot\text{in}^2$$

PROBLEM 12-33

Statement: Figure P12-13 shows a fourbar linkage and its dimensions in inches. Links 2 and 4 are rectangular steel with a 1-in wide x 0.12-in thick cross-section and 0.5-in radius ends. The coupler is 0.25-in-thick aluminum with 0.5-in radii at points A, B, and P. Design counterweights to force balance the linkage using the method of Berkof and Lowen.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad L_2 := 5.00 \cdot in \quad \text{Link 3 (A to B)} \quad L_3 := 7.40 \cdot in$$

$$\text{Link 4 (B to O}_4) \quad L_4 := 8.00 \cdot in \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad L_1 := 9.50 \cdot in$$

$$\text{Link cross section: } h := 0.50 \cdot in \quad t := 0.20 \cdot in \quad r := 0.25 \cdot in$$

$$\begin{aligned} \text{Mass center: } b_2 &:= 0.5 \cdot L_2 & b_4 &:= 0.5 \cdot L_4 & \text{Coupler point data: } p &:= 8.9 \cdot in & \delta &:= 56 \cdot deg \\ \phi_2 &:= 0 \cdot deg & \phi_4 &:= 0 \cdot deg \end{aligned}$$

$$\text{Mass density of steel and aluminum: } \rho_1 := 7.8 \cdot 10^3 \cdot kg \cdot m^{-3} \quad \rho_2 := 2.8 \cdot 10^3 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-13 and Mathcad file P1233.

1. Determine the location of the mass center and the mass of link 3. Take AB as the base of the triangle. Then,

$$\text{The height is: } h := p \cdot \sin(\delta) \quad h = 7.378 \cdot in \quad t_3 := 0.25 \cdot in$$

$$\text{The horizontal distance from A to P is: } AP_x := p \cdot \cos(\delta) \quad AP_x = 4.977 \cdot in$$

$$xbar := \frac{L_3 + AP_x}{3} \quad xbar = 4.126 \cdot in \quad ybar := \frac{h}{3} \quad ybar = 2.459 \cdot in$$

$$b_3 := \sqrt{xbar^2 + ybar^2} \quad b_3 = 4.803 \cdot in \quad \phi_3 := \text{atan}\left(\frac{ybar}{xbar}\right) \quad \phi_3 = 30.801 \cdot deg$$

$$m_3 := \rho_2 \cdot t_3 \cdot \frac{L_3 \cdot h}{2} \quad m_3 = 1.788 \times 10^{-3} \cdot blob$$

2. Calculate the mass of links 2 and 4.

$$m_2 := (L_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho_1 \quad m_4 := (L_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho_1$$

$$m_2 = 5.414 \times 10^{-3} \cdot blob \quad m_4 = 8.645 \times 10^{-3} \cdot blob$$

2. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \cos(\phi_3) - L_2 \right) \quad mb_{2x} = -3.956 \times 10^{-3} \cdot blob \cdot in$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 2.972 \times 10^{-3} \cdot blob \cdot in$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -7.976 \times 10^{-3} \cdot blob \cdot in$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = -4.755 \times 10^{-3} \cdot blob \cdot in$$

3. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -0.017 \cdot blob \cdot in$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2)$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4)$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4)$$

$$mR_{2y} = 2.972 \times 10^{-3} \text{ blob} \cdot \text{in}$$

$$mR_{4x} = -0.043 \text{ blob} \cdot \text{in}$$

$$mR_{4y} = -4.755 \times 10^{-3} \text{ blob} \cdot \text{in}$$

4. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2}$$

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y})$$

$$mR_{b2} = 0.0177 \text{ blob} \cdot \text{in}$$

$$\theta_{b2} = 170.358 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2}$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y})$$

$$mR_{b4} = 0.0428 \text{ blob} \cdot \text{in}$$

$$\theta_{b4} = -173.625 \text{ deg}$$

PROBLEM 12-34

Statement: Use the data of Problem 12-33, changing link 3 to be steel with the same cross-section dimensions as links 2 and 4, to design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment the linkage exerts on the ground link.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a_2 := 5.00 \cdot in \quad \text{Link 3 (A to B)} \quad a_3 := 7.40 \cdot in$$

$$\text{Link 4 (B to O}_4) \quad a_4 := 8.00 \cdot in \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad a_1 := 9.50 \cdot in$$

$$\text{Link cross-section dims: } h := 0.50 \cdot in \quad t := 0.20 \cdot in \quad r := 0.25 \cdot in$$

$$\text{Material density: steel } \rho := 7800 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-13 and Mathcad file P1234.

- Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \cdot \sqrt{3 \cdot \left(\frac{a_3}{h}\right)^2 - 1} - \frac{a_3}{2} \quad e = 2.704 \cdot in$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 12.807 \cdot in$$

- Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 2.500 \cdot in$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 3.700 \cdot in$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 4.000 \cdot in$$

- Determine the mass of each link.

$$m_2 := (a_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := (a_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho$$

$$m_2 = 3.936 \times 10^{-4} \cdot blob \quad m_3 = 9.348 \times 10^{-4} \cdot blob \quad m_4 = 6.126 \times 10^{-4} \cdot blob$$

- Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 3.700 \cdot in$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3}\right) \quad mr_2 = 2.337 \times 10^{-3} \cdot blob \cdot in$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3}\right) \quad mr_4 = 3.739 \times 10^{-3} \cdot blob \cdot in$$

- Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 1.353 \times 10^{-3} \cdot blob \cdot in$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 1.289 \times 10^{-3} \cdot blob \cdot in$$

- Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 2 \cdot in$$

$$R_4 := 1.905 \cdot in$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 6.765 \times 10^{-4} \text{ blob}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 6.766 \times 10^{-4} \text{ blob}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 1.070 \times 10^{-3} \text{ blob} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 2.184 \text{ in}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 1.289 \times 10^{-3} \text{ blob} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 2.900 \text{ in}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 9.4862 \times 10^{-3} \text{ blob} \cdot in^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 2.977 \text{ in}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 0.0331 \text{ blob} \cdot in^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 5.069 \text{ in}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2^2 + r_2^2 + a_2 \cdot r_2) \quad I_{cw2} = 0.0263 \text{ blob} \cdot in^2$$

$$I_{cw4} := m'_4(k'_4^2 + r_4^2 + a_4 \cdot r_4) \quad I_{cw4} = 0.0739 \text{ blob} \cdot in^2$$

PROBLEM 12-35

Statement: Figure P12-14 shows a fourbar linkage and its dimensions in inches. All links are 0.08-in-thick steel and have a uniform cross-section of 0.26-in wide x 0.12-in thick. Links 2 and 4 have rounded ends with a 0.13-in radius. Link 3 has a squared ends that extend 0.13-in from the pivot point centers. Design counterweights to force balance the linkage using the method of Berkof and Lowen.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

Link 2 (O_2 to A)	$L_2 := 5.52 \cdot in$	Link 3 (A to B)	$L_3 := 4.88 \cdot in$
Link 4 (B to O_4)	$L_4 := 6.48 \cdot in$	Link 1 (O_2 to O_4)	$L_1 := 2.72 \cdot in$
Link cross section:	$h := 0.26 \cdot in$	$t := 0.08 \cdot in$	$r := 0.13 \cdot in$
Mass center:	$b_2 := 0.5 \cdot L_2$	$b_3 := 0.5 \cdot L_3$	$b_4 := 0.5 \cdot L_4$
	$\phi_2 := 0 \cdot deg$	$\phi_3 := 0 \cdot deg$	$\phi_4 := 0 \cdot deg$

Mass density of steel: $\rho := 7.8 \cdot 10^3 \cdot kg \cdot m^{-3}$

Solution: See Figure P12-14 and Mathcad file P1235.

1. Calculate the mass of each link.

$$m_2 := (L_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_3 := (L_3 + 2 \cdot r) \cdot h \cdot t \cdot \rho \quad m_4 := (L_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho$$

$$m_2 = 8.690 \times 10^{-5} \text{ blob} \quad m_3 = 7.803 \times 10^{-5} \text{ blob} \quad m_4 = 1.015 \times 10^{-4} \text{ blob}$$

2. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \cos(\phi_3) - L_2 \right) \quad mb_{2x} = -2.154 \times 10^{-4} \text{ blob} \cdot in$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.000 \text{ blob} \cdot in$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -2.528 \times 10^{-4} \text{ blob} \cdot in$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.000 \text{ blob} \cdot in$$

3. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -4.552 \times 10^{-4} \text{ blob} \cdot in$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.000 \text{ blob} \cdot in$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -5.816 \times 10^{-4} \text{ blob} \cdot in$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 0.000 \text{ blob} \cdot in$$

4. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 4.552 \times 10^{-4} \text{ blob} \cdot in$$

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 5.816 \times 10^{-4} \text{ blob} \cdot in$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 180.000 \text{ deg}$$

PROBLEM 12-36

Statement: Use the data of Problem 12-35 to design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment the linkage exerts on the ground link.

Units: $\text{blob} := \text{lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a_2 := 5.52 \cdot \text{in} \quad \text{Link 3 (A to B)} \quad a_3 := 4.88 \cdot \text{in}$$

$$\text{Link 4 (B to O}_4) \quad a_4 := 6.48 \cdot \text{in} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad a_1 := 2.72 \cdot \text{in}$$

$$\text{Link cross-section dims: } h := 0.26 \cdot \text{in} \quad t := 0.12 \cdot \text{in} \quad r := 0.13 \cdot \text{in}$$

$$\text{Material density: steel } \rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3}$$

Solution: See Figure P12-14 and Mathcad file P1236.

- Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \cdot \sqrt{3 \cdot \left(\frac{a_3}{h}\right)^2 - 1} - \frac{a_3}{2} \quad e = 1.784 \text{ in}$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 8.448 \text{ in}$$

- Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 2.760 \text{ in}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 2.440 \text{ in}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 3.240 \text{ in}$$

- Determine the mass of each link.

$$m_2 := \left(a_2 \cdot h + \pi \cdot r^2\right) \cdot t \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := \left(a_4 \cdot h + \pi \cdot r^2\right) \cdot t \cdot \rho$$

$$m_2 = 1.304 \times 10^{-4} \text{ blob} \quad m_3 = 1.924 \times 10^{-4} \text{ blob} \quad m_4 = 1.522 \times 10^{-4} \text{ blob}$$

- Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 2.440 \text{ in}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3}\right) \quad mr_2 = 5.310 \times 10^{-4} \text{ blob} \cdot \text{in}$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3}\right) \quad mr_4 = 6.233 \times 10^{-4} \text{ blob} \cdot \text{in}$$

- Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 1.712 \times 10^{-4} \text{ blob} \cdot \text{in}$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 1.302 \times 10^{-4} \text{ blob} \cdot \text{in}$$

- Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 2 \cdot \text{in} \quad R_4 := 1.52 \cdot \text{in}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 8.561 \times 10^{-5} \text{ blob}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 8.563 \times 10^{-5} \text{ blob}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 2.160 \times 10^{-4} \text{ blob} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 2.459 \text{ in}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 2.378 \times 10^{-4} \text{ blob} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 2.621 \text{ in}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 3.900 \times 10^{-3} \text{ blob}\cdot\text{in}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 4.250 \text{ in}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 5.865 \times 10^{-3} \text{ blob}\cdot\text{in}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 4.966 \text{ in}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2^2 + r_2^2 + a_2 r_2) \quad I_{cw2} = 0.00814 \text{ blob}\cdot\text{in}^2$$

$$I_{cw4} := m'_4(k'_4^2 + r_4^2 + a_4 r_4) \quad I_{cw4} = 0.0115 \text{ blob}\cdot\text{in}^2$$

PROBLEM 12-37

Statement: A manufacturing company makes 5-blade ceiling fans. Before assembling the fan blades onto the hub the blades are weighed and the location of the CG is determined as a distance from the center of rotation and an angular offset from the geometric center of the blade. At final assembly a technician is provided with the weight and CG data for the 5 blades. Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to calculate the required weight and angular position of a balance weight that is attached to the hub at a radius of 2.5 in. Use the geometric center of blade one as a reference axis. Test your program with the data given in Table P12-3.

Given: Number of blades $N := 5$ Blade counter $i := 1, 2 \dots N$ Hub radius $r_{hub} := 2.5 \cdot \text{in}$
Blade test data:

Blade	Weight	Rad to CG	CG Ang Offset
$i =$	$W_i :=$	$r_i :=$	$\delta_i :=$
1	1.50 · lbf	12.01 · in	-0.25 · deg
2	1.48 · lbf	11.97 · in	0.75 · deg
3	1.54 · lbf	11.95 · in	0.25 · deg
4	1.55 · lbf	12.03 · in	-1.00 · deg
5	1.49 · lbf	12.04 · in	-0.50 · deg

Solution: See Mathcad file P1237.

- Resolve the position vectors into xy components with respect to an x-y frame whose positive x axis is coincident with the geometric center of blade one.

$$R_{x_i} := r_i \cdot \cos \left[(i-1) \cdot \frac{360 \cdot \text{deg}}{N} + \delta_i \right] \quad R_{y_i} := r_i \cdot \sin \left[(i-1) \cdot \frac{360 \cdot \text{deg}}{N} + \delta_i \right]$$

$$R_{x_i} = \begin{array}{|c|} \hline 12.010 \\ \hline 3.550 \\ \hline -9.698 \\ \hline -9.854 \\ \hline 3.620 \\ \hline \end{array} \text{ in} \quad R_{y_i} = \begin{array}{|c|} \hline -0.052 \\ \hline 11.432 \\ \hline 6.982 \\ \hline -6.900 \\ \hline -11.483 \\ \hline \end{array} \text{ in}$$

- Solve equation 12.2c for the mass-radius product components.

$$mR_{bx} := - \sum_{i=1}^N \left(\frac{W_i}{g} \cdot R_{x_i} \right) \quad mR_{by} := - \sum_{i=1}^N \left(\frac{W_i}{g} \cdot R_{y_i} \right)$$

$$mR_{bx} = 1.547 \text{ in} \cdot \text{lb} \quad mR_{by} = 0.212 \text{ in} \cdot \text{lb}$$

- Solve equations 12.2d and 12.2e for the position angle (with respect to the centerline of blade one) and mass-radius product required.

$$\theta_b := \text{atan2}(mR_{bx}, mR_{by}) \quad \theta_b = 7.818 \text{ deg}$$

$$mR_b := \sqrt{mR_{bx}^2 + mR_{by}^2} \quad mR_b = 1.561 \text{ in} \cdot \text{lb}$$

- Determine the balance weight required at the hub radius.

$$W_b := \frac{mR_b \cdot g}{r_{hub}} \quad W_b = 10.0 \text{ ozf}$$

PROBLEM 12-38

Statement: The motor rotor shown in Figure P12-14 has been tested on a dynamic balance machine at 1800 rpm and shows unbalanced forces of $F_1 = 2.43 \text{ lb}$ @ $\theta_1 = 34.5 \text{ deg}$ in the x-y plane at 1 and $F_4 = 5.67 \text{ lb}$ @ $\theta_4 = 198 \text{ deg}$ in the x-y plane at 4. Balance weights consisting of cylindrical disks whose center of rotation is a drilled hole located at a distance e from the center of the disk. The net weight of each disk is 0.50 lb and the disks are located on planes 2 and 3. Determine the angular locations (of the line through the drilled hole and the center of the disk with respect to the reference axis) and the eccentric distances e to dynamically balance the system.

Units: $\text{blob} := \text{lb} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Forces and locations:

$$F_1 := 2.43 \cdot \text{lb} \quad \theta_1 := 34.5 \cdot \text{deg} \quad l_1 := -1.75 \cdot \text{in}$$

$$F_4 := 5.67 \cdot \text{lb} \quad \theta_4 := 198 \cdot \text{deg} \quad l_4 := 8.25 \cdot \text{in}$$

$$\text{Distance between correction planes: } l_B := 6.00 \cdot \text{in}$$

$$\text{Correction weight: } W_c := 0.50 \cdot \text{lb}$$

$$\text{Test speed: } \omega := 1800 \cdot \text{rpm}$$

Solution: See Figure P12-14 and Mathcad file P1238.

1. Resolve the force vectors into xy components in the arbitrary coordinate system associated with the test apparatus.

$$\begin{aligned} F_{1x} &:= F_1 \cdot \cos(\theta_1) & F_{1x} &= 2.003 \text{ lbf} & F_{1y} &:= F_1 \cdot \sin(\theta_1) & F_{1y} &= 1.376 \text{ lbf} \\ F_{4x} &:= F_4 \cdot \cos(\theta_4) & F_{4x} &= -5.392 \text{ lbf} & F_{4y} &:= F_4 \cdot \sin(\theta_4) & F_{4y} &= -1.752 \text{ lbf} \end{aligned}$$

2. Solve equations 12.3 for summation of moments about O, which is at plane 2.

$$mR_{Bx} := -\frac{F_{1x} \cdot l_1 + F_{4x} \cdot l_4}{l_B \cdot \omega^2} \quad mR_{Bx} = 2.251 \times 10^{-4} \text{ blob} \cdot \text{in}$$

$$mR_{By} := -\frac{F_{1y} \cdot l_1 + F_{4y} \cdot l_4}{l_B \cdot \omega^2} \quad mR_{By} = 7.910 \times 10^{-5} \text{ blob} \cdot \text{in}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (3). Also, solve for the eccentricity of the drilled hole in the balance weight.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 19.360 \text{ deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 2.3862 \times 10^{-4} \text{ blob} \cdot \text{in}$$

$$e_B := mR_B \cdot \frac{g}{W_c} \quad e_B = 0.184 \text{ in}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (2).

$$mR_{Ax} := \frac{-F_{1x} - F_{4x}}{\omega^2} - mR_{Bx} \quad mR_{Ax} = -1.2972 \times 10^{-4} \text{ blob} \cdot \text{in}$$

$$mR_{Ay} := \frac{-F_{1y} - F_{4y}}{\omega^2} - mR_{By}$$

$$mR_{Ay} = -6.853 \times 10^{-5} \text{ blob}\cdot\text{in}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (2). Also, solve for the eccentricity of the drilled hole in the balance weight.

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay})$$

$$\theta_A = -152.153 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2}$$

$$mR_A = 1.4671 \times 10^{-4} \text{ blob}\cdot\text{in}$$

$$e_A := mR_A \cdot \frac{g}{W_c}$$

$$e_A = 0.113 \text{ in}$$

PROBLEM 12-39

Statement: The motor rotor shown in Figure P12-14 has been tested on a dynamic balance machine at 1450 rpm and shows unbalanced forces of $F_1 = 4.82 \text{ lb}$ @ $\theta_1 = 163 \text{ deg}$ in the x-y plane at 1 and $F_4 = 7.86 \text{ lb}$ @ $\theta_4 = 67.8 \text{ deg}$ in the x-y plane at 4. Balance weights consisting of cylindrical disks whose center of rotation is a drilled hole located at a distance e from the center of the disk. The net weight of each disk is 0.375 lb and the disks are located on planes 2 and 3. Determine the angular locations (of the line through the drilled hole and the center of the disk with respect to the reference axis) and the eccentric distances e to dynamically balance the system.

Units: $\text{blob} := \text{lb} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Forces and locations:

$$\begin{aligned} F_1 &:= 4.82 \cdot \text{lb} & \theta_1 &:= 163 \cdot \text{deg} & l_1 &:= -1.75 \cdot \text{in} \\ F_4 &:= 7.86 \cdot \text{lb} & \theta_4 &:= 67.8 \cdot \text{deg} & l_4 &:= 8.25 \cdot \text{in} \end{aligned}$$

Distance between correction planes: $l_B := 6.00 \cdot \text{in}$

Correction weight: $W_c := 0.375 \cdot \text{lb}$

Test speed: $\omega := 1450 \cdot \text{rpm}$

Solution: See Figure P12-14 and Mathcad file P1239.

1. Resolve the force vectors into xy components in the arbitrary coordinate system associated with the test apparatus.

$$\begin{aligned} F_{1x} &:= F_1 \cdot \cos(\theta_1) & F_{1x} &= -4.609 \text{ lbf} & F_{1y} &:= F_1 \cdot \sin(\theta_1) & F_{1y} &= 1.409 \text{ lbf} \\ F_{4x} &:= F_4 \cdot \cos(\theta_4) & F_{4x} &= 2.970 \text{ lbf} & F_{4y} &:= F_4 \cdot \sin(\theta_4) & F_{4y} &= 7.277 \text{ lbf} \end{aligned}$$

2. Solve equations 12.3 for summation of moments about O, which is at plane 2.

$$\begin{aligned} mR_{Bx} &:= -\frac{F_{1x} \cdot l_1 + F_{4x} \cdot l_4}{l_B \cdot \omega^2} & mR_{Bx} &= -2.354 \times 10^{-4} \text{ blob} \cdot \text{in} \\ mR_{By} &:= -\frac{F_{1y} \cdot l_1 + F_{4y} \cdot l_4}{l_B \cdot \omega^2} & mR_{By} &= -4.162 \times 10^{-4} \text{ blob} \cdot \text{in} \end{aligned}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (3). Also, solve for the eccentricity of the drilled hole in the balance weight.

$$\begin{aligned} \theta_B &:= \text{atan2}(mR_{Bx}, mR_{By}) & \theta_B &= -119.496 \text{ deg} \\ mR_B &:= \sqrt{mR_{Bx}^2 + mR_{By}^2} & mR_B &= 4.7814 \times 10^{-4} \text{ blob} \cdot \text{in} \\ e_B &:= mR_B \cdot \frac{g}{W_c} & e_B &= 0.492 \text{ in} \end{aligned}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (2).

$$mR_{Ax} := \frac{-F_{1x} - F_{4x}}{\omega^2} - mR_{Bx} \quad mR_{Ax} = 3.0653 \times 10^{-4} \text{ blob} \cdot \text{in}$$

$$mR_{Ay} := \frac{-F_{1y} - F_{4y}}{\omega^2} - mR_{By}$$

$$mR_{Ay} = 3.941 \times 10^{-5} \text{ blob}\cdot\text{in}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (2). Also, solve for the eccentricity of the drilled hole in the balance weight.

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay})$$

$$\theta_A = 7.327 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2}$$

$$mR_A = 3.0905 \times 10^{-4} \text{ blob}\cdot\text{in}$$

$$e_A := mR_A \cdot \frac{g}{W_c}$$

$$e_A = 0.318 \text{ in}$$

PROBLEM 12-40a

Statement: Table P12-4 gives the geometry and kinematic data for several fourbar linkages similar to that shown in Figure P12-11. For row *a* in Table P12-4, design counterweights of the type shown in Figure P12-15 for links 2 and 4 to completely force balance the linkage by the method of Berkof and Lowen. The square ends of link 3 extend a distance, e , from the hole center. The other links' ends are full round with a radius, r , about the hole center. All pin holes have the same diameter, d , and all links have the same width, $2r$, and thickness, t . The hole-to-hole link lengths are L_1, L_2, L_3 , and L_4 .

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad L_2 := 100 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad L_3 := 300 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4) \quad L_4 := 200 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4) \quad L_1 := 375 \cdot \text{mm}$$

$$\text{Link features:} \quad r := 13 \cdot \text{mm} \quad e := 13 \cdot \text{mm} \quad d := 6 \cdot \text{mm} \quad t := 4 \cdot \text{mm}$$

$$\text{Mass density of steel:} \quad \rho := 7.8 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^{-3}$$

Solution: See Table P12-4, Figure P12-11, Figure P12-15, and Mathcad file P1240a.

1. Calculate the mass of each link.

$$m_2 := \left(L_2 \cdot 2 \cdot r + \pi \cdot r^2 - 0.5 \cdot \pi \cdot d^2 \right) \cdot t \cdot \rho \quad m_2 = 0.096 \text{ kg}$$

$$m_3 := \left[(L_3 + 2 \cdot e) \cdot 2 \cdot r - 0.5 \cdot \pi \cdot d^2 \right] \cdot t \cdot \rho \quad m_3 = 0.263 \text{ kg}$$

$$m_4 := \left(L_4 \cdot 2 \cdot r + \pi \cdot r^2 - 0.5 \cdot \pi \cdot d^2 \right) \cdot t \cdot \rho \quad m_4 = 0.177 \text{ kg}$$

2. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4. For these links, $b = 0.5L$ and $\phi = 0$.

$$b_3 := 0.5 \cdot L_3 \quad \phi_3 := 0$$

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \cos(\phi_3) - L_2 \right) \quad mb_{2x} = -13.134 \cdot \text{kg} \cdot \text{mm}$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.000 \cdot \text{kg} \cdot \text{mm}$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -26.269 \cdot \text{kg} \cdot \text{mm}$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.000 \cdot \text{kg} \cdot \text{mm}$$

3. Determine the additional mR product components for links 2 and 4.

$$b_2 := 0.5 \cdot L_2 \quad \phi_2 := 0 \quad b_4 := 0.5 \cdot L_4 \quad \phi_4 := 0$$

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -17.930 \cdot \text{kg} \cdot \text{mm}$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.000 \cdot \text{kg} \cdot \text{mm}$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -43.973 \cdot \text{kg} \cdot \text{mm}$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 0.000 \cdot \text{kg} \cdot \text{mm}$$

4. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 17.9304 \cdot \text{kg} \cdot \text{mm}$$

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \cdot \text{deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 43.973 \cdot \text{kg} \cdot \text{mm}$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 180.000 \cdot \text{deg}$$

5. Determine the required dimensions for the counterweights on links 2 and 4. There are two independent variables that define the mR product of the counterweight, α and R . For each link, make a design choice for R and then solve for α . Repeat the process until you have satisfactory values for each.

Link 2: $R_2 := 8 \cdot r \quad R_2 = 104 \text{ mm}$

$$\alpha_2 := \frac{2 \cdot mR_{b2}}{R_2 \cdot (R_2^2 - r^2) \cdot t \cdot \rho} \quad \alpha_2 = 59 \cdot \text{deg}$$

$$m_{cw2} := \frac{\alpha_2}{2} \cdot (R_2^2 - r^2) \cdot t \cdot \rho \quad m_{cw2} = 0.172 \text{ kg}$$

Link 4: $R_4 := 10 \cdot r \quad R_4 = 130 \text{ mm}$

$$\alpha_4 := \frac{2 \cdot mR_{b4}}{R_4 \cdot (R_4^2 - r^2) \cdot t \cdot \rho} \quad \alpha_4 = 74 \cdot \text{deg}$$

$$m_{cw4} := \frac{\alpha_4}{2} \cdot (R_4^2 - r^2) \cdot t \cdot \rho \quad m_{cw4} = 0.338 \text{ kg}$$

PROBLEM 13-1

Statement: A slider-crank linkage has the dimensions and speed at $t = 0$ given below. Its initial crank angle is zero. Calculate the piston acceleration at the time specified below. Use two methods, the exact solution, and the approximate Fourier series solution and compare the results.

Given: Link lengths: $r := 3 \cdot \text{in}$ $l := 12 \cdot \text{in}$

Initial angular velocity: $\omega := 200 \cdot \text{rad} \cdot \text{sec}^{-1}$ Time span: $t := 1.0 \cdot \text{sec}$

Solution: See Mathcad file P1301.

1. Calculate the exact acceleration using equation 13.1f.

$$a_{\text{exact}} := -r \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) - \frac{r \cdot \left[l^2 \cdot (1 - 2 \cdot \cos(\omega \cdot t)^2) - r^2 \cdot \sin(\omega \cdot t)^4 \right]}{\left(l^2 - r^2 \cdot \sin(\omega \cdot t)^2 \right)^{\frac{3}{2}}} \right]$$

$$a_{\text{exact}} = -42679.3 \frac{\text{in}}{\text{sec}^2}$$

2. Calculate the approximate acceleration using equation 13.3e.

$$a_{\text{approx}} := -r \cdot \omega^2 \cdot \left(\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos(2 \cdot \omega \cdot t) \right) \quad a_{\text{approx}} = -42703.6 \frac{\text{in}}{\text{sec}^2}$$

3. Compare the results by calculating the error in the approximation as a percent of the exact.

$$\text{error} := \frac{a_{\text{approx}} - a_{\text{exact}}}{a_{\text{exact}}} \quad \text{error} = 0.057 \%$$

The approximate solution is a little less than 0.06% high.

PROBLEM 13-2

Statement: A slider-crank linkage has the dimensions and speed at $t = 0$ given below. Its initial crank angle is zero. Calculate the piston acceleration at the time specified below. Use two methods, the exact solution, and the approximate Fourier series solution and compare the results.

Given: Link lengths: $r := 4 \cdot \text{in}$ $l := 15 \cdot \text{in}$
 Initial angular velocity: $\omega := 200 \cdot \text{rad} \cdot \text{sec}^{-1}$ Time span: $t := 0.9 \cdot \text{sec}$

Solution: See Mathcad file P1302.

1. Calculate the exact acceleration using equation 13.1f.

$$a_{\text{exact}} := -r \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) - \frac{r \cdot \left[l^2 \cdot (1 - 2 \cdot \cos(\omega \cdot t)^2) - r^2 \cdot \sin(\omega \cdot t)^4 \right]}{\left(l^2 - r^2 \cdot \sin(\omega \cdot t)^2 \right)^{\frac{3}{2}}} \right]$$

$$a_{\text{exact}} = 107395.7 \frac{\text{in}}{\text{sec}^2}$$

2. Calculate the approximate acceleration using equation 13.3e.

$$a_{\text{approx}} := -r \cdot \omega^2 \cdot \left(\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos(2 \cdot \omega \cdot t) \right) \quad a_{\text{approx}} = 107857.8 \frac{\text{in}}{\text{sec}^2}$$

3. Compare the results by calculating the error in the approximation as a percent of the exact.

$$\text{error} := \frac{a_{\text{approx}} - a_{\text{exact}}}{a_{\text{exact}}} \quad \text{error} = 0.430 \%$$

The approximate solution is a little less than 0.5% high.

PROBLEM 13-3

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the gas force and gas torque at this position.

Given: Link lengths: $r := 3 \cdot \text{in}$ $l := 12 \cdot \text{in}$

Piston bore: $B := 2 \cdot \text{in}$ Peak pressure: $p_g := 1000 \cdot \text{psi}$ Crank angle: $\theta := 10 \cdot \text{deg}$

Assumptions: An approximate solution is acceptable.

Solution: See Mathcad file P1303.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \quad F_g = 3142 \text{ lbf}$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21} := F_g \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta) \right) \quad T_{g21} = 2040 \text{ in} \cdot \text{lbf}$$

PROBLEM 13-4

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the gas force and gas torque at this position.

Given: Link lengths: $r := 4 \cdot \text{in}$ $l := 15 \cdot \text{in}$

Piston bore: $B := 3 \cdot \text{in}$ Peak pressure: $p_g := 600 \cdot \text{psi}$ Crank angle: $\theta := 5 \cdot \text{deg}$

Assumptions: An approximate solution is acceptable.

Solution: See Mathcad file P1304.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \quad F_g = 4241 \text{ lbf}$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21} := F_g \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta) \right) \quad T_{g21} = 1871 \text{ in} \cdot \text{lbf}$$

PROBLEM 13-5

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the exact gas torque at this position and compare it to that obtained by the approximate expression in equation 13.8b. What is the percent error?

Given: Link lengths: $r := 3 \cdot \text{in}$ $l := 12 \cdot \text{in}$
 Piston bore: $B := 2 \cdot \text{in}$ Peak pressure: $p_g := 1000 \cdot \text{psi}$ Crank angle: $\theta := 10 \cdot \text{deg}$

Solution: See Mathcad file P1305.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \quad F_g = 3142 \text{ lbf}$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21a} := F_g \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta) \right) \quad T_{g21a} = 2039.53 \text{ in} \cdot \text{lbf}$$

3. Calculate the exact gas torque on the crank using equations 13.7b, 13.1d, and 13.6d.

$$\phi := \text{atan} \left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2}} \right] \quad \phi = 2.488 \text{ deg}$$

$$x := r \cdot \cos(\theta) + l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2} \quad x = 14.943 \text{ in}$$

$$T_{g21e} := F_g \cdot \tan(\phi) \cdot x \quad T_{g21e} = 2039.91 \text{ in} \cdot \text{lbf}$$

4. Compare the results by calculating the error in the approximation as a percent of the exact.

$$\text{error} := \frac{T_{g21a} - T_{g21e}}{T_{g21e}} \quad \text{error} = -0.0186 \%$$

PROBLEM 13-6

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the exact gas torque at this position and compare it to that obtained by the approximate expression in equation 13.8b. What is the percent error?

Given: Link lengths: $r := 4 \cdot \text{in}$ $l := 15 \cdot \text{in}$

Piston bore: $B := 3 \cdot \text{in}$ Peak pressure: $p_g := 600 \cdot \text{psi}$ Crank angle: $\theta := 5 \cdot \text{deg}$

Solution: See Mathcad file P1306.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \quad F_g = 4241 \text{ lbf}$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21a} := F_g \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta) \right) \quad T_{g21a} = 1871.35 \text{ in} \cdot \text{lbf}$$

3. Calculate the exact gas torque on the crank using equations 13.7b, 13.1d, and 13.6d.

$$\phi := \text{atan} \left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2}} \right] \quad \phi = 1.332 \text{ deg}$$

$$x := r \cdot \cos(\theta) + l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2} \quad x = 18.981 \text{ in}$$

$$T_{g21e} := F_g \cdot \tan(\phi) \cdot x \quad T_{g21e} = 1871.45 \text{ in} \cdot \text{lbf}$$

4. Compare the results by calculating the error in the approximation as a percent of the exact.

$$\text{error} := \frac{T_{g21a} - T_{g21e}}{T_{g21e}} \quad \text{error} = -0.00567 \%$$

PROBLEM 13-7

Statement: The dimensions and mass properties of a connecting rod are given below.

- Calculate an exact dynamic model using two lumped masses, one at the wrist pin, point B, and one at whatever point is required. Define the lumped masses and their locations.
- Calculate an approximate dynamic model using two lumped masses, one at the wrist pin, point B, and one at the crank pin, point A. Define the lumped masses and their locations.
- Calculate the error in the mass moment of inertia of the approximate model as a percentage of the original mass moment of inertia.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 \cdot blob$

Mass moment of inertia: $I_{G3} := 0.620 \cdot blob \cdot in^2$

Distance to CG (as fraction of l): $r_a := 0.4$

Solution: See Mathcad file P1307.

1. Determine the exact model using equations 13.9d and 13.9e.

Distance from point A to CG: $l_a := r_a \cdot l$ $l_a = 4.800 \text{ in}$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 7.200 \text{ in}$

Distance from CG to lumped mass at P: $l_p := \frac{I_{G3}}{m_3 \cdot l_b}$ $l_p = 4.306 \text{ in}$

Masses: $m_p := m_3 \cdot \frac{l_b}{l_p + l_b}$ $m_p = 0.0125 \text{ blob}$

$m_b := m_3 \cdot \frac{l_p}{l_p + l_b}$ $m_b = 0.00748 \text{ blob}$

2. Determine the approximate model using equations 13.9d, letting $l_p = l_a$.

Let $l_p := l_a$

Masses: $m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b}$ $m_{3a} = 0.0120 \text{ blob}$

$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b}$ $m_{3b} = 0.00800 \text{ blob}$

3. Calculate the mass moment of inertia of the approximate model and compare it to the original.

$I_{G3approx} := m_{3a} \cdot l_a^2 + m_{3b} \cdot l_b^2$ $I_{G3approx} = 0.691 \text{ blob} \cdot in^2$

$error := \frac{I_{G3approx} - I_{G3}}{I_{G3}}$ $error = 11.48 \%$

PROBLEM 13-8

- Statement:** The dimensions and mass properties of a connecting rod are given below.
- Calculate an exact dynamic model using two lumped masses, one at the wrist pin, point B, and one at whatever point is required. Define the lumped masses and their locations.
 - Calculate an approximate dynamic model using two lumped masses, one at the wrist pin, point B, and one at the crank pin, point A. Define the lumped masses and their locations.
 - Calculate the error in the mass moment of inertia of the approximate model as a percentage of the original mass moment of inertia.

Units: $blob := \text{lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Conrod length: $l := 15 \cdot \text{in}$ mass: $m_3 := 0.025 \cdot blob$

Mass moment of inertia: $I_{G3} := 1.020 \cdot blob \cdot \text{in}^2$

Distance to CG (as fraction of l): $r_a := 0.25$

Solution: See Mathcad file P1308.

1. Determine the exact model using equations 13.9d and 13.9e.

Distance from point A to CG: $l_a := r_a \cdot l$ $l_a = 3.750 \text{ in}$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 11.250 \text{ in}$

Distance from CG to lumped mass at P: $l_p := \frac{I_{G3}}{m_3 \cdot l_b}$ $l_p = 3.627 \text{ in}$

Masses: $m_p := m_3 \cdot \frac{l_b}{l_p + l_b}$ $m_p = 0.0189 \text{ blob}$

$m_b := m_3 \cdot \frac{l_p}{l_p + l_b}$ $m_b = 0.00609 \text{ blob}$

2. Determine the approximate model using equations 13.9d, letting $l_p = l_a$.

Let $l_p := l_a$

Masses: $m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b}$ $m_{3a} = 0.0188 \text{ blob}$

$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b}$ $m_{3b} = 0.00625 \text{ blob}$

3. Calculate the mass moment of inertia of the approximate model and compare it to the original.

$I_{G3approx} := m_{3a} \cdot l_a^2 + m_{3b} \cdot l_b^2$ $I_{G3approx} = 1.055 \text{ blob} \cdot \text{in}^2$

$error := \frac{I_{G3approx} - I_{G3}}{I_{G3}}$ $error = 3.40 \%$

PROBLEM 13-9

Statement: The dimensions and mass properties of a crank are given below. Calculate a statically equivalent, two-lumped mass dynamic model with the lumps placed at the main pin and crankpin. What is the percent error in the model's moment of inertia about the crank pivot?

Units: $blob := \text{lb} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Crank length: $r := 3.5 \cdot \text{in}$ mass: $m_2 := 0.060 \cdot blob$

Mass moment of inertia: $I_2 := 0.300 \cdot blob \cdot \text{in}^2$

Distance to CG (as fraction of r): $r_a := 0.3$

Solution: See Mathcad file P1309.

1. Determine the statically equivalent model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_a \cdot r$ $r_{G2} = 1.050 \text{ in}$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $m_{2a} = 0.0180 \text{ blob}$

2. Calculate the mass moment of inertia of the statically equivalent model and compare it to the original.

$I_{O2model} := m_{2a} \cdot r^2$ $I_{O2model} = 0.2205 \text{ blob} \cdot \text{in}^2$

$error := \frac{I_{O2model} - I_2}{I_2}$ $error = -26.50 \%$

PROBLEM 13-10

Statement: The dimensions and mass properties of a crank are given below. Calculate a statically equivalent, two-lumped mass dynamic model with the lumps placed at the main pin and crankpin. What is the percent error in the model's moment of inertia about the crank pivot?

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$

Mass moment of inertia: $I_2 := 0.400 \cdot blob \cdot in^2$

Distance to CG (as fraction of r): $r_a := 0.4$

Solution: See Mathcad file P13010.

1. Determine the statically equivalent model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_a \cdot r$ $r_{G2} = 1.600 \text{ in}$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $m_{2a} = 0.0200 \text{ blob}$

2. Calculate the mass moment of inertia of the statically equivalent model and compare it to the original.

$I_{O2model} := m_{2a} \cdot r^2$ $I_{O2model} = 0.3200 \text{ blob} \cdot in^2$

$error := \frac{I_{O2model} - I_2}{I_2}$ $error = -20.00 \%$

PROBLEM 13-11

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.4$
 Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.3$
 Piston mass: $m_4 := 0.012 \cdot blob$
 Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 45 \cdot deg$

Solution: See Mathcad file P1311.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $l_a = 4.800 \text{ in}$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 7.200 \text{ in}$

Let $l_p := l_a$ $m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b}$ $m_{3a} = 0.0120 \text{ blob}$

$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b}$ $m_{3b} = 0.00800 \text{ blob}$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $r_{G2} = 1.050 \text{ in}$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $m_{2a} = 0.0180 \text{ blob}$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.0300 \text{ blob}$

$m_B := m_{3b} + m_4$ $m_B = 0.0200 \text{ blob}$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right]$ $F_{ix} = 5428 \text{ lbf}$

$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta))$ $F_{iy} = 3257 \text{ lbf}$

$F_i := \sqrt{F_{ix}^2 + F_{iy}^2}$ $F_i = 6330 \text{ lbf}$ at $atan2(F_{ix}, F_{iy}) = 30.964 \text{ deg}$

$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot \sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + \cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot \cos(2 \cdot \theta) \right)$ $T_{i21} = -6482 \text{ in} \cdot \text{lbf}$

PROBLEM 13-12

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 \cdot blob$

Distance to CG (as fraction of l): $r_{3a} := 0.4$

Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.4$

Piston mass: $m_4 := 0.019 \cdot blob$

Crank speed: $\omega := 3000 \cdot rpm$ Crank angle: $\theta := 30 \cdot deg$

Solution: See Mathcad file P1312.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $l_a = 4.800 \text{ in}$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 7.200 \text{ in}$

Let $l_p := l_a$ $m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b}$ $m_{3a} = 0.0120 \text{ blob}$

$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b}$ $m_{3b} = 0.00800 \text{ blob}$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $r_{G2} = 1.600 \text{ in}$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $m_{2a} = 0.0200 \text{ blob}$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.0320 \text{ blob}$

$m_B := m_{3b} + m_4$ $m_B = 0.0270 \text{ blob}$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right]$ $F_{ix} = 21948 \text{ lbf}$

$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta))$ $F_{iy} = 6317 \text{ lbf}$

$F_i := \sqrt{F_{ix}^2 + F_{iy}^2}$ $F_i = 22839 \text{ lbf}$ at $\text{atan2}(F_{ix}, F_{iy}) = 16.055 \text{ deg}$

$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot \sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + \cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot \cos(2 \cdot \theta) \right)$ $T_{i21} = -27345 \text{ in} \cdot \text{lbf}$

PROBLEM 13-13

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.25$
 Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.3$
 Piston mass: $m_4 := 0.023 \cdot blob$
 Crank speed: $\omega := 2500 \cdot rpm$ Crank angle: $\theta := 24 \cdot deg$

Solution: See Mathcad file P1313.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $l_a = 3.750 \text{ in}$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 11.250 \text{ in}$

Let $l_p := l_a$ $m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b}$ $m_{3a} = 0.0188 \text{ blob}$

$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b}$ $m_{3b} = 0.00625 \text{ blob}$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $r_{G2} = 1.050 \text{ in}$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $m_{2a} = 0.0180 \text{ blob}$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.0368 \text{ blob}$

$m_B := m_{3b} + m_4$ $m_B = 0.0293 \text{ blob}$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right]$ $F_{ix} = 15559 \text{ lbf}$

$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta))$ $F_{iy} = 3586 \text{ lbf}$

$F_i := \sqrt{F_{ix}^2 + F_{iy}^2}$ $F_i = 15967 \text{ lbf}$ at $atan2(F_{ix}, F_{iy}) = 12.978 \text{ deg}$

$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot \sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + \cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot \cos(2 \cdot \theta) \right)$ $T_{i21} = -12630 \text{ in} \cdot \text{lbf}$

PROBLEM 13-14

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lb \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 \cdot blob$

Distance to CG (as fraction of l): $r_{3a} := 0.25$

Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.4$

Piston mass: $m_4 := 0.015 \cdot blob$

Crank speed: $\omega := 4000 \cdot rpm$ Crank angle: $\theta := 18 \cdot deg$

Solution: See Mathcad file P1314.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $l_a = 3.750 \text{ in}$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 11.250 \text{ in}$

Let $l_p := l_a$ $m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b}$ $m_{3a} = 0.0188 \text{ blob}$

$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b}$ $m_{3b} = 0.00625 \text{ blob}$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $r_{G2} = 1.600 \text{ in}$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $m_{2a} = 0.0200 \text{ blob}$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.0388 \text{ blob}$

$m_B := m_{3b} + m_4$ $m_B = 0.0213 \text{ blob}$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right]$ $F_{ix} = 43267 \text{ lbf}$

$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta))$ $F_{iy} = 8404 \text{ lbf}$

$F_i := \sqrt{F_{ix}^2 + F_{iy}^2}$ $F_i = 44075 \text{ lbf}$ at $\text{atan2}(F_{ix}, F_{iy}) = 10.992 \text{ deg}$

$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot \sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + \cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot \cos(2 \cdot \theta) \right)$ $T_{i21} = -25956 \text{ in} \cdot \text{lbf}$

PROBLEM 13-15

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.4$
 Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.3$
 Piston mass: $m_4 := 0.022 \cdot blob$ Gas force: $F_g := 300 \cdot lbf$
 Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 45 \cdot deg$

Solution: See Mathcad file P1315.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 4.800 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 7.200 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.0120 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.00800 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.050 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0180 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.0300 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0300 \text{ blob}$$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := \text{atan} \left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2}} \right] \quad \phi = 11.902 \text{ deg}$$

$$a_B := -r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \quad a_B = -108560.1 \frac{\text{in}}{\text{sec}^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -[(m_4 + m_{3b}) \cdot a_B + F_g] \cdot \tan(\phi) \quad F_{41} = 623.2 \text{ lbf}$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$F_{34x} := F_g + m_4 \cdot a_B$$

$$F_{34x} = -2088.3 \text{ lbf}$$

$$F_{34y} := -[F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$F_{34y} = 623.2 \text{ lbf}$$

$$F_{34} := \sqrt{F_{34x}^2 + F_{34y}^2}$$

$$F_{34} = 2179.3 \text{ lbf}$$

$$\theta_{34} := \text{atan2}(F_{34x}, F_{34y})$$

$$\theta_{34} = 163.384 \text{ deg}$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\theta) - (m_{3b} + m_4) \cdot a_B - F_g$$

$$F_{32x} = 4259.5 \text{ lbf}$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + [F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$F_{32y} = 679.5 \text{ lbf}$$

$$F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2}$$

$$F_{32} = 4313.4 \text{ lbf}$$

$$\theta_{32} := \text{atan2}(F_{32x}, F_{32y})$$

$$\theta_{32} = 9.064 \text{ deg}$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot \cos(\theta) + F_{32x}$$

$$F_{21x} = 6213.6 \text{ lbf}$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + F_{32y}$$

$$F_{21y} = 2633.6 \text{ lbf}$$

$$F_{21} := \sqrt{F_{21x}^2 + F_{21y}^2}$$

$$F_{21} = 6748.7 \text{ lbf}$$

$$\theta_{21} := \text{atan2}(F_{21x}, F_{21y})$$

$$\theta_{21} = 22.969 \text{ deg}$$

PROBLEM 13-16

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.4$
 Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.4$
 Piston mass: $m_4 := 0.019 \cdot blob$ Gas force: $F_g := 600 \cdot lbf$
 Crank speed: $\omega := 3000 \cdot rpm$ Crank angle: $\theta := 30 \cdot deg$

Solution: See Mathcad file P1316.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 4.800 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 7.200 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.0120 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.00800 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.600 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0200 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.0320 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0270 \text{ blob}$$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := \text{atan} \left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2}} \right] \quad \phi = 9.594 \text{ deg}$$

$$a_B := -r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \quad a_B = -407690.5 \frac{\text{in}}{\text{sec}^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -[(m_4 + m_{3b}) \cdot a_B + F_g] \cdot \tan(\phi) \quad F_{41} = 1759.2 \text{ lbf}$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$\begin{aligned}
 F_{34x} &:= F_g + m_4 \cdot a_B & F_{34x} &= -7146.1 \text{ lbf} \\
 F_{34y} &:= -[F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi) & F_{34y} &= 1759.2 \text{ lbf} \\
 F_{34} &:= \sqrt{F_{34x}^2 + F_{34y}^2} & F_{34} &= 7359.5 \text{ lbf} \\
 \theta_{34} &:= \text{atan2}(F_{34x}, F_{34y}) & \theta_{34} &= 166.170 \text{ deg}
 \end{aligned}$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$\begin{aligned}
 F_{32x} &:= m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\theta) - (m_{3b} + m_4) \cdot a_B - F_g & F_{32x} &= 14510.4 \text{ lbf} \\
 F_{32y} &:= m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + [F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi) & F_{32y} &= 609.5 \text{ lbf} \\
 F_{32} &:= \sqrt{F_{32x}^2 + F_{32y}^2} & F_{32} &= 14523.2 \text{ lbf} \\
 \theta_{32} &:= \text{atan2}(F_{32x}, F_{32y}) & \theta_{32} &= 2.405 \text{ deg}
 \end{aligned}$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$\begin{aligned}
 F_{21x} &:= m_{2a} \cdot r \cdot \omega^2 \cdot \cos(\theta) + F_{32x} & F_{21x} &= 21348.2 \text{ lbf} \\
 F_{21y} &:= m_{2a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + F_{32y} & F_{21y} &= 4557.3 \text{ lbf} \\
 F_{21} &:= \sqrt{F_{21x}^2 + F_{21y}^2} & F_{21} &= 21829.2 \text{ lbf} \\
 \theta_{21} &:= \text{atan2}(F_{21x}, F_{21y}) & \theta_{21} &= 12.050 \text{ deg}
 \end{aligned}$$

PROBLEM 13-17

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.25$
 Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.3$
 Piston mass: $m_4 := 0.032 \cdot blob$ Gas force: $F_g := 900 \cdot lbf$
 Crank speed: $\omega := 2500 \cdot rpm$ Crank angle: $\theta := 24 \cdot deg$

Solution: See Mathcad file P1317.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 3.750 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 11.250 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.0188 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.00625 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.050 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0180 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.0368 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0383 \text{ blob}$$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := \text{atan} \left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2}} \right] \quad \phi = 5.446 \text{ deg}$$

$$a_B := -r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \quad a_B = -256600.5 \frac{\text{in}}{\text{sec}^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -[(m_4 + m_{3b}) \cdot a_B + F_g] \cdot \tan(\phi) \quad F_{41} = 849.9 \text{ lbf}$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$\begin{aligned}
 F_{34x} &:= F_g + m_4 \cdot a_B & F_{34x} &= -7311.2 \text{ lbf} \\
 F_{34y} &:= -[F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi) & F_{34y} &= 849.9 \text{ lbf} \\
 F_{34} &:= \sqrt{F_{34x}^2 + F_{34y}^2} & F_{34} &= 7360.5 \text{ lbf} \\
 \theta_{34} &:= \text{atan2}(F_{34x}, F_{34y}) & \theta_{34} &= 173.369 \text{ deg}
 \end{aligned}$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$\begin{aligned}
 F_{32x} &:= m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\theta) - (m_{3b} + m_4) \cdot a_B - F_g & F_{32x} &= 13024.0 \text{ lbf} \\
 F_{32y} &:= m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + [F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi) & F_{32y} &= 979.5 \text{ lbf} \\
 F_{32} &:= \sqrt{F_{32x}^2 + F_{32y}^2} & F_{32} &= 13060.8 \text{ lbf} \\
 \theta_{32} &:= \text{atan2}(F_{32x}, F_{32y}) & \theta_{32} &= 4.301 \text{ deg}
 \end{aligned}$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$\begin{aligned}
 F_{21x} &:= m_{2a} \cdot r \cdot \omega^2 \cdot \cos(\theta) + F_{32x} & F_{21x} &= 16968.6 \text{ lbf} \\
 F_{21y} &:= m_{2a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + F_{32y} & F_{21y} &= 2735.8 \text{ lbf} \\
 F_{21} &:= \sqrt{F_{21x}^2 + F_{21y}^2} & F_{21} &= 17187.7 \text{ lbf} \\
 \theta_{21} &:= \text{atan2}(F_{21x}, F_{21y}) & \theta_{21} &= 9.159 \text{ deg}
 \end{aligned}$$

PROBLEM 13-18

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.25$
 Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.4$
 Piston mass: $m_4 := 0.014 \cdot blob$ Gas force: $F_g := 1200 \cdot lbf$
 Crank speed: $\omega := 4000 \cdot rpm$ Crank angle: $\theta := 18 \cdot deg$

Solution: See Mathcad file P1318.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 3.750 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 11.250 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.0188 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.00625 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.600 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0200 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.0388 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0203 \text{ blob}$$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := \text{atan} \left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2}} \right] \quad \phi = 4.727 \text{ deg}$$

$$a_B := -r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \quad a_B = -818901.3 \frac{\text{in}}{\text{sec}^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -[(m_4 + m_{3b}) \cdot a_B + F_g] \cdot \tan(\phi) \quad F_{41} = 1271.9 \text{ lbf}$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$F_{34x} := F_g + m_4 \cdot a_B \quad F_{34x} = -10264.6 \text{ lbf}$$

$$F_{34y} := -[F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi) \quad F_{34y} = 1271.9 \text{ lbf}$$

$$F_{34} := \sqrt{F_{34x}^2 + F_{34y}^2} \quad F_{34} = 10343.1 \text{ lbf}$$

$$\theta_{34} := \text{atan2}(F_{34x}, F_{34y}) \quad \theta_{34} = 172.936 \text{ deg}$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\theta) - (m_{3b} + m_4) \cdot a_B - F_g \quad F_{32x} = 27898.2 \text{ lbf}$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + [F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi) \quad F_{32y} = 2794.6 \text{ lbf}$$

$$F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2} \quad F_{32} = 28037.8 \text{ lbf}$$

$$\theta_{32} := \text{atan2}(F_{32x}, F_{32y}) \quad \theta_{32} = 5.720 \text{ deg}$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot \cos(\theta) + F_{32x} \quad F_{21x} = 41247.9 \text{ lbf}$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + F_{32y} \quad F_{21y} = 7132.2 \text{ lbf}$$

$$F_{21} := \sqrt{F_{21x}^2 + F_{21y}^2} \quad F_{21} = 41860.0 \text{ lbf}$$

$$\theta_{21} := \text{atan2}(F_{21x}, F_{21y}) \quad \theta_{21} = 9.810 \text{ deg}$$

PROBLEM 13-19

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.
 a. Exactly balance the crank and recalculate the inertia force.
 b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius $-r$ on the crank and recalculate the inertia force.
 c. Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.4$
 Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.3$
 Piston mass: $m_4 := 0.012 \cdot blob$
 Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 45 \cdot deg$

Solution: See Mathcad file P1319.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 4.800 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 7.200 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.0120 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.00800 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.050 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0180 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.0300 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0200 \text{ blob}$$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \quad F_{ix} = 2171 \text{ lbf}$$

$$F_{iy} := 0 \cdot lbf \quad F_{iy} = 0 \text{ lbf}$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{ib} = 2171 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = 0.000 \text{ deg}$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \quad m_p = 6.6667 \times 10^{-3} \text{ blob}$$

$$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \dots \quad F_{ix} = 1447 \text{ lbf}$$

$$+ (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \cos(\theta))$$

$$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) + (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) \quad F_{iy} = -724 \text{ lbf}$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{iob} = 1618 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = -26.565 \text{ deg}$$

6. Compare the results to those for an unbalanced crank. From Problem 13-11, the inertia force for the unbalanced crank is $F_i := 6330 \cdot \text{lbf}$. The percent differences for the balanced and overbalanced cranks are:

Exactly balanced: $\Delta := \frac{F_{ib} - F_i}{F_i} \quad \Delta = -65.7 \%$

Overbalanced: $\Delta := \frac{F_{iob} - F_i}{F_i} \quad \Delta = -74.4 \%$

PROBLEM 13-20

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.

- Exactly balance the crank and recalculate the inertia force.
- Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius $-r$ on the crank and recalculate the inertia force.
- Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.4$
 Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.4$
 Piston mass: $m_4 := 0.019 \cdot blob$
 Crank speed: $\omega := 3000 \cdot rpm$ Crank angle: $\theta := 30 \cdot deg$

Solution: See Mathcad file P1320.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 4.800 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 7.200 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.0120 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.00800 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.600 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}^2}{r} \quad m_{2a} = 0.0200 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.0320 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0270 \text{ blob}$$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \quad F_{ix} = 11008 \text{ lbf}$$

$$F_{iy} := 0 \cdot lbf \quad F_{iy} = 0 \text{ lbf}$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{ib} = 11008 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = 0.000 \text{ deg}$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \quad m_p = 9.0000 \times 10^{-3} \text{ blob}$$

$$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \dots \quad F_{ix} = 7931 \text{ lbf}$$

$$+ (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \cos(\theta))$$

$$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) + (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) \quad F_{iy} = -1777 \text{ lbf}$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{iob} = 8127 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = -12.626 \text{ deg}$$

6. Compare the results to those for an unbalanced crank. From Problem 13-12, the inertia force for the unbalanced crank is $F_i := 22839 \cdot \text{lbf}$. The percent differences for the balanced and overbalanced cranks are:

$$\text{Exactly balanced:} \quad \Delta := \frac{F_{ib} - F_i}{F_i} \quad \Delta = -51.8 \%$$

$$\text{Overbalanced:} \quad \Delta := \frac{F_{iob} - F_i}{F_i} \quad \Delta = -64.4 \%$$

PROBLEM 13-21

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.

- Exactly balance the crank and recalculate the inertia force.
- Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius $-r$ on the crank and recalculate the inertia force.
- Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.25$
 Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.3$
 Piston mass: $m_4 := 0.023 \cdot blob$
 Crank speed: $\omega := 2500 \cdot rpm$ Crank angle: $\theta := 24 \cdot deg$

Solution: See Mathcad file P1321.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 3.750 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 11.250 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.0188 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.00625 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.050 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}^2}{r} \quad m_{2a} = 0.0180 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.0368 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0293 \text{ blob}$$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \quad F_{ix} = 7506 \text{ lbf}$$

$$F_{iy} := 0 \cdot lbf \quad F_{iy} = 0 \text{ lbf}$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{ib} = 7506 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = 0.000 \text{ deg}$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \quad m_p = 9.7500 \times 10^{-3} \text{ blob}$$

$$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \dots \quad F_{ix} = 5369 \text{ lbf}$$

$$+ (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \cos(\theta))$$

$$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) + (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) \quad F_{iy} = -951 \text{ lbf}$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{iob} = 5453 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = -10.048 \text{ deg}$$

6. Compare the results to those for an unbalanced crank. From Problem 13-13, the inertia force for the unbalanced crank is $F_i := 15967 \cdot \text{lbf}$. The percent differences for the balanced and overbalanced cranks are:

$$\text{Exactly balanced:} \quad \Delta := \frac{F_{ib} - F_i}{F_i} \quad \Delta = -53.0 \%$$

$$\text{Overbalanced:} \quad \Delta := \frac{F_{iob} - F_i}{F_i} \quad \Delta = -65.9 \%$$

PROBLEM 13-22

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.

- Exactly balance the crank and recalculate the inertia force.
- Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius $-r$ on the crank and recalculate the inertia force.
- Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.25$
 Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.4$
 Piston mass: $m_4 := 0.015 \cdot blob$
 Crank speed: $\omega := 4000 \cdot rpm$ Crank angle: $\theta := 18 \cdot deg$

Solution: See Mathcad file P1322.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 3.750 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 11.250 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.0188 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.00625 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.600 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}^2}{r} \quad m_{2a} = 0.0200 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.0388 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0213 \text{ blob}$$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \quad F_{ix} = 17402 \text{ lbf}$$

$$F_{iy} := 0 \cdot lbf \quad F_{iy} = 0 \text{ lbf}$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{ib} = 17402 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = 0.000 \text{ deg}$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \quad m_p = 7.0833 \times 10^{-3} \text{ blob}$$

$$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \dots \quad F_{ix} = 12674 \text{ lbf}$$

$$+ (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \cos(\theta))$$

$$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) + (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) \quad F_{iy} = -1536 \text{ lbf}$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{iob} = 12766 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = -6.911 \text{ deg}$$

6. Compare the results to those for an unbalanced crank. From Problem 13-14, the inertia force for the unbalanced crank is $F_i := 44075 \cdot \text{lbf}$. The percent differences for the balanced and overbalanced cranks are:

$$\text{Exactly balanced:} \quad \Delta := \frac{F_{ib} - F_i}{F_i} \quad \Delta = -60.5 \%$$

$$\text{Overbalanced:} \quad \Delta := \frac{F_{iob} - F_i}{F_i} \quad \Delta = -71.0 \%$$

PROBLEM 13-23

Statement: Combine the necessary equations to develop expressions that show how each of these dynamic parameters varies as a function of the crank/conrod ratio alone:

- Piston acceleration.
- Inertia force.
- Inertia torque.
- Pin forces.

Plot the functions. Check your conclusions with program ENGINE.

Solution: See Mathcad file P1323.

- Note that the stroke volume V of the cylinder must be held at some constant value while varying the r/l ratio in order to isolate the effects of that parameter. We must hold all other factors constant for this analysis as well, such as all masses, crank angle, and bore/stroke ratio. We will assume that the crank radius r is held constant as we vary the r/l ratio in order to keep the stroke volume constant. Note that we can vary the r/l ratio independently of the crank radius r since for any r we can choose a value of l to maintain r/l constant.
- (Part a.) The expression for the piston acceleration is shown in equation *a*. Note that the r/l ratio only appears in the second harmonic term. If we allow r/l to become very small (long conrod), then this term diminishes toward zero, leaving a pure harmonic primary component. Thus, as the conrod length approaches infinity, the acceleration approaches a pure cosine. The r/l term acts as a distortion factor that changes the shape of this harmonic function in a way that increases its peak negative value. The optimum value for r/l is zero. The longer the conrod, the lower the peak acceleration.

$$a_p := -r \cdot \omega^2 \cdot \left(\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos(2 \cdot \omega \cdot t) \right) \quad (a)$$

- (Part b.) The expression for inertia force is shown in equation *b*. Note that only the x -component contains an r/l term and it is within the piston acceleration term. Thus the effects of the r/l ratio on inertia force are the same as those described for piston acceleration above. The optimum value for r/l is then zero. The longer the conrod, the lower the peak inertia force in the x -direction.

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot \cos(\omega \cdot t) \right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left[\cos(\theta) + \frac{r}{l} \cdot \cos[2 \cdot (\omega \cdot t)] \right] \right] \quad (b)$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot \sin(\omega \cdot t) \right)$$

- (Part c.) The expression for inertia torque is shown in equation *c*. Note that only the primary and tertiary components contain an r/l term. Thus, the effects of the r/l ratio on inertia torque are similar to those described for inertia force above. The dominant term in this equation is the second harmonic as it has the largest possible coefficient of 1. If the r/l ratio were zero (infinite conrod) then the inertia torque would be a pure harmonic of twice crank frequency. A nonzero r/l adds distortion to this curve in the form of the 1st and 3rd harmonics. The optimum value for r/l is zero. The longer the conrod, the lower the peak inertia torque.

$$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot \sin(\omega \cdot t) \cdot \left[\frac{r}{2 \cdot l} + \cos(\omega \cdot t) + \frac{3 \cdot r}{2 \cdot l} \cdot \cos[2 \cdot (\omega \cdot t)] \right] \quad (c)$$

- (Part d.) The equation for the wrist pin force is shown in equation *d*. It contains both the acceleration of the piston (*e*) and $\tan \phi$ (*f*). These are substituted into *d* to get *g*. This expression shows that the wrist pin force is directly proportional to both r/l and the sum of that ratio squared to form the fourth power as shown in *h*. The optimum value of r/l is then zero. The longer the conrod, the lower the wrist pin force.

$$F_{34x} := F_g + m_4 \cdot a_B \quad (d)$$

$$F_{34y} := -\left[F_g + (m_4 + m_{3b}) \cdot a_B \right] \cdot \tan(\phi)$$

$$a_B := -r \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos[2 \cdot (\omega \cdot t)] \right] \quad (e)$$

$$\phi := \operatorname{atan} \left[\frac{r \cdot \sin(\omega \cdot t)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\omega \cdot t) \right)^2}} \right] \quad (f)$$

$$F_{34x} := F_g + m_4 \cdot \left[-r \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos[2 \cdot (\omega \cdot t)] \right] \right] \quad (g)$$

$$F_{34y} := - \left[F_g + (m_4 + m_{3b}) \cdot \left[-r \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos[2 \cdot (\omega \cdot t)] \right] \right] \right] \cdot \left[\frac{r}{l} \cdot \frac{\sin(\omega \cdot t)}{\sqrt{1 - \left(\frac{r}{l} \cdot \sin(\omega \cdot t) \right)^2}} \right]$$

$$\begin{aligned} F_{34x} & \text{ proportional to } \frac{r}{l} \\ F_{34y} & \text{ proportional to } - \left(\frac{r^2}{l^2} + \frac{r^4}{l^4} \right) \end{aligned} \quad (h)$$

6. (Part d.) The equation for the crankpin force is shown in equation *i*. It contains both the acceleration of the piston (*e*) and $\tan \phi$ (*f*). These are substituted into *i* to get *j*. This expression shows that the crankpin force is directly proportional to both r/l and the sum of that ratio squared and to the fourth power as shown in *k*. The optimum value of r/l is then zero. The longer the conrod, the lower the crankpin force.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\omega \cdot t) - (m_{3b} + m_4) \cdot a_B - F_g \quad (i)$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\omega \cdot t) + [F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\omega \cdot t) - (m_{3b} + m_4) \cdot \left[-r \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos[2 \cdot (\omega \cdot t)] \right] \right] - F_g$$

$$\begin{aligned} F_{32y} := & m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\omega \cdot t) \dots \\ & + \left[F_g + (m_4 + m_{3b}) \cdot \left[-r \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos[2 \cdot (\omega \cdot t)] \right] \right] \right] \cdot \left[\frac{r}{l} \cdot \frac{\sin(\omega \cdot t)}{\sqrt{1 - \left(\frac{r}{l} \cdot \sin(\omega \cdot t) \right)^2}} \right] \end{aligned} \quad (j)$$

$$\begin{aligned} F_{32x} & \text{ proportional to } \frac{r}{l} \\ F_{32y} & \text{ proportional to } \left(\frac{r^2}{l^2} + \frac{r^4}{l^4} \right) \end{aligned} \quad (k)$$

7. (Part d.) The equation for the mainpin force is shown in equation *l*. It is equal to the crankpin force just analyzed above plus another term. Note that the second term does not contain r/l , so the mainpin force has the same dependence on r/l as does the crankpin force.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot \cos(\omega \cdot t) + F_{32x}$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot \sin(\omega \cdot t) + F_{32y}$$
(l)

$$F_{21x} \quad \text{proportional to} \quad \frac{r}{l}$$

$$F_{21y} \quad \text{proportional to} \quad \left(\frac{r^2}{l^2} + \frac{r^4}{l^4} \right)$$
(m)

PROBLEM 13-24

Statement: Combine the necessary equations to develop expressions that show how each of these dynamic parameters varies as a function of the bore/stroke ratio alone:

- Gas force.
- Gas torque.
- Inertia force.
- Inertia torque.
- Pin forces.

Plot the functions. Check your conclusions with program ENGINE.

Solution: See Mathcad file P1324.

- The stroke volume V of the cylinder must be held at some constant value as shown in equation a while varying the B/S ratio in order to isolate the effects of that parameter. This relationship is rearranged to introduce the bore/stroke ratio B/S in equation b . We must hold all other factors constant for this analysis as well, such as all masses, crank angle, and crank/conrod ratio. Note that we can vary the B/S ratio independently of the crank/ conrod ratio r/l since for any r we can choose a value of l to maintain r/l constant.

$$V := \frac{\pi}{4} \cdot B^2 \cdot S = \text{constant} \quad (a)$$

$$\text{but, } B^2 := \left(\frac{B}{S}\right)^2 \cdot S^2 \quad \text{so, } V := \frac{\pi}{4} \cdot \left(\frac{B}{S}\right)^2 \cdot S^3 \quad (b)$$

- (Part a.) The gas force equation is shown in equation c and the expression for $(B/S)^2$ from b is substituted in equation c . Expressions b and c are each then solved for S and combined in d . Gas force F_g is proportional to B/S as shown in e . Any increase in the B/S ratio will increase the gas force and vice versa.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \quad \text{or} \quad F_g := \frac{\pi}{4} \cdot p_g \cdot \left(\frac{B}{S}\right)^2 \cdot S^2 \quad (c)$$

$$\text{combine equations } b \text{ and } c \text{ to get: } F_g := 0.922635 \cdot p_g \cdot \left[V \cdot \left(\frac{B}{S}\right)\right]^{\frac{2}{3}} \quad (d)$$

$$\text{so, for a given } p_g \text{ and } V, F_g \text{ is proportional to } (B/S)^{2/3}. \quad (e)$$

- (Part b.) The gas torque equation is shown in equation f . Expressions for its F_g and r terms are shown in equation g . These are substituted into f and shown as h . The first expression in parentheses is the stroke volume V , which is constant as shown in i . Thus, as shown in j , the gas torque is independent of B/S .

$$T_{g21} := F_g \cdot r \cdot \sin(\omega \cdot t) \cdot \left(1 + \frac{r}{l} \cdot \cos(\omega \cdot t)\right) \quad (f)$$

$$F_g := -\frac{\pi}{4} \cdot p_g \cdot B^2 \quad r := \frac{S}{2} \quad (g)$$

$$T_{g21} := -\frac{1}{2} \cdot p_g \cdot \left(\frac{\pi}{4} \cdot B^2 \cdot S\right) \cdot \sin(\omega \cdot t) \cdot \left(1 + \frac{r}{l} \cdot \cos(\omega \cdot t)\right) \quad (h)$$

$$V := \frac{\pi}{4} \cdot B^2 \cdot S = \text{constant} \quad (i)$$

$$T_{g21} := -\frac{1}{2} \cdot p_g \cdot V \cdot \sin(\omega \cdot t) \cdot \left(1 + \frac{r}{l} \cdot \cos(\omega \cdot t)\right) \quad (j)$$

4. (Part c.) The expression for inertia force is shown in equation *k* with the expression $S = 2r$ substituted and the S factored out. These are simplified by grouping all other factors as parameters K_1 , K_2 , and K_3 and the magnitude of the force found. The dimensionless parameter of B/S ratio is introduced in *l*. The stroke volume constraint is then introduced, combined with *l* and substituted in *k* to get the nonlinear inverse relationship of inertia force to the B/S ratio shown in *m*.

$$F_{ix} := S \cdot \left[\frac{\omega^2}{2} \cdot \left[m_A \cdot \cos(\omega \cdot t) + m_B \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \omega \cdot t) \right) \right] \right] = S \cdot K_1$$

$$F_{iy} := S \cdot \left(\frac{m_A}{2} \cdot \omega^2 \cdot \sin(\omega \cdot t) \right) = S \cdot K_2$$

$$F_i = \sqrt{F_{ix}^2 + F_{iy}^2} = \sqrt{(S \cdot K_1)^2 + (S \cdot K_2)^2} = \left(S \cdot \sqrt{K_1^2 + K_2^2} = S \cdot K_3 \right) \quad (k)$$

$$S := B \cdot \left(\frac{B}{S} \right)^{-1} \quad F_i := K_3 \cdot B \cdot \left(\frac{B}{S} \right)^{-1} \quad (l)$$

introduce volume constraint,

$$V = \frac{\pi}{4} \cdot B^2 \cdot S = \frac{\pi}{4} \cdot B^3 \cdot \left(\frac{B}{S} \right)^{-1} \quad B = \left[\frac{4}{\pi} \cdot V \cdot \left(\frac{B}{S} \right) \right]^{\frac{1}{3}} = 1.083852 \cdot V^{\frac{1}{3}} \cdot \left(\frac{B}{S} \right)^{\frac{1}{3}}$$

substitute this into the force equation

$$F_i := 1.083852 \cdot K_3 \cdot V^{\frac{1}{3}} \cdot \left(\frac{B}{S} \right)^{-\frac{2}{3}} \quad (m)$$

5. (Part d.) The inertia torque equation is shown simplified in equation *n*. Equation *l* is substituted into *n* to show the relationship of inertia torque to the B/S ratio in *o*.

$$T_{i21} := S^2 \cdot \left[\frac{m_B \cdot \omega^2}{8} \cdot \left(\frac{r}{2 \cdot l} \cdot \sin(\omega \cdot t) - \sin(2 \cdot \omega \cdot t) - \frac{3 \cdot r}{2 \cdot l} \cdot \sin(3 \cdot \omega \cdot t) \right) \right] = S^2 \cdot K_4 \quad (n)$$

$$T_{i21} = K_4 \cdot \left[B \cdot \left(\frac{B}{S} \right)^{-1} \right]^2 = K_4 \cdot B^2 \cdot \left(\frac{B}{S} \right)^{-2} = K_4 \cdot \left[\frac{4}{\pi} \cdot V \cdot \left(\frac{B}{S} \right) \right]^{\frac{2}{3}} \cdot \left(\frac{B}{S} \right)^{-2} \quad (o)$$

6. (Part e.) The equation for the wrist pin force is shown in equation *p*. It contains both the gas force (*c*) and the acceleration of the piston (*q*). These are substituted along with *l* into *p* to get *r*. This expression shows that the wrist pin force is directly proportional to both B^2 and S as shown in *s*. This means that there may be an optimum value of B/S , which will minimize the wrist pin force. Note that the $\tan \phi$ term contains only r/l terms so is constant in this analysis.

$$F_{34x} := F_g + m_4 \cdot a_B \quad (p)$$

$$F_{34y} := -[F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$a_B := -r \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos[2 \cdot (\omega \cdot t)] \right] \quad (q)$$

$$F_{34x} := -\frac{\pi}{4} \cdot p_g \cdot B^2 + m_4 \cdot \left[-\frac{S}{2} \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos[2 \cdot (\omega \cdot t)] \right] \right] \quad (r)$$

$$F_{34y} := \left[-\frac{\pi}{4} \cdot p_g \cdot B^2 + (m_4 + m_{3b}) \cdot \left[-\frac{S}{2} \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos[2 \cdot (\omega \cdot t)] \right] \right] \right] \cdot \tan(\phi)$$

$$F_{34} \quad \text{is proportional to} \quad B^2 - S \quad (s)$$

7. (Part e.) The equation for the crankpin force is shown in equation *t*. It contains both the gas force F_g (*c*) and the acceleration of the piston (*q*). These are substituted along with l into *t* to get *u*. This expression shows that the crankpin force is directly proportional to both B^2 and S as shown in *v*. This means that there may be an optimum value of B/S , which will minimize the crankpin force. Note that the $\tan\phi$ term contains only r/l terms so is constant in this analysis.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\omega \cdot t) - (m_{3b} + m_4) \cdot a_B - F_g \quad (t)$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\omega \cdot t) + [F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$F_{32x} := \left[m_{3a} \cdot \frac{S}{2} \cdot \omega^2 \cdot \cos(\omega \cdot t) - (m_{3b} + m_4) \cdot \left[-\frac{S}{2} \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos[2 \cdot (\omega \cdot t)] \right] \right] \right] + \frac{\pi}{4} \cdot p_g \cdot B^2$$

$$F_{32y} := m_{3a} \cdot \frac{S}{2} \cdot \omega^2 \cdot \sin(\omega \cdot t) + \left[-\frac{\pi}{4} \cdot p_g \cdot B^2 + (m_4 + m_{3b}) \cdot \left[-\frac{S}{2} \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos[2 \cdot (\omega \cdot t)] \right] \right] \right] \cdot \tan(\phi) \quad (u)$$

$$F_{32x} \quad \text{proportional to} \quad B^2 + S$$

$$F_{32y} \quad \text{proportional to} \quad -B^2 \quad (v)$$

8. (Part e.) The equation for the mainpin force is shown in equation *w*. It is equal to the crankpin force just analyzed above plus another term. The result from *u* is substituted along with l into *w* to get *x*. This expression shows that the crankpin force is directly proportional to both B^2 and S as shown in *x*. This means that there may be an optimum value of B/S , which will minimize the mainpin force.

$$F_{21x} := m_{2a} \cdot \frac{S}{2} \cdot \omega^2 \cdot \cos(\omega \cdot t) + F_{32x} \quad (w)$$

$$F_{21y} := m_{2a} \cdot \frac{S}{2} \cdot \omega^2 \cdot \sin(\omega \cdot t) + F_{32y}$$

$$F_{21x} \quad \text{proportional to} \quad B^2 + S$$

$$F_{21y} \quad \text{proportional to} \quad B^2 + S \quad (x)$$

PROBLEM 13-25

Statement: Develop an expression to determine the optimum bore/stroke ratio to minimize the wrist pin force. Plot the function.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Assumptions: Use the data from Problem 13-15 for the unvarying parameters.

Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 \cdot blob$

Distance to CG (as fraction of l): $r_{3a} := 0.4$

Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.3$

Piston mass: $m_4 := 0.022 \cdot blob$ Gas pressure: $p_g := 300 \cdot psi$

Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 45 \cdot deg$

Stroke volume: $V := 1 \cdot in^3$ R/L ratio: $RoverL := 0.2917$

Solution: See Problem 13-15 and Mathcad file P1325.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $l_a = 4.800 \text{ in}$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 7.200 \text{ in}$

Let $l_p := l_a$ $m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b}$ $m_{3a} = 0.0120 \text{ blob}$

$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b}$ $m_{3b} = 0.00800 \text{ blob}$

2. Determine the stroke and crank radius as functions of B/S .

Stroke as function of B/S : $S(BoverS) := \left(\frac{4 \cdot V}{\pi} \right)^{\frac{1}{3}} \cdot BoverS^{-\frac{2}{3}}$

Crank length as a function of stroke: $r(BoverS) := 0.5 \cdot S(BoverS)$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$\phi := atan \left[\frac{RoverL \cdot \sin(\theta)}{\sqrt{1 - (RoverL \cdot \sin(\theta))^2}} \right]$ $\phi = 11.903 \text{ deg}$

$a_B(BoverS) := -r(BoverS) \cdot \omega^2 \cdot (\cos(\theta) + RoverL \cdot \cos(2 \cdot \theta))$

5. Write an expression for the gas force as a function of the B/S ratio.

$F_g(BoverS) := \frac{\pi}{4} \cdot p_g \cdot BoverS^2 \cdot S(BoverS)^2$

6. Determine the wrist pin force F_{34} using equation 13.21.

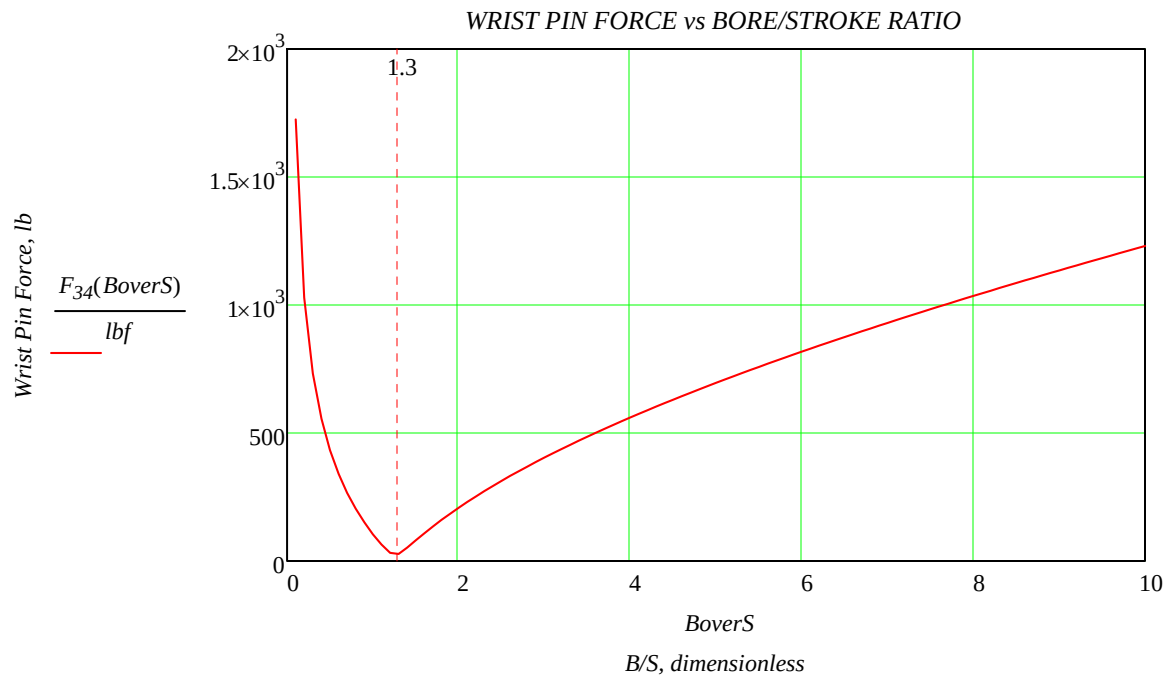
$F_{34x}(BoverS) := F_g(BoverS) + m_4 \cdot a_B(BoverS)$

$$F_{34y}(BoverS) := -[F_g(BoverS) + (m_4 + m_{3b}) \cdot a_B(BoverS)] \cdot \tan(\phi)$$

$$F_{34}(BoverS) := \sqrt{F_{34x}(BoverS)^2 + F_{34y}(BoverS)^2}$$

7. Plot the wrist pin force F_{34} as a function of B/S .

$$BoverS := 0.1, 0.2 \dots 10$$



8. The graph shows that there is an optimum value of B/S that will minimize the wrist pin force, which in this case, is about $B/S = 1.3$. However, the optimum value is dependent on the other design choices (especially peak gas pressure) and can vary considerably from this value. The bore, stroke, and crank radius that would result from choosing this value are:

$$S_{opt} := S(1.3)$$

$$S_{opt} = 0.910 \text{ in}$$

$$r_{opt} := r(1.3)$$

$$r_{opt} = 0.455 \text{ in}$$

$$B_{opt} := \sqrt{\frac{4 \cdot V}{\pi \cdot S_{opt}}}$$

$$B_{opt} = 1.183 \text{ in}$$

$$l_{opt} := \frac{r_{opt}}{RoverL}$$

$$l_{opt} = 1.560 \text{ in}$$

This is probably not a practical design.

PROBLEM 13-26

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the main pin of a one-cubic-inch displacement, single-cylinder engine with bore = 1.1283 in for the following situations.

- Piston, conrod and crank masses = 0.
- Piston mass = 1 blob, conrod and crank masses = 0.
- Conrod mass = 1 blob, piston and crank masses = 0.
- Crank mass = 1 blob, conrod and piston masses = 0.

Place the CG of the crank at $0.5r$ and the conrod at $0.33l$. Compare and explain the differences in the main pin force under these different conditions with reference to the governing equations.

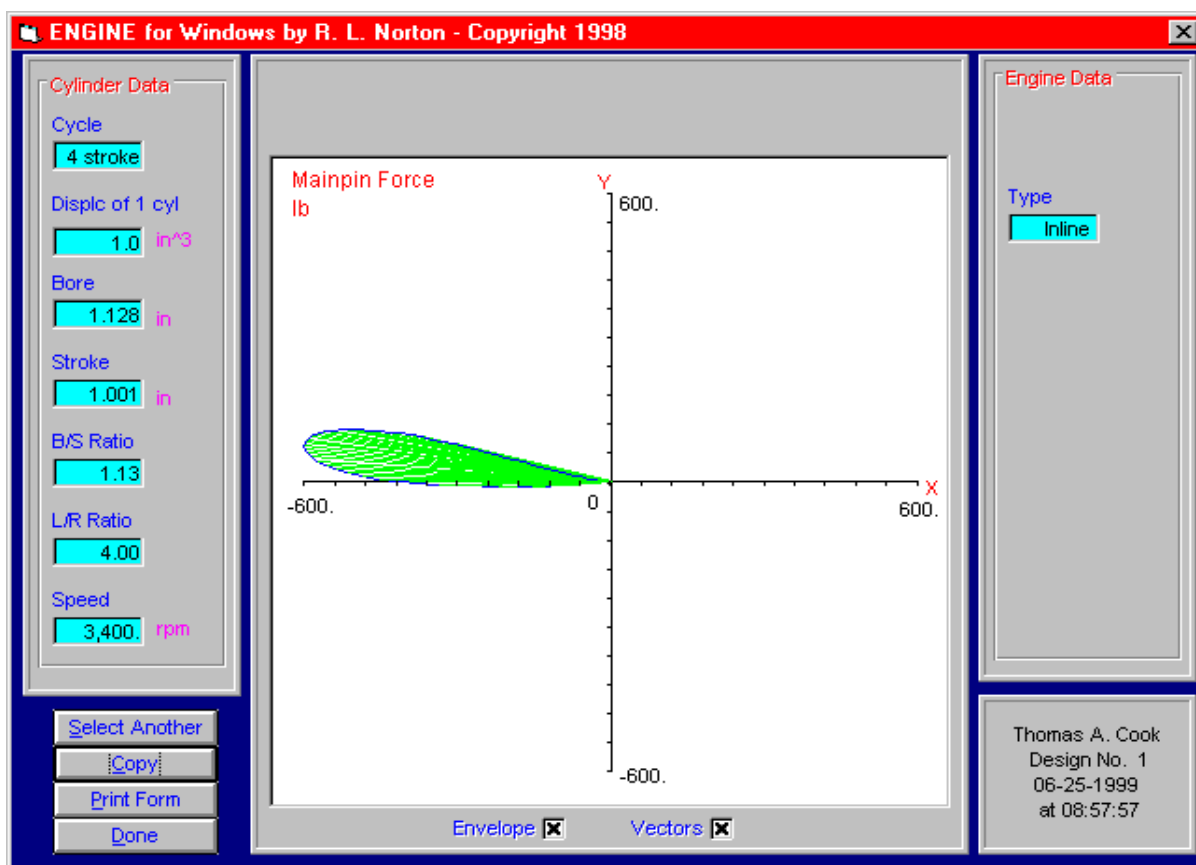
- Assumptions:**
- Peak cylinder pressure is 600 psi.
 - Link length ratio is $L/R = 4$.
 - Main pin dia is 2.00 in.
 - Crankpin dia is 1.500 in.
 - Friction coefficient is zero.

Solution: See Mathcad file P1326.

- Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

$$a. \quad m_2 := 0 \quad m_3 := 0 \quad m_4 := 0 \quad F_{21max} := 604.3 \cdot lbf$$

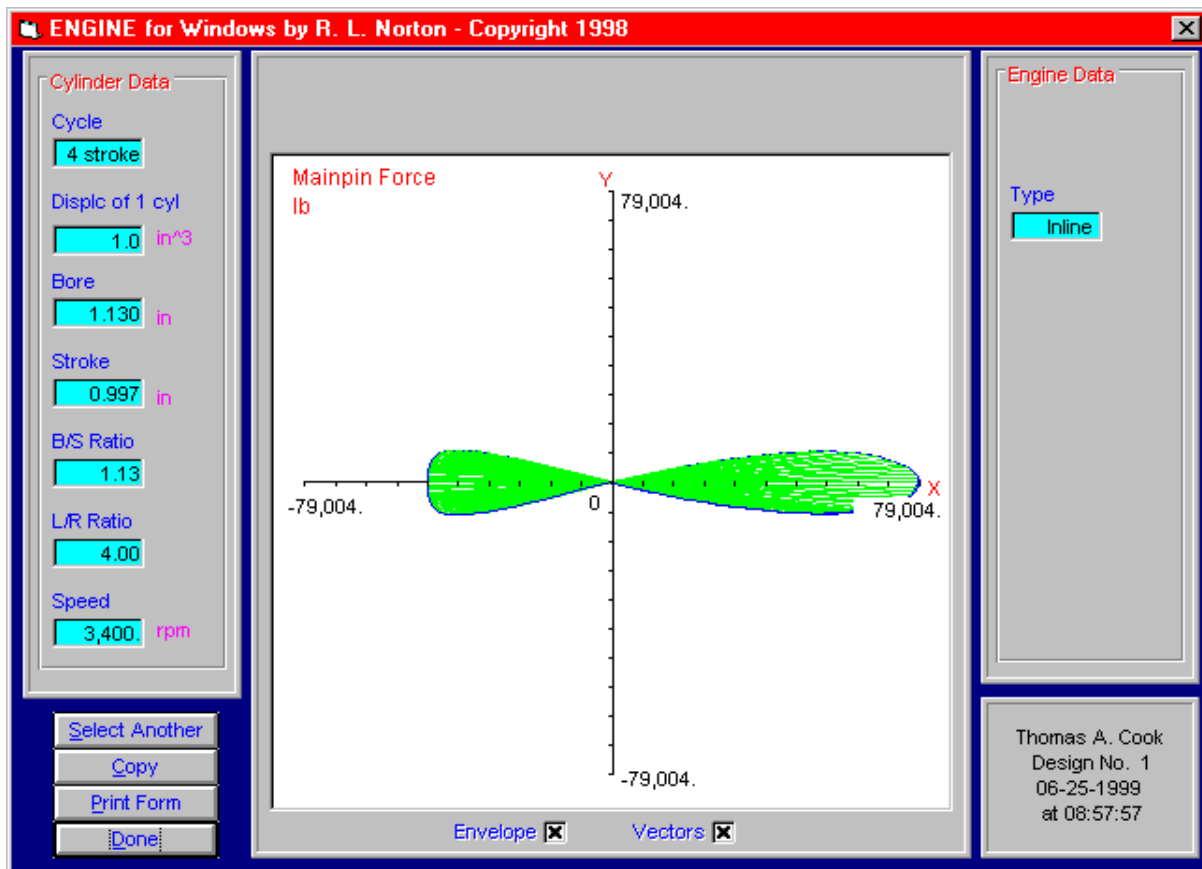
Polar plot of main pin force for part *a*.



- Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

$$b. \quad m_2 := 0 \quad m_3 := 0 \quad m_4 := 1 \quad F_{21max} := 79004 \cdot lbf$$

Polar plot of main pin force for part *b*.



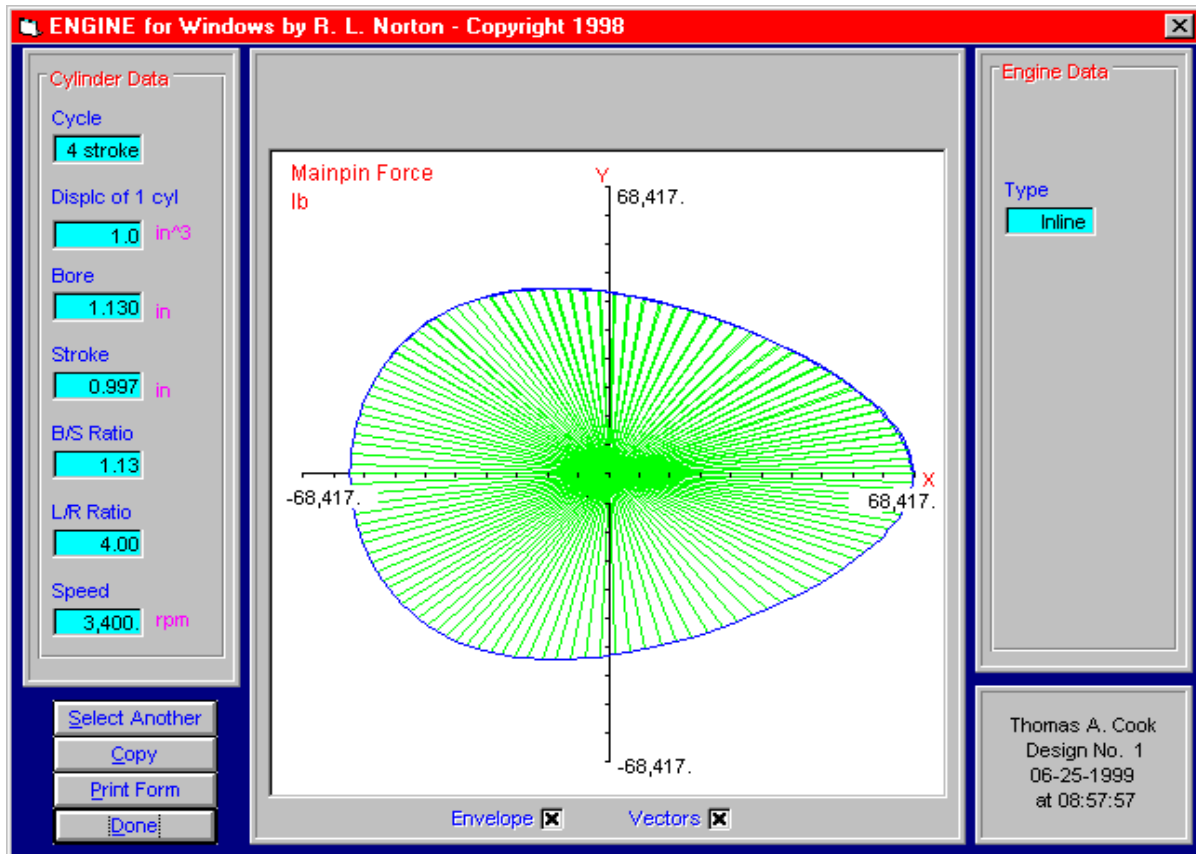
3. Enter the above data and the masses for part *c* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

$$c. \quad m_2 := 0 \quad m_3 := 1 \quad m_4 := 0 \quad F_{21max} := 68417 \cdot \text{lb}f$$

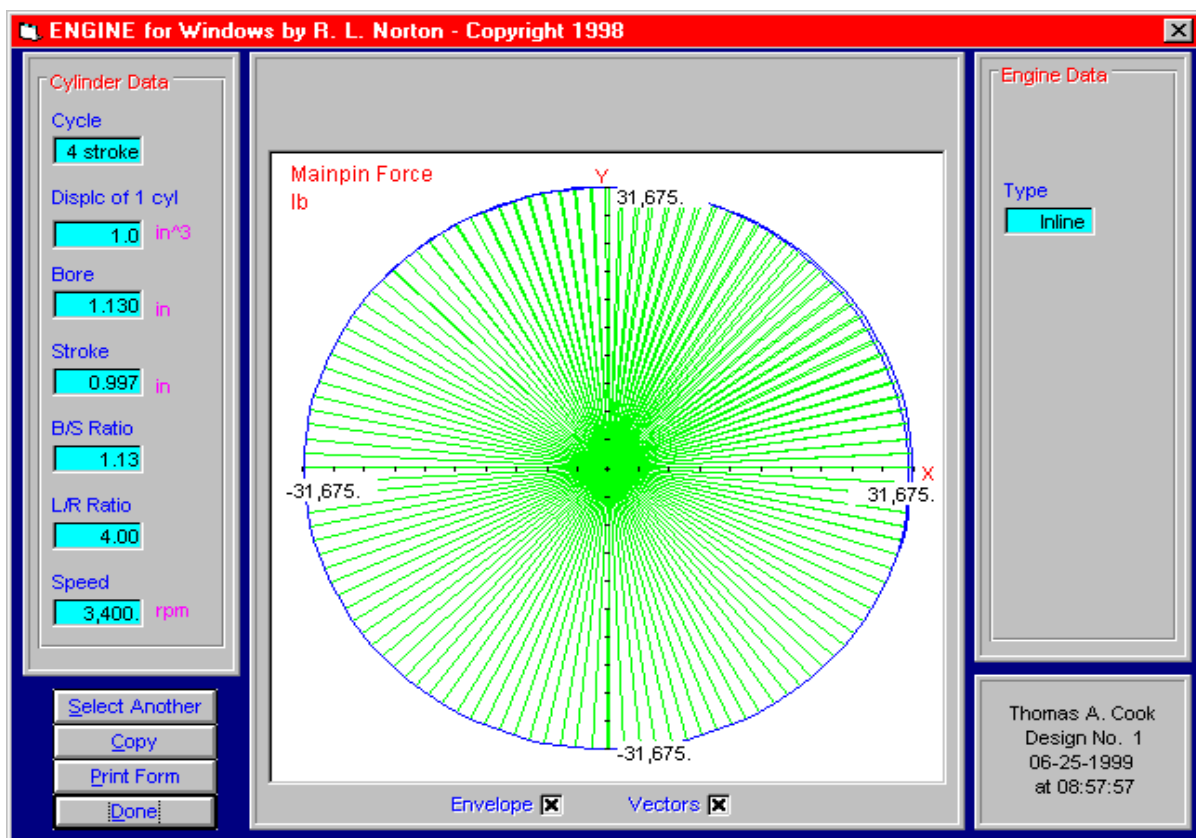
4. Enter the above data and the masses for part *d* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

$$d. \quad m_2 := 1 \quad m_3 := 0 \quad m_4 := 0 \quad F_{21max} := 31696 \cdot \text{lb}f$$

Polar plot of main pin force for part c.



Polar plot of main pin force for part d.



PROBLEM 13-27

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the crankpin of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

- Piston, conrod and crank masses = 0.
- Piston mass = 1 blob, conrod and crank masses = 0.
- Conrod mass = 1 blob, piston and crank masses = 0.
- Crank mass = 1 blob, conrod and piston masses = 0.

Place the CG of the crank at $0.5r$ and the conrod at $0.33l$. Compare and explain the differences in the crankpin force under these different conditions with reference to the governing equations.

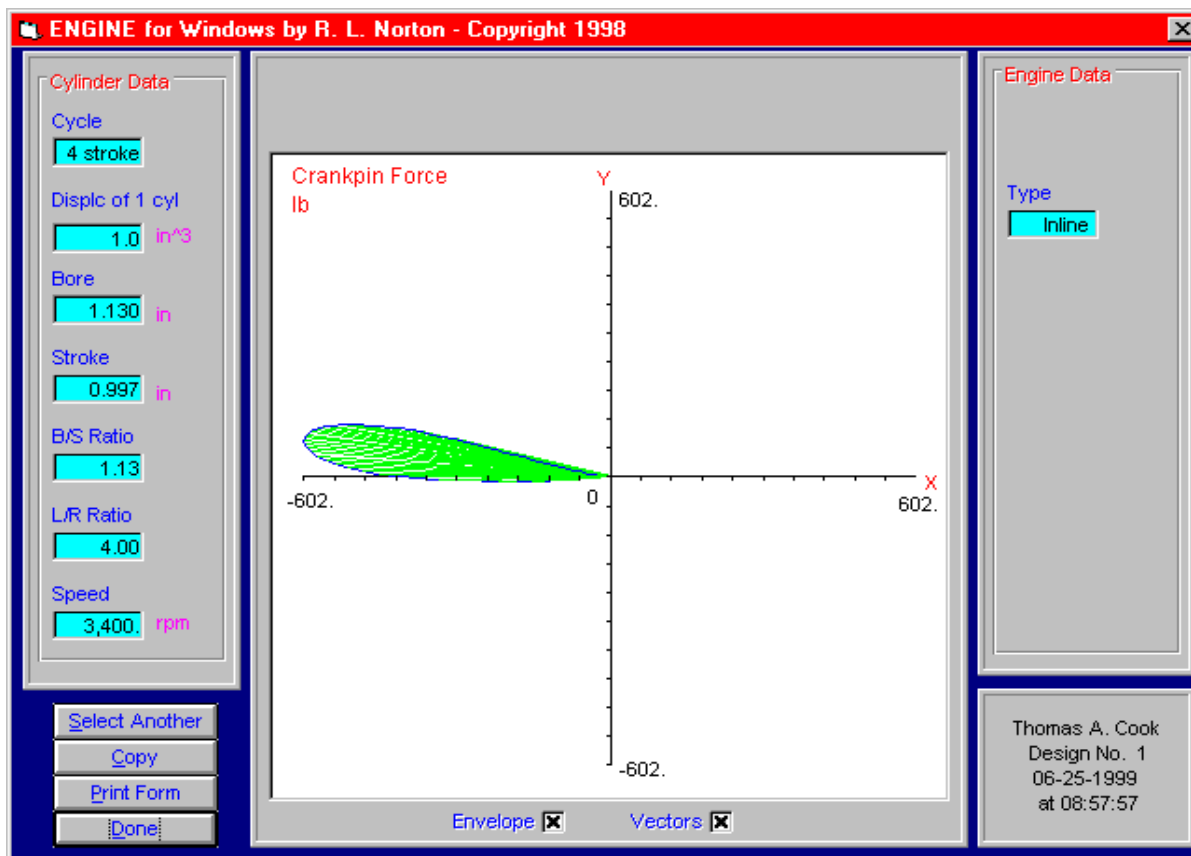
- Assumptions:**
- Peak cylinder pressure is 600 psi.
 - Link length ratio is $L/R = 4$.
 - Main pin dia is 2.00 in.
 - Crankpin dia is 1.500 in.
 - Friction coefficient is zero.

Solution: See Mathcad file P1327.

- Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

$$a. \quad m_2 := 0 \quad m_3 := 0 \quad m_4 := 0 \quad F_{32max} := 602 \cdot \text{lbf}$$

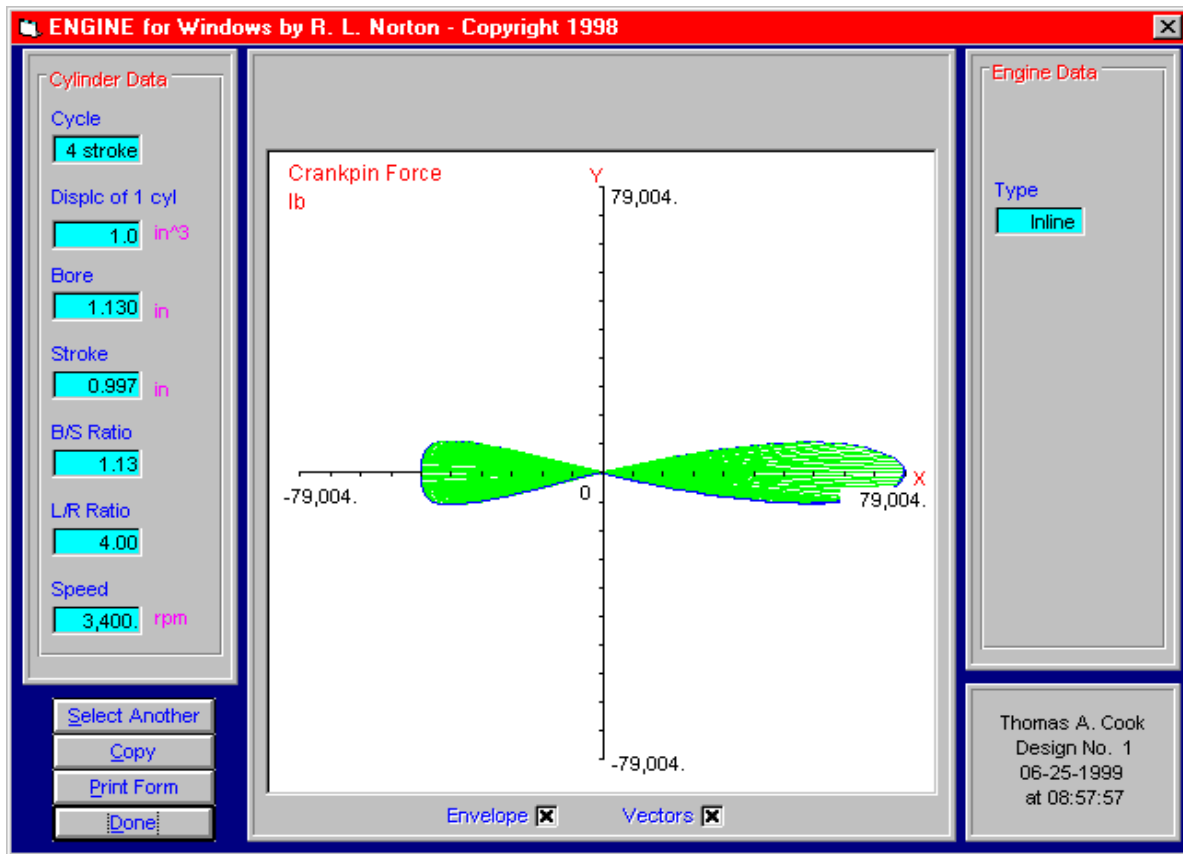
Polar plot of crankpin force for part *a*.



- Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

$$b. \quad m_2 := 0 \quad m_3 := 0 \quad m_4 := 1 \quad F_{32max} := 79004 \cdot \text{lbf}$$

Polar plot of crankpin force for part *b*.



3. Enter the above data and the masses for part *c* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

$$c. \quad m_2 := 0 \quad m_3 := 1 \quad m_4 := 0 \quad F_{32max} := 68417 \cdot \text{lb}f$$

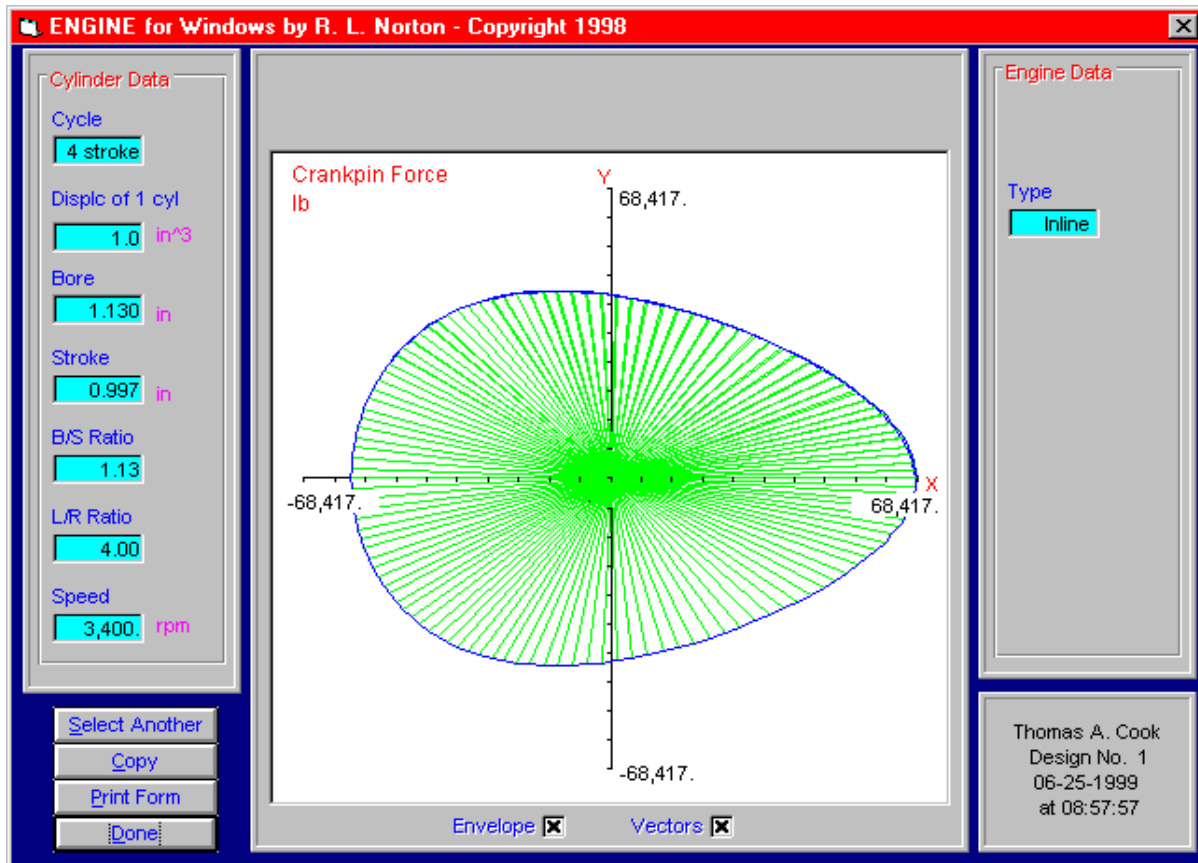
See polar plot on next page.

4. Enter the above data and the masses for part *d* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

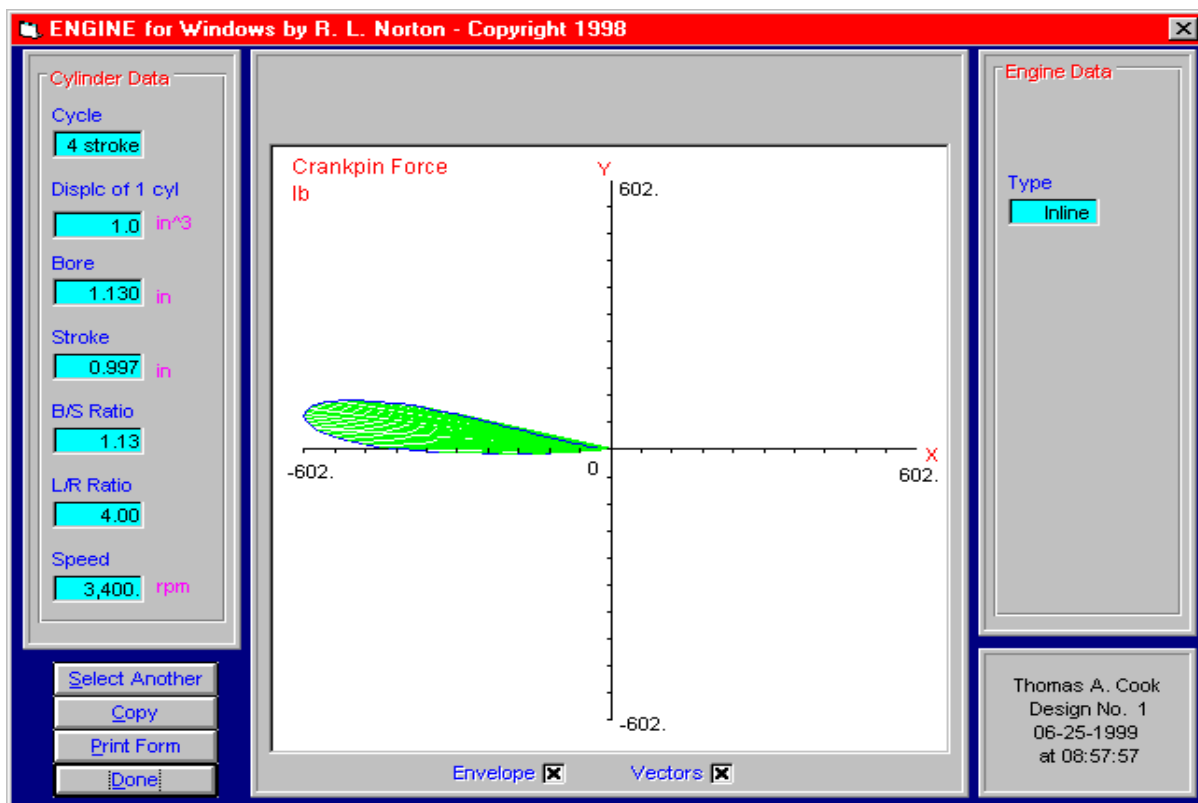
$$d. \quad m_2 := 1 \quad m_3 := 0 \quad m_4 := 0 \quad F_{32max} := 602 \cdot \text{lb}f$$

See polar plot on next page.

Polar plot of crankpin force for part c.



Polar plot of crankpin force for part d.



PROBLEM 13-28

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the wrist pin of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

- Piston, conrod and crank masses = 0.
- Piston mass = 1 blob, conrod and crank masses = 0.
- Conrod mass = 1 blob, piston and crank masses = 0.
- Crank mass = 1 blob, conrod and piston masses = 0.

Place the CG of the crank at $0.5r$ and the conrod at $0.33l$. Compare and explain the differences in the wrist pin force under these different conditions with reference to the governing equations.

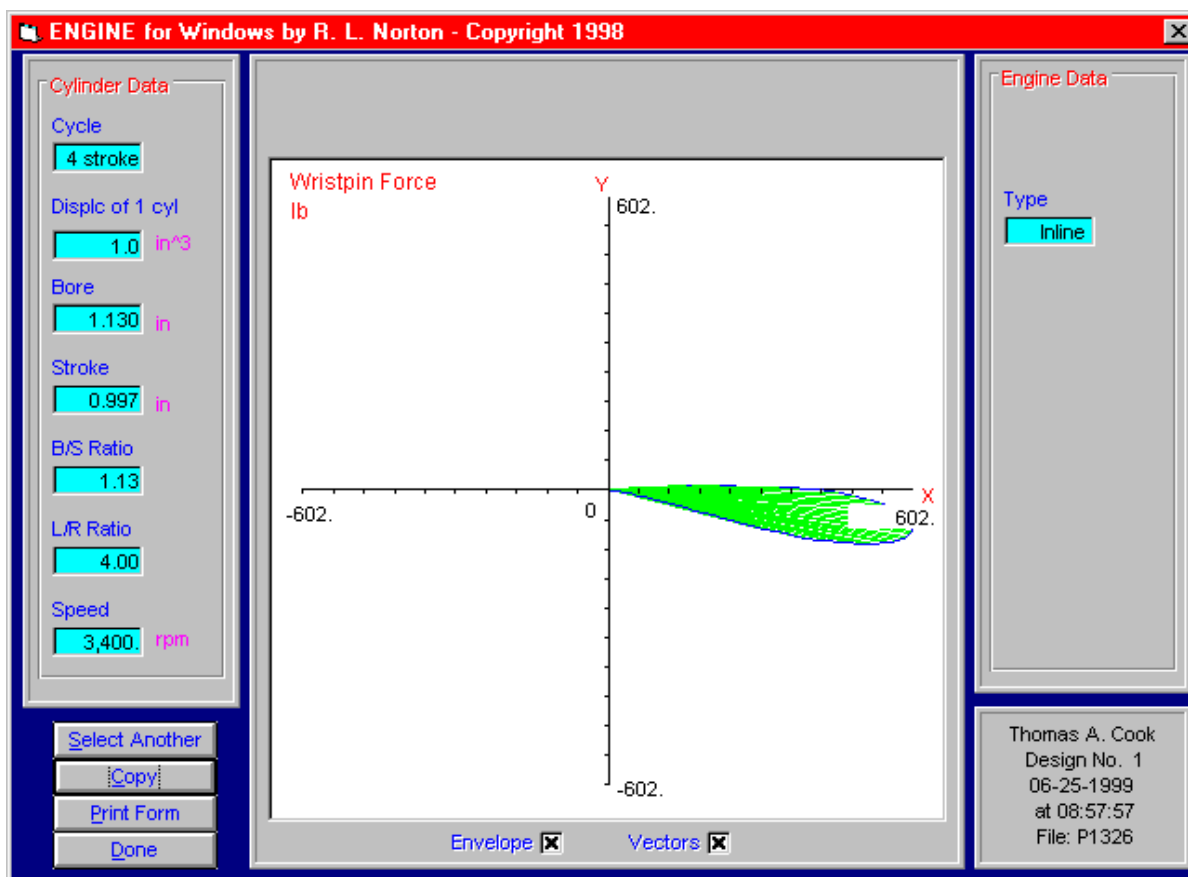
- Assumptions:**
- Peak cylinder pressure is 600 psi.
 - Link length ratio is $L/R = 4$.
 - Main pin dia is 2.00 in.
 - Crankpin dia is 1.500 in.
 - Friction coefficient is zero.

Solution: See Mathcad file P1328.

- Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

$$a. \quad m_2 := 0 \quad m_3 := 0 \quad m_4 := 0 \quad F_{34max} := 602 \cdot \text{lb}$$

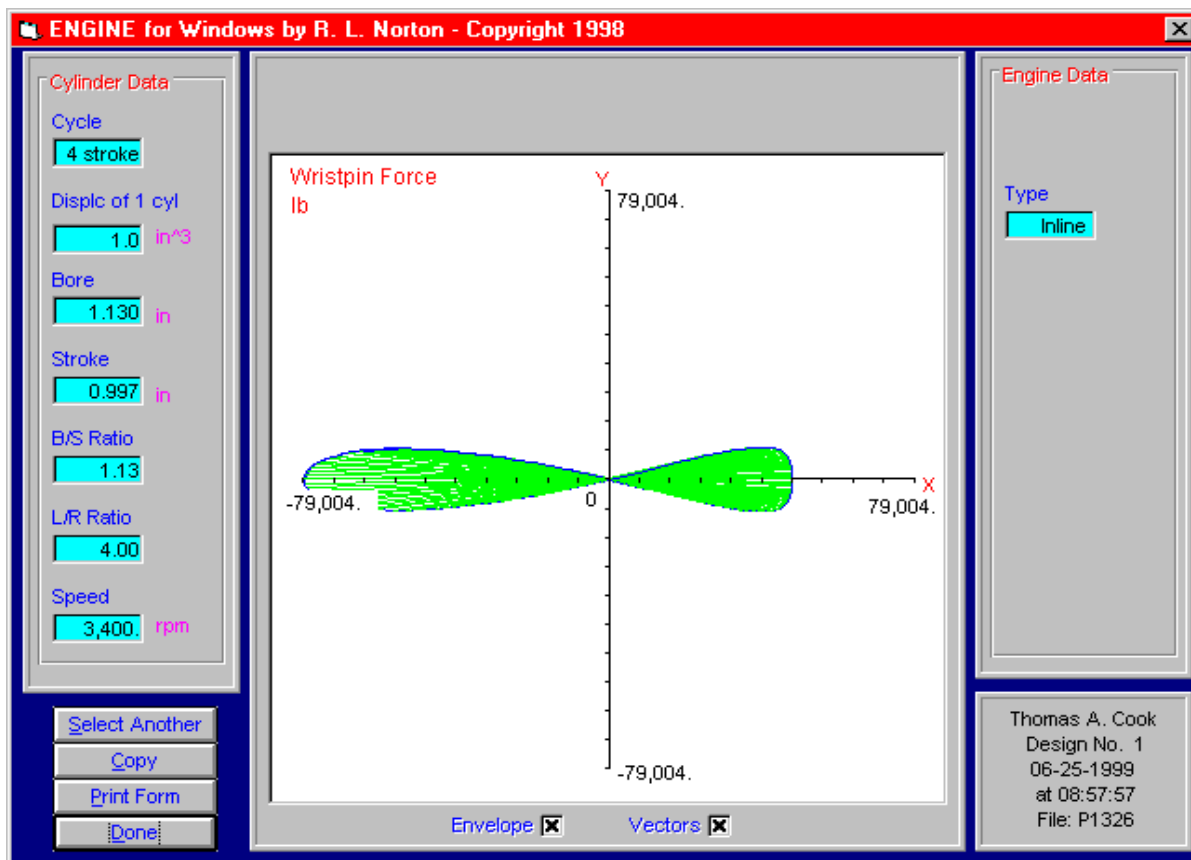
Polar plot of wrist pin force for part *a*.



- Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

$$b. \quad m_2 := 0 \quad m_3 := 0 \quad m_4 := 1 \quad F_{34max} := 79004 \cdot \text{lb}$$

Polar plot of wrist pin force for part *b*.



3. Enter the above data and the masses for part *c* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

$$c. \quad m_2 := 0 \quad m_3 := 1 \quad m_4 := 0 \quad F_{34max} := 2914 \cdot lbf$$

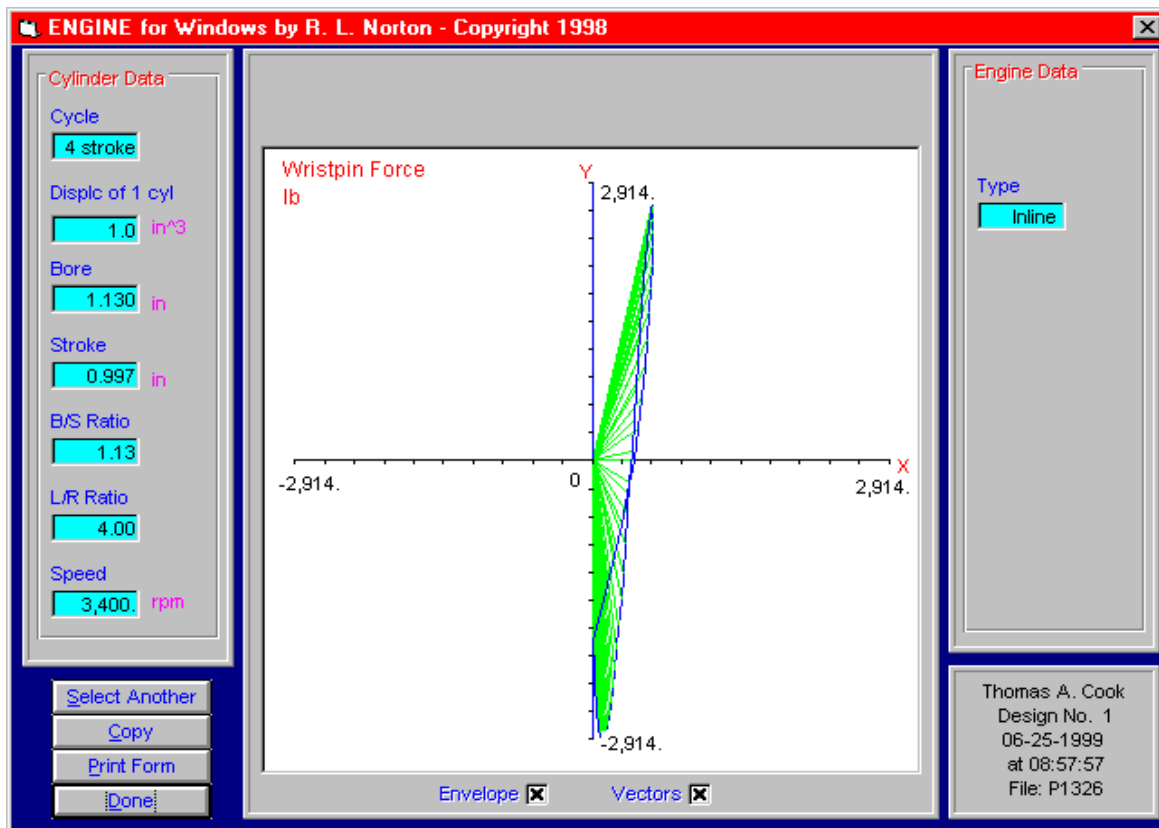
See polar plot on next page.

4. Enter the above data and the masses for part *d* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

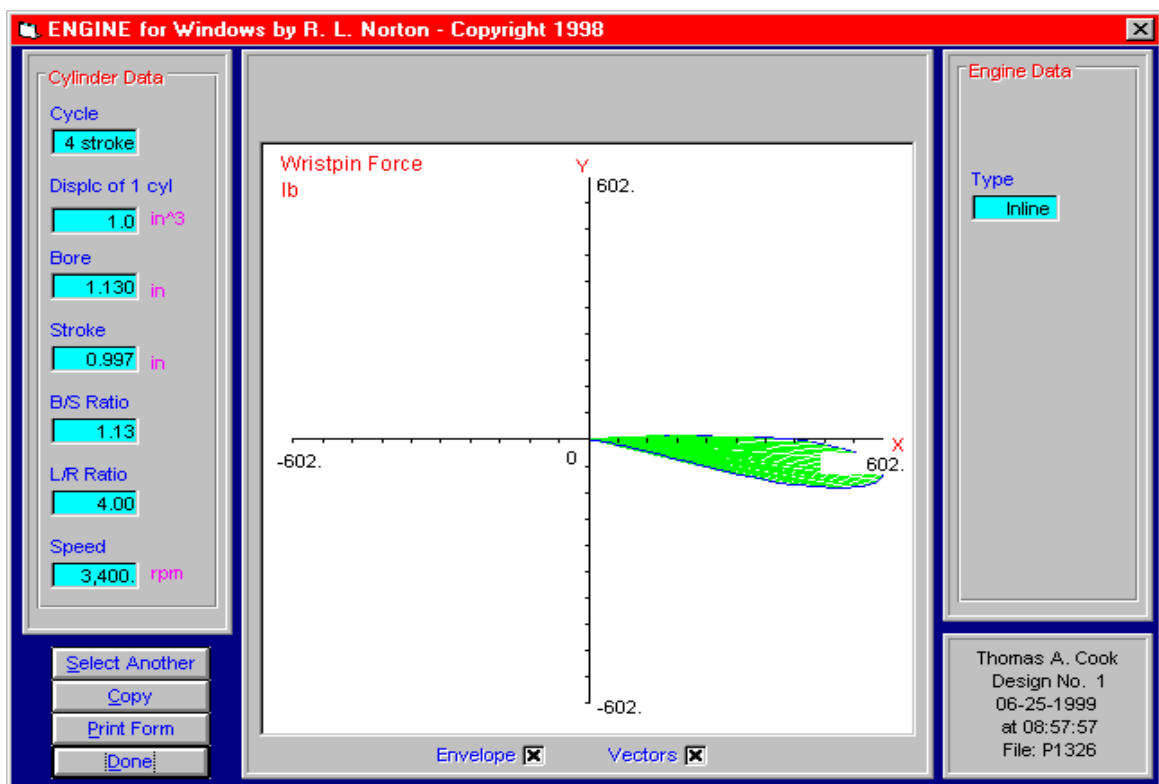
$$d. \quad m_2 := 1 \quad m_3 := 0 \quad m_4 := 0 \quad F_{34max} := 602 \cdot lbf$$

See polar plot on next page.

Polar plot of wrist pin force for part c.



Polar plot of wrist pin force for part d.



PROBLEM 13-29

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the main pin of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

- Engine unbalanced.
- Crank exactly balanced against mass at crankpin.
- Crank optimally overbalanced against masses at crankpin and wrist pin.

Piston, conrod, and crank masses = 1. Place the CG of the crank at $0.5r$ and the conrod at $0.33l$. Compare and explain the differences in the main pin force under these different conditions with reference to the governing equations.

Assumptions:

1. Peak cylinder pressure is 600 psi.
2. Link length ratio is $L/R = 4$.
3. Main pin dia is 2.00 in.
4. Crankpin dia is 1.500 in.
5. Friction coefficient is zero.

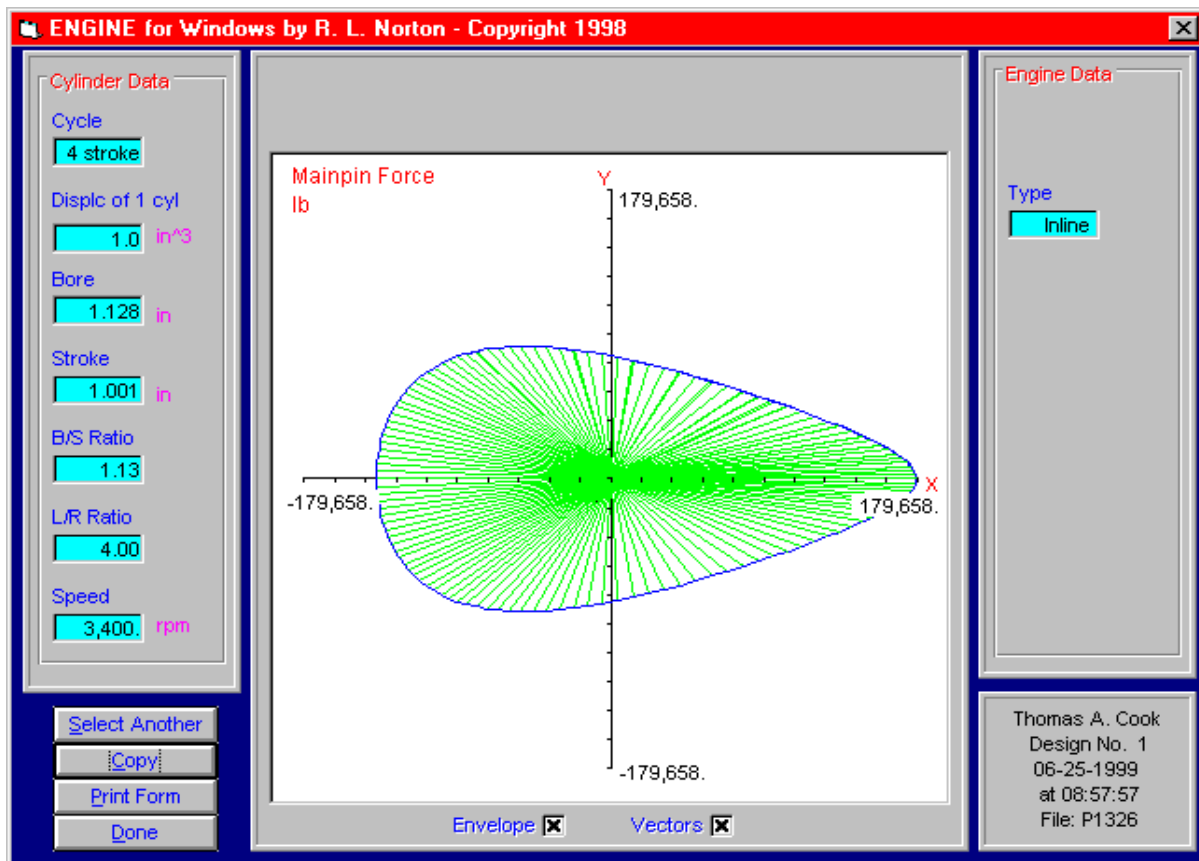
Solution: See Mathcad file P1329.

1. Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

a. Engine unbalanced

$$F_{21max} := 179658 \text{ lbf}$$

Polar plot of main pin force for part *a*.

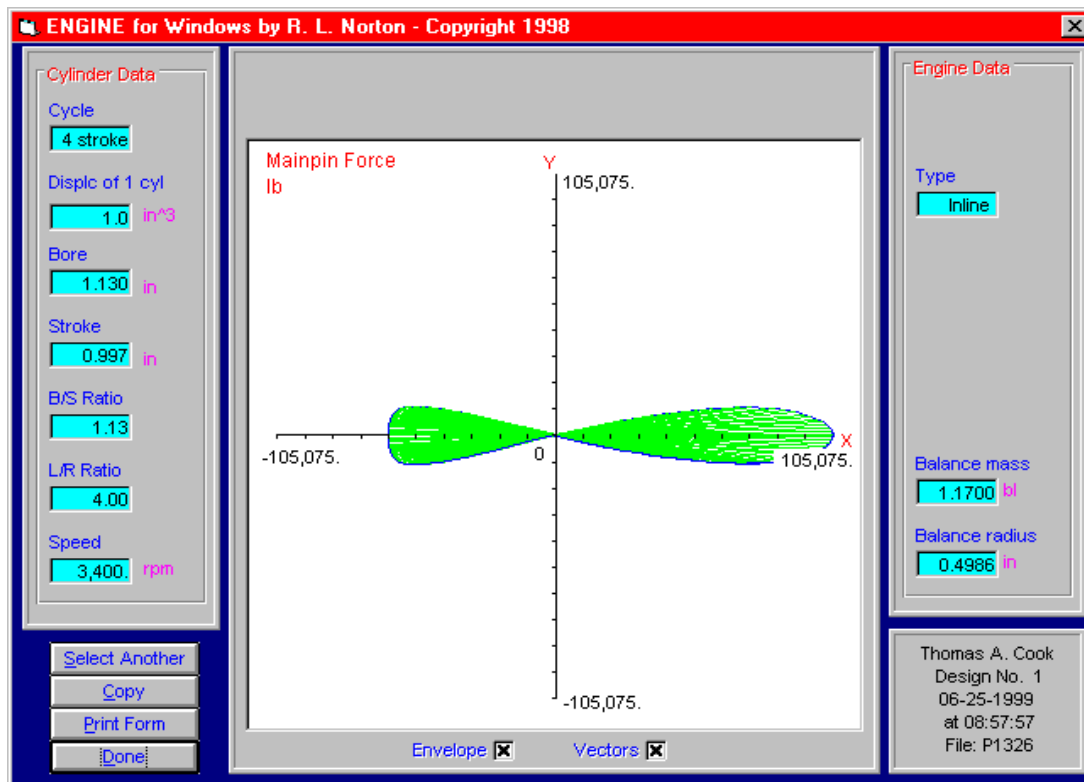


2. Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

b. Crank exactly balanced against mass at crankpin

$$F_{21max} := 105075 \cdot \text{lb}_f$$

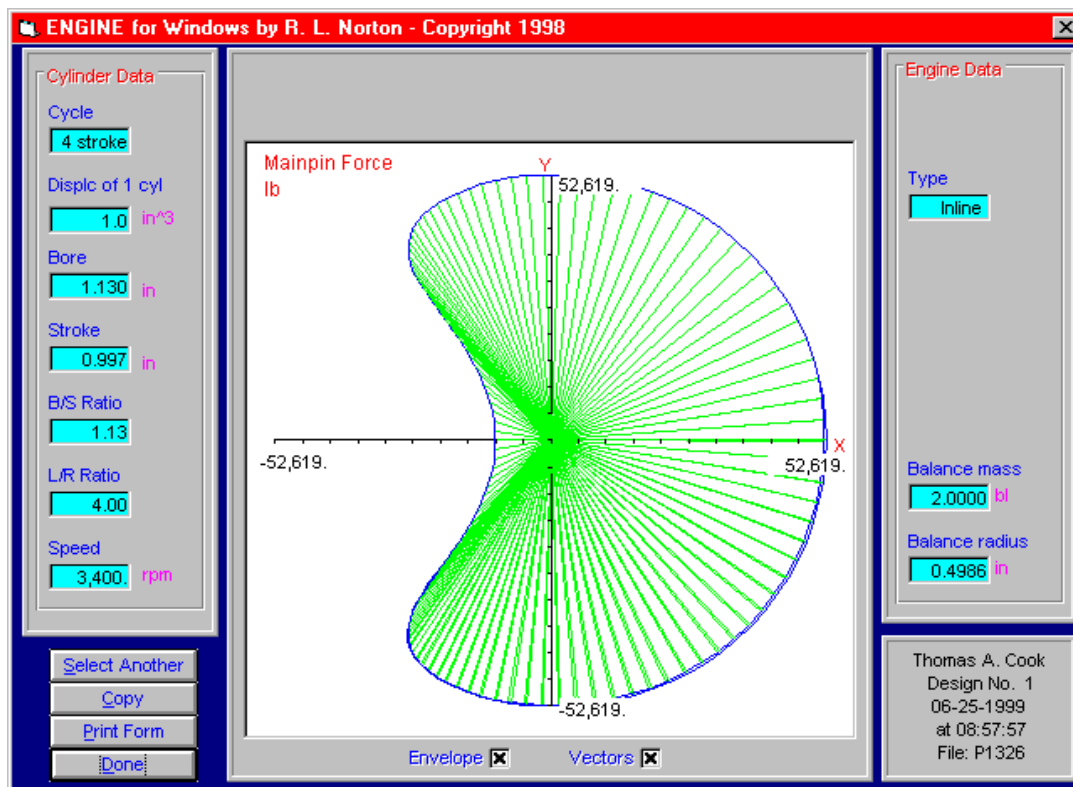
Polar plot of main pin force for part b.



3. Enter the above data and the masses for part c into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

c. Crank optimally overbalanced.

$$F_{21max} := 52619 \cdot \text{lb}_f$$



PROBLEM 13-30

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the crankpin of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

- Engine unbalanced.
 - Crank exactly balanced against mass at crankpin.
 - Crank optimally overbalanced against masses at crankpin and wrist pin.
- Piston, conrod, and crank masses = 1. Place the CG of the crank at $0.5r$ and the conrod at $0.33l$. Compare and explain the differences in the crankpin force under these different conditions with reference to the governing equations.

- Assumptions:**
1. Peak cylinder pressure is 600 psi.
 2. Link length ratio is $L/R = 4$.
 3. Main pin dia is 2.00 in.
 4. Crankpin dia is 1.500 in.
 5. Friction coefficient is zero.

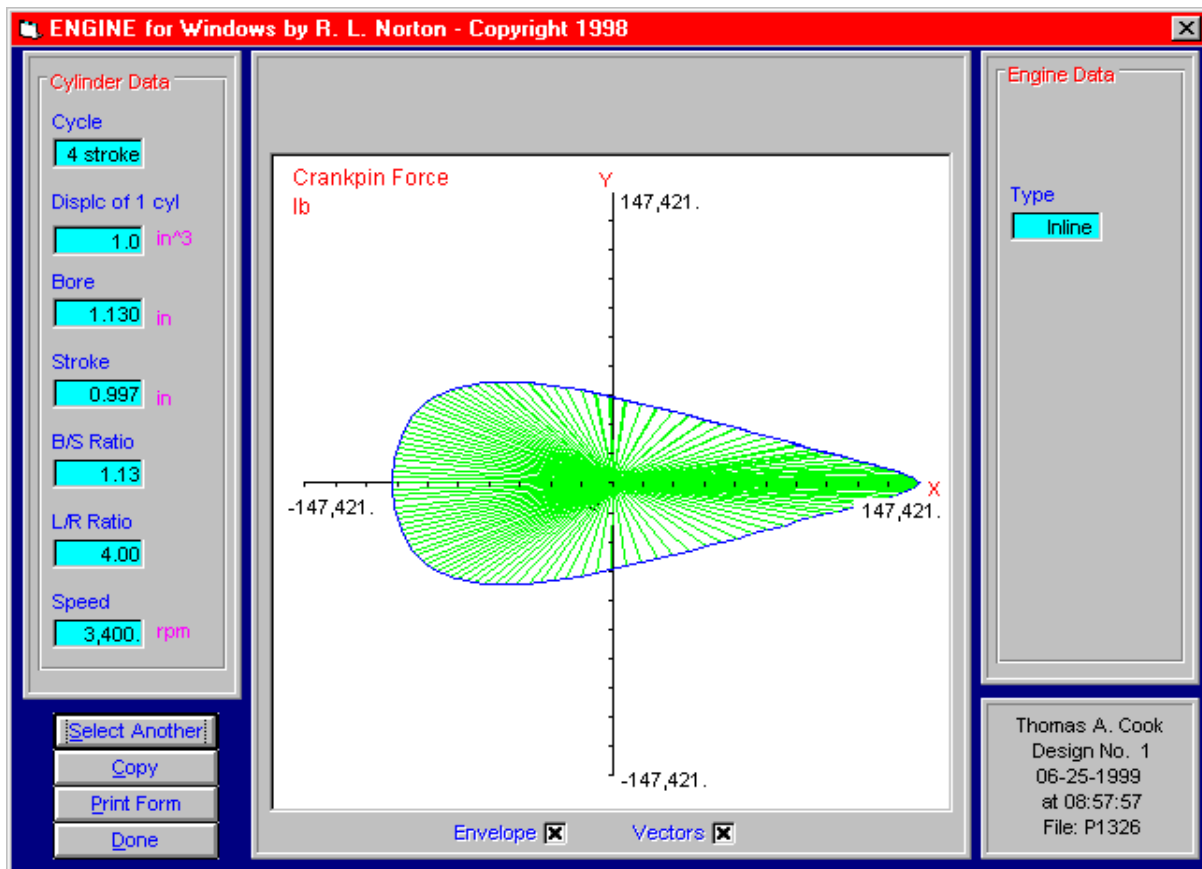
Solution: See Mathcad file P1330.

1. Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

a. Engine unbalanced

$$F_{21max} := 147421 \cdot \text{lbf}$$

Polar plot of crankpin force for part *a*.

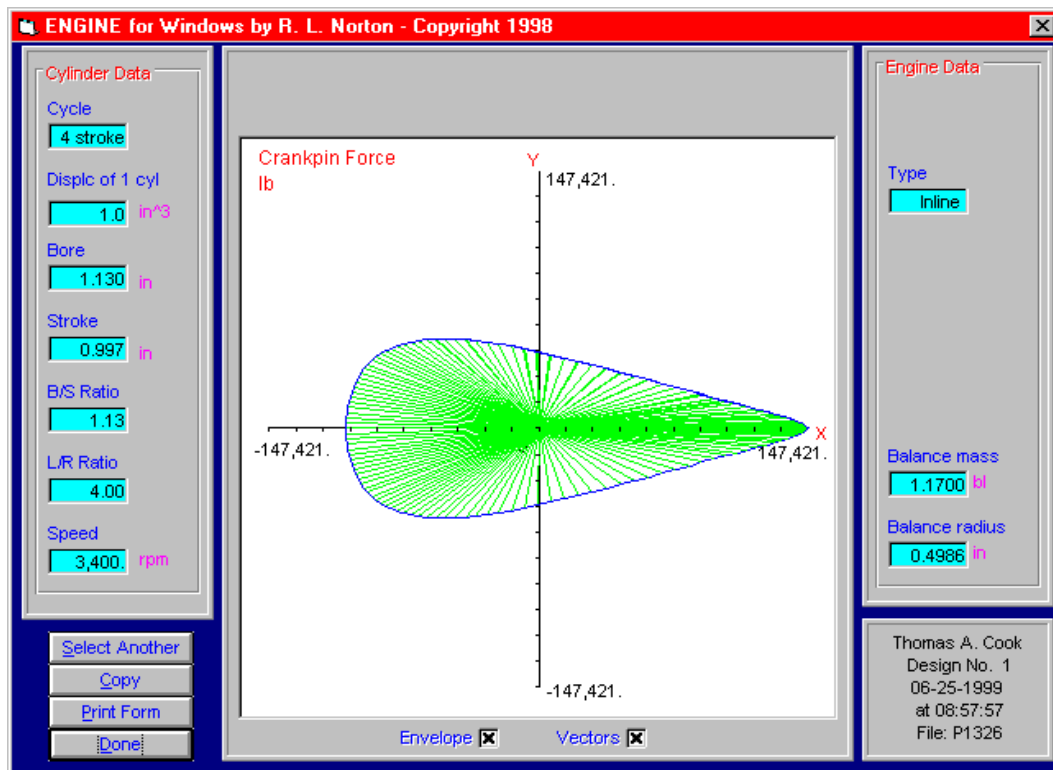


2. Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

b. Crank exactly balanced against mass at crankpin

$$F_{21max} := 147421 \cdot lbf$$

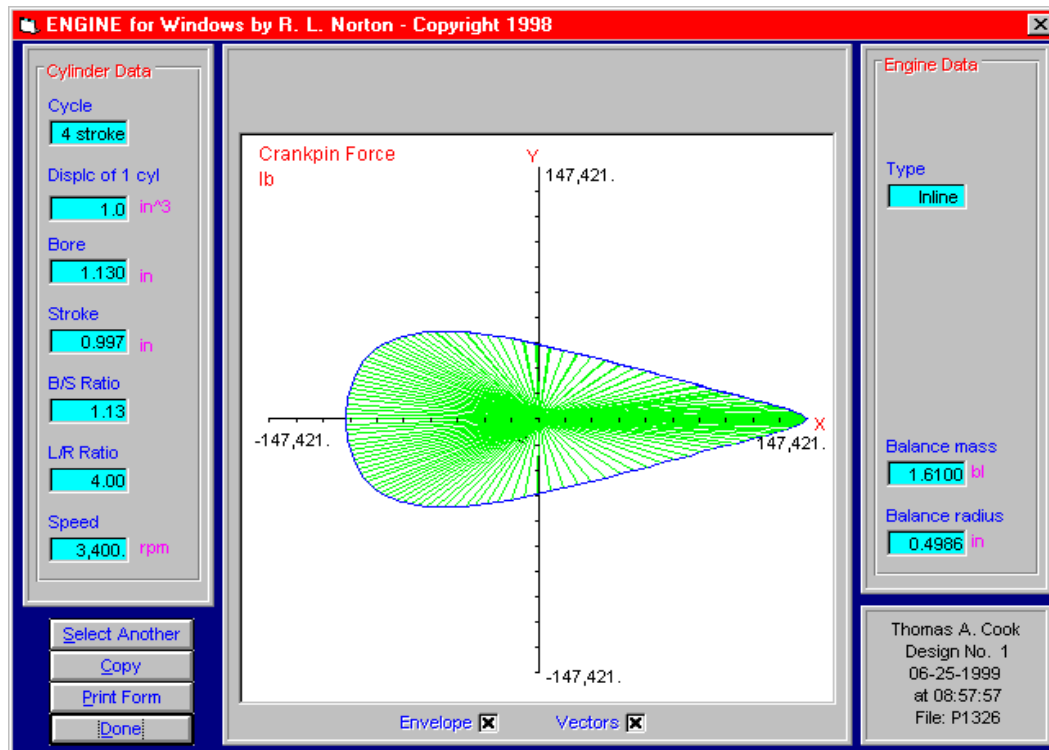
Polar plot of crankpin force for part b.



3. Enter the above data and the masses for part c into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

c. Crank optimally overbalanced.

$$F_{21max} := 147421 \cdot lbf$$



PROBLEM 13-31

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the wrist pin of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

- Engine unbalanced.
 - Crank exactly balanced against mass at crankpin.
 - Crank optimally overbalanced against masses at crankpin and wrist pin.
- Piston, conrod, and crank masses = 1. Place the CG of the crank at $0.5r$ and the conrod at $0.33l$. Compare and explain the differences in the wrist pin force under these different conditions with reference to the governing equations.

Assumptions:

1. Peak cylinder pressure is 600 psi.
2. Link length ratio is $L/R = 4$.
3. Main pin dia is 2.00 in.
4. Crankpin dia is 1.500 in.
5. Friction coefficient is zero.

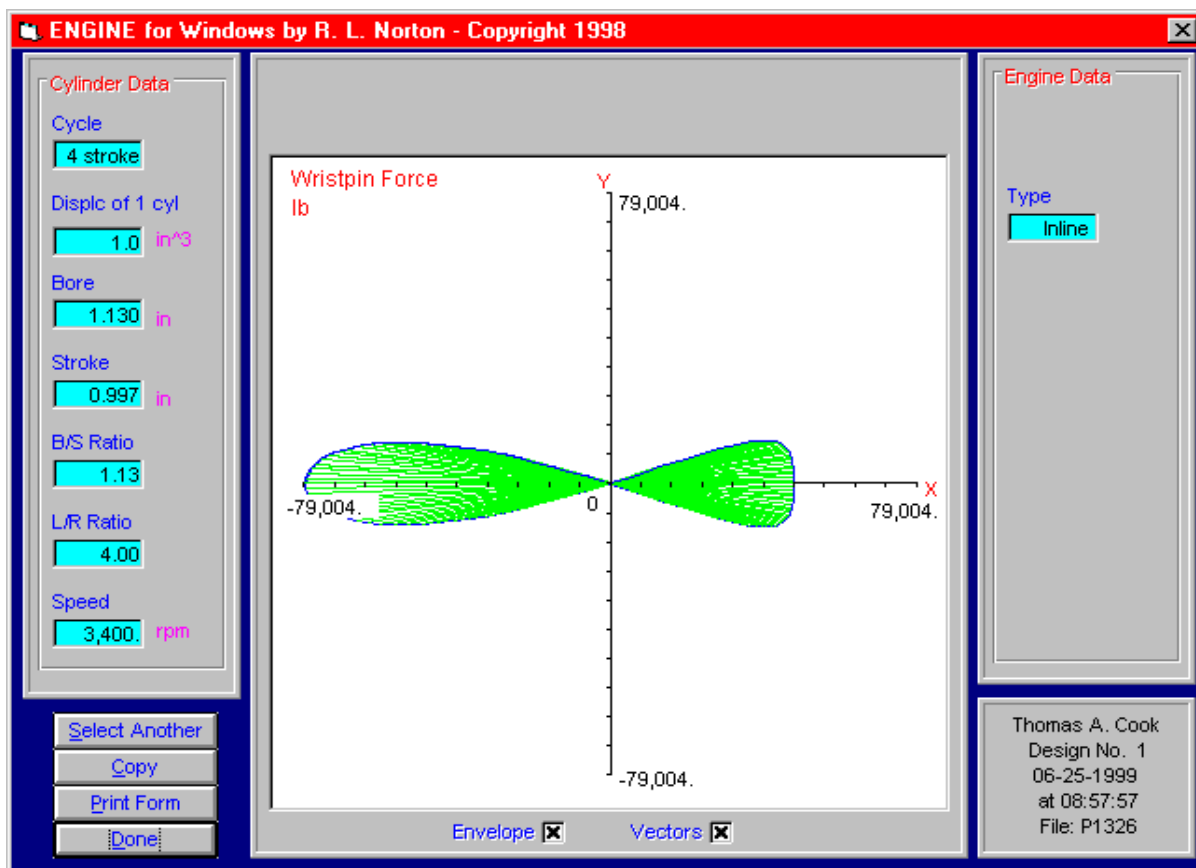
Solution: See Mathcad file P1331.

1. Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

a. Engine unbalanced

$$F_{21max} := 79004 \cdot \text{lbf}$$

Polar plot of wrist pin force for part *a*.

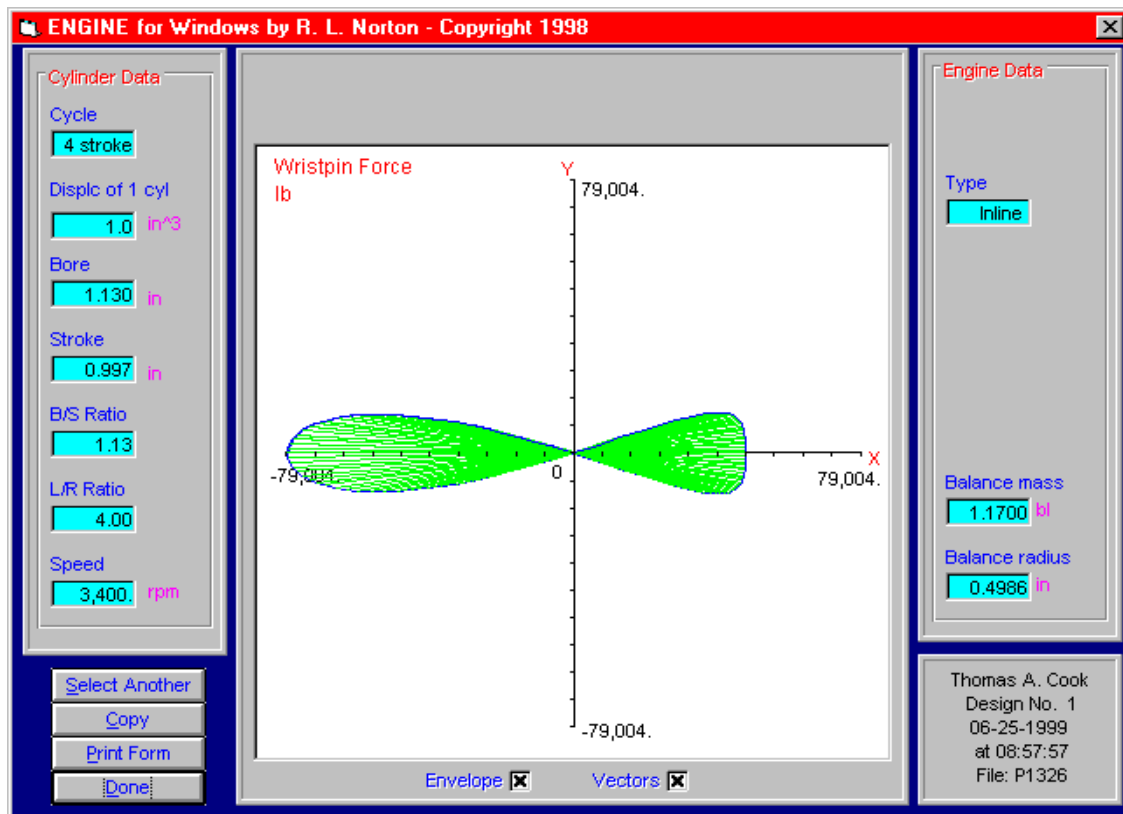


2. Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

b. Crank exactly balanced against mass at crankpin

$$F_{21max} := 79004 \cdot \text{lbf}$$

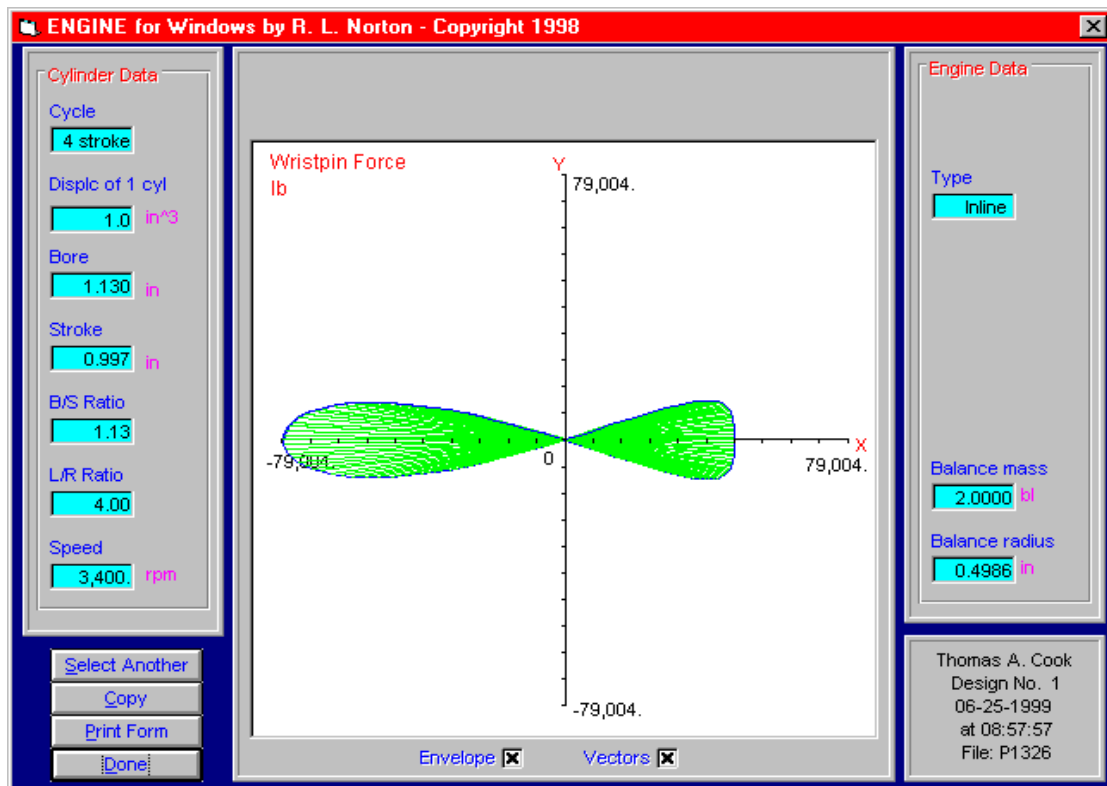
Polar plot of wrist pin force for part *b*.



3. Enter the above data and the masses for part c into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

c. Crank optimally overbalanced.

$$F_{21max} := 79004 \cdot lb_f$$



PROBLEM 13-32

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the shaking force of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

- Engine unbalanced.
 - Crank exactly balanced against mass at crankpin.
 - Crank optimally overbalanced against masses at crankpin and wrist pin.
- Piston, conrod, and crank masses = 1. Place the CG of the crank at $0.5r$ and the conrod at $0.33l$. Compare and explain the differences in the shaking force force under these different conditions with reference to the governing equations.

Assumptions:

1. Peak cylinder pressure is 600 psi.
2. Link length ratio is $L/R = 4$.
3. Main pin dia is 2.00 in.
4. Crankpin dia is 1.500 in.
5. Friction coefficient is zero.

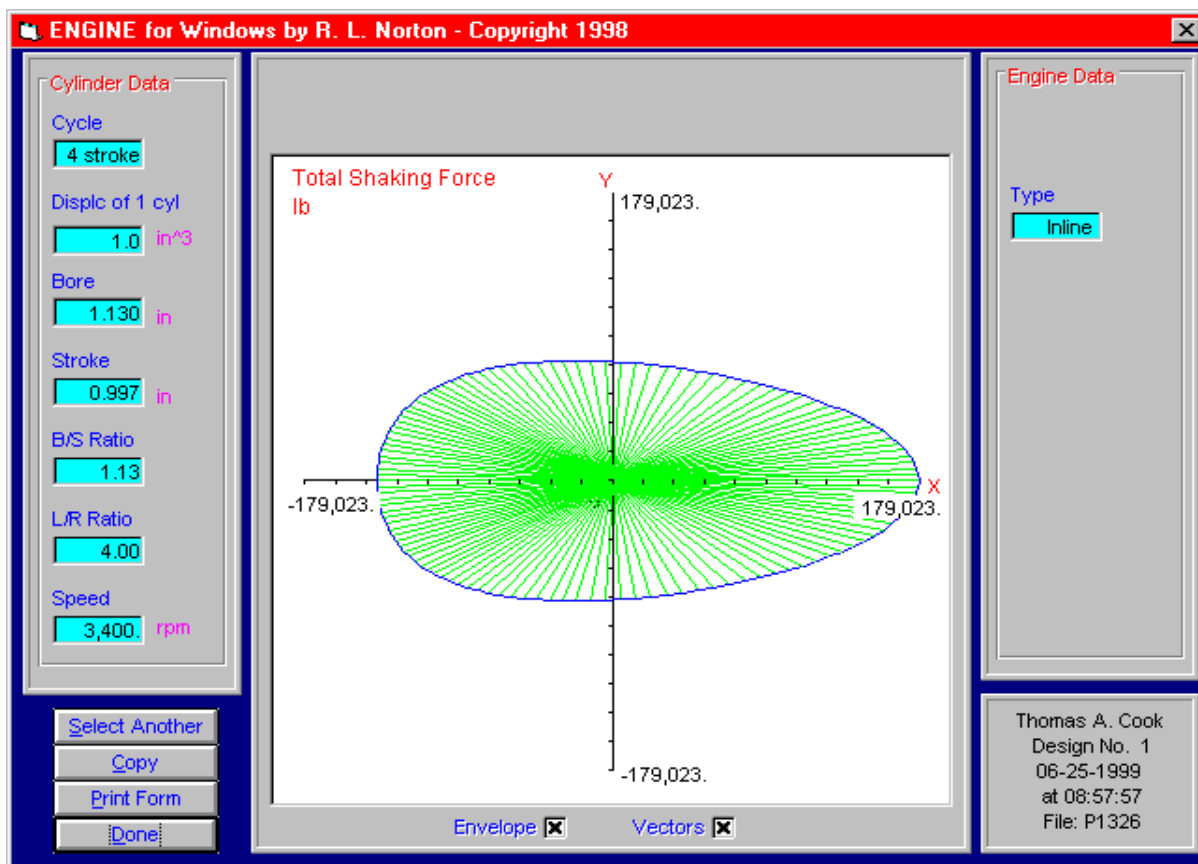
Solution: See Mathcad file P1332.

1. Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the shaking force and to get the polar plot of this force.

a. Engine unbalanced

$$F_{21max} := 179023 \cdot \text{lb}$$

Polar plot of shaking force for part *a*.

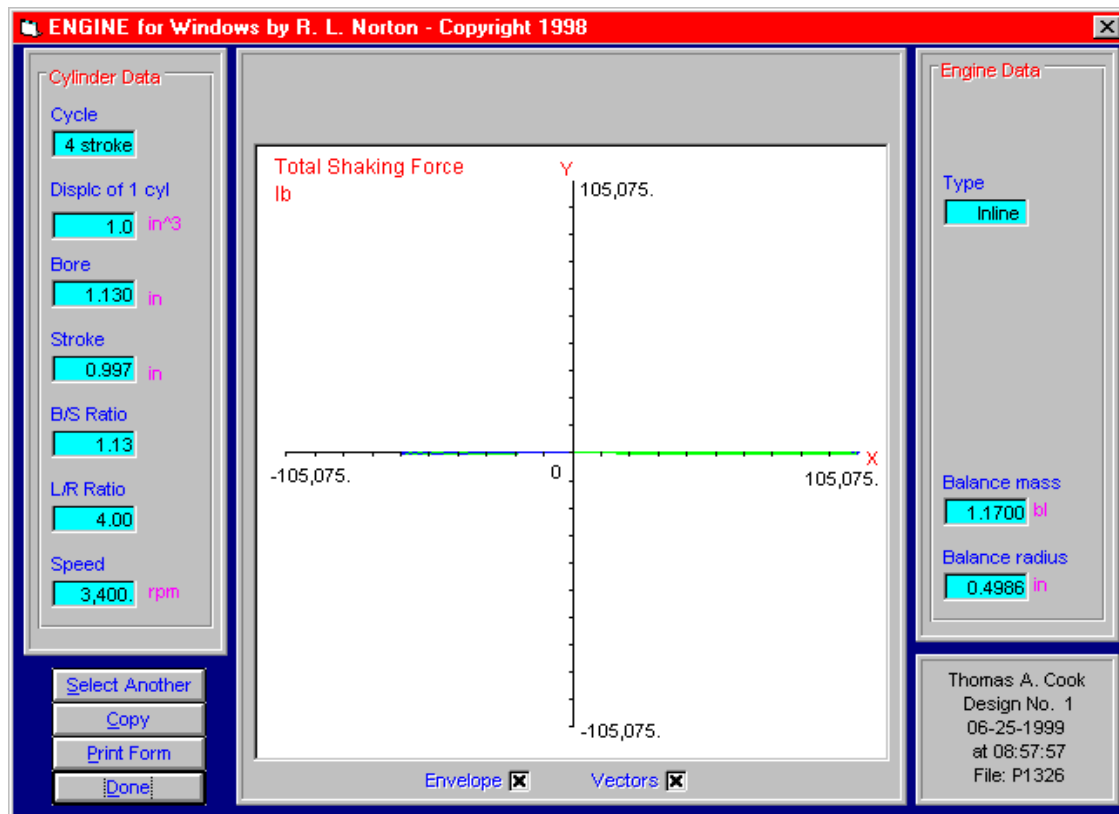


2. Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the shaking force and to get the polar plot of this force.

b. Crank exactly balanced against mass at crankpin

$$F_{21max} := 105075 \cdot \text{lb}$$

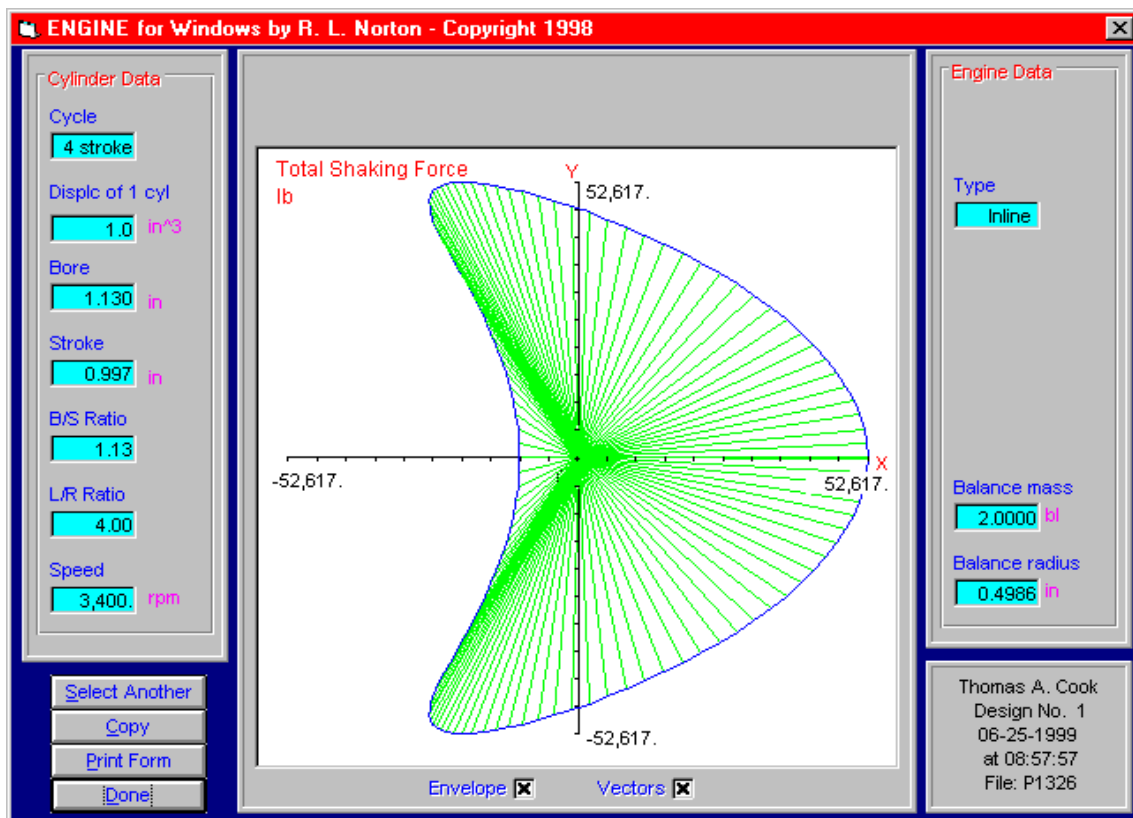
Polar plot of shaking force for part *b*.



3. Enter the above data and the masses for part c into program ENGINE to determine the maximum value of the shaking force and to get the polar plot of this force.

c. Crank optimally overbalanced.

$$F_{21max} := 52617 \cdot \text{lb}$$



PROBLEM 13-33

Statement: Figure P13-1 shows a single-cylinder air compressor stopped at top dead center (TDC). There is a static pressure $P = 100$ psi trapped in the 3-in-bore cylinder. The entire assembly weighs 30 lb. Draw the necessary free-body diagrams to determine the forces at points A, B, and C, and the supports R_1 and R_2 , which are symmetrically located about the piston centerline. Assume that the piston remains stationary.

Given: Pressure: $p := 100 \cdot \text{psi}$ Bore: $d := 3.00 \cdot \text{in}$

Solution: See Figure P13-1 and Mathcad file P1333.

1. Calculate the gas force:

$$F_g := p \cdot \left(\frac{\pi \cdot d^2}{4} \right) \quad F_g = 706.9 \text{ lbf}$$

2. Draw the FBDs. There will be four FBDs: the piston, the conrod, the crank, and the housing or body.

The piston will have two downward forces; the gas force F_g and the piston weight; and one upward force F_{34} acting at point C, whose magnitude is the sum of the gas force and the piston weight.

The conrod will have two downward forces; F_{43} and the conrod weight; and one upward force F_{23} acting at point B, whose magnitude is the sum of the gas force and the weights of the piston and the conrod.

The crank will have two downward forces; F_{32} and the crank weight; and one upward force F_{12} acting at point A, whose magnitude is the sum of the gas force and the weights of the piston, conrod, and crank.

The housing will have two downward forces; F_{21} acting at A and the weight of the housing acting at its CG; and three upward forces; F_g acting at the top of the cylinder, and the reaction forces R_1 and R_2 , whose magnitudes will be one-half the total weight of the assembly, which is 15 lb each. The gas force causes compression in piston, conrod, and crank; and tension in the housing. The net force at the supports is the total assembly weight.

PROBLEM 13-34

Statement: Calculate and plot the position, velocity, and acceleration of a slider-crank linkage with $r = 3$, $l = 12$, and $\omega = 200$ rad/sec over one cycle using the exact solution and the approximate Fourier solution. Also, calculate and plot the percent difference between the exact and approximate solutions for acceleration.

Given: Link lengths: $r := 3$ $l := 12$
 Angular velocity: $\omega := 200 \cdot \text{rad} \cdot \text{sec}^{-1}$

Solution: See Mathcad file P1334.

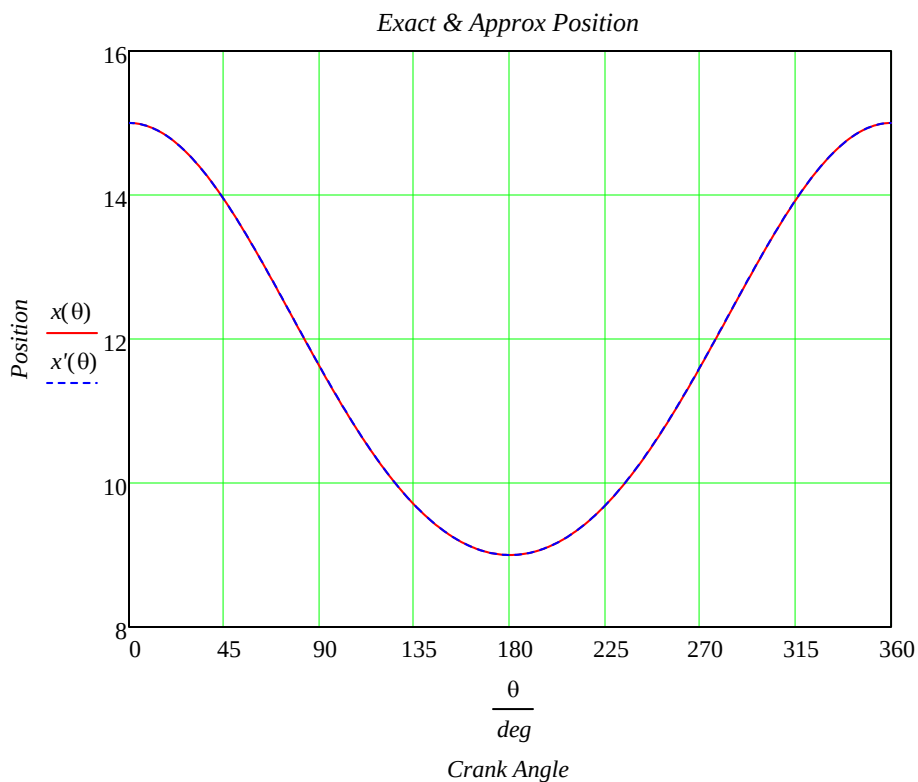
1. Define the crank angle over one cycle.

$$\theta := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

2. Calculate and plot the exact and approximate position using equations 13.1d and 13.3c.

$$\text{Exact:} \quad x(\theta) := r \cdot \cos(\theta) + l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta)\right)^2}$$

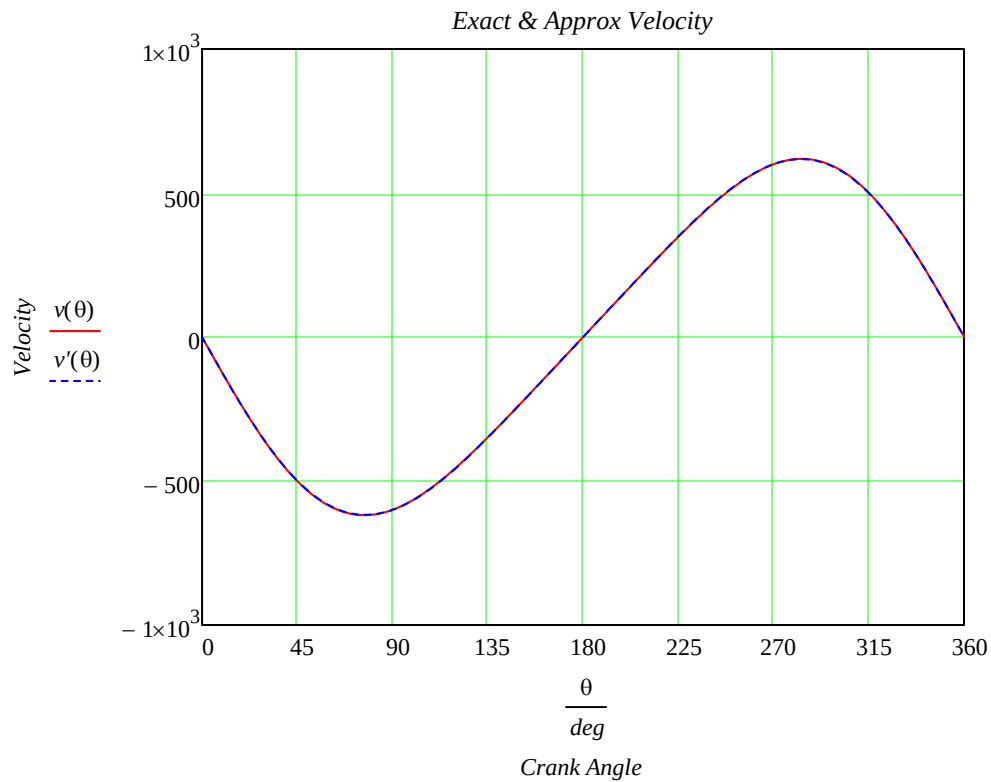
$$\text{Approximate:} \quad x'(\theta) := l - \frac{r^2}{4 \cdot l} + r \cdot \left(\cos(\theta) + \frac{r}{4 \cdot l} \cdot \cos(2 \cdot \theta) \right)$$



3. Calculate and plot the exact and approximate velocity using equations 13.1e and 13.3d.

$$\text{Exact:} \quad v(\theta) := -r \cdot \omega \cdot \left[\sin(\theta) + \frac{r}{2 \cdot l} \cdot \frac{\sin(2 \cdot \theta)}{\sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta)\right)^2}} \right]$$

Approximate:
$$v'(\theta) := -r \cdot \omega \cdot \left(\sin(\theta) + \frac{r}{2 \cdot l} \cdot \sin(2 \cdot \theta) \right)$$



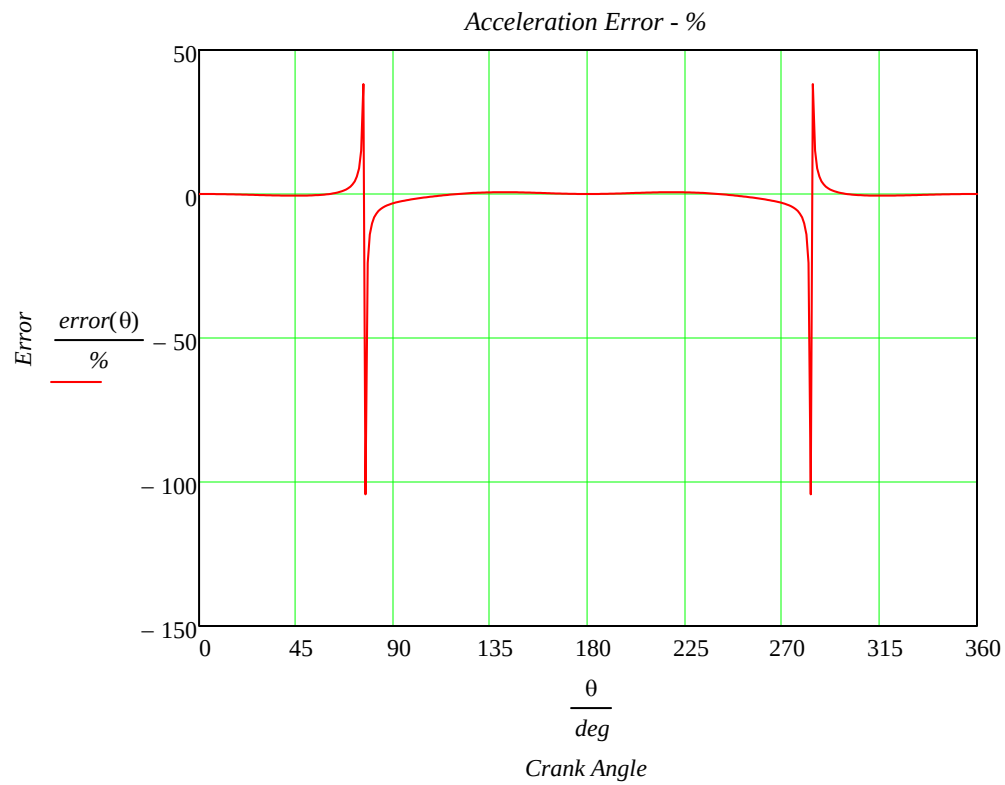
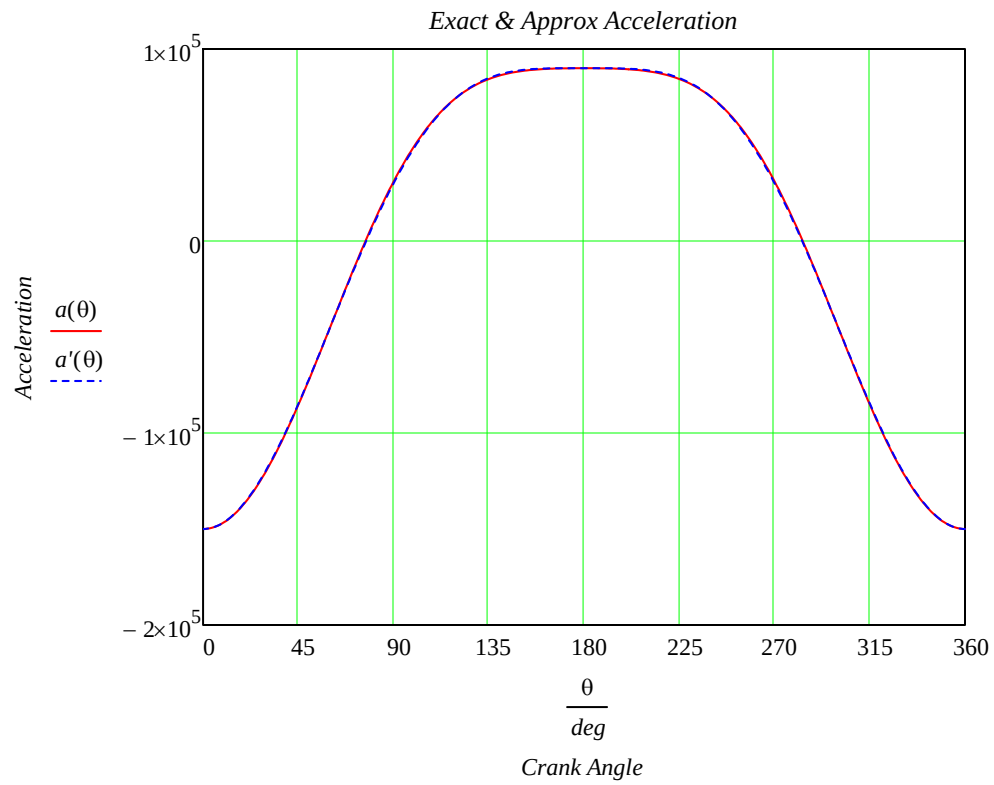
4. Calculate and plot the exact and approximate acceleration using equations 13.1f and 13.3e.

Exact:
$$a(\theta) := -r \cdot \omega^2 \cdot \left[\cos(\theta) - \frac{r \cdot \left[l^2 \cdot (1 - 2 \cdot \cos(\theta)^2) - r^2 \cdot \sin(\theta)^4 \right]}{\left(l^2 - r^2 \cdot \sin(\theta)^2 \right)^{\frac{3}{2}}} \right]$$

Approximate:
$$a'(\theta) := -r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right)$$

5. Compare the results by calculating the error in the approximation as a percent of the exact.

$$error(\theta) := \frac{a'(\theta) - a(\theta)}{a(\theta)}$$



PROBLEM 13-35

Statement: Calculate and plot the position, velocity, and acceleration of a slider-crank linkage with $r = 3$, $l = 15$, and $\omega = 100$ rad/sec over one cycle using the exact solution and the approximate Fourier solution. Also, calculate and plot the percent difference between the exact and approximate solutions for acceleration.

Given: Link lengths: $r := 3$ $l := 15$
 Angular velocity: $\omega := 100 \cdot \text{rad} \cdot \text{sec}^{-1}$

Solution: See Mathcad file P1335.

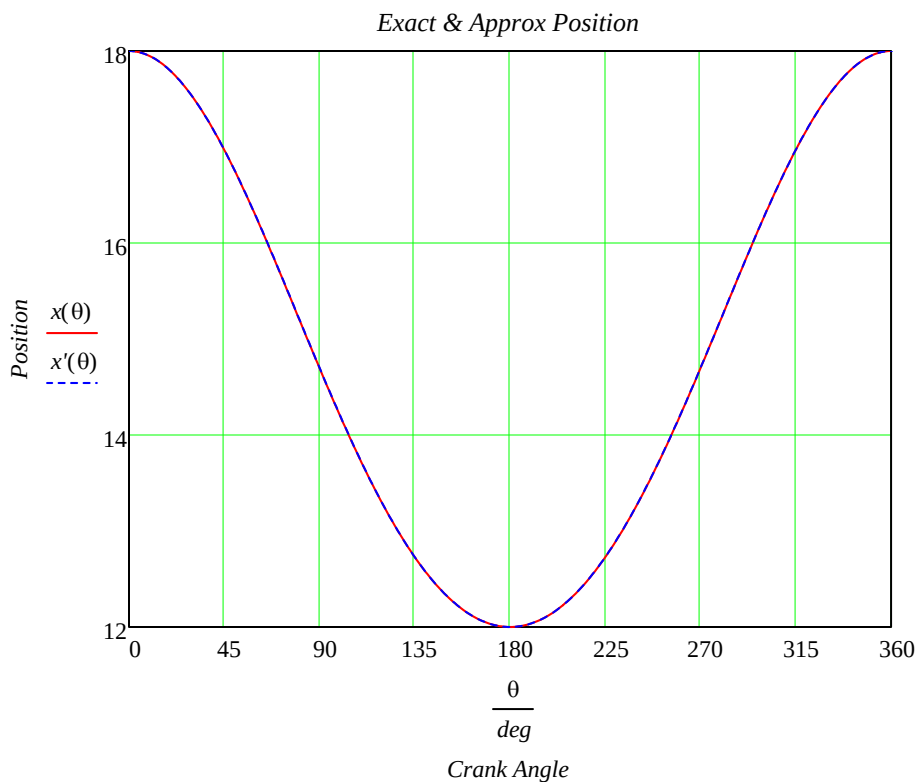
1. Define the crank angle over one cycle.

$$\theta := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 360 \cdot \text{deg}$$

2. Calculate and plot the exact and approximate position using equations 13.1d and 13.3c.

$$\text{Exact:} \quad x(\theta) := r \cdot \cos(\theta) + l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta)\right)^2}$$

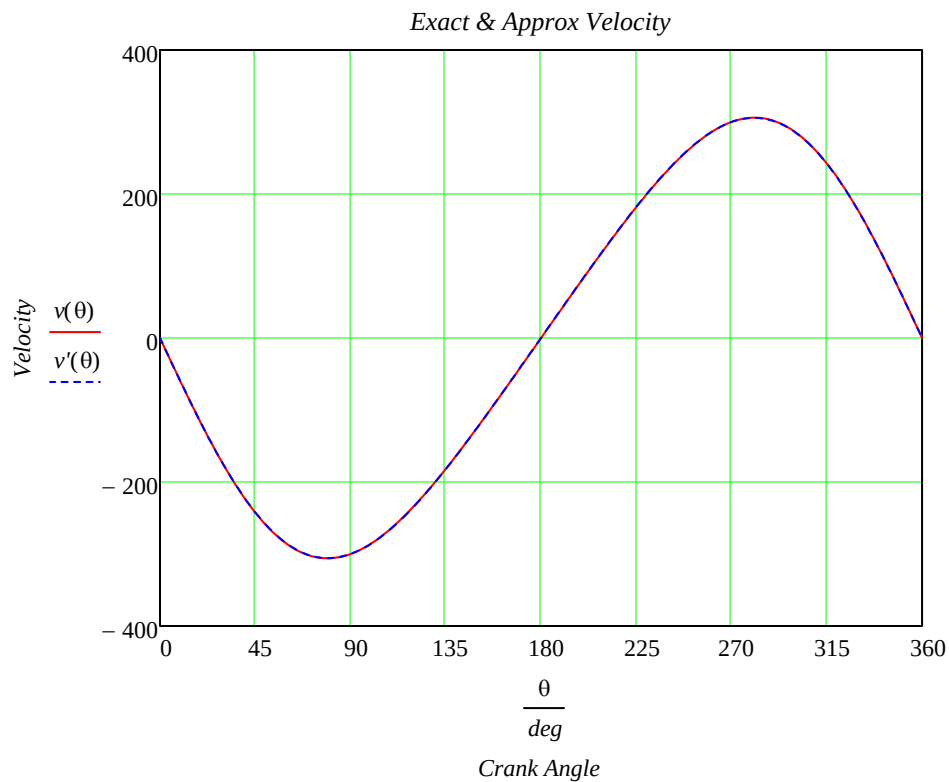
$$\text{Approximate:} \quad x'(\theta) := l - \frac{r^2}{4 \cdot l} + r \cdot \left(\cos(\theta) + \frac{r}{4 \cdot l} \cdot \cos(2 \cdot \theta) \right)$$



3. Calculate and plot the exact and approximate velocity using equations 13.1e and 13.3d.

$$\text{Exact:} \quad v(\theta) := -r \cdot \omega \cdot \left[\sin(\theta) + \frac{r}{2 \cdot l} \cdot \frac{\sin(2 \cdot \theta)}{\sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta)\right)^2}} \right]$$

Approximate:
$$v'(\theta) := -r \cdot \omega \cdot \left(\sin(\theta) + \frac{r}{2 \cdot l} \cdot \sin(2 \cdot \theta) \right)$$



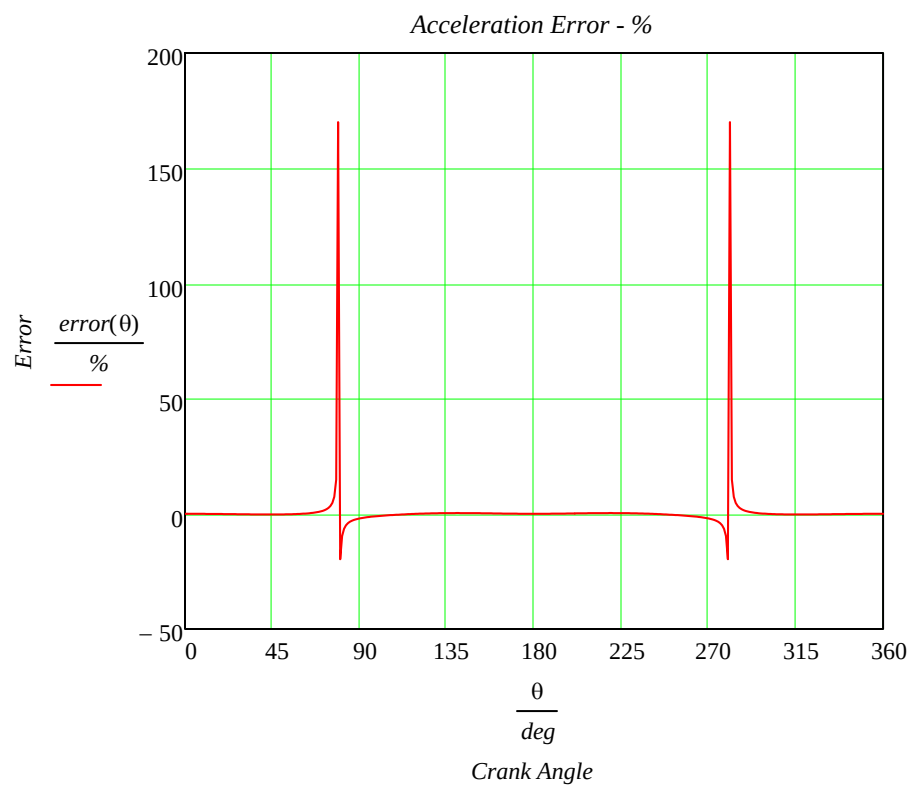
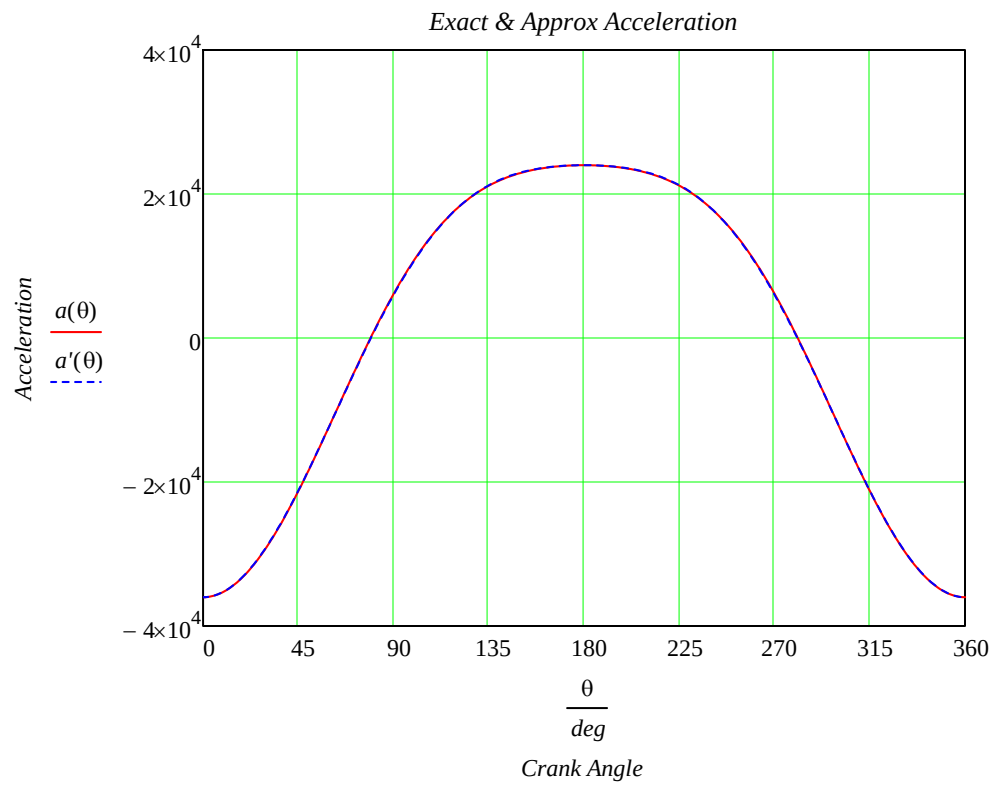
4. Calculate and plot the exact and approximate acceleration using equations 13.1f and 13.3e.

Exact:
$$a(\theta) := -r \cdot \omega^2 \cdot \left[\cos(\theta) - \frac{r \cdot \left[l^2 \cdot (1 - 2 \cdot \cos(\theta)^2) - r^2 \cdot \sin(\theta)^4 \right]}{\left(l^2 - r^2 \cdot \sin(\theta)^2 \right)^{\frac{3}{2}}} \right]$$

Approximate:
$$a'(\theta) := -r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right)$$

5. Compare the results by calculating the error in the approximation as a percent of the exact.

$$\text{error}(\theta) := \frac{a'(\theta) - a(\theta)}{a(\theta)}$$



PROBLEM 13-36

Statement: A slider-crank linkage has the dimensions and speed at $t = 0$ given below. Its initial crank angle is zero. Calculate the piston acceleration at the time specified below. Use two methods, the exact solution, and the approximate Fourier series solution and compare the results.

Given: Link lengths: $r := 3 \cdot \text{in}$ $l := 9 \cdot \text{in}$

Initial angular velocity: $\omega := 100 \cdot \text{rad} \cdot \text{sec}^{-1}$ Time span: $t := 0.01 \cdot \text{sec}$

Solution: See Mathcad file P1336.

1. Calculate the exact acceleration using equation 13.1f.

$$a_{\text{exact}} := -r \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) - \frac{r \cdot \left[l^2 \cdot (1 - 2 \cdot \cos(\omega \cdot t)^2) - r^2 \cdot \sin(\omega \cdot t)^4 \right]}{\left(l^2 - r^2 \cdot \sin(\omega \cdot t)^2 \right)^{\frac{3}{2}}} \right]$$

$$a_{\text{exact}} = -12133.3 \frac{\text{in}}{\text{sec}^2}$$

2. Calculate the approximate acceleration using equation 13.3e.

$$a_{\text{approx}} := -r \cdot \omega^2 \cdot \left(\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos(2 \cdot \omega \cdot t) \right) \quad a_{\text{approx}} = -12047.6 \frac{\text{in}}{\text{sec}^2}$$

3. Compare the results by calculating the error in the approximation as a percent of the exact.

$$\text{error} := \frac{a_{\text{approx}} - a_{\text{exact}}}{a_{\text{exact}}} \quad \text{error} = -0.706 \%$$

PROBLEM 13-37

Statement: A slider-crank linkage has the dimensions and speed at $t = 0$ given below. Its initial crank angle is zero. Calculate the piston acceleration at the time specified below. Use two methods, the exact solution, and the approximate Fourier series solution and compare the results.

Given: Link lengths: $r := 3 \cdot \text{in}$ $l := 15 \cdot \text{in}$

Initial angular velocity: $\omega := 100 \cdot \text{rad} \cdot \text{sec}^{-1}$ Time span: $t := 0.02 \cdot \text{sec}$

Solution: See Mathcad file P1337.

1. Calculate the exact acceleration using equation 13.1f.

$$a_{\text{exact}} := -r \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) - \frac{r \cdot \left[l^2 \cdot (1 - 2 \cdot \cos(\omega \cdot t)^2) - r^2 \cdot \sin(\omega \cdot t)^4 \right]}{\left(l^2 - r^2 \cdot \sin(\omega \cdot t)^2 \right)^{\frac{3}{2}}} \right]$$

$$a_{\text{exact}} = 16436.6 \frac{\text{in}}{\text{sec}^2}$$

2. Calculate the approximate acceleration using equation 13.3e.

$$a_{\text{approx}} := -r \cdot \omega^2 \cdot \left(\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos(2 \cdot \omega \cdot t) \right) \quad a_{\text{approx}} = 16406.3 \frac{\text{in}}{\text{sec}^2}$$

3. Compare the results by calculating the error in the approximation as a percent of the exact.

$$\text{error} := \frac{a_{\text{approx}} - a_{\text{exact}}}{a_{\text{exact}}} \quad \text{error} = -0.185 \%$$

PROBLEM 13-38

Statement: The equation given below is an approximation of the gas force over 180 deg of crank angle. Using this equation with $\beta = 15^\circ$ and $F_{gmax} = 1200 \text{ lb}$, calculate and plot the approximate gas torque for $r = 4 \text{ in}$ and $l = 12 \text{ in}$. What is the total energy delivered by the gas force over the 180 deg of motion? What is the average power delivered if the crank rotates at a constant speed of 1500 rpm?

Given: Link lengths: $r := 4 \cdot \text{in}$ $l := 12 \cdot \text{in}$ Speed: $n := 1500 \cdot \text{rpm}$
 Maximum gas force: $F_{gmax} := 1200 \cdot \text{lbf}$ Gas force parameter: $\beta := 15 \cdot \text{deg}$
 Gas force equation:

$$F_{g1}(\theta) := F_{gmax} \cdot \sin\left(\frac{\theta}{\beta} \cdot \frac{\pi}{2}\right) \quad F_{g2}(\theta) := \frac{F_{gmax}}{2} \cdot \left(1 + \cos\left(\pi \cdot \frac{\theta - \beta}{\pi - \beta}\right)\right)$$

$$F_g(\theta) := \text{if}(\theta \leq \beta, F_{g1}(\theta), F_{g2}(\theta))$$

Solution: See Figure P2-17 and Mathcad file P0236.

1. Define the crank angle over 180 deg.

$$\theta := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 180 \cdot \text{deg}$$

2. Using equation 13.8b, calculate and plot the approximate gas torque as a function of crank angle.

$$T_{g21}(\theta) := F_g(\theta) \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta)\right)$$

3. Integrate the gas torque over the range of crank motion (180 deg) to determine the energy delivered.

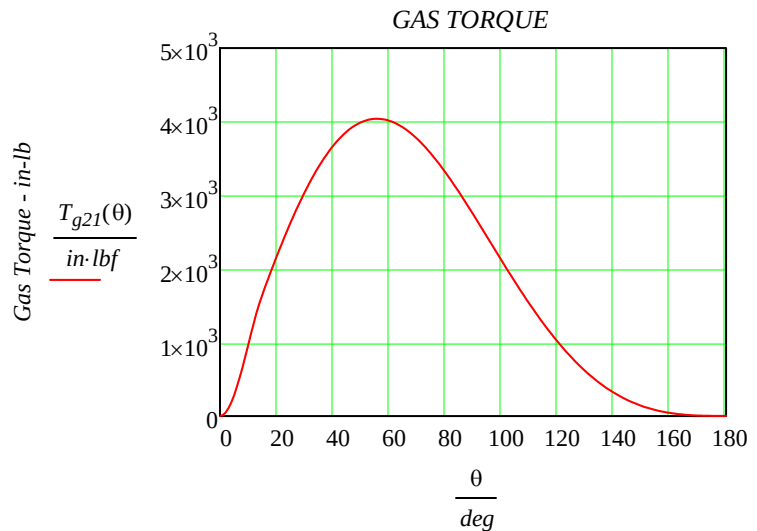
$$E := \int_0^\pi T_{g21}(\theta) d\theta$$

$$E = 485.7 \text{ lbf} \cdot \text{ft}$$

4. Divide the energy delivered by the time for the crank to travel 180 deg to get the average power.

$$t := \frac{180 \cdot \text{deg}}{n} \quad t = 0.020 \text{ s}$$

$$P_{avg} := \frac{E}{t} \quad P_{avg} = 44.2 \text{ hp}$$



PROBLEM 13-39

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the gas force and gas torque at this position.

Given: Link lengths: $r := 3.75 \cdot in$ $l := 11 \cdot in$

Piston bore: $B := 2.5 \cdot in$ Peak pressure: $p_g := 1150 \cdot psi$ Crank angle: $\theta := 12 \cdot deg$

Assumptions: An approximate solution is acceptable.

Solution: See Mathcad file P1339.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \quad F_g = 5645 \text{ lbf}$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21} := F_g \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta) \right) \quad T_{g21} = 5869 \text{ in} \cdot \text{lbf}$$

PROBLEM 13-40

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the exact gas torque at this position and compare it to that obtained by the approximate expression in equation 13.8b. What is the percent error?

Given: Link lengths: $r := 3.75 \cdot \text{in}$ $l := 11 \cdot \text{in}$
 Piston bore: $B := 2.5 \cdot \text{in}$ Peak pressure: $p_g := 1150 \cdot \text{psi}$ Crank angle: $\theta := 12 \cdot \text{deg}$

Solution: See Mathcad file P1340.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \quad F_g = 5645 \text{ lbf}$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21a} := F_g \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta) \right) \quad T_{g21a} = 5868.91 \text{ in} \cdot \text{lbf}$$

3. Calculate the exact gas torque on the crank using equations 13.7b, 13.1d, and 13.6d.

$$\phi := \text{atan} \left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2}} \right] \quad \phi = 4.064 \text{ deg}$$

$$x := r \cdot \cos(\theta) + l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2} \quad x = 14.640 \text{ in}$$

$$T_{g21e} := F_g \cdot \tan(\phi) \cdot x \quad T_{g21e} = 5872.61 \text{ in} \cdot \text{lbf}$$

4. Compare the results by calculating the error in the approximation as a percent of the exact.

$$\text{error} := \frac{T_{g21a} - T_{g21e}}{T_{g21e}} \quad \text{error} = -0.0630 \%$$

PROBLEM 13-41

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the gas force and gas torque at this position.

Given: Link lengths: $r := 4.12 \cdot in$ $l := 14.5 \cdot in$

Piston bore: $B := 2.25 \cdot in$ Peak pressure: $p_g := 1325 \cdot psi$ Crank angle: $\theta := 9 \cdot deg$

Assumptions: An approximate solution is acceptable.

Solution: See Mathcad file P1341.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \quad F_g = 5268 \text{ lbf}$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21} := F_g \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta) \right) \quad T_{g21} = 4348 \text{ in} \cdot \text{lbf}$$

PROBLEM 13-42

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the exact gas torque at this position and compare it to that obtained by the approximate expression in equation 13.8b. What is the percent error?

Given: Link lengths: $r := 4.12 \cdot in$ $l := 14.5 \cdot in$
 Piston bore: $B := 2.25 \cdot in$ Peak pressure: $p_g := 1325 \cdot psi$ Crank angle: $\theta := 9 \cdot deg$

Solution: See Mathcad file P1342.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \quad F_g = 5268 \text{ lbf}$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21a} := F_g \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta) \right) \quad T_{g21a} = 4348.38 \text{ in} \cdot \text{lbf}$$

3. Calculate the exact gas torque on the crank using equations 13.7b, 13.1d, and 13.6d.

$$\phi := \operatorname{atan} \left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2}} \right] \quad \phi = 2.548 \text{ deg}$$

$$x := r \cdot \cos(\theta) + l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2} \quad x = 18.555 \text{ in}$$

$$T_{g21e} := F_g \cdot \tan(\phi) \cdot x \quad T_{g21e} = 4349.32 \text{ in} \cdot \text{lbf}$$

4. Compare the results by calculating the error in the approximation as a percent of the exact.

$$\text{error} := \frac{T_{g21a} - T_{g21e}}{T_{g21e}} \quad \text{error} = -0.0217 \%$$

PROBLEM 13-43

Statement: The dimensions and mass properties of a slider-crank linkage are given below. Determine the approximately dynamically equivalent two-mass lumped parameter model for this linkage with the masses placed at the crank and wrist pins.

Units: $blob := lb \cdot sec^2 \cdot in^{-1}$

Given: Masses: $m_2 := 0.045 \cdot blob$ $m_3 := 0.120 \cdot blob$ $m_4 := 0.15 \cdot blob$
 Crank CG location as fraction of its length: $r_2 := 0.4$

Assumption: Two-thirds of the conrod mass is placed at the crank pin and one-third at the wrist pin.

Solution: See Mathcad file P1343.

1. Determine the lumped masses for link 3 using equations 13.10a and 13.10b and the assumption as to mass distribution.

$$\text{Masses:} \quad m_{3a} := m_3 \cdot \frac{2}{3} \quad m_{3a} = 0.0800 \, blob$$

$$m_{3b} := m_3 \cdot \frac{1}{3} \quad m_{3b} = 0.04000 \, blob$$

2. Determine the lumped mass at point A due to the crank alone using equation 13.11.

$$\text{Masses:} \quad m_{2a} := m_2 \cdot r_2 \quad m_{2a} = 0.0180 \, blob$$

3. Determine the two-mass model with masses placed at the crank and wrist pins using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.098 \, blob$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.190 \, blob$$

PROBLEM 13-44

Statement: The dimensions and mass properties of a connecting rod are given below. Calculate the sizes of two dynamically equivalent masses and the location of one if the other is placed at point *B* (see Figure 13-10).

Units: $blob := lb \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12.5 \cdot in$ mass: $m_3 := 0.120 \cdot blob$
 Mass moment of inertia: $I_{G3} := 0.15 \cdot blob \cdot in^2$
 Distance to CG: $l_a := 4.5 \cdot in$

Solution: See Mathcad file P1344.

- Determine the exact model using equations 13.9d and 13.9e.

$$\text{Distance from point } B \text{ to CG: } l_b := l - l_a \quad l_b = 8.000 \text{ in}$$

$$\text{Distance from CG to lumped mass at P: } l_p := \frac{I_{G3}}{m_3 \cdot l_b} \quad l_p = 0.156 \text{ in}$$

$$\text{Masses: } m_p := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_p = 0.118 \text{ blob}$$

$$m_b := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_b = 0.00230 \text{ blob}$$

PROBLEM 13-45

Statement: The dimensions and mass properties of a slider-crank linkage are given below. Determine the approximately dynamically equivalent two-mass lumped parameter model for this linkage with the masses placed at the crank and wrist pins.

Units: $blob := lb \cdot sec^2 \cdot in^{-1}$

Given: Masses: $m_2 := 0.060 \cdot blob$ $m_3 := 0.180 \cdot blob$ $m_4 := 0.160 \cdot blob$

Crank CG location as fraction of its length: $r_2 := 0.38$

Assumption: Two-thirds of the conrod mass is placed at the crank pin and one-third at the wrist pin.

Solution: See Mathcad file P1345.

1. Determine the lumped masses for link 3 using equations 13.10a and 13.10b and the assumption as to mass distribution.

$$\text{Masses:} \quad m_{3a} := m_3 \cdot \frac{2}{3} \quad m_{3a} = 0.1200 \, blob$$

$$m_{3b} := m_3 \cdot \frac{1}{3} \quad m_{3b} = 0.06000 \, blob$$

2. Determine the lumped mass at point A due to the crank alone using equation 13.11.

$$\text{Masses:} \quad m_{2a} := m_2 \cdot r_2 \quad m_{2a} = 0.0228 \, blob$$

3. Determine the two-mass model with masses at the crank and wrist pins using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.143 \, blob$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.220 \, blob$$

PROBLEM 13-46

Statement: The dimensions and mass properties of a connecting rod are given below. Calculate the sizes of two dynamically equivalent masses and the location of one if the other is placed at point *B* (see Figure 13-10).

Units: $blob := lb \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 10.4 \cdot in$ mass: $m_3 := 0.180 \cdot blob$
 Mass moment of inertia: $I_{G3} := 0.12 \cdot blob \cdot in^2$
 Distance to CG: $l_a := 4.16 \cdot in$

Solution: See Mathcad file P1346.

- Determine the exact model using equations 13.9d and 13.9e.

$$\text{Distance from point } B \text{ to CG: } l_b := l - l_a \quad l_b = 6.240 \text{ in}$$

$$\text{Distance from CG to lumped mass at P: } l_p := \frac{I_{G3}}{m_3 \cdot l_b} \quad l_p = 0.107 \text{ in}$$

$$\text{Masses: } m_p := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_p = 0.177 \text{ blob}$$

$$m_b := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_b = 0.00303 \text{ blob}$$

PROBLEM 13-47

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12.5 \cdot in$ mass: $m_3 := 0.120 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.36$
 Crank length: $r := 3.13 \cdot in$ mass: $m_2 := 0.045 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.4$
 Piston mass: $m_4 := 0.015 \cdot blob$
 Crank speed: $\omega := 1800 \cdot rpm$ Crank angle: $\theta := 30 \cdot deg$

Solution: See Mathcad file P1347.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $l_a = 4.500 \text{ in}$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 8.000 \text{ in}$

Let $l_p := l_a$ $m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b}$ $m_{3a} = 0.0768 \text{ blob}$

$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b}$ $m_{3b} = 0.04320 \text{ blob}$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $r_{G2} = 1.252 \text{ in}$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $m_{2a} = 0.0180 \text{ blob}$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.0948 \text{ blob}$

$m_B := m_{3b} + m_4$ $m_B = 0.0582 \text{ blob}$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right]$ $F_{ix} = 15546 \text{ lbf}$

$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta))$ $F_{iy} = 5271 \text{ lbf}$

$F_i := \sqrt{F_{ix}^2 + F_{iy}^2}$ $F_i = 16415 \text{ lbf}$ at $atan2(F_{ix}, F_{iy}) = 18.731 \text{ deg}$

$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot \sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + \cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot \cos(2 \cdot \theta) \right)$ $T_{i21} = -11943 \text{ in} \cdot \text{lbf}$

PROBLEM 13-48

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 10.4 \cdot in$ mass: $m_3 := 0.180 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.40$
 Crank length: $r := 2.60 \cdot in$ mass: $m_2 := 0.060 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.38$
 Piston mass: $m_4 := 0.016 \cdot blob$
 Crank speed: $\omega := 1850 \cdot rpm$ Crank angle: $\theta := 20 \cdot deg$

Solution: See Mathcad file P1348.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 4.160 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 6.240 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.1080 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.07200 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 0.988 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0228 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.1308 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0880 \text{ blob}$$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \quad F_{ix} = 21708 \text{ lbf}$$

$$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) \quad F_{iy} = 4365 \text{ lbf}$$

$$F_i := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_i = 22143 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = 11.371 \text{ deg}$$

$$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot \sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + \cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot \cos(2 \cdot \theta) \right) \quad T_{i21} = -10324 \text{ in} \cdot \text{lbf}$$

PROBLEM 13-49

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 10.4 \cdot in$ mass: $m_3 := 0.120 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.36$
 Crank length: $r := 2.60 \cdot in$ mass: $m_2 := 0.045 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.40$
 Piston mass: $m_4 := 0.015 \cdot blob$
 Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 25 \cdot deg$

Solution: See Mathcad file P1349.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $l_a = 3.744 \text{ in}$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 6.656 \text{ in}$

Let $l_p := l_a$ $m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b}$ $m_{3a} = 0.0768 \text{ blob}$

$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b}$ $m_{3b} = 0.04320 \text{ blob}$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $r_{G2} = 1.040 \text{ in}$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $m_{2a} = 0.0180 \text{ blob}$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.0948 \text{ blob}$

$m_B := m_{3b} + m_4$ $m_B = 0.0582 \text{ blob}$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right]$ $F_{ix} = 16881 \text{ lbf}$

$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta))$ $F_{iy} = 4569 \text{ lbf}$

$F_i := \sqrt{F_{ix}^2 + F_{iy}^2}$ $F_i = 17489 \text{ lbf}$ at $atan2(F_{ix}, F_{iy}) = 15.145 \text{ deg}$

$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot \sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + \cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot \cos(2 \cdot \theta) \right)$ $T_{i21} = -9280 \text{ in} \cdot \text{lbf}$

PROBLEM 13-50

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12.5 \cdot in$ mass: $m_3 := 0.180 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.40$
 Crank length: $r := 3.13 \cdot in$ mass: $m_2 := 0.060 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.38$
 Piston mass: $m_4 := 0.015 \cdot blob$
 Crank speed: $\omega := 1500 \cdot rpm$ Crank angle: $\theta := 22 \cdot deg$

Solution: See Mathcad file P1350.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 5.000 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 7.500 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.1080 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.07200 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.189 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0228 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.1308 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0870 \text{ blob}$$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \quad F_{ix} = 16806 \text{ lbf}$$

$$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) \quad F_{iy} = 3784 \text{ lbf}$$

$$F_i := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_i = 17227 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = 12.689 \text{ deg}$$

$$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot \sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + \cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot \cos(2 \cdot \theta) \right) \quad T_{i21} = -10419 \text{ in} \cdot \text{lbf}$$

PROBLEM 13-51

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces torque on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12.5 \cdot in$ mass: $m_3 := 0.120 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.36$
 Crank length: $r := 3.13 \cdot in$ mass: $m_2 := 0.045 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.40$
 Piston mass: $m_4 := 0.150 \cdot blob$ Gas force: $F_g := 600 \cdot lbf$
 Crank speed: $\omega := 1800 \cdot rpm$ Crank angle: $\theta := 30 \cdot deg$

Solution: See Mathcad file P1351.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 4.500 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 8.000 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.0768 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.04320 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.252 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0180 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.0948 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.1932 \text{ blob}$$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := \text{atan} \left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2}} \right] \quad \phi = 7.192 \text{ deg}$$

$$a_B := -r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \quad a_B = -110234.9 \frac{\text{in}}{\text{sec}^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -[(m_4 + m_{3b}) \cdot a_B + F_g] \cdot \tan(\phi) \quad F_{41} = 2612 \text{ lbf}$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$F_{34x} := F_g + m_4 \cdot a_B$$

$$F_{34x} = -15935 \text{ lbf}$$

$$F_{34y} := -[F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$F_{34y} = 2611.9 \text{ lbf}$$

$$F_{34} := \sqrt{F_{34x}^2 + F_{34y}^2}$$

$$F_{34} = 16147.9 \text{ lbf}$$

$$\theta_{34} := \text{atan2}(F_{34x}, F_{34y})$$

$$\theta_{34} = 170.692 \text{ deg}$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\theta) - (m_{3b} + m_4) \cdot a_B - F_g$$

$$F_{32x} = 28094 \text{ lbf}$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + [F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$F_{32y} = 1658.6 \text{ lbf}$$

$$F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2}$$

$$F_{32} = 28143 \text{ lbf}$$

$$\theta_{32} := \text{atan2}(F_{32x}, F_{32y})$$

$$\theta_{32} = 3.379 \text{ deg}$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot \cos(\theta) + F_{32x}$$

$$F_{21x} = 29828 \text{ lbf}$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + F_{32y}$$

$$F_{21y} = 2659.5 \text{ lbf}$$

$$F_{21} := \sqrt{F_{21x}^2 + F_{21y}^2}$$

$$F_{21} = 29946 \text{ lbf}$$

$$\theta_{21} := \text{atan2}(F_{21x}, F_{21y})$$

$$\theta_{21} = 5.095 \text{ deg}$$

PROBLEM 13-52

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces torque on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 10.4 \cdot in$ mass: $m_3 := 0.180 \cdot blob$

Distance to CG (as fraction of l): $r_{3a} := 0.40$

Crank length: $r := 2.60 \cdot in$ mass: $m_2 := 0.060 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.38$

Piston mass: $m_4 := 0.160 \cdot blob$ Gas force: $F_g := 600 \cdot lbf$

Crank speed: $\omega := 1850 \cdot rpm$ Crank angle: $\theta := 20 \cdot deg$

Solution: See Mathcad file P1352.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $l_a = 4.160 \cdot in$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 6.240 \cdot in$

Let $l_p := l_a$ $m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b}$ $m_{3a} = 0.1080 \cdot blob$

$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b}$ $m_{3b} = 0.07200 \cdot blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $r_{G2} = 0.988 \cdot in$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $m_{2a} = 0.0228 \cdot blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.1308 \cdot blob$

$m_B := m_{3b} + m_4$ $m_B = 0.2320 \cdot blob$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$\phi := \text{atan} \left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2}} \right]$ $\phi = 4.905 \cdot deg$

$a_B := -r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right)$ $a_B = -110386.2 \frac{in}{sec^2}$

5. Determine the sidewall force F_{41} using equation 13.20.

$F_{41} := -[(m_4 + m_{3b}) \cdot a_B + F_g] \cdot \tan(\phi)$ $F_{41} = 2146 \cdot lbf$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$F_{34x} := F_g + m_4 \cdot a_B$$

$$F_{34x} = -17062 \text{ lbf}$$

$$F_{34y} := -[F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$F_{34y} = 2146.3 \text{ lbf}$$

$$F_{34} := \sqrt{F_{34x}^2 + F_{34y}^2}$$

$$F_{34} = 17196.3 \text{ lbf}$$

$$\theta_{34} := \text{atan2}(F_{34x}, F_{34y})$$

$$\theta_{34} = 172.830 \text{ deg}$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\theta) - (m_{3b} + m_4) \cdot a_B - F_g$$

$$F_{32x} = 34913 \text{ lbf}$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + [F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$F_{32y} = 1458.2 \text{ lbf}$$

$$F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2}$$

$$F_{32} = 34943 \text{ lbf}$$

$$\theta_{32} := \text{atan2}(F_{32x}, F_{32y})$$

$$\theta_{32} = 2.392 \text{ deg}$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot \cos(\theta) + F_{32x}$$

$$F_{21x} = 37004 \text{ lbf}$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + F_{32y}$$

$$F_{21y} = 2219.2 \text{ lbf}$$

$$F_{21} := \sqrt{F_{21x}^2 + F_{21y}^2}$$

$$F_{21} = 37070 \text{ lbf}$$

$$\theta_{21} := \text{atan2}(F_{21x}, F_{21y})$$

$$\theta_{21} = 3.432 \text{ deg}$$

PROBLEM 13-53

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces torque on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 10.4 \cdot in$ mass: $m_3 := 0.120 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.36$
 Crank length: $r := 2.60 \cdot in$ mass: $m_2 := 0.045 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.40$
 Piston mass: $m_4 := 0.150 \cdot blob$ Gas force: $F_g := 350 \cdot lbf$
 Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 25 \cdot deg$

Solution: See Mathcad file P1353.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 3.744 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 6.656 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.0768 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.04320 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.040 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0180 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.0948 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.1932 \text{ blob}$$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := \text{atan} \left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2}} \right] \quad \phi = 6.065 \text{ deg}$$

$$a_B := -r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \quad a_B = -121690.6 \frac{\text{in}}{\text{sec}^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -[(m_4 + m_{3b}) \cdot a_B + F_g] \cdot \tan(\phi) \quad F_{41} = 2461 \text{ lbf}$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$F_{34x} := F_g + m_4 \cdot a_B$$

$$F_{34x} = -17904 \text{ lbf}$$

$$F_{34y} := -[F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$F_{34y} = 2460.8 \text{ lbf}$$

$$F_{34} := \sqrt{F_{34x}^2 + F_{34y}^2}$$

$$F_{34} = 18071.9 \text{ lbf}$$

$$\theta_{34} := \text{atan2}(F_{34x}, F_{34y})$$

$$\theta_{34} = 172.174 \text{ deg}$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\theta) - (m_{3b} + m_4) \cdot a_B - F_g$$

$$F_{32x} = 31099 \text{ lbf}$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + [F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$F_{32y} = 1240.9 \text{ lbf}$$

$$F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2}$$

$$F_{32} = 31124 \text{ lbf}$$

$$\theta_{32} := \text{atan2}(F_{32x}, F_{32y})$$

$$\theta_{32} = 2.285 \text{ deg}$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot \cos(\theta) + F_{32x}$$

$$F_{21x} = 32959 \text{ lbf}$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + F_{32y}$$

$$F_{21y} = 2108.5 \text{ lbf}$$

$$F_{21} := \sqrt{F_{21x}^2 + F_{21y}^2}$$

$$F_{21} = 33027 \text{ lbf}$$

$$\theta_{21} := \text{atan2}(F_{21x}, F_{21y})$$

$$\theta_{21} = 3.660 \text{ deg}$$

PROBLEM 13-54

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces torque on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12.5 \cdot in$ mass: $m_3 := 0.180 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.40$
 Crank length: $r := 3.13 \cdot in$ mass: $m_2 := 0.060 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.38$
 Piston mass: $m_4 := 0.150 \cdot blob$ Gas force: $F_g := 550 \cdot lbf$
 Crank speed: $\omega := 1500 \cdot rpm$ Crank angle: $\theta := 22 \cdot deg$

Solution: See Mathcad file P1354.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 5.000 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 7.500 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.1080 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.07200 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.189 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0228 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.1308 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.2220 \text{ blob}$$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := \text{atan} \left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta) \right)^2}} \right] \quad \phi = 5.382 \text{ deg}$$

$$a_B := -r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \quad a_B = -85516.9 \frac{\text{in}}{\text{sec}^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -[(m_4 + m_{3b}) \cdot a_B + F_g] \cdot \tan(\phi) \quad F_{41} = 1737 \text{ lbf}$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$F_{34x} := F_g + m_4 \cdot a_B$$

$$F_{34x} = -12278 \text{ lbf}$$

$$F_{34y} := -[F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$F_{34y} = 1736.9 \text{ lbf}$$

$$F_{34} := \sqrt{F_{34x}^2 + F_{34y}^2}$$

$$F_{34} = 12399.8 \text{ lbf}$$

$$\theta_{34} := \text{atan2}(F_{34x}, F_{34y})$$

$$\theta_{34} = 171.948 \text{ deg}$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\theta) - (m_{3b} + m_4) \cdot a_B - F_g$$

$$F_{32x} = 26168 \text{ lbf}$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + [F_g + (m_4 + m_{3b}) \cdot a_B] \cdot \tan(\phi)$$

$$F_{32y} = 1387.7 \text{ lbf}$$

$$F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2}$$

$$F_{32} = 26205 \text{ lbf}$$

$$\theta_{32} := \text{atan2}(F_{32x}, F_{32y})$$

$$\theta_{32} = 3.035 \text{ deg}$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot \cos(\theta) + F_{32x}$$

$$F_{21x} = 27801 \text{ lbf}$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + F_{32y}$$

$$F_{21y} = 2047.3 \text{ lbf}$$

$$F_{21} := \sqrt{F_{21x}^2 + F_{21y}^2}$$

$$F_{21} = 27876 \text{ lbf}$$

$$\theta_{21} := \text{atan2}(F_{21x}, F_{21y})$$

$$\theta_{21} = 4.212 \text{ deg}$$

PROBLEM 13-55

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.
 a. Exactly balance the crank and recalculate the inertia force.
 b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius $-r$ on the crank and recalculate the inertia force.
 c. Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12.5 \cdot in$ mass: $m_3 := 0.120 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.36$
 Crank length: $r := 3.13 \cdot in$ mass: $m_2 := 0.045 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.4$
 Piston mass: $m_4 := 0.015 \cdot blob$
 Crank speed: $\omega := 1800 \cdot rpm$ Crank angle: $\theta := 30 \cdot deg$

Solution: See Problem 13-47 and Mathcad file P1355.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 4.500 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 8.000 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.0768 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.04320 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.252 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0180 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.0948 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0582 \text{ blob}$$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \quad F_{ix} = 6416 \text{ lbf}$$

$$F_{iy} := 0 \cdot lbf \quad F_{iy} = 0 \text{ lbf}$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{ib} = 6416 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = 0.000 \text{ deg}$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \quad m_p = 0.0194 \text{ blob}$$

$$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \dots \quad F_{ix} = 4547 \text{ lbf}$$

$$+ (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \cos(\theta))$$

$$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) + (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) \quad F_{iy} = -1079 \text{ lbf}$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{iob} = 4673 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = -13.346 \text{ deg}$$

6. Compare the results to those for an unbalanced crank. From Problem 13-47, the inertia force for the unbalanced crank is $F_i := 16415 \cdot \text{lbf}$. The percent differences for the balanced and overbalanced cranks are:

$$\text{Exactly balanced:} \quad \Delta := \frac{F_{ib} - F_i}{F_i} \quad \Delta = -60.9 \%$$

$$\text{Overbalanced:} \quad \Delta := \frac{F_{iob} - F_i}{F_i} \quad \Delta = -71.5 \%$$

PROBLEM 13-56

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.
 a. Exactly balance the crank and recalculate the inertia force.
 b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius $-r$ on the crank and recalculate the inertia force.
 c. Compare these results to those for the unbalanced crank.

Units: $\text{blob} := \text{lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Conrod length: $l := 10.4 \cdot \text{in}$ mass: $m_3 := 0.180 \cdot \text{blob}$
 Distance to CG (as fraction of l): $r_{3a} := 0.40$
 Crank length: $r := 2.60 \cdot \text{in}$ mass: $m_2 := 0.060 \cdot \text{blob}$
 Distance to CG (as fraction of r): $r_{2a} := 0.38$
 Piston mass: $m_4 := 0.016 \cdot \text{blob}$
 Crank speed: $\omega := 1850 \cdot \text{rpm}$ Crank angle: $\theta := 20 \cdot \text{deg}$

Solution: See Problem 13-48 and Mathcad file P1356.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 4.160 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 6.240 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.1080 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.07200 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 0.988 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0228 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.1308 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0880 \text{ blob}$$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \quad F_{ix} = 9714 \text{ lbf}$$

$$F_{iy} := 0 \cdot \text{lbf} \quad F_{iy} = 0 \text{ lbf}$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{ib} = 9714 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = 0.000 \text{ deg}$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \quad m_p = 0.0293 \text{ blob}$$

$$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \dots \quad F_{ix} = 7024 \text{ lbf}$$

$$+ (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \cos(\theta))$$

$$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) + (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) \quad F_{iy} = -979 \text{ lbf}$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{iob} = 7092 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = -7.935 \text{ deg}$$

6. Compare the results to those for an unbalanced crank. From Problem 13-48, the inertia force for the unbalanced crank is $F_i := 22143 \cdot \text{lbf}$. The percent differences for the balanced and overbalanced cranks are:

$$\text{Exactly balanced:} \quad \Delta := \frac{F_{ib} - F_i}{F_i} \quad \Delta = -56.1 \%$$

$$\text{Overbalanced:} \quad \Delta := \frac{F_{iob} - F_i}{F_i} \quad \Delta = -68.0 \%$$

PROBLEM 13-57

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.
 a. Exactly balance the crank and recalculate the inertia force.
 b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius $-r$ on the crank and recalculate the inertia force.
 c. Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 10.4 \cdot in$ mass: $m_3 := 0.120 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.36$
 Crank length: $r := 2.60 \cdot in$ mass: $m_2 := 0.045 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.40$
 Piston mass: $m_4 := 0.015 \cdot blob$
 Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 25 \cdot deg$

Solution: See Problem 13-49 and Mathcad file P1357.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 3.744 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 6.656 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.0768 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.04320 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.040 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0180 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.0948 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0582 \text{ blob}$$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \quad F_{ix} = 7082 \text{ lbf}$$

$$F_{iy} := 0 \cdot lbf \quad F_{iy} = 0 \text{ lbf}$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{ib} = 7082 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = 0.000 \text{ deg}$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \quad m_p = 0.0194 \text{ blob}$$

$$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \dots \quad F_{ix} = 5077 \text{ lbf}$$

$$+ (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \cos(\theta))$$

$$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) + (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) \quad F_{iy} = -935 \text{ lbf}$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{iob} = 5163 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = -10.435 \text{ deg}$$

6. Compare the results to those for an unbalanced crank. From Problem 13-49, the inertia force for the unbalanced crank is $F_i := 17489 \cdot \text{lbf}$. The percent differences for the balanced and overbalanced cranks are:

$$\text{Exactly balanced:} \quad \Delta := \frac{F_{ib} - F_i}{F_i} \quad \Delta = -59.5 \%$$

$$\text{Overbalanced:} \quad \Delta := \frac{F_{iob} - F_i}{F_i} \quad \Delta = -70.5 \%$$

PROBLEM 13-58

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.
 a. Exactly balance the crank and recalculate the inertia force.
 b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius $-r$ on the crank and recalculate the inertia force.
 c. Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12.5 \cdot in$ mass: $m_3 := 0.180 \cdot blob$
 Distance to CG (as fraction of l): $r_{3a} := 0.36$
 Crank length: $r := 2.60 \cdot in$ mass: $m_2 := 0.045 \cdot blob$
 Distance to CG (as fraction of r): $r_{2a} := 0.40$
 Piston mass: $m_4 := 0.015 \cdot blob$
 Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 25 \cdot deg$

Solution: See Problem 13-50 and Mathcad file P1358.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

$$\text{Distance from point A to CG: } l_a := r_{3a} \cdot l \quad l_a = 4.500 \text{ in}$$

$$\text{Distance from point B to CG: } l_b := l - l_a \quad l_b = 8.000 \text{ in}$$

$$\text{Let } l_p := l_a \quad m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \quad m_{3a} = 0.1152 \text{ blob}$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \quad m_{3b} = 0.06480 \text{ blob}$$

2. Determine the statically equivalent crank model using equations 13.11.

$$\text{Distance from } O_2 \text{ to CG: } r_{G2} := r_{2a} \cdot r \quad r_{G2} = 1.040 \text{ in}$$

$$\text{Mass: } m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \quad m_{2a} = 0.0180 \text{ blob}$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \quad m_A = 0.1332 \text{ blob}$$

$$m_B := m_{3b} + m_4 \quad m_B = 0.0798 \text{ blob}$$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \quad F_{ix} = 9465 \text{ lbf}$$

$$F_{iy} := 0 \cdot lbf \quad F_{iy} = 0 \text{ lbf}$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{ib} = 9465 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = 0.000 \text{ deg}$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \quad m_p = 0.0266 \text{ blob}$$

$$F_{ix} := -m_A \cdot (-r \cdot \omega^2 \cdot \cos(\theta)) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \dots \quad F_{ix} = 6716 \text{ lbf}$$

$$+ (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \cos(\theta))$$

$$F_{iy} := -m_A \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) + (m_A + m_p) \cdot (-r \cdot \omega^2 \cdot \sin(\theta)) \quad F_{iy} = -1282 \text{ lbf}$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \quad F_{iob} = 6837 \text{ lbf} \quad \text{at} \quad \text{atan2}(F_{ix}, F_{iy}) = -10.808 \text{ deg}$$

6. Compare the results to those for an unbalanced crank. From Problem 13-50, the inertia force for the unbalanced crank is $F_i := 17227 \cdot \text{lbf}$. The percent differences for the balanced and overbalanced cranks are:

$$\text{Exactly balanced:} \quad \Delta := \frac{F_{ib} - F_i}{F_i} \quad \Delta = -45.1 \%$$

$$\text{Overbalanced:} \quad \Delta := \frac{F_{iob} - F_i}{F_i} \quad \Delta = -60.3 \%$$

PROBLEM 14-1

Statement: Draw a crank phase diagram for a three-cylinder inline engine with a 0, 120, 240-deg crankshaft and determine all possible firing orders for

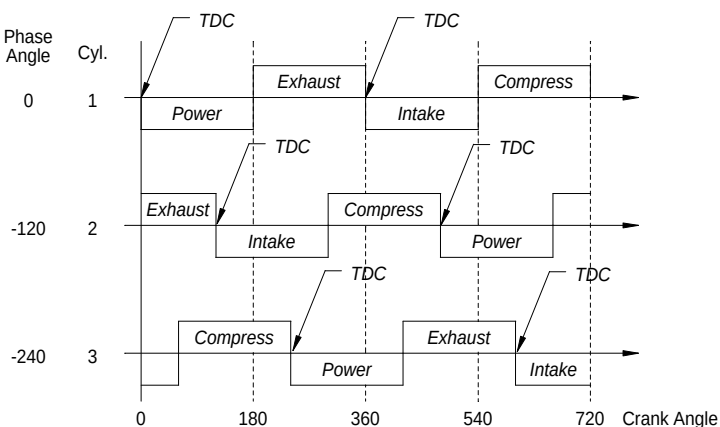
- Four-stroke cycle.
- Two-stroke cycle.

Select the best arrangement to give even firing for each stroke cycle.

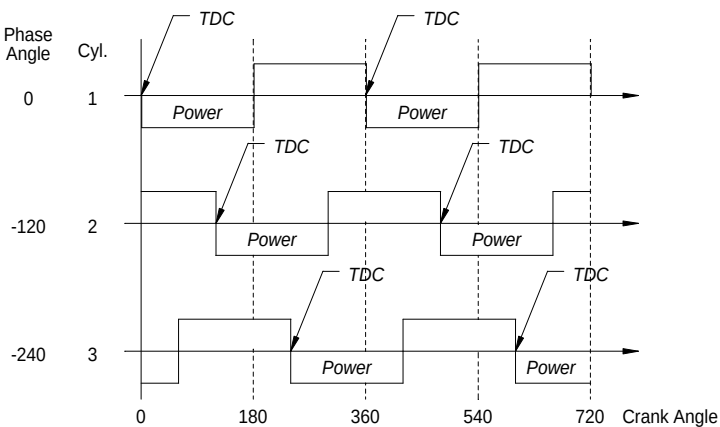
Solution: See Mathcad file P1401.

- Even firing is possible with both two and four stroke designs. A 1, 2, 3 firing order is required for the two-stroke and a 1, 3, 2 firing order for the four-stroke.

a. Four-stroke cycle



b. Two-stroke cycle



PROBLEM 14-2

Statement: Draw a crank phase diagram for an inline four-cylinder inline engine with a 0, 90, 270, 180-deg crankshaft and determine all possible firing orders for

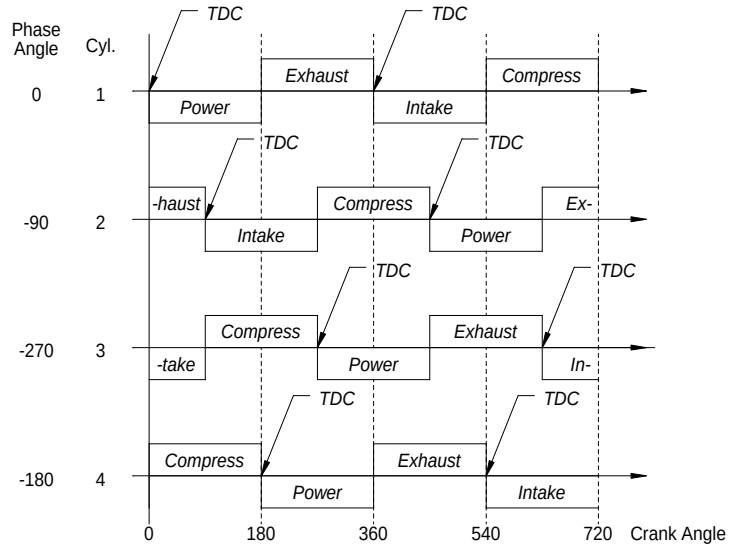
- Four-stroke cycle.
- Two-stroke cycle.

Select the best arrangement to give even firing for each stroke cycle.

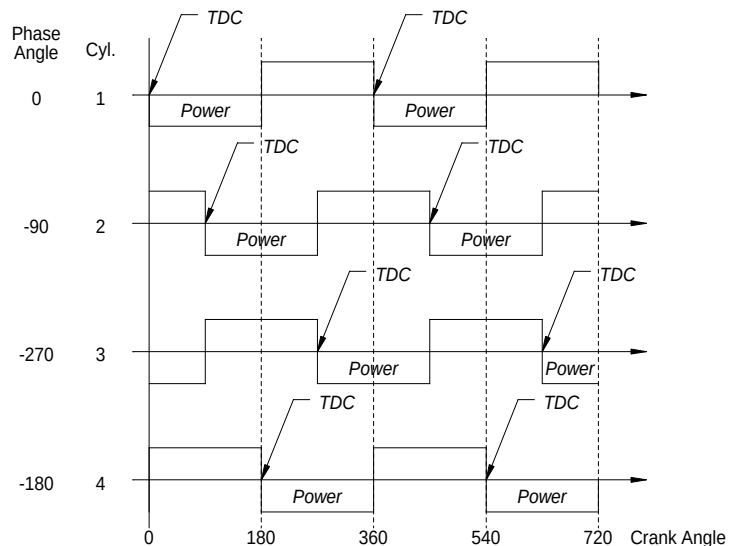
Solution: See Mathcad file P1402.

- The two-stroke has even firing but even firing is not possible with this four stroke design. The best firing order is 1, 2, 4, 3 for the two-stroke and 1, 4, 3, 2 for the four-stroke.

a. Four-stroke cycle



b. Two-stroke cycle



PROBLEM 14-3

Statement: Draw a crank phase diagram for a 45-deg vee, four-cylinder engine with a 0, 90, 270, 180-deg crankshaft and determine all possible firing orders for

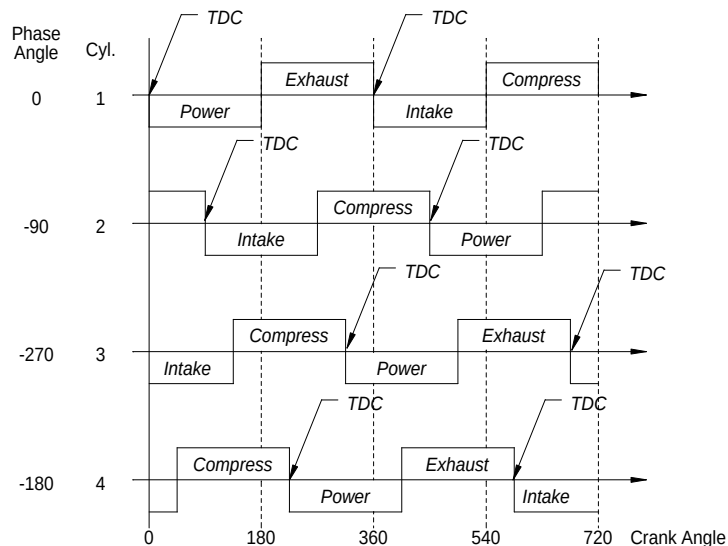
- Four-stroke cycle.
- Two-stroke cycle.

Select the best arrangement to give even firing for each stroke cycle.

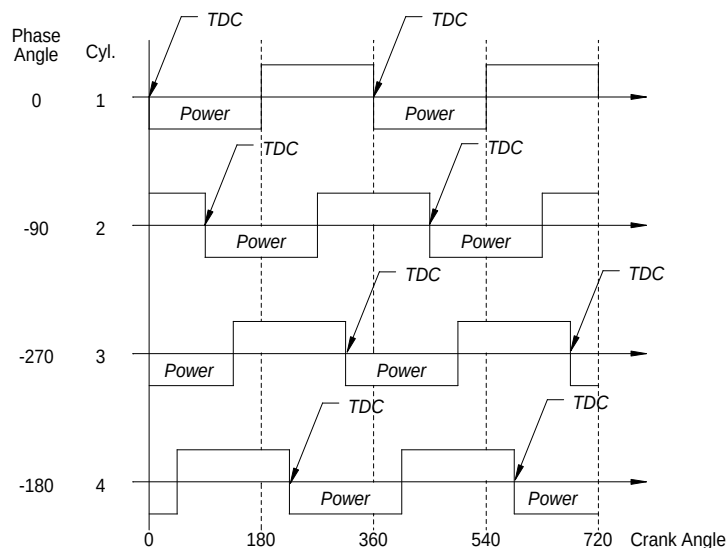
Solution: See Mathcad file P1403.

- Neither stroke has even firing with this design. The best firing order is 1, 2, 4, 3 for the two-stroke and 1, 4, 3, 2 for the four-stroke.

a. Four-stroke cycle



b. Two-stroke cycle



PROBLEM 14-4

Statement: Draw a crank phase diagram for a 45-deg vee, two-cylinder engine with a 0, 90-deg crankshaft and determine all possible firing orders for

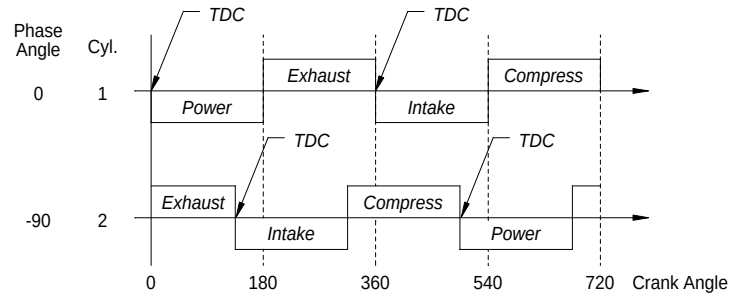
- Four-stroke cycle.
- Two-stroke cycle.

Select the best arrangement to give even firing for each stroke cycle.

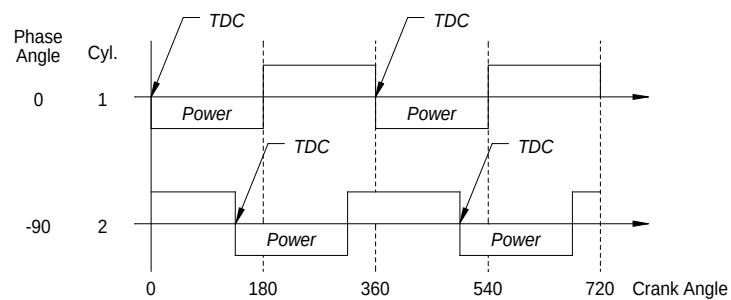
Solution: See Mathcad file P1404.

- Neither stroke has even firing with this design. The only possible firing order is 1, 2 for either stroke-cycle.

a. Four-stroke cycle



b. Two-stroke cycle



PROBLEM 14-5

Statement: Draw a crank phase diagram for a 90-deg vee, two-cylinder engine with a 0, 180-deg crankshaft and determine all possible firing orders for

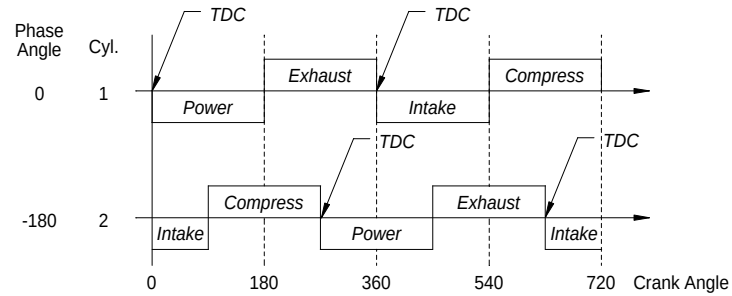
- Four-stroke cycle.
- Two-stroke cycle.

Select the best arrangement to give even firing for each stroke cycle.

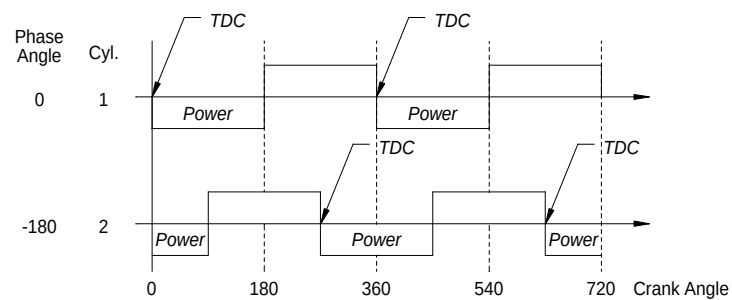
Solution: See Mathcad file P1405.

- Neither stroke has even firing with this design. The only possible firing order is 1, 2 for either stroke-cycle.

a. Four-stroke cycle



b. Two-stroke cycle



PROBLEM 14-6

Statement: Draw a crank phase diagram for a 180-deg opposed, two-cylinder engine with a 0, 180-deg crankshaft and determine all possible firing orders for

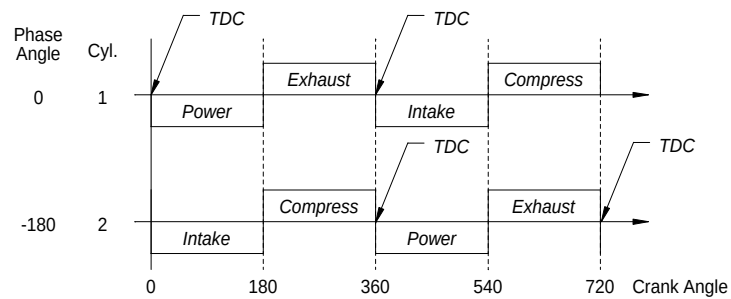
- Four-stroke cycle.
- Two-stroke cycle.

Select the best arrangement to give even firing for each stroke cycle.

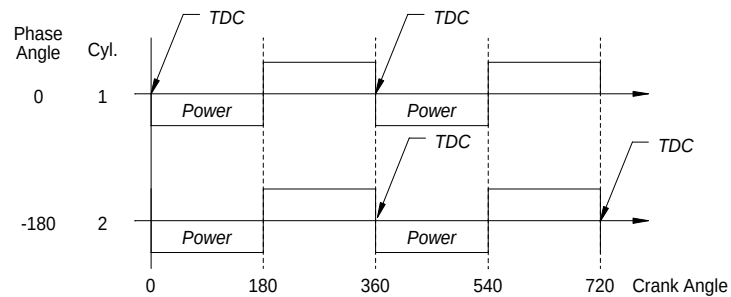
Solution: See Mathcad file P1406.

- Both stroke-cycles are even firing but the two-stroke looks like a four-stroke in its firing pattern since the second set of firing pulses is on top of the first set. The only possible firing order is 1, 2 for either stroke-cycle.

a. Four-stroke cycle



b. Two-stroke cycle



PROBLEM 14-7

Statement: Draw a crank phase diagram for a 180-deg opposed, four-cylinder engine with a 0, 180, 180, 0-deg crankshaft and determine all possible firing orders for

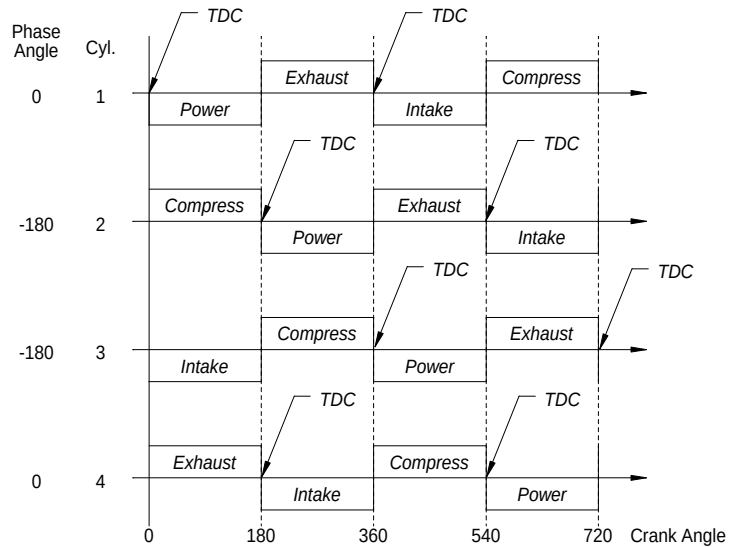
- Four-stroke cycle.
- Two-stroke cycle.

Select the best arrangement to give even firing for each stroke cycle.

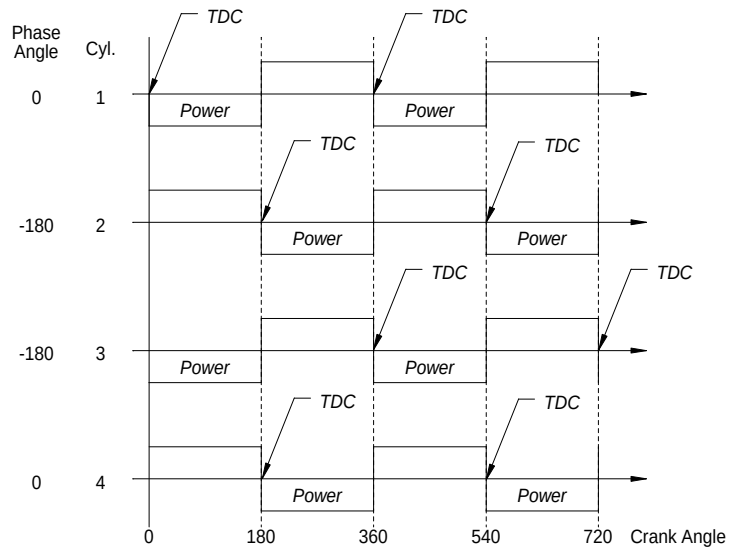
Solution: See Mathcad file P1407.

- Both stroke-cycles are even firing but the two-stroke looks like a four-stroke in its firing pattern since the second set of firing pulses is on top of the first set. The best firing order is 1, 2, 4, 3 for the two-stroke-cycle and 1, 2, 3, 4 for the four-stroke.

a. Four-stroke cycle



b. Two-stroke cycle



PROBLEM 14-8

Statement: Calculate the shaking force, torque and moment balance conditions through the second harmonic for an inline three-cylinder engine with a 0, 120, 240 deg crankshaft.

Assumptions: Use File F14-12.eng for the cylinder data required on the cylinder input screen.

Solution: See program ENGINE files P14082.eng, P14084.eng, and Mathcad file P1408.mcd.

1. Open file P14082.eng in program ENGINE for the two-stroke cycle or file P14084.eng for the four-stroke cycle. The problem setup data is input from the disk file. Click the *Calculate* button and then the *Done* button, which will return you to the *Home* screen. Click the *Balance* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Assemble* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. View the calculated results by clicking on the *Charts* drop-down menu, from the main option bar, or use the *Plot* or *Print* buttons on the *Home* screen.

PROBLEM 14-9

Statement: Calculate the shaking force, torque and moment balance conditions through the second harmonic for an inline four-cylinder engine with a 0, 90, 270, 180 deg crankshaft.

Assumptions: Use File F14-12.eng for the cylinder data required on the cylinder input screen.

Solution: See program ENGINE files P14092.eng, P14094.eng, and Mathcad file P1409.mcd.

1. Open file P14092.eng in program ENGINE for the two-stroke cycle or file P14094.eng for the four-stroke cycle. The problem setup data is input from the disk file. Click the *Calculate* button and then the *Done* button, which will return you to the *Home* screen. Click the *Balance* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Assemble* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. View the calculated results by clicking on the *Charts* drop-down menu, from the main option bar, or use the *Plot* or *Print* buttons on the *Home* screen.

PROBLEM 14-10

Statement: Calculate the shaking force, torque and moment balance conditions through the second harmonic for an 45-deg vee, four-cylinder engine with a 0, 90, 270, 180 deg crankshaft.

Assumptions: Use File F14-12.eng for the cylinder data required on the cylinder input screen.

Solution: See program ENGINE files P14102.eng, P14104.eng, and Mathcad file P1410.mcd.

1. Open file P14102.eng in program ENGINE for the two-stroke cycle or file P14104.eng for the four-stroke cycle. The problem setup data is input from the disk file. Click the *Calculate* button and then the *Done* button, which will return you to the *Home* screen. Click the *Balance* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Assemble* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. View the calculated results by clicking on the *Charts* drop-down menu, from the main option bar, or use the *Plot* or *Print* buttons on the *Home* screen.

PROBLEM 14-11

Statement: Calculate the shaking force, torque and moment balance conditions through the second harmonic for an 45-deg vee, two-cylinder engine with a 0, 90 deg crankshaft.

Assumptions: Use File F14-12.eng for the cylinder data required on the cylinder input screen.

Solution: See program ENGINE files P14112.eng, P14114.eng, and Mathcad file P1411.mcd.

1. Open file P14112.eng in program ENGINE for the two-stroke cycle or file P14114.eng for the four-stroke cycle. The problem setup data is input from the disk file. Click the *Calculate* button and then the *Done* button, which will return you to the *Home* screen. Click the *Balance* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Assemble* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. View the calculated results by clicking on the *Charts* drop-down menu, from the main option bar, or use the *Plot* or *Print* buttons on the *Home* screen.

PROBLEM 14-12

Statement: Calculate the shaking force, torque and moment balance conditions through the second harmonic for an 90-deg vee, two-cylinder engine with a 0, 180 deg crankshaft.

Assumptions: Use File F14-12.eng for the cylinder data required on the cylinder input screen.

Solution: See program ENGINE files P14122.eng, P14124.eng, and Mathcad file P1412.mcd.

1. Open file P14122.eng in program ENGINE for the two-stroke cycle or file P14124.eng for the four-stroke cycle. The problem setup data is input from the disk file. Click the *Calculate* button and then the *Done* button, which will return you to the *Home* screen. Click the *Balance* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Assemble* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. View the calculated results by clicking on the *Charts* drop-down menu, from the main option bar, or use the *Plot* or *Print* buttons on the *Home* screen.

PROBLEM 14-13

Statement: Calculate the shaking force, torque and moment balance conditions through the second harmonic for an 180-deg opposed, two-cylinder engine with a 0, 180 deg crankshaft.

Assumptions: Use File F14-12.eng for the cylinder data required on the cylinder input screen.

Solution: See program ENGINE files P14132.eng, P14134.eng, and Mathcad file P1413.mcd.

1. Open file P14132.eng in program ENGINE for the two-stroke cycle or file P14134.eng for the four-stroke cycle. The problem setup data is input from the disk file. Click the *Calculate* button and then the *Done* button, which will return you to the *Home* screen. Click the *Balance* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Assemble* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. View the calculated results by clicking on the *Charts* drop-down menu, from the main option bar, or use the *Plot* or *Print* buttons on the *Home* screen.

PROBLEM 14-14

Statement: Calculate the shaking force, torque and moment balance conditions through the second harmonic for an 180-deg opposed, four-cylinder engine with a 0, 180, 180, 0 deg crankshaft.

Assumptions: Use File F14-12.eng for the cylinder data required on the cylinder input screen.

Solution: See program ENGINE files P14142.eng, P14144.eng, and Mathcad file P1414.mcd.

1. Open file P14142.eng in program ENGINE for the two-stroke cycle or file P14144.eng for the four-stroke cycle. The problem setup data is input from the disk file. Click the *Calculate* button and then the *Done* button, which will return you to the *Home* screen. Click the *Balance* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Assemble* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. View the calculated results by clicking on the *Charts* drop-down menu, from the main option bar, or use the *Plot* or *Print* buttons on the *Home* screen.

PROBLEM 14-15

Statement: Derive expressions, in general terms, for the magnitude and angular location with respect to the first crank throw, of the mass-radius products needed on the crankshaft to balance the shaking moment in a 90-deg vee-eight engine with a 0, 90, 270, 180-deg crankshaft.

Solution: No solution is provided to this algebraic exercise.

PROBLEM 14-16

Statement: Derive expressions, in general terms, for the magnitude and angular location with respect to the first crank throw, of the mass-radius products needed on the crankshaft to balance the shaking moment in a 90-deg vee-six engine with a 0, 240, 120-deg crankshaft.

Solution: No solution is provided to this algebraic exercise.

PROBLEM 14-17

Statement: Derive expressions, in general terms, for the magnitude and angular location with respect to the first crank throw, of the mass-radius products needed on the crankshaft to balance the shaking moment in a 90-deg vee-four engine with a 0, 180-deg crankshaft.

Solution: No solution is provided to this algebraic exercise.

PROBLEM 14-18

Statement: Design a pair of Nakamura balance shafts to cancel the shaking force and reduce oscillations in the engine shown in Figure 14-18.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: From Figure 14-18 and file F14-18.eng:

$$\text{Stroke: } S := 3.537 \cdot in \quad \text{L/R ratio: } LoverR := 3.50$$

$$\text{Effective wrist pin mass: } m_B := 0.0116 \cdot blob$$

Solution: See Figure 14-18, program ENGINE file F14-18.eng, and Mathcad file P1418.

1. Calculate the crank radius and conrod length.

$$\text{Crank radius: } r := 0.5 \cdot S \quad r = 1.768 \cdot in$$

$$\text{Conrod length: } l := LoverR \cdot r \quad l = 6.190 \cdot in$$

2. Use equation 14.18 to calculate the mr product needed for the balance shafts.

$$mr_{bal} := \frac{r}{2 \cdot l} \cdot m_B \cdot r \quad mr_{bal} = 2.931 \times 10^{-3} \cdot blob \cdot in$$

$$wr_{bal} := mr_{bal} \cdot g \quad wr_{bal} = 1.131 \cdot lbf \cdot in$$

3. The specific location of the two balance shafts will depend on the specific geometry of the engine being balanced. The y-dimensions must be symmetric with respect to the engine longitudinal center plane ($y_2 = -y_1$), and the vertical dimensions should be such that $x_1 - x_2 = 0.7 \cdot l$.

PROBLEM 14-19

Statement: Using program ENGINE, data in Table P14-1 and the crank phase diagram from Problem 14-1, determine the maximum force magnitudes on main pin, crank pin, wrist pin, and piston for a 2-stroke engine with even firing. Over balance the crank, if necessary, to bring the balanced shaking force down to at least half of the unbalanced value.

Solution: See program ENGINE file P1419.eng and Mathcad file P1419.mcd.

1. Open file P1419.eng in program ENGINE. The problem setup data is input from the disk file. Click the *Calculate* button and then the *Done* button, which will return you to the *Home* screen. Click the *Balance* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Assemble* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Flywheel* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. View the calculated results by clicking on the *Charts* drop-down menu, from the main option bar, or use the *Plot* or *Print* buttons on the *Home* screen.

PROBLEM 14-20

Statement: Using program ENGINE, data in Table P14-1 and the crank phase diagram from Problem 14-2, determine the maximum force magnitudes on main pin, crank pin, wrist pin, and piston for a 4-stroke engine with even firing. Over balance the crank, if necessary, to bring the balanced shaking force down to at least half of the unbalanced value.

Solution: See program ENGINE file P1420.eng and Mathcad file P1420.mcd.

1. Open file P1420.eng in program ENGINE. The problem setup data is input from the disk file. Click the *Calculate* button and then the *Done* button, which will return you to the *Home* screen. Click the *Balance* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Assemble* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Flywheel* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. View the calculated results by clicking on the *Charts* drop-down menu, from the main option bar, or use the *Plot* or *Print* buttons on the *Home* screen.

PROBLEM 14-21

Statement: Using program ENGINE, data in Table P14-1 and the crank phase diagram from Problem 14-3, determine the maximum force magnitudes on main pin, crank pin, wrist pin, and piston for a 4-stroke engine with even firing. Over balance the crank, if necessary, to bring the balanced shaking force down to at least half of the unbalanced value.

Solution: See program ENGINE file P1421.eng and Mathcad file P1421.mcd.

1. Open file P1421.eng in program ENGINE. The problem setup data is input from the disk file. Click the *Calculate* button and then the *Done* button, which will return you to the *Home* screen. Click the *Balance* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Assemble* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Flywheel* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. View the calculated results by clicking on the *Charts* drop-down menu, from the main option bar, or use the *Plot* or *Print* buttons on the *Home* screen.

PROBLEM 14-22

Statement: Using program ENGINE, data in Table P14-1 and the crank phase diagram from Problem 14-4, determine the maximum force magnitudes on main pin, crank pin, wrist pin, and piston for a 2-stroke engine with even firing. Over balance the crank, if necessary, to bring the balanced shaking force down to at least half of the unbalanced value.

Solution: See program ENGINE file P1422.eng and Mathcad file P1422.mcd.

1. Open file P1422.eng in program ENGINE. The problem setup data is input from the disk file. Click the *Calculate* button and then the *Done* button, which will return you to the *Home* screen. Click the *Balance* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Assemble* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. Click the *Flywheel* button, accept the values shown and click *Calculate* and then *Done*, which will return you to the *Home* screen. View the calculated results by clicking on the *Charts* drop-down menu, from the main option bar, or use the *Plot* or *Print* buttons on the *Home* screen.

PROBLEM 14-23

Statement: A four-cylinder inline engine with a 0, 180, 180, 0-deg crankshaft has a stroke of $S = 3.500$ in, a conrod length to crank radius ratio of $L/R = 3.750$, and an effective wrist pin mass of $m_B = 0.0215$ blob. Design a pair of Nakamura balance shafts to cancel the shaking force and reduce the oscillations in the engine.

Units: $\text{blob} := \text{lb} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Stroke: $S := 3.500 \cdot \text{in}$ L/R ratio: $L/R := 3.750$

Effective wrist pin mass: $m_B := 0.0215 \cdot \text{blob}$

Solution: See Figure 14-31 and Mathcad file P1423.

1. Calculate the crank radius and conrod length.

$$\text{Crank radius: } r := 0.5 \cdot S \quad r = 1.750 \text{ in}$$

$$\text{Conrod length: } l := L/R \cdot r \quad l = 6.563 \text{ in}$$

2. Use equation 14.18 to calculate the mr product needed for the balance shafts.

$$mr_{bal} := \frac{r}{2 \cdot l} \cdot m_B \cdot r \quad mr_{bal} = 5.017 \times 10^{-3} \text{ blob} \cdot \text{in}$$

$$wr_{bal} := mr_{bal} \cdot g \quad wr_{bal} = 1.937 \text{ lb} \cdot \text{in}$$

3. The specific location of the two balance shafts will depend on the specific geometry of the engine being balanced. The y-dimensions must be symmetric with respect to the engine longitudinal center plane ($y_2 = -y_1$), and the vertical dimensions should be such that $x_1 - x_2 = 0.7 l$.

PROBLEM 14-24

Statement: A four-cylinder inline engine with a 0, 180, 180, 0-deg crankshaft has a stroke of $S = 2.750$ in, a conrod length to crank radius ratio of $L/R = 3.00$, and an effective wrist pin mass of $m_B = 0.0125$ blob. Design a pair of Nakamura balance shafts to cancel the shaking force and reduce the oscillations in the engine.

Units: $\text{blob} := \text{lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Stroke: $S := 2.750 \cdot \text{in}$ L/R ratio: $LoverR := 3.00$

Effective wrist pin mass: $m_B := 0.0125 \cdot \text{blob}$

Solution: See Figure 14-31 and Mathcad file P1424.

1. Calculate the crank radius and conrod length.

$$\text{Crank radius: } r := 0.5 \cdot S \quad r = 1.375 \text{ in}$$

$$\text{Conrod length: } l := LoverR \cdot r \quad l = 4.125 \text{ in}$$

2. Use equation 14.18 to calculate the mr product needed for the balance shafts.

$$mr_{bal} := \frac{r}{2 \cdot l} \cdot m_B \cdot r \quad mr_{bal} = 2.865 \times 10^{-3} \text{ blob} \cdot \text{in}$$

$$wr_{bal} := mr_{bal} \cdot g \quad wr_{bal} = 1.106 \text{ lbf} \cdot \text{in}$$

3. The specific location of the two balance shafts will depend on the specific geometry of the engine being balanced. The y-dimensions must be symmetric with respect to the engine longitudinal center plane ($y_2 = -y_1$), and the vertical dimensions should be such that $x_1 - x_2 = 0.7 l$.

PROBLEM 15-1

Statement: Design a double-dwell cam to meet the specifications given below. Size a return spring and specify its preload to maintain contact between cam and follower. Calculate and plot the dynamic force and torque. Repeat for a form-closed cam. Compare the dynamic force, torque, and natural frequency for the form-closed design and the force-closed design.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 60 \cdot \text{deg}$	$\beta_2 := 120 \cdot \text{deg}$	$\beta_3 := 30 \cdot \text{deg}$	$\beta_4 := 150 \cdot \text{deg}$
$h_1 := 2.5 \cdot \text{in}$	$h_2 := 0 \cdot \text{in}$	$h_3 := 2.5 \cdot \text{in}$	$h_4 := 0 \cdot \text{in}$
Mod. sine		Cycloidal	
Cycle time:	$\tau := 4 \cdot \text{sec}$	Damping ratio:	$\zeta := 0.2$ Follower mass: $m_f := 2.2$

Solution: See DYNACAM files P1501a and P1501b.

- The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.571 \cdot \frac{\text{rad}}{\text{sec}} \quad \omega = 15.000 \cdot \text{rpm}$$

- Load the noted files into program DYNACAM. P1501a is the force-closed solution and P1501b is the form-closed solution. In each case, after opening the file, click on the *Size Cam* button and then click *Calculate* to see the pressure angle and minimum radius of curvature. Then click the *Done* button and you will see the cam drawing. After clicking the *Done* button on that screen, click the *Dynamics* button on the *Home* screen. Click its *Calculate* button to see the fully developed *Dynamics* screen. Once this sequence of commands has been entered, you may see further data by clicking on the *Print* or *Plot* buttons on the home screen. Accept all default values, which have been brought in from the file. It is not necessary to go to the *SVAJ* screen as that data has been brought in from the files.
- The cam is sized with an 11.125-in prime circle radius and -2.64 in eccentricity, which gives balanced pressure angles of 29.8 deg and a minimum radius of curvature of 2.94 in. The roller radius is 1.00 in.
- For the force-closed case, a spring constant of 100 lb/in with a preload of 170 lb is chosen to keep the follower force positive and prevent jump. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient are shown on the *Cam Dynamics* screen, which is shown below on the next page. The system natural frequency is about 4.3 times the driving frequency. This ratio could be increased at the expense of larger follower force by increasing the spring constant.
- For the form-closed case, it is only necessary to define spring constant and preload values of zero. The "b" diskfile does this. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient for this case are shown on the second *Cam Dynamics* screen, which is shown below on the next page.
- Comparing these two cases shows that in the force-closed case, the peak follower force is greater and the driving torque excursions are lower than in the form-closed case. This gives the form-closed cam the advantage in terms of lower stresses in cam and follower but a larger drive motor. However, this advantage comes at the expense of a more difficult and costly cam to make, since two cam surfaces must be machined instead of one.

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Cam Dynamics**

Effective Mass: lb

Spring Constant: $\frac{\text{lb}}{\text{in}}$

Spring Preload: lb

Damping Ratio ζ : $\frac{\text{lb-s}}{\text{in}}$

Coefficient C: $\frac{\text{lb-s}}{\text{in}}$

Thomas A. Cook
Design No. 2
06-28-1999
at 14:10:22

Wn: rad/s Cc: Wd: rad/s

Fmax: lb Fmin: lb Tmax: lb-in

Cam Journal Dia: in

Journal Friction:

Follower Friction:

Torque Calcs: ☒ Start Fresh ☐ Accumulate

Coef Fluctuation:

Areas of Camshaft Torque Pulses in Order over 1 cycle

in Energy Units of : lb-in - Radians

Order	Neg Area	Pos Area
1	-471.9	69287.4
2	-68390.9	0

Average Torque: 413.46 lb-in

Calculate Copy Print Form Done Cancel

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Cam Dynamics**

Effective Mass: lb

Spring Constant: $\frac{\text{lb}}{\text{in}}$

Spring Preload: lb

Damping Ratio ζ : $\frac{\text{lb-s}}{\text{in}}$

Coefficient C: $\frac{\text{lb-s}}{\text{in}}$

Thomas A. Cook
Design No. 2
06-28-1999
at 14:15:40
File: P1501a

Wn: rad/s Cc: Wd: rad/s

Fmax: lb Fmin: lb Tmax: lb-in

Cam Journal Dia: in

Journal Friction:

Follower Friction:

Torque Calcs: ☒ Start Fresh ☐ Accumulate

Coef Fluctuation:

Areas of Camshaft Torque Pulses in Order over 1 cycle

in Energy Units of : lb-in - Radians

Order	Neg Area	Pos Area
1	-69.2	5807.9
2	-10262.3	23385.8
3	-8051.4	-6.9
4	-10670.4	0

Average Torque: 70.93 lb-in

Calculate Copy Print Form Done Cancel

PROBLEM 15-2

Statement: Design a double-dwell cam to meet the specifications given below. Size a return spring and specify its preload to maintain contact between cam and follower. Calculate and plot the dynamic force and torque. Repeat for a form-closed cam. Compare the dynamic force, torque, and natural frequency for the form-closed design and the force-closed design.

Given:

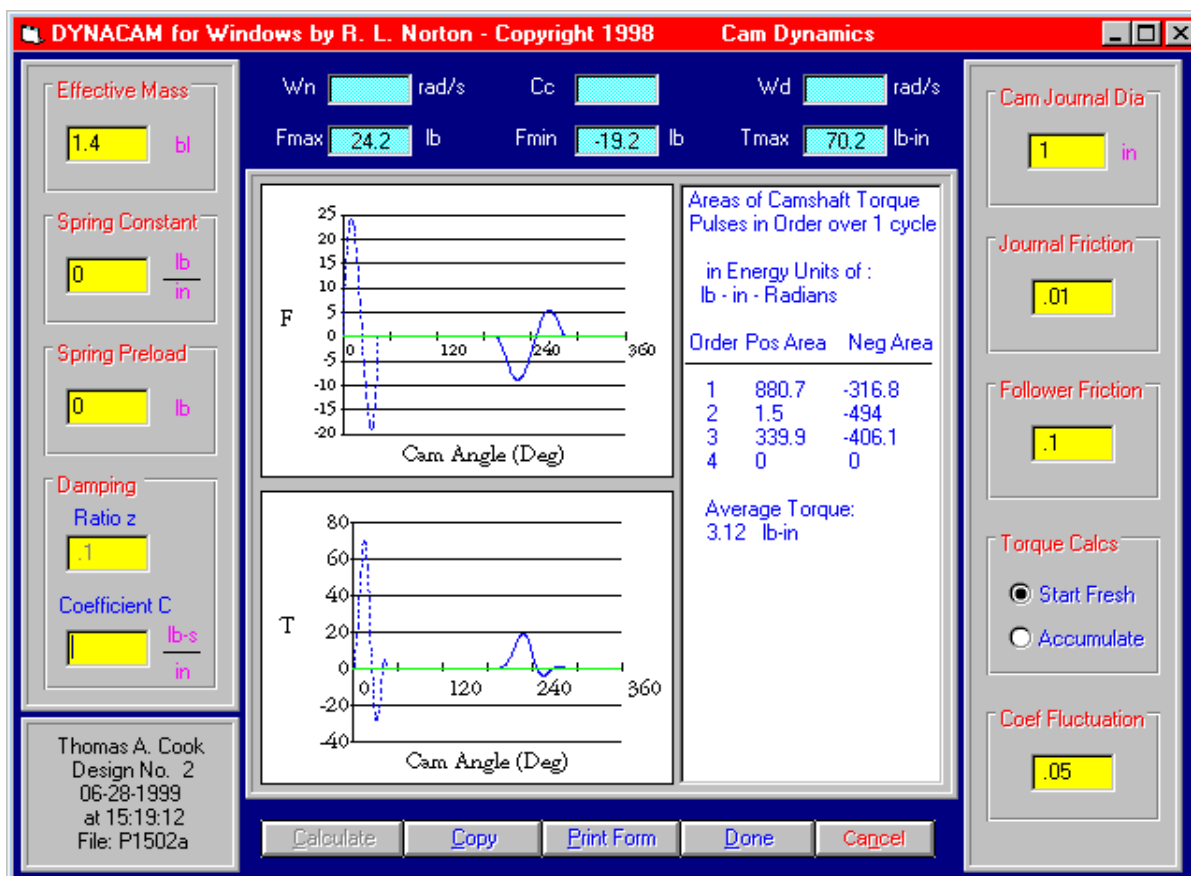
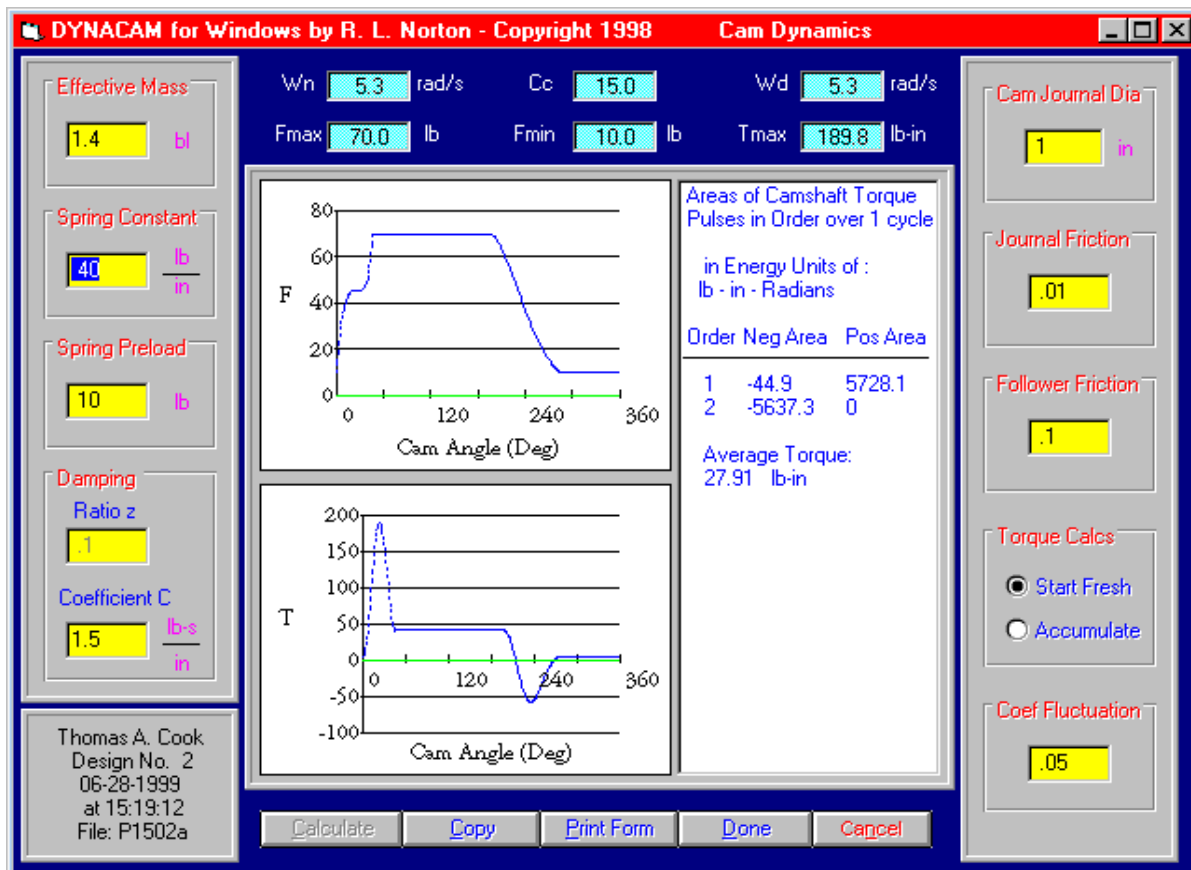
RISE	DWELL	FALL	DWELL
$\beta_1 := 45 \cdot \text{deg}$	$\beta_2 := 150 \cdot \text{deg}$	$\beta_3 := 90 \cdot \text{deg}$	$\beta_4 := 75 \cdot \text{deg}$
$h_1 := 1.5 \cdot \text{in}$	$h_2 := 0 \cdot \text{in}$	$h_3 := 1.5 \cdot \text{in}$	$h_4 := 0 \cdot \text{in}$
3-4-5 poly		4-5-6-7 poly	
Cycle time: $\tau := 6 \cdot \text{sec}$	Damping ratio: $\zeta := 0.1$	Follower mass: $m_f := 1.4$	

Solution: See DYNACAM files P1502a and P1502b.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 1.047 \frac{\text{rad}}{\text{sec}} \quad \omega = 10.000 \text{ rpm}$$

2. Load the noted files into program DYNACAM. Diskfile P1502a contains the force-closed solution and P1502b the form-closed solution. In each case, after opening the file, click on the *Size Cam* button and then click *Calculate* to see the pressure angle and minimum radius of curvature. Then click the *Done* button and you will see the cam drawing. After clicking the *Done* button on that screen, click the *Dynamics* button on the *Home* screen. Click its *Calculate* button to see the fully developed *Dynamics* screen. Once this sequence of commands has been entered, you may see further data by clicking on the *Print* or *Plot* buttons on the home screen. Accept all default values, which have been brought in from the file. It is not necessary to go to the *SVAJ* screen as that data has been brought in from the files.
3. The cam is sized with an 4.25-in prime circle radius and 0.75 in eccentricity, which gives balanced pressure angles of 30 deg and a minimum radius of curvature of 1.75 in. The roller radius is 1.00 in.
4. For the force-closed case, a spring constant of 40 lb/in with a preload of 10 lb is chosen to keep the follower force positive and prevent jump. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient are shown on the *Cam Dynamics* screen, which is shown below on the next page. The system natural frequency is about 5.3 times the driving frequency. This ratio could be increased at the expense of larger follower force by increasing the spring constant.
5. For the form-closed case, it is only necessary to define spring constant and preload values of zero. The "b" diskfile does this. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient for this case are shown on the second *Cam Dynamics* screen, which is shown below on the next page.
6. Comparing these two cases shows that in the force-closed case both the peak follower force and driving torque excursions are greater than in the form-closed case. This gives the form-closed cam the advantage in terms of lower stresses in cam and follower and a smaller drive motor. However, this advantage comes at the expense of a more difficult and costly cam to make, since two cam surfaces must be machined instead of one.



PROBLEM 15-3

Statement: Design a single-dwell cam to meet the specifications given below. Size a return spring and specify its preload to maintain contact between cam and follower. Calculate and plot the dynamic force and torque. Repeat for a form-closed cam. Compare the dynamic force, torque, and natural frequency for the form-closed design and the force-closed design.

Given:

RISE	FALL	DWELL
$\beta_{11} := 60 \cdot \text{deg}$	$\beta_{12} := 90 \cdot \text{deg}$	$\beta_2 := 210 \cdot \text{deg}$
$h_1 := 2 \cdot \text{in}$		$h_2 := 0 \cdot \text{in}$
Seventh degree polynomial		
Cycle time: $\tau := 5 \cdot \text{sec}$	Damping ratio: $\zeta := 0.15$	Follower mass: $m_f := 3.2$

Solution: See DYNACAM files P1503a and P1503b.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \qquad \omega = 1.257 \frac{\text{rad}}{\text{sec}} \qquad \omega = 12.000 \text{ rpm}$$

2. Load the noted files into program DYNACAM. Diskfile P1503a contains the force-closed solution and P1503b the form-closed solution. In each case, after opening the file, click on the *Size Cam* button and then click *Calculate* to see the pressure angle and minimum radius of curvature. Then click the *Done* button and you will see the cam drawing. After clicking the *Done* button on that screen, click the *Dynamics* button on the *Home* screen. Click its *Calculate* button to see the fully developed *Dynamics* screen. Once this sequence of commands has been entered, you may see further data by clicking on the *Print* or *Plot* buttons on the home screen. Accept all default values, which have been brought in from the file. It is not necessary to go to the *SVAJ* screen as that data has been brought in from the files.
3. The cam is sized with an 4.40-in prime circle radius and 0.22 in eccentricity, which gives balanced pressure angles of 29.9 deg and a minimum radius of curvature of 2.53 in. The roller radius is 1.00 in.
4. For the force-closed case, a spring constant of 60 lb/in with a preload of 15 lb is chosen to keep the follower force positive and prevent jump. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient are shown on the *Cam Dynamics* screen, which is shown below on the next page. The system natural frequency is about 3.4 times the driving frequency. This ratio could be increased at the expense of larger follower force by increasing the spring constant.
5. For the form-closed case, it is only necessary to define spring constant and preload values of zero. The "b" diskfile does this. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient for this case are shown on the second *Cam Dynamics* screen, which is shown below on the next page.
6. Comparing these two cases shows that in the force-closed case both the peak follower force and driving torque excursions are greater than in the form-closed case. This gives the form-closed cam the advantage in terms of lower stresses in cam and follower and a smaller drive motor. However, this advantage comes at the expense of a more difficult and costly cam to make, since two cam surfaces must be machined instead of one.

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Effective Mass: lb

Spring Constant: $\frac{\text{lb}}{\text{in}}$

Spring Preload: lb

Damping Ratio ζ :

Coefficient C: $\frac{\text{lb-s}}{\text{in}}$

Thomas A. Cook
Design No. 2
06-28-1999
at 15:32:40

Wn: rad/s Cc: Wd: rad/s

Fmax: lb Fmin: lb Tmax: lb-in

Cam Journal Dia: in

Journal Friction:

Follower Friction:

Torque Calcs: ☒ Start Fresh ☐ Accumulate

Coef Fluctuation:

Areas of Camshaft Torque Pulses in Order over 1 cycle

in Energy Units of :
lb - in - Radians

Order	Neg Area	Pos Area
1	-22.6	11425.3
2	-11364.3	0

Average Torque: 26.85 lb-in

Calculate Copy Print Form Done Cancel

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Cam Dynamics**

Effective Mass: lb

Spring Constant: $\frac{\text{lb}}{\text{in}}$

Spring Preload: lb

Damping Ratio ζ :

Coefficient C: $\frac{\text{lb-s}}{\text{in}}$

Thomas A. Cook
Design No. 2
06-28-1999
at 15:32:40
File: P1503a

Wn: rad/s Cc: Wd: rad/s

Fmax: lb Fmin: lb Tmax: lb-in

Cam Journal Dia: in

Journal Friction:

Follower Friction:

Torque Calcs: ☒ Start Fresh ☐ Accumulate

Coef Fluctuation:

Areas of Camshaft Torque Pulses in Order over 1 cycle

in Energy Units of :
lb - in - Radians

Order	Neg Area	Pos Area
1	-12.3	2622.4
2	-814.1	2268
3	-4037.8	0
4	0	0

Average Torque: 14.06 lb-in

Calculate Copy Print Form Done Cancel

PROBLEM 15-4

Statement: Design a three-dwell cam to meet the specifications given below. Size a return spring and specify its preload to maintain contact between cam and follower. Calculate and plot the dynamic force and torque. Repeat for a form-closed cam. Compare the dynamic force, torque, and natural frequency for the form-closed design and the force-closed design.

Given:

RISE/FALL	DWELL	FALL	DWELL
$\beta_1 := 40 \cdot \text{deg}$	$\beta_2 := 100 \cdot \text{deg}$	$\beta_3 := 90 \cdot \text{deg}$	$\beta_4 := 20 \cdot \text{deg}$
$h_1 := 2.5 \cdot \text{in}$	$h_2 := 0.0 \cdot \text{in}$	$h_3 := 1.5 \cdot \text{in}$	$h_4 := 0.0 \cdot \text{in}$
$\beta_5 := 30 \cdot \text{deg}$	$\beta_6 := 80 \cdot \text{deg}$		
$h_5 := 1.0 \cdot \text{in}$	$h_6 := 0.0 \cdot \text{in}$		

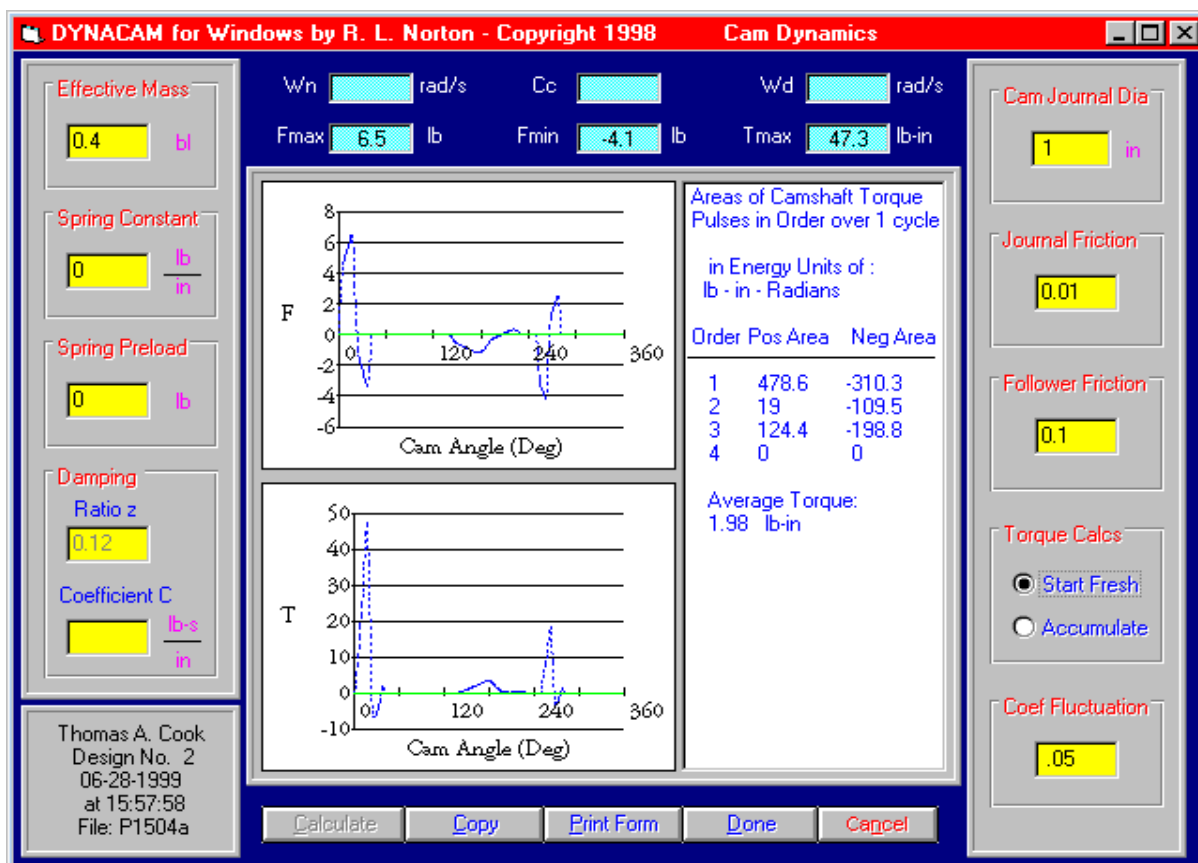
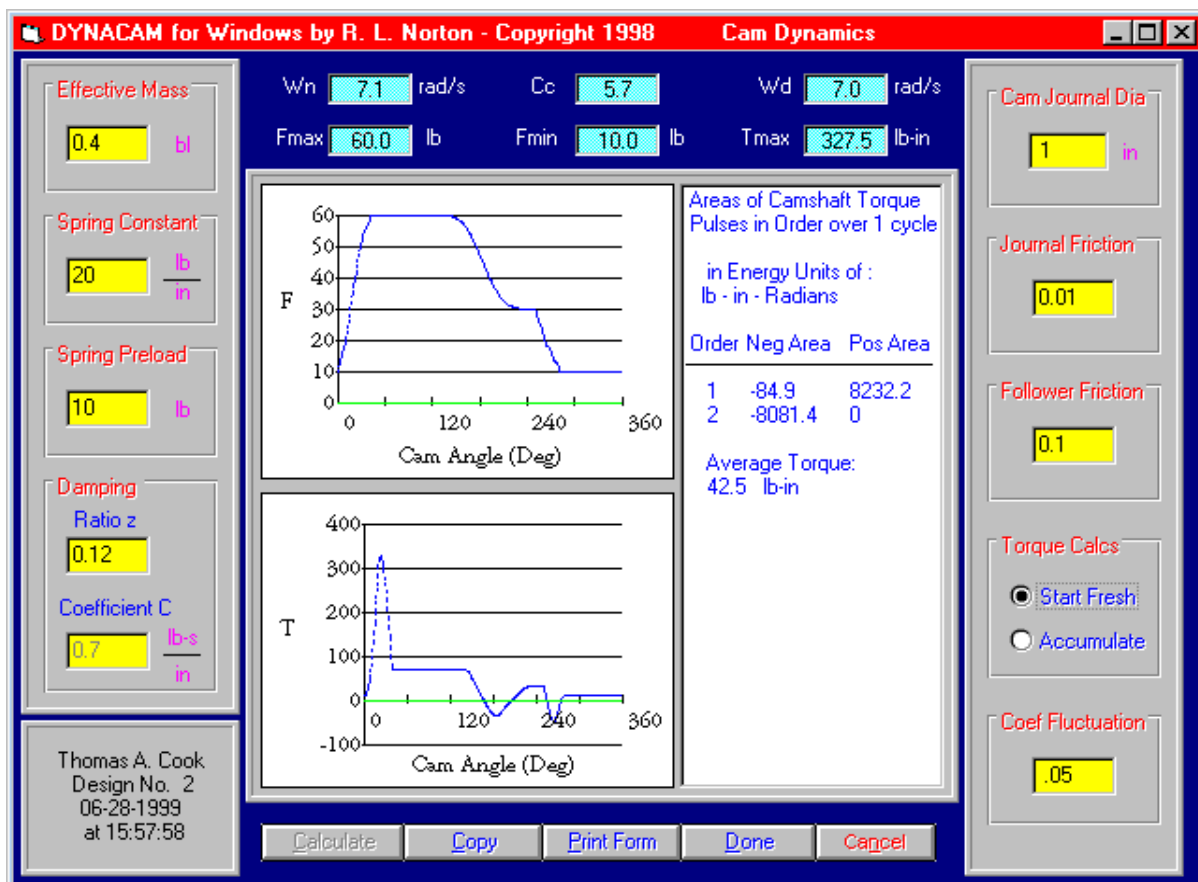
Cycle time: $\tau := 10 \cdot \text{sec}$ Damping ratio: $\zeta := 0.12$ Follower mass: $m_f := 0.4$

Solution: See DYNACAM files P1504a and P1504b.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \qquad \omega = 0.628 \frac{\text{rad}}{\text{sec}} \qquad \omega = 6.000 \text{ rpm}$$

2. Load the noted files into program DYNACAM. Diskfile P1504a contains the force-closed solution and P1504b the form-closed solution. In each case, after opening the file, click on the *Size Cam* button and then click *Calculate* to see the pressure angle and minimum radius of curvature. Then click the *Done* button and you will see the cam drawing. After clicking the *Done* button on that screen, click the *Dynamics* button on the *Home* screen. Click its *Calculate* button to see the fully developed *Dynamics* screen. Once this sequence of commands has been entered, you may see further data by clicking on the *Print* or *Plot* buttons on the home screen. Accept all default values, which have been brought in from the file. It is not necessary to go to the *SVAJ* screen as that data has been brought in from the files.
3. Modified trapezoidal acceleration was used for the rise and both falls. The cam is sized with an 8.875-in prime circle radius and 1.45 in eccentricity, which gives balanced pressure angles of 29.7 deg and a minimum radius of curvature of 3.59 in. The roller radius is 1.00 in.
4. For the force-closed case, a spring constant of 20 lb/in with a preload of 10 lb is chosen to keep the follower force positive and prevent jump. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient are shown on the *Cam Dynamics* screen, which is shown below on the next page. The system natural frequency is about 11.3 times the driving frequency. This ratio could be increased at the expense of larger follower force by increasing the spring constant.
5. For the form-closed case, it is only necessary to define spring constant and preload values of zero. The "b" diskfile does this. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient for this case are shown on the second *Cam Dynamics* screen, which is shown below on the next page.
6. Comparing these two cases shows that in the force-closed case both the peak follower force and driving torque excursions are greater than in the form-closed case. This gives the form-closed cam the advantage in terms of lower stresses in cam and follower and a smaller drive motor. However, this advantage comes at the expense of a more difficult and costly cam to make, since two cam surfaces must be machined instead of one.



PROBLEM 15-5

Statement: Design a four-dwell cam to meet the specifications given below. Size a return spring and specify its preload to maintain contact between cam and follower. Calculate and plot the dynamic force and torque. Repeat for a form-closed cam. Compare the dynamic force, torque, and natural frequency for the form-closed design and the force-closed design.

Given:	RISE/FALL	DWELL	FALL	DWELL
	$\beta_1 := 40 \cdot \text{deg}$	$\beta_2 := 100 \cdot \text{deg}$	$\beta_3 := 90 \cdot \text{deg}$	$\beta_4 := 20 \cdot \text{deg}$
	$h_1 := 2.5 \cdot \text{in}$	$h_2 := 0.0 \cdot \text{in}$	$h_3 := 1.5 \cdot \text{in}$	$h_4 := 0.0 \cdot \text{in}$
	$\beta_5 := 30 \cdot \text{deg}$	$\beta_6 := 40 \cdot \text{deg}$	$\beta_7 := 30 \cdot \text{deg}$	$\beta_8 := 10 \cdot \text{deg}$
	$h_5 := 0.5 \cdot \text{in}$	$h_6 := 0.0 \cdot \text{in}$	$h_7 := 0.5 \cdot \text{in}$	$h_8 := 0.0 \cdot \text{in}$

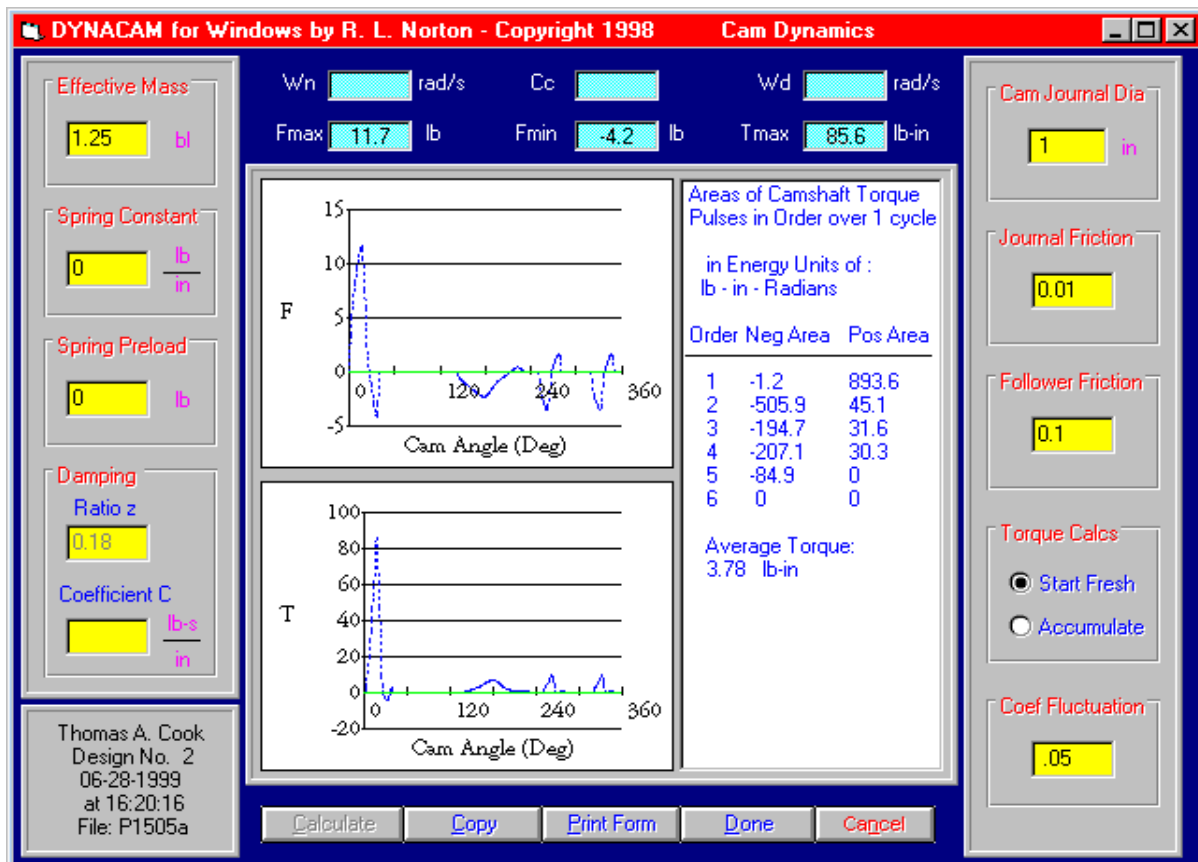
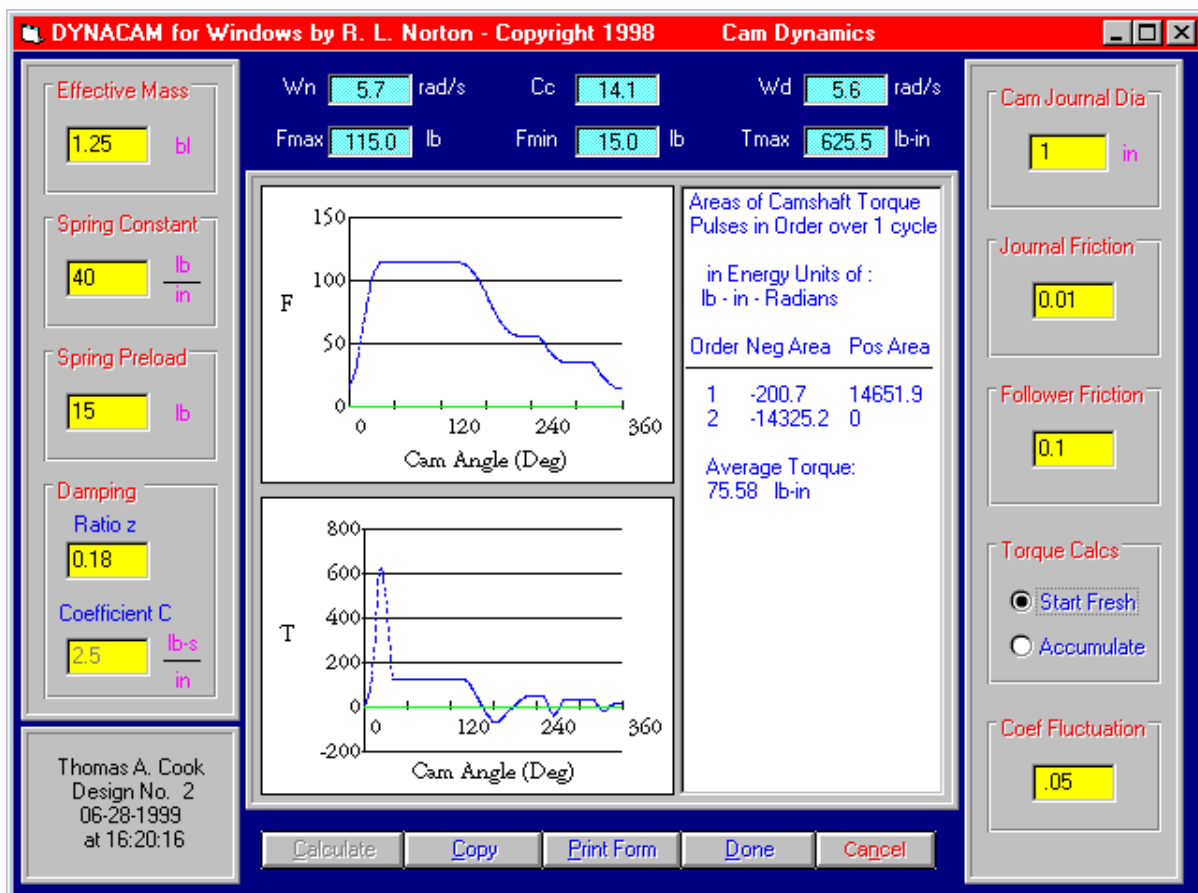
Cycle time: $\tau := 15 \cdot \text{sec}$ Damping ratio: $\zeta := 0.18$ Follower mass: $m_f := 1.25$

Solution: See DYNACAM files P1505a and P1505b.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \qquad \omega = 0.419 \frac{\text{rad}}{\text{sec}} \qquad \omega = 4.000 \text{ rpm}$$

2. Load the noted files into program DYNACAM. Diskfile P1505a contains the force-closed solution and P1505b the form-closed solution. In each case, after opening the file, click on the *Size Cam* button and then click *Calculate* to see the pressure angle and minimum radius of curvature. Then click the *Done* button and you will see the cam drawing. After clicking the *Done* button on that screen, click the *Dynamics* button on the *Home* screen. Click its *Calculate* button to see the fully developed *Dynamics* screen. Once this sequence of commands has been entered, you may see further data by clicking on the *Print* or *Plot* buttons on the home screen. Accept all default values, which have been brought in from the file. It is not necessary to go to the *SVAJ* screen as that data has been brought in from the files.
3. Modified trapezoidal acceleration was used for the rise and all falls. The cam is sized with an 7.500-in prime circle radius and 2.35 in eccentricity, which gives balanced pressure angles of 30 deg and a minimum radius of curvature of 3.01 in. The roller radius is 1.00 in.
4. For the force-closed case, a spring constant of 40 lb/in with a preload of 15 lb is chosen to keep the follower force positive and prevent jump. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient are shown on the *Cam Dynamics* screen, which is shown below on the next page. The system natural frequency is about 13.6 times the driving frequency. This ratio could be increased at the expense of larger follower force by increasing the spring constant.
5. For the form-closed case, it is only necessary to define spring constant and preload values of zero. The "b" diskfile does this. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient for this case are shown on the second *Cam Dynamics* screen, which is shown below on the next page.
6. Comparing these two cases shows that in the force-closed case both the peak follower force and driving torque excursions are greater than in the form-closed case. This gives the form-closed cam the advantage in terms of lower stresses in cam and follower and a smaller drive motor. However, this advantage comes at the expense of a more difficult and costly cam to make, since two cam surfaces must be machined instead of one.



PROBLEM 15-6a

Statement: A mass-spring-damper system as shown in Figure 15-1b has the values shown in Table P15-1. Find the undamped and damped natural frequencies and the value of critical damping for the system in row *a* of the table.

Given: $M := 1.2 \cdot \text{kg}$ $k := 14 \cdot \text{N} \cdot \text{m}^{-1}$ $c := 1.1 \cdot \text{N} \cdot \text{sec} \cdot \text{m}^{-1}$

Solution: See Figure 15-1b, Table P15-1, and Mathcad file P1506a.

1. Calculate the undamped natural frequency using equation 15.1d.

$$\omega_n := \sqrt{\frac{k}{M}} \qquad \omega_n = 3.416 \cdot \frac{\text{rad}}{\text{sec}}$$

2. Calculate the damped natural frequency using equation 15.3c.

$$\omega_d := \sqrt{\frac{k}{M} - \left(\frac{c}{2 \cdot M}\right)^2} \qquad \omega_d = 3.385 \cdot \frac{\text{rad}}{\text{sec}}$$

3. Calculate the critical damping factor using equation 15.2i.

$$c_c := 2 \cdot M \cdot \omega_n \qquad c_c = 8.198 \cdot \frac{\text{N} \cdot \text{sec}}{\text{m}}$$

PROBLEM 15-7

Statement: Figure P15-1 shows a cam-follower system. The dimensions and other data are given below. Find the arm's mass, center of gravity location and mass moment of inertia about both its CG and arm pivot. Create a linear, one-DOF lumped mass model of the dynamic system referenced to the cam follower and calculate the cam-follower force for one revolution. The cam is a pure eccentric with eccentricity and speed given below. Ignore damping.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given:	Arm dimensions:	Cutout dimensions:
	Width (Z-direction): $a := 2 \cdot in$	Width (Z-direction): $a' := 1.5 \cdot in$
	Height (Y-direction): $b := 2.5 \cdot in$	Height (Y-direction): $b' := 2.5 \cdot in$
	Length (X-direction): $c := 28 \cdot in$	Length (X-direction): $c' := 3 \cdot in$
	Distance from arm pivot to spring: $L_s := 19 \cdot in$	
	Distance from arm pivot to roller center: $L_r := 12 \cdot in$	
	Cam eccentricity and speed: $e := 0.5 \cdot in$	$\omega_2 := 500 \cdot rpm$
	Cam radius and thickness: $r_c := 3 \cdot in$	$w_c := 0.75 \cdot in$ (measured from Figure)
	Roller follower dimensions: $r_f := 1 \cdot in$	$w_f := 1.5 \cdot in$
	Spring rate and preload: $k_s := 123 \cdot lbf \cdot in^{-1}$	$F_{50} := 173 \cdot lbf$

Solution: See Figure P15-1 and Mathcad file P1507.

1. The effective mass and spring constant of the arm referenced to the cam follower were determined in Problem 10-16 as:

$$m_{eff} := 0.04652 \cdot blob \quad k_{aef} := 1.0131 \cdot 10^5 \cdot \frac{lbf}{in}$$

2. The spring rate and preload referenced to the cam follower are:

$$k_{eff} := \left[\frac{1}{k_{aef}} + \left(\frac{L_r}{L_s} \right)^2 \cdot \frac{1}{k_s} \right]^{-1} \quad k_{eff} = 307.418 \frac{lbf}{in} \quad F_{pl} := \frac{L_s}{L_r} \cdot F_{50} \quad F_{pl} = 273.917 lbf$$

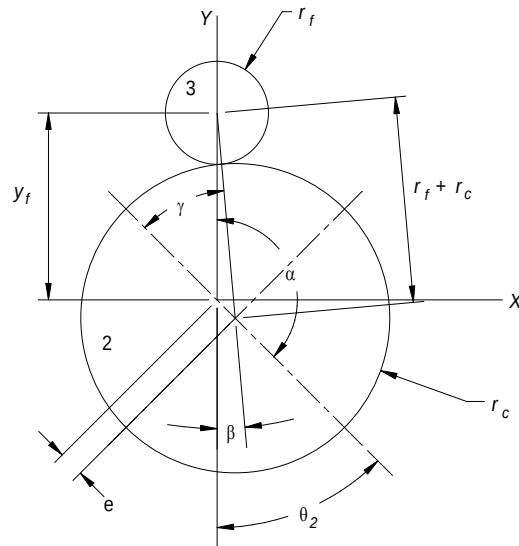
3. Calculate the motion of the roller follower center with respect to the X axis, which goes through the center of rotation of the cam.

The cam and follower are shown at right. The origin of the coordinate frame is at the center of rotation. The cam, link 2, is shown rotated an amount θ_2 from the position at which the follower, link 3, is at its lowest position. A triangle is formed with sides e , $r_f + r_c$, and y_f . This triangle will be used to determine the displacement of the follower, y_f , with respect to the cam rotation angle, θ_2 .

Using the law of sines,

$$\beta(\theta_2) := \text{asin} \left(\frac{e}{r_f + r_c} \cdot \sin(\theta_2) \right)$$

From the relationship among the angles of a triangle,



$$\gamma(\theta_2) := \theta_2 - \beta(\theta_2)$$

And, using the law of cosines,
$$y_f(\theta_2) := \left[\frac{e^2 + (r_f + r_c)^2 - 2 \cdot e \cdot (r_f + r_c) \cdot \cos(\gamma(\theta_2))}{+ e - (r_f + r_c)} \right]^{\frac{1}{2}} \dots$$

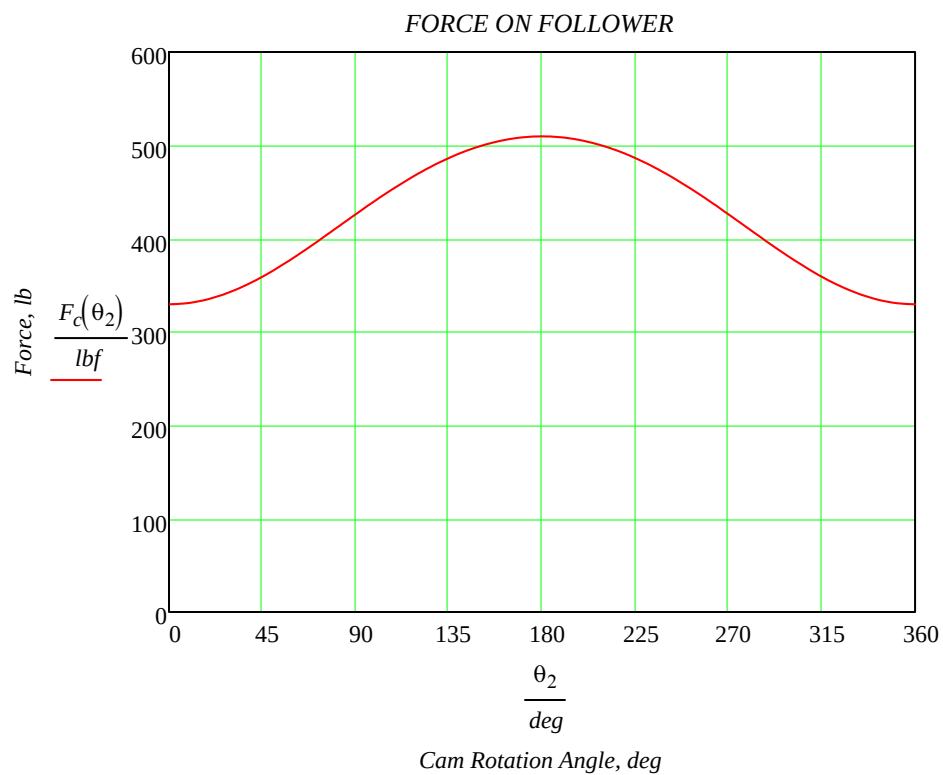
4. Differentiate the displacement function twice to get the velocity and acceleration functions.

$$v_f(\theta_2) := \left(\frac{d}{d\theta_2} y_f(\theta_2) \right) \cdot \omega_2 \qquad a_f(\theta_2) := \left(\frac{d}{d\theta_2} v_f(\theta_2) \right) \cdot \omega_2$$

5. Substitute the expressions for displacement, acceleration and spring preload into equation 15.9 and solve for the force on the follower as a function of cam angle.

$$F_c(\theta_2) := m_{eff} \cdot a_f(\theta_2) + k_{eff} \cdot y_f(\theta_2) + F_{pl}$$

6. Plot the force on the follower for one revolution of the cam. $\theta_2 := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$



PROBLEM 15-8

Statement: Repeat Problem 15-7 for a double-dwell cam to move the roller follower from 0 to 2.5 inches in 60 deg with modified sine acceleration, dwell for 120 deg, fall 2.5 inches in 30 deg with cycloidal motion, and dwell for the remainder. Cam speed is 100 rpm. Choose a suitable spring rate and preload to maintain follower contact. Select a spring from Appendix D. Assume a damping ratio of 0.10.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given:

Arm dimensions:

Width (Z-direction): $a := 2 \cdot in$

Height (Y-direction): $b := 2.5 \cdot in$

Length (X-direction): $c := 28 \cdot in$

Distance from arm pivot to spring:

Distance from arm pivot to roller center:

Cam lift and speed: $h := 2.5 \cdot in$

Roller follower dimensions: $r_f := 1 \cdot in$

Cutout dimensions:

Width (Z-direction): $a' := 1.5 \cdot in$

Height (Y-direction): $b' := 2.5 \cdot in$

Length (X-direction): $c' := 3 \cdot in$

$L_s := 19 \cdot in$

$L_r := 12 \cdot in$

$\omega_2 := 100 \cdot rpm$

$w_f := 1.5 \cdot in$

Solution: See Figure P15-1, Problem 15-7, and DYNACAM file P1508.cam.

1. The effective mass and spring constant of the arm referenced to the cam follower were determined in Problem 10-16 as:

$$m_{eff} := 0.04652 \cdot blob \quad k_{aeff} := 1.0131 \cdot 10^5 \cdot \frac{lbf}{in}$$

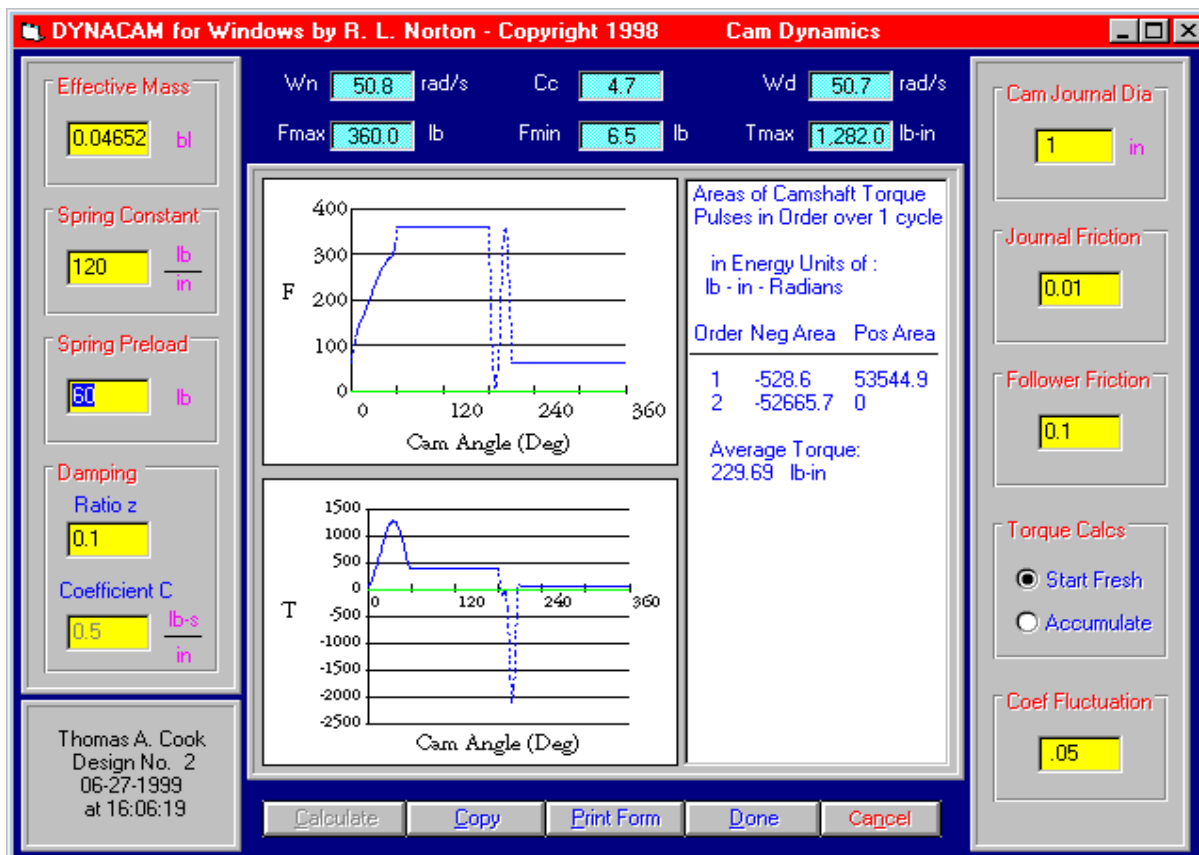
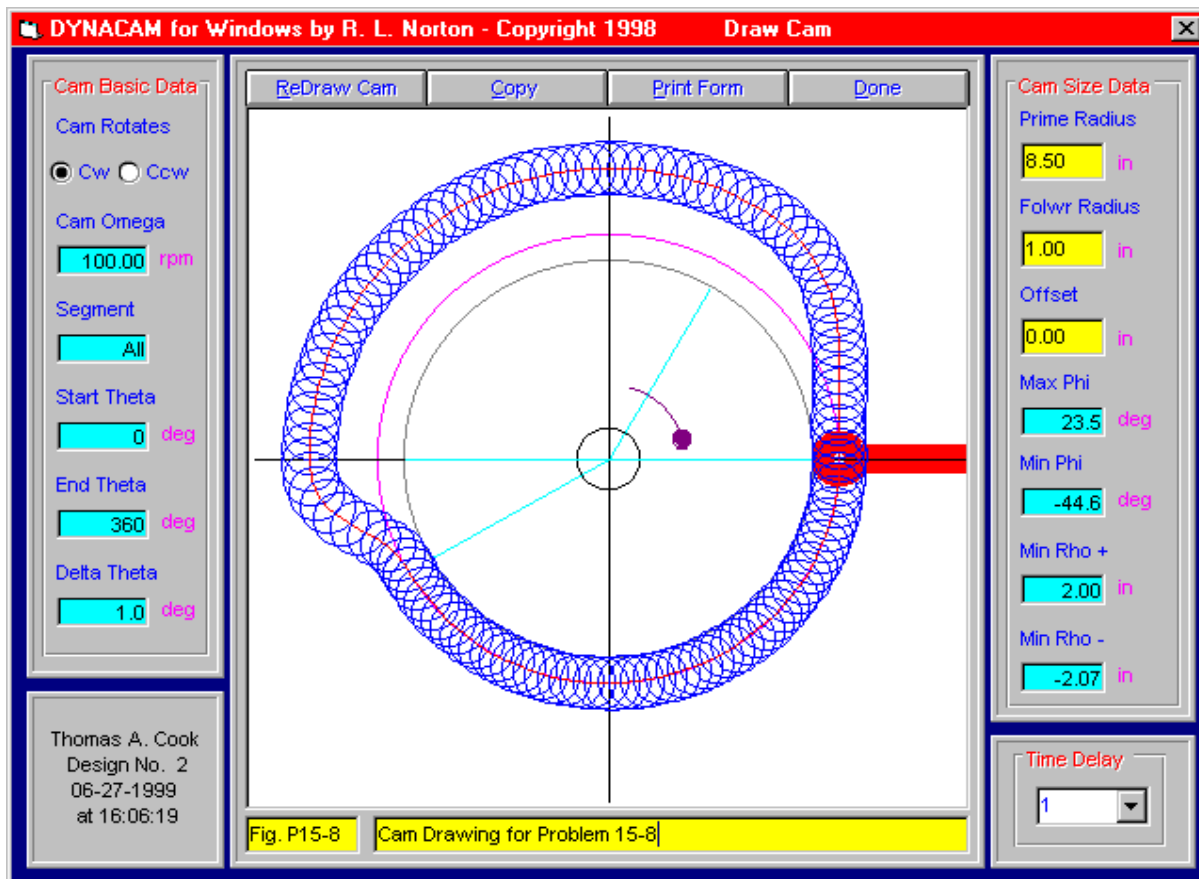
2. There is no extension spring listed in Appendix D that is acceptable. A 2.5-in follower lift is nearly 4 in at the end of the arm where the spring attaches. There is no spring that has a safe maximum load that is 4 times the spring rate. We will, therefore, have to have a custom made spring with the following rate and preload when installed.

$$\text{Spring rate and preload: } k_s := 75.8 \cdot lbf \cdot in^{-1} \quad F_{50} := 37.9 \cdot lbf$$

3. The spring rate and preload referenced to the cam follower are:

$$k_{eff} := \left[\frac{1}{k_{aeff}} + \left(\frac{L_r}{L_s} \right)^2 \cdot \frac{1}{k_s} \right]^{-1} \quad k_{eff} = 190 \cdot \frac{lbf}{in} \quad F_{pl} := \frac{L_s}{L_r} \cdot F_{50} \quad F_{pl} = 60 \cdot lbf$$

4. Enter the given cam data into the program DYNACAM and create a cam with acceptable pressure angles and radius of curvature. A cam drawing from DYNACAM that meets the requirements is shown on the next page. Find a suitable spring rate and preload to keep the minimum cam force positive and minimize the maximum cam force. One solution is shown in the Cam Dynamics screen shown below the cam drawing.



PROBLEM 15-9

Statement: Repeat Problem 15-7 for a double-dwell cam to move the roller follower from 0 to 1.5 inches in 45 deg with 3-4-5 polynomial motion, dwell for 150 deg, fall 1.5 inches in 90 deg with 4-5-6 polynomial motion, and dwell for the remainder. Cam speed is 250 rpm. Choose a suitable spring rate and preload to maintain follower contact. Select a spring from Appendix D. Assume a damping ratio of 0.15.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given:

Arm dimensions:

Width (Z-direction): $a := 2 \cdot in$

Height (Y-direction): $b := 2.5 \cdot in$

Length (X-direction): $c := 28 \cdot in$

Distance from arm pivot to spring:

Distance from arm pivot to roller center:

Cam lift and speed: $h := 2.5 \cdot in$

Roller follower dimensions: $r_f := 1 \cdot in$

Cutout dimensions:

Width (Z-direction): $a' := 1.5 \cdot in$

Height (Y-direction): $b' := 2.5 \cdot in$

Length (X-direction): $c' := 3 \cdot in$

$L_s := 19 \cdot in$

$L_r := 12 \cdot in$

$\omega_2 := 100 \cdot rpm$

$w_f := 1.5 \cdot in$

Solution: See Figure P15-1, Problem 15-7, and DYNACAM file P1509.cam.

1. The effective mass and spring constant of the arm referenced to the cam follower were determined in Problem 10-16 as:

$$m_{eff} := 0.04652 \cdot blob \quad k_{aeff} := 1.0131 \cdot 10^5 \cdot \frac{lbf}{in}$$

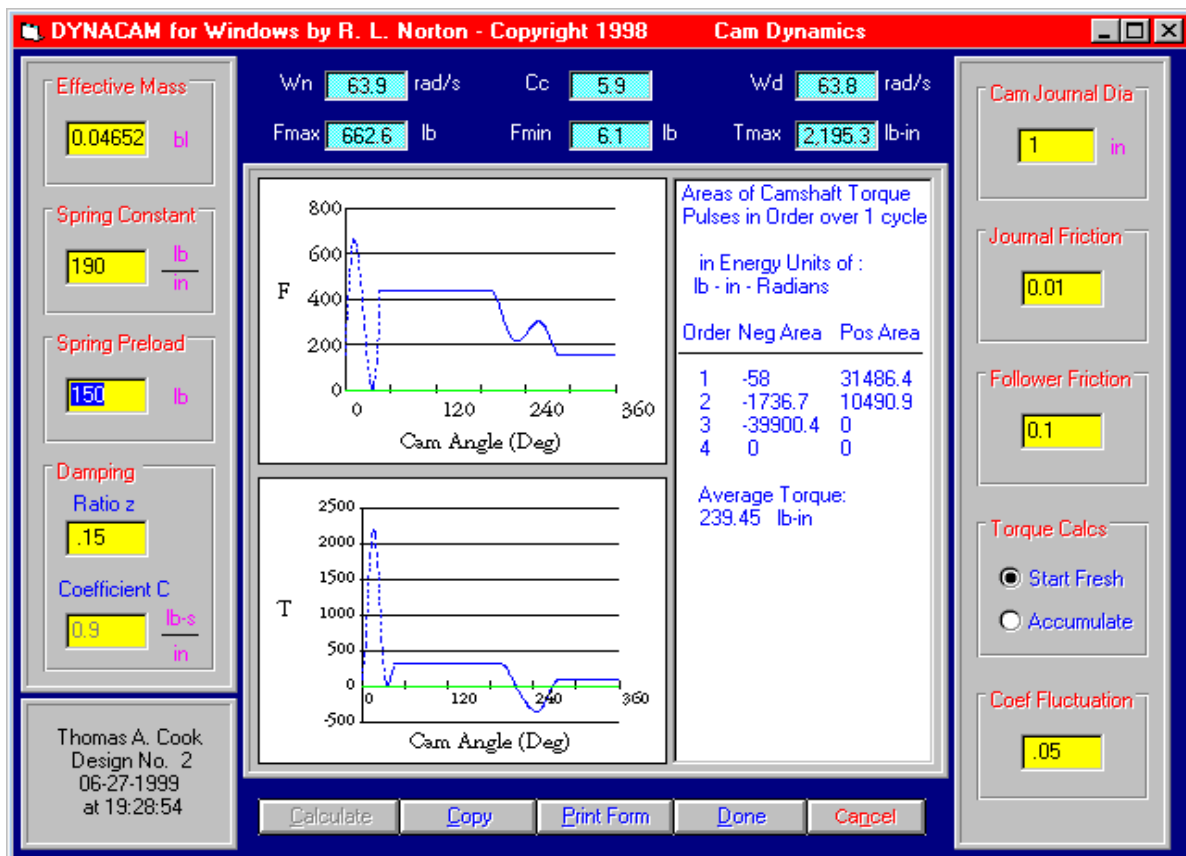
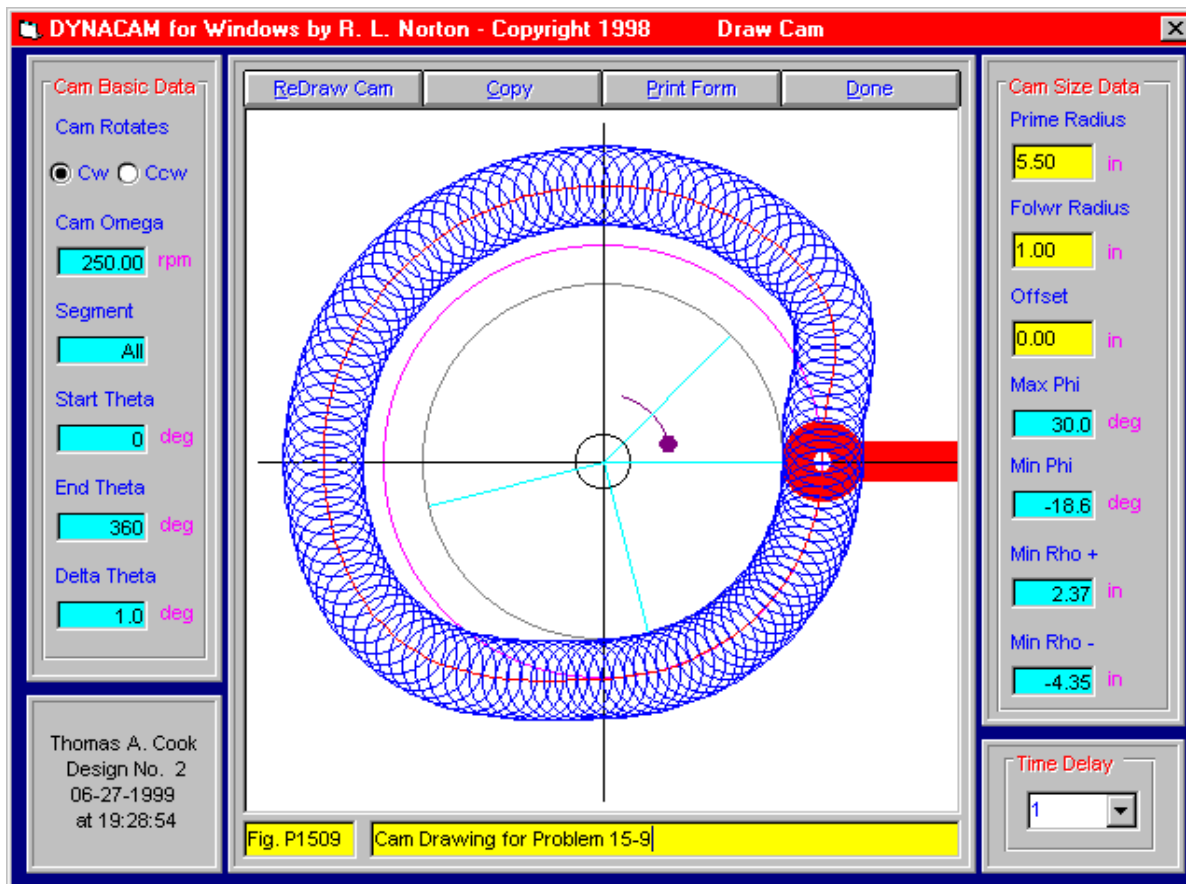
2. There is no extension spring listed in Appendix D that is acceptable. A 1.5-in follower lift is nearly 2.5 in at the end of the arm where the spring attaches. There is no spring that has a safe maximum load that is 2.5 times the spring rate. We will, therefore, have to have a custom made spring with the following rate and preload when installed.

$$\text{Spring rate and preload: } k_s := 120 \cdot lbf \cdot in^{-1} \quad F_{50} := 94.7 \cdot lbf$$

3. The spring rate and preload referenced to the cam follower are:

$$k_{eff} := \left[\frac{1}{k_{aeff}} + \left(\frac{L_r}{L_s} \right)^2 \cdot \frac{1}{k_s} \right]^{-1} \quad k_{eff} = 300 \frac{lbf}{in} \quad F_{pl} := \frac{L_s}{L_r} \cdot F_{50} \quad F_{pl} = 150 \cdot lbf$$

4. Enter the given cam data into the program DYNACAM and create a cam with acceptable pressure angles and radius of curvature. A cam drawing from DYNACAM that meets the requirements is shown on the next page. Find a suitable spring rate and preload to keep the minimum cam force positive and minimize the maximum cam force. One solution is shown in the Cam Dynamics screen shown below the cam drawing.



PROBLEM 15-10

Statement: Repeat Problem 15-7 for a single-dwell cam to move the roller follower from 0 to 2 inches in 60 deg, fall 2 inches in 90 deg, and dwell for the remainder. Use a seventh degree polynomial. Cam speed is 250 rpm. Choose a suitable spring rate and preload to maintain follower contact. Select a spring from Appendix D. Assume a damping ratio of 0.15.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given:	Arm dimensions:	Cutout dimensions:
	Width (Z-direction): $a := 2 \cdot in$	Width (Z-direction): $a' := 1.5 \cdot in$
	Height (Y-direction): $b := 2.5 \cdot in$	Height (Y-direction): $b' := 2.5 \cdot in$
	Length (X-direction): $c := 28 \cdot in$	Length (X-direction): $c' := 3 \cdot in$
	Distance from arm pivot to spring: $L_s := 19 \cdot in$	
	Distance from arm pivot to roller center: $L_r := 12 \cdot in$	
	Cam lift and speed: $h := 2.5 \cdot in$	$\omega_2 := 100 \cdot rpm$
	Roller follower dimensions: $r_f := 1 \cdot in$	$w_f := 1.5 \cdot in$

Solution: See Figure P15-1, Problem 15-7, and DYNACAM file P1510.cam.

1. The effective mass and spring constant of the arm referenced to the cam follower were determined in Problem 10-16 as:

$$m_{eff} := 0.04652 \cdot blob \quad k_{aeff} := 1.0131 \cdot 10^5 \cdot \frac{lbf}{in}$$

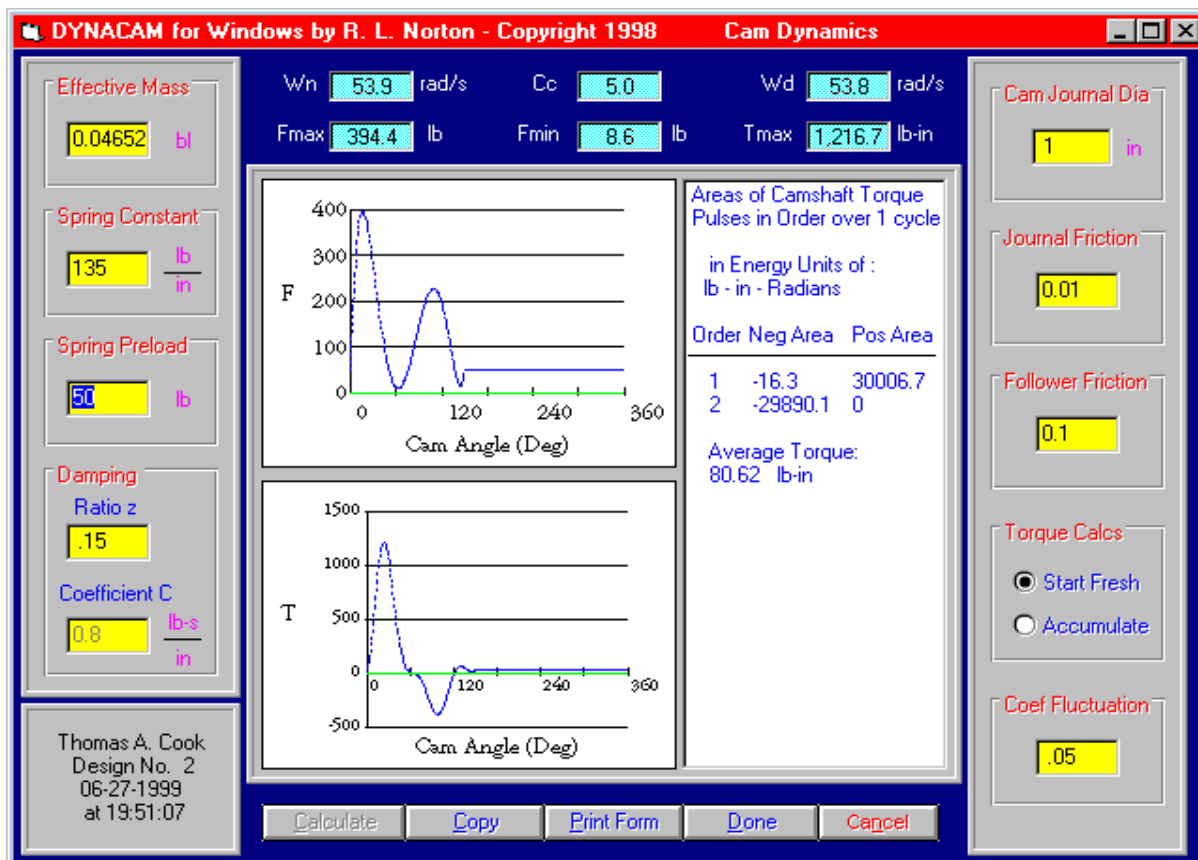
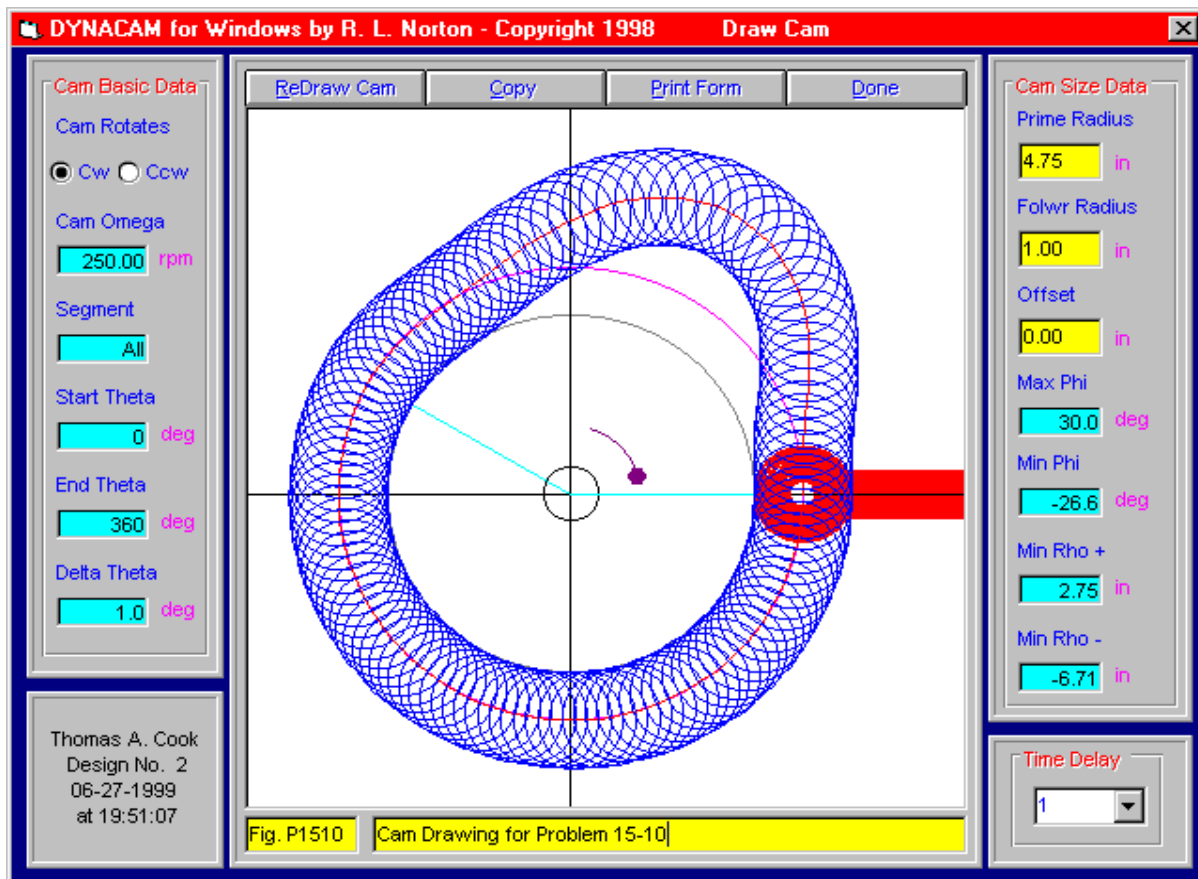
2. There is no extension spring listed in Appendix D that is acceptable. A 2-in follower lift is over 3 in at the end of the arm where the spring attaches. There is no spring that has a safe maximum load that is 3 times the spring rate. We will, therefore, have to have a custom made spring with the following rate and preload when installed.

$$\text{Spring rate and preload: } k_s := 85.3 \cdot lbf \cdot in^{-1} \quad F_{50} := 31.6 \cdot lbf$$

3. The spring rate and preload referenced to the cam follower are:

$$k_{eff} := \left[\frac{1}{k_{aeff}} + \left(\frac{L_r}{L_s} \right)^2 \cdot \frac{1}{k_s} \right]^{-1} \quad k_{eff} = 213 \frac{lbf}{in} \quad F_{pl} := \frac{L_s}{L_r} \cdot F_{50} \quad F_{pl} = 50 \cdot lbf$$

4. Enter the given cam data into the program DYNACAM and create a cam with acceptable pressure angles and radius of curvature. A cam drawing from DYNACAM that meets the requirements is shown on the next page. Find a suitable spring rate and preload to keep the minimum cam force positive and minimize the maximum cam force. One solution is shown in the Cam Dynamics screen shown below the cam drawing.



PROBLEM 15-11

Statement: Repeat Problem 15-7 for a double-dwell cam to move the roller follower from 0 to 2 inches in 60 deg with cycloidal motion, dwell for 150 deg, fall 2 inches in 90 deg with modified sine acceleration, and dwell for the remainder. Cam speed is 200 rpm. Choose a suitable spring rate and preload to maintain follower contact. Select a spring from Appendix D. Assume a damping ratio of 0.15.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given:

Arm dimensions:	Cutout dimensions:
Width (Z-direction): $a := 2 \cdot in$	Width (Z-direction): $a' := 1.5 \cdot in$
Height (Y-direction): $b := 2.5 \cdot in$	Height (Y-direction): $b' := 2.5 \cdot in$
Length (X-direction): $c := 28 \cdot in$	Length (X-direction): $c' := 3 \cdot in$
Distance from arm pivot to spring:	$L_s := 19 \cdot in$
Distance from arm pivot to roller center:	$L_r := 12 \cdot in$
Cam lift and speed: $h := 2.5 \cdot in$	$\omega_2 := 100 \cdot rpm$
Roller follower dimensions: $r_f := 1 \cdot in$	$w_f := 1.5 \cdot in$

Solution: See Figure P15-1, Problem 15-7, and DYNACAM file P1511.cam.

1. The effective mass and spring constant of the arm referenced to the cam follower were determined in Problem 10-16 as:

$$m_{eff} := 0.04652 \cdot blob \quad k_{aeff} := 1.0131 \cdot 10^5 \cdot \frac{lbf}{in}$$

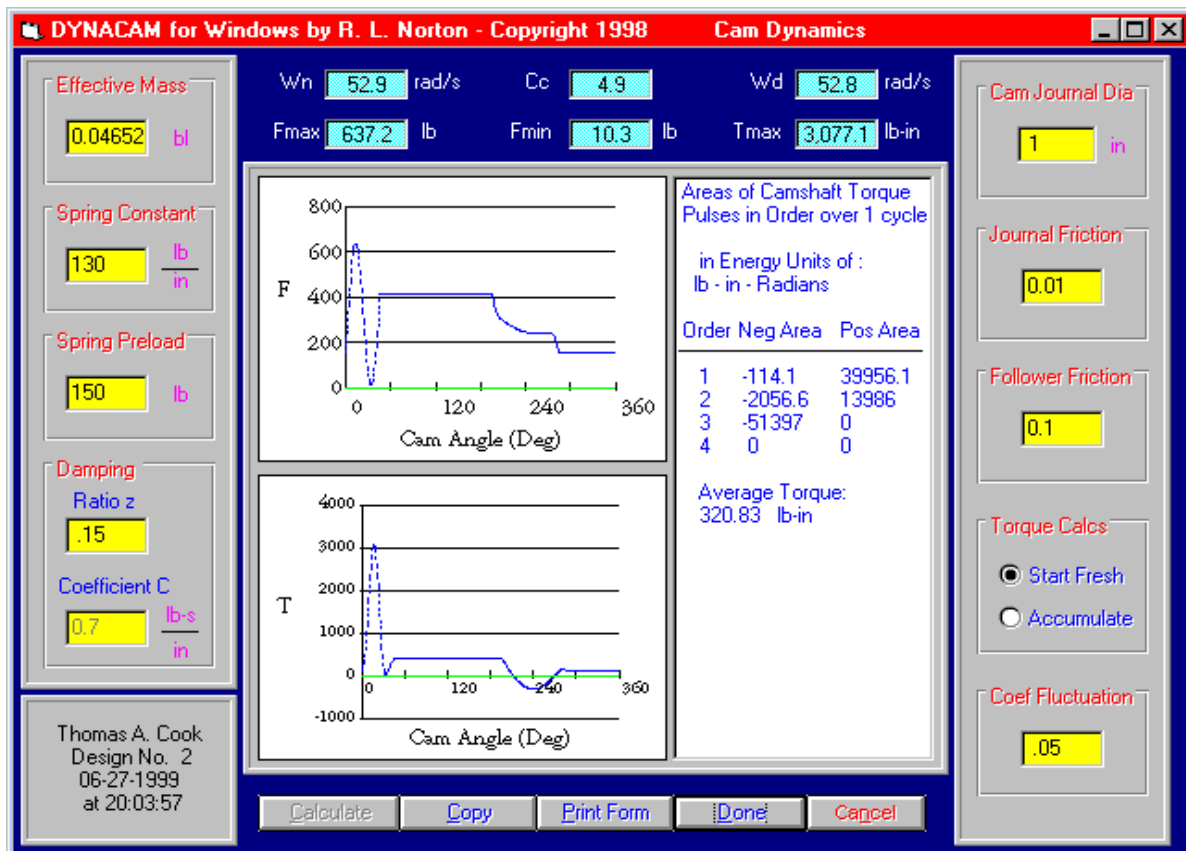
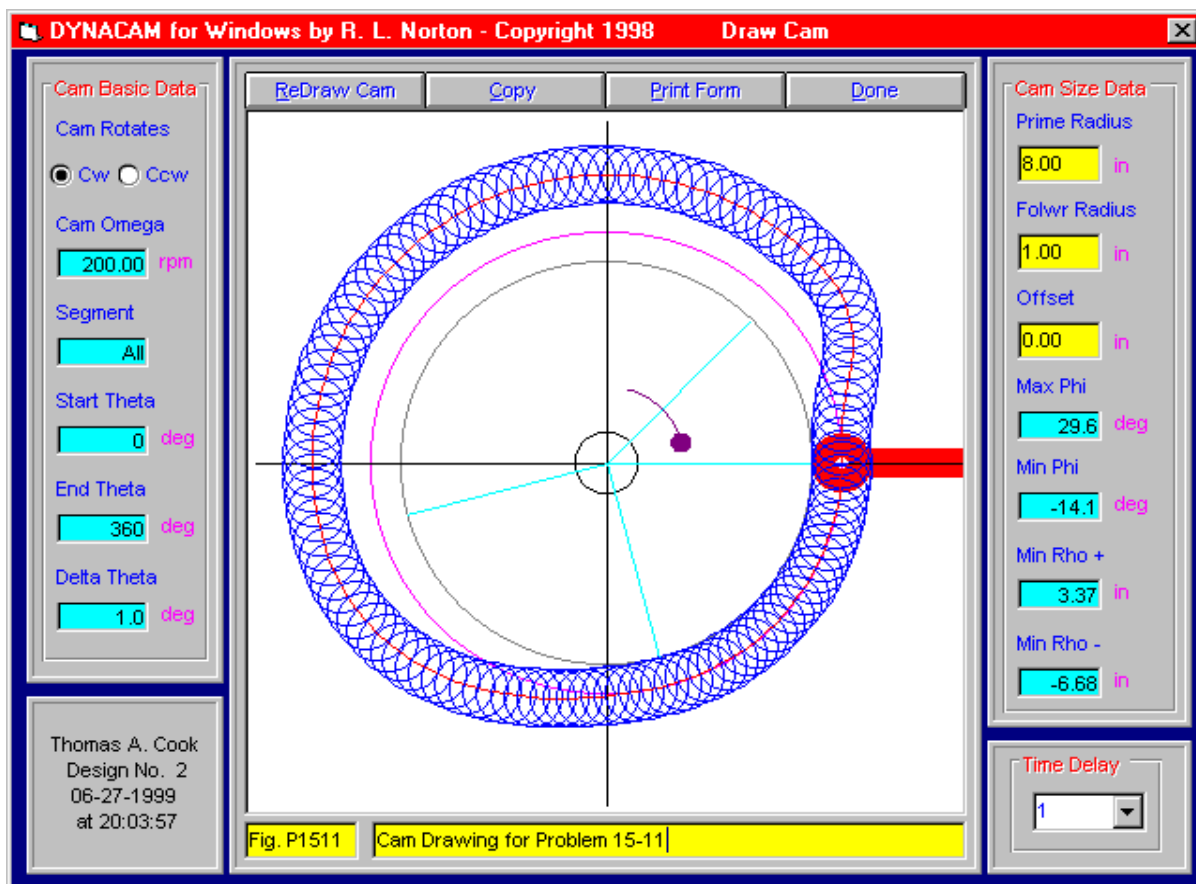
2. There is no extension spring listed in Appendix D that is acceptable. A 2-in follower lift is over 3 in at the end of the arm where the spring attaches. There is no spring that has a safe maximum load that is 3 times the spring rate. We will, therefore, have to have a custom made spring with the following rate and preload when installed.

$$\text{Spring rate and preload: } k_s := 82.1 \cdot lbf \cdot in^{-1} \quad F_{50} := 94.7 \cdot lbf$$

3. The spring rate and preload referenced to the cam follower are:

$$k_{eff} := \left[\frac{1}{k_{aeff}} + \left(\frac{L_r}{L_s} \right)^2 \cdot \frac{1}{k_s} \right]^{-1} \quad k_{eff} = 205 \frac{lbf}{in} \quad F_{pl} := \frac{L_s}{L_r} \cdot F_{50} \quad F_{pl} = 150 \cdot lbf$$

4. Enter the given cam data into the program DYNACAM and create a cam with acceptable pressure angles and radius of curvature. A cam drawing from DYNACAM that meets the requirements is shown on the next page. Find a suitable spring rate and preload to keep the minimum cam force positive and minimize the maximum cam force. One solution is shown in the Cam Dynamics screen shown below the cam drawing.



PROBLEM 15-12

Statement: The cam in Figure P15-2 is a pure eccentric with eccentricity = 20 mm and turns at 200 rpm. The mass of the follower is 1 kg. The spring has a rate of 10 N/m and a preload of 0.2 N. Find the follower force over one revolution. Assume a damping ratio of 0.10. If there is follower jump, respecify the spring rate and preload to eliminate it.

Given: Cam eccentricity and speed: $a := 20 \cdot \text{mm}$ $\omega := 200 \cdot \text{rpm}$
 Follower mass: $m_f := 1 \cdot \text{kg}$ Damping ratio: $\zeta := 0.10$
 Spring: $k := 10 \cdot \text{N} \cdot \text{m}^{-1}$ $F_{pl} := 0.2 \cdot \text{N}$

Solution: See Figure P15-2 and Mathcad file P1512.

- Using the equation given in the figure, write functions for the displacement, velocity, and acceleration of the follower. Note that the displacement function is written such that at it is zero at $t = 0$.

$$\text{Displacement: } s(\theta) := a \cdot (1 - \cos(\theta))$$

$$\text{Velocity: } v(\theta) := a \cdot \omega \cdot \sin(\theta)$$

$$\text{Acceleration: } a(\theta) := a \cdot \omega^2 \cdot \cos(\theta)$$

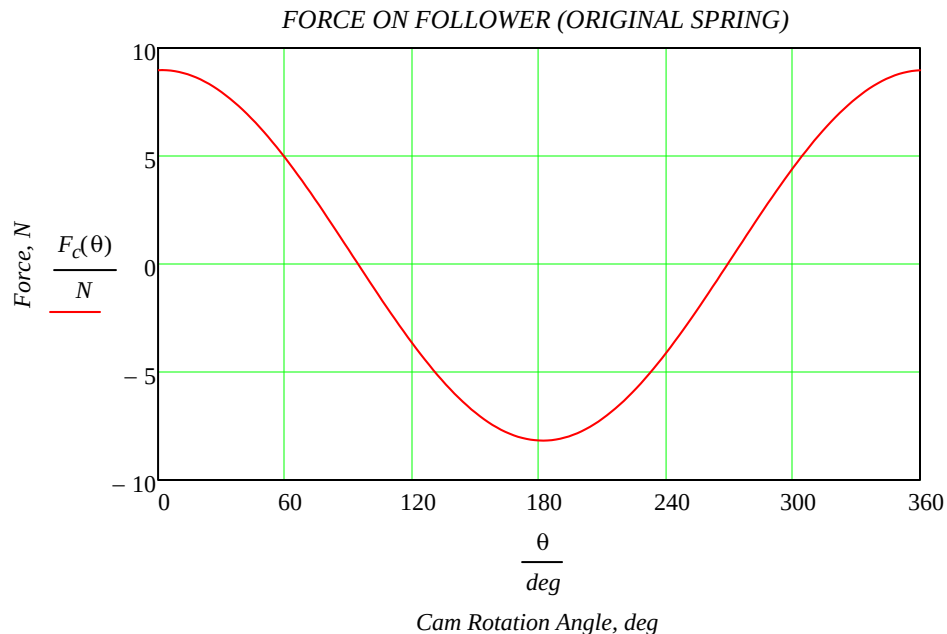
- Calculate the damping coefficient using equations 15.2i and 15.3a.

$$c := 2 \cdot \zeta \cdot \sqrt{m_f \cdot k} \quad c = 0.632 \frac{\text{N} \cdot \text{sec}}{\text{m}}$$

- Substitute the expressions for displacement, velocity, acceleration, and spring preload into equation 15.9 and solve for the force on the follower as a function of cam angle.

$$F_c(\theta) := m_f \cdot a(\theta) + c \cdot v(\theta) + k \cdot s(\theta) + F_{pl}$$

- Plot the force on the follower for one revolution of the cam. $\theta := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$

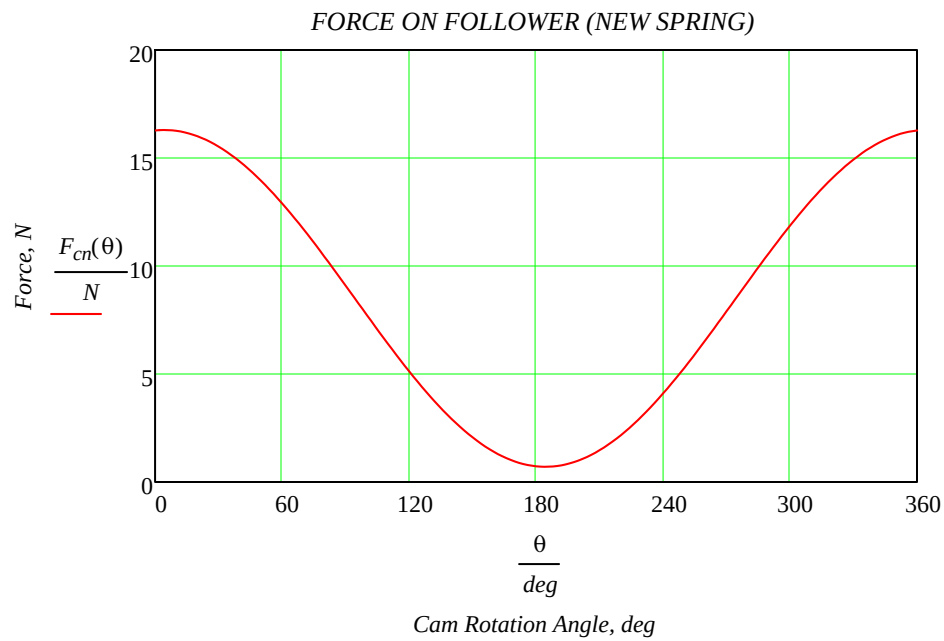


5. The cam force becomes negative, which indicates that the follower will jump or lose contact with the cam. New values of spring rate and/or preload will be tried iteratively to make the force always positive.

Let the new spring parameters be: $k := 50 \cdot N \cdot m^{-1}$ $F_{pl} := 7.5 \cdot N$

$$c := 2 \cdot \zeta \cdot \sqrt{m_f \cdot k} \quad c = 1.414 \frac{N \cdot sec}{m}$$

$$F_{cn}(\theta) := m_f \cdot a(\theta) + c \cdot v(\theta) + k \cdot s(\theta) + F_{pl}$$



PROBLEM 15-13

Statement: The cam in Figure P15-2 has a symmetric double harmonic rise and fall of 20 mm and turns at 200 rpm. The mass of the follower is 1 kg. The spring has a rate of 10 N/m and a preload of 0.2 N. Find the follower force over one revolution. Assume a damping ratio of 0.10. If there is follower jump, respecify the spring rate and preload to eliminate it.

Given: Cam rise/fall and speed: $h := 20 \cdot \text{mm}$ $\omega := 200 \cdot \text{rpm}$
 Follower mass: $m_f := 1 \cdot \text{kg}$ Damping ratio: $\zeta := 0.10$
 Spring: $k := 10 \cdot \text{N} \cdot \text{m}^{-1}$ $F_{pl} := 0.2 \cdot \text{N}$

Solution: See Figure P15-2 and Mathcad file P1513.

- Using equations 8.25, write functions for the displacement, velocity, and acceleration of the follower. The value of β for both the rise and the fall is $\beta := 180 \cdot \text{deg}$. To write the global equations, define a range function that has a value of unity between the limits of a and b and is zero elsewhere.

$$R(\theta, a, b) := \text{if}[(\theta > a) \cdot (\theta \leq b), 1, 0]$$

$$\begin{aligned} \text{Displacement: } s(\theta) := & R(\theta, 0, \beta) \cdot \frac{h}{2} \cdot \left[\left(1 - \cos\left(\pi \cdot \frac{\theta}{\beta}\right) \right) - \frac{1}{4} \cdot \left(1 - \cos\left(2 \cdot \pi \cdot \frac{\theta}{\beta}\right) \right) \right] \dots \\ & + R(\theta, \beta, 2 \cdot \pi) \cdot \left[\frac{h}{2} \cdot \left[1 + \cos\left(\pi \cdot \frac{\theta - \beta}{\beta}\right) \right] - \frac{1}{4} \cdot \left(1 - \cos\left(2 \cdot \pi \cdot \frac{\theta - \beta}{\beta}\right) \right) \right] \end{aligned}$$

$$\begin{aligned} \text{Velocity: } v(\theta) := & \omega \cdot \frac{\pi}{\beta} \cdot \frac{h}{2} \cdot \left[R(\theta, 0, \beta) \cdot \left(\sin\left(\pi \cdot \frac{\theta}{\beta}\right) - \frac{1}{2} \cdot \sin\left(2 \cdot \pi \cdot \frac{\theta}{\beta}\right) \right) \dots \right. \\ & \left. + -R(\theta, \beta, 2 \cdot \pi) \cdot \left(\sin\left(\pi \cdot \frac{\theta - \beta}{\beta}\right) + \frac{1}{2} \cdot \sin\left(2 \cdot \pi \cdot \frac{\theta - \beta}{\beta}\right) \right) \right] \end{aligned}$$

$$\begin{aligned} \text{Acceleration: } a(\theta) := & \omega^2 \cdot \frac{\pi^2}{\beta^2} \cdot \frac{h}{2} \cdot \left[R(\theta, 0, \beta) \cdot \left(\cos\left(\pi \cdot \frac{\theta}{\beta}\right) - \cos\left(2 \cdot \pi \cdot \frac{\theta}{\beta}\right) \right) \dots \right. \\ & \left. + -R(\theta, \beta, 2 \cdot \pi) \cdot \left(\cos\left(\pi \cdot \frac{\theta - \beta}{\beta}\right) + \cos\left(2 \cdot \pi \cdot \frac{\theta - \beta}{\beta}\right) \right) \right] \end{aligned}$$

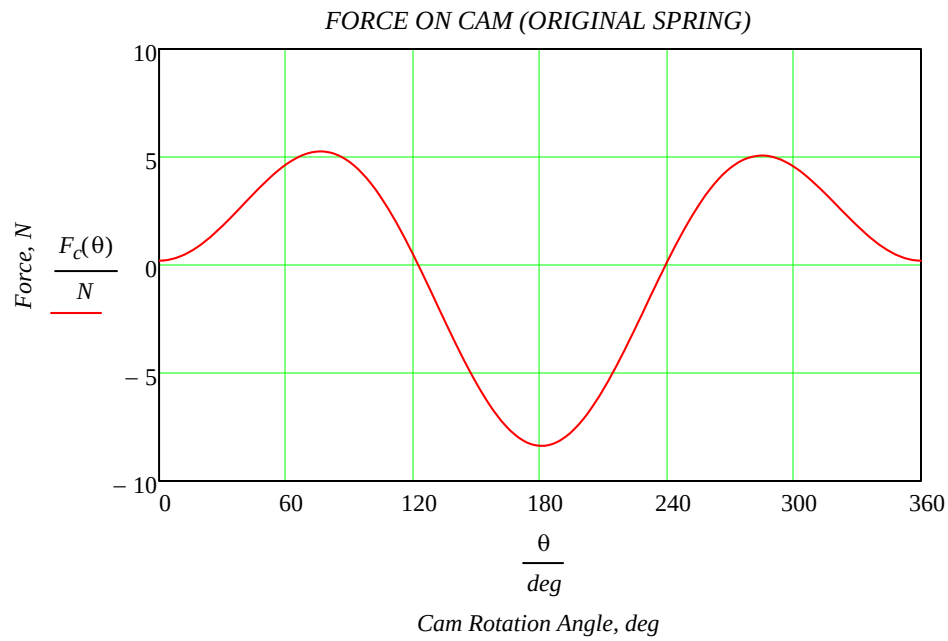
- Calculate the damping coefficient using equations 15.2i and 15.3a.

$$c := 2 \cdot \zeta \cdot \sqrt{m_f \cdot k} \qquad c = 0.632 \frac{\text{N} \cdot \text{sec}}{\text{m}}$$

- Substitute the expressions for displacement, velocity, acceleration, and spring preload into equation 15.9 and solve for the force on the cam as a function of cam angle.

$$F_c(\theta) := m_f \cdot a(\theta) + c \cdot v(\theta) + k \cdot s(\theta) + F_{pl}$$

- Plot the force on the cam for one revolution of the cam. $\theta := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$

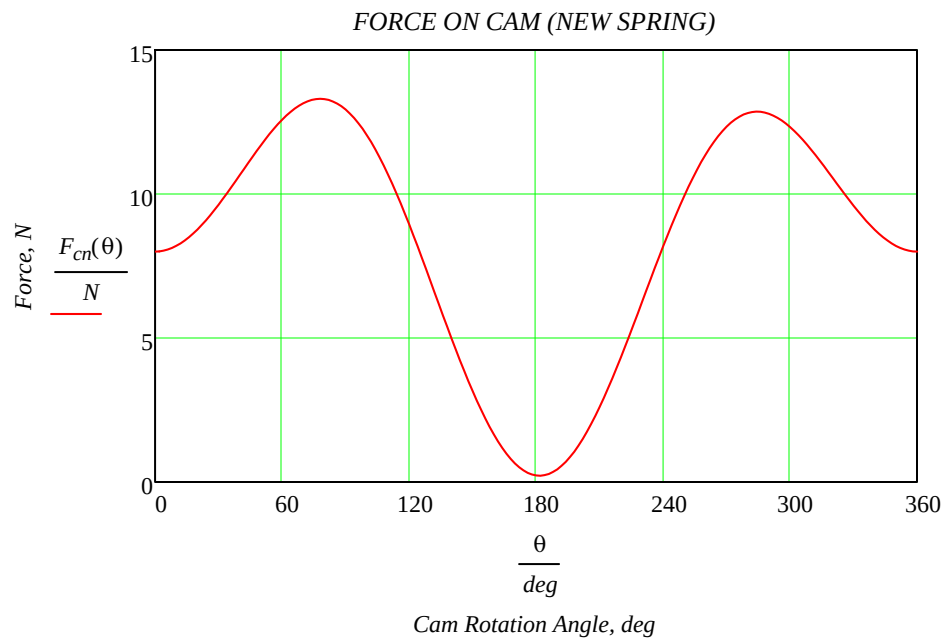


5. The cam force becomes negative, which indicates that the follower will jump or lose contact with the cam. New values of spring rate and/or preload will be tried iteratively to make the force always positive.

Let the new spring parameters be: $k := 50 \cdot \text{N} \cdot \text{m}^{-1}$ $F_{pl} := 8 \cdot \text{N}$

$c := 2 \cdot \zeta \cdot \sqrt{m_f \cdot k}$ $c = 1.414 \frac{\text{N} \cdot \text{sec}}{\text{m}}$

$$F_{cn}(\theta) := m_f \cdot a(\theta) + c \cdot v(\theta) + k \cdot s(\theta) + F_{pl}$$



PROBLEM 15-14

Statement: The cam in Figure P15-2 has a symmetric 3-4-5-6 polynomial rise and fall of 20 mm and turns at 200 rpm. The mass of the follower is 1 kg. The spring has a rate of 10 N/m and a preload of 0.2 N. Find the follower force over one revolution. Assume a damping ratio of 0.10. If there is follower jump, respecify the spring rate and preload to eliminate it.

Given: Cam rise/fall and speed: $h := 20 \cdot \text{mm}$ $\omega := 200 \cdot \text{rpm}$
 Follower mass: $m_f := 1 \cdot \text{kg}$ Damping ratio: $\zeta := 0.10$
 Spring: $k := 10 \cdot \text{N} \cdot \text{m}^{-1}$ $F_{pl} := 0.2 \cdot \text{N}$

Solution: See Figure P15-2 and Mathcad file P1514.

- Using equations 8.26, write functions for the displacement, velocity, and acceleration of the follower. The value of β for the total rise and fall is $\beta := 360 \cdot \text{deg}$.

$$\text{Displacement: } s(\theta) := h \cdot \left[64 \cdot \left(\frac{\theta}{\beta} \right)^3 - 192 \cdot \left(\frac{\theta}{\beta} \right)^4 + 192 \cdot \left(\frac{\theta}{\beta} \right)^5 - 64 \cdot \left(\frac{\theta}{\beta} \right)^6 \right]$$

$$\text{Velocity: } v(\theta) := \omega \cdot \frac{h}{\beta} \cdot \left[192 \cdot \left(\frac{\theta}{\beta} \right)^2 - 768 \cdot \left(\frac{\theta}{\beta} \right)^3 + 960 \cdot \left(\frac{\theta}{\beta} \right)^4 - 384 \cdot \left(\frac{\theta}{\beta} \right)^5 \right]$$

$$\text{Acceleration: } a(\theta) := \omega^2 \cdot \frac{h}{\beta^2} \cdot \left[384 \cdot \left(\frac{\theta}{\beta} \right) - 2304 \cdot \left(\frac{\theta}{\beta} \right)^2 + 3840 \cdot \left(\frac{\theta}{\beta} \right)^3 - 1920 \cdot \left(\frac{\theta}{\beta} \right)^4 \right]$$

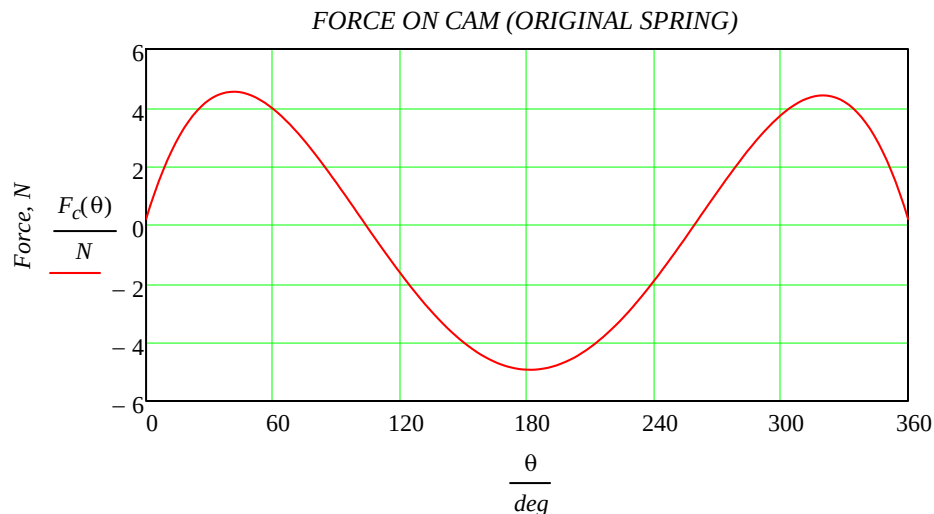
- Calculate the damping coefficient using equations 15.2i and 15.3a.

$$c := 2 \cdot \zeta \cdot \sqrt{m_f \cdot k} \quad c = 0.632 \frac{\text{N} \cdot \text{sec}}{\text{m}}$$

- Substitute the expressions for displacement, velocity, acceleration, and spring preload into equation 15.9 and solve for the force on the cam as a function of cam angle.

$$F_c(\theta) := m_f \cdot a(\theta) + c \cdot v(\theta) + k \cdot s(\theta) + F_{pl}$$

- Plot the force on the cam for one revolution of the cam. $\theta := 0 \cdot \text{deg}, 2 \cdot \text{deg} \dots 360 \cdot \text{deg}$

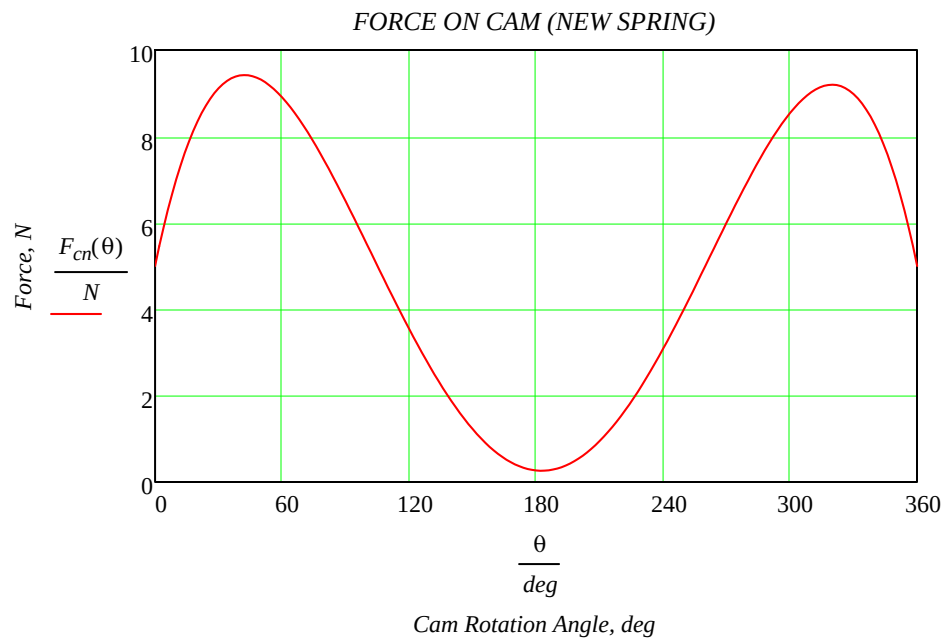


5. The cam force becomes negative, which indicates that the follower will jump or lose contact with the cam. New values of spring rate and/or preload will be tried iteratively to make the force always positive.

Let the new spring parameters be: $k := 30 \cdot N \cdot m^{-1}$ $F_{pl} := 5 \cdot N$

$$c := 2 \cdot \zeta \cdot \sqrt{m_f \cdot k} \quad c = 1.095 \frac{N \cdot sec}{m}$$

$$F_{cn}(\theta) := m_f \cdot a(\theta) + c \cdot v(\theta) + k \cdot s(\theta) + F_{pl}$$



PROBLEM 15-15

Statement: Design a double-dwell cam to meet the specifications given below. Size a return spring, selected from Appendix D, and specify its preload to maintain contact between cam and follower. Calculate and plot the dynamic force and torque. Repeat for a form-closed cam. Compare the dynamic force, torque, and natural frequency for the form-closed design and the force-closed design.

Given:

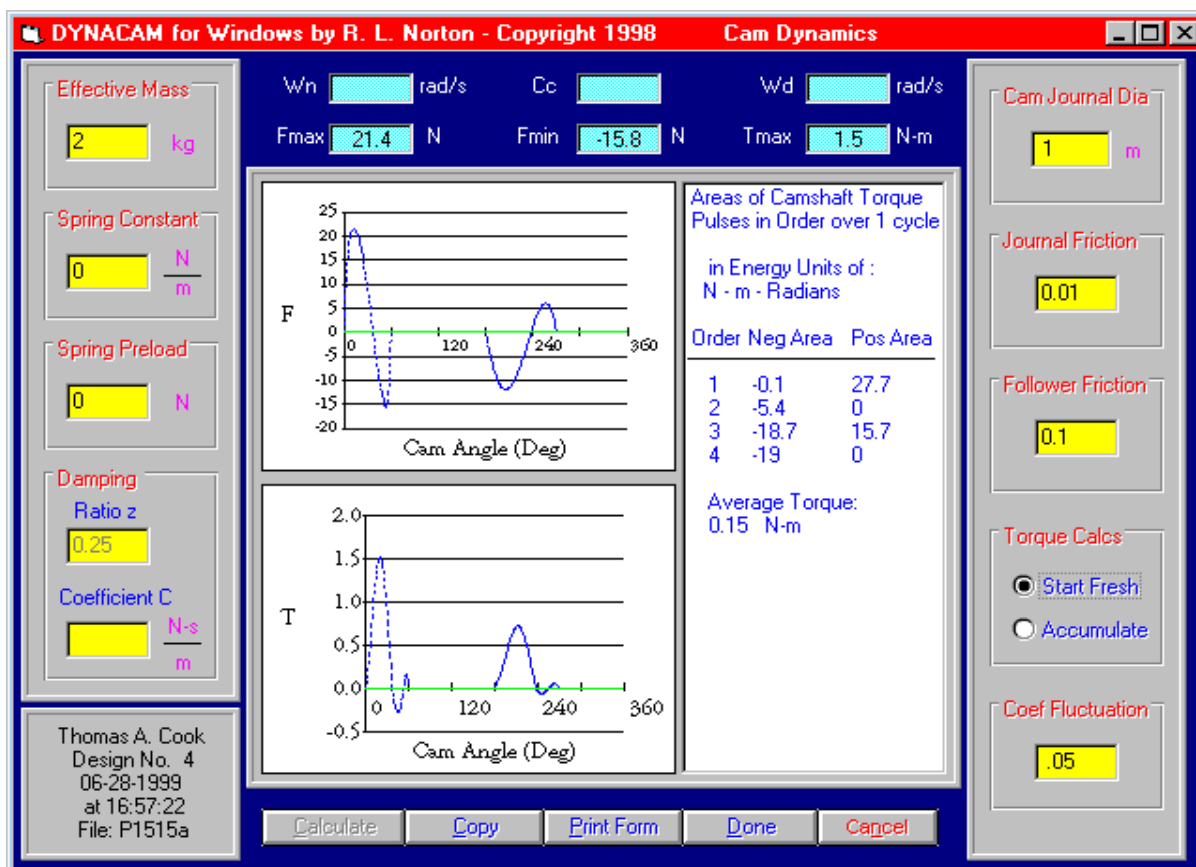
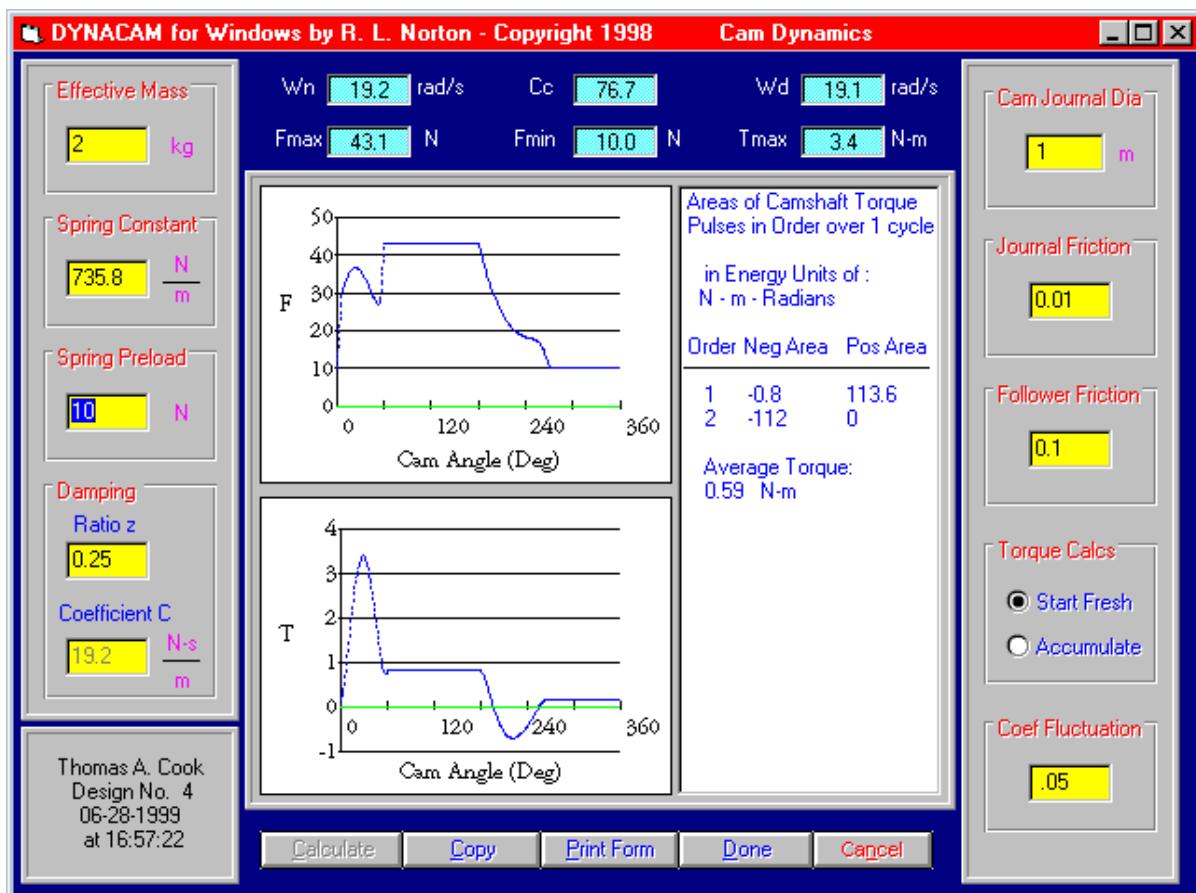
RISE	DWELL	FALL	DWELL
$\beta_1 := 60 \cdot \text{deg}$	$\beta_2 := 120 \cdot \text{deg}$	$\beta_3 := 90 \cdot \text{deg}$	$\beta_4 := 90 \cdot \text{deg}$
$h_1 := 45 \cdot \text{mm}$	$h_2 := 0 \cdot \text{mm}$	$h_3 := 45 \cdot \text{mm}$	$h_4 := 0 \cdot \text{mm}$
Mod. sine		3-4-5 poly	
Cycle time: $\tau := 1 \cdot \text{sec}$	Damping ratio: $\zeta := 0.25$	Follower mass: $m_f := 2 \cdot \text{kg}$	

Solution: See DYNACAM files P1515a and P1515b.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \quad \omega = 6.283 \frac{\text{rad}}{\text{sec}} \quad \omega = 60.000 \text{ rpm}$$

2. Load the noted files into program DYNACAM. Diskfile P1515a contains the force-closed solution and P1515b the form-closed solution. In each case, after opening the file, click on the *Size Cam* button and then click *Calculate* to see the pressure angle and minimum radius of curvature. Then click the *Done* button and you will see the cam drawing. After clicking the *Done* button on that screen, click the *Dynamics* button on the *Home* screen. Click its *Calculate* button to see the fully developed *Dynamics* screen. Once this sequence of commands has been entered, you may see further data by clicking on the *Print* or *Plot* buttons on the home screen. Accept all default values, which have been brought in from the file. It is not necessary to go to the *SVAJ* screen as that data has been brought in from the files.
3. The cam is sized with an 92.5-mm prime circle radius and 10.8-mm eccentricity, which gives balanced pressure angles of 29.9 deg and a minimum radius of curvature of 52.4 mm. The roller radius is 25 mm.
4. For the force-closed case, choose catalog number 447 from with a spring constant of 4.2 lb/in (735.8 N/m) with a preload of 10 N to keep the follower force positive and prevent jump. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient are shown on the *Cam Dynamics* screen, which is shown below on the next page. The system natural frequency is about 5.3 times the driving frequency. This ratio could be increased at the expense of larger follower force by increasing the spring constant.
5. For the form-closed case, it is only necessary to define spring constant and preload values of zero. The "b" diskfile does this. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient for this case are shown on the second *Cam Dynamics* screen, which is shown below on the next page.
6. Comparing these two cases shows that in the force-closed case both the peak follower force and driving torque excursions are greater than in the form-closed case. This gives the form-closed cam the advantage in terms of lower stresses in cam and follower and a smaller drive motor. However, this advantage comes at the expense of a more difficult and costly cam to make, since two cam surfaces must be machined instead of one.



PROBLEM 15-16

Statement: Design a single-dwell cam to meet the specifications given below. Size a return spring, selected from Appendix D, and specify its preload to maintain contact between cam and follower. Calculate and plot the dynamic force and torque. Repeat for a form-closed cam. Compare the dynamic force, torque, and natural frequency for the form-closed design and the force-closed design.

Given:

RISE	FALL	DWELL
$\beta_{11} := 60 \cdot \text{deg}$	$\beta_{12} := 90 \cdot \text{deg}$	$\beta_2 := 210 \cdot \text{deg}$
$h_1 := 25 \cdot \text{mm}$		$h_3 := 0 \cdot \text{mm}$
Seventh-degree polynomial		
Cycle time: $\tau := 2 \cdot \text{sec}$	Damping ratio: $\zeta := 0.15$	Follower mass: $m_f := 10 \cdot \text{kg}$

Solution: See DYNACAM files P1516a and P1516b.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \qquad \omega = 3.142 \frac{\text{rad}}{\text{sec}} \qquad \omega = 30.000 \text{ rpm}$$

2. Load the noted files into program DYNACAM. Diskfile P1516a contains the force-closed solution and P1516b the form-closed solution. In each case, after opening the file, click on the *Size Cam* button and then click *Calculate* to see the pressure angle and minimum radius of curvature. Then click the *Done* button and you will see the cam drawing. After clicking the *Done* button on that screen, click the *Dynamics* button on the *Home* screen. Click its *Calculate* button to see the fully developed *Dynamics* screen. Once this sequence of commands has been entered, you may see further data by clicking on the *Print* or *Plot* buttons on the home screen. Accept all default values, which have been brought in from the file. It is not necessary to go to the *SVAJ* screen as that data has been brought in from the files.
3. The cam is sized with an 85-mm prime circle radius and 2.8-mm eccentricity, which gives balanced pressure angles of 21.5 deg and a minimum radius of curvature of 50.0 mm. The roller radius is 25 mm.
4. For the force-closed case, choose catalog number 447 from with a spring constant of 4.2 lb/in (735.8 N/m) with a preload of 10 N to keep the follower force positive and prevent jump. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient are shown on the *Cam Dynamics* screen, which is shown below on the next page. The system natural frequency is about 2.7 times the driving frequency. This ratio could be increased at the expense of larger follower force by increasing the spring constant.
5. For the form-closed case, it is only necessary to define spring constant and preload values of zero. The "b" diskfile does this. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient for this case are shown on the second *Cam Dynamics* screen, which is shown below on the next page.
6. Comparing these two cases shows that in the force-closed case both the peak follower force and driving torque excursions are greater than in the form-closed case. This gives the form-closed cam the advantage in terms of lower stresses in cam and follower and a smaller drive motor. However, this advantage comes at the expense of a more difficult and costly cam to make, since two cam surfaces must be machined instead of one.

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Cam Dynamics**

Effective Mass: kg

Spring Constant: $\frac{\text{N}}{\text{m}}$

Spring Preload: N

Damping Ratio z :

Coefficient C : $\frac{\text{N}\cdot\text{s}}{\text{m}}$

Thomas A. Cook
Design No. 2
06-28-1999
at 18:54:58

Wn: rad/s Cc: Wd: rad/s

Fmax: N Fmin: N Tmax: N-m

Cam Journal Dia: m

Journal Friction:

Follower Friction:

Torque Calcs:
☒ Start Fresh
☐ Accumulate

Coef Fluctuation:

Areas of Camshaft Torque Pulses in Order over 1 cycle
in Energy Units of :
N - m - Radians

Order	Neg Area	Pos Area
1	0	37.6
2	-37.5	0

Average Torque: 0.2 N-m

Calculate Copy Print Form Done Cancel

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Cam Dynamics**

Effective Mass: kg

Spring Constant: $\frac{\text{N}}{\text{m}}$

Spring Preload: N

Damping Ratio z :

Coefficient C : $\frac{\text{N}\cdot\text{s}}{\text{m}}$

Thomas A. Cook
Design No. 2
06-28-1999
at 18:54:58
File: P1516a

Wn: rad/s Cc: Wd: rad/s

Fmax: N Fmin: N Tmax: N-m

Cam Journal Dia: m

Journal Friction:

Follower Friction:

Torque Calcs:
☒ Start Fresh
☐ Accumulate

Coef Fluctuation:

Areas of Camshaft Torque Pulses in Order over 1 cycle
in Energy Units of :
N - m - Radians

Order	Neg Area	Pos Area
1	0	9.1
2	-2	8.1
3	-15.1	0
4	0	0

Average Torque: 0.06 N-m

Calculate Copy Print Form Done Cancel

PROBLEM 15-17

Statement: Design a four-dwell cam to meet the specifications given below. Size a return spring and specify its preload to maintain contact between cam and follower. Calculate and plot the dynamic force and torque. Repeat for a form-closed cam. Compare the dynamic force, torque, and natural frequency for the form-closed design and the force-closed design.

Given:	RISE/FALL	DWELL	FALL	DWELL
	$\beta_1 := 40\text{-deg}$	$\beta_2 := 100\text{-deg}$	$\beta_3 := 90\text{-deg}$	$\beta_4 := 20\text{-deg}$
	$h_1 := 40\text{-mm}$	$h_2 := 0.0\text{-in}$	$h_3 := 20\text{-mm}$	$h_4 := 0.0\text{-in}$
	$\beta_5 := 30\text{-deg}$	$\beta_6 := 40\text{-deg}$	$\beta_7 := 30\text{-deg}$	$\beta_8 := 10\text{-deg}$
	$h_5 := 10\text{-mm}$	$h_6 := 0.0\text{-in}$	$h_7 := 10\text{-mm}$	$h_8 := 0.0\text{-in}$
Cycle time: $\tau := 10\text{-sec}$ Damping ratio: $\zeta := 0.25$ Follower mass: $m_f := 3\text{-kg}$				

Solution: See DYNACAM files P1517a and P1517b.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau} \qquad \omega = 0.628 \frac{\text{rad}}{\text{sec}} \qquad \omega = 6.000 \text{ rpm}$$

2. Load the noted files into program DYNACAM. Diskfile P1517a contains the force-closed solution and P1517b the form-closed solution. In each case, after opening the file, click on the *Size Cam* button and then click *Calculate* to see the pressure angle and minimum radius of curvature. Then click the *Done* button and you will see the cam drawing. After clicking the *Done* button on that screen, click the *Dynamics* button on the *Home* screen. Click its *Calculate* button to see the fully developed *Dynamics* screen. Once this sequence of commands has been entered, you may see further data by clicking on the *Print* or *Plot* buttons on the home screen. Accept all default values, which have been brought in from the file. It is not necessary to go to the *SVAJ* screen as that data has been brought in from the files.
3. Modified trapezoidal acceleration was used for the rise and all falls. The cam is sized with an 130-mm prime circle radius and 34-mm eccentricity, which gives balanced pressure angles of 29 deg and a minimum radius of curvature of 50 mm. The roller radius is 25 mm.
4. For the force-closed case, a spring constant of 150 N/mm with a preload of 5 N is chosen to keep the follower force positive and prevent jump. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient are shown on the *Cam Dynamics* screen, which is shown below on the next page. The system natural frequency is about 11.3 times the driving frequency. This ratio could be increased at the expense of larger follower force by increasing the spring constant.
5. For the form-closed case, it is only necessary to define spring constant and preload values of zero. The "b" diskfile does this. The maximum and minimum follower force, maximum driving torque, undamped and damped natural frequencies, and damping coefficient for this case are shown on the second *Cam Dynamics* screen, which is shown below on the next page.
6. Comparing these two cases shows that in the force-closed case both the peak follower force and driving torque excursions are greater than in the form-closed case. This gives the form-closed cam the advantage in terms of lower stresses in cam and follower and a smaller drive motor. However, this advantage comes at the expense of a more difficult and costly cam to make, since two cam surfaces must be machined instead of one.

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Cam Dynamics**

Effective Mass: kg

Spring Constant: $\frac{\text{N}}{\text{m}}$

Spring Preload: N

Damping: Ratio z : Coefficient C : $\frac{\text{N}\cdot\text{s}}{\text{m}}$

Thomas A. Cook
Design No. 2
06-28-1999
at 19:34:33

Wn: rad/s Cc: Wd: rad/s
Fmax: N Fmin: N Tmax: N-m

Cam Journal Dia: m

Journal Friction:

Follower Friction:

Torque Calcs: ☒ Start Fresh ☐ Accumulate

Coef Fluctuation:

Areas of Camshaft Torque Pulses in Order over 1 cycle
in Energy Units of : N - m - Radians

Order	Neg Area	Pos Area
1	-0.2	25.2
2	-24.8	0

Average Torque: 0.19 N-m

Calculate Copy Print Form Done Cancel

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Cam Dynamics**

Effective Mass: kg

Spring Constant: $\frac{\text{N}}{\text{m}}$

Spring Preload: N

Damping: Ratio z : Coefficient C : $\frac{\text{N}\cdot\text{s}}{\text{m}}$

Thomas A. Cook
Design No. 2
06-28-1999
at 19:34:33
File: P1517a

Wn: rad/s Cc: Wd: rad/s
Fmax: N Fmin: N Tmax: N-m

Cam Journal Dia: m

Journal Friction:

Follower Friction:

Torque Calcs: ☒ Start Fresh ☐ Accumulate

Coef Fluctuation:

Areas of Camshaft Torque Pulses in Order over 1 cycle
in Energy Units of : N - m - Radians

Order	Pos Area	Neg Area
1	1.5	-0.9
2	0	-0.4
3	0.1	-0.4
4	0.1	-0.1
5	0	0

Average Torque: 0.01 N-m

Calculate Copy Print Form Done Cancel

PROBLEM 15-18

Statement: Design a cam to drive an automotive valve train whose parameters are given below. Select a spring constant and preload to avoid jump to 3500 rpm. Fast opening and closing and maximum open time are desired.

Units: $kN := 10^3 \cdot N$

Given: Cam rise/fall and speed: $h := 12 \cdot mm$ $\omega := 3500 \cdot rpm$

Valve train mass: $m_v := 0.2 \cdot kg$ Damping ratio: $\zeta := 0.30$

Roller follower: $R_f := 5 \cdot mm$

Open-close event is 160 deg, low dwell is 200 deg.

Solution: See Mathcad file P1518.

1. Use a 3-4-5 polynomial for both the rise and the fall. Although peak jerk is higher with this choice than with the 4-5-6-7 polynomial, peak acceleration is considerably lower. Using equation 8.24, write functions for the displacement, velocity, and acceleration of the follower. The value of β for the both the rise and fall is $\beta := 40 \cdot deg$. To write the global equations, define a range function that has a value of unity between the limits of a and b and is zero elsewhere.

$$R(\theta, a, b) := \text{if}[(\theta > a) \cdot (\theta \leq b), 1, 0]$$

Displacement:

$$\text{Rise: } \beta_1 := \beta \quad \beta_1 = 40.000 \text{ deg}$$

$$s_1(\theta) := h \cdot \left[10 \cdot \left(\frac{\theta}{\beta_1} \right)^3 - 15 \cdot \left(\frac{\theta}{\beta_1} \right)^4 + 6 \cdot \left(\frac{\theta}{\beta_1} \right)^5 \right]$$

$$\text{High dwell: } \beta_2 := 160 \cdot \text{deg} - 2 \cdot \beta \quad \beta_2 = 80.000 \text{ deg} \quad s_2(\theta) := h$$

$$\text{Fall: } \beta_3 := \beta \quad \beta_3 = 40.000 \text{ deg}$$

$$s_3(\theta) := h \cdot \left[1 - \left[10 \cdot \left(\frac{\theta - \beta_1 - \beta_2}{\beta_3} \right)^3 - 15 \cdot \left(\frac{\theta - \beta_1 - \beta_2}{\beta_3} \right)^4 + 6 \cdot \left(\frac{\theta - \beta_1 - \beta_2}{\beta_3} \right)^5 \right] \right]$$

$$\text{Global equation: } s(\theta) := R(\theta, 0, \beta_1) \cdot s_1(\theta) + R(\theta, \beta_1, \beta_1 + \beta_2) \cdot s_2(\theta) \dots \\ + R(\theta, \beta_1 + \beta_2, \beta_1 + \beta_2 + \beta_3) \cdot s_3(\theta)$$

Velocity:

$$\text{Rise: } v_1(\theta) := \omega \cdot \frac{h}{\beta_1} \cdot \left[30 \cdot \left(\frac{\theta}{\beta_1} \right)^2 - 60 \cdot \left(\frac{\theta}{\beta_1} \right)^3 + 30 \cdot \left(\frac{\theta}{\beta_1} \right)^4 \right]$$

$$\text{High dwell: } v_2(\theta) := 0 \cdot \frac{m}{\text{sec}}$$

$$\text{Fall: } v_3(\theta) := -\omega \cdot \frac{h}{\beta_3} \cdot \left[30 \cdot \left(\frac{\theta - \beta_1 - \beta_2}{\beta_3} \right)^2 - 60 \cdot \left(\frac{\theta - \beta_1 - \beta_2}{\beta_3} \right)^3 + 30 \cdot \left(\frac{\theta - \beta_1 - \beta_2}{\beta_3} \right)^4 \right]$$

Global equation:
$$v(\theta) := R(\theta, 0, \beta_1) \cdot v_1(\theta) + R(\theta, \beta_1, \beta_1 + \beta_2) \cdot v_2(\theta) \dots$$

$$+ R(\theta, \beta_1 + \beta_2, \beta_1 + \beta_2 + \beta_3) \cdot v_3(\theta)$$

Acceleration:

Rise:
$$a_1(\theta) := \omega^2 \cdot \frac{h}{\beta_1^2} \cdot \left[60 \cdot \left(\frac{\theta}{\beta_1} \right) - 180 \cdot \left(\frac{\theta}{\beta_1} \right)^2 + 120 \cdot \left(\frac{\theta}{\beta_1} \right)^3 \right]$$

High dwell:
$$a_2(\theta) := 0 \cdot \frac{m}{\text{sec}^2}$$

Fall:
$$a_3(\theta) := -\omega^2 \cdot \frac{h}{\beta_3^2} \cdot \left[60 \cdot \left(\frac{\theta - \beta_1 - \beta_2}{\beta_3} \right) - 180 \cdot \left(\frac{\theta - \beta_1 - \beta_2}{\beta_3} \right)^2 \dots \right.$$

$$\left. + 120 \cdot \left(\frac{\theta - \beta_1 - \beta_2}{\beta_3} \right)^3 \right]$$

Global equation:
$$a(\theta) := R(\theta, 0, \beta_1) \cdot a_1(\theta) + R(\theta, \beta_1, \beta_1 + \beta_2) \cdot a_2(\theta) \dots$$

$$+ R(\theta, \beta_1 + \beta_2, \beta_1 + \beta_2 + \beta_3) \cdot a_3(\theta)$$

2. Iteratively choose spring parameters that will prevent follower jump and then, calculate the damping coefficient using equations 15.2i and 15.3a.

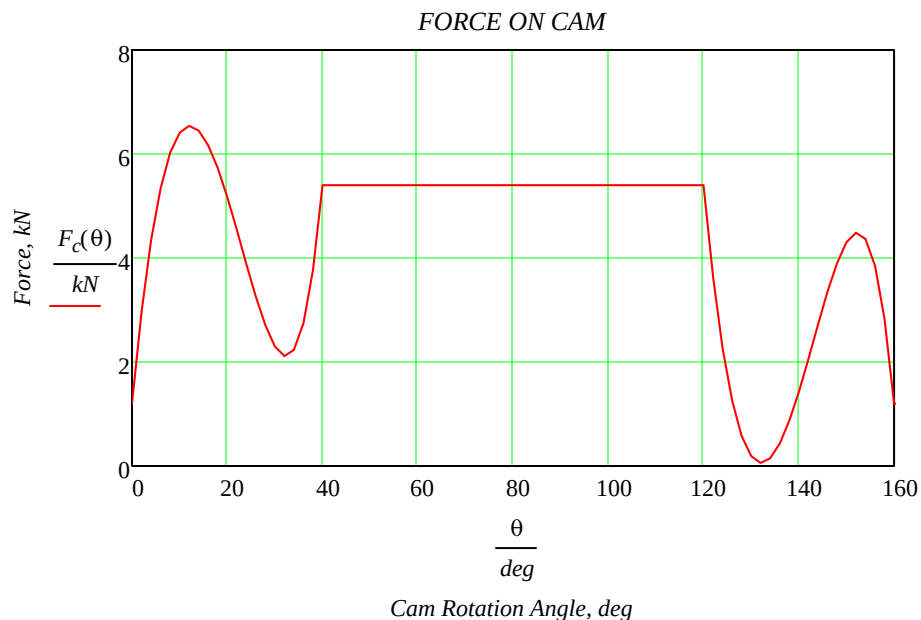
Spring: $k := 350 \cdot N \cdot mm^{-1}$ $F_{pl} := 1200 \cdot N$

$c := 2 \cdot \zeta \cdot \sqrt{m_v \cdot k}$ $c = 158.745 \frac{N \cdot sec}{m}$

3. Substitute the expressions for displacement, velocity, acceleration, and spring preload into equation 15.9 and solve for the force on the cam as a function of cam angle.

$$F_c(\theta) := m_v \cdot a(\theta) + c \cdot v(\theta) + k \cdot s(\theta) + F_{pl}$$

4. Plot the force on the cam for one revolution of the cam. $\theta := 0 \cdot deg, 2 \cdot deg \dots 360 \cdot deg$



PROBLEM 15-19

Statement: Figure P15-3 shows a cam-follower system that drives slider 6 through an external output arm 3. Arms 2 and 3 are both rigidly attached to the 0.75-in-dia shaft X-X, which rotates in bearings that are supported by the housing. The pin-to-pin dimensions of the links are shown. The cross-section of arms 2, 3, and 5 are solid, rectangular 1.5 x 0.75 in steel. The ends of these links have a full radius equal to one half the link width. Link 4 is 1-in-dia x 0.125 in-wall, round steel tubing. Link 6 is a 2-in-dia x 6-in long solid steel cylinder. Find the effective mass and effective spring constant of the follower train referenced to the cam-follower roller if the spring at A has a rate of 150 lb/in with a preload of 60 lb. Then determine and plot the kinetostatic follower force and camshaft torque over one cycle if the cam provides a 3-4-5 polynomial double dwell angular motion to roller arm 2 with a rise of 10 deg in 90 camshaft degrees, dwell for 90 deg, fall 10 deg in 90 deg and dwell for the remainder. The camshaft turns 100 rpm.

Units: $\text{blob} := \text{lb} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Follower rise: $\sigma := 10 \cdot \text{deg}$ Cam speed: $n := 100 \cdot \text{rpm}$ Roller arm radius: $r := 8 \cdot \text{in}$

Link 6 (solid cylinder): $d_6 := 2.00 \cdot \text{in}$ $L_6 := 6.00 \cdot \text{in}$

Rocker arm 5: $w := 0.75 \cdot \text{in}$ $h := 1.50 \cdot \text{in}$ $a := 10.0 \cdot \text{in}$ $b := 8.0 \cdot \text{in}$

Pushrod 4 (hollow cylinder): $d_{4od} := 1.00 \cdot \text{in}$ $d_{4id} := 0.75 \cdot \text{in}$ $L_4 := 22 \cdot \text{in}$

Output arm 3: $w := 0.75 \cdot \text{in}$ $h := 1.50 \cdot \text{in}$ $L_3 := 16 \cdot \text{in}$

Spring: $k_s := 150 \cdot \text{lb} \cdot \text{in}^{-1}$ $F_{so} := 60 \cdot \text{lb}$

Roller arm 2: $w := 0.75 \cdot \text{in}$ $h := 1.50 \cdot \text{in}$ $L_2 := 8 \cdot \text{in}$

All parts are steel: Modulus of elasticity $E := 30 \cdot 10^6 \cdot \text{psi}$ Spec. weight $\gamma := 0.28 \cdot \frac{\text{lb}}{\text{in}^3}$

Solution: See Figure P15-3 and Mathcad file P1519.

1. Use the method of Chapter 10 to find the equivalent lumped mass system. Break the system into individual elements as shown in Figure 10-9b.
2. Define the individual spring constants of each of the six elements.

Roller arm (cantilever beam with end load):

$$\begin{aligned} \text{Moment of inertia} \quad I_2 &:= \frac{w \cdot h^3}{12} \\ \text{Spring constant} \quad k_2 &:= \frac{3 \cdot E \cdot I_2}{L_2^3} \quad k_2 = 3.708 \times 10^4 \frac{\text{lb}}{\text{in}} \end{aligned}$$

Output arm:

$$\begin{aligned} \text{Moment of inertia} \quad I_3 &:= \frac{w \cdot h^3}{12} \\ \text{Spring constant} \quad k_3 &:= \frac{3 \cdot E \cdot I_3}{L_3^3} \quad k_3 = 4.635 \times 10^3 \frac{\text{lb}}{\text{in}} \end{aligned}$$

Pushrod (hollow cylinder):

$$\begin{aligned} \text{Area} \quad A_4 &:= \frac{\pi \cdot (d_{4od}^2 - d_{4id}^2)}{4} \\ \text{Spring constant} \quad k_4 &:= \frac{A_4 \cdot E}{L_4} \quad k_4 = 4.686 \times 10^5 \frac{\text{lb}}{\text{in}} \end{aligned}$$

Rocker arm 5 (side B):

$$\text{Moment of inertia} \quad I_5 := \frac{w \cdot h^3}{12}$$

$$\text{Spring constant} \quad k_{5a} := \frac{3 \cdot E \cdot I_5}{a^3} \quad k_{5a} = 1.898 \times 10^4 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

Rocker arm 5 (side C):

$$\text{Spring constant} \quad k_{5b} := \frac{3 \cdot E \cdot I_5}{b^3} \quad k_{5b} = 3.708 \times 10^4 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

3. Determine the mass of each of the elements.

Roller arm 2 (cantilever beam with end load):

$$\text{Volume} \quad V_2 := w \cdot h \cdot L_2$$

$$\text{Mass} \quad m_2 := \frac{\gamma \cdot V_2}{g} \quad m_2 = 6.527 \times 10^{-3} \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Outout arm 3 (cantilever beam with end load):

$$\text{Volume} \quad V_3 := w \cdot h \cdot L_3$$

$$\text{Mass} \quad m_3 := \frac{\gamma \cdot V_3}{g} \quad m_3 = 0.013 \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Pushrod 4 (hollow cylinder):

$$\text{Volume} \quad V_4 := A_4 \cdot L_4$$

$$\text{Mass} \quad m_4 := \frac{\gamma \cdot V_4}{g} \quad m_4 = 5.482 \times 10^{-3} \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Rocker arm 5 (side B):

$$\text{Volume} \quad V_{5a} := w \cdot h \cdot a$$

$$\text{Mass} \quad m_{5a} := \frac{\gamma \cdot V_{5a}}{g} \quad m_{5a} = 8.159 \times 10^{-3} \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Rocker arm 5 (side C):

$$\text{Volume} \quad V_{5b} := w \cdot h \cdot b$$

$$\text{Mass} \quad m_{5b} := \frac{\gamma \cdot V_{5b}}{g} \quad m_{5b} = 6.527 \times 10^{-3} \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Cylinder 6:

$$\text{Volume} \quad V_6 := \frac{\pi \cdot d_6^2}{4} \cdot L_6$$

$$\text{Mass} \quad m_6 := \frac{\gamma \cdot V_6}{g} \quad m_6 = 0.014 \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Omit the spring because no data are available.

4. Determine the effective mass and spring constant on either side of the rocker arm 5, reflected to point A.

Upper side:

$$m_U := m_4 + m_{5a}$$

$$m_U = 0.014 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_U := \left(\frac{1}{k_s} + \frac{1}{k_4} + \frac{1}{k_{5a}} \right)^{-1}$$

$$k_U = 148.777 \frac{\text{lbf}}{\text{in}}$$

Lower side:

$$m_L := m_6 + m_{5b}$$

$$m_L = 0.020 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_L := k_{5b}$$

$$k_L = 3.708 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

$$m_{Aeff} := m_U + \left(\frac{b}{a} \right)^2 \cdot m_L$$

$$m_{Aeff} = 0.027 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_{Aeff} := k_U + \left(\frac{b}{a} \right)^2 \cdot k_L$$

$$k_{Aeff} = 2.388 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

5. Determine the effective mass and spring constant on either side of the output and roller arms, reflected to the roller.

Output side:

$$m_O := m_{Aeff} + m_3$$

$$m_O = 0.040 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_O := \left(\frac{1}{k_{Aeff}} + \frac{1}{k_3} \right)^{-1}$$

$$k_O = 3.881 \times 10^3 \frac{\text{lbf}}{\text{in}}$$

Roller side:

$$m_R := m_2$$

$$m_R = 6.527 \times 10^{-3} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_R := k_2$$

$$k_R = 3.708 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

$$m_{eff} := m_R + \left(\frac{L_3}{L_2} \right)^2 \cdot m_O$$

$$m_{eff} = 0.165 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_{eff} := k_R + \left(\frac{L_3}{L_2} \right)^2 \cdot k_O$$

$$k_{eff} = 5.260 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

6. The program DYNACAM will be used for this solution. Calculate the speed in rad/sec and the effective lift in inches for input to the SVAJ screen of DYNACAM.

$$\omega := n$$

$$\omega = 10.472 \frac{\text{rad}}{\text{sec}}$$

$$h := r \cdot \sigma$$

$$h = 1.4 \text{ in}$$

Choose poly functions for the rise and fall segments and, when calculating these segments, choose 6 BCs, setting V and A to zero at the beginning and end of the segments. This will result in 3-4-5 polynomials for the rise and fall.

7. Click *Done* and then, at the *Home* screen, click *Size Cam*. Choose a follower radius and then a prime circle radius that will make the radius of curvature, $\rho_{\min}$, significantly larger than the follower radius.

DYNACAM for Windows by R. L. Norton - Copyright 1998 **Input Screen**

Cam Data

Cam Omega (rad/sec)
10.47

No. Segmts
4

Delta Theta (deg)
1

Follower

☒ Trans
☐ Oscit

Calc All
Clear Segs
Clear All
Copy
Print Form
Done
Cancel

Segment Data

Seg	Angles			Cam Contour		Position (in)	
	Beta	Start	End	Motion	Program	Start	End
1	90	0	90	Poly	PO - Polynomial	0	1.4
2	90	90	180	Dwell	DW - Dwell	1.4	1.4
3	90	180	270	Poly	PO - Polynomial	1.4	0
4	90	270	360	Dwell	DW - Dwell	0	0
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

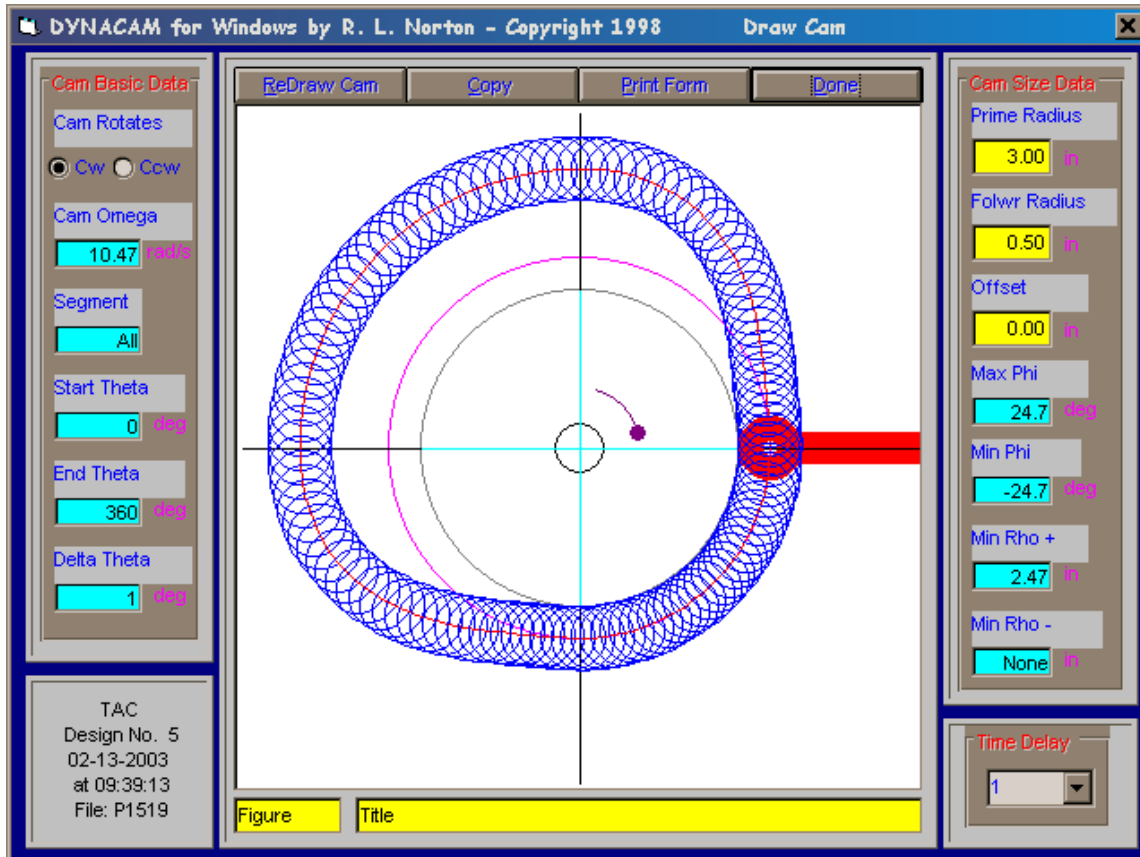
Examples

Calc Plot Print

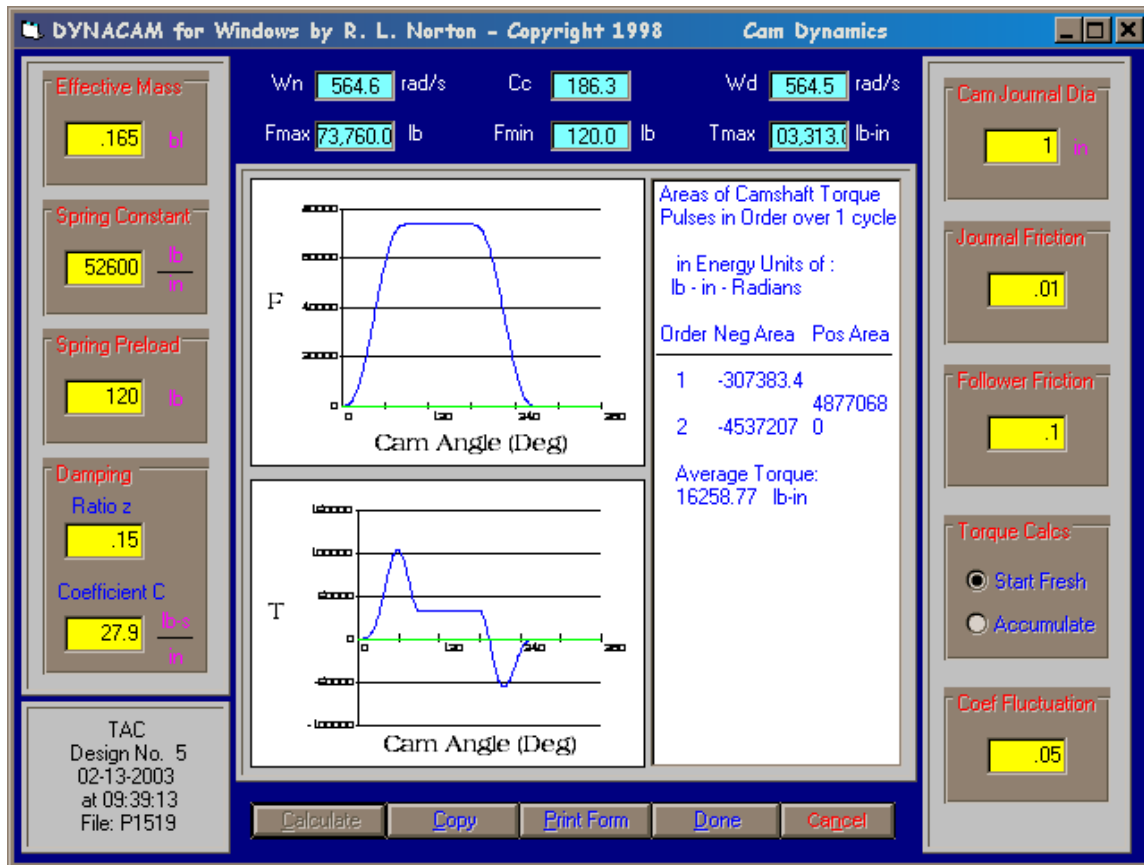
Calc Plot Print

Calc Plot Print

Calc Plot Print



8. Click *Done* and then, at the *Home* screen, click *Dynamics*. Enter the effective mass, spring rate, and spring preload. The results are shown below.



PROBLEM 15-20

Statement: Figure P15-3 shows a cam-follower system that drives slider 6 through an external output arm 3. Arms 2 and 3 are both rigidly attached to the 0.75-in-dia shaft X-X, which rotates in bearings that are supported by the housing. The pin-to-pin dimensions of the links are shown. The cross-section of arms 2, 3, and 5 are solid, rectangular 1.5 x 0.75 in steel. The ends of these links have a full radius equal to one half the link width. Link 4 is 1-in-dia x 0.125 in-wall, round steel tubing. Link 6 is a 2-in-dia x 6-in long solid steel cylinder. Find the effective mass and effective spring constant of the follower train referenced to the cam-follower roller if the spring at A has a rate of 150 lb/in with a preload of 60 lb. Then determine and plot the kinetostatic follower force and camshaft torque over one cycle if the cam provides a cycloidal displacement double dwell angular motion to roller arm 2 with a rise of 10 deg in 90 camshaft degrees, dwell for 90 deg, fall 10 deg in 90 deg and dwell for the remainder. The camshaft turns 100 rpm.

Units: $\text{blob} := \text{lb} \cdot \text{sec}^2 \cdot \text{in}^{-1}$

Given: Follower rise: $\sigma := 10 \cdot \text{deg}$ Cam speed: $n := 100 \cdot \text{rpm}$ Roller arm radius: $r := 8 \cdot \text{in}$

Link 6 (solid cylinder): $d_6 := 2.00 \cdot \text{in}$ $L_6 := 6.00 \cdot \text{in}$

Rocker arm 5: $w := 0.75 \cdot \text{in}$ $h := 1.50 \cdot \text{in}$ $a := 10.0 \cdot \text{in}$ $b := 8.0 \cdot \text{in}$

Pushrod 4 (hollow cylinder): $d_{4od} := 1.00 \cdot \text{in}$ $d_{4id} := 0.75 \cdot \text{in}$ $L_4 := 22 \cdot \text{in}$

Output arm 3: $w := 0.75 \cdot \text{in}$ $h := 1.50 \cdot \text{in}$ $L_3 := 16 \cdot \text{in}$

Spring: $k_s := 150 \cdot \text{lb} \cdot \text{in}^{-1}$ $F_{so} := 60 \cdot \text{lb}$

Roller arm 2: $w := 0.75 \cdot \text{in}$ $h := 1.50 \cdot \text{in}$ $L_2 := 8 \cdot \text{in}$

All parts are steel: Modulus of elasticity $E := 30 \cdot 10^6 \cdot \text{psi}$ Spec. weight $\gamma := 0.28 \cdot \frac{\text{lb}}{\text{in}^3}$

Solution: See Figure P15-3 and Mathcad file P1520.

1. Use the method of Chapter 10 to find the equivalent lumped mass system. Break the system into individual elements as shown in Figure 10-9b.
2. Define the individual spring constants of each of the six elements.

Roller arm (cantilever beam with end load):

$$\begin{aligned} \text{Moment of inertia} \quad I_2 &:= \frac{w \cdot h^3}{12} \\ \text{Spring constant} \quad k_2 &:= \frac{3 \cdot E \cdot I_2}{L_2^3} \quad k_2 = 3.708 \times 10^4 \frac{\text{lb}}{\text{in}} \end{aligned}$$

Output arm:

$$\begin{aligned} \text{Moment of inertia} \quad I_3 &:= \frac{w \cdot h^3}{12} \\ \text{Spring constant} \quad k_3 &:= \frac{3 \cdot E \cdot I_3}{L_3^3} \quad k_3 = 4.635 \times 10^3 \frac{\text{lb}}{\text{in}} \end{aligned}$$

Pushrod (hollow cylinder):

$$\begin{aligned} \text{Area} \quad A_4 &:= \frac{\pi \cdot (d_{4od}^2 - d_{4id}^2)}{4} \\ \text{Spring constant} \quad k_4 &:= \frac{A_4 \cdot E}{L_4} \quad k_4 = 4.686 \times 10^5 \frac{\text{lb}}{\text{in}} \end{aligned}$$

Rocker arm 5 (side B):

$$\text{Moment of inertia} \quad I_5 := \frac{w \cdot h^3}{12}$$

$$\text{Spring constant} \quad k_{5a} := \frac{3 \cdot E \cdot I_5}{a^3} \quad k_{5a} = 1.898 \times 10^4 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

Rocker arm 5 (side C):

$$\text{Spring constant} \quad k_{5b} := \frac{3 \cdot E \cdot I_5}{b^3} \quad k_{5b} = 3.708 \times 10^4 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

3. Determine the mass of each of the elements.

Roller arm 2 (cantilever beam with end load):

$$\text{Volume} \quad V_2 := w \cdot h \cdot L_2$$

$$\text{Mass} \quad m_2 := \frac{\gamma \cdot V_2}{g} \quad m_2 = 6.527 \times 10^{-3} \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Outout arm 3 (cantilever beam with end load):

$$\text{Volume} \quad V_3 := w \cdot h \cdot L_3$$

$$\text{Mass} \quad m_3 := \frac{\gamma \cdot V_3}{g} \quad m_3 = 0.013 \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Pushrod 4 (hollow cylinder):

$$\text{Volume} \quad V_4 := A_4 \cdot L_4$$

$$\text{Mass} \quad m_4 := \frac{\gamma \cdot V_4}{g} \quad m_4 = 5.482 \times 10^{-3} \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Rocker arm 5 (side B):

$$\text{Volume} \quad V_{5a} := w \cdot h \cdot a$$

$$\text{Mass} \quad m_{5a} := \frac{\gamma \cdot V_{5a}}{g} \quad m_{5a} = 8.159 \times 10^{-3} \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Rocker arm 5 (side C):

$$\text{Volume} \quad V_{5b} := w \cdot h \cdot b$$

$$\text{Mass} \quad m_{5b} := \frac{\gamma \cdot V_{5b}}{g} \quad m_{5b} = 6.527 \times 10^{-3} \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Cylinder 6:

$$\text{Volume} \quad V_6 := \frac{\pi \cdot d_6^2}{4} \cdot L_6$$

$$\text{Mass} \quad m_6 := \frac{\gamma \cdot V_6}{g} \quad m_6 = 0.014 \text{lb} \cdot \text{f} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

Omit the spring because no data are available.

4. Determine the effective mass and spring constant on either side of the rocker arm 5, reflected to point A.

Upper side:

$$m_U := m_4 + m_{5a}$$

$$m_U = 0.014 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_U := \left(\frac{1}{k_s} + \frac{1}{k_4} + \frac{1}{k_{5a}} \right)^{-1}$$

$$k_U = 148.777 \frac{\text{lbf}}{\text{in}}$$

Lower side:

$$m_L := m_6 + m_{5b}$$

$$m_L = 0.020 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_L := k_{5b}$$

$$k_L = 3.708 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

$$m_{Aeff} := m_U + \left(\frac{b}{a} \right)^2 \cdot m_L$$

$$m_{Aeff} = 0.027 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_{Aeff} := k_U + \left(\frac{b}{a} \right)^2 \cdot k_L$$

$$k_{Aeff} = 2.388 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

5. Determine the effective mass and spring constant on either side of the output and roller arms, reflected to the roller.

Output side:

$$m_O := m_{Aeff} + m_3$$

$$m_O = 0.040 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_O := \left(\frac{1}{k_{Aeff}} + \frac{1}{k_3} \right)^{-1}$$

$$k_O = 3.881 \times 10^3 \frac{\text{lbf}}{\text{in}}$$

Roller side:

$$m_R := m_2$$

$$m_R = 6.527 \times 10^{-3} \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_R := k_2$$

$$k_R = 3.708 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

$$m_{eff} := m_R + \left(\frac{L_3}{L_2} \right)^2 \cdot m_O$$

$$m_{eff} = 0.165 \text{ lbf} \cdot \text{sec}^2 \cdot \text{in}^{-1}$$

$$k_{eff} := k_R + \left(\frac{L_3}{L_2} \right)^2 \cdot k_O$$

$$k_{eff} = 5.260 \times 10^4 \frac{\text{lbf}}{\text{in}}$$

6. The program DYNACAM will be used for this solution. Calculate the speed in rad/sec and the effective lift in inches for input to the SVAJ screen of DYNACAM.

$$\omega := n \quad \omega = 10.472 \frac{\text{rad}}{\text{sec}} \quad h := r \cdot \sigma \quad h = 1.4 \text{ in}$$

Choose full cycloidal for the rise and fall segments. Set the units to in-lb-s.

7. Click *Done* and then, at the *Home* screen, click *Size Cam*. Choose a follower radius and then a prime circle radius that will make the radius of curvature, $\rho_{\min}$, significantly larger than the follower radius.

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Help Units

Cam Data

Cam Omega (rad/sec)
10.47

No. Segments
4

Delta Theta (deg)
1.0

Follower

☒ Translating
☐ Oscillating

Starting Angle
0 deg

☐ External Force

Function

☒ Motion
☐ Force

Calc All
Clear Segs
Clear All
Copy Print
< Back Next >

Seg	Angles			Cam Contour		Position (in)	
	Beta	Start	End	Motion	Program	Start	End
1	90	0	90	Rise	CF - Cycloid Full	0.000	1.40
2	90	90	180	Dwell	DW - Dwell	1.40	1.40
3	90	180	270	Fall	CF - Cycloid Full	1.40	0.000
4	90	270	360	Dwell	DW - Dwell	0.000	0.000
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Examples

Calc Plot Print

Status: S-V-A-J in-lb-bl-s

DYNACAM - Student Edition - by R. L. Norton - Copyright 2006 Release 9.0 Rev 4.8 8/23/06 Draw Cam

Help Zoom Redraw

Cam Basic Data

Cam Rotates
☐ Cw ☒ Ccw

Cam Omega
10.47 rad/s

Delta Theta
1.0 deg

Cam Angle Data

At Machine Zero, Keyway is at:
12 O'clock

Cam Start Angle
0 deg

Follower Angle
0 deg

☒ Show Cam
☐ Show Summary
☐ Show Conjugate

Animation Speed

<< Slow Fast >>

Range Cycles
10 1

☒ Inner Surface ☐ Outer Surface ☐ Follower Path ☐ Cutter Path ☐ Cam vs Machine Zero

Note: Initial follower position is shown at cam zero, not at machine zero. Key is at machine zero.

<< Start Time ----> End -->

Cam Size Data

Prime Radius
3.000 in

Folwr Radius
0.500 in

Offset
0.00 in

Max Phi
26.0 deg

Min Phi
-26.0 deg

Min Rho +
2.37 in

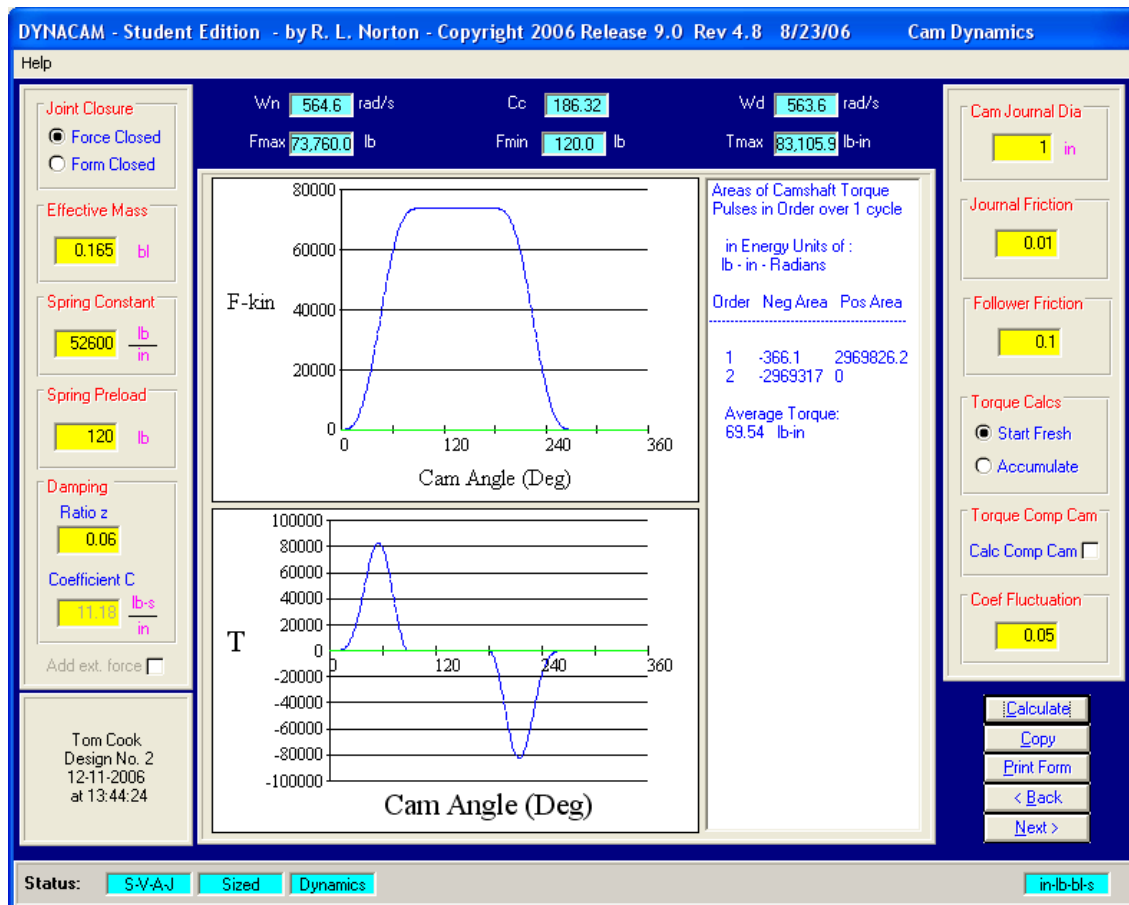
Min Rho -
-122.12 in

Cam Theta
0 deg

Draw Cam
Step
Run
Copy
Print Form
< Back
Next >

Status: S-V-A-J Sized in-lb-bl-s

8. Click *Done* and then, at the *Home* screen, click *Dynamics*. Enter the effective mass, spring rate, and spring preload. The results are shown below.



PROBLEM 15-21

Statement: A single-dwell cam-follower system similar to that shown in Figure 15-1a provides a two-segment polynomial for a rise of 35 mm in 75 deg, a fall of 35 mm in 120 deg, and a dwell for the remainder of the cycle. Using equations 15.8 and 15.10, calculate and plot the dynamic force and torque for one cycle if the roller follower train weighs 2.34 N, the system has a damping ratio of $\zeta = 0.06$, and the spring has a rate of 1.5 N/mm with a preload of 10 N. The cam turns at 20 rpm.

Given: RISE/FALL DWELL
 $\beta_3 := 195 \cdot \text{deg}$ $\beta_3 := 165 \cdot \text{deg}$ Cycle time: $\tau := 3 \cdot \text{sec}$
 $h := 35 \cdot \text{mm}$ $h_3 := 0.0 \cdot \text{mm}$
 Roller-follower train weight $W := 2.34 \cdot \text{N}$ Damping ratio $\zeta := 0.06$
 Spring rate and preload $k := 1.5 \cdot \text{N} \cdot \text{mm}^{-1}$ $F_{pl} := 10 \cdot \text{N}$

Solution: See Mathcad file P1521.

1. The camshaft speed is $\omega := 20 \cdot \text{rpm}$ $\omega = 2.094 \frac{\text{rad}}{\text{sec}}$

The damping coefficient (see equation 15.3) is

$$c := 2 \cdot \sqrt{k \cdot \frac{W}{g}} \cdot \zeta \quad c = 2.270 \times 10^{-3} \frac{\text{N} \cdot \text{s}}{\text{mm}}$$

2. Using a two-segment polynomial, let the rise and fall, together, be one segment and the dwell be the second segment. Then, the boundary conditions are:

$$\begin{aligned} \text{at } \theta = 0: \quad s &= 0, \quad v = 0, \quad a = 0 \\ \theta = \beta_1: \quad s &= h, \quad v = 0 \\ \theta = \beta: \quad s &= 0, \quad v = 0, \quad a = 0 \end{aligned}$$

This is a minimum set of 8 BCs. The $v = 0$ condition at $\theta = \beta_1$ is required to keep the displacement from overshooting the lift, h . Define the total lift, the rise interval, the fall interval, and the ratio of rise to the total interval.

$$\text{Total lift:} \quad h = 35.000 \text{ mm}$$

$$\text{Rise interval:} \quad \beta_1 := 75 \cdot \text{deg} \quad A := \frac{\beta_1}{\beta} \quad A = 0.385$$

$$\text{Fall interval:} \quad \beta_2 := 120 \cdot \text{deg}$$

Solving the polynomial equation for these boundary conditions yields the following coefficients:

$$\begin{aligned} c_3 &:= 5609.280 \cdot \text{mm} & c_4 &:= -24548.849 \cdot \text{mm} & c_5 &:= 39990.867 \cdot \text{mm} \\ c_6 &:= -28772.307 \cdot \text{mm} & c_7 &:= 7721.009 \cdot \text{mm} \end{aligned}$$

3. Write the SVA equations for the rise/fall segment.

$$\begin{aligned} S_I(\theta) &:= c_3 \left(\frac{\theta}{\beta} \right)^3 + c_4 \left(\frac{\theta}{\beta} \right)^4 + c_5 \left(\frac{\theta}{\beta} \right)^5 + c_6 \left(\frac{\theta}{\beta} \right)^6 + c_7 \left(\frac{\theta}{\beta} \right)^7 \\ V_I(\theta) &:= \frac{1}{\beta} \left[3 \cdot c_3 \left(\frac{\theta}{\beta} \right)^2 + 4 \cdot c_4 \left(\frac{\theta}{\beta} \right)^3 + 5 \cdot c_5 \left(\frac{\theta}{\beta} \right)^4 + 6 \cdot c_6 \left(\frac{\theta}{\beta} \right)^5 + 7 \cdot c_7 \left(\frac{\theta}{\beta} \right)^6 \right] \\ A_I(\theta) &:= \frac{1}{\beta^2} \left[6 \cdot c_3 \left(\frac{\theta}{\beta} \right) + 12 \cdot c_4 \left(\frac{\theta}{\beta} \right)^2 + 20 \cdot c_5 \left(\frac{\theta}{\beta} \right)^3 + 30 \cdot c_6 \left(\frac{\theta}{\beta} \right)^4 + 42 \cdot c_7 \left(\frac{\theta}{\beta} \right)^5 \right] \end{aligned}$$

4. Write the SVA equations for the dwell segment.

$$S_2(\theta) := 0 \cdot \text{mm} \quad V_2(\theta) := 0 \cdot \text{mm} \quad A_2(\theta) := 0 \cdot \text{mm}$$

5. To define the SVA curves for the complete cycle define a range function that has a value of one between the values of a and b and zero elsewhere.

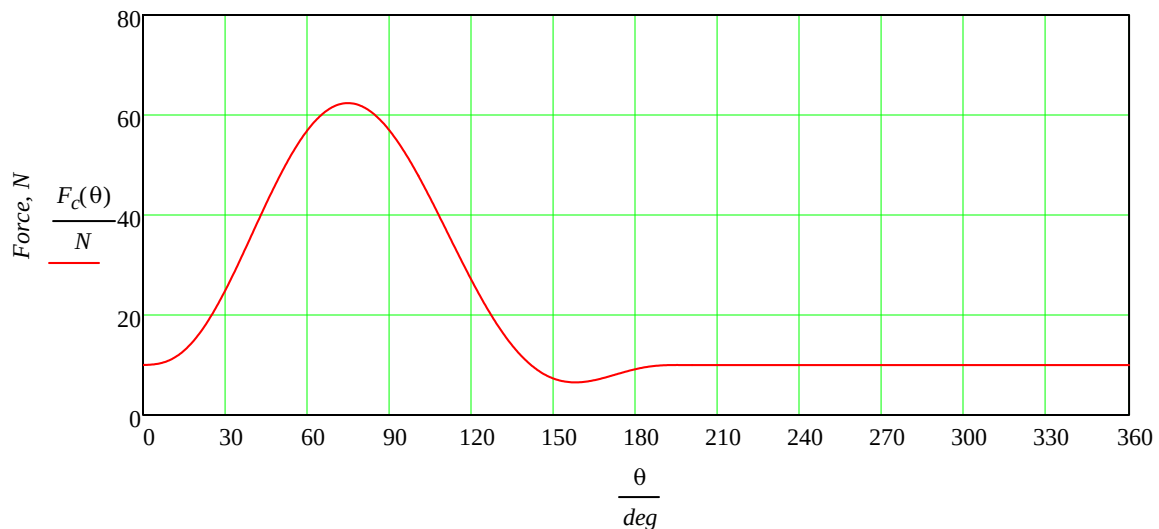
$$R(x, a, b) := \text{if}[(x > a) \wedge (x \leq b), 1, 0]$$

6. Write the SVA equations for the entire cycle in absolute units.

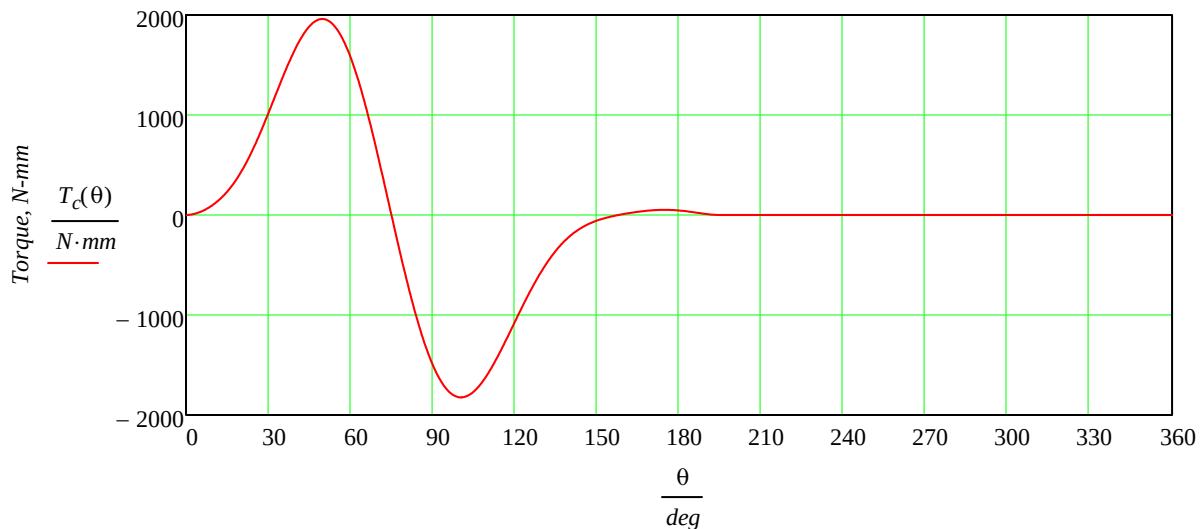
$$\begin{aligned} S(\theta) &:= R(\theta, 0 \cdot \text{deg}, \beta) \cdot S_1(\theta) + R(\theta, \beta, 360 \cdot \text{deg}) \cdot S_2(\theta) \\ V(\theta) &:= \omega \cdot (R(\theta, 0 \cdot \text{deg}, \beta) \cdot V_1(\theta) + R(\theta, \beta, 360 \cdot \text{deg}) \cdot V_2(\theta)) \\ A(\theta) &:= \omega^2 \cdot (R(\theta, 0 \cdot \text{deg}, \beta) \cdot A_1(\theta) + R(\theta, \beta, 360 \cdot \text{deg}) \cdot A_2(\theta)) \end{aligned}$$

7. Plot the force on the cam using equation 15.8.

$$F_c(\theta) := \frac{W}{g} \cdot A(\theta) + c \cdot V(\theta) + k \cdot S(\theta) + F_{pl} \quad \theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



8. Plot the driving torque on the cam using equation 15.10. $T_c(\theta) := \frac{F_c(\theta) \cdot V(\theta)}{\omega}$



PROBLEM 15-22

Statement: A single-dwell cam-follower system similar to that shown in Figure 15-1a provides a two-segment polynomial for a rise of 35 mm in 75 deg, a fall of 35 mm in 120 deg, and a dwell for the remainder of the cycle. Using equation 15.8, size a return spring and specify its preload to maintain contact between cam and follower. Then calculate and plot the dynamic force for one cycle if the roller follower train weighs 3.55 N, the system has a damping ratio of $\zeta = 0.06$, and the cam turns at 100 rpm.

Given: RISE/FALL DWELL
 $\beta := 195 \cdot \text{deg}$ $\beta_3 := 165 \cdot \text{deg}$
 $h := 35 \cdot \text{mm}$ $h_3 := 0.0 \cdot \text{mm}$
 Roller-follower train weight $W := 3.55 \cdot \text{N}$ Damping ratio $\zeta := 0.06$

Solution: See Mathcad file P1522.

1. The camshaft speed is $\omega := 100 \cdot \text{rpm}$ $\omega = 10.472 \frac{\text{rad}}{\text{sec}}$

Interactively chose a spring rate and preload, do the calculations and return to this point until satisfactory values are chosen.

$$k := 0.25 \cdot \text{N} \cdot \text{mm}^{-1} \qquad F_{pl} := 2.0 \cdot \text{N}$$

The damping coefficient (see equation 15.3) is

$$c := 2 \cdot \sqrt{k \cdot \frac{W}{g}} \cdot \zeta \qquad c = 1.142 \times 10^{-3} \frac{\text{N} \cdot \text{s}}{\text{mm}}$$

2. Using a two-segment polynomial, let the rise and fall, together, be one segment and the dwell be the second segment. Then, the boundary conditions are:

$$\begin{aligned} \text{at } \theta = 0: \quad s &= 0, \quad v = 0, \quad a = 0 \\ \theta = \beta_1: \quad s &= h, \quad v = 0 \\ \theta = \beta: \quad s &= 0, \quad v = 0, \quad a = 0 \end{aligned}$$

This is a minimum set of 8 BCs. The $v = 0$ condition at $\theta = \beta_1$ is required to keep the displacement from overshooting the lift, h . Define the total lift, the rise interval, the fall interval, and the ratio of rise to the total interval.

$$\text{Total lift:} \qquad h = 35.000 \text{ mm}$$

$$\text{Rise interval:} \quad \beta_1 := 75 \cdot \text{deg} \qquad A := \frac{\beta_1}{\beta} \qquad A = 0.385$$

$$\text{Fall interval:} \quad \beta_2 := 120 \cdot \text{deg}$$

Solving the polynomial equation for these boundary conditions yields the following coefficients:

$$\begin{aligned} c_3 &:= 5609.280 \cdot \text{mm} & c_4 &:= -24548.849 \cdot \text{mm} & c_5 &:= 39990.867 \cdot \text{mm} \\ c_6 &:= -28772.307 \cdot \text{mm} & c_7 &:= 7721.009 \cdot \text{mm} \end{aligned}$$

3. Write the SVA equations for the rise/fall segment.

$$\begin{aligned} S_I(\theta) &:= c_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + c_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + c_5 \cdot \left(\frac{\theta}{\beta}\right)^5 + c_6 \cdot \left(\frac{\theta}{\beta}\right)^6 + c_7 \cdot \left(\frac{\theta}{\beta}\right)^7 \\ V_I(\theta) &:= \frac{1}{\beta} \cdot \left[3 \cdot c_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot c_4 \cdot \left(\frac{\theta}{\beta}\right)^3 + 5 \cdot c_5 \cdot \left(\frac{\theta}{\beta}\right)^4 + 6 \cdot c_6 \cdot \left(\frac{\theta}{\beta}\right)^5 + 7 \cdot c_7 \cdot \left(\frac{\theta}{\beta}\right)^6 \right] \end{aligned}$$

$$A_1(\theta) := \frac{1}{\beta^2} \left[6 \cdot c_3 \cdot \left(\frac{\theta}{\beta} \right) + 12 \cdot c_4 \cdot \left(\frac{\theta}{\beta} \right)^2 + 20 \cdot c_5 \cdot \left(\frac{\theta}{\beta} \right)^3 + 30 \cdot c_6 \cdot \left(\frac{\theta}{\beta} \right)^4 + 42 \cdot c_7 \cdot \left(\frac{\theta}{\beta} \right)^5 \right]$$

4. Write the SVA equations for the dwell segment.

$$S_2(\theta) := 0 \cdot mm \quad V_2(\theta) := 0 \cdot mm \quad A_2(\theta) := 0 \cdot mm$$

5. To define the SVA curves for the complete cycle define a range function that has a value of one between the values of a and b and zero elsewhere.

$$R(x, a, b) := \text{if}[(x > a) \wedge (x \leq b), 1, 0]$$

6. Write the SVA equations for the entire cycle in absolute units.

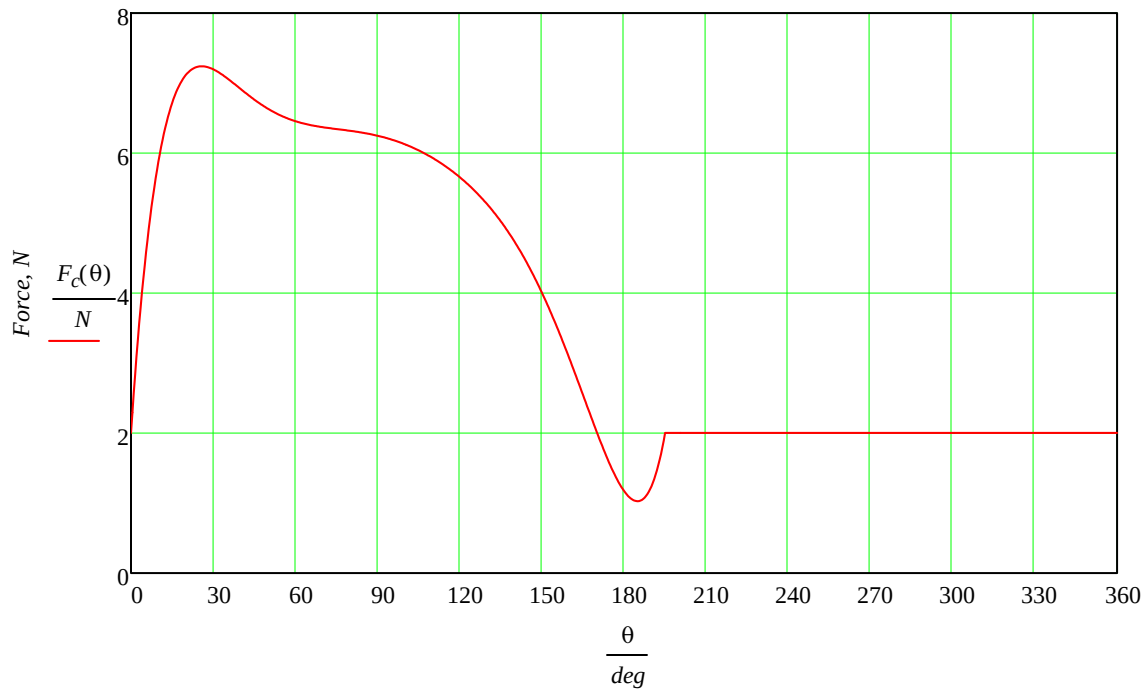
$$S(\theta) := R(\theta, 0 \cdot deg, \beta) \cdot S_1(\theta) + R(\theta, \beta, 360 \cdot deg) \cdot S_2(\theta)$$

$$V(\theta) := \omega \cdot (R(\theta, 0 \cdot deg, \beta) \cdot V_1(\theta) + R(\theta, \beta, 360 \cdot deg) \cdot V_2(\theta))$$

$$A(\theta) := \omega^2 \cdot (R(\theta, 0 \cdot deg, \beta) \cdot A_1(\theta) + R(\theta, \beta, 360 \cdot deg) \cdot A_2(\theta))$$

7. Plot the force on the cam using equation 15.8.

$$F_c(\theta) := \frac{W}{g} \cdot A(\theta) + c \cdot V(\theta) + k \cdot S(\theta) + F_{pl} \quad \theta := 0 \cdot deg, 0.5 \cdot deg .. 360 \cdot deg$$



PROBLEM 15-23

Statement: A single-dwell cam-follower system similar to that shown in Figure 15-1a provides a constant velocity to the follower of 100 mm/sec for 2 sec then returns to its starting position with a total cycle time of 3 sec. Using equations 15.8 and 15.10, calculate and plot the dynamic force and torque for one cycle if the roller follower train weighs 4.5 N, the system has a damping ratio of $\zeta = 0.06$, and the spring has a rate of 2.5 N/mm with a preload of 50 N.

Given: Constant velocity: $v_c := 100 \cdot \text{mm} \cdot \text{sec}^{-1}$
 Time duration of cv segment: $t_{cv} := 2 \cdot \text{sec}$ Cycle time: $\tau := 3 \cdot \text{sec}$
 Roller-follower train weight $W := 4.5 \cdot \text{N}$ Damping ratio $\zeta := 0.06$
 Spring rate and preload $k := 2.5 \cdot \text{N} \cdot \text{mm}^{-1}$ $F_{pl} := 50 \cdot \text{N}$

Solution: See Mathcad file P1523.

1. The camshaft speed is $\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau}$ $\omega = 2.094 \frac{\text{rad}}{\text{sec}}$

The damping coefficient (see equation 15.3) is

$$c := 2 \cdot \sqrt{k \cdot \frac{W}{g}} \cdot \zeta \quad c = 4.064 \times 10^{-3} \frac{\text{N} \cdot \text{s}}{\text{mm}}$$

2. Use a two-segment polynomial as demonstrated in Example 8-12. The lift during the first segment and the sva equations for the first segment are:

$$\begin{aligned} \text{Normalized velocity: } v_{cv} &:= \frac{v_c}{\omega} & v_{cv} &= 47.746 \text{ mm} \\ h_{cv} &:= v_c \cdot t_{cv} & h_{cv} &= 200.000 \text{ mm} & \beta_1 &:= \frac{t_{cv}}{\tau} \cdot 360 \cdot \text{deg} & \beta_1 &= 240 \text{ deg} \\ s_1(\theta) &:= h_{cv} \cdot \frac{\theta}{\beta_1} & v_1(\theta) &:= v_{cv} & a_1(\theta) &:= 0 \cdot \text{mm} \end{aligned}$$

3. The boundary conditions for the second segment are:

$$\begin{aligned} \text{at } \theta = \beta_1: & \quad s = h_{cv}, \quad v = v_{cv}, \quad a = 0 \\ \theta = 360 \text{ deg: } & \quad s = 0, \quad v = v_{cv}, \quad a = 0 \end{aligned}$$

This is a minimum set of 6 BCs. Define the total interval and the constant velocity interval, and the ratio of constant velocity interval to the total interval.

$$\begin{aligned} \text{Total interval: } \beta &:= 360 \cdot \text{deg} \\ \text{CV interval: } \beta_1 &= 240 \text{ deg} & A &:= \frac{\beta_1}{\beta} & A &= 0.667 \end{aligned}$$

4. Use the 6 BCs and equation 8.23 to write 6 equations in s , v , and a similar to those in example 8-9 but with 6 terms in the equation for s (the highest term will be fifth degree). The resulting coefficients are:

$$\begin{aligned} c_0 &:= 153600 \cdot \text{mm} & c_1 &:= -971700 \cdot \text{mm} & c_2 &:= 2430000 \cdot \text{mm} \\ c_3 &:= -2997000 \cdot \text{mm} & c_4 &:= 1822500 \cdot \text{mm} & c_5 &:= -437400 \cdot \text{mm} \end{aligned}$$

5. Write the sva equations for the return segment.

$$\begin{aligned} s_2(\theta) &:= c_0 + c_1 \left(\frac{\theta}{\beta} \right) + c_2 \left(\frac{\theta}{\beta} \right)^2 + c_3 \left(\frac{\theta}{\beta} \right)^3 + c_4 \left(\frac{\theta}{\beta} \right)^4 + c_5 \left(\frac{\theta}{\beta} \right)^5 \\ v_2(\theta) &:= \frac{1}{\beta} \left[c_1 + 2 \cdot c_2 \left(\frac{\theta}{\beta} \right) + 3 \cdot c_3 \left(\frac{\theta}{\beta} \right)^2 + 4 \cdot c_4 \left(\frac{\theta}{\beta} \right)^3 + 5 \cdot c_5 \left(\frac{\theta}{\beta} \right)^4 \right] \end{aligned}$$

$$a_2(\theta) := \frac{1}{\beta^2} \cdot \left[2 \cdot c_2 + 6 \cdot c_3 \cdot \left(\frac{\theta}{\beta} \right) + 12 \cdot c_4 \cdot \left(\frac{\theta}{\beta} \right)^2 + 20 \cdot c_5 \cdot \left(\frac{\theta}{\beta} \right)^3 \right]$$

6. To define the SVA curves for the complete cycle define a range function that has a value of one between the values of a and b and zero elsewhere.

$$R(x, a, b) := \text{if}[(x > a) \wedge (x \leq b), 1, 0]$$

7. Write the SVA equations for the entire cycle in absolute units.

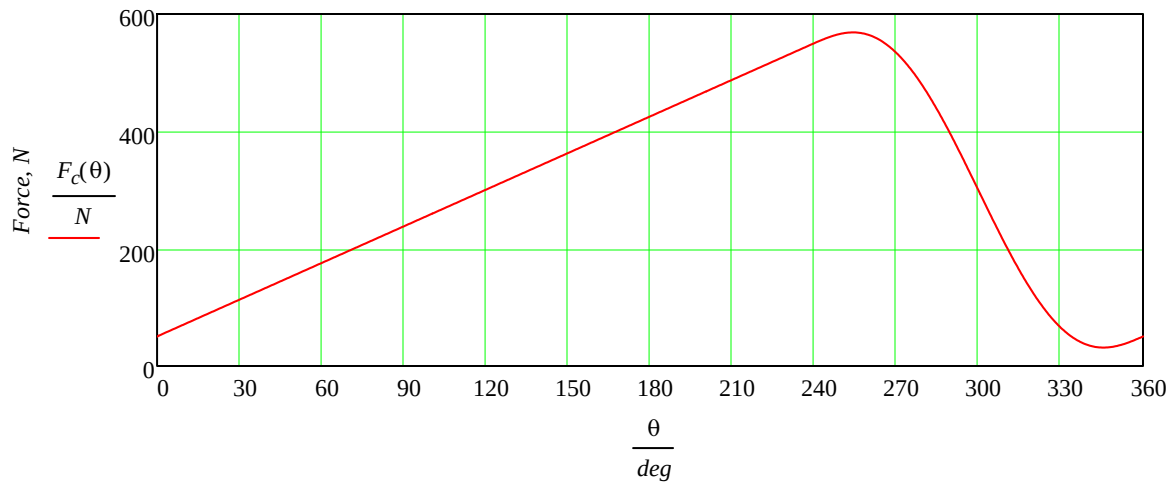
$$S(\theta) := R(\theta, 0 \cdot \text{deg}, \beta_1) \cdot s_1(\theta) + R(\theta, \beta_1, 360 \cdot \text{deg}) \cdot s_2(\theta)$$

$$V(\theta) := \omega \cdot (R(\theta, 0 \cdot \text{deg}, \beta_1) \cdot v_1(\theta) + R(\theta, \beta_1, 360 \cdot \text{deg}) \cdot v_2(\theta))$$

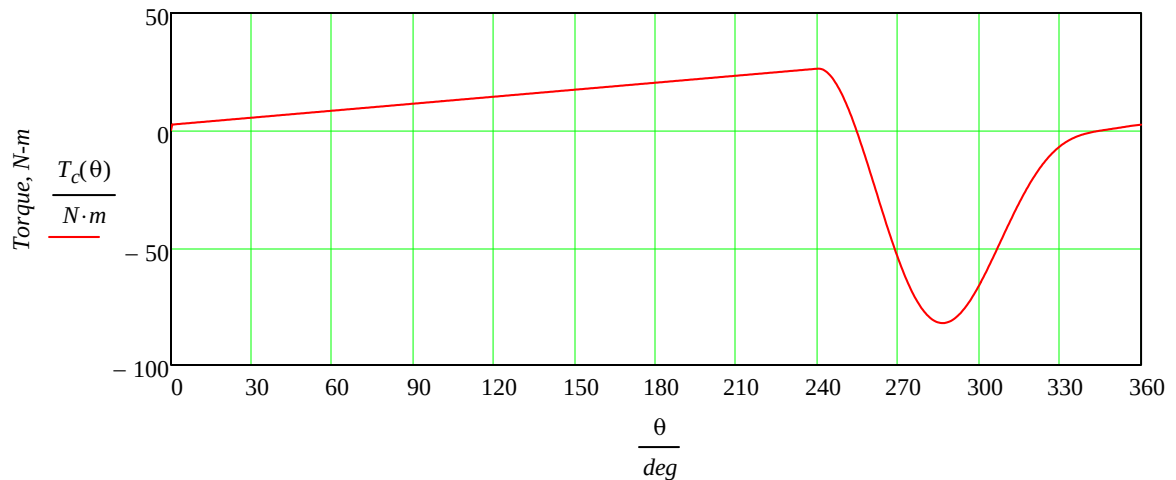
$$A(\theta) := \omega^2 \cdot (R(\theta, 0 \cdot \text{deg}, \beta_1) \cdot a_1(\theta) + R(\theta, \beta_1, 360 \cdot \text{deg}) \cdot a_2(\theta))$$

8. Plot the force on the cam using equation 15.8.

$$F_c(\theta) := \frac{W}{g} \cdot A(\theta) + c \cdot V(\theta) + k \cdot S(\theta) + F_{pl} \quad \theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. 360 \cdot \text{deg}$$



9. Plot the driving torque on the cam using equation 15.10. $T_c(\theta) := \frac{F_c(\theta) \cdot V(\theta)}{\omega}$



PROBLEM 15-24

Statement: A single-dwell cam-follower system similar to that shown in Figure 15-1a provides a constant velocity to the follower of 100 mm/sec for 2 sec then returns to its starting position with a total cycle time of 3 sec. Using equation 15.8, size a return spring and specify its preload to maintain contact between cam and follower. Then calculate and plot the dynamic force for one cycle if the roller follower train weighs 5.0 N and the system has a damping ratio of $\zeta = 0.06$.

Given: Constant velocity: $v_c := 100 \cdot \text{mm} \cdot \text{sec}^{-1}$
 Time duration of cv segment: $t_{cv} := 2 \cdot \text{sec}$ Cycle time: $\tau := 3 \cdot \text{sec}$
 Roller-follower train weight $W := 5.0 \cdot \text{N}$ Damping ratio $\zeta := 0.06$

Solution: See Mathcad file P1524.

1. The camshaft speed is $\omega := \frac{2 \cdot \pi \cdot \text{rad}}{\tau}$ $\omega = 2.094 \frac{\text{rad}}{\text{sec}}$

Interactively chose a spring rate and preload, do the calculations and return to this point until satisfactory values are chosen.

$$k := 0.25 \cdot \text{N} \cdot \text{mm}^{-1} \quad F_{pl} := 4.0 \cdot \text{N}$$

The damping coefficient (see equation 15.3) is

$$c := 2 \cdot \sqrt{k \cdot \frac{W}{g}} \cdot \zeta \quad c = 1.355 \times 10^{-3} \frac{\text{N} \cdot \text{s}}{\text{mm}}$$

2. Use a two-segment polynomial as demonstrated in Example 8-12. The lift during the first segment and the sva equations for the first segment are:

$$\begin{aligned} \text{Normalized velocity: } v_{cv} &:= \frac{v_c}{\omega} & v_{cv} &= 47.746 \text{ mm} \\ h_{cv} &:= v_c \cdot t_{cv} & h_{cv} &= 200.000 \text{ mm} & \beta_1 &:= \frac{t_{cv}}{\tau} \cdot 360 \cdot \text{deg} & \beta_1 &= 240 \text{ deg} \\ s_1(\theta) &:= h_{cv} \cdot \frac{\theta}{\beta_1} & v_1(\theta) &:= v_{cv} & a_1(\theta) &:= 0 \cdot \text{mm} \end{aligned}$$

3. The boundary conditions for the second segment are:

$$\begin{aligned} \text{at } \theta = \beta_1: & \quad s = h_{cv}, \quad v = v_{cv}, \quad a = 0 \\ \theta = 360 \text{ deg: } & \quad s = 0, \quad v = v_{cv}, \quad a = 0 \end{aligned}$$

This is a minimum set of 6 BCs. Define the total interval and the constant velocity interval, and the ratio of constant velocity interval to the total interval.

$$\text{Total interval: } \beta := 360 \cdot \text{deg}$$

$$\text{CV interval: } \beta_1 = 240 \text{ deg} \quad A := \frac{\beta_1}{\beta} \quad A = 0.667$$

4. Use the 6 BCs and equation 8.23 to write 6 equations in s , v , and a similar to those in example 8-9 but with 6 terms in the equation for s (the highest term will be fifth degree). The resulting coefficients are:

$$\begin{aligned} c_0 &:= 153600 \cdot \text{mm} & c_1 &:= -971700 \cdot \text{mm} & c_2 &:= 2430000 \cdot \text{mm} \\ c_3 &:= -2997000 \cdot \text{mm} & c_4 &:= 1822500 \cdot \text{mm} & c_5 &:= -437400 \cdot \text{mm} \end{aligned}$$

5. Write the sva equations for the return segment.

$$s_2(\theta) := c_0 + c_1 \cdot \left(\frac{\theta}{\beta}\right) + c_2 \cdot \left(\frac{\theta}{\beta}\right)^2 + c_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + c_4 \cdot \left(\frac{\theta}{\beta}\right)^4 + c_5 \cdot \left(\frac{\theta}{\beta}\right)^5$$

$$v_2(\theta) := \frac{1}{\beta} \cdot \left[c_1 + 2 \cdot c_2 \cdot \left(\frac{\theta}{\beta} \right) + 3 \cdot c_3 \cdot \left(\frac{\theta}{\beta} \right)^2 + 4 \cdot c_4 \cdot \left(\frac{\theta}{\beta} \right)^3 + 5 \cdot c_5 \cdot \left(\frac{\theta}{\beta} \right)^4 \right]$$

$$a_2(\theta) := \frac{1}{\beta^2} \cdot \left[2 \cdot c_2 + 6 \cdot c_3 \cdot \left(\frac{\theta}{\beta} \right) + 12 \cdot c_4 \cdot \left(\frac{\theta}{\beta} \right)^2 + 20 \cdot c_5 \cdot \left(\frac{\theta}{\beta} \right)^3 \right]$$

6. To define the SVA curves for the complete cycle define a range function that has a value of one between the values of a and b and zero elsewhere.

$$R(x, a, b) := \text{if}[(x > a) \wedge (x \leq b), 1, 0]$$

7. Write the SVA equations for the entire cycle in absolute units.

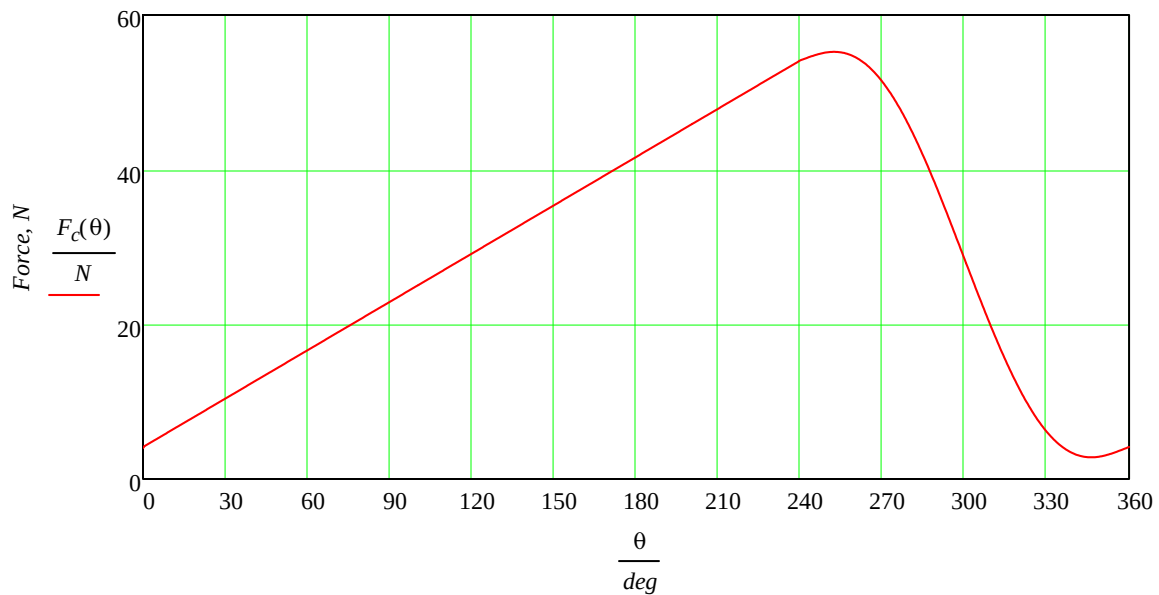
$$S(\theta) := R(\theta, 0 \cdot \text{deg}, \beta_1) \cdot s_1(\theta) + R(\theta, \beta_1, 360 \cdot \text{deg}) \cdot s_2(\theta)$$

$$V(\theta) := \omega \cdot (R(\theta, 0 \cdot \text{deg}, \beta_1) \cdot v_1(\theta) + R(\theta, \beta_1, 360 \cdot \text{deg}) \cdot v_2(\theta))$$

$$A(\theta) := \omega^2 \cdot (R(\theta, 0 \cdot \text{deg}, \beta_1) \cdot a_1(\theta) + R(\theta, \beta_1, 360 \cdot \text{deg}) \cdot a_2(\theta))$$

8. Plot the force on the cam using equation 15.8.

$$F_c(\theta) := \frac{W}{g} \cdot A(\theta) + c \cdot V(\theta) + k \cdot S(\theta) + F_{pl} \quad \theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} .. 360 \cdot \text{deg}$$



PROBLEM 15-25

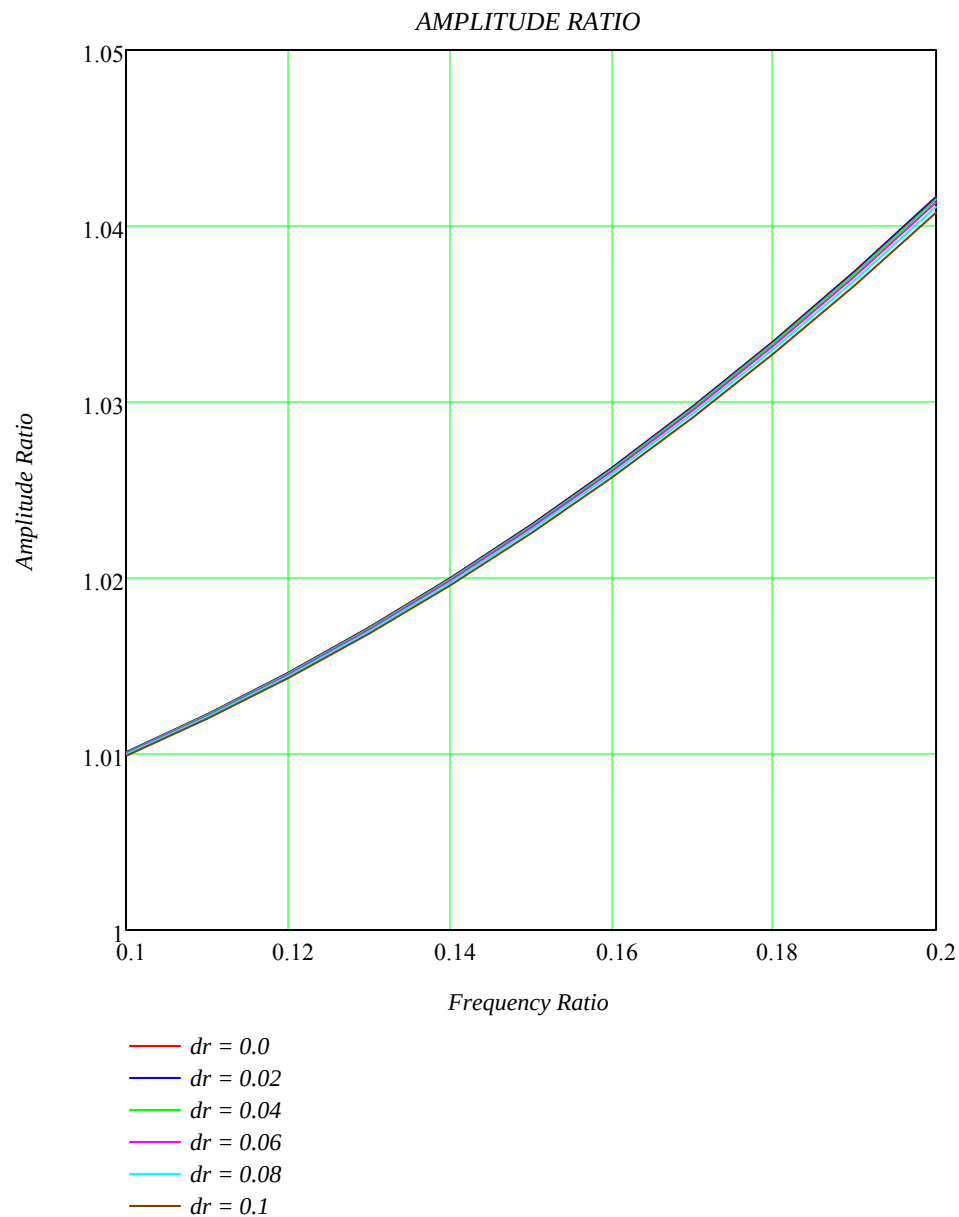
Statement: As stated in Section 15.2, "A common rule of thumb is to design the (cam-follower) system to have a fundamental frequency ω_n at least ten times the highest forcing frequency expected in service..." Since this is not always possible, calculate and plot the amplitude ratio from $5 \leq \omega_n/\omega_f \leq 10$ for damping ratios of 0, 0.02, 0.04, 0.06, 0.08, and 0.10.

Given: Frequency ratio: $fr := 0.1, 0.11 \dots 0.2$ (frequency ratio is the reciprocal of ω_n/ω_f)

Solution: See Mathcad file P1525.

1. Calculate the amplitude ratio as a function of frequency ratio and damping ratio using equation 15.6e.

$$AR(fr, dr) := \frac{1}{\sqrt{(1 - fr^2)^2 + (2 \cdot dr \cdot fr)^2}}$$



PROBLEM 15-26

Statement: A cam-follower system similar to that shown in Figure P15-2 has a lumped mass of 0.1 kg, lumped stiffness of 0.10 N/mm, and a damping coefficient of 0.40 kg/s. If the forcing frequency is 50 rpm, determine the resulting amplitude ratio.

Given: $M := 0.1 \cdot \text{kg}$ $k := 0.10 \cdot \text{N} \cdot \text{mm}^{-1}$ $c := 0.40 \cdot \text{kg} \cdot \text{s}^{-1}$ $\omega_f := 50 \cdot \text{rpm}$

Solution: See Figure P15-2 and Mathcad file P1526.

1. Calculate the undamped natural frequency using equation 15.1d.

$$\omega_n := \sqrt{\frac{k}{M}} \qquad \omega_n = 31.623 \cdot \frac{\text{rad}}{\text{sec}} \qquad \omega_f = 5.236 \cdot \frac{\text{rad}}{\text{sec}}$$

2. Calculate the damping ratio using equation 15.3a.

$$\zeta := \frac{c}{2 \cdot M \cdot \omega_n} \qquad \zeta = 0.063$$

3. Calculate the amplitude ratio using equation 15.6e.

$$\text{ampratio} := \frac{1}{\sqrt{\left[1 - \left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left(2 \cdot \zeta \cdot \frac{\omega_f}{\omega_n}\right)^2}} \qquad \text{ampratio} = 1.028$$

PROBLEM 15-27

Statement: A cam-follower system similar to that shown in Figure P15-1 has a lumped mass of 2.5 kg, lumped stiffness of 4 N/mm, and a damping coefficient of 12.0 kg/s. If the forcing frequency is 80 rpm, determine the resulting amplitude ratio.

Given: $M := 2.5 \cdot \text{kg}$ $k := 4.0 \cdot \text{N} \cdot \text{mm}^{-1}$ $c := 12.0 \cdot \text{kg} \cdot \text{s}^{-1}$ $\omega_f := 80 \cdot \text{rpm}$

Solution: See Figure P15-2 and Mathcad file P1526.

1. Calculate the undamped natural frequency using equation 15.1d.

$$\omega_n := \sqrt{\frac{k}{M}} \qquad \omega_n = 40.000 \cdot \frac{\text{rad}}{\text{sec}} \qquad \omega_f = 8.378 \cdot \frac{\text{rad}}{\text{sec}}$$

2. Calculate the damping ratio using equation 15.3a.

$$\zeta := \frac{c}{2 \cdot M \cdot \omega_n} \qquad \zeta = 0.060$$

3. Calculate the amplitude ratio using equation 15.6e.

$$\text{ampratio} := \frac{1}{\sqrt{\left[1 - \left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left(2 \cdot \zeta \cdot \frac{\omega_f}{\omega_n}\right)^2}} \qquad \text{ampratio} = 1.046$$

PROBLEM 16-1

Statement: Design a double-dwell motion to translate a slider from 0 to 75 mm in 60-deg (timing angle) with modified sine acceleration, dwell for 120-deg, fall 75 mm in 30-deg with cycloidal motion, and dwell for the remainder. Calculate and plot the displacement function of the slider using DYNACAM and modify that function to drive the crank of a non-offset crank-slider with the dimensions: crank = 100 mm, coupler = 300 mm such that at maximum stroke of the slider, the crank is at zero degrees. The slider must follow the specified motion. Plot the resulting crank input motion.

Given: Crank-slider geometry:

$$L_2 := 100 \cdot \text{mm} \quad L_3 := 300 \cdot \text{mm} \quad \text{Offset} := 0 \cdot \text{mm}$$

Motion data:

RISE	DWELL	FALL	DWELL
$\beta_1 := 60 \cdot \text{deg}$	$\beta_2 := 120 \cdot \text{deg}$	$\beta_3 := 30 \cdot \text{deg}$	$\beta_4 := 150 \cdot \text{deg}$
$h_1 := 75 \cdot \text{mm}$	$h_2 := 0 \cdot \text{mm}$	$h_3 := 75 \cdot \text{mm}$	$h_4 := 0 \cdot \text{mm}$
modified sine		cycloidal	

Table 8-2 Parameters and coefficients for the modified sine acceleration:

$$b := 0.25 \quad c := 0.00 \quad d := 0.75 \quad C_a := 5.5280$$

Solution: See Table 8-2 and Mathcad file P1601.

- Using equations 8.12d and 8.14 through 8.19, write the displacement equations for the rise and fall of the slider

Rise: modified sine

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left(\frac{\pi}{b} \cdot x \right) \right]$$

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2} \right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \right]$$

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2} \right) \cdot x + \left(\frac{d}{\pi} \right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2} \right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi} \right)^2 \cdot \cos \left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2} \right) \right] \right]$$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + (2 \cdot d^2 - b^2) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot (d^2 - b^2)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin \left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

Fall: cycloidal

$$y(x) := \left(x - \frac{1}{2 \cdot \pi} \cdot \sin(2 \cdot \pi \cdot x) \right)$$

- The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the displacement curve, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

- The global displacement equation is composed of four intervals (rise, dwell, fall, and dwell). The local equation above must be assembled into a single equation for the displacement that applies over the range $0 \leq \theta \leq 360$ deg of the timing angle.
- Write the local displacement equation for the first interval, $0 \leq \theta \leq \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 \leq \theta \leq \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \dots \right. \\ \left. + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

For $\beta_1 \leq \theta \leq \beta_1 + \beta_2$ (dwell)

$$s_2(x) := h_1$$

For $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$ (fall)

$$s_3(x) := h_3 \cdot (1 - y(x))$$

For $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq 2\pi$ (dwell)

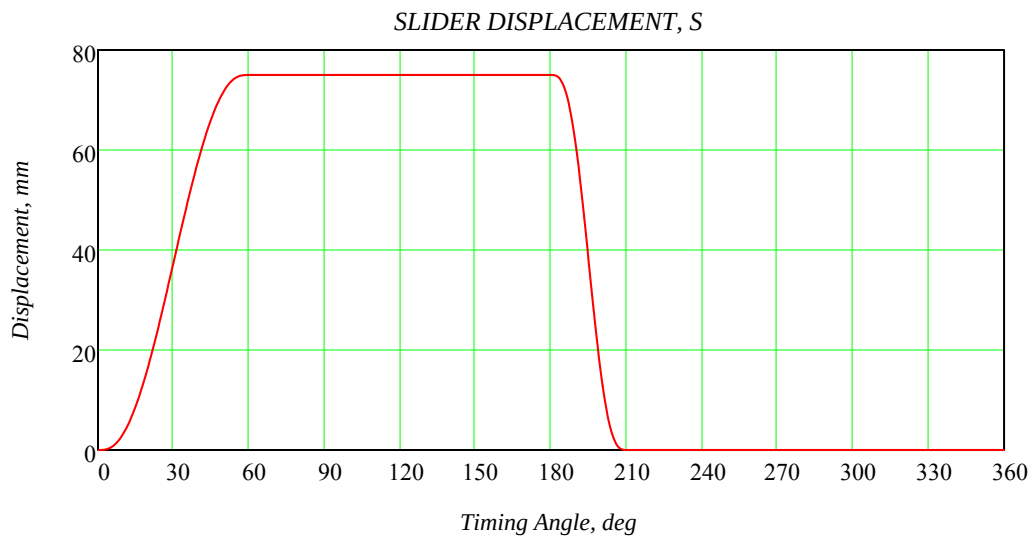
$$s_4(x) := 0 \cdot \text{mm}$$

- Write the complete global equation for the displacement and plot it over one timing cycle, which is the sum of the four intervals defined above.

$$\text{Let } \theta_1 := \beta_1 \quad \theta_2 := \theta_1 + \beta_2 \quad \theta_3 := \theta_2 + \beta_3 \quad \theta_4 := \theta_3 + \beta_4$$

$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2) \cdot s_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \dots \\ + R(\theta, \theta_2, \theta_3) \cdot s_3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4) \cdot s_4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$

$$\theta := 0 \cdot \text{deg}, 0.5 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



6. Write the equation for the slider position as a function of the timing angle, θ

Top dead center ($\theta_2 = 0\text{-deg}$): $TDC := L_2 + L_3$ $TDC = 400.000\text{ mm}$

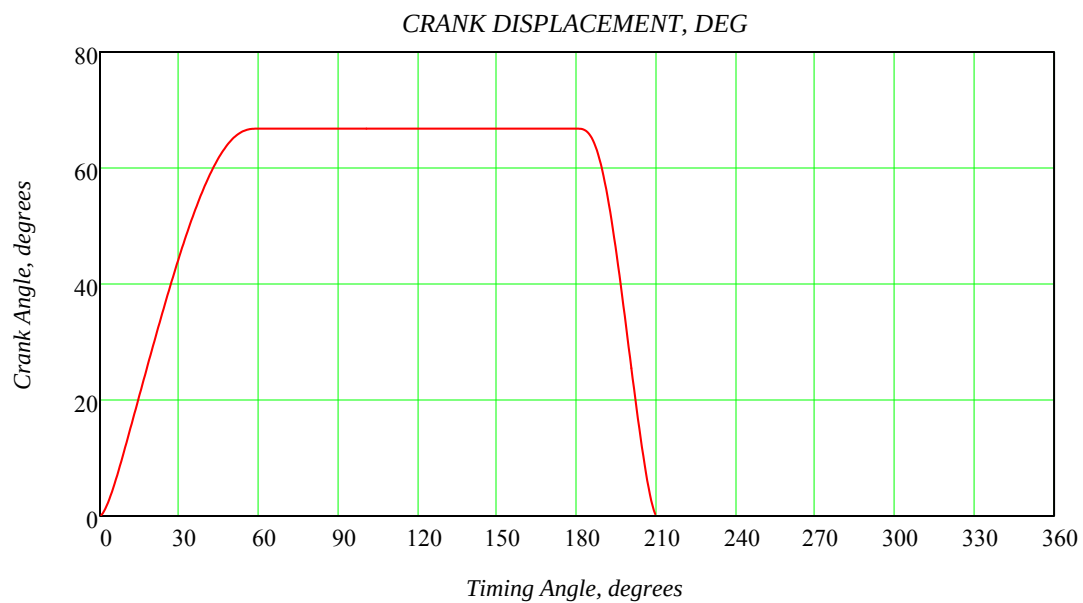
Slider position: $d(\theta) := TDC - S(\theta)$

7. Using equations 4.20 and 4.21, write the equation for the crank angle, θ_2 as a function of the timing angle, θ

$$K_1(\theta) := L_2^2 - L_3^2 + (d(\theta))^2 \quad K_2 := 0 \cdot \text{mm}^2 \quad K_3(\theta) := -2 \cdot L_2 \cdot d(\theta)$$

$$A(\theta) := K_1(\theta) - K_3(\theta) \quad B := 2 \cdot K_2 \quad C(\theta) := K_1(\theta) + K_3(\theta)$$

$$\theta_2(\theta) := 2 \cdot \text{atan2}\left(2 \cdot A(\theta), -B + \sqrt{B^2 - 4 \cdot A(\theta) \cdot C(\theta)}\right)$$



Maximum crank angle:

$$\theta_{2max} := \theta_2(60 \cdot \text{deg})$$

$$\theta_{2max} = 66.782 \cdot \text{deg}$$

PROBLEM 16-2

Statement: Design a double-dwell motion to translate a slider from 0 to 65 mm in 80-deg (timing angle) with 3-4-5 polynomial, dwell for 100-deg, fall 65 mm in 40-deg with 3-4-5 polynomial motion, and dwell for the remainder. Calculate and plot the displacement function of the slider using DYNACAM and modify that function to drive the crank of a non-offset crank-slider with the dimensions: crank = 100 mm, coupler = 300 mm such that at maximum stroke of the slider, the crank is at zero degrees. The slider must follow the specified motion. Plot the resulting crank input motion.

Given: Crank-slider geometry:

$$L_2 := 100 \cdot \text{mm} \quad L_3 := 300 \cdot \text{mm} \quad \text{Offset} := 0 \cdot \text{mm}$$

Motion data:

RISE	DWELL	FALL	DWELL
$\beta_1 := 80 \cdot \text{deg}$	$\beta_2 := 100 \cdot \text{deg}$	$\beta_3 := 40 \cdot \text{deg}$	$\beta_4 := 140 \cdot \text{deg}$
$h_1 := 65 \cdot \text{mm}$	$h_2 := 0 \cdot \text{mm}$	$h_3 := 65 \cdot \text{mm}$	$h_4 := 0 \cdot \text{mm}$
3-4-5 poly		3-4-5 poly	

Solution: See Mathcad file P1602.

- Using equation 8.24, write the displacement equation for the rise and fall of the slider.

Rise and fall: 3-4-5 polynomial

$$y(x) := 10 \cdot x^3 - 15 \cdot x^4 + 6 \cdot x^5$$

- The above equations can be used for a rise or fall by using the proper values of θ , β , and h . To plot the displacement curve, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x_1, x_2) := \text{if}[(x > x_1) \wedge (x \leq x_2), 1, 0]$$

- The global displacement equation is composed of four intervals (rise, dwell, fall, and dwell). The local equation above must be assembled into a single equation for the displacement that applies over the range $0 \leq \theta \leq 360$ deg of the timing angle.
- Write the local displacement equation for each interval.

$$\text{For } 0 \leq \theta \leq \beta_1 \text{ (Rise)} \quad s_1(x) := h_1 \cdot y(x)$$

$$\text{For } \beta_1 \leq \theta \leq \beta_1 + \beta_2 \text{ (dwell)} \quad s_2(x) := h_1$$

$$\text{For } \beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3 \text{ (fall)} \quad s_3(x) := h_3 \cdot (1 - y(x))$$

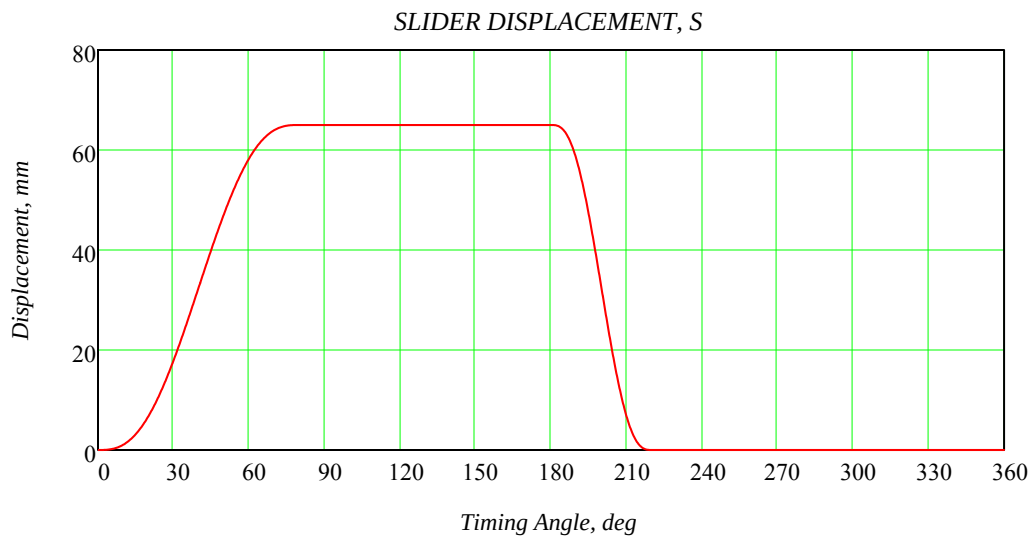
$$\text{For } \beta_1 + \beta_2 + \beta_3 \leq \theta \leq 2\pi \text{ (dwell)} \quad s_4(x) := 0 \cdot \text{mm}$$

- Write the complete global equation for the displacement and plot it over one timing cycle, which is the sum of the four intervals defined above.

$$\text{Let} \quad \theta_1 := \beta_1 \quad \theta_2 := \theta_1 + \beta_2 \quad \theta_3 := \theta_2 + \beta_3 \quad \theta_4 := \theta_3 + \beta_4$$

$$S(\theta) := R(\theta, 0, \theta_1) \cdot s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2) \cdot s_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \dots \\ + R(\theta, \theta_2, \theta_3) \cdot s_3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4) \cdot s_4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$

$$\theta := 0 \cdot \text{deg}, 0.2 \cdot \text{deg} \dots 360 \cdot \text{deg}$$



6. Write the equation for the slider position as a function of the timing angle, θ

Top dead center ($\theta_2 = 0\text{-deg}$): $TDC := L_2 + L_3$ $TDC = 400.000\text{ mm}$

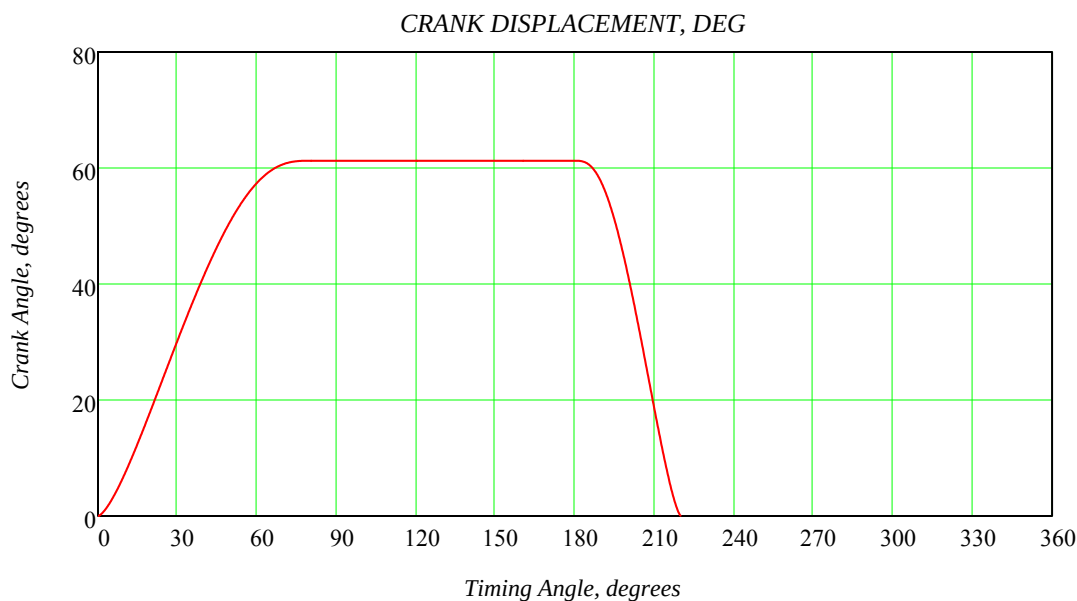
Slider position: $d(\theta) := TDC - S(\theta)$

7. Using equations 4.20 and 4.21, write the equation for the crank angle, θ_2 as a function of the timing angle, θ

$$K_1(\theta) := L_2^2 - L_3^2 + (d(\theta))^2 \quad K_2 := 0 \cdot \text{mm}^2 \quad K_3(\theta) := -2 \cdot L_2 \cdot d(\theta)$$

$$A(\theta) := K_1(\theta) - K_3(\theta) \quad B := 2 \cdot K_2 \quad C(\theta) := K_1(\theta) + K_3(\theta)$$

$$\theta_2(\theta) := 2 \cdot \text{atan2}\left(2 \cdot A(\theta), -B + \sqrt{B^2 - 4 \cdot A(\theta) \cdot C(\theta)}\right)$$



Maximum crank angle: $\theta_{2max} := \theta_2(65\text{-deg})$ $\theta_{2max} = 59.468\text{-deg}$

PROBLEM 16-3

Statement: Design a single-dwell polynomial motion to translate a slider from 0 to 75 mm and return to 0 in 120-deg (timing angle) and dwell for the remainder. Calculate and plot the displacement function of the slider using DYNACAM and modify that function to drive the crank of a non-offset crank-slider with the dimensions: crank = 90 mm, coupler = 270 mm such that at maximum stroke of the slider, the crank is at 180 degrees. The slider must follow the specified motion. Plot the resulting crank input motion.

Given: Crank-slider geometry:
 $L_2 := 90 \cdot \text{mm}$ $L_3 := 270 \cdot \text{mm}$ $\text{Offset} := 0 \cdot \text{mm}$

Motion data:

RISE-FALL DWELL

$\beta_1 := 120 \cdot \text{deg}$ $\beta_2 := 240 \cdot \text{deg}$

$h_1 := 75 \cdot \text{mm}$ $h_2 := 0 \cdot \text{mm}$

polynomial

Solution: See Example 8-8 and Mathcad file P1603.

- Using equation (a) from Example 8-8, write the displacement equation for the rise and fall of the slider.

Rise and fall: siingle dwell polynomial

$$y(x) := 64 \cdot x^3 - 192 \cdot x^4 + 192 \cdot x^5 - 64 \cdot x^6$$

- The above equation can be used for a rise or fall by using the proper values of θ , β , and h . To plot the displacement curve, first define a range function that has a value of one between the values of $x1$ and $x2$ and zero elsewhere.

$$R(x, x1, x2) := \text{if}[(x > x1) \wedge (x \leq x2), 1, 0]$$

- The global displacement equation is composed of two intervals (rise-fall, and dwell). The local equation above must be assembled into a single equation for the displacement that applies over the range $0 \leq \theta \leq 360$ deg of the timing angle.
- Write the local displacement equation for each of the two intervals.

$$\text{For } 0 \leq \theta \leq \beta_1 \text{ (Rise-Fall)} \quad s_1(x) := h_1 \cdot y(x)$$

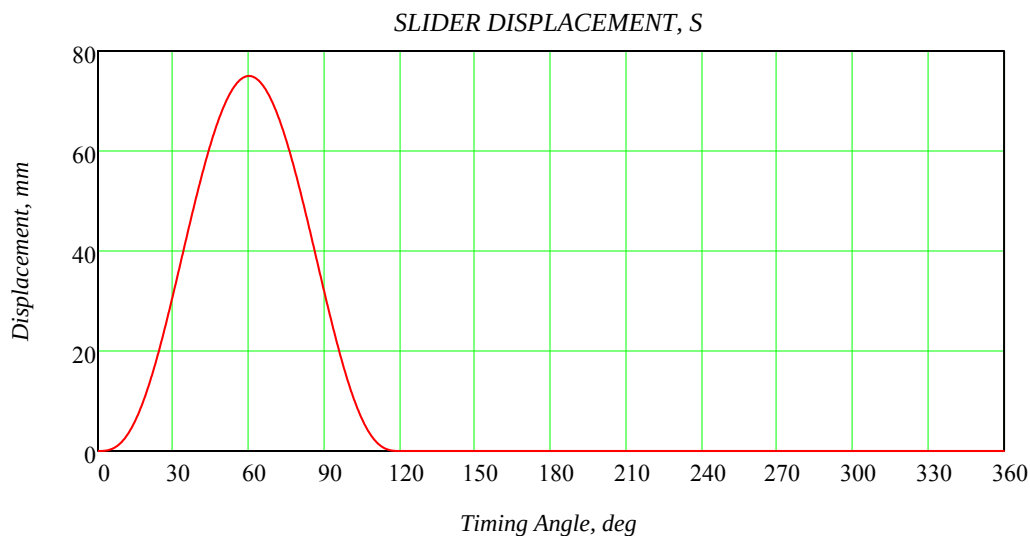
$$\text{For } \beta_1 \leq \theta \leq 360 \text{ (dwell)} \quad s_2(x) := 0 \cdot \text{mm}$$

- Write the complete global equation for the displacement and plot it over one timing cycle, which is the sum of the four intervals defined above.

$$\text{Let} \quad \theta_1 := \beta_1 \quad \theta_2 := 360 \cdot \text{deg}$$

$$S(\theta) := R(\theta, 0, \theta_1) \cdot s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2) \cdot s_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right)$$

$$\theta := 0 \cdot \text{deg}, 0.2 \cdot \text{deg} .. 360 \cdot \text{deg}$$



6. Write the equation for the slider position as a function of the timing angle, θ

Bottom dead center ($\theta_2 = 180\text{-deg}$): $BDC := L_3 - L_2$

$BDC = 180.000\text{ mm}$

Slider position:

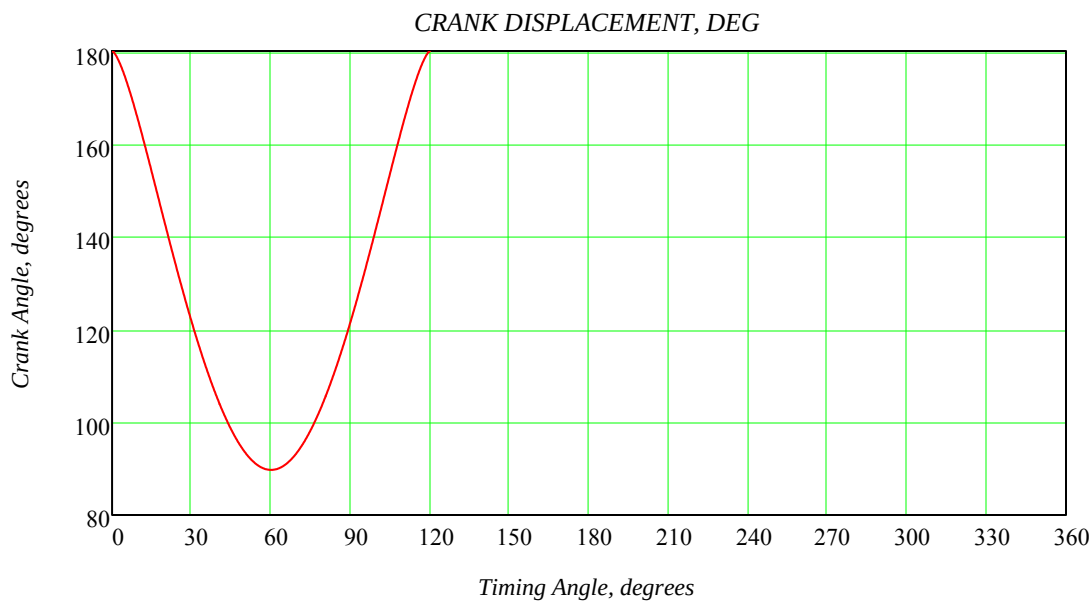
$d(\theta) := BDC + S(\theta)$

7. Using equations 4.20 and 4.21, write the equation for the crank angle, θ_2 as a function of the timing angle, θ

$K_1(\theta) := L_2^2 - L_3^2 + (d(\theta))^2 \quad K_2 := 0 \cdot \text{mm}^2 \quad K_3(\theta) := -2 \cdot L_2 \cdot d(\theta)$

$A(\theta) := K_1(\theta) - K_3(\theta) \quad B := 2 \cdot K_2 \quad C(\theta) := K_1(\theta) + K_3(\theta)$

$\theta_2(\theta) := 2 \cdot \text{atan2}\left(2 \cdot A(\theta), -B + \sqrt{B^2 - 4 \cdot A(\theta) \cdot C(\theta)}\right)$



Minimum crank angle:

$\theta_{2max} := \theta_2(60 \cdot \text{deg})$

$\theta_{2max} = 89.719 \cdot \text{deg}$

PROBLEM 16-4

Statement: Using the constant velocity motion program of Example 16-1 applied to the slider of a nonoffset fourbar crank-slider with constant velocity stroke of 150 mm at 40 mm/s, calculate and plot the s , v , a , j functions for this motion applied directly to the slider using DYNACAM. Calculate the modified program needed to drive the crank of the linkage to obtain the specified motion at the slider. Crank = 200 mm, coupler = 600 mm, crank start angle = 110 deg. The cam rotates at a constant 10 rpm.

Given: Crank-slider geometry:
 $a := 200 \cdot \text{mm}$ $b := 600 \cdot \text{mm}$ $c := 0 \cdot \text{mm}$

Cam data: $\omega := 10 \cdot \text{rpm}$

Slider data: $V_{CV} := 40 \cdot \frac{\text{mm}}{\text{s}}$ $\Delta d_{CV} := 150 \cdot \text{mm}$

Crank data: $\theta_{2\text{start}} := 110 \cdot \text{deg}$

Solution: See Example 8-12 and Mathcad file P1604.

- Determine the cam rotation angle for the constant velocity and return segments.

$$\text{Constant velocity period:} \quad t_{CV} := \frac{\Delta d_{CV}}{V_{CV}} \quad t_{CV} = 3.750 \text{ s}$$

$$\text{CV cam rotation angle:} \quad \beta_{CV} := \omega t_{CV} \quad \beta_{CV} = 225.000 \cdot \text{deg}$$

$$\text{Return cam rotation angle:} \quad \beta_{ret} := 2 \cdot \pi - \beta_{CV} \quad \beta_{ret} = 135.000 \cdot \text{deg}$$

- Using equation 8.28a from Example 8-12, write the s , v , a equation for the constant velocity and return segments of the slider.

$$\text{Constant velocity segment:} \quad s_1(\theta) := \Delta d_{CV} \cdot \frac{\theta}{\beta_{CV}} \quad v_1 := V_{CV} \quad a_1 := 0 \cdot \frac{\text{mm}}{\text{sec}^2}$$

Return segment:

$$s(\theta) = C_0 + C_1 \cdot \frac{\theta}{\beta_{ret}} + C_2 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^2 + C_3 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^3 + C_4 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^4 + C_5 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^5$$

$$v(\theta) = \frac{\omega}{\beta_{ret}} \cdot \left[C_1 + 2 \cdot C_2 \cdot \left(\frac{\theta}{\beta_{ret}} \right) + 3 \cdot C_3 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^2 + 4 \cdot C_4 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^3 + 5 \cdot C_5 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^4 \right]$$

$$a(\theta) = \left(\frac{\omega}{\beta_{ret}} \right)^2 \cdot \left[2 \cdot C_2 + 6 \cdot C_3 \cdot \left(\frac{\theta}{\beta_{ret}} \right) + 12 \cdot C_4 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^2 + 20 \cdot C_5 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^3 \right]$$

- Applying the BCs given in Example 8-12:

$$C_0 := \Delta d_{CV} \quad C_1 := \frac{V_{CV} \cdot \beta_{ret}}{\omega} \quad C_2 := 0 \cdot \text{mm}$$

$$A := \begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{pmatrix} \quad B := \begin{pmatrix} -C_0 - C_1 \\ 0 \cdot \text{mm} \\ 0 \cdot \text{mm} \end{pmatrix} \quad C := A^{-1} \cdot B \quad C = \begin{pmatrix} -2.400 \times 10^3 \\ 3.600 \times 10^3 \\ -1.440 \times 10^3 \end{pmatrix} \text{ mm}$$

$$C_3 := -2400 \cdot mm \quad C_4 := 3600 \cdot mm \quad C_5 := -1440 \cdot mm$$

$$s_2(\theta) := C_0 + C_1 \cdot \frac{\theta}{\beta_{ret}} + C_2 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^2 + C_3 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^3 + C_4 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^4 + C_5 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^5$$

$$v_2(\theta) := \frac{\omega}{\beta_{ret}} \cdot \left[C_1 + 2 \cdot C_2 \cdot \left(\frac{\theta}{\beta_{ret}} \right) + 3 \cdot C_3 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^2 + 4 \cdot C_4 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^3 + 5 \cdot C_5 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^4 \right]$$

$$a_2(\theta) := \left(\frac{\omega}{\beta_{ret}} \right)^2 \cdot \left[2 \cdot C_2 + 6 \cdot C_3 \cdot \left(\frac{\theta}{\beta_{ret}} \right) + 12 \cdot C_4 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^2 + 20 \cdot C_5 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^3 \right]$$

4. To plot the sva curves, first define a range function that has a value of one between the values of x1 and x2 and zero elsewhere.

$$R(x, x1, x2) := \text{if}[(x > x1) \wedge (x \leq x2), 1, 0]$$

5. The global displacement equation is composed of two intervals (constant velocity and return). The local equations above must be assembled into a single equation for the displacement that applies over the range $0 \leq \theta \leq 360$ deg of the timing angle.

6. Write the local sva equations for each of the two intervals.

For $0 \leq \theta \leq \beta_{CV}$ (Constant velocity) $s_1(\theta) := s_1(\theta)$

$$v_1(\theta) := V_{CV}$$

$$a_1(\theta) := 0 \cdot \frac{mm}{sec^2}$$

For $\beta_{CV} < \theta \leq 360$ (Return)

$$s_2(\theta) := s_2(\theta)$$

$$v_2(\theta) := v_2(\theta)$$

$$a_2(\theta) := a_2(\theta)$$

7. Write the complete global equation for the displacement and plot it over one timing cycle, which is the sum of the four intervals defined above.

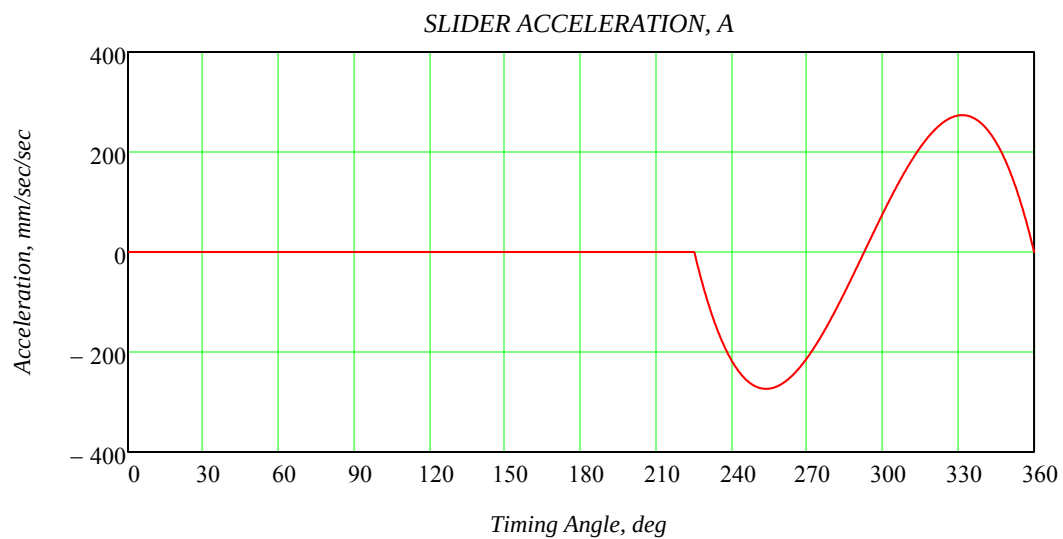
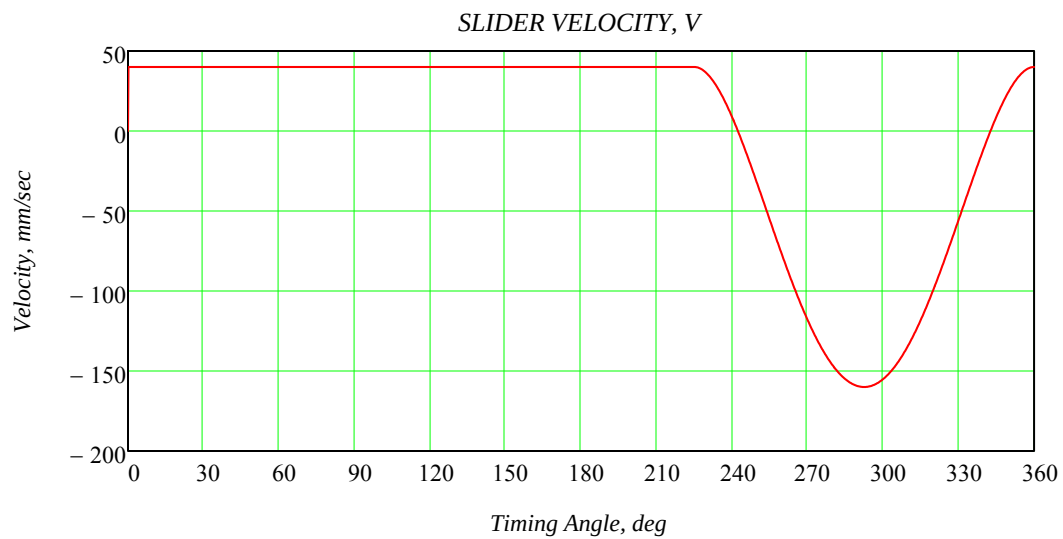
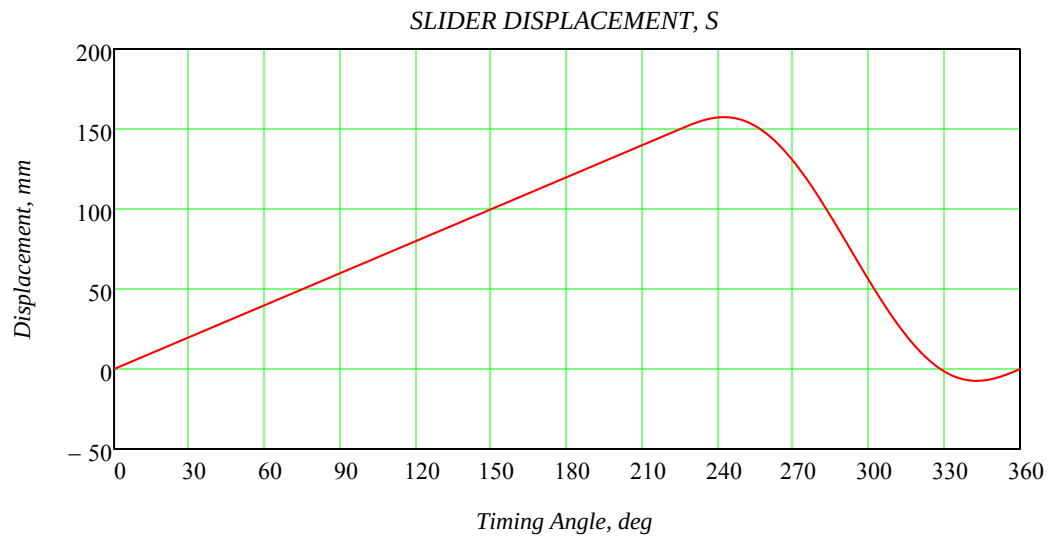
Let $\theta_1 := \beta_{CV} \quad \theta_2 := 360 \cdot deg$

$$S(\theta) := R(\theta, 0, \theta_1) \cdot s_1(\theta) + R(\theta, \theta_1, \theta_2) \cdot s_2(\theta - \theta_1)$$

$$V(\theta) := R(\theta, 0, \theta_1) \cdot v_1(\theta) + R(\theta, \theta_1, \theta_2) \cdot v_2(\theta - \theta_1)$$

$$A(\theta) := R(\theta, 0, \theta_1) \cdot a_1(\theta) + R(\theta, \theta_1, \theta_2) \cdot a_2(\theta - \theta_1)$$

$$\theta := 0 \cdot deg, 0.2 \cdot deg .. 360 \cdot deg$$



8. Write the equation for the slider position as a function of the timing angle, θ

$$\text{Let} \quad A := 1 \quad B := -2 \cdot a \cdot \cos(\theta_{2start}) \quad C := a^2 - b^2$$

$$\text{then} \quad d_0 := \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \quad d_0 = 501.402 \text{ mm}$$

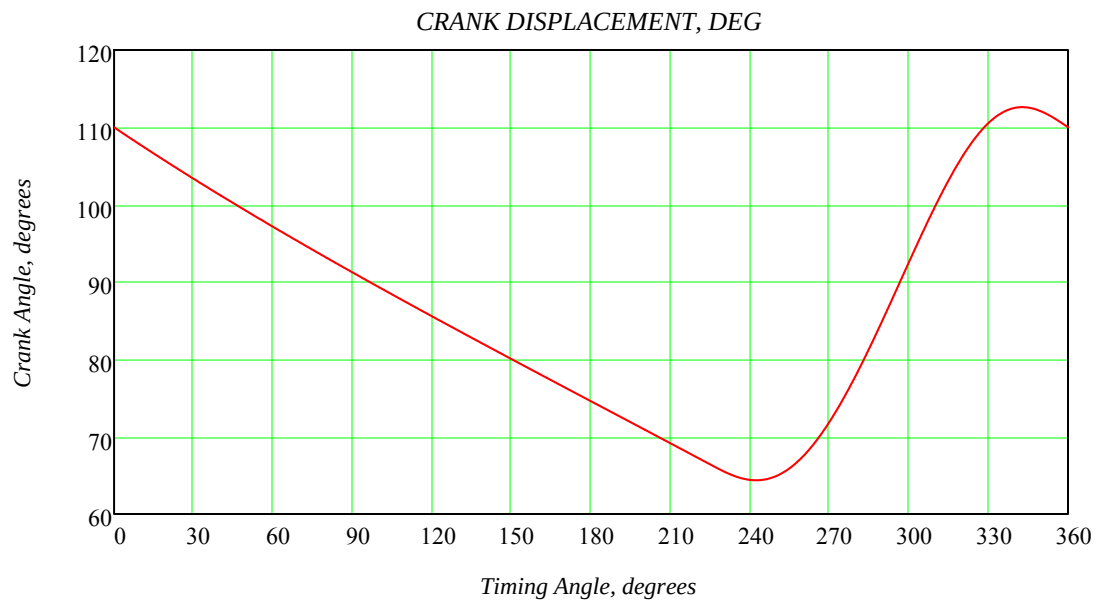
$$\text{Slider position:} \quad d(\theta) := d_0 + S(\theta)$$

9. Using equations 4.20 and 4.21, write the equation for the crank angle, θ_2 as a function of the timing angle, θ

$$K_1(\theta) := a^2 - b^2 + (d(\theta))^2 \quad K_2 := 0 \cdot \text{mm}^2 \quad K_3(\theta) := -2 \cdot a \cdot d(\theta)$$

$$A(\theta) := K_1(\theta) - K_3(\theta) \quad B := 2 \cdot K_2 \quad C(\theta) := K_1(\theta) + K_3(\theta)$$

$$\theta_2(\theta) := 2 \cdot \text{atan2}\left(2 \cdot A(\theta), -B + \sqrt{B^2 - 4 \cdot A(\theta) \cdot C(\theta)}\right)$$



PROBLEM 16-5

Statement: Using the constant velocity motion program of Example 16-1 applied to the slider of a nonoffset fourbar crank-slider with constant velocity stroke of 5.5 in at 0.75 in/s, calculate and plot the s , v , a , j functions for this motion applied directly to the slider using DYNACAM. Calculate the modified program needed to drive the crank of the linkage to obtain the specified motion at the slider. Crank = 7.5 in, coupler = 18 in, crank start angle = 120 deg. The cam rotates at a constant 5 rpm.

Given: Crank-slider geometry:
 $a := 7.5 \cdot \text{in}$ $b := 18.0 \cdot \text{in}$ $c := 0 \cdot \text{in}$

Cam data: $\omega := 5 \cdot \text{rpm}$

Slider data: $V_{CV} := 0.75 \cdot \frac{\text{in}}{\text{s}}$ $\Delta d_{CV} := 5.5 \cdot \text{in}$

Crank data: $\theta_{2\text{start}} := 120 \cdot \text{deg}$

Solution: See Example 8-12 and Mathcad file P1605.

1. Determine the cam rotation angle for the constant velocity and return segments.

$$\text{Constant velocity period:} \quad t_{CV} := \frac{\Delta d_{CV}}{V_{CV}} \quad t_{CV} = 7.333 \text{ s}$$

$$\text{CV cam rotation angle:} \quad \beta_{CV} := \omega t_{CV} \quad \beta_{CV} = 220.000 \cdot \text{deg}$$

$$\text{Return cam rotation angle:} \quad \beta_{ret} := 2 \cdot \pi - \beta_{CV} \quad \beta_{ret} = 140.000 \cdot \text{deg}$$

2. Using equation 8.28a from Example 8-12, write the sva equation for the constant velocity and return segments of the slider.

$$\text{Constant velocity segment:} \quad s1(\theta) := \Delta d_{CV} \cdot \frac{\theta}{\beta_{CV}} \quad v1 := V_{CV} \quad a1 := 0 \cdot \frac{\text{mm}}{\text{sec}^2}$$

Return segment:

$$s(\theta) = C_0 + C_1 \cdot \frac{\theta}{\beta_{ret}} + C_2 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^2 + C_3 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^3 + C_4 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^4 + C_5 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^5$$

$$v(\theta) = \frac{\omega}{\beta_{ret}} \cdot \left[C_1 + 2 \cdot C_2 \cdot \left(\frac{\theta}{\beta_{ret}} \right) + 3 \cdot C_3 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^2 + 4 \cdot C_4 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^3 + 5 \cdot C_5 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^4 \right]$$

$$a(\theta) = \left(\frac{\omega}{\beta_{ret}} \right)^2 \cdot \left[2 \cdot C_2 + 6 \cdot C_3 \cdot \left(\frac{\theta}{\beta_{ret}} \right) + 12 \cdot C_4 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^2 + 20 \cdot C_5 \cdot \left(\frac{\theta}{\beta_{ret}} \right)^3 \right]$$

3. Applying the BCs given in Example 8-12:

$$C_0 := \Delta d_{CV} \quad C_1 := \frac{V_{CV} \cdot \beta_{ret}}{\omega} \quad C_2 := 0 \cdot \text{mm}$$

$$A := \begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{pmatrix} \quad B := \begin{pmatrix} -C_0 - C_1 \\ 0 \cdot \text{mm} \\ 0 \cdot \text{mm} \end{pmatrix} \quad C := A^{-1} \cdot B \quad C = \begin{pmatrix} -90.000 \\ 135.000 \\ -54.000 \end{pmatrix} \text{ in}$$

$$\begin{aligned}
 C_3 &:= -90 \cdot \text{in} & C_4 &:= 135 \cdot \text{in} & C_5 &:= -54 \cdot \text{in} \\
 s_2(\theta) &:= C_0 + C_1 \cdot \frac{\theta}{\beta_{\text{ret}}} + C_2 \cdot \left(\frac{\theta}{\beta_{\text{ret}}} \right)^2 + C_3 \cdot \left(\frac{\theta}{\beta_{\text{ret}}} \right)^3 + C_4 \cdot \left(\frac{\theta}{\beta_{\text{ret}}} \right)^4 + C_5 \cdot \left(\frac{\theta}{\beta_{\text{ret}}} \right)^5 \\
 v_2(\theta) &:= \frac{\omega}{\beta_{\text{ret}}} \cdot \left[C_1 + 2 \cdot C_2 \cdot \left(\frac{\theta}{\beta_{\text{ret}}} \right) + 3 \cdot C_3 \cdot \left(\frac{\theta}{\beta_{\text{ret}}} \right)^2 + 4 \cdot C_4 \cdot \left(\frac{\theta}{\beta_{\text{ret}}} \right)^3 + 5 \cdot C_5 \cdot \left(\frac{\theta}{\beta_{\text{ret}}} \right)^4 \right] \\
 a_2(\theta) &:= \left(\frac{\omega}{\beta_{\text{ret}}} \right)^2 \cdot \left[2 \cdot C_2 + 6 \cdot C_3 \cdot \left(\frac{\theta}{\beta_{\text{ret}}} \right) + 12 \cdot C_4 \cdot \left(\frac{\theta}{\beta_{\text{ret}}} \right)^2 + 20 \cdot C_5 \cdot \left(\frac{\theta}{\beta_{\text{ret}}} \right)^3 \right]
 \end{aligned}$$

4. To plot the sva curves, first define a range function that has a value of one between the values of x1 and x2 and zero elsewhere.

$$R(x, x1, x2) := \text{if}[(x > x1) \wedge (x \leq x2), 1, 0]$$

5. The global displacement equation is composed of two intervals (constant velocity and return). The local equations above must be assembled into a single equation for the displacement that applies over the range $0 \leq \theta \leq 360$ deg of the timing angle.
6. Write the local sva equations for each of the two intervals.

For $0 \leq \theta \leq \beta_{\text{CV}}$ (Constant velocity) $s_1(\theta) := s_1(\theta)$

$$v_1(\theta) := V_{\text{CV}}$$

$$a_1(\theta) := 0 \cdot \frac{\text{mm}}{\text{sec}^2}$$

For $\beta_{\text{CV}} < \theta \leq 360$ (Return)

$$s_2(\theta) := s_2(\theta)$$

$$v_2(\theta) := v_2(\theta)$$

$$a_2(\theta) := a_2(\theta)$$

7. Write the complete global equation for the displacement and plot it over one timing cycle, which is the sum of the four intervals defined above.

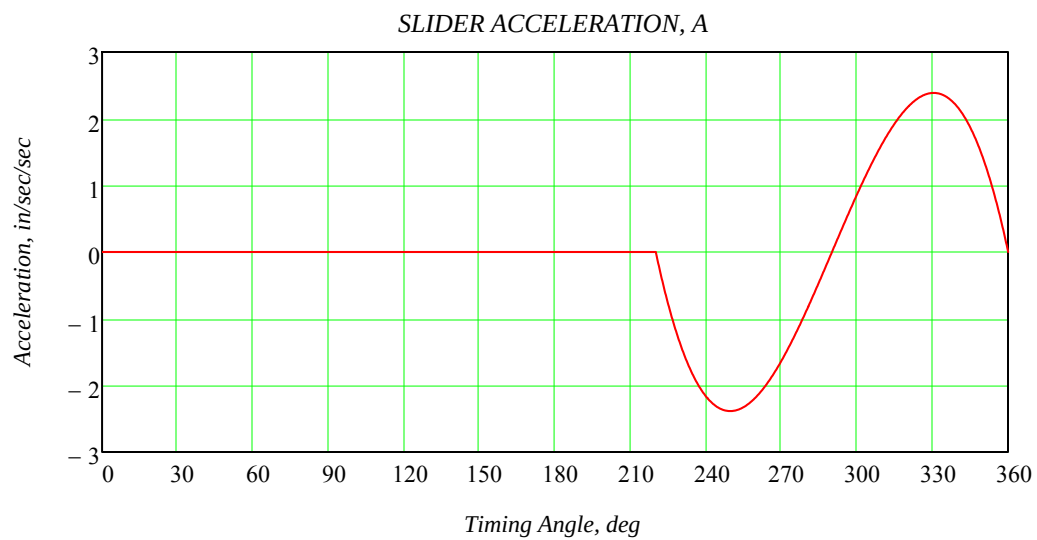
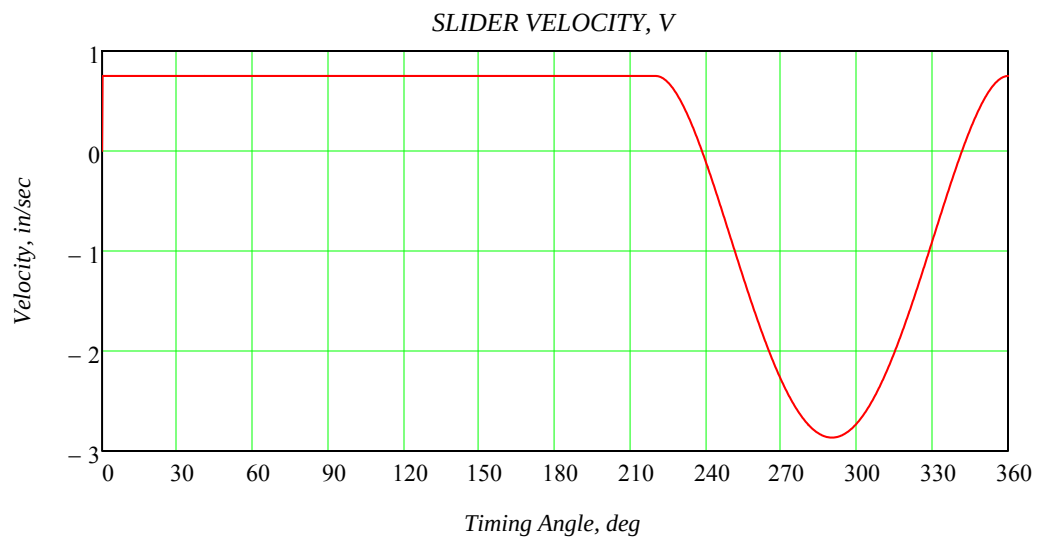
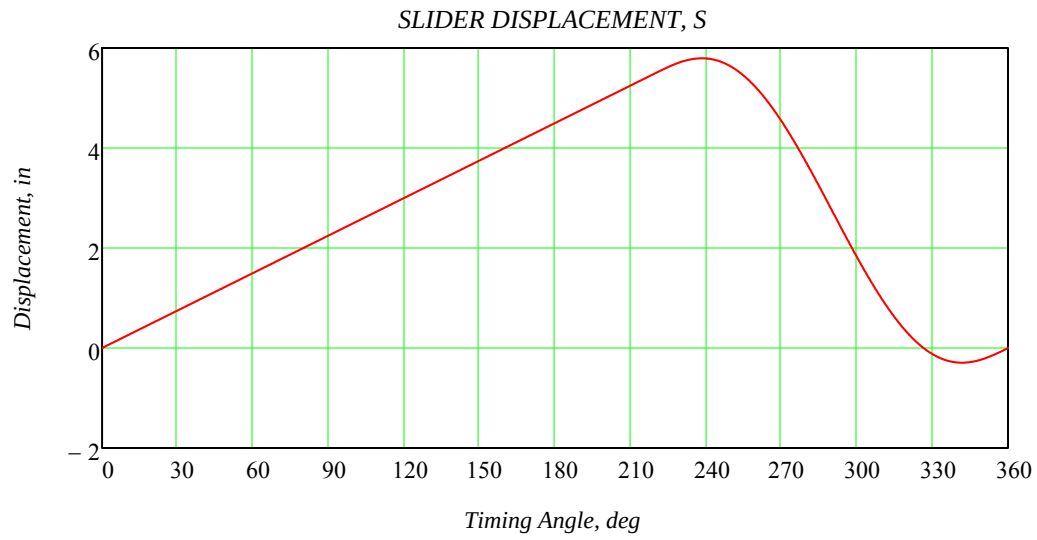
Let $\theta_1 := \beta_{\text{CV}}$ $\theta_2 := 360 \cdot \text{deg}$

$$S(\theta) := R(\theta, 0, \theta_1) \cdot s_1(\theta) + R(\theta, \theta_1, \theta_2) \cdot s_2(\theta - \theta_1)$$

$$V(\theta) := R(\theta, 0, \theta_1) \cdot v_1(\theta) + R(\theta, \theta_1, \theta_2) \cdot v_2(\theta - \theta_1)$$

$$A(\theta) := R(\theta, 0, \theta_1) \cdot a_1(\theta) + R(\theta, \theta_1, \theta_2) \cdot a_2(\theta - \theta_1)$$

$$\theta := 0 \cdot \text{deg}, 0.2 \cdot \text{deg} .. 360 \cdot \text{deg}$$



8. Write the equation for the slider position as a function of the timing angle, θ

$$\text{Let} \quad A := 1 \quad B := -2 \cdot a \cdot \cos(\theta_{2start}) \quad C := a^2 - b^2$$

$$\text{then} \quad d_0 := \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \quad d_0 = 13.037 \text{ in}$$

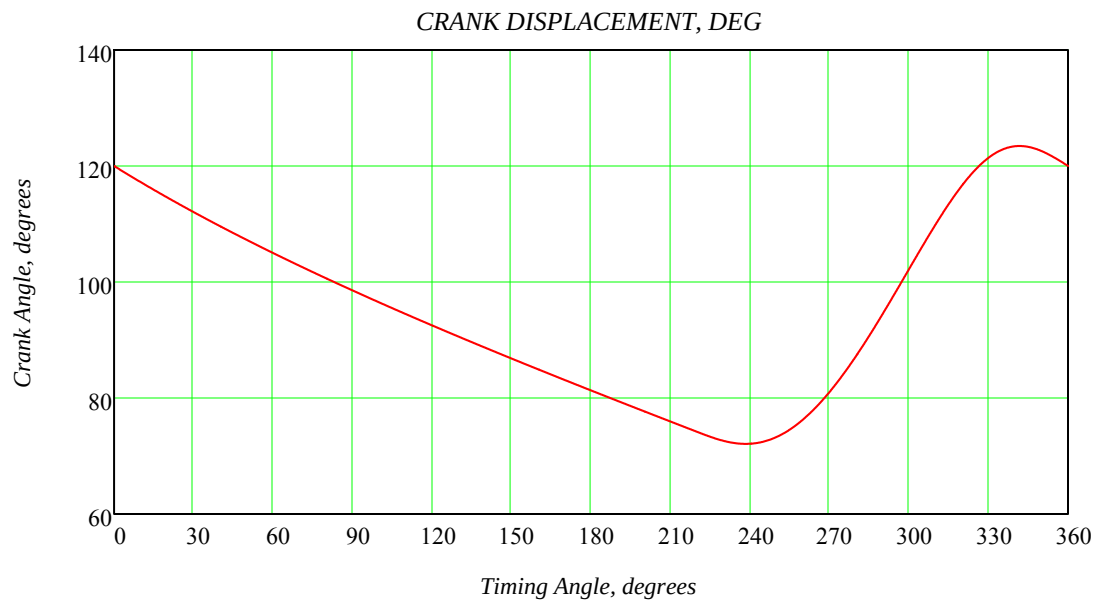
$$\text{Slider position:} \quad d(\theta) := d_0 + S(\theta)$$

9. Using equations 4.20 and 4.21, write the equation for the crank angle, θ_2 as a function of the timing angle, θ

$$K_1(\theta) := a^2 - b^2 + (d(\theta))^2 \quad K_2 := 0 \cdot \text{mm}^2 \quad K_3(\theta) := -2 \cdot a \cdot d(\theta)$$

$$A(\theta) := K_1(\theta) - K_3(\theta) \quad B := 2 \cdot K_2 \quad C(\theta) := K_1(\theta) + K_3(\theta)$$

$$\theta_2(\theta) := 2 \cdot \text{atan2}\left(2 \cdot A(\theta), -B + \sqrt{B^2 - 4 \cdot A(\theta) \cdot C(\theta)}\right)$$



PROBLEM 16-6

Statement: A non-offset crank-slider with crank and coupler lengths of 3.25 in and 10.875 in, respectively, is to be driven by a servo through a gear reducer. The required slider timing diagram is shown in Figure P16-1. Choose a servo from Table 16-2 to drive the crank with the lowest resulting vibration and calculate the resulting crank displacement for one cycle. Based on this crank input, calculate the resulting slider motion and compare the two by plotting their normalized displacements. The crank angle at $t = 0$ is 70-deg, and the crank turns clockwise to accomplish the first motion of the slider.

Given: Crank-slider geometry:

$$a := 3.25 \cdot \text{in} \quad b := 10.875 \cdot \text{in} \quad c := 0 \cdot \text{in}$$

Motion data:

RISE	DWELL	FALL	DWELL
$\tau_1 := 10 \cdot \text{sec}$	$\tau_2 := 10 \cdot \text{sec}$	$\tau_3 := 12 \cdot \text{sec}$	$\tau_4 := 8 \cdot \text{sec}$
$h_1 := 2.0 \cdot \text{in}$	$h_2 := 0 \cdot \text{in}$	$h_3 := 2.0 \cdot \text{in}$	$h_4 := 0 \cdot \text{in}$

Crank angle at $t = 0$: $\theta_{20} := 70 \cdot \text{deg}$

Cycle time: $\tau := 40 \cdot \text{sec}$

Solution: See Figure P16-1, Table 16-2 and Mathcad file P1606.

- From Table 16-2, choose the S-curve servo (cycloidal motion).
- Using equations 4.16a and b, calculate the position of the slider at $t = 0$ and the displacement of the crank equivalent to the stroke of the slider using the law of cosines.

$$\theta_{30} := \text{asin}\left(\frac{a \cdot \sin(\theta_{20}) - c}{b}\right) + \pi \quad \theta_{30} = 163.690 \cdot \text{deg}$$

$$d_0 := a \cdot \cos(\theta_{20}) - b \cdot \cos(\theta_{30}) \quad d_0 = 11.549 \text{ in}$$

$$\theta_{21} := \text{acos}\left[\frac{(d_0 + h_1)^2 + a^2 - b^2}{2 \cdot a \cdot (d_0 + h_1)}\right] \quad \theta_{21} = 30.515 \cdot \text{deg}$$

$$\Delta\theta := \theta_{20} - \theta_{21} \quad \Delta\theta = 39.485 \cdot \text{deg}$$

- Using equation 8.12d, for the crank angle during the slider extension and return.

$$y(x) := \left(x - \frac{1}{2 \cdot \pi} \cdot \sin(2 \cdot \pi \cdot x)\right)$$

- To plot the crank displacement curve, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x1, x2) := \text{if}[(x > x1) \wedge (x \leq x2), 1, 0]$$

- The global displacement equation is composed of four intervals (rise, dwell, fall, and dwell). The local equation above must be assembled into a single equation for the displacement that applies over the range $0 \leq \theta \leq 360$ deg of the timing angle.
- Write the local displacement equation for the intervals τ_1 , τ_2 , τ_3 , and τ_4 :

For $0 \leq t \leq \tau_1$ (Rise)

$$s_1(x) := \Delta\theta \cdot y(x)$$

For $\beta_1 \leq \theta \leq \beta_1 + \beta_2$ (dwell)

$$s_2(x) := \Delta\theta$$

For $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$ (fall)

$$s_3(x) := \Delta\theta \cdot (1 - y(x))$$

For $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq 2\pi$ (dwell)

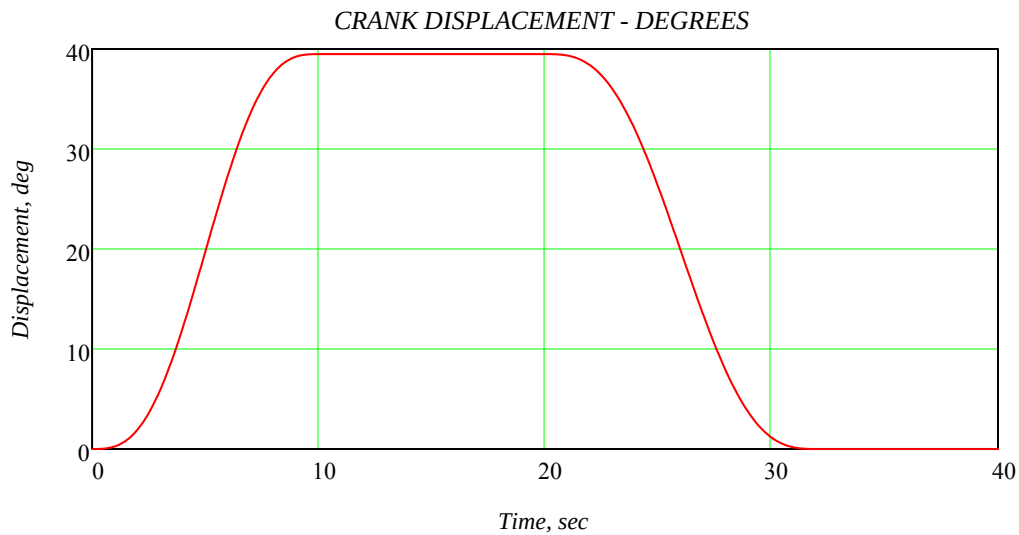
$$s_4(x) := 0 \cdot \text{deg}$$

5. Write the complete global equation for the displacement and plot it over one timing cycle, which is the sum of the four intervals defined above.

$$\text{Let} \quad t_1 := \tau_1 \quad t_2 := t_1 + \tau_2 \quad t_3 := t_2 + \tau_3 \quad t_4 := t_3 + \tau_4$$

$$S(t) := R(t, 0 \cdot \text{sec}, t_1) \cdot s_1\left(\frac{t}{t_1}\right) + R(t, t_1, t_2) \cdot s_2\left(\frac{t - t_1}{t_2 - t_1}\right) \dots \\ + R(t, t_2, t_3) \cdot s_3\left(\frac{t - t_2}{t_3 - t_2}\right) + R(t, t_3, t_4) \cdot s_4\left(\frac{t - t_3}{t_4 - t_3}\right)$$

$$t := 0 \cdot \text{sec}, 0.05 \cdot \text{sec} \dots \tau$$

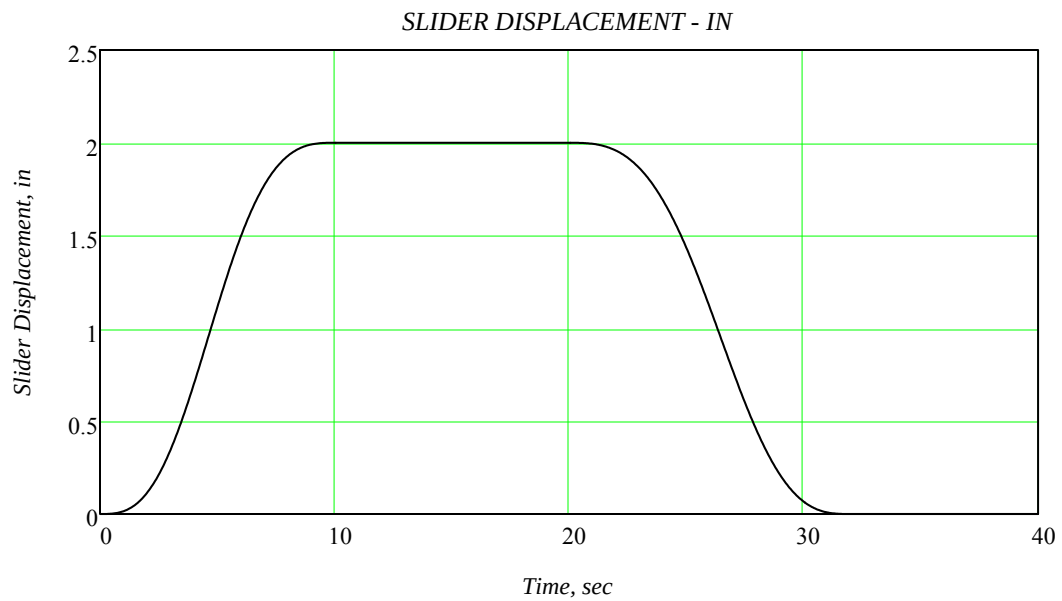


6. Write the equation for the slider position as a function of the crank angle, θ_2 ($S(t)$) using the law of cosines.

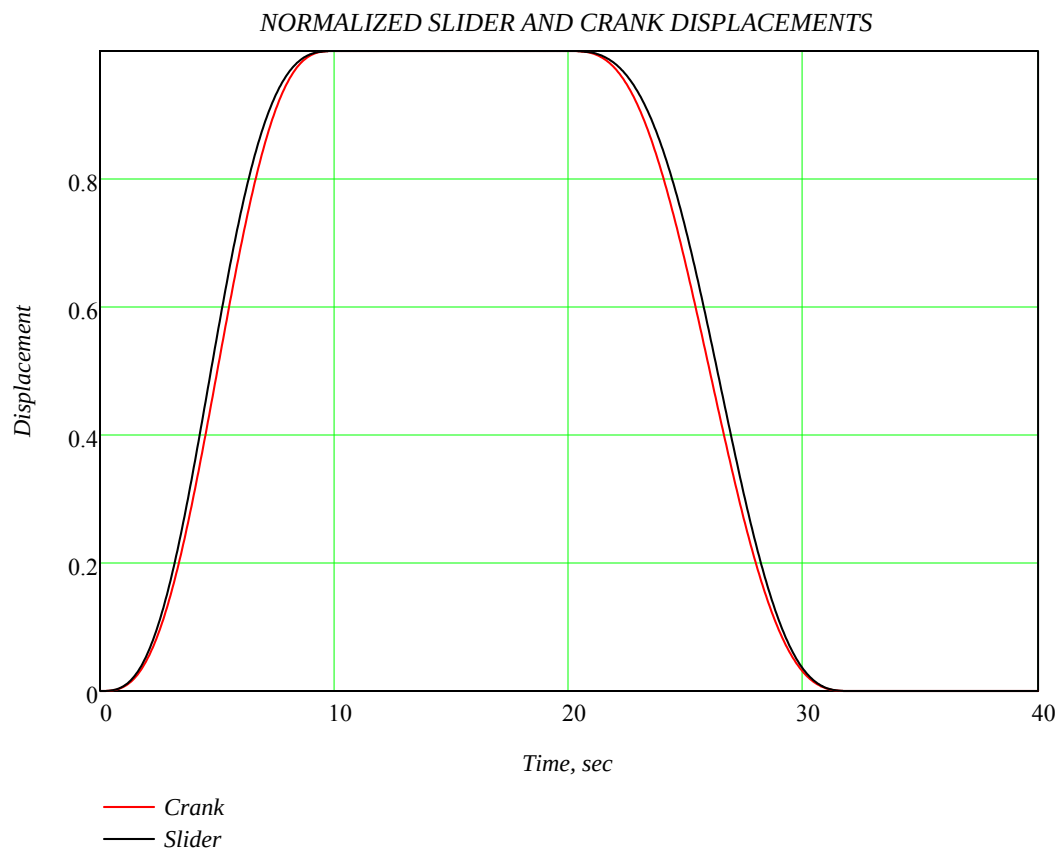
$$d^2 - 2 \cdot a \cdot d \cdot \cos(\theta_2) + (a^2 - b^2) = 0$$

$$\text{Let} \quad A := 1 \quad B(t) := -2 \cdot a \cdot \cos(\theta_{20} - S(t)) \quad C := a^2 - b^2$$

$$\text{Then} \quad d(t) := \frac{-B(t) + \sqrt{B(t)^2 - 4 \cdot A \cdot C}}{2 \cdot A} - d_0$$



7. Plot the normalized crank and slider displacements by dividing each by its total stroke.



PROBLEM 16-7

Statement: Repeat Problem 16-6 using the timing diagram of Figure P16-1 but with a total stroke of 75 mm. The crank length is 110 mm, the coupler length is 275 mm, and the initial crank angle is 90 deg.

Given: Crank-slider geometry:

$$a := 110 \cdot \text{mm} \quad b := 275 \cdot \text{mm} \quad c := 0 \cdot \text{mm}$$

Motion data:

RISE DWELL FALL DWELL

$$\tau_1 := 10 \cdot \text{sec} \quad \tau_2 := 10 \cdot \text{sec} \quad \tau_3 := 12 \cdot \text{sec} \quad \tau_4 := 8 \cdot \text{sec}$$

$$h_1 := 75 \cdot \text{mm} \quad h_2 := 0 \cdot \text{mm} \quad h_3 := 75 \cdot \text{mm} \quad h_4 := 0 \cdot \text{mm}$$

$$\text{Crank angle at } t = 0: \quad \theta_{20} := 90 \cdot \text{deg}$$

$$\text{Cycle time:} \quad \tau := 40 \cdot \text{sec}$$

Solution: See Figure P16-1, Table 16-2 and Mathcad file P1606.

1. From Table 16-2, choose the S-curve servo (cycloidal motion).
2. Using equations 4.16a and b, calculate the position of the slider at $t = 0$ and the displacement of the crank equivalent to the stroke of the slider using the law of cosines.

$$\theta_{30} := \text{asin}\left(-\frac{a \cdot \sin(\theta_{20}) - c}{b}\right) + \pi \quad \theta_{30} = 156.422 \cdot \text{deg}$$

$$d_0 := a \cdot \cos(\theta_{20}) - b \cdot \cos(\theta_{30}) \quad d_0 = 252.042 \text{ mm}$$

$$\theta_{21} := \text{acos}\left[\frac{(d_0 + h_1)^2 + a^2 - b^2}{2 \cdot a \cdot (d_0 + h_1)}\right] \quad \theta_{21} = 52.869 \cdot \text{deg}$$

$$\Delta\theta := \theta_{20} - \theta_{21} \quad \Delta\theta = 37.131 \cdot \text{deg}$$

3. Using equation 8.12d, for the crank angle during the slider extension and return.

$$y(x) := \left(x - \frac{1}{2 \cdot \pi} \cdot \sin(2 \cdot \pi \cdot x)\right)$$

4. To plot the crank displacement curve, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x, x1, x2) := \text{if}[(x > x1) \wedge (x \leq x2), 1, 0]$$

3. The global displacement equation is composed of four intervals (rise, dwell, fall, and dwell). The local equation above must be assembled into a single equation for the displacement that applies over the range $0 \leq \theta \leq 360$ deg of the timing angle.
4. Write the local displacement equation for the intervals τ_1 , τ_2 , τ_3 , and τ_4 :

For $0 \leq t \leq \tau_1$ (Rise)

$$s_1(x) := \Delta\theta \cdot y(x)$$

For $\beta_1 \leq \theta \leq \beta_1 + \beta_2$ (dwell)

$$s_2(x) := \Delta\theta$$

For $\beta_1 + \beta_2 \leq \theta \leq \beta_1 + \beta_2 + \beta_3$ (fall)

$$s_3(x) := \Delta\theta \cdot (1 - y(x))$$

For $\beta_1 + \beta_2 + \beta_3 \leq \theta \leq 2\pi$ (dwell)

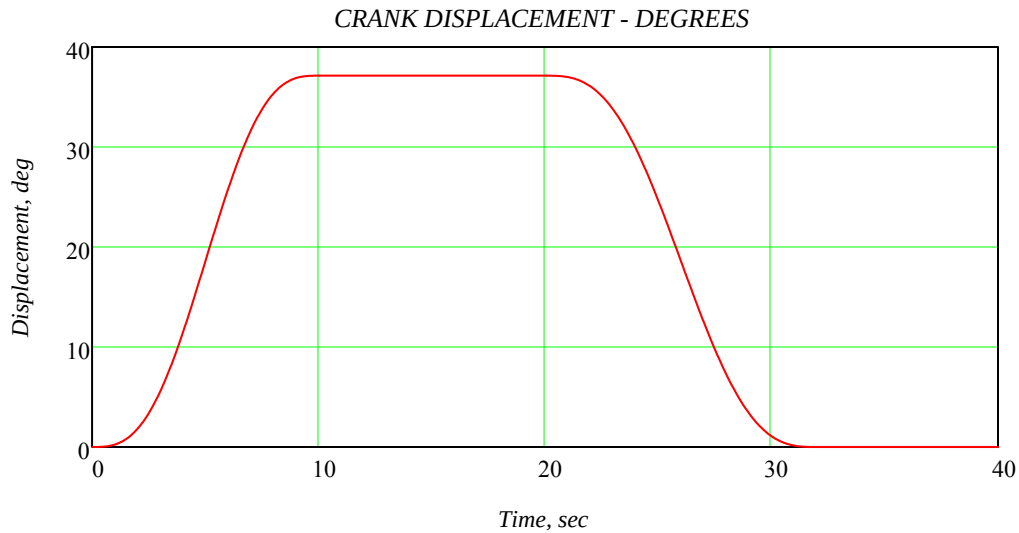
$$s_4(x) := 0 \cdot \text{deg}$$

5. Write the complete global equation for the displacement and plot it over one timing cycle, which is the sum of the four intervals defined above.

$$\text{Let} \quad t_1 := \tau_1 \quad t_2 := t_1 + \tau_2 \quad t_3 := t_2 + \tau_3 \quad t_4 := t_3 + \tau_4$$

$$S(t) := R(t, 0 \cdot \text{sec}, t_1) \cdot s_1\left(\frac{t}{t_1}\right) + R(t, t_1, t_2) \cdot s_2\left(\frac{t - t_1}{t_2 - t_1}\right) \dots \\ + R(t, t_2, t_3) \cdot s_3\left(\frac{t - t_2}{t_3 - t_2}\right) + R(t, t_3, t_4) \cdot s_4\left(\frac{t - t_3}{t_4 - t_3}\right)$$

$$t := 0 \cdot \text{sec}, 0.05 \cdot \text{sec} \dots \tau$$

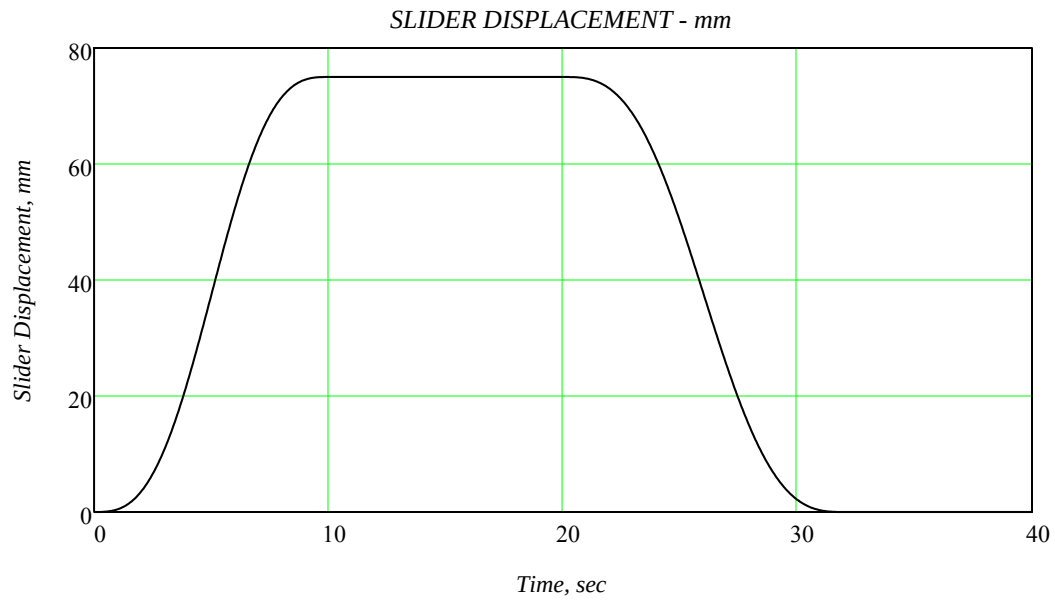


6. Write the equation for the slider position as a function of the crank angle, θ_2 ($S(t)$) using the law of cosines.

$$d^2 - 2 \cdot a \cdot d \cdot \cos(\theta_2) + (a^2 - b^2) = 0$$

$$\text{Let} \quad A := 1 \quad B(t) := -2 \cdot a \cdot \cos(\theta_{20} - S(t)) \quad C := a^2 - b^2$$

$$\text{Then} \quad d(t) := \frac{-B(t) + \sqrt{B(t)^2 - 4 \cdot A \cdot C}}{2 \cdot A} - d_0$$



7. Plot the normalized crank and slider displacements by dividing each by its total stroke.

