9.1: PROBLEM DEFINITION

Situation:

In which case is the flow caused by a pressure gradient?

a. Couette flow

b. Hele-Shaw flow

SOLUTION

The correct answer is
b) Hele-Shaw flow

9.2: PROBLEM DEFINITION

Situation:

Couette flow of liquid with temperature distribution between plates.

Find:

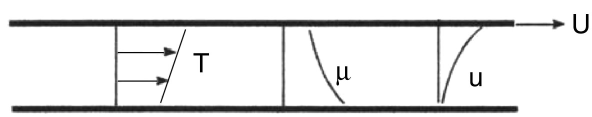
Qualitative description of velocity profile.

SOLUTION

In a Couette flow, the shear stress is constant across the flow so

$$\tau = \mu \frac{du}{dy}$$

so the product of the viscosity and velocity gradient must be a constant. Thus the velocity gradient will be the highest where the viscosity is the smallest. The velocity profile is shown below.



9.3: PROBLEM DEFINITION

Situation:

a. Liquid flow between parallel plates and the viscosity is constant in the flow direction; compare this with a case where the viscosity decreased in the flow direction, such as due to a temperature rise.

b. Gas flow between two flat plates and temperature increases in flow direction and density decreases. Assume viscosity is unaffected.

Find:

a. How will the pressure distribution change in the flow direction due to the temperature rise in the flow direction.

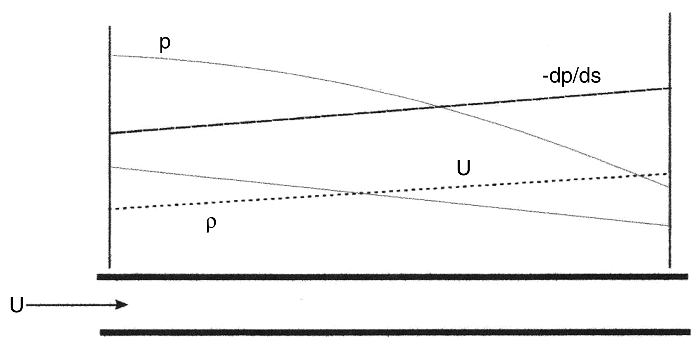
b. How will the velocity and pressure distribution change from the case with constant density? Sketch the pressure distribution and give the rationale for your result.

SOLUTION

Part a. The discharge for flow between parallel plates is

$$q = -\frac{B^3}{12\mu}\frac{dp_z}{ds}$$

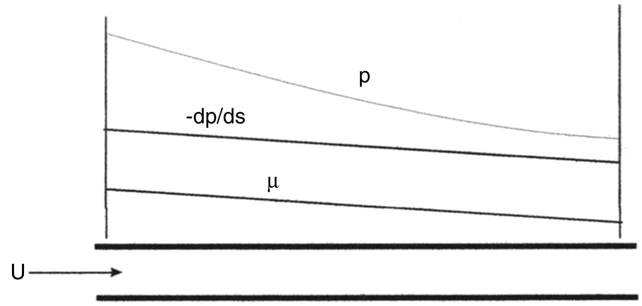
(where B is the distance between the two plates) and the discharge is constant so dp_z/ds must vary directly as μ.



SOLUTION

Part b.

Because of the continuity equation, the product ρu must be constant, so as density decreases, velocity must increase. However to effect an acceleration, the pressure has to increase. The qualitative changes are shown in figure below.


p
-dp/ds
μ
U

9.4: PROBLEM DEFINITION

Situation:

A 39 cm block on side weighing 110 N slides on an oil film with thickness of 0.11 mm.

Find:

Terminal velocity of block.

Properties:

Viscosity is 10^{-2} N·s/m^2

PLAN

Apply equilibrium. Then relate shear force (viscous drag force) to viscosity and solve the resulting equation.

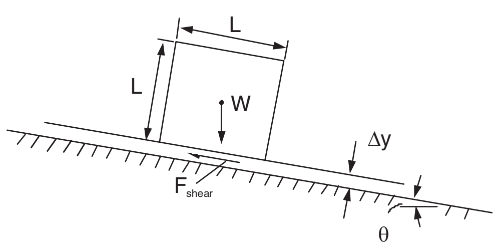
SOLUTION

Force equilibrium

$$\begin{aligned} F_{\text{shear}} &= W \sin\theta \\ \tau &= \frac{F_{\text{shear}}}{A_s} = \frac{W\sin\theta}{L^2} \end{aligned}$$

Shear stress

$$\tau = \mu \frac{dV}{dy} = \mu \times \frac{V}{\Delta y}$$



or

$$V = \frac{\tau \Delta y}{\mu}$$

Then

$$\begin{aligned} V &= \frac{W\sin\theta}{L^2}\frac{\Delta y}{\mu} \\ V &= \left(\frac{110\sin 10^\circ}{(0.39\,\text{m})^2}\right) \times 1.1 \times 10^{-4}/10^{-2} \end{aligned}$$

$$\boxed{V = 1.38 \text{ m/s}}$$

9.5: PROBLEM DEFINITION

Situation:

A board 1 m by 1 m weighing 180 N slides on an inclined surface at 0.15 m/s on a film of oil 0.05 cm thick.

Find:

Dynamic viscosity of oil.

SOLUTION

Equating the gravitational force and shear force

$$W\sin\theta = \mu\frac{V}{\Delta y}L^2$$

$$\begin{aligned}\mu &= \left(\frac{W\sin\theta}{L^2}\right)\frac{\Delta y}{V};\ \text{where }\ \sin\theta = 5/13\\ \mu &= \frac{180\text{ N}\times(5/13)}{(1\text{ m})^2}\times\frac{(0.0005\,\text{m})}{0.15\,\text{m/ s}}\\ &\boxed{\mu = 0.23\text{ N-s/m}^2}\end{aligned}$$

9.6: PROBLEM DEFINITION

Situation:

A board 1 m by 1 m weighing 30 N slides down inclined ramp at 17 cm/s on 0.8 mm layer of oil.

Find:

Dynamic viscosity of oil.

SOLUTION

Equating gravitational and shear force

$$\begin{aligned} W\sin\theta &= \mu\frac{V}{\Delta y}L^2 \\ \mu &= \left(\frac{W\sin\theta}{L^2}\right)\frac{\Delta y}{V};\quad \text{where } \sin\theta = 5/13 \end{aligned}$$

$$\mu = \frac{30\text{ N}\times(5/13)}{(1\text{ m}^2)}\times\frac{8\times10^{-4}\text{ m}}{0.17\text{ m/s}}$$

$$\boxed{\mu = 5.43\times10^{-2}\text{ N}\cdot\text{s/m}^2}$$

9.7: PROBLEM DEFINITION

Situation:

Uniform, steady flow is occurring between horizontal parallel plates.

Find:

a. The flow is Hele-Shaw, therefore what is causing the fluid to move?
b. Where is the maximum velocity located?
c. Where is the maximum shear stress located?
d. Where is the minimum shear stress located?

SOLUTION

a. By definition, if the flow is Hele-Shaw, it is caused by a pressure gradient.

b. In this figure, which is symmetrical about the centerline, the maximum velocity is located at the centerline.

c. In this figure, which is symmetrical about the centerline, the maximum shear stress is located at the 2 walls.

d. In this figure, which is symmetrical about the centerline, the minimum shear stress is located at the centerline.

9.8: PROBLEM DEFINITION

Situation:
Uniform, steady flow occurs between two plates.

Find:
(a) Conditions present to cause odd velocity distribution.
(b) Location of minimum shear stress.

SOLUTION

(a) Answer ⟹Pressure distribution causing flow in x-direction but upper plate is moving to the left relative to the lower plate.

(b) Answer ⟹Minimum shear stress occurs where the maximum velocity occurs (where $du/dy = 0$).

9.9: PROBLEM DEFINITION

Situation:
A asymmetrical laminar velocity distribution

Find:
Whether statements (a) through (d) are true or false.

SOLUTION

a). False   b). False   c). False   d). True

9.10: PROBLEM DEFINITION

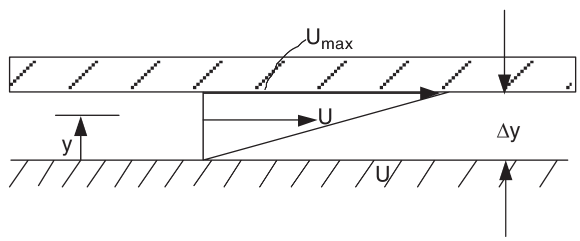
Situation:
A plate 1 m long and 30 cm wide is being pulled over layer of oil 2 mm thick.

Find:
(a) Express the velocity mathematically in terms of the coordinate system shown.
(b) Whether flow is rotation or irrotational.
(c) Whether continuity is satisfied.
(d) Force required to produce plate motion.

Properties:
Viscosity is 4 N·s/m^2

SOLUTION

a) By similar triangles $u/y = u_{\max}/\Delta t$



or

$$
\begin{aligned}
u &= \left(\frac{u_{\max}}{\Delta y}\right) y \\
u &= \left(\frac{0.3\,\mathrm{m/\,s}}{0.002\,\mathrm{m}}\right) y \text{ m} \\
&\boxed{u = 150y \text{ (m/s)}} \\
v &= 0
\end{aligned}
$$

b) For flow to be irrotational $\partial u/\partial y = \partial V/\partial x$ here $\partial u/\partial y = 150$ and $\partial V/\partial x = 0$. The equation is not satisfied; $\boxed{\text{flow is rotational}}$.
c) $\partial u/\partial x + \partial v/\partial y = 0$ (continuity equation) $\partial u/\partial x = 0$ and $\partial v/\partial y = 0$
so $\boxed{\text{continuity is satisfied.}}$
d) The force is the product of the area and the shear stress for a Couette flow

$$
\begin{aligned}
F_s &= \frac{A\mu V}{t} \\
F_s &= \frac{0.3 \times (1 \times 0.3) \times 4}{0.002} \\
&\boxed{F_s = 180\text{N}}
\end{aligned}
$$

9.11: PROBLEM DEFINITION

Situation:

The figure shows a Hele-Shaw flow.

Find:

Determine which of the statements (a) through (e) are true.

SOLUTION

Valid statements are (c), (e).

9.12: PROBLEM DEFINITION

Situation:

A plate is separated by a layer of oil between and moving upper plate and a fixed lower plate.

Find:

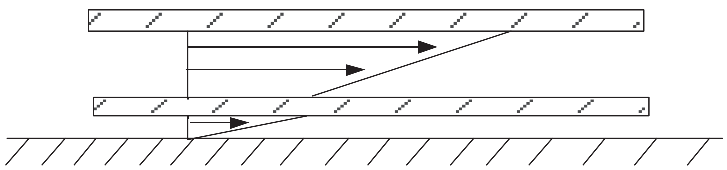
Derive an equation for the velocity of the intermediate plate.

Assumptions:

A linear velocity distribution within the oil.

SOLUTION

The velocity distribution will appear as below:



$$\begin{aligned}
(\text{Force on top of middle plate}) &= (\text{Force on bottom of middle plate}) \\
\tau_1 A &= \tau_2 A \\
\tau_1 &= \tau_2
\end{aligned}$$

$$\begin{aligned}
\frac{\mu_1 \Delta V_1}{t_1} &= \frac{\mu_2 \Delta V_2}{t_2} \\
\mu_1 \times \frac{V - V_{\text{lower}}}{t_1} &= \frac{\mu_2 V_{\text{lower}}}{t_2} \\
\frac{\mu_1 V}{t_1} - \frac{\mu_1 V_{\text{lower}}}{t_1} &= \frac{\mu_2 V_{\text{lower}}}{t_2} \\
V_{\text{lower}} \left(\frac{\mu_2}{t_2} + \frac{\mu_1}{t_1} \right) &= V \frac{\mu_1}{t_1} \\
& \boxed{V_{\text{lower}} = \frac{V \mu_1 / t_1}{\mu_2 / t_2 + \mu_1 / t_1}}
\end{aligned}$$

9.13: PROBLEM DEFINITION

Situation:

A 27 cm-diameter disk in oil is rotated at 31 rad/s above a fixed plate over a layer of oil 3 mm thick.

Find:

Torque required to rotate disk.

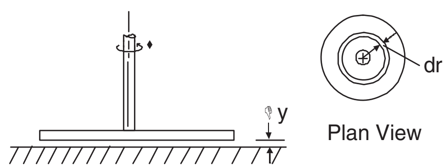
Properties:

Viscosity is 8 N·s/m^2

SOLUTION

Assume a Couette flow between plate and disk.

$$\begin{aligned}
\tau &= \mu \frac{dv}{dy} \\
\tau &= \frac{\mu r \omega}{\Delta y} \\
dT &= r dF \\
dT &= r \tau dA \\
dT &= r \left(\frac{\mu \omega}{\Delta y} \right) 2\pi r dr
\end{aligned}$$



Then

$$\begin{aligned}
T &= \int_0^r dT = \int_0^{r_0} (\frac{\mu r \omega}{\Delta y}) 2\pi r^3 dr \\
T &= \frac{(2\pi \mu \omega / \Delta y) r^4}{4} |_0^{r_0} = \frac{2\pi \mu \omega r_0^4}{4 \Delta y}
\end{aligned}$$

$$T = \frac{2\pi \mu \omega r_0^4}{4 \Delta y}$$

where

$$\begin{aligned}
r_o &= 0.135 \text{ m} \\
\Delta y &= 3 \times 10^{-3} \text{ m} \\
\omega &= 31 \text{ rad/s} \\
\mu &= 8 \text{ N} \cdot \text{s/m}^2 \\
T &= 2\pi \times 8 \text{ N} \cdot \text{s/m}^2 \times 31 \text{ rad/s} \times \frac{(0.135 \text{ m})^4}{4 \times (3 \times 10^{-3} \text{ m})}
\end{aligned}$$

$$\boxed{T = 43.1 \text{ N·m}}$$

9.14: PROBLEM DEFINITION

Situation:

A 2 mm thick plate and 1 m wide is pulled at 0.4 m/s between two walls and the space is occupied by glycerine at 20^{o}C in glycerin is described in the problem statement.

Find:

a) Sketch the velocity distribution at section $A - A$.

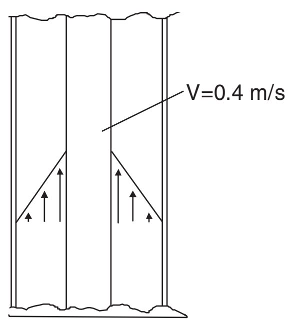
b) Force required to pull plate.

Properties:

Glycerin (Table A.4): $\mu = 1.41\,\mathrm{N \cdot s/\,m^2}$.

SOLUTION

Velocity distribution:



$$
\begin{aligned}
F &= \tau A \\
&= \mu \frac{dV}{dy} A \\
&= \left(1.41\,\mathrm{N \cdot s/\,m^2}\right)\left(\frac{0.4\,\mathrm{m/\,s}}{0.002\,\mathrm{m}}\right) \times 1\,\mathrm{m} \times 2\,\mathrm{m} \times 2 \text{ sides} \\
&= 1128\,\mathrm{N} \\
& \boxed{F = 1128 \text{ N}}
\end{aligned}
$$

9.15: PROBLEM DEFINITION

Situation:

A bearing turns at 200 rad/s inside a 30-mm diameter cylinder 1 cm long. The distance between the shaft and cylinder is 1 mm and filled with SAE 30 oil.

Find:

Torque required to turn bearing.

Properties:

Viscosity from Table A.4 is 0.1 N·s/m^2

SOLUTION

$$\begin{aligned} \tau &= \frac{\mu V}{\delta} \\ T &= \tau A r \end{aligned}$$

where T = torque, A = bearing area = $2\pi r b$

$$\begin{aligned} T &= \tau 2\pi r b r = \tau 2\pi r^2 b \\ &= \frac{\mu V}{\delta}(2\pi r^2 b) \end{aligned}$$

where V=$r\omega$. Then

$$\begin{aligned} T &= \frac{\mu}{\delta}(r\omega)(2\pi r^2 b) \\ &= \frac{\mu}{\delta}(2\pi\omega)r^3 b \\ &= \left(\frac{0.1 \text{ N}\cdot\text{s/m}^2}{0.001 \text{ m}}\right)(2\pi)(200 \text{ rad/s})(0.014 \text{ m})^3(0.01 \text{ m}) \\ & \boxed{T = 3.45 \times 10^{-3}\,\text{N}\cdot\text{m}} \end{aligned}$$

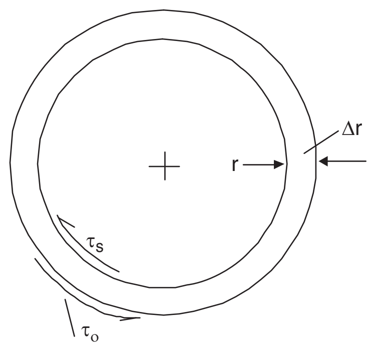
9.16: PROBLEM DEFINITION

Situation:
A shaft turning inside a cylinder is described in the problem statement.

Find:
Show that $d(\tau r)/dr = 0$ and that the torque per unit length acting on the inner cylinder is given by $T = 4\pi\mu\omega r_s^2/(1 - r_s^2/r_o^2)$.

SOLUTION

Subscript s refers to inner cylinder. Subscript o refers to outer cylinder. The cylinder is unit length into page.



$$
\begin{aligned}
T_s &= \tau(2\pi r)(r) \\
T_o &= \tau(2\pi r)(r) + \frac{d}{dr}(\tau 2\pi r \cdot r)\Delta r \\
T_s - T_o &= 0 \\
\frac{d}{dr}(\tau 2\pi r^2)\Delta r &= 0; \frac{d}{dr}(\tau r^2) = 0
\end{aligned}
$$

Since there is no net angular acceleration, the net torque must be zero so

$$
\begin{aligned}
T_s - T_o &= 0 \\
\frac{d}{dr}(\tau r^2) &= 0
\end{aligned}
$$

Then

$$
\begin{aligned}
\tau r^2 &= C_1 \\
\tau &= \mu r \frac{d}{dr}\left(\frac{V}{r}\right)
\end{aligned}
$$

so

$$
\begin{aligned}
\mu r^3 (\frac{d}{dr}\left(\frac{V}{r}\right)) &= C_1 \\
\mu \frac{d}{dr}\left(\frac{V}{r}\right) &= C_1 r^{-3}
\end{aligned}
$$

Integrating,

$$\frac{\mu V}{r} = (-1/2)C_1 r^{-2} + C_2$$

At $r = r_o$, $V = 0$ and at $r = r_s$, $V = r_s\omega$ so

$$\begin{aligned} C_1 &= 2C_2 r_0^2 \\ \mu\omega &= C_2\left(1 - \frac{r_0^2}{r_s^2}\right) \\ C_2 &= \frac{\mu\omega}{1 - r_0^2/r_s^2} \end{aligned}$$

Then

$$\tau_s = C_1 r_s^{-2} = 2C_2\left(\frac{r_0^2}{r_s^2}\right)^2 = \frac{2\mu\omega r_0^2}{r_s^2 - r_0^2} = \frac{2\mu\omega}{(r_s^2/r_0^2) - 1}$$

So

$$T_s = \tau 2\pi r_s^2 = \frac{4\pi\mu\omega r_s^2}{(r_s^2/r_0^2) - 1}$$

which is the torque per unit length on the fluid.

$$T = \frac{4\pi\mu\omega r_s^2}{1 - (r_s^2/r_0^2)}$$

9.17: PROBLEM DEFINITION

Situation:

A 2-cm shaft 3 cm long turning inside a 2.2-cm casing at 60 rad/s with SAE 30 oil as lubricant.

Find:

Power necessary to rotate shaft.

Properties:

Viscosity from Table A.4 is 0.1 N·s/m^2

PLAN

Apply the equation developed in Problem 9.15 (10e).

SOLUTION

$$
\begin{aligned}
T &= \frac{4\pi\mu\omega r_s^2}{1-(r_s^2/r_0^2)} \\
&= \frac{4\pi \times 0.1 \text{ N}\cdot\text{s/m}^2 \times 60 \text{ rad/s} \times (0.01 \text{ m})^2 \times 0.03 \text{ m}}{1-(1 \text{ cm}/1.1 \text{ cm})^2} \\
&= 0.00130 \text{ N}\cdot\text{m} \\
P &= T\omega \\
&= (0.00130 \text{ N}\cdot\text{m})\,(60\,\text{s}^{-1}) \\
& \boxed{P = 0.00780 \text{ W}}
\end{aligned}
$$

9.18: PROBLEM DEFINITION

Situation:

A viscosity measuring device is consists of measuring torque on bearing. There is a 10-cm cylinder with 4-cm shaft and 4.5-cm bearing. A force of 0.6 N force on inner cylinder rotates system at 20 rpm.

Find:

Viscosity of fluid.

PLAN

Apply the equation developed in Problem 9.16 (EFM 10e).

SOLUTION

$$
\begin{aligned}
T &= 0.6(0.02) = 0.012 \text{ N} \cdot \text{m} \\
\omega &= 20 \text{ rev/min=}20 \times 2\pi \text{ rad/rev/60 s/min} = 2.09 \text{ rad/s} \\
\mu &= \frac{T(1 - r_s^2/r_0^2)}{4\pi\omega\ell r_s^2} \\
&= \frac{0.012 \text{ N} \cdot \text{m} \times \left[1 - (2 \text{ cm})^2 / (2.25 \text{ cm})^2\right]}{4\pi \times 2.09 \text{ rad/s} \times 0.1 \text{ m} \times (0.02 \text{ m})^2} \\
& \boxed{\mu = 2.40 \text{ N·s/m}^2}
\end{aligned}
$$

9.19: PROBLEM DEFINITION

Situation:

Flow occurs between two plates separated by 0.0045 m has a pressure gradient of −3650 Pa/m.

Find:

Maximum fluid velocity in x−direction.

Properties:

Viscosity is 0.05 N-s/m^2

PLAN

Use Eq. (9.7a, in 10e) with no change in elevation

SOLUTION

$$
\begin{aligned}
u_{\max} &= -\left(\frac{B^2}{8\mu}\right)\left(\frac{dp}{ds}\right) \\
&= -\left[\frac{(0.0045 \text{ m})^2}{8 \times 0.05 \text{ N-s/m}^2}\right] \times 3650 \text{ N/m}^3
\end{aligned}
$$

$$
\boxed{u_{\max} = 0.18 \text{ m/s}}
$$

9.20: PROBLEM DEFINITION

Situation:

Flow occurs between two plates with pressure and elevation given at two points.

Find:

Direction of flow.

PLAN

Flow will move in direction of decreasing piezometric head. Evaluate piezometric head at both locations and determine flow direction

SOLUTION

$$\begin{aligned} h_A &= \frac{p_A}{\gamma} + z_A = (7 \text{ kN/m}^2/15.7 \text{ kN/m}^3) + 0 = 0.45 \text{ m} \\ h_B &= \frac{p_B}{\gamma} + z_B = (5 \text{ kN/m}^2/15.7 \text{ kN/m}^3) + 0.3 \text{ m} = 0.6 \text{ m} \\ h_B &> h_A \end{aligned}$$

Therefore flow is from B to A: downward

9.21: PROBLEM DEFINITION

Situation:

Glycerin flows downward between two plates spaced 0.4 cm apart. Both ends open — no pressure gradient.

Find:

Discharge per unit width.

Properties:

Table A.4 (Glycerin) $\mu = 1.41 \text{ N}\cdot\text{s/m}^2, \nu = 1.12 \text{ m}^2\text{/s}$ and $\gamma = 12,300 \text{ N/m}^3$.

Assumptions:

Flow will be laminar.

SOLUTION

Use Eq. (9.8, in 10e)

$$q = -\frac{B^3\gamma}{12\mu}\frac{dh}{ds}$$

$$\begin{aligned}\frac{dh}{ds} &= \frac{d}{ds}\left(\frac{p}{\gamma}+z\right) \\ &= \left(\frac{1}{\gamma}\right)\frac{dp}{ds}+\frac{dz}{ds} \\ &= -1\end{aligned}$$

Then

$$\begin{aligned}q &= -\left(\frac{B^3\gamma}{12\mu}\right)(-1) \\ &= -\left[\frac{(0.004 \text{ m})^3 \times 12,300 \text{ N/m}^3}{12 \times 1.41 \text{ N}\cdot\text{s/m}^2}\right](-1)\end{aligned}$$

$$\boxed{q = 4.65 \times 10^{-5} \text{ m}^2\text{/s}}$$

Now check to see if the flow is laminar (Reynolds number $< 1,000$)

$$\begin{aligned}\text{Re} &= \frac{VB}{v} = \frac{q}{v} \\ &= \frac{4.65 \times 10^{-5} \text{ m}^2\text{/s}}{1.12 \times 10^{-3} \text{ m}^2\text{/s}} \\ \text{Re} &= 0.0415 \leftarrow \text{Laminar}\end{aligned}$$

Therefore, the original assumption of laminar flow was correct.

9.22: PROBLEM DEFINITION

Situation:

Flow occurs between two plates separated by 0.003 m and the pressure decreases at rate of 9400 Pa/m in vertical direction.

Find:

Maximum fluid velocity in z-direction.

Properties:

$\mu = 0.05$ Pa-s and $S = 0.8$.

SOLUTION

Use Eq. (9.7a)

$$u_{\max} = -\frac{B^2}{8\mu}\frac{d}{ds}(p + \gamma z)$$

where

$$\begin{aligned} \frac{d}{ds}(p + \gamma z) &= \frac{dp}{ds} + \gamma \\ &= -9400 \text{ N/m}^3 + 0.8 \times 9.8 \text{ kN/m}^3 \\ &= -1560 \text{ N/m}^3 \end{aligned}$$

Then

$$u_{\max} = -\frac{(0.003 \text{ m})^2}{8 \times 0.05 \text{ N-s/m}^2} \times (-1560 \text{ N/m}^3)$$

$$\boxed{u_{\max} = +0.035 \text{ m/s}}$$

The flow is upward.

9.23: PROBLEM DEFINITION

Situation:

Flow of SAE 30 at 38 °C occurs between two plates spaced 0.002 m apart and inclined at 60° with a rate of 0.00025 m^3/s per meter. Flow is downward.

Find:

Pressure gradient in the direction of flow.

Properties:

From Table A.4 $\mu = 0.1$ N-s/m^2; $\gamma = 8630$ N/m^3.

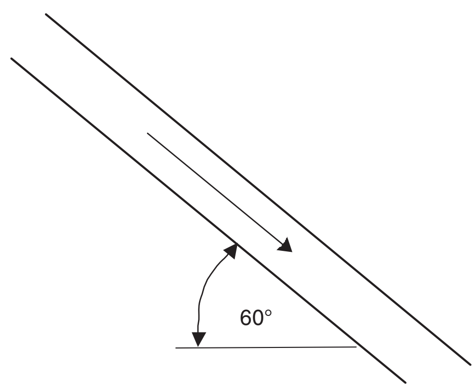
Assumptions:

Assumption: unit width

SOLUTION

Flow rate and maximum velocity

$$
\begin{aligned}
\bar{V} &= \frac{q}{B} \\
&= \frac{0.00025 \text{ m}^2/\text{s}}{0.002 \text{ m} \times 0.3 \text{ m}} \\
&= 0.42 \text{ m/s} \\
u_{\max} &= (3/2)\bar{V} = 0.63 \text{ m/s}
\end{aligned}
$$



$$
\begin{aligned}
u_{\max} &= -\frac{B^2\gamma}{8\mu}\frac{dh}{ds} \\
\frac{dh}{ds} &= -\left(\frac{8\mu u_{\max}}{\gamma B^2}\right) \\
&= -\left(\frac{8 \times \left(0.1 \text{ N-s/m}^2\right) \times 0.63 \text{ m/s}}{8630 \text{ N/m}^3 \times (0.002 \text{ m})^2}\right) \\
&= -14.6
\end{aligned}
$$

But

$$\frac{dh}{ds} = \left(\frac{1}{\gamma}\right)\frac{dp}{ds} + \frac{dz}{ds}$$

where $dz/ds = -0.866$. Then

$$\begin{aligned} -14.6 &= \left(\frac{1}{\gamma}\right)\frac{dp}{ds} - 0.866 \\ \frac{dp}{ds} &= \gamma(-14.6 + 0.866) \\ &= 8630(-14.6 + 0.866) \text{ N/m}^2 \\ &= -118,524 \text{ N/m}^3 \end{aligned}$$

$$\boxed{\frac{dp}{ds} = -119 \text{ kPa/m}}$$

9.24: PROBLEM DEFINITION

Situation:

Glycerin at 20 °C flows downward between two cylinders with outside diameter of 3 cm and inside diameter of 2.8 cm. Pressure is constant.

Find:

Discharge.

Properties:

Table A.4 (Glycerin) $\mu = 1.41$ N·s/m^2 and $\nu = 1.12 \times 10^{-3}\,\mathrm{m}^2/\,\mathrm{s}$.

SOLUTION

Treat flow between cylinders as flow between flat plates. Discharge per unit width between two stationary plates is

$$q = -\left(\frac{B^3\gamma}{12\mu}\right)\left(\frac{dh}{ds}\right)$$

Multiple this by the average width of the channel $\left(\pi\overline{D}\right)$ to give

$$Q = -\left(\frac{B^3\gamma}{12\mu}\right)\left(\frac{dh}{ds}\right)\pi\overline{D}$$

The change in piezometric head (h) with position (s) is given by

$$\begin{aligned} \frac{dh}{ds} &= \frac{d(\frac{P}{\gamma}+z)}{ds} \\ &= \frac{dz}{ds} \\ &= -1 \end{aligned}$$

Combining equations gives

$$\begin{aligned} Q &= \left(\frac{B^3\gamma}{12\mu}\right)\pi\overline{D} \\ &= \left(\frac{(0.001^3\,\mathrm{m}^3)\,(12,300\,\mathrm{N}/\,\mathrm{m}^3)}{12\times(1.41\,\mathrm{N}\cdot\,\mathrm{s}/\,\mathrm{m}^2)}\right)\times\pi\times(0.029\,\mathrm{m}) \\ &= 6.62\times10^{-8}\,\mathrm{m}^3/\,\mathrm{s} \end{aligned}$$

$$\boxed{Q = 6.62\times10^{-8}\,\mathrm{m}^3/\,\mathrm{s}}$$

9.25: PROBLEM DEFINITION

Situation:

A bearing consists of flow discharging from central source outward between two parallel surfaces. Pressure distribution is linear. Bearing is 43 cm wide and clearance is 1.5 mm. The load is 190 kN per bearing length.

Find:

Amount of oil pumped per hour per meter of bearing length.

Properties:

$\mu = 0.20$ N·s/m^2

SOLUTION

Force is average pressure times the area

$$\begin{aligned} F &= p_{\text{avg.}} \times A \\ &= 1/2\, p_{\max} \times A \\ &= 1/2\, p_{\max} \times 0.43 \text{ m} \times 1 \text{ m} \end{aligned}$$

or

$$\begin{aligned} p_{\max} &= \frac{2F}{0.43\,\text{m}^2} = \frac{2 \times 190,000}{0.43} \\ &= 8.84 \times 10^5 \text{ N/m}^2 \end{aligned}$$

Then $dp/ds = -8.84 \times 10^5$ N/m^2/0.215 m $= -4.11 \times 10^6$ N/m^3. For flow between walls with no elevation change

$$\begin{aligned} q_{\text{per side}} &= -(1/12)\left(\frac{B^3}{\mu}\right)\frac{dp}{ds} \\ q_{\text{total}} &= -(1/6)\left(\frac{B^3}{\mu}\right)\frac{dp}{ds} \\ &= -\frac{1}{6}\frac{(15 \times 10^{-4}\text{ m})^3}{0.2 \text{ N}\cdot\text{ s/m}^2}\left(-4.11 \times 10^6 \text{N/m}^3\right) \\ &= 1.16 \times 10^{-2} \text{ m}^3/\text{s} \end{aligned}$$

$$\boxed{q = 41.6 \text{ m}^3/\text{hr}}$$

9.26: PROBLEM DEFINITION

Situation:

Couette flow of liquid with temperature and viscosity distribution. The viscosity varies as

$$\mu = \mu_o \exp\left(-0.1\frac{y}{L}\right)$$

Find:

Find shear stress in the form $\tau = C(U\mu_o/L)$

SOLUTION

In a Couette flow the shear stress is constant between plates so

$$\tau = \mu\frac{du}{dy} = \mu_o \exp\left(-0.1\frac{y}{L}\right)\frac{du}{dy} = const$$

Separating variables

$$\exp\left(0.1\frac{y}{L}\right) d\left(\frac{y}{L}\right) = \frac{\mu_o}{\tau L}du$$

Integrating

$$\begin{aligned}
\int_0^1 \exp\left(0.1\frac{y}{L}\right) d\left(\frac{y}{L}\right) &= \frac{\mu_o}{\tau L}\int_0^U du \\
10\left[\exp(0.1) - 1\right] &= \frac{\mu_o U}{\tau L} \\
1.052 &= \frac{\mu_o U}{\tau L}
\end{aligned}$$

$$\boxed{\tau = 0.951\frac{\mu_o U}{L}}$$

9.27: PROBLEM DEFINITION

Situation:

Gas in a Couette flow with viscosity distribution between plates as

$$\mu = \mu_o \left(1 + 0.1\frac{y}{L}\right)^{1/2}$$

Find:

The shear stress in the form $\tau = C(\mu_o U/L)$

SOLUTION

The shear stress is constant across a Couette flow so

$$\tau = \mu\frac{du}{dy} = \mu_o \left(1 + 0.1\frac{y}{L}\right)^{1/2} \frac{du}{dy} = const$$

Separating variables

$$\left(1 + 0.1\frac{y}{L}\right)^{-1/2} d\left(\frac{y}{L}\right) = \frac{\mu_o}{\tau L}du$$

Integrating

$$\begin{aligned}
\int_0^1 \left(1 + 0.1\frac{y}{L}\right)^{-1/2} d\left(\frac{y}{L}\right) &= \frac{\mu_o}{\tau L}\int_0^U du \\
20\left(1.1^{1/2} - 1\right) &= \frac{\mu_o U}{\tau L} \\
0.976 &= \frac{\mu_o U}{\tau L}
\end{aligned}$$

$$\boxed{\tau = 1.024\frac{\mu_o U}{L}}$$

9.28: PROBLEM DEFINITION

Situation:

A channel 2 cm wide, 5 cm long and 0.2 mm spacing is used for electronic cooling. Average velocity is 5 cm/s.

Find:

Pressure drop in channel and power requirements for operation.

Properties:

$\mu = 1.2$ cp (1.2×10^{-3} N·s/m^2) and $\rho = 800$ kg/m^3.

SOLUTION

The relationship for average velocity and pressure drop is

$$\begin{aligned} V &= -\left(\frac{B^2}{12\mu}\right)\frac{dp}{ds} \\ \frac{dp}{ds} &= -\left(\frac{12\mu V}{B^2}\right) \\ &= -\left(\frac{12\times1.2\times10^{-3}\text{ N}\cdot\text{s/m}^2\times0.05\text{ m/s}}{(0.0002\text{ m})^2}\right) \\ \frac{dp}{ds} &= -18000\text{ Pa/m} \end{aligned}$$

$$\boxed{\frac{dp}{ds} = -1.80\times10^4\text{ Pa/m}}$$

Power

$$\begin{aligned} P &= Q\Delta p \\ &= 0.05\text{ m/s}\times0.0002\text{ m}\times0.02\text{ m}\times1.8\times10^4\text{ N/m}^3\times0.05\text{ m} \\ &= 1.8\times10^{-4}\text{ N}\cdot\text{m/s} \end{aligned}$$

$$\boxed{P = 1.8\times10^{-4}\text{ W}}$$

9.29: PROBLEM DEFINITION

Situation:

Viscosity changes with distance in heated channel according to

$$\mu = \mu_{\rm o} \exp(-0.1\frac{s}{L})$$

Find:

Percentage change in pressure drop due to viscosity variation.

SOLUTION

Discharge is constant through the channel so

$$\begin{aligned} q &= -\left(\frac{B^3}{12\mu}\right)\frac{dp}{ds} \\ \frac{12q}{B^3}\mu &= -\frac{dp}{ds} \\ \frac{12qL}{B^3}\mu_{\rm o}\exp(-0.1\frac{s}{L})d\left(\frac{s}{L}\right) &= -dp \end{aligned}$$

Integrating each side

$$\begin{aligned} \frac{12qL}{B^3}\mu_{\rm o}\int_0^1 \exp(-0.1\eta)d\eta &= -\int_{p(0)}^{p(L)} dp \\ \frac{12qL}{B^3}\mu_{\rm o}10\left[\exp(-0.1)-1\right] &= p(0)-p(L) = \Delta p \\ -0.952\frac{12qL}{B^3}\mu_{\rm o} &= \Delta p \\ q &= -1.051\left(\frac{B^3}{12\mu}\right)\frac{\Delta p}{L} \end{aligned}$$

So

$$\begin{aligned} 1.051\Delta p_{\text{heat}} &= \Delta p_{\text{no heat}} \\ \Delta p_{\text{heat}} &= 0.952\Delta p_{\text{no heat}} \end{aligned}$$

and the pressure drop is reduced by $\boxed{4.84\%}$

9.30: PROBLEM DEFINITION

Find:

a. Definition of boundary layer, in your own words.

b. Definition of boundary layer thickness.

SOLUTION

a. The boundary layer is the thin layer of fluid between a surface and the free stream over which the velocity changes from the wall velocity to the free stream velocity.

b. The boundary layer thickness is the distance at which the velocity in the boundary layer has reached 99% of the difference between the wall and free stream velocities.

9.31: PROBLEM DEFINITION

Situation:

Which of the following are features of a laminar boundary layer? (Select all that are correct.)

a. Flow is smooth.

b. The boundary layer thickness increases in the downstream direction.

c. A decreasing boundary layer thickness correlates with decreased shear stress.

d. An increasing boundary layer thickness correlates with decreased shear stress

SOLUTION

The correct statements are a, b, and d.

9.32: PROBLEM DEFINITION

Situation:

Effect of a heated wall on the thickness and shear stress in laminar boundary layer.

SOLUTION

The reduced viscosity near the wall will reduce the velocity gradient and lead to a thicker boundary layer. The reduced viscosity will lead to a reduced shear stress near the wall.

9.33: PROBLEM DEFINITION

Situation:

A thin plate 2 m long and 1 m wide is held stationary in a 1.5 m/s stream of water at 15.5 °C.

Find:

For $\text{Re}_x = 5 \times 10^5$ Find (a) Thickness of boundary layer, (b) distance from leading edge and (c) shear stress.

Properties:

From Table A.5 $\nu = 1.14 \times 10^{-6}$ m²/s, $\mu = 1.14 \times 10^{-3}$ N-s/m².

PLAN

Calculate the boundary layer thickness and shear stress with the appropriate correlations

SOLUTION

b) Reynolds number

$$\begin{aligned} \text{Re} &= \frac{U_0 x}{\nu} \\ x &= \frac{\text{Re}\,\nu}{U_0} \\ &= \frac{500,000 \times 1.14 \times 10^{-6}\,\text{m}^2/\,\text{s}}{1.5} \\ &\boxed{x = 0.38 \text{ m}} \end{aligned}$$

a) Boundary layer thickness correlation

$$\begin{aligned} \delta &= \frac{5x}{\text{Re}_x^{1/2}} \quad \text{(laminar flow)} \\ &= \frac{5 \times 0.38}{(500,000)^{1/2}} \\ &= 0.00269 \text{ m} \\ &\boxed{\delta = 2.69 \text{ mm}} \end{aligned}$$

c) Local shear stress correlation

$$\begin{aligned} \tau_0 &= 0.332\mu \left(\frac{U_0}{x}\right) \text{Re}_x^{1/2} \\ &= 0.332 \times 1.14 \times 10^{-3} \text{ N-s/m}^2 (1.5 \text{ m/s}/0.38 \text{ m}) \times (500,000)^{1/2} \\ &\boxed{\tau_0 = 1.06 \text{ N/m}^2} \end{aligned}$$

9.34: PROBLEM DEFINITION

Situation:

Laminar flow over a smooth, flat plate.

Find:

Ratio of the boundary layer thickness to the distance from leading edge just before transition.

SOLUTION

Transition occurs at $\mathrm{Re}_x = 5 \times 10^5$. Boundary layer thickness

$$\begin{aligned} \frac{\delta}{x} &= \frac{5}{\mathrm{Re}_x^{1/2}} \text{(laminar flow)} \\ &= \frac{5}{(500,000)^{1/2}} \\ &\boxed{\frac{\delta}{x} = 0.0071} \end{aligned}$$

9.35: PROBLEM DEFINITION

Situation:
Flow over a 12.5 cm chord, 1 m span model airplane wing flying in air at 15 °C.

Find:
(a) Speed at which turbulent boundary layer appears.
(b) Total drag at this speed.

Properties:
From Table A.3 $\nu = 1.79 \times 10^{-5}$ m^2/s, $\rho = 1.23$ kg/m^3.

SOLUTION

Reynolds number for transition, $\mathrm{Re}_x = 5 \times 10^5$.

$$\begin{aligned} \mathrm{Re}_{\mathrm{turb}} &= 5 \times 10^5 \\ &= \frac{Uc}{\nu} \\ U &= \frac{(5 \times 10^5)\nu}{c} \\ &= \frac{(5 \times 10^5)(1.79 \times 10^{-5}\ \mathrm{m^2/s})}{0.125\ \mathrm{m}} \\ &= 71.6\,\mathrm{m/\,s} \\ &\boxed{U = 72\ \mathrm{m/s}} \end{aligned}$$

Average shear stress coefficient

$$\begin{aligned} C_f &= \frac{1.33}{(5 \times 10^5)^{0.5}} \\ &= 0.00188 \end{aligned}$$

Surface resistance (drag force) for two surfaces (top and bottom)

$$\begin{aligned} F_s &= C_f \left(\frac{\rho U^2}{2} \right) A \\ &= (0.00188) \left(\frac{(1.23\ \mathrm{kg/m^3})(72\ \mathrm{m/s})^2}{2} \right) (2)(1\ \mathrm{m})\,(0.125\,\mathrm{m}) \\ &\boxed{F_s = 1.44\ \mathrm{N}} \end{aligned}$$

9.36: PROBLEM DEFINITION

Situation:

Oil flows over a smooth flat plate at 4 m/s.

Find:

Ratio of shear stress at edge of boundary layer to shear stress at the plate surface: τ_δ/τ_0

SOLUTION

At the edge of the boundary layer the shear stress, τ_δ, is approximately zero. Therefore, $\tau_\delta/\tau_0 \approx 0$. Choice $\boxed{\text{(a)}}$ is the correct one.

9.37: PROBLEM DEFINITION

Situation:

A liquid flows past a smooth flat plate at $U_0 = 2$ m/s.

Find:

Liquid velocity at a location $x = 1.0$ m downstream from the leading edge and $y =$ 0.8 mm from surface.

Properties:

$\nu = 2 \times 10^{-5} \text{m}^2/\text{s}$, $\mu = 2 \times 10^{-2}$ N·s/m^2, $\rho = 1000$ kg/m^3

PLAN

Calculate Reynolds number and then use Figure 9.6 (EFM10e) for velocity distribution in laminar boundary layer.

SOLUTION

Reynolds number

$$\text{Re}_x = \frac{Vx}{\nu} = \frac{2\,\text{m/s} \times 1\,\text{m}}{2 \times 10^{-5}\,\text{m}^2/\text{s}} = 100,000$$

The boundary layer is laminar as stated in the problem, (a safe assumption because a boundary layer transitions from laminar to turbulent at about $\text{Re}_x = 500,000$, see §9.4).

$$\frac{y\text{Re}_x^{0.5}}{x} = \frac{0.0008\,\text{m}(1 \times 10^5)^{0.5}}{1.0\,\text{m}} = 0.253$$

Use Figure 9.6 (EFM10e) to obtain u/U_0. $u/U_0 \approx 0.08$; $\boxed{u = 0.16 \text{ m/s}}$

9.38: PROBLEM DEFINITION

Situation:
Flow over a thin, flat plate 3 m long and 1 m wide. Same properties as problem 9.37.

Find:
Skin friction drag on one side of plate.

SOLUTION

Reynolds number

$$
\begin{aligned}
\mathrm{Re}_L &= \frac{UL}{\nu} = \frac{1 \text{ m/s} \times 3 \text{ m}}{2 \times 10^{-5} \text{ m}^2/\text{s}} \\
\mathrm{Re}_L &= 1.5 \times 10^5 \\
C_f &= \frac{1.33}{\mathrm{Re}_L^{0.5}} \\
&= 0.00343
\end{aligned}
$$

Surface resistance (drag force)

$$
\begin{aligned}
F_x &= C_f B L \frac{\rho U^2}{2} \\
&= .00343 \times 1\,\mathrm{m} \times 3\,\mathrm{m} \times \frac{1,000\,\mathrm{kg/\,m^3} \times (1\,\mathrm{m/\,s})^2}{2} \\
& \boxed{F_x = 5.15 \text{ N}}
\end{aligned}
$$

9.39: PROBLEM DEFINITION

Situation:
Oil flows over a smooth, flat plate at 5 m/s.

Find:
Velocity 1 m downstream and 3 mm from plate.

Properties:
$\nu = 10^{-4}$ m^2/s

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re}_x &= \frac{Ux}{\nu} \\ &= \frac{5 \text{ m/s} \times 1 \text{ m}}{10^{-4} \text{ m}^2/\text{s}} \\ &= 5 \times 10^4 \end{aligned}$$

Since $\text{Re}_x \leq 500,000$, the boundary layer is laminar.

Use laminar velocity profile to obtain u/U_0

$$\begin{aligned} \frac{y\text{Re}_x^{0.5}}{x} &= \frac{(0.003 \text{ m})(5 \times 10^4)^{0.5}}{1 \text{ m}} \\ &= 0.671 \end{aligned}$$

From Fig. 9.6 (in EFM 10e) $u/U_0 = 0.23$. Therefore

$$\begin{aligned} u &= 5 \text{ m/s} \times 0.23 \\ &\boxed{u = 1.15 \text{ m/s}} \end{aligned}$$

9.40: PROBLEM DEFINITION

Situation:
Oil flows over a flat plate at 0.85 m/s.

Find:
Oil velocity 1.6 m from leading edge and 10 cm from surface.

Properties:
$\nu = 10^{-4}$ m^2/s

PLAN

Calculate Reynolds number and apply Figure 9.6 (EFM10e) for velocity distribution in laminar boundary layer.

SOLUTION

Reynolds number

$$\text{Re}_x = \frac{0.85 \text{ m/s} \times 1.6 \text{ m}}{10^{-4} \text{ m}^2/\text{s}} = 1.36 \times 10^3$$

The boundary layer is laminar.

$$\frac{y\text{Re}_x^{0.5}}{x} = \frac{0.10\,\text{m} \times (1.36 \times 10^3)^{0.5}}{0.85\,\text{m}} = 4.34$$

Use Figure 9.6 (EFM10e) to obtain u/U_0. $u/U_0 \approx 1.0$ so $\boxed{u = U_0 = 0.85 \text{ m/s.}}$

9.41: PROBLEM DEFINITION

Situation:

Water at 10 °C flows over a submerged flat plate 0.7 m long and 1.5 m wide at 1.5 m/s.

Find:

(a) Thickness of boundary layer at the location where $Re_x = 500,000$.
(b) Distance from leading edge.where the Reynolds number reaches 500,000.
(c) Local shear stress at the location where $Re_x = 500,000$.

Properties:

Table A.5 (water at 10 °C): $\rho = 1000\,\mathrm{kg/m^3}$.
$\mu = 1.31 \times 10^{-3}\,\mathrm{N \cdot s/m^2}$, $\nu = 1.31 \times 10^{-6}\,\mathrm{m^2/s}$.

PLAN

Calculate Reynolds number. Next calculate boundary layer thickness and local shear stress.

SOLUTION

Reynolds number

$$\begin{aligned} Re_x &= 500,000 \\ 500,000 &= \frac{U_0 x}{\nu} \\ x &= \frac{500,000\nu}{U_0} \\ &= \frac{500000 \times (1.31 \times 10^{-6}\,\mathrm{m^2/s})}{1.5\,\mathrm{m/s}} \\ &= 0.4367\,\mathrm{m} \end{aligned}$$

$$\boxed{\text{b.)} \quad x = 0.437\,\mathrm{m}}$$

Boundary layer thickness correlation

$$\begin{aligned} \delta &= \frac{5x}{\mathrm{Re}_x^{1/2}} \text{ ....Laminar flow} \\ &= \frac{5 \times 0.4367\,\mathrm{m}}{\sqrt{500000}} \\ &= 3.09 \times 10^{-3}\ \mathrm{m} \end{aligned}$$

$$\boxed{\text{a.)}\ \delta = 3.09\ \mathrm{mm}}$$

Local shear stress correlation

$$\tau_0 = 0.332\mu \left(\frac{U_0}{x}\right) \mathrm{Re}_x^{1/2}$$

$$= 0.332 \times 1.31 \times 10^{-3}\ \mathrm{N \cdot s/m^2} \left(\frac{1.5\ \mathrm{m/s}}{0.437\ \mathrm{m}}\right) \times (500,000)^{1/2}$$

$$\boxed{\text{c.)}\ \tau_0 = 1.06\ \mathrm{N/m^2}}$$

9.42: PROBLEM DEFINITION

Situation:

Water at 20 °C flows over a flat plate 1.5 m long and 1.0 m wide at 15 cm/s.

Find:

(a) Resistance of plate.

(b) Boundary layer thickness at trailing edge.

Properties:

Table A.5 (water at 20 °C): $\rho = 998\,\text{kg/m}^3$.

$\mu = 1.00 \times 10^{-3}\,\text{N}\cdot\text{s/m}^2$, $\nu = 1.00 \times 10^{-6}\,\text{m}^2/$

SOLUTION

Reynolds number

$$\begin{aligned}\text{Re}_L &= \frac{U_0 L}{\nu} \\ &= \frac{0.15\text{ m/s} \times 1.5\text{ m}}{10^{-6}\text{ m}^2/\text{s}} \\ &= 2.25 \times 10^5\end{aligned}$$

$Re_L \leq 500{,}000$; therefore, laminar boundary layer

Boundary layer thickness

$$\begin{aligned}\delta &= \frac{5x}{\text{Re}_x^{1/2}} \\ &= \frac{5 \times 1.5\text{ m}}{(2.25 \times 10^5)^{1/2}} = 1.58 \times 10^{-2}\text{ m} \\ &\boxed{\delta = 15.8\text{ mm}}\end{aligned}$$

Average shear stress coefficient

$$\begin{aligned}C_f &= \frac{1.33}{\text{Re}_L^{1/2}} \\ &= \frac{1.33}{(2.25 \times 10^5)^{1/2}} \\ &= 0.00280\end{aligned}$$

Surface resistance (drag force) - 2 surfaces (top and bottom)

$$\begin{aligned}F_s &= C_f A \frac{\rho U_0^2}{2} \\ &= 0.00280 \times (1.0\text{ m} \times 1.5\text{ m} \times 2) \times \frac{998\,\text{kg/m}^2 \times (0.15\text{ m/s})^2}{2} \\ &\boxed{F_s = 0.0943\text{ N}}\end{aligned}$$

9.43: PROBLEM DEFINITION

Situation:

Transition between laminar and turbulent boundary layer occurs between 10^5 and 3×10^6. The thickness for a turbulent boundary layer is $\delta/x = 0.16/\operatorname{Re}_x^{1/7}$.

Find:

The ratio of the boundary layer thickness at the end to transition to that at the beginning.

SOLUTION

The thickness of the laminar boundary layer is $\delta/x = 5/\operatorname{Re}_x^{1/2}$ .Thus

$$
\begin{aligned}
\frac{\delta_{\text{turb}}}{\delta_{\text{lam}}} &= \frac{0.16 \times x_{\text{turb}}}{(3 \times 10^6)^{1/7}} \frac{\sqrt{10^5}}{5 \times x_{\text{lam}}} \\
&= \frac{x_{\text{turb}}}{x_{\text{lam}}} \times 1.20
\end{aligned}
$$

But

$$\frac{x_{\text{turb}}}{x_{\text{lam}}} = \frac{3 \times 10^6}{10^5} = 30$$

So

$$\frac{\delta_{\text{turb}}}{\delta_{\text{lam}}} = 30 \times 1.2 = 36$$

$$\boxed{\frac{\delta_{\text{turb}}}{\delta_{\text{lam}}} = 36}$$

9.44: PROBLEM DEFINITION

Situation:

Classify the following into one of 2 categories: laminar boundary layer (L), or turbulent boundary layer (T).

a. Flow is smooth
b. Three differently shaped velocity distributions in 3 zones
c. Velocity profile that follows a power law
d. Velocity profile that is a function of $\sqrt{\text{Re}}$
e. Logarithmic velocity distribution
f. Thickness is inversely related to the 7th root of Re
g. Thickness is inversely related to $\sqrt{\text{Re}}$
h. Velocity defect region
i. Mixing action causes locally unsteady velocities
j. Shear stress is a function of a natural log
k. Shear stress is a function of $\sqrt{\text{Re}}$

SOLUTION

a. Flow is smooth **L**
b. Three differently shaped velocity distributions in 3 zones **T**
c. Velocity profile that follows a power law **T**
d. Velocity profile that is a function of $\sqrt{\text{Re}}$ **L**
e. Logarithmic velocity distribution **T**
f. Thickness is inversely related to the 7th root of Re **T**
g. Thickness is inversely related to $\sqrt{\text{Re}}$ **L**
h. Velocity defect region **T**
i. Mixing action causes locally unsteady velocities **T**
j. Shear stress is a function of a natural log **T**
k. Shear stress is a function of $\sqrt{\text{Re}}$ **L**

9.45: PROBLEM DEFINITION

Situation:

The viscosity of a turbulent boundary layer is reduced near the wall.

Find:

The change in the turbulent boundary layer.

SOLUTION

The laminar sublayer will be thinner and the law of the wall will not extend as far.

9.46: PROBLEM DEFINITION

Situation:

Air flows over a flat plate.
U_o= 30 m/s.
Sensing element: 1 cm by 1 cm; situated 1 m from the leading edge.
The boundary layer is tripped.

Find:

Force due to shear stress on the sensing element.

Properties:

Air (properties given in problem statement): $\rho = 1.2\ \text{kg/m}^3, \nu = 1.5 \times 10^{-5}\,\text{m}^2/\text{s}$

Assumptions:

Over the length of the device (1 cm), assume that the local shear stress coefficient (c_f) equals the average shear stress coefficient (C_f).

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re}_x &= \frac{U_o x}{\nu} \\ &= \frac{(30\,\text{m/s}) \times (1\,\text{m})}{(1.5 \times 10^{-5}\,\text{m}^2/\text{s})} \\ &= 2.0 \times 10^6 \end{aligned}$$

Reynolds number less than 10^7 so

$$\begin{aligned} c_f &= \frac{0.027}{\text{Re}_x^{1/7}} \\ &= \frac{0.027}{(2 \times 10^6)^{1/7}} \\ &= 0.003398 \end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned} F_s &= c_f \frac{\rho U_o^2}{2} A \\ &= c_f \frac{\rho U_o^2}{2} A \\ &= 0.003398 \times \frac{(1.2\,\text{kg/m}^3)\,(30\,\text{m/s})^2}{2} (0.01\,\text{m})^2 \\ &= 1.83 \times 10^{-4}\,\text{N} \end{aligned}$$

$$\boxed{F_s = 1.83 \times 10^{-4}\,\text{N}}$$

9.47: PROBLEM DEFINITION

Situation:

Water at 10 °C flows over a submerged flat plate 0.7 m long and 1.5 m wide at 1.5 m/s.

Find:

(a) Shear resistance (drag force) for the portion of the plate that is exposed to laminar boundary layer flow.

(b) Ratio of laminar shearing force to total shearing force.

Properties:

Table A.5 (water at 10 °C): $\rho = 1000\,\mathrm{kg/m^3}, \mu = 1.31 \times 10^{-3}\,\mathrm{N \cdot s/m^2}$, $\nu = 1.31 \times 10^{-6}\,\mathrm{m^2/s}$.

SOLUTION

For the part of the plate exposed to laminar boundary layer flow, the average shear stress coefficient (C_f) is

$$\begin{aligned} C_f &= \frac{1.33}{\sqrt{\mathrm{Re}_L}} \quad \text{(laminar BL flow)} \\ &= \frac{1.33}{\sqrt{500000}} \\ &= 0.00188 \end{aligned}$$

Transition occurs when Reynolds number is 500,000.

$$\begin{aligned} 500000 &= \frac{U_o x_{\text{transition}}}{\nu} \\ 500000 &= \frac{(1.5\,\mathrm{m/s}) \times (x_{\text{transition}})}{1.31 \times 10^{-6}\,\mathrm{m^2/s}} \end{aligned}$$

Solving for the transition location gives

$$x_{\text{transition}} = 0.436\,7\,\mathrm{m}$$

Surface resistance (drag force) for the part of the plate exposed to laminar boundary layer is

$$\begin{aligned} F_s &= C_f \frac{\rho U_o^2}{2} A \\ &= 0.00188 \left(\frac{1000\,\mathrm{kg/m^3} \times (1.5\,\mathrm{m/s})^2}{2} \right) (0.436\,7\,\mathrm{m} \times 1.5\,\mathrm{m}) \\ &= 1.385\,\mathrm{N} \end{aligned}$$

$$\boxed{F_s = 1.385\,\mathrm{N}}$$

Reynolds number for the plate

$$\begin{aligned}
\mathrm{Re}_L &= U_0 \times \frac{L}{\nu} \\
&= \frac{1.5\text{ m/s} \times 0.7\text{ m}}{1.31 \times 10^{-6}\text{ m}^2/\text{s}} \\
&= 8.015 \times 10^5
\end{aligned}$$

Thus, the boundary layer is mixed. The average shear stress coefficient (C_f) is

$$\begin{aligned}
C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \quad \text{(mixed BL flow)} \\
&= \frac{0.523}{\ln^2(0.06 \times 8.015 \times 10^5)} - \frac{1520}{8.015 \times 10^5} \\
&= 0.00260
\end{aligned}$$

Surface resistance (drag force) for the whole plate is

$$\begin{aligned}
F_{s_{\text{total}}} &= C_f \left(\frac{\rho U_0^2}{2} \right) A \\
&= 0.00260 \left(\frac{1000\,\text{kg/m}^3 \times (1.5\,\text{m/s})^2}{2} \right) (0.7\,\text{m} \times 1.5\,\text{m}) \\
&= 3.071\,\text{N}
\end{aligned}$$

The ratio of drag forces is

$$\begin{aligned}
\frac{F_s\,(\text{laminar flow})}{F_s\,(\text{total})} &= \frac{1.385\,\text{N}}{3.071\,\text{N}} \\
&= 0.4510
\end{aligned}$$

$$\boxed{\frac{F_{s_{\text{lam.}}}}{F_{s_{\text{total}}}} = 0.451}$$

9.48: PROBLEM DEFINITION

Situation:

Air at 30 °C flows over an airplane wing with 2 m chord and 11 m span at 200 km/hr.

Find:

(a) Friction drag on wing.
(b) Power to overcome friction drag.
(c) Fraction of chord which is laminar flow.
(d) Change in drag if boundary tripped at leading edge.

Properties:

From Table A.3 $\nu = 1.6 \times 10^{-5}$ m²/s and $\rho = 1.17$ kg/m³.

PLAN

(a) Calculate friction drag.
(b) Find power as the product of drag force and speed: $P = F_s V$
(c) Calculate the critical length at a Reynolds number of $\text{Re} = 5 \times 10^5$.
(d) Compare the average shear stress coefficients for a mixed boundary layer and all-turbulent boundary layer.

SOLUTION

$$\begin{aligned} U_0 &= (200 \text{ km/hr})(1{,}000 \text{ m/km})/(3{,}600 \text{ s/hr}) \\ U_0 &= 55.56 \text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{U_0 L}{\nu} \\ &= \frac{(55.56 \text{ m/s})(2 \text{ m})}{1.6 \times 10^{-5} \text{ m}^2/\text{s}} \\ &= 6.9 \times 10^6 \end{aligned}$$

The flow is mixed laminar and turbulent.

Surface resistance (drag force)

$$\begin{aligned} F_s &= C_f B L \frac{\rho U_o^2}{2} \\ C_f &= \frac{0.523}{\ln^2(0.06\text{Re})} - \frac{1520}{\text{Re}} \\ &= 0.00290 \end{aligned}$$

a) Total resistance. Wing has two surfaces so

$$\begin{aligned} F_{s,\text{wing}} &= 2 \times C_f B L \frac{\rho U_o^2}{2} \\ &= (0.00290)(11 \text{ m})(2 \text{ m})(1.17 \text{ kg/m}^3)(55.56 \text{ m/s})^2 \\ &\boxed{F_{s,\text{wing}} = 230 \text{ N}} \end{aligned}$$

b) Power

$$\begin{aligned} P &= F_{\text{s,wing}} U_0 \\ &= 230\,\text{N} \times 55.56\,\text{m/s} \\ &\boxed{P = 12.8 \ \text{kW}} \end{aligned}$$

c) Critical length. $\text{Re} = 5 \times 10^5 = U_0 x/\nu$

$$\begin{aligned} x_{cr} &= \frac{5 \times 10^5 \nu}{U_0} \\ &= \frac{(5 \times 10^5)(1.6 \times 10^{-5}\,\text{m}^2/\text{s})}{55.56\,\text{m/s}} \\ &\boxed{x_{cr} = 14.4 \ \text{cm}} \end{aligned}$$

d) If all of boundary layer is turbulent then

$$\begin{aligned} C_f &= \frac{0.032}{\text{Re}^{1/7}} \\ C_f &= 0.00337 \end{aligned}$$

Then

$$\begin{aligned} \frac{F_{\text{tripped B.L.}}}{F_{\text{normal}}} &= \frac{0.00337}{0.00290} \\ &= 1.162 \end{aligned}$$

Change in drag with tripped B.L. is $\boxed{16.2\% \text{ increase.}}$

9.49: PROBLEM DEFINITION

Situation:

Water at 20 °C flows over creating a turbulent boundary layer. Local shear stress at point is 0.2 N/m^2.

Find:

Velocity 0.52 cm above plate surface.

Properties:

From Table A.5 $\rho = 998$ kg/m^3; $\nu = 10^{-6}$ m^2/s.

SOLUTION

Local shear velocity and nondimensional wall distance.

$$
\begin{aligned}
u_* &= \left(\frac{\tau_0}{\rho}\right)^{0.5} = \left(\frac{0.2 \text{ N/m}^2}{998 \text{ kg/m}^3}\right)^{0.5} = 0.014 \text{ m/s} \\
\frac{u_* y}{v} &= \frac{(0.014 \text{ m/s})(0.0052 \text{ m})}{10^{-6} \text{ m}^2\text{/s}} = 73.6
\end{aligned}
$$

The point is in the law of the wall region since $11.6 < u_* y/\nu < 500$ so

$$
\begin{aligned}
\frac{u}{u_*} &= 2.44 \ln\left(\frac{y u_*}{v}\right) + 5.56 \\
&= 2.44 \ln(73.6) + 5.56 = 16.0 \\
u &= u_* \times 16.0 = 0.014 \text{ m/s} \times 16.0 \\
&\boxed{u = 0.225 \text{ m/s}}
\end{aligned}
$$

9.50: PROBLEM DEFINITION

Situation:

Liquid flows over a flat plate 2.5 m long with velocity of 16 m/s.

Find:

(a) Skin friction drag per unit width of plate.

(b) Velocity gradient at surface 1 m downstream from leading edge.

Properties:

$\mu = 10^{-5}$ N·s/m^2, $\rho = 1.5$ kg/m^3.

SOLUTION

Calculate Reynolds number

$$\begin{aligned}\mathrm{Re}_L &= \frac{U_0 L \rho}{\mu} \\ &= \frac{16\text{ m/s} \times 2.5\text{ m} \times 1.5\text{ kg/m}^3}{10^{-5}\text{N}\cdot\text{s/m}^2} \\ &= 6 \times 10^6\end{aligned}$$

Average shear stress coefficient

$$\begin{aligned}C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re})} - \frac{1520}{\mathrm{Re}} \\ &= 0.00294\end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned}\frac{F_s}{B} &= C_f(2BL)\frac{\rho U_0^2}{2};\text{ where } B = \text{ unit width } = 1 \\ &= 0.00294 \times (2\,\text{m} \times 2.5\text{ m}) \times \frac{1.5\text{ kg/m}^3 \times (16\text{ m/s})^2}{2} \\ & \boxed{F_s = 2.82\text{ N/m}}\end{aligned}$$

Reynolds number

$$\begin{aligned}\mathrm{Re}_{1m} &= 6 \times 10^6 \times (1/2.5) \\ &= 2.4 \times 10^6\end{aligned}$$

Local shear stress coefficient

$$\begin{aligned}c_f &= \frac{0.455}{\ln^2(0.06\mathrm{Re}_{1m})} \\ &= \frac{0.455}{\ln^2(0.06 \times 2.4 \times 10^6)} \\ &= 0.00323\end{aligned}$$

Local shear stress

$$\begin{aligned}\tau_0 &= c_f \frac{\rho U_0^2}{2} \\ &= 0.00323 \times \frac{1.5\,\mathrm{kg/\,m^3} \times (16\,\mathrm{m/\,s})^2}{2} \\ &= 0.619\ \mathrm{N/m^2}\end{aligned}$$

$$\tau_0 = \mu \frac{du}{dy}$$

or

$$\begin{aligned}\frac{du}{dy} &= \frac{\tau_0}{\mu} \\ &= \frac{0.619\,\mathrm{N/\,m^2}}{10^{-5}\,\mathrm{N\,s/\,m^2}} \\ &\boxed{\frac{du}{dy} = 6.19 \times 10^4\ \mathrm{s}^{-1}}\end{aligned}$$

9.51: PROBLEM DEFINITION

Situation:
Flow over a flat plate with linear velocity profile at trailing edge.

Find:
Skin friction drag on top per unit width stress on plate at downstream end.

Properties:
$\mu = 1.8 \times 10^{-5}$ N·s/m^2

PLAN

Relate velocity profile and shear stress at trailing edge.

SOLUTION

Local shear stress

$$\begin{aligned} \tau_0 &= \mu \frac{dV}{dy} = \mu \frac{V}{\Delta y} \\ &= \frac{1.8 \times 10^{-5}\ \mathrm{N \cdot s/m^2} \times 40\ \mathrm{m/s}}{3 \times 10^{-3}\ \mathrm{m}} \\ & \boxed{\tau_0 = 0.24\ \mathrm{N/m^2}} \end{aligned}$$

9.52: PROBLEM DEFINITION

Situation:

The velocity profile in a boundary layer is replaced by a step profile.

Find:

Derive an equation for displacement thickness.

SOLUTION

$$\dot{m} = \int_0^\delta \rho u dy = \int_{\delta^*}^\delta \rho_\infty U_\infty dy = \rho_\infty U_\infty (\delta - \delta^*)$$

$$\rho_\infty U_\infty \delta^* = \rho_\infty U_\infty \delta - \int_0^\delta \rho u dy$$

$$= \rho_\infty U_\infty \int_0^\delta \left(1 - \frac{\rho u}{\rho_\infty U_\infty}\right) dy$$

$$\therefore \delta^* = \int_0^\delta \left(1 - \frac{\rho u}{\rho_\infty U_\infty}\right) dy$$

9.53: PROBLEM DEFINITION

Situation:

Displacement thickness for a linear velocity profile.

Find:

Magnitude of displacement thickness.

SOLUTION

The equation for displacement thickness is

$$\delta^* = \int_0^\delta \left(1 - \frac{\rho u}{\rho_\infty U_\infty}\right) dy$$

For constant density

$$\delta^* = \int_0^\delta (1 - \frac{u}{U})dy$$

For linear velocity profile, $u = (y/\delta)U$

$$\begin{aligned} \delta^* &= \delta \int_0^1 (1 - \frac{y}{\delta})d\left(\frac{y}{\delta}\right) = \delta \int_0^1 (1-\eta)d\eta \\ &= \delta \left[\eta - \frac{\eta^2}{2}\right]_0^1 = \frac{\delta}{2} \end{aligned}$$

Displacement thickness

$$\delta^* = \frac{3 \text{ mm}}{2}$$

$$\boxed{\delta^* = 1.5 \text{ mm}}$$

9.54: PROBLEM DEFINITION

Situation:

Boundary layer profile is $u/U_o = (y/\delta)^{1/7}$.

Find:

The ratio δ_*/δ.

PLAN

Use the equation for displacement thickness.

SOLUTION

Equation for displacement thickness with constant density

$$\begin{aligned}
\delta^* &= \int_0^\delta (1-\frac{u}{U})dy \\
&= \delta \int_0^\delta (1-\frac{u}{U})dy = \delta \int_0^1 \left(1-\left(\frac{y}{\delta}\right)^{1/7}\right) d\left(\frac{y}{\delta}\right) \\
\frac{\delta^*}{\delta} &= \int_0^1 (1-\eta^{1/7})d\eta = \left[\eta - \frac{7}{8}\eta^{8/7}\right]_0^1
\end{aligned}$$

$$\boxed{\frac{\delta^*}{\delta} = \frac{1}{8}}$$

9.55: PROBLEM DEFINITION

Situation:

Two flat plates, one 30 m long and 5 m wide and the other 10 m long and 5 m wide, are towed through water at 10 m/s.

Find:

Ratio of skin friction drag on two plates.

Properties:

From Table A.5 $\nu = 10^{-6}$ m^2/s

SOLUTION

Surface resistance (drag force)

$$F_s = C_f BL \frac{\rho U_0^2}{2}$$

where

$$C_f = \frac{0.523}{\ln^2(0.06 \times \text{Re}_L)} - \frac{1520}{\text{Re}_L}$$

Reynolds number

$$\begin{aligned} \text{Re}_{L,30} &= 30 \text{ m} \times 10 \text{ m/s}/10^{-6} \text{ m}^2\text{/s} = 3 \times 10^8 \\ \text{Re}_{L,10} &= 10^8 \end{aligned}$$

Then

$$\begin{aligned} C_{f,30} &= 0.00187 \\ C_{f,10} &= 0.00213 \end{aligned}$$

Then

$$\frac{F_{s,30}}{F_{s,10}} = \left(\frac{0.00187}{0.00213}\right) \times 3$$

$$\boxed{\frac{F_{s,30}}{F_{s,10}} = 2.63}$$

9.56: PROBLEM DEFINITION

Situation:

A sign 30 m long and 2 m wide is being pulled through air at 40 m/s.

Find:

Power required to pull sign.

Properties:

From Table A.3 $\nu = 1.41 \times 10^{-5}$ m^2/s and $\rho = 1.25$ kg/m^3.

PLAN

Find the average shear stress coefficient (C_f) and then calculate the surface resistance (drag force). Find power using the product of speed and drag force ($P = F_s V$).

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re}_L &= \frac{V_0 L}{\nu} \\ &= \frac{40 \text{ m/s} \times 30 \text{ m}}{1.41 \times 10^{-5} \text{ m}^2/\text{s}} \\ \text{Re}_L &= 8.51 \times 10^7 \end{aligned}$$

Average shear stress coefficient

$$\begin{aligned} C_f &= \frac{0.523}{\ln^2(0.06\,\text{Re}_L)} - \frac{1520}{\text{Re}_L} \quad \text{(turbulent flow)} \\ &= \frac{0.523}{\ln^2(0.06 \times 8.51 \times 10^7)} - \frac{1520}{8.51 \times 10^7} \\ &= 0.00217 \end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned} A &= (2\,\text{m})(30\,\text{m})(2\text{ sides}) = 120\,\text{m}^2 \\ F_s &= C_f A \frac{\rho U_0^2}{2} \\ F_s &= 0.00217\left(120\,\text{m}^2\right)\frac{(1.25\,\text{kg/m}^3)(40\,\text{m/s})^2}{2} \\ &= 260.9\,\text{N} \end{aligned}$$

Power equation

$$\begin{aligned} P &= F_s V = (260.9\,\text{N})(40\,\text{m/s}) \\ & \boxed{P = 10.4 \text{ kW}} \end{aligned}$$

9.57: PROBLEM DEFINITION

Situation:

A plastic panel 3 mm thick and weighing 300 N is being lowered in the ocean at 3 m/s.

Find:

Tension in cable.

Properties:

From Table A.4 $\nu = 1.4 \times 10^{-6}$ m^2/s and $\gamma = 10,070$ N/m^3, .$\rho = 1026$ kg/m^3

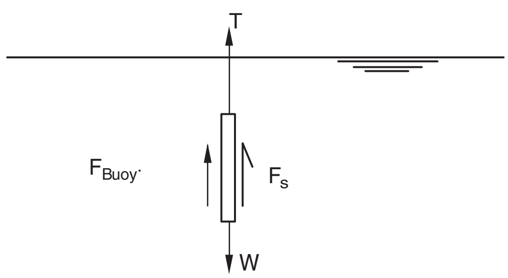
PLAN

Apply equilibrium to the panel. Apply the surface resistance equation and the buoyancy force equation to calculate the unknown forces.

SOLUTION

Equilibrium

$$\sum F_z = 0$$
$$T + F_s + F_{\text{Buoy.}} - W = 0$$
$$T = W - F_s - F_{\text{Buoy.}} \tag{1}$$



Buoyancy force

$$\begin{aligned} F_{\text{Buoy.}} &= \gamma_{\text{water}} \forall \\ &= 0.003 \text{ m} \times 3 \text{ m} \times 1\text{m} \times 10,070 \text{ N/m}^3 \\ &= 90.63 \text{ N} \end{aligned}$$

Surface resistance (drag force)

$$F_s = C_f A \frac{\rho U_0^2}{2}$$

Reynolds number

$$\begin{aligned} R_{e_L} &= \frac{VL}{\nu} \\ &= \frac{3 \text{ m/s} \times 1 \text{ m}}{1.4 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= 2.143 \times 10^6 \end{aligned}$$

Equation for average shear stress, Eq. (9.38, in 10e)

$$\begin{aligned}
C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \\
&= \frac{0.523}{\ln^2(0.06 \times 2.143 \times 10^6)} - \frac{1520}{2.143 \times 10^6} \\
&= 0.00307
\end{aligned}$$

So for both sides

$$\begin{aligned}
F_s &= 0.00307 \times 2 \times 3\text{m} \times 1\text{ m} \times \frac{1026\text{ kg/m}^3 \times (3\text{ m/s})^2}{2} \\
&= 85.05\text{ N}
\end{aligned}$$

Eq. (1) gives

$$\begin{aligned}
T &= 300 - 85.05 - 90.63 \\
& \boxed{T = 124\text{ N}}
\end{aligned}$$

9.58: PROBLEM DEFINITION

Situation:

A 1 m by 2 m plate with a volume of 0.002 m^3 is falling though water.

Find:

Falling speed in fresh water.

Properties:

From Table A.5 $\nu = 10^{-6}$ m^2/s and $\rho = 998$ kg/m^3

PLAN

Apply equilibrium with the weight, buoyancy and drag force.

SOLUTION Equilibrium

$$W - B = F_s$$

$$\begin{aligned} W - \gamma_{\text{water}} \forall &= \frac{1}{2} C_f A_T \rho U_0^2 \\ 23.5 \text{ N} - 998 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 0.002 \text{ m}^3 &= \frac{1}{2} \times 998 \text{ kg/m}^3 \times 2 \times 2 \text{ m}^2 \times C_f \times U_0^2 \end{aligned}$$

or

$$U_0^2 = \frac{0.001963}{C_f}$$

Using the equation for the average resistance coefficient, Eq. (9.38, in 10e)

$$C_f = \frac{0.523}{\ln^2 (0.06 \, \text{Re}_L)} - \frac{1520}{\text{Re}_L}$$

and iterate. As a first guess assume $C_f = 0.002$, then $U_0 = 0.99$ m/s. Then calculate the Reynolds number and a new C_f.

$$\begin{aligned} \text{Re} &= \frac{UL}{\nu} = \frac{0.99 \text{ m/s} \times 2\text{m}}{10^{-6} \text{ m}^2/\text{s}} = 1.98 \times 10^6 \\ C_f &= \frac{0.523}{\ln^2 (0.06 \times 1.98 \times 10^6)} - \frac{1520}{1.98 \times 10^6} = .00306 \end{aligned}$$

The new velocity estimate is $U_0 = 0.801$ m/s. One further iteration is sufficient and gives

$$\boxed{U_0 = 0.805 \text{ m/s}}$$

9.59: PROBLEM DEFINITION

Situation:

Turbulent boundary on flat plate in 7.7 m/s flow of water at 20 °C.

Find:

Thickness of viscous sublayer 7.8 m downstream from leading edge.

Properties:

From Table A.5 $\nu = 10^{-6}$ m^2/s.

SOLUTION

Thickness of viscous sublayer

$$\delta' = \frac{5\nu}{u_*}$$

where $u_* = (\tau_0/\rho)^{0.5}$.

Local shear stress

$$\begin{aligned}\tau_0 &= c_f \frac{\rho U_0^2}{2} \\ \frac{\tau_0}{\rho} &= \left[\frac{0.455}{\ln^2(0.06\mathrm{Re}_x)}\right] \frac{U_0^2}{2}\end{aligned}$$

Reynolds number

$$\begin{aligned}\mathrm{Re}_x &= \frac{U_0 x}{\nu} \\ &= \frac{(7.7 \text{ m/s})(7.8 \text{ m})}{10^{-6} \text{ m}^2/\text{s}} \\ &= 6 \times 10^7\end{aligned}$$

Then

$$\begin{aligned}\frac{\tau_0}{\rho} &= \left[\frac{0.455}{\ln^2(0.06\mathrm{Re}_x)}\right]\left(\frac{(7.7\,\text{m/s})^2}{2}\right) \\ \frac{\tau_0}{\rho} &= 0.0592 \text{ m}^2/\text{s}^2 \\ u_* &= \left(\frac{\tau_0}{\rho}\right)^{0.5} = 0.243 \text{ m/s}\end{aligned}$$

Finally

$$\delta' = \frac{5\nu}{u_*} = \frac{(5)(10^{-6} \text{ m}^2/\text{s})}{(0.243\,\text{m/s})}$$

$$\boxed{\delta' = 20.6 \times 10^{-6} \text{ m}}$$

9.60: PROBLEM DEFINITION

Situation:

A model airplane with 1 m span and 10 cm chord and weighing 3 N dives vertically though air at 20 °C.

Find:

Falling speed.

Properties:

From Table A.3 $\rho = 1.2$ kg/m^3; $\nu = 1.51 \times 10^{-5}$ m^2/s.

PLAN

Determine the drag force (surface resistance) and apply equilibrium.

SOLUTION

Surface resistance (drag force)

$$\begin{aligned} F_s &= C_f \rho \frac{U_0^2}{2} A \\ C_f &= \frac{0.032}{\text{Re}^{1/7}} \end{aligned}$$

Equilibrium

$$\begin{aligned} W &= F_s \\ 3\,\text{N} &= \frac{0.032}{(U_0 \times 0.1 \text{ m}/(1.51 \times 10^{-5} \text{ m}^2/\text{s}))^{1/7}} (1.2 \text{ kg/m}^3) \left(\frac{U_0^2}{2}\right) (2 \times 1 \text{ m} \times 0.1 \text{ m}) \\ 3\,\text{N} &= 5.463 \times 10^{-4} U_0^{13/7} \end{aligned}$$

Solving for U_0 yields $\boxed{U_0 = 103 \text{ m/s.}}$

9.61: PROBLEM DEFINITION

Situation:

A 24 m/s flow in air over a flat plate 3 m long and 0.5 m wide. Boundary layer tripped on one side but not on other.

Find:

Total drag force on plate.

Properties:

From Table A.3 $\rho = 1.2$ kg/m^3; $\nu = 1.51 \times 10^{-5}$ m^2/s.

SOLUTION

The drag force (due to shear stress) is

$$F_s = C_f \frac{1}{2} \rho U_o^2 BL$$

The Reynolds number based on the plate length is

$$\text{Re}_L = \frac{24 \text{ m/s} \times 3 \text{ m}}{1.51 \times 10^{-5} \text{ m}^2/\text{s}} = 4.77 \times 10^6$$

which is less than 10^7.The average shear stress coefficient on the "tripped" side of the plate is

$$C_f = \frac{0.032}{(4.77 \times 10^6)^{1/7}} = 0.00356$$

The average shear stress coefficient on the "untripped" side is

$$C_f = \frac{0.523}{\ln^2(0.06 \times 4.77 \times 10^6)} - \frac{1520}{4.77 \times 10^6} = 0.00299$$

The total force is

$$F_s = \frac{1}{2} \times 1.2 \, \text{kg/m}^3 \times (24 \, \text{m/s})^2 \times 3 \times 0.5 \times (0.00356 + 0.00299)$$

$$\boxed{F_s = 3.40 \text{ N}}$$

9.62: PROBLEM DEFINITION

Situation:

A horizontal plate (part of an engineered system for fish bypass) divides a flow of water at 5 °C into two streams. The plate is 1.8 m long and 1.2 m wide and water velocity is 3 m/s.

Find:

Calculate the viscous drag force on the plate (both sides).

Properties:

From Table A.5. $\nu = 1.51 \times 10^{-6}\,\mathrm{m^2/s}, \rho = 1000\,\mathrm{kg/m^3}$.

PLAN

Find the Reynolds number to establish whether the boundary layer is laminar or mixed. Select the appropriate correlation for average resistance coefficient (C_f). Then, calculate the shear (i.e. drag) force (F_s).

SOLUTION

Reynolds Number.

$$\begin{aligned} \mathrm{Re}_L &= \frac{U_o L}{\nu} \\ &= \frac{(3\,\mathrm{m/s})\,(1.8\,\mathrm{m})}{(1.51 \times 10^{-6}\,\mathrm{m^2/s})} = 3.6 \times 10^6 \end{aligned}$$

Thus, the boundary layer is mixed.

Average shear stress coefficient

$$\begin{aligned} C_f &= \frac{0.523}{\ln^2 (0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \\ &= \frac{0.523}{\ln^2 (0.06 \times 3.6 \times 10^6)} - \frac{1520}{3.6 \times 10^6} = 0.00304 \end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned} F_s &= C_f \frac{\rho V^2}{2} A \\ &= 0.00304 \frac{(1000\,\mathrm{kg/m^3})\,(3\,\mathrm{m/s})^2}{2} (2 \times 1.8\,\mathrm{m} \times 1.2\,\mathrm{m}) \\ &= 59.098\,\mathrm{N} \end{aligned}$$

$$\boxed{F_s = 59.1\,\mathrm{N}}$$

9.63: PROBLEM DEFINITION

Situation:

Entrance region consists of two flat plates 4 mm apart. Water flow at 10 m/s and boundary layer tripped.

Find:

(a) Length where boundary layers merge.

(b) Shearing force per unit depth.

Properties:

From Table A.5 $\rho = 1000$ kg/m^3, $\nu = 1.31 \times 10^{-6}$ m^2/s.

PLAN

Apply the correlation for boundary layer thickness for a tripped leading edge.

SOLUTION

Boundary layer thickness

$$\begin{aligned} \delta &= \frac{0.16x}{\text{Re}_x^{1/7}} \text{ (boundary layer tripped at leading edge)} \\ &= \frac{0.16x^{6/7}}{\left(\frac{U_o}{\nu}\right)^{1/7}} \end{aligned}$$

Setting $\delta = 0.002$ m and $x = L$ gives

$$L^{6/7} = \frac{0.002}{0.16}\left(\frac{10}{1.31 \times 10^{-6}}\right)^{1/7} = 0.12026$$

or

$$\boxed{L = 0.0845 \text{ m}}$$

Check the Reynolds number

$$\begin{aligned} \text{Re}_x &= \frac{0.0845 \times 10}{1.31 \times 10^{-6}} \\ &= 6.45 \times 10^5 \end{aligned}$$

so the equations for the tripped boundary layer ($Re_x < 10^7$) are valid.

Average shear stress coefficient

$$\begin{aligned} C_f &= \frac{0.032}{(6.45 \times 10^5)^{1/7}} \\ &= 0.00473 \end{aligned}$$

Surface resistance (drag force).

$$\begin{aligned}\frac{F_s}{B} &= 2 \times \frac{1}{2}\rho U_o^2 C_f L \\ &= 1000\ \mathrm{kg/m^3} \times (10\ \mathrm{m/s})^2 \times 0.00473 \times 0.0845\,\mathrm{m} \\ &\boxed{\tfrac{F_s}{B} = 40.0\ \mathrm{N/m}}\end{aligned}$$

9.64: PROBLEM DEFINITION

Situation:

A boat with a width of 1 m and length of 2.5 m planes in water at a temperature of 15 °C . Boat speed is $U_0 = 110$ km/h $= 30.8\,\mathrm{m/s}$.

Find:

Power required to overcome skin friction drag.

Properties:

Table A.5 $\nu = 1.14 \times 10^{-6}$ m^2/s, $\rho = 999\,\mathrm{kg/m^3}$.

Assumptions:

Friction only comes from the bottom surface (ignore side plate effects).

PLAN

Power is the product of drag force and speed $(P = F_s U_0)$. Find the drag force using the appropriate correlation.

SOLUTION

Boat speed

$$U_0 = 110\,\mathrm{km/h} \times (1000\,\mathrm{m/km})/(3600\,\mathrm{s/h}) = 30.8\,\mathrm{m/s}$$

Reynolds number

$$\begin{aligned} \mathrm{Re}_L &= \frac{U_0 L}{\nu} \\ &= \frac{(30.8\,\mathrm{m/s})\,(2.5\,\mathrm{m})}{(1.14 \times 10^{-6}\,\mathrm{m^2/s})} \\ &= 6.73 \times 10^7 \end{aligned}$$

Thus, the boundary layer is mixed. Using the equation for average shear stress, Eq. (9.38, in 10e)

$$\begin{aligned} C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \\ &= 0.00224 \end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned} F_s &= C_f \left(\frac{\rho U_0^2}{2} \right) A \\ &= 0.00224 \left(\frac{(999\,\mathrm{kg/m^3})\,(30.8\,\mathrm{m/s})^2}{2} \right) (2.5\,\mathrm{m} \times 0.9\,\mathrm{m}) \\ &= 2388\ \mathrm{N} \end{aligned}$$

Power

$$\begin{aligned} P &= F_s U_0 \\ &= (2388 \text{ N})\,(30.8\,\text{m/ s}) \\ &= 73,550 \text{ Nm/s} \\ &\boxed{P = 73,550 \text{ W}} \end{aligned}$$

9.65: PROBLEM DEFINITION

Situation:
A 0.5 m diameter log 50 m long is being pulled through water at 1.7 m/s. Boundary layer tripped at leading edge

Find:
Force required to overcome surface resistance.

Properties:
Table A.5 $\nu = 1.31 \times 10^{-6}$ m^2/s, $\rho = 1000$ kg/m^3.

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re}_L &= \frac{1.7 \text{ m/s} \times 50 \text{ m}}{1.31 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= 6.49 \times 10^7 \end{aligned}$$

Reynolds number larger than 10^7 so use

$$\begin{aligned} C_f &= \frac{0.523}{\ln^2(0.06\,\text{Re}_L)} \\ &= \frac{0.523}{\ln^2(0.06 \times 6.49 \times 10^7)} \\ &= 0.00227 \end{aligned}$$

Surface resistance

$$\begin{aligned} F_s &= C_f A_s \frac{\rho V_0^2}{2} \\ &= 0.00227 \times \pi \times 0.5 \text{ m} \times 50 \text{ m} \times \frac{1{,}000 \text{ kg/m}^3 \times (1.7 \text{ m/s})^2}{2} \\ &\boxed{F_s = 258 \text{ N}} \end{aligned}$$

9.66: PROBLEM DEFINITION

Situation:

A passenger train 81 m long with a 10 m perimeter moving through air at 81.1 km/hr and 204 km/hr. Boundary layer is tripped.

Find:

Power required at both speeds.

Surface resistance at both speeds.

Properties:

Table A.3 $\nu = 1.41 \times 10^{-5}$ m^2/s, $\rho = 1.25$ kg/m^3.

SOLUTION

Speeds

$$\begin{aligned} U_{81.1\text{ km/hr}} &= 81.1\frac{\text{km}}{\text{hr}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ hr}}{3600\text{ s}} = 22.5\text{ m/s} \\ U_{204\text{ km/hr}} &= 56.7\text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re}_L &= \frac{U_0 L}{\nu} \\ \text{Re}_{81.1} &= 1.29 \times 10^8 \\ \text{Re}_{204} &= 3.26 \times 10^8 \end{aligned}$$

Reynolds number larger than 10^7 so use

$$\begin{aligned} C_f &= \frac{0.523}{\ln^2(0.06\,\text{Re}_L)} \\ C_{f_{81.1}} &= 0.00208 \\ C_{f_{204}} &= 0.00186 \end{aligned}$$

Surface resistance equation, Eq. 9.17 (10e)

$$\begin{aligned} F_s &= C_f A \frac{\rho V_0^2}{2} \\ F_{s_{100}} &= 0.00208 \times 10\text{ m} \times 81\text{ m} \times \frac{1.25\text{ kg/m}^3 \times (22.5\text{ m/s})^2}{2} \end{aligned}$$

$$\boxed{F_{s_{81.1}} = 534\text{N}}$$

$$\boxed{F_{s_{204}} = 3020\text{ N}}$$

Power

$$\begin{aligned} P_{81.1} &= 534\text{ N} \times 22.5\text{ m/s} \\ &\boxed{P_{81.1} = 12.1\text{ kW}} \\ P_{204} &= 3017\text{ N} \times 56.7\text{ m/s} \\ &\boxed{P_{204} = 171\text{ kW}} \end{aligned}$$

9.67: PROBLEM DEFINITION

Situation:

A boundary layer next to the smooth hull of a ship that moves at 15 m/s in fresh water at 15 °C.

Find:

(a) Thickness of boundary layer at $x = 30\,\text{m}$.
(b) Velocity of water at $y/\delta = 0.5$.
(c) Shear stress on hull at $x = 30\,\text{m}$.

Properties:

Table A.5 (water at 15 °C): $\rho = 999\,\text{kg/m}^3$, $\gamma = 9800\,\text{N/m}^3$, $\mu = 1.14 \times 10^{-3}$ $\text{N}\cdot\text{s/m}^2$, $\nu = 1.14 \times 10^{-6}\,\text{m}^2/\text{s}$.

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re}_x &= \frac{Ux}{\nu} \\ &= \frac{(15\text{ m/s})(30\text{ m})}{1.14 \times 10^{-6}\text{ m}^2/\text{s}} = 3.690 \times 10^8 \end{aligned}$$

Local shear stress coefficient

$$\begin{aligned} c_f &= \frac{0.455}{\ln^2(0.06\,\text{Re}_x)} = \frac{0.455}{\ln^2(0.06 \times 3.690 \times 10^8)} \\ &= 0.001591 \end{aligned}$$

Local shear stress

$$\begin{aligned} \tau_0 &= c_f \left(\frac{\rho U_0^2}{2}\right) \\ &= (0.001591)\left(\frac{999\text{ kg/m}^3 \times (15\text{ m/s})^2}{2}\right) \end{aligned}$$

$$\boxed{\tau_0 = 179\text{ N/m}^2\text{ (c)}}$$

Shear velocity

$$\begin{aligned} u_* &= (\tau_0/\rho)^{0.5} \\ &= (179\text{ N/m}^2/999\text{ kg/m}^3)^{0.5} \\ &= 0.42\text{ m/s} \end{aligned}$$

Boundary layer thickness (turbulent flow)

$$\begin{aligned}
\delta/x &= 0.16\,\mathrm{Re}_x^{-1/7} = 0.16\left(3.690 \times 10^8\right)^{-1/7} \\
&= 0.009556 \\
\delta &= (0.009556)(30\,\mathrm{m}) \\
& \boxed{\delta = 0.29 \text{ m (a)}} \\
\frac{\delta}{2} &= 0.15 \text{ m}
\end{aligned}$$

From figure for velocity defect law (Fig 9.12 in 10e), at $y/\delta = 0.50$, $(U_0 - u)/u_* \approx 3$
Then

$$\frac{15 - u}{0.42} = 3$$

$$\boxed{u\,(y = \delta/2) = 13.7 \text{ (b)}}$$

9.68: PROBLEM DEFINITION

Situation:

This problem involves an Eiffel-type wind tunnel.

Test section width (square) is $W = 457\,\mathrm{mm}$. Test section length is $L = 914\,\mathrm{mm}$.

Find:

Find the ratio of maximum boundary layer thickness to test section width $(\delta\,(x = L)\,/W)$ for two cases:

(a) Minimum operating velocity ($U_o = 1\,\mathrm{m/\,s}$).

(b) Maximum operating velocity ($U_o = 70\,\mathrm{m/\,s}$).

Properties:

Air properties from Table A.3. At $T = 20\,^\circ\mathrm{C}$ and $p = 1\,\mathrm{atm}$, $\nu = 15.1 \times 10^{-6}\,\mathrm{m^2/\,s}$.

PLAN

Calculate the Reynolds number to establish if the boundary layer flow is laminar or turbulent. Then, apply the appropriate correlation for boundary layer thickness (i.e. for δ).

SOLUTION

(a) Reynolds number for minimum operating velocity

$$\begin{aligned} \mathrm{Re}_L &= \frac{U_o L}{\nu} \\ &= \frac{(1\,\mathrm{m/\,s})\,(0.914\,\mathrm{m})}{(15.1 \times 10^{-6}\,\mathrm{m^2/\,s})} \\ &= 60,530 \text{ (minimum operating velocity)} \end{aligned}$$

Since $\mathrm{Re}_L \quad \leq 500,000$, the boundary layer is laminar.

Correlation for boundary layer thickness (laminar flow)

$$\begin{aligned}\delta &= \frac{5x}{\mathrm{Re}_x^{1/2}}\\ &= \frac{5\times(0.914\,\mathrm{m})}{\sqrt{60,530}}\\ &= 18.57\,\mathrm{mm}\end{aligned}$$

Ratio of boundary layer thickness to width of the test section

$$\frac{\delta}{W} = \frac{18.57\,\mathrm{mm}}{457\,\mathrm{mm}}$$

$$\boxed{\frac{\delta}{W} = 0.0406 = 4.1\%\ \text{(minimum operating velocity)}}$$

(b) Reynolds number (maximum operating velocity)

$$\begin{aligned}\mathrm{Re}_L &= \frac{U_o L}{\nu}\\ &= \frac{(70\,\mathrm{m/\,s})\,(0.914\,\mathrm{m})}{(15.1\times10^{-6}\,\mathrm{m^2/\,s})}\\ &= 4,237,000\ \text{(maximum operating velocity)}\end{aligned}$$

Since $\mathrm{Re}_L \geq 500,000$, the boundary layer is turbulent.

Correlation for boundary layer thickness (turbulent flow):

$$\begin{aligned}\delta &= \frac{0.16x}{\mathrm{Re}_x^{1/7}}\\ &= \frac{0.16\times(0.914\,\mathrm{m})}{(4,237,000)^{1/7}}\\ &= 16.53\,\mathrm{mm}\end{aligned}$$

Ratio of boundary layer thickness to width of the test section

$$\frac{\delta}{W} = \frac{16.53\,\mathrm{mm}}{457\,\mathrm{mm}}$$

$$\boxed{\frac{\delta}{W} = 0.036 = 3.6\%\ \text{(maximum operating velocity)}}$$

REVIEW

1. Notice that the boundary layer is slightly thinner for the maximum velocity.
2. In both cases (maximum and minimum velocity), the boundary layer thickness is only a small fraction of the width ($\simeq 4\%$).

9.69: PROBLEM DEFINITION

Situation:
A 180 m long ship moving at 8 m/s through fresh water. Submerged area is 4600 m^2.

Find:
Skin friction drag on ship.

Properties:
Table A.5 $\nu = 1.31 \times 10^{-6}$ m^2/s and $\rho = 1000$ kg/m^3.

SOLUTION

Reynolds number

$$\begin{aligned} \mathrm{Re}_L &= \frac{U_0 L}{\nu} \\ &= \frac{(8 \text{ m/s})(180 \text{ m})}{1.31 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= 1.06 \times 10^9 \end{aligned}$$

Average skin friction coefficient

$$\begin{aligned} C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \\ &= \frac{0.523}{\ln^2(0.06 \times 1.06 \times 10^9)} - \frac{1520}{1.06 \times 10^9} \\ &= .00162 \end{aligned}$$

This agrees well with Fig. 9.13 (EFM10e).

Surface resistance equation.

$$\begin{aligned} F_s &= C_f A_s \frac{\rho U_0^2}{2} \\ &= (0.00162)(4600 \text{ m}^2)\frac{(1000 \text{ kg/m}^3)(8 \text{ m/s})^2}{2} \\ & \boxed{F_s = 238,464} \end{aligned}$$

9.70: PROBLEM DEFINITION

Situation:
A barge in a river draws 0.6 m water and towed at 3 m/s.

Find:
Shear (drag) force.

Properties:
Table A.5 $\nu = 1.14 \times 10^{-6}$ m^2/s and $\rho = 999$ kg/m^3.

SOLUTION

Barge length = 60 m, submerged surface area $\simeq 822$ m^2
Reynolds number

$$\begin{aligned}\text{Re}_L &= \frac{VL}{\nu} \\ &= \frac{3 \text{ m/s} \times 60 \text{ m}}{1.14 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= 1.6 \times 10^8\end{aligned}$$

Average shear stress coefficient

$$\begin{aligned}C_f &= \frac{0.523}{\ln^2(0.06\,\text{Re}_L)} - \frac{1520}{\text{Re}_L} \\ &= \frac{0.523}{\ln^2(0.06 \times 1.6 \times 10^8)} - \frac{1520}{1.6 \times 10^8} \\ &= 0.00200\end{aligned}$$

This agrees well with Fig. 9-14 in EFM 10e.
Surface resistance (drag force)

$$\begin{aligned}F_s &= C_f BL \frac{\rho V_0^2}{2} \\ &= (0.00200)(822 \text{ m}^2) \frac{999 \text{ kg/m}^3 (3 \text{ m/s})^2}{2} \\ & \boxed{F_s = 7390 \text{ N}}\end{aligned}$$

9.71: PROBLEM DEFINITION

Situation:

A supertanker with length 325 m, breadth 48 m and draught 19 m sails in open seas at 9.27 m/s.

Find:

(a) Skin friction drag.
(b) Power required.
(c) Boundary layer thickness 300 m from bow.

Properties:

From Table A.4 $\nu = 1.4 \times 10^{-6}$ m^2/s and $\rho = 1026$ kg/m^3.

PLAN

Find Reynolds number, and then calculate the average shear stress coefficient (C_f). Next, find the drag force and calculate power as the product of drag force and speed $(P = F_s \times V)$. To find boundary layer thickness, apply the correlation for a turbulent boundary layer.

SOLUTION

Reynolds number

$$\begin{aligned} \mathrm{Re}_L &= \frac{U_0 L}{\nu} \\ &= \frac{9.27 \text{ m/s} \times 325 \text{ m}}{1.4 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= 2.15 \times 10^9 \end{aligned}$$

Average shear stress coefficient

$$\begin{aligned} C_f &= \frac{0.523}{\ln^2 (0.06 \, \mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \quad \text{(turbulent flow)} \\ &= \frac{0.523}{\ln^2 (0.06 \times 2.15 \times 10^9)} - \frac{1520}{2.15 \times 10^9} \\ &= 0.00150 \end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned} F_s &= C_f A \frac{\rho U_0^2}{2} \\ &= 0.00150 \times 325 \text{ m} \times (48 \text{ m} + 38 \text{ m}) \times \frac{1026 \text{ kg/m}^3 \times (9.27 \text{ m/s})^2}{2} \\ &= 1.848 \times 10^6 \, \text{N} \\ & \boxed{F_s = 1.85 \text{ MN}} \end{aligned}$$

Power

$$P = 1.85 \times 10^6 \text{ MN} \times 9.27 \text{ m/s}$$
$$\boxed{P = 17.1 \text{ MW}}$$

Reynolds number

$$\begin{aligned}
\text{Re}_{300} &= \frac{U_0 x}{\nu} \\
&= \frac{9.27 \text{ m/s} \times 300 \text{ m}}{1.4 \times 10^{-6} \text{m}^2/\text{s}} \\
&= 1.986 \times 10^9
\end{aligned}$$

Thus, turbulent boundary layer

Correlation for boundary layer thickness (turbulent flow)

$$\begin{aligned}
\frac{\delta}{x} &= \frac{0.16}{\text{Re}_x^{1/7}} \\
&= \frac{0.16}{(1.986 \times 10^9)^{1/7}} \\
&= 7.513 \times 10^{-3} \\
\delta &= 300\,\text{m} \times .007513
\end{aligned}$$
$$\boxed{\delta = 2.25 \text{ m}}$$

9.72: PROBLEM DEFINITION

Situation:

A 1:100 model test used to predict the drag on a prototype ship which is 150 m long, has wetted area of 2300 m^2 and operates at 9 m/s in sea water. Measured model drag is 0.5 N.

Find:

Wave drag on actual ship.

Properties:

Table A.5 $\nu = 1.14 \times 10^{-6}$ m^2/s and $\rho = 999$ kg/m^3.

SOLUTION

Match Froude numbers

$$\begin{aligned}
\text{Fr}_{\text{model}} &= \text{Fr}_{\text{prototype}} \\
\left(\frac{V}{\sqrt{gL}}\right)_{\text{model}} &= \left(\frac{V}{\sqrt{gL}}\right)_{\text{prototype}} \\
\frac{V_{\text{model}}}{V_{\text{prototype}}} &= \sqrt{\frac{L_{\text{model}}}{L_{\text{prototype}}}} \\
\frac{V_{\text{model}}}{9\,\text{m/s}} &= \sqrt{\frac{1}{100}} \\
V_{\text{model}} &= 0.9 \text{ m/s}
\end{aligned}$$

Reynolds number on model

$$\begin{aligned}
\text{Re}_L &= \left(\frac{VL}{\nu}\right)_{\text{model}} \\
&= \frac{(0.9\,\text{m/s})\,(1.5\,\text{m})}{1.14 \times 10^{-6}\,\text{m}^2/\text{s}} \\
&= 1.18 \times 10^6
\end{aligned}$$

Average shear stress coefficient on model

$$\begin{aligned}
C_f &= \frac{0.523}{\ln^2(0.06\,\text{Re}_L)} - \frac{1520}{\text{Re}_L} \quad \text{(mixed BL flow)} \\
&= \frac{0.523}{\ln^2(0.06 \times 1.18 \times 10^6)} - \frac{1520}{1.18 \times 10^6} \\
&= 0.0029
\end{aligned}$$

Surface resistance (drag force) on model

$$\begin{aligned} F_{s,m} &= C_f A_{\text{wet}} \frac{\rho V^2}{2} \\ &= 0.0029 \left(\frac{2300\,\text{m}^2}{100^2} \right) \frac{(999\,\text{kg/m}^3)\,(0.9\,\text{m/s})^2}{2} \\ &= 0.27\,\text{N} \end{aligned}$$

Wave drag on model

$$\begin{aligned} F_{\text{total}} &= F_{\text{wave}} + F_{\text{viscous}} \\ 0.5\,\text{N} &= F_{\text{wave}} + (0.27\,\text{N}) \\ F_{\text{wave}} &= 0.23\,\text{N} \end{aligned}$$

Equating force coefficients

$$\frac{F_m}{F_p} = \frac{\rho_m}{\rho_p} \left(\frac{V_m}{V_p} \right)^2 \left(\frac{L_m}{L_p} \right)^2$$

Inserting values

$$\left(\frac{0.23\,\text{N}}{F_{\text{prototype}}} \right)_{\text{wave drag}} = \left(\frac{1}{1.03} \right) \left(\frac{1}{10} \right)^2 \left(\frac{1}{100} \right)^2$$

Thus

$$F\,(\text{wave drag on prototype}) = 236870\ \text{N}$$

$$\boxed{\text{Wave Drag on Prototype} = 236870\,\text{N}}$$

9.73: PROBLEM DEFINITION

Situation:

A 1:40 scale model of ship 250 m long, with 30 m beam and 12 m draft and surface area at waterline of 8800 m^2. A drag of 26 N measured on model at 1.45 m/s.

Find:

(a) Speed of prototype.
(b) Model skin friction and wave drag.
(c) Ship drag in salt water.

Properties:

From Table A.5 $\nu_m = 1.00 \times 10^{-6}$ m^2/s and $\rho_m = 998$ kg/m^3.
From Table A.4 $\nu_p = 1.4 \times 10^{-6}$ m^2/s and $\rho_m = 1026$ kg/m^3.

SOLUTION

$$
\begin{aligned}
V_m &= 1.45 \text{ m/s} \\
V_p &= \left(\frac{L_p}{L_m}\right)^{1/2} \times V_m \\
&= \sqrt{40} \times 1.45 \\
& \boxed{V_m = 9.17 \text{ m/s}} \\
\text{Re} &= \frac{UL}{\nu} \\
\text{Re}_m &= \frac{1.45 \text{ m/s}(250 \text{ m}/40)}{1.00 \times 10^{-6} \text{m}^2/\text{s}} = 9.06 \times 10^6 \\
\text{Re}_p &= \frac{9.17 \times 250}{1.4 \times 10^{-6}} = 1.63 \times 10^9 \\
C_f &= \frac{0.523}{\ln^2(0.06\,\text{Re})} - \frac{1520}{\text{Re}} \\
C_{fm} &= 0.00283 \\
C_{fp} &= 0.00154
\end{aligned}
$$

Surface resistance (drag force)

$$
\begin{aligned}
F_{sm} &= C_{fm} A \frac{\rho V^2}{2} \\
&= 0.00283 \left(\frac{8,800 \text{ m}^2}{40^2} \right) \frac{998 \text{ kg/m}^3 \times (1.45 \text{ m/s})^2}{2} \\
& \boxed{F_{sm} = 16.3 \text{ N}} \\
F_{\text{wave}_m} &= 26.0 - 16.3 \\
& \boxed{F_{\text{wave}_m} = 9.7 \text{ N}} \\
F_{\text{wave}_p} &= \frac{\rho_p}{\rho_m} \left(\frac{L_p}{L_m} \right)^3 F_{\text{wave}_m} = \left(\frac{1,026 \, \text{kg/ m}^3}{998 \, \text{kg/ m}^3} \right) (40)^3 \, (9.7 \, \text{N}) = 638 \text{ kN} \\
F_{sp} &= C_{f_p} A \frac{\rho V^2}{2} = 0.00154(8,800 \text{ m}^2) \frac{1,026 \text{ kg/m}^3 \times (9.17 \text{ m/s})^2}{2} \\
&= 585 \text{ kN} \\
F_p &= F_{\text{wave}_p} + F_{sp} = 638 + 585 \\
& \boxed{F_p = 1,220 \text{ kN}}
\end{aligned}
$$

9.74: PROBLEM DEFINITION

Situation:

A hydroplane 3 m long skims at 15 m/s across a lake.

Find:

Minimum shear stress on smooth bottom.

Properties:

From Table A.5 $\nu = 10^{-6}$ m^2/s and $\rho = 998$ kg/m^3.

PLAN

Minimum τ_0 occurs where c_f is minimum. Two points to check: (1) where Re_x is highest; i.e., $\mathrm{Re}_x = \mathrm{Re}_L$ and (2) Transition point at $\mathrm{Re}_x = 5 \times 10^5$ (this is the end of the laminar boundary layer).

SOLUTION

(1) Check end of plate

$$\begin{aligned}
\mathrm{Re}_L &= \frac{U_0 L}{\nu} \\
&= 15 \text{ m/s} \times 3 \text{ m}/10^{-6} \text{ m}^2\text{/s} \\
&= 4.5 \times 10^7 \\
c_f &\approx \frac{0.455}{\ln^2(0.06\,\mathrm{Re}_x)} = 0.00207
\end{aligned}$$

(2) Check transition

$$\mathrm{Re}_x = 5 \times 10^5$$

$$\begin{aligned}
c_f &= \frac{0.664}{\mathrm{Re}_x^{1/2}} \\
&= 0.00094
\end{aligned}$$

Minimum shear stress at end of plate

$$\begin{aligned}
\tau_{0_{\min}} &= c_{f_{\min}} \frac{\rho U_0^2}{2} \\
&= 0.00094 \times \frac{998\,\mathrm{kg/\,m^3} \times (15\,\mathrm{m/\,s})^2}{2}
\end{aligned}$$

$$\boxed{\tau_{0_{\min}} = 106 \text{ N/m}^2}$$

9.75: PROBLEM DEFINITION

Situation:
A water skier with skis 1.2 m long and 0.15 m wide moving at 50 km/h.

Find:
Power to overcome surface resistance.

Properties:
Table A.5 $\nu = 1.14 \times 10^{-6}$ m^2/s and $\rho = 999$ kg/m^3.

SOLUTION

Speed, 50 km/h = 14 m/s
Reynolds number

$$\begin{aligned} \mathrm{Re}_L &= \frac{VL}{v} \\ &= \frac{14 \text{ m/s} \times 1.2 \text{ m}}{1.14 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= 1.4 \times 10^7 \end{aligned}$$

Average shear stress coefficient.

$$\begin{aligned} C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \\ &= \frac{0.523}{\ln^2(0.06 \times 1.4 \times 10^7)} - \frac{1520}{1.44 \times 10^7} \\ &= 0.00269 \end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned} F_D \text{ (per ski)} &= 0.00269(1.2 \text{ m})(0.15 \text{ m})\frac{(999 \text{ kg/m}^3)(14 \text{ m/s})^2}{2} = 47.4 \text{ N} \\ F_D \text{ (2 skis)} &= 94.8 \text{ N} \end{aligned}$$

Power

$$\begin{aligned} P(\mathrm{W}) &= 94.8 \text{ N} \times 14 \text{ m/s} \\ &\boxed{P = 1327 \text{ W}} \end{aligned}$$

9.76: PROBLEM DEFINITION

Situation:
A 80-m ship with 1500 m^2 wetted area travels at 15 m/s.

Find:
(a) Surface drag.
(b) Thickness of boundary layer at stern.

Properties:
Table A.4 $\nu = 1.4 \times 10^{-6}$ m^2/s, $\rho = 1026$ kg/m^3.

PLAN

Apply the surface resistance equation by first finding Reynolds number and C_f. Then apply the correlation for boundary layer thickness.

SOLUTION

Reynolds number

$$\begin{aligned} \mathrm{Re}_L &= \frac{U_0 L}{\nu} = \frac{15 \text{ m/s} \times 80 \text{ m}}{1.4 \times 10^{-6} \text{ m}^2/\text{s}} \\ \mathrm{Re}_L &= 8.57 \times 10^8 \end{aligned}$$

Average shear stress coefficient

$$\begin{aligned} C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \\ &= \frac{0.523}{\ln^2(0.06 \times 8.57 \times 10^8)} - \frac{1520}{8.57 \times 10^8} \\ &= 0.00166 \end{aligned}$$

Surface resistance

$$\begin{aligned} F_D &= C_f A \frac{\rho U_0^2}{2} \\ &= 0.00166 \times 1,500 \text{ m}^2 \times \frac{1,026 \text{ kg/m}^3 \times (15 \text{ m/s})^2}{2} \\ & \boxed{F_D = 287 \text{ kN}} \end{aligned}$$

Boundary layer thickness

$$\begin{aligned} \frac{\delta}{x} &= \frac{0.16}{\mathrm{Re}_x^{1/7}} \\ \frac{\delta}{x} &= 0.00847 \\ \delta &= 80 \times 0.00847 \\ & \boxed{\delta = 0.678 \text{ m}} \end{aligned}$$