15.1: PROBLEM DEFINITION

Situation:

Comparison of full pipes versus open channels.

Find:

Why is the Reynolds number for open channels different than the one for pipes?

PLAN

Consider the different definitions for Re for the two cases.

SOLUTION

For fully flowing pipes, $\mathrm{Re} = \frac{VD}{\nu}$.
For partly flowing pipes and open channels, $\mathrm{Re} = \frac{VR_h}{\nu}$
Diameter is not equal to R_h.
For turbulence in open channels, if you wanted to use the criterion Re = 2000, you would need to define Re as: $\mathrm{Re} = \frac{V4R_h}{\nu}$.
Instead of inserting a "4" in the definition of Re for open channels, practioners established the convention of defining $\mathrm{Re} = \frac{VR_h}{\nu}$ in open channels, for which it has been found that $\mathrm{Re} > 500$.
Summary Answer: **Because Re_{oc} is in terms of R_h, and Re_{pipe} is in terms of D**

15.2: PROBLEM DEFINITION

Situation:

A rectangular open channel has a base of length 2b, and the water is flowing with a depth of b.

Find:

a. Sketch of channel.

b. Hydraulic radius of channel

PLAN

a. Sketch not shown

b. Use equation for hydraulic radius.

SOLUTION

Hydraulic radius

$$\begin{aligned} R_h &= A/P \\ &= \frac{2b \times b}{b + 2b + b} \\ &= \frac{2b^2}{4b} \\ & \boxed{\mathrm{R}_h = \frac{b}{2}} \end{aligned}$$

15.3: PROBLEM DEFINITION

Situation:

Two channels have the same cross-sectional area, but different geometry, as shown in problem statement.

Find:

a. Which channel has the largest wetted perimeter?
b. Which channel has more contact between water and channel-wall?
c. Which channel will have more energy loss to friction?

SOLUTION

a. Compare wetted perimeters

$P_A = 5 + 10 + 5 = 20;\qquad P_B = 3 \times 7.07 = 21.21;$

Conclusion: **Channel B has largest wetted perimeter**

b. **Channel B has the most contact between water and channel-wall.** This corresponds to its having the largest wetted perimeter.

c. **Channel B will have the most energy loss to friction**, because it has the largest wetted perimeter. This assumes that the two channels are of the same roughness.

15.4: PROBLEM DEFINITION

Situation:

Uniform flow of water in two channels.
Same slope, same wall roughness, same cross-sectional area.

Find:

Relate flow rates of two channels.

PLAN

Use Manning Equation for steady uniform open channel flow.

SOLUTION

$$
\begin{aligned}
Q &= \frac{1.0}{n} A R_h^{2/3} S^{1/2} \\
\frac{Q_A}{Q_B} &= \frac{R_{h,A}^{2/3}}{R_{h,B}^{2/3}} = \left(\frac{R_{h,A}}{R_{h,B}}\right)^{2/3} \\
\text{where } R_{h,A} &= \frac{4.5}{6} = 0.75 \text{ m};\ R_{h,B} = \frac{4.6}{3 \times 2.15} = 0.7 \text{ m} \\
R_{h,A} &> R_{h,B} \\
&\therefore \boxed{Q_A > Q_B}
\end{aligned}
$$

The correct choice is $\boxed{\text{(c).}}$

REVIEW

Note that it doesn't matter whether you use the Manning Equation for SI or Traditional units, because all of the constants cancel when you make a ratio of the two flow rates.

15.5: PROBLEM DEFINITION

Situation:

A wood flume of triangular cross-section (90^o interior angle).

Depth is 1 m, $S_o = .0019$

Wood, assumed to be planed, $n = 0.012$

Find:

Discharge of water ($\mathrm{m^3/s}$).

PLAN

Apply Manning's equation.

SOLUTION

Manning's equation

$$\begin{aligned}
Q &= \frac{1}{n} A R_h^{2/3} S_0^{1/2} \\
A &= \frac{(1)(2)}{2} = 1 \text{ m}^2 \\
R_h &= \frac{A}{P} \\
R_h &= \frac{1^2}{2(1^2 + 1^2)^{0.5}} = 0.35 \text{ m} \\
Q &= \frac{1}{.012}(1)(0.35)^{2/3}(0.0019)^{0.5} \\
& \boxed{Q = 1.82 \text{ m}^3/\text{s}}
\end{aligned}$$

REVIEW

Make sure to use the Manning Equation for SI units when your data are in SI units.

15.6: PROBLEM DEFINITION

Situation:

A rock-bedded stream, $d_{84} = 30\,\text{cm}$
Average depth = 1.80 m.
Channel slope of 0.0037.
Width = 52 m.

Find:

Discharge (m^3/ s) .

Assumptions:

$k_s = d_{84} = 30$ cm $= 0.3\,\text{m}$.
Width >> average depth, so $R_h = \frac{A}{P} \approx depth$.

PLAN

1. Use the form of f developed by Limerinos for a rocky stream bed.
2. Calculate Q using the Chezy Equation.

SOLUTION

1. Calculate f for rocky stream bed

$$\begin{aligned} R_h &= \frac{A}{P} \approx 1.80 \text{ m} \\ f &= \frac{1}{\left[1.2 + 2.03\log(\frac{R_h}{d_{84}})\right]^2} \\ f &= \frac{1}{\left[1.2 + 2.03\log(\frac{1.80}{0.3})\right]^2} \\ f &= 0.129 \end{aligned}$$

2. Calculate C and Q using Chezy equation

$$C = \sqrt{8g/f}$$

$$C = \sqrt{\frac{8 \times 9.81}{.129}}$$

$$= 24.6 \text{ m}^{1/2}\text{s}^{-1}$$

$$Q = CA\sqrt{R_h S_0}$$

$$Q = 24.6 \times 52 \times 1.80\sqrt{1.80 \times 0.0037}$$

$$\boxed{Q = 188 \text{ m}^3/\text{s}}$$

REVIEW

Remember: for a wide channel, $R_h \approx depth$.

15.7: PROBLEM DEFINITION

Situation:

Rectangular concrete channel
Water at 10 °C
Depth 1.5 m
$S_o = .001$

Find:

Discharge (m^3/s) .

Assumptions:

$\nu = 1.31 \times 10^{-6}\ \text{m}^2/\text{s}$; $k_s = 10^{-3}$ m

PLAN

Solve for Q with Darcy-Weisbach equation (note: could also use Manning equation).

SOLUTION

$$\begin{aligned} A &= 4.5\ \text{m}^2 \\ P &= 6\ \text{m} \\ R_h &= A/P = 0.75\ \text{m} \\ k_s/4R_h &= 0.333 \times 10^{-3} \end{aligned}$$

From Fig. 10.14 (EFM10e), $f = 0.016$

$$\begin{aligned} \frac{h_f}{L} &= \frac{fV^2}{2g4R_h} \\ V &= \sqrt{(8g/f)R_h S_o} = 1.92\ \text{m/s} \\ \text{Re} &= 1.92 \times 3/(1.31 \times 10^{-6}) = 4.4 \times 10^6 \end{aligned}$$

From Fig. 10.14 (EFM10e), $f = 0.015$,using the k_s assumed above
Then

$$V = 1.92 \times \sqrt{0.016/0.015} = 1.98\ \text{m/s}$$

Finally,

$$\begin{aligned} Q &= 1.98 \times 4.5 \\ & \boxed{\text{Q} = 8.91\ \text{m}^3/\text{s}} \end{aligned}$$

15.8: PROBLEM DEFINITION

Situation:

Concrete channel 4.3 m wide, uniform flow
Depth is 1.2 m.
Slope is $\frac{1.8}{2400} = 0.00075$.

Find:

Discharge $(\mathrm{m^3/s})$.

Assumptions:

$k_s = 0.003$

PLAN

1. Use Darcy-Weisbach equation to solve for Q.
2. Alternate Solution: use Manning equation.

SOLUTION

1. Darcy-Weisbach equation

$$R_h = A/P = 1.2 \times 4.3/(4.3 + 2 \times 1.2) = 0.8 \text{ m}$$
$$k_s/(4R_h) = 0.003/(4 \times 0.8) = 0.00094$$
$$\text{Re } f^{1/2} = \frac{(4R_h)^{3/2}}{\nu}\left[2gS_o\right]^{1/2}$$
$$\text{Re } f^{1/2} = 5.9 \times 10^5$$

From Fig. 10.14 (EFM10e), $f = 0.015$

$$\begin{aligned} V &= \sqrt{8gR_hS_o/f} \\ &= \sqrt{8g \times 0.8 \times \frac{1.8}{2400 \times 0.015}} \\ V &= 1.8 \\ Q &= 1.8(1.2)4.3 \\ &\boxed{Q = 9.3\,\mathrm{m^3/s}} \end{aligned}$$

2. Alternate solution: Manning equation
Assume $n = 0.015$

$$\begin{aligned} Q &= (1.0/n)AR_h^{2/3}S_o^{1/2} \\ &= \frac{1.0}{0.015} \times 1.2 \times 4.3(0.8)^{2/3}\left(\frac{1.8}{2400}\right)^{1/2} \\ &\boxed{Q = 8.1\,\mathrm{m^3/s}} \end{aligned}$$

15.9: PROBLEM DEFINITION

Situation:

Channels of rectangular cross section

For each channel, the following applies: $Q = 3\,\text{m}^3/\,\text{s} \quad S_o = 0.001$

Find:

Cross-sectional areas for widths of 0.6, 1.2, 1.8, 2.4, 3, and 4.5 m.

Assumptions:

$n = 0.015$ for unfinished concrete

PLAN

Explore best hydraulic section for a rectangular channel.
Best hydraulic section is minimum wetted perimeter for a given cross-sectional area.

1. Use Manning equation to relate area and hydraulic radius
2. Solve to find different values of y as a function of b
3. Find when wetted perimeter is a minimum

SOLUTION

1. Mannings equation in form of y and b

$$\begin{aligned}
Q &= (\frac{1.0}{n})AR_h^{0.667}S_o^{0.5} \\
\text{or } \frac{Qn}{1.0S_o^{0.5}} &= AR_h^{0.667} \\
\frac{3 \times 0.015}{1.0 \times .001^{0.5}} &= AR_h^{0.667} \\
1.42 &= (by)(\frac{by}{b+2y})^{0.667}
\end{aligned}$$

2. For different values of b one can compute y and the wetted perimeter, P. The following table results.

b (m)	y (m)	P (m^2)	y/b
0.6	4.95	3.15	8.2
1.2	1.8	1.44	1.5
1.8	1.14	1.22	0.63
2.4	0.84	1.22	0.35
3	0.69	1.31	0.23
4.5	0.51	1.66	0.11

3. The wetted perimeter is a minimum between 0.63 and 0.35, which verifies that the best hydraulic section for a rectangular cross-section is $\boxed{\frac{y}{b} = 0.5.}$

15.10: PROBLEM DEFINITION

Situation:

Sewer partially fills a concrete pipe.
The slope is 0.3 m of drop per 240 m of length.
Pipe diameter is $D = 0.8\,\mathrm{m}$.
Depth of sewer is $y = 0.4\,\mathrm{m}$.

Find:

The discharge (m^3/s).

Assumptions:

1. The properties of the sewer are those of clean water.
2. Assume concrete pipe; Manning's n-value of $n = 0.013$ is a middle value for concrete.

PLAN

1. Find flow area of sewer for Manning equation, given that the diameter is half full.
2. Find hydraulic radius for Manning equation, given that the diameter is half full.
2. Using Manning's equation (traditional units) to find

SOLUTION

1. Flow area

$$\begin{aligned} A &= \frac{\pi D^2}{8} = \frac{\pi\,(0.8\,\mathrm{m})^2}{8} \\ &= 0.25\ \mathrm{m}^2 \end{aligned}$$

2. Hydraulic radius

$$\begin{aligned} R_h &= \frac{A_c}{P_{\mathrm{wet}}} = \frac{\pi D^2/8}{\pi D/2} = \frac{D}{4} \\ &= \frac{0.8\,\mathrm{m}}{4} = 0.2\,\mathrm{m} \end{aligned}$$

.

3. Manning's equation (traditional units)

$$\begin{aligned} Q &= \frac{1.0}{n} A R_h^{2/3} \sqrt{S_o} \\ &= \frac{1.0}{0.013} \times 0.25 \times 0.2^{2/3} \sqrt{\frac{0.3\,\mathrm{m}}{240\,\mathrm{m}}} \end{aligned}$$

$$\boxed{Q = 0.23\,\mathrm{m}^3/\mathrm{s}}$$

15.11: PROBLEM DEFINITION

Situation:

Concrete sewer pipe
$D = 2\,\text{m}$, $S_o = 0.001$
Depth, $y = 1\,\text{m}$

Find:

The discharge (m^3/s).

Assumptions:

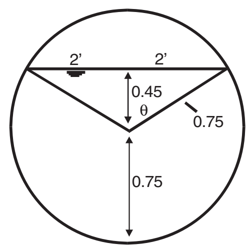
Smooth concrete, $n = 0.012$

PLAN

Use Manning's equation for a case where the depth is 4/5 of the diameter.

SOLUTION

$$Q = (1.0/n)AR_h^{0.667}S_0^{0.5}$$



$$\begin{aligned}
\cos\theta &= 0.45 \text{ m}/0.75 \text{ m} \\
\theta &= 53.13^\circ \\
A &= \pi r^2((360^\circ - 2\times 53.13^\circ)/360) + 0.5\times 1.2 \text{ m}\times 0.45 \text{ m} \\
A &= 1.51 \text{ m}^2 \\
P &= \pi D((360^\circ - 2\times 53.13^\circ)/360) = 3.3 \text{ m} \\
R_h &= A/P = 0.46 \text{ m} \\
R_h^{0.667} &= 0.596 \\
\text{Then } Q &= (1.0/0.012)(1.51)(0.596)(0.001)^{0.5} \\
& \boxed{Q = 2.4 \text{ m}^3/\text{s}}
\end{aligned}$$

15.12: PROBLEM DEFINITION

Situation:

Trapezoidal, concrete-lined channel - figure in the problem statement
Depth = 2 m
$S_o = 0.3$ m in 450 m

Find:

Average velocity (m/ s) .
Discharge (m^3/ s).

Assumptions:

$k_s = 0.0009$ m or $n = 0.015$
$\nu = 1.31 \times 10^{-6}$ m^2/s

PLAN

1. Use Darcy-Weisbach equation to solve for Q.
2. Alternate Solution: use Manning equation.

SOLUTION

1. Darcy-Weisbach equation

$$R_h = \frac{A}{P} = \frac{(3+2)2}{3+2(\sqrt{4+2})} = 1.3$$

$$\begin{aligned}(k_s/4R_h) &= 0.0009/(4\times 1.3) = 0.00017 \\ \text{Re}f^{1/2} &= ((4R_h)^{3/2}/\nu)(2gS_o)^{1/2} = [(4\times 1.3)^{3/2}/1.31\times 10^{-6}](2g\times 0.3/450)^{1/2} \\ &= 1.04\times 10^6\end{aligned}$$

From Fig. 10.14 (EFM10e), $f = 0.013$. Then

$$\begin{aligned}V &= \sqrt{8gR_hS_o/f} \\ &= \sqrt{8g\times 1.3\times 0.3/(450\times 0.014)} \\ &\boxed{V = 1.9 \text{ m/s}} \\ Q &= VA \\ &= 1.9\times 12 \\ &\boxed{Q = 22.8\,\text{m}^3/\text{s}}\end{aligned}$$

2. Manning equation

$$\begin{aligned}V &= \frac{1.0}{n}R_h^{2/3}S_o^{1/2} \\ &= (1.0/0.015)(1.3)^{2/3}(0.3/450)^{1/2} \\ &= \boxed{V = 2.05 \text{ m/s}} \\ Q &= 2.05(12.57) \\ &\boxed{Q = 25.76\,\text{m}^3/\text{s}}\end{aligned}$$

15.13: PROBLEM DEFINITION

Situation:

Trapezoidal, concrete-lined channel
$Q = 30\ \text{m}^3/\text{s}$
Slope is 0.3 m in 150 m
Bottom width = 3 m, and side slopes are 1:1

Find:

Depth of flow (m).

Assumptions:

$n = 0.012$

PLAN

Use Manning's equation (traditional units).

SOLUTION

Flow area

$$\begin{aligned} A_c &= \left(\frac{3\,\text{m} + (3\,\text{m} + 2d)}{2}\right) d \\ &= 3d + d^2 \end{aligned}$$

Wetted perimeter

$$\begin{aligned} P_{\text{wet}} &= 3\,\text{m} + 2 \times \sqrt{2d^2} \\ &= 3 + 2\sqrt{2}d \end{aligned}$$

Hydraulic radius

$$\begin{aligned} R_h &= \frac{A_c}{P_{\text{wet}}} \\ &= \frac{3d + d^2}{3 + 2\sqrt{2}d} \end{aligned}$$

Manning's equation (traditional units)

$$\begin{aligned} Q &= \frac{1.0}{n} A_c R_h^{2/3} \sqrt{S_o} \\ 30 &= \frac{1.0}{0.012} \times \left(3d + d^2\right) \times \left(\frac{3d + d^2}{3 + 2\sqrt{2}d}\right)^{2/3} \sqrt{\frac{0.3\,\text{m}}{150\,\text{m}}} \end{aligned}$$

Solve this equation (use a computer program such as MathCAD) to give $d = 1.601\,\text{m}$.

$$\boxed{d = 1.6\,\text{m}}$$

15.14: PROBLEM DEFINITION

Situation:

Trapezoidal, concrete-lined channel
Slope is 0.8 m/km
Bottom width = 6 m, and side slopes are 1:1
$d = 1.5\,\mathrm{m}$

Find:

Discharge $(\mathrm{m}^3/\mathrm{s})$.

Assumptions:

$n = 0.012$

PLAN

Use Manning's equation (traditional units).

SOLUTION

$$
\begin{aligned}
Q &= \frac{1.0}{n} A R_h^{2/3} S_o^{1/2} \\
A &= (6 \times 1.5) + 1.5 = 10.5\,\mathrm{m}^2, \quad P = 6 + 2\sqrt{1.5 + 1.5} = 9.46 \\
R_h &= \frac{A}{P} = \frac{10.5\,\mathrm{m}^2}{9.46\,\mathrm{m}} = 1.11 \text{ m}
\end{aligned}
$$

Then

$$
\begin{aligned}
Q &= (1.0/0.012)(10.5)(1.11)^{2/3}(0.8/1000)^{1/2} \\
&\boxed{Q = 26.53 \text{ m}^3/\text{s}}
\end{aligned}
$$

15.15: PROBLEM DEFINITION

Situation:

Rectangular, concrete-lined channel
Width is 4 m
Slope is .004
$Q = 25\,\mathrm{m^3/s}$

Find:

The uniform flow depth (m).

Assumptions:

$n = 0.015$

PLAN

Use the Manning equation to solve for d, given that $A = d \times 4\,\mathrm{m}$

SOLUTION

$$\begin{aligned} Q &= (1/n) A R_h^{2/3} {S_o}^{1/2} \\ 25 &= (1.0/0.015) 4d (4d/(4+2d))^{2/3} \times 0.004^{1/2} \end{aligned}$$

Solving for d yields: $\boxed{d = 1.6 \text{ m}}$

15.16: PROBLEM DEFINITION

Situation:

Rectangular, troweled concrete channel
Width is 2.4 m
Slope is 3 m in 900 m
$Q = 11.3\,\mathrm{m}^3/\,\mathrm{s}$

Find:

The depth of flow (m) .

Assumptions:

$n = 0.012$

PLAN

Use the Manning equation

SOLUTION

$$
\begin{aligned}
Q &= \frac{1.0}{n} A R_h^{2/3} {S_o}^{1/2} \\
400 &= (1.0/0.012) 2.4d (2.4/(2.4+2d))^{2/3} \times (3/900)^{1/2}
\end{aligned}
$$

Solving for d yields: $\boxed{d = 1.3 \text{ m}}$

15.17: PROBLEM DEFINITION

Situation:

Trapezoidal, concrete-lined channel
Slope is .001
Bottom width = 3 m, and side slopes are 1:2
$Q = 85\,\mathrm{m}^3/\,\mathrm{s}$

Find:

Depth of flow in channel (m).

Assumptions:

$n = 0.015$

PLAN

Use the Manning equation

SOLUTION

$$
\begin{aligned}
Q &= \frac{1.0}{n} A\, R_h^{2/3} S_0^{1/2} \\
85 &= ((1.0)/(0.015))(3d+2d^2)((3d+2d^2)/(3+2\sqrt{5d}))^{2/3}(0.001)^{1/2} \\
40 &= (3d+2d^2)((3d+2d^2)/(3+2\sqrt{5d}))^{2/3}
\end{aligned}
$$

Solving for d gives $\boxed{d = 3.03 \text{ m}}$

15.18: PROBLEM DEFINITION

Situation:

Canal, trapezoidal.
Design $Q = 25$ m^3/s
$S_o = 0.002$

Find:

Design with the best hydraulic section (m).

Assumptions:

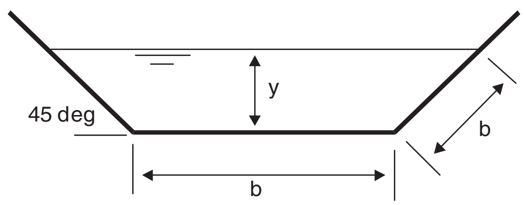
$n = 0.015$ (concrete, wood forms unfinished - Table 15.1, EFM10e)

PLAN

1. Best hydraulic section for a trapezoidal cross-section is a half-filled hexagon (note, for depiction below, the dimension b should be the same length on all limbs).
2. Use Manning equation
3. Relate to geometry of hexagon

SOLUTION

1. Sketch of best cross-section, noting the necessary angles for a hexagon.



2. Manning's equation

$$Q = \frac{1.0}{n} A R_h^{0.667} S_0^{0.5}$$

Then

$$25 = (1.0/0.015) A R_h^{0.667} (0.002)^{0.5}$$

$$A R_h^{0.667} = 7.5$$

3. For half-filled hexagon, $A = by + y^2$ where $y = b \cos 45^\circ = 0.707\text{b}$

$$\begin{aligned} A &= 0.707\text{b}^2 + 0.50b^2 = 1.207\text{b}^2 \\ R_h &= A/P = 1.207\text{b}^2/3b = 0.4024b \end{aligned}$$

Thus

$$AR_h^{0.667} = 7.5 = 1.207b^2(0.4024b)^{0.667}$$

$$\begin{aligned} b^{2.667} &= 15.5 \\ & \boxed{b = 2.79 \text{ m}} \end{aligned}$$

15.19: PROBLEM DEFINITION

How are head loss and slope related for non-uniform flow, as compared to uniform flow? Consider both rapidly and gradually varying non-uniform flow.

SOLUTION

- In uniform flow, velocity is constant along a streamline. Practically speaking, this requires a design of constant cross-section and slope.

  For uniform flow, as shown in Figure 15.4 (EFM10e), the slope of the HGL will be the same as the channel slope, because the velociy and depth are the same in the 2 sections that are compared. The HGL, and thus the slope of the water surface, is controlled by head loss. That is,

  $S_o = \frac{h_f}{L}$, where, from the Darcy-Weisbach equation: $h_L = f\frac{LV^2}{4R_h 2g}$.

  Therefore, for uniform flow, h_L is a function of viscosity

- For non-uniform flow, one must consider whether the situation at hand is rapidly varying, or gradually varying.

* For rapidly varied flow, one can neglect the resistance of the channel walls and bottom because it occurs over a short distance. Therefore, h_L is not a function of viscosity. For rapidly varying flow, let us consider a hydraulic jump. One determines the head loss by comparing the specific energy before the hydraulic jump to the specific energy after the jump; the head loss is the difference between the 2 specific energies. This yields

  $$h_L = \frac{(y_2-y_1)^3}{4y_1y_2}$$

  Therefore, for rapidly varying non-uniform flow, h_L is NOT a function of viscosity

* For gradually varied flow, because of the long distance involved, the surface resistance is a significant variable in the energy balance. In the introduction of "Quantitative Evaluation of the Water-Surface Profile", the following form for h_L over some short distance Δx is given by:

  $$h_L = f\frac{\Delta x V^2}{4R_h 2g}$$

- **Therefore, for gradually varying non-uniform flow, h_L is a function of viscosity**

15.20: PROBLEM DEFINITION

Situation:

Critical flow in general.

Find:

Is critical flow a desirable or undesirable flow condition? Why?

SOLUTION

Critical flow is undesirable because it is very unstable.
It is unstable because it occurs when specific energy is a minimum for a given discharge.
This means that only a slight change in specific energy wall cause the depth to increase or decrease a significant amount.
This manifests as a a series of standing waves - not a desirable engineered condition.
Therefore, channels are designed with a normal depth either much smaller than, or much larger than, the critical depth.

15.21: PROBLEM DEFINITION

Situation:

Critical flow __________. (Select all of the following that are correct.)

a. occurs when specific energy is a minimum for a given discharge.

b. occurs when the discharge is maximum for a given specific energy.

c. occurs when $\text{Fr} < 1$.

d. occurs when $\text{Fr} = 1$.

SOLUTION

a. Yes

b. Yes

c. No

d. Yes

15.22: PROBLEM DEFINITION

Situation:

Water flows through a rectangular channel.

$V = 10\,\mathrm{m/\,s}$, $y = 0.2\,\mathrm{m}$.

Find:

(a) Determine if the flow is subcritical or supercritical.

(b) Calculate the alternate depth (m).

PLAN

Check the Froude number, then apply the specific energy equation to calculate the alternative depth.

SOLUTION

Froude number

$$\begin{aligned} Fr &= V/\sqrt{gy} \\ &= 10\sqrt{9.81 \times 0.2} \\ Fr &= 7.13 \end{aligned}$$

The Froude number is greater than 1 so the flow is $\boxed{\text{supercritical}}$.

Specific Energy Equation

$$\begin{aligned} E &= y + V^2/2g \\ E &= 0.2 + 10/(2 \times 9.81) \\ E &= 5.3 \text{ m} \end{aligned}$$

Let the alternate depth $= y_2$, then

$$\begin{aligned} E &= y_2 + \frac{V_2^2}{2g} \\ &= y_2 + \frac{Q^2}{2g\,(y_2 \times 0.9)^2} \end{aligned}$$

Solving for the alternative depth for $E = 5.3\,\mathrm{m}$ yields $\boxed{y_2 = 5.5 \text{ m.}}$

15.23: PROBLEM DEFINITION

Water flows through a rectangular channel.
$Q = 25\,\mathrm{m}^3/\,\mathrm{s}$, $y = 1\,\mathrm{m}$
width $= 5\,\mathrm{m}$.

Find:
Determine if the flow is subcritical or supercritical.

PLAN

Calculate average velocity by applying the flow rate equation. Then check the Froude number.

SOLUTION

Flow rate equation

$$\begin{aligned} Q &= VA \\ 25 &= V \times 5 \times 1 \\ V &= 5\,\mathrm{m}/\,\mathrm{s} \end{aligned}$$

Froude number

$$\begin{aligned} Fr &= V/\sqrt{gy} \\ &= 5/\sqrt{9.81 \times 1} \\ Fr &= 1.6 \end{aligned}$$

The Froude number is greater than 1 so the flow is $\boxed{\text{supercritical}}$.

15.24: PROBLEM DEFINITION

Situation:

Water flows through a rectangular channel.
$Q = 12\,\text{m}^3/\,\text{s}$, $V = 3\,\text{m}/\,\text{s}$.
width $= 6\,\text{m}$.

Find:

Determine if the flow is subcritical or supercritical.

PLAN

Calculate y by applying the flow rate equation. Then check the Froude number.

SOLUTION

Flow rate equation

$$\begin{aligned} Q &= VA \\ 12 &= 3 \times 6 \times y \\ y &= 0.67 \text{ m} \end{aligned}$$

Froude number

$$\begin{aligned} Fr &= \frac{V}{\sqrt{gy}} \\ &= \frac{3\,\text{m}/\,\text{s}}{\sqrt{9.81 \times 0.67}} \\ Fr &= 1.2 \end{aligned}$$

Since $Fr > 1$, the flow is **supercritical**

15.25: PROBLEM DEFINITION

Situation:

Water flows through a rectangular channel.

$Q = 8\,\mathrm{m}^3/\,\mathrm{s}$.

width $= 2\,\mathrm{m}$.

Three depths of flow are of interest: $y = 0.3,\ 1.0$, and $2.0\,\mathrm{m}$.

Find:

(a) For each specified depth:

(i) Calculate the Froude number.

(ii) Determine if the flow is subcritical or supercritical.

(b) Calculate the critical depth (m) .

PLAN

Calculate average velocities by applying the flow rate equation. Then check the Froude numbers. Then apply the critical depth equation.

SOLUTION

Flow rate equation to find average velocities

$$\begin{aligned}
Q &= VA \\
8 \text{ m}^3/\text{s} &= V(2 \times y) \\
V_{0.30} &= 8 \text{ m}^3/\text{s } /(2 \text{ m} \times 0.30 \text{ m}) = 13.3 \text{ m/s}; \\
V_{1.0} &= 8 \text{ m}^3/\text{s } /(2 \text{ m } \times 1 \text{ m}) = 4 \text{ m/s} \\
V_{2.0} &= 8 \text{ m}^3/\text{s}/(2 \text{ m} \times 2 \text{ m}) = 2 \text{ m/s}
\end{aligned}$$

Froude numbers for each depth

$$\begin{aligned}
Fr_{0.3} &= 13.33 \text{ m/s}/(9.81 \text{ m/s}^2 \times 0.30 \text{ m})^{1/2} \\
&\boxed{Fr_{0.3} = 7.77 \text{ (supercritical)}} \\
Fr_{1.0} &= 4 \text{ m/s}/(9.81 \text{ m/s}^2 \times 1.0 \text{ m})^{1/2} \\
&\boxed{Fr_{1.0} = 1.28 \text{ (supercritical)}} \\
Fr_{2.0} &= 2 \text{ m/s } /9.81 \text{ m/s}^2 \times 1.0 \text{ m})^{1/2} = \\
&\boxed{Fr_{2.0} = 0.452 \text{ (subcritical)}}
\end{aligned}$$

Critical depth equation

$$\begin{aligned}
y_c &= (q^2/g)^{1/3}, \text{ where } q = Q/w \\
&= ((4 \text{ m}^2/\text{s})^2/(9.81 \text{ m/s}^2))^{1/3} \\
&\boxed{y_c = 1.18 \text{ m}}
\end{aligned}$$

15.26: PROBLEM DEFINITION

Situation:
Water flows through a rectangular channel.
$Q = 12\,\mathrm{m}^3/\,\mathrm{s}$, $y = 0.3\,\mathrm{m}$.
width $= 3\,\mathrm{m}$.

Find:
(a) Alternate depth (m).
(b) Specific energy (m).

PLAN

Apply the flow rate equation to find the average velocity. Then calculate specific energy and alternate depth.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{12}{3 \times 0.3} \\ &= 13.3 \text{ m/s} \end{aligned}$$

Specific Energy Equation

$$\begin{aligned} E &= y + V^2/2g \\ &= 0.30 + 9.06 \\ &\boxed{E = 9.36 \text{ m}} \end{aligned}$$

Let the alternate depth $= y_2$, then

$$\begin{aligned} E &= y_2 + \frac{V_2^2}{2g} \\ &= y_2 + \frac{Q^2}{2g\,(y_2 \times 3)^2} \end{aligned}$$

Substitute numerical values

$$9.36 = y_2 + \frac{12^2}{2 \times 9.81\,(y_2 \times 3)^2}$$

Solving for y_2 gives the alternate depth.

$$\boxed{y_2 = 9.35 \text{ m}}$$

15.27: PROBLEM DEFINITION

Situation:

Water flows at the critical depth in a channel; $V = 10\,\text{m/s}$.

Find:

Depth of flow (critical depth) (m).

PLAN

Calculate the critical depth by setting Froude number equal to 1.

SOLUTION

Froude number

$$\begin{aligned} Fr_c &= \frac{V}{\sqrt{gy_c}} \\ 1 &= \frac{10\ \text{m/s}}{\sqrt{9.81\ \text{m/s}^2 \times y_c}} \end{aligned}$$

Critical depth

$$\begin{aligned} y_c &= \frac{V^2}{g} \\ &= \frac{(10\ \text{m/s})^2}{9.81\ \text{m/s}^2} \\ &\boxed{y_c = 10.19\ \text{m}} \end{aligned}$$

15.28: PROBLEM DEFINITION

Situation:

Water flows in a rectangular channel.
Bottom slope = 0.005.
$n = 0.014$, $Q = 9.6 \text{ m}^3/\text{s}$.
width = 3.6 m.

Find:

Determine if the flow is subcritical or supercritical.

PLAN

Calculate y, then calculate the average velocity by applying the flow rate equation. Then check the Froude number.

SOLUTION

$$\begin{aligned}
Q &= \frac{1.0}{n} A R^{2/3} S_o^{1/2} \\
&= \frac{1.0}{n} A (A/P)^{2/3} S_o^{1/2} \\
&= \frac{1.0}{n} By(By/(b+2y))^{2/3} S_o^{1/2} \\
&= \frac{1.0}{n} 3.6y(3.6y/(3.6+2y))^{2/3} S_o^{1/2} \\
9.6 &= \frac{1.0}{0.014} 3.6y(3.6y/(3.6+2y))^{2/3}(0.005)^{1/2}
\end{aligned}$$

Solving for y yields: $y = 0.75$ m.
Flow rate equation

$$\begin{aligned}
V &= \frac{Q}{A} \\
&= 9.6 \text{ m}^3/\text{s} \;/(3.6 \text{ m} \times 0.75 \text{ m}) \\
V &= 3.6 \text{ m/s}
\end{aligned}$$

Froude number

$$\begin{aligned}
Fr &= V/\sqrt{gy} \\
&= 3.6/(9.81 \times 0.75)^{1/2} \\
Fr &= 1.33 \; \boxed{\text{supercritical}}
\end{aligned}$$

15.29: PROBLEM DEFINITION

Situation:

Water flows in a trapezoidal channel–additional details are provided in the problem statement.

Find:

Determine if the flow is subcritical or supercritical.

PLAN

Calculate Froude number by first applying the flow rate equation to find average velocity and the hydraulic depth equation to find the depth.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{10 \text{ m}^3/\text{s}}{(3 \times 1 \text{ m}^2) + 1^2 \text{ m}^2} \\ V &= 2.50 \text{ m/s} \end{aligned}$$

Calculate hydraulic depth

$$\begin{aligned} D &= \frac{A}{T} \\ &= \frac{4\,\text{m}^2}{5\,\text{m}} \\ D &= 0.80\,\text{m} \end{aligned}$$

Froude number

$$\begin{aligned} Fr &= \frac{V}{\sqrt{gD}} \\ &= \frac{2.50}{\sqrt{9.81 \times 0.80}} \\ Fr &= 0.89 \end{aligned}$$

Since $Fr < 1$, the flow is **subcritical**

15.30: PROBLEM DEFINITION

Situation:

Water flows in a trapezoidal channel—additional details are provided in the problem statement.

Find:

The critical depth (m).

PLAN

Calculate the critical depth by setting Froude number equal to 1, and simultaneously solving it along with the flow rate equation and the hydraulic depth equation.

SOLUTION

For the critical flow condition, Froude number = 1.

$$V/\sqrt{gD} = 1$$

or

$$(V/\sqrt{D}) = \sqrt{g}$$

Flow rate equation

$$\begin{aligned} V &= Q/A = 20/(3y + y^2) \\ D &= A/T = (3y + y^2)/(3 + 2y) \end{aligned}$$

Combine equations

$$(20/(3y + y^2))/((3y + y^2)/(3 + 2y))^{0.5} = \sqrt{9.81}$$

Solve for y

$$\boxed{y_{cr} = 1.40 \text{ m}}$$

15.31: PROBLEM DEFINITION

Situation:
Water flows in a rectangular channel–additional details are provided in the problem statement.

Find:
(a) Plot depth versus specific energy.
(b) Calculate the alternate depth (m).
(c) Calculate the sequent depth (m).

PLAN

Apply the specific energy equation.

SOLUTION

Specific Energy Equation for a rectangular channel.

$$E = y + q^2/(2gy^2)$$

For this problem

$$q = Q/B = 18/6 = 3 \text{ m}^2/\text{s}$$

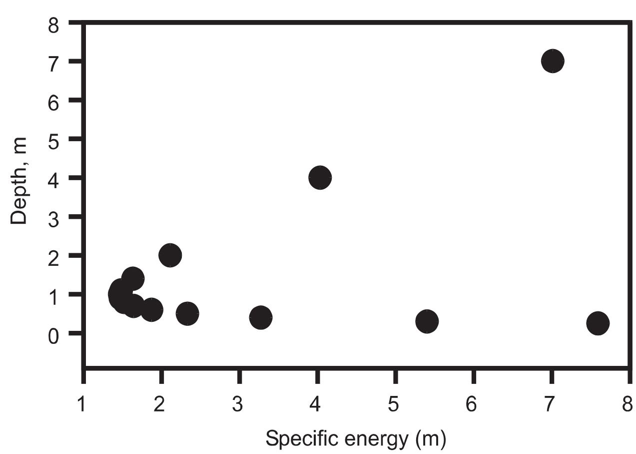
so

$$\begin{aligned} E &= y + 3^2/(2gy^2) \\ &= y + 0.4587/y^2 \end{aligned}$$

The calculated E versus y is shown below

y (m)	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.4	2.0	4.0	7.0
E (m)	7.59	5.4	3.27	2.33	1.87	1.64	1.52	1.47	1.46	1.48	1.63	2.11	4.03	7.01

(a) The corresponding plot is



(b) The alternate depth to $y = 0.30$ is $\boxed{y = 5.38 \text{ m}}$

(c) Sequent depth:

$$\begin{aligned} y_2 &= (y_1/2)(\sqrt{1+8F_1^2} - 1) \\ Fr_1 &= V/\sqrt{gy_1} \\ &= (3/0.3)/\sqrt{9.81 \times 0.30} \\ &= 5.83 \end{aligned}$$

Hydraulic jump equation

$$\begin{aligned} y_2 &= (0.3/2)(\sqrt{1+8 \times 5.83^2} - 1) = \boxed{2.33 \text{ m}} \\ &\boxed{y_2 = 2.33 \text{ m}} \end{aligned}$$

15.32: PROBLEM DEFINITION

Situation:

A rectangular channel ends in a free outfall
width = 8 m
depth at brink is 0.55 m

Find:

Discharge in the channel (m^3/s).

PLAN

Calculate the critical depth by setting Froude number equal to 1, and simultaneously solve it along with the brink depth equation. Then apply the flow rate equation.

SOLUTION

At the brink, the depth is 71% of the critical depth

$$d_{\text{brink}} \approx 0.71 y_c \tag{1}$$

Just before the brink where the flow is critical, $Fr = 1$

$$\begin{aligned} 1 &= \frac{V}{\sqrt{g y_c}} \\ &= \frac{q}{\sqrt{g y_c^3}} \end{aligned} \tag{2}$$

Combine Eqs. (1) and (2)

$$d_{\text{brink}} = 0.71 \left(\frac{q^2}{g} \right)^{\frac{1}{3}}$$

Or

$$\begin{aligned} q &= g^{1/2} \left(\frac{d_{\text{brink}}}{0.71} \right)^{3/2} \\ &= (9.81)^{1/2} \left(\frac{0.55}{0.71} \right)^{3/2} \\ &= 2.135 \text{ m}^2/\text{s} \end{aligned}$$

Discharge is

$$\begin{aligned} Q &= qw \\ &= \left(2.135 \text{ m}^2/\text{s} \right) (8 \text{ m}) \\ & \boxed{Q = 17.1 \text{ m}^3/\text{s}} \end{aligned}$$

15.33: PROBLEM DEFINITION

Situation:

A rectangular channel ends in a free outfall—additional details are provided in the problem statement.

Find:

Discharge in the channel ($\mathrm{m^3/s}$).

PLAN

Same solution procedure applies as in Prob. 15.32.

SOLUTION

From the solution to Prob. 15.32, we have

$$\begin{aligned} q &= (0.36 \times 9.81^{1/3}/0.71)^{3/2} \\ q &= 1.13 \text{ m}^2/\text{s} \end{aligned}$$

Then

$$\begin{aligned} Q &= 4.5 \times 1.13 \\ & \boxed{Q = 5.09 \text{ m}^3/\text{s}} \end{aligned}$$

15.34: PROBLEM DEFINITION

Situation:

A rectangular channel ends in a free outfall.
$Q = 15$ m^3/s.
Width = 4.2 m.

Find:

Depth of water at the brink of the outfall (m).

PLAN

Calculate the depth at the brink by setting Froude number equal to 1, and simultaneously solve this equation along with the brink depth equation.

SOLUTION

At the brink, the depth is 71% of the critical depth

$$d_{\text{brink}} \approx 0.71 y_c \tag{1}$$

Just before the brink where the flow is critical, $Fr = 1$

$$\begin{aligned} 1 &= \frac{V}{\sqrt{g y_c}} \\ &= \frac{q}{\sqrt{g y_c^3}} \end{aligned} \tag{2}$$

Combine Eqs. (1) and (2)

$$d_{\text{brink}} = 0.71 \left(\frac{q^2}{g} \right)^{\frac{1}{3}}$$

where

$$\begin{aligned} q &= \frac{Q}{w} \\ &= \frac{15\,\text{m}^3/\,\text{s}}{4.2\,\text{m}} \\ &= 3.6\,\text{m}^2/\,\text{s} \end{aligned}$$

Thus

$$\begin{aligned} d_{\text{brink}} &= 0.71 \left(\frac{(3.6\,\text{m}^2/\,\text{s})^2}{9.81\,\text{m}/\,\text{s}^2} \right)^{\frac{1}{3}} \\ &\boxed{d_{\text{brink}} = 0.78\ \text{m}} \end{aligned}$$

15.35: PROBLEM DEFINITION

Situation:

Water flows over a broad-crested weir
height = 1 m
$H = 0.5$ m

Find:

Discharge of water (m^3/s).

PLAN

Apply the Broad crested weir–Discharge equation.

SOLUTION

To look up the discharge coefficient, we need the parameter $\frac{H}{H+P}$,where P = height of weir, as in Fig. 15.13 (EFM10e).

$$\frac{H}{H+P} = (0.5/1.5)$$
$$\frac{H}{H+P} = 0.33$$

From Fig. 15.13 (EFM10e) $C = 0.88$.
Broad crested weir–Discharge equation

$$Q = 0.385\ CL\sqrt{2g}H^{1.5}$$
$$Q = 0.385(0.88)(3)\sqrt{2 \times 9.81}(0.5)^{1.5}$$
$$\boxed{Q = 1.6\ \text{m}^3/\text{s}}$$

15.36: PROBLEM DEFINITION

Situation:

Water flows over a broad-crested weir.
The weir height is $P = 2\,\mathrm{m}$.
The height of water above the weir is $H = 0.6\,\mathrm{m}$.
The length of the weir is $L = 5\,\mathrm{m}$.

Find:

Discharge ($\mathrm{m}^3/\,\mathrm{s}$).

PLAN

Apply the Broad crested weir–Discharge equation.

SOLUTION

To look up the discharge coefficient, we need the parameter $\frac{H}{H+P}$, where $P =$ height of weir, as in Fig. 15.13 (EFM10e).

$$\begin{aligned} \frac{H}{H+P} &= \frac{0.6}{0.6+2} \\ &= 0.23 \end{aligned}$$

From Fig. 15.13 (EFM10e)

$$C \approx 0.865$$

Broad crested weir–Discharge equation

$$\begin{aligned} Q &= 0.385\,CL\sqrt{2g}H^{3/2} \\ &= (0.385)\,(0.865)(5)\sqrt{2 \times 9.81}(0.60)^{1.5} \\ &\boxed{Q = 3.43\ \mathrm{m}^3/\mathrm{s}} \end{aligned}$$

15.37: PROBLEM DEFINITION

Situation:

Water flows over a broad-crested weir.

Additional details are given in the problem statement.

Find:

The water surface elevation in the reservoir upstream (m).

PLAN

Apply the Broad crested weir–Discharge equation.

SOLUTION

From Fig. 15.13 (EFM10e), $C \approx 0.85$

Broad crested weir–Discharge equation

$$Q = 0.385\, CL\sqrt{2g}H^{3/2}$$
$$25 = 0.385(0.85)(10)\sqrt{2 \times 9.81}H^{3/2}$$

Solve for H

$$(H)^{3/2} = 1.725$$
$$H = 1.44\,\text{m}$$

Water surface elevation

$$\text{Elev.} = 100\,\text{m} + 1.44\,\text{m}$$
$$\boxed{\text{Elev.} = 101.44 \text{ m}}$$

15.38: PROBLEM DEFINITION

Situation:

Water flows over a broad-crested weir.
Additional details are given in the problem statement.

Find:

The water surface elevation in the upstream reservoir (m).

PLAN

Apply the Broad crested weir–Discharge equation.

SOLUTION

From Fig. 15.13 (EFM10e), $C \approx 0.85$
Broad crested weir–Discharge equation

$$\begin{aligned} Q &= 0.385C\, L\sqrt{2g}H^{3/2} \\ 36 &= 0.385(0.85)(12)\sqrt{19.62}H^{3/2} \\ H &= 1.62 \text{ m} \end{aligned}$$

Water surface elevation = **91.62 m**

15.39: PROBLEM DEFINITION

Situation:

Water flows in a rectangular channel.

Two situations are of interest: an upstep and a downstep in bottom elev, each of 30 cm.

$V = 3$ m/s

Initial depth = 3m

Find:

(a) Change in depth and water surface elevation for upstep of 30cm (m).

(b) Change in depth and water surface elevation for the downstep of 30cm (m).

(c) Maximum size of upstep so that no change in upstream depth occurs (m).

PLAN

Apply the specific energy equation and check the Froude number.

SOLUTION

(a) Specific Energy Equation prior to upstep

$$\begin{aligned} E_1 &= y_1 + V_1^2/2g \\ &= 3 + 3^2/(2 \times 9.81) \\ E_1 &= 3.46 \text{ m} \end{aligned}$$

Froude number

$$\begin{aligned} Fr_1 &= V_1/\sqrt{gy_1} \\ &= 3/\sqrt{9.81 \times 3} \\ Fr_1 &= 0.55 \text{ (subcritical)} \end{aligned}$$

Apply upstep

$$\begin{aligned} E_2 &= E_1 - \Delta z_{\text{step}} = 3.46 - 0.30 \\ & \boxed{E_2 = 3.16 \text{ m}} \end{aligned}$$

Specific Energy Equation

$$\begin{aligned} y_2 + q^2/(2gy_2^2) &= 3.16 \text{ m} \\ y_2 + 9^2/(2gy_2^2) &= 3.16 \\ y_2 + 4.13/y_2^2 &= 3.16 \end{aligned}$$

Solving for y_2 yields

$$y_2 = 2.49 \, \text{m}$$

Then, for change in depth

$$\begin{aligned}\Delta y &= y_2 - y_1\\ &= 2.49 - 3.00\\ &\boxed{\Delta y = \text{-}0.51 \text{ m}}\end{aligned}$$

So water surface drops from 3.0 to 2.49, which is a drop of $\boxed{0.21 \text{ m.}}$

(b) For a downstep

$$\begin{aligned}E_2 &= E_1 + \Delta z_{\text{step}}\\ &= 3.46 + 0.3\\ &\boxed{E_2 = 3.76 \text{ m}}\\ y_2 + 4.13/y_2^2 &= 3.76\end{aligned}$$

Solving for y_2 gives

$$y_2 = 3.40\text{m}$$

Then

$$\begin{aligned}\Delta y &= y_2 - y_1\\ &= 3.40 - 3\\ &\boxed{\Delta y = 0.40 \text{ m}}\end{aligned}$$

Water surface elevation change is from 3.0 to 3.4, which is an increase of $\boxed{+0.10 \text{ m}}$

(c) Max. upward step before altering upstream conditions:

$$\begin{aligned}y_c &= y_2 = \sqrt[3]{q^2/g} = \sqrt[3]{9^2/9.81} = 2.02\\ E_1 &= \Delta z_{\text{step}} + E_2\end{aligned}$$

where

$$E_2 = 1.5 y_c = 1.5 \times 2.02 = 3.03 \text{ m}$$

Maximum size of step

$$\begin{aligned}z_{\text{step, max}} &= E_1 - E_2 = 3.46 - 3.03\\ &\boxed{z_{\text{step, max}} = 0.43 \text{ m}}\end{aligned}$$

15.40: PROBLEM DEFINITION

Situation:

Water flows in a rectangular channel.
Two situations are of interest: an upstep and a downstep.
Additional details are provided in the problem statement.

Find:

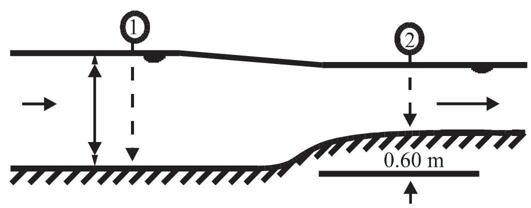
(a) Change in depth and water surface elevation for the upstep (m).
(b) Change in depth and water surface elevation for the downstep (m).
(c) Maximum size of upstep so that no change in upstream depth occurs (m).

PLAN

Apply the specific energy equation by first calculating Froude number and critical depth.

SOLUTION

For the upstep



$$\begin{aligned} E_2 &= E_1 - 0.60 \\ V_1 &= 2 \text{ m/s} \end{aligned}$$

Froude number

$$\begin{aligned} Fr_1 &= V_1/\sqrt{gy_1} \\ &= 2/\sqrt{9.81 \times 3} \\ &= 0.369 \end{aligned}$$

Specific Energy Equation

$$\begin{aligned} E_2 &= 3 + (2^2/(2 \times 9.81)) - 0.60 = 2.60 \text{ m} \\ y_2 + q^2/(2gy_2^2) &= 2.60 \end{aligned}$$

where $q = 2 \times 3 = 6$ m^3/s/m. Then

$$\begin{aligned} y_2 + 6^2/(2 \times 9.81 \times y_2^2) &= 2.60 \\ y_2 + 1.83/y_2^2 &= 2.60 \end{aligned}$$

Solving, one gets $y_2 = 2.24$ m. Then

$$\begin{aligned} \Delta y &= y_2 - y_1 = 2.34 - 3.00 \\ & \boxed{\Delta y = -0.76 \text{ m}} \end{aligned}$$

Water surface drops $\boxed{0.16 \text{ m}}$
For downward step of 15 cm we have

$$\begin{aligned} E_2 &= (3 + (2^2/(2 \times 9.81)) + 0.15 = 3.35 \text{ m} \\ y_2 + 6^2/(2 \times 9.81 \times y_2^2) &= 3.35 \\ y_2 + 1.83/y_2^2 &= 3.35 \end{aligned}$$

Solving: $y_2 = 3.17$ m or

$$\begin{aligned} y_2 - y_1 &= 3.17 - 3.00 \\ & \boxed{y_2 - y_1 = 0.17 \text{ m}} \end{aligned}$$

Water surface rises $\boxed{0.02 \text{ m}}$
The maximum upstep possible before affecting upstream water surface levels is for $y_2 = y_c$
Critical depth equation

$$y_c = \sqrt[3]{q^2/g} = 1.54 \text{ m}$$

Then

$$\begin{aligned} E_1 &= \Delta z_{\text{step}} + E_{2,\text{crit}} \\ \Delta z_{\text{step}} &= E_1 - E_{2,\text{crit}} = 3.20 - (y_c + V_c^2/2g) = 3.20 - 1.5 \times 1.54 \\ & \boxed{\Delta z_{\text{step}} = 0.89 \text{ m}} \end{aligned}$$

15.41: PROBLEM DEFINITION

Situation:
Water flows over a gradual upstep, shown in figure.
Desired: Unit flowrate $q = 6$ m^2/s.
Upstream depth $y_1 = 3$m

Find:
Maximum value of Δz to permit a unit flow rate of 6 m^2/s without increasing the upstream depth (m).

Assumptions:
No energy loss.

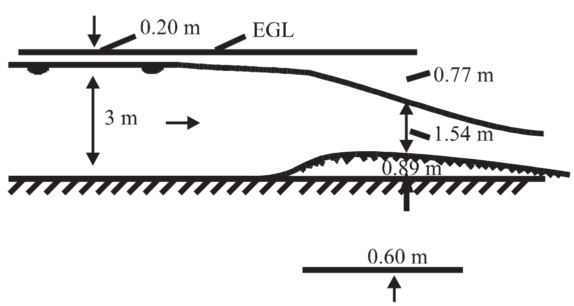
SOLUTION

Critical depth equation

$$\begin{aligned} y_c &= (q^2/g)^{1/3} \\ &= (6^2/9.81)^{0.333} \\ y_c &= 1.542 \text{ m} \end{aligned}$$

where y_c is depth allowed over the hump for the given conditions.
Specific Energy Equation

$$\begin{aligned} E_1 &= E_2 \\ V_1 &= q/y_1 = 6/3 = 2 \text{ m/s} \\ V_2 &= q/y_2 = 6/1.542 = 3.891 \text{ m/s} \\ V_1^2/2g + y_1 &= V_2^2/2g + y_2 + \Delta z \\ 2^2/2g + 3 &= (3.891^2/(2 \times 9.81)) + 1.542 + \Delta z \\ \Delta z &= 3.204 - 0.772 - 1.542 \\ &\boxed{\Delta z = 0.89 \text{ m}} \end{aligned}$$



15.42: PROBLEM DEFINITION

Situation:

A rectangular channel has a gradual contraction in width—additional details are provided in the problem statement.

Find:

(a) Change in depth (m).
(b) Change in water surface elevation (m).
(c) Greatest contraction allowable so that upstream conditions are not altered (m).

SOLUTION

(a) Froude number

$$\begin{aligned} Fr_1 &= V_1/\sqrt{gy_1} \\ &= 3/\sqrt{9.81 \times 3} \\ Fr_1 &= 0.55 \text{ (subcritical)} \end{aligned}$$

(b) Specific Energy Equation to find y_2, and thus Δz

$$\begin{aligned} E_1 &= E_2 \\ &= y_1 + V_1^2/2g \\ &= 3 + 3^2/2 \times 9.81 = 3.46 \text{ m} \\ q_2 &= Q/B_2 = 27/2.6 = 10.4 \text{ m}^3/\text{s/m} \end{aligned}$$

Then

$$\begin{aligned} & y_2 + q^2/(2gy_2^2) \\ &= y_2 + (10.4)^2/(2 \times 9.81 \times y_2^2) = 3.46 \\ y_2 + 5.50/y_2^2 &= 3.46 \end{aligned}$$

Solving: $\boxed{y_2 = 2.71 \text{ m.}}$

$$\begin{aligned} \Delta z_{\text{water surface}} &= \Delta y = y_2 - y_1 = 2.71 - 3.00 \\ & \boxed{\Delta z_{\text{water surface}} = 0.29 \text{ m}} \end{aligned}$$

(c) Max. contraction without altering the upstream depth will occur with $y_2 = y_c$

$$E_2 = 1.5y_c = 3.45; \; y_c = 2.31 \text{ m}$$

Then

$$\begin{aligned} V_c^2/2g &= y_c/2 = 2.31/2 \text{ or } V_c = 4.76 \text{ m/s} \\ Q_1 &= Q_2 = 27 = B_2 y_c V_c \\ B_2 &= 27/(2.31 \times 4.76) = 2.46 \text{ m} \end{aligned}$$

The width for max. contraction = $\boxed{2.46 \text{ m}}$

15.43: PROBLEM DEFINITION

Situation:

"Ship squat" problem when the draft of vessels approaches the depth of the ship channel.

Ship reduces the cross-sectional area available for flow in the channel.

Ship is lower when moving than it would be if it were stationary.

$V_{ship} = 5\text{kt} = 2.575\text{m/s}$; $y_1 = 35\text{m}$; channel width = 200 m

Draft of ship is 29m when fully loaded; width ship = 63m; length ship = 414m

Find:

The change in elevation or "ship squat" of a fully loaded supertanker (m).

PLAN

Reference the water velocity to the ship, and apply the energy equation.
Apply the specific energy equation from a section in the channel upstream of the ship to a section where the ship is located.
Then apply the flow rate equation and solve for y_2.

SOLUTION

Specific Energy Equation

$$\begin{aligned}
E_1 &= E_2 \\
V_1^2/2g + y_1 &= V_2^2/2g + y_2 \\
A_1 &= 35 \times 200 = 7,000 \text{ m}^2 \\
V_1 &= 5 \times 0.515 = 2.575 \text{ m/s} \\
2.575^2/(2 \times 9.81) + 35 &= (Q/A_2)^2/(2 \times 9.81) + y_2 \quad (1) \\
\text{where } Q &= V_1 A_1 = 2.575 \times 7,000 \text{ m}^3/\text{s} \\
A_2 &= (200 \text{ m } \times y_2) - (29 \times 63)
\end{aligned}$$

Flow rate equation

$$\begin{aligned}
Q &= V_1 A_1 \quad (2) \\
&= 2.575 \times 7,000 \text{ m}^3/\text{s} \\
A_2 &= (200 \text{ m } \times y_2) - (29 \times 63) \quad (3)
\end{aligned}$$

Substituting Eqs. (2) and (3) into Eq. (1) and solving for y_2 yields $y_2 = 34.70$ m. Therefore, the ship squat is

$$\begin{aligned}
y_1 - y_2 &= 35.0 - 34.7 \\
&\boxed{\text{ship squat} = 0.30 \text{ m}}
\end{aligned}$$

15.44: PROBLEM DEFINITION

Situation:

A rectangular channel has a small reach that is roughened with angle irons—additional details are provided in the problem statement.

Find:

Determine the depth of water downstream of angle irons (m).

PLAN

Apply the momentum principle for a unit width.

SOLUTION Momentum principle

$$\begin{aligned}
\sum F_x &= \sum \dot{m}_o V_o - \sum \dot{m}_i V_i \\
\gamma y_1^2/2 - \gamma y_2^2/2 - 8900 &= -\rho V_1^2 y_1 + \rho V_2^2 y_2
\end{aligned}$$

Let $V_1 = q/y_1$ and $V_2 = q/y_2$ and divide by γ

$$\begin{aligned}
y_1^2/2 - y_2^2/2 - 8900/\gamma &= -q^2 y_1/(g y_1^2) + q^2 y_2/(g y_2^2) \\
1/2 - y_2^2/2 - 0.91 &= (+(5.7)^2/9.81)(-1 + 1/y_2)
\end{aligned}$$

Solving for y_2 yields: $\boxed{y_2 = 0.5 \text{ m}}$

15.45: PROBLEM DEFINITION

Situation:

Water flows out of a reservoir into a steep rectangular channel—additional details are provided in the problem statement.

Find:

Discharge (m^3/s).

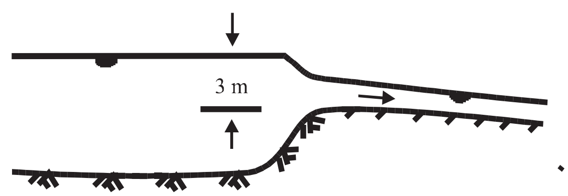
Assumptions:

Negligible velocity in the reservoir and negligible energy loss. Then the channel entrance will act like a broad crested weir.

PLAN

Apply the Broad crested weir–Discharge equation.

SOLUTION



Broad crested weir–Discharge equation

$$Q = 0.545\sqrt{g}LH^{3/2}$$

where $L = 4$ m and $H = 3$ m. Then

$$Q = 0.545\sqrt{9.81} \times 4 \times 3^{3/2}$$

$$\boxed{Q = 35.5 \text{ m}^3/\text{s}}$$

15.46: PROBLEM DEFINITION

Situation:

A small wave is produced in a pond.
Pond depth = 0.15 m.

Find:

Speed of the wave (m/ s).

PLAN

Apply the wave celerity equation.

SOLUTION

Wave celerity

$$\begin{aligned} V &= \sqrt{gy} \\ &= \sqrt{9.81\,\text{m/s}^2 \times 0.15\,\text{m}} \\ & \boxed{V = 1.21 \text{ m/s}} \end{aligned}$$

15.47: PROBLEM DEFINITION

Situation:

A small wave travels in a pool of water.
Depth of water is constant.
Wave speed = 1.5 m/ s.

Find:

Depth of water (m).

PLAN

Apply the wave celerity equation.

SOLUTION

Wave celerity

$$\begin{aligned} V &= \sqrt{gy} \\ 1.5 &= \sqrt{9.81y} \\ & \boxed{y = 0.23 \text{ m}} \end{aligned}$$

15.48: PROBLEM DEFINITION

Situation:

As ocean waves approach a sloping beach, they curve so that they are aligned parallel to the beach.

Find:

Explain the observed phenomena.

PLAN

Apply the wave celerity equation.

SOLUTION

As the waves travel into shallower water their speed is decreased.
Wave celerity

$$V = \sqrt{gy}$$

Therefore, the wave in shallow water lags that in deeper water. Thus, the wave crests tend to become parallel to the shoreline.

15.49: PROBLEM DEFINITION

Situation:

For a hydraulic jump, ____________________. (Select all of the following that are correct.)

a. the flow changes from subcritical to supercritical.

b. the flow changes from supercritical to subcritical.

c. significant energy is lost.

d. the height of the water abruptly increases from the upstream to the downstream cross-section.

e. the downstream and upstream depth are related quantitatively in terms of the upstream Fr.

f. the energy equation is a better tool for analysis than the momentum equation.

SOLUTION

a. No
b. Yes
c. Yes
d. Yes
e. Yes
f. No

15.50: PROBLEM DEFINITION

Situation:

A baffled ramp is used to dissipate energy in an open channel—additional details are provided in the problem statement.

Find:

(a) Head that is lost (m).
(b) Power that is dissipated (W).
(c) Horizontal component of the force exerted by ramp on the water (N).

Assumptions:

The kinetic energy correction factors are all $\alpha \approx 1.0$.
x positive in the direction of flow.

PLAN

Let the upstream section (where $y = 1$ m) be section 1 and the downstream section ($y = 0.6$ m) be section 2. Solve for the velocities at 1 and 2 using the flow rate equation. Then apply the energy equation and power equation. Determine the force of ramp by writing the momentum equation between section 1 and 2. Let F_x be the force of the ramp on the water.

SOLUTION

(a) Flow rate equation

$$\begin{aligned} V &= Q/A \\ V_1 &= 2/1 \\ &= 2 \text{ m/s} \\ V_2 &= 0.6 \\ V_2 &= 3.3 \text{ m/s} \end{aligned}$$

Energy equation

$$\begin{aligned} y_1 + \alpha_1 V_1^2/2g + z_1 &= y_2 + \alpha_2 V_2^2/2g + z_2 + h_L \\ 1 + 2^2/(2 \times 9.81) + 0.6 &= 0.6 + 3.3^2/(2 \times 9.81) + h_L \\ & \boxed{h_L = 0.65 \text{ m}} \end{aligned}$$

(b) Power equation

$$\begin{aligned} P &= Q\gamma h_L \\ &= 2 \times 9810 \times 0.65 \\ & \boxed{P = 12{,}753 \text{ W}} \end{aligned}$$

(c) Momentum principle, for horizontal component

$$\begin{aligned}
\sum F_x &= \rho q(V_{2x} - V_{1x}) \\
\gamma y_1^2/2 - \gamma y_2^2/2 + F_x &= 999 \times 2(3.3 - 2) \\
(9810/2)(1^2 - 0.6) + F_x &= 2597.4 \\
F_x &= -542 \text{ N}
\end{aligned}$$

The ramp exerts a force of **542 N opposite to the direction of flow.**

15.51: PROBLEM DEFINITION

Situation:

Water flows out a reservoir, down a spillway and then forms a hydraulic jump near the base of the spillway.

Flow rate is $q = 2.9\,\mathrm{m}^3/\,\mathrm{s}$ per m of width.

Additional details are provided in the problem statement.

Find:

Depth downstream of hydraulic jump (m).

PLAN

Apply the specific energy equation to calculate y_1. Then calculate Froude number in order to apply the Hydraulic jump equation.

SOLUTION

Specific Energy

$$\begin{aligned} y_0 + q^2/(2gy_0^2) &= y_1 + q^2/(2gy_1^2) \\ 5 + 2.9^2/(2(9.81)5^2) &= y_1 + 2.9^2/(2(9.81)y_1^2) \\ y_1 &= 0.3015 \text{ m} \end{aligned}$$

Froude number

$$\begin{aligned} Fr_1 &= \frac{q}{\sqrt{gy_1^3}} \\ &= \frac{2.9}{\sqrt{9.81(0.3015)^3}} \\ Fr_1 &= 5.593 \end{aligned}$$

Hydraulic jump equation

$$\begin{aligned} y_2 &= (y_1/2)\left(\sqrt{1+8F_1^2}-1\right) \\ &= (0.3015/2)\left(\sqrt{1+8(5.593^2)}-1\right) \\ & \boxed{y_2 = 2.24 \text{ m}} \end{aligned}$$

15.52: PROBLEM DEFINITION

Situation:

Water flows out a sluice gate

Depth = 32 cm

$Q = 5.2\ \mathrm{m^3/s}$ per meter

Find:

(a) Determine if a hydraulic jump can exist.

(b) If the hydraulic jump can exist, calculate the depth downstream of the jump (m).

PLAN

Calculate Froude number, then apply the hydraulic jump equation.

SOLUTION

Froude number

$$\begin{aligned} Fr_1 &= \frac{V}{\sqrt{gy}} \\ &= \frac{q}{\sqrt{gy^3}} \\ &= \frac{5.2\,\mathrm{m^2/\,s}}{\sqrt{9.81 \times 0.32^3}\,\mathrm{m^2/\,s}} \\ Fr_1 &= 9.17 \end{aligned}$$

Thus, a $\boxed{\text{hydraulic jump can occur.}}$

Hydraulic jump equation

$$\begin{aligned} y_2 &= (y_1/2)\left(\sqrt{1+8Fr_1^2}-1\right) \\ &= (0.32/2)\left(\sqrt{1+8\times 9.17^2}-1\right) \\ &\boxed{y_2 = 3.99\ \mathrm{m}} \end{aligned}$$

15.53: PROBLEM DEFINITION

Situation:
A hydraulic jump is described in the problem statement.

Find:
Depth upstream of the hydraulic jump (m).

PLAN

Apply the hydraulic jump equation.

SOLUTION

Hydraulic jump equation

$$y_2 = (y_1/2)((1 + 8Fr_1^2)^{0.5} - 1)$$

where Froude number

$$Fr_1^2 = V_1^2/(gy_1) = q^2/(gy_1^3)$$

Then

$$\begin{aligned} y_2 &= (y_1/2)((1 + 8q^2/(gy_1^3))^{0.5} - 1) \\ y_2 - y_1 &= (y_1/2)[((1 + 8q^2/(gy_1^3))^{0.5} - 1 - 2] \end{aligned}$$

However

$$\begin{aligned} y_2 - y_1 &= 4 \text{ m (given)} \\ q &= 6 \text{ m}^2/\text{s} \end{aligned}$$

Therefore

$$\begin{aligned} 4 \text{ m} &= (y_1/2)[((1 + 8 \times 6^2/(gy_1^3))^{0.5} - 1) - 2] \\ & \boxed{y_1 = 0.32 \text{ m}} \end{aligned}$$

15.54: PROBLEM DEFINITION

Situation:

An obstruction in a channel causes a hydraulic jump.

On the upstream side of the jump: $V_1 = 8\,\mathrm{m/s}$, $y_1 = 0.40\,\mathrm{m}$.

Find:

Depth of flow downstream of the jump (m).

PLAN

Calculate the upstream Froude number. Then apply the Hydraulic jump equation to find the downstream depth.

SOLUTION

Froude number

$$\begin{aligned} Fr_1 &= \frac{V}{\sqrt{gy_1}} \\ &= \frac{8}{\sqrt{9.81 \times 0.4}} \\ Fr_1 &= 4.039 \end{aligned}$$

Hydraulic jump equation

$$\begin{aligned} y_2 &= \frac{y_1}{2}\left[\sqrt{1 + 8Fr_1^2} - 1\right] \\ &= \frac{0.40}{2}\left[\sqrt{1 + 8 \times 4.039^2} - 1\right] \\ &\boxed{y_2 = 2.09 \ \mathrm{m}} \end{aligned}$$

15.55: PROBLEM DEFINITION

Situation:

A hydraulic jump is described in the problem statement.
$\gamma = 9,810 \text{ N/m}^2$, $B = 5$ m.
$y_1 = 40$ cm $= 0.40$ m, $V = 10\,\text{m/s}$.

Find:

Depth of flow downstream of jump (m).

SOLUTION

Check Fr upstream to see if the flow is really supercritical flow. Then apply the momentum principle.

$$\begin{aligned} Fr &= V/(gD)^{0.5} \\ D &= A/T \\ &= (By + y^2)/(B + 2y) \\ D_{y\,=\,0.4} &= (5 \times 0.4 + 0.4^2)/(5 + 2 \times 0.4) \\ &= 0.372 \text{ m} \end{aligned}$$

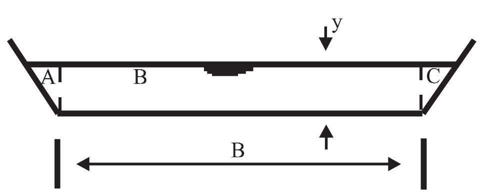
Then

$$\begin{aligned} Fr_1 &= 10 \text{ m/s}/((9.81 \text{ m/s}^2)(0.372))^{0.5} \\ Fr_1 &= 5.23 \end{aligned}$$

so flow is supercritical and a jump will form. Applying the momentum equation (Eq. 15.37 EFM10e):

$$\bar{p}_1 A_1 + \rho Q V_1 = \bar{p}_2 A_2 + \rho Q V_2 \qquad (1)$$

Evaluate $\bar{p}_1$ by considering the hydrostatic forces on the trapezoidal section divided into rectangular plus triangular areas as shown below:



Then

$$\begin{aligned} \bar{p}_1 A_1 &= \bar{p}_A A_A + \bar{p}_B A_B + \bar{p}_C A_C \\ &= (\gamma y_1/3)(y_1^2/2) + (\gamma y_1/2)\ B y_1 + (\gamma y_1/3)(y_1^2/2) \\ &= \gamma(y_1^3/6) + \gamma B(y_1^2/2) + \gamma(y_1^3/6) \\ &= \gamma(y_1^3/3) + \gamma B(y_1^2/2) \\ \bar{p}_1 A_1 &= \gamma((y_1^3/3) + B(y_1^2/2)) \end{aligned}$$

Also

$$\rho Q V_1 = \rho Q Q / A_1 = \rho Q^2 / A_1$$

Equation (1) is then written as

$$\gamma((y_1^3/3) + (B(y_1^2/2)) + \rho Q^2/A_1 = \gamma((y_2^3/3) + B(y_2^2/2)) + \rho Q^2/(By_2 + y_2^2)$$

Flow rate equation

$$\begin{aligned} Q &= V_1 A_1 = 21.6 \text{ m}^3/\text{s} \\ A_1 &= 5 \times 0.4 + 0.4^2 = 2.16 \text{ m}^2 \end{aligned}$$

Solving for y_2 yields: $\boxed{y_2 = 2.45 \text{ m}}$

15.56: PROBLEM DEFINITION

Situation:

A hydraulic jump occurs in a wide rectangular channel.
The upstream depth is $y_1 = 0.15\,\text{m}$.
The downstream depth is $y_2 = 3\,\text{m}$.

Find:

Discharge per foot of width of channel ($\text{m}^3/\,\text{s}/\,\text{m}$).

PLAN

Apply the Hydraulic jump equation to solve for the Froude number. Next, use the value of the Froude number to solve for the discharge q.

SOLUTION

Hydraulic jump equation

$$
\begin{aligned}
y_2 &= \frac{y_1}{2}\left[\sqrt{1+8Fr_1^2}-1\right] \\
3 &= \frac{0.15}{2}\left[\sqrt{1+8\times Fr_1^2}-1\right]
\end{aligned}
$$

Solve the above equation for Froude number.

$$Fr_1 = 14.49$$

Froude number

$$
\begin{aligned}
Fr_1 &= \frac{q}{\sqrt{gy_1^3}} \\
14.49 &= \frac{q}{\sqrt{9.81\times 0.15^3}}
\end{aligned}
$$

Solve the above equation for q

$$\boxed{q = 2.9\ \text{m}^2/\text{s}}$$

15.57: PROBLEM DEFINITION

Situation:

A rectangular channel has three different reaches—additional details are provided in the problem statement.

Find:

(a) Calculate the critical depth and normal depth in reach 1 (m).
(b) Classify the flow in each reach (subcritical, critical or supercritical).
(c) For each reach, determine if a hydraulic jump can occur.

PLAN

Apply the critical depth equation. Determine jump height and location by applying the hydraulic jump equation.

SOLUTION

Critical depth equation

$$\begin{aligned} y_c &= (q^2/g)^{1/3} \\ q &= 14/6 = 2.3\,\mathrm{m}^3/\,\mathrm{s}\text{ per meter} \\ y_c &= (2.3^2/9.81)^{1/3} \\ &\boxed{y_c = 0.83\text{ m}} \end{aligned}$$

Solving for $y_{n,1}$yields $\boxed{0.56\text{ m.}}$
Thus one concludes that the normal depth in each reach is

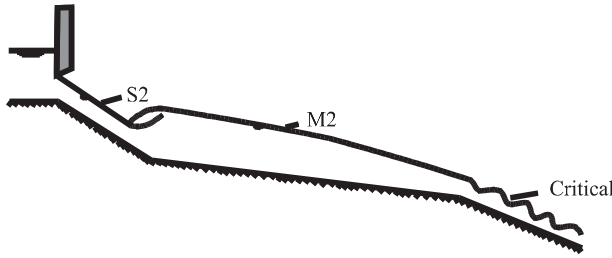
- Supercritical in reach 1
- Subcritical in reach 2
- Critical in reach 3

If reach 2 is long then the flow would be near normal depth in reach 2. Thus, the flow would probably go from supercritical flow in reach 1 to subcritical in reach 2. In going from sub to supercritical a hydraulic jump would form.

Hydraulic jump equation

$$\begin{aligned} y_2 &= (y_1/2)((1+8Fr_1^2)^{0.5}-1) \\ Fr_1 &= V_1/(gy_1)^{0.5} = (2.3/0.56)/(9.81\times 0.56)^{0.5} = 1.75 \\ y_2 &= (0.56/2)((1+8\times 1.75^2)^{0.5}-1) \\ &\boxed{y_2 = 0.81\text{ m}} \end{aligned}$$

Because y_2 is less than the normal depth in reach 2 **the jump will probably occur in reach 1**. The water surface profile could occur as shown below.


S2
M2
Critical

15.58: PROBLEM DEFINITION

Situation:
Water flows out a sluice gate and then over a free overfall—additional details are provided in the problem statement.

Find:
(a) Determine if a hydraulic jump will form.
(i) If a jump forms, locate the position.
(ii)If a jump does not form, sketch the full profile and label each part.
(b) Sketch the EGL

PLAN

Check Froude numbers. Then determine y_1 for a y_2 of 1.1 m by applying the hydraulic jump equation.

SOLUTION

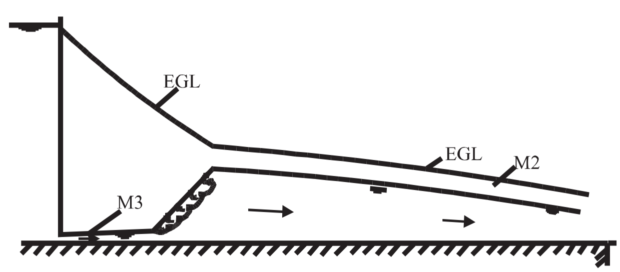
Froude number

$$\begin{aligned} Fr_1 &= V_1/\sqrt{gy_1} = 10/\sqrt{9.81 \times 0.10} = 10.1 \qquad \text{(supercritical)} \\ V_2 &= q/y_2 = (0.10 \text{ m}) \ (10 \text{ m/s})/(1.1 \text{ m}) = 0.91 \text{ m/s} \\ Fr_2 &= V_2/(gy_2)^{0.5} = 0.91/(9.81 \times 1.1)^{0.5} = 0.277 \end{aligned}$$

A hydraulic jump will form because flow goes from supercritical to subcritical.
Hydraulic jump equation

$$\begin{aligned} y_1 &= (y_2/2)((1 + 8Fr_2^2)^{0.5} - 1) \\ &= (1.1/2)((1 + 8 \times .277^2)^{0.5} - 1) \\ & \boxed{y_1 = 0.14 \text{ m}} \end{aligned}$$

Therefore, based on the figure provided in the problem statement, the jump will start at about the 29 m distance downstream of the sluice gate. Profile and energy grade line:



15.59: PROBLEM DEFINITION

Situation:

Water flows out of sluice gate and then through a hydraulic jump—additional details are provided in the problem statement.

Find:

Power in kilowatt in the hydraulic jump.

Assumptions:

Negligible energy loss for flow under the sluice gate.

PLAN

Apply the Bernoulli equation from a location upstream of the sluice gate to a location downstream. Then, calculate the Froude number and apply the equations that govern a hydraulic jump. Calculate the power using $P = Q\gamma h_L$.

SOLUTION

Bernoulli equation

$$\begin{aligned} y_0 + V_0^2/2g &= y_1 + V_1^2/2g \\ 20 + \text{neglig.} &= 0.3 + V_1^2/2g \\ V_1 &= \sqrt{19.7 \times 19.62} = 19.66 \text{ m/s} \end{aligned}$$

Froude number

$$\begin{aligned} Fr_1 &= V_1/\sqrt{gy_1} \\ &= 19.66/\sqrt{9.81 \times 0.3} \\ Fr_1 &= 11.3 \end{aligned}$$

Hydraulic jump equations

$$\begin{aligned} y_2 &= (y_1/2)(\sqrt{1 + 8F_1^2} - 1) \\ &= (0.3/2)(\sqrt{1 + 8 \times 11.3^2} - 1) \\ &= 4.6 \text{ m} \\ h_L &= (y_2 - y_1)^3/(4y_1y_2) \\ &= (4.3)^3/(4 \times 0.3 \times 4.6) \\ &\boxed{y_2 = 14.4 \text{ m}} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_L \\ &= (19.66 \times 0.3 \times 1.5) \times 9810 \times 14.4 \\ &\boxed{P = 1250 \text{ kW}} \end{aligned}$$

15.60: PROBLEM DEFINITION

Situation:

Water flows in a rectangular channel.
A sill installed on the bottom of the channel forces a hydraulic jump to occur.
Additional details are provided in the problem statement.
channel width = 8 m
y_1 = 32 cm

Find:

Estimate the height of hydraulic jump (the height is the change in elevation of the water surface) (m).

Assumptions:

$n = 0.012.$

PLAN

Calculate Froude number in order to apply the Hydraulic jump equation.

SOLUTION

$$\begin{aligned} V &= (1/n)R^{2/3}S_0^{1/2} \\ R &= A/P = (0.32 \times 8)/(2 \times 0.32 + 8) = 0.296 \text{ m} \\ V &= (1/0.012)(0.296)^{2/3} \times (0.04)^{1/2} = 7.40 \text{ m/s} \end{aligned}$$

Froude number

$$\begin{aligned} Fr_1 &= V/\sqrt{gy_1} \\ &= 7.40/\sqrt{9.81 \times 0.32} \\ Fr_1 &= 4.18 \text{ (supercritical)} \end{aligned}$$

Hydraulic jump equation

$$\begin{aligned} y_2 &= (y_1/2)(\sqrt{1 + 8 \times F_1^2} - 1) \\ &= (0.32/2)(\sqrt{1 + 8 \times (4.18)^2} - 1) \\ & y_2 = 1.74 \text{ m} \\ y_1 - y_2 &= 0.32 - 1.74 = -1.42 \\ & \boxed{\Delta \text{ Elev} = 1.42 \text{ m (increase)}} \end{aligned}$$

15.61: PROBLEM DEFINITION

Situation:

Water flows in a rectangular channel.

A sill installed on the bottom of the channel forces a hydraulic jump to occur.

Additional details are provided in the problem statement.

Find:

(a) Estimate the shear force associated with the jump ($\mathrm{N/m^2}$).

(b) Calculate the ratio F_s/F_H, where F_s is shear force and F_H is the net hydrostatic force acting on the jump.

Assumptions:

(a) The shear stress will be the average of τ_{0_1} (associated with uniform flow approaching the jump), and τ_{0_2} (associated with uniform flow leaving the jump).

(b) The flow may be idealized as normal flow in a channel.

PLAN

Apply the local shear stress Equation (10.11, EFM10e) and calculate the Reynolds numbers. Then find V_2 by applying the same solution procedure from Problem 15.60 (EFM10e). Then estimate the total shear force by using an average shear stress.

SOLUTION

Local shear stress

$$\tau_0 = f\rho V^2/8$$

where $f = f(\mathrm{Re},\ k_s/4R)$

$$R_{e_1} = V_1(4R_1)/\nu \quad R_{e_2} = V_2 \times (4R_2)/\nu$$

Using the same solution process as Prob. 15.60 (EFM 10e), one finds $y_2 = 2.26\,\mathrm{m}$.

$$V_2 = V_1 \times 0.4/2.26 = 1.52 \text{ m/s}$$

$$\begin{array}{ll} R_{e_1} = 8.59 \times (4 \times 0.37)/10^{-6} & R_2 = A/P = (2.26 \times 10)/(2 \times 2.26 + 10) = 1.31 \text{ m} \\ R_{e_1} = 1.3 \times 10^7 & R_{e_2} = 1.52 \times (4 \times 1.56)/10^{-6} = 9.5 \times 10^6 \end{array}$$

Assume $k_s = 3 \times 10^{-3}$ m

$$\begin{array}{ll} k_s/4R_1 = 3 \times 10^{-3}/(4 \times 0.37) & k_s/4R_2 = 3 \times 10^{-3}/(4 \times 1.56) \\ k_s/4R_1 = 2 \times 10^{-3} & k_s/4R_2 = 4.8 \times 10^{-4} \end{array}$$

From Fig. 10.14 (EFM10e), $f_1 = 0.024$ and $f_2 = 0.017$. Then

$$\begin{array}{ll} \tau_{0_1} = 0.024 \times 1,000 \times (6.87)^2/8 & \tau_{0_2} = 0.017 \times 1,000 \times (1.52)^2/8 \\ \tau_{0_1} = 142 \text{ N/m}^2 & \tau_{0_2} = 4.9 \text{ N/m}^2 \end{array}$$

$$\begin{aligned} \tau_{\text{avg}} &= (142 + 4.9)/2 \\ &\boxed{\tau_{\text{avg}} = 73 \ \ \text{N/m}^2} \end{aligned}$$

Then

$$F_s = \tau_{\text{avg}} A_s = \tau_{\text{avg}} PL$$

where $L \approx 6y_2, P \approx B + (y_1 + y_2)$. Then

$$\begin{aligned} F_2 &\approx 73(10 + (0.40 + 2.26))(6 \times 2.26) \\ &= 10,790 \text{ N} \end{aligned}$$

$$\begin{aligned} F_H &= (\gamma/2)(y_2^2 - y_1^2)B \\ &= (9,810/2)((2.26)^2 - (0.40)^2) \times 10 \\ &= 242,680 \text{ N} \end{aligned}$$

Thus

$$\begin{aligned} F_s/F_H &= 10,790/242,680 \\ &\boxed{F_s/F_H = 0.044} \end{aligned}$$

REVIEW

The above estimate is probably influenced too much by τ_{0_1} because shear stress will not be linearly distributed. A better estimate might be to assume a linear distribution of velocity with an average f and then integrate $\tau_0 dA$ from one end to the other.

15.62: PROBLEM DEFINITION

Situation:

Water flows out of a sluice gate—additional details are provided in the problem statement.

Find:

(a) Determine the type of water surface profile that occurs downstream of the sluice gate.

(b) Calculate the shear stress on bottom of the channel at a horizontal distance of 0.5 m downstream from the sluice gate ($\mathrm{N/m^2}$).

Assumptions:

The flow can be idealized as a boundary layer flow over a flat plate, with the leading edge of the boundary layer located at the sluice gate.

PLAN

Apply the hydraulic jump equation by first calculating q applying the flow rate equation. Then apply the local shear stress equation.

SOLUTION

Flow rate equation

$$\begin{aligned} q &= 0.40 \times 10 \\ &= 4.0 \ \frac{\mathrm{m}^2}{\mathrm{s}} \end{aligned}$$

Hydraulic jump equation

$$\begin{aligned} y_c &= \sqrt[3]{q^2/g} \\ &= \sqrt[3]{(4.0)^2/9.81} \\ &= 1.18\,\mathrm{m} \end{aligned}$$

Then we have $y < y_n < y_c$; therefore, the water surface profile will be an $\boxed{S3.}$

Reynolds number

$$\begin{aligned} \mathrm{Re}_x &\approx V \times 0.5/\nu \\ \mathrm{Re}_x &= 10 \times 0.5/10^{-6} \\ &= 5 \times 10^6 \end{aligned}$$

The local shear stress coefficient is

$$\begin{aligned} c_f &= \frac{0.455}{\ln^2(0.06\,\mathrm{Re}_x)} \\ &= \frac{0.455}{\ln^2(0.06 \times 5 \times 10^6)} \\ &= 0.00286 \end{aligned}$$

Local shear stress

$$\begin{aligned}\tau_0 &= c_f \frac{\rho V_0^2}{2} \\ &= 0.00286 \frac{998 \times 10^2}{2} \\ &\boxed{\tau_0 = 142.7 \text{ N/m}^2}\end{aligned}$$

15.63: PROBLEM DEFINITION

Situation:

Water flows in a rectangular channel

$Q = 3 \text{ m}^3/\text{s}$

$y_n = 0.6$ m $\quad y_{actual} = 1.2$ m $\quad$ width $= 3$ m

Find:

Classify the water surface profile as

a.) S1
b.) S2
c.) M1
d.) M2

PLAN

Use principles of water classification for gradually-varied flow

SOLUTION

$$\begin{aligned} y_n &= 0.6 \text{ m} \\ y_c &= (q^2/g)^{1/3} = (0.3^2/9.81)^{1/3} = 0.21 \text{ m.} \\ y &> y_n > y_c \end{aligned}$$

From Fig. 15.29 and 15.30 (EFM10e) the profile is M1. Thus, the correct choice is **c.**

15.64: PROBLEM DEFINITION

Situation:

Water surface is labeled with a question mark in figure in text.

Find:

Classify the water surface profile as one of the following:

a.) M2
b.) S2
c.) H2
d.) A2

PLAN

Use principles of water classification for gradually-varied flow

SOLUTION

The correct choice is **d).**

15.65: PROBLEM DEFINITION

Situation:

The problem statement shows a partial sketch of a water-surface profile.

Find:

(a) Sketch the missing part of the water profile.

(b) Identify the various types of profiles.

PLAN

Check the Froude number at points 1 and 2. Apply the Broad crested weir–Discharge equation to calculate y_2 for the second Froude number.

SOLUTION

Froude number

$$\begin{aligned} Fr_1 &= q/\sqrt{gy^3} \\ &= (5/3)/\sqrt{9.81(0.3)^3} \\ &= 3.24 > 1 \text{ (supercritical)} \end{aligned}$$

Broad crested weir–Discharge equation

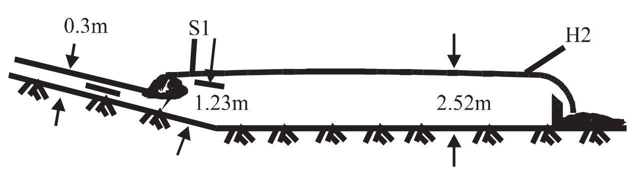
$$\begin{aligned} Q &= (0.40 + 0.05H/P)L\sqrt{2g}H^{3/2} \\ 5 &= (0.40 + 0.05H/1.6) \times 3\sqrt{2(9.81)}H^{3/2} \end{aligned}$$

Solving by iteration gives $H = 0.917$ m. Depth upstream of weir $= 0.917 + 1.6 = 2.52$ m

$$Fr_2 = (5/3)/\sqrt{9.81(2.52)^3} = 0.133 < 1 \text{ (subcritical)}$$

Therefore a hydraulic jump forms.
Hydraulic jump equation

$$\begin{aligned} y_2 &= (y_1/2)(\sqrt{1 + 8Fr_1^2} - 1) \\ y_2 &= (0.3/2)(\sqrt{1 + 8(3.24)^2} - 1) \\ y_2 &= 1.23\,\text{m} \end{aligned}$$



15.66: PROBLEM DEFINITION

Situation:

Rectangular channel ends with a free overfall

Channel is very long. width = 3 m $S_o = .0001$ $Q = 120$ cfs

One mile upstream the flow is uniform

Find:

Determine the classification of the water surface just before the brink of the overfall.

SOLUTION

The profile might be an M profile or an S_o profile depending upon whether the slope is mild or steep. However, if it is a steep slope the flow would be uniform right to the brink. Check to see if M or S_o slope. Assume $n = 0.012$.

$$
\begin{aligned}
Q &= (1.0/n)AR^{0.667}S_o^{0.5} \\
AR^{2/3} &= Q/((1.0/n)(S_o^{0.5})); \\
&= 4/((1.0/0.012)(0.0001)^{0.5}) \\
(by)(by/(3+2y))^{.667} &= 4.8
\end{aligned}
$$

With $b = 3$ m we can solve for y to obtain $y = 1.6$ m.

Flow rate equation

$$
\begin{aligned}
V &= Q/A \\
&= 4/4.8 \\
&= 0.8 \text{ m/s}
\end{aligned}
$$

Froude number

$$
\begin{aligned}
Fr &= V/\sqrt{gy} \\
&= 0.8/(\sqrt{9.81 \times 1.6}) \\
&= 0.2 \text{ (subcritical)}
\end{aligned}
$$

Therefore, the water surface profile will be an M2.

15.67: PROBLEM DEFINITION

Situation:
Water flows out a sluice gate and thorough a rectangular channel.
A weir will be added to the channel.
Additional details are provided in the problem statement.

Find:
(a) Determine if a hydraulic jump will occur.
(b) If a jump form, calculate the location.
(c) Label any water surface profiles that may be classified.

SOLUTION

Rectangular weir equation

$$Q = K\sqrt{2g}LH^{3/2}$$

where $K = 0.40 + 0.05H/P$. By trial and error (first assume K then solve for H, etc.) yields $H = 0.62$ m.
Flow rate equation

$$\begin{aligned} V_1 &= Q/A \\ &= 3/(0.62 \times 3) \\ V_1 &= 1.6 \text{ m/s} \end{aligned}$$

Froude number

$$\begin{aligned} Fr_1 &= V/\sqrt{gy} \\ &= 1.6/(9.81 \times (0.62))^{0.5} \\ Fr_1 &= 0.64 \text{ (subcritical)} \end{aligned}$$

The Froude number just downstream of the sluice gate will next be determined:
Flow rate equation

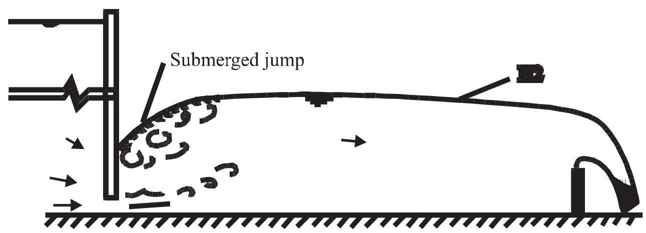
$$\begin{aligned} V_2 &= Q/A \\ &= 3/(3 \times 0.12) \\ V_2 &= 8.3 \text{ m/s} \end{aligned}$$

Froude number

$$\begin{aligned} Fr_2 &= V/\sqrt{gy} \\ &= 8.3/\sqrt{9.81 \times 0.12} \\ Fr_2 &= 7.52 \text{ (supercritical)} \end{aligned}$$

Because the flow is supercritical just downstream of the sluice gate and subcritical upstream of the weir a jump will form someplace between these two sections.

Now determine the approximate location of the jump. Let y_2 = depth downstream of the jump and assume it is approximately equal to the depth upstream of the weir ($y \approx 1.22$ m). By trial and error (applying the hydraulic jump equation) it can be easily shown that a depth of 0.12 m is required to produce the given y_2. Thus the jump will start immediately downstream of the sluice gate and it will be approximately 7.5 m long. Actually, because of the channel resistance y_2 will be somewhat greater than $y_2 = 1.22$ m; therefore, the jump may be submerged against the sluice gate and the water surface profile will probably appear as shown below.



15.68: PROBLEM DEFINITION

Situation:
A rectangular channel is described in the problem statement.

Find:
(a) Sketch all possible water-surface profiles.
(b) Label each part of the water-surface profile with its classification.

PLAN

Apply the critical depth equation to determine if a hydraulic jump will form.

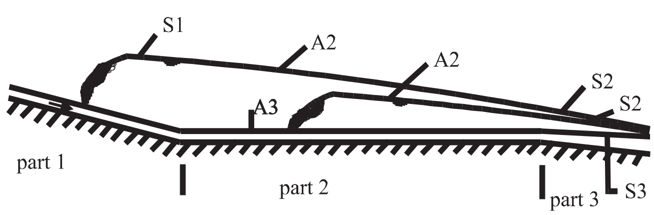
SOLUTION

Critical depth equation

$$\begin{aligned} y_c &= \sqrt[3]{q^2/g} \\ &= \sqrt[3]{2^2/9.81} \\ y_c &= 0.76\,\mathrm{m} \end{aligned}$$

Thus the slopes in parts 1 and 3 are steep.

If part 2 is very long, then a depth greater than critical will be forced in part 2 (the part with adverse slope). In that case a hydraulic jump will be formed and it may occur on part 2 or it may occur on part 1. The other possibility is for no jump to form on the adverse part. These three possibilities are both shown below.



15.69: PROBLEM DEFINITION

Situation:

Water flow through a sluice gate and down a rectangular channel is described in the problem statement.

Find:

Sketch the water surface profile until a depth of 60 cm. is reached.

Assumptions:

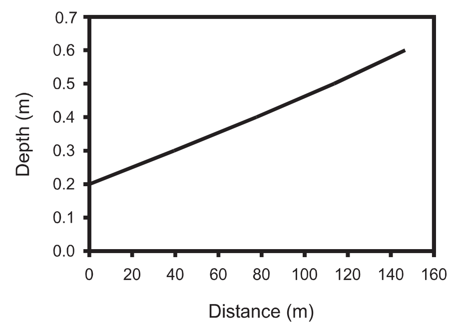
The value of f is constant with a value of 0.02 (given).

SOLUTION

Froude number

$$\begin{aligned} Fr_1 &= q/\sqrt{gy^3} \\ &= 3/\sqrt{9.81(0.2)^3} = 10.71 \\ Fr_2 &= 3/\sqrt{9.81(0.6)^3} = 2.06 \end{aligned}$$

Therefore the profile is a continuous $H3$ profile.



y	$\bar{y}$	V	$\bar{V}$	E	ΔE	S_f	Δx	x
0.2		15		11.6678				0
	0.25		12.5		6.2710	0.1593	39.4	
0.3		10		5.3968				39.4
	0.35		8.75		2.1298	0.0557	38.2	
0.4		7.5		3.2670				77.6
	0.45		6.75		0.9321	0.0258	36.1	
0.5		6.0		2.3349				113.7
	0.55		5.5		0.4607	0.0140	32.9	
0.6		5.0		1.8742				146.6

15.70: PROBLEM DEFINITION

Situation:

A horizontal channel ends in a free outfall—additional details are provided in the problem statement.

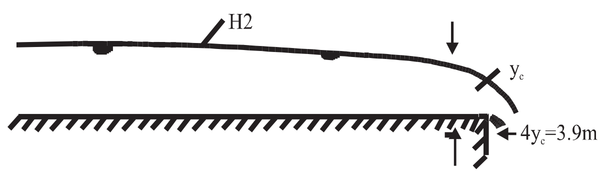
Find:

Water depth 300 m upstream of the outfall (m).

PLAN

Apply the critical depth equation. Then carry out a step solution for the profile upstream from the brink.

SOLUTION



$$\begin{aligned} q &= Q/B \\ &= 12/4 = 3 \text{ m}^3/\text{s/m} \\ y_c &= \sqrt[3]{q^2/g} \\ y_c &= 0.972 \text{ m (This depth occurs near brink.)} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &\approx V \times 4R/\nu \approx 3 \times 1/10^{-6} \approx 3 \times 10^6 \\ k_s/4R &\approx 0.3 \times 10^{-3}/4 \approx 0.000075 \\ f &\approx 0.010 \end{aligned}$$

See solution table below.

Solution Table for Problem 15.51

Section number upstream of y_c	Depth y,m	Velocity at section V,m/s	Mean Velocity in reach $(V_1+V_0)/2$	V^2	Hydraulic Radius R=A/P,m	Mean Hydraulic Radius $R_m=(R_1+R_2)/2$	$s_f=fV^2_{mean}/8gR_{mean}$	$\Delta x=((y_2+V^2_2/2g)-(y_1+V_1/2g))/S_f$	Distance upstream from brink x,m
1 at $y=y_c$	0.972	3.086			0.654				3.9m
			3.073	9.443		0.656	1.834×10^{-3}	0.1m	4.0m
2	0.980	3.060			0.658				
			3.045	9.272		0.660	1.790×10^{-3}	0.4m	4.4m
3	0.990	3.030			0.662				
			2.986	8.916		0.669	1.698×10^{-3}	1.7m	6.1m
4	1.020	2.941			0.675				
			2.886	8.327		0.684	1.551×10^{-3}	4.7m	10.9m
5	1.060	2.830			0.693				
			2.779	7.721		0.701	1.403×10^{-3}	7.7m	18.6m
6	1.100	2.727			0.710				
			2.613	6.828		0.730	1.192×10^{-3}	33.2m	51.8m
7	1.200	2.500			0.750				
			2.404	5.779		0.769	9.576×10^{-4}	55.3m	107.1m
8	1.300	2.308			0.788				
			2.225	4.951		0.806	7.83×10^{-4}	80.0m	187.1m
9	1.400	2.143			0.824				
			2.072	4.291		0.841	6.501×10^{-4}	107.4m	294.5m
10	1.500	2.000			0.857				

The depth 300 m upstream is approximately 1.51 m

15.71: PROBLEM DEFINITION

Situation:

Water flows through a sluice gate, down a channel and across a hydraulic jump.
Additional details are provided in the problem statement.

Find:

(a) Determine the water-surface profile classification
- i) Upstream of the jump.
- ii) Downstream of the jump.

(b) Determine how the addition of baffle block will effect the jump.

SOLUTION

Upstream of the jump, the profile will be an $\boxed{H3.}$

Downstream of the jump, the profile will be an $\boxed{H2.}$

The baffle blocks will cause the depth upstream of A to increase; therefore, the jump will move towards the sluice gate.

15.72: PROBLEM DEFINITION

Situation:

Water flows out of a reservoir, down a spillway and then over an outfall.
Additional details are provided in the problem statement.

Find:

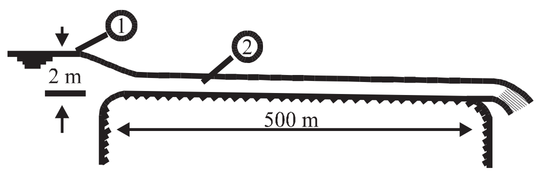
Discharge in the channel (m^3/s).

Assumptions:

$V_1 = 0$ and $\alpha_2 = 1.0$.

PLAN Apply the energy equation from the reservoir, (1), to the entrance section (2) and set the Froude number equal to 1 (critical flow) to solve for y_c and V_c. Then calculate the discharge by applying the flow rate equation.

SOLUTION



The channel is steep; therefore, critical depth will occur just inside the channel entrance.

Energy equation

$$y_1 + \alpha_1 V_1^2/2g = y_2 + \alpha_2 V_2^2/2g$$

Then

$$2 = y_2 + V_2^2/2g$$

Froude number

$$\begin{aligned} V_2^2/2g &= V_c^2/2g \\ &= 0.5y_c \end{aligned} \quad (1)$$

The energy equation becomes

$$y_1 = y_c + 0.5y_c$$

Let $y_1 = 2\,\text{m}$ and solve for y_c

$$y_c = 2\,\text{m}/1.5 = 1.33 \text{ m}$$

From Eq. (1)

$$\begin{aligned} V_c^2/g &= y_c \\ &= 1.33 \\ \text{or } V_c &= 3.62 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_c A_2 \\ &= 3.62 \times 1.33 \times 4 \\ &\boxed{Q = 19.3 \text{ m}^3/\text{s}} \end{aligned}$$

15.73: PROBLEM DEFINITION

Situation:

Water flows out a reservoir and down a channel.

Find:

(a) Estimate the discharge (m^3/s).

(b) Describe a procedure for calculating the discharge if the channel length was 100 m.

Assumptions:

Uniform flow is established in the channel except near the downstream end. $n = 0.012$.

PLAN

Apply the energy equation from the reservoir to a section near the upstream end of the channel to solve for V. Then apply the flow rate equation to calculate the discharge.

SOLUTION

(a) Energy equation

$$2.5 \approx V_n^2/2g + y_n \tag{1}$$

Also

$$\begin{aligned} V_n &= (1/n)R^{2/3}S_o^{1/2} \\ V_n^2/2g &= (1/n^2)R^{4/3}S_o/2g \end{aligned} \tag{2}$$

where

$$R = A/P = 3.5y_n/(2y_n + 3.5) \tag{3}$$

Then combining Eqs. (1), (2) and (3) we have

$$2.5 = ((1/n^2)((3.5y_n/(2y_n + 3.5))^{4/3}S_o/2g) + y_n \tag{4}$$

Assuming $n = 0.012$ and solving Eq. (4) for y_n yields: $y_n = 2.16$ m; also solving (2) yields $V_n = 2.58$ m/s. Then

$$\begin{aligned} Q &= VA \\ &= 2.58 \times 3.5 \times 2.15 \\ &\boxed{Q = 19.4 \text{ m}^3/\text{s}} \end{aligned}$$

(b) With only a 100 m-long channel, uniform flow will not become established in the channel; therefore, a trial-and-error solution is required. Critical depth will occur just upstream of the brink, so assume a value of y_c, then calculate Q and calculate the water surface profile back to the reservoir. Repeat the process for different values of y_c until a match between the reservoir water surface elevation and the computed profile is achieved.

15.74: PROBLEM DEFINITION

Situation:

During flood flow, water flows out of a reservoir.

Find:

Calculate the water surface profile upstream from the dam until the depth is six meters.

PLAN

Apply the critical depth equation. Then carry out a step solution for the profile upstream from the dam.

SOLUTION

$$
\begin{aligned}
q &= 10\ \mathrm{m^3/s/m} \\
y_c &= \sqrt[3]{q^2/g} \\
&= \sqrt[3]{10^2/9.81} \\
&= 2.17\ \mathrm{m}
\end{aligned}
$$

y	$\bar{y}$	V	$\bar{V}$	E	ΔE	$S_f\times 10^4$	Δx	x	elev.
52.17		0.1917		52.170				0	52.17
	51.08		0.1958		2.168	0.00287	-5,429		
50		0.20		50.002				-5,430	52.17
	45		0.2222		9.999	0.00419	-25,024		
40		0.25		40.003				-30,450	52.18
	35		0.2857		9.997	0.00892	-25,048		
30		0.333		30.006				-55,550	52.22
	25		0.400		9.993	0.02447	-25,146		
20		0.50		20.013				-80,650	52.26
	15		0.6667		9.962	0.11326	-25,631		
10		1.00		10.051				-106,280	52.51
	9		1.1111		1.971	0.5244	-5,671		
8		1.25		8.080				-111,950	52.78
	7		1.4286		1.938	1.1145	-6,716		
6		1.667		6.142				-118,670	53.47

15.75: PROBLEM DEFINITION

Situation:

Water flows in a wide rectangular concrete channel.
Additional details are provided in the problem statement.

Find:

Determine the water surface profile from section 1 to section 2.

Assumptions:

$n = 0.015$, $K = 0.42$, $k_s = 0.0003$ m so $k_s/4R = 0.00034$.

PLAN

Determine whether the uniform flow in the channel is super or subcritical. Determine y_n and then see if for this y_n the Froude number is greater or less than unity. Then apply the hydraulic jump equation to get y_2. Then apply the Rectangular weir equation to find the head on the weir. A rough estimate for the distance to where the jump will occur may be found by applying Eq. (15.43, EFM10e) with a single step computation. A more accurate calculation would include several steps.

SOLUTION

Froude number

$$\begin{aligned} Q &= (1.0/n)AR^{2/3}S_o^{1/2} \\ 1 &= (1.0/0.015) \times y \times y^{2/3} \times (0.04)^{1/2} \\ y_n &= 0.2 \text{ m and } V = Q/y_n = 5 \text{ m/s} \\ F &= V/\sqrt{gy_n} = 3.6 \end{aligned}$$

Solving for y_n gives $y_n = 0.2$ m and

$$V = Q/y_n = 5 \text{ m/s}$$

Therefore, uniform flow in the channel is supercritical and one can surmise that a hydraulic jump will occur upstream of the weir. One can check this by determining what the sequent depth is. If it is less than the weir height plus head on the weir height plus head on the weir then the jump will occur.

Now find sequent depth:

$$\begin{aligned} y_2 &= (y_1/2)(\sqrt{1 + 8F_1^2} - 1) \\ &= (0.2/2)(\sqrt{1 + 8 \times 3.6^2} - 1) \\ & \boxed{y_2 = 0.92 \text{ m}} \end{aligned}$$

Rectangular weir equation

$$
\begin{aligned}
Q &= K\sqrt{2g}LH^{3/2} \\
1 &= 0.42\sqrt{19.62} \times 0.3 \times H^{3/2} \\
H &= 1.5 \text{ m} \\
H/P &= 1.5/0.9 = 1.66
\end{aligned}
$$

so

$$K = 0.40 + 0.05 \times 1.66$$

A better estimate is

$$H = 1.2 \text{ m} \quad K = 0.44$$

Then depth upstream of weir $= 0.9 + 1.2 = 2.1$ m. Therefore, it is proved that a jump will occur.

The single-step calculation is given below:

$$\Delta x = ((y_1 - y_2) + (V_1^2 - V_2^2))/2g/(S_f - S_0)$$

where $y_1 = 0.92$; $V_1 = q/y_1 = 1/0.92 = 1.1$ m/s; $V_1^2 = 1.21$ m^2/s^2 and $y_2 = 2.1$ m; $V_2 = 0.65$ m/s.

$$
\begin{aligned}
V_2^2 &= 0.42 \text{ m}^2/\text{s}^2 \\
S_f &= fV_{\text{avg}}^2/(8gR_{\text{avg}}) \\
V_{\text{avg}} &= 0.9 \text{ m/s} \\
R_{\text{avg}} &= 1.3 \text{ m}
\end{aligned}
$$

Assuming $k_s = 0.0003$ m so $k_s/4R = 0.00034$.

$$\text{Re} = V \times 4R/\nu = ((1.1 + 0.65)/2) \times 4 \times 1.3/(1.14 \times 10^{-6}) = 4.0 \times 10^6$$

Then

$$f = 0.015$$

and

$$
\begin{aligned}
S_f &= 0.015 \times 0.9^2/(8 \times 9.81 \times 1.3) = 0.00012 \\
\Delta x &= ((0.92 - 2.1) + (1.21 - 0.42)/(19.62))/(0.00012 - 0.04) \\
&\boxed{\Delta x = 17.1 \text{ m}}
\end{aligned}
$$

Thus, the water surface profile is shown below: