Problem 13.1 no solution provided; answers will vary.

13.2: PROBLEM DEFINITION

Situation: A straw (stagnation tube) and a water-filled, u-tube manometer are used to measure the speed of an automobile.

Find:
a. sketch the apparatus.
b. determine the lowest speed that can be measured.

Assumptions:

- The straw will be situated far from the car body. Thus, the free stream velocity will be measured; not the velocity that is influenced by the body of the car.
- A 2 mm water column deflection is measurable.

Properties:
Water (20 °C), Table A.5, $\gamma = 9790 \text{ N/m}^3$.
Air (20 °C), Table A.3, $\rho = 1.2 \text{ kg/m}^3$.

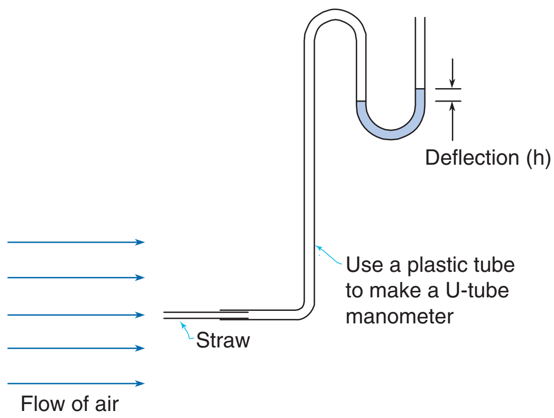
PLAN

1. To develop a concept design, select materials that are commonly available. Also, consider a design that can be built quickly and adapted to use in an automobile.
2. To find the minimum velocity, determine the relationship between the deflection of the water column and the speed of the air.

SOLUTION

1. Concept design.

- The straw will be positioned outside a window.
- The air will travel at speed V_o relative to the car.
- The speed of the car equals V_o.
- The deflection h will be measured by a person riding in the car.
- A plastic tube will allow the stagnation tube to be placed outside the car window and the manometer to be placed inside the car.
- The stagnation tube can be (if needed) connected to a rod in order to position the tube away from the car body.



Problem 13.3

2. Derivation

- Bernoulli equation (point 1 located 10 cm in front of the straw; point 2 at tip of straw)

$$p_1 + \frac{\rho V_1^2}{2} = p_2 + \frac{\rho V_2^2}{2}$$

$$0 + \frac{\rho_{\text{air}} V_o^2}{2} = p_2 + 0 \tag{1}$$

- Hydrostatic equation (apply to water column)

$$p_2 = \gamma_{H20} h \tag{2}$$

- Combine Eqs. (1) and (2)

$$\begin{aligned} \frac{\rho_{\text{air}} V_o^2}{2} &= \gamma_{H20} h \\ V_o &= \sqrt{\frac{2\gamma_{H20} h}{\rho_{\text{air}}}} \\ &= \sqrt{\frac{2\,(9790\,\text{N/m}^3)\,(0.002\,\text{m})}{(1.2\,\text{kg/m}^3)}} \\ &= 5.7\,\text{m/s} \end{aligned}$$

$$\boxed{\text{Minimum speed} = 5.7\,\text{m/s}}$$

REVIEW

- Measurements need to be corrected for head winds and tail winds.
- A similar device is used in airplanes to measure airspeed.

13.3: PROBLEM DEFINITION

Situation: A stagnation tube ($d = 1$ mm) is used to measure air speed.

Find: Velocity such that the measurement error is $\leq 2.5\%$.

Properties: $\nu = 1.46 \times 10^{-5}$ m^2/s.

SOLUTION Algebra using the coefficient of pressure gives

$$V_o = \sqrt{2\Delta p/(\rho C_p)}$$

The allowable error is 2.5%, thus

$$V_o = \sqrt{\frac{2\Delta p}{\rho C_p}} = (1 - 0.025)\sqrt{\frac{2\Delta p}{\rho}}$$

Thus

$$\begin{aligned} \sqrt{\frac{1}{C_p}} &= 0.975 \\ \frac{1}{C_p} &= 0.975^2 \\ C_p &= \frac{1}{0.975^2} = 1.052 \end{aligned}$$

Thus when $C_p \approx 1.05$, there will be a 2.5% error in V_o.

From Fig. 13.1, when $C_p = 1.05$, then Re ≈ 35

$$\begin{aligned} \frac{V_o d}{\nu} &= \text{Re} \\ \frac{V_o d}{\nu} &= 35 \\ V_o &= \frac{35\nu}{d} \\ &= \frac{35 \times (1.46 \times 10^{-5}\,\text{m}^2/\,\text{s})}{0.001\,\text{m}} \\ &= 0.511\,\text{m}/\,\text{s} \end{aligned}$$

$$\boxed{V_o = 0.511\,\text{m}/\,\text{s}}$$

13.4: PROBLEM DEFINITION

Situation: A stagnation tube ($d = 1.5$ mm) is used to measure the speed of water.

Find: Velocity such that the measurement error is $\leq 1\%$.

SOLUTION Algebra using the coefficient of pressure gives $V_o = \sqrt{2\Delta p/(\rho C_p)}$. The allowable error is 1%, thus

$$V_o = \sqrt{\frac{2\Delta p}{\rho C_p}} = 0.99\sqrt{\frac{2\Delta p}{\rho}}$$

This simplifies to

$$\begin{aligned} \sqrt{\frac{1}{C_p}} &= 0.99 \\ \frac{1}{C_p} &= 0.99 \\ C_p &= \frac{1}{0.99^2} = 1.020 \end{aligned}$$

Thus when $C_p \approx 1.02$, there will be a 1% error in V_o.

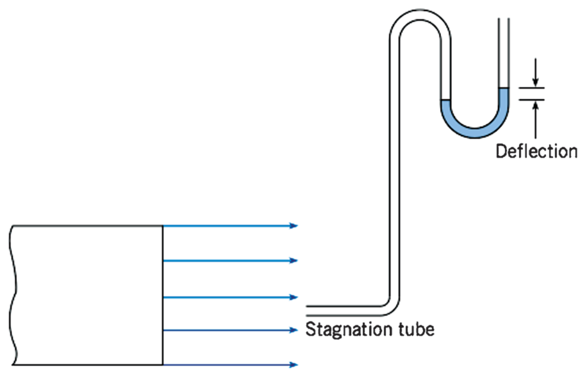
From Fig. 13.1, when $C_p = 1.02$, then $\text{Re} \approx 60$. Thus

$$\begin{aligned} \text{Re} &= \frac{Vd}{\nu} = 60 \\ V &= \frac{60\nu}{d} \\ &= \frac{60 \times (10^{-6}\,\text{m}^2/\,\text{s})}{0.0015\,\text{m}} \\ &= 0.04\,\text{m}/\,\text{s} \end{aligned}$$

$$\boxed{V \geq 0.04 \text{ m/s}}$$

13.5: PROBLEM DEFINITION

Situation:
A stagnation tube ($d = 2$ mm) is used to measure air speed.
Manometer deflection is 1 mm-H_2O.



Find: Air Velocity: V

Assumptions: Neglect viscous effects

SOLUTION

Bernoulli equation applied to a stagnation tube

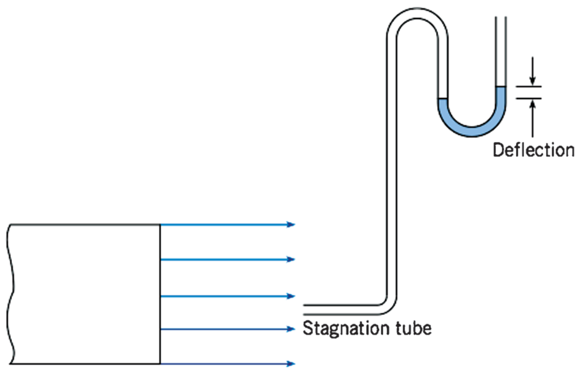
$$V = \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2\gamma_w \Delta h}{\rho}} = \sqrt{\frac{2\,(9810\,\mathrm{N/\,m^3})\,(0.001\,\mathrm{m})}{1.25\,\mathrm{kg/\,m^3}}}$$

$$\boxed{V = 3.96\,\mathrm{m/\,s}}$$

Note that the diameter d does not matter when determining the velocity with a stagnation tube.

13.6: PROBLEM DEFINITION

Situation:
A stagnation tube ($d = 2$ mm) is used to measure air speed
$V = 24$ m/s



Find: Deflection on a water manometer: Δh

Assumptions: Neglect viscous effects

Properties: For air, $\nu = 1.4 \times 10^{-5}$ m^2/s.

SOLUTION

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{98,000}{287 \times (273 + 10)} \\ &= 1.21\,\text{kg/m}^3\end{aligned}$$

Bernoulli equation applied to a stagnation tube

$$\Delta p = \rho V^2/2 \qquad (1)$$

Manometer equation

$$\Delta p = \gamma_{\text{H}_2\text{O}} \Delta h \qquad (2)$$

Combine Eqs. (1) and (2)

$$\begin{aligned}\gamma_{\text{H}_2\text{O}} \Delta h &= \rho V^2/2 \\ \Delta h &= \frac{\rho V^2}{2\gamma_{\text{H}_2\text{O}}} = \frac{(1.21\,\text{kg/m}^3)\,(24\,\text{m/s})^2}{2\,(9810\,\text{N/m}^3)}\end{aligned}$$

$$\boxed{\Delta h = 35.5\,\text{mm}}$$

13.7: PROBLEM DEFINITION

Situation:

A stagnation tube ($d = 2$ mm) is used to measure air speed.

Air kinematic viscosity is 1.55×10^{-5}

Find: Error in velocity if $C_p = 1$ is used for the calculation.

Properties: Stagnation pressure is $\Delta p = 5$ Pa.

PLAN Calculate density of air by applying the ideal gas law. Calculate speed of air by applying the Bernoulli equation to a stagnation tube. Then calculate Reynolds number in order to check C_p.

SOLUTION Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{100,000}{287 \times 298} \\ &= 1.17 \text{ kg/m}^3\end{aligned}$$

Bernoulli equation applied to a stagnation tube

$$\begin{aligned}V &= \sqrt{\frac{2\Delta p}{\rho}} \\ &= \sqrt{\frac{2 \times 5}{1.17}} \\ &= 2.92 \text{ m/s}\end{aligned}$$

Reynolds number

$$\begin{aligned}\text{Re} &= \frac{Vd}{\nu} \\ &= \frac{2.92 \times 0.002}{1.55 \times 10^{-5}} \\ &= 377\end{aligned}$$

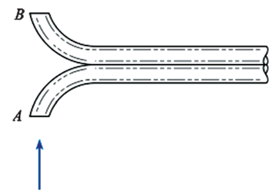
Thus, $C_p = 1.002$

$$\% \text{ error} = (1 - 1/\sqrt{1.002}) \times 100$$

$$\boxed{\% \text{ error} = 0.1\%}$$

13.8: PROBLEM DEFINITION

Situation: A probe for measuring velocity of a stack gas is described in the problem statement.



Find: Stack gas velocity: V_o

SOLUTION Pressure coefficient

$$\begin{aligned} C_p &= 1.4 = \Delta p/(\rho V_0^2/2) \\ \text{Thus } V_0 &= \sqrt{\frac{2\Delta p}{1.4\rho}} \end{aligned}$$

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{100,000}{410 \times 573} \\ &= 0.426\,\mathrm{kg/m^3} \end{aligned}$$

Calculate pressure difference

$$\begin{aligned} \Delta p &= 0.01 \text{ m} \times 9810 \frac{\mathrm{N}}{\mathrm{m}^3} \\ &= 98.1 \text{ Pa} \end{aligned}$$

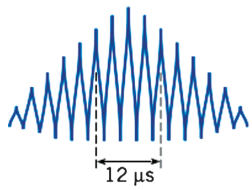
Substituting values

$$\begin{aligned} V_0 &= \sqrt{\frac{2\Delta p}{1.4\rho}} \\ &= \sqrt{\frac{2 \times 98.1}{1.4 \times 0.426}} \\ &\boxed{V_0 = 18.1 \text{ m/s}} \end{aligned}$$

Problem 13.9 no solution provided

13.10: PROBLEM DEFINITION

Situation:
Velocity of air is measured with an LDV.
$\lambda = 4880$ Å, $2\phi = 20°$.
On the Doppler burst, 5 peaks occur in 12 μs.



Find: Velocity (m/s).

SOLUTION

1. Fringe spacing

$$\begin{aligned}\Delta x &= \frac{\lambda}{2\sin\phi} \\ &= \frac{4880 \times 10^{-10}}{2 \times \sin 10°} \\ &= 1.405 \times 10^{-6}\,\mathrm{m}\end{aligned}$$

2. Velocity

$$\begin{aligned}\Delta t &= 12\ \mu\mathrm{s}/4 = 3\ \mu\mathrm{s} \\ V &= \frac{\Delta x}{\Delta t} \\ &= \frac{1.405 \times 10^{-6}\,\mathrm{m}}{3 \times 10^{-6}\,\mathrm{s}}\end{aligned}$$

$$\boxed{V = 0.468\ \mathrm{m/s}}$$

13.11: PROBLEM DEFINITION

Situation:

Classify the following devices as to whether they are used to measure velocity (V), pressure (P), or discharge (Q).
a) hot-wire anemometer
b) venturi meter
c) differential manometer
d) orifice meter
e) stagnation tube
f) rotameter
g) ultrasonic flow meter
h) Bourdon-tube gage
i) weir
j) laser-Doppler anemometer

SOLUTION

The following devices are classified with a V, P or Q to indicated whether they are used to measure
velocity (V), pressure (P), or discharge (Q).
V - hot-wire anemometer
Q - venturi meter
P - differential manometer
Q - orifice meter
P - stagnation tube
Q - rotameter
Q - ultrasonic flow meter
P - Bourdon-tube gage
Q - weir
V - laser-Doppler anemometer

Problem 13.12 no solution provided; answers will vary

13.13: PROBLEM DEFINITION

Situation: In 3 minutes, 8 kN of water flows into a weigh tank.

Find: Discharge (in m^3/s).

Properties: Water (20 °C), Table A.5, $\gamma = 9790\,N/m^3$.

SOLUTION

1. Weight per unit time

$$\begin{aligned}\dot{W} &= \frac{\Delta W}{\Delta t} \\ &= \frac{8000\,N}{3 \times 60\,s} = 44.4\,N/s\end{aligned}$$

2. Flow rate

$$\begin{aligned}Q &= \frac{\dot{W}}{\gamma} \\ &= \frac{44.4\,N/s}{9790\,N/m^3}\end{aligned}$$

$$\boxed{Q = 4.54 \times 10^{-3}\ m^3/s}$$

13.14: PROBLEM DEFINITION

Situation: In 6 minutes, 67 m^3 of water flows into a weigh tank.

Find: Discharge: Q in units of m^3/s.

SOLUTION

$$\begin{aligned} Q &= \frac{V\!\!\!-}{t} \\ &= \frac{67}{360} \end{aligned}$$

$$\boxed{Q = 0.186 \ \mathrm{m^3/s}}$$

13.15: PROBLEM DEFINITION

Situation: Velocity data in a 24 cm oil pipe are given in the problem statement.

Find:
(a) Discharge.
(b) Mean velocity.
(c)Ratio of maximum to minimum velocity.

SOLUTION

Numerical integration

r(m)	V(m/s)	$2\pi V r$	Local Q (by trapezoidal rule)
0	8.7	0	
0.01	8.6	0.54	0.0027
0.02	8.4	1.06	0.0080
0.03	8.2	1.55	0.0130
0.04	7.7	1.94	0.0175
0.05	7.2	2.26	0.0210
0.06	6.5	2.45	0.0236
0.07	5.8	2.55	0.0250
0.08	4.9	2.46	0.0250
0.09	3.8	2.15	0.0231
1.10	2.5	1.57	0.0186
0.105	1.9	1.25	0.0070
0.11	1.4	0.97	0.0056
0.115	0.7	0.51	0.0037
0.12	0	0	0.0013

Summing the values in the last column in the above table gives $Q = 0.196$ m^3/s. Then,

$$\begin{aligned} V_{\text{mean}} &= Q/A \\ &= 0.196/(\frac{\pi}{4}(0.24)^2) \end{aligned}$$

$$\boxed{V_{\text{mean}} = 4.33\,\text{m/s}}$$

Velocity ratio

$$V_{\max}/V_{\text{mean}} = 8.7/4.33$$

$$\boxed{V_{\max}/V_{\text{mean}} = 2}$$

This ratio indicates the flow is laminar. The discharge is

$$\boxed{Q = 0.196\,\text{m}^3/\text{s}}$$

13.16: PROBLEM DEFINITION

Situation: Velocity data in a 40 cm circular air duct are given in the problem statement.
$p = 98.6$ kPa, $T = 21\,°\text{C}$

Find: (a) Flow rate: Q in m^3/s and m^3/min.
(b) Ratio of maximum to mean velocity.
(c) Whether the flow is laminar or turbulent.
(d) Mass flow rate: $\dot{m}$.

PLAN Perform numerical integration to find flow rate (Q). Apply the ideal gas law to calculate density. Find mass flow rate using $\dot{m} = \rho Q$.

SOLUTION Numerical integration

y(cm)	r(cm)	V(m/s)	$2\pi rV$(m²/s)	Local Q (m³/s) (by trapezoidal rule)
0.0	20.0	0	0	
0.25	19.75	21.9	26.8	0.03
0.5	19.5	24.1	29.0	0.07
1.0	19.0	26.8	31.5	0.16
1.5	18.5	28.3	32.4	0.17
2.5	17.5	30.5	33	0.34
3.75	16.25	32.3	32.5	0.42
5.0	15.0	33.5	31.1	0.41
7.5	12.5	35.7	27.6	0.76
10.0	10.0	37.2	23	0.66
12.5	7.5	38.4	17.8	0.53
15.0	5.0	39.3	12.2	0.39
17.5	2.5	40.2	6.2	0.24
20.0	0.0	41.1	0	0.08

Summing values in the last column of the above table gives $\boxed{Q = 4.3\ \text{m}^3/\text{s} \ = \ 258\ \text{m}^3/\text{min}}$

Flow rate equation

$$\begin{aligned}
V_{\text{mean}} &= Q/A \\
&= 4.3/(\frac{\pi}{4}(0.4)^2) \\
&= 34.2\ \text{m/s} \\
V_{\max}/V_{\text{mean}} &= 41.1/34.2 \\
& \boxed{V_{\max}/V_{\text{mean}} = 1.2}
\end{aligned}$$

which suggests turbulent flow.

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{(98.6\,\text{kPa})}{(287)\,(294)} \\ &= 1.17 \text{ kg/m}^3\end{aligned}$$

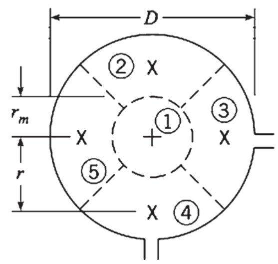
Flow rate

$$\begin{aligned}\dot{m} &= \rho Q \\ &= (1.17)(4.3) \\ & \boxed{\dot{m} = 5.03 \text{ kg/s}}\end{aligned}$$

13.17: PROBLEM DEFINITION

Situation: A heated gas flows through a cylindrical stack—additional information is provided in the problem statement.

Station	Δh (cm)
1	1.2
2	1.1
3	1.1
4	0.9
5	1.05



Find: (a) The ratio r_m/D such that the areas of the five measuring segments are equal.
(b) The location of the probe expressed as a ratio of r_c/D that corresponds to the centroid of the segment
(c) Mass flow rate

SOLUTION

$$\begin{aligned}
\pi r_m^2 &= (\pi/4)\left[(D/2)^2 - r_m^2\right] \\
(r_m/D)^2 &= 1/16 - (r_m/D)^2(1/4) \\
5/4(r_m/D)^2 &= 1/16 \\
5(r_m/D)^2 &= 1/4 \\
r_m/D &= \sqrt{\frac{1}{20}}
\end{aligned}$$

$$\boxed{r_m/D = 0.224}$$

b)

$$r_c A = \int_{0.2236D}^{D/2} [r\sin(\alpha/2)/(\alpha/2)](\pi/4)2r dr = 0.9(\pi/2)(r^3/3)|_{0.2236D}^{0.5D}$$

$$(r_c)(\pi/4)[(D/2)^2 - (0.2236D)^2] = 0.90(\pi/6)[(0.5D)^3 - (0.2236D)^3]$$

$$\boxed{r_c/D = 0.341}$$

c)Ideal gas law

$$\begin{aligned}\rho &= p/(RT) \\ &= 110\times 10^3/(400\times 573) \\ &= 0.480 \text{ kg/m}^3\end{aligned}$$

Bernoulli equation applied to a stagnation tube

$$\begin{aligned}V &= \sqrt{2\Delta p/\rho_g} \\ &= \sqrt{(2)\rho_w g\Delta h/\rho_g} \\ &= \sqrt{(2)(1,000)(9.81)/0.48}\sqrt{\Delta h} \\ &= 202.2\sqrt{\Delta h}\end{aligned}$$

Values for each section are

Station	Δh	V
1	0.012	7.00
2	0.011	6.71
3	0.011	6.71
4	0.009	6.07
5	0.0105	6.55

Mass flow rate is given by

$$\begin{aligned}\dot{m} &= \sum A_{\text{sector}}\rho V_{\text{sector}} = A_T\rho(\sum V/5) \\ &= (\pi 2^2/4)(0.480)(6.61)\end{aligned}$$

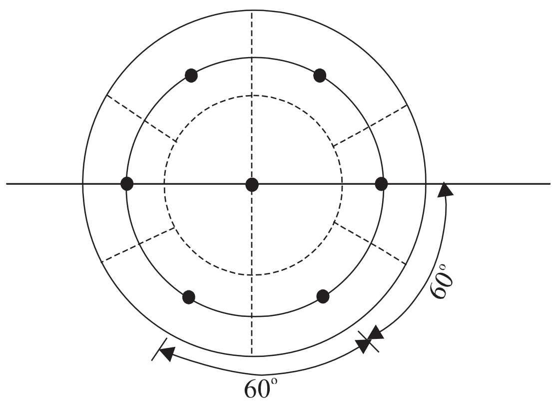
$$\boxed{\dot{m} = 9.96\,\text{kg/ s}}$$

13.18: PROBLEM DEFINITION

Situation: A heated gas flows through a cylindrical stack—additional information is provided in the problem statement.

Find: (a) The ratio r_m/D such that the areas of the measuring segments are equal
(b) The location of the probe expressed as a ratio of r_c/D that corresponds to the centroid of the segment
(c) Mass flow rate

SOLUTION Schematic of measurement locations



a)

$$
\begin{aligned}
\pi r_m^2 &= (\pi/6)[(D/2)^2 - r_m^2] \\
7/6(r_m/D)^2 &= (1/6)(1/4) \\
(r_m/D)^2 &= 1/28 \\
&\boxed{r_m/D = 0.189}
\end{aligned}
$$

b)

$$
\begin{aligned}
r_c A &= 1/6 \int_{0.189D}^{0.5D} [r \sin(\alpha/2)/(\alpha/2)] 2\pi r \, dr \\
(\pi r_c/6)[(D/2)^2 - (r_m)^2] &= 0.955(\pi/3)(r^3/3)|_{0.189D}^{0.50D} \\
r_c(0.5^2 - 0.189^2) &= 0.955(6/9)[0.5^3 - 0.189^3]D \\
r_c/D &= (0.955)6(0.118)/(9(0.2143)) \\
&\boxed{r_c/D = 0.351}
\end{aligned}
$$

c)

$$
\begin{aligned}
\rho &= p/RT = 115 \times 10^3/((420)(250 + 273)) = 0.523 \text{ kg/m}^3 \\
V &= \sqrt{2g\rho_w \Delta h/\rho_g} = \sqrt{(2)(9.81)(1,000)/0.523}\sqrt{\Delta h} = 193.7\sqrt{\Delta h}
\end{aligned}
$$

Calculating velocity from Δh data gives

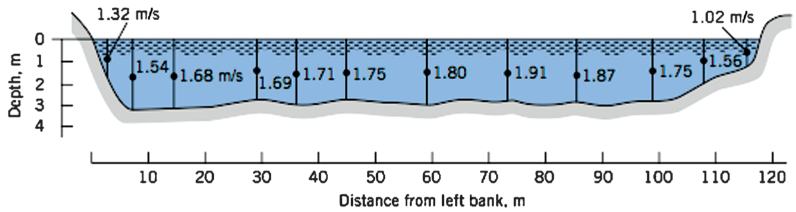
Station	Δh(mm)	V
1	8.2	17.54
2	8.6	17.96
3	8.2	17.54
4	8.9	18.27
5	8.0	17.32
6	8.5	17.86
7	8.4	17.75

From the above table, $V_{avg} = 17.75$ m/s, Then
Flow rate equation

$$\begin{aligned}\dot{m} &= (\pi D^2/4)\rho V_{\text{avg.}} \\ &= ((\pi)(1.5)^2/4)(0.523)(17.75) \\ &\boxed{\dot{m} = 16.4 \ \text{kg/s}}\end{aligned}$$

13.19: PROBLEM DEFINITION

Situation: Velocity data for a river is described in the problem statement.



Find: Discharge: Q

SOLUTION Flow rate equation

$$Q = \sum V_i A_i$$

V	A	VA
1.32 m/s	7.6 m^2	10.0
1.54	21.7	33.4
1.68	18.0	30.2
1.69	33.0	55.8
1.71	24.0	41.0
1.75	39.0	68.2
1.80	42.0	75.6
1.91	39.0	74.5
1.87	37.2	69.6
1.75	30.8	53.9
1.56	18.4	28.7
1.02	8.0	8.2

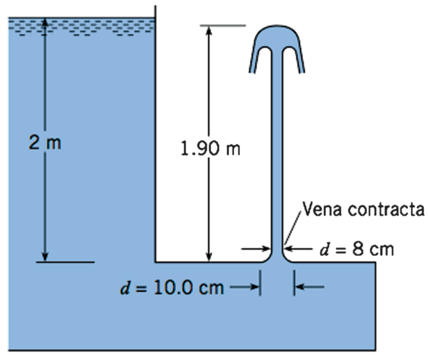
Summing the last column gives

$$\boxed{Q = 549 \text{ m}^3/\text{s}}$$

Problem 13.20 no solution provided

13.21: PROBLEM DEFINITION

Situation: Water jets out of an orifice.



Find: Coefficients for an orifice: C_v, C_c, C_d.

Assumptions: $V_j = \sqrt{2g \times 1.90}$

SOLUTION

$$\begin{aligned} C_v &= V_j/V_{\text{theory}} = \sqrt{2g \times 1.90}/\sqrt{2g \times 2} \\ C_v &= \sqrt{1.90/2.0} \end{aligned}$$

$$\boxed{C_v = 0.975}$$

$$C_c = A_j/A_0 = (8/10)^2$$

$$\boxed{C_c = 0.640}$$

$$C_d = C_v C_c = 0.975 \times 0.64$$

$$\boxed{C_d = 0.624}$$

13.22: PROBLEM DEFINITION

Situation:
A fluid jet discharges from a 4 cm orifice.
At the vena contracta, $d = 3.7$ cm.

Find: Coefficient of contraction: C_c

SOLUTION

$$\begin{aligned} C_c &= A_j/A_0 \\ &= (3.7/4)^2 \\ &\boxed{C_c = 0.856} \end{aligned}$$

13.23: PROBLEM DEFINITION

Situation: Geometry of a sharp edged orifice is modified as described in the problem statement.

Find: Is the flow coefficient the same?

SOLUTION If the angle is 90°, the orifice and expected flow pattern is shown below in Fig. A.

A B

We presume that the flow would separate at the sharp edge just as it does for the orifice with a knife edge. Therefore, the flow pattern and flow coefficient K should be the same as with the knife edge.

However, if the orifice were very thick relative to the orifice diameter (Fig. B), then the flow may reattach to the metal of the orifice thus creating a different flow pattern and different flow coefficient K than the knife edge orifice.

13.24: PROBLEM DEFINITION

Situation: Aging changes in an orifice are described in the problem statement.

Find: Explain the changes and how they effect the flow coefficients.

SOLUTION Some of the possible changes that might occur are listed below:

1. Blunting (rounding) of the sharp edge might occur because of erosion or corrosion. This would probably increase the value of the flow coefficient because C_c would probably be increased.

2. Because of corrosion or erosion the face of the orifice might become rough. This would cause the flow next to the face to have less velocity than when it was smooth. With this smaller velocity in a direction toward the axis of the orifice it would seem that there would be less momentum of the fluid to produce contraction of the jet which is formed downstream of the orifice. Therefore, as in case A, it appears that K would increase but the increase would probably be very small.

3. Some sediment might lodge in the low velocity zones next to and upstream of the face of the orifice. The flow approaching the orifice (lower part at least) would not have to change direction as abruptly as without the sediment. Therefore, the C_c would probably be increased for this condition and K would also be increased.

13.25: PROBLEM DEFINITION

Situation:

Water (15 °C, $Q = 0.127\,\mathrm{m^3/s}$) flows through an orifice ($d = 15$ cm) in a pipe ($D = 25$ cm).

A mercury manometer is connected across the orifice.

Find: Manometer deflection

Properties:

Table A.5 (water at 15 °C): $\rho = 999\,\mathrm{kg/m^3}$, $\gamma = 9800\,\mathrm{N/m^3}$, $\nu = 1.14\times10^{-6}\,\mathrm{m^2/s}$.

Table A.4 (mercury at 20 °C): $S = 13.55$.

PLAN Find K, and then apply the orifice equation to find the pressure drop across the orifice meter. Apply the manometer equation to relate the pressure drop to the deflection of the mercury manometer.

SOLUTION Find K

$$\begin{aligned} d/D &= 0.60 \\ \mathrm{Re}_d &= \frac{4Q}{\pi d \nu} \\ &= \frac{4 \times 0.127}{\pi \times 0.15 \times 1.14 \times 10^{-6}} \\ &= 9.4 \times 10^5 \end{aligned}$$

from Fig. 13.15 in EFM10e:

$$K = 0.65$$

Orifice section area

$$A_o = (\pi/4) \times (0.15)^2 = 0.018 \text{ m}^2$$

Orifice equation

$$\begin{aligned} \Delta h &= \left(\frac{Q}{KA_o}\right)^2 \frac{1}{2g} \\ &= \left(\frac{0.127}{0.65 \times 0.018}\right)^2 \left(\frac{1}{2 \times 9.81}\right) \\ &= 6\,\mathrm{m} \text{ (deflection of water height)} \end{aligned}$$

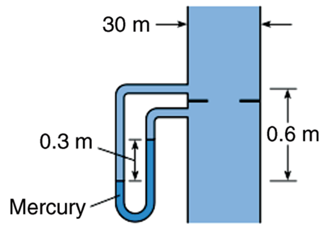
Manometer equation

$$\begin{aligned} \Delta h &= h_{\text{mercury}}\left(S_{\text{mercury}} - 1\right) \\ 6\,\mathrm{m} &= h_{\text{mercury}}\,(13.55 - 1) \end{aligned}$$

$$\boxed{h_{\text{mercury}} = 0.48\,\mathrm{m} = 48\,\mathrm{cm}}$$

13.26: PROBLEM DEFINITION

Situation: Water flows through a 17.5 cm orifice in a 30 cm pipe. Assume $T = 15\,^\circ\text{C}$, $\nu = 1.14 \times 10^{-6}$ m^2/s.



Find: Discharge: Q

PLAN Calculate piezometric head. Then find K and apply the orifice equation.

SOLUTION Piezometric head

$$\Delta h = (0.3)(13.6 - 1) = 3.8 \text{ m}$$

Find parameters needed to use Fig. 13.15 in EFM10e.

$$\begin{aligned} (d/D) &= 0.583 \\ (2g\Delta h)^{0.5} d/\nu &= (2g \times 3.8)^{0.5}(0.175)/(1.14 \times 10^{-6}\,\text{m}^2/\,\text{s}) \\ &= 1.36 \times 10^6 \end{aligned}$$

Look up K on Fig. 13.15 in EFM10e

$$K = 0.66$$

Orifice equation

$$\begin{aligned} Q &= KA_0(2g\Delta h)^{0.5} \\ Q &= 0.625(\pi/4 \times (0.175 \text{ m})^2)(19.62 \text{ m/s}^2 \times 3.8 \text{ m})^{0.5} \\ & \boxed{Q = 0.13\,\text{m}^3/\,\text{s}} \end{aligned}$$

13.27: PROBLEM DEFINITION

Situation: A rough orifice is described in the problem statement.

Find: Applicability of Figure 13.15 in EFM10e

SOLUTION A rough pipe will have a greater maximum velocity at the center of the pipe relative to the mean velocity than would a smooth pipe. Because more flow is concentrated near the center of the rough pipe less radial flow is required as the flow passes through the orifice; therefore, there will be less contraction of the flow. Consequently the coefficient of contraction will be larger for the rough pipe. So, using K from Fig. 13.15 in EFM10e would probably result in an estimated discharge that is too small.

13.28: PROBLEM DEFINITION

Situation:

Water flows through an orifice in a pipe.
$d = 7.5\,\text{cm} = 0.075$ m, $D = 15\,\text{cm} = 0.15$ m.
$h = 1.2\,\text{m}$.

Find: Discharge: Q

Properties: Table A.5 (water at 15 °C): $\nu = 1.14 \times 10^{-6}\,\text{m}^2/\,\text{s}$.

PLAN

1. Find K using the upper horizontal region of Fig. 13.15 (EFM10e).
2. Find Q by applying the orifice equation.

SOLUTION

1. Flow coefficient.

Reynolds number.

$$\begin{aligned}\frac{\text{Re}_d}{K} &= \sqrt{2g\Delta h}\frac{d}{\nu} \\ &= \sqrt{2 \times (9.81\,\text{m}/\,\text{s}^2) \times (1.2\,\text{m})}\left(\frac{0.075\,\text{m}}{1.14 \times 10^{-6}\,\text{m}^2/\,\text{s}}\right) \\ &= 3.2\,\times 10^5\end{aligned}$$

For an aspect ratio of $d/D = 0.5$, Fig. 13.15 (EFM10e) gives

$$K \approx 0.63$$

2. Orifice equation.

$$A_o = \frac{\pi\,(0.075\,\text{m})^2}{4} = 4.\,42 \times 10^{-3}\,\text{m}^2$$

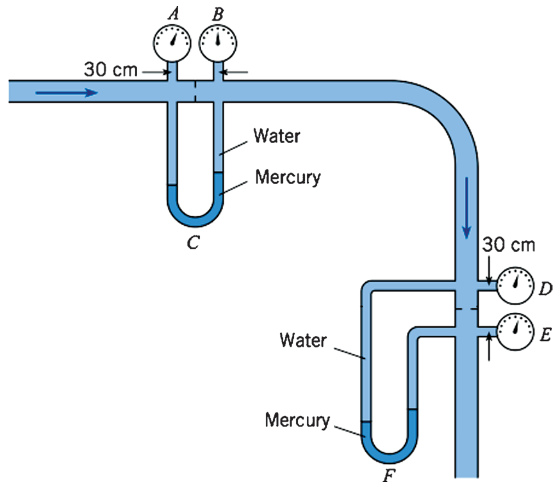
$$\begin{aligned}Q &= KA_o\sqrt{2g\Delta h} \\ &= 0.63 \times \left(4.\,42 \times 10^{-3}\,\text{m}^2\right)\sqrt{2 \times (9.81\,\text{m}/\,\text{s}^2) \times (1.2\,\text{m})} \\ &= 0.0135\,\text{m}^3/\,\text{s}\end{aligned}$$

$$\boxed{Q = 0.0135\,\text{m}^3/\,\text{s}}$$

13.29: PROBLEM DEFINITION

Situation:

Water at 20 °C flows in a pipe containing two orifices.
For each orifice, $D = 30$ cm and $d = 10$ cm.
$Q = 0.1 \text{ m}^3/\text{s}$.



Find:

(a) Pressure differential across each orifice: Δp_C, Δp_F.
(b) Deflection for each mercury-water manometer: Δh_C, Δh_F

SOLUTION

Reynolds number

$$\begin{aligned} 4Q/(\pi d\nu) &= 4 \times 0.10/(\pi \times 0.10 \times 1.31 \times 10^{-6}) \\ &= 9.7 \times 10^5 \end{aligned}$$

From Fig. 13.15 for $d/D = 0.333$

$$K = 0.60$$

Orifice section area

$$\begin{aligned} A_o &= (\pi/4)(0.10)^2 \\ &= 7.85 \times 10^{-3} \text{ m}^2 \end{aligned}$$

Orifice equation

$$Q = KA_o\sqrt{2g\Delta h}$$

Thus

$$\begin{aligned} \Delta h &= Q^2/(K^2A_o^2 2g) = 0.1^2/(0.6^2 \times (7.85 \times 10^{-3})^2 \times 2 \times 9.81) \\ &\Delta h_C = \Delta h_F = 22.97 \text{ m} - \text{H}_2\text{O} \end{aligned}$$

The deflection across the manometers is (using $S_{\text{Hg}} = 13.6$)

$$\begin{aligned} h_C &= h_F = 22.97/(S_{\text{Hg}} - S_{\text{water}}) \\ &\boxed{h_C = h_F = 1.82 \text{ m}} \end{aligned}$$

The deflection will be the same on each manometer

Find Δp

$$\begin{aligned} p_A - p_B &= \gamma \Delta h = 9790 \times 22.97 = 224.9 \text{ kPa} \\ &\boxed{\Delta p_C = 225 \text{ kPa}} \end{aligned}$$

For manometer F

$$((p_D/\gamma) + z_D) - ((p_E/\gamma) + z_E) = \Delta h = 22.97 \text{ m}$$

Thus,

$$\begin{aligned} \Delta p_F &= p_D - p_E = \gamma \Delta h - \gamma (z_D - z_E) \\ &= 9790(22.97 - 0.3) \\ &\boxed{\Delta p_F = 222 \text{ kPa}} \end{aligned}$$

Because of the elevation difference for manometer F, $\Delta p_C \neq \Delta p_F$

13.30: PROBLEM DEFINITION

Situation: A pipe ($D = 30$ cm) is terminated with an orifice. The orifice size is increased from 15 to 20 cm with pressure drop ($\Delta p = 50$ kPa) held constant.

Find: Percentage increase in discharge.

Assumptions: Large Reynolds number.

SOLUTION Find K values

Assuming large Re, so K depends only on d/D. From Fig. 13.15 (EFM10e)

$$\begin{aligned} K_{15} &= 0.62 \\ K_{20} &= 0.685 \end{aligned}$$

Orifice equation

$$\begin{aligned} Q_{15} &= K_{15} A_{15} \sqrt{2g\Delta h} \\ Q_{15} &= 0.62 \times (\pi/4)(0.15)^2 \sqrt{2g\Delta h} \\ Q_{15} &= 0.01395(\pi/4)\sqrt{2g\Delta h} \end{aligned}$$

For the 20 cm orifice

$$\begin{aligned} Q_{20} &= 0.685 \times (\pi/4)(0.20)^2 \sqrt{2g\Delta h} \\ Q_{20} &= 0.0274(\pi/4)\sqrt{2g\Delta h} \end{aligned}$$

Thus the % increase is

$$(0.0274 - 0.01395)/0.01395) \times 100 = 96\%$$

$$\boxed{\text{Percent increase in discharge} = 96\%}$$

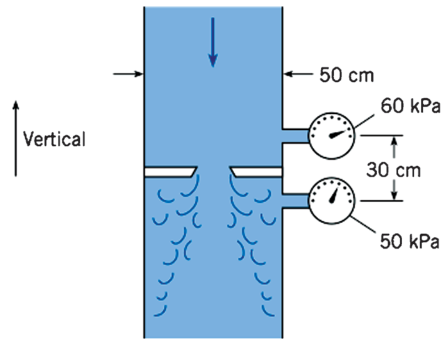
13.31: PROBLEM DEFINITION

Situation:

Water flows through an orifice.

$D = 0.5\,\text{m}$, $d = 0.043\,\text{m}$.

$\Delta p = 10$ kPa, $\Delta z = 0.3\,\text{m}$.



Find: Flow rate (m^3/s)

Properties: Water (20 °C), Table A.5, $\gamma = 9790\,\text{N/m}^3$, $\nu = 1.00 \times 10^{-6}\,\text{m}^2/\text{s}$.

PLAN

1. Find Δh.
2. Find the flow coefficient K.
3. Find Q by applying the orifice equation.

SOLUTION

1. Piezometric head

$$\begin{aligned} \Delta h &= (p_1/\gamma + z_1) - (p_2/\gamma + z_2) \\ &= \Delta p/\gamma + \Delta z \\ &= 10000\,\text{Pa}/9790\,\text{N/m}^3 + 0.3\,\text{m} \\ &= 1.321 \text{ m of water} \end{aligned}$$

2. Flow Coefficient

$$\begin{aligned} d/D &= 0.043\,\text{m}/0.5\,\text{m} = 0.086 \\ \frac{\text{Re}_d}{K} &= \sqrt{2g\Delta h}\frac{d}{\nu} \\ &= \sqrt{2(9.81\,\text{m/s}^2)(1.321\,\text{m})}\frac{0.043\,\text{m}}{10^{-6}\,\text{m}^2/\text{s}} \\ &= 2.19 \times 10^5 \end{aligned}$$

From Fig. 13.15

$$K = 0.60$$

3. Orifice equation

$$\begin{aligned}
Q &= KA_o\sqrt{2g\Delta h} \\
&= 0.60 \times (\pi/4) \times (0.043\,\mathrm{m})^2\sqrt{2\,(9.81\,\mathrm{m/\,s^2})\,(1.321\,\mathrm{m})} \\
&\boxed{Q = 4.44 \times 10^{-3}\,\mathrm{m^3/\,s}}
\end{aligned}$$

13.32: PROBLEM DEFINITION

Situation: Flow through an orifice is described in the problem statement.

Find: Show that the difference in piezometric pressure is given by the pressure difference across the transducer.

SOLUTION

Hydrostatic equation

$$\begin{aligned} p_{T,1} &= p_1 + \gamma \ell_1 \\ p_{T,2} &= p_2 - \gamma \ell_2 \end{aligned}$$

so

$$\begin{aligned} p_{T,1} - p_{T,2} &= p_1 + \gamma \ell_1 - p_2 + \gamma \ell_2 \\ &= p_1 - p_2 + \gamma(\ell_1 + \ell_2) \end{aligned}$$

But

$$\ell_1 + \ell_2 = z_1 - z_2$$

or

$$p_{T,1} - p_{T,2} = p_1 - p_2 + \gamma(z_1 - z_2)$$

Thus,

$$\boxed{p_{T,1} - p_{T,2} = (p_1 + \gamma z_1) - (p_2 + \gamma z_2)}$$

13.33: PROBLEM DEFINITION

Situation: Water ($T = 50\,°\mathrm{C}$, $Q = 0.57\,\mathrm{m}^3/\mathrm{s}$) flows in the system shown in the textbook. $f = 0.015$.

Find:
(a) Pressure change across the orifice.
(b) Power delivered to the flow by the pump.
(c) Sketch the HGL and EGL.

PLAN Calculate pressure change by applying the orifice equation. Then calculate the head of the pump by applying the energy equation from section 1 to 2 (section 1 is the upstream reservoir water surface, section 2 is the downstream reservoir surface). Then, apply the power equation.

SOLUTION

$$\begin{aligned} \mathrm{Re} &= 4Q/(\pi d\nu) \\ &= 4 \times 0.57/(\pi \times 0.3 \times 1.31 \times 10^{-6}) = 1.8 \times 10^6 \end{aligned}$$

Then for $d/D = 0.50$, $K = 0.625$

Orifice equation

$$Q = KA\sqrt{2g\Delta h} \text{ or } \Delta h = (Q/(KA))^2/2g$$

where $A = \pi/4 \times 0.3$. Then

$$\begin{aligned} \Delta h &= (0.57/(0.625 \times (\pi/4)))^2/2g \\ \Delta h &= 8.5 \text{ m} \\ \Delta p &= \gamma \Delta h = 9800 \times 8.5 = 83,300 \text{ Pa} \\ & \boxed{\Delta p = 83,300 \text{ Pa}} \end{aligned}$$

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 + h_p &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 3 + h_p &= 0 + 0 + 1.5 + \sum h_L \\ h_p &= -5 + V^2/2g(K_e + K_E + fL/D) + h_{L,\mathrm{orifice}} \\ K_e &= 0.5;\ K_E = 1.0 \end{aligned}$$

The orifice head loss will be like that of an abrupt expansion:

$$h_{L,\text{ orifice}} = (V_j - V_{\mathrm{pipe}})^2/(2g)$$

Here, V_j is the jet velocity as the flow comes from the orifice.

$$V_j = Q/A_j \text{ where } A_j = C_c A_0$$

Assume

$$C_c \approx 0.65 \text{ then } V_j = 0.57/((\pi/4) \times 0.3 \times 0.65) = 13 \text{ m/s}$$

Also

$$V_p = Q/A_p = 0.57/(\pi/4 \times 0.6^2) = 2 \text{ m/s}$$

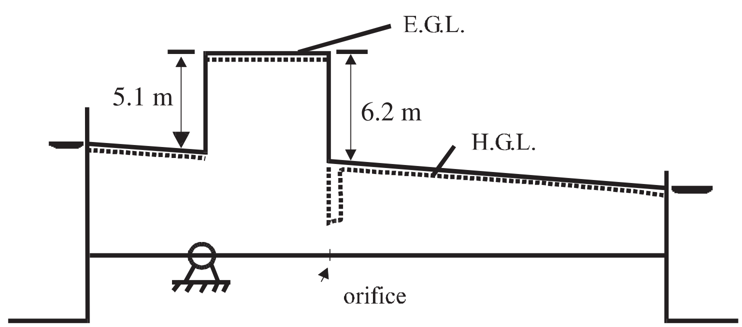
Then

$$h_{L,\text{orifice}} = (13\text{m/s})^2/(2g) = 6.2 \text{ m}$$

Finally,

$$\begin{aligned} h_p &= 1.5 + (2/(2g))(0.5 + 1.0 + (0.015 \times 90/2)) + 6.2 \\ h_p &= 5.1 \text{ m} \\ P &= Q\gamma h_p \\ &= 0.57 \times 999 \times 5.1 \\ & \boxed{P = 2904 \text{ W}} \end{aligned}$$

The HGL and EGL are shown below:



13.34: PROBLEM DEFINITION

Situation: Water flows ($Q = 0.03$ m^3/s) through an orifice. Pipe diameter, $D = 15$ cm. Manometer deflection is 1 m-Hg.

Find: Orifice size: d

PLAN Guess a value of K. Apply the orifice equation to solve for orifice diameter. Then calculate Reynolds number and d/D in order to find a new value of K. Iterate until the value of K does not change.

SOLUTION

Piezometric head

$$\Delta h = 12.6 \times 1.0\,\text{m} = 12.6\,\text{m of water}$$

Orifice equation

$$\begin{aligned} Q &= KA_o\sqrt{2g\Delta h} \\ &= K\left(\frac{\pi d^2}{4}\right)\sqrt{2g\Delta h} \end{aligned}$$

Algebra

$$d = \left[\left(\frac{4Q}{\pi K}\right)\left(\frac{1}{\sqrt{2g\Delta h}}\right)\right]^{1/2}$$

Guess $K \approx 0.65$

$$\begin{aligned} d &= \left[\left(\frac{4 \times .03}{\pi \cdot 0.65}\right)\left(\frac{1}{\sqrt{2 \times 9.81 \times 12.6}}\right)\right]^{1/2} \\ &= 0.0611\,\text{m} \end{aligned}$$

Calculate values needed for Fig. 13.15

$$\begin{aligned} \frac{d}{D} &= \frac{0.0611}{0.15} = 0.41 \\ \text{Re} &= \frac{4Q}{\pi d\nu} \\ &= \frac{4 \times 0.03}{\pi \times 0.0611 \times (1.14 \times 10^{-6})} \\ &= 5.48 \times 10^5 \end{aligned}$$

From Fig. 13.15 (EFM10e) with $d/D = 0.41$ and $\text{Re} = 5.48 \times 10^5$, the value of K is

$$K = 0.62$$

When the calculation is iterated again with the new K, the resulting value for d is:

$$\boxed{d = 0.0626\,\text{m} = 6.26\text{ cm}}$$

and (to check for sufficient iterations) recalculation of Re and d/D yields no change in K.

13.35: PROBLEM DEFINITION

Situation: Gasoline ($S = 0.68$) flows through an orifice ($d = 6$ cm) in a pipe ($D = 12$ cm).
$\Delta p = 50$ kPa.

Find: Discharge: Q

Properties: $\nu = 4 \times 10^{-7}$ m^2/s (Fig. A-3)

Assumptions: $T = 20\,^\circ$C.

SOLUTION Piezometric head

$$\begin{aligned}\Delta h &= \Delta p/\gamma \\ &= 50,000/(0.68 \times 9,810) \\ &= 7.50 \text{ m}\end{aligned}$$

Find K using Fig. 13.15 (EFM10e) for the region of the figure that refers to orifices

$$\begin{aligned}d/D &= 0.50 \\ \sqrt{2g\Delta h}d/\nu &= \sqrt{2 \times 9.81 \times 7.50} \times 0.06/(4 \times 10^{-7}) = 1.82 \times 10^6 \\ K &= 0.66\end{aligned}$$

Orifice equation

$$\begin{aligned}Q &= KA_o\sqrt{2g\Delta h} \\ &= 0.66 \times (\pi/4)(0.06)^2\sqrt{2g \times 7.50} \\ &\boxed{Q = 0.0226 \text{ m}^3/\text{s}}\end{aligned}$$

13.36: PROBLEM DEFINITION

Situation: Water flows ($Q = 2\ \text{m}^3/\text{s}$) through an orifice in a pipe ($D = 1$ m). $\Delta h = 6$ m-H_2O.

Find: Orifice size: d

PLAN Guess a value of K. Apply the orifice equation to solve for orifice diameter. Then calculate Reynolds number and d/D in order to find a new value of K. Iterate until the value of K does not change.

SOLUTION Orifice equation

$$\begin{aligned} Q &= KA_o\sqrt{2g\Delta h} \\ &= K\left(\frac{\pi d^2}{4}\right)\sqrt{2g\Delta h} \end{aligned}$$

Algebra

$$d = \left[\left(\frac{4Q}{\pi K}\right)\left(\frac{1}{\sqrt{2g\Delta h}}\right)\right]^{1/2}$$

Guess $K \approx 0.65$

$$\begin{aligned} d &= \left[\left(\frac{4\times 2}{\pi\cdot 0.65}\right)\left(\frac{1}{\sqrt{2\times 9.81\times 6}}\right)\right]^{1/2} \\ &= 0.601\,\text{m} \end{aligned}$$

Calculate values needed for Fig. 13.15

$$\begin{aligned} \frac{d}{D} &= \frac{0.601}{1.0} = 0.6 \\ \text{Re} &= \frac{4Q}{\pi d\nu} \\ &= \frac{4\times 2}{\pi\times 0.601\times(1.14\times 10^{-6})} \\ &= 3.72\times 10^6 \end{aligned}$$

From Fig. 13.15 (EFM10e) with $d/D = 0.6$ and $\text{Re} = 3.72\times 10^6$, the value of K is

$$K = 0.65$$

Since this is the guessed value, there is no need to iterate.

$$\boxed{d = 0.601\,\text{m}}$$

13.37: PROBLEM DEFINITION

Situation: Water flows ($Q = 4$ m^3/s) through an orifice in a pipe ($D = 1.2$ m). $\Delta p = 48$ kPa.

Find: Orifice size: d

Assumptions: $K = 0.7$; $T = 20\,^\circ$C.

SOLUTION Piezometric head

$$\begin{aligned}\Delta h &= \Delta p/\gamma \\ &= 48,000\,\text{Pa}/9790\,\text{N/m}^3 \\ &= 4.90\,\text{m}\end{aligned}$$

Orifice equation

$$\begin{aligned}d^2 &= (4/\pi) \times (4.0\,\text{m}^3/\text{s})/[0.7\sqrt{(2)(9.81\,\text{m/s}^2)(4.90\,\text{m})}] = 0.742\,\text{m}^2 \\ d &= 0.861\,\text{m}\end{aligned}$$

Check K:

$$\begin{aligned}Re_d &= 4Q/(\pi d\nu) \\ &= 4 \times (4.0\,\text{m}^3/\text{s})/(\pi \times 0.861 \times 10^{-6}) \\ &= 5.7 \times 10^6\end{aligned}$$

From Fig. 13.15 (region of figure that refers to orifices) for $d/D = 0.861/1.2 = 0.72$, $K = 0.710$
Try again:

$$d = \sqrt{(0.70/0.71)} \times 0.861\,\text{m} = 0.855\,\text{m}$$

Check K: $Re_d = 5 \times 10^6$ and $d/D = 0.71$. From Fig. 13.15, $K = 0.71$ so

$$\begin{aligned}d &= \sqrt{(0.71/0.71)} \times 0.855\,\text{m} \\ & \boxed{d = 0.855\text{ m}}\end{aligned}$$

13.38: PROBLEM DEFINITION

Situation: Water flows through a hemicircular orifice as shown in the textbook.

Find:
(a) Develop a formula for discharge.
(b) Calculate Q.

PLAN Apply the flow rate equation, continuity principle, and the Bernoulli equation to solve for Q.

SOLUTION Bernoulli equation

$$p_1 + \rho V_1^2/2 = p_2 + \rho V_2^2/2$$

Continuity principle

$$\begin{aligned} V_1 A_1 &= V_2 A_2;\ V_1 = V_2 A_2/A_1 \\ V_2 &= \sqrt{2(p_1 - p_2)/p}/\sqrt{1 - (A_2^2/A_1^2)} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= A_2 V_2 \\ &= \left[A_2/\sqrt{1 - (A_2^2/A_1^2)}\right]\sqrt{2\Delta p/\rho} \end{aligned}$$

but $A_2 = C_c A_0$ where A_0 is the section area of the orifice. Then

$$Q = \left[C_c A_0/\sqrt{1 - (A_2^2/A_1^2)}\right]\sqrt{2\Delta p/\rho}$$

or orifice equation

$$\boxed{Q = K A_0 \sqrt{2\Delta p/\rho}}$$

where K is the flow coefficient. Assume $K = 0.65$; Also $A = (\pi/8) \times 0.30^2 = 0.0353$ m^2. Then

$$\begin{aligned} Q &= 0.65 \times 0.0353\sqrt{2 \times 80,000/1,000} \\ &\boxed{Q = 0.290 \text{ m}^3/\text{s}} \end{aligned}$$

Problem 13.39

What is the main advantage of a venturi meter versus an orifice meter? Main disadvantage?

Advantages:

- the head loss of a venturi meter is less than the head loss for an orifice meter.

Disadvantages:

- a venturi meter is more expensive than an orifice meter

13.40: PROBLEM DEFINITION

Situation: Water (20 °C, $Q = 0.75$ m^3/s) flows through a venturi meter ($d = 40$ cm) in a pipe ($D = 70$ cm).

Find: Deflection on a mercury manometer.

SOLUTION Reynolds number

$$\begin{aligned} \mathrm{Re}_d &= (4)(0.75\,\mathrm{m}^3/\,\mathrm{s})/(\pi \times 0.40\,\mathrm{m} \times 1 \times 10^{-6}\,\mathrm{m}^2/\,\mathrm{s}) \\ &= 2.39 \times 10^6 \end{aligned}$$

For $d/D = 0.57$, find K from Fig. 13.15 (EFM10e)

$$K = 1.05$$

Venturi equation

$$\begin{aligned} \Delta h &= [Q/(KA_t)]^2/(2g) \\ &= \left[.75/(1.05 \times (\pi/4) \times 0.4^2)\right]^2/(2 \times 9.81) \\ &= 1.65 \text{ m H}_2\text{O} \end{aligned}$$

Manometer equation

$$h_{H_g} = \Delta h_{H_2O}/\left(\frac{\gamma_{H_g}}{\gamma_{H_2O}} - 1\right)$$

$$h = 1.65\,\mathrm{m}/12.6$$

$$\boxed{h = 0.13 \text{ m}}$$

13.41: PROBLEM DEFINITION

Situation: Water ($Q = 0.76$ m^3/s) flows through a venturi meter in a horizontal pipe ($D = 0.61$ m). $\Delta p = 200$ kPa.

Find: Venturi throat diameter.

Assumptions: $T = 20\,^\circ$C.

SOLUTION Guess that $K = 1.02$, and then proceed with calculations

$$Q = KA_o/\sqrt{2g\Delta h}$$

where $\Delta h = 200,000$ Pa/(9,790 N/m^3) = 20.4 m. Then
Venturi equation

$$\begin{aligned} A_t &= Q/(K\sqrt{2g\Delta h}) \\ &\text{or} \\ \pi d^2/4 &= Q/(K\sqrt{2g\Delta h}) \\ d &= (4Q/(\pi K\sqrt{2g\Delta h})^{1/2} \\ d &= (4 \times (0.76\,\mathrm{m}^3/\,\mathrm{s})/(\pi \times 1.02\sqrt{2g \times 20.4\,\mathrm{m}})^{1/2} = 0.217\,\mathrm{m} \end{aligned}$$

Calculate K and compare with the assumed value

$$\mathrm{Re} = 4Q/(\pi d\nu) = 4.4 \times 10^6$$

Also $d/D = 0.36$ so from Fig. 13.15 (EFM10e) $K \approx 0.99$. Try again:

$$\begin{aligned} d &= (1.02/0.99)^{1/2} \times 0.217\,\mathrm{m} \\ &\boxed{d = 0.22 \text{ m}} \end{aligned}$$

13.42: PROBLEM DEFINITION

Situation: A venturi meter is described in the problem statement.

Find: Rate of flow: Q

SOLUTION Find K.

$$\begin{aligned} \Delta h &= 1.2 \text{ m and } d/D = 0.33 \\ \text{Re}_d/K &= (0.33)\sqrt{2 \times 9.81 \times 1.2}/1.14 \times 10^{-6}) = 1.4 \times 10^6 \\ K &= 0.97 \text{ (Estimated from Fig. 13.15)} \end{aligned}$$

Venturi equation

$$\begin{aligned} Q &= KA\sqrt{2gh} \\ &= 0.97(\pi/4 \times 0.1^2)\sqrt{2 \times 9.81 \times 1.2} \end{aligned}$$

$$\boxed{Q = 0.04 \text{ m}^3/\text{s}}$$

13.43: PROBLEM DEFINITION

Situation: A venturi meter is described in the problem statement.

Find: Range that the venturi meter would read: Δp

SOLUTION The answer is 70 kPa $< \Delta$ p $<$ 0 so the correct choice is b).

13.44: PROBLEM DEFINITION

Situation: Water flows through a horizontal venturi meter. $\Delta p = 92$ kPa, $d = 1$ m, $D = 2$ m.

Find: Discharge: Q

Properties: $\nu = 10^{-6}$ m^2/s.

SOLUTION

$$\Delta p = 92 \text{ kPa so } \Delta h = \Delta p/\gamma = 92,000/9790 = 9.40\,\text{m}$$

Find Re_d to compute K.

$$\begin{aligned} \sqrt{2g\Delta h}d/\nu &= \sqrt{(2)(9.81\,\text{m/s}^2)(9.40\,\text{m})} \times 1/(10^{-6}\,\text{m}^2/\text{s}) \\ &= 1.36 \times 10^7 \end{aligned}$$

Then $K \approx 1.025$ (extrapolated from Fig. 13.15, (EFM10e, in the venturi region of this figure).

Venturi equation

$$\begin{aligned} Q &= KA\sqrt{2g\Delta h} \\ &= 1.025 \times (\pi/4) \times (1\,\text{m})^2 \sqrt{2g \times 9.40\,\text{m}} \end{aligned}$$

$$\boxed{Q = 10.9\,\text{m}^3/\text{s}}$$

13.45: PROBLEM DEFINITION

Situation: A poorly designed venturi meter is described in the problem statement.

Find: Correction factor: K

SOLUTION Because of the streamline curvature (concave toward wall) near the pressure tap, the pressure at point 2 will be less than the average pressure across the section. Therefore, Q_0 will be too large as determined by the formula. Thus, $K < 1$.

13.46: PROBLEM DEFINITION

Situation: Water (10 °C) flows through a vertical venturi meter. $\Delta p = 37.2$ kPa, $d = 18$ cm, $D = 30$ cm, $\nu = 1.31 \times 10^{-6}$ m^2/s.

Find: Discharge: Q

SOLUTION

$$\Delta p = 37.2 \text{ kPa} = 37,200 \,\text{N/m}^2$$

Thus

$$\Delta h = 37,200/9810 = 3.8 \,\text{m}$$

Find K

$$\begin{aligned} \frac{\text{Re}_d}{K} &= \sqrt{2g\Delta h}\frac{d}{\nu} \\ &= \sqrt{2 \times 9.81 \,\text{m/s}^2 \times 3.8 \,\text{m}}\left(\frac{0.18 \,\text{m}}{1.31 \times 10^{-6} \,\text{m}^2/\text{s}}\right) \\ &= 1.2 \times 10^6 \end{aligned}$$

So $K = 1.02$.
Venturi equation

$$\begin{aligned} Q &= KA_t\sqrt{2g\Delta h} \\ &= 1.02 \times (\pi/4) \times (0.18 \,\text{m})^2 \sqrt{2 \times 19.62 \,\text{m/s}^2 \times 3.8 \,\text{m}} \\ & \boxed{Q = 0.22 \ \text{m}^3/\text{s}} \end{aligned}$$

13.47: PROBLEM DEFINITION

Situation:
Gasoline ($S = 0.69$) flows through a venturi meter. A differential pressure gage indicates $\Delta p = 40$ kPa.
$d = 20$ cm, $D = 40$ cm, $\mu = 3 \times 10^{-4}$ N·s/m^2.

Assumptions:
Neglect height of transducer, h. Extraneous information.

Find:
Discharge: Q

SOLUTION

$$\begin{aligned}\Delta h &= 40,000\,\text{Pa}/(0.69 \times 9,810\frac{\text{N}}{\text{m}^3}) = 5.91 \text{ m} \\ \nu &= \mu/\rho = 3 \times 10^{-4}/690 = 4.3 \times 10^{-7} \frac{\text{m}^2}{\text{s}}\end{aligned}$$

Then

$$\sqrt{2g\Delta h}d/\nu = \sqrt{2 \times 9.81\frac{\text{m}}{\text{s}^2} \times 5.91\,\text{m}} \times 0.20\,\text{m}/(4.3 \times 10^{-7}\frac{\text{m}^2}{\text{s}}) = 5.0 \times 10^6$$

From Fig. 13.15 (EFM10e)

$$K = 1.02$$

Venturi equation

$$\begin{aligned}Q &= KA\sqrt{2g\Delta h} \\ &= 1.02 \times (\pi/4) \times (0.20\,\text{m})^2\sqrt{2 \times 9.81\frac{\text{m}}{\text{s}^2} \times 5.91\,\text{s}}\end{aligned}$$

$$\boxed{Q = 0.345 \text{ m}^3/\text{s}}$$

13.48: PROBLEM DEFINITION

Situation: Water passes through a flow nozzle. $\Delta p = 8$ kPa. $d = 2$ cm, $d/D = 0.5$, $\nu = 10^{-6}$ m^2/s, $\rho = 1000$ kg/m^3.

Find: Discharge: Q

PLAN Find K, and then apply the orifice equation.

SOLUTION Find K.

$$\begin{aligned}
\text{Re}_d/K &= (2\Delta p/\rho)^{0.5}(d/\nu) \\
&= ((2 \times 8 \times 10^3)/(1,000))^{0.5}(0.02/10^{-6}) \\
&= 8.0 \times 10^4
\end{aligned}$$

From Fig. 13.15 (EFM10e) with $d/D = 0.5$; $K = 0.99$.
Venturi equation

$$\begin{aligned}
Q &= KA(2\Delta p/\rho)^{0.5} \\
&= (0.99)(\pi/4)(0.02^2)(2 \times 8 \times 10^3/10^3)^{0.5} \\
&\boxed{Q = 0.00124 \text{ m}^3/\text{s}}
\end{aligned}$$

13.49: PROBLEM DEFINITION

Situation: Water flows through the annular venturi that is shown in the textbook.

Find: Discharge

Assumptions: $C_d = 0.98$

SOLUTION

Estimate K using Eq. (13.6) in EFM10e

$$\begin{aligned} K &= C_d/\sqrt{1-(A_2/A_1)^2} \\ &= 0.98/\sqrt{1-0.75^2} \\ K &= 1.48 \end{aligned}$$

Venturi equation

$$\begin{aligned} A &= 0.00147\,\mathrm{m}^2 \\ Q &= KA(2g\Delta h)^{0.5} \\ Q &= (1.48)(0.00147)(2.0 \times 9.81 \times 1)^{0.5} \\ &\boxed{Q = 0.00964 \ \mathrm{m}^3/\mathrm{s}} \end{aligned}$$

13.50: PROBLEM DEFINITION

Situation: The problem statement describes a flow nozzle with $d/D = 1.3$.

Find: Develop an expression for head loss.

PLAN Apply the sudden expansion head loss equation and the continuity principle.

SOLUTION

Continuity principle

$$\begin{aligned} V_0 A_0 &= V_j A_j \\ V_j &= V_0 A_0 / A_j \\ &= V_0 \times (3/1)^2 = 9V_0 \end{aligned}$$

Head loss (sudden expansion)

$$h_L = (V_j - V_0)^2 / 2g$$

Combining equations

$$h_L = (9V_0 - V_0)^2 / 2g$$

$$\boxed{h_L = 64V_0^2/2g}$$

13.51: PROBLEM DEFINITION

Situation: A vortex meter (1 cm shedding element) is used in a 5 cm diameter duct. For shedding on one side of the element, $St = 0.2$ and $f = 50$ Hz.

Find: Discharge: Q

PLAN

Find velocity from the Strouhal number $(St = nD/V)$. Then, find the discharge using the flow rate equation.

SOLUTION

$$\begin{aligned} St &= nD/V \\ V &= nD/St \\ &= (50)(0.01)/(0.2) \\ &= 2.5 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= (2.5)(\pi/4)(0.05^2) \\ & \boxed{Q = 0.0049 \text{ m}^3/\text{s}} \end{aligned}$$

13.52: PROBLEM DEFINITION

Situation: A rotameter is described in the problem statement.

Find: Describe how the reading on the rotameter would be corrected for nonstandard conditions.

SOLUTION

Equilibrium (drag force balances weight):

$$\begin{aligned} F_D &= W \\ C_D A \rho V^2/2 &= mg \end{aligned}$$

Solve for velocity

$$V = \sqrt{2gm/(\rho A C_D)}$$

Since all terms are constant except density

$$V/V_{\text{std.}} = (\rho_{\text{std.}}/\rho)^{0.5}$$

Introduce flow rate

$$\begin{aligned} Q &= VA \\ \therefore & \boxed{Q/Q_{\text{std.}} = (\rho_{\text{std.}}/\rho)^{0.5}} \end{aligned} \quad (1)$$

Correct by calculating ρ for the actual conditions and then correct Q for the actual conditions.

13.53: PROBLEM DEFINITION

Situation:

A rotameter is calibrated for gas with $\rho_{\text{standard}} = 1.2\ \text{kg/m}^3$, but is used for $\rho = 1.0\ \text{kg/m}^3$.

The rotameter indicates $Q = 5\ \text{L/s}$.

Find: Actual gas flow rate (Q) in liters per second.

PLAN Apply equilibrium, drag force, and the flow rate equation.

SOLUTION

The deflection of the rotameter is a function of the drag on the rotating element. Equilibrium of the drag force with the weight of the float gives

$$\begin{aligned} F_D &= W \\ C_D A \frac{\rho V^2}{2} &= mg \end{aligned}$$

Use the above equation to derive a ratio of standard to nonstandard conditions gives

$$\frac{V}{V_{\text{std.}}} = \sqrt{\frac{\rho_{\text{std.}}}{\rho}}$$

also

$$Q = VA$$

Therefore

$$\frac{Q}{Q_{\text{std.}}} = \sqrt{\frac{\rho_{\text{std.}}}{\rho}}$$

Thus

$$\begin{aligned} Q &= (5\ L/\text{s})\sqrt{\frac{1.2}{1.0}} \\ & \boxed{Q = 5.48\ \text{L/s}} \end{aligned}$$

13.54: PROBLEM DEFINITION

Situation: One mode of operation of an ultrasonic flow meter involves the time for a wave to travel between two measurement stations—additional details are provided in the problem statement.

Find:
(a) Derive an expression for the flow velocity.
(b) Express the flow velocity as a function of L, c and t.
(c) Calculate the flow velocity for the given data.

SOLUTION

(a)

$$\begin{aligned} t_1 &= L/(c+V) \\ t_2 &= L/(c-V) \\ \Delta t &= t_2 - t_1 \\ &= \frac{L}{c-V} - \frac{L}{c+V} \\ &= \frac{2LV}{c^2 - V^2} \quad (1) \end{aligned}$$

Thus

$$\begin{aligned} (c^2 - V^2)\Delta t &= 2LV \\ & V^2 \Delta t + 2LV - c^2 \Delta t = 0 \\ & V^2 + (2LV/\Delta t) - c^2 = 0 \end{aligned}$$

Solving for V:

$$[(-2L/\Delta t) \pm \sqrt{(2L/\Delta t)^2 + 4c^2}]/2 = (-L/\Delta t) \pm \sqrt{(L/\Delta t)^2 + c^2}$$

Selecting the positive value for the radical

$$\boxed{V = (L/\Delta t)[-1 + \sqrt{1 + (c\Delta t/L)^2}}$$

(b) From Eq. (1)

$$\Delta t = \frac{2LV}{c^2} \quad \text{for } c >> V$$

$$\boxed{V = \tfrac{c^2 \Delta t}{2L}}$$

(c)

$$V = \frac{(300)^2 (10 \times 10^{-3})}{2 \times 20}$$

$$\boxed{V = 22.5\,\mathrm{m/s}}$$

Problem 13.55

Part a. On the Internet, locate quality resources relevant to weirs, skim these resources, and write down five important findings.

No solution provided; answers will vary.

Part b. What variables influence flow rate through a rectangular weir?

PLAN

To identify the factors, apply *logical reasoning* to the rectangular weir equation:

$$Q = K\sqrt{2g}LH^{3/2}$$

SOLUTION

The variables that influence Q appear on the right side of the equation. Thus, flow rate is influenced by

- Flow coefficient K which depends on H and P.
- The head H on the weir which is the vertical distance from the top of weir to the top of the water surface.
- The height P of the weir.
- The length L of the weir.

Notice that all these variables are lengths.

13.56: PROBLEM DEFINITION

Situation:

Water flows over a rectangular weir.
$L = 2$ m, $H = 0.10$ m, $P = 0.30$ m.

Find: Discharge (in $\mathrm{m^3/s}$).

SOLUTION

1. Flow coefficient:

$$\begin{aligned} K &= 0.40 + 0.05\left(\frac{H}{P}\right) \\ &= 0.40 + 0.05\left(\frac{0.10\,\mathrm{m}}{0.30\,\mathrm{m}}\right) \\ &= 0.417 \end{aligned}$$

2. Rectangular weir equation:

$$\begin{aligned} Q &= K\sqrt{2g}LH^{3/2} \\ &= 0.417 \times \sqrt{2 \times 9.81\frac{\mathrm{m}}{\mathrm{s}^2}} \times 2\,\mathrm{m} \times (0.1\,\mathrm{m})^{3/2} \\ &= 0.117\,\mathrm{m^3/s} \end{aligned}$$

$$\boxed{Q = 0.117\,\mathrm{m^3/s}}$$

13.57: PROBLEM DEFINITION

Situation:

Water flows over a 60^o triangular weir.

$H = 0.25$ m.

Find: Discharge ($\mathrm{m^3/s}$)

SOLUTION

Triangular weir equation

$$
\begin{aligned}
Q &= 0.179\sqrt{2g}H^{5/2} \\
Q &= 0.179\sqrt{2 \times 9.81\frac{\mathrm{m}}{\mathrm{s}^2}}(0.25\,\mathrm{m})^{5/2} \\
& \boxed{Q = 0.0248\,\mathrm{m^3/s}}
\end{aligned}
$$

13.58: PROBLEM DEFINITION

Situation: Two weirs (A and B) are described in the problem statement.

Find: Relationship between the flow rates: Q_A and Q_B

SOLUTION Correct choice is $\boxed{\text{c) } Q_A < Q_B}$ because of the side contractions on A.

13.59: PROBLEM DEFINITION

Situation: A rectangular weir is described in the problem statement.

Find: The height ratio: H_1/H_2

SOLUTION

Correct choice is (b) $(H_1/H_2) < 1$ because K is larger for smaller height of weir; therefore, less head is required for the smaller P value.

13.60: PROBLEM DEFINITION

Situation: A rectangular weir is being designed for $Q = 4\ \text{m}^3/\text{s}$, $L = 3$ m. Water depth upstream of weir is 2 m.

Find: Weir height: P

SOLUTION First guess $H/P = 0.60$. Then

$$K = 0.40 + 0.05(0.60) = 0.43.$$

Rectangular weir equation (solve for H)

$$\begin{aligned} H &= (Q/(K\sqrt{2g}L))^{2/3} \\ &= (4\frac{\text{m}^3}{\text{s}}/(0.43\sqrt{(2)(9.81\frac{\text{m}}{\text{s}^2})}(3\,\text{m})))^{2/3} = 0.788\ \text{m} \end{aligned}$$

Iterate:

$$\begin{aligned} H/P &= 0.788\,\text{m}/(2\,\text{m} - 0.788\,\text{m}) = 0.65;\ K = 0.40 + .05(.65) = 0.433 \\ H &= (4\frac{\text{m}^3}{\text{s}}/(0.433\sqrt{(2)(9.81\frac{\text{m}}{\text{s}^2}(3\,\text{m}))})^{2/3} = 0.785\ \text{m} \end{aligned}$$

Thus:

$$\begin{aligned} P &= 2.0\,\text{m} - H = 2.00\,\text{m} - 0.785\,\text{m} \\ & \boxed{P = 1.22\ \text{m}} \end{aligned}$$

13.61: PROBLEM DEFINITION

Situation: The head of the rectangular weir (described in the preceding problem) is doubled.

Find: The discharge.

SOLUTION

Rectangular weir equation

$$Q = K\sqrt{2g}LH^{3/2}$$

Correct choice is (c) because flow varies as head to the 3/2 power.

13.62: PROBLEM DEFINITION

Situation: A basin is draining over a rectangular weir. $L = 0.6$ m, $P = 0.6$ m. Initially, $H = 0.3$ m.

Find: Time for the head to decrease from $H = 0.3\,\text{m}$ to $0.05\,\text{m}$ (2).

SOLUTION With a head of $H = 0.3\,\text{m}$

$$\frac{H}{P} = \frac{0.3\,\text{m}}{0.6\,\text{m}} = 0.5$$

thus

$$\begin{aligned} K_i &= 0.40 + 0.05 \times 0.5 \\ &= 0.425 \end{aligned}$$

With a head of $H = 0.05\,\text{m}$

$$\frac{H}{P} = \frac{0.05 \text{ m}}{0.6 \text{ m}} = 0.0833$$

and

$$\begin{aligned} K_f &= 0.40 + 0.05 \times 0.0833 \\ &= 0.404 \end{aligned}$$

As a simplification, assume K is constant at

$$\begin{aligned} K &= (.425 + .404)/2 \\ &= 0.415 \end{aligned}$$

Rectangular weir equation

$$Q = 0.415\sqrt{2g}LH^{3/2}$$

For a period of dt the volume of water leaving the basin is equal to $A_B dH$ where $A_B = 9\,\text{m}^2$ is the plan area of the basin. Also this volume is equal to Qdt. Equating these two volumes yields:

$$\begin{aligned} Qdt &= A_B dH \\ \left(0.415\sqrt{2g}LH^{3/2}\right) dt &= A_B\, dH \end{aligned}$$

Separate variables

$$\begin{aligned} dt &= \frac{A_B\, dH}{\left(0.415\sqrt{2g}LH^{3/2}\right)} \\ &= \frac{(9\,\text{m}^2)\ dH}{\left(0.415\sqrt{2 \times (9.81\,\text{m/s}^2)}\,(0.6\,\text{m})\,H^{3/2}\right)} \\ &= \left(8.2\sqrt{\text{m}} \cdot \text{s}\right) \frac{dH}{H^{3/2}} \end{aligned}$$

Integrate

$$\begin{aligned}
\int_0^{\Delta t} dt &= (8.2)\int_{0.05}^{0.3} \frac{dH}{H^{3/2}} \\
\Delta t &= (-8.2)\left(\frac{2}{\sqrt{H}}\right)_{0.05}^{0.3} = (-8.2)\left(\frac{2}{\sqrt{0.3}} - \frac{2}{\sqrt{0.05}}\right) \\
&= 43.44\,\mathrm{s}
\end{aligned}$$

$$\boxed{\Delta t = 43.4\,\mathrm{s}}$$

13.63: PROBLEM DEFINITION

Situation: A piping system and channel are described in the textbook. The channel empties over a rectangular weir.
Weir crest height $P = 3$
Pipe is 10 cm steel, $L = 30$ m
Two bends with $r/D = 1$,so $K_b = 0.35$ for each bend (from Table 10.5 of EFM10e)
$K_E = 1$, as usual
$K_e = 0.5$ (from Table 10.5 of EFM10e for pipe entrance)

Find: (a) Water surface elevation in the channel.
(b) Discharge.

SOLUTION Rectangular weir equation

$$Q = K\sqrt{2g}LH^{3/2}$$

Assume $H = 0.15$ m. Then $K = 0.4 + 0.05(\frac{1}{2}/3) = 0.41$, then

$$\begin{aligned} Q &= 0.41\sqrt{19.62} \times 0.6H^{3/2} \\ Q &= 1.1H^{3/2} \end{aligned} \qquad (1)$$

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 30 &= 0 + 0 + 0.9 + H + \sum h_L \end{aligned} \qquad (2)$$

Combined head loss

$$\begin{aligned} \sum h_L &= (V^2/2g)(K_e + fL/D + 2K_b + K_E) \\ &= (V^2/2g)(0.5 + f(30/(0.1)) + 2 \times 0.35 + 1) \end{aligned}$$

Assume $f = 0.02$ (first try). Then

$$\sum h_L = 8.2V^2/2g$$

Eq. (2) then becomes

$$29.1 = H + 8.2V^2/2g \qquad (3)$$

But $V = Q/A$ so Eq. (3) is written as

$$29.1 = H + 8.2Q^2/(2gA^2)$$

where

$$\begin{aligned} A^2 &= ((\pi/4)(0.1)^2)^2 = 6.2 \times 10^{-5} \text{ m}^4 \\ 29.1 &= H + 8.2Q^2/(2g \times 6.2 \times 10^{-5}) \\ 29.1 &= H + 6741 \end{aligned} \qquad (4)$$

Solve for Q and H between Eqs. (1) and (4)

$$
\begin{aligned}
29.1 &= H + 6741Q^2 \\
29.1 &= H + 6741(6.58H^{3/2})^2 \\
H &= 0.15 \text{ m and } Q = 0.066 \text{ m}^3/\text{s}
\end{aligned}
$$

Now check Re and f

Flow rate equation

$$
\begin{aligned}
V &= Q/A \\
&= 8.3 \text{ m/s}
\end{aligned}
$$

Reynolds number

$$
\begin{aligned}
\text{Re} &= VD/\nu = 8.3 \times (0.1)/(1.31 \times 10^{-6}) \\
\text{Re} &= 6.5 \times 10^5
\end{aligned}
$$

From Fig. 10.14 (EFM 10e) and Table 10.5 (EFM 10e) $f = 0.017$. Then Eq. (3) becomes

$$29.1 = H + 7.3V^2/2g$$

and Eq. (4) is

$$29.1 = H + 6001$$

Solve for H and Q again:

$$\boxed{H = 0.16 \text{ m}, \; Q = 0.08 \text{ m}^3/\text{s}}$$

13.64: PROBLEM DEFINITION

Situation: Water flows into a tank at a rate $Q = 0.1\ \mathrm{m}^3/\mathrm{s}$. The tank has two outlets: a rectangular weir ($P = 1$ m, $L = 1$ m) on the side, and an orifice ($d = 10$ cm) on the bottom.

Find: Water depth in tank.

PLAN Apply the rectangular weir equation and the orifice equation by guessing the head on the orifice and iterating.

SOLUTION Guess the head on the orifice is 1.05 m.

Orifice equation

$$\begin{aligned} Q_{\text{orifice}} &= KA_0\sqrt{2gh};\ K \approx 0.595 \\ Q_{\text{orifice}} &= 0.595 \times (\pi/4) \times (0.10)^2\sqrt{2 \times 9.81 \times 1.05} = 0.0212\ \mathrm{m}^3/\mathrm{s} \end{aligned}$$

Rectangular weir equation

$$\begin{aligned} Q_{\text{weir}} &= K\sqrt{2g}LH^{3/2};\ H_{\text{weir}} = (Q/(K\sqrt{2g}L)^{2/3} \text{ where } K \approx 0.405 \\ H_{\text{weir}} &= ((0.10 - 0.0212)/(0.405\sqrt{2 \times 9.81} \times 1))^{2/3} = 0.124\ \mathrm{m} \end{aligned}$$

Try again, using $P + H_{\text{weir}} = 1$ m $+0.124$ m$= 1.124$ m

$$\begin{aligned} Q_{\text{orifice}} &= (1.124/1.05)^{1/2} \times 0.0212\ \mathrm{m}^3/\mathrm{s} = 0.0219\ \mathrm{m}^3/\mathrm{s} \\ H_{\text{weir}} &= ((0.10 - 0.0219)/(0.405\sqrt{2 \times 9.81} \times 1))^{2/3} = \boxed{0.124\ \mathrm{m}} \end{aligned}$$

H_{weir} is same as before, so iteration is complete. Depth of water in tank is $\boxed{1.124\ \mathrm{m}}$

13.65: PROBLEM DEFINITION

Situation:

Water flows over a rectangular weir.
$L = 3$ m, $P = 0.9$ m, and $H = 0.5$ m.

Find: Discharge: Q

PLAN

1. Find the flow coefficient K.
2. Apply the rectangular weir equation.

SOLUTION

1. Flow coefficient

$$\begin{aligned} K &= 0.40 + 0.05\left(\frac{H}{P}\right) \\ &= 0.40 + 0.05\left(\frac{0.5}{0.9}\right) \\ &= 0.428 \end{aligned}$$

2. Rectangular weir equation

$$\begin{aligned} Q &= K\sqrt{2g}LH^{3/2} \\ &= 0.428\left(\sqrt{2 \times 9.81}\right) 3 \times 0.5^{3/2} \end{aligned}$$

$$\boxed{Q = 2.01\,\mathrm{m}^3/\,\mathrm{s}}$$

13.66: PROBLEM DEFINITION

Situation: Water (15 °C) flows into a reservoir through a venturi meter ($K = 1$, $A_o = 0.09$ m^2, $\Delta p = 70$ kPa). Water flows out of the reservoir over a 60^o triangular weir.

Find: Head of weir: H

SOLUTION Venturi equation

$$\begin{aligned} Q &= K A_o \sqrt{2\Delta p/\rho} \\ &= 1 \times (0.0078)\sqrt{2 \times 70,000/999} \\ &= 0.09 \text{ m}^3/\text{s} \end{aligned}$$

Rectangular weir equation

$$\begin{aligned} Q &= 0.179\sqrt{2g}H^{5/2} \\ 0.09 &= 0.179\sqrt{19.62}H^{5/2} \\ & \boxed{H = 0.42 \text{ m} = 42 \text{ cm}} \end{aligned}$$

13.67: PROBLEM DEFINITION

Situation: Water enters a tank through two pipes A and B. Water exits the tank through a rectangular weir.

Find: Is water level rising, falling or staying the same?

PLAN Calculate Q_{in} and Q_{out} and compare the values. Apply the rectangular weir equation to calculate Q_{out} and the flow rate equation to calculate Q_{in}.

SOLUTION Rectangular weir equation

$$\begin{aligned} Q_{\text{out}} &= K(2g)^{0.5}LH^{3/2} \\ K &= 0.40 + 0.05(1/2) = 0.425 \\ Q_{\text{out}} &= 0.425(4.43)(0.6)(0.2) \\ &= 0.23 \text{ m}^3/\text{s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q_{\text{in}} &= V_A A_A + V_B A_B \\ &= 1.2(\pi/4)(0.3^2) + 1.2(\pi/4)(0.15^2) \\ &= 0.11 \text{ m}^3/\text{s} \end{aligned}$$

$Q_{\text{in}} < Q_{\text{out}}$; therefore, **water level is falling**

13.68: PROBLEM DEFINITION

Situation: Water exits an upper reservoir across a rectangular weir ($L/H_R = 3$, $P/H_R = 2$) and then into a lower reservoir. The water exits the lower reservoir through a 60° triangular weir.

Find: Ratio of head for the rectangular weir to head for the triangular weir: H_R/H_T

Assumptions: Steady flow.

PLAN Apply continuity principle by equating the discharge in the two weirs.

SOLUTION Rectangular weir equation

$$Q = (0.40 + .05(1/2))\sqrt{2g}(3H_R)H_R^{1.5} \tag{1}$$

Triangular weir equation

$$Q = 0.179\sqrt{2g}H_T^{2.5} \tag{2}$$

Equate Eqs. (1) and (2)

$$\begin{aligned}
(0.425\sqrt{2g}(3)H_R^{2.5} &= 0.179\sqrt{2g}H_T^{2.5} \\
(H_R/H_T)^{2.5} &= 0.179/(3 \times 0.425) \\
& \boxed{H_R/H_T = 0.456}
\end{aligned}$$

13.69: PROBLEM DEFINITION

Situation: For Problem 13.68 (EFM 10e), the flow entering the upper reservoir is increased by 50%.

Find: Describe what will happen, both qualitatively and quantitatively.

PLAN Apply the rectangular and triangular weir equations.

SOLUTION As soon as the flow is increased, the water level in the first reservoir will start to rise. It will continue to rise until the outflow over the rectangular weir is equal to the inflow to the reservoir. The same process will occur in the second reservoir until the outflow over the triangular weir is equal to the inflow to the first reservoir.

Calculations

Determine the increase in head on the rectangular weir with an increase in discharge of 50%. Initial conditions: $H_R/P = 0.5$ so

$$K = 0.4 + .05 \times .5 = 0.425$$

Then

$$Q_{Ri} = 0.425\sqrt{2g}LH_{Ri}^{3/2} \tag{1}$$

Assume

$$K_f = K_i = 0.425 \text{ (first try)}$$

Then

$$Q_{Rf} = 0.425\sqrt{2g}LH_{Rf}^{3/2} \text{ (where } Q_{Rf} = 1.5Q_i) \tag{2}$$

Divide Eq. (2) by Eq. (1)

$$\begin{aligned} Q_{Rf}/Q_{Ri} &= (0.425L/0.425L)(H_{Rf}/H_{Ri})^{3/2} \\ H_{Rf}/H_{Ri} &= (1.5)^{2/3} = 1.31 \end{aligned}$$

Check K_i :

$$K = 0.40 + .05 \times 0.5 \times 1.31 = 0.433$$

Recalculate H_{Rf}/H_{Ri}.

$$H_{Rf}/H_{Ri} = ((0.425/0.433) \times 1.5)^{2/3} = 1.29$$

The final head on the rectangular weir will be 29% greater than the initial head. Now determine the increase in head on the triangular weir with a 50% increase in discharge.

$$\begin{aligned} Q_{Tf}/Q_{Ti} &= (H_{Tf}/H_{Ti})^{5/2} \\ \text{so } H_{Tf}/H_{Ti} &= (Q_{Tf}/Q_{Ti}) \\ &= (1.5)^{2/5} \\ & \boxed{H_{Tf}/H_{Ti} = 1.18} \end{aligned}$$

The head on the triangular weir will be 18% greater with the 50% increase in discharge.

13.70: PROBLEM DEFINITION

Situation:

Water flows over a 60^o triangular weir.

$H = 0.55$ m.

Find: Discharge (in m^3/s).

PLAN Apply the triangular weir equation.

SOLUTION

$$\begin{aligned} Q &= 0.179\sqrt{2g}H^{5/2} \\ Q &= 0.179\sqrt{2 \times (9.81\,\mathrm{m/\,s^2})} \times (0.55\,\mathrm{m})^{5/2} \\ &\boxed{Q = 0.18\ \mathrm{m^3/s}} \end{aligned}$$

13.71: PROBLEM DEFINITION

Situation: Water flows over a 45^o triangular weir. $Q = 0.0028$ m^3/s $\quad C_d = 0.6$.

Find: Head on the weir: H

SOLUTION

$$\begin{aligned}
Q &= (8/15)C_d(2g)^{0.5}\tan(\theta/2)H^{5/2} \\
Q &= (8/15)(0.60)(19.62\,\mathrm{m/s^2})^{0.5}\tan(22.5^\circ)H^{5/2} \\
Q &= 0.58H^{5/2} \\
H &= (Q/0.58)^{2/5} \\
&= \left(0.0028\,\mathrm{m^3/s}\middle/0.58\right)^{2/5}
\end{aligned}$$

$$\boxed{H = 0.12 \text{ m}}$$

13.72: PROBLEM DEFINITION

Situation:

A pump transports water from a well to a tank.
The tank empties through a 60° triangular weir.
Additional details are provided in the problem statement.

Find: Water level in the tank: h

Assumptions: $f = 0.02$

PLAN

Apply the triangular weir equation to calculate h. Apply the flow rate equation and the energy equation from well water surface to tank water surface to relate Q and h.

SOLUTION

1. Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 + h_p &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 0 + h_p &= 0 + 0 + (2 + h) + (V^2/2g)(K_e + (fL/D) + K_E) \end{aligned}$$

Inserting parameter values

$$\begin{aligned} 20 &= (2 + h) + (V^2/2g)(0.5 + (0.02 \times 2.5/0.05) + 1) \\ 18 &= h + 0.127V^2 \\ V &= ((18 - h)/0.127)^{0.5} \end{aligned}$$

$$\begin{aligned} Q &= VA \\ &= ((18 - h)/0.127)^{0.5}(\pi/4)(0.05)^2 \\ &= 0.00551(18 - h)^{0.5} \end{aligned} \quad (1)$$

2. Triangular weir equation

$$Q = 0.179\sqrt{2g}H^{2.5}$$

where $H = h - 1$. Then

$$Q = 0.179\sqrt{2g}(h - 1)^{2.5} = 0.793(h - 1)^{2.5} \quad (2)$$

3. Continuity (apply to tank)

$$Q_{\text{pump}} = Q_{\text{weir}}$$

Introduce Eq. (1) and Eq. (2)

$$\begin{aligned} 0.00551(18 - h)^{0.5} &= 0.793(h - 1)^{2.5} \\ 0.00695(18 - h)^{0.5} &= (h - 1)^{2.5} \end{aligned}$$

Solve for h:

$$\boxed{h = 1.24 \text{ m}}$$

Also, upon checking Re we find our assumed f is OK.

13.73: PROBLEM DEFINITION

Situation: A Pitot tube is used to record data in subsonic flow. $p_t = 140$ kPa, $p = 100$ kPa, $T_t = 300$ K.

Find: (a) Mach number: M
(b) Velocity: V

SOLUTION Use total pressure to find the Mach number

$$\begin{aligned} p_t/p_1 &= (1 + \frac{k-1}{2}M^2)^{\frac{k}{k-1}} \\ &= (1 + 0.2M^2)^{3.5} \text{ for air} \\ (140/100) &= (1 + 0.2M^2)^{3.5} \\ & \boxed{M = 0.710} \end{aligned}$$

Total temperature

$$\begin{aligned} T_t/T &= 1 + 0.2M^2 \\ T &= 300/1.10 = 273 \end{aligned}$$

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{(1.4)(287)(273)} \\ &= 331 \text{ m/s} \end{aligned}$$

Velocity from Mach number

$$\begin{aligned} V &= Mc \\ &= (0.71)(331) \\ & \boxed{V = 235 \text{ m/s}} \end{aligned}$$

13.74: PROBLEM DEFINITION

Find: Use the normal shock wave relationships from Chapter 12 to derive the Rayleigh supersonic Pitot formula.

SOLUTION

The purpose of the algebraic manipulation is to express p_1/p_{t_2} as a function of M_1 only.

For convenience, express the group of variables below as

$$\begin{aligned} F &= 1 + ((k-1)/2)M^2 \\ G &= kM^2 - ((k-1)/2) \\ p_1/p_{t_2} &= (p_1/p_{t_1})(p_{t_1}/p_{t_2}) = (p_1/p_{t_1})(p_1/p_2)(F_1/F_2)^{k/k-1} \end{aligned}$$

From Eq. (12.37) in EFM10e,

$$p_1/p_2 = (1 + kM_2^2)/(1 + kM_1^2)$$

So

$$p_1/p_{t_2} = (p_1/p_{t_1})((1 + kM_2^2)/(1 + kM_1^2))(F_1/F_2)^{k/k-1}$$

From Eq. (12.39) in EFM10e

$$(M_1/M_2) = ((1 + kM_1^2)/(1 + kM_2^2))(F_2/F_1)^{1/2}$$

Thus, we can write

$$(p_1/p_{t_2}) = (p_1/p_{t_1})(M_2/M_1)(F_1/F_2)^{k+1/(2(k-1))}$$

But, from Eq. (12.40) in EFM10e

$$M_2 = (F_1/G_1)^{1/2}$$

Also, $p_1/p_{t_1} = 1/(F_1^{k/k-1})$. So

$$\begin{aligned} p_1/p_{t_2} &= 1/(F_1^{k/k-1})(F_1^{1/2}/G_1^{1/2})(1/M_1)(F_1/F_2)^{k+1/(2(k-1))} \\ &= (G_1^{-1/2}/M_1)F_2^{-(k+1)/2(k-1)} \end{aligned}$$

However,

$$F_2 = 1 + ((k-1)/2)M_2^2 = 1 + ((k-1)/2)(F_1/G_2) = (((k+1)/2)M_1)^2/G_1$$

Substituting for F_2 in expression for p_1/p_{t_2} gives

$$p_1/p_{t_2} = (1/M_1)(G_1^{1/k-1})/((k+1)/2M_1)^{k+1/k-1}$$

Multiplying numerator and denominator by $(2/k+1)^{1/k-1}$ gives

$$p_1/p_{t_2} = \frac{\{[2kM_1^2/(k+1)] - (k-1)/(k+1)\}^{1/(k-1)}}{\{[(k+1)/2]M_1^2\}^{k/(k-1)}}$$

13.75: PROBLEM DEFINITION

Situation: A Pitot tube is used in supersonic airflow. $p = 54$ kPa, $p_t = 200$ kPa, $T_t = 350$ K.

Find: (a) Mach number: M_1
(b) Velocity: V_1

PLAN Apply the Rayleigh Pitot tube formula to calculate the Mach number. Then apply the Mach number equation and the total temperature equation to calculate the velocity.

SOLUTION

$$\begin{aligned} p_1/p_{t_2} &= \frac{\{[2kM_1^2/(k+1)] - (k-1)/(k+1)\}^{1/(k-1)}}{\{[(k+1)/2]M_1^2\}^{k/(k-1)}} \\ 54/200 &= (1.1667M_1^2 - 0.1667)^{2.5}/(1.2M_1^2)^{3.5} \end{aligned}$$

and solving for M_1 gives $\boxed{M_1 = 1.79}$

$$\begin{aligned} T_1 &= T_t/\left[1 + 0.5(k-1)M_1^2\right] \\ T_1 &= 350/(1 + 0.2(1.79)^2) \\ &= 213 \text{ K} \\ c_1 &= \sqrt{kRT} \\ &= \sqrt{(1.4)(287)(213)} \\ &= 293 \text{ m/s} \\ V_1 &= M_1 c_1 \\ &= 1.79 \times 293 \\ & \boxed{V_1 = 521 \text{ m/s}} \end{aligned}$$

13.76: PROBLEM DEFINITION

Situation: A venturi meter is used to measure flow of helium—additional details are provided in the problem statement.
$p_1 = 120$ kPa $p_2 = 80$ kPa $k = 1.66$ $D_2/D_1 = 0.5$, $T_1 = 17\,^\circ\text{C}$ $R = 2077$ J/kg·K.

Find: Mass flow rate: $\dot{m}$

PLAN Apply the ideal gas law and Eq. 13.23 in EFM10e to solve for the density and velocity at section 2. Then find mass flow rate $\dot{m} = \rho_2 A_2 V_2$.

SOLUTION Ideal gas law

$$\begin{aligned} \rho_1 &= p_1/(RT_1) \\ &= 120 \times 10^3/(2,077 \times 290) \\ &= 0.199 \text{ kg/m}^3 \\ p_1/\rho_1 &= 6.03 \times 10^5 \end{aligned}$$

Eq. (13.23) in EFM10e

$$\begin{aligned} V_2 &= ((5)(6.03 \times 10^5)(1 - 0.666^{0.4})/(1 - (0.666^{1.2} \times 0.54)))^{1/2} = 686 \text{ m/s} \\ \rho_2 &= (p_2/p_1)^{1/k}\rho_1 = (0.666)^{0.6}\rho_1 = 0.784\rho_1 = 0.156 \text{ kg/m}^3 \end{aligned}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho_2 A_2 V_2 \\ &= (0.156)(\pi/4 \times 0.005^2)(686) \end{aligned}$$

$$\boxed{\dot{m} = 0.0021 \text{ kg/s}}$$

13.77: PROBLEM DEFINITION

Situation:
Hydrogen (100 kPa, 15 °C) flows through an orifice ($d/D = 0.5$, $K = 0.62$) in a 2 cm pipe. The pressure drop across the orifice is 1 kPa.

Find:
Mass flow rate

Assumptions:
Y = compressibility = 1

SOLUTION

$$\begin{aligned} d/D &= 0.50 \\ d &= 0.5 \times 0.02 \text{ m} = 0.01 \text{ m} \end{aligned}$$

From Table A.2 for hydrogen ($T = 15°C = 288K$) : $k = 1.41$, and $\rho = 0.0851$ kg/m^3.

$$\begin{aligned} A_0 &= (\pi/4)(0.01)^2 = 7.85 \times 10^{-5} \text{ m}^2 \\ \dot{m} &= YA_0K(2\rho_1\Delta p) \\ \dot{m} &= (1)(7.85 \times 10^{-5})(0.62)(2(0.0851)(1000))^{0.5} \\ & \boxed{\dot{m} = 6.35 \times 10^{-4} \text{ kg/s}} \end{aligned}$$

13.78: PROBLEM DEFINITION

Situation: Natural gas (345 kPa, 21 °C) flows in a pipe.
A hole ($d = 0.005$ m) leaks gas.
$p_{atm} = 96$ kPa

Find: Rate of mass flow out of the leak: $\dot{m}$

Properties: For natural gas: $k = 1.31,\ R = 518$ J/kg-K.

Assumptions: The hole shape is like a truncated nozzle

SOLUTION

Hole area

$$\begin{aligned} A &= \frac{\pi d^2}{4} = \frac{\pi (0.005)^2}{4} \\ &= 1.96 \times 10^{-5} \end{aligned}$$

Pressure and temperature conversions.

$$\begin{aligned} p_t &= (345 + 96) = 441 \text{ kPa} \\ T &= (273 + 21) = 294 \text{ K} \end{aligned}$$

To determine if the flow is sonic or subsonic, calculate the critical pressure ratio

$$\begin{aligned} \frac{p_*}{p_t} &= \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \\ &= \left(\frac{2}{1.31+1}\right)^{\frac{1.31}{1.31-1}} \\ &= 0.544 \end{aligned}$$

Compare this to the ratio of back pressure to total pressure:

$$\begin{aligned} \frac{p_b}{p_t} &= \frac{96 \text{ kPa}}{441 \text{ kPa}} \\ &= 0.219 \end{aligned}$$

Since, $p_b/p_t < p_*/p_t$, the exit flow must be sonic (choked). Calculate the critical mass flow rate.

$$\begin{aligned} \dot{m} &= \frac{p_t A_*}{\sqrt{RT_t}} \sqrt{k} \left(\frac{2}{k+1}\right)^{\frac{(k+1)}{2(k-1)}} \\ &= \frac{441,000 \times 1.96 \times 10^{-5}}{\sqrt{518 \times 294}} \sqrt{1.31} \left(\frac{2}{1.31+1}\right)^{\frac{(1.31+1)}{2(1.31-1)}} \\ &= 0.015 \text{ kg/s} \end{aligned}$$

$$\boxed{\dot{m} = 0.015 \text{ kg/s}}$$

13.79: PROBLEM DEFINITION

Situation:
A stagnation tube is used to measure air speed
$\rho_{\text{air}} = 1.25$ kg/m^3, $d = 2$ mm, $C_p = 1.00$
Deflection on an air-water manometer, $h = 1$ mm.
The only uncertainty in the manometer reading is $U_h = 0.1$ mm.

Find:
(a) Air Speed: V
(b) Uncertainty in air speed: U_V

Assumptions: Neglect viscous effects ($C_p = 1$)

SOLUTION

$$\begin{aligned} V &= \left(\frac{2\Delta p}{\rho_{air}C_p}\right)^{1/2} \\ \Delta p &= h\gamma_w \end{aligned}$$

Combining equations

$$V = \left(\frac{2\gamma_w h}{\rho_{air}C_p}\right)^{1/2} = \left(\frac{(2)(9,810)(0.001)}{(1.25)(1.00)}\right)^{1/2}$$

$$\boxed{V = 3.96 \text{ m/s}}$$

Uncertainty equation

$$U_V = \frac{\partial V}{\partial h}U_h$$

The derivative is

$$\frac{\partial V}{\partial h} = \sqrt{\frac{2\gamma_w}{\rho_a C_p}}\frac{1}{2\sqrt{h}}$$

Combining equations gives

$$\begin{aligned} \frac{U_V}{V} &= \frac{U_h}{2h} \\ &= \frac{0.1\,\text{mm}}{2\times 1.0\,\text{mm}} \\ &= 0.05 \end{aligned}$$

So

$$\begin{aligned} U_V &= 0.05V \\ &= 0.05\times 3.96\,\text{m/s} \end{aligned}$$

$$\boxed{U_V = 0.198 \text{ m/s}}$$

13.80: PROBLEM DEFINITION

Situation: Water flows through a 15 cm orifice situated in a 30 cm pipe. On a mercury manometer, $\Delta h = 0.3$ m-Hg. The uncertainty values are $U_K = 0.03$, $U_H = 1.25$ cm-Hg, $U_d = 0.125$ cm.

Find: (a) Discharge: Q
(b)Uncertainty in discharge: U_Q

PLAN Calculate discharge by first calculating Δh (apply piezometric head and manometer equation) and to apply the orifice equation. Then apply the uncertainty equation.

SOLUTION Piezometric head

$$\Delta h = \left(\frac{p_1}{\gamma_w} + z_1\right) - \left(\frac{p_2}{\gamma_w} + z_2\right)$$

Manometer equation

$$\begin{aligned} p_1 + \gamma_w z_1 - \gamma_{H_g}(0.3\text{ m}) - \gamma_w(z_2 - 0.3\text{ m}) &= p_2 \\ \frac{p_1 - p_2}{\gamma_w} &= -(z_1 - z_2) + \left(\frac{\gamma_{Hg}}{\gamma_w}\right) 0.3\text{ m} - 0.3\text{ m} \end{aligned}$$

Combining equations

$$\begin{aligned} \Delta h &= (0.3\text{ m})\left(\frac{\gamma_{Hg}}{\gamma_w} - 1\right) \\ &= 0.3(13.6 - 1) = 3.8\text{ m of water} \end{aligned}$$

Uncertainty equation for Δh

$$\begin{aligned} U_{\Delta h} &= (0.0125\text{ m})\left(\frac{\gamma_{Hg}}{\gamma_w} - 1\right) = (0.0125)(13.6 - 1) \\ &= 0.158\text{ m of water} \end{aligned}$$

Orifice equation

$$\begin{aligned} Q &= K\frac{\pi}{4}d^2\sqrt{2g\Delta h} \\ \text{where } K &= 0.625 \text{ (from Problem 13.26 in EFM 10e)} \\ \text{Thus, } Q &= 0.625 \times \frac{\pi}{4} \times 0.15^2\sqrt{2 \times 9.81 \times 3.8} \end{aligned}$$

$$\boxed{Q = 0.095\,\text{m}^3/\text{s}}$$

Uncertainty equation applied to the discharge relationship

$$\begin{aligned}
\left(\frac{U_Q}{Q}\right)^2 &= \left(\frac{\frac{\partial Q}{\partial K}U_K}{Q}\right)^2 + \left(\frac{\frac{\partial Q}{\partial d}U_d}{Q}\right)^2 + \left(\frac{\frac{\partial Q}{\partial \Delta h}U_{\Delta h}}{Q}\right)^2 \\
\left(\frac{U_Q}{Q}\right)^2 &= \left(\frac{U_K}{K}\right)^2 + \left(\frac{2U_d}{d}\right)^2 + \left(\frac{U_{\Delta h}}{2\Delta h}\right)^2 \\
\left(\frac{U_Q}{Q}\right)^2 &= \left(\frac{.03}{0.625}\right)^2 + \left(\frac{2\times 0.00125}{0.15}\right)^2 + \left(\frac{0.158}{2\times 3.8}\right)^2 \\
\frac{U_Q}{Q} &= 0.055 \\
U_Q &= 0.055\times 0.095 = \boxed{5.23\times 10^{-3}\,\mathrm{m^3/s}}
\end{aligned}$$

$$\boxed{U_Q = 0.00523\,\mathrm{m^3/s}}$$

13.81: PROBLEM DEFINITION

Situation: A rectangular weir ($L = 3$ m, $P = 0.9$ m, $H = 0.5$ m) is used to measure discharge. The uncertainties are $U_K = 5\%$, $U_H = 7.5$ cm, $U_L = 2.5$ cm.

Find: (a) Discharge: Q
(b) Uncertainty in discharge: U_Q

PLAN Calculate K and apply the rectangular weir equation to find discharge. Then apply the uncertainty equation.

SOLUTION Rectangular weir equation

$$\begin{aligned} K &= 0.4 + 0.05\frac{H}{P} = 0.4 + 0.05 \times \left(\frac{0.5}{0.9}\right) \\ &= 0.428 \\ Q &= K\sqrt{2g}LH^{3/2} \\ &= (0.428)\sqrt{2 \times 9.81}(3)(0.5)^{3/2} \\ &\boxed{Q = 2.01 \text{ m}^3/\text{s}} \end{aligned}$$

Uncertainty equation

$$\begin{aligned} U_Q^2 &= \left(\frac{\partial Q}{\partial K}U_K\right)^2 + \left(\frac{\partial Q}{\partial L}U_L\right)^2 + \left(\frac{\partial Q}{\partial H}U_H\right)^2 \\ \left(\frac{U_Q}{Q}\right)^2 &= \left(\frac{U_K}{K}\right)^2 + \left(\frac{U_L}{L}\right)^2 + \left(\frac{3}{2} \times \frac{U_H}{H}\right)^2 \\ &= (.05)^2 + \left(\frac{0.025}{3}\right)^2 + \left(\frac{3}{2} \times \frac{0.075}{0.5}\right)^2 \\ &= 0.0532 \\ \text{Thus, } U_Q &= \sqrt{0.0532}\ \text{Q} \\ &= (0.231)(2.01) \\ &\boxed{U_Q = 0.46\,\text{m}^3/\text{s}} \end{aligned}$$