6.1: PROBLEM DEFINITION

Situation:
Identify the surface and body forces acting on a glider in flight. Also, sketch a free body diagram and explain how Newton's laws of motion apply.

Find:
Surface and body forces acting on a glider in flight.

PLAN

Make use of a sketch with a free-body diagram.

SOLUTION

The forces acting on glider in flight are:

Surface forces:
1. Lift - a surface force because the wing must touch the air to generate lift.
2. Drag - both form drag and friction drag result from air touching the glider.
3. Thrust - the air and propellors must touch in order to generate thrust from the engine's energy.

Body force:
1. Weight - the pull of the earth on the glider through the action of a gravity field. Earth and the glider do not need to touch in order for the force of gravity to act, so weight is a body force

Sketch:
An acceptable sketch would be Fig. 6.2 in §6.1 of EFM10e.

Newton's Laws:
Newton's first law (i.e. force equilibrium) tells us that the sum of forces must balance. Therefore the glider stays at the same elevation when lift is equal to weight. The glider stays at the same speed when thrust balances drag. For the glider to accelerate, thrust must be greater than drag.

6.2: PROBLEM DEFINITION

Situation:

Interpretation of Newton's second law.

$F = \frac{d(mv)}{dt}$, $F = m\frac{dv}{dt} + v\frac{dm}{dt}$

Find:

Relationship between momentum and acceleration.

SOLUTION

Expressing Newton's second law as

$$F = \frac{d}{dt}(mv)$$

is correct. However, Newton's second law is valid only for a system of constant mass so differentiation yields

$$F = m\frac{dv}{dt}$$

In the differentiation by parts, the dm/dt term is zero.

6.3: PROBLEM DEFINITION

Situation:

Which of the following are correct with respect to the derivation of the Momentum Equation? (Select all that apply.)

a. Reynold's Transport Theorem is applied to Fick's Law.
b. The extensive property is momentum.
c. The intensive property is mass.
d. The velocity is assumed to be uniformly distributed across each inlet and outlet.
e. The net momentum flow is the "ins" minus the "outs".
f. The net force is the sum of forces acting on the matter inside the CV

SOLUTION

The correct statements are b, d and f.

6.4: PROBLEM DEFINITION

Situation:

When making a force diagram (FD) and its partner momentum diagram (MD) in order to set up the equations for a momentum equation problem (see Fig. 6.10 in §6.3), which of the following elements should be in the FD, and which should be in the MD?

(Classify all below as either FD or MD.)

a. Each mass stream with product $\dot{m}_o v_o$ or product $\dot{m}_i v_i$ crossing a control surface boundary.

b. Reaction forces required to hold walls, vanes, or pipes in place.

c. Weight of a solid body that contains or contacts the fluid.

d. Weight of the fluid.

e. Pressure force caused by a fluid flowing across a control surface boundary.

SOLUTION

a. All products of the form $\dot{m}v$ should be in the MD
b. Reaction forces should be in the FD
c. Solid body weights should be in the FD if they must be borne by a reaction force
d. Fluid weight should be in the FD (if significant)
e. Pressure forces should be in the FD

6.5: PROBLEM DEFINITION

Situation:
Examples of jets and how they are used in practice.

Find:
Give 5 examples of jets and applications.

SOLUTION

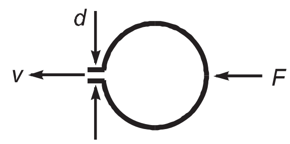
1. Water jet from a fire hose - fire suppression
2. Ink jet in a printer - produce ink letters on page
3. High pressure water jet - used from cutting in manufacturing
4. Jet engine nozzle - produce thrust
5. Nozzle on lawn sprinkler - used to distribute water for agricultural needs.

6.6: PROBLEM DEFINITION

Situation:
A balloon is held stationary by a force F.
$d = 8\,\text{mm}$, $v = 45\,\text{m/s}$.

Find: Force required to hold balloon stationary (N).

Sketch:



Assumptions:
Steady flow, constant density.

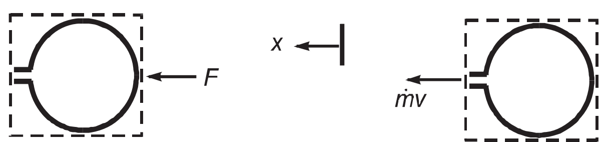
Properties:
$\rho = 1.2\,\text{kg/m}^3$.

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams (x-direction terms)



Momentum equation (x-direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ F &= \dot{m}v \\ &= \rho A v^2 \\ &= (1.2)\left(\frac{\pi \times 0.008^2}{4}\right)(45^2)\end{aligned}$$

$$\boxed{F = 0.122\ N}$$

6.7: PROBLEM DEFINITION

Situation:

A balloon is held stationary by a force.

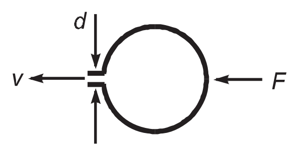
$d = 1$ cm, $p = 20$ cm H_2O.

Find:

x-component of force required to hold balloon stationary (N).

Exit velocity (m/s).

Sketch:



Assumptions:

Steady, irrotational, constant density flow.

Properties:

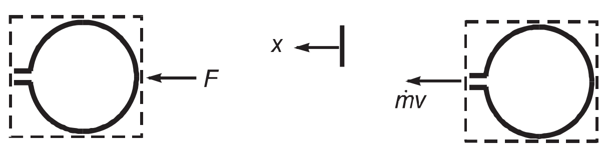
$\rho = 1.2\,\mathrm{kg/\,m^3}$.

PLAN

To find the exit velocity, apply the Bernoulli equation. To find the force, apply the momentum equation.

SOLUTION

Force and momentum diagrams (x-direction terms)



Bernoulli equation applied from inside the balloon to nozzle exit

$$
\begin{aligned}
p &= 20 \text{ cm } H_2O = 1960\,\mathrm{Pa} \\
\frac{p}{\rho} &= \frac{v^2}{2} \\
v &= \sqrt{\frac{2p}{\rho}} = \sqrt{\frac{2 \times 1960\,\mathrm{Pa}}{1.2\,\mathrm{kg/\,m^3}}}
\end{aligned}
$$

$$\boxed{v = 57.15 \ \mathrm{m/\,s}}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
F &= \dot{m} v = \rho A v^2 = \left(1.2\,\mathrm{kg/\,m^3}\right)\left(\frac{\pi}{4} \times (0.01\,\mathrm{m})^2\right)(57.15\,\mathrm{m/\,s})^2 \\
&\boxed{F = 0.31\,\mathrm{N}}
\end{aligned}$$

6.8: PROBLEM DEFINITION

Situation:

For Example 6.2 in §6.4, the situation diagram shows concrete being "shot" at an angle into a cart that is tethered by a cable, and sitting on a scale. Determine whether the following two statements are "true" or "false."

a. Mass is being accumulated in the cart.
b. Momentum is being accumulated in the cart.

SOLUTION

a. True
b. False

6.9: PROBLEM DEFINITION

Situation:

A water jet is filling a tank.

$m = 25\,\text{kg}$, $\forall = 25$ L.

$d = 30$ mm, $v = 25$ m/s.

Find:

Force on the bottom of the tank (N).

Force acting on the stop block (N).

Assumptions:

Steady flow.

Properties:

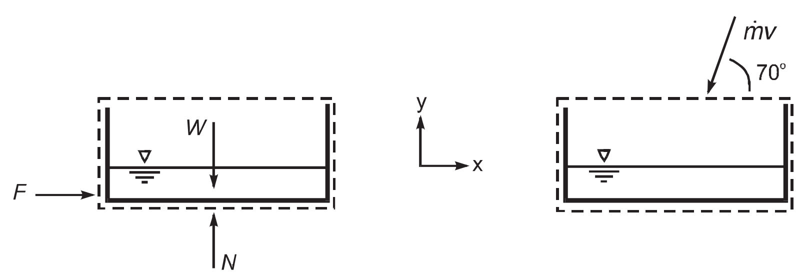
Water (15 °C), Table A.5: $\rho = 999\,\text{kg/m}^3$, $\gamma = 9800\,\text{N/m}^3$.

PLAN

Apply the momentum equation in the x-direction and in the y-direction.

SOLUTION

Force and momentum diagrams



Momentum equation (x-direction)

$$\sum F_x = \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix}$$
$$F = -(-\dot{m}v \cos 70^o)$$
$$= \rho A v^2 \cos 70^o$$

Calculations

$$\rho A v^2 = \left(999\,\text{kg/m}^3\right)\left(\frac{\pi \times (0.03\,\text{m})^2}{4}\right)(25\,\text{m/s})^2$$
$$= 441.3 \text{ N}$$

$$F = (441.3 \text{ N})(\cos 70^o)$$
$$= 150.9\,\text{N}$$

$$\boxed{F = 150.9\,\text{N pushing to the left on the stop block}}$$

y-direction

$$\begin{aligned}
\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\
N - W &= -(-\dot{m} v \sin 70^o) \\
N &= W + \rho A v^2 \sin 70^o
\end{aligned}$$

Calculations:

$$\begin{aligned}
W &= W_{\text{tank}} + W_{\text{water}} \\
&= (25\,\text{kg})\left(9.81\,\text{m/s}^2\right) + (0.025\,\text{m}^3)(9800\,\text{N/m}^3) \\
&= 490.3\ \text{N}
\end{aligned}$$

$$\begin{aligned}
N &= W + \rho A v^2 \sin 70^o \\
&= (490.3\ \text{N}) + (441.3\ \text{N}) \sin 70^o
\end{aligned}$$

$$\boxed{N = 905\ N \text{ acting upward}}$$

6.10: PROBLEM DEFINITION

Situation:

Water jet is filling a tank.
$m = 12\,\text{kg} = 118\,\text{N}$, $V\!\!\!/ = 20\,\text{liters} = 0.02\,\text{m}^3$.
$d = 0.05$ m, $v = 18$ m/s.

Find:

Minimum coefficient of friction so force on stop block is zero.

Assumptions:

Steady flow, constant density, steady and irrotational flow.

Properties:

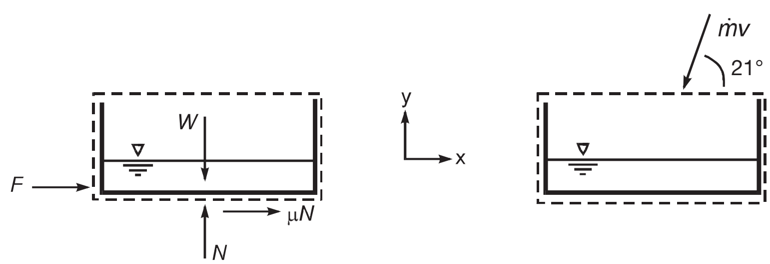
Water (21 °C), Table A.5: $\rho = 1000\,\text{kg/m}^3$, $\gamma = 9810\,\text{N/m}^3$

PLAN

Apply the momentum equation in the x- and y-directions.

SOLUTION

Force and momentum diagrams



Momentum equation (y-direction)

$$\begin{aligned}\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\ N - W &= -(-\dot{m} v \sin 21^\circ) \\ N &= W + \rho A v^2 \sin 21^\circ\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}\mu N &= -(-\dot{m} v \cos 21^\circ) = \rho A v^2 \cos 21^\circ \\ \mu &= \frac{(\rho A v^2 \cos 21^\circ)}{N}\end{aligned}$$

Calculations

$$
\begin{aligned}
\rho A v^2 &= \left(1000\,\mathrm{kg/m^3}\right)\left(\pi \times \frac{(0.05^2\,\mathrm{m}^2)}{4}\right)(18\,\mathrm{m/s})^2 \\
&= 636\ \mathrm{N} \\
W_{H20} &= \gamma \forall \\
&= (9810\,\mathrm{N/m^3})(0.02\,\mathrm{m^3}) \\
&= 196\ \mathrm{N} \\
W &= (196 + 118)\ \mathrm{N} \\
&= 314\ \mathrm{N} \\
N &= 314\,\mathrm{N} + 636\,\mathrm{N} \times \sin 21^\circ \\
&= 542\ \mathrm{N} \\
\mu &= \frac{636\,\mathrm{N} \times \cos 21^\circ}{542\,\mathrm{N}}
\end{aligned}
$$

$$\boxed{\mu = 1.095}$$

6.11: PROBLEM DEFINITION

Situation:

A design contest features a submarine powered by a water jet.

$V_{\text{sub}} = 1.0\,\text{m/s}$, $D_1 = 25\,\text{mm}$.

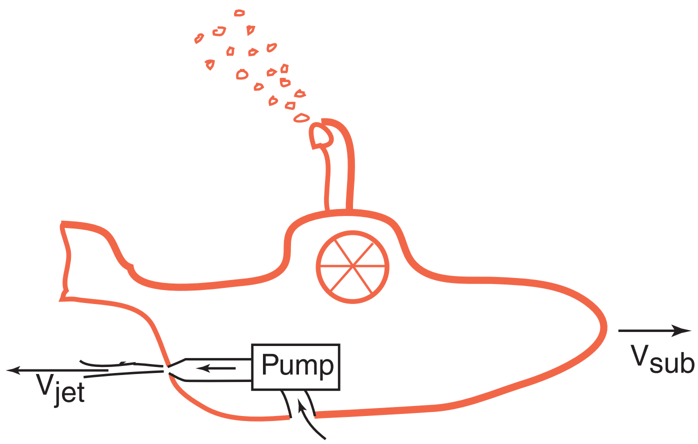
$D_2 = 5\,\text{mm}$, $F_D = C_D\left(\frac{\rho V_{\text{sub}}^2}{2}\right)A_p$

$C_D = 0.3$, $A_p = 0.28\,\text{m}^2$.

Find:

Speed of the fluid jet (m/ s).

Sketch:



Assumptions:

Assume steady flow so that the accumulation of momentum term is zero.

Properties:

Water (15 °C), Table A.5: $\rho = 999\,\text{kg/m}^3$.

PLAN

The speed of the fluid jet can be found from the momentum equation because the drag force will balance with the net rate of momentum outflow.

SOLUTION

Momentum equation. Select a control volume that surrounds the sub. Select a reference frame located on the submarine. Let section 1 be the outlet (water jet) and section 2 be the inlet. The momentum equation is

$$\begin{aligned}\sum \mathbf{F} &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\ F_{\text{Drag}} &= \dot{m}_2 v_2 - \dot{m}_1 v_{1x}\end{aligned}$$

By continuity, $\dot{m}_1 = \dot{m}_2 = \rho A_{\text{jet}} V_{\text{jet}}$. The outlet velocity is $v_2 = V_{\text{jet}}$. The x-component of the inlet velocity is $v_{1x} = V_{\text{sub}}$. The momentum equation simplifies to

$$F_{\text{Drag}} = \rho A_{\text{jet}} V_{\text{jet}} (V_{\text{jet}} - V_{\text{sub}})$$

The drag force is

$$\begin{aligned} F_{\text{Drag}} &= C_D \left(\frac{\rho V_{\text{sub}}^2}{2} \right) A_p \\ &= 0.3 \left(\frac{(999\,\text{kg/m}^3)\,(1.0\,\text{m/s})^2}{2} \right) (0.28\,\text{m}^2) \\ &= 42.0\,\text{N} \end{aligned}$$

The momentum equation becomes

$$\begin{aligned} F_{\text{Drag}} &= \rho A_{\text{jet}} V_{\text{jet}} \left[V_{\text{jet}} - V_{\text{sub}} \right] \\ 42.0\,\text{N} &= \left(999\,\text{kg/m}^3\right) \left(1.96 \times 10^{-5}\,\text{m}^2\right) V_{\text{jet}} \left[V_{\text{jet}} - (1.0\,\text{m/s}) \right] \end{aligned}$$

Solving for the jet speed gives

$$\boxed{V_{\text{jet}} = 46.8\,\text{m/s}}$$

REVIEW

1. The jet speed (46.8 m/s) is above 160 km/h. This presents a safety issue. Also, this would require a pump that can produce a large pressure rise.

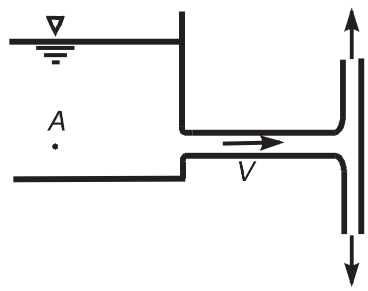
2. It is recommended that the design be modified to produce a lower jet velocity. One way to accomplish this goal is to increase the diameter of the jet.

6.12: PROBLEM DEFINITION

Situation:
Horizontal round jet strikes a plate.
$Q = 0.05$ m^3/s, $F_x = 900$ N.

Find:
Speed of water jet (m/s).

Sketch:



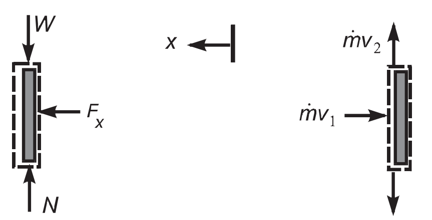
Properties:
Water (21°C), Table A.5: $\rho = 1000$ kg/m^3.

PLAN

Apply the momentum equation to a control volume surrounding the plate.

SOLUTION

Force and momentum diagrams



Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= -\dot{m}v_{1x} \\
F_x &= -(-\dot{m}v_1) = \rho Q v_1 \\
v_1 &= \frac{F_x}{\rho Q} \\
&= \frac{900\,\text{N}}{1000\,\text{kg/m}^3 \times 0.05\,\text{m}^3/\text{s}}
\end{aligned}$$

$$\boxed{v_1 = 18 \text{ m/s}}$$

6.13: PROBLEM DEFINITION

Situation:

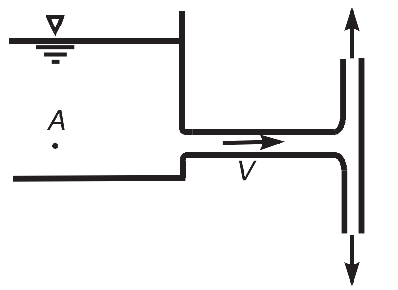
Horizontal round jet strikes a plate.

$F_x = 2.7$ kN

Find:

Diameter of jet (m).

Sketch:



Properties:

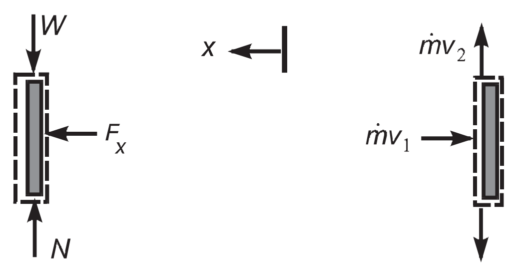
$p_A = 170$ kPa.

Water (21 °C), Table A.5: $\rho = 997.8$ kg/m^3.

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION



Force and momentum diagrams

Bernoulli equation applied from inside of tank to nozzle exit

$$\begin{aligned}
\frac{p_A}{\rho} &= \frac{v_1^2}{2} \\
v_1 &= \sqrt{\frac{2p_A}{\rho}} \\
&= \sqrt{\frac{2 \times 170 \text{ kPa}}{997.8\,\text{kg/m}^3}} \\
&= 18.5 \text{ m/s}
\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= -\dot{m}v_{1x} \\
F_x &= -(-\dot{m}v_1) = \rho A v_1^2 \\
A &= \frac{F_x}{\rho v_1^2} = \frac{2700\,\text{N}}{997.8\,\text{kg/m}^3 \times (18.5\,\text{m/s})^2} \\
A &= 0.0079 \text{ m}^2 \\
d &= \sqrt{\frac{4A}{\pi}} \\
&= \sqrt{\frac{4 \times 0.0079\,\text{m}^2}{\pi}}
\end{aligned}$$

$$\boxed{d = 0.1\,\text{m}}$$

6.14: PROBLEM DEFINITION

Situation:

An engineer is designing a toy to create a jet of water.

$D = 80\,\text{mm}$, $d = 15\,\text{mm}$.

$V_{\text{piston}} = 300\,\text{mm/s}$.

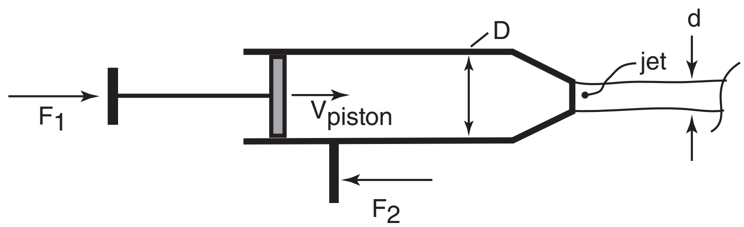
Find:

Which force (F_1 versus F_2) is larger? Explain your answer using concepts of the momentum equation.

Calculate F_1.

Calculate F_2.

Sketch:



Assumptions:

Neglect friction between the piston and the wall.

Assume the Bernoulli equation applies (neglect viscous effects; neglect unsteady flow effects).

Properties:

Table A.5 (water at 20 °C): $\rho = 998\,\text{kg/m}^3$.

PLAN

To find the larger force, recognize that the net force must be in the direction of acceleration. To solve the problem, apply the momentum equation, continuity equation, equilibrium equation, and the Bernoulli equation.

SOLUTION

Finding the larger force (F_1 versus F_2). Since the fluid is accelerating to the right the net force must act to the right. Thus, F_1 is larger than F_2. This can also be seen by application of the momentum equation.

Momentum equation (x-direction) applied to a control volume surrounding the toy.

$$\begin{aligned} \sum F_x &= \dot{m} v_{\text{out}} \\ F_1 - F_2 &= \dot{m} v_{\text{out}} \\ F_1 - F_2 &= \rho \left(\frac{\pi d^2}{4} \right) V_{\text{out}}^2 \qquad (1) \end{aligned}$$

Notice that Eq. (1) shows that $F_1 > F_2$.

Continuity equation applied to a control volume situated inside the toy.

$$\begin{aligned}
Q_{\text{in}} &= Q_{\text{out}} \\
\left(\frac{\pi D^2}{4}\right) V_{\text{piston}} &= \left(\frac{\pi d^2}{4}\right) V_{\text{out}} \\
V_{\text{out}} &= V_{\text{piston}} \frac{D^2}{d^2} \\
&= (0.3\,\text{m/s}) \left(\frac{80\,\text{mm}}{15\,\text{mm}}\right)^2 \\
V_{\text{out}} &= 8.533\,\text{m/s}
\end{aligned}$$

Bernoulli equation applied from inside the toy to the nozzle exit plane.

$$\begin{aligned}
p_{\text{inside}} + \frac{\rho V_{\text{piston}}^2}{2} &= \frac{\rho V_{\text{out}}^2}{2} \\
p_{\text{inside}} &= \frac{\rho \left(V_{\text{out}}^2 - V_{\text{piston}}^2\right)}{2} \\
&= \frac{(998\,\text{kg/m}^3)\left((8.533\,\text{m/s})^2 - (0.3\,\text{m/s})^2\right)}{2} \\
&= 36.33\,\text{kPa}
\end{aligned}$$

Equilibrium applied to the piston (the applied force F_1 balances the pressure force).

$$\begin{aligned}
F_1 &= p_{\text{inside}} \left(\frac{\pi D^2}{4}\right) \\
&= (36330\,\text{Pa}) \left(\frac{\pi\,(0.08\,\text{m})^2}{4}\right) \\
&\boxed{F_1 = 182.6\,\text{N}}
\end{aligned}$$

Momentum equation (Eq. 1)

$$\begin{aligned}
F_2 &= F_1 - \rho \left(\frac{\pi d^2}{4}\right) V_{\text{out}}^2 \\
&= 182.6\,\text{N} - \left(998\,\text{kg/m}^3\right) \left(\frac{\pi\,(0.015\,\text{m})^2}{4}\right) (8.533\,\text{m/s})^2 \\
&\boxed{F_2 = 169.8\,\text{N}}
\end{aligned}$$

REVIEW

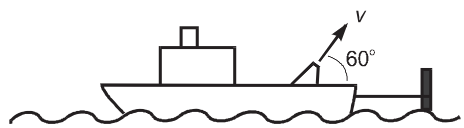
1. The force F_1 is only slightly larger than F_2.
2. The forces (F_1 and F_2) are each about 180 N. This magnitude of force may be too large for users of a toy. Or, this magnitude of force may lead to material failure (it breaks!). It is recommended that the specifications for this product be modified.

6.15: PROBLEM DEFINITION

Situation:
Water jet from a fire hose on a boat.
$d = 10$ cm, $V = 100$ km/h $= 27.8$ m/s.

Find:
Tension in cable (N).

Sketch:



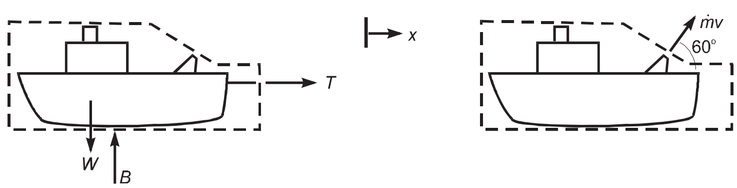
Properties:
Water (10 °C), Table A.5: $\rho = 1000\,\text{kg/m}^3$.

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



Flow rate

$$\begin{aligned}\dot{m} &= \rho A V \\ &= \left(1000\,\text{kg/m}^3\right)\left(\pi \times (0.05\,\text{m})^2\right)(27.8\,\text{m/s}) \\ &= 218.3\,\text{kg/s}\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}\sum F &= \dot{m}\,(v_o)_x \\ T &= \dot{m}V\cos 60^\circ \\ T &= (218.3\,\text{kg/s})(27.8\,\text{m/s})\cos 60^\circ \\ &= 3034.37\,\text{N}\end{aligned}$$

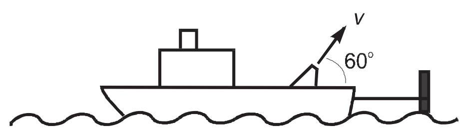
$$\boxed{T = 3034\ \text{N}}$$

6.16: PROBLEM DEFINITION

Situation:
Water jet from a fire hose on a boat.
$T = 5.0$ kN, $v = 50$ m/s.

Find:
Mass flow rate of jet (kg/s).
Diameter of jet (cm).

Sketch:



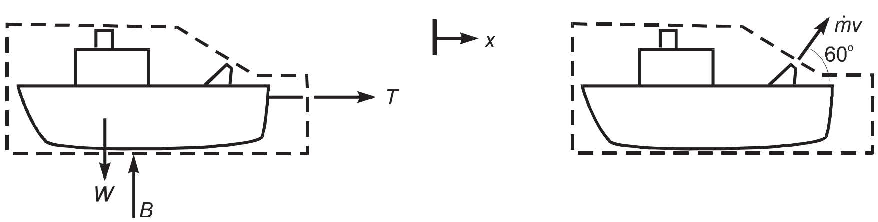
Properties:
Water (5 °C), Table A.5: $\rho = 1000\,\text{kg/m}^3$.

PLAN

Apply the momentum equation to find the mass flow rate. Then, calculate diameter using the flow rate equation.

SOLUTION

Force and momentum diagrams



Momentum equation (x-direction)

$$\begin{aligned} \sum F &= \dot{m}\,(v_o)_x \\ T &= \dot{m} v \cos 60^o \\ \dot{m} &= \frac{T}{v\cos 60^o} = \frac{5000\,\text{N}}{(50 \times \cos 60^o)\,\text{m/s}} \\ & \boxed{\dot{m} = 200\,\text{kg/s}} \end{aligned}$$

Flow rate

$$
\begin{aligned}
\dot{m} &= \rho A v = \frac{\rho \pi d^2 v}{4} \\
d &= \sqrt{\frac{4\dot{m}}{\rho \pi v}} \\
&= \sqrt{\frac{4 \times 200\,\mathrm{kg/\,s}}{1000\,\mathrm{kg/\,m^3} \times \pi \times 50\,\mathrm{m/\,s}}} \\
&= 7.136 \times 10^{-2}\,\mathrm{m}
\end{aligned}
$$

$$
\boxed{d = 7.14\,\mathrm{cm}}
$$

6.17: PROBLEM DEFINITION

Situation:
Piston water guns used to shoot water from one raft to another.
After volleys, rafts are drifting apart.
Water is ejected from piston water gun at a flow rate of 3.8 L/s.
Diameter of gun exit is 4 cm.

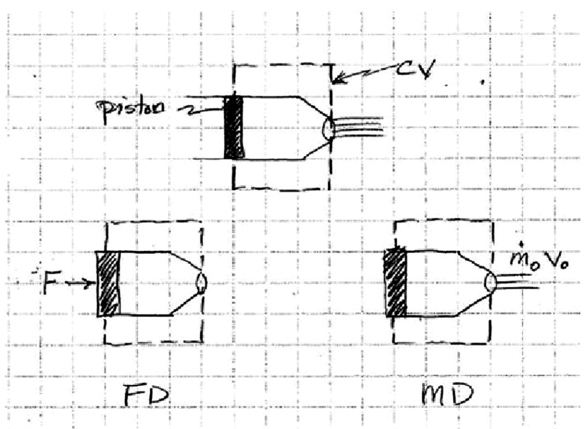
Find:
The momentum flux (N) generated by ejecting water from a water gun.

PLAN

a. Sketch a CV, a FD and a MD as preparation for employing the momentum equation.
b. Employ the momentum equation (also called the Impulse-Momentum Eqn).

SOLUTION

a. Sketch a CV, a FD and a MD.



b. Employ the Linear Momentum Equation.

Determine the velocity of the jet

$$\begin{aligned} V &= \frac{Q}{A} = \frac{4Q}{\pi d^2} \\ &= \left(\frac{3.8\,\mathrm{L}}{\mathrm{s}}\right)\left(0.001\,\mathrm{m}^3\right)\left(\frac{4}{\pi\left(0.04\,\mathrm{m}\right)^2\ \mathrm{m}^2}\right) \\ &= 3.02\,\mathrm{m/s} \end{aligned}$$

Determine the mass flux, $\dot{m} = \rho Q$

$$\begin{aligned}\dot{m} &= \rho Q \\ &= \left(\frac{1000\,\text{kg}}{\text{m}^3}\right) 0.0038\,\text{m}^3/\,\text{s} \\ &= 3.8\,\text{kg}/\,\text{s}\end{aligned}$$

Using the FD and the MD, we set up the following equation, where F = force of the person compressing the piston.

$$\begin{aligned}\sum \mathbf{F} &= \dot{m}_o \mathbf{v}_o \\ F &= \left(\frac{3.8\,\text{kg}}{\text{s}}\right)\left(\frac{3.02\,\text{m}}{\text{s}}\right) \\ & \boxed{F = 11.5\,\text{N}}\end{aligned}$$

6.18: PROBLEM DEFINITION

Situation:

Pressurized air drives a water jet out of a tank. The thrust of the water jet reduces the tension in a supporting cable.

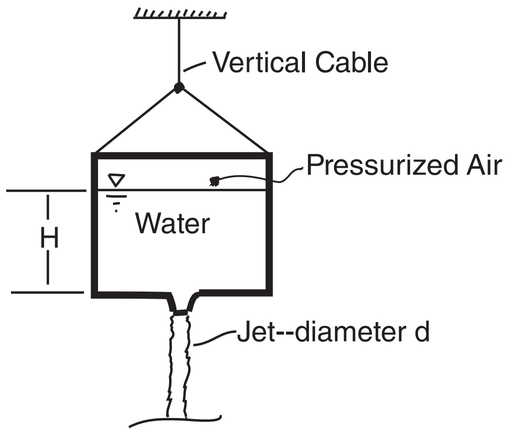
$W = 200\,\text{N}$, $T = 10\,\text{N}$.

$d = 12\,\text{mm}$, $H = 425\,\text{mm}$.

Find:

The pressure in the air that is situated above the water.

Sketch:



Assumptions:

Assume that the Bernoulli equation can be applied (i.e. assume irrotational and steady flow).

Properties:

Water (15 °C), Table A.5: $\rho = 999\,\text{kg/m}^3$.

PLAN

Apply the momentum equation to find the exit velocity. Then, apply the Bernoulli equation to find the pressure in the air.

SOLUTION

Section area of jet

$$\begin{aligned} A_2 &= \frac{\pi d^2}{4} \\ &= \frac{\pi\,(0.012\,\text{m})^2}{4} \\ &= 1.131\times 10^{-4}\,\text{m}^2 \end{aligned}$$

Momentum equation (cv surrounding the tank; section 2 at the nozzle)

$$\begin{aligned}
\sum \mathbf{F} &= \dot{m}_o \mathbf{v}_o \\
-T + W &= \dot{m} v_2 \\
(-10 + 200)\ \text{N} &= \rho A_2 v_2^2
\end{aligned}$$

Solve for exit speed (v_2)

$$\begin{aligned}
190\,\text{N} &= \left(999\,\text{kg/m}^3\right)\left(1.131 \times 10^{-4}\,\text{m}^2\right) v_2^2 \\
v_2 &= 41.01\,\text{m/s}
\end{aligned}$$

Bernoulli equation (location 1 is on the water surface, location 2 is at the water jet).

$$p_{\text{air}} + \frac{\rho v_1^2}{2} + \rho g z_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g z_2$$

Let $v_1 \approx 0$, $p_2 = 0$ gage and $\Delta z = 0.425\,\text{m}$.

$$\begin{aligned}
p_{\text{air}} &= \frac{\rho v_2^2}{2} - \rho g \Delta z \\
&= \frac{(999\,\text{kg/m}^3)(41.01\,\text{m/s})^2}{2} - \left(999\,\text{kg/m}^3\right)(9.81\,\text{m/s}^2)(0.425\,\text{m}) \\
&= (835,900\,\text{Pa})\left(\frac{1.0\,\text{atm}}{101.3\,\text{kPa}}\right)
\end{aligned}$$

$$\boxed{p_{\text{air}} = 8.25\,\text{atm}}$$

6.19: PROBLEM DEFINITION

Situation:

Free water jet from upper tank to lower tank, lower tank supported by scales A and B.

$Q = 0.05\,\mathrm{m^3/s}$, $d_1 = 10$ cm.

$h = 0.3$ m, $H = 2.7$ m

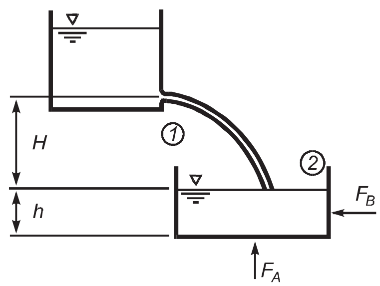
$W_T = 1.4$ kN, $A_2 = 0.37\,\mathrm{m^2}$.

Find:

Force on scale A (N).

Force on scale B (N).

Sketch:



Properties:

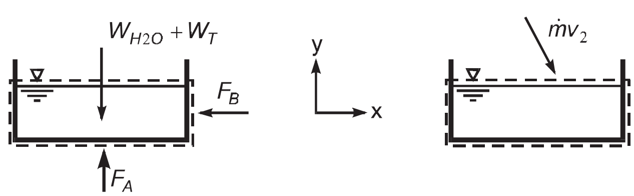
Water (15.5 °C): $\rho = 998.9$ kg/m^3, $\gamma = 9799$ N/m^3.

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



Flow rate

$$
\begin{aligned}
\dot{m} &= \rho Q \\
&= 998.9\,\mathrm{kg/m^3} \times 0.05\,\mathrm{m^3/\,s} \\
&= 49.95 \text{ kg/s} \\
v_1 &= \frac{Q}{A_1} = \frac{4Q}{\pi D^2} \\
&= \frac{4 \times 0.05\,\mathrm{m^3/\,s}}{\pi \times (0.1)^2\,\mathrm{m^2}} \\
&= 6.4 \text{ m/s}
\end{aligned}
$$

Projectile motion equations

$$
\begin{aligned}
v_{2x} &= v_1 = 6.4 \text{ m/s} \\
v_{2y} &= \sqrt{2gH} \\
&= \sqrt{2 \times 9.81\,\mathrm{m/s} \times 2.7\,\mathrm{m}} \\
&= 7.3 \text{ m/s}
\end{aligned}
$$

Momentum equation (x-direction)

$$
\begin{aligned}
\sum F_x &= \dot{m}\left[(v_o)_x - (v_i)_x\right] \\
-F_B &= -\dot{m}\,(v_{2x}) \\
-F_B &= -49.95\,\mathrm{kg/s} \times 6.4\,\mathrm{m/s} \\
& \boxed{F_B = 319.7 \text{ N}}
\end{aligned}
$$

Momentum equation (y-direction)

$$
\begin{aligned}
\sum F_y &= \dot{m}\left[(v_o)_y - (v_i)_y\right] \\
F_A - W_{H2O} - W_T &= -\dot{m}\,(v_{2y}) \\
F_A &= W_{H2O} + W_T - \dot{m}\,(v_{2y}) \\
F_A &= (9799\,\mathrm{N/m^3} \times 0.37\,\mathrm{m^3/\,s} \times 0.3\,\mathrm{m}) + 1400\,\mathrm{N} - (49.95\,\mathrm{kg/s} \times (-7.3\,\mathrm{m/s})) \\
& \boxed{F_A = 2852.3 \text{ N}}
\end{aligned}
$$

6.20: PROBLEM DEFINITION

Situation:

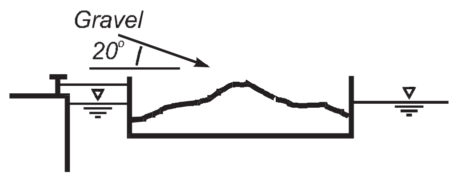
Gravel flows into a barge that is secured with a hawser.

$Q = 38 \text{ m}^3/\text{min} = 0.63 \text{ m}^3/\text{s}$, $v = 3$ m/s.

Find:

Tension in hawser: T

Sketch:



Assumptions:

i. Steady flow.

ii. The difference in height between the conveyor belt and barge can be neglected.

Properties:

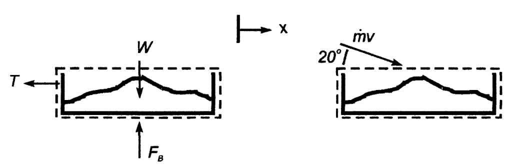
$\gamma = 18,900 \text{ N/m}^3$

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}\,(v_o)_x - \dot{m}\,(v_i)_x \\ -T &= -\dot{m}(v\cos 20) = -(\gamma/g)Q(v\cos 20) \\ T &= \frac{18,900\,\text{N/m}^3}{9.81\,\text{m/s}^2} \times 0.63\,\text{m}^3/\text{s} \times 3\,\text{m/s} \times \cos(20) = 3422 \text{ N} \\ & \boxed{T = 3422 \text{ N}} \end{aligned}$$

6.21: PROBLEM DEFINITION

Situation:

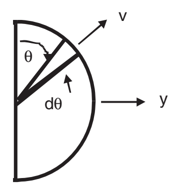
A hemispherical nozzle sprays a sheet of liquid through an arc.

Find:

An expression for the force in y-direction to hold the nozzle stationary.

$F_y = F_y(\rho, v, r, t)$.

Sketch:



PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (y-direction)

$$\begin{aligned} F_y &= \int_{cs} v_y \rho \mathbf{V} \cdot \mathbf{dA} \\ &= \int_0^\pi (v \sin\theta) \rho v (t r d\theta) \\ &= \rho v^2 t r \int_0^\pi \sin\theta d\theta \end{aligned}$$

$$\boxed{F_y = 2\rho v^2 t r}$$

6.22: PROBLEM DEFINITION

Situation:

The design of a conical rocket nozzle.

Find:

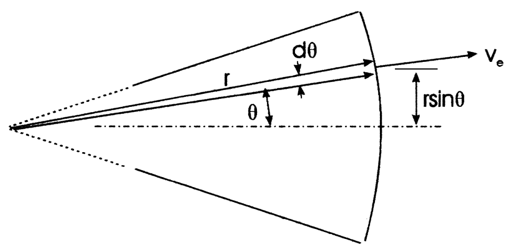
Show that $T = \dot{m}V_e\frac{1+\cos\alpha}{2}$.

PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (x-direction)



$$\begin{aligned}
\sum \mathbf{F} &= \int \mathbf{v}\rho\mathbf{v}\cdot\mathbf{dA} \\
T &= \int_0^\alpha v_e\cos\theta\rho v_e 2\pi r\sin\theta r d\theta \\
T &= 2\pi r^2\rho v_e^2\int_0^\alpha \cos\theta\sin\theta d\theta \\
&= 2\pi r^2\rho v_e^2\sin^2\alpha/2 \\
&= \rho v_e^2 2\pi r^2\frac{(1-\cos\alpha)(1+\cos\alpha)}{2}
\end{aligned}$$

Exit Area

$$A_e = \int_0^\alpha 2\pi r\sin\theta r d\theta = 2\pi r^2(1-\cos\alpha)$$

$$T = \rho v_e^2 A_e(1+\cos\alpha)/2$$

$$\boxed{T = \dot{m}v_e(1+\cos\alpha)/2}$$

6.23: PROBLEM DEFINITION

Situation:

A fixed vane in the horizontal plane.

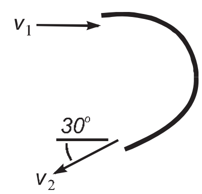
$v_1 = 22$ m/s, $v_2 = 21$ m/s.

$Q = 0.15$ m^3/s, $S = 0.9$.

Find:

Components of force to hold vane stationary (kN).

Sketch:

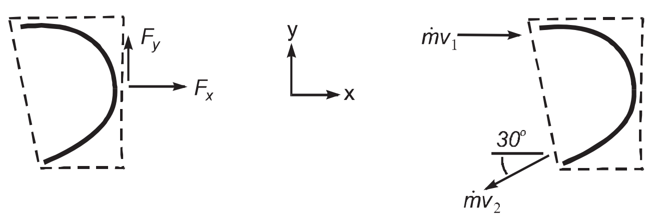


PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



Mass flow rate

$$\begin{aligned} \dot{m} &= \rho Q \\ &= 0.9 \times 1000\,\mathrm{kg/\,m^3} \times 0.15\,\mathrm{m^3/\,s} \\ &= 135 \text{ kg/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}\,(v_o)_x - \dot{m}\,(v_i)_x \\ F_x &= \dot{m}(-v_2 \cos 30^\circ) - \dot{m} v_1 \\ F_x &= -135\,\mathrm{kg/\,s}(21\,\mathrm{m/\,s} \cos 30^\circ + 22\,\mathrm{m/\,s}) \end{aligned}$$

$$\boxed{F_x = -5.43\,\mathrm{kN} \text{ (acts to the left)}}$$

Momentum equation (y-direction)

$$\begin{aligned}
\sum F_y &= \dot{m}\,(v_o)_y - \dot{m}\,(v_i)_y \\
F_y &= \dot{m}\,(-v_2 \sin 30^\circ) \\
&= 135\,\mathrm{kg/\,s}\,(-21\,\mathrm{m/\,s} \sin 30^\circ) \\
&= -1.42\ \mathrm{kN}
\end{aligned}$$

$$\boxed{F_y = -1.42\,\mathrm{kN}\ \text{(acts downward)}}$$

6.24: PROBLEM DEFINITION

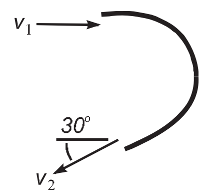
Situation:

A fixed vane in the horizontal plane.
$v_1 = 21$ m/s, $v_2 = 19.8$ m/s.
$Q = 0.043$ m^3/s, $S = 0.9$.

Find:

Components of force to hold vane stationary (N).

Sketch:

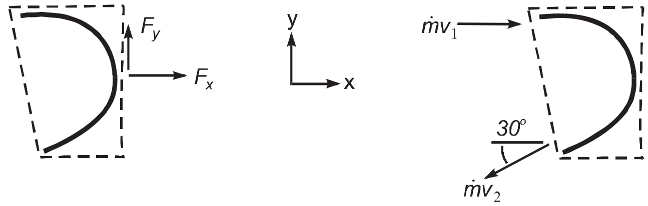


PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



Mass flow rate

$$\dot{m} = \rho Q = 0.9 \times 1000\,\text{kg/m}^3 \times 0.043\,\text{m}^3/\,\text{s} = 38.7 \text{ kg/s}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}\,(v_o)_x - \dot{m}\,(v_i)_x \\ F_x &= \dot{m}(-v_2\cos 30°) - \dot{m}v_1 \\ F_x &= 38.7\,\text{kg/s}(19.8\,\text{m/s}\cos 30° + 21\,\text{m/s}) \\ &\boxed{F_x = 1476 \text{ N (acts to the left)}} \end{aligned}$$

y-direction

$$\begin{aligned} \sum F_y &= \dot{m}\,(v_o)_y - \dot{m}\,(v_i)_y \\ F_y &= \dot{m}\,(-v_2\sin 30°) = 38.7\,\text{kg/s}\,(-19.8\,\text{m/s}\sin 30°) = -383 \text{ N} \\ &\boxed{F_y = 383 \text{ N (acts downward)}} \end{aligned}$$

6.25: PROBLEM DEFINITION

Situation:

A horizontal, two-dimensional water jet deflected by a fixed vane.
$v_1 = 12$ m/s, $w_2 = 0.06$ m, $w_3 = 0.03$ m.

Find:

Components of force, per meter of width, to hold the vane stationary (N/m).

Assumptions:

Neglect elevation changes.
Neglect viscous effects.

Properties:

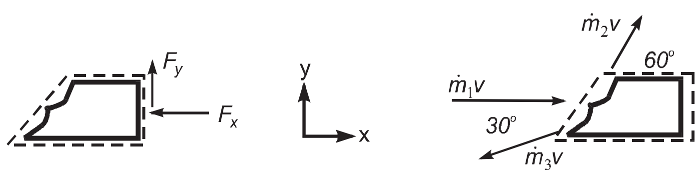
Water, Table A.5: $\rho = 998.9$ kg/m^3.

PLAN

Apply the Bernoulli equation, the continuity equation, and finally the momentum equation.

SOLUTION

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v_3 = v = 12 \text{ m/s}$$

Continuity equation

$$\begin{aligned} w_1 v_1 &= w_2 v_2 + w_3 v_3 \\ w_1 &= w_2 + w_3 = (0.06 + 0.03) = 0.1 \text{ m} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\ -F_x &= \dot{m}_2 v \cos 60^\circ + \dot{m}_3(-v \cos 30^\circ) - \dot{m}_1 v \\ F_x &= \rho v^2(-A_2 \cos 60^\circ + A_3 \cos 30^\circ + A_1) \\ F_x &= 998.9 \,\text{kg/m}^3 \times (12 \,\text{m/s})^2 \times (-0.06 \,\text{m} \cos 60^\circ + 0.03 \,\text{m} \cos 30^\circ + 0.1 \,\text{m}) \\ & \boxed{F_x = 13{,}806 \text{ N (acts to the left)}} \end{aligned}$$

Momentum equation (y-direction)

$$
\begin{aligned}
\sum F_y &= \sum \dot{m}_o \, (v_o)_y \\
F_y &= \dot{m}_2 v \sin 60^\circ + \dot{m}_3(-v \sin 30^\circ) \\
&= \rho v^2 (A_2 \sin 60^\circ - A_3 \sin 30^\circ) \\
&= 998.9\,\mathrm{kg/m^3} \times (12\,\mathrm{m/s})^2 \times (0.06\,\mathrm{m} \sin 60^\circ - 0.03\,\mathrm{m} \sin 30^\circ) \\
& \boxed{F_y = 5317 \text{ N/m (acts upward)}}
\end{aligned}
$$

6.26: PROBLEM DEFINITION

Situation:

A water jet is deflected by a fixed vane.

$v_1 = 9$ m/s, $\dot{m} = 16$ kg/s.

Find:

Force of the water on the vane (N).

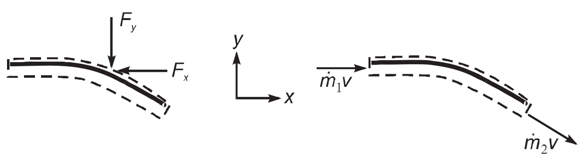
Sketch:

PLAN

Apply the Bernoulli equation, and then the momentum equation.

SOLUTION

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v = 9 \text{ m/s}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\ -F_x &= \dot{m}v\cos 30^\circ - \dot{m}v \\ F_x &= \dot{m}v(1-\cos 30^\circ) = 16\,\text{kg/s} \times 9\,\text{m/s} \times (1-\cos 30^\circ) \\ F_x &= 19.3 \text{ N to the left} \end{aligned}$$

y-direction

$$\begin{aligned} \sum F_y &= \dot{m}_o (v_o)_y \\ -F_y &= \dot{m}(-v\cos 60^\circ) = -16\,\text{kg/s} \times 9\,\text{m/s} \times \sin 30^\circ \\ F_y &= 72 \text{ N downward} \end{aligned}$$

Since the forces acting on the vane represent a state of equilibrium, the force of water on the vane is equal in magnitude & opposite in direction.

$$\begin{aligned} \mathbf{F} &= -F_x\mathbf{i} - F_y\mathbf{j} \\ &= \boxed{(19.3 \text{ N})\,\mathbf{i} + (72 \text{ N})\,\mathbf{j}} \end{aligned}$$

6.27: PROBLEM DEFINITION

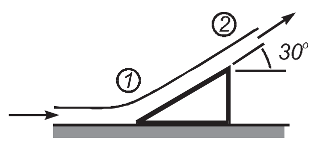
Situation:

A water jet strikes a block and the block is held in place by friction.
$v_1 = 10$ m/s, $\dot{m} = 1.5$ kg/s.
$\mu = 0.1$, $m = 1$ kg.

Find:

Will the block slip?
Force of the water jet on the block (N).

Sketch:



Assumptions:

Neglect weight of water.
Neglect elevation changes.
Neglect viscous forces.

Properties:

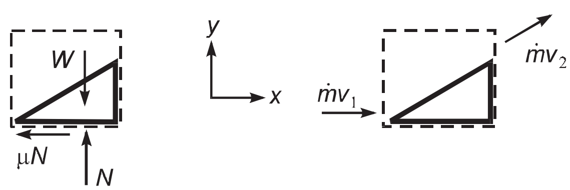
$\rho = 1000\,\text{kg/m}^3$.

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v = 10 \text{ m/s}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\ -F_f &= \dot{m}v \cos 30^\circ - \dot{m}v \\ F_f &= \dot{m}v(1 - \cos 30^\circ) \\ &= 1.5\,\text{kg/s} \times 10\,\text{m/s} \times (1 - \cos 30^\circ) \\ &\boxed{F_f = 2.01\,\text{N}} \end{aligned}$$

y-direction

$$\begin{aligned}
\sum F_y &= \dot{m}_o \left(v_o\right)_y \\
N - W &= \dot{m}(v \sin 30^\circ) \\
N &= mg + \dot{m}(v \sin 30^\circ) \\
&= 1.0\,\text{kg} \times 9.81\,\text{m/s}^2 + 1.5\,\text{kg/s} \times 10\,\text{m/s} \times \sin 30^\circ \\
& \boxed{N = 17.3\ \text{N}}
\end{aligned}$$

Analyze friction:

- F_f (required to prevent block from slipping) = 2.01 N
- F_f (maximum possible value) $= \mu N = 0.1 \times 17.3 = 1.73$ N

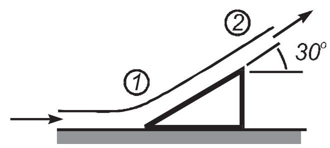
$$\boxed{\text{block will move}}$$

6.28: PROBLEM DEFINITION

Situation:
A water jet strikes a block and the block is held in place by friction.
$\dot{m} = 1$ kg/s, $m = 1$ kg.
$\mu = 0.1$, $\theta = 30\,°$.

Find:
Maximum velocity such that the block will not slip.

Sketch:



Assumptions:
Neglect weight of water.

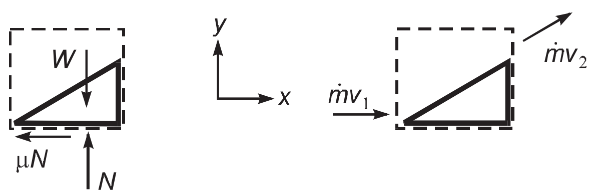
Properties:
$\rho = 1000\,\text{kg/m}^3$.

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v$$

Momentum equation (x-direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ -\mu N &= \dot{m} v \cos 30° - \dot{m} v \\ N &= \dot{m} v \left(1 - \cos 30°\right)/\mu\end{aligned}$$

y-direction

$$\begin{aligned}\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\ N - W &= \dot{m}(v \sin 30°) \\ N &= mg + \dot{m}(v \sin 30°)\end{aligned}$$

Combine previous two equations

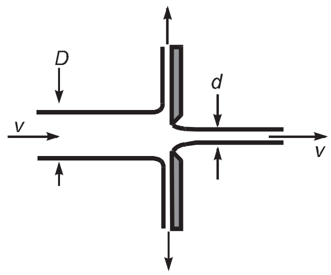
$$\begin{aligned}
\frac{\dot{m}v\,(1-\cos 30^\circ)}{\mu} &= mg + \dot{m}(v\sin 30^\circ) \\
v &= \frac{mg}{[\dot{m}\,(1/\mu - \cos 30^\circ/\mu - \sin 30^\circ)]} \\
v &= \frac{1\,\mathrm{kg}\times 9.81\,\mathrm{m/\,s^2}}{[1.5\,\mathrm{kg/\,s}\times(1/0.1-\cos 30^\circ/0.1-\sin 30^\circ)]} \\
& \boxed{v = 7.79\ \mathrm{m/\,s}}
\end{aligned}$$

6.29: PROBLEM DEFINITION

Situation:

A water jet strikes a plate with a sharp edged orifice at its center.

$v = 90$ m/s, $D = 10$ cm, $d = 3.5$ cm



Find:

Force required to hold plate stationary (N).

Assumptions:

Neglect gravity.

Properties:

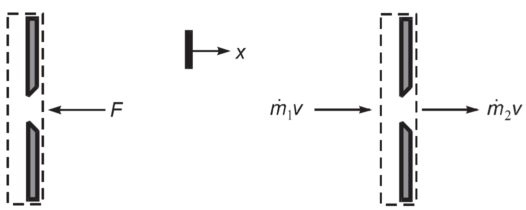
$\rho = 1000$ kg/m^3

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams (only x-direction vectors shown)



Momentum equation (x-direction)

$$\begin{aligned}
\sum \mathbf{F} &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\
-F &= \dot{m}_2 v - \dot{m}_1 v \\
F &= \rho A_1 v^2 - \rho A_2 v^2 \\
&= \rho v^2 \left(\frac{\pi}{4}\right)(D^2 - d^2) \\
&= 1000\,\mathrm{kg/m^3} \times (90\,\mathrm{m/s})^2 \times \frac{\pi}{4} \times ((0.10\,\mathrm{m})^2 - (0.035\,\mathrm{m})^2)
\end{aligned}$$

$$\boxed{F = 55.8 \text{ kN (to the left)}}$$

6.30: PROBLEM DEFINITION

Situation:

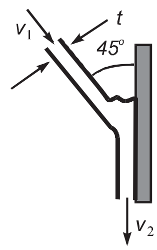
A 2D liquid jet impinges on a vertical wall.

$v_1 = v_2 = v$, $\theta = 45\,°$.

Find:

Calculate the force acting on the wall.

Sketch and explain the shape of the liquid surface.



Assumptions:

Steady flow.

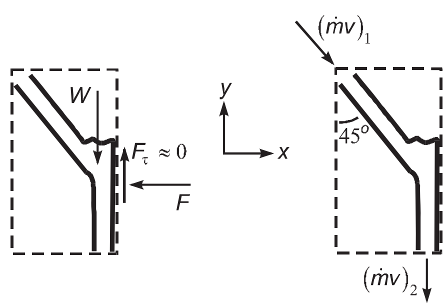
Force associated with shear stress is negligible.

PLAN

Apply the momentum equation.

SOLUTION

Let w = the width of the jet in the z-direction. Force and momentum diagrams



Momentum equation (x-direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ -F &= -\dot{m}v_1 \sin 45^o \\ F &= \rho w t v^2 \sin 45^o\end{aligned}$$

The force on that acts on the wall is in the opposite direction to force pictured on the force diagram, thus

$$\boxed{F_{\text{on wall}} = \rho t v^2 \sin 45^o \text{ (acting to the right)}}$$

y-direction

$$\begin{aligned}
\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\
-W &= \dot{m}\,(-v) - \dot{m}\,(-v)\cos 45^o \\
W &= \dot{m}v(1 - \cos 45^o)
\end{aligned}$$

REVIEW

Thus, weight provides the force needed to increase y-momentum flow. This weight is produced by the fluid swirling up to form the shape show in the above sketches.

6.31: PROBLEM DEFINITION

Situation:

A cone is supported by a vertical jet of water.
$W = 30\,\text{N}$, $V_1 = 15\,\text{m/s}$.
$d_1 = 2\,\text{cm}$, $\theta = 60\,°$.

Find:

Height to which cone will rise (m).

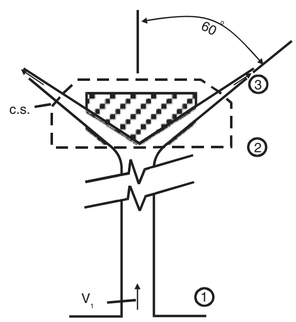
Assumptions:

Speed of the fluid as it passes by the cone is constant $(V_2 = V_3)$.

PLAN

Apply the Bernoulli equation and the momentum equation.

SOLUTION



Bernoulli equation

$$\begin{aligned} \frac{V_1^2}{2g} + 0 &= \frac{V_2^2}{2g} + h \\ V_2^2 &= (V_1)^2 - 2gh \\ V_2^2 &= 225 - 19.62h \end{aligned}$$

Momentum equation (y-direction). Select a control volume surrounding the cone.

$$\begin{aligned} \sum F_y &= \dot{m}_o v_{oy} - \dot{m}_i v_{iy} \\ -W &= \dot{m}(v_{3y} - v_2) \\ -30\,\text{N} &= 1000\,\text{kg/m}^3 \times 15\,\text{m/s} \times \pi \times (0.01\,\text{m})^2 (V_2 \sin 30° - V_2) \end{aligned}$$

Solve for the V_2

$$V_2 = 12.73 \text{ m/s}$$

Complete the Bernoulli equation calculation

$$\begin{aligned} V_2^2 &= 225 - 19.62h \\ (12.73\,\mathrm{m/s})^2 &= 225 - 19.62h \end{aligned}$$

$$\boxed{h = 3.21\,\mathrm{m}}$$

6.32: PROBLEM DEFINITION

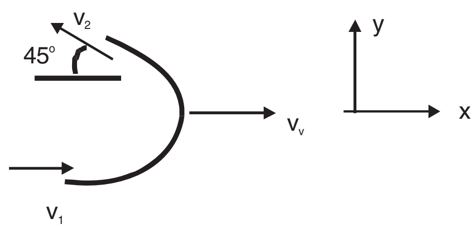
Situation:

A fluid jet strikes a vane that is moving at a speed.
$v_1 = 20\,\mathrm{m/s}$, $v_v = 7$ m/s.
$D_1 = 6$ cm.

Find:

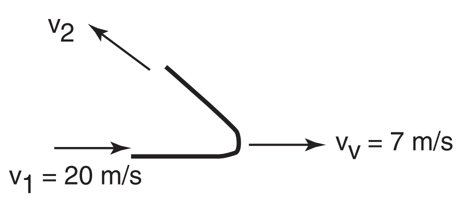
Force of the water on the vane.

Sketch:



SOLUTION

Force and momentum diagrams
Select a control volume surrounding and moving with the vane. Select a reference frame attached to the moving vane.



Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}v_{2x} - \dot{m}v_{1x} \\ -F_x &= -\dot{m}v_2\cos 45^\circ - \dot{m}v_1 \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= \dot{m}v_{2y} - \dot{m}v_{1y} \\ F_y &= \dot{m}v_2\sin 45^\circ \end{aligned}$$

Velocity analysis

- v_1 is relative to the reference frame $= (20-7) = 13\,\mathrm{m/s}$.
- in the term $\dot{m} = \rho A v$ use v which is relative to the control surface. In this case $v = (20-7) = 13$ m/s

- v_2 is relative to the reference frame $v_2 = v_1 = 13$ m/s

Mass flow rate

$$\begin{aligned} \dot{m} &= \rho A v \\ &= (1,000 \text{ kg})(\pi/4 \times (0.06\,\mathrm{m})^2)(13\,\mathrm{m/\,s}) \\ &= 36.76 \text{ kg/s} \end{aligned}$$

Evaluate forces

$$\begin{aligned} F_x &= \dot{m} v_1 (1 + \cos 45^\circ) \\ &= 36.76\,\mathrm{kg/\,s} \times 13\,\mathrm{m/\,s}(1 + \cos 45^\circ) = 816 \text{ N} \end{aligned}$$

which is in the negative $x-$direction.

$$\begin{aligned} F_y &= \dot{m} v_2 \sin 45^\circ \\ &= 36.76\,\mathrm{kg/\,s} \times 13\,\mathrm{m/\,s} \sin 45^\circ = 338 \text{ N} \end{aligned}$$

The force of the water on the vane is the negative of the force of the vane on the water. Thus the force of the water on the vane is

$$\boxed{\mathbf{F} = (816\mathbf{i} - 338\mathbf{j}) \text{ N}}$$

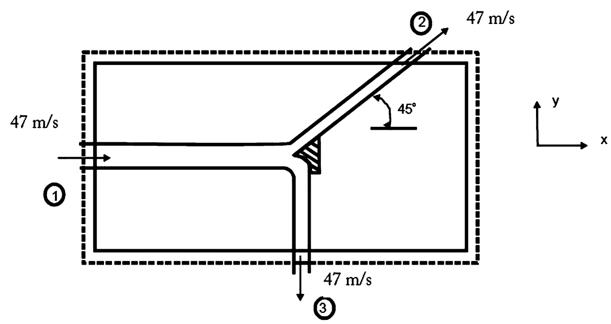
6.33: PROBLEM DEFINITION

Situation:

Cart, moving with a steady speed of 3m/s.

Jet, $V = 50$ m/s, $D = 15$ cm, is being sprayed from behind the cart, and the jet is divided and deflected by a vane situated on the cart.

Speed of jet as it impacts the vane is 47 m/s, as shown.



Find:

Force exerted by the vane on the jet: **F**

PLAN

Apply the momentum equation.

SOLUTION

Make the flow steady by referencing all velocities to the moving vane and let the c.v. move with the vane as shown in the figure above.

Momentum equation (x-direction)

$$\begin{aligned}
F_x &= \dot{m}_2 v_{2x} - \dot{m}_1 v_1 \\
\dot{m} &= \rho A V = 1000\,\mathrm{kg/\,m^3} \times (\pi/4) \times (0.15\,\mathrm{m})^2 \times 47\,\mathrm{m/\,s} = 830.1 \text{ kg/s} \\
F_x &= (\frac{\dot{m}}{2} v \cos 45^{\mathrm{o}} - \dot{m} v) = \dot{m} v (\frac{\cos 45^{\mathrm{o}}}{2} - 1) \\
&= 830.1 \text{ kg/s} \times 47 \text{ m/s} \times (0.3535 - 1) \\
&= -25200 \text{ N}
\end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned}
F_y &= \dot{m}_2 v_{2y} - \dot{m} v_{3y} \\
&= \frac{\dot{m}}{2} v \sin 45^{\text{o}} - \frac{\dot{m}}{2} v = \frac{\dot{m}}{2} v (\sin 45^{\text{o}} - 1) \\
&= \frac{830.1 \text{ kg/s}}{2} \times 47 \text{ m/s} \times (0.707 - 1) \\
&= -5720 \text{ N} \\
& \boxed{\mathbf{F}(\text{water on vane}) = (25200\mathbf{i} + 5720\mathbf{j}) \text{ N}}
\end{aligned}$$

6.34: PROBLEM DEFINITION

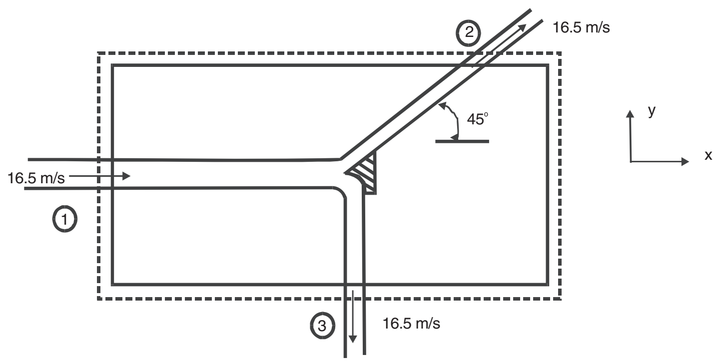
Situation:

A cart is moving with steady speed—additional details are provided in the problem statement.

Find:

Rolling resistance of the cart: $F_{rolling}$

SOLUTION Let the control surface surround the cart and let it move with the cart at 1.5 m/s. Then we have a steady flow situation and the relative jet velocities are shown below.



Momentum equation (x-direction)

$$\sum F_x = \dot{m}_2 v_{2_x} - \dot{m}_1 v_1$$

Calculations

$$\begin{aligned}
\dot{m}_1 &= \rho A_1 V_1 \\
&= (1000 \text{ kg/m}^3)(\pi/4 \times (0.045^2 \text{m}^2) \times 16.5 \text{ m/s} \\
&= 26.23 \text{ kg/s} \\
\dot{m}_2 &= \dot{m}_3 = (26.23 \text{ kg/s})/2 \\
&= 13.115 \text{ kg/s} \\
F_{\text{rolling}} &= \dot{m}_1 v_1 - \dot{m}_2 v_2 \cos 45° \\
&= 26.23 \, \text{kg/s} \times 16.5 \, \text{m/s} - 13.115 \, \text{kg/s} \times 16.5 \, \text{m/s} \cos 45° \\
& \boxed{F_{\text{rolling}} = 280 \, \text{N (acting to the left)}}
\end{aligned}$$

6.35: PROBLEM DEFINITION

Situation:

A water is deflected by a moving cone.

Speed of the water jet is $60\,\mathrm{m/s}$ (to the right). Speed of the cone is $5\,\mathrm{m/s}$ (to the left). Diameter of the jet is $D = 10\,\mathrm{cm}$.

Angle of the cone is $\theta = 50^o$.

Find:

Calculate the external horizontal force needed to move the cone: F_x

Assumptions:

As the jet passes over the cone (a) assume the Bernoulli equation applies, and (b) neglect changes in elevation.

PLAN

Apply the momentum equation.

SOLUTION

Select a control volume surrounding the moving cone. Select a reference frame fixed to the cone. Section 1 is the inlet. Section 2 is the outlet.
Inlet velocity (relative to the reference frame and surface of the control volume).

$$\begin{aligned} v_1 &= V_1 = (60+5)\ \mathrm{m/s} \\ & \quad 65\,\mathrm{m/s} \end{aligned}$$

Bernoulli equation. Pressure and elevation terms are zero, so

$$V_1 = V_2 = v_2 = 65\,\mathrm{m/s}$$

Momentum equation (x-direction)

$$\begin{aligned} F_x &= \dot{m}(v_{2x} - v_1) \\ &= \rho A_1 V_1 \left(v_2 \cos\theta - v_1\right) \\ &= \rho A_1 V_1^2 \left(\cos\theta - 1\right) \\ &= \left(1000 \frac{\mathrm{kg}}{\mathrm{m}^3}\right) \times \left(\frac{\pi \times (0.1\,\mathrm{m})^2}{4}\right) \times (65\,\mathrm{m/s})^2 \left(\cos 50^o - 1\right) \\ &= -11.85\ \mathrm{kN} \end{aligned}$$

$$\boxed{F_x = 11.85\ \mathrm{kN}\ \text{(acting to the left)}}$$

6.36: PROBLEM DEFINITION

Situation:

A jet of water is deflected by a moving vane—additional details are provided in the problem statement.

Find:

Power (per meter of width of the jet) transmitted to the vane: P

PLAN

Apply the momentum equation.

SOLUTION

Select a control volume surrounding the moving cone. Select a reference frame fixed to the cone.

Density

$$\rho = \frac{9810\,\text{N/m}^3}{9.81\,\text{m/s}^2} = 1000\ \text{kg/m}^3$$

Velocity analysis

$$\begin{aligned} v_1 &= V_1 = 12\ \text{m/s} \\ v_2 &= 12\ \text{m/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}(v_{2x} - v_1) \\ F_x &= 1000\,\text{kg/m}^3 \times 12\,\text{m/s} \times 0.09\,\text{m} \times (12\,\text{m/s}\cos 50^\circ - 12\,\text{m/s}) \\ &= 4629.5\ \text{N/m} \end{aligned}$$

Calculate power

$$\begin{aligned} P &= Fv \\ &= 4629.5\,\text{N/m} \times 18\,\text{m/s} \\ &\boxed{P = 83,331\ \text{N-m/s/m} = 83,331\ \text{W/m}} \end{aligned}$$

6.37: PROBLEM DEFINITION

Situation:

A sled of mass $m_s = 1000$ kg is decelerated by placing a scoop of width $w = 20$ cm into water at a depth $d = 8$ cm.

Find:

Deceleration of the sled: a_s

SOLUTION

Select a moving control volume surrounding the scoop and sled. Select a stationary reference frame.

Momentum equation (x-direction)

$$0 = \frac{d}{dt}(m_s v_s) + \dot{m} v_{2_x} - \dot{m} v_{1_x}$$

Velocity analysis

$$\begin{aligned} v_{1_x} &= 0 \\ V_1 &= 100 \text{ m/s} \\ V_2 &= 100 \text{ m/s} \\ \mathbf{v}_2 &= 100 \text{ m/s}[-\cos 60^\circ \mathbf{i} + \sin 60^\circ \mathbf{j}] + 100\mathbf{i} \text{ m/s} \\ v_{2x} &= 50 \text{ m/s} \end{aligned}$$

The momentum equation equation simplifies to

$$0 = m_s a_s + \dot{m} v_{2_x} \tag{1}$$

Flow rate

$$\begin{aligned} \dot{m} &= \rho A_1 V_1 \\ &= 1000 \,\text{kg/m}^3 \times 0.2 \,\text{m} \times 0.08 \,\text{m} \times 100 \,\text{m/s} \\ &= 1600 \text{ kg/s} \end{aligned}$$

From Eq. (1).

$$\begin{aligned} a_s &= -\frac{\dot{m} v_{2x}}{m_s} \\ &= \frac{(-1600 \,\text{kg/s})(50 \,\text{m/s})}{1000 \,\text{kg}} \end{aligned}$$

$$\boxed{a_s = -80 \text{ m/s}^2}$$

6.38: PROBLEM DEFINITION

Situation:

A snowplow is described in the problem statement.

Find:

Power required for snow removal: P

PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (x-direction)

Select a control volume surrounding the snow-plow blade. Attach a reference frame to the moving blade.(Snow is 0.1 m deep)

$$\sum F_x = \rho Q(v_{2x} - v_1)$$

Velocity analysis

$$\begin{aligned} V_1 &= v_1 = 12 \text{ m/s} \\ v_{2_x} &= 12\,\text{m/s}\cos 60^\circ \cos 30^\circ \\ &= 5.2 \text{ m/s} \end{aligned}$$

Calculations

$$\begin{aligned} \sum F_x &= \rho V dWS(v_{2x} - v_1) \\ &= 1000\,\text{kg/m}^3 \times 0.2 \times 12\,\text{m/s} \times 0.6\,\text{m} \times 0.1\,\text{m}(5.2\,\text{m/s} - 12\,\text{m/s}) \\ &= 2477 \text{ N} \end{aligned}$$

Power

$$\begin{aligned} P &= FV \\ &= 2477\,\text{N} \times 12\,\text{m/s} \\ &= 29,724 \text{ N-m/s} \\ & \boxed{P = 29,724 \text{ W}} \end{aligned}$$

6.39: PROBLEM DEFINITION

Situation:

The flow over an airfoil is modeled as the flow in a circular stream tube which has a diameter equal to the wing span and is deflected by an angle of 2°.

Find:

The lift and the drag forces.

Assumptions:

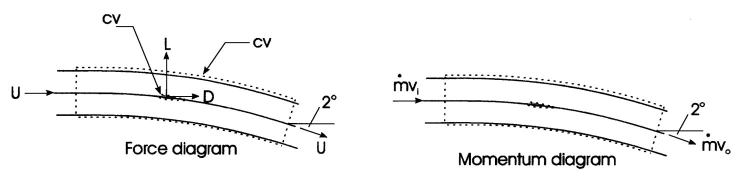
Assume the pressure is constant far from the airfoil.

PLAN

Apply the component form of the momentum equation.

SOLUTION

Draw an outer volume that encloses the airfoil far from the airfoil and one around the airfoil as shown in the diagram. The space between the two volumes is the control volume



The force diagram shows a lift force and drag force produced by the airfoil and act on the control surface. There is no net pressure force on the outer surface since the pressure is constant. The momentum diagram shows an inflow and outflow of momentum. From continuity, the mass flow rate in is equal to the mass flow rate out. The sum of the forces in the x-direction is

$$\sum F_x = D$$

and in the y-direction

$$\sum F_y = L$$

The component momentum equation in the x-direction for steady flow is

$$\sum F_x = \dot{m}\left(U\cos\theta - U\right)$$

The component momentum equation in the y-direction is

$$\sum F_y = \dot{m}(-U\sin\theta - 0)$$

The mass flow rate is

$$\dot{m} = \rho U A = \rho U\left(\frac{\pi b^2}{4}\right)$$

The density is obtained from the ideal gas law

$$\rho = \frac{p}{RT} = \frac{101.3\,\text{kPa}}{287 \text{ J/kg-K} \times (273 + 15.5) \text{ K}} = 1.22 \text{ kg/m}^3$$

The mass flow rate is

$$\dot{m} = 1.22 \text{ kg/m}^3 \times 90 \text{ m/s} \times \frac{\pi \times 9^2 \text{ m}^2}{4} = 6982 \text{ kg/s}$$

Solving for the drag force

$$\begin{aligned} D &= \dot{m}U(\cos\theta - 1) \\ &= 6982 \text{ kg/s} \times 90 \text{ m/s} \times (\cos 2^\text{o} - 1) \\ &= -383 \text{ N} \end{aligned}$$

Solving for lift force

$$\begin{aligned} L &= -\dot{m}U \sin\theta \\ &= -6982 \times 90 \times \sin 2^\text{o} \\ &= -21,930 \text{ N} \end{aligned}$$

The values calculated are the force of the airfoil in the fluid. The force of the fluid on the airfoil which are the actual definitions of lift and drag would have the opposite sign so

$$\boxed{D = 383 \text{ N}}$$

$$\boxed{L = 21,930 \text{ N}}$$

6.40: PROBLEM DEFINITION

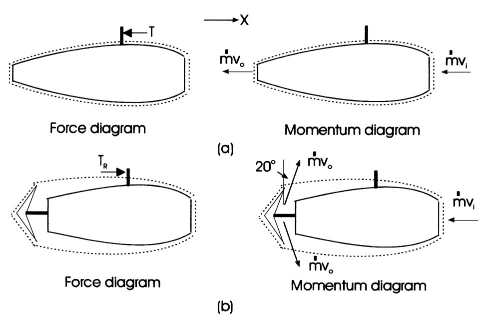
Situation:
A clam shell thrust reverser is deployed on an aircraft engine.

Find:
(a) The thrust under normal operation.
(b) The reverse thrust.

Assumptions:
Engine is stationary.
Exit gas velocity unchanged at deployment.
Pressure is atmospheric at exhaust plane.

PLAN

Apply the component momentum equation.



SOLUTION

The control volumes for both cases are shown in the diagram. For case (a) the sum of the forces in the x-direction is

$$\sum F_x = -T$$

and for case (b)

$$\sum F_x = T_R$$

From the momentum diagrams for case (a) there is an influx and outflow of momentum in the same direction. For case (b), the outlet direction of the momentum is altered by the vane of the thrust reverser.
The component momentum equation in the x-direction is

$$\sum F_x = \frac{d}{dt}\int_{cv} \rho v d\forall + \sum \dot{m}_o v_{xo} - \sum \dot{m}_i v_{xi}$$

The motor is stationary, so there is no unsteady term. Also the mass flow rate in is equal to the mass flow rate out, $\dot{m}_i = \dot{m}_o = \dot{m}$. For case (a)

$$\begin{aligned} -T &= \dot{m}[-U_o - (-U_i)] \\ T &= \dot{m}(U_o - U_i) \end{aligned}$$

The mass flow rate is

$$\dot{m} = 68 \text{ kg/s}$$

The thrust for case (a) is

$$\begin{aligned} T &= 68\,\text{kg/s}(425\,\text{m/s} - 90\,\text{m/s}) \\ &\boxed{T = 22,780 \text{ N}} \end{aligned}$$

For case (b)

$$\begin{aligned} T_R &= \dot{m}[U_o \sin 20^o - (-U_i)] \\ &= \dot{m}(U_o \sin 20^o + U_i) \end{aligned}$$

The reverse thrust is

$$\begin{aligned} T_R &= 68\,\text{kg/s}(425\,\text{m/s} \times \sin 20^o + 90\,\text{m/s}) \\ &\boxed{T = 16{,}004 \text{ N}} \end{aligned}$$

6.41: PROBLEM DEFINITION

Situation:
Information of fire hoses and nozzles

Find:
Information of operational conditions and typical hose sizes and nozzles.

SOLUTION

Information depends on source.

6.42: PROBLEM DEFINITION

Situation:
High speed water jets.

Find:
Estimate water speed for 4.14×10^5 kPa (gage) pressure.

Assumptions:
Inlet velocity is negligible and viscous effects are not important. Assume the exit pressure is atmospheric.

PLAN

Apply the Bernoulli equation.

SOLUTION

The Bernoulli equation between the chamber and nozzle exit

$$p_o + \gamma z_o + \rho \frac{V_o^2}{2} = p_e + \gamma z_e + \rho \frac{V_e^2}{2}$$

The pressure difference is much larger than the pressure due to elevation change so

$$\begin{aligned} V_e^2 &= \frac{2p_c}{\rho} \\ V_e &= \sqrt{\frac{2 \times 414000000 \, \text{N/m}^2}{1000 \text{ kg/m}^3}} \end{aligned}$$

$$\boxed{V_e = 910 \text{ m/s}}$$

This velocity is less than the speed of sound in water (~ 1500 m/s) so the exit velocity is subsonic and the exit pressure will equal the atmospheric pressure.

6.43: PROBLEM DEFINITION

Situation:

Water (15.5 °C) flows through a nozzle.
$d_1 = 0.075\,\text{m}$, $d_2 = 0.025$ m
$p_1 = 120$ kPa, $p_2 = 0$ kPa

Find:

(a) Speed at nozzle exit: v_2
(b) Force to hold nozzle stationary: F

Assumptions:

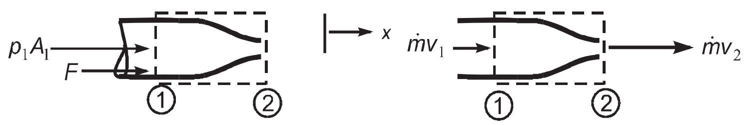
Neglect weight, steady flow.

PLAN

Apply the continuity equation, then the Bernoulli equation, and finally the momentum equation.

SOLUTION

Force and momentum diagrams



Continuity equation

$$\begin{aligned} A_1 v_1 &= A_2 v_2 \\ v_1 &= v_2 \left(\frac{d_2}{d_1}\right)^2 \end{aligned} \quad (1)$$

Bernoulli equation applied from 1 to 2

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} = \frac{v_2^2}{2} \quad (2)$$

Combining Eqs. (1) and (2)

$$\begin{aligned} p_1 &= \rho \left(\frac{v_2^2}{2}\right)\left(1 - \left(\frac{d_2}{d_1}\right)^4\right) \\ 120 \text{ kPa} &= 998.9\,\text{kg/m}^3 \times \left(\frac{v_2^2}{2}\right) \times \left(1 - \left(\frac{0.025}{0.075}\right)^4\right) \\ v_2 &= v_e = 15.6 \text{ m/s} \end{aligned}$$

From Eq. (1)

$$
\begin{aligned}
v_1 &= v_2 \left(\frac{d_2}{d_1}\right)^2 \\
&= 15.6\,\text{m/s} \times \left(\frac{0.025}{0.075}\right)^2 \\
&= 1.73 \text{ m/s}
\end{aligned}
$$

Flow rate

$$
\begin{aligned}
\dot{m}_1 &= \dot{m}_2 = \dot{m} \\
&= (\rho A v)_2 \\
&= 998.9\,\text{kg/m}^3 \times \left(\frac{\pi}{4} \times (0.025\,\text{m})^2\right) \times 15.6\,\text{m/s} \\
&= 7.6 \text{ kg/s}
\end{aligned}
$$

Momentum equation (x-direction)

$$
\begin{aligned}
\sum F_x &= \dot{m}\left[(v_o)_x - (v_i)_x\right] \\
F + p_1 A_1 &= \dot{m}\,(v_2 - v_1) \\
F &= -p_1 A_1 + \dot{m}\,(v_2 - v_1) \\
F &= -(120,000\,\text{N/m}^2) \times \left(\frac{\pi}{4} \times (0.075)^2\right) \text{ m}^2 \\
&\quad + (7.6\,\text{kg/s}) \times (15.6 - 1.73)\,\text{m/s} \\
&= -424.46
\end{aligned}
$$

$$
\boxed{\text{Force on nozzle} = 424.5 \text{ to the left}}
$$

6.44: PROBLEM DEFINITION

Situation:

Water (15 ^oC) flows through a nozzle.

$d_1 = 10$ cm., $d_2 = 2$ cm., $v_2 = 25$ m/s, $\rho = 999$ kg/m^3

Find:

(a)Pressure at inlet: p_1

(b)Force to hold nozzle stationary: F

Assumptions:

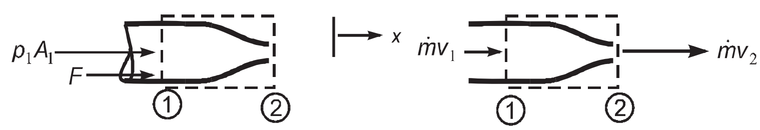
Neglect weight, steady flow, $p_2 = 0$ kPa-gage.

PLAN

Apply the continuity equation, then the Bernoulli equation, and finally the momentum equation.

SOLUTION

Force and momentum diagrams



Continuity equation

$$\begin{aligned}
A_1 v_1 &= A_2 v_2 \\
v_1 &= v_2 \left(\frac{d_2}{d_1}\right)^2 \\
&= 25 \times \left(\frac{2}{10}\right)^2 \\
&= 1.0 \text{ m/s} \\
\dot{m}_1 &= \dot{m}_2 \\
&= (\rho A v)_2 \\
&= 999\,\text{kg/m}^3 \times \left(\frac{\pi \times (0.02\,\text{m})^2}{4}\right) \times 25\,\text{m/s} \\
&= 7.85 \text{ kg/s}
\end{aligned}$$

Bernoulli equation applied from 1 to 2

$$
\begin{aligned}
\frac{p_1}{\rho} + \frac{v_1^2}{2} &= \frac{v_2^2}{2} \\
p_1 &= \frac{\rho}{2}\left(v_2^2 - v_1^2\right) \\
&= \frac{999\,\mathrm{kg/\,m^3}}{2}((25\,\mathrm{m/\,s})^2 - (1\,\mathrm{m/\,s})^2) \\
&= 3.\,117 \times 10^5\,\mathrm{Pa}
\end{aligned}
$$

$$
\boxed{p_1 = 312\ \text{kPa}}
$$

Momentum equation (x-direction)

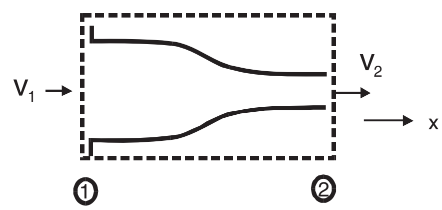
$$
\begin{aligned}
\sum F_x &= \dot{m}\left[(v_o)_x - (v_i)_x\right] \\
F + p_1 A_1 &= \dot{m}\,(v_2 - v_1) \\
F &= -p_1 A_1 + \dot{m}\,(v_2 - v_1) \\
F &= -(311.7 \times 10^3\,\mathrm{Pa})\left(\frac{\pi \times (0.1\,\mathrm{m})^2}{4}\right) + (7.85\,\mathrm{kg/\,s})\,(25 - 1)\,\mathrm{m/\,s} \\
&= -2259.\,7\,\mathrm{N}
\end{aligned}
$$

$$
\boxed{\text{Force on nozzle} = 2.26\ \text{kN to the left}}
$$

6.45: PROBLEM DEFINITION

Situation:

Water flows through a converging nozzle—additional details are provided in the problem statement.



Find:

Force at the flange to hold the nozzle in place: F

PLAN

Apply the Bernoulli equation to establish the pressure at section 1, and then apply the momentum equation to find the force at the flange.

SOLUTION

Continuity equation (select a control volume that surrounds the nozzle).

$$Q_1 = Q_2 = Q = 0.56\,\mathrm{m^3/\,s}$$

Flow rate equations

$$\begin{aligned}
v_1 &= \frac{Q}{A_1} = \frac{4 \times Q}{\pi D_1^2} = \frac{4 \times (0.56\,\mathrm{m^3/\,s})}{\pi\,(0.65\,\mathrm{m})^2} \\
&= 1.7\ \mathrm{m/s} \\
v_2 &= \frac{Q}{A_2} = \frac{4 \times Q}{\pi D_2^2} = \frac{4 \times (0.56\,\mathrm{m^3/\,s})}{\pi\,(0.225\,\mathrm{m})^2} \\
&= 14.1\,\mathrm{m/s}
\end{aligned}$$

Bernoulli equation

$$\begin{aligned}
p_1 + \frac{\rho v_1^2}{2} &= p_2 + \frac{\rho v_2^2}{2} \\
p_1 &= 0 + \frac{\rho(v_2^2 - v_1^2)}{2} \\
&= \frac{1000\,\mathrm{kg/m^3}((14.1\,\mathrm{m/s})^2 - (1.7\,\mathrm{m/s})^2)}{2} \\
&= 97,960\,\mathrm{N/m^2}
\end{aligned}$$

Momentum equation (x-direction)

$$p_1 A_1 + F = \dot{m} v_2 - \dot{m} v_1$$

Calculations

$$
\begin{aligned}
p_1 A_1 &= (97,960\,\mathrm{N/m^2})(\pi/4)(0.65\,\mathrm{m})^2 \\
&= 32,490\,\mathrm{N} \\
\dot{m}v_2 - \dot{m}v_1 &= \rho Q\,(v_2 - v_1) \\
&= (1000\,\mathrm{kg/m^3})(0.56\,\mathrm{m^3/\,s})\,(14.1 - 1.7)\ \mathrm{m/s} \\
&= 6944\,\mathrm{N}
\end{aligned}
$$

Substituting numerical values into the momentum equation

$$
\begin{aligned}
F &= -p_1 A_1 + (\dot{m}v_2 - \dot{m}v_1) \\
&= -32,490\,\mathrm{N} + 6944\,\mathrm{N} \\
&= -25,546\,\mathrm{N}
\end{aligned}
$$

$$
\boxed{F = -25,546\ \ \mathrm{N}\ \text{(acts to left)}}
$$

6.46: PROBLEM DEFINITION

Situation:

Water flows through a converging nozzle—additional details are provided in the problem statement.

Find:

Force at the flange to hold the nozzle in place: F_x

PLAN

Apply the Bernoulli equation, and then the momentum equation.

SOLUTION

Velocity calculation

$$\begin{aligned} v_1 &= \frac{0.3\,\mathrm{m}^3}{\pi \times 0.15\,\mathrm{m} \times 0.15\,\mathrm{m}} = 4.244 \text{ m/s} \\ v_2 &= 4.244\,\mathrm{m/\,s} \times 9 = 38.196 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$p_1 = 0 + \frac{1000\,\mathrm{kg/\,m}^3}{2}(38.196^2 - 4.244^2)\,\mathrm{m}^2/\,\mathrm{s}^2 = 720 \text{ kPa}$$

Momentum equation (x-direction)

$$\begin{aligned} F_x &= -720,000\,\mathrm{Pa} \times \pi \times (0.15\,\mathrm{m})^2 + 1,000\,\mathrm{kg/\,m}^3 \times 0.3\,\mathrm{m}^3/\,\mathrm{s} \times (38.196 - 4.244)\,\mathrm{m/s} \\ &\boxed{F_x = -40.7 \text{ kN (acts to the left)}} \end{aligned}$$

6.47: PROBLEM DEFINITION

Situation:

Water flows through a nozzle with two openings—additional details are provided in the problem statement.

Find:

x-component of force through flange bolts to hold nozzle in place.

PLAN

Apply the Bernoulli equation, and then the momentum equation.

SOLUTION

Velocity calculation

$$\begin{aligned} v_A &= v_B = \frac{0.45\,\text{m}^3/\,\text{s}}{[(\pi/4)(0.1\,\text{m}\times 0.1\,\text{m} + 0.12\,\text{m}\times 0.12\,\text{m})]} \\ &= 23.4\,\text{m/s} \\ v_1 &= \frac{0.45\,\text{m}^3/\,\text{s}}{\pi\times 0.15\,\text{m}\times 0.15\,\text{m}} \\ &= 6.4\,\text{m/s} \end{aligned}$$

Bernoulli equation

$$p_1 = 0 + \frac{1000\,\text{kg/m}^3}{2}(23.4\,\text{m/s}\times 23.4\,\text{m/s} - 6.4\,\text{m/s}\times 6.4\,\text{m/s}) = 253,300\,\text{Pa}$$

Momentum equation (x-direction)

$$F_x + \rho_1 A_1 \sin 30\,^\circ = -\dot{m}_A v_A - \dot{m}_B v_B \sin 30\,^\circ$$

$$F_x = -253,300 \text{ Pa} \times \pi \times 0.15\,\text{m}\times 0.15\,\text{m}\times \sin 30^\circ - 23.4\,\text{m/s}\times 1000\,\text{kg/m}^3 \times 23.4\,\text{m/s}\times \pi$$
$$\times 0.05\,\text{m}\times 0.05\,\text{m} - 6.4\,\text{m/s}\times 1000\,\text{kg/m}^3 \times 0.45\,\text{m}^3/\,\text{s}\sin 30^\circ$$

$$\boxed{F_x = -14,693 \text{ N}}$$

6.48: PROBLEM DEFINITION

Situation:

Water flows through a nozzle with two openings—additional details are provided in the problem statement.

Find:

x-component of force through flange bolts to hold nozzle in place: F_x

PLAN

Apply the Bernoulli equation, and then the momentum equation.

SOLUTION

Velocity calculation

$$\begin{aligned} v_A &= v_B = Q/(A_1 + A_2) \\ &= \frac{0.65\,\mathrm{m^3/s}}{\pi \times 0.04\,\mathrm{m} \times 0.04\,\mathrm{m} + \pi \times 0.045\,\mathrm{m} \times 0.045\,\mathrm{m}} = 57.1 \text{ m/s} \\ v_1 &= \frac{0.65\,\mathrm{m^3/s}}{\pi \times 0.15\,\mathrm{m} \times 0.15\,\mathrm{m}} = 9.20 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$p_1 = \frac{1000\,\mathrm{kg/m^3}}{2}\left((57.1\,\mathrm{m/s})^2 - (9.20\,\mathrm{m/s})^2\right) = 1,586,570 \text{ Pa}$$

Momentum equation (x-direction)

$$\sum F_x = \dot{m}_o v_{ox} - m_i v_{ix}$$

$$F_x + p_1 A_1 \sin 30^\circ = -\dot{m} v_A - \dot{m} v_i \sin 30^\circ$$

$$F_x = -1,586,570\,\mathrm{Pa} \times \pi \times (0.15\,\mathrm{m})^2 \times \sin 30^\circ - 57.1\,\mathrm{m/s} \times 1,000\,\mathrm{kg/m^3} \times 57.1\,\mathrm{m/s}$$

$$\times \pi \times (0.04\,\mathrm{m})^2 - 9.20\,\mathrm{m/s} \times 1000\,\mathrm{kg/m^3} \times 0.65\,\mathrm{m^3/s} \times \sin 30^\circ = -75,438\mathrm{N}$$

$$\boxed{F_x = -75.4 \text{ kN}}$$

6.49: PROBLEM DEFINITION

Situation:

A rocket nozzle is connected to a combustion chamber.

Mass flow: $\dot{m} = 220\,\mathrm{kg/\,s}$. Ambient pressure: $p_o = 100\,\mathrm{kPa}$.

Nozzle inlet conditions: $A_1 = 1\,\mathrm{m}^2$, $u_1 = 100\,\mathrm{m/\,s}$, $p_1 = 1.5\,\mathrm{MPa}$-abs.

Nozzle exit condition? $A_2 = 2\,\mathrm{m}^2$, $u_2 = 2000\,\mathrm{m/\,s}$, $p_2 = 80\,\mathrm{kPa}$-abs.

Assumptions:

The rocket is moving at a steady speed.

Find:

Force on the connection between the nozzle and the chamber.

PLAN

Apply the momentum equation to a control volume situated around the nozzle.

SOLUTION

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} \\ F + p_1 A_1 - p_2 A_2 &= \dot{m}(v_2 - v1) \end{aligned}$$

where F is the force carried by the material that connects the rocket nozzle to the rocket chamber.

Calculations (note the use of gage pressures).

$$\begin{aligned} F &= \dot{m}(v_2 - v1) + p_2 A_2 - p_1 A_1 \\ &= (220\,\mathrm{kg/\,s})\,(2000 - 100)\,\mathrm{m/\,s} + \left(-20,000\,\mathrm{N/\,m^2}\right)\left(2\,\mathrm{m}^2\right) \\ &\quad - \left(1,400,000\,\mathrm{N/\,m^2}\right)\left(1\,\mathrm{m}^2\right) \\ &= -1.022 \times 10^6\ \mathrm{N} \\ &= -1.022\ \mathrm{MN} \end{aligned}$$

The force on the connection will be

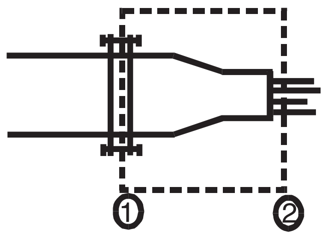
$$\boxed{F = 1.02\ \mathrm{MN}}$$

The material in the connection is in tension.

6.50: PROBLEM DEFINITION

Water flows through a nozzle.
The nozzle is bolted to a pipe flange with 6 bolts.
$D_1 = 0.30\,\text{m}$, $D_2 = 0.15\,\text{m}$, $p_1 = 200\,\text{kPa}$ gage.

Sketch:

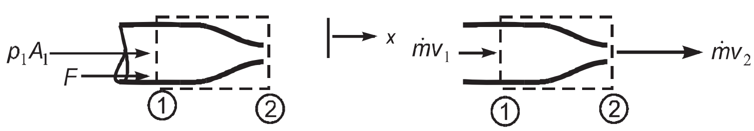


Find:
Tension in each bolt (in Newtons)

PLAN Since force is the goal, start with the momentum equation. Then, apply continuity and the Bernoulli equations to find terms needed to calculate force. The steps are.
1. Apply the momentum equation to relate force to properties at 1 and 2.
2. Relate v_2 and v_1 using continuity.
3. Solve for v_2, v_1, and Q using the Bernoulli equation and the flow rate equation.
4. Calculate force.

SOLUTION

1. Momentum equation (x-direction)



$$\begin{aligned}\sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} \\ F_{\text{bolts}} + p_1 A_1 &= \rho Q (v_2 - v_1)\end{aligned}$$

2. Continuity equation (apply to cv shown above; accumulation is zero).

$$v_2 = \frac{A_1}{A_2} v_1 = \left(\frac{0.30\,\text{m}}{0.15\,\text{m}}\right)^2 v_1 = 4v_1$$

3. Bernoulli equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 \\
\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + 0 &= 0 + \frac{16v_1^2}{2g} + 0 \\
\frac{200000\,\mathrm{Pa}}{9810\,\mathrm{N/\,m^3}} &= \frac{15v_1^2}{2g} \\
v_1 &= 5.16 \text{ m/s}
\end{aligned}$$

$$v_2 = 4v_1 = 20.66 \text{ m/s}$$

Flow rate equation.

$$Q = A_1 v_1 = \frac{\pi\,(0.3\,\mathrm{m})^2}{4}\,(5.16\,\mathrm{m/\,s}) = 0.365\,\mathrm{m^3/\,s}$$

4. Calculate force

$$\begin{aligned}
F_{\text{bolts}} &= -p_1 A_1 + \rho Q(v_2 - v_1) \\
F_{\text{bolts}} &= -\,(200,000\,\mathrm{Pa})\,\pi\,(0.15\,\mathrm{m})^2 + \left(1000\,\mathrm{kg/\,m^3}\right)\left(0.365\,\mathrm{m^3/\,s}\right)(20.66\,\mathrm{m/\,s} - 5.16\,\mathrm{m/\,s}) \\
&= -8480 \text{ N}
\end{aligned}$$

$$\boxed{\text{Force per bolt} = 1410 \text{ N}}$$

6.51: PROBLEM DEFINITION

Situation:

Water jets out of a two dimensional slot.

Flow rate is $Q = 0.23\,\mathrm{m^3/s}$ per meter of slot width. Slot spacing is $H = 20\,\mathrm{cm}$. Jet height is $b = 10\,\mathrm{cm}$.

Find:

(a)Pressure at the gage.

(b)Force (per meter of length of slot) of the water acting on the end plates of the slot.

PLAN

To find pressure at the centerline of the flow, apply the Bernoulli equation. To find the pressure at the gage (higher elevation), apply the hydrostatic equation. To find the force required to hold the slot stationary, apply the momentum equation.

SOLUTION

Continuity. Select a control volume surrounding the nozzle. Locate section 1 across the slot. Locate section 2 across the water jet.

$$Q_1 = Q_2 = Q = \frac{0.23\,\mathrm{m^3/\,s}}{0.3\,\mathrm{m}}$$

Flow rate equations

$$\begin{aligned}
V_1 &= \frac{Q}{A_1} = \frac{0.76\,\mathrm{m^2/\,s}}{(0.2)\ \mathrm{m}} \\
&= 3.8\,\mathrm{m/s} \\
V_2 &= \frac{Q}{A_2} = \frac{0.76\,\mathrm{m^2/\,s}}{(0.10)\ \mathrm{m}} \\
&= 7.6\,\mathrm{m/s}
\end{aligned}$$

Bernoulli equation

$$\begin{aligned}
p_1 &= \frac{\rho}{2}(V_2^2 - V_1^2) \\
&= \frac{1000\,\mathrm{kg/m^3}}{2}(7.6^2 - 3.8^2)\frac{\mathrm{m^2}}{\mathrm{s^2}} \\
p_1 &= 21,660\,\mathrm{N/m^2}
\end{aligned}$$

Hydrostatic equation. Location position 1 at the centerline of the slot. Locate position 3 at the gage.

$$\begin{aligned}
\frac{p_1}{\gamma} + z_1 &= \frac{p_3}{\gamma} + z_3 \\
\frac{21,660\,\mathrm{N/m^2}}{9810\,\mathrm{N/m^3}} + 0 &= \frac{p_3}{9810\,\mathrm{N/m^3}} + \frac{(0.2)\ \mathrm{m}}{2} \\
p_3 &= 20,686\ \mathrm{Pa}
\end{aligned}$$

$$\boxed{p_3 = 21\,\text{kPa}}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}V_2 - \dot{m}V_1 \\
F_x + p_1 A_1 &= \rho Q(V_2 - V_1) \\
F_x &= -p_1 A_1 + \rho Q(V_2 - V_1) \qquad (1)
\end{aligned}$$

Calculations

$$\begin{aligned}
p_1 A_1 &= (21,660\,\text{Pa})\,(0.2\,\text{m}) \\
&= 4332\,\text{N/m} \qquad (\text{a}) \\
\rho Q(V_2 - V_1) &= \left(1000\,\text{kg/m}^3\right)\left(0.76\,\text{m}^2/\,\text{s}\right)(7.6\,\text{m/s} - 3.8\,\text{m/s}) \\
&= 2888\,\text{N/m} \qquad (\text{b})
\end{aligned}$$

Substitute (a) and (b) into Eq. (1)

$$\begin{aligned}
F_x &= -\,(4332\,\text{N/m}) + 2888\,\text{N/m} \\
&= -1444\,\text{N/m}
\end{aligned}$$

The force acting on the end plates is equal in magnitude and opposite in direction (Newton's third law).

$$\boxed{F_{\text{water on the end plates}} = 1444\,\text{N/m acting to the right}}$$

6.52: PROBLEM DEFINITION

Situation:

Water is discharged from a two-dimensional slot—additional details are provided in the problem statement

Find:

(a)Pressure at the gage.

(b)Force (per meter of length of slot) on the end plates of the slot.

PLAN

Apply the Bernoulli equation, then the hydrostatic equation, and finally the momentum equation.

SOLUTION

Velocity calculation

$$\begin{aligned} v_b &= \frac{0.4\,\mathrm{m}^3/\,\mathrm{s}}{0.07\,\mathrm{m}^2} = 5.71 \text{ m/s} \\ v_B &= \frac{0.4\,\mathrm{m}^3/\,\mathrm{s}}{0.2\,\mathrm{m}^2} = 2.00 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$p_B = \frac{1000\,\mathrm{kg}/\,\mathrm{m}^3}{2}((5.71\,\mathrm{m}/\,\mathrm{s})^2 - (2.00\,\mathrm{m}/\,\mathrm{s})^2) = 14,302 \text{ kPa}$$

Hydrostatic equation

$$\begin{aligned} p_{\text{gage}} &= 14,302\,\mathrm{kPa} - 9810\,\mathrm{N}/\,\mathrm{m}^3 \times 0.1\,\mathrm{m}^2 \\ &\boxed{p_{\text{gage}} = 13.3 \text{ kPa}} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} \\ F_x + p_B A_B &= \rho Q(v_b - v_B) \end{aligned}$$

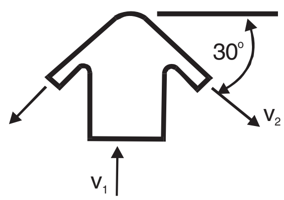
thus

$$\begin{aligned} F_x &= -14,302\,\mathrm{kPa} \times 0.2\,\mathrm{m}^2 + 1000\,\mathrm{kg}/\,\mathrm{m}^3 \times 0.4\,\mathrm{m}^3/\,\mathrm{s} \times (5.71 - 2.00)\,\mathrm{m}/\,\mathrm{s} \\ &= -1,376 \text{ N} \\ &\boxed{F_x = -1.38 \text{ kN/m}} \end{aligned}$$

6.53: PROBLEM DEFINITION

Situation:

Water flows through a spray head—additional details are provided in the problem statement.



Find:

Force acting through the bolts needed to hold the spray head on: F_y

PLAN

Apply the Bernoulli equation, and then the momentum equation.

SOLUTION

Velocity calculation

$$v_1 = \frac{Q}{A_1} = \frac{0.12\,\mathrm{m}^3/\,\mathrm{s}}{\pi/4 \times (0.15\,\mathrm{m})^2} = 6.8 \text{ m/s}$$

Bernoulli equation

$$\begin{aligned} p_1 &= \frac{\rho}{2}\left(v_2^2 - v_1^2\right) \\ &= \frac{1000\,\mathrm{kg/m}^3}{2}\left((20\,\mathrm{m/s})^2 - (6.8\,\mathrm{m/s})^2\right) \\ &= 176,880 \text{ Pa} \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= \dot{m}_o v_{oy} - \dot{m}_i v_{iy} \\ F_y + p_1 A_1 &= \rho Q(-v_2 \sin 30^\circ - v_1) \\ F_y &= (-176,880 \text{ Pa})(\pi/4 \times (0.15\,\mathrm{m})^2) \\ &\quad +1000\,\mathrm{kg/m}^3 \times 0.12\,\mathrm{m}^3/\,\mathrm{s} \times (-20\,\mathrm{m/s}\sin 30^\circ - 6.8\,\mathrm{m/s}) \\ &\quad \boxed{F_y = -5140 \text{ N}} \end{aligned}$$

6.54: PROBLEM DEFINITION

Situation:

An unusual nozzle creates two jets of water.
$d = 0.012\,\text{m}, v_2 = v_3 = 25\,\text{m/s}$.
$D = 0.09\,\text{m}\ p = 345\ \text{kPa}$.

Find:

Force required at the flange to hold the nozzle in place: **F**

PLAN

Apply the continuity equation, then the momentum equation.

SOLUTION

Continuity equation

$$\begin{aligned} v_1 &= \frac{Q}{A} \\ &= \frac{2 \times 25\,\text{m/s} \times \pi/4 \times (0.012\,\text{m})^2}{\pi/4 \times (0.09\,\text{m})^2} \\ &= 0.9\,\text{m/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum \dot{m}_{ox} - \dot{m}_i v_{ix} \\ p_1 A_1 + F_x &= \dot{m}_2 v_{2x} + \dot{m}_3 v_{3x} - \dot{m}_1 v_{1x} \\ F_x &= -345\,\text{kPa} \times \pi\,(0.045\,\text{m})^2 + (1000\,\text{kg/m}^3)\,(25\,\text{m/s})^2 \times \pi\,(0.006\,\text{m})^2 \\ &\quad -(1000\,\text{kg/m}^3 \times 25\,\text{m/s} \times \pi \times (0.006\,\text{m})^2 \times 25\,\text{m/s}) \times \sin 30^\circ \\ &\quad -(1000\,\text{kg/m}^3 \times 0.9\,\text{m/s} \times \pi \times (0.006\,\text{m})^2) \times 0.9\,\text{m/s} \\ &= -2158.8\ \text{N} \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= \dot{m}_{oy} - \dot{m}_i v_{iy} \\ F_y &= \dot{m}_3 v_{3y} = \rho A v_3(-v_3 \cos 30^\circ) \\ &= -1000\,\text{kg/m}^3(\pi/4 \times (0.012)^2\,\text{m}^2)\,(25\,\text{m/s})^2 \cos 30^\circ \\ &= 61.2\ \text{N} \end{aligned}$$

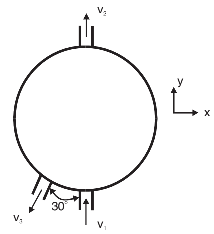
Net force

$$\boxed{\mathbf{F} = (2158.8 - 61.2)\ \text{N}}$$

6.55: PROBLEM DEFINITION

Situation:

Liquid flows through a "black sphere" —additional details are provided in the problem statement.



Find:

Force in the inlet pipe wall required to hold sphere stationary: **F**

PLAN

Apply the continuity equation, then the momentum equation.

SOLUTION

Continuity equation

$$\begin{aligned} A_1 v_1 &= A_2 v_2 + A_3 v_3 \\ v_3 &= v_1 \frac{A_1}{A_3} - v_2 \frac{A_2}{A_3} \\ &= 15\,\mathrm{m/s}\left(\frac{0.05^2}{0.025^2}\right) - 30\,\mathrm{m/s}\left(\frac{0.025^2}{0.025^2}\right) \\ &= 30\,\mathrm{m/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} F_x &= \dot{m}_3 v_{3x} \\ &= -\rho A_3 v_3^2 \sin 30^\circ \\ &= -(1000\,\mathrm{kg/m^3} \times 1.2)\left(\frac{\pi 0.025\,\mathrm{m}^2}{4}\right)(30\,\mathrm{m/s})^2 \sin 30^\circ \\ &= -265\ \mathrm{N} \end{aligned}$$

y-direction

$$F_y - W + p_1 A_1 = \dot{m}_2 v_{2y} + \dot{m}_3 v_{3y} - \dot{m}_1 v_{1y}$$

thus

$$F_y = W - p_1 A_1 + \dot{m}_2 v_2 - m_3 v_3 \cos 30^\circ - \dot{m}_1 v_1$$

Calculations

$$
\begin{aligned}
W - p_1 A_1 &= 900 - 400,000 \times \pi \times 0.025^2 \\
&= 115 \text{ N} \\
\dot{m}_2 v_2 &= \rho A_2 v_2^2 \\
&= (1.2 \times 1000\,\text{kg/m}^3) \left(\frac{\pi \left(0.025\right)^2 \ \text{m}^2}{4} \right) (30\,\text{m/s})^2 \\
&= 530 \text{ N} \\
\dot{m}_3 v_3 \cos 30^\circ &= \rho A_3 v_3^2 \cos 30^\circ \\
&= (1.2 \times 1000\,\text{kg/m}^3) \left(\frac{\pi \left(0.025\right)^2 \ \text{m}^2}{4} \right) (30\,\text{m/s})^2 \cos 30^\circ \\
&= 459 \text{ N} \\
\dot{m}_1 v_1 &= \rho A_1 v_1^2 \\
&= (1.2 \times 1000\,\text{kg/m}^3) \left(\frac{\pi \left(0.05\right)^2 \ \text{m}^2}{4} \right) (15)^2 \\
&= 530 \text{ N}
\end{aligned}
$$

thus,

$$
\begin{aligned}
F_y &= (W - p_1 A_1) + \dot{m}_2 v_2 - (m_3 v_3 \cos 30^\circ) - \dot{m}_1 v_1 \\
&= (115)\,\text{N} + 530\,\text{N} - (459)\ \text{N} - 530\,\text{N} \\
&= -344 \text{ N}
\end{aligned}
$$

Net Force

$$
\boxed{\mathbf{F} = (-265\mathbf{i} - 344\mathbf{j})\ \text{N}}
$$

6.56: PROBLEM DEFINITION

Situation:

Liquid flows through a "black sphere"—additional details are provided in the problem statement.

Find:

Force required in the pipe wall to hold the sphere in place: **F**

PLAN

Apply the continuity equation, then the momentum equation.

SOLUTION

Continuity equation

$$\begin{aligned} v_3 &= \frac{10 \times 5^2 - 30 \times 2.5^2}{(2.5)^2} \\ &= 10 \text{ m/s} \\ \rho &= S\rho_w = 1.5 \times 1000\,\text{kg/m}^3 = 1500\,\text{kg/m}^3 \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} F_x &= -\dot{m}_j v_j \sin 30^\circ = -\rho_j A_j v_j^2 \sin 30^\circ \\ &= -10 \sin 30^\circ \times 1500\,\text{kg/m}^3 \times 10\,\text{m/s} \times \pi \times (0.0125\,\text{m})^2 \\ &= -36.8 \text{ N} \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} F_y &= -P_i A_i + W_t - \dot{m}_i v_i + \dot{m}_o v_o - \dot{m}_j v_j \cos 30^\circ \\ &= -400,000\,\text{Pa} \times \frac{\pi}{4} \times (0.025\,\text{m})^2 + 600\,\text{N} + (1500\,\text{kg/m}^3 \times \pi) \\ &\quad \times (-(10\,\text{m/s})^2 \times (0.025\,\text{m})^2 + (30\,\text{m/s})^2 \times (0.0125\,\text{m})^2 \\ &\quad - (10\,\text{m/s})^2 \times (0.0125\,\text{m})^2 \cos 30^\circ) \\ &= 119 \text{ N} \end{aligned}$$

Net Force

$$\boxed{\mathbf{F} = (-36.8\mathbf{i} + 119\mathbf{j}) \text{ N}}$$

6.57: PROBLEM DEFINITION

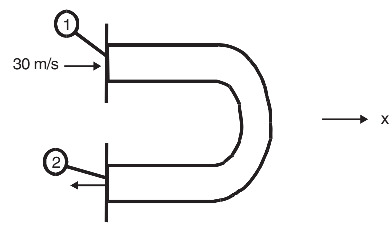
Situation:
Hot gas flows through a return bend—additional details in problem statement.

Find:
Force required to hold the bend in place: F_x

PLAN

Apply the continuity equation, then the momentum equation.

SOLUTION



$$\dot{m} = 0.45 \text{ kg/s}$$

At section (1):

$$\begin{aligned} v_1 &= 30 \text{ m/s} \\ \rho_1 &= 0.32 \text{ kg/m}^3 \end{aligned}$$

At section (2):

$$\rho_2 = 1 \text{ kg/m}^3$$

Continuity equation

$$\begin{aligned} \rho_1 v_1 A_1 &= \rho_2 v_2 A_2 \\ v_2 &= \frac{\rho_1}{\rho_2}\left(\frac{A_1}{A_2}\right) v_1 \\ v_2 &= \left(\frac{0.32 \text{ kg/m}^3}{1 \text{ kg/m}^3}\right)\left(\frac{0.45}{0.45}\right) 30 \\ &= 9.6 \text{ m/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{\subset s} \dot{m}_o v_{o_x} - \sum_{cs} \dot{m}_i v_{i_x} \\ &= \dot{m}(v_2 - v_1) \\ F_x &= 0.45\,\text{kg/s}(-9.6\,\text{m/s} - 30\,\text{m/s}) \\ & \boxed{F_x = -17.8 \text{ N}} \end{aligned}$$

6.58: PROBLEM DEFINITION

Situation:

Fluid (density ρ, discharge Q, and velocity V) flows through a 180^o pipe bend—additional details are provided in the problem statement.. Cross sectional area of pipe is A.

Find:

Magnitude of force required at flanges to hold the bend in place.

Assumptions:

Gage pressure is same at sections 1 and 2. Neglect gravity.

PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ p_1 A_1 + p_2 A_2 + F_x &= \dot{m}(v_2 - v_1) \end{aligned}$$

thus

$$\boxed{F_x = -2pA - 2\dot{m}V}$$

$$\boxed{F_x = -2pA - 2\rho QV}$$

Correct choice is (d)

6.59: PROBLEM DEFINITION

Situation:

Water flows through a 180^o pipe bend—additional details are provided in the problem statement.

Find:

External force required to hold bend in place.

PLAN

Apply the momentum equation.

SOLUTION

Flow rate equation

$$v = \frac{Q}{A} = \frac{0.6\,\mathrm{m^3/\,s}}{\pi \times 0.15\,\mathrm{m} \times 0.15\,\mathrm{m}} = 8.5\,\mathrm{m/s}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ p_1 A_1 + p_2 A_2 + F_x &= \dot{m}(v_2 - v_1) \end{aligned}$$

thus

$$\begin{aligned} F_x &= -2pA - 2\dot{m}v \\ &= -2(105,000\,\mathrm{N/m^2} \times \frac{\pi}{4} \times (0.3\,\mathrm{m})^2 + 1000\,\mathrm{kg/m^3} \times 0.6\,\mathrm{m^3/\,s} \times 8.5\,\mathrm{m/s}) \\ &= -12,518\ \mathrm{N} \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= 0 \\ -W_{\mathrm{bend}} - W_{H_20} + F_y &= 0 \\ F_y &= 900\,\mathrm{N} + 0.085\,\mathrm{m^3} \times 9810\,\mathrm{N/m^3} = 1734\ \mathrm{N} \end{aligned}$$

Force required

$$\boxed{\mathbf{F} = (-12,518\mathbf{i} + 1734\mathbf{j})\ \mathrm{N}}$$

6.60: PROBLEM DEFINITION

Situation:

Water flows through a 180° pipe bend—additional details are provided in the problem statement.

Find:

Force that acts on the flanges to hold the bend in place.

PLAN

Apply the continuity and momentum equations.

SOLUTION

Flow rate

$$\begin{aligned} v_1 &= \frac{Q}{A} \\ &= \frac{4 \times 0.35\,\mathrm{m}^3/\,\mathrm{s}}{\pi \times (0.2\,\mathrm{m})^2} \\ &= 11.14\ \mathrm{m/s} \end{aligned}$$

Continuity. Place a control volume around the pipe bend. Let section 2 be the exit and section 1 be the inlet

$$\begin{aligned} Q &= A_1 v_1 = A_2 v_2 \\ \text{thus } v_1 &= v_2 \end{aligned}$$

Momentum equation (x-direction). Place a control volume around the pipe bend. Let section 2 be the exit and section 1 be the inlet.

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ 2pA + F_x &= \rho Q\,(-v_2) - \rho Q v_1 \\ F_x &= -2pA - 2\rho Q v \end{aligned}$$

Calculations

$$\begin{aligned} 2pA &= (2)(100,000\,\mathrm{Pa})(\frac{\pi}{4})(0.2\,\mathrm{m})^2 \\ &= 6283\,\mathrm{N} \\ 2\rho Q v &= (2)(1000\,\mathrm{kg/\,m^3})(0.35\,\mathrm{m^3/\,s})(11.14\,\mathrm{m/\,s}) \\ &= 7798\,\mathrm{N} \\ F_x &= -\left(2pA + 2\rho Q v\right) \\ &= -\left(6283\,\mathrm{N} + 7798\,\mathrm{N}\right) \\ &= -14.1\,\mathrm{kN} \end{aligned}$$

Momentum equation (z-direction). There are no momentum flow terms so the momentum equation simplifies to

$$\begin{aligned} F_z &= W_{\text{bend}} + W_{\text{water}} \\ &= 400\,\text{N} + (0.1\,\text{m}^3)(9810\,\text{N}/\,\text{m}^3) \\ &= 1.381 \text{ kN} \end{aligned}$$

The force that acts on the flanges is

$$\boxed{\mathbf{F} = (-14.1\mathbf{i} + 0\mathbf{j} + 1.38\mathbf{k}) \text{ kN}}$$

6.61: PROBLEM DEFINITION

Situation:

Set up the solution for the preceeding problem, and answer the following questions:

Find:

a. Do the 2 pressure forces from the inlet and exit act in the same direction, or in opposite directions?

b. For the data given, which term has the larger magnitude (in N), the pressure force term, or the net momentum flux term?

SOLUTION

a. The 2 pressure forces acting on the inlet and the exit are both acting **compressively**, so they are both acting to the right.

b. For the data given, the pressure force term, at 6283 N, is slightly larger than the momentum term, at 5730 N. The force that acts on the flanges is the sum of these 2 terms.

6.62: PROBLEM DEFINITION

Situation:

A 90° pipe bend is described in the problem statement.

Find:

Force on the upstream flange to hold the bend in place.

PLAN

Apply the momentum equation.

SOLUTION

Velocity calculation

$$v = \frac{Q}{A} = \frac{0.34\,\mathrm{m^3/\,s}}{\pi/4 \times (0.3\,\mathrm{m})^2} = 4.8\ \mathrm{m/s}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
pA + F_x &= \rho Q(0 - v) \\
F_x &= 1000\,\mathrm{kg/m^3} \times 0.34\,\mathrm{m^3/\,s}(0 - 4.8\,\mathrm{m/s}) - 28,000\,\mathrm{N/m^2} \times \frac{\pi}{4} \times (0.3\,\mathrm{m})^2 = -3610\ \mathrm{N}
\end{aligned}$$

y-direction

$$\begin{aligned}
F_y &= \rho Q(-v - 0) \\
F_y &= -(1000\,\mathrm{kg/\,m^3} \times 0.34\,\mathrm{m^3/\,s} \times 4.8\,\mathrm{m/\,s}) = -1632\ \mathrm{N}
\end{aligned}$$

z-direction

$$\sum F_z = 0$$

$$-445\,\mathrm{N} - 0.1\,\mathrm{m^3} \times 9810\,\mathrm{N/\,m^3} + F_z = 0$$

$$F_z = +1426\ \mathrm{N}$$

The force is

$$\boxed{\mathbf{F} = (-3610\mathbf{i}\ - 1632\mathbf{j}\ + 1426\mathbf{k})\ \mathrm{N}}$$

6.63: PROBLEM DEFINITION

Situation:
A 90^0 pipe bend is described in the problem statement.

Find:
$x-$component of force applied to bend to hold it in place: F_x

PLAN

Apply the momentum equation.

SOLUTION

Velocity calculation

$$v = \frac{Q}{A} = \frac{10\,\mathrm{m}^3/\,\mathrm{s}}{\pi/4 \times (1\,\mathrm{m})^2} = 12.73 \text{ m/s}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}v_{ox} - \sum_{cs} \dot{m}v_{ix} \\ pA + F_x &= \rho Q(0 - v) \end{aligned}$$

$$\begin{aligned} 300,000\,\mathrm{Pa} \times \pi \times (0.5\,\mathrm{m})^2 + F_x &= 1000\,\mathrm{kg}/\,\mathrm{m}^3 \times 10\,\mathrm{m}^3/\,\mathrm{s} \times (0 - 12.73\,\mathrm{m}/\,\mathrm{s}) \\ F_x &= -362,919 \text{ N} \\ & \boxed{F_x = -363 \text{ kN}} \end{aligned}$$

6.64: PROBLEM DEFINITION

Situation:

Water flows through a 30^o pipe bend—additional details are provided in the problem statement.

Find:

Vertical component of force exerted by the anchor on the bend: F_a

PLAN

Apply the momentum equation.

SOLUTION

Velocity calculation

$$\begin{aligned} v &= \frac{Q}{A} \\ &= \frac{0.9\,\mathrm{m}^3/\,\mathrm{s}}{\pi \times 0.3\,\mathrm{m} \times 0.3\,\mathrm{m}} \\ &= 3.2\ \mathrm{m/s} \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= \rho Q(v_{2y} - v_{1y}) \\ F_a - W_{\text{water}} - W_{\text{bend}} - p_2 A_2 \sin 30^\circ &= \rho Q(v \sin 30^\circ - v \sin 0^\circ) \\ F_a &= \pi \times 0.3\,\mathrm{m} \times 0.3\,\mathrm{m} \times 1.2\,\mathrm{m} \times 9810\,\mathrm{N/m}^3 + 1350\,\mathrm{N} \\ & \quad 60,000\,\mathrm{N/m}^2 \times 1000\,\mathrm{kg/m}^3 \\ & \quad 1000\,\mathrm{kg/m}^3 \times 0.9\,\mathrm{m}^3/\,\mathrm{s} \times (3.2\,\mathrm{m/s} \times 0.5 - 0) \\ & \quad \boxed{F_a = 14,595\ \ \mathrm{N}} \end{aligned}$$

6.65: PROBLEM DEFINITION

Situation:

Water flows through a 60^o pipe bend and jets out to atmosphere—additional details are provided in the problem statement.

Find:

Magnitude and direction of external force components to hold bend in place.

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION

Flow rate equation

$$\begin{aligned} \left(\frac{D_2}{D_1}\right)^2 v_2 &= \left(\frac{30\,\text{cm}}{60\,\text{cm}}\right)^2 10\,\text{m/s} = 2.5\,\text{m/s} \\ Q &= A_1 v_1 = \pi \times 0.3\,\text{m} \times 0.3\,\text{m} \times 2.5\,\text{m/s} = 0.707\ \text{m}^3/\text{s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_1 &= p_2 + \frac{\rho}{2}(v_2^2 - v_1^2) \\ &= 0 + \frac{1000\,\text{kg/m}^3}{2}\left(\left(\frac{10\,\text{m}}{\text{s}}\right)^2 - \left(\frac{2.5\,\text{m}}{\text{s}}\right)^2\right) \\ &= 46,875\ \text{Pa gage} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} F_x + p_1 A_1 &= \rho Q(-v_2 \cos\ 60^\circ - v_1) \\ F_x &= -46,875\,\text{Pa} \times \pi \times 0.3\,\text{m} \times 0.3\,\text{m} \\ &\quad +1000\,\text{kg/m}^3 \times 0.707\,\text{m}^3/\text{s} \times (-10\,\text{m/s}\cos 60^\circ - 2.5\,\text{m/s}) \\ &= -18,560\ \text{N} \end{aligned}$$

y-direction

$$\begin{aligned} F_y &= \rho Q(-v_2 \sin 60^\circ - v_1) \\ F_y &= 1000\,\text{kg/m}^3 \times 0.707 \times (-10\,\text{m/s}\sin 60^\circ - 0) \\ &= -6120\ \text{N} \end{aligned}$$

z-direction

$$\begin{aligned} F_z - W_{\text{H}_20} - W_{\text{bend}} &= 0 \\ F_z &= (0.25\,\text{m}^3 \times 9,810\,\text{N/m}^3) + (250\,\text{kg} \times 9.81\,\text{m/s}^2) = 4,905\ \text{N} \end{aligned}$$

Net force

$$\boxed{\mathbf{F} = (-18.6\mathbf{i} - 6.12\mathbf{j} + 4.91\mathbf{k})\ \text{kN}}$$

6.66: PROBLEM DEFINITION

Situation:

Water flows through a nozzle—additional details are provided in the problem statement.

Find:

Vertical force applied to the nozzle at the flange: F_y

PLAN

Apply the continuity equation, then the Bernoulli equation, and then the momentum equation.

SOLUTION

Continuity equation

$$\begin{aligned} v_1 A_1 &= v_2 A_2 \\ v_1 &= v_2 \frac{A_2}{A_1} = 20 \text{ m/s} \\ Q &= v_2 A_2 = (40 \text{ m/s})(0.05 \text{ m}^2) \\ &= 2 \, \text{m}^3/\text{s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 \\ \frac{p_1}{\gamma} &= 0 + \frac{(40 \, \text{m/s})^2}{2g} + 0.6 - \frac{(20 \, \text{m/s})^2}{2g} \\ p_1 &= 9810 \, \text{N/m}^3 (81.55 + 0.6 - 20.4) \, \text{m} \\ p_1 &= 605,768 \text{ N/m}^2 \end{aligned}$$

Momentum equation (y-direction)

$$p_1 A_1 - W_{H_20} - W_{\text{nozzle}} + F_y = \rho Q (v_2 \sin 30^\circ - v_1) \tag{1}$$

Momentum flow terms

$$\begin{aligned} \rho Q (v_2 \sin 30^\circ - v_1) &= (1000 \, \text{kg/m}^3)(2 \, \text{m}^3/\text{s}) \left[(40 \, \text{m/s} \sin 30^\circ) - 20 \, \text{m/s} \right] \\ &= 0 \text{ N} \end{aligned}$$

Thus, Eq. (1) becomes

$$\begin{aligned} F_y &= W_{H_20} + W_{\text{nozzle}} - p_1 A_1 \\ &= (0.05 \, \text{m}^3 \times 9810 \, \text{N/m}^3) + (445 \, \text{N}) - (605,768 \, \text{N/m}^2 \times 0.1 \, \text{m}^2) \\ &= -59,641.5 \text{ N} \end{aligned}$$

$$\boxed{F_y = 59,640 \, \text{N (acting downward)}}$$

6.67: PROBLEM DEFINITION

Situation:

Gasoline flows through a 135^o pipe bend—additional details are provided in the problem statement.

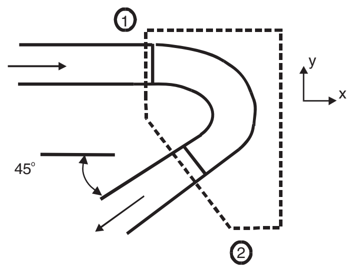
Find:

External force required to hold the bend: F

PLAN

Apply the momentum equation.

SOLUTION



Flow rate

$$\begin{aligned} Q &= vA = 6\,\text{m/s} \times \pi/4 \times (0.3\,\text{m})^2 \\ &= 0.424\,\text{m}^3/\text{s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \rho Q(v_{2x} - v_{1x}) \\ p_1A_1 + p_2A_2\cos 45^\circ + F_x &= \rho Q(-v_2\cos 45^\circ - v_1) \end{aligned}$$

$$\begin{aligned} F_x &= -pA(1+\cos 45^\circ) - \rho Qv(1+\cos 45^\circ) \\ &= -(70,000\,\text{N/m}^2) \times (\pi/4 \times (0.3\,\text{m})^2)(1+\cos 45^\circ) \\ &\quad -(0.8 \times 1000\,\text{kg/m}^3)(0.424\,\text{m}^3/\,\text{s})(6\,\text{m/s})(1+\cos 45^\circ) \\ &= -11,917\ \text{N} \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= \rho Q(v_{2y} - v_{1y}) \\ p_2A_2\sin 45^\circ + F_y &= \rho Q(-v_2\sin 45^\circ - 0) \end{aligned}$$

$$\begin{aligned} F_y &= -pA\sin 45^\circ - \rho Qv_2\sin 45^\circ \\ F_y &= -(70,000\,\text{N/m}^2)(\pi/4 \times (0.3\,\text{m})^2)\sin 45^\circ - (0.8 \times 1000\,\text{kg/m}^3)(0.424\,\text{m}^3/\,\text{s})(6\,\text{m/s})\sin 45^\circ \\ F_y &= -4936\ \text{N} \end{aligned}$$

Net force

$$\boxed{\mathbf{F} = (-11,917\mathbf{i} - 4936\mathbf{j})\ \text{N}}$$

6.68: PROBLEM DEFINITION

Situation:

A 180^o pipe bend (0.15 m diameter) carries water.

$Q = 0.06$ m^3/s $p = 140$ kPa gage

Find:

Force needed to hold the bend in place: F_x (the component of force in the direction parallel to the inlet flow)

Assumptions:

The weight acts perpendicular to the flow direction; the pressure is constant throughout the bend.

PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (x-direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ 2pA - F_x &= -2\dot{m}v\end{aligned}$$

Calculations

$$\begin{aligned}pA &= (140,000\,\mathrm{N/m^2})\left(\pi/4 \times (0.15\,\mathrm{m})^2\right) = 2475 \text{ N} \\ \dot{m}v &= \frac{\rho Q^2}{A} = \frac{1000\,\mathrm{kg/m^3} \times (0.06\,\mathrm{m^3/\,s})^2}{\pi/4 \times (0.15\,\mathrm{m})^2} = 205 \text{ N} \\ F_x &= 2(pA + \dot{m}v) = 2 \times (2475 + 205) \text{ N}\end{aligned}$$

$$\boxed{F_x = 5360 \text{ N (acting to the left, opposite of inlet flow)}}$$

6.69: PROBLEM DEFINITION

Situation:

Gasoline flows through a 135^o pipe bend—additional details are provided in the problem statement.

Find:

External force required to hold the bend in place: F

PLAN

Apply the momentum equation.

SOLUTION

Discharge

$$\begin{aligned} Q &= 8 \times \pi/4 \times 0.15\,\mathrm{m} \times 0.15\,\mathrm{m} \\ &= 0.141\ \mathrm{m}^3/\mathrm{s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}(v_{2x} - v_{1x}) \\ p_1A_1 + p_2A_2\cos 45^\circ + F_x &= \rho Q(-v_2\cos 45^\circ - v_1) \\ F_x &= -pA(1+\cos 45^0) - \rho Q v(1+\cos 45^\circ) \\ &= -(100,000\,\mathrm{Pa})(\pi/4 \times (0.15\,\mathrm{m})^2)(1+\cos 45^\circ) \\ &\quad -(1000\,\mathrm{kg/\,m^3} \times 0.8)(0.141\,\mathrm{m^3/\,s})(8\,\mathrm{m/\,s})(1+\cos 45^\circ) \\ &= -4557\ \mathrm{N} \end{aligned}$$

Momentum equation y-direction

$$\begin{aligned} \sum F_y &= \rho Q(v_{2y} - v_{1y}) \\ p_2A_2\sin 45^\circ + F_y &= -\rho Q v_2 \sin 45^\circ \\ &= -(100,000\,\mathrm{Pa})(\pi/4 \times (0.15\,\mathrm{m})^2)\sin 45^\circ \\ &\quad -(1,000\,\mathrm{kg/\,m^3} \times 0.8)(0.141\,\mathrm{m^3/\,s})(8\,\mathrm{m/\,s})\sin 45^\circ \\ &= -1,888\ \mathrm{N} \end{aligned}$$

Net force

$$\boxed{\mathbf{F} = (-4.56\mathbf{i} - 1.89\mathbf{j})\ \mathrm{kN}}$$

6.70: PROBLEM DEFINITION

Situation:

Water flows through a 60^o reducing bend—additional details are provided in the problem statement.

Find:

Horizontal force required to hold bend in place: F_x

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION

Bernoulli equation

$$\begin{aligned} v_1 &= v_2 \frac{A_2}{A_1} \\ &= 50\,\mathrm{m/\,s}\frac{1}{10} \\ &= 5\ \mathrm{m/s} \\ p_1 + \frac{\rho v_1^2}{2} &= p_2 + \frac{\rho v_2^2}{2} \end{aligned}$$

Let $p_2 = 0$, then

$$\begin{aligned} p_1 &= -\left(\frac{1,000\,\mathrm{kg/\,m^3}}{2}\right)(5\,\mathrm{m/\,s})^2 + \left(\frac{1,000\,\mathrm{kg/\,m^3}}{2}\right)(50\,\mathrm{m/\,s})^2 \\ p_1 &= 1237\ \mathrm{kPa} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}(v_{2x} - v_{1x}) \\ p_1 A_1 + F_x &= \rho A_2 v_2 (v_2 \cos 60^\circ - v_1) \\ F_x &= -1,237,000\,\mathrm{Pa/\,m^2} \times 0.001\,\mathrm{m^2} \\ &\quad +1,000\,\mathrm{kg/\,m^3} \times 0.0001\,\mathrm{m^2} \times 50\,\mathrm{m/\,s}(50\,\mathrm{m/\,s}\cos 60^\circ - 5\,\mathrm{m/\,s}) \\ &\quad \boxed{F_x = 1140\ \mathrm{N}} \end{aligned}$$

6.71: PROBLEM DEFINITION

Situation:

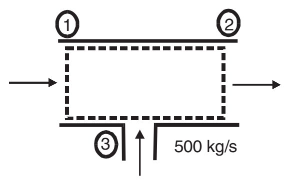
Water flows through a tee——additional details are provided in the problem statement.

Find:

Pressure difference between sections 1 and 2.

PLAN

Apply the continuity equation, then the momentum equation.



SOLUTION

Continuity equation

$$
\begin{aligned}
\dot{m}_1 + 500 \text{ kg/s} &= \dot{m}_2 \\
\dot{m}_1 &= (10 \text{ m/s})(0.10 \text{ m}^2)(1000 \text{ kg/m}^3) = 1000 \text{ kg/s} \\
\dot{m}_2 &= 1000\,\text{kg/s} + 500\,\text{kg/s} = 1500 \text{ kg/s} \\
v_2 &= \frac{\dot{m}_2}{\rho A_2} = \frac{1500\,\text{kg/s}}{1000\,\text{kg/m}^3 \times 0.1\,\text{m}^2} = 15 \text{ m/s}
\end{aligned}
$$

Momentum equation (x-direction)

$$
\begin{aligned}
\sum F_x &= \dot{m}_2 v_{2x} - \dot{m}_1 v_{1x} - \dot{m}_3 v_{3x} \\
p_1 A_1 + p_2 A_2 &= \dot{m}_2 v_2 - \dot{m}_1 v_1 - 0 \\
A(p_1 - p_2) &= (1500\,\text{kg}/s)(15\,\text{m/s}) - (1000\,\text{kg/s})(10\,\text{m/s}) \\
p_1 - p_2 &= \frac{(22,500 - 10,000)\,\text{Pa}}{0.10\,\text{m}^2} \\
&= 125,000 \text{ Pa} \\
& \boxed{p_1 - p_2 = 125 \text{ kPa}}
\end{aligned}
$$

6.72: PROBLEM DEFINITION

Situation:

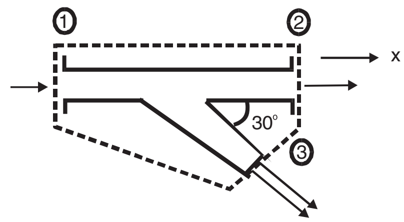
Water flows through a wye—additional details are provided in the problem statement.

Find:

$x-$component of force to hold wye in place.

PLAN

Apply the momentum equation.



Flow rate

$$\begin{aligned} v_1 &= \frac{Q_1}{A_1} = 6 \text{ m/s} \\ v_2 &= \frac{Q_2}{A_2} = 3.6 \text{ m/s} \\ Q_3 &= 0.6 - 0.36 = 0.24 \text{ m}^3/\text{s} \\ v_3 &= \frac{Q_3}{A_3} = 9.6 \text{ m/s} \end{aligned}$$

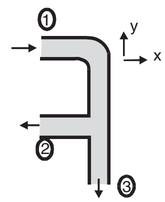
Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}_2 v_2 + \dot{m}_3 v_3 \cos 30^\circ - \dot{m}_1 v_1 \\ F_x + p_1 A_1 - p_2 A_2 &= (6\rho)(-0.6) + (3.6\rho)(+0.36) \\ &\quad +(9.6 \cos 30^\circ)(\rho)(8) \\ F_x + (48,000\,\text{N/m}^2)(0.1\,\text{m}^2) - (43,000\,\text{N/m}^2)(0.1\,\text{m}^2) &= -3.6\rho\,\text{N} + 1.3\rho\,\text{N} + \rho(1.995) \\ F_x &= -500\,\text{N} + 1000\,\text{kg/m}^3(-0.305\,\text{m}^4/\text{s}^2) \\ &\boxed{F_x = -805 \text{ N (acting to the left)}} \end{aligned}$$

6.73: PROBLEM DEFINITION

Situation:

Water flow through a horizontal bend and T section—additional details are provided in the problem statement.



$$
\begin{aligned}
\dot{m}_1 &= 6 \text{ kg/s} \\
\dot{m}_2 &= \dot{m}_3 = 3 \text{ kg/s} \\
A_1 &= A_2 = A_3 = 32 \text{ cm}^2 \\
p_1 &= 35,000\,\text{N/m}^2 \\
p_2 &= p_3 = 0
\end{aligned}
$$

Find:

Horizontal component of force to hold fitting stationary: F_x

PLAN

Apply the momentum equation.

SOLUTION

Velocity calculations

$$
\begin{aligned}
v_1 &= \frac{\dot{m}_1}{\rho A_1} \\
&= \frac{(6\,\text{kg/s})}{\left[(1000\,\text{kg/m}^3)(0.0032\,\text{m}^2)\right]} \\
&= 1.88 \text{ m/s} \\
v_2 &= \frac{\dot{m}_2}{\rho A_2} \\
&= \frac{(3\,\text{kg/s})}{\left[(1000\,\text{kg/m}^3)(0.0032\,\text{m}^2)\right]}. \\
&= 0.94 \text{ m/s}
\end{aligned}
$$

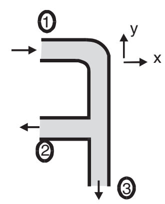
Momentum equation (x-direction)

$$
\begin{aligned}
\sum F_x &= -\dot{m}_2 v_2 - \dot{m}_1 v_1 \\
p_1 A_1 + F_x &= -\dot{m}_2 v_2 - \dot{m}_1 v_1 \\
F_x &= -p_1 A_1 - \dot{m}_2 v_2 - \dot{m}_1 v_1 \\
&= -(35,000\,\mathrm{N/m^2} \times 0.0032\,\mathrm{m^2}) - (3\,\mathrm{kg/s})(0.94\,\mathrm{m/s}) \\
&\quad -(6\,\mathrm{kg/s})(1.88\,\mathrm{m/s}) \\
&\quad \boxed{F_x = -126.1\ \mathrm{N}}
\end{aligned}
$$

6.74: PROBLEM DEFINITION

Situation:

Water flows through a horizontal bend and T section—additional details are provided in the problem statement.



$$\begin{aligned} v_1 &= 6 \text{ m/s} \qquad p_1 = 4.8 \text{ kPa} \\ v_2 &= v_3 = 3 \text{ m/s} \qquad p_2 = p_3 = 0 \\ A_1 &= A_2 = A_3 = 0.20 \text{ m}^2 \end{aligned}$$

Find:

Components of force (F_x, F_y) needed to hold bend stationary.

PLAN

Apply the momentum equation.

SOLUTION

Discharge

$$\begin{aligned} Q_1 &= A_1 v_1 = 0.2 \times 6 = 1.2 \text{ m}^3/\text{s} \\ Q_2 &= Q_3 = A_2 v_2 = 0.2 \times 3 = 0.6 \text{ m}^3/\text{s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= -\dot{m}_2 v_2 - \dot{m}_1 v_1 \\ p_1 A_1 + F_x &= -\rho(Q_2 v_2 + Q_1 v_1) \\ F_x &= -p_1 A_1 - \rho(Q_2 v_2 + Q_1 v_1) \\ &= -4800 \times 0.2 - 1000(0.6 \times 3 + 1.2 \times 6) \\ & \boxed{F_x = -9.96 \text{ kN (acts to the left)}} \end{aligned}$$

y-direction

$$\begin{aligned} \sum F_y &= \dot{m}_3(-v_3) \\ F_y &= -\rho Q_3 v_3 = -1000 \times 0.6 \times 3 \\ & \boxed{F_y = -1.8 \text{ kN (acts downward)}} \end{aligned}$$

6.75: PROBLEM DEFINITION

Situation:
Water flows through a horizontal tee—additional details are provided in the problem statement.

Find:
Components of force (F_x, F_y) needed to hold the tee in place.

PLAN

Apply the momentum equation.

SOLUTION

Velocity calculations

$$\begin{aligned} V_1 &= \frac{0.25\,\mathrm{m}^3/\,\mathrm{s}}{(\pi \times 0.075\,\mathrm{m} \times 0.075\,\mathrm{m})} \\ &= 14.15\ \mathrm{m/s} \\ V_2 &= \frac{0.10\,\mathrm{m}^3/\,\mathrm{s}}{(\pi \times 0.035\,\mathrm{m} \times 0.035\,\mathrm{m})} \\ &= 25.98\ \mathrm{m/s} \\ V_3 &= \frac{(0.25 - 0.10)\,\mathrm{m}^3/\,\mathrm{s}}{(\pi \times 0.075\,\mathrm{m} \times 0.075\,\mathrm{m})} \\ &= 8.49\ \mathrm{m/s} \end{aligned}$$

Momentum equation (x-direction)

$$F_x + p_1 A_1 - p_3 A_3 = \dot{m}_3 V_3 - \dot{m}_1 V_1$$

$$F_x = -p_1 A_1 + p_3 A_3 + \rho V_3 Q - \rho V_1 Q$$

$$F_x = -\,(100,000\,\mathrm{Pa} \times \pi \times 0.075\,\mathrm{m} \times 0.075\,\mathrm{m}) + (80,000\,\mathrm{Pa} \times \pi \times 0.075\,\mathrm{m} \times 0.075\,\mathrm{m})$$
$$+ \left(1000\,\mathrm{kg}/\,\mathrm{m}^3 \times 8.49\,\mathrm{m}/\,\mathrm{s} \times 0.15\,\mathrm{m}^3/\,\mathrm{s}\right) - \left(1000\,\mathrm{kg}/\,\mathrm{m}^3 \times 14.15\,\mathrm{m}/\,\mathrm{s} \times 0.25\,\mathrm{m}^3/\,\mathrm{s}\right)$$

$$F_x = -2617\,\mathrm{N}$$

Momentum equation y-direction

$$\begin{aligned} F_y + p_3 A_3 &= -\rho V_3 Q \\ F_y &= -\rho V_3 Q - p_3 A_3 \\ F_y &= -1000\,\mathrm{kg}/\,\mathrm{m}^3 \times 25.98\,\mathrm{m}/\,\mathrm{s} \times 0.10\,\mathrm{m}^3/\,\mathrm{s} - 70,000\,\mathrm{Pa} \times \pi \times 0.035\,\mathrm{m} \times 0.035\,\mathrm{m} \\ &= -2867\ \mathrm{N} \end{aligned}$$

Net force

$$\boxed{\mathbf{F} = (-2.62\mathbf{i} - 2.87\mathbf{j})\ \mathrm{kN}}$$

6.76: PROBLEM DEFINITION

Situation:

Risk of windows being broken by the force of jet from a firehose.

A 0.2-m (D_1) firehose attached to a nozzle with a 0.1-m (d_2) outlet.

The free jet from the nozzle is deflected by 90° when it hits the window.

Water from firehose flows at a rate of 0.15 m^3/s.

Find:

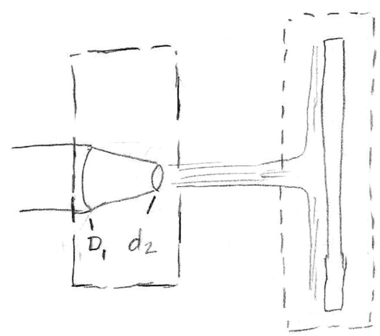
Force the window must withstand due to the impact of the jet.

PLAN

1. Assess the CV and set up the problem
2. Apply the momentum equation

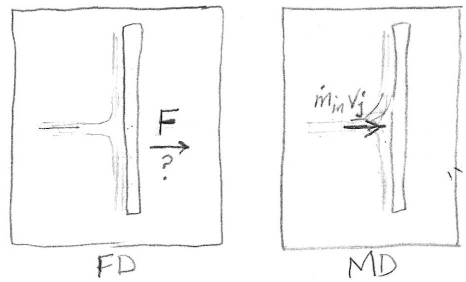
SOLUTION

Problem setup.
Choose coordinate system.
Make a sketch of the system with CV, and then another sketch of the FD and MD.
Draw reaction force to the right.



Consider: Where does one draw the CV? The big dashed line, or the small dashed line? Some problem-solvers may think that they need to include information about both D_1 and d_2, and thus might use the CV described with the large dashes. However the only thing causing the force on the window is the force of the jet. Therefore the appropriate CV is the one drawn with the small dashed line, and the only velocity of interest is v_j.

Since the force of interest is in the x-direction, it is only necessary to assess the forces and momentum acting in the x-axis. The momentum and force diagrams are thus:



$$\sum F_x = \sum(\text{momentum out})_x - \sum(\text{momentum in})_x$$

$$F_x = 0 - \dot{m}_j v_j$$

$$F_x = -\rho Q_j v_j = \rho Q_j \left(\frac{Q_j}{A_j}\right); \text{ where } A = \frac{\pi d^2}{4} = \frac{\pi(0.1m)^2}{4} = 0.0079\,\text{m}^2$$

$$F_x = -\left(\frac{1000\,\text{kg}}{\text{m}^3}\right)\left(\frac{0.15\,\text{m}^3}{s}\right)^2\left(\frac{1}{0.0079\,\text{m}^2}\right)$$

$$\boxed{F_x = -2.86 \text{ kN (acts to the left)}}$$

6.77: PROBLEM DEFINITION

Situation:

Fireman wants to throttle down his flow rate so that it will not break windows.
Assumption is given that typical window withstands a force up to 110 N.
20 cm (D_1) firehose discharging through a nozzle with 10 cm (d_2) outlet.

Find:

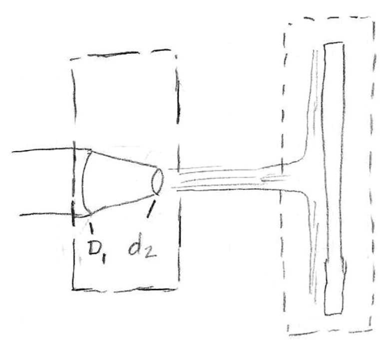
The largest volumetric flow rate fireman should allow to exit his nozzle (L/min).

PLAN

1. Assess the CV and set up the problem
2. Apply the momentum equation

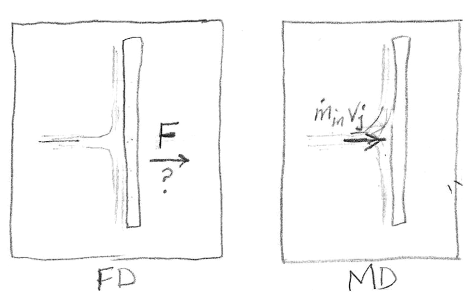
SOLUTION

Problem setup.
Choose coordinate system.
Make a sketch of the system with CV, and then another sketch of the FD and MD.
Draw reaction force to the right.



Consider: Where does one draw the CV? The big dashed line, or the small dashed line? Some problem-solvers may think that they need to include information about both D_1 and d_2, and thus might use the CV described with the large dashes. However the only thing causing the force on the window is the force of the jet. Therefore the CV is the one drawn with the small dashed line, and the only velocity of interest is v_j.

Since the force of interest is in the x-direction, it is only necessary to assess the forces and momentum acting in the x-axis. The momentum and force diagrams are thus:



$$
\begin{aligned}
\sum F_x &= \sum(\text{momentum out})_x - \sum(\text{momentum in})_x \\
F_x &= 0 - \dot{m}_j v_j \\
F_x &= -\rho Q_j v_j = -\rho Q_j \left(\frac{Q_j}{A_j}\right);\ \text{where } A = \frac{\pi d^2}{4} = \frac{\pi(0.1\,\mathrm{m})^2}{4} = 0.00785\,\mathrm{m}^2 \\
F_x &= \rho\frac{Q_j^2}{A_j} \text{to the left, and thus } Q = \sqrt{\frac{FA}{\rho}} \\
Q &= \sqrt{\frac{(110\,\mathrm{N})\,(0.00785\,\mathrm{m}^2)}{1000\,\mathrm{kg/m}^3}} \\
& \boxed{Q = 0.0294\,\mathrm{m}^3/\,\mathrm{s}}
\end{aligned}
$$

6.78: PROBLEM DEFINITION

Situation:

A flow in a pipe is laminar and fully developed—additional details are provided in the problem statement.

Find:

Derive a formula for the resisting shear force (F_τ) as a function of the parameters $D,\ p_1, p_2,\ \rho$, and U.

PLAN

Apply the momentum equation, then the continuity equation.

SOLUTION

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \int_{cs} \rho v(v \cdot dA) \\
p_1 A_1 - p_2 A_2 - F_\tau &= \int_{A_2} \rho u_2^2 dA - (\rho A u_1) u_1 \\
p_1 A - p_2 A - F_\tau &= -\rho u_1^2 A + \int_{A_2} \rho u_2^2 dA \qquad (1)
\end{aligned}$$

Integration of momentum outflow term

$$\begin{aligned}
u_2 &= u_{\max}(1-(r/r_0)^2)^2 \\
u_2^2 &= u_{\max}^2(1-(r/r_0)^2)^2 \\
\int_{A_2} \rho u_2^2 dA &= \int_0^{r_0} \rho u_{\max}^2 (1-(r/r_0)^2)^2 2\pi r dr \\
&= -\rho u_{\max}^2 \pi r_0^2 \int_0^{r_0} (1-(r/r_0)^2)^2(-2r/r_0^2) dr
\end{aligned}$$

To solve the integral, let

$$u = 1 - \left(\frac{r}{r_o}\right)^2$$

Thus

$$du = \left(-\frac{2r}{r_o^2}\right) dr$$

The integral becomes

$$\begin{aligned}
\int_{A_2} \rho u_2^2 dA &= -\rho u_{\max}^2 \pi r_0^2 \int_1^0 u^2 du \\
&= -\rho u_{\max}^2 \pi r_0^2 \left(\frac{u^3}{3}|_1^0 \right) \\
&= -\rho u_{\max}^2 \pi r_0^2 \left(0 - \frac{1}{3} \right) \\
&= \frac{+\rho u_{\max}^2 \pi r_0^2}{3} \qquad (2)
\end{aligned}$$

Continuity equation

$$\begin{aligned}
UA &= \int u dA \\
&= \int_0^{r_0} u_{\max}(1 - (r/r_0)^2) 2\pi \text{ rdr} \\
&= -u_{\max} \pi r_0^2 \int_0^{r_0} (1 - (r/r_0)^2)(-2r/r_0^2) \text{ dr} \\
&= -u_{\max} \pi r_0^2 (1 - (r/r_0)^2)^2 / 2|_0^{r_0} \\
&= u_{\max} \pi r_0^2 / 2
\end{aligned}$$

Therefore

$$u_{\max} = 2U$$

Substituting back into Eq. 2 gives

$$\int_{A_2} \rho u_2^2 dA = 4\rho U^2 \pi r_0^2 / 3$$

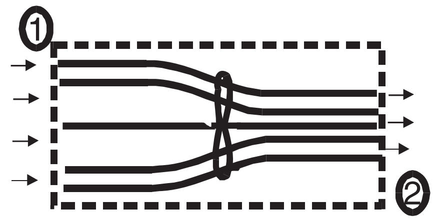
Finally substituting back into Eq. 1, and letting $u_1 = U$, the shearing force is given by

$$\boxed{F_\tau = \frac{\pi D^2}{4} \left[p_1 - p_2 - (1/3)\rho U^2 \right]}$$

6.79: PROBLEM DEFINITION

Situation:

A swamp boat is powered by a propeller—additional details are provided in the problem statement.



Find:

(a) Propulsive force when the boat is not moving.

(b) Propulsive force when the boat is moving at 9 m/s.

Assumptions:

When the boat is stationary, neglect the inlet flow of momentum–that is, assume $v_1 \approx 0$.

PLAN

Apply the momentum equation.

SOLUTION

From Table A.3, $\rho = 1.18$ kg/m^3

a.) Boat is stationary

Momentum equation (x-direction) Select a control volume that surrounds the boat.

$$\begin{aligned} \sum F_x &= \dot{m}v_2 - \dot{m}v_1 \\ F_{\text{stop}} &\approx \dot{m}v_2 \end{aligned}$$

Mass flow rate

$$\begin{aligned} \dot{m} &= \rho A_2 v_2 \\ &= \left(1.18\,\text{kg/m}^3\right)\left(\frac{\pi\,(0.9\,\text{m})^2}{4}\right)(30\,\text{m/s}) \\ &= 22.5\,\text{kg/s} \end{aligned}$$

Thus

$$\begin{aligned} F_{\text{stop}} &= \dot{m}v_2 \\ &= (22.5\,\text{kg/s})\,(30\,\text{m/s}) \\ &= 675\,\text{N} \end{aligned}$$

$$\boxed{\text{Force (stationary boat)} = 675 \text{ N}}$$

b.) Boat is moving

Momentum equation (x-direction). Select a control volume that surrounds the boat and moves with the speed of the boat. The inlet velocity is $v_1 = 9$ m/s

$$\begin{aligned} \sum F_x &= \dot{m}\,(v_2 - v_1) \\ &= (22.5\,\text{kg/s})\,(30 - 9)\ \text{m/s} \\ &= 473\ \text{N} \end{aligned}$$

$$\boxed{\text{Force (moving boat)} = 473 \text{ N}}$$

6.80: PROBLEM DEFINITION

Situation:

Air flows through a windmill—additional details are provided in the problem statement.

Find:

Thrust on windmill.

PLAN

Apply the continuity equation, then the momentum equation.

SOLUTION

Continuity equation

$$v_2 = 12\,\mathrm{m/\,s} \times \left(\frac{3\,\mathrm{m}}{4.5\,\mathrm{m}}\right)^2 = 5.33\ \mathrm{m/s}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}(v_2 - v_1) \\
F_x &= \dot{m}(v_2 - v_1) \\
&= (1.2\,\mathrm{kg/\,m^3})(\pi/4 \times (3\,\mathrm{m})^2)(12\,\mathrm{m/\,s})(5.33 - 12)\,\mathrm{m/\,s} \\
F_x &= -687.6\ \mathrm{N}\ \text{(acting to the left)} \\
& \boxed{T = 688\ \mathrm{N}\ \text{(acting to the right)}}
\end{aligned}$$

6.81: PROBLEM DEFINITION

Situation:

A jet pump is described in the problem statement.

Find:

(a) Derive a formula for pressure increase across a jet pump.

(b) Evaluate the pressure change for water if $A_j/A_o = 1/3$, $v_j = 15$ m/s and $v_o = 2$ m/s.

PLAN

Apply the continuity equation, then the momentum equation.

SOLUTION

Continuity equation

$$v_1 = \frac{v_0 D_0^2}{D_0^2 - D_j^2} \tag{1}$$

$$v_2 = \frac{v_0 D_0^2 + v_j D_j^2}{D_0^2} \tag{2}$$

Momentum equation (x-direction)

$$\sum F_x = \dot{m}(v_2 - v_1)$$

$$(p_1 - p_2)\frac{\pi D_0^2}{4} = -\frac{\rho v_1^2 \pi (D_0^2 - D_j^2)}{4} - \frac{\rho v_j^2 \pi D_j^2}{4} + \frac{\rho v_2^2 \pi D_0^2}{4}$$

thus,

$$\boxed{(p_2 - p_1) = \frac{\rho v_1^2 (D_0^2 - D_j^2)}{D_0^2} + \rho v_j^2 \times \frac{D_j^2}{D_0^2} - \rho v_2^2} \tag{3}$$

Calculations

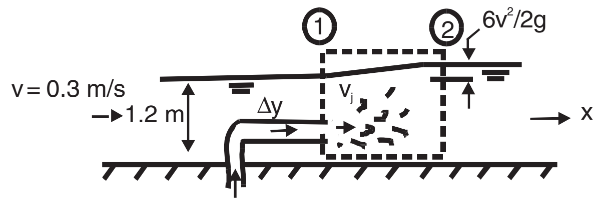
$$\begin{aligned} v_1 &= \frac{v_0}{1 - (D_j/D_0)^2} \\ &= \frac{2}{1 - (1/3)} \\ &= 3 \text{ m/s} \\ v_2 &= v_0 + v_j \frac{D_j^2}{D_0^2} \\ &= 2 + 15\left(\frac{1}{3}\right) \\ &= 7 \text{ m/s} \end{aligned}$$

from Eq. (3)

$$
\begin{aligned}
p_2 - p_1 &= \rho\left[v_1^2\left(1-\frac{D_j^2}{D_0^2}\right)+v_j^2\frac{D_j^2}{D_0^2}-v_2^2\right] \\
&= 1000\,\mathrm{kg/\,m^3}\left[(3\,\mathrm{m/\,s})^2\left(1-\frac{1}{3}\right)+(15\,\mathrm{m/\,s})^2\left(\frac{1}{3}\right)-(7\,\mathrm{m/\,s})^2\right] \\
&\boxed{p_2 - p_1 = 32\ \mathrm{kPa}}
\end{aligned}
$$

6.82: PROBLEM DEFINITION

Situation:
The problem statement describes a jet pump.



Find:
Develop a preliminary design by calculating basic dimensions for a jet pump.

PLAN

Apply the momentum equation, then the continuity equation.

SOLUTION

Momentum equation (x-direction)
Carry out the analysis for a section 0.3 m wide (unit width) and neglect bottom friction.

$$\begin{aligned} \sum F_x &= \dot{m}_2 v_2 - \dot{m}_1 v_1 - \dot{m}_j v_j \\ \gamma y_1^2/2 - \gamma y_2^2/2 &= 0.3\rho(0.3 \times (1.2 - \Delta y)) - v_j \rho(v_j \Delta y) + v_2 \rho(v_2 y_2) \qquad (1) \\ \text{but } y_2 &= 1.2 \text{ m} + 6 \ v^2/2g \\ &= 1.2 + 6/2g = 1.5 \text{ m} \end{aligned}$$

Continuity equation

$$\begin{aligned} v_2 y_2 &= v_1(1.2 - \Delta y) + v_j \Delta y \\ v_2 &= v_1 \frac{(1.2 - \Delta y)}{y_2} + \frac{v_j \Delta y}{y_2} \end{aligned}$$

Assume

$$\Delta y = 0.03 \text{ m}$$

Then

$$v_2 = 0.3(1.17)/(1.5) \times 0.03/1.5 = 0.234 + 0.02 v_j \qquad (2)$$

Combine Eqs. (1) and (2)

$$\begin{aligned} -(0.234 + 0.024 v_j)^2 \times 1.5 &= 2g(y_2^2 - y_1^2) - 3.51 \\ &= 12.38 \text{ m}_2^2/\text{ s} \end{aligned}$$

Solving:

$$v_j = 3.52 \text{ m/s} \quad A_j = 0.01 \text{ m}^2$$

If circular nozzles were used, then $A_j = (\pi/4)d_j^2$; $d_j = 0.11$ m. Therefore, one could use $\boxed{\text{8 nozzles of about 0.1 m}}$ in diameter discharging water at $\boxed{\text{3.5 m/s}}$.

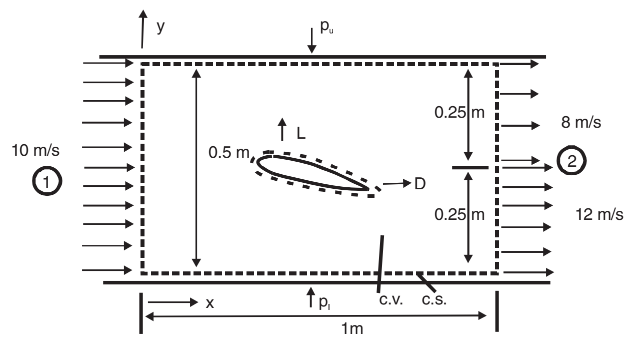
REVIEW

Like most design problems, this problem has more than one solution. That is, other combinations of d_j, v_j and the number of jets are possible to achieve the desired result.

6.83: PROBLEM DEFINITION

Situation:

Lift and drag forces are being measured on an airfoil that is situated in a wind tunnel—additional details are provided in the problem statement.



Find:

(a) Lift force: L

(b) Drag force: D

PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}v_0 - \dot{m}_1 v_1 \\
-D + p_1 A_1 - p_2 A_2 &= v_1(-\rho v_1 A) + v_a \frac{\rho v_a A}{2} + v_b \frac{\rho v_b A}{2} \\
-\frac{D}{A} &= p_2 - p_1 - \rho v_1^2 + \frac{\rho v_a^2}{2} + \frac{\rho v_b^2}{2}
\end{aligned}$$

where

$$\begin{aligned}
p_1 &= p_u(x=0) = p_\ell(x=0) = 100 \text{ Pa, gage} \\
p_2 &= p_u(x=1) = p_\ell(x=1) = 90 \text{ Pa, gage}
\end{aligned}$$

then

$$\begin{aligned}
-\frac{D}{A} &= 90\,\text{Pa} - 100\,\text{Pa} + 1.2\,\text{kg/m}^3 \times (-100 + 32 + 72)\,\text{m}^2/\text{s}^2 \\
-\frac{D}{A} &= -5.2\,\text{Pa} \\
D &= 5.2\,\text{Pa} \times (0.5\,\text{m})^2 \\
&\boxed{D = 1.3 \text{ N}}
\end{aligned}$$

Momentum equation (y-direction)

$$\sum F_y = 0$$

$$-L + \int_1^2 p_\ell B dx - \int_0^1 p_u B dx = 0 \text{ where } B \text{ is depth of tunnel}$$

$$-L + \int_0^1 (100 - 10x + 20x(1-x))0.5dx - \int_0^1 (100 - 10x - 20x(1-x))0.5dx = 0$$

$$-L + 0.5(100x - 5x^2 + 10x^2 - (20/3)x^3)|_0^1 - 0.5(100x - 5x^2 - 10x^2 + (20/3)x^3|_0^1 = 0$$

thus,

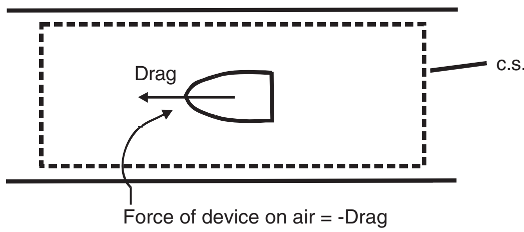
$$-L + 0.5 \times (20 - \frac{40}{3}) = 0$$

$$\boxed{L = 3.33 \text{ N}}$$

6.84: PROBLEM DEFINITION

Situation:

A torpedo-like device is being tested in a wind tunnel—additional details are provided in the problem statement.



Find:

(a) Mass rate of flow.
(b) Maximum velocity at the outlet section.
(c) Drag on the device and support vanes.

PLAN

Apply the momentum equation.

SOLUTION

Mass flow rate

$$\begin{aligned}\dot{m} &= \rho v A \\ &= \left(1.35\,\text{kg/m}^3\right) \times (36\,\text{m/s}) \times \left(\frac{\pi(0.9\,\text{m})^2}{4}\right) \\ &= 30.9\ \text{kg/s}\end{aligned}$$

$$\boxed{\dot{m} = 30.9\ \text{kg/s}}$$

At the outlet section

$$\int v dA = Q$$

But v is linearly distributed, so $v = v_{\max}(r/r_0)$. Thus

$$\begin{aligned}\int_0^{r_0} \left(v_{\max}\frac{r}{r_o}\right) 2\pi r dr &= \overline{v}A \\ \frac{2v_{\max}r_0^2}{3} &= \overline{v}r_0^2 \\ v_{\max} &= \frac{3\overline{v}}{2} \\ &= \frac{3\,(36\,\text{m/s})}{2} \\ v_{\max} &= 54\,\text{m/s}\end{aligned}$$

$$\boxed{v_{\max} = 54\,\mathrm{m/s}}$$

Momentum equation (x-direction)

$$\sum F_x = \int_0^{r_0} \rho v_2^2 dA - \dot{m} v_1 \tag{1}$$

a) Forces analysis

$$\sum F_x = p_1 A_1 - p_2 A_2 - D \tag{a}$$

b) Outlet velocity profile

$$\begin{aligned} v_2 &= v_{\max} \frac{r}{r_o} \\ &= \left(\frac{3\overline{v}}{2}\right)\left(\frac{r}{r_o}\right) \end{aligned} \tag{b}$$

c) Outlet momentum flow

$$\begin{aligned} \int_0^{r_0} \rho v_2^2 dA &= \int_0^{r_0} \rho \left[\left(\frac{3\overline{v}}{2}\right)\left(\frac{r}{r_o}\right)\right]^2 2\pi r dr \\ &= 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \int_0^{r_0} \left(\frac{r}{r_o}\right)^2 r dr \\ &= 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \left(\frac{r_o^2}{4}\right) \end{aligned} \tag{c}$$

Substituting Eqns. (a) and (c) into the momentum equation (1) gives

$$\begin{aligned} \sum F_x &= \int_0^{r_0} \rho v_2^2 dA - \dot{m} v_1 \\ p_1 A_1 - p_2 A_2 - D &= 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \left(\frac{r_o^2}{4}\right) - \dot{m} v_1 \\ D &= p_1 A_1 - p_2 A_2 - 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \left(\frac{r_o^2}{4}\right) + \dot{m} v_1 \end{aligned} \tag{2}$$

Calculations (term by term)

$$
\begin{aligned}
p_1 A_1 &= (1.65\,\text{kPa}) \times \left(\frac{\pi \times 0.9^2}{4} \right) \\
&= 1050\,\text{N} \\
p_2 A_2 &= (100) \times \left(\frac{\pi \times 0.9^2}{4} \right) \\
&= 445\,\text{N} \\
\int_0^{r_0} \rho v_2^2 dA &= 2\pi\rho \left(\frac{3\overline{v}}{2} \right)^2 \left(\frac{r_o^2}{4} \right) \\
&= 2\pi\,(1.35) \left(\frac{3\,(36)}{2} \right)^2 \left(\frac{0.45}{4} \right) \\
&= 1251.5\,\text{N} \\
\dot{m} v_1 &= (30.9)\,(36) \\
&= 1112.4\,\text{N}
\end{aligned}
$$

Substituting numerical values into Eq. (2)

$$
\begin{aligned}
D &= p_1 A_1 - p_2 A_2 - 2\pi\rho \left(\frac{3\overline{v}}{2} \right)^2 \left(\frac{r_o^2}{4} \right) + \dot{m} v_1 \\
&= 1050\,\text{N} - 445\,\text{N} - 1251.5\,\text{N} + 1112.4\,\text{N} \\
&= 465.9\,\text{N}
\end{aligned}
$$

$$
\boxed{D = 466\,\text{N}}
$$

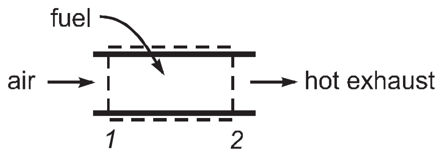
6.85: PROBLEM DEFINITION

Situation:

A jet engine (ramjet) takes in air, adds fuel, and then exhausts the hot gases produced by combustion.

$v_1 = 225$ m/s

$\rho_2 = 0.25$ kg/m^3, $A_2 = 0.5$ m^2



Find:

Thrust force produced by the ramjet: T

Assumptions:

Neglect the mass addition due to the fuel (that is, $\dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \dot{m} = 60$ kg/s).

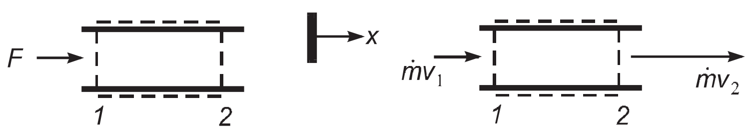
2.) Assume steady flow.

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



where F is the force required to hold the ramjet stationary.

Calculate exit velocity

$$\begin{aligned} \dot{m}_2 &= \rho_2 A_2 v_2 \\ v_2 &= \frac{\dot{m}_2}{\rho_2 A_2} = \frac{60\,\text{kg/s}}{0.25\,\text{kg/m}^3 \times 0.5\,\text{m}^2} = 480 \text{ m/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ F &= \dot{m}(v_2 - v_1) = 60\,\text{kg/s}(480\,\text{m/s} - 225\,\text{m/s}) \\ & \boxed{T = 15.3 \text{ kN (to the left)}} \end{aligned}$$

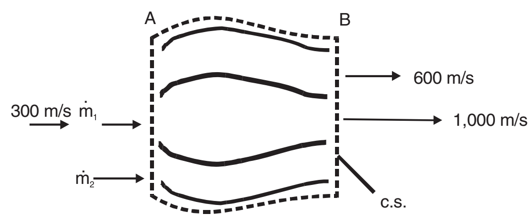
6.86: PROBLEM DEFINITION

Situation:

Air flows through a turbofan engine. Inlet mass flow is $300\,\mathrm{kg/s}$.

Bypass ratio is 2.5. Speed of bypass air is $600\,\mathrm{m/s}$.

Speed of air that passes through the combustor is $1000\,\mathrm{m/s}$.



Additional details are given in the problem statement.

Find:

Thrust (T) of the turbofan engine.

Assumptions:

Neglect the mass flow rate of the incoming fuel.

PLAN

Apply the continuity and momentum equations.

SOLUTION

Continuity equation

$$\dot{m}_A = \dot{m}_B = 300\,\mathrm{kg/s}$$

also

$$\begin{aligned}
\dot{m}_B &= \dot{m}_{\text{combustor}} + \dot{m}_{\text{bypass}} \\
&= \dot{m}_{\text{combustor}} + 2.5\dot{m}_{\text{combustor}} \\
\dot{m}_B &= 3.5\dot{m}_{\text{combustor}}
\end{aligned}$$

Thus

$$\begin{aligned}
\dot{m}_{\text{combustor}} &= \frac{\dot{m}_B}{3.5} = \frac{300\,\mathrm{kg/s}}{3.5} \\
&= 85.71\,\mathrm{kg/s} \\
\dot{m}_{\text{bypass}} &= \dot{m}_B - \dot{m}_{\text{combustor}} \\
&= 300\,\mathrm{kg/s} - 85.71\,\mathrm{kg/s} \\
&= 214.3\,\mathrm{kg/s}
\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \sum \dot{m}v_{\text{out}} - \dot{m}v_{\text{in}} \\
F_x &= [\dot{m}_{\text{bypass}}V_{\text{bypass}} + \dot{m}_{\text{combustor}}V_{\text{combustor}}] - \dot{m}_A V_A \\
&= [(214.3\,\text{kg/s})\,(600\,\text{m/s}) + (85.71\,\text{kg/s})\,(1000\,\text{m/s})] - (300\,\text{kg/s})\,(300\,\text{m/s}) \\
&= 124,290\,\text{N}
\end{aligned}$$

$$\boxed{T = 124\ \text{kN}}$$

6.87: PROBLEM DEFINITION

Situation:

Inertial reference frame.

Find:

Definition of inertial reference frame.

SOLUTION

The inertial reference is any frame in which Newton's first and second laws are valid. It is any frame which is neither rotating nor accelerating with respect to the sun.

6.88: PROBLEM DEFINITION

Situation:

Centrifugal acceleration on the surface of earth.
$t = 24\,\mathrm{h},\ D = 12,900\,\mathrm{km}.$

Find:

Value of centrifugal acceleration on earth's surface and comparison to acceleration to gravity.

SOLUTION

The acceleration is

$$a_r = \omega^2 r$$

The angular velocity is

$$\omega = \frac{2\pi \text{ rad}}{24 \text{ hr} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ s}}{1 \text{ min}}} = 7.27 \times 10^{-5} \text{ rad/s}$$

Acceleration

$$a_r = (7.27 \times 10^{-5} \text{ rad/s})^2 \times 6450 \times 10^3 \text{ m}$$

$$\boxed{a_r = 0.034\,\mathrm{m/s^2}}$$

The acceleration due to gravity is 9.81 m/s^2 so

$$\frac{a_r}{g_c} = \frac{0.034}{9.81}$$

$$\boxed{\frac{a_r}{g_c} = 0.0035}$$

or less than 0.5%.

6.89: PROBLEM DEFINITION

Maximum force occurs at the beginning; hence, the tank will accelerate immediately after opening the cap. However, as water leaves the tank the force will decrease, but acceleration may decrease or increase because mass will also be decreasing. In any event, the tank will go faster and faster until the last drop leaves, assuming no aerodynamic drag.

6.90: PROBLEM DEFINITION

Situation:

Open water tank on a frictionless plane.

Capped orifice on side has a 4-cm diameter exit pipe located 3 m below the water surface.

Ignore all friction effects.

Find:

The force needed to keep the tank from moving when cap is removed from orifice.

PLAN

Consider the physics, and apply the momentum equation.

a. When cap is removed, a jet will result.
b. Apply Bernoulli's eqn: find velocity of jet from head of water in tank.
c. Use momentum equation to find force needed to balance the momentum.
d. Jet and force will be x-direction only.

SOLUTION

1. Bernoulli's equation relating 2 locations - in reservoir at depth 3m, and jet:

$$\begin{aligned}
\frac{v_j^2}{2g} &= h;\text{ therefore } v_j = \sqrt{2gh} \\
v_j &= \sqrt{2\left(9.81\,\mathrm{m/\,s^2}\right)\left(3\,\mathrm{m}\right)} \\
v_j &= 7.672\,\mathrm{m/\,s}
\end{aligned}$$

2. Momentum equation:

$$\begin{aligned}
\sum F_x &= \sum(\text{momentum out})_x - \sum(\text{momentum in})_x \\
F_x &= \dot{m}_j v_j - 0 \\
F_x &= \rho\left(v_j A_j\right)\left(-v_j\right) A_j = \text{where } A = \frac{\pi d^2}{4} = \frac{\pi(0.04\,\mathrm{m})^2}{4} = 0.00126\,\mathrm{m}^2 \\
F_x &= -\left(\frac{1000\,\mathrm{kg}}{\mathrm{m}^3}\right)\left(7.672\frac{\mathrm{m}}{\mathrm{s}}\right)^2\left(0.00126\,\mathrm{m}^2\right)
\end{aligned}$$

$$\boxed{F_x = -73.97\text{ N (must act to the left)}}$$

6.91: PROBLEM DEFINITION

Situation:

A tank of water rests on a sled—additional details are provided in the problem statement.

Find:

Acceleration of sled at time t

PLAN

Apply the momentum equation.

SOLUTION

This type of problem is directly analogous to the rocket problem except that the weight does not directly enter as a force term and $p_e = p_{\text{atm}}$. Therefore, the appropriate equation is

$$\begin{aligned} M\ dv_s/dt &= \rho v_e^2 A_e - F_f \\ a &= (1/M)(\rho v_e^2 (\pi/4) d_e^2 - \mu W) \end{aligned}$$

where μ =coefficient of sliding friction and W is the weight

$$\begin{aligned} W &= 350 + 0.1 \times 1000 \times 9.81 = 1331 \text{ N} \\ a &= (g/W)(1,000 \times 25^2 (\pi/4) \times 0.015^2 - (1331 \times 0.05)) \\ &= (9.81/1,331)(43.90) \text{ m/s}^2 \\ &\boxed{a = 0.324 \text{ m/s}^2} \end{aligned}$$

6.92: PROBLEM DEFINITION

Situation:

A cart is moving with a steady speed along a track.
Speed of cart is 5 m/ s (to the right). Speed of water jet is 10 m/ s.
Nozzle area is $A = 0.002\,\text{m}^2$.

Find:

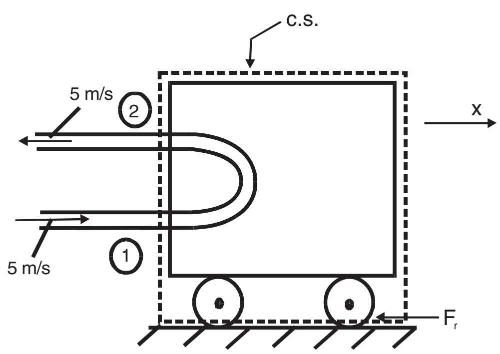
Resistive force on cart: F_r

PLAN

Apply the momentum equation.

SOLUTION

Assume the resistive force (F_r) is caused primarily by rolling resistance (bearing friction, etc.); therefore, the resistive force will act on the wheels at the ground surface. Select a reference frame fixed to the moving cart. The velocities and resistive force are shown below.



Velocity analysis

$$
\begin{aligned}
V_1 &= v_1 = v_2 = 5 \text{ m/s} \\
\dot{m} &= \rho A_1 V_1 \\
&= (1000\,\text{kg/ m}^3)(0.002\,\text{m}^2)(5\,\text{m/ s}) \\
&= 10 \text{ kg/s}
\end{aligned}
$$

Momentum equation (x-direction)

$$
\begin{aligned}
\sum F_x &= \dot{m}(v_2 - v_1) \\
-F_r &= 10\,\text{kg/ s}(-5\,\text{m/ s} - 5\,\text{m/ s}) = -100 \text{ N}
\end{aligned}
$$

$$\boxed{F_r = 100 \text{ N (acting to the left)}}$$

6.93: PROBLEM DEFINITION

Situation:

A water jet ($\rho = 1000\,\text{kg/m}^3$) accelerates a cart
$Q = 0.1\ \text{m}^3/\text{s}$
Jet speed: $v_j = 10$ m/s.
Cart Mass $M = 10$ kg
Deflection of the jet is normal to the cart.

Find:

(a) Develop an expression for the acceleration of the cart.
(b) Calculate the acceleration when $v_c = 5$ m/s.

Assumptions:

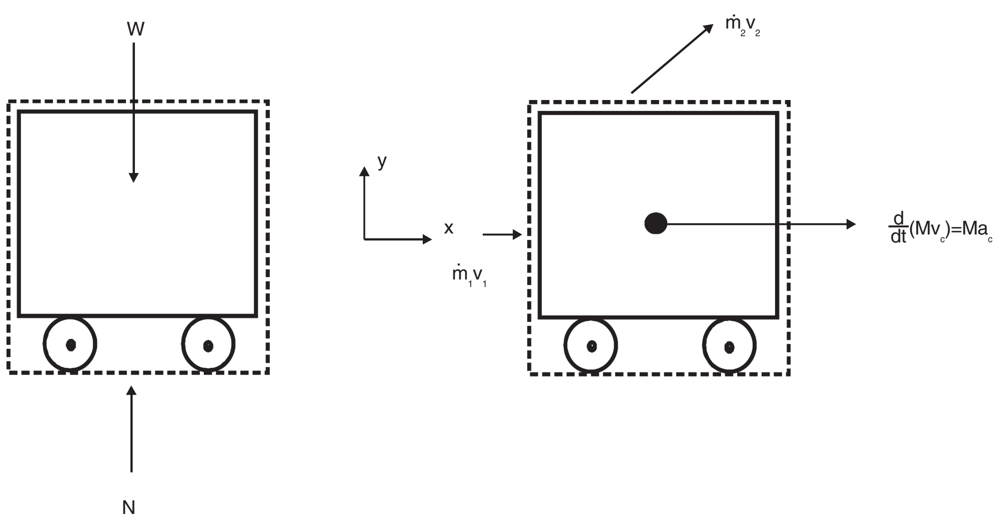
Neglect rolling resistance.
Mass of water << mass of cart.

PLAN

To develop an equation for acceleration of the cart, apply the momentum equation to a cv surround the cart. Select a inertial reference frame fixed to the ground because the cart is accelerating. Then, use continuity and other equations to solve for the acceleration.

SOLUTION

1. Force and momentum diagrams



2. Momentum equation (x-direction)

$$\begin{aligned}\sum F_x &= \frac{d}{dt}(Mv_c) + \dot{m}_2 v_{2_x} - \dot{m}_1 v_1 \\ 0 &= Ma_c + \dot{m}_2 v_{2_x} - \dot{m}_1 v_1 \qquad (1)\end{aligned}$$

3. Continuity equation.

$$\dot{m}_2 = \dot{m}_1 = \dot{m} \tag{2}$$

4. Flow Rate Eqn (for flow crossing cs, use velocity relative to cs)

$$\begin{aligned} \dot{m} &= \rho A_j (v_j - v_c) \\ &= \rho \left(\frac{Q}{v_j} \right) (v_j - v_c) \end{aligned} \tag{3}$$

5. Velocity analysis (velocity is relative to fixed reference frame)

$$v_1 = v_j \tag{4a}$$

$$v_{2_x} = v_c \tag{4b}$$

6. Combine Eqs (1) to (4)

$$\begin{aligned} 0 &= M a_c + \dot{m} (v_{2_x} - v_1) \\ 0 &= M a_c + \left[\left(\frac{\rho Q}{v_j} \right) (v_j - v_c) \right] (v_c - v_j) \end{aligned}$$

Solving for acceleration

$$\boxed{a_c = \frac{\rho Q (v_j - v_c)^2}{v_j M}}$$

7. Calculations

$$a_c = \frac{(1000\,\text{kg/m}^3)\,(0.1\,\text{m}^3/\text{s})\,(10\,\text{m/s} - 5\,\text{m/s})^2}{(10\,\text{m/s})\,(10\,\text{kg})}$$

$$\boxed{a_c = 25\ \text{m/s}^2 \quad (\text{when } v_c = 5\,\text{m/s})}$$

6.94: PROBLEM DEFINITION

Situation:

A jet strikes a cart and accelerates the cart from zero to one-half the jet velocity.

Find:

Time (s) to accelerate to one-half jet velocity.

Assumptions:

No resistance to cart motion and mass of water jet moving with cart is negligible.

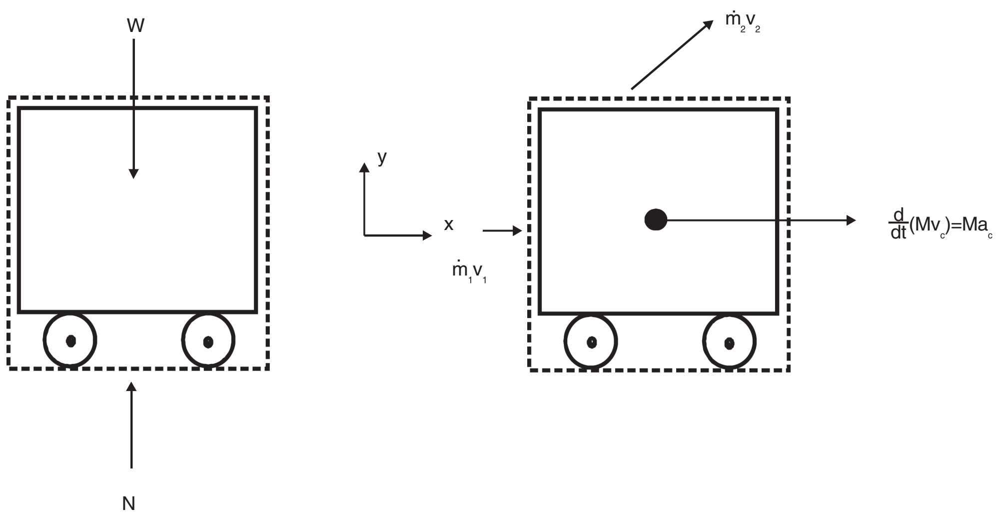
PLAN

Apply the momentum equation to obtain equation of motion for cart and integrate to obtain time.

SOLUTION

Select a control surface surrounding the moving cart. Select a reference frame fixed to the nozzle. Note that a reference frame fixed to the cart would be non-inertial.

Force and momentum diagrams



Momentum equation (x-direction)

$$\sum F_x = \frac{d}{dt}(mv_c) + \dot{m}_2 v_{2_x} = -\dot{m}_1 v_1$$

Momentum accumulation

Note that the cart is accelerating. Thus,

$$\begin{aligned}\frac{d}{dt}\int_{cv} v_x \rho dV &= \frac{d}{dt} v_c \int_{cv} \rho dV = \frac{d}{dt}(Mv_c) \\ &= M\frac{dv_c}{dt}\end{aligned}$$

where M is the mass of the cart (mass of water moving with cart is negligible)
From conservation of mass

$$\begin{aligned} v_{2y} &= (v_j - v_c) \\ v_{2_x} &= v_c \\ \dot{m}_2 &= \dot{m}_1 \end{aligned}$$

Combining terms

$$\begin{aligned} \sum F_x &= \frac{d}{dt}(Mv_c) + \dot{m}(v_{2x} - v_1) \\ 0 &= M\frac{dv_c}{dt} + \rho A_1 (v_j - v_c)(v_c - v_j) \\ M\frac{dv_c}{dt} &= \rho A_1 v_j^2 (1 - \frac{v_c}{v_j})^2 \\ &= \dot{m} v_j (1 - \frac{v_c}{v_j})^2 \end{aligned}$$

Since the jet velocity is constant

$$\begin{aligned} \frac{d}{dt}\left(\frac{v_c}{v_j}\right) &= \frac{\dot{m}}{M}(1 - \frac{v_c}{v_j})^2 \\ \frac{d\left(\frac{v_v}{v_j}\right)}{(1 - \frac{v_c}{v_j})^2} &= \frac{\dot{m}}{M}dt \end{aligned}$$

Integrating and substituting in the limits, $v_c/v_j = 0$ at $t = 0$ and $v_c/v_j = 0.5$ at $t = \Delta t$ gives

$$\begin{aligned} \Delta t &= \frac{M}{\dot{m}} \\ &= \frac{100 \text{ kg}}{45 \text{ kg/s}} \\ & \boxed{\Delta t = 2.22 \text{ s}} \end{aligned}$$

6.95: PROBLEM DEFINITION

Situation:

A problem in rocket-trajectory analysis is described in the problem statement.

Find:

Initial mass of a rocket needed to place the rocket in orbit.

SOLUTION

$$\begin{aligned} M_0 &= M_f \exp\left(\frac{V_{b0}\lambda}{T}\right) \\ &= 50\,\text{kg} \exp\left(\frac{7200\,\text{m/s}}{3000\,\text{m/s}}\right) \end{aligned}$$

$$\boxed{M_0 = 551 \text{ kg}}$$

6.96: PROBLEM DEFINITION

Situation:

A toy rocket is powered by a jet of water—additional details are provided in the problem statement.

Find:

Maximum velocity of the rocket.

Assumptions:

Neglect hydrostatic pressure; Inlet kinetic pressure is negligible.

SOLUTION

Newtons 2^{nd} law.

$$\begin{aligned} \sum F &= ma \\ T - W &= ma \end{aligned}$$

where T =thrust and W =weight

$$\begin{aligned} T &= \dot{m}v_e \\ \dot{m}v_e - mg &= m dv_R/dt \\ dv_R/dt &= (T/m) - g \\ &= (T/(m_i - \dot{m}t)) - g \\ dv_R &= ((Tdt)/(m_i - \dot{m}t)) - gdt \\ v_R &= (-T/\dot{m})\ell n(m_i - \dot{m}t) - gt + \text{const.} \end{aligned}$$

where $v_R = 0$ when $t = 0$. Then

$$\begin{aligned} \text{const.} &= (T/\dot{m})\ln(m_i) \\ v_R &= (T/\dot{m})\ln((m_i)/(m_i - \dot{m}t)) - gt \\ v_{R\max} &= (T/\dot{m})\ln(m_i/m_f) - gt_f \\ T/\dot{m} &= \dot{m}v_e/\dot{m} = v_e \end{aligned}$$

Bernoulli equation
(neglecting hydrostatic pressure)

$$p_i + \rho_f v_i^2/2 = p_e + \rho_f v_e^2/2$$

The exit pressure is zero (gage) and the inlet kinetic pressure is negligible. So

$$\begin{aligned} v_e^2 &= 2p_i/\rho_f \\ &= 2 \times 100 \times 10^3/1000 \\ &= 200 \text{ m}^2/\text{s}^2 \\ v_e &= 14.14 \text{ m/s} \\ \dot{m} &= \rho_e v_e A_e \\ &= 1000 \times 14.14 \times 0.1 \times 0.05^2 \times \pi/4 \\ &= 2.77 \text{ kg/s} \end{aligned}$$

Time for the water to exhaust:

$$\begin{aligned} t &= m_w/\dot{m} \\ &= 0.10/2.77 \\ &= 0.036s \end{aligned}$$

Thus

$$\begin{aligned} v_{\max} &= 14.14\ln((100+50)/50) - (9.81)(0.036) \\ &\boxed{v_{\max} = 15.2 \text{ m/s}} \end{aligned}$$

6.97: PROBLEM DEFINITION

Situation:

Water is discharged from a slot in a pipe—additional details are provided in the problem statement.

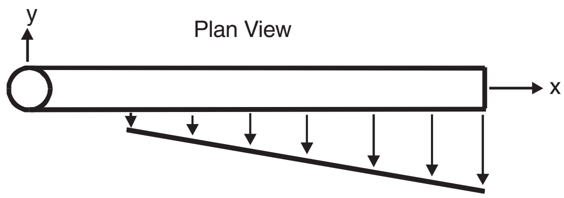
Find:

Reaction (Force and Moment) at station $A - A$

PLAN

Apply the momentum equation and the moment of momentum equation.

SOLUTION



$$v_y = -(3.1 + 3x) \text{ m/s}$$

Momentum equation (y-direction)

$$\begin{aligned}
\sum F_y &= \int v_y \rho \mathbf{v} \cdot \mathbf{dA} \\
F_y &= -\int_{0.3}^{1.3} (3.1 + 3x) \times 1,000 \times (3.1 + 3x) \times 0.015 dx = -465 \text{ N} \\
R_y &= 465 \text{ N}
\end{aligned}$$

Flow rate

$$\begin{aligned}
Q &= \int v dA = 0.015 \int_{0.3}^{1.3} (3.1 + 3x) dx = 0.0825 \text{ m}^3/\text{s} \\
v_1 &= Q/A = 0.0825/(\pi \times 0.04^2) = 16.4 \text{ m/s}
\end{aligned}$$

Momentum equation (z-direction)

$$\sum F_z = -\dot{m}_1 v_1$$

$$F_z - p_A A_A - W_f = -\dot{m} v_1$$

$$F_z = 30,000 \times \pi \times 0.04^2 + 0.08 \times \pi \times 0.04^2 \times 9,810$$

$$+1.3 \times \pi \times 0.025^2 \times 9,810 + 1000 \times 0.0825 \times 16.4$$

$$= 1530 \text{ N}$$

$$R_z = -1530 \text{ N}$$

Moment-of-momentum ($z-$direction)

$$\begin{aligned} T_z &= \int_{cs} rv\rho\mathbf{v}\cdot\mathbf{dA} \\ &= 15\int_{0.3}^{1.3}(3.1+3r)^2 rdr = 413.2\ \text{N}\cdot\text{m} \end{aligned}$$

Moment-of-momentum (y-direction)

$$T_y + Wr_{cm} = 0$$

where W=weight, r_{cm}=distance to center of mass

$$T_y = -1.3\pi\times 0.025^2\times 9810\times 0.65 = -16.28\ \text{N}\cdot\text{m}$$

Net reaction at A-A

$$\boxed{\mathbf{F} = (465\mathbf{j} - 1530\mathbf{k})\,\text{N}}$$

$$\boxed{\mathbf{T} = (16.3\mathbf{j} - 413\mathbf{k})\,\text{N}\cdot\ \text{m}}$$

6.98: PROBLEM DEFINITION

Situation:

A helicopter rotor uses two small rockets motors—details are provided in the problem statement.

Find:

Power provided by rocket motors.

PLAN

Apply the momentum equation. Select a control volume that encloses one motor, and select a stationary reference frame.

SOLUTION

Velocity analysis

$$\begin{aligned} v_i &= 0 \\ V_i &= rw \\ &= 3.5 \times 2\pi \\ &= 21.991 \text{ m/s} \\ V_0 &= 500 \text{ m/s} \\ v_0 &= (500 - 21.99) \text{ m/s} \\ &= 478.01 \text{ m/s} \end{aligned}$$

Flow rate

$$\begin{aligned} \dot{m} &= \rho A_i V_i \\ &= 1.2 \times .002 \times 21.991 \\ &= 0.05278 \text{ kg/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} F_x &= \dot{m}v_0 - \dot{m}v_i \\ &= \dot{m}v_0 \\ &= 0.05278 \times 478 \\ &= 25.23 \text{ N} \end{aligned}$$

Power

$$\begin{aligned} P &= 2Frw \\ &= 2 \times 25.23 \times 3.5 \times 2\pi \\ &= 1110 \text{ W} \end{aligned}$$

$$\boxed{P = 1.11 \text{ kW}}$$

6.99: PROBLEM DEFINITION

Situation:

A rotating lawn sprinkler is to be designed.

The design target is 0.00625 m of water per hour over a circle of 15 m radius.

Find:

Determine the basic dimensions of the lawn sprinkler.

Assumptions:

The Bernoulli equation applies.

Assume mechanical friction is present.

PLAN

Apply the momentum equation.

SOLUTION

Flow rate. To supply water to a circle 15 m in diameter at a 0.00625 m per hour requires a discharge of

$$\begin{aligned} Q &= \dot{h}A \\ &= (0.00625)\pi(15^2/4)/3600 \\ &= 3.07 \times 10^{-4}\,\mathrm{m}^3/\,\mathrm{s} \end{aligned}$$

Bernoulli equation. Assuming no losses between the supply pressure and the sprinkler head would give and exit velocity at the head of

$$\begin{aligned} V &= \sqrt{\frac{2p}{\rho}} \\ &= \sqrt{\frac{2 \times 350,000}{1000}} \\ &= 26.5 \text{ m/s} \end{aligned}$$

If the water were to exit the sprinkler head at the angle for the optimum trajectory (45^o), the distance traveled by the water would be

$$s = \frac{V_e^2}{2g}$$

The velocity necessary for a 7.5 m distance (radius of the spray circle) would be

$$\begin{aligned} V_e^2 &= 2gs = 2 \times 9.81 \times 7.5 = 147.2 \\ V_e &= 12.1 \text{ m/s} \end{aligned}$$

This means that there is ample pressure available to do the design. There will be losses which will affect the design. As the water spray emerges from the spray head, atomization will occur which produces droplets. These droplets will experience aerodynamic drag which will reduce the distance of the trajectory. The size distribution of droplets will lead to small droplets moving shorter distances and larger droplets farther which will contribute to a uniform spray pattern.

The sprinkler head can be set in motion by having the water exit at an angle with respect to the radius. For example if the arm of the sprinkler is 0.1 m and the angle of deflection at the end of the arm is 10 degrees, the torque produced is

$$\begin{aligned} M &= \rho Q r V_e \sin\theta \\ &= 1000 \times 0.000307 \times 0.1 \times 12.1 \times \sin 10^\circ \\ &= 0.07\,\mathrm{N-m} \end{aligned}$$

The downward load on the head due to the discharge of the water is

$$\begin{aligned} F_y &= \rho Q V_e \sin 45^\circ \\ &= 1000 \times 0.000307 \times 12.1 \times \sin 45^\circ \\ &= 2.6\ \mathrm{N} \end{aligned}$$

The moment necessary to overcome friction on a flat plate rotating on another flat plate is

$$M = (2/3)\mu F_n r_o$$

where μ is the coefficient of friction and r_o is the radius of the plate. Assuming a 0.0125 m radius, the limiting coefficient of friction would be

$$\begin{aligned} \mu &= \frac{3}{2}\frac{M}{F_n r_o} \\ &= \frac{3}{2}\frac{0.07}{2.6 \times (0.0125)} \\ &= 3.2 \end{aligned}$$

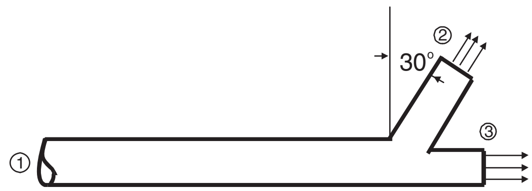
This is very high, which means there is adequate torque to overcome friction.

These are initial calculations showing the feasibility of the design. A more detailed design would now follow.

6.100: PROBLEM DEFINITION

Situation:

Water flows out a pipe with two exit nozzles—additional details are provided in the problem statement.



Find:

Reaction (Force and Moment) at section 1.

PLAN

Apply the continuity equation, then the momentum equation and the moment of momentum equation.

SOLUTION

Continuity equation

$$v_1 = (0.01 \times 15 + 0.02 \times 15)/0.06 = 7.5 \text{ m/s}$$

Momentum equation (x-direction)

$$\sum F_x = \dot{m}_3 v_{3x} + \dot{m}_2 v_{2x}$$
$$F_x + p_1 A_1 = -\rho v_1^2 A_1 + \rho v_3^2 A_3 + \rho v_2^2 A_2 \cos 60^\circ$$
$$F_x = -135000 \times 0.06 - 1000 \times 7.5^2 \times 0.06 + 1000 \times 15^2 \times 0.02$$
$$+1000 \times 15^2 \times 0.001 \times \cos 60^\circ = 5850 \, \text{N}$$

Momentum equation (y-direction)

$$\sum F_y = \dot{m}_2 v_{2y}$$
$$F_y = 1000 \times 15 \times 15 \times 0.01 \times \cos 30^\circ = 1948.5 \text{ N}$$

Moment-of-momentum ($z-$direction)

$$r_2 \dot{m}_2 v_{2y} = (0.9)(1000 \times 0.01 \times 15) 15 \sin 60^\circ = 1753 \, \text{N} - \text{m}$$

Reaction at section 1

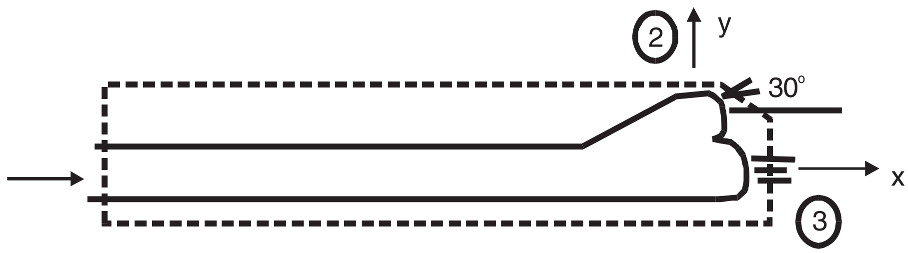
$$\boxed{\mathbf{F} = (5850\mathbf{i} - 1949\mathbf{j})\text{N}}$$

$$\boxed{\mathbf{M} = (-1753\mathbf{k}) \text{ N-m}}$$

6.101: PROBLEM DEFINITION

Situation:

Water flows out a pipe with two exit nozzles—additional details are provided in the problem statement.



Find:

Reaction (Force and Moment) at section 1.

PLAN

Apply the continuity equation, then the momentum equation and the moment of momentum equation.

SOLUTION

Continuity equation equation

$$V_1 = (0.01 \times 20 + 0.02 \times 20)/0.1 = 6 \text{ m/s}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \sum \dot{m}_o v_{ox} - \sum \dot{m}_i v_{ix} \\
F_x + p_1 A_1 &= \dot{m}_3 v_3 + \dot{m}_2 v_2 \cos 30^\circ - \dot{m}_1 v_1 \\
F_x &= -200,000 \times 0.1 - 1000 \times 6^2 \\
&\quad \times 0.1 + 1000 \times 20^2 \times 0.02 \\
&\quad +1000 \times 20^2 \times 0.01 \times \cos 30^\circ \\
&= \boxed{F_x = \text{-}12,100 \text{ N}}
\end{aligned}$$

Momentum equation (y-direction)

$$F_y - W = \dot{m}_2 v_2 \sin 30^\circ$$

Weight

$$\begin{aligned}
W &= W_{\mathrm{H_2O}} + W_{\text{pipe}} \\
&= (0.1)(1)(9810) + 90 \\
&= 1071 \text{ N}
\end{aligned}$$

thus

$$
\begin{aligned}
F_y &= 1000 \times 20^2 \times 0.01 \times \sin 30^\circ + 1,071 \\
&= \boxed{F_y = 3070 \text{ N}}
\end{aligned}
$$

Moment-of-momentum ($z-$direction)

$$
\begin{aligned}
M_z - W r_{cm} &= r_2 \dot{m}_2 v_{2y} \\
M_z &= (1071 \times 0.5) + (1.0)(1000 \times 0.01 \times 20)(20 \sin 30^\circ) \\
&= 2535 \text{ N} \cdot \text{m}
\end{aligned}
$$

Reaction at section 1

$$
\boxed{\mathbf{F} = (12.1\mathbf{i} - 3.1\mathbf{j}) \,\text{kN}}
$$

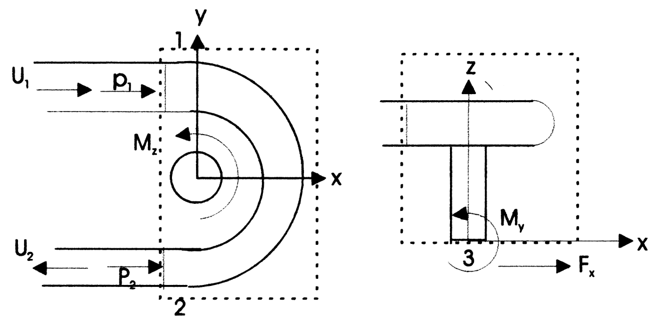
$$
\boxed{\mathbf{M} = (-2.54\mathbf{k}) \,\text{kN} \cdot \text{ m}}
$$

6.102: PROBLEM DEFINITION

Situation:

A reducing pipe bend held in place by a pedestal. Water flow. No force transmission through the pipe at sections 1 and 2.

Assume irrotational flow. Neglect weight



Find:

(a) Force needed to hold bend stationary: **F**

(b) Moment needed to hold bend stationary: **M**

PLAN

Apply the Bernoulli equation, then the momentum equation, and then the moment of momentum equation.

SOLUTION

Bernoulli equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{v_1^2}{2g} &= \frac{p_2}{\gamma} + \frac{v_2^2}{2g} \\
v_1 &= Q/A_1 = 0.06/(\pi/4 \times 0.15^2) = 3.4 \text{ m/s} \\
v_2 &= Q/A_1 = 0.06/(\pi/4 \times 0.1^2) = 7.6 \text{ m/s} \\
p_1 &= 140,000 \text{ Pa} \\
p_2 &= p_1 + \rho(v_1^2 - v_2^2)/2 \\
&= 140,000 \,\text{Pa} + 1000(3.4^2 - 7.6^2)/2 \\
&= 116,900 \text{ N}
\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}
F_x + p_1 A_1 + p_2 A_2 &= \dot{m} v_{2_x} - \dot{m} v_{1_x} \\
F_x &= -p_1 A_1 - p_2 A_2 - \dot{m}(v_2 + v_1)
\end{aligned}$$

where

$$\begin{aligned}
A_1 &= \pi/4 \times 0.15 = 0.018 \text{ m}^2 \\
A_2 &= \pi/4 \times 0.1 = 0.008 \text{ m}^2 \\
\dot{m} &= \rho A_1 v_1 = 1000 \times 0.018 \times 3.4 = 61.2 \text{ kg/s}
\end{aligned}$$

thus

$$F_x = -140,000 \times 0.018 - 116,900 \times 0.008 - 61.2(3.4 + 7.6) = 4128 \text{ N}$$

Moment-of-momentum ($z-$direction)

$$\begin{aligned} m_z - rp_1A_1 + rp_2A_2 &= -r\dot{m}v_2 + r\dot{m}v_1 \\ m_z &= r(p_1A_1 - p_2A_2) - r\dot{m}(v_2 - v_1) \end{aligned}$$

where $r = 0.3$ m

$$\begin{aligned} M_z &= 0.3(140,000 \times 0.018 - 116,900 \times 0.008) - 0.3 \times 61.2(7.6 - 3.4) \\ &= 398 \text{ N-m} \end{aligned}$$

Moment-of-momentum (y-direction)

$$M_y + p_1A_1r_3 + p_2A_2r_3 = -r_3\dot{m}v_2 - r_3\dot{m}_1v_1$$

where $r_3 = 0.6$ m

$$\begin{aligned} M_y &= -r_3[p_1A_1 + p_2A_2 + \dot{m}(v_1 + v_2)] \\ &= -0.6 \times 4128 \\ M_y &= -2477 \text{ N-m} \end{aligned}$$

Net force and moment at 3

$$\boxed{\mathbf{F} = -4128\mathbf{i}\,\text{N}}$$

$$\boxed{\mathbf{M} = (-2477 + 398\mathbf{k}) \text{ N-m}}$$

6.103: PROBLEM DEFINITION

Situation:
Centrifugal fan is used to pump air

Find:
Power (kW) required to operate fan

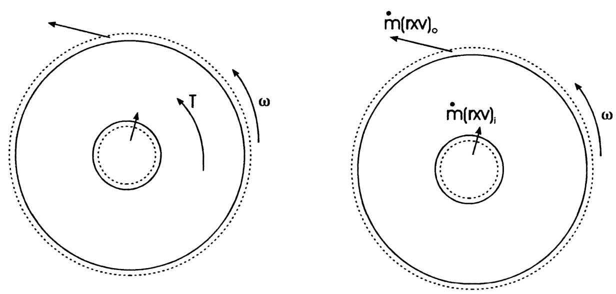
Assumptions:
Neglect the compressibility of air.

PLAN

Apply the moment of momentum equation between inlet and outlet.

SOLUTION

The control volume enclosed the rotor but does not rotate. The flow is steady within the control volume. Assume positive direction comes out of the page, the $\mathbf{e}_z$ direction.



The moment diagram shows one moment (torque)

$$\sum \mathbf{M} = T\mathbf{e}_z$$

There is no moment of momentum inflow because the inlet velocity is radial (or the fluid enters with zero radius). There is a moment of momentum outflow.
The moment of momentum equation is

$$\sum \mathbf{M} = \frac{d}{dt}\int_{cv} \rho(\mathbf{r} \times \mathbf{v})dV + \sum \dot{m}_o(\mathbf{r} \times \mathbf{v})_o - \sum \dot{m}_i(\mathbf{r} \times \mathbf{v})_i$$

Since the flow is steady and there is not inflow of moment of moment, the equation reduces to

$$T\mathbf{e}_z = \dot{m}_o(\mathbf{r} \times \mathbf{v})_o$$

The exit radial velocity is

$$v_r = \frac{Q}{\pi D\ell} = \frac{0.8\,\mathrm{m}^3/s}{\pi \times 0.3\ \mathrm{m} \times (0.05\ \mathrm{m})} = 16.98\ \mathrm{m/s}$$

The density of the air is

$$\rho = \frac{p}{RT} = \frac{101.3\,\text{kPa}}{287\ \text{J/kg-K} \times 288\ \text{K}} = 1.23\ \text{kg/m}^3$$

At the outlet

$$(\mathbf{r} \times \mathbf{v})_o = \frac{D}{2}\omega\frac{D}{2}\mathbf{e}_z$$

The torque is

$$\begin{aligned} T &= \rho Q\omega\frac{D^2}{4} = 1.23\,\text{kg/m}^3 \times 0.8\,\text{m}^3/\text{s} \times 377\,\text{rad/s} \times \frac{0.3^2\text{m}^2}{4} \\ &= 8.3\ \text{N-m} \end{aligned}$$

The power is

$$\begin{aligned} P &= T\omega = 8.3\ \text{N-m} \times 377\,\text{rad/s} = 3129\ \text{N-m/s} \\ & \boxed{P = 3.13\ \text{kW}} \end{aligned}$$