5.1: PROBLEM DEFINITION

Situation:

Consider an automobile gas tank being filled by a nozzle.

Find:

(a) Discharge (L/min).
(b) Time to put 190 liters in the tank (min).
(c) Cross-sectional area (m^2) of the nozzle and velocity at the exit (m/s).

Assumptions:

Assume diameter of the nozzle to be 2.5 cm.

SOLUTION

a)

$$Q = \frac{1\text{ L}}{10\text{s}} \times \frac{60\text{ s}}{1\text{ min}}$$

$$\boxed{Q = 6\text{ L/min}}$$

b)

$$t = \frac{190\text{ L}}{6\text{ L/min}}$$

$$\boxed{t = 31.6\,\text{min}}$$

c)

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.025\,\text{m})^2$$

$$\boxed{A = 4.91 \times 10^{-4}\,\text{m}^2}$$

Discharge in cfs (m^3/s).

$$\frac{6\text{ L}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{0.001\text{ m}^3}{\text{L}} = 0.006\text{ m}^3/\text{s}$$

Discharge velocity.

$$V = \frac{Q}{A} = \frac{0.006\text{ m}^3/\text{s}}{4.91 \times 10^{-4}\text{ m}^2}$$

$$\boxed{V = 12.2\,\text{m/s}}$$

5.2: PROBLEM DEFINITION

Situation:

Water is released through Grand Coulee Dam.

$A = Wd$, $W = 90\,\mathrm{m}$.

$Q = 3100\,\mathrm{m}^3/\mathrm{s}$.

Find:

Calculate river depth (m).

Assumptions:

Make a reasonable estimate of the river velocity ($V = 2.2\,\mathrm{m/s}$).

PLAN

Apply flow rate equation.

SOLUTION

The discharge is given by

$$Q = AV$$

Solving for depth

$$d = \frac{Q}{VW} = \frac{3100\,\mathrm{m}^3/\mathrm{s}}{2.2\ \mathrm{m/s} \times 90\ \mathrm{m}}$$

$$\boxed{d = 15.7\,\mathrm{m}}$$

5.3: PROBLEM DEFINITION

Situation:
Fill a jar with water and measure the time to fill.

Find:
Calculate discharge (m^3/s).
Calculate velocity (m/s).

Assumptions:
Make an estimate of the cross-sectional area for the faucet ($d = 0.0125\,\text{m}$).
$\forall = 2$ L, $t = 13\,\text{s}$.

PLAN

Apply flow rate equation.

SOLUTION

The discharge is

$$Q = \frac{2\text{ L}}{13\text{ s}} \times \frac{0.001\text{ m}^3}{1\text{ L}}$$

$$\boxed{Q = 0.000154\,\text{m}^3/\text{s}}$$

Faucet outlet area

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.0125\text{ m})^2 = 0.000127\text{ m}^2$$

Discharge velocity

$$V = \frac{Q}{A} = \frac{0.000154\text{ m}^3/\text{s}}{0.000127\text{ m}^2}$$

$$\boxed{V = 1.2\,\text{m}/\text{s}}$$

5.4: PROBLEM DEFINITION

Situation:
Another name for the volume flow rate equation could be:
a. the discharge equation
b. the mass flow rate equation
c. either a or b

SOLUTION

Correct answer is (a), the discharge equation.

5.5: PROBLEM DEFINITION

Situation:
Liquid flows through a pipe at constant velocity.

Find:
If a pipe twice the size is used with the same flow rate, find whether the flow rate is (a) halved, (b) doubled, or (c) quadrupled.

SOLUTION

Use discharge equation, $Q = AV$.
Since the diameter is doubled, the area is quadrupled so correct answer is $\boxed{\text{c)}}$.

5.6: PROBLEM DEFINITION

Situation:

For flow of a gas in a pipe, which form of the continuity equation is more general?

a. $V_1 A_1 = V_2 A_2$

b. $\rho_1 V_1 A_1 = \rho_2 V_2 A_2$

c. both are equally applicable

SOLUTION

The correct answer is (b). Equation (b) is more general, because it allows density to vary. If the density is different at location 1 from the density at location 2, then Eq. (a) is not applicable.

5.7: PROBLEM DEFINITION

Situation:

Water flows in a pipe.

$Q = 0.06\ \text{m}^3/\text{s},\ D = 0.35\,\text{m}.$

Find:

Mean velocity (m/s).

PLAN

Apply the flow rate equation.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.06\,\text{m}^3/\,\text{s}}{\frac{\pi}{4}\,(0.35\,\text{m})^2} \\ &\boxed{V = 0.624\ \text{m/s}} \end{aligned}$$

5.8: PROBLEM DEFINITION

Situation:

Water flows in a pipe.
$V = 1.2$ m/s, $D = 0.46\,\text{m}$.

Find:

Discharge in m^3/s and L/min.

PLAN

Apply the flow rate equation.

SOLUTION

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= (1.2\,\text{m/s})(\frac{\pi}{4}(0.46\,\text{m})^2) \\ & \boxed{Q = 0.1993 \text{ m}^3/\text{s}} \\ &= (0.1993 \text{ m}^3/\text{s})(1000 \text{ L/m})^3/(1/60\,\text{min/s}) \\ & \boxed{Q = 11,958 \text{ L/min}} \end{aligned}$$

5.9: PROBLEM DEFINITION

Situation:

Water flows in a pipe.

$V = 4$ m/s, $D = 2$ m.

Find:

Discharge in m^3/s.

PLAN

Apply the flow rate equation.

SOLUTION

Flow rate equation

$$
\begin{aligned}
Q &= VA \\
&= (4\,\text{m/s})\left(\frac{\pi}{4}(2\,\text{m})^2\right) \\
&\boxed{Q = 12.6\ \text{m}^3/\text{s}}
\end{aligned}
$$

5.10: PROBLEM DEFINITION

Situation:

A pipe carries air.

$V = 19\ \mathrm{m/s}$, $D = 0.06\,\mathrm{m}$.

Find:

Mass flow rate ($\mathrm{kg/m^3}$).

Properties:

Air (20 °C, 180 kPa) Table A.2: $R = 287\,\mathrm{J/kg\,K}$.

PLAN

1. Use Ideal Gas Law to find density.
2. Use Mass Flow Rate equation to find $\dot{m}$.

SOLUTION

1. Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{180{,}000\,\mathrm{Pa}}{(287\,\mathrm{J/kg\,K})\,(273+20)\ \mathrm{K}} \\ \rho &= 2.141\ \mathrm{kg/m^3}\end{aligned}$$

2. Flow rate equation

$$\begin{aligned}\dot{m} &= \rho V A \\ &= 2.141 \times 19 \times (\pi \times 0.06^2/4) \\ &\boxed{\dot{m} = 0.115\ \mathrm{kg/s}}\end{aligned}$$

5.11: PROBLEM DEFINITION

Situation:

A pipe carries natural gas.

$V = 25$ m/s, $D = 0.84$ m.

Find:

Mass flow rate (kg/m^3).

Properties:

Methane (15 °C, 160 kPa gage) Table A.4: $R = 518$ J/kg K.

PLAN

1. Apply the ideal gas law to find ρ.
2. Use the flow rate equation to find $\dot{m}$.

SOLUTION

1. Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{(101+160)10^3 \text{ Pa}}{(518 \text{ J/kg K})(273+15) \text{ K}} \\ &= 1.749 \text{ kg/m}^3\end{aligned}$$

2. Flow rate equation

$$\begin{aligned}\dot{m} &= \rho V A \\ &= 1.749 \times 25 \times \pi \times (0.84)^2/4 \\ &\boxed{\dot{m} = 24.2 \text{ kg/s}}\end{aligned}$$

5.12: PROBLEM DEFINITION

Situation:

A duct is attached to an aircraft engine.
$\dot{m} = 180 \text{ kg/s}$, $V = 255 \text{ m/s}$.

Find:

Pipe diameter (m).

Properties:

Air (−18 °C, 50 kPa) Table A.2: $R = 287 \text{ J/kg K}$.
$p = 50 \text{ kPa}$.

PLAN

1. Apply the ideal gas law to find ρ.
2. Use the flow rate equation to find A from $\dot{m}$ and then find D.

SOLUTION

1. Ideal gas law

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{(50 \times 10^3 \text{ Pa})}{(287 \text{ J/kg K})(273 - 18) \text{ K}} \\ &= 0.683 \text{ kg/m}^3 \end{aligned}$$

2. Flow rate equation

$$\dot{m} = \rho A V$$

So

$$\begin{aligned} A &= \frac{\dot{m}}{\rho V} \\ &= \frac{180 \text{ kg/s}}{(0.683 \text{ kg/m}^3)(255 \text{ m/s})} \\ &= 1.03 \text{ m}^2 \\ A &= (\pi/4) D^2 = 1.03 \text{ m}^2 \\ D &= \left(\frac{4 \times 1.03 \text{ m}^2}{\pi} \right)^{1/2} \end{aligned}$$

$$\boxed{D = 1.15 \text{ m}}$$

5.13: PROBLEM DEFINITION

Situation:

Air flows in a rectangular air duct.

$A = 1.0\,\text{m} \times 0.2\ \text{m}$, $Q = 1000\ \text{m}^3/\,\text{h}$.

Find:

Air velocity (m/s).

Properties:

Air (30 °C, 100 kPa).

PLAN

Apply the flow rate equation.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{1000\,\text{m}^3/\,\text{h}\,(1\,\text{h}/3600\,\text{s})}{(1.0\,\text{m} \times 0.2\,\text{m})} \\ &\boxed{V = 1.39\ \text{m}/\,\text{s}} \end{aligned}$$

5.14: PROBLEM DEFINITION

Situation:

In a circular duct the velocity profile is $v(r) = V_0 \left(1 - \frac{r}{R}\right)$.

Find:

Ratio of mean velocity to center line velocity, $\frac{\bar{V}}{V_0}$.

PLAN

Apply the integral form of the flow rate equation, because velocity is not constant across the cross-section.

SOLUTION

Flow rate equation

$$Q = \int v dA$$

where $dA = 2\pi r dr$. Then

$$\begin{aligned} Q &= \int_0^R V_0 \left(1 - \left(\frac{r}{R}\right)\right) 2\pi r dr \\ &= V_0(2\pi) \left(\frac{r^2}{2} - \frac{r^3}{3R}\right) |_0^R \\ &= 2\pi V_0 \left(\frac{R^2}{2} - \frac{R^2}{3}\right) \\ &= (2/6)\pi V_0 R^2 \end{aligned}$$

Average Velocity

$$\begin{aligned} \bar{V} &= \frac{Q}{A} \\ \frac{\bar{V}}{V_0} &= \frac{Q}{A} \frac{1}{V_0} \\ &= \frac{(2/6)\pi V_0 R^2}{\pi R^2} \frac{1}{V_0} \end{aligned}$$

$$\boxed{\frac{\bar{V}}{V_o} = \frac{1}{3}}$$

5.15: PROBLEM DEFINITION

Situation:

Two dimensional flow in a channel of width W and depth D.

$V(x,y) = V_S\left(1 - \frac{4x^2}{W^2}\right)\left(1 - \frac{y^2}{D^2}\right)$.

Find:

An expression for the discharge: $Q = Q(V_S,\, D,\, W)$.

PLAN

Apply the integral form of the flow rate equation, because v is not constant over the cross-section.

SOLUTION

Flow rate equation

$$
\begin{aligned}
Q &= \int \mathbf{V}\cdot d\mathbf{A} = \int\int V(x,y)dxdy \\
&= \int_{-W/2}^{W/2}\int_{y=0}^{D} V_S\left(1 - \frac{4x^2}{W^2}\right)\left(1 - \frac{y^2}{D^2}\right)dydx \\
&\boxed{Q = (\tfrac{4}{9})V_S W D}
\end{aligned}
$$

5.16: PROBLEM DEFINITION

Situation:

Water flows in a pipe with a linear velocity profile.
$V_{\max} = 4.5$ m/s, $V_{\min} = 3.6$ m/s.
$D = 1.2\,\mathrm{m}$.

Find:

Discharge in m^3/s and L/min.

PLAN

Apply the integral form of the flow rate equation with area expressed as a function of radius.

SOLUTION

Flow rate equation

$$\begin{aligned} Q &= \int_A V dA \\ &= \int_0^{r_0} V 2\pi r dr \end{aligned}$$

The equation for the velocity distribution is a straight line in the form $V = mr + b$ with $V = 4.5$ m/s at $r = 0$ and $V = 3.6$ m/s at $r = r_0$ yielding $V = 4.5$ m/s$-0.9r/r_0$.

$$\begin{aligned} Q &= \int_0^{r_0} \left(4.5 - \left(\frac{0.9r}{r_0}\right)\right) 2\pi r dr \\ &= 2\pi r_0^2 \left(\frac{4.5}{2} - \frac{0.9}{3}\right) \\ &= 2\pi \left[0.36 \left(\frac{4.5}{2} - \frac{0.9}{3}\right)\right] \\ & \boxed{Q = 4.41\,\mathrm{m}^3/\mathrm{s}} \\ &= 4.41\,\mathrm{m}^3/\mathrm{s} \left(\frac{1000 \times 60\,\mathrm{L/m}^3}{\mathrm{min/s}}\right) \\ & \boxed{Q = 264{,}600 \text{ L/min}} \end{aligned}$$

5.17: PROBLEM DEFINITION

Situation:

Water flows in a pipe with a linear velocity profile.
$V_{\max} = 8\ \text{m/s}$, $V_{\min} = 6\ \text{m/s}$.
$D = 2\,\text{m}$.

Find:

Discharge (m^3/s).
Mean velocity (m/s).

PLAN

Apply the integral form of the flow rate equation with area expressed as a function of radius.

SOLUTION

Flow rate equation

$$\begin{aligned} Q &= \int_A V dA \\ &= \int_0^{r_0} V 2\pi r dr \end{aligned}$$

The equation for the velocity distribution is a straight line in the form $V = mr + b$ with $V = 8$ m/s at $r = 0$ and $V = 6$ m/s at $r = r_0$ yielding $V = 8\ \text{m/s}{-}2r/r_0$.

$$\begin{aligned} Q &= \int_0^{r_0} \left(8 - \left(\frac{2r}{r_0}\right)\right) 2\pi r dr \\ &= 2\pi r_0^2 \left(\frac{8}{2} - \frac{2}{3}\right) \\ &= 2\pi \times 1.0 \left(\frac{8}{2} - \frac{2}{3}\right) \\ & \boxed{Q = 20.9\ \text{m}^3/\text{s}} \end{aligned}$$

Mean velocity

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{20.9\ \text{m}^3/\text{s}}{\pi\,(1\ \text{m})^2} \\ & \boxed{V = 6.67\,\text{m/s}} \end{aligned}$$

5.18: PROBLEM DEFINITION

Situation:

Air flows in a square duct with velocity profile shown in the figure.
$D = 1\,\text{m}$, $V_{\text{max}} = 10\,\text{m/s}$.

Find:

(a) Volume flow rate (m^3/s).
(b) Mean velocity (m/s).
(c) Mass flow rate (kg/s).

Properties:

Air: $\rho = 1.2\,\text{kg/m}^3$.

PLAN

Use various form of the flow rate equation.
Use the integral form of the flow rate equation because velocity is not constant of the area.

SOLUTION

The velocity profile is $V = 20y$. For $y = 0$ to $y = 0.5$,

$$\begin{aligned} dQ &= VdA \\ dQ &= (20y)dy \\ Q &= 2\int_0^{0.5} VdA \\ &= 2\int_0^{0.5} 20ydy \\ &= \frac{40y^2}{2}\Big|_0^{0.5} \\ &= 20 \times 0.25 \\ & \boxed{Q = 5\ \text{m}^3/\text{s}} \end{aligned}$$

Mean velocity

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{5\,\text{m}^3/\text{s}}{1\,\text{m}^2} \\ & \boxed{V = 5\ \text{m/s}} \end{aligned}$$

Mass flow rate

$$\begin{aligned} \dot{m} &= \rho Q \\ &= (1.2\ \text{kg/m}^3)(5\ \text{m}^3/\text{s}) \\ & \boxed{\dot{m} = 6.0\ \text{kg/s}} \end{aligned}$$

5.19: PROBLEM DEFINITION

Situation:

An open channel flow has a 30^o incline.
$V = 4.5$ m/s.
Depth $= y = 1.2\,\mathrm{m}$.
Width $= x = 8.5\,\mathrm{m}$.

Find:

Discharge ($\mathrm{m^3/s}$).

PLAN

Apply the flow rate equation.

SOLUTION

Flow rate equation

$$\begin{aligned} Q &= V \times A \\ &= (4.5\,\mathrm{m/s})\,(1.2\,\mathrm{m}\cos 30^\circ)\,(8.5\,\mathrm{m}) \\ &\boxed{Q = 39.8\,\mathrm{m^3/s}} \end{aligned}$$

5.20: PROBLEM DEFINITION

Situation:

A rectangular channel has a 30º incline.
$u = y^{1/3}$ m/s.
Depth $= y = 1$ m.
Width $= x = 1.2$ m.
$d = 1\,\text{m} \times \cos(30^o) = 0.866$ m

Find:

Discharge (m^3/s).

PLAN

Apply the integral form of the flow rate equation becuse velocity is not constant over the area.

SOLUTION

Flow rate equation

$$\begin{aligned} Q &= \int_0^{0.866} y^{1/3}(1.2 \text{ dy}) \\ &= 1.2 \int_0^{0.866} y^{1/3}\text{dy} \\ &= \left(\frac{1.2}{4/3}\right) y^{4/3}|_0^{0.866 \text{ m}} \end{aligned}$$

$$\boxed{Q = 0.743 \text{ m}^3/\text{s}}$$

5.21: PROBLEM DEFINITION

Situation:

A rectangular channel has a 30° incline.

$u = 8[\exp(y) - 1]$ m/s.

$y = 1$ m, $x = 2$ m.

Find:

Discharge (m^3/s).

Mean velocity (m/s).

PLAN

Apply the integral form of the flow rate equation becuse velocity is not constant over the area.

SOLUTION

Discharge.

$$
\begin{aligned}
Q &= \int_0^{0.866} V dA \qquad \text{where } dA = 2dy \\
Q &= \int_0^{0.866} (8)(e^y - 1)2 \text{ dy} \\
&= [(2)(8)(e^y - y)]_0^{0.866} \\
&\boxed{Q = 8.18 \text{ m}^3/\text{s}}
\end{aligned}
$$

Mean velocity

$$
\begin{aligned}
\bar{V} &= \frac{Q}{A} \\
&= \frac{8.18 \text{ m}^3/\text{s}}{2 \times 0.866 \text{ m}^2} \\
&\boxed{\bar{V} = 4.72 \text{ m/s}}
\end{aligned}
$$

5.22: PROBLEM DEFINITION

Situation:

Water enters a weigh tank from a pipe.
$t = 20\,\text{min}$, $W = 20\,\text{kN}$.

Find:

Discharge (m^3/s).

Properties:

Water ($20\,°\text{C}$), Table A.5: $\gamma = 9790\ \text{N/m}^3$.

PLAN

Definition of discharge is a volume/time.

SOLUTION

$$\begin{aligned}
Q &= \frac{V\!\!\!/}{\Delta t} \\
&= \frac{W}{\gamma \Delta t} \\
&= \frac{20,000\,\text{N}}{9790\,\text{N/ m}^3 \times 20\,\text{min} \times 60\,\text{s/ min}}
\end{aligned}$$

$$\boxed{Q = 1.70 \times 10^{-3}\ \text{m}^3/\text{s}}$$

5.23: PROBLEM DEFINITION

Situation:

Water enters a lock for a ship canal through 180 ports.
$A_p = 0.6\,\text{m} \times 0.6\,\text{m}$, $A_{rise} = 32\,\text{m} \times 275\,\text{m}$.
$V_{rise} = 1.8\,\text{m/min}$.

Find:

Mean velocity in each port (m/s).

PLAN

Apply the continuity equation.

SOLUTION

Continuity equation

$$\begin{aligned} \sum V_p A_p &= V_{\text{rise}} \times A_{\text{rise}} \\ 180 \times V_p \times (0.6 \times 0.6)\,\text{m}^2 &= \left(\frac{1.8}{60}\right)\,\text{m/s} \times (32 \times 275)\,\text{m}^2 \\ & \boxed{V_{\text{port}} = 4.1 \text{ m/s}} \end{aligned}$$

5.24: PROBLEM DEFINITION

Situation:

Water flows through a rectangular and horizontal open channel.
$u = u_{\max}(y/d)^n$, $u_{\max} = 3$ m/s.
$d = 1.2$ m, and $n = 1/6$.

Find:

Discharge per meter of channel width ($\mathrm{m^2/s}$).
Mean velocity (m/s).

PLAN

Apply the flow rate equation, considering that velocity is not constant of the cross-sectional area.

SOLUTION

Discharge per meter

$$\begin{aligned} q &= \int_0^d u_{\max}\left(\frac{y}{d}\right)^n dy = \frac{u_{\max} d}{n+1} \\ &= \frac{3\,\mathrm{m/s} \times 1.2\,\mathrm{m}}{\frac{1}{6}+1} \\ &\boxed{q = 3.09\ \mathrm{m^2/s}} \end{aligned}$$

Mean velocity

$$\begin{aligned} V &= \frac{q}{d} \\ &= \frac{3.09\,\mathrm{m^2/s}}{1.2\,\mathrm{m}} \\ &\boxed{V = 2.57\ \mathrm{m/s}} \end{aligned}$$

5.25: PROBLEM DEFINITION

Situation:

A flow with a linear velocity profile occurs in a triangular-shaped open channel.
$V_{\text{max}} = 1.8\,\text{m/s}$, $d = 0.3\,\text{m}$, $w_{\text{max}} = 0.15\,\text{m}$.

Find:

Discharge (m^3/s).

PLAN

Apply the flow rate equation, considering that velocity is not constant of the cross-sectional area.

SOLUTION

Flow rate equation

$$Q = \int V dA$$

where $V = 1.8y$ m/s, $dA = xdy = 0.15\ ydy$ m^2

$$\begin{aligned} q &= \int_0^{0.3} (1.8y) \times (0.15ydy) \\ &= \left.\frac{0.27y^3}{3}\right|_0^{0.3} \\ & \boxed{q = 0.00243\ \text{m}^3/\text{s}} \end{aligned}$$

5.26: PROBLEM DEFINITION

Situation:

Flow in a circular pipe.

$V = V_c(1 - (r/r_o)^2)^n$.

Find:

Mean velocity of the form $V = V(V_c, n)$.

PLAN

Apply the flow rate equation, considering that velocity is not constant across the cross-sectional area.

SOLUTION

Flow rate equation

$$\begin{aligned} Q &= \int_A V dA \\ &= \int_0^{r_0} V_c \left[1 - \left(\frac{r}{r_0} \right)^2 \right]^n 2\pi r dr \\ &= -\pi r_0^2 V_c \int_0^{r_o} \left(1 - \left(\frac{r}{r_0} \right)^2 \right)^n \left(\frac{-2r}{r_0^2} \right) dr \end{aligned}$$

This integral is in the form of

$$\int_0^U u^n du = \frac{U^{n+1}}{n+1}$$

so the result is

$$\begin{aligned} Q &= -\pi r_0^2 V_c \frac{\left(1 - \left(\frac{r}{r_0} \right)^2 \right)^{n+1}}{n+1} \Big|_0^{r_0} \\ &= \left(\frac{1}{n+1} \right) V_c \pi r_0^2 \\ V &= \frac{Q}{A} \\ & \boxed{V = \left(\frac{1}{n+1} \right) V_c} \end{aligned}$$

5.27: PROBLEM DEFINITION

Situation:

Flow in a pipe.
$V = 12(1 - r^2/r_0^2)$.
$D = 1\,\text{m}$, $V_c = 12\,\text{m/s}$.

Find:

Plot the velocity profile.
Mean velocity (m/s).
Discharge (m^3/s).

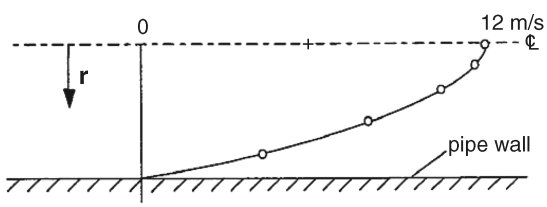
PLAN

1. Velocity profile is $V = f(r)$.
2. Apply the flow rate equation, considering that velocity is not constant across the cross-sectional area.

SOLUTION

1. Velocity

r/r_0	$1-(r/r_0)^2$	V(m/s)
0.0	1.00	12.0
0.2	0.96	11.5
0.4	0.84	10.1
0.6	0.64	7.68
0.8	0.36	4.32
1.0	0.00	0.0



2. Flow rate equation

$$
\begin{aligned}
Q &= \int_A V dA \\
&= \int_0^{r_0} 12\left(1-\left(\frac{r}{r_0}\right)^2\right) 2\pi r dr \\
&= 24\pi \int_0^{r_0}\left(r-\frac{r^3}{r_0^2}\right) dr \\
&= 24\pi\left(\frac{r^2}{2}-\frac{r^4}{4r_0^2}\right)\Big|_0^{r_0} \\
&= 2V_c\pi\left(\frac{r_0^2}{2}-\frac{r_0^2}{4}\right) \\
&= 2V_c\pi\left(\frac{r_0^2}{4}\right) \\
V &= \frac{Q}{A} \\
& \boxed{V=(1/2)V_c} \\
& \boxed{V=6 \text{ m/s}}
\end{aligned}
$$

$$
\begin{aligned}
Q &= VA \\
&= (6\,\text{m/s})\,(\pi/4)\,(1\,\text{m})^2 \\
& \boxed{Q=4.71 \text{ m}^3/\text{s}}
\end{aligned}
$$

5.28: PROBLEM DEFINITION

Situation:

Water flows in a pipe.

$D = 0.1\,\text{m}$, $\dot{m} = 0.57\ \text{kg/s}$.

Find:

Mean velocity (m/s).

Properties:

Water (15.5 °C), Table A.5: $\rho = 1000\ \text{kg/m}^3$.

PLAN

Apply the flow rate equation.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{\dot{m}}{\rho A} \\ V &= \frac{0.57\,\text{kg/s}}{1000\,\text{kg/m}^3\left(\frac{\pi}{4}\times(0.1\,\text{m})^2\right)} \\ &\boxed{V = 0.073\ \text{m/s}} \end{aligned}$$

5.29: PROBLEM DEFINITION

Situation:

Water flows in a pipe.

$D = 0.15\,\text{m}$, $\dot{m} = 700\ \text{kg/min}$.

Find:

Mean velocity (m/s).

Properties:

Water (20 °C), Table A.5: $\rho = 998\ \text{kg/m}^3$.

PLAN

Apply the flow rate equation.

SOLUTION

Flow rate equation

$$
\begin{aligned}
V &= \frac{\dot{m}}{\rho A} \\
&= \frac{\frac{700\,\text{kg/min}}{60\,\text{s/min}}}{998\,\text{kg/m}^3(\frac{\pi}{4}\times(0.15\,\text{m})^2)} \\
&\boxed{V = 0.662\ \text{m/s}}
\end{aligned}
$$

5.30: PROBLEM DEFINITION

Situation:
Water enters a weigh tank.
$W = 21,000\,\text{N}$, $t = 15\,\text{min}$.

Find:
Discharge in units of m^3/s and L/min.

Properties:
Water (15.5 °C), Table A.5: $\gamma = 9810\,\text{N/m}^3$.

SOLUTION

$$
\begin{aligned}
Q &= \frac{\mathcal{V}}{\Delta t} \\
&= \frac{\Delta W}{\gamma \Delta t} \\
&= \frac{21,000\,\text{N}}{\left(9810\,\text{N/m}^3\right)(15\,\text{min})(60\,\text{s/min})} \\
& \boxed{Q = 0.0024\,\text{m}^3/\text{s}} \\
&= 0.0024\,\text{m}^3/\text{s} \times 1000\,\text{L/m}^3 \times \frac{60\ \text{s}}{1\ \text{min}} \\
& \boxed{Q = 144\ \text{L/min}}
\end{aligned}
$$

5.31: PROBLEM DEFINITION

Situation:

A shell and tube heat exchanger with one pipe inside another pipe. Liquids flow in opposite directions.

$V_o = V_i$, $Q_o = Q_i$.

Find:

Find ratio of diameters.

PLAN

Use discharge equation $Q = AV$ and neglect pipe wall thickness.

SOLUTION

Discharge and velocity the same so

$$Q = A_{\text{inner}}V = A_{\text{outer}}V$$

Therefore

$$\frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}D_i^2$$

so

$$\boxed{\frac{D_o}{D_i} = \sqrt{2}}$$

5.32: PROBLEM DEFINITION

Situation:

A heat exchanger has three pipes enclosed in a larger pipe.
$V = 10\,\text{m/s}$, $D_{small} = 2.5\,\text{cm}$ with wall thickness $3\,\text{mm}$.
$D_{\text{large}} = 8\,\text{cm}$.

Find:

Discharge inside larger pipe.

PLAN

Use discharge equation, $Q = AV$ where A is net area inside large pipe.

SOLUTION

The outside cross-sectional area of each smaller pipe is

$$A = \frac{\pi}{4}(2.5 + 0.6)^2 = 7.55 \text{ cm}^2$$

The cross-sectional area between the large pipe and the small pipes is

$$A = \frac{\pi}{4}8^2 - 3 \times 7.55 = 27.61 \text{ cm}^2 = 0.002761 \text{ m}^2$$

The discharge is

$$Q = AV = 0.002761\,\text{m}^2 \times 10\,\text{m/s}$$

$$\boxed{Q = 0.0276\,\text{m}^3/\text{s}}$$

5.33: PROBLEM DEFINITION

Situation:

Water flows in a pipe.

$V = 2.6\ \text{m/s}$, $D = 0.15\,\text{m}$.

Find:

Discharge in units of m^3/s and L/min.

Mass flow rate (kg/s).

Properties:

Water (15.5 °C), Table A.5: $\rho = 1000\ \text{kg/m}^3$.

PLAN

Apply the flow rate equation.

SOLUTION

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= (2.6\,\text{m/s})\left(\frac{\pi}{4}\times(0.15\,\text{m})^2\right) \\ & \boxed{Q = 0.046\ \text{m}^3/\text{s}} \\ &= 0.046\,\text{m}^3/\text{s}\times\frac{1000\ \text{L}}{\text{m}^3}\times\frac{60\ \text{s}}{1\ \text{min}} \\ & \boxed{Q = 2760\ \text{L/min}} \end{aligned}$$

Mass flow rate

$$\begin{aligned} \dot{m} &= \rho Q \\ &= 0.046\text{m}^3/\text{s}\times 1000\ \text{kg/m}^3 \\ & \boxed{\dot{m} = 46\ \text{kg/s}} \end{aligned}$$

5.34: PROBLEM DEFINITION

Read §4.2, §5.2 of EFM10e, and the internet to find answers to the following questions.

Find:
a. What does the Lagrangian approach mean? What are three real-world examples that illustrate the Lagrangian approach? [use examples that are not in the text].

SOLUTION

Generally, a Lagrangian approach means to observe or describe the motion of a body of matter of fixed identity. In fluid mechanics, the Lagrangian approach means monitoring the movement of a fluid particle, or clump of particles, along a given streamline. Answers may vary for the real-world examples, but they could include looking at the density change of a given mass of air in a balloon as a function of temperature, predicting the motion of a leaf floating in a river by knowing the location of a given streamline, watching a charged particle being moved from one battery pole to another through electrophoresis gel, or a person riding on a roller coaster saying: "my velocity now is fast and downward..... now it's slow and upward...."

Find:
b. What does the Eulerian approach mean? What are three real world examples that illustrate the Eulerian approach? [use examples that are not in the text].

SOLUTION

The Eulerian approach involves selecting a region in space and then describing the motion that is occurring at points in that region. In addition, the Eulerian approach allows properties to be evaluated at spatial locations as a function of time. This is because the Eulerian approach uses fields. In fluid mechanics, this means that you keep track of the whole velocity or pressure field; defining the field for all x and t (1D), or x, y, and t (2D), or x, y, z and t (3D). Mass can flow from one "cell" in a flow field to the next one. Answers may vary for the real-world examples, but examples could include air being allowed to exit a balloon, the flow in and out of reservoir based on a function of head differences and assuming that the inlet and outlet are on different streamlines, or a groundwater flow model for which you know the pressure gradient in a sandstone aquifer that extends from a mountain range to a river. The Eulerian approach to describing the person on the roller coaster is to imagine a big cloud of latitude, longitude, and elevation points (x, y, z field) through which the person is traveling, and then predicting the location of the person as being pulled from a high point to a low point by the gravity field through which they are also traveling.

Find:
c. What are three important differences between the Eulerian and the Lagrangian approach?

SOLUTION

Answers will vary. Some differences are given here.
1. L: defined unit of mass moving through space E: A field is quantified in space, and any amount of mass moves according to that field.
2. L: Doesn't use fields. E: Uses fields
3. L: Used in solid mechanics, and closed systems. E: Used for control volume (open system) solutions.
4. L: In fluids, v = f(s, t), where s is location on a streamline. E: in fluids, v = f (x, y, z, t)

Find:
d. Why use an Eulerian approach? What are the benefits?

SOLUTION

The benefit of using a Eulerian approach is that you just need to be able to describe the flow field, such as the pressure distribution, everywhere throughout the system, which is often possible if you know the flow field conditions at various boundaries and times. When using the Lagrangian description in fluids, in order to evaluate properties at an arbitrary point in a flow field, the pathline that starts where the flow field is known and passes through the desired point must be located. This is not a straight-forward technique. The problem is accentuated with unsteady flow where different pathlines may pass through the same point at different times.

Find:
e. What is a field? How is a field related to the the Eulerian approach?

SOLUTION

A field is some variable that has been defined over some spatial region, such as the pressure gradient as a function of x, y and z at some time t. If the field is unsteady, it changes for different times t. For the Eulerian approach, we define gravity fields, pressure fields, velocity fields, magnetic fields, etc., in a x,y,z spatial system, and then predict the movement of matter through the field given certain boundary conditions and initial conditions.

Find:
f. What are the shortcomings of describing a flow field using the Lagrangian description?

SOLUTION

One must know where the streamlines are at all times, which is difficult in many systems, especially those with turbulence.

5.35: PROBLEM DEFINITION

Situation:

Properties.

Find:

What is the difference between an intensive and extensive property?

SOLUTION

The value of an extensive property depends on the amount matter and an intensive property is independent of the amount of matter. An example of an extensive property is volume because the volume depends on the amount of matter contained in the volume. Volume per unit mass (reciprocal density) is independent of the amount of mass.

5.36: PROBLEM DEFINITION

Situation:

State whether each of the following quantities is extensive or intensive:

a. mass
b. volume
c. density
d. energy
e. specific energy

SOLUTION

a. mass – extensive, because it depends upon amount

b. volume – extensive, because it depends upon amount

c. density – intensive, because density doesn't vary if you have more or less of a substance (assuming T and P are the same). Also, when you divide and extensive by an extensive property, you get an intensive property.

d. energy – extensive, because it depends upon amount

e. specific energy – intensive, because it is energy per mass. When you divide and extensive by an extensive property, you get an intensive property.

5.37: PROBLEM DEFINITION

Situation:

What type of property do you get when you divide an extensive property by another extensive property – extensive or intensive?

Hint: Consider density.

SOLUTION

When you divide an extensive property by another extensive property, you get an intensive property.

5.38: PROBLEM DEFINITION

Situation:
Control surface and volume.

Find:
What is a control surface and control volume?
Can mass pass through a control surface?

SOLUTION

A control volume is volume defined in space and the control surface encloses the control volume. The control volume can translate, rotate and dilate or contract with time.

Mass can pass through the control surface and, hence, through the control volume.

5.39: PROBLEM DEFINITION

Situation:

In Fig 5.11 in §5.2 of EFM10e,

a. the CV is passing through the system.

b. the system is passing through the CV.

SOLUTION

The answer is (b), the system, which is a defined mass (think of the system as a dyed collection of molecules), is moving through the CV.

5.40: PROBLEM DEFINITION

Situation:

Reynolds transport theorem.

Find:

The purpose of Reynolds transport theorem.

SOLUTION

The Reynolds transport theorem is used to relate Lagrangian equations to their Eulerian counterpart forms. The Lagrangian equations for conservation of mass, momentum and energy can be converted to their Eulerian forms by application of the Reynolds transport theorem.

5.41: PROBLEM DEFINITION

Situation:

Mass is flowing into and out of a tank.
$V_1 = 10\,\text{m/s},\ A_1 = 0.10\,\text{m}^2,\ \rho_1 = 3.00\,\text{kg/m}^3$.
$V_2 = 5\,\text{m/s},\ A_2 = 0.20\,\text{m}^2,\ \rho_2 = 2.00\,\text{kg/m}^3$.

Find:

Select the statement(s) that are true.

SOLUTION

Mass flow out

$$\begin{aligned}\dot{m}_{\text{o}} &= (\rho A V)_2 \\ &= \left(2\,\text{kg/m}^3\right)(0.2\,\text{m})(5\,\text{m/s}) \\ &= 2\ \text{kg/s}\end{aligned}$$

Mass flow in

$$\begin{aligned}\dot{m}_{\text{i}} &= (\rho A V)_1 \\ &= \left(3\,\text{kg/m}^3\right)(0.1\,\text{m})(10\,\text{m/s}) \\ &= 3\ \text{kg/s}\end{aligned}$$

Since the mass flow in is not equal to the mass flow out, the flow is unsteady. **Only selection (b) is valid**.

5.42: PROBLEM DEFINITION

Situation:

A piston in a cylinder is moving up and control consists of volume in cylinder.

Find:

Indicate which of the statements are true.

SOLUTION

a) True, there is no flow entering or leaving across the control surface.

b) True, since there is no mass flux across the control surfaces, the mass in the control volume does not change with time.

c) True, since the mass in the control volume is constant, $\rho \overline{V}_{cv}$=constant so ρ increases as volume decreases.

d) True, assuming the piston is moving rapidly, there is no time for heat transfer so temperaure must increaese.

e) True, due to piston motion the velocity of the gases in the cylinder will be changing with time.

5.43: PROBLEM DEFINITION

Situation:

Two flow cases: a closed tank is filled with a fluid and a pipe contracts.
$V = 3.6\,\mathrm{m/s},\ A = 0.14\,\mathrm{m}^2,\ \rho = 1030\,\mathrm{kg/m}^3.$
$V_1 = 0.3\,\mathrm{m/s},\ A_1 = 0.18\,\mathrm{m}^2,\ \rho_1 = 1030\,\mathrm{kg/m}^3.$
$V_2 = 0.6\,\mathrm{m/s},\ A_2 = 0.09\,\mathrm{m}^2,\ \rho_2 = 1030\,\mathrm{kg/m}^3.$

Find:

(a) Value of b.
(b) Value of dB_{sys}/dt.
(c) Value of $\sum b\rho\mathbf{V}\cdot\mathbf{A}$
(d) Value of $d/dt\int_{\mathrm{cv}} b\rho d\forall$

SOLUTION

Case (a)

1) $\boxed{b = 1}$

2) $\boxed{dB_{\mathrm{sys}}/dt = 0}$

3) $\sum b\rho\mathbf{V}\cdot\mathbf{A} = \sum \rho\mathbf{V}\cdot\mathbf{A}$
$= -1030 \times 3.6 \times 0.14$

$$\boxed{\sum b\rho\mathbf{V}\cdot\mathbf{A} = -519\ \mathrm{kg/s}}$$

4) $\boxed{\frac{d}{dt}\int_{\mathrm{cv}} b\rho d\forall = +519\ \mathrm{kg/s}}$

Case (b)

1) $\boxed{B = 1}$

2) $\boxed{dB_{\mathrm{sys}}/dt = 0}$

3) $\sum b\rho\mathbf{V}\cdot\mathbf{A} = \sum \rho\mathbf{V}\cdot\mathbf{A}$
$= 1030 \times 0.3 \times 0.18 - 1030 \times 0.6 \times 0.09$

$$\boxed{\sum b\rho\mathbf{V}\cdot\mathbf{A} = 0}$$

4) $\boxed{\frac{d}{dt}\int_{\mathrm{cv}} b\rho d\forall = 0}$

5.44: PROBLEM DEFINITION

Situation:

The law of conservation of mass for a closed system requires that the mass of the system is:

a. constant

b. zero

SOLUTION

The answer is (a), constant, by definition.

5.45: PROBLEM DEFINITION

Situation:

Consider the simplified form of the continuity equation, Eq. 5.29 of EFM10e. An engineer is using this equation to find the Q_C of a creek at the confluence with a large river, because she has automatic electronic measurements of the river discharge upstream (Q_{Ru}) and downstream (Q_{Rd}) of the creek confluence.

Find:

a. Which of the 3 terms on the left-hand side of Eq. 5.29 will the engineer assume is zero? Why?

b. Sketch the creek and the river and sketch the CV you would select to solve this problem.

SOLUTION

a. The engineer will assume that the $\frac{d}{dt}m_{cv}$ term is zero. This is because there can't be any water stored at the confluence, which is the fork where 2 drainages join together (con=with; fluence=flow) so flow in must equal flow out. Make a sketch (part b) to see how this works.

b. Sketch not shown here. One possible sketch consists of 2 tubes coming into the confluence, one for the creek, and one for the river upstream of the confluence. You could make a small box for the confluence – that box would be your control volume, or CV. You would then sketch one tube leaving the box, for the downstream reach of the river. Your CV doesn't have a place (e.g. a lake or a dam) for storage, so $\frac{d}{dt}m_{cv}$ is zero.

5.46: PROBLEM DEFINITION

Situation:

Pipe flows full with water.

Find:

Is it possible for the volume flow rate into the pipe to be different than the flow rate out of the pipe?

SOLUTION

Application of the continuity equation to a control volume passing through the inlet section and outlet section shows

$$0 = \frac{d}{dt}\int_{cv} \rho dV + \dot{m}_{\text{out}} - \dot{m}_{\text{in}}$$

Since the density is constant

$$0 = \frac{d}{dt}(\rho V_{cv}) + \dot{m}_{\text{out}} - \dot{m}_{\text{in}}$$

Since the volume of the control volume is constant the unsteady term is zero so $\dot{m}_{\text{out}} = \dot{m}_{\text{in}}$.

5.47: PROBLEM DEFINITION

Situation:

Air flows in a tube.

Find:

Is it possible for the mass flow rate into the tube to be different than the flow rate out of the tube?

Air is pumped into one end of a tube at a certain mass flow rate. Is it necessary that the same mass flow rate of air comes out the other end of the tube?

Application of the continuity equation over a control surface that includes the inlet and outlet shows

$$0 = \frac{d}{dt}\int_{cv} \rho dV + \dot{m}_{\text{out}} - \dot{m}_{\text{in}}$$

The density of the air in the control volume can change with time, the unsteady term may not be zero and $\dot{m}_{\text{out}} \neq \dot{m}_{\text{in}}$

5.48: PROBLEM DEFINITION

Situation:
Tire develops a leak.

Find:
How do air and density change with time?
How is air density related to tire pressure?

Assumptions:
Constant temperature.

SOLUTION

If an automobile tire develops a leak, how does the mass of air and density change inside the tire with time?
Assume the effective volume of the tire is unchanged. The air mass in the tire will decrease. Also, since the tire volume is constant, the air density will also decrease.

Assuming the temperature remains constant, how is the change in density related to the tire pressure?
From the ideal gas law, a decrease in density relates directly to a decrease in pressure.

5.49: PROBLEM DEFINITION

Situation:

Two pipes are connected in series.
$D_1 = 2D_2$, $V_1 = 4\,\text{m/s}$.

Find:

Velocity in smaller pipe (m/ s).

SOLUTION

Use continuity equation for discharge. $Q = AV$ which is valid since density is constant.

$$\begin{aligned}
A_{\text{large}}V_{\text{large}} &= A_{\text{small}}V_{\text{small}} \\
V_{\text{small}} &= V_{\text{large}}\left(\frac{A_{\text{large}}}{A_{\text{small}}}\right) \\
&= 4\text{ m/s} \times 2^2 \\
&\boxed{V_{\text{small}} = 16\,\text{m/s}}
\end{aligned}$$

5.50: PROBLEM DEFINITION

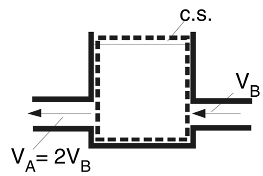
Situation:

The level in the tank is influenced by the motion of pistons A and B moving left. $V_A = 2V_B$, $D_A = 0.075\,\text{m}$, $D_B = 0.15\,\text{m}$.

Find:

Determine whether the water level is rising, falling or staying the same.

Sketch:



PLAN

Apply the continuity equation. Select a control volume as shown above. Assume it is coincident with and moves with the water surface.

SOLUTION

Continuity equation

$$
\begin{aligned}
\dot{m}_o - \dot{m}_i &= -\frac{d}{dt}\int_{\text{cv}} \rho d\forall \\
\rho 2V_B A_A - \rho V_B A_B &= -\rho \frac{d}{dt}\int_{\text{cv}} d\forall
\end{aligned}
$$

where $A_A = (\pi/4)0.075^2$; $A_B = (\pi/4)0.15^2$ and $A_A = (1/4)A_B$. Then

$$
\begin{aligned}
2V_B(1/4)A_B - V_B A_B &= -\frac{d}{dt}\int_{CV} d\forall \\
V_B A_B((1/2) - 1) &= -\frac{d}{dt}\int_{CV} d\forall \\
\frac{d}{dt}\int_{CV} d\forall &= (1/2)V_B A_B \\
\frac{d}{dt}(Ah) &= (1/2)V_B A_B \\
A\frac{dh}{dt} &= (1/2)V_B A_B
\end{aligned}
$$

Because $(1/2)V_B A_B$ is positive dh/dt is positive; therefore, one concludes that the water surface is **rising.**

5.51: PROBLEM DEFINITION

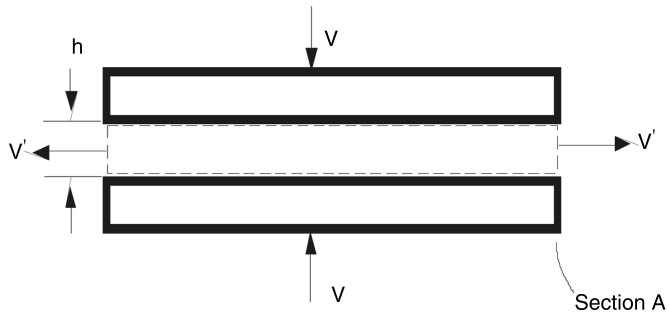
Situation:

Two round plates move together. At the instant shown, the plate spacing is h. Air flows across section A with a speed V'.

Find:

An expression for the radial component of convective acceleration at section A.

Sketch:



Assumptions:

Assume V' is constant across section A.

Assume the air has constant density.

PLAN

Apply the continuity equation to the control volume defined in the problem sketch.

SOLUTION

Continuity equation

$$\begin{aligned}\dot{m}_o - \dot{m}_i &= -\frac{d}{dt}\int_{\text{c.v.}} \rho d\forall \\ \rho V' A' &= -(-2\rho V A) \\ 2VA &= V'A'\end{aligned}$$

The control volume has radius r so

$$V' = \frac{2VA}{A'} = \frac{2V\pi r^2}{2\pi r h} = \frac{Vr}{h}$$

Convective acceleration

$$\begin{aligned}a_c &= V' \frac{\partial}{\partial r}(V') \\ &= \left(\frac{Vr}{h}\right)\left(\frac{\partial}{\partial r}\right)\left(\frac{Vr}{h}\right) \\ &= \frac{V^2 r}{h^2}\end{aligned}$$

$$\boxed{a_c = \frac{V^2 D}{2h^2}}$$

5.52: PROBLEM DEFINITION

Situation:

Pipe flows A and B merge into a single pipe.
$Q_A = 0.04t\ \text{m}^3/\text{s}$, $Q_B = 0.006t^2\,\text{m}^3/\text{s}$.
$A_{\text{exit}} = 0.01\ \text{m}^2$, $t = 1\,\text{s}$.

Find:

Velocity at the exit, V_{exit}.
Acceleration at the exit, a_{exit}.

Assumptions:

Incompressible flow.

PLAN

Apply the continuity equation.

SOLUTION

Since the flow is incompressible, the unsteady term is zero. Continuity equation

$$\begin{aligned}
Q_{\text{exit}} &= Q_A + Q_B \\
V_{\text{exit}} &= \left(\frac{1}{A_{\text{exit}}}\right)(Q_A + Q_B) \\
&= \left(\frac{1}{0.01\,\text{m}^2}\right)(.04t\ \text{m}^3/\text{s}\ + 0.006t^2\ \text{m}^3/\text{s}) \\
&= 4t\ \text{m/s}\ + 0.6t^2\ \text{m/s}
\end{aligned}$$

Then at $t = 1$ sec,

$$\boxed{V_{\text{exit}} = 4.6\ \text{m/s}}$$

The acceleration along a pathline at the $(s \to x)$ exit is

$$a_{\text{exit}} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}$$

Since V varies with time, but not with position, there is no convective acceleration so

$$a_{\text{exit}} = \frac{\partial V}{\partial t} = 4 + 1.2t\ \text{m/s}^2$$

Then at $t = 1$ sec

$$\boxed{a_{\text{exit}} = 5.2\ \text{m/s}^2}$$

5.53: PROBLEM DEFINITION

Situation:

Air flow downward through a pipe and then outward between two parallel disks.
$Q = 0.380\,\mathrm{m^3/\,s}$, $r = 20\,\mathrm{cm}$
$D = 0.1\,\mathrm{m}$, $h = 0.6\,\mathrm{cm}$.

Find:

(a) Expression for acceleration at point A.
(b) Value of acceleration at point A.
(c) Velocity in the pipe.

PLAN

Apply the flow rate equation.

SOLUTION

---

a)
Flow rate equation

$$V_r = \frac{Q}{A} = \frac{Q}{2\pi rh}$$

Evaluate convective acceleration along a radial pathline ($s \to r$)

$$\begin{aligned} a_c &= \frac{V_r \partial V_r}{\partial r} \\ &= \left(\frac{Q}{2\pi rh}\right)(-1)\left(\frac{Q}{2\pi r^2 h}\right) \end{aligned}$$

$$\boxed{a_c = \frac{-Q^2}{r(2\pi rh)^2}}$$

---

b)

$$\begin{aligned} V_{\text{pipe}} &= \frac{Q}{A_{\text{pipe}}} \\ &= \frac{(0.380\,\mathrm{m^3/\,s})}{\frac{\pi}{4}\,(0.1\,\mathrm{m})^2} \end{aligned}$$

$$\boxed{V_{\text{pipe}} = 48.4\ \mathrm{m/s}}$$

---

c)

$$a_c = -\frac{(0.38\,\mathrm{m^3/\,s})^2}{(0.2\,\mathrm{m})(2\pi\,(0.2\,\mathrm{m})\,(0.006\,\mathrm{m}))^2}$$

$$\boxed{a_c = -12,700\ \mathrm{m/s^2}}$$

5.54: PROBLEM DEFINITION

Situation:

Air flow downward through a pipe and then outward between two parallel disks.
$Q = Q_0(t/t_0)$, $r = 20\,\text{cm}$.
$D = 10\,\text{cm}$, $h = 1\,\text{cm}$.
$t_0 = 1\,\text{s}$, $Q_0 = 0.1\,\text{m}^3/\,\text{s}$.

Find:

(a) At $t = 2$ s, acceleration at point A: a_2.
(b) At $t = 3$ s, acceleration at point A: a_3.

SOLUTION

Local acceleration

$$\begin{aligned}
a_\ell &= \frac{\partial V}{\partial t} = \frac{\partial}{\partial t}\left(\frac{Q}{2\pi r h}\right) \\
a_\ell &= \frac{\partial}{\partial t}\frac{Q_0(t/t_0)}{2\pi r h} \\
a_\ell &= \frac{Q_0/t_0}{2\pi r h} \\
a_{\ell;2,3} &= \frac{(0.1\,\text{m}^3/\,\text{s}/1\,\text{s})}{2\pi \times 0.20\,\text{m} \times 0.01\,\text{m}} = 7.958\ \text{m/s}^2
\end{aligned}$$

From solution to Problem 5.53 in EFM10e

$$a_c = \frac{-Q^2}{r(2\pi r h)^2}$$

At $t = 2s, Q = 0.2\ \text{m}^3/\text{s}$

$$\begin{aligned}
a_{c,2s} &= -5,066\ \text{m/s}^2 \\
a_{2s} &= a_\ell + a_c = 7.958 - 5,066 \\
& \boxed{a_{2s} = -5,058\ \text{m/s}^2}
\end{aligned}$$

At $t = 3\text{s}$, $Q = 0.3\ \text{m}^3/\text{s}$

$$\begin{aligned}
a_{c,3s} &= -11,398\ \text{m/s}^2 \\
a_{3s} &= -11,398 + 7.958 \\
& \boxed{a_{3s} = -11,390\ \text{m/s}^2}
\end{aligned}$$

5.55: PROBLEM DEFINITION

Situation:

Water flows into a tank through a pipe on the side and then out a pipe on the bottom of the tank.

$A_{out} = A_{in} = 0.0025\,\mathrm{m}^2$, $A_{\text{tank}} = 0.1\,\mathrm{m}^2$,
At $h = 1\,\mathrm{m}$, $dh/dt = 0.1\,\mathrm{m}$, $V = \sqrt{2gh}$.

Find:

Velocity in the inlet: V_{in}.

Assumptions:

Incompressible flow.

PLAN

Apply the continuity equation. Let the control surface surround the liquid in the tank and let it follow the liquid surface at the top.

SOLUTION

Continuity equation

$$\dot{m}_o - \dot{m}_i = -\frac{d}{dt}\int_{\text{cv}} \rho d\forall$$

$$-\rho V_{\text{in}} A_{\text{in}} + \rho V_{\text{out}} A_{\text{out}} = -\frac{d}{dt}(\rho A_{\text{tank}} h)$$

$$-V_{\text{in}} A_{\text{in}} + V_{\text{out}} A_{\text{out}} = -A_{\text{tank}}\left(\frac{dh}{dt}\right)$$

$$-V_{\text{in}}(.0025) + \sqrt{2\text{g}(1)}(.0025) = -0.1(0.1) \times 10^{-2}$$

$$V_{\text{in}} = \frac{\sqrt{19.62}(.0025) + 10^{-4}}{0.0025}$$

$$\boxed{V_{\text{in}} = 4.47 \text{ m/s}}$$

5.56: PROBLEM DEFINITION

Situation:

A bicycle tire is inflated with air.The density of the air in the inflated tire is 6.4 kg/m^3.

$\forall = 0.0009\ \text{m}^3$, $Q_{\text{in}} = 0.02\,\text{m}^3/\,\text{min}$.

Find:

Time needed to inflate the tire.

Properties:

$\rho_{in} = 1.2\,\text{kg/m}^3$, $\rho_{\text{CV}} = 6.4\,\text{kg/m}^3$.

PLAN

Apply the continuity equation. Select a control volume surrounding the air within tire.

SOLUTION

Continuity equation

$$(\rho Q)_{\text{in}} = \frac{d}{dt} M_{\text{cv}}$$

This equation may be integrated to give

$$(\rho Q)_{\text{in}}\, t = M_{\text{CV}}$$

or

$$\begin{aligned} t &= \frac{M_{\text{CV}}}{(\rho Q)_{\text{in}}} \\ &= \frac{0.0009\,\text{m}^3 \times 6.4\,\text{kg/m}^3}{1.2\,\text{kg/m}^3 \times 0.02\,\text{m}^3/\,\text{min} \times (1\ \text{min}/60\,\text{s})} \\ & \boxed{t = 14.4\ \text{s}} \end{aligned}$$

5.57: PROBLEM DEFINITION

Situation:

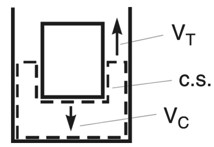
A cylinder falls in a tube containing a liquid.

$V_C = 1.2$ m/s, $D_{tube} = 0.2\,\text{m}$, $D_{cylinder} = 0.15\,\text{m}$.

Find:

Mean velocity of the liquid in between the cylinder and the wall.

Sketch:



PLAN

Apply continuity equation and let the c.s. be fixed except at the bottom of the cylinder where the c.s. follows the cylinder as it moves down. The top of the control volume is stationary with resepct to the wall.

SOLUTION

Continuity equation

$$
\begin{aligned}
0 &= \frac{d}{dt}\int \rho dV + \dot{m}_o - \dot{m}_i \\
0 &= \frac{d}{dt}(V) + V_T A_A \\
0 &= V_C A_C + V_T\left(\frac{\pi}{4}\right)\left[(0.2\,\text{m})^2 - (0.15\,\text{m})^2\right] \\
0 &= -1.2\,\text{m/s} \times \left(\frac{\pi}{4}\right)(0.15\,\text{m})^2 + V_T\left(\frac{\pi}{4}\right)\left[(0.2\,\text{m})^2 - (0.15\,\text{m})^2\right] \\
V_T &= \frac{0.02\,\text{m}^3/\text{s}}{(0.0137\,\text{m}^2)} \\
& \boxed{V_T = 1.46 \text{ m/s (upward)}}
\end{aligned}
$$

5.58: PROBLEM DEFINITION

Situation:

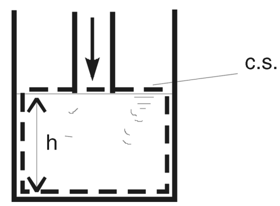
A round tank is being filled with water.

$V_p = 3\,\text{m/s}$, $D_T = 1.2\,\text{m}$, $D_p = 0.3\,\text{m}$.

Find:

Rate at which the water surface is rising.

Sketch:



PLAN

Apply the continuity equation and let the c.s. move up with the water surface in the tank.

SOLUTION

Continuity equation

$$\begin{aligned} 0 &= \frac{d}{dt}\int_{CV} \rho dV + \dot{m}_o - \dot{m}_i \\ 0 &= \frac{d}{dt}(hA_T) - ((3 + V_R)A_p) \end{aligned}$$

where A_T = tank area, V_R =rise velocity and A_p =pipe area.

$$0 = A_T\frac{dh}{dt} - 3A_p - V_RA_p$$

but $dh/dt = V_R$ so

$$\begin{aligned} 0 &= A_TV_R - 3A_p - V_RA_p \\ V_R &= \frac{3A_p}{A_T - A_p} = \frac{(3\,\text{m/s})\,(\pi/4)(0.3\,\text{m})^2}{\left[(\pi/4)\,(1.2\,\text{m})^2 - (\pi/4)\,(0.3\,\text{m})^2\right]} \\ & \boxed{V_R = 0.2\text{ m/s}} \end{aligned}$$

5.59: PROBLEM DEFINITION

Situation:

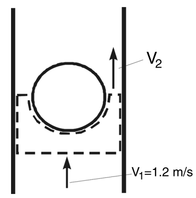
A sphere is falling in a cylinder filled with water.

$D_1 = 20\,\text{cm}$, $D_2 = 0.3\,\text{m}$, $V_1 = 1.2\,\text{m/s}$.

Find:

Velocity of water at the midsection of the sphere.

Sketch:



PLAN

Apply the continuity equation.

SOLUTION

As shown in the above sketch, select a control volume that is attached to the falling sphere. Relative to the sphere, the velocity entering the control volume is V_1 and the velocity exiting is V_2

Continuity equation

$$
\begin{aligned}
-\frac{d}{dt}\int_{CV}\rho dV &= 0 = \dot{m}_i - \dot{m}_o \\
A_1V_1 &= A_2V_2 \\
\left(\frac{\pi}{4}(0.3\,\text{m})^2\right)(1.2\,\text{m}) &= V_2\frac{\pi\left[(0.3\,\text{m})^2 - (0.2\,\text{m})^2\right]}{4} \\
V_2 &= 2.18\ \text{m/s}
\end{aligned}
$$

The velocity of the water relative to a stationary observer is

$$
\begin{aligned}
V &= V_2 - V_{sphere} \\
V &= 2.18 - 1.2 \\
& \boxed{V = 0.98\ \text{m/s}}
\end{aligned}
$$

5.60: PROBLEM DEFINITION

Situation:

Air flows in a rectangular duct.
$Q = 1.44 \text{ m}^3/\text{s}$, $A_1 = 20\,\text{cm} \times 60\ \text{cm}$.
$A_2 = 10\,\text{cm} \times 40\ \text{cm}$.

Find:

Air speed for initial duct area, V_1.
Air speed for latter duct area, V_2.

Assumptions:

Constant air density.

PLAN

Apply the flow rate equation.

SOLUTION

Flow rate equation

$$\begin{aligned} V_1 &= \frac{Q}{A_1} \\ &= \frac{(1.44\,\text{m}^3/\,\text{s})}{(0.2\,\text{m} \times 0.6\,\text{m})} \\ & \boxed{V_1 = 12\ \text{m/s}} \\ V_2 &= \frac{(1.44\,\text{m}^3/\,\text{s})}{(0.1\,\text{m} \times 0.4\,\text{m})} \\ & \boxed{V_2 = 36\ \text{m/s}} \end{aligned}$$

5.61: PROBLEM DEFINITION

Situation:

A pipe divides into two outlets.

$D_{30cm} = 30\,\text{cm},\ D_{20cm} = 20\,\text{cm},\ D_{18cm} = 18\,\text{cm}.$

$V_{20cm} = V_{18cm},\ Q = 0.4\ \text{m}^3/\text{s}.$

Find:

Discharge in each branch.

PLAN

Apply the flow rate equation.

SOLUTION

Flow rate equation

$$
\begin{aligned}
V &= \frac{Q_{30\,\text{cm}}}{A_{combined}} \\
V &= \frac{(0.4\,\text{m}^3/s)}{\frac{\pi}{4}\left((0.2\,\text{m})^2 + (0.18\,\text{m})^2\right)} \\
&= 7.03\ \text{m/s} \\
Q_{20\ \text{cm}} &= VA_{20} \\
&= (7.03\,\text{m/s})\,(\pi\,(0.1\,\text{m})\,(0.1\,\text{m})) \\
&\boxed{Q_{20\ \text{cm}} = 0.221\ \ \text{m}^3/\text{s}} \\
Q_{18\ \text{cm}} &= VA_{18} \\
&= (7.03\,\text{m/s})\,(\pi\,(0.09\,\text{m})\,(0.09\,\text{m})) \\
&\boxed{Q_{18\ \text{cm}} = 0.179\ \text{m}^3/\text{s}}
\end{aligned}
$$

5.62: PROBLEM DEFINITION

Situation:

A pipe divides into two outlets.

$D_{30cm} = 30\,\text{cm},\ D_{20cm} = 20\,\text{cm},\ D_{15cm} = 15\,\text{cm}.$

$Q_{20cm} = 2Q_{15cm},\ Q = 0.4\ \text{m}^3/\,\text{s}.$

Find:

Mean velocity in each outlet branch.

SOLUTION

Continuity equation

$$Q_{\text{tot.}} = 0.40\ \text{m}^3/\text{s} = Q_{20} + Q_{15}$$

Since $Q_{20} = 2Q_{15}$

$$\begin{aligned} 0.40 &= 2Q_{15} + Q_{15} \\ Q_{15} &= 0.138\ \text{m}^3/\text{s}; \\ Q_{20} &= 0.267\ \text{m}^3/\text{s}; \end{aligned}$$

Flow rate equation

$$\begin{aligned} V_{15} &= \frac{Q_{15}}{A_{15}} \\ & \boxed{V_{15} = 7.36\ \text{m/s}} \\ V_{20} &= \frac{Q_{20}}{A_{20}} \\ & \boxed{V_{20} = 8.50\ \text{m/s}} \end{aligned}$$

5.63: PROBLEM DEFINITION

Situation:

Water flows through pipe that is in series with a narrower pipe.
A pipe divides into two outlets.
$D_{0.25} = 0.25\,\text{m}$, $D_{0.15} = 0.15\,\text{m}$.
$V_{0.25\,\text{m}} = V_{0.15\,\text{m}}$, $Q = 3400$ liters per minute.

Find:

Mean velocity in each pipe.

PLAN

Apply the flow rate equation.

SOLUTION

Flow rate equation

$$
\begin{aligned}
Q &= 3400 \text{ L/min} = 0.06 \text{ m}^3/\text{s} \\
V_{0.25} &= \frac{Q}{A_{0.25}} \\
&= \frac{0.06\,\text{m}^3/\,\text{s}}{\frac{\pi}{4}\,(0.25\,\text{m})^2} \\
& \boxed{V_{0.25} = 1.22\,\text{m/s}} \\
V_{0.15} &= \frac{Q}{A_{0.15}} \\
&= \frac{0.06\,\text{m}^3/\,\text{s}}{\frac{\pi}{4}\,(0.15\,\text{m})^2} \\
& \boxed{V_{0.15} = 3.4\,\text{m/s}}
\end{aligned}
$$

5.64: PROBLEM DEFINITION

Situation:

Water flows through a tee.

$D_A = D_B = 4\,\text{m}$, $D_C = 2\,\text{m}$.

$V_A = 6\,\text{m/s}$, $V_C = 4\,\text{m/s}$.

Find:

Mean velocity in outlet B.

PLAN

Apply the continuity equation.

SOLUTION

Continuity equation

$$\begin{aligned} V_B &= \frac{V_A A_A - V_c A_c}{A_B} \\ &= \frac{[(6\,\text{m/s})(\pi/4)(4\,\text{m})^2 - (4\,\text{m/s})(\pi/4)(2\,\text{m})^2)]}{(\pi/4\,(4\,\text{m})^2)} \end{aligned}$$

$$\boxed{V_B = 5.00\ \text{m/s}}$$

5.65: PROBLEM DEFINITION

Situation:

Gas flows in a round conduit that tapers to a smaller diameter.
$D_1 = 1.2\,\text{m}$, $D_2 = 0.6\,\text{m}$, $V_1 = 15\,\text{m/s}$.

Find:

Mean velocity at section 2.

Properties:

$\rho_1 = 2.0\,\text{kg/m}^3$, $\rho_2 = 1.5\,\text{kg/m}^3$.

PLAN

Apply the continuity equation.

SOLUTION

Continuity equation

$$\begin{aligned} V_2 &= \frac{\rho_1 A_1 V_1}{\rho_2 A_2} \\ &= \frac{\rho_1 D_1^2 V_1}{\rho_2 D_2^2} \\ &= \frac{(2.0\,\text{kg/m}^3)(1.2\,\text{m})^2(15\,\text{m/s})}{(1.5\,\text{kg/m}^3)(0.6\,\text{m})^2} \end{aligned}$$

$$\boxed{V_2 = 80.0\ \text{m/s}}$$

5.66: PROBLEM DEFINITION

Situation:

Pipes A and B are connected to an open tank.

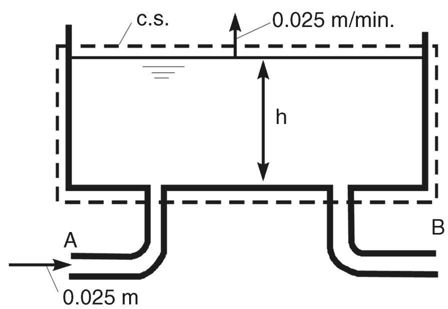
$Q_A = 0.28\,\text{m}^3/\,\text{min}$, $A = 7.4\,\text{m}^2$, $dh/dt = 0.025\,\text{m}/\,\text{min}$.

Find:

Discharge in pipe B.

If flow in pipe B is entering or leaving the tank.

Sketch:



PLAN

Apply the continuity equation. Define a control volume as shown in the above sketch. Let the c.s. move upward or downward with the water surface.

SOLUTION

Continuity equation

$$
\begin{aligned}
0 &= \frac{d}{dt}\int_{CV} \rho dV + \sum \rho \mathbf{V}\cdot\mathbf{A} \\
0 &= A\frac{dh}{dt} + Q_B - Q_A \\
Q_B &= Q_A - A\frac{dh}{dt} \\
&= 0.28\,\text{m}^3/\,\text{min} - (7.4\,\text{m}^2)(0.0025\,\text{m/min}) \\
& \boxed{Q_B = 0.095\ m^3/\,\text{min}}
\end{aligned}
$$

Because Q_B is positive $\boxed{\text{flow is leaving the tank through pipe } B.}$

5.67: PROBLEM DEFINITION

Situation:

A tank has one inflow and two outflows.
$D_{4in} = 0.1\,\text{m}$, $V_{0.1\,\text{m}} = 3\,\text{m/s}$.
$D_{6in} = 0.15\,\text{m}$, $V_{0.15\,\text{m}} = 2.1\,\text{m/s}$.
$D_{3in} = 0.075\,\text{m}$, $V_{0.075\,\text{m}} = 1.2\,\text{m/s}$.
$r_{\text{tank}} = 0.9\,\text{m}$.

Find:

Is the tank filling or emptying.
Rate at which the tank level is changing: $\frac{dh}{dt}$

SOLUTION

$$\begin{aligned}
\text{Inflow} &= (3\,\text{m/s})\left(\frac{\pi}{4}\right)(0.1\,\text{m})^2 = 0.024\ \text{m}^3/\text{s} \\
\text{Outflow} &= (2.1\,\text{m/s})\left(\frac{\pi}{4}\right)(0.15\,\text{m})^2 + (1.2\,\text{m/s})\left(\frac{\pi}{4}\right)(0.075\,\text{m})^2 = 0.042\ \text{m}^3/\text{s}
\end{aligned}$$

Outflow > Inflow, Thus, $\boxed{\text{tank is emptying}}$

$$\begin{aligned}
\frac{dh}{dt} &= -\frac{Q}{A} \\
&= -\frac{(0.042 - 0.024)\ \text{m}^3/\,\text{s}}{\pi\,(0.9\,\text{m})^2}
\end{aligned}$$

$$\boxed{\frac{dh}{dt} = -0.0071\ \text{m/s}}$$

5.68: PROBLEM DEFINITION

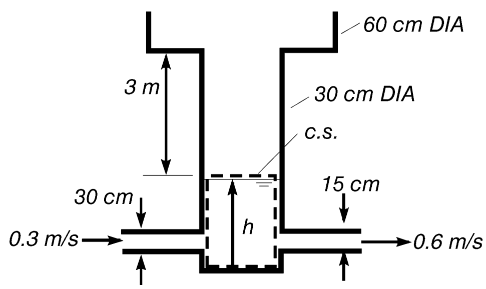
Situation:

A tank is filled with water over time.
$D_i = 30\,\text{cm} = 0.3\,\text{m}$, $V_i = 0.3\,\text{m/s}$.
$D_o = 15\,\text{cm} = 0.15\,\text{m}$, $V_o = 0.6\,\text{m/s}$.
$h_1 = 0\,\text{m}$, $D_{\text{tank1}} = 0.3\,\text{m}$.
$D_{\text{tank2}} = 0.6\,\text{m}$, $h_2 = 3\,\text{m}$.

Find:

At $t = 22$ s, if the the water surface will be rising or falling.
Rate at which the tank level is changing: $\frac{dh}{dt}$

Sketch:



PLAN

Apply the continuity equation. Define a control volume in which the control surface (c.s.) is coincident with the water surface and moving with it.

SOLUTION

Continuity equation

$$\begin{aligned}
\frac{d}{dt}\int_{cv}\rho dV &= \dot{m}_i - \dot{m}_o \\
\frac{d}{dt}(\rho Ah) &= (\rho AV)_{\text{in}} - (\rho AV)_{\text{out}} \\
\frac{d}{dt}(\rho Ah) &= \rho\left(\frac{\pi}{4}\times(0.3\,\text{m})^2\right)(0.3\,\text{m/s}) - \rho\left(\frac{\pi}{4}\times(0.15\,\text{m/s})^2\right)(0.6\,\text{m/s}) \\
A\frac{dh}{dt} &= \frac{0.027\pi}{4} - \frac{0.0135\pi}{4} \\
A\frac{dh}{dt} &= \frac{0.0135\pi}{4}
\end{aligned}$$

Since $Adh/dt > 0$, the water level must be rising. While the water column occupies the 30 cm. section, the rate of rise is

$$\begin{aligned}
\frac{dh}{dt} &= \frac{0.0135\pi/4}{A} \\
&= \frac{0.0135\pi}{4\times\pi/4\times(0.3)^2} \\
&= 0.15\ \ \text{m/s}
\end{aligned}$$

Determine the time it takes for the water surface to reach the 0.6 m section:

$$
\begin{aligned}
3 &= \left(\frac{dh}{dt}\right) t; \\
t &= \frac{3}{0.15} = 20 \text{ s.}
\end{aligned}
$$

Therefore, at the end of 20 s. the water surface will be in the 0.6 cm section. Then the rise velocity will be:

$$
\begin{aligned}
\frac{dh}{dt} &= \frac{0.0135\pi}{4A} \\
&= \frac{0.0135\pi}{4 \times \pi/4 \times (0.6)^2} \\
& \boxed{\tfrac{dh}{dt} = 0.04 \text{ m/s}}
\end{aligned}
$$

5.69: PROBLEM DEFINITION

Situation:

A lake is fed by an inlet and has no outlet. Lake surface area is $A(h)$, where h is depth in meters.

$Q_{in} = 34\,\mathrm{m}^3/\,\mathrm{s}$, $A(h) = 4.5 + 5.5h\,\mathrm{km}^2$.

$Q_{Evap} = 0.36\,\mathrm{m}^3/\,\mathrm{s}$ per km^2.

Find:

Equilibrium depth of lake.

The minimum discharge to prevent the lake from drying up.

PLAN

Apply the continuity equation.

SOLUTION

Continuity equation

$$\begin{aligned} Q_{\text{Evap.}} &= Q_{\text{in.}} \\ \left(0.36\ \mathrm{m}^3/\mathrm{s}/\mathrm{km}^2\right)(4.5 + 5.5h)\ \mathrm{km}^2 &= 34\ \mathrm{m}^3/\mathrm{s} \end{aligned}$$

Solve for depth h :

$$\boxed{h = 16.4\ \mathrm{m}} \text{ at equilibrium}$$

The lake will dry up when $h = 0$ and $Q_{\text{Evap.}} = Q_{\text{in.}}$. For $h = 0$,

$$0.36(4.5 + 5.5 \times 0) = Q_{\text{in.}}$$

Lake will dry up when $\boxed{Q_{\text{in.}} = 1.62\ \mathrm{m}^3/\mathrm{s}}$

5.70: PROBLEM DEFINITION

Situation:

A nozzle discharges water onto a plate moving towards the nozzle. Plate speed equals half the jet speed.

$Q_{in} = 0.14\,\text{m}^3/\text{s}$, $V_{in} = 2V_p$.

Find:

Rate at which the plate deflects water.

PLAN

Apply the continuity equation. Select a control volume surrounding the plate and moving with the plate.

SOLUTION

Continuity equation

$$Q_{in} = Q_p$$

Reference velocities to the moving plate. Let V_o be the speed of the water jet relative to the nozzle. From the moving plate, the water has a speed of $V_o + 1/2V_o = 3V_o/2$. Thus

$$\begin{aligned} Q_p &= Q_{in} \\ &= V_{in}A_o \\ &= \left(\frac{3V_o}{2}\right)(A_o) = \frac{3}{2}(V_oA_o) \\ &= \frac{3}{2}Q_o \\ &\boxed{Q_p = 0.21\ \text{m}^3/\text{s}} \end{aligned}$$

5.71: PROBLEM DEFINITION

Situation:

A tank with a depth h has one inflow (m^3/s) and one outflow through a 0.3 m diameter pipe. The outflow velocity is

$Q = 0.57\,\text{m}^3/\text{s}$, $V_{out} = \sqrt{2gh}$, $D_{out} = 0.3\,\text{m}$.

Find:

Equilibrium depth of liquid.

PLAN

Apply the continuity equation and the flow rate equation.

SOLUTION

Continuity equation

$$Q_{\text{in.}} = Q_{\text{out}} \qquad \text{at equilibrium}$$
$$Q_{\text{out}} = 0.57\,\text{m}^3/\text{s}$$

Flow rate equation

$$\begin{aligned} Q_{\text{out}} &= V_{\text{out}} A_{\text{out}} \\ 0.57 &= (\sqrt{2gh})(\pi/4 \times D_{\text{out}}^2) \text{ where } D = 0.3 \text{ m} \end{aligned}$$

Solving for h yields

$$\boxed{h = 3.3 \text{ m}}$$

5.72: PROBLEM DEFINITION

Situation:

Flows with different specific weights enter a closed tank through ports A and B and exit the tank through port C. Assume steady flow. Details are provided on figure with problem statement.

$D_A = 15\,\text{cm}$, $Q_A = 0.085\,\text{m}^3/\text{s}$.

$S_A = 0.95$, $D_C = 15\,\text{cm}$.

$Q_B = 0.028\,\text{m}^3/\text{s}$, $S_B = 0.85$, $D_B = 10\,\text{cm}$.

Find:

Mass flow rate at C.

Average velocity at C.

Specific gravity of the mixture.

Assumptions:

Steady state.

PLAN

Apply the continuity equation and the flow rate equation.

SOLUTION

Continuity equation

$$\begin{aligned}
\sum \dot{m}_i - \sum \dot{m}_o &= 0 \\
-\rho_A V_A A_A - \rho_B V_B A_B + \rho_C V_C A_C &= 0 \\
\rho_C V_C A_C &= 0.95 \times 1000\,\text{kg/m}^3 \times 0.085\,\text{m}^3/\,\text{s} \\
&\quad +0.85 \times 1000\,\text{kg/m}^3 \times 0.028\,\text{m}^3/\,\text{s} \\
&\boxed{\dot{m} = 104.55 \text{ kg/s}}
\end{aligned}$$

Continuity equation, assuming incompressible flow

$$\begin{aligned}
V_C A_C &= V_A A_A + V_B A_B \\
&= 0.085 + 0.028 = 0.113 \text{ m}^3/\text{s}
\end{aligned}$$

Flow rate equation

$$\begin{aligned}
V_C &= \frac{Q}{A} = \frac{0.113\,\text{m}^3/\,\text{s}}{\frac{\pi}{4}\,(0.15\,\text{m})^2} \\
&\boxed{V_C = 6.4 \text{ m/ s}} \\
\rho_C &= \frac{104.55}{0.113} = 925.22 \text{ kg/m}^3 \\
S &= \frac{925.22}{1000} \\
&\boxed{S = 0.925}
\end{aligned}$$

5.73: PROBLEM DEFINITION

Situation:

O_2 and CH_4 are mixed in a mixer before exiting.
$V_{O_2} = V_{CH_4} = 5\,\text{m/s}$.
$A_{CH_4} = 1\,\text{cm}^2$, $A_{O_2} = 3\,\text{cm}^2$.

Find:

Exit velocity of the gas mixture, V_e.

Properties:

From Table A.2: $R_{O_2} = 260\,\text{J/kg K}$, $R_{CH_4} = 518\,\text{J/kg K}$.
$T = 100\,°\text{C}$, $\rho = 1.9\,\text{kg/m}^3$, $p = 200\,\text{kPa}$.

PLAN

Apply the ideal gas law to find inlet density. Then apply the continuity equation.

SOLUTION

Ideal gas law

$$\begin{aligned}\rho_{0_2} &= \frac{p}{RT} \\ &= \frac{200,000\,\text{Pa}}{(260\,\text{J/kg K})\,(273+100)\ \text{K}} \\ &= 2.06\ \text{kg/m}^3 \\ \rho_{CH_4} &= \frac{200,000\,\text{Pa}}{(518\,\text{J/kg K})\,(273+100)\ \text{K}} \\ &= 1.03\ \text{kg/m}^3\end{aligned}$$

Continuity equation

$$\begin{aligned}\sum \dot{m}_i &= \sum \dot{m}_o \\ \rho_e V_e A_e &= \rho_{O_2} V_{O_2} A_{O_2} + \rho_{CH_4} V_{CH_4} A_{CH_4} \\ V_e &= \frac{2.06\,\text{kg/m}^3 \times 5\,\text{m/s} \times 3\,\text{cm}^2 + 1.03\,\text{kg/m}^3 \times 5\,\text{m/s} \times 1\,\text{cm}^2}{1.9\,\text{kg/m}^3 \times 3\,\text{cm}^2} \\ &\boxed{V_e = 6.33\ \text{m/s}}\end{aligned}$$

5.74: PROBLEM DEFINITION

Situation:

A pipe with a series of holes is used to distribute air.

$Q_{hole} = 0.67A_0 \left(\frac{2\Delta p}{r}\right)^{1/2}$.

$n_{hole} = 50/\,\text{m}$, $L = 10\,\text{m}$.

$D_{pipe} = 0.5\,\text{m}$, $D_{hole} = 2.5\,\text{cm}$.

Find:

Velocity of air entering the pipe.

Properties:

From Table A.2: $R = 287\,\text{J/kg K}$.

$T = 20\,°\text{C}$, $p_{pipe} = 100\,\text{Pa}$ gage

PLAN

The total discharge out of the holes is equal to the inlet discharge.

$$Q_{\text{in}} = AV_{\text{in}} = NQ_{\text{hole}}$$

where N is the number of holes and $Q_{\text{hole}}$is the discharge for each hole.

SOLUTION

The total number of holes

$$N = 50 \times 10 = 500 \text{ holes}$$

The density in the pipe is

$$\rho = \frac{p}{RT} = \frac{100,000 + 100\,\text{Pa}}{(287\,\text{J/kg K})\,(273 + 20)\text{ K}} = 1.19 \text{ kg/m}^3$$

The flow rate through the holes is

$$\begin{aligned} Q_{\text{hole}} &= 0.67A_o \left(\frac{2\Delta p}{\rho}\right)^{1/2} \\ Q_{\text{hole}} &= 0.67 \left(\frac{\pi}{4}\,(0.025\,\text{m})^2\right) \left(\frac{2 \times 100\,\text{Pa}}{1.19\,\text{kg/m}^3}\right)^{1/2} = 0.00426 \text{ m}^3/\text{s} \end{aligned}$$

The velocity at the pipe entrance is

$$V = \frac{NQ_{\text{hole}}}{A} = \frac{500 \times 0.00426\,\text{m}^3/\text{s}}{\frac{\pi}{4} \times (0.5\,\text{m})^2}$$

$$\boxed{V = 10.8\,\text{m/s}}$$

5.75: PROBLEM DEFINITION

Situation:

Water through a globe valve.
$Q = 38\,\text{L/min}$, $\Delta r_2 = 0.0032\,\text{m}$.
$D_1 = 0.025\,\text{m}$, $D_2 = 0.0125\,\text{m}$

Find:

Pressure drop across the valve.

Properties:

$T = 15.5\,°\text{C}$.

PLAN

Apply the Bernoulli equation between the 0.025 m upstream pipe and the opening at the seat of the valve.

SOLUTION

The pressure drop across the globe valve is

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g}$$

assuming negligible change in elevation. The p_1 is the pressure in the pipe and p_2 at the valve seat. From continuity $V_2 A_2 = V_1 A_1$ so

$$\frac{p_1 - p_2}{\gamma} = \frac{V_1^2}{2g}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

The area between the disc and the seat is

$$A_2 = 2\pi r \Delta r = 2\pi \times 0.0064 \text{ m} \times 0.0032 \text{ m} = 0.000129 \text{ m}^2$$

The upstream pipe area is

$$A_1 = \pi r^2 = \pi \times (0.0125 \text{ m})^2 = 0.000491 \text{ m}^2$$

Thus $A_1/A_2 = 4.0$. The volume flow rate is

$$Q = 38 \text{ L/min} \frac{0.001 \text{ m}^3\text{L}}{60 \text{ s/min}} = 0.000633 \text{ m}^3/\text{s}$$

and the velocity in the pipe is

$$V_1 = \frac{Q}{A_1} = \frac{0.000633 \text{ m}^3/\text{s}}{0.000491 \text{ m}^2} = 1.28 \text{ m/s}$$

Thus

$$\frac{p_1 - p_2}{\gamma} = \frac{(1.28\,\text{m/s})^2}{2 \times 9.81\,\text{m/s}^2}\left[4^2 - 1\right] = 1.25 \text{ m}$$

The specific weight of water at $15.5\,°\text{C}$ is 9799 N/m^3. The pressure drop is

$$p_2 - p_1 = 9799 \text{ N/m}^3 \times 1.25 \text{ m} = 12,248.75\,\text{Pa}$$

$$\boxed{p_2 - p_1 = 12 \text{ kPa}}$$

5.76: PROBLEM DEFINITION

Situation:

Water flow through an orifice in a pipe.

$D_1 = 2.5\,\text{cm},\ D_o = 1.5\,\text{cm}.$

$0.64A_o = A_2,\ Q = CA_o\left(\frac{2\Delta p}{\rho}\right)^{1/2}.$

$\Delta p = 10\,\text{kPa},\ Q = 1000\,\text{kg/m}^3.$

Find:

a) Derive equation for discharge.

b) Evaluate discharge across orifice.

PLAN

Apply the continuity equation and the Bernoulli equation between pipe and vena contracta. Neglect elevation change.

SOLUTION

Let point 1) be at the centerline of the upstream pipe and point 2) at the vena contracta. The Bernoulli equation gives

$$\begin{aligned} p_1 - p_2 &= \frac{\rho}{2}(V_2^2 - V_1^2) \\ &= \frac{\rho}{2}V_1^2\left(\frac{V_2^2}{V_1^2} - 1\right) \\ &= \frac{\rho}{2}V_1^2\left(\frac{A_1^2}{A_2^2} - 1\right) \\ &= \frac{\rho}{2}(V_1A_1)^2\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) \\ &= \frac{\rho}{2}(V_1A_1)^2\left(\frac{A_1^2 - A_2^2}{A_1^2A_2^2}\right) \end{aligned}$$

Solving for $V_1A_1 = Q$

$$Q = A_2\sqrt{\frac{2(p_1 - p_2)}{\rho}}\left(\frac{A_1^2}{A_1^2 - A_2^2}\right)^{1/2}$$

The ratio of the cross-sectional area at the vena contracta to the area of the orifice is

$$\frac{A_2}{A_o} = 0.64$$

and

$$\begin{aligned} \frac{A_2}{A_1} &= \frac{0.64A_o}{A_1} = 0.64\left(\frac{D_o}{D_1}\right)^2 \\ \frac{A_2}{A_1} &= 0.64\left(\frac{D_o}{D_1}\right)^2 = 0.64 \times \left(\frac{1.5\text{ cm}}{2.5\text{ cm}}\right)^2 \\ &= 0.2304 \end{aligned}$$

Substitute into discharge equation

$$Q = 0.64 A_o \sqrt{\frac{2(p_1 - p_2)}{\rho}} (\frac{1}{1 - A_2^2/A_1^2})^{1/2}$$

$$\begin{aligned} Q &= 0.64 A_o \sqrt{\frac{2(p_1 - p_2)}{\rho}} (\frac{1}{1 - 0.2304^2})^{1/2} \\ &\boxed{Q = 0.658 A_o \sqrt{\frac{2(p_1 - p_2)}{\rho}}} \end{aligned}$$

For a $\Delta p = 10$ kPa the discharge is

$$\begin{aligned} Q &= 0.658 \times (\frac{\pi}{4} \times 0.015^2 \text{ m}^2) \sqrt{\frac{2 \times 10000 \text{ Pa}}{1000 \text{ kg/m}^3}} \\ &= 0.000520 \text{ m}^3/\text{ s} \\ &\boxed{Q = 5.20 \times 10^{-4} \text{ m}^3/\text{ s}} \end{aligned}$$

5.77: PROBLEM DEFINITION

Situation:

A tank is filled with air from a compressor.

$V\!\!\!/ = 10\,\text{m}^3$, $\dot{m} = 0.5\frac{\rho_0}{\rho}\,\text{kg/s}$.

Find:

Time to increase the density of the air in the tank by a factor of 2.

Properties:

$\rho_0 = 2\,\text{kg/m}^3$

PLAN

Apply the continuity equation.

SOLUTION

Continuity equation

$$\begin{aligned}
\dot{m}_o - \dot{m}_i &= -\frac{d}{dt}\int_{CV}\rho dV\!\!\!/ \\
-\frac{d}{dt}(\rho V\!\!\!/) &= -\dot{m}_i \\
V\!\!\!/(\frac{d\rho}{dt}) &= 0.5\rho_0/\rho
\end{aligned}$$

Separating variables and integrating

$$\begin{aligned}
\rho d\rho &= \frac{0.5\rho_0 dt}{V\!\!\!/} \\
\frac{\rho^2}{2}\Big|_0^f &= \frac{0.5\rho_0 dt}{V\!\!\!/} \\
\frac{\rho_f^2 - \rho_0^2}{2} &= \frac{0.5\rho_0 \Delta t}{V\!\!\!/} - \\
\Delta t &= V\!\!\!/\rho_0\left[\left(\frac{\rho_f^2}{\rho_0^2}\right) - 1\right] \\
&= \left(10\,\text{m}^3\right)(2\,\text{kg/m}^3)((2)^2 - 1) \\
&\boxed{\Delta t = 60\text{s}}
\end{aligned}$$

5.78: PROBLEM DEFINITION

Situation:

A tire develops a slow leak.
$\dot{m} = 0.68pA/\sqrt{RT}$, $\forall = 0.015\,\text{m}^3$, $t = 3\,\text{h}$.

Find:

Area of the leak.

Properties:

From Table A.2: 287 J/kg K.
$T = 15.5\,^\circ\text{C}$, $p_1 = 205$ kPa, $p_2 = 170$ kPa.

PLAN

Apply the continuity equation.

SOLUTION

Continuity equation

$$\dot{m}_{out} = -\frac{d}{dt}(\rho\forall)$$

Ideal gas law

$$\rho = \frac{p}{RT}$$

Combining previous 2 equations

$$\dot{m}_{out} = -\frac{\forall}{RT}\left(\frac{dp}{dt}\right)$$

Let $\dot{m}_{out} = 0.68pA/\sqrt{RT}$ in the above equation

$$\frac{0.68pA}{\sqrt{RT}} = -\frac{\forall}{RT}\left(\frac{dp}{dt}\right)$$

Separating variables and integrating

$$\begin{aligned}\frac{1}{p}\left(\frac{dp}{dt}\right) &= -\frac{(0.68A\sqrt{RT})}{\forall} \\ \ell n\left(\frac{p_0}{p}\right) &= \frac{(0.68A\sqrt{RT})}{\forall}\end{aligned}$$

Finding area

$$\begin{aligned}A &= \left(\frac{\forall}{0.68t\sqrt{RT}}\right)\ell n\left(\frac{p_0}{p}\right) \\ &= \left(\frac{0.015}{(0.68)\,(3\,\text{h})\,(3,600\,\text{s/ h})\,\sqrt{287\,\text{J/kg K} \times 288\text{ K}}}\right)\ell n(306/271)\end{aligned}$$

$$\boxed{A = 8.59 \times 10^{-10}\,\text{m}^2 = 8.59 \times 10^{-4}\,\text{mm}^2}$$

5.79: PROBLEM DEFINITION

Situation:

An O_2 bottle leaks oxygen through a small orifice causing the pressure to drop.
$\dot{m} = 0.68pA/\sqrt{RT}$, $\forall = 0.1\,\text{m}^3$.
$D = 0.12\,\text{mm}$.

Find:

Time required for the specified pressure change.

Properties:

From Table A.2: $R = 260\,\text{J/kg K}$.
$T = 18\,°\text{C}$, $p_0 = 10\,\text{MPa}$.
$p = 5\,\text{MPa}$.

PLAN

Apply the continuity equation and the ideal gas law.

SOLUTION

Continuity equation

$$\dot{m}_{out} = -\frac{d}{dt}(\rho \forall)$$

Ideal gas law

$$\rho = \frac{p}{RT}$$

Combining previous 2 equations

$$\dot{m}_{out} = -\left(\frac{\forall}{RT}\right)\frac{dp}{dt}$$

Let $\dot{m}_{out} = 0.68A/\sqrt{RT}$ in the above equation

$$\frac{0.68pA}{\sqrt{RT}} = -\left(\frac{\forall}{RT}\right)\frac{dp}{dt}$$

Separating variables and integrating

$$\begin{aligned} \frac{1}{p}\left(\frac{dp}{dt}\right) &= -\frac{(0.68A\sqrt{RT})}{\forall} \\ \ell n\left(\frac{p_0}{p}\right) &= \frac{(0.68A\sqrt{RT})}{\forall}t \end{aligned}$$

Finding time

$$
\begin{aligned}
t &= \left(\frac{\forall}{0.68A\sqrt{RT}}\right)\ell n\left(\frac{p_0}{p}\right) \\
A &= \frac{\pi}{4}(0.12\times 10^{-3}\ \mathrm{m})^2 = 1.131\times 10^{-8}\ \mathrm{m}^2 \\
\sqrt{RT} &= \sqrt{260\times 291} = 275.1\ \mathrm{m/s} \\
t &= \frac{0.1\ \mathrm{m}^3\times \ell n\,(10/5)}{0.68\times 1.131\,\mathrm{m}^2\times 10^{-8}\ \mathrm{m}^2\times 275\ \mathrm{m/s}} = 3.28\times 10^4\ \mathrm{s} \\
&\boxed{t = 9\,\mathrm{h}\ 6\ \mathrm{min.}}
\end{aligned}
$$

5.80: PROBLEM DEFINITION

Situation:

A tank is draining through an orifice.

$h_1 = 3\,\text{m},\ h = 0.5\,\text{m}.$

$D_T = 0.6\,\text{m},\ D_2 = 3\,\text{cm}$

Find:

Time required for the water surface to drop the specified distance (3 to 0.5 m).

SOLUTION

From Example 5-6 the time to decrease the elevation from h_1 to h is

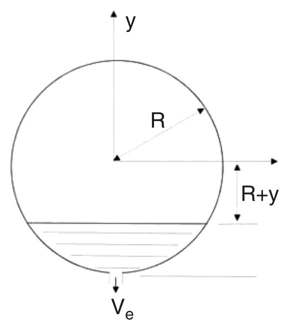
$$\begin{aligned} t &= \left(\frac{2A_T}{\sqrt{2g}A_2}\right)(h_1^{1/2} - h^{1/2}) \\ &= \frac{2 \times \left(\pi/4 \times (0.6\,\text{m})^2\right)\left(\sqrt{3} - \sqrt{0.5}\right)\ \text{m}^{1/2}}{\sqrt{2 \times 9.81\,\text{m/s}^2} \times (\pi/4) \times (0.03\,\text{m})^2} \\ & \boxed{t = 185\ \text{s}} \end{aligned}$$

5.81: PROBLEM DEFINITION

Situation:

A cylindrical drum of water is emptying through a pipe on the bottom.
$D = 0.6$ m, $R = 0.3$ m.
$V = \sqrt{2gh}$, $L = 1.2$ m.
$d = 0.05$ m, $h_0 = 0.3$ m.

Sketch:



Find:

Time to empty the drum.

PLAN

Apply the continuity equation. Let the control surface surround the water in the tank. Let the c.s. be coincident with the moving water surface. Thus, the control volume will decrease in volume as the tank empties. Situate the origin at the center of the tank.

SOLUTION

Continuity equation

$$\dot{m}_o - \dot{m}_i = -\frac{d}{dt}\int_{\text{cv}} \rho d\forall$$

$$+\rho VA = -\frac{d}{dt}\int_{\text{cv}} \rho d\forall \tag{1}$$

$$\rho\sqrt{2gh}A = -\rho\frac{d}{dt}(\forall) \tag{2}$$

$$dt\sqrt{2gh}A = -d\forall \tag{3}$$

Let $d\forall = L(2x)dy$. Substituted into Eq. (3) we have

$$dt\sqrt{2gh}A = 2Lxdy \tag{4}$$

But h can be expressed as a function of y:

$$h = R + y$$

or

$$dt\sqrt{2g(R+y)}A = 2Lxdy$$

Also

$$\begin{aligned}
R^2 &= x^2 + y^2 \\
x &= \sqrt{R^2 - y^2} = \sqrt{(R-y)(R+y)} \\
dt\sqrt{2g(R+y)}A &= -2L\sqrt{(R-y)(R+y)}dy \\
dt &= -\left(\frac{2L}{\sqrt{2g}A}\right)\sqrt{(R-y)}dy \qquad (5)
\end{aligned}$$

Integrate Eq. (5)

$$\begin{aligned}
t|_0^t &= -\left(\frac{2L}{\sqrt{2g}A}\right)\int_0^R \sqrt{R-y}dy \\
&= \left(\frac{2L}{\sqrt{2g}A}\right)[(2/3)(R-y)^{3/2}]_0^{-R} \\
t &= \left(\frac{2L}{\sqrt{2g}A}\right)(2/3)\left[(2R)^{3/2} - R^{3/2}\right]
\end{aligned}$$

For $R = 1$

$$t = \left(\frac{2L}{\sqrt{2g}A}\right)(2/3)(2^{3/2} - 1) \qquad (6)$$

In Eq. (5) $A = (\pi/4)d^2 = 0.00196$ m^2. Therefore

$$t = \left(2 \times \frac{1.2\,\mathrm{m}}{\sqrt{19.62\,\mathrm{m/s}^2 \times 0.00196\,\mathrm{m}^2}}\right)(2/3)(1.828)$$

$$\boxed{t = 332\ \mathrm{s}}$$

REVIEW

The above solution assumes that the velocity of water is uniform across the jet just as it leaves the tank. This is not exactly so, but the solution should yield a reasonable approximation.

5.82: PROBLEM DEFINITION

Situation:

Water drains from a pressurized tank.

$V_e = \sqrt{\frac{2p}{\rho} + 2gh}$, $h_o = 2$ m.

$A = 1\,\text{m}^2$, $A_e = 10\,\text{cm}^2$.

Find:

Time for the tank to empty with given supply pressure.

Time for the tank to empty if supply pressure is zero.

Properties:

$p = 10$ kPa.

Water, Table A.5: $\rho = 1000$ kg/m^3.

PLAN

Apply the continuity equation. Define a control surface coincident with the tank walls and the top of the fluid in the tank.

SOLUTION

Continuity equation

$$\rho \frac{dV}{dt} = -\rho A_e V_e$$

Density is constant. The differential volume is Adh so the above equation becomes

$$-\frac{Adh}{A_e V_e} = -dt$$

or

$$-\frac{Adh}{A_e\sqrt{\frac{2p}{\rho} + 2gh}} = dt$$

Integrating this equation gives

$$-\frac{A}{A_e}\frac{1}{g}\left(\frac{2p}{\rho} + 2gh\right)^{1/2} |_{h_o}^{0} = \Delta t$$

or

$$\Delta t = \frac{A}{A_e}\frac{1}{g}\left[\left(\frac{2p}{\rho} + 2gh_o\right)^{1/2} - \left(\frac{2p}{\rho}\right)^{1/2}\right]$$

and for $A = 1$ m^2, $A_e = 10^{-3}$ m^2, $h_o = 2$ m, $p = 10$ kPa and $\rho = 1000$ kg/m^3 results in

$$\boxed{\Delta t = 329\,\text{s or } 5.48 \text{ min}} \text{ (supply pressure of 10 kPa)}$$

For zero pressure in the tank, the time to empty is

$$\Delta t = \frac{A}{A_e}\sqrt{\frac{2h_o}{g}} = 639 \text{ s or}$$

$$\boxed{\Delta t = 10.6 \text{ min}} \text{ (supply pressure of zero)}$$

5.83: PROBLEM DEFINITION

Situation:

A tapered tank drains through an orifice at bottom of tank.
$V_e = \sqrt{2gh}$, $D = d + C_1 h$.
$h_0 = 1\,\text{m}$, $h = 20\,\text{cm}$, $d = 20\,\text{cm}$.
$C_1 = 0.3$, $d_j = 5\,\text{cm}$.

Find:

Derive a formula for the time to drain.
Calculate the time to drain.

PLAN

Apply the continuity equation.

SOLUTION

From continuity equation

$$\begin{aligned} Q &= -A_T\left(\frac{dh}{dt}\right) \\ dt &= -A_T\frac{dh}{Q} \end{aligned}$$

where $Q = \sqrt{2gh}A_j = \sqrt{2gh}(\pi/4)d_j^2$

$$\begin{aligned} A_T &= \frac{\pi}{4}(d + C_1 h)^2 = \frac{\pi}{4}(d^2 + 2dC_1 h + C_1^2 h^2) \\ dt &= \frac{-(d^2 + 2dC_1 h + C_1 h^2)dh}{\sqrt{2g}h^{1/2}d_j^2} \\ t &= -\int_{h_0}^{h}\frac{(d^2 + 2dC_1 h + C_1^2 h^2)dh}{\sqrt{2g}h^{1/2}d_j^2} \\ t &= \frac{1}{d_j^2\sqrt{2g}}\int_{h}^{h_0}(d^2 h^{-1/2} + 2dC_1 h^{1/2} + C_1^2 h^{3/2})dh \\ t &= \frac{2}{d_j^2\sqrt{2g}}\left[d^2 h^{1/2} + (2/3)dC_1 h^{3/2} + (1/5)C_1^2 h^{5/2}\right]_h^{h_0} \end{aligned}$$

Evaluating the limits of integration gives

$$\boxed{t = \frac{2}{d_j^2\sqrt{2g}}\left[(d^2(h_0^{1/2} - h^{1/2}) + \frac{2}{3}dC_1(h_0^{3/2} - h^{3/2}) + \frac{1}{5}C_1^2(h_0^{5/2} - h^{5/2})\right]}$$

$$\boxed{t = 13.8\text{ s}}$$

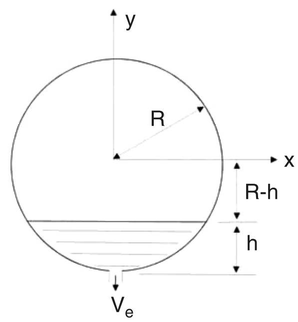
5.84: PROBLEM DEFINITION

Situation:

Water drains out of a spherical tank that begins at half full.

$V_e = \sqrt{2gh}$, $R = 0.5\,\text{m}$, $d_e = 1\,\text{cm}$.

Sketch:



Find:

Time required to empty the tank.

PLAN

Apply the continuity equation. Select a control volume that is inside of the tank and level with the top of the liquid surface.

SOLUTION

Continuity equation

$$\rho \frac{d\mathcal{V}}{dt} = -\rho A_e V_e$$

Let

$$\frac{d\mathcal{V}}{dt} = A\frac{dh}{dt}$$

Continuity becomes

$$\frac{dh}{dt} = -\frac{A_e}{A}\sqrt{2gh}$$

The cross-sectional area in terms of R and h is

$$A = \pi[R^2 - (R-h)^2] = \pi(2Rh - h^2)$$

Substituting into the differential equation gives

$$\frac{\pi(-2Rh + h^2)}{A_e\sqrt{2gh}} dh = dt$$

or

$$\frac{\pi}{\sqrt{2g}A_e}\left(-2Rh^{1/2} + h^{3/2}\right) dh = dt$$

Integrating this equation results in

$$\frac{\pi}{\sqrt{2g}A_e}\left(-\frac{4}{3}Rh^{3/2}+\frac{2}{5}h^{5/2}\right)|_R^0 = \Delta t$$

Substituting in the limits yields

$$\frac{\pi}{\sqrt{2g}A_e}\frac{14}{15}R^{5/2} = \Delta t$$

For $R = 0.5$ m and $A_e = 7.85 \times 10^{-5}$ m^2, the time to empty the tank is

$$\boxed{\Delta t = 1491\,\text{s or } 24.8 \text{ min}}$$

5.85: PROBLEM DEFINITION

Situation:

A tank containing oil is draining from the bottom.
$D_T = 2\,\mathrm{m}$, $h_o = 5\,\mathrm{m}$.
$L = 6\,\mathrm{m}$, $d_e = 2\,\mathrm{cm}$.
$p = (p_o + p_{atm}) \times (L - h_o)/(L - h) - p_{atm}$.
$dh/dt = -A_e/A_T \times \sqrt{2gh + 2p/\rho}$.

Find:

Predict the depth of the oil with time for a one hour period.

Properties:

$\rho = 880\,\mathrm{kg/\,m^3}$, $p_o = 300\,\mathrm{kPa}$, $p_{atm} = 100\,\mathrm{kPa}$.

SOLUTION

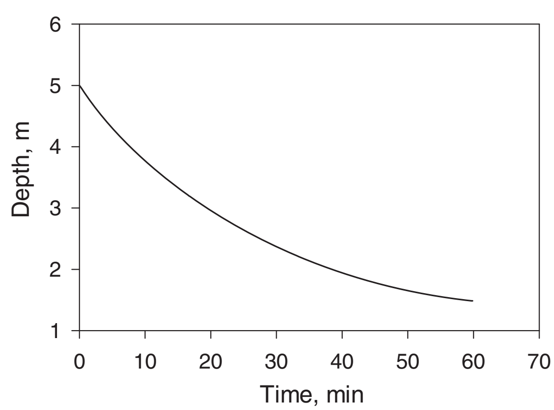
From the continuity equation

$$\begin{aligned} \frac{dM_{cv}}{dt} &= \rho A_T \frac{dh}{dt} = -\dot{m}_o \\ \frac{dh}{dt} &= -\frac{\rho V_e A_o}{\rho A_T} = \frac{A_o}{A_T}\sqrt{2gh + \frac{2p}{\rho}} \end{aligned}$$

where

$$p = (p_0 + p_{atm})\left(\frac{L - h_0}{L - h}\right) - p_{atm}$$

A numerical program was developed and the numerical solution provides the following results:



5.86: PROBLEM DEFINITION

Situation:

Rocket Propulsion. To prepare for problems 5.87, 5.88, and 5.89 in EFM10e, use the internet or other resources and define the following terms in the context of rocket propulsion: (a) solid fuel, (b) grain, and (c) surface regression.
Also explain how a solid fuel rocket engine works.

SOLUTION

Answers will vary, but should include the following elements:
(a) Solid fuel, or solid propellants, are used in rockets
(b) grain is the initial or starting amount of solid, expressed as a length (because it is a solid, and has some given diameter), also called "charge", of the solid fuel.
(c) surface regression, also called burn rate, is the rate at which the solid fuel is burned. Since the grain is defined as a length, the surface regression is a length/time.

5.87: PROBLEM DEFINITION

Situation:

Propellant fuels an end-burning rocket motor.
$D_c = 0.1\,\text{m}$, $D_e = 0.08\,\text{m}$, $\dot{r} = 1.5\,\text{cm/s}$.

Find: Gas velocity at nozzle exit plane.

Properties:

$\rho = 1800\,\text{kg/m}^3$, $R = 415\,\text{J/kg K}$.
$p_e = 10\,\text{kPa}$, $T = 2200\,^\circ\text{C}$.

PLAN Apply the continuity equation and the ideal gas law.

SOLUTION Ideal gas law

$$\begin{aligned} \rho_e &= \frac{p}{RT} \\ &= \frac{10000\,\text{Pa}}{415\,\text{J/kg K} \times 2473\,\text{K}} = 0.00974\ \text{kg/m}^3 \end{aligned}$$

The rate of mass decease of the solid propellant is $\rho_p A_c \dot{r}$ where ρ_p is the propellant density, A_c is the chamber cross-sectional area and $\dot{r}$ is the regression rate. This is equal to the mass flow rate supplied to the chamber or across the control surface. From the continuity equation

$$\begin{aligned} V_e &= \frac{\rho_p A_c \dot{r}}{\rho_e A_e} \\ A_c &= \pi/4 \times (0.1\ \text{m})^2 = 0.00785\ \text{m}^2 \\ A_e &= \pi/4 \times (0.08\ \text{m})^2 = 0.00503\ \text{m}^2 \\ V_e &= 1800\ \text{kg/m}^3 \times 0.00785\ \text{m}^2 \times \frac{0.015\ \text{m/s}}{[0.00974\ \text{kg/m}^3 \times 0.00503\ \text{m}^2]} \\ & \boxed{V_e = 4330\ \text{m/s}} \end{aligned}$$

5.88: PROBLEM DEFINITION

Situation:

Propellant fuels a cylindrical-port rocket motor.
$D_0 = 0.2\,\text{m},\ D = 0.12\,\text{m}.$
$D_e = 0.2\,\text{m},\ V_e = 1800\,\text{m/s}.$
$p_e = 10\,\text{kPa},\ \dot{r} = 1.0\,\text{cm/s}.$
$L = 0.4\,\text{m}.$

Find:

Gas density at the exit.

Properties:

$\rho_g = 2000\,\text{kg/m}^3,\ R = 415\,\text{J/kg K}.$

SOLUTION Area of the grain surface (internal surface and two ends)

$$
\begin{aligned}
A_g &= \pi DL + 2(\pi/4)(D_0^2 - D^2) \\
&= \pi \times 0.12\,\text{m} \times 0.4\,\text{m} + \frac{\pi}{2}((0.2\,\text{m})^2 - (0.12\,\text{m})^2) = 0.191\ \text{m}^2 \\
\rho_e &= \frac{V_g \rho_g A_g}{V_e A_e} \\
A_e &= (\pi/4) \times (0.20\ \text{m})^2 = 0.03142\ \text{m}^2 \\
\rho_e &= \frac{\left(0.01\ \text{m/s} \times 2,000\ \text{kg/m}^3 \times 0.191\ \text{m}^2\right)}{1,800\ \text{m/s} \times 0.03142\ \text{m}^2} \\
& \boxed{\rho_e = 0.0676\ \text{kg/m}^3}
\end{aligned}
$$

5.89: PROBLEM DEFINITION

Situation:

Propellant flows through a nozzle in a rocket chamber.

$\dot{m} = \frac{0.65 p_c A_t}{\sqrt{RT_c}}$, $\dot{r} = a p_c^n$.

$n = 0.3$, $p_c =$ kPa.

$p_c = \left(\frac{a\rho_p}{0.65}\right)^{1/(1-n)} \left(\frac{A_g}{A_t}\right)^{1/(1-n)} (RT_c)^{1/[2(1-n)]}$.

Find:

Derive a formula for chamber pressure.

Calculate the increase in chamber pressure if a crack increases burn area by 20%.

PLAN

Apply the flow rate equation.

SOLUTION

Continuity equation. The mass flux off the propellant surface equals flow rate through nozzle.

$$\begin{aligned}
\rho_p \dot{r} A_g &= \dot{m} \\
\rho_p a p_c^n A_g &= \frac{0.65 p_c A_t}{\sqrt{RT_c}} \\
p_c^{1-n} &= \frac{a\rho_p}{0.65} \left(\frac{A_g}{A_t}\right) (RT_c)^{1/2} \\
& \boxed{p_c = \left(\frac{a\rho_p}{0.65}\right)^{1/(1-n)} \left(\frac{A_g}{A_t}\right)^{1/(1-n)} (RT_c)^{1/(2(1-n))}} \\
\Delta p_c &= 3.5 \text{ MPa} (1 + 0.20)^{1/(1-0.3)} \\
& \boxed{\Delta p_c = 4.54 \text{ MPa}}
\end{aligned}$$

5.90: PROBLEM DEFINITION

Situation:

A piston moves in a cylinder and drives exhaust gas out an exhaust port in a four cycle engine.

$\dot{m} = \frac{0.65 p_c A_v}{\sqrt{RT_c}}$, $d_{bore} = 0.1\,\text{m}$.

$L = 0.1\,\text{m}$, $A_v = 1\,\text{cm}^2$.

$V = 30\,\text{m/s}$.

Find:

Rate at which the gas density is changing in the cylinder.

Assumptions:

The gas in the cylinder is ideal and has a uniform density and pressure.

Properties:

$T = 600\,^\circ\text{C}$, $R = 350\,\text{J/kg K}$, $p = 300\,\text{kPa}$.

SOLUTION

Continuity equation. Control volume is defined by piston and cylinder.

$$\begin{aligned}
\frac{d}{dt}(\rho V) + \frac{0.65 p_c A_v}{\sqrt{RT_c}} &= 0 \\
V\frac{d\rho}{dt} + \rho\frac{dV}{dt} + \frac{0.65 p_c A_v}{\sqrt{RT_c}} &= 0 \\
\frac{d\rho}{dt} &= -\left(\frac{\rho}{V}\right)\frac{dV}{dt} - \frac{0.65 p_c A_v}{V\sqrt{RT_c}} \\
V &= (\pi/4)(0.1\,\text{m})^2(0.1\,\text{m}) = 7.854\times10^{-4}\ \text{m}^3 \\
\frac{dV}{dt} &= -(\pi/4)(0.1\,\text{m})^2(30\,\text{m/s}) = -0.2356\ \text{m}^3/\text{s} \\
\rho &= \frac{p}{RT} = \frac{300,000\,\text{Pa}}{(350\,\text{J/kg K}\times 873\,\text{K})} \\
&= 0.982\ \text{kg/m}^3 \\
\frac{d\rho}{dt} &= -\frac{0.982\,\text{kg/m}^3}{7.854\times10^{-4}\,\text{m}^3}\times(-0.2356\,\text{m}^3/\text{s}) \\
&\quad -\frac{0.65\times 300,000\,\text{Pa}\times 1\times10^{-4}\,\text{m}^2}{7.854\times10^{-4}\,\text{m}^3\times\sqrt{350\,\text{J/kg/K}\times 873\,\text{K}}}
\end{aligned}$$

$$\boxed{\frac{d\rho}{dt} = 250\ \text{kg/m}^3\cdot\text{s}}$$

5.91: PROBLEM DEFINITION

Situation:
Gas is flowing from Location 1 to 2 in a pipe expansion. The inlet density, diameter and velocity are ρ_1, D_1, and V_1 respectively. If D_2 is $2D_1$, and V_2 is $\frac{1}{2}V_1$, what is the magnitude of ρ_2?
a. $\rho_2 = 4\rho_1$
b. $\rho_2 = 2\rho_1$
c. $\rho_2 = \frac{1}{2}\rho_1$
d. $\rho_2 = \rho_1$

PLAN

Apply the continuity equation, and utilize the definition of volume flowrate.

SOLUTION

$$\begin{aligned}
\rho_1 \frac{\pi}{4} D_1^2 V_1 &= \rho_2 \frac{\pi}{4} D_2^2 V_2 \\
\rho_1 D_1^2 V_1 &= \rho_2 (2D_1)^2 (\frac{1}{2} V_1) \\
\rho_1 &= \rho_2 (4)(\frac{1}{2}) \\
\rho_1 &= 2\rho_2 \\
\rho_2 &= \frac{1}{2}\rho_1
\end{aligned}$$

The correct answer is (c).

5.92: PROBLEM DEFINITION

Situation:

Air is flowing from a ventilation duct (cross-section 1) as shown, and is expanding to be released into a room at cross-section 2. The area at cross-section 2, A_2, is 3 times A_1. Assume that the density is constant. The relation between $Q_1$and Q_2 is:

a. $Q_2 = \frac{1}{3}Q_1$
b. $Q_2 = Q_1$
c. $Q_2 = 3\ Q_1$
d. $Q_2 = 9\ Q_1$

PLAN

Apply the continuity equation.

SOLUTION

According to the continuity equation, $Q_2 = Q_1$, because there is no storage in the duct.
Therefore the correct answer is (b).

Note: It is also true that $V_2 = \frac{1}{3}V_1$, but the relationship between the two velocities is not the subject of this question.

5.93: PROBLEM DEFINITION

Situation:

Water is flowing from Location 1 to 2 in this pipe expansion. D_1 and V_1 are known at the inlet. D_2 and P_2 are known at the outlet. What equation(s) do you need to solve for the inlet pressure P_1? Neglect viscous effects.

a. The continuity equation

b. The continuity equation and the flow rate equation.

c. The continuity equation, the flow rate equation, and the Bernoulli equation

d. There is insufficient information to solve the problem

PLAN

We know that we need to apply the continuity equation, utilize the definition of volume flowrate, and use the Bernoulli equation which relates pressure, elevation, and velocity. To verify that there is sufficient information to solve the problem, set up the equations and document the knowns and unknowns.

SOLUTION

$$A_1 V_1 = A_2 V_2$$

A_1, V_1 and A_2 are known, so one can solve for V_2

$$\frac{P_1}{\gamma} + \frac{V_2^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

V_1, P_2 and V_2 are known, so one can solve for P_1

Thus, we have demonstrated that there is sufficient info to solve the problem

Therefore the correct answer is (c).

5.94: PROBLEM DEFINITION

Situation:

Water flows in a pipe with a contraction.

$Q = 1.7\,\mathrm{m^3/s}$, $d = 0.6\,\mathrm{m}$, $D = 1.8\,\mathrm{m}$.

Find:

Pressure at point B.

Assumptions:

Water temperature is 10 °C.

Properties:

Water (10 °C), Table A.5: $\gamma = 9810\,\mathrm{N/m^3}$.

$p_A = 153$ kPa.

PLAN

Apply the Bernoulli equation and the continuity equation.

SOLUTION

Continuity equation

$$\begin{aligned} V_A &= \frac{Q}{A_A} = \frac{1.7\,\mathrm{m^3/s}}{\pi/4 \times (1.8\,\mathrm{m})^2} = 0.67 \text{ m/s} \\ V_B &= \frac{Q}{A_B} = \frac{1.7\,\mathrm{m^3/s}}{\pi/4 \times (0.6\,\mathrm{m})^2} = 6.02 \text{ m/s} \end{aligned}$$

The Bernoulli equation

$$\begin{aligned} \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B \\ \frac{p_B}{\gamma} &= \frac{153 \text{ kPa}}{9810 \text{ N/m}^3} + \frac{(0.67 \text{ m/s})^2}{19.62 \text{ m/s}^2} - \frac{(6.02 \text{ m/s})^2}{19.62 \text{ m/s}^2} - 1.2 \text{ m} \\ p_B &= 123,606 \text{ N/m}^2 \\ & \boxed{p_B = 124\,\mathrm{kPa}} \end{aligned}$$

5.95: PROBLEM DEFINITION

Situation:

Water flows through a contraction section of circular pipe.

$V_{\text{in}} = (10 \text{ m/s})\,[1 - \exp(-t/10)],\ D_i = 2D_o$.

Find:

Velocity variation at outlet.

PLAN

Apply the continuity equation.

SOLUTION

Because water is incompressible, there is no unsteady term in continuity equation, so

$$V_{\text{out}} = V_{\text{in}}\left(\frac{A_{\text{in}}}{A_{\text{out}}}\right) = \left(\frac{2}{1}\right)^2 V_{\text{in}}$$

$$\boxed{\text{V}_{\text{out}}\text{=}(40 \text{ m/s}) \left[1 - \exp\left(-\frac{t}{10}\right)\right]}$$

5.96: PROBLEM DEFINITION

Situation:

An annular venturimeter is mounted in a pipe with air flow at standard conditions. $D = 15\,\text{cm} = 0.15\,\text{m}$, $d = 0.8D$.

Find:

Find the volume flow rate.

Assumptions:

Flow is incompressible, inviscid, steady and velocity is uniformly distributed.

Properties:

$\Delta p = 0.05\,\text{m H}_2\text{O}$, $\rho = 1.22\,\text{kg/m}^3$.

PLAN

Apply the Bernoulli equation.

SOLUTION

Take point 1 as upstream in pipe and point 2 in annular section. The flow is incompressible, steady and inviscid so the Bernoulli equation applies

$$p_1 + \gamma z_1 + \rho\frac{V_1^2}{2} = p_2 + \gamma z_2 + \rho\frac{V_2^2}{2}$$

Also $z_1 = z_2$. From the continuity equation

$$A_1V_1 = A_2V_2$$

But

$$A_2 = \frac{\pi}{4}(D^2 - d^2)$$

so

$$\begin{aligned}\frac{A_2}{A_1} &= 1 - \frac{d^2}{D^2} \\ &= 0.36\end{aligned}$$

Therefore

$$V_2 = \frac{V_1}{0.36} = 2.78V_1$$

Substituting into the Bernoulli equation

$$\begin{aligned}p_1 - p_2 &= \frac{\rho}{2}(V_2^2 - V_1^2) \\ &= \frac{\rho}{2}V_1^2(2.78^2 - 1) \\ &= 3.36\rho V_1^2\end{aligned}$$

The standard density is 1.22 kg/m^3 and the pressure difference is

$$\begin{aligned}\Delta p &= (0.05\,\text{m})\left(9810\,\text{N/m}^3\right)\\ &= 490.5\text{ Pa}\end{aligned}$$

Solving for V_1

$$\begin{aligned}V_1^2 &= \frac{490.5\,\text{N/m}^2}{3.36\left(1.22\,\text{kg/m}^3\right)}\\ &= 119.66\,\text{m}^2/\,\text{s}\\ V_1 &= 10.94\text{ m/s}\end{aligned}$$

The discharge is

$$\begin{aligned}Q &= A_1 V_1\\ &= (10.94\,\text{m/s})\left(\frac{\pi}{4}\right)(0.15\,\text{m})^2\\ &= 0.193\text{ m}^3/\text{s}\\ &\boxed{Q = 0.2\text{ m}^3/\text{s}}\end{aligned}$$

5.97: PROBLEM DEFINITION

Situation:

A venturi-type applicator is used to spray liquid fertilizer.

$D_2 = 1\,\text{cm}$, $A_2/A_1 = 2$, $Q = 8$ L/ min.

$z_3 = -0.1\,\text{m}$, $Q_l = 0.5\sqrt{\Delta h}$.

Find:

The flow rate of liquid fertilizer.

The mixture ratio of fertilizer to water at exit.

Properties:

$T = 20\,^\circ\text{C}$.

PLAN

Use the continuity and Bernoulli equation to find the pressure at the throat and use this pressure to find the difference in piezometric head and flow rate.

SOLUTION

The Bernoulli equation is applicable between stations 1 (the throat) and 2 (the exit).

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

From the continuity equation

$$\begin{aligned} V_1 &= \frac{A_2}{A_1} V_2 \\ &= 2V_2 \end{aligned}$$

Also $z_1 = z_2$ so

$$\begin{aligned} \frac{p_1}{\gamma} - \frac{p_2}{\gamma} &= \frac{V_2^2}{2g}(1 - 2^2) \\ &= -3\frac{V_2^2}{2g} \end{aligned}$$

At the exit $p_2 = 0$ (gage)

$$\frac{p_1}{\gamma} = -3\frac{V_2^2}{2g}$$

The flow rate is 8 L/min or

$$\begin{aligned} Q &= 8\ \frac{\text{L}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} \times \frac{10^{-3}\ \text{m}^3}{1\ \text{L}} \\ &= 0.133 \times 10^{-3}\ \text{m}^3/\text{s} \end{aligned}$$

The exit diameter is 1 cm so

$$\begin{aligned} A_2 &= \frac{\pi}{4}\,(0.01\,\text{m})^2 \\ &= 7.85 \times 10^{-5}\ \text{m}^2 \end{aligned}$$

The exit velocity is

$$\begin{aligned} V_2 &= \frac{Q}{A_2} = \frac{0.133 \times 10^{-3}\,\mathrm{m}^3/\,\mathrm{s}}{7.85 \times 10^{-5}\,\mathrm{m}^2} \\ &= 1.692 \text{ m/s} \end{aligned}$$

Therefore

$$\begin{aligned} \frac{p_1}{\gamma} &= -3 \times \frac{(1.692\,\mathrm{m}/\,\mathrm{s})^2}{2 \times 9.81\,\mathrm{m}/\,\mathrm{s}^2} \\ &= -0.259 \text{ m} \end{aligned}$$

Let point 3 be the entrance to the feed tube. Then

$$\begin{aligned} \Delta h &= h_3 - h_1 \\ &= \frac{p_3}{\gamma} + z_3 - \left(\frac{p_1}{\gamma} + z_1\right) \\ &= \frac{p_3}{\gamma} - \frac{p_1}{\gamma} + (z_3 - z_1) \\ &= 0.05 - (-0.259) - 0.1 \\ &= 0.209 \text{ m} \end{aligned}$$

a) The flow rate in the feed tube is

$$\begin{aligned} Q_f &= 0.5\sqrt{0.209} \\ & \boxed{Q_f = 0.228 \text{ L/min}} \end{aligned}$$

b) Concentration in the mixture

$$\begin{aligned} \frac{Q_l}{Q_l + Q_w} &= \frac{0.228 \text{ L/min}}{(8 + 0.228) \text{ L/min}} \\ & \boxed{\frac{Q_l}{Q_l+Q_w} = 0.028 \quad (\text{or } 2.8\%)} \end{aligned}$$

5.98: PROBLEM DEFINITION

Situation:

Air flows upward in a vertical venturi.

$V_1 = 24\,\text{m/s}$, $A_2/A_1 = 0.5$.

Find:

Deflection of manometer.

Assumptions:

Uniform air density.

Properties:

$\rho = 1\,\text{kg/m}^3$, $\gamma = 18.9\,\text{kN/m}^3$.

PLAN

Apply the Bernoulli equation from 1 to 2 and then the continuity equation. Let section 1 be in the large duct where the manometer pipe is connected and section 2 in the smaller duct at the level where the upper manometer pipe is connected.

SOLUTION

Continuity equation

$$\begin{aligned} V_1 A_1 &= V_2 A_2 \\ V_2 &= V_1 \left(\frac{A_1}{A_2} \right) \\ &= (24\,\text{m/s})\,(2) \\ &= 48 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_{z1} + \frac{\rho V_1^2}{2} &= p_{z2} + \frac{\rho V_2^2}{2} \\ p_{z1} - p_{z2} &= (1/2)\rho(V_2^2 - V_1^2) \\ p_{z1} - p_{z2} &= (1/2)\left(1\,\text{kg/m}^3\right)\left[(48\,\text{m/s})^2 - (24\,\text{m/s})^2\right] \\ &= 864 \text{ N/m}^2 \end{aligned}$$

Manometer equation

$$\begin{aligned} p_{z1} - p_{z2} &= \Delta h(\gamma_{\text{liquid}} - \gamma_{\text{air}}) \\ 864\,\text{N/m}^2 &= \Delta h(18,900\,\text{N/m}^3 - 9.80\,\text{N/m}^3) \\ & \boxed{\Delta h = 0.046 \text{ m}} \end{aligned}$$

5.99: PROBLEM DEFINITION

Situation:
An atomizer utilizing a constriction in an air duct.

Find:
Design an operable atomizer.

SOLUTION

Assume the bottom of the tube through which water will be drawn is 12.5 cm. below the neck of the atomizer. Therefore if the atomizer is to operate at all, the pressure in the necked down portion must be low enough to draw water 12.5 cm. up the tube. In other words p_{neck} must be $-(0.125)\gamma_{\text{water}} = -1226$ Pa. Let the outlet diameter of the atomizer be 1.25 cm. and the neck diameter be 0.625 cm. Assume that the change in area from neck to outlet is gradual enough to prevent separation so that the Bernoulli equation will be valid between these sections. Thus

$$p_n + \frac{\rho V_n^2}{2} = p_0 + \frac{\rho V_0^2}{2}$$

were n and 0 refer to the neck and outlet sections respectively. But

$$p_n = -1226 \text{ Pa and } p_0 = 0$$

or

$$-1226 + \frac{\rho V_n^2}{2} = \frac{\rho V_0^2}{2} \tag{1}$$

$$\begin{aligned} V_n A_n &= V_0 A_0 \\ V_n &= \frac{V_0 A_0}{A_n} \\ &= V_0 \left(\frac{1.25}{0.625}\right)^2 \\ V_n &= 4V_0 \end{aligned} \tag{2}$$

Eliminate V_n between Eqs. (1) and (2)

$$\begin{aligned} -1226 + \frac{\rho(4V_0)^2}{2} &= \frac{\rho V_0^2}{2} \\ -1226 + \frac{16\rho V_0{}^2}{2} &= \frac{\rho V_0^2}{2} \\ \frac{15\rho V_0{}^2}{2} &= 1226 \\ V_0 &= \left(\frac{2452/15}{\rho}\right)^{1/2} \end{aligned}$$

Assume $\rho = 1.27$ kg/m^3

$$\begin{aligned}
V_0 &= \left(\frac{2452/15}{1.27}\right)^{1/2} \\
&= 11.3 \text{ m/s} \\
Q &= VA = (11.3\,\text{m/s})\,(\pi/4)(0.0125\,\text{m})^2 \\
&= 1.39 \times 10^{-3} \text{ m}^3\text{/s} \\
&= 0.083 \text{ m}^3\text{/min}
\end{aligned}$$

One could use a vacuum cleaner (one that you can hook the hose to the discharge end) to provide the air source for such an atomizer.

5.100: PROBLEM DEFINITION

Situation:

A suction device based on a venturi lifts objects submerged in water.
$A_e = 10^{-3}\,\text{m}^2$, $A_t = 0.25A_e$, $A_s = 0.1\,\text{m}^2$.

Find:

(a) Velocity of water at exit for maximum lift.
(b) Discharge.
(c) Maximum load supportable by suction cup.

Properties:

Water (15 °C) Table A.5: $p_v = 1,700\,\text{Pa}$, $\rho = 999\ \text{kg/m}^3$.
$p_{atm} = 100\,\text{kPa}$.

PLAN

Apply the Bernoulli equation and the continuity equation.

SOLUTION

Venturi exit area, $A_e = 10^{-3}\ \text{m}^2$, Venturi throat area, $A_t = (1/4)A_e$, Suction cup area, $A_s = 0.1\ \text{m}^2$

$$\begin{aligned} p_{\text{atm}} &= 100\ \text{kPa} \\ T_{\text{water}} &= 15^\circ\ \text{C} \end{aligned}$$

Bernoulli equation for the Venturi from the throat to exit with the pressure at the throat equal to the vapor pressure of the water. This will establish the maximum lift condition. Cavitation would prevent any lower pressure from developing at the throat.

$$\frac{p_v}{\gamma} + \frac{V_t^2}{2g} + z_t = \frac{p_e}{\gamma} + \frac{V_{e\max}^2}{2g} + z_e \tag{1}$$

Continuity equation

$$\begin{aligned} V_t A_t &= V_e A_e \\ V_t &= V_e \frac{A_e}{A_t} \\ V_t &= 4V_e \end{aligned} \tag{2}$$

Then Eq. (1) can be written as

$$\begin{aligned} \frac{1,700}{\gamma} + \frac{(4V_{e\max})^2}{2g} &= \frac{100,000}{\gamma} + \frac{V_{e\max}^2}{2g} \\ V_{e\max} &= \left[\left(\frac{1}{15}\right)\left(\frac{2g}{\gamma}\right)(98,300)\right]^{1/2} \\ &= \left[\left(\frac{1}{15}\right)\left(\frac{2}{\rho}\right)(98,300)\right]^{1/2} \\ &\boxed{V_{e\max} = 3.62\ \text{m/s}} \end{aligned}$$

$$
\begin{aligned}
Q_{\max} &= V_e A_e \\
&= (3.62 \text{ m/s})(10^{-3}\,\text{m}^2) \\
&\boxed{Q_{\max} = 0.00362 \text{ m}^3/\text{s}}
\end{aligned}
$$

Find pressure in the suction cup at the level of the suction cup.

$$
\begin{aligned}
p_t + \gamma \Delta h &= p_{\text{suction}} \\
p_{\text{suction}} &= 1,700 \text{ Pa} + 9,800\,\text{N/ m}^3 \times 2\,\text{m} \\
&= 21,300 \text{ Pa}
\end{aligned}
$$

But the pressure in the water surrounding the suction cup will be $p_{\text{atm}} + \gamma \times 1 = (100 + 9.80)$ kPa, or

$$
\begin{aligned}
p_{\text{water}} - p_{\text{suction}} &= (109,800 - 21,300) \text{ Pa} \\
&= 88,500 \text{ Pa}
\end{aligned}
$$

Thus the maximum lift will be:

$$
\begin{aligned}
\text{Lift}_{\max} &= \Delta p A_s = (p_{\text{water}} - p_{\text{suction}}) A_s \\
&= (88,500 \text{ N/m}^2)(0.1 \text{ m}^2) \\
&\boxed{\text{Lift}_{\max} = 8,850 \text{ N}}
\end{aligned}
$$

5.101: PROBLEM DEFINITION

Situation:

A hovercraft is supported by air pressure.
$l = 4.5\,\text{m}$, $w = 2.1\,\text{m}$, $W = 8.9\,\text{kN}$,

Find:

Air flow rate necessary to support the hovercraft.

Assumptions:

Air is incompressible.
Steady flow.
Viscous effects are negligible.
Air in the chamber is at stagnation conditions ($V = 0,\ p =$ uniform)
Just under the skirt $p = p_{\text{atm}}$

Properties:

Air $(T = 15.5\,°\text{C},\ p = 1\,\text{atm})$, $\rho = 1.22\,\text{kg/m}^3$, Table A.3.

PLAN

Because flow rate is the goal, apply $Q = VA$. The steps are:
1. Find the pressure in the chamber by apply force equilibrium in the vertical direction.
2 Find V by applying the Bernoulli equation from inside the chamber to just under the skirt.
3. Apply the flow rate equation.

SOLUTION

1. Force equilibrium (vertical direction)

$$\begin{aligned} \Delta pA &= W \\ \Delta p &= \frac{W}{A} = \frac{8.9\text{ kN}}{(4.5\times 2.1)\text{ m}^2} = 942\,\text{N/m}^2 \end{aligned}$$

2. Bernoulli equation (elevations terms are neglected; point 1 is in the chamber; point 2 is underneath the skirt)

$$\begin{aligned} p_1 + \rho\frac{V_1^2}{2} &= p_2 + \rho\frac{V_2^2}{2} \\ \left(942\,\text{N/m}^2\right) + 0 &= 0 + \left(1.22\,\text{kg/m}^3\right)\frac{V_2^2}{2} \\ V_2 &= \sqrt{\frac{2\left(942\,\text{N/m}^2\right)}{\left(1.22\,\text{kg/m}^3\right)}} = 39.3\,\text{m/s} \end{aligned}$$

3. Flow rate equation

$$Q = VA = (39.3\,\text{m/s})\,(0.075\,\text{m})\,(9\,\text{m} + 4.2\,\text{m}) = 39\,\text{m}^3/\,\text{s}$$

$$\boxed{Q = 2340\ \text{m}^3/\,\text{min}}$$

5.102: PROBLEM DEFINITION

Situation:

Water forced out of a cylinder by a piston.
$d = 0.05\,\text{m}$, $D = 0.1\,\text{m}$, $V = 1.8\,\text{m/s}$.

Find:

Efflux velocity and force required to drive piston.

Properties:

$T = 15.5\,^\circ\text{C}$.

PLAN

Apply the Bernoulli equation and the continuity equation.

SOLUTION

Continuity equation

$$\begin{aligned} V_1 A_1 &= V_2 A_2 \\ V_2 &= V_1 \left(\frac{D}{d}\right)^2 = 1.8 \times \left(\frac{0.1\,\text{m}}{0.05\,\text{m}}\right)^2 \\ &\boxed{V_2 = 7.2\,\text{m/s}} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} &= \frac{V_2^2}{2g} \\ p_1 &= \frac{\rho}{2}(V_2^2 - V_1^2) \\ &= \frac{1000\,\text{kg/m}^3}{2}\left[(7.2\,\text{m/s})^2 - (1.8\,\text{m/s})^2\right] \\ &= 24,300\ \text{Pa} \end{aligned}$$

Then

$$\begin{aligned} F_{\text{piston}} &= p_1 A_1 = 24,300\ \text{Pa} \times (\pi/4) \times (0.1\,\text{m})^2 \\ &\boxed{F = 191\,\text{N}} \end{aligned}$$

5.103: PROBLEM DEFINITION

Situation:

Air flows through constant area, heated pipe.
$D = 10\,\text{cm}$, $V = 10\,\text{m/s}$.
$p_2 = 80\,\text{kPa}$, $p_1 = 100\,\text{kPa}$.

Find:

Velocity at exit.

Determine if the Bernoulli equation be used to relate the pressure and velocity changes.

Properties:

$T_1 = 20\,^\circ\text{C}$, $T_2 = 50\,^\circ\text{C}$.

PLAN

Apply the continuity equation.

SOLUTION

The flow is steady so the continuity equation for constant area pipe yields

$$\begin{aligned} \rho_1 V_1 &= \rho_2 V_2 \\ V_2 &= V_1 \left(\frac{\rho_1}{\rho_2} \right) \end{aligned}$$

From ideal gas law

$$\frac{\rho_1}{\rho_2} = \frac{p_1}{p_2} \frac{T_2}{T_1}$$

so

$$\begin{aligned} V_2 &= 10 \text{ m/s} \frac{100 \text{ kPa}}{80 \text{ kPa}} \times \frac{323 \text{ K}}{293 \text{ K}} \\ & \boxed{V_2 = 13.8\,\text{m/s}} \end{aligned}$$

The Bernoulli equation is not applicable because the density is not constant.

5.104: PROBLEM DEFINITION

Situation:

On a hot day a fuel pump can cavitate.

Find:

What is happening to the gasoline?
How does this affect pump operation?

Properties:

$p_2 = 80\,\text{kPa}$, $p_1 = 100\,\text{kPa}$.

SOLUTION

Sometimes driving your car on a hot day, you may encounter a problem with the fuel pump called pump cavitation. What is happening to the gasoline?
The temperature of a hot day causes the vapor pressure to increase. The high fluid velocities in a pump can cause the pressure to decrease to the point that cavitation occurs.

How does this affect the operation of the pump?
At this point, the pump designed to pump liquid is no longer effective.

5.105: PROBLEM DEFINITION

Situation:
Cavitation.

Find:
What is cavitation?
Why does tendency for cavitation in a liquid increase with temperature?

SOLUTION

What is cavitation?
Cavitation occurs when the liquid pressure reaches the vapor pressure and local boiling occurs.

Why does the tendency for cavitation in a liquid increase with increased temperatures?
The tendency of cavitation to increase with temperature is the result of the vapor pressure increasing with temperature.

5.106: PROBLEM DEFINITION

Situation:
The following questions have to do with cavitation.

Find:
a. Is it more correct to say that cavitation has to do with
i) vacuum pressures, or
ii) vapor pressures?

SOLUTION

It is most correct to say that cavitation has to do with vapor pressures. Vapor pressure is a function of the ambient temperature. It may a occur at a pressure that is not a vacuum with respect to atmospheric pressure at that location. For example, boiling of water occurs at atmospheric pressure (in an open pot) at 100 °C.

Find:
b. Is cavitation more likely to occur on the low pressure (suction) side of a pump, or the high pressure (discharge) side? Why?

SOLUTION

Cavitation is more likely to occur on the suction side of a pump. When the pump adds energy to the water, continuity still must be satisfied, so $Q_{in} = Q_{out}$, which is also $A_1V_1 = A_2V_2$. Therefore, both the upstream and downstream velocities may be very high. According to the energy equation, before the pump, if V_1 is large, then P_1 will be small. P_1 may be reduced to the vapor pressure.

Find:
c. What does the word cavitation have to do with cavities, like the ones we get in our teeth? Is this aspect of cavitation the
(i) cause, or the
(ii) result of the phenomenon?

SOLUTION

Cavitation can cause small holes, or cavities, to be knocked out of the surface of structures that are adjacent to where it occurs. These small pits grow larger and larger if the cavitation continues, in the following manner. When cavitation occurs, small bubbles of vapor are created. As soon as the fluid containing such bubbles is swept into a location of slightly higher pressure, the bubbles implode again. When the bubbles implode, a shock wave is created that can pit the surface of adjacent structures, such as the metal of a pump, or the concrete walls of a dam. Therefore the cavities, are (ii) the result of the phenomenon, not the cause.

Find:

d. When water goes over a waterfall, and one can see lots of bubbles in the water, is that due to cavitation?

Why, or why not?

SOLUTION

When water goes over a waterfall, the bubbles you see are entrained air. Cavitation does not occur here because the temperatures are not high, nor are the pressures low (conditions that would be required to cause the vapor pressure to be reached).

5.107: PROBLEM DEFINITION

Situation:
Cavitation in a venturi section.
$D = 50\,\text{cm}$, $d = 10\,\text{cm}$.

Find:
Discharge for incipient cavitation.

Properties:
Water ($10\,^\circ\text{C}$), Table A.5: $\rho = 1000\,\text{kg/m}^3$.
$p_A = 130\,\text{kPa}$, $p_{atm} = 100\,\text{kPa}$.

PLAN

Apply the continuity equation and the Bernoulli equation.

SOLUTION

Cavitation will occur when the pressure reaches the vapor pressure of the liquid ($p_V = 1,230$ Pa abs).
Bernoulli equation

$$p_A + \frac{\rho V_A^2}{2} = p_{\text{throat}} + \frac{\rho V_{\text{throat}}^2}{2}$$

where $V_A = Q/A_A = Q/((\pi/4) \times 0.50^2)$
Continuity equation

$$V_{\text{throat}} = \frac{Q}{A_{\text{throat}}} = \frac{Q}{\pi/4 \times (0.10\,\text{m})^2}$$

$$\frac{\rho}{2}(V_{\text{throat}}^2 - V_A^2) = p_A - p_{\text{throat}}$$

$$\frac{\rho Q^2}{2}\left[\frac{1}{((\pi/4) \times (0.10\,\text{m})^2)^2} - \frac{1}{((\pi/4) \times (0.50\,\text{m})^2)^2}\right] = 230,000\,\text{Pa} - 1,230\,\text{Pa}$$

$$500Q^2(16,211 - 25) = 228,770\,\text{Pa}$$

$$\boxed{Q = 0.168\ \text{m}^3/\text{s}}$$

5.108: PROBLEM DEFINITION

Situation:

A sphere moves below the surface in water.
$D = 0.3\,\mathrm{m}$, $h = 3.6\,\mathrm{m}$.

Find:

Speed at which cavitation occurs.

Properties:

Water (10 °C), Table A.5: $\rho = 1000\,\mathrm{kg/m^3}$.

PLAN

Apply the Bernoulli equation between the free stream and the maximum width.

SOLUTION

Let p_o be the pressure on the streamline upstream of the sphere. The minimum pressure will occur at the maximum width of the sphere where the velocity is 1.5 times the free stream velocity.
Bernoulli equation

$$p_o + \frac{1}{2}\rho V_o^2 + \gamma h_o = p + \frac{1}{2}\rho(1.5V_o)^2 + \gamma(h_o + 0.15)$$

Solving for the pressure p gives

$$p = p_o - 0.625\rho V_o^2 - 0.15\gamma$$

The pressure at a depth of 3.6 m is 35,862 $\mathrm{N/m^2}$. The density of water is 1000 $\mathrm{kg/m^3}$ and the specific weight is 9810 $\mathrm{N/m^3}$. At a temperature of 10 °C, the vapor pressure is 1.23 kPa. Substituting into the above equation

$$\begin{aligned} 1230 \text{ Pa} &= 35,862 \text{ Pa} - (0.625)\,(1000)\,V_o^2 - (0.15)\,(9810) \\ 33,160 &= 625V_o^2 \end{aligned}$$

Solving for V_o gives

$$\boxed{V_o = 7.3 \text{ m/s}}$$

5.109: PROBLEM DEFINITION

Situation:
A hydrofoil is tested in water.
$h = 1.8\,\text{m}$, $V = 8\,\text{m/s}$.

Find:
Speed that cavitation occurs.

Assumptions:
$p_{\text{atm}} = 101$ kPa abs; $p_{\text{vapor}} = 1,230$ Pa abs.

Properties:
$T = 10\,^\circ\text{C}$, $p_0 = 70\,\text{kPa}$.

PLAN

Consider a point ahead of the foil (at same depth as the foil) and the point of minimum pressure on the foil, and apply the pressure coefficient definition between these two points.

SOLUTION

Pressure coefficient

$$C_p = \frac{(p_{\text{min}} - p_0)}{\rho V_0^2/2}$$

where

$$\begin{aligned} p_0 &= p_{\text{atm}} + 1.8\gamma = 101,000\,\text{Pa} + 1.8\,\text{m} \times 9,810\,\text{N/m}^3 = 118,658 \text{ Pa abs.} \\ p_{\text{min}} &= 70,000 \text{ Pa abs; } V_0 = 8 \text{ m/s} \end{aligned}$$

Then

$$C_p = \frac{70,000\,\text{Pa} - 118,658\,\text{Pa}}{500 \times (8\,\text{m/s})^2} = -1.521$$

Now use $C_p = -1.521$ (constant) for evaluating V for cavitation where p_{min} is now p_{vapor}:

$$-1.521 = \frac{(1,230\,\text{Pa} - 118,658\,\text{Pa})}{(1,000/2)V_0^2}$$

$$\boxed{V_0 = 12.4 \text{ m/s}}$$

5.110: PROBLEM DEFINITION

Situation:
A hydrofoil is tested in water.
$h = 3\,\mathrm{m}$, $V = 8\,\mathrm{m/s}$.

Find:
Speed that cavitation begins.

Properties:
$T = 10\,^\circ\mathrm{C}$, $p_0 = 70\,\mathrm{kPa}$.

PLAN

Same solution procedure applies as in Prob. 5.98.

SOLUTION

From the solution to Prob. 5.98, we have the same C_p, but $p_0 = 101,000 + 3\gamma = 130,430$. Then:

$$-1.521 = \frac{1,230\,\mathrm{Pa} - 130,430\,\mathrm{Pa}}{(1,000/2)V_0^2}$$

$$\boxed{V_0 = 13.0 \text{ m/s}}$$

5.111: PROBLEM DEFINITION

Situation:

A hydrofoil is tested in water.
$h = 1.2\,\text{m}$, $V = 7.5\,\text{m/s}$.

Find:

Speed that cavitation begins.

Properties:

$p_0 = 17$ kPa vacuum.
Water (10 °C) Table A.5, $p_v = 1230$ N/m^2.

PLAN

Consider a point ahead of the foil (at same depth as the foil) and the point of minimum pressure on the foil, and apply the pressure coefficient definition between these two points.

SOLUTION

$$\begin{aligned} p_{\text{min}} &= -17,000 \text{ Pa gage} \\ p_0 &= 1.2\gamma = 1.2 \times 9810 = 11,772 \text{ Pa} \end{aligned}$$

Then

$$\begin{aligned} C_p &= \frac{(p_{\text{min}} - p_0)}{\rho V_0^2/2} = \frac{(-17,000 - 11,772)\,\text{Pa}}{(1000\,\text{kg/m}^3/2) \times (7.5\,\text{m/s})^2} \\ C_p &= -1.02 \end{aligned}$$

Now let $p_{\text{min}} = p_{\text{vapor}} = 1230$ N/m^2
Then

$$-1.02 = \frac{1230\,\text{Pa} - 11,772\,\text{Pa}}{(1000\,\text{kg/m}^3/2)V_0^2}$$

$$\boxed{V_0 = 4.55 \text{ m/s}}$$

5.112: PROBLEM DEFINITION

Situation:
A hydrofoil is tested in water.
$h = 3\,\text{m}$, $V = 7.5\,\text{m/s}$.

Find:
Speed that cavitation begins when depth is 3 m.

Properties:
$T = 10\,^\circ\text{C}$, $p_0 = 17$ kPa vacuum.

PLAN Same solution procedure applies as in Prob. 5.111 in EFM10e.

SOLUTION From solution of Prob. 5.111 in EFM10e we have $C_p = -1.02$ but now $p_0 = 3\gamma = 29,430$ Pa. Then:

$$-1.02 = -\frac{29,430 \text{ Pa} + 100,070 \text{ Pa}}{(1000/2)V_0^2}$$

$$\boxed{V_0 = 15.9 \text{ m/s}}$$

5.113: PROBLEM DEFINITION

Situation:
A sphere moving in water.
$V = 1.5V_0$.

Find: Speed at which cavitation occurs.

Properties:
Water (10 °C) Table A.5: $p_v = 1230$ N/m^2, $\gamma = 9810$ N/m^3.
$p_0 = 124$ kPa.

PLAN

Apply the Bernoulli equation between a point in the free stream to the 90° position where $V = 1.5V_0$. The free stream velocity is the same as the sphere velocity (reference velocities to sphere).

SOLUTION

Bernoulli equation

$$\begin{aligned}
\frac{\rho V_0^2}{2} + p_0 &= p + \frac{\rho(1.5V_0)^2}{2} \\
\text{where } p_0 &= 124 \text{ kPa} \\
\frac{\rho V_0^2(2.25-1)}{2} &= (124,000 \text{ Pa} - 1230 \text{ Pa}) \\
V_0^2 &= \frac{2(122,770 \text{ N/m}^2)}{(1.25)1000 \text{ kg/m}^2} \\
&\boxed{V_0 = 14 \text{ m/s}}
\end{aligned}$$

5.114: PROBLEM DEFINITION

Situation:

A cylinder is moving in water.
$h = 1\,\text{m}$, $V_0 = 5\,\text{m/s}$.

Find:

Velocity at which cavitation occurs.

Properties:

Water (10 °C) Table A.5 $p_v = 1,230$ Pa, $\rho = 1000\,\text{kg/m}^3$.
$p_0 = 80\,\text{kPa}$, $p_{atm} = 100\,\text{kPa}$.

PLAN

Apply the definition of pressure coefficient.

SOLUTION

Pressure coefficient

$$\begin{aligned}
C_p &= \frac{(p - p_0)}{(\rho V_0^2/2)} \\
p_0 &= 100,000\,\text{Pa} + 1 \times 9,810\ \text{Pa} = 109,810\ \text{Pa} \\
p &= 80,000\ \text{Pa} \\
C_p &= \frac{(80,000 - 109,810)\,\text{Pa}}{\left[(1000\,\text{kg/m}^3)\,(5\,\text{m/s})^2/2\right]} \\
C_p &= -2.385
\end{aligned}$$

For cavitation to occur $p = 1,230$ Pa

$$\begin{aligned}
-2.385 &= (1,230 - 109,810)/(1,000V_0^2/2) \\
&\boxed{V_0 = 9.54\ \text{m/s}}
\end{aligned}$$