

---

**10.1: PROBLEM DEFINITION**

Situation:

Kerosene flows in a pipe.

$$Q = 0.02 \text{ m}^3/\text{s}, \quad D = 17.7 \text{ cm}.$$

Find:

Is the flow laminar or turbulent?

Entrance length (meters).

Properties:

Kerosene (20 °C), Table A.4,  $\nu = 2.37 \times 10^{-6} \text{ m}^2/\text{s}$ .

**SOLUTION**

Reynolds number:

$$\text{Re} = \frac{4Q}{\pi D \nu} = \frac{4(0.02 \text{ m}^3/\text{s})}{\pi(0.177 \text{ m})(2.37 \times 10^{-6} \text{ m}^2/\text{s})} = 60704$$

Flow is turbulent

Entrance length:

$$\frac{L_e}{D} = 50$$

$$L_e = 50D = 50(0.177 \text{ m}) = \boxed{8.85 \text{ m}}$$

---

**10.2: PROBLEM DEFINITION**Situation:

A compressor is drawing ambient air through a duct.

$$Q = 0.3 \text{ m}^3/\text{s}, \quad D = 0.15 \text{ m}.$$

$$L = 10 \text{ m}, \quad T = 20^\circ\text{C}.$$

Find:

Determine if the flow is laminar or turbulent.

Entrance length (meters).

Assumptions:

Smooth inlet, so  $L_e$  correlations apply.

Pressure in duct is 1.0 atm.

Properties:

Air ( $20^\circ\text{C}$ ,  $p = 1 \text{ atm}$ ), Table A.3,  $\nu = 15.1 \times 10^{-6} \text{ m}^2/\text{s}$ .

**SOLUTION**

Reynolds number:

$$\text{Re} = \frac{4Q}{\pi D \nu} = \frac{4 (0.3 \text{ m}^3/\text{s})}{\pi (0.15 \text{ m}) (15.1 \times 10^{-6} \text{ m}^2/\text{s})} = 169,600$$

Since  $\text{Re} > 3000$ , the flow is turbulent.

Flow is turbulent

Entrance length:

$$\begin{aligned} \frac{L_e}{D} &= 50 \\ L_e &= 50D = 50 (0.15 \text{ m}) = 7.5 \text{ m} \end{aligned}$$

$$L_e = 7.5 \text{ m}$$

---

**10.3: PROBLEM DEFINITION****Situation:**

A tube carries SAE 10W-30 oil.

$$Q = 0.1 \text{ L/s} = 0.0001 \text{ m}^3/\text{s}, T = 38^\circ\text{C}.$$

Flow must be laminar and fully developed.

**Find:**

Specify a tube length (millimeters).

Specify a tube diameter (millimeters).

**Assumptions:**

Smooth inlet, so  $L_e$  correlations apply.

**Properties:**

SAE 10W-30 oil ( $38^\circ\text{C}$ ), Table A.4,  $\nu = 7.6 \times 10^{-5} \text{ m}^2/\text{s}$ .

**PLAN**

1. Find diameter  $D$  by specifying a Reynolds number of 1500 (laminar).
2. Find length  $L$  by specifying a length so  $L > 0.05D \text{ Re}$ .

**SOLUTION**

Reynolds number:

$$\text{Re} = \frac{4Q}{\pi D \nu} = \frac{4(0.0001 \text{ m}^3/\text{s})}{\pi D (7.6 \times 10^{-5} \text{ m}^2/\text{s})} = 1500$$

$$D = 1.12 \text{ mm}$$

Entrance length:

$$L > 0.05D \text{ Re}$$

$$L > 0.05(0.00112 \text{ m})(1500) = 0.084 \text{ m}$$

Select

$$L = 100 \text{ mm}$$

**REVIEW**

Notice that the answer will change if a different value of  $\text{Re}$  is selected.

---

**Problem 10.4**

Answer the questions below.

---

a.) What is pipe head loss? How is pipe head loss related to total head loss?

- Pipe head loss  $\implies$  head loss associated with fully developed flow in straight sections of pipe.
  - Total head loss = (Pipe head loss) + (Component head loss).
- 

b.) What is the friction factor  $f$ ? How is  $f$  related to wall shear stress?

- $f$  is a  $\pi$ -group
- $f$  is defined as the ratio of wall shear stress to kinetic pressure (with an extra constant of 0.25 included)

$$f \equiv \frac{\tau_{\text{wall}}}{0.25 \times \rho V^2 / 2}$$

- $f$  can be thought of as a dimensionless wall shear stress
- 

c.) What assumptions need to be satisfied to apply the Darcy Weisbach equation?

- the conduit needs to be completely full of the flowing fluid
- fully developed flow
- steady flow

---

**Problem 10.5**

Apply the grid method to each situation described below. Apply the Darcy Weisbach equation. Note: Unit cancellations are not shown in this solution.

---

Situation: (a)

Water is flowing in a pipe.

$Q = 75 \text{ lpm}$ ,  $V = 55 \text{ m/min}$ .

$L = 60 \text{ m}$ ,  $f = 0.02$ .

Find:

Head loss (m).

Solution:

Flow rate eqn:

$$Q = \left( \frac{\pi D^2}{4} \right) V$$
$$D = \sqrt{\frac{4Q}{\pi V}} = \sqrt{\frac{4}{\pi} \left( \frac{75 \text{ L}}{\text{min}} \right) \left( \frac{\text{min}}{55 \text{ m}} \right) \left( \frac{0.001 \text{ m}^3}{\text{L}} \right)} = 0.042 \text{ m}$$

Darcy Weisbach eqn:

$$h_f = f \frac{L V^2}{D 2g} = 0.02 \left( \frac{60 \text{ m}}{0.042 \text{ m}} \right) \left( \frac{55 \text{ m}}{\text{min}} \right)^2 \left( \frac{1.0}{2(9.81 \text{ m/s}^2)} \right) \left( \frac{1.0 \text{ min}}{60 \text{ s}} \right)^2$$

$$\boxed{h_f = 1.22 \text{ m}}$$

---

Situation: (b)

Flow in a PVC pipe.

$h_f = 0.8 \text{ m}$ ,  $f = 0.012$ ,  $L = 15 \text{ m}$   $Q = 0.028 \text{ m}^3/\text{s}$ .

Find:

Pipe diameter (meters).

Solution:

$$h_f = f \frac{L V^2}{D 2g} = f \frac{L (Q/A)^2}{D 2g} = f \frac{L 16Q^2}{D 2g\pi^2 D^4}$$
$$D = \sqrt[5]{f \left( \frac{L}{h_f} \right) \frac{16Q^2}{2g\pi^2}} = \sqrt[5]{0.012 \left( \frac{15 \text{ m}}{0.8 \text{ m}} \right) \frac{16}{2(9.81 \text{ m/s}^2) \pi^2} \left( \frac{0.028 \text{ m}^3}{\text{s}} \right)^2}$$

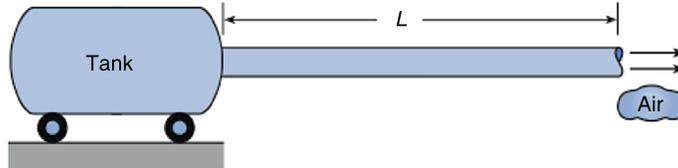
$$\boxed{D = 0.108 \text{ m}}$$

## 10.6: PROBLEM DEFINITION

Situation:

### SITUATION

Air is flowing from a large tank to ambient through a horizontal pipe. Pipe is 25 mm Schedule 40.  $D = 0.0266$  m.  $V = 10$  m/s.  $f = 0.015$ ,  $L = 50$  m.



Assumptions:

Air has constant density (look up properties at 1 atm).  
KE correction factor is  $\alpha_2 = 1.0$ .

Properties:

Air ( $20^\circ\text{C}$ , 1 atm, Table A.3):  $\rho = 1.2$  kg/m<sup>3</sup>.

### PLAN

1. Relate pressure in tank to head loss using the energy equation.
2. Describe head loss using the Darcy-Weisbach equation.
3. Combine steps 1 & 2.

### SOLUTION

1. Energy eqn. (location 1 inside the tank, location 2 at the pipe exit)

$$\begin{aligned}\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ \frac{p_1}{\gamma} + 0 + 0 + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_f\end{aligned}\quad (1)$$

2. Darcy-Weisbach eqn.:

$$h_f = f \frac{L}{D} \frac{V^2}{2g}\quad (2)$$

3. Combine Eqs. (1) and (2).

$$\begin{aligned}p_1 &= \gamma \frac{V_2^2}{2g} \left(1 + f \frac{L}{D}\right) = \frac{\rho V_2^2}{2} \left(1 + f \frac{L}{D}\right) \\ &= \frac{(1.2 \text{ kg/m}^3) (10 \text{ m/s})^2}{2} \left(1 + (0.015) \frac{(50 \text{ m})}{(0.0266 \text{ m})}\right) \\ &= 1.75 \text{ kPa-gage}\end{aligned}$$

$$p_{\text{tank}} = 1.75 \text{ kPa gage}$$

**REVIEW** The constant density assumption is valid because the pressure in the tank is less than 2% of atmospheric pressure.

---

**10.7: PROBLEM DEFINITION**Situation:

Water is flowing through a horizontal pipe (garden hose).

$$D = 0.022 \text{ m}, \quad L = 20 \text{ m}.$$

$$f = 0.012, \quad V = 2.0 \text{ m/s}.$$

Find:

Pressure drop (Pa) for 20 m of hose.

Properties:

Water (15 °C), Table A.5,  $\rho = 999 \text{ kg/m}^3$ .

Assumptions:

Assume turbulent, so  $\alpha = 1.0$  everywhere.

**PLAN**

1. Relate pressure drop to head loss using the energy equation.
2. Describe head loss using the Darcy-Weisbach equation.
3. Combine steps 1 & 2.

**SOLUTION**

1. Energy eqn. (location 1 upstream; location 2 is 20 m downstream)

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + 0 + 0 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + 0 + h_L \\ &\text{since } V_1 = V_2, \text{ KE terms cancel} \\ \Delta p &= \gamma h_L \end{aligned} \tag{1}$$

2. Darcy-Weisbach eqn.:

$$h_L = h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{2}$$

3. Combine Eqs. (1) and (2):

$$\begin{aligned} \Delta p &= \gamma \left( f \frac{L}{D} \frac{V^2}{2g} \right) = \frac{\rho V^2}{2} \left( f \frac{L}{D} \right) \\ &= \frac{(999 \text{ kg/m}^3) (2 \text{ m/s})^2}{2} \left( 0.012 \times \frac{20 \text{ m}}{0.022 \text{ m}} \right) \end{aligned}$$

$$\boxed{\Delta p = 21.8 \text{ kPa}}$$

## 10.8: PROBLEM DEFINITION

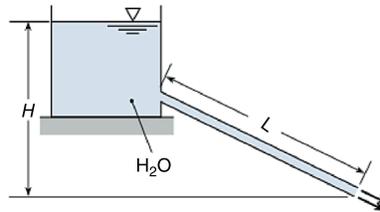
### Situation:

Water is flowing from a tank through a tube & then discharging to ambient.

$$D = 0.008 \text{ m}, \quad L = 6 \text{ m}.$$

$$H = 3 \text{ m}, \quad f = 0.015.$$

### Sketch:



### Find:

Exit velocity (m/s).

Discharge (L/s).

Sketch the HGL & EGL.

### Assumptions:

The only head loss is in the tube.

Turbulent flow so  $\alpha_2 = 1.0$ .

### Properties:

Water (15 °C), Table A.5 (EFM10e),  $\rho = 999 \text{ kg/m}^3$   $\nu = 1.14 \times 10^{-6} \text{ m}^2/\text{s}$ .

## PLAN

1. Relate  $H$  to head loss using the energy equation.
2. Describe head loss using the Darcy-Weisbach equation.
3. Find  $V$  by combining steps 1 & 2.
4. Find  $Q$  by using the flow rate equation.

## SOLUTION

1. Energy eqn. (location 1 at the free surface, location 2 at the pipe exit)

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + H + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_L \end{aligned} \quad (1)$$

2. Darcy-Weisbach eqn.:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \quad (2)$$

3. Combine Eqs. (1) and (2):

$$H = \frac{V_2^2}{2g} \left( 1 + f \frac{L}{D} \right)$$

$$V_2 = \sqrt{\frac{2gH}{1 + f \frac{L}{D}}}$$

$$= \sqrt{\frac{2(9.81 \text{ m/s}^2)(3 \text{ m})}{1 + 0.015 \frac{(6 \text{ m})}{(0.008 \text{ m})}}}$$

$V = 2.19 \text{ m/s}$

4. Flow rate equation:

$$Q = VA = V \frac{\pi D^2}{4} = (2.192 \text{ m/s}) \frac{\pi (0.008 \text{ m})^2}{4}$$

$Q = 0.110 \text{ L/s}$

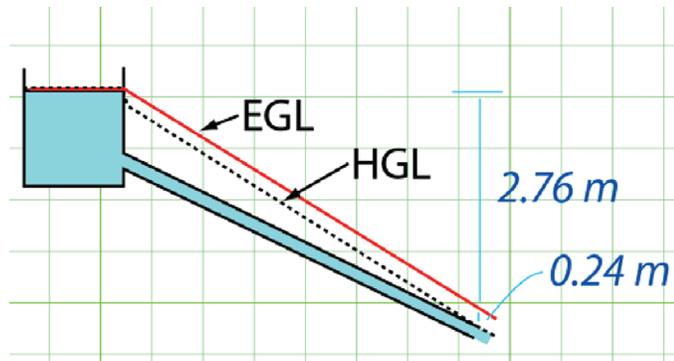
5. Sketch HGL & EGL

- Locate the EGL & HGL on free surface of tank.
- Velocity head and head loss:

$$\frac{V^2}{2g} = \frac{(2.192 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 0.24 \text{ m}$$

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = 0.015 \frac{(6 \text{ m})}{(0.008 \text{ m})} \frac{(2.192 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 2.76 \text{ m}$$

- Locate the EGL and HGL at the end of the pipe. Sketch lines.



**REVIEW** Check the turbulent flow assumption.

$$\text{Re} = \frac{VD}{\nu} = \frac{(2.192 \text{ m/s})(0.008 \text{ m})}{(1.14 \times 10^{-6} \text{ m}^2/\text{s})}$$

$$\text{Re} = 15400 > 3000$$

Thus, the assumption of turbulent flow is valid.

## 10.9: PROBLEM DEFINITION

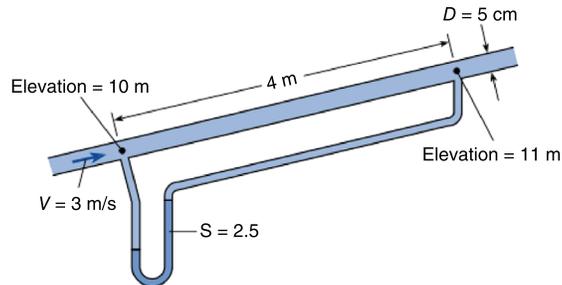
Situation:

Water flows through a pipe.

$$V = 3 \text{ m/s}, z_1 = 10 \text{ m.}$$

$$z_2 = 11 \text{ m}, D = 5 \text{ cm}$$

Sketch:



Find: Resistance coefficient,  $f$ .

## SOLUTION

Manometer equation

$$\begin{aligned} h_f &= \Delta h_{\text{manometer}} (\gamma_m / \gamma_{\text{H}_2\text{O}} - 1) \\ h_f &= (0.90 \text{ m}) (2.5 - 1) = 1.35 \text{ m of water} \end{aligned}$$

Darcy Weisbach

$$h_f = f(L/D)V^2/2g$$

Solve for  $f$

$$\begin{aligned} f &= h_f \left( \frac{D}{L} \right) \left( \frac{2g}{V^2} \right) \\ &= (1.35 \text{ m}) \left( \frac{0.05 \text{ m}}{4 \text{ m}} \right) \left( \frac{2(9.81 \text{ m/s}^2)}{(3 \text{ m/s})^2} \right) \end{aligned}$$

$$\boxed{f = 0.037}$$

---

**10.10: PROBLEM DEFINITION**

---

Answer the questions below.

---

a. What are the main characteristics of laminar flow?

- Flow in layers
  - Smooth flow; visualize flow of honey.
  - Low rates of mixing.
  - Low values of Reynolds number.
- 

b. What is the meaning of each variable that appears in Eq. (10.27)?

- $\bar{V}$  is the area-averaged velocity in the pipe; also called the mean velocity
  - $r_o$  is the pipe radius.
  - $\mu$  is the dynamic viscosity of the fluid.
  - $\gamma$  is the specific weight of the fluid.
  - $\Delta h$  is the change in piezometric head from location 1 to location 2.
  - $\Delta L$  is the length between location 1 to location 2.
- 

c. In Eq. (10.33), what is the meaning of  $h_f$ ?

- $h_f$  is the head loss associated with flow in a conduit.

---

**10.11: PROBLEM DEFINITION**

Situation:

Fluid flowing in a pipe.

$$V = 0.04 \text{ m/s}, \quad D = 0.1 \text{ m.}$$

Find:

- (a) Reynolds number.
- (b) Maximum velocity in the pipe.
- (c) Friction factor  $f$ .
- (d) Shear stress at the wall.
- (e) Shear stress 25 mm from pipe center.

Properties:  $\mu = 10^{-2} \text{ Pa} \cdot \text{s}$ ,  $\rho = 800 \text{ kg/m}^3$

**SOLUTION**

Reynolds number:

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{0.04 \times 0.1 \times 800}{0.01} \end{aligned}$$

$$\boxed{\text{Re} = 320}$$

Therefore, the flow is laminar:

$$\begin{aligned} V_{\max} &= 2V = \boxed{8 \text{ cm/s}} \\ f &= 64/\text{Re} \\ &= 64/320 \\ &= \boxed{0.20} \end{aligned}$$

Wall shear stress (from definition of  $f$ ) :

$$\begin{aligned} f &= \frac{4\tau_o}{\rho V^2/2} \\ \tau_o &= \left(\frac{f}{4}\right) \left(\frac{\rho V^2}{2}\right) = \frac{0.2}{4} \left(\frac{(800 \text{ kg/m}^3)(0.04 \text{ m/s})^2}{2}\right) = 0.032 \text{ Pa} \end{aligned}$$

$$\boxed{\tau_o = 0.032 \text{ Pa}}$$

Get  $\tau_{r=0.025}$  by using proportions. Rationale: shear stress varies linearly from  $\tau = 0$  at  $r = 0$  to  $\tau = \tau_o$  at  $r = r_o$ .

$$\begin{aligned}0.025/0.05 &= \tau/\tau_0 \\ \tau &= 0.5\tau_0 \\ \tau &= 0.5(0.032 \text{ Pa}) \\ &= \boxed{0.016 \text{ N/m}^2}\end{aligned}$$

---

**10.12: PROBLEM DEFINITION****Situation:**

Water is flowing in a pipe.  $Re = 1000$ ,  $T = 15^\circ\text{C}$ .  
Nominal diameter = 15 mm Schedule 40.  $D = 0.0158$  m.

**Find:**

- Mass flow rate (kg/s).
- Friction factor  $f$ .
- Head loss per meter of pipe length.
- Pressure drop per meter of head length.

Properties: Water ( $15^\circ\text{C}$ , 1 atm), Table A.5,  $\rho = 999$  kg/m<sup>3</sup>,  $\gamma = 9800$  N/m<sup>3</sup>,  $\nu = 1.14 \times 10^{-6}$  m<sup>2</sup>/s,  $\mu = 1.14 \times 10^{-3}$  N · s/m<sup>2</sup>.

**PLAN**

- Find  $V$  by using known  $Re$ .
- Find  $\dot{m}$  by using  $\dot{m} = \rho AV$ .
- Find  $f$  by using  $64/Re$ .
- Find  $h_L$  by using Darcy-Weisbach eqn.
- Find  $\Delta p$  by using the energy eqn.

**SOLUTION**

- Reynolds Number:

$$Re = \frac{VD}{\nu}$$
$$V = \frac{\nu Re}{D} = \frac{(1.14 \times 10^{-6} \text{ m}^2/\text{s})(1000)}{(0.0158 \text{ m})}$$
$$V = 0.0722 \text{ m/s}$$

- Mass flow rate:

$$\begin{aligned}\dot{m} &= \rho AV \\ &= (999 \text{ kg/m}^3) \left( \frac{\pi (0.0158 \text{ m})^2}{4} \right) (0.0722 \text{ m/s}) \\ &= 0.0141 \text{ kg/s}\end{aligned}$$

$$\boxed{\dot{m} = 0.0141 \text{ kg/s}}$$

- Friction factor:

$$f = \frac{64}{Re} = \frac{64}{1000}$$

$$\boxed{f = 0.064}$$

4. Darcy-Weisbach Eqn.:

$$\frac{h_f}{L} = \frac{f V^2}{D 2g} = \frac{(0.064)}{(0.0158 \text{ m})} \left( \frac{(0.0722 \text{ m/s})^2}{2 (9.81 \text{ m/s}^2)} \right)$$

$$\boxed{\frac{h_f}{L} = 0.00108 \frac{\text{m}}{\text{m}} \quad \text{that is, m of } h_L \text{ per m of pipe length}}$$

5. Energy eqn.

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

- KE terms cancel.
- Assume horizontal pipe.
- $h_p = h_t = 0$ .  $h_L = h_f$

$$\begin{aligned} \frac{p_1}{\gamma} + 0 + 0 + 0 &= \frac{p_2}{\gamma} + 0 + 0 + 0 + h_f \\ \Delta p &= \gamma h_f = (9800 \text{ N/m}^3) (0.00108) \end{aligned}$$

$$\boxed{\frac{\Delta p}{L} = 10.6 \frac{\text{Pa}}{\text{m}} \quad \text{that is, Pa of pressure head drop per m of pipe length}}$$

---

**10.13: PROBLEM DEFINITION**Situation:

Liquid flows through a smooth pipe.

$V = 0.5 \text{ m/s}$ .  $h_f = 2 \text{ m per m}$ ,  $D = 0.025 \text{ m}$ .

Find:

Friction factor.

Reynolds number.

Prove that doubling the flow will double the head loss.

Assumptions:

Laminar flow.

Fully developed flow.

**PLAN**

1. Find  $f$  using the Darcy-Weisbach eqn.
2. Find  $\text{Re}$  using  $f = 64/\text{Re}$ .
3. Determine the effect of doubling  $Q$  by logical reasoning with the head loss eqn.

**SOLUTION**

1. Darcy-Weisbach:

$$\begin{aligned}h_f &= f \frac{L V^2}{D 2g} \\f &= \frac{h_f 2gD}{L V^2} \\&= \left( \frac{2 \text{ m}}{1 \text{ m}} \right) \frac{2 (9.81 \text{ m/s}^2) (0.025 \text{ m})}{(0.5 \text{ m/s})^2} = 3.92\end{aligned}$$

2. Assume laminar flow:

$$\begin{aligned}f &= \frac{64}{\text{Re}} \\ \text{Re} &= \frac{64}{f} = \frac{64}{3.92} = 16.3\end{aligned}$$

Since  $\text{Re} \ll 2000$ , we conclude that the flow is laminar.

3. Head loss in laminar flow

$$h_f = \frac{32\mu LV}{\gamma D^2}$$

is linear with  $V$ . Thus, doubling  $Q$  will increase  $V$  by a factor of 2 and increase  $h_f$  by a factor of 2.

## 10.14: PROBLEM DEFINITION

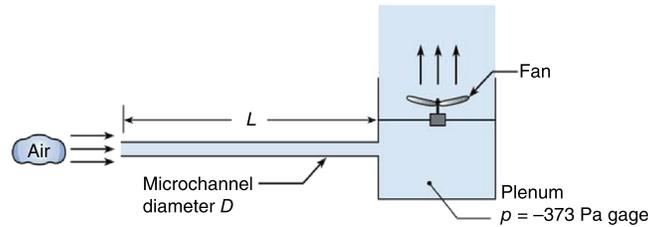
### Situation:

Air flows through a round tube.

$D = 0.0005$  m,  $L = 0.75$  m.

$p_2 = -373$  Pa gage.

### Sketch:



### Find:

Velocity in the tube (m/s).

### Assumptions:

Fully developed flow.

Laminar flow.

Only source of head loss is flow in tube.

### Properties:

Air (20°C, 1 atm), Table A.3,  $\mu = 1.81 \times 10^{-5}$  N · s/m<sup>2</sup>,  $\nu = 15.1 \times 10^{-6}$  m<sup>2</sup>/s.

## PLAN

1. Relate velocity to pressure using the energy equation.
2. Find head loss.
3. Find velocity by combining steps 1 and 2.
4. Check laminar flow assumption by calculating Re.
5. Assume flow is laminar, therefore use  $\alpha = 2$  in energy equation.

## SOLUTION

1. Energy eqn (point 1 back from tube inlet; point 2 at tube outlet):

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + 0 + 0 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + 0 + 0 + h_f \end{aligned} \quad (1)$$

2. Head loss (laminar flow):

$$h_f = \frac{32\mu LV}{\gamma D^2} \quad (2)$$

3. Combine Eq. (1) and (2), :

$$\begin{aligned} -p_2 &= \alpha_2 \frac{\rho V_2^2}{2} + \frac{32\mu L V_2}{D^2} \\ &\text{where } p_2 \text{ is } -373 \text{ Pa gage, and with } \alpha_2 = 2 \\ (+373 \text{ Pa}) &= 2 \frac{(1.2 \text{ kg/m}^3) V_2^2}{2} + \frac{32 (1.81 \times 10^{-5} \text{ N} \cdot \text{s/m}^2) (0.75 \text{ m}) V_2}{(0.0005 \text{ m})^2} \end{aligned}$$

Solve using quadratic equation:

$$\boxed{V_2 = 0.215 \text{ m/s}}$$

4. Reynolds number:

$$\text{Re} = \frac{VD}{\nu} = \frac{(0.215 \text{ m/s}) (0.0005 \text{ m})}{(15.1 \times 10^{-6} \text{ m}^2/\text{s})} = 7.1$$

Since  $\text{Re} < 2000$ , the laminar flow assumption is valid.

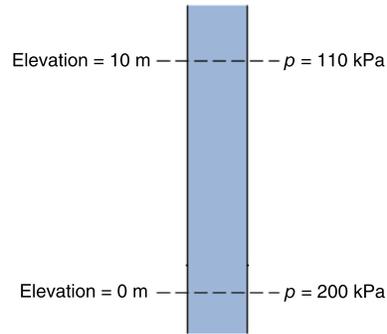
## 10.15: PROBLEM DEFINITION

### Situation:

Liquid flows in a vertical pipe; flow direction is unknown.

$$D = 0.008 \text{ m.}$$

### Sketch:



### Find:

Direction of flow.

Velocity (m/s).

### Properties:

$$\gamma = 10 \text{ kN/m}^3, \quad \mu = 3.0 \times 10^{-3} \text{ N} \cdot \text{s/m}^2.$$

## PLAN

1. Find the flow direction by calculating the piezometric head at locations 1 and 2. Rationale: using the HGL shows that flow goes from high to low piezometric head in a constant diameter pipe.
2. Find head loss using the energy eqn.
3. Find velocity by using the equation for head loss in laminar flow.

## SOLUTION

1. Piezometric Head (location 1 at  $z = 10 \text{ m}$ ):

$$h_1 = \frac{p_1}{\gamma} + z_1 = \frac{110 \text{ kPa}}{10 \text{ kN/m}^3} + 10 \text{ m} = 21 \text{ m}$$

$$h_2 = \frac{p_2}{\gamma} + z_2 = \frac{200 \text{ kPa}}{10 \text{ kN/m}^3} + 0 \text{ m} = 20 \text{ m}$$

Since  $h_1 > h_2$ , the direction of flow is downward.

2. Energy equation:

$$\begin{aligned}\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ \frac{p_1}{\gamma} + 0 + z_1 + 0 &= \frac{p_2}{\gamma} + 0 + z_2 + h_f \\ 21 \text{ m} &= 20 \text{ m} + h_f \\ h_f &= 1 \text{ m}\end{aligned}$$

3. Head loss (laminar flow):

$$\begin{aligned}h_f &= \frac{32\mu LV}{\gamma D^2} \\ V &= \frac{h_f \gamma D^2}{32\mu L} \\ &= \frac{(1 \text{ m})(10000 \text{ N/m}^3)(0.008 \text{ m})^2}{32(3.0 \times 10^{-3} \text{ N} \cdot \text{s/m}^2)(10 \text{ m})} \\ &= 0.667 \text{ m/s}\end{aligned}$$

$$\boxed{V = 0.667 \text{ m/s}}$$

---

**10.16: PROBLEM DEFINITION****Situation:**

Oil is pumped through a horizontal pipe.  $Q = 1.2 \times 10^{-4} \text{ m}^3/\text{s}$ .  
Nominal diameter = 25 mm Schedule 80.  $D = 0.024 \text{ m}$ .

**Find:**

Head loss (meters) per 30 m of pipe.

**Properties:**

Oil,  $S = 0.97$ ,  $\mu = 0.479 \text{ N}\cdot\text{s}/\text{m}^2$ .

**PLAN**

1. Find  $V$  using the flow rate equation.
2. Find flow regime by calculating  $\text{Re}$ .
3. Find  $h_f$ .

**SOLUTION**

1. Flow rate equation:

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4(1.2 \times 10^{-4} \text{ m}^3/\text{s})}{\pi (0.024 \text{ m})^2} = 0.27 \text{ m/s}$$

2. Reynolds number:

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} = \frac{(0.27 \text{ m/s})(0.024 \text{ m})(0.97 \times 999 \text{ kg/m}^3)}{(0.479 \text{ N}\cdot\text{s}/\text{m}^2)} \\ &= 12 \text{ (thus, flow is laminar)} \end{aligned}$$

3. Head loss (laminar flow):

$$\begin{aligned} h_f &= \frac{32\mu LV}{\gamma D^2} \\ &= \frac{32(0.479 \text{ N}\cdot\text{s}/\text{m}^2)(30 \text{ m})(0.27 \text{ m/s})}{(0.97 \times 9810 \text{ N}/\text{m}^3)} \end{aligned}$$

$$h_f = 22.7 \text{ m per 30 m run of pipe}$$

---

**10.17: PROBLEM DEFINITION**Situation:

A liquid flows in a pipe.  
 $D = 0.1 \text{ m}$ ,  $V = 1.5 \text{ m/s}$ .

Find:

Show that the flow is laminar.  
Friction factor  $f$ .  
Head loss per meter of pipe length. .

Properties:

$\rho = 1000 \text{ kg/m}^3$ ,  $\mu = 10^{-1} \text{ N} \cdot \text{s/m}^2$ ,  $\nu = 10^{-4} \text{ m}^2/\text{s}$ .

**SOLUTION**

1. Reynolds number:

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{(1.5 \text{ m/s})(0.1 \text{ m})}{(10^{-4} \text{ m}^2/\text{s})} \\ &= 1500 \end{aligned}$$

Since  $\text{Re} < 2000$ , the **flow is laminar**.

2. Friction factor:

$$f = \frac{64}{\text{Re}} = \frac{64}{1500}$$
$$\boxed{f = 0.043}$$

3. Darcy Weisbach eqn.:

$$\frac{h_f}{L} = \frac{f V^2}{D 2g} = \frac{0.043}{(0.1 \text{ m}) 2(9.81 \text{ m/s}^2)} (1.5 \text{ m/s})^2$$

$$\boxed{\frac{h_f}{L} = 0.049 \frac{\text{m}}{\text{m}} \quad \text{that is, m of } h_L \text{ per m of pipe length}}$$

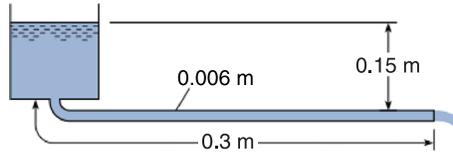
## 10.18: PROBLEM DEFINITION

### Situation:

Kerosene flows out a tank and through a tube.

$D = 0.006$  m,  $L = 0.3$  m.

$z_1 = 0.15$  m.



### Find:

Mean velocity in the tube.

Discharge.

### Assumptions:

Laminar flow so  $\alpha = 2$ .

Only head loss is in the tube.

### Properties:

Kerosene (20 °C):  $S = 0.8$ .

## PLAN

Apply the energy equation from the surface of the reservoir to the pipe outlet.

## SOLUTION

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V^2/2g + z_2 + 32\mu LV/(\gamma D^2) \\ (\text{where } \alpha_2 &= 2), \text{ thus} \\ 0 + 0 + 0.15 &= 0 + V^2/g + 32\mu LV/(\gamma D^2) \end{aligned}$$

Thus

$$\begin{aligned} V^2/g + 32\mu LV/(\gamma D^2) - 0.15 &= 0 \\ V^2/9.81 + 32(1.9 \times 10^{-3})(0.3)V/(0.80 \times 9810 \times (0.006)^2) - 0.15 &= 0 \\ V^2 + 0.628V - 1.5 &= 0 \end{aligned}$$

Solving the above quadratic equation for  $V$  yields:

$$V = 0.95 \text{ m/s}$$

Check Reynolds number to see if flow is laminar

$$\begin{aligned} \text{Re} &= VD\rho/\mu \\ &= 0.95 \times (0.006)(999 \times 0.8)/(1.9 \times 10^{-3}) \\ \text{Re} &= 2398 \text{ (laminar)} \\ Q &= VA \\ &= 0.95 \times (\pi/4)(0.006)^2 = 2.7 \times 10^{-5} \text{ m}^3/\text{s} \end{aligned}$$

$$\boxed{Q = 2.7 \times 10^{-5} \text{ m}^3/\text{s}}$$

---

**10.19: PROBLEM DEFINITION****Situation:**

Oil is pumped through a horizontal pipe.

$$D = 0.05 \text{ m}, \quad V = 0.5 \text{ m/s}.$$

**Find:**

Head loss per 10 m of pipe.

**Properties:**

$$S = 0.94, \quad \mu = 0.048 \text{ N} \cdot \text{s/m}^2.$$

**PLAN**

1. Determine flow regime (laminar or turbulent) by finding the Reynolds number.
2. Relate pressure drop to head loss using the energy eqn.
3. Apply eqn. for head loss in laminar flow.
4. Combine steps 2 and 3.

**SOLUTION**

1. Reynolds number:

$$\begin{aligned} \text{Re} &= \frac{\rho V D}{\mu} = \frac{(0.94 \times 1000 \text{ kg/m}^3) (0.5 \text{ m/s}) (0.05 \text{ m})}{(0.048 \text{ N} \cdot \text{s/m}^2)} \\ &= 490 \text{ (laminar flow)} \end{aligned}$$

2. Energy equation:

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ \frac{p_1}{\gamma} + 0 + 0 + 0 &= \frac{p_2}{\gamma} + 0 + 0 + 0 + h_f \\ \Delta p &= \gamma h_f \end{aligned} \tag{1}$$

3. Head loss (laminar flow):

$$h_f = \frac{32\mu LV}{\gamma D^2} \tag{2}$$

4. Combine Eqs. (1) and (2):

$$\Delta p = \gamma h_f = \frac{32\mu LV}{D^2} = \frac{32 (0.048 \text{ N} \cdot \text{s/m}^2) (10 \text{ m}) (0.5 \text{ m/s})}{(0.05 \text{ m})^2}$$

$$\Delta p = 3.07 \text{ kPa per 10 m of pipe length}$$

## 10.20: PROBLEM DEFINITION

### Situation:

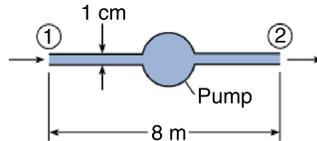
SAE 10W-30 oil is pumped through a tube.

$$L = 8 \text{ m}, D = .01 \text{ m}.$$

$$Q = 7.85 \times 10^{-4} \text{ m}^3/\text{s}$$

Pump efficiency:  $\eta = 1.0$

Pressures at points 1 and 2 are equal.



**Find:** Power to operate the pump.

**Properties:** SAE 10W-30 Oil (given in problem statement)  $\nu = 7.6 \times 10^{-5} \text{ m}^2/\text{s}$ ,  
 $\gamma = 8630 \text{ N/m}^3$ .

**PLAN** Power can be found using  $P = \gamma Q h_p$ . To find the terms on the right side, the step are:

1. Develop an equation for  $h_p$  by applying the energy equation.
2. Find velocity  $V$  by apply the flow rate equation.
3. Find the Reynolds number so we know flow regime (laminar or turbulent).
4. Calculate  $f$ .
5. Combine steps 1 to 4 and calculate  $h_p$ .
6. Calculate power.

### SOLUTION

1. Energy equation (from 1 to 2)

$$p_1/\gamma + z_1 + \alpha_1 V_1^2/2g + h_p = p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_L$$

Simplify

$$h_p = h_L = f(L/D)(V^2/2g)$$

2. Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= \frac{(7.85 \times 10^{-4} \text{ m}^3/\text{s})}{(\pi/4)(0.01 \text{ m})^2} \\ &= 10 \text{ m/s} \end{aligned}$$

3. Reynolds number

$$\text{Re} = VD/\nu = (10)(0.01)/(7.6 \times 10^{-5}) = 1316 \text{ (laminar)}$$

Therefore, select a friction factor for laminar flow

4. Friction factor ( $f$ )

$$\begin{aligned} f &= \frac{64}{\text{Re}} \\ &= \frac{64}{1316} \\ &= 0.0486 \end{aligned}$$

5. Head of the pump

$$\begin{aligned} h_p &= f(L/D)(V^2/2g) \\ &= 0.0486(8/0.01)(10^2/((2)(9.81))) \\ &= 198 \text{ m} \end{aligned}$$

6. Power equation

$$\begin{aligned} P &= h_p \gamma Q \\ &= (198 \text{ m}) (8630 \text{ N/m}^3) (7.85 \times 10^{-4} \text{ m}^3/\text{s}) \\ &= 1341 \text{ W} \end{aligned}$$

$$\boxed{P = 1340 \text{ W}}$$

### **REVIEW**

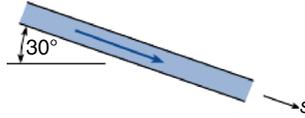
1. The head of the pump (198 m) is quite high because the velocity is high (10 m/s).
2. If the velocity was reduced, for example by specifying a larger tube, the required power would be reduced. For example, sizing the tube so  $V = 1 \text{ m/s}$  would reduce  $h_p$  by more than 100. This would reduce the pump power by more than 100!

### 10.21: PROBLEM DEFINITION

Situation:

Oil flows downward in a pipe.  
 $D = 0.03 \text{ m}$ ,  $V = 1 \text{ m/s}$ , Slope of  $30^\circ$ .

Sketch:



Find:

Pressure gradient along the pipe (Pa/m).

Properties:

Oil,  $S = 0.8$ ,  $\mu = 0.479 \text{ Pa} \cdot \text{s}$ ,  $\nu = 0.00053 \text{ m}^2/\text{s}$ .

### SOLUTION

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{(1 \text{ m/s})(0.03 \text{ m})}{0.00053 \text{ m}^2/\text{s}} \\ &= 56.6 \text{ (laminar)} \\ -\frac{d}{ds}(p + \gamma z) &= \frac{32\mu V}{D^2} \\ -\frac{dp}{ds} - \gamma \frac{dz}{ds} &= \frac{(32)(0.479 \text{ N}\cdot\text{s}/\text{m}^2)(1 \text{ m/s})}{(0.03 \text{ m})^2} \\ -\frac{dp}{ds} - \gamma(-1 \sin 30^\circ) &= 17,031 \text{ N}/\text{m}^3 \\ \frac{dp}{ds} &= (0.5)(0.8)(9810 \text{ N}/\text{m}^3) - 17,031 \text{ N}/\text{m}^3 \\ \frac{dp}{ds} &= 3924 - 17,031 \\ \boxed{\frac{dp}{ds} = -13,107 \text{ Pa}/\text{m}} \end{aligned}$$

## 10.22: PROBLEM DEFINITION

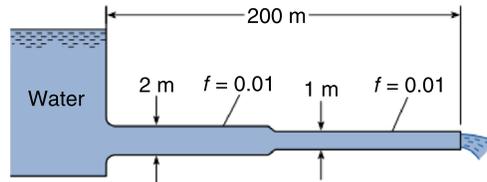
Situation:

Fluid flows out of a tank through a pipe with an abrupt contraction.

$$L_1 = L_2 = 100 \text{ m}, f = 0.01.$$

$$D_1 = 2 \text{ m}, D_2 = 1 \text{ m}.$$

Sketch:



Find:

Ratio of head loss.

$$\frac{h_L (1\text{-m pipe})}{h_L (2\text{-m pipe})}$$

## SOLUTION

$$\begin{aligned} h_L &= f \frac{L V^2}{D 2g} \\ \frac{h_L (1\text{-m pipe})}{h_L (2\text{-m pipe})} &= \left( \frac{f_1 L_1 V_1^2 / (D_1)}{f_2 L_2 V_2^2 / (D_2)} \right) \\ &= \left( \frac{D_2}{D_1} \right) \left( \frac{V_1^2}{V_2^2} \right) \\ V_1 A_1 &= V_2 A_2 \\ \frac{V_1}{V_2} &= \frac{A_2}{A_1} = \left( \frac{D_2}{D_1} \right)^2 \\ \left( \frac{V_1}{V_2} \right)^2 &= \left( \frac{D_2}{D_1} \right)^4 \end{aligned}$$

Thus

$$\begin{aligned} \frac{h_L (1\text{-m pipe})}{h_L (2\text{-m pipe})} &= \left( \frac{D_2}{D_1} \right) \left( \frac{D_2}{D_1} \right)^4 \\ &= \left( \frac{D_2}{D_1} \right)^5 = 2^5 = 32 \end{aligned}$$

Correct choice is (d)

---

**10.23: PROBLEM DEFINITION**Situation:

Glycerine flows in a pipe  
 $D = 0.15 \text{ m}$ ,  $\bar{V} = 0.45 \text{ m/s}$ .

Find:

Determine if the flow is laminar or turbulent.  
Plot the velocity distribution.

Properties:

Glycerine at  $20^\circ\text{C}$  from Table A.4:  
 $\mu = 1.41 \text{ N}\cdot\text{s/m}^2$ ,  $\nu = 1.12 \times 10^{-3} \text{ m}^2/\text{s}$ .

**SOLUTION**

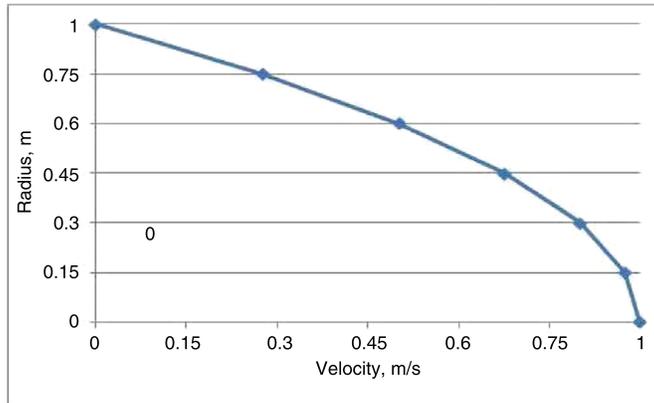
$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{0.45 \times 0.15}{1.12 \times 10^{-3} \text{ m}^2/\text{s}} \\ &= 60.26 \quad \boxed{\text{laminar}} \end{aligned}$$

To plot the velocity distribution, begin with Eq. (10.23) from EFM10e. Recall also for laminar flow that  $V_{\max} = 2\bar{V}$ .

$$\begin{aligned} V(r) &= V_{\max} \left( 1 - \frac{r^2}{r_o^2} \right) \\ &= (1 \text{ m/s}) \left( 1 - \left( \frac{r}{r_o} \right)^2 \right) \end{aligned}$$

Create a table of values and then plot

| $r$ (m) | $r/r_o$ | $V(r)$ (m/s) |
|---------|---------|--------------|
| 0       | 0       | 1            |
| 0.15    | 1/6     | 0.9          |
| 0.3     | 1/3     | 0.8          |
| 0.45    | 1/2     | 0.7          |
| 0.60    | 2/3     | 0.5          |
| 0.75    | 5/6     | 0.3          |
| 1       | 1       | 0            |

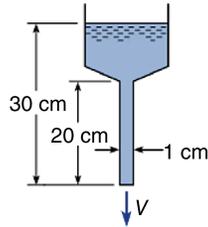


## 10.24: PROBLEM DEFINITION

### Situation:

Glycerin flows through a funnel  
 $D = 1 \text{ cm}$ ,  $L = 20 \text{ cm}$ .

### Sketch:



**Find:** Mean velocity (in m/s) at the exit.

### Assumptions:

Laminar flow ( $\alpha_2 = 2.0$ ).  
The only head loss is due to friction in tube.

### Properties:

Glycerin (20 °C), Table A.4:  
 $\rho = 1260 \text{ kg/m}^3$ ,  $\gamma = 12,300 \text{ N/m}^3$ .  
 $\mu = 1.41 \text{ N} \cdot \text{s/m}^2$ ,  $\nu = 1.12 \times 10^{-3} \text{ m}^2/\text{s}$ .

## SOLUTION

Energy equation (Let section 1 be the surface of the liquid and section 2 be the exit plane of the funnel).

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$0 + 0 + 0.30 = 0 + 2.0 \left( \frac{V_2^2}{2g} \right) + 0 + \frac{32\mu LV_2}{\gamma D^2}$$
$$0.30 = 2.0 \left( \frac{V_2^2}{2 \times 9.81} \right) + \left( \frac{32 \times 1.41 \text{ N s/m}^2 \times 0.2 \text{ m} \times V_2}{12300 \text{ N/m}^3 \times (0.01 \text{ m})^2} \right)$$

Solve quadratic equation.

$$V_2 = -72.01 \text{ m/s}$$

or,  $V_2 = 4.087 \times 10^{-2} \text{ m/s}$

Select the positive root

$$V_2 = 0.0409 \text{ m/s}$$

Check the laminar flow assumption

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{0.0409 \text{ m/s} \times 0.01 \text{ m} \times 1260 \text{ kg/m}^3}{1.41 \text{ N s/m}^2} \\ &= 0.365 \end{aligned}$$

Since  $\text{Re} \leq 2000$ , the laminar flow assumption is valid.

---

**10.25: PROBLEM DEFINITION**Situation:

Castor oil flows through a steel pipe.  
 $Q = 0.006 \text{ m}^3/\text{s}$ ,  $L = 0.8 \text{ km} = 800 \text{ m}$ .  
Allowable pressure drop is 70 kPa.

Find:

Nominal diameter of pipe ( m).

Assumptions:

Laminar flow.  
Horizontal pipe.

Properties:

Castor oil (32 °C):  $\mu = 4 \text{ Pa}\cdot\text{s}$ ,  $S = 0.85$ .

**SOLUTION**

1. Energy eqn.

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

• KE terms cancel.

•  $h_p = h_t = 0$ .  $h_L = h_f$

$$\begin{aligned} \frac{p_1}{\gamma} + 0 + 0 + 0 &= \frac{p_2}{\gamma} + 0 + 0 + 0 + h_f \\ \Delta p &= \gamma h_f \end{aligned} \tag{1}$$

2. Head loss (laminar flow)

$$h_f = \frac{32\mu LV}{\gamma D^2} \tag{2}$$

3. Combine Eq. (1) and (2)

$$\Delta p = \frac{32\mu LV}{D^2}$$

4. Let  $V = Q/A$

$$\Delta p = \frac{32\mu LQ}{(\pi/4) \times D^4}$$

5. Solve for diameter.

$$\begin{aligned} D^4 &\geq \frac{128\mu LQ}{\pi \Delta p} \\ &\geq \frac{128 (4 \text{ Pa}\cdot\text{s}) (800 \text{ m}) (0.006 \text{ m}^3/\text{s})}{\pi (70,000 \text{ Pa})} \\ D &\geq 0.33 \text{ m} \end{aligned}$$

6. Select a nominal pipe size.

$$D > 33 \text{ cm}$$

Thus, select a 35 cm nominal diameter NPS schedule 40 pipe ( $ID = 34 \text{ cm}$ )

7. Check Laminar flow assumption:

Velocity:

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{(0.006 \text{ m}^3/\text{s})}{\pi/4 (0.33 \text{ m})^2} \\ &= 0.07 \text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{(0.07 \text{ m/s}) (0.33 \text{ m}) (0.85 \times 999 \text{ kg/m}^3)}{(4 \text{ Pa}\cdot\text{s})} \\ &= 4.9 \end{aligned}$$

Thus, the initial assumption of laminar flow is valid.

---

**10.26: PROBLEM DEFINITION**Situation:

Velocity measurements are made in a pipe.

$$D = 0.3 \text{ m}, V_{center} = V_{max} = 1.5 \text{ m/s.}$$

$$\Delta p = 1.9 \text{ kPa per } 100 \text{ m of pipe.}$$

Find:

Kinematic viscosity of fluid ( $\text{m}^2/\text{s}$ ).

Assumptions:

Laminar flow (since velocity profile is parabolic).

Horizontal pipe.

Properties:

$$S = 0.8$$

**SOLUTION**

1. Energy eqn.

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

- KE terms cancel.
- Assume horizontal pipe.
- $h_p = h_t = 0$ .  $h_L = h_f$

$$\begin{aligned} \frac{p_1}{\gamma} + 0 + 0 + 0 &= \frac{p_2}{\gamma} + 0 + 0 + 0 + h_f \\ \Delta p &= \gamma h_f \end{aligned} \tag{1}$$

2. Head loss (laminar flow)

$$h_f = \frac{32\mu LV}{\gamma D^2} \tag{2}$$

3. Combine Eq. (1) and (2)

$$\begin{aligned} \Delta p &= \frac{32\mu LV}{D^2} \\ \nu &= \frac{\mu}{\rho} = \frac{\Delta p D^2}{32\rho LV}; \text{ where } V_{avg} = \frac{1}{2}V_{max} = 0.75 \text{ m/s} \\ &= \frac{(1900 \text{ Pa})(0.3 \text{ m})^2}{32(0.8 \times 1000 \text{ kg/m}^3)(100 \text{ m})(0.75 \text{ m/s})} \end{aligned}$$

$$\boxed{\nu = 8.91 \times 10^{-5} \text{ m}^2/\text{s}}$$

## 10.27: PROBLEM DEFINITION

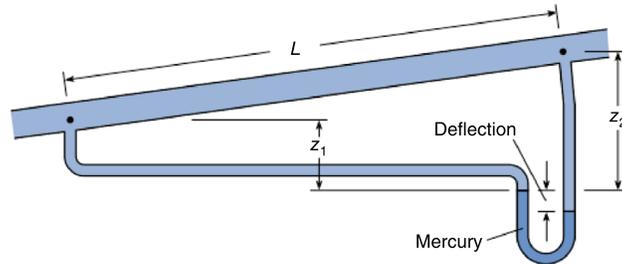
### Situation:

Oil flows through a smooth pipe.

$$L = 12 \text{ m}, z_1 = 1 \text{ m}, z_2 = 2 \text{ m}.$$

$$V = 1.2 \text{ m/s}, D = 5 \text{ cm}.$$

### Sketch:



### Find:

Flow direction.

Resistance coefficient.

Nature of flow (laminar or turbulent).

Viscosity of oil ( $\text{N s/m}^2$ ).

### Properties:

$$S_{oil} = 0.8; S_{Hg} = 13.6.$$

## SOLUTION

Based on the deflection on the manometer, the piezometric head (and HGL) on the right side of the pipe is larger than that on the left side.

Thus, the flow is downward (from right to left).

Energy principle

$$\frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_L$$

Assume  $\alpha_1 V_1 = \alpha_2 V_2$ . Let  $z_2 - z_1 = 1 \text{ m}$ . Also the head loss is given by the Darcy Weisbach equation:  $h_f = f(L/D)V^2/(2g)$ . The energy principle becomes

$$\frac{p_2 - p_1}{\gamma_{oil}} = (-1 \text{ m}) + f \frac{L}{D} \frac{V^2}{2g} \quad (1)$$

Manometer equation

$$p_2 + (2 \text{ m}) \gamma_{oil} + (0.1 \text{ m}) \gamma_{oil} - (0.1 \text{ m}) \gamma_{Hg} - (1 \text{ m}) \gamma_{oil} = p_1$$

Algebra gives

$$\begin{aligned}
\frac{p_2 - p_1}{\gamma_{\text{oil}}} &= -(2 \text{ m}) - (0.1 \text{ m}) + (0.1 \text{ m}) \frac{\gamma_{\text{Hg}}}{\gamma_{\text{oil}}} + (1 \text{ m}) \\
&= -(1 \text{ m}) + (0.1 \text{ m}) \left( \frac{S_{\text{Hg}}}{S_{\text{oil}}} - 1 \right) \\
&= -(1 \text{ m}) + (0.1 \text{ m}) \left( \frac{13.6}{0.8} - 1 \right) \\
\frac{p_2 - p_1}{\gamma_{\text{oil}}} &= 0.6 \text{ m} \tag{2}
\end{aligned}$$

Substituting Eq. (2) into (1) gives

$$\begin{aligned}
(0.6 \text{ m}) &= (-1 \text{ m}) + f \frac{L V^2}{D 2g} \\
&\text{or} \\
f &= 1.6 \left( \frac{D}{L} \right) \left( \frac{2g}{V^2} \right) \\
&= 1.6 \left( \frac{0.05 \text{ m}}{12 \text{ m}} \right) \left( \frac{2 \times 9.81 \text{ m/s}^2}{(1.2 \text{ m/s})^2} \right) \\
&\boxed{f = 0.0908}
\end{aligned}$$

Since the resistance coefficient is now known, this value can be used to find viscosity. To perform this calculation, assume the flow is laminar.

$$\begin{aligned}
f &= \frac{64}{\text{Re}} \\
0.0908 &= \frac{64\mu}{\rho V D} \\
&\text{or} \\
\mu &= \frac{0.0908 \rho V D}{64} \\
&= \frac{0.0908 \times (0.8 \times 1000) \times 1.2 \times 0.05}{64} \\
&\boxed{\mu = 0.068 \text{ N} \cdot \text{s} / \text{m}^2}
\end{aligned}$$

Now, check Reynolds number to see if laminar flow assumption is valid

$$\begin{aligned}
\text{Re} &= \frac{VD\rho}{\mu} \\
&= \frac{1.2 \times 0.05 \times (0.8 \times 1000)}{0.068} \\
&= 706
\end{aligned}$$

Thus, flow is laminar.

## 10.28: PROBLEM DEFINITION

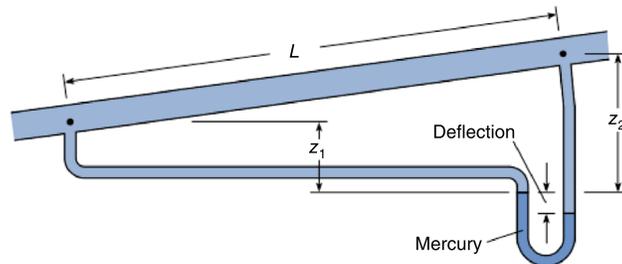
### Situation:

Oil flows through a smooth pipe.

$$D = 0.05 \text{ m}, \quad V = 1.5 \text{ m/s.}$$

$$L = 9 \text{ m}, \quad z_1 = 0.6 \text{ m}, \quad z_2 = 1.2 \text{ m.}$$

### Sketch:



### Find:

The direction of the flow.

Resistance coefficient.

Nature of the flow (laminar or turbulent).

Viscosity of oil ( $\text{Ns/m}^2$ ).

### Properties:

Oil,  $S = 0.8$ ,  $S_{Hg} = 13.6$ .

## SOLUTION

Based on the deflection on the manometer, the piezometric head (and HGL) on the right side of the pipe is larger than that on the left side.

Thus, the flow is downward (from right to left).

Energy principle

$$\frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_L$$

Term by term analysis

$$\alpha_1 V_1 = \alpha_2 V_2; \quad z_2 - z_1 = 2 \text{ ft}$$

Darcy Weisbach equation

$$h_L = f(L/D)V^2/(2g)$$

Combine equations

$$\frac{p_2 - p_1}{\gamma_{\text{oil}}} = (-0.6 \text{ m}) + f \frac{L}{D} \frac{V^2}{2g} \quad (1)$$

Manometer equation

$$p_2 + (1.2 \text{ m}) \gamma_{\text{oil}} + (0.099 \text{ m}) \gamma_{\text{oil}} - (0.099 \text{ m}) \gamma_{\text{Hg}} - (0.6 \text{ m}) \gamma_{\text{oil}} = p_1$$

Calculate values

$$\begin{aligned}
 \frac{p_2 - p_1}{\gamma_{\text{oil}}} &= -(1.2 \text{ m}) - (0.099 \text{ m}) + (0.099 \text{ m}) \frac{\gamma_{\text{Hg}}}{\gamma_{\text{oil}}} + (0.6 \text{ m}) \\
 &= -(0.6 \text{ m}) + (0.099 \text{ m}) \left( \frac{S_{\text{Hg}}}{S_{\text{oil}}} - 1 \right) \\
 &= -(0.6 \text{ m}) + (0.099 \text{ m}) \left( \frac{13.6}{0.8} - 1 \right) \\
 \frac{p_2 - p_1}{\gamma_{\text{oil}}} &= 0.984 \text{ m} \tag{2}
 \end{aligned}$$

Substitute Eq. (2) into (1)

$$\begin{aligned}
 (0.984 \text{ m}) &= (-0.6 \text{ m}) + f \frac{L V^2}{D 2g} \\
 &\text{or} \\
 f &= 1.584 \left( \frac{D}{L} \right) \left( \frac{2g}{V^2} \right) \\
 &= 1.584 \left( \frac{0.05 \text{ m}}{9 \text{ m}} \right) \left( \frac{2 \times 9.81 \text{ m/s}^2}{(1.5 \text{ m/s})^2} \right) \\
 &\boxed{f = 0.076}
 \end{aligned}$$

Since the resistance coefficient ( $f$ ) is now known, use this value to find viscosity.

Resistance coefficient ( $f$ ) (assume laminar flow)

$$\begin{aligned}
 f &= \frac{64}{\text{Re}} \\
 0.076 &= \frac{64\mu}{\rho V D} \\
 &\text{or} \\
 \mu &= \frac{0.076 \rho V D}{64} \\
 &= \frac{0.076 \times (0.8 \times 999 \text{ kg/m}^3) \times 1.5 \text{ m/s} \times (0.05 \text{ m})}{64} \\
 &\boxed{\mu = 0.071 \text{ N} \cdot \text{s/m}^2}
 \end{aligned}$$

Check laminar flow assumption

$$\begin{aligned}
 \text{Re} &= \frac{VD\rho}{\mu} \\
 &= \frac{1.5 \text{ m/s} \times (0.05 \text{ m}) \times (0.8 \times 999 \text{ kg/m}^3)}{0.071 \text{ Ns/m}^2} \\
 &= 840
 \end{aligned}$$

Answer  $\Rightarrow$   $\boxed{\text{Flow is laminar}}$ .

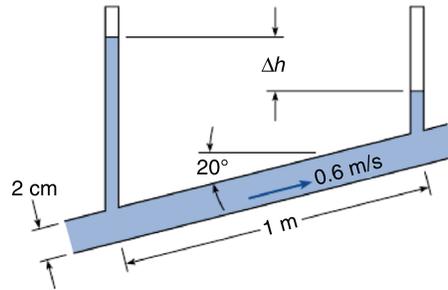
## 10.29: PROBLEM DEFINITION

### Situation:

Glycerin flows through a commercial steel pipe connected to two piezometers.

$$D = 2 \text{ cm}, V = 0.6 \text{ m/s}.$$

### Sketch:



### Find:

Height differential ( in m).

### Properties:

Glycerin (20°C), Table A.4,  $\mu = 1.41 \text{ N} \cdot \text{s} / \text{m}^2$ ,  $\nu = 1.12 \times 10^{-3} \text{ m}^2 / \text{s}$ .

## SOLUTION

Energy equation (apply from one piezometer to the other)

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_L \\ p_1/\gamma + z_1 &= p_2/\gamma + z_2 + h_L \\ ((p_1/\gamma) + z_1) - ((p_2/\gamma) + z_2) &= h_L \\ \Delta h &= h_L \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{(0.6)(0.02)}{1.12 \times 10^{-3}} \\ &= 10.71 \end{aligned}$$

Since  $\text{Re} < 2000$ , the flow is laminar. The head loss for laminar flow is

$$\begin{aligned} h_L &= \frac{32\mu LV}{\gamma D^2} \\ &= \frac{(32)(1.41)(1)(0.6)}{12300 \times 0.02^2} \\ &= \boxed{5.502 \text{ m}} \end{aligned}$$

Energy equation

$$\Delta h = h_L$$

$\Delta h = 5.502 \text{ m}$

### 10.30: PROBLEM DEFINITION

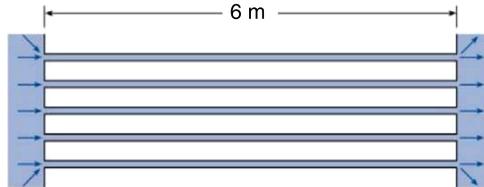
Situation:

Water is pumped through tubes in a heat exchanger

$D = 6 \text{ mm}$ ,  $L = 6 \text{ m}$ ,  $V = 0.12 \text{ m/s}$ .

$T_1 = 20^\circ\text{C}$ ,  $T_2 = 30^\circ\text{C}$ .

Sketch:



Find:

Pressure difference across heat exchanger.

Properties:

Water ( $20^\circ\text{C}$ ), Table A.5:  $\nu = 10^{-6} \text{ m}^2/\text{s}$ .

### SOLUTION

Reynolds number (based on temperature at the inlet)

$$\text{Re}_{20^\circ} = \frac{VD}{\nu} = \frac{0.12 \text{ m/s} \times 0.006 \text{ m}}{10^{-6} \text{ m}^2/\text{s}} = 720$$

Since  $\text{Re} \leq 2000$ , the flow is laminar. Thus,

$$\Delta p = \frac{32\mu LV}{D^2}$$

Assume linear variation in  $\mu$  and use the temperature at  $25^\circ\text{C}$ . From Table A.5

$$\begin{aligned}\mu_{\text{avg.}} &= \mu_{25^\circ} \\ &= 8.91 \times 10^{-4} \text{ N} \cdot \text{s}/\text{m}^2\end{aligned}$$

and

$$\begin{aligned}\Delta p &= \frac{32\mu LV}{D^2} \\ &= \frac{32 \times 8.91 \times 10^{-4} \text{ N s}/\text{m}^2 \times 6 \text{ m} \times 0.12 \text{ m/s}}{(0.006 \text{ m})^2} = 570 \text{ Pa}\end{aligned}$$

$$\boxed{\Delta p = 570 \text{ Pa}}$$

---

**10.31: PROBLEM DEFINITION**

Find:

Use Figure 10.3, Table 10.3, and Table 10.4 (in §10.6 of EFM10e) to assess the following statements as True or False:

- a. If  $k_s/D$  is 0.05 or larger, and the flow is turbulent, the value of  $f$  is not dependent on  $Re_D$ .
- b. For smooth pipes and turbulent flow,  $f$  depends on  $k_s/D$  and not  $Re_D$ .
- c. For laminar flow,  $f$  is always given by  $f = 64/Re_D$ .
- d. If  $Re_D = 2 \times 10^7$  and  $k_s/D = 0.00005$ , then  $f = 0.012$ .
- e. If  $Re_D = 1000$  and the pipe is smooth,  $f = 0.04$ .
- f. The sand roughness height  $k_s$  for wrought iron is 0.002 mm.

**SOLUTION**

- a. If  $k_s/D$  is 0.05 or larger, and the flow is turbulent, the value of  $f$  is not dependent on  $Re_D$ . **True**
- b. For smooth pipes and turbulent flow,  $f$  depends on  $k_s/D$  and not  $Re_D$ . **False**
- c. For laminar flow,  $f$  is always given by  $f = 64/Re_D$ . **True**
- d. If  $Re_D = 2 \times 10^7$  and  $k_s/D = 0.00005$ , then  $f = 0.012$ . **False**;  $f = .011$ , or, more precisely,  $f = .0108$ . Student would only select this as true if they incorrectly read the figure for  $Re_D = 2 \times 10^6$
- e. If  $Re_D = 1000$  and the pipe is smooth,  $f = 0.04$ . **False**;  $f = 64/Re_D = 0.06$
- f. The sand roughness height  $k_s$  for wrought iron is 0.002 mm. **False**; it is 0.002 inches, or 0.046 mm.

---

**10.32: PROBLEM DEFINITION**Situation:

Water flows through a PVC pipe.  
100 mm Schedule 40.  $D = 0.1023$  m.  
 $Q = 0.03$  m<sup>3</sup>/s,  $k_s = 0$ .

Find:

Resistance coefficient  $f$ .

Properties:

Water (21 °C), Table A.5,  $\nu = 9.8 \times 10^{-7}$  m<sup>2</sup>/s.

**SOLUTION**

1. Reynolds number.

$$\begin{aligned} \text{Re} &= \frac{4Q}{\pi D \nu} = \frac{4(0.03 \text{ m}^3/\text{s})}{\pi(0.1023 \text{ m})(9.8 \times 10^{-7} \text{ m}^2/\text{s})} \\ &= 3.81 \times 10^5 \text{ (turbulent flow)} \end{aligned}$$

2. Swamee and Jain Eqn (10.39 in EFM 10e)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10} \left( 0 + \frac{5.74}{(3.81 \times 10^5)^{0.9}} \right)\right]^2} = 0.0139 \end{aligned}$$

$$\boxed{f = 0.0139}$$

---

**10.33: PROBLEM DEFINITION**

Situation:

Water flows through a brass tube.

$$k_s = 0, \quad D = 2 \text{ cm.}$$

$$Q = 0.003 \text{ m}^3/\text{s}.$$

Find:

Resistance coefficient,  $f$ .

**SOLUTION**

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.003 \text{ m}^3/\text{s}}{\pi/4 \times (0.02 \text{ m})^2} \\ &= 9.55 \text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{9.55 \text{ m/s} \times 0.02 \text{ m}}{10^{-6} \text{ m}^2/\text{s}} \\ &= 1.91 \times 10^5 \end{aligned}$$

Friction factor (Swamee-Jain correlation)

$$\begin{aligned} f &= \frac{0.25}{\left[ \log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \\ &= \frac{0.25}{\left[ \log_{10} \left( 0 + \frac{5.74}{(1.91 \times 10^5)^{0.9}} \right) \right]^2} \\ &= \boxed{f = 0.0157} \end{aligned}$$

---

**10.34: PROBLEM DEFINITION**

Situation:

Water flows through a smooth pipe.

$$D = 0.25 \text{ m}, Q = 0.05 \text{ m}^3/\text{s}.$$

$$k_s = 0.$$

Find:

Resistance coefficient,  $f$ .

Properties:

Water (10 °C), Table A.5,  $\nu = 1.31 \times 10^{-6} \text{ m}^2/\text{s}$ .

**SOLUTION** a

1. Reynolds number

$$\begin{aligned} \text{Re} &= \frac{4Q}{\pi D \nu} \\ &= \frac{4(0.05 \text{ m}^3/\text{s})}{\pi(0.25 \text{ m})(1.31 \times 10^{-6} \text{ m}^2/\text{s})} = 1.94 \times 10^5 \end{aligned}$$

2. Moody diagram:

$$f = 0.016$$

---

**10.35: PROBLEM DEFINITION****Situation:**

Water flows through a cast-iron pipe.

$$D = 10 \text{ cm}, V = 4 \text{ m/s}.$$

**Find:**

Calculate the resistance coefficient.

Plot the velocity distribution.

**Properties:**

Water (10 °C), Table A.5:  $\nu = 1.31 \times 10^{-6} \text{ m}^2/\text{s}$ .

**SOLUTION**

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{4 \text{ m/s}(0.1 \text{ m})}{1.31 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= 3.053 \times 10^5 \end{aligned}$$

Sand roughness height

$$\begin{aligned} \frac{k_s}{D} &= \frac{0.00026}{0.1} \\ &= 0.0026 \end{aligned}$$

Resistance coefficient (Swamee-Jain correlation; turbulent flow)

$$\begin{aligned} f &= \frac{0.25}{\left[ \log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \\ &= \frac{0.25}{\left[ \log_{10} \left( \frac{0.0026}{3.7} + \frac{5.74}{(3.053 \times 10^5)^{0.9}} \right) \right]^2} \\ &= 0.0258 \end{aligned}$$

$$\boxed{f = 0.0258}$$

---

**10.36: PROBLEM DEFINITION****Situation:**

A fluid flows in a smooth pipe.

$$D = 100 \text{ mm}, \bar{V} = 500 \text{ mm/s}.$$

**Find:**

- (a) Maximum velocity (m/s).
- (b) Resistance coefficient.
- (c) Shear velocity (m/s).
- (d) Shear stress 25 mm from pipe center (N/m<sup>2</sup>).
- (e) Determine if the head loss will double if discharge is doubled.

**Properties:**

$$\mu = 10^{-2} \text{ N} \cdot \text{s/m}^2, \rho = 800 \text{ kg/m}^3.$$

**SOLUTION**

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{(0.5 \text{ m/s})(0.1 \text{ m})(800 \text{ kg/m}^3)}{10^{-2}} \\ &= 4000 \end{aligned}$$

Because  $\text{Re} > 2000$ , assume the flow is turbulent.

a) Table 10.2 relates mean and centerline velocity. From this table,

$$\begin{aligned} V_{\max} &= \frac{\bar{V}}{0.791} \\ &= \frac{0.50 \text{ m/s}}{0.791} \\ &\boxed{V_{\max} = 0.632 \text{ m/s}} \end{aligned}$$

b) Resistance coefficient (from Moody diagram)

$$\boxed{f = 0.041}$$

c) Shear velocity is defined as

$$u_* = \sqrt{\frac{\tau_o}{\rho}} \tag{1}$$

Wall shear stress

$$\tau_o = \frac{f}{4} \frac{\rho V^2}{2}$$

Combine equations

$$\begin{aligned}u_* &= V \left( \frac{f}{8} \right)^{0.5} \\&= (0.5 \text{ m/s}) \left( \frac{0.041}{8} \right)^{0.5} \\&= \sqrt{\frac{0.041 \times 0.5^2}{8}} \\&\boxed{u_* = 0.0358 \text{ m/s}}\end{aligned}$$

d) In a pipe flow, shear stress is linear with distance from the wall. The distance of 25 mm from the center of the pipe is half way between the wall and the centerline. Thus, the shear stress is 1/2 of the wall value:

$$\tau_{25 \text{ mm}} = \frac{\tau_o}{2}$$

The shear stress at the wall is given by Eq. (1)

$$\begin{aligned}\tau_o &= \rho u_*^2 \\&= 800 \text{ kg/m}^3 \times (0.0358 \text{ m/s})^2 \\&= 1.025 \text{ N/m}^2\end{aligned}$$

Thus

$$\begin{aligned}\tau_{25 \text{ mm}} &= \frac{\tau_o}{2} \\&= \frac{1.025 \text{ N/m}^2}{2} \\&\boxed{\tau_{25 \text{ mm}} = 0.513 \text{ N/m}^2}\end{aligned}$$

e) If flow rate ( $Q$ ) is doubled, the velocity will also double. Thus, head loss will be given by

$$h_f = f_{\text{new}} \left( \frac{L}{D} \right) \frac{(2V)^2}{2g}$$

The increase in velocity will increase Reynolds number, thereby decreasing the friction factor so that  $f_{\text{new}} < f_{\text{original}}$ . Overall the head loss will increase by slightly less than a factor of 4.0.

**No, the increase in head loss will be closer to a factor of 4.0**

---

**10.37: PROBLEM DEFINITION****Situation:**

Water flows in a cast iron pipe.

$$D = 0.16 \text{ m}, \quad Q = 0.1 \text{ m}^3/\text{s}.$$

$$k_s = 0.26 \text{ mm}.$$

**Find:**

Reynolds number.

Friction factor,  $f$ .

Shear stress at the wall ( $Pa$ ).

**Properties:**

Water (20 °C), Table A.5:  $\rho = 998 \text{ kg/m}^3$ ,  $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$ .

**SOLUTION**

1. Flow rate eqn.

$$\begin{aligned} V &= \frac{Q}{A} = \frac{(0.1 \text{ m}^3/\text{s})}{(\pi/4)(0.16 \text{ m})^2} \\ &= 4.974 \text{ m/s} \end{aligned}$$

2. Reynolds number

$$\text{Re} = \frac{VD}{\nu} = \frac{(4.974 \text{ m/s})(0.16 \text{ m})}{(1.00 \times 10^{-6} \text{ m}^2/\text{s})} = 7.96 \times 10^5$$

$$\boxed{\text{Re} = 7.96 \times 10^5}$$

3. Relative roughness

$$\frac{k_s}{D} = \frac{0.26 \text{ mm}}{160 \text{ mm}} = 1.625 \times 10^{-3}$$

4. Swamee Jain eqn.

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10} \left( \frac{1.625 \times 10^{-3}}{3.7} + \frac{5.74}{(7.96 \times 10^5)^{0.9}} \right)\right]^2} = 0.0225 \end{aligned}$$

$$\boxed{f = 0.0225}$$

5. Definition of  $f$ :

$$\begin{aligned} \tau_0 &= \frac{f\rho V^2}{8} = \frac{0.023(998 \text{ kg/m}^3)(4.974 \text{ m/s})^2}{8} \\ &= 69.55 \text{ Pa} \end{aligned}$$

$$\boxed{\tau_0 = 69.6 \text{ Pa}}$$

---

**10.38: PROBLEM DEFINITION**

Situation:

Water flows in a uncoated cast iron pipe.

$$D = 0.1 \text{ m}, Q = 0.0006 \text{ m}^3/\text{s}.$$

Find: Resistance coefficient  $f$ .

Properties:

From Table A.5 (15 °C):  $\nu = 1.14 \times 10^{-6} \text{ m}^2/\text{s}$ .

From Table 10.4:  $k_s = 0.00026 \text{ m}$ .

**SOLUTION**

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{4Q}{\pi D \nu} \\ &= \frac{4 \times 0.0006 \text{ m}^3/\text{s}}{\pi \times 0.1 \text{ m} \times (1.14 \times 10^{-6} \text{ m}^2/\text{s})} \\ &= 6.7 \times 10^3 \end{aligned}$$

Sand roughness height

$$\begin{aligned} \frac{k_s}{D} &= \frac{0.00026}{0.1} \\ &= 0.0026 \end{aligned}$$

Friction factor (from Moody diagram)

$$f = 0.038$$

---

**10.39: PROBLEM DEFINITION**

Situation:

Fluid flows in a concrete pipe.

$D = 15 \text{ cm}$ ,  $L = 270 \text{ m}$ .

$Q = 0.085 \text{ m}^3/\text{s}$ ,  $k_s = 0.00006 \text{ m}$ .

Find:

Head loss (m).

Properties:

$\rho = 773 \text{ kg/m}^3$ ,  $\nu = 3 \times 10^{-4} \text{ m}^2/\text{s}$ .

**SOLUTION**

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{4Q}{\pi D \nu} \\ &= \frac{4(0.085 \text{ m}^3/\text{s})}{\pi(0.15 \text{ m})3 \times 10^{-4} \text{ m}^2/\text{s}} \\ &= 2406 \text{ (unpredictable range; assume laminar)} \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{\pi D^2/4} \\ &= \frac{0.085 \text{ m}^3/\text{s}}{\pi/4 \times (0.15 \text{ m})^2} \\ &= 4.8 \text{ m/s} \end{aligned}$$

Head loss (laminar flow)

$$\begin{aligned} h_f &= \frac{32\mu LV}{\gamma D^2} \\ &= \frac{32 \times (0.24 \text{ N/m}^2) \times 270 \text{ m} \times 4.8 \text{ m/s}}{773 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times (0.15 \text{ m})^2} = 58.3 \text{ m} \end{aligned}$$

$$\boxed{h_f = 58 \text{ m}}$$

---

**10.40: PROBLEM DEFINITION**

Situation:

Crude oil flows through a steel pipe.

$$D = 15 \text{ cm}, Q = 0.03 \text{ m}^3/\text{s}.$$

$$p_B = 300 \text{ kPa}, L = 1.5 \text{ km}.$$

$$z_B = 20 \text{ m}$$

Find:

Pressure at point  $A$  (kPa).

Properties:

$$S = 0.82, \mu = 10^{-2} \text{ N s/m}^2.$$

$$\text{From Table 10.4: } k_s = 4.6 \times 10^{-5} \text{ m}.$$

$$\rho_{\text{water}} \text{ at } 40^\circ\text{C} = 1000 \text{ kg/m}^3$$

**SOLUTION**

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{4Q}{\pi D\nu} \\ &= \frac{4 \times 0.03 \text{ m}^3/\text{s}}{\pi \times 0.15 \text{ m} \times (10^{-2} \text{ N s/m}^2 / (\rho \times 0.82))} \\ &= 2.09 \times 10^4 \text{ (turbulent)} \end{aligned}$$

Sand roughness height

$$\begin{aligned} \frac{k_s}{D} &= \frac{4.6 \times 10^{-5}}{0.15} \\ &= 3.1 \times 10^{-4} \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.03 \text{ m}^3/\text{s}}{\pi \times (0.15 \text{ m})^2 / 4} \\ &= 1.698 \text{ m/s} \end{aligned}$$

Resistance coefficient (Swamee-Jain correlation; turbulent flow)

$$\begin{aligned} f &= \frac{0.25}{\left[ \log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \\ &= \frac{0.25}{\left[ \log_{10} \left( \frac{4.6 \times 10^{-5}}{3.7(0.15)} + \frac{5.74}{(2.09 \times 10^4)^{0.9}} \right) \right]^2} \\ &= 0.026 \end{aligned}$$

Darcy Weisbach equation

$$\begin{aligned}h_f &= f \frac{L V^2}{D 2g} \\&= 0.026 \left( \frac{1500 \text{ m}}{0.15 \text{ m}} \right) \left( \frac{(1.698 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} \right) \\&= 38.19 \text{ m}\end{aligned}$$

Energy equation

$$p_A/\gamma + \alpha_A V_A^2/2g + z_A = p_B/\gamma + \alpha_B V_B^2/2g + z_B + h_f$$

$$p_A = 0.82 \times 9810[(300000/(0.82 \times 9810)) + 20 + 38.19] = 768 \text{ kPa}$$

$$\boxed{p_A = 768 \text{ kPa}}$$

---

**10.41: PROBLEM DEFINITION**

Situation:

A pipe is being using to measure viscosity of a fluid.

$D = 1.5 \text{ cm}$ ,  $L = 1 \text{ m}$ .

$V = 4 \text{ m/s}$ ,  $h_f = 50 \text{ cm}$ .

Find:

Kinematic viscosity.

**SOLUTION**

$$\begin{aligned}h_f &= f \left( \frac{L}{D} \right) \left( \frac{V^2}{2g} \right) \\0.50 &= f \left( \frac{1.0 \text{ m}}{0.015 \text{ m}} \right) \left( \frac{(4 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} \right) \\f &= 0.0092\end{aligned}$$

At this value of friction factor, Reynolds number can be found from the Moody diagram. The result is:

$$\text{Re} = 1.0 \times 10^8$$

Thus

$$\begin{aligned}\nu &= \frac{VD}{\text{Re}} \\&= \frac{(4 \text{ m/s})(0.015 \text{ m})}{1.0 \times 10^8} = 6.0 \times 10^{-10} \text{ m}^2/\text{s}\end{aligned}$$

$$\boxed{\nu = 6.0 \times 10^{-10} \text{ m}^2/\text{s}}$$

---

**10.42: PROBLEM DEFINITION**

Situation:

For a selected pipe:

$$f = 0.06, D = 40 \text{ cm.}$$

$$V = 3 \text{ m/s}, \nu = 10^{-5} \text{ m}^2/\text{s}.$$

Find:

Change in head loss per unit meter if the velocity were doubled.

**SOLUTION**

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{3 \text{ m/s} \times 0.4 \text{ m}}{10^{-5} \text{ m}^2/\text{s}} \\ &= 1.2 \times 10^5 \end{aligned}$$

Since  $\text{Re} > 3000$ , the flow is turbulent and obviously the conduit is very rough ( $f = 0.06$ ); therefore, one would expect  $f$  to be virtually constant with increased velocity. Since  $h_f = f(L/D)(V^2/2g)$ , we expect,  $h_f \sim V^2$ , so if the velocity is doubled, the head loss will be **quadrupled.**

---

**10.43: PROBLEM DEFINITION**Situation:

Water flows through a horizontal run of PVC pipe

$$V = 1.5 \text{ m/s}, \quad L = 30 \text{ m.}$$

Nominal diameter 65 mm Schedule 40  $D = 0.0622 \text{ m}$ .

Find:

- (a) Pressure drop in kPa.
- (b) Head loss in meters.
- (c) Power in kilowatts needed to overcome the head loss.

Assumptions:

Assume  $k_s = 0$ .

Assume  $\alpha_1 = \alpha_2$ , where subscripts 1 and 2 denote the inlet and exit of the pipe.

Properties:

Water ( $10^\circ\text{C}$ ), Table A.5:

$$\rho = 1000 \text{ kg/m}^3, \quad \gamma = 9810 \text{ N/m}^3, \quad \nu = 1.31 \times 10^{-6} \text{ m}^2/\text{s}.$$

**PLAN**

To establish laminar or turbulent flow, calculate the Reynolds number. Then find the appropriate friction factor ( $f$ ) and apply the Darcy-Weisbach equation to find the head loss. Next, find the pressure drop using the energy equation. Lastly, find power using  $P = \dot{m}gh_f$ .

**SOLUTION**

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{(1.5 \text{ m/s})(0.0622 \text{ m})}{(1.31 \times 10^{-6} \text{ m}^2/\text{s})} \\ &= 71,221 \end{aligned}$$

Thus, flow is turbulent.

Friction factor ( $f$ ) (Swamee-Jain correlation)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10} \left( \frac{5.74}{71,221^{0.9}} \right)\right]^2} \\ &= 0.0192 \end{aligned}$$

Darcy-Weisbach equation

$$\begin{aligned}h_f &= f \frac{L V^2}{D 2g} \\&= 0.0192 \left( \frac{30 \text{ m}}{0.0622 \text{ m}} \right) \frac{(1.5 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} \\&= 1.06 \text{ m}\end{aligned}$$

$$\boxed{h_f = 1.1 \text{ m (part b)}}$$

Energy equation (section 2 located 30 m downstream of section 1).

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Term by term analysis

$$\begin{aligned}\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ \frac{p_1}{\gamma} + \alpha \frac{V^2}{2g} + 0 + 0 &= \frac{p_2}{\gamma} + \alpha \frac{V^2}{2g} + 0 + 0 + h_f \\ \frac{p_1}{\gamma} &= \frac{p_2}{\gamma} + h_f \\ \text{or } \Delta p &= \gamma h_f \\ &= (9810 \text{ N/m}^3) (1.1 \text{ m}) \\ &= 10,791 \text{ N/m}^2\end{aligned}$$

$$\boxed{\Delta p = 11 \text{ kPa (part a)}}$$

Flow rate equation

$$\begin{aligned}\dot{m} &= \rho AV \\ &= (1000 \text{ kg/m}^3) \left( \frac{\pi (0.0622 \text{ m})^2}{4} \right) (1.5 \text{ m/s}) \\ &= 4.6 \text{ kg/s}\end{aligned}$$

Power equation

$$\begin{aligned}\dot{W} &= \dot{m} g h_f \\ &= (4.6 \text{ kg/s}) (9.81 \text{ m/s}^2) (1.1 \text{ m}) \\ &= 49.6 \text{ N-m/s}\end{aligned}$$

$$\boxed{\text{Power to overcome head loss} = 50 \text{ W (part c)}}$$

**REVIEW**

1. The pressure drop for a 30 m run of pipe ( $\Delta p = 10,791 \text{ Pa} \approx 11 \text{ kPa}$ ) could be decreased by selecting a larger pipe diameter.
2. The power to overcome the frictional head loss is about 50 W.

---

**10.44: PROBLEM DEFINITION**Situation:

Water flows through a horizontal run of PVC pipe

$$V = 2 \text{ m/s}, L = 50 \text{ m.}$$

Nominal diameter 65 mm Schedule 40.  $D = 0.0622 \text{ m.}$

Find:

- (a) Pressure drop in kPa.
- (b) Head loss in meters.
- (c) Power in watts needed to overcome the head loss.

Assumptions:

- 1.) Assume  $k_s = 0$ .
- 2.) Assume  $\alpha_1 = \alpha_2$ , where subscripts 1 and 2 denote the inlet and exit of the pipe.

Properties:

Water ( $10^\circ\text{C}$ ), Table A.5:

$$\rho = 1000 \text{ kg/m}^3, \gamma = 9810 \text{ N/m}^3, \nu = 1.31 \times 10^{-6} \text{ m}^2/\text{s}.$$

**PLAN**

To establish laminar or turbulent flow, calculate the Reynolds number. Then find the appropriate friction factor ( $f$ ) and apply the Darcy-Weisbach equation to find the head loss. Next, find the pressure drop using the energy equation. Lastly, find power using  $P = \dot{m}gh_f$ .

**SOLUTION**

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{(2 \text{ m/s})(0.0622 \text{ m})}{(1.31 \times 10^{-6} \text{ m}^2/\text{s})} \\ &= 94,960 \end{aligned}$$

Thus, flow is turbulent.

Friction factor ( $f$ ) (Swamee-Jain equation)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10} \left( \frac{5.74}{94,960^{0.9}} \right)\right]^2} \\ &= 0.0181 \end{aligned}$$

Darcy-Weisbach equation

$$\begin{aligned}h_f &= f \frac{L V^2}{D 2g} \\&= 0.0181 \left( \frac{50 \text{ m}}{0.0622 \text{ m}} \right) \frac{(2 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} \\&= 2.966 \text{ m}\end{aligned}$$

$$\boxed{h_f = 2.97 \text{ m (part b)}}$$

Energy equation

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Select a control volume surrounding the pipe. After analysis of each term, the energy equation simplifies to

$$\begin{aligned}\frac{p_1}{\gamma} &= \frac{p_2}{\gamma} + h_f \\ \text{or } \Delta p &= \gamma h_f \\ &= (9810 \text{ N/m}^3) (2.966 \text{ m}) \\ &= 29,096 \text{ kPa}\end{aligned}$$

$$\boxed{\Delta p = 29.1 \text{ kPa (part a)}}$$

Flow rate equation

$$\begin{aligned}\dot{m} &= \rho AV \\ &= (1000 \text{ kg/m}^3) \left( \frac{\pi (0.0622 \text{ m})^2}{4} \right) (2 \text{ m/s}) \\ &= 6.077 \text{ kg/s}\end{aligned}$$

Power equation

$$\begin{aligned}\dot{W} &= \dot{m} g h_f \\ &= (6.077 \text{ kg/s}) (9.81 \text{ m/s}^2) (2.966 \text{ m}) \\ &= 176.8 \text{ W}\end{aligned}$$

$$\boxed{\text{Power to overcome head loss} = 177 \text{ W (part c)}}$$

## REVIEW

1. The pressure drop (29 kPa) is about 1/3 of an atmosphere. This value could be decreased by increasing the pipe diameter to lower the speed of the water.
2. The power to overcome the frictional head loss is small; 177 W is about 1/4 of a horsepower.

---

**10.45: PROBLEM DEFINITION****Situation:**

Air flows through a smooth tube.

$$Q = 0.015 \text{ m}^3/\text{s}, D = 3 \text{ cm.}$$

$$p = 110 \text{ kPa-absolute.}$$

**Find:**

Pressure drop per meter of tube length.

**Properties:**

Air (20°C) Table A.3:  $\mu = 1.81 \times 10^{-5} \text{ N}\cdot\text{s}/\text{m}^2$ ,  $\rho = 1.2 \text{ kg}/\text{m}^3$ .

**Assumptions:**

1. Pipe is horizontal.
2. Fully developed flow so  $V$  and  $\alpha$  are constant.

**PLAN**

Solve the problem by applying the energy equation. The steps are:

1. Develop an equation for  $\Delta p$  by applying the energy equation.
2. Calculate  $V$  using the flow rate equation.
3. Calculate  $\rho$  using the ideal gas law
4. Calculate Reynolds number.
5. Look up  $f$  on the Moody diagram using the Re from step 4.
6. Calculate  $h_f$  using the Darcy-Wiesbach equation.
7. Combine results.

**SOLUTION**

1. Energy equation (cv surrounding a 1-m length of pipe)

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Since the velocity head terms cancel,  $z_1 = z_2$ , and  $h_p = h_t = 0$ , the energy equation simplifies to

$$\begin{aligned} \frac{p_1}{\gamma} &= \frac{p_2}{\gamma} + h_L \\ \Delta p &= \gamma h_L = \rho g h_f \end{aligned}$$

2. Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.015 \text{ m}^3/\text{s}}{\pi/4 \times (0.03 \text{ m})^2} \\ &= 21.2 \text{ m/s} \end{aligned}$$

3. Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{110,000 \text{ Pa}}{287 \text{ J/kg K} \times 293 \text{ K}} \\ &= 1.31 \text{ kg/m}^3\end{aligned}$$

4. Reynolds number

$$\begin{aligned}\text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{21.2 \text{ m/s} \times 0.03 \text{ m} \times 1.20 \text{ kg/m}^3}{1.81 \times 10^{-5} \text{ N s/m}^2} \\ &= 42166\end{aligned}$$

5. Friction factor ( $f$ ) (Moody diagram)

$$f = 0.0212$$

6. Darcy Weisbach equation

$$\begin{aligned}h_f &= f \frac{L V^2}{D 2g} \\ &= 0.0212 \left( \frac{1 \text{ m}}{0.03 \text{ m}} \right) \left( \frac{(21.2 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} \right) \\ &= 16.19 \text{ m for a 1.0 m length of pipe}\end{aligned}$$

7. Combine results

$$\begin{aligned}\Delta p &= h_f \rho g \\ &= (16.19 \text{ m}) (1.31 \text{ kg/m}^3) (9.81 \text{ m/s}^2) \\ &= 207.6 \text{ Pa for a 1.0 m length of pipe}\end{aligned}$$

$$\boxed{\Delta p/L = 208 \text{ Pa/m}}$$

---

**10.46: PROBLEM DEFINITION**Situation:

Water flows from point A to B in a cast iron pipe.

$$L = 4.8 \text{ km}, D = 0.6 \text{ m}.$$

$$\Delta p_{B-A} = 140 \text{ kPa}, \Delta h_{A-B} = 9 \text{ m}.$$

Find:

Direction and rate of flow ( $\text{m}^3/\text{s}$ ).

Assumptions:

Flow is from A to B.

Properties:

Water ( $10^\circ\text{C}$ ), Table A.5:  $\nu = 1.31 \times 10^{-6} \text{ m}^2/\text{s}$ .

Pipe Roughness, Table 10.4 (EFM10e),  $k_s = 0.26 \text{ mm} = 0.00026 \text{ m}$ .

**SOLUTION**

$$\begin{aligned} h_f &= \Delta(p/\gamma + z) \\ &= (-140000/9810) + 9 \\ &= 5.3 \text{ m} \end{aligned}$$

Therefore, flow is from B to A.

Parameters for the Moody diagram, Fig. 10.14 in 10e

$$\begin{aligned} \text{Re } f^{1/2} &= (D^{3/2}/\nu)(2gh_f/L)^{1/2} \\ &= (2^{0.6/2}/(1.31 \times 10^{-6}) \times (19.62 \times 5.3/4600))^{1/2} \\ &= 5.3 \times 10^4 \\ k_s/D &= 4.2 \times 10^{-4} \end{aligned}$$

Resistance coefficient (from the Moody diagram, Fig. 10.14 in 10e)

$$f = 0.0175$$

Darcy Weisbach equation

$$\begin{aligned} V &= \sqrt{h_f 2gD/fL} \\ &= \sqrt{(5.3 \times 0.6 \times 19.62)/(0.0175 \times 4800)} \\ &= 0.86 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} q &= VA \\ q &= 0.86 \times (\pi/4) \times 0.6^2 \end{aligned}$$

$$\boxed{q = 0.24 \text{ m}^3/\text{s}}$$

---

**10.47: PROBLEM DEFINITION**

Situation:

Air flows through a smooth tube.

$$D = 0.025 \text{ m}, Q = 0.0142 \text{ m}^3/\text{s}.$$

$$p = 103.4 \text{ kPa}.$$

Find:

Pressure drop per meter of tube.

Properties:

Air (25°C) Table A.3:  $\mu = 1.81 \times 10^{-5} \text{ N}\cdot\text{s}/\text{m}^2$ .

**SOLUTION**

$$V = \frac{Q}{A} = \frac{0.0142 \text{ m}^3/\text{s} \times 4}{\pi \times (0.025 \text{ m})^2} = 29 \text{ m/s}$$

$$\rho = \frac{p}{RT} = \frac{103.4 \text{ kPa}}{287 \text{ J/kg}\cdot\text{K} \times 298 \text{ K}} = 1.21 \text{ kg/m}^3$$

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} = \frac{29 \text{ m/s} \times (0.025) \text{ m} \times 1.21 \text{ kg/m}^3}{1.81 \times 10^{-5} \text{ N}\cdot\text{s}/\text{m}^2} \\ &= 4.84 \times 10^4 \end{aligned}$$

Resistance coefficient ( $f$ ) (Swamee-Jain correlation; turbulent flow)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10} \left( \frac{5.74}{(4.84 \times 10^4)^{0.9}} \right)\right]^2} \\ &= 0.021 \end{aligned}$$

Pressure drop per foot of tube

$$\Delta p = f \frac{L}{D} \left( \frac{\rho V^2}{2} \right)$$

$$\Delta p = 0.021 \left( \frac{0.3 \text{ m}}{0.025 \text{ m}} \right) \left( \frac{1.21 \text{ kg/m}^3 \times (29 \text{ m/s})^2}{2} \right)$$

$$\Delta p = 128 \text{ N/m}^2$$

$$\boxed{\frac{\Delta p}{L} = 128 \text{ Pa per meter of tube}}$$

---

**10.48: PROBLEM DEFINITION**

Situation:

Water is pumped through a vertical steel pipe to an elevated tank.

$$D = 10 \text{ cm}, p_1 = 1.6 \text{ MPa.}$$

$$L = 110 \text{ m}, Q = 0.02 \text{ m}^3/\text{s}.$$

$$\Delta z = 110 \text{ m}$$

Find:

Pressure at point 110 m above pump.

Properties:

Water (20 °C), Table A.5:  $\gamma = 9790 \text{ N/m}^3, \nu = 10^{-6} \text{ m}^2/\text{s}$

**SOLUTION**

$$\begin{aligned} \text{Re} &= \frac{4Q}{\pi D \nu} \\ &= \frac{4 \times 0.02 \text{ m}^3/\text{s}}{\pi \times 0.10 \text{ m} \times 10^{-6} \text{ m}^2/\text{s}} = 2.55 \times 10^5 \\ \frac{k_s}{D} &= \frac{4.6 \times 10^{-2}}{0.1} = 4.6 \times 10^{-1} \end{aligned}$$

Resistance coefficient (Swamee-Jain correlation; turbulent flow)

$$\begin{aligned} f &= \frac{0.25}{\left[ \log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \\ &= \frac{0.25}{\left[ \log_{10} \left( \frac{4.6 \times 10^{-2}}{3.7(0.10)} + \frac{5.74}{(2.55 \times 10^5)^{0.9}} \right) \right]^2} \\ &= 0.0183 \end{aligned}$$

Then

$$h_f = f \frac{L V^2}{D 2g}$$

where

$$\begin{aligned} V &= \frac{0.02 \text{ m}^3/\text{s}}{(\pi/4) \times (0.1 \text{ m})^2} = 2.546 \text{ m/s} \\ h_f &= 0.0183 \times \frac{110 \text{ m}}{0.10 \text{ m}} \times \frac{(2.546 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} = 6.65 \text{ m} \end{aligned}$$

Energy equation (from pump to location 110 m higher)

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_f$$

$$\frac{1.6 \times 10^6 \text{ Pa}}{9,790 \text{ N/m}^3} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + 110 + 6.65$$

$$V_1 = V_2$$

$$\boxed{p_2 = 458 \text{ kPa}}$$

---

**10.49: PROBLEM DEFINITION****Situation:**

Basement siphoning situation.

Case 1: length  $L$  and height  $h$

Case 2: length  $2L$  and height  $2h$

Case 3: length  $L$ , height  $h$ , and  $D_3 = 2D_1$

**Find:**

Part a. Assume  $h_L = 0$ , find whether  $Q$  doubles when the  $L$  is doubled from Case 1 to Case 2.

Part b. Assume  $h_L = 0.025(L/D)(V^2/2g)$ , find  $Q$  for Cases 1 and 2, where  $D = 0.025$  m,  $L = 15$  m, and  $h = 6$  m. How much does  $Q$  increase for Case 2 as compared to Case 1?

Part c. Case 3 with  $D_3 = 2D_1$ , find  $Q$  where  $L = 15$  m,  $h = 6$  m, and  $D = 0.05$  m. Use  $h_L$  as in part (b). How much of an improvement in  $Q$  is accomplished in Case 3 as compared to Case 1 in part (b)?

**PLAN**

Employ energy equation

**SOLUTION**

Part a. Assume  $h_L = 0$ . The following is true for each case:

$$\begin{aligned}0 + 0 + h &= 0 + 0 \frac{V_2^2}{2g} + 0 \\ \frac{V^2}{2g} &= h \\ V &= \sqrt{2gh}\end{aligned}$$

Now, solving for the 2 cases:

$$\begin{array}{cc} \text{Case 1} & \text{Case 2} \\ V = \sqrt{2gh} & V = \sqrt{4gh} \end{array}$$

Result, Part a: flowrate does not double when you have a larger head drop. Instead,  $Q$  increases by a factor of  $\frac{2}{\sqrt{2}} = \sqrt{2}$ , or 1.414. (Note: because flowrate is  $Q = VA$ , and area is the same for both cases, we can just compare velocities.)

**SOLUTION**

Part b. Assume  $h_L = 0.025(\frac{L}{D})(\frac{V^2}{2g})$

$$\begin{array}{cc} \text{Case 1} & \text{Case 2} \\ D = 0.025 \text{ m} & D = 0.025 \text{ m} \\ L = 15 \text{ m} & L = 30 \text{ m} \\ h = 6 \text{ m} & h = 12 \text{ m} \end{array}$$

The following is true for each case:

$$\begin{aligned}
 0 + 0 + h &= 0 + 0 \frac{V_2^2}{2g} + 0 + h_L \\
 \frac{V^2}{2g} &= h - h_L \\
 \frac{V^2}{2g} &= h - 0.025 \left( \frac{L}{D} \right) \left( \frac{V^2}{2g} \right)
 \end{aligned}$$

Solve for velocity for each of the two cases:

|  |   |
|--|---|
| Case 1   | Case 2  |
| $\frac{V^2}{2g} = 6 - 0.025 \left( \frac{15}{0.025} \right) \left( \frac{V^2}{2g} \right)$ | $\frac{V^2}{2g} = 12 - 0.025 \left( \frac{30}{0.025} \right) \left( \frac{V^2}{2g} \right)$ |
| $\frac{V^2}{2g} = 0.375 \text{ m}$   | $\frac{V^2}{2g} = 0.39 \text{ m}$   |
| $V = 2.7 \text{ m/s}$  | $V = 2.8 \text{ m/s}$   |

Result, Part b: there is only a small  $\uparrow V$  (and therefore  $Q$ ) by lengthening the  $\Delta h$ ; this is because by increasing the  $L$ , one increases the head loss. With these data,  $Q$  would increase by a factor of only  $\frac{2.8}{2.7} = 1.02$ . Almost all the  $\uparrow Q$  gained by increasing the head difference from the top to bottom of the siphon is lost due to the increased head loss in the longer hose.

**SOLUTION**

Part c. Increase hose diameter (Case 3).

Compare Case 1 to Case 3; Use the same form of  $h_L$  as in Part b.

|                       |                      |
|-----------------------|----------------------|
| Case 1                | Case 3               |
| $D = 0.025 \text{ m}$ | $D = 0.05 \text{ m}$ |
| $L = 15 \text{ m}$    | $L = 15 \text{ m}$   |
| $h = 6 \text{ m}$     | $L = 6 \text{ m}$    |

|   |   |
|---|---|
| Case 1  | Case 3  |
| $\frac{V^2}{2g} = 6 - 0.025 \left( \frac{15}{0.25} \right) \left( \frac{V^2}{2g} \right)$ | $\frac{V^2}{2g} = 6 - 0.025 \left( \frac{15}{0.05} \right) \left( \frac{V^2}{2g} \right)$ |
| $\frac{V^2}{2g} = 0.375 \text{ m}$  | $\frac{V^2}{2g} = 0.69 \text{ m}$   |
| $V = 2.7 \text{ m/s}$   | $V = 3.7 \text{ m/s}$   |

Result, Part c: With a larger diameter, we have less head loss (than by lengthening the hose) so more of the head difference provided by the siphon can result in velocity. Also, area doubles. With these data,  $Q$  would increase by a factor of  $\frac{Q_3}{Q_1} = \frac{V_3 A_3}{V_1 A_1} = \frac{3.7(2A_1)}{2.7(A_1)} =$  factor of 1.3. This is the best improvement of all.

**REVIEW** If you need to increase the flow from a siphon, get a bigger hose, or get more hoses. Don't lengthen the hose.

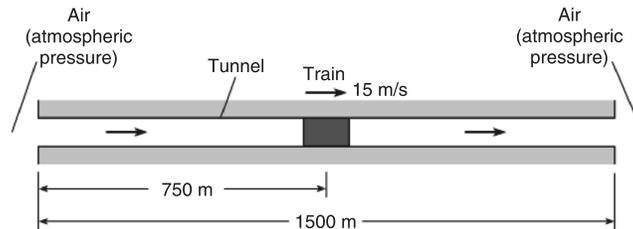
## 10.50: PROBLEM DEFINITION

### Situation:

A train travels through a tunnel.

$$D = 3 \text{ m}, k_s = 0.015 \text{ m}.$$

$$V = 15 \text{ m/s}, L = 750 \text{ m}$$



### Find:

- Change in pressure between the front and rear of the train.
- Power required to produce the air flow in the tunnel.
- Sketch an EGL and a HGL.

### Properties:

Air (15°C) Table A.3:  $\gamma = 12 \text{ N/m}^3$ ,  $\nu = 1.46 \times 10^{-5} \text{ m}^2/\text{s}$ .

## PLAN

Apply the energy equation from front of train to outlet of tunnel.

## SOLUTION

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_f \\ \frac{p_1}{\gamma} + \frac{V_1^2}{2g} &= 0 + 0 + 0 + \frac{V_2^2}{2g} + f \frac{L}{D} \frac{V_2^2}{2g} \\ &\text{(assume the KE terms are zero)} \\ \frac{p_1}{\gamma} &= f \frac{L}{D} \frac{V_2^2}{2g} \end{aligned}$$

$$\begin{aligned} \frac{k_s}{D} &= \frac{0.015}{3} = 0.005 \\ \text{Re} &= \frac{VD}{\nu} = \frac{15 \text{ m/s} \times 3 \text{ m}}{1.46 \times 10^{-5} \text{ m}^2/\text{s}} = 3.2 \times 10^6 \end{aligned}$$

Resistance coefficient (from Moody diagram)

$$f = 0.030$$

Darcy Weisbach equation

$$\begin{aligned} p_1 &= \gamma f \frac{L V_2^2}{D 2g} \\ &= (12 \text{ N/m}^3)(0.03) \left( \frac{750 \text{ m}}{3 \text{ m}} \right) \left( \frac{(15 \text{ m/s})^2}{19.62 \text{ m/s}^2} \right) \\ p_1 &= 1032 \text{ N/m}^2 \end{aligned}$$

Energy equation (from outside entrance to rear of train)

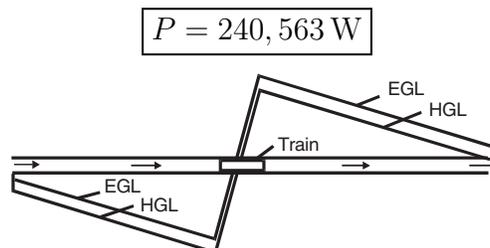
$$\begin{aligned} \frac{p_3}{\gamma} + \alpha_3 \frac{V_3^2}{2g} + z_3 &= \frac{p_4}{\gamma} + \alpha_4 \frac{V_4^2}{2g} + z_4 + \sum h_L \\ 0 + 0 + 0 &= \frac{p_4}{\gamma} + \frac{V_4^2}{2g} + 0 + K_e + f \frac{L V^2}{D 2g} \\ \frac{p_4}{\gamma} &= - \left( \frac{V^2}{2g} \right) \left( 1.5 + f \frac{L}{D} \right) \\ &= - \frac{(15 \text{ m/s})^2}{2g} \left( 1.5 + 0.03 \left( \frac{750 \text{ m}}{3 \text{ m}} \right) \right) \\ p_4 &= -\gamma(103.2 \text{ m}) = -1238.4 \text{ N/m}^2 \end{aligned}$$

$$\begin{aligned} \Delta p &= p_1 - p_4 \\ &= 1032 - (-1238.4) = 2270.4 \text{ N/m}^2 \end{aligned}$$

$$\boxed{\Delta p = 2270 \text{ N/m}^2}$$

Power equation

$$\begin{aligned} P &= FV \\ &= (\Delta p A)(15 \text{ m/s}) \\ &= (2270 \text{ N/m}^2 \times \pi/4 \times (3 \text{ m})^2)(15 \text{ m/s}) \\ &= 240,563 \text{ Nm/s} = 240,563 \text{ W} \end{aligned}$$



---

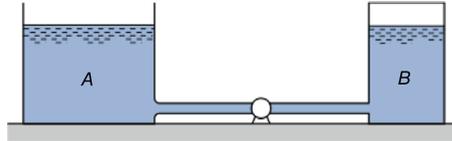
**10.51: PROBLEM DEFINITION**Situation:

Water is pumped from a reservoir to a tank.

$$D = 0.1 \text{ m}, L = 90 \text{ m}.$$

$$Q = 0.03 \text{ m}^3/\text{s}, \eta = 0.9.$$

$$p_B = 70 \text{ kPa}, p_A = 0 \text{ kPa}.$$

Sketch:Find:

Power to operate the pump.

Assumptions:

Assume the entrance is smooth.

Properties:

Water (15 °C) Table A.5:  $\nu = 1.14 \times 10^{-6} \text{ m}^2/\text{s}$ .

Pipe roughness, Table 10.4 (EFM10e),  $k_s = 0.046 \text{ mm} = 4.6 \times 10^{-5} \text{ m}$ .

Loss Coefficients, Table 10.5 (EFM10e),  $K_e = 0.03$ ,  $K_E = 1$ .

**SOLUTION**

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} = \frac{0.03 \text{ m}^3/\text{s}}{(\pi/4)D^2} \\ &= \frac{0.03 \text{ m}^3/\text{s}}{(\pi/4)(0.1 \text{ m})^2} \\ &= 3.8 \text{ m/s} \end{aligned}$$

Then

$$\begin{aligned} \text{Re} &= \frac{3.8 \text{ m/s} \times (0.1) \text{ m}}{1.14 \times 10^{-6} \text{ m}^2/\text{s}} = 3.33 \times 10^5 \\ \frac{k_s}{D} &= 4.6 \times 10^{-4} \end{aligned}$$

Resistance coefficient (from Moody diagram)

$$f = 0.0165$$

Then

$$f \frac{L}{D} = 0.0165 \frac{90 \text{ m}}{(0.1) \text{ m}} = 14.85$$

Energy equation (from water surface  $A$  to water surface  $B$ )

$$\begin{aligned} \frac{p_A}{\gamma} + \alpha_A \frac{V_A^2}{2g} + z_A + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 0 + 0 + 0 + h_p &= \frac{70,000 \text{ N/m}^2}{19.62 \text{ m/s}^2} + 0 + \left( K_e + K_E + f \frac{L}{D} \right) \frac{V^2}{2g} \end{aligned}$$

Thus

$$\begin{aligned} h_p &= 6.9 \text{ m} + (0.009 + 0.3 + 14.85) \text{ m} \frac{(3.8 \text{ m/s})^2}{19.62 \text{ m/s}^2} \\ &= 18.1 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= \frac{Q\gamma h_p}{\eta} \\ &= \frac{0.03 \text{ m}^3/\text{s} \times 9810 \text{ N/m}^3 \times 18.1 \text{ m}}{0.9} \\ &= 5919 \text{ N-m/s} \\ &\boxed{P = 5919 \text{ W}} \end{aligned}$$

---

**Problem 10.52**

Classify problems as case 1, 2, or 3.

---

a. Problem 10.51 (EFM 10e)

- Classification: Case 1.
  - Rationale for classification: Flow rate is specified. Solution path involves finding head loss and then finding pump power.
- 

b. Problem 10.54 (EFM 10e)

- Classification: Case 3.
  - Rationale for classification: Hose diameter is the goal. Flow rate is specified.
- 

c. Problem 10.57 (EFM 10e)

- Classification: Case 3.
- Rationale for classification: Pipe diameter is the goal. Flow rate is specified and head loss can be related to the specified pressure drop.

### 10.53: PROBLEM DEFINITION

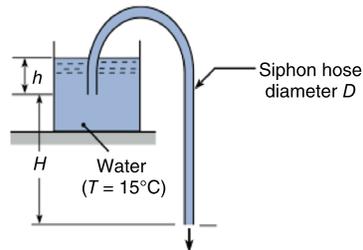
#### Situation:

Water is flowing out a plastic siphon hose.

$$D = 0.012 \text{ m}, H = 3 \text{ m}.$$

$$h = 1.0 \text{ m}, L = 5.5 \text{ m}.$$

$$k_s = 0.$$



#### Find:

Velocity (assume the Bernoulli equation applies).

Velocity (include the head loss in the hose).

#### Assumptions:

Steady flow.

Neglect all head loss (part 1 of problem).

Neglect component head loss (part 2 of problem).

Turbulent flow. Also,  $\alpha_2 = 1.0$ .

#### Properties:

Water (15 °C), Table A.5:  $\nu = 1.14 \times 10^{-6} \text{ m}^2/\text{s}$ .

### PLAN

1. Use the Bernoulli equation to find velocity  
Classify this problem as case 2 ( $V$  is unknown), then
2. Write the energy eqn., the Darcy-Weisbach eqn., etc. to produce a set of 4 equations with 4 unknowns.
3. Solve the set of equations using a computer program (we used TK Solver).

### SOLUTION

1. Bernoulli equation (point 1 on tank surface; point 2 on exit plane of hose):

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$
$$0 + (H + h) + 0 = 0 + 0 + \frac{V_2^2}{2g}$$

$$V = \sqrt{2g(H + h)} = \sqrt{2(9.81 \text{ m/s}^2)(3 \text{ m} + 1 \text{ m})}$$

$$V = 8.86 \text{ m/s}$$

2. Equations for finding velocity:

- Energy equation:

$$\begin{aligned}\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + (H + h) + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_f\end{aligned}\quad (1)$$

- Darcy-Weisbach:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \quad (2)$$

- Swamee-Jain:

$$f = \frac{0.25}{\left[ \log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \quad (3)$$

- Reynolds number:

$$\text{Re} = \frac{VD}{\nu} \quad (4)$$

3. Solution of Eqs. (1) to (4):

$$\begin{aligned}h_f &= 3.66 \text{ m} \\ \text{Re} &= 26930 \\ f &= 0.024\end{aligned}$$

$$\boxed{V = 2.56 \text{ m/s}}$$

**REVIEW**

1. Notice that the turbulent flow assumption is valid because  $\text{Re} > 2300$ .
2. An easy way to solve case 2 and case 3 problems is to acquire a computer program that can solve coupled, non-linear equations.

## 10.54: PROBLEM DEFINITION

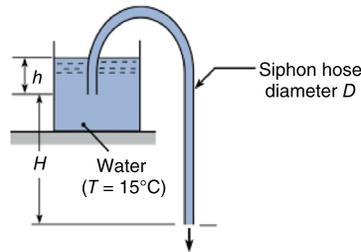
### Situation:

Water is flowing out a plastic siphon hose.

$$Q = 0.0015 \text{ m}^3/\text{s}, \quad H = 5 \text{ m.}$$

$$h = 0.5 \text{ m}, \quad L = 7 \text{ m}, \quad k_s = 0.$$

### Sketch:



### Find:

Diameter of hose (meters).

### Assumptions:

Steady flow.

Component head loss is zero.

Turbulent flow. Also,  $\alpha_2 = 1.0$ .

### Properties:

Water (15°C), Table A.5,  $\nu = 1.14 \times 10^{-6} \text{ m}^2/\text{s}$ .

### PLAN

Classify this problem as case 3 ( $D$  is unknown), then

1. Write the energy eqn., the Darcy-Weisbach eqn., etc. to produce a set of 5 equations with 5 unknowns.
2. Solve the set of equations using a computer program (we used TK Solver).

### SOLUTION

1. Governing equations:

- Flow rate equation:

$$Q = V \left( \frac{\pi D^2}{4} \right) \quad (1)$$

- Energy equation (section 1 on water surface, section 2 at exit plane)

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + (H + h) + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_f \end{aligned} \quad (2)$$

- Darcy-Weisbach:

$$h_f = f \frac{L V^2}{D 2g} \quad (3)$$

- Swamee-Jain:

$$f = \frac{0.25}{\left[ \log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \quad (4)$$

- Reynolds number:

$$\text{Re} = \frac{VD}{\nu} \quad (5)$$

3. Solution of Eqs. (1) to (5):

$$\begin{aligned} h_f &= 4.715 \text{ m} \\ \text{Re} &= 75900 \\ f &= 0.0189 \\ V &= 3.92 \text{ m/s} \end{aligned}$$

$$\boxed{D = 0.022 \text{ m}}$$

## **REVIEW**

- Notice that the turbulent flow assumption is valid because  $\text{Re} > 2300$ .
- An easy way to solve case 2 and case 3 problems is to acquire a computer program that can solve coupled, non-linear equations.
- Notice that most of the elevation head (5.5 m) is converted to head loss (4.72 m).

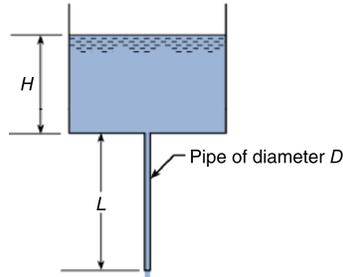
## 10.55: PROBLEM DEFINITION

### Situation:

Water is draining out of a tank through a galvanized iron pipe.

$k_s = 0.015 \text{ cm} = 0.00015 \text{ m}$ ,  $L = 3 \text{ m}$ ,  $H = 1.2 \text{ m}$ .

The pipe is 25 mm schedule 40 NPS,  $D = 0.0266 \text{ m}$ .



### Find:

Velocity in the pipe (m/s).

Flow rate ( $\text{m}^3/\text{s}$ )

### Assumptions:

Steady flow.

Component head loss is zero.

Turbulent flow. Also,  $\alpha_2 = 1.0$ .

### Properties:

Water ( $21^\circ \text{C}$ ), Table A.5,  $\rho = 997.8 \text{ kg/m}^3$ ,  $\gamma = 9788.2 \text{ N/m}^3$ ,  $\nu = 9.8 \times 10^{-7} \text{ m}^2/\text{s}$ .

**PLAN** Classify this problem as case 2 ( $V$  is unknown), then

1. Write the energy eqn., the Darcy-Weisbach eqn., etc. to produce a set of 4 equations with 4 unknowns.
2. Solve the set of equations using a computer program (we used TK Solver).
3. Find the flow rate with  $Q = VA$ .

### SOLUTION

1. Governing equations:

- Energy equation (section 1 on water surface, section 2 at exit plane)

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + (H + L) + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_f \end{aligned} \quad (1)$$

- Darcy-Weisbach:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \quad (2)$$

- Swamee-Jain:

$$f = \frac{0.25}{\left[\log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right)\right]^2} \quad (3)$$

- Reynolds number:

$$\text{Re} = \frac{VD}{\nu} \quad (4)$$

2. Solution of Eqs. (1) to (4):

$$\begin{aligned} h_f &= 3.3 \text{ m} \\ \text{Re} &= 113800 \\ f &= 0.0326 \end{aligned}$$

$$\boxed{V = 4.2 \text{ m/s}}$$

3. Flow rate equation:

$$\begin{aligned} Q &= \frac{\pi D^2}{4} V \\ &= \left( \frac{\pi (0.0266 \text{ m})^2}{4} \right) (4.2 \text{ m/s}) = 2.33 \times 10^{-3} \text{ m}^3/\text{s} \end{aligned}$$

$$\boxed{Q = 0.00233 \text{ m}^3/\text{s}}$$

### **REVIEW**

- Notice that the turbulent flow assumption is valid because  $\text{Re} > 2300$ .
- An easy way to solve case 2 and case 3 problems is to acquire a computer program that can solve coupled, nonlinear equations.
- Notice that most of the elevation head (4.2 m) is converted to head loss (3.3 m).

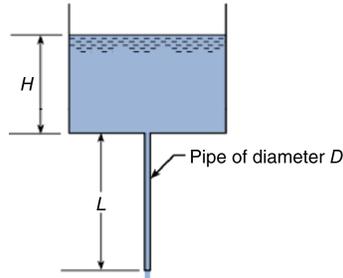
## 10.56: PROBLEM DEFINITION

### Situation:

Water is draining out of a tank through a galvanized iron pipe.

$$L = 2 \text{ m}, \quad H = 1 \text{ m}, \quad k_s = 0.15 \times 10^{-3} \text{ m}$$

The pipe is 15 mm schedule 40 NPS,  $D = 0.0158 \text{ m}$ .



### Find:

Velocity in the pipe (m/s).

### Assumptions:

Steady flow.

Component head loss is zero.

Turbulent flow, so  $\alpha_2 = 1.0$ .

### Properties:

Water ( $15^\circ\text{C}$ ), Table A.5,  $\nu = 1.14 \times 10^{-6} \text{ m}^2/\text{s}$ .

## PLAN

Classify this problem as case 2 ( $V$  is unknown), then

1. Write the energy eqn., the Darcy-Weisbach eqn., etc. to produce a set of 4 equations with 4 unknowns.
2. Solve the set of equations using a computer program (we used TK Solver).

## SOLUTION

1. Governing equations:

- Energy equation (section 1 on water surface, section 2 at exit plane)

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + (H + L) + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_f \end{aligned} \quad (1)$$

- Darcy-Weisbach:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \quad (2)$$

- Swamee-Jain:

$$f = \frac{0.25}{\left[\log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right)\right]^2} \quad (3)$$

- Reynolds number:

$$\text{Re} = \frac{VD}{\nu} \quad (4)$$

2. Solution of Eqs. (1) to (4):

$$h_f = 2.50 \text{ m}$$

$$\text{Re} = 43600$$

$$f = 0.039$$

$$\boxed{V = 3.15 \text{ m/s}}$$

### **REVIEW**

1. Notice that the turbulent flow assumption is valid because  $\text{Re} > 2300$ .
2. An easy way to solve case 2 and case 3 problems is to acquire a computer program that can solve coupled, nonlinear equations.

---

**10.57: PROBLEM DEFINITION**Situation:

Air is flowing in a horizontal copper tube.

$L = 150$  m,  $Q = 0.1$  m<sup>3</sup>/s,  $k_s = 1.5 \times 10^{-6}$  m.

Pressure drop in the tube cannot exceed  $\Delta p = 1493$  Pa.

Find:

Tube diameter (meters).

Assumptions:

Steady flow. Fully developed flow.

Component head loss is zero.

Turbulent flow.

Properties:

Air (40°C, 1 atm), Table A.5,  $\gamma = 11.1$  N/m<sup>3</sup>,  $\nu = 1.69 \times 10^{-5}$  m<sup>2</sup>/s.

**PLAN**

Classify this problem as case 3 ( $D$  is unknown), then

1. Write the energy eqn., the Darcy-Weisbach eqn., etc. to produce a set of 5 equations with 5 unknowns.
2. Solve the set of equations using a computer program (we used TK Solver).

**SOLUTION**

1. Governing equations:

- Flow rate equation:

$$Q = V \left( \frac{\pi D^2}{4} \right) \quad (1)$$

- Energy equation (section 1 is located 150 m upstream from section 2).

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

– Since the flow is fully developed, the KE terms cancel out.

– let  $p_1 - p_2 = \Delta p$

–  $z_1 = z_2 = h_p = h_t = 0$

$$\Delta p = \gamma h_f \quad (2)$$

- Darcy-Weisbach:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \quad (3)$$

- Swamee-Jain:

$$f = \frac{0.25}{\left[ \log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \quad (4)$$

- Reynolds number:

$$\text{Re} = \frac{VD}{\nu} \quad (5)$$

3. Solution of Eqs. (1) to (5):

$$\begin{aligned} h_f &= 134.5 \text{ m} \\ \text{Re} &= 67000 \\ f &= 0.0195 \\ V &= 10.1 \text{ m/s} \end{aligned}$$

$$\boxed{D > 0.112 \text{ m}}$$

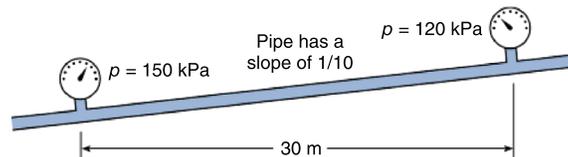
**REVIEW**. The turbulent flow assumption is valid because  $\text{Re} > 2300$ .

**10.58: PROBLEM DEFINITION**Situation:

A fluid flows through a galvanized iron pipe.

$D = 8$  cm.

Pipe slope is 1 Horizontal to 10 Vertical.

Sketch:Find:

Flow rate.

Properties:

From Table 10.4  $k_s = 0.15$  mm.

$\rho = 800$  kg/m<sup>3</sup>,  $\nu = 10^{-6}$  m<sup>2</sup>/s.

Assumptions:

$\alpha_1 = \alpha_2 = 1.0$

**SOLUTION**

Energy equation

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_f$$

$$\frac{150000 \text{ Pa}}{800 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2} + \frac{V_1^2}{2g} + 0 = \frac{120000 \text{ Pa}}{800 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2} + \frac{V_2^2}{2g} + 3 \text{ m} + h_f$$

$$h_f = 0.823 \text{ m}$$

$$\left( \frac{D^{3/2}}{\nu} \right) \times (2gh_f/L)^{1/2} = \left( \frac{(0.08)^{3/2}}{10^{-6}} \right) \times (2 \times 9.81 \times 0.823/30.14)^{1/2}$$

$$= 1.66 \times 10^4$$

Relative roughness

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-4}}{0.08} = 1.9 \times 10^{-3}$$

Resistance coefficient. From Fig. 10.14 (in 10e)  $f = 0.025$ . Then

$$h_f = f \frac{L V^2}{D 2g}$$

Solving for  $V$

$$\begin{aligned} V &= \sqrt{\left(\frac{h_f}{f}\right) \left(\frac{D}{L}\right) 2g} \\ &= \sqrt{\left(\frac{0.823 \text{ m}}{0.025}\right) \left(\frac{0.08 \text{ m}}{30.14 \text{ m}}\right) \times 2 \times 9.81 \text{ m/s}^2} = 1.312 \text{ m/s} \\ Q &= VA \\ &= 1.312 \text{ m/s} \times (\pi/4) \times (0.08 \text{ m})^2 = 6.59 \times 10^{-3} \text{ m}^3/\text{s} \end{aligned}$$

$$\boxed{Q = 6.59 \times 10^{-3} \text{ m}^3/\text{s}}$$

---

**10.59: PROBLEM DEFINITION**Situation:

Commercial steel pipe will convey water.

$h_L = 0.3$  m per 300 m of pipe length.

$Q = 8.5$  m<sup>3</sup>/s.

Find:

Pipe diameter to produce specified head loss.

Assumptions:

The pipes are available in even mm sizes (e.g. 250 mm, 300 mm, 350 mm, etc.)

Properties:

Water (15 °C), Table A.5:  $\nu = 1.14 \times 10^{-6}$  m<sup>2</sup>/s.

From Table 10.4:  $k_s = 0.005$  cm =  $5 \times 10^{-5}$  m.

**SOLUTION**

Darcy Weisbach equation

$$\begin{aligned}h_f &= f \frac{L V^2}{D 2g} \\&= f \frac{L Q^2}{D 2g A^2} \\&= f \frac{8LQ^2}{g\pi^2 D^5}\end{aligned}$$

Solve for diameter

$$D = \left( f \frac{8LQ^2}{g\pi^2 h_f} \right)^{1/5}$$

Assume  $f = 0.015$  (initial guess)

$$\begin{aligned}D &= \left( 0.015 \frac{8 (300 \text{ m}) (8.5 \text{ m}^3/\text{s})^2}{9.81 \text{ m/s}^2 \times \pi^2 \times 0.3 \text{ m}} \right)^{1/5} \\&= 2.5 \text{ m}\end{aligned}$$

Now get a better estimate of  $f$ :

$$\begin{aligned}\text{Re} &= \frac{4Q}{\pi D \nu} = 3.9 \times 10^6 \\f &= \frac{0.25}{\left[ \log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \\&= \frac{0.25}{\left[ \log_{10} \left( \frac{0.00005}{3.7 \times 2.5} + \frac{5.74}{(3.9 \times 10^6)^{0.9}} \right) \right]^2} \\&= 0.0104\end{aligned}$$

Compute  $D$  again:

$$\begin{aligned} D &= \left( 0.0104 \frac{8 (300 \text{ m}) (8.5 \text{ m}^3/\text{s})^2}{9.81 \text{ m/s}^2 \times \pi^2 \times 0.3 \text{ m}} \right)^{1/5} \\ &= 2.28 \text{ m} \end{aligned}$$

Third iteration, recalculate  $Re$ , verify that  $f = 0.0104$ , which means that the solution has converged.

Thus, specify a pipe with  $D = 2.28 \text{ m} \approx 228 \text{ cm}$

---

**10.60: PROBLEM DEFINITION**Situation:

A steel pipe will carry crude oil.

$$Q = 0.1 \text{ m}^3/\text{s}.$$

$$h_L = 50 \text{ m per km of pipe length.}$$

Find:

- (a) Diameter of pipe for a head loss of 50 m.
- (b) Pump power.

Assumptions:

Available pipe diameters are  $D = 20, 22,$  and  $24 \text{ cm}.$

Properties:

From Table 10.4:  $k_s = 0.046 \text{ mm}.$

$$S = 0.93, \nu = 10^{-5} \text{ m}^2/\text{s}.$$

**SOLUTION**

Darcy Weisbach equation

$$\begin{aligned} h_f &= f \frac{L V^2}{D 2g} \\ &= f \frac{L Q^2}{D 2gA^2} \\ &= f \frac{8LQ^2}{g\pi^2 D^5} \end{aligned}$$

Solve for diameter

$$D = \left( f \frac{8LQ^2}{g\pi^2 h_f} \right)^{1/5}$$

Assume  $f = 0.015$

$$\begin{aligned} D &= \left( 0.015 \frac{8(1000 \text{ m})(0.1 \text{ m}^3/\text{s})^2}{9.81 \text{ m/s}^2 \times \pi^2 \times 50 \text{ m}} \right)^{1/5} \\ &= 0.19 \text{ m} \end{aligned}$$

Calculate a more accurate value of  $f$

$$\begin{aligned}\text{Re} &= \frac{4Q}{\pi D\nu} \\ &= \frac{4 \times 0.1 \text{ m}^3/\text{s}}{\pi \times 0.19 \text{ m} \times 10^{-5} \text{ m}^2/\text{s}} \\ &= 6.7 \times 10^4 \\ f &= \frac{0.25}{\left[\log_{10} \left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10} \left(\frac{0.046}{3.7 \times 190} + \frac{5.74}{67000^{0.9}}\right)\right]^2} \\ &= 0.021\end{aligned}$$

Recalculate diameter using new value of  $f$

$$\begin{aligned}D &= \left(\frac{0.021}{0.015}\right)^{1/5} \times 0.19 \\ &= 0.203 \text{ m} = 20.3 \text{ cm}\end{aligned}$$

Do a third iteration, to verify that  $f$  does not change, and that  $D = 20.3$  cm.  
Use the next larger size of pipe;  $D = 22$  cm

Power equation (assume the head loss is remains at  $h_L \approx 50$  m/1,000 m)

$$\begin{aligned}P &= Q\gamma h_f \\ &= 0.1 \times (0.93 \times 9810) \times 50 = 45.6 \text{ kW}\end{aligned}$$

$$P = 45.6 \text{ kW for each kilometer of pipe length}$$

---

**10.61: PROBLEM DEFINITION**

Find:

Use Table 10.5 (in §10.8) to select loss coefficients,  $K$ , for the following transitions and fittings.

- a. A threaded pipe 90° elbow.
- b. A 90° smooth bend with  $r/d = 2$ .
- c. A pipe entrance with  $r/d$  of 0.3.
- d. An abrupt contraction, with  $\theta = 180^\circ$ , and  $D_2/D_1 = 0.60$ .
- e. A gate valve, wide open.

**SOLUTION**

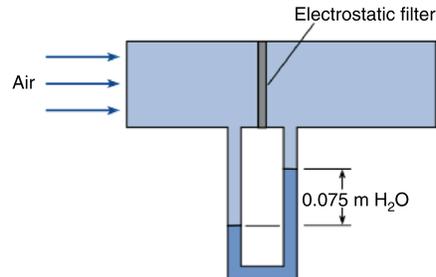
- a. Answer: 0.9
- b. Answer: 0.19
- c. Answer: 0.03
- d. Answer: 0.27
- e. Answer: 0.2

## 10.62: PROBLEM DEFINITION

### Situation:

An electrostatic air filter is being tested.

Pressure drop is  $\Delta p = 0.075 \text{ m H}_2\text{O}$ . Air speed is  $V = 10 \text{ m/s}$ .



**Find:** The minor loss coefficient ( $K$ ) for the filter.

### Properties:

Air ( $20^\circ\text{C}$ ) Table A.3:  $\nu = 15.1 \times 10^{-6} \text{ m}^2/\text{s}$ .

$\rho = 1.2 \text{ kg/m}^3$ ,  $\gamma = 11.8 \text{ N/m}^3$ .

### PLAN

Apply the energy equation to relate the pressure drop to head loss. Then, find the minor loss coefficient using  $h_L = KV^2/2g$ .

### SOLUTION

Energy equation (select a control volume surrounding the filter)

$$\left(\frac{p}{\gamma}\right)_1 = \left(\frac{p}{\gamma}\right)_2 + h_L$$

Thus

$$\begin{aligned} h_L &= \frac{\Delta p}{\gamma_{\text{air}}} \\ &= \frac{(0.075 \text{ m-H}_2\text{O}) \left(249.2 \frac{\text{Pa}}{\text{m-H}_2\text{O}}\right)}{11.8 \text{ N/m}^3} \\ &= 63.36 \text{ m} \end{aligned}$$

Head loss

$$h_L = \frac{KV^2}{2g}$$
$$K = \frac{2gh_L}{V^2} \quad (1)$$

$$= \frac{2(9.81 \text{ m/s}^2)(63.36 \text{ m})}{(10 \text{ m/s})^2}$$
$$= 12.43$$

$$\boxed{K = 12.4} \quad (2)$$

### **REVIEW**

- 1.) This minor loss coefficient is larger than the coefficient for any components listed in Table 10.5.
- 2.) Combining Eqs. (1) and (2) gives  $K = \Delta p/(\rho V^2/2)$ . Thus, the pressure drop for the filter is about 12 times larger than the pressure change that results when the flow is brought to rest.

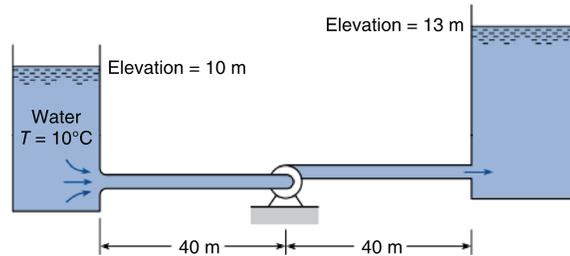
### 10.63: PROBLEM DEFINITION

#### Situation:

Water is pumped between reservoirs through a steel pipe.

$$Q = 0.1 \text{ m}^3/\text{s}, \quad D = 15 \text{ cm}.$$

#### Sketch:



#### Find:

Power that is supplied to the system by the pump.

#### Properties:

From Table 10.4:  $k_s = 0.046 \text{ mm}$ .

Water ( $10^\circ\text{C}$ ), Table A.5:  $\nu = 1.3 \times 10^{-6} \text{ m}^2/\text{s}$ ,  $\gamma = 9810 \text{ N/m}^3$ .

### SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.1 \text{ m}^3/\text{s}}{(\pi/4) \times (0.15 \text{ m})^2} \\ &= 5.66 \text{ m/s} \\ \frac{V^2}{2g} &= 1.63 \text{ m} \\ \frac{k_s}{D} &= \frac{0.0046}{15} = 0.0003 \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} = \frac{5.66 \text{ m/s} \times 0.15 \text{ m}}{1.3 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= 6.4 \times 10^5 \end{aligned}$$

Resistance coefficient (from the Moody diagram, Fig. 10.14 in EFM10e)

$$f = 0.016$$

Energy equation (between the reservoir surfaces), with a  $K_e$  and  $K_E$

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L$$

assume the two KE terms are equal and thus cancel, so

$$\begin{aligned} h_p &= z_2 - z_1 + \frac{V^2}{2g} (K_e + f \frac{L}{D} + K_E) \\ &= 13 \text{ m} - 10 \text{ m} + 1.63 \text{ m} \times (0.1 + 0.016 \times \frac{80 \text{ m}}{0.15 \text{ m}} + 1) \\ &= 3 + 15.7 = 18.7 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ &= 0.10 \text{ m}^3/\text{s} \times 9810 \text{ N}/\text{m}^3 \times 18.7 \text{ m} \\ &= 18,345 \text{ W} \end{aligned}$$

$$\boxed{P = 18.3 \text{ kW}}$$

### 10.64: PROBLEM DEFINITION

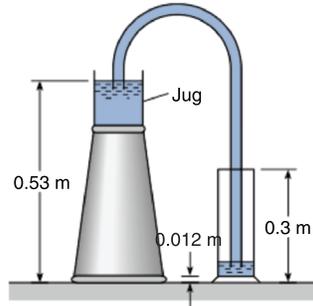
#### Situation:

A siphon tube is used to drain water from a jug into a graduated cylinder.

$$D_{\text{tube}} = 5 \text{ mm} = 0.005 \text{ m.}$$

$$L_{\text{tube}} = 1.2 \text{ m, } V = 500 \text{ mL.}$$

#### Sketch:



#### Find:

Time to fill cylinder.

#### Assumptions:

$$T \simeq 15^\circ\text{C} \text{ with } \nu = 1.14 \times 10^{-6} \text{ m}^2/\text{s.}$$

Neglect head loss associated with any bend in the Tygon tube.

### SOLUTION

Energy equation (from the surface of the water in the jug to the surface in the graduated cylinder)

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \quad (1)$$

Assume that the entrance loss coefficient is equal to 0.5. It could be larger than 0.5, but this should yield a reasonable approximation. Therefore

$$\sum h_L = \left(0.5 + f \frac{L}{D} + K_E\right) \frac{V^2}{2g}$$

The exit loss coefficient,  $K_E$ , is equal to 1.0. Therefore, Eq. 1 becomes

$$\begin{aligned} \Delta z &= z_j - z_c = \left(\frac{V^2}{2g}\right) \left(1.5 + f \frac{L}{D}\right) \\ \text{or } V &= \sqrt{\frac{2g\Delta z}{1.5 + f \frac{L}{D}}} \\ &= \sqrt{\frac{2g\Delta z}{1.5 + f \times \left(\frac{1.2 \text{ m}}{0.005 \text{ m}}\right)}} \end{aligned} \quad (1)$$

Initial guess: assume  $f = 0.03$  and let  $\Delta z = (0.53 - 0.064) \text{ m} = 0.466 \text{ m}$ . Then

$$\begin{aligned}
 V &= \sqrt{\frac{2g \times 0.466 \text{ m}}{1.5 + 7.2}} \\
 &= 1.03 \text{ m/s} \\
 \text{Re} &= \frac{VD}{\nu} \\
 &= \frac{1.03 \text{ m/s} \times 0.005 \text{ m}}{1.14 \times 10^{-6} \text{ m}^2/\text{s}} \\
 &= 4518
 \end{aligned}$$

Resistance coefficient (recalculate)

$$\begin{aligned}
 f &= \frac{0.25}{\left[\log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right)\right]^2} \\
 &= \frac{0.25}{\left[\log_{10} \left( 0 + \frac{5.74}{4518^{0.9}} \right)\right]^2} \\
 &= 0.039
 \end{aligned}$$

Repeat calculations with a new value of friction factor.

$$\begin{aligned}
 V &= \sqrt{\frac{2g \times 0.466}{1.5 + 10.68}} \\
 &= 0.87 \text{ m/s} \\
 \text{Re} &= \frac{VD}{\nu} \\
 &= 3815
 \end{aligned}$$

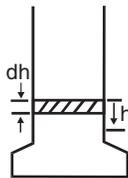


Fig. A

Use  $f = 0.040$  for final solution. As a simplifying assumption assume that as the cylinder fills the level of water in the jug has negligible change. As the cylinder is being filled one can visualize (see figure) that in time  $dt$  a volume of water equal to  $Qdt$  will enter the cylinder and that volume in the cylinder can be expressed as  $A_c dh$ , that is

$$\begin{aligned}
 Qdt &= A_c dh \\
 dt &= \frac{A_c}{Q} dh
 \end{aligned}$$

But

$$Q = V_t A_t \quad (3)$$

so

$$dt = \frac{A_c/A_t}{V} dh$$

Substitute  $V$  of Eq. (2) into Eq. (3):

$$dt = \frac{A_c/A_t}{(2g\Delta z/(1.5 + 260))^{1/2}} dh$$
$$V_c = 0.5 \text{ liter} = 0.0005 \text{ m}^3$$

or

$$0.0005 \text{ m}^3 = A_c \times (0.29 \text{ m})$$
$$A_c = 0.001724 \text{ m}^2$$
$$A_{\text{tube}} = (\pi/4)(0.005 \text{ m})^2 = 0.00001963 \text{ m}^2$$
$$\frac{A_c}{A_t} = 87.8$$

The differential equation becomes

$$dt = \frac{87.8}{(2g\Delta z/(1.5 + 10.4))^{1/2}} dh$$

Let  $h$  be measured from the level where the cylinder is 0.05 m full. Then

$$\Delta z = (0.53 \text{ m} - 0.064 \text{ m}) - h$$
$$\Delta z = 0.466 - h$$

Now we have

$$dt = \frac{87.8}{(2g(68.6 - h)/11.9)^{1/2}} dh$$
$$dt = \frac{68.6}{(0.466 - h)^{1/2}} dh$$
$$dt = -\frac{68.6}{(0.466 - h)^{1/2}} (-dh)$$

Integrate:

$$\begin{aligned}t &= -\frac{68.6(0.466 - h)^{1/2}}{1/2} \Big|_0^h \\&= -137.2(0.466 - h)^{1/2} \Big|_0^{0.2} \\&= -137.2[(0.266)^{1/2} - (0.466)^{1/2}] \\&= -137.2(0.52 - 68) \\&\quad \boxed{t = 22 \text{ s}}\end{aligned}$$

### **REVIEW**

Possible problems with this solution: The Reynolds number is very close to the point where turbulent flow will occur and this would be an unstable condition. The flow might alternate between turbulent and laminar flow.

## 10.65: PROBLEM DEFINITION

### Situation:

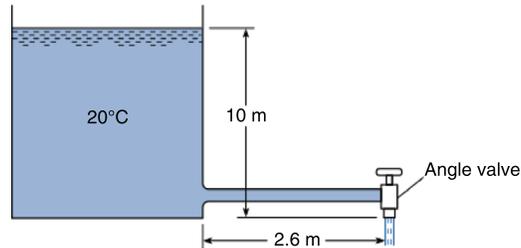
Water exits a tank through a short galvanize iron pipe.

$$D_{\text{tank}} = 2 \text{ m}, D_{\text{pipe}} = 26 \text{ mm}.$$

$$L_{\text{pipe}} = 2.6 \text{ m}, z_1 = 10 \text{ m}.$$

Fully open angle valve:  $K_v = 5.0$ .

### Sketch:



### Find:

Time required for the water level in tank to drop from 10 m to 2 m.

### Assumptions:

The pipe entrance is smooth:  $K_e \approx 0$

The kinetic energy correction factor in the pipe is  $\alpha_2 = 1.0$

## PLAN

Apply the energy equation from the top of the tank (location 1) to the exit of the angle valve (location 2).

## SOLUTION

Energy equation

$$h = \alpha_2 \frac{V^2}{2g} + \frac{V^2}{2g} (K_e + K_v + f \frac{L}{D})$$

Term by term analysis

$$\alpha_2 = 1.0$$

$$K_e \approx 0, K_v = 5.0$$

$$L/D = 2.6/0.026 = 100.0$$

Combine equation and express  $V$  in terms of  $h$

$$V = \sqrt{\frac{2gh}{6 + 100 \times f}}$$

Sand roughness height

$$\frac{k_s}{D} = \frac{0.15}{26} = 5.8 \times 10^{-3}$$

Reynolds number

$$\text{Re} = \frac{V \times 0.026}{10^{-6}} = 2.6 \times 10^4 V$$

Rate of decrease of height

$$\frac{dh}{dt} = -\frac{Q}{A} = -\frac{D_{\text{pipe}}^2}{D_{\text{tank}}^2} V = -\frac{0.000531}{3.14} V = -0.000169V$$

A program was written to first find  $V$  iteratively for a given  $h$  using the Swamee-Jain equation for the friction factor. Then a new  $h$  was found by

$$h_n = h_{n-1} - 0.000169V\Delta t$$

where  $\Delta t$  is the time step. The result was 1424 s or 23.7 minutes.

$$\boxed{t = 23.7 \text{ min}}$$

### **REVIEW**

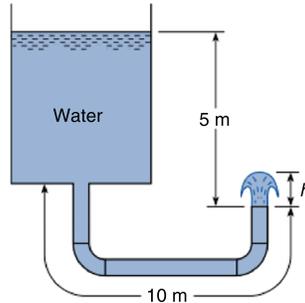
1. When valves are tested to evaluate  $K_{\text{valve}}$  the pressure taps are usually connected to pipes both upstream and downstream of the valve. Therefore, the head loss in this problem may not actually be  $5V^2/2g$ .
2. The velocity exiting the valve will probably be highly non-uniform; therefore, this solution should be considered as an approximation only.

**10.66: PROBLEM DEFINITION**Situation:

Water drains from a tank, passes through a pipe and then jets upward.

$D = 1.5 \text{ cm}$ ,  $L = 10 \text{ m}$ ,  $\Delta z = 5 \text{ m}$ .

Two  $90^\circ$  elbows in pipe.

Find:

- (a) Exit velocity of water (m/s).
- (b) Height of water jet (cm).

Assumptions:

The pipe is galvanized iron.

The water temperature is  $20^\circ\text{C}$  so  $\nu = 10^{-6} \text{ m}^2/\text{s}$ .

Relative roughness  $k_s/D = .015/1.5 = 0.01$ . Start iteration at  $f = 0.035$ .

Turbulent, so  $\alpha_2 = 1.0$

Properties:

From Table 10.4  $k_s = 0.15 \text{ mm} = 0.015 \text{ cm}$ .

From Table 10.5  $K_b = 0.9$  and  $K_e = 0.5$ .

**PLAN**

Apply the energy equation from the water surface in the tank to the pipe outlet.

**SOLUTION**

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 0 + 0 + 5 &= 0 + \alpha_2 \frac{V_2^2}{2g} + 0 + (K_e + 2K_b + f \frac{L}{D}) \frac{V_2^2}{2g} \\ 5 &= \left( \frac{V_2^2}{2g} \right) \left( 1 + 0.5 + 2 \times 0.9 + .035 \times \frac{10 \text{ m}}{0.015 \text{ m}} \right) \\ 5 &= \left( \frac{V_2^2}{2 \times 9.81} \right) (26.6) \\ V_2 &= 1.920 \text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned}\text{Re} &= VD/\nu \\ &= 1.92 \times 0.015/10^{-6} \\ &= 2.88 \times 10^4.\end{aligned}$$

Resistance coefficient (new value)

$$\begin{aligned}f &= \frac{0.25}{\left[\log_{10} \left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10} \left(\frac{0.01}{3.7} + \frac{5.74}{28800^{0.9}}\right)\right]^2} \\ &= 0.0404\end{aligned}$$

Recalculate  $V_2$  with this new value of  $f$

$$\boxed{V_2 = 1.80 \text{ m/s}}$$

Energy equation (from the pipe outlet to the top of the water jet)

$$\begin{aligned}h &= \frac{V^2}{2g} \\ &= \frac{(1.80 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} \\ &= 0.1650 \text{ m} \\ &\quad \boxed{h = 16.5 \text{ cm}}\end{aligned}$$

**10.67: PROBLEM DEFINITION**

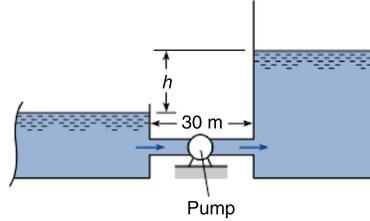
Situation:

A pump operates between a reservoir and a tank.

$$h_p = h_o(1 - Q^2/Q_{\max}^2), \quad h_o = 50 \text{ m.}$$

$$Q_{\max} = 2 \text{ m}^3/\text{s}, \quad f = 0.018.$$

$$D = 90 \text{ cm}, \quad A_{\text{tank}} = 100 \text{ m}^2.$$



Find:

Time to fill tank to 40 meters.

Properties:

From Table 10.5:  $K_e = 0.5$  and  $K_E = 1.0$ .

**PLAN**

Apply the energy equation from the reservoir water surface to the tank water surface. The head losses will be due to entrance, pipe resistance, and exit.

**SOLUTION**

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 0 + 0 + z_1 + h_p &= 0 + 0 + z_2 + (K_e + f \frac{L}{D} + K_E) \frac{V^2}{2g} \\ h_p &= (z_2 - z_1) + \left( 0.5 + \left( 0.018 \times \frac{30}{0.9} \right) + 1.0 \right) \frac{V^2}{2g} \\ h_p &= h + (2.1) \frac{V^2}{2g} \end{aligned}$$

But the head supplied by the pump is  $h_o(1 - (Q^2/Q_{\max}^2))$  so

$$\begin{aligned} h_o \left( 1 - \frac{Q^2}{Q_{\max}^2} \right) &= h + 1.05 \frac{V^2}{g} \\ 50 \left( 1 - \frac{Q^2}{4} \right) &= h + 1.05 \frac{Q^2}{gA^2} \\ 50 - 12.5Q^2 &= h + 1.05 \frac{Q^2}{gA^2} \end{aligned}$$

Area

$$A = (\pi/4)D^2 = (\pi/4)(0.9^2) = 0.63 \text{ m}^2$$

So

$$\begin{aligned} 50 - 12.5Q^2 &= h + 0.270Q^2 \\ 50 - h &= 127.77Q^2 \\ \sqrt{50 - h} &= 3.57Q \end{aligned}$$

The discharge into the tank and the rate of water level increase is related by

$$Q = A_{\text{tank}} \frac{dh}{dt}$$

so

$$\sqrt{50 - h} = 3.57 A_{\text{tank}} \frac{dh}{dt}$$

or

$$dt = 3.57 A_{\text{tank}} (50 - h)^{-1/2} dh$$

Integrating

$$t = 2 \times 3.57 A_{\text{tank}} (50 - h)^{1/2} + C$$

when  $t = 0$ ,  $h = 0$  and  $A_{\text{tank}} = 100 \text{ m}^2$  so

$$t = 714(7.071 - (50 - h)^{1/2})$$

When  $h = 40 \text{ m}$

$$t = 2791 \text{ s}$$

$$\boxed{t = 46.5 \text{ min}}$$

## 10.68: PROBLEM DEFINITION

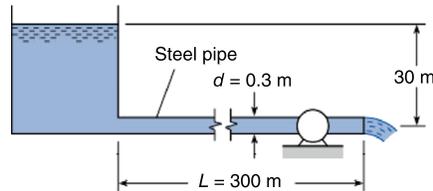
### Situation:

Water flows out of reservoir, through a steel pipe and a turbine.

$$Q = 0.3 \text{ m}^3/\text{s}, \eta = 0.8, \Delta z = 30 \text{ m}.$$

$$D = 0.3 \text{ m}, L = 300 \text{ m}.$$

### Sketch:



### Find:

Power delivered by turbine.

### Assumptions:

Turbulent flow, so  $\alpha_2 \approx 1$ .

### Properties:

Water ( $21^\circ\text{C}$ ), Table A.5:  $\nu = 9.8 \times 10^{-7} \text{ m}^2/\text{s}$

## PLAN

Apply the energy equation from the reservoir water surface to the jet at the end of the pipe.

## SOLUTION

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_T + \sum h_L \\ 0 + 0 + z_1 &= 0 + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_T + \left( K_e + f \frac{L}{D} \right) \frac{V^2}{2g} \\ z_1 - z_2 &= h_T + \left( 1 + 0.5 + f \frac{L}{D} \right) \frac{V^2}{2g} \\ 30 \text{ m} &= h_T + \left( 1.5 + f \frac{L}{D} \right) \frac{V^2}{2g} \end{aligned}$$

But

$$\begin{aligned}V &= \frac{Q}{A} = \frac{0.14 \text{ m}^3/\text{s}}{(\pi/4)(0.3 \text{ m})^2} = 1.98 \text{ m/s} \\ \frac{V^2}{2g} &= 0.199 \text{ m} \\ \text{Re} &= \frac{VD}{\nu} = 6.0 \times 10^5\end{aligned}$$

From Fig. 10.14 (in EFM10e)  $f = 0.015$  for  $k_s/D = 0.000167$ . Then

$$\begin{aligned}30 \text{ m} &= h_T + \left(1.5 + 0.0150 \times \frac{300 \text{ m}}{0.3 \text{ m}}\right) (0.199 \text{ m}) \\ h_T &= (30 - 3.3) \text{ m} = 26.7 \text{ m}\end{aligned}$$

Power equation

$$\begin{aligned}P &= Q\gamma h_T \eta \\ &= (0.14 \text{ m}^3/\text{s}) (9810 \text{ N/m}^3) (26.7 \text{ m}) (0.80) \\ &= 29,336 \text{ Nm/s} \\ &\boxed{P = 29,336 \text{ W}}\end{aligned}$$

## 10.69: PROBLEM DEFINITION

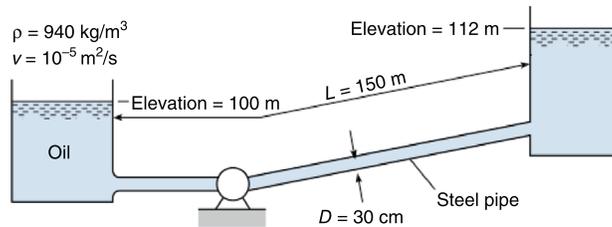
### Situation:

Oil is pumped from a lower reservoir to an upper reservoir through a steel pipe.

$$D = 30 \text{ cm}, Q = 0.20 \text{ m}^3/\text{s}.$$

$$z_1 = 100 \text{ m}, z_2 = 112 \text{ m}, L = 150 \text{ m}.$$

### Sketch:



### Find:

- Pump power.
- Sketch an EGL and HGL.

### Properties:

$$\rho = 940 \text{ kg/m}^3, \nu = 10^{-5} \text{ m}^2/\text{s}.$$

$$\text{From Table 10.4 } k_s = 0.046 \text{ mm}$$

## PLAN

Apply the energy equation between reservoir surfaces .

## SOLUTION

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 100 + h_p &= 112 + \frac{V^2}{2g} (K_e + f \frac{L}{D} + K_E) \\ h_p &= 12 + \left( \frac{V^2}{2g} \right) \left( 0.03 + f \frac{L}{D} + 1 \right) \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.20 \text{ m}^3/\text{s}}{(\pi/4) \times (0.30 \text{ m})^2} \\ &= 2.83 \text{ m/s} \\ \frac{V^2}{2g} &= 0.408 \text{ m} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{2.83 \text{ m/s} \times 0.30 \text{ m}}{10^{-5} \text{ m}^2/\text{s}} \\ &= 8.5 \times 10^4 \\ \frac{k_s}{D} &= \frac{4.6 \times 10^{-5} \text{ m}}{0.3 \text{ m}} \\ &= 1.5 \times 10^{-4} \end{aligned}$$

Resistance coefficient (from the Moody diagram, Fig. 10.14 in EFM10e)

$$f = 0.019$$

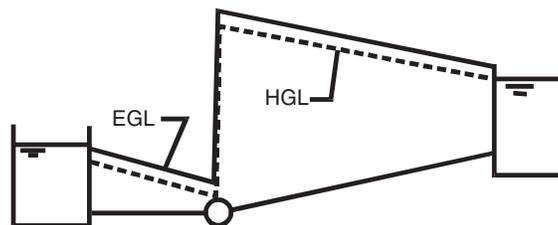
Then

$$\begin{aligned} h_p &= 12 \text{ m} + 0.408 \text{ m} \left( 0.03 + \left( 0.019 \times \frac{150 \text{ m}}{0.3 \text{ m}} \right) \right) + 1.0 \\ &= 16.3 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ &= 0.20 \text{ m}^3/\text{s} \times (940 \text{ kg}/\text{m}^3 \times 9.81 \text{ m}/\text{s}^2) \times 16.3 \text{ m} = 30100 \text{ W} \end{aligned}$$

$$P = 30.1 \text{ kW}$$



---

**10.70: PROBLEM DEFINITION****Situation:**

A cast iron pipe joins two reservoirs.

$$D = 0.3 \text{ m}, L = 12 \text{ m}.$$

$$z_1 = 45 \text{ m}, z_2 = 12 \text{ m}.$$

$$z_{\text{pipe1}} = 36 \text{ m}, z_{\text{pipe2}} = 9 \text{ m}.$$

**Find:**

- Calculate the discharge in the pipe.
- Sketch the EGL and HGL.

**Properties:**

From Table 10.4:  $k_s = 0.00026 \text{ m}$

Water (15°C), Table A.5:

$$\nu = 1.14 \times 10^{-6} \text{ m}^2/\text{s}, \mu = 1.14 \times 10^{-3} \text{ N} \cdot \text{s}/\text{m}^2, \rho = 999 \text{ kg}/\text{m}^3.$$

**PLAN**

Apply the energy equation from the water surface in the upper reservoir to the water surface in the lower reservoir.

**SOLUTION**

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 0 + 0 + 45 &= 0 + 0 + 12 + \left( K_e + 2K_v + K_E + f \frac{L}{D} \right) \frac{V^2}{2g} \\ 45 \text{ m} &= 12 \text{ m} + \left( 0.5 + 2 \times 0.2 + 1.0 + f \times \frac{60 \text{ m}}{0.3 \text{ m}} \right) \frac{V^2}{2g} \end{aligned}$$

The equation for  $V$  becomes

$$\frac{V^2}{2g} = \frac{33}{1.9 + 200f} \quad (1)$$

Relative roughness

$$\begin{aligned} \frac{k_s}{D} &= \frac{0.00026 \text{ m}}{0.3 \text{ m}} \\ &= 8.67 \times 10^{-4} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{V \times 0.3 \text{ m}}{1.14 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= (2.6 \times 10^5 \times V) \end{aligned} \quad (2)$$

Friction factor (Swamee-Jain Eq. (10.39 in EFM10e))

$$f = \frac{0.25}{\left[ \log_{10} \left( \frac{8.67 \times 10^{-4}}{3.7 \times 0.3} + \frac{5.74}{(2.6 \times 10^5 \times V)^{0.9}} \right) \right]^2} \quad (3)$$

Solve Eqs. (1) to (3) simultaneously (we applied a computer program, TK Solver)

$$\begin{aligned} V &= 10.6 \text{ m/s} \\ \text{Re} &= 2,891,000 \\ f &= 0.019 \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= 10.6 \text{ m/s} (\pi/4 \times (0.3 \text{ m})^2) \\ &\boxed{Q = 0.75 \text{ m}^3/\text{s}} \end{aligned}$$

## 10.71: PROBLEM DEFINITION

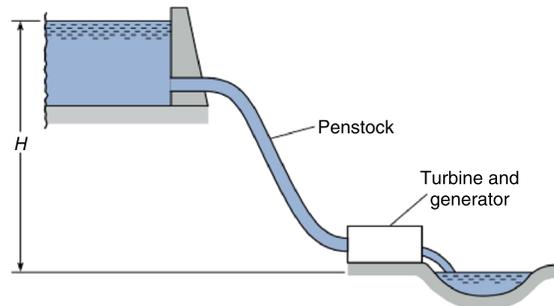
### Situation:

A small stream fills a reservoir—water from this reservoir is used to create electrical power.

$$Q = 0.06 \text{ m}^3/\text{s}, \quad H = 10 \text{ m.}$$

$$h_f = 0.9 \text{ m}, \quad L = 26 \text{ m.}$$

### Sketch:



### Find:

Find the minimum diameter for the penstock pipe.

### Assumptions:

Neglect minor losses associated with flow through the penstock.

Assume that pipes are available in following sizes— 50 mm, 100 mm, 150 mm, etc.

Assume a smooth, plastic pipe—  $k_s = 0$ .

Assume turbulent flow (check this after the calculation is done).

### Properties:

Water ( $5^\circ\text{C}$ ), Table A.5:  $\nu = 1.51 \times 10^{-6} \text{ m}^2/\text{s}$ .

## PLAN

Apply the Darcy-Weisbach equation to relate head loss ( $h_f$ ) to pipe diameter. Apply the Swamee-Jain correlation to relate friction factor ( $f$ ) to flow velocity. Also, write equations for the Reynolds number and the flow rate. Solve these four equations simultaneously to give values of  $D$ ,  $V$ ,  $f$ , and  $\text{Re}$ .

## SOLUTION

Darcy-Weisbach equation

$$h_f = f \frac{L V^2}{D 2g} \quad (1)$$

Resistance coefficient (Swamee-Jain correlation; turbulent flow)

$$f = \frac{0.25}{\left[ \log_{10} \left( \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \quad (2)$$

Reynolds number

$$\text{Re} = \frac{VD}{\nu} \quad (3)$$

Flow rate equation

$$Q = V \frac{\pi D^2}{4} \quad (4)$$

Solve Eqs. (1) to (4) simultaneously. The computer program TKSolver was used for our solution.

$$\begin{aligned} f &= 0.01448 \\ V &= 2.7 \text{ m/s} \\ D &= 15.9 \text{ cm} \\ \text{Re} &= 289,000 \end{aligned}$$

Recommendation

Select a pipe with  $D = 20 \text{ cm}$ .

### REVIEW

With an 20-cm-diameter pipe, the head loss associated with flow in the pipe will be less than 10% of the total available head (10 m). If an engineer selects a pipe that is larger than 20 cm, then cost goes up.

---

**10.72: PROBLEM DEFINITION****Situation:**

A pipe runs from a reservoir to an open drain.

$z_{\text{reservoir}} = 45 \text{ m}$ ,  $z_{\text{pipe1}} = 30 \text{ m}$ ,  $z_{\text{pipe2}} = 18 \text{ m}$ .

$D = 0.15 \text{ m}$ ,  $L = 30 \text{ m}$ ,  $p_1 = p_2 = 0 \text{ kPa}$ .

**Find:**

Discharge ( $\text{m}^3/\text{s}$ ).

**Properties:**

From Table 10.4:  $k_s = 1.2 \times 10^{-4} \text{ m}$ .

Water ( $10^\circ\text{C}$ ), Table A.5:  $\nu = 1.31 \times 10^{-6} \text{ m}^2/\text{s}$ .

From Table 10.5:  $K_e = 0.5$ .

**PLAN**

Apply the energy equation from water surface in reservoir to the outlet.

**SOLUTION**

Energy equation

$$\begin{aligned}\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \\ 0 + 0 + 45 \text{ m} &= 0 + \frac{V^2}{2g} + 18 \text{ m} + (K_e + K_E + f \frac{L}{D}) \frac{V^2}{2g} \\ \frac{V^2}{2g} \left( 1.5 + f \frac{L}{D} \right) &= 27 \text{ m} \\ \frac{V^2}{2g} &= \frac{27 \text{ m}}{1.5 + 60f}\end{aligned}\tag{1}$$

Sand roughness height

$$\frac{k_s}{D} = \frac{1.2 \times 10^{-4} \text{ m}}{0.15 \text{ m}} = 0.0008$$

Reynolds number

$$\text{Re} = 3.54 \times 10^4 \times V\tag{2}$$

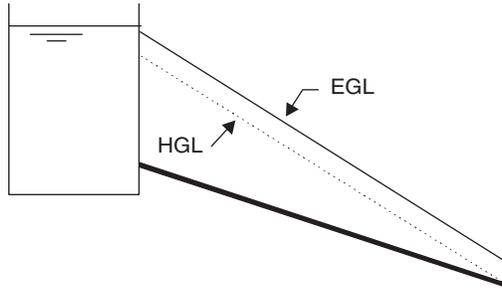
Solve a set of three simultaneous equations: Eq. (1) Eq. (2), and the Swamee-Jain correction. The result is

$$V = 9.93 \text{ m/s}$$

Flow rate equation

$$\begin{aligned}Q &= VA \\ &= 9.93(\pi/4)(0.15^2)\end{aligned}$$

$$Q = 0.18 \text{ m}^3/\text{s}$$



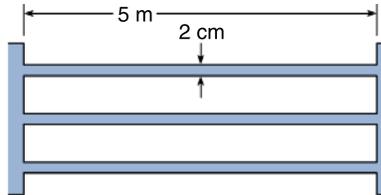
**10.73: PROBLEM DEFINITION**Situation:

A shell and tube heat exchanger is used in a geothermal power system  
Clean fluid inside the tubes; brine outside of the tubes.

100 tubes total. Galvanized iron.

$D = 2 \text{ cm}$ ,  $L = 5 \text{ m}$ ,  $\dot{m} = 50 \text{ kg/s}$ .

After continued used, 2 mm of build up,  $k_s = .5 \text{ mm}$ .

Find:

Power required to operate heat exchanger with:

- (a) clean tubes.
- (b) scaled tubes.

Properties:

Pipe roughness (galvanized iron), Table 10.4 (EFM10e),  $k_s = 0.15 \text{ mm}$ .

Given fluid properties ( $T = 200 \text{ }^\circ\text{C}$ )  $\rho = 860 \text{ kg/m}^3$ ,  $\mu = 1.35 \times 10^{-4} \text{ N s/m}^2$ .

**SOLUTION**

$$\dot{m}/\text{tube} = 0.50 \text{ kg/s}$$

$$Q/\text{tube} = \frac{0.50 \text{ kg/s}}{860 \text{ kg/m}^3} = 5.8139 \times 10^{-4} \text{ m}^3/\text{s}$$

$$V = \frac{Q}{A} = \frac{5.8139 \times 10^{-4} \text{ m}^3/\text{s}}{(\pi/4) \times (0.02 \text{ m})^2} = 1.851 \text{ m/s}$$

$$\text{Re} = \frac{VD\rho}{\mu} = \frac{1.851 \text{ m/s} \times 0.02 \text{ m} \times 860 \text{ kg/m}^3}{1.35 \times 10^{-4} \text{ N s/m}^2} = 2.35 \times 10^5$$

$$\frac{k_s}{D} = \frac{0.15 \text{ mm}}{20 \text{ mm}} \approx 0.007$$

From the Moody diagram,  $f = 0.034$ . Then

$$h_f = f \frac{L V^2}{D 2g} = 0.034 \times \frac{5 \text{ m}}{0.02 \text{ m}} \times \frac{(1.851 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} = 1.48 \text{ m}$$

$$\text{a) } P = Q\gamma h_f = 5.8139 \times 10^{-4} \text{ m}^3/\text{s} \times 860 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 1.48 \text{ m} \times 100 = 726 \text{ W}$$

$$P = 726 \text{ W (clean tubes)}$$

Part (b)

$$\begin{aligned}\frac{k_s}{D} &= \frac{0.5}{16} \\ &= 0.031\end{aligned}$$

From the Moody diagram,  $f = 0.058$

$$P = 726 \text{ W} \times \frac{0.058}{0.034} \times \left(\frac{20}{16}\right)^4 = 3.03 \text{ kW}$$

$$\boxed{P = 3.03 \text{ kW (scaled tubes)}}$$

### 10.74: PROBLEM DEFINITION

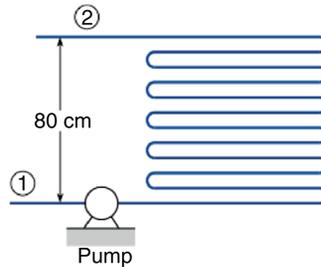
#### Situation:

Water flows through a heat exchanger.

$$L = 10 \text{ m}, D = 2 \text{ cm}, Q = 3.4 \times 10^{-4} \text{ m}^3/\text{s}.$$

$$T_1 = 20^\circ\text{C}, T_2 = 80^\circ\text{C}, \Delta z = 0.8 \text{ m}, p_1 = p_2.$$

#### Sketch:



#### Find:

Pump power required.

#### Assumptions:

$K = 2 \times K$  for smooth bends of  $90^\circ$ ,  $r/d \approx 1$ ,  $K_b \approx 2 \times 0.35 = 0.7$

Properties can be found at the average temperature in the heat exchanger.

Smooth tubes ( $k_s = 0.0 \text{ m}$ )

#### Properties:

Water ( $50^\circ\text{C}$ ), Table A.5:  $\nu = 5.53 \times 10^{-7} \text{ m}^2/\text{s}$ ,  $\rho = 998 \text{ kg}/\text{m}^3$ ,  $\gamma = 9693 \text{ N}/\text{m}^3$ .

### SOLUTION

Energy equation (section 1 at inlet, section 2 at exit)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Since  $V_1 = V_2$  and  $p_1 = p_2$

$$h_p = h_L + (z_2 - z_1)$$

Velocity

$$V = \frac{Q}{A} = \frac{3 \times 10^{-4} \text{ m}^3/\text{s}}{\pi/4 \times (0.02 \text{ m})^2} = 0.955 \text{ m/s}$$

Reynolds number and resistance coefficient

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} = \frac{0.955 \text{ m/s} \times (0.02 \text{ m})}{5.53 \times 10^{-7} \text{ m}^2/\text{s}} = 34539 \\ f &= 0.022 \end{aligned}$$

Head loss

$$\begin{aligned} h_L &= \left( f \frac{L}{D} + 19K_b \right) \frac{V^2}{2g} = \left( 0.022 \frac{10 \text{ m}}{0.02 \text{ m}} + 19 \times 0.7 \right) \frac{(0.955 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} \\ &= 1.13 \text{ m} \end{aligned}$$

Final calculations

$$\begin{aligned} h_p &= z_2 - z_1 + h_L = 0.8 + 1.13 = 1.93 \text{ m} \\ P &= \gamma h_p Q = 9693 \text{ N/m}^3 \times 1.93 \text{ m} \times 3 \times 10^{-4} \\ &\boxed{P = 5.61 \text{ W}} \end{aligned}$$

## 10.75: PROBLEM DEFINITION

### Situation:

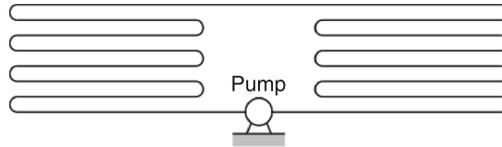
A heat exchanger is described in the problem statement.

$$V = 8 \text{ m/s}, D = 2 \text{ cm.}$$

$$\eta = 0.85, L = 10 \text{ m.}$$

$$14 \text{ - } 180^\circ \text{ elbows, } K_L = 2.2.$$

### Sketch:



### Find:

Power required to operate pump.

### Properties:

Water (40°C), Table A.5:  $\nu = 6.58 \times 10^{-7} \text{ m}^2/\text{s}$ ,  $\gamma = 9732 \text{ N/m}^3$

From Table 10.4  $k_s = 0.0015 \text{ mm}$ .

## SOLUTION

Reynolds number

$$\text{Re} = \frac{0.02 \times 8}{6.58 \times 10^{-7}} = 2.43 \times 10^5$$

Flow rate equation

$$Q = \frac{\pi}{4} \times (0.02 \text{ m})^2 \times 8 \text{ m/s} = 0.00251 \text{ m}^3/\text{s}$$

Relative roughness (copper tubing)

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-3} \text{ mm}}{20 \text{ mm}} = 7.5 \times 10^{-5}$$

Resistance coefficient (from Moody diagram)

$$f = 0.0155$$

Energy equation

$$\begin{aligned} h_p &= \frac{V^2}{2g} \left( f \frac{L}{D} + \sum K_L \right) \\ &= \frac{(8 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} \left( 0.0155 \times \frac{10 \text{ m}}{0.02 \text{ m}} + 14 \times 2.2 \right) = 126 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= \frac{\gamma Q h_p}{\eta} \\ &= \frac{9732 \text{ N/m}^3 \times 0.00251 \text{ m}^3/\text{s} \times 126 \text{ m}}{0.85} \\ &= 3618 \text{ W} \end{aligned}$$

$$\boxed{P = 3.62 \text{ kW}}$$

## 10.76: PROBLEM DEFINITION

Situation:

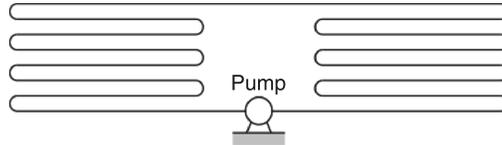
A heat exchanger is described in the problem statement.

$L = 15$  m,  $D = 15$  mm.

$Q_{\max} = 10^{-3}$  m<sup>3</sup>/s.

$h_p = h_{p0} [1 - (Q/Q_{\max})^3]$ .

14 - 180° elbows,  $K_L = 2.2$ .



Find:

System operating points for  $h_{p0}$  of 2 m, 10 m and 20 m.

Properties:

From Table 10.4:  $k_s = 1.5 \times 10^{-3}$  mm.

## SOLUTION

Energy equation

$$h_p = \frac{V^2}{2g} \left( \sum K_L + f \frac{L}{D} \right)$$

Substitute in the values for loss coefficients,  $L/D$  and the equation for  $h_p$

$$h_{p0} \left[ 1 - \left( \frac{Q}{Q_{\max}} \right)^3 \right] = \frac{V^2}{2g} (14 \times 2.2 + f \times 1000)$$

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= 1.767 \times 10^{-4} V \end{aligned}$$

Combine equations

$$h_{p0} \left[ 1 - \left( \frac{Q}{Q_{\max}} \right)^3 \right] = 1.632 \times 10^6 Q^2 (30.8 + f \times 1000) \quad (1)$$

Relative roughness

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-3}}{15} = 10^{-4}$$

Reynolds number

$$\begin{aligned}\text{Re} &= \frac{VD}{\nu} \\ &= \frac{V \times 0.015}{6.58 \times 10^{-7}} = 2.28 \times 10^4 V = 1.29 \times 10^8 Q\end{aligned}$$

Eq. (1) becomes

$$F(Q) = h_{p0} \left[ 1 - \left( \frac{Q}{Q_{\max}} \right)^3 \right] - 1.632 \times 10^6 Q^2 (30.8 + f \times 1000)$$

A program was written to evaluate  $F(Q)$  by inputting a value for  $Q$  and trying different  $Q$ 's until  $F(Q) = 0$ . The results are

| $h_{p0}$ (m) | $Q$ (m <sup>3</sup> /s) |
|--------------|-------------------------|
| 2            | 0.000356                |
| 10           | 0.000629                |
| 20           | 0.000755                |

## 10.77: PROBLEM DEFINITION

### Situation:

Gasoline being pumped from a gas tank.

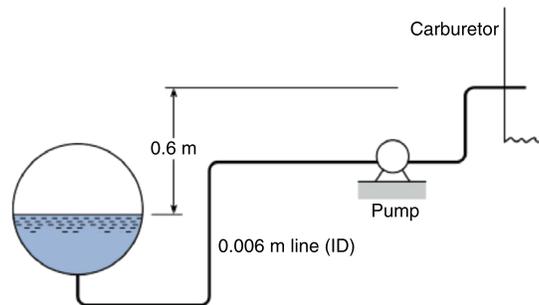
Pressure in the tank = 101.3 kPa, pressure in the carburetor = 96.5 kPa.

$D_{\text{tube}} = 0.6 \text{ cm} = 0.006 \text{ m}$ ,  $D_{\text{jet}} = 0.075 \text{ cm} = 0.00075 \text{ m}$ .

$L = 3 \text{ m}$ ,

$\eta = 0.80$ ,  $Q = 0.45 \text{ L/min} = 7.5 \times 10^{-6} \text{ m}^3/\text{s}$

### Sketch:



### Find:

Pump power.

### Properties:

Gasoline Fig. A.2 (EFM10e):  $S = 0.68$ ,  $\nu = 5.12 \times 10^{-7} \text{ m}^2/\text{s}$ ,  $\gamma = 9800 \text{ N/m}^3 \times 0.68 = 6664 \text{ N/m}^3$

Loss coefficient, Table 10.5 (EFM10e),  $90^\circ$  smooth bends,  $r/d = 6$ ,  $K_b = 0.21$ .

## SOLUTION

Velocity values

$$V_{\text{tube}} = \frac{4Q}{\pi D_{\text{tube}}^2} = \frac{4(7.5 \times 10^{-6} \text{ m}^3/\text{s})}{\pi (0.006 \text{ m})^2} = 0.27 \text{ m/s}$$

$$V_{\text{jet}} = V_{\text{tube}} \left( \frac{0.006 \text{ m}}{0.00075 \text{ m}} \right)^2 = (0.27 \text{ m/s}) (64) = 17.3 \text{ m/s}$$

$$\frac{V_{\text{tube}}^2}{2g} = 0.0037 \text{ m}$$

$$\frac{V_{\text{jet}}^2}{2g} = 15.3 \text{ m}$$

Reynolds number (fuel line)

$$\begin{aligned}\text{Re} &= \frac{VD}{\nu} \\ &= \frac{0.27 \text{ m/s} \times 0.006 \text{ m}}{5.12 \times 10^{-7} \text{ m}^2/\text{s}} \\ &= 3164\end{aligned}$$

From Moody diagram, Fig 10.14 (EFM10e)

$$f \approx 0.040$$

Energy equations

$$\begin{aligned}h_L &= \left( f \frac{L}{D} + 5K_b \right) \frac{V_1^2}{2g} \\ &= \left( 0.040 \times \frac{3 \text{ m}}{0.006 \text{ m}} + 5 \times 0.21 \right) 0.0037 \text{ m} = 0.08 \text{ m}\end{aligned}$$

$$\begin{aligned}h_p &= \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2}{2g} + h_L \\ &= \frac{(96.5 \text{ kPa} - 101.3 \text{ kPa})}{6664 \text{ N/m}^3} + 0.6 + 15.3 + 0.08 \text{ m} = 15.25 \text{ m}\end{aligned}$$

Power equation

$$P = \frac{\gamma h_p Q}{\eta} = \frac{6664 \text{ N/m}^3 (15.25 \text{ m}) 0.0000075 \text{ m}^3/\text{s}}{0.8} = 0.953 \text{ N-m/s}$$

$$\boxed{P = 0.953 \text{ W}}$$

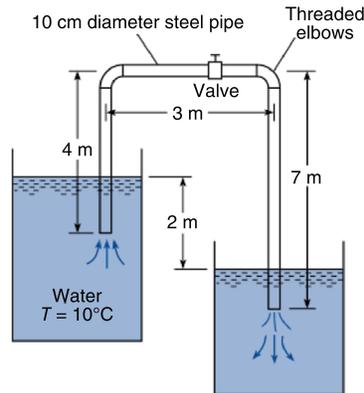
## 10.78: PROBLEM DEFINITION

### Situation:

A partially-closed valve on a steel pipeline between two reservoirs.

$D = 10$  cm,  $\Delta z = 2$  m,  $L = 14$  m.

### Sketch:



### Find:

Loss coefficient for valve,  $K_v$ .

### Properties:

From Table 10.4:  $k_s = 0.046$  mm

Water (10°C), Table A.5:  $\nu = 1.31 \times 10^{-6}$  m<sup>2</sup>/s.

## PLAN

First find  $Q$  for valve wide open. Assume valve is a gate valve.

## SOLUTION

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 2 &= 0 + 0 + 0 + \frac{V^2}{2g} (0.5 + 0.9 + 0.2 + 0.9 + 1 + f \frac{L}{D}) \\ V^2 &= \frac{4g}{3.5 + f \frac{L}{D}} \end{aligned}$$

Assume  $f = 0.015$ . Then

$$V = \left( \frac{4 \times 9.81 \text{ m/s}^2}{3.5 + 0.015 \times 14 \text{ m}/0.1 \text{ m}} \right)^{1/2} = 2.65 \text{ m/s}$$
$$\frac{k_s}{D} \simeq 0.0005$$
$$\text{Re} = \frac{2.65 \text{ m/s} \times 0.10 \text{ m}}{1.3 \times 10^{-6} \text{ m}^2/\text{s}} = 2.0 \times 10^5$$

From the Moody diagram,  $f = 0.019$ . Then

$$V = \left( \frac{4 \times 9.81 \text{ m/s}^2}{3.5 + 0.019 \times 14 \text{ m}/0.1 \text{ m}} \right)^{1/2} = 2.52 \text{ m/s}$$
$$\text{Re} = 2.0 \times 10^5 \times \frac{2.52}{2.65} = 1.9 \times 10^5; \text{ O.K.}$$

This is close to  $2.0 \times 10^5$  so no further iterations are necessary. For one-half the discharge

$$V = 1.26 \text{ m/s}$$
$$\text{Re} = 9.5 \times 10^4$$

and from the Moody diagram  $f = 0.021$ . So

$$V^2 = 1.588 = \frac{4 \times 9.81 \text{ m/s}^2}{3.3 + K_v + 0.021 \times 14 \text{ m}/0.1 \text{ m}}$$
$$3.3 + K_v + 2.94 = 24.7$$
$$\boxed{K_v = 18.5}$$

---

**10.79: PROBLEM DEFINITION**Situation:

A galvanized steel pipe connects a water main to a factory.

$$p_1 = 350 \text{ kPa}, Q = 0.025 \text{ m}^3/\text{s}.$$

$$L = 160 \text{ m}, z_2 = 8 \text{ m}, p_2 = 70 \text{ kPa}.$$

Find:

The pipe size.

Properties:

From Table 10.4  $k_s = 0.15 \text{ mm}$ .

Water (10 °C), Table A.5:  $\gamma = 9810 \text{ N/m}^3$ ,  $\nu = 1.31 \times 10^{-6} \text{ m}^2/\text{s}$ .

**SOLUTION**

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f \\ \frac{350000 \text{ Pa}}{9810 \text{ N/m}^3} + 0 &= \frac{70000 \text{ Pa}}{9810 \text{ N/m}^3} + 8 + h_f \end{aligned}$$

$$h_f = 20.54 \text{ m}$$

$$f \frac{L}{D} \frac{Q^2/A^2}{2g} = 20.54$$

$$f \frac{L}{D} \frac{Q^2}{(\pi/4 \times D^2)^2 / 2g} = 20.54$$

$$\frac{4^2 f L Q^2 / \pi^2}{2g D^5} = 20.54$$

$$D = \left[ \frac{(8/20.54) f L Q^2}{\pi^2 g} \right]^{1/5}$$

Assume  $f = 0.020$ . Then

$$\begin{aligned} D &= \left[ \frac{(8 \text{ m}/20.54 \text{ m}) \times 0.02 \times 160 \text{ m} \times (0.025 \text{ m}^3/\text{s})^2}{\pi^2 \times 9.81 \text{ m/s}^2} \right]^{1/5} \\ &= 0.0957 \text{ m} \end{aligned}$$

Relative roughness

$$\begin{aligned} \frac{k_s}{D} &= \frac{0.15}{96} \\ &= 0.00157 \end{aligned}$$

Reynolds number

$$\begin{aligned}\text{Re} &= \frac{4Q}{\pi D\nu} \\ &= \frac{4 \times (0.025 \text{ m}^3/\text{s})}{\pi \times (0.0957 \text{ m}) \times (1.31 \times 10^{-6} \text{ m}^2/\text{s})} \\ &= 2.538 \times 10^5\end{aligned}$$

Friction factor ( $f$ ) (Swamee-Jain correlation)

$$\begin{aligned}f &= \frac{0.25}{\left[\log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10} \left( \frac{.00157}{3.7} + \frac{5.74}{(2.538 \times 10^5)^{0.9}} \right)\right]^2} \\ &= 2.2966 \times 10^{-2}\end{aligned}$$

Recalculate pipe diameter

$$D = 0.0957 \times \left( \frac{0.0297}{0.020} \right)^{1/5} = 0.098 \text{ m}$$

Specify a 10-cm pipe

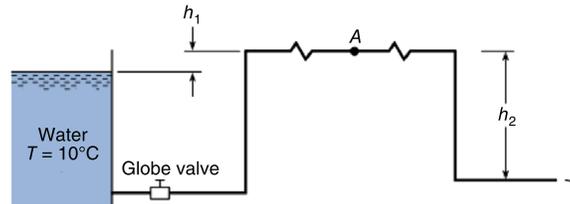
## 10.80: PROBLEM DEFINITION

### Situation:

A steel pipe discharges into the atmosphere.

$D = 12 \text{ cm}$ ,  $L = 800 \text{ m}$ ,  $z_1 = 12 \text{ m}$ .

### Sketch:



### Find:

Discharge ( $\text{m}^3/\text{s}$ ).

Pressure at point A.

### Assumptions:

Water temperature is  $10^\circ\text{C}$ .

### Properties:

Water ( $10^\circ\text{C}$ ), Table A.5:  $\nu = 1.31 \times 10^{-6} \text{ m}^2/\text{s}$ .

From Table 10.5:  $K_v = 10$ ,  $K_b = 0.9$ ,  $K_e = 0.5$ .

## SOLUTION

Energy equation

$$\begin{aligned}\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 0 + 12 + 0 &= 0 + 0 + \frac{V_2^2}{2g}(1 + K_e + K_v + 4K_b + f \times \frac{L}{D})\end{aligned}$$

Using a pipe diameter of 10 cm and assuming  $f = 0.025$

$$24g = V^2(1 + 0.5 + 10 + 4(0.9) + 0.025 \times \frac{800 \text{ m}}{0.12 \text{ m}})$$

$$V^2 = 24g/265.1 = 1.0668 \text{ m}^2/\text{s}^2$$

$$V = 1.138 \text{ m/s}$$

$$Q = VA$$

$$= 1.138 \text{ m/s} \times \pi/4 \times (0.12 \text{ m})^2$$

$$\boxed{Q = 0.0129 \text{ m}^3/\text{s}}$$

$$\text{Re} = 1.138 \times 0.12 / 1.31 \times 10^{-6} = 1.04 \times 10^5$$

From Fig. 10.8  $f \approx 0.025$

$$\begin{aligned}\frac{p_A}{\gamma} + \frac{V^2}{2g} + z_A &= \frac{p_B}{\gamma} + \frac{V^2}{2g} + z_B + \sum h_L \\ \frac{p_A}{\gamma} + 15 &= \frac{V^2}{2g} (2K_b + f \times \frac{L}{D}) \\ \frac{p_A}{\gamma} &= \frac{1.067 \text{ m}^2/\text{s}^2}{2g} (2 \times 0.9 + 0.025 \times \frac{400 \text{ m}}{0.12 \text{ m}}) - 15 \text{ m} = -9.38 \text{ m} \\ p_A &= 9810 \text{ N/m}^3 \times (-9.38 \text{ m}) \\ &\boxed{p_A = -92.0 \text{ kPa}}\end{aligned}$$

Note that this is not a good installation because the pressure at  $A$  is near cavitation level.

### 10.81: PROBLEM DEFINITION

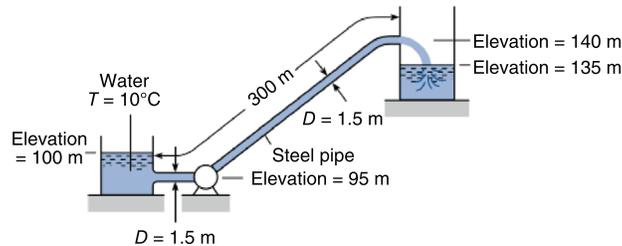
#### Situation:

Water is pumped from one reservoir to another reservoir.

$$D = 1.5 \text{ m}, Q = 25 \text{ m}^3/\text{s}.$$

$$L = 300 \text{ m}, z_2 = 140 \text{ m}, z_1 = 100 \text{ m}.$$

#### Sketch:



#### Find:

Pump power.

#### Properties:

From Table 10.4:  $k_s = 0.046 \text{ mm}$ .

Water ( $10^\circ\text{C}$ ), Table A.5  $\nu = 1.31 \times 10^{-6} \text{ m}^2/\text{s}$ .

### SOLUTION

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 0 + 0 + 100 + h_p &= 0 + \frac{V_2^2}{2g} + 140 + \frac{V_2^2}{2g} \left( 0.03 + f \frac{L}{D} \right) \end{aligned}$$

Flow rate

$$\begin{aligned} V_2 &= \frac{Q}{A_p} \\ &= \frac{25 \text{ m}^3/\text{s}}{(\pi/4) \times (1.5 \text{ m})^2} \\ &= 14.15 \text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{14.15 \text{ m/s} \times 1.5 \text{ m}}{1.31 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= 1.620 \times 10^7 \\ \frac{k_s}{D} &= \frac{0.046}{1500} \\ &= 0.00003 \end{aligned}$$

Friction factor (Moody Diagram) or the Swamee-Jain correlation:

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10} \left( \frac{0.00003}{3.7} + \frac{5.74}{(1.620 \times 10^7)^{0.9}} \right)\right]^2} \\ &= 0.009995 \\ &\approx 0.01 \end{aligned}$$

Then

$$\begin{aligned} h_p &= 140 \text{ m} - 100 \text{ m} + \frac{V_2^2}{2g} \left( 1.03 + 0.010 \times \frac{300 \text{ m}}{1.5 \text{ m}} \right) \\ &= 140 \text{ m} - 100 \text{ m} + \frac{(14.15 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} \left( 1.03 + 0.010 \times \frac{300 \text{ m}}{1.5 \text{ m}} \right) \\ h_p &= 70.92 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ &= (25 \text{ m}^3/\text{s}) \times (9810 \text{ N/m}^3) \times (70.92 \text{ m}) = 17.4 \text{ MW} \end{aligned}$$

$$\boxed{P = 17.4 \text{ MW}}$$

## 10.82: PROBLEM DEFINITION

### Situation:

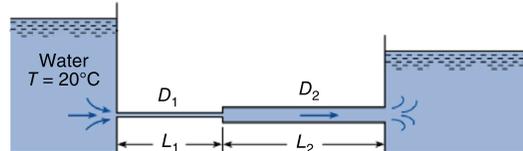
A system with two pipe sizes connects two reservoirs.

$$k_s = 0.1 \text{ mm}, Q = 0.1 \text{ m}^3/\text{s}.$$

$$D_1 = 12 \text{ cm}, L_1 = 60 \text{ m}.$$

$$D_2 = 24 \text{ cm}, L_2 = 120 \text{ m}.$$

### Sketch:



### Find:

Difference in water surface between two reservoirs.

### Properties:

Water (20°C), Table A.5:  $\nu = 10^{-6} \text{ m}^2/\text{s}$ .

## SOLUTION

$$\frac{k_s}{D_{12}} = \frac{0.1}{120} = 0.00083$$

$$\frac{k_s}{D_{24}} = \frac{0.1}{240} = 0.00042$$

$$V_{12} = \frac{Q}{A_{12}} = \frac{0.1 \text{ m}^3/\text{s}}{\pi/4 \times (0.12 \text{ m})^2} = 8.842 \text{ m/s}$$

$$V_{24} = 2.210 \text{ m/s}$$

$$\text{Re}_{12} = \frac{VD}{\nu} = \frac{8.842 \text{ m/s} \times 0.12 \text{ m}}{10^{-6} \text{ m}^2/\text{s}} = 1.06 \times 10^6$$

$$\text{Re}_{24} = \frac{2.210 \text{ m/s} \times 0.24 \text{ m}}{10^{-6} \text{ m}^2/\text{s}} = 5.37 \times 10^5$$

Resistance Coefficient (from the Moody diagram, Fig. 10.8)

$$f_{12} = 0.0190$$

$$f_{24} = 0.0170$$

Energy equation

$$z_1 - z_2 = \sum h_L$$

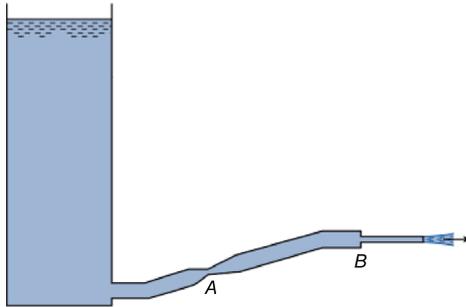
$$\begin{aligned}
z_1 - z_2 &= \frac{V_{12}^2}{2g} \left( 0.5 + 0.019 \times \frac{60}{0.12} \right) \\
&\quad + \frac{V_{24}^2}{2g} \left( 1 + 0.017 \times \frac{120 \text{ m}}{0.24 \text{ m}} \right) + \frac{(V_{12} - V_{24})^2}{2g} \\
z_1 - z_2 &= \left( \frac{(8.842 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} \right) (10.0) \\
&\quad + \left( \frac{(2.210 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} \right) (9.5) + \frac{(8.842 \text{ m/s} - 2.210 \text{ m/s})^2}{2 \times 9.81} \\
\boxed{z_1 - z_2 = 44.5 \text{ m}}
\end{aligned}$$

### 10.83: PROBLEM DEFINITION

Situation:

A tank discharges to atmosphere through a piping system.

Sketch:

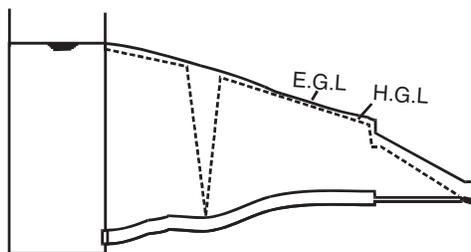


Find:

Sketch the EGL and HGL.

Identify where cavitation might occur.

### SOLUTION



Answer ==> Cavitation could occur in the venturi throat section or just downstream of the abrupt contraction (where there will be a contraction of the flow area).

### 10.84: PROBLEM DEFINITION

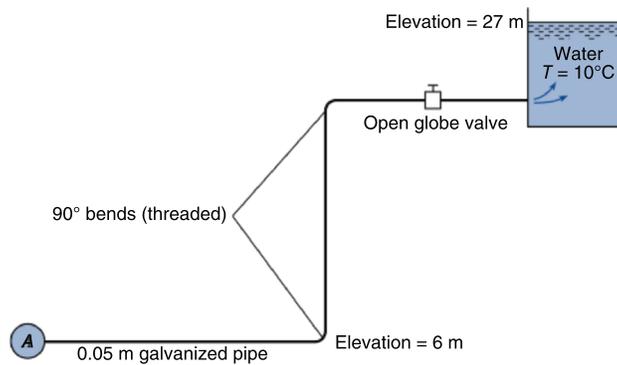
#### Situation:

A steel pipe carries water from a main pipe to a reservoir.

$$z_1 = 6 \text{ m}, z_2 = 27 \text{ m}.$$

$$Q = 190 \text{ lpm}, D = 0.05 \text{ m}, L = 72 \text{ m}.$$

#### Sketch:



#### Find:

Pressure at point A.

#### Properties:

From Table 10.5:  $K_b = 0.9$ ,  $K_v = 10$ .

From Table 10.4:  $k_s = 1.5 \times 10^{-4} \text{ m}$ .

Water (10°C), Table A.5:  $\nu = 1.31 \times 10^{-6} \text{ m}^2/\text{s}$ .

### SOLUTION

Energy equation

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L$$

$$\frac{p_A}{\gamma} + 6 \text{ m} + 0 = 0 + 27 \text{ m} + 0 + \frac{V^2}{2g} (0.5 + 2K_b + K_v + f \frac{L}{D} + 1)$$

$$V = \frac{Q}{A} = \frac{190 \text{ L/min} \times (0.001 \text{ m}^3/\text{L}) / (60 \text{ s/min})}{\pi/4 \times (0.05 \text{ m})^2} = 1.6 \text{ m/s}$$

$$\frac{V^2}{2g} = \frac{(1.6 \text{ m/s})^2}{19.62 \text{ m/s}^2} = 0.13 \text{ m}$$

$$\text{Re} = \frac{1.6 \text{ m/s} \times 0.05 \text{ m}}{1.31 \times 10^{-6} \text{ m}^2/\text{s}} = 6 \times 10^4$$

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-4}}{0.05} = 0.003$$

Resistance coefficient (from Moody diagram)

$$f = 0.028$$

Energy equation becomes

$$\begin{aligned} p_A &= \gamma \left[ 21 \text{ m} + 0.13 \text{ m} \left( 0.5 + 2 \times 0.9 + 10 + \left( 0.028 \times \frac{72}{0.05} \right) + 1.0 \right) \right] \\ &= 9810 \text{ N/m}^3 \times 27.97 \text{ m} = 274,386 \text{ Pa} \\ &\quad \boxed{p_A = 274 \text{ kPa}} \end{aligned}$$

## 10.85: PROBLEM DEFINITION

### Situation:

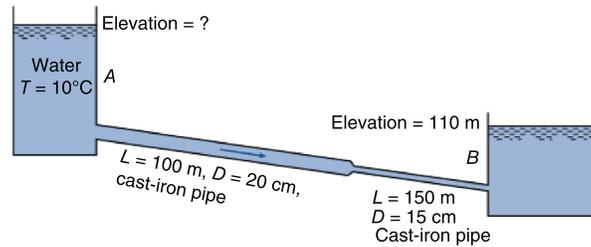
Two reservoirs are connected by a cast-iron pipe of varying diameter.

$$z_2 = 110 \text{ m}, Q = 0.3 \text{ m}^3/\text{s}.$$

$$D_1 = 20 \text{ cm}, L_1 = 100 \text{ m}.$$

$$D_2 = 15 \text{ cm}, L_2 = 150 \text{ m}.$$

### Sketch:



### Find:

Water surface elevation in reservoir A.

### Properties:

From Table 10.4:  $k_s = 0.26 \text{ mm}$ .

Water ( $10^\circ\text{C}$ ), Table A.5:  $\nu = 1.3 \times 10^{-6} \text{ m}^2/\text{s}$ .

## SOLUTION

$$\frac{k_s}{D_{20}} = \frac{0.26}{200} = 0.0013$$

$$\frac{k_s}{D_{15}} = 0.0017$$

$$V_{20} = \frac{Q}{A_{20}} = \frac{0.03 \text{ m}^3/\text{s}}{\pi/4 \times (0.20 \text{ m})^2} = 0.955 \text{ m/s}$$

$$\frac{Q}{A_{15}} = 1.697 \text{ m/s}$$

$$\text{Re}_{20} = \frac{VD}{\nu} = \frac{0.955 \text{ m/s} \times 0.2 \text{ m}}{1.3 \times 10^{-6} \text{ m}^2/\text{s}} = 1.5 \times 10^5$$

$$\text{Re}_{15} = \frac{1.697 \text{ m/s} \times 0.15 \text{ m}}{1.3 \times 10^{-6} \text{ m}^2/\text{s}} = 1.9 \times 10^5$$

From Fig. 10.14 (EFM 10e):  $f_{20} = 0.022$ ;  $f_{15} = 0.024$

$$\begin{aligned}z_1 &= z_2 + \sum h_L \\z_1 &= 110 + \frac{V_{20}^2}{2g} \left( 0.5 + 0.022 \times \frac{100 \text{ m}}{0.2 \text{ m}} + 0.19 \right) \\&\quad + \frac{V_{15}^2}{2g} \left[ \left( 0.024 \times \frac{150 \text{ m}}{0.15 \text{ m}} \right) + 1.0 + 0.19 \right] \\&= 110 \text{ m} + 0.0465 \text{ m}(11.7) + 0.1468 \text{ m}(25.19) \\&= 110 + 0.535 + 3.70 = 114.2 \text{ m}\end{aligned}$$

$$\boxed{z_1 = 114 \text{ m}}$$

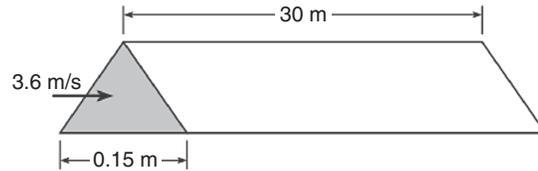
**10.86: PROBLEM DEFINITION**

Situation:

Air flowing through a triangle shaped horizontal duct.

$L = 30$  m,  $V = 3.6$  m/s.

$k_s = 0.00015$  m, Triangle side = 0.15 m.



Find:

Pressure drop over 30 m length.

Properties:

Air (15°C), Table A.3:  $\nu = 1.46 \times 10^{-5}$  m<sup>2</sup>/s and  $\rho = 1.23$  kg/m<sup>3</sup>.

**SOLUTION**

$$\begin{aligned}
 h &= (0.15 \text{ m})(\cos 30^\circ) = 0.13 \text{ m} \\
 A &= (0.15 \text{ m})(0.13 \text{ m})/2 = 0.00975 \text{ m}^2 \\
 R_h &= \frac{A}{P} = \frac{0.00975 \text{ m}^2}{3 \times 0.15 \text{ m}} = 0.023 \text{ m} \\
 4R_h &= 0.092 \text{ m} \\
 \frac{k_s}{4R_h} &= \frac{0.00015}{0.092} = 0.00173 \\
 \text{Re} &= \frac{(V)(4R_h)}{\nu} = \frac{(3.6 \text{ m/s})(0.092 \text{ m})}{1.46 \times 10^{-5} \text{ m}^2/\text{s}} = 2.2 \times 10^4
 \end{aligned}$$

From the Moody diagram (Fig 10.14, EFM10e),  $f = 0.030$  so the pressure drop is

$$\begin{aligned}
 \Delta p_f &= f \left( \frac{L}{4R_h} \right) \frac{\rho V^2}{2} \\
 \Delta p_f &= 0.030 \left( \frac{30 \text{ m}}{0.092 \text{ m}} \right) \left( \frac{1.23 \times (3.6 \text{ m/s})^2}{2} \right)
 \end{aligned}$$

$$\boxed{\Delta p_f = 77.97 \text{ N/m}^2}$$

---

**10.87: PROBLEM DEFINITION**Situation:

Air moves through a galvanized iron cold-air duct.

$$b = 100 \text{ cm}, A = 100 \text{ cm} \times 15 \text{ cm}.$$

$$Q = 6 \text{ m}^3/\text{s}.$$

Find:

Power loss in duct.

Assumptions:

$$k_s = .15 \text{ mm} = 1.5 \times 10^{-4} \text{ m}.$$

Properties:

Air ( $15^\circ\text{C}$ ), Table A.3:  $\nu = 1.46 \times 10^{-5} \text{ m}^2/\text{s}$ .

From Table A.2:  $\rho = 1.22 \text{ kg/m}^3$ .

**SOLUTION**

Hydraulic radius

$$\begin{aligned} A &= 0.15 \text{ m}^2 \\ P &= 2.30 \text{ m} \\ R &= \frac{A}{P} = 0.0652 \text{ m} \\ 4R &= 0.261 \text{ m} \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{6 \text{ m}^3/\text{s}}{0.15 \text{ m}^2} \\ &= 40 \text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= V \times \frac{4R}{\nu} \\ &= \frac{40 \text{ m/s} \times 0.261 \text{ m}}{1.46 \times 10^{-5} \text{ m}^2/\text{s}} \\ &= 7.15 \times 10^5 \end{aligned}$$

Friction factor ( $f$ ) (turbulent flow: Swamee-Jain equation)

$$\begin{aligned} f &= \frac{0.25}{\left[ \log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \\ &= \frac{0.25}{\left[ \log_{10} \left( \frac{1.5 \times 10^{-4}}{3.7 \times 0.261} + \frac{5.74}{(7.15 \times 10^5)^{0.9}} \right) \right]^2} \\ &= 0.01797 \approx 0.018 \end{aligned}$$

Darcy Weisbach equation

$$\begin{aligned}h_f &= f \frac{L V^2}{D 2g} \\&= 0.018 \times \left( \frac{100 \text{ m}}{0.261 \text{ m}} \right) \left( \frac{(40 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} \right) \\&= 562.4 \text{ m}\end{aligned}$$

Power equation

$$\begin{aligned}P_{\text{loss}} &= Q\gamma h_f \\&= 6 \text{ m}^3/\text{s} \times 1.22 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 562.4 \text{ m} = 40400 \text{ W}\end{aligned}$$

$$\boxed{P_{\text{loss}} = 40.4 \text{ kW}}$$

---

**10.88: PROBLEM DEFINITION****Situation:**

Air flows through a horizontal, rectangular, air-conditioning duct.

$L = 20$  m, Section area is 0.01 m.

$V = 10$  m/s,  $k_s = 0.004$  mm.

**Find:**

- (a) The pressure drop in meters of water.
- (b) The power in watts needed to overcome head loss.

**Assumptions:**

Neglect all head loss associated with minor losses.

$$\alpha_1 = \alpha_2$$

**Properties:**

Air at 20°C from Table A.3:  $= 15.1 \times 10^{-6}$  m<sup>2</sup>/s.

$\rho = 1.2$  kg/m<sup>3</sup>,  $\gamma = 11.8$  N/m<sup>3</sup>.

**PLAN**

To account for the rectangular section, use hydraulic diameter. Calculate Reynolds number and then choose a suitable correlation for the friction factor ( $f$ ). Apply the Darcy-Weisbach equation to find the head loss ( $h_f$ ). Apply the energy equation to find the pressure drop, and calculate power using  $P = \dot{m}gh_f$ .

**SOLUTION**

Hydraulic diameter ( $D_H$ ) (four times the hydraulic radius)

$$\begin{aligned} D_H &= \frac{4A}{P} \\ &= \frac{4(0.102 \text{ m})(0.254 \text{ m})}{(0.102 \text{ m} + 0.102 \text{ m} + 0.254 \text{ m} + 0.254 \text{ m})} \\ &= 0.1456 \text{ m} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD_H}{\nu} \\ &= \frac{(10 \text{ m/s})(0.1456 \text{ m})}{(15.1 \times 10^{-6} \text{ m}^2/\text{s})} \\ &= 96,390 \end{aligned}$$

Friction factor ( $f$ ) (Swamee-Jain correlation)

$$\begin{aligned} f &= \frac{0.25}{\left[ \log_{10} \left( \frac{k_s}{3.7D_H} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \\ &= \frac{0.25}{\left[ \log_{10} \left( \frac{4 \times 10^{-6} \text{ m}}{3.7 \times (0.1456 \text{ m})} + \frac{5.74}{96,390^{0.9}} \right) \right]^2} \\ &= 0.0182 \end{aligned}$$

Darcy-Weisbach equation

$$\begin{aligned}h_f &= f \frac{L V^2}{D 2g} \\&= 0.0182 \left( \frac{20 \text{ m}}{0.1456 \text{ m}} \right) \left( \frac{10^2 \text{ m}^2/\text{s}^2}{2 \times 9.81 \text{ m/s}^2} \right) \\&= 12.72 \text{ m}\end{aligned}$$

Energy equation (section 1 and 2 are the inlet and exit of the duct)

$$\left( \frac{p}{\gamma} \right)_1 = \left( \frac{p}{\gamma} \right)_2 + h_L$$

Thus

$$\begin{aligned}\Delta p &= \gamma_{\text{air}} h_f \\&= (11.8 \text{ N/m}^3) (12.72 \text{ m}) \\&= 150 \text{ Pa} \\&= 150 \text{ Pa} \left( \frac{0.01 \text{ m-H}_2\text{O}}{248.8 \text{ Pa}} \right)\end{aligned}$$

$$\boxed{\Delta p = 0.015 \text{ mH}_2\text{O}}$$

Power equation

$$\begin{aligned}P &= \gamma Q h_f \\&= \Delta p A V \\&= (150 \text{ Pa}) (0.102 \text{ m} \times 0.254 \text{ m}) (10 \text{ m/s})\end{aligned}$$

$$\boxed{P = 38.9 \text{ W}}$$

### **REVIEW**

The power to overcome head loss is small (39 W)—this is equivalent to the power required to light a small light bulb.

---

**10.89: PROBLEM DEFINITION**

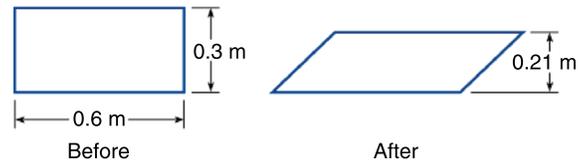
Situation:

A rectangular duct (initial state).

A trapezoidal duct (after being run over by a truck).

Rectangular area is 0.3 by 0.6 m. Trapezoidal area is 0.21 by 0.6 m.

Sketch:



Find:

Ratio of velocity in trapezoidal to rectangular duct.

**SOLUTION**

$$\Delta h_{\text{rect}} = \Delta h_{\text{trap}}$$

$$\therefore h_{f,\text{rect}} = h_{f,\text{trap}}$$

$$\left( \frac{f_b L}{4R_b} \right) \frac{V_b^2}{2g} = \left( \frac{f_a L}{4R_a} \right) \frac{V_a^2}{2g}$$

$$R_b = \frac{A_b}{P_b} = \frac{0.18 \text{ m}^2}{1.8 \text{ m}} = 0.1 \text{ m}$$

$$R_a = \frac{A_a}{P_a} = \frac{0.13 \text{ m}^2}{1.8 \text{ m}} = 0.07 \text{ m}$$

$$\frac{V_a^2}{V_b^2} = \frac{R_a}{R_b} = 0.70$$

$$\boxed{\frac{V_{\text{trap}}}{V_{\text{rect}}} = 0.84}$$

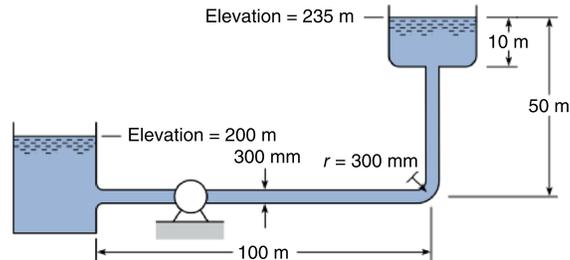
## 10.90: PROBLEM DEFINITION

### Situation:

Water is pumped through a steel pipe from one tank to another.

$D = 300$  mm,  $L = 140$  m,  $Q = 0.4$  m<sup>3</sup>/s.

$z_1 = 200$  m,  $z_2 = 235$  m, Elbow radius is 300 mm.



### Find:

The pump power.

### Assumptions:

Pipe entrance is well-rounded:  $r/D > 0.2$ .

### Properties:

From Table 10.5:  $K_e = 0.03$ ;  $K_b = 0.35$ ;  $K_E = 1.0$ .

Water (20 °C), Table A.5:  $\nu = 10^{-6}$  m<sup>2</sup>/s.

From Table 10.4:  $k_s = 0.046$  mm.

## PLAN

Apply the energy equation from the water surface in the lower reservoir to the water surface in the upper reservoir.

## SOLUTION

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 0 + 0 + 200 \text{ m} + h_p &= 0 + 0 + 235 \text{ m} + \frac{V_2^2}{2g} (K_e + K_b + K_E + f \frac{L}{D}) \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.4 \text{ m}^3/\text{s}}{(\pi/4) \times (0.3 \text{ m})^2} \\ &= 5.66 \text{ m/s} \\ \frac{V^2}{2g} &= 1.63 \text{ m} \end{aligned}$$

Reynolds number

$$\begin{aligned}\text{Re} &= \frac{VD}{\nu} \\ &= \frac{5.66 \text{ m/s} \times 0.3 \text{ m}}{10^{-6} \text{ m}^2/\text{s}} \\ &= 1.70 \times 10^6 \\ \frac{k_s}{D} &\approx 0.00015\end{aligned}$$

Resistance coefficient (from the Moody diagram)

$$f = 0.015$$

So

$$\begin{aligned}f \frac{L}{D} &= 0.015 \times \frac{140 \text{ m}}{0.3 \text{ m}} = 7 \\ h_p &= 235 \text{ m} - 200 \text{ m} + 1.63 \text{ m}(0.03 + 0.35 + 1 + 7) \\ &= 47.9 \text{ m}\end{aligned}$$

Power equation

$$\begin{aligned}P &= Q\gamma h_p \\ &= 0.4 \text{ m}^3/\text{s} \times 9,790 \text{ N/m}^3 \times 47.9 \text{ m} \\ &\boxed{P = 188 \text{ kW}}\end{aligned}$$

### 10.91: PROBLEM DEFINITION

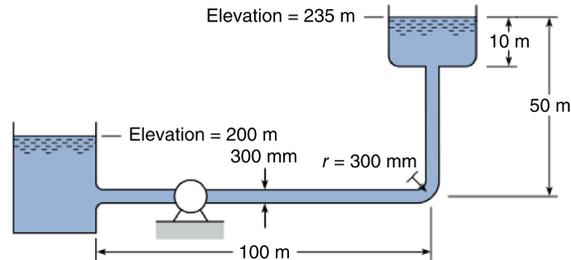
Situation:

Water is pumped through the system sketched below.

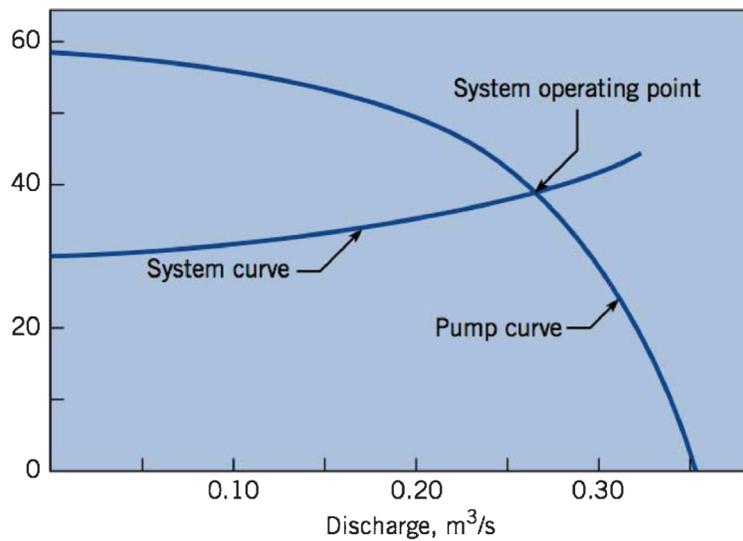
$D = 300$  mm,  $L = 140$  m.

$z_1 = 200$  m,  $z_2 = 235$  m.

Elbow radius is 300 mm.



The pump curve is shown on the figure below.



Find:

Discharge.

### PLAN

For the system curve, follow the solution for problem 10.90 in EFM10e. Then plot the system curve on the above diagram to find the operating point.

### SOLUTION

The solution to Prob. 10.90 in EFM10e, gives the system curve

$$0 + 0 + 200 \text{ m} + h_p = 0 + 0 + 235 \text{ m} + \frac{V^2}{2g} (K_e + K_b + K_E + f \frac{L}{D})$$

$$h_p = 35 \text{ m} + \frac{V^2}{2g} (0.03 + 0.35 + 1 + 6.53)$$

$$h_p = 35 \text{ m} + 7.91 \frac{V^2}{2g}$$

$$h_p = 35 \text{ m} + 7.91 \left[ \frac{Q^2 / ((\pi/4) \times 0.3^2)^2}{2g} \right] = 35 + 85.6Q^2$$

System data computed and shown below:

|                          |               |      |      |      |      |      |
|--------------------------|---------------|------|------|------|------|------|
| $Q(\text{m}^3/\text{s})$ | $\rightarrow$ | 0.05 | 0.10 | 0.15 | 0.20 | .30  |
| $h_p(\text{m})$          | $\rightarrow$ | 35.2 | 35.8 | 36.9 | 38.4 | 42.7 |

Then, plotting the system curve on the pump performance curve of Fig.10.20b in EFM10e (shown above) yields the operating point

$$\boxed{Q = 0.25 \text{ m}^3/\text{s}}$$

## 10.92: PROBLEM DEFINITION

### Situation:

Water is pumped from one tank to another.

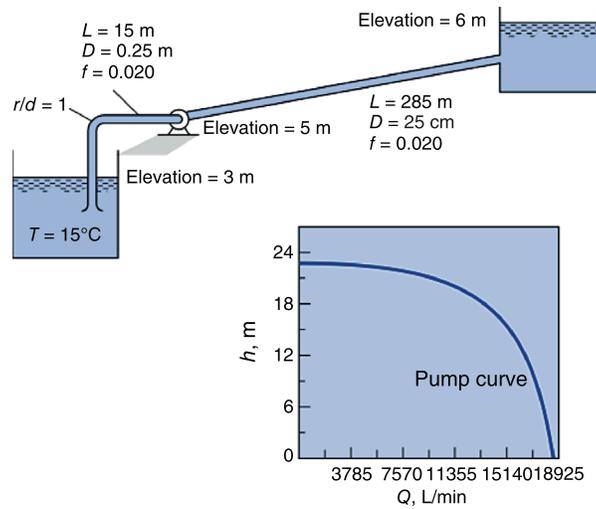
$$r/d = 1, D_1 = 0.25 \text{ m}, L_1 = 15 \text{ m}.$$

$$f_1 = 0.020, z_1 = 3 \text{ m}.$$

$$D_2 = 0.25 \text{ m}, L_2 = 285 \text{ m}, f_2 = 0.020.$$

$$z_2 = 6 \text{ m}, T = 15^\circ\text{C}.$$

### Sketch:



### Find:

Discharge ( $\text{m}^3/\text{s}$ ).

### Properties:

From Table 10.5:  $K_e = 0.03$ ;  $K_b = 0.35$ ;  $K_E = 1.0$ .

## SOLUTION

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 0 + 0 + 3 + h_p &= 0 + 0 + 6 + \frac{V_2^2}{2g} \left( K_e + f \frac{L}{D} + K_b + K_E \right) \\ h_p &= 3 + \frac{Q^2}{2gA^2} \left( 0.03 + 0.02 \times \frac{300 \text{ m}}{0.25 \text{ m}} + 0.35 + 1 \right) \\ A &= (\pi/4) \times (0.25 \text{ m})^2 = 0.05 \text{ m}^2 \end{aligned}$$

$$\begin{aligned}
 h_p &= 3 + 508Q_{\text{m}^3/\text{s}} \\
 1 \text{ m}^3/\text{s} &= 44960,000 \text{ L/min} \\
 h_p &= 3 + \frac{508Q_{\text{L/min}}^2}{(60,000)^2} \\
 h_p &= 3 + 1.41 \times 10^{-7}Q_{\text{L/min}}^2
 \end{aligned}$$

|                           |      |      |       |
|---------------------------|------|------|-------|
| $Q$ (L/min) $\rightarrow$ | 3785 | 7570 | 11355 |
| $h$ (m) $\rightarrow$     | 4.95 | 10.8 | 20.58 |

Plotting this on pump curve figure yields  $Q \approx 11,350 \text{ L/min}$

### 10.93: PROBLEM DEFINITION

#### Situation:

A liquid is pumped through a pipe from one tank to another.

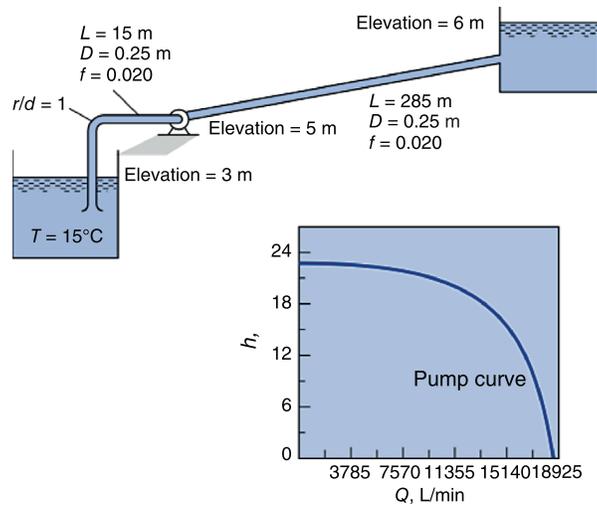
$$r/d = 1, D_1 = 0.25 \text{ m}, L_1 = 15 \text{ m.}$$

$$f_1 = 0.020, z_1 = 3 \text{ m.}$$

$$D_2 = 0.25 \text{ m}, L_2 = 285 \text{ m.}$$

$$f_2 = 0.020, z_2 = 6 \text{ m.}$$

#### Sketch:



#### Find:

Pumping rate (lpm).

#### Assumptions:

No head loss for this liquid.

**SOLUTION**  $h_p = 6 \text{ m} - 3 \text{ m} = 3 \text{ m}$

Then from the pump curve above one finds  $Q = 17,813 \text{ lpm}$ .

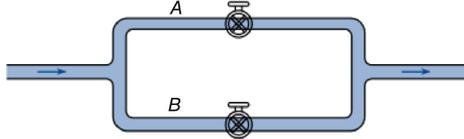
### 10.94: PROBLEM DEFINITION

Situation:

Two pipes are connected in parallel.

$$K_{vA} = 0.2, K_{vB} = 10, 2 \times A_A = A_B.$$

Sketch:



Find:

Ratio of discharge in line  $B$  to that in line  $A$ .

Assumptions:

Head loss due to valves overshadows losses due to junctions, elbows and friction.

### SOLUTION

$$h_{LA} = h_{LB}$$
$$0.2 \frac{V_A^2}{2g} = 10 \frac{V_B^2}{2g} \quad (1)$$

$$V_A = \sqrt{50} V_B$$
$$\frac{Q_B}{Q_A} = \frac{V_B A_B}{V_A A_A}$$
$$= \frac{V_B A_B}{V_A \times \frac{1}{2} A_B} \quad (2)$$

$$\frac{Q_B}{Q_A} = \frac{2V_B}{V_A}$$

Solve Eqs. (1) and (2) for  $Q_B/Q_A$ :

$$\frac{Q_B}{Q_A} = \frac{2 \times V_B}{\sqrt{50} V_B}$$

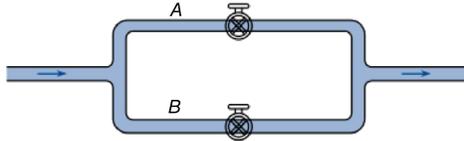
$$\boxed{\frac{Q_B}{Q_A} = 0.283}$$

---

**10.95: PROBLEM DEFINITION**Situation:

Two pipes are connected in parallel.

Line A has a half open gate valve, line B a fully open globe valve.

Sketch:Find:

Ratio of velocity in line A to B.

Assumptions:

Head loss due to friction is negligible.

Properties:

From Table 10.5:  $K_{vA} = 5.6$ ,  $K_{vB} = 10$ ,  $K_b = 0.9$ .

**SOLUTION**

$$\begin{aligned}\sum h_{LB} &= \sum h_{LA} \\ h_{L,\text{globe}} + 2h_{L,\text{elbow}} &= h_{L,\text{gate}} + 2h_{L,\text{elbow}} \\ 10 \frac{V_B^2}{2g} + 2 \left( 0.9 \frac{V_B^2}{2g} \right) &= 5.6 \frac{V_A^2}{2g} + 2 \left( 0.9 \frac{V_A^2}{2g} \right) \\ 11.8 \frac{V_B^2}{2g} &= 7.4 \frac{V_A^2}{2g}\end{aligned}$$

$$\boxed{\frac{V_A}{V_B} = 1.26}$$

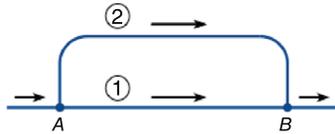
---

**10.96: PROBLEM DEFINITION**Situation:

Two pipes are connected in parallel.

$$L_1 = 1200 \text{ m}, D_1 = 50 \text{ cm}, L_2 = 1500 \text{ m}.$$

$$D_2 = 35 \text{ cm}, Q = 1.2 \text{ m}^3/\text{s}.$$

Sketch:Find:

Division of flow of water.

Assumptions:

Friction factor,  $f$ , is equal in both lines.

**SOLUTION**

$$\frac{V_1}{V_2} = \left[ \left( \frac{f_2}{f_1} \right) \left( \frac{L_2}{L_1} \right) \left( \frac{D_1}{D_2} \right) \right]^{1/2}$$

Substituting values, and assumption that  $f_1 = f_2$

$$\frac{V_1}{V_2} = \left[ \left( \frac{1500 \text{ m}}{1200 \text{ m}} \right) \left( \frac{0.5 \text{ m}}{0.35 \text{ m}} \right) \right]^{1/2}$$
$$= 1.336$$

$$V_1 = 1.34V_2$$

$$1.2 = V_1A_1 + V_2A_2$$

$$1.2 = 1.34V_2 \times (\pi/4) \times (0.5 \text{ m})^2 + V_2 \times (\pi/4) \times (0.35 \text{ m})^2$$

$$\boxed{V_2 = 3.35 \text{ m/s}}$$

Then  $V_1 = 1.336 \times 3.346$

$$\boxed{V_1 = 4.47 \text{ m/s}}$$

$$Q_1 = V_1A_1$$

$$= 4.47 \text{ m/s} \times (\pi/4) \times (0.5 \text{ m})^2$$

$$\boxed{Q_1 = 0.878 \text{ m}^3/\text{s}}$$

$$\boxed{Q_2 = 0.322 \text{ m}^3/\text{s}}$$

---

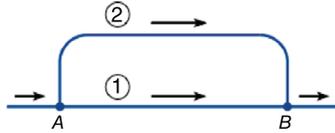
**10.97: PROBLEM DEFINITION**

Situation:

Two pipes are connected in parallel.

$$L_2 = 3L_1, D_1 = D_2, Q_2 = 0.03 \text{ m}^3/\text{s}.$$

Sketch:



Find:

Discharge in pipe 1.

Assumptions:

Friction factor,  $f$ , is equal in both lines.

**SOLUTION**

$$h_{f,1} = h_{f,2}$$

$$\begin{aligned} f \left( \frac{L}{D} \right) \frac{V_1^2}{2g} &= f \left( \frac{3L}{D} \right) \frac{V_2^2}{2g} \\ V_1^2 &= 3V_2^2 \\ V_1 &= 1.73V_2 \end{aligned}$$

Thus

$$Q_1 = 1.73Q_2$$

$$Q_1 = 0.052 \text{ m}^3/\text{s}$$

**10.98: PROBLEM DEFINITION**

Situation:

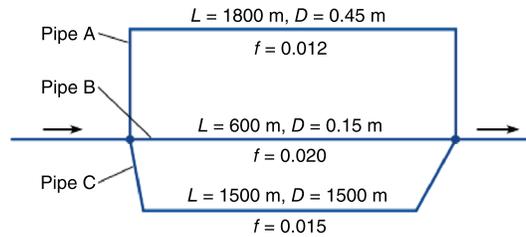
Three pipes are connected in parallel

$$L_A = 1800 \text{ m}, D_A = 0.45 \text{ m}, f_A = 0.012.$$

$$L_B = 600 \text{ m}, D_B = 0.15 \text{ m}, f_B = 0.020.$$

$$L_C = 1500 \text{ m}, D_C = 0.3 \text{ m}, f_C = 0.015.$$

Sketch:



Find:

The pipe having the greatest velocity.

**SOLUTION**

$$h_{p,A} = h_{f,B} = h_{f,C}$$

$$f \frac{L}{D} \left( \frac{V^2}{2g} \right)_A = f \frac{L}{D} \left( \frac{V^2}{2g} \right)_B = f \frac{L}{D} \left( \frac{V^2}{2g} \right)_C$$

$$0.012 \left( \frac{1800 \text{ m}}{0.45 \text{ m}} \right) V_A^2 = 0.02 \left( \frac{600 \text{ m}}{0.15 \text{ m}} \right) V_B^2 = 0.015 \left( \frac{1500 \text{ m}}{0.3 \text{ m}} \right) V_C^2$$

$$48V_A^2 = 80V_B^2 = 75V_C^2$$

Therefore,  $V_A$  will have the greatest velocity. Correct choice is (a)

---

**10.99: PROBLEM DEFINITION****Situation:**

Two pipes are connected in parallel.

$$L_1 = 4L_2, D_1 = 2D_2.$$

$$f_1 = 0.010, f_2 = 0.012.$$

**Find:**

Ratio of discharges in two pipes.

**SOLUTION**

$$\frac{V_1}{V_2} = \left[ \left( \frac{f_2}{f_1} \right) \left( \frac{L_2}{L_1} \right) \left( \frac{D_1}{D_2} \right) \right]^{1/2}$$

Let pipe 1 be large pipe and pipe 2 be smaller pipe. Then

$$\frac{V_1}{V_2} = \left[ \left( \frac{0.012}{0.01} \right) \left( \frac{L}{4L} \right) \left( \frac{2D}{D} \right) \right]^{1/2} = 0.775$$

$$\frac{Q_1}{Q_2} = \frac{V_1 A_1}{V_2 A_2} = 0.775 \times \left( \frac{2D}{D} \right)^2 = 3.10$$

$$\boxed{(Q_{\text{large}}/Q_{\text{small}}) = 3.1}$$

### 10.100: PROBLEM DEFINITION

#### Situation:

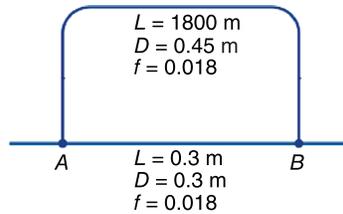
Two pipes are connected in parallel.

$$Q = 0.4 \text{ m}^3/\text{s}, L_1 = 1800 \text{ m.}$$

$$D_1 = 0.45 \text{ m}, f_1 = 0.018.$$

$$L_2 = 600 \text{ m}, D_2 = 0.3 \text{ m}, f_2 = 0.018.$$

#### Sketch:



#### Find:

Division of flow ( $\text{m}^3/\text{s}$ ).

Head loss (m).

### SOLUTION

$$\begin{aligned} Q_{0.45} + Q_{0.3} &= 0.4 \text{ m}^3/\text{s} \\ h_{L_{0.45}} &= h_{L_{0.3}} \\ f_{0.45} \frac{L_{0.45}}{D_{0.45}} \frac{V_{0.45}^2}{2g} &= f_{0.3} \frac{L_{0.3}}{D_{0.3}} \frac{V_{0.3}^2}{2g} \\ f_{0.45} &= 0.018 = f_{0.3} \end{aligned}$$

so

$$\begin{aligned} \frac{L_{0.45} Q_{0.45}^2}{D_{0.45}^5} &= \frac{L_{0.3} Q_{0.3}^2}{D_{0.3}^5} \\ Q_{0.45}^2 &= \left( \frac{D_{0.45}}{D_{0.3}} \right)^5 \left( \frac{L_{0.3}}{L_{0.45}} \right) Q_{0.3}^2 \\ &= \left( \frac{0.45 \text{ m}}{0.3 \text{ m}} \right)^5 \left( \frac{600 \text{ m}}{1800 \text{ m}} \right) Q_{0.3}^2 \\ &= 2.53 Q_{0.3}^2 \\ Q_{0.45} &= 1.59 Q_{0.3} \end{aligned}$$

$$1.59 Q_{0.3} + Q_{0.3} = 0.4$$

$$2.59 Q_{0.3} = 0.4$$

$$Q_{0.3} = 0.15 \text{ m}^3/\text{s}$$

$$\begin{aligned} Q_{0.45} &= 1.59Q_{0.3} \\ &= 1.59(0.15) \end{aligned}$$

$$\boxed{Q_{0.45} = 0.24 \text{ m}^3/\text{s}}$$

$$V_{0.3} = \frac{0.15 \text{ m}^3/\text{s}}{\pi/4 \times (0.3 \text{ m})^2} = 2.12 \text{ m/s}$$

$$V_{0.45} = \frac{0.24 \text{ m}^3/\text{s}}{\pi/4 \times (0.45 \text{ m})^2} = 1.5 \text{ m/s}$$

$$h_{L_{0.3}} = 0.018 \left( \frac{600 \text{ m}}{0.3 \text{ m}} \right) \frac{(2.12 \text{ m/s})^2}{19.62 \text{ m/s}^2} = 8.25 \text{ m}$$

$$h_{L_{0.45}} = 0.018 \left( \frac{1800 \text{ m}}{0.45 \text{ m}} \right) \frac{(1.5 \text{ m/s})^2}{19.62 \text{ m/s}^2} = 8.25 \text{ m}$$

Thus,  $\boxed{h_{L_{A-B}} = 8.25 \text{ m}}$

## 10.101: PROBLEM DEFINITION

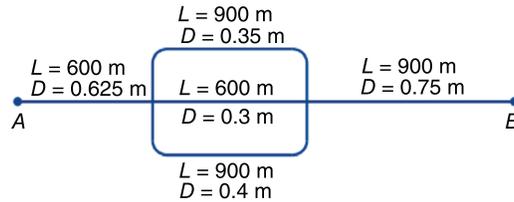
Situation:

A concrete piping system is described in the problem statement.

$$Q = 0.7 \text{ m}^3/\text{s}, f = 0.030, L_{0.625} = 600 \text{ m}, D_{0.625} = 0.625 \text{ m}.$$

$$L_{0.75} = 900 \text{ m}, D_{0.75} = 0.75 \text{ m}, L_{0.35} = 900 \text{ m}, D_{0.35} = 0.35 \text{ m}.$$

$$L_{0.3} = 600 \text{ m}, D_{0.3} = 0.3 \text{ m}, L_{0.4} = 900 \text{ m}, D_{0.4} = 0.4 \text{ m}.$$



Find:

Division of flow.

Head loss.

## SOLUTION

Sketch:

$$Q = Q_{0.35} + Q_{0.3} + Q_{0.4}$$

$$0.7 = V_{0.35} \times (\pi/4) (0.35 \text{ m})^2 + V_{0.3} \times (\pi/4) (0.3 \text{ m})^2 + V_{0.4} \times (\pi/4) (0.4 \text{ m})^2 \quad (1)$$

Also,  $h_{f_{0.35}} = h_{f_{0.3}} = h_{f_{0.4}}$  and assuming  $f = 0.03$  for all pipes

$$\left( \frac{900 \text{ m}}{0.35 \text{ m}} \right) V_{0.35}^2 = \left( \frac{600 \text{ m}}{0.3 \text{ m}} \right) V_{0.3}^2 = \left( \frac{900 \text{ m}}{0.4 \text{ m}} \right) V_{0.4}^2 \quad (2)$$

$$V_{0.35}^2 = 0.778 V_{0.3}^2 = 0.875 V_{0.4}^2$$

From Eq.(1)

$$0.7 = 0.096 V_{0.35} + 0.080 V_{0.35} + 0.134 V_{0.35}$$

$$V_{0.35} = 2.26 \text{ m/s}$$

and  $V_{0.3} = 2.6 \text{ m/s}$ ,  $V_{0.35} = 2.26 \text{ m/s}$ ;  $V_{0.4} = 2.41 \text{ m/s}$ . Calculate flow rate using  $Q = VA$

$$Q (0.3 \text{ m pipe}) = 0.18 \text{ m}^3/\text{s}$$

$$Q (0.35 \text{ m pipe}) = 0.22 \text{ m}^3/\text{s}$$

$$Q (0.4 \text{ m pipe}) = 0.3 \text{ m}^3/\text{s}$$

$$\begin{aligned}
V_{0.6} &= \frac{Q}{A_{0.6}} = \frac{0.7}{\pi/4 \times (0.6 \text{ m})^2} = 2.48 \text{ m/s}; \\
V_{0.75} &= 1.59 \text{ m/s} \\
h_{LAB} &= \frac{0.03}{19.62 \text{ m/s}^2} \left[ \left( \frac{600 \text{ m}}{0.6 \text{ m}} \right) (2.48 \text{ m/s})^2 + \left( \frac{600 \text{ m}}{0.3 \text{ m}} \right) \times (2.6 \text{ m/s})^2 \right. \\
&\quad \left. + \left( \frac{900 \text{ m}}{0.75 \text{ m}} \right) \times (1.59 \text{ m/s})^2 \right]
\end{aligned}$$

$$\boxed{h_{LAB} = 27 \text{ m}}$$

## 10.102: PROBLEM DEFINITION

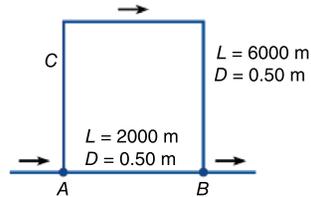
### Situation:

Two pipes are connected in parallel with the pump from Fig 10.20b (EFM10e) at point C.

$$L_1 = 2000 \text{ m}, D_1 = 0.50 \text{ m}, L_2 = 6000 \text{ m}.$$

$$D_2 = 0.50 \text{ m}, Q = 0.60 \text{ m}^3/\text{s}.$$

### Sketch:



### Find:

Division of flow between pipes ( $\text{m}^3/\text{s}$ ).

Head loss (m).

### Properties:

From Table 10.4  $k_s = 0.046 \text{ mm}$ .

## SOLUTION

Call pipe A-B pipe 1 and pipe ACB pipe 2. Then

$$\begin{aligned} h_{f,1} + h_p &= h_{f,2} \\ \frac{k_s}{D} &= \frac{0.046}{500} \simeq 0.0001 \end{aligned}$$

Assume  $f_1 = f_2 = 0.013$  (guess from Fig. 10-8)

$$\begin{aligned} f \frac{L_1}{D_1} \frac{V_1^2}{2g} + h_p &= f \frac{L_2}{D_2} \frac{V_2^2}{2g} \\ 0.013 \left( \frac{2000 \text{ m}}{0.5 \text{ m}} \right) \frac{V_1^2}{2g} + h_p &= 0.013 \left( \frac{6000 \text{ m}}{0.5 \text{ m}} \right) \frac{V_2^2}{2g} \\ 2.65V_1^2 + h_p &= 7.951V_2^2 \end{aligned} \quad (1)$$

Continuity principle

$$\begin{aligned} (V_1 + V_2)A &= 0.60 \text{ m}^3/\text{s} \\ V_1 + V_2 &= \frac{0.6 \text{ m}^3/\text{s}}{A} = \frac{0.6 \text{ m}^3/\text{s}}{\pi/4 \times (0.5 \text{ m})^2} = 3.0558 \\ V_1 &= 3.0558 - V_2 \end{aligned} \quad (2)$$

By iteration (Eqs. (1), (2) and pump curve) one can solve for the division of flow:

$$\boxed{Q_1 = 0.27 \text{ m}^3/\text{s}}$$

$$\boxed{Q_2 = 0.33 \text{ m}^3/\text{s}}$$

Head loss determined along pipe 1

$$h_L = f \frac{L}{D} \frac{V_1^2}{2g}$$

$$V_1 = \frac{Q_1}{A} = \frac{0.27}{\pi/4 \times (0.5 \text{ m})^2} = 1.38 \text{ m/s}$$

$$h_l = 0.013 \left( \frac{2000 \text{ m}}{0.5 \text{ m}} \right) \frac{(1.38 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2}$$

$$\boxed{h_l = 5.05 \text{ m}}$$

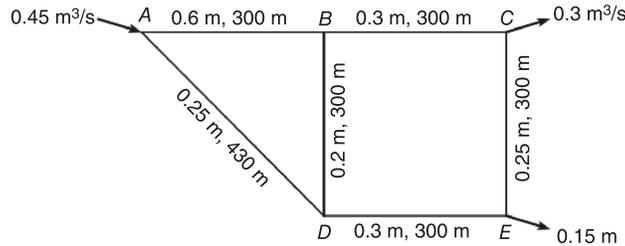
### 10.103: PROBLEM DEFINITION

Situation:

A piping network with sources and loads is specified.

$$f = 0.012, p_A = 400 \text{ kPa}, Q_A = 0.45 \text{ m}^3/\text{s}.$$

$$Q_C = 0.3 \text{ m}^3/\text{s}, Q_E = 0.15 \text{ m}^3/\text{s}.$$



Find:

Load distribution and pressure at load points.

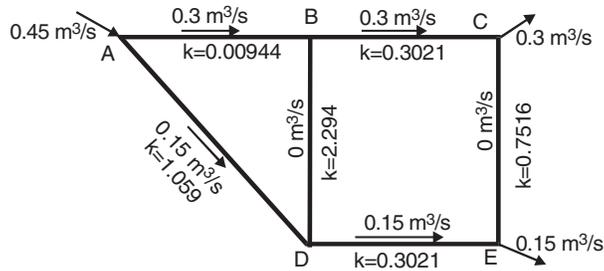
Properties:

Water, Table A.5:  $\gamma = 9810 \text{ N/m}^3$ .

### SOLUTION

An assumption is made for the discharge in all pipes making certain that the continuity equation is satisfied at each junction. The following figure shows the network with assumed flows.

Fig. A

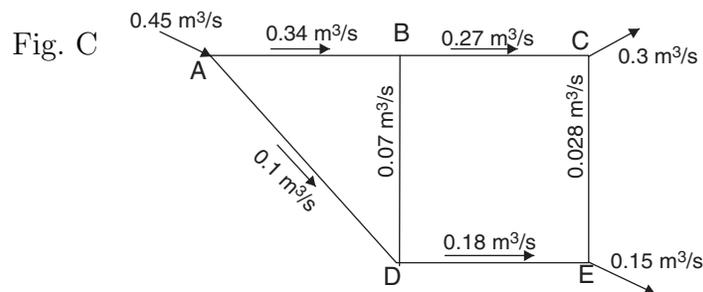
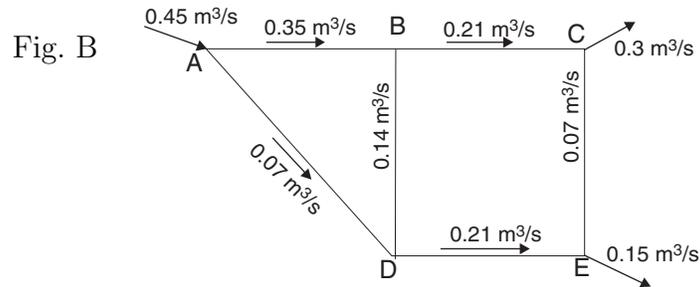


Darcy-Weisbach equation

$$\begin{aligned} h_f &= f \left( \frac{L}{D} \right) \left( \frac{V^2}{2g} \right) \\ &= 8 \left( \frac{fL}{gD^5\pi^2} \right) Q^2 \\ &= kQ^2. \end{aligned}$$

where  $k = 8 \left( \frac{fL}{gD^5\pi^2} \right)$ . The loss coefficient,  $k$ , for each pipe is computed and shown in Fig. A. Next, the flow corrections for each loop are calculated as shown in the accompanying table. Since  $n = 2$  (exponent on  $Q$ ),  $nkQ^{n-1} = 2kQ$ . When the correction obtained in the table are applied to the two loops, we get the pipe discharges shown in Fig. B. Then with additional iterations, we get the final distribution of flow as shown in Fig. C. Finally, the pressures at the load points are calculated.

| Loop <i>ABC</i>   |              |                    |
|---|--------------|--------------------|
| Pipe  | $h_f = kQ^2$ | $2kQ$              |
| <i>AB</i>   | +0.00085     | 0.00567            |
| <i>AD</i>   | -0.024       | 0.32               |
| <i>BD</i>   | 0            | 0                  |
| $\sum kQ_c^2 - \sum kQ_{cc}^2$                          | -0.0232      | $\sum 2KQ = 0.326$ |
| $\Delta Q = -0.0232/0.326 = 0.071 \text{ m}^3/\text{s}$ |              |                    |
| Loop <i>BCDE</i>  |              |                    |
| Pipe  | $h_f$        | $2kQ$              |
| <i>BC</i>   | +0.027       | 0.18               |
| <i>BD</i>   | 0            | 0                  |
| <i>CE</i>   | 0            | 0                  |
| <i>DE</i>   | -0.0068      | 0.09               |
|   | +0.0202      | 0.27               |
| $\Delta Q = -0.0202/0.27 = -0.075 \text{ m}^3/\text{s}$ |              |                    |



$$\begin{aligned}
p_C &= p_A - \gamma(k_{AB}Q_{AB}^2 + k_{BC}Q_{BC}^2) \\
&= 400 \text{ kPa} - 9810 \text{ N/m}^3(0.00944 \times 0.34^2 + 0.3021 \times 0.27^2) \\
&= 400,000 \text{ N/m}^2 - 227 \text{ N/m}^2 \\
&= 399,773 \text{ Pa}
\end{aligned}$$

$$\boxed{p_C = 399.7 \text{ kPa}}$$

$$\begin{aligned}
p_E &= 400,000 \text{ kPa} - \gamma(k_{AD}Q_{AD}^2 + k_{DE}Q_{DE}^2) \\
&= 400,000 \text{ N/m}^2 - 9810(1.059 \times 0.1^2 + 0.3021 \times 0.18^2) \\
&= 399.8 \text{ kPa}
\end{aligned}$$

$$\boxed{p_E = 399.8 \text{ kPa}}$$

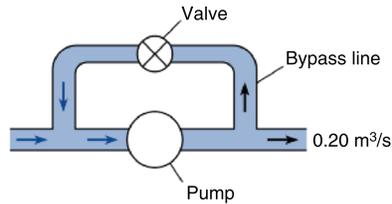
### 10.104: PROBLEM DEFINITION

Situation:

Two pipes are in parallel. One has a bypass valve and the other a pump.

$$h_p = 100 - 100Q_p, \quad Q_p = Q_v + 0.2 \text{ m}^3/\text{s}.$$

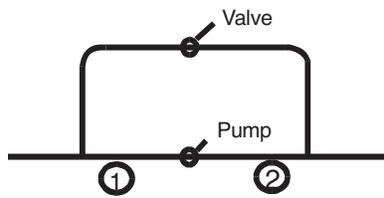
$$D_v = 10 \text{ cm}, \quad K_v = 0.2.$$



Find:

Discharge through pump and bypass line.

### SOLUTION



$$Q_p = Q_v + 0.2$$

$$\frac{p_2 - p_1}{\gamma} = h_p$$

$$A = \pi/4 \times (0.1 \text{ m})^2 = 0.00785 \text{ m}^2$$

$$\frac{K_v V_v^2}{2g} = \frac{K_v Q_v^2}{2g A^2} = h_p$$

$$h_p = 100 - 100(Q_v + 0.2)$$

$$\frac{(0.2)Q_v^2}{2 \times 9.81 \text{ m/s}^2 \times (0.00785 \text{ m}^2)^2} = 100 - 100Q_v - 20$$

$$165Q_v^2 + 100Q_v - 80 = 0$$

Solve by quadratic formula

$$Q_v = 0.456 \text{ m}^3/\text{s}$$

$$Q_p = 0.456 + 0.2$$

$$Q_p = 0.656 \text{ m}^3/\text{s}$$