8.1: PROBLEM DEFINITION

Situation:

Dimensions of density, viscosity and pressure.

Find:

Primary dimensions of density, viscosity and pressure.

SOLUTION

Density

$$[\rho] = \frac{M}{L^3}$$

Viscosity

$$[\mu] = \frac{M}{LT}$$

Pressure

$$[p] = \frac{M}{LT^2}$$

8.2: PROBLEM DEFINITION

Situation:

Application of the Buckingham Π theorem.
6 dimensional variables.
3 primary dimensions.

Find:

Number of dimensionless variables.

SOLUTION

Six dimensional variables, three primary variables (6-3)=3 dimensionless variables (or π groups)

8.3: PROBLEM DEFINITION

Situation:

Dimensional homogeneity.

Find:

Definition of dimensional homogeneity.

SOLUTION

An equation is dimensionally homogeneous if all the terms have the same dimensions.

8.4: PROBLEM DEFINITION

Situation:

Consider equations:

(a) $Q = (2/3)CL\sqrt{2g}H^{3/2}$.

(b) $V = (1.49/n)R^{2/3}S^{1/2}$.

(c) $h_f = f(L/D)V^2/2g$.

(d) $D = 0.074R_e^{-0.2}Bx\rho V^2/2$.

Find:

Determine which equations are homogeneous.

SOLUTION

(a)

$$\begin{aligned} Q &= (2/3)CL\sqrt{2g}H^{3/2} \\ [Q] &= \frac{L^3}{T} = L\left(\frac{L}{T^2}\right)^{1/2} L^{3/2} \\ \frac{L^3}{T} &= \frac{L^3}{T} \quad \boxed{\text{homogeneous}} \end{aligned}$$

(b)

$$\begin{aligned} V &= \left(\frac{1.49}{n}\right) R^{2/3}S^{1/2} \\ [V] &= L/T = L^{-1/6}L^{2/3} \quad \boxed{\text{not homogeneous}} \end{aligned}$$

(c)

$$\begin{aligned} h_f &= f(L/D)V^2/2g \\ [h_f] &= L = \frac{(L/L)\,(L/T)^2}{L/T^2} \boxed{\text{homogeneous}} \end{aligned}$$

(d)

$$\begin{aligned} D &= \frac{0.074R_e^{-0.2}Bx\rho V^2}{2} \\ [D] &= \frac{ML}{T^2} = L \times L \times \left(\frac{M}{L^3}\right)\left(\frac{L}{T}\right)^2 \boxed{\text{homogeneous}} \end{aligned}$$

8.5: PROBLEM DEFINITION

Situation:

Consider variables:
(a) T (torque).
(b) $\rho V^2/2$.
(c) $\sqrt{\tau/\rho}$.
(d) Q/ND^3.

Find:

Determine the dimensions of the variables.

SOLUTION

(a) $[T] = \frac{ML}{T^2} \times L = \boxed{\frac{ML^2}{T^2}}$

(b) $[\rho V^2/2] = \left(\frac{M}{L^3}\right)\left(\frac{L}{T}\right)^2 = \boxed{\frac{M}{LT^2}}$

(c) $[\sqrt{\tau/\rho}] = \sqrt{\left(\frac{ML/T^2}{L^2}\right) / \left(\frac{M}{L^3}\right)} = \boxed{L/T}$

(d) $[Q/ND^3] = \left(\frac{L^3}{T}\right) / (T^{-1}L^3) = 1 \rightarrow \boxed{\text{Dimensionless}}$

8.6: PROBLEM DEFINITION

Situation:

Liquid is draining out of a tank through an orifice at the bottom.

Find:

The $\pi-$groups in the form.

$$\frac{\Delta h}{d} = f(\pi_1, \pi_2, \pi_3)$$

.

PLAN

The fluid density and specific weight must also be included so functional form is

$$\Delta h = f(d, D, \gamma, t, h_i, \rho)$$

Use the step-by-step method.

SOLUTION

Δh	L	$\frac{\Delta h}{d}$	0	$\frac{\Delta h}{d}$	0	$\frac{\Delta h}{d}$	0
t	T	t	T	t	T		
ρ	$\frac{M}{L^3}$	ρd^3	M				
D	L	$\frac{D}{d}$	0	$\frac{D}{d}$	0	$\frac{D}{d}$	0
d	L						
γ	$\frac{M}{L^2T^2}$	γd^2	$\frac{M}{T^2}$	$\frac{\gamma}{\rho d}$	$\frac{1}{T^2}$	$\frac{\gamma t^2}{\rho d}$	0
h_1	L	$\frac{h_1}{d}$	0	$\frac{h_1}{d}$	0	$\frac{h_1}{d}$	0

In the first step, length is taken out with d. In the second step, mass is taken out with ρd^3. In the third step, time is taken out with t. The functional relationship is

$$\boxed{\frac{\Delta h}{d} = f(\frac{D}{d}, \frac{\gamma t^2}{\rho d}, \frac{h_1}{d})}$$

This can also be written as

$$\frac{\Delta h}{d} = f(\frac{d}{D}, \frac{gt^2}{d}, \frac{h_1}{d})$$

8.7: PROBLEM DEFINITION

Situation:

Liquid rises in a capillary tube.

Find:

The $\pi-$groups.

PLAN

Use the step-by-step method.

SOLUTION

$$\begin{array}{llllll} h & L & \frac{h}{d} & 0 & \frac{h}{d} & 0 \\ d & L & & & & \\ \sigma & \frac{M}{T^2} & \sigma & \frac{M}{T^2} & \frac{\sigma}{\gamma d^2} & 0 \\ \gamma & \frac{M}{L^2T^2} & \gamma d^2 & \frac{M}{T^2} & & \end{array}$$

In the first step, d was used to remove length and in the second γd^2 was used to remove both length and time. The final functional form is

$$\boxed{\frac{h}{d} = f(\frac{\sigma}{\gamma d^2})}$$

8.8: PROBLEM DEFINITION

Situation:
Drag force on a small sphere.

Find:
The relevant $\pi-$groups.

PLAN

The functional form of the equation is

$$F_D = f(V, d, \mu)$$

Use the step-by-step method.

SOLUTION

$$\begin{array}{llllllll}
F_D & \frac{ML}{T^2} & \frac{F_D}{d} & \frac{M}{T^2} & \frac{F_D}{\mu d^2} & \frac{1}{T} & \frac{F_D}{\mu V d} & 0 \\
V & \frac{L}{T} & \frac{V}{d} & \frac{1}{T} & \frac{V}{d} & \frac{1}{T} & & \\
\mu & \frac{M}{LT} & \mu d & \frac{M}{T} & & & & \\
d & L & & & & & &
\end{array}$$

In the first step, length is removed with d. In the second, mass is removed with μd and in the third time is removed with V/d. Finally

$$\boxed{\frac{F_D}{\mu V d} = C}$$

8.9: PROBLEM DEFINITION

Situation:

The side thrust of a rough spinning ball in a fluid.

Find:

The $\pi-$groups in the form

$$\frac{F_s}{\rho V_o^2 D^2} = f(\pi_1, \pi_2, \pi_3)$$

.

PLAN

Use the step-by-step method.

SOLUTION

F	$\frac{ML}{T^2}$	$\frac{F}{D}$	$\frac{M}{T^2}$	$\frac{F}{\rho D^4}$	$\frac{1}{T^2}$	$\frac{F}{\rho V^2 D^2}$	0
D	L						
V	$\frac{L}{T}$	$\frac{V}{D}$	$\frac{1}{T}$	$\frac{V}{D}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	ρD^3	M				
μ	$\frac{M}{LT}$	μD	$\frac{M}{T}$	$\frac{\mu}{\rho D^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho V D}$	0
k_s	L	$\frac{k_s}{D}$	0	$\frac{k_s}{D}$	0	$\frac{k_s}{D}$	0
ω	$\frac{1}{T}$	ω	$\frac{1}{T}$	ω	$\frac{1}{T}$	$\frac{\omega D}{V}$	0

Length is removed in the first step with D, mass in the second step with ρD^3 and time in the third step with V/D. The functional form is

$$\boxed{\frac{F}{\rho V^2 D^2} = f(\frac{\rho V D}{\mu}, \frac{k_s}{D}, \frac{\omega D}{V})}$$

There are other possible forms.

8.10: PROBLEM DEFINITION

Situation:

Steady, viscous flow through a small horizontal tube.

Find:

The $\pi-$groups.

PLAN

The functional form is

$$\frac{\Delta p}{\Delta \ell} = f(V, \mu, D)$$

There are four dimensional variables so there will only be one π-group. Use the step-by-step method.

SOLUTION

$\frac{\Delta p}{\Delta \ell}$	$\frac{M}{L^2T^2}$	$\frac{\Delta p}{\Delta \ell}D^2$	$\frac{M}{T^2}$	$\frac{\Delta p}{\Delta \ell}\frac{D}{\mu}$	$\frac{1}{T}$	$\frac{\Delta p}{\Delta \ell}\frac{D^2}{\mu V}$	0
μ	$\frac{M}{LT}$	μD	$\frac{M}{T}$				
V	$\frac{L}{T}$	$\frac{V}{D}$	$\frac{1}{T}$	$\frac{V}{D}$	$\frac{1}{T}$		
D	L						

Length is removed in the first step with D, mass is removed in the second with μD and time is removed in the third with V/D. Finally we have

$$\boxed{\frac{\Delta p}{\Delta \ell}\frac{D^2}{\mu V} = C}$$

or

$$\frac{\Delta p}{\Delta \ell} = C\frac{\mu V}{D^2}$$

8.11: PROBLEM DEFINITION

Situation:

Vortex meter for flow rate measurement.

Find:

The functional relation in the form

$$\frac{Q}{\omega D^3} = f(\pi_1, \pi_2)$$

PLAN The functional form of the equation is

$$Q = (\omega, D, l, \rho, \mu)$$

Use the step-by-step method.

SOLUTION Setting up the table

Q	$\frac{L^3}{T}$	$\frac{Q}{\omega}$	L^3	$\frac{Q}{\omega D^3}$		$\frac{Q}{\omega D^3}$
ω	$\frac{1}{T}$					
D	L	D	L			
l	L	l	L	$\frac{l}{D}$		$\frac{l}{D}$
ρ	$\frac{M}{L^3}$	ρ	$\frac{M}{L^3}$	ρD^3	M	$\frac{\omega\rho D^2}{\mu}$
μ	$\frac{M}{LT}$	$\frac{\mu}{\omega}$	$\frac{M}{L}$	$\frac{\mu D}{\omega}$	M	

The time was eliminated with ω, the length with D and the mass with ρD^3. The function form is

$$\boxed{\frac{Q}{\omega D^3} = f(\frac{l}{D}, \frac{\omega\rho D^2}{\mu})}$$

8.12: PROBLEM DEFINITION

Situation:
A centrifugal pump develops a pressure change when in operation.

Find:
The $\pi-$groups.

PLAN

The functional form is

$$\Delta p = f(D, n, Q, \rho)$$

There are 5 dimensional variables so there should be 2 π groups. Use the step-by-step method.

SOLUTION

Δp	$\frac{M}{LT^2}$	$\Delta p D$	$\frac{M}{T^2}$	$\frac{\Delta p}{\rho D^2}$	$\frac{1}{T^2}$	$\frac{\Delta p}{n^2 \rho D^2}$	0
D	L						
n	$\frac{1}{T}$	n	$\frac{1}{T}$	n	$\frac{1}{T}$		
Q	$\frac{L^3}{T}$	$\frac{Q}{D^3}$	$\frac{1}{T}$	$\frac{Q}{D^3}$	$\frac{1}{T}$	$\frac{Q}{nD^3}$	0
ρ	$\frac{M}{L^3}$	ρD^3	M				

In the first step, length is removed with D. In the second step, mass is removed with ρD^3 and time is removed in the third step with n. The functional form is

$$\boxed{\frac{\Delta p}{n^2 \rho D^2} = f\left(\frac{Q}{nD^3}\right)}$$

8.13: PROBLEM DEFINITION

Situation:

The force on a satellite in the earth's upper atmosphere.

Find:

The nondimensional form of equation.

PLAN

The dimensional form of the equation is

$$F = f(\lambda, D, \rho, c)$$

Use the exponent method.

SOLUTION

$$\begin{aligned} F &= \lambda^a \rho^b D^c c^d \\ ML/T^2 &= L^a \left(\frac{M}{L^3}\right)^b L^c \left(\frac{L}{T}\right)^d \\ &= L^{a-3b+c+d} M^b T^{-d} \end{aligned}$$

Equating powers of M, L and T, we have

$$\begin{aligned} T : d &= 2 \\ M : b &= 1 \\ L : 1 &= a - 3 + c + c \\ 1 &= a - 3 + c + 2 \\ a + c &= 2 \\ a &= 2 - c \end{aligned}$$

Therefore,

$$F = \lambda^{(2-c)} \rho D^c c^2$$

$$\boxed{F/(\rho c^2 \lambda^2) = f(D/\lambda)}$$

Another valid answer would be

$$\boxed{F/(\rho c^2 D^2) = f(D/\lambda)}$$

8.14: PROBLEM DEFINITION

Situation:

A study involves capillary rise of a liquid in a tube.

Find:

The $\pi-$groups in the form

$$\frac{h}{d} = f(\pi_1, \pi_2, \pi_3)$$

PLAN

The dimensional form of the equation is

$$h = f(t, d, \sigma, \rho, \gamma, \mu)$$

There are 7 dimensional variables so there should be 4 π groups. Use the step-by-step method.

SOLUTION

h	L	$\frac{h}{d}$	0	$\frac{h}{d}$	0	$\frac{h}{d}$	0
t	T	t	T	t	T		
σ	$\frac{M}{T^2}$	σ	$\frac{M}{T^2}$	$\frac{\sigma}{\rho d^3}$	$\frac{1}{T^2}$	$\frac{\sigma t^2}{\rho d^3}$	0
ρ	$\frac{M}{L^3}$	ρd^3	M				
γ	$\frac{M}{L^2T^2}$	γd^2	$\frac{M}{T^2}$	$\frac{\gamma}{\rho d}$	$\frac{1}{T^2}$	$\frac{\gamma t^2}{\rho d}$	0
μ	$\frac{M}{LT}$	μd	$\frac{M}{T}$	$\frac{\mu}{\rho d^2}$	$\frac{1}{T}$	$\frac{\mu t}{\rho d^2}$	0
d	L						

In the first step, length is removed with d. In the second step, mass is removed with ρd^3 and in the final step, time is removed with t. The final functional form is

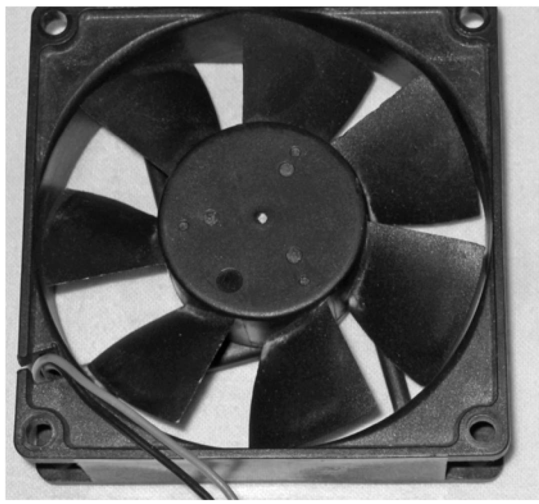
$$\boxed{\frac{h}{d} = f\left(\frac{\sigma t^2}{\rho d^3}, \frac{\gamma t^2}{\rho d}, \frac{\mu t}{\rho d^2}\right)}$$

8.15: PROBLEM DEFINITION

Situation:

An engineer characterizing power consumed by a fan.
Power depends on four variables: $P = f(\rho, D, Q, n)$.

- ρ is the density of air
- D is the diameter of the fan impeller
- Q is the flow rate produced by the fan
- n is the rotation rate of the fan.



Find:

(a) Find the relevant π-groups.
(b) Suggest a way to plot the data.

PLAN

Apply the π-Buckingham theorem to establish the number of π-groups that need to be found. Apply the step-by-step method to find these groups and then use the π-groups to decide how a plot should be made.

SOLUTION

π-Buckingham theorem. The number of variables is $n = 5$. The number of primary dimensions is $m = 3$.

$$\begin{aligned}\text{Number of } \pi\text{-group} = & \quad n - m \\ = & \quad 5 - 3 \\ = & \quad 2\end{aligned}$$

Step by step method. The variable of interest are $P = f(\rho, D, Q, n)$. The step-by-step process is given in the table below. In the first step, the length dimension is eliminated with D. In the second step, the mass dimension is eliminated with ρD^3. In the third step, the time dimension is eliminated with $1/n$.

P	$\frac{ML^2}{T^3}$	$\frac{P}{D^2}$	$\frac{M}{T^3}$	$\frac{P}{\rho D^5}$	$\frac{1}{T^3}$	$\frac{P}{\rho D^5 n^3}$	0
ρ	$\frac{M}{L^3}$	ρD^3	M				
D	L						
Q	$\frac{L^3}{T}$	$\frac{Q}{D^3}$	$\frac{1}{T}$	$\frac{Q}{D^3}$	$\frac{1}{T}$	$\frac{Q}{nD^3}$	0
n	$\frac{1}{T}$	n	$\frac{1}{T}$	n	$\frac{1}{T}$		

The functional form of the equation using π-groups to characterize the variables is:

$$\boxed{\frac{P}{\rho D^5 n^3} = f\left(\frac{Q}{nD^3}\right)}$$

Answer part b ==> Plot dimensionless power $(P/\rho D^5 n^3)$on the vertical axis, dimensionless flow rate (Q/nD^3)on the horizontal axis.

8.16: PROBLEM DEFINITION

Situation:

A liquid in a pipe flows through an abrupt contraction.

Find:

The π-groups that characterize pressure drop. Express the answer as

$$\frac{\Delta p d^4}{\rho Q^2} = f(\pi_1,\ \pi_2)$$

PLAN

The pressure change should depend on the upstream and downstream diameters, the discharge, density, viscosity The functional form of the equation is

$$\Delta p = f(Q, \rho, \mu, d, D)$$

Use the step-by-step method.

SOLUTION

Δp	$\frac{M}{LT^2}$	$\Delta p d$	$\frac{M}{T^2}$	$\frac{\Delta p}{\rho d^2}$	$\frac{1}{T^2}$	$\frac{\Delta p d^4}{\rho Q^2}$	0
Q	$\frac{L^3}{T}$	$\frac{Q}{d^3}$	$\frac{1}{T}$	$\frac{Q}{d^3}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	ρd^3	M				
μ	$\frac{M}{LT}$	μd	$\frac{M}{T}$	$\frac{\mu}{\rho d^2}$	$\frac{1}{T}$	$\frac{\mu d}{\rho Q}$	0
D	L	$\frac{D}{d}$	0	$\frac{D}{d}$	0	$\frac{D}{d}$	0
d	L						

Length is removed with d in the first step, mass with ρd^3 in the second step and time with Q/d^3 in the third step. The final form is

$$\boxed{\frac{\Delta p d^4}{\rho Q^2} = f\left(\frac{\mu d}{\rho Q}, \frac{D}{d}\right)}$$

8.17: PROBLEM DEFINITION

Situation:

A solid particle falls through a viscous fluid.

The functional form is

$$V = f(\rho_f, \rho_p, \mu, D, g)$$

Find:

Find the $\pi-$groups—express the answer in the form:

$$\frac{V}{\sqrt{gD}} = f\left(\pi_1,\ \pi_2\right)$$

PLAN

Use the exponent method.

SOLUTION

$$V^a = \rho_f^b \rho_p^c \mu^d D^e g^f$$

Writing out the dimensions

$$\left(\frac{L}{T}\right)^a = \left(\frac{M}{L^3}\right)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{LT}\right)^d (L)^e \left(\frac{L}{T^2}\right)^f$$

Setting up the equations for dimensional homogeneity

$$\begin{array}{ll} L: & a = -3b - 3c - d + e + f \\ M: & 0 = b + c + d \\ T: & a = d + 2f \end{array}$$

Substituting the equation for T into the one for L gives

$$\begin{aligned} 0 &= -3b - 3c - 2d + e - f \\ 0 &= b + c + d \end{aligned}$$

Solving for e from the first equation and c from the second equation

$$\begin{aligned} e &= 3b + 3c + 2d + f \\ c &= -d - b \end{aligned}$$

and the equation for e becomes

$$e = -d + f$$

Substituting into the original equation

$$V^{d+2f} = \rho_f^b \rho_p^{-d-b} \mu^d D^{-d+f} g^f$$

Collecting terms

$$\left(\frac{V\rho_p D}{\mu}\right)^d = \left(\frac{Dg}{V^2}\right)^f \left(\frac{\rho_f}{\rho_p}\right)^b$$

The functional equation can be written as

$$\boxed{\frac{V}{\sqrt{gD}} = f\left(\frac{V\rho_p D}{\mu}, \frac{\rho_f}{\rho_p}\right)}$$

8.18: PROBLEM DEFINITION

Situation:

Calibration of a flow meter to measure mass flow through a pipe.

The dimensional functional form:

$$\dot{m} = f(D, \mu, \Delta p, \rho)$$

Find:

The $\pi-$groups in the form

$$\frac{\dot{m}}{\sqrt{\rho \Delta p D^4}} = f(\pi)$$

PLAN

Use the exponent method.

SOLUTION

The functional relationship is

$$\dot{m} = f(D, \mu, \Delta p, \rho)$$

Using the exponent method, we have

$$\dot{m}^a = D^b \mu^c \Delta p^d \rho^e$$

Writing out the dimensional equation

$$\frac{M}{T}^a = L^b \left(\frac{M}{LT}\right)^c \left(\frac{M}{LT^2}\right)^d \left(\frac{M}{L^3}\right)^e$$

and the equations for the dimensions are

$$\begin{aligned} L: &\quad 0 = b - c - d - 3e \\ M: &\quad a = c + d + e \\ T: &\quad a = c + 2d \end{aligned}$$

Substituting the equation for time into the equation for mass yields two equations

$$\begin{aligned} 0 &= b - c - d - 3e \\ 0 &= -d + e \quad \text{or} \quad d = e \end{aligned}$$

and the first equation becomes

$$0 = b - c - 4d \qquad \text{or} \qquad b = c + 4d$$

Substituting back into the original equation

$$\dot{m}^{c+2d} = D^{c+4d} \mu^c \Delta p^d \rho^d$$

Collecting like powers gives

$$\left(\frac{\dot{m}^2}{D^4\rho\Delta p}\right)^d = \left(\frac{\mu D}{\dot{m}}\right)^c$$

A functional relationship is

$$\boxed{\frac{\dot{m}}{\sqrt{\rho\Delta p}D^2} = f\left(\frac{\mu D}{\dot{m}}\right)}$$

8.19: PROBLEM DEFINITION

Situation:
A torpedo-like device travels just below the water surface.

Find:
Identify which $\pi-$groups are significant.
Justify the answer.

SOLUTION

- Viscous stresses influence drag. Thus, Reynolds number is significant.
- Because the body in near the surface, the motion will produce waves. These waves will influence drag. Thus, the Froude number is important.
- A major design consideration is the drag force on the object. The appropriate $\pi-$group is the coefficient of drag (C_D)which is defined by

$$C_D = \frac{F_{\text{drag}}}{\rho V^2/2A_r}$$

Answer ==> Significant $\pi-$groups are Reynolds number, Froude number and the coefficient of drag.

8.20: PROBLEM DEFINITION

Situation:

Drag forces on an oscillating fin is being tested in a wind tunnel.

$$F_D = f(\rho, V, S, \omega)$$

Find:

The $\pi-$groups in the form

$$\frac{F_D}{\rho V^2 S} = f(\pi)$$

PLAN

Since there are 5 dimensional variables, there are only 2 π-groups. Use the exponent method.

SOLUTION

The functional relationship is

$$F_D = f(\rho, V, S, \omega)$$

Writing out the dimensional parameters using the exponent method

$$F_D^a = \rho^b V^c S^d \omega^e$$

Including the dimensions

$$\left(\frac{ML}{T^2}\right)^a = \left(\frac{M}{L^3}\right)^b \left(\frac{L}{T}\right)^c L^{2d} \left(\frac{1}{T}\right)^e$$

Writing the equations for dimensional homogeneity,

$$\begin{aligned} M: &\quad a = b \\ L: &\quad a = -3b + c + 2d \\ T: &\quad 2a = c + e \end{aligned}$$

Solving for a, b and c in terms of d, and e gives

$$\begin{aligned} a &= d - e/2 \\ b &= d - e/2 \\ c &= 2d - 2e \end{aligned}$$

Substituting into the original equation

$$F_D^{d-e/2} = \rho^{d-e/2} V^{2d-2e} S^d \omega^e$$

$$\left(\frac{F_D}{\rho V^2 S}\right)^d = \left(\frac{F_D^{1/2}\omega}{\rho^{1/2} V^2}\right)^e$$

so

$$\frac{F_D}{\rho V^2 S} = f\left(\frac{F_D^{1/2}\omega}{\rho^{1/2} V^2}\right)$$

It is standard practice to eliminate F_D from the right side of the equation. To do this, we may use the concept that π-groups may be combined by multiplication or division. The result is

$$\boxed{\frac{F_D}{\rho V^2 S} = f\left(\frac{\omega^2 S}{V^2}\right)}$$

8.21: PROBLEM DEFINITION

Situation:

Biofluid mechanics involves flow through tubes that change in size over time.

Find:

The π-groups in the form

$$\frac{Q}{\omega D^3} = f(\pi_1, \pi_2)$$

PLAN

The dimensional functional form is

$$Q = f(\omega, D, \rho, \mu, \Delta p/\Delta l)$$

Since there are 6 dimensional variables, there are 3 π-groups. Use the step-by-step method.

SOLUTION

Set up the table

Q	$\frac{L^3}{T}$	$\frac{Q}{\omega}$	L^3	$\frac{Q}{\omega D^3}$	0	$\frac{Q}{\omega D^3}$	0
ω	$\frac{1}{T}$						
D	L	D	L				
ρ	$\frac{M}{L^3}$	ρ	$\frac{M}{L^3}$	ρD^3	M		
μ	$\frac{M}{LT}$	$\frac{\mu}{\omega}$	$\frac{M}{L}$	$\frac{\mu D}{\omega}$	M	$\frac{\mu}{\omega\rho D^2}$	0
$\Delta p/\Delta l$	$\frac{M}{L^2T^2}$	$\frac{\Delta p/\Delta l}{\omega^2}$	$\frac{M}{L^2}$	$\frac{\Delta p/\Delta l}{\omega^2}D^2$	M	$\frac{\Delta p/\Delta l}{\rho\omega^2 D}$	0

The time was eliminated with ω, the length with D and the mass with ρD^3. The final functional form is

$$\boxed{\frac{Q}{\omega D^3} = f\left(\frac{\mu}{\omega\rho D^2}, \frac{\Delta p/\Delta l}{\rho\omega^2 D}\right)}$$

8.22: PROBLEM DEFINITION

Situation:

A bubble rises in a fluid.

Find:

The π-groups in the form

$$\frac{V_b}{\sqrt{gD}} = f(\pi_1, \pi_2)$$

PLAN

The dimensional form of the equation is

$$V_b = f(D, \rho_f, \mu, g, \rho_l - \rho_b)$$

There are 6 dimensional variables so there are 3 π-groups. Use the step-by-step method.

SOLUTION

Setting up the table.

V_b	$\frac{L}{T}$	$\frac{V_b}{D}$	$\frac{1}{T}$	$\frac{V_b}{D}\left(\frac{D}{g}\right)^{1/2}$	0	$\frac{V_b}{\sqrt{gD}}$	0
D	L						
ρ_f	$\frac{M}{L^3}$	$\rho_f D^3$	M	$\rho_f D^3$	M		
μ	$\frac{M}{LT}$	μD	$\frac{M}{T}$	$\mu D\left(\frac{D}{g}\right)^{1/2}$	M	$\frac{\mu}{\rho_f g^{1/2} D^{3/2}}$	0
g	$\frac{L}{T^2}$	$\frac{g}{D}$	$\frac{1}{T^2}$				
$\rho_f - \rho_b$	$\frac{M}{L^3}$	$(\rho_f - \rho_b)D^3$	M	$(\rho_f - \rho_b)D^3$	M	$\frac{\rho_f - \rho_b}{\rho_f}$	0

The length was eliminated with D, the time with $(D/g)^{1/2}$ and the mass with $\rho_f D^3$.The dimensionless form is

$$\frac{V_b}{\sqrt{gD}} = f\left(\frac{\mu}{\rho_f g^{1/2} D^{3/2}}, \frac{\rho_f - \rho_b}{\rho_f}\right)$$

8.23: PROBLEM DEFINITION

Situation:

Discharge through a centrifugal pump.

The dimensional functional form:

$$Q = f(N, D, h_p, \mu, \rho, g)$$

Find:

The π-groups in the $\pi-$groups in the form

$$\frac{Q}{ND^3} = f(\pi_1, \pi_2, \pi_3)$$

.

PLAN

There are 7 dimensional groups so there should be 4 π-groups. Use the step-by-step method.

Q	$\frac{L^3}{T}$	$\frac{Q}{D^3}$	$\frac{1}{T}$	$\frac{Q}{D^3}$	$\frac{1}{T}$	$\frac{Q}{ND^3}$	0
N	$\frac{1}{T}$	N	$\frac{1}{T}$	$\frac{1}{T}$	$\frac{1}{T}$		
D	L						
h_p	L	$\frac{h_p}{D}$	0	$\frac{h_p}{D}$	0	$\frac{h_p}{D}$	0
μ	$\frac{M}{LT}$	μD	$\frac{M}{T}$	$\frac{\mu}{\rho D^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho N D^2}$	0
ρ	$\frac{M}{L^3}$	ρD^3	M				
g	$\frac{L}{T^2}$	$\frac{g}{D}$	$\frac{1}{T^2}$	$\frac{g}{D}$	$\frac{1}{T^2}$	$\frac{g}{N^2 D}$	0

The functional relationship is

$$\boxed{\frac{Q}{ND^3} = f\left(\frac{h_p}{D}, \frac{\mu}{\rho N D^2}, \frac{g}{N^2 D}\right)}$$

Some dimensionless variables can be combined to yield a different form

$$\frac{Q}{ND^3} = f\left(\frac{h_p g}{N^2 D^2}, \frac{\mu}{\rho N D^2}, \frac{g}{N^2 D}\right)$$

8.24: PROBLEM DEFINITION

Situation:

Drag of a square plate placed normal to a free stream velocity.

Find:

The $\pi-$groups in the form

$$\frac{F_D}{\rho V^2 B^2} = f(\pi_1, \pi_2, \pi_3).$$

.

PLAN

The functional form of the dimensional equation is

$$F_D = f(\rho, V, B, \mu, u_{rms}, L_x)$$

There are 7 dimensional parameters, so there are 4 π-groups. Use the step-by-step method.

SOLUTION

Setting up the table,

F_D	$\frac{ML}{T^2}$	$\frac{F_D}{B}$	$\frac{M}{T^2}$	$\frac{F_D}{\rho B^4}$	$\frac{1}{T^2}$	$\frac{F_D}{\rho V^2 B^2}$	0
V	$\frac{L}{T}$	$\frac{V}{B}$	$\frac{1}{T}$	$\frac{V}{B}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	ρB^3	M				
B	L						
μ	$\frac{M}{LT}$	μB	$\frac{M}{T}$	$\frac{\mu}{\rho B^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho V B}$	0
u_{rms}	$\frac{L}{T}$	$\frac{u_{rms}}{B}$	$\frac{1}{T}$	$\frac{u_{rms}}{B}$	$\frac{1}{T}$	$\frac{u_{rms}}{V}$	0
L_x	L	$\frac{L_x}{B}$	0	$\frac{L_x}{B}$	0	$\frac{L_x}{B}$	0

Length is removed in first step with B, mass is removed in second with ρB^3 and time is removed in the third with V/B. The function form is

$$\boxed{\frac{F_D}{\rho V^2 B^2} = f\left(\frac{\mu}{\rho V B}, \frac{u_{rms}}{V}, \frac{L_x}{B}\right)}$$

Other forms are possible. The π-group u_{rms}/V is referred to as "turbulence intensity."

8.25: PROBLEM DEFINITION

Situation:
Using Wikipedia, read about the Womersley Number (α) and answer the following questions.

Find:
a. Is α dimensionless? How do you know? Show that all the terms in fact cancel out.
b. Like other independent π-groups, α is the ratio of two forces. Of what two forces is it the ratio?
c. What does the velocity profile in a blood vessel look like for $\alpha \leq 1$? For $\alpha \geq 10$?
d. What is the aorta and where in the human body is it located? What is a typical value for α in the aorta? What might you conclude about the velocity profile there?

SOLUTION

a. To check whether α is dimensionless, use the relationship $\alpha = r\sqrt{\frac{\omega\rho}{\mu}}$,and insert the dimensions of these variables.

$$\begin{aligned}[\alpha] &= \left[r\sqrt{\frac{\omega\rho}{\mu}}\right] = L\sqrt{\frac{1}{T}\frac{1}{L^3}\frac{LT}{M}} \\ [\alpha] &= L\sqrt{\frac{1}{L^2}} = \frac{L}{L} = \text{dimensionless}\end{aligned}$$

b. From the internet, α is given as a ratio of [pulsatile transient force]/[viscous force].
c. In the range $\alpha \leq 1$, a parabolic (laminar) velocity distribution has time to develop in a tube during each heartbeat cycle. When $\alpha \geq 10$, the velocity profile is relatively flat (called plug flow) in the blood vessel.
d. The aorta is the largest artery, which delivers oxygenated blood from the lungs to the rest of the body. According to the Wikipedia definition for Womersley Number (accessed Feb 12, 2012), a typical value for α in a dog is in the range $11-13$. Sources in medical journals give values for humans of $\alpha \approx 20$.

8.26: PROBLEM DEFINITION

The Womersley Number (α) is a π-group given by the ratio of [pulsatile transient force]/[viscous force]. Biomedical engineers have applied this to characterize flow in blood vessels. The Womersley Number is given by:

$$\alpha = r\sqrt{\frac{\omega\rho}{\mu}}$$

where r = blood vessel radius, and ω = frequency, typically the heart rate. Like Re, α has different practical implications in critical ranges. In the range $\alpha \leq 1$, a parabolic (laminar) velocity distribution has time to develop in a tube during each heartbeat cycle. When $\alpha \geq 10$, the velocity profile is relatively flat (called plug flow) in the blood vessel.

Situation:

Assume a human subject with the following features:
$\omega = 70$ beats/s, or 70 s^{-1}
$\text{radius}_{aorta} = 4$ mm
$\rho = 1060\ \text{kg/m}^3$
$\text{radius}_{capillary} = 7\ \mu\text{m}$
$\mu = 3 \times 10^{-3}\ \text{Pa} \cdot \text{s}$

Find:

a. Find α for the aorta of this subject.
b. Find α for the capillary of this subject.
c. Does either the aorta or the capillary have an α that would predict plug flow? Does either have an α indicating a parabolic velocity distribution?

PLAN

Use the equation for Womersley Number (α)

$$\alpha = r\sqrt{\frac{\omega\rho}{\mu}}$$

SOLUTION

a. To find α for the aorta,

$$\begin{aligned} \alpha_{aorta} &= 0.004\,\text{m}\sqrt{\left(\frac{70}{\text{s}}\right)\left(\frac{1060\,\text{kg}}{\text{m}^3}\right)\left(\frac{1}{3\times 10^{-3}\,\text{Pa}\cdot\text{s}}\right)} \\ \alpha_{aorta} &= (0.004\,\text{m})\left(4793\,\text{m}^{-1}\right) \\ & \boxed{\alpha_{aorta} = 19.9} \end{aligned}$$

b. To find α for the capillary,

$$\alpha_{capillary} = 7 \times 10^{-6}\,\text{m}\sqrt{\left(\frac{70}{\text{s}}\right)\left(\frac{1060\,\text{kg}}{\text{m}^3}\right)\left(\frac{1}{3 \times 10^{-3}\,\text{Pa}\cdot\text{s}}\right)}$$

$$\alpha_{capillary} = \left(7 \times 10^{-6}\,\text{m}\right)\left(4793\,\text{m}^{-1}\right)$$

$$\boxed{\alpha_{capillary} = 0.035}$$

c. The aorta was found to have an $\alpha \geq 10$, indicating that plug flow is occurring in the aorta. The capillary was found to have an $\alpha \leq 1$, which indicates that a parabolic (laminar) velocity distribution has time to develop in this blood vessel during each heartbeat cycle.

8.27: PROBLEM DEFINITION

Find:

For each item below, which π-group (Re, We, M or Fr) would best match the given description?

a. (Kinetic force)/(Surface-tension force)
b. (Kinetic force)/(Viscous force)
c. (Kinetic force)/(Gravitational force)
d. (Kinetic force)/(Compressive force)
e. Used for modeling water flowing over a spillway on a dam
f. Used for designing laser jet printers
g. Used for analyzing the drag on a car in a wind tunnel
h. Used to analyze the flight of supersonic jets

SOLUTION

Answers are provided to the right, bolded.

a. (Kinetic force)/(Surface-tension force) **We**
b. (Kinetic force)/(Viscous force) **Re**
c. (Kinetic force)/(Gravitational force) **Fr**
d. (Kinetic force)/(Compressive force) **M**
e. Used for modeling water flowing over a spillway on a dam **Fr**
f. Used for designing laser jet printers **We**
g. Used for analyzing the drag on a car in a wind tunnel **Re**
h. Used to analyze the flight of supersonic jets **M**

8.28: PROBLEM DEFINITION

Situation:
Geometric similitude.

Find:
Definition of geometric similitude.

SOLUTION

Geometric similitude exists when the model is a scale model of the prototype. All the ratios of corresponding dimensions are the same for the model and prototype.

8.29: PROBLEM DEFINITION

Situation:

Gather information on wind tunnel testing of automobiles.

SOLUTION

The information will depend on the sources utilized.

8.30: PROBLEM DEFINITION

Situation:

Testing of an automobile in a wind tunnel.

Find:

Discuss the effect of the ground on the aerodynamic performance of automobiles.

SOLUTION

The effects of the road surface may effect the automobile drag increasing the velocity on the underside and larger drag force. The rotating tires may promote turbulent flow fields which enhance the drag force.

8.31: PROBLEM DEFINITION

Situation:

Wind tunnel at NASA Facility at Moffat Field, CA

Find:

Summarize information on the facility.

SOLUTION

The information will depend on the sources utilized.

8.32: PROBLEM DEFINITION

Situation:

Hydrodynamics of a sailboat.

Find:

Information on simulations of sailboat performance.

SOLUTION

The information will depend on the sources utilized.

8.33: PROBLEM DEFINITION

Situation:

Drag force on a submarine is studied using a scale model in water tunnel.
$V = 3\,\mathrm{m/s}$, $\frac{1}{18}$ scale model.

Find:

Speed of water in the tunnel for dynamic similitude.
The ratio of drag forces (ratio of drag force on the model to that on the prototype).

Properties:

Sea Water (20 °C): $\rho = 1015\,\mathrm{kg/m^3}$, $\nu = 1.4 \times 10^{-6}\,\mathrm{m^2/s}$.

PLAN

Dynamic similarity is achieved when the Reynolds numbers are the same. With similitude, the force coefficients are the same.

SOLUTION

Match Reynolds number for dynamic similitude.
Equating Reynolds number of model and prototype,

$$\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
\frac{U_m L_m}{\nu_m} &= \frac{U_p L_p}{\nu_p} \\
U_m &= U_p \frac{L_p}{L_m} \frac{\nu_m}{\nu_p} \\
&= 3\ \mathrm{m/s}\ \frac{18}{1} \frac{1 \times 10^{-6}\ \mathrm{m^2/s}}{1.4 \times 10^{-6}\ \mathrm{m^2/s}} \\
&\boxed{U_m = 38.57\ \mathrm{m/s}}
\end{aligned}$$

The ratio of the drag force on the model to that on the prototype is

$$\begin{aligned}
C_{Fp} &= C_{Fm} \\
\frac{F_{D,m}}{F_{D,p}} &= \frac{\rho_m}{\rho_p}\left(\frac{V_m}{V_p}\right)^2 \left(\frac{l_m}{l_p}\right)^2 \\
&= \frac{998}{1015}\left(\frac{38.57}{3}\right)^2 \left(\frac{1}{18}\right)^2 \\
&\boxed{\frac{F_{D,m}}{F_{D,p}} = 0.502}
\end{aligned}$$

8.34: PROBLEM DEFINITION

Situation:

Water flows through a pipe.

$V = 0.5\,\text{m/s}$, $D = 4\,\text{cm}$.

Find:

Velocity of water for dynamic similarity with oil ($\nu = 10^{-5}$ m^2/s) at 0.5 m/s.

Properties:

Water: $\nu = 10^{-6}\,\text{m}^2/\text{s}$.

Oil: $\nu = 10^{-5}\,\text{m}^2/\text{s}$.

PLAN

Dynamic similarity is achieved when the Reynolds numbers are the same.

SOLUTION

Match Reynolds number

$$\begin{aligned}
\text{Re}_w &= \text{Re}_0 \\
\frac{V_w d}{\nu_w} &= \frac{V_0 d}{\nu_0} \\
V_w &= \frac{V_0 \nu_w}{\nu_0} \\
&= 0.5 \text{ m/s} \left(\frac{10^{-6}\,\text{m}^2/\text{s}}{10^{-5}\,\text{m}^2/\text{s}} \right) \\
&\boxed{V_w = 0.05 \text{ m/s}}
\end{aligned}$$

8.35: PROBLEM DEFINITION

Situation:

Oil flows through a smooth pipe.

$D_{12} = 12\,\text{cm}$, $V_{12} = 2.3\,\text{m/s}$.

$D_5 = 5\,\text{cm}$.

Find:

Velocity of water at 5 cm pipe for dynamic similarity.

Properties:

Oil (20 °C): $\nu_{12} = 4 \times 10^{-6}\,\text{m}^2/\text{s}$.

Water (20 °C): $\nu_5 = 10^{-6}\,\text{m}^2/\text{s}$.

PLAN

Dynamic similarity is achieved when the Reynolds numbers are the same.

SOLUTION

Match Reynolds number

$$
\begin{aligned}
\text{Re}_5 &= \text{Re}_{12} \\
\frac{V_5 D_5}{\nu_5} &= \frac{V_{12} D_{12}}{\nu_{12}} \\
V_5 &= V_{12}\left(\frac{D_{12}}{D_5}\right)\left(\frac{\nu_5}{\nu_{12}}\right) \\
&= (2.3\text{ m/s})\left(\frac{12\,\text{cm}}{5\,\text{cm}}\right)\left(\frac{10^{-6}\,\text{m}^2/\text{s}}{4\times 10^{-6}\,\text{m}^2/\text{s}}\right) \\
& \boxed{V_5 = 1.38\ \text{m/s}}
\end{aligned}
$$

8.36: PROBLEM DEFINITION

Situation:

A large venturi meter is calibrated with a scale model using a prototype liquid.
$\frac{1}{10}$ scale model, $p = 400\,\text{kPa}$.

Find:

The discharge ratio (Q_m/Q_p)

Pressure difference (Δp_p) expected for the prototype.

Assumptions:

Same fluids used; densities and viscosities are the same in model and prototype.

SOLUTION

Match Reynolds number

$$\begin{aligned}
\text{Re}_m &= \text{Re}_p \\
\frac{V_m L_m}{\nu_m} &= \frac{V_p L_p}{\nu_p} \\
\frac{V_m}{V_p} &= \left(\frac{L_p}{L_m}\right)\left(\frac{\nu_m}{\nu_p}\right) = \left(\frac{L_p}{L_m}\right)(1) \qquad (1)
\end{aligned}$$

Multiply both sides of Eq. (1) by $A_m/A_p = L_m^2/L_p^2$:

$$\begin{aligned}
\frac{V_m A_m}{V_p A_p} &= \left(\frac{L_p}{L_m}\right) \times (1) \times \left(\frac{L_m}{L_p}\right)^2 \\
\frac{Q_m}{Q_p} &= \left(\frac{L_m}{L_p}\right) \\
&\boxed{\frac{Q_m}{Q_p} = \frac{1}{10}} \\
C_{p_m} &= C_{p_p} \\
\left(\frac{\Delta p}{\rho V^2}\right)_m &= \left(\frac{\Delta p}{\rho V^2}\right)_p \\
\Delta p_p &= \Delta p_m \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2;\ \text{where } \left(\frac{\rho_p}{\rho_m}\right) = 1^* \\
&= \Delta p_m (1) \left(\frac{L_m}{L_p}\right)^2 \\
&= 400\,\text{kPa} \times \left(\frac{1}{10}\right)^2 \\
&\boxed{\Delta p_p = 4.0\ \text{kPa}}
\end{aligned}$$

* because the same fluid is assumed to be used in both model and prototype

8.37: PROBLEM DEFINITION

Situation:

Drag is to be measured with a scale model of a bathysphere.

Find:

The ratio of towing speeds (ratio of speed of the model to the speed of the prototype).

PLAN

Dynamic similarity based on matching Reynolds number of the model and prototype.

SOLUTION

Reynolds number matching

$$\text{Re}_m = \text{Re}_p$$
$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

Assume $\nu_m = \nu_p$

$$V_m L_m = V_p L_p$$
$$\frac{V_m}{V_p} = \frac{L_p}{L_m} = 5$$

$$\boxed{V_m/V_p = 5}$$

8.38: PROBLEM DEFINITION

Situation:

A spherical balloon is tested by towing a scale model in a lake.
$D_m = 0.43$ m, $D_p = 5.2$ m.
$V_m = 1.5$ m, $F_m = 165\,\text{N}$.

Find:

Drag force on the prototype (N).

Properties:

Air (15.5 °C), Table A.3: $\nu_p = 1.47 \times 10^{-5}$ m^2/s, $\rho_p = 1.22\,\text{kg/m}^3$.
Water (15.5 °C), Table A.5: $\nu_m = 1.14 \times 10^{-6}$ m^2/s, $\rho_m = 999\,\text{kg/m}^3$.

PLAN

Dynamic similarity based on Reynolds number and same force coefficient.

SOLUTION

Match Reynolds numbers

$$\begin{aligned} \text{Re}_m &= \text{Re}_p \\ V_m D_m/\nu_m &= V_p D_p/\nu_p \end{aligned}$$

or

$$\begin{aligned} \frac{V_p}{V_m} &= \left(\frac{D_m}{D_p}\right)\left(\frac{\nu_p}{\nu_m}\right) = \left(\frac{0.43}{5.2}\right)\left(\frac{1.47 \times 10^{-5}\,\text{m}^2/\text{s}}{1.14 \times 10^{-6}\,\text{m}^2/\text{s}}\right) \\ &= 1.079 \end{aligned}$$

Match force coefficients

$$\begin{aligned} C_{Fm} &= C_{Fp} \\ \frac{F_p}{F_m} &= \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{D_p}{D_m}\right)^2 \end{aligned}$$

$$\begin{aligned} \frac{F_p}{F_m} &= \left(\frac{1.22\,\text{kg/m}^3}{999\,\text{kg/m}^3}\right)(1.079)^2\,(12)^2 \\ &= 0.20 \\ F_p &= 165 \times 0.20 \\ &\boxed{F_p = 33.72\,\text{N}} \end{aligned}$$

8.39: PROBLEM DEFINITION

Situation:

Lift force for an airplane.
The $\pi-$group for lift coefficient is

$$C_L = 2\frac{F_L}{\rho V^2 S}$$

Air: $\rho = 1.1\,\mathrm{kg/\,m^3}$, $C_L = 0.4$.
$V = 80\,\mathrm{m/\,s}$, $S = 15\,\mathrm{m^2}$.

Find:

Lift force (N).

PLAN

Use the specified value of $C_L = 0.4$ along with the definition of this π-group.

SOLUTION

From the definition of C_L:

$$\begin{aligned} F_L &= C_L\left(\frac{\rho V^2}{2}\right)S \\ &= 0.4\left(\frac{1.1\,\mathrm{kg/\,m^3}\times(80\,\mathrm{m/\,s})^2}{2}\right)\left(15\,\mathrm{m^2}\right) \\ &= 21,100\,\mathrm{N} \\ &\boxed{F_L = 21.1\,\mathrm{kN}} \end{aligned}$$

8.40: PROBLEM DEFINITION

Situation:

A scale model of a plane is tested in a wind tunnel.

$\frac{1}{8}$ scale model.

Find:

Density of the air in tunnel.

Properties:

Air (10 °C, 100 kPa), Table A.4: $R = 287\,\mathrm{J/\,kg\,K}$.

Air (10 °C, 100 kPa), Table A.3: $\mu_p = 1.76 \times 10^{-5}\ \mathrm{N\,s/\,m^2}$.

Air (25 °C, 100 kPa), Table A.3: $\mu_m = 1.83 \times 10^{-5}\ \mathrm{N\,s/\,m^2}$.

PLAN

Dynamic similarity based on matching Reynolds number and Mach number.

SOLUTION

Match Reynolds number

$$\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
\left(\frac{VD}{\nu}\right)_m &= \left(\frac{VD}{\nu}\right)_p \\
\frac{V_m}{V_p} &= \left(\frac{D_p}{D_m}\right)\left(\frac{\upsilon_m}{\upsilon_p}\right) \\
\frac{\nu_m}{\nu_p} &= \frac{V_m D_m}{V_p D_p} \\
\frac{\mu_m \rho_p}{\mu_p \rho_m} &= \frac{V_m D_m}{V_p D_p} \\
\rho_m &= \rho_p \left(\frac{\mu_m}{\mu_p}\right)\left(\frac{V_p}{V_m}\right)\left(\frac{D_p}{D_m}\right) \qquad (1)
\end{aligned}$$

Match Mach number

$$\begin{aligned}
M_m &= M_p \\
\left(\frac{V}{c}\right)_m &= \left(\frac{V}{c}\right)_p \\
\frac{V_m}{V_p} &= \frac{c_m}{c_p} \\
&= \left(\frac{T_m}{T_p}\right)^{1/2} \\
&= \left(\frac{298}{283}\right)^{1/2} \qquad (2) \\
&= 1.026 \qquad (1)
\end{aligned}$$

Density of air at 100 kPa and 10^{o}C

$$
\begin{aligned}
\rho &= \frac{p}{RT} \\
&= \frac{100 \times 1000 \text{ Pa}}{287 \text{ J/kg-K} \times 283 \text{ K}} \\
&= 1.23 \text{ kg/m}^3
\end{aligned}
$$

Combining Eqs. (1) and (2):

$$
\rho_m = 1.23 \left(\frac{1.83 \times 10^{-5}\,\mathrm{N\,s/\,m^2}}{1.76 \times 10^{-5}\,\mathrm{N\,s/\,m^2}} \right) (1.026)(8)
$$

$$
\boxed{\rho_m = 10.51\,\mathrm{kg/\,m^3}}
$$

8.41: PROBLEM DEFINITION

Situation:

A windtunnel is used to do tests for the Airbus.
$z = 10000\,\text{m}$.
$L_m = 1\,\text{m}$, $L_p = 79.8\,\text{m}$.

Find:

Wind tunnel pressure to have Reynolds number similitude.

Properties:

$p_{atm} = 101.3\,\text{kPa}$.
Air (20 °C), Table A.4: $R = 287\,\text{J/kg K}$.

PLAN

Find the velocity ratio for the same Mach number, set the Reynolds numbers the same and solve for the pressure.

SOLUTION

The properties at 10,000 m. From Chapter 3, Eq. (3.18) of EFM10e.

$$\begin{aligned} T &= T_o - \alpha z \\ T &= 296 \text{ K} - 5.87 \times 10^{-3} \text{ K/m } \times 10,000 \text{ m} \\ &= 237.3 \text{ K} \end{aligned}$$

$$\begin{aligned} p &= p_o \left(\frac{T}{T_o} \right)^{5.823} \\ &= 101.3 \text{ kPa} \times \left(\frac{237.3}{296} \right)^{5.823} \\ &= 27.97 \text{ kPa} \end{aligned}$$

The density at 10000 m is

$$\rho = \frac{p}{RT} = \frac{27.97 \times 10^3 \text{ Pa}}{287 \text{ J/kg-K} \times 237.3 \text{ K}} = 0.4107 \text{ kg/m}^3$$

For Mach number similitude

$$\begin{aligned} M_m &= M_p \\ \frac{V_m}{c_m} &= \frac{V_p}{c_p} \\ \frac{V_m}{V_p} &= \frac{c_m}{c_p} = \sqrt{\frac{T_m}{T_p}} \end{aligned}$$

For Reynolds number similitude

$$\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
\frac{\rho_m L_m V_m}{\mu_m} &= \frac{\rho_p L_p V_p}{\mu_p} \\
\frac{\rho_m}{\rho_p} &= \left(\frac{L_p}{L_m}\right)\left(\frac{V_p}{V_m}\right)\left(\frac{\mu_m}{\mu_p}\right) \\
&= \left(\frac{L_p}{L_m}\right)\sqrt{\frac{T_p}{T_m}}\sqrt{\frac{T_m}{T_p}} \\
&= \frac{79.8\ \mathrm{m}}{1\ \mathrm{m}} = 79.8
\end{aligned}$$

The model density must be

$$\rho_m = 79.8 \times 0.4107\ \mathrm{kg/m^3} = 32.77\ \mathrm{kg/m^3}$$

From the ideal gas law

$$\begin{aligned}
p &= \rho R T \\
&= 32.77\ \mathrm{kg/m^3} \times 293\ \mathrm{K} \times 287\ \mathrm{J/kg\text{-}K} \\
&= 2.76\ \mathrm{MPa} \\
& \boxed{p = 27.6\,\mathrm{kbar}}
\end{aligned}$$

8.42: PROBLEM DEFINITION

Situation:

Wind tunnel test done for a Beoing 787 to simulate forces on model with only Mach number similitude.

$z = 10,000\,\mathrm{m}$, $L_m = 1\,\mathrm{m}$.

$L_p = 52\,\mathrm{m}$, $V = 945\,\mathrm{km/\,h}$.

$c_m = 340\,\mathrm{m/\,s}$, $M = 0.85$.

Find:

The ratio of force on prototype to force on model.

Properties:

$p_{atm} = 101.3\,\mathrm{kPa}$.

Air, Table A.4: $R = 287\,\mathrm{J/\,kg\,K}$.

Air, Table A.3: $\rho = 0.98\,\mathrm{kg/\,m^3}$.

PLAN

Assume Reynolds number similitude is not important and find velocity ratio using Mach number similitude. Equate force coefficients to find force ratio.

SOLUTION

From Chapter 3, the temperature at 10,000 m is

$$\begin{aligned} T &= T_o - 5.87 \times 10^{-3}\ \mathrm{K/m} \times h \\ &= 296\,\mathrm{K} - 5.87 \times 10^{-3}\,\mathrm{K/\,m} \times 10,000\,\mathrm{m} \\ &= 237.3\ \mathrm{K} \end{aligned}$$

The pressure is

$$\begin{aligned} p &= p_o \left(\frac{T}{T_o}\right)^{5.823} \\ &= 101.3\ \mathrm{kPa} \times \left(\frac{237\,\mathrm{K}}{296\,\mathrm{K}}\right)^{5.823} \\ &= 27.967\ \mathrm{kPa} \end{aligned}$$

The density is

$$\begin{aligned} \rho &= \frac{p}{RT} = \frac{27.967 \times 10^3\ \mathrm{Pa}}{287\ \mathrm{J/kg\text{-}K} \times 237.3\ \mathrm{K}} \\ &= 0.4106\ \mathrm{kg/m^3} \end{aligned}$$

The velocity in the wind tunnel is

$$\begin{aligned} V_m &= 0.85 c_m = 0.85 \times 340\ \mathrm{m/s} \\ &= 289\ \mathrm{m/s} \end{aligned}$$

Speed of the airplane

$$\begin{aligned} V_p &= 945 \text{ km/hr} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \\ &= 262.5 \text{ m/s} \end{aligned}$$

Equating the force coefficients

$$\begin{aligned} C_{Fp} &= C_{Fm} \\ \frac{F_p}{F_m} &= \frac{\rho_p}{\rho_m} \left(\frac{V_p}{V_m}\right)^2 \left(\frac{L_p}{L_m}\right)^2 \\ &= \frac{0.4106\,\text{kg/m}^3}{0.98\,\text{kg/m}^3} \times \left(\frac{262.5\,\text{m/s}}{289\,\text{m/s}}\right)^2 \left(\frac{52\,\text{m}}{1\,\text{m}}\right)^2 \\ & \boxed{\tfrac{F_p}{F_m} = 935} \end{aligned}$$

8.43: PROBLEM DEFINITION

Situation:
Flow in a pipe is being tested with air and water.

Find:
Velocity ratio: $V_{\text{air}}/V_{\text{water}}$ for dynamic similitude.

Assumptions:
$T = 20\,^\circ\text{C}$.

Properties:
Air (20 °C), Table A.3: $\nu_A = 1.51 \times 10^{-4}\,\text{m}^2/\,\text{s}$.
Water (20 °C), Table A.5: $\nu_W = 1.00 \times 10^{-6}\,\text{m}^2/\,\text{s}$.

SOLUTION

Match Reynolds number

$$\text{Re}_A = \text{Re}_W$$

$$\frac{V_A L_A}{\nu_A} = \frac{V_W L_W}{\nu_W} \;\; ; \;\; \text{but } L_A/L_W = 1$$

$$\therefore \frac{V_A}{V_W} = \frac{v_A}{v_W} \approx \frac{1.51 \times 10^{-4}}{1.00 \times 10^{-6}}$$

$$V_A/V_W > 1$$

The correct choice is $\boxed{\text{(c)}}$

8.44: PROBLEM DEFINITION

Situation:

Crude oil flows in a smooth pipe.
$d_p = 1.2\,\text{m}$, $d_m = 0.1\,\text{m}$.
$V_p = 0.6\,\text{m/s}$.

Find:

Mean velocity of water in model to insure dynamic similarity.

Properties:

Crude Oil: $\rho_p = 900\,\text{kg/m}^3$, $\mu_p = 0.02\,\text{Pa-s}$
Water (15 °C), Table A.5: $\rho_m = 999\,\text{kg/m}^3$, $\mu_m = 1.14 \times 10^{-3}\,\text{Ns/m}^2$.

SOLUTION

Match Reynolds number

$$\begin{aligned}
\text{Re}_m &= \text{Re}_p \\
\frac{V_m d_m \rho_m}{\mu_m} &= \frac{V_p d_p \rho_p}{\mu_p} \\
V_m &= V_p \left(\frac{d_p}{d_m}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{\mu_m}{\mu_p}\right) \\
V_m &= (0.6\text{ m/s})\left(\frac{1.2\,\text{m}}{0.1\,\text{m}}\right)\left(\frac{900\,\text{kg/ m}^3}{999\,\text{kg/ m}^3}\right)\left(\frac{1.14\times 10^{-3}\,\text{Ns/ m}^2}{0.02\,\text{N s/ m}^2}\right) \\
&\boxed{V_m = 0.37\,\text{m/s}}
\end{aligned}$$

8.45: PROBLEM DEFINITION

Situation:

A student team is designing a radio-controlled blimp.
Drag force is characterized with a coefficient of drag:.

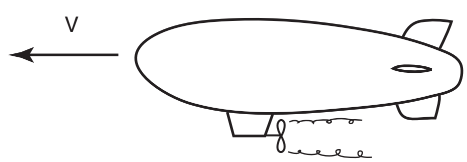
$$C_D = 2\frac{F_D}{\rho V^2 A_p} = 0.3$$

$V = 800\,\text{mm/s}$, $D = 0.475\,\text{m}$.
$A_p = \pi D^2/4$.

Find:

a.) Reynolds number.
b.) Force of drag (N).
c.) Power in watts (W).

Sketch:



Assumptions:

Assume the blimp cross section is round.

Properties:

Air (20 °C), Table A.3: $\rho = 1.2\,\text{kg/m}^3$, $\mu = 18.1 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2$.

PLAN

Find the Reynolds number by direct calculation. Find the drag force using the definition of C_D. Find power (P) by using the product of force and speed: $P = F_{\text{Drag}}V$.

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{(0.8\,\text{m/s})\,(0.475\,\text{m})\,(1.2\,\text{kg/m}^3)}{(1.81 \times 10^{-5}\,\text{N}\cdot\text{s/m}^2)} \\ &\boxed{\text{Re} = 25,200} \end{aligned}$$

Projected area

$$
\begin{aligned}
A_p &= \frac{\pi D^2}{4} = \frac{\pi \left(0.475\,\mathrm{m}\right)^2}{4} \\
&= 0.177\,\mathrm{m}^2
\end{aligned}
$$

Drag force

$$
\begin{aligned}
F_D &= C_D \left(\frac{\rho V^2}{2}\right) A_p \\
&= (0.3)\,\frac{(1.2\,\mathrm{kg/\,m^3})\,(0.8\,\mathrm{m/\,s})^2}{2}\left(0.177\,\mathrm{m}^2\right) \\
& \boxed{F_D = 20.4 \times 10^{-3}\,\mathrm{N}}
\end{aligned}
$$

Power

$$
\begin{aligned}
P &= F_D V \\
&= \left(20.4 \times 10^{-3}\,\mathrm{N}\right)(0.8\,\mathrm{m/\,s}) \\
& \boxed{P = 16.3 \times 10^{-3}\,\mathrm{W}}
\end{aligned}
$$

REVIEW

1. The drag force is about $1/50^{\text{th}}$ of a Newton.
2. The power is about 16 milliwatts. The supplied power would need to be higher to account for factors such as propeller efficiency and motor efficiency.

8.46: PROBLEM DEFINITION

Situation:

Flow in a conduit (on earth) to be used to characterize a prototype that will be build on the moon.

Find:

Kinematic viscosity of fluid for model on earth.

Properties:

Fluid: $\nu = 0.5 \times 10^{-5}\,\text{m}^2/\,\text{s}$.

PLAN

Dynamic similarity based on Reynolds number and Froude number.

SOLUTION

Match Froude number

$$
\begin{aligned}
Fr_{\text{moon}} &= Fr_{\text{earth}} \\
\left(\frac{V}{\sqrt{gL}}\right)_m &= \left(\frac{V}{\sqrt{gL}}\right)_e \\
\frac{V_e}{V_m} &= \left(\frac{g_e}{g_m}\right)^{0.5}\left(\frac{L_e}{L_m}\right)^{0.5} \\
&= (5)^{0.5}(1)
\end{aligned}
$$

Match Reynolds number

$$
\begin{aligned}
\text{Re}_m &= \text{Re}_e \\
\left(\frac{VL}{\nu}\right)_m &= \left(\frac{VL}{\nu}\right)_e \\
\nu_e &= \left(\frac{V_e}{V_m}\right)\nu_m = (5)^{0.5}\left(0.5 \times (10^{-5})\ \text{m}^2/\text{s}\right)
\end{aligned}
$$

$$
\boxed{\nu_e = 1.12 \times 10^{-5}\ \text{m}^2/\text{s}}
$$

8.47: PROBLEM DEFINITION

Situation:

A scale model of a drying tower is to be tested with water.

$V_p = 12\,\text{m/s}$.

$\frac{1}{15}$scale model.

Find:

Entry velocity of the model for similitude.

Properties:

Air: $\nu_p = 4 \times 10^{-5}\,\text{m}^2/\text{s}$.

Water: $\nu_m = 10^{-6}\,\text{m}^2/\text{s}$.

PLAN

Similitude based on matching Reynolds numbers.

SOLUTION

Match Reynolds number

$$
\begin{aligned}
\text{Re}_m &= \text{Re}_p \\
\frac{V_m L_m}{\nu_m} &= \frac{V_p L_p}{\nu_p} \\
V_m &= \left(\frac{L_p}{L_m}\right)\left(\frac{\nu_m}{\nu_p}\right) V_p \\
&= (15)\left(\frac{1 \times 10^{-6}}{4 \times 10^{-5}}\right)(12\text{ m/s}) \\
&\boxed{V_m = 4.50\text{ m/s}}
\end{aligned}
$$

8.48: PROBLEM DEFINITION

Situation:

A scale model is of a discharge meter for oil is tested using water.
$\frac{1}{9}$ scale model.
$V_m = 1.6\,\text{m/s}$, $\Delta p_m = 3\,\text{kPa}$.

Find:

Velocity for the prototype for dynamic similitude (m/ s).
Pressure difference for the prototype (kPa).

Properties:

Oil (20 °C): $\nu_p = 10^{-5}\,\text{m}^2/\text{s}$, $\rho_p = 860\,\text{kg/m}^3$.
Water: $\nu_m = 10^{-6}\,\text{m}^2/\text{s}$, $\rho_m = 998\,\text{kg/m}^3$.

PLAN

Dynamic similarity based on Reynolds number and pressure coefficients.

SOLUTION

Match Reynolds number

$$\begin{aligned}\text{Re}_p &= \text{Re}_m \\ V_p &= V_m\left(\frac{L_m}{L_p}\right)\left(\frac{\nu_p}{\nu_m}\right) \\ V_p &= 1.6\left(\frac{1}{9}\right)\left(\frac{10^{-5}\,\text{m}^2/\text{s}}{10^{-6}\,\text{m}^2/\text{s}}\right) \\ &\boxed{V_p = 1.78\ \text{m/s}}\end{aligned}$$

Match pressure coefficients

$$\begin{gathered}C_{p,m} = C_{p,p} \\ \left(\frac{\Delta p}{\rho V^2}\right)_m = \left(\frac{\Delta p}{\rho V^2}\right)_p \\ \Delta p_p = \Delta p_m\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2 \\ = 3.0 \times \left(\frac{860\,\text{kg/m}^3}{998\,\text{kg/m}^3}\right) \times \left(\frac{1.78\,\text{m/s}}{1.6\,\text{m/s}}\right)^2 \\ \boxed{\Delta p_p = 3.19\,\text{kPa}}\end{gathered}$$

8.49: PROBLEM DEFINITION

Situation:

Water flowing through a rough pipe is modeled with air.
$D = 10\,\text{cm}$, $V = 1.5\,\text{m/s}$.
$p_{\text{pipe}} = 150\,\text{kPa}$, $\Delta p_a = 780\,\text{Pa}$.

Find:

Air velocity to achieve dynamic similarity (m/ s).
Pressure difference for the water flow (kPa).

Properties:

Water (10 °C), Table A.5: $\mu_w = 1.31 \times 10^{-3}\,\text{N s/m}^2$, $\rho_w = 1000\,\text{kg/m}^3$.
Air (20 °C), Table A.5: $\mu_w = 1.81 \times 10^{-5}\,\text{N s/m}^2$, $\rho_a = 1.2\,\text{kg/m}^3$.

PLAN

Dynamic similitude based on Reynolds number and pressure coefficients.

SOLUTION

Match Reynolds number

$$
\begin{aligned}
\text{Re}_{\text{air}} &= \text{Re}_{\text{water}} \\
\left(\frac{VD\rho}{\mu}\right)_{\text{air}} &= \left(\frac{VD\rho}{\mu}\right)_{\text{water}} \\
V_a &= V_w \left(\frac{D_w}{D_a}\right)\left(\frac{\rho_w}{\rho_a}\right)\left(\frac{\mu_a}{\mu_w}\right) \\
\rho_w &= 1,000\ \text{kg/m}^3 \\
\rho &= \rho_{a,\ \text{std. atm.}} \times \left(\frac{150\,\text{kPa}}{101\,\text{kPa}}\right) \\
&= 1.20 \times \left(\frac{150}{101}\right) = 1.78\ \text{kg/m}^3 \\
\mu_a &= 1.81 \times 10^{-5}\ \text{N}\cdot\text{s/m}^2 \\
\mu_w &= 1.31 \times 10^{-3}\ \text{N}\cdot\text{s/m}^2
\end{aligned}
$$

Then

$$
V_a = 1.5\ \text{m/s}\left(\frac{1000\,\text{kg/m}^3}{1.78\,\text{kg/m}^3}\right)\left(\frac{1.81 \times 10^{-5}\,\text{N s/m}^2}{1.31 \times 10^{-3}\,\text{N s/m}^2}\right)
$$

$$
\boxed{V_a = 11.6\ \text{m/s}}
$$

Match pressure coefficients

$$
\begin{aligned}
C_{p_w} &= C_{p_a} \\
\left(\frac{\Delta p}{\rho V^2}\right)_w &= \left(\frac{\Delta p}{\rho V^2}\right)_a \\
\Delta p_w &= \Delta p_a \left(\frac{\rho_w}{\rho_a}\right)\left(\frac{V_w}{V_a}\right)^2 \\
&= 780\,\mathrm{Pa}\left(\frac{1000\,\mathrm{kg/\,m^3}}{1.78\,\mathrm{kg/\,m^3}}\right)\left(\frac{1.5\,\mathrm{m/\,s}}{11.6\,\mathrm{m/\,s}}\right)^2 \\
&= 7,330 \ \mathrm{Pa} \\
& \boxed{\Delta p_w = 7.33 \ \mathrm{kPa}}
\end{aligned}
$$

8.50: PROBLEM DEFINITION

Situation:

An "acoustic" minesweeper (a noisemaker) will be studied by using a scale model in a water tunnel.

$\frac{1}{5}$scale model.

$V_{\text{prot.}} = 5\,\text{m/s}$.

Find:

Velocity to use in the water tunnel (m/ s).

Drag force on the prototype (N).

Assumptions:

Sea water under the same conditions is used in both tests.

PLAN

Dynamic similarity based on matching Reynolds number and drag force with force coefficient.

SOLUTION

Match Reynolds number

$$\begin{aligned}
\text{Re}_{\text{tunnel}} &= \text{Re}_{\text{prototype}} \\
V_{\text{tunnel}} &= V_{\text{prot.}} \left(\frac{5}{1}\right) \left(\frac{\nu_{\text{tunnel}}}{\nu_{\text{prot.}}}\right) \\
V_{\text{tunnel}} &= 5\,\text{m/s} \left(\frac{5}{1}\right) (1) \\
& \boxed{V_{\text{tunnel}} = 25\,\text{m/s}}
\end{aligned}$$

Match force coefficients

$$C_{Fm} = C_{Fp}$$

.

$$\begin{aligned}
\frac{D_{\text{tunnel}}}{D_{\text{prot}}} &= \left(\frac{\rho_{\text{tunnel}}}{\rho_{\text{prot.}}}\right) \times \left(\frac{V_{\text{tunnel}}^2}{V_{\text{prot.}}^2}\right) \times \left(\frac{L_{\text{tunnel}}}{L_{\text{prot}}}\right)^2 \\
&= \left(\frac{1}{1}\right) (5)^2 \left(\frac{1}{5}\right)^2 = 1 \\
& \boxed{F_{\text{tunnel}} = F_{\text{prot.}} = 2400\ N}
\end{aligned}$$

8.51: PROBLEM DEFINITION

Situation:

Forces due to wind on a building are to be modeled by using a scale model in wind tunnel.

$\frac{1}{500}$scale model.

$V_p = 14.5\,\text{m/s}$, $V_m = 90\,\text{m/s}$.

$F_m = 200\,\text{N}$.

Find:

Density needed for the air in the wind tunnel $\left(\text{kg/m}^3\right)$.

Force on the full-scale building (prototype) (N).

Properties:

Air, $\rho_p = 1.24\,\text{kg/m}^3$.

PLAN

Equate the Reynolds number for dynamic similitude and equate force coefficients for force ratio.

SOLUTION

Reynolds number

$$
\begin{aligned}
\text{Re}_m &= \text{Re}_p \\
\left(\frac{\rho L V}{\mu}\right)_m &= \left(\frac{\rho L V}{\mu}\right)_p \\
\frac{\rho_m}{\rho_p} &= \left(\frac{V_p}{V_m}\right)\left(\frac{L_p}{L_m}\right)\left(\frac{\mu_m}{\mu_p}\right) \\
&= \left(\frac{14.5\,\text{m/s}}{90\,\text{m/s}}\right)(500)(1) \\
&= 80 \\
\rho_m &= 80\rho_p \\
& \boxed{\rho_m = 99.2\,\text{kg/m}^3}
\end{aligned}
$$

Equating force coefficients

$$
\begin{aligned}
C_{Fp} &= C_{Fm} \\
\frac{F_p}{F_m} &= \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2 \\
&= \left(\frac{1}{80}\right)\times\left(\frac{14.5\,\text{m/s}}{90\,\text{m/s}}\right)^2\times\left(\frac{500}{1}\right)^2 \\
&= 80
\end{aligned}
$$

Force on prototype

$$\begin{aligned} F_p &= 80F_m \\ &= 80 \times 200\,\mathrm{N} \\ &\boxed{F_p = 16{,}000\,\mathrm{N}} \end{aligned}$$

8.52: PROBLEM DEFINITION

Situation:

Performance of a large valve in petroleum pipeline is characterized by recording data on a scale model in water.

$D = 60\,\text{cm}$, $Q = 0.5\,\text{m}^3/\,\text{s}$.

$\frac{1}{3}$scale model, $C_p = 1.07$.

Find:

Flow rate to be used in the model (laboratory) test ($\text{m}^3/\,\text{s}$).

The pressure coefficient for the prototype.

Properties:

Petroleum: $S = 0.82$, $\mu = 3 \times 10^{-3}\,\text{N}\,\text{s}/\,\text{m}^2$.

Water: $\mu = 10^{-3}\,\text{N}\,\text{s}/\,\text{m}^2$.

PLAN

Equate flow rates by Reynolds number matching With dynamic similitude, the pressure coefficients will be the same for model and prototype.

SOLUTION

Matching Reynolds number

$$\begin{aligned}
\text{Re}_m &= \text{Re}_p \\
\left(\frac{VD\rho}{\mu}\right)_m &= \left(\frac{VD\rho}{\mu}\right)_p \\
\frac{V_m}{V_p} &= \left(\frac{D_p}{D_m}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{\mu_m}{\mu_p}\right)
\end{aligned}$$

Multiply both sides of above equation by $A_m/A_p = (D_m/D_p)^2$

$$\begin{aligned}
\left(\frac{A_m}{A_p}\right)\left(\frac{V_m}{V_p}\right) &= \left(\frac{D_p}{D_m}\right)\left(\frac{D_m}{D_p}\right)^2\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{\mu_m}{\mu_p}\right) \\
\frac{Q_m}{Q_p} &= \left(\frac{D_m}{D_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{\mu_m}{\mu_p}\right) \\
&= \left(\frac{1}{3}\right)(0.82)\left(\frac{10^{-3}}{3 \times 10^{-3}}\right) \\
\frac{Q_m}{Q_p} &= 0.0911 \\
Q_m &= Q_p \times 0.0911 \\
&= 0.50 \times 0.0911\ \text{m}^3/\text{s} \\
&\boxed{Q_m = 0.0455\ \text{m}^3/\,\text{s}} \\
&\boxed{C_p = 1.07}
\end{aligned}$$

8.53: PROBLEM DEFINITION

Situation:

The moment acting on the rudder of submarine in sea water is studied using a 1/50 scale model in a water tunnel.

$\frac{1}{40}$ scale model.

$V_m = 6.6\,\text{m/s}$, $M_m = 2\,\text{N m}$.

Find:

Speed of the prototype that corresponds to the speed in the water tunnel (m/s).

Moment that corresponds to the data from the model (N m).

Properties:

Sea Water (10 °C), Table A.4: $\nu_p = 1.4 \times 10^{-6}\,\text{m}^2/\text{s}$.

Water (10 °C), Table A.5: $\nu_m = 1.31 \times 10^{-6}\,\text{m}^2/\text{s}$.

PLAN

Match the Reynolds number for dynamic similitude and extend force coefficient to model the moment.

SOLUTION

Match Reynolds numbers

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$
$$\frac{V_m}{V_p} = \left(\frac{L_p}{L_m}\right)\left(\frac{\nu_m}{\nu_p}\right)$$

so

$$V_p = 6.6\left(\frac{1}{40}\right)\left(\frac{1.41 \times 10^{-6}\,\text{m}^2/\text{s}}{1.31 \times 10^{-6}\,\text{m}^2/\text{s}}\right)$$
$$\boxed{V_p = 0.176\,\text{m/s}}$$

In the same way as a force coefficient was defined there is a corresponding moment coefficient in the form

$$C_M = \frac{M}{\frac{1}{2}\rho V^2 L^2}$$

which is the same for the model and the prototype when conditions for similitude are satisfied. Thus

$$\frac{M_p}{M_m} = \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2$$

Using the velocity ratio from the Reynolds number matching

$$\frac{V_p}{V_m} = \left(\frac{L_m}{L_p}\right)\left(\frac{\nu_p}{\nu_m}\right)$$

gives

$$
\begin{aligned}
\frac{M_p}{M_m} &= \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{\nu_p}{\nu_m}\right)^2\left(\frac{L_p}{L_m}\right) \\
&= \frac{1,026 \text{ kg/m}^3}{1000 \text{ kg/m}^3} \times \left(\frac{1.4 \times 10^{-6} \text{ m}^2/\text{s}}{1.31 \times 10^{-6} \text{ m}^2/\text{s}}\right)^2 \times \left(\frac{40}{1}\right) \\
&= 46.875
\end{aligned}
$$

Moment on the submarine

$$
\begin{aligned}
M_p &= 46.875 \times 2 \text{ N} \cdot \text{m} \\
& \boxed{M_p = 93.8 \, \text{N} \cdot \text{ m}}
\end{aligned}
$$

8.54: PROBLEM DEFINITION

Situation:

A scale model hydrofoil is tested in a water tunnel.

$V_m = 15\,\text{m/s}$, $F_m = 25\,\text{kN}$.

Find:

Lift force on the prototype.

PLAN

Match the Reynolds number to obtain model-prototype velocity ratio and match force coefficients for force ratio.

SOLUTION

Match Reynolds numbers

$$\begin{aligned} \left(\frac{VL\rho}{\mu}\right)_m &= \left(\frac{VL\rho}{\mu}\right)_p \\ \left(\frac{V_p}{V_m}\right)^2 &= \left(\frac{L_m}{L_p}\right)^2 \left(\frac{\rho_m}{\rho_p}\right)^2 \left(\frac{\mu_p}{\mu_m}\right)^2 \end{aligned}$$

Match force coefficients

$$\begin{aligned} C_{Fp} &= C_{Fm} \\ \frac{F_p}{F_m} &= \left(\frac{\rho_p}{\rho_m}\right) \left(\frac{V_p}{V_m}\right)^2 \left(\frac{L_p}{L_m}\right)^2 \end{aligned}$$

Using velocity ratio from Reynolds number matching

$$\begin{aligned} \frac{F_p}{F_m} &= \left(\frac{\rho_p}{\rho_m}\right) \left(\frac{V_p}{V_m}\right)^2 \left(\frac{L_p}{L_m}\right)^2 \\ &= \frac{\rho_m}{\rho_p} \left(\frac{\mu_p}{\mu_m}\right)^2 \end{aligned}$$

When the same fluid is used for the model and prototype

$$\begin{aligned} F_p &= F_m \\ &\boxed{F_p = 25\,\text{kN}} \end{aligned}$$

8.55: PROBLEM DEFINITION

Situation:

A scale model of an automobile will be tested in a pressurized wind tunnel.

$\frac{1}{10}$ scale model.

$V_p = 100\,\mathrm{km/h}$, $c = 345\,\mathrm{m/s}$.

Find:

Pressure in tunnel test section for same Mach number and Reynolds numbers.

PLAN

Match the Mach number and Reynolds number and solve for pressure.

SOLUTION

Match Mach number

$$\begin{aligned} M_m &= M_p \\ \frac{V_m}{c_m} &= \frac{V_p}{c_p}; \frac{V_m}{V_p} = \frac{c_m}{c_p} \end{aligned}$$

The speed of sound the same for both model and prototype so

$$\frac{V_m}{V_p} = 1$$

Match Reynolds number

$$\begin{aligned} \mathrm{Re}_m &= \mathrm{Re}_p \\ \frac{V_m L_m \rho_m}{\mu_m} &= \frac{V_p L_p \rho_p}{\mu_p} \\ \frac{\rho_m}{\rho_p} &= \frac{V_p}{V_m}\frac{L_p}{L_m}\frac{\mu_m}{\mu_p} \\ &= \frac{L_p}{L_m}\frac{\mu_m}{\mu_p} \end{aligned}$$

The dynamic viscosity is the same for model and prototype with same temperature so

$$\frac{\rho_m}{\rho_p} = \frac{L_p}{L_m} = 10$$

From ideal gas law

$$\begin{aligned} p &= \rho R T \\ \frac{p_m}{p_p} &= \frac{\rho_m}{\rho_p} = 10 \end{aligned}$$

so

$$\begin{aligned} p_m &= 10 \text{ atm} \\ & \boxed{p_m = 1010\,\mathrm{kPa}} \end{aligned}$$

8.56: PROBLEM DEFINITION

Situation:

A scale model of an automobile will be tested in a wind tunnel at atmospheric pressure.

$\frac{1}{10}$scale model.

$V_m = 80\,\mathrm{km/\,h}$, $c_m = 345\,\mathrm{m/\,s}$.

Find:

Speed of air in the wind tunnel to match the Reynolds number of the prototype.

Determine if Mach number effects would be important in the wind tunnel.

PLAN

Match the Reynolds number and find velocity for model. Evaluate Mach number for compressibility effects.

SOLUTION

Match Reynolds number

$$\begin{aligned} \mathrm{Re}_m &= \mathrm{Re}_p \\ \frac{V_m L_m \rho_m}{\mu_m} &= \frac{V_p L_p \rho_p}{\mu_p};\ \text{But } \frac{\rho_m}{\mu_m} = \frac{\rho_p}{\mu_p} \end{aligned}$$

so

$$V_m = V_p \left(\frac{L_p}{L_m} \right) = 80\,\mathrm{km/\,h} \times 10 = 800 \text{ km/hr}$$

$$\boxed{V_m = 222 \ \mathrm{m/\,s}}$$

Mach number

$$M = \frac{V}{c} = \frac{222\,\mathrm{m/\,s}}{345\,\mathrm{m/\,s}} = 0.644$$

Because $M \geq 0.3$, **Mach number effects would be important**.

8.57: PROBLEM DEFINITION

Situation:

Water droplets are in an air stream.

$W_e/\sqrt{\mathrm{Re}} = 0.5$, $V_{\mathrm{air}} = 12\,\mathrm{m/s}$.

Find:

Droplet diameter for break up (mm).

Properties:

$p_{\mathrm{air}} = 1.01\,\mathrm{kPa}$, $\sigma = 0.041\,\mathrm{N/m}$.

Air (20 °C), Table A.3: $\rho = 1.2\,\mathrm{kg/m^3}$, $\mu = 1.81 \times 10^{-5}\,\mathrm{N\,s/m^2}$.

PLAN

Apply the $W_e/\sqrt{\mathrm{Re}} = 0.5$ criteria, combined with the equations for Weber number and Reynolds number.

SOLUTION

Weber number and Reynolds number

$$\begin{aligned}\frac{W_e}{\sqrt{\mathrm{Re}}} &= \frac{\rho d V^2 \sqrt{\nu}}{\sigma\sqrt{Vd}} \\ &= \frac{V^{3/2}\sqrt{\rho d \mu}}{\sigma}\end{aligned}$$

So breakup occurs when

$$\frac{V^{3/2}\sqrt{\rho d \mu}}{\sigma} = 0.5$$

Solve for diameter

$$\begin{aligned}d &= \left[\frac{0.5\sigma}{V^{3/2}\sqrt{\rho\mu}}\right]^2 \\ &= \frac{0.25\sigma^2}{V^3\rho\mu}\end{aligned}$$

Calculations

$$\begin{aligned}d &= \frac{0.25\sigma^2}{V^3\rho\mu} \\ &= \frac{0.25 \times (0.041\,\mathrm{N/m})^2}{(12\,\mathrm{m/s})^3 \times (1.2\,\mathrm{kg/m^3}) \times (18.1 \times 10^{-6}\,\mathrm{N\,s/m^2})} \\ &\boxed{d = 11.2\,\mathrm{mm}}\end{aligned}$$

8.58: PROBLEM DEFINITION

Situation:

Breakup of a jet of water into droplets in airstream.
$V = 30\,\mathrm{m/s}$, $W_e = 6.0$.

Find:

Estimated diameter of droplets.

Properties:

Table A.3: $\rho = 1.20$ kg/m^3.
Table A.5: $\sigma = 0.073$ N/m.

PLAN

Set the Weber number equal to 6 and solve for droplet size.

SOLUTION

Weber number

$$W_e = 6.0 = \frac{\rho d V^2}{\sigma}$$

$$d = \frac{6\sigma}{\rho V^2} = \frac{6 \times 0.073\,\mathrm{N/m^3}}{(1.2\,\mathrm{kg/m^3} \times (30\,\mathrm{m/s})^2)} = 4.055 \times 10^{-4}\ \mathrm{m}$$

$$\boxed{d = 0.406\ \ \mathrm{mm}}$$

8.59: PROBLEM DEFINITION

Situation:

Satisfying both Froude number and Reynolds number criteria for model testing.

Find:

Relationship between kinematic viscosity ratio and scale ratio.

PLAN

Match Froude number and Reynolds numbers and solve for ratios.

SOLUTION

Match Froude numbers

$$\begin{aligned} F_m &= F_p \\ \left(\frac{V}{\sqrt{gL}}\right)_m &= \left(\frac{V}{\sqrt{gL}}\right)_p \\ \frac{V_m}{V_p} &= \sqrt{\frac{g_m L_m}{g_p L_p}} \end{aligned}$$

However, because

$$g_m = g_p$$

then

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$$

Match Reynolds numbers

$$\begin{aligned} \mathrm{Re}_m &= \mathrm{Re}_p \\ \left(\frac{VL}{\nu}\right)_m &= \left(\frac{VL}{\nu}\right)_p \\ \frac{V_m}{V_p} &= \left(\frac{L_p}{L_m}\right)\left(\frac{\nu_m}{\nu_p}\right) \end{aligned}$$

Eliminate V_m/V_p between equations

$$\sqrt{\frac{L_m}{L_p}} = \left(\frac{L_p}{L_m}\right)\left(\frac{\nu_m}{\nu_p}\right)$$

Therefore: $\boxed{\nu_m/\nu_p = (L_m/L_p)^{3/2}}$

8.60: PROBLEM DEFINITION

Situation:

The spillway of a dam is simulated using a scale model.

$\frac{1}{20}$scale model.

$t_m = 2\,\text{s}$, $L_m = 8\,\text{cm}$.

Find:

Wave height (prototype).

Wave period (prototype).

PLAN

Dynamic similarity based on Froude number.

SOLUTION

Since it is a scale model, the wave height will scale directly

$$\begin{aligned} L_p &= 20L_m \\ \text{Wave height}_p &= 20 \times 8 \text{ cm} \\ & \boxed{L_p = 1.6\,\text{m}} \end{aligned}$$

Match Froude number

$$\begin{aligned} \frac{V_m}{\sqrt{gL_m}} &= \frac{V_p}{\sqrt{gL_p}} \\ \frac{V_p}{V_m} &= \left(\frac{L_p}{L_m}\right)^{1/2} \\ &= \sqrt{20} = 4.472 \end{aligned}$$

The time will scale as

$$t \sim \frac{L}{V}$$

so

$$\begin{aligned} \frac{t_p}{t_m} &= \frac{L_p}{L_m}\frac{V_m}{V_p} \\ &= 20/\sqrt{20} = \sqrt{20} \end{aligned}$$

Thus

$$\begin{aligned} t_p &= 4.472 \times 2 \text{ s} \\ & \boxed{t_p = 8.94\,\text{s}} \end{aligned}$$

8.61: PROBLEM DEFINITION

Situation:

A prototype of a dam spillway is represented with a scale model.

$\frac{1}{25}$scale model.

$V = 2.5\,\text{m/s}$, $Q = 0.10\,\text{m}^3/\text{s}$.

Find:

Velocity for prototype (m/ s).

Discharge for prototype (m^3/ s).

PLAN

Dynamic similarity based on Froude number.

SOLUTION

Match Froude number

$$\begin{aligned} Fr_m &= Fr_p \\ \frac{V_m}{(g_m L_m)^{0.5}} &= \frac{V_p}{(g_p L_p)^{0.5}} \\ \frac{V_p}{V_m} &= \left(\frac{L_p}{L_m}\right)^{0.5} = 5 \\ V_p &= (2.5)(5)\ \text{m/s} \\ &\boxed{V_p = 12.5\ \ \text{m/s}} \end{aligned}$$

The discharge ratio is

$$\begin{aligned} \frac{Q_p}{Q_m} &= \frac{V_p}{V_m}\left(\frac{L_p}{L_m}\right)^2 \\ &= \left(\frac{L_p}{L_m}\right)^{1/2}\left(\frac{L_p}{L_m}\right)^2 \\ &= \left(\frac{L_p}{L_m}\right)^{2.5} \end{aligned}$$

Discharge for prototype

$$\begin{aligned} Q_p &= 0.1\ \text{m}^3/\text{s} \times 25^{2.5} \\ &\boxed{Q_p = 312\,\text{m}^3/\text{s}} \end{aligned}$$

8.62: PROBLEM DEFINITION

Situation:

A seaplane model has a scale to simulate take-off.
$\frac{1}{6}$ scale model.
$V_p = 117\,\text{km/h}$.

Find:

Model speed to simulate a takeoff conditions (m/s).

PLAN

Use Froude number scaling.

SOLUTION

Match Froude number

$$\begin{aligned} V_m &= V_p\sqrt{\frac{L_m}{L_p}} \\ &= 117 \ \text{km/h} \times \left(\frac{1\,\text{h}}{3600\,\text{s}}\right) \times \left(\frac{1000\,\text{m}}{1\,\text{km}}\right) \times \sqrt{\frac{1}{6}} \\ &\boxed{V_m = 13.27 \ \text{m/s} = 47.8 \ \text{k m/hr}} \end{aligned}$$

8.63: PROBLEM DEFINITION

Situation:

A model spillway has a scale.
$\frac{1}{36}$scale model.
$Q = 3000\,\mathrm{m^3/s}$.

Find:

Velocity ratio.
Discharge ratio.
Model discharge $(\mathrm{m^3/s})$.

PLAN

Dynamic similarity based on Froude number.

SOLUTION

Match Froude number

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} \tag{1}$$

or for this case

$$\boxed{\frac{V_m}{V_p} = \sqrt{\frac{1}{36}} = \frac{1}{6}}$$

Multiply both sides of Eq. (1) by $A_m/A_p = (L_m/L_p)^2$

$$\frac{V_m A_m}{V_p A_p} = \left(\frac{L_m}{L_p}\right)^{1/2}\left(\frac{L_m}{L_p}\right)^2$$

$$\frac{Q_m}{Q_p} = \left(\frac{L_m}{L_p}\right)^{5/2}$$

or for this case

$$\frac{Q_m}{Q_p} = \left(\frac{1}{36}\right)^{5/2}$$

$$\boxed{\frac{Q_m}{Q_p} = \frac{1}{7776}}$$

$$Q_m = \frac{3000\,\mathrm{m^3/s}}{7776}$$

$$\boxed{Q_m = 0.386\ \mathrm{m^3/s}}$$

8.64: PROBLEM DEFINITION

Situation:

Flow in a river is to be studied using a scale model.
$\frac{1}{64}$scale model.
$d_p = 6\,\text{m}$, $V_p = 4.5\,\text{m/s}$.

Find:

Model velocity (m/s).
Model depth (m/s).

PLAN

Use Froude number scaling.

SOLUTION

Match Froude number

$$
\begin{aligned}
Fr_m &= Fr_p \\
\frac{V_m}{(g_m L_m)^{0.5}} &= \frac{V_p}{(g_p L_p)^{0.5}} \\
V_m &= V_p \left(\frac{L_m}{L_p}\right)^{0.5} = 4.5 \text{ m/s} \times (1/8) \\
&\boxed{V_m = 0.56\,\text{m/s}}
\end{aligned}
$$

Geometric similitude

$$
\begin{aligned}
\frac{d_m}{d_p} &= \frac{1}{64} \\
d_m &= \frac{1}{64} d_p \\
&= \frac{1}{64}(6) \\
&\boxed{d_m = 0.094\,\text{m}}
\end{aligned}
$$

8.65: PROBLEM DEFINITION

Situation:

A model of spillway modeled in laboratory.

$\frac{1}{40}$ scale model.

$V_m = 1\,\text{m/s}$, $Q_m = 0.1\,\text{m}^3/\text{s}$.

Find:

Prototype velocity (m/s).

Prototype discharge (m^3/s).

PLAN

Use Froude number scaling.

SOLUTION

Match Froude number

$$\begin{aligned} V_p &= V_m\sqrt{\frac{L_p}{L_m}} \qquad (1) \\ &= 1\sqrt{40} \\ &\boxed{V_p = 6.3\,\text{m/s}} \end{aligned}$$

Multiply both sides of Eq. (1) by $A_p/A_m = (L_p/L_m)^2$

$$\frac{V_pA_p}{V_mA_m} = \left(\frac{L_p}{L_m}\right)^{5/2}$$

So

$$\begin{aligned} \frac{Q_p}{Q_m} &= \left(\frac{L_p}{L_m}\right)^{5/2} \\ Q_p &= 0.1\,\text{m}^3/\text{s} \times (40)^{5/2} \\ &\boxed{Q_p = 1012\,\text{m}^3/\text{s}} \end{aligned}$$

8.66: PROBLEM DEFINITION

Situation:

Flow around a bridge pier is studied using a scale model.

$\frac{1}{12}$scale model.

$V_m = 0.9\,\text{m/s}$, $L_m = 2.5\,\text{cm}$.

Find:

(a) Velocity.

(b) Wave height in prototype.

PLAN

Use Froude number matching.

SOLUTION

Match Froude numbers

$$V_p = V_m\sqrt{\frac{L_p}{L_m}} = 0.90\sqrt{12}$$

$$\boxed{V_p = 3.12\ \text{m/s}}$$

$L_p/L_m = 12$; therefore, wave height$_{\text{prot.}} = \boxed{L_p = 12 \times 2.5\ \text{cm} = 30\ \text{cm}}$

8.67: PROBLEM DEFINITION

Situation:

A scale model of a spillway is tested.

$\frac{1}{25}$scale model.

$Q_m = 0.1\,\text{m}^3/\,\text{s}$, $t_m = 1\,\text{min}$.

Find:

Time for a particle to move along a corresponding path in the prototype (min).

Prototype discharge ($\text{m}^3/\,\text{s}$).

PLAN

Use Froude number matching.

SOLUTION

Match Froude numbers

$$\frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}}$$

or

$$\frac{L_p/t_p}{L_m/t_m} = \left(\frac{L_p}{L_m}\right)^{1/2}$$

Then

$$\begin{aligned}
\frac{t_p}{t_m} &= \left(\frac{L_p}{L_m}\right)\left(\frac{L_m}{L_p}\right)^{1/2} \\
\frac{t_p}{t_m} &= \left(\frac{L_p}{L_m}\right)^{1/2} \\
t_p &= 1 \times \sqrt{25} \\
& \boxed{t_p = 5\,\text{min}}
\end{aligned}$$

Also

$$\begin{aligned}
\frac{Q_p}{Q_m} &= \left(\frac{L_p}{L_m}\right)^{5/2} \\
Q_p &= 0.10\,\text{m}^3/\,\text{s} \times (25)^{5/2} \\
& \boxed{Q_p = 312\ \ \text{m}^3/\,\text{s}}
\end{aligned}$$

8.68: PROBLEM DEFINITION

Situation:

A tidal estuary is modeled using a scale model.

$\frac{1}{600}$scale model.

$V_p = 3.6\,\text{m/s}$, $t_p = 12.5\,\text{h}$.

Find:

Velocity and period in the model.

PLAN

Use Froude number matching.

SOLUTION

Match Froude number

$$Fr_m = Fr_p$$

or

$$\left(\frac{V}{\sqrt{gL}}\right)_m = \left(\frac{V}{\sqrt{gL}}\right)_p$$

$$\text{Because } g_m = g_p,$$

$$\frac{V_m}{V_p} = \left(\frac{L_m}{L_p}\right)^{1/2}$$

$$V_m = 3.6 \text{ m/s} \times \left(\frac{1}{600}\right)^{1/2}$$

$$\boxed{V_m = 0.147\,\text{m/s}}$$

The time varies as $t \sim L/V$ so

$$\begin{aligned} \frac{t_m}{t_p} &= \frac{L_m}{L_p}\frac{V_p}{V_m} \\ &= \frac{L_m}{L_p}\left(\frac{L_m}{L_p}\right)^{-1/2} \\ &= \left(\frac{L_m}{L_p}\right)^{1/2} \end{aligned}$$

The model period is

$$\begin{aligned} t_m &= 12.5 \text{ hr} \times \left(\frac{1}{600}\right)^{1/2} \\ &= 0.510 \text{ hr} \\ &\boxed{t_m = 30.6\,\text{min}} \end{aligned}$$

8.69: PROBLEM DEFINITION

Situation:

A scale sea wall tests maximum wave force.
$\frac{1}{36}$ scale model.
$F_m = 80\,\text{N}$.

Find:

Force on the wall for the prototype (N).

Assumptions:

Fresh water (model) and seawater (prototype).

Properties:

Sea Water (10 °C), Table A.4: $\rho_p = 1026\,\text{kg/m}^3$.
Water (10 °C), Table A.5: $\rho_m = 1000\,\text{kg/m}^3$.

PLAN

Dynamic similarity based on force coefficient and Froude number.

SOLUTION

Match Froude numbers

$$\begin{aligned} Fr_m &= Fr_p \\ \frac{V_m}{V_p} &= \sqrt{\frac{L_m}{L_p}} \end{aligned}$$

Match force coefficients.

$$\begin{aligned} C_{Fm} &= C_{Fp} \\ \frac{F_p}{F_m} &= \frac{\rho_p}{\rho_m}\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2 \end{aligned}$$

Using result from Froude number scaling

$$\begin{aligned} \frac{F_p}{F_m} &= \frac{\rho_p}{\rho_m}\frac{L_p}{L_m}\left(\frac{L_p}{L_m}\right)^2 \\ &= \frac{\rho_p}{\rho_m}\left(\frac{L_p}{L_m}\right)^3 \\ &= \frac{1026\ \text{kg/m}^3}{1000\ \text{kg/m}^3}\times 36^3 \\ &= 4.787\times 10^4 \end{aligned}$$

Force on wall

$$\begin{aligned} F_p &= 80\ \text{N}\times 4.787\times 10^4 \\ &\boxed{F_p = 3.83\,\text{MN}} \end{aligned}$$

8.70: PROBLEM DEFINITION

Situation:

A scale model of a spillway is built to test flow conditions.
$\frac{1}{80}$ scale model.
$Q_p = 800\,\text{m}^3/\,\text{s},\ F_m = 51\,\text{N}.$

Find:

Water discharge in model for dynamic similarity ($\text{m}^3/\,\text{s}$).
Force on the prototype (kN).

PLAN

Dynamic similitude based on matching force coefficients and Froude numbers.

SOLUTION

Match Froude number

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$$

The discharge ratio is

$$\frac{Q_m}{Q_p} = \frac{V_m}{V_p}\left(\frac{L_m}{L_p}\right)^2$$

Using velocity ratio from Froude number equality

$$\begin{aligned}\frac{Q_m}{Q_p} &= \left(\frac{L_m}{L_p}\right)^{2.5} \\ &= \left(\frac{1}{80}\right)^{2.5} = 3.2 \times 10^{-4}\end{aligned}$$

Discharge for model

$$Q_m = 800\text{ m}^3/\text{s} \times 1.75 \times 10^{-5}$$

$$\boxed{Q_m = 0.014\,\text{m}^3/\,\text{s}}$$

Match force coefficients,

$$\begin{aligned}C_{Fp} &= C_{Fm} \\ \frac{F_p}{F_m} &= \frac{\rho_p}{\rho_m}\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2\end{aligned}$$

Using Froude number matching

$$\begin{aligned}\frac{F_p}{F_m} &= \frac{\rho_p}{\rho_m}\left(\frac{L_p}{L_m}\right)\left(\frac{L_p}{L_m}\right)^2 \\ &= \frac{\rho_p}{\rho_m}\left(\frac{L_p}{L_m}\right)^3 \\ &= 1 \times 80^3 = 51.2 \times 10^4\end{aligned}$$

Force on prototype

$$F_p = 51 \text{ N} \times 51.2 \times 10^4$$
$$\boxed{F_p = 26,112 \text{ kN}}$$

8.71: PROBLEM DEFINITION

Situation:

A scale model of a dam will be constructed in a laboratory.

$L_p = 1200\,\text{m}$, $W_p = 300\,\text{m}$.

$Q_p = 5000\,\text{m}^3/\,\text{s}$, $Q_m = 0.9\,\text{m}^3/\,\text{s}$.

$A_m = 50\,\text{m} \times 20\,\text{m} = 1000\,\text{m}^2$.

Find:

The largest feasible scale ratio.

SOLUTION

Check the scale ratio as dictated by Q_m/Q_p

$$\frac{Q_m}{Q_p} = \frac{0.90\,\text{m}^3/\,\text{s}}{5,000\,\text{m}^3/\,\text{s}} = \left(\frac{L_m}{L_p}\right)^{5/2}$$

or

$$\frac{L_m}{L_p} = 0.0318$$

Then with this scale ratio

$$\begin{aligned} L_m &= 0.0318 \times 1,200 \text{ m} = 38.1 \text{ m} \\ W_m &= 0.0318 \times 300 \text{ m} = 9.54 \text{ m} \end{aligned}$$

Therefore, model will fit into the available space, so use

$$\boxed{\frac{L_m}{L_p} = \frac{1}{31.4} = 0.0318}$$

8.72: PROBLEM DEFINITION

Situation:

A scale model of a ship is tested in a towing tank.
$L_m = 1.2\,\text{m}$, $L_p = 30\,\text{m}$.
$V_m = 1.5\,\text{m/s}$.

Find:

Speed for the prototype that corresponds to the model test.

PLAN

Dynamic similarity based on Froude number.

SOLUTION

Match Froude number

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

$$V_p = \frac{V_m \sqrt{L_p}}{\sqrt{L_m}}$$

$$= (1.5\,\text{m/s}) \left(\frac{30\,\text{m}}{1.2} \right)^{1/2}$$

$$\boxed{V_p = 7.5\,\text{m/s}}$$

8.73: PROBLEM DEFINITION

Situation:
A scale model of a ship is tested in tank.
$\frac{1}{25}$scale model.
$V_m = 1.5\,\text{m/s},\ F_m = 9\,\text{N}.$

Find:
Velocity of the prototype (m/s).
Wave resistance of the prototype (N).

Assumptions:
Same fluids used, therefore $\rho_p = \rho_m$.

PLAN

Dynamic similarity based on Froude number.

SOLUTION

Match the Froude number

$$\begin{aligned}
\frac{V_m}{V_p} &= \sqrt{\frac{L_m}{L_p}} \\
V_p &= 1.5 \times \sqrt{25} \\
&\boxed{V_p = 7.5 \text{ m/s}}
\end{aligned}$$

Equating the force coefficients

$$\begin{aligned}
C_{Fp} &= C_{Fm} \\
\frac{F_p}{F_m} &= \frac{\rho_p}{\rho_m}\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2
\end{aligned}$$

With Froude number matching

$$\begin{aligned}
\frac{F_p}{F_m} &= \frac{\rho_p}{\rho_m}\left(\frac{L_p}{L_m}\right)\left(\frac{L_p}{L_m}\right)^2 \\
&= \frac{\rho_p}{\rho_m}\left(\frac{L_p}{L_m}\right)^3 \\
&= 25^3
\end{aligned}$$

Force on prototype

$$\begin{aligned}
F_p &= 9 \text{ N} \times 25^3 \\
&\boxed{F_p = 140{,}625\,\text{N}}
\end{aligned}$$

8.74: PROBLEM DEFINITION

Situation:

A scale model of a building is being tested.
$\frac{1}{20}$scale model.
$F_m = 200\,\text{N}$, $V_m = 20\,\text{m/s}$.
$V_p = 40\,\text{m/s}$.

Find:

Drag on the prototype building.

Assumptions:

Reynolds sufficiently high that dynamic similitude is satisfied.

SOLUTION

Match force coefficients

$$\begin{aligned}
C_{Fp} &= C_{Fm} \\
\frac{F_p}{F_m} &= \frac{\rho_p}{\rho_m}\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2 \\
&= 1 \times \left(\frac{40\,\text{m/s}}{20\,\text{m/s}}\right)^2 20^2 \\
&= 1600 \\
F_p &= 1600 \times F_m \\
&= 1600 \times 200\,\text{N} \\
F_p &= 320\,\text{kN}
\end{aligned}$$

Choice (d) is correct.

8.75: PROBLEM DEFINITION

Situation:

A scale model of a building is being tested in a wind tunnel.

$\frac{1}{550}$scale model.

$V_m = 20\,\text{m/s}$, $V_p = 200\,\text{km/h}$.

$F_m = 20\,\text{N}$, $C_{p_m} = 1, -2.7, -0.8$.

Find:

Pressure values on the prototype.

- windward wall
- side wall
- leeward wall

Lateral force on the prototype in a 150 km/hr wind.

Properties:

Model: Air at $20°C$, $\rho_m = 1.20\,\text{kg/m}^3$

Prototype: Air at $10°C$, $\rho_m = 1.25\,\text{kg/m}^3$

Assumptions:

Assume Reynolds number sufficiently large that dynamic similitude is satisfied.

PLAN

Match the pressure coefficients and the force coefficient.

SOLUTION

Match pressure coefficients

$$C_{p,\text{model}} = C_{p,\text{prot.}}$$

then

$$\frac{\Delta p_p}{(1/2)\rho_p V_p^2} = C_{p_p} = C_{p_m}$$

Velocity for prototype is

$$\begin{aligned} V_p &= 200\ \frac{\text{km}}{\text{hr}} \times \frac{1000\text{ m}}{\text{km}} \times \frac{1\text{ hr}}{3600\text{ s}} \\ &= 55.56\text{ m/s} \end{aligned}$$

or

$$\begin{aligned} \Delta p_p &= C_{p_m}((1/2)\rho_p V_p^2) \\ &= C_{p_m} \times (1/2) \times 1.25 \times 55.56^2 \\ p - p_0 &= 1929.0 C_{p_m} \end{aligned}$$

but $p_0 = 0$ gage so

$$p = 1929 C_{p_m} \text{ Pa}$$

Extremes of pressure are therefore:

$$\boxed{p_{\text{windward wall}} = 1.93\,\text{kPa}}$$
$$\boxed{p_{\text{side wall}} = 1929\,\text{Pa} \times (-2.7) = -5.21\,\text{kPa}}$$
$$\boxed{p_{\text{leeward wall}} = 1929\,\text{Pa} \times (-0.8) = -1.54\,\text{kPa}}$$

Match force coefficient

$$\begin{aligned}
C_{Fm} &= C_{Fp} \\
\frac{F_p}{F_m} &= \frac{\rho_p}{\rho_m}\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2 \\
&= \frac{1.25\,\text{kg/m}^3}{1.2\,\text{kg/m}^3}\left(\frac{55.56\,\text{m/s}}{20\,\text{m/s}}\right)^2\left(\frac{550}{1}\right)^2 \\
&= 2.431 \times 10^6
\end{aligned}$$

Lateral Force:

$$\begin{aligned}
F_p &= 20\text{ N} \times 2.431 \times 10^6 \\
&\boxed{F_p = 48.6\,\text{MN}}
\end{aligned}$$

8.76: PROBLEM DEFINITION

Situation:

Drag force is measured in a water tunnel and a wind tunnel on three different models.

Model sizes: 5 cm, 8 cm, 15 cm.

Find:

Find the relevant π-groups.

Write a computer program and reduce the given data.

Plot the data using the relevant π-groups.

SOLUTION

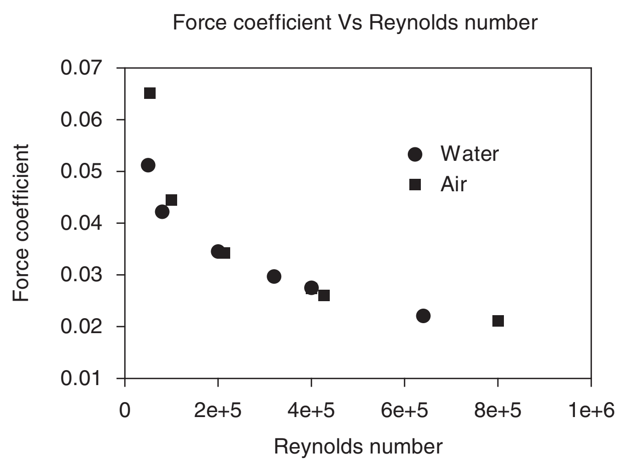
The drag force on an object is quantified by the force coefficient. The only significant π-group is Reynolds number so

$$C_F = f(\text{Re})$$

or

$$\frac{F}{\frac{1}{2}\rho V^2 L^2} = f\left(\frac{VL}{\nu}\right)$$

The data are reduced and plotted below.



8.77: PROBLEM DEFINITION

Situation:

Pressure drop is measured in a pipe with water and oil.

Find:

Find the relevant π-groups.
Write a computer program and reduce the given data.
Plot the data using the relevant π-groups.

SOLUTION

Performing a dimensional analysis on the equation for pressure drop shows which is

$$\boxed{\frac{\Delta p D}{L\rho V^2} = f\left(\frac{\rho V D}{\mu}\right)}$$

where the independent parameter is Reynolds number. Plotting the data using the dimensionless parameters gives the following graph.