12.1: PROBLEM DEFINITION

Situation:

The speed of sound in an ideal gas ________. (Select all that are correct):

a. depends upon $\sqrt{T}$ where T is absolute temperature

b. depends upon $\sqrt{T}$ where T is temperature in °C

c. depends upon $\sqrt{k}$, where $k = \frac{c_p}{c_v}$, a ratio of specific heats for a given gas

SOLUTION

The correct answers are (a) and (c).

12.2: PROBLEM DEFINITION

Part a.

Situation:

Speed of sound in air is 340 m/s.

Find:

Speed in miles per hour.

SOLUTION

This requires a unit conversion.

$$V = 340\frac{\text{m}}{\text{s}} \times \frac{1 \text{ mi}}{1609 \text{ m}} \times \frac{3600 \text{ s}}{1 \text{ hr}}$$

$$\boxed{V = 761 \text{ mph}}$$

Part b.

Situation:

4 seconds between seeing lightening and hearing thunder when T=10 °C....

Find:

Distance to lightning strike.

Properties:

Table A.2 (EFM 10e), $R_{air} = 287$ J/kg K

SOLUTION

Thunder travels at the speed of sound. The speed of sound in a gas is given by:

$$\begin{aligned} c &= \sqrt{kRT} = \sqrt{1.4 \times 287 \text{ J/kg K} \times 288 \text{ K}} = 340 \text{ m/s} \\ &= 340 \text{ m/s} \times 1 \text{ km}/1000 \text{ m} = 0.34 \text{ km/s} \end{aligned}$$

Therefore, the distance for this case is:

$$s = c\Delta t = 0.34 \text{ km/s} \times 4 \text{ s}$$

$$\boxed{s = 1.36 \text{ km}}$$

12.3: PROBLEM DEFINITION

Situation:

The Mach number ______. (Select all that are correct).

a. is the ratio V/c, where $c =$ specific heat

b. is the ratio V/c, where $c =$ the speed of sound

c. depends on the velocity, V, of the moving body relative to the fluid flow

d. has a magnitude of $M < 1$ for subsonic flow

e. has a magnitude of $M > 1$ for supersonic flow

SOLUTION

The correct choices are b, c, d and e.

12.4: PROBLEM DEFINITION

Situation: Investigate cruise Mach number of conventional airliners and possible regions of supersonic flow.

SOLUTION Nominally commercial transport aircraft fly at a Mach number of 0.8 to 0.85. Supersonic flows can occur in areas where velocity is higher.

12.5: PROBLEM DEFINITION

Situation: Satellite re-entry of earth's atmosphere at $-60\,^{\circ}$C.

Find: Mach number

Properties: Table A.2 (EFM 10e), $R_{air} = 287$ J/kg-K

SOLUTION Orbital velocity

$$\begin{aligned}
\frac{V^2}{r} &= g \\
V^2 &= gr = 9.81\ \text{m/s}^2 \times 6400\ \text{km} \times 1000\ \text{m/km} \\
V &= 7,920\ \text{m/s}
\end{aligned}$$

Speed of sound

$$c = \sqrt{kRT} = \sqrt{1.4 \times 287\ \text{J/kg-K} \times 213\ \text{K}} = 292\ \text{m/s}$$

Mach number

$$\begin{aligned}
M &= (7,920\ \text{m/s})\ /\ (292\ \text{m/s}) \\
& \boxed{M = 27.1}
\end{aligned}$$

12.6: PROBLEM DEFINITION

Situation: A sound wave travels in methane at $-5\,^\circ$C.

Find: Speed of wave.

Properties: Table A.2 (EFM 10e), $R_{\text{methane}} = 513$ J/kg-K, $k = 1.31$

SOLUTION Speed of sound in a gas

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{1.31 \times 518 \text{ J/kg-K} \times 268 \text{ K}} \\ &\boxed{c = 427 \text{ m/s}} \end{aligned}$$

12.7: PROBLEM DEFINITION

Situation: A sound wave travels in helium at 45 °C.

Find: Speed of wave.

Properties: Table A.2 (EFM 10e), $R_{\text{He}} = 2077$ J/kg-K, $k = 1,66$

SOLUTION

Speed of sound in a gas

$$
\begin{aligned}
c &= \sqrt{kRT} \\
&= \sqrt{1.66 \times 2077 \text{ J/kg-K} \times (45 + 273)\text{K}} \\
&\boxed{c = 1047 \text{ m/s}}
\end{aligned}
$$

12.8: PROBLEM DEFINITION

Situation: A sound wave travels in hydrogen at 3 °C.

Find: Speed of wave.

Properties: Table A.2 (EFM 10e), $R_{\mathrm{H_2}} = 4127$ J/kg K, $k = 1.41$

SOLUTION Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{1.41 \times 4127 \text{ J/kg K} \times (3 + 273)\text{K}} \\ &\boxed{c = 1267 \text{ m/s}} \end{aligned}$$

12.9: PROBLEM DEFINITION

Situation: A sound wave travels in helium and another in nitrogen both at 20 °C.

Find: Difference in speed of sound.

Properties: Table A.2 (EFM 10e), $R_{\mathrm{He}} = 2077$ J/kg-K, $k = 1.66$, $R_{\mathrm{N_2}} = 297$ J/kg-K, $k = 1.40$

SOLUTION Speed of sound

$$\begin{aligned} c_{\mathrm{He}} &= \sqrt{(kR)_{\mathrm{He}}T} \\ &= \sqrt{1.66 \times 2077 \times 293} \\ &= 1005 \text{ m/s} \end{aligned}$$

$$\begin{aligned} c_{\mathrm{N_2}} &= \sqrt{(kR)_{\mathrm{N_2}}T} \\ &= \sqrt{1.40 \times 297 \times 293} \\ &= 349 \text{ m/s} \end{aligned}$$

$$c_{\mathrm{He}} - c_{\mathrm{N_2}} = \boxed{656 \text{ m/s}}$$

12.10: PROBLEM DEFINITION

Situation: A sound wave travels in an ideal gas.

Find: Speed of sound for an isothermal process.

SOLUTION

$$c^2 = \partial p/\partial \rho; \ p = \rho RT$$

If isothermal, T=const.

$$\therefore \ \partial p/\partial \rho = RT$$
$$\therefore \ c^2 = RT$$
$$\boxed{c = \sqrt{RT}}$$

12.11: PROBLEM DEFINITION

Situation: The relationship between pressure and density for sound traveling through a fluid is

$$p - p_0 = E_v \ln(\rho/\rho_0)$$

Find: Speed of sound in water.

Properties: $E_v = 2.2$ GPa, $\rho = 1000$ kg/m^3 .

SOLUTION

$$\begin{aligned} p - p_o &= E_v \ln(\rho/\rho_o) \\ c^2 &= \frac{\partial p}{\partial \rho} = \frac{E_v}{\rho} \\ c &= \sqrt{E_v/\rho} \\ c &= \sqrt{2.20 \times 10^9/10^3} \\ & \boxed{c = 1483 \text{ m/s}} \end{aligned}$$

12.12: PROBLEM DEFINITION

Situation: An aircraft flying in air at 30 °C at Mach 1.6.

Find: Surface temperature.

Properties: Table A.2 (EFM 10e), data for air, $k = 1.4$

SOLUTION

Total temperature will develop at exposed surface.Total temperature

$$
\begin{aligned}
T_t &= T(1 + \frac{k-1}{2} M^2) \\
&= 303 \text{ K} \times (1 + 0.2 \times 1.6^2) \\
&= 458 \text{ K} \\
& \boxed{T_t = 185\,^{\circ}\text{C}}
\end{aligned}
$$

12.13: PROBLEM DEFINITION

Situation: A fighter is flying at Mach 3 though air at $-20\,^{\circ}\mathrm{C}$.

Find: Temperature on nose.

Properties: From Table A.2 (EFM 10e), $k = 1.4$

SOLUTION Total temperature will develop at exposed surface. Total temperature

$$\begin{aligned} T_t &= T(1+\frac{k-1}{2}M^2) \\ &= 253\,\mathrm{K} \times (1+0.2\times 3^2) \\ &= 708\,\mathrm{K} \\ &\quad \boxed{T_t = 435\,^{\circ}\mathrm{C}} \end{aligned}$$

12.14: PROBLEM DEFINITION

Situation: An aircraft is flying at Mach 1.8 through air at 10000 m, 30.5 kPa, and −44 °C.

Find: (a) Speed of aircraft.
(b) Total temperature.
(c) Speed for $M = 1$.

Properties: From Table A.2 (EFM 10e), $k = 1.4$, $R_{air} = 287$ J/kg-K

SOLUTION Speed of sound (at 10,000 m)

$$\begin{aligned} c &= \sqrt{kRT} \\ c &= \sqrt{(1.4)(287 \text{ J/kg-K})(229 \text{ K})} \\ c &= 303.3 \,\text{m/s} \end{aligned}$$

a) Speed

$$\begin{aligned} V &= (1.8)(303.3 \text{ m/s})(3,600 \text{ s/hr})/(1,000 \text{ m/km}) \\ &= \boxed{1965 \text{ km/hr}} \end{aligned}$$

b) Total temperature

$$\begin{aligned} T_t &= T(1 + \frac{k-1}{2} M^2) \\ T_t &= 229 \text{ K}(1 + ((1.4 - 1)/2) \times 1.8^2) \\ &= 377K = \boxed{104 \text{ °C}} \end{aligned}$$

c) Speed for Mach number=1

$$\begin{aligned} M &= 1;\ V = 1 \times c = c \\ V &= (303.3 \text{ m/s})(3,600 \text{ s/hr})/(1,000 \text{ m/km}) \\ &= \boxed{1090 \text{ km/hr}} \end{aligned}$$

12.15: PROBLEM DEFINITION

Situation: An airplane is travelling at 850 km/hr at sea level where temperature is 10 °C.

Find: Speed of aircraft with same Mach number at altitude where $T = -50\,^{\circ}\text{C}$.

Properties: From Table A.2 (EFM 10e), $k = 1.4$, $R_{air} = 287$ J/kg-K

PLAN Apply the Mach number equation and the speed of sound equation.

SOLUTION At sea level

Speed of sound (sea level)

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{(1.4)(287 \text{ J/kg-K})(283 \text{ K})} \\ &= 337.2 \text{ m/s} \end{aligned}$$

Mach number(sea level)

$$\begin{aligned} V &= 850 \text{ km/hr } = 236.1 \text{ m/s} \\ M &= 236.1/337.2 = 0.700 \end{aligned}$$

Speed of sound (at altitude)

$$\begin{aligned} c &= \sqrt{(1.4)(287)(223)} \\ &= 299.3 \text{ m/s} \end{aligned}$$

Mach number (at altitude)

$$\begin{aligned} V &= Mc \\ &= 0.700 \times 299.3 \\ & \boxed{V = 210\,\text{m/s} = 755\,\text{km/h}} \end{aligned}$$

12.16: PROBLEM DEFINITION

Situation: An aircraft flying at M=0.95 through air at 10,000 m where T = −44 °C and p=30 kPa, abs. Lift coefficient is 0.05.

Find: Wing loading.

SOLUTION

Kinetic pressure

$$\begin{aligned} q &= (k/2)pM^2 \\ &= (1.4/2)(30)(0.95)^2 \\ &= 18.95 \text{ kPa} \end{aligned}$$

Lift force

$$\begin{aligned} F_L &= C_L q S \\ W &= F_L/S = C_L q \\ &= (0.05)(18.95) \\ &= 0.948 \text{ kPa} \\ & \boxed{W = 948 \text{ Pa}} \end{aligned}$$

12.17: PROBLEM DEFINITION

Situation: Early passenger aircraft flew at 400 km/h at 4500 m.

Find: Importance of compressible flow effects.

SOLUTION

Compressible flow effects not important for $M < 0.3$.Temperature decreases at 5.87 K/km. The temperature drop for 4500 m altitude would be $5.87 \times 4.5 = 26.4\,^{\circ}\mathrm{C}$. If ambient temperature were 273 K, then the absolute temperature would be 246 K. The speed of sound is

$$c = \sqrt{kRT} = \sqrt{1.4 \times 287 \times 246} = 314 \text{ m/s}$$

The aircraft speed is

$$V = 400\,\text{km/h} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 111 \text{ m/s}$$

Mach number

$$M = \frac{111}{314} = 0.35$$

Compressible flow effects could have occurred but would have been insignificant.

12.18: PROBLEM DEFINITION

Situation: Difference between total and stagnation pressure.

SOLUTION Total pressure is the pressure resulting when a flow is brought to rest isentropically (adiabatic and reversible). Stagnation pressure corresponds to the pressure resulting when a flow is brought to rest arbitrarily, such as with a total head tube.

12.19: PROBLEM DEFINITION

Situation: An object immersed in a 250 m/s airflow at 200 kPa, abs and 20 °C.

Find: (a) Pressure.
(b) Temperature at stagnation point.

Properties: From Table A.2 (EFM 10e), $k = 1.4$, $R_{air} = 287$ J/kg-K

SOLUTION Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{(1.4)(287 \text{ J/kg-K})(293 \text{ K})} \\ &= 343 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} M &= 250/343 \\ &= 0.729 \end{aligned}$$

Total properties
Temperature

$$\begin{aligned} T_t &= T(1 + \frac{k-1}{2} M^2) \\ T_t &= (293 \text{ K})(1 + 0.2 \times (0.729)^2) \\ &= 293 \text{ K} \times 1.106 \\ &= 324 \text{ K} \\ &\boxed{T_t = 51\,^\circ\text{C}} \end{aligned}$$

Pressure

$$\begin{aligned} p_t &= p\left(1 + \frac{k-1}{2} M^2\right)^{k/(k-1)} \\ p_t &= (200) \text{ kPa}(1.106)^{3.5} \\ &\boxed{p_t = 285\,\text{kPa}} \end{aligned}$$

12.20: PROBLEM DEFINITION

Situation: An airflow with Mach number of 0.85 through a 60 cm^2-conduit with 360 kPa absolute pressure and total temperature of 10 °C.

Find: Mass flow rate through conduit.

Properties: From Table A.2 (EFM 10e), $k = 1.4$, $R_{\text{air}} = 287$ J/kg-K

PLAN Apply the flow rate equation, the ideal gas law, Mach number, speed of sound, and the total properties equations.

SOLUTION Total temperature equation

$$\begin{aligned} T &= \frac{T_t}{\left(1 + \frac{k-1}{2} M^2\right)} \\ &= \frac{283 \text{ K}}{\left(1 + \left(\frac{1.4-1}{2}\right) 0.85^2\right)} \\ &= 247\,\text{K} \end{aligned}$$

Total pressure equation

$$\begin{aligned} p &= \frac{p_t}{\left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}} \\ &= \frac{360\,\text{kPa}}{\left(1 + \left(\frac{1.4-1}{2}\right) 0.85^2\right)^{\frac{1.4}{1.4-1}}} \\ &= 224 \text{ kPa} \end{aligned}$$

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ c &= \sqrt{(1.4)(287 \text{ J/kg-K})(247 \text{ K})} \\ &= 315\,\text{m/s} \end{aligned}$$

Velocity

$$\begin{aligned} V &= Mc \\ V &= (0.85)(315) \\ &= 268 \text{ m/s} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{224 \times 10^3 \text{ Pa}}{287 \text{ J/kg-K} \times 247 \text{ K}} \\ &= 3.16 \text{ kg/m}^3 \end{aligned}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= V\rho A \\ &= (268 \text{ m/s})(3.16 \text{ kg/m}^3)(0.0060 \text{ m}^2) \\ &\boxed{\dot{m} = 5.08 \text{ kg/s}} \end{aligned}$$

12.21: PROBLEM DEFINITION

Situation: Oxygen flows from a reservoir with temperature of 200 °C and 300 kPa, abs. through conduit with Mach number of 0.9.

Find: (a) Velocity.
(b) Pressure.
(c) Temperature.

Properties: From Table A.2 (EFM 10e), $k = 1.4$, $R_{O_2} = 260$ J/kg-K

SOLUTION

Total properties

$$\begin{aligned} T_t &= 200^\circ\text{C} = 473 \text{ K} \\ T &= T_t/(1 + \frac{k-1}{2}M^2) \\ T &= 473 \text{ K}/(1 + 0.2 \times 0.9^2) \\ &= 473 \text{ K}/1.162 \\ &\boxed{T = 407 \text{ K}} \\ p_t &= 300 \text{ kPa} \\ p &= p_t/(1 + \frac{k-1}{2}M^2)^{k/(k-1)} \\ p &= 300 \text{ kPa}/(1.162)^{3.5} \\ &\boxed{p = 177 \text{ kPa}} \end{aligned}$$

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ c &= [(1.4)(260 \text{ J/kg-K})(407 \text{ K})]^{1/2} \\ &= 385 \text{ m/s} \end{aligned}$$

Velocity

$$\begin{aligned} V &= Mc \\ &= (0.9)(385 \text{ m/s}) \\ &\boxed{V = 346 \text{ m/s}} \end{aligned}$$

12.22: PROBLEM DEFINITION

Situation: High Mach number flow of air from a reservoir at 300 K. Condensation occurs when T=50 K.

Find: Mach number where condensation will occur.

Properties: From Table A.2 (EFM 10e), $k = 1.4$

PLAN Apply total temperature equation setting the T to 50 K and T_t to 300 K.

SOLUTION Total temperature equation

$$\begin{aligned} T_0/T &= 1 + ((k-1)/2)M^2 \\ 300\text{ K}/50\text{ K} &= 6 = 1 + 0.2\ M^2 \\ &\boxed{\text{M=5}} \end{aligned}$$

12.23: PROBLEM DEFINITION

Situation: Hydrogen flow from a reservoir with temperature 20 °C and pressure 500 kPa, abs through a 2-cm diameter duct where velocity is 250 m/s.

Find:
(a) Temperature.
(b) Pressure.
(c) Mach number.
(d) Mass flow rate.

Properties: From Table A.2 (EFM 10e), $k = 1.41$, $R_{\text{H}_2} = 4127$ J/kg-K.

SOLUTION Evaluate c_p from

$$\begin{aligned} c_p &= \frac{k}{k-1}R \\ &= \frac{1.41}{0.41} \times 4127 \text{ J/kg-K} \\ &= 14,200 \text{ J/kg-K} \end{aligned}$$

Stagnation conditions

$$\begin{aligned} T_t &= 20^\circ\text{C} = 293 \text{ K} \\ P_t &= 500 \text{ kPa} \end{aligned}$$

Energy equation

$$\begin{aligned} c_p T + V^2/2 &= c_p T_t \\ T &= T_t - V^2/(2c_p) \\ &= 293 \text{ K} - (250 \text{ m/s})^2/((2)(14,200 \text{ J/kg-K})) \\ &\boxed{T = 291 \text{ K}} \end{aligned}$$

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{(1.41)(4,127 \text{ J/kg-K})(291 \text{ K})} \\ &= 1301 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} M &= (250 \text{ m/s})/(1300 \text{ m/s}) \\ &\boxed{\text{M=0.192}} \end{aligned}$$

Local pressure

$$
\begin{aligned}
p &= p_t/(1+\frac{k-1}{2}M^2)^{k/(k-1)} \\
p &= 500/[1+(0.41/2)\times 0.192^2]^{(1.41/0.41)} \\
& \boxed{p = 487 \text{ kPa}}
\end{aligned}
$$

Ideal gas law

$$
\begin{aligned}
\rho &= p/RT \\
&= (487\times 10^3 \text{ Pa})/(4,127 \text{ J/kg-K}\times 291 \text{ K}) \\
&= 0.406 \text{ kg/m}^3
\end{aligned}
$$

Flow rate equation

$$
\begin{aligned}
\dot{m} &= \rho A V \\
&= (0.406 \text{ kg/m}^3)(0.02 \text{ m})^2(\pi/4)(250 \text{ m/s}) \\
& \boxed{\dot{m} = 0.032 \text{ kg/s}}
\end{aligned}
$$

12.24: PROBLEM DEFINITION

Situation: A 3 cm diameter sphere in a Mach-2.5 wind tunnel with total pressure of 547 kPa, abs has drag coefficient of 0.95.

Find: Drag on the sphere.

Properties: From Table A.2 (EFM 10e), $k = 1.4$

SOLUTION Find static pressure and then dynamic pressure.

$$\begin{aligned} p &= p_t/[1 + ((k-1)/2)M^2]^{k/(k-1)} \\ &= 547 \text{ kPa}/[1 + 0.2(2.5)^2]^{3.5} \\ &= 32.0 \text{ kPa} \\ (1/2)\rho U^2 &= kpM^2/2 \\ &= 1.4 \times 32 \times 2.5^2/2 \\ &= 140.0 \text{ kPa} \end{aligned}$$

Drag force

$$\begin{aligned} F_D &= C_D(1/2)\rho U^2 A \\ &= (0.95)(140 \times 10^3 \text{ Pa})(0.03 \text{ m})^2(\pi/4) \\ &\boxed{F_D = 94.01\,\text{N}} \end{aligned}$$

12.25: PROBLEM DEFINITION

Situation:
Using equation for total pressure to calculate pressure coefficient, $C_p = (p_t - p)/(\frac{1}{2}\rho V^2)$
$k = 1.4$

Find:
(a) Expression for pressure coefficient.
(b) Values for pressure coefficient for different values of M

SOLUTION

$$\begin{aligned} p_t &= (p)[1 + (k-1)/2 \times M^2]^{(k/(k-1))} \\ C_p &= (p_t - p)/\rho U^2/2 \\ &= (p_t - p)/(kpM^2/2) \\ &= (2/kM^2)[(p_t/p) - 1] \end{aligned}$$

$$\boxed{C_p = 2/(kM^2)[(1 + (k-1)M^2/2)^{(k/(k-1))} - 1]}$$

$$\boxed{C_p(0) = \text{undefined}}$$

$$\boxed{C_p(2) = 2.43}$$

$$\boxed{C_p(4) = 13.47}$$

$$\boxed{C_{p_{\text{inc.}}} = 1.0}$$

12.26: PROBLEM DEFINITION

Situation: With low velocities, the pressure ratio can be approximated with $p_t/p = 1+\varepsilon$.

Find: Show that Mach number goes to zero as ϵ goes to zero, and that Eq. (12.31) in EFM 10e reduces to $M = [(2/k)(p_t/p-1)]^{1/2}$

SOLUTION

$$
\begin{aligned}
p_t/p &= [1+(k-1)M^2/2]^{k/(k-1)} \\
M &= \sqrt{(2/(k-1))[(p_t/p)^{(k-1)/k}-1]} \\
p_t/p &= 1+\varepsilon; (p_t/p)^{(k-1)/k} = (1+\varepsilon)^{(k-1)/k} = 1+((k-1)/k)\varepsilon + 0(\varepsilon^2) \\
(p_t/p)^{(k-1)/k} - 1 &\backsimeq ((k-1)/k)\varepsilon + 0(\varepsilon^2)
\end{aligned}
$$

Assume $\varepsilon \to 0$, then the higher order terms can be neglected.

$$
\begin{aligned}
M &= [(2/(k-1))((k-1)/k)\varepsilon]^{1/2} \\
M &= [2\varepsilon/k]^{1/2} \\
\text{then, applying } p_t/p &= 1+\varepsilon \\
M &= [(2/k)(p_t/p-1)]^{1/2}
\end{aligned}
$$

12.27: PROBLEM DEFINITION

Situation: True and false statements concerning shock waves.

SOLUTION Shock waves occur only in supersonic flows (T) Static pressure increases across normal shock wave (T) and Mach number downstream of normal shock wave is supersonic (F).

12.28: PROBLEM DEFINITION

Situation: Occurrence or normal shock waves in subsonic flows

SOLUTION An occurrence of a shock wave in subsonic flow would violate 2nd law of thermodynamics so impossible.

12.29: PROBLEM DEFINITION

Situation: A normal shock wave occurs in 500 m/s stream of nitrogen with static pressure of 70 kPa and static temperature of $-50\,^\circ$C.

Find: (a) Mach number.
(b) Pressure downstream of wave.
(c) Temperature downstream of wave.
(d) Entropy increase.

Properties: From Table A.2 (EFM 10e), $k = 1.4$, $R_{\mathrm{N_2}} = 297$ J/kg-K.

SOLUTION Speed of sound

$$\begin{aligned} c_1 &= \sqrt{kRT} \\ &= \sqrt{(1.4)(297 \text{ J/kg-K})(223 \text{ K})} \\ &= 305 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} M_1 &= V/c \\ &= 500/305 \\ &= 1.64 \end{aligned}$$

Normal shock wave (Mach number)

$$\begin{aligned} M_2^2 &= [(k-1)M_1^2+2]/[2kM_1^2-(k-1)] \\ &= [(0.4)(1.64)^2+2]/[(2)(1.4)(1.64)^2-0.4] \\ &= .4313 \\ & \boxed{M_2 = 0.657} \end{aligned}$$

Normal shock wave (Pressure ratio)

$$\begin{aligned} p_2 &= p_1(1+k_1M_1^2)/[(1+k_1M_2^2)] \\ &= (70 \text{ kPa})(1+1.4\times 1.64^2)/(1+1.4\times 0.657^2) \\ & \boxed{p_2 = 208 \text{ kPa}} \end{aligned}$$

Temperature ratio

$$\begin{aligned} T_2 &= T_1(1+((k-1)/2)M_1^2)/(1+((k-1)/2)M_2^2) \\ &= 223[1+0.2\times 1.64^2]/[1+0.2\times 0.657^2] \\ & \boxed{T_2 = 316\,\text{K} = 43\,^\circ\text{C}} \end{aligned}$$

Entropy increase

$$\begin{aligned}\Delta s &= R\ell n[(p_1/p_2)(T_2/T_1)^{k/(k-1)}] \\ &= R[\ell n(p_1/p_2) + (k/(k-1))\ell n(T_2/T_1)] \\ &= 297 \text{ J/kg-K}[\ell n(70 \text{ kPa}/208 \text{ kPa}) + 3.5\ell n(315 \text{ K}/223 \text{ K})] \\ &\boxed{\Delta s = 35.6 \text{ J/kg K}}\end{aligned}$$

12.30: PROBLEM DEFINITION

Situation: A normal shock wave occurs in a Mach 3 flow of air with static temperature of 2 °C and static pressure of 207 kPa.

Find: (a) Mach number downstream of shock wave.
(b) Pressure downstream of shock wave.
(c) Temperature downstream of shock wave.

Properties: From Table A.2 (EFM 10e), $k = 1.4$.

SOLUTION

Mach number (downstream)

$$M_2^2 = [(k-1)M_1^2 + 2]/[2kM_1^2 - (k-1)]$$

$$\boxed{M_2 = 0.475}$$

Temperature ratio

$$\begin{aligned}
(T_2/T_1) &= [1 + ((k-1)/2)M_1^2]/[1 + ((k-1)/2)M_2^2] \\
&= (1 + (0.2)(9))/(1 + (0.2)(0.475)^2) = 2.68 \\
T_2 &= 275 \times 2.68
\end{aligned}$$

$$\boxed{T_2 = 737\,\text{K} = 464\,°\text{C}}$$

Pressure ratio

$$\begin{aligned}
p_2/p_1 &= (1 + kM_1^2)/(1 + kM_2^2) \\
&= (1 + 1.4 \times 9)/(1 + 1.4 \times (0.475)^2) \\
&= 10.333 \\
p_2 &= (10.33)(207)
\end{aligned}$$

$$\boxed{p_2 = 2138 \text{ kPa}}$$

12.31: PROBLEM DEFINITION

Situation: Pitot-static tube used to measure Mach number in supersonic flow. Total pressure downstream of shock is 150 kPa and static pressure upstream of shock is 40 kPa.

Find: Mach number.

Properties: From Table A.2 (EFM 10e), $k = 1.41$.

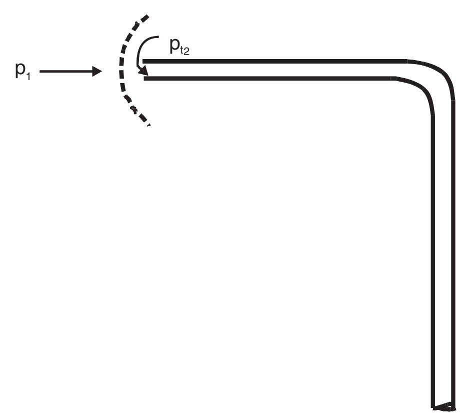
PLAN Find pressure ratios and apply the compressible flow tables.

SOLUTION

$$p_{t_2}/p_1 = 150/40 = 3.75 = (p_{t_2}/p_{t_1})(p_{t_1}/p_1)$$

Using compressible flow tables, Table A.1 (EFM 10e),:

M	p_{t_2}/p_{t_1}	p_1/p_{t_1}	p_{t_2}/p_1
1.60	0.8952	0.2353	3.80
1.50	0.9278	0.2724	3.40
1.40	0.9582	0.3142	3.04
1.35	0.9697	0.3370	2.87



Therefore, interpolating, $\boxed{M = 1.59}$

12.32: PROBLEM DEFINITION

Situation: A shock wave occurs in a Mach 3 stream of methane where static pressure is 89 kPa and temperature is 20 °C.

Find: (a)The downstream Mach number.
(b) Static pressure.
(c) Static temperature.
(d) Density.

Properties: From Table A.2 (EFM 10e), $k = 1.31, R_{\text{methane}}$=518 J/kg-K.

PLAN Apply the normal shock wave equations to find Mach number, pressure, and temperature. Apply the ideal gas law to find density.

SOLUTION Normal shock wave
Mach number

$$\begin{aligned} M_2^2 &= [(k-1)M_1^2+2]/[2kM_1^2-(k-1)] \\ &= ((0.31)(9)+2)/((2)(1.31)(9)-0.31) = \\ & \quad 0.2058 \\ & \quad \boxed{M_2 = 0.454} \end{aligned}$$

Pressure ratio

$$\begin{aligned} p_2/p_1 &= (1+kM_1^2)/(1+kM_2^2) \\ &= (1+1.31\times 9)/(1+1.31\times 0.2058) = 10.1 \\ & \quad \boxed{p_2 = 680 \text{ kPa, abs}} \end{aligned}$$

Temperature ratio

$$\begin{aligned} T_2/T_1 &= [1+((k-1)/2)M_1^2]/[1+((k-1)/2)M_2^2] \\ &= 2.32 \\ T_2 &= (293 \text{ K})(2.32) \\ & \quad \boxed{T_2 = 680\,\text{K} = 407\,^\circ\text{C}} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho_2 &= p_2/(RT_2) \\ &= (680\times 10^3 \text{ Pa})/(518 \text{ J/kg-K}\times 680 \text{ K}) \\ & \quad \boxed{\rho_2 = 2.55 \text{ kg/m}^3} \end{aligned}$$

12.33: PROBLEM DEFINITION

Situation: A shock wave occurs in helium where downstream Mach number is 0.85 and static temperature is 110 °C.

Find: Velocity upstream of wave

Properties: From Table A.2 (EFM 10e), $k = 1.66$; $R_{\text{He}} = 2,077$ J/kg-K.

SOLUTION

Normal shock wave

Mach number

$$\begin{aligned} M_1^2 &= [(k-1)M_2^2 + 2]/[2kM_2^2 - (k-1)] \\ &= 1.194 \\ &\quad M_1 = 1.19 \end{aligned}$$

Temperature ratio

$$\begin{aligned} T_1/T_2 &= [1 + ((k-1)/2)M_2^2]/[1 + ((k-1)/2)M_1^2] \\ &= 0.8423 \\ T_1 &= (0.8423)(383 \text{ K}) = 322.6 \text{ K} \end{aligned}$$

Speed of sound

$$\begin{aligned} c_1 &= \sqrt{kRT} \\ &= (1.66 \times 2,077 \times 322.6)^{1/2} \\ &\quad c_1 = 1,055 \text{ m/s} \end{aligned}$$

Velocity upstream

$$\begin{aligned} V_1 &= c_1 M_1 \\ &= (1,055)(1.19) \\ &\quad \boxed{V_1 = 1,255 \text{ m/s}} \end{aligned}$$

12.34: PROBLEM DEFINITION

Situation: Expressions for minimum Mach number and largest density downstream of normal shock wave.

Find: (a) Lowest Mach number possible downstream of shock wave.
(b) Largest density ratio possible.
(c) Limiting values of M_2 and ρ_2/ρ_1 for air.

SOLUTION

$$M_2^2 = ((k-1)M_1^2 + 2)/(2kM_1^2 - (k-1))$$

Because

$$\begin{aligned} M_1 >> 1, (k-1)M_1^2 &\gg 2 \\ 2kM_1^2 &\gg (k-1) \end{aligned}$$

So in limit

$$\begin{aligned} M_2^2 &\to ((k-1)M_1^2)/2kM_1^2 = (k-1)/2k \\ \therefore & \boxed{M_2 \to \sqrt{(k-1)/2k}} \end{aligned}$$

$$\begin{aligned} \rho_2/\rho_1 &= \frac{(p_2/p_1)}{(T_1/T_2)} \\ &= ((1+kM_1^2)/(1+kM_2^2))(1+((k-1)/2)M_2^2)/(1+((k-1)/2)M_1^2) \end{aligned}$$

in limit $M_2^2 \to (k-1)/2k$ and $M_1 \to \infty$

$$\frac{\rho_2}{\rho_1} \to \frac{\left[\frac{kM_1^2}{1+\frac{k-1}{2}}\right]\left[1+\frac{(k-1)^2}{4k}\right]}{(\frac{k-1}{2})M_1^2}$$

$$\frac{\rho_2}{\rho_1} \to \frac{\left[\frac{2k}{k+1}\right]\left[\frac{(k+1)^2}{4k}\right]}{(\frac{k-1}{2})}$$

$$\boxed{\rho_2/\rho_1 \to \frac{(k+1)}{(k-1)}}$$

For air $k = 1.4$, so

$$\boxed{\text{M}_2(\text{air}) = 0.378}$$

$$\boxed{\rho_2/\rho_1(\text{air}) = 6.0}$$

12.35: PROBLEM DEFINITION

Situation: Mach number downstream of weak shock wave.

Find: (a) Approximation for Mach number downstream of wave.
(b) Compare M_2 computed with equation from (a) with values in Table A.1 (EFM 10e) for $M_1 = 1$, 1.05, 1.1, and 1.2.

SOLUTION

$$\begin{aligned}
M_2^2 &= [(k-1)M_1^2 + 2]/[2kM_1^2 - (k-1)] \\
&= [(k-1)(1+\varepsilon) + 2]/[2k(1+\varepsilon) - (k-1)] = [k+1+(k-1)\varepsilon]/[k+1+2k\varepsilon] \\
&= [1 + (k-1)\varepsilon/(k+1)]/[1 + (2k\varepsilon)/(k+1)] \\
&\approx [1 + (k-1)\varepsilon/(k+1)][1 - (2k\varepsilon)/(k+1)] \\
&\approx 1 + (k-1-2k)\varepsilon/(k+1) \\
&\approx 1 - \varepsilon \\
&\approx 1 - (M_1^2 - 1) \\
M_2^2 &\approx 2 - M_1^2
\end{aligned}$$

M_1	M_2	M_2 (Table A.1)
1.0	1.0	1.0
1.05	0.947	0.953
1.1	0.889	0.912
1.2	0.748	0.842

12.36: PROBLEM DEFINITION

Situation: Meaning of back pressure.

SOLUTION Back pressure is the pressure of the surroundings or environment to which an nozzle exhausts.

12.37: PROBLEM DEFINITION

Situation: Computer program for flow through a truncated nozzle. Inputs: total pressure, total temperature, back pressure, ratio of specific heats, gas constant, and nozzle diameter.

Find: (a) Develop a computer program for calculating the mass flow.
(b) Compare program with Example 12.12 with back pressures of 80, 90, 100, 110, 120, and 130 kPa and make a table.

SOLUTION

The computer program shows the flow is subsonic at the exit and the mass flow rate is 0.239 kg/s. The flow rate as a function of back pressure is given in the following table.

Back pressure, kPa	Flow rate, kg/s
80	0.243
90	0.242
100	0.239
110	0.229
120	0.215
130	0.194

REVIEW

One notes that the mass flow rate begins to decrease more quickly as the back pressure approaches the total pressure.

12.38: PROBLEM DEFINITION

Situation: A truncated nozzle has an area of 3 cm^2; the total temperature and pressure are 20°C and 300 kPa, abs. and the back pressure is 90 kPa, abs. as described in the problem statement.

Find: Mass flow rate.

Properties: From Table A.2 (EFM 10e), $k = 1.4$, $R_{\text{air}} = 287$ J/kg-K.

SOLUTION Pressures,pressure ratio and nozzle area.

$$\begin{aligned} A_T &= 3 \text{ cm}^2 = 3 \times 10^{-4} \text{ m}^2 \\ p_t &= 300 \text{ kPa; } T_t = 20^\circ = 293 \text{ K} \\ p_b &= 90 \text{ kPa} \\ p_b/p_t &= 90/300 = 0.3 \end{aligned}$$

Because $p_b/p_t < 0.528$, sonic flow at exit.
Laval nozzle flow rate equation

$$\begin{aligned} \dot{m} &= 0.685 p_t A_T / \sqrt{RT_t} \\ &= (0.685)(3 \times 10^5 \text{ Pa})(3 \times 10^{-4} \text{ m}^2)/\sqrt{(287 \text{ J/kg-K})(293 \text{ K})} \\ &\boxed{\dot{m} = 0.213 \text{ kg/s}} \end{aligned}$$

12.39: PROBLEM DEFINITION

Situation: A 3 cm^2 truncated nozzle in a 12 cm^2 pipe used to measure mass flow rate of methane. The upstream total temperature and pressure are 30 °C and 150 kPa, abs. Back pressure is 100 kPa.

Find: (a)Mass flow rate of methane.
(b) Mass flow rate if Bernoulli equation is used.

Properties: From Table A.2 (EFM 10e), $k = 1.31$; $R_{\text{methane}} = 518$ J/kgK.

SOLUTION

Areas, pressures and temperatures.

$$\begin{aligned}
A_T &= 3 \text{ cm}^2 = 3 \times 10^{-4}\text{m}^2 \\
A_p &= 12 \text{ cm}^2 = 12 \times 10^{-4}\text{m}^2 \\
p_t &= 150 \text{ kPa}; T_t = 303 \text{ K} \\
p_b &= 100 \text{ kPa}; \\
p_b/p_t &= 100 \text{ kPa}/150 \text{ kPa} = 0.667 \\
p_*/p_{t|_{\text{methane}}} &= (2/(k+1))^{k/(k-1)} = 0.544 \\
p_b &> p_*
\end{aligned}$$

The back pressure is greater than the critical pressure so subsonic flow at exit.
Mach number

$$\begin{aligned}
M_e &= \sqrt{(2/(k-1))[(p_t/p_b)^{(k-1)/k} - 1]} \\
&= \sqrt{6.45[(1.5)^{0.2366} - 1]} \\
&= 0.806
\end{aligned}$$

Temperature

$$\begin{aligned}
T_e &= T_t/\left[1 + (k-1)M_e^2/2\right] \\
T_e &= 303 \text{ K}/(1 + (0.31/2) \times (0.806)^2) \\
&= 275 \text{ K}
\end{aligned}$$

Speed of sound

$$\begin{aligned}
c_e &= \sqrt{kRT_e} \\
&= \sqrt{(1.31)(518)(275)} \\
&= 432 \text{ m/s}
\end{aligned}$$

Ideal gas law

$$\begin{aligned}
\rho_e &= p_b/(RT_e) \\
&= 100 \times 10^3 \text{ Pa}/(518 \text{ J/kg-K} \times 275 \text{ K}) \\
&= 0.702 \text{ kg/m}^3
\end{aligned}$$

Flow rate equation

$$
\begin{aligned}
\dot{m} &= \rho_e V_e A_T \\
&= (0.702 \text{ kg/m}^3)(0.806)(432 \text{ m/s})(3 \times 10^{-4} \text{ m}^2) \\
& \boxed{\dot{m} = 0.0733 \text{ kg/s}}
\end{aligned}
$$

Assume the Bernoulli equation is valid,

$$
p_t - p_b = (1/2)\rho V_e^2
$$

$$
\begin{aligned}
V_e &= \sqrt{2(150 - 100)10^3 \text{ Pa}/0.702 \text{ kg/m}^3} \\
&= 377 \text{ m/s} \\
\dot{m} &= (0.702 \text{ kg/m}^3)(377 \text{ m/s})(3 \times 10^{-4} \text{ m}^2) \\
& \boxed{\dot{m} = 0.0794 \text{ kg/s}} \\
\text{Error} &= 8.3\% \text{ (too high)}
\end{aligned}
$$

12.40: PROBLEM DEFINITION

Situation:

A 8 cm^2 truncated nozzle is used to measure a mass flow rate of air of 0.40 kg/s.
Static temperature of air at exit is 0 °C
Back pressure is 100 kPa.

Find: The total pressure.

Properties: From Table A.2 (EFM 10e), $k = 1.4$, $R_{\text{air}} = 287$ J/kg-K.

SOLUTION Speed of sound at exit.

$$\begin{aligned} c_e &= \sqrt{kRT_e} \\ &= \sqrt{(1.4)(287 \text{ J/kg-K})(273 \text{ K})} \\ &= 331 \text{ m/s} \end{aligned}$$

Ideal gas law (assume sonic flow at the exit so $p_e = 100\,\text{kPa}$)

$$\begin{aligned} \rho_e &= p_e/RT_e \\ &= 100 \times 10^3 \text{ Pa}/(287 \text{ J/kg-K} \times 273 \text{ K}) \\ &= 1.28 \text{ kg/m}^3 \end{aligned}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho_e A_e c_e \\ &= (1.28 \text{ kg/m}^3)(8 \times 10^{-4} \text{ m}^2)(331 \text{ m/s}) \\ &= 0.338 \text{ kg/s} \end{aligned}$$

Because the mass flow is too low, flow must exit sonically at pressure higher than the back pressure.
Flow rate equation

$$\begin{aligned} \rho_e &= \frac{\dot{m}}{c_e A_e} \\ &= \frac{0.40 \text{ kg/s}}{337 \text{ m/s} \times (8 \times 10^{-4} \text{ m}^2)} \\ &= 1.510 \text{ kg/m}^3 \end{aligned}$$

Pressure at exit from ideal gas law

$$\begin{aligned} p_e &= \rho_e R T_e \\ &= 1.510 \text{ kg/m}^3 \times 287 \text{ J/kg-K} \times 273 \text{ K} = 1.18 \times 10^5 \text{ Pa} \end{aligned}$$

Then from equation for total pressure

$$\begin{aligned}
\frac{p_t}{p_e} &= ((k+1)/2)^{k/(k-1)} \\
&= (1.2)^{3.5} = 1.893 \\
p_t &= 1.893 \times 1.18 \times 10^5 \text{ Pa} \\
p_t &= 2.24 \times 10^5 \text{ Pa} \\
& \boxed{p_t = 223 \text{ kPa}}
\end{aligned}$$

12.41: PROBLEM DEFINITION

Situation: A 10 cm^2 truncated nozzle is connected to helium reservoir at 28 °C with a pressure first of 130 kPa and then 350 kPa and a back pressure of 100 kPa.

Find: Mass flow rate of helium at both pressures.

Properties: From Table A.2 (EFM 10e), $k = 1.66$, $R_{\text{He}} = 2077$ J/kg-K.

SOLUTION (a) $p_t = 130$ kPa

If sonic at exit,

$$\begin{aligned} p_* &= [2/(k+1)]^{k/(k-1)} p_t \\ &= 0.487 \times 130\,\text{kPa} \\ &= 63.3\,\text{kPa} \end{aligned}$$

Back pressure is higher so flow must exit subsonically.
Find Mach number

$$\begin{aligned} M_e^2 &= (2/(k-1))[(p_t/p_b)^{(k-1)/k} - 1] \\ &= 3.03[(130 \text{ kPa}/100 \text{ kPa})^{0.4} - 1] = 0.335 \\ M_e &= 0.579 \end{aligned}$$

Exit temperature

$$\begin{aligned} \therefore T_e &= T_t/(1 + ((k-1)/2)M^2) \\ &= 301 \text{ K}/(1 + (0.33)(0.335)) \\ &= 271\,\text{K} \end{aligned}$$

Density at exit from ideal gas law

$$\begin{aligned} \rho_e &= p/RT_e \\ &= 100 \times 10^3 \text{ Pa}/[(2,077 \text{ J/kg-K})(271 \text{ K})] \\ &= 0.178 \text{ kg/m}^3 \end{aligned}$$

Velocity at exit

$$\begin{aligned} V_e &= M_e\sqrt{kRT_e} \\ &= 0.579 \times \sqrt{1.66 \times 2077 \text{ J/kg-K} \times 271\text{K}} \\ &= 560 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho_e A_e V_e \\ &= 0.178 \text{ kg/m}^3 \times 10 \times 10^{-4} \text{ m}^2 \times 560 \text{ m/s} \\ & \boxed{\dot{m} = 0.100 \text{ kg/s}} \end{aligned}$$

b) Exit pressure is 350 kPa.

$$
\begin{aligned}
p_t &= 350 \text{ kPa} \\
&\therefore \; p_* = (0.487)(350) = 170 \text{ kPa} \\
&\therefore \;\; \text{Flow exits sonically}
\end{aligned}
$$

Flow rate equation for sonic flow at exit

$$
\begin{aligned}
\dot{m} &= 0.727 p_t A_* / \sqrt{R T_t} \\
&= (0.727)(350 \times 10^3 \text{ Pa})(10 \times 10^{-4} \text{ m}^2) / \sqrt{2,077 \text{ J/kg-K} \times 301 \text{ K}} \\
& \boxed{\dot{m} = 0.322 \text{ kg/s}}
\end{aligned}
$$

12.42: PROBLEM DEFINITION

Situation: A sampling probe consists of a 2 mm diameter nozzle in a 4 mm diameter pipe. Probe mounted in 600 °C air stream with velocity of 60 m/s. Back pressure for nozzle is 100 kPa, abs.

Find: Pressure required for isokinetic sampling.

Properties: From Table A.2 (EFM 10e), $R = 287$ J/kgK; $k = 1.4$.

SOLUTION Gas density in stream from ideal gas law

$$\begin{aligned}\rho &= p/RT \\ &= 100\times 10^3 \text{ Pa}/(287 \text{ J/kg-K})(873 \text{ K}) \\ &= 0.399 \text{ kg/m}^3\end{aligned}$$

Flow rate equation

$$\begin{aligned}\dot m &= \rho V A \\ &= (0.399 \text{ kg/m}^3)(60 \text{ m/s})(\pi/4)(4\times 10^{-3}\text{ m})^2 \\ &\quad \dot m = 0.000301 \text{ kg/s}\end{aligned}$$

Mach number

$$\begin{aligned}M &= V/\sqrt{kRT} \\ &= 60/\sqrt{(1.4)(287 \text{ J/kg-K})(873 \text{ K})} \\ &= 0.101\end{aligned}$$

Total properties

$$\begin{aligned}p_t &= p\left[1+\frac{k-1}{2}M^2\right]^{k/(k-1)} \\ p_t &= (100 \text{ kPa})[1+(0.2)(0.101)^2]^{3.5} \\ &= 100.7 \text{ kPa} \\ T_t &= T\left(1+\frac{k-1}{2}M^2\right) \\ &= 873\times(1+0.2\times 0.101^2) \\ T_t &= 875 \text{ K}\end{aligned}$$

Laval nozzle flow rate equation (assume sonic flow)

$$\begin{aligned}\dot m &= 0.685 p_t A_*/\sqrt{RT_t} \\ &= 0.685(100.7\times 10^3 \text{ Pa})(\pi/4)(2\times 10^{-3}\text{ m})^2/\sqrt{(287 \text{ J/kg-K})(875\text{K})} \\ &\quad \dot m = 0.000432 \text{ kg/s}\end{aligned}$$

This is higher than mass flux entering for isokinetic sampling, so flow must be subsonic at constriction and solution must be found iteratively. Assume M at constriction and solve for $\dot{m}$ in terms of M.
Exit conditions

$$\begin{aligned} \rho_e &= \rho_t(1+((k-1)/2)M^2)^{(-1/(k-1)} = \rho_t(1+0.2M_e^2)^{-2.5} \\ c_e &= c_t(1+((k-1)/2)M^2)^{-1/2} = c_t(1+0.2M_e^2)^{-0.5} \end{aligned}$$

Flow rate

$$\begin{aligned} \dot{m} &= \rho_e A_e c_e M_e \\ &= A_e M_e \rho_t c_t (1+0.2M_e^2)^{-3} \end{aligned}$$

Total properties

$$\begin{aligned} \rho_t &= \rho\left(1+\frac{k-1}{2}M^2\right)^{1/(k-1)} \\ \rho_t &= (0.399\ \text{kg/m}^3)[1+(0.2)(0.101)^2]^{2.5} = 0.401\ \text{kg/m}^3 \end{aligned}$$

$$\begin{aligned} c_t &= \sqrt{kRT_t} \\ &= \sqrt{(1.4)(287\ \text{J/kg-K})(875\ \text{K})} = 593\ \text{m/s} \end{aligned}$$

The mass flow in terms of exit Mach number is

$$\begin{aligned} \dot{m} &= (2\times 10^{-3}\ \text{m})^2\left(\frac{\pi}{4}\right)(0.401\ \text{kg/m}^3)(593\ \text{m/s})M_e(1+0.2M_e^2)^{-3} \\ &= 7.47\times 10^{-4}\ \text{kg/s}\ M_e(1+0.2M_e^2)^{-3} \end{aligned}$$

Create a table for mass flow as function of exit Mach number.

M_e	$\dot{m}\times 10^4$
0.5	3.22
0.4	2.71
0.45	2.98
0.454	3.004
0.455	3.01 (correct flow rate)

Use Mach number to calculate back pressure

$$\begin{aligned} p_e &= p_b = p_t(1+\frac{k-1}{2}M_e^2)^{-k/(k-1)} \therefore \\ p_b &= (100.7)(1+0.2\times 0.455^2)^{-3.5} \\ & \boxed{p_b = 87.2\ \text{kPa}} \end{aligned}$$

12.43: PROBLEM DEFINITION

Situation: The design of a test system using truncated nozzles to measure performance of compressor rated at 0.05 m^3/s at 830 kPa.

Find: Explain how to carry out the test program to generate a performance curve, pressure vs. flow rate.

Properties: From Table A.2 (EFM 10e), $R_{\text{air}} = 287$ J/kg-K; $k = 1.4$.

SOLUTION A truncated nozzle is attached to a storage tank supplied by the compressor. The temperature and pressure will be measured in the tank. These represent the total conditions. The nozzles will be sonic provided that the tank pressure is greater than $101.3/3.6 = 28.1$ kPa.

Density at standard conditions using ideal Gas Law

$$\rho = \frac{p}{RT} = \frac{101.3}{287 \times 288} = 1.2 \text{ kg/m}^3$$

A mass flow rate of 0.05 m^3/s corresponds to

$$\dot{m} = 0.05 \,\text{m}^3/\,\text{s} \times 1.22 = 0.061 \text{ kg/s}$$

The flow rate is given by

$$\dot{m} = 0.685 \frac{p_t A_*}{\sqrt{RT_t}}$$

Using 830 kPa and a flow rate of 0.05 m^3/s gives a throat area of

$$\begin{aligned} A_* &= \frac{\dot{m}\sqrt{RT_t}}{0.685 p_t} \\ &= \frac{0.061 \times \sqrt{287 \times 288}}{0.685 \times 931,000} \\ &= 2.75 \times 10^{-5} \text{ m}^2 \end{aligned}$$

This area corresponds to an opening of

$$\begin{aligned} D &= \sqrt{\frac{4}{\pi} \times 2.75 \times 10^{-5}} \\ &= 0.0059 \text{ m} = 60 \text{ cm} \end{aligned}$$

REVIEW

1. This would represent the maximum nozzle size. A series of truncated nozzles would be used which would yield mass flows of 1/4, 1/2, and 3/4 of the maximum flow rate. The suggested nozzle diameters would be 27.5 cm, 37.5 cm and 47.5 cm. Another point would be with no flow which represents another data point.

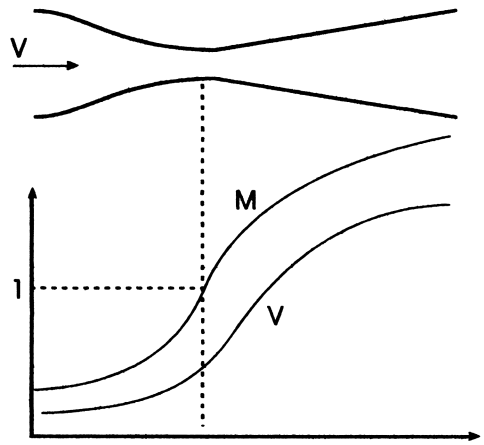
2. Each nozzle would be attached to the tank and the pressure and temperature measured. For each nozzle the pressure in the tank must exceed 28.1 kPa to insure sonic flow in the nozzle. The mass flow rate would be calculated for each nozzle size and these data would provide the pump curve, the variation of pressure with flow rate. More data can be obtained by using more nozzles.

12.44: PROBLEM DEFINITION

Situation: Mach number and velocity variation through a Laval nozzle.

Find: Sketch how Mach number and velocity vary through nozzle.

SOLUTION



The Mach number and velocity continuously increase through a Laval nozzle with the Mach number being unity at the throat (minimum area). In a venturi configuration the velocity would reach a maximum at the minimum area and then decrease in the expansion section.

12.45: PROBLEM DEFINITION

Situation: Definition of expansion ratio in a Laval nozzle.

SOLUTION The expansion ratio is the ratio of the cross-sectional area at the exit to that at the throat. A nozzle with an expansion ratio of 4 means the exit area is four times the throat area.

12.46: PROBLEM DEFINITION

Situation: Variation of flow properties in a Laval nozzle with Mach number and ratio of specific heats.

Find: Develop a computer program with Mach number and ratio of specific heats and outputs: area ratio, static to total pressure ratio, static to total temperature ratio, density to total density ratio and pressure ratio across a normal shock wave. Run the program for $M = 2$ and $k = 1.3$, 1.4 and 1.67.

SOLUTION The following results are obtained from the computer program for a Mach number of 2:

A/A_*	1.69	1.53	1.88
T/T_t	0.555	0.427	0.714
p/p_t	0.128	0.120	0.132
ρ/ρ_t	0.230	0.281	0.186
M_2	0.577	0.607	0.546
p_2/p_1	4.5	4.75	4.27

12.47: PROBLEM DEFINITION

Situation: Mach number in a Laval nozzle varies with area ratio and the ratio of specific heats.

Find: Develop a computer program that with area ratio outputs Mach number. Run the program for an area ratio of 5.

SOLUTION

The following results are obtained for an area ratio of 5:

k	M_{subsonic}	$M_{\text{supersonic}}$
1.4	0.117	3.17
1.67	0.113	3.81
1.31	0.118	2.99

12.48: PROBLEM DEFINITION

Situation: A supersonic wind tunnel with air is designed to have a Mach number of 3, a static pressure of 10 kPa and a static temperature of $-23\,^\circ$C described in the problem statement.

Find: The area ratio and reservoir conditions.

Properties: From Table A.2 (EFM 10e), $k = 1.4$, $R_{\text{air}} = 287$ J/kg-K.

SOLUTION Mach number-area ratio relationship

$$\begin{aligned} A/A_* &= (1/M)[(1+((k-1)/2)M^2)/((k+1)/2)]^{(k+1)/(2(k-1))} \\ &= (1/3)[(1+0.2\times 3^2)/1.2]^3 \\ &\boxed{A/A_* = 4.23} \end{aligned}$$

Total temperature

$$\begin{aligned} T_t &= T\left(1+\frac{k-1}{2}M^2\right) \\ &= 250\text{ K}\times(1+0.2\times 3^2) \\ &\boxed{\text{T}_t = 700\ \text{ K} = 427\,^\circ\text{C}} \end{aligned}$$

Total pressure

$$\begin{aligned} p_t &= p\left(1+\frac{k-1}{2}M^2\right)^{k/(k-1)} \\ &= 10\text{ kPa}\times(1+0.2\times 3^2)^{3.5} \\ &\boxed{p_t = 367.3\text{ kPa}} \end{aligned}$$

12.49: PROBLEM DEFINITION

Situation: Nitrogen in a Laval nozzle expands ideally to a pressure of 25 kPa, abs. from a stagnation pressure of 1 MPa.

Find: a) Nozzle area ratio, b) throat area for a mass flow of 10 kg/s and stagnation temperature of 550 K.

Properties: From Table A.2 (EFM 10e), $k = 1.4$; $R = 297\,\text{J/kgK}$.

SOLUTION

Calculate exit Mach number

$$\begin{aligned} M_e &= \sqrt{(2/(k-1))[(p_t/p_e)^{(k-1)/k} - 1]} \\ &= \sqrt{5[(1,000 \text{ kPa}/25 \text{ kPa})^{0.286} - 1]} \\ &= 3.79 \end{aligned}$$

Mach number-area ratio relationship

$$\begin{aligned} A_e/A_* &= (1/M)[(1 + ((k-1)/2)M^2)/((k+1)/2)]^{(k+1)/(2(k-1))} \\ &= (1/3.79)[(1 + (0.2)(3.79)^2)/1.2]^3 \\ & \boxed{A_e/A_* = 4.45} \end{aligned}$$

Flow rate equation for Laval nozzle

$$\begin{aligned} \dot{m} &= 0.685 p_t A_T/\sqrt{RT_t} \\ A_T &= \dot{m}\sqrt{RT_t}/(0.685 \times p_t) \\ &= 5 \times \sqrt{(297 \text{ J/kg-K})(550 \text{ K})}/(0.685 \times 10^6 \text{ Pa}) \\ &= 0.00295 \text{ m}^2 \\ & \boxed{A_T = 29.5 \text{ cm}^2} \end{aligned}$$

12.50: PROBLEM DEFINITION

Situation: A rocket nozzle with area ratio of 4 and total pressure of 1.3 MPa exhausts to a back pressure of 30 kPa, abs.

Find: The state of exit conditions.

Properties: From Table A.1, (EFM 10e),$k = 1.4$.

SOLUTION From Table A.1:

$$
\begin{aligned}
M_e &\approx 2.94 => p_e/p_t \approx 0.030 \\
\therefore & \;\; p_e = 39 \text{ kPa} \\
\therefore & \;\; p_e > p_b \boxed{\text{under expanded}}
\end{aligned}
$$

12.51: PROBLEM DEFINITION

Situation: A rocket nozzle with area ratio of 4 and total pressure of 1.3 MPa exhausts to a back pressure of 30 kPa, abs.

Find: State of exit conditions.

Properties: $k = 1.2$.

SOLUTION The Mach number-area ratio equation for $k = 1.2$ is

$$\begin{aligned} \frac{A}{A_*} &= \frac{1}{M}\left[\frac{1+\frac{k-1}{2}M^2}{(k+1)/2}\right]^{(k+1)/2(k-1)} \\ &= \frac{1}{M}\left(\frac{1+0.1M^2}{1.1}\right)^{5.5} \end{aligned}$$

Carrying out calculations for area ratio for a range of Mach number yields

M	A/A_*
2.60	3.893
2.61	3.950
2.615	3.976
2.619	3.998
2.620	4.003

From interpolation, the exit Mach number is 2.6195. The exit pressure is

$$\begin{aligned} p &= p_t/(1+\frac{k-1}{2}M^2)^{k/(k-1)} \\ &= 1.3 \text{ MPa}/(1+0.1\times 2.6195^2)^6 \\ &= 56.6 \text{ kPa} \end{aligned}$$

Therefore the nozzle is underexpanded.

12.52: PROBLEM DEFINITION

Situation: A Laval nozzle with expansion ratio of 1.688 exhausts air into 100 kPa back pressure.

Find: (a) Show reservoir pressure must be 782.5 kPa for ideal expansion at Mach 2.
(b) Static pressure and temperature at throat for total temperature of 17 °C.
(c) Exit conditions for reservoir pressure of 700 kPa..
(d) Reservoir pressure for normal shock at exit.

SOLUTION a) $p = p_t$ in reservoir because $V = 0$ in reservoir
From Table A.1 (EFM 10e), $p/p_t = 0.1278$ for $A/A_* = 1.688$ and $M = 2$

$$\begin{aligned} p_t &= p/0.1278 \\ &= 100 \text{ kPa}/0.1278 \\ &\boxed{p_t = 782.5 \text{ kPa}} \end{aligned}$$

b) Throat conditions for $M = 1$:

$$\begin{aligned} p/p_t &= 0.5283 \\ T/T_t &= 0.8333 \\ p &= 0.5283(782.5) \\ &\boxed{p = 413\,\text{kPa}} \\ T &= 0.8333(17 + 273) \\ &= 242\,\text{K} \\ &\boxed{T = -31\,°\text{C}} \end{aligned}$$

c) Conditions for $p_t = 700$ kPa:

$$\begin{aligned} p/p_t &= 0.1278 \\ p &= 0.1278(700) = 89.5 \text{ kPa} \Longrightarrow 89.5 \text{ kPa} < 100 \text{ kPa} \end{aligned}$$

overexpanded exit condition

d) p_t for normal shock at exit: Assume shock exists at $M = 2$. From Table A.1 (EFM 10e) for pressure across normal shock, $p_2/p_1 = 4.5$. Nozzle exhausts to 100 kPa.

$$\begin{aligned} p_{M=2} &= 100 \text{ kPa}/4.5 = 22.2 \text{ kPa} \\ p/p_t &= 0.1278 \\ p_t &= p/0.1278 \\ &= 22.2 \text{ kPa}/0.1278 \\ &\boxed{p_t = 174 \text{ kPa}} \end{aligned}$$

12.53: PROBLEM DEFINITION

Situation: Flow in a Laval nozzle with air.

Find: (a) Mach number and (b) area ratio where dynamic pressure is maximized.

Properties: $k = 1.4$.

SOLUTION Find Mach number

$$\begin{aligned}
q &= (k/2)pM^2 \\
&= (k/2)p_t[1 + ((k-1)/2)M^2]^{-k/(k-1)}M^2 \\
\ell nq &= \ell n(kp_t/2) - (k/(k-1))\ell n(1 + ((k-1)/2)M^2) + 2\ell nM \\
(\partial/\partial M)\ell nq &= (1/q)(\partial q/\partial M) \\
&= (-k/(k-1))[1/(1 + ((k-1)/2)M^2)][(k-1)M] + 2/M \\
0 &= [-kM]/[1 + ((k-1)/2)M^2] + (2/M) \\
&= [(-kM^2 + 2 + (k-1)M^2)/[(1 + ((k-1)/2)M^2)M] \\
0 &= 2 - M^2 \\
& \boxed{M = \sqrt{2}}
\end{aligned}$$

Mach number-area ratio relationship

$$\begin{aligned}
A/A_* &= (1/M)[1 + ((k-1)/2)M^2]/[(k+1)/2]^{(k+1)/2(k-1)} \\
&= (1/\sqrt{2})[(1 + 0.2(2))/1.2]^3 \\
& \boxed{A/A_* = 1.12}
\end{aligned}$$

12.54: PROBLEM DEFINITION

Situation: A rocket motor with expansion ratio of 4 operates where back pressure is 30 kPa The chamber pressure and temperature are 1.2 MPa and 3000 °C.

Find: (a) Mach number, pressure and density at exit.
(b) Mass flow rate.
(c) Thrust.
(d) Chamber pressure for ideal expansion.

Properties: $k = 1.2, .R = 400$ J/kg-K

SOLUTION Mach number-area ratio relationship

$$\begin{aligned} A/A_* &= (1/M_e)\left[\frac{1+\frac{k-1}{2}M_e^2}{\frac{k+1}{2}}\right]^{\frac{k+1}{2(k-1)}} \\ &= (1/M_e)((1+0.1\times M_e^2)/1.1)^{5.5} = 4 \end{aligned}$$

a) Solve for M by iteration:

M_e	A/A_*
3.0	6.73
2.5	3.42
2.7	4.45
2.6	3.90
2.62	4.0

$$\therefore \boxed{M_e = 2.62}$$

Total properties
Pressure

$$\begin{aligned} p_e/p_t &= (1+\frac{k-1}{2}M_e^2)^{-k/(k-1)} \\ p_e/p_t &= (1+0.1\times 2.62^2)^{-6} = 0.0434 \\ &\therefore\ p_e = (0.0434)(1.2\times 10^6 \text{ Pa}) \\ &\boxed{p_e = 52.1\times 10^3 \text{ Pa}} \end{aligned}$$

Temperature

$$\begin{aligned} T_e/T_t &= (1+0.1\times 2.62^2)^{-1} = 0.593 \\ T_e &= (3,273 \text{ K}\times 0.593) \\ &= 1,941 \text{ K} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho_e &= p_e/(RT_e) \\ &= (52.1\times 10^3 \text{ Pa})/(400 \text{ J/kg-K}\times 1,941 \text{ K}) \\ &\boxed{\rho_e = 0.0671\text{kg/m}^3} \end{aligned}$$

Speed of sound

$$\begin{aligned} c_e &= \sqrt{kRT} \\ &= \sqrt{(1.2 \times 400 \text{ J/kg-K} \times 1,941 \text{ K})} \\ &= 965 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} V_e &= (965 \text{ m/s})(2.62) \\ & \boxed{V_e = 2528 \text{ m/s}} \end{aligned}$$

b) Flow rate equation

$$\begin{aligned} \dot{m} &= \rho_e A_e V_e \\ &= (0.0671 \text{ kg/m}^3)(4 \times 10^{-2} \text{ m}^2)(2,528 \text{ m/s}) \\ & \boxed{\dot{m} = 6.78 \text{ kg/s}} \end{aligned}$$

c) Thrust

$$\begin{aligned} T &= \dot{m}V_e + (p_e - p_0)A_e \\ &= (6.78 \text{ kg/s})(2,528 \text{ m/s}) + (52.1 - 30) \times 10^3 \text{ Pa} \times 4 \times 10^{-2} \text{ m}^2 \\ & \boxed{T = 18.0 \text{ kN}} \end{aligned}$$

d) Chamber pressure

$$\begin{aligned} p_t &= 30 \text{ kPa}/0.0434 \\ & \boxed{p_t = 691 \text{ kPa}} \end{aligned}$$

Mass flow rate and thrust

$$\begin{aligned} \rho_e &= p_e/RT_e \\ &= 30 \times 10^3 \text{ Pa}/(400 \text{ J/kg-K} \times 1941 \text{ K}) \\ &= 0.0386 \text{ kg/m}^3 \\ \dot{m} &= \rho_e A_e V_e \\ &= 0.0386 \text{ kg/m}^3 \times 4 \times 10^{-2} \text{ m}^2 \times 2528 \text{ m/s} \\ &= 3.90 \text{ kg/s} \\ T &= (3.90 \text{ kg/s})(2,528 \text{ m/s}) \\ & \boxed{T = 9.86 \text{ kN}} \end{aligned}$$

12.55: PROBLEM DEFINITION

Situation: A rocket motor with 2 MPa and 3300 K chamber pressure and temperature and 10 cm^2 throat area designed for sea-level operation where back pressure is 100 kPa.

Find: (a) Nozzle expansion ratio for ideal expansion and thrust.
(b) Thrust if expansion ratio reduced by 10%.

Properties: $k = 1.2, .R = 400$ J/kg-K

SOLUTION Total pressure ratio and exit Mach number

$$\begin{aligned} p_t/p_e &= (1 + ((k-1)/2)M_e^2)^{k/(k-1)} \\ &= (1 + 0.1M_e^2)^6 \\ M_e &= \sqrt{10[(p_t/p_e)^{1/6} - 1]} \\ &= \sqrt{10[(2,000/100)^{1/6} - 1]} \\ &= 2.54 \end{aligned}$$

Mach number-area ratio relationship

$$\begin{aligned} A_e/A_* &= (1/M_e)[(1 + \frac{k-1}{2}M_e^2)/\frac{k+1}{2}]^{(k+1)/2(k-1)} \\ &= (1/M_e)[(1 + 0.1M_e^2)/1.1]^{5.5} \\ & \boxed{A_e/A_* = 3.60} \end{aligned}$$

Total properties (temperature)

$$\begin{aligned} T_e &= T_t/(1 + \frac{k-1}{2}M_e^2) \\ T_e &= 3,300 \text{ K}/(1 + (0.1)(2.54)^2) \\ &= 2006 \text{ K} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho_e &= 100 \times 10^3/(400 \times 2,006) \\ &= 0.125 \text{ kg/m}^3 \end{aligned}$$

Speed of sound

$$\begin{aligned} c_e &= \sqrt{(1.2)(400 \text{ J/kg-K})(2006 \text{ K})} \\ &= 981 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned}\dot{m} &= \rho_e A_e V_e \\ &= (0.125)(3.60)(10^{-3})(981)(2.54) \\ &= 1.12 \text{ kg/s}\end{aligned}$$

Thrust equation for ideal expansion

$$\begin{aligned}T &= \dot{m} V_e \\ &= (1.12 \text{ kg/s})(981 \text{ m/s})(2.54) \\ & \boxed{T = 2790 \text{ N}}\end{aligned}$$

(b) Expansion ratio reduced by 10%

$$\begin{aligned}A_e/A_* &= (0.9)(3.60) = 3.24 \\ 3.42 &= (1/M_e)((1 + 0.1M_e^2)/1.1)^{5.5}\end{aligned}$$

Solve by iteration:

M_e	A/A_*
2.4	3.011
2.5	3.420
2.45	3.204
2.455	3.228
2.458	3.241

$$\begin{aligned}&\therefore \quad M_e = 2.46 \\ p_e/p_t &= (1 + 0.1M_e^2)^{-6} = 0.0585 \\ p_e &= (0.0585)(2.0 \times 10^6) = 117 \text{ kPa} \\ T_e &= 3,300/(1 + 0.1 \times 2.46^2) = 2,056 \text{ K}\end{aligned}$$

Speed of sound

$$\begin{aligned}c_e &= \sqrt{kRT_e} \\ &= \sqrt{(1.2)(400)(2056)} \\ &= 993 \text{ m/s}\end{aligned}$$

Thrust

$$\begin{aligned}T &= \dot{m} V_e + (p_e - p_0) A_e \\ &= (1.12 \text{ kg/s})(993 \text{ m/s})(2.46) + (117 - 100) \times 10^3 \text{ Pa} \times 3.24 \times 10^{-3} \text{ m}^2 \\ & \boxed{T = 2790 \text{ N}}\end{aligned}$$

12.56: PROBLEM DEFINITION

Situation: Air flows through Laval nozzle with expansion ratio of 4. The total pressure is 200 kPa and back pressure 100 kPa.

Find: Area ratio where shock occurs in nozzle.

SOLUTION Back pressure/total pressure ratio

$$p_b/p_t = 0.5$$

Solution by iteration:

1. Choose M.
2. Determine the area ratioA/A^* from Table A.1 (EFM 10e).
3. Find ratio of total pressures across shock, p_{t_2}/p_t ,at same Mach number.
4. Because the mass flow rate is the same on each side of shock

$$\dot{m} = 0.685\frac{p_t A_*}{\sqrt{RT_t}}$$

and T_t is constant across shock

$$\frac{p_{t_2}}{p_{t_1}} = \frac{A_{*_1}}{A_{*_2}}$$

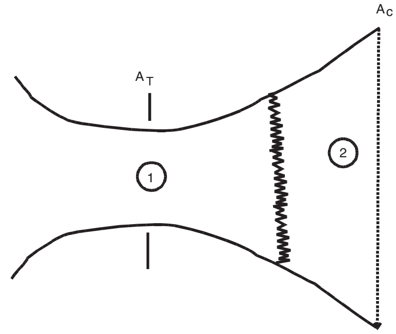
Because $\left(\frac{A_e}{A_*}\right)_1 = 4$ then $\left(\frac{A_e}{A_*}\right)_2 = \left(\frac{A_e}{A_*}\right)_1 \frac{A_{*1}}{A_{*2}} = 4\frac{p_{t2}}{p_{t_1}}$.

5. From Table A.1 for subsonic flow Mach number at exit, M_e.
6. Evaluate p_e/p_{t_1}using

$$\frac{p_e}{p_{t_1}} = \frac{p_e}{p_{t_2}}\frac{p_{t_2}}{p_{t_1}}$$

where p_e/p_{t_2} is available from subsonic table for exit Mach number.

7. Continue table until converge on $p_e/p_{t_1} = 0.5$.



M	A/A_*	p_{t_2}/p_{t_1}	(A_e/A_*)	M_e	p_e/p_{t_1}
2	1.69	0.721	2.88	0.206	0.7
2.5	2.63	0.499	2.00	0.305	0.468
2.4	2.40	0.540	2.16	0.28	0.511
2.43	2.47	0.527	2.11	0.287	0.497
2.425	2.46	0.530	2.12	0.285	0.50

Therefore

$$\boxed{A/A_* = 2.46}$$

12.57: PROBLEM DEFINITION

Situation: A shock wave occurs in a expansion ratio 4 rocket nozzle. Total pressure is 250 kPa and back pressure is 100 kPa. The ratio of specific heats is 1.2. The nozzle has a linear shape with expansion angle of 15°.

Find: Area ratio and location of shock wave.

SOLUTION

Exit pressure and area ratio

$$\begin{aligned} p_b/p_t &= 100/250 = 0.4 \\ A_e/A_* &= 4 \end{aligned}$$

An iterative method must be used to find area ratio at which shock occurs. Table A.1 (EFM 10e) is not applicable since the gas has different ratio of specific heats.

1. Create a subsonic flow table for later using

$$\frac{A}{A_*} = \frac{1}{M}\left(\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2(k-1)}} = \frac{1}{M}\left(\frac{1+0.1M^2}{1.1}\right)^{5.5}$$

Mach number	Area ratio
0.36	1.7652
0.37	1.7243
0.38	1.6858
0.39	1.6494
0.40	1.6151

2. Choose M (supersonic)

3. Determine the area ratioA/A^* from equation for Step 1

4. Find Mach number and pressure ratio across shock at same Mach number.

$$\begin{aligned} M_2 &= \left(\frac{(k-1)M_1^2+2}{2kM_1^2-(k-1)}\right)^{1/2} = \left(\frac{0.2M_1^2+2}{2.4M_1^2-0.2}\right)^{1/2} \\ \frac{p_2}{p_1} &= \frac{1+kM_1^2}{1+kM_2^2} = \frac{1+1.2M_1^2}{1+1.2M_2^2} \end{aligned}$$

and the ratio of total pressures across the shock wave

$$\frac{p_{t_2}}{p_{t_1}} = \frac{p_2}{p_1} \times \left(\frac{1+\frac{k-1}{2}M_2^2}{1+\frac{k-1}{2}M_1^2}\right)^{k/(k-1)} = \frac{p_2}{p_1} \times \left(\frac{1+0.1M_2^2}{1+0.1M_1^2}\right)^6$$

5. Because the mass flow rate is the same on each side of shock

$$\dot{m} = 0.685\frac{p_t A_*}{\sqrt{RT_t}}$$

and T_t is constant across shock

$$\frac{p_{t_2}}{p_{t_1}} = \frac{A_{*1}}{A_{*2}}$$

Because $\left(\frac{A_e}{A_*}\right)_1 = 4$ then $\left(\frac{A_e}{A_*}\right)_2 = \left(\frac{A_e}{A_*}\right)_1 \frac{A_{*1}}{A_{*2}} = 4\frac{p_{t2}}{p_{t_1}}$.

6. Solve iteratively for Mach number at exit, M_e,using table generated in step 1. Then

$$\frac{p_e}{p_{t_2}} = (1+\frac{k-1}{2}M_e^2)^{-1} = (1+0.1M_e^2)^{-1}$$

7. Evaluate p_e/p_{t_1} using

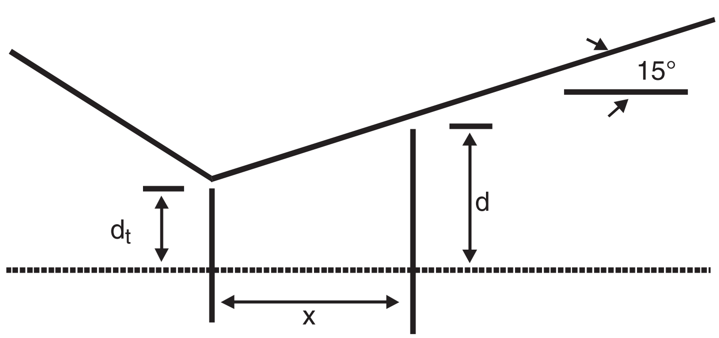
$$\frac{p_e}{p_{t_1}} = \frac{p_e}{p_{t_2}}\frac{p_{t_2}}{p_{t_1}}$$

8. Continue table until converge on $p_e/p_{t_1} = 0.4$.

M	A/A_*	p_{t_2}/p_{t_1}	$(A_e/A_*)_2$	M_e	p_e/p_{t_1}
2.0	1.88	0.600	2.40	0.255	0.596
2.4	3.01	0.403	1.61	0.400	0.396
2.35	2.82	0.425	1.70	0.366	0.419
2.39	2.97	0.407	1.63	0.396	0.400

The area ratio of the shock is $\boxed{A/A_* = 2.97}$.
From geometry: $d = d_t + 2 \times \tan 15°$

$$\begin{aligned}
d/d_t &= 1 + (2x/d_t)\tan 15° \\
A/A_* &= (d/d_t)^2 = 2.97 \\
&= [1 + (2x/d_t)(0.268)]^2 \\
&= [1 + (0.536x/d_t)]^2 \\
\therefore x/d_t &= 1.349 \\
x &= (1.349)(4) \\
&\boxed{x = 5.40 \text{ cm}}
\end{aligned}$$



12.58: PROBLEM DEFINITION

Situation: A normal shock wave occurs in a nozzle with hydrogen at area ratio of 5.

Find: Entropy increase.

Properties: From Table A.2 (EFM 10e), $k = 1.41$.

SOLUTION

$$\frac{A}{A_*} = (1/M)((1 + 0.205 \times M^2)/1.205)^{2.939}$$

Solve iteratively for M (to give $A/A_* = 5$)

M	A/A_*
3.0	2.61
3.1	4.57
3.15	4.79
3.196	5.0

$$\begin{aligned}
M_1 &= 3.196 \\
M_2^2 &= ((k-1)M_1^2 + 2)/(2kM_1^2 - (k-1)) \\
M_2 &= 0.4668 \\
p_2/p_1 &= (1 + kM_1^2)/(1 + kM_2^2) = 11.77 \\
p_t/p_1 &= (1 + ((k-1)/2)M_1^2)^{k/(k-1)} = 48.56 \\
p_t/p_2 &= 1.162 \\
p_{t_2}/p_{t_1} &= (p_{t_2}/p_2)(p_2/p_1)(p_1/p_{t_1}) = 0.2816 \\
\Delta s &= R\ln(p_{t_1}/p_{t_2}) = 4127 \text{ J/kgK} \ln(1/0.2816)
\end{aligned}$$

$$\boxed{\Delta s = 5230 \text{ J/kgK}}$$

12.59: PROBLEM DEFINITION

Situation: Supersonic airflow with Mach number 2.1 enters in a variable-area channel with shock wave at throat. Inlet area is 100 cm^2, throat area is 0.75 cm^2 and downstream area is 120 cm^2. Upstream static pressure is 65 kPa.

Find: At station 3, find (a) Mach number, (b) static pressure and (c) stagnation pressure.

Properties: Air with $k = 1.4$.

SOLUTION From Table A.1 (EFM 10e), $M = 2.1$, $A/A_* = 1.837$, $p/p_t = 0.1094$

$$\begin{aligned} A_* &= 100/1.837 = 54.4 \text{ cm}^2 \\ p_t &= 65/0.1094 = 594 \text{ kPa} \\ A_2/A_* &= 75/54.4 = 1.379 \\ M &= 1.74 \rightarrow p_2/p_t = 0.1904 \rightarrow p_2 = 0.1904(594 \text{ kPa}) = 113 \text{ kPa} \end{aligned}$$

after shock, $M_2 = 0.630$; $p_2 = 3.377(113 \text{ kPa}) = 382 \text{ kPa}$

$$\begin{aligned} A_2/A_* &= (1/M)((1+0.2M^2)/1.2)^3 \\ &= 1.155 \\ p_t/p_2 &= (1+0.2M^2)^{3.5} = 1.307 \\ A_* &= 75/1.155 = 64.9;\ p_t = 382(1.307) = \boxed{499 \text{ kPa}} \\ A_3/A_* &= 120/64.9 = 1.849;\ \text{from Table A.1 (EFM 10e)}, \boxed{M_3 = 0.336} \\ p_3/p_t &= 0.9245;\ p_3 = 0.9245(499) = \boxed{461 \text{ kPa}} \end{aligned}$$

12.60: PROBLEM DEFINITION

Situation: A shock wave in air exists in a Laval nozzle where cross-sectional area is 120 cm^2.The inlet Mach number, area and static pressure are 0.3, 200 cm^2 and 400 kPa. Exit area is 140 cm^2.

Find: Back pressure for shock position.

SOLUTION

From Table A.1 (EFM 10e)

$$\begin{aligned} M &= 0.3 \\ A/A_* &= 2.0351 \\ p/p_t &= 0.9395 \end{aligned}$$

Therefore

$$\begin{aligned} A_* &= 200 \text{ cm}^2/2.0351 = 98.3 \text{ cm}^2 \\ p_t &= 400 \text{ kPa}/0.9395 = 426 \text{ kPa} \end{aligned}$$

The area ratio at the shock location

$$A_s/A_* = 120/98.3 = 1.2208$$

By interpolation from Table A.1 (EFM 10e):

$$\begin{aligned} M_{s1} &= 1.562;\ p_1/p_t = 0.2490 \rightarrow p_1 = 0.249(426) = 106 \text{ kPa} \\ M_{s2} &= 0.680;\ p_{s2}/p_1 = 2.679 \rightarrow p_{s2} = 2.679(106) = 284 \text{ kPa} \\ A_s/A_{*2} &= 1.1097 \rightarrow A_{*2} = 120/1.1097 = 108 \text{ cm}^2 \\ p_{s2}/p_{t2} &= 0.7338;\ p_{t2} = 284/0.7338 = 387 \text{ kPa} \\ A_2/A_{*2} &= 140/108 = 1.296 \rightarrow M_2 = 0.525 \\ p_2/p_{t2} &= 0.8288 \\ p_2 &= 0.8288(387 \text{ kPa}) \\ & \boxed{p_2 = 321 \text{ kPa}} \end{aligned}$$

12.61: PROBLEM DEFINITION

Situation: Design of a supersonic wind tunnel with a Mach number of 1.5 in a 5 cm by 5 cm test section. The tunnel operates by drawdown into a vacuum tank and must operate for 30 seconds.

Find: Carry out a preliminary design of a the system.

SOLUTION The area of the test section is

$$A_T = 0.05 \times 0.05 = 0.0025 \text{ m}^2$$

From Table A.1 (EFM 10e), the conditions for a Mach number of 1.5 are

$$p/p_t = 0.2724, \quad T/T_t = 0.6897 \quad A/A_* = 1.176$$

The area of the throat is

$$A_* = 0.0025/1.176 = 0.002125 \text{ m}^2$$

Since the air is being drawn in from the atmosphere, the total pressure and total pressure are 293 K and 100 kPa. The static temperature and pressure at the test section will be

$$T = 0.6897 \times 293 = 202 \text{ K}, \;\; p = 0.2724 \times 100 = 27.24 \text{ kPa}$$

The speed of sound and velocity in the test section is

$$\begin{aligned} c &= \sqrt{kRT} = \sqrt{1.4 \times 287 \times 202} = 285 \text{ m/s} \\ v &= 1.5 \times 285 = 427 \text{ m/s} \end{aligned}$$

The mass flow rate is obtained using

$$\begin{aligned} \dot{m} &= 0.685 \frac{p_t A_*}{\sqrt{RT_t}} \\ &= 0.685 \frac{10^5 \times 0.002125}{\sqrt{287 \times 293}} \\ &= 0.502 \text{ kg/s} \end{aligned}$$

The pressure and temperature in the vacuum tank can be analyzed using the relationships for an open, unsteady system. The system consists of a volume (the vacuum tank) and an inlet coming from the test section. In this case, the first law of thermodynamics gives

$$m_2 u_2 - m_1 u_1 = m_{in}(h_{in} + v_{in}^2/2) + Q_2$$

Assume that the heat transfer is negligible and that the tank is initially evacuated. Then

$$m_2 u_2 = m_2(h_{in} + v_{in}^2/2)$$

since $m_{in} = m_2$. Thus the temperature in the tank will be constant and given by

$$\begin{aligned} c_v T &= c_p T_{in} + v_{in}^2/2 \\ 717 \times T &= 1004 \times 202 + 427^2/2 \\ T &= 410 \text{ K} \end{aligned}$$

The continuity equation applied to the vacuum tank is

$$V\frac{d\rho}{dt} = \dot{m}$$

The density from the ideal gas law is

$$\rho = \frac{p}{RT}$$

which gives

$$V\frac{dp}{dt} = \dot{m}RT$$

or

$$V = \frac{\dot{m}RT}{dp/dt}$$

Assume the final pressure in the tank is the pressure in the test section. Thus the rate of change of pressure will be

$$\frac{dp}{dt} = \frac{27.24}{30} = 0.908 \text{ kPa/s}$$

The volume of the tank would then be

$$\begin{aligned} V &= \frac{0.502 \times 0.287 \times 410}{0.908} \\ &= 65 \text{ m}^3 \end{aligned}$$

This would be a spherical tank with a diameter of

$$D = \sqrt[3]{\frac{6V}{\pi}} = 5.0 \text{ m}$$

REVIEW

1. The tank volume could be reduced if the channel was narrowed after the test section to reduce the Mach number and increase the pressure. This would reduce the temperature in the tank and increase the required rate of pressure increase.

2. The tunnel would be designed to have a contour between the throat and test section to generate a uniform velocity profile. Also a butterfly valve would have to be used to open the channel in minium time.