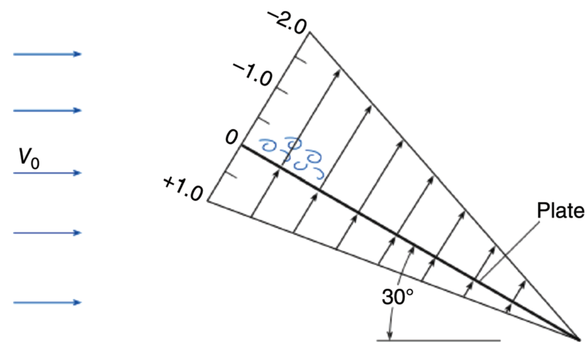
11.1: PROBLEM DEFINITION

Situation:
A hypothetical pressure-coefficient distribution acts on a plate.
Plate length = c, Plate width (into paper) = 1.0 unit.



Find: Coefficient of drag.

Assumptions: Viscous effects are negligible.

SOLUTION

1. Find the force normal to the plate

$$\begin{aligned} F_n &= \Delta p_{\text{average}} \times A \\ &= C_{p,\text{ average}} \left(\frac{\rho V_0^2}{2} \right) \times c \times 1 \\ &= 1.5 \times \rho V_0^2/2 \times c \times 1 \end{aligned}$$

2. Find the component of the force that is aligned with the direction of the free stream. This force is the drag force

$$\begin{aligned} F_D &= F_{\text{normal}} \sin 30^\circ \\ &= (1.5 \rho V_0^2/2) \times c \times 1 \times \sin 30^\circ \end{aligned}$$

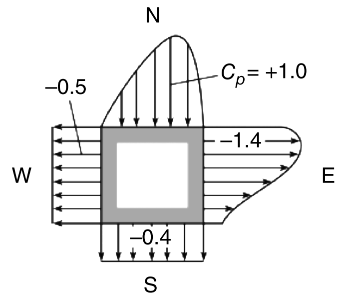
3. Find the coefficient of drag.

$$C_D = \frac{F_D}{\frac{1}{2}\rho V_0^2 A} = \frac{(1.5\rho V_0^2/2) \times c \times \sin 30^\circ}{\frac{1}{2}\rho V_0^2 \times c \times 1}$$

$$\boxed{C_D = 0.5}$$

11.2: PROBLEM DEFINITION

Situation: Fluid flow past a square rod.



Find: Direction from which the flow is coming.

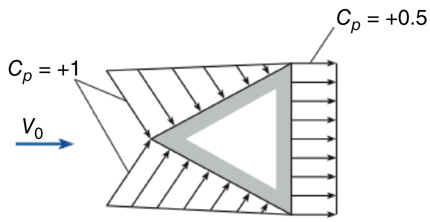
SOLUTION

Flow is from the N.E. direction because the C_p values are highest on this corner of the rod.
Correct choice is d)

11.3: PROBLEM DEFINITION

Situation:

A pressure distribution acts on a triangular rod.



Find: Drag coefficient for rod.

PLAN

Find the total force on the rod by adding up the forces on the three sides. Once total force is known, find C_D by using the definition.

SOLUTION

1. Define the reference area

$$A_R = A_p = \text{ area of one side of the rod}$$

2. Force contributing to drag on the back (downstream) face

$$\begin{aligned} F_b &= p_{\text{average}} A = \left(C_{p,\text{avg}} \frac{\rho V^2}{2} \right) A_p \\ &= 0.5 A_p \rho V_o^2 / 2 \end{aligned}$$

3. The force on each side face

$$F_s = 0.5 A_p \rho V_o^2 / 2$$

Component of this force in horizontal direction

$$F_s \sin \alpha = 0.5 A_p \rho V_o^2 / 2 \times 0.5$$

4 Total drag force

$$F_D = 2 F_s \sin \alpha + F_b$$

$$F_D = 2((0.5 A_p \rho V_o^2 / 2) \times 0.5) + 0.5 A_p \rho V_o^2 / 2 = A_p \rho V_o^2 / 2$$

5. Coefficient of drag

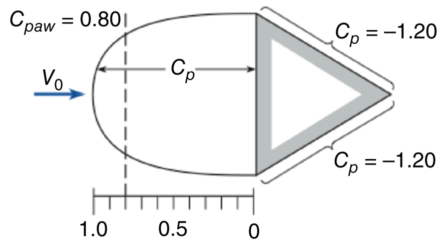
$$C_D = \frac{F_D}{A_p \rho V^2 / 2} = \frac{A_p \rho V_o^2 / 2}{A_p \rho V^2 / 2} = 1.0$$

$$\boxed{C_D = 1.0}$$

11.4: PROBLEM DEFINITION

Situation:

A pressure distribution acts on a triangular rod.



Find: Drag coefficient.

SOLUTION

1. Reference area

$$\text{(Reference area)} = \text{(Projected area)} = A_p = \text{(length of a side)}\,\text{(1 unit)}$$

2. Drag force on the windward (front) side

$$F_w = 0.8 \times \frac{1}{2}\rho V_0^2 A_p$$

3. Force on each back side

$$F_s = -1.2 \times \frac{1}{2}\rho V_0^2 A_p$$

4. Total drag force on the rod

$$F_D = \left(\begin{array}{c} \text{Force on} \\ \text{windward side} \end{array} \right) + 2 \left(\begin{array}{c} \text{Force on a} \\ \text{back side} \end{array} \right) \sin 30^o$$

$$\begin{aligned} F_D &= 0.8 \times \frac{1}{2}\rho V_0^2 A_p - 2(-1.2 \times \frac{1}{2}\rho V_0^2 A_p) \sin 30^o \\ &= \frac{1}{2}\rho V_0^2 A_p (0.8 + 1.2) = 2.0 A_p \frac{\rho V_0^2}{2} \end{aligned}$$

The drag coefficient is

$$C_D = \frac{F_D}{\frac{1}{2}\rho V_0^2 A_p}; \;\; \boxed{C_D = 2.0}$$

Problem 11.5

Find:
A. ___________ is associated with the viscous shear-stress distribution.
a. Form drag
b. Friction drag
B. ___________ is associated with the pressure distribution.
a. Form drag
b. Friction drag.

SOLUTION

A. [b. Friction drag] is associated with the viscous shear-stress distribution.
B. [a. Form drag] is associated with the pressure distribution.

Problem 11.6

Situation:
The coefficient of drag for a body (select all that apply):
a. is dimensionless
b. is usually determined by experiment
c. depends upon thrust
d. depends upon the body's shape
e. requires an updraft

SOLUTION

Correct answers are: a, b and d.

Problem 11.7

Apply the grid method to each situation described below. Note: Unit cancellations are not shown in this solution.

a.)

Situation:

Automobile. $V = 100$ km/h.
$A_p = 2\,\mathrm{m}^2$, $C_D = 0.4$.

Find: Drag force on automobile (Newtons).

Assumptions: Air temperature on a typical summer day is 30 °C.

Properties: Air (30 °C, 1 atm), Table A.5 (EFM 10e), $\rho = 1.17\,\mathrm{kg/m^3}$.

Solution:

$$F_D = C_D A_{\mathrm{Ref}} \frac{\rho V^2}{2} = (0.4)\left(2\,\mathrm{m}^2\right) \frac{(1.17\,\mathrm{kg/m^3})\,(100\ \mathrm{km/h})^2}{2} \left(\frac{1000\,\mathrm{m/km}}{3600\ \mathrm{s/h}}\right)^2$$

$$\boxed{F_D = 361\,\mathrm{N}}$$

b.)

Situation:

Bicycle. $F_D = 22\,\mathrm{N}$.
$A_p = 0.5\,\mathrm{m}^2$, $C_D = 0.3$.

Find: Speed of the bike rider (km/h).

Assumptions: Air temperature on a typical summer day is 30 °C.

Properties: Air (30 °C, 1 atm), Table A.5 (EFM 10e), $\rho = 1.17\,\mathrm{kg/m^3}$

Solution:

$$\begin{aligned} F_D &= C_D A_{\mathrm{Ref}} \frac{\rho V^2}{2} \\ V &= \sqrt{\frac{2F_D}{C_D A_{\mathrm{Ref}} \rho}} = \sqrt{\frac{2\,(22\,\mathrm{N})}{(0.3)\,(0.5\,\mathrm{m}^2)\,(1.17\,\mathrm{kg/m^3})}} = 15.8\,\mathrm{m/s} \\ &= (15.8\,\mathrm{m/s})\left(\frac{1/1000\ \mathrm{km/m}}{1/3600\,\mathrm{h/min}}\right) = 56.9\ \mathrm{km/h} \end{aligned}$$

$$\boxed{V = 56.9\ \mathrm{km/h}}$$

Review. The solution (56.9 km/h) corresponds a bicycle rider who is sprinting.

Problem 11.8

Answer the questions below.

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a. What are the four most important factors that influence drag force?

To identify the factors, apply *logical reasoning* to the drag force equation:

$$F_D = C_D A_{\text{Ref}} \frac{\rho V^2}{2}$$

The right side of this equation reveals that

- Drag force is influenced by the shape of the object. The shape of the object is characterized by the value of C_D.
- Drag force is influenced by the size of the object. Size is characterized by the value of the reference area A_{Ref}.
- Drag force varies as the speed of the fluid squared. That is, $F_D \sim V^2$. The speed of the fluid is determined relative to an observer on the object.
- Drag force is influence by the density ρ of the ambient fluid.

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b. How are stress and drag related?

- Integrating stress (force/area) over area gives the net force.
- The net force can be resolved into a lift force plus a drag force.
- Need to use unit vectors (or sines & cosines) to track direction because both stress and force are vectors.
- In mathematical form, the result is given by Eq. (11.2) in EFM 10e.

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c.) What is form drag? What is friction drag?

- Form drag is that portion of the drag force that is caused by the pressure distribution.
- Friction drag is that portion of the drag force that is caused by the shear stress distribution.

Problem 11.9

Situation:
Objects are immersed in flowing fluids.

Find:
a. C_D for a sphere that is falling through water. Re = 10000.
b. C_D for air blowing normal to a very long circular cylinder. Re = 7000.
c. C_D for air blowing normal to a billboard that is 6 m wide by 3 m high.

PLAN

First, find the right table, equation, or chart. Then, find C_D.

SOLUTION

a. Sphere fall through water.

From Fig. 11.8 (EFM10e) ==> $C_D \approx 0.40$.
From the Clift and Gauvin equation:

$$\begin{aligned} C_D &= \frac{24}{\mathrm{Re}_D}\left(1+0.15\,\mathrm{Re}_D^{0.687}\right)+\frac{0.42}{1+4.25\times 10^4\,\mathrm{Re}^{-1.16}} \\ &= \frac{24}{10000}\left(1+0.15\,(10000)^{0.687}\right)+\frac{0.42}{1+4.25\times 10^4\,(10000)^{-1.16}} \\ &= 0.42 \end{aligned}$$

$$\boxed{\text{a. } C_D \approx 0.40}$$

Review. The two answers are within about 5%. This difference is within the stated accuracy (-4% to 6%) of the Clift & Gauvin correlation.

b. Air is blowing normal to a cylinder.

From Fig. 11.5 (EFM10e):

$$\boxed{\text{b. } C_D \approx 0.98}$$

c. Wind is blowing normal to a billboard that is 6 m wide by 3 m high.

Use Table 11.1 (EFM 10e) and represent the billboard as a rectangular plate with an aspect ratio of $l/b = 2$.
Assume Re $> 10^4$ and interpolate.

$$\boxed{\text{c. } C_D \approx 1.18 \approx 1.2}$$

11.10: PROBLEM DEFINITION

Situation: Wind acts on a billboard—additional details are provided in the problem statement.
$b = 4\,\text{m}, w = 12\,\text{m}$
$V_0 = 100\,\text{km/h}$

Find: Force of the wind.

Properties: Air, Table A.3 (EFM 10e) $\nu = 1.46 \times 10^{-5}$ m^2/s; $\rho = 1.23$ kg/m^3.

SOLUTION

Reynolds number

$$\begin{aligned} V_0 &= 100 \text{ km/h} = 27.8 \text{ m/s} \\ \text{Re} &= V_0 b/\nu \\ &= 27.8 \times 4/(1.46 \times 10^{-5}) \\ &= 7.6 \times 10^6 \end{aligned}$$

Drag force. From Table 11.1 (EFM 10e) $C_D = 1.19$. Then

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ &= 1.19 \times 48 \times 1.23 \times 27.8^2/2 \\ & \boxed{F_D = 27{,}148 \text{ N}} \end{aligned}$$

11.11: PROBLEM DEFINITION

Situation: Assume Stoke's law is valid for a Reynolds number below 0.5.

Find: Largest raindrop that will fall in the Stokes' flow regime.

Assumptions:
spherical rain drop.
ambient temperature is 15 °C.

Properties:
Air (15 °C), Table A.3 (EFM 10e), $\rho = 1.23$ kg/m^3, $\mu_{\text{air}} = 1.79 \times 10^{-5}$ N-s/m^2.
Water (15 °C), Table A.5 (EFM 10e), $\gamma = 9810\,\text{N/m}^3$.

PLAN

Apply Stoke's law and the equilibrium principle.

SOLUTION

Drag force

$$F_D = 3\pi\mu V_0 D$$

Equilibrium

$$\text{Weight} = \text{Drag Force}$$

$$\begin{aligned}\frac{\pi D^3 \gamma_{\text{water}}}{6} &= 3\pi\mu_{\text{air}} V_0 D \\ D^2\gamma_{\text{water}} &= 18\mu_{\text{air}} V_0\end{aligned}$$

Reynolds number limit for Stokes flow

$$\begin{aligned}V_0 D/\nu &= 0.5 \\ V_0 &= \frac{0.5\ \nu_{\text{air}}}{D}\end{aligned}$$

Combining equations

$$\begin{aligned}D^2\gamma_{\text{water}} &= 18\mu_{\text{air}}\left(\frac{0.5\ \nu_{\text{air}}}{D}\right) \\ D^3 &= 9\mu_{\text{air}}\frac{\nu_{\text{air}}}{D\gamma_{\text{water}}}\end{aligned}$$

Solving for D

$$\begin{aligned}D^3 &= \frac{9\mu_{\text{air}}^2}{\rho_{\text{air}}\gamma_{\text{water}}} \\ &= \frac{9 \times (1.79 \times 10^{-5})^2}{1.23 \times 9810} \\ &= 2.39 \times 10^{-13}\ \text{m}^3 \\ D &= 6.2 \times 10^{-5}\ \text{m} \\ &= \boxed{0.062\ \text{mm}}\end{aligned}$$

11.12: PROBLEM DEFINITION

Situation:

A sheet of plywood (0.6 m by 1.2 m) is oriented normal to the wind.
$V_o = 56\,\text{km/h} = 15.6\,\text{m/s}$.

Find: Drag force on the plate.

Properties: Air (15 °C), Table A.3 (EFM 10e), $\rho = 1.23$ kg/m^3.

PLAN

1. Estimate C_D from Table 11.1 (EFM 10e).
2. Find the drag force with the drag force equation.

SOLUTION

1. Coefficient of drag.

Represent the billboard as a rectangular plate with an aspect ratio of $l/b = 2$. Assume Re $> 10^4$ and interpolate in Table 11.1 (EFM 10e).

$$\boxed{C_D \approx 1.18 \approx 1.2}$$

2. Drag force.

$$\begin{aligned} F_D &= C_D A_p \left(\frac{\rho V_0^2}{2}\right) \\ F_D &= (1.2)(0.6 \times 1.2)\frac{(1.23\text{ kg/m}^3)(15.6\,\text{m/s})^2}{2} \end{aligned}$$

$$\boxed{F_D = 130\text{ N}}$$

11.13: PROBLEM DEFINITION

Situation:
A 3 m by 4 m plate is towed through water.
$V = 5\,\mathrm{m/s}$.

Find:
a. Drag force (in Newtons) for plate oriented for minimum drag.
b. Drag force (in Newtons) for plate oriented for maximum drag.

Properties: Water (10 °C), Table A.5 (EFM 10e), $\rho = 1000\,\mathrm{kg/m^3}$, $\nu = 1.31 \times 10^{-6}\ \mathrm{m^2/s}$.

SOLUTION

Minimum drag force (case a)

- Select a plate orientation that is aligned with the flow; flow direction aligned with the 4 m length.
- Reynolds number:

$$\mathrm{Re} = \frac{VL}{\nu} = \frac{(5\,\mathrm{m/s})\,(4\,\mathrm{m})}{1.31 \times 10^{-6}\ \mathrm{m^2/s}} = 1.53 \times 10^{7}$$

- Shear stress coefficient (from Chapter 9):

$$\begin{aligned} C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \\ &= \frac{0.523}{\ln^2(0.06\,(1.53 \times 10^7))} - \frac{1520}{1.53 \times 10^7} \\ &= 0.0027 \end{aligned}$$

- Drag force (from Chapter 9)

$$\begin{aligned} F_s &= C_f A \frac{\rho U_o^2}{2} \\ &= (0.0027)\,(2 \times 3\,\mathrm{m} \times 4\,\mathrm{m})\,\frac{(1000\,\mathrm{kg/m^3})\,(5\,\mathrm{m/s})^2}{2} \end{aligned}$$

$$\boxed{F_s = 810\,\mathrm{N} \quad \text{(case a)}}$$

Maximum drag force (case b)

- Select a plate orientation that is normal with the flow.
- Coefficient of drag (Table 11.1 (EFM 10e); rectangular plate). $C_D \approx 1.18$

- Drag Force

$$F_D = C_D A_{\text{Ref}} \frac{\rho V^2}{2} = (1.18)\,(3\,\text{m} \times 4\,\text{m})\,\frac{(1000\,\text{kg/m}^3)\,(5\,\text{m/s})^2}{2}$$

$$\boxed{F_D = 177\,\text{kN} \quad \text{(case b)}}$$

REVIEW

Notice that the ratio of drag forces is

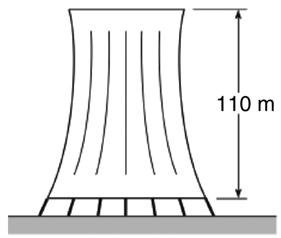
$$\frac{\text{Drag force (maximum)}}{\text{Drag force (minimum)}} = \frac{177000\,\text{N}}{810\,\text{N}} = 218.5$$

In this problem, case a is all friction drag and case b is all form drag.

11.14: PROBLEM DEFINITION

Situation:

Wind acts on a cooling tower.
$H = 110\,\text{m}, D = 60\,\text{m}$
$V_o = 240$ km/h $= 66.7\,\text{m/s}$.



Find: Drag force (in N)

Properties: Air (15 °C), Table A.3 (EFM 10e), $\rho = 1.23$ kg/m^3; $\nu = 1.46 \times 10^{-5}$ m^2/s.

Assumptions:

- Coefficient of drag of the tower is similar to the coefficient of drag for a circular cylinder of infinite length (see Fig. 11.5 (EFM10e).
- Coefficient of drag for a cylinder is constant at high Reynolds numbers.

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{V_o D}{\nu} \\ &= \frac{66.7 \times 60}{1.46 \times 10^{-5}} \\ &= 2.7 \times 10^8 \end{aligned}$$

From Fig. 11.5 EFM10e (extrapolated) $C_D \approx 0.70$. The drag force is given by

$$\begin{aligned} F_D &= C_D A_{\text{Ref}} \frac{\rho V^2}{2} \\ &= 0.70 \times (60\,\text{m} \times 110\,\text{m}) \frac{\left(1.23 \text{ kg/m}^3\right)(66.7\,\text{m/s})^2}{2} \\ &= 1.26 \times 10^7\,\text{kg} \cdot \text{m/s}^2 \end{aligned}$$

$$\boxed{F_D = 1.26 \times 10^7\,\text{N}}$$

11.15: PROBLEM DEFINITION

Situation: The problem statement describes the wind force on a person.

Find: Wind force (the person is you).

Assumptions:
C_D is like a rectangular plate: $C_D \approx 1.20$.
Height is 1.83 meters; width is .3 meters.

PLAN

Apply the ideal gas law, then the drag force equation.

SOLUTION

Ideal gas law

$$\begin{aligned}\rho &= p/RT \\ &= 96,000/(287 \times (273+20)) \\ &= 1.14 \text{ kg/m}^3\end{aligned}$$

Drag force

$$\begin{aligned}F_D &= C_D A_p \rho V^2/2 \\ &= 1.2 \times 1.83 \times 0.30 \times 1.14 \times 30^2/2 \\ &\boxed{F_D = 338 \text{ N}}\end{aligned}$$

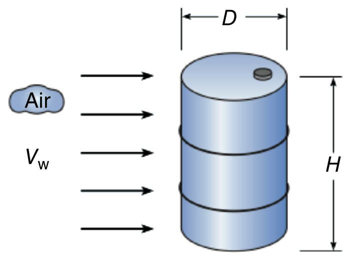
REVIEW

F_D depends on C_D and on dimensions assumed.

11.16: PROBLEM DEFINITION

Situation:

Wind is blowing on a 210 L drum.
$m = 22$ kg.
$H = 0.88\,\text{m}$.
$D = 0.57\,\text{m}$.



Find: Wind speed (m/s) that will cause the drum to tip over.

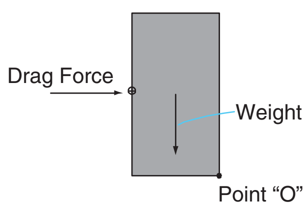
Assumptions:

The drum will tip over before it slips.
Air temperature is 20 °C.
C_D is the same as that of a rectangular plate. Thus, $C_D \approx 1.20$ from Table 11.1 (EFM 10e).

Properties: Air (20 °C), Table A.3 (EFM 10e), $\rho = 1.2\,\text{kg/m}^3, \nu = 15.1 \times 10^{-6}\,\text{m}^2/\text{s}$.

PLAN

1. Relate the drag force to wind speed using the drag force equation.
2. To determine when the drum will tip, sum moments about point O.



3. Find the wind speed by combining results from steps 1 and 2.

SOLUTION

1. Drag force:

$$F_D = C_D A_{\text{Ref}} \frac{\rho V^2}{2} = C_D H D \frac{\rho V^2}{2} \tag{1}$$

2. Moment equilibrium (about 0).

$$F_D \frac{H}{2} = mg \frac{D}{2} \tag{2}$$

3. Combine Eqs. (1) and (2):

$$C_D H D \frac{\rho V^2}{2} = mg \frac{D}{H}$$

Solve for the wind speed

$$\begin{aligned} V &= \sqrt{mg \frac{D}{H} \frac{2}{\rho C_D H D}} = \sqrt{\frac{2mg}{H^2 \rho C_D}} \\ &= \sqrt{\frac{2\,(22\,\mathrm{kg})\,(9.81\,\mathrm{m/\,s^2})}{(0.88\,\mathrm{m})^2\,(1.2\,\mathrm{kg/\,m^3})\,(1.2)}} \end{aligned}$$

$$\boxed{V = 19.7\,\mathrm{m/\,s}}$$

REVIEW

This wind speed is about 71 km/h.

11.17: PROBLEM DEFINITION

Situation:

A round disk ($D = 0.75\,\text{m}$) is towed in water ($V = 4\,\text{m/s}$).
The disk is oriented normal to the direction of motion.

Find: Drag force (in newtons).

Properties: Water ($10\,^\circ\text{C}$), Table A.5 (EFM 10e), $\rho = 1000\,\text{kg/m}^3$.

PLAN

1. Look up a suitable coefficient of drag.
2. Calculate F_D by using the drag force equation.

SOLUTION

1. From Table 11.1 (EFM10e) (circular cylinder with $l/d = 0$)

$$C_D = 1.17$$

2. Drag force equation

$$\begin{aligned} F_D &= C_D A_p \left(\frac{\rho V_0^2}{2}\right) \\ &= 1.17 \left(\frac{\pi\,(0.75\,\text{m})^2}{4}\right)\left(\frac{(1000\,\text{kg/m}^3)\,(4\ \text{m/s})^2}{2}\right) \\ &= 4140\,\text{N} \end{aligned}$$

$$\boxed{F_D = 4.14\,\text{kN}}$$

REVIEW

- Fig. 11.5 (EFM 10e) gives $C_D \approx 1$. Using this value gives $F_D = 3.54\,\text{kN}$.

11.18: PROBLEM DEFINITION

Situation: A circular billboard is described in the problem statement.

Find: Force on the billboard.

Properties: From Table A.3 (EFM 10e), $\rho = 1.25$ kg/m^3.

SOLUTION

From Table 11.1 (EFM 10e)

$$C_D = 1.17$$

Drag force

$$\begin{aligned} F_D &= C_D A_p \rho V^2/2 \\ &= 1.17 \times (\pi/4) \times 7^2 \times 1.25 \times 50^2/2 = \boxed{70{,}354 \text{ N}} \\ &\boxed{F_D = 70.4 \text{ kN}} \end{aligned}$$

11.19: PROBLEM DEFINITION

Situation:

A large rock is located on the bottom of a river.

The current is strong enough so the rock moves along the bottom of the river.

Find: The speed (m/s) of the current required for the rock to roll.

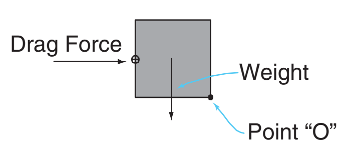
Assumptions:

- The rock tumbles along the bottom (rationale: rocks will tumble rather than slide).
- The rock can be idealized as a cube of size $L \times L \times L$, where $L = 0.3$ m.(rationale: large rocks are block like; the selected size is typical).
- Uniform velocity variation in the river (rationale: turbulent flow has a relatively uniform velocity distribution).
- $C_D = 1.1$ (value for a cube from Table 11.1, EFM 10e).
- The river bottom is horizontal (rationale: neglect the small slope that is typically found in rivers).
- The specific gravity of the rock is $S = 2.8$ (rationale: a typical value because $S_{\text{granite}} = 2.6$ to 2.7, $S_{\text{basalt}} = 2.8$ to 3.0, $S_{\text{shale}} = 2.4$ to 2.8).

Properties: Water (10 °C), Table A.5 (EFM 10e), $\rho = 1000\,\text{kg/m}^3$.

PLAN

1. Relate the drag force to speed of the current using the drag force equation.
2. To determine when the rock will tip, sum moments about point O.



3. Find the speed of the current by combining results from steps 1 and 2.

SOLUTION

1. Drag force:

$$F_D = C_D A_{\text{Ref}} \frac{\rho V^2}{2} = C_D L^2 \frac{\rho V^2}{2} \tag{1}$$

2. Moment equilibrium (about O).

$$\begin{aligned} F_D \frac{L}{2} &= mg\frac{L}{2} = \left(S\rho L^3\right) g \frac{L}{2} \\ F_D &= S\rho g L^3 \end{aligned} \tag{2}$$

3. Combine Eqs. (1) and (2):

$$
\begin{aligned}
C_D L^2 \frac{\rho V^2}{2} &= S\rho g L^3 \\
V &= \sqrt{S\rho g L^3 \frac{2}{\rho C_D L^2}} = \sqrt{\frac{2SgL}{C_D}} \\
&= \sqrt{\frac{2\,(2.8)\,(9.81\,\mathrm{m/\,s^2})\,(0.3\,\mathrm{m})}{1.1}}
\end{aligned}
$$

$$
\boxed{V = 3.9\,\mathrm{m/\,s}}
$$

REVIEW

- This speed is about 4 m/s.
- The major variable is the size of the rock (L). Larger rocks require faster rivers. Notice that the speed needed to transport a rock is proportional to the square root of a typical dimension. $V \sim \sqrt{L}$.

11.20: PROBLEM DEFINITION

Situation:
Wind $V_o = 35\,\text{m/s}$ acts on a tall smokestack.
Height is $h = 75\,\text{m}$. Diameter is $D = 2.5\,\text{m}$.

Find: Overturning moment at the base.

Assumptions: Neglect end effects–that is the coefficient of drag from a cylinder of infinite length is applicable.

Properties: Air (20 °C) ,Table A.3 (EFM 10e): $\rho = 1.2 \times 99/101.3 = 1.17\,\text{kg/m}^3$, $\nu = 1.51 \times 10^{-5}\,\text{m}^2/\text{s}$.

SOLUTION

Reynolds number

$$\begin{aligned}\text{Re} &= \frac{V_o D}{\nu} \\ &= \frac{(35\,\text{m/s}) \times (2.5\,\text{m})}{1.51 \times 10^{-5}\,\text{m}^2/\text{s}} \\ &= 5.79 \times 10^6\end{aligned}$$

Drag force. From Fig. 11.5 (EFM 10e) $C_D \approx 0.62$ so

$$\begin{aligned}F_D &= C_D A_p \frac{\rho V_0^2}{2} \\ &= 0.62 \times (2.5 \times 75\,\text{m}^2) \times \frac{(1.17\,\text{kg/m}^3) \times (35\,\text{m/s})^2}{2} \\ &= 83.31 \text{ kN}\end{aligned}$$

Overturning moment

$$\begin{aligned}M_o &= h/2 \times F_D \\ M_o &= (75/2) \text{ m} \times (83.31\,\text{kN})\end{aligned}$$

$$\boxed{M_o = 3.12\,\text{MN} \cdot \text{m}}$$

11.21: PROBLEM DEFINITION

Situation:

Wind acts on a flag pole.
$V = 37.5\,\mathrm{m/s}$.
$H = 20\,\mathrm{m}$, $D = 0.08\,\mathrm{m}$

Find: Moment at bottom of flag pole.

Properties: Air (20 °C), Table A.3 (EFM 10e), $\rho = 1.2\,\mathrm{kg/m^3}, \nu = 15.1 \times 10^{-6}\,\mathrm{m^2/s}$.

SOLUTION

1. Reynolds number

$$\begin{aligned} \mathrm{Re} &= \frac{VD}{\nu} \\ &= \frac{(37.5\,\mathrm{m/s})\,(0.08\,\mathrm{m})}{(15.1 \times 10^{-6}\,\mathrm{m^2/s})} \\ &= 1.99 \times 10^5 \end{aligned}$$

2. From Fig. 11.5 (EFM10e), $C_D = 0.8$.

3. Drag Force

$$\begin{aligned} F_D &= C_D A_p \frac{\rho V_0^2}{2} = C_D \pi D H \frac{\rho V_0^2}{2} \\ &= 0.8\,(0.08\,\mathrm{m})\,(20\,\mathrm{m}) \times \frac{(1.2\,\mathrm{kg/m^3}) \times (37.5\,\mathrm{m/s})^2}{2} \\ &= 1.08\,\mathrm{kN} \end{aligned}$$

4. Moment

$$\begin{aligned} M &= F_D \frac{H}{2} \\ &= (1.08\,\mathrm{kN})\,\frac{(20\,\mathrm{m})}{2} \\ &\boxed{M = 10.8\,\mathrm{kN \cdot m}} \end{aligned}$$

11.22: PROBLEM DEFINITION

Situation:

A man in a boat is pulling up an anchor made of concrete.
$V = 1\,\text{m/s}$
$D = 0.3\,\text{m}$, $L = 0.3\,\text{m}$

Find: Tension in rope (in newtons).

Properties:

Water (10 °C), Table A.5 (EFM 10e), $\gamma = 9.81\,\text{kN/m}^3$.
Concrete, $\gamma = 15\,\text{kN/m}^3$.

PLAN

Apply equilibrium. The forces are tension, weight, drag force and buoyancy force.

SOLUTION

Equilibrium

$$\begin{aligned}\sum F_y &= 0 \\ T - W - F_D + F_B &= 0\end{aligned}$$

Solve for T

$$T = W - F_B + F_D$$

Substitute equations and values for weight, buoyant force, and drag force

$$T = (\pi/4) \times 0.3^2 \times 0.3(15000 - 9810) + C_D(\pi/4) \times 0.3^2 \times 1000 \times 1.0^2/2$$

From Table 11.1 in EFM10e, $C_D = 0.90$. Then

$$T = 110 + 31.8$$

$$\boxed{T = 142\,\text{N}}$$

11.23: PROBLEM DEFINITION

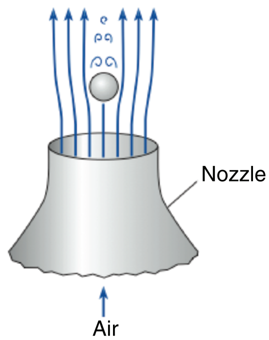
Situation:

A ping-pong ball is supported by an air jet.

$m = 2.6 \times 10^{-3}\,\text{kg}, \ D = .038\,\text{m}.$

$T = 18\,^\circ\text{C} = 291.2\,\text{K}.$

$p = 91.4\,\text{kPa}.$



Find: The speed of the air jet.

Properties: Air, Tables A.2 & A.3 (EFM 10e), $R = 287\,\text{J/kg}\cdot\text{K}$, $\mu = 1.80 \times 10^{-5}\,\text{N}\cdot\text{s/m}^2$.

Assumptions: Assume the ping-pong ball is stationary (stable equilibrium).

PLAN

Find density of the air jet. Then, to find V, balance the weight of the ball with the drag force. Solve the resulting set of equations using a computer program.

SOLUTION

Ideal gas law

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{91,400\,\text{Pa}}{(287\,\text{J/kg}\cdot\text{K})\,(291.2\,\text{K})} \\ &= 1.094\,\text{kg/m}^3 \end{aligned}$$

Equilibrium

$$mg = F_{\text{Drag}} \tag{1}$$

Drag force

$$\begin{aligned} F_{\text{Drag}} &= C_D A_{\text{Ref}} \left(\frac{\rho V^2}{2}\right) \\ &= C_D \left(\frac{\pi D^2}{4}\right)\left(\frac{\rho V^2}{2}\right) \end{aligned} \tag{2}$$

Cliff and Gauvin correlation (drag on a sphere)

$$C_D = \frac{24}{\mathrm{Re}_D}\left(1 + 0.15\,\mathrm{Re}_D^{0.687}\right) + \frac{0.42}{1 + 4.25 \times 10^4\,\mathrm{Re}^{-1.16}} \tag{3}$$

Reynolds Number

$$\mathrm{Re} = \frac{VD\rho}{\mu} \tag{4}$$

Solve Eqs. (1) to (4) simultaneously. The computer program TKSolver was used for our solution.

$$\begin{aligned}
\mathrm{Re} &= 21,717 \\
F_{\mathrm{Drag}} &= 0.026\,\mathrm{N} \\
C_D &= 0.46 \\
V &= 9.45\,\mathrm{m/\,s}
\end{aligned}$$

$$\boxed{V_{\mathrm{jet}} = 9.45\,\mathrm{m/\,s}}$$

11.24: PROBLEM DEFINITION

Situation:
Wind ($V = 35\,\mathrm{m/s}$)is acting on a sign $3\,\mathrm{m} \times 2\,\mathrm{m}$.
Center of the sign is $4\,\mathrm{m}$ above the ground.

Find: Moment at ground level.

Assumptions: Neglect the drag force on the sign post.

Properties: Air ($10\,^\circ\mathrm{C},\ 1.0\,\mathrm{atm}$), Table A.3 (EFM 10e), $\rho = 1.25\,\mathrm{kg/m^3}$.

SOLUTION

1. Coefficient of drag (from Table 11.1 in EFM 10e).

$$C_D = 1.18$$

2. Drag force.

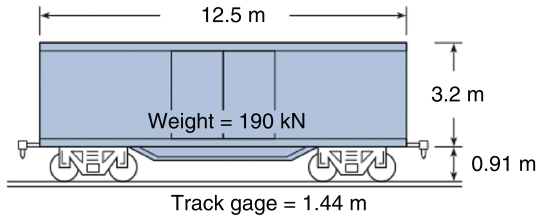
$$F_D = C_D A \frac{\rho V_o^2}{2}$$

3. Moment at the base of the sign.

$$\begin{aligned} M &= LF_D = LC_D A \frac{\rho V_o^2}{2} \\ &= (4\,\mathrm{m})\,(1.18)\,(3\,\mathrm{m} \times 2\,\mathrm{m})\,\frac{(1.25\,\mathrm{kg/m^3})\,(35\,\mathrm{m/s})^2}{2} \\ & \boxed{M = 21.7\,\mathrm{kN \cdot m}} \end{aligned}$$

11.25: PROBLEM DEFINITION

Situation: Wind is blowing normal to a boxcar.



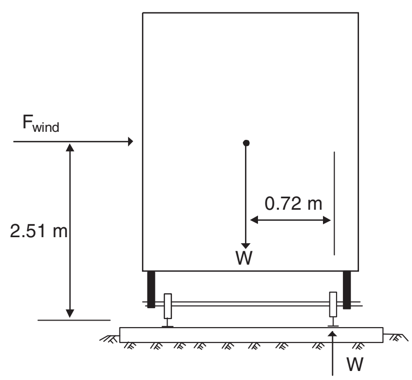
Find: Speed of wind required to blow boxcar over.

Assumptions: $T = 10\,^{\circ}\mathrm{C}$; $\rho = 1.25$ kg/m^3.

SOLUTION

Take moments about one wheel for impending tipping.

$$\sum M = 0$$



$$\begin{aligned} W \times 0.72 - F_D \times 2.51 &= 0 \\ F_D = (190{,}000 \times 0.72)/2.51 = 54{,}500 \text{ N} &= C_D A_p \rho V^2/2 \end{aligned}$$

From Table 11.1 (EFM 10e), assume $C_D = 1.20$. Then

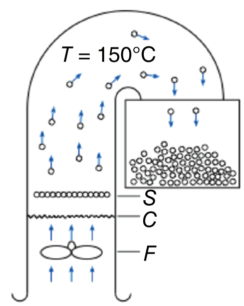
$$\begin{aligned} V^2 &= 54{,}500 \times 2/(1.2 \times 12.5 \times 3.2 \times 1.25) \\ & \boxed{V = 42.6 \text{ m/s}} \end{aligned}$$

11.26: PROBLEM DEFINITION

Situation:

Air speed is being determined in a popcorn popper.

Additional information is provided in problem statement.



Find: Range of airspeeds for popcorn popper operation.

Properties: Air, Table A.3 (EFM 10e), 150 °C, $\rho = 0.83$ kg/m^3 and $\nu = 2.8 \times 10^{-5}$ m^2/s.

PLAN

This device should lift popped kernels while leaving unpopped kernels on the screen.

1. Find the maximum air speed by equating the drag force on an unpopped kernel its weight.
2. Find the minimum air speed by equating the drag force on a popped kernel with its weight.

SOLUTION

1. Before popcorn is popped, it should not be thrown out by the air. Thus, let

$$V_{\max} = \sqrt{\frac{2F_D}{C_D A_p \rho_{\text{air}}}}$$

where F_D is equal to the weight of an unpopped kernel.

$$\begin{aligned} F_D &= mg \\ &= 0.15 \times 10^{-3} \times 9.81 \\ &= 1.472 \times 10^{-3}\,\text{N} \end{aligned}$$

Section area of a kernel

$$\begin{aligned} A_p &= (\pi/4) \times (0.006)^2 \text{ m}^2 \\ &= 2.83 \times 10^{-5}\,\text{m}^2 \end{aligned}$$

Assume $C_D \backsimeq 0.4$. Then

$$\begin{aligned} V_{\max} &= \sqrt{\frac{2F_D}{C_D A_p \rho_{\text{air}}}} \\ &= \sqrt{\frac{2 \times 1.472 \times 10^{-3}}{0.4 \times 2.83 \times 10^{-5} \times 0.83}} \\ &= 17.7 \ \text{m/s} \end{aligned}$$

Check Reynolds number and C_D:

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{17.7 \times 0.006}{2.8 \times 10^{-5}} \\ &= 3800 \end{aligned}$$

From Fig. 11.9 (EFM 10e) $C_D \approx 0.4$ so solution is valid.

2. For minimum velocity let popped corn be suspended by stream of air. Assume only that diameter changes. So

$$\begin{aligned} V_{\min} &= V_{\max} \times (A_u/A_p)^{1/2} \\ &= V_{\max} \frac{D_u}{D_p} \end{aligned}$$

where D_p = diameter of popped corn and D_u = diameter of unpopped corn

$$\begin{aligned} V_{\min} &\backsimeq V_{\max} \frac{D_u}{D_p} \\ &= 17.7 \left(\frac{6\,\text{mm}}{18\,\text{mm}} \right) \\ & V_{\min} = 5.9\,\text{m/s} \end{aligned}$$

$$\boxed{(5.9\,\text{m/s}) \leq V \leq (17.7\,\text{m/s})}$$

11.27: PROBLEM DEFINITION

Situation: Wind loads act on a flag pole that is carrying an 2 m high American Flag.

Find: Determine a diameter for the pole.

Assumptions: The failure mechanism is yielding due to static loading.

SOLUTION

An American flag is 1.9 times as long as it is high. Thus

$$A = 2^2 \times 1.9 = 7.6\,\text{m}^2$$

Assume

$$\begin{aligned} T &= 15\,^\circ\text{C}, \rho = 1.23 \text{ kg/m}^3 \\ V_0 &= 160 \text{ km/h} = 44 \text{ m/s} \end{aligned}$$

Compute drag force on flag

$$\begin{aligned} F_D &= C_D A \rho V_0^2/2 \\ &= 0.14 \times 7.6 \times 1.23 \times 44^2/2 \\ &= 1267 \text{ N} \end{aligned}$$

Make the flag pole of steel using one size for the top half and a larger size for the bottom half. To start the determination of d for the top half, assume that the pipe diameter is 15 cm. Then

$$\begin{aligned} F_{\text{on pipe}} &= C_D A_p \rho V_0^2/2 \\ \text{Re} = VD/\nu &= 44 \times 0.15/(1.46 \times 10^{-5}) \\ &= 4.5 \times 10^5 \end{aligned}$$

With an Re of $4.5 \times 10^5, C_D$ may be as low as 0.3 (Fig. 11.5 in EFM 10e); however, for conservative design purposes, assume $C_D = 1.0$. Then

$$\begin{aligned} F_{\text{pipe}} &= 1 \times 15 \times 0.15 \times 1.23 \times 44^2/2 = 2679 \text{ N} \\ M &= 1267 \times 15 + 2679 \times 7.5 = 39{,}098 \text{ N-m} \end{aligned}$$

Assume that the allowable stress is 207 kPa.

$$\begin{aligned} \frac{I}{c} &= \frac{M}{\sigma_{\max}} \\ &= \frac{39{,}098}{207{,}000} \\ &= 0.19 \text{ m}^3 \end{aligned}$$

From a handbook it is found that a 0.15 m double extra-strength pipe will be adequate.

Bottom half, Assume bottom pipe will be 0.3 m in diameter.

$$\begin{aligned} F_{\text{flag}} &= 1267 \text{ N} \\ F_{0.15 \text{ m pipe}} &= 2679 \text{ N} \\ F_{0.3 \text{ m pipe}} &= 1 \times 15 \times 0.3 \times 1.23 \times 44^2/2 \\ &= 5358 \text{ N} \end{aligned}$$

$$\begin{aligned} M &= 1267 \times 30 + 2679 \times 23 + 5358 \times 7.5 \\ &= 139{,}812 \text{ N-m} \\ M_s &= 0.68 \text{ m}^3 = I/c \end{aligned}$$

Handbook shows that **0.3 m extra-strength pipe** should be adequate.

REVIEW

Many other designs are possible.

11.28: PROBLEM DEFINITION

Situation:
A spherical ($D = 1.5\,\text{m}$) submarine is moving through seawater.
$V = 18.5\ \text{km/h} = 5.14\ \text{m/s}$.

Find: Power (in kW) to propel the sub.

Assumptions:
All power is being used to overcome drag.
Skin friction is negligible.

Properties: Seawater (10 °C), Table A.4 (EFM 10e), $\rho = 1026\,\text{kg/m}^3$, $\nu = 1.4 \times 10^{-6}\,\text{m}^2/\text{s}$.

PLAN

1. Find Re, then find C_D using the Clift & Gauvin correlation.
2. Find the drag force.
3. Find power using $P = F_D V$.

SOLUTION

1. Reynolds Number

$$\text{Re} = \frac{VD}{\nu} = \frac{(5.14\,\text{m/s})(1.5\,\text{m})}{(1.4 \times 10^{-6}\,\text{m}^2/\text{s})} = 5.51 \times 10^6$$

Clift and Gauvin correlation (drag on a sphere):

$$\begin{aligned} C_D &= \frac{24}{\text{Re}_D}\left(1 + 0.15\,\text{Re}^{0.687}\right) + \frac{0.42}{1 + 4.25 \times 10^4\,\text{Re}^{-1.16}} \\ &= \frac{24}{(5.51 \times 10^6)}\left(1 + 0.15\left(5.51 \times 10^6\right)^{0.687}\right) + \frac{0.42}{1 + (4.25 \times 10^4)\left(5.51 \times 10^6\right)^{-1.16}} \\ &= 0.45 \end{aligned}$$

2. Drag Force (use projected area as reference area).

$$\begin{aligned} F_D &= C_D A \frac{\rho V_o^2}{2} = C_D\left(\frac{\pi D^2}{4}\right)\frac{\rho V_o^2}{2} \\ &= 0.45\left(\frac{\pi\,(1.5\,\text{m})^2}{4}\right)\left(\frac{(1026\,\text{kg/m}^3)\,(5.14\,\text{m/s})^2}{2}\right) \\ &= 10.78\,\text{kN} \end{aligned}$$

3. Power equation.

$$\begin{aligned} P &= F_D V \\ &= (10.78\,\text{kN})\,(5.14\,\text{m/s}) \\ & \boxed{P = 55.5\ \text{kW}} \end{aligned}$$

11.29: PROBLEM DEFINITION

Situation:

The problem statement describes a dirigible.
$V_0 = 9$ m/s
$d = 24$ m

Find:

Power required for dirigible.

SOLUTION

Reynolds number

$$\text{Re} = V_0 d/\nu = (9)(24)/(1.2 \times 10^{-5}) = 1.8 \times 10^7$$

Drag force

From Fig. 11.9 in EFM 10e (extrapolated) $C_D = 0.05$

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ &= (0.05)(\pi/4)(24^2)(11/9.81)(9^2)/2 \\ &= 1027 \text{ N} \end{aligned}$$

Power

$$\begin{aligned} P &= F_D V_0 \\ &= (1027)(9) \\ &= 9243 \text{ N-m/s} = \boxed{9243 \text{ W}} \end{aligned}$$

11.30: PROBLEM DEFINITION

Situation:

- A runner is competing in a 10 km race.
- Running speed is a 4:30 pace (i.e. each kilometer takes 4 minutes and 30 seconds). Thus, $V = 3.7\,\mathrm{m/s}$.
- The product of frontal area and coefficient of drag is $C_D A = 0.74\,\mathrm{m}^2$.
- One "food calorie" is equivalent to 4186 J.

Find: Estimate the energy in joules and kcal (food calories) that the runner needs to supply to overcome aerodynamic drag.

Properties: Density of air is $1.22\,\mathrm{kg/m}^3$.

Assumptions: Assume that the air is still–that is, there is no wind.

PLAN

Energy is related to power (P) and time (t) by $E = Pt$. Find power using the product of speed and drag force $(P = VF_{\mathrm{Drag}})$. Find time by using distance (d) and speed $(d = Vt)$.

SOLUTION

Find the time to run 10 km.

$$\begin{aligned} t &= \frac{d}{V} \\ &= \frac{10,000\,\mathrm{m}}{3.7\,\mathrm{m/s}} \\ &= 2703\,\mathrm{s} \quad (45\,\mathrm{min\ and}\ 3\,\mathrm{s}) \end{aligned}$$

Drag force

$$\begin{aligned} F_{\mathrm{Drag}} &= C_D A_{\mathrm{Ref}} \left(\frac{\rho V^2}{2}\right) \\ &= \left(0.743\,\mathrm{m}^2\right)\left(\frac{(1.22\,\mathrm{kg/m}^3)\,(3.7\,\mathrm{m/s})^2}{2}\right) \\ &= 6.18\,\mathrm{N} \end{aligned}$$

Power

$$\begin{aligned} P &= F_{\mathrm{Drag}} V \\ &= (6.18\,\mathrm{N})\,(3.7\,\mathrm{m/s}) \\ &= 22.9\,\mathrm{W} \end{aligned}$$

Energy

$$\begin{aligned} E &= Pt \\ &= (22.9\,\mathrm{J/\,s})\,(2703\,\mathrm{s}) \\ &= 61.9 \end{aligned}$$

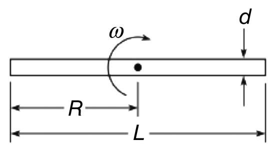
$$\boxed{\text{Energy} = 61.9 = 14.8 \text{ Food Calories}}$$

REVIEW

1. The drag force is small, about 6.18 N.
2. The power to overcome drag is small (22.9 W). Based on one of the author's (DFE) experience in sports, a fit runner might supply 180 W to run at a 4:30 pace. Thus, the power to overcome drag is about 1/7 of the total power that the runner supplies.
3. The energy that the runner expends (14.8 Food Calories) can be acquired by eating a small amount of food. For example, a small piece of candy.

11.31: PROBLEM DEFINITION

Situation: A cylindrical rod is rotated about its midpoint—additional details are provided in the problem statement.



Find:
a) Derive an equation for the power to rotate rod.
b) Calculate the power.

SOLUTION

For an infinitesimal element, dr, of the rod

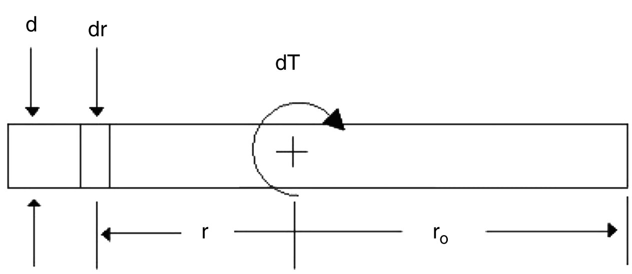
$$dF_D = C_D(dr)d\rho V_{\text{rel.}}^2/2$$

where $V_{\text{rel.}} = r\omega$. Then

$$\begin{aligned} dT &= rdF_D = C_D\rho d(V_{\text{rel.}}^2/2)rdr \\ T_{\text{total}} &= 2\int_0^{r_0} dT = 2\int_0^{r_0} C_D d\rho((r\omega)^2/2)rdr \\ T_{\text{total}} &= C_D d\rho\omega^2 \int_0^{r_0} r^3 dr = C_D d\rho\omega^2 r_0^4/4 \end{aligned}$$

but $r_0 = L/2$ so

$$\begin{aligned} T_{\text{total}} &= C_D d\rho\omega^2 L^4/64 \\ &\quad \text{or} \\ P &= T\omega = \boxed{C_D d\rho\omega^3 L^4/64} \end{aligned}$$



Then for the given conditions:

$$P = 1.2 \times 0.02 \times 1.2 \times (50)^3 \times 1.5^4/64$$

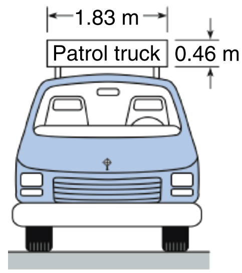
$$\boxed{P = 285 \text{ W}}$$

11.32: PROBLEM DEFINITION

Situation:

A truck carries a rectangular sign.

$V = 30\,\mathrm{m/s}$.



Find: Additional power required to carry the sign (as compared to not carrying the sign).

Assumptions:

Density of air is $\rho = 1.2\,\mathrm{kg/m^3}$.

The coefficient of drag is not influenced by the airflow over the truck.

PLAN

1. Find C_D.
2. Find the force of drag.
3. Calculate power as the product of drag force and speed.

SOLUTION

1. Drag coefficient. From Table 11.1 (EFM 10e) for a rectangular plate with an aspect ratio of $l/d = 3.98$:

$$C_D \approx 1.20$$

2. Drag force equation.

$$\begin{aligned} F_D &= C_D A \frac{\rho V_o^2}{2} \\ &= (1.2)\,(1.83\,\mathrm{m} \times 0.46\,\mathrm{m})\,\frac{(1.2\,\mathrm{kg/m^3})\,(30\,\mathrm{m/s})^2}{2} \\ &= 545.5\,\mathrm{N} \end{aligned}$$

3. Power equation.

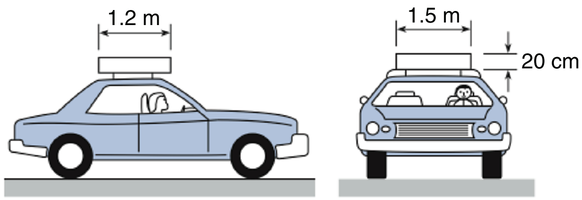
$$\begin{aligned} P &= F_D V \\ &= (545.5\,\mathrm{N})\,(30\,\mathrm{m/s}) \\ &= 16.4\,\mathrm{kW} \end{aligned}$$

Additional power = 16.4 kW

11.33: PROBLEM DEFINITION

Situation:

A cartop carrier is used on an automobile.
$V_c = 100\,\text{km/h} = 27.8\,\text{m/s}$,
$V_w = 25\,\text{km/h} = 6.94\,\text{m/s}$.



Find: Additional power required due to the carrier.

Assumptions:

Density of air is $\rho = 1.2\,\text{kg/m}^3$.
The coefficient of drag is not influenced by the car.
C_D is best approximated as a rectangular plate.

PLAN

1. Find C_D.
2. Find the force of drag.
3. Calculate power as the product of drag force and speed.

SOLUTION

1. Coefficient of drag. The aspect ratio is

$$\frac{l}{d} = \frac{1.5\,\text{m}}{0.2\,\text{m}} = 7.5$$

From Table 11.1 (EFM 10e)

$$C_D \approx 1.25$$

2. Drag force equation.

$$\begin{aligned} F_D &= C_D A \frac{\rho V_o^2}{2} \\ &= (1.25)(1.5\,\text{m} \times 0.2\,\text{m}) \frac{(1.2\,\text{kg/m}^3)(27.8\,\text{m/s} + 6.94\,\text{m/s})^2}{2} \\ &= 271.6\ \text{N} \end{aligned}$$

3. Power equation.

$$\begin{aligned} P &= F_D V \\ &= (271.6\ \text{N})(27.8\,\text{m/s}) \\ &= 7.55\,\text{kW} \end{aligned}$$

$$\boxed{\text{Additional power} = 7.55\,\text{kW}}$$

11.34: PROBLEM DEFINITION

Situation: The problem statement describes motion of an automobile.

Find: Percentage savings in gas mileage when travelling at 90 km/h instead of 105 km/h.

SOLUTION

Work is force times distance. Thus, energy is

$$E = F \times s$$

The energy, E, per unit distance is simply the force or

$$E/s = F$$

Substituting drag force

$$\begin{aligned} E/s &= \mu \times W + C_D A_p \rho V^2/2 \\ E/s &= 0.02 \times 13,350\,\text{N} + 0.3 \times 1.86 \times (1.23/2)V^2 \end{aligned}$$

For

$$\begin{aligned} V &= 90 \text{ km/h} = 25 \text{ m/s} \\ E/s &= 481 \text{ N} \end{aligned}$$

For

$$\begin{aligned} V &= 105 \text{ km/h} = 29.2 \text{ m/s} \\ E/s &= 559 \text{ N} \end{aligned}$$

Then energy savings are

$$(559 - 481)/559 = (559 - 481)/559 \text{ or } \boxed{13.95\%}$$

11.35: PROBLEM DEFINITION

Situation:

A car ($W = 8900\,\mathrm{N}$) coasting down a hill (Slope $= 6\%$) ; has reached steady speed.
$\mu_{\text{rolling}} = \mu = 0.01$
$C_D = 0.29 \quad A_P = 1.67\,\mathrm{m}^2$
$\rho_{\text{air}} = \rho = 1.03\,\mathrm{kg/\,m^3}$

Find: Maximum coasting speed.

SOLUTION

Slope of a hill is rise over run, so the angle of the hill is

$$\begin{aligned} \tan\theta &= 0.06 \\ &\text{or} \\ \theta &= \arctan(0.06) \\ &= 0.0599\,\text{rad} = 3.43^o \end{aligned}$$

Equate forces

$$F_D + F_r = W \times \sin 3.43^{\mathrm{o}}$$

where F_D =drag force, F_r =rolling friction and W =weight of car.

Insert expressions for drag force and rolling friction.

$$C_D A_p \rho V^2/2 + W \times 0.01 \times \cos 3.43^o = W \times \sin 3.43^o$$

$$V^2 = \frac{2W(\sin 3.43^o - .01 \times \cos 3.43^o)}{C_D A_p \rho}$$

$$V^2 = \frac{2 \times 8900(0.0599 - 0.00998)}{0.29 \times 1.67 \times 1.03}$$

$$= 1781\ \mathrm{m}^2/\mathrm{s}^2$$

$$V = 42.2\ \mathrm{m/s} = \boxed{152\ \mathrm{km/h}}$$

11.36: PROBLEM DEFINITION

Situation: The problem statement describes a car being driven up a hill

Find: Power required.

SOLUTION

The power required is the product of the forces acting on the automobile in the direction of travel and the speed. The drag force is

$$F_D = \frac{1}{2}\rho V^2 C_D A = \frac{1}{2} \times 1.2 \times 30^2 \times 0.4 \times 4 = 864 \text{ N}$$

The force due to gravity is

$$F_g = Mg \sin 3^{\text{o}} = 1000 \times 9.81 \times \sin 3^{\text{o}} = 513 \text{ N}$$

The force due to rolling friction is

$$F_r = \mu Mg \cos 3^{\text{o}} = 0.02 \times 1000 \times 9.81 \times \cos 3^{\text{o}} = 196 \text{ N}$$

The power required is

$$P = (F_D + F_r + F_f)V = 1573 \times 30; \boxed{P = 47.2 \text{ kW}}$$

11.37: PROBLEM DEFINITION

Situation: A bicyclist is coasting down a hill—additional details are provided in the problem statement

Find: Speed of the bicycle.

SOLUTION

Equilibrium (direction parallel to motion of the bicyclist)

$$\sum F = 0$$

$$\text{(Weight component)} - \text{(Drag force)} - \text{(Rolling resistance)} = 0$$

$$\begin{aligned} F_W - F_D - Ff &= 0 \\ W \sin 4^\circ - C_D A_p \rho V_{R.}^2/2 - 0.02\ W \cos 4^\circ &= 0 \\ W \sin 4^\circ - 0.5 \times 0.5 \times 1.2 V_R^2/2 - 0.02 W \cos 4^\circ &= 0 \end{aligned}$$

$$\begin{aligned} W &= 80\,\text{kg} = 784.8 \text{ N} \\ W \sin 4^\circ &= 54.7 \text{ N} \\ W \cos 8^\circ &= 782.9 \text{ N} \end{aligned}$$

Then

$$54.7 - 0.15 V_R^2 - .02 \times 782.9 = 0$$

$$V_R = 16.14 \text{ m/s} = V_{\text{bicycle}} + 7 \text{ m/s}$$

Note that 7 m/s is the head wind so the relative speed is $V_{\text{bicycle}} + 7$.

$$\boxed{V_{\text{bicycle}} = 9.14 \text{ m/s}}$$

11.38: PROBLEM DEFINITION

Situation:
A bicyclist is traveling into a 3 m/s head wind.
$P = 275\,\text{W}, \quad A_p = 0.5\,\text{m}^3, \quad C_D = 0.3.$

Find: Speed of the bicyclist.

Assumptions:
Neglect rolling resistance.
All power is used to overcome drag. (neglect power losses in the chain and gears)

Properties: Air. $\rho = 1.2\,\text{kg/m}^3$.

PLAN

1. Relate the cyclist's speed V_c to the drag force.
2. Relate cyclist's speed and power using the power equation.
3. Solve the resulting cubic equation using a numerical solver.

SOLUTION

1. Drag equation (use velocity of air relative to the cyclist).

$$\begin{aligned} F_D &= C_D A \left(\frac{\rho V_R^2}{2} \right) \\ V_R &= (\text{speed of cyclist}) + (\text{speed of head wind}) \\ &= (V_c + 3\,\text{m/s}) \\ F_D &= C_D A_p \left(\frac{\rho (V_c + 3)^2}{2} \right) \end{aligned}$$

2. Power equation.

$$\begin{aligned} P &= F_D \times V_c \\ &= C_D A_p \left(\frac{\rho (V_c + 3)^2}{2} \right) V_c \\ 275\,\text{W} &= 0.3 \left(0.5\,\text{m}^2\right) \left(\frac{(1.2\,\text{kg/m}^3)(V_c + 3)^2}{2} \right) V_c \left(\frac{\text{m}}{\text{s}} \right)^3 \end{aligned}$$

3. Solving the cubic equation (we used a computer program) for speed gives

$$\boxed{V_c = 12.6 \ \text{m/s}}$$

11.39: PROBLEM DEFINITION

Situation: The problem statement describes a 1932 Fiat Balillo that is "souped up" by the addition of a 164 brake kW engine.

Find: Maximum speed of a "souped up" Balillo.

SOLUTION

From Table 11.2 (EFM 10e), $C_D = 0.60$.

$$\begin{aligned} P &= (F_D + F_r)V \\ V &= 100 \text{ km/h} = 28 \text{ m/s} \\ F_r &= (P/V) - F_D = (P/V) - C_D A_p \rho V^2/2 \\ &= (30,000/28) - (0.60)(2.8)(1.23)(28^2)/2 \\ &= 1071 - 810 = 261 \text{ N} \end{aligned}$$

"Souped up" version:

$$\begin{aligned} (F_D + 261)V &= 164,000 \\ ((C_D A_p \rho V^2/2) + 261)V &= 164,000 \\ (C_D A_p \rho V^3/2) + 261V &= 164,000 \\ 2.1V^3 + 261V - 164,000 &= 0 \end{aligned}$$

Solve this cubic equation for V using a computer program

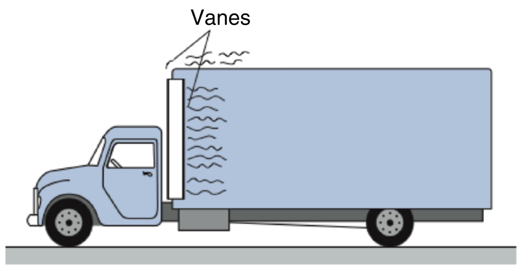
$$V = 43.7 \text{ m/s}$$

$$\boxed{V = 157.3 \text{ km/h}}$$

11.40: PROBLEM DEFINITION

Situation:

To reduce drag, vanes are added to a truck.
C_D (no vanes) = 0.78.
Vanes reduce drag by 25%.
$A_p = 8.36\,\mathrm{m}^2$, $V = 100\,\mathrm{km/h} = 27.8\,\mathrm{m/s}$.



Find: Reduction in drag force due to the vanes.

Properties: Air (20 °C, 1 atm), Table A.3 (EFM 10e), $\rho = 1.2\,\mathrm{kg/m}^3$.

SOLUTION

$$\begin{aligned} F_D &= C_D A \frac{\rho V_o^2}{2} \\ F_{D_{\text{reduction}}} &= (0.25 \times 0.78)\left(8.36\,\mathrm{m}^2\right) \frac{(1.2\,\mathrm{kg/m}^3)(27.8\,\mathrm{m/s})^2}{2} \\ F_{D_{\text{reduction}}} &= \boxed{756 \text{ N}} \end{aligned}$$

11.41: PROBLEM DEFINITION

Situation:

To reduce drag, vanes are added to a truck.

C_D (no vanes) = 0.78.

Vanes reduce drag by 25%.

$A_p = 8.36\,\mathrm{m}^2$, $V = 100\,\mathrm{km/h} = 27.8\,\mathrm{m/s}$.

Total resistance is given by $R = F_D + C$ where $C = 350\,\mathrm{N}$ accounts for bearing friction.

Find: Percentage savings in fuel.

Assumptions: Fuel savings are directly proportional to power savings

Properties: Air (20 °C, 1 atm), Table A.3 (EFM 10e), $\rho = 1.2\,\mathrm{kg/m^3}$.

SOLUTION

1. Power (no vanes).

$$\begin{aligned} P &= (F_D + C)\,V = C_D A \frac{\rho V_o^3}{2} + CV_o \\ &= (0.78)\left(8.36\,\mathrm{m}^2\right)\frac{(1.2\,\mathrm{kg/m^3})\,(27.8\,\mathrm{m/s})^3}{2} + (350\,\mathrm{N})\,(27.8\,\mathrm{m/s}) \\ &= 93.8\,\mathrm{kW} \end{aligned}$$

2. Power (vanes are installed)

$$\begin{aligned} P &= (F_D + C)\,V = C_D A \frac{\rho V_o^3}{2} + CV_o \\ &= (0.78 \times 0.75)\left(8.36\,\mathrm{m}^2\right)\frac{(1.2\,\mathrm{kg/m^3})\,(27.8\,\mathrm{m/s})^3}{2} + (350\,\mathrm{N})\,(27.8\,\mathrm{m/s}) \\ &= 72.8\,\mathrm{kW} \end{aligned}$$

3. Fuel savings.

$$\text{Fuel savings} = \frac{93.8\,\mathrm{kW} - 72.8\,\mathrm{kW}}{93.8\,\mathrm{kW}}$$

$$\boxed{\text{Fuel savings} = 22.4\%}$$

Problem 11.42

Situation: An engineer is designing an object to fall at a speed of 1 m/s in seawater.

Find: Identify the variables that have the most influence on terminal velocity.

PLAN

1. Derive the governing equation by applying force equilibrium.
2. Use the governing equation to identify which variables influence terminal velocity.

SOLUTION

1. Force equilibrium.

$$\begin{aligned} F_{\text{Drag}} + F_{\text{Buoyancy}} &= W \\ C_D A \frac{\rho V_o^2}{2} + \gamma_{\text{seawater}} \forall &= \gamma_{\text{obj}} \forall \end{aligned}$$

$$\begin{aligned} V_o &= \sqrt{\frac{2\left(\gamma_{\text{obj}} - \gamma_{\text{seawater}}\right)(\forall/A)}{C_D \rho_{\text{seawater}}}} \\ &= \sqrt{\frac{2g\left(S_{\text{obj}} - 1\right)(\forall/A)}{C_D}} \end{aligned}$$

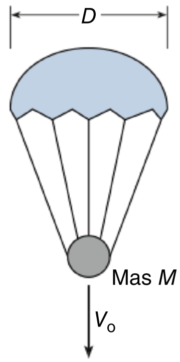
2. Variables that influence terminal velocity:

- Specific gravity of the material used to build the object: S_{obj}
- Ratio of the volume of the object to the reference area: $(\forall/A)$.
- Coefficient of drag of the object: C_D.

11.43: PROBLEM DEFINITION

Situation:

A parachute ($D = 0.35\,\text{m}$) is falling through the air.
$m = 0.02\,\text{kg},\ C_D = 2.2,\ A_p = \pi D^2/4.$



Find: Terminal velocity (in m/s).

Assumptions: Neglect mass of the chute.

Properties: Air (20 °C, 1 atm), Table A.3 (EFM 10e), $\rho = 1.2\,\text{kg/m}^3$.

PLAN

Develop an equation for terminal velocity by applying force equilibrium.

SOLUTION

1. Force equilibrium.

$$W = F_D$$

Since $W = mg$

$$mg = F_D \tag{1}$$

2. Drag force equation.

$$F_D = C_D A \frac{\rho V_0^2}{2} = C_D \left(\frac{\pi D^2}{4}\right)\left(\frac{\rho V_0^2}{2}\right) \tag{2}$$

3. Combine Eq. (1) and (2).

$$\begin{aligned} W &= F_D \\ mg &= C_D \left(\frac{\pi D^2}{4}\right)\left(\frac{\rho V_0^2}{2}\right) \\ V_0 &= \sqrt{\frac{8mg}{C_D \pi D^2 \rho}} = \sqrt{\frac{8\,(0.02\,\text{kg})\,(9.81\,\text{m/s}^2)}{(2.2)\,\pi\,(0.35\,\text{m})^2\,(1.2\,\text{kg/m}^3)}} \end{aligned}$$

$$\boxed{V_0 = 1.24\,\text{m/s}}$$

11.44: PROBLEM DEFINITION

Situation: A small air bubble is rising in a very tall column of liquid–additional details are provided in the problem statement.

Find:
(a)Acceleration of the bubble.
(b)Form of the drag (mostly skin-friction or form).

SOLUTION

Equating the drag force and the buoyancy force.

$$F_D = C_1 \gamma_{\text{liq.}} D^3 = C_2 D^3$$

Also

$$F_D = C_D A_p \rho V^2/2 = C_3 D^2 V^2$$

Eliminating F_D between these two equations yields

$$V^2 = C_4 D \text{ or } V = \sqrt{C_4 D}$$

As the bubble rises it will expand because the pressure decreases with an increase in elevation; thus, the bubble will **accelerate** as it moves upward. The drag will be **form drag** because there is no solid surface to the bubble for viscous shear stress to act on.

REVIEW

As a matter of interest, the surface tension associated with contaminated fluids creates a condition which acts like a solid surface.

11.45: PROBLEM DEFINITION

Situation:
A ball ($D = 0.08\,\text{m}$) falls through water.
Weight of the ball in air is $15\,\text{N}$.

Find: Terminal velocity (in m/s) of the ball.

Properties: Water ($10\,^\circ\text{C}$), Table A.5 (EFM 10e), $\rho = 1000\,\text{kg/m}^3$, $\gamma = 9810\,\text{N/m}^3$, $\nu = 1.31 \times 10^{-6}\ \text{m}^2/\text{s}$.

PLAN

1. Determine if the ball is falling or rising by comparing the buoyant force and the weight.
2. Relate terminal velocity to weight and buoyancy.
3. Develop equations for C_D and Re.
4. Solve the resulting set of equations using a computer program.

SOLUTION

1. Compare buoyant force and weight

$$\begin{aligned} F_{\text{buoyant}} &= \gamma\left(\frac{\pi D^3}{6}\right) = \left(9810\,\text{N/m}^3\right)\left(\frac{\pi\,(0.08\,\text{m})^3}{6}\right) \\ &= 2.63\,\text{N} \end{aligned}$$

Since the buoyant force is less than the weight, the ball will travel downward.

2. Equilibrium

$$\begin{aligned} F_D + F_{\text{buoyant}} &= W \\ F_D &= W - F_{\text{buoyant}} = 15\,\text{N} - 2.63\,\text{N} \\ &= 12.37\,\text{N} \end{aligned} \tag{1}$$

Drag force equation.

$$F_D = C_D A \frac{\rho V_o^2}{2} = C_D\left(\frac{\pi D^2}{4}\right)\left(\frac{\rho V_o^2}{2}\right) \tag{2}$$

Combine Eqs. (1) and (2)

$$12.37\,\text{N} = C_D\left(\frac{\pi D^2}{4}\right)\left(\frac{\rho V_o^2}{2}\right) \tag{3}$$

3. Clift and Gauvin correlation (drag on a sphere)

$$C_D = \frac{24}{\text{Re}_D}\left(1 + 0.15\,\text{Re}_D^{0.687}\right) + \frac{0.42}{1 + 4.25 \times 10^4\,\text{Re}^{-1.16}} \tag{4}$$

Reynolds number

$$\mathrm{Re} = \frac{V_o D}{\nu} \tag{5}$$

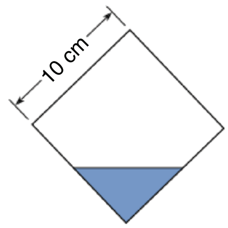
4. Solve Eqs. (3), (4) and (5) simultaneously (we applied TK Solver).

$$\begin{aligned} C_D &= 0.49 \\ \mathrm{Re} &= 1.94 \times 10^5 \end{aligned}$$

$$\boxed{V_o = 3.18\,\mathrm{m/\,s}}$$

11.46: PROBLEM DEFINITION

Situation: A weighted cube falls through water (see the problem statement for all the details).



Find: Terminal velocity in water.

Assumptions: Density of water: $\rho = 1000$ kg/m^3.

SOLUTION

From Table 11.1 (EFM 10e), $C_D = 0.81$. The drag force is

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ A_p &= (2)(L\cos 45^\circ)(L) = 1.414L^2 \end{aligned}$$

Equilibrium

$$\begin{aligned} F_D &= W - F_{\text{buoy}} \\ &= 22.2\,\text{N} - 9,810L^3 = 22.2\,\text{N} - 9,810 \times (10^{-1})^3 = 12.39\ \text{N} \\ 12.39\,\text{N} &= (0.81\,\text{N})(1.414 \times 10^{-2}\,\text{m}^2)(1,000\,\text{kg/}\,\text{m}^3)(V_0^2)/2 \end{aligned}$$

$$\boxed{V_0 = 1.47\ \text{m/s}}$$

11.47: PROBLEM DEFINITION

Situation:
A spherical rock fall in water.
$W = 30\,\mathrm{N}$. Weight in water is $5\,\mathrm{N}$.

Find: Terminal velocity (in m/s) of the rock.

Properties: Water (20 °C), Table A.5 (EFM 10e), $\rho = 998\,\mathrm{kg/\,m^3}$, $\gamma = 9790\,\mathrm{N/\,m^3}$, $\nu = 1.00 \times 10^{-6}\,\mathrm{m^2/\,s}$.

PLAN

1. Find diameter by using known buoyant force.
2. Relate terminal velocity to C_D using the drag force equation.
3. Solve the resulting equation using an iterative approach.

SOLUTION

1. Buoyant force equation

$$\begin{aligned} W - F_B &= \text{Weight in water} \\ 30\,\mathrm{N} - F_B &= 5\,\mathrm{N} \\ F_B &= 25\,\mathrm{N} \end{aligned}$$

Now solve for diameter

$$\begin{aligned} F_B &= \gamma_{H20}\frac{\pi D^3}{6} \\ D &= \left(\frac{6F_B}{\pi\gamma_{H20}}\right)^{1/3} = \left(\frac{6\,(25\,\mathrm{N})}{\pi\,(9790\,\mathrm{N/\,m^3})}\right)^{1/3} \\ D &= 0.1696\,\mathrm{m} \end{aligned}$$

2. Drag force equation

$$F_D = C_D A\frac{\rho V_o^2}{2} = C_D\left(\frac{\pi D^2}{4}\right)\left(\frac{\rho V_o^2}{2}\right) \tag{1}$$

Equilibrium

$$\begin{aligned} W &= F_D + F_B \\ 30\,\mathrm{N} &= F_D + 25\,\mathrm{N} \\ F_D &= 5\,\mathrm{N} \qquad (2) \end{aligned}$$

Combine Eqs. (1) and (2)

$$\begin{aligned} 5\,\mathrm{N} &= C_D\left(\frac{\pi D^2}{4}\right)\left(\frac{\rho V_o^2}{2}\right) \\ 5\,\mathrm{N} &= C_D\left(\frac{\pi\,(0.1696\,\mathrm{m})^2}{4}\right)\left(\frac{(998\,\mathrm{kg/\,m^3})\,V_o^2}{2}\right) \\ 0.4436 &= C_D V_o^2 \qquad (3) \end{aligned}$$

3. Solve Eq. (3). Begin by rewriting the equation

$$V_o = \frac{0.666}{\sqrt{C_D}}$$

Now, apply the iterative solution approach that is described in Example 11.4. The result is

$$\begin{aligned} C_D &= 0.49 \\ \text{Re} &= 161000 \\ V_o &= 0.953\,\text{m/s} \end{aligned}$$

$$\boxed{V = 0.95 \text{ m/s}}$$

11.48: PROBLEM DEFINITION

Situation:

A helium-filled balloon is ascending in air.
Standard atmosphere.
$W_{\text{Balloon}}(\text{empty}) = W_B = 3\,\text{N}$.

Find: Terminal velocity (in m/s) of balloon.

Properties: Air (15 °C), Table A.2 (EFM 10e), $\rho = 1.22\,\text{kg/m}^3$, $\nu = 1.46\times10^{-5}\,\text{m}^2/\text{s}$.

PLAN

The goal (velocity) appears in the drag force equation. Thus, find the drag force using equilibrium and then solve for the velocity. This requires an iterative or a numerical approach. The solution below is an iterative approach. The steps are
1. Relate the drag force to other forces using equilibrium.
2. Develop an algebraic equation for terminal velocity (V_o) using the drag force equation.
3. Guess a value of C_D and calculate V_o.
4. Using the value of V_o, calculate Re.
5. Calculate C_D and then recalculate V_o. Repeat until convergence.

SOLUTION

1. Force equilibrium (vertical direction).

$$\begin{aligned} F_D &= -W_{\text{balloon}} - W_{\text{He}} + F_{\text{buoy}} \\ F_D &= -3 + (1/6)\pi D^3(\gamma_{\text{air}} - \gamma_{\text{He}}) \\ &= -3 + (1/6)\pi \times 2^3 \times \gamma_{\text{air}}(1 - 287/2077) \\ &= -3 + (1/6)\pi \times 8 \times 1.225 \times 9.81(1 - 0.138) \\ &= 40.391\ \text{N} \end{aligned}$$

2. Drag force equation (to find equation for terminal velocity).

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ V_0 &= \sqrt{40.391 \times 2/((\pi/4) \times 2^2 \times 1.22 C_D)} \\ &= \sqrt{21.077/C_D} \end{aligned}$$

3. Assume $C_D = 0.4$ then

$$V_0 = \sqrt{21.077/0.4} = 7.2590\ \text{m/s}$$

4. Reynolds number

$$\text{Re} = VD/\nu = 7.2590 \times 2/(1.46 \times 10^{-5}) = 9.944 \times 10^5$$

5. From the Clift and Gauvin correlation (Eq. (11.9) in EFM10e) $C_D = 0.47$ so

$$V_0 = \sqrt{21.077/0.47} = 6.6966\,\text{m/s}$$

Reynolds number

$$\mathrm{Re} = VD/\nu = 6.6966 \times 2/(1.46 \times 10^{-5}) = 9.1734 \times 10^{5}$$

The Clift and Gauvin correlation (Eq. (11.9) in EFM10e) gives a value of $C_D = 0.47$ so no further iterations are necessary

$$\boxed{V_0 = 6.70\,\mathrm{m/s}}$$

11.49: PROBLEM DEFINITION

Situation: A sphere 2 cm in diameter rises in oil at a velocity of 1.5 cm/s.

Find: Specific weight of the sphere material.

SOLUTION

Equilibrium

$$\begin{aligned} \sum F &= 0 = -F_D - W + F_{\text{buoyancy}} \\ F_D &= F_{\text{buoyancy}} - W \end{aligned} \quad (1)$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{0.015 \times 0.02 \times 900}{0.096} \\ &= 2.812 \end{aligned}$$

Then from Fig. 11.8

$$C_D \approx 10.0$$

Substitute drag force, weight and buoyancy force into Eq. (1)

$$C_D A_p \rho V_0^2/2 = V(\gamma_{\text{oil}} - \gamma_{\text{sphere}}) \quad (2)$$

Sphere volume is

$$\begin{aligned} V &= (4/3)\pi r^3 \\ &= 4.19 \times 10^{-6} \text{ m}^3 \end{aligned}$$

Eq. (2) becomes

$$\begin{aligned} 10 \times \pi \times 0.01^2 \times 900 \times 0.015^2/2 &= 4.19 \times 10^{-6}(900 \times 9.81 - \gamma_{\text{sphere}}) \\ \gamma_{\text{sphere}} &= 8753 \, \text{N/m}^3 \end{aligned}$$

$$\boxed{\gamma_{\text{sphere}} = 8753 \, \text{N/m}^3}$$

11.50: PROBLEM DEFINITION

Situation: The problem statement describes a 1.5-mm sphere moving in oil.

Find: Terminal velocity of the sphere.

PLAN Apply the equilibrium principle. To find the drag force, assume Stokes drag.

SOLUTION

Equilibrium. Since the ball moves at a steady speed, the sum of forces is zero.

$$W = F_B + F_D \tag{1}$$

where W is weight, F_B is the buoyant force and F_D is drag.

Because the viscosity is large, it is expected that the sphere will fall slowly, so assume that Stoke's law applies. Thus, the drag force is

$$\begin{aligned} F_D &= 3\pi\mu V_0 D \\ &= 3\pi\nu\rho V_0 D \end{aligned}$$

Buoyant force

$$F_B = \gamma_{\text{oil}} \left(\frac{\pi D^3}{6} \right)$$

Eq. (1) becomes

$$\begin{aligned} W &= F_B + F_D \\ \gamma_{\text{sphere}} \left(\frac{\pi D^3}{6} \right) &= \gamma_{\text{oil}} \left(\frac{\pi D^3}{6} \right) + 3\pi\nu\rho V_0 D \\ \left(\frac{\pi D^3 \gamma_{\text{water}}}{6} \right) (S_{\text{sphere}} - S_{\text{oil}}) &= 3\pi\nu\rho V_0 D \\ \left(\frac{\pi\,(0.0015\,\text{m})^3 \times 9810\,\text{N/m}^3}{6} \right) (1.07 - 0.95) &= 3\pi\nu\rho V_0 D \\ 2.080 \times 10^{-6}\,\text{N} &= 3\pi \left(10^{-4}\,\text{m}^2/\text{s}\right) \left(950\,\text{kg/m}^3\right) V_o\,(0.0015\,\text{m}) \\ 2.080 \times 10^{-6}\,\text{N} &= \left(1.343 \times 10^{-3}\,\text{kg/s}\right) V_o \end{aligned}$$

The solution is

$$\boxed{V_o = 1.55\,\text{mm/s}}$$

Check Reynolds number

$$\begin{aligned} \text{Re} &= \frac{V_0 D}{\nu} \\ &= \frac{(0.00155\,\text{m/s}) \times (0.0015\,\text{m})}{10^{-4}\,\text{m}^2/\text{s}} \\ &= 0.023\,25 \end{aligned}$$

REVIEW

The value of Re is within Stokes' range ($\text{Re} \leq 0.5$), so the use of Stokes' law is valid.

11.51: PROBLEM DEFINITION

Situation:

- A 534 N skydiver is free-falling at an altitude of 1981 m.
- Maximum drag conditions: $C_DA = 0.743\,\mathrm{m}^2$.
- Minimum drag conditions, $C_DA = 0.0929\,\mathrm{m}^2$.
- Pressure and temperature at sea level are 101 kPa and 15 °C.
- Lapse rate for the U.S. Standard atmosphere is $\alpha = 0.00587\,\mathrm{K/m}$.

Find:
Estimate the terminal velocity in mph.
a.) Case A (maximum drag) $C_DA = 0.743\,\mathrm{m}^2$.
b.) Case B (minimum drag) $C_DA = 0.0929\,\mathrm{m}^2$.

PLAN

At terminal velocity, the force of drag will balance weight. The only unknown is fluid density–this can be found by using the ideal gas law along with the equations from chapter 3 that describe the US Standard atmosphere. Use SI units throughout.

SOLUTION

Atmospheric pressure variation (troposphere)

$$\begin{aligned} T &= T_o - \alpha(z - z_o) \\ &= (288.1\,\mathrm{K}) - (0.00587\,\mathrm{K/m}) \times (1981 - 0)\ \mathrm{m} \\ &= 276.5\,\mathrm{K} \\ \frac{p}{p_o} &= \left[\frac{T_o - \alpha(z - z_o)}{T_o}\right]^{\frac{g}{\alpha R}} \\ \frac{p}{101\,\mathrm{kPa}} &= \left[\frac{276.5\,\mathrm{K}}{288.1\,\mathrm{K}}\right]^{\frac{9.81}{(0.00587)(287)}} \end{aligned}$$

so

$$p = 79.45\,\mathrm{kPa}$$

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{79,450}{287 \times 276.5} \\ &= 1.001\,\mathrm{kg/\,m^3}\end{aligned}$$

Equilibrium

$$\text{Weight} = \text{Drag}$$

Case A

$$\begin{aligned}W &= C_D A \frac{\rho V_0^2}{2} \\ 534\,\mathrm{N} &= \left(0.743\,\mathrm{m^2}\right) \frac{(1.001\,\mathrm{kg/\,m^3})\,V_0^2}{2}\end{aligned}$$

Calculations give

$$V_O = 37.9\,\mathrm{m/\,s}$$

$$\boxed{V_O = 136.4 \text{ km/h for maximum drag conditions}}$$

Case B.
Since $C_D A$ decreases by a factor of 8, the speed will increase by a factor of $\sqrt{8}$.

$$V_O = (136.4 \text{ km/h})\,\sqrt{8}$$

$$\boxed{V_O = 386 \text{ km/h for minimum drag conditions}}$$

11.52: PROBLEM DEFINITION

Situation: A falling hail stone is described in the problem statement.

Find: Terminal velocity of hail stone.

PLAN

Apply the ideal gas law, then calculate the drag force and apply the equilibrium principle.

SOLUTION

Ideal gas law

$$\rho = p/RT = 96,000/(287 \times 273) = 1.23 \text{ kg/m}^3$$

Equilibrium

$$\begin{aligned} \sum F &= 0 = F_D - W \\ F_D &= W \end{aligned}$$

Substitute for drag force and weight

$$C_D A_p \rho V^2/2 = Vol \times 6,000$$

Assume $C_D = 0.5$

$$\begin{aligned} 0.5 \times (\pi d^2/4) \times 1.23V^2/2 &= (1/6)\pi d^3 \times 6,000 \\ V &= \sqrt{d \times 1,000 \times 16/1.23} \\ V &= \sqrt{5 \times 16/1.23} = 8.06 \text{ m/s} \end{aligned}$$

Check Reynolds number

$$\begin{aligned} \text{Re} &= 8.06 \times 0.005/(1.3 \times 10^{-5}) \\ &= 3100 \end{aligned}$$

From Fig. 11.9 (EFM 10e) $C_D = 0.39$ so

$$\begin{aligned} V &= 8.06 \times (0.5/0.39)^{1/2} \\ &\boxed{V = 9.13 \text{ m/s}} \end{aligned}$$

REVIEW

The drag coefficient will not change with further iterations.

11.53: PROBLEM DEFINITION

Situation: A drag chute is used to decelerate an airplane—additional details are provided in the problem statement.

Find: Initial deceleration of aircraft.

Assumptions: Density, $\rho = 1.2 \text{ kg/m}^3$.

SOLUTION

Drag force

$$F_D = C_D A_p \rho V_0^2/2 = Ma$$

then

$$a = C_D A_p \rho V_0^2/(2M)$$

where $M = 9000$ kg. From Table 11.1 (EFM 10e) $C_D = 1.20$.

$$A_p = (\pi/4)D^2 = 10.2 \text{ m}^2$$

Then

$$\begin{aligned} a &= 1.20 \times 10.2 \times 1.2 \times 60^2/(2 \times 9000) \\ & \boxed{2.9 \text{ m/s}^2} \end{aligned}$$

11.54: PROBLEM DEFINITION

Situation: A paratrooper falls using a parachute–additional details are provided in the problem statement.

Find: Descent rate of paratrooper.

Assumptions: Density, $\rho = 1.2 \text{ kg/m}^3$

PLAN

In equilibrium, drag force balances weight of the paratrooper.

SOLUTION

Equilibrium

$$W = F_D$$

Drag Force

$$F_D = C_D A_p \frac{\rho V_0^2}{2}$$

From Table 11.1 (EFM 10e) $C_D = 1.20$. Thus

$$\begin{aligned} W &= F_D = C_D A_p \rho V_0^2/2 \\ V_0 &= \sqrt{2W/(C_D A_p \rho)} \\ &= \sqrt{2 \times 900/(1.2 \times (\pi/4) \times 49 \times 1.2)} \\ &\boxed{V_0 = 5.70 \text{ m/s}} \end{aligned}$$

REVIEW

1. Reynolds number based on terminal velocity is approximately:

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{(5.70 \text{ m/s})(7 \text{ m})}{14.6 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= 2.73 \times 10^6 \end{aligned}$$

2. While a Reynolds number of 2.7×10^6 is an order of magnitude less than the value given in Table 11.1 (EFM 10e), the drag coefficient is not strongly effected by Reynolds number for Reynolds numbers exceeding 10^4. Note the drag on a hemispherical shell in Table 11.1 (EFM 10e).

11.55: PROBLEM DEFINITION

Situation: A helium-filled balloon moves through air
$W = 0.10$ N
$D = 0.6$ m

Find: Terminal velocity of the balloon.

Properties: at $T = 15°C$: $\rho_{\text{air}} \approx 1.22$ kg/m^3; $\rho_{\text{He}} = 0.169$ kg/m^3

PLAN

Apply equilibrium with the weight, drag force and buoyancy force.

SOLUTION

Equilibrium (based upon a FBD that assumes the ballon is falling downward)

$$\begin{aligned} F_{\text{net}} &= F_D - W_{\text{balloon}} - W_{\text{helium}} + F_{\text{buoy}} = 0 \\ F_D &= +0.10 - (1/6)\pi D^3(\gamma_{\text{air}} - \gamma_{\text{He}}) \\ &= +0.10 - (1/6)\pi \times (0.60)^3 9.81(\rho_{\text{air}} - \rho_{\text{He}}) \end{aligned}$$

Solve for drag force

$$F_D = +0.10 - (1/6)\pi(0.60)^3 \times 9.81(1.22 - 0.169) = -1.067 \text{ N}$$

Ahah! Having calculated a negative F_D, we realize that the balloon must have been moving upward! We are in fact calculating the terminal velocity for an upward-rising balloon.
Assume $C_D \approx 0.40$ Then, for velocity from drag force equation

$$V_0 = (2F_D/(C_D A\rho))^{1/2}$$

$$\begin{aligned} V_0 &= ((2 \times 1.067/(0.40 \times (\pi/4) \times 0.6^2 \times 1.22))^{1/2} \\ &= 3.93 \text{ m/s} \end{aligned}$$

Check Re and C_D:

$$\text{Re} = VD/\nu = 3.93 \times 0.6/(1.46 \times 10^{-5}) = 1.62 \times 10^5$$

From Fig. 11.9 (EFM 10e), $C_D \approx 0.42$ so one additional iteration is necessary.

$$\boxed{V_0 = 3.83 \text{ m/s upward}}$$

11.56: PROBLEM DEFINITION

Situation:

A 2-cm plastic ball with specific gravity of 1.2 is released from rest in water (T=20 °C)

Additional details are provided in the problem statement.

Find: Time and distance to achieve 99% of terminal velocity.

SOLUTION

The equation of motion for the plastic sphere is

$$m\frac{dv}{dt} = -F_D + W - F_B$$

The drag force can is expressed as

$$F_D = \frac{1}{2}\rho v^2 C_D \frac{\pi}{4} d^2 = \frac{C_D \text{Re}}{24} 3\pi\mu d v$$

The equation of motion becomes

$$m\frac{dv}{dt} = -\frac{C_D \text{Re}}{24} 3\pi\mu d v + \rho_b \forall g - \rho_w g \forall$$

Dividing through by the mass of the ball gives

$$\frac{dv}{dt} = -\frac{C_D \text{Re}}{24}\frac{18\mu}{\rho_b d^2} v + g(1 - \frac{\rho_w}{\rho_b})$$

Substituting in the values

$$\frac{dv}{dt} = -0.0375\frac{C_D \text{Re}}{24} v + 1.635$$

The Clift and Gauvin correlation for drag on a sphere can be rewritten as:

$$\frac{C_D \text{Re}}{24} = 1 + 0.15\text{Re}^{0.687} + \frac{0.0175\text{Re}}{1 + 4.25 \times 10^4 \text{Re}^{-1.16}}$$

This equation can be integrated using the Euler method

$$\begin{aligned} v_{n+1} &= v_n + \left(\frac{dv}{dt}\right)_n \Delta t \\ s_{n+1} &= s_n + 0.5(v_n + v_{n+1})\Delta t \end{aligned}$$

The terminal velocity is 0.362 m/s. The time to reach 99% of the terminal velocity is $\boxed{0.54 \text{ seconds}}$ and travels $\boxed{14.2 \text{ cm}}$.

Problem 11.57

No solution provided.

Problem 11.58

Apply the grid method to each situation described below. Note: Unit cancellations are not shown in this solution.

a.)

Situation:

Lift force acts on a rotating baseball.

Lift force equation:

$$F_L = C_L A \frac{\rho V_o^2}{2}$$

$C_L = 1.2,\ V_o = 145 \text{ km/h} = 40.3 \text{ m/s}$
$A = \pi r^2, \quad r = 37\,\text{mm} = 0.037\,\text{m}.$

Find: Lift force (newtons).

Assumptions: Air temperature on a typical summer day is 30 °C.

Properties: Air (30 °C, 1 atm), Table A.5 (EFM 10e), $\rho = 1.17\,\text{kg/m}^3$.

Solution:

a.)

$$\begin{aligned} F_L &= C_L A \frac{\rho V_o^2}{2} = C_L \left(\pi r^2\right) \left(\frac{\rho V_o^2}{2}\right) \\ &= (1.2)\,\pi (0.037\,\text{m}^2) \frac{(40.3\,\text{m/s}^2)}{2} \end{aligned}$$

$$\boxed{F_L = 4.9 \text{ N}}$$

b.) Situation:

A model ($m = 570\,\text{g}$) aircraft is flying in straight & level flight.

Lift force equation:

$$F_L = C_L A \frac{\rho V_o^2}{2}$$

$C_L = 1.2,\ V_o = 130 \text{ km/h} = 36.1 \text{ m/s}.$

Find: Wing size (area from a top view in mm^2).

Assumptions: Straight and level flight so weight balances lift force.

Properties: Air (30 °C, 1 atm), Table A.5 (EFM 10e), $\rho = 1.17\,\text{kg/m}^3$.

Solution:

Calculate weight

$$\begin{aligned} W &= mg \\ &= \left(\frac{570\,\text{g}}{1.0}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{1.0\,\text{kg}}{1000\,\text{g}}\right)\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right) = 5.59\,\text{N} \end{aligned}$$

Equilibrium

$$\text{Weight} = \text{Lift force}$$

$$W = F_L = 5.59\,\text{N}$$

Lift force equation

$$\begin{aligned}
F_L &= C_L A \frac{\rho V_o^2}{2} \\
A &= \frac{2F_L}{C_L \rho V_o^2} \\
&= \left(\frac{2.0}{1.2}\right)\left(\frac{5.59\,\text{N}}{1.0}\right)\left(\frac{\text{m}^3}{1.17\,\text{kg}}\right)(36.1\,\text{m/s})^2\left(\frac{1000\,\text{mm}}{1.0\,\text{m}}\right)^2
\end{aligned}$$

$$\boxed{A = 6.1 \times 10^3\,\text{mm}^2}$$

Problem 11.59

Answer the questions below.

---

a. What is circulation? Why is it important?

- Circulation is a measure of rotation of the fluid particles that lies on an area bounded by a closed curve
- Circulation equals the average rate of rotation of fluid particles situated on the area times the area

  $\Gamma =$ (average rate of rotation of fluid particles) (area within the bounding curve)

- When lift is nonzero for a body, then circulation is nonzero for a curve that surrounds the body. That is, the theory of lift relates circulation to lift.

---

b. What is lift force?

- When fluid flows past a body, the fluid exerts normal and tangential stresses on the body. When these stresses are integrated over area, the result is a force vector. The component of force that is perpendicular to the flowing fluid is defined as the lift force.

---

c.) What variables influence the magnitude of the lift force?

To identify the variables, apply *logical reasoning* to the lift force equation:

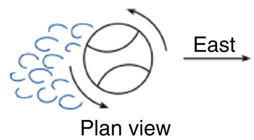
$$F_L = C_L A \frac{\rho V_o^2}{2}$$

That is, the equation reveals that four variables influence lift.

- Lift depends on the coefficient of lift (C_L) which is influenced by the shape of the body, the angle of attack, and Re. Also, C_L is sometimes influence by angular speed as in the rotation of golf ball.
- Lift depends on the reference area (A) of the body. Reference areas characterizes size.
- Lift depends on the density of the ambient fluid.
- Lift depends on the free-stream speed (V_o) squared. This speed is measured relative to an observer on the body.

Problem 11.60

Situation:
A spinning baseball is thrown from west to east.



Find: Direction the baseball will "break."

SOLUTION

It will "break" toward the north. The correct answer is a).

11.61: PROBLEM DEFINITION

Situation: A rotating baseball is described in the problem statement.

Find:
(a) Lift force on the baseball.
(b) Deflection of the ball from its original path.

Properties: From Table A.3 (EFM 10e), $\rho = 1.23$ kg/m^3.

Assumptions: Axis of rotation is vertical, standard atmospheric conditions ($T = 21\,^\circ$C).

SOLUTION

Rotational parameter

$$\begin{aligned} V_0 &= 137 \text{ km/h} = 38.1 \text{ m/s} \\ r\omega/V_0 &= 0.225/2\pi \times 35 \times 2\pi/38.1 = 0.21 \end{aligned}$$

Lift force. From Fig. 11.18 (EFM 10e)

$$C_L = 3 \times 0.05 = 0.15$$

Then

$$\begin{aligned} F_L &= C_L A \rho V_0^2/2 \\ &= 0.15 \times (0.225^2\pi) \times (\pi/4) \times 1.23 \times 38.4^2/2 \\ &\boxed{F_L = 0.44 \text{ N}} \end{aligned}$$

Deflection will be $\delta = 1/2\ at^2$ where a is the acceleration

$$\begin{aligned} a &= F_L/M \\ t &= L/V_0 = 18/38.4 = 0.48 \text{ s} \\ a &= F_L/M = 0.44/(150 \times 0.001) = 2.9 \text{ m/s}^2 \end{aligned}$$

Then

$$\delta = (1/2) \times 2.9 \times 0.48^2; \ \boxed{\delta = 0.33 \text{ m}}$$

11.62: PROBLEM DEFINITION

Situation: A glider will move along glide path with an angle of

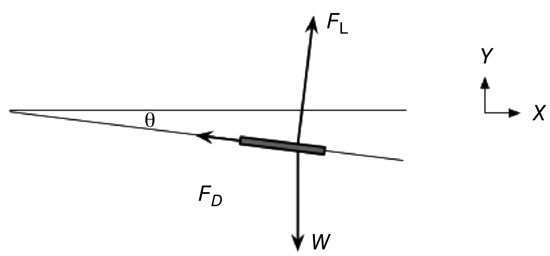
$$\theta = \frac{C_D}{C_L}$$

Find: Prove the above statement.

Assumptions:
Steady speed.
Straight path.
Glide angle is shallow so $\tan\theta \approx \theta$.
The lift and drag equations use the same reference area.

PLAN

The motion of the glider is determined by three forces: weight, the lift force, and the drag force



1. Since the glider is traveling with constant velocity, apply force equilibrium.
2. Apply the lift force and drag force equations.
3. Combine results.

SOLUTION

1. Equilibrium (horizontal direction; this eliminates weight).

$$\begin{aligned} F_L \sin\theta &= F_D \cos\theta \\ \tan\theta &= \frac{F_D}{F_L} \qquad (1) \end{aligned}$$

2. Lift and drag force equations.

$$F_L = C_L A \frac{\rho V_o^2}{2} \qquad (2)$$

$$F_D = C_D A \frac{\rho V_o^2}{2} \qquad (3)$$

3. Combine Eqs. (1) to (3).

$$\tan\theta = \frac{C_D}{C_L}$$

Assume $\tan\theta \approx \theta$

$$\boxed{\theta = \frac{C_D}{C_L}}$$

REVIEW

The glide angle can be as small as 1:60. This means that the glider will travel 60 m of horizontal distance for every meter of vertical distance. The ratio is small (because lift is in the denominator), but the outcome is a large distance.

11.63: PROBLEM DEFINITION

Situation:
A rotating sphere ($D = 0.1\,\mathrm{m}$) is situated in a flow of water
$\omega = 286$ rpm $= 30\,\mathrm{s}^{-1}$, $V_o = 1.5\,\mathrm{m/s}$.

Find: Lift force (in newtons) on the sphere.

Properties: Water (15 °C), Table A.5 (EFM 10e), $\rho = 999\,\mathrm{kg/m}^3$.

PLAN

1. Find C_L using Fig. 11.18 (EFM 10e).
2. Calculate F_L using the lift force equation.

SOLUTION

1. Coefficient of lift

$$\begin{aligned}\frac{r\omega}{V_0} &= \frac{(0.05\,\mathrm{m})\,(30\,\mathrm{rad/s})}{1.5\,\mathrm{m/s}} \\ &= 1\end{aligned}$$

From Fig. 11.18 (EFM 10e) $C_L = 0.32$

2. Lift force equation

$$\begin{aligned}F_L &= C_L A\frac{\rho V_o^2}{2} = C_L\left(\frac{\pi D^2}{4}\right)\frac{\rho V_o^2}{2} \\ F_L &= (0.32)\left(\frac{\pi\,(0.1\,\mathrm{m})^2}{4}\right)\frac{(999\,\mathrm{kg/m}^3)\,(1.5\,\mathrm{m/s})^2}{2} \\ &\boxed{F_L = 2.82\ \mathrm{N}}\end{aligned}$$

11.64: PROBLEM DEFINITION

Situation:

An airplane wing has a chord of 1.1 m. Air speed is $V_o = 60\,\mathrm{m/s}$.
The lift is 8000 N. The angle of attack is 3^o.
The coefficient of lift is specified by the data on Fig. 11.24 in EFM10e.

Find: The span of the wing.

Properties: Density of air is 1.24 kg/m^3.

PLAN

Guess an aspect ratio, look up a coefficient of lift and then calculate the span. Then, iterate to find the span.

SOLUTION

Lift force. From Fig. 11.24 (EFM10e) assume $C_L \approx 0.60$.Then

$$\begin{aligned} F_L &= C_L A \frac{\rho V_0^2}{2} \\ 8000 &= (0.60)(1.1)\frac{(1.24)(60^2)}{2} \\ b &= 5.4 \text{ m} \\ b/c &= 5.4/1.2 = 4.5 \end{aligned}$$

From Fig. 11.24 (EFM10e), $C_L = 0.50$. Recalculate the span

$$\begin{aligned} b &= (5.4\,\mathrm{m})\left(\frac{0.60}{0.50}\right) \\ &= 6.48\,\mathrm{m} \end{aligned}$$

$$\boxed{b = 6.48\,\mathrm{m}}$$

11.65: PROBLEM DEFINITION

Situation: A lifting vane for a boat of the hydrofoil type is described in the problem statement.

Find: Dimensions of the foil needed to support the boat.

SOLUTION

Use Fig. 11.24 (in EFM10e) for characteristics; $b/c = 4$ so $C_L = 0.55$

$$\begin{aligned} F_L &= C_L A \rho V_0^2/2 \\ 44,500 &= 0.55 \times 4c^2 \times (1000/2) \times 324 \\ c^2 &= 0.125 \text{ m} \\ c &= 0.35 \text{ m} \\ b &= 4c = 1.4 \text{ m} \end{aligned}$$

Use a foil $\boxed{0.35 \text{ m wide} \times 1.4 \text{ m long}}$

11.66: PROBLEM DEFINITION

Situation: Two wings, A and B, are described in the problem statement.

Find: Total lift of wing B compared to wing A.

SOLUTION

C_L increases with increase in aspect ratio. The correct choice is $\boxed{\text{(d)}}$.

11.67: PROBLEM DEFINITION

Situation: An aircraft increases speed in level flight.

Find: What happens to the induced drag coefficient.

SOLUTION

$$C_{Di} = \frac{C_L^2}{\pi(b^2/S)}$$

In the equation for the induced drag coefficient (above) the only variable for a given airplane is C_L; therefore, one must determine if C_L varies for the given conditions. If the airplane is in level flight the lift force must be constant. Because $F_L = C_L A \rho V^2/2$ it is obvious that C_L must decrease with increasing V. This would be accomplished by decreasing the angle of attack. If C_L decreases, then Eq. (11.24) in EFM10e shows that C_{Di} also must decrease. The correct answer is $\boxed{\text{(b)}}$.

11.68: PROBLEM DEFINITION

Situation: An airplane wing is described in the problem statement.

Find:
(a) An expression for V for which the power is a minimum.
(b) V for minimum power

SOLUTION

$$W/S = \frac{1}{2}\rho C_L V^2$$

or

$$\begin{aligned}
C_L &= (2/\rho)(1/V^2)(W/S) \\
P &= F_D V \\
&= (C_{Do} + C_L^2/\pi\Lambda)(1/2)\rho V^3 S \\
P &= \frac{1}{2}\rho V^3 S C_{Do} + (4/\rho^2)(1/V^4)W^2/S^2)(1/(\pi\Lambda))(\frac{1}{2}\rho V^3 S) \\
P &= \left[\frac{1}{2}V^3 C_{Do} + (2/\rho)(1/(\pi\Lambda V)(W^2/S^2)\right] S \\
dP/dV &= ((3/2)\rho V^2 C_{Do} - (2/\rho)(1/(\pi\Lambda V^2))(W/S)^2)S
\end{aligned}$$

For minimum power $dP/dV = 0$ so

$$\begin{aligned}
(3/2)\rho V^2 C_{Do} &= (2/\rho)(1/(\pi\Lambda V^2)(W/S)^2 \\
& \boxed{V=\left[\tfrac{4}{3}(W/S)^2(1/(\pi\Lambda\rho^2 C_{D_0}))\right]^{1/4}}
\end{aligned}$$

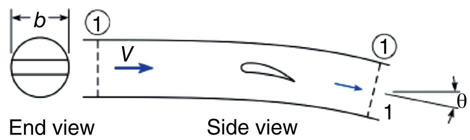
For $\rho = 1$ kg/m$^3, \Lambda = 10$, $W/S = 600$ and $C_{Do} = 0.2$

$$\begin{aligned}
V &= \left[\frac{4}{3}(600^2)(1/(\pi \times 10 \times 1^2 \times 0.02))\right]^{1/4} \\
& \boxed{V = 29.6 \text{ m/s}}
\end{aligned}$$

11.69: PROBLEM DEFINITION

Situation:

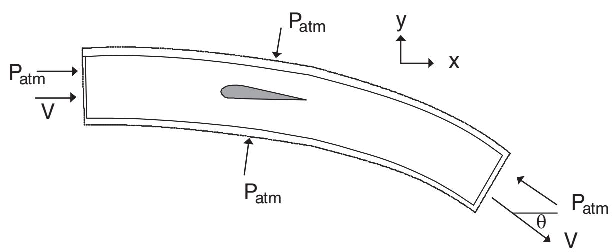
An airstream affected by the wing of an airplane is described in the problem statement.



Find: Show that $C_{Di} = C_L^2/(\pi\Lambda)$.

SOLUTION

Take the stream tube between sections 1 and 2 as a control volume and apply the momentum principle



For steady flow the momentum equation is

$$\sum F_y = \dot{m}_2 V_{2y} - \dot{m}_1 V_{1y}$$

Also $V_1 = V_2 = V$. The only F_y, is the force of the wing on the fluid in the control volume:

$$\begin{aligned} F_y &= (-V\sin\theta)\dot{m} = (-V\sin\theta)\rho V A \\ &= -\rho V^2 A \sin\theta \end{aligned}$$

But the fluid acting on the wing in the y direction is the lift F_L and it is the negative of F_y. So

$$\begin{aligned} F_L &= \rho V^2 A \sin\theta \\ C_L &= 2F_L/(\rho V^2 S) \end{aligned}$$

Eliminate F_L between the two equations yields

$$\begin{aligned} C_L &= 2\rho V^2 A \sin\theta/(\rho V^2 S) \\ C_L &= 2A\sin\theta/S \\ &= 2(\pi/4)b^2 \sin\theta/S \\ C_L &= (\pi/2)\sin\theta(b^2/S) \end{aligned}$$

But $\sin\theta \approx \theta$ for small angles. Therefore

$$C_L = (\pi/2)\theta(b^2/S)$$

or

$$\begin{aligned} \theta &= 2C_L/(\pi b^2/S) \\ C_{Di}\rho V^2 S/2 &= (C_L \rho V^2 S/2)(\theta/2) \end{aligned}$$

Eliminating θ between the two equations gives

$$C_{Di}\rho V^2 S/2 = (C_L \rho V^2 S/2)(C_L/(\pi b^2/S))$$
$$\boxed{C_{Di} = C_L^2/(\pi\Lambda)}$$

11.70: PROBLEM DEFINITION

Situation: The problem statement provides data describing aircraft takeoff and landing.

Find:
(a) Landing speed.
(b) Stall speed.

SOLUTION

$C_{L\max} = 1.40$ which is the C_L at stall. Thus, for stall

$$\begin{aligned} W &= C_{L\max} S\rho V_s^2/2 \\ &= 1.4 S\rho V_s^2/2 \end{aligned}$$

For landing

$$W = 1.2 S\rho V_L^2/2$$

But

$$V_L = V_s + 8$$

so

$$W = 1.2A\rho(V_s + 8)^2/2$$

Therefore

$$1.2(V_s + 8)^2 = 1.4V_s^2$$

$$\boxed{V_s = 99.8 \text{ m/s}}$$

$$V_L = V_s + 8$$

$$\boxed{V_L = 107.8 \text{ m/s}}$$

11.71: PROBLEM DEFINITION

Situation:

An aircraft wing is described in the problem statement.
$m = 1000$ kg
Area= $S = 16$ m^2
wingspan = $b = 10$ m
$V = 50$ m/s
Altitude = 3000 m

Find: Total drag on wing and power to overcome drag.

SOLUTION

Calculate p and then ρ:

$$\begin{aligned} p &= p_0[(T_0 - \alpha(z - z_0))/T_0]^{g/\alpha R} \\ p &= 101.3[(296 - (5.87 \times 10^{-3})(3,000))/296]^{(9.81/(5.87\times10^{-3}\times287))} = 70.1 \text{ kPa} \\ T &= 296 - 5.87 \times 10^{-3} \times 3,000 = 278.4 \text{ K} \end{aligned}$$

Then

$$\begin{aligned} \rho &= p/RT \\ &= 70,100/(287 \times 278.4) \\ &= 0.877 \text{ kg/m}^3 \end{aligned}$$

$$\begin{aligned} C_L &= (F_L/S)/(\rho V_0^2/2) \\ &= (1,000 \times 9.81/16)/(0.877 \times 50^2/2) \\ &= 0.553 \end{aligned}$$

Then

$$\begin{aligned} C_{D_i} &= C_L^2/(\pi(b^2/S)) \\ &= 0.553^2/(\pi/(10^2/16)) \\ &= 0.0156 \end{aligned}$$

Then the total drag coefficient

$$\begin{aligned} C_D &= C_{D_i} + 0.01 \\ &= 0.0256 \end{aligned}$$

Total wing drag

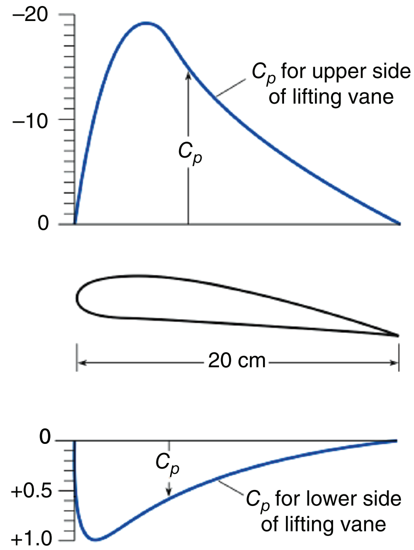
$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ F_D &= 0.0256 \times 16 \times 0.877 \times 50^2/2 \\ & \boxed{F_D = 454 \text{ N}} \end{aligned}$$

Power

$$\begin{aligned} P &= 50 \times 454 \\ & \boxed{P = 22.7 \text{ kW}} \end{aligned}$$

11.72: PROBLEM DEFINITION

Situation: The problem statement provides data for a Gottingen 387-FB lifting vane.



Find:
(a) Speed at which cavitation begins.
(b) Lift per unit length on foil.

SOLUTION

Cavitation will start at point where C_p is minimum, or in this case, where

$$\begin{aligned} C_p &= -1.95 \\ C_p &= (p - p_0)/(\rho V_0^2/2) \end{aligned}$$

Also

$$p_0 = 0.70 \times 9,810 \text{ Pa gage}$$

and for cavitation

$$\begin{aligned} p &= p_{\text{vapor}} = 1,230 \text{ Pa abs} \\ p_0 &= 0.7 \times 9,810 + 101,300 \text{ Pa abs} \end{aligned}$$

So

$$\begin{aligned} -1.95 &= [1,230 - (0.7 \times 9,810 + 101,300)]/(1,000V_0^2/2) \\ & \boxed{V_0 = 10.5 \text{ m/s}} \end{aligned}$$

By approximating the C_p diagrams by triangles, it is found that $C_{p_{\text{avg.}}}$ on the top of the lifting vane is approx. -1.0 and $C_{p_{\text{avg.,bottom}}} \approx +0.45$

Thus, $\Delta C_{p_{\text{avg.}}} \approx 1.45$. Then

$$\begin{aligned} F_L &= C_L A_p \rho V_0^2/2 \\ F_{L/\text{length}} &= 1.45 \times 0.20 \times 1,000 \times (10.5)^2/2 \\ & \boxed{F_{L/\text{length}}\text{=16,000 N/m}} \end{aligned}$$

11.73: PROBLEM DEFINITION

Situation: The distribution of C_p on the wing section in problem 11.72 (EFM10e) is described in the problem statement.

Find: Range that C_L will fall within.

SOLUTION

The correct choice is (b).

11.74: PROBLEM DEFINITION

Situation: The drag coefficient for a wing is described in the problem statement.

Find: Derive an expression for the C_L that corresponds to minimum C_D/C_L and the corresponding C_L/C_D.

SOLUTION

$$C_D/C_L = (C_{D_0}/C_L) + (C_L/(\pi\Lambda))$$

$$d/dC_L(C_D/C_L) = (-C_{D_0}/C_L^2) + (1/(\pi\Lambda)) = 0$$

$$\boxed{C_L = \sqrt{\pi\Lambda C_{D_0}}}$$

$$C_D = C_{D_0} + \pi\Lambda C_{D_0}/(\pi\Lambda) = 2C_{D_0}$$

Then

$$\boxed{C_L/C_D = (1/2)\sqrt{\pi\Lambda/C_{D_0}}}$$

11.75: PROBLEM DEFINITION

Situation:

A glider at elevation of 800 m descends to sea level
mass = 180 kg
$S = 20 \text{ m}^2$
glide ange is 1.7°
$\rho = 1.2 \text{ kg/m}^3$
$C_L = 0.83$

Find: Time in minutes for the descent.

SOLUTION

$$\begin{aligned}
\ell &= 800/(\sin 1.7^\circ) = 26,967 \text{ m} \\
F_L &= W = (1/2)\rho V^2 C_L S \\
180 \times 9.81 &= 0.5 \times 1.2 \times V^2 \times 0.83 \times 20 \\
&\text{so} \\
V &= 13.32 \text{ m/s}
\end{aligned}$$

Then

$$\begin{aligned}
t &= 26,967 \text{ m}/(13.32 \text{ m/s}) \\
&= 2025 \text{ s} \\
&= \boxed{t = 33.7 \text{ min}}
\end{aligned}$$

11.76: PROBLEM DEFINITION

Situation: An aircraft wing is described in the problem statement.

Find: Drag force on the wing.

SOLUTION

Lift force

$$F_L = C_L S \frac{\rho V_0^2}{2}$$
$$F_L/S = C_L \frac{\rho V_0^2}{2}$$

Thus

$$\frac{\rho V_0^2}{2} = \frac{F_L/S}{C_L}$$
$$\frac{\rho V_0^2}{2} = \frac{2000\,\mathrm{N/\,m^2}}{0.3} = 6667\,\mathrm{N/\,m^2}$$

From Fig. 11.25 (EFM 10e) at $C_L = 0.30,\ C_D \approx 0.06$

Drag force

$$F_D = C_D S \frac{\rho V_0^2}{2} = (0.06)\left(10\,\mathrm{m^2}\right)\left(6667\,\mathrm{N/\,m^2}\right)$$
$$\boxed{F_D = 4000\ \mathrm{N}}$$

11.77: PROBLEM DEFINITION

Situation: The problem statement describes an ultralight airplane.

Find:
(a) Angle of attack.
(b) Drag force on wing.

SOLUTION

Lift force

$$
\begin{aligned}
W &= C_L S \rho V_0^2/2 \\
C_L &= W/(S\rho V_0^2/2) = (1800)/((18.6)(1.03)(15^2)/2) = 0.80
\end{aligned}
$$

From Fig. 11.24 in EFM10e $C_D = 0.06$ and

$$\boxed{\alpha = 7^\circ}$$

The drag force is

$$
\begin{aligned}
F_D &= C_D S \rho V_0^2/2 \\
&= (0.06)(18.6)(1.03)(15^2)/2 \\
&= \boxed{F_D = 130 \text{ N}}
\end{aligned}
$$

11.78: PROBLEM DEFINITION

Situation: The parameters for a human-powered aircraft are given in the problem statement.

Find: Design the human-powered aircraft using the characteristics of the wing in Fig. 11.24 (EFM10e).

SOLUTION

There are several ways to address this design problem. One approach would be to consider the wing area and velocities necessary to meet the power constraint. That is,

$$373\,\mathrm{W} = (0.05 + C_D)\frac{1}{2}(1.23\ \mathrm{kg/m^3})V_0^3 S$$

Make plots of V_0 versus S with C_D as a parameter. Then use the constraint of the lift equaling the weight.

$$180\,\mathrm{N} + 5.7\,\mathrm{N/\,m^2} \times S = C_L\frac{1}{2}(1.23\ \mathrm{kg/m^3})V_0^2 S$$

Make plots of V_0 versus S with C_L as a parameter. Where these curves intersect would give values where both constraints are satisfied. Next you can plot the curve for the pairs of C_D and C_L where the curves cross. You can also plot C_D versus C_L (drag polar) for the airfoil and see if there is a match. If there is no match, the airfoil will not work. If there is a match, you should try to find the configuration that will give the minimum weight.