4.1: PROBLEM DEFINITION

Situation:

Path of a fluid particle.

Find:

If a light was attached to a fluid particle and take a time exposure, would the image you photographed be a pathline or streakline?

SOLUTION

The pathline is defined as the path taken by a fluid particle moving through a field. The photograph would yield a pathline.

4.2: PROBLEM DEFINITION

Situation:

Smoke rising from a chimney.

Find:

The pattern produced by smoke rising from a chimney on a windy day is analogous to a pathline or streakline?

SOLUTION

The streakline is defined as a line generated by a tracer injected into flow at starting point. The tracer is the smoke and the starting point is the chimney so the smoke's pattern is analogous to a streakline. The diffusion of the smoke prevents achieving a fine line. The wind also causes disperson, which is another reason that keeps the line from being fine.

4.3: PROBLEM DEFINITION

Situation:

A windsock is a sock-shaped device attached to a swivel on top of a pole. Windsocks at airports are used by pilots to see instantaneous shifts in the direction of the wind. If one drew a line co-linear with a windsock's orientation at any instant, the line would be best approximate a (a) pathline (b) streakline (c) streamline.

SOLUTION

Answer is (c) streamline. A windsock shows you the wind direction (flow field) at a given location at one instant in time - it is not a parade of a series of particles (streakline), or a path recorded on some medium (pathline). So the windsock reveals the streamline.

4.4: PROBLEM DEFINITION

Situation:
For pathlines, streaklines, and streamlines to all be co-linear, the flow must be
a) dividing
b) stagnant
c) steady
d) a tracer

SOLUTION

The answer is (c) steady.

4.5: PROBLEM DEFINITION

Situation:

Dye is injected into a flow field and produces a streakline.

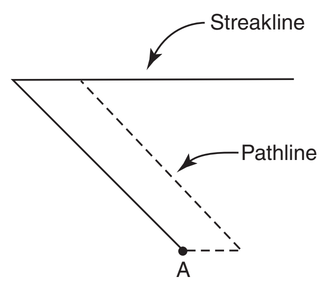
Pathline starts at $t = 4\,\text{s}$, ends at $t = 10\,\text{s}$. Flow speed is constant.

Find:

Draw a pathline of the particle.

SOLUTION

The streakline shows that the velocity field was originally in the horizontal direction to the right and then the flow field changed upward to the left. The pathline starts off to the right and then continues upward to the left.



4.6: PROBLEM DEFINITION

Situation:

A dye streak was started, and a particle was released.

For $0 \leq t \leq 5$ s, $u = 2$ m/s, $v = 0$.

For $5 < t \leq 10$ s, $u = 3$ m/s, $v = -4$ m/s.

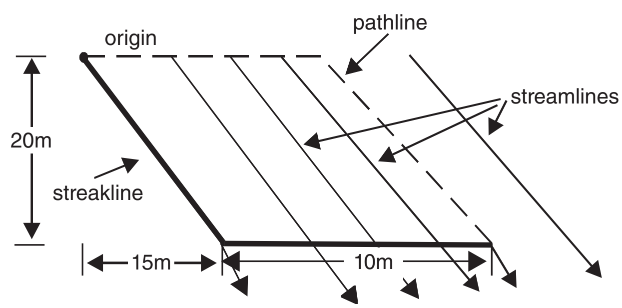
Find:

For $t = 10$ s, draw to scale the streakline, pathline of the particle, and streamlines.

SOLUTION

From 0<t<5, the dye in the streakline moved to the right for a distance of 10 m. At the same time a particle is released from the origin and travels 10 m to the right. Then from 5<t<10, the original line of dye is transported in whole downward to the right while more dye is released from the origin. The pathline of the particle proceeds from its location at t=5 sec downward to the right.

At 10 sec, the streamlines are downward to the right.



4.7: PROBLEM DEFINITION

Situation:

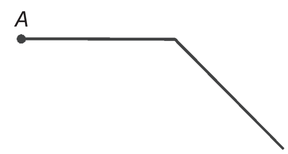
A dye streak is produced in a flow that has a constant speed.

Find:

Sketch a streamline at $t = 8$ s.

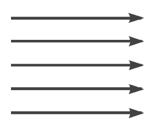
Sketch a particle pathline at $t = 10$ s for a particle that was released from point A at time $t = 2$ s.

Sketch:



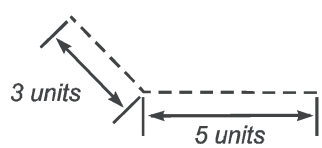
SOLUTION

At 8 seconds (near 10 sec) the streamlines of the flow are horizontal to the right.



Streamlines at $t = 8$ s

Initially the flow is downward to the right and then switches to the horizontal direction to the right. Thus one has the following pathline.



Particle pathline for a particle released at $t = 2$ s

4.8: PROBLEM DEFINITION

Situation:

A velocity field is given mathematically as $\mathbf{V} = 2\mathbf{i} + 4y\mathbf{j}$. The velocity field is:

a. 1D in x

b. 1D in y

c. 2D in x and y

SOLUTION

The answer is (c). Because there is an **i** component and a **j** component, the vector is representing a field that varies in 2 dimensions.

4.9: PROBLEM DEFINITION

Situation:

Gasoline spill in a major river.

The mayor of a large downstream city demands an estimate of hours for the spill to get to reach supply plant intake.

Emergency responders measure the speed of the leading edge of the spill, effectively focusing on one particle of fluid.

Environmental engineers employ a computer model which simulates the velocity field for any stage of the river, and for all locations (including steep narrow canyon sections with fast velocities, and an extremely wide reach with slow velocities).

To compare these two mathematical approaches, which statement is most correct?

a. The responders have an Eulerian approach, and the engineers have a Lagrangian one

b. The responders have a Lagrangian approach, and the engineers have an Eulerian one.

SOLUTION

Answer is (b). The responders have a Lagrangian approach, because they are making a measurement based on following one particle. The engineers are using a Eulerian approach, because they are defining velocity fields for many locations in space.

4.10: PROBLEM DEFINITION

Situation:
Unsteady flow.

Find:
Identify five examples of an unsteady flow, and explain what features classify them as unsteady.

SOLUTION

Answers will vary, but typical student answers might include:

1. Gust of wind blowing past a pole - velocity direction and magnitude vary with time; the word gust means sudden burst of speed, rather than constant speed.

2. Flow next to a rock in a natural river - local velocities (direction and magnitude) vary with time in the vicinity of the rock, even if the average river flowrate is unchanging.

3. Flow past the lips due to inhaling and exhaling - direction keeps alternating, speed goes to zero each time the direction changes.

4. The motion of water at the center of a boiling pot - in convection cells, fluid is rising generally, but at any location there is some turbulent mixing as new water flows in from the side to be heated and then rise.

5. At the outlet hose of a manual tire pump - with each stroke of the pump handle flowrate and velocity are first increasing, then decreasing to zero.

4.11: PROBLEM DEFINITION

Situation:

Pouring a heavy syrup on pancakes.

Find:

Would the thin film of syrup be a laminar or turbulent flow?

SOLUTION

The velocity is very low, the viscosity is high and the thickness of the layer is thin. These conditions favor laminar flow.

4.12: PROBLEM DEFINITION

Situation:
A velocity field is given by $\mathbf{V} = 10xy\mathbf{i}$. It is
a. 1D and steady
b. 1D and unsteady
c. 2D and steady
d. 2D and unsteady

SOLUTION

Answer is (a), 1D and steady. It is steady because the variable t does not appear anywhere. It is 1D because there is only one unit operator: $\mathbf{i}$.

4.13: PROBLEM DEFINITION

Situation:
Which is the most correct way to characterize turbulent flow?
a. 1D
b. 2D
c. 3D

SOLUTION

There are 2 possible answers.
The most general answer is (c) 3D; because it is generally applicable to water flow in a river, or air moving in the atmosphere.
However, if one is modeling turbulent flow in a pipe, it is safe to represent it as (a) 1D, because there is very little variation of velocity across the cross-section of turbulent flow in a pipe, see Fig 4.13b, EFM 10e.
Turbulent flow in a pipe is referred to as plug flow, because it has a blunt-ended velocity profile instead of laminar (bullet-shaped) profile.

4.14: PROBLEM DEFINITION

Situation:

The valve in a system is gradually opened to have a constant rate of increase in discharge.

Find:

Describe the flow at points A and B.

SOLUTION

A: Unsteady, uniform.

B: Non-uniform, unsteady.

4.15: PROBLEM DEFINITION

Situation:
Water flows in a widening passage with flow rate decreasing with time.

Find:
Describe the flow.

PLAN

Steady or unsteady has to do with a time rate of change. Uniform or non-uniform has to do with whether the streamlines are parallel or not.

SOLUTION

(b) Unsteady and (d) non-uniform.

4.16: PROBLEM DEFINITION

Situation:
A flow pattern has converging streamlines.

Find:
Classify the flow.

SOLUTION

Definately non-uniform; could be either steady or unsteady.

4.17: PROBLEM DEFINITION

Situation:

A fluid flows in a straight conduit. The conduit has a section with constant diameter, followed by a section with changing diameter.

Find:

Match the given flow labels with the mathematical descriptions.

SOLUTION

Steady flow corresponds to $\partial V_s/\partial t = 0$.
Unsteady flow corresponds to $\partial V_s/\partial t \neq 0$.
Uniform flow corresponds to $\partial V_s/\partial s = 0$.
Non-uniform flow corresponds to $\partial V_s/\partial s \neq 0$.

4.18: PROBLEM DEFINITION

Situation:

A series of flows are either one, two or three dimensional.

Find:

Classify the flows as one, two or three dimensional.

(a) Water flow over the crest of a long spillway of a dam.

(b) Flow in a straight horizontal pipe.

(c) Flow in a constant-diameter pipeline that follows the contour of the ground in hilly country.

(d) Airflow from a slit in a plate at the end of a large rectangular duct.

(e) Airflow past an automobile.

(f) Air flow past a house.

(g) Water flow past a pipe that is laid normal to the flow across the bottom of a wide rectangular channel.

SOLUTION

a.	Two dimensional	e.	Three dimensional
b.	One dimensional	f.	Three dimensional
c.	One dimensional	g.	Two dimensional
d.	Two dimensional		

4.19: PROBLEM DEFINITION

Situation:
Acceleration.

Find:
Is the acceleration vector always aligned with the velocity vector?

SOLUTION

No. For flow along a curved path, there is a centripetal acceleration which is normal to the velocity vector.

4.20: PROBLEM DEFINITION

Situation:

Rotating bodies.

Find:

Is the acceleration toward the center of rotation a centripetal or centrifugal acceleration?

SOLUTION

The acceleration toward the center of rotation is centripetal acceleration. “Petal” comes from Latin word “petere” which means to move toward so “centripetal” means moving toward center. “Fugal” comes from Latin “fugere” which means to flee so “centrifugal” means moving from center.

4.21: PROBLEM DEFINITION

Situation:

In a flowing fluid, acceleration means that a fluid particle is

a. changing direction

b. changing speed

c. changing both speed and direction

d. any of the above

SOLUTION

The correct answer is d.

4.22: PROBLEM DEFINITION

Situation:

Flow through a nozzle is steady.

V increases between the entrance and the exit of the nozzle.

The acceleration halfway between the entrance and the nozzle is:

a. convective

b. local

c. both

SOLUTION

If the flow is steady, there is no local acceleration.

Therefore the answer is (a) convective acceleration.

4.23: PROBLEM DEFINITION

Situation:
Local acceleration
a. is close to the origin
b. is quasi-nonuniform
c. occurs in unsteady flow

SOLUTION

The answer is c.

4.24: PROBLEM DEFINITION

Situation:

Flow past a circular cylinder with constant approach velocity.

Find:

Describe the flow as:

(a) Steady or unsteady.

(b) One dimensional, two dimensional, or three dimensional.

(c) Locally accelerating or not, and is so, where.

(d) Convectively accelerating or not, and if so, where.

SOLUTION

(a) Steady.

(b) Two-dimensional; velocity is changing in the x and y direction (but we assume it is not changing in the axis that comes out of the page)

(c) No; the flow is steady, so by definition there is no local acceleration.

(d) Convective acceleration is present at each where a fluid particles changes speed as it moves along the streamline. Centripetal acceleration, which is a form of convective acceleration, occurs where there is streamline curvature.

4.25: PROBLEM DEFINITION

Situation:

A path line is given with velocity as a function of distance and time.
$V = s^2 t^{1/2}$, $r = 0.4\,\text{m}$.
$s = 1.5\,\text{m}$, $t = 0.5$ s.

Find:

Acceleration along and normal to pathline (m/s^2).

PLAN

Apply Eq. 4.11 of EFM 10e for acceleration along pathline.

SOLUTION

Equation 4.5

$$\mathbf{a} = (V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t})\mathbf{u}_t + \left(\frac{V^2}{r}\right)\mathbf{u}_n$$

Evaluation of velocity and derivatives at $s = 2$ m and $t = 0.5$ sec.

$$\begin{aligned} V &= s^2 t^{1/2} = 1.5^2 \times 0.5^{1/2} = 1.59 \text{ m/s} \\ \frac{\partial V}{\partial s} &= 2st^{1/2} = 2 \times 1.5 \times 0.5^{1/2} = 2.121/\text{s} \\ \frac{\partial V}{\partial t} &= \frac{1}{2}s^2 t^{-1/2} = \frac{1}{2} \times 1.5^2 \times 0.5^{-1/2} = 1.59 \text{ m/s}^2 \end{aligned}$$

Evaluation of the acceleration

$$\mathbf{a} = (1.59 \times 2.12 + 1.59)\mathbf{u}_t + \left(\frac{1.59^2}{0.4}\right)\mathbf{u}_n$$

$$\boxed{\mathbf{a} = 4.96\mathbf{u}_t + 6.32\mathbf{u}_n \;\; (\text{m/s}^2)}$$

4.26: PROBLEM DEFINITION

Situation:

Air is flowing around a sphere in a wind tunnel.

$u = -U_o(1 - r_o^3/x^3)$.

Find:

An expression for the acceleration of a fluid particle on the x-axis. The form of the answer should be $a_x = a_x\left(x, r_o, U_o\right)$.

PLAN

Use Eq. 4.5 along x-axis which is a pathline. Replace V with u and s with x.

SOLUTION

$$\begin{aligned}
a_x &= u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} \\
&= -U_0\left(1 - \frac{r_0^3}{x^3}\right)\frac{\partial}{\partial x}\left(-U_0\left(1 - \frac{r_0^3}{x^3}\right)\right) + \frac{\partial}{\partial t}\left(-U_0\left(1 - \frac{r_0^3}{x^3}\right)\right) \\
&= -U_0^2\left(1 - \frac{r_0^3}{x^3}\right)\left(-3\frac{r_0^3}{x^4}\right) + 0
\end{aligned}$$

$$\boxed{a_x = (3\,U_0^2\,\frac{r_0^3}{x^4})(1 - \frac{r_0^3}{x^3})}$$

4.27: PROBLEM DEFINITION

Situation:

Flow occurs in a tapered passage.
$V = 5\,\mathrm{m/s} - 2.25\frac{t}{t_o}\,\mathrm{m/s}$,
$\partial V/\partial s = +2\,\mathrm{s}^{-1}$ at $t = 0.5\,\mathrm{s}$.

Find:

(a) Local acceleration at section AA ($\mathrm{m/s^2}$).
(b) Convective acceleration at section AA ($\mathrm{m/s^2}$).

SOLUTION

a) Local acceleration

$$\begin{aligned} a_l &= \frac{\partial V}{\partial t} = -\frac{2.25}{t_0} \\ &= -\frac{2.25}{0.5} \\ & \boxed{a_l = -4.5 \text{ m/s}^2} \end{aligned}$$

b) Convective acceleration

$$\begin{aligned} a_c &= V\frac{\partial V}{\partial s} \\ &= \left((5-2.25)\times\frac{0.5}{0.5}\right) \text{ m/s} \times 2 \text{ s}^{-1} \\ & \boxed{a_c = 5.5 \text{ m/s}^2} \end{aligned}$$

4.28: PROBLEM DEFINITION

Situation:

One-dimensional flow occurs in a nozzle.

$V_{tip} = 1.2\,\text{m/s}$, $V_{base} = 0.3\,\text{m/s}$, $L = 0.5\,\text{m}$.

Find:

Convective acceleration (m/s^2).

SOLUTION

Velocity gradient.

$$
\begin{aligned}
\frac{dV}{ds} &= \frac{V_{\text{tip}} - V_{\text{base}}}{L} \\
&= \frac{(1.2 - 0.3)\ \text{m/s}}{0.5\,\text{m}} \\
&= 2\ \text{s}^{-1}
\end{aligned}
$$

Acceleration at mid-point

$$
\begin{aligned}
V &= \frac{(0.3 + 1.2)\,\text{m/ s}}{2} \\
&= 0.75\ \text{m/s} \\
a_c &= V\frac{dV}{ds} \\
&= 0.75\,\text{m/ s} \times 2 \\
& \boxed{a_c = 1.5\,\text{m/s}^2}
\end{aligned}
$$

4.29: PROBLEM DEFINITION

Situation:

One-dimensional flow occurs in a nozzle and the velocity varies linearly with distance along the nozzle.

$V_{tip} = 6t\,\mathrm{m/s}$, $V_{base} = 2t\,\mathrm{m/s}$, $t = 2\,\mathrm{s}$.

Find:

Local acceleration midway in the nozzle ($\mathrm{m/s^2}$).

SOLUTION

$$
\begin{aligned}
a_\ell &= \frac{\partial V}{\partial t} \\
V &= \frac{2t + 6t}{2} \\
&= 4t \text{ (m/s)}
\end{aligned}
$$

Then

$$
a_\ell = \frac{\partial}{\partial t}(4t)
$$

$$
\boxed{a_\ell = 4.0 \text{ m/s}^2}
$$

4.30: PROBLEM DEFINITION

Situation:

Flow in a two-dimensional slot.
$V = 2\left(\frac{q_o}{b}\right)\left(\frac{t}{t_o}\right)$, $x = 2B$, $y = 0\,\text{in.}$

Find:

An expression for local acceleration midway in nozzle.

SOLUTION

$$\begin{aligned} V &= 2\left(\frac{q_o}{b}\right)\left(\frac{t}{t_o}\right) \text{ but } b = B/2 \\ V &= \left(\frac{4q_o}{B}\right)\left(\frac{t}{t_o}\right) \\ a_l &= \frac{\partial V}{\partial t} \end{aligned}$$

$$\boxed{a_l = \frac{4q_o}{Bt_o}}$$

4.31: PROBLEM DEFINITION

Situation:

Flow in a two-dimensional slot and velocity varies as
$V = 2\left(\frac{q_o}{b}\right)\left(\frac{t}{t_o}\right)$, $x = 2B$, $y = 0\,\text{cm}$.

Find:

An expression for convective acceleration midway in nozzle.

SOLUTION

$$a_c = \frac{V\partial V}{\partial x}$$

The width varies as

$$b = B - \frac{x}{8}$$

$$\begin{aligned} V &= \left(\frac{q_0}{t_0}\right) 2t \left(B - \frac{x}{8}\right)^{-1} \\ \frac{\partial V}{\partial x} &= \left(\frac{q_0}{t_0}\right) 2t \left(\frac{1}{8}\right) \left(B - \frac{x}{8}\right)^{-2} \\ a_c &= \frac{V\partial V}{\partial x} = \frac{(q_0/t_0)^2 4t^2\,(1/8)}{(B-(1/8)x)^3} \end{aligned}$$

At $x = 2B$

$$a_c = (1/2)\left(\frac{q_0}{t_0}\right)^2 \frac{t^2}{((3/4)B)^3}$$

$$\boxed{a_c = 32/27\left(\frac{q_0}{t_0}\right)^2 \frac{t^2}{B^3}}$$

4.32: PROBLEM DEFINITION

Situation:

Water flow in a nozzle with

$$V = \frac{2t}{(1-0.5x/L)^2}$$

$L = 1.2\,\text{m}$, $x = 0.5L$, $t = 3\,\text{s}$.

Find:

Local acceleration (m/ s²).

Convective acceleration (m/ s²).

SOLUTION

$$\begin{aligned}
a_\ell &= \partial V/\partial t \\
&= \partial/\partial t[2t/(1-0.5x/L)^2] \\
&= 2/(1-0.5x/L)^2 \\
&= 2/(1-0.5\times 0.5L/L)^2 \\
& \boxed{a_\ell = 3.56 \text{ m/s}^2} \\
a_c &= V(\partial V/\partial x) \\
&= [2t/(1-0.5x/L)^2]\partial/\partial x[2t/(1-0.5x/L)^2] \\
&= \frac{4t^2}{(1-0.5x/L)^5}(-2)\left(-\frac{0.5}{L}\right) \\
&= \frac{4\times 3^2}{(1-0.5\times 0.5L/L)^5}\left(\frac{1}{L}\right) \\
& \boxed{a_c = 63 \text{ m/s}^2}
\end{aligned}$$

4.33: PROBLEM DEFINITION

Situation:

State Newton's second law of motion.

Find:

Are there any limitations on the use of Newton's second law?

SOLUTION

Newtons second law states

$$\vec{F} = m\vec{a}$$

where m is the mass of the system. The velocity (and acceleration) must be measured with respect to an inertial reference frame and the mass must be constant.

4.34: PROBLEM DEFINITION

Situation:

Force weight and force pressure.

Find:

What is the difference between a force due to weight and a force due to pressure?

SOLUTION

The force due to weight is the gravitational attraction on the mass and the magnitude of the force depends on the mass. The force due to pressure is the force acting on a surface and depends on the magnitude of the pressure and the area of the surface.

4.35: PROBLEM DEFINITION

Situation:

Flow through an inclined pipe at 30° from horizontal.

$a_\ell = -0.4$g.

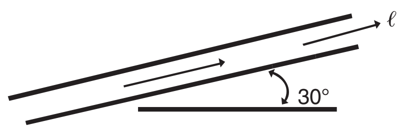
Find:

Pressure gradient in flow direction.

PLAN

Apply Euler's equation.

SOLUTION



Euler's equation

$$
\begin{aligned}
\frac{\partial}{\partial \ell}(p + \gamma z) &= -\rho a_\ell \\
\frac{\partial p}{\partial \ell} + \gamma \frac{\partial z}{\partial \ell} &= -\rho a_\ell \\
\frac{\partial p}{\partial \ell} &= -\rho a_\ell - \gamma \frac{\partial z}{\partial \ell} \\
&= -\frac{\gamma}{g} \times (-0.40g) - \gamma \sin 30^\circ \\
&= \gamma(0.40 - 0.50) \\
& \boxed{\frac{\partial p}{\partial \ell} = -0.10\gamma}
\end{aligned}
$$

4.36: PROBLEM DEFINITION

Situation:

Kerosene is accelerated upward in vertical pipe.
$S = 0.81$, $a_z = 0.5\text{g}$.

Find:

Pressure gradient required to accelerate flow (N/m^3).

Properties:

$\gamma = 9810\,\text{N/m}^3$.

PLAN

Apply Euler's equation.

SOLUTION

Applying Euler's equation in the z direction.

$$
\begin{aligned}
\frac{\partial(p+\gamma z)}{\partial z} &= -\rho a_z = -\frac{\gamma}{g} \times 0.50g \\
\frac{\partial p}{\partial z} + \gamma &= -0.50\gamma \\
\frac{\partial p}{\partial z} &= \gamma(-1-0.50) \\
&= 0.81\left(9810\,\text{N/m}^3\right)(-1.50) \\
& \boxed{\frac{\partial p}{\partial z} = -11.92\ \text{kN/m}^3}
\end{aligned}
$$

4.37: PROBLEM DEFINITION

Situation:
A hypothetical liquid flows through a vertical tube.
$v = 0$.

Find:
Direction of acceleration.

Properties:
$\gamma = 10\,\text{kN/m}^3$, $p_B - p_A = 12\,\text{kPa}$.

PLAN Apply Euler's equation.

SOLUTION Euler's equation

$$\begin{aligned} \rho a_\ell &= -\frac{\partial}{\partial \ell}(p + \gamma z) \\ a_\ell &= \frac{1}{\rho}\left(-\frac{\partial p}{\partial \ell} - \gamma \frac{\partial z}{\partial \ell}\right) \end{aligned}$$

Let ℓ be positive upward. Then $\partial z/\partial \ell = +1$ and $\partial p/\partial \ell = (p_A - p_B)/1 = -12,000$ Pa/m. Thus

$$\begin{aligned} a_\ell &= \frac{g}{\gamma}(12,000 - \gamma) \\ a_\ell &= g\left(\frac{12,000}{\gamma} - 1\right) \\ a_\ell &= g(1.2 - 1.0)\ \text{m/s}^2 \end{aligned}$$

a_ℓ has a positive value; therefore, acceleration is upward. Correct answer is **a).**

4.38: PROBLEM DEFINITION

Situation:
A piston and water accelerating upward at 0.4g.
$a = 0.4\text{g}$, $z = 0.6\,\text{m}$.

Find:
Pressure in water column (Pa).

Properties:
$\rho = 1000\ \text{kg/m}^3$, $\gamma = 9810\,\text{N/m}^3$

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation

$$\rho a_\ell = -\frac{\partial}{\partial \ell}(p + \gamma z)$$

Let ℓ be positive upward.

$$\begin{aligned}
\rho(0.4g) &= -\frac{\partial p}{\partial \ell} - \gamma\frac{\partial z}{\partial \ell} \\
(\frac{\gamma}{g})(0.4g) &= -\frac{\partial p}{\partial \ell} - \gamma(0.3) \\
\frac{\partial p}{\partial \ell} &= -\gamma(0.4 + 0.3) = -0.7\gamma
\end{aligned}$$

Thus the pressure decreases upward at a rate of 0.7γ. The pressure at the top is atmospheric. At a depth of 0.6 m.:

$$\begin{aligned}
p_2 &= (0.7\gamma)(0.6) = 0.42\gamma \\
&= 0.42\ \text{m.} \times 9810\ \text{N/m}^3 \\
&\boxed{p_2 = 4120\ \text{Pa}}
\end{aligned}$$

4.39: PROBLEM DEFINITION

Situation:

Water stands with depth of 3 m in a vertical pipe open at top and supported by piston at the bottom.

$z = 0\,\text{m}$, $z_2 = 3\,\text{m}$.

Find:

Acceleration of piston (m/s^2).

Properties:

$\gamma = 9810\,\text{N/m}^3$, $\rho = 1000\,\text{kg/m}^3$.

$p_1 = 55$ kPa, $p_2 = 0$ kPa.

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation

$$\frac{\partial}{\partial s}(p + \gamma z) = -\rho a_s$$

Take s as vertically upward with point 1 at piston surface and point 2 at water surface.

$$\begin{aligned}
-\Delta(p + \gamma z) &= \rho a_s \Delta s \\
-(p_2 - p_1) - \gamma(z_2 - z_1) &= \rho a_s \Delta s \\
-(0 - 55\text{ kPa}) - 9810\,\text{N/m}^3 \times 3\,\text{m} &= 1000\,\text{kg/m}^3 \times 3a_s \\
a_s &= \frac{(55,000\,\text{N/m}^2 - 9810\,\text{N/m}^3 \times 3\,\text{m})}{3000\,\text{kg/m}^3} \\
& \boxed{a_s = 8.5\text{ m/s}^2}
\end{aligned}$$

4.40: PROBLEM DEFINITION

Situation:

Water accelerates in a horizontal pipe.

$a_s = 8\,\mathrm{m/s^2}$, $\rho = 1000\,\mathrm{kg/m^3}$.

Find:

Pressure gradient ($\mathrm{N/m^3}$).

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation with no change in elevation

$$\begin{aligned}\frac{\partial p}{\partial s} &= -\rho a_s \\ &= -1,000\,\mathrm{kg/m^3} \times 8\,\mathrm{m/s^2} \\ &\boxed{\frac{\partial p}{\partial s} = -8,000\ \mathrm{N/m^3}}\end{aligned}$$

4.41: PROBLEM DEFINITION

Situation:

Water accelerated from rest in horizontal pipe.

$L = 80\,\text{m}$, $D = 30\,\text{cm}$, $a_s = 5\,\text{m/s}^2$.

Find:

Pressure at upstream end (kPa).

Properties:

$\rho = 1000\,\text{kg/m}^3$, $p_{downstream} = 90\,\text{kPa}$.

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation with no change in elevation

$$
\begin{aligned}
\frac{\partial p}{\partial s} &= -\rho a_s \\
&= -1,000\,\text{kg/m}^3 \times 5\,\text{m/s}^2 \\
&= -5,000\ \text{N/m}^3 \\
p_{\text{downstream}} - p_{\text{upstream}} &= \frac{\partial p}{\partial s}\Delta s \\
p_{\text{upstream}} &= 90,000\,\text{Pa} + \left(5,000\,\text{N/m}^3\right)(80\,\text{m}) \\
&= 490,000\ \text{Pa gage} \\
&\boxed{p_{\text{upstream}} = 490\ \text{kPa gage}}
\end{aligned}
$$

4.42: PROBLEM DEFINITION

Situation:

Water stands in a vertical pipe closed at the bottom by a piston.

$z = 3\,\text{m}$.

Find:

Maximum downward acceleration before vaporization (m/s^2).

Assumptions:

Vapor pressure is zero.

Properties:

$\rho = 1000\ \text{kg/m}^3$, $\gamma = 9810\,\text{N/m}^3$.

PLAN

Apply Euler's equation.

SOLUTION

Applying Euler's equation in the z-direction with $p = 0$ at the piston surface

$$\begin{aligned}
\frac{\partial}{\partial z}(p + \gamma z) &= -\rho a_z \\
\Delta(p + \gamma z) &= -\rho a_z \Delta z \\
(p + \gamma z)_{\text{at water surface}} - (p + \gamma z)_{\text{at piston}} &= -\rho a_z (z_{\text{surface}} - z_{\text{piston}}) \\
p_{\text{atm}} - p_v + \gamma(z_{\text{surface}} - z_{\text{piston}}) &= -3\ \rho a_z \\
101300\,\text{N/m}^2 - 0 + \left(9810\,\text{N/m}^3\right)(3\,\text{m}) &= -3 \times 1000\,\text{kg/m}^3 \times a_z
\end{aligned}$$

$$\boxed{a_z = -43.6\ \text{m/s}^2}$$

4.43: PROBLEM DEFINITION

Situation:
A liquid flows through a conduit.

Find:
Which statements can be discerned with certainty:
(a) The velocity is in the positive ℓ direction.
(b) The velocity is in the negative ℓ direction.
(c) The acceleration is in the positive ℓ direction.
(d) The acceleration is in the negative ℓ direction.

Assumptions:
Viscosity is zero.

Properties:
$p_A = 8.1$ kPa, $p_B = 4.8$ kPa, $\gamma = 15,700\,\mathrm{N/m^3}$.

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation

$$-\frac{\partial}{\partial \ell}(p + \gamma z) = \rho a_\ell$$
$$-\frac{\partial p}{\partial \ell} - \gamma \frac{\partial z}{\partial \ell} = \rho a_\ell$$

where $\partial p/\partial \ell = (p_B - p_A)/\ell = (4800 - 8100)/0.6 = -5500$ N/m^3 and $\partial z/\partial \ell = \sin 30° = 0.5$. Then

$$a_\ell = \frac{1}{\rho}(5500\,\mathrm{N/m^3} - (4800)(0.5))$$
$$= \frac{1}{\rho}(-3100\ )\ \mathrm{N/m^3}$$

- Because a_ℓ has a negative value we conclude that Answer $\Rightarrow$ (d) the acceleration is in the negative ℓ direction .
- Answer $\Rightarrow$ The flow direction cannot be established; so answer (d) is the only answer that can be discerned with certainty.

4.44: PROBLEM DEFINITION

Situation:

Velocity varies linearly with distance in water nozzle.
$L = 0.3\,\text{m}$, $V_1 = 9\,\text{m/s}$, $V_2 = 24\,\text{m/s}$.

Find: Pressure gradient midway in the nozzle (kPa/m).

Properties:
$\rho = 1000\,\text{kg/m}^3$.

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation

$$\frac{\partial}{\partial x}(p + \gamma z) = -\rho a_x$$

but z =const.; therefore

$$\frac{\partial p}{\partial x} = -\rho a_x$$

$$a_x = a_{\text{convective}} = V\frac{\partial V}{\partial x}$$

$$\frac{\partial V}{\partial x} = (24 - 9)/0.3 = 50\ \text{s}^{-1}$$

$$V_{\text{mid}} = (24\,\text{m/s} + 9\,\text{m/s})/2 = 16.5\ \text{m/s}$$

$$a_x = (16.5\ \text{m/s})(50\ \text{m/s/m}) = 825\ \text{m/s}^2$$

Finally

$$\frac{\partial p}{\partial x} = (1000\ \text{kg/m}^3)(825\ \text{m/s}^2)$$

$$\boxed{\frac{\partial p}{\partial x} = -825\ \text{kPa/m}}$$

4.45: PROBLEM DEFINITION

Situation:

Closed tank is full of liquid.
$L = 0.9\,\text{m}$, $H = 1.2\,\text{m}$, $a_x = 0.9g$.
$a_\ell = 1.5g$, $S = 1.2$.

Find:

(a) $p_C - p_A$ (Pa).
(b) $p_B - p_A$ (Pa).

Properties:

$\rho = 1000\,\text{kg/m}^3$.

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation. Take ℓ in the z-direction.

$$-\frac{dp}{d\ell} - \gamma\frac{d\ell}{d\ell} = \rho a_l$$

$$\begin{aligned}
\frac{dp}{d\ell} &= -\rho(g + a_\ell) \\
&= -1.2\left(1000\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2 - 1.5\left(9.81\,\text{m/s}^2\right)\right) \\
&= 5886\ \text{Pa/m} \\
p_B - p_A &= -5886\ \text{Pa/m} \times 1.2\,\text{m} \\
& \boxed{p_B - p_A = -7063.2\ \text{Pa}}
\end{aligned}$$

Take ℓ in the x-direction. Euler's equation becomes

$$\begin{aligned}
-\frac{dp}{dx} &= \rho a_x \\
p_C - p_B &= \rho a_x L \\
&= 1.2 \times 1000\,\text{kg/m}^3 \times 0.9g \times 0.9\,\text{m} \\
&= 9535.3\ \text{Pa} \\
p_C - p_A &= p_C - p_B + (p_B - p_A) \\
p_C - p_A &= 9535.3\ \text{Pa} - 7063.2\ \text{Pa} \\
& \boxed{p_C - p_A = 2472.1\ \text{Pa}}
\end{aligned}$$

4.46: PROBLEM DEFINITION

Situation:

Closed tank is full of liquid.

$L = 2.5\,\mathrm{m}$, $H = 3\,\mathrm{m}$, $a_\ell = 2/3g$, $a_x = 1.0g$, $S = 1.3$.

Find:

(a) $p_C - p_A$ (kPa).

(b) $p_B - p_A$ (kPa).

Properties:

$\rho = 1000\,\mathrm{kg/\,m^3}$.

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation in z direction

$$\begin{aligned}
\frac{dp}{dz} + \gamma &= -\rho a_z \\
\frac{dp}{dz} &= -\rho(g + a_z) \\
\frac{dp}{dz} &= -1.3\left(1,000\,\mathrm{kg/\,m^3}\right)\left(9.81\,\mathrm{m/\,s^2} - 6.54\,\mathrm{m/\,s^2}\right) \\
&= -4,251\ \mathrm{N/m^3} \\
p_B - p_A &= \left(4,251\,\mathrm{N/\,m^3}\right)(3\,\mathrm{m}) \\
&= 12,753\ \mathrm{Pa} \\
&\boxed{p_B - p_A = 12.7\ \mathrm{kPa}}
\end{aligned}$$

Euler's equation in x-direction

$$\begin{aligned}
-\frac{dp}{dx} &= \rho a_x \\
p_C - p_B &= \rho a_x L \\
&= 1.3 \times 1,000 \times 9.81 \times 2.5 \\
&= 31,882\ \mathrm{Pa} \\
p_C - p_A &= p_C - p_B + (p_B - p_A) \\
p_C - p_A &= 31,882 + 12,753 \\
&= 44,635\ \mathrm{Pa} \\
&\boxed{p_C - p_A = 44.6\ \mathrm{kPa}}
\end{aligned}$$

4.47: PROBLEM DEFINITION

Situation:
Aspirators.

Find:
How does an aspirator work?

SOLUTION

Air is forced through a constriction in a duct There is a port at the smallest area connected to a reservoir of fluid to be aspirated. The Bernoulli equation predicts a minimum pressure at the contraction which pulls fluid into the air flow from the reservoir and breaks it up into droplets that emerge from the aspirator.

4.48: PROBLEM DEFINITION

Situation:

When the Bernoulli Equation applies to a venturi, such as in Fig. 4.27 (EFM 10e) in §4.6, which of the following are true? (Select all that apply.)

a. if the velocity head and elevation head increase, then the pressure head must decrease

b. pressure always decreases in the direction of flow along a streamline

c. the total head of the flowing fluid is constant along a streamline

SOLUTION

The correct answers are a and c.

REVIEW

If you selected b, you were probably thinking of a pipe of constant diameter. Because of the diameter changes in a venturi, at the location where the diameter is small, velocity must increase (due to continuity), and thus pressure must drop at the narrow location according to the Bernoulli Equation.

4.49: PROBLEM DEFINITION

Situation:

A water jet fires vertically from a nozzle.

$V = 18\,\text{m/s}$.

Find:

Height jet will rise.

PLAN

Apply the Bernoulli equation from the nozzle to the top of the jet. Let point 1 be in the jet at the nozzle and point 2 at the top.

SOLUTION

Bernoulli equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = p_2 = 0$ gage

$$\begin{aligned} V_1 &= 18 \text{ m/s} \\ V_2 &= 0 \\ 0 + \frac{(18\,\text{m/s})^2}{2g} + z_1 &= 0 + 0 + z_2 \\ z_2 - z_1 &= h = \frac{324\,\text{m}^2/\text{s}^2}{19.62\,\text{m/s}^2} \\ & \boxed{h = 16.5\,\text{m}} \end{aligned}$$

4.50: PROBLEM DEFINITION

Situation:

Water discharges from a pressurized tank.

$z_1 = 0.5\,\text{m}$, $z_2 = 0\,\text{m}$, $V_1 = 0\,\text{m/s}$.

Find:

Velocity of water at outlet (m/s).

Properties:

Water (20 °C, 10 kPa), Table A.5: $\rho = 998\,\text{kg/m}^3$, $\gamma = 9790\,\text{N/m}^3$.

SOLUTION

Apply the Bernoulli equation between the water surface in the tank (1) and the outlet (2)

$$p_1 + \gamma z_1 + \rho\frac{V_1^2}{2} = p_2 + \gamma z_2 + \rho\frac{V_2^2}{2}$$

Neglect V_1 ($V_1 \ll V_2$).Also $p_2 = 0$ gage. The Bernoulli equation reduces to

$$\begin{aligned} \rho\frac{V_2^2}{2} &= p_1 + \gamma(z_1 - z_2) \\ V_2 &= \sqrt{\frac{2\left(p_1 + \gamma(z_1 - z_2)\right)}{\rho}} \end{aligned}$$

Elevation difference $z_1 - z_2 = 0.5$ m. For water at 20^{o}C, $\rho = 998\text{kg/m}^3$ and $\gamma = 9790$ N/m^3.Therefore

$$V_2 = \sqrt{\frac{2(10,000\,\text{Pa} + 9790\,\text{N/m}^3\,(0.5\,\text{m}))}{998\,\text{kg/m}^3}}$$

$$\boxed{V_2 = 5.46\,\text{m/s}}$$

4.51: PROBLEM DEFINITION

Situation:

Water flows through a vertical venturi configuration.

$V_1 = 3\,\text{m/s}$, $\Delta z = 0.15\,\text{m}$.

Find:

Velocity at minimum area (m/s).

Properties:

$T = 20\,^\circ\text{C}$.

SOLUTION

Apply the Bernoulli equation between the pipe (1) and the minimum area (2)

$$p_1 + \gamma z_1 + \rho\frac{V_1^2}{2} = p_2 + \gamma z_2 + \rho\frac{V_2^2}{2}$$

From problem statement, $V_1 = 3$ m/s. Rewriting equation

$$\rho\frac{V_2^2}{2} = \rho\frac{V_1^2}{2} + (p_1 + \gamma z_1) - (p_2 + \gamma z_2)$$

The difference in the elevation in piezometers gives the change in piezometric pressure, $(p_1 + \gamma z_1) - (p_2 + \gamma z_2) = \gamma \Delta h$ so

$$\begin{aligned} V_2 &= \sqrt{V_1^2 + \frac{2\gamma\Delta h}{\rho}} = \sqrt{V_1^2 + 2g\Delta h} \\ &= \sqrt{3^2\ (\text{m/s})^2 + 2\left(9.81\ \text{m/s}^2\right)(0.15\ \text{m})} \\ &\boxed{V_2 = 3.5\,\text{m/s}} \end{aligned}$$

4.52: PROBLEM DEFINITION

Situation:

Kerosene flows through a contraction section and a pressure is measured between pipe and contraction section.

$V_2 = 10\,\text{m/s}$.

Find:

Velocity in upstream pipe (m/s).

Properties:

Table A.4: $\rho = 814\,\text{kg/m}^3$.

$T = 20\,^\circ\text{C}$, $\Delta p = 20\,\text{kPa}$.

SOLUTION

Apply the Bernoulli equation between pipe (1) and contraction section (2)

$$
\begin{aligned}
p_1 + \gamma z_1 + \rho \frac{V_1^2}{2} &= p_2 + \gamma z_2 + \rho \frac{V_2^2}{2} \\
p_{z1} + \rho \frac{V_1^2}{2} &= p_{z2} + \rho \frac{V_2^2}{2}
\end{aligned}
$$

The pressure gage measures the difference in piezometric pressure, $p_{z1} - p_{z2} = 20$ kPa. Rewrite the Bernoulli equation for V_1

$$
\begin{aligned}
\rho \frac{V_1^2}{2} &= \rho \frac{V_2^2}{2} - (p_{z1} - p_{z2}) \\
V_1 &= \sqrt{V_2^2 - \frac{2(p_{z1} - p_{z2})}{\rho}}
\end{aligned}
$$

The density of kerosene at $20\,^\circ\text{C}$ is 814 kg/m^3.Solving for V_1

$$
V_1 = \sqrt{(10\,\text{m/s})^2 - \frac{2(20,000\,\text{Pa})}{(814\,\text{kg/m}^3)}}
$$

$$
\boxed{V_1 = 7.13\,\text{m/s}}
$$

4.53: PROBLEM DEFINITION

Situation:

A stagnation tube placed in a river (select all that apply)

a. can be used to determine air pressure

b. can be used to determine fluid velocity

c. measures kinetic pressure

SOLUTION

Correct answers are b and c.

4.54: PROBLEM DEFINITION

Situation:

A Pitot tube on an airplane is used to measure airspeed
$z_2 = 3048\,\text{m}$, $h_{H_2O} = 0.25\,\text{m}$.
$T = -5\,^\circ\text{C} = 268$ K, $p = 69$ kPa.

Find:

Airspeed (m/ s).

Properties:

Water ($-5\,^\circ$C), Table A.5: $\gamma = 9810\,\text{N/m}^3$.
Air. Table A.2: $R = 287\,\text{J/kg K}$.

PLAN

Since the airspeed can be found by applying the Pitot-static tube equation, the steps to reach the goal are:

1. Find Δp_z by using the hydrostatic equation.
2. Find density by applying the ideal gas law.
3. Substitute values into the Pitot-static tube equation.

SOLUTION

1. Hydrostatic equation.

$$\begin{aligned}\Delta p_z &= \gamma_{\text{H}_2\text{O}} h_{\text{H}_2\text{O}} \\ &= 9810\,\text{N/m}^3 \times 0.25\,\text{m} \\ &= 2453 \text{ Pa}\end{aligned}$$

2. Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{69 \text{ kPa}}{(287 \text{ J/kg} \cdot \text{K})(268 \text{ K})} \\ &= 0.897 \text{ kg/m}^3\end{aligned}$$

3. Pitot-Static Tube equation.

$$\begin{aligned}V &= \sqrt{\frac{2\Delta p_z}{\rho}} \\ V &= \sqrt{\frac{2 \times 2453 \text{ N/m}^2}{(0.897 \text{ kg/m}^3)}}\end{aligned}$$

$$\boxed{V = 74 \text{ m/s}}$$

4.55: PROBLEM DEFINITION

Situation:

A glass tube with 90° bend inserted into a stream of water.

$V = 5\,\text{m/s}$.

Find:

Rise in vertical leg above water surface (m).

PLAN

Apply the Bernoulli equation.

SOLUTION

Hydrostatic equation (between stagnation point and water surface in tube)

$$\frac{p_s}{\gamma} = h + d$$

where d is depth below surface and h is distance above water surface.

Bernoulli equation (between free stream and stagnation point)

$$\begin{aligned}
\frac{p_s}{\gamma} &= d + \frac{V^2}{2g} \\
h + d &= d + \frac{V^2}{2g} \\
h &= \frac{V^2}{2g}
\end{aligned}$$

$$\begin{aligned}
h &= \frac{(5\,\text{m/s})^2}{2\,(9.81\,\text{m/s}^2)} \\
& \boxed{h = 1.27\,\text{m}}
\end{aligned}$$

4.56: PROBLEM DEFINITION

Situation:

A Bourdon tube gage attached to plate in an air stream.

$D = 0.3$, $V_0 = 12\,\mathrm{m/s}$.

Find:

Pressure read by gage (>, =, <) $\rho V_0^2/2$.

SOLUTION

Because it is a Bourdon tube gage, the difference in pressure that is sensed will be between the stagnation point and the separation zone downstream of the plate. Therefore

$$\begin{aligned}
\Delta C_p &= 1 - (C_{p,\text{back of plate}}) \\
\Delta C_p &= 1 - (\text{neg. number}) \\
\therefore \frac{\Delta p}{\rho V_0^2/2} &= 1 + \text{positive number} \\
\Delta p &= \left(\frac{\rho V_0^2}{2}\right)(1 + \text{positive number})
\end{aligned}$$

Case (c) is the correct choice.

4.57: PROBLEM DEFINITION

Situation:

An air-water manometer is connected to a Pitot-static tube to measure air velocity.
$T = 15.5\,^\circ\text{C}$, $\Delta h = 5\,\text{cm}$.

Find:

Velocity (m/ s).

Properties:

Table A.2: $R = 287\,\text{J/kg K}$.
Water (15.5 °C, 103 kPa), Table A.5: $\gamma = 9810\,\text{N/m}^3$.

PLAN Apply the Pitot tube equation calculate velocity. Apply the ideal gas law to solve for density.

SOLUTION Ideal gas law

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{103\,\text{kPa}}{(287\,\text{J/kg K})(15.5 + 273)\,\text{K}} \\ &= 1.24\ \text{kg/m}^3 \end{aligned}$$

Pitot tube equation

$$V = \left(\frac{2\Delta p_z}{\rho}\right)^{1/2}$$

From the manometer equation

$$\Delta p_z = \gamma_w \Delta h \left(1 - \frac{\gamma_a}{\gamma_w}\right)$$

but $\gamma_a/\gamma_w \ll 1$ so

$$\begin{aligned} V &= \left(\frac{2\gamma_w \Delta h}{\rho}\right)^{1/2} \\ &= \left[\frac{2\,(9810\,\text{N/m}^3)\,(0.05\,\text{m})}{1.24\,\text{kg/m}^3}\right]^{1/2} \end{aligned}$$

$$\boxed{V = 28.1\,\text{m/s}}$$

4.58: PROBLEM DEFINITION

Situation:

A flow-metering device is described in the problem.

$V_2 = 1.5V_1$, $\Delta h = 10\,\text{cm}$.

Find:

Velocity at station 2 (m/ s).

Properties:

$\rho = 1.2\,\text{kg/m}^3$.

PLAN

Apply the Bernoulli equation and the manometer equation.

SOLUTION

Bernoulli equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_t}{\gamma}$$

Manometer equation

$$\begin{aligned}
p_1 + 0.1 \times 9810 - \overbrace{0.1 \times 1.2 \times 9.81}^{\text{neglect}} &= p_t \\
p_t - p_1 &= 981\ \text{N/m}^2 = \frac{\rho V_1^2}{2} \\
V_1^2 &= \frac{2\,(981\,\text{N/m}^2)}{1.2\,\text{kg/m}^3} \\
V_1 &= 40.4\,\text{m/s} \\
V_2 &= 1.5V_1 \\
&\boxed{V_2 = 60.7\,\text{m/s}}
\end{aligned}$$

4.59: PROBLEM DEFINITION

Situation:

A spherical Pitot tube is used to measure the flow velocity in water.
$V_2 = 1.5V_0$, $\Delta h = 10\,\text{cm}$.

Find:

Free stream velocity (m/ s).

Properties:

$\rho = 1000\,\text{kg/ m}^3$, $\Delta p = 2\,\text{kPa}$.

PLAN

Apply the Bernoulli equation between the two points. Let point 1 be the stagnation point and point 2 at 90° around the sphere.

SOLUTION

Bernoulli equation

$$\begin{aligned} p_{z1} + \frac{\rho V_1^2}{2} &= p_{z2} + \frac{\rho V_2^2}{2} \\ p_{z1} + 0 &= p_{z2} + \frac{\rho (1.5V_0)^2}{2} \\ p_{z1} - p_{z2} &= 1.125 \rho V_0^2 \\ V_0^2 &= \frac{2,000\,\text{Pa}}{1.125\,(1,000\,\text{kg/ m}^3)} = 1.778\ \text{m}^2/\text{s}^2 \\ & \boxed{V_0 = 1.33\,\text{m/ s}} \end{aligned}$$

4.60: PROBLEM DEFINITION

Situation:

A device for measuring the water velocity in a pipe consists of a cylinder with pressure taps at forward stagnation point and at the back on the cylinder.

$\rho = 1000\,\text{kg/m}^3$, $\Delta p = 500\,\text{Pa}$, Pressure Coefficient is -0.3.

Find:

Water velocity (m/ s).

PLAN

Apply the Bernoulli equation between the location of the two pressure taps. Let point 1 be the forward stagnation point and point 2 in the wake of the cylinder.

SOLUTION

The piezometric pressure at the forward pressure tap (stagnation point, $C_p = 1$) is

$$p_{z1} = p_{z0} + \rho\frac{V^2}{2}$$

At the rearward pressure tap

$$\frac{p_{z2} - p_{z0}}{\rho\frac{V_0^2}{2}} = -0.3$$

or

$$p_{z2} = p_{z0} - 0.3\rho\frac{V_0^2}{2}$$

The pressure difference is

$$p_{z1} - p_{z2} = 1.3\rho\frac{V_0^2}{2}$$

The pressure gage records the difference in piezometric pressure so

$$\begin{aligned} V_0 &= \left(\frac{2}{1.3\rho}\Delta p_z\right)^{1/2} \\ &= \left[\frac{2}{1.3\,(1000\,\text{kg/m}^3)}\,(500\,\text{Pa})\right]^{1/2} \\ &= 0.88\,\text{m/s} \end{aligned}$$

$$\boxed{V_0 = 0.88\,\text{m/s}}$$

4.61: PROBLEM DEFINITION

Situation:

A Pitot tube measures the flow direction and velocity in water.

Find:

Explain how to design the Pitot tube.

SOLUTION

Three pressure taps could be located on a sphere at an equal distance from the nominal stagnation point. The taps would be at intervals of 120^o. Then when the probe is mounted in the stream, its orientation could be changed in such a way as to make the pressure the same at the three taps. Then the axis of the probe would be aligned with the free stream velocity.

4.62: PROBLEM DEFINITION

Situation:

Two Pitot tubes are connected to air-water manometers to measure air and water velocities.

Find:

The relationship between V_A and V_W .

$$V = \sqrt{2g\Delta h} = \sqrt{\frac{2\Delta p_z}{\rho}}$$

SOLUTION

The Δp_z is the same for both; however,

$$\rho_w >> \rho_a$$

Therefore $V_A > V_W$. The correct choice is **b)**.

4.63: PROBLEM DEFINITION

Situation:

A Pitot tube measures the velocity of kerosene at center of a pipe.
$D = 30\,\text{cm}$, $\Delta h = 10\,\text{cm}$,

Find:

Velocity (m/ s).

Properties:

From Table A.4: $\rho_{\text{ker}} = 814\ \text{kg/m}^3$.
$T = 20\,^\circ\text{C}$, $\gamma_{\text{ker}} = 8010\ \text{N/m}^3$, $\gamma_{\text{HG}} = 133,000\ \text{N/m}^3$.

PLAN

Apply the Pitot tube equation and the hydrostatic equation.

SOLUTION

Hydrostatic equation

$$\begin{aligned}\Delta p_z &= \Delta h(\gamma_{\text{HG}} - \gamma_{\text{ker}}) \\ &= 0.1\,\text{m}(133,000 - 8010)\,\text{N/m}^3 \\ &= 12,499\,\text{N/m}^2\end{aligned}$$

Pitot tube equation

$$\begin{aligned}V &= \left(\frac{2\Delta p_z}{\rho}\right)^{1/2} \\ &= \left[\frac{2\left(12,499\,\text{N/m}^2\right)}{814\,\text{kg/m}^3}\right]^{1/2}\end{aligned}$$

$$\boxed{V = 5.5\,\text{m/s}}$$

4.64: PROBLEM DEFINITION

Situation:
A Pitot tube for measuring velocity of air.

Find:
Air velocity (m/ s).

Properties:
Air (20 °C), Table A.3: $\rho = 1.2$ kg/m^3.
$\Delta p_z = 2\,\text{kPa}$.

PLAN

Apply the Pitot tube equation.

SOLUTION

Pitot tube equation

$$
\begin{aligned}
V &= \left(\frac{2\Delta p_z}{\rho}\right)^{1/2} \\
&= \left[\frac{2\,(2,000\,\text{Pa})}{1.2\,\text{kg/}\,\text{m}^3}\right]^{1/2} \\
&\boxed{V = 57.7\,\text{m/}\,\text{s}}
\end{aligned}
$$

4.65: PROBLEM DEFINITION

Situation:

A Pitot tube is used to measure the velocity of air.

$\Delta p_z = 718$ Pa, $T = 15.5\,^{\circ}$C.

Find:

Air velocity (m/ s).

Properties:

Air (15.5 °C), Table A.3: $\rho = 1.22\ \text{kg/m}^3$.

PLAN

Apply the Pitot tube equation.

SOLUTION

Pitot tube equation

$$V = \sqrt{\frac{2\Delta p_z}{\rho}}$$

$$V = \left[\frac{2\left(718\,\text{N/m}^2\right)}{1.22\,\text{kg/m}^3}\right]^{1/2}$$

$$\boxed{V = 34.3\,\text{m/s}}$$

4.66: PROBLEM DEFINITION

Situation:

A Pitot tube measures gas velocity in a duct.

Find:

Gas velocity in duct (m/ s).

Properties:

$\Delta p_z = 13.8$ kPa, $\rho = 2.25\,\text{kg/m}^3$.

PLAN

Apply the Pitot tube equation.

SOLUTION

Pitot tube equation. The density is 2.25 kg/m^3.

$$\begin{aligned} V &= \sqrt{\frac{2\Delta p_z}{\rho}} \\ &= \left[\frac{2\,(13.8\text{ kPa})}{2.25\,\text{kg/m}^3}\right]^{1/2} \end{aligned}$$

$$\boxed{V = 111\text{ m/s}}$$

4.67: PROBLEM DEFINITION

Situation:

A sphere moving horizontally through still water.

$V_0 = 3.35\,\mathrm{m/\,s}$, $V_A = 0.3\,\mathrm{m/\,s}$.

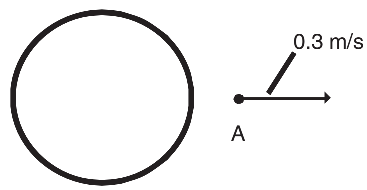
Find:

Pressure ratio: p_A/p_0

PLAN

Apply the Bernoulli equation.

SOLUTION



By referencing velocities to the spheres a steady flow case will be developed. Thus, for the steady flow case $V_0 = 3.35$ m/s and $V_A = 3$ m/s. Then when Bernoulli's equation is applied between points 0 and A it will be found that $\boxed{p_A/p_0 > 1 \text{ (case c)}}$

4.68: PROBLEM DEFINITION

Situation:

A body moving horizontally through still water.
$V_A = 13\,\mathrm{m/s}$, $V_B = 5\,\mathrm{m/s}$, $V_C = 3\,\mathrm{m/s}$.

Find:

$p_B - p_C$ (kPa).

SOLUTION

Apply the Bernoulli equation.

$$p_B - p_C = \frac{\rho}{2}(V_C^2 - V_B^2) \tag{1}$$

Reference all velocities to an observer situated on the sphere. From this reference frame, the flow is steady and the Bernoulli equation is applicable.

$$V_C = 13\,\mathrm{m/s} - 3\,\mathrm{m/s} = 10\,\mathrm{m/s} \tag{2}$$

$$V_B = 13\,\mathrm{m/s} - 5\,\mathrm{m/s} = 8\,\mathrm{m/s} \tag{3}$$

Combine Eqs. (1) to (3)

$$\begin{aligned} p_B - p_C &= \frac{\rho}{2}(V_C^2 - V_B^2) \\ p_B - p_C &= (\frac{1,000\,\mathrm{kg/m^3}}{2})[(10\,\mathrm{m/s})^2 - (8\,\mathrm{m/s})^2] \\ &= 18,000\,\mathrm{Pa} \\ & \boxed{p_B - p_C = 18\text{ kPa}} \end{aligned}$$

4.69: PROBLEM DEFINITION

Situation:

Water is in a flume with a pressure gage along the bottom.

$D_a = D_b$, $V_a = 0\,\mathrm{m/s}$, $V_b = 3\,\mathrm{m/s}$.

Find:

If gage A will read greater or less than gage B.

SOLUTION

Both gage A and B will read the same, due to hydrostatic pressure distribution in the vertical in both cases. There is no acceleration in the vertical direction.

4.70: PROBLEM DEFINITION

Situation:

An apparatus is used to measure the air velocity in a duct. It is connected to a slant tube manometer with a 30º leg with the indicated deflection.

$D = 10\,\text{cm}$, $D_{stagn} = 2\,\text{mm}$

$\ell_1 = 6.7\,\text{cm}$, $\ell_2 = 2.3\,\text{cm}$.

Find:

Air velocity (m/ s).

Properties:

Table A.2: $R = 287\,\text{J/ kg K}$.

$T = 20\,°\text{C}$, $p_{stagn} = 150\,\text{kPa}$, $S = 0.7$

PLAN

Apply the Bernoulli equation.

SOLUTION

The side tube samples the static pressure for the undisturbed flow and the central tube senses the stagnation pressure.
Bernoulli equation

$$\begin{aligned} p_0 + \frac{\rho V_0^2}{2} &= p_{\text{stagn.}} + 0 \\ \text{or } V_0 &= \sqrt{\frac{2}{\rho}(p_{\text{stagn.}} - p_0)} \end{aligned}$$

But

$$\begin{aligned} p_{\text{stagn.}} - p_0 &= (\ell_1 - \ell_2)\sin\theta(\gamma_m - \gamma_{air}) \\ \text{but } \gamma_m &\gg \gamma_{air} \\ p_{\text{stagn.}} - p_0 &= (0.067\,\text{m} - 0.023\,\text{m})\sin 30°\,(0.7)\left(9,810\,\text{N/ m}^3\right) = 151.1 \text{ Pa} \\ \rho &= \frac{p}{RT} = \frac{150,000\,\text{Pa}}{(287\,\text{J/ kg K})\,(273+20)\,\text{K}} = 1.784 \text{ kg/m}^3 \end{aligned}$$

Then

$$V_0 = \sqrt{\frac{2}{1.784\,\text{kg/ m}^3}(151.1\,\text{Pa})}$$

$$\boxed{V_0 = 13.0\,\text{m/ s}}$$

4.71: PROBLEM DEFINITION

Situation:

An instrument used to find gas velocity in smoke stacks.
$C_{pA} = 1$, $C_{pB} = -0.3$, $\Delta h = 5\,\mathrm{mm}$.

Find:

Velocity of stack gases (m/ s).

Properties:

$T = 20\,^{\circ}\mathrm{C}$, $R = 200\,\mathrm{J/\,kg\,K}$.
$T_{gas} = 250\,^{\circ}\mathrm{C}$, $p_{gas} = 101\,\mathrm{kPa}$.

SOLUTION

Ideal gas law

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{101,000\,\mathrm{Pa}}{(200\,\mathrm{J/\,kg\,K})\,(250+273)\,\mathrm{K}} \\ &= 0.966\ \mathrm{kg/m^2} \end{aligned}$$

Manometer equation

$$\Delta p_z = (\gamma_w - \gamma_a)\Delta h$$

but $\gamma_w \gg \gamma_a$ so

$$\begin{aligned} \Delta p_z &= \gamma_w \Delta h \\ &= 9790\,\mathrm{N/\,m^3}\,(0.005\,\mathrm{m}) \\ &= 48.95\ \mathrm{Pa} \end{aligned}$$

$$\begin{aligned} (p_A - p_B)_z &= (C_{pA} - C_{pB})\frac{\rho V_0^2}{2} \\ (p_A - p_B)_z &= 1.3\frac{\rho V_0^2}{2} \\ V_0^2 &= \frac{2\,(48.95\,\mathrm{Pa})}{1.3\,(0.966\,\mathrm{kg/\,m^2})} \end{aligned}$$

$$\boxed{V_0 = 8.83\ \mathrm{m/s}}$$

4.72: PROBLEM DEFINITION

Situation:

The wake of a sphere which separates at 120°.

$V_0 = 100\,\text{m/s}$.

$V = 1.5V_0$, $\theta = 120°$.

Find:

(a) Gage pressure (kPa).

(b) Pressure coefficient.

Properties:

$\rho = 1.2\,\text{kg/m}^3$.

PLAN

Apply the Bernoulli equation from the free stream to the point of separation and the pressure coefficient equation.

SOLUTION

Pressure coefficient

$$C_p = \frac{p - p_0}{\rho V^2/2}$$

Bernoulli equation

$$\begin{aligned} p_0 + \frac{\rho U^2}{2} &= p + \frac{\rho u^2}{2} \\ p - p_0 &= \frac{\rho}{2}(U^2 - u^2) \end{aligned}$$

or

$$\frac{p - p_0}{\rho U^2/2} = (1 - \left(\frac{u}{U}\right)^2)$$

but

$$\begin{aligned} u &= 1.5U \sin\theta \\ u &= 1.5U \sin 120° \\ u &= 1.5U \times 0.866 \end{aligned}$$

At the separation point

$$\begin{aligned}
\frac{u}{U} &= 1.299 \\
\left(\frac{u}{U}\right)^2 &= 1.687 \\
C_p &= 1 - 1.687 \\
&\boxed{C_p = -0.687} \\
p_{\text{gage}} &= C_p \left(\frac{\rho}{2}\right) U^2 \\
&= (-0.687)(1.2\,\text{kg/m}^3/2)(100\,\text{m/s})^2 \\
&= -4,122 \text{ Pa} \\
&\boxed{p_{\text{gage}} = -4.12 \text{ kPa gage}}
\end{aligned}$$

4.73: PROBLEM DEFINITION

Situation:

An airplane uses a Pitot-static tube to measure airspeed.

$z_2 = 3000\,\text{m}$, $V_{ind} = 70\,\text{m/s}$.

Find:

True air-speed (m/s).

Properties:

$T_{SL} = 17\,^\circ\text{C}$, $T = -6.3\,^\circ\text{C}$.

$p_{SL} = 101\,\text{kPa}$, $p = 70\,\text{kPa}$.

PLAN

Apply the Pitot-tube equation and correct for density change.

SOLUTION

The Pitot-static tube equation is

$$V = \left(\frac{2\Delta p}{\rho}\right)^{1/2}$$

Multiplying and dividing by the sea level density

$$V = \left(\frac{2\Delta p}{\rho_{SL}}\right)^{1/2}\left(\frac{\rho_{SL}}{\rho}\right)^{1/2}$$

The factor $\left(\frac{2\Delta p}{\rho_{SL}}\right)^{1/2}$ is the indicated airspeed so

$$V_{true} = V_{ind}\left(\frac{\rho_{SL}}{\rho}\right)^{1/2}$$

From the ideal gas law

$$\frac{\rho_{SL}}{\rho} = \frac{p_{SL}}{T_{SL}}\frac{T}{p} = \frac{101\,\text{kPa}}{70\,\text{kPa}}\frac{(273-6.3)\ \text{K}}{(273+17)\ \text{K}} = 1.327$$

True air speed

$$V_{true} = 70\ \text{m/s} \times \sqrt{1.327}$$

$$\boxed{V_{true} = 80.6\,\text{m/s}}$$

4.74: PROBLEM DEFINITION

Situation:

An airplane uses a Pitot-static tube to measure airspeed.

$z = 3048\,\text{m}$.

Find:

Speed of aircraft (km/s).

Properties:

$T_2 = 3.9\,^\circ\text{C}$, $p = 67.6$ kPa, $\Delta p = 3.5$ kPa.

SOLUTION

The temperature is 3.9 degrees Celcius and the pressure is 67.6 kPa (gage). The pressure difference is 3.5 kPa. The pressure is 67.6 kPa + 101.3 kPa = 168.9 kPa (abs). The temperature is 3.9 + 273.14 = 277.04 K. The gas constant is 8314/10 = 831.4 J/kmol K.

The density is

$$\rho = \frac{p}{RT} = \frac{168900 \text{ N/m}^2}{8314 \text{ J/kmol K} \times 277.04\,\text{K}} = 0.73 \text{ kg/m}^3$$

The differential pressure is 3.5 kPa.

The pitot equation is

$$V = \left(\frac{2\Delta p}{\rho}\right)^{1/2} = \left[\frac{2\left(3500 \text{ N/m}^2\right)}{0.73\,\text{kg/ m}^3}\right]^{1/2} = (9.6 \times 10^3)^{1/2} = 98 \text{ m/s}$$

$$\boxed{V = 0.098 \text{ km/s}}$$

4.75: PROBLEM DEFINITION

Situation:

Check equations for pitot tube velocity measurement provided by instrument company.

$V = 1096.7\sqrt{h_v/d}$, $d = 1.325P_a/T$.

Find:

Validity of Pitot tube equations provided.

PLAN

Apply the Bernoulli equation

SOLUTION

Applying the Bernoulli equation to the Pitot tube, the velocity is related to the change in piezometric pressure by

$$\Delta p_z = \rho\frac{V^2}{2}$$

where Δp_z is in Pa, ρ is in kg/m^3 and V is in m/s. The piezometric pressure difference is related to the "velocity pressure" by

$$\begin{aligned}\Delta p_z(\text{N/m}^2) &= \gamma_w(\text{kg/m}^3)h_v(\text{m}) \\ &= 9810 \times h_v \\ &= 9810h_v\end{aligned}$$

The density in kg/m^3 is given by

$$\begin{aligned}\rho(\text{kg/m}^3) &= \frac{d\ (\text{N/m}^3)}{g_c(\text{m/s}^2)} \\ &= \frac{d}{9.81} \\ &= 0.102d\end{aligned}$$

The velocity in m/min is obtained by multiplying the velocity in m/s by 60. Thus

$$\begin{aligned}V &= 60\sqrt{\frac{2 \times 9810h_v}{0.102d}} \\ &= 26314.8\sqrt{\frac{h_v}{d}}\end{aligned}$$

This differs by less than 0.1% from the manufacturer's recommendations. This could be due to the value used for g_c but the difference is probably not significant compared to accuracy of "velocity pressure" measurement.

From the ideal gas law, the density is given by

$$\rho = \frac{p}{RT}$$

where ρ is in kg/m^3, p in N/m^2 and T in K. The gas constant for air is 287 J/kg-K. The pressure in N/m^2 is given by

$$\begin{aligned} p\,(\mathrm{N/m}^2) &= P_a(\text{m-Hg}) \times 13.6 \times 9810(\mathrm{N/m}^3) \\ &= 133,416P_a \end{aligned}$$

where 13.6 is the specific gravity of mercury. The density in kg/m^3 is

$$\begin{aligned} d &= g_c\rho \\ &= 9.81 \times \frac{133,416P_a}{287 \times T} \\ &= 4560.32\frac{P_a}{T} \end{aligned}$$

which is within 0.2% of the manufacturer's recommendation.

4.76: PROBLEM DEFINITION

Situation:

The flow of water over different surfaces.

Find:

Relationship of pressures.

(a) $p_C > p_B > p_{A.}$

(b) $p_B > p_C > p_{A.}$

(c) $p_C = p_B = p_{A.}$

(d) $p_B < p_C < p_{A.}$

(e) $p_A < p_B < p_{C.}$

SOLUTION

The flow curvature requires that $p_B > p_D + \gamma d$ where d is the liquid depth. Also, because of hydrostatics $p_C = p_D + \gamma d$. Therefore $p_B > p_C$. Also $p_A < p_D + \gamma d$ so $p_A < p_C$, and thus $p_B > p_C > p_A$.

The valid statement is $\boxed{\text{(b).}}$

4.77: PROBLEM DEFINITION

Situation:
Fluid element rotation.

Find:
What is meant by rotation of a fluid element?

SOLUTION

An arbitrary cubical element is selected in a flow. One side lies along the x-axis. As the element moves through the flow it will be deformed. If the angle between the bisectors of the sides and the x-axis does not change, there is no rotation.

4.78: PROBLEM DEFINITION

Situation:

A spherical fluid element in an inviscid fluid.

Find:

If pressure and gravitational forces are the only forces acting on the element, can they cause the element to rotate?

SOLUTION

The result force due to pressure passes through the center of the sphere so no moment arm to create rotation. The resultant forces due to gravity also pass though the center so cannot cause rotation.

4.79: PROBLEM DEFINITION

Situation:

A two-dimensional velocity field is represented by the vector $\mathbf{V} = 10x\mathbf{i} - 10y\mathbf{j}$.

Find:

Is the flow irrotational?

SOLUTION

In a two dimensional flow in the $x - y$ plane, the flow is irrotational if (Eq. 4.34a)

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

The velocity components and derivatives are

$$\begin{aligned} u &= 10x \qquad \frac{\partial u}{\partial y} = 0 \\ v &= -10y \qquad \frac{\partial v}{\partial x} = 0 \end{aligned}$$

Therefore $\boxed{\text{the flow is irrotational}}$

4.80: PROBLEM DEFINITION

Situation:

A flow field has velocity components described by $u = -\omega y$ and $v = \omega x$.

Find:

Vorticity.

Rate of rotation.

SOLUTION

Rate of rotation

$$\begin{aligned}\omega_z &= (1/2)(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) \\ &= \frac{1}{2}(\omega - (-\omega)) \\ &= \frac{1}{2}(2\omega) \\ &\boxed{\omega_z = \omega}\end{aligned}$$

Vorticity is twice the average rate of rotation; therefore, the $\boxed{\text{vorticity} = 2\omega}$

4.81: PROBLEM DEFINITION

Situation:

A two-dimensional velocity field is given by:
$u = \frac{Cx}{(x^2+y^2)}$, $v = \frac{Cy}{(x^2+y^2)}$.

Find:

Check if flow is irrotational.

SOLUTION

Apply equations for flow rotation in $x - y$ plane.

$$\begin{aligned} \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} &= \frac{-2xCy}{(x^2+y^2)^2} - \left[-\frac{2yCx}{(x^2+y^2)^2} \right] \\ &= 0 \\ &\boxed{\text{The flow is irrotational}} \end{aligned}$$

4.82: PROBLEM DEFINITION

Situation:

A two-dimensional flow field is defined by:
$u = x^2 - y^2,\ v = -2xy$.

Find:

If the flow is rotational or irrotational.

SOLUTION

Rate of flow rotation about the z-axis,

$$\begin{aligned}\Omega_z &= \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) \\ &= \frac{1}{2}(-2y + 2y) = 0\end{aligned}$$

Therefore, the flow is **irrotational.**

4.83: PROBLEM DEFINITION

Situation:

Fluid flows between two stationary plates.

$u = 2(1 - 4y^2)$, $V_{\text{max}} = 2\,\text{cm/s}$.

Find:

Find rotation of fluid element when it moves 1 cm downstream

PLAN

Apply equations for rotation rate of fluid element.

SOLUTION

The rate of rotation for this planar (two-dimensional) flow is

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

In this problem, $v = 0$ so

$$\begin{aligned} \omega_z &= -\frac{1}{2}\frac{\partial u}{\partial y} \\ &= 8y \end{aligned}$$

The time to travel 1 cm is

$$\begin{aligned} \Delta t &= \frac{1}{u} \\ &= \frac{1}{2(1 - 4y^2)} \end{aligned}$$

The amount of rotation in 1 cm travel is

$$\Delta\theta = \omega_z \Delta t$$

$$\boxed{\Delta\theta = \frac{4y}{(1-4y^2)}}$$

4.84: PROBLEM DEFINITION

Situation:

A velocity distribution is provided for a combination of free and forced vortex.

$v_\theta = \frac{1}{r}\left[1 - \exp(-r^2)\right]$, $r = 0.5,\ 1.0,\ 1.5.$

$2\dot{\theta}_z = \frac{dv_\theta}{dr} + \frac{v_\theta}{r} = \frac{1}{r}\frac{d}{dr}(v_\theta r).$

Find:

Find how much a fluid element rotates in one circuit around the vortex as a function of radius.

SOLUTION

The rate of rotation is given by

$$\begin{aligned}\dot{\theta} &= \frac{1}{2}\frac{1}{r}\frac{d}{dr}(v_\theta r)\\ &= \frac{1}{2}\frac{1}{r}\frac{d}{dr}[1-\exp(-r^2)]\\ &= \exp(-r^2)\end{aligned}$$

The time to complete one circuit is

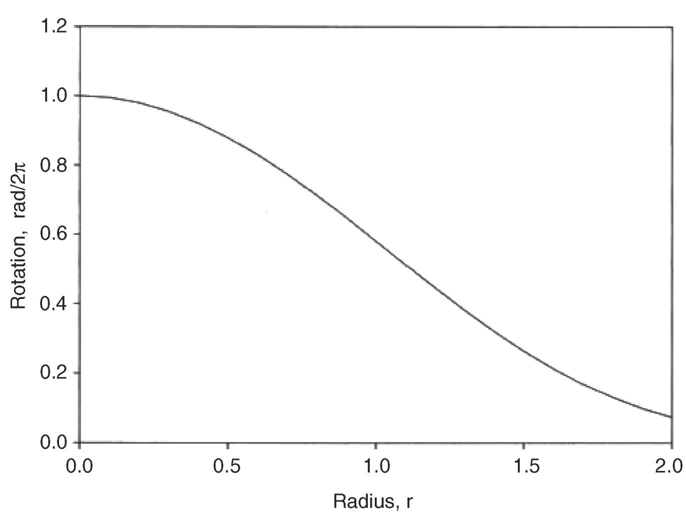
$$\begin{aligned}\Delta t &= \frac{2\pi r}{v_\theta}\\ &= \frac{2\pi r^2}{[1-\exp(-r^2)]}\end{aligned}$$

So, the total rotation in one circuit is given by

$$\Delta\theta = \dot{\theta}\Delta t$$

$$\boxed{\frac{\Delta\theta}{2\pi}\ (\text{rad}) = r^2\frac{\exp(-r^2)}{1-\exp(-r^2)}}$$

A plot of the rotation in one circuit is shown. Note that the rotation is 2π for r→ 0 (rigid body rotation) and approaches zero (irrotational) as r becomes larger.



4.85: PROBLEM DEFINITION

Situation:

Incompressible and inviscid liquid flows around a bend.

$V = \frac{1}{r}\,\mathrm{m/s}$, $r_i = 1\,\mathrm{m}$, $r_o = 3\,\mathrm{m}$.

Find:

Depth of liquid from inside to outside radius (m).

PLAN

Flow field is irotational so apply the Bernoulli equation across streamlines between the outside of the bend at the surface (point 2) and the inside of the bend at the surface (point 1).

SOLUTION

Bernoulli equation

$$
\begin{aligned}
\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 &= \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \\
0 + \frac{V_2^2}{2g} + z_2 &= 0 + \frac{V_1^2}{2g} + z_1 \\
z_2 - z_1 &= \frac{V_1^2}{2g} - \frac{V_2^2}{2g}
\end{aligned}
$$

where $V_2 = (1/3)$ m/s; $V_1 = (1/1)$ m/s. Then

$$
z_2 - z_1 = \frac{1}{2g}\left((1\,\mathrm{m/s})^2 - (0.33\,\mathrm{m/s})^2\right)
$$

$$
\boxed{z_2 - z_1 = 0.045\ \mathrm{m}}
$$

4.86: PROBLEM DEFINITION

Situation:

An outlet pipe from a reservoir.

$V = 9\,\text{m/s}$, $h = 5.5\,\text{m}$.

Find:

Pressure at point A (Pa).

PLAN

Apply the Bernoulli equation.

SOLUTION

Bernoulli equation. Let point 1 be at surface in reservoir.

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A \\
0 + 0 + 5.5 &= \frac{p_A}{9810\,\text{N/m}^3} + \frac{(9\,\text{m/s})^2}{2 \times 9.81\,\text{m/s}^2} + 0 \\
p_A &= 1.37\,\text{m} \times 9810\,\text{N/m}^3 \\
p_A &= 13439.7\ \text{N/m}^2 \\
&\boxed{p_A = 13440\ \text{Pa}}
\end{aligned}$$

4.87: PROBLEM DEFINITION

Situation:

An outlet pipe from a reservoir.

$V = 8\,\mathrm{m/s}$, $h = 19\,\mathrm{m}$.

Find:

Pressure at point A (kPa).

Assumptions:

Flow is irrotational.

PLAN

Apply the Bernoulli equation.

SOLUTION

Bernoulli equation. Let point 1 be at reservoir surface.

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A \\
0 + 0 + 19 &= \frac{p_A}{9810\,\mathrm{N/m^3}} + \frac{(8\,\mathrm{m/s})^2}{2 \times 9.81\,\mathrm{m/s^2}} + 0 \\
p_A &= (19\,\mathrm{m} - 3.26\,\mathrm{m})\left(9810\,\mathrm{N/m^3}\right) \\
p_A &= 154{,}390 \text{ Pa, gage} \\
& \boxed{p_A = 154.39 \text{ kPa, gage}}
\end{aligned}$$

4.88: PROBLEM DEFINITION

Situation:

Air flows past a cylinder. Highest velocity at the maximum width of sphere is twice the free stream velocity.

$V_0 = 40\,\text{m/s}$, $V_{\max} = 2V_0$.

Find:

Pressure difference between highest and lowest pressure (kPa).

Assumptions:

Hydrostatic effects are negligible and the wind has density of 1.2 kg/m^3.

PLAN

Apply the Bernoulli equation between points of highest and lowest pressure.

SOLUTION

The maximum pressure will occur at the stagnation point where $V = 0$ and the point of lowest pressure will be where the velocity is highest ($V_{\max} = 80$ m/s).

Bernoulli equation

$$\begin{aligned} p_h + \frac{\rho V_h^2}{2} &= p_\ell + \frac{\rho V_\ell^2}{2} \\ p_h + 0 &= p_\ell + \frac{\rho}{2}(V_{\max}^2) \\ p_h - p_\ell &= \frac{1.2\,\text{kg/m}^3}{2}(80\,\text{m/s})^2 \\ &= 3,840 \text{ Pa} \\ & \boxed{p_h - p_\ell = 3.84 \text{ kPa}} \end{aligned}$$

4.89: PROBLEM DEFINITION

Situation:

Velocity and pressure given at two points in a duct.
$V_1 = 1\,\mathrm{m/s}$, $V_2 = 2\,\mathrm{m/s}$.

Find:

Determine which is true:
(a) Flow in contration in nonuniform and irrotational.
(b) Flow in contration is uniform and irrotational.
(c) Flow in contration is nonuniform and rotational.
(d) Flow in contration is uniform and rotational.

Assumptions:

Elevations are equal.

Properties:

$p_1 = 10\,\mathrm{kPa}$, $p_2 = 7\,\mathrm{kPa}$.
$\rho = 1000\,\mathrm{kg/m^3}$.

PLAN

Check to see if it is irrotational by seeing if it satisfies Bernoulli's equation.

SOLUTION

The flow is **non-uniform.**
Bernoulli equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \\
\frac{10,000\,\mathrm{Pa}}{9,810\,\mathrm{N/m^3}} + \frac{(1\,\mathrm{m/s})^2}{2\,(9.81\,\mathrm{m/s^2})} + 0 &= \frac{7,000\,\mathrm{Pa}}{9,810\,\mathrm{N/m^3}} + \frac{(2\,\mathrm{m/s})^2}{2\,(9.81\,\mathrm{m/s^2})} + 0 \\
1.070 &\neq 0.917
\end{aligned}$$

Flow is **rotational.** The correct choice is **c.**

4.90: PROBLEM DEFINITION

Situation:

Water flowing from a large orifice at the bottom of a tank.

$V_A = 1.2\,\text{m/s}$, $V_B = 3.6\,\text{m/s}$.

$z_A = 0.3\,\text{m}$, $z_B = 0\,\text{m}$.

Find:

$p_A - p_B$ (Pa).

Properties:

$\rho = 1000\,\text{kg/m}^3$.

PLAN

Apply the Bernoulli equation.

SOLUTION

Bernoulli equation

$$\begin{aligned}
\frac{p_A}{\gamma} + z_A + \frac{V_A^2}{2g} &= \frac{p_B}{\gamma} + z_B + \frac{V_B^2}{2g} \\
p_A - p_B &= \gamma \left[\frac{(V_B^2 - V_A^2)}{2g} - z_A \right] \\
&= 9810\,\text{N/m}^3 \left[\frac{(12.96 - 1.44)\text{m}^2/\text{s}^2}{2\,(9.81\,\text{m/s}^2)} - 0.3\,\text{m} \right] \\
&\boxed{p_A - p_B = 2817 \text{ Pa}}
\end{aligned}$$

4.91: PROBLEM DEFINITION

Situation:

A flow pattern past an airfoil.

$V_0 = 80\,\mathrm{m/s}$, $V_1 = 85\,\mathrm{m/s}$, $V_2 = 75\,\mathrm{m/s}$.

Find:

Pressure difference between bottom and top (kPa).

Assumptions:

The pressure due to elevation difference between points is negligible.

Properties:

$\rho = 1.2\,\mathrm{kg/m^3}$.

SOLUTION

The flow is ideal and irrotational so the Bernoulli equation applies between any two points in the flow field

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \\
p_2 - p_1 &= \frac{\rho}{2}(V_1^2 - V_2^2) \\
p_2 - p_1 &= \frac{1.2\,\mathrm{kg/m^3}}{2}(85^2 - 75^2)\,\mathrm{m/s} \\
&= 960 \text{ Pa} \\
&\boxed{p_2 - p_1 = 0.96 \text{ kPa}}
\end{aligned}$$

4.92: PROBLEM DEFINITION

Situation:

Flow of water between parallel plates.

Find:

Is the Bernoulli equation valid between plates?

SOLUTION

The flow between the two plates is rotational. The Bernoulli equation cannot be applied across streamlines in rotational flows.

4.93: PROBLEM DEFINITION

Situation:

A two dimensional flow in the xy plane is described in the problem statement.

Find:

(a) Show that $d(\frac{u^2+v^2}{2}+gh)=0$.

(b) Show $\frac{V^2}{2g+h}$ is constant in all directions.

SOLUTION

a) Substituting the equation for the streamline into the Euler equation gives

$$\begin{array}{c} u\frac{\partial u}{\partial x}dx+u\frac{\partial u}{\partial y}dy=-g\frac{\partial h}{\partial x}dx \\ v\frac{\partial v}{\partial x}dx+v\frac{\partial v}{\partial y}dy=-g\frac{\partial h}{\partial y}dy \end{array}$$

or

$$\begin{array}{c} \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right)dx+\frac{\partial}{\partial y}(\frac{u^2}{2})dy=-g\frac{\partial h}{\partial x}dx \\ \frac{\partial}{\partial x}\left(\frac{v^2}{2}\right)dx+\frac{\partial}{\partial y}(\frac{v^2}{2})dy=-g\frac{\partial h}{\partial y}dy \end{array}$$

Adding both equations

$$\frac{\partial}{\partial x}\left(\frac{u^2+v^2}{2}\right)dx+\frac{\partial}{\partial y}\left(\frac{u^2+v^2}{2}\right)dy=-g(\frac{\partial h}{\partial x}dx+\frac{\partial h}{\partial y}dy)$$

or

$$d(\frac{u^2+v^2}{2}+gh)=0$$

b) Substituting the irrotationality condition into Euler's equation gives

$$\begin{array}{c} u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial x}=-g\frac{\partial h}{\partial x} \\ v\frac{\partial v}{\partial y}+u\frac{\partial u}{\partial y}dy=-g\frac{\partial h}{\partial y} \end{array}$$

or

$$\begin{array}{c} \frac{\partial}{\partial x}(\frac{u^2+v^2}{2}+gh)=0 \\ \frac{\partial}{\partial y}(\frac{u^2+v^2}{2}+gh)=0 \end{array}$$

4.94: PROBLEM DEFINITION

Situation:
A fluid is flowing around a cylinder as shown in Fig 4.37 in §4.10. A favorable pressure gradient can be found:
a. upstream of the stagnation point
b. at the stagnation point
c. between the stagnation point and separation point

SOLUTION

A favorable pressure gradient is defined as where the flow is accelerating.
Therefore the correct answer is (c); flow is accelerating from $v = 0$ at the stagnation point to a positive velocity.
Upstream of the stagnation point the flow is decelerating.
At the stagnation point the flow has stopped, so it is neither decelerating or accelerating.

4.95: PROBLEM DEFINITION

Situation:

Flow over a sphere.

$u_\theta = 1.5U \sin\theta$, $p = -6.35\,\text{cm H}_2\text{O}$.

$V = 30\,\text{m/s}$.

Find:

Angle of separation point.

Properties:

$\rho = 1.12\,\text{kg/m}^3$.

SOLUTION

Since the fluid is air, neglect the contribution of hydrostatic in the Bernoulli equation The pressure coefficient defined by

$$C_p = \frac{(p - p_\infty)}{\frac{1}{2}\rho U^2}$$

can be expressed in terms of velocities as

$$C_p = 1 - \left(\frac{V}{U}\right)^2$$

by application of the Bernoulli equation. The pressure in N/m^2 at the stagnation point is

$$\begin{aligned} p_{sep} &= -0.0635 \text{ m-H}_2\text{O} \times 9810 \text{ N/m}^3 \\ &= -623 \text{ N/m}^2 \end{aligned}$$

The dynamic pressure is

$$\frac{1}{2}\rho V^2 = \frac{1}{2} \times 1.12 \text{ kg/m}^3 \times 30^2 \text{ m}^2/\text{s}^2 = 504 \text{ N/m}^2$$

The pressure coefficient at the separation point is

$$C_p = \frac{-623}{504} = -1.2$$

so

$$-1.2 = 1 - \left(\frac{V}{U}\right)^2 = 1 - 1.5^2 \sin^2\theta$$

Solving for $\sin\theta$ gives

$$\sin\theta = 0.988$$

There are two solutions

$$\theta = 81.1^{\text{o}},\ 98.9^{\text{o}}$$

Separation occurs on windward side so

$$\boxed{\theta_{sep} = 81.1^{\text{o}}}$$

4.96: PROBLEM DEFINITION

Situation:

Application of the Bernoulli equation between a point upstream and in the wake of a sphere.

Find:

Is the Bernoulli equation valid between these two points?

SOLUTION

The flow in the wake is irrotational, so the Bernoulii equation cannot be applied between two arbitrary points.

4.97: PROBLEM DEFINITION

Situation:
Stirring a liquid in a cup.

Find:
Report on the contour of the surface. Provide an explanation for the observed shape.

SOLUTION

Stirring the cup of liquid creates a surface depressed at the center and higher at the wall of the cup. The difference in depth between the wall and the cup center creates an inward radial force to keep the fluid moving in a circle.

4.98: PROBLEM DEFINITION

Situation:

A closed tank filled with water is rotated about a vertical axis.
$D = 1.2\,\text{m}$, $\omega = 10\ \text{rad/s}$.

Find:

Pressure at bottom center of tank (Pa).

Properties:

$\rho = 1000\ \text{kg/m}^3$.

PLAN

Apply the equation for pressure variation equation-rotating flow.

SOLUTION

Pressure variation equation-rotating flow

$$p + \gamma z - \frac{\rho r^2 \omega^2}{2} = p_p + \gamma z_p - \frac{\rho r_p^2 \omega^2}{2}$$

where $p_p = 0$, $r_p = 0.9$ m and $r = 0$, then

$$\begin{aligned} p &= -\frac{\rho}{2}(r_p\omega)^2 + \gamma(z_p - z) \\ &= (2)\,(0.9\,\text{m} \times 10)^2 + \left(9810\,\text{N/m}^3\right)(0.75\,\text{m}) \\ &= -33,142.5\ \text{Pa} = -33,143\ \text{Pa} \end{aligned}$$

$$\boxed{p = -33,143\ \text{Pa}}$$

4.99: PROBLEM DEFINITION

Situation:

A tank of liquid is rotated on an arm.
$S = 0.80,\ D = 0.3\,\text{m}$.
$h = 0.3\,\text{m},\ r = 0.6\,\text{m}$.
$V_A = 6\,\text{m/s},\ p_A = 1.2$ kPa.

Find:

Pressure at B (kPa).

Properties:

$\rho = 1000\ \text{kg/m}^3,\ \gamma = 9810\,\text{N/m}^3$.

PLAN

Apply the pressure variation equation- rotating flow from point A to point B.

SOLUTION

Pressure variation equation- rotating flow

$$\begin{aligned} p_A + \gamma z_A - \frac{\rho r_A^2 \omega^2}{2} &= p_B + \gamma z_B - \frac{\rho r_B^2 \omega^2}{2} \\ p_B &= p_A + \frac{\rho}{2}(\omega^2)(r_B^2 - r_A^2) + \gamma(z_A - z_B) \end{aligned}$$

where $\omega = V_A/r_A = 6/0.45 = 13.333$ rad/s and $\rho = 0.8 \times 1000\ \text{kg/m}^3$. Then

$$\begin{aligned} p_B &= 1.2\ \text{kPa} + \left[1000\,\text{kg/m}^3\,(0.80/0.6)\right](13.33\,\text{rad/s})^2\left[(0.75\,\text{m})^2 - (0.45\,\text{m})^2\right] \\ &\quad +9810\,\text{N/m}^3\,(0.8)\,(-0.3) \\ &= 1200 + 85077 - 2354 \\ &\boxed{p_B = 83,923\ \text{Pa}} \end{aligned}$$

4.100: PROBLEM DEFINITION

Situation:

A cream separator is in operation.
$D = 20\,\text{cm},\ f = 9000$ rpm.

Find:

Centripetal acceleration (m/s^2).
RCF.

SOLUTION

The centripetal acceleration is

$$a_r = \frac{V^2}{r} = \omega^2 r$$

The rotational rate of the separator is

$$\omega = 2\pi \left(\frac{9000\text{rpm}}{60\,\text{s/ min}} \right) = 942.5 \text{ rad/s}$$

The radius of the separator is 10 cm or 0.1 m. The acceleration is

$$a_r = (942.5\,\text{rad/ s})^2 (0.1\,\text{m})$$

$$\boxed{a_r = 88800 \text{ m/s}^2}$$

The RCF is

$$RCF = 88831 \text{ m/s}^2 / 9.81 \text{ m/s}^2$$

$$\boxed{RCF = 9060}$$

4.101: PROBLEM DEFINITION

Situation:

A closed tank with liquid is rotated about the vertical axis.

$\omega = 10$ rad/s, $r_B = 0.5\,\text{m}$, $a_z = 4\,\text{m/s}^2$.

Find:

Difference in pressure between points A and B (kPa).

Properties:

$\rho = 1000\,\text{kg/m}^3$, $S = 1.2$.

PLAN

Apply the pressure variation equation for rotating flow between points B & C. Let point C be at the center bottom of the tank.

SOLUTION

Pressure variation equation- rotating flow

$$p_B - \frac{\rho r_B^2 \omega^2}{2} = p_C - \frac{\rho r_C^2 \omega^2}{2}$$

where $r_B = 0.5$ m, $r_C = 0$ and $\omega = 10$ rad/s. Then

$$\begin{aligned} p_B - p_C &= \frac{\rho}{2}(\omega^2)(r^2) \\ &= \frac{1200\,\text{kg/m}^3}{2}(100\,\text{rad}^2/\text{s}^2)(0.25\,\text{m}^2) \\ &= 15,000\ \text{Pa} \\ p_C - p_A &= 2\gamma + \rho a_z \ell \\ &= 2\left(11,772\,\text{N/m}^3\right) + \left(1,200\,\text{kg/m}^3\right)\left(4\,\text{m/s}^2\right)(2) \\ &= 33.1\ \text{kPa} \end{aligned}$$

Then

$$\begin{aligned} p_B - p_A &= p_B - p_C + (p_C - p_A) \\ &= 15,000\,\text{Pa} + 33,144\,\text{Pa} \\ &= 48,144\,\text{Pa} \\ &\boxed{p_B - p_A = 48.1\ \text{kPa}} \end{aligned}$$

4.102: PROBLEM DEFINITION

Situation:

A U-tube rotating about the leg on the right side.
$r_1 = 0.5\,\text{m}$, $z_1 = 0.5\,\text{m}$.
$z_2 = 0\,\text{m}$, $r_2 = 0\,\text{m}$.

Find:

Maximum rotational speed so that no liquid escapes from the leg on the left side (rad/s).

PLAN

Since the fluid is in rigid body rotation, apply the pressure variation equation for rotating flow. At the condition of imminent spilling, the liquid will be to the top of the left leg and at the bottom of the right leg. Thus, locate point 1 be at top of the left (outside) leg. Locate point 2 at the bottom of the right (inside) leg.

SOLUTION

Pressure variation equation- rotating flow

$$p_1 + \gamma z_1 - \frac{\rho r_1^2 \omega^2}{2} = p_2 + \gamma z_2 - \frac{\rho r_2^2 \omega^2}{2} \tag{1}$$

Term-by-term analysis

$$\begin{aligned} p_1 &= p_2 = 0\,\text{kPa-gage} \\ z_1 &= 0.5\,\text{m} \\ r_1 &= 0.5\,\text{m} \\ z_2 &= 0\,\text{m} \\ r_2 &= 0\,\text{m} \end{aligned}$$

Substitute values into Eq. 1.

$$\begin{gathered} p_1 + \gamma z_1 - \frac{\rho r_1^2 \omega^2}{2} = p_2 + \gamma z_2 - \frac{\rho r_2^2 \omega^2}{2} \\ 0 + \rho g\,(0.5\,\text{m}) - \frac{\rho\,(0.5^2\,\text{m}^2)\,\omega^2}{2} = 0 + 0 - 0 \\ g\,(0.5\,\text{m}) - \frac{(0.5^2\,\text{m}^2)\,\omega^2}{2} = 0 \\ \omega^2 = 4g \\ \omega = 2\sqrt{g} \\ \boxed{\omega = 6.26 \text{ rad/s}} \end{gathered}$$

4.103: PROBLEM DEFINITION

Situation:

A stagnation tube in a tank is rotated.

$\omega = 100$ rad/ s, $r = 20\,\text{cm}$, $\gamma = 10000\,\text{N/ m}^3$.

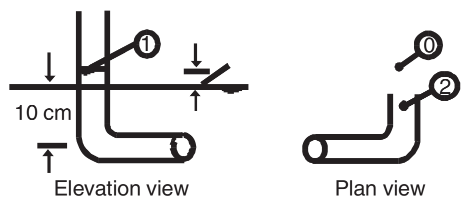
Find:

Location of liquid surface in central tube.

PLAN

Pressure variation equation for rotating flow from pt. 1 to pt. 2 where pt. 1 is at liquid surface in vertical part of tube and pt. 2 is just inside the open end of the Pitot tube.

SOLUTION



Pressure variation equation- rotating flow

$$\begin{aligned}\frac{p_1}{\gamma} - \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} - \frac{V_2^2}{2g} + z_2 \\ 0 - 0 + (0.10 + \ell) &= \frac{p_2}{\gamma} - \frac{r^2\omega^2}{2g} - 0 \qquad (1)\end{aligned}$$

where $z_1 = z_2$. If we reference the velocity of the liquid to the tip of the Pitot tube then we have steady flow and Bernoulli's equation will apply from pt. 0 (point ahead of the Pitot tube) to point 2 (point at tip of Pitot tube).

$$\begin{aligned}\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \\ \frac{0.1\gamma}{\gamma} + \frac{r^2\omega^2}{2g} &= \frac{p_2}{\gamma} + 0 \qquad (2)\end{aligned}$$

Solve Eqs. (1) & (2) for ℓ

$\boxed{\ell = 0}$ liquid surface in the tube is the same as the elevation as outside liquid surface.

4.104: PROBLEM DEFINITION

Situation:

A U-tube partially full of liquid is rotating about one leg.

$f = 5.2$ rad/s, $S = 3.0$, $r_2 = 0.3\,\text{m}$.

Find:

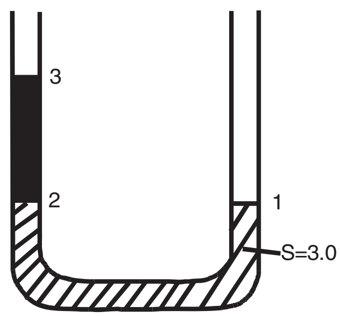
Specific gravity of other fluid.

PLAN

Apply the pressure variation equation for rotating flow between points 1 & 2.

SOLUTION

Pressure variation equation-rotating flow



$$p_2 + \gamma z_2 - \frac{\rho r_2^2 \omega^2}{2} = p_1 + \gamma z_1 - \frac{\rho r_1^2 \omega^2}{2}$$

where $z_2 = z_1$, $r_1 = 0$, $r_2 = 0.3$ m. and $\omega =$ rad/s. Then

$$p_2 = \left[3\left(1000\,\text{kg/ m}^3\right)\right](0.3\,\text{m})^2 \frac{(5.2\,\text{rad/ s})^2}{2} = 3650 \text{ Pa} \quad (1)$$

Also, by hydrostatics, because there is no acceleration in the vertical direction

$$p_2 = 0 + \frac{0.3}{2} \times \gamma_f \quad (2)$$

where γ_f is the specific weight of the other fluid. Solve for γ_f between Eqs. (1) & (2)

$$\begin{aligned} \gamma_f &= 24,333 \text{ N/m}^3 \\ S &= \frac{\gamma_f}{\gamma_{H_2O}} \\ &= \frac{24,333}{9810} \\ & \boxed{S = 2.5} \end{aligned}$$

4.105: PROBLEM DEFINITION

Situation:

A manometer is rotated about one leg.

$\Delta z = 20\,\text{cm}$, $r = 10\,\text{cm}$, $S = 0.8$.

Find:

Acceleration in g's in leg with greatest amount of oil.

PLAN

Apply the pressure variation equation for rotating flow between the liquid surfaces of 1 & 2 Let leg 1 be the leg on the axis of rotation. Let leg 2 be the other leg of the manometer.

SOLUTION

Pressure variation equation- rotating flow

$$\begin{aligned}
p_1 + \gamma z_1 - \frac{\rho r_1^2 \omega^2}{2} &= p_2 + \gamma z_2 - \frac{\rho r_2^2 \omega^2}{2} \\
0 + \gamma z_1 - 0 &= \gamma z_2 - \frac{\gamma}{g} \frac{r_2^2 \omega^2}{2} \\
\frac{r_2^2 \omega^2}{2g} &= z_2 - z_1 \\
a_n &= r\omega^2 \\
&= \frac{(z_2 - z_1) 2g}{r_2} \\
&= \frac{(0.20)(2\text{g})}{0.1} \\
& \boxed{a_n = 4g}
\end{aligned}$$

4.106: PROBLEM DEFINITION

Situation:

A fuel tank rotated in zero-gravity environment.

$f = 3$ rpm, $r_1 = 1.5\,\mathrm{m}$, $z_A = 1\,\mathrm{m}$.

Find:

Pressure at exit (Pa).

Properties:

$\rho = 800\,\mathrm{kg/\,m^3}$, $p_1 = 0.1\,\mathrm{kPa}$.

PLAN

Apply the pressure variation equation for rotating flow from liquid surface to point A. Call the liquid surface point 1.

SOLUTION

Pressure variation equation- rotating flow

$$
\begin{aligned}
p_1 + \gamma z_1 - \frac{\rho r_1^2 \omega^2}{2} &= p_A + \gamma z_A - \frac{\rho r_A^2 \omega^2}{2} \\
p_A &= p_1 + \frac{\rho \omega^2}{2}(r_A^2 - r_1^2) + \gamma(z_1 - z_A)
\end{aligned}
$$

However $\gamma(z_1 - z_A) = 0$ in zero-g environment. Thus

$$
\begin{aligned}
p_A &= p_1 + \frac{800 \text{ kg/m}^3}{2}\left(\frac{6\pi}{60 \text{ rad/s}}\right)^2 ((1.5\,\mathrm{m})^2 - (1\,\mathrm{m})^2) \\
&= 100 \text{ Pa} + 49.3 \text{ Pa} \\
&\boxed{p_A = 149.3 \text{ Pa}}
\end{aligned}
$$

4.107: PROBLEM DEFINITION

Situation:

A rotating set of tubes has liquid in the bottom of it.

$D_1 = 2d,\ D_2 = d.$

$r_2 = \ell,\ z_2 = 4\ell.$

Find:

Derive a formula for the angular speed when the water will begin to spill.

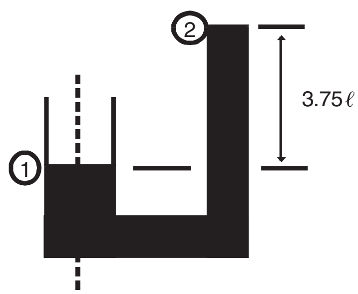
PLAN

Start with pressure variation equation for rotating flow. Let point 1 be at the liquid surface in the large tube and point 2 be at the liquid surface in the small tube.

SOLUTION

Pressure variation equation- rotating flow

$$p_1 + \gamma z_1 - \frac{\rho r_1^2 \omega^2}{2} = p_2 + \gamma z_2 - \frac{\rho r_2^2 \omega^2}{2}$$



The change in volume in leg 1 has to be the same as leg 2. So

$$\begin{aligned} \Delta h_1 d_1^2 &= \Delta h_2 d_2^2 \\ \Delta h_1 &= \Delta h_2 \left(\frac{d_2^2}{d_1^2}\right) \\ &= \frac{\Delta h_2}{4} \end{aligned}$$

The elevation difference between 1 and 2 will be

$$\begin{aligned} z_2 - z_1 &= 3\ell + \frac{3\ell}{4} \\ &= 3.75\ell \end{aligned}$$

Then $p_1 = p_2 = 0$ gage, $r_2 = \ell$, and $z_2 - z_1 = 3.75\ell$ so

$$\begin{aligned}
\frac{\rho r_2^2 \omega^2}{2} &= \gamma(3.75\ell) \\
\frac{\gamma}{2g}(\ell^2)\omega^2 &= 3.75\gamma\ell \\
\omega^2 &= \frac{7.5g}{\ell}
\end{aligned}$$

$$\boxed{\omega = \sqrt{\frac{7.5g}{\ell}}}$$

4.108: PROBLEM DEFINITION

Situation:

Water fills a tube that is closed at one end.

$D = 1\,\text{cm}$, $r = 40\,\text{cm}$, $\omega = 50$ rad/s.

Find:

Force exerted on closed end (N).

Properties:

$\rho = 1000\,\text{kg/m}^3$

PLAN

Apply the pressure variation equation for rotating flow from the open end of the tube to the closed end.

SOLUTION

Pressure variation equation- rotating flow

$$p_1 + \gamma z_1 - \frac{\rho r_1^2 \omega^2}{2} = p_2 + \gamma z_2 - \frac{\rho r_2^2 \omega^2}{2}$$

where $z_1 = z_2$. Also let point 2 be at the closed end; therefore $r_1 = 0$ and $r_2 = 0.40$ m.

$$\begin{aligned} p_2 &= \frac{\rho}{2}(0.4\,\text{m})^2(50\text{ rad/s})^2 \\ &= 500\,\text{kg/m}^3\left(0.16\,\text{m}^2\right)\left(2500\,\text{rad}^2/\text{s}^2\right) \\ &= 200\text{ kPa} \end{aligned}$$

Then

$$\begin{aligned} F &= p_2 A = 200,000\,\text{Pa}(\pi/4)(.01\,\text{m})^2 \\ & \boxed{F = 15.7\text{ N}} \end{aligned}$$

4.109: PROBLEM DEFINITION

Situation:

Water sits in a U-tube that is closed at one end.
$D = 1\,\text{cm}$, $\ell = 2\,\text{cm}$.

Find:

Rotational speed when water will begin to spill from open tube (rad/s).

Properties:

$\rho = 1000\,\text{kg/m}^3$, $\gamma = 9810\,\text{N/m}^3$.

PLAN

Apply the pressure variation equation for rotating flow between water surface in leg A-A to water surface in open leg after rotation.

SOLUTION

When the water is on the verge of spilling from the open tube, the air volume in the closed part of the tube will have doubled. Therefore, we can get the pressure in the air volume with this condition.

$$p_i \forall_i = p_f \forall_f$$

and i and f refer to initial and final conditions

$$\begin{aligned} p_f &= p_i \frac{\forall_i}{\forall_f} = 101 \text{ kPa} \times \frac{1}{2} \\ p_f &= 50.5 \text{ kPa, abs} = -50.5 \text{ kPa, gage} \end{aligned}$$

Pressure variation equation- rotating flow

$$p_A + \gamma z_A - \frac{\rho r_A^2 \omega^2}{2} = p_{\text{open}} + \gamma z_{\text{open}} - \frac{\rho r_{\text{open}}^2 \omega^2}{2}$$

$$p_A + 0 - 0 = 0 + \gamma \times 6\ell - \frac{\rho (6\ell)^2 \omega^2}{2}$$

$$-50.5 \times 10^3\,\text{Pa} = 9810\,\text{N/m}^3\,(6)\,(0.02\,\text{m}) - \left(1000\,\text{kg/m}^3\right)(6 \times 0.02\,\text{m})^2 \left(\frac{\omega^2}{2}\right)$$

$$\boxed{\omega = 84.72 \text{ rad/s}}$$

4.110: PROBLEM DEFINITION

Situation:

Water is pumped from a reservoir by a centrifugal pump consisting of a disk with radial ports.

$r = 5\,\text{cm},\ f = 3000$ rpm, $z_1 = 0\,\text{m}$.

Find:

Maximum operational height (m).

PLAN

Apply the pressure variation equation for rotating flow

Locate point 1 at the liquid surface where $z = 0$.

Locate point 2 at the outer edge of the rotating disk.

SOLUTION

Pressure variation equation

$$\begin{aligned} p_1 + \gamma z_1 - \frac{\rho r_1^2 \omega^2}{2} &= p_2 + \gamma z_2 - \frac{\rho r_2^2 \omega^2}{2} \\ 0 + 0 - 0 &= 0 + \gamma z_2 - \frac{\rho r_2^2 \omega^2}{2} \\ z_2 &= \frac{r_2^2 \omega^2}{2g} \end{aligned}$$

Rotational Rate

$$\omega = (3000 \text{ rev/min})(1\text{min}/60 \text{ s})(2\pi \text{ rad/rev}) = 314.1 \text{ rad/s}$$

Find z_2

$$z_2 = = \frac{r_2^2 \omega^2}{2g} = \frac{(0.05\,\text{m})^2 (314.1\,\text{rad/ s})^2}{2\,(9.81\,\text{m/ s}^2)}$$

$$\boxed{z_2 = 12.6 \text{ m}}$$

4.111: PROBLEM DEFINITION

Situation:

A tank rotated about the horizontal axis and water in tank rotates as a solid body.
$V = r\omega$, $z = -1, 0, +1\,\mathrm{m}$, $\omega = 5$ rad/ s.

Find:

Pressure gradient each value of z (kPa/m).

Properties:

$\rho = 1000\,\mathrm{kg/\,m^3}$.

PLAN

Apply the pressure variation equation for rotating flow.

SOLUTION

Pressure variation equation- rotating flow.

$$\begin{aligned}\frac{\partial p}{\partial z} + \gamma\frac{\partial z}{\partial z} &= -\rho r\omega^2 \\ \frac{\partial p}{\partial z} &= -\gamma - \rho r\omega^2\end{aligned}$$

when $z = -1$ m

$$\begin{aligned}\frac{\partial p}{\partial z} &= -\gamma - \rho\omega^2 \\ &= -\gamma\left(1 + \frac{\omega^2}{g}\right) \\ &= -9,810\,\mathrm{N/\,m^3}(1 + \frac{25}{9.81\,\mathrm{m/\,s^2}})\end{aligned}$$

$$\boxed{\frac{\partial p}{\partial z} = -34.8\text{ kPa/m}}$$

when $z = +1$ m

$$\begin{aligned}\frac{\partial p}{\partial z} &= -\gamma + \rho\omega^2 \\ &= -\gamma\left(1 + \frac{\omega^2}{g}\right) \\ &= -9810\,\mathrm{N/\,m^3} \times \left(1 - \frac{25}{9.81\,\mathrm{m/\,s^2}}\right)\end{aligned}$$

$$\boxed{\frac{\partial p}{\partial z} = 15.2\text{ kPa/m}}$$

At $z = 0$

$$\frac{\partial p}{\partial z} = -\gamma$$

$$\boxed{\frac{\partial p}{\partial z} = -9.81 \text{ kPa/m}}$$

4.112: PROBLEM DEFINITION

Situation:

A tank 1.2 m in diameter and 3.6 m long rotated about horizontal axis and water in tank rotates as a solid body. Maximum velocity is 7.5 m/s.

$V = r\omega$, $V_{\max} = 7.5 \text{ m/s}$.

$D = 1.2\,\text{m}$, $L = 3.6\,\text{m}$.

Find:

Maximum pressure difference in tank (Pa).

Point of minimum pressure (m).

Properties:

$\rho = 1000\,\text{kg/m}^3$.

PLAN

SOLUTION

Below the axis both gravity and acceleration cause pressure to increase with decrease in elevation; therefore, the maximum pressure will occur at the bottom of the cylinder. Above the axis the pressure initially decreases with elevation (due to gravity); however, this is counteracted by acceleration due to rotation. Where these two effects completely counter-balance each other is where the minimum pressure will occur ($\partial p/\partial z = 0$). Thus, above the axis:

$$\frac{\partial p}{\partial z} = 0 = -\gamma + r\omega^2\rho \text{ minimum pressure condition}$$

Solving: $r = \gamma/\rho\omega^2$; $p_{\min}$ occurs at $z_{\min} = +g/\omega^2$.Using the equation for pressure variation in rotating flows between the tank bottom where the pressure is a maximum ($z_{\max} = -r_0$) and the point of minimum pressure.

$$\begin{aligned} p_{\max} + \gamma z_{\max} - \frac{\rho r_0^2\omega^2}{2} &= p_{\min} + \gamma z_{\min} - \frac{\rho r_{\min}^2\omega^2}{2} \\ p_{\max} - \gamma r_0 - \frac{\rho r_0^2\omega^2}{2} &= p_{\min} + \frac{\gamma g}{\omega^2} - \frac{\rho(g/\omega^2)^2\omega^2}{2} \end{aligned}$$

$$p_{\max} - p_{\min} = \Delta p_{\max} = \frac{\rho\omega^2}{2}\left[r_0^2 - \left(\frac{g}{\omega^2}\right)^2\right] + \gamma\left(r_0 + \frac{g}{\omega^2}\right)$$

Rewriting

$$\boxed{\Delta p_{\max} = \frac{\rho\omega^2 r_0^2}{2} + \gamma r_0 + \frac{\gamma g}{2\omega^2}}$$

SOLUTION

$p_{\min}$occurs at $z = \gamma/\rho\omega^2$ where $\omega = (7.5 \text{ m/s})/0.6 \text{ m} = 12.5 \text{ rad/s}$. Then

$$
\begin{aligned}
z_{\min} &= \frac{\gamma}{\rho\omega^2} \\
&= \frac{g}{\omega^2} \\
&= \frac{9.81\,\mathrm{m/\,s^2}}{(12.5\,\mathrm{rad/\,s})^2} \\
& \boxed{z_{\min} = 0.063 \text{ m above axis}}
\end{aligned}
$$

From the analysis section above,

$$
\begin{aligned}
\Delta p_{\max} &= \frac{\rho\omega^2 r_0^2}{2} + \gamma r_0 + \frac{\gamma g}{2\omega^2} \\
&= \frac{1000\,\mathrm{kg/\,m^3}\,(12.5\,\mathrm{rad/\,s})^2\,(0.6\,\mathrm{m})^2}{2} + (9810\,\mathrm{N/m^3})\,(0.6\,\mathrm{m}) + \frac{(9810\,\mathrm{N/\,m^3})\,(9.81\,\mathrm{m/\,s^2})}{2\,(12.5\,\mathrm{rad/\,s})^2} \\
&= 28,125 + 5886 + 308 \\
& \boxed{\Delta p_{\max} = 34,319 \text{ Pa}}
\end{aligned}
$$